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A COMPARISON OF TRADITIONAL AND REFORM STYLES IN TEACHING APPLIED CALCULUS

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A COMPARISON OF TRADITIONAL AND REFORM STYLES IN TEACHING APPLIED CALCULUS

A Dissertation
APPROVED FOR THE DEPARTMENT OF
INSTRUCTIONAL LEADERSHIP AND ACADEMIC CURRICULUM

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ABSTRACT

This study compared the use of reform methods in the teaching of applied calculus at a large comprehensive public university during the Spring 1997 semester. Fifty-nine students, mostly freshmen and sophomores enrolled in two sections of the second semester of a two-semester sequence of applied calculus. The experimental section used a reform textbook, graphing calculators, and small group activities. The control section used a traditional textbook, scientific calculators, and lectures. Common examination questions were used to compare the two groups. Students in the experimental section scored significantly better on conceptual questions, and showed no significant difference on computational question. Students in the experimental section had better affective responses to questions about the usefulness of mathematics and their ability to solve mathematical problems, especially non-routine problems.

CHAPTER ONE

INTRODUCTION

In the mid-1980's, some collegiate mathematics educators began to articulate their belief that the typical approach to teaching calculus was not effective (e.g., see Douglas, 1986; Steen, 1989). They believed that the then-current teaching strategies had many weaknesses. In particular, traditional approaches were considered to include:

- emphasizing rote memorization and application of formulas,
- bland lectures and homework assignments that parroted lecture examples,
- ignoring graphical and numerical representations of functions, and
- showing little, if any, regard for applications from real life.

As a result, many students seemed to learn only enough to get through examinations, without trying to make sense of the material presented.

At about the same time, new advances in useful technology became far more readily available to mathematicians, mathematics instructors, and students. In a surprisingly short time, calculators by which students could graph functions with only a few keystrokes became widely available at low cost. Other innovations emerging into common use were computer algebra systems (CAS). Not just calculating devices, these systems could perform mathematical transformations on symbolic expressions (e.g., factoring or simplifying algebraic expressions, solving equations, finding derivatives and integrals). Many skills and techniques traditionally taught in elementary calculus now could be done easily by relatively inexpensive, widely available technological devices.

Background

In 1985, a panel discussion "Calculus Instruction, Crucial but Ailing" (Smith, 1994) drew a large crowd at the Joint Mathematical Association of America - American Mathematical Society (MAA-AMS) Winter Meeting in Anaheim, California. A conference at Tulane University in 1986 produced a report (Douglas, 1986) stressing a new "lean and lively" way of teaching. Those early efforts produced thinking and research that set the stage for attempts to reform the way calculus is taught.

Reformers typically focused their efforts on one or more of several learning goals. These goals included:

- involving students more actively,
- using applications from "client" disciplines,
- working in teams with others rather than seeking only individual effort,
- requiring students to explain answers in writing assignments, and
- using emerging technological advances appropriately in both learning and doing mathematics (Committee on the Mathematical Sciences in the Year 2000 [hereafter, MS2000], 1991; Steen, 1989).

One favored strategy for many calculus reformers was group projects (Smith, 1994). Such projects incorporated each of the other four goals, and these elements have become common in many current reform calculus courses.

Most research done during the first decade of recent reform (about 1985 to 1995) centered on calculus taught to college students majoring in mathematics, engineering, and the physical sciences. Considerably less effort was directed to calculus students in the fields of business, life and social sciences. However, at some institutions, this latter group is larger than the former group. Almost always, a separate calculus sequence is taught for each group.

The Mathematics Department at the University of Oklahoma recognized the need to consider reforming the way in which calculus is taught to business, life and social science majors (that is, in its MATH 1743 and MATH 2123 courses). Nationally, and to a lesser extent locally, this one-year sequence began to come under the same kinds of criticism that the two-year sequence for math, engineering and physical science majors endured.

An experimental section of MATH 1743 (Calculus I for Business, Life and Social Sciences) was established for the Fall 1996 semester to explore potential reform at the University of Oklahoma. A new, reform-oriented textbook, *Applied Calculus* by Hughes-Hallett et al., (1996) was adopted for this section. Students were required to purchase and use a graphing calculator such as the TI-85. During the Spring 1997 semester, students were able to complete this calculus sequence in an experimental section of MATH 2123 (Calculus II for Business, Life and Social Sciences). These sections were regarded by the Mathematics Department as an exploration of whether these changes (i.e., reform) should be more widely implemented.

Since those sections were experimental, comparison sections were established during the same semesters. This researcher also taught sections of the same courses in a traditional text and a lecture format. Students in the traditional (control) section used

Calculus by Bittinger (1996), the text used in all other sections of the course. These students were required to use a non-graphing calculator.

Some topics are common to both approaches. These common topics form the foundation for a careful empirical comparison reported here. Specifically, these topics include:

- consumer and producer surplus,
- present and future value,
- probability density functions,
- introductory differential equations,
- slope fields,
- multivariate differential calculus,
- simple linear regression,
- optimization using Lagrange multipliers, and
- Cobb-Douglas production functions.

These topics provide the basis for comparison criterion test items.

Problem Statement

This study is a comparison of two philosophical approaches to teaching calculus to undergraduate business, life, and social science majors. To reduce variability, the two sections in this study were taught in the same semester by the same instructor. Both sections covered topics traditionally found in the second half of a two-semester applied calculus course.

The sections differed in these respects:

- The experimental (reform) section had fewer tests. Group projects replaced two one-hour exams.
- The experimental section used graphing calculators such as the TI-85. The traditional section used scientific (non-graphing) calculators.
- The experimental section spent more class time working in small groups. The traditional section was almost exclusively lecture format.
- The experimental section used algebraic, graphical and numerical representations of functions. The traditional section used primarily algebraic representations.

This study examined not only the existence of differences attributable to the two approaches, but also specific differences. These include differences in conceptual understanding as measured by both qualitative and quantitative measures. Differences in affective factors including student perceptions about mathematics and ability to learn and do mathematics were also examined.

Establishing clear differences between the two approaches would provide a basis for further research and analysis of these two approaches to teaching calculus. Additionally, a rational choice of teaching philosophies could be implemented.

Research Questions

1. To what extent did students in the reform (experimental) section perform differently on written tests compared to students in the traditional (control) section?

   1.1. To what extent did students in the experimental section perform differently from students in the control section on items for topics common to the two approaches?

1.2. To what extent did students in the experimental section perform differently from students in the control section on items for topics emphasized or taught directly only in the more traditional approach?

1.3. To what extent did students in the experimental section perform differently from students in the control section on items taught directly only in the experimental approach or less emphasized in the traditional approach?

2. To what extent do students in the experimental section have a higher level of comprehension of calculus concepts than students in the control section as measured by more qualitative, in-depth methods rather than by typical pencil and paper test items?

2.1. To what extent do students in the experimental section have a higher level of comprehension of calculus concepts on items for topics common to the two approaches than students in the control section as measured by more qualitative, in-depth methods rather than by typical pencil and paper test items?

2.2. To what extent do students in the experimental section have a higher level of comprehension of calculus concepts on topics emphasized or taught directly only in the more traditional approach than students in the control section as measured by more qualitative, in-depth methods rather than by typical pencil and paper tests?

2.3. To what extent do students in the experimental section have a higher level of comprehension of calculus concepts on topics less emphasized or not taught directly in the more traditional approach than students in the control section as measured by more qualitative, in-depth methods rather than by typical pencil and paper tests?

3. Do students in the experimental section show affective differences compared to students in the control section?

   3.1. Do students in the experimental section differ in their perception of the nature of mathematics compared to students in the control section?

   3.2. Do students in the experimental section differ in their perception of the usefulness of mathematics compared to students in the control section?

   3.3. Do students in the experimental section differ in their perception of how successful they are doing and learning mathematics compared to students in the control section?

CHAPTER TWO

REVIEW OF THE LITERATURE

History

Current calculus reforms actually constitute just the latest round in a series of reforms dating back to the very beginnings of calculus and instruction related to it. The most recent reform began in 1985 at the mathematics meetings with a panel discussion. So much interest was shown in the teaching of calculus and how to improve it that a movement was born. It is interesting to note that these beginnings predate the most recent innovations in technology, now an integral part of calculus reform.

The teaching of calculus in the 1980s was seen largely as a series of mechanical operations, memorization, and contrived applications that were easily solved in one class period (Hughes-Hallett et al., 1996). Most of the time in class was spent in lecture with emphasis on the "how-to", and little, if any time on the "when" and "why". Students were seen as passive receptacles into which the teacher was to dump as much knowledge as possible (Dossey, 1992).

Current calculus reform efforts have taken many forms, depending on the attitudes and beliefs of the instructors involved. Reforms vary from minimal alterations in teacher attitude and use of technology to complete reorganization of the course curriculum and pedagogy, with many shades of reform in between.

Most proponents in calculus reform agree on several points. The first is the use of modern technology. This can take several different forms: hand-held calculators, graphing calculators, or computers with various kinds of software. Logically, the use of appropriate technology would seem to make certain topics moot in teaching calculus. For example, learning to create graphs of functions is a topic that changes dramatically when graphing calculators are used. Instead of focusing on how to find features such as critical points or inflection points, students can quickly program a graphing calculator to get an accurate representation of the function as a graph.

This use of technology can free students from drudgery and give them the opportunity to investigate the graph more intensively. Traditionally-taught students spend considerable effort on the "how-to" of graphing, they had less opportunity to think about the "why" of graphing. The potential of these changes can go unrealized if reformers emphasize using new technology in rote and otherwise inappropriate methods (National Council of Teachers of Mathematics, [NCTM], 1989), but the changes create possibilities and perhaps an implicit impetus toward more complete reforms and away from simplistic use of technology.

Traditional approaches to calculus instruction emphasized the symbolic approach, with virtually no use of numerical or graphical approaches to functions. Reform has emphasized the importance of a three-fold approach. Outside the classroom, most applications requiring calculus do not occur in symbolic form; they occur as tables of numbers or graphs. Reform calculus stresses the value of each approach, and the interplay of these three ways to represent functions. When graphs are incorporated into learning, for example, the derivative becomes the rate of change, and integral really can be seen to be area under the curve.

A third area of calculus reform is that of student empowerment. This reform is hardly unique to calculus or mathematics in general. Educators in many fields have come to see that students should not be passive receivers in the classroom who are then told to "Go and do likewise." Students should be actively involved in their own learning; the teacher should be a facilitator in that process. In the slope field instance, the ease of graphing makes it possible for a student to consider questions beyond those presented by the teacher. The student now can control the direction of his/her own learning, without excessive reliance on the teacher.

Empowerment of learning also appears in the form of projects. Life is not simple and problems are not always solved by the end of the class period. Projects give students the opportunity to explore a scenario and develop new approaches to exploring everyday problems. It also shows that there isn't always exactly one correct answer. Projects also show that answers are not always easily obtained. If done in small groups, projects foster cooperative learning rather than competitive learning. In many settings beyond the classroom, students will be required to work in teams, and not as loners. Learning should be a social activity, both inside and outside the classroom.

Of course, calculus reform brings a new set of problems for teachers to cope with. One of the most obvious is that of technology. Assessment is very different with graphing calculators. Students can do a great deal of work and have only an answer to show for it. If errors are made, they can be virtually invisible to everyone, including both the student and the teacher. Just an answer on paper makes it difficult to determine the level of student comprehension. Did the student understand the problem, or was it just a lucky guess?

How then to assess student learning? One way is to change the type of questions asked. Traditional questions that emphasize the routine application of formulas or algorithms must be reconsidered. Asking a question such as "find the derivative of a given function" usually involves application of formulas. With technology, a student doesn't even have to know the formulas, but simply how to enter the function into the computer, which does all the work. A new type of question must be created for this situation, one in which the student must express understanding of the concept in question, not just provide a numerical answer. To this end, the teacher must write questions that require a higher order of student thinking than typical of the past.

Another way to test student comprehension is by exploiting the weaknesses of technology. For example, calculators tend to graph noncontinuous functions in a way that makes the function appear continuous. Also, calculators don't always recognize the full domain of some functions. Questions can be written to take advantage of these weaknesses. Such questions measure the extent to which students understand the concept, and not the ability to use a calculator.

An excellent approach to assessing student understanding of concepts is to ask directly. This may lead to students using words and sentences to explain a concept, rather than just a formula with numbers. For example, instead of just asking students to find the derivative of $y=x^2$, ask them to explain with a graph why this derivative is negative on some regions and not others.

One change in the teaching of calculus is the target audience. At times in the past, calculus was considered a requirement of an educated person; at some universities, all students were required to study calculus. More recently, calculus has been the province of students in the engineering, physical science, and mathematics fields.

In the past ten to fifteen years, a new calculus course has appeared at many universities. Called *Applied Calculus*, this course has been modified to fit the needs of students in the fields of business, life sciences and social sciences. Students in these fields tend to be much less interested in the study of calculus for its own sake, and more interested in its practical application. Narasimhan (1993) argues that reform is at least as important for these students as for the traditional calculus student. Applications become even more important in motivating students to learn, because the connections to business and social science are not obvious to most students in these fields. Contrived examples do not encourage learning, but carefully chosen projects can go a long way toward showing students how calculus can be used in their fields. At this university, business students now spend a semester working in groups to create and market a product. Group projects in calculus class can give students an introduction to that cooperative undertaking.

Technology is also important to these students because they will be using technology outside the classroom. It is important to equip students with the tools as they learn the concepts, not after the fact.

Implementing Reform

One of the characteristics of calculus reform is the use of appropriate technology. One of the ways to implement this reform is the use of a computer algebra system. Although several such systems exist, one of the most popular is Mathematica by Wolfram Research. Holdener (1997) reports an experiment at the United States Air Force Academy on the use of Mathematica in a multivariate calculus course. Students in the experimental sections learned concepts through a series of interactive Mathematica notebooks. Students in the traditional sections learned concepts through a combination of lectures and small-group work.

At the end of the semester, all students answered a series of multiple-choice questions on the final examination. Students in the Mathematica-based sections scored significantly better ($p=0.016$) than students in the traditional sections. The questions were then categorized as computational or conceptual. The only significant difference between the groups appeared on the conceptual questions. The Mathematica students averaged 89.9% correct, while the traditional students averaged 71.8%.

A less successful attempt to use Mathematica in a calculus curriculum is outlined by Alarcon and Stoudt (1997). Mathematica was incorporated into the calculus curriculum for science and science education majors at a medium-sized regional state university. The authors identify numerous problems associated with the implementation of Mathematica; not the least of which is resistance on the part of both students and instructors. Students did not want to take an active role in their learning, but seemed to want more lectures and less personal exploration. Several instructors found it hard to be facilitators rather than teachers dispensing facts.

The authors identified several topics that seemed to be particularly well-suited to being presented with Mathematica. They included early use of real-life data, numerical approximation, introduction of complicated functions such as Erf(x) and Gamma(x), the Gauss-Green Theorem, and spherical coordinates. The ability of

Mathematica to accurately draw three-dimensional figures was also an asset.

At the end of the sequence, students and instructors were interviewed about the use of Mathematica in the calculus sequence. The consensus was that Mathematica was best used as a supplement to a more traditional course. Alarcon and Stoudt report that this approach is exactly how calculus is now taught at their university.

Cooley (1997) compared students who used Mathematica as an enhancement to a traditional calculus text to those who used only the traditional text. She emphasized six topics for the computer students. They included limits, instantaneous growth rates, curve sketching, maxima/minima of a function, the derivative, and the integral. The students who used Mathematica showed significantly higher scores on overall achievement ($p=0.02$). They also scored significantly better on the individual concepts of the limit, the derivative, and curve sketching ($p<0.022$ in all cases). Qualitatively, Cooley found that the Mathematica students had a better understanding of the geometric relationship between a function and its derivative. These students were more likely to use graphs to answer questions.

Narasimhan (1993) studied traditional and reform calculus for non-science students at a large metropolitan university. She found that the emphasis on the multiple representations of functions, the use of technology, and the use of real data for applications were the most positive features of the reform course. Students in the traditional section were less confident and less able to solve word problems that differed from the examples presented in the text. This study, while interesting, is flawed because business students were enrolled in the traditional section, and information systems and computer science majors enrolled in the reform section. No apparent effort was made to consider if differences in major contributed to differences in results.

Gordon (1997) reported on the types of questions asked by traditional and reform calculus students. He found that reform students' questions showed deeper reasoning and more insight that the traditional students' questions. One of the most interesting examples Gordon included was the student who asked if it was acceptable to use the third derivative to locate a change in concavity of a polynomial graph. Essentially, this student had correctly extended the idea of the first and second derivative tests to the third derivative.

Bookman and Blake (1996) described the implementation of Project CALC at Duke University. Project CALC students used computer algebra systems, worked in groups and had fewer lectures than the traditional students. Gateway examinations were introduced to guarantee computational mastery of derivatives and integrals. Analysis showed that Project CALC students were unhappy at first, but became more confident as the semester progressed. Faculty were concerned about the workload associated with reform. Project CALC students also did significantly better on a test composed of word problems. Traditional students were better on tests of computational skills.

Overall, calculus reform has moved in fits and starts, showing both promise and problems. Most experiments illustrate both positive results such as greater conceptual understanding, and negative results such as faculty and student dissatisfaction with work loads. As more colleges and universities try reform, each trial will produce results that will refine calculus reform efforts.

CHAPTER THREE

METHOD

This quasi-experiment is a classical comparison study. The study compares two styles of teaching applied calculus to undergraduates at a large midwestern university. One group received the experimental treatment, involving "reform" methods, while the second group functioned as the control. The control group received a "traditional" method of teaching. Data will be analyzed for differences in learning due to the treatment.

Subjects

The subjects of this study were students at the University of Oklahoma enrolled in Calculus for Business, Life and Social Sciences II (MATH 2123). Enrollment in this course requires successful completion of Calculus for Business, Life and Social Sciences I (MATH 1743). Students enrolled in the sections following normal enrollment procedures. The researcher explained the study to both groups at the start of the semester. Of the 23 students in the experimental section, 12 had been exposed to the reform approach in the prerequisite course.

The distribution of students enrolled in this study included 41 males and 18 females. The average mathematics ACT score for the subjects was 25.88. Distribution by classification yielded 30 freshmen, 17 sophomores, 7 juniors and 5 seniors. Table 1 and Table 2 give the distribution by treatment.

Table 1
Frequency Distribution for Gender by Treatment

	Reform	**Traditional**	**Total**
Male	17	24	41
Female	6	12	18
Total	23	36	59

Table 2
Frequency Distribution for Classification by Treatment

	Reform	**Traditional**	**Total**
Freshman	12	18	30
Sophomore	7	10	17
Junior	3	4	7
Senior	1	4	5
Total	23	36	59

Treatment

Both groups studied topics frequently covered in the second semester of an applied calculus course. These topics included:

- integration,
- probability,
- differential equations, and
- multivariate calculus.

Both texts presented the material in approximately the same order. An outline of the specific topics and their order can be found in Appendix A.

Control Section

Subjects in the traditional (control) section were assigned homework on a regular basis, and were required to complete three tests as well as a comprehensive final examination. All grading was based on individual effort; no group grades were assigned.

The text for this section was *Calculus* by Bittinger (1996). Students were required to use a scientific calculator without graphing capabilities.

Classes consisted primarily of lecture by the instructor, with occasional small group work during class time. No group work was required outside of class time.

Experimental Section

Subjects in the reform (experimental) section were assigned homework on a regular basis, and completed one test as well as a comprehensive final exam. Additionally, these subjects were assigned three group projects related to the material presented. Homework, tests, and the final examination were graded individually. The group projects were graded as a group effort; each student in a group received the same grade.

The text for this section was *Applied Calculus for Business, Social Sciences and Life Sciences* by Hughes-Hallett et al.(1996). Students were required to use a graphing calculator. Although not specifically required, every student opted to use a TI-85 (the one used by the instructor). Class time was split between lecture by the instructor and frequent small group work. Students worked both individually and in groups outside of class. A summary of the characteristics is shown in Table 3.

All of the previously-listed (Chapter One) learning goals of reformers were incorporated into the treatment. As a result, several aspects of the class management changed. Traditionally, only one variable should be changed within an study. This researcher felt that all aspects must be changed in order to properly test the effect of the reform approach. The collection of changes will be considered as one variable.

The use of lab sheets and group projects required

Table 3. Comparison of Class Characteristics

Characteristics	**Reform**	**Traditional**
Homework	weekly	weekly
Tests	one	three
Final Exam	yes	yes
Group Projects	three	zero
Calculator	graphing required	scientific only
Class Format	even mixture of lecture and group work	mostly lecture, occasional group work

active learning. Students didn't just listen to the instructor lecture, they actively developed concepts.

Since almost all the students were business majors, the projects and homework problems were chosen to include applications from business whenever possible. The appropriate use of technology was an additional consideration in the choice of group projects and homework assignments.

Students were assigned a group grade on the projects to encourage a group effort. The groups were either three or four students to allow all members a chance to participate. A large portion of the grade for the projects was based on the written report. Several correct answers existed for the projects, so students were primarily graded on the quality of their work and their ability to communicate their results.

The final examination and the homework accounted for one-half the course grade in both sections. In the control section, the three tests accounted for the other half. In the experimental section, the projects and one test accounted for half the grade. The number of tests in the experimental section was reduced to compensate for the extra work associated with the projects.

Experimental Measures

Each student filled out a background questionnaire during the first week of the semester. Some of the questions requested basic demographic information and information about extracurricular activities. The remainder of the questions related to previous mathematics courses and the student's perception of his/her aptitude and interest in mathematics. The questionnaire is in Appendix B.

Each student completed a pretest based on the major concepts of MATH 1743, the prerequisite course. These concepts included:

- limits determined both graphically and analytically,
- the definition of the derivative,
- finding derivatives using standard formulas,
- absolute maximum and minimum values,

- using rectangles to approximate area,
- evaluating simple integrals, and
- applications of those concepts.

The pretest is in Appendix C.

Tests for each section were written to include material appropriate for that section. Whenever suitable, the same questions were used for both sections. Appendix D includes the test and examination questions common to both sections.

At the end of the course, students completed a questionnaire. Two questions examined the way in which students analyzed and solved non-routine problems. The second part of the questionnaire measured student attitudes about mathematics and the treatment provided to their section. A copy of the questionnaire is in Appendix E.

Individual interviews were conducted with volunteers after the semester ended. The interviews were used to measure student attitudes and problem-solving skills in more depth than possible in the written questionnaire. A sample question protocol for the interviews is in Appendix F.

Data Analysis

The results of the background questionnaire and pretest were analyzed using a t-test of means to measure differences in student preparation, as well as differences in student commitments away from school. The scores on the common test and examination questions were analyzed using a t-test of means to measure differences in student comprehension of the material presented.

Data concerning the demographic composition of all sections of the course over a two-year period were collected from the University. Information concerning the gender, classification, and college of the students was compiled. The proportions of each category were determined and compared to the same proportions within the experimental sections. Hypothesis tests of sample proportions were used to examine differences between the sample and the population. The two sections were then compared to determine if the individual sections differed in any of the characteristics considered in this study.

The results of the second questionnaire were analyzed using t-tests of means to compare students' attitudes about mathematics, the use of calculators, and group projects. The interviews measured student attitudes in greater depth than possible on a written questionnaire.

Supplemental Analysis

Shortly after the study began, this researcher noticed that the students in the reform section could be classified as students who had taken Calculus I in the reform style (henceforth called "veterans") and students who had taken Calculus I in the traditional style (henceforth called "novices"). This situation provided the opportunity to compare student performance in reform second-semester calculus as a function of previous college calculus exposure.

Mean scores on examination and survey questions for veterans and novices were computed and compared using t-tests of means.

CHAPTER FOUR

RESULTS

The analysis of data was carried out as described in Chapter Three.

Demographics

In order to analyze the profile of students participating in this experiment, data concerning students enrolled in all sections of MATH 2123 were collected for the four-semester period 1996-1997. This group will be considered the population. The proportions of students by gender, classification, and college enrollment are shown in Table 4. The first two columns refer to the two sections taught by the researcher, while the third column refers to all students who participated in the experiment.

The proportions relating to the experiment were analyzed using a large-sample hypothesis test about a population proportion as outlined by Mendenhall and

Table 4
Proportionate Comparison of Experimental Sections and Total Enrollment

CHARACTERISTIC	REFORM	TRADITIONAL	EXPERIMENT	POPULATION
GENDER				
Male	.739	.667	.695	.619
Female	.261	.333	.305	.381
CLASSIFICATION				
Freshman	.522	.500	.508	.169
Sophomore	.304	.278	.288	.379
Junior	.130	.111	.119	.262
Senior	.043	.111	.085	.188
COLLEGE				
Arts and Sciences	.043	.028	.034	.031
Business	.565	.472	.508	.802
University	.391	.500	.458	.150
Other	.000	.000	.000	.016
n	23	36	59	1359

Sincich (1995). The results are shown in Table 5. There is no significant difference in the proportion of students in the sample when gender is considered.

The proportions by classification are significant for all classes except sophomores. This is likely due to the nature of scheduling classes and enrollment procedures at the University. The experimental sections

Table 5
Hypothesis Tests of Proportion Comparing the Experiment to the Population

CHARACTERISTIC	H_o/H_a	OBSERVED Z-VALUE	SIGNIFICANCE LEVEL
Male	p=.619	1.202	.2302
	p≠.619		
Female	p=.381	-1.202	.2303
	p≠.381		
Freshman	p=.169	6.948	<.001
	p≠.169		
Sophomore	p=.379	-1.441	.1498
	p≠.379		
Junior	p=.262	-2.498	<.001
	p≠.262		
Senior	p=.188	2.025	.0434
	p≠.188		
Arts and	p=.031	.133	.8996
Sciences	p≠.031		
Business	p=.802	5.667	<.001
	p≠.802		
University	p=.150	-6.626	<.001
	p≠.150		
Other	p=.016	.979	.3270
	p≠.016		

were not listed in the published class schedule. Students tend to enroll in published sections first. Since upperclass students are allowed to enroll before underclass students, they are less likely to choose the experimental sections. When the published sections were full, later-enrolling students (typically freshmen and transfer students) were encouraged to enroll in the experimental sections. Hence, the large proportion of freshmen enrolled in the experimental sections.

Similar results occur in the analysis of college enrollment. Significant differences are seen in the proportion of students enrolled in the College of Business Administration and the University College. This result is directly related to the number of freshmen enrolled in the experimental sections. All new students are automatically admitted to the University College when they enter the university. Therefore, a large proportion of freshmen and transfer students will produce a large proportion of students in the University College. In fact, when surveyed by the researcher, 92% of the students indicated a major that would involve admission to the College of Business Administration. This actually exceeds the proportion of population that is enrolled in the College of Business Administration. The observed proportion is close to the population proportion that is enrolled in the University College or the College of Business Administration (95.2%).

The proportions were also analyzed to compare the two experimental sections to each other, using a hypothesis test of the difference between two population proportions. For each test, the hypotheses are: H_o: $p_{reform} - p_{traditional} = 0$ and

H_a: $p_{reform} - p_{traditional} \neq 0$.

The results are shown in Table 6.

Table 6
Results of the Hypothesis Tests of the Difference Between the Experimental Sections

CHARACTERISTIC	Z-VALUE	SIGNIFICANCE
Male	.597	.5486
Female	-.597	.5486
Freshman	.167	.8650
Sophomore	.213	.8336
Junior	.217	.8258
Senior	-1.01	.3124
Arts and Sciences	.297	.7642
Business	.701	.4840
University	.829	.4066

In this case, no significant differences in the proportions were found. In terms of gender, classification, and college, these two sections were similar.

Each student in this experiment completed a survey at the beginning of the semester. This survey is in Appendix B. Each student also answered a pretest covering topics from the prerequisite course. The pretest is in Appendix C. The results of these instruments were used to compare the two groups. Each question of the survey was analyzed using a two-tailed t-test to compare the means of the groups for several characteristics. They include:

- high school GPA,
- ACT Mathematics score,
- proportion taking calculus in high school,
- proportion taking the prerequisite course at this university,
- average number of hours working at a job,
- self-reported interest in mathematics,

* self-reported aptitude in mathematics, and
* pretest scores.

Results are shown in Table 7.

These results indicate that no significant differences exist among the measured variables at the $\alpha=0.05$ level. The only statistic that even comes close to being significant is the ACT Mathematics score. Since no significant differences were detected between the two experimental groups, analysis of covariance was not used to analyze the test measures.

Table 7
Comparison of the Treatment and Control Groups

CHARACTERISTIC	REFORM	TRADITIONAL	OBSERVED T-VALUE	SIGNIFICANCE
HS GPA	3.542	3.637	0.741	0.464
ACT Math score	26.95	25.167	-1.749	0.087
HS Calculus	0.381	0.250	-0.974	0.335
Prerequisite	0.800	0.929	1.324	0.192
Job Hours	10.0	10.6	0.150	0.881
Interest	2.762	2.57	-0.662	0.511
Aptitude	2.905	2.857	-0.176	0.861
Pretest Score	43.85	37.53	-1.371	0.177

Notes:
1. High school GPA is reported on a 4 point scale.
2. Job Hours is average hours per week.
3. Mathematical interest and aptitude are based on a seven point scale with 1 = very high and 7 = very low.

Examination Questions Results

Eight questions on the final examination were the same for both groups of students. They are listed in Appendix D. The numbering of the questions matches that of the final examination for both groups. The questions were presented in the same order and in the same relative position on both examinations.

To measure consistency of the grading process, an additional grader graded a sample of the common examination questions using a grading rubric constructed by the researcher. A correlation coefficient was calculated for the scores assigned by the researcher to the scores assigned by the additional grader. this analysis resulted in a value of r

Question 4 was explicitly covered in the traditional course. Students in the reform section covered probability density functions, but did not learn about uniform distributions. Students in the traditional section had a mean score of 11.2 out of 15 points possible, while the students in the reform section had a mean score of 5.0. A t-test of means was performed; the results are shown in Table 8. This difference is significant with t=3.37 (p=0.00142).

Question 5 asked students to apply a supply and demand function to consumer surplus and producer surplus. These concepts are part of basic economics, and are a common application in applied calculus. Both sections explicitly studied the concept and how to use supply and demand functions to calculate the producer and consumer surplus. The traditional students had a mean score of 13.3/15 on this question, while the reform students had a mean score of 13.7. The results

Table 8
T-test of Means - Examination Question 4

EXAMINATION QUESTION 4	TRADITIONAL	REFORM
Mean	11.18182	5
Variance	37.34091	52.5
Observations	33	21
Pooled Variance	43.17133	
Hypothesized Mean Difference	0	
df	52	
t Stat	3.370453	
P(T<=t) one-tail	0.000711	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.001421	
t Critical two-tail	2.006645	

Table 9
T-test of Means - Examination Question 5

EXAMINATION QUESTION 5	TRADITIONAL	REFORM
Mean	13.33333	13.66667
Variance	6.104167	7.433333
Observations	33	21
Pooled Variance	6.615385	
Hypothesized Mean Difference	0	
df	52	
t Stat	-0.46427	
P(T<=t) one-tail	0.322196	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.644392	
t Critical two-tail	2.006645	

of the t-test of means is shown in Table 9. The calculated t value is -0.464, with a significance level of 0.644.

Question 8 asks students to compute the first and second partials of a multivariate function. Both groups studied partial derivatives extensively. The reform section stressed the geometric interpretations of partial derivatives. The traditional section focused on the computational aspect. In fact, this examination question is very similar to one assigned as homework to the traditional students. The traditional students had a mean score of 8.76 out of 15 points, while the reform students had a mean score of 8.57. The calculated t-value is 0.127, with a significance level of 0.899. The complete results of the t-test are shown in Table 10.

Question 9 asks students to write a formula for revenue as a function of price and demand. These concepts are frequently used in applied calculus courses as an application designed to appeal to business majors. The traditional students had a mean score of 11.33 out of 15 points, while the reform students scored a mean of 2.62 points. The observed t-

Table 10
T-test of Means - Examination Question 8

EXAMINATION QUESTION 8	TRADITIONAL	REFORM
Mean	8.757576	8.571429
Variance	28.31439	26.15714
Observations	33	21
Pooled Variance	27.48468	
Hypothesized Mean Difference	0	
df	52	
t Stat	0.127198	
P(T<=t) one-tail	0.449637	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.899274	
t Critical two-tail	2.006645	

Table 11
T-test of Means - Examination Question 9

EXAMINATION QUESTION 9	TRADITIONAL	REFORM
Mean	11.33333	2.619048
Variance	16.91667	14.04762
Observations	33	21
Pooled Variance	15.81319	
Hypothesized Mean Difference	0	
df	52	
t Stat	7.850401	
P(T<=t) one-tail	1.1E-10	
t Critical one-tail	1.674689	
P(T<=t) two-tail	2.2E-10	
t Critical two-tail	2.006645	

value is 7.85, with a significance level less than 0.00001.

This problem was stressed in the traditional section, but not in the reform section. As expected, students in the traditional section did much better.

Question 10 asks students to solve a differential equation. This equation models exponential growth, and is a rich source of applications to everyday life. A few of those applications include population growth, continuous compounding of interest, and cost of living. The reform textbook stresses this equation because of the applications, and students were assigned several homework problems based on this equation. Students in the traditional section were exposed to this differential equation, but only worked one problem in their homework. Results of the t-test of means for this problem appear in Table 12. Students in the reform section had a mean score of 11.2 out of 15 possible, while the mean score for the traditional section was only 4.58. The calculated t-value is -4.07 with a significance level equal to 0.000159.

Question 12 asks students to calculate present value of an investment. This application was stressed

Table 12
T-test of Means - Examination Question 10

EXAMINATION QUESTION 10	TRADITIONAL	REFORM
Mean	4.575758	11.2381
Variance	32.37689	37.49048
Observations	33	21
Pooled Variance	34.34366	
Hypothesized Mean Difference	0	
df	52	
t Stat	-4.07261	
P(T<=t) one-tail	7.95E-05	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.000159	
t Critical two-tail	2.006645	

in both sections of the experiment. The major difference was in the proportion of time spent on the derivation of the formula and the time spent on the mechanics of using the formula. Students in the reform section spent more time on the derivation, while students in the traditional section spent more time on the mechanics. The results of the t-test of means for this question are shown in Table 13. The results indicate that the students in the reform section scored 13.6 points out of 15 possible, while the students in the traditional section scored 11.2 points. The

Table 13
T-test of Means - Examination Question 12

EXAMINATION QUESTION 12	TRADITIONAL	REFORM
Mean	11.15152	13.61905
Variance	37.82008	14.64762
Observations	33	21
Pooled Variance	28.90759	
Hypothesized Mean Difference	0	
df	52	
t Stat	-1.64409	
P(T<=t) one-tail	0.053095	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.10619	
t Critical two-tail	2.006645	

calculated t-value is -1.64 with a significance level of 0.106; this is a non-significant result. The most common error on this problem was confusing the present value with the future value. This mistake was more common in the traditional section; although the results are not significantly different. Perhaps, this indicates that the time spent on learning the derivation of the formula improves the students' mechanical ability in this application.

Question 13 asks students to find all relative extreme points (maxima and minima) and saddle points for a bivariate function. This topic was presented to both sections as a major topic. The difference in the presentation was in the emphasis on the geometric representation of the function. Students in the reform section spent time studying this type of function using a computer program to graph in three dimensions. These students were given classtime and a project to explore these functions. Students in the traditional section were given a brief presentation on the geometric aspects of these functions; most of the lecture related to the algebraic techniques to find the extremes and saddle points.

The results of the t-test of means on this question can be found in Table 14. Students in the traditional section had a mean score of 13.1 out of 15, while students in the reform section had a mean score of 10.0. The calculated t-value is 2.69 with a significance level of 0.00949. This result indicates that the traditional students did significantly better. In this case, the emphasis on the interpretation did not improve the mechanical ability of students in the

Table 14
T-test of Means - Examination Question 13

EXAMINATION QUESTION 13	TRADITIONAL	REFORM
Mean	13.06061	10.04762
Variance	8.683712	27.84762
Observations	33	21
Pooled Variance	16.05445	
Hypothesized Mean Difference	0	
df	52	
t Stat	2.693825	
P(T<=t) one-tail	0.004744	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.009487	
t Critical two-tail	2.006645	

traditional section.

Question 15 is a conceptual question about the partial derivatives of a bivariate function. Students were expected not only to determine the sign of the partial derivatives, but also to explain their answer. Interpretation of the partial derivatives was stressed in the reform section, but mentioned only in passing in the traditional section.

The results of the t-test of means are shown in Table 15. Students in the reform section had a mean score of 6.43 out of 15, while students in the

Table 15
T-test of means - Examination Question 15

EXAMINATION QUESTION 15	TRADITIONAL	REFORM
Mean	0.909091	6.428571
Variance	10.08523	45.35714
Observations	33	21
Pooled Variance	23.65135	
Hypothesized Mean Difference	0	
df	52	
t Stat	-4.06574	
P(T<=t) one-tail	8.13E-05	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.000163	
t Critical two-tail	2.006645	

traditional section had a mean score of 0.91. The calculated t-value is -4.07, with a significance level of 0.000163. Neither section did particularly well on this question, but students in the reform section did do significantly better.

The questions used in the comparison constituted just over half the final examination in each section. The remainder of each examination was specific to the material covered in that section of the experiment. These questions were over topics that were stressed in that section and not mentioned at all in the other section. The mean score for the complete examination was calculated for each section and compared. The results are shown in Table 16.

Students in the traditional section had a mean score of 151.7 out of 225 possible, while students in the reform section had a mean score of 143.2. The calculated t-value is 0.856 with a significance level of 0.396. This non-significant result indicates that the differing scores on the common questions did not contribute to a significant difference in the overall

Table 16
T-test of Means - Total Score on Final Examination

FINAL EXAMINATION SCORE	TRADITIONAL	REFORM
Mean	151.6667	143.2381
Variance	969.2292	1687.59
Observations	33	21
Pooled Variance	1245.522	
Hypothesized Mean Difference	0	
df	52	
t Stat	0.855555	
P(T<=t) one-tail	0.198084	
t Critical one-tail	1.674689	
P(T<=t) two-tail	0.396169	
t Critical two-tail	2.006645	

scores.

Of the eight common questions on the final examination, five produced significantly different results and three produced non-significant results. A summary of the results is shown in Table 17.

Students in the experimental "reform" section did better overall on the conceptual questions with only slight deficits in the computational questions. These results suggest that reform can be implemented to increase conceptual understanding without losing the computational abilities stressed in traditional approaches.

Table 17
Summary of T-test Results

QUESTION	STRESSED IN:	T-VALUE	SIGNIFICANT?
4	traditional	3.37	yes
5	both	-0.46	no
8	both	0.13	no
9	traditional	7.85	yes
10	reform	-4.07	yes
12	both	-1.64	no
13	both	2.69	yes
15	reform	-4.07	yes

Post-treatment Survey Results

Each student completing the course was given the opportunity to answer a survey about mathematics and calculus. All students were asked fourteen questions. Students in the reform section were asked six more questions. Students chose the response that matched their feelings from a scale of 1 (strongly agree) to 7 (strongly disagree). Two problems were included for both sections. The entire survey is in Appendix E.

The survey questions can be sorted into several categories:

- mathematics in business settings,
- applications,

- affective beliefs,
- the use of technology, and
- group projects.

The last category includes the six questions asked only of the reform students.

The questions relating to mathematics in business settings include numbers one, nine and thirteen. A t-test of means was performed on the responses for each question. Results for Question 1 are shown in Table 18. Results for Question 9 are shown in Table 19, and for Question 13 in Table 20.

Table 18
T-test of Means - Survey Question 1

SURVEY QUESTION 1	REFORM	TRADITIONAL
Mean	1.88235	2.07692
Variance	0.86029	0.63385
Observations	17	26
Pooled Variance	0.72222	
Hypothesized Mean Difference	0	
df	41	
t Stat	-0.73404	
P(T<=t) one-tail	0.23355	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.4671	
t Critical two-tail	2.01954	

Table 19
T-test of Means - Survey Question 9

SURVEY QUESTION 9	REFORM	TRADITIONAL
Mean	2.125	2.5
Variance	0.78333	0.58
Observations	16	26
Pooled Variance	0.65625	
Hypothesized Mean Difference	0	
df	40	
t Stat	-1.45686	
P(T<=t) one-tail	0.07648	
t Critical one-tail	1.68385	
P(T<=t) two-tail	0.15296	
t Critical two-tail	2.02107	

Table 20
T-test of Means - Survey Question 13

SURVEY QUESTION 13	REFORM	TRADITIONAL
Mean	2	2.15385
Variance	0.8	0.53538
Observations	16	26
Pooled Variance	0.63462	
Hypothesized Mean Difference	0	
df	40	
t Stat	-0.60779	
P(T<=t) one-tail	0.27338	
t Critical one-tail	1.68385	
P(T<=t) two-tail	0.54676	
t Critical two-tail	2.02107	

None of these three questions showed a significant difference in attitudes between the sections, although in each case, students in the reform section agreed more strongly with the statement than students in the traditional section. The t-values for these three questions are -0.734 (p = 0.467), -1.457 (p = 0.153), and -0.608 (p = 0.547).

The second category of questions related to applications of calculus to other disciplines, especially business. The questions in this category are numbers three, four, and six. A t-test of means was performed on each set of results. Results for Question 3 are shown in Table 21, for Question 4 in Table 22, and for Question 6 in Table 23.

Significant differences were found in the mean of the responses to question 3. Figure 1 shows the responses for each section in a relative frequency histogram. This question was written in negative form, and asks students to assess the relevance of what was studied.

Table 21
T-test of Means - Survey Question 3

SURVEY QUESTION 3	REFORM	TRADITIONAL
Mean	3.1875	4.16
Variance	1.629167	2.306667
Observations	16	25
Pooled Variance	2.04609	
Hypothesized Mean Difference	0	
df	39	
t Stat	-2.12356	
P(T<=t) one-tail	0.020052	
t Critical one-tail	1.684875	
P(T<=t) two-tail	0.040105	
t Critical two-tail	2.022689	

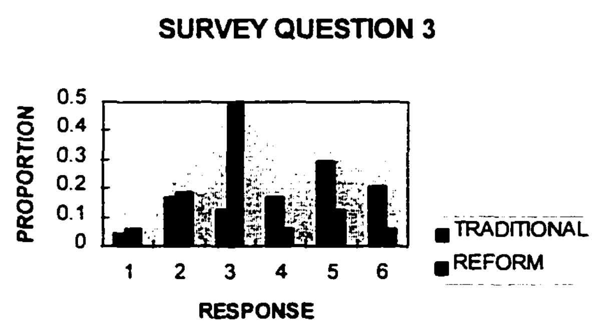

Figure 1.

Frequency of Answers - Survey Question 3

Students in the traditional section were more likely to disagree with the statement than students in the reform section, that is, to note the content as relevant. This can be seen in Figure 1. Note the shift of responses from the reform section to the negative direction (left), compared to the responses from the traditional section. The calculated t-value is -2.124 with a significance level of 0.040. The mean values suggest that students did not feel strongly about the relevance of the material studied (mean equal to 3.2 and 4.2). However, the variance on this question is

Table 22
T-test of Means - Survey Question 4

SURVEY QUESTION 4	REFORM	TRADITIONAL
Mean	3.35294	3.69231
Variance	1.49265	1.98154
Observations	17	26
Pooled Variance	1.79075	
Hypothesized Mean Difference	0	
df	41	
t Stat	-0.81307	
P(T<=t) one-tail	0.21044	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.42087	
t Critical two-tail	2.01954	

Table 23
T-test of Means - Survey Question 6

SURVEY QUESTION 6	REFORM	TRADITIONAL
Mean	1.875	1.96154
Variance	0.78333	0.67846
Observations	16	26
Pooled Variance	0.71779	
Hypothesized Mean Difference	0	
df	40	
t Stat	-0.32146	
P(T<=t) one-tail	0.37477	
t Critical one-tail	1.68385	
P(T<=t) two-tail	0.74953	
t Critical two-tail	2.02107	

much larger for this question than for any other question on the survey. This suggests a wide difference of opinion. It is possible that a relatively large proportion of the students felt strongly about this question in both directions. It is also possible that some students did not read the question carefully.

Question 4 asked students about previous exposure to the applications covered in this course. No significant difference was seen between the sections, with a t-value of -0.813, and a significance level of 0.421. The means (3.56 overall) indicated that students as a group did not have strong responses to this question. However, the large variance on this question is also significant. An analysis of the responses showed that upperclassmen tended to give a stronger positive response than underclassmen. This suggests that the large number of freshmen in the experiment had an influence on this question, and suggests that these applications are encountered by students.

Question 6 asked students about the usefulness of the applications in making the techniques clearer. There was no significant difference in the means, with a calculated t-value of -0.321, and a significance level of 0.750. The strength of the responses is very interesting. The overall mean is 1.93, indicating that most students felt strongly that the applications helped make the mathematics clearer. This result suggests that the applications are important to the success of the course.

Questions two, five, seven, and twelve relate to the affective aspects of the survey. A t-test of means was performed for each question. Table 24 shows the results for question 2, and Table 25 shows the results for question 5. The results for question 7 are in Table 26, and those for question 12 are in Table 27. In each case, students generally agreed with the question, but only somewhat. No significant differences were found in the means for each section, and the variances were not large.

Table 24
T-test of Means - Survey Question 2

SURVEY QUESTION 2	REFORM	TRADITIONAL
Mean	2.05882	2.07692
Variance	0.93382	0.63385
Observations	17	26
Pooled Variance	0.75091	
Hypothesized Mean Difference	0	
df	41	
t Stat	-0.06697	
P(T<=t) one-tail	0.47347	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.94693	
t Critical two-tail	2.01954	

Table 25
T-test of Means - Survey Question 5

SURVEY QUESTION 5	REFORM	TRADITIONAL
Mean	2.05882	2.24
Variance	0.93382	1.77333
Observations	17	25
Pooled Variance	1.43753	
Hypothesized Mean Difference	0	
df	40	
t Stat	-0.48069	
P(T<=t) one-tail	0.31668	
t Critical one-tail	1.68385	
P(T<=t) two-tail	0.63336	
t Critical two-tail	2.02107	

Table 26
T-test of Means - Survey Question 7

SURVEY QUESTION 7	REFORM	TRADITIONAL
Mean	2.4375	2.19231
Variance	0.92917	0.56154
Observations	16	26
Pooled Variance	0.6994	
Hypothesized Mean Difference	0	
df	40	
t Stat	0.92271	
P(T<=t) one-tail	0.18084	
t Critical one-tail	1.68385	
P(T<=t) two-tail	0.36169	
t Critical two-tail	2.02107	

Table 27
T-test of Means - Survey Question 12

SURVEY QUESTION 12	REFORM	TRADITIONAL
Mean	2.11765	2.38462
Variance	0.48529	0.64615
Observations	17	26
Pooled Variance	0.58338	
Hypothesized Mean Difference	0	
df	41	
t Stat	-1.12063	
P(T<=t) one-tail	0.13448	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.26897	
t Critical two-tail	2.01954	

The next category of questions on the survey dealt with the use of technology in this course. The questions in this category include numbers eight, ten, eleven, and fourteen. A t-test of means was performed for each question. Table 28 contains the results for Question 8, while Table 29 has the results for Question 10. Question 11 results appear in Table 30, and those for Question 14 in Table 31.

Results of the t-tests show that each of the questions had significantly different means between the two sections. In each case, students in the reform

Table 28
T-test of Means - Survey Question 8

SURVEY QUESTION 8	REFORM	TRADITIONAL
Mean	5.29412	2.80769
Variance	1.09559	2.08154
Observations	17	26
Pooled Variance	1.69678	
Hypothesized Mean Difference	0	
df	41	
t Stat	6.11984	
P(T<=t) one-tail	1.5E-07	
t Critical one-tail	1.68288	
P(T<=t) two-tail	2.9E-07	
t Critical two-tail	2.01954	

Table 29
T-test of Means - Survey Question 10

SURVEY QUESTION 10	REFORM	TRADITIONAL
Mean	1.94118	2.88462
Variance	0.30882	1.30615
Observations	17	26
Pooled Variance	0.91695	
Hypothesized Mean Difference	0	
df	41	
t Stat	-3.15877	
P(T<=t) one-tail	0.00149	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.00297	
t Critical two-tail	2.01954	

Table 30
T-test of Means - Survey Question 11

SURVEY QUESTION 11	REFORM	TRADITIONAL
Mean	1.11765	1.46154
Variance	0.11029	0.41846
Observations	17	26
Pooled Variance	0.2982	
Hypothesized Mean Difference	0	
df	41	
t Stat	-2.01903	
P(T<=t) one-tail	0.02503	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.05005	
t Critical two-tail	2.01954	

Table 31
T-test of Means - Survey Question 14

SURVEY QUESTION 14	REFORM	TRADITIONAL
Mean	1.52941	2.26923
Variance	0.26471	1.16462
Observations	17	26
Pooled Variance	0.81343	
Hypothesized Mean Difference	0	
df	41	
t Stat	-2.62992	
P(T<=t) one-tail	0.00599	
t Critical one-tail	1.68288	
P(T<=t) two-tail	0.01197	
t Critical two-tail	2.01954	

section had more positive responses compared to students in the traditional section.

Question 8 asked about the usefulness of technology in the course. Students in the reform section felt that the calculators helped them understand the mathematical concepts and also made the course more enjoyable. Question 8 is notable because of the extreme difference in the means: 5.29 for the reform section and 2.81 for the traditional section. The calculated t-value is 6.12 with a significance level of 0.000000292. This question does not ask if the calculators were useful in solving problems, but if they were useful in understanding concepts. Students in the reform section found calculators helpful in understanding the concepts, while students in the traditional section did not find the use of calculators helpful in understanding the concepts.

Question 10 asked students if technology made the course more enjoyable. One would expect that most students would agree with this statement because the alternative would be to solve problems by hand. As expected, students tended to agree. The overall mean was 2.51. A significant difference in the means was found, with a calculated t-value of -3.16 with a significance level of 0.00298. Students in the reform section felt more strongly that technology made the course more enjoyable. This result is interesting because there were many days when these students were frustrated and stymied with figuring out how to use their calculators.

Question 11 was more general in nature, asking students about their need to know how to use modern technological tools. Both sections agreed with the statement, but the reform section students felt significantly more so. The calculated t-value for this question was -2.02 with a significance level of 0.050.

Overall, the mean was 1.33 for this question. This result indicates that students are aware of the need to learn how to use modern technological tools. It appears that the use of such tools makes students stronger believers.

Question 14 asked students about the

appropriateness of the technology used in the course. Students in the reform section agreed with this statement more strongly than those in the traditional section. The calculated t-value for this question is -2.63 with a significance level of 0.0120. Most interesting is the fact that students in the reform section generally felt that their technology was more appropriate than students in the traditional section.

The last category of questions dealt with the use of group projects. Since students in the reform section were the only ones to participate in group projects, they were the only ones to answer these questions. The mean values and standard deviations for these questions are shown in Table 32.

Table 32
Means and Variances - Survey Questions 15-20

QUESTION	MEAN	STD DEVIATION
15	2.625	0.2562
16	4.125	0.2213
17	1.250	0.1118
18	1.938	0.2809
19	5.375	0.1548
20	5.250	0.3354

Some interesting observations can be made from these data. These students recognize the importance of group work as a skill important in their career field (question 17). Students found that the projects related to the major concepts of the course (question 16), and that the projects made the mathematical concepts clearer (question 15). They agreed with the question that the course was more enjoyable because of the projects (question 18) and that group projects are a realistic part of the course (question 20). Students felt that the technology was useful in doing the projects (question 19).

Supplemental Analysis Results

Students in the reform section were classified by the Calculus I course they used as a prerequisite. Of the 21 students who took the final examination, 13 had taken the first-semester reform course, and 8 had taken a non-reform course.

Mean scores for each examination question were calculated for each group and compared using t-tests of means. The null hypothesis was that no difference exists in the mean scores. Results for each question are shown in Table 33.

For $\alpha = 0.05$, none of the questions had mean score that were significantly different between the two groups. Only one question, number 8 came close, with $\underline{t} = -1.39$ ($\underline{p} = .091$). Question 8 asked students to calculate the first and second partial derivatives of a function of two variables. The students not enrolled in

Table 33.

Comparison of Mean Scores - Reform and Non-reform Prerequisite

Examination Question	**4**	**5**	**8**	**9**
Mean Score Reform	4.62	13.85	7.38	2.69
Mean Score Non-reform	5.63	13.38	10.50	2.50
t Stat	-0.303	0.376	-1.39	0.111
P(T<=t) two-tail	0.765	0.711	0.091	0.456

Examination Question	**10**	**12**	**13**	**15**
Mean Score Reform	11.31	12.92	9.54	7.69
Mean Score Non-reform	11.13	14.75	10.88	4.375
t Stat	0.065	-1.07	-0.554	1.10
P(T<=t) two-tail	0.475	0.150	0.293	0.142

a reform-style course for the prerequisite scored better than those enrolled in the reform-style prerequisite course.

Qualitative Results

A total of four students were interviewed after the semester was over. Each student volunteered and was chosen because s/he was available after semester grades were submitted. Three of the students were in the experimental section, and one in the traditional section. They answered questions about the format of the course, including teaching style and the use of calculators. They also were asked about their feelings concerning the course, the weak and strong points, and how it could be improved. Sample questions for the interviews can be found in Appendix F. The listed questions should be considered the starting point for various topics to be covered in the interviews. Specific questions were tailored to fit the responses given to previous questions.

Three of the students were male, one was female. Each of these students was enrolled in a section of the prerequisite course taught by this researcher. Two students earned a grade of A in the course, one earned a B, and one withdrew to avoid failing the course.

Three of the students answered questions on the topic of graphing calculators. All three expressed their frustrations with learning to use the calculator, and felt that more classtime should be spent learning how to use the calculator. One student commented: "What good is a fancy calculator if you can't use it?" The two students in the reform section who passed the course felt that they had mastered the calculator and also classified the use of the graphing calculator as more of a help than a hindrance. One of these students did state that he really didn't know *everything* there was to know about his calculator, but that he felt comfortable using the owner's manual to figure out new commands. Both of these students endorsed the idea of using their calculators again. Both felt that they would use the calculators in other classes, although neither was sure exactly which courses. Interestingly, one of these students changed his major and enrolled in a pre-calculus course one year later. He expressed frustration that this course did not permit the use of graphing calculators: "After I got used to using this one all the time, it's hard to go back to a regular calculator."

In contrast, the third student, who withdrew, said that she never felt comfortable with the calculator: "I was always worried about how the calculator should be used, even when the problem didn't even really need a calculator." This student felt that she would have probably passed the course if she had chosen a traditional section.

Predictably, the two reform-section students who passed the course endorsed the expansion of reform calculus to all sections. One of these students rated this course as the best he had ever taken. The other student suggested that the use of computers be expanded to exploit the greater graphing capabilities in computer algebra systems. He mentioned the Texas

Instruments TI-92 as a better calculator for this course because "it does more things for you." He acknowledged that it is important to be able to perform basic operations such as differentiating or integrating by hand, but felt that there was too much emphasis on hand work.

The fourth student was enrolled in the traditional section. He felt satisfied with the course the way it was taught, but expressed interest in the use of graphing calculators when introduced to one during the interview. He expressed some desire to have been in the experimental course. A junior in classification, he had completed courses that required the use of computers, and felt that any preparation he could get for those courses would have been helpful.

On the subject of projects, the three reform-section students were all generally enthusiastic. The female student commented that she was concurrently enrolled in the Integrated Business Course in the College of Business, and that "this [calculus] course would have helped some of the people in my group be better group members." She felt that the short, two-week life of each project was good preparation for a semester-long project.

Each of these students expressed concerns about the use of group grades. "If one person isn't coming to meetings and stuff, why should we do their work for them and they still get a good grade, when we're the ones who busted for them?" None of the three liked the instructor assigning groups, and felt strongly that students should be allowed to form groups of their own choosing. One student felt that groups stifled his creativity, but acknowledged that he gained from the experience of having to cooperate with others.

The traditional-section student liked the textbook (Bittinger, 1996) used in his section because the explanations and examples helped in working the homework problems. He claimed to read the book before lectures in order to understand the material presented in lecture better. He liked the fact that each even-numbered problem had an odd-numbered clone. This meant that if he had trouble, he could check the solutions manual for ideas about the assigned problem. He also acknowledged that this technique may not have been the best for long-term learning, but also noted that it made him successful in this course.

The reform-section students liked the text (Hughes-Hallett, 1996) as well. One student expressed admiration for the problems that used data from journals: "These things [applications] really came from someone, and weren't just a bunch of numbers." Two of the students did complain that the homework was hard because the problems weren't just like the examples. They felt that the time spent in class reviewing problems helped in this regard. One student did recognize that life is not always predictable and like the examples: "When I worked at [an internship location], every day was different, and we were constantly trying to figure out how to deal with the latest crisis. I guess the homework was sorta like that."

In conclusion, the reform-section students liked the calculus course, and were glad they "stuck it out."

One commented that he enjoyed this calculus much more than the calculus course he took in high school because he could work at figuring out concepts better with fewer lectures. Even the student who withdrew expressed disappointment that it didn't work out. "Maybe if my semester hadn't been so crazy with IBC [the Integrated Business Course], I could have handled this course better." All four students endorsed the wider use of reform calculus.

CHAPTER FIVE

DISCUSSION

The purpose of this study was to compare two ways of teaching applied calculus to students in non-science disciplines.

Summary

Research question one attempted to measure the extent to which students in the reform section differed on written questions compared to students in the traditional section. The common questions on the final examination were used to answer this question. There were eight questions on the final examination common to both courses. Of these, two, questions four and nine can be classified as stressed in the traditional section and not stressed in the reform section. Two questions, numbers ten and fifteen, can be classified as not stressed in the traditional course and stressed in the reform section. The remaining four questions, numbers five, eight, twelve, and thirteen were stressed in both sections.

Students in the traditional section did significantly better on the questions stressed in their section compared to students in the reform section. Question 4 required the recall of the formula for probability in an interval. Both sections used that formula, but the traditional section explicitly covered the uniform distribution. Most students in the traditional section evaluated the resulting integral; a few students recalled that the appropriate area under the curve is a rectangle and found the area using the geometric formula $A=lw$. A few of the students in the reform section correctly recalled the integral formula, most did not. Some who did not, tried to draw the appropriate graph, but did not recognize the area as a rectangle.

Question 9 required the use of the formula Revenue=Price x Quantity. Although both sections saw this formula in class, a note was added to the examination question to help the students. The first part of the question asked students to create a formula for revenue in terms of price only. This involved the use of formulas for quantity as a function of price given in the problem. The second part of the problem required students to find the prices that would maximize revenue. Traditional-section students generally did well on both parts of this problem. The reform-section students did less well on both parts of this problem.

It is no surprise that students in the traditional section did well on these questions; they had seen virtually identical problems in homework assignments. Clearly, despite efforts to improve students' analytical skills in the reform section, they still have difficulty with new questions in a testing situation.

Questions 10 and 15 are classified as stressed in the reform section. As expected, students in the reform section did better than students in the traditional section. Question 10 asked students to solve a differential equation, specifically the equation for exponential growth. Both sections worked this type of problem, and spent approximately the same amount of time on this topic in class.

Students in the traditional section had difficulty not only in recognizing the type of differential equation, but also that P was the function name, and not the independent variable. Many of them integrated with respect to P. Students in the reform section fared much better on this question. They tended to recognize that the independent variable was not shown, and was assumed to be t. The most common mistake for these students was computational; students who lost points were unable to correctly incorporate the initial condition.

Question 15 was designed to analyze students' understanding of the practical interpretation of the partial derivative. Students in the reform section did much better than students in the traditional section.

Almost none of the students in the traditional section earned any points for this problem. Most of them responded that the derivatives must be positive because the problem dealt with either the number of people, or the cost of a ticket, both of which must be positive. Approximately one-third of the reform section students recognized that the derivative could be negative, and several of these students were able to determine that the signs of the two partial derivatives must be opposite.

Question 15 along with Question 8 measured students' understanding of the partial derivative. Question 8 is computational in nature, while Question 15 is conceptual. Students in the reform section did almost as well on the computational problem, and much better on the conceptual problem. This pair of problems suggests that students can still learn the conceptual part of a mathematical idea, without sacrificing learning the computational part of the idea. This result is especially important, given the rapid proliferation of technological devices that can perform the computational parts of these problems.

The remaining questions, numbers five, eight, twelve, and thirteen were stressed in both sections.

Question 5 and Question 8 had clearly non-significant differences in scores. Both groups of students were able to compute in these routine problems.

What is most interesting is that both of these questions came (with slight changes of numbers) from the traditional textbook assignments. This means that students in the traditional section saw virtually the exact same problem in homework assignments. Students in the reform section studied these concepts, but had not seen the exact problem before the examination. This suggests that they were capable of applying a concept to a problem that was somewhat different than problems seen before. By scoring the same on these questions, students in the reform section showed a higher level of comprehension on these questions.

Question 12 was a routine problem about present and future value. Students in the reform section did slightly better than students in the traditional section. The difference in scores has a borderline significance level of 0.106. The most common mistake on this problem was confusing present and future value.

Both groups of students were mostly able to recall the formula, but not all were able to correctly apply the formula to this problem. Since the mistakes were so similar for both groups, it is not clear that there was a difference in comprehension. It is possible that this is a typical score for this question, no matter which way the topic is presented.

Students in the traditional section did much better on Question 13. This question asked students to find the relative extremes and saddle points of a bivariate function. Both groups of students learned the mechanics of this problem, although students in the reform section were less likely to remember every step of the process. Students in the traditional section had more homework that stressed only the mechanics of the process. Students in the reform section had homework that required them not only to compute, but also to interpret their results. Perhaps they needed more drill on this topic.

The supplemental analysis was undertaken after it became clear that a significant number of students enrolled in the reform section without having completed the prerequisite course in reform style. Preliminary results indicate no significant differences in mean scores on the final examination questions, with one possible exception. Students without the reform-style prerequisite did slightly better on one computational question.

The qualitative results of this experiment suggest that some students are very receptive to the use of modern technology, but that some students may have trouble adapting. The experimental section was taught in a way that was different from most mathematics courses. Some students like the familiar and do not want to change, even if the change is recognized as probably good.

Given the pervasiveness of handheld graphing calculators, and the inexorable march of technological progress, it is incumbent on instructors to adapt and teach in ways that maximize student comprehension. Reform calculus appears to have many features that will do just that.

Threats to Validity

Campbell and Stanley (1966) and Cook and Campbell (1979) have identified several threats to validity in research in the social sciences. These threats can be categorized as internal or external. This discussion will focus on internal threats to validity. They include history, maturation, testing; instrumentation, statistical regression, selection, experimental mortality, interactions with selection, ambiguity about the direction of causal influence, diffusion of treatments, compensatory equalization of treatments, compensatory rivalry by subjects, and resentful demoralization of subjects. Each of these threats must be considered, but not all of these threats are significant in this experiment.

The first threat is **history**. This occurs when an event not part of the treatment takes place between the pretest and the posttest. Since this study continued over a sixteen week semester, many such events may have occurred. On a global level, both groups experienced the same external events. On a local level, it is possible that events in the classroom on specific days may have had an influence on the subjects. No extraordinary events, such as fire drills, occurred during class time. Other, less-significant, events may well have occurred during the semester.

The second threat is **maturation**. This refers to changes that occur within subjects merely as a function of time. This threat is hard to quantify, and is most likely to occur as an interaction with selection.

The third threat is **testing**. This refers to repeated testing of the subjects and familiarity caused by such testing. Although none of the subjects had seen the specific questions on the posttest, this threat may have occurred because both groups had tests during the semester. The control group was given three tests before the posttest, while the experimental group was given only one test during the semester. It is plausible to believe that this differential testing policy contributed to decreased internal validity.

The fourth threat is **instrumentation**. This threat occurs when the measuring instrument is changed between pretest and posttest. Although the specific questions that comprised the instruments in this study were different between the pretest and the posttest, the form and evaluation were the same on both tests. This threat is minimal in this study.

The fifth threat is **statistical regression**. This threat occurs when subjects are classified and placed into experimental groups based on a measure such as the pretest. This threat is most pronounced when one group scores much higher or lower than the other on a measure. In this study, the groups did not differ significantly on any of the variables measured at the beginning of the study. This threat is not considered to be serious in this study.

The sixth threat is **selection**. This threat occurs when different types of people enter each experimental group. Cook and Campbell (1979) describe this threat as "pervasive in quasi-experimental research" (p. 53). This is probably the most serious threat occurring in this study.

During the course of the interviews, two subjects noted that an academic advisor suggested enrollment in the experimental section only to high-achieving students. Additionally, students were given the opportunity to change sections without penalty at the start of the semester. It is reasonable to assume that students who are averse to using technology would self-select out of the experimental section. It is also reasonable to assume that students who are keenly interested in technology would self-select into the experimental section.

It should be noted that two measures of achievement, math ACT score and high school GPA, did not significantly differ between the two groups. These measures are not exhaustive, but do suggest that selection is not a fatal threat in this study.

The seventh threat is **experimental mortality**. This threat occurs when subjects that do not complete the study differ in some characteristic between the two groups. This threat is plausible in this study because subjects were able to drop the course during the first two-thirds of the study. Each student who dropped the course was failing the course at the time of the drop, but no additional measures of these subjects were made.

The eighth threat to validity is **interactions with selection**. These may include selection-maturation, selection-history, and selection-instrumentation interactions. The latter two interactions appear not to be significant in this study. The interactions of selection and maturation might be significant in this study. Since some self-selection occurred, it is plausible that students in the experimental section may have matured as students of calculus at a different rate than students in the control section. If the experimental section students matured more quickly, this threat might explain some of the difference in scores. It is worth noting that neither group consistently did better than the other group on all measures, suggesting that this threat may not be serious.

The ninth threat is **ambiguity about the direction**

of causal influence. This threat is not of concern in studies where the order of temporal precedence is well established. This threat is not of concern for this study.

The tenth threat is **diffusion of treatments**. This threat may occur when subjects in the two groups communicate with each other. Such communication may have occurred, but no evidence of any communication affecting the subjects was seen by the researcher, and interviews failed to detect any such diffusion.

The eleventh threat to validity **is compensatory equalization of treatments**. This threat occurs when the treatment is seen as desirable, and there is a desire by the researcher to compensate the control group for not receiving the treatment. In this study, no such compensation occurred.

The twelfth threat is **compensatory rivalry by subjects**. This threat occurs when subjects in the control group becomes motivated to diminish the expected differences between the experimental and control groups. In this study, no evidence of this threat appeared to the researcher. The control group was not given any information suggesting that the experimental treatment was better, just that it was different. It is also the case that a student in the control section was free to switch to the experimental section at the beginning of the semester, thus providing any student the opportunity to join the experimental treatment.

The thirteenth threat is **resentful demoralization of subjects**. This threat is just the opposite of the previous threat; in this case, subjects in the control group intentionally perform poorly. No evidence of these threats appeared to this researcher.

In sum, there are some threats to the internal validity of this study, specifically, history, maturation, selection, mortality, and the interactions that are possible with the listed threats. Selection appears to be the most serious, and the one most difficult to control in this type of study. The best way to reduce these threats would be to repeat this study in a true experimental format, specifically with random assignment of subjects. While not impossible to accomplish in this setting, random assignment of subjects would be difficult, while maintaining the other characteristics of this study.

Suggestions for Future Research

This experiment involved a small number of students compared to the total population. Students in both sections were told of the experiment and the role of each section in the study. Students in each section were able to change their enrollment during the first two weeks of the semester for any reason. Some chose to leave the experimental section, especially those who had not been in the experimental section for the prerequisite course. The ability to leave the section may have biased the sample toward students who are particularly interested in the use of technology and group work. This is an issue that needs further examination. How does an instructor effectively include technology when faced with students who are averse to the idea of technology? What techniques, what examples, what homework problems will ease the transition?

The researcher designed this experiment with several variables that differed between the control and experimental sections. It is possible that some of these variables could be isolated and examined individually. For example, how would student attitudes and learning change if graphing calculators were used in a traditional setting? How would student attitudes and learning change if group projects were incorporated into a traditional setting?

The supplemental analysis yielded interesting material for future study. Approximately one-third of the students enrolling in MATH 2123 at this university have completed the prerequisite course at another institution. Since reform methods are not used universally, students without reform backgrounds will be part of this population for some time. Further research could study the possible need to help such students adapt to a reform course, and how to provide such need, if required.

Another issue important to this university setting is the size of the sections. Most students enroll in sections with enrollment of two hundred or more. Some of the techniques used in the reform section may not be possible in such a large section. The use of group projects is much more difficult when there are fifty groups instead of eight. Even the use of small group discussions during class time is problematic when one instructor is responsible for fifty groups or more. What modifications will be necessary when large numbers of students are exposed to reform? Which aspects can be successfully incorporated, and which aspects are not possible when large sections are taught?

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APPENDIX A

Comparison of Topics

REFORM COURSE | **TRADITIONAL COURSE**

3.4 Definite Integral as Average Value

- Def'n: $\frac{1}{b-a}\int_a^b f(x)dx$
- Average on a graph

3.5 Interpretations of the Definite Integral

- Notations and units
- Definite integral as total change
- Integral of marginal cost

3.6 Fundamental Theorem of Calculus (FTC)

- Statement of the theorem
- Using FTC to learn about F(x)

5.5 Integration: Substitution (Traditional Course)

- Basic formulas

5.6 Integration: Parts (Traditional Course)

- Formula: $\int udv = uv - \int vdu$
- Tabular integration by parts

6.1 Definite Integral Revisited

- Interpretations
- Fundamental Theorem of Calculus
- Graphing a function given the graph of f'
- Applications of FTC

6.2 Applications to Life Sciences

- Populations growth rates
- Bioavailability of drugs

6.3 Present and Future Value

- Def'ns: present value, future value

6.2 Applications of $\int_0^T P_0 e^{\pm kt}dt$ (Traditional Course)

- Def'n: continuous money

- Formulas: $B = Pe^{\pm kt}$
- Income streams
- Formulas for present value, future value

6.4 Consumer/Producer Surplus

- Def'ns: consumer and producer surplus
- Formulas
- Effect of price controls

6.5 Applications to Distribution Functions

- Example: age distribution
- Smoothing the histogram
- Def'ns: density function, cumulative distribution function

6.6 Probability and More on Distributions

- Probability as an integral
- Def'n: median: $\int_{-\infty}^{T} p(x)dx = 0.5$
- Def'n: mean: $\int_{-\infty}^{\infty} xp(x)dx$
- Normal distribution

7.1 What is a Differential Equation?

- Finding a solution numerically

flow

- Future value
- Present value
- Accumulated present value

6.1 Consumer/Producer Surplus

- Def'ns: consumer and producer surplus
- Formulas

6.3 Improper Integrals

- Def'n using limits
- Def'n: convergent/divergent
- Perpetual money flow

6.4 Probability Part I

- Def'ns: continuous random variable, probability density function (pdf)
- Constructing pdf's
- Uniform distributions
- Exponential distribution

6.5 Probability Part II

- Expected value
- Def'n: $\mu = E(x)$
- Variance
- Standard deviation
- Normal distribution

6.7 Differential Equations

- Def'n: differential equation
- General solutions

- Using formulas to find a solution
- Using initial conditions
- Verifying solutions

7.2 Slope Fields
- Setting up a slope field
- Visualizing a solution
- Existence/uniqueness of solutions

7.3 Growth and Decay
- General solution of $y' = ky$
- Modeling population growth
- Continuously compounded interest
- Example: pollution dissipation
- Example: quantity of a drug in the body

7.4 Applications and Modeling
- Newton's Law of Heating
- General solution of $y' = k(y - A)$
- Equilibrium solutions
- Net worth of a company

8.1 Functions of Many Variables
- Example: weather maps/isotherms
- Example: beef consumption
- Formula representations
- Varying one variable

8.2 A Tour of 3-Dimensional Space

- Particular solutions
- Verifying solutions
- Separation of variables
- Elasticity

B.2 First Order Differential Equations
- Equations of the form: $a_1(x)y' = g(x)$
- Solving the general equation
- Integrating factors

B.3 Slope Fields
- Slope and concavity
- Slope fields
- Stable/unstable solutions

7.1 Functions of Several Variables
- Definition
- Geometric interpretation

- Cartesian coordinates in 3-space
- Graphing equations

8.3 Graphs of Functions of Two Variables
- Visualizing surfaces
- Plotting $f(x,y) = x^2 + y^2$
- Shifts
- Cross-sections
- Linear functions

8.4 Contour Diagrams
- Topographical maps
- Contour diagrams
- Contour diagrams and graphs
- Finding contours algebraically
- Contour diagrams and tables

8.5 Linear Functions
- What makes a plane flat?
- Numerical point of view
- Contour diagram

8.6 Cobb-Douglas Production Functions
- Contour diagrams of production functions
- Formulas for productions functions
- Def'n: Cobb-Douglas production function
- Returns to scale

7.2 Partial Derivatives (covered elsewhere)
- Cobb-Douglas production function

9.1 The Partial Derivative
- Definitions
- Visualizing on a graph
- Estimating with contour diagrams
- Using units to interpret

9.2 Computing Algebraically
- Estimating a small change

7.2 Partial Derivatives
- Finding partial

- Functions of more than two variables
- Second-order partial derivatives
- Mixed partials are equal

9.3 Local and Global Extrema

- Def'ns: local and global maxima and minima
- Detecting extremes
- Def'n: saddle points
- Second derivative test

9.4 Unconstrained Optimization

- Example: maximizing profit
- Fitting a line to data
- Simple linear regression

9.5 Constrained Optimization - Lagrange Multipliers

- Graphical approach
- Lagrange multipliers
- Distinguishing maxima and minima
- Meaning of l
- Lagrangian function
- More general problems

derivatives

- Geometrical interpretation
- Cobb-Douglas production function

7.3 Higher-Order Partial Derivatives

- Second-order partial derivatives
- Equality of mixed partials

7.4 Maximum - Minimum Problems

- Def'ns: relative maximum, relative minimum
- Finding extremes
- The D-test

7.5 Application: Least-Squares

- Least-squares assumption
- Finding the regression line

7.6 Constrained Maximum - Minimum Values: Lagrange Multipliers

- Method of Lagrange multipliers

7.7 Multiple Integration

- Double integrals
- Geometric interpretation
- Joint Probability density functions

APPENDIX B

Pre-treatment Questionnaire

MATH 2123 QUESTIONNAIRE
SPRING 1997

Name:______________________________ Section:__________

ID #:____________________ Major:______________________

Classification:__________________ Age:___________

What math classes have you taken at OU? What grades did you earn in them? Include classes you took more than once.

What math classes have you taken elsewhere? Where? What grades did you earn?

Do you have a job? If so, how many hours do you work in a typical week?

Do you participate in organized sports? If so, what?

Are you involved in extracurricular activities? If so, what?

How would you rate you interest in math? (circle one)

very high	high	somewhat high	average	somewhat low	low	very low

How would you rate you aptitude in math? (circle one)

very high	high	somewhat high	average	somewhat low	low	very low

On the back of this sheet, please include any other information about yourself that you would like me to know.

APPENDIX C

Pretest

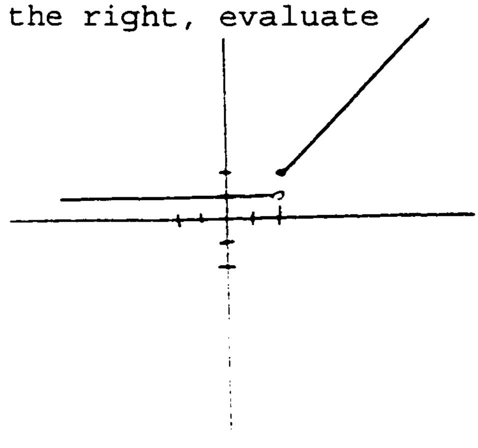
PRETEST
MATH 2123
SPRING 1997

NAME__________________

ID #__________________

Please answer each question to the best of your ability. There may be some problems you cannot answer; just do the best you can on the rest. Circle your answer. Put all work on this sheet.

1. For the graph of f(x) at the right, evaluate



A) $\lim_{x \to 0} f(x)$

B) $\lim_{x \to 2^+} f(x)$

C) $\lim_{x \to 2^-} f(x)$

D) $\lim_{x \to 2} f(x)$

E) Is f continuous at x=2?

2. Find dy/dx for the following functions:

A) $y=e^{3x} - \ln x^2$

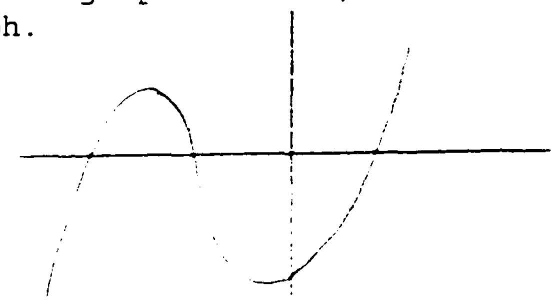
B) $y=0.5x^4 + x^3 - 7x^2 + 4x^{-2}$

C) $x^2 + y = y^3$

D) $y = x^3e^x$

E) Find y'' for the function in part B.

3. For the function graphed below, draw its derivative on the same graph.



4. Find the absolute maximum and minimum values of $f(x) = x^3 + x^2 - 5x$ on the interval $[0,3]$.

5. Of all numbers whose sum is 17, find the two whose product is a maximum.

6. If you invest \$1000 at 6% interest compounded continuously, how much will you have in 7 years?

7. Using 3 rectangles (n=3), find the left-hand and right-hand approximations of the area under the curve $f(x) = x^4$ on the interval [0,3]. Show your work.

8. Evaluate:

$$\int x^3 + 4x^2 - 3 \, dx$$

$$\int e^{7x} \, dx$$

$$\int_{-1}^{1} x^2 + 2x \, dx$$

9. Define the derivative of a function, and tell me what it means.

APPENDIX D

Common Examination Questions

4. A number is selected at random from the interval [15,35]. The probability density function for x, a number chosen from this interval is f(x) = 1/20 for $15 \le x \le 35$. find the probability that a number selected is in the subinterval [18,27].

5. Given the demand function D(x) = 500 - 5x, and the supply function S(x) = 44 + 7x, find the equilibrium point, the consumer's surplus at the equilibrium point, and the producer's surplus at the equilibrium point.

8. Given $f(x) = e^{x2+y2}$, find f_x, f_y, f_{xx}, f_{xy}, f_{yy}.

9. A company markets two products which compete with each other. Their demand functions are expressed by the following: $q_1 = 57 - 10p_1 + 20p_2$ and $q_2 = 81 + 24p_1 - 50p_2$, where p_1 and p_2 are the prices for the two products and q_1 and q_2 are the quantities of each product. Recall that revenue is given by price *x* quantity. Write a formula for revenue in terms of price only. What prices should be charged to maximize revenue?

10. Find P where P′ = 0.05P and P(0) = 3000.

12. You receive an inheritance and decide to invest some of it for a down payment on a house in 5 years. You want to have $10,000 and estimate that you can earn 6% interest, compounded continuously. How much should you invest today (one-time deposit)?

13. Find all relative extreme values and saddle points for $f(x,y) = 2x^2 - 3xy + y^2 - 5x$.

15. Commuters to the town of Metropolis can either take the train or the bus. Let N(b,t) be the number of people who ride the bus to commute, where b is the price of a bus ticket and t is the price of a train ticket. What would you expect the signs of $\delta N/\delta b$ and $\delta N/\delta t$ to be? Why?

APPENDIX E

Post-treatment Questionnaire

SURVEY FOR MATH 2123
SPRING 1997

Read each statement and then circle the response that matches your feelings. Use the following choices:

1 = strongly agree	4 = slightly disagree
2 = somewhat agree	5 = somewhat disagree
3 = slightly agree	6 = strongly disagree

1. Mathematical techniques are important tools in modern business practice.	1	2	3	4	5	6
2. Since taking this mathematics course, I believe I have a much clearer understanding of mathematical concepts that will be important in my career.	1	2	3	4	5	6
3. Much of what we studied seem to me not really relevant to what I need to be successful.	1	2	3	4	5	6
4. I have previously encountered many of the applications we studied in this course.	1	2	3	4	5	6
5. I can now solve mathematical problems that I never believed I would be able to solve before this course.	1	2	3	4	5	6
6. The applications we studied made techniques clearer.	1	2	3	4	5	6
7. The things I learned in this course seem likely to be important to my career plans.	1	2	3	4	5	6

8. Calculators did not really help me understand the mathematical concepts.	1	2	3	4	5	6
9. Mathematical concepts make it easier to understand business concepts.	1	2	3	4	5	6
10. The technology we used made the course more enjoyable.	1	2	3	4	5	6
11. It is important that I know how to use modern technological tools.	1	2	3	4	5	6
12. I now have skills that are important to my career.	1	2	3	4	5	6
13. The applications we studied are important in the business world.	1	2	3	4	5	6
14. The technology we used is appropriate for this course.	1	2	3	4	5	6

15. The projects helped make the mathematical concepts clearer.	1	2	3	4	5	6
16. The projects did not relate to the major concepts of this course.	1	2	3	4	5	6
17. Being able to complete a project important in the business world.	1	2	3	4	5	6
18. Working in groups made this course more enjoyable.	1	2	3	4	5	6
19. Using technology didn't help in doing the projects.	1	2	3	4	5	6
20. Working in groups is unrealistic for this class.	1	2	3	4	5	6

APPENDIX F

Qualitative Interview Questions

1. (Subject is presented with a problem.) Please solve this problem, telling me out loud what you are doing in each step.

2. Do you feel that you have mastered the use of the graphing calculator in this course? Why or why not?

3. Has using the graphing calculator been more of a help or a hindrance to you? Why?

4. Do you feel that the calculator you used was helpful in this course? Would you recommend using this calculator when the course is taught again?

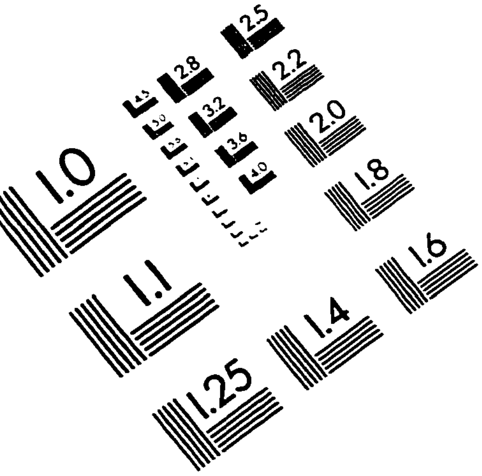
5. How would you rate this textbook compared to other mathematics textbooks you have used? Would you recommend the continued use of this textbook for this course?

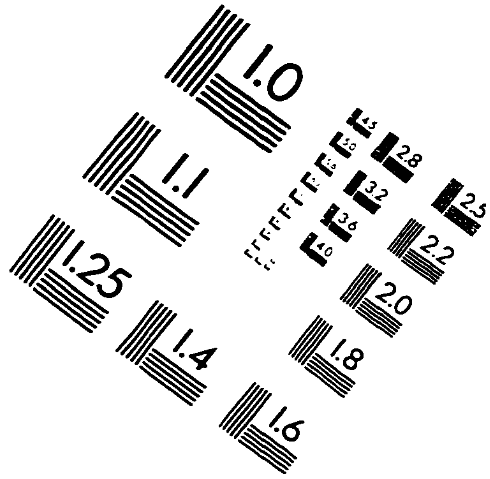
6. Having completed the experimental course, do you have any suggestions or comments to the mathematics department as it considers permanent adoption of this style of teaching calculus?

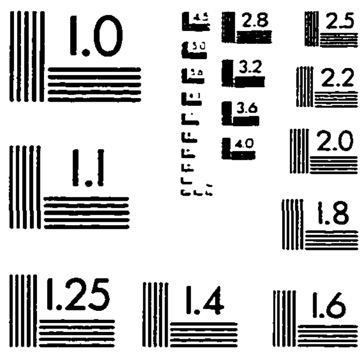
7. What was your reaction to the group projects? Did you like them? Did they help you understand the material better?

8. Here are the responses you gave on the questionnaire. Did any of the questions confuse you? Please explain any responses of 1 or 6.

IMAGE EVALUATION TEST TARGET (QA-3)







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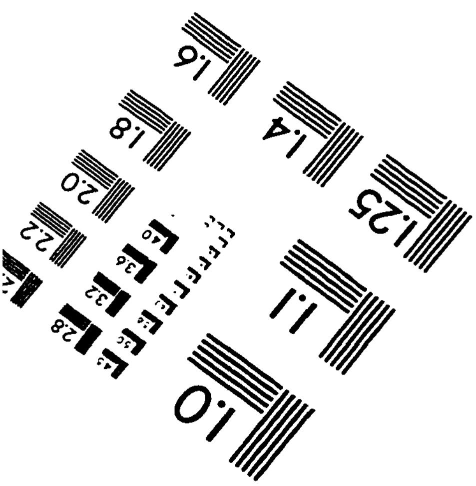

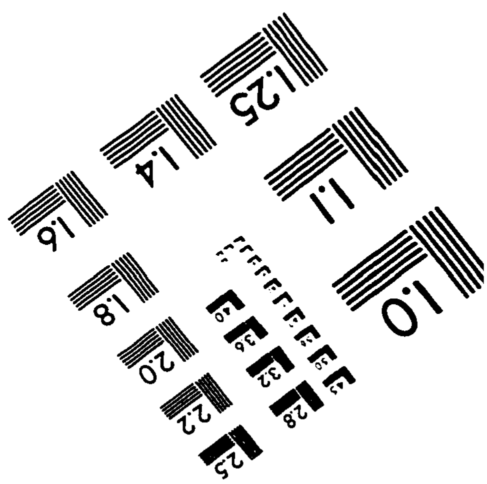

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