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SPECIAL RELATIVITY AND THE SECOND LAW OF THERMODYNAMICS

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PROLOGUE

After Einstein's initial presentation of special relativity in 1905, there was a great deal of interest in developing Lorentz invariant forms for each of the branches of classical physics. Hence, it is not suprising that, due mainly to the work of Planck, relativistic thermodynamics was molded into a fairly complete theory by as early as 1908[1]. This first formulation of relativistic thermodynamics was based on the assumption that the first and second laws of thermodynamics retained their classical mathematical forms when comoving bodies interacted thermally and on a particular definition of heat transfer. From this starting point Planck was able to deduce the following relationship between temperature, T_0, of a body as measured in the body's rest frame and the temperature, T, measured by a moving observer:

$$T = T_0/\gamma. \tag{1i}$$

Since the Lorentz factor, γ, is always greater than one, Eq. (1i) predicts that the moving temperature is less than the rest temperature.*

The above theory was attacked in 1963 by H. Ott[2] because Planck's definition of heat transfer forced part of the energy emitted by a black body radiator to be classified as work. Therefore, Ott introduced a new definition of heat transfer which caused the moving

*Schmid[10] has provided an interesting study of the early history of relativistic thermodynamics.

temperature and the rest temperature to be related by Eq. (2i).

$$T = \gamma T_0 \qquad (2i)$$

Ott's work went unnoticed for a few years, but about 1967 a number of papers[2,11], most of which supported Ott, were published. The most interesting of these papers is the one authored by T. Kibble. Although Kibble seems aware of the arbitrariness contained in the definition of thermodynamic quantities, he gives Ott's definition a stronger thermodyanmic basis and comes out in favor of the modern thermodynamics. Kibble's work is followed by that of C. Møller[3,5] in which the framework of modern thermodynamics is still more firmly grounded. In addition, Møller has taken a position strongly opposed to Planck thermodynamics (Møller states emphatically that Ott has corrected a mistake in the older thermodynamics).

Even though modern thermodynamics lends itself more readily to covariant notation, these two formulations of relativistic thermodynamics are physicallly equivalent and all differences are purely formalistic. Before the development of modern thermodynamics, a number of authors[12] pointed out that Planck's formulation was not unique and discussed other possible formalisms; however, these works have been largely ignored.

P. T. Landsberg and K. A. Johns[8] have presented a third equivalent development of comoving relativistic thermodynamics: they argue that temperature should be an invariant because of difficulties involved in thermometrically measuring the temperature of moving bodies. The assumption of temperature invariance forces Landsberg and Johns to alter the mathematical form of the second law.

Thus, at the present there is some confusion as to what constitutes different physical theories and what is just a difference in language. In the introduction of this thesis a particularly simple demonstration of the physical equivalence of Planck thermodynamics and Ott thermodynamics is presented. The main task of the thesis is the development of a relativistic thermodynamics for systems in relative motion (The earlier studies are concerned only with processes which can be described by classical thermodynamics in some inertial frame). Since the meaning of the second law in the form $dS \geq dQ/T$ is subject to interpretation in processes involving relatively moving bodies, the second law will be expressed verbally in the form of a generalized Clausius' postulate and a generalized Kelvin's postulate. Using classical thermodynamics as a guide, a definition of temperature and the existence of an exact differential, called the generalized entropy, will be derived from these verbal statements of the second law.

If two bodies having different rest temperatures and different four-velocities are allowed to interact until final equilibrium is reached, their rest temperatures and four-velocities will adjust until they are equal. In this thesis the possibility of having interactions which will allow the systems to relax to quasi-equilibrium states is explored. That is, it will be postulated that there exists an interaction, called frictionless heat transfer, which allows the rest temperatures of the systems to equalize, but not the four-velocities. Likewise, systems interacting by means of friction will relax to the same four-velocity, but not necessarily the same rest temperature. Friction will be shown to be a completely symmetric interaction, while frictionless heat transfer will be completely antisymmetric. After demonstrating that frictionless heat transfer between relatively moving bodies at the same rest temperature can be a reversible processes, a family of reversible Carnot cycles will be developed. The existence of these Carnot cycles will be crucial to relativistic thermodynamics as presented in this thesis.

J. Lindhard(7) has published a paper in which the exixtence of an entropy is derived from a postulate similar to the generalized Kelvin's postulate. However, he does not present a Clausius' postulate and he does not apply himself to processes involving relatively moving systems.

CHAPTER I

INTRODUCTION

Prior to this paper relativistic thermodynamics involved only heat transfer between comoving bodies. Classical thermodynamics already describes this process in the rest frame of the comoving bodies, thus no new thermal physics is presented in any of the earlier formulations. That is, everyone agrees on what has happened in the rest frame and it is only a matter of interpreting the phenomena from a moving frame. In this paper a truly relativistic thermal phenomenon, heat conduction between moving bodies, will be studied.

Most formulations of relativistic thermodynamics are based on the assumption that the first and second laws of thermodynamics retain their classical form when thermal processes are viewed from a moving frame, i.e.

$$dE = dW + dQ \tag{1}$$

and

$$dS \geq dQ/T, \tag{2}$$

where E is the energy, W is the work, Q is the heat transferred, T is the temperature, and S is the entropy. Mechanical forces can be used to reversibly accelerate a body from rest to a given velocity, without altering its rest properties. By assuming that no heat transfer is involved when only mechanical forces are present and that $dS = dQ/T$ for reversible processes, it can easily be seen that the entropy depends only on the rest properties of a body. Thus, implicit in the assumption that the second law retains its classical form is the tacit assumption that entropy is an invariant. There are other arguments for entropy invariance, but they employ statistical definitions of thermodynamic quantities. Since statistical mechanics is based on thermodynamics, it is preferable to keep thermodynamics independent of statistical arguments.

If Eqs. (1) and (2) are to be ratained, one must decide how to separate heat transfer from work when viewing moving processes. In other words, a definition of heat transfer must be selected. Naturally, all acceptable definitions of heat transfer must reduce to the classical value when viewed from the rest frame. When a process involving heat transfer is viewed from a moving inertial frame, part of the energy transfer has some of the properties of heat and some of the properties of work; therefore, any division of energy transfer into dQ and dW in Eqs. (1) and (2) must be somewhat arbitrary. In all calculations involving only the first law, it obviously makes no physical difference how energy transfer terms are classified.

Using entropy invariance and Eq. (2) for reversible processes, one can establish a relationship between the moving temperature, T, and the rest temperature, T_0.

$$dQ_0/T_0 = dQ/T \quad . \tag{3}$$

If heat trnasfer is defined so that the ratio of dQ to dQ_0 is the same for all processes, Eq. (3) yields a unique relationship between T and T_0. This relationship will vary with different definitions of dQ; however, any acceptable formulation should be required to give a unique value for the moving temperature. At first it seems that different definitions of dQ might lead to variances in entropy calculations; however, this possibility does not materialize because all agree that entropy is an invariant. That is, since comoving processes are the only type considered, entropy can be calculated in the rest frame using classical thermodynamics. Since all formulations of comoving relativistic thermodynamics which proceed in the above manner agree on energy conservation and entropy invariance, they are physically equivalent and any disagreement is entirely formalistic.

The two prominent formulations of relativistic thermodynamics rest on the basis just outlined. The first formulation[(1)], which shall be called Planck thermodynamics, defines heat transfer, dQ_p, as the total energy transfer due to non-work processes.* The second formulation[(2)], which shall be called modern thermodynamics, defines heat transfer, dQ_m, as the total energy transfer due to non-mechanical processes. Planck thermodynamics was presented shortly after Einstein developed special relativity, while modern thermodynamics was presented in the 1960's to sl w an alleged mistake in Planck thermodynamics. These two formulations wil. now be discussed in conjunction with a particularly simple process.

An isolated body is a system the energy and momentum of which form a four-vector, e.g. an ideal gas and its rigid container. Work

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*The monogram by Tolman[(6)] contains a good presentation of Planck thermodynamics.

caused by accelerating forces can be performed on isolated bodies, but stresses, forces which tend to deform bodies, cannot act on isolated bodies. Since deformation due to Lorentz contractions cannot result in work being done on an isolated body, an isolated body must be kept in a zero pressure environment when being accelerated. Consider the case in which two comoving, isolated bodies exchange an amount of heat, dQ_0, as seen in their rest frame. If this is observed from a frame moving at a velocity, $v = -\beta c$, in the x-direction relative to the isolated bodies, this energy trnasfer is seen as a four-momentum exchange, dQ^i, of the following magnitude:

$$dQ^i = (\gamma dQ_0, \gamma\beta dQ_0, 0, 0) \quad , \tag{4}$$

where γ is the Lorentz factor, $(1-\beta^2)^{-\frac{1}{2}}$. The relationship between the four-momentum exchange, dQ^i, the energy exchange, dE, and the momentum exchange, $d\vec{P}$, is

$$dQ^i = (dE, cd\vec{P}) \quad , \tag{5}$$

where c denotes the speed of light. According to Planck thermodynamics, the amount of heat transfer, as seen in the moving frame, is the total energy transfer less the work done, which in this case is $dE - \vec{v}\cdot d\vec{P}$, so

$$dQ_p = \gamma dQ_0 - \gamma\beta^2 dQ_0 = dQ_0/\gamma \quad . \tag{6}$$

In modern thermodynamics, since this interaction is entirely thermal (non-mechanical), the heat transfer is equal to the energy transfer, thus

$$dQ_m = \gamma dQ_0 \quad . \tag{7}$$

If heat transfer has the same transformation equations ($dQ_p = dQ_0/\gamma$ and

$dQ_m = \gamma dQ_0$) for all processes, the following relationships between moving temperatures, T_p and T_m, and the rest temperature, T_0, can be taken from Eq. (3):

$$T_p = T_0/\gamma \tag{9}$$

and

$$T_m = \gamma T_0 \; . \tag{10}$$

The term, $- \gamma\beta^2 dQ_0$, in Eq. (6), which shall be called the Planck work, arises because of the relativistic equivalence of mass and energy. Due to the mass associated with heat transfer, an observer in a moving frame sees a momentum change. Since in special relativity force is defined as the time rate of change of momentum, a moving observer perceives some Planck work. Modern thermodynamics considers the Planck work to be part of the heat transfer, while Planck thermodynamics calls it work. Since both agree that Planck work is present and both agree that entropy is equal to the value calculated in the rest frame using classical thermodynamics, it makes no physical difference which formalism one picks when examining comoving processes. In other words, there is no experiment which could decide between Planck thermodynamics and modern thermodynamics.

In this paper thermodynamics will be extended to describe a truly relativistic phenomenon, heat conduction between relatively moving bodies. Since there is no rest frame for this type of interaction, entropy calculations cannot be done in a rest frame using classical thermodynamics. Entropy invariance, which is assumed in Planck thermodynamics and modern thermodynamics, will be derived from physically testable postulates. A mathematical form of the first and second laws will not be assumed. Of course conservation of energy will be accepted, but it will not be necessary to compel the first and second laws to be in the form, $dE = dQ + dW$ and $dS \geq dQ/T$. Relativistic thermodynamics will be developed in a manner which is identical to the procedure used to derive classical thermodynamics. Two verbal statements of the relativistic second law, the generalized Clausius' postulate and the generalized Kelvin's postulate, will be introduced. With the help of a new type of relativistic Carnot cycle, the two postulates will be shown to be equivalent and a unique definition of temperature, which is equal to the temperature measured with a frictionless ideal gas thermometer, will be presented. Finally, an exact differential, called the generalized entropy, will be described and shown to be an invariant.

CHAPTER II

FRICTIONLESS HEAT TRANSFER

Before proceeding with the development of the generalized second law, it is necessary to present a theory of frictionless interaction between moving, isolated bodies. In classical thermodynamics moving bodies can transfer heat in a completely non-mechanical manner, if they are sliding frictionlessly across each other. Here it is desired to determine what conditions must be imposed upon an interaction between relativistically moving bodies in order that the interaction consist only of heat transfer. That is, a relativistic theory of non-mechanical interactions between moving, isolated bodies is needed. This type of interaction should occur when a smooth, hot body slides over a smooth, cold body.

An isolated body cannot undergo work caused by deformations, thus an accelerating force is the only means by which energy and momentum can be mechanically transferred to an isolated body. Since the energy and momentum gained from an accelerating force form a four-vector and since the energy and momentum of an isolated body form a four-vector, it is easily seen using conservation of energy and momentum that heat transfer between isolated bodies must result in a four-momentum exchange. In this section the four-momentum property of heat transfer is needed only for isolated bodies, but later in the thesis it will be assumed that heat transfer forms a four-vector for arbitrary thermal interactions.*

Consider the case of two isolated bodies, S_1 and S_2, with rest temperatures, T_{01} and T_{02}. Let S_2 be moving at a velocity of βc in the x-direction relative to S_1. Now let an infinitesimal amount of four-momentum, dQ^i, [$dQ^i = (dE, cdP_x, 0, 0)$ in the rest frame of S_1] be carried from S_1 to S_2 by heat transfer. A thermal momentum transfer in the y or z direction can lead to no change in the rest properties of either body; therefore, dP_y and dP_z are of little interest and for simplicity they are taken to be zero. The energy, E, of an isolated body can be written in terms of its rest mass, m_0, and its velocity, βc.

$$E = \gamma m_0 c^2 = (\gamma - 1) m\ c\gamma^2 + m_0 c^2 \qquad (11)$$

m_0c^2 shall be called the internal energy and shall be denoted by U. The kinetic energy, K.E., of an isolated body is $(\gamma-1)m_0c^2$, so

$$E = K.E. + U \quad . \qquad (12)$$

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*C. Møller[3] demonstrates that this assumption is valid for perfect fluids.

In this paper an interaction between S_1 and S_2 is considered to be frictionless only if the total kinetic energy, $K.E._1 + K.E._2$, is conserved. Heat transfer is a non-mechanical interaction, thus a totally thermal process should conserve mechanical energy. Since this frictionless interaction involves only S_1 and S_2,

$$dK.E._1 + dK.E._2 + dU_1 + dU_2 = 0 \quad . \tag{13}$$

This means that the change in total kinetic energy is zero if and only if the change in total internal energy is zero. U is a relativistic invariant, so $dU_1 + dU_2 = 0$ is a coordinate independent statement. From this it follows that kinetic energy conservation is an invariant requirement.

The form of the infinitesimal four-momentum exchange will now be extracted from the conservation of internal energy requirements, $dU_1 + dU_2 = 0$. If the signature of the metric is taken as -2 and the four-velocities of S_1 and S_2 are denoted by V_1^i and V_2^i, the internal energy changes of S_1 and S_2 can be written as

$$dU_1 = - dQ^i V_{1i} \tag{14}$$

and

$$dU_2 = dQ^i V_{2i} \quad . \tag{15}$$

By performing these summations in the rest frame of S_1, it is found that

$$dU_1 = - dE \tag{16}$$

and

$$dU_2 = \gamma dE - \gamma\beta c dP_x \ . \quad (17)$$

Using conservation of internal energy, the following relationship between dP_x and dE is discovered:

$$dP_x = (\gamma-1)dE/\gamma\beta c = \gamma\beta dE/c(\gamma+1) \ . \quad (18)$$

This means that the four-momentum exchange as seen from the rest frame of S_1 must be of the form $[dE,(\gamma-1)dE/\gamma\beta,0,0]$. Since the absolute value of $(\gamma-1)/\gamma\beta$ is less than one, heat transfer forms a timelike four-vector. By making a Lorentz transformation, it is found that $[dE,(\gamma'-1)dE/\gamma'\beta',0,0]$ is the form of the four-momentum transfer as seen by S_2. $\beta'c$ equals $-\beta c$ and is the velocity of S_1 as seen by S_2. Notice that when $\beta = 0$ internal energy is conserved regardless of the ratio of dP_x to dE. Assuming that Eq. (18) is continuous, it is found that the four-momentum exchange is of the form $(dE,0,0,0)$ when β is zero.

Since the nature of an infinitiesimal four-momentum exchange should not depend on extensive properties, such as the masses of S_1 and S_2, the center of mass frame should be of no particular interest in this type of interaction. The symmetry of the situation does however pick out one inertial frame of particular interest. A reasonable requirement for an interaction to be frictionless is that dP_x be zero in the frame which sees S_1 moving at a velocity $-w$ and S_2 moving at a velocity w. It can be easily shown that the requirement of conserved kinetic energy causes the four-momentum exchange to be of the form $(dE',0,0,0)$ in the frame relativistically midway between the two bodies.

Three possible covariant generalizations of frictionless interaction between isolated bodies have been given. They are: (1) The total kinetic energy is conserved; (2) the total internal energy is conserved; (3) the interaction is forceless as seen by an observer relativistically midway between the two bodies. These three conditions are equivalent and no other conflicting, physically reasonable, covariant statements are apparent. In the remainder of this paper it will be assumed that if heat transfer takes place between two isolated bodies, it must be a four-momentum exchange of the form $[dE,(\gamma-1)dE/\gamma\beta,0,0]$ as seen in the momentary rest frame of one of the bodies, where βc is the velocity in the x-direction of the other body.

This theory leads to an interesting, perhaps unexpected, relativistic effect viz., the motion of a small hot body sliding frictionlessly upon an infinitely massive cold body. The cold body will be called S_1, the hot body will be called S_2, and all calculations will be done in the rest frame of S_1, where $v_1 = \beta_1 c = 0$ and $v_2 = \beta_2 c$.

$$dK.E._1 = d\{(\gamma_1-1)U_1\} = (\gamma_1-1)dU_1 + U_1 d\gamma_1 \;, \tag{19}$$

but

$$\gamma_1 = 1 \text{ and } d\gamma_1 = \beta_1\gamma^3{}_1 d\beta = 0. \tag{20}$$

So,

$$dK.E._1 = 0 \quad \text{and} \quad dK.E._2 = 0 \;. \tag{21}$$

$$- dU_2/U_2 = d\gamma_2/(\gamma_2-1) = \beta_2\gamma_2^3 d\beta_2/(\gamma_2-1) \;, \tag{22}$$

so

$$d\beta_2 = - \frac{(\gamma_2-1)dU_2}{\beta_2\gamma_2^3 U_2} \;. \tag{23}$$

It is assumed that when hot and cold bodies exchange heat the one at the higher rest temperature will lose internal energy and the one at the lower rest temperature will gain internal energy (the validity of this assumption will follow from the generalized Clausius' postulate which will be presented later in this paper). Thus, dU_2 is less than zero, and if β_2 is greater than zero, $d\beta_2$ must be greater than zero. Since S_1 is infinitely massive, a finite four-momentum transfer cannot change its four-velocity. This means that S_1 sees S_2 speed up as S_2 slides over S_1.

Although they are not essential to the development of the generalized second law, some ideas concerning relativistic friction are presented in the next section.

Friction

A description of the processes involving relativistic friction will now be presented for the sake of completeness and because it provides another argument in favor of the theory of frictionless heat conduction. Let S_1 and S_2 be two isolated bodies with identical rest properties, but in relative motion. Now let S_1 and S_2 scrape against each other momentarily. Using conservation of energy and momentum, it is easily seen that this interaction must result in a four-momentum exchange $dP^i = (dE, cdP_x, cdP_y, cdP_z)$, from S_1 to S_2. Look at this process in the rest frame of S_1; next, make a Lorentz transformation to the rest frame of S_2; and finally, make a spatial inversion. As demonstrated in Fig. 1, this process looks the same in the final rest frame as it did in the original frame except S_1 (not S_2) is now the moving body, thus the four-

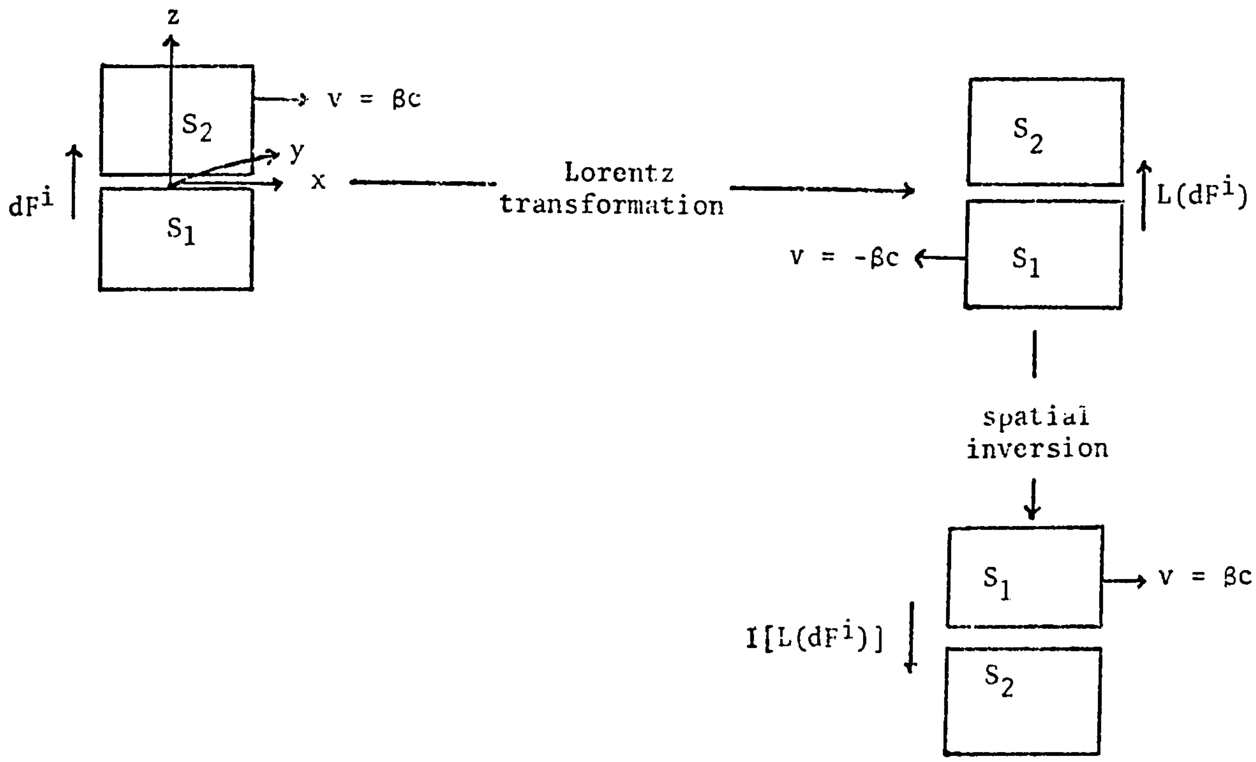

Figure 1. Transformation properties of friction.

momentum transferred from S_1 to S_2 in the original frame should equal that transferred from S_2 to S_1 in the final frame. The four-momentum transferred from S_1 to S_2 in the final frame can be found by operating on dF^i with a Lorentz transformation followed by an inversion, so

$$I\{L(dF^i)\} = -dF^i \tag{24}$$

or

$$(\gamma dE-\gamma\beta c dP_x, -\gamma c dP_x+\gamma\beta dE, -cdP_y, -cdP_z) = (-dE, -cdP_x, -cdP_y, -cdP_z) \tag{25}$$

So,

$$-\gamma dE + \gamma\beta c dP_x = dE\ , \tag{26}$$

$$\gamma c dP_x - \gamma\beta dE = c dP_x, \tag{27}$$

$$dP_y = dP_y\ , \tag{28}$$

and

$$dP_z = dP_z\ . \tag{29}$$

From Eq. (26) or (27) the following relationship between dE and dP_x can be derived for the rest frame of S_1:

$$c dP_x = \frac{(\gamma+1)}{\gamma\beta}\, dE\ . \tag{30}$$

Since the dP_y and dP_z terms are produced by static friction, they are of little interest and for simplicity will be put to zero. Notice that Eq. (30) is derived solely from symmetry arguments and as yet no physics has been injected.

So that relativistic friction will be consistent with the classical idea of kinetic energy being converted into heat, the momentum gained by one of the identical bodies as seen by the other will be assumed to be opposite in sign to the momentum of the first body. This is equivalent to requiring that dE be less than or equal to zero.

Since the initial frictional interaction, the four-momentum exchange caused by the scraping of the two surfaces, only depends on the regions very near the points of contact, the form of the initial frictional interaction should not depend on the two isolated bodies being identical. If two non-identical isolated bodies at the same rest temperature rub against each other, the interaction is assumed to begin with a four-momentum exchange in the form of Eq. (30). Since the two bodies are not identical, the initial frictional interaction could cause one of the bodies to have a higher rest temperature in the region near contact, and this temperature difference could result in heat transfer. So, the total frictional interaction between two isolated bodies at the same rest temperature consists of a four-momentum exchange in the form of Eq. (30) followed by an induced heat transfer. Since the initial frictional interaction causes both bodies to gain internal energy, since internal energy gain results in temperature gain, and since internal energy should flow from high to low rest temperature during the induced heat transfer, neither body can be allowed to lose internal energy during the total frictional interaction; therefore, the amount of induced heat transfer must be restricted by requiring the amount of internal energy lost by a body due to induced heat transfer to be less than the internal energy gained in the initial frictional interaction. Since internal energy is usually a monotonically increasing function of rest temperature, this requirement insured that a common type of thermodynamic system cannot lose rest temperature in a totally frictional interaction. It can easily be seen from Eqs. (19) and (30) that four-momentum from the initial frictional interaction is normal to the four-momentum exchange resulting from heat transfer ($dF^i dQ_i = 0$); therefore, since in the frame where S_2 has a velocity $-\beta'c$ and S_2 has a velocity $\beta'c$ dQ^i is of the form $(dE,0,0,0)$, dF^i must be of the form $(0,dP,0,0)$. So, in the midway frame a four-momentum transfer of the form $(0,dP,0,0)$ is followed by one of the form $(dE,0,0,0)$ when friction occurs. The internal energy change due to $(dE,0,0,0)$ must be less than that due to $(0,dP,0,0)$, so

$$|\gamma' dE| \leq |\gamma'\beta' dP| \ , \tag{31}$$

and the total frictional interaction must lie in the shaded portion of the energy-momentum plane in Fig. 2.

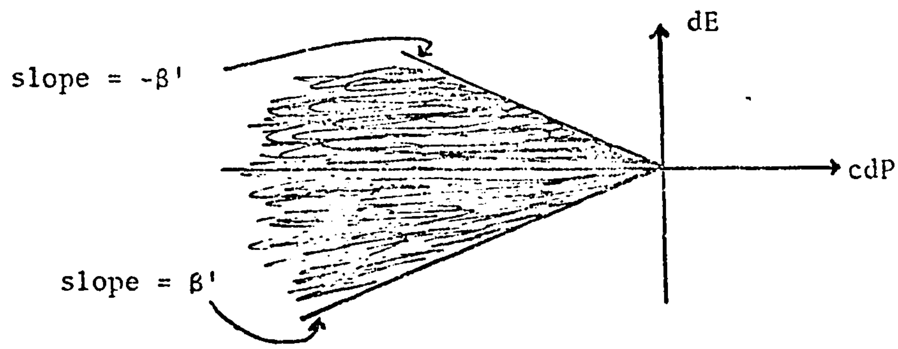

Figure 2. The frictional four-momentum transfer from S_1 to S_2, as seen in the inertial frame where S_1 and S_2 have equal and opposite velocities ($-\beta'c$ and $\beta'c$).

Another way to obtain the theory of frictionless interaction is to require that it contain no friction. In the midway frame the initial frictional interaction is of the form $(0,dP,0,0)$, so if heat transfer is to contain no friction it must be of the form $(dE,0,0,0)$ in this frame. This, of course, is the same result that was obtained earlier.

The distinction between an isolated body and a more general system is that stresses cannot act on an isolated body. It seems unlikely that work done by stresses could have an effect on the form of interactions due to friction or heat transfer; therefore, heat transfer and friction should be expected to be in their above forms when interactions between general bodies are considered.

Now that the digression into relativistic friction is complete, we will return to the main task of the thesis, the development of the generalized second law.

CHAPTER III

CARNOT CYCLES

As developed in this thesis, heat conduction between isolated bodies conserves total internal energy; therefore, if a differential amount of heat conduction occurs, one isolated body will gain an amount of rest mass and the other will lose an equal amount. Imagine a situation in which two relatively moving isolated bodies with identical rest properties come into frictionless thermal contact. If heat flow were to take place, all observers would agree that one particular body gained rest mass and the other lost rest mass. The symmetry of this situation destroys any possibility of deciding which of the bodies is the one that gains rest mass, thus if heat were transferred spontaneously between two such bodies, the symmetry of the situation would be denied. Now take two relatively moving bodies which are at the same rest temperature, but not necessarily identical or isolated, and let each body be in thermal contact with a comoving isolated body also at the same rest temperature. Suppose the two isolated bodies are given identical rest properties and allowed to come into frictionless thermal contact with each other. Since the nature of a differential thermal interaction can only depend on the properties of the interacting material near the points of contact, allowing the identical isolated bodies to be in thermal contact with more general bodies will not alter the fact that the isolated bodies cannot spontaneously transfer heat. Of course, classical thermodynamics outlaws any spontaneous heat transfer between a more general body and its comoving isolated body. So, since thermal contact can occur between the two more general bodies without spontaneous flow, it follows, using the same arguments as found in classical thermodynamics, that any infinitesimal transfer of heat between bodies at the same rest temperature is a reversible process.

Heat sources, bodies which have the ability to interact with their surroundings only by heat transfer, are assumed to exist in classical thermodynamics. It is also assumed that there exists a heat source or series of heat sources which can replace the thermal part of any process. That is, when a system interacts with its surroundings, the interaction can be divided into a mechanical and a non-mechanical part and the effect on the system due to the non-mechanical part could be provided by a heat source. It will now be assumed that heat sources exist in the thermodynamics of moving bodies, and since work done by stresses is not a thermal phenomenon, it will be assumed that heat sources can be isolated bodies. So, a heat source is an isolated body which has the ability to only transfer heat and which can perform all heat transfer functions.

When a body exchanges an infinitesimal amount of heat with a heat source at the same rest temperature, the process shall be envisioned as occurring in the following manner: First the body is isolated from its mechanical environment, placed in a rigid container, and allowed to exchange an infinitesimal amount of four-momentum of the form $[dE,(\gamma-1)dE/\gamma\beta,0,0]$ with the heat source; the body is separated from the heat source and put back in contact with the environment; stresses can do an infinitesimal amount of work on the body and the environment can produce a mechanical accelerating force which transfers an infinitesimal amount of four-momentum to the body. The magnitudes of these two interactions with the environment can be adjusted at will. The energy and momentum supplied in the above way by a heat source always forms a timelike four-vector. Thus, if heat sources are to be capable of performing all thermal roles, it must be assumed that the thermal part of the energy and momentum gained by a system always forms a timelike four-vector.

In this paper ideal gases will be used as thermometers and as working fluids for Carnot cycles. So, in order to show that the theory developed in this paper can be used at temperatures so high that the individual particles have relativistic velocities, it is necessary to review certain properties of the relativistic ideal gas. For a detailed discussion of this subject, see THE RELATIVISTIC GAS by Synge(4). The ideal gas law ($PV_0 = nRT_0$) is also obeyed at relativistic temperatures. Since an ideal gas and a relatively moving body at the same rest temperature and in frictionless thermal contact will not spontaneously interact, it will be assumed that an ideal gas and a relatively moving body in frictionless contact, but not at the same rest temperature, will interact until they come to the same rest temperature. Thus, a frictionless ideal gas can be used to measure the rest temperature of a moving body. In other words, an ideal gas enclosed in a massless, smooth, rigid, container will be used as a thermometer, and such a thermometer will read the rest temperature of a relatively moving body.

In order to operate Carnot cycles between bodies at all temperatures, a relationship between the changes in rest volume and rest temperature must be derived for adiabatic transformations of ideal gases. When an ideal gas is adiabatically transformed, it is found, using the first law in the rest frame of the gas, that $dU + PdV_0 = 0$. The specific heat of an ideal gas is a function of rest temperature $\{dU/dT_0 = f(T_0)\}$. $f(T_0)$ is $(3/2)nR$ at normal temperatures, but at relativistic temperatures it is a non-constant function of rest temperature. So, the first law becomes

$$\frac{dV_0}{V_0} = -\frac{f(T_0)dT_0}{nRT_0} \quad . \tag{32}$$

Integrating Eq. (32) yields Eq. (33).

$$\frac{V_0(\text{final})}{V_0(\text{initial})} = G\{T_0(\text{initial}, T_0(\text{final})\} \ , \tag{33}$$

where $G\{T_0(\text{initial}, T_0(\text{final})\}$ is a function only of the final and initial rest temperatures. This demonstrates that the ratio of the final and initial rest volumes of an ideal gas which undergoes an adiabatic transformation depends only on the initial and final rest temperatures.

Now the discussion of the two Carnot cycles necessary for the development of the generalized second law will be presented.

The Six Step Carnot Cycle

Carnot cycles will be developed in terms of infinitesimal interactions between finite mass heat sources, though this can be done with finite interactions between infinite mass heat sources. The six step Carnot cycle used here is identical to the one presented by C. Møller[5] except an ideal gas will be used in place of a perfect fluid. The Carnot cycle will be operated between two heat sources, S_1 and S_2, with rest temperatures, T_{01} and T_{02}. S_2 will have a velocity βc in the x-direction relative to S_1. The six step will now be described as seen from the rest frame of S_1.

(1) One mole of an ideal gas is compressed (or expanded) isothermally while in thermal contact with S_1. The velocity of the gas is maintained at zero.

(2) The ideal gas is adiabatically compressed (or expanded) at zero velocity until the rest temperature goes from T_{01} to T_{02}.

(3) The gas is accelerated by mechanical forces from velocity zero to βc without changing its rest properties.

(4) The gas is isothermally expanded (or compressed) at a constant velocity, βc, while in thermal contact with S_2. The magnitude of this interaction is adjusted so that the Carnot cycle will return the ideal gas to its original state at the end of step (6).

(5) The ideal gas is adiabatically expanded (or compressed) at a constant velocity, βc, until its rest temperature goes from T_{02} to T_{01}.

(6) The ideal gas is decelerated by a mechanical force to zero velocity without changing its rest properties. The ideal gas has now returned to its original state.

One mole of an ideal gas has the following properties:

$$U = f(T_0) \;; \qquad E = \gamma(U+\beta^2 RT_0) \;; \tag{34}$$

$$PV_0 = RT_0 \;; \quad \text{and} \quad M = \frac{\gamma\beta}{c}(U+RT_0) \;, \tag{35}$$

where E is the total energy of the ideal gas, M is the total momentum, and U is the internal energy. As seen by S_1 the four-momentum, dQ^i_{11}, produced by S_1 in step (1) is of the form $(dQ_0,0,0,0)$. Using the classical first law in the rest frame of S_1, it follows that

$$dQ_0 = dW_0 = RT_0 \ln(V_{02}/V_{01}) \;, \tag{36}$$

where V_{01} is the rest volume of the gas before step (1), V_{02} is the rest volume before step (2), etc. Since steps (2) and (5) are adiabatic processes operating between the same rest temperatures.

$$V_{03}/V_{02} = G(T_{01},T_{02}) = V_{05}/V_{06} \;. \tag{37}$$

The four-momentum produced by S_2 in step (4) will be calculated first in the rest frame of S_2. As seen by S_2, S_2 produces a four-momentum of the form $(dQ^*,0,0,0)$. Using the classical first law in the rest frame of S_2, it is found that

$$dQ^* = RT_{02}\ln(V_{05}/V_{04}) \;. \tag{38}$$

Since $V_{03} = V_{04}$ and $V_{06} = V_{01}$, Eq. (38) goes to Eq. (39).

$$dQ^* = -\,RT_{02}\ln(V_{02}/V_{01}) \tag{39}$$

or

$$dQ^* = (-T_{02}/T_{01})dQ_0 \ . \tag{40}$$

A Lorentz transformation of this four-momentum to the rest frame of S_1 reveals that the four-momentum produced by S_2, as seen by S_1, is $dQ^i_{21} = [-\gamma(T_{02}/T_{01})dQ_0, -\gamma\beta(T_{02}/T_{01})dQ_0, 0, 0]$. Since all the other interactions between the ideal gas and the environment are expansions, compressions, and accelerations, they can be supplied by mechanical forces. Thus, the sole result of this Carnot cycle, as seen by S_1, is that S_1 produces an amount of four-momentum, $(dQ_0, 0, 0, 0)$, and S_2 produces an amount, $[-\gamma(T_{02}/T_{01})dQ_0, -\gamma\beta(T_{02}/T_{01})dQ_0, 0, 0]$.

In order to prove some of the theorems in the later sections, it is necessary to have at least two different types of Carnot cycles; therefore, the theory of frictionless heat transfer will now be used to develope a new type of Carnot cycle.

The Four-Step Carnot Cycle

This reversible carnot cycle is the same as the six step cycle except steps (3) and (6) are omitted. That is, the ideal gas is maintained at zero velocity relative to S_1 throughout the entire cycle, and the contact during step (4) is frictionless and reversible in the sense described earlier. The form of the four-momentum produced by S_2 in step (4), as seen by S_1, is $dQ^i_{21} = [dQ', (\gamma-1)dQ'/\gamma\beta, 0, 0]$. The interaction between the ideal gas and the mechanical environment in step (4) is adjusted so that the temperature and velocity of the gas remain unchanged. This means that the environment must do some compressional type work to keep the rest temperature constant and must supply the gas with an amount of momentum, $dP = -(\gamma-1)dQ'/\gamma\beta c$ as seen by S_1, to keep the velocity constant. The internal energy transferred from S_2 to the gas is dQ', thus doing a calculation identical to the one done for the six step cycle, it is found that $dQ' = -(T_{02}/T_{01})dQ_0$. In summary, as seen by S_1 the sole result of the four step Carnot cycle is that S_1 produces a four-momentum, $(dQ_0,0,0,0)$, and S_2 produces a four-momentum, $\{-(T_{02}/T_{01})dQ_0,-(T_{02}/T_{01})(\gamma-1)dQ_0/\gamma\beta,0,0\}$.

CHAPTER IV

THE GENERALIZED SECOND LAW OF THERMODYNAMICS

In classical thermodynamics heat transfer is considered to be an exchange of energy, but in relativistic thermodynamics heat transfer must be viewed as an exchange of four-momentum. Relativistically heat transfers are four-momentum transfers, while work is a single quantity and does not by itself form a four-vector. Thus, statements such as, "The sole result of a process at a uniform temperature cannot be a change of heat into work.", do not have a clear meaning in relativistic thermodynamics. For this reason the two verbal statements of the second law must be generalized and put into relativistic language.

The Generalized Kelvin's Postulate

In classical thermodynamics it is assumed that by using friction it is always possible to transform work into heat at any temperature.* In relativistic language this assumption becomes: As seen in the rest frame of a body, it is always possible for a body to accept

*See Thermodynamics by Fermi.[9]

four-momentum of the form $(dE,0,0,0)$, $dE \geq 0$, when the sole result is an increase in internal energy of the body. The term, sole result, means that there is no change in the rest thermodynamic properties of any of the bodies which compose the environment. This type on interaction could be supplied by a series of processes of the type described in the section on friction. For example, the gravitational potential energy of a system could be used to push a brush back and forth over an isolated body; as seen in the rest frame of the isolated body, the momentum change will cancel in the back and forth motion and the result will be a gain of four-momentum of the form $(dE,0,0,0)$, $dE \geq 0$. Mechanical accelerating forces are another sole result process which can transfer four-momentum to an isolated body. When an accelerating force is applied, the isolated body undergoes a four-momentum change of the form $(0,cdP_x,cdP_y,cdP_z)$, as seen in the rest frame of the isolated body. Arbitrary values of dP_x, dP_y, and dP_z can be produced by varying the magnitude, direction, and duration of the force. By making combinations of friction and mechanical forces, it is possible to construct processes the sole result being that an isolated body accepts any differential amount of four-momentum which the body observes to be future pointing.

Now that some possible four-momentum interactions which do not alter the rest thermodynamic properties of the environment have been discussed, Kelvin's postulate will be that all such four-momentum productions by heat sources must be of this form. The generalized Kelvin's postulate is: A process the sole result of which is that a heat source produces an amount of four-momentum which is future pointing in the rest frame of the heat source is impossible.

The Generalized Clausius' Postulate

The classical Clausius' postulate is that a process, the only result being that heat is conducted from a given body at a uniform temperature to another body at a higher uniform temperature, is impossible. The theory of frictionless heat conduction predicts that one isolated body gains and amount of internal energy during heat transfer and the other body loses an equal amount of internal energy. It is natural to expect the body at the higher rest temperature to lose internal energy; therefore, the generalized Clausius postulate is: A process the only result of which is that a heat source transfers an amount of four-momentum of the form $[dE, (\gamma-1)dE/\gamma\beta, 0, 0]$, $dE > 0$, as seen in its rest frame, to a heat source at a higher rest temperature and a velocity, βc, is impossible. In an only result process there is no change in the environment of the two bodies. Notice that an only result process is more restrictive than a sole result process.

The Equivalence of the Clausius' and Kelvin's Postulates

First it will be shown that the generalized Kelvin's postulate is valid if the generalized Clausius' postulate is valid. If the generalized Kelvin's postulate were false, a process the sole result of which is that a heat source produces an amount of four-momentum, $(dE, cdP_x, cdP_y, cdP_z)$, $dE > 0$, as seen in the heat source's rest frame, would be possible. Now a mechanical force of such magnitude and duration that the four-momentum transfer is $(0, cdP_x, cdP_y, cdP_z)$ is applied to the heat source. Heat sources are bodies with the ability to interact only thermally, but, if desired, mechanical accelerating forces can be applied to them. Thus, there now exists a process the sole result of which is that a heat source can produce an amount of four-momentum (dE,0,0,0), $dE > 0$, as seen in the heat source's rest frame. Notice that this is a violation of the classical Kelvin's postulate; therefore, the generalized Kelvin's postulate is valid if the classical Kelvin's postulate is valid. Since the above production of (dE,0,0,0) is a sole result process, dE must not alter the rest properties of any of the bodies in the environment. This means that dE must go into the environment as a kinetic or potential energy. By means of back and forth friction this kinetic or potential energy can be turned into heat, and this heat can be absorbed by a second heat source which is at rest with respect to the first heat source. If the rest temperature of the second heat source is taken to be greater than that of the first, it is possible to create a process whose only result is that four-momentum of the form $[dE,(\gamma-1)dE/\gamma\beta,0,0]$, $dE > 0$, [here $\beta = 0$ and $(\gamma-1)/\gamma\beta = 0$] is transferred from a heat source to another heat source at a higher rest temperature. So, the generalized Kelvin's postulate is valid if the generalized Clausius' postulate is valid.

This part of the equivalence could have been proved using the kinetic or potential energy to drive a Carnot cycle, instead of using it to produce heat. The result would be the same, i.e. a process the only result of which is a transfer of heat from one heat source to another at a higher rest temperature.

Now it will shown that the generalized Kelvin's postulate is valid only if the generalized Clausius' postulate is valid. Imagine two heat source, S_1 and S_2, at rest temperatures, T_{01} and T_{02}, $T_{02}>T_{01}$. S_2 is given a velocity βc in the x-direction relative to S_1. Now assume that in violation of the generalized Clausius' postulate S_1 transfers an amount of four-momentum $dQ^i = [dE,(\gamma-1)dE/\gamma\beta,0,0]$, $dE > 0$, as seen in the rest frame of S_1, to S_2. Earlier in this paper a six-step Carnot cycle, which will be called cycle #1, and a four-step Carnot cycle, which will be called cycle #2, were discussed. If the two cycles are allowed to operate between S_1 and S_2, the four-momentum productions shown in Table #1 will occur. Notice that the ratio of the energy to the momentum produced by S_2 is different in cycle #1 and cycle #2, so by adjusting the values of dE_1 and dE_2, any ratio of energy to momentum produced by cycle #1 and cycle #2 combined can be realized. The magnitudes of dE_1 and dE_2 are now adjusted so that the Carnot cycles combined with the assumed four-momentum transfer will result in S_1 remaining unchanged. Thus,

$$dE + \gamma\frac{T_{02}}{T_{01}}dE_1 + \frac{T_{02}}{T_{01}}dE_2 = 0 \quad , \tag{41}$$

and

$$\frac{(\gamma-1)}{\gamma\beta}dE + \frac{T_{02}}{T_{01}}\gamma\beta dE_1 + \frac{T_{02}}{T_{01}}\frac{(\gamma-1)}{\gamma\beta}dE_2 = 0 \ . \tag{42}$$

Equations (41) and (42) reduce to $dE_1 = 0$ and $dE_2 = -(T_{01}/T_{02})dE$. The sole result of this series of processes is that S_1 produces some four-momentum. If $dE + dE_1 + dE_2 > 0$, the generalized Kelvin's postulate is violated.

$$dE + dE_1 + dE_2 = (1-\frac{T_{01}}{T_{02}})dE \quad . \tag{43}$$

By assumption T_{01}/T_{02} is less than one and dE is greater than zero, so $(1-T_{01}/T_{02})dE$ is positive and the generalized Kelvin's postulate is

Table 1. Four-momentum Productions for Cycle #1 and Cycle #2.

	Cycle #1	Cycle #2
The four-momentum produced by S_1	$(dE_1,0,0,0)$	$(dE_2,0,0,0)$
The four-momentum produced by S_2	$(-\gamma \frac{T_{02}}{T_{01}} dE_1, -\gamma\beta \frac{T_{02}}{T_{01}} dE_1, 0, 0)$	$(- \frac{T_{02}}{T_{01}} dE_2, - \frac{(\gamma-1)}{\gamma\beta} \frac{T_{02}}{T_{01}} dE_2, 0, 0)$

defied. This means that the generalized Kelvin's postulate is valid only if the generalized Clausius' postulate is valid.

It has now been demonstrated that the two verbal statements of the generalized second law of thermodynamics are equivalent. Since the classical Kelvin's postulate implies the generalized Kelvin's postulate, it also implies the generalized Clausius' postulate, if the existence of frictionless interactions is accepted. Now that the generalized second law has been established, we will, following the example of classical thermodynamics, use the second law to define temperature and entropy.

CHAPTER V

TEMPERATURE AS DEFINED BY THE SECOND LAW

Referring to Table #1, it is found that the internal energy changes suffered by the two heat sources when cycle #1 is operated are

$$dU_1 = - dE_1 \quad , \tag{44}$$

and

$$dU_2 = \frac{T_{02}}{T_{01}} (\gamma^2-\gamma^2\beta^2)dE_1 = \frac{T_{02}}{T_{01}} dE_1 \quad . \tag{45}$$

When cycle #2 is operated, the internal energy changes are

$$dU_1 = - dE_2 \tag{46}$$

and

$$dU_2 = \frac{T_{02}}{T_{01}} (\gamma-\gamma+1)dE_2 = \frac{T_{02}}{T_{01}} dE_2 \quad . \tag{47}$$

Notice that the ratio of the internal energy changes of the heat sources is the same in cycles #1 and #2, i.e.

$$- \frac{dU_1}{dU_2} = \frac{T_{01}}{T_{02}} \tag{48}$$

for both cycles.

<u>Theorem #1</u>: Let there be two isolated bodies, S_1 and S_2, at rest temperatures, T_{01} and T_{02}, $T_{02} > T_{01}$, and let S_2 have a velocity βc in the x-direction relative to S_1. If there is an arbitrary cyclic heat engine operating between S_1 and S_2 such that S_1 and S_2 produce the amounts of four-momenta, $dQ_1^i = (dL_1, cdN_1, 0, 0)\,(dL_1 < 0)$ and $dQ_2^i = (dL\ , dN\ , 0, 0)$, as seen by S_1, during the operation of the cycle, then

$$- dU_2^*/dU_1^* \leq - dU_2/dU_1 \ , \tag{49}$$

where dU_1^* and dU_2^* are changes in the internal energy caused by the arbitrary cycle and dU_2/dU_1 is the ratio of the internal energy changes of the heat sources resulting from the operation of either of the two reversible Carnot cycles presented earlier. The equality sign will hold in the case of a reversible cycle.

In order to prove this theorem, the two reversible Carnot cycles, cycle #1 and cycle #2, are adjusted so that there is no change in S_2 after all three cycles have operated. As viewed from the rest frame of S_1, the momentum and energy productions must obey the following formulae:

$$dL_2 + \gamma \frac{T_{02}}{T_{01}} dE_1 + \frac{T_{02}}{T_{01}} dE_2 = 0 \tag{50}$$

and

$$dN_2 + \gamma\beta \frac{T_{02}}{T_{01}} dE_1 + \frac{(\gamma-1)}{\gamma\beta} \frac{T_{02}}{T_{01}} dE_2 = 0 \quad . \tag{51}$$

Solving these two simultaneous equations for dE_1 and dE_2, it is found that

$$dE_2 = \frac{T_{01}}{T_{02}} \frac{(\gamma^2 \beta dN_2 - \gamma^2 \beta^2 dL_2)}{(\gamma-1)} \tag{52}$$

and

$$dE_1 = \frac{T_{01}}{T_{02}} \frac{(dL_2 - \gamma\beta dN_2)}{(\gamma-1)} \quad . \tag{53}$$

The sole result of these three cyclic processes is that S_1 produces an amount of four-momentum. If this four-momentum is future pointing in the rest frame of S_1, the generalized Kelvin's postulate is violated. This means $dL_1 + dE_1 + dE_2$ must be less than or equal to zero, i.e.

$$\begin{aligned} dL_1 + dE_1 + dE_2 &= dL_1 - \frac{T_{01}}{T_{02}} (\gamma-1)\{\gamma^2\beta dN_2 - \gamma^2\beta^2 dL_2 - \gamma\beta dN_2\} \\ &\quad + \{(1/\gamma) - 1 + \gamma\beta^2\} dL_2 \leq 0 \quad . \end{aligned} \tag{54}$$

Simplifying Eq. (54) yields Eq. (55).

$$dL_1 + dE_1 + dE_2 = dL_1 + \frac{T_{01}}{T_{02}} \gamma(dL_2 - \beta dN_2) \leq 0 \quad . \tag{55}$$

Remembering that $dU_1^* = -dL_1$ and $dU_2^* = -\gamma(dL_2 - \beta dN_2)$, it is discovered that

$$-dU_2^*/dU_1^* \leq \frac{T_{02}}{T_{01}} = -dU_2/dU_1 \quad . \tag{56}$$

Since a reversible cycle can be operated in the opposite direction, it can be shown by the same method as above that

$$-dU_2^*(\text{rev.})/dU_1^*(\text{rev.}) \geq -dU_2/dU_1 \quad . \tag{57}$$

This means the equality sign must hold in the case of reversible cycles, Q.E.D.

Since all reversible cycles operating between S_1 and S_2 have the same ratio of internal energy changes, by using the same arguments as found in classical thermodynamics, it can be shown that the temperature, T, can be defined by Eq. (58).

$$T_2/T_1 = - dU_2(\text{rev.})/dU_1(\text{rev.}) \quad . \tag{58}$$

Recalling that $- dU_2(\text{rev.})/dU_1(\text{rev.}) = T_{02}/T_{01}$, it is seen that $T = T_0$. So, as in classical thermodynamics, the temperature measured by a reversible cycle is equal to the temperature measured by an ideal gas thermometer. It should be mentioned that other authors[7,8] have also argued that $T = T_0$.

CHAPTER VI

THE GENERALIZED ENTROPY

Theorem #2:

Let there be a system which undergoes a cyclic transformation in such a way that all of the thermal energy and momentum transfers come from N different heat sources. If the rest temperatures of the heat sources are denoted by $T_{02}, T_{02}, \cdots, T_{0N}$, if the four-momenta produced by the heat sources are denoted by $dQ_1^i, dQ_2^i, \cdots, dQ_N^i$, and if the four-velocities of the heat sources are denoted by $V_{1i}, V_{2i}, \cdots, V_{Ni}$, then

$$\sum_{j=1}^{N} dQ_j^i V_{ji}/T_{0j}$$

must be less than or equal to zero and the equality condition must hold for reversible cycles.

In order to prove theorem #2, a cycle #1 and a cycle #2 will be operated between that source #1 and each of the remaining N-1 heat sources. The magnitude of the 2(N-1) cycles will be adjusted so that there is no change in each of the N-1 heat sources. Doing a series of calculations identical to the one done in the proof of theorem #1 and using the generalized Kelvin's postulate, it is found that

$$\sum_{j=1}^{N} dQ_j^i V_{ji}/(T_{0j}/T_{01}) \leq 0 \quad . \tag{59}$$

Multiplying Eq. (59) by $1/T_{01}$, gives that

$$\sum_{j=1}^{N} dQ_j^i V_{ji}/T_{0j} \leq 0 \quad . \tag{60}$$

As in theorem #1, the equality sign must hold for a reversible cycle, Q.E.D.

Since $dQ_j^i V_{ji} = - dU_j$, where dU_j is the change in internal energy of the jth heat source, theorem #2 can be written

$$\sum_{j=1}^{N} - dU_j/T_{0j} \leq 0 \quad . \tag{61}$$

Now take an arbitrary cyclic process which does not necessarily accept all of its heat from heat sources as did the cycle in theorem #2. Since by assumption there exists a heat source or series of heat sources which is capable of performing any thermal process, an identical cycle could be operated with heat sources supplying all the non-mechanical energy and momentum. Hence, the arguments used in theorem #2 can be applied to an arbitrary cycle. This means that

$$\sum_{j=1}^{N} (dU_{n.m.})_j/T_{0j} = \sum_{j=1} (dQ^i_{n.m.})_j V_{ji}/T_{0j} \leq 0 \ , \tag{62}$$

where $(dU_{n.m.})_j$ is the change in internal energy of the working fluid due to non-mechanical occurences in the jth interaction, $(dQ^i_{n.m.})_j$ is the non-mechanical four-momentum provided by the environment during the jth interaction, T_{0j} is the rest temperature of the heat source which replaces the non-mechanical part of the jth interaction, and V_{ji} is the four-velocity of that heat source. When there is a continuous sequence of differential processes, the summation can be changed to an integral.

Using theorem #2, it is seen that $dU_{n.m.}/T_0$ is an exact differential for reversible processes; therefore, the generalized entropy can be defined by the following equation:

$$dS = (dU_{n.m.}/T_0)_{rev.} \quad . \tag{63}$$

The second law can now be written

$$dS \geq dU_{n.m.}/T_0 \quad , \tag{64}$$

where T_0 is the rest temperature of the environment. From Eq. (63) it is observed that the second law has been developed in a manner which causes the entropy to be an invariant.

As an application of Eq. (64), let's investigate the entropy change when two identical isolated bodies, S_1 and S_2, having rest temperature T_0 interact by means of friction. In this case, the four-momentum transfer from S_1 to S_2, as seen by S_1, is $[dE,(\gamma+1)dE/\gamma\beta,0,0]$. For a system made by combining S_1 and S_2 Eq. (64) becomes

$$dS = \frac{dU_1}{T_0} + \frac{dU_2}{T_0} \geq 0 \quad . \tag{65}$$

Now let's calculate dU_1 and dU_2.

$$dU_1 = - dE \tag{66}$$

$$dU_2 = \gamma dE - \gamma\beta \frac{(\gamma+1)}{\gamma\beta} dE = - dE \quad . \tag{67}$$

So, Eq. (65) becomes

$$- 2 \frac{dE}{T_0} \geq 0 \tag{68}$$

or

$$dE \leq 0 \quad . \tag{69}$$

Equation (69) is the same result as was obtained from the physical argument in the section on friction.

CHAPTER VII

THE RELATIONSHIP TO OTHER THERMODYNAMICS

In Planck thermodynamics the heat transfer, dQ_p, is defined for a perfect fluid by the following equation:

$$dQ_p = dE + PdV - \vec{v} \cdot d\vec{M} \quad , \tag{70}$$

where - PdV is the work done on the perfect fluid due to deformations, $d\vec{M}$ is the change in momentum of the fluid, and $\vec{v}$ is the velocity of the fluid. In special relativity force, $\vec{F}$, is defined as the time rate of change of momentum; therefore, $\vec{v} \cdot d\vec{M}$ is $\vec{F} \cdot d\vec{r}$ and is equal to the work done on the fluid by accelerating forces. This means that dQ_p is the change in energy less the total amount of work done on the fluid, i.e. the total energy gained due to non-work processes. In the 1960's Ott, Kibble, Møller, and others(2) correctly pointed out that the term $d\vec{M}$ in Planck's definition of dQ contains momentum changes due to heat transfer. The modern authors now argue that, since heat transfer is a non-mechanical process, only the work due to mechanical forces should be subtracted from the energy change in determining dQ. The modern authors have in effect redefined dQ as the total energy change less the mechanical work, i.e. the total amount of non-mechanical energy transferred. Notice that these are not the only plausible definitions of dQ, e.g. dQ could be defined as $dU_{n.m.}$ and the same statements about entropy and temperature as given in this paper would be deduced.

Since $dS = (dU_{n.m.}/T_0)_{rev.}$ reduces to the classical value of the entropy change when comoving processes are studied, the thermodynamics presented in this thesis predicts the same entropy change for comoving processes as does Planck thermodynamics and the modern thermodynamics. So, since all compute the same comoving entropy changes and since everyone accepts conservation of energy, Planck thermodynamics, modern thermodynamics, and the thermodynamics presented here are physically equivalent for comoving processes. In order to be consistent with the theory presented here, Planck and modern thermodynamics must agree with this paper when entropy changes suffered during either of the two moving processes described earlier (frictionless heat conduction and friction) are calculated. In order to check this consistency, let's first calculate, using each of the three theories, the entropy change suffered by S_2, as seen by S_1, when S_1 and S_2 exchange heat frictionlessly. If $T_{02} > T_{01}$ and βc is the velocity of S_2 relative to S_1, the four-momentum transfer from S_1 to S_2 is $[dE,(\gamma-1)dE/\gamma\beta,0,0]$, $dE \leq 0$. So, using Planck thermodynamics,

$$dS_2 = \frac{dQ_p}{T_{02}/\gamma} = \gamma\frac{(dE-dW)}{T_{02}} = \left\{dE-\beta\,\frac{(\gamma-1)}{\gamma\beta}\,dE\right\}\frac{\gamma}{T_{02}} = dE/T_{02}\ . \tag{71}$$

Since heat transfer is a completely non-mechanical interaction, modern thermodynamics predicts Eq. (71).

$$dS_2 = dQ_m/\gamma T_{02} = dE/\gamma T_{02}\ . \tag{72}$$

For the theory presented in this thesis,

$$dS_2 = dU_{n.m.}/T_{02} = dE/T_{02}\ . \tag{73}$$

From Eqs. (71), (72) and (73), it is seen that Planck thermodynamics and the theory presented here disagree with modern thermodynamics on entropy changes for this non-comoving phenomenon as observed from this particular rest frame. It can easily be shown that Planck thermodynamics agrees with this thesis on entropy calculations when heat transfer is observed from any inertial frame. By a similar calculation, it can be shown that Planck thermodynamics is consistent with this thesis when friction is encountered, while modern thermodynamics is not consistent unless a particular interpretation of reversible heat transfer is used. The difficulty in modern thermodynamics is that dQ_m does not equal $\gamma(dQ_m)_0$ for some processes in which a body suffers non-mechanical momentum changes in its rest frame (friction and frictionless heat transfer are this type of process). C. Møller[3,5] introduces a redefinition of dQ_m for processes in which a body suffers non-mechanical momentum changes in its rest frame. Entropy calcuations made using Møller's revised definition of dQ are in agreement with the theory presented in this thesis. Though it is of no great importance, it is interesting that modern thermodynamics must adopt a somewhat contrived redefinition of dQ, while dQ_p can remain unaltered for processes in which there is a non-mechanical momentum transfer in the rest frame.

It can easily be shown that $-(dQ_p)_2/(dQ_p)_1$ is the same for the four- and six-step reversible Carnot cycles. If $dQ_{r.m.}$ is used to denote the revised modern definition of heat transfer, the quantity $-(dQ_{r.m.})_2/(dQ_{r.m.})_1$ can also be shown to be the same for cycle #1 and cycle #2. Thus, either of these ratios could have been used to define temperature, and the entropy could have been called dQ_p/T_p or $dQ_{r.m.}/T_m$.

That is, using the procedure developed in this paper, entropy and temperature could have been put in the form of Planck thermodynamics or the modern revised thermodynamics. However, it is desirable to have the temperature an invariant so that the temperature defined by the second law is equal to the temperature measured by thermometric means. Also, if one accepts the generalized Clausius' postulate, making the temperature an invariant insures that bodies at the same temperature do not exchange heat spontaneously and that heat (internal energy) flows from a higher to a lower temperature.

J. Lindhard[7] has published a paper which in spirit is closely akin to the work done in this thesis. That is, Lindhard does not assume that the second law retains its classical mathematical form, $dS \geq dQ/T$, when processes are viewed from a moving frame. Instead, he developes a definition of temperature and proves the existence of an exact integral, the entropy, by postulating a physically testable law which is closely related to the generalized Kelvin's postulate. Lindhard does not explicitly treat moving processes and he does not develope anything similar to the generalized Clausius' postulate. Lindhard assumes that there exists a means by which the energy and momentum extracted from a cycle operating between two heat sources can be converted into kinetic energy in the heat sources. This assumption is similar to the assumption to be used in the next section of this paper, i.e. there exists a means by which four-momentum can be accepted by a heat source if the four-momentum is produced by a cycle and future pointing in the rest frame of the heat source.

CHAPTER VIII

EXTENSION OF THE GENERALIZED CLAUSIUS' POSTULATE

S_1 and S_2 are heat sources at rest temperatures T_{01} and T_{02}, $T_{02} > T_{01}$. S_2 has a velocity βc in the x-direction relative to S_1. Four-momentum transfers from S_1 to S_2 of the form $(dE, cdP, 0, 0)$ can be viewed as a plane, the E-P plane, of four-momentum exchanges. The generalized Clausius' postulate says that four-momentum transfers from S_1 to S_2 of the form $[dE, (\gamma-1)dE/\gamma\beta, 0, 0]$, $dE > 0$, as seen by S_1, are impossible. Heat transfer in the opposite direction, i.e. a four-momentum transfer of the same form, but $dE \leq 0$, is assumed to be possible. By means of frictional interactions, four-momentum exchanges represented by the shaded region in Fig. 2 can be realized. So, the theories of heat conduction and friction have caused limited parts of the E-P plane to be classified as allowed or unallowed areas of four-momentum exchange, but nothing has been said about the other regions of the E-P plane.

The second law in the form of Eq. (64) will now be used to show that at least half the E-P must represent impossible four-momentum exchanges. Imagine that there is a four-momentum transfer from S_1 to S_2 of the form $(dE, cdP, 0, 0)$. If Eq. (64) is applied to the isolated system formed by mentally combining S_1 and S_2, the following result can be obtained:

$$dS = -\frac{dE}{T_{01}} + \frac{\gamma dE}{T_{02}} - \frac{\gamma\beta cdP}{T_{02}} \geq 0 \qquad (74)$$

or

$$cdP \leq \left(\frac{\gamma - T_{02}/T_{01}}{\gamma\beta}\right) dE \quad . \tag{75}$$

Hence, region I in Fig. 3 and Fig. 4 represents unallowed four-momentum exchanges. Eq. (75) can now be used to extend the generalized Clausius' postulate to the following statement: A process the only result of which is that a heat source, S_1, at rest temperature T_{01} transfers an amount of four-momentum of the form (dE,cdP,0,0), $cdP > (\gamma - T_{02}/T_{01})dE/\gamma\beta$, as seen by S_1, to another heat source, S_2, at a higher rest temperature T_{02} and a velocity βc in the x-direction is impossible. It should be noticed that this postulate reduces to the classical Clausius' postulate as $\beta \rightarrow 0$ or when $T_{02}/T_{01} >> \gamma$.

Now, what about the other half of the E-P plane; can it be divided into allowed and unallowed regions? If it is assumed that it is possible for a heat source to accept four-momentum produced by a Carnot cycle when that four-momentum is future pointing as seen by the heat source, this question can be partially answered. The above assumption is based on the possibility that the momentum part of the four-momentum transfer might be achieved by a mechanical accelerating force and the energy part might be transmitted by back and forth friction. The theory of frictionless interactions provides the means to mentally construct a family of reversible Carnot cycles (cycles #1 and #2 are members of this family). The Carnot cycles are devised by forcing the operating fluid to interact by isothermal heat transfer with S_1 while at an arbitrary velocity $\beta_1 c$ relative to S_1 and to interact by isothermal heat transfer with S_2 while at an arbitrary velocity, $\beta_2 c$ relative to S_2. The magnitudes of these interactions must be adjusted so that the process is cyclic. A cycle is now operated at every combination of β_1 and β_2. If for a given β_1 and β_2 the four-momentum produced by the cycle is future pointing as seen by S_2 (or S_1), S_2 (or S_1) is allowed to absorb this four-momentum. The only result of this combination of processes, Carnot cycle plus four-momentum absorption, is a four-momentum transfer from S_1 to S_2 equal to the four-momentum that S_1 produces (or minus the four-momentum that S_2 produces) during the operation of the cycle. By carrying out the calculation described above, new regions of the E-P plane will now be shown to represent possible four-momentum exchanges.

Let S_1 and S_2 be two heat sources at rest temperatures, T_{01} and T_{02}, $T_{02} > T_{01}$. S_2 is given a velocity, βc, in the x-direction relative to S_1. Now a cycle of the type just described is operated between S_1 and S_2. The four-momentum produced by S_1, as seen by S_1, is of the form $[dE_1,(\gamma_1-1)dE_1/\gamma_1\beta_1,0,0]$, and the four-momentum produced by S_2, as seen by S_2, is of the form $[dE_2,(\gamma_2-1)dE_2/\gamma_2\beta_2,0,0]$. In order to make the process cyclic dE_2 must be equal to $- dE_1T_{02}/T_{01}$. If the total four-momentum produced by the Carnot cycle is future pointing in the rest frame of S_2, S_2 is allowed to absorb that four-momentum. If for the moment it is assumed that $dE_1 \leq 0$, the four-momentum produced by the cycle is future pointing, as seen by S_2, if the following inequality is satisfied:

$$\frac{T_{02}}{T_{01}} - \gamma + \gamma\beta\frac{(\gamma_1-1)}{\gamma_1\beta_1} \geq 0 \quad . \tag{76}$$

Equation (76) is fulfilled if

$$\frac{\gamma_1-1}{\gamma_1\beta_1} \geq \left(\frac{\gamma-T_{02}/T_{01}}{\gamma\beta}\right) \quad . \tag{77}$$

The final result of this combination of Carnot cycle plus four-momentum absorption by S_2 is that an amount of four-momentum, $[dE_1,(\gamma_1-1)dE_1/\gamma_1\beta_1, 0,0]$, $dE_1 \leq 0$, is transferred from S_1 to S_2, as seen by S_1. The above combination of processes is performed at all β_1 which satisfy Eq. (77). The result is that region II in Figs. 3 and 4 represents possible four-momentum transfers from S_1 to S_2. Now it is assumed that $dE_1 \geq 0$. This reverses the inequality sign in Eqs. (76) and (77) and causes the four-momentum transferred from S_1 to S_2 to be future pointing. Thus, region III in Fig. 3 now represents possible four-momentum transfers. By making combinations of exchanges in regions II and III, any four-momentum transfer in region IV of Fig. 3 can be realized. The above procedure could be altered by allowing S_1 (not S_2) to absorb the four-momentum produced by the cycle; however, this calculation only leads to a repeat of the results in Figs. 3 and 4.

Irreversible Carnot cycles can be constructed by allowing either S_1 or S_2 to interact with the operating fluid by means of friction in the form of Eq. (30) instead of frictionless heat transfer. By running cycles of this nature, region V in Fig. 4 can be shown to be a possible area for four-momentum exchanges. More cycles can be formed by allowing induced heat transfer into the frictional interaction, but this does not result in any new information. A number of other processes have been studied; however, as yet no differential four-momentum transfers in the unmarked regions of Fig. 4 have been discovered. Nevertheless, it is suspected that the line, $[dE,(\gamma-T_{02}/T_{01})dE/\gamma\beta,0,0]$, divides the allowed from the unallowed regions in Fig. 4 as well as Fig. 3.

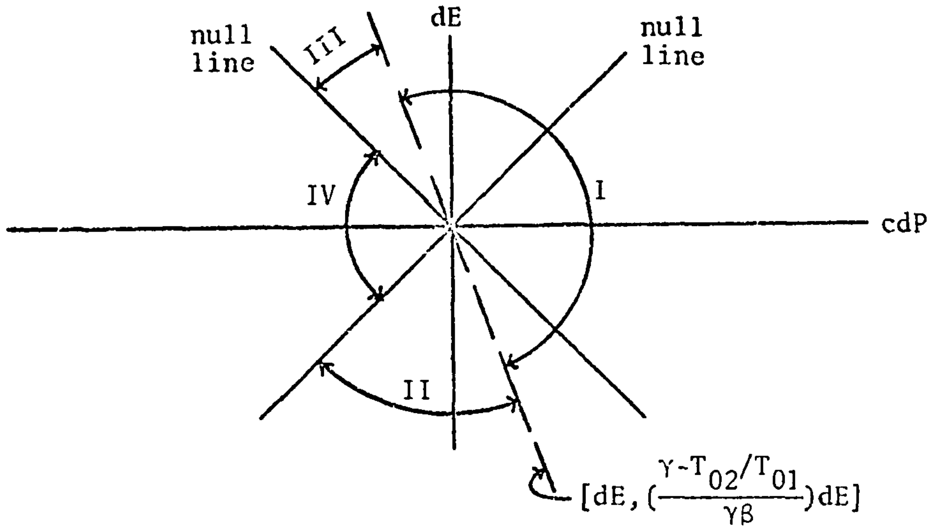

Figure 3. Allowed and unallowed four-momentum exchanges from S_1 to S_2 for the case that $T_{02} > T_{01}$ and $|\frac{\gamma - T_{02}/T_{01}}{\gamma\beta}| < 1$, as seen by S_1.

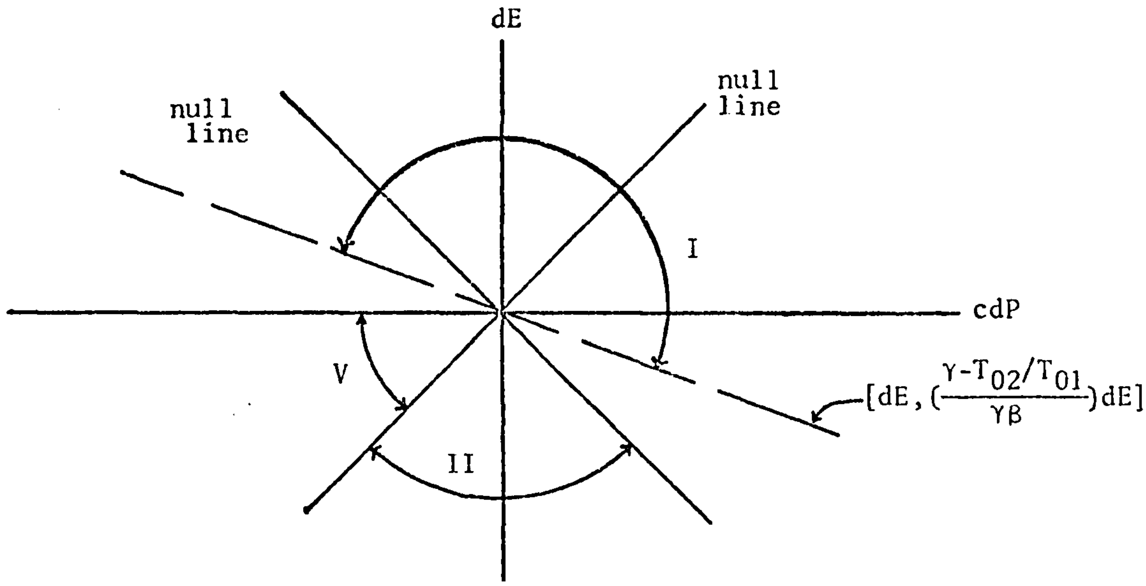

Figure 4. Allowed and unallowed four-momentum exchanges from S_1 to S_2 for the case that $T_{02} > T_{01}$ and $|\frac{\gamma - T_{02}/T_{01}}{\gamma\beta}| \geq 1$, as seen by S_1.

CHAPTER IX

THOUGHT EXPERIMENT

Although it is well out of the range of experimental capabilities, an experiment similar to the one described below could in principle be used to verify the theory of frictionless heat conduction and the theory of relativistic friction. Let there be two thin rings, R_1 and R_2, with identical rest properties spinning about their centers with angular velocities, ω and $-\omega$. Now let the two spinning rings touch momentarily. Since the rings are identical, this interaction could result in a frictional four-momentum exchange in the form of Eq. (30). The laboratory frame is the midway frame; therefore, as seen in the laboratory frame, the interaction is a pure momentum exchange, i.e. $dP^i = (0,cdP,0,0)$ is the four-momentum transfer from R_1 to R_2, as seen in the laboratory frame. This exchange causes R_1 to have the internal energy change given in Eq. (78).

$$dU_1 = C_V dT_{01} = -\gamma\beta(-cdP) = \gamma\beta c dP,$$

where C_V is dU_1/dT_{01}, β is $\omega r/c$, and r is the radius of the rings. By using the fact that $-rdP$ is the change in angular momentum, Eq. (79) can be derived.

$$-rdP = d(\gamma r^2 U_1 \omega/c^2) \quad . \tag{79}$$

The following relationship between $d\omega$ and dT_{01} can be reached by substituting for dP from Eq. (78) and simplifying:

$$d\omega = - \frac{C_v dT_{01}}{\gamma^2 \omega r^2 U_1 / c^2} \quad . \tag{80}$$

Equation (80) differs from the Newtonian expression by a factor of $(1/\gamma^2)$.

Now let's increase the rest temperature of R_2 and let the two rings touch in the same manner as before. In addition to the friction, there will be some heat transfer. The thermal four-momentum exchange will be of the form (-dE,0,0,0) in the laboratory frame. For the heat transfer part of the interaction, Eq. (78) becomes Eq. (80) and Eq. (79) becomes Eq. (82).

$$dU_1 = C_v dT_{01} = \gamma dE \quad , \tag{81}$$

$$0 = d(\gamma r^2 U_1 \omega / c^2) \quad . \tag{82}$$

Equations (81) and (82) can be used to derive the following relationship between $d\omega$ and dT_{01}:

$$\frac{d\omega}{\omega} = - \frac{C_v dT_{01}}{U_1 \gamma^2} \quad . \tag{83}$$

If it is assumed that the frictional part of the four-momentum exchange is the same in both interactions, the change in the rest temperature and the change in angular velocity in the second interaction less those in the first interaction should be in the form of Eq. (81). The Newtonian version of Eq. (81) is $d\omega = 0$.

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