

Let  $\Gamma$  be a directed graph consisting of  $V$ , a set of vertices,  $A$ , a set of arrows, and  $s, t : A \rightarrow V$  the source and target maps respectively. We construct an algebra on the doubled graph where

$$V_{double} = V \quad E_{double} = E \cup E^*$$

where  $E^* := \{e^* | e \in E\}$  and

$$\begin{aligned} s|_E &= s & t|_E &= t \\ s e^* &= t e & t e^* &= s e \quad \text{for all } e^* \in E^* \end{aligned}$$

On the doubled graph, we create  $L(\Gamma)$ , the Leavitt path algebra over some field  $\mathbb{F}$  where we impose the following relations on the non-unital algebra generated by words in  $V \cup E_{double}$  with the following relations imposed:

- (1)  $uv = \delta_{u,v}v$  for all  $u, v \in V$
- (2)  $s e e = e$  and  $e t e = e$  for all  $e \in E_{double}$
- (3)  $e^* f = \delta_{e,f} t e$  for all  $e \in E, f^* \in E^*$
- (4)  $\sum_{e \in s^{-1}v} e e^* = v$  for all  $v \in V$  such that  $s^{-1}v \neq \emptyset$

We will now construct a module on which  $L(\Gamma)$  acts. For this, we will consider the case where  $\Gamma$  contains a directed cycle. Let  $C = e_0 e_1 \cdots e_n$  be a prime cycle (it is not realizable as some power of a shorter cycle).

$\mathbb{F}[C^\infty]$  is a vector space whose basis are paths that extend infinitely to the right (in  $\Gamma$ , not the doubled graph) that are eventually  $\cdots C \cdot C \cdot C \cdots$ . Also, let  $f(x)$  be an irreducible polynomial over  $\mathbb{F}$  with  $f(0) = 1$ .

Then  $M := \mathbb{F}[x, x^{-1}] / (f(x)) \otimes \mathbb{F}[C^\infty]$

Anyone caught using formulas such as  $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$  or  $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$  will fail.

The binomial theorem is

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

A favorite sum of most mathematicians is

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Likewise a popular integral is

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

**Theorem 0.1.** *The square of any real number is non-negative.*

*Proof.* Any real number  $x$  satisfies  $x > 0$ ,  $x = 0$ , or  $x < 0$ . If  $x = 0$ , then  $x^2 = 0 \geq 0$ . If  $x > 0$  then as a positive time a positive is positive we have  $x^2 = xx > 0$ . If  $x < 0$  then  $-x > 0$  and so by what we have just done  $x^2 = (-x)^2 > 0$ . So in all cases  $x^2 \geq 0$ .  $\square$

## 1. INTRODUCTION

This is a new section.

**1.1. Things that need to be done.** Prove theorems.

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