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RESILIENCE-BASED RESTORATION OF SYSTEMS OF INTERDEPENDENT INFRASTRUCTURE NETWORKS: IMPORTANCE MEASURES, OPTIMIZATION, AND SOLUTION APPROACHES

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RESILIENCE-BASED RESTORATION OF SYSTEMS OF INTERDEPENDENT INFRASTRUCTURE NETWORKS: IMPORTANCE MEASURES, OPTIMIZATION, AND SOLUTION APPROACHES

A DISSERTATION APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

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To
My wife Arwa
My sons Amjad and Asseel
My father, my mother, and my brothers and sisters

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Abstract

Critical infrastructure networks such as electric power, water distribution, natural gas, transportation, and telecommunications, among others, are the backbone of modern societies as they provide them with the fundamental services that support the economic productivity, security, and quality of life of citizens. However, these infrastructure networks are not isolated from each other but instead, most of them rely on one another for their proper functioning, i.e., they are interconnected and mutually interdependent. Hence, they are highly vulnerable to any disruptive event (e.g., random failures, malevolent attacks, or natural disasters), where the occurrence of a disruption in one infrastructure network could affect other dependent infrastructure networks leading to a more significant adverse impact on societies. Moreover, the proliferation of interdependencies among infrastructure networks has increased the complexity associated with recovery planning after a disruptive event, which becomes a more challenging task for decision makers.

Recognizing the inevitability of large-scale disruptions and their impacts to societies, the research objective of this work is to study the recovery of systems of interdependent infrastructure networks following a disruptive event. Accordingly, the purpose of the research components, i.e., main contribution, is to develop: (i) importance measures, and (ii) restoration modeling approaches, that enhance the resilience of the system of interdependent infrastructure networks considering the physical interdependency among the infrastructure networks.

The first research component is *measuring the importance of the interdependent infrastructure networks components*. Hence, the first contribution in this dissertation is

developing measures of importance or criticality of components of a system of interdependent infrastructure networks that determine which components adversely affect the resilience of the entire system of interdependent infrastructure networks when disrupted; and prioritize their restoration tasks accordingly. The goal of the proposed importance measures is to identify the critical network components that influence not only (i) the performance of their networks the most when disrupted and restored, but also (ii) the performance of other networks due to their interdependent nature. The proposed importance measures are illustrated through generated interdependent power-water networks. The proposed importance measures represent a useful tool that can help decision makers to identify critical components in their networks following a disruptive event and prioritize their resolution accordingly.

The second research component is *optimizing the restoration of interdependent infrastructure networks*. Hence, the second contribution in this dissertation is developing restoration modeling approaches using mixed-integer programming with the objective of maximizing the resilience of the system of interdependent infrastructure networks while minimizing the total cost associated with the restoration process considering the availability of limited time and resources. The proposed modeling approaches aim to: (i) identify the set of disrupted components to be restored according to their influence on the resilience of the system of interdependent infrastructure networks, and (ii) assign and schedule the restoration tasks to the available work crews. The proposed modeling approaches are illustrated through generated interdependent power-water networks with multiple disruptions scenarios as well as a system of interdependent infrastructure networks after multiple hypothetical earthquakes in

Shelby County, TN, United States. The proposed modeling approaches represent a useful tool for decision makers that can help them finding the optimal restoration strategy for their networks following a disruptive event.

Moreover, we extend our two research components to address the restoration problem of community structures in a system of interdependent infrastructure networks following a disruptive event to enhance their resilience considering the interdependencies among the infrastructure networks. Accordingly, the third contribution in this dissertation if proposing a restoration model, using mixed-integer programming, to restore community structures of interdependent infrastructure networks with the objective of maximizing the resilience of the system interdependent infrastructure networks. Furthermore, we propose and discuss some community structures importance measures to priorities their restoration process. The proposed community structures importance measures are categorized into two groups: (i) prior to disruption importance measures, and (ii) post disruption importance measures. The proposed restoration model and importance measures for community structures in a system of interdependent infrastructure networks are illustrated through generated interdependent power-water networks.

Finally, though the work in this dissertation discusses systems of interdependent infrastructure networks, the developed importance measures and restoration modelling approaches in this dissertation could be applied to any set of physically interdependent networks.

Chapter 1 : Introduction

1.1 Overview

A critical infrastructure network is defined as a network of independent, mostly privately-owned, man-made systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services [The Report of the President's Commission on Critical Infrastructure Protection 1997]. Hence, critical infrastructure networks such as electric power, water distribution, natural gas, transportation and telecommunications, among others, are the backbone of modern societies, which depend on their continuous and proper functioning. Such critical infrastructure networks provide the fundamental services that support the economic productivity, security, and quality of life of citizens.

However, infrastructure networks are subjected to be affected by different types of disruptive events, including random failures, malevolent attacks, and natural disasters, that could affect their performance differently due to their uncertainty and have direct consequences on communities and people's daily lives. Hence, for several years, the United States, as well as many countries around the globe, have been interested in effectively preparing for and responding in a timely manner to such disruptive events (e.g., "secure, functioning, and resilient critical infrastructures" [White House 2013]). Therefore, it is increasingly important to not only protect current infrastructure networks against disruption, but to be able to restore them once they have been disrupted.

The study of critical infrastructure networks under disruption has matured considerably in the past fifteen years, where emphasis has been given to identifying descriptors that enable the study of critical network components that lead to network vulnerability. Such descriptors include topological-based descriptors such as average path length [Newman 2003], network efficiency [Nagurney and Qiang 2009], resilience-based descriptors such as resilience worth [Barker et al. 2013] and resilience [Whitson and Ramirez-Marqez 2009]), flow-based descriptors such as flow vulnerability [Ouyang et al. 2014] and flow capacity rate [Nicholson et al. 2016]), reliability-based descriptors such as reliability achievement worth [Ramirez-Marquez and Coit 2005] and availability [Barabady and Kumar 2007], other descriptors such as travel time and distance [Erath et al. 2009, Rodríguez-Núñez and García-Palomares 2014], cost of travel time [Jenelius et al. 2006, Sullivan et al. 2010], and accessibility [Sohn 2006, Chen et al. 2007], among others.

On the other hand, other studies provide methods and algorithms for restoring critical infrastructure networks following the occurrence of a disruptive event by: (i) determining the set of disrupted network components that need to be restored to maximize the performance of the network, (ii) assigning these components to work crews, (iii) determining the restoration sequence of these components, or (iv) an integrated approach [e.g., Xu et al. 2007, Yan and Shih 2009, Matisziw et al. 2010, Nurre et al. 2012, Aksu and Ozdamar 2014, Vugrin et al. 2014, Kamamura et al. 2015, Fang et al. 2016, Hu et al. 2016, Fang and Sansavini 2017, Fu et al. 2017].

However, infrastructure networks are not isolated from each other, but rather they rely on one another in different ways for their proper functioning. Hence, they exhibit interdependency, where two infrastructure networks are said to be interdependent if there is a bidirectional relationship between them through which the state of each infrastructure is dependent on the state of the other [Rinaldi et al. 2001, Peerenboom et al. 2002].

In general, interdependencies across critical infrastructure networks can improve their operational efficiency since they lead to greater centralization of control, hence they play a significant role in the continuous, reliable operation of infrastructure network [Rinaldi et al. 2001]. However, the proliferation of interdependencies among infrastructure networks may potentially cause them to be highly vulnerable to disruption. Consequently, if the operability of an infrastructure network is affected by the occurrence of a disruptive event, this could lead to cascading inoperability in some or all dependent infrastructure networks due to their interdependencies; hence, a much more significant impact on society and its economy [Little 2002, Wallace et al. 2003, Buldyrev et al. 2010, Eusgeld et al. 2011, Wang et al. 2013, Ouyang 2014, Loggins and Wallace 2015, Danziger et al. 2016, Wu et al. 2016, Ouyang 2016]. The high vulnerability of the infrastructure networks, due to their increased interdependencies, has been shown through several recent worldwide events, including the 1998 Canada ice storm [Chang et al. 2007], the 2001 US World Trade Center attack [Mendonça and Wallace 2006], the 2003 North American blackout [U.S.-Canada Power System Outage Task Force 2004], and the 2010 Chile earthquake and tsunami [Wen et al. 2011], among others. Therefore, it is crucial for decision makers to account for interdependencies between infrastructure networks when preparing the plans for their recoverability to obtain a realistic analysis of their performance [Holden et al. 2013]. In addition, performing restoration activities for each infrastructure network independently could lead to improper utilization of available resources, wasted time, and may even cause further disruptions when improperly scheduled [Baidya and Sun 2017]. As a result, the restoration of such interdependent infrastructure networks following a disruptive event has become more challenging for decision makers as the increase in interdependency among infrastructure networks magnifies the complexity associated with planning for their post-disruption recovery and operation.

Several models and techniques that consider interdependencies among infrastructure networks are proposed in the literature. Rinaldi [2004] categorized such models and techniques into six broad categories: (i) aggregate supply and demand tools [e.g., Lee et al. 2007, Min et al. 2007], (ii) dynamic simulations [e.g., Hernandez-Fajardo and Dueñas-Osorio 2013, Zhang et al. 2016], (iii) agent-based models [e.g., Panzieri et al. 2004, Oliva et al. 2010], (iv) physics-based models [e.g., An et al. 2003, Unsihuay et al. 2007], (v) population mobility models [e.g., Casalicchio et al. 2009], and (vi) Leontief input-output models [e.g., Haimes and Jiang 2001, Reed et al. 2009]. The models and techniques proposed in this dissertation fall in the aggregate supply and demand tools category by Rinaldi [2004], which evaluates the total demands, in the form of services or commodities, for an infrastructure network in a region and the ability to supply them. In addition, it is classified as a network-based approach according to a similar categorization by Ouyang [2014] for the approaches of modeling and simulation in infrastructure networks considering their interdependencies. The network-based approach [Ouyang 2014] describes the infrastructures as networks of nodes, links, and inter-links (i.e., nodes represent the different components of the infrastructures, links represent the physical relationship between the nodes, and inter-links represent the interdependencies among different infrastructures).

1.2 Problem Statement

The ubiquitous nature of critical infrastructure networks has made them highly vulnerable due to the different types of interdependencies among them. Moreover, the proliferation of interdependencies among infrastructure networks has increased the complexity associated with recovery planning after a disruptive event, which becomes a more challenging task for decision makers.

Recognizing the inevitability of large-scale disruptions, emphasis shifted from reliability-driven perspective of protection to a perspective of resilience, or the ability to withstand, adapt to, and recover in a timely manner from the effects of a disruptive event [Turnquist and Vugrin 2013]. Hence, US federal planning documents suggest (as do many across the globe) the importance of addressing critical infrastructure network resilience in such a way that reflects their “interconnectedness and interdependency” [White House 2013]. The National Infrastructure Protection Plan [DHS 2013] highlights the importance of addressing the risks associated with the interdependencies among different infrastructure networks as being “essential to enhancing critical infrastructure security and resilience”. That is, the study of the recoverability of infrastructure systems or the importance of their components should account for their interdependent nature. For example, an electric power substation may be more important when its influence on not only the rest of the electric power network is considered, but also when its influence on other infrastructures, such as water distribution or telecommunications, are included. Hence, it is important to have resilient infrastructure networks accounting for the interdependencies between them, thus the motivation of this dissertation. The Infrastructure Security Partnership [2011] defined resilient infrastructure networks as the infrastructure networks that would "prepare for, prevent, protect against, respond or mitigate any anticipated or unexpected significant threat or event" and "rapidly recover and reconstitute critical assets, operations, and services with minimum damage and disruption".

Hence, the research objective of the work in this dissertation is to study the recovery of systems of interdependent infrastructure networks following a disruptive event. Accordingly, we develop: (i) importance measures, and (ii) restoration modeling approaches, that enhance the resilience of the system of interdependent infrastructure networks considering the physical interdependency among the infrastructure networks.

First, we develop two components importance measures (CIMs), namely Optimal Recovery Time (ORT) and Resilience Reduction Worth (ЯRW). The two proposed CIMs determine which components adversely affect the resilience of the entire system of interdependent infrastructure networks when disrupted; and prioritize their restoration tasks accordingly. Hence, the goal of the proposed CIMs is to identify the critical network components that influence not only (i) the performance of their networks the most when disrupted and restored, but also (ii) the performance of other networks due to their interdependent nature. The proposed CIMs prioritize disrupted components of a system of interdependent infrastructure networks based on multiple interdependent networks resilience optimization models using mixed-integer programming (MIP) with the objective of enhancing their resilience considering the interdependences among the infrastructure networks. The purpose of the proposed CIMs, i.e., ORT and ЯRW, is to (i) quantify the effect of the disrupted components on the resilience of the interdependent infrastructure networks once they are recovered, and

(ii) measure the potential impact on the resilience of the interdependent infrastructure networks caused by a specific disrupted network component, respectively.

Second, we study the interdependent network restoration problem (INRP), which seeks to find the minimum-cost restoration strategy of a system of interdependent networks following the occurrence of a disruptive event that enhances its resilience considering the availability of time and resources. Accordingly, we propose restoration optimization models using MIP to solve this problem and suggest some solution approaches for large scale disruptions. In particular, the proposed models: (i) prioritize the restoration of the disrupted components for each infrastructure network, and (ii) assign and schedule the prioritized networks components to the available work crews, such that the resilience of the system of interdependent infrastructure networks is enhanced considering the physical interdependency among them. The proposed optimization models for solving the INRP consider partial and complete: (i) disruptions for the disrupted network components, (ii) recovery of the disrupted network components, and (iii) dependence between nodes in different networks. Furthermore, four different recovery strategies considering different assumptions regarding work crews assignment and recovery process have been explored. These strategies include: (i) recovery acceleration (i.e., assigning more than one work crew to restore the same disrupted component at the same time), (ii) network component functionality (i.e., recovering a disrupted component partially), (iii) recovery tasks assignment (i.e., assigning the same work crew to recover a disrupted component at any time), and (iv) recovery process (i.e., considering a preemptive or non-preemptive recovery process).

Third, we address the restoration problem of community structures in a system of interdependent infrastructure networks following a disruptive event to enhance their resilience considering the interdependencies among the infrastructure networks. We extend our proposed restoration model, using MIP, to restore community structures of interdependent infrastructure networks with the objective of maximizing the resilience of the system interdependent infrastructure networks. Furthermore, we propose some community structures importance measures (CSIMs) to priorities their restoration process. The proposed CSIM are categorized into two groups: (i) prior to disruption CSIMs, and (ii) post disruption CSIMs.

Finally, though the work in this dissertation discusses systems of interdependent infrastructure networks, the developed importance measures and restoration modelling approaches in this dissertation could be applied to any set of physically interdependent networks. Moreover, by studying the resilience of systems of interdependent infrastructure networks in this dissertation, we unveil the effects on their performance of both the magnitude of the disruptive event (i.e., network vulnerability) and the trajectory of recovery of their disrupted components (i.e., network recoverability).

1.3 Dissertation Organization

This dissertation consists of seven chapters where the content of each chapter is briefly described in this section.

In Chapter 1, we give an overview about the studies of critical infrastructure networks under disruptions and introduce the interdependencies among infrastructure networks. In addition, we address the problem statement with the objective of this dissertation as well as the main contributions.

In Chapter 2, we present some backgrounds about network definition, resilience modeling, mathematical programming, and infrastructure networks interdependencies. Also, we discuss the previous studies in which similar kind of research is done.

In Chapter 3, we present two component importance measures for systems of interdependent infrastructure networks based on four different developed resilience optimization models using MIP. Additionally, we present and discuss the results of a numerical experiment. Moreover, we discuss the effects of uncertainty on the proposed measures and also compare them with other two non-resilience-based measures.

In Chapter 4, we present a restoration model for systems of interdependent infrastructure networks using MIP. Also, we present and discuss the results of a numerical experiment. Furthermore, we present a progressive restoration approach for large scale networks disruption.

In Chapter 5, we present another restoration model for systems of interdependent infrastructure networks using MIP. In addition, we explore some recovery strategies for optimal interdependent infrastructure network resilience. Also, we present and discuss the results of a numerical experiment.

In Chapter 6, we address the restoration problem of community structure in systems of interdependent infrastructure networks. We present a community structures restoration model using MIP. Furthermore, we present multiple importance measures for community structures. Also, we present and discuss the results of a numerical experiment.

In Chapter 7, we summarize the contributions of the work in this dissertation and present some recommendations for future work and possible extensions.

Chapter 2 : Background and Related Work

2.1 Background

In this section, we discuss network definition, network resilience modelling, mathematical programming, and the different types of infrastructure networks interdependencies.

2.1.1 Network Definition

In this work, we consider networks that are classified as undirected graphs. Each network is denoted by $G = (N, L)$ where N is a set of n nodes and $L \subset \{(i,j): i,j \in N, i \neq j\}$ is a set of undirected links with a limited capacity on each link. The flow and capacity on link $(i,j) \in L$ are denoted by x_{ij} and o_{ij}, respectively. Each network has one or multiple supply nodes as well as one or multiple demand nodes, where supply and demand nodes are connected by finite directed paths. Let nodes s and t represent source and demand nodes, respectively. Nodes s and t are connected by a finite *directed path*, P, through a set of internal nodes in N and one or more links in L. The *maximum capacity of a path* equals the minimum capacity of all the links within that path (i.e., $min_{(i,j) \in P} o_{ij}$) [Ford and Fulkerson 1956].

The objective of the $s - t$ *max flow problem* is to find the maximum flow from node s to node t by utilizing a subset of all possible paths between them accounting for links capacity. Hence, the problem can be formulated as a linear programing (LP) model [Bazaraa et al. 2011], as shown in Eqs. (2-1) – (2-3). The objective of the model, Eq.(2-1), is to maximize the flow from node s to node t, φ_{st}, for any source and demand node pair such that where $s, t \in N$ where $s \neq t$, otherwise $\varphi_{st} = 0$ if $s = t$. Eq. (2-2) represents the flow conservation constraints, which ensure that the flow into and out of internal nodes are equal and the flow out of node s and into node t equal the maximum flow between them, φ_{st}. The capacity constraints in Eq. (2-3) ensure that there is no negative flow as well as flow through any link does not exceed its capacity. We solve the $s-t$ max flow problem using the push–relabel maximum flow algorithm [Goldberg and Tarjan 1988] for a more CPU efficient solution.

$$\max \varphi_{st} \tag{2-1}$$

subject to

$$\sum_{(i,j)\in L} x_{ij} - \sum_{(j,i)\in L} x_{ji} = \begin{cases} \varphi_{st} & \text{if } i = s \\ 0 & \text{if } i \in N\backslash\{s,t\} \\ -\varphi_{st} & \text{if } i = t \end{cases} \tag{2-2}$$

$$0 \le x_{ij} \le o_{ij}, \quad i,j \in N \tag{2-3}$$

In this dissertation, we consider infrastructure networks with multiple source and demand nodes, which represent reality in many infrastructure networks (e.g., electric power networks) [Rocco et al. 2018]. The multiple source, multiple demand network can be reduced to a single source, single demand network by using the Ford-Fulkerson [1962] algorithm as follows:

(i) Add a new source node s^* (i.e., super-source), and connect it to all source nodes by adding a link (s^*, s) from s^* to every source node s.

(ii) Add a new demand node t^* (i.e., super-demand), and connect it to all demand nodes by adding a link (t, t^*) from every node t to t^*.

(iii) Assign a capacity to each link (s^*, s) equal to the capacity of node s

(iv) Assign a capacity to each link (t, t^*) equal to the demand of node t.

The maximum flow from the super-source node, s^*, to the super-demand node, t^*, represents the total flow that can be supplied from the source nodes to the demand nodes within the network.

2.1.2 Resilience Modeling

Resilience is generally defined as the ability of an entity or system to withstand, adapt to, and recover from a disruptive event to a desired level of performance in a timely manner [Barker et al. 2017]. Resilience has been quantified by several different approaches that exist in the literature [Hosseini et al. 2016], including: the normalized shaded area underneath the performance function curve of a system [Cimellaro et al. 2010], topological measures [Rosenkrantz et al. 2009], the ratio of the probability of failure and recovery [Li and Lence 2007], the ratio of the expected degradation and the maximum possible degradation in the performance of the system due to a disruption [Rose 2007], the loss of the system caused by a disruption [Bruneau et al. 2003], among others. In this work, we consider the paradigm proposed by Henry and Ramirez-Marquez [2012] to describe and quantify the resilience of a system or a network based on its performance, as shown in Figure 2-1 and Figure 2-2, which is considered by several papers in the literature [e.g., Ramirez-Marquez et al. 2018, Baroud et al. 2015, Pant et al. 2014, Baroud et al. 2014, Barker et al. 2013].

Figure 2-1 and Figure 2-2 show three transition states with regards to the operation within a network: (i) the original state, S_0, which is the state of the network from time t_0 until the occurrence of a disruptive event, e^j at time t_e, (ii) the disrupted state, S_d, which is the state resulted following the maximum disruption that occurred during the period $(t_\mathrm{e}, t_\mathrm{d})$ and will last until the recovery process starts at time t_s, and

(iii) the recovered state, S_f, which is the state of the network upon the completion of the recovery process at time t_f, which is not necessarily be same as S_0 as it could be lower or higher than S_0.

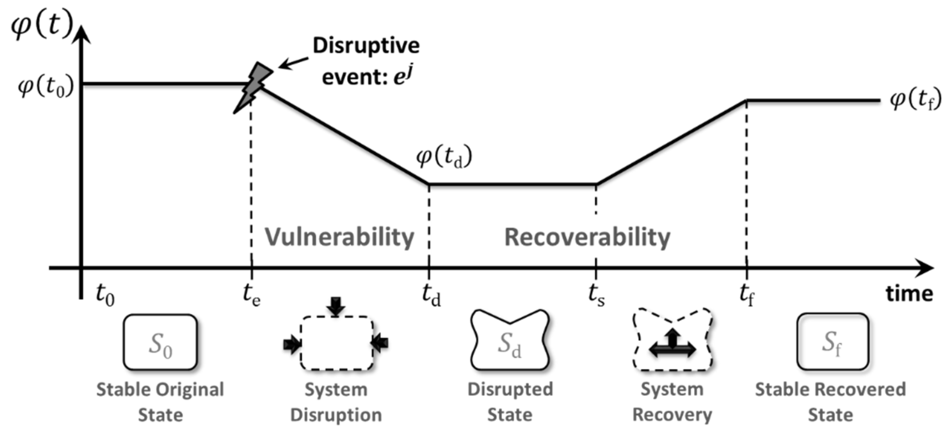


Figure 2-1. Illustration of decreasing network performance, $\varphi(t)$, across different transition states over time (adapted from Henry and Ramirez-Marquez et al. [2012])

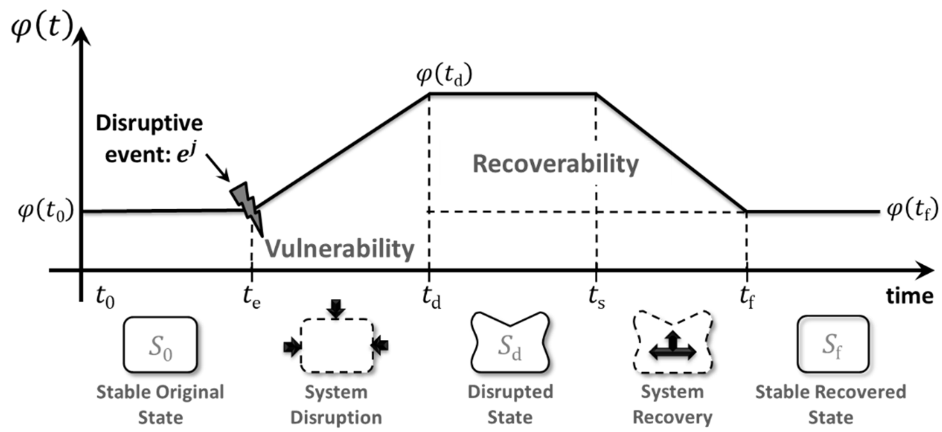


Figure 2-2. Illustration of increasing network performance, $\varphi(t)$, across different transition states over time (adapted from Ramirez-Marquez et al. [2017])

The performance of the network (e.g., flow, connectivity, unsatisfied customers, or delay) across these different states over time is measured by the function $\varphi(t)$, which describes the behavior of the network: (i) prior to the occurrence of a disruptive event, $\varphi(t_0)$, (ii) after being disrupted, $\varphi(t_\mathrm{d})$, and (iii) after being recovered to a desired level, $\varphi(t_\mathrm{f})$. Note that the performance of the system (network) following a disruptive event,

$\varphi(t_\mathrm{d})$, could decrease as a result of the disruption (e.g., flow, connectivity, utilization of asset), as illustrated in Figure 2-1 [Henry and Ramirez-Marquez 2012], or increase (e.g., unsatisfied customers, delays in flow or service), as illustrated in Figure 2-2 [Ramirez-Marquez et al. 2017], depending on how "performance" is measured.

Henry and Ramirez-Marquez [2012] define *network resilience*, denoted by Я, as the time dependent ratio of the recovered performance of the network over the maximum loss in its performance following a disruptive event, e^j, from a set J of possible disruptive events (i.e., $Я(t) = \mathrm{Recovery}(t)/\mathrm{Loss}(t_\mathrm{d}),\, t_\mathrm{d} < t$). Hence, $Я(t)$ quantifies the resilience of the network at time t, $t_\mathrm{d} < t < t_\mathrm{f}$, as shown in Figure 2-1 and Figure 2-2. Two primary dimensions of the system (network) resilience are illustrated in Figure 2-1 and Figure 2-2: (i) *vulnerability*, or the magnitude of damage to a network caused by a disruptive event [Jönsson et al. 2008], and (ii) *recoverability*, or the speed at which a disrupted network recovers to a desired level of performance following the occurrence of a disruptive event [Rose 2007].

Hence, network resilience can be demonstrated when the performance of the network at S_0, $\varphi(t_0)$, is affected by a disruptive event, e^j, at time t_e. Starting at this time, the network performance degrades until time t_d. Then, the network will stay at the disrupted state S_d, which has an associated performance level of $\varphi(t_\mathrm{d})$, until the restoration process commences at time t_s. The restoration process continues until the network reaches the desired state S_f, which has an associated performance level of $\varphi(t_\mathrm{f})$. Thus, the resilience at time t (i.e., $t_\mathrm{s} < t < t_\mathrm{f}$), $Я(t)$, for networks with decreasing performance when disrupted depicted in Figure 2-1, can be mathematically represented by Eq. (2-4), where $\varphi\left(t|e^j\right) - \varphi\left(t_\mathrm{d}|e^j\right)$ represent the recovery of the network performance at time t, and $\varphi(t_\mathrm{o}) - \varphi(t_\mathrm{d}|e^j)$ represent the loss (degradation) of the network performance up to time t_d.

$$Я_\varphi(t|e^j) = \frac{\varphi(t|e^j) - \varphi(t_\mathrm{d}|e^j)}{\varphi(t_\mathrm{o}) - \varphi(t_\mathrm{d}|e^j)}, t \in (t_\mathrm{s}, t_\mathrm{f}) \tag{2-4}$$

Similarly the resilience at a time t (i.e., $t_\mathrm{s} < t < t_\mathrm{f}$), $Я(t)$, for networks with increasing performance when disrupted depicted in Figure 2-2, can be mathematically represented by Eq. (2-5), where $\varphi(t_\mathrm{d}|e^j) - \varphi(t|e^j)$ represent the recovery of the network performance at time t, and $(t_\mathrm{d}|e^j) - \varphi(t_\mathrm{o})$ represent the loss (degradation) of the network performance up to time t_d.

$$Я_\varphi(t|e^j) = \frac{\varphi(t_\mathrm{d}|e^j) - \varphi(t|e^j)}{\varphi(t_\mathrm{d}|e^j) - \varphi(t_\mathrm{o})}, t \in (t_\mathrm{s}, t_\mathrm{f}) \tag{2-5}$$

According to both Eqs. (2-4) and (2-5), the value of the network resilience, $Я_\varphi(t|e^j)$, at time t given the occurrence of a disruptive event, e^j, is between 0 and 1 (i.e., $Я_\varphi(t|e^j) \in [0,1]$), where $Я_\varphi(t|e^j) = 1$ indicates the network is fully resilient.

In this work, we consider the flow as the measure for the performance of networks. Hence, the performance of the networks in or study decreases following the occurrence of a disruptive event, as shown in Figure 2-1. That is, the maximum flow of an interdependent infrastructure network from its multiple supply nodes to its multiple demand nodes is considered to be the function by which the network performance is measured, and its resilience is determined, using Eq. (2-4).

In this work, the maximum flow of an interdependent infrastructure network from its multiple supply nodes to its multiple demand nodes is considered to be the function by which the network performance is measured, and its resilience is determined.

2.1.3 Mathematical Programming

Mathematical programming is a modeling approach used for decision-making problems. Formulations of mathematical programming include a set of *decision variables*, which represent the decisions that need to be found and an *objective function*, a function of the decision variables, which assesses the quality of the solution. A mathematical program will then either minimize or maximize the value of this objective function.

The decisions of the model are subject to certain requirements and restrictions which can be included as a set of *constraints* in the model. Each constraint can be described as a function of the decision variables which bounds the feasible region of the solution and it is either equal to, not less than, or not more than, a certain value. Also, another type of constraint can simply restrict the set of values to which a variable might be assigned.

Throughout this dissertation, we use Mixed Integer Programming (MIP) for constructing our models which is a subset of mathematical programming. We use MIP where the constraints and objective function are all linear with the restriction that some of the variables must be integer-valued. Several applications for MIP involve decisions that are discrete, while some other decisions are continuous in nature. In this thesis, we will refer to the form of MIP as the standard form which is described as:

$$Min/Max \quad f(x)$$

$$\text{subject to} \quad g_i(x) \leq 0$$

$$h_j(x) = 0$$

where:

$f(x)$ is the objective function to be minimized or maximized

$g_i(x)$ are the inequality constraints to the problem for $i = 1,2,3, \dots, m$

$h_j(x)$ are the equality constraints to the problem for $j = 1,2,3, \dots, n$

m and n are the number of the constraints for the inequalities and the equalities, respectively [Murty 1995, Smith and Taşkin 2008, Murty 2010].

2.1.4 Infrastructure Networks Interdependencies

There are multiple classifications for the interdependencies among infrastructure networks provided in the literature [e.g., Rinaldi et al. 2001, Zimmerman 2001, Wallace et al. 2003, Dudenhoeffer et al. 2006, Lee et al. 2007, Zhang and Peeta 2011]. Rinaldi et al. [2001] classified the interdependencies between infrastructure networks into four categories: (i) physical interdependency, an output from an infrastructure network is an input to another one and vice versa, (ii) cyber interdependency, if an infrastructure network depends on information transmitted through an information infrastructure, (iii) geographical interdependency, if two infrastructure networks are affected by the same local disruptive event, and (iv) logical interdependency, all other types of interdependencies. Figure 2-3 shows an example of the interdependencies between different infrastructure networks. In this chapter, we consider the physical interdependency among different critical infrastructure networks. This physical interdependence by Rinaldi et al. [2001] is equivalent to functional or input interdependency in other interdependence classifications. However, the work in this dissertation could be extended to consider other types of interdependencies such as geographical interdependency, which could be incorporated in this work by considering the co-location of disrupted components from multiple interdependent infrastructure networks.

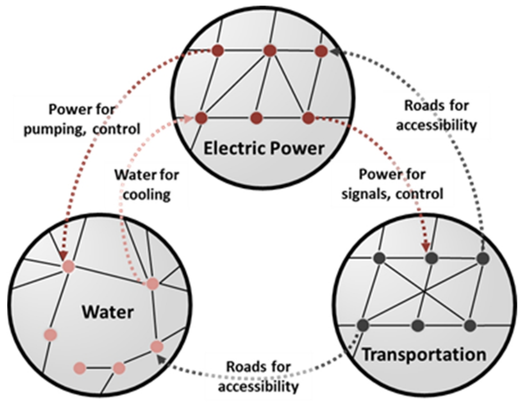


Figure 2-3. An example of interdependences among different infrastructure networks (adapted from Rinaldi et al. [2001])

2.2 Related Work

In this section, we present and discuss some previous studies in the literature that are related to the work in this dissertation, i.e., component importance measures and restoration models for interdependent infrastructure networks.

2.2.1 Component Importance Measures

Multiple component importance measures (CIMs) are proposed in the literature in which the interdependences between critical infrastructure networks are considered to identify their critical components. Dueñas-Osorio et al. [2007] prioritized the restoration process for the nodes of interdependent infrastructure networks to reduce their fragility according to a proposed CIM, interdependent rank ordering (IRO), considering physical interdependency among the infrastructure networks. IRO accounts for multiple criteria for each node (i.e., network connectivity, flow traversal, flow transfer at the interface with other infrastructures, and network vulnerability), where each node has its own rank in each one of these criteria. The final prioritized list of nodes can be obtained by considering the rank of the nodes in each criterion along with a weighting factor for each criterion. Patterson and Apostolakis [2007] developed a new approach, geographic valued worth (GVW), to rank geographical locations that can affect multiple infrastructure networks the most when disrupted considering the geographical interdependency between the infrastructure networks. First, the worth of each network component in each infrastructure network is calculated. Then, a generic grid is developed and laid across the map of the interdependent infrastructure networks in order to determine the geographical locations for the study. Finally, the GVW value is determined for each geographical location which consists of combined valued worth of the network components of all interdependent infrastructure networks within that location then rank the geographical locations accordingly. However, the rank of the geographical locations is dependent of their size. Johansson and Hassel [2010] proposed a CIM that identifies the critical components of interdependent infrastructure networks by quantifying the contribution of their synergistic consequences to the total synergistic consequences for a specific size of the failure set (e.g., single failures, two simultaneous failures, three simultaneous failures, and so on) taking into account the physical interdependency between the infrastructure networks. They also identified the critical locations within the interdependent infrastructure networks by evaluating the consequences on the interdependent infrastructure networks performance when components in close proximity to each other and could be from different infrastructure networks are removed considering physical and geographical interdependencies between infrastructure networks where two infrastructure networks are said to be geographically interdependent if they are affected by the same local disruption. Wang et al. [2012] suggested a new approach to identify the critical components in an infrastructure network considering its physical interdependency with other infrastructure networks. They evaluated the importance of a network component through assessing the relative drop in the performance of the infrastructure network when this component is disrupted (i.e., not working). Hence, the global safety efficiency of the infrastructure network is obtained when each component is not working. Accordingly, the network components are ranked with respect to the proportion of degradation in the global safety efficiency of their network where the higher degradation indicates the most vulnerable network component.

However, by recognizing the inevitability of large-scale disruptions, emphasis shifted from reliability-driven perspective of protection [Patterson and Apostolakis 2007, Johansson and Hassel 2010, Wang et al. 2012] to a perspective of resilience, or the ability to withstand, adapt to, and recover in a timely manner from the effects of a disruptive event [Turnquist and Vugrin 2013]. Therefore, it is important to have resilient infrastructure networks accounting for the interdependencies between them.

2.2.2 Restoration Models

The literature has recently addressed the restoration problem of interdependent infrastructure networks. Accordingly, several approaches have been developed that could best be described with two groups: (i) infrastructure-specific approaches, which consider the physics of different infrastructures are considered (e.g., DC power flow model) and hence could be applied on these infrastructure networks only, and (ii) general approaches, which could be applied to any system of interdependent infrastructure networks.

As for the infrastructure-specific approaches for the interdependent network restoration, Coffrin et al. [2012] studied the problem of restoring two physically interdependent infrastructure networks, power and gas networks. They integrated two network-specific flow models (i.e., a linearized DC flow model for the power network and a maximum flow model for the gas network) using MIP with the objective of maximizing the weighted sum of interdependent demand over the restoration time horizon and solved them using a randomized adaptive decomposition approach. The proposed models aim to find: (i) the set of disrupted components to be restored, and (ii) the restoration order of the selected disrupted components. However, the proposed model did not consider different restoration durations for the disrupted networks components in addition for being developed for specific types of infrastructure networks. Baidya and Sun [2017] provided an optimization-based restoration strategy that aims to prioritize the restoration activities between two physically interdependent infrastructure networks, power system and communication networks, considering their physics-based properties. The proposed approach is formulated using MIP with the objective of activating every node in both networks with the minimum number of activation/energization of branches. Tootaghaj et al. [2017] focused on the cascading disruptions impact on the physically interdependent power grid and communication network considering disruptions in power networks only. Accordingly, they proposed a two-phase recovery approach: (i) avoid further cascade, for which they formulate the minimum cost flow assignment problem using linear programming (LP) with the objective of finding a DC power flow setting that stops the cascading failure at minimum cost, and (ii) provide a recovery schedule, for which they formulate the recovery problem using MIP with the objective of maximizing the total amount of delivered power over the recovery horizon and solve it using two heuristic approaches: a shadow-pricing heuristic and a backward algorithm.

Regarding the general approaches for setting up the restoration of interdependent infrastructure networks, Lee et al. [2007] proposed an interdependent layer network model using MIP that accounts for different interdependencies among the infrastructure networks. The objective of the model is to minimize the flow costs along with the slack costs but not including the cost associated with the restoration process of the disrupted components. Moreover, it focuses only on determining the set of disrupted components (i.e., links) of the interdependent infrastructure networks that need to be recovered to restore the performance of each of the infrastructure networks to the functionality level prior to the occurrence of a disruptive event. Hence, the proposed model does not specify the time at which they need to be restored (i.e., the prioritizing of the restoration process for the disrupted components) or which work crew is assigned to restore which disrupted component. In addition, the model assumes binary status of network components (i.e., disrupted or not disrupted). On the other hand, Gong et al. [2009] focused only on the scheduling problem of a predetermined set of disrupted components for interdependent infrastructure networks with predefined due dates for them. They provided a multi-objective restoration planning model, using MIP, to find the optimal restoration schedule for disrupted components and solved it using a logic-based benders decomposition approach. The objective of the model is to minimize the weighted sum of the cost, tardiness, and makespan that are associated with the restoration process of the disrupted components. Cavdaroglu et al. [2013] and Sharkey et al. [2015] integrated the two approaches by Lee et al. [2007] and Gong et al. [2009] by providing a MIP model that integrates: (i) determining the set of disrupted components (i.e., links) to be restored, along with (ii) assigning and scheduling them to work crews, and solved it using a suggested heuristic solution method. The objective of this model is to minimize the total cost of flow cost, unsatisfied demand, and installation and assignment that is associated with the full restoration of a set of infrastructure networks accounting for the interdependencies among them. However, they assumed binary status of network components (i.e., disrupted or not disrupted) which could be restored with a non-preemptive recovery process. Holden et al. [2013] proposed an extended network-flow approach to simulate the performance of a set of infrastructure networks at a local scale (i.e., community scale) considering the physical interdependency among them. Hence, they provided an optimization model using LP that aims to find the optimal performance of the infrastructure networks such that the total cost associated with production, storage, commodity flow, discharge, and shortage (i.e., unsatisfied demand) is minimized. However, the proposed approach does not explicitly discuss what are the set of disrupted networks components, their restoration durations, and their restoration priorities. Also, the approach does not consider the availability of the work crews; hence determine their restoration schedule. Di Muro et al. [2016] studied the recovery problem of the system of physically interdependent infrastructure networks in the presence of cascading failures to mitigate its breakdown. They considered restoring the disrupted network components (i.e., nodes) that are located at the boundary of the largest connected component (i.e., functional network) and reconnect them to it considering the probability of recovery that halts the cascade for interdependent infrastructure networks. They developed a stochastic model for the competition between the cascading failures and the restoration strategy for the disrupted components and solved it theoretically using random node percolation theory. However, they considered a random recovery strategy for the disrupted nodes. In addition, they have not considered the availability of work crews. González et al. [2016] studied the interdependent network design problem considering their physical and geographical interdependencies. They formulated an MIP model to determine: (i) the set of disrupted components to be restored, and (ii) the order of their restoration, with the objective of minimizing the overall cost associated with preparing geographical locations, restoration of disrupted components, unbalance from disconnection, and flow. However, the model does not specify which work crews should restore particular disrupted components. Moreover, they assumed binary status of network components (i.e., disrupted or not disrupted). Zhang et al. [2018] provided an optimization model that determines the optimal allocation of restoration resources for a set infrastructure networks that are physically interdependent such that its resilience is enhanced. The proposed model aims to: (i) allocate limited resources to interdependent infrastructure networks, and (ii) determine the optimal budget for restoration following a specific disruptive event, solved using a genetic algorithm approach. However, their work focuses only on the allocation of restoration resources (i.e., budget) for a set of infrastructure networks following a disruptive event.

Chapter 3 : Component Importance Measures for Interdependent Network Resilience

3.1 Introduction

In this chapter, we propose two resilience-based CIMs that (i) quantify the effect of the disrupted components on the resilience of the interdependent infrastructure networks once they are recovered, and (ii) measure the potential impact on the resilience of the interdependent infrastructure networks caused by a specific disrupted network element, respectively. Hence, they provide a set of prioritized restoration tasks for the disrupted components (nodes and links) in the interdependent infrastructure networks aiming to enhance their resilience taking into account their physical interdependency. The two proposed CIMs are based on multiple interdependent infrastructure networks resilience optimization models using MIP and considering complete or proportional disruptions for disrupted network components with a fixed or variable recovery durations for each one of them.

3.2 Resilience Optimization Models

In this section, we propose four different resilience optimization models for interdependent infrastructure networks using MIP considering the disruption status for the disrupted network components as well as their recovery durations. The objective of the proposed resilience optimization models is to maximize the resilience of the system of interdependent infrastructure networks considering the physical interdependency among them. The four proposed resilience optimization models have different considerations regarding disruption status and recovery duration of the disrupted network component, see Table 3-1.

Table 3-1. Different considerations for resilience optimization models

Model	Disruption		Recovery duration	
	Partial	Full	Fixed	Different
Model I		√	√	
Model II	√		√	
Model III		√		√
Model IV	√			√

3.2.1 Assumptions

There are several assumptions for the proposed optimization models:

- Each infrastructure network consists of a set of components (used generally to refer to nodes and/or links) that are subjected to disruptions.
- Disrupted network components could be either: (i) completely disrupted with a binary status for each network component following a disruptive event (i.e., 0 if completely disrupted and 1 if undisrupted), or (ii) partially disrupted, where a disrupted component is not completely disrupted but instead functioning partially.
- Each disrupted component in each infrastructure network can be restored.
- Recovery durations for disrupted network components are either: (i) fixed, where all disrupted components are assumed to have the same recovery duration (i.e., of one time unit for each disrupted component), or (ii) varying, where recovery durations are not fixed for all disrupted components.
- Partially disrupted network components, with different recovery durations, could be recovered faster than completely disrupted network components due to their status (size) of disruption (i.e., the recovery durations for partially disrupted network

components is proportional of the recovery durations of the completely disrupted network components).

- Each supply node, demand node, and link in each infrastructure network has a known supply capacity, demand, and flow capacity, respectively.
- The physical interdependence among different infrastructure networks is considered. That is, for a dependent node in an infrastructure network to be operational, it requires a specific node from another infrastructure network to also be operational.
- The models, in which partial disruptions are considered, allow for partial interdependencies considering the partial status of disruption (i.e., partial functioning of dependent nodes). That is, a node could be functioning partially if the other node upon which it depends is also functioning partially.

3.2.2 Notation

The sets, parameters, and decision variables of the proposed resilience optimization models are shown in Table 3-2, Table 3-3, and Table 3-4, respectively.

Table 3-2. Sets of the proposed resilience optimization models

T	Time periods in the restoration horizon, $T = \{1, \ldots, \tau\}$
K	Interdependent infrastructure networks, K
N^k	Nodes in network $k \in K$
L^k	Links in network $k \in K$
N_s^k	Supply nodes in network $k \in K$, $N_s^k \subseteq N^k$
N_d^k	Demand nodes in network $k \in K$, $N_d^k \subseteq N^k$
N'^k	Disrupted nodes in network $k \in K$, $N'^k \subseteq N^k$
L'^k	Disrupted links in network $k \in K$, $L'^k \subseteq L^k$
Ψ	Interdependent nodes (i.e., $\left((i,k),(\bar{i},\bar{k})\right) \in \Psi$ indicates that node $\bar{i} \in N^{\bar{k}}$ in network $\bar{k} \in K$ requires node $i \in N^k$ in network $k \in K$ to be operational)

Table 3-3. Parameters of the proposed resilience optimization models for network $k \in K$

o_{ij}^k	Capacity of link $(i,j) \in L^k$
ω^k	Weight, $\sum_{k \in K} \omega^k = 1$
U_o^k	Total maximum flow from all supply nodes to all demand nodes prior to the disruption
U_d^k	Total maximum flow from all supply nodes to all demand nodes after the disruption
λ_i^k	Restoration duration of node $i \in N_k'$
π_{ij}^k	Restoration duration of link $(i,j) \in L'^k$
$\hat{\lambda}_i^k$	Restoration duration of partially disrupted node $i \in N'^k$
$\hat{\pi}_{ij}^k$	Restoration duration of partially disrupted link $(i,j) \in L'^k$
ϑ_i^k	Size of partial disruption in node $i \in N'^k$
ξ_i^k	Size of partial disruption in link $(i,j) \in L'^k$

Table 3-4. Decision variables of the proposed resilience optimization models for network $k \in K$ at time $t \in T$

u_{it}^k	Amount of supply and demand at node $i \in N_s^k$ and node $i \in N_d^k$, respectively
x_{ijt}^k	Amount of flow through link $(i,j) \in L^k$
y_{it}^k	A binary variable that equals 1 if node $i \in N^k$ is operational; and 0 otherwise
z_{ijt}^k	A binary variable that equals 1 if link $(i,j) \in L'^k$ is operational; and 0 otherwise
v_{it}^k	A binary variable that equals 1 if node $i \in N'^k$ is restored; and 0 otherwise
w_{ijt}^k	A binary variable that equals 1 if link $(i,j) \in L'^k$ is restored; and 0 otherwise

3.2.3 Objective

The amount of flow supplied from node $i \in N_s^k$ in network $k \in K$, u_{it}^k, is considered in the proposed resilience optimization model to be the maximum flow from supply node $i \in N_s^k$ to all demand nodes in network $k \in K$ after recovery at time period $t \in T$. As such, u_{it}^k is obtained by solving the maximum flow problem described earlier (i.e., Eqs. (2-1) – (2-3)). Hence, the total maximum flow supplied from all supply nodes and received by all demand nodes in network $k \in K$ after recovery at time period $t \in T$ equals $\sum_{i \in N_s^k} u_{it}^k$. Moreover, U_o^k refers to the original maximum flow at time t_o, and U_d^k refers to the maximum flow at time t_d following a disruptive event, e^j, see Figure 2-1.

Accordingly, the resilience of the system of interdependent infrastructure networks, denoted by $Я_{sys}$, can be represented mathematically by Eq. (3-1). Hence, $\sum_{t=1}^{\tau} \left[\sum_{i \in N_s^k} u_{it}^k - \sum_{i \in N_s^k} u_{i(t-1)}^k \right]$ represents the recovery of network $k \in K$ over the restoration time horizon, where $\sum_{i \in N_s^k} u_{i0}^k$ equals to U_d^k, while the total loss in network $k \in K$ is represented by $(U_o^k - U_d^k)$.

$$Я_{sys} = \sum_{k \in K} \omega^k \left[\frac{\sum_{t=1}^{\tau} \left[\sum_{i \in N_s^k} u_{it}^k - \sum_{i \in N_s^k} u_{i(t-1)}^k \right]}{U_o^k - U_d^k} \right] \tag{3-1}$$

3.2.4 Model I

The first resilience optimization model considers binary status of each network component following a disruptive event (i.e., 0 if completely disrupted and 1 if undisrupted). All disrupted components (nodes or links) are assumed to have the same recovery duration. In other words, this model assumes a fixed time duration for recovery of one-time unit for each disrupted component. Accordingly, Model I can be mathematically formulated with objective (3-2) and constraints (3-3) – (3-18).

$$\max \ Я_{sys} \tag{3-2}$$

subject to

$$\sum_{(i,j) \in L^k} x_{ijt}^k - \sum_{(j,i) \in L^k} x_{jit}^k = u_{it}^k, \quad \forall i \in N_s^k, k \in K, t \in T \tag{3-3}$$

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k = 0, \quad \forall i \in N^k \backslash \{N_s^k, N_d^k\}, k \in K, t \in T \tag{3-4}$$

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k = -u_{it}^k, \quad \forall i \in N_d^k, k \in K, t \in T \tag{3-5}$$

$$x_{ijt}^k - o_{ij}^k \leq 0, \quad \forall (i,j) \in L^k, k \in K, t \in T \tag{3-6}$$

$$x_{ijt}^k - o_{ij}^k y_{it}^k \leq 0, \quad \forall (i,j) \in L^k, i \in N^k, k \in K, t \in T \tag{3-7}$$

$$x_{ijt}^k - o_{ij}^k y_{jt}^k \leq 0, \quad \forall (i,j) \in L^k, j \in N^k, k \in K, t \in T \tag{3-8}$$

$$x_{ijt}^k - o_{ij}^k z_{ijt}^k \leq 0, \quad \forall (i,j) \in L'^k, k \in K, t \in T \tag{3-9}$$

$$y_{\bar{\iota}t}^{\bar{k}} - y_{it}^k \leq 0, \quad \forall \left((i,k), (\bar{\iota}, \bar{k})\right) \in \Psi, t \in T \tag{3-10}$$

$$z_{ijt}^k - z_{ijt+1}^k \leq 0, \qquad \forall (i,j) \in L^k, k \in K, t \in T \tag{3-11}$$

$$y_{it}^k - y_{it+1}^k \leq 0, \qquad \forall i \in N^k, k \in K, t \in T \tag{3-12}$$

$$\sum_{i\in N'^k} \left[y_{it}^k - y_{i(t-1)}^k\right] + \sum_{(i,j)\in L'^k} \left[z_{ijt}^k - z_{ij(t-1)}^k\right] \leq 1, \quad \forall k \in K, t \in T \tag{3-13}$$

$$z_{ij0}^k = 0, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{3-14}$$

$$y_{i0}^k = 0, \qquad \forall i \in N^k, k \in K, t \in T \tag{3-15}$$

$$x_{ijt}^k \geq 0, \qquad \forall (i,j) \in L^k, k \in K, t \in T \tag{3-16}$$

$$z_{ijt}^k \in \{0,1\}, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{3-17}$$

$$y_{it}^k \in \{0,1\}, \qquad \forall i \in N'^k, k \in K, t \in T \tag{3-18}$$

Objective (3-2) maximizes the resilience of the interdependent infrastructure networks over the recovery time horizon. Constraints (3-3) – (3-5) are flow conservation constraints at node $i \in N^k$ in network $k \in K$ at time $t \in T$. Constraints (3-6) – (3-9) are capacity constraints on link $(i,j) \in L^k$ in network $k \in K$ at time $t \in T$ considering undisrupted network components in (3-6), disrupted or non-operational nodes in (3-7) and (3-8), and disrupted links in constraints (3-9). The physical interdependence between the networks is represented in (3-10), which ensure that node $\bar{\imath} \in N^{\bar{k}}$ in network $\bar{k} \in K$ cannot be operational at time $t \in T$ unless node $i \in N^k$ in network $k \in K$ is operational at time $t \in T$ as well. Constraints (3-11) and (3-12) ensure that once a link or node, respectively, in network $k \in K$ is recovered or operational at time $t \in T$, it will remain operational thereafter. Constraints (3-13) ensure that at most one component (node or link) in network $k \in K$ can be recovered during time $t \in T$. Constraints (3-14) and (3-15) reflect the initial status of the disrupted links and nodes, respectively, in network $k \in K$. Constraints (3-16) – (3-18) deal with the nature of the decision variables.

3.2.5 Model II

Though this second resilience optimization model is similar to Model I in assuming a fixed time duration for recovery of one time unit for each disrupted component, it considers proportional disruptions for the disrupted components, meaning that a disrupted node or link is not completely disrupted but instead functioning partially. Considering proportional disruption allows for partial functioning of dependent nodes. Hence, the Model II can be mathematically formulated as Model I but with constraints (3-7), (3-8), and (3-9) replaced by constraints (3-19), (3-20), and (3-21), respectively, to account for the proportional disruptions.

$$x_{ijt}^k - o_{ij}^k\left(\left(1 - y_{it}^k\right) \times \vartheta_i^k + y_{it}^k\right) \le 0, \quad \forall (i,j) \in L^k, i \in N^k, k \in K, t \in T \qquad (3\text{-}19)$$

$$x_{ijt}^k - o_{ij}^k\left(\left(1 - y_{jt}^k\right) \times \vartheta_i^k + y_{jt}^k\right) \le 0, \quad \forall (i,j) \in L^k, j \in N^k, k \in K, t \in T \qquad (3\text{-}20)$$

$$x_{ijt}^k - o_{ij}^k\left(\left(1 - z_{ijt}^k\right) \times \xi_i^k + z_{ijt}^k\right) \le 0, \quad \forall (i,j) \in L'^k, k \in K, t \in T \qquad (3\text{-}21)$$

3.2.6 Model III

The third resilience optimization model considers binary status of each network component following a disruptive event (i.e., 0 if completely damaged and 1 if unaffected). However, the recovery duration for disrupted nodes and links are different: the recovery duration is not fixed for all disrupted components. Accordingly, the Model III can be mathematically formulated by objective (3-2), constraints (3-3) – (3-10) and (3-16) – (3-18), along with constraints (3-22) – (3-30).

$$\sum_{t=1}^{\lambda_i^k - 1} y_{it}^k = 0, \quad \forall i \in N'^k, k \in K \tag{3-22}$$

$$\sum_{t=1}^{\pi_{ij}^k - 1} z_{ijt}^k = 0, \quad \forall (i,j) \in L'^k, k \in K \tag{3-23}$$

$$\sum_{t=1}^{\lambda_i^k - 1} v_{it}^k = 0, \quad \forall i \in N'^k, k \in K \tag{3-24}$$

$$\sum_{t=1}^{\pi_{ij}^k - 1} w_{ijt}^k = 0, \quad \forall (i,j) \in L'^k, k \in K \tag{3-25}$$

$$\sum_{i \in N'^k} \sum_{l=t}^{\min\{\tau, t+\lambda_i^k - 1\}} v_{il}^k + \sum_{(i,j) \in L^{k\prime}} \sum_{l=t}^{\min\{\tau, t+\pi_{ij}^k - 1\}} w_{ijl}^k \leq 1, \quad \forall k \in K, t \in T \tag{3-26}$$

$$y_{it}^k \leq \sum_{l=1}^{t} v_{il}^k, \quad \forall i \in N'^k, t \in T, k \in K \tag{3-27}$$

$$z_{ijt}^k \leq \sum_{l=1}^{t} w_{ijl}^k, \quad \forall (i,j) \in L'^k, t \in T, k \in K \tag{3-28}$$

$$v_{it}^k \in \{0,1\}, \quad \forall i \in N'^k, k \in K, t \in T \tag{3-29}$$

$$w_{ijt}^k \in \{0,1\}, \quad \forall (i,j) \in L'^k, k \in K, t \in T \tag{3-30}$$

Constraints (3-22) and (3-23) ensure that no disrupted component (node or link) is operational prior to its required recovery time (i.e., node i and link (i,j) in network $k \in K$ cannot be operational prior to λ_i^k and π_{ij}^k time periods, respectively which are their required recovery durations). Similarly, constraints (3-24) and (3-25) ensure that no disrupted component (node or link) is recovered before its required recovery duration (i.e., node i and link (i,j) in network $k \in K$ require λ_i^k and π_{ij}^k time periods, respectively for them to be recovered and be operational; hence they cannot be recovered prior to these times). Constraints (3-26) ensure that at most one component (node or link) is recovered at time $t \in T$ (i.e., if v_{il}^k equals 1, it means that node i in network $k \in K$ is recovered during the time period $(l - \lambda_i^k + 1, l)$ and if w_{ijl}^k equals 1, it means that link (i,j) in network $k \in K$ is recovered during the time period $(l - \pi_{ij}^k + 1, l)$). Constraints (3-27) and (3-28) ensure that if a node or link, respectively is operational at time $t \in T$, it must also be recovered at that time. Constraints (3-29) and (3-30) are constraints on the nature of the decision variables.

3.2.7 Model IV

The fourth resilience optimization model considers proportional disruptions for the disrupted components, where a disrupted component is not completely disrupted but functioning partially, thus allowing for partial functioning of dependent nodes. Model IV also considers different recovery durations for disrupted nodes and links. However, they could be recovered faster than their required recovery duration, as they are proportionally disrupted. As such, let the time to recover the partial disruption in node $i \in N'^k$ in network $k \in K$ denoted by $\hat{\lambda}_i^k$ (i.e., $\hat{\lambda}_i^k = \vartheta_i^k \times \lambda_i^k$). Likewise, let the time to recover the proportional disruption in link $(i,j) \in L'^k$ in network $k \in K$ denoted by $\hat{\pi}_{ij}^k$ (i.e., $\hat{\pi}_{ij}^k = \xi_i^k \times \pi_{ij}^k$). Model IV can be mathematically formulated as Model III with constraints (3-7), (3-8), and (3-9) replaced by constraints (3-19), (3-20), and (3-21), respectively, to account for the proportional disruptions and constraints (3-22) – (3-26) replaced by constraints (3-31) – (3-35), respectively, to account for the recovery of the proportional disruptions. Note that constraints (3-31) – (3-35) are similar to constraints (3-22) – (3-26) but with the difference of the recovery duration of the disrupted networks components due to the consideration of a different disruption assumption (i.e., proportional disruptions).

$$\sum_{t=1}^{\hat{\lambda}_i^k - 1} z_{it}^k = 0, \quad \forall i \in N'^k, k \in K \tag{3-31}$$

$$\sum_{t=1}^{\hat{\pi}_{ij}^k - 1} y_{ijt}^k = 0, \quad \forall (i,j) \in L'^k, k \in K \tag{3-32}$$

$$\sum_{t=1}^{\hat{\lambda}_i^k - 1} \alpha_{it}^k = 0, \quad \forall i \in N'^k, k \in K \tag{3-33}$$

$$\sum_{t=1}^{\hat{\pi}_{ij}^k - 1} \beta_{ijt}^k = 0, \quad \forall (i,j) \in L'^k, k \in K \tag{3-34}$$

$$\sum_{i \in N'^k} \sum_{l=t}^{\min\{\tau, t+\hat{\lambda}_i^k - 1\}} \alpha_{il}^k + \sum_{(i,j) \in L'^k} \sum_{l=t}^{\min\{\tau, t+\hat{\pi}_{ij}^k - 1\}} \beta_{ijl}^k \leq 1, \quad \forall k \in K, t \in T \tag{3-35}$$

Similar to Model III but with the consideration of the proportional disruptions, constraints (3-31) and (3-32) ensure that no proportionally disrupted component (node or link) is operational prior to the required recovery duration for its proportional disruption (i.e., node i and link (i,j) in network $k \in K$ cannot be operational prior to $\hat{\lambda}_i^k$ and $\hat{\pi}_{ij}^k$ time periods, respectively which are their required recovery time). Similarly, constraints (3-33) and (3-34) ensure that no partially disrupted component (node or link) is recovered before its required recovery duration (i.e., node i and link (i,j) in network $k \in K$ require $\hat{\lambda}_i^k$ and $\hat{\pi}_{ij}^k$ time periods, respectively for them to be recovered from their proportional disruption and be operational; hence they cannot be recovered prior to these times). Constraints (3-35) ensure that at most one component (node or link) is recovered at time $t \in T$ (i.e., if v_{il}^k equals 1, it means that node i in network $k \in K$ is recovered during the time period $(l - \hat{\lambda}_i^k + 1, l)$ and if w_{ijl}^k equals 1, it means that link (i,j) in network $k \in K$ is recovered during the time period $(l - \hat{\pi}_{ij}^k + 1, l)$).

3.3 Component Importance Measures

In this dissertation, we propose two resilience-based component importance measures (CIMs), namely Optimal Recovery Time (ORT) and Resilience Reduction Worth (ЯRW). These measures will prioritize the disrupted components, be they nodes and/or links, according to their criticality and importance based on their effect on the resilience of the interdependent infrastructure networks. The two resilience-based CIM are defined as follows.

3.3.1 Optimal Recovery Time

The *optimal recovery time* CIM is defined as the optimal time to recover a disrupted network component (node or link) such that the resilience of the interdependent infrastructure networks is maximized over the recovery time horizon. It quantifies the effect of the disrupted components on the resilience of the interdependent infrastructure networks once they are recovered and prioritizes them accordingly where the lower value of the ORT indicates the extent to which the component is more important to the resilience of the interdependent infrastructure networks. This CIM is similar to the importance measure proposed by Fang et al. [2016] but extended for interdependent infrastructure networks, as well as it is based on the multiple interdependent network resilience optimization formulations presented in Section 3. This CIM provides decision makers with the restoration priorities of only the disrupted networks components that have influence on the resilience of the interdependent infrastructure networks, which satisfy the objective of the multiple proposed models. As such, there could be some disrupted components that do not enhance the resilience of the interdependent infrastructure networks when restored; hence, their restoration priorities are left to the preferences of decision makers.

Definition 3.3.2. *(ORT)***.** The ORT of a disrupted network component $e \in E'^k = N'^k \cup L'^k$ in network $k \in K$, denoted as I_e^{ORT}, is defined in Eq. (3-36), where μ_{et}^k represents the status of each component at time period $t \in T$ such that μ_{et}^k equals 1 if component $e \in E'^k$ in network $k \in K$ is operational at time period $t \in T$ and 0 otherwise.

$$I_e^{ORT} = 1 + \sum_{t=1}^{\tau}(1 - \mu_{et}^k) \tag{3-36}$$

where:

$$\mu_{et}^k = \begin{cases} z_{it}^k, & \text{if } e \text{ is a node}, e = i \\ y_{ijt}^k, & \text{if } e \text{ is a link}, e = (i,j) \end{cases}$$

3.3.2 Resilience Reduction Worth

The *resilience reduction worth* CIM is defined as the ratio of the optimal system resilience at recovery time τ to the optimal system resilience when a disrupted network component (node or link) is not recovered at recovery time τ. It measures the potential impact on the resilience of the interdependent infrastructure networks caused by a specific disrupted network element (i.e., when this specific disrupted network element is not recovered during the recovery time horizon). The higher value of ЯRW indicates the more critical the component is to the resilience of the interdependent infrastructure networks. This CIM is inspired by the performance reduction worth importance measure [Levitin et al. 2003] and the reliability reduction worth importance measure [Espiritu et al. 2007], both of which are defined by the ratio of actual system performance to the system performance when a specific component is always considered to be failed or not working.

Definition 3.3.2. *(ЯRW)*. The ЯRW of a disrupted network component $e \in E'^k = N'^k \cup L'^k$ in network $k \in K$, denoted as $I_e^{ЯRW}$, is defined in Eq. (3-37), where $Я(\tau)$ is the optimal resilience of the interdependent infrastructure networks at time τ and $Я(\tau | \sum_{t \in T} \mu_{et}^k = 0)$ is the optimal resilience of the interdependent infrastructure networks at time τ when network component $e \in E'^k$ is not recovered.

$$I_e^{ЯRW} = \frac{Я_{sys}(\tau)}{Я_{sys}(\tau | \sum_{t \in T} \mu_{et}^k = 0)} \quad (3\text{-}37)$$

3.4 Numerical Experiment

In this section, we illustrate our proposed resilience-based CIMs considering the multiple interdependent network resilience optimization models with some generated interdependent infrastructure networks.

3.4.1 Networks Data

Due to the difficulty of obtaining real data for interdependent infrastructure networks [Johansson and Hassel 2010, Bagchi et al. 2010], the proposed CIMs are illustrated with realistic fictional interdependent infrastructure networks. These fictional interdependent infrastructure networks are generated using the extended algorithm for proximal topology generator proposed by Xin-Jian [2007] which was initially introduced by Casey [2005]. Hence, we generate the fictional interdependent infrastructure networks in two stages: (i) generating individual networks; and (ii) building the interdependencies across them [Zhang et al. 2016].

In this dissertation, we illustrate our restoration model considering two infrastructure networks, namely simulated power and water networks. For the power network, power generators and substations represent the supply and demand nodes, respectively and the lines between the nodes within this network represent the links. For the water network, water pumps and storage tanks represent the supply and demand nodes, respectively and the pipelines between the nodes within this network represent the links. These two networks are interdependent as the water network needs power for operation and the power network requires water for cooling and emission control [Dueñas-Osorio et al. 2007, Zhang et al. 2016].

For the first phase of the generation process of the interdependent infrastructure networks, each network will initially be seeded with independent and randomly distributed source nodes (i.e., no links between them). At each time step, a new randomly distributed node is added to the network and connected to the nearest existing node based on Euclidean distance by adding a new undirected link between them. Then, a sparse random graph is added after the final time step to the generated network.

In the dissertation, the physical dependence between the infrastructure networks is considered to describe their interdependence, that is the functionality of a node in one network is dependent on the functionality of a node in another network. Hence, the two infrastructure networks considered in this illustrative example are interdependent as described earlier. Accordingly, we build the interdependencies between these two interdependent infrastructure networks, representing the second stage of the fictional interdependent infrastructure networks generation process. Hence, for the second phase of the generation process of the interdependent infrastructure networks, each water pump and storage tank in the water network will depend on the nearest power generator or substation in the power network (i.e., power generators or substation), based on Euclidean distance, for their functionality. Likewise, each power generator in the power network will depend on the nearest water pump in the water network, based on Euclidean distance, for its functionality.

Figure 3-1 shows the generated two fictional interdependent infrastructure networks using the *igraph* library in the R platform. Furthermore, Table 3-5 depicts the general properties for the interdependent power and water networks, that includes number of nodes, number of undirected links, number of supply nodes, number of demand nodes, and average node degree, respectively.

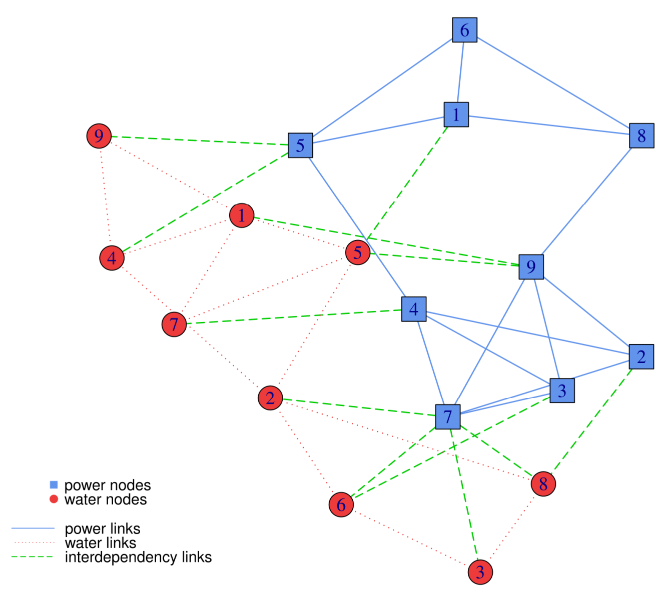


Figure 3-1. An example of interdependent infrastructure networks

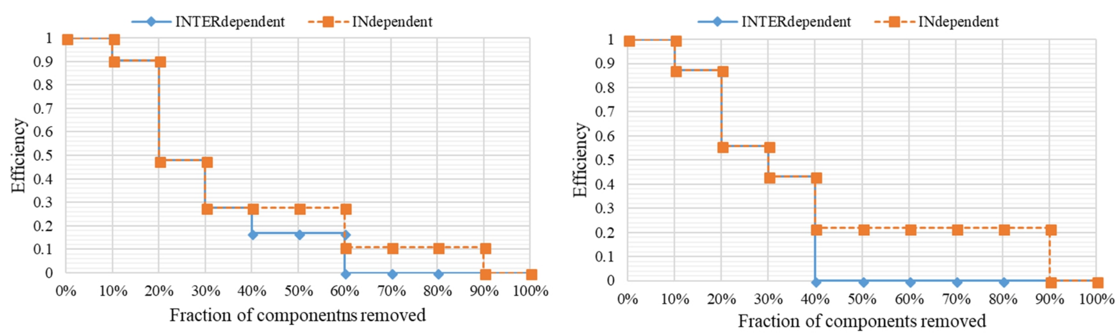
Table 3-5. General properties of the interdependent infrastructure networks

Network	N	L	N^s	N^d	$\langle deg \rangle$
Power	9	15	3	3	3.33
Water	9	12	3	3	2.67

In this work, the two generated infrastructure networks, shown in Figure 3-1, are utilized to illustrate our proposed resilience-based CIMs which aim to enhance the resilience of the system for interdependent infrastructure network based on the maximum flow from its multiple supple nodes to its multiple demand nodes. However, the two generated infrastructure networks could also be utilized with other topologies (e.g., shortest paths) and considering other type of interdependencies among the infrastructure networks (e.g., geographical interdependence) that we are considering for a future work.

3.4.2 Experimental Results

In this work, we measure the efficiency of an infrastructure network as the ratio of the current maximum flow from all its multiple supply nodes to all its demand nodes over the original maximum flow prior to a disruption or network components removals. Figure 3-2 shows the drop in the efficiency of each of the two interdependent infrastructure networks associated with the fractional removal of some components (nodes and links) from both interdependent infrastructure networks. Hence, the effect of these removed network components is shown in both interdependent infrastructure networks due to their interdependencies. The drop in efficiency of each infrastructure network associated with the fractional removal of some components from each infrastructure network independently (i.e., without considering the interdependence between the two infrastructure networks) is also illustrated in Figure 3-2. Comparing the effect of the removal of fraction of components from the two infrastructure networks on each individual one independently and interdependently, it can be concluded as expected that the efficiency of both infrastructure networks declines faster when considering the interdependencies between the two infrastructure networks rather than when the interdependencies between them are not considered, see Figure 3-2.



(a) (b)

Figure 3-2. Efficiency with fractional removal of networks components for the (a) power network, and (b) water network

The two interdependent infrastructure networks were disrupted randomly, targeting the same number and type of disrupted components in both network, 10 network components in each network (i.e., 3 nodes and 7 links). The set of disrupted components in power and water networks are shown in Table 3-6 and Table 3-7, respectively. Table 3-6 and Table 3-7 show the importance of each disrupted component in power and water, respectively according to the two proposed CIM, ORT and ЯRW, considering the multiple interdependent network resilience optimization formulations discussed in Section 3.2 which are solved using LINGO 17.0. Without loss of generality, we consider the following parameter distributions and values for illustrative purposes: $\omega^k = 1/|K|$, $o_{ij}^k \sim U(20,50)$, and λ_i^k, π_{ij}^k, $\hat{\lambda}_i^k$, and $\hat{\pi}_{ij}^k \sim U(1,3)$.

In Model IV, if the recovery duration of a partially disrupted network component is not integer, it is rounded up to the nearest integer number since we are dealing with time periods. The disrupted components for both networks are considered completely disrupted in Model I and Model III whereas they are considered partially disrupted in Model II and Model IV with the same partial disruptions.

According to the ORT importance measure, the disrupted components in the two networks have different ranks based on their importance and effect on the resilience of the interdependent infrastructure networks when based on different models, as shown in Table 3-6 and Table 3-7. For example, node 4 in the power network is the most important component with the highest restoration priority when considering complete disruption (based on Model I and Model III), and the most important component when considering proportional disruption (based on Model II and Model IV), as shown in Table 3-6. Similarly, link (3,8) in the water network is the most important component with the highest restoration priority based on Model I, while it is not the most important component when based on Model II, Model III, or Model IV, as shown in Table 3-7.

Table 3-6. Restoration priorities for the disrupted components in Power network according to the two CIMS based on multiple formulations

Disrupted component	Restoration order							
	ORT				ЯRW			
	Model I	Model II	Model III	Model IV	Model I	Model II	Model III	Model IV
node 4	1	8	1	8	1	8	1	8
node 5	4	5	4	5	2	3	2	3
node 6	6	4	7	4	3	4	3	4
link (1,8)	5	1	5	1	5	2	5	2
link (2,4)	9	9	10	9	9	9	9	9
link (3,9)	2	3	2	3	6	5	6	5
link (4,7)	3	7	3	7	8	7	8	7
link (5,6)	8	6	8	6	7	6	7	6
link (6,8)	7	2	6	2	4	1	4	1
link (8,9)	10	10	9	10	10	10	10	10

Table 3-7. Restoration priorities for the disrupted components in Water network according to the two CIMS based on multiple formulations

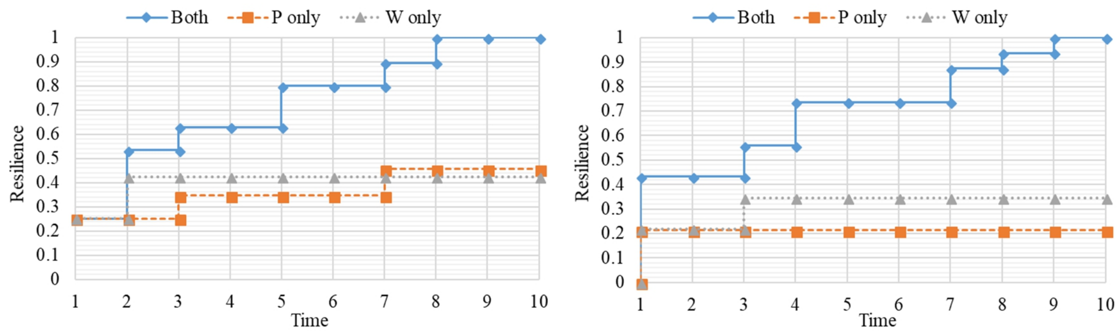
Disrupted component		Restoration order ORT Model I	ORT Model II	ORT Model III	ORT Model IV	ЯRW Model I	ЯRW Model II	ЯRW Model III	ЯRW Model IV
node	4	7	6	6	5	5	4	5	4
node	5	5	2	7	2	1	1	1	1
node	6	2	9	1	8	2	8	2	8
link	(1,4)	10	8	10	9	10	9	10	9
link	(1,9)	4	5	4	4	4	7	4	7
link	(2,5)	9	10	9	10	9	10	9	10
link	(2,8)	3	1	2	1	7	2	7	2
link	(3,8)	1	4	3	7	3	6	3	6
link	(4,9)	6	7	5	6	6	5	6	5
link	(5,7)	8	3	8	3	8	3	8	3

As for the ЯRW importance measure, the disrupted components in both interdependent infrastructure networks have different ranks according to their importance and influence on the resilience of the interdependent infrastructure networks when considering complete or proportional disruptions, as shown in Table 3-6 and Table 3-7. That is, when the ЯRW importance measure is based on Model I and Model II provides different restoration priorities than when it is based on Model III and Model IV. However, this CIM gives the same rank when fixed or different recovery durations are considered for restoring the disrupted components in the interdependent infrastructure networks as the recovery durations are not taken into account in this CIM, see Table 3-6 and Table 3-7. Table 3-6 and Table 3-7 show that the ЯRW importance measure provides the same restoration priorities when it is based on Model I and Model III and also the same restoration priorities when it is based on Model II and Model IV, where the difference between each two models is the recovery durations of the disrupted network components. That is, the restoration priorities according to the ЯRW importance measure are different when considering different disruption assumptions (i.e., partial or complete disruption) regardless of the recovery durations of the disrupted network components.

In general, CIMs produce useful information for decision makers. They assist infrastructure decision makers by identifying the critical and influential components in their respective networks that affect the performance of their infrastructure networks the most when disrupted or restored following a disruptive event. For example, in this work, CIMs identify the most important disrupted network components to be restored according to their effect on the resilience of the interdependent infrastructure networks once they are recovered, ORT importance measure, or by the potential impact on the resilience of the interdependent infrastructure networks caused by a specific disrupted network element, ЯRW importance measure, as discussed earlier in Section 4. Such information could be useful for decision makers in enabling more effective restoration priorities of the disrupted networks components or expediting the recovery process for some of the critical and influential ones. However, different CIMs are generally defined based on different perspectives of network performance that produce different information for decision makers. Hence, they could provide different restoration priorities of the disrupted network components based on their influence on the resilience of the interdependent infrastructure networks as shown in Table 3-6 and Table 3-7. Thus, decision makers might face a challenge of combining such information from different CIMs to obtain a unique set of restoration priorities of the disrupted network components. One means to identify the most important disrupted network components to be restored through a unique set of restoration priorities based on multiple CIMs with different perspectives is with the use of a multi-criteria decision analysis tool such as TOPSIS [Almoghathawi et al. 2017a].

Moreover, the uncertain nature of disruptive events could lead to different prioritizations for the restoration of the disrupted network components when considering partial disruptions for the disrupted networks components (i.e., when the CIMs are based on Model II and Model IV). Such uncertainty in the nature of disruptive events is discussed in Section 3.4.3. In addition, we are considering the uncertainty in the recovery durations of the disrupted networks components for a future work.

Figure 3-3 shows the improvement in the resilience of the each of the two considered interdependent infrastructure networks by restoring the disrupted components in both interdependent infrastructure networks according to their importance (i.e. restoration priorities) by the ORT importance measure based on Model I. Three restoration scenarios for restoring the disrupted components in the power and water networks are considered in Figure 3-3 which are: (i) restoring all the disrupted components in both network (“Both”), (ii) restoring the disrupted components in the power network only (“P only”), and (iii) restoring the disrupted components in the water network only (“W only”). Though there is a slight improvement in the resilience of the interdependent infrastructure networks individually when restoring the disrupted components in one network only, as in scenario (ii) or (iii), the resilience will only reach a certain level of improvement, as shown in Figure 3-3, unless the components of one network do not depend on any of the disrupted components in the other one in order to be functional (e.g., we consider restoration scenario (ii) and all the components in the power network do not depend on any of the disrupted components of the water network). Figure 3-3 serves to illustrate the importance of considering the interdependent nature of the two networks when making restoration decisions.



(a) (b)

Figure 3-3. Network resilience considering different restoration scenarios based on the ORT with Model I for the (a) power network, and (b) water network

Figure 3-4 shows the improvement in the resilience of each of the two generated infrastructure networks along with the system of the interdependent infrastructure networks by restoring the partially disrupted components in both interdependent infrastructure networks according to their importance (i.e., restoration priorities or ranks) based on the ORT importance measure. Two restoration strategies for restoring the partially disrupted components in the power and water networks are considered in Figure 3-4 which are: (i) restoring all the partially disrupted components in both network according to their rank by ORT based on Model II, i.e., considering their partial disruptions ("P.M II" for power network, "W.M II" for water network, and "S.M II" for the system of interdependent infrastructure networks), see Table 3-6 and Table 3-7, and (ii) restoring the partially disrupted components in both network according to their rank by ORT based on Model I, i.e., assuming a complete disruption for the disrupted components ("P.M I" for power network, "W.M I" for water network, and "S.M I" for the system of interdependent infrastructure networks), see Table 3-6 and

Table 3-7. Figure 3-4 shows the difference in the improvement of the resilience when restoring partially disrupted networks components according to their priorities based on the worst-case disruption scenario (i.e., completely disrupted) and current disruption scenario considering their partial disruptions. As a result, the rank of the disrupted components considering CD disruption scenario is not necessarily the best rank for them in all disruptions scenarios as shown in Figure 3-4, which illustrates the importance of considering the uncertain nature of the disruptive events when making restoration decisions.

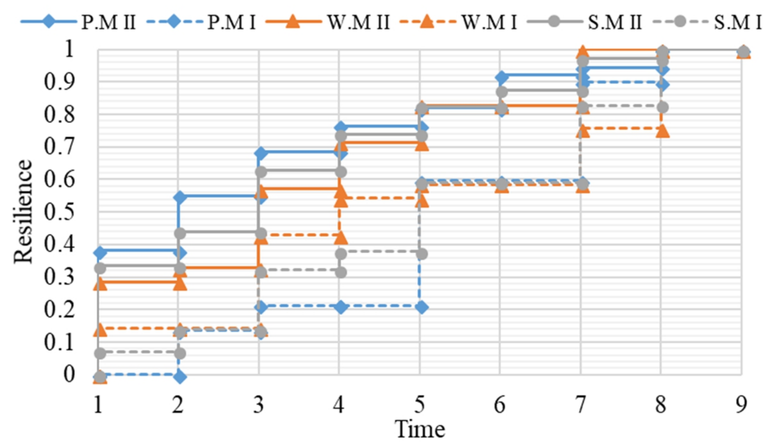
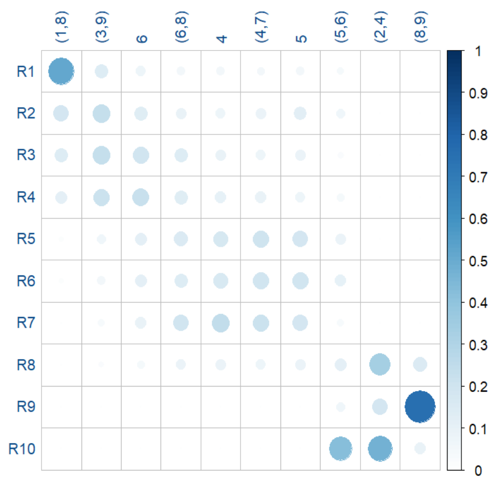


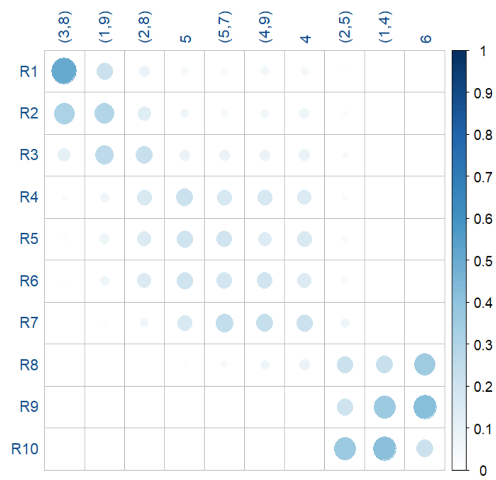
Figure 3-4. Network resilience considering different restoration scenarios by ORT

3.4.3 Effects of Uncertainty

In general, different disruption scenarios could lead to having different priorities for the restoration process of the disrupted components in the interdependent infrastructure networks, presented in Almoghathawi et al. [2017b]. To illustrate how the uncertain nature of the disruptive events could affect the ranking of the disrupted components in the interdependent infrastructure networks, we have considered a large number of different partial disruption scenarios generated randomly to find the probability of the occurrence of each disrupted component in each priority position among other disrupted components.

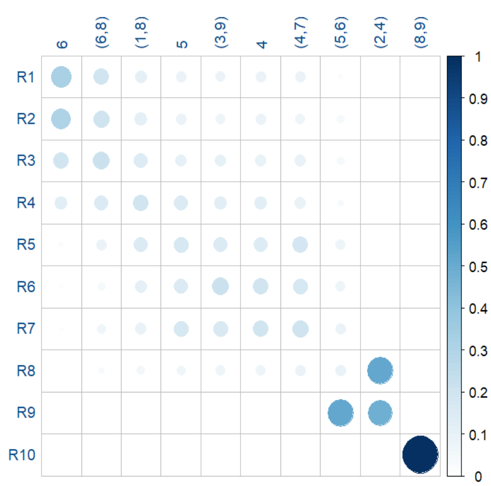


(a) Power network components by ORT

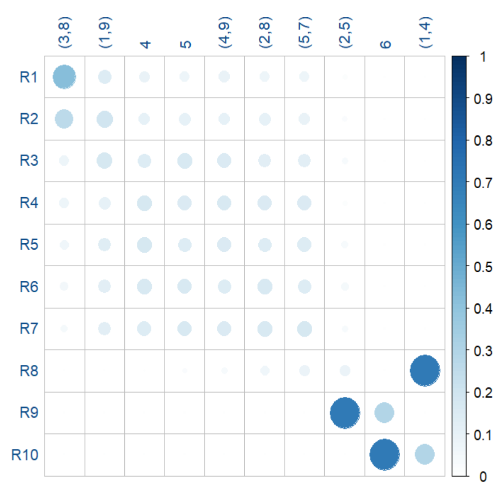


(b) Water network components by ORT

Figure 3-5. Heat Maps for the interdependent infrastructure networks by ORT



(a) Power network components by ЯRW



(b) Water network components by ЯRW

Figure 3-6. Heat Maps for the interdependent infrastructure networks by ЯRW

Figure 3-5 and Figure 3-6 show the heat maps for the disrupted components in the power and water networks according to the two proposed resilience-based CIMs, ORT and ЯRW, respectively. Each heat map displays the disrupted components where the darker circle represents the higher probability of that disrupted component being ranked in that position. For example, link (1,8) in the power network has a higher probability by ORT than other disrupted components in the same network to be ranked in the first position, R1, as the network component with the highest priority to be restored first, see Figure 3-5(a). Similarly, link (8,9) in the power network is always ranked by ЯRW in the last position, R10, as the least important network component to be restored as shown in Figure 3-6(a).

3.4.4 Comparison with Non-Resilience-Based Measures

In this section, we compare the rank of the disrupted components in the interdependent infrastructure networks found from the two proposed resilience-based CIMs with two other network centrality measures, flow centrality (FC) [Nicholson et al. 2016] and betweenness centrality (BC) [Freeman et al. 1991, Girvan and Newman 2002]. The comparison is made based on Model I. Furthermore, the trajectory of resilience is compared when restoration is guided by each of the four measures.

The *flow centrality* CIM measures the contribution of a given network component to the maximum flow from all the supply nodes to all the demand nodes within the network. Accordingly, FC is defined as the ratio of the total volume of flow through a given network component to the maximum flow from all supply nodes to all demand nodes in the same network.

Definition 5.3.2.1. *(FC)*. The FC of a disrupted network component $e \in E'^k = N'^k \cup L'^k$ in network $k \in K$, denoted as I_e^{FC}, is defined in Eq. (3-38), where $w_{sd}(e)$ is the flow through component $e \in E'^k$ when determining the maximum flow from supply node $s \in N_s^k$ to demand node $d \in N_d^k$ in network $k \in K$ and w_{sd} is the maximum flow from supply node $s \in N_s^k$ to demand node $d \in N_d^k$ in network $k \in K$.

$$I_e^{FC} = \frac{\sum_{s \in N_s^k, d \in N_d^k} w_{sd}(e)}{\sum_{s \in N_s^k, d \in N_d^k} w_{sd}} \tag{3-38}$$

The *betweenness centrality* CIM is similar to the *flow centrality* CIM, though it measures the contribution of a given network component to the number of geodesic paths, rather than maximum flow, from all the supply nodes to all the demand nodes within the network. Accordingly, BC is defined as the ratio of the total number of geodesic paths that go through a given network component to the total number of geodesic paths from all supply nodes to all demand nodes in the same network.

Definition 5.3.2.2. *(BC)*. The BC of a disrupted network component $e \in E'^k = N'^k \cup L'^k$ in network $k \in K$, denoted as I_e^{BC}, is defined in Eq. (3-39), where $g_{sd}(e)$ is the number of geodesic paths from supply node $s \in N_s^k$ to demand node $d \in N_d^k$ in network $k \in K$ that go through component $e \in E_k'$ and g_{sd} is the total number of geodesic paths from supply node $s \in N_s^k$ to demand node $d \in N_d^k$ in network $k \in K$.

$$I_e^{BC} = \sum_{s \in N_s^k, d \in N_d^k} \frac{g_{sd}(e)}{g_{sd}} \tag{3-39}$$

Table 3-8 and Table 3-9 show the importance of each disrupted component in power and water, respectively, according to the two proposed resilience-based CIMs and the two network centrality measures, based on Model I. Note from Table 3-8 and Table 3-9 that each CIM has a different rank for prioritizing the restoration of the disrupted components for each infrastructure network. However, the more important aspect is the improvement in the resilience of the system of interdependent infrastructure networks with the recovery of the disrupted components according to each priority rank.

Table 3-8. Restoration priorities for the disrupted components in the power network according to the two CIMs based on Model I

Disrupted component		Restoration order			
		ORT	ЯRW	FC	BC
node	4	1	1	1	1
node	5	4	2	2	3
node	6	6	3	3	10
link	(1,8)	5	5	6	8
link	(2,4)	9	9	10	4
link	(3,9)	2	6	7	7
link	(4,7)	3	8	8	6
link	(5,6)	8	7	4	9
link	(6,8)	7	4	5	2
link	(8,9)	10	10	9	5

Table 3-9. Restoration priorities for the disrupted components in the water network according to the two CIMs based on Model I

Disrupted component		Restoration order			
		ORT	ЯRW	FC	BC
node	4	7	5	3	4
node	5	5	1	5	1
node	6	2	2	10	10
link	(1,4)	10	10	9	7
link	(1,9)	4	4	2	5
link	(2,5)	9	9	6	2
link	(2,8)	3	7	7	3
link	(3,8)	1	3	1	9
link	(4,9)	6	6	4	8
link	(5,7)	8	8	8	6

Figure 3-7 shows the resilience of the two interdependent infrastructure networks individually with the restoration order for the disrupted components in each network according to different CIMs (i.e., ORT, ЯRW, FC, and BC) based on Model I, considering a complete disruption for the disrupted network components with a fixed restoration time of one unit for each one of them. The resilience of the water network improves faster based on the FC restoration order for the disrupted components, as shown Figure 3-7. However, the resilience of the system of the interdependent infrastructure networks improves faster when considering the restoration order by the two proposed resilience-based CIMs, ORT and ЯRW, as illustrated in Figure 3-8. Figure 3-8 shows the resilience of the system of the interdependent infrastructure networks over the restoration time horizon according restoration priorities for the disrupted components according to different CIMs (i.e., ORT, ЯRW, FC, and BC) based on Model I.

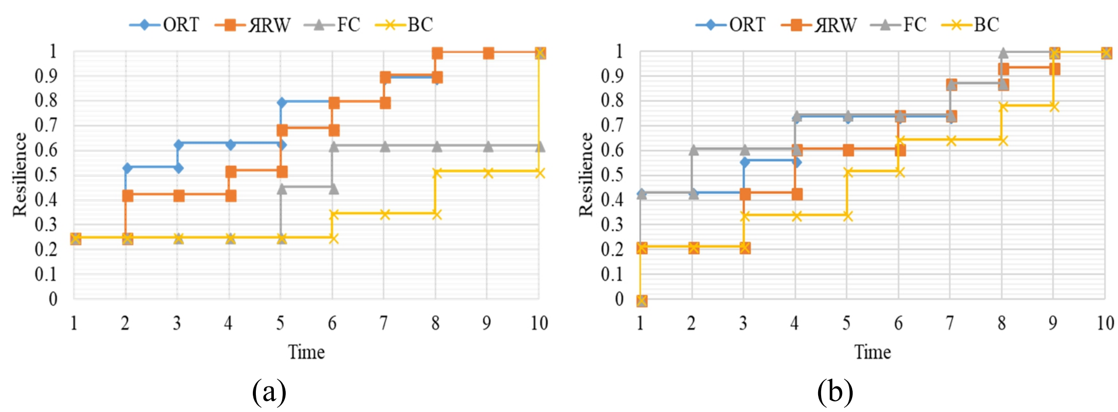


Figure 3-7. Network resilience based on different CIM based on Model I for the (a) power network, and (b) water network

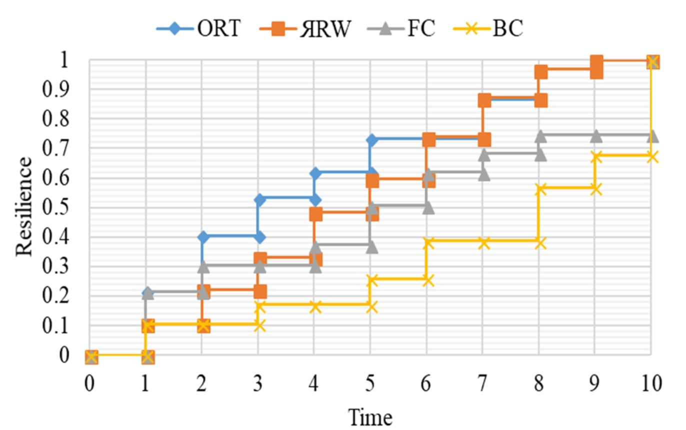


Figure 3-8. Resilience of the set of interdependent infrastructure networks based on different CIM based on Model I

The objective of this work is to enhance the resilience of the system of interdependent infrastructure networks given the physical interdependencies among them. As stated earlier, the maximum flow of an interdependent infrastructure network from its multiple supply nodes to its multiple demand nodes is considered to be the function by which the network performance is measured, and its resilience is determined. Accordingly, the two CIMs, ORT and ЯRW, are proposed to help decision makers finding the best restoration priorities for the disrupted networks components that could achieve the highest level of resilience of the system of interdependent infrastructure networks. As a result, ORT and ЯRW should provide more effective restoration priorities that enhance the resilience of the system of interdependent infrastructure networks than when considering other CIMs, e.g., FC and BC, or a random restoration order.

Chapter 4 : Resilience-Driven Restoration Model for Interdependent Networks

4.1 Introduction

In this chapter, we study the interdependent network restoration problem (INRP) following the occurrence of a disruptive event considering different disruption scenarios. We propose a resilience-driven multi-objective optimization model using MIP with the objectives of (i) maximizing the resilience of the interdependent infrastructure networks and (ii) minimizing the costs associated with the restoration process, including flow, disruption, and restoration costs. Moreover, the proposed MIP restoration model takes into account the availability of the time and resources considering that there is a set of available resources or work crews or that are specific to each network. There are two main assumptions for the proposed model: (i) the components of the interdependent infrastructure networks are either fully disrupted or undisrupted, (ii) the disrupted networks components have different restoration times for each one of them (i.e., the restoration time is not fixed or the same for all disrupted networks components). The model provides a set of prioritized restoration tasks to which to allocate and schedule available work crews considering the physical interdependence between the infrastructure networks such that the resilience of the interdependent infrastructure networks is maximized while the restoration cost is minimized. The proposed restoration model focuses on maximizing the resilience of the interdependent infrastructure networks to retain their performance level prior to the disruption. Hence, the disrupted components might not be all restored, especially if they do not have an effect on the resilience of the other networks. While this work addresses interdependent network restoration, a resilience measure in the objective function will enable future explorations of the balance between "withstanding a disruption" and "recovering from a disruption."

4.2 Restoration Model

In this section, we present the assumptions, notation, objectives and constraints of the proposed multi-objective restoration optimization model.

4.2.1 Assumptions

There are several assumptions and considerations for the proposed restoration optimization model:

- Each infrastructure network consists of a set of components (used to generally refer to nodes and links) that are subjected to be completely disrupted.
- Each disrupted component in each infrastructure network can be restored with different restoration durations (i.e., recovery durations are not fixed for all disrupted components).
- Each disrupted component in each infrastructure network cannot be operational until it is completely restored (i.e., this model does not consider partial functioning).
- A single work crew can work on restoring a single disrupted network component at a time, where they cannot leave the disrupted component until it is completely restored (i.e., this model considers a non-preemptive recovery process)
- Each supply node, demand node, and link in each infrastructure network has a known supply capacity, demand, and flow capacity, respectively.

- The flow costs through each link, disruption (i.e., unmet demand) costs, and restoration costs for disrupted components in each infrastructure network are known and fixed.
- The physical interdependence among different infrastructure networks is considered. That is, for a node in an infrastructure network to be operational, it requires a specific node from another infrastructure network to also be operational.
- The number of available work crews for each infrastructure network (i.e., infrastructure-specific resources) for the restoration of its disrupted components is known and could be different from one infrastructure network to another.

4.2.2 Notation

The sets, parameters, and decision variables of the proposed optimization model to solve the INRP are shown in Table 4-1, Table 4-2, and Table 4-3, respectively.

Table 4-1. Sets of the proposed restoration model

T	Time periods in the restoration horizon, $T = \{1, \dots, \tau\}$
K	Interdependent infrastructure networks, K
N^k	Nodes in network $k \in K$
L^k	Links in network $k \in K$
R^k	Available resources for network $k \in K$
N_s^k	Supply nodes in network $k \in K$, $N_s^k \subseteq N^k$
N_d^k	Demand nodes in network $k \in K$, $N_d^k \subseteq N^k$
N'^k	Disrupted nodes in network $k \in K$, $N'^k \subseteq N^k$
L'^k	Disrupted links in network $k \in K$, $L'^k \subseteq L^k$
Ψ	Interdependent nodes (i.e., $\left((i,k),(\bar{\imath},\bar{k})\right) \in \Psi$ indicates that node $\bar{\imath} \in N^{\bar{k}}$ in network $\bar{k} \in K$ requires node $i \in N^k$ in network $k \in K$ to be operational)

The amount of supply at node $i \in N_s^k$ in network $k \in K$, u_i^k, is considered in this model to be the maximum flow from node $i \in N_s^k$ to all demand nodes in network $k \in$

K. As such, u_i^k, assumed to be time independent, can be obtained by solving the model (2-1) – (2-3).

Table 4-2. Parameters of the proposed restoration model for network $k \in K$

u_i^k	Amount of supply and demand at node $i \in N_s^k$ and node $i \in N_d^k$, respectively
o_{ij}^k	Capacity of link $(i,j) \in L^k$
ω^k	Weight, $\sum_{k \in K} \omega^k = 1$
Q_o^k	Total slacks at all demand nodes in before the disruption
Q_d^k	Total slacks at all demand nodes in after the disruption
f_{ij}^k	Unitary flow cost through link $(i,j) \in L^k$
p_i^k	Penalty of unmet demand in node $i \in N_d^k$
g_i^k	Fixed restoration cost for node $i \in N'^k$
h_{ij}^k	Fixed restoration cost for link $(i,j) \in L'^k$
λ_i^k	Restoration duration of node $i \in N'^k$
π_{ij}^k	Restoration duration of link $(i,j) \in L'^k$

Table 4-3. Decision variables of the proposed restoration model for network $k \in K$ at time $t \in T$

q_{it}^k	Amount of unmet demand, called slack, at node $i \in N_d^k$
x_{ijt}^k	Amount of flow through link $(i,j) \in L^k$
y_{it}^k	A binary variable that equals 1 if node $i \in N^k$ is operational; and 0 otherwise
z_{ijt}^k	A binary variable that equals 1 if link $(i,j) \in L'^k$ is operational; and 0 otherwise
$\hat{y}_i^k$	A binary variable that equals 1 if node $i \in N'^k$ is to be restored; and 0 otherwise
$\hat{z}_{ij}^k$	A binary variable that equals 1 if link $(i,j) \in L'^k$ is to be restored; and 0 otherwise
v_{it}^{kr}	A binary variable that equals 1 if node $i \in N'^k$ is restored by work crew $r \in R^k$; and 0 otherwise
w_{ijt}^{kr}	A binary variable that equals 1 if link $(i,j) \in L'^k$ is restored by work crew $r \in R^k$; and 0 otherwise

4.2.3 Objectives

There are two objectives for the proposed restoration model: (i) maximizing the resilience of the system of interdependent infrastructure networks over the restoration time horizon, and (ii) minimizing the costs associated with the restoration process. The two objectives are discussed in the following sections.

4.2.3.1 Resilience Objective

The resilience, in this dissertation, is assumed to be a function of unmet demand, q_{it}^k, or the extent to which demand in node i of network k is not being met at time t (as opposed to using u_i^k, which is a fixed desired performance level of the interdependent infrastructure networks for our proposed model). Accordingly, slacks in the model represent the loss in the maximum flow and reducing them to a desired level represents a means to measure the effectiveness of the restoration process. Hence, the first objective function, the resilience of the system of interdependent infrastructure networks, is represented mathematically by Eq. (4-1), where μ^k is the weight of network $k \in K$ such that $\sum_{k \in K} \mu^k = 1$ and Q_o^k and Q_d^k represent the total slacks at all demand nodes in network $k \in K$ before and after a disruption, respectively (i.e., Q_o^k refers to the total original slacks at all demand nodes in network $k \in K$ at time t_0 and Q_d^k refers to the total slacks at all demand nodes in network $k \in K$ at time t_d following a disruptive event, e^j, as shown in Figure 2-1). Moreover, $\sum_{t=1}^{\tau} \left[t \left(Q_d^k - \sum_{i \in N_d^k} q_{it}^k \right) - (t-1) \left(Q_d^k - \sum_{i \in N_d^k} q_{i(t-1)}^k \right) \right]$ represents the cumulative *recovery* of network $k \in K$ over the restoration time horizon, where the recovery of the network at time $t \in T$ is determined by $Q_d^k - \sum_{i \in N_d^k} q_{it}^k$, while the total *loss* in network $k \in K$ is represented by $(Q_d^k - Q_o^k)$.

$$\max \sum_{k \in K} \omega^k \left[\frac{\sum_{t=1}^{\tau} \left[t \left(Q_d^k - \sum_{i \in N_d^k} q_{it}^k \right) - (t-1) \left(Q_d^k - \sum_{i \in N_d^k} q_{i(t-1)}^k \right) \right]}{\tau (Q_d^k - Q_o^k)} \right] \tag{4-1}$$

4.2.3.2 Cost Objective

Three different costs associated with the restoration process are considered in the restoration model: (i) flow cost, (ii) disruption cost (i.e., penalties of unmet demand), and (iii) restoration cost. The flow cost is a unitary cost for the flow through link $(i,j) \in L^k$ in network $k \in K$. The disruption cost is a unitary cost of unmet demand at node $i \in N_d^k$ in network $k \in K$. The restoration cost is a fixed cost for restoring node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$. Hence, the second objective function, the system cost, can be represented mathematically by Eq. (4-2).

$$\min \sum_{k \in K} \left(\sum_{i \in N'_k} g_i^k \hat{y}_i^k + \sum_{(i,j) \in L'^k} h_{ij}^k \hat{z}_{ij}^k + \sum_{t \in T} \left[\sum_{(i,j) \in L^k} c x_{ijt}^k + \sum_{i \in N_d^k} p_i^k q_{it}^k \right] \right) \tag{4-2}$$

4.2.4 *Constraints*

Several sets of constraints are considered in the proposed restoration model: (i) network flow constraints, (ii) restoration constraints, (iii) interdependence constraints, (iv) logical link constraints for the network flow with restoration, and (v) constraints governing the nature of the decision variables. All sets of constraints are explained and formulated in the following sections.

4.2.4.1 Network Flow Constraints

For each infrastructure network, the flow conservation at any (i) supply node, $i \in N_s^k$, (ii) transshipment node, $i \in N^k \backslash \{N_s^k, N_d^k\}$, and (iii) demand node, $i \in N_d^k$

is represented by constraints (4-3), (4-4), and (4-5), respectively. Constraints (4-6) ensure that the flow through link $(i,j) \in L^k$ in network $k \in K$ at time $t \in T$ does not exceed its capacity.

$$\sum_{(i,j)\in L^k} x_{ijt}^k \leq u_i^k, \quad \forall i \in N_s^k, k \in K, t \in T \tag{4-3}$$

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k = 0, \quad \forall i \in N^k \backslash \{N_s^k, N_d^k\}, k \in K, t \in T \tag{4-4}$$

$$\sum_{(j,i)\in L^k} x_{jit}^k + q_{it}^k = u_i^k, \quad \forall i \in N_d^k, k \in K, t \in T \tag{4-5}$$

$$x_{ijt}^k - o_{ij}^k \leq 0, \quad \forall (i,j) \in L^k, k \in K, t \in T \tag{4-6}$$

4.2.4.2 Restoration Constraints

For node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$, if it is selected to be restored, it is scheduled to be restored by work crew $r \in R^k$ at time $t \in T$, as shown in constraints (4-7) and (4-8), respectively. Work crew $r \in R^k$ in infrastructure network $k \in K$ can work on the restoration of a single disrupted network component, i.e., node $i \in N'^k$ or link $(i,j) \in L'^k$, at time $t \in T$, as shown in constraints (4-9). Constraints (4-10) and (4-11) ensure that node $i \in N'^k$ and link $(i,j) \in L'^k$, respectively in network $k \in K$ is operational at time $t \in T$ if it is restored by work crew $r \in R^k$. Constraints (4-12) and (4-13) ensure that node $i \in N'^k$ and link $(i,j) \in L'^k$, respectively in network $k \in K$ cannot be operational prior to its restoration duration. Similarly, work crew $r \in R^k$ cannot complete the restoration of node $i \in N'^k$ and link $(i,j) \in L'^k$ prior to its restoration duration, as shown in constraints (4-14) and (4-15), respectively.

$$\hat{y}_{ij}^k = \sum_{r\in R^k} \sum_{t\in T} w_{ijt}^{kr}, \quad \forall (i,j) \in L'^k, k \in K \tag{4-7}$$

$$\hat{z}_i^k = \sum_{r \in R^k} \sum_{t \in T} v_{it}^{kr}, \qquad \forall i \in N'^k, k \in K \tag{4-8}$$

$$\sum_{(i,j) \in L'^k} \sum_{l=t}^{\min\{\tau, t+\pi_{ij}^k - 1\}} w_{ijl}^{kr} + \sum_{i \in N'^k} \sum_{l=t}^{\min\{\tau, t+\lambda_i^k - 1\}} v_{il}^{kr} \leq 1, \tag{4-9}$$

$\forall k \in K, r \in R^k, t \in T$

$$z_{ijt}^k \leq \sum_{r \in R^k} \sum_{l=1}^{t} w_{ijl}^{kr}, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{4-10}$$

$$y_{it}^k \leq \sum_{r \in R^k} \sum_{l=1}^{t} v_{il}^{kr}, \qquad \forall i \in N'^k, k \in K, t \in T \tag{4-11}$$

$$\sum_{t=1}^{dl_{ij}^k - 1} z_{ijt}^k = 0, \qquad \forall (i,j) \in L'^k, k \in K \tag{4-12}$$

$$\sum_{t=1}^{dn_i^k - 1} y_{it}^k = 0, \qquad \forall i \in N'^k, k \in K \tag{4-13}$$

$$\sum_{r \in R^k} \sum_{t=1}^{dl_{ij}^k - 1} w_{ijt}^{kr} = 0, \qquad \forall (i,j) \in L'^k, k \in K \tag{4-14}$$

$$\sum_{r \in R^k} \sum_{t=1}^{dn_i^k - 1} v_{it}^{kr} = 0, \qquad \forall i \in N'^k, k \in K \tag{4-15}$$

4.2.4.3 Interdependence Constraints

The physical interdependence among the different infrastructure networks is captured by constraints (4-16). This set of constraints ensure that for a node $\bar{\imath} \in N^{\bar{k}}$ in network $\bar{k} \in K$ to be operational at time $t \in T$, node $i \in N^k$ in network $k \in K$ must be operational at time $t \in T$ as well, where $\left((i,k),(\bar{\imath},\bar{k})\right) \in \Psi$.

$$y_{\bar{\imath}t}^{\bar{k}} - y_{it}^{k} \leq 0, \quad \forall \left((i,k),(\bar{\imath},\bar{k})\right) \in \Psi, t \in T \tag{4-16}$$

In this work considerate is assumed that for a dependent node to be operational, the other node or nodes upon which it depends must be operational. However, the proposed model could be easily generalized by adding a new parameter that captures all different cases of interdependencies [González et. al. 2016]: (i) a node can be operational if the other node or set of nodes that it depends on is operational, (ii) a node can be operational if at least one of the nodes that it depends on is operational, (iii) a node can be operational if a specific node or group of nodes from the set of the nodes that it depends on is operational, and (iv) a node depends partially on the functionality of a set of nodes.

4.2.4.4 Logical Link Constraints of Network Flow to Restoration

The flow through link $(i,j) \in L^k$ in network $k \in K$ is determined by the capacity of the link as well as the functionality status of the nodes at both ends on that link as shown in constraints (5-17) and (5-18). Furthermore, the capacity of link $(i,j) \in L'^k$ in network $k \in K$ is determined by functionality status of the link itself which is captured by constraints (5-19).

$$x_{ijt}^{k} - o_{ij}^{k} y_{it}^{k} \leq 0, \quad \forall (i,j) \in L^k, i \in N^k, k \in K, t \in T \tag{4-17}$$

$$x_{ijt}^{k} - o_{ij}^{k} y_{jt}^{k} \leq 0, \quad \forall (i,j) \in L^k, j \in N^k, k \in K, t \in T \tag{4-18}$$

$$x_{ijt}^{k} - o_{ij}^{k} z_{ijt}^{k} \leq 0, \quad \forall (i,j) \in L'^k, k \in K, t \in T \tag{4-19}$$

4.2.4.5 Constraints on the Nature of Decision Variables

The amount of unmet demand (slack), q_{it}^{k}, at node $i \in N_d^k$ and flow through link $(i,j) \in L^k$, x_{ijt}^{k}, in network $k \in K$ must be non-negative at time $t \in T$, as shown in constraints (4-20) and (4-21), respectively. Constraints (4-22) and (4-23) represent the restoration decision of node $i \in N'^k$ and link $(i,j) \in L'^k$, respectively in network $k \in K$. The functionality status of node $i \in N^k$ and link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$ is represented by constraints (4-24) and (4-25), respectively. Finally, constraints (4-26) and (4-27) represent the binary restoration variables for node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$, respectively.

$$q_{it}^k \geq 0, \quad \forall i \in N_d^k, k \in K, t \in T \tag{4-20}$$

$$x_{ijt}^k \geq 0, \quad \forall (i,j) \in L^k, k \in K, t \in T \tag{4-21}$$

$$\hat{z}_{ij}^k \in \{0,1\}, \quad \forall (i,j) \in L'^k, k \in K \tag{4-22}$$

$$\hat{y}_i^k \in \{0,1\}, \quad \forall i \in N^k, k \in K \tag{4-23}$$

$$z_{ijt}^k \in \{0,1\}, \quad \forall (i,j) \in L'^k, k \in K, t \in T \tag{4-24}$$

$$y_{it}^k \in \{0,1\}, \quad \forall i \in N^k, k \in K, t \in T \tag{4-25}$$

$$w_{ijt}^{kr} \in \{0,1\}, \quad \forall (i,j) \in L'^k, k \in K, t \in T, r \in R^k \tag{4-26}$$

$$v_{it}^{kr} \in \{0,1\}, \quad \forall i \in N'^k, k \in K, t \in T, r \in R^k \tag{4-27}$$

4.2.5 Multi-Objective Optimization Technique

The proposed optimization model has multiple objectives which could be difficult to solve since many tradeoff solutions between the multiple objectives must be identified for consideration in the restoration of interdependent infrastructure networks. Hence, different multi-objective optimization techniques can be applied to find tradeoff solutions. In this dissertation, we use ε-constraint method proposed by Haimes et al. [1971] to generate Pareto-optimal solutions for our restoration model as it does not aggregate the multiple objectives but instead minimizes one of them while the remaining objectives are constrained within given target values specified by decision makers. Accordingly, the multiple objectives of our proposed model, (4-1) and (4-2) can be substituted by the new objective function (4-28) and the additional constraint (4-29), where $\varepsilon \in [0,1]$ since we are dealing with the resilience of the interdependent infrastructure networks, and the value of the network resilience, $Я(t)$, at time t is between 0 and 1 (see Section 2.2).

$$\min \sum_{k \in K} \left(\sum_{i \in N_k'} g_i^k \hat{y}_i^k + \sum_{(i,j) \in L'^k} h_{ij}^k \hat{z}_{ij}^k + \sum_{t \in T} \left[\sum_{(i,j) \in L^k} c x_{ijt}^k + \sum_{i \in N_d^k} p_i^k q_{it}^k \right] \right) \quad (4\text{-}28)$$

$$\sum_{k \in K} \omega^k \left[\frac{\sum_{t=1}^{\tau} \left[t \left(Q_d^k - \sum_{i \in N_d^k} q_{it}^k \right) - (t-1) \left(Q_d^k - \sum_{i \in N_d^k} q_{i(t-1)}^k \right) \right]}{\tau (Q_d^k - Q_o^k)} \right] \geq \varepsilon \quad (4\text{-}29)$$

4.3 Numerical Experiment

4.3.1 Networks Data

We illustrate our proposed restoration model in this work with fictional interdependent infrastructure networks (i.e., power and water), using the extended algorithm for proximal topology generator proposed by Xin-Jian [2007] described in Section 3.4.1. The two interdependent infrastructure networks are generated using R platform. Accordingly, the two interdependent infrastructure networks are generated as illustrated in Figure 4-1. The general properties for each network are shown in Table 4-4 which includes the number of nodes, number of undirected links, number of supply nodes, number of demand nodes, and average node degree, respectively.

Table 4-4. General properties of the interdependent infrastructure networks

Network	N	L	N^s	N^d	$\langle deg \rangle$
Power	25	31	5	5	2.48
Water	25	35	5	5	2.8

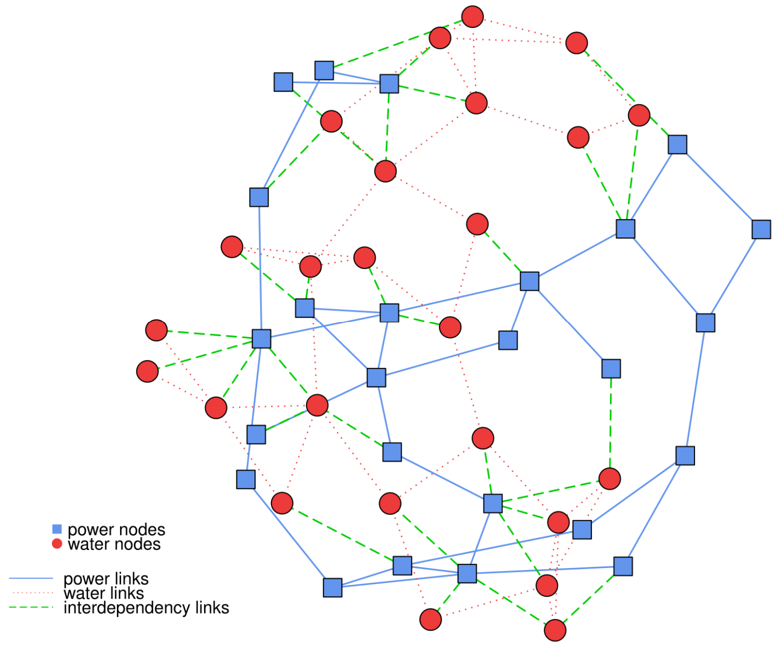


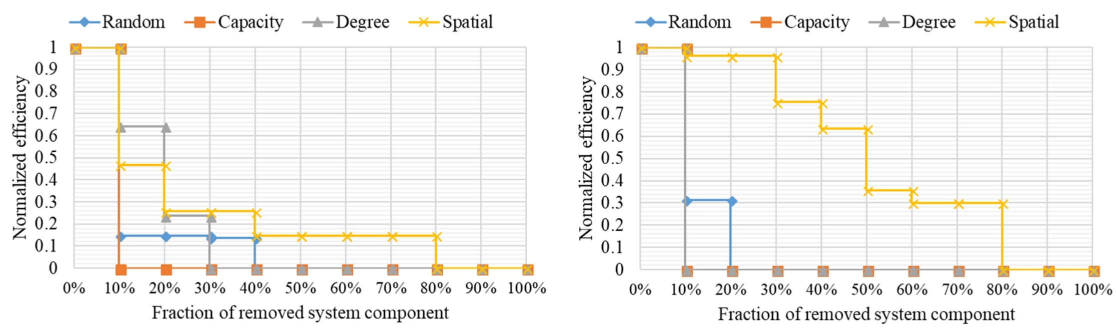
Figure 4-1. An example of interdependent infrastructure networks

4.3.2 Disruption Scenarios

Interdependent infrastructure networks are subjected to different scenarios of disruptions which could affect their performances differently. These disruption scenarios can be categorized into three groups [Wang et al. 2013]: random failures, malevolent attacks, and spatial failures. Random failures include common failures and manmade accidents such as aging, operating errors, and poor maintenance. For this scenario of disruptions, interdependent network components (nodes or links) are removed randomly with equal failure probability for all components. Malevolent attacks reflect intelligent attacks such as terrorism where important network components are targeted. Two scenarios are considered for malevolent attacks: capacity-based, where components with higher capacity are targeted, and degree-based, where components with higher degree (i.e., connections with other components) are targeted. For the capacity-based scenario, the capacities of the internal nodes are determined as the min of the sum of capacities for the incoming and outgoing links, respectively (i.e., $c_i^k = \min\{\sum_{(j,i)\in L^k} c_{ji}^k, \sum_{(i,l)\in L^k} c_{il}^k\}$, where c_i^k is the capacity of node i in network k and L^k is the set of links in network k). For the degree-based scenario, the degree of link (i,j) is defined as the average of the degree of node i and node j (i.e., $\deg_{ij} = \frac{1}{2}(\deg_i + \deg_j)$, where the degree of node i is the number of connections it has with other nodes in the network). Hence, interdependent network components are removed from their networks according to their capacity or degree where the components with the highest capacity or degree have higher failure probabilities than others. Finally, spatial failures capture natural disasters, such as earthquakes and hurricanes, that disrupt geographical locations. Consequently, the spatial disruptions affect the components of the interdependent infrastructure networks that are spatially closed to each other (i.e., can be affected by the same local disruption). In this work, the area of the interdependent infrastructure networks is divided into multiple regions where if a disruption occurs in a region, all the interdependent infrastructure networks components within that region will be disrupted and hence removed from their networks.

4.3.3 Experimental Results

Considering the different possible scenarios of disruptions discussed in Section 4.3.2 (“Random” for random failures, “Capacity” for capacity-based malevolent attacks, “Degree” for degree-based malevolent attacks, and “Spatial” for spatial failures), the efficiency of each interdependent infrastructure network with the removal of a fraction of components (nodes or links) of the interdependent infrastructure networks is illustrated in Figure 4-2. Efficiency is measured as the ratio of the current max flow over the original max flow. It can be observed from Figure 4-2 that the removal of the components of the interdependent infrastructure networks with the highest capacity result in the largest decline in the individual network efficiencies as the fraction of components removal increases. On the other hand, the spatial removal of the components of the interdependent infrastructure networks mostly result in the smallest drop in the individual networks efficiencies among other disruptions scenarios which could because of the existence of alternative routes within the networks since it affects a specific area in the network.



(a) (b)

Figure 4-2. Network efficiency with fraction components removals of the system of interdependent networks considering four disruption scenarios for (a) power network, and (b) water network

To assess the proposed multi-objective restoration model, a subset of the Pareto optimal set (i.e. non-dominated solutions) were obtained using Python 2.7 with Gurobi 7.5, see Figure 4-3, by varying the value of ε and solve the optimization model again for each value of ε. Figure 4-3 illustrates the generation of different points of the subset of Pareto front using different values of ε (i.e., $\varepsilon \in [0.5,1]$) considering the availability of one work crew for each network during the restoration process with different possible scenarios of disruptions, see Section 4.3.2. In addition, the restoration cost is considered for Figure 4-3 to be higher than the unmet demand cost for node $i \in N'^{k}$ in network $k \in K$ (i.e.,

$g_i^k > p_i^k$). Otherwise, the resilience will always be 1 given that there is enough time to recover the essential components because both objectives are focused on unmet demand, and by doing so, the resilience of the interdependent infrastructure networks will be maximized, and the total cost associated with the restoration will be minimized as well. In Figure 4-3, the horizontal axis represents the resilience of the interdependent infrastructure networks for which a maximum is sought, while the vertical axis represents total cost (i.e., restoration cost, flow cost, and disruption cost) that we would like to minimize. As observed in Figure 4-3, the total cost associated with the restoration process increases as the value of ε (i.e., the minimum value of the interdependent infrastructure networks resilience desired) increases. The lowest value of objective 1 (i.e. resilience) for all different scenarios of disruptions in Figure 4-3 is when $\varepsilon = 0.5$ while the highest value is when $\varepsilon = 1$. Hence, Figure 4-3 serves to illustrate the tradeoffs between the two objectives of the restoration model where when a higher level of resilience is desired for the systems of interdependent infrastructure networks, the total cost associated with the restoration process will be higher.

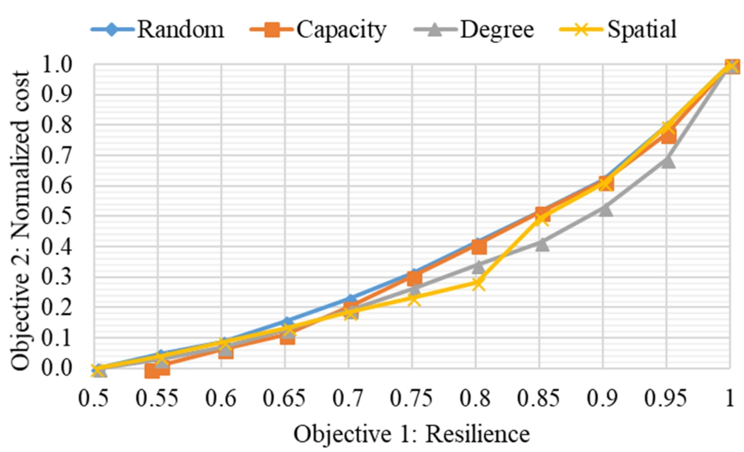
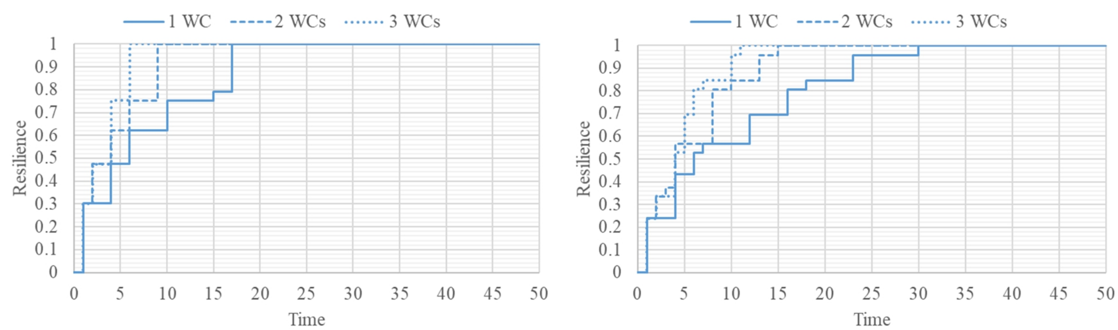


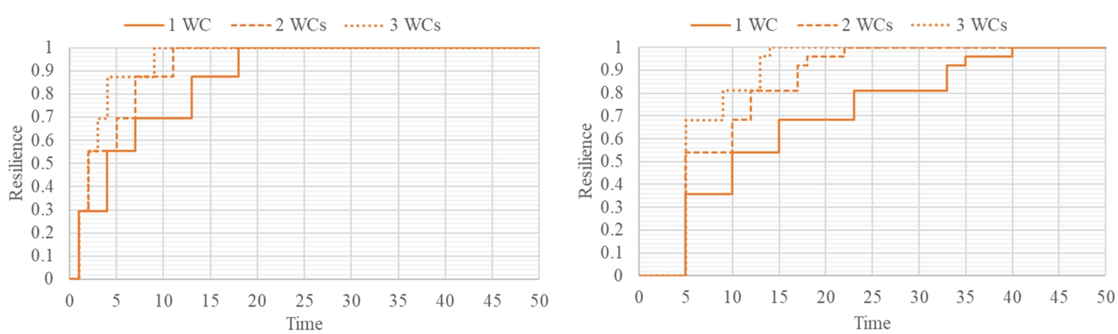
Figure 4-3. Objectives tradeoffs

Without loss of generality, we consider the following parameter distributions and values for illustrative purposes of the proposed multi-objective restoration model: $\omega^k = 1/|K|$, $\tau = 50$, $g_i^k, h_{ij}^k, o_{ij}^k \sim U(20{,}50)$, $f_{ij}^k \sim U(1{,}10)$, $p_i^k = 60$, and $\lambda_i^k, \pi_{ij}^k \sim U(1{,}5)$. In this chapter, the disruption cost for node $i \in N'^k$ in network $k \in K$, p_i^k, is considered higher than its restoration cost, g_i^k, since we are aiming to maximize the resilience of the system of interdependent infrastructure networks. Hence, both objectives will be focusing on minimizing the unmet demand at node $i \in N'^k$ in network $k \in K$. The proposed restoration model was solved using Python 2.7 with Gurobi 7.5. Accordingly, Figure 4-4 through Figure 4-7 depict the trajectory of interdependent infrastructure network resilience considering the three different scenarios for availability of the work crews (WC): one, two, and three work crews with random, capacity-based, degree-based, and spatial disruptions, respectively which could help decision makers when developing their restoration plans following a disruptive event (e.g., considering more work crews for one network than the other). The percentages of disrupted components in the interdependent power-water networks are: 21%, 21%, 21%, and 32% for the scenarios of random, capacity-based, degree-based, and spatial disruptions, respectively. The disrupted components in the first three disruptions scenarios are: 10 nodes, 5 from each network, and 14 bi-directional links, 7 from each network. For the spatial disruptions: 15 nodes, 9 from the power network and 6 from the water network, and 21 bi-directional links, 12 from the power network and 9 from the water network. Moreover, the number of removed components from the interdependent infrastructure networks in the first three disruption scenarios are equal, 10 nodes and 14 links, as we are removing them individually according to specific criteria (i.e., random, highest capacity, highest degree). However, for the spatial disruption scenario, they are removed according to their locations (i.e., if a disruption occurs in a region, all the interdependent infrastructure networks components within that region will be disrupted and hence removed from their networks).



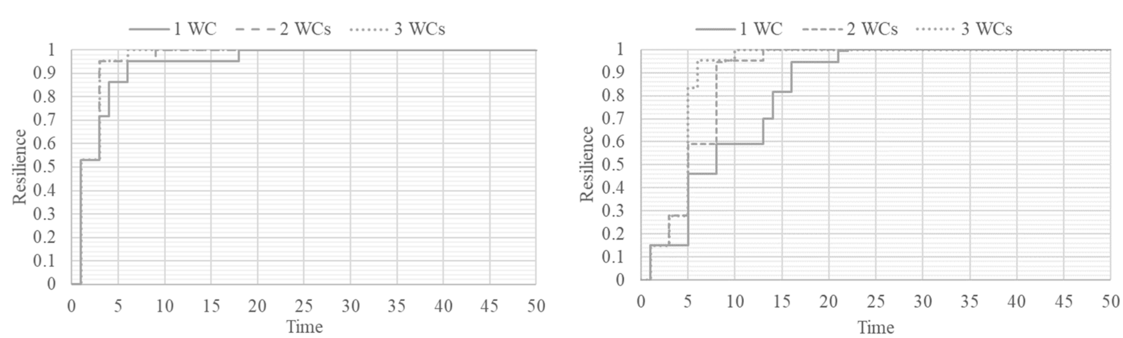
(a) (b)

Figure 4-4. Trajectory of network resilience considering random disruption and different number of work crews for (a) power network, and (b) water network



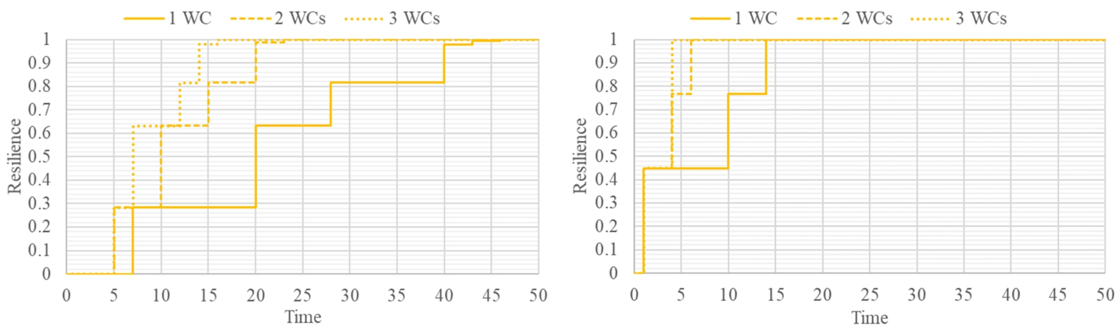
(a) (b)

Figure 4-5. Trajectory of network resilience considering capacity-based disruption and different number of work crews for (a) power network, and (b) water network



(a) (b)

Figure 4-6. Trajectory of network resilience considering degree-based disruption and different number of work crews for (a) power network, and (b) water network



(a) (b)

Figure 4-7. Trajectory of network resilience considering spatial disruption and different number of work crews for (a) power network, and (b) water network

As can be observed from Figure 4-4 through Figure 4-7, considering more work crews helps in achieving full resilience of the interdependent infrastructure networks earlier. However, it could take longer time in an individual network than the others though they are assigned same number of work crews because of their interdependencies. For example, considering the random, capacity-based, and degree-based disruptions scenarios, water network took longer time to be fully resilient than power network as it depends on some nodes on power network which were disrupted and need to be restored first, see Figure 4-4 through Figure 4-6. Similarly, power network took longer time to be fully resilient than water network considering spatial disruption scenario due to its dependency on some disrupted nodes on water network, see Figure 4-7. Hence, assigning more work crews to restore the disrupted components in one network than the other could help in reaching the maximum level of resilience of the system of interdependent infrastructure networks faster considering the available time periods. In general, there are three factors that affect the progress of improvement for the resilience of the system of interdependent infrastructure networks: (i) the set of disrupted components in the interdependent infrastructure networks, (ii) the nature of the interdependencies among the infrastructure networks, and (iii) the number of available work crews for each infrastructure network during the restoration process. Furthermore, the available time period and budget for the restoration process can decide what will be the maximum level of resilience that the system of interdependent infrastructure networks can reach. Accordingly, what are the disrupted components in the interdependent infrastructure networks need to be restored.

The proposed resilience-driven multi-objective restoration model focuses on maximizing the resilience of the interdependent infrastructure networks to retain their performance level prior to the disruption. Hence, the disrupted components might not be all restored, especially if they do not have an effect on the resilience of the other networks. Accordingly, the full resilience of the interdependent infrastructure networks could be achieved prior to complete restoration of these networks (i.e., time to full resilience (TFR) ≤ time to complete restoration (TCR) [Barker et al. 2013, Baroud et al. 2014]). Table 4-5 shows a comparison between the time when the interdependent power-water networks are fully resilient and the time when all the disrupted components are restored considering the different disruptions scenarios discussed earlier.

Table 4-5. Comparison between time to full resilience (TFR) and time to complete restoration (TCR) for the system of interdependent infrastructure networks considering different disruption scenarios with different number of work crews

Disruption Scenario	One work crew		Two work crews		Three work crews	
	TFR	TCR	TFR	TCR	TFR	TCR
Random	30	45	21	31	11	16
Capacity	41	50	26	31	14	18
Degree	22	41	16	29	10	20
Spatial	46	83	26	45	16	29

4.4 Progressive Restoration Approach

For large-scale networks or disruptions, it is difficult to obtain optimal restoration decisions for the disrupted components (i.e., prioritization, assignment and scheduling). Accordingly, our primary question is how can we make restoration decisions in a timely manner based on the proposed optimization model for the disrupted components of the system of interdependent infrastructure networks considering their interdependencies? Hence, in this section, we propose a progressive restoration approach (PRA) for the restoration model of the system of interdependent infrastructure networks considering: (i) fixed recovery durations, and (ii) different recovery durations, for the disrupted components.

4.5.1 PRA – I (Fixed Recovery Durations)

In this approach, we reduce the scale of the problem by restoring the disrupted components in the system of interdependent infrastructure networks in multiple restoration stages. At each stage, a set of disrupted components (nodes and links), $\mathcal{H}$, is restored for each infrastructure network within a practical time considering the availability of time and resources (i.e., work crews). Hence, $\mathcal{H}$ represents the maximum number of disrupted components that can be restored at a single stage of the restoration process. Accordingly, $\mathcal{H}$ is a function of m and n, where m is the number of components restored simultaneously in a subset, hence, m can be determined by the number of available work crews; and n is the number of subsets in one stage, hence, n can be determined by how many components can be restored by a work crew in a single restoration stage. A depiction of the set $\mathcal{H}$ is represented in Figure 4-8.

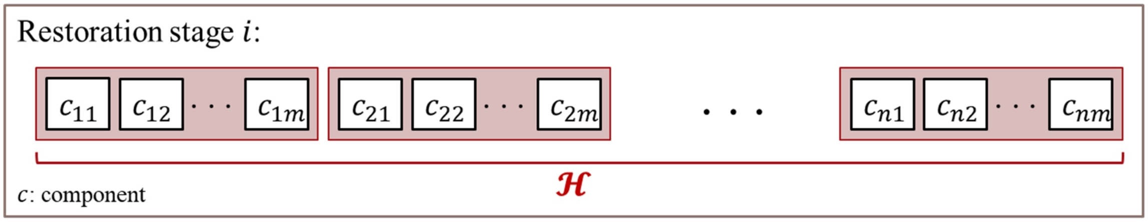


Figure 4-8. Illustration of the set of disrupted network components to be restored at each stage

4.5.2 PRA – II (Different Recovery Durations)

There are two approaches to solve the restoration model of the system of interdependent infrastructure networks considering different recovery durations for the disrupted components.

4.5.2.1 <u>PRA – II (A)</u>

In this approach, we solve the restoration problem in two phases:

Phase I: *Prioritize* the disrupted components according to their criticality and importance based on their effect on the resilience of the interdependent infrastructure networks with the following considerations:

- Priorities are based on the optimal recovery time (ORT) component importance measure.
- The recovery time horizon is reduced to π time periods for obtaining the disrupted components restoration priorities.
- The applicable restoration optimization model for the ORT CIM, presented earlier in Section 3.2, is solved iteratively until the priorities of all disrupted components are obtained.

Phase II: *Assign and schedule* the disrupted components in the system of interdependent infrastructure networks to the available work crews for each infrastructure network according to their priorities.

4.5.2.2 PRA – II (B)

In this approach, we also solve the restoration problem in two phases, where the main difference with PRA – II (A) is the second phase. Accordingly, the two phases for this approach are:

Phase I: same as in Phase I in PRA – II (A).

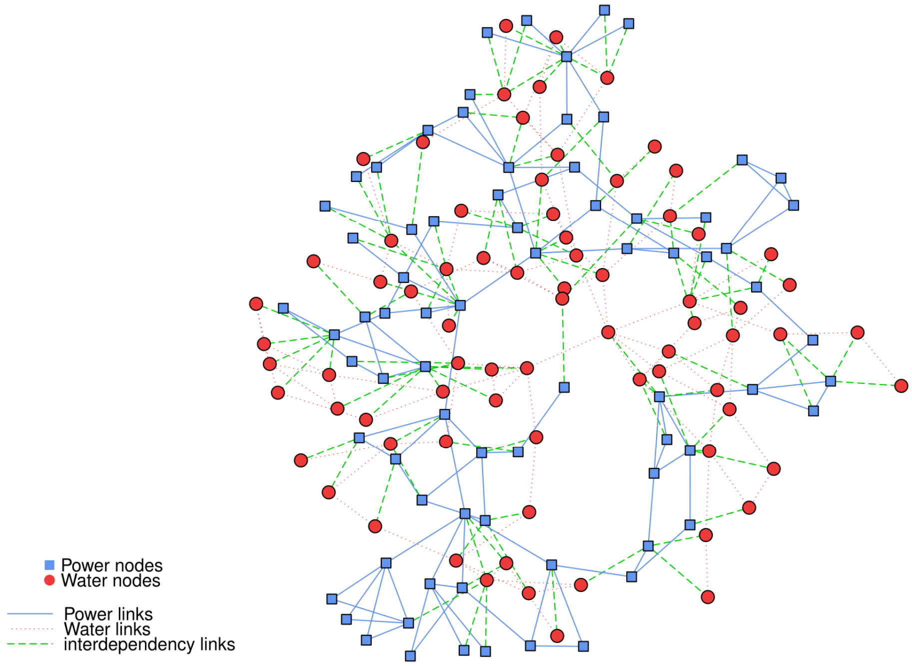
Phase II: *Solve* the original restoration model of the system of interdependent infrastructure networks considering the restoration priorities of the disrupted components.

4.5.3 Illustrative Example

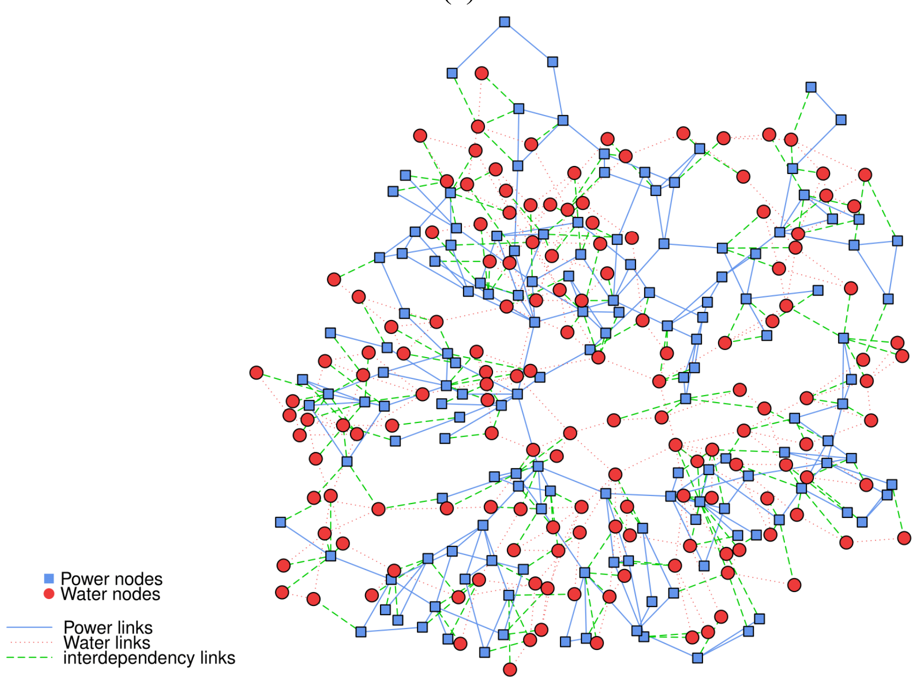
We consider two examples of a system of generated interdependent infrastructure networks: (i) System 1 (150 nodes), and (ii) System 2 (300 nodes), as shown in Figure 4-9. The general properties for each system of interdependent infrastructure networks are shown in Table 4-6 which includes number of: nodes, undirected links, source nodes, and demand nodes, as well as the average node degree for each network. Three scenarios of disruptions are considered: degree-based, capacity-based, and spatial, which are discussed earlier in Section 4.3.2. Moreover, we consider three disruption sizes: 10%, 20%, and 30% disruptions. The number of disrupted components for both interdependent infrastructure networks examples are shown in Table 4-6. The two approaches were solved using Python 2.7 with Gurobi 7.5.

Table 4-6. General properties of the interdependent infrastructure networks

System	N	L	N^s	N^d	$\langle deg \rangle$	Disrupted components		
						10%	20%	30%
1	150	188	50	50	2.51	35	69	103
2	300	382	100	100	2.55	69	137	205



(a)



(b)

Figure 4-9. An interdependent infrastructure networks example with (a) 150 nodes, and (b) 300 nodes

When considering fixed recovery durations for disrupted network components, the optimal solution was easily obtained for System 1 of interdependent infrastructure networks (i.e., 150-node) and the trajectory of system resilience considering different disruption sizes and average of disruption scenarios is shown in Figure 4-10. However, for System 2 of interdependent infrastructure networks (i.e., 300-node), the optimal solution was obtained but after a long running time for the model. Hence, we need to solve the model with PRA – I.

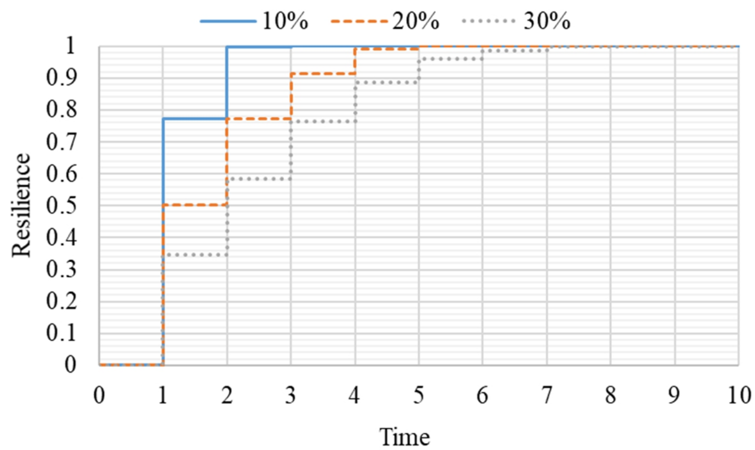


Figure 4-10. Network resilience considering different disruption sizes and average of disruption scenarios

To apply PRA – I, first we need to determine the set $\mathcal{H}$. In this work, we consider the availability of six work crews, hence, m equals 6. The number of components to be restored by each work crew, n, is obtained by finding the best combination of computational time and optimality gap for the worst restoration scenario, spatial disruption in this example. Hence, we consider a set of feasible realistic options for n, (i.e., 1, 2, or 3) as shown in Figure 4-11, after which the computational time was not practical. Accordingly, we consider n to be equal 2 (i.e., each work crew can restore at most two components in one stage). As a result, $\mathcal{H}$ would be 12 (i.e., at most 12 disrupted components can be restored at each stage).

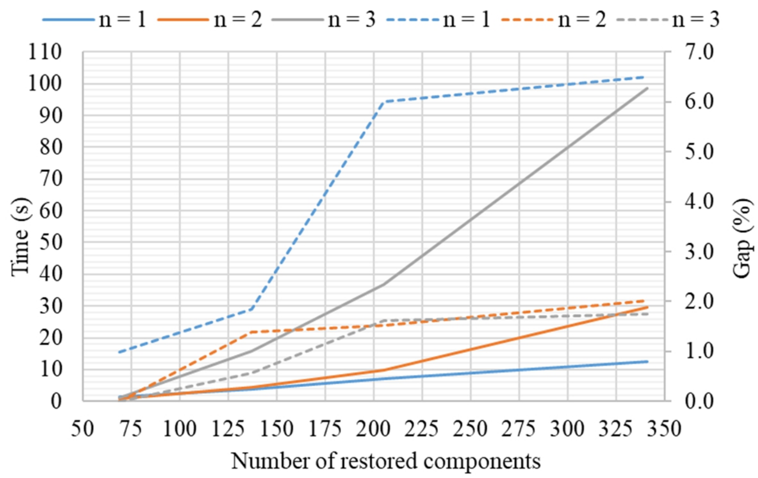


Figure 4-11. Computational time and optimality gap for the worst restoration scenario, spatial disruption, considering fixed recovery durations

Figure 4-12 shows the trajectory of the resilience enhancement for System 2 obtained by the MIP restoration model and the proposed solution approach, PRA – I, considering the average of the three disruptions scenarios (i.e., degree-based, capacity-based, and spatial) with the disruption size of 30% of the whole system. As it can be observed from Figure 4-12, both solutions are almost identical, and the proposed approach provided close results to the optimal solution of the restoration model.

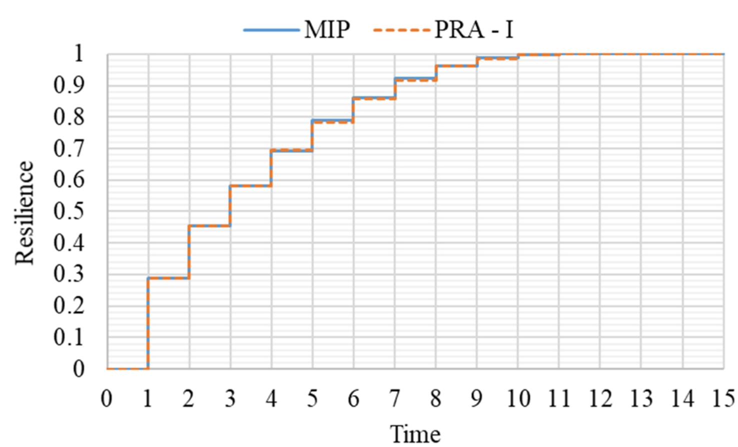


Figure 4-12. Network resilience considering disruption size of 30% and average of disruption scenarios for System 2

In addition, Table 4-7 shows the computational time of the MIP restoration model and PRA – I and optimality gap with the restoration model for System 2 considering fixed recovery durations for disrupted components and different disruption scenarios (i.e., degree-based, capacity-based, spatial) along with different disruption sizes (i.e., 10%, 20%, 30%).

Table 4-7. PRA – I computational results for System 2

Disruption Scenario	Disruption Size	MIP		PRA – I	
		Time (s)	Gap (%)	Time (s)	Gap (%)
Degree	10%	2.45	0.00	1.30	0.00
	20%	23.06	0.00	3.39	0.00
	30%	1479.85	0.00	11.77	0.01
Capacity	10%	2.54	0.00	0.67	0.00
	20%	12.70	0.00	1.72	0.00
	30%	54.58	0.00	3.50	0.19
Spatial	10%	1.98	0.00	0.78	0.00
	20%	113.55	0.00	4.28	1.38
	30%	4160.90	0.00	9.88	1.53

On the other hand, the optimal solution was difficult to obtain considering different recovery durations for disrupted networks components. Hence, we need to apply the proposed approach, PRA – II to get a solution in practical time. Accordingly, for phase I, we need to determine the reduced time horizon, π, to obtain the priorities of the disrupted networks components. So, we consider a similar technique to the determination of the number of subset in each restoration stage, n, for PRA – I (i.e., when considering fixed recovery durations for disrupted network components). Thus, the reduced time horizon, π, is obtained by finding the best combination of computational time and optimality gap for the worst restoration scenario, spatial disruption in this example. Hence, we consider a set of feasible realistic options for π, (i.e., 5, 6, or 7) as shown in Figure 4-13, after which the computational time was not practical. The minimum value of the set of feasible options is set to 5 which is equal to the maximum recovery time of any disrupted network component in this example. Accordingly, we consider π to be equal 7 (i.e., the model will run iteratively considering 7 time periods in each iteration until the priorities of all disrupted network components are obtained).

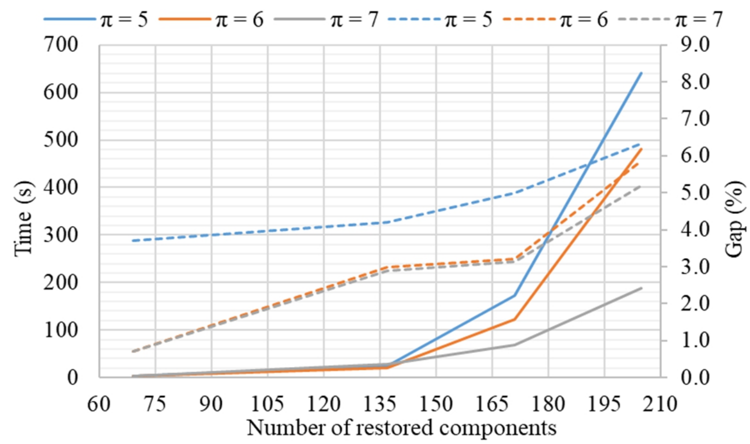


Figure 4-13. Computational time and optimality gap for the worst restoration scenario, spatial disruption, considering different recovery durations

Figure 4-14 shows the trajectory of the resilience enhancement for System 1 obtained by the MIP restoration model and the proposed solution approaches for different recovery durations, PRA – II (A) and (B), considering the average of the three disruptions scenarios (i.e., degree-based, capacity-based, and spatial) with the disruption size of 30% of the whole system. Similarly, Figure 4-15 shows the trajectory of the resilience enhancement for System 2 obtained by the MIP restoration model, PRA – II (A), and PRA – II (B) considering the average of the three disruptions scenarios with the disruption size of 20% of the whole system since the optimal solution could not be obtained, within 21600 seconds, for some disruption scenarios.

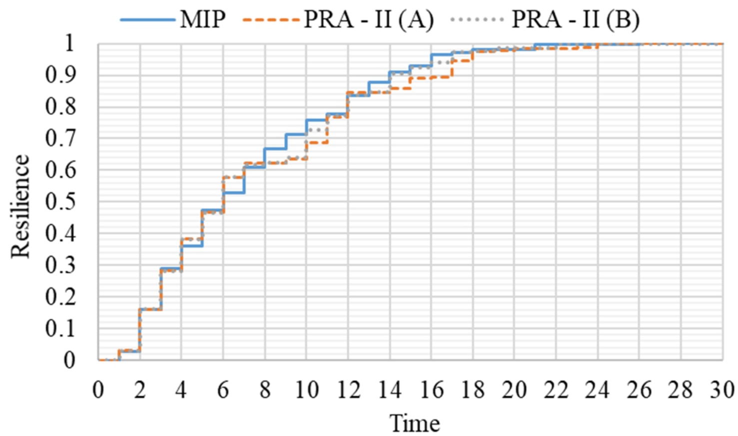


Figure 4-14. Network resilience considering disruption size of 30% and average of disruption scenarios for System 1

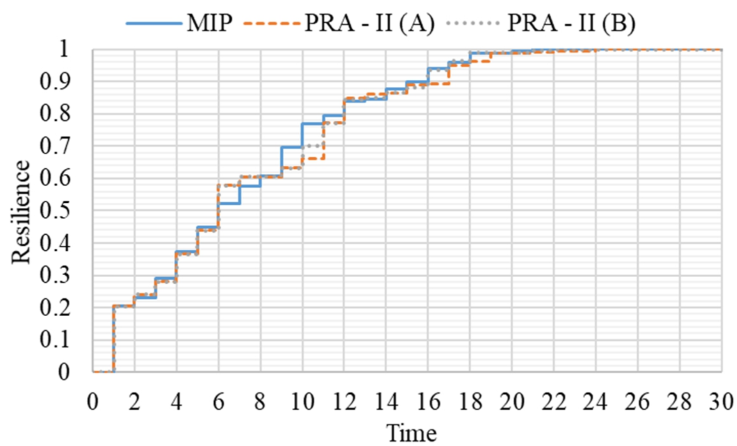


Figure 4-15. Network resilience considering disruption size of 20% and average of disruption scenarios for System 2

As it can be observed from Figure 4-14 and Figure 4-15, both PRA – II (A) and PRA – II (B) perform well with respect to the optimal solution obtained by the MIP restoration model since the resilience curves are close to each other. However, PRA – II (B) provide better results than PRA – II (A) for both System 1 and System 2, see Figure 4-14 and Figure 4-15.

Moreover, Table 4-8 shows the computational time of PRA – II (A) and (B) and their optimality gap with respect to the restoration model for System 1 considering different disruption scenarios (i.e., degree-based, capacity-based, spatial) along with different disruption sizes (i.e., 10%, 20%, 30%). Likewise, Table 4-9 shows the computational time of PRA – II (A) and (B) along with their optimality gap with respect to the restoration model for System 2 considering different disruption scenarios along with different disruption sizes. Furthermore, for some instances, the optimal solution is not obtained by the MIP restoration model during the set run time for the model (i.e., 21600 seconds) as shown in Table 4-9 for degree-based and spatial disruption scenarios with 30% disruption size for System 2. For such cases, we compare the solutions obtained by the proposed approaches, PRA – II (A) and (B), with the lower bound of the MIP restoration model when the model terminates (i.e., after 21600 seconds). Hence, the optimality gap of obtained solutions by PRA – II (A) and (B) are the upper bounds of the actual optimality gap of these solutions.

Table 4-8. PRA – II (A) and (B) computational results for System 1

Disruption Scenario	Disruption Size	MIP		PRA – II (A)		PRA – II (B)	
		Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)
Degree	10%	3.93	0.00	1.99	0.99	0.99	0.00
	20%	11.25	0.00	3.02	3.49	1.58	0.92
	30%	355.88	0.00	7.95	4.64	5.00	2.08
Capacity	10%	1.93	0.00	1.12	0.00	0.52	0.00
	20%	31.04	0.00	3.89	3.88	2.53	2.14
	30%	242.02	0.00	6.19	4.97	8.08	3.24
Spatial	10%	3.78	0.00	1.93	3.05	1.02	0.00
	20%	32.40	0.00	3.98	8.55	3.53	3.17
	30%	445.85	0.00	8.09	7.83	9.16	2.10

Table 4-9. PRA – II (A) and (B) computational results for System 2

Disruption Scenario	Disruption Size	MIP		PRA – II (A)		PRA – II (B)	
		Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)
Degree	10%	12.96	0.00	4.20	2.69	1.58	1.38
	20%	2262.40	0.00	12.22	3.85	23.23	1.95
	30%	21600.00	1.20	23.86	5.75	148.82	3.85
Capacity	10%	9.78	0.00	3.99	6.14	1.77	1.90
	20%	241.42	0.00	11.39	3.53	24.33	1.77
	30%	1765.14	0.00	16.70	4.24	149.33	2.34
Spatial	10%	10.37	0.00	4.08	2.15	2.17	0.66
	20%	1434.20	0.00	10.49	10.35	25.33	2.91
	30%	21600.00	5.00	23.80	8.71	177.00	5.20

Though PRA – II (A) gives results faster the PRA – II (B), PRA – II (B) provides better solutions (i.e., smaller optimality gap with respect to the optimal solution), as shown in Table 4-8 and Table 4-9. Hence, it is a tradeoff, that can be decided by decision makers, between the run time of the model and the gap with the optimal solution.

Chapter 5 : Exploring Recovery Strategies for Optimal Interdependent Network Resilience

4.3 Introduction

In this chapter, we propose a general resilience-driven multi-objective optimization model to solve the INRP using MIP with the objectives of: (i) maximizing the resilience of the system of interdependent infrastructure networks, and (ii) minimizing the total costs associated with the restoration process (i.e., flow, restoration, and disruption costs). The proposed model expands on the restoration model discussed in Chapter 4, by not only considering: (i) binary status of the networks components (i.e., either fully disrupted or undisrupted), (ii) complete dependence between nodes (i.e., a dependent node cannot be functioning unless the node or nodes that it depends on are completely functioning), and (iii) non-preemptive recovery process, but also considering: (iv) partial disruptions for the disrupted network components, (v) partial recovery of the disrupted network components considering their different restoration rates which allows for a preemptive recovery process , and (vi) partial dependence between nodes (i.e., a dependent node could be partially functioning if the node or nodes it depends on are partially functioning as well). Furthermore, the proposed optimization model takes into account the availability of the time and network-specific resources (i.e., a set of available resources or work crews or that are specific to each network). Different recovery strategies are explored considering different assumptions for work crews and disrupted component functionality. The proposed optimization model focuses on maximizing the resilience of the interdependent infrastructure networks to retain their performance level prior to the disruption. Hence, the disrupted networks components might: (i) not be all restored, especially if they do not influence the resilience of the other networks, or (ii) restored partially, if they could be functioning partially. Next section gives and overview regarding network resilience and how it can be quantified

4.4 Optimization Model

In this section, we present the assumptions, notation, objectives, and constraints of the proposed optimization model for solving the INRP.

4.4.1 Assumptions

There are several assumptions for the proposed optimization model:

- Each infrastructure network consists of a set of components (used to generally refer to nodes and links) that are subjected to be partially or completely disrupted.
- Each disrupted component in each infrastructure network can be restored with different restoration rates (i.e., recovery durations are not fixed).
- Each disrupted component in each infrastructure network could be partially recovered according to their restoration rates, which allow for a preemptive recovery process. Accordingly, different work crews can work to restore the same disrupted network component at different time periods.
- A single work crew can work on restoring a disrupted network component at a time.
- Each supply node, demand node, and link in each infrastructure network has a known supply capacity, demand, and flow capacity, respectively.
- The flow costs through each link, disruption costs, and restoration costs for disrupted components in each infrastructure network are known and fixed.

- The physical interdependence among different infrastructure networks is considered. That is, for a node in an infrastructure network to be operational, it requires a specific node from another infrastructure network to also be operational.
- The model allows for partial interdependencies considering: (i) partial status of disruption, or (ii) partial recovery of a disrupted component, i.e., a node could be operating partially if the other node upon which it depends is operating partially too.
- The number of available work crews for each infrastructure network (i.e., infrastructure-specific resources) for the restoration of its disrupted components is known and could be different from one infrastructure network to another.

4.4.2 Notation

The sets, parameters, and decision variables of the proposed optimization model to solve the INRP are shown in Table 1, Table 2, and Table 3, respectively.

Table 5-1. Sets of the proposed optimization model

T	Time periods in the restoration horizon, $T = \{1, \dots, \tau\}$
K	Interdependent infrastructure networks, K
N	Nodes
L	Links
N'	Disrupted nodes
L'	Disrupted links
N^k	Nodes in network $k \in K$, $\bigcup_{k \in K} N^k = N$
L^k	Links in network $k \in K$, $\bigcup_{k \in K} L^k = L$
R^k	Available resources for network $k \in K$
N_s^k	Supply nodes in network $k \in K$, $N_s^k \subseteq N^k$
N_d^k	Demand nodes in network $k \in K$, $N_d^k \subseteq N^k$
N'^k	Disrupted nodes in network $k \in K$, $N'^k \subseteq N^k$, $\bigcup_{k \in K} N'^k = N'$
L'^k	Disrupted links in network $k \in K$, $L'^k \subseteq L^k$, $\bigcup_{k \in K} L'^k = L'$
Ψ	Interdependent nodes (i.e., $\left((i,k),(\bar{\iota},\bar{k})\right) \in \Psi$ indicates that node $\bar{\iota} \in N^{\bar{k}}$ in network $\bar{k} \in K$ requires node $i \in N^k$ in network $k \in K$ to be operational)

Table 5-2. Parameters of the proposed optimization model for network $k \in K$

s_i^k	Supply capacity at node $i \in N_s^k$
d_i^k	Demand at node $i \in N_d^k$
o_{ij}^k	Capacity of link $(i,j) \in L^k$
ω^k	Weight, $\sum_{k \in K} \omega^k = 1$
Q_o^k	Total slacks at all demand nodes in before the disruption
Q_d^k	Total slacks at all demand nodes in after the disruption
f_{ij}^k	Unitary flow cost through link $(i,j) \in L^k$
p_{it}^k	Penalty of unmet demand in node $i \in N_d^k$ at time $t \in T$
$\hat{g}_{it}^k$	Fixed restoration cost for node $i \in N'^k$ at time $t \in T$
$\hat{h}_{ijt}^k$	Fixed restoration cost for link $(i,j) \in L'^k$ at time $t \in T$
γ_{it}^k	Restoration rate of node $i \in N'^k$ at time $t \in T$
δ_{ijt}^k	Restoration rate of link $(i,j) \in L'^k$ at time $t \in T$
y_{i0}^k	Initial operational status of node $i \in N_k'$ after a disruption
z_{ij0}^k	Initial operational status of link $(i,j) \in L'^k$ after a disruption
a_i^k	Number of units in node $i \in N^k$, $a_i^k \in \mathbb{Z}^+$
b_{ij}^k	Number of units in link $(i,j) \in L^k$, $b_{ij}^k \in \mathbb{Z}^+$

Terms a_i^k and b_{ij}^k refer to the number of units in node $i \in N^k$ and link$(i,j) \in L^k$, respectively, that can work independently from each other. For example, the number of units in a highway (i.e., a link in a transportation network) could be represented by the number of lanes in that highway. Consequently, the status of nodes and links is represented by the operational units in each one of them. That is, if a network component has more than one unit, it could be functioning partially depending on the number of operational units in that component following a disruption in two cases: (i) if it is not completely disrupted, or (ii) after a partial recovery. On the other hand, in case if a disrupted network component cannot be operational unless it is fully recovered, the number of units in this network component is assumed to be one, since the component cannot be functioning partially.

Table 5-3. Decision variables of the proposed optimization model for network $k \in K$ at time $t \in T$

u_{it}^{k}	Amount of supply at node $i \in N_s^k$
q_{it}^{k}	Amount of unmet demand, called slack, at node $i \in N_d^k$
x_{ijt}^{k}	Amount of flow through link $(i,j) \in L^k$
y_{it}^{k}	Status of node $i \in N^k$
z_{ijt}^{k}	Status of link $(i,j) \in L'^k$
α_{it}^{k}	Number of operational units in node $i \in N^k$
β_{ijt}^{k}	Number of operational units in link $(i,j) \in L'^k$
v_{it}^{kr}	A binary variable that equals 1 if node $i \in N'^k$ is restored by work crew $r \in R^k$; and 0 otherwise
w_{ijt}^{kr}	A binary variable that equals 1 1 if link $(i,j) \in L'^k$ is restored by work crew $r \in R^k$; and 0 otherwise

4.4.3 Objectives

The proposed mathematical model for solving the INRP focuses on optimizing two main objectives: (i) maximizing a measure of resilience for the collective set of networks, and (ii) minimizing the total costs associated with the restoration process. The two objectives are explained in more detail in the following sections.

4.4.3.1 Resilience Objective

We assume that resilience is a function of unmet demand, q_{it}^{k}, or the extent to which demand in node i of network k is not being met at time t. Accordingly, slacks represent the loss in the maximum flow, and reducing them to a desired level represents a means to measure the effectiveness of the restoration process. Hence, the first objective function, the resilience of the system of interdependent infrastructure networks, is represented mathematically by Eq. (5-1). Moreover, Q_o^k refers to the total original slacks at all demand nodes in network $k \in K$ at time t_0 and Q_d^k refers to the total slacks at all demand nodes in network $k \in K$ at time t_d following a disruptive event, e^j, as shown in Figure 2-1. Also, $(q_{i(t-1)}^k - q_{it}^k)$ determines the recovery at node $i \in N_d^k$ in network $k \in K$ at time $t \in T$. Hence, $\sum_{i \in N_d^k}\left(q_{i(t-1)}^k - q_{it}^k\right)$ represents the *recovery* of network $k \in K$ at time $t \in T$ and $(Q_d^k - Q_0^k)$ represents the total *loss* in network $k \in K$ following a disruptive event

$$\max \sum_{k \in K} \omega^k \left[\frac{\sum_{t=1}^{\tau} \sum_{i \in N_d^k}\left(q_{i(t-1)}^k - q_{it}^k\right)}{Q_d^k - Q_o^k}\right] \tag{5-1}$$

4.4.3.2 <u>Cost Objective:</u>

Three different costs associated with the restoration process are considered in the optimization model for solving the INRP: (i) flow cost, (ii) disruption cost (i.e., penalties of unmet demand), and (iii) restoration cost. The flow cost is a unitary cost for the flow through link $(i,j) \in L^k$ in network $k \in K$. The disruption cost is a unitary cost of unmet demand at node $i \in N_d^k$ in network $k \in K$. The restoration cost is a fixed cost for restoring node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$ based on their restoration rates, γ_{it}^k and δ_{ijt}^k, respectively. Hence, the system cost (second objective function) can be represented mathematically by Eq. (5-2).

$$\min \sum_{k \in K} \sum_{t \in T} \left(\sum_{(i,j) \in L^k} f_{ij}^k x_{ijt}^k + \sum_{i \in N_d^k} p_{it}^k q_{it}^k + \sum_{r \in R^k} \left[\sum_{i \in N_k'} \hat{g}_{it}^k \gamma_{it}^k v_{it}^{kr} + \sum_{(i,j) \in L'^k} \hat{h}_{ijt}^k \delta_{ijt}^k w_{ijt}^{kr} \right] \right) \tag{5-2}$$

4.4.4 Constraints

Several sets of constraints are considered in the proposed optimization model for solving the INRP: (i) network flow constraints, (ii) restoration constraints, (iii) interdependence constraints, (iv) logical link constraints for the network flow with restoration, and (v) constraints governing the nature of the decision variables. All sets of constraints are explained and formulated in the following sections.

4.4.4.1 Network Flow Constraints

For each infrastructure network, the flow conservation at each of its (i) supply nodes, $i \in N_s^k$, (ii) transshipment nodes, $i \in N^k \backslash \{N_s^k, N_d^k\}$, and (iii) demand nodes, $i \in N_d^k$ is represented by constraints (5-3), (5-4), and (5-5), respectively. Constraints (5-6) ensure that the flow through link $(i,j) \in L^k$ in network $k \in K$ at time $t \in T$ does not exceed its capacity. Constraints (5-7) ensure that the amount of slack or unmet demand, q_{it}^k, at node $i \in N_d^k$ in network $k \in K$ at time $t \in T$ does not exceed the required demand at that node.

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k = u_{it}^k, \qquad \forall i \in N_s^k, k \in K, t \in T \tag{5-3}$$

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k = 0, \qquad \forall i \in N^k \backslash (N_s^k \cup N_d^k), k \in K, t \in T \tag{5-4}$$

$$\sum_{(i,j)\in L^k} x_{ijt}^k - \sum_{(j,i)\in L^k} x_{jit}^k - q_{it}^k = -d_i^k, \qquad \forall i \in N_d^k, k \in K, t \in T \tag{5-5}$$

$$x_{ijt}^k - o_{ij}^k \le 0, \qquad \forall (i,j) \in L^k, k \in K, t \in T \tag{5-6}$$

$$q_{it}^k - d_i^k \le 0, \qquad \forall i \in N_s^k, k \in K, t \in T \tag{5-7}$$

4.4.4.2 Restoration Constraints:

Work crew $r \in R^k$ in infrastructure network $k \in K$ can work on the restoration of a single disrupted network component, node $i \in N'^k$ or link $(i,j) \in L'^k$, as shown in constraints (5-8). Constraints (5-9) and (5-10) ensure that for network $k \in K$, only a single work crew is assigned to work on the restoration of node $i \in N'^k$ and link $(i,j) \in$

L'^k, respectively, at time $t \in T$. The recovery status of node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$ is determined by constraints (5-11) and (5-12), respectively, which represent the status of the disrupted components after the occurrence of a disruptive event along with the recovery progress of these disrupted components by the available work crews in that network.

$$\sum_{i \in N'^k} v_{it}^{kr} + \sum_{(i,j) \in L'^k} w_{ijt}^{kr} \leq 1, \qquad \forall k \in K, t \in T, r \in R^k \tag{5-8}$$

$$\sum_{r \in R^k} v_{it}^{kr} \leq 1, \qquad \forall i \in N'^k, k \in K, t \in T \tag{5-9}$$

$$\sum_{r \in R^k} w_{ijt}^{kr} \leq 1, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-10}$$

$$y_{it}^k \leq y_{i0}^k + \sum_{r \in R^k} \sum_{l=1}^{t} \gamma_{il}^k v_{il}^{kr}, \qquad \forall i \in N'^k, k \in K, t \in T \tag{5-11}$$

$$z_{ijt}^k \leq z_{ij0}^k + \sum_{r \in R^k} \sum_{l=1}^{t} \delta_{ijl}^k w_{ijl}^{kr}, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-12}$$

4.4.4.3 Interdependence Constraints:

The physical interdependence among the different infrastructure networks is captured by constraints (5-13). This set of constraints ensure that for a node $\bar{\imath} \in N^{\bar{k}}$ in network $\bar{k} \in K$ to be operational at time $t \in T$, node $i \in N^k$ in network $k \in K$ must be operational at time $t \in T$ as well, where $\left((i,k),(\bar{\imath},\bar{k})\right) \in \Psi$.

$$y_{\bar{\imath}t}^{\bar{k}} - y_{it}^k \leq 0, \qquad \forall \left((i,k),(\bar{\imath},\bar{k})\right) \in \Psi, t \in T \tag{5-13}$$

In this work considerate is assumed that for a dependent node to be operational, the other node or nodes upon which it depends must be operational. However, the proposed model could be easily generalized by adding a new parameter that captures all different cases of interdependencies [González et. al. 2016]: (i) a node can be operational if the other node or set of nodes that it depends on is operational, (ii) a node can be operational if at least one of the nodes that it depends on is operational, (iii) a node can be operational if a specific node or group of nodes from the set of the nodes that it depends on is operational, and (iv) a node depends partially on the functionality of a set of nodes.

4.4.4.4 Logical Link Constraints of Network Flow to Restoration:

The number of operational units, α_{it}^k and β_{ijt}^k, in node $i \in N'^k$ and link $(i, j) \in L'^k$ in network $k \in K$ at time $t \in T$, respectively, are based on their operational state and determined by constraints (5-14) and (5-15), respectively. For example, in a transportation network, if a highway has four lanes, then the number of units in this highway will be four where each lane represents 25% of that highway. So, if the highway is completely disrupted and then recovered 50%, then two lanes will be operational. However, if it is 60% recovered, then again still two lanes will be working until the link is 75% recovered such that a third lane will then be available, and so on. Hence, the amount of supply at node $i \in N_s^k$ in network $k \in K$ could be affected by how many units are operational at that node, as governed by constraints (5-16). Also, the flow through link $(i, j) \in L^k$ in network $k \in K$ is determined by the capacity of the link as well as the number of the operational units in the nodes at both ends on that link as shown in constraints (5-17) and (5-18). Furthermore, the capacity of link $(i, j) \in L'^k$ in network $k \in K$ is determined by the number of the operational units in the link itself which is captured by constraints (5-19).

$$a_i^k y_{it}^k \geq \alpha_{it}^k, \qquad \forall i \in N'^k, k \in K, t \in T \tag{5-14}$$

$$b^k_{ij} z^k_{ijt} \geq \beta^k_{ijt}, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-15}$$

$$u^k_{it} - s^k_i(\alpha^k_{it}/a^k_i) \leq 0, \qquad \forall i \in N^k_s, k \in K, t \in T \tag{5-16}$$

$$x^k_{ijt} - o^k_{ij}(\alpha^k_{it}/a^k_i) \leq 0, \qquad \forall (i,j) \in L^k, i \in N^k, k \in K, t \in T \tag{5-17}$$

$$x^k_{ijt} - o^k_{ij}(\alpha^k_{jt}/a^k_j) \leq 0, \qquad \forall (i,j) \in L^k, j \in N^k, k \in K, t \in T \tag{5-18}$$

$$x^k_{ijt} - o^k_{ij}\left(\beta^k_{ijt}/b^k_{ij}\right) \leq 0, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-19}$$

4.4.4.5 Constraints on the Nature of Decision Variables:

For infrastructure network $k \in K$, the amount of supply, s^k_{it}, slack for unmet demand, sl^k_{it}, and flow through link $(i,j) \in L^k$, x^k_{ijt}, must be non-negative at time $t \in T$, as shown in constraints (5-20), (5-21), and (5-22), respectively. Constraints (5-23) and (5-24) represent the status of node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$, respectively, which is continuous depending on the magnitude of damage occurred at each one of them and their recovery progress. The number of operational units in node $i \in N^k$ and link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$ must be non-negative integer, see constraints (5-25) and (5-26), respectively. Finally, constraints (5-27) and (5-28) represent the binary restoration variables for node $i \in N'^k$ and link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$, respectively.

$$u^k_{it} \geq 0, \qquad \forall i \in N^k_s, k \in K, t \in T \tag{5-20}$$

$$q^k_{it} \geq 0, \qquad \forall i \in N^k_s, k \in K, t \in T \tag{5-21}$$

$$x^k_{ijt} \geq 0, \qquad \forall (i,j) \in L^k, k \in K, t \in T \tag{5-22}$$

$$0 \leq y^k_{it} \leq 1, \qquad \forall i \in N^k, k \in K, t \in T \tag{5-23}$$

$$0 \leq z^k_{ijt} \leq 1, \qquad \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-24}$$

$$\alpha^k_{it} \in \{0\} \cup \mathbb{Z}^+, \qquad \forall i \in N^k, k \in K, t \in T \tag{5-25}$$

$$\beta_{ijt}^{k} \in \{0\} \cup \mathbb{Z}^{+}, \qquad \forall (i,j) \in L'^{k}, k \in K, t \in T \tag{5-26}$$

$$v_{it}^{kr} \in \{0,1\}, \qquad \forall i \in N'^{k}, k \in K, t \in T, r \in R^{k} \tag{5-27}$$

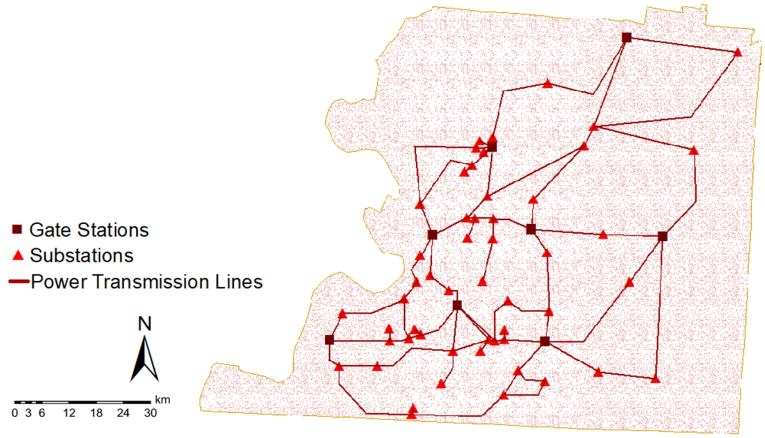
$$w_{ijt}^{kr} \in \{0,1\}, \qquad \forall (i,j) \in L'^{k}, k \in K, t \in T, r \in R^{k} \tag{5-28}$$

4.5 Numerical Experiment

In this section, the proposed optimization model to solve the INRP is illustrated through a realistic, well-known case in the literature, system of interdependent infrastructure networks in Shelby County, TN, in the United States. This county, which contains the city of Memphis, is constantly under earthquake hazard due to its proximity to the New Madrid Seismic Zone. Hence, in this example, we study the restoration strategies of such system considering the impact on it by multiple hypothetical earthquakes.

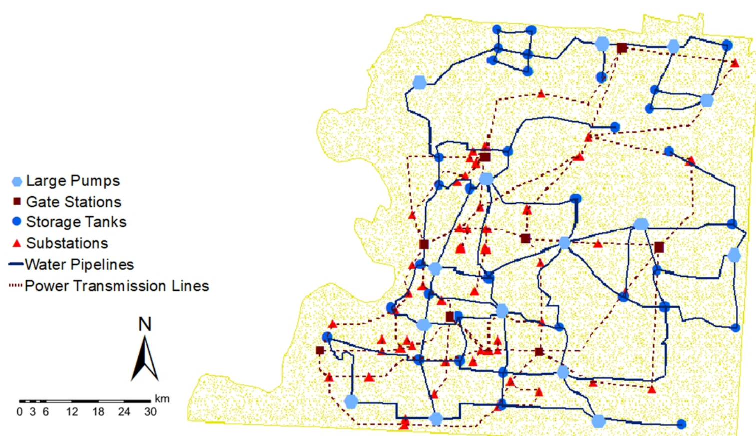
4.5.1 Networks Data

The system of networks considered in this study consists of two interdependent infrastructure networks in Shelby County, TN: water and power, see Figure 5-1. The topologies used were adapted from González et al. [2016] and Hernandez-Fajardo and Dueñas-Osorio [2011]. In particular, there are 256 network components that form this system of interdependent infrastructure networks (i.e., 109 nodes and 147 links). The water network is composed of 49 nodes and 71 links, while the power network is composed of 60 nodes and 76 links.



(a)

(b)



(c)

Figure 5-1. Graphical representations of the, (a) power, (b) water, and (c) interdependent water and power networks in Shelby County, TN (adapted from González et al. [2016])

4.5.2 Experimental Results

This work explores the four different magnitudes for hypothetical earthquake scenarios in Shelby County, TN presented by González et al. [2016], $M_w \in \{6,7,8,9\}$, considering the different failure probabilities of each component (node or link) in the system of interdependent infrastructure networks with each hypothetical earthquake scenario. Accordingly, the average number of the disrupted network components, as well as their percentage of the total number of components for the system of interdependent infrastructure networks, for each hypothetical earthquake scenario, considering a large number of disaster realizations for each magnitude, are shown in Table 5-4.

Table 5-4. Disruption size with hypothetical earthquake scenarios of different magnitudes

Magnitude	$N' \cup L'$	Disruption percentage
6	13	5.08%
7	31	12.11%
8	58	22.66%
9	90	31.16%

In this work, the demands at node $i \in N_d^k$ in network $k \in K$ is assumed proportional to the population surrounding it [González et al. 2016]. Also, the unitary flow cost and fixed restoration cost for link $(i,j) \in L^k$ and $(i,j) \in L^k$, respectively, are assumed proportional to their lengths. Moreover, the cost of unmet demand (i.e., disruption cost) in node $i \in N_d^k$ is considered to be greater than the maximum feasible total flow and restoration costs to set the priorities for the restoration strategy of the proposed model (i.e., satisfying the unmet demand first). In addition, the number of units in each of the network components is considered to be equal 1, (i.e., a_i^k, b_{ij}^k =1). That is, a disrupted network component will not be operational unless it is fully restored. It is assumed that $\mu^k = 1/|K|$, $\tau = 18$, $R^k = 6$, and $\gamma_{it}^k, \delta_{ijt}^k \sim U(0,1)$. Naturally, the chosen values of the parameters considered in this work could easily accommodate other assumptions to reflect more realistic operating and accounting scenarios. The proposed optimization model was solved using Python 2.7 with Gurobi 7.5. Accordingly, Figure 5-2 illustrates the improvement of the interdependent network resilience measure throughout the restoration process for the four different scenarios from Table 5-4.

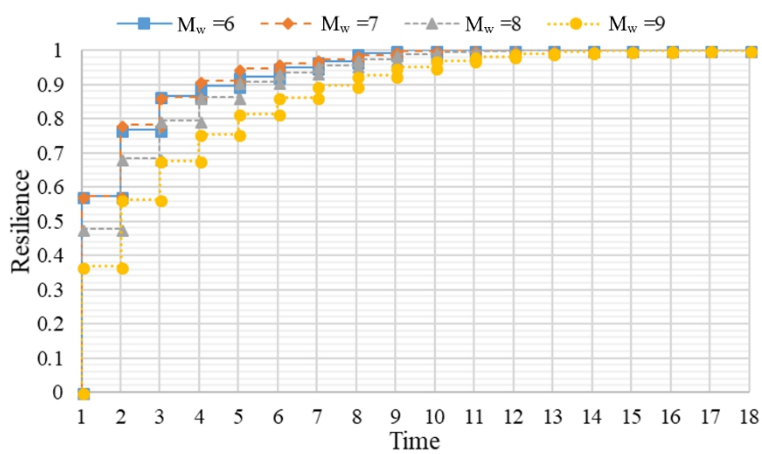
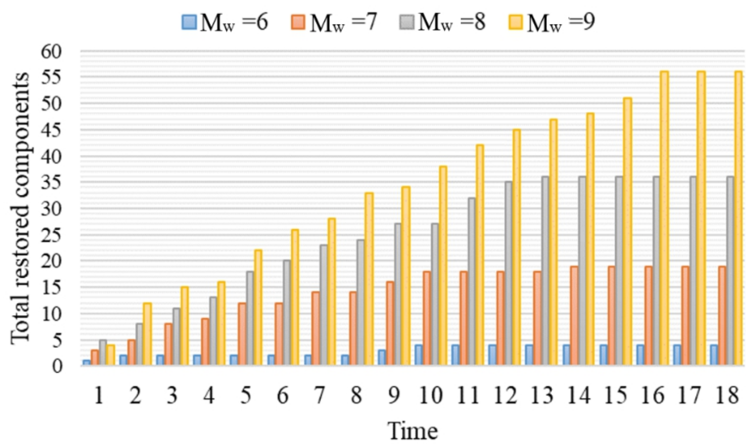
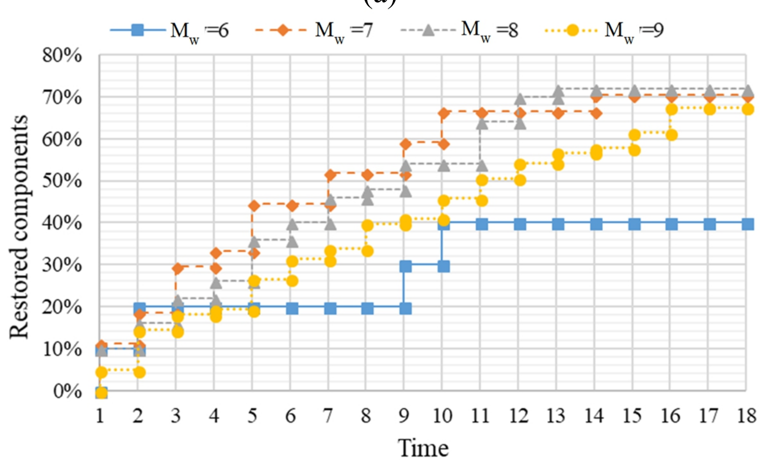


Figure 5-2. Network resilience with hypothetical earthquake scenarios of different magnitudes

As stated earlier in Section 5.1, the proposed optimization model for solving the INRP focuses on enhancing the resilience of the interdependent infrastructure networks to regain their performance level prior to the disruption. Hence, the disrupted networks components might: (i) not all be restored, especially if they do not influence the resilience of the other networks, or (ii) restored partially, if they could be functioning partially. This point is illustrated in Figure 5-3 for the example of the system of interdependent infrastructure networks in Shelby County, TN, with one of the disaster realizations for each of the different magnitudes of hypothetical earthquake scenarios, $M_w \in \{6,7,8,9\}$. Figure 5-3 shows: (i) the cumulative number of restored components over the restoration time horizon, and (ii) the percentage of the number of restored components to the number of disrupted components, shown in Table 4. Observed from Figure 5-3 is that not all the disrupted components are restored for the system of interdependent infrastructure networks (i.e., 4 components are restored (40.0%) with M_w = 6, 19 components are restored (70.4%) with M_w = 7, 36 components are restored (72.0%) with M_w = 8, and 56 components are restored (67.5%) with M_w = 9).



(a)



(b)

Figure 5-3. Restored network components over time in terms of (a) magnitude (bars) and (b) percentage (lines) with hypothetical earthquake scenarios of different magnitudes

4.6 Exploring Different Recovery Considerations

As shown in Section 5.2, the proposed optimization model for solving the INRP takes into account some assumptions and considerations related to the assignment of work crews and the functionality of network components. However, this section offers some extensions, considerations, and strategies to those assumptions and considerations, that could be incorporated in the proposed optimization model.

4.6.1 Recovery Acceleration

In the proposed optimization model, it is assumed that only a single work crew can work on restoring a disrupted component at time $t \in T$. However, since some network components could be critical and have high influence on their performance (or the performance of other networks), having multiple work crews working on restoring them at the same time could help in expediting the restoration process for the components themselves as well as their networks. In addition, the number of work crews that can work at the same time could differ from one time to another according to the criticality and the need as determined by decision makers. Hence, to allow for such consideration, constraints (5-9) and (5-10) are replaced by constraints (5-29) and (5-30), respectively, where θ_i^k is the maximum number of work crews allowed to work at the same time on node $i \in N'^k$ in network $k \in K$ at $t \in T$. Similarly, ρ_{ij}^k is the maximum number of work crews allowed to work at the same time on link $(i, j) \in L'^k$ in network $k \in K$ at $t \in T$.

$$\sum_{r \in R^k} v_{it}^{kr} \leq \theta_{it}^k, \forall i \in N'^k, k \in K, t \in T \qquad (5\text{-}29)$$

$$\sum_{r \in R^k} w_{ijt}^{kr} \le \rho_{ijt}^k, \forall (i,j) \in L'^k, k \in K, t \in T \tag{5-30}$$

Figure 5-4 shows the trajectory of the resilience of the interdependent infrastructure networks with the recovery progress considering two scenarios where (i) a single work crew, “one WC”, and (ii) multiple work crews, “multiple WCs”, can work on node $i \in N'^k$ or link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$. For illustrative purposes, each disrupted network component is assumed to have the option of having any number of the available work crews to work on its restoration at the same time at any time, that is θ_{it}^k and ρ_{ijt}^k are equal to the number of the available work crews in network $k \in K$ (i.e., θ_{it}^k, $\rho_{ijt}^k = \kappa$). Moreover, four different magnitudes for hypothetical earthquake scenarios are considered (i.e., $M_w \in \{6,7,8,9\}$) see Figure 5-4.

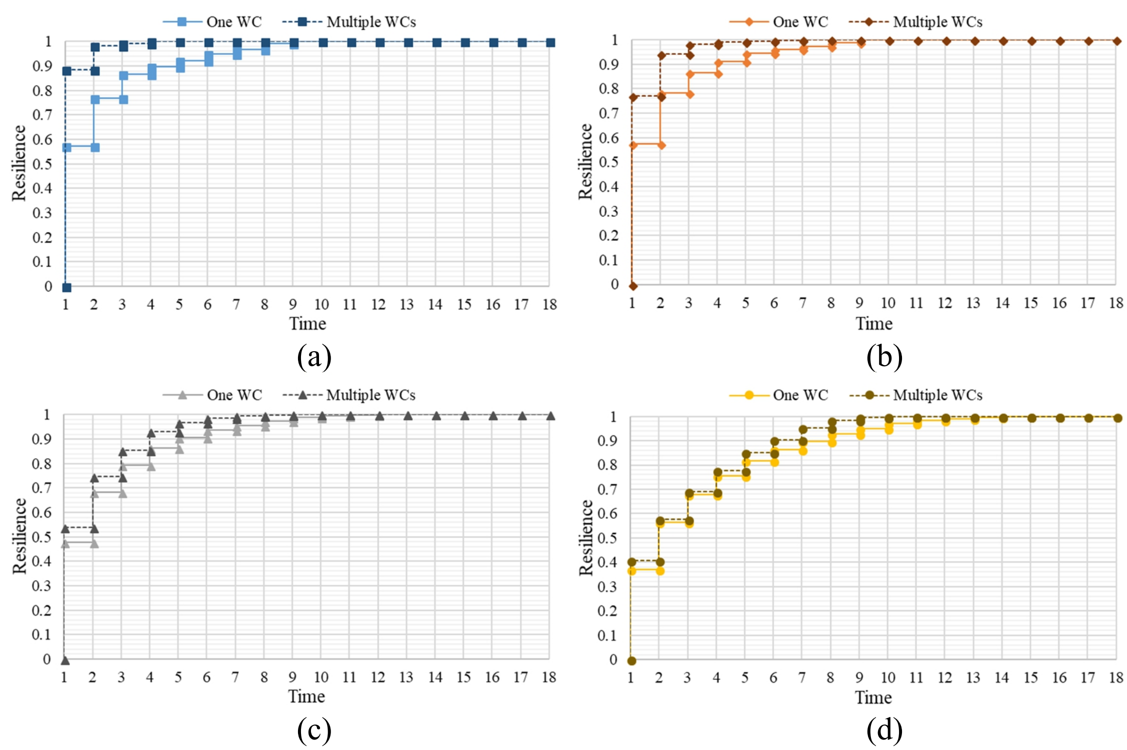


Figure 5-4. Network resilience considering different work crew scenarios with hypothetical earthquake scenarios of magnitude (a) M_w= 6, (b) M_w = 7, (c) M_w = 8, and (d) M_w = 9

As it can be observed from Figure 5-4, the difference in the resilience measure of the interdependent infrastructure networks between the two work crew assignment strategies reduces as the disruption is larger. Hence, though assigning multiple work crews to the same disrupted network component could aid in faster recovery, there are more critical network components that need to be restored to achieve a higher level of resilience. Therefore, different work crews are assigned to different disrupted network components, not the same component.

4.6.2 Network Components Functionality

Recall that a_i^k and b_{ij}^k represent the number of units in node $i \in N^k$ and link $(i, j) \in L^k$ in network $k \in K$, respectively. Such numbers of units could be one or multiple depending on the nature of the network and the functionality of its components. Accordingly, the number of unit in a network component could be one if the network component cannot be operational until it is completely restored. On the other hand, there could be multiple units in a network component if the component can be functioning partially following a disrupted event if it is not completely disrupted or after a partial recovery. While the initial illustration in Figure 5-2 assumed that the number of units in each component was 1, it could be assumed that $a_i^k, b_{ij}^k \sim U(1,4)$ such that they could be functioning partially when they are partially disrupted or partially recovered. Although the values of these parameters (i.e., for a_i^k and b_{ij}^k) are considered for illustrative purpose, other assumptions could be captured by the proposed model to reflect a more realistic network scenario. Figure 5-5 shows the improvement in the resilience of the interdependent infrastructure networks with the recovery progress considering two assumptions: (i) $a_i^k, b_{ij}^k = 1$, “one unit”, and (ii)

$a_i^k, b_{ij}^k \sim U(1,4)$, “multiple units”, for node $i \in N^k$ and link $(i,j) \in L^k$ in network $k \in K$, respectively.

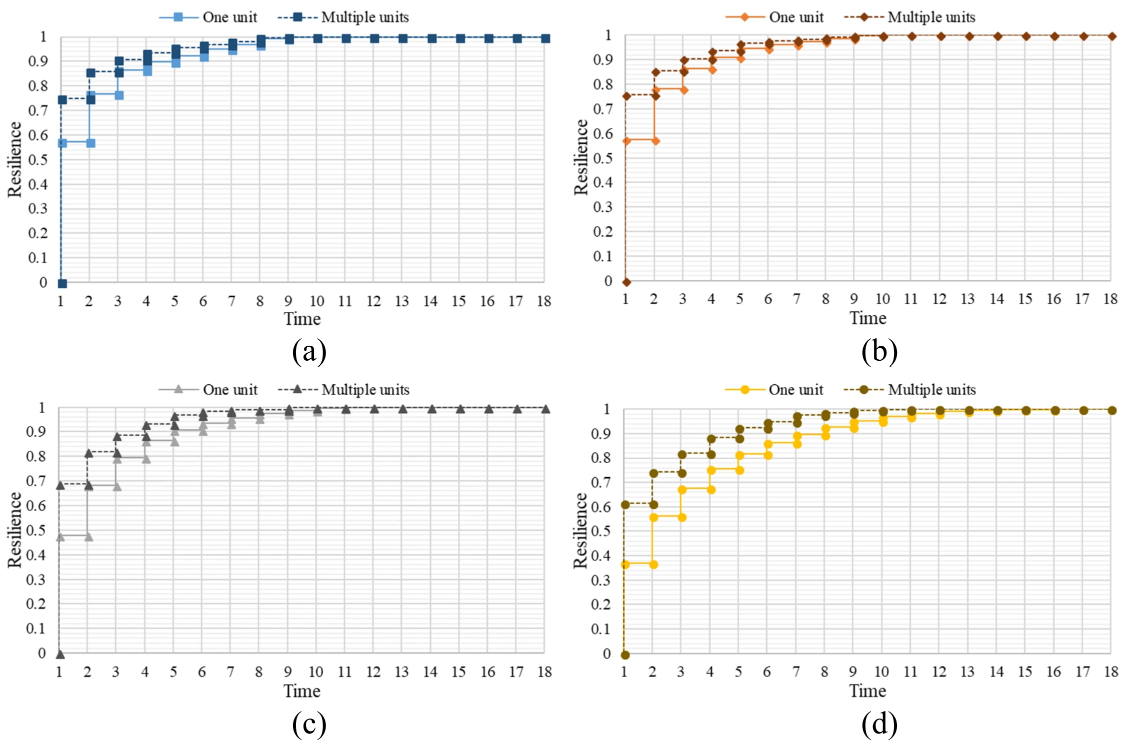


Figure 5-5. Network resilience considering different recovery assumptions with hypothetical earthquake scenarios of magnitude (a) M_w= 6, (b) M_w = 7, (c) M_w = 8, and (d) M_w = 9

As shown in Figure 5-5, considering partial functioning of the disrupted networks components results in a better level of resilience for the of the system of interdependent infrastructure networks through the recovery time horizon. However, the two different assumptions reach to the level of having a fully resilient system of interdependent infrastructure networks at the same time. It should be noted that the notion of a “units” is a function of the type of network not a recovery strategy, and that the illustration in Figure 5-5 may not be appropriate for actual water and electric power networks.

4.6.3 *Recovery Task Assignment*

The proposed optimization model for solving the INRP assures that only one work crew is working to restore node $i \in N'^{k}$ or link $(i, j) \in L'^{k}$ in network $k \in K$ at time $t \in T$. However, there could be different work crews working on the same network component at different time periods, especially when the restoration rate and cost are specific to the work crew. To illustrate this idea, we consider work crew-based restoration costs and rates shown in Table 5-5.

Table 5-5. Modified restoration parameters for work crew $r \in R^k$ in network $k \in K$

g_{it}^{kr}	Fixed restoration cost for node $i \in N'^{k}$ at time $t \in T$
h_{ijt}^{kr}	Fixed restoration cost for link $(i, j) \in L'^{k}$ at time $t \in T$
γ_{it}^{kr}	Restoration rate of node $i \in N'_k$ at time $t \in T$
δ_{ijt}^{kr}	Restoration rate of link $(i, j) \in L'^{k}$ at time $t \in T$

To assign restoration tasks of a network component that requires multiple time periods for its restoration to the same work crew, new assignment variables must be added. These decision variables are used to assign the recovery tasks of node $i \in N^k$ or link $(i, j) \in L^k$ in network $k \in K$ to the available work crews. Hence, $\hat{v}_i^{kr}$ is a binary variable that equals 1 if node $i \in N'^{k}$ in network $k \in K$ is assigned to work crew $r \in R^k$; and 0 otherwise. Similarly, $\widehat{w}_{ijl}^{kr}$ is a binary variable that equals 1 if link $(i, j) \in L'^{k}$ in network $k \in K$ is assigned to work crew $r \in R^k$; and 0 otherwise. As such, constraints (5-31) – (5-34) are added to the proposed model.

$$v_{it}^{kr} \leq \hat{v}_i^{kr}, \qquad \forall i \in N'^{k}, k \in K, t \in T, r \in R^k \tag{5-31}$$

$$w_{ijt}^{kr} \leq \widehat{w}_{ij}^{kr}, \qquad \forall (i, j) \in L'^{k}, k \in K, t \in T, r \in R^k \tag{5-32}$$

$$\sum_{r \in R^k} \hat{v}_i^{kr} \leq 1, \qquad \forall i \in N'^k, k \in K \tag{5-33}$$

$$\sum_{r \in R^k} \hat{w}_{ij}^{kr} \leq 1, \qquad \forall (i,j) \in L'^k, k \in K \tag{5-34}$$

Though only a single work crew can work on node $i \in N'^k$ or link $(i,j) \in L'^k$ in network $k \in K$ at time $t \in T$ (i.e., the original assumption of the proposed optimization model), two strategies are considered for the work crew assignment: (i) the same work crew works on the same disrupted network component at any time (referred to as "same WC"), and (ii) different work crews could work on the same disrupted network component at different time periods (or "different WCs"). Figure 5-6 illustrates the improvement in the resilience of the interdependent infrastructure networks with the recovery progress for these two strategies considering $\gamma_{it}^{kr}, \delta_{ijt}^{kr} \sim U(0,1)$. The resilience measure for the system of interdependent infrastructure networks is very similar for both strategies, as shown in Figure 5-6, and that is due to the number of units being equal to 1 (i.e., a_i^k, b_{ij}^k =1). That is, a disrupted network component will not be operational unless it is fully restored. However, since we are considering different restoration rates for different work crews, which result in different restoration cost for the disrupted network components accordingly, the restoration cost for both strategies are compared. Hence, Figure 5-7 shows the restoration cost for both strategies, normalized by the lowest restoration cost (i.e., considering the original assumption, different work crews, of the proposed optimization model), where the steady state of the cost indicates that the system of interdependent infrastructure networks has reached the maximum level of resilience (i.e., $Я = 1$ for the example). Considering different work crews to restore a network component at different time periods could result in a lower restoration cost due to the different recovery rates of the available work crews, as shown in Figure 10. The difference in the restoration cost between the two strategies for work crew assignment reduces as the disruption worsens, which is due to the size of the disruption (i.e., number of disrupted network components) and the number of available work crews during the recovery process.

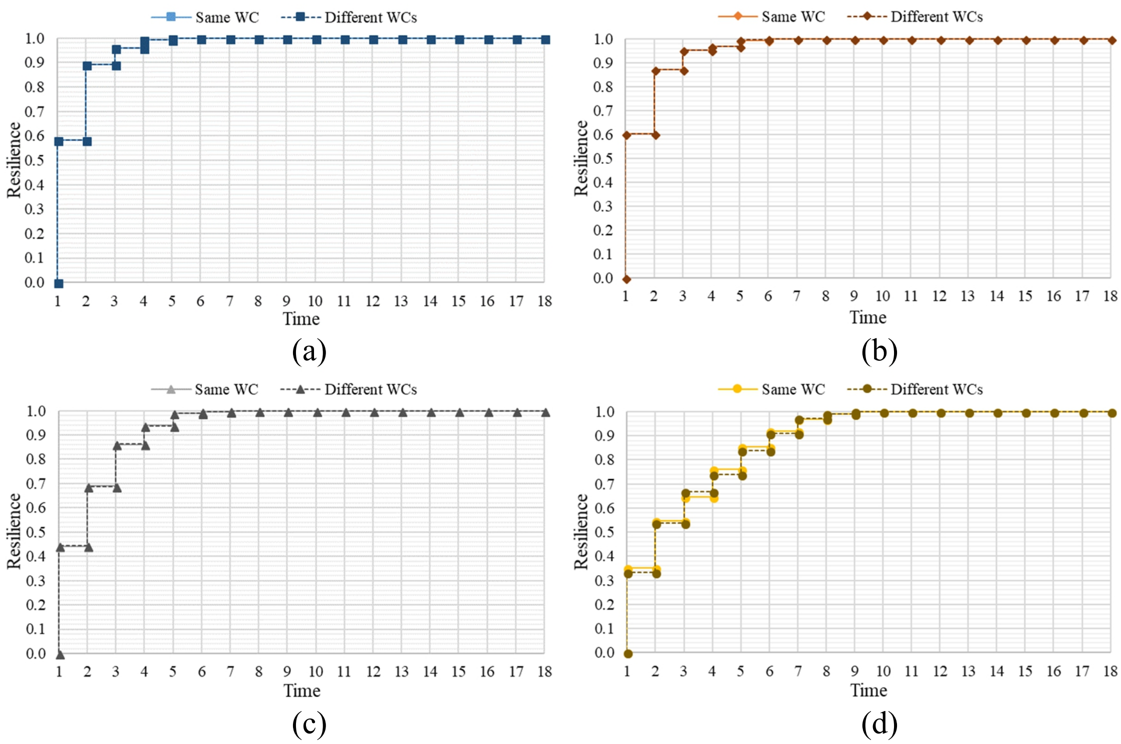


Figure 5-6. Network resilience considering different work crew assignment strategies with hypothetical earthquake scenarios of magnitude (a) M_w= 6, (b) M_w = 7, (c) M_w = 8, and (d) M_w = 9

In general, the variation in the improvement of the interdependent network resilience measure depends on: (i) the status (i.e., disruption size) of the disrupted networks components as well as their networks, (ii) the nature of the interdependency among the infrastructure networks, (iii) the number of available work crews for each infrastructure network, and (iv) and variation in the restoration rates for the work crews; hence the variation in restoration costs of the disrupted networks components.

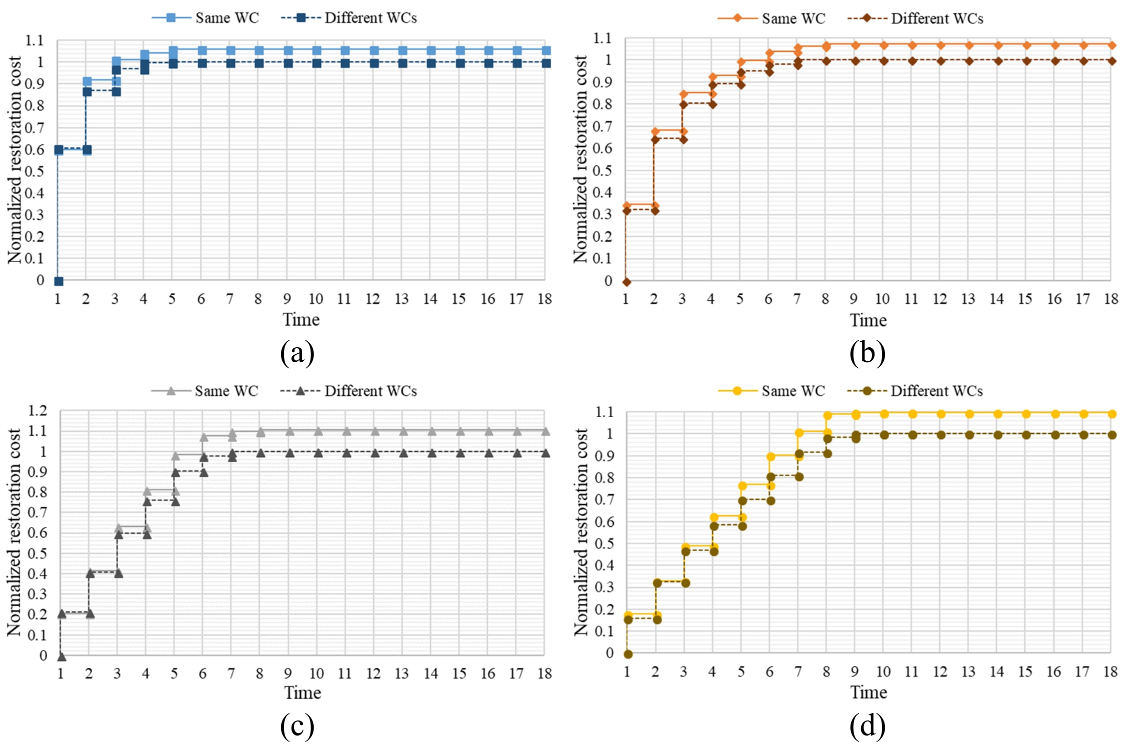


Figure 5-7. Normalized restoration cost considering different work crew assignment strategies with hypothetical earthquake scenarios of magnitude (a) M_w= 6, (b) M_w = 7, (c) M_w = 8, and (d) M_w = 9

4.6.4 Recovery Process

There are two different cases for the recovery process of the disrupted network component regarding the work crews: (i) preemptive recovery, and (ii) non-preemptive recovery. The proposed optimization model for solving the INRP considers the preemptive recovery process, where a work crew can move from one disrupted component to another in different time periods without having achieved full restoration of the previous disrupted component (e.g., a work crew can work on the restoration of node $i \in N'^k$ in network $k \in K$ at time $t \in T$ and then work on the restoration of node $j \in N'^k$ in network $k \in K$ at time $t + 1 \in T$). However, for a non-preemptive recovery process, a work crew is not allowed to move from a disrupted component to another unless they complete the restoration of the previous one. To consider a non-preemptive recovery process with the assumption that the disrupted network components need to reach a desired level of functionality, new parameters are added to represent the recovery durations of the disrupted components to reach a desired level of recovery. Hence, m_i^k is the recovery duration for node $i \in N'^k$ in network $k \in K$ (i.e., $m_i^k = \lceil (\zeta_i^k - y_{i0}^k)/\gamma_{it}^{kr} \rceil$) where $\zeta_i^k \in [0,1]$ is the desired level of functionality for node $i \in N'^k$ in network $k \in K$ (i.e., $\zeta_i^k \geq y_{i0}^k$). Likewise, n_{ij}^k is the recovery duration for link $(i,j) \in L'^k$ in network $k \in K$ (i.e., $n_{ij}^k = \lceil (\eta_{ij}^k - z_{ij0}^k)/\delta_{ijt}^{kr} \rceil$) where $\eta_{ij}^k \in [0,1]$ is the desired level of functionality for link $(i,j) \in L'^k$ in network $k \in K$ (i.e., $\eta_{ij}^k \geq z_{ij0}^k$). Since the proposed optimization model is dealing with time periods for the restoration duration of the disrupted components, the recovery durations for the disrupted nodes and links (i.e., m_i^k and n_{ij}^k, respectively) are rounded up to the nearest integer value. Moreover, constraint (5-8) is replaced by constraint (5-35) with the consideration of the recovery tasks assignment constraints in Section 5.3.

$$\sum_{i \in N'^k} \sum_{l=t}^{\min\{\tau, t+m_i^k-1\}} v_{it}^{kr}/m_i^k + \sum_{(i,j) \in L'^k} \sum_{l=t}^{\min\{\tau, t+n_{ij}^k-1\}} w_{ijt}^{kr}/n_{ij}^k \leq 1, \qquad \forall k \in K, t \in T, r \in R^k \tag{5-35}$$

Similar to the result in Section 5.3, the interdependent network resilience measure is very similar for the two different recovery process assumptions due to the number of units being equal to 1 (i.e., a_i^k, b_{ij}^k =1). Figure 5-8 shows the restoration cost considering the two different cases for the recovery process (preemptive and non-preemptive recovery processes) normalized by the restoration cost resulting from the original preemptive assumption. Hence, considering a preemptive recovery assumption during the recovery process could lead to a lower restoration cost over time, as shown in Figure 5-8. Moreover, the difference in the restoration cost of the two assumptions by the work crew is small as each of the disrupted network components in this example has one unit only (i.e., a disrupted network component cannot be operational unless it is completely restored).

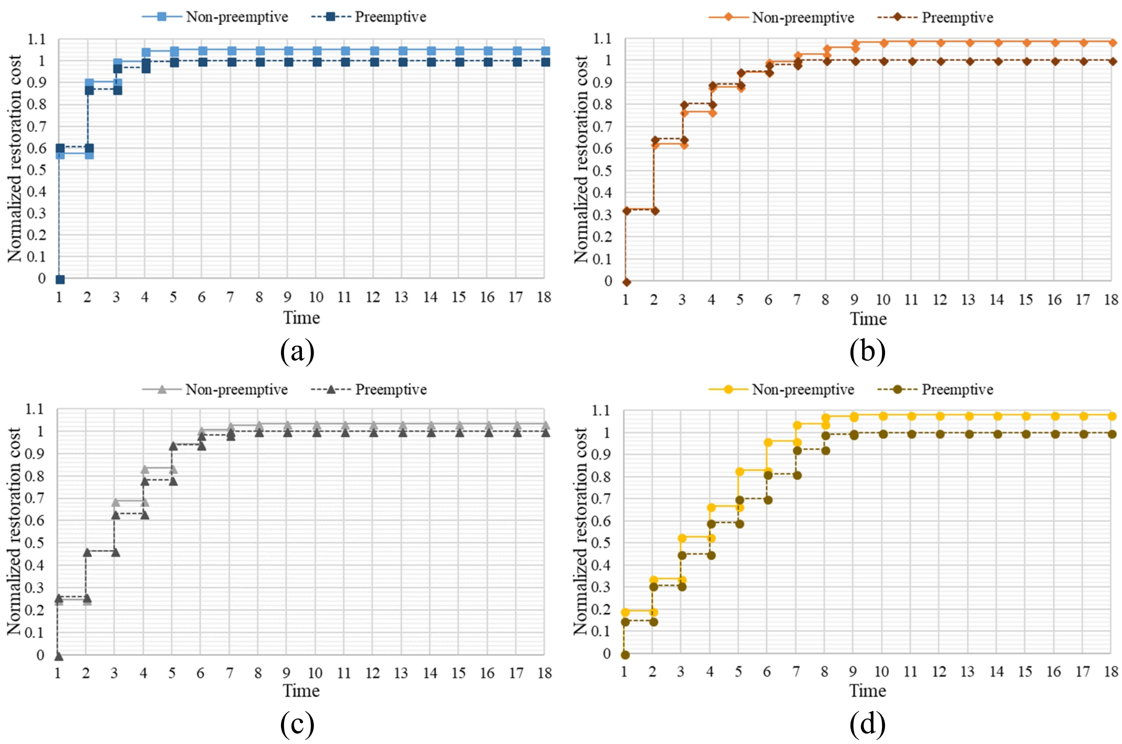


Figure 5-8. Normalized restoration cost considering different assumptions for the recovery process with hypothetical earthquake scenarios of magnitude (a) M_w= 6, (b) M_w = 7, (c) M_w = 8, and (d) M_w = 9

On the other hand, when the disrupted network components have multiple units each (i.e., they can be functioning partially), the difference in the restoration cost could be substantial. In addition, the resilience measure for the system of interdependent infrastructure networks could be different when considering preemptive and non-preemptive assumptions since the disrupted network components could be functioning partially or have some partial recovery.

Chapter 6 : Restoring Community Structures in Interdependent Networks

6.1 Introduction

Community structures exist in many critical infrastructure networks (e.g., electric power, water distribution, transportation), where each network is partitioned into sets of densely connected components with sparse connections between them. Such community structures are formed in infrastructure networks based on physical connections within each network or their spatial characteristics, among others. In this chapter, we address the restoration problem of community structures in interdependent infrastructure networks following a disruption to enhance their resilience considering the interdependencies among the infrastructure networks.

Several approaches are provided in the literature to identify community structures in networks [Fortunato 2010]. In this work, we consider the Fast Modularity algorithm proposed by Clauset et al. [2004], which is available in the *igraph* library in the R coding platform.

6.2 Community Structures Restoration Model

We extend the proposed restoration model, discussed in Section 4.2, to account for the community structures of interdependent infrastructure networks. However, the new model (i.e., community structure restoration model, CSRM) deals with a single objective of maximizing the resilience of the interdependent infrastructure networks.

6.2.1 Assumptions

There are several assumptions and considerations for the proposed community structure restoration model:

- Each infrastructure network consists of a set of community structures, which could be of different sizes, identified according to the Fast Modularity algorithm.
- Each community structure in each infrastructure network consists of a set of components (nodes and links) that are subjected to be completely disrupted.
- Community structures within an infrastructure network are connected through different number of inter-community links, with known capacities, which are subjected to be completely disrupted.
- Each disrupted component in each infrastructure network can be restored with different restoration durations (i.e., recovery durations are not fixed for all disrupted components).
- Each disrupted component in each infrastructure network cannot be operational until it is completely restored (i.e., this model does not consider partial functioning).
- A single work crew can work on restoring a single disrupted network component at a time, where they cannot leave the disrupted component until it is completely restored (i.e., this model considers a non-preemptive recovery process)
- Each supply node, demand node, and link in community structure in each infrastructure network has a known supply capacity, demand, and flow capacity, respectively.
- The flow costs through each link, unmet demand penalties, and restoration costs for disrupted components in community structure in each infrastructure network are known and fixed.
- The physical interdependence among different infrastructure networks is considered. That is, for a node in a community structure in an infrastructure network

to be operational, it requires a specific node in a community structure in another infrastructure network to also be operational.

- The number of available work crews for each infrastructure network (i.e., infrastructure-specific resources) for the restoration of its disrupted components is known and could be different from one infrastructure network to another.

6.2.2 Notation

The sets, parameters, and decision variables of the proposed optimization model to solve the INRP are shown in Table 6-1, Table 6-2, and Table 6-3, respectively.

Table 6-1. Sets of the proposed CSRM

T	Time periods in the restoration horizon, $T = \{1, \dots, \tau\}$
K	Interdependent infrastructure networks, K
R^k	Available resources for network $k \in K$
C^k	Community structures in network $k \in K$
N^{ck}	Nodes in community structure $c \in C^k$ in network $k \in K$
L^{ck}	Links in community structure $c \in C^k$ in network $k \in K$
N_s^{ck}	Supply nodes in community structure $c \in C^k$ in network $k \in K$, $N_s^{ck} \subseteq N^{ck}$
N_d^{ck}	Demand nodes in community structure $c \in C^k$ in network $k \in K$, $N_d^{ck} \subseteq N^{ck}$
N'^{ck}	Disrupted nodes in community structure $c \in C^k$ in network $k \in K$, $N'^{ck} \subseteq N^{ck}$
L'^{ck}	Disrupted links in community structure $c \in C^k$ in network $k \in K$, $L'^{ck} \subseteq L^{ck}$
Ψ	Interdependent nodes (i.e., $\left((i, c, k), (\bar{\imath}, \bar{c}, \bar{k})\right) \in \Psi$ indicates that node $\bar{\imath} \in N^{\overline{ck}}$ in community structure $\bar{c} \in C^k$ in network $\bar{k} \in K$ requires node $i \in N^{ck}$ in community structure $c \in C^k$ in network $k \in K$ to be operational)

Table 6-2. Parameters of the proposed CSRM for community structure $c \in C^k$ in network $k \in K$

u_i^{ck}	Amount of supply and demand at node $i \in N_s^{ck}$ and node $i \in N_d^{ck}$, respectively
o_{ij}^{ck}	Capacity of link $(i, j) \in L^{ck}$
ω^{ck}	Weight, $\sum_{k \in K} \sum_{c \in C^k} \omega^{ck} = 1$
Q_o^{ck}	Total slacks at all demand nodes in prior to the disruption
Q_d^{ck}	Total slacks at all demand nodes in after the disruption
λ_i^{ck}	Restoration duration of node $i \in N'^{ck}$
π_{ij}^{ck}	Restoration duration of link $(i, j) \in L'^{ck}$

Table 6-3. Decision variables of the proposed CSRM for community structure $c \in C^k$ in network $k \in K$ at time $t \in T$

q_{it}^{ck}	Amount of unmet demand, called slack, at node $i \in N_d^{ck}$
x_{ijt}^{ck}	Amount of flow through link $(i,j) \in L^{ck}$
y_{it}^{ck}	A binary variable that equals 1 if node $i \in N^{ck}$ is operational; and 0 otherwise
z_{ijt}^{ck}	A binary variable that equals 1 if link $(i,j) \in L'^{ck}$ is operational; and 0 otherwise
v_{it}^{ckr}	A binary variable that equals 1 if node $i \in N'^{k}$ is restored by work crew $r \in R^k$; and 0 otherwise
w_{ijt}^{ckr}	A binary variable that equals 1 if link $(i,j) \in L'^{k}$ is restored by work crew $r \in R^k$; and 0 otherwise

6.2.3 Objective

The resilience of the community structures in a system of interdependent infrastructure networks is assumed to be a function of unmet demand (slack), q_{it}^{k}. Accordingly, reducing the slacks to a desired level represents a means to measure the effectiveness of the restoration process. Hence, the resilience of the system of interdependent infrastructure networks, i.e., the objective function of the CSRM, can be represented mathematically by Eq. (6-1).

$$\max \sum_{k \in K} \sum_{c \in C^k} \omega^{ck} \left[\frac{\sum_{t=1}^{\tau} \left[\sum_{i \in N_d^{ck}} q_{i(t-1)}^{ck} - \sum_{i \in N_d^{ck}} q_{it}^{ck} \right]}{Q_d^{ck} - Q_o^{ck}} \right] \tag{6-1}$$

where Q_o^{ck} refers to the total original slacks at all demand nodes in network $k \in K$ at time t_0 and Q_d^{ck} refers to the total slacks at all demand nodes in network $k \in K$ at time t_d following a disruptive event, e^j, as shown in Figure 2-1. Also, $(q_{i(t-1)}^{ck} - q_{it}^{ck})$ determines the recovery at node $i \in N_d^{ck}$ in community structure $c \in C^k$ in network $k \in K$ at time $t \in T$. Hence, $\sum_{i \in N_d^{ck}} \left(q_{i(t-1)}^{ck} - q_{it}^{ck} \right)$ represents the *recovery* of community structure $c \in C^k$ in network $k \in K$ at time $t \in T$ and $(Q_d^{ck} - Q_0^{ck})$

represents the total *loss* in community structure $c \in C^k$ in network $k \in K$ following a disruptive event.

6.2.4 Constraints

Several sets of constraints are considered in the proposed restoration model: (i) network flow constraints, (ii) restoration constraints, (iii) interdependence constraints, (iv) logical link constraints for the network flow with restoration, and (v) constraints governing the nature of the decision variables. All sets of constraints are explained and formulated in the following sections.

6.2.4.1 Network Flow Constraints

For community structure in each infrastructure network, the flow conservation at any (i) supply node, $i \in N_s^{ck}$, (ii) transshipment node, $i \in N^{ck} \backslash \{N_s^{ck}, N_d^{ck}\}$, and (iii) demand node, $i \in N_d^{ck}$ is represented by constraints (6-2), (6-3), and (6-4), respectively. Constraints (6-5) ensure that the flow through link $(i,j) \in L^{ck}$ in community structure $c \in C^k$ in network $k \in K$ at time $t \in T$ does not exceed its capacity.

$$\sum_{(i,j)\in L^{ck}} x_{ijt}^{ck} \leq u_i^{ck}, \quad \forall i \in N_s^{ck}, c \in C^k, k \in K, t \in T \tag{6-2}$$

$$\sum_{(i,j)\in L^{ck}} x_{ijt}^{ck} - \sum_{(j,i)\in L^{ck}} x_{jit}^{ck} = 0, \quad \forall i \in N^{ck} \backslash \{N_s^{ck}, N_d^{ck}\}, c \in C^k, k \in K, t \in T \tag{6-3}$$

$$\sum_{(j,i)\in L^{ck}} x_{jit}^{ck} + q_{it}^{ck} = u_i^{ck}, \quad \forall i \in N_d^{ck}, c \in C^k, k \in K, t \in T \tag{6-4}$$

$$x_{ijt}^{ck} - o_{ij}^{ck} \leq 0, \quad \forall (i,j) \in L^{ck}, c \in C^k, k \in K, t \in T \tag{6-5}$$

6.2.4.2 Restoration Constraints

Work crew $r \in R^k$ in infrastructure network $k \in K$ can work on the restoration of a single disrupted network component, i.e., node $i \in N'^{ck}$ or link $(i,j) \in L'^{ck}$, at time

$t \in T$, as shown in constraints (6-6). Constraints (6-7) and (6-8) ensure that node $i \in N'^{ck}$ and link $(i,j) \in L'^{ck}$, respectively in community structure $c \in C^k$ in network $k \in K$ is operational at time $t \in T$ if it is restored by work crew $r \in R^k$. Constraints (6-9) and (6-10) ensure that node $i \in N'^{ck}$ and link $(i,j) \in L'^{ck}$, respectively in community structure $c \in C^k$ in network $k \in K$ cannot be operational prior to its restoration duration. Similarly, work crew $r \in R^k$ cannot complete the restoration of node $i \in N'^{ck}$ and link $(i,j) \in L'^{ck}$ in community structure $c \in C^k$ in network $k \in K$ prior to its restoration duration, as shown in constraints (6-11) and (6-12), respectively.

$$\sum_{(i,j)\in L'^{ck}} \sum_{l=t}^{\min\{\tau, t+dl_{ij}^{ck}-1\}} z_{ijl}^{ckr} + \sum_{i\in N'^{ck}} \sum_{l=t}^{\min\{\tau, t+dn_i^{ck}-1\}} y_{il}^{ckr} \leq 1, \quad \forall k \in K, r \in R^k, t \in T \tag{6-6}$$

$$y_{it}^{ck} \leq \sum_{r\in R^k} \sum_{l=1}^{t} \gamma_{il}^{ckr}, \qquad \forall i \in N'^k, c \in C^k, k \in K, t \in T \tag{6-7}$$

$$z_{ijt}^{ck} \leq \sum_{r\in R^k} \sum_{l=1}^{t} \delta_{ijl}^{ckr}, \qquad \forall (i,j) \in L'^k, c \in C^k, k \in K, t \in T \tag{6-8}$$

$$\sum_{t=1}^{dn_i^{ck}-1} y_{it}^{ck} = 0, \qquad \forall i \in N'^{ck}, c \in C^k, k \in K \tag{6-9}$$

$$\sum_{t=1}^{dl_{ij}^{ck}-1} z_{ijt}^{ck} = 0, \qquad \forall (i,j) \in L'^{ck}, c \in C^k, k \in K \tag{6-10}$$

$$\sum_{r\in R^k} \sum_{t=1}^{dn_i^{ck}-1} \gamma_{it}^{ckr} = 0, \qquad \forall i \in N'^k, c \in C^k, k \in K \tag{6-11}$$

$$\sum_{r \in R^k} \sum_{t=1}^{dl_{ij}^{ck}-1} \delta_{ijt}^{ckr} = 0, \quad \forall (i,j) \in L'^{ck}, c \in C^k, k \in K \tag{6-12}$$

6.2.4.3 Interdependence Constraints

The physical interdependence among the community structures of different infrastructure networks is captured by constraints (6-13). This set of constraints ensure that for a node $\bar{\iota} \in N^{\overline{ck}}$ in community structure $\bar{c} \in C^k$ in network $\bar{k} \in K$ to be operational at time $t \in T$, node $i \in N^{ck}$ in community structure $c \in C^k$ in network $k \in K$ must be operational at time $t \in T$ as well, where $\left((i, \mathrm{c}, k), (\bar{\iota}, \bar{c}, \bar{k})\right) \in \Psi$.

$$y_{\bar{\iota}t}^{\overline{ck}} - y_{it}^{ck} \leq 0, \quad \forall \left((i,c,k), (\bar{\iota}, \bar{c}, \bar{k})\right) \in \Psi, t \in T \tag{6-13}$$

6.2.4.4 Logical Link Constraints of Network Flow to Restoration

The flow through link $(i,j) \in L^{ck}$ in community structure $c \in C^k$ in network $k \in K$ is determined by the capacity of the link as well as the functionality status of the nodes at both ends on that link as shown in constraints (6-14) and (6-15). Furthermore, the capacity of link $(i,j) \in L'^{ck}$ in community structure $c \in C^k$ in network $k \in K$ is determined by functionality status of the link itself which is captured by constraints (6-16).

$$x_{ijt}^{ck} - o_{ij}^{ck} y_{it}^{ck} \leq 0, \quad \forall (i,j) \in L^{ck}, i \in N^{ck}, c \in C^k, k \in K, t \in T \tag{6-14}$$

$$x_{ijt}^{ck} - o_{ij}^{ck} y_{jt}^{ck} \leq 0, \quad \forall (i,j) \in L^{ck}, j \in N^{ck}, c \in C^k, k \in K, t \in T \tag{6-15}$$

$$x_{ijt}^{ck} - o_{ij}^{ck} z_{ijt}^{ck} \leq 0, \quad \forall (i,j) \in L'^{ck}, c \in C^k, k \in K, t \in T \tag{6-16}$$

6.2.4.5 Constraints on the Nature of Decision Variables

The amount of unmet demand (slack), q_{it}^{ck}, at node $i \in N_d^{ck}$ and flow through link $(i,j) \in L^{ck}$, x_{ijt}^{ck}, in community structure $c \in C^k$ in network $k \in K$ must be non-negative at time $t \in T$, as shown in constraints (6-17) and (6-18), respectively. The functionality status of node $i \in N^{ck}$ and link $(i, j) \in L'^{ck}$ in community structure $c \in C^k$ in network $k \in K$ at time $t \in T$ is represented by constraints (6-19) and (6-20), respectively. Finally, constraints (6-21) and (6-22) represent the binary restoration variables for node $i \in N'^{ck}$ and link $(i, j) \in L'^{ck}$ in community structure $c \in C^k$ in network $k \in K$ at time $t \in T$, respectively.

$$q_{it}^{ck} \geq 0, \quad \forall i \in N^k, c \in C^k, k \in K, t \in T \tag{6-17}$$

$$x_{ijt}^{ck} \geq 0, \quad \forall (i, j) \in L^k, c \in C^k, k \in K, t \in T \tag{6-18}$$

$$y_{it}^{ck} \in \{0,1\}, \quad \forall i \in N^k, c \in C^k, k \in K, t \in T \tag{6-19}$$

$$z_{ijt}^{ck} \in \{0,1\}, \quad \forall (i, j) \in L'^k, c \in C^k, k \in K, t \in T \tag{6-20}$$

$$\gamma_{it}^{ckr} \in \{0,1\}, \quad \forall i \in N'^k, c \in C^k, k \in K, t \in T, r \in R^k \tag{6-21}$$

$$\delta_{ijt}^{ckr} \in \{0,1\}, \quad \forall (i, j) \in L'^k, c \in C^k, k \in K, t \in T, r \in R^k \tag{6-22}$$

6.3 Community Structures Importance Measures

In this section, we propose and discuss multiple community structures importance measures (CSIMs) that could be utilized to prioritize the recovery process of community structures in a system of interdependent infrastructure networks according to multiple different factors. These CSIMs can be categorized into two groups: (i) prior to disruptions CSIMs, that is prioritizing community structures in a system of interdependent infrastructure network without considering the effect of a disruptive event on them, and (ii) post disruptions CSIMs, which consider the effect of a disruptive event on community structures in a system of interdependent infrastructure network when prioritizing them for restoration. The two groups of CSIMs are presented in the following sections.

6.3.1 Prior to disruption CSIMs

6.3.1.1 Inter-Community Links (ICL)

The *inter-community links* CSIM quantifies the importance of a community structure based on the number of links between that community structure and other community structures within the same infrastructure network. ICL is inspired by Rocco and Ramirez-Marques [2011]. The higher value of ICL indicates the more critical the community structure is for its network.

Definition 6.3.1.1. *(ICL)*. The ICL of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{ICL}, is defined in Eq. (6-23), where V_c^k is the set of links between community structure $c \in C^k$ and other community structures in network $k \in K$.

$$I_c^{ICL} = \frac{1}{|V_c^k|}, \qquad V_c^k \neq \emptyset \tag{6-23}$$

6.3.1.2 Interdependency Links (IL)

The *interdependency links* CSIM measures the importance of a community structure based on the number of interdependency links between that community structure and other community structures in other infrastructure networks. IL can be defined as the ratio of the number of interdependency links for a community structure with other community structures in other infrastructure networks to the total number of interdependency links exists for other community structures within the same infrastructure networks. The higher value of IL indicates the more critical the community structure is for the system of interdependent networks.

Definition 6.3.1.2. *(IL)*. The IL of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{IL}, is defined in Eq. (6-24), where W_c is the set of interdependency links between a community structure $c \in C^k$ in network $k \in K$ and other community structures in network $\bar{k} \in K$ where $k \neq \bar{k}$.

$$I_c^{IL} = \frac{|W_c|}{\sum_{c \in C^k} |W_c|} \quad (6\text{-}24)$$

6.3.1.3 Interdependency and Inter-Community Links (IICL)

The *interdependency and inter-community links* CSIM is simply the sum of the normalized scores for each community structure by ICL and IL CSIMs. Hence, the higher value of IICL indicates the more critical the community structure is for the system of interdependent networks.

Definition 6.3.1.3. *(IICL)*. The IICL of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{IICL}, is defined in Eq. (6-25), where $\hat{I}_c^{ICL}$ and $\hat{I}_c^{IL}$ are the normalized scores for community structure $c \in C^k$ in network $k \in K$ by ICL and IL, respectively.

$$I_c^{IICL} = \hat{I}_c^{ICL} + \hat{I}_c^{IL} \quad (6\text{-}25)$$

6.3.1.4 Community Demand (CD)

The *community demand* CSIM prioritizes community structures of an infrastructure network based on the total amount of demands required by each community structure in each network. CD can be defined as the ratio of the demand required by a community structure in an infrastructure network to the total demand required by all community structures within the same infrastructure network. The higher value of CD indicates the more critical the community structure is for its network.

Definition 6.3.1.4. *(CD)*. The CD of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{CD}, is defined in Eq. (6-26), where D_c is the demand units required by community structure $c \in C^k$ in network $k \in K$.

$$I_c^{CD} = \frac{D_c}{\sum_{c \in C^k} D_c} \tag{6-26}$$

6.3.1.5 Community Reduction Worth (CRW)

The *community reduction worth* CSIM is similar to the ЯRW CIM discussed earlier in Section 3.3.2. Hence, CRW can be defined as the ratio of the optimal system resilience at recovery time τ to the optimal system resilience when disrupted network components in a community structure are not recovered at recovery time τ. As this CSIM does not consider an actual disruption size or scenario, we consider a complete disruption of the system of interdependent infrastructure networks and obtain the priorities of community structures accordingly. The higher value of CRW indicates the more critical the community structure is for the system of interdependent networks.

Definition 6.3.1.5. *(CRW)*. The CRW of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{CRW}, is defined in Eq. (6-27), where $Я(\tau)$ is the optimal resilience of the interdependent infrastructure networks at time τ, $e \in E'^{ck} = N'^{ck} \cup L'^{ck}$, and $Я(\tau | \sum_{t \in T} \sum_{e \in E'^{ck}} \mu_{et}^k = 0)$ is the optimal resilience of the interdependent infrastructure networks at time τ when all disrupted network components in community structure $c \in C^k$ in network $k \in K$ are not recovered.

$$I_c^{CRW} = \frac{Я_{sys}(\tau)}{Я_{sys}(\tau | \sum_{t \in T} \sum_{e \in E'^{ck}} \mu_{et}^k = 0)} \tag{6-27}$$

6.3.2 Post disruption CSIMs

6.3.2.1 Community Slacks (CS)

The *community slacks* CSIM prioritizes community structures of a networks based on the total number of unmet demand units (i.e., slack) at each community structure in each infrastructure network following a disruptive event. CS is defined as the ratio of the slack at a community structure in an infrastructure network to the total slack at all community structures within the same infrastructure network. The higher value of CS indicates the more critical the community structure is for its network.

Definition 6.3.2.1. *(CS)*. The CS of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{CS}, is defined in Eq. (6-28), where S_c is the unmet demand units (i.e., slacks) in community structure $c \in C^k$ in network $k \in K$.

$$I_c^{CS} = \frac{S_c}{\sum_{c \in C^k} S_c} \tag{6-28}$$

6.3.2.2 Post Disruption Reduction Worth (PDRW)

The *post disruption reduction worth* CSIM is similar to CRW CSIM with the main difference of accounting for the real size and scenario of a disruptive event. The higher value of PDRW indicates the more critical the community structure is for the system of interdependent networks.

Definition 6.3.2.2. *(PDRW)*. The PDRW of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{CRW}, is defined in Eq. (6-29), where Я(τ) is the optimal resilience of the interdependent infrastructure networks at time τ, $e \in E'^{ck} = N'^{ck} \cup L'^{ck}$, and Я$(\tau | \sum_{t \in T} \sum_{e \in E'^{ck}} \mu_{et}^k = 0)$ is the optimal resilience of the interdependent infrastructure networks at time τ when all disrupted network components in community structure $c \in C^k$ in network $k \in K$ are not recovered.

$$I_c^{PDRW} = \frac{Я_{sys}(\tau)}{Я_{sys}(\tau | \sum_{t \in T} \sum_{e \in E'^{ck}} \mu_{et}^k = 0)} \tag{6-29}$$

6.3.2.3 Weighted Resilience Improvement (WRI)

The *weighted resilience improvement* CSIM ranks the community structures in a network according to ratio of their weighted improvement to the resilience of the system of interdependent infrastructure networks when their disrupted components are restored. That is, the proportional resilience enhancement by restoring the disrupted components in a community structure (i.e., having a fully resilient community structure) is divided by the sum of their recovery durations. The higher value of WRI indicates the more critical the community structure is for the system of interdependent networks.

Definition 6.3.2.3. *(WRI)*. The WRI of a community structure $c \in C^k$ in network $k \in K$, denoted as I_c^{WRI}, is defined in Eq. (6-30), where $Я_{sys}\left(\tau \middle| \sum_{t\in T}\sum_{e\in E'^{ck}} \mu_{et}^k = 1\right)$ is the optimal resilience of the interdependent infrastructure networks at time τ when disrupted components in community structure $c \in C^k$ only are restored, $e \in E'^{ck} = N'^{ck} \cup L'^{ck}$, and $Я(0)$ is the resilience of the interdependent networks after the disruption at time 0 (i.e., when all disrupted network components are not restored)

$$I_c^{WRI} = \frac{Я_{sys}\left(\tau \middle| \sum_{t\in T}\sum_{e\in E'^{ck}} \mu_{et}^k = 1\right) - Я_{sys}(0)}{\sum_{e\in E'^{ck}} r_e} \tag{6-30}$$

6.4 Numerical Experiment

6.4.1 Networks Data

We illustrate our proposed restoration model with fictional interdependent infrastructure networks (i.e., power and water), which are generated using R platform as described earlier in Section 3.4.1. Accordingly, the two interdependent infrastructure networks are generated as illustrated in Figure 6-1. The general properties for each network are shown in Table 6-4.

Table 6-4. General properties of the interdependent infrastructure networks

Network	$\boldsymbol{N}$	$\boldsymbol{L}$	$\boldsymbol{N^s}$	$\boldsymbol{N^d}$	$\langle \boldsymbol{deg} \rangle$
Power	60	76	17	24	2.53
Water	40	49	12	15	2.45

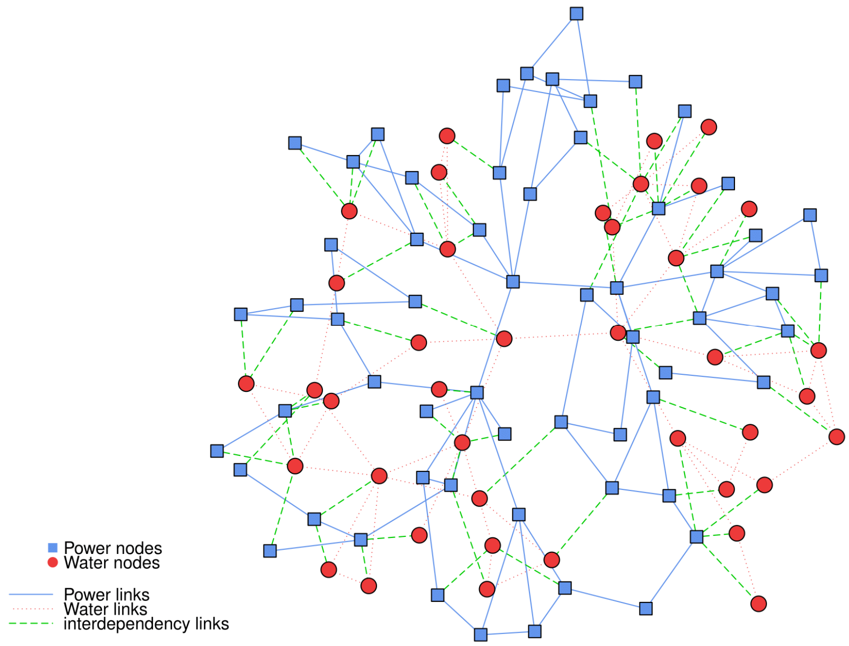
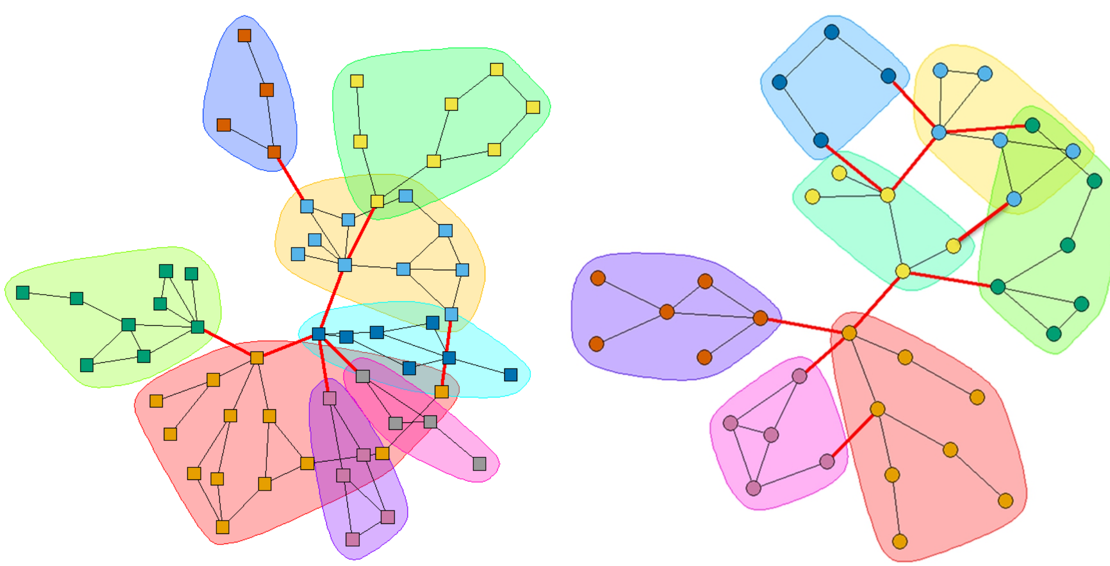


Figure 6-1. An example of interdependent infrastructure networks

Figure 6-2 shows the community structures in the system of interdependent infrastructure networks identifies by the Fast Modularity algorithm using the *igraph* library in the R platform. Accordingly, there are 8 community structures in power network and 7 community structures in water network.



(a) (b)

Figure 6-2. Community structures in (a) power network, and (b) water network

The supply and demand in each community structure in power and water networks are shown in Figure 6-3. In some community structures, the supply is more than the demand such as community structures 6 and 8 in power network and community structures 2 and 3 in water network, see Figure 6-3. Such community structures could be less affected by a disruptive event than others. On the other hand, the demand in some community structures is more than their supply such as community structures 2 and 5 in power network and community structures 1 and 4 in water network, see Figure 6-3. Hence, such communities could be more affected by affected by a disruption than other community structures.

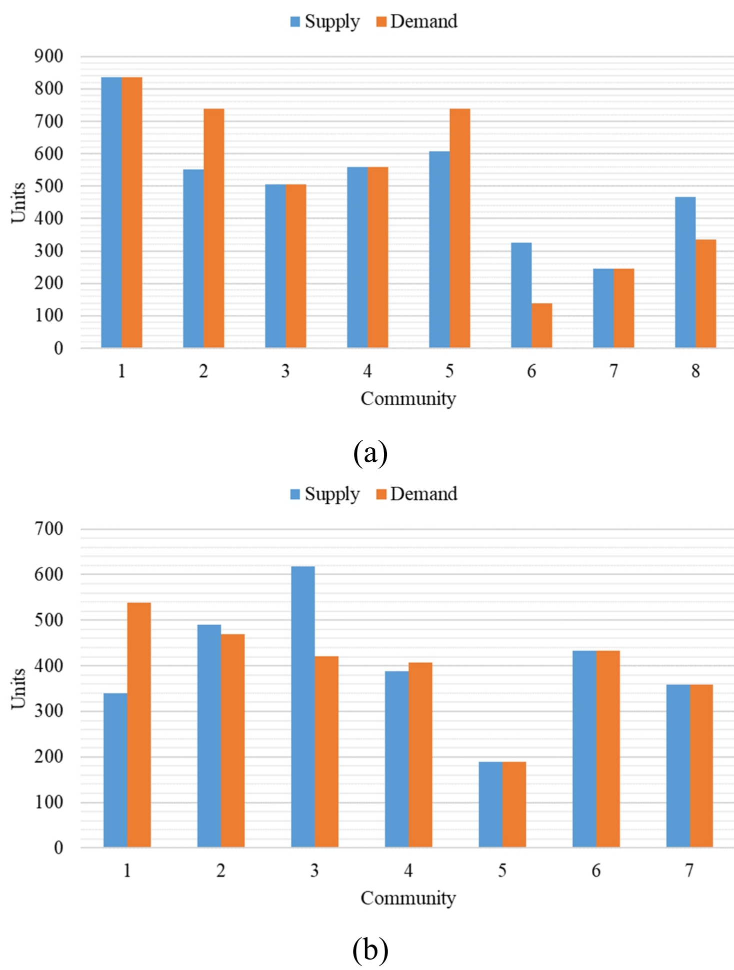
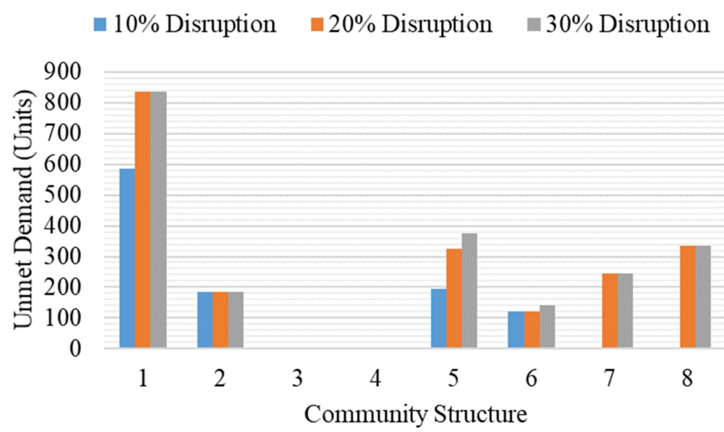


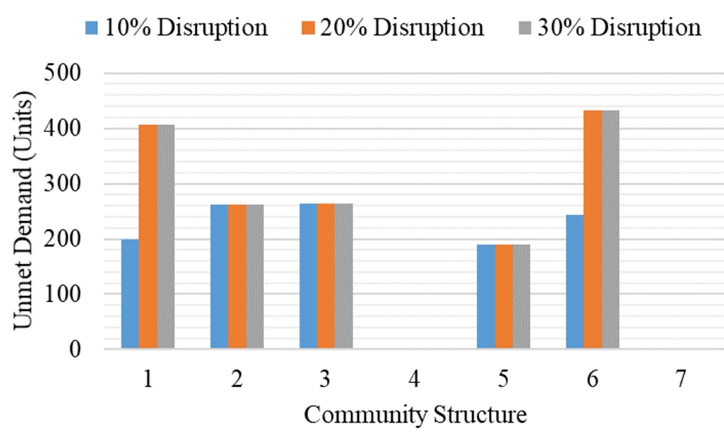
Figure 6-3. Supply and demand in each community structure in (a) power network, and (b) water network

6.4.2 Experimental Results

For illustrative purposes, we consider a spatial disruption scenario with three different disruption sizes (i.e., disrupted network components): 10%, 20%, and 30% of the total number of components (nodes and links) in the system if interdependent in restructure networks. Consequently, the unmet demand after disruption in each community structure in both power and water networks is shown in Figure 6-4.



(a)



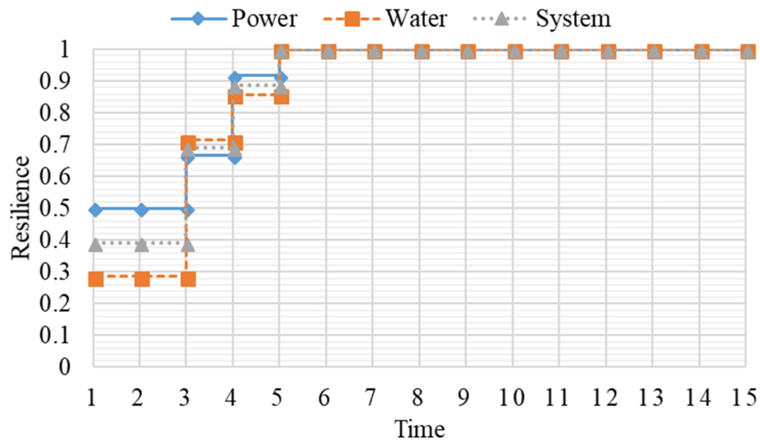
(b)

Figure 6-4. Unmet demand after disruption in each community structure in (a) power network, and (b) water network

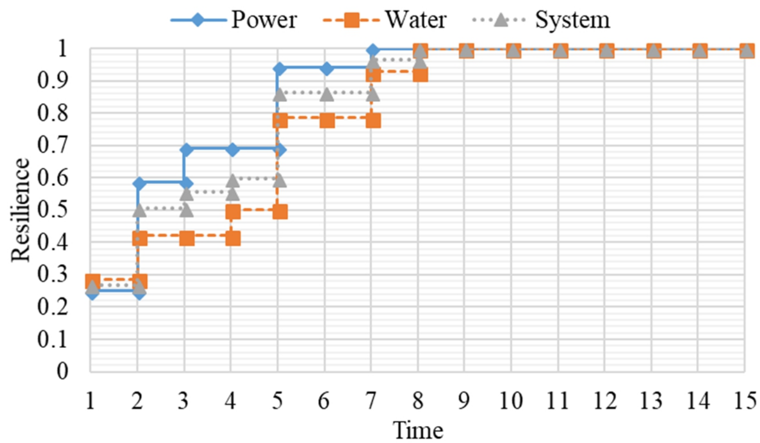
As it can be observed from Figure 6-4, some community structures were not affected by any considered disruption scenario though there are in different sizes such as community structures 3 and 4 in power network and community structures 4 and 7 in water network, see Figure 6-4. This could be due to their location since we are considering spatial disruption scenario; hence they might be far from the disruption location. On the other hand, some community structures were having the same affect after disruptions regardless of their sizes such as community structures 2 in power network and community structures 2, 3, and 5 in water network, as shown in Figure 6-4. Other community structures were affected differently after different disruptions due to their different sizes, as illustrated in Figure 6-4.

6.4.2.1 CSRM

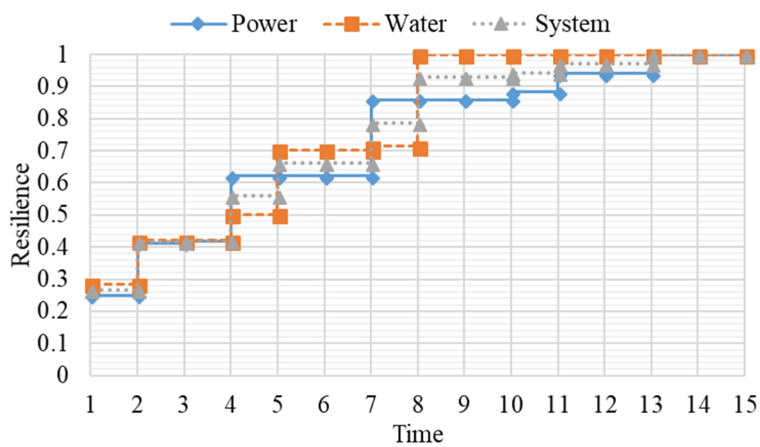
As for CSRM, Figure 6-5(a), (b), and (c) show the trajectory of resilience improvement, obtained when apply CSRM using Python 2.7 with Gurobi 7.5, for both power and water networks and the system of interdependent infrastructure networks considering different sizes of spatial disruptions, i.e., 10% spatial disruption, 20% spatial disruption, and 30% spatial disruption, respectively, with the availability of six work crews. As expected, as the disruption size increases, the time to reach to fully resilient system of interdependent infrastructure network increases, see Figure 6-5. In some cases, the power network reaches to a fully resilient network status before the water network, as shown in Figure 6-5 (a) and (b) considering 10% and 20% spatial disruption sizes. However, when considering 30% spatial disruption size, the water network to a fully resilient network status before the power network which could be resulted due to two factors: (i) the interdependency between both networks, and (ii) the recovery durations of the disrupted components in both networks, given the same number of work crews are considered to be available for all disruption scenarios.



(a)



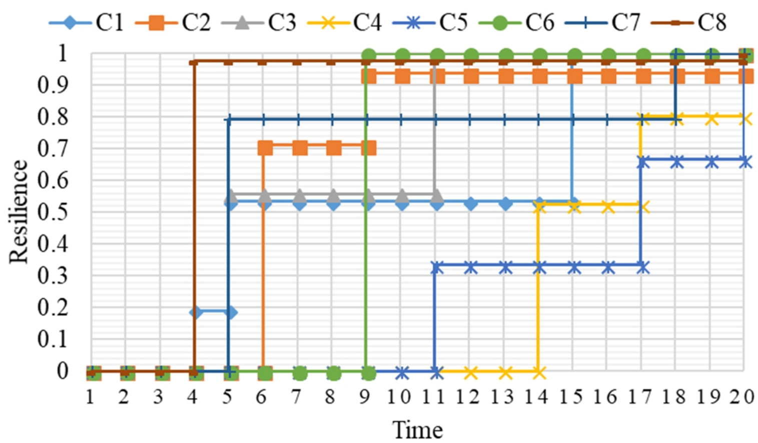
(b)



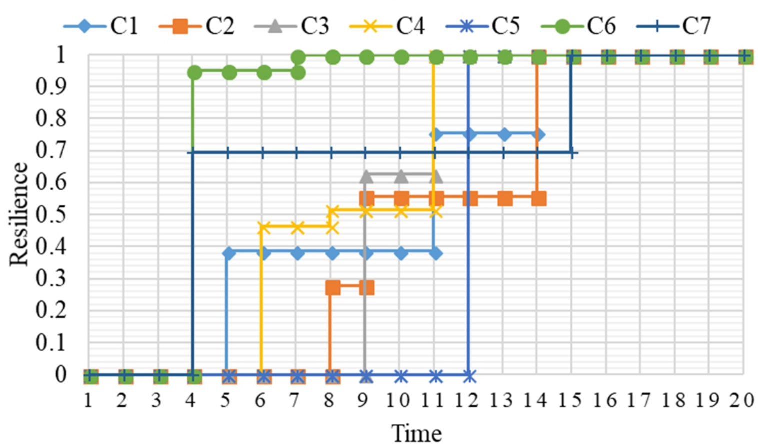
(c)

Figure 6-5. Network resilience considering (a) 10%, (b) 20%, and (c) 30%, spatial disruption

To observe the effect of a disruptive event on each community structure in each network and how long does it take to be a fully resilient community structure, we consider a complete disruption of the whole system of interdependent network (i.e., 100% disruption). Accordingly, the trajectory of the resilience of each community structure in both power and water network is illustrated in Figure 6-6, where community structure 1 is represented by "C1", and community structure 2 is represented by "C2", and so on.



(a)



(b)

Figure 6-6. Resilience of community structures in (a) power network, and (b) water network, considering 100% network disruption

As shown in Figure 6-6, some community structures reach the status of being fully resilient earlier than other such as community structure 6 in power network and community structure 6 in water network. On the other hand, some community structure longer time than others to be fully resilient such as community structure 5 in power network and community structure 7 in water networks. Hence, there are three factors that could affect the trajectory of the resilience improvement for a community structure: (i) the size of the community structure (i.e., number of components in the community structure), as community structures might be in different sizes as shown earlier in Figure 6-2, (ii) the interdependency between community structures (i.e., within a single network or from different networks), and (iii) the recovery durations of the disrupted components in the community structure.

6.4.2.2 CSIMs

The restoration priorities for all community structures in the generated system of interdependent infrastructure networks were obtained by the multiple prior to and post disruptions CSIMs discussed earlier in Section 6.3. Accordingly, Figure 6-7, Figure 6-8, and Figure 6-9 show the restoration priorities by multiple CSIMs for community structures in both power and water networks considering different spatial disruption sizes (i.e., 10%, 20%, and 30%, respectively).

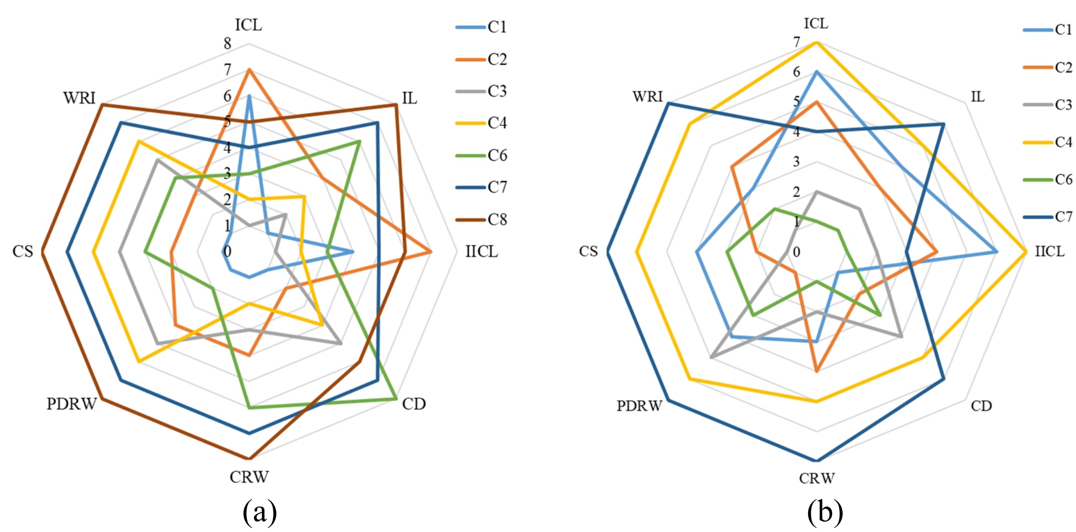
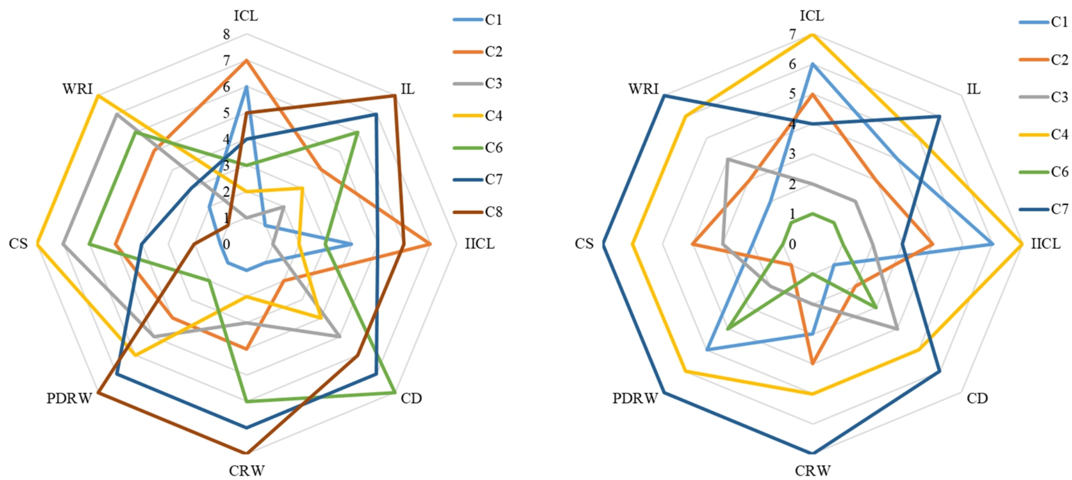
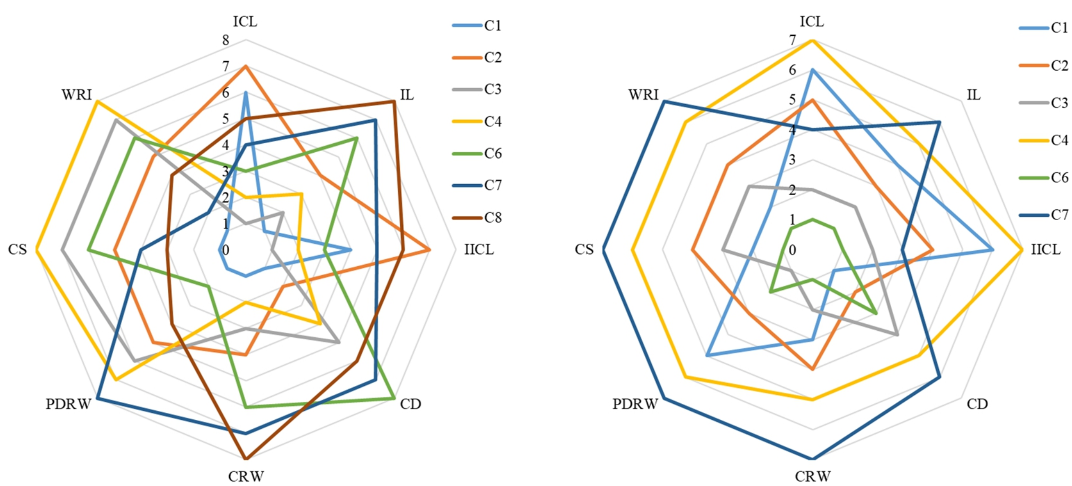


Figure 6-7. Restoration priorities of community structures in (a) power network, and (b) water network by multiple CSIMs considering 10% spatial disruption



(a) (b)

Figure 6-8. Restoration priorities of community structures in (a) power network, and (b) water network by multiple CSIMs considering 20% spatial disruption

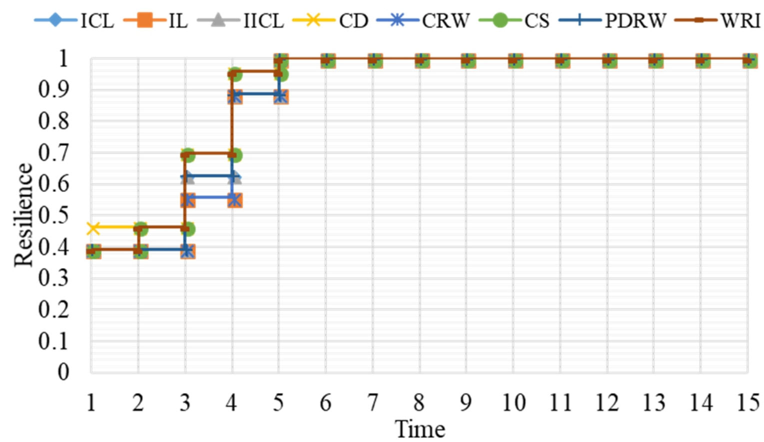


(a) (b)

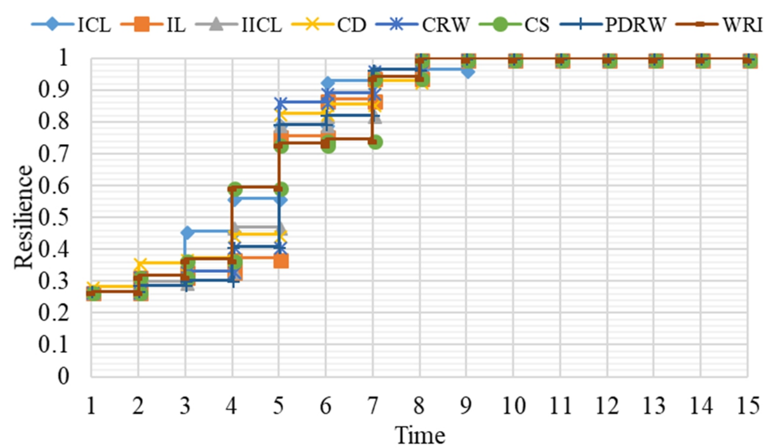
Figure 6-9. Restoration priorities of community structures in (a) power network, and (b) water network by multiple CSIMs considering 30% spatial disruption

For each prior to disruption CSIM, the priorities of the community structures in the system of the interdependent infrastructure networks are the same regardless of the disruption size or scenario as they are not accounted for, see Figure 6-7, Figure 6-8, and Figure 6-9. However, when prioritizing community structures in the system of the interdependent infrastructure networks with post disruption CSIMs, their restoration priorities could be different as such CSIMs account for the disruption affects, as shown in Figure 6-7, Figure 6-8, and Figure 6-9. For example, community structure 8 in power network was ranked 8th (i.e., the least important community structure) by WRI when considering 10% spatial disruption, see Figure 6-7(a). However, it was ranked 1st (i.e., the most important community structure) and 4th by the same CSIM when considering 20% spatial disruption and 30% spatial disruption, respectively, see Figure 6-8(a) and Figure 6-9(a). Similarly, community structure 3 in water network was ranked 5th by PDRW when considering 10% spatial disruption, see Figure 6-7(b). On the other hand, it was ranked 2nd and 1st when considering 20% spatial disruption and 30% spatial disruption, respectively, see Figure 6-8(b) and Figure 6-9(b).

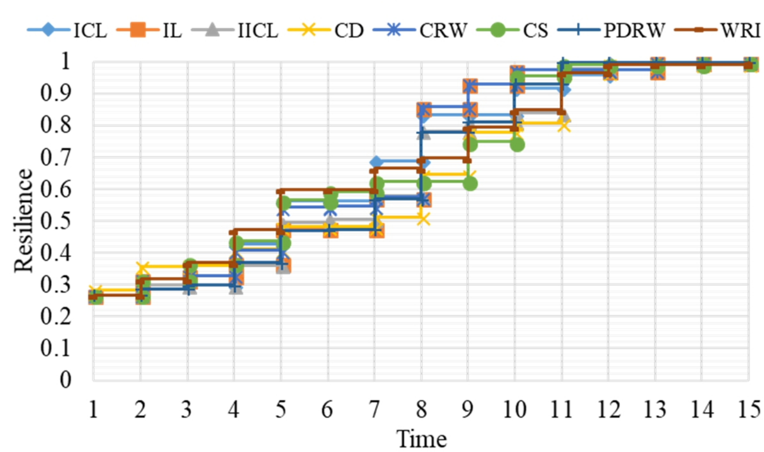
As a solution approach to the CSRM, we consider restoring community structures according to their priorities obtained by the multiple, prior to disruption and post disruption, CSIMs discussed earlier in Section 6.3. Accordingly, the most important community structure by a CSIM in each network in the system of the interdependent infrastructure networks is restored first, then the second most important community structure by the same CSIM is restored second, and the recovery process continues until all the community structures are restored according to their priorities. That is, the recovery process does not start in a community structure until the demands of all community structures with higher importance, considering the same CSIM, are satisfied. Hence, Figure 6-10 shows the trajectory of the resilience of the system of interdependent infrastructure networks when restoring community structures in each network according to their priorities by multiple CSIMs considering different spatial disruption sizes (i.e., 10%, 20%, and 30% spatial disruptions).



(a)



(b)



(c)

Figure 6-10. Network resilience considering community structures priorities by multiple CSIMs with (a) 10%, (b) 20%, and (c) 30%, spatial disruption

Another approach of utilizing the priorities of community structures obtained by the multiple CSIMs discussed earlier is that considering these priorities as the weights of community structures in Eq. (6-1), instead of assuming equal weights for all community structures in each network, and solve the CSRM accordingly. Hence, Figure 6-11 shows the trajectory of the resilience of the system of interdependent infrastructure networks considering different weights for community structures in each network based on their priorities obtained by multiple CSIMs with 30% spatial disruption scenario. That is, the disrupted components in different community structures were restored according to the importance of their community structures by different CSIMs. Accordingly, the trajectory of the resilience considering different restoration priorities by different CSIMs was compared using a common resilience measure, Eq. (4-1), considering equal weights for each network. Hence, considering different weights for community structures results in different trajectory of resilience enhancement of the system of interdependent networks than when assuming equal weights for them, as shown in Figure 6-11, which could be considered by decision makers. In addition, since different CSIMs measure the importance from different perspectives, priorities obtained by them could be combined using a multi-criteria decision-making tool such as TOPSIS [Almoghathawi et al. 2017a] to have a unique rank for the community structures.

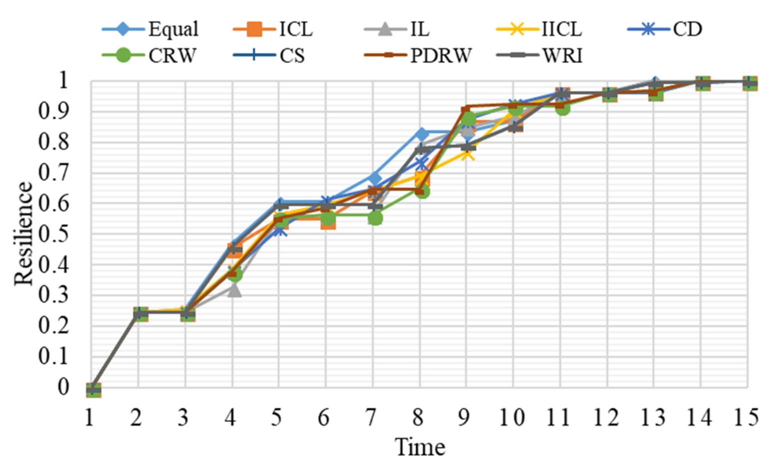


Figure 6-11. Network resilience considering priorities by CSIMs as weights with 30% spatial disruption

Chapter 7 : Conclusion

In this chapter, we summarize the contributions of the work in this dissertation and present some recommendations for future work.

7.1 Concluding Remarks

Critical infrastructure networks are the backbone of modern societies, providing the fundamental services that support their continuous operation. However, the ubiquitous nature of such infrastructure networks has made them highly vulnerable due to the different types of interdependencies among them. Moreover, the proliferation of interdependencies among infrastructure networks has increased the complexity associated with recovery planning after a disruptive event, which becomes a more challenging task for decision makers.

Recognizing the inevitability of large-scale disruptions and their impacts to societies, the research objective of this work is to study the recovery of systems of interdependent infrastructure networks following a disruptive event. Accordingly, the main contribution in this dissertation is developing: (i) importance measures, and (ii) restoration modeling approaches, that enhance the resilience of a system of interdependent infrastructure networks considering the physical interdependency among the infrastructure networks. Though the work in this dissertation discusses systems of interdependent infrastructure networks, the developed importance measures and restoration modelling approaches in this dissertation could be applied to any set of physically interdependent networks.

7.1.1 Component Importance Measures

The goal of the importance measures is to identify the critical network components that influence not only (i) the performance of their networks the most when disrupted and restored, but also (ii) the performance of other networks due to their interdependent nature. Hence, we propose two component importance measures (CIMs), to prioritize the disrupted components of interdependent infrastructure networks based on multiple interdependent networks resilience optimization models using mixed-integer programming (MIP) with the objective of enhancing their resilience considering their interdependences. The purpose of the two proposed CIMs is to (i) quantify the effect of the disrupted components on the resilience of the interdependent infrastructure networks once they are recovered, and (ii) measure the potential impact on the resilience of the interdependent infrastructure networks caused by a specific disrupted network element, respectively, and prioritize the set of disrupted components accordingly. Multiple factors could affect the rank of the disrupted networks components: (i) the nature of the disruptive event, (ii) the set of disrupted components, and (iii) the interdependency between infrastructure networks.

The proposed CIMs represent a useful tool that can help decision makers to identify critical components in their networks following a disruptive event according to their impact on resilience of the system of interdependent network. Hence, managerial decisions guided by this exercise could involve the preplacement of restoration resources near important components, or perhaps investments in hardening or redundancy to reduce the vulnerability of those components prior to a disruption.

7.1.2 Restoration Models

We study the interdependent network restoration problem (INRP), which seeks to find the minimum-cost restoration strategy of a system of interdependent networks following the occurrence of a disruptive event that enhances its resilience considering the availability of time and resources. Accordingly, we propose optimization models using MIP to solve this problem and suggest some solution approaches for large scale disruptions. In particular, the proposed model: (i) prioritizes the restoration of the disrupted components for each infrastructure network, and (ii) assigns and schedule the prioritized networks components to the available work crews, such that the resilience of the system of interdependent infrastructure networks is enhanced considering the physical interdependency among them. The proposed optimization models for solving the INRP consider partial and complete: (i) disruptions for the disrupted network components, (ii) recovery of the disrupted network components, and (iii) dependence between nodes in different networks. Furthermore, four different recovery strategies considering different assumptions regarding work crew assignment and recovery process have been explored. These strategies include: (i) recovery acceleration (i.e., assigning more than one work crew to restore the same disrupted component at the same time), (ii) network component functionality (i.e., recovering a disrupted component partially), (iii) recovery tasks assignment (i.e., assigning the same work crew to recover a disrupted component at any time), and (iv) recovery process (i.e., considering a preemptive or non-preemptive recovery process).

Since the proposed optimization models focuses on enhancing the resilience of the system of interdependent networks to retain their performance level prior to the disruption, not all the disrupted networks components might be restored. In addition, several factors could affect the progress of improvement for the resilience of the system of interdependent infrastructure networks and the total cost associated with the recovery process: (i) the disruption size, (i.e., number of disrupted components in each infrastructure network), (ii) the nature of the interdependencies among the infrastructure networks, and (iii) the available work crews for each infrastructure network during the restoration process (i.e., the number of available work crews, the restoration rate of each work crew). Furthermore, the available time and budget for the restoration process can decide the maximum level of resilience that the system of interdependent infrastructure networks can reach.

7.1.3 Restoring Community Structures

We address the restoration problem of community structures in a system of interdependent infrastructure networks following a disruptive event to enhance their resilience considering the interdependencies among the infrastructure networks. We propose a restoration model using MIP to restore community structures of interdependent infrastructure networks with the objective of maximizing the resilience of the system interdependent infrastructure networks. We also, propose some community structures importance measures (CSIMs) to priorities their restoration process. The proposed CSIM are categorized into two groups: (i) prior to disruption CSIMs, and (ii) post disruption CSIMs. Such measures could be used in solving the restoration model of community structures either as weights for the community structure in the objective function or solving the model by restoring community structures according to their importance.

7.2 Future Research

Multiple recommendations for possible extensions to the work of this dissertation are presented in the coming sections.

7.2.1 Component Importance Measures

The two proposed CIMs consider only the physical interdependency between infrastructure networks, which could be extended to incorporate other interdependencies such as geographical interdependency. Furthermore, the proposed CIMs could be extended to incorporate the parameters uncertainty such as recovery duration, disruption size, supply, demand, available number of work crews, among others. Also, another extension could be considering the restoration up to a specific desired level of performance or resilience, supplies and demands that are a function of time, and other adaptive capacity measures that could assist in meeting demand (e.g., alternate supply units).

7.2.2 Restoration Models

Studying the vulnerability of the components in each infrastructure network could help in identifying the critical ones to reinforce or protect prior to any disruption, thus potentially leading a shorter time to achieve full resilience as well as a lower cost associated with the restoration process. Therefore, a tradeoff between the vulnerability and restoration of the interdependent infrastructure networks could be studied to find the optimal strategy for investment. Moreover, the proposed models could be extended to consider the location of facilities from which work crews dispatch to the locations of their assigned disrupted networks components. Hence, accounting for the movement of the available work crews within the networks. That is, finding the optimal location of these facilities from a set of candidate sites considering the cost of establishing such facilities along with the travel distance and cost for the work crews. Also, the model could be extended to account for the accessibility of the roads. In addition, the proposed models consider only the physical interdependency among infrastructure networks. However, other types of interdependency could be considered such as geographical interdependency. Geographical interdependency could be incorporated in the proposed model by considering the preparation of spaces that are shared by disrupted components from multiple interdependent infrastructure networks prior to the commencement of their restoration activities. Furthermore, instead of assigning the same weight for each infrastructure network to determine the resilience of the system of interdependent networks, a new method could be utilized for trading off one infrastructure network versus another and their weights could be adjusted accordingly. Also, the proposed model could be extended to incorporate the parameters uncertainty. Finally, the proposed models could be extended to quantify objectives related not just to infrastructure resilience but also to the resilience of the communities by considering the vulnerability of the society that interacts with these infrastructure networks.

7.2.3 Restoring Community Structures

In this work, the Fast Modularity algorithm is considered to identify community structure in an infrastructure network. However, other methods based on different interesting aspects could be used such as identifying community structure based on geographical location, social vulnerability index, among others. In addition, other interdependencies could be considered instead of the physical one only. Moreover, the proposed model could be extended to incorporate the parameters uncertainty.

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