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ABSTRACT

Time Geography provides an elegant theoretical framework for examining movement as information. The Time Geographic perspective conceptualizes observed movement as a signal encoding an emergent description of the mover. An actor's movements are shown to capture latent behavioral responses to pressures in the environmental context and to the presence of other movers. Considered as a set of spatiotemporal relocations constrained by what is accessible or desirable to the mover, Time Geographic methods allow us to extract evidence of the mover's evolving state and agency from trajectory data. This thinking has proven applicable in studies of animal movement, individual access in transportation systems, mover identity and the lived experience of a varied social space, psychiatric healthcare outcomes and tourism.

Despite its established and fast-growing literature, Time Geographic innovation is focused on the action of discrete, thinking, living individuals. Displacement or relocation in the action of non-point, invisible or distributed moving phenomena is understudied in this literature. Time Geographic treatments for questions surrounding the movements of aerosol plumes, aggregate or censored disease and crime event counts, samples of invasive plant presences and neighborhood-level price responses to rent control policies are not, for example, available in the literature. This limitation is due in part to the discipline's epistemological construction of the mover and in part to instrumentation. Individual movement captured by location-aware technologies readily translates to the common trajectory data structure facilitating Time Geographic analysis. However, corresponding routines characterizing the displacements of non-point, spatially indistinct or invisible phenomena as trajectory data are not available. Confronting

the notion of a non-point or invisible *mover* raises immediate questions about their reliable delineation and the location of their representative space-time anchors or location fixes within the trajectory structure. Additionally, Time Geographic conceptualizations of the mover depend, at least in part, on a grasp of the mover's agency, their behaviors, preferences and lifespans. An ontological bridge between the prevailing conception of an individual's movement and the metric displacements of a distributed actor has eluded the attention and perhaps the imagination of Time Geographers.

We offer an invitation to consider the ways such an entity might move. This dissertation contributes directly to the building of this ontology with the introduction of an adapter capable of producing space-time trajectories from change observed in area-aggregated spatiotemporal observations. Termed *Moving Local Indicators of Spatial Association* (Moving LISA), this novel probabilistic method avails a new class of subject matter to the existing traditions of Time Geographic analysis, to movement ecology and mobility science. Essentially, the Moving LISA constructs an entity-linking over space and time for the appearance and persistence of extreme observations, or locations clustering in a single variable. Trajectories are generated from the lineages of these clusters and returned to the user for further analysis and comparison. Chapter 2 expands on the research gap addressed by Moving LISA, along with its theoretical and mechanistic procedure as a sequencing and spatiotemporal entity-linking of cross-sectional LISA results. A prototypical formulation for the Moving LISA procedure is discussed, along with relevant pseudocode reproducing the method. The approach is further illustrated across both a contrived simulated dataset and empirical US Census data for the State of Oklahoma.

Chapter 3 engages with Moving LISA results as a motivating visualization accompanying a series of corresponding, cross-sectional spatially autoregressive model results concerned with

the proliferation of specialty vape stores in Oklahoma, from 2013-2023. This chapter represents Moving LISA as a companion to existing methods from spatiotemporal public health research, while simultaneously delivering the first known temporal examination of specialty vape store presence and socioeconomic vulnerability conducted for the state of Oklahoma. Over the course of the study period, a statewide dispersal in clusters of specialty vape store presence were observed alongside evolving associations between the expectation for vape store counts and socioeconomic covariates at the Zip Code level. Covariates capturing dimensions of social vulnerability (example, *Households With SNAP Benefits Last 12 Months*, *Percent Latino of Any Race*, and *Percent 25-34 Years*) were shown as significant positive effects on the expected count of specialty vape stores at the Zip Code level over the 3 study periods.

Chapter 4 demonstrates the application of existing analytics from movement ecology to output Moving LISA trajectories directly, with an investigation of the emergent gentrification process in the Oklahoma City Metropolitan Statistical Area for 2009-2023. Recent theoretical and empirical developments from Urban Geography offer a parsimonious identifier for the onset of gentrification, where median home prices and incomes are reframed obtaining a signal locating newly gentrifying areas. Drawing from this development, I have analyzed the respective Moving LISA trajectories of the covariates as a *dyad*; an interacting pair of movers relevant for questions developed in movement ecology. Where respective time-series of median home price hotspots and median income cold spots were reconstructed as Moving LISA trajectories, I found confirmation in a set of theoretical expectations for the temporal character of the gentrification process. With the application of trajectory comparison metrics from movement ecology, attraction or cohesion in *dyadic movement* among *aggregate movers* representing high home prices and high median incomes was observed. A corresponding repulsion between high home

prices and low median incomes was also identified. Further, in concert with a theoretical expectation where these trends would intensify within urban areas, non-parametric tests for difference in pseudomedian location in dyadic movement metric results showed significant intensification of these respective attractive and repulsive movements, for trajectories found inside versus outside of the Oklahoma City limits. Chapter 4 represents the first direct analysis of Moving LISA trajectories and suggests for the procedure's utility in further analyzing the movements of distributed spatial phenomena.

This dissertation is directed in large part to expanding the set of phenomena subject to Time Geographic analysis. The notion of action in a non-point or invisible spatial process and its construction as a moving object forms a nascent literature within Time Geographic thought, dealing foremost with the method gap for its proper characterization. The Moving LISA functionally bridges this epistemological and methodological gap and further avails new research questions to examination in the Time Geographic tradition. This dissertation is meant as a primer for a growing branch of geographic inquiry. It provides a theoretical narrative, tooling, and a few example applications. Future research would call first for a protocol assisting the incorporation of domain knowledge into Moving LISA applications. Forthcoming studies might directly address the method's sensitivities, explain caveats and the quasi-agency and pseudo-biology of *aggregate movers* in terms of impact to analysis as fully qualified Time Geographic objects.

1 Chapter One: Introduction

1.1 *Time Geography and Movement as Information*

Time Geographic thought is centered on a notion where observed movements are understood to capture latent information describing the mover and its environment. Movement is in this sense a signal; a coded, evolving response to stimuli. Sequences of recorded movements referred to as *trajectories* capture behaviors and reflect the pressures and preferences imposed on the focal mover by its spatial context and by the presence of other movers (Anderson & Yang, 2025; Dodge, 2021; J. Downs, Horner, Hyzer, et al., 2014; Loraamm, 2019). Revealing the relationships held in trajectory data is valuable for studies concerned with the many intersections of space and decision-making in their subject individual and the contribution of the individual to its enclosing emergent phenomena. Examining questions involving the drivers of these patterns could inform on the parameters of larger movement processes, spatiotemporal regimes, or ecological understandings developed from atomic individual behaviors.

Geographers, for their direct attention to space, scale and the relationships within as subject matter, are well positioned to address questions of meaning in movement (Pred, 1979; Thrift & Pred, 1981). Departing from a worldview focused on the effects and relations of space, Time Geographic thought expanded the theoretical boundaries of Geography to include consideration of space-time as the medium in which emergent phenomena are constructed and realized. Hägerstrand's landmark work introduced the Time Geographic framework first as a means to understand observed trade-offs in human mobility across urban environments (Hägerstrand, 1970). Through this early conception as a constraints-based approach for

identifying the feasible set of movement opportunities for an individual at any given time, Time Geography has since enjoyed steady growth and adoption. The appearance of applied quantitative contributions drawing on this theory accelerated with the arrival of ubiquitous computing, when new routes to the analysis and visualization of Hägerstrand's original elements of Time Geography were realized (Huisman & Forer, 1998; Miller, 2005, 2017).

Along with computing power, a proliferation of location-aware technologies has resulted in the availability of trajectory data in excess of our means to analyze it (Anderson & Yang, 2025; Cagnacci et al., 2010; Dodge, 2021; Dodge et al., 2020; J. A. Long et al., 2018; Loraamm, 2019; Miller et al., 2019). The literature tasked with decoding focal movement to inform on spatiotemporal processes is vast, increasingly interdisciplinary and fast developing. Examinations engaging with Time Geographic theory include those at the intersection of animal movement and transportation infrastructure (J. Downs, Horner, Loraamm, et al., 2014; Ho & Loraamm, 2023; Loraamm et al., 2018, 2021), for animal interactions (J. Downs, Lamb, et al., 2014), human mobility in urban spaces (Hägerstrand, 1970; J. Lee & Miller, 2019; Miller, 1991; Wang et al., 2024), psychiatric care outcomes (Sunnqvist et al., 2020), epidemiological concerns (Yin et al., 2021), crime (Yin et al., 2023) and the lived experience and identity of the mover in an uneven spatial context (Dodge & Nelson, 2023; Kwan, 1999).

Alongside these applied additions to the scope of what is considered Time Geographic analysis, the literature reflects commensurate methodological expansion from the original tenets of Hägerstrand's framework. The original elements of Time Geography include the concept of the *space-time anchor*, a three-dimensional point having spatial location and associated time-of-capture for a moving object. These anchors are sequenced in ascending time of occurrence and connected with a straight-line path referred to as the *space-time path* (Miller, 2005; Winter &

Yin, 2011). Between each anchor pair we are afforded a capture of the mover's displacement, an observed speed, and a relative turning angle. Together, these elements form the basis of trajectory datasets, from which derivative measures such as the *space-time prism* (STP) bounding feasibly accessible locations and the 2-dimensional *potential path area* (PPA) amounting to the shadow of this prism become possible (Figure 1).

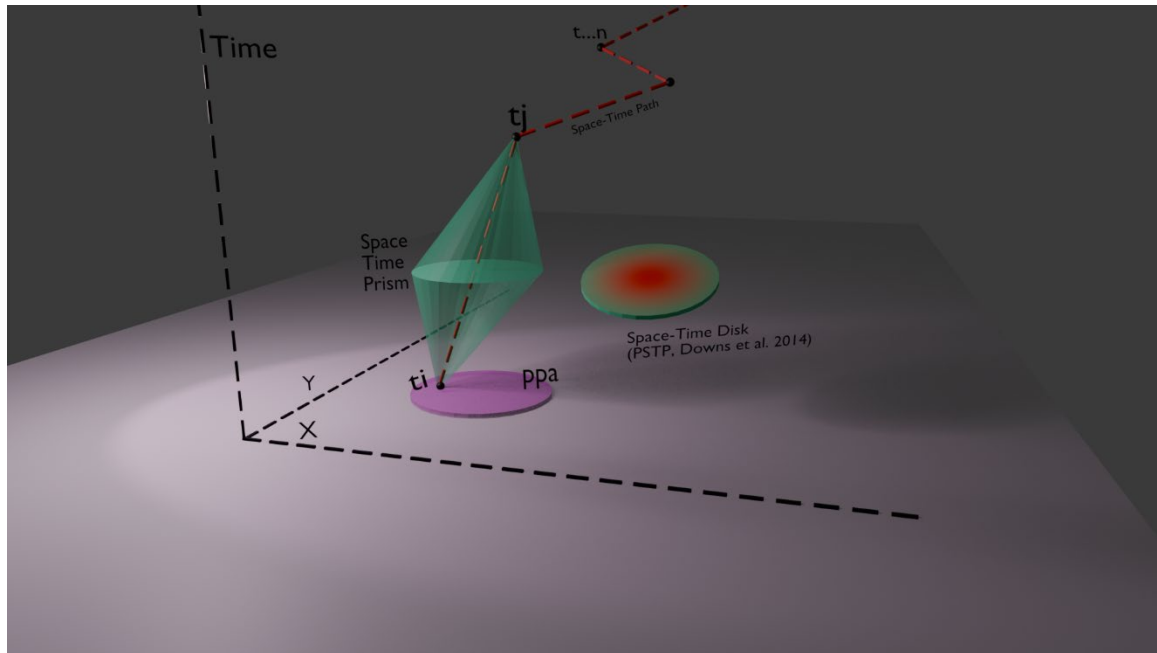


Figure 1. The Elements of Time Geography

Space-time anchors t_i and t_j are instantaneous observations of a moving object. The space-time path represents the fastest possible traversal between t_i and t_j , the space-time prism encloses the feasible set of locations traversable for the object for the time elapsed between t_i and t_j . This figure first appeared in (Anderson & Yang, 2025).

These original elements have been refined and expanded toward the development of probabilistic interior regions suggesting for the mover's unobserved locations within the space-

time prism as a function of resistance or conductance posed by its context (J. Downs, Horner, Hyzer, et al., 2014; J. Downs, Lamb, et al., 2014; Ho & Loraamm, 2023; Loraamm, 2019), similarly probabilistic realizations of the PPA (J. Downs et al., 2018; J. A. Downs & Horner, 2014; J. A. Long, 2018; Wang et al., 2024), methods addressing kinematics and GPS or anchor-uncertainty (Kuijpers et al., 2017; Kuijpers & Othman, 2017; J. A. Long, 2016), approaches summarizing bundles of input trajectories (Dodge et al., 2021; Gao et al., 2019) and those reconstructing space-time as a four-dimensional object (Demšar & Long, 2019).

Throughout these contributions, a common requirement for a discrete, directly observable and individual focal mover's trajectory is maintained. The subject matter and corresponding theoretical developments offered in these treatments are applicable only to movement drawn from the activity of individuals. A compatible theoretical construction of movement drawn from the activity of invisible, intangible or spatially indistinct entities has so far escaped exploration in the Time Geographic literature. These phenomena are categorized by practical constraints on the scale of their observation. Their distributed, atomic action is often reported in aggregate at spatial units. Spatial processes including for example change in prevailing neighborhood rents, disease reporting, crowdsourced sightings of invasive species or the market expansion of a disruptive product or service fall within this category of *aggregate movers* (Anderson & Yang, 2025). These entities, as observed, lack immediate spatial form or identifying boundaries as necessary for Time Geographic analysis. A strategy for the fair characterization of their spatial dimensions and temporal displacements would represent the first methodological goal in this analysis. The foundational idea of this dissertation focuses on the development of a method adapting the observed dynamics and spatial evolution of these processes into valid inputs for Time Geographic analysis.

1.2 The Moving LISA: An Entity-Linking for Spatially Indistinct Phenomena

The Moving LISA is essentially an entity-linking approach for spatial panel data, where cross-sectionally identified, probabilistically extreme observations are evaluated for their persistence and are then connected over time. The set of connected entities yields approximate space-time trajectories for the displacement or movement action of the observed spatial process. These returned trajectories are mechanistically qualified as inputs to a wide range of existing analytics from Time Geography, Movement Ecology and movement science. However, as demonstrated in the applied chapters of this research, further development in the meaning and informed identification of the parameters of *aggregate mover* movements (maximum velocity, influences of context, lifespans) demanded by this class of metrics remains an open question. Movement in spatially indistinct, invisible, or distributed phenomena are understudied, and the literature addressing adjacent questions surrounding their characterization has only recently emerged (Yin et al., 2021, 2023). With its introduction and demonstration of the Moving LISA, this dissertation contributes a procedure operationalizing a Time Geographic ontology for the capture and representation of *aggregate movers*.

With thorough reflection on the theory and with careful implementation of this extension, I hope the materials presented in this dissertation propel the nascent literature on the Time Geography of *aggregate movers* on to new questions. With the arrival of the Moving LISA, we are met with a corresponding arrival of new considerations, apparent limitations and caveats in application not immediately discussed from the safety of the proof-of-concept. Still, the opportunity to connect existing Time Geographic treatments to new spatial questions presents as a motivating proposal for the method's adoption and refinement.

1.3 Dissertation Layout and Contribution

This dissertation is comprised of three main chapters including a formal introduction of the Moving LISA procedure, followed by two illustrations for the method realized as applied studies in public health and Urban Geography, respectively. Chapter 2 relays this introduction for the Moving LISA as a novel, probabilistic and explicitly spatial approach to the characterization of Time Geographic movement trajectories from the displacements of clustering locations over time. Chapter 2 starts with discussion surrounding the state of Time Geography's treatment of non-point or invisible movers and where their consideration would intersect with the prevailing theoretical tenets of Time Geography. A review of the conceptual elements of Time Geography is then provided in context with formal explanation on how the Moving LISA procedure emits compatible objects capturing the movements of distributed phenomena. Two sample demonstrations are included, one evaluating the Moving LISA across a highly specified simulated dataset and another involving population estimates from US Census data. Psuedocode reproducing the method is provided in an appendix. This chapter has been co-authored with Dr. Anni Yang and appears in *Transactions in GIS*, December 2025.

Chapter 3 introduces the first applied work involving the Moving LISA and is directed toward a contemporary and evolving issue from public health. In this chapter, Moving LISA trajectory results are positioned as a motivating visualization of spatial autocorrelation depicting the progression in specialty vape shop retail establishment hot spots in Oklahoma from 2013-2023 at the Zip Code level (ZCTA). This result is discussed in context with spatially autoregressive generalized linear models examining the influence of contemporaneous socioeconomic conditions on expected counts of specialty vape stores at Oklahoma ZCTAs for each of 3 American Community Survey 5-Year estimates end-years, 2013, 2018 and 2023.

Specialty vapor retail establishment represents an emerging threat to public health, reminiscent in its spatial expression of the socially uneven and harmful past establishment patterns of commercial tobacco retailers (Berg et al., 2020; Bostean et al., 2018; Giovenco et al., 2016). The presence of specialty vape shops poses a complex pressure on both existing smokers for vaping's positioning as harm reduction or a cessation aid, and for youth vulnerable to initiation of vaping as vape shops normalize and socialize the behavior (Chido-Amajuoyi et al., 2020; J. G. L. Lee et al., 2018; Y. O. Lee & Kim, 2015; Venugopal et al., 2022; Walley et al., 2019). Distinguishing themselves from the well understood strategies of commercial tobacco, specialty vape retailers have demonstrated an effective marketing practice with consumers across new channels such as social media and vape-themed social events (Berg et al., 2020; J. G. L. Lee et al., 2018; Y. O. Lee & Kim, 2015). Further, evidence for new socioeconomic associations and reversals of previous findings demonstrate the vapor retail process' departure and uniqueness from corresponding, existing studies from tobacco control (Dai et al., 2017; Dai & Hao, 2017). Because of these differences, more research is necessary to further inform positive action for public health policy and intervention surrounding vaping. In addition to representing the first applied public health study involving the Moving LISA, Chapter 3 is also the first known longitudinal investigation of the socioeconomic associations of vape shop establishments for Oklahoma. Chapter 3 is under preparation to submit for publication in *The International Journal of Health Geographics*.

Chapter 4 moves beyond the illustrative use of Moving LISA results toward a direct examination of their metric attributes in exploration of a recent theoretical finding from Urban Geography for the Oklahoma Metropolitan Statistical Area (MSA) over the years 2009-2023. By comparing Moving LISA trajectories generated for socioeconomic covariates as members of a

dyad, or paired trajectories representing related or interacting movers, Chapter 4 is a demonstration in the compatibility of Moving LISA outputs with existing methods from Movement Ecology (Joo et al., 2018). From the perspective of Movement Ecology, the characteristics and interactions expressed among animal movements capture latent information about the influences and operation of ecological processes. Chapter 4 extends this thinking to meet the social and urban geographic issue of gentrification, for which interesting movements and interactions are embedded in patterns of displacement and change in associated socioeconomic variables.

While there is consensus surrounding a qualitative definition for the gentrification process in literature, empirical methods for its identification are numerous and vary greatly in their approach and interpretation (Barton, 2016; Finio, 2022; Preis et al., 2021). Gentrification generally refers to a period of inflow of higher-earning residents to a neighborhood following a pronounced rise in property values and the vacating of the existing, often socially vulnerable residents (Brown–Saracino, 2016; Halasz, 2018; Rucks-Ahidiana, 2022). Recent findings treating the process from an alternative theoretical framework offer a parsimonious signal for the onset of gentrification as a function of median incomes and median home values over time (Bunten et al., 2024). These variables are theorized to oppose one another over time and space, a process which is shown to intensify with proximity to the central business district (CBD) of the gentrifying study area.

Engaging with Moving LISA trajectories generated for median income hotspots, cold spots, and hotspots of median home values as the set from which dyadic pairs are selected, Chapter 4 evaluates their cohesion and repulsion in movement with application of the Dynamic Interaction *DI* (J. Long, 2014; J. A. Long & Nelson, 2013) and *Shirabe's Correlation (Cr)*

(Shirabe, 2006) metrics. Confirming the theorized dynamic, median home value hotspot trajectories were shown to repel median income cold spot trajectories in the OKC MSA over the study period. The direction of this relationship as captured in $DI\theta$ and Cr values was found to significantly intensify where trajectories originating within the Oklahoma City limits were compared to those originating outside of the city limits in nonparametric tests for difference of medians. Likewise, attractions were observed between median income and median home value hotspots. As the first direct analysis of the metric attributes of Moving LISA trajectories for Urban Geography, Chapter 4 is targeted for publication in *Cities* or *Urban Studies*.

Chapter 5 closes with expansion on the broader impact potential of a burgeoning Time Geography for Distributed Phenomena, to which this dissertation serves as an invitation. Recounting the component contributions and ontological connections brought forward by the Moving LISA to the Time Geographic subdiscipline, I further discuss the empirical findings facilitated by the method across the dissertation's two applied studies. Specific attention is given to the method's known limitations and the logical difficulties posed by supplying certain biologically-informed parameters to existing movement metrics for use with Moving LISA trajectory outputs.

1.4 References

- Anderson, J., & Yang, A. (2025). Moving Local Indicators of Spatial Association for Time Geography. *Transactions in GIS*, 29(8), e70158. <https://doi.org/10.1111/tgis.70158>
- Barton, M. (2016). An exploration of the importance of the strategy used to identify gentrification. *Urban Studies*, 53(1), 92–111. <https://doi.org/10.1177/0042098014561723>

- Berg, C. J., Schleicher, N. C., Johnson, T. O., Barker, D. C., Getachew, B., Weber, A., Park, A. J., Patterson, A., Dorvil, S., Fairman, R. T., Meyers, C., & Henriksen, L. (2020). Vape shop identification, density and place characteristics in six metropolitan areas across the US. *Preventive Medicine Reports*, 19, 101137. <https://doi.org/10.1016/j.pmedr.2020.101137>
- Bostean, G., Sanchez, L., & Lippert, A. M. (2018). Sociodemographic disparities in e-cigarette retail environment: Vape stores and census tract characteristics in Orange County, CA. *Health & Place*, 50, 65–72. <https://doi.org/10.1016/j.healthplace.2017.12.004>
- Brown–Saracino, J. (2016). An Agenda for the Next Decade of Gentrification Scholarship. *City & Community*, 15(3), 220–225. <https://doi.org/10.1111/cico.12187>
- Bunten, D. M., Preis, B., & Aron-Dine, S. (2024). Re-measuring gentrification. *Urban Studies*, 61(1), 20–39. <https://doi.org/10.1177/00420980231173846>
- Cagnacci, F., Boitani, L., Powell, R. A., & Boyce, M. S. (2010). Animal ecology meets GPS-based radiotelemetry: A perfect storm of opportunities and challenges. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 365(1550), Article 1550. <https://doi.org/10.1098/rstb.2010.0107>
- Chido-Amajuoyi, O. G., Ozigbu, C. E., & Zhang, K. (2020). School proximity and census tract correlates of e-cigarette specialty retail outlets (vape shops) in central Texas. *Preventive Medicine Reports*, 18, 101079. <https://doi.org/10.1016/j.pmedr.2020.101079>
- Dai, H., & Hao, J. (2017). Geographic density and proximity of vape shops to colleges in the USA. *Tobacco Control*, 26(4), 379–385. <https://doi.org/10.1136/tobaccocontrol-2016-052957>

- Dai, H., Hao, J., & Catley, D. (2017). Vape Shop Density and Socio-Demographic Disparities: A US Census Tract Analysis. *Nicotine & Tobacco Research*, 19(11), 1338–1344.
<https://doi.org/10.1093/ntr/ntx063>
- Demšar, U., & Long, J. A. (2019). Potential path volume (PPV): A geometric estimator for space use in 3D. *Movement Ecology*, 7(1), Article 1. <https://doi.org/10.1186/s40462-019-0158-4>
- Dodge, S. (2021). A Data Science Framework for Movement. *Geographical Analysis*, 53(1), 92–112.
- Dodge, S., Gao, S., Tomko, M., & Weibel, R. (2020). Progress in computational movement analysis – towards movement data science. *International Journal of Geographical Information Science*, 34(12), 2395–2400.
<https://doi.org/10.1080/13658816.2020.1784425>
- Dodge, S., & Nelson, T. A. (2023). A framework for modern time geography: Emphasizing diverse constraints on accessibility. *Journal of Geographical Systems*, 25(3), 357–375.
<https://doi.org/10.1007/s10109-023-00404-1>
- Dodge, S., Su, R., Johnson, J., Simcharoen, A., Goulías, K., Smith, J. L. D., & Ahearn, S. C. (2021). ORTEGA: An object-oriented time-geographic analytical approach to trace space-time contact patterns in movement data. *Computers, Environment and Urban Systems*, 88, 101630. <https://doi.org/10.1016/j.compenvurbsys.2021.101630>
- Downs, J. A., & Horner, M. W. (2014). Adaptive-velocity time-geographic density estimation for mapping the potential and probable locations of mobile objects. *Environment and Planning B: Planning and Design*, 41, 1006–1021.
- Downs, J., Horner, M., Lamb, D., Loraamm, R., Anderson, J., & Wood, B. (2018). Testing time-geographic density estimation for home range analysis using an agent-based model of

- animal movement. *International Journal of Geographical Information Science*, 32(7), Article 7. <https://doi.org/10.1080/13658816.2017.1421764>
- Downs, J., Horner, M., Loraamm, R., Anderson, J., Kim, H., & Onorato, D. (2014). Strategically Locating Wildlife Crossing Structures for Florida Panthers Using Maximal Covering Approaches: Strategically Locating Wildlife Crossing Structures for Florida Panthers. *Transactions in GIS*, 18(1), Article 1. <https://doi.org/10.1111/tgis.12005>
- Downs, J., Horner, M. W., Hyzer, G., Lamb, D., & Loraamm, R. (2014). Voxel-based probabilistic space-time prisms for analysing animal movements and habitat use. *International Journal of Geographical Information Science*, 28(5), Article 5. <https://doi.org/10.1080/13658816.2013.850170>
- Downs, J., Lamb, D., Hyzer, G., Loraamm, R., Smith, Z. J., & O'Neal, B. M. (2014). Quantifying spatio-temporal interactions of animals using probabilistic space-time prisms. *Applied Geography*, 55, 1–8. <https://doi.org/10.1016/j.apgeog.2014.08.010>
- Finio, N. (2022). Measurement and Definition of Gentrification in Urban Studies and Planning. *Journal of Planning Literature*, 37(2), 249–264. <https://doi.org/10.1177/08854122211051603>
- Gao, Y., Cheng, J., Meng, H., & Liu, Y. (2019). Measuring spatio-temporal autocorrelation in time series data of collective human mobility. *Geo-Spatial Information Science*, 22(3), Article 3. <https://doi.org/10.1080/10095020.2019.1643609>
- Giovenco, D. P., Duncan, D. T., Coups, E. J., Lewis, M. J., & Delnevo, C. D. (2016). Census tract correlates of vape shop locations in New Jersey. *Health & Place*, 40, 123–128. <https://doi.org/10.1016/j.healthplace.2016.05.008>

- Hägerstrand, T. (1970). What about people in Regional Science? *Papers of the Regional Science Association*, 24(1), Article 1. <https://doi.org/10.1007/BF01936872>
- Halasz, J. R. (2018). The super-gentrification of Park Slope, Brooklyn. *Urban Geography*, 39(9), 1366–1390. <https://doi.org/10.1080/02723638.2018.1453454>
- Ho, K., & Loraamm, R. (2023). Using a Cost-Distance Time-Geographic Approach to Identify Red Deer Habitat Use in Banff National Park, Alberta, Canada. *ISPRS International Journal of Geo-Information*, 12(8), 339. <https://doi.org/10.3390/ijgi12080339>
- Huisman, O., & Forer, P. (1998). Computational agents and urban life spaces: A preliminary realisation of the time-geography of student lifestyles. *Proceedings of the Third International Conference on GeoComputation, 17-19 September 1998, Bristol, UK* 18 p.
- Joo, R., Etienne, M.-P., Bez, N., & Mahévas, S. (2018). Metrics for describing dyadic movement: A review. *Movement Ecology*, 6(1), 26. <https://doi.org/10.1186/s40462-018-0144-2>
- Kuijpers, B., Miller, H. J., & Othman, W. (2017). Kinetic prisms: Incorporating acceleration limits into space–time prisms. *International Journal of Geographical Information Science*, 31(11), Article 11. <https://doi.org/10.1080/13658816.2017.1356462>
- Kuijpers, B., & Othman, W. (2017). The geometry of space-time prisms with uncertain anchors. *International Journal of Geographical Information Science*, 31(9), Article 9. <https://doi.org/10.1080/13658816.2017.1319950>
- Kwan, M.-P. (1999). Gender and Individual Access to Urban Opportunities: A Study Using Space–Time Measures. *The Professional Geographer*, 51(2), Article 2. <https://doi.org/10.1111/0033-0124.00158>

- Lee, J. G. L., Orlan, E. N., Sewell, K. B., & Ribisl, K. M. (2018). A new form of nicotine retailers: A systematic review of the sales and marketing practices of vape shops. *Tobacco Control*, 27(e1), e70–e75. <https://doi.org/10.1136/tobaccocontrol-2017-054015>
- Lee, J., & Miller, H. J. (2019). Analyzing collective accessibility using average space-time prisms. *Transportation Research Part D: Transport and Environment*, 69, 250–264. <https://doi.org/10.1016/j.trd.2019.02.004>
- Lee, Y. O., & Kim, A. E. (2015). ‘Vape shops’ and ‘E-Cigarette lounges’ open across the USA to promote ENDS. *Tobacco Control*, 24(4), 410–412. <https://doi.org/10.1136/tobaccocontrol-2013-051437>
- Long, J. (2014). *wildlifeDI: Calculate Indices of Dynamic Interaction for Wildlife Tracking Data* (p. 1.0.1) [Dataset]. <https://doi.org/10.32614/CRAN.package.wildlifeDI>
- Long, J. A. (2016). Kinematic interpolation of movement data. *International Journal of Geographical Information Science*, 30(5), Article 5. <https://doi.org/10.1080/13658816.2015.1081909>
- Long, J. A. (2018). Modeling movement probabilities within heterogeneous spatial fields. *Journal of Spatial Information Science*, (16), Article 16. <https://doi.org/10.5311/JOSIS.2018.16.372>
- Long, J. A., & Nelson, T. A. (2013). Measuring Dynamic Interaction in Movement Data. *Transactions in GIS*, 17(1), 62–77. <https://doi.org/10.1111/j.1467-9671.2012.01353.x>
- Long, J. A., Weibel, R., Dodge, S., & Laube, P. (2018). Moving ahead with computational movement analysis. *International Journal of Geographical Information Science*, 32(7), Article 7. <https://doi.org/10.1080/13658816.2018.1442974>

- Loraamm, R. (2019). Incorporating behavior into animal movement modeling: A constrained agent-based model for estimating visit probabilities in space-time prisms. *International Journal of Geographical Information Science*, 1–21.
<https://doi.org/10.1080/13658816.2019.1658875>
- Loraamm, R., Anderson, J., & Burch, C. (2021). Identifying road avoidance behavior using time-geography for red deer in Banff National Park, Alberta, Canada. *Transactions in GIS*, 25(3), Article 3. <https://doi.org/10.1111/tgis.12748>
- Loraamm, R., Downs, J., & Lamb, D. (2018). A time-geographic approach to quantifying wildlife–road interactions. *Transactions in GIS*, 23(1), Article 1.
<https://doi.org/10.1111/tgis.12497>
- Miller, H. J. (1991). Modelling accessibility using space-time prism concepts within geographical information systems. *International Journal of Geographical Information System*, 5(3), Article 3.
- Miller, H. J. (2005). A Measurement Theory for Time Geography. *Geographical Analysis*, 37(1), Article 1. <https://doi.org/10.1111/j.1538-4632.2005.00575.x>
- Miller, H. J. (2017). Time Geography and Space-Time Prism. In D. Richardson, N. Castree, M. F. Goodchild, A. Kobayashi, W. Liu, & R. A. Marston (Eds.), *International Encyclopedia of Geography: People, the Earth, Environment and Technology* (pp. 1–19). John Wiley & Sons, Ltd. <https://doi.org/10.1002/9781118786352.wbieg0431>
- Miller, H. J., Dodge, S., Miller, J., & Bohrer, G. (2019). Towards an integrated science of movement: Converging research on animal movement ecology and human mobility science. *International Journal of Geographical Information Science*, 33(5), 855–876.
<https://doi.org/10.1080/13658816.2018.1564317>

- Pred, A. (1979). THE ACADEMIC PAST THROUGH A TIME-GEOGRAPHIC LOOKING GLASS. *Annals of the Association of American Geographers*, 69(1), 175–180.
<https://doi.org/10.1111/j.1467-8306.1979.tb01249.x>
- Preis, B., Janakiraman, A., Bob, A., & Steil, J. (2021). Mapping gentrification and displacement pressure: An exploration of four distinct methodologies. *Urban Studies*, 58(2), 405–424.
<https://doi.org/10.1177/0042098020903011>
- Rucks-Ahidiana, Z. (2022). Theorizing Gentrification as a Process of Racial Capitalism. *City & Community*, 21(3), 173–192. <https://doi.org/10.1177/15356841211054790>
- Shirabe, T. (2006). Correlation Analysis of Discrete Motions. In M. Raubal, H. J. Miller, A. U. Frank, & M. F. Goodchild (Eds.), *Geographic Information Science* (Vol. 4197, pp. 370–382). Springer Berlin Heidelberg. https://doi.org/10.1007/11863939_24
- Sunnqvist, C., Rämgård, M., & Örmon, K. (2020). Time Geography, a Method in Psychiatric Nursing Care. *Issues in Mental Health Nursing*, 41(11), 1004–1010.
<https://doi.org/10.1080/01612840.2020.1757795>
- Thrift, N., & Pred, A. (1981). Time-geography: A new beginning. *Progress in Human Geography*, 5(2), 277–286. <https://doi.org/10.1177/030913258100500209>
- Venugopal, P. D., Morse, A. L., Alrefai-Kirkpatrick, R., Tworek, C., & Chang, H. W. (2022). The Co-occurrence of Specialty Vape Shops, Social Disadvantage, and Poor Air Quality in the United States: An Assessment of Cumulative Risks to Youth. *Health Equity*, 6(1), 132–141. <https://doi.org/10.1089/heq.2021.0151>
- Walley, S. C., Wilson, K. M., Winickoff, J. P., & Groner, J. (2019). A Public Health Crisis: Electronic Cigarettes, Vape, and JUUL. *Pediatrics*, 143(6), e20182741.
<https://doi.org/10.1542/peds.2018-2741>

- Wang, A.-S., Yin, Z.-C., & Ying, S. (2024). Probabilistic Time Geographic Modeling Method Considering POI Semantics. *ISPRS International Journal of Geo-Information*, 13(1), 22. <https://doi.org/10.3390/ijgi13010022>
- Winter, S., & Yin, Z.-C. (2011). The elements of probabilistic time geography. *GeoInformatica*, 15(3), Article 3. <https://doi.org/10.1007/s10707-010-0108-1>
- Yin, Z., Chen, Y., & Ying, S. (2023). Construction and Analysis of Space–Time Paths for Moving Polygon Objects Based on Time Geography: A Case Study of Crime Events in the City of London. *ISPRS International Journal of Geo-Information*, 12(6), Article 6. <https://doi.org/10.3390/ijgi12060210>
- Yin, Z., Huang, W., Ying, S., Tang, P., Kang, Z., & Huang, K. (2021). Measuring of the COVID-19 Based on Time-Geography. *International Journal of Environmental Research and Public Health*, 18(19), 10313. <https://doi.org/10.3390/ijerph181910313>

2 Chapter Two: Moving Local Indicators of Spatial Association for Time Geography

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Abstract

Recently, the Time Geographic literature has shown rapid expansion and the inspiration to address and explore new factors affecting the observed movement of the individual. Movement is a message, decoded for the internal states and external influences contributing to its expression. Exciting questions examining mover identity, experience and access are subject to this expanding Time Geographic perspective. However, despite these developments, Time Geography is largely formulated to examine individual movers. The movements of nonpoint entities, invisible and distributed spatial processes are understudied. Displacement in space and time among these phenomena forms a basis for a nascent set of methods characterizing their movements. With this research, we introduce the Moving Local Indicators of Spatial Association (Moving LISA), a novel, probabilistic way to extend Time Geographic thinking to the action of distributed spatial phenomena. Given a spatial time series, Moving LISA returns trajectories characterizing movement among clustering locations.

Keywords: LISA, Trajectory, Movement, Time Geography, Spatiotemporal.

2.1 Introduction

Movement is a signal. It encodes and encapsulates both visible and latent characteristics of the mover. Movement reflects an actor's internal state, the pressures of interaction with their spatial context, and their encounters with others (Ahearn et al., 2017; J. Downs et al., 2014; Ho & Loraamm, 2023; Long et al., 2018; Loraamm, 2019). The movement concept maintains a durable and important impact in science. Researchers and Theorists have constructed movement first as the simple case of an object's observed displacement over time, towards a landmark relativistic

view revealing an interdependence between space and time (Gutfreund et al., 2015). Both accounts are equally accurate to the multifaceted reality of movement. Patterns in movement stand to inform on causalities inherent to spatial processes challenging our world.

In addition to the directly observable movements of individuals, the movements of distributed phenomena such as disease, aerosol plumes, housing markets, invasive plant encroachments, and group-movements of migratory animals all embed information about their driving processes. Significant study has been directed to understanding the implications of movement for individual objects, animals or people in Geography. However, the movements of nonpoint, invisible or distributed phenomena are understudied; a new and growing literature addresses these moving objects (Z. Yin et al., 2021, 2023). The present study extends the emerging literature on nonpoint movers by introducing a novel probabilistic method for the characterization and the analysis of their movements, drawing from both the Time Geographic (Hägerstrand, 1970) and Local Indicators of Spatial Association (LISA) (Anselin, 1995) traditions. The approach leverages LISA measures across aggregate time-series data to evaluate and suggest a space-time trajectory for subsequent movement analysis. The method is not itself an alternative spatiotemporal LISA nor a novel approach to trajectory data, but a strategy for the characterization of new subject matter as moving objects compatible with the Time Geographic framework. We term the resulting approach the “Moving LISA”, and the generated class of nonpoint moving entities returned by the method as *aggregate movers*.

Geographers are uniquely positioned to contemplate movement as a driver of spatial processes. The Time Geographic tradition within Geography offers both its distinctive constraints-based perspective on the examination of movement and a fast-growing applied literature (Dodge et al., 2020). By analyzing observed movements of the individual, Time

Geography provides a route to understanding the influences of space, time, behavior and context in the expression of emergent spatial phenomena across scales. Time Geography's primary concerns also include the reconstruction or estimation of a mover's position at unobserved times or between available observations, often captured as trajectory data. No other geographic perspective parallels the direct attention to movement, context and positional uncertainty afforded by Time Geography and its treatment of trajectories (Thrift & Pred, 1981).

Location-aware technologies continue to generate trajectory data in unprecedented amount, accuracy and availability (Cagnacci et al., 2010; Dodge et al., 2020; Dodge, 2021; Long, 2018; Loraamm, 2019; Loraamm, Downs, et al., 2020; Miller et al., 2019). In response, Time Geographers have built upon Hägerstrand's fundamental theory of Time Geographic analysis extending its applicability and juxtaposing new ideas into the framework (Hägerstrand, 1970; Miller, 2005). These contributions have improved understanding in both the role of the moving individual in the expression of spatial processes and of the influence of space on the individual mover. Recent work includes probabilistic reconstructions of space-time suggesting the most probable unobserved object positions in a trajectory (J. Downs et al., 2014; J. A. Downs, 2010; Winter & Yin, 2011; Z.-C. Yin et al., 2018), extensions further accounting for the resistance and conductance of the mover's spatial context (Ho & Loraamm, 2023; Long, 2018; Loraamm, 2019; Loraamm et al., 2021; Miller & Bridwell, 2009), methods accounting for kinematics and GPS uncertainty (Kobayashi et al., 2011; Kuijpers et al., 2010; Kuijpers & Othman, 2017; Long et al., 2014), movements in network spaces (Kuijpers & Othman, 2009), even methods capable of trajectory reconstruction in four dimensions (Demšar & Long, 2019). Additionally, we see developments applicable to materializations of metric space separate from and beyond our familiar, real environmental context (Buonaccorsi et al., 2001; Liebhold et al., 2004). It is shown

abstract and virtual (*in silico*) spaces or simulated environments also host movement subject to a Time Geographic interpretation (Rey, 2016; Rey et al., 2011; Soler-Domínguez et al., 2019). Additional perspectives have incorporated the Time Geographic progression of patients' life events as noted of interest to the practice of psychiatric nursing (Sunnqvist et al., 2020) and the influences of identity and the lived experience of the mover across an uneven social space (Kar et al., 2023; Kwan, 1999). Applied Time Geographic analysis has assisted in animal space-use studies valuable to wildlife management (Dodge et al., 2021; Ho & Loraamm, 2023, 2023; Loraamm, Goodenough, et al., 2020; Su et al., n.d.), human concerns for transportation, identity, and associated mobility and accessibility (Dodge & Nelson, 2023; Kwan, 1999; Lee & Miller, 2019; Miller, 1991; A.-S. Wang et al., 2024), crime (Nakaya & Yano, 2010; Z. Yin et al., 2023) and public health (Z. Yin et al., 2021).

Despite the rapid progress and high applicability of the Time Geographic approach, we note the subject matter focus has historically been on the individual mover. Methods from the subdiscipline are generally formulated for the treatment of trajectories describing the discrete individual (Hägerstrand, 1970), to compare individual trajectories for interactions with obstacles in the environment or other individuals (Long, 2018; Loraamm et al., 2018; Miller & Bridwell, 2009), or to summarize bundles of trajectories (Gao et al., 2010, 2019), describing the general movements of a group of individuals (Dodge et al., 2021; Laube et al., 2005). The explicit examination of movement in invisible phenomena, event distributions captured as point-sets, nonpoint entities or spatial processes where observations are obtained in aggregate are all understudied in this literature (Z. Yin et al., 2021, 2023).

Trajectory data capturing the movement of the individual is a basic input and requirement for Time Geographic methods addressing the individual mover. The tendency to examine a

discrete individual mover both originates with the human geographic and urban mobility focus of classical Time Geography in context with notions of human agency in the mover, and the discrete nature of point location data respectively (Hägerstrand, 1970; Miller, 2005). The question of how to best identify and capture the trajectory of a nonpoint entity, or the progression of extreme observations for a distributed spatial process for Time Geographic analysis remains both open and complex. We argue a Time Geography for nonpoint phenomena does not pose a theoretical incompatibility versus the focus on the individual. Instead, it is an opportunity to present novel approaches treating movement in distributed phenomena, rooted in the Time Geographic tradition. This study presents an approach enabling both the delineation of moving nonpoint entities and an output characterizing their movements compatible with existing Time Geographic methods. Delineation is a function of sequential comparisons among LISA measures, movement trajectories are derived from a filtration of displacements in extreme values as indicated by LISAs. We further provide our *aggregate mover* construct with a theoretical notion of agency, where their movements may be regarded as the combined response of component actors to complex and unobserved stimuli or effects across the study area. Following from ideas endemic to swarm intelligence, complexity theory and agent-based modeling, we suggest the pattern of displacement in extreme values exhibited by *aggregate movers* represents a summary response to many forces, influences, and decision-making among individual actors and their latent component processes from which aggregate observations are drawn (Anderson et al., 2017; Epstein, 1999; Kar, 2016; Tang & Bennett, 2010).

Defined with boundaries, movement and agency, the *aggregate mover* offers an ontological bridge enabling us to recognize Time Geographic movement in distributed spatial phenomena. Incorporation of nonpoint or *aggregate movers* as objects of study into the Time

Geographic tradition would expand the subject matter and avail new classes of problems to the framework, relaxing practical limitations surrounding classical Time Geography and the sampling of distributed phenomena (Z. Yin et al., 2021, 2023). For example, instrumentation and sampling strategies for phenomena such as aerosol concentrations unavoidably lead to observations capturing a combined effect of many individual point-source polluters at-once. Beyond this uncertainty in sampling, social protections such as individual privacy concerns may result in similar aggregation in sensitive spatial datasets, for example in Time Geographic examinations of disease incidence or housing foreclosures. Once provided with a relevant and meaningful characterization of movement in these distributed spatial phenomena, existing Time Geographic methods may be applied. For example, we note the space-time path or trajectory of the individual may be reimagined as the spatiotemporal paths of greatest observed effect for phenomena such as epidemic disease, where the individual space-time prism could instead relate uncertainty in the spatiotemporal range of effect for a distributed process identified and later examined as a mover (Z. Yin et al., 2021).

The Moving LISA represents a next step owing its inspiration in part to a nascent group of methodologies characterizing the movements of nonpoint spatial entities (Z. Yin et al., 2021, 2023). Departing from these available methods, the Moving LISA approach is novel in its identification of the central features or space-time anchors for each time step in the trajectory as a function of stochastically extreme values occurring in a neighborhood structure (clustering geographies), as opposed to the existing strategy leveraging absolute increase or decrease in a variable of interest taken globally over the study area. Additionally, the Moving LISA characterizes clustering geographies as discrete, persistent moving objects each having a respective trajectory, along with an accounting for the initial identities, divergences,

convergences and terminations among clustering geographies over the study area and period. For clarity, the method introduced in this research is meant to generate or suggest trajectories from observations at aggregate spatial units for a single variable; the Moving LISA does not aggregate bundles of trajectories observed for individuals as in (Gao et al., 2010, 2019). The Moving LISA approximates a trajectory where no individual-level telemetry data are available or where individual observation is not feasible.

A wealth of existing methodologies and prior work treat subject matter adjacent to Moving LISA. Our method does not directly compete with most of these as the Moving LISA itself is a juxtaposition of available perspectives in service to an expanded subject matter for Time Geography. The overall goal is to avail new, distributed phenomena as subject matter for the Time Geographic approach. Compared to adjacent contributions from exploratory space-time data analysis (ESTDA), The Moving LISA in this original formulation relies on cross-sectional LISA measures and does not use a 3-dimensional or otherwise augmented for-space-time weights matrix (Lee & Li, 2017; Z. Wang & Lam, 2020a) and it does not address issues of stationarity in the global expected values or intensities for observations supplied to LISAs (Tao & Chen, 2022). The Moving LISA also arrives alongside Emerging Hot Spot Analysis (EMHSA) (Parry & Locke, 2022; Purwanto et al., 2021; Xu et al., 2022) but differs in its fundamental treatment of distributed phenomena as moving objects. While both EMHSA and Moving LISA incorporate LISA measures, Moving LISA tracks movers in their free traversal of the study area, rather than accounting for the emergence and disappearance of hot spots at an array of static locations. Compared to the class of LISA measures dealing specifically with origin-destination flow data, the distinction is similar, where the goal of Moving LISA is not to identify extreme flows among networked locations over time, but to characterize displacements in extreme values as movers

themselves (Tao & Thill, 2016, 2020, 2020). Further, the Moving LISA departs from the literature on space-time scan statistics (Birant & Kut, 2007; Kulldorff, 1997, 2001; P. A. Rogerson, 2022) similarly to its departure from spatiotemporal LISAs having space-time aware weights matrices, as the LISAs employed in Moving LISA are themselves cross-sectional. Finally, the Moving LISA differs from applications incorporating space-time cubes in terms of its inputs (Mo et al., 2020; Nakaya & Yano, 2010). Input geographies for Moving LISA are not constructed from a regular grid and do not represent a binning of values. Instead, input geographies will generally carry some geographic meaning in terms of jurisdiction, zoning, ecotone, or microclimate such that information or the spatial meaning assigned to boundaries themselves are not stratified out of consideration.

Following background on the component methodological perspectives comprising the Moving LISA in Section 2, we discuss the Moving LISA procedure itself in Section 3, followed by a demonstration consuming both simulated and empirical datasets in Section 4. Results from the demonstration are considered for their indication or capture of relevant spatiotemporal relationships by the method in Section 4. We close with Section 5 detailing our thoughts on applicability and the Moving LISA's broader impacts.

2.2 *Background*

2.2.1 *The Elements of Time Geography and the Space-Time Prism*

Time Geography offers a straightforward operationalization of its constraints-based theory on movement. Fundamentally, the limit of uncertainty for the location of an individual moving object at times between observations for can be bounded and represented as a function of the object's known maximum speed and the time elapsed between the available observations

(Brunsdon et al., 2015; Hägerstrand, 1970; Miller, 2005). This approach represents the simplest case among time geographic methods, returning a boundary in space and time from which the mover could not have theoretically escaped while it was unobserved. Departing from this central idea, the methodological *elements of Time Geography* include instantaneous spatiotemporal point-observations of the focal mover as-captured in *trajectory datasets* termed *space-time anchors*, and *the space-time path*, an estimated straight-line path drawn between sequential space-time anchors in three dimensions (Loraamm, Downs, et al., 2020; Miller, 2005, 2017). The space-time path forms the center line of the *space-time prism (STP)*, a volume enclosing the uncertainty in the mover's position at unobserved times occurring between a sequential pair of anchors (Figure 1). The sequence of STP's fit for a mover's overall trajectory dataset has been metaphorically referred to as its *lifeline necklace*; itself an object suspended among the three dimensions of the *space-time aquarium* or space-time domain of the study area (Kuijpers et al., 2011; Winter & Yin, 2010). Additionally, we note the set of the mover's accessible locations at any specific interval of time over the period of observation is termed a *space-time disk*. The set of disks accounting for the time elapsed between anchors may be stacked or integrated to construct the space-time prism, with disks exhibiting their greatest X/Y area at the temporal midpoint between anchors, having a convergence to zero-area at the space-time anchor locations, in the classical formulation. The projection or shadow of a space-time prism in X/Y space is referred to as the *potential path area (PPA)* of the mover, relating uncertainty in the object's position over the entire course of the prism. The overall STP is at-once a time budget for the mover and a map of its spatiotemporal position. Use of the STP as an analytic tool is a unifying characteristic among Time Geographic studies and traces its appearance in the literature to Hägerstrand's early contribution (Long et al., 2018; Loraamm, Downs, et al., 2020; Loraamm et

al., 2021). The Moving LISA's role in context with these elements is to suggest space-time paths capturing displacements in distributed spatial phenomena as an indication of movement.

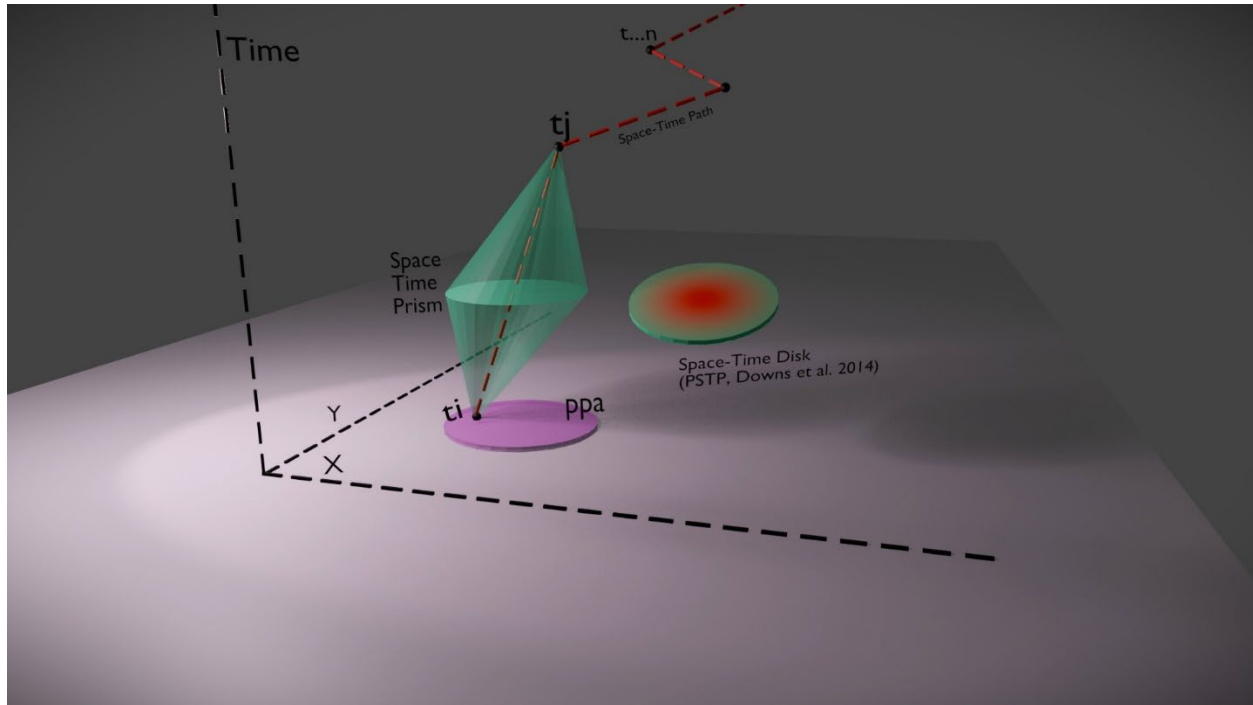


Figure 1. The Elements of Time Geography and the Space-Time Prism, with representation of a single PSTP space-time disk having greatest visit-probability at its center. Note the dotted-red space-time path provides a straight-line reconstruction of the object's spatiotemporal position between each space-time anchor (black spheres). The feasible region of an object's departure from the space-time path is bounded by the green Space-Time Prism's volume.

Prevailing formulations for the classical STP and its descendant methods owe their popularity in part to the ubiquity of modern computing. While continuous or analytic (as opposed to numeric, discrete) formulations of the measures have been made available (Kuijpers et al., 2011; Winter & Yin, 2011) a simple and popular option for the discretization and comparison of space-time is offered by use of the Voxel (also known as *volume element*) (J. Downs et al., 2014; Huisman & Forer, 1998; Loraamm, Downs, et al., 2020). The Voxel is a

straightforward discretization of space-time; the atomic voxel can be thought of as a cube of a fixed size with one face representing space (X/Y) and a vertical edge (Z) corresponding to a duration of time. Any number of attributes or values can be assigned to the spatiotemporal midpoint of the voxel. These values are generalized to the entire space of the voxel, introducing a trade-off consideration in terms of spatiotemporal resolution (Loraamm, Downs, et al., 2020). Altogether, a tensor of voxel disks sharing the same spatial and temporal resolution offer a greatly simplified construction of the *space-time aquarium* for use in Time Geographic analysis. The piecewise function shown reflects one approach to bounding the classical STP volume in terms of voxels (Equation 1). Evaluating this function at each voxel location for all voxels falling within the duration of a pair of space-time anchors will construct a voxel representation of the space time prism. Each voxel is evaluated to store a binary determination on whether the voxel's space-time location was feasibly accessible to the mover.

$$STP_{x_a} = \begin{cases} 1, & \text{if } \|x_a - x_i\| \leq (t_a - t_i)s_{ij} \wedge \|x_j - x_a\| \leq (t_j - t_a)s_{ij} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Where:

$\|x_x - x_x\|$ is the Euclidean distance between the current voxel centroid location x_a and either the starting x_i , or ending x_j , space-time anchor locations in a given, sequential anchor pair.

t_a is the temporal midpoint associated with x_a , and the $(t_a - t_i)s_{ij}$, $(t_a - t_j)s_{ij}$ expressions calculate the distance the mover could have successfully traversed between the anchors, given the time elapsed and time remaining between x_i and x_j , respectively, considering the object's expected maximum speed between the anchors, s_{ij} .

2.2.2 *Local Indicators of Spatial Association*

The co-occurrence of like observations in space has long been regarded as a source of information on the action of spatial processes (Cliff et al., 1981; Cliff & Ord, 1973; Moran, 1950). Collectively, measures revealing spatial structure and dependence in mapped observations assisting in the development of research questions are called *Exploratory Spatial Data Analysis* (ESDA) methods (Anselin, 1988; Rey et al., 2023). Spatial structure in data, or apparent dependence in the distribution of their values across space evidences that nonrandom influences are at work in the phenomena we examine. ESDA follows in Tukey's practice of Exploratory Data Analysis (EDA) where the approach aids researchers in suggesting and exploring hypotheses for their observations. Researchers leverage ESDA in concert with their existing scientific grasp of the process of interest to produce informative summaries and to choose next steps for their analysis (Anselin, 1996; Johnson et al., 2022; McElreath, 2020; Tukey, 1977). The literature on ESDA is vast and of its own merit. For an early and detailed treatment of these methods from first principles, the textbooks by Cliff and Ord, and Anselin are suggested (Anselin, 1988; Cliff et al., 1981). Specific to the Moving LISA procedure, a particular extension to ESDA has been identified as a probabilistic mechanism to characterize moving objects from sequences of observations assigned to nonpoint geographies. These methods are known as *Local Indicators of Spatial Association* or LISA statistics (Anselin, 1995, 2019; Anselin et al., 2006).

Establishing the plausible presence of spatial structure and dependence among observations is the goal of a LISA statistic. What makes LISAs unique among ESDA measures is their capability for identification and assessment for significance of individual clustering geographies in a study area (Anselin, 1995, 2019). Where a *global* measure of spatial association may indicate whether an observed spatial pattern is plausibly random or nonrandom overall for

the entire study area, a *local* measure further identifies the locations of *clusters*, or extreme observations. The general construction of a LISA for each observed location follows as a ratio. In the numerator, an expression is calculated at each focal geography weighted by the collection of other values for the expression in the focal geography's *neighborhood*. In the denominator, a similar expression is evaluated for *all values in the study area*. For example, this strategy of appears in the *Local Moran's I* where the expression identifies deviations from the mean in one variable (Anselin, 1995), or in the case of the *Getis-Ord Local G* where the expression is a weighted sum of observations (Getis & Ord, 1992; Ord & Getis, 1995). Other LISAs address concepts outside of these summary measures, such as quantifying Local Spatial Heteroskedasticity (Ord & Getis, 2012). For LISAs, the *neighborhood* is realized as a matrix (the "spatial weights matrix") reporting a strength of association among all possible pairs of observations in the study area. Authoring the weights matrix is a critical and sensitive step in LISA methods and can be used to reflect the researcher's *a priori* knowledge about spatial process under study (Getis & Aldstadt, 2004; P. Rogerson, 2024). Defining connectivity in a spatial weights matrix may follow a range of strategies. For example, immediate or first-order neighbors may be equally weighted among spatial units, or weights may be determined using distance thresholds, K-nearest neighbors or the presence of common attributes among neighboring geographies.

With respect to establishing significance for a given LISA statistic result, popular implementations use a conditional randomization approach. This randomization generates a reference distribution for the statistic by repeated shuffling of available observations and recalculation of the statistic (Anselin et al., 2022; Rey et al., 2022). For the present demonstration of the Moving LISA, the Getis-Ord Local *G* statistic has been selected for its

simplicity (Equation 2), although other LISAs are theoretically compatible with the procedure (Anselin et al., 2022; Getis & Ord, 1992; Ord & Getis, 1995).

$$G_i = \frac{\sum_{j \neq i} w_{ij} x_j}{\sum_{j \neq i} x_j} \quad (2)$$

Where:

G_i is the Local G statistic result, indicating positive spatial autocorrelation if G_i is above the mean of the variable of interest, and vice versa.

w_{ij} is the weights matrix element suggesting the influence on the variable of interest between geographies i and j .

x_j is the value of the variable of interest at location j .

2.3 Methods

2.3.1 The Moving LISA Procedure

The Moving LISA is a procedure returning a trajectory which suggests the spatiotemporal progression of a distributed or nonpoint spatial process. The space-time anchors in this trajectory are identified as a function of clustering locations, or locations with extreme values at each time step. Clustering locations are delineated as *aggregate movers*, and compared between time steps to establish “contiguity,” or the continuation and persistence of the cluster as a moving object over the course of the trajectory.

Departing from recent Time Geographic work on the emerging subject of nonpoint movers, the Moving LISA presents as a novel and probabilistic means of characterizing movers from extreme observations, contrary to available methods relying on presences at locations, or absolute increase or decrease in a variable, evaluated globally (Z. Yin et al., 2021, 2023).

Moving LISA is also novel in its further use of probabilistic tools allowing the method to mark the appearances, terminations, divergences (where one mover splits into more than one mover) and convergences (where more than one movers collide and move as a group) of clusters over time.

The Moving LISA procedure has 5 phases (Figure 2). These phases are repeated in sequence for time steps t_i in the input time series $\mathbf{T}$ with each time step having a set of nonpoint observations assigned to $\mathbf{G}$ projected lattice (vector polygon) geographies. The geographies are indexed g_{it} for a specific location in space and time. As groups of clustering geographies m_{i,t_i} are shown to qualify for the set of movers $\mathbf{M}$ per the Moving LISA method over the course of the time series, an output set of nonpoint mover trajectories $\mathbf{R}$ is constructed. For each time step t_i , all active movers $m_{i,t_i} \in \mathbf{M}$ are subject to an appropriate measure from centrography returning their respective approximate space-time anchor locations $R_{m_{t_i}}$ to be included in the output set of trajectories.

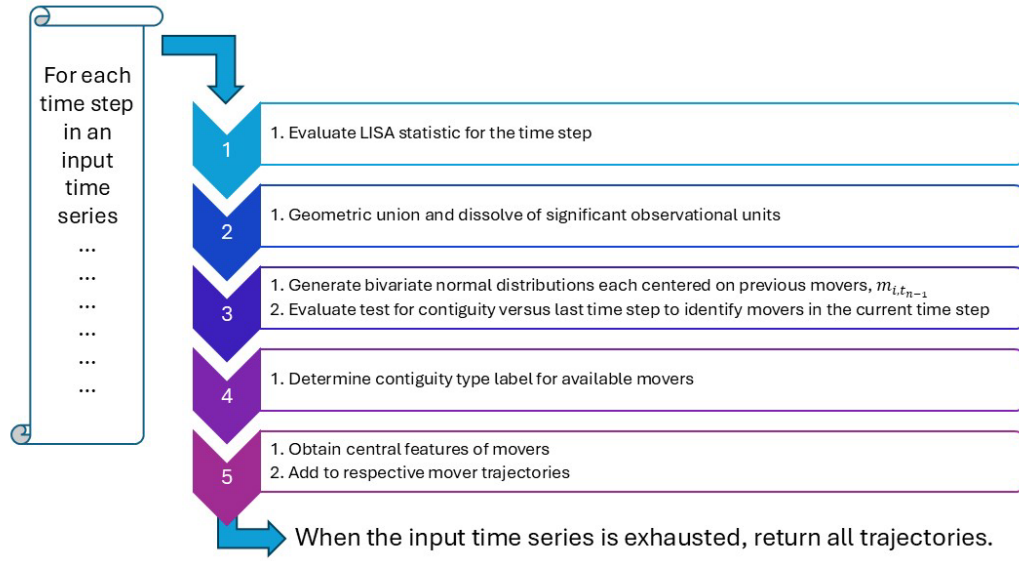


Figure 2. The 5 Phases of the Moving LISA Approach.

For each time step, these 5 phases of the Moving LISA are: (1) application of an otherwise ordinary and cross-sectional LISA statistic, (2) taking the geometric union of significant or clustering geographies for the statistic to dissolve their shared boundaries resulting in single entities passed to step (3) assessment of clusters establishing contiguity using an overlay approach and (4) tracking lineage for clusters found in the current m_{i,t_i} and previous $m_{i,t_{i-1}}$ time step and (5) application of a user-selected measure from centrography returning space-time anchor locations for each active mover in the time step. The results returned upon exhausting the entire input time series T include a collection of nonpoint mover trajectories R , with an associated table recording the ancestry and LISA statistic results for each mover m_i over the study period. Given this result set of probabilistically suggested space-time paths of nonpoint moving objects, existing measures from Time Geography may subsequently be applied to the

result for further analysis. The Moving LISA characterizes movement trajectories for spatial processes observed or sampled in aggregate.

Phases one (1) and two (2) are straightforward. They include the routine application of a cross-sectional LISA statistic across all geographies or observational units from the focal time step $g_{i...n_{t_i}}$ followed by taking a geometric union of the geographies found to be significant for the statistic. This phase results in geographies grouped for their significance and dissolved along their shared boundaries, returning individual clustering regions from groups of adjacent clustering geographies. Spatially disjoint groups which are significant for the statistic are returned as distinct dissolved geographies or *movers* $m_{i...n}$ each bearing a unique label. In the input time series T , at t_0 the geographies resulting from the geometric union proceed directly to Phase 5. For this initial time step t_0 , phases 3 and 4 are omitted since a comparable LISA result for the next time step (t_1) is not yet available. Selecting the Getis-Ord Local G as our LISA of choice (Equation 2) for the present introduction, the action of phases 1 (LISA) and 2 (Geometric Union) can be written as (Equation 3)

$$\left\{ y_{i_{t_i}} \mid LISA(g_{i_{t_i}}) = \begin{cases} 1, & \text{if } p < a \\ 0, & \text{otherwise} \end{cases} \right\} \Rightarrow y_{i_{t_i}} \cup y_{i...n_{t_i}} \Rightarrow m_{i...n} \quad (3)$$

$$\forall g_{i_{t_i}} \in G_{t_i}$$

Where:

$y_{i_{t_i}} \mid LISA(g_{i_{t_i}}) = \begin{cases} 1, & \text{if } p < a \\ 0, & \text{otherwise} \end{cases}$ returns the set of all geographies scoring as significant for the LISA statistics. For the present introduction, these are Getis-Ord Local G High-High clustering geographies. Local G Low-Low clusters, or further, Local Moran's I High-Low clusters etc. may be selected here.

p is the (pseudo) p-value for the LISA statistic, obtained by conditional randomization. This is a proportion of the amount of simulated values for the test statistic which exceeded the observed test statistic.

a is the significance threshold for the LISA statistic, supplied by the user.

$\Rightarrow Y_{i,t_i} \cup Y_{i \dots n, t_i} \Rightarrow m_{i \dots n}$ Geographies scoring as significant are passed to a geometric union function dissolving their shared boundaries, and assigning each resulting feature a unique *mover* identity, m_i .

Phases 3 and 4 establish contiguity and lineage in discovered movers $m_{i \dots n}$ between the current (t_n) and previous (t_{n-1}) time step. These phases are critical in the overall effort to characterize persistent nonpoint movers and their respective trajectories from the input time series. Here we answer the question, “Can we treat a given mover present in t_{n-1} as the *same individual moving entity* in the current t_n time step?” Phase 3 relies on an overlay technique to suggest continuity among movers between time steps. Samples are drawn from a bivariate normal distribution centered on the central feature of $m_{i,t_{n-1}}$ and are then examined for their proportion of overlap with the geometry of m_{i,t_n} . For the present introduction to the method, the axes of this distribution are uncorrelated. If a user-determined threshold π is met, *contiguity*, or the continuation and persistence in the identity of a mover $m_{i,t_{n-1}}$ to m_{i,t_n} is established.

Contiguity is a status; it suggests we may treat a given mover $m_{i,t_{n-1}}$ as belonging to the same moving object as m_{i,t_n} . The (spatial) uncorrelated bivariate normal distribution for sampled points A has its mean corresponding to the centroid coordinates of $m_{i,t_{n-1}}$ and has a variance based on the standard distance of the $m_{i,t_{n-1}}$ geometry, multiplied by a user-determined scaling factor φ . Contiguity status is tracked for a given m_{i,t_n} as $m_{i,t_n}C$, an associative array of identifiers for movers shown to be contiguous with the focal mover. Phase 3 can be expressed as Equation 4.

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} \sim \text{norm}_2 \left[\begin{pmatrix} \mu = m_{i,t_{n-1}}: \text{Centroid}_x \\ \mu = m_{i,t_{n-1}}: \text{Centroid}_y \end{pmatrix}, \begin{pmatrix} \sigma^2 = m_{i,t_{n-1}}: \text{Std Dist}^2 \times \varphi & 0 \\ 0 & \sigma^2 = m_{i,t_{n-1}}: \text{Std Dist}^2 \times \varphi \end{pmatrix} \right]$$

$$\forall a_{x,y} \in A$$

$$\text{Threshold}_{(m_{i,t_{n-1}}, m_{i,t_n})} = \begin{cases} m_{i,t_{n-1}} \text{ is contiguous with } m_{i,t_n}, & \text{if } P(A \cap m_{i,t_n}) \geq \pi \\ m_{i,t_{n-1}} \text{ is not contiguous with } m_{i,t_n}, & \text{otherwise} \end{cases} \Rightarrow m_{i,t_n} C \quad (4)$$

Where:

$\text{Threshold}_{(m_{i,t_{n-1}}, m_{i,t_n})}$ evaluates to 1 or 0, with 1 suggesting that $m_{i,t_{n-1}}$ can be considered the same *mover* as m_{i,t_n} . We label this contiguity trait for the mover using $m_{i,t_n} C$, where C stores the 1/0 result.

π is the user's choice of probability threshold; in practice this value is a minimum acceptable count of coincident points drawn from A which spatially intersect the geometry of m_{i,t_n} .

$a_{x,y}$ are point locations generated from a bivariate normal distribution having its mean(s) at

$\text{Centroid}(m_{i,t_{n-1}})$ and standard deviation $\text{Standard Distance}(m_{i,t_{n-1}})^2 \times \varphi$

φ is a user-specified scaling factor.

The axes for the distribution are uncorrelated.

C is an associative array of mover identifiers where each item in $m_{i,t_n} C$ refers to a mover contiguous with m_{i,t_n} .

Phase 4 applies a logical test assigning a *contiguity type* to all movers $m_{i...n,t_{n-1}}$ available at the focal time step t_n . This contiguity type assignment is a function of the count and identities of movers available at the focal time step t_n found to be contiguous with movers from the prior time step. All new clustering geographies from phases 1 and 2, along with all existing movers are subject to this test at each time step. A contiguity type is recorded for each active mover at each time step. New movers identified at the focal time step t_n result from cases where no earlier mover from $m_{i...n,t_{n-1}}$ met a contiguity threshold evaluated for their geometry. These new movers

are labeled “appearances” for their contiguity type. When a mover from $m_{i\dots n,t_n}$ is contiguous with more than one ancestor from $m_{i\dots n,t_{n-1}}$ it carries the “convergence” contiguity type, as it traces its appearance to the presence of more than one previously identified mover. Conversely, when more than one mover from $m_{i\dots n,t_n}$ is contiguous with only one mover from $m_{i\dots n,t_{n-1}}$ the “divergence” label is applied, as each divergent mover traces their existence to a single ancestor. Any movers from $m_{i\dots n,t_{n-1}}$ which may have expanded or contracted in terms of clustering geographic coverage but have otherwise not resulted in any spatially disjoint, unique descendants are labeled “persistent”. Finally, any movers from $m_{i\dots n,t_{n-1}}$ which did not meet any of these continuity types, for example, where their component geographies are no longer significant for clustering at the current time step t_n are considered “terminations.” Terminated movers’ trajectories end with the space-time anchor generated at the time step immediately prior to their termination. The logical test for Phase 4 can be expressed as Equation 5.

$$\forall m_i \in m_{i\dots n,t_n}$$

$$Contiguty\ Type_{(m_{i,t_{n-1}},\ contiguous(m_{i,t_n}))} = \begin{cases} Appearance, count(contiguous) == 0 \\ Convergence, count(contiguous) > 1 \wedge count\ of\ unique(contiguous) > 1 \\ Divergence, count(contiguous) > 1 \wedge count\ of\ unique(contiguous) = 1 \\ Persistence, count(contiguous) = 1 \wedge count\ of\ unique(contiguous) = 1 \\ Termination, otherwise \end{cases} \Rightarrow m_{i,t_n} C, F \quad (5)$$

Where:

$Contiguty\ Type_{(m_{i,t_{n-1}},\ contiguous(m_{i,t_n}))}$ evaluates the set of movers from $m_{i,t_{n-1}}$ found to be contiguous with m_{i,t_n} to return a contiguity type label applied to m_{i,t_n} . Contiguity types are assigned as a piecewise function applying conditional logic to the focal mover m_{i,t_n} and those movers from $m_{i,t_{n-1}}$ shown to be contiguous with the focal mover. We store the resulting F contiguity type label with each processed mover $m_{i,t_n} C, F$.

Phase 5 incrementally constructs the practical output of the Moving LISA method. The output is a bundle of trajectories $\mathbf{R}$ reflecting the spatiotemporal progression of identified

nonpoint movers R_{m_i} over the period of observation. Each individual trajectory in the output bundle $R_{m_i} \ni R$ is comprised of space-time anchor points x_i indexed R_{m_i, x_i} . Space-time anchors are located for all active movers at each time step using routine measures from centrography. While a range of measures and alternative formulations are capable of returning a central feature $C_{x,y}$ for the coordinates of a polygon geometry, for the present introduction we choose the geometric centroid (Bourke, 1988), applied in phase 5 and expressed in Equation 6.

$$\forall m_i \in m_{i \dots n, t_n} \quad (6)$$

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i)$$

$$m_{iC_x} = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1}) (x_i y_{i+1} - x_{i+1} y_i)$$

$$m_{iC_y} = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1}) (x_i y_{i+1} - x_{i+1} y_i)$$

$$R_{m_i, x_i} = m_{iC_{x,y}}$$

Where:

Expressions evaluating to m_{iC_x} and m_{iC_y} follow well-known formulations for the centroid coordinate of a polygon, based on it's area A . The centroid coordinate of a mover translates to the space-time anchor location for the mover at a given time step.

For pseudocode detailing the sequence and application of the Moving LISA's 5 phases in terms of equations 2-6, please see Appendix A, "Moving LISA Pseudocode".

2.4 *Synthetic and Empirical Data, Results*

To demonstrate the behavior of the Moving LISA procedure, we examine one simulated and one empirical dataset under a range of parameter choices. Simulating input data allows for guaranteed stationarity in the expected value/intensity and variance of the simulated process, such that comparisons among cross-sectional LISA results are easier to interpret. We might imagine the simulated dataset represents aggregated observations like aerosol concentrations for a hazardous chemical plume in the wind, incidence counts for a communicable disease, total housing vacancies in a metro area or presences of an invasive plant observed over time. The selected empirical dataset contains total population estimates by US Zip Code Tabulation Areas (ZCTA) in Oklahoma, taken from the US Census Bureau’s American Community Survey 5-year estimates *demographic profiles* product, for end-years 2013, 2018, and 2023, respectively (U.S. Census Bureau, 2023; US Census Bureau, 2024). For each of these datasets, the Moving LISA is positioned to suggest trajectories for the progression of persistent and extreme values in time and space.

The Moving LISA procedure is implemented as a computer program, using open libraries enabling its component methods. The present research is realized in part due to the open availability and reproducibility of these open tools, as the method itself is a juxtaposition of existing concepts toward new spatial questions. The *Python Spatial Analysis Library* (PySAL) ecosystem, its dependencies and allied packages, and the grander package ecosystem supporting general scientific computing in Python all deserve attention here (Hunter, 2007; Joris Van den Bossche et al., 2023; Rey et al., 2022; van der Walt et al., 2011; Waskom, 2021). Retrievals from

the US Census Bureau API for this work were assisted by the excellent *census* package (Carbaugh et al., 2024).

The following tests use a “parameter sweep” strategy to reveal the range of responses to varied inputs for each respective dataset. Once summary measures accounting for the appearances, convergences, divergences and terminations of movers are available, we further select a set of representative parameter choices whose outputs are visualized for further discussion.

For the simulated dataset, event counts were generated across a subset of ZCTAs in central Oklahoma over two phases. First, a series of 26 circular areas were created for use as point process simulation boundaries. Each individual zone had a diameter of 10 US miles and the set was arranged in a circular pattern having its center at Zip Code 73069 in Norman, Oklahoma, USA. This pattern of simulation boundaries repeated for 3 cycles, with each boundary carrying a strictly increasing timestamp value assigned clockwise, yielding a helical progression of 78 simulation boundaries in space and time at the study area. These point process simulation zones cover only a subset of the overall study area, as shown in Figure 3 A (left). Additional background events were simulated for each zip code location as a random draw from a half-normal distribution with mean of 3 and standard deviation of 1, truncated to whole integer values.

For each simulation boundary, a sampling from a two-stage Neyman-Scott Poisson Point Process was used to generate a moving progression of events for the theoretical process (Neyman & Scott, 1957, 1958). In a given realization taken from a Neyman-Scott Poisson Point Process, a “parent” location is drawn from a Poisson distribution within the circular boundary, followed by an additional sampling of a fixed number of points within a smaller circular region having a

defined radius, centered on the parent location. For each circular simulation boundary, parameters governing the shape of the Neyman-Scott distribution were fixed and shared among all simulation boundaries. A sampling of 200 total point locations was generated within 1500 meters of 10 randomly assigned parent locations within each simulation boundary. Additionally, the fixed mean and variance for background case counts drawn from a half-normal distribution did not vary with time step. The *pointpats* submodule of the PySAL software package provided a readily available implementation of the point process applicable to this work (Rey, 2015; Rey et al., 2022; Rey & Anselin, 2007; Sergio Rey et al., 2024). Using an overlay technique, the total count of Neyman-Scott and background case counts were assigned to each zip code observational unit by time step in the study area.

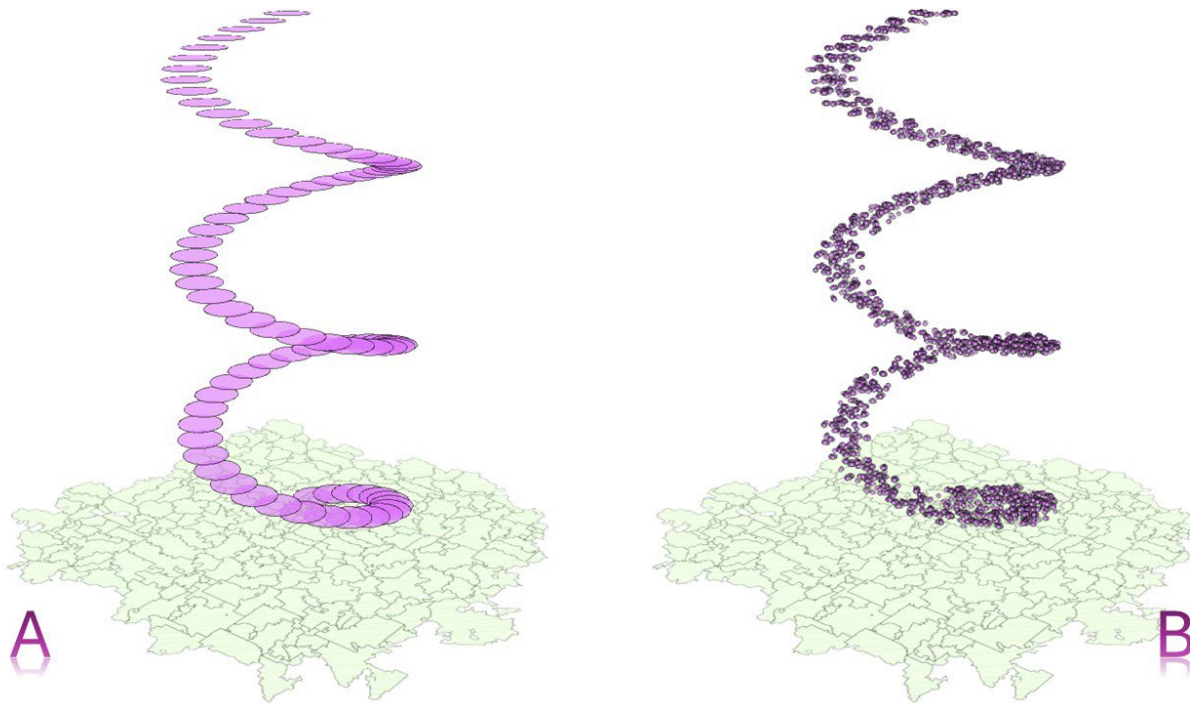


Figure 3. Helical Simulation Boundaries (A) and Simulated Event Points (B) in Central Oklahoma. A Neyman-Scott point process is simulated at each circular boundary shown in (A), with results shown in (B).

We note these choices for the intensity and count of parent locations emit a highly specified and controlled realization from a poisson-distributed spatial process. This guarantees a stationary mean or intensity for the data generating process used in the demonstration, contributing to the comparability of LISA results across time steps. Recent work surrounding the comparability of cross-sectional LISA measurements between time frames notes the potential complications posed by shifts in the expected value of real-world data-generating spatial processes over time (Tao & Chen, 2022). Tao and Chen note that without comparison to a fixed mean for LISA calculations at subsequent time steps, only relative examination of distributional dynamics may be drawn from cross-sectional LISA results. For the present research, we acknowledge the potential for a nonstationary expected value/intensity in the US Census Bureau population data examined, and adopt the *Relative Perspective* on the global mean for the purposes of this demonstration.

2.4.1 Test Results

To illustrate the response and sensitivity among parameters for Moving LISA method, we hold most parameters constant and execute the procedure across both the synthetic and empirical datasets for each of 90 increasing values for the contiguity threshold π , from 0.10 to 1.0. Across these 90 executions, we set the standard distance scaling factor φ to 2 and the significance threshold α to 0.05 for the Local G statistic, equating to a one-tailed test for high-high clusters in the study area. As the contiguity threshold increases, the amount of spatial overlap necessary to be considered as the same spatial entity among temporally neighboring clusters will increase. “Contiguity” becomes more difficult to achieve as the contiguity threshold π increases. Other inputs to the method remain constant, including the input dataset and a LISA weights matrix for

queen's contiguity. At each execution, we collect the total counts of entities by contiguity type generated over the course of each run of the procedure. The pattern in response for this strategy applied to the datasets are shown in Figures 4 and 5. While this type of response will vary uniquely with the structure and values in the input dataset and the user's parameter choices, we note a critical point at approximately $\pi = 0.3$ for both datasets, where the relative shares of *persistent* and *terminating* clusters cross over in their proportions. This effect could be attributed to the influence of the spatial arrangement of observational units, noting that identical ZCTAs form the spatial substrate for both the synthetic and empirical datasets. With respect to our synthetic dataset, the location of highest intensity for the process moves or is displaced at each time step. As π approaches 1.0, we should expect to see counts of persistent clusters converge to 1, since the only allowable *persistent* clusters in that case would be those that have not moved.

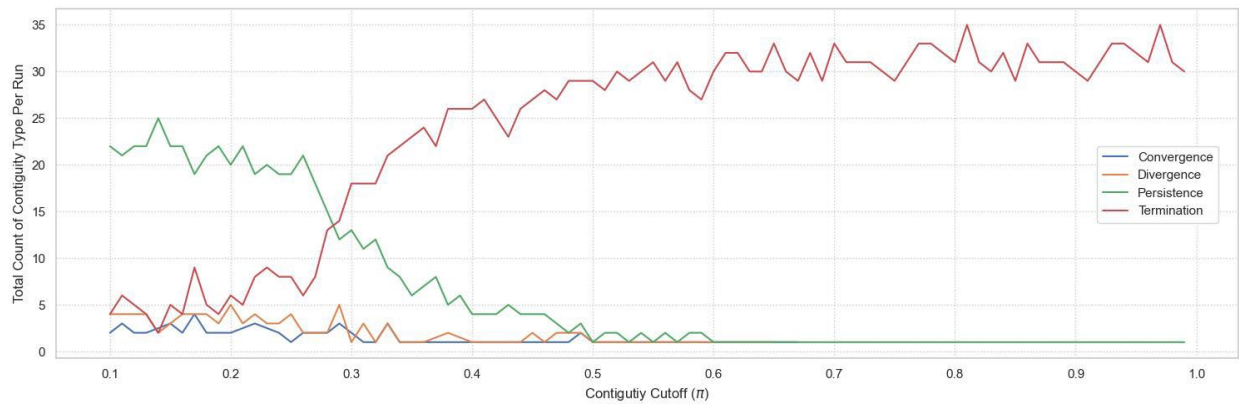


Figure 4. Response to changing Contiguity Threshold parameter. As the threshold value for contiguity increases, the overall count of persistent clusters identified in the procedure will decrease. To be considered contiguous, a higher contiguity threshold demands a higher amount of overlapping probability density between clusters at the current and previous time steps.

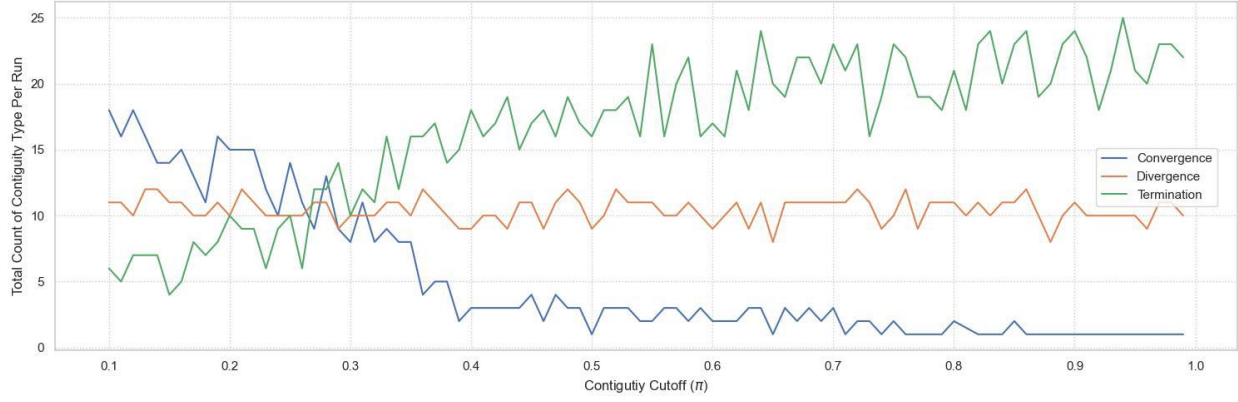


Figure 5. Response to changing Contiguity Threshold parameter for 5-year population estimates at ZCTAs.

Attenuating the standard distance scaling factor φ affects the resulting overall count of “contiguities” generated during the procedure. This effect is due to the action of the scaling factor on the spatial spread or width (the variance) of a bivariate normal distribution used to evaluate a threshold for contiguity (Equation 6). For example, selecting a large value for φ and thus creating a large simulation boundary far exceeding the spatial arrangement or configuration of an observational unit $m_{i,t_{i-1}}$ and its target m_{i,t_i} can lead to situations where the required proportion of overlap for contiguity is not met. Here, in evaluating the overlap of probability mass and the geometry of m_{i,t_i} , a very wide simulation boundary may indeed completely cover the target m_{i,t_i} geometry. However, the required probability mass would not be met, as much of the bivariate distribution’s coverage would fall on areas other than m_{i,t_i} . Consequently, this relationship becomes counter-intuitive and difficult to interpret where the reflex expectation would align with an increase in the parameter to increase the proportion of overlap. Locating a value for φ which acknowledges the spatial arrangement of the input data may relax the difficulty. This selection can be an *a priori* informed selection or the result of an additional

parameter search or “sweep” by the user. Setting the contiguity cutoff to 0.3 (the general location at which persistence and terminations show change from the first test) and increasing the standard distance scaling factor ϕ by 0.1 in each of 100 runs of the procedure over a range from 0.5 to 3.0 results in the response shown in Figures 6 and 7, for the synthetic and empirical datasets, respectively. Reducing the standard distance scaling factor ϕ decreases the effective spatial width of the bivariate normal simulation by attenuating the distribution’s variance parameter, resulting in an interaction with contiguity cutoff where fewer clusters achieve contiguity as the scaling parameter decreases.

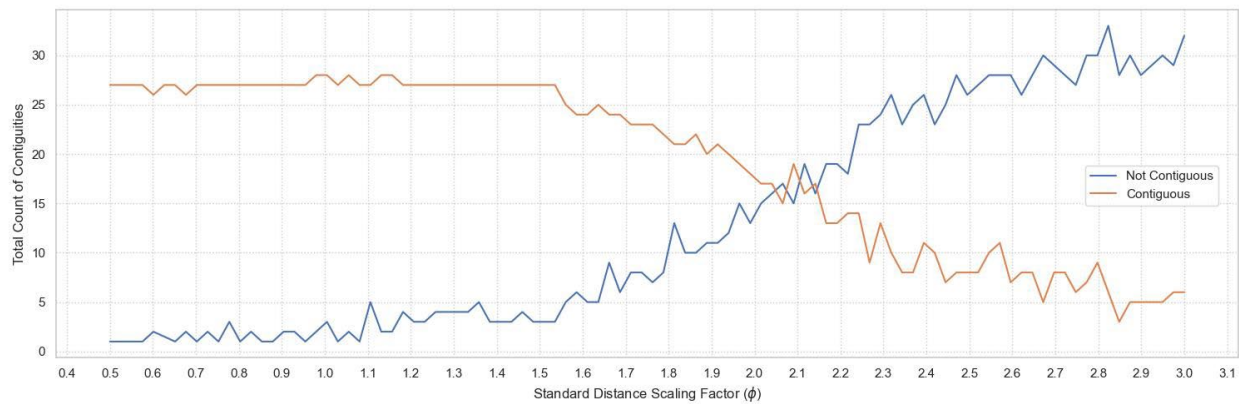


Figure 6. Simulated-Data response to changing Standard Distance Scaling Factor parameter. As the Standard Distance Scaling Factor is increased, the variance or width of the bivariate normal simulation increases. Since contiguity is a function of meeting a minimum of overlapping probability mass, increasing the variance of the distribution spreads the available probability mass over a larger area.

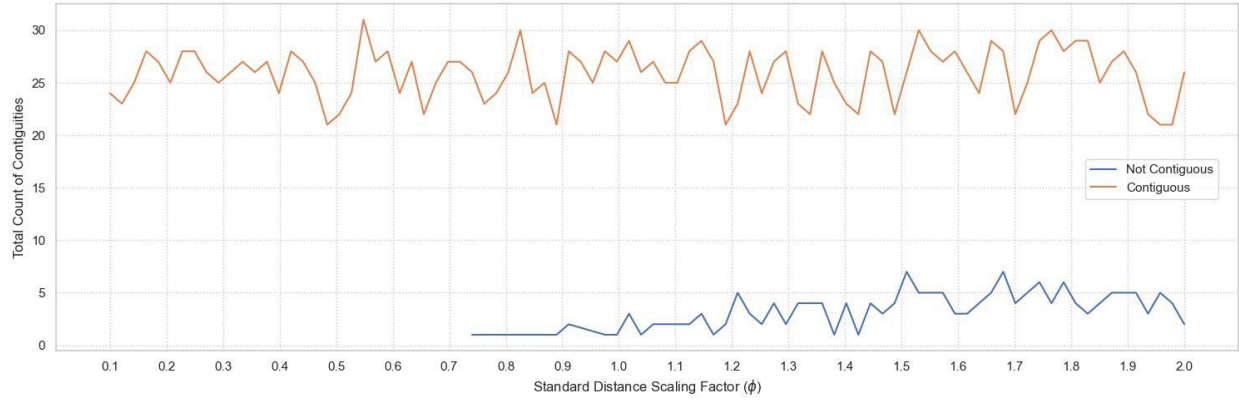


Figure 7. Response to changing Standard Distance Scaling Factor parameter for 5-year population estimates at ZCTAs.

Decreasing the Getis-Ord Local G significance threshold α will also decrease counts of “contiguities”, by raising the criteria of what is considered a significant value for the statistic. This behavior is a function of identifying clusters or extreme observations at each time step, rather than a function of comparisons among clusters between time steps. As α decreases, the observed Local G statistic value for a given location must, in the example of identifying High-High clusters, have a higher normalized value for the Local G statistic to be considered significant. Holding the contiguity threshold constant at approximately $\pi = 0.3$ and the standard distance scaling factor at $\phi = 2$ and testing for all α $[0.01, 0.50]$ at a 0.01 resolution, we see the response in terms of count of contiguities in Figure 8. For the synthetic dataset, decreasing incidence in terms of Local Getis-Ord G high-high clusters corresponds to a decrease in the count of *True* or successful “contiguities” generated during each run of the procedure.

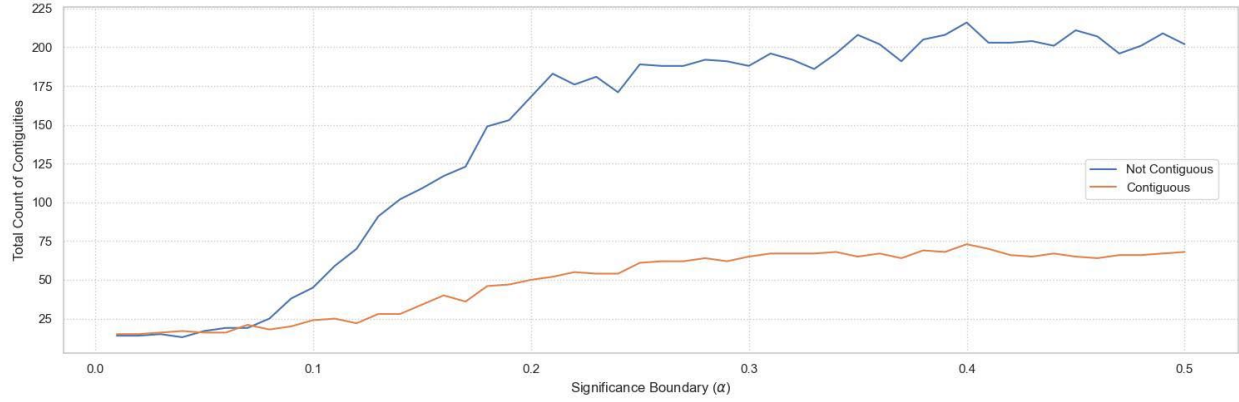


Figure 8. Simulated-Data response to changing Getis-Ord Local G significance boundary. Decreasing the significance boundary value for the Getis-Ord Local G statistic will demand that G statistic values must be progressively more extreme to score as significant. For example, a hot-spot cluster found to be significant at the $\alpha = 0.10$ level will have shown a normalized G statistic value exceeding of 90% of values generated during conditional permutations.

Noting the relationships among the parameters discussed in this section, to conclude the demonstration we obtained the **R** result set of mover space-time anchors for both datasets, and visualized the space-time paths (B) of *aggregate movers* (a) in the study area in Figure 9. Figures 10-12 visualize a result of the empirical test dataset, illustrating the action of convergences (Figure 12) and terminations (Figure 13, B) per the cluster tracking dimensions of Moving LISA across a real-world dataset. The parameterization yielding the visualized results for both datasets is: $\pi = 0.3, \varphi = 2, a = 0.05$.

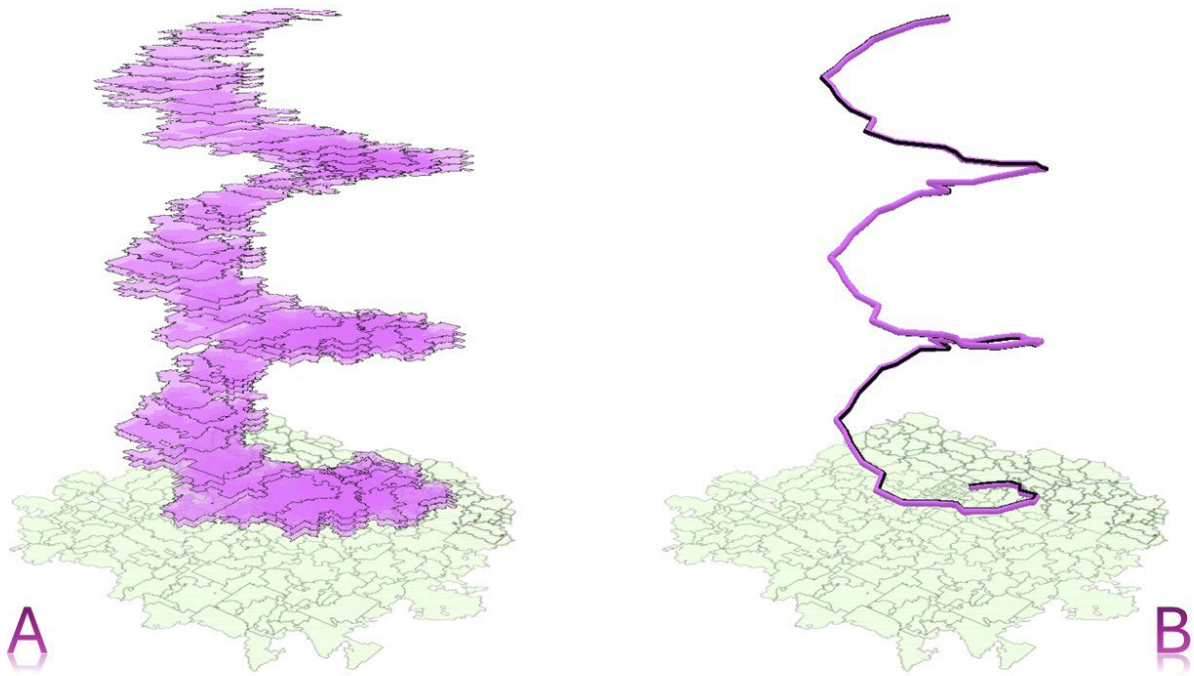


Figure 9. A sample Moving LISA mover progression (A) and output trajectory (B). Collecting the centroid locations of singular mover shown in (A), a space-time trajectory for the Moving LISA mover is suggested by constructing a line from these centroids in (B).

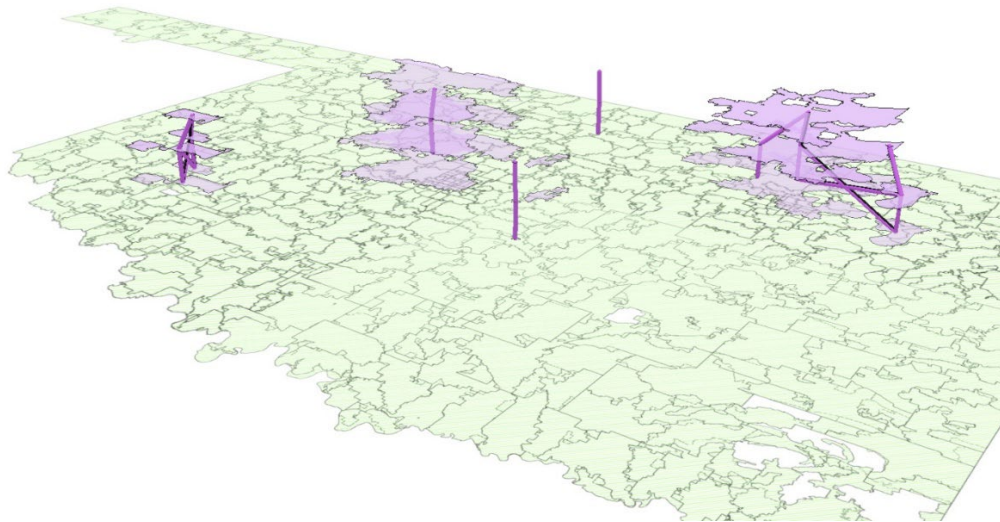


Figure 10. A statewide result for ACS 5-year population estimates at Oklahoma ZCTAs, calculated at $\pi = 0.3$ and $\varphi = 2.0$.

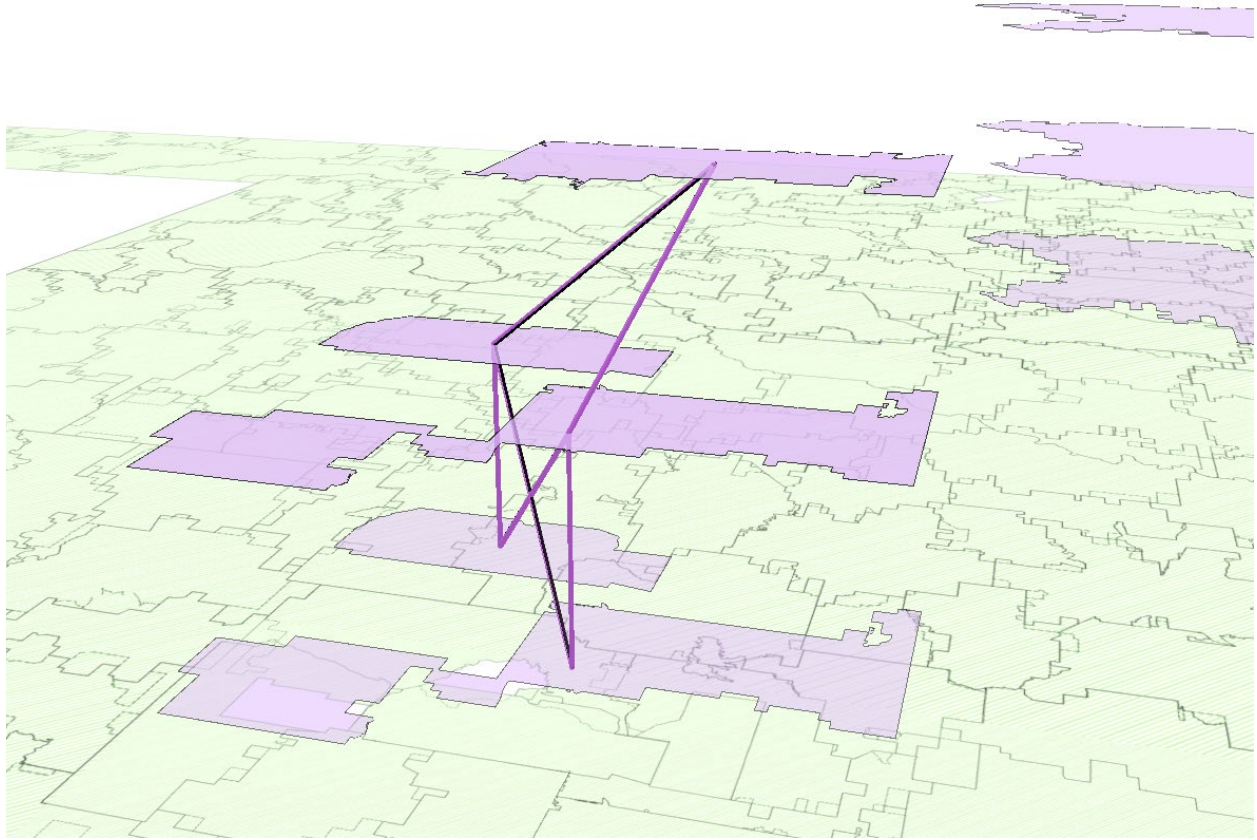


Figure 11. Shared convergences at timestep 2, further convergence to a single mover in timestep 3 where earlier clustering zip codes were no longer found significant for the Local G statistic. Clusters at timesteps 1 and 2 were spatially disjoint, but passed contiguity tests at $\pi = 0.3$ and $\varphi = 2.0$ identifying two movers at timestep 2.

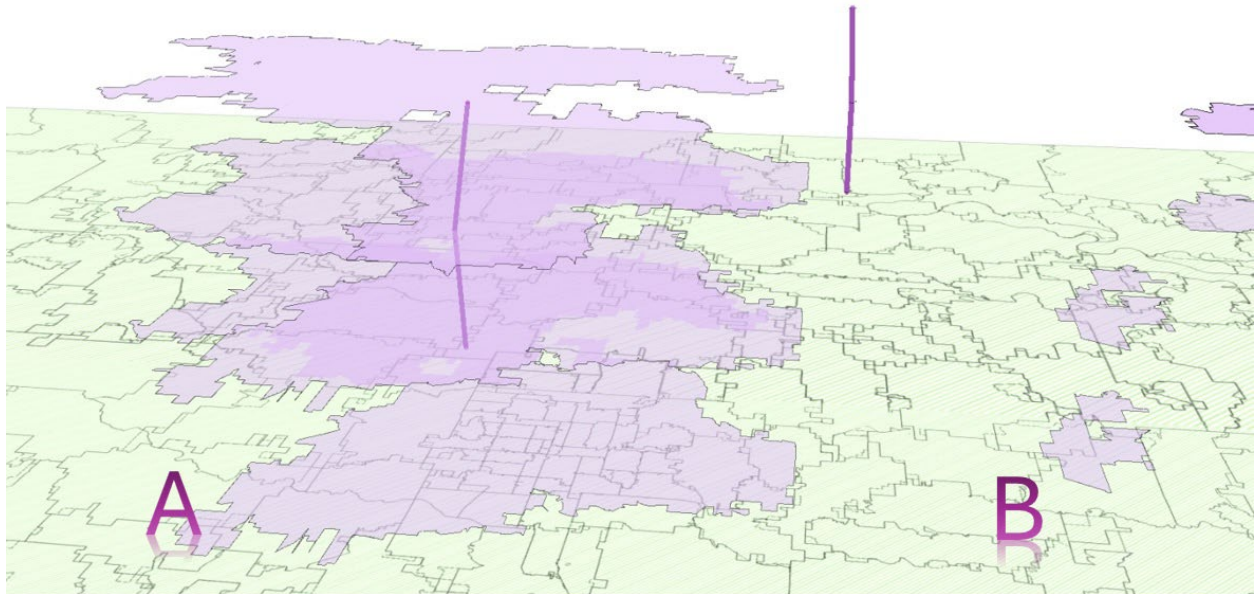


Figure 12. Persistent *aggregate mover* shown across 3 timeframes of population data (A), adjacent to a *terminated mover* (B) which appeared significant for the Local G in 2013 and 2023, but was not for 2018. No *aggregate mover* trajectory is returned unless clustering is persistent and sequential.

2.5 Discussion and Conclusion

Time geographic methods examining movement in nonpoint, invisible or distributed spatial phenomena are novel. However, the notion of monitoring a stationary clustering location over time or to consider an observation within a spatiotemporal neighborhood for its status as a cluster represent known strategies from recent exploratory space-time data analysis (ESTDA) literature. To be clear, the Moving LISA seeks to evaluate and aggregate observations of distributed spatial phenomena to suggest movement trajectories characterizing change over time. Movement trajectories comprise the basic input for Time Geographic methods, where those methods are traditionally interested in the movement of individual, discrete and directly observable entities.

Moving LISA provides one avenue toward the treatment of distributed spatial phenomenon as Time Geographic moving entities.

The Moving LISA inherits both the sensitivities of LISA approaches and the generalization tradeoffs inherent to locating space-time anchors in Time Geography. Important considerations include the careful construction of weights matrices reflecting known spatial structure in a process and parameter choices encoding our null hypotheses about what constitutes an extreme observation (Sauer et al., 2022). Variability in input data relates to additional concerns raised in recent work surrounding the application of LISAs in situations where data are nonstationary with respect to their generating processes or are highly temporally correlated (Arribas-Bel & Tranos, 2018; Tao & Chen, 2022). The selected datasets for our demonstration are themselves an acknowledgement that Moving LISA, by its reliance on LISA measures, would be similarly confounded in such situations. Further still, the effects of seasonality, or edge effects related to the selection of study areas are not currently addressed for the Moving LISA approach. Overall, phenomena lending themselves to observation at meaningful areal units, sampled at a regular interval are ideal for the method as presented. Ideal inputs for the Moving LISA would most likely include data supporting studies in public health equity, public transportation demand planning and vacancy in housing markets; essentially human and urban geographic pursuits where the driving processes are in theory invariant to natural influences on variability, but at the same time demand observation in aggregate at human-defined observational units. The Moving LISA also carries the limitations of classical time geographic analyses, where a shortest-path distance in space and time is defaulted to as the space-time path of the mover and where space is treated as homogeneously traversable along the space time path.

Recent work in Time Geography dissolves some of these difficulties, but no such advancement has been incorporated into the trajectory-generating action of the Moving LISA thus far.

From the demonstration, it is evident that an initial parameter sweep is advisable in obtaining a departure point for parameter choices for researchers using the Moving LISA. Additional options attenuating the behavior of the method are available in the present implementation but were not explored in this research, such as setting the number of realizations drawn from bivariate normal-based comparisons for contiguity, an adjustment which would interact with both the standard distance multiplier and the contiguity cutoff parameters. Ultimately, the unknowns for users center on these Moving LISA-specific parameters, which are additional to routine choices made for the cross-sectional application of LISAs. Should a given researcher be confident in their LISA results for a sampling of cross-sectional measures taken from a larger panel dataset, they are encouraged to maintain their LISA parameter choices and experiment with π and φ from Moving LISA respectively.

Future work could both address some of the limitations associated with the Moving LISA or more readily provide researchers with tools to manage associated drawbacks. Some of the most direct examples from the literature include spatiotemporal extensions to the LISA statistics themselves, such as a Local Getis-Ord G statistic having a 3-dimensional weights matrix (Lee, 2023; Z. Wang & Lam, 2020b). Measures of spatiotemporal trend could also be incorporated alongside improvements to the weights matrix, for example EMHSA incorporates a Mann-Kendall test as part of its procedure characterizing the emergence, oscillation or collapse of hot spots over time. An open software package implementing the Moving LISA method is also forthcoming. Introducing a Moving LISA in this research also demands its utility should be further substantiated in future applied research. The Authors look to longitudinal questions from

Public Health, specifically on the emergent pattern of specialty Vape Shops in Oklahoma, and to questions of long-term housing supply as applications in the short term. For the Time Geographic study of nonpoint phenomena and indeed for Time Geography in general, the literature is entering an exciting time; where new ideas about movement, access, equity, identity and their causalities are met with expanding access to movement data. Commensurate with the appearance of Moving LISA, we can expect and anticipate expanded and useful research activity in all forms of Time Geography.

2.6 References

- Ahearn, S. C., Dodge, S., Simcharoen, A., Xavier, G., & Smith, J. L. D. (2017). A context-sensitive correlated random walk: A new simulation model for movement. *International Journal of Geographical Information Science*, 31(5), 867–883.
<https://doi.org/10.1080/13658816.2016.1224887>
- Anderson, J. H., Downs, J. A., Loraamm, R., & Reader, S. (2017). Agent-based simulation of Muscovy duck movements using observed habitat transition and distance frequencies. *Computers, Environment and Urban Systems*, 61, 49–55.
<https://doi.org/10.1016/j.compenvurbsys.2016.09.002>
- Anselin, L. (1988). *Spatial Econometrics: Methods and Models* (Vol. 4). Springer Science & Business Media.
- Anselin, L. (1995). Local Indicators of Spatial Association—LISA. *Geographical Analysis*, 27(2), 93–115. <https://doi.org/10.1111/j.1538-4632.1995.tb00338.x>
- Anselin, L. (1996). The Moran scatterplot as an ESDA tool to assess local instability in spatial association. In *Spatial analytical perspectives on GIS* (pp. 111–126). Routledge.

- Anselin, L. (2019). A Local Indicator of Multivariate Spatial Association: Extending Geary's c. *Geographical Analysis*, 51(2), 133–150. <https://doi.org/10.1111/gean.12164>
- Anselin, L., Li, X., & Koschinsky, J. (2022). GeoDa, From the Desktop to an Ecosystem for Exploring Spatial Data. *Geographical Analysis*, 54(3), 439–466. <https://doi.org/10.1111/gean.12311>
- Anselin, L., Syabri, I., & Kho, Y. (2006). GeoDa: An Introduction to Spatial Data Analysis. *Geographical Analysis*, 38(1), 5–22. <https://doi.org/10.1111/j.0016-7363.2005.00671.x>
- Arribas-Bel, D., & Tranos, E. (2018). Characterizing the Spatial Structure(s) of Cities “on the fly”: The Space-Time Calendar. *Geographical Analysis*, 50(2), 162–181. <https://doi.org/10.1111/gean.12137>
- Birant, D., & Kut, A. (2007). ST-DBSCAN: An algorithm for clustering spatial–temporal data. *Data & Knowledge Engineering*, 60(1), Article 1. <https://doi.org/10.1016/j.datak.2006.01.013>
- Bourke, P. (1988, July 1). *Calculating the area and centroid of a polygon*. <https://paulbourke.net/geometry/polygonmesh/>
- Brunsdon, C., Singleton, A., & Miller, H. J. (2015). Spatiotemporal Knowledge Discovery. In *Geocomputation: A Practical Primer*. SAGE Publications, Inc. <https://doi.org/10.4135/9781473916432>
- Buonaccorsi, J. P., Elkinton, J. S., Evans, S. R., & Liebhold, A. M. (2001). MEASURING AND TESTING FOR SPATIAL SYNCHRONY. *Ecology*, 82(6), 1668–1679. [https://doi.org/10.1890/0012-9658\(2001\)082\[1668:MATFSS\]2.0.CO;2](https://doi.org/10.1890/0012-9658(2001)082[1668:MATFSS]2.0.CO;2)
- Cagnacci, F., Boitani, L., Powell, R. A., & Boyce, M. S. (2010). Animal ecology meets GPS-based radiotelemetry: A perfect storm of opportunities and challenges. *Philosophical*

- Transactions of the Royal Society B: Biological Sciences*, 365(1550), Article 1550.
<https://doi.org/10.1098/rstb.2010.0107>
- Carbaugh, J., Gregg, F., & Contributors. (2024). *Census* (Version 0.8.24) [Computer software].
<https://github.com/datamade/census>
- Cliff, A. D., & Ord, J. K. (1973). *Spatial autocorrelation*. Pion.
<https://archive.org/details/spatialautocorre0000clif>
- Cliff, A. D., Ord, J. K., & Cliff, A. D. (1981). *Spatial processes: Models & applications*. Pion.
- Demšar, U., & Long, J. A. (2019). Potential path volume (PPV): A geometric estimator for space use in 3D. *Movement Ecology*, 7(1), Article 1. <https://doi.org/10.1186/s40462-019-0158-4>
- Dodge, S. (2021). A Data Science Framework for Movement. *Geographical Analysis*, 53(1), 92–112.
- Dodge, S., Gao, S., Tomko, M., & Weibel, R. (2020). Progress in computational movement analysis – towards movement data science. *International Journal of Geographical Information Science*, 34(12), 2395–2400.
<https://doi.org/10.1080/13658816.2020.1784425>
- Dodge, S., & Nelson, T. A. (2023). A framework for modern time geography: Emphasizing diverse constraints on accessibility. *Journal of Geographical Systems*, 25(3), 357–375.
<https://doi.org/10.1007/s10109-023-00404-1>
- Dodge, S., Su, R., Johnson, J., Simcharoen, A., Goulías, K., Smith, J. L. D., & Ahearn, S. C. (2021). ORTEGA: An object-oriented time-geographic analytical approach to trace space-time contact patterns in movement data. *Computers, Environment and Urban Systems*, 88, 101630. <https://doi.org/10.1016/j.compenvurbsys.2021.101630>

- Downs, J. A. (2010). Time-Geographic Density Estimation for Moving Point Objects. In S. I. Fabrikant, T. Reichenbacher, M. van Kreveld, & C. Schlieder (Eds.), *Geographic Information Science* (Vol. 6292, pp. 16–26). Springer Berlin Heidelberg.
https://doi.org/10.1007/978-3-642-15300-6_2
- Downs, J., Horner, M. W., Hyzer, G., Lamb, D., & Loraamm, R. (2014). Voxel-based probabilistic space-time prisms for analysing animal movements and habitat use. *International Journal of Geographical Information Science*, 28(5), Article 5.
<https://doi.org/10.1080/13658816.2013.850170>
- Epstein, J. M. (1999). Agent-Based Computational Models And Generative Social Science. *Complexity*, 4(5), Article 5.
- Gao, Y., Cheng, J., Meng, H., & Liu, Y. (2019). Measuring spatio-temporal autocorrelation in time series data of collective human mobility. *Geo-Spatial Information Science*, 22(3), Article 3. <https://doi.org/10.1080/10095020.2019.1643609>
- Gao, Y., Zheng, B., Chen, G., & Li, Q. (2010). Algorithms for constrained k-nearest neighbor queries over moving object trajectories. *GeoInformatica*, 14(2), Article 2.
<https://doi.org/10.1007/s10707-009-0084-5>
- Getis, A., & Aldstadt, J. (2004). Constructing the Spatial Weights Matrix Using a Local Statistic. *Geographical Analysis*, 36(2), 90–104. <https://doi.org/10.1111/j.1538-4632.2004.tb01127.x>
- Getis, A., & Ord, J. K. (1992). The Analysis of Spatial Association by Use of Distance Statistics. *Geographical Analysis*, 24(3), 189–206. <https://doi.org/10.1111/j.1538-4632.1992.tb00261.x>

- Hägerstrand, T. (1970). What about people in Regional Science? *Papers of the Regional Science Association*, 24(1), Article 1. <https://doi.org/10.1007/BF01936872>
- Harris, N. L., Goldman, E., Gabris, C., Nordling, J., Minnemeyer, S., Ansari, S., Lippmann, M., Bennett, L., Raad, M., Hansen, M., & Potapov, P. (2017). Using spatial statistics to identify emerging hot spots of forest loss. *Environmental Research Letters*, 12(2), 024012. <https://doi.org/10.1088/1748-9326/aa5a2f>
- Ho, K., & Loraamm, R. (2023). Using a Cost-Distance Time-Geographic Approach to Identify Red Deer Habitat Use in Banff National Park, Alberta, Canada. *ISPRS International Journal of Geo-Information*, 12(8), 339. <https://doi.org/10.3390/ijgi12080339>
- Hunter, J. D. (2007). Matplotlib: A 2D Graphics Environment. *Computing in Science & Engineering*, 9(3), Article 3. <https://doi.org/10.1109/MCSE.2007.55>
- Johnson, A. A., Ott, M. Q., & Dogucu, M. (2022). *Bayes rules! An introduction to Bayesian modeling with R*. CRC Press.
- Joris Van den Bossche, Kelsey Jordahl, Martin Fleischmann, James McBride, Jacob Wasserman, Matt Richards, Adrian Garcia Badaracco, Alan D. Snow, Jeff Tratner, Jeffrey Gerard, Brendan Ward, Matthew Perry, Carson Farmer, Geir Arne Hjelle, Mike Taves, Ewout ter Hoeven, Micah Cochran, rraymondgh, Sean Gillies, ... Aleksey Bilogur. (2023). *geopandas/geopandas: V0.14.1* (Version v0.14.1) [Computer software]. Zenodo. <https://doi.org/10.5281/ZENODO.2585848>
- Kar, A. K. (2016). Bio inspired computing – A review of algorithms and scope of applications. *Expert Systems with Applications*, 59, 20–32. <https://doi.org/10.1016/j.eswa.2016.04.018>

- Kobayashi, T., Miller, H. J., & Othman, W. (2011). Analytical methods for error propagation in planar space–time prisms. *Journal of Geographical Systems*, 13(4), Article 4.
<https://doi.org/10.1007/s10109-010-0139-z>
- Kuijpers, B., Grimson, R., & Othman, W. (2011). An analytic solution to the alibi query in the space–time prisms model for moving object data. *International Journal of Geographical Information Science*, 25(2), 293–322. <https://doi.org/10.1080/13658810902967397>
- Kuijpers, B., Miller, H. J., Neutens, T., & Othman, W. (2010). Anchor uncertainty and space-time prisms on road networks. *International Journal of Geographical Information Science*, 24(8), Article 8. <https://doi.org/10.1080/13658810903321339>
- Kuijpers, B., & Othman, W. (2009). Modeling uncertainty of moving objects on road networks via space–time prisms. *International Journal of Geographical Information Science*, 23(9), Article 9. <https://doi.org/10.1080/13658810802097485>
- Kuijpers, B., & Othman, W. (2017). The geometry of space-time prisms with uncertain anchors. *International Journal of Geographical Information Science*, 31(9), Article 9.
<https://doi.org/10.1080/13658816.2017.1319950>
- Kulldorff, M. (1997). A spatial scan statistic. *Communications in Statistics - Theory and Methods*, 26(6), 1481–1496. <https://doi.org/10.1080/03610929708831995>
- Kulldorff, M. (2001). Prospective Time Periodic Geographical Disease Surveillance Using a Scan Statistic. *Journal of the Royal Statistical Society Series A: Statistics in Society*, 164(1), 61–72. <https://doi.org/10.1111/1467-985X.00186>
- Kwan, M.-P. (1999). Gender and Individual Access to Urban Opportunities: A Study Using Space–Time Measures. *The Professional Geographer*, 51(2), Article 2.
<https://doi.org/10.1111/0033-0124.00158>

- Laube, P., Imfeld, S., & Weibel, R. (2005). Discovering relative motion patterns in groups of moving point objects. *International Journal of Geographical Information Science*, 19(6), Article 6. <https://doi.org/10.1080/13658810500105572>
- Lee, J. (Ed.). (2023). *Spatiotemporal analytics*. CRC Press.
- Lee, J., & Li, S. (2017). Extending Moran's Index for Measuring Spatiotemporal Clustering of Geographic Events. *Geographical Analysis*, 49(1), 36–57. <https://doi.org/10.1111/gean.12106>
- Lee, J., & Miller, H. J. (2019). Analyzing collective accessibility using average space-time prisms. *Transportation Research Part D: Transport and Environment*, 69, 250–264. <https://doi.org/10.1016/j.trd.2019.02.004>
- Liebhold, A., Koenig, W. D., & Bjørnstad, O. N. (2004). Spatial Synchrony in Population Dynamics. *Annual Review of Ecology, Evolution, and Systematics*, 35(1), 467–490. <https://doi.org/10.1146/annurev.ecolsys.34.011802.132516>
- Long, J. A. (2018). Modeling movement probabilities within heterogeneous spatial fields. *Journal of Spatial Information Science*, 16, Article 16. <https://doi.org/10.5311/JOSIS.2018.16.372>
- Long, J. A., Nelson, T. A., & Nathoo, F. S. (2014). Toward a kinetic-based probabilistic time geography. *International Journal of Geographical Information Science*, 28(5), Article 5. <https://doi.org/10.1080/13658816.2013.818151>
- Long, J. A., Weibel, R., Dodge, S., & Laube, P. (2018). Moving ahead with computational movement analysis. *International Journal of Geographical Information Science*, 32(7), Article 7. <https://doi.org/10.1080/13658816.2018.1442974>

- Loraamm, R. (2019). Incorporating behavior into animal movement modeling: A constrained agent-based model for estimating visit probabilities in space-time prisms. *International Journal of Geographical Information Science*, 1–21.
<https://doi.org/10.1080/13658816.2019.1658875>
- Loraamm, R., Anderson, J., & Burch, C. (2021). Identifying road avoidance behavior using time-geography for red deer in Banff National Park, Alberta, Canada. *Transactions in GIS*, 25(3), Article 3. <https://doi.org/10.1111/tgis.12748>
- Loraamm, R., Downs, J., Anderson, J., & Lamb, D. S. (2020). PySTPrism: Tools for voxel-based space–time prisms. *SoftwareX*, 12, 100499. <https://doi.org/10.1016/j.softx.2020.100499>
- Loraamm, R., Downs, J., & Lamb, D. (2018). A time-geographic approach to quantifying wildlife–road interactions. *Transactions in GIS*, 23(1), Article 1.
<https://doi.org/10.1111/tgis.12497>
- Loraamm, R., Goodenough, K. S., Burch, C., Davenport, L. C., & Haugaasen, T. (2020). A time-geographic approach to identifying daily habitat use patterns for Amazonian Black Skimmers. *Applied Geography*, 118, 102189.
<https://doi.org/10.1016/j.apgeog.2020.102189>
- McElreath, R. (2020). *Statistical rethinking: A Bayesian course with examples in R and Stan* (Second edition). Chapman & Hall/CRC.
- Miller, H. J. (1991). Modelling accessibility using space-time prism concepts within geographical information systems. *International Journal of Geographical Information System*, 5(3), Article 3.
- Miller, H. J. (2005). A Measurement Theory for Time Geography. *Geographical Analysis*, 37(1), Article 1. <https://doi.org/10.1111/j.1538-4632.2005.00575.x>

- Miller, H. J. (2017). Time Geography and Space-Time Prism. In D. Richardson, N. Castree, M. F. Goodchild, A. Kobayashi, W. Liu, & R. A. Marston (Eds.), *International Encyclopedia of Geography: People, the Earth, Environment and Technology* (pp. 1–19). John Wiley & Sons, Ltd. <https://doi.org/10.1002/9781118786352.wbieg0431>
- Miller, H. J., & Bridwell, S. A. (2009). A Field-Based Theory for Time Geography. *Annals of the Association of American Geographers*, 99(1), Article 1. <https://doi.org/10.1080/00045600802471049>
- Miller, H. J., Dodge, S., Miller, J., & Bohrer, G. (2019). Towards an integrated science of movement: Converging research on animal movement ecology and human mobility science. *International Journal of Geographical Information Science*, 33(5), 855–876. <https://doi.org/10.1080/13658816.2018.1564317>
- Mo, C., Tan, D., Mai, T., Bei, C., Qin, J., Pang, W., & Zhang, Z. (2020). An analysis of spatiotemporal pattern for COVID-19 in China based on space-time cube. *Journal of Medical Virology*, 92(9), 1587–1595. <https://doi.org/10.1002/jmv.25834>
- Moran, P. A. P. (1950). Notes on Continuous Stochastic Phenomena. *Biometrika*, 37(1/2), Article 1/2. <https://doi.org/10.2307/2332142>
- Morckel, V., & Durst, N. (2023). Using Emerging Hot Spot Analysis to Explore Spatiotemporal Patterns of Housing Vacancy in Ohio Metropolitan Statistical Areas. *Urban Affairs Review*, 59(1), 309–328. <https://doi.org/10.1177/10780874211065014>
- Nakaya, T., & Yano, K. (2010). Visualising Crime Clusters in a Space-time Cube: An Exploratory Data-analysis Approach Using Space-time Kernel Density Estimation and Scan Statistics. *Transactions in GIS*, 14(3), 223–239. <https://doi.org/10.1111/j.1467-9671.2010.01194.x>

- Neyman, J., & Scott, E. L. (1957). On a Mathematical Theory of Populations Conceived as Conglomerations of Clusters. *Cold Spring Harbor Symposia on Quantitative Biology*, 22(0), 109–120. <https://doi.org/10.1101/SQB.1957.022.01.014>
- Neyman, J., & Scott, E. L. (1958). Statistical Approach to Problems of Cosmology. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 20(1), 1–29. <https://doi.org/10.1111/j.2517-6161.1958.tb00272.x>
- Ord, J. K., & Getis, A. (1995). Local Spatial Autocorrelation Statistics: Distributional Issues and an Application. *Geographical Analysis*, 27(4), 286–306. <https://doi.org/10.1111/j.1538-4632.1995.tb00912.x>
- Ord, J. K., & Getis, A. (2012). Local spatial heteroscedasticity (LOSH). *The Annals of Regional Science*, 48(2), 529–539. <https://doi.org/10.1007/s00168-011-0492-y>
- Parry, J., & Locke, D. (2022). *sfdep: Spatial Dependence for Simple Features* (p. 0.2.5) [Dataset]. <https://doi.org/10.32614/CRAN.package.sfdep>
- Purwanto, P., Utaya, S., Handoyo, B., Bachri, S., Astuti, I. S., Utomo, K. S. B., & Aldianto, Y. E. (2021). Spatiotemporal Analysis of COVID-19 Spread with Emerging Hotspot Analysis and Space–Time Cube Models in East Java, Indonesia. *ISPRS International Journal of Geo-Information*, 10(3), 133. <https://doi.org/10.3390/ijgi10030133>
- Rey, S. J. (2015). Python Spatial Analysis Library (Pysal): An Update and Illustration. In pages 233–253, *Geocomputation: A Practical Primer* (First Edition). SAGE Publications, Inc. <https://doi.org/10.4135/9781473916432>
- Rey, S. J. (2016). Space–Time Patterns of Rank Concordance: Local Indicators of Mobility Association with Application to Spatial Income Inequality Dynamics. *Annals of the*

- American Association of Geographers*, 106(4), 788–803.
- <https://doi.org/10.1080/24694452.2016.1151336>
- Rey, S. J., & Anselin, L. (2007). PySAL: A Python Library of Spatial Analytical Methods. *The Review of Regional Studies*, 37(1), 5–27.
- Rey, S. J., Anselin, L., Amaral, P., Arribas-Bel, D., Cortes, R. X., Gaboardi, J. D., Kang, W., Knaap, E., Li, Z., Lumnitz, S., Oshan, T. M., Shao, H., & Wolf, L. J. (2022). The PySAL Ecosystem: Philosophy and Implementation. *Geographical Analysis*, 54(3), 467–487.
- <https://doi.org/10.1111/gean.12276>
- Rey, S. J., Arribas-Bel, D., & Wolf, L. J. (2023). *Geographic data science with Python*. CRC Press, Taylor & Francis Group. <https://geographicdata.science/book/intro.html>
- Rey, S. J., Murray, A. T., & Anselin, L. (2011). Visualizing regional income distribution dynamics. *Letters in Spatial and Resource Sciences*, 4(1), 81–90.
- <https://doi.org/10.1007/s12076-010-0048-2>
- Rogerson, P. (2024). The Effects of Weight Choices on the Power of the GETIS–ORD Statistic. *Geographical Analysis*, 56(1), 26–39. <https://doi.org/10.1111/gean.12361>
- Rogerson, P. A. (2022). Scan Statistics Adjusted for Global Spatial Autocorrelation. *Geographical Analysis*, 54(4), 739–751. <https://doi.org/10.1111/gean.12301>
- Sauer, J., Oshan, T., Rey, S., & Wolf, L. J. (2022). The Importance of Null Hypotheses: Understanding Differences in Local Moran’s under Heteroskedasticity. *Geographical Analysis*, 54(4), 752–768. <https://doi.org/10.1111/gean.12304>
- Sergio Rey, Wei Kang, Levi John Wolf, eli knaap, Hu Shao, Martin Fleischmann, Mridul Seth, James Gaboardi, Sugam Srivastava, privateAGI, Giovanni Palla, Dani Arribas-Bel,

- Qiusheng Wu, & stevee404. (2024). *pysal/pointpats: V2.5.1* (Version v2.5.1) [Computer software]. Zenodo. <https://doi.org/10.5281/ZENODO.13886730>
- Soler-Domínguez, J. L., Contero, M., & Alcañiz, M. (2019). Workflow and tools to track and visualize behavioural data from a Virtual Reality environment using a lightweight GIS. *SoftwareX*, 10, 100269. <https://doi.org/10.1016/j.softx.2019.100269>
- Su, R., Liu, Y., & Dodge, S. (n.d.). *ORTEGA v1.0: An open-source Python package for context-aware interaction analysis using movement data*.
- Sunnqvist, C., Rämgård, M., & Örmon, K. (2020). Time Geography, a Method in Psychiatric Nursing Care. *Issues in Mental Health Nursing*, 41(11), 1004–1010. <https://doi.org/10.1080/01612840.2020.1757795>
- Tang, W., & Bennett, D. A. (2010). The Explicit Representation of Context in Agent-Based Models of Complex Adaptive Spatial Systems. *Annals of the Association of American Geographers*, 100(5), Article 5. <https://doi.org/10.1080/00045608.2010.517739>
- Tao, R., & Chen, Y. (2022). Applying Local Indicators of Spatial Association to Analyze Longitudinal Data: The Absolute Perspective. *Geographical Analysis*, n/a(0), 1–14. <https://doi.org/10.1111/gean.12323>
- Tao, R., & Thill, J.-C. (2016). Spatial Cluster Detection in Spatial Flow Data. *Geographical Analysis*, 48(4), 355–372. <https://doi.org/10.1111/gean.12100>
- Tao, R., & Thill, J.-C. (2020). BiFlowLISA: Measuring spatial association for bivariate flow data. *Computers, Environment and Urban Systems*, 83, 101519. <https://doi.org/10.1016/j.compenvurbsys.2020.101519>
- Thrift, N., & Pred, A. (1981). Time-geography: A new beginning. *Progress in Human Geography*, 5(2), 277–286. <https://doi.org/10.1177/030913258100500209>

- Tukey, J. W. (1977). *Exploratory data analysis*. Addison-Wesley Pub. Co.
- U.S. Census Bureau. (2023). *American Community Survey 5-Year Estimates: Demographic Profile Product* [Dataset].
[https://api.census.gov/data/2023/acs/acs5/profile?get=group\(DP02\)&for=us:1](https://api.census.gov/data/2023/acs/acs5/profile?get=group(DP02)&for=us:1)
- US Census Bureau. (2024). *TL_2024_us_zcta520* [TIGER/Line Shapefiles].
<https://www.census.gov/cgi-bin/geo/shapefiles/index.php>
- van der Walt, S., Colbert, S. C., & Varoquaux, G. (2011). The NumPy Array: A Structure for Efficient Numerical Computation. *Computing in Science & Engineering*, 13(2), Article 2.
<https://doi.org/10.1109/MCSE.2011.37>
- Wang, A.-S., Yin, Z.-C., & Ying, S. (2024). Probabilistic Time Geographic Modeling Method Considering POI Semantics. *ISPRS International Journal of Geo-Information*, 13(1), 22.
<https://doi.org/10.3390/ijgi13010022>
- Wang, Z., & Lam, N. S. N. (2020a). Extending Getis–Ord Statistics to Account for Local Space–Time Autocorrelation in Spatial Panel Data. *The Professional Geographer*, 72(3), 411–420. <https://doi.org/10.1080/00330124.2019.1709215>
- Wang, Z., & Lam, N. S. N. (2020b). Extending Getis–Ord Statistics to Account for Local Space–Time Autocorrelation in Spatial Panel Data. *The Professional Geographer*, 72(3), 411–420. <https://doi.org/10.1080/00330124.2019.1709215>
- Waskom, M. L. (2021). seaborn: Statistical data visualization. *Journal of Open Source Software*, 6(60), 3021. <https://doi.org/10.21105/joss.03021>
- Winter, S., & Yin, Z.-C. (2010). Directed movements in probabilistic time geography. *International Journal of Geographical Information Science*, 24(9), Article 9.
<https://doi.org/10.1080/13658811003619150>

- Winter, S., & Yin, Z.-C. (2011). The elements of probabilistic time geography. *GeoInformatica*, 15(3), Article 3. <https://doi.org/10.1007/s10707-010-0108-1>
- Xu, B., Qi, B., Ji, K., Liu, Z., Deng, L., & Jiang, L. (2022). Emerging hot spot analysis and the spatial–temporal trends of NDVI in the Jing River Basin of China. *Environmental Earth Sciences*, 81(2), 55. <https://doi.org/10.1007/s12665-022-10175-5>
- Yin, Z., Chen, Y., & Ying, S. (2023). Construction and Analysis of Space–Time Paths for Moving Polygon Objects Based on Time Geography: A Case Study of Crime Events in the City of London. *ISPRS International Journal of Geo-Information*, 12(6), Article 6. <https://doi.org/10.3390/ijgi12060210>
- Yin, Z., Huang, W., Ying, S., Tang, P., Kang, Z., & Huang, K. (2021). Measuring of the COVID-19 Based on Time-Geography. *International Journal of Environmental Research and Public Health*, 18(19), 10313. <https://doi.org/10.3390/ijerph181910313>
- Yin, Z.-C., Wu, Y., Winter, S., Hu, L.-F., & Huang, J.-J. (2018). Random encounters in probabilistic time geography. *International Journal of Geographical Information Science*, 32(5), Article 5. <https://doi.org/10.1080/13658816.2018.1428748>

2.7 Appendix A: Moving LISA Pseudocode

THE MOVING LISA ($t_i \in \mathbf{T}$, $g_{i_{t_i}} \in \mathbf{G}_{t_i}$, $y_{i \dots n_{t_i}} \ni \mathbf{y}$, $m_{i \dots n_{t_i}} \ni \mathbf{m}$, w_{ij} , a , π , $\varphi \Rightarrow R_{m_{i,x_i}} \ni \mathbf{R}$)

Note sequence $\mathbf{T}$ is an input time-series and consists of t_i time steps each containing $\mathbf{G}$ geographies or observational units indexed $g_{i_{t_i}}$ for a specific observed location in space and time. User-specified parameters include a weights matrix w_{ij} and significance boundary a for use with the choice of LISA statistic, in addition to a user-specified contiguity threshold π [0,1] and standard distance multiplier φ . These user-specified parameters are held constant for all time steps for the present introduction. Intermediate objects include $y_{i \dots n_{t_i}} \ni \mathbf{y}$ clusters identified by the choice of LISA statistic, subject to a geometric union step yielding *movers* $m_{i \dots n_{t_i}} \ni \mathbf{m}$. The result $\mathbf{R}$ is a set of trajectories describing movers' progression in space and time having space-time anchors $R_{m_{i,x_i}} \ni \mathbf{R}$.

1 USER-SELECTED LISA: GETIS-ORD LOCAL G $(w_{ij}, a, g_{i...t_i} \Rightarrow g_{i...n_{t_i}} \text{ stat, sig})$

Referring to Equation 4, a cross-sectional Getis-Ord Local G is supplied the geographies of the current time step, along with a weights matrix and significance threshold. For the Local G, this demonstration isolates only high-high clustering locations (locations significant at a and positive for the Z-normalized result of the statistic). This Local G function in pseudocode accepts all geometries $g_{i...n_{t_i}}$ returns each individual input geometry $g_{i_{t_i}}$ with its associated z-normalized score (*stat*) and p-value (*sig*) for the statistic.

2 GEOMETRIC UNION AND DISSOLVE $(y_{i...n_{t_i}} \Rightarrow m_{i...n})$

// Referring to Equation 5, the clustering geographies for a given time step are subject to a geometric union and dissolving of their shared boundaries. This function in pseudocode accepts the entire set of clustering geographies identified at the current time step and applies the geometric union and dissolve steps to this set. Spatially disjoint groups of $y_{i...n_{t_i}}$ are each assigned unique $m_{i...n}$ identifiers.

3 CONTIGUITY THRESHOLD CHECK $(m_{i...n, t_{n-1}}, m_{i...n, t_n}, \pi, \phi, cov \Rightarrow m_{i, t_n} C)$

// Referring to Equation 6, movers from the current and prior time steps are compared for contiguity on the basis of meeting a threshold proportion of overlap where realizations from a bivariate normal distribution centered on $m_{i...n, t_{n-1}}$ are overlaid with the geometry of $m_{i...n, t_n}$. For illustration, this function in pseudocode is supplied the entire set of movers to show each iteration of the logic over the input set.

4 CONTIGUITY TYPE ASSIGNMENT $(m_{i...n, t_n} C \Rightarrow m_{i...n, t_n} C, F)$

// Referring to Equation 7, contiguous movers from the current and prior time steps are examined for their *contiguity type*, a status label referring to the divergence and convergence behavior of movers among these time steps. The pseudocode for this function uses convenience functions *count* and *count_unique* which return the number of members listed in a $m_{i...n, t_i} C$ associative array, and the number of unique identifiers in this array, respectively. Sometimes, repeat identifiers will appear in C indicating a situation where more than one mover at $m_{i...n, t_{n-1}}$ is contiguous with a mover from the focal time step. In these situations for example, the *convergence* contiguity type is suggested.

5 CONSTRUCT SPACE-TIME ANCHOR $(m_{i...n, t_n} C, F \Rightarrow R_{m_{i...n, x_i}} \ni R)$

// Referring to Equation 8, a well-known formulation for the centroid of a polygon is used to identify a space-time anchor for each available mover in the time step. This space-time anchor is included in the overall trajectory for the focal mover in the R result set of mover trajectories. This function in pseudocode simply returns the centroid coordinate for the mover's geometry based on the action of Equation X or an equivalent implementation.

```

6    $R = \{ \}, user\ arguments(T, w_{ij}, a, \pi, \varphi)$ 
   // Initialization, an empty result set and user-supplied arguments to the method.
6   for each time step  $t_i \in T$ 
7        $y = \{ \}$ 
       // Initializing an empty container for clustering geographies
8       for each observation  $g_{i,t_i} \in G$ 
9            $g_{i...n,t_i\ stat,sig} = GETIS - ORD\ LOCAL\ G(w_{ij}, a, g_{i...n,t_i})$ 
10          if  $g_{i...n,t_i\ stat} > 0$  and  $g_{i...n,t_i\ sig} < a$ 
11               $y \cup g_{i,t_i}$ 
              // The high-high (Local G) clustering geography is included in Y
12          else
13              pass
14           $m_{i...n,t_i} = GEOMETRIC\ UNION\ AND\ DISSOLVE(y_{i...n,t_i})$ 
          // Spatially disjoint groups of clustering geographies are returned as distinct  $m_{i,t_i}$ 
15          if  $t_i \neq 0$ 
16               $m_{i...n,t_i}C = CONTIGUITY\ THRESHOLD\ CHECK(m, \pi, \varphi) \gg$ 
17              for each mover  $m_{i,t_i}, m_{i,t_{i-1}} \in m$ 
18                   $m_{i,t_{i-1}}: C_{x,y} = Centroid(m_{i,t_{i-1}})$ 
19                   $m_{i,t_{i-1}}: Std = Std\_Distance(m_{i,t_{i-1}})^2$ 
20                   $a_x \sim norm(\mu = C_x, \sigma^2 = Std \times \varphi)$ 
21                   $a_y \sim norm(\mu = C_y, \sigma^2 = Std \times \varphi)$ 
22                  if  $P\left(\binom{a_x}{a_y} \cap m_{i,t_i}\right) \geq \pi$  // proportion of overlap meets threshold
23                       $m_{i...n,t_i}C \mid m_{i,t_{i-1}}$  is Contiguous (True)
24                  else
25                       $m_{i...n,t_i}C \mid m_{i,t_{i-1}}$  Is Not Contiguous (False)
          // add an array element noting the contiguity status
26           $m_{i...n,t_i}F = CONTIGUITY\ TYPE\ ASSIGNMENT(m) \gg$ 
27          for each mover  $m_{i,t_i} \in m$ 
28              if  $count(m_{i...n,t_i}C) = 0$ 
29                   $m_{i...n,t_i}F = Appearance$ 
30              else if  $count(m_{i...n,t_i}C) > 1$  and  $count\_unique(m_{i...n,t_i}C) > 1$ 
31                   $m_{i...n,t_i}F = Convergence$ 
32              else if  $count(m_{i...n,t_i}C) > 1$  and  $count\_unique(m_{i...n,t_i}C) = 1$ 

```

```

27          $m_{i...n,t_i}F = Divergence$ 
28     else if  $count(m_{i...n,t_i}C) = 1$  and  $count\_unique(m_{i...n,t_i}C) = 1$ 
29          $m_{i...n,t_i}F = Persistence$ 
30     else
31          $m_{i...n,t_i}F = Termination$ 
32 else
33      $m_{i...n,t_i}F = Appearance$ 
34     // On the initial time step, there are no comparable movers, assign appearance
35     for each mover  $m_{i,t_i}$ 
36          $R_{m_{i...n,x_i}} = CONSTRUCT\ SPACE - TIME\ ANCHOR(m_{i,t_i}) \gg$ 
37          $x_i = Centroid(m_{i,t_i})$ 
38          $R \cup R_{m_{i...n,x_i}}$ 
39 return  $R$  // The result set of nonpoint mover trajectories as sequences of space-time anchors

```

3 Chapter Three: Spatial Emergence and Socioeconomic Associations of Vape Shops in Oklahoma (2013-2023)

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Abstract

With the arrival of Electronic Nicotine Delivery Systems (ENDS) in US markets, the corresponding establishment pattern of specialty vape shops has raised questions surrounding public health equity and exposure to retail pressure. Having distinct marketing practices and a focus on the normalization of vaping, specialty vape store presence in Oklahoma is noteworthy for its rapid onset and density of these specialty retailers versus the US. We examined the development of specialty vape store retail presence in Oklahoma, US at 5-year intervals with end-years 2013, 2018 and 2023, in terms of their emergent spatial arrangement and socioeconomic covariates at the Zip Code Tabulation Area (ZCTA) level. We developed a longitudinal dataset of specialty vape shop establishment counts at ZCTAs in Oklahoma for the years 2013-2023. We used spatially autoregressive, generalized linear models (GLM) to suggest cross-sectional associations with expected specialty vape store counts at each end-year. In context with these model results, we applied the Moving LISA, a recent approach enabling the construction of movement trajectories from spatial panel data as a visualization of the progression of specialty vape retail in Oklahoma. Results were largely consistent with prevailing understandings of specialty vape store establishment patterns in both Oklahoma and the US at large; areas hosting young people, people with minority status and socioeconomic disadvantage continue to correlate with increased expected counts of specialty vape shops. From literature, this is the first study examining specialty vape shop presence spatially over time in Oklahoma and the first to employ the Moving LISA for this purpose. Findings could inform policy discussion on licensing, quotas or limits for specialty vape retail density with respect to ongoing apparent targeting of youth and disadvantaged populations.

Keywords: Vaping, ENDS Retail, Trajectory, Time Geography, Public Health.

3.1 Introduction

Increasing prevalence and normalization of vaping and e-cigarette use in the United States (US) represents a complex and evolving public health concern, especially in terms of use, availability, awareness and initiation in youth populations (Birdsey et al., 2023; Dai et al., 2017; DiCasmirro et al., 2025). In the US, vape or e-cigarettes represent the most used nicotine-containing tobacco product among youth, where 5.9% of middle and high school students reported current use of e-cigarettes in 2024 (Jamal et al., 2024; Park-Lee et al., 2024). Despite a vast and successful history of public health intervention addressing other forms of commercial tobacco use for both youth and adults, studies specific to vaping as a function of presence and access suggest e-cigarette markets and vaping behavior present unique characteristics which often diverge from those observed in prior work on commercial tobacco control (Dai et al., 2017; Levy et al., 2021; Venugopal et al., 2022). This has demanded a corresponding departure in research on vaping originating from strategies developed in prior tobacco control literature. New and creative questions examining vaping in terms of its spatially uneven market presence, public conceptions of vaping as harm-reducing or as a smoking cessation aid, gaps in regulation and the clinical discovery of the fatal disease *E-Cigarette or Vaping Product Use-Associated Lung Injury* (EVALI) all further indicate that vaping presents a unique and complex public health issue (Belok et al., 2020; Kalininskiy et al., 2019). Additionally, while vaping is associated with pulmonary and cardiac complications similar to combustible tobacco, research examining e-cigarette use as a harm-reducing alternative to smoking or as a cessation tool raises the need for nuanced messaging and greater understanding in the development of the Electronic Nicotine Delivery Systems (ENDS)-control discourse in public health (Berg et al., 2020; Bullen et al., 2013; Kong et al., 2017; J. G. L. Lee et al., 2018; Levy et al., 2021).

With the arrival of vaping in the US circa 2007, a distinct class of specialized vapor product retailer has emerged, referred to as the “Specialty Vape Shop” (Giovenco, 2018; Kim et al., 2016; J. G. L. Lee et al., 2018; Y. O. Lee & Kim, 2015). Specialty vape shops are dedicated retailers of ENDS, e-cigarettes and allied products (such as e-liquids, system modifications, etc.) having unique marketing characteristics different from alternative vape product retailers including convenience stores and pharmacies. The recent proliferation of vape shops in the US has been difficult to quantify, as existing regulatory frameworks do not generally cover their activities. No comprehensive listing or roster of registered vapor product retailers is currently available nationally. Additionally, varied regulatory responses at the State level have further resulted in incomplete or inconsistent measures of vape shop prevalence across all 50 United States. Some estimates place the total number of these vapor retail outlets as high as 35,000 establishments operating nationwide (Berg, 2018). The unique business strategies of specialty vape shops are shown to include marketing vaping as a safer alternative to combustible tobacco products in-person and in social media, normalizing vaping by providing social settings for vaping and product sampling (the *vapor lounge*) and in some cases seeking to promote vaping by assuming a messaging stance opposed to combustible tobacco altogether (Y. O. Lee & Kim, 2015). It is further demonstrated in prior study that specialty vape shops have located in and marketed to areas of lower socioeconomic status (Huang et al., 2016) and seek to target their marketing strategies to college students (M. Cheney et al., 2015).

With the rapid emergence of specialty vape shops, accounting for potential health equity concerns related to their locations and their role in normalizing and initiation for vaping behaviors presents an understudied question in ENDS-control literature. This question is inherently spatial; the locations of specialty vape shops may encode their associations with

socioeconomic conditions indicative of prevailing marketing and business practices. The understudied status of this question is in part attributable to how quickly the specialty vape shop business model has established itself in the US. Where available, prior research on vape shop locations provide for analysis and characterization of specialty vape shop establishment patterns at various study areas and scales (Berg et al., 2020; Dai et al., 2017). While this literature on vape shop establishment patterns asserts that exposures to public health-harming effects of retail pressure are uneven across space and disproportionately assigned to areas with greater socioeconomic disadvantage, it has also identified certain divergences or even reversals away from the corresponding spatial character of commercial tobacco retail presence (Dai et al., 2017; Kong & Henriksen, 2022). It is evident that not only is the specialty vape shop a distinctive and understudied retail establishment pattern subject to examination for public health; it also represents a divergence in marketing strategy compared to other commercial tobacco retailers.

In Oklahoma, the arrival of specialty vape store establishments has been both rapid and uniquely prevalent. Compared with the market entry and establishment histories of specialty vape stores elsewhere in the US, Oklahoma has emerged as a market anomaly; it has proven to be a surprisingly successful location for the development of a specialty vapor retail market. In 2014, Oklahoma held an estimated 6 percent of the 5000 vape shops operating nationally, in a state where only 1 percent of the US population resides (Richtel, 2014). In the same year, it was shown 18 percent of Oklahoma high school students had initiated vaping behavior, and analyses of marketing practices raised concerns where vape shops were engaging in strategies which were correspondingly restricted for tobacco retailers (M. Cheney et al., 2015; Richtel, 2014).

One explanation for Oklahoma's proliferation of vapor retail presence is suggested by a high existing tobacco user status in the state, providing a potential customer base for vaping.

Oklahoma has held a relatively high population of smokers versus other US states. In 2023, it was found 15.8% of Oklahoma adults were currently combustible tobacco smokers in combination with 11.3% of adults using E-cigarettes (Centers for Disease Control and Prevention et al., 2023). Per the 2023 US CDC's Behavioral Risk Factor Surveillance System survey, Oklahoma has the highest rate of adult current E-cigarette use in the US. Mechanistically, this population drives demand from both dual-use consumers of combustible tobacco and vapor products as well as demand from commercial tobacco users seeking to quit, since ENDS are represented, and in some contexts marketed, as a cessation tool (Berg et al., 2015; Bullen et al., 2013). In Oklahoma, high use of vapor products, rapid development of specialty vapor retail and a declining but serious youth initiation rate in vaping behavior all combine to demand greater understanding in the spatial pattern and process of vapor market presence in the state. Oklahoma is objectively, particularly susceptible to the known and unknown harms associated with vaping.

Examining the emergence of specialty vape shops across the state as a spatial pattern offers a unique perspective toward understanding the process of their retail establishment. Allying with strategies developed in the tobacco control literature, the present research cross-sectionally examines change over time in the expected count of specialty vape shops at the Zip Code Tabulation Area (ZCTA) level in Oklahoma, in context with a range of socioeconomic covariates. This approach is applied for three US Census Bureau American Community Survey 5-year (ACS 5-Year) estimate periods, ending in 2013, 2018 and 2023, respectively. Further, this research applies the Moving LISA (Anderson & Yang, 2025), a novel method visualizing the progression of Oklahoma zip codes having extreme vape shop prevalence over space and time to further identify spatiotemporal patterning in specialty vape store establishment. To our review of literature, the present research is the first to employ exploratory spatial data analysis (ESDA)

methods longitudinally to reconstruct the market encroachment of specialty vapor retail in any US State. Further, this study is the first to longitudinally examine change in a cross-sectional model of expected vape store counts.

3.2 *Methods and Materials*

3.2.1 *Data*

The study area includes all US Census Bureau Zip Code Tabulation Areas having their geographic centroid within the US state boundary of Oklahoma. This collection of ZCTAs do not fully coincide with the state boundary of Oklahoma and often cover some areas in neighboring states. Therefore, it is possible that covariates collected for these bordering areas represent conditions from more than one US State, within a given zip code. The study period captures the years 2008-2023, with relevant sociodemographic variables and vape store business establishment events collected at three end-year labeled intervals each having a five-year time span.

3.2.1.1 *Vape Shop Business Data*

We queried the *AtoZ Academic Business and Consumer Data* database (AtoZ) for all businesses having North American Industrial Classification System (NAICS) code of 459991 (Tobacco, Electronic Cigarette, and Other Smoking Supplies Retailers) and a physical address in the State of Oklahoma. This search returned an initial set of (n=596) business locations (as street addresses and WGS 1984 coordinates) with associated business name and year established. Searching the *AtoZ* database for NAICS 459991 together with the more ENDS-specific SIC Code of 5993009 (Electronic Cigarette Stores) returned a proper subset of the businesses

carrying NAICS 459991 in this database. Noting omissions in the data for the SIC 5993009 attribute, where known specialty vape shops operating in the Norman, Oklahoma area were not shown or represented, we opted to retain the initial set of business locations (n=596) for further filtration.

Consequently, within this larger set of NAICS 459991 businesses, we found that establishments which could plausibly operate as Specialty Vape Shops may lack the appropriate SIC code label. A filtration of the initial set (n=596) of retail businesses was necessary. Following business name search term and business type recommendations from (Kim et al., 2016) business types including Pharmacies, Convenience Stores, Tobacco Stores & Stands, Smoke Shops, Tobacco Wholesalers, Hookah Bars and Medical Marijuana Dispensaries were removed from the initial set. Following filtration a random sample (n=30) of the remaining businesses (n=308) were evaluated to confirm their status as specialty vape shops with a review of storefront images public-facing web presence in context with literature describing specialty vape store marketing characteristics (Berg et al., 2020; M. Cheney et al., 2015; M. K. Cheney et al., 2016; Huang et al., 2016). The remaining businesses (n=308) were stratified by the ACS 5-year span covering their year of establishment and were then overlaid with corresponding sets of ZCTA geometries (decennial census 2010 or 2020) yielding a count of vape shops by ZCTA by ACS 5-year span. It is assumed that a business present in the *AtoZ* search return is still operating at the time the search was conducted. This assumption is supported by the *AtoZ* database product documentation, where *AtoZ* database maintenance efforts seek to remove defunct businesses from the dataset.

The specialty vape shop locations dataset developed for this study showed a sample variance (of count per ZCTA) exceeding the average and had a high relative presence of zeros,

where zeros represent ZCTAs hosting zero vape shops within their boundaries. In the data collected, all three study periods (ACS 5-Year end-years of 2013, 2018, 2023, respectively) showed overdispersion in specialty vape store counts at Oklahoma ZCTAs (Table 1). Additionally, these data show positive and significant spatial autocorrelation for each of the three study periods, per results of the Global Moran's *I* test for spatial autocorrelation (Table 2). Spatial autocorrelation as identified here conceptualizes adjacency (also called *Queen's adjacency*, from the game of chess) between locations as equally influential among all first-order neighbors of a focal location, where those neighbors share a boundary or vertex in terms of their geometry (Anselin, 1996; Moraga, 2020; Pebesma & Bivand, 2023).

Variance and Mean of ZCTA Vape Store Counts by ACS5 End-Year

<i>ACS5 End-Year</i>	<i>Variance</i>	<i>Mean</i>
2013	0.05459	0.05093
2018	0.403	0.22994
2023	1.39477	0.45633

Table 1. Variances and means of ZCTA vape store counts for each study period, counts are overdispersed at each ACS 5 end-year considered.

3.2.1.2 US Census Bureau TIGER ZCTA Geometries

The assignment of US Zip Codes is a function of the United States Postal Service (USPS) meant to improve and facilitate mail delivery. USPS Zip Codes represent the service areas of postal offices in terms of surrounding street networks; their boundaries are not directly a function of

population density, or natural or artificial features of the landscape. The US Census Bureau TIGER ZCTA product is an estimate of the areal geometries of USPS Zip Code networks. Since our approach is longitudinal, consideration of ZCTAs for their change in terms of labeling and geographic shape over time is needed.

During the time elapsed between US Census Bureau Decennial Census releases, the configuration, labeling, and number of ZCTAs in the study area is subject to change. For example, a particular spatial location may fall within a given ZCTA 5-digit label in Census year 2010 and could fall within a different number in year 2020. Alternatively, a location may receive a constant ZCTA 5-digit label over 2010-2020, but the geometry or shape of that ZCTA may change.

For socioeconomic variables retrieved for ACS 5-Year end years 2013 and 2018, the TIGER ZCTA 2019 data release was used to provide spatial representation. Here, ACS 5-Year end-year 2013 and 2018 sociodemographic data were assigned to TIGER ZCTAs from 2019. The 2019 TIGER ZCTA data release is an updated set derived from the 2010 US Decennial Census. For socioeconomic data from the ACS 5-Year period ending in 2023, the TIGER ZCTA 2024 release was used. The 2024 TIGER ZCTA data release is derived from the 2020 US Decennial Census.

3.2.1.3 American Community Survey 5-Year Variables

In total, 24 US ACS 5-year variables were initially retrieved by their respective Demographic Profile product variable names at the Oklahoma ZCTA level for ACS 5-year end-years 2013, 2018 and 2023. These variables were selected in part for their appearance and examination in prior work associating them with the presence of vape shops (Berg et al., 2020; Dai et al., 2017; Kong et al., 2017). Due to the potential for changes in the thematic meaning and calculation

approach for ACS estimates between end-years, only those variables having a consistent measurement and name across all study end-years were retained. For example, new variables reporting for any new concept at end-year 2023, when that variable was not available in 2013 and 2018, respectively, were not included in this study. The selected ACS 5-year variables capture socioeconomic themes including educational attainment, employment, age, housing characteristics, income, poverty, race and ethnicity. For a correspondence table of this initial set of ACS 5 Demographic Profile variable names by end-year, see Appendix, Table 1. Of the 24 variables collected, a subset of 21 variables was retained following checks for multicollinearity using Variance Inflation Factor (VIF). The VIF relates the presence of multicollinearity for a single predictor, among the overall set of predictors, by returning a value for this predictor's variance as explained by the other predictors in an Ordinary Least Squares regression (Marquardt, 1970; Salmerón et al., 2020). This is repeated for all predictors under study. Variables which were dropped from consideration are shown in **bold text** in Appendix, Table 1.

“Percent of Female”, “Percent of White One Race” and “Percent of American Indian and White Two Races” were all removed from consideration due to high VIF values. Following removal, all remaining VIF values for all study years were less than an acceptable reference value of 5.0 (Marcoulides & Raykov, 2019). We note further given definitions of these variables in the ACS this result is to be expected, where for example “Percent of Female” as a label for the birth sex of a respondent was highly negatively correlated to “Percent of Male” and carried overlapping information. “Percent of White” was similarly negatively correlated with “Percent of Black” potentially due to ongoing patterns of residential segregation in the US, and “Percent of American Indian and White Two Races” was removed to improve clarity, as this variable

overlaps with concepts captured by others already selected for this research (US Census Bureau, 2023).

3.2.2 *The Moving LISA Procedure*

Mapping the spatial progression of zip codes hosting extreme counts of specialty vape shops in Oklahoma may aid in the identification of any spatiotemporal patterning in vape shop establishment. This mapping may serve as both a visual summary of the encroachment of specialty vape shops and as an accessible departure point for policy discussion. As an ESDA approach borrowing ideas from both Time Geography and Local Indicators of Spatial Association (LISAs), the Moving LISA (Anderson & Yang, 2025) seeks to identify and track the growth, reduction and displacements of persistent local clusters in a single variable over time in the study area. From spatial panel data recorded at aggregate spatial units, the Moving LISA returns a set of trajectories representing the respective traversals or spatiotemporal progression of persistent clusters across the study area and period. Trajectories are a unifying data structure and theoretical construct in Time Geographic analysis, they formalize and relate a metric capture of movement for further examination (Miller, 2005, 2017).

The Moving LISA trajectory is produced as a function of both the user's choice and parameterization of a cross-sectional LISA statistic, for the present research the Getis-Ord Local G (Getis & Ord, 1992; Ord & Getis, 1995), combined with a probabilistic overlay strategy providing for an entity-linking of clusters found at times t_n and t_{n+1} respectively. At each time step, the overlay technique will establish, confirm (as a cluster persisting into t_{n+1}) or dissolve the identity value for a given cluster as an *aggregate mover*; a discrete and durable time-geographic entity derived from non-point, invisible or otherwise spatially indistinct observation. The overlay strategy is further parameterized in terms of a minimum probability mass-overlap

threshold needed to consider clusters as *linked*, or representing the same durable entity from one timestep to the next. As part of this overlay strategy, a user-defined multiplier attenuates behavior by increasing or decreasing the variance of a bivariate normal distribution centered on each respective cluster found at t_n , where the base variance before applying this multiplier is the standard distance of the cluster's geometry (Anderson & Yang, 2025). Over the course of an input time-series, at each time step *aggregate movers* may diverge (result in more than one mover), converge (geometries of two or more movers merge to share a boundary), persist (may have moved, but have not interacted with other movers) or terminate (the mover fails to maintain significance since last time step) as determined by a comparison of mover geometries and significance. Resulting Moving LISA trajectories are constructed from *space-time-anchors* (nodes) generated using centrography measures returning a representative point location for a given cluster's overall shape at each time step. The series of space time anchors relating the spatiotemporal locations of a cluster are connected with straight-line *space-time-paths* (edges) to construct the overall trajectory dataset. The trajectory dataset returned by Moving LISA is thereafter compatible with trajectory summary and comparison measures from Time Geography. Further specifics surrounding the Moving LISA procedure can be found in Anderson & Yang (2025).

3.2.3 Modeling of Expected Vape Store Counts at ZCTAs

Exploring the associations among observed counts of specialty vape shops at Oklahoma ZCTAs and their corresponding socioeconomic characteristics may reveal new information about patterns of establishment and persistence of local clusters. As a set of observations potentially capturing the character of the specialty vape shop establishment process in Oklahoma, the data examined in this research are comprised of counts of businesses found to be plausibly operating

as specialty vape shops at the ZCTA level along with relevant socioeconomic covariates (Dai et al., 2017; Kim et al., 2016).

3.2.3.1 Global Moran's I test for Spatial Autocorrelation

There exists a natural tendency for similar observations to appear near to one another in space (Tobler, 1970). Materializing among the observations in a sample, spatial autocorrelation indicates this spatial non-randomness could be at work in the data generating process. Whether the study area is spatially heterogeneous in its production of realizations within its boundaries, or if those realizations are spatially dependent upon one another for their character, spatial autocorrelation will appear in the sample (Anselin, 1988; Cliff et al., 1981; Cliff & Ord, 1973). This structure in the spatial arrangement of observations complicates the interpretation of models expecting independently and identically distributed input data (IID). Unaddressed spatial autocorrelation is a confound, it is shown to reduce the variance of modeled explanatory variables (marginal posterior distributions), falsely inflating the significance of effects incorporated into IID models (Fortin & Dale, 2005; Legendre & Fortin, 1989; Mets et al., 2017).

To identify spatial autocorrelation in the pattern of vape shop counts at ZCTA level, we employed the Global Moran's *I* test, which is an established measure for its identification (Anselin, 1996; Cliff et al., 1981; Moran, 1948, 1950). For a single variable, the statistic weights the product of deviations from the mean in all paired locations by a matrix capturing what is known about the spatial structure of the study area and compares the result to the total deviation of all locations. A positive result for the statistic relates to positive spatial autocorrelation or the nonrandom co-occurrence of like values in the study area. A negative result for the statistic suggests a repulsion among like values. Inference about the statistic is obtained, usually, through

a conditional randomization of the input values yielding a sampling distribution of the statistic (Anselin et al., 2006, 2010). The expression in Equation 1 follows (Yang et al., 2023).

$$I = \frac{n \sum_i \sum_j w_{ij} (y_i - \bar{y})(y_j - \bar{y})}{W \sum_i (y_i - \bar{y})^2} \quad (1)$$

Where:

I is the Moran's I statistic result, indicating positive spatial autocorrelation if I is positive, and vice versa.

n is the total number of observations.

w_{ij} is the weights matrix where each element suggests the influence on the variable of interest between geographies i and j .

$W = \sum_i \sum_j w_{ij}$ is the sum of all weights.

$(y_i - \bar{y})(y_j - \bar{y})$ is the product of deviations from the sample mean at locations i and j .

<i>ACS5 End-Year</i>	<i>Moran's I Statistic</i>	<i>p</i>
2013	0.0910	8.508e-05
2018	0.14278	3.172e-09
2023	0.2072	2.2e-16

Table 2. Moran's *I* result for vape store patterns at Oklahoma ZCTAs for each study period. Weak positive and significant spatial autocorrelation was present in each input data subset.

Given that the Moran's *I* statistics reported in Table 2 are positive with p-values less than 0.001, the Vape Store patterns at Oklahoma ZCTAs shows significantly positive spatial autocorrelation. Spatial autocorrelation embedded in the data can significantly bias statistical inferences. Consequently, a spatial model is required to accurately represent the factors influencing vape store incidence.

3.2.3.2 *Besag-York-Mollié (BYM) Modeling and its Extensions*

To account for spatial effects, we selected the Besag-York-Mollié 2 (BYM2) approach to further identify both spatial and aspatial associations on specialty vape store counts, modeled as informing a negative binomial (NB) response variable (Riebler et al., 2016; Simpson et al., 2017). A model construction having a Poisson distributed response was also tested.

The BYM2 model is a reformulation of a popular hierarchical Bayesian approach capable of distinguishing between the effects of spatial neighborhood structures and unstructured spatial noise in concert with otherwise aspatial or independently distributed covariates. An Intrinsic

Conditional Autoregressive (ICAR) prior distribution, which is a special type of multivariate normal distribution that incorporates notions of adjacency and influence among spatial observations, is constructed for each observation (Moraga, 2020). The joint distribution of the ICARs found in this lowest level of the hierarchy is usually reported in the result as the fit of the spatially structured component of the model (Blangiardo & Cameletti, 2015; Riebler et al., 2016). The BYM2 often appears in literature as parcel to generalized linear models seeking to identify the associations of both spatial and independently distributed influences on the posterior distribution of a single dependent variable (Riebler et al., 2016; Simpson et al., 2017). The BYM and its derivatives has been used in studies from spatial epidemiology, bicycle-automobile crash risk assessment (Chen, 2015) and examinations of electric vehicle adoption rates (Yang et al., 2023).

Departing from the initial formulation introduced by Besag et al. (Besag et al., 1991) where techniques from image restoration were redirected toward archaeological and disease risk spatial smoothing questions, extensions to the method have addressed limitations surrounding separability and interpretation in the relative contributions of BYM's spatially structured and spatially unstructured components to the overall spatial effect (Dean et al., 2001; Leroux et al., 2000). The prevailing *BYM2* formulation (Riebler et al., 2016; Simpson et al., 2017) is novel in that it incorporates parameters for scaling and mixing of the spatial effects, enabling a comparability in the returned size of effect for model comparison where adjacency structures have changed (Freni-Sterrantino et al., 2018; Moraga, 2020). The BYM2 notation below is adapted from (Yang et al., 2023) and (Moraga, 2020).

$$y_i \sim \text{NegativeBinomial}(p_i, r) \tag{2}$$

$$p_i = \frac{\exp(\eta_i)}{1 + \exp(\eta_i)} \quad (3)$$

$$\log(\eta_i) = a + \beta \times X_i + b_i \quad (4)$$

$$b_i = \frac{1}{\sqrt{\tau_b}} (\sqrt{1 - \phi} u_* + \sqrt{\phi} v_*) \quad (5)$$

$$u_i | u_{\neq i} \sim \text{Norm} \left(\frac{\sum_{j \in N(i)} u_j}{\#N(i)}, \frac{\tau_u^{-1}}{\#N(i)} \right) \quad (6)$$

$$v_i \stackrel{iid}{\sim} \text{Norm}(0, \tau_v^{-1}) \quad (7)$$

Where:

i is the i^{th} ZCTA in Oklahoma, y_i is the count of specialty vape stores at the i^{th} ZCTA, p_i is the probability parameter of the negative binomial distribution function (2) seeking count of r vape shops, calculated from the log-link (3) of the η_i linear predictor. For the (4) linear predictor η_i , a is the intercept, $\beta \times X_i$ represents scaling the set of X_i covariates for a location by its corresponding regression coefficient from the vector β . For the (5, 6, 7) spatial components, b_i represents the influence of the ICAR and independent spatial effects, which are proportioned by a mixing parameter (ϕ) and have precisions of τ_b , τ_u , and τ_v respectively. Regression coefficients, precisions, scaling and mixing parameters are all estimated during model approximation or by a sampling method.

For the present research, following multicollinearity checks examining each covariate's VIF on the overall set of ACS 5-year variables for the given study period, we fit a full Besag-York-Mollié 2 model with all remaining variables to explore the effects of socioeconomic covariates on vape shop incidence at the ZCTA level. The NB distribution was chosen due to overdispersion in the input data. An alternative construction of this model having a Poisson-distributed response was also tested. The models were run using the R-INLA software package (Rue et al., 2009), a widely used library for approximating Bayesian models with support for examining spatial dependence (Bivand, 2022; Pebesma & Bivand, 2023). As a departure in strategy from the intensive sampling approaches associated with Markov Chain Monte Carlo (MCMC), the Integrated Nested Laplace Approximation (INLA) allows for a fast, deterministic approximation of the marginal posterior distributions for individual parameters for the class of

latent Gaussian models. In part to enable INLA’s gains in speed, the model is constructed as a Gaussian Markov Random Field (GMRF) where the set of latent or unknown quantities (the parameters and hyperparameters) are estimated as conditionally independent of each y_i , and then the joint distribution of y given those estimated quantities is reported for the result (Blangiardo & Cameletti, 2015; Riebler et al., 2016; Rue et al., 2009). In latent Gaussian models, the linear predictor (Equation 3) is constructed not only from a set of coefficients estimating the influence of the covariates on the response, but with an additional set of functions of these covariates (b_i , Equation 3). This component (Equation 4, for BYM2 spatial effects), allowing use of functions of the covariates in the linear predictor, provides flexibility to capture a range of relationships among the input data. These may be functions of the input data accounting for known trends, seasonal effects, or in our case, spatial effects (Blangiardo & Cameletti, 2015). In the BYM2, Equation 5 expresses a function where neighborhood-structured spatial dependence (Equation 6), along with spatially random noise (Equation 7) are considered for each observation.

3.3 Results

3.3.1 Expected Vape Store Counts at ZCTAs

The results for BYM2 regression models for each of ACS 5 end-years are reflected in Tables 3, 4 and 5, respectively. In these tables, only those covariates which were found significant in their effect on the target parameter are listed, to save space. In these results, the variable “Percent Households with SNAP Benefits Last 12 Months” captures the percent of households receiving food-purchasing assistance through the US Supplemental Nutrition Assistance Program (SNAP). In the US, SNAP is administered by the US Department of Agriculture and distributed in our

study area by the Oklahoma Department of Human Services (OKDHS). For reference, the overall set of ACS 5-year variables considered for all models appears in Appendix, Table 1.

BYM2 Significant Effects for End-Year 2013							
	mean	sd	0.025quant	0.5quant	0.975quant	mode	kld
Percent Households with SNAP Benefits Last 12 Months	0.169	0.127	0.017	0.128	0.481	0.111	1.412e-05
Percent of Vacant Housing Units	-0.188	0.072	-0.370	-0.186	-0.043	-0.188	1.094e-06
Percent of Male	-0.190	0.147	-0.571	-0.142	-0.020	-0.123	2.302e-05
Phi for region id	0.333	0.275	0.009	0.257	0.916	0.012	N/A

Table 3. BYM2 Results for ACS End-Year 2013

BYM2 Significant Effects for End-Year 2018							
	mean	sd	0.025quant	0.5quant	0.975quant	mode	kld
Percent Population in College or Graduate School	0.037	0.015	0.007	0.037	0.066	0.037	8.462e-09
Percent Households with SNAP Benefits Last 12 Months	0.065	0.028	0.010	0.065	0.120	0.065	3.074e-09
Percent of Vacant Housing Units	-0.105	0.018	-0.141	-0.105	-0.069	-0.105	2.358e-09
Percent 25-34 Years	0.079	0.036	0.008	0.079	0.150	0.079	3.975e-09
Percent of Latino Any Race	0.029	0.014	0.001	0.029	0.056	0.029	8.983e-09
Phi for region id	0.416	0.258	0.029	0.390	0.901	0.081	N/A

Table 4. BYM2 Results for ACS End-Year 2018

BYM2 Significant Effects for End-Year 2023							
	mean	sd	0.025quant	0.5quant	0.975quant	mode	kld
Percent Bachelors Degree 25 and Older	0.056	0.021	0.015	0.056	0.098	0.056	4.096 e-09
Percent Households with SNAP Benefits Last 12 Months	0.076	0.023	0.031	0.076	0.120	0.076	1.867 e-09
Percent of Vacant Housing Units	-0.097	0.016	-0.128	-0.097	-0.066	-0.097	3.775 e-09
Percent 25-34 Years	0.068	0.031	0.008	0.067	0.129	0.067	2.439 e-08
Percent 35-44 Years	0.073	0.033	0.009	0.073	0.138	0.073	5.184 e-09
Percent of Black One Race	-0.033	0.016	-0.065	-0.033	-0.001	-0.033	2.69934677230003 e-09
Percent of Latino Any Race	0.036	0.011	0.014	0.036	0.057	0.036	1.68526739346343 e-08
Phi for region id	0.364	0.283	0.011	0.298	0.934	0.016	N/A

Table 5. BYM2 Results for ACS End-Year 2023

For all three ACS 5 end-years considered (2013, 2018, 2023), significant and positive association between the “Percent of Households With SNAP Benefits Last 12 Months” indicates a tendency for the co-location of specialty vape stores in ZCTAs having more families requiring

food assistance, a known indication of socioeconomic disadvantage. Young people (25-34 years) and Latino populations positively and significantly associated with specialty vape store counts in end-years 2018 and 2023, commensurate with findings from literature (Dai et al., 2017; Venugopal et al., 2022). In addition to corroborating the direction and significance of covariate associations as identified in previous studies, deliberate attention to spatial structure in this study reveals further associations for each ACS 5 end-year's pattern of specialty vape shop appearance. First, the contribution of the ϕ (phi) parameter accounting for the neighborhood structured, or conditionally autoregressive spatial effect in the model is shown to positive and significant for all time periods. This is both confirmation in the need for a spatially aware modeling approach and an indication of some agglomeration effect in the siting of specialty vape stores. In Oklahoma, specialty vape stores appear to co-locate, where examined for the presence of neighboring vape stores in their first-order neighboring ZCTAs.

3.3.2 *Moving LISA Trajectories for Observed Vape Store Counts*

For observed counts of operating vape shops at Oklahoma ZCTAs for the three US Census Bureau ACS 5-year periods (the timesteps) available, a parameter sweep approach was employed to examine responses for the *contiguity cutoff* and *standard distance scaling factor* parameters of the Moving LISA method. This approach involves two stages. First, the *standard distance scaling factor* was held constant at $\phi = 2.0$, and *contiguity cutoff* values were evaluated over the interval of $\pi = [0.1, 1.0]$. Next, the *contiguity cutoff* value indicated as an inflection point among the response in terms of counts of cluster *contiguities* was selected and held constant for the subsequent parameter sweep of *standard distance scaling factor* values.

Per the original formulation of the Moving LISA, *contiguity* among clustering geographies at future time steps was determined as a function of boundary overlap, where a

bivariate normal distribution is centered on each clustering geography at the focal time step and a proportion of overlap is calculated among clusters present at the next time step (Anderson & Yang, 2025). Should the proportion of probability mass meet the *contiguity cutoff*, then respective clusters found at the focal and next time steps are considered *contiguous*, or acting as a persistent, moving spatial entity. Contiguity tests involved 1000 draws from this distribution at each time step, with the distribution having a variance equal to the standard distance of the focal cluster's geography multiplied by the *standard distance scaling* factor. From these results (Figures 1 and 2) specific parameter choices were selected for further visualization (Figures 3, 4 and 5). Evaluating the parameter sweep result, parameters selected for the visualization were a *contiguity cutoff* value of $\pi = 0.2$ and a *standard distance scaling factor* value of $\phi = 1.1$. This approach to parameter selection subjectively produces a kind of balance in visualization, where significant clusters at each time step can be effectively brought into focus and given visual contiguity (Anderson & Yang, 2025). We note that our Moving LISA for the present research utilized the Getis-Ord Local G statistic as the user's choice of cross-sectional LISA at each timestep, itself having a constant significance boundary or *alpha* of $a = 0.05$.

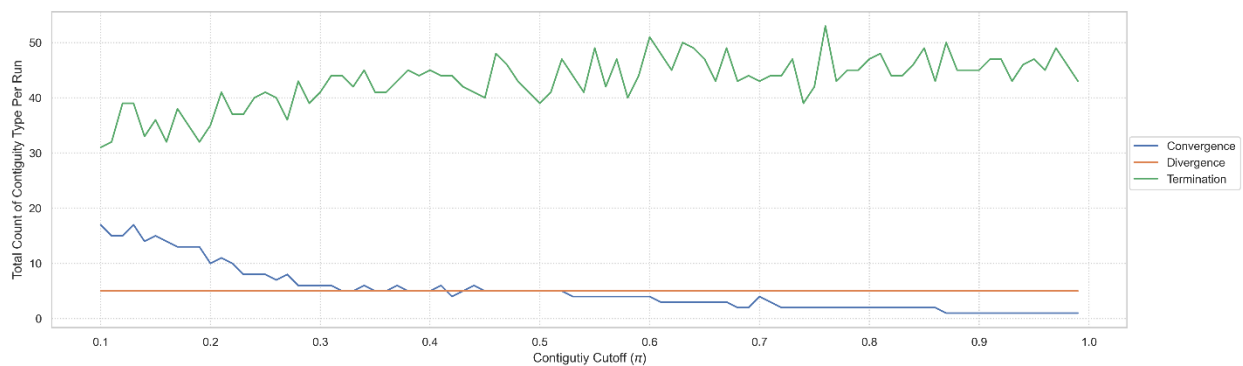


Figure 1. Parameter sweep results for *contiguity cutoff* selection in the Moving LISA procedure.

Note the departure among counts of terminations and convergences beginning at the selected value of $\pi = 0.2$. Each *contiguity cutoff* value (X-axis) reports its corresponding total count of

mover *contiguities* by type (Y-axis) for each Moving LISA run comprising the parameter sweep. Convergences, divergences, and terminations represent a subset of potential outcome states for Moving LISA *aggregate movers* at each time step.

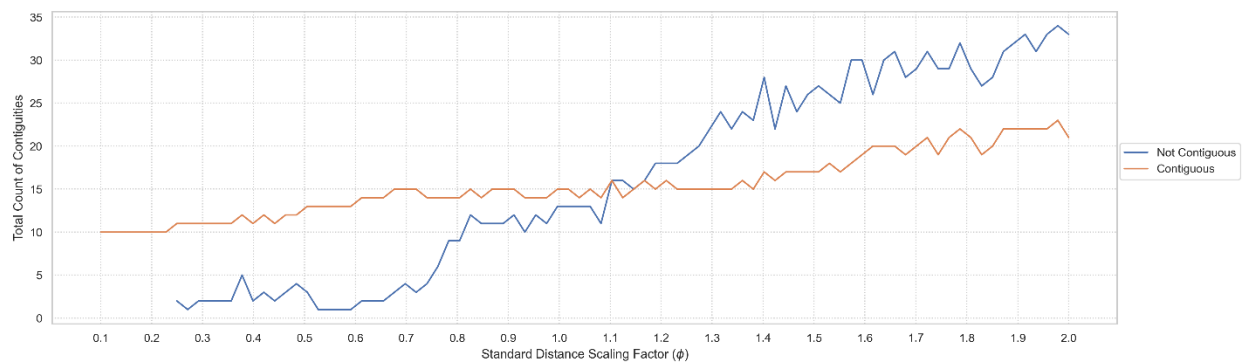


Figure 2. Parameter sweep results for *standard distance scaling factor* in the Moving LISA procedure. Note the inflection of contiguity counts at $\phi = 1.1$, the value selected for further Moving LISA visualization. All convergences, divergences, and persistence of movers between time steps are counted as “Contiguous”. Here, “Not Contiguous” are counts of mover terminations.

The visualization in Figure 3 shows the spatiotemporal progression of specialty vape store establishment in Oklahoma from 2013-2023, where lines represent the *trajectories* of persistent and distinctive clusters of ZCTAs having extreme counts of vape store establishment. A two-dimensional view of the spatial arrangement of Moving LISA clusters at each timestep is available in Figure 4. Figure 5 shows the resulting Moving LISA trajectories without their corresponding clustering geography boundaries. Note the termination points of some clusters at each ACS 5 end-year, where a given cluster was not able to maintain LISA significance between time steps. Additionally, note the amount or count of individual Moving LISA *aggregate movers*

at each time step, where a divergence, spreading or splitting of spatially distinct clusters is evident between ACS5 end-years 2013 and 2018, respectively.

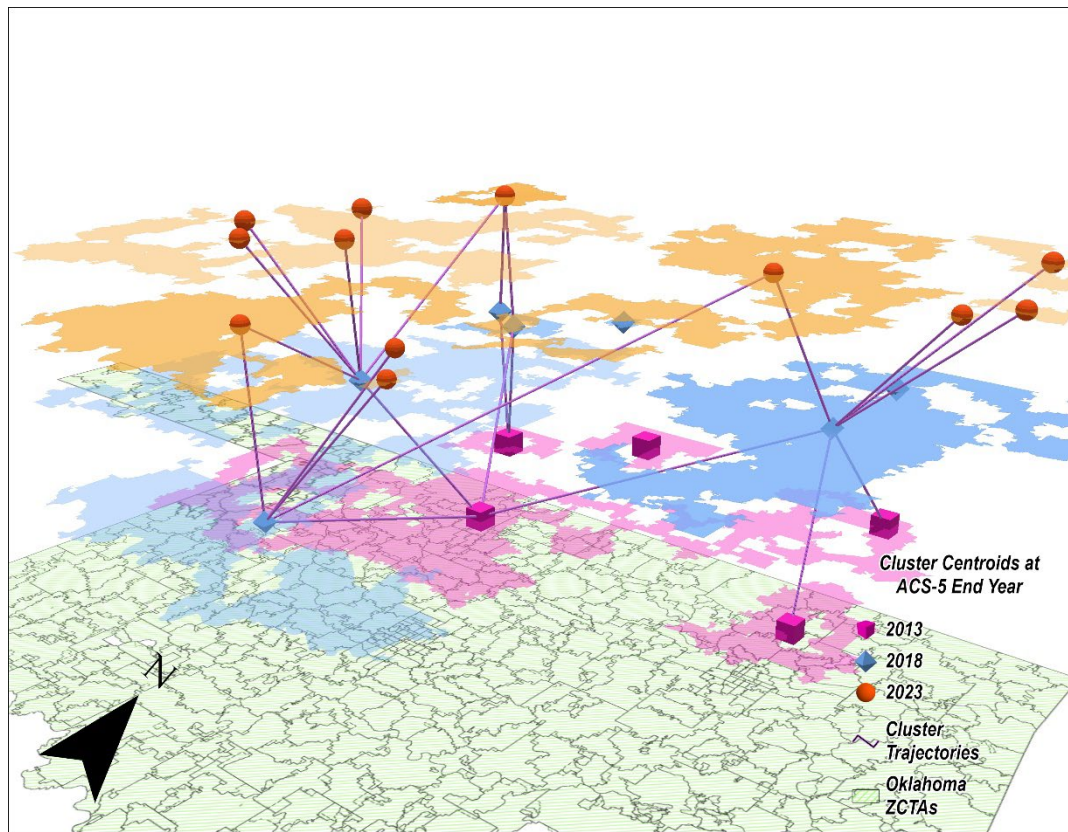


Figure 3. Moving LISA visualization for vape store counts at Oklahoma ZCTAs, shown are vape shop hotspots and corresponding Moving LISA trajectories over three time periods. Nodes which fail to connect to edges in later time periods represent movers which have terminated, or failed to meet the LISA significance boundary and contiguity constraints necessary to persist into the next time period.

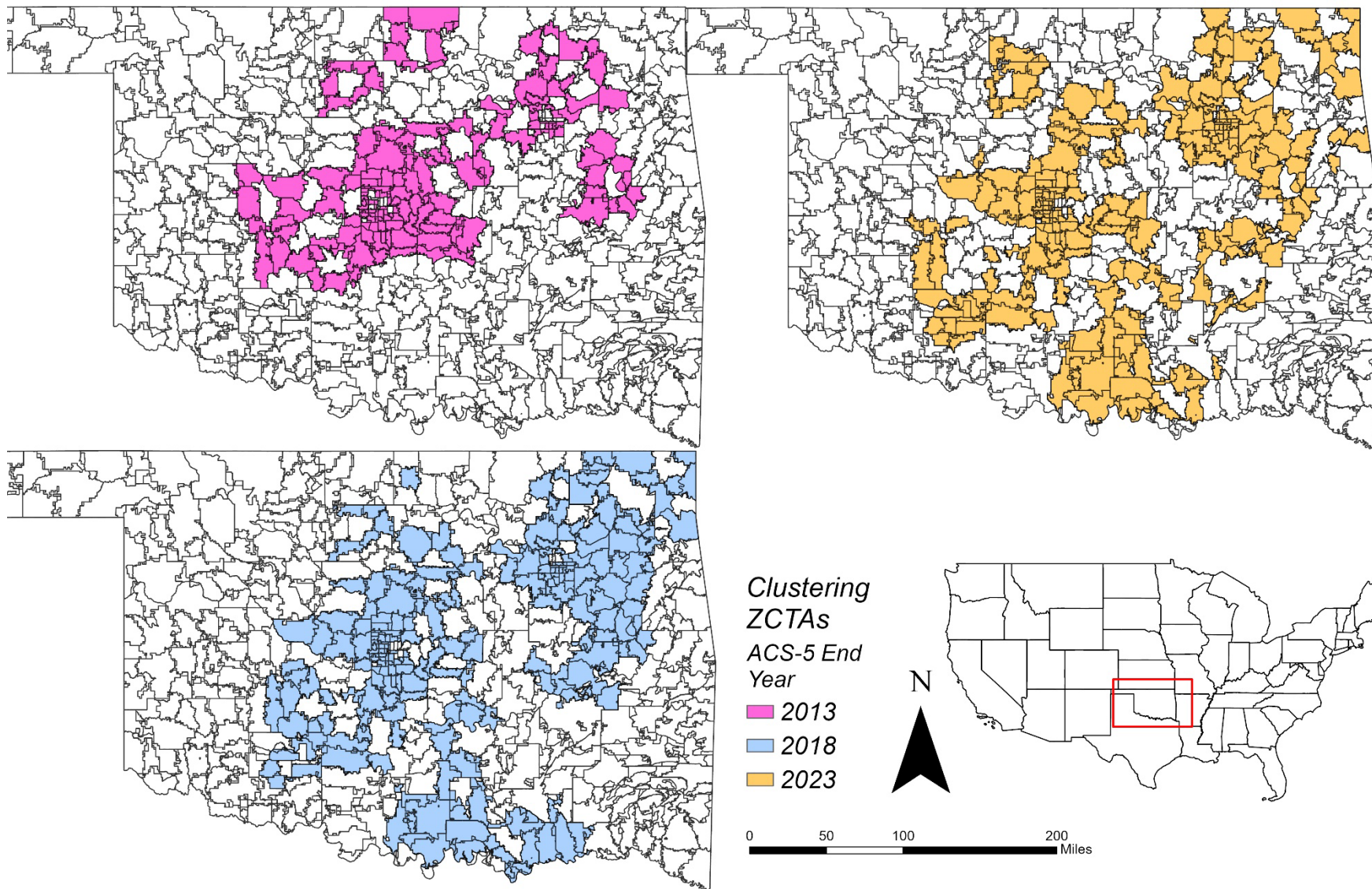


Figure 4. Clustering ZCTAs at each ACS 5 end-year for vape store counts.

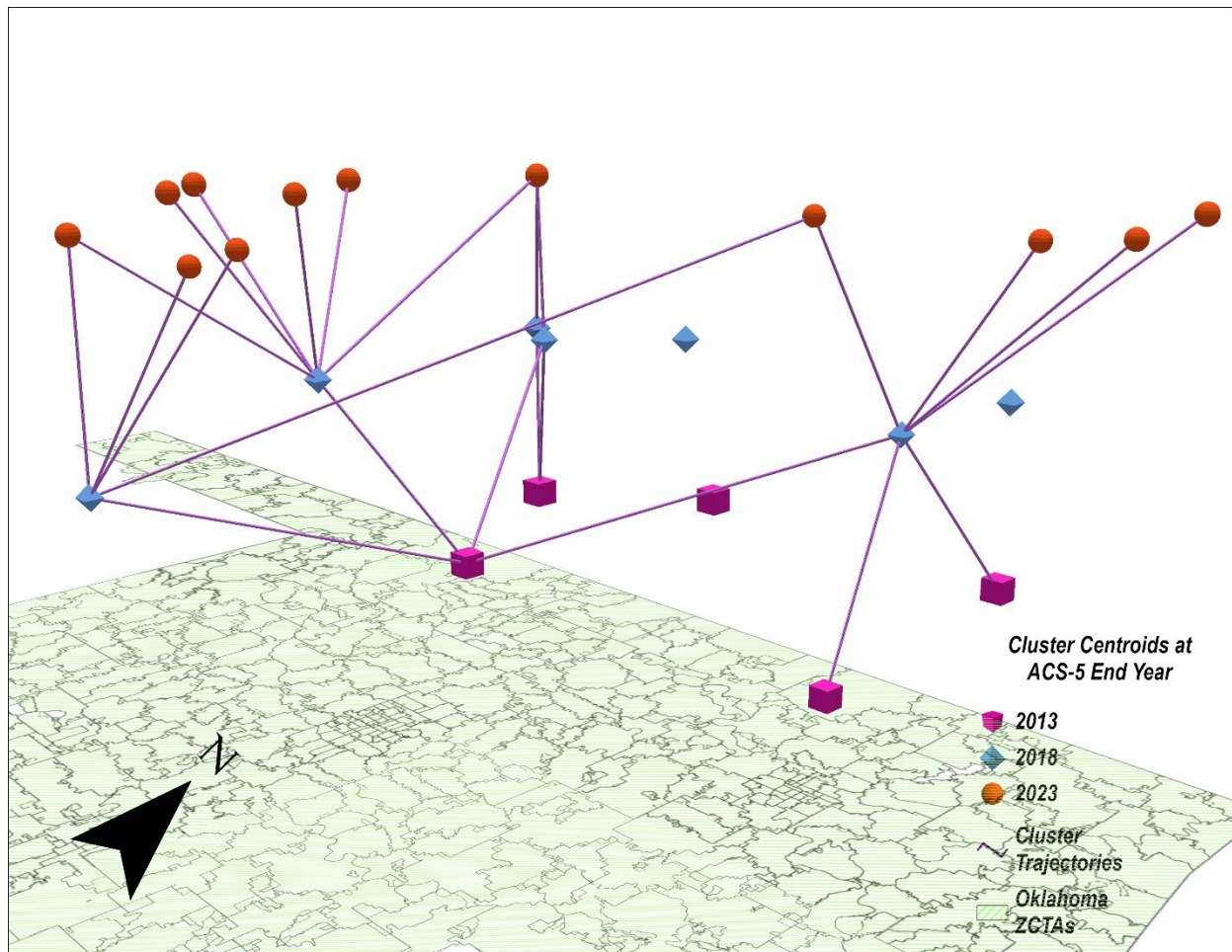


Figure 5. Trajectories of *aggregate movers* at each ACS 5 end-year for vape store counts.

3.4 Discussion

Overall, the extent and speed of Oklahoma’s specialty vape shop development during the years analyzed in this study had been raised in news media as nationally unique and unexpected (Richtel, 2014). In 2014 it was shown Oklahoma hosted “...6 percent of vape shops...and 1 percent of the (US) population...” nationally, with Oklahoma driving the national trend of vaping “...from Oklahoma outward...” among areas in the interior of the US which is “...somewhat anomalous...” in terms of then-expectations for vape retail market diffusion (Richtel, 2014).

Oklahoma and the Oklahoma City MSA have also been a focus of recent work on vaping from public health, where the Oklahoma City metropolitan statistical area (MSA) was found to have the highest proportion of specialty vape shops versus other vapor retail outlets, along with the highest reported prevalence of past-month vaping behavior considered among five other MSA's in the US (Berg et al., 2020; M. Cheney et al., 2015). Vaping, the social normalization of the behavior and its implications for health, the impact of its uneven market presence and further the many unknowns surrounding its long-term effect could acutely manifest for Oklahomans in the coming years. The pattern and process of specialty vape store establishment appears to flourish in the state.

In this study we have sought to examine and better understand the spatial structure and sociodemographic associations presented by the emergence of Oklahoma's specialty vape stores. Our Moving LISA results (Figures 3, 4 and 5) present foremost as an illustration of the rapid proliferation in specialty vape store retail presence in Oklahoma, from urban centers outward, especially for the first two ACS 5 End-Years of 2013 and 2018. Visual review of the trajectories reflected in Figure 5 demonstrates how *aggregate movers* both increased in number and diffused into peri-urban and rural space over the three study periods. In terms of basic count or incidence of specialty vape stores, 33 vape stores were identified in this research statewide in 2013, with 149 vape stores statewide in 2018 (a 4.5x increase over last ACS 5 end-year). Further divergences and repositioning among cluster entities present in 2018 reflect maintained increases in vape store count overall approaching 2023 (303 vape stores statewide in 2023, a 2.03x increase over last ACS 5 end-year). The visually apparent stabilization of overall clustering areas between 2018 and 2023 (Figure 4) might be an indication of either the onset of market saturation in urban areas, or a decrease in the supply of applicable retail space for specialty vape stores.

Moving LISA results, a consequence of the spatial arrangement and density of specialty vape shops at each study period, ally with BYM2 modeling outputs by identifying the geographic areas clustering for vape store presence and suggesting for their spatial dynamics and evolution. Provided with this trajectory constructing the spatiotemporal progression of high incidence of specialty vape shops, it follows that Moving LISA trajectories embed an identification of areas hosting positive sociodemographic associations with the specialty vape retail establishment process. Here, a Moving LISA trajectory allows for the arrangement and qualitative evaluation of cross-sectional BYM2 findings across time and space, enabling policymakers and stakeholders to examine where clusters of vape shops may have co-located with concentrations of social disadvantage identified by its indicating covariates.

Considering the BYM2 modeling result by itself, the significant and positive associations found for “Percent of Households With SNAP Benefits Last 12 Months”, “Percent Latino of Any Race”, and “Percent 25-34 Years” generally agree with and confirm the prevailing empirical evidence for emerging equity issues posed by specialty vape retail (Berg et al., 2020; M. Cheney et al., 2015; M. K. Cheney et al., 2016). This evidence arrives alongside a longstanding body of work identifying very similar mechanisms of inequality and health harm in the practices of commercial tobacco retail (Friedman & Horn, 2019; Kong, Ganz, et al., 2023, 2023; Kong & Henriksen, 2022; Sun et al., 2022; Wheeler et al., 2020). Early warning with respect to these associations can help to inform policy regulating the permissible location of new specialty vape shops. If a material and motivating link can be argued between the observed practices of specialty vape store establishment and the widely examined history of commercial tobacco retail, it is possible existing policy advancements from tobacco control could be retooled to apply to specialty vape shops. Several previous studies have also suggested that youth populations are

associated with the process of specialty vape shop establishment (M. Cheney et al., 2015; Dai et al., 2017; Kong & Henriksen, 2022). However, this relationship was not observed in our study in OKC, which may be a function of our selection of a more recent study period, as well as the unique public health context and retail space availability in Oklahoma (Richtel, 2014).

While there are resemblances identified between the emergence of specialty vape retail research and findings yielded in the pursuit of tobacco control, a set of reversals in trend have been demonstrated by specialty vape shops versus commercial tobacco outlets (Dai et al., 2017). Namely per the BYM2, the negative and significant association of “Percent of Black One Race” in ACS 5 end-year 2023 captures one such departure from the established patterns known to tobacco control research and is raised in recent studies on vape store establishment patterns (Dai et al., 2017; Dai & Hao, 2017; Wheeler et al., 2020). Where commercial tobacco in the locations of its outlets is shown to be racialized and attracted to social disadvantage (Kong, Herbert, et al., 2023; Kong & Henriksen, 2022), there is evidence suggesting specialty vape retail seeks to co-locate with populations attending college (Dai & Hao, 2017; Sun et al., 2022). Correspondingly, in the BYM2 result for 2018 “Percent Population in College or Graduate School” was found to be positive and significant, followed in this trend by “Percent Bachelors Degree 25 and Older” in end-year 2023. These results support the suggestion from recent literature where strategies and trends in the market pattern of specialty vape store establishment are not yet well-understood. Per the present study, there is indication of these gaps in understanding at work in Oklahoma.

In terms of limitations of our approach, it should be noted that young people and the presence of institutions of higher education (“Percent 25-34 Years”, “Percent Population in College or Graduate School”) may be causally related or collinear in a manner not captured by routine diagnostics; Oklahoma’s major research universities fall within and neighboring to

ZCTA's clustering for high counts of vape shops in the input data. Further, the sample size for 2013 of ZCTAs hosting at least one specialty vape store may be smaller than necessary for the analytic tooling selected. It is theoretically true the minimum sample size for a Bayesian posterior update is one observation (McElreath, 2020). However, the spatial autocorrelation metric and conditional autoregressive prior involved in this work demand a formalization of spatial adjacency and a sample size assumed to be informative on the overall spatial structure of the process at each focal location. For the 2013 ACS 5 end-year, it's possible the specialty vape sector was not well enough established in the state of Oklahoma to signal its pattern. Finally, since there does not appear to be standard or consensus technique for positively identifying specialty vape stores versus other ENDS retailers, this study's construction of the dependent variable will vary versus the approaches presented in prior work and may arise as a limitation on study comparisons (Kim et al., 2016; Kong et al., 2017).

This study follows the contributions of prior work examining the emerging public health issue of vaping, with an explicitly spatial investigation of specialty vape store establishment patterns over time. This study is the first to examine the process longitudinally for Oklahoma, with specific attention to the spatial arrangement of high incidence areas for specialty vape shops at the ZCTA level. Our results indicate confirmation of a pattern of specialty vape shop establishment in areas hosting youth, minority or socially disadvantaged populations. The long-term effects of vaping, for use as a smoking cessation/harm reduction strategy or as an addictive behavior are currently revealing themselves through ongoing research in public health. It is known that vapor retail control research, while similar in its targets to commercial tobacco control, is examining a relatively new market entity having characteristics separating it from

commercial tobacco retail. For all of these reasons, additional research is encouraged and necessary.

3.5 References

- Anderson, J., & Yang, A. (2025). Moving Local Indicators of Spatial Association for Time Geography. *Transactions in GIS*, 29(8), e70158. <https://doi.org/10.1111/tgis.70158>
- Anselin, L. (1988). *Spatial Econometrics: Methods and Models* (Vol. 4). Springer Science & Business Media.
- Anselin, L. (1996). The Moran scatterplot as an ESDA tool to assess local instability in spatial association. In *Spatial analytical perspectives on GIS* (pp. 111–126). Routledge.
- Anselin, L., Syabri, I., & Kho, Y. (2006). GeoDa: An Introduction to Spatial Data Analysis. *Geographical Analysis*, 38(1), 5–22. <https://doi.org/10.1111/j.0016-7363.2005.00671.x>
- Anselin, L., Syabri, I., & Kho, Y. (2010). GeoDa: An Introduction to Spatial Data Analysis. In M. M. Fischer & A. Getis (Eds.), *Handbook of Applied Spatial Analysis* (pp. 73–89). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-03647-7_5
- Belok, S. H., Parikh, R., Bernardo, J., & Kathuria, H. (2020). E-cigarette, or vaping, product use-associated lung injury: A review. *Pneumonia*, 12(1), 12. <https://doi.org/10.1186/s41479-020-00075-2>
- Berg, C. J. (2018). Vape shop location and marketing in the context of the Food and Drug Administration regulation. *Public Health*, 165, 142–145. <https://doi.org/10.1016/j.puhe.2018.09.002>
- Berg, C. J., Haardoerfer, R., Escoffery, C., Zheng, P., & Kegler, M. (2015). Cigarette Users' Interest in Using or Switching to Electronic Nicotine Delivery Systems for Smokeless

- Tobacco for Harm Reduction, Cessation, or Novelty: A Cross-Sectional Survey of US Adults. *Nicotine & Tobacco Research*, 17(2), 245–255. <https://doi.org/10.1093/ntr/ntu103>
- Berg, C. J., Schleicher, N. C., Johnson, T. O., Barker, D. C., Getachew, B., Weber, A., Park, A. J., Patterson, A., Dorvil, S., Fairman, R. T., Meyers, C., & Henriksen, L. (2020). Vape shop identification, density and place characteristics in six metropolitan areas across the US. *Preventive Medicine Reports*, 19, 101137. <https://doi.org/10.1016/j.pmedr.2020.101137>
- Besag, J., York, J., & Mollié, A. (1991). Bayesian image restoration, with two applications in spatial statistics. *Annals of the Institute of Statistical Mathematics*, 43(1), 1–20. <https://doi.org/10.1007/BF00116466>
- Birdsey, J., Cornelius, M., Jamal, A., Park-Lee, E., Cooper, M. R., Wang, J., Sawdey, M. D., Cullen, K. A., & Neff, L. (2023). Tobacco Product Use Among U.S. Middle and High School Students—National Youth Tobacco Survey, 2023. *MMWR. Morbidity and Mortality Weekly Report*, 72(44), 1173–1182. <https://doi.org/10.15585/mmwr.mm7244a1>
- Bivand, R. (2022). R Packages for Analyzing Spatial Data: A Comparative Case Study with Areal Data. *Geographical Analysis*, 54(3), 488–518. <https://doi.org/10.1111/gean.12319>
- Blangiardo, M., & Cameletti, M. (2015). *Spatial and spatio-temporal Bayesian models with R-INLA*. John Wiley and Sons, Inc.
- Bullen, C., Howe, C., Laugesen, M., McRobbie, H., Parag, V., Williman, J., & Walker, N. (2013). Electronic cigarettes for smoking cessation: A randomised controlled trial. *The Lancet*, 382(9905), 1629–1637. [https://doi.org/10.1016/S0140-6736\(13\)61842-5](https://doi.org/10.1016/S0140-6736(13)61842-5)
- Centers for Disease Control and Prevention, National Center for Chronic Disease Prevention and Health Promotion, & Division of Population Health. (2023). *BRFSS Prevalence & Trends Data*. <https://www.cdc.gov/brfss/brfssprevalence/>

- Chen, P. (2015). Built environment factors in explaining the automobile-involved bicycle crash frequencies: A spatial statistic approach. *Safety Science*, 79, 336–343.
<https://doi.org/10.1016/j.ssci.2015.06.016>
- Cheney, M., Gowin, M., & Wann, T. F. (2015). Marketing Practices of Vapor Store Owners. *American Journal of Public Health*, 105(6), e16–e21.
<https://doi.org/10.2105/AJPH.2015.302610>
- Cheney, M. K., Gowin, M., & Wann, T. F. (2016). Vapor Store Owner Beliefs and Messages to Customers. *Nicotine & Tobacco Research*, 18(5), 694–699.
<https://doi.org/10.1093/ntr/ntv129>
- Cliff, A. D., & Ord, J. K. (1973). *Spatial autocorrelation*. Pion.
<https://archive.org/details/spatialautocorre0000clif>
- Cliff, A. D., Ord, J. K., & Cliff, A. D. (1981). *Spatial processes: Models & applications*. Pion.
- Dai, H., & Hao, J. (2017). Geographic density and proximity of vape shops to colleges in the USA. *Tobacco Control*, 26(4), 379–385. <https://doi.org/10.1136/tobaccocontrol-2016-052957>
- Dai, H., Hao, J., & Catley, D. (2017). Vape Shop Density and Socio-Demographic Disparities: A US Census Tract Analysis. *Nicotine & Tobacco Research*, 19(11), 1338–1344.
<https://doi.org/10.1093/ntr/ntx063>
- Dean, C. B., Ugarte, M. D., & Militino, A. F. (2001). Detecting Interaction Between Random Region and Fixed Age Effects in Disease Mapping. *Biometrics*, 57(1), 197–202.
<https://doi.org/10.1111/j.0006-341X.2001.00197.x>
- DiCasmirro, J., Tranmer, J., Davison, C., Woo, K., Toon, D., Hubeny, M., Ross-White, A., & Goldie, C. (2025). Public Health Interventions Targeting the Prevention of Adolescent

- Vaping: A Scoping Review. *Public Health Nursing*, 42(1), 604–614.
<https://doi.org/10.1111/phn.13464>
- Fortin, M.-J., & Dale, M. R. T. (2005). *Spatial Analysis: A Guide for Ecologists* (1st ed.). Cambridge University Press. <https://doi.org/10.1017/CBO9780511542039>
- Freni-Sterrantino, A., Ventrucci, M., & Rue, H. (2018). A note on intrinsic conditional autoregressive models for disconnected graphs. *Spatial and Spatio-Temporal Epidemiology*, 26, 25–34. <https://doi.org/10.1016/j.sste.2018.04.002>
- Friedman, A. S., & Horn, S. J. L. (2019). Socioeconomic Disparities in Electronic Cigarette Use and Transitions from Smoking. *Nicotine & Tobacco Research*, 21(10), 1363–1370.
<https://doi.org/10.1093/ntr/nty120>
- Getis, A., & Ord, J. K. (1992). The Analysis of Spatial Association by Use of Distance Statistics. *Geographical Analysis*, 24(3), 189–206. <https://doi.org/10.1111/j.1538-4632.1992.tb00261.x>
- Giovenco, D. P. (2018). Smoke Shop Misclassification May Cloud Studies on Vape Shop Density. *Nicotine & Tobacco Research*, 20(8), 1025–1026.
<https://doi.org/10.1093/ntr/ntx136>
- Huang, J., Kim, Y., Vera, L., & Emery, S. L. (2016). Electronic Cigarettes Among Priority Populations. *American Journal of Preventive Medicine*, 50(2), 199–209.
<https://doi.org/10.1016/j.amepre.2015.06.032>
- Jamal, A., Park-Lee, E., Birdsey, J., West, A., Cornelius, M., Cooper, M. R., Cowan, H., Wang, J., Sawdey, M. D., Cullen, K. A., & Navon, L. (2024). Tobacco Product Use Among Middle and High School Students—National Youth Tobacco Survey, United States, 2024.

- MMWR. Morbidity and Mortality Weekly Report*, 73(41), 917–924.
<https://doi.org/10.15585/mmwr.mm7341a2>
- Kalininskiy, A., Bach, C. T., Nacca, N. E., Ginsberg, G., Marraffa, J., Navarette, K. A., McGraw, M. D., & Croft, D. P. (2019). E-cigarette, or vaping, product use associated lung injury (EVALI): Case series and diagnostic approach. *The Lancet Respiratory Medicine*, 7(12), 1017–1026. [https://doi.org/10.1016/S2213-2600\(19\)30415-1](https://doi.org/10.1016/S2213-2600(19)30415-1)
- Kim, A. E., Loomis, B., Rhodes, B., Eggers, M. E., Liedtke, C., & Porter, L. (2016). Identifying e-cigarette vape stores: Description of an online search methodology. *Tobacco Control*, 25(e1), e19–e23. <https://doi.org/10.1136/tobaccocontrol-2015-052270>
- Kong, A. Y., Eaddy, J. L., Morrison, S. L., Asbury, D., Lindell, K. M., & Ribisl, K. M. (2017). Using the Vape Shop Standardized Tobacco Assessment for Retail Settings (V-STARS) to Assess Product Availability, Price Promotions, and Messaging in New Hampshire Vape Shop Retailers. *Tobacco Regulatory Science*, 3(2), 174–182.
<https://doi.org/10.18001/TRS.3.2.5>
- Kong, A. Y., Ganz, O., & Villanti, A. C. (2023). Exploring the Presence and Type of Premium Cigar Retailers With Neighborhood Sociodemographic Correlates in the United States, 2019–2021. *Nicotine and Tobacco Research*, 25(Supplement_1), S65–S68.
<https://doi.org/10.1093/ntr/ntad042>
- Kong, A. Y., & Henriksen, L. (2022). Retail endgame strategies: Reduce tobacco availability and visibility and promote health equity. *Tobacco Control*, 31(2), 243–249.
<https://doi.org/10.1136/tobaccocontrol-2021-056555>
- Kong, A. Y., Herbert, L., Feldman, J. M., Trangenstein, P. J., Fakunle, D. O., & Lee, J. G. L. (2023). Tobacco and Alcohol Retailer Availability and Neighborhood Racialized,

- Economic, and Racialized Economic Segregation in North Carolina. *Journal of Racial and Ethnic Health Disparities*, 10(6), 2861–2871. <https://doi.org/10.1007/s40615-022-01463-8>
- Lee, J. G. L., Orlan, E. N., Sewell, K. B., & Ribisl, K. M. (2018). A new form of nicotine retailers: A systematic review of the sales and marketing practices of vape shops. *Tobacco Control*, 27(e1), e70–e75. <https://doi.org/10.1136/tobaccocontrol-2017-054015>
- Lee, Y. O., & Kim, A. E. (2015). ‘Vape shops’ and ‘E-Cigarette lounges’ open across the USA to promote ENDS. *Tobacco Control*, 24(4), 410–412. <https://doi.org/10.1136/tobaccocontrol-2013-051437>
- Legendre, P., & Fortin, M. J. (1989). Spatial pattern and ecological analysis. *Vegetatio*, 80(2), 107–138. <https://doi.org/10.1007/BF00048036>
- Leroux, B. G., Lei, X., & Breslow, N. (2000). Estimation of Disease Rates in Small Areas: A new Mixed Model for Spatial Dependence. In M. E. Halloran & D. Berry (Eds.), *Statistical Models in Epidemiology, the Environment, and Clinical Trials* (Vol. 116, pp. 179–191). Springer New York. https://doi.org/10.1007/978-1-4612-1284-3_4
- Levy, D. T., Tam, J., Sanchez-Romero, L. M., Li, Y., Yuan, Z., Jeon, J., & Meza, R. (2021). Public health implications of vaping in the USA: The smoking and vaping simulation model. *Population Health Metrics*, 19(1), 19. <https://doi.org/10.1186/s12963-021-00250-7>
- Marcoulides, K. M., & Raykov, T. (2019). Evaluation of Variance Inflation Factors in Regression Models Using Latent Variable Modeling Methods. *Educational and Psychological Measurement*, 79(5), 874–882. <https://doi.org/10.1177/0013164418817803>

- Marquardt, D. W. (1970). Generalized Inverses, Ridge Regression, Biased Linear Estimation, and Nonlinear Estimation. *Technometrics*, 12(3), 591–612.
<https://doi.org/10.1080/00401706.1970.10488699>
- McElreath, R. (2020). *Statistical rethinking: A Bayesian course with examples in R and Stan* (Second edition). Chapman & Hall/CRC.
- Mets, K. D., Armenteras, D., & Dávalos, L. M. (2017). Spatial autocorrelation reduces model precision and predictive power in deforestation analyses. *Ecosphere*, 8(5), e01824.
<https://doi.org/10.1002/ecs2.1824>
- Miller, H. J. (2005). A Measurement Theory for Time Geography. *Geographical Analysis*, 37(1), Article 1. <https://doi.org/10.1111/j.1538-4632.2005.00575.x>
- Miller, H. J. (2017). Time Geography and Space-Time Prism. In D. Richardson, N. Castree, M. F. Goodchild, A. Kobayashi, W. Liu, & R. A. Marston (Eds.), *International Encyclopedia of Geography: People, the Earth, Environment and Technology* (pp. 1–19). John Wiley & Sons, Ltd. <https://doi.org/10.1002/9781118786352.wbieg0431>
- Moraga, P. (2020). *Geospatial health data: Modeling and visualization with R-INLA and Shiny*. CRC Press/Taylor & Francis Group.
- Moran, P. A. P. (1948). The Interpretation of Statistical Maps. *Journal of the Royal Statistical Society. Series B (Methodological)*, 10(2), Article 2. JSTOR.
- Moran, P. A. P. (1950). Notes on Continuous Stochastic Phenomena. *Biometrika*, 37(1/2), Article 1/2. <https://doi.org/10.2307/2332142>
- Ord, J. K., & Getis, A. (1995). Local Spatial Autocorrelation Statistics: Distributional Issues and an Application. *Geographical Analysis*, 27(4), 286–306. <https://doi.org/10.1111/j.1538-4632.1995.tb00912.x>

- Park-Lee, E., Jamal, A., Cowan, H., Sawdey, M. D., Cooper, M. R., Birdsey, J., West, A., & Cullen, K. A. (2024). *Notes from the Field: E-Cigarette and Nicotine Pouch Use Among Middle and High School Students—United States, 2024. MMWR. Morbidity and Mortality Weekly Report*, 73(35), 774–778. <https://doi.org/10.15585/mmwr.mm7335a3>
- Pebesma, E., & Bivand, R. S. (2023). *Spatial Data Science With Applications in R*. Chapman & Hall. <https://r-spatial.org/book/>
- Richtel, M. (2014). Where vapor comes sweeping down the plain. *New York Times*.
- Riebler, A., Sørbye, S. H., Simpson, D., & Rue, H. (2016). An intuitive Bayesian spatial model for disease mapping that accounts for scaling. *Statistical Methods in Medical Research*, 25(4), 1145–1165. <https://doi.org/10.1177/0962280216660421>
- Rue, H., Martino, S., & Chopin, N. (2009). Approximate Bayesian Inference for Latent Gaussian models by using Integrated Nested Laplace Approximations. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 71(2), 319–392. <https://doi.org/10.1111/j.1467-9868.2008.00700.x>
- Salmerón, R., García, C., & García, J. (2020). *Overcoming the inconsistencies of the variance inflation factor: A redefined VIF and a test to detect statistical troubling multicollinearity* (Version 1). arXiv. <https://doi.org/10.48550/ARXIV.2005.02245>
- Simpson, D., Rue, H., Riebler, A., Martins, T. G., & Sørbye, S. H. (2017). Penalising Model Component Complexity: A Principled, Practical Approach to Constructing Priors. *Statistical Science*, 32(1). <https://doi.org/10.1214/16-STS576>
- Sun, D. L., Schleicher, N. C., Recinos, A., & Henriksen, L. (2022). Spatial Clustering of Hookah Lounges, Vape Shops, and All Tobacco Retailers Near Colleges. *Nicotine & Tobacco Research*, 24(6), 834–839. <https://doi.org/10.1093/ntr/ntac007>

- Tobler, W. R. (1970). A Computer Movie Simulating Urban Growth in the Detroit Region. *Economic Geography*, 46, 234–240. <https://doi.org/10.2307/143141>
- US Census Bureau. (2023). *American Community Survey and Puerto Rico Community Survey 2023 Subject Definitions*. https://www2.census.gov/programs-surveys/acs/tech_docs/subject_definitions/2023_ACSSubjectDefinitions.pdf
- Venugopal, P. D., Morse, A. L., Alrefai-Kirkpatrick, R., Tworek, C., & Chang, H. W. (2022). The Co-occurrence of Specialty Vape Shops, Social Disadvantage, and Poor Air Quality in the United States: An Assessment of Cumulative Risks to Youth. *Health Equity*, 6(1), 132–141. <https://doi.org/10.1089/heq.2021.0151>
- Wheeler, D. C., Do, E. K., Hayes, R. B., Fugate-Laue, K., Fallavollita, W. L., Hughes, C., & Fuemmeler, B. F. (2020). Neighborhood Disadvantage and Tobacco Retail Outlet and Vape Shop Outlet Rates. *International Journal of Environmental Research and Public Health*, 17(8), 2864. <https://doi.org/10.3390/ijerph17082864>
- Yang, A., Liu, C., Yang, D., & Lu, C. (2023). Electric vehicle adoption in a mature market: A case study of Norway. *Journal of Transport Geography*, 106, 103489. <https://doi.org/10.1016/j.jtrangeo.2022.103489>

3.6 Appendix

American Community Survey 5-Year Variables Collected

ACS 5 Demographic Profile Product Variable Name	ACS 5 End-Year 2013	ACS 5 End-Year 2018	ACS 5 End-Year 2023	Renamed To
Estimate!!HOUSEHOLDS BY TYPE!!Total households!!Average household size	DP02_0015E	DP02_0015E	DP02_0016E	Average Household Size
Percent!!SCHOOL ENROLLMENT!!Population 3 years and over enrolled in school!!High school (grades 9-12)	DP02_0056PE	DP02_0056PE	DP02_0057PE	Percent Population in High School
Percent!!SCHOOL ENROLLMENT!!Population 3 years and over enrolled in school!!College or graduate school	DP02_0057PE	DP02_0057PE	DP02_0058PE	Percent Population in College or Graduate School
Percent!!EDUCATIONAL ATTAINMENT!!Population 25 years and over!!9th to 12th grade, no diploma	DP02_0060PE	DP02_0060PE	DP02_0061PE	Percent Not a High School Graduate (25 and Older)
Percent!!EDUCATIONAL ATTAINMENT!!Population 25 years and over!!High school graduate (includes equivalency)	DP02_0061PE	DP02_0061PE	DP02_0062PE	Percent High School Graduate (25 and Older)

American Community Survey 5-Year Variables Collected

ACS 5 Demographic Profile Product Variable Name	ACS 5 End-Year 2013	ACS 5 End-Year 2018	ACS 5 End-Year 2023	Renamed To
Percent!!EDUCATIONAL ATTAINMENT!!Population 25 years and over!!Bachelor's degree	DP02_0064PE	DP02_0064PE	DP02_0065PE	Percent Bachelors Degree (25 and Older)
Percent!!EMPLOYMENT STATUS!!Population 16 years and over!!In labor force!!Civilian labor force!!Unemployed	DP03_0005PE	DP03_0005PE	DP03_0005PE	Percent in Labor Force and Unemployed (16 or Over)
Percent!!!INCOME AND BENEFITS (IN 2023 INFLATION-ADJUSTED DOLLARS)!!Total households!!With Food Stamp/SNAP benefits in the past 12 months	DP03_0074PE	DP03_0074PE	DP03_0074PE	Percent Households with SNAP Benefits Last 12 Months
Percent!!PERCENTAGE OF FAMILIES AND PEOPLE WHOSE INCOME IN THE PAST 12 MONTHS IS BELOW THE POVERTY LEVEL!!All families	DP03_0119PE	DP03_0119PE	DP03_0119PE	Percent Families Below Poverty Level Last 12 Months
Percent!!HOUSING OCCUPANCY!!Total housing units!!Vacant housing units	DP04_0003PE	DP04_0003PE	DP04_0003PE	Percent of Vacant Housing Units

American Community Survey 5-Year Variables Collected

<i>ACS 5 Demographic Profile Product Variable Name</i>	<i>ACS 5 End-Year 2013</i>	<i>ACS 5 End-Year 2018</i>	<i>ACS 5 End-Year 2023</i>	<i>Renamed To</i>
Percent!!SEX AND AGE!!Total population!!Male	DP05_0002PE	DP05_0002PE	DP05_0002PE	Percent of Male
Percent!!SEX AND AGE!!Total population!!Female	DP05_0003PE	DP05_0003PE	DP05_0003PE	Percent of Female
Percent!!SEX AND AGE!!Total population!!15 to 19 years	DP05_0007PE	DP05_0008PE	DP05_0008PE	Percent 15-19 Years
Percent!!SEX AND AGE!!Total population!!20 to 24 years	DP05_0009PE	DP05_0009PE	DP05_0009PE	Percent 20-24 Years
Percent!!SEX AND AGE!!Total population!!25 to 34 years	DP05_0010PE	DP05_0010PE	DP05_0010PE	Percent 25-34 Years
Percent!!SEX AND AGE!!Total population!!35 to 44 years	DP05_0011PE	DP05_0011PE	DP05_0011PE	Percent 35-44 Years
Percent!!SEX AND AGE!!Total population!!45 to 54 years	DP05_0012PE	DP05_0012PE	DP05_0012PE	Percent 45-54 Years
Percent!!RACE!!Total population!!One race!!White	DP05_0032PE	DP05_0037PE	DP05_0037PE	Percent of White One Race
Percent!!RACE!!Total population!!One race!!Black or African American	DP05_0033PE	DP05_0038PE	DP05_0038PE	Percent of Black One Race

American Community Survey 5-Year Variables Collected

<i>ACS 5 Demographic Profile Product Variable Name</i>	<i>ACS 5 End-Year 2013</i>	<i>ACS 5 End-Year 2018</i>	<i>ACS 5 End-Year 2023</i>	<i>Renamed To</i>
Percent!!RACE!!Total population!!One race!!American Indian and Alaska Native	DP05_0034PE	DP05_0039PE	DP05_0039PE	Percent of American Indian One Race
Percent!!RACE!!Total population!!One race!!Asian	DP05_0039PE	DP05_0044PE	DP05_0047PE	Percent of Asian One Race
Percent!!RACE!!Total population!!One race!!Some Other Race	DP05_0055PE	DP05_0060PE	DP05_0060PE	Percent of American Indian and White Two Races
Percent!!RACE!!Total population!!Two or More Races!!Black or African American and American Indian and Alaska Native	DP05_0061PE	DP05_0066PE	DP05_0066PE	Percent of American Indian One or More Races
Percent!!HISPANIC OR LATINO AND RACE!!Total population!!Hispanic or Latino (of any race)	DP05_0066PE	DP05_0071PE	DP05_0076PE	Percent of Latino Any Race

Table 1. American Community Survey 5-Year Estimates Demographic Profile variable names, end-years 2013, 2018, 2023. Covariates which were removed during multicollinearity checks are shown in **bold text**.

4 Chapter Four: Low Incomes Diverge from High Home Values: Gentrification in the Oklahoma City Metropolitan Area (2009-2023) as Dyadic Movement.

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Abstract

Despite a clear theoretical construction and a vast history of qualitative observation for the gentrification process, the literature has not reached consensus on a quantitative characterization. Recent contributions studying gentrification have sought to catalog and categorize the abundance of measures available for diagnosing the onset of the capital and social changes observed, still others have successfully reframed known associations under fresh theoretical interpretation towards simpler, user-tractable measures. Following from the recent theoretical reframing and corresponding quantitative contributions of Bunten et al. (2024), this study reimagines flows of people and capital in gentrifying areas as a function of dyadic movement between respective trajectories of median home values and incomes. As the first direct analysis of Moving Local Indicators of Spatial Association (Moving LISA) trajectories, our results show spatiotemporal, moving-object repulsion between hotspots of home values and cold spots of incomes. Conversely, Moving LISA trajectories for hotspots of home values attracted the movements of high incomes, in the Oklahoma City Metropolitan Statistical Area, for the years of 2009-2023.

Keywords: Gentrification, Moving LISA, Trajectory, Dyadic Movement, Time Geography.

4.1 Introduction

Consensus on measuring gentrification has proven elusive in the literature despite clear qualitative understandings of the identifying characteristics, potential drivers and timeline of development for gentrification (Bunten et al., 2024; Liu et al., 2019). Gentrification as an urban process can be loosely defined as a movement of capital whereby vulnerable areas are rapidly invested in and migrated to by high-earning individuals, resulting in rising rents and property

values. This transition routinely follows with an outflow of lower-earning, often socially disadvantaged people from gentrifying neighborhoods (Corrigan et al., 2021; Halasz, 2018; Lees, 2012; Raymond et al., 2021; Rucks-Ahidiana, 2021). The pioneering sociologist Ruth Glass coined the term gentrification and set the discourse surrounding the process and its implications in motion (Glass, 1960). Theoretical constructions of gentrification, depending on the disciplinary lens of their origin, have since raised many possible causes for the process. These causes include reinvestment driven by financialization of already-gentrified housing (Halasz, 2018; Lees, 2003), a reaction to climate drivers (Gould & Lewis, 2021; Immergluck & Balan, 2018) and social disparity from racial ascription in housing market valuation (Fielder, 2018; Rucks-Ahidiana, 2022). Gentrification has political, economic, social and cultural dimensions within which its effects are realized, where those effects are often detrimental to those with less influence on capital. The process is a practical concern for urban geographers and planners who seek objectivity in its measurement to develop early warning systems informing policies such as constraints on redevelopment to reduce social harm (Barton, 2016; Finio, 2022; Liu et al., 2019).

With respect to the availability and variety of approaches to measuring gentrification, recent assessments have cataloged more than 100 methods (Finio, 2022). Additionally, among these it is shown that methodologically adjacent approaches, despite sharing similar theory and inputs, can yield contradictory results when applied to the same study area and timeframe (Barton, 2016; Bunten et al., 2024; Preis et al., 2021). These contradictions complicate efforts to separate those neighborhoods which are categorically gentrifying versus their surroundings. As a novel method offering parsimony and ease of interpretation, Bunten et al. developed an expectation-based approach to identifying a threshold for the onset of neighborhood-level gentrification in context with conditions present in the enclosing US Census Metropolitan

Statistical Area (MSA). Central to their development of a gentrification *signal* is a theoretical framework which distinguishes variables whose values represent present conditions (median income, population) versus variables which incorporate information on future conditions, such as property values. For example, drawing from asset valuation theory, property values today reflect market expectations for return on investment (Bunten et al., 2024). This approach uses property values as a proxy measurement incorporating the local knowledge of residents about conditions for future real estate investment, valuation and transactions in their neighborhoods.

Mechanistically for Bunten et al., gentrifiable neighborhoods are located in the inner-third of a given MSA's area, where the *signal* for the onset of gentrification arises with a 25-point gap in a neighborhood's percentile rank for incomes versus home property values, respectively (Bunten et al., 2024). Gentrification under this straightforward characterization is shown to reflect a temporal lag between changes in these variables, where initial incomes are generally maintained while property values and rents will rise. This amounts to increased outmigration pressure on lower-income residents who may already struggle to meet prevailing rents. After property values have outpaced the earnings of these residents, the rate of outmigration is shown to increase. Bunten et al. also found, among the data empirically examined, that gentrification per their signal materializes faster in neighborhoods which were closer to downtown, those actively adding to housing supply and those which were majority Black in their initial demographic composition (Bunten et al., 2024). Figure 1 illustrates Bunten's Gentrification Signal (BGS) as directed acyclic graph identifying the component concepts.

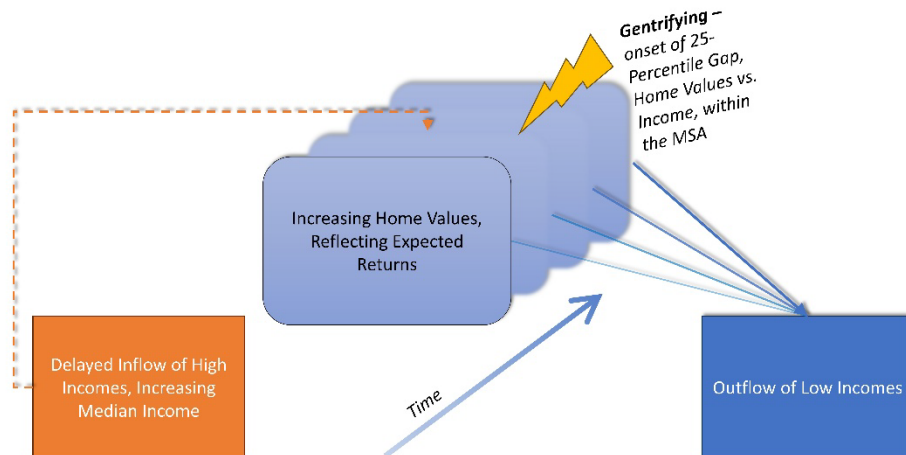


Figure 1. Bunten et. al's theoretical implication of current and future property value expectations, incomes, and the onset of gentrification in the urban US. A candidate neighborhood is actively gentrifying once a 25-percentile gap in incomes versus home values is observed, based on percentiles calculated at the MSA-level.

The BGS brings progress for both gentrification measurement and for its communication among stakeholders. The theoretical framework for the signal casts readily available data into a refreshed perspective which is tractable for the varied users and disciplines engaged in understanding urban spaces. In any science, arriving at this kind of simplicity and knowing the solution works is both highly desirable and very difficult (Akaike, 1974; Forster & Sober, 1994; Sober, 2002; Vehtari et al., 2017). In the original publication the ideas surrounding the signal are leveraged to specify a difference-in-differences model to confirm the temporal dynamics between its selected expectation-based and present-oriented themes for data longitudinally describing centrally located urban geographies. The authors further applied the signal measure to reconstruct and map the onset of gentrification for cities and timeframes noted as gentrifying in the literature.

However, in these applications spatial interaction among the variables is not directly considered, and the temporal link between observations is a matter of map comparison. As a point of departure for the present research, we borrow the core insights of the BGS toward an explicitly spatiotemporal construction in the spirit of crossing “methodological divides” to encourage gentrification scholarship (Brown–Saracino, 2016). We apply the Moving LISA, a novel technique generating space-time trajectories from aggregate spatial observations toward characterizing the present-vs-expectation based view of BGS as a matter of dyadic movement (Anderson & Yang, 2025). Specifically, we use the Oklahoma City MSA as a case study to compare the movements of space-time trajectories for median home value hotspots versus median income cold spots and hot spots, respectively, to study city gentrification from 2009 – 2023.

4.2 Methods

4.2.1 Trajectories and Time Geography

Time Geography presents as both a distinct methodological tradition and subdiscipline within Geography (Hägerstrand, 1970; Miller, 2005; Thrift & Pred, 1981). With its unique and constraints-based treatment of movement as subject matter, the Time Geographic approach is primarily concerned with interpreting the observed space-time trajectories of objects toward understanding in the causes of their movements. Individual movements are regarded as contributions to larger spatiotemporal processes, where for example focal animal avoidance of the built environment, patterns in crime or the daily movement economies of urban residents collect into larger spatial expressions (Ho & Loraamm, 2023; Wang et al., 2024; Yin et al., 2023). Factors including a mover’s environmental context, behavioral responses and pressures

arising from interactions with other movers all comprise the variables Time Geographers see as encoded in movement trajectories (Anderson & Yang, 2025; J. Downs, Lamb, et al., 2014; Loraamm, 2019; Loraamm et al., 2021).

Time Geographic literature is expanding quickly alongside the emergence and ubiquity of location aware technologies (Dodge, 2021; Dodge et al., 2020). New questions expanding the set of influences considered for analysis toward questions of access, identity, and experience abound during this exciting time of innovation and data availability in the subdiscipline. However, the methodology is shown as largely specified to treat the movements of discrete, visible and tangible moving objects. The movements of invisible, spatially indistinct or distributed phenomena, or those phenomena which are reported in aggregate due to circumstance (disease reporting, censored data), are understudied in this literature (Anderson & Yang, 2025; Yin et al., 2021). The open question of how to best identify and relate movement in these distributed spatial phenomena as discrete trajectories positions an emerging set of methods in the subdiscipline.

4.2.1.1 Time-Geographic Entity-Linking with the Moving LISA

One new approach to characterizing movement from the dynamics and evolution of distributed spatial phenomena as trajectories compatible with the Time Geographic framework is made available through the Moving Local Indicators of Spatial Association (Moving LISA) procedure (Anderson & Yang, 2025). While we refer readers to the original paper for a detailed formulation, the Moving LISA accepts a time series of spatially referenced observations in a single variable and returns representative trajectories for displacements in clustering locations. Inputs to the Moving LISA may for example be event counts or median values of a socioeconomic variable assigned to aerial or lattice geographies. Departing from the discrete

space-time locations of an individual animal or city taxi as captured in traditional trajectories, a Moving LISA trajectory is a suggestion of movement for an abstract entity whose boundaries are either indistinct or lost upon observation at an aggregate spatial unit. The returned trajectories are generated as a function of probabilistic spatial comparisons applied to cross-sectional LISA results, establishing the respective boundaries, lineages and space-time progressions of *aggregate movers*. These artificial moving entities are drawn from dynamics of the target spatial process, as a sequence of appearance, disappearance and change in extreme-valued observations.

With this action, the Moving LISA is a type of entity-linking approach among locations positive for a user-selected measure of local clustering. The procedure is sensitive to the selection of LISA for this purpose, and to parameter selections affecting behavior for both this LISA measurement and the spatial comparison step (*contiguity cutoff* and *standard scaling factor*) within the method. Further, the Moving LISA must be understood as inheriting all known limitations and considerations of its component methods, namely, construction of the LISA weights matrix to represent known spatial structure in the study area is a critical step (Anselin, 1988, 1995; Getis & Aldstadt, 2004).

4.2.1.2 *Getis-Ord Local G*

Per the Moving LISA procedure, a user-selected LISA statistic is used to establish the presence of clustering geographies at each time step. For this research, we have selected the Getis-Ord Local G (Getis & Ord, 1992; Ord & Getis, 1995). For the Local G, hotspots in a single variable represent geographies where neighboring G-statistic values exceed expected values in the study area. Conversely, cold spot geographies represent a significant dearth in the values of a single variable, in a user-defined neighborhood. While the Local G cannot identify spatial outliers

(individual locations which significantly depart from the values of their neighbors), the interpretation and calculation of the Local G metric is straightforward and aligns with the goals of this research.

$$G_i = \frac{\sum_{j \neq i} w_{ij} x_j}{\sum_{j \neq i} x_j} \quad (1)$$

Where:

G_i is the Local G statistic result, indicating positive spatial autocorrelation if G_i is above the mean of the variable of interest, and vice versa.

w_{ij} is the weights matrix element suggesting the influence on the variable of interest between geographies i and j .

x_j is the value of the variable of interest at location j .

4.2.2 Measures of Dyadic Movement for Moving LISA Trajectories

Identifying interaction between the trajectories of moving objects is of interest to a range of behavioral and ecological research questions outside of urban geography. Patterns of avoidance (predator avoidance), cohesion (herding, flocking, migration) and time-delayed interactions (territoriality) among individuals may reveal or describe more general behaviors or ecological trends in the population (Bennett & Tang, 2006; Bonnell et al., 2013; J. A. Long, 2018; Ringelman, 2014). With this perspective, interaction pressures become an additional factor to the environmental context of the mover, in which their movements express their spatiotemporal preferences, decisions and behaviors (Ho & Loraamm, 2023; J. A. Long, 2018; Loraamm, 2019; Loraamm et al., 2021). The literature from Movement Ecology, Time Geography and Mobility Science also share an understanding where movers' local actions are thought to generate macroscopic patterns, suggesting for analysis of these interactions from the basic case of paired trajectories forming a *dyad* (Camazine et al., 2020; Joo et al., 2018). A tradition of methods

applied to better summarize and understand the action of dyads, usually for ecological questions, has grown alongside the proliferation of location aware technologies and methods from the movement sciences (Joo et al., 2018; Kenward et al., 1993; J. A. Long & Nelson, 2013; Minta, 1992; Shirabe, 2006).

Beyond this interest in dyads, a sizeable methodology surrounds the analysis of moving object trajectories in isolation and in comparison, inclusive of individual trajectories, and groupings exceeding trajectory pairs such as herds, flocks, and crowds (Dodge et al., 2008, 2021; Fan et al., 2023; Zhang et al., 2018). This methodology is further expanded with extensions to the Time Geographic concept of the *space-time-prism* enabling comparisons among probabilistic reconstructions of the unobserved positions and movements within the duration of a mover's trajectory (J. Downs et al., 2018; J. Downs, Horner, et al., 2014; J. Downs, Lamb, et al., 2014; Ho & Loraamm, 2023; J. A. Long, 2018). These approaches represent a subset of a thriving literature and share in a common characteristic where the sequential nature of the space-time-trajectory is regarded as part of the overall information it captures. This perspective departs from some of the existing strategies for trajectory analysis, namely, those which treat trajectories as disconnected, time-series of point patterns (Burt, 1943; J. A. Downs et al., 2012; J. A. Downs & Horner, 2009; J. A. Long et al., 2014). Depending on the goals of the analysis, when data describing the sequence, displacement distances and relative turning angles of trajectories are disregarded for use in point-pattern logic, analytic opportunities may be missed (J. A. Downs et al., 2012; Joo et al., 2018; J. A. Long et al., 2014). Distinguishing between these two methodological lineages, the present research seeks to treat Moving LISA trajectories as fully qualified space-time-paths subject to analysis for their relocation characteristics in space and time, to be paired and examined for their *dynamic interactions* as *dyads*.

Pursuant to this methodological genre, the *WildlifeDI* package for R provides not only an accessible implementation for the *dynamic interaction* metric of its namesake (J. Long, 2014), but an additional set of allied measurements including the Coefficient of Sociality (Kenward et al., 1993), Minta's Spatiotemporal Interaction (Minta, 1992) and Correlation Analysis of Discrete Motions (Shirabe, 2006) are found in the package. Wildlife DI also conveniently accepts popular trajectory dataset formats including *AdehabitatLT* class "ltraj" and *move2* trajectories, respectively (Calenge, 2011; Kranstauber et al., 2023). Once the user has imported their Moving LISA trajectories into this ecosystem, a great deal of subsequent analytic opportunities become available. Once Moving LISA trajectories were generated for the ACS 5-Year variables of *median home value* (hotspots) and *median income* (hotspots and coldspots, respectively) these three themes were examined for *DI* (J. A. Long & Nelson, 2013), *Cr* (Shirabe, 2006), and *Cs* (Kenward et al., 1993) in the OKC Metropolitan Statistical Area, USA.

4.2.2.1 *Dynamic Interaction (DI) and the DI 9 Component*

The Dynamic Interaction (DI) metric captures cohesion or avoidance in two contemporaneous trajectories as a deterministic function of relative turning angles and distances traveled between paired relocations. DI incorporates several factors of movement at once in a tractable and straightforward way, there is no inference step nor permutation for a reference distribution. Long and Nelson provided the original formulation in 2013 with a demonstration applying DI the movements of ultimate frisbee athletes and grizzly bears, respectively (J. A. Long & Nelson, 2013). The *DI* metric has a result range of $[-1, 1]$, where negative values suggest avoidance behavior and positive values suggest attraction or cohesion in dyadic movement. The aggregation

of local di results from each respective relocation in paired trajectories to the DI value for the entire dyad is also simple to calculate.

While we will not explicitly discuss the local di metric for Moving LISA dyads in the present research, di has a unique property among dyadic movement metrics where it can be examined for trend at the *local* or single relocation (space-time anchor pair) level. This is contrary to similar metrics dependent on terms representing the overall trajectory in their formulations. This allows researchers to identify periods of cohesion or avoidance for di within the overall duration of larger observed trajectories, a property unique to this approach. For completeness, the presentation of di in Equation 2 demonstrates its role as an atomic measurement summarized in the DI expression relating the strength of cohesion in movement for the input dyad.

$$di = \left(1 - \frac{|d_t^a - d_t^\beta|}{d_t^a + d_t^\beta}\right) \times \cos(\theta_t^a - \theta_t^\beta) \quad (2)$$

$$DI = \frac{1}{n-1} \sum_{i=1}^{n-1} di$$

Where:

di is the local Dynamic Interaction result for a given timestep in the input dyad.

DI is the global or overall average Dynamic Interaction result for the input dyad.

a, β identify the two trajectories comprising the input dyad, respectively.

d_t^a is the distance traversed during the last relocation of trajectory a at time t .

θ_t^a is the relative turning angle or pivot since last relocation of the direction of trajectory a at time t .

n is the number of contemporaneous segments or relocations in the input dyad, their length in terms of relocations.

$$di_{\theta} = \cos(\theta_t^a - \theta_t^{\beta}) \quad (3)$$

$$DI_{\theta} = \frac{1}{n-1} \sum_{i=1}^{n-1} di_{\theta}$$

Where:

di_{θ} is the local Dynamic Interaction result for a given timestep in the input dyad.

DI_{θ} is the global or overall average Dynamic Interaction result for the input dyad.

a, β identify the two trajectories comprising the input dyad, respectively.

θ_t^a is the relative turning angle or pivot since last relocation of the direction of trajectory a at time t .

n is the number of contemporaneous segments or relocations in the input dyad, their length in terms of relocations.

4.2.2.2 Correlation Analysis of Discrete Motions (*Shirabe's Correlation, Cr*)

The trajectory correlation coefficient originally presented in (Shirabe, 2006) is straightforward and can be readily interpreted as a measure of dyadic trajectory association in a domain of [-1,

1], with -1 suggesting aversion or avoidance, and 1 suggesting cohesion or attraction. This

metric's structure and its interpretation is reminiscent of Pearson's correlation, and renders the measure (hereafter, *Shirabe's Cr*) accessible and inherently familiar for users. While

straightforward, use of the measure has been cautioned on its sensitivity to the overall trajectory mean direction and distance vectors used in the comparison. Here, it is noted that incorporating a mean direction and distance component might confound or obscure the detection of temporary or

shorter-length interactions within the span of the input trajectories in the dyad (J. A. Long, 2015; J. A. Long et al., 2014).

$$Cr = \frac{\sum_{t=1}^{n-1} (v_t - \bar{v}) \cdot (w_t - \bar{w})}{\sqrt{\sum_{t=1}^{n-1} |v_t - \bar{v}|^2} \sqrt{\sum_{t=1}^{n-1} |w_t - \bar{w}|^2}} \quad (4)$$

Where:

Cr is Shirabe's Trajectory Correlation Coefficient.

v_t, w_t are movement vectors for the trajectories comprising the dyad of interest.

$\bar{v}, \bar{w}$ are the mean vectors for these trajectories (average direction and distance).

4.2.2.3 Coefficient of Sociality (Cs)

The Coefficient of Sociality follows from the original formulation found in (Kenward et al., 1993).

$$CS = \frac{D_E - D_O}{D_O + D_E} \quad (5)$$

Where:

Cs is the Coefficient of Sociality statistic result.

D_o is the mean distance between contemporaneous space-time anchors in a dyad (based on a user-specified time-threshold).

D_E is the expectation for mean distance based on permutations of these contemporaneous space-time anchors.

4.2.3 Stratifying by the Oklahoma City MSA versus the City of Oklahoma City

Municipal boundaries are legal determinations voted on by residents and have implications for the provision of city services and amenities. Municipally incorporated areas tend to comprise a social and physical space more strongly associated with the characteristics, economics and

identity of a regional city. Additionally, an emerging consensus characterization of *gentrifiable* neighborhoods spatially identify the most vulnerable areas as within the inner-third of the host MSA's area, by distance from the central business district of the MSA's major city (Bunten et al., 2024; Finio, 2022). The US Census Metropolitan Statistical Area (MSA) is a larger scope of classification to include associated county geographies representing overall metropolitan areas. Urbanized areas hosting at least one large population center along with other neighboring, economically and socially connected communities have their county boundaries included in the MSA (US Census Bureau, 2023). MSA's are not legal definitions and are used to prepare representative statistics about populations and economics.

For the present research, each analysis approach presented was conducted at two geographic levels of classification. First, Moving LISA trajectories were examined for a parameterization and then generated for analysis, across the entire Oklahoma City Metropolitan Area. Next, any Moving LISA trajectories having *their first space-time anchor location within the City of Oklahoma City's City limits* were filtered into a 2nd dataset to be analyzed as *Municipal* trajectories. The distinction assists in meeting the research questions of this study, to isolate and compare economic effects for the identification of gentrification signals as dyadic movements.

4.2.3.1 Non-Parametric Tests for Difference of Medians

A Wilcoxon rank-sum test with continuity correction (R Core Team, 2020; Wilcoxon, 1992) for difference in (pseudo)medians was evaluated for the Moving LISA result sets respective distributions of dyadic movement metrics (example, *DI 9* and *DI*) for Moving LISA trajectories generated from patterns of median home values and median incomes at the US census tract level

for the study area and period. Selected as the Moving LISA's local autocorrelation statistic for this research, the Getis-Ord Local G metric spatially identifies statistically significant clusters for a single input variable and allows for one-tailed significance testing establishing either high-value (hotspots) or low-value (cold spots) (Equation 1). Generation of trajectories for hotspot and cold spot displacements in each of the input variables is then possible.

For each dyadic pairing of Home Value Hotspots versus Median Income Value Cold Spots and Home Value Hotspots versus Median Income Value Hot Spots Moving LISA trajectories, and additionally for each of the *All Trajectories* and *Municipal Trajectories* groups, respectively, Wilcox tests were evaluated on dyadic movement metric results. Here, for example among two dyadic pairings of output trajectories, each of two spatial subsets were evaluated for a total of four Wilcox test results, in each of the *DI 9* and *DI* metrics. While distributional visualizations of *DI 9* may suggest normality in a given result, we note the process of obtaining *DI 9* values warranted a non-parametric test for differences.

Obtaining *DI 9* for a dyad of Moving LISA result trajectories is a deterministic calculation evaluated on the characteristics of their space-time anchors. These anchors result from a Moving LISA procedure which is sensitive to parameter selection and the arrangement of its input geographies, it is not known or guaranteed that *DI 9* values would follow and be comparable to any parametric distribution when calculated from Moving LISA result trajectories. For example, the action of a spatial process could produce different Moving LISA results when its observations are aggregated to different input areal units, where Moving LISA input parameters are held constant. This renders the *distance* component of a given relocation with a Moving LISA trajectory dependent in large part on the shapes of the input geographies similar to Modifiable Aerial Unit Problem concerns (Openshaw, 1981). In the context of examining the

dyadic movements of Moving LISA trajectories, since relative turning angles are less dependent on input shape than relocation distances, we note angles serve as a less sensitive measure of attraction or repulsion for analysis.

To better understand distinctions between the *DI 9* results returned by the *Municipal* and *All-Trajectories* result sets, one-tailed nonparametric tests for difference in (pseudo)medians of sets of *DI 9* values were evaluated. Specifically, we note *DI 9* values calculated between high median home value and low median income value trajectories show a significant and negative shift in (pseudo)median location, for comparison of the *All-Trajectories* set versus the *Municipal* subset. Here, the “repulsion” or divergence in direction for relocation turning angles is shown to intensify for trajectories originating *inside the OKC City limits*. Conversely, increased “attraction” is suggested between high median home value and high median income value trajectories, between their respective *Municipal* and *All Trajectories* subsets.

4.2.3.2 *Guidance on Interpreting Divergences in Moving LISA Trajectories*

Trajectory analytics, as presented in Movement Ecology, Time Geography and Mobility Science tend to focus on the relocations of discrete and visible actors across a meaningful spatiotemporal context. The subject matter for the Moving LISA procedure departs somewhat from this focus to incorporate and characterize processes referred to as invisible, spatially indistinct or distributed phenomena. A given Moving LISA trajectory also departs, with respect to certain traits, from the prevailing character of a mover’s trajectory as may be expected in these disciplines. It may be surprising or counterintuitive to approach Moving LISA trajectories under the same rules and expectations which would be cogent for animals, pedestrians or city taxis. For example, where a Time Geographic study might track the movements of a migratory bird, due to biological factors

we do not expect this single animal to spontaneously clone or divide itself into two, proceeding from this spacetime point as if two animals had moved coincidentally and then diverged paths. Further, in terms of Moving LISA output trajectories, these divergences are expected and generate a representation of two distinct copies of a common space-time path, separating at the time of divergence. Here, the Moving LISA’s routine for capturing divergences and ancestry in *aggregate movers* is *amoebic* and hereditary, like the movements of the *Entamoeba histolytica* being reconstructed as two distinct lineages upon the parasite’s reproduction by binary fission. Both entities in this case are assigned evidence of their common ancestry. Mechanistically, this strategy arises from the LIFO (*last in, first out*) routine for connecting the space-time-anchors yielded by the Moving LISA procedure and accounting for trajectory ancestries (Anderson & Yang, 2025). Here, the lineages of output Moving LISA trajectories are traced in reverse from t_{max} such that every entity present is provided with a space-time-path capturing the evolution of clustering locations resulting in their current state.

4.3 Study Area and Data

For this research, the ACS 5-year variables “B25077_001E” *Estimated Median Value (dollars) for Owner-Occupied Housing Units* and “B19013_001E” *Estimated Median Household Income in the Past 12 Months* were collected for our analysis, pursuant to the theoretical characterization of gentrification found in (Bunten et al., 2024). These data were collected at the tract level for ACS 5-year results having end-years of 2009, 2013, 2018 and 2023, respectively. Collection of this data was facilitated using the *tidycensus* package for R (Walker & Herman, 2025). Recent municipal boundaries for the City of Oklahoma City and neighboring municipalities were obtained from the City of Oklahoma City’s *Open Data Portal*,

(<https://data.okc.gov/portal/page/start>). Following the application of Moving LISA to these input spatial panel data, three representative space-time trajectories were retained for further analysis. These include trajectories of median income hotspots, median income coldspots, and median home value hotspots, respectively. Moving LISA trajectories are then paired and analyzed as dyads using methods from movement ecology.

4.4 Results

4.4.1 Moving LISA Parameterization and Trajectories

4.4.1.1 Parameterization

Parameterization of the Moving LISA consisted of iterative execution of the procedure, where values selected for further visualization were identified using a “parameter sweep” strategy. This approach reveals the range of responses to varied parameter inputs in the evaluation of the Moving LISA approach across each respective input dataset. Once summary visualizations accounting for the appearances, convergences, divergences and terminations of *aggregate movers* under alternative parameters are available, we further selected a set of representative parameter choices whose outputs are visualized for further discussion. Figure 2 reflects the results of the parameter sweep step for each of the median income hotspots, median income coldspots, and median home value hotspots, respectively.

For further analysis in context with the results of this parameter sweep, Moving LISA trajectories were subset for the results generated using a *contiguity cutoff* (π) value of 0.3, a *standard distance scaling factor* (ϕ) of 2.0, and a *significance boundary* (α) of 0.05 (one-tailed) for evaluations of the Getis-Ord Local G in both the hotspot and cold spot specifications. This set of parameter selections was applied to the generation of all three output trajectory types.

Contiguity comparisons for entity-linking of clusters over time relied on 1000 draws from a bivariate normal distribution at each time step in concert with the geometries of clusters and the *scaling factor* ϕ for each candidate cluster of type to be included in an output trajectory (Anderson & Yang, 2025). Figures (3 and 4) show the overall set of Moving LISA trajectories generated at this parameter selection for the three thematic variables of interest.



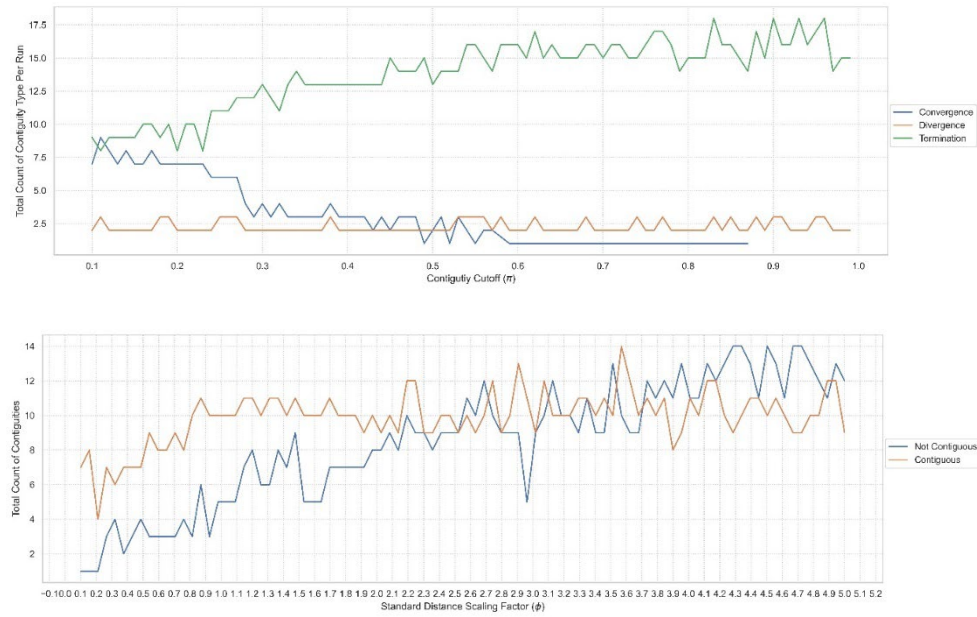


Figure 2. Parameter sweep results for each of contiguity cutoff and standard distance scaling factor values, for each of the median income hotspot, median income cold spot and home value hotspot Moving LISA runs, respectively.

4.4.1.2 Moving LISA output Trajectories

Moving LISA output trajectories for the three themes of interest (Home Value Hot Spots and Income Hot and Cold Spots) were produced and visualized using ArcGIS Pro 3.5.4. Practical visualization of a given Moving LISA output trajectory involves constructing Z-aware vector polylines which trace the lineage of a mover's appearance and relocation over a sequence of unique identifiers for "self" and for "immediate ancestor" assigned to space-time-anchors. The result is shown in Figures 3, 4, 5 and 6. In Figures 3 and 4, we show the entire result set across these three variables. In Figures 5 and 6 respectively, representative dyads are shown to illustrate the concept of avoidance between low median incomes and high home values, and the reverse of cohesion among high incomes and high home values.

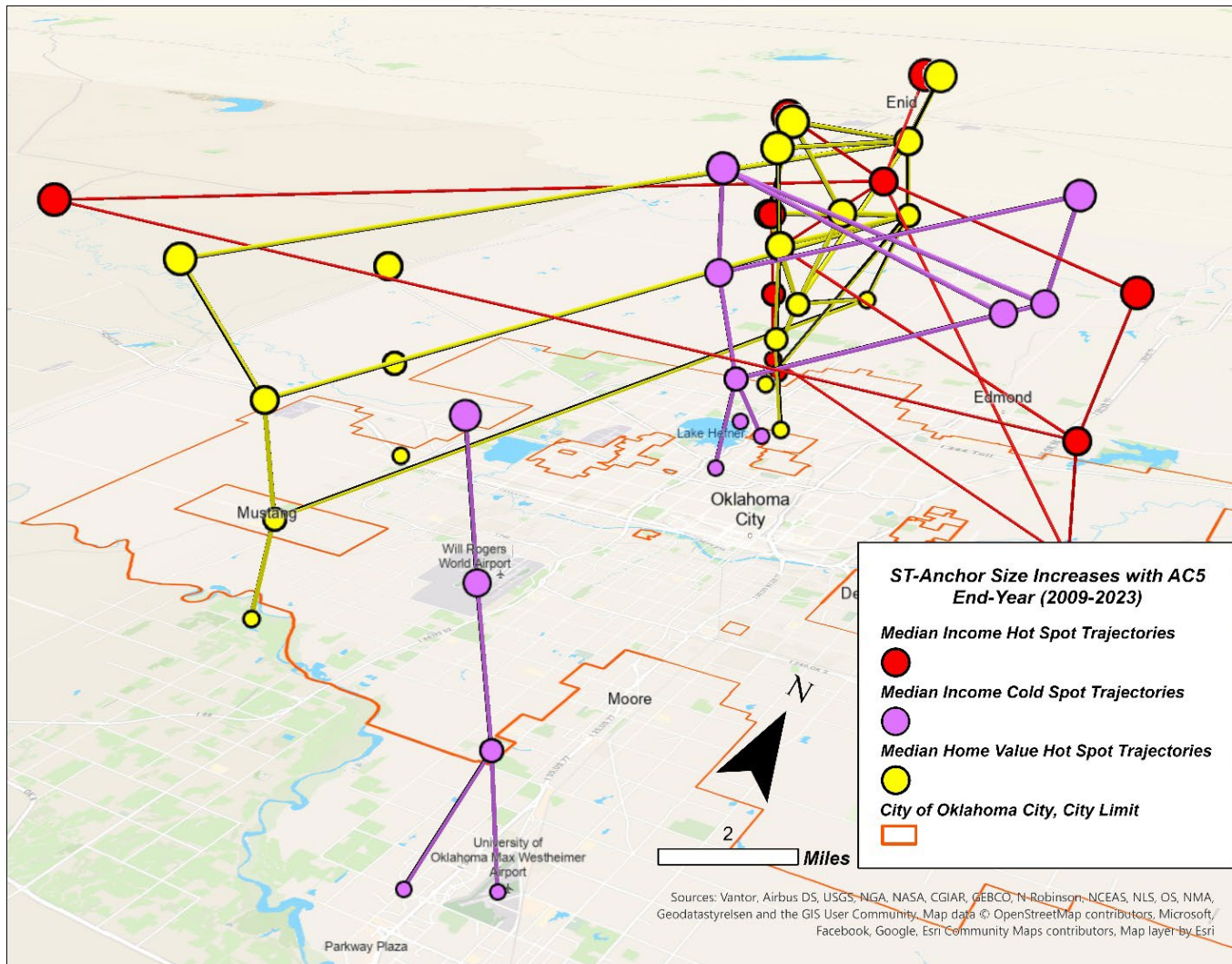


Figure 3. Visualization of the overall set of home value hotspots, median income cold spots, and median income hot spots trajectories.

For each of four study periods, the Z-axis plots the Moving LISA output trajectories across the X/Y space of the Oklahoma City metropolitan statistical area.

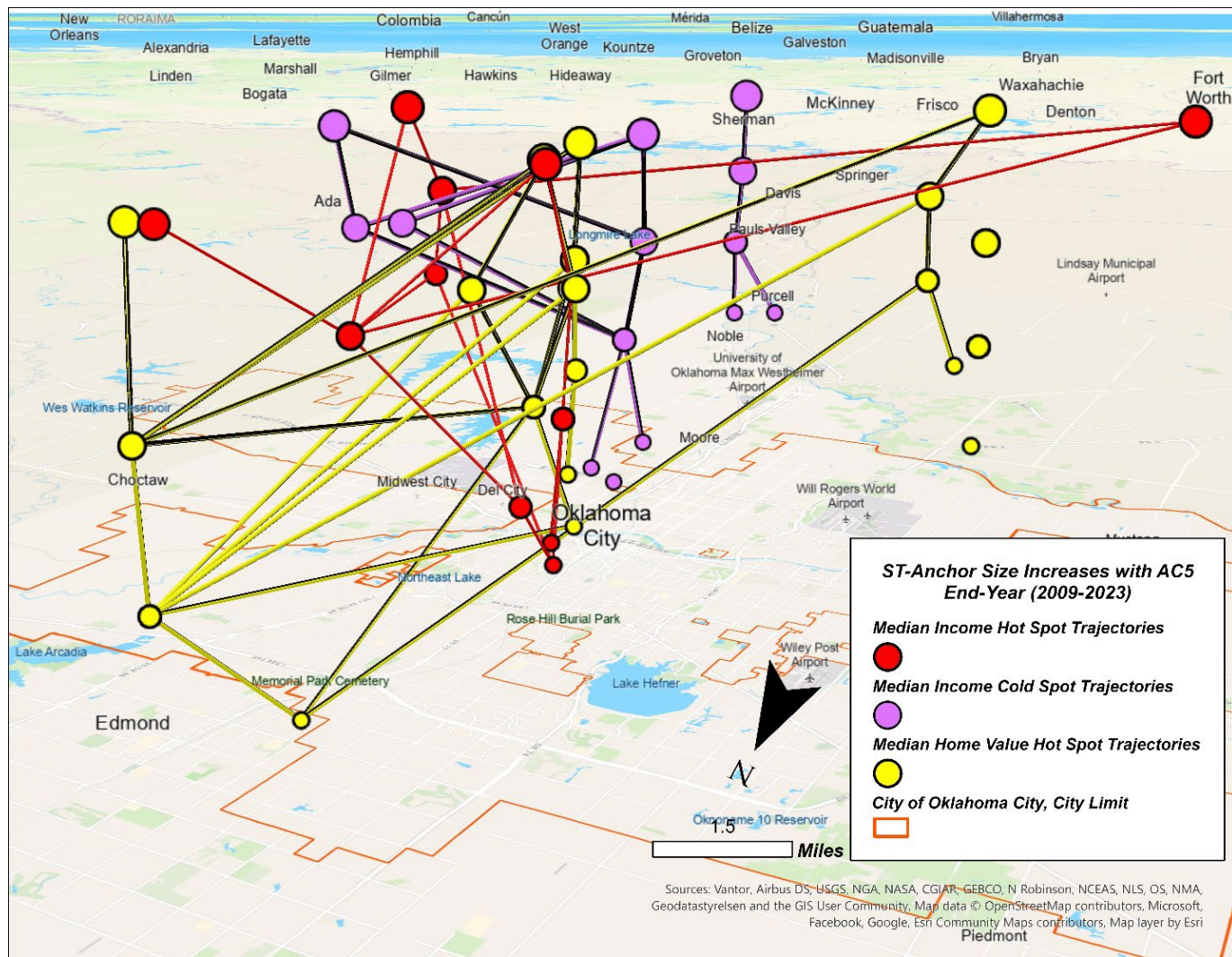


Figure 4. Rotated view of Figure 3, showing the set of Moving LISA result trajectories as viewed from areas north of the Oklahoma City metropolitan statistical area.

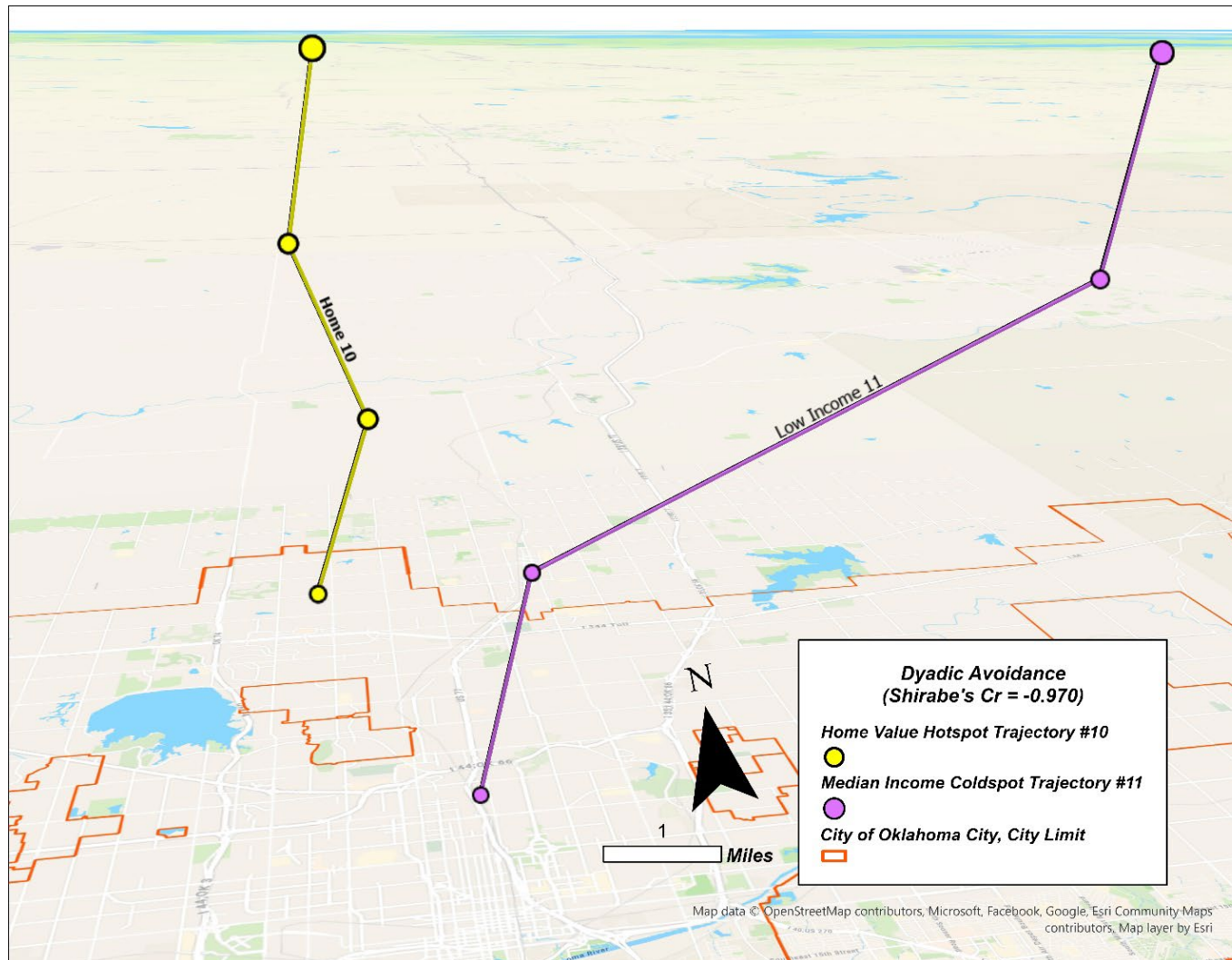


Figure 5. Example of a repulsed or avoidant dyad of home value hotspots and median income cold spots trajectories.

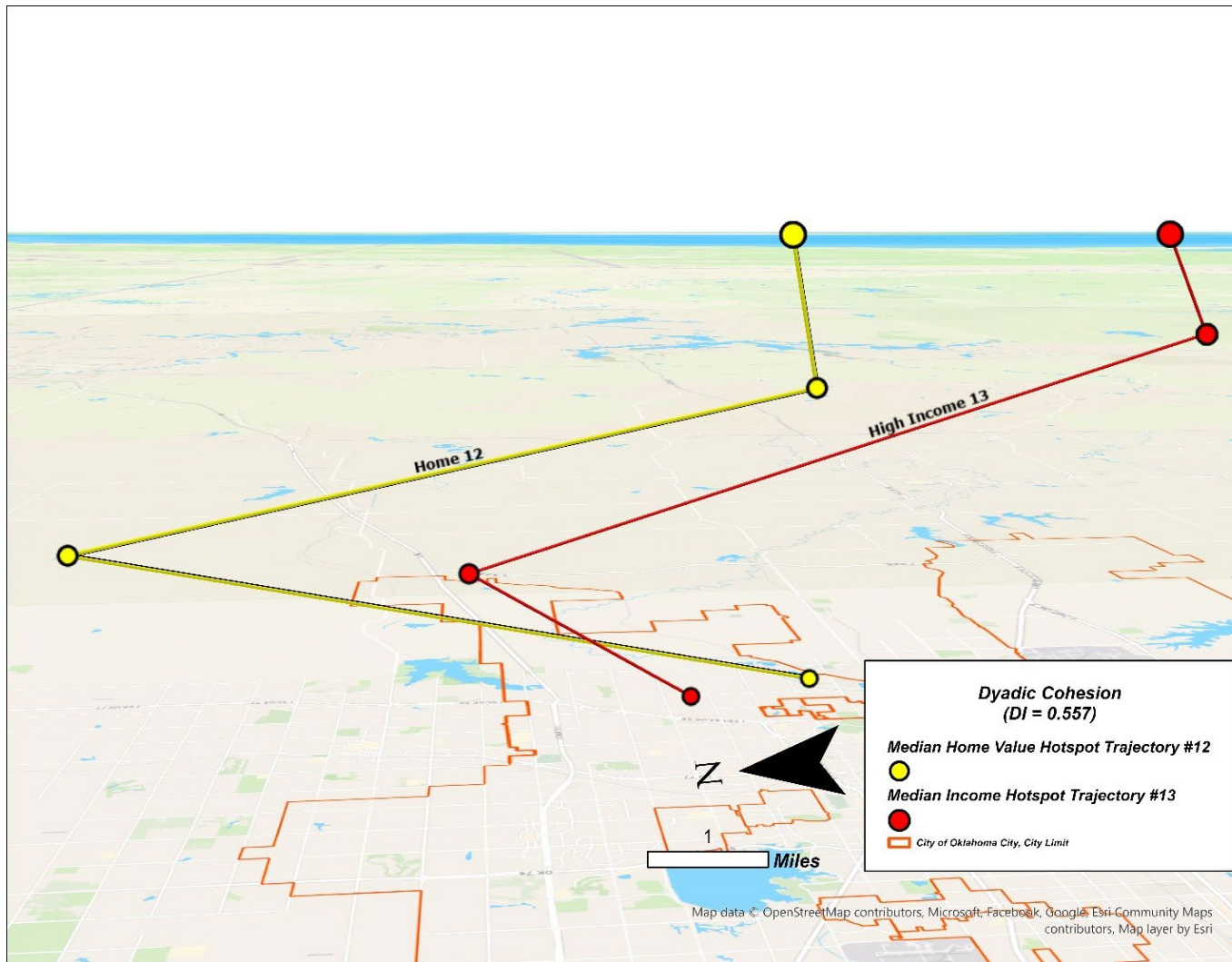


Figure 6. Example of an attracted or cohesive dyad of home value hotspots and median income hot spots trajectories.

4.4.2 Dyadic Movement in Moving LISA Trajectories

4.4.2.1 Dynamic Interaction

Visualizations for the respective distributions of DI and $DI \theta$ were constructed from values calculated pairwise for all Moving LISA output trajectories of types median home value hot spot versus median income cold spot, and median income hot spot, respectively. These comparisons were also stratified and re-calculated to include only those median home value hot spot trajectories which have their initial space-time-anchor within the OKC city limits versus the median income sets. Figures 7 and 8 display histograms for these result sets. Tables (1, 2, 3 and 4) relate top five the most repelled and most cohesive trajectory pairings by DI and $DI \theta$ for these input sets.

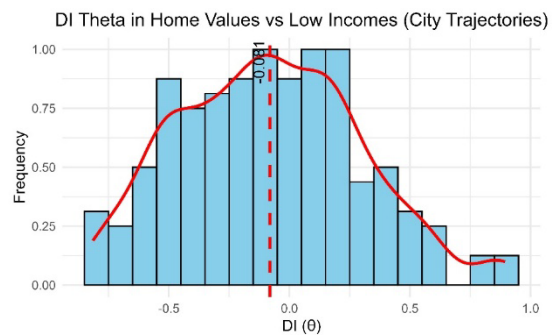
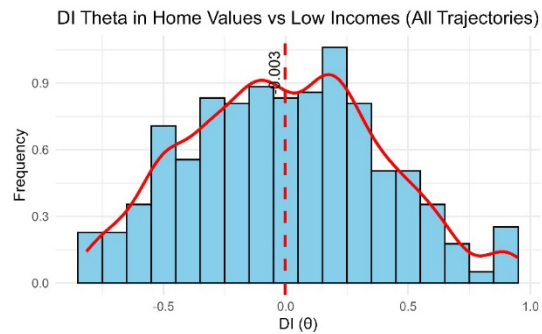
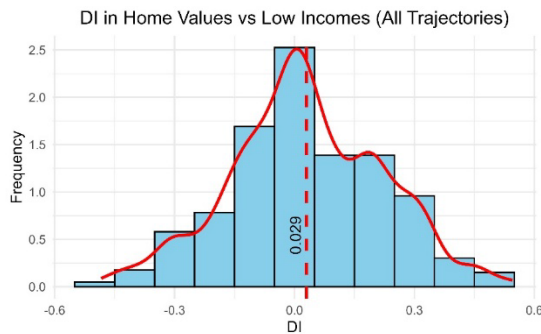


Figure 7. Distribution of DI and $DI \theta$ results for trajectories comprised of home value hotspots and median income cold spots outside of, and within OKC, respectively.

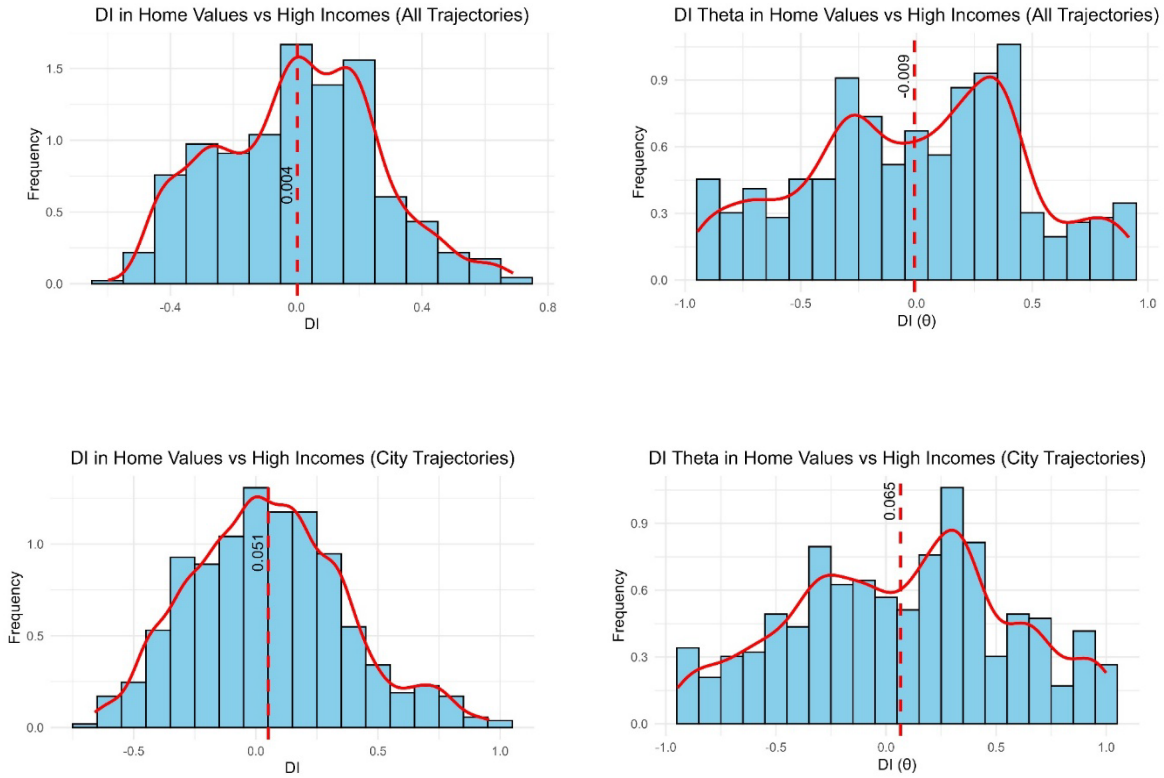


Figure 8. Distribution of DI results for trajectories comprised of home value hotspots and median income hot spots outside of, and within OKC, respectively.

Most Repelled Moving LISA Dyads by DI

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Coldspot Traj ID	DI	$DI \theta$	$DI.d$
home_33	Oklahoma City	lowinc_1	-0.338	-0.527	0.348
home_4	Nichols Hills	lowinc_12	-0.312	-0.751	0.405

Most Repelled Moving LISA Dyads by DI

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Coldspot Traj ID	DI	$DI \theta$	$DI.d$
home_32	Nichols Hills	lowinc_2	-0.284	-0.812	0.306
home_20	Nichols Hills	lowinc_12	-0.278	-0.631	0.549
home_33	Oklahoma City	lowinc_11	-0.268	-0.390	0.376

Table 1. Identities of the most-repelled *Municipal* trajectories comprised of home value hotspots and median income cold spots by DI results.

Most Repelled Moving LISA Dyads by $DI \theta$

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Coldspot Traj ID	DI	$DI \theta$	$DI.d$
home_20	Nichols Hills	lowinc_10	-0.238	-0.815	0.266
home_32	Nichols Hills	lowinc_2	-0.284	-0.812	0.306
home_24	Nichols Hills	lowinc_4	-0.263	-0.776	0.336
home_4	Nichols Hills	lowinc_10	-0.236	-0.769	0.248
home_4	Nichols Hills	lowinc_12	-0.312	-0.751	0.405

Table 2. Identities of the most-repelled *Municipal* trajectories comprised of home value hotspots and median income cold spots by $DI \theta$ results.

Most Attracted Moving LISA Dyads by DI

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Hotspot Traj ID	DI	$DI \theta$	$DI.d$
home_12	Nichols Hills	highinc_13	0.557	0.886	0.632
home_6	Nichols Hills	highinc_2	0.515	0.796	0.567
home_26	Nichols Hills	highinc_12	0.500	0.786	0.560
home_26	Nichols Hills	highinc_2	0.459	0.761	0.536
home_14	Nichols Hills	highinc_12	0.454	0.777	0.517

Table 3. Identities of the most-attracted *Municipal* trajectories comprised of home value hotspots and median income hot spots by DI results.

Most Attracted Moving LISA Dyads by $DI \theta$

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Hotspot Traj ID	DI	$DI \theta$	$DI.d$
home_20	Nichols Hills	highinc_3	0.383	0.918	0.421
home_4	Nichols Hills	highinc_3	0.420	0.898	0.499
home_12	Nichols Hills	highinc_13	0.557	0.886	0.632

Most Attracted Moving LISA Dyads by $DI \theta$

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Hotspot Traj ID	DI	$DI \theta$	$DI.d$
home_22	Nichols Hills	highinc_12	0.307	0.812	0.342
home_6	Nichols Hills	highinc_8	0.327	0.811	0.362

Table 4. Identities of the most-attracted *Municipal* trajectories comprised of home value hotspots and median income hot spots by $DI \theta$ results.

4.4.2.2 Difference in Pseudo-medians, $DI \theta$ Municipal versus $DI \theta$ All-Trajectories

The *Municipal* high median home value and high median income value trajectories show a significant and positive shift in $DI \theta$ values versus the *All Trajectories* set (Table 5).

Wilcoxon Rank Sum Tests for Moving LISA $DI \theta$

Estimate	Statistic	P-value	Low Interval	High Interval	Alternative H0
<i>High Home Values and High Incomes, Municipal vs. All Trajectories</i>					
0.0683	2,110,000	0.00000511	0.0415	Inf	greater
<i>High Home Values and Low Incomes, Municipal vs. All Trajectories</i>					
-0.0791	447,000	0.00000727	-Inf	-0.0498	less

Table 5. Results of one-tailed Wilcoxon rank-sum tests with continuity correction, for $DI \theta$ values generated by Moving LISA trajectories.

Conversely, and potentially due to the sensitivity of relocation distances discussed in this section, we note equivalent tests for *DI* results were not significant for either of the one-tailed tests evaluated. Corresponding test results for the *DI* metric are shown in Table 6.

Wilcoxon Rank Sum Tests for Moving LISA <i>DI</i>					
Estimate	Statistic	P-value	Low Interval	High Interval	Alternative H0
<i>High Home Values and High Incomes, Municipal vs. All Trajectories</i>					
-0.0238	781,000	0.992	-0.0405	Inf	greater
<i>High Home Values and Low Incomes, Municipal vs. All Trajectories</i>					
-0.00171	503,000	0.379	-Inf	0.0112	less

Table 6. Results of one-tailed Wilcoxon rank-sum tests with continuity correction, for *DI* values generated by Moving LISA trajectories.

4.4.2.3 Correlation Analysis of Discrete Motions (Shirabe's Correlation, *Cr*)

Visualizations for the distributions of *Cr* were constructed from values calculated pairwise for all Moving LISA output trajectories of types median home value hot spot versus median income cold spot, and median income hot spot, respectively. These comparisons were also stratified and re-calculated to include only those median home value hot spot trajectories which have their initial space-time-anchor within the OKC city limits versus the median income sets. Figure 9 displays histograms for these result sets. Tables (7 and 8) relate top five the most repelled and most cohesive trajectory pairings by *Cr* for these input sets.

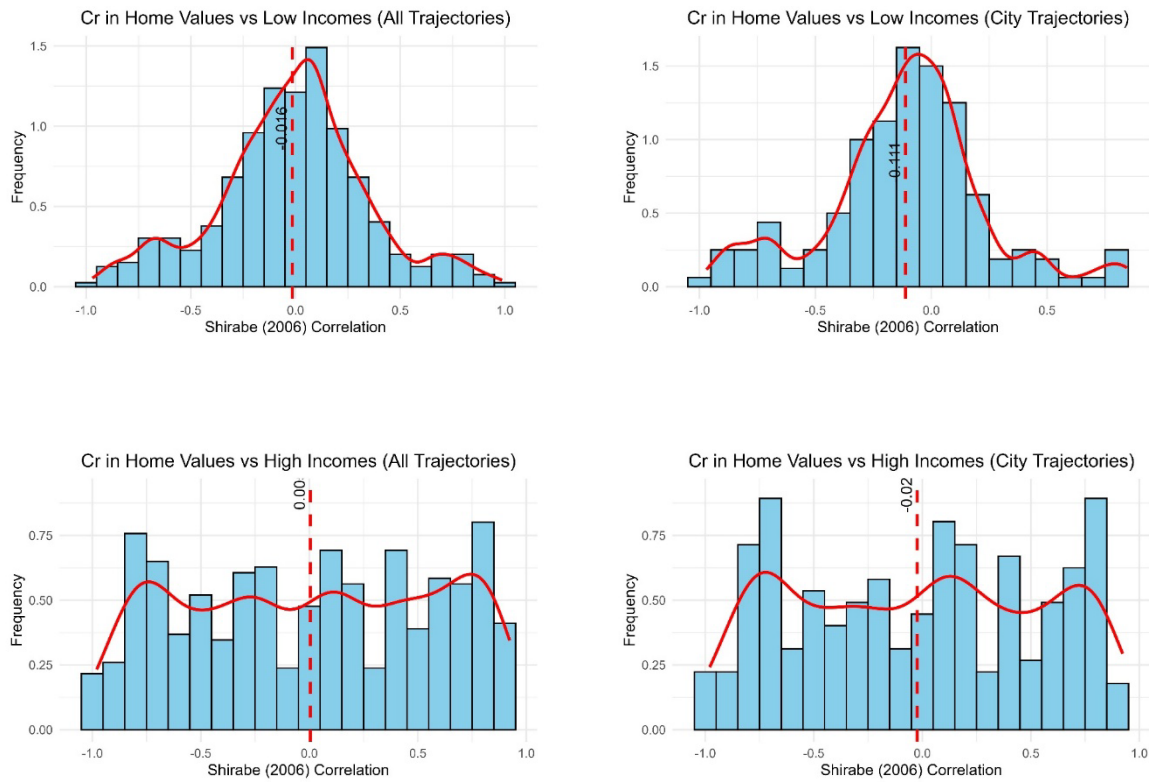


Figure 9. Distribution of Shirabe's Cr results for trajectories comprised of home value hotspots and median income cold spots.

Most Repelled Moving LISA Dyads, Shirabe's Correlation

Home Value Hotspot Traj	City Name of Home Value Trajectory	Med Income Coldspot Traj	
ID	Origin	ID	Cr
home_10	Nichols Hills	lowinc_11	-0.970
home_24	Nichols Hills	lowinc_2	-0.907
home_20	Nichols Hills	lowinc_2	-0.895

Most Repelled Moving LISA Dyads, Shirabe's Correlation

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Coldspot Traj ID	<i>Cr</i>
home_10	Nichols Hills	lowinc_12	-0.871
home_18	Nichols Hills	lowinc_9	-0.864

Table 7. Identities of the most-repelled *Municipal* trajectories comprised of home value hotspots and median income cold spots by Shirabe's *Cr* results.

Most Attracted Moving LISA Dyads, Shirabe's Correlation

Home Value Hotspot Traj ID	City Name of Home Value Trajectory Origin	Med Income Coldspot Traj ID	<i>Cr</i>
home_22	Nichols Hills	highinc_12	0.923
home_8	Nichols Hills	highinc_1	0.911
home_28	Nichols Hills	highinc_11	0.880
home_22	Nichols Hills	highinc_2	0.860
home_30	Nichols Hills	highinc_13	0.849

Table 8. Identities of the most-attracted *Municipal* trajectories comprised of home value hotspots and median income hot spots by Shirabe's *Cr* results.

4.4.2.4 Difference in Pseudo-medians, *Cr* Municipal versus *Cr* All-Trajectories

Following from the application of a Wilcoxon Rank Sum Test on *DI 9* and *DI* metrics, equivalent tests for *Cr* suggest that high median home values repel the movements of low median income values more within OKC than outside of OKC. In a one-tailed test for difference in pseudo-medians, the median value for *Cr* is significantly less (more repelled, less cohesive) within the City Limits of OKC than outside (Table 9).

Wilcoxon Rank Sum Tests for Moving LISA <i>Shirabe Cr</i>					
Estimate	Statistic	P-value	Low Interval	High Interval	Alternative H0
<i>High Home Values and High Incomes, Municipal vs. All Trajectories</i>					
-0.0247	803,000	0.896	-0.0601	Inf	greater
<i>High Home Values and Low Incomes, Municipal vs. All Trajectories</i>					
-0.0946	418,000	0.0000000000555	-Inf	-0.0704	less

Table 9. Results of one-tailed Wilcoxon rank-sum tests with continuity correction, for *Shriabe's Cr* values generated by Moving LISA trajectories.

4.4.2.5 Coefficient of Sociality (*Cs*)

Coefficient of Sociality (*Cs*) results were generally inconclusive and largely failed to meet a significance boundary of $p=0.05$ for each respective analyzed pair of Moving LISA trajectories. Of the metrics evaluated in this research, only *Cs* attempts to permute a null distribution for the expected distances between trajectory space-time anchors (Kenward et al., 1993). This approach has since been critiqued as highly sensitive to the length of the input trajectory in terms of the

count of available space-time anchors or relocations and has been identified as difficult for users to interpret (Joo et al., 2018). In the present study we are analyzing a set of procedurally suggested Moving LISA trajectories, where each trajectory only contains four relocations of a generated entity tracked over 5-year periods. These may not provide a candidate sample of movement for use with the Coefficient of Sociality method in context with the structure and strategies of the metric. Despite this potential mismatch in data versus analytic approach, one Moving LISA dyad (*home_4* vs. *lowinc_1*) was significant for avoidance or repulsion at a 90% confidence level ($p < 0.10$). It should be noted for future analysis that regardless of the input being a generated Moving LISA trajectory or otherwise directly observed trajectory using telemetry or GPS, that caveats and limitations present in interaction metrics need to carefully considered in context with the data generating process. Specifically, with respect to *Cs*, the length of a Moving LISA trajectory in terms of relocations is a direct reflection of the number of time steps considered in the Moving LISA procedure.

4.5 Discussion

This research has first generated movement trajectories from observed change in socioeconomic variables associated with gentrification and has followed with their paired analysis from the perspective of dyadic movement. The goal is twofold; (1) to provide for the first known direct examination of Moving LISA trajectories as dyadic movers, and (2) to carry out this examination with a view to understanding gentrification in the OKC MSA. The results set is spatiotemporally descriptive of gentrification in the OKC study area from 2009-2023 and supports an analysis affirmative of prevailing theoretical constructions of gentrification observed at the level of US Census MSA's. This research demonstrates an alternative approach to capturing and analyzing

the dynamics of gentrification as expressed in median home values and incomes in terms of their spatial emergence.

Bunten et al. showed that theorized capital and human movements parcel to the gentrification process may be captured by a small set of socioeconomic variables, once certain variables were reframed as containing future-oriented information (Bunten et al., 2024). The core theoretical contribution of this approach is both tractable and elegant in this clear temporal implication between incomes and home values. Namely in Bunten et al. 2024, as a time-lagged series of events approaching categorical gentrification, market action in a susceptible area will first raise home values, then vulnerable people will begin to move away, followed by inflow of higher-earning people. Upon operationalizing the theory as a signal, additional findings including an acceleration of the gentrification process for areas closer to the central business district were observed (Bunten et al., 2024).

The present research embarked on a Time Geographic study of these same variables seeking further evidence for the future-oriented variable framework and findings presented with the BGS. We believe our results suggest a Time Geographic assessment of the gentrification process, for example as theorized in Bunten et al. (2024), is meaningful and possible through its examination as dyadic movement. By establishing an ontological bridge between the dynamics and emergence of spatial phenomena reported in aggregate and existing analytics for movement, the Moving LISA avails new treatments to open spatial questions.

The results further provide for a demonstration in the direct examination of Moving LISA trajectories as input to methods from movement ecology. Commensurate with the trends identified in Bunten et al., paired Moving LISA trajectories for high median home values and low median incomes were avoidant in their movements, with dyads having a significantly lower

pseudo-median value for distributions of $DI \theta$ inside of city limits than in all trajectories, based on one-tailed Wilcoxon tests. Conversely, higher pseudo-medians were shown in distributions of $DI \theta$ for attracted or cohesively moving dyads of high home values and high median incomes. Further, results for Shirabe's Cr showed significantly lower pseudomedian values in-city-limits than in all trajectories for high home values and low median incomes, signaling an intensifying avoidance for trajectories originating within the OKC city limits. For reported city of origin labels for the top-scoring home value trajectories (Tables 1, 2, 3, 4, 7, and 8), we note the City of Nichols Hills origin returned is a representative location for the origin of these high-scoring *aggregate movers*. The resulting origin space-time anchor location for these median home value hotspot trajectories is a function of spatial comparisons between clustering geographies over time, followed by a reduction in dimension to a centroid point per the Moving LISA procedure (Anderson & Yang, 2025). While the origin space-time anchor is representative and should be viewed as an approximated location, it also falls within an area of the OKC MSA reported to experience residential gentrification events during the study period (Fleming, 2016; Riley, 2016). Aside from Nichols Hills and reinvestment surrounding Lake Hefner, the OKC MSA's storied Deep Deuce district is an area with pronounced gentrification occurring over a period exceeding the timeframe examined in this study (Payne & Greiner, 2019). Potentially, the focus on home values versus rents or values of multifamily units in the present research may have led to a failure to highlight gentrification pressures still at work in Deep Deuce.

In terms of contribution to Time Geography, the present research represents the first study to analyze Moving LISA result trajectories directly as inputs to animal movement metrics. The methods employed treat the trajectories generated not only as a mapping of change in extreme-valued locations for variables of interest, but instead as information encoded in relocation

distances and turning movements interpretable as dyadic movement interaction. This component of the present research has arrived with both a novel analysis perspective and a corresponding influx of new considerations and limitations. First, while the literature from Movement Ecology is well developed in its treatment of trajectories as information, constituent methods are inherently geared towards living, individual animals. The Moving LISA trajectory conversely describes an abstract variety of mover, an entity delineated from census geographies which moves and reacts independently of influences cogent to the natural world. At the time of this writing for example, no such ecology of the *aggregate mover* for estimated median incomes is available or discussed. Moving LISA entities do not have a known top speed, or meaningful habitat preference. Many of the dyadic movement metrics evaluated for inclusion in this study were not compatible with Moving LISA trajectories due to requirements such as parameter choices for known maximum velocity, biologically informed interaction distance thresholds, or *a priori* delineation of a shared reference area or resource. The study was limited to selecting dyadic movement measures which strictly relied on information held in trajectory relocations.

Ostensibly until a body of research involving Moving LISA trajectories for socioeconomic variables develops, the commonalities among studies necessary to suggest for these biologically-adjacent and meaningful parameter selections would not yet be available. Second, as the first study to directly examine Moving LISA trajectories as dyadic movers, Moving LISA's own sensitivity to selections for *contiguity cutoff* (π), *standard distance scaling factor* (ϕ), and the internal LISA's *significance boundary* (α) are highlighted as influential to the result. Thus far, a parameter-sweep approach followed by a qualitative parameter selection based on divergences in results is the only such tuning procedure coupled to use of the Moving LISA (Anderson & Yang, 2025). A threshold calculated for a moving window along the axes of

these three parameters could provide for a more reproducible strategy for parameter selection between studies. Finally, future research dealing with temporally lagged dyadic movement, where generated trajectories contained more relocations, could shift and or lag the relative space-time anchors among individual dyads to reveal a type of spatiotemporal cross-correlation function in metrics like the *DI* and *Cr* as presented group-wise in this research. This could provide an additional and uniquely Time Geographic perspective adjacent to the BGS's 25-point percentile gap strategy, where strong changes in *DI* could signal the onset of outmigration. The Moving LISA is new and has appeared in the literature so far only for an introduction of the approach. The proper and meaningful analysis of Moving LISA results is a developing and open question which will demand adequate domain knowledge in each future application of the method.

4.6 References

- Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6), 716–723. <https://doi.org/10.1109/TAC.1974.1100705>
- Anderson, J., & Yang, A. (2025). Moving Local Indicators of Spatial Association for Time Geography. *Transactions in GIS*, 29(8), e70158. <https://doi.org/10.1111/tgis.70158>
- Anselin, L. (1988). *Spatial Econometrics: Methods and Models* (Vol. 4). Springer Science & Business Media.
- Anselin, L. (1995). Local Indicators of Spatial Association—LISA. *Geographical Analysis*, 27(2), 93–115. <https://doi.org/10.1111/j.1538-4632.1995.tb00338.x>
- Barton, M. (2016). An exploration of the importance of the strategy used to identify gentrification. *Urban Studies*, 53(1), 92–111. <https://doi.org/10.1177/0042098014561723>

- Bennett, D. A., & Tang, W. (2006). Modelling adaptive, spatially aware, and mobile agents: Elk migration in Yellowstone. *International Journal of Geographical Information Science*, 20, 1039–1066.
- Bonnell, T. R., Dutilleul, P., Chapman, C. A., Reyna-Hurtado, R., Hernández-Sarabia, R. U., & Sengupta, R. (2013). Analysing small-scale aggregation in animal visits in space and time: The ST-BBD method. *Animal Behaviour*, 85(2), Article 2.
<https://doi.org/10.1016/j.anbehav.2012.12.014>
- Brown–Saracino, J. (2016). An Agenda for the Next Decade of Gentrification Scholarship. *City & Community*, 15(3), 220–225. <https://doi.org/10.1111/cico.12187>
- Bunten, D. M., Preis, B., & Aron-Dine, S. (2024). Re-measuring gentrification. *Urban Studies*, 61(1), 20–39. <https://doi.org/10.1177/00420980231173846>
- Burt, W. H. (1943). Territoriality and Home Range Concepts as Applied to Mammals. *Journal of Mammalogy*, 24(3), Article 3. JSTOR. <https://doi.org/10.2307/1374834>
- Calenge, C. (2011). *adehabitatLT: Analysis of Animal Movements* (p. 0.3.29) [Dataset].
<https://doi.org/10.32614/CRAN.package.adehabitatLT>
- Camazine, S., Bonabeau, E., Theraula, G., Deneubourg, J.-L., & Franks, N. R. (2020). *Self-Organization in Biological Systems*. Princeton University Press.
<https://doi.org/10.1515/9780691212920>
- Corrigan, A. E., Curriero, F. C., & Linton, S. L. (2021). Characterizing clusters of gentrification in metro Atlanta, 2000 to 2016. *Applied Geography*, 137, 102597.
<https://doi.org/10.1016/j.apgeog.2021.102597>
- Dodge, S. (2021). A Data Science Framework for Movement. *Geographical Analysis*, 53(1), 92–112.

- Dodge, S., Gao, S., Tomko, M., & Weibel, R. (2020). Progress in computational movement analysis – towards movement data science. *International Journal of Geographical Information Science*, 34(12), 2395–2400.
<https://doi.org/10.1080/13658816.2020.1784425>
- Dodge, S., Su, R., Johnson, J., Simcharoen, A., Goulías, K., Smith, J. L. D., & Ahearn, S. C. (2021). ORTEGA: An object-oriented time-geographic analytical approach to trace space-time contact patterns in movement data. *Computers, Environment and Urban Systems*, 88, 101630. <https://doi.org/10.1016/j.compenvurbsys.2021.101630>
- Dodge, S., Weibel, R., & Lautenschütz, A.-K. (2008). Towards a taxonomy of movement patterns. *Information Visualization*, 7(3–4), Article 3–4.
<https://doi.org/10.1057/PALGRAVE.IVS.9500182>
- Downs, J. A., Heller, J. H., Loraamm, R., Stein, D. O., McDaniel, C., & Onorato, D. (2012). Accuracy of home range estimators for homogeneous and inhomogeneous point patterns. *Ecological Modelling*, 225, 66–73. <https://doi.org/10.1016/j.ecolmodel.2011.11.010>
- Downs, J. A., & Horner, M. W. (2009). A Characteristic-Hull Based Method for Home Range Estimation: A Characteristic-Hull Based Method for Home Range Estimation. *Transactions in GIS*, 13(5–6), Article 5–6. <https://doi.org/10.1111/j.1467-9671.2009.01177.x>
- Downs, J., Horner, M., Lamb, D., Loraamm, R., Anderson, J., & Wood, B. (2018). Testing time-geographic density estimation for home range analysis using an agent-based model of animal movement. *International Journal of Geographical Information Science*, 32(7), Article 7. <https://doi.org/10.1080/13658816.2017.1421764>

- Downs, J., Horner, M. W., Hyzer, G., Lamb, D., & Loraamm, R. (2014). Voxel-based probabilistic space-time prisms for analysing animal movements and habitat use. *International Journal of Geographical Information Science*, 28(5), Article 5. <https://doi.org/10.1080/13658816.2013.850170>
- Downs, J., Lamb, D., Hyzer, G., Loraamm, R., Smith, Z. J., & O'Neal, B. M. (2014). Quantifying spatio-temporal interactions of animals using probabilistic space–time prisms. *Applied Geography*, 55, 1–8. <https://doi.org/10.1016/j.apgeog.2014.08.010>
- Fan, Y., Wang, H., Wang, L., Guo, S., & Liu, J. (2023). A novel trajectory similarity measurement method based on node-sequence hierarchical digraph. *Transactions in GIS*, 27(8), 2320–2342. <https://doi.org/10.1111/tgis.13121>
- Fielder. (2018). Designing Latinidad: Gulf South Migration and Contemporary Gentrification in Ybor City, Florida. *The Global South*, 12(1), 89. <https://doi.org/10.2979/globalsouth.12.1.06>
- Finio, N. (2022). Measurement and Definition of Gentrification in Urban Studies and Planning. *Journal of Planning Literature*, 37(2), 249–264. <https://doi.org/10.1177/08854122211051603>
- Fleming, M. (2016, January 19). Millionaire's row: Developer to replace rental homes in Nichols Hills. *The Journal Record*. <https://journalrecord.com/2016/01/29/millionaires-row-developer-to-replace-rental-homes-in-nichols-hills-real-estate/>
- Forster, M., & Sober, E. (1994). How to Tell When Simpler, More Unified, or Less *Ad Hoc* Theories will Provide More Accurate Predictions. *The British Journal for the Philosophy of Science*, 45(1), 1–35. <https://doi.org/10.1093/bjps/45.1.1>

- Getis, A., & Aldstadt, J. (2004). Constructing the Spatial Weights Matrix Using a Local Statistic. *Geographical Analysis*, 36(2), 90–104. <https://doi.org/10.1111/j.1538-4632.2004.tb01127.x>
- Getis, A., & Ord, J. K. (1992). The Analysis of Spatial Association by Use of Distance Statistics. *Geographical Analysis*, 24(3), 189–206. <https://doi.org/10.1111/j.1538-4632.1992.tb00261.x>
- Glass, R. L. (1960). *London: Aspects of change*. MacGibbon & Kee.
- Gould, K. A., & Lewis, T. L. (2021). Resilience Gentrification: Environmental Privilege in an Age of Coastal Climate Disasters. *Frontiers in Sustainable Cities*, 3, 687670. <https://doi.org/10.3389/frsc.2021.687670>
- Hägerstrand, T. (1970). What about people in Regional Science? *Papers of the Regional Science Association*, 24(1), Article 1. <https://doi.org/10.1007/BF01936872>
- Halasz, J. R. (2018). The super-gentrification of Park Slope, Brooklyn. *Urban Geography*, 39(9), 1366–1390. <https://doi.org/10.1080/02723638.2018.1453454>
- Ho, K., & Loraamm, R. (2023). Using a Cost-Distance Time-Geographic Approach to Identify Red Deer Habitat Use in Banff National Park, Alberta, Canada. *ISPRS International Journal of Geo-Information*, 12(8), 339. <https://doi.org/10.3390/ijgi12080339>
- Huisman, O., & Forer, P. (1998). Computational agents and urban life spaces: A preliminary realisation of the time-geography of student lifestyles. *Proceedings of the Third International Conference on GeoComputation, 17-19 September 1998, Bristol, UK* 18 p.
- Immergluck, D., & Balan, T. (2018). Sustainable for whom? Green urban development, environmental gentrification, and the Atlanta Beltline. *Urban Geography*, 39(4), 546–562. <https://doi.org/10.1080/02723638.2017.1360041>

- Joo, R., Etienne, M.-P., Bez, N., & Mahévas, S. (2018). Metrics for describing dyadic movement: A review. *Movement Ecology*, 6(1), 26. <https://doi.org/10.1186/s40462-018-0144-2>
- Kenward, R. E., Marcström, V., & Karlbom, M. (1993). Post-nestling behaviour in goshawks, *Accipiter gentilis*: II. Sex differences in sociality and nest-switching. *Animal Behaviour*, 46(2), 371–378. <https://doi.org/10.1006/anbe.1993.1199>
- Kranstauber, B., Safi, K., & Scharf, A. K. (2023). *move2: Processing and Analysing Animal Trajectories* (p. 0.4.5) [Dataset]. <https://doi.org/10.32614/CRAN.package.move2>
- Lees, L. (2003). Super-gentrification: The Case of Brooklyn Heights, New York City. *Urban Studies*, 40(12), 2487–2509. <https://doi.org/10.1080/0042098032000136174>
- Lees, L. (2012). The geography of gentrification: Thinking through comparative urbanism. *Progress in Human Geography*, 36(2), 155–171. <https://doi.org/10.1177/0309132511412998>
- Liu, C., Deng, Y., Song, W., Wu, Q., & Gong, J. (2019). A comparison of the approaches for gentrification identification. *Cities*, 95, 102482. <https://doi.org/10.1016/j.cities.2019.102482>
- Long, J. (2014). *wildlifeDI: Calculate Indices of Dynamic Interaction for Wildlife Tracking Data* (p. 1.0.1) [Dataset]. <https://doi.org/10.32614/CRAN.package.wildlifeDI>
- Long, J. A. (2015). Quantifying spatial-temporal interactions from wildlife tracking data: Issues of space, time, and statistical significance. In A. Stein & D. Allard (Eds.), *Spatial Statistics Conference 2015* (Part 1, pp. 3–10).
- Long, J. A. (2018). Modeling movement probabilities within heterogeneous spatial fields. *Journal of Spatial Information Science*, (16), Article 16. <https://doi.org/10.5311/JOSIS.2018.16.372>

- Long, J. A., & Nelson, T. A. (2013). Measuring Dynamic Interaction in Movement Data. *Transactions in GIS*, 17(1), 62–77. <https://doi.org/10.1111/j.1467-9671.2012.01353.x>
- Long, J. A., Nelson, T. A., Webb, S. L., & Gee, K. L. (2014). A critical examination of indices of dynamic interaction for wildlife telemetry studies. *Journal of Animal Ecology*, 83(5), 1216–1233. <https://doi.org/10.1111/1365-2656.12198>
- Loraamm, R. (2019). Incorporating behavior into animal movement modeling: A constrained agent-based model for estimating visit probabilities in space-time prisms. *International Journal of Geographical Information Science*, 1–21. <https://doi.org/10.1080/13658816.2019.1658875>
- Loraamm, R., Anderson, J., & Burch, C. (2021). Identifying road avoidance behavior using time-geography for red deer in Banff National Park, Alberta, Canada. *Transactions in GIS*, 25(3), Article 3. <https://doi.org/10.1111/tgis.12748>
- Loraamm, R., Downs, J., Anderson, J., & Lamb, D. S. (2020). PySTPrism: Tools for voxel-based space–time prisms. *SoftwareX*, 12, 100499. <https://doi.org/10.1016/j.softx.2020.100499>
- Miller, H. J. (2005). A Measurement Theory for Time Geography. *Geographical Analysis*, 37(1), Article 1. <https://doi.org/10.1111/j.1538-4632.2005.00575.x>
- Miller, H. J. (2017). Time Geography and Space-Time Prism. In D. Richardson, N. Castree, M. F. Goodchild, A. Kobayashi, W. Liu, & R. A. Marston (Eds.), *International Encyclopedia of Geography: People, the Earth, Environment and Technology* (pp. 1–19). John Wiley & Sons, Ltd. <https://doi.org/10.1002/9781118786352.wbieg0431>
- Minta, S. C. (1992). Tests of Spatial and Temporal Interaction Among Animals. *Ecological Applications*, 2(2), 178–188. <https://doi.org/10.2307/1941774>

- Openshaw, S. (1981). The modifiable areal unit problem. *Quantitative Geography: A British View*, 60–69.
- Ord, J. K., & Getis, A. (1995). Local Spatial Autocorrelation Statistics: Distributional Issues and an Application. *Geographical Analysis*, 27(4), 286–306. <https://doi.org/10.1111/j.1538-4632.1995.tb00912.x>
- Payne, A. A., & Greiner, A. L. (2019). New-Build Development and the Gentrification of Oklahoma City’s Deep Deuce Neighborhood. *Geographical Review*, 109(1), 108–130. <https://doi.org/10.1111/gere.12294>
- Preis, B., Janakiraman, A., Bob, A., & Steil, J. (2021). Mapping gentrification and displacement pressure: An exploration of four distinct methodologies. *Urban Studies*, 58(2), 405–424. <https://doi.org/10.1177/0042098020903011>
- R Core Team. (2020). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. <https://www.R-project.org>
- Raymond, E. L., Miller, B., McKinney, M., & Braun, J. (2021). Gentrifying Atlanta: Investor Purchases of Rental Housing, Evictions, and the Displacement of Black Residents. *Housing Policy Debate*, 31(3–5), 818–834. <https://doi.org/10.1080/10511482.2021.1887318>
- Riley, P. (2016, February 2). Nichols Hills is being gentrified... *The Lost Ogle*. <https://thelostogle.com/2016/02/02/nichols-hills-is-being-gentrified>
- Ringelman, K. M. (2014). Predator foraging behavior and patterns of avian nest success: What can we learn from an agent-based model? *Ecological Modelling*, 272, 141–149.
- Rucks-Ahidiana, Z. (2021). Racial composition and trajectories of gentrification in the United States. *Urban Studies*, 58(13), 2721–2741. <https://doi.org/10.1177/0042098020963853>

- Rucks-Ahidiana, Z. (2022). Theorizing Gentrification as a Process of Racial Capitalism. *City & Community*, 21(3), 173–192. <https://doi.org/10.1177/15356841211054790>
- Shirabe, T. (2006). Correlation Analysis of Discrete Motions. In M. Raubal, H. J. Miller, A. U. Frank, & M. F. Goodchild (Eds.), *Geographic Information Science* (Vol. 4197, pp. 370–382). Springer Berlin Heidelberg. https://doi.org/10.1007/11863939_24
- Sober, E. (2002). Instrumentalism, Parsimony, and the Akaike Framework. *Philosophy of Science*, 69(S3), S112–S123. <https://doi.org/10.1086/341839>
- Thrift, N., & Pred, A. (1981). Time-geography: A new beginning. *Progress in Human Geography*, 5(2), 277–286. <https://doi.org/10.1177/030913258100500209>
- US Census Bureau. (2023, July 25). *Metropolitan and Micropolitan Statistical Areas—About*. <https://www.census.gov/programs-surveys/metro-micro/about.html>
- Vehtari, A., Gelman, A., & Gabry, J. (2017). Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. *Statistics and Computing*, 27(5), 1413–1432. <https://doi.org/10.1007/s11222-016-9696-4>
- Walker, K., & Herman, M. (2025). *tidycensus: Load US Census Boundary and Attribute Data as “tidyverse” and ‘sf’-Ready Data Frames*. <https://walker-data.com/tidycensus/>
- Wang, A.-S., Yin, Z.-C., & Ying, S. (2024). Probabilistic Time Geographic Modeling Method Considering POI Semantics. *ISPRS International Journal of Geo-Information*, 13(1), 22. <https://doi.org/10.3390/ijgi13010022>
- Wilcoxon, F. (1992). Individual Comparisons by Ranking Methods. In S. Kotz & N. L. Johnson (Eds.), *Breakthroughs in Statistics* (pp. 196–202). Springer New York. https://doi.org/10.1007/978-1-4612-4380-9_16

- Winter, S., & Yin, Z.-C. (2011). The elements of probabilistic time geography. *GeoInformatica*, 15(3), Article 3. <https://doi.org/10.1007/s10707-010-0108-1>
- Yin, Z., Chen, Y., & Ying, S. (2023). Construction and Analysis of Space–Time Paths for Moving Polygon Objects Based on Time Geography: A Case Study of Crime Events in the City of London. *ISPRS International Journal of Geo-Information*, 12(6), Article 6. <https://doi.org/10.3390/ijgi12060210>
- Yin, Z., Huang, W., Ying, S., Tang, P., Kang, Z., & Huang, K. (2021). Measuring of the COVID-19 Based on Time-Geography. *International Journal of Environmental Research and Public Health*, 18(19), 10313. <https://doi.org/10.3390/ijerph181910313>
- Zhang, D., Lee, K., & Lee, I. (2018). Hierarchical trajectory clustering for spatio-temporal periodic pattern mining. *Expert Systems with Applications*, 92, 1–11. <https://doi.org/10.1016/j.eswa.2017.09.040>

5 Chapter Five: Conclusion

5.1 *Toward a Time Geography of Spatially Indistinct Phenomena*

This dissertation seeks to expand the applicable subject matter for the Time Geographic methodology. The existing scope of Time Geographic inquiry and method is focused on the meaning of movement observed for the individual agent or actor as captured in a movement trajectory. There is not a consensus approach for the characterization and analysis of movement in non-point, distributed or invisible spatial phenomena in the subdiscipline. Such phenomena while measurable and spatiotemporal in their action are disconnected from available Time Geographic treatments due in part to the lack of a cogent representation of their movement trajectories. Phenomenon whose observations are reported at aggregate spatial units, for example processes captured in disease reporting, neighborhood-level rent pricing, samples or sightings of invasive plants and concentrations of aerosol plumes all arrive as opportunities for Time Geographic analysis if compatible trajectories for their movements become available.

The central contribution from this dissertation consists of the introduction and implementation of an ontological bridge offering compatibility in analysis between existing Time Geographic methods and a new construction of movement trajectory derived from emergent patterns in observations reported at aggregate spatial units. The implementation returns movement trajectories compatible with the Time Geographic methodology given input time series of these observations. At each time step in the input series, adjacent locations shown to be clustering for a single variable are subject to comparisons against corresponding clusters from the previous time step. Based on the results of a routine of spatial and probabilistic comparisons, the geometries and displacements of so-called *aggregate movers* are suggested, constructing

space-time trajectories of their progressions. The method, termed *Moving Local Indicators of Spatial Association* (Moving LISA, Chapter 2) represents the first known probabilistic approach to characterizing space-time trajectories from observations reported in aggregate at spatial units, arriving as part of a nascent literature treating related and understudied questions further informing a new Time Geography of Distributed Phenomena (Anderson & Yang, 2025; Yin et al., 2021, 2023).

This dissertation also provides for a functional demonstration of the Moving LISA with application in two studies (Chapters 3 and 4). While each of these efforts represent respective, standalone spatial inquiries aimed at improving understanding of their target processes through a distinctively spatiotemporal lens, these chapters also act as case studies in the application potential of the Moving LISA.

The first application examines the recent emergence of specialty vape store businesses in Oklahoma (Chapter 3) where it engages with Moving LISA results as a motivating visualization of the spatiotemporal progression of vapor retail in Oklahoma. This is discussed in context with results from corresponding, spatially autoregressive cross-sectional models fit to contemporaneous socioeconomic data. The goal of Chapter 3 as an applied study for public health is to deliver the first spatiotemporally explicit examination of vapor retail establishments considered for their socioeconomic associations in the State of Oklahoma. Work completed with this chapter also produced a spatial dataset depicting the point locations of Oklahoma specialty vape stores with their date of business establishment. In Chapter 3, vapor retail establishment is approached as a public health concern theorized in relevant literature to target vulnerable populations, a well-studied and proven theme from Tobacco Control practice. Chapter 3 allies with the ongoing work of this public health function by examining the spatial and temporal

pattern of vapor retail establishment for socioeconomic associations indicative of a socially uneven risk to health (Berg et al., 2020; Dai et al., 2017; Dai & Hao, 2017; Giovenco et al., 2016; Kim et al., 2016; Venugopal et al., 2020).

The second study (Chapter 4) engages with the metric attributes of Moving LISA trajectory results directly, where generated trajectories serve as input to existing summary and comparison metrics from Movement Ecology (Long, 2014; Long et al., 2014; Long & Nelson, 2013; Shirabe, 2006). As qualified Time Geographic objects, Moving LISA output trajectories contain relative turning angle and distance measurements measured between space-time anchors (mover relocations, trajectory “fixes”) which form the input for a broad class of methods from Movement Ecology. Chapter 4, while its findings offer a confirming opinion in recent theoretical and methodological advancements in the study of the gentrification process (Bunten et al., 2024), it serves also as the first empirical analysis of Moving LISA trajectories. By examining Moving LISA result trajectories as a substitute for animal movement trajectories, Chapter 4 reveals additional evidence for a prevailing theoretical argument placing high home values and low incomes in spatiotemporal opposition over the landscape. The repulsed action of these patterns forms a component signal of gentrification (Bunten et al., 2024; Preis et al., 2021; Rucks-Ahidiana, 2021, 2022). While the relationship is not a novel notion, its confirmation with non-parametric tests, applied to the results from animal movement analysis, is new. The materials in Chapter 4 contribute toward the methods compatibility goal set by the Moving LISA and this dissertation. With this chapter I sought to demonstrate the value of an adapter availing new data types to treatment with existing methods.

5.2 *Specific Contributions and Findings*

This dissertation presents both an extension to the Time Geographic methodology and two applications demonstrating its use pursuant to an expanded Time Geographic subject matter. The theoretical motivation and functional specification of the Moving LISA provided in Chapter 2 is positioned as an early contribution in an emerging literature addressing notions of movement in distributed phenomena (Anderson & Yang, 2025; Yin et al., 2021, 2023). Moving LISA is the first known probabilistic method to appear in this specific and germinating literature on non-point movement.

Methodologically adjacent approaches are shown to either rely on a deterministic characterization of the space-time anchor position for the mover (Yin et al., 2021) at each time step, or a summary of point-event locations within a space-time window achieving the mover's characterization (Yin et al., 2023). Departing from the few available constructions of the mover in this new literature, Moving LISA's approach identifies space-time anchor locations as representative points taken as central locations in areas of spatially autoregressive extreme effect. Further, these areas are given a theorized lifespan and a notion of agency (Anderson & Yang, 2025). They may only persist to the next time-step if an overlap threshold is met in comparison to corresponding candidate mover geometries found in the next time step. Existing approaches can be highly dependent on the selection of the space-time window (Yin et al., 2023), outliers, and spatial autocorrelation in observations driving the critical metric of, for example, *max disease case increase* from last time step (Yin et al., 2021). Additionally, these mechanisms operate globally over the study area and identify only a single mover for the study area. Moving LISA tracks a complex lineage of appearances, divergences, convergences and terminations of movers over the course of the input study period and is the only known method seeking to

address this dimension of the concept. This is somewhat critical in that natural patterns like dispersion, encroachment and distributed community establishment across a study area would be masked or lost to methods returning only a single trajectory summarizing their input.

Chapter 3's application of the Moving LISA comprises a motivating visualization and reference for discussion of Besag-York-Mollié (BYM2) results examining patterns of specialty vape store emergence in Oklahoma from 2013-2023 (Anderson & Yang, 2025; Riebler et al., 2016; Simpson et al., 2017). Chapter 3 presents the first such mixed methods approach to involve the Moving LISA, geared toward spatial questions from public health and tobacco control. Additionally for Oklahoma, Chapter 3 represents the first model construction directed at the vape store emergence process to smooth the influence of space as an intrinsic conditionally random effect (ICAR) (Besag et al., 1991; Dean et al., 2001; Leroux et al., 2000; Riebler et al., 2016; Simpson et al., 2017). In Chapter 3, it was shown expected counts of specialty vape stores at Oklahoma Zip Code Tabulation Areas were significantly and positively associated with the "Percent of Households with SNAP Benefits in the Last 12 Months" and negatively for "Percent of Vacant Housing Units" for all study periods. This expectation of higher incidence of vape stores in areas where housing is less available and where resident populations have greater need for basic nutrition assistance speaks to longstanding findings suggesting that vapor retail unevenly co-locates with social disadvantage or vulnerability (Bostean et al., 2018; Chido-Amajuoyi et al., 2020; Venugopal et al., 2020, 2022; Wheeler et al., 2020). In concert with these findings relating associations at the level of a representative ZCTA, the Moving LISA visualizations and component LISA results from Chapter 3 reflect Oklahoma recent and nationally unprecedented spatial emergence of vape store presence (Berg et al., 2020; Richtel, 2014).

Chapter 4 shows that Moving LISA results are qualified inputs to existing trajectory analysis methods and are capable of producing a meaningful result, this time borrowing analytics from Movement Ecology (Long, 2014; Long et al., 2014; Long & Nelson, 2013; Shirabe, 2006). Chapter 4 is carried forward with analysis of the Moving LISA trajectory results for Hotspot Median Home Value and Hot/Cold spot locations for Median Income, respectively, for the American Community Survey 5-Year Estimates end-years of 2008, 2013, 2018 and 2023 in the Oklahoma City Metropolitan Statistical Area at the census tract level. Moving LISA trajectories for these variables are compared to illustrate existing theoretical and empirical characterizations of the gentrification process, where a spatiotemporal opposition between low incomes and high property values form an identifying signal for gentrifying areas (Bunten et al., 2024; Finio, 2022; Preis et al., 2021).

Results from Chapter 4 confirm the general expectation of spatiotemporal repulsion between areas of high property value and low income in the US metropolitan context from literature. Specific to a theoretical and empirical understanding from Urban Geography where the intensities and rate of gentrification is shown to increase with proximity to the central business district (Bunten et al., 2024), Chapter 4 applies a series of non-parametric tests for difference in median location between Moving LISA wildlife interaction metrics taken from *inside of municipal limits* and *outside of municipal limits* in the OKC MSA, respectively. Significant differences in the locations of pseudo median values for metrics relating repulsion or coherence among movement trajectories suggest that Moving LISA trajectories for low incomes and high home values are more repelled when found close to downtown areas, compared to those interacting outside of city limits. In Chapter 4, by leveraging the ontological bridge supplied by

the Moving LISA, relevant methods from individual movement analysis may be brought to bear on observations of change across the social and economic landscape of housing.

5.3 *Limitations and Open Questions*

Several limitations in the method which suggest for appropriate sensitivity analyses appear in the chapters delivered with this dissertation. The Moving LISA parameter sweep approach employed in each chapter of this dissertation is rudimentary and only examines the effects of attenuation in each input parameter in isolation. This parameter sweep does not directly address the implications of co-variation among the inputs in terms of model outputs. The Moving LISA as described in Chapter 2 is sensitive to these user-selected values for its required parameters. This includes the user's choice of *contiguity cutoff*, *standard distance scaling factor*, and *significance boundary* (the alpha-level for the selected LISA).

As a novel method, no guidance exists on parameter selection for the Moving LISA, save for assistance regarding the chosen LISA component from the relevant literature (Getis & Aldstadt, 2004; Ord & Getis, 1995; Sauer et al., 2022; Wolf, 2024). Changes in each of these parameters could affect the persistence, membership and area of the *aggregate movers* constructed at each time step. These changes in identification and entity-linking of movers at any time step will translate to changes in the output set of trajectories. A sensitivity analysis conducted in terms of Moving LISA results, where each parameter choice is modeled as a response to all other parameters as covariates might offer further quantitative insight into the interactions between user parameter choices for the method. Future work applying the Moving

LISA could uncover a more generalized strategy for sensitivity analysis and potentially yield a set of guidelines for parameter selection drawn from a sampling of Moving LISA use cases.

With respect to cluster identification, this dissertation only discusses specifications of Moving LISA where the Getis-Ord Local G (Getis & Ord, 1992; Ord & Getis, 1995) is chosen as the driving LISA statistic. Other choices of LISA identify clustering locations using strategies different from the Local G 's ratio of neighborhood contribution versus a global sum. For example, the Local Moran's I identifies clustering in terms of neighborhood departure from a global mean enabling identification of local spatial outliers (Anselin, 1995). While it does not represent theoretical incompatibility with the overall method of constructing space-time trajectories from clustering location, changing the component LISA could introduce these additional cluster types which have not been addressed in this dissertation. It should also be noted that Moving LISA inherits all of the limitations already present in the choice of LISA statistic including sensitivity to the user's construction of the spatial weights matrix and the problem of multiple comparisons affecting LISAs' inference by conditional permutation (Anselin, 2019).

5.4 *Future Directions*

Outside of improvements in application including sensitivity analyses and use-case guidance, the Moving LISA's notions of agency, the underlying mechanisms and behavior in the theoretical construction of *aggregate movers* should be further explored. For example, recent developments in the Time Geographic literature further developing space-time-prisms focus on reconstructing the unobserved movements of individuals as a probabilistic function of their context along with a

priori knowledge of their behaviors (Ho & Loraamm, 2023; Loraamm, 2019; Loraamm et al., 2021). Essentially, knowledge about the focal mover's ecology or biology is incorporated into the suggestion of plausible locations which the mover may have occupied at unobserved times.

For the Moving LISA, *aggregate movers* and their trajectories as described in this dissertation lack the biotic character of movers comprising most Time Geographic subject matter. *Aggregate movers* are a derivative of distributed spatial dynamics and emergence, rather than discrete, living and thinking individuals. Consequently, several of the core parameters of individual movement analysis are difficult to obtain. For example, displacements in high home values do not immediately report a meaningful value for their maximum velocity, though this is a required input for the most basic space-time prism and potential path area calculations from the subdiscipline (Downs et al., 2014; Miller, 2005; Winter & Yin, 2011). Further still, if we are to consider the maximum speed of a moving cluster of high home values, this speed would vary with context and market forces. Maximum speed is in this case an abstract observation generated from coincident spatial effects rather than a biologically dictated upper limit based on species. Without corresponding *a priori* knowledge for the focal *aggregate mover*, the scope of movement analytics which can be applied to Moving LISA results is limited to those requiring only the basic attributes of trajectories (time, location, distance, direction).

To counter this deadlock in the knowable dimensions of the target *aggregate mover*, I suggest a thorough accounting of domain knowledge describing the focal process in future interpretations of Moving LISA results. Where in the example of a “top speed” for shifts in housing market values may arrive as nonsensical, appropriate findings from Real Estate could place functional limits on the spatial reach of market areas, allowing us to ultimately infer such a “maximum velocity” among movers within the boundary, as a proportion of available distance

covered. As a further alternative, it may not become necessary to treat the *aggregate mover* as a biotic entity for relevant Time Geographic analysis. Perhaps instead it may be possible to extract summary behaviors by collecting and analyzing samples from the atomic action in the target process, such as the generative activity of home buyers transacting across the study area (Anderson et al., 2017; Epstein, 1999; Tobler, 1970).

Finally, it would serve the emerging literature surrounding the Moving LISA to prepare the implementation used during the writing of this dissertation for release as open software (Loraamm et al., 2020). I am heartedly grateful for the contributions of those before me who have provided the generalized, highly compatible and openly available spatial libraries which were vital to the quantitative dimensions of this research (Hunter, 2007; Joris Van den Bossche et al., 2023; Long, 2014; Rey et al., 2022; van der Walt et al., 2011).

5.5 References

- Anderson, J. H., Downs, J. A., Loraamm, R., & Reader, S. (2017). Agent-based simulation of Muscovy duck movements using observed habitat transition and distance frequencies. *Computers, Environment and Urban Systems*, 61, 49–55.
<https://doi.org/10.1016/j.compenvurbsys.2016.09.002>
- Anderson, J., & Yang, A. (2025). Moving Local Indicators of Spatial Association for Time Geography. *Transactions in GIS*, 29(8), e70158. <https://doi.org/10.1111/tgis.70158>
- Anselin, L. (1995). Local Indicators of Spatial Association—LISA. *Geographical Analysis*, 27(2), 93–115. <https://doi.org/10.1111/j.1538-4632.1995.tb00338.x>
- Anselin, L. (2019). A Local Indicator of Multivariate Spatial Association: Extending Geary's c. *Geographical Analysis*, 51(2), 133–150. <https://doi.org/10.1111/gean.12164>

- Berg, C. J., Schleicher, N. C., Johnson, T. O., Barker, D. C., Getachew, B., Weber, A., Park, A. J., Patterson, A., Dorvil, S., Fairman, R. T., Meyers, C., & Henriksen, L. (2020). Vape shop identification, density and place characteristics in six metropolitan areas across the US. *Preventive Medicine Reports*, 19, 101137. <https://doi.org/10.1016/j.pmedr.2020.101137>
- Besag, J., York, J., & Mollié, A. (1991). Bayesian image restoration, with two applications in spatial statistics. *Annals of the Institute of Statistical Mathematics*, 43(1), 1–20. <https://doi.org/10.1007/BF00116466>
- Bostean, G., Sanchez, L., & Lippert, A. M. (2018). Sociodemographic disparities in e-cigarette retail environment: Vape stores and census tract characteristics in Orange County, CA. *Health & Place*, 50, 65–72. <https://doi.org/10.1016/j.healthplace.2017.12.004>
- Bunten, D. M., Preis, B., & Aron-Dine, S. (2024). Re-measuring gentrification. *Urban Studies*, 61(1), 20–39. <https://doi.org/10.1177/00420980231173846>
- Chido-Amajuoyi, O. G., Ozigbu, C. E., & Zhang, K. (2020). School proximity and census tract correlates of e-cigarette specialty retail outlets (vape shops) in central Texas. *Preventive Medicine Reports*, 18, 101079. <https://doi.org/10.1016/j.pmedr.2020.101079>
- Dai, H., & Hao, J. (2017). Geographic density and proximity of vape shops to colleges in the USA. *Tobacco Control*, 26(4), 379–385. <https://doi.org/10.1136/tobaccocontrol-2016-052957>
- Dai, H., Hao, J., & Catley, D. (2017). Vape Shop Density and Socio-Demographic Disparities: A US Census Tract Analysis. *Nicotine & Tobacco Research*, 19(11), 1338–1344. <https://doi.org/10.1093/ntr/ntx063>

- Dean, C. B., Ugarte, M. D., & Militino, A. F. (2001). Detecting Interaction Between Random Region and Fixed Age Effects in Disease Mapping. *Biometrics*, 57(1), 197–202.
<https://doi.org/10.1111/j.0006-341X.2001.00197.x>
- Downs, J., Horner, M. W., Hyzer, G., Lamb, D., & Loraamm, R. (2014). Voxel-based probabilistic space-time prisms for analysing animal movements and habitat use. *International Journal of Geographical Information Science*, 28(5), Article 5.
<https://doi.org/10.1080/13658816.2013.850170>
- Epstein, J. M. (1999). Agent-Based Computational Models And Generative Social Science. *Complexity*, 4(5), Article 5.
- Finio, N. (2022). Measurement and Definition of Gentrification in Urban Studies and Planning. *Journal of Planning Literature*, 37(2), 249–264.
<https://doi.org/10.1177/08854122211051603>
- Getis, A., & Aldstadt, J. (2004). Constructing the Spatial Weights Matrix Using a Local Statistic. *Geographical Analysis*, 36(2), 90–104. <https://doi.org/10.1111/j.1538-4632.2004.tb01127.x>
- Getis, A., & Ord, J. K. (1992). The Analysis of Spatial Association by Use of Distance Statistics. *Geographical Analysis*, 24(3), 189–206. <https://doi.org/10.1111/j.1538-4632.1992.tb00261.x>
- Giovenco, D. P., Duncan, D. T., Coups, E. J., Lewis, M. J., & Delnevo, C. D. (2016). Census tract correlates of vape shop locations in New Jersey. *Health & Place*, 40, 123–128.
<https://doi.org/10.1016/j.healthplace.2016.05.008>

- Ho, K., & Loraamm, R. (2023). Using a Cost-Distance Time-Geographic Approach to Identify Red Deer Habitat Use in Banff National Park, Alberta, Canada. *ISPRS International Journal of Geo-Information*, 12(8), 339. <https://doi.org/10.3390/ijgi12080339>
- Hunter, J. D. (2007). Matplotlib: A 2D Graphics Environment. *Computing in Science & Engineering*, 9(3), Article 3. <https://doi.org/10.1109/MCSE.2007.55>
- Joris Van den Bossche, Kelsey Jordahl, Martin Fleischmann, James McBride, Jacob Wasserman, Matt Richards, Adrian Garcia Badaracco, Alan D. Snow, Jeff Tratner, Jeffrey Gerard, Brendan Ward, Matthew Perry, Carson Farmer, Geir Arne Hjelle, Mike Taves, Ewout ter Hoeven, Micah Cochran, rraymondgh, Sean Gillies, ... Aleksey Bilogur. (2023). *geopandas/geopandas: V0.14.1* (Version v0.14.1) [Computer software]. Zenodo. <https://doi.org/10.5281/ZENODO.2585848>
- Kim, A. E., Loomis, B., Rhodes, B., Eggers, M. E., Liedtke, C., & Porter, L. (2016). Identifying e-cigarette vape stores: Description of an online search methodology. *Tobacco Control*, 25(e1), e19–e23. <https://doi.org/10.1136/tobaccocontrol-2015-052270>
- Leroux, B. G., Lei, X., & Breslow, N. (2000). Estimation of Disease Rates in Small Areas: A new Mixed Model for Spatial Dependence. In M. E. Halloran & D. Berry (Eds.), *Statistical Models in Epidemiology, the Environment, and Clinical Trials* (Vol. 116, pp. 179–191). Springer New York. https://doi.org/10.1007/978-1-4612-1284-3_4
- Long, J. (2014). *wildlifeDI: Calculate Indices of Dynamic Interaction for Wildlife Tracking Data* (p. 1.0.1) [Dataset]. <https://doi.org/10.32614/CRAN.package.wildlifeDI>
- Long, J. A., & Nelson, T. A. (2013). Measuring Dynamic Interaction in Movement Data. *Transactions in GIS*, 17(1), 62–77. <https://doi.org/10.1111/j.1467-9671.2012.01353.x>

- Long, J. A., Nelson, T. A., Webb, S. L., & Gee, K. L. (2014). A critical examination of indices of dynamic interaction for wildlife telemetry studies. *Journal of Animal Ecology*, 83(5), Article 5. <https://doi.org/10.1111/1365-2656.12198>
- Loraamm, R. (2019). Incorporating behavior into animal movement modeling: A constrained agent-based model for estimating visit probabilities in space-time prisms. *International Journal of Geographical Information Science*, 1–21. <https://doi.org/10.1080/13658816.2019.1658875>
- Loraamm, R., Anderson, J., & Burch, C. (2021). Identifying road avoidance behavior using time-geography for red deer in Banff National Park, Alberta, Canada. *Transactions in GIS*, 25(3), Article 3. <https://doi.org/10.1111/tgis.12748>
- Loraamm, R., Downs, J., Anderson, J., & Lamb, D. S. (2020). PySTPrism: Tools for voxel-based space–time prisms. *SoftwareX*, 12, 100499. <https://doi.org/10.1016/j.softx.2020.100499>
- Miller, H. J. (2005). A Measurement Theory for Time Geography. *Geographical Analysis*, 37(1), Article 1. <https://doi.org/10.1111/j.1538-4632.2005.00575.x>
- Ord, J. K., & Getis, A. (1995). Local Spatial Autocorrelation Statistics: Distributional Issues and an Application. *Geographical Analysis*, 27(4), 286–306. <https://doi.org/10.1111/j.1538-4632.1995.tb00912.x>
- Preis, B., Janakiraman, A., Bob, A., & Steil, J. (2021). Mapping gentrification and displacement pressure: An exploration of four distinct methodologies. *Urban Studies*, 58(2), 405–424. <https://doi.org/10.1177/0042098020903011>
- Rey, S. J., Anselin, L., Amaral, P., Arribas-Bel, D., Cortes, R. X., Gaboardi, J. D., Kang, W., Knaap, E., Li, Z., Lumnitz, S., Oshan, T. M., Shao, H., & Wolf, L. J. (2022). The PySAL

- Ecosystem: Philosophy and Implementation. *Geographical Analysis*, 54(3), 467–487.
<https://doi.org/10.1111/gean.12276>
- Richtel, M. (2014). Where vapor comes sweeping down the plain. *New York Times*.
- Riebler, A., Sørbye, S. H., Simpson, D., & Rue, H. (2016). An intuitive Bayesian spatial model for disease mapping that accounts for scaling. *Statistical Methods in Medical Research*, 25(4), 1145–1165. <https://doi.org/10.1177/0962280216660421>
- Rucks-Ahidiana, Z. (2021). Racial composition and trajectories of gentrification in the United States. *Urban Studies*, 58(13), 2721–2741. <https://doi.org/10.1177/0042098020963853>
- Rucks-Ahidiana, Z. (2022). Theorizing Gentrification as a Process of Racial Capitalism. *City & Community*, 21(3), 173–192. <https://doi.org/10.1177/15356841211054790>
- Sauer, J., Oshan, T., Rey, S., & Wolf, L. J. (2022). The Importance of Null Hypotheses: Understanding Differences in Local Moran's under Heteroskedasticity. *Geographical Analysis*, 54(4), 752–768. <https://doi.org/10.1111/gean.12304>
- Shirabe, T. (2006). Correlation Analysis of Discrete Motions. In M. Raubal, H. J. Miller, A. U. Frank, & M. F. Goodchild (Eds.), *Geographic Information Science* (Vol. 4197, pp. 370–382). Springer Berlin Heidelberg. https://doi.org/10.1007/11863939_24
- Simpson, D., Rue, H., Riebler, A., Martins, T. G., & Sørbye, S. H. (2017). Penalising Model Component Complexity: A Principled, Practical Approach to Constructing Priors. *Statistical Science*, 32(1). <https://doi.org/10.1214/16-STS576>
- Tobler, W. R. (1970). A Computer Movie Simulating Urban Growth in the Detroit Region. *Economic Geography*, 46, 234–240. <https://doi.org/10.2307/143141>

- van der Walt, S., Colbert, S. C., & Varoquaux, G. (2011). The NumPy Array: A Structure for Efficient Numerical Computation. *Computing in Science & Engineering*, 13(2), Article 2. <https://doi.org/10.1109/MCSE.2011.37>
- Venugopal, P. D., Morse, A. L., Alrefai-Kirkpatrick, R., Tworek, C., & Chang, H. W. (2022). The Co-occurrence of Specialty Vape Shops, Social Disadvantage, and Poor Air Quality in the United States: An Assessment of Cumulative Risks to Youth. *Health Equity*, 6(1), 132–141. <https://doi.org/10.1089/heq.2021.0151>
- Venugopal, P. D., Morse, A. L., Tworek, C., & Chang, H. W. (2020). Socioeconomic Disparities in Vape Shop Density and Proximity to Public Schools in the Conterminous United States, 2018. *Health Promotion Practice*, 21(1_suppl), 9S-17S. <https://doi.org/10.1177/1524839919887738>
- Wheeler, D. C., Do, E. K., Hayes, R. B., Fugate-Laue, K., Fallavollita, W. L., Hughes, C., & Fuemmeler, B. F. (2020). Neighborhood Disadvantage and Tobacco Retail Outlet and Vape Shop Outlet Rates. *International Journal of Environmental Research and Public Health*, 17(8), 2864. <https://doi.org/10.3390/ijerph17082864>
- Winter, S., & Yin, Z.-C. (2011). The elements of probabilistic time geography. *GeoInformatica*, 15(3), Article 3. <https://doi.org/10.1007/s10707-010-0108-1>
- Wolf, L. J. (2024). Confounded Local Inference: Extending Local Moran Statistics to Handle Confounding. *Annals of the American Association of Geographers*, 114(6), 1216–1231. <https://doi.org/10.1080/24694452.2024.2326541>
- Yin, Z., Chen, Y., & Ying, S. (2023). Construction and Analysis of Space–Time Paths for Moving Polygon Objects Based on Time Geography: A Case Study of Crime Events in the City of

London. *ISPRS International Journal of Geo-Information*, 12(6), Article 6.

<https://doi.org/10.3390/ijgi12060210>

Yin, Z., Huang, W., Ying, S., Tang, P., Kang, Z., & Huang, K. (2021). Measuring of the COVID-19 Based on Time-Geography. *International Journal of Environmental Research and Public Health*, 18(19), 10313. <https://doi.org/10.3390/ijerph181910313>