251 (2025) 213877

Contents lists available at ScienceDirect

Geoenergy Science and Engineering

journal homepage: www.sciencedirect.com/journal/geoenergy-science-and-engineering

Hybrid ROP modeling: Combining analytical and data-driven approaches for drilling

Ashutosh Sharma [a,*], Tunc Burak [a], Runar Nygaard [a], Espen Hoel [b], Tron Kristiansen [b], Morten Welmer [c]

[a] Mewbourne School of Petroleum and Geological Engineering, University of Oklahoma, OK, USA
[b] Aker BP ASA, Stavanger, Norway
[c] Sekal AS, Stavanger, Norway

ARTICLE INFO

Keywords:
ROP modeling
Machine learning
Real-time drilling
Lithology prediction
Rock strength estimation and drilling optimization

ABSTRACT

Rate of penetration (ROP) modeling has been widely employed to improve drilling efficiency, aiming to reduce both operational costs and risks. This study presents a hybrid model for predicting ROP by combining analytical and data-driven approaches, aimed at enhancing drilling efficiency in challenging formations. The model integrates operational parameters, rock strength properties, and bit design factors, using machine learning (ML) to estimate real-time rock strength at the bit for each depth interval by predicting compressional wave velocity and lithology. These predictions facilitate the calculation of uniaxial compressive strength (UCS) and confined compressive strength (CCS), inputs for ROP estimation. The study included datasets from five wells in the Norwegian Continental Shelf, with four wells used for model training and one for model testing/validation (blind test). The Random Forest regression model achieved an R^2 value of 93% for compressional wave velocity predictions, while the Random Forest classification model attained 96% accuracy in lithology prediction during blind testing. Model validation showed a strong correlation between calculated and measured ROP values, underscoring its accuracy. Sensitivity analysis was performed to evaluate the influence of various parameters, such as weight on bit (WOB), revolutions per minute (RPM), drilling fluid density, flow rate, bit hydraulics, and CCS, highlighting their interdependent effects on ROP. The sensitivity analysis indicated that CCS, WOB, RPM and bit diameter have the greatest impact on ROP. Existing ROP models lack real-time integration of lithology and compressional wave velocity at the bit, limiting their ability to estimate rock strength. This study addresses these gaps by incorporating ML to predict these parameters at each depth interval, enhancing unconfined and confined compressive strength calculations used in the ROP model. The results highlight the importance of optimizing drilling parameters to maximize ROP and operational efficiency, indicating the hybrid model's potential for real-time applications in complex drilling environments.

1. Introduction

One of the primary objectives in drilling is the improvement of the ROP, which directly impacts the effectiveness and cost of the drilling operation. The parameters that affect the ROP are mainly divided into four groups, as shown in Fig. 1. The controllable parameters are, WOB, RPM, drilling fluid density (i.e., mud weight, MW), and drilling fluid flow rate (FR), referred to collectively as operational parameters. The uncontrollable variables include rock strength, formation characteristics, and drilling fluid requirements, which are determined by the characteristics of the subsurface formation to be drilled and borehole stability analysis. In addition, configuration and design of a Polycrystalline diamond compact (PDC) bit, as well as hydraulic factors, play an important role in ROP determination (Mazen et al., 2020). The challenge of drilling optimization is that these parameters have integrated and interdependent effects. For instance, the properties of the drilling fluid affect the bit hydraulics, which then influences the efficiency of cuttings removal and cooling at the bit face. Similarly, rock strength and WOB are interconnected, and both ultimately influence ROP. Since these variables are related to each other in a complex way, it is vital to develop an explicit, correct relationship between ROP and the influencing factors involved within the four categories: operational

* Corresponding author.
E-mail address: ashutosh.sharma@ou.edu (A. Sharma).

https://doi.org/10.1016/j.geoen.2025.213877
Received 17 December 2024; Received in revised form 23 March 2025; Accepted 5 April 2025
Available online 8 April 2025
2949-8910/© 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

Downloaded from SHAREOK. Persistent Handle URL: https://hdl.handle.net/11244/343073

parameters (WOB, RPM, drilling fluid density (MW), FR, UCS and CCS, bit hydraulics (hydraulic horsepower per square inch (HSI), pressure losses at the bit (P_{bit})), and PDC bit profile and design (bit diameter (D_b), cutter side rake (SR) and back rake (BR) angles, PDC bit number of blades (NOB) and PDC bit junk slot area (JSA)). Through the interactions of these influential parameters with ROP, this relationship has become one of the most important fields of study in the last few decades in drilling engineering. This has, in turn, led to the development of both empirical and data-driven models aiming to predict and optimize ROP under variable conditions.

These previously reported ROP models provide pre-drilling planning and post-drilling analysis in optimizing the operations. These models, which were developed either analytically or empirically from drilling laboratories experiments, aim to simulate field conditions for better accuracy (Burak, 2018; Burak and Akin, 2021). The ROP behavior is often characterized to comprise three phases, as shown in Fig. 2. If WOB is increased during Phase I, the rate of penetration would increase slowly. This slow response occurs because the WOB is insufficient to achieve significant depth of cut (DOC), resulting in grinding and inefficient cuttings creation (Sharma et al., 2021). When WOB reaches a critical value, Phase II denoted Effective drilling starts. In phase II drilling is characterized by cutter crushing into the rock and developing a shear failure in the rock in front of the cutter and is chipping the cuttings away (Sharma et al., 2021). A linear relationship develops between WOB and ROP, since the rock no longer resists normal stress, allowing increased WOB to develop higher cutting forces, thus increasing drilling efficiency. Phase II is commonly referred to as the efficient drilling phase. However, beyond the threshold value of WOB, the efficiency of drilling starts falling, beginning Phase III, which corresponds to the Founder point, as shown in Fig. 2. The Founder point is inferred as where cutting removal replaces cutter generation as the limiting factor. This is due to a number of dysfunctions such as poor bit hydraulics, drilling vibrations, and other inefficiencies. At this stage, the system becomes less efficient, and further increases in WOB do not result in proportional gains in ROP (Sharma et al., 2021, 2024; Sharma, 2024).

Fig. 3 outlines a timeline of some of the key analytical ROP models, presented in chronological order. The timeline begins with the Bingham (1965) ROP model, which applies to various drill bit types and offers a simple equation for estimating the rate of penetration (ROP). In the Bingham model, the coefficients indicate the drillability of the formation and determined for each rock type (Bingham, 1965). Although Bingham's model laid the foundation for future ROP models, it is considered outdated due to the advancements in PDC bit design and drilling technologies. The variables, such as bit profile and design, bit wear, hydraulics, and formation characteristics, were not considered in this model, which have since been shown to improve accuracy in ROP modeling.

Bourgoyne and Young (1974) exponential ROP model incorporated eight key factors that influenced drilling performance. Bourgoyne and

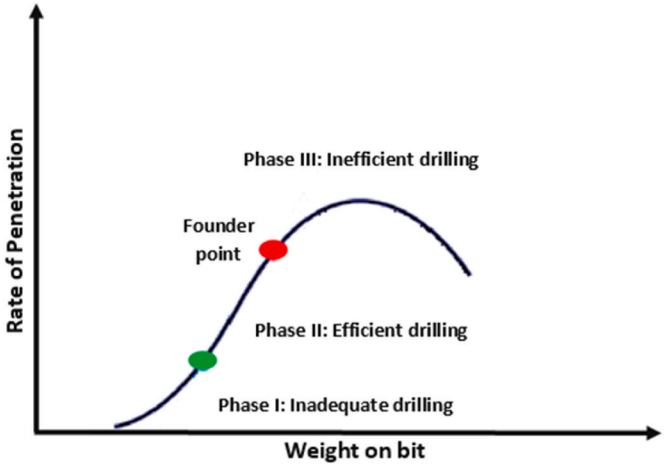


Fig. 2. Three phases of drilling operations are shown. The target region is Phase II (green dot to red dot region), where efficient drilling operations occur. Inefficient drilling operations start with the red dot, also called as founder point, due to drilling dysfunctions and inefficient bit hydraulics (Sharma et al., 2021). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

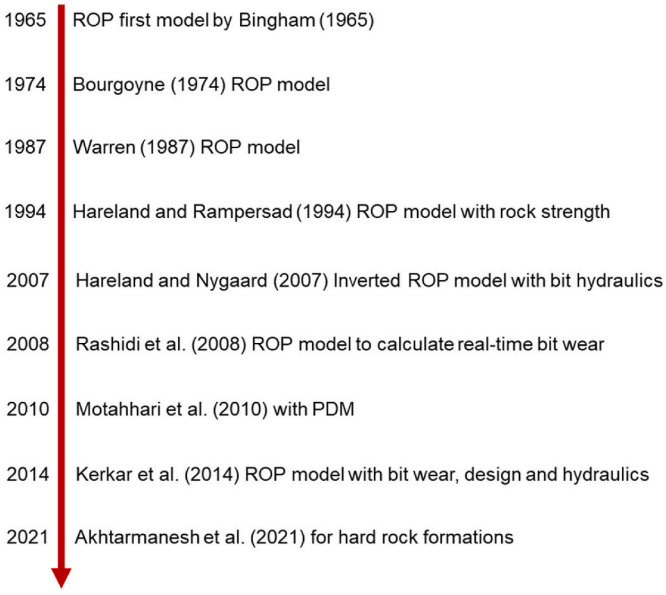


Fig. 3. Timeline for ROP analytical modeling.

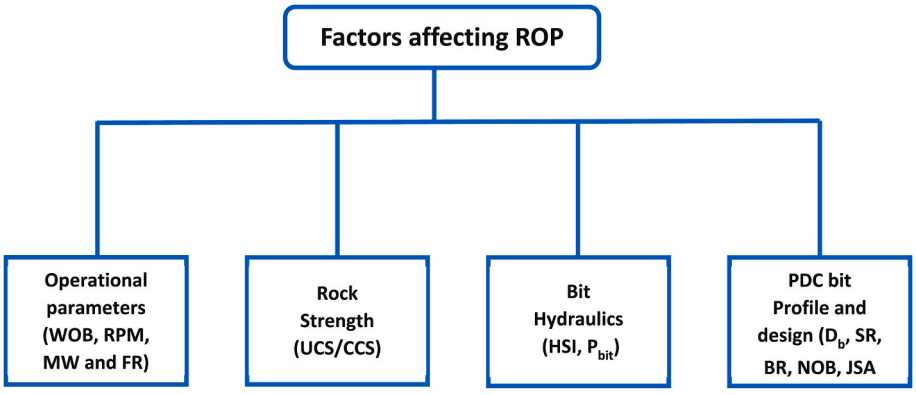


Fig. 1. Different factors and parameters affecting ROP for PDC bits.

Young (1974) provided a general linear equation for the rate of penetration, which accounted for both controllable and uncontrollable variables. They used multiple linear regression analysis to develop a mathematical model for predicting ROP, considering eight independent variables: formation strength, normal compaction, under-compaction, differential pressure, weight on bit, rotary speed, bit teeth wear, and bit hydraulics. This model incorporated all the key parameters to be included to model ROP. Initially, the model was designed for vertical wells drilled with roller-cone bits, and it was later adapted for predicting ROP in deviated wells using PDC bits. They concluded that using simple drilling optimization equations could potentially reduce operational costs by about 10% (Bourgoyne and Young, 1974).

Warren (1987) introduced a ROP model for roller-cone bits that incorporates both chip formation and cutting removal processes. The model explains how ROP is limited by either the generation or removal of cuttings and highlights the impact of high borehole pressure on reducing ROP due to localized and global cleaning effects. Under steady-state drilling conditions, the rate of chip formation is equal to the rate of cuttings removal, which means that ROP is controlled by these two factors. Perfect cleaning model, applicable to tricone bits, assumes that the rate of new chip formation equals the rate of cuttings removal, making ROP dependent on either or both processes. The model uses mechanical factors and generalized response curves but does not account for cuttings-removal inefficiencies, limiting its practical application. Where imperfect cleaning model showed that with increase in depth of the well, ROP often decreases due to factors like differential pressure and changing rock properties. In real-world conditions, the efficiency of cuttings removal becomes critical. To address this, Warren updated the original equation to include additional parameters such as fluid viscosity, bit constants, and jet impact force. This revised model provides a more realistic prediction of ROP, accounting for the imperfect cleaning commonly encountered in field drilling (Warren, 1987).

Hareland and Rampersad (1994) developed a ROP model for single PDC cutters, which linked the rate of penetration to the rock cutting process. The model included correction factors for rock properties, RPM, and WOB but assumed efficient hole cleaning. The model focused on bit design, cutter geometry, formation characteristics, and operating conditions, using data from lab tests, drill-off testing, or field performance. A key feature was its ability to adapt to evolving conditions caused by bit wear, enhancing its utility for real-time optimization and cost-effective bit selection. Its flexibility and ease of use made it a significant advancement in drilling technology, offering strategies for both planning and real-time drilling operations (Hareland and Rampersad, 1994).

Hareland and Nygaard (2007) developed a method for calculating uniaxial compressive rock strength (UCS) using drilling data. They employed Inverted ROP models that incorporated various operational drilling parameters, such as WOB, RPM, and drilling fluid properties, along with formation characteristics. These models were validated using data from both onshore and offshore fields, showing that rock strength estimates derived from drilling data aligned well with traditional methods like sonic logs and laboratory tests. This approach provided a cost-effective means of obtaining continuous rock strength profiles along the wellbore, particularly useful in cases where core samples or logs were unavailable. It also proved valuable for wellbore stability assessments, completion design, and managing sand production (Hareland and Nygaard, 2007).

Rashidi et al. (2008) proposed a method for predicting real-time drill bit wear by combining the concepts of mechanical specific energy (MSE) and drilling strength. The study aimed to improve drilling efficiency and cost by developing a real-time analysis tool that monitors bit wear during drilling operations. The existing MSE model was modified to account for factors such as bit wear and changes in drilling fluid density, which are often not included in standard MSE calculations. The results showed that incorporating MSE into real-time monitoring could help determine the optimal time to pull the bit, potentially enhancing overall drilling economics (Rashidi et al., 2008).

Motahhari et al. (2010) expanded on the Hareland and Rampersad (1994) model to enhance ROP predictions when using Positive Displacement Motors with PDC bits. A key feature of their model is the bit coefficient (G), which incorporates factors such as bit geometry, cutter size, back and side rake angles, and friction between the rock and cutter. They introduced a non-dimensional wear function, W_f, to account for the gradual reduction in ROP caused by bit wear during drilling. The model assumes ideal cleaning conditions, with ROP dependent on bit wear, bit geometry, WOB, and RPM. Additionally, they developed a correlation to estimate confined rock strength based on empirical constants linked to lithology. This approach provides a more dynamic ROP prediction, particularly under variable bit wear and wellbore conditions (Motahhari et al., 2010).

Kerkar et al. (2014) developed a method that uses initial rock core analysis and standard drilling data to estimate UCS and Young's modulus (YM). They combined wellbore friction analysis with a torque and drag model, integrating a ROP model for PDC bits, which accounts for bit wear and force balance. The CCS was calculated and correlated to UCS and YM using regression constants from lab data. The method was applied to horizontal wells in Alberta, Canada, and compared to log data, highlighting potential uncertainties from drilling data. This was the first model which incorporates the combined effect of operational parameters, bit wear, bit profile and design, hydraulics and rock strength on ROP modeling (Kerkar et al., 2014).

Akhtarmanesh et al. (2021) developed an improved ROP model for hard and abrasive formations, particularly in geothermal drilling. Their model identified three drilling phases as shown in Fig. 2, focusing on the transition between inefficient (Phase I) and efficient drilling (Phase II), using parameters like bit geometry, UCS, and bit wear. The model was validated with field data from PDC bits and incorporates empirical correlations for better accuracy in predicting ROP under varying conditions (Akhtarmanesh et al., 2021).

Hybrid ROP modeling has gained significant attention as an effective approach to enhance predictive accuracy and interpretability by integrating physics-based models with ML techniques. Traditional empirical models, such as the Bourgoyne and Young equation, provide a solid theoretical foundation but often struggle with complex nonlinear drilling conditions (Gan et al., 2019). While purely data-driven models offer high accuracy, they lack generalization and interpretability (Ren et al., 2023). Hybrid models address these challenges by combining deterministic and ML approaches, leading to improved performance in adaptability, noise reduction, and predictive accuracy (Hegde et al., 2018; Gan et al., 2019). Anemangely et al. (2018) developed a hybrid model combining a multilayer perceptron neural network with metaheuristic optimization techniques, demonstrating significant improvements in accuracy and convergence speed for ROP prediction. Their study also emphasized the importance of denoising and feature selection in enhancing model reliability. Advanced optimization techniques, including genetic algorithms and artificial neural networks, have further enhanced hybrid model efficiency in real-time drilling applications (Li and Cheng, 2020). Additionally, ensemble learning, error compensation modeling, and weighted averaging have demonstrated superior accuracy in heterogeneous formations (Ren et al., 2023; Zhou et al., 2023). Automated machine learning has also been explored to dynamically optimize hybrid models, ensuring a balance between accuracy and computational efficiency (Ma et al., 2024). These advancements underscore the superiority of hybrid models in ROP prediction, making them an essential tool for modern drilling optimization (Jiao et al., 2024).

Several models were developed in the past to predict ROP using analytical, machine learning, and hybrid methods. Although these models contributed to optimizing drilling performance, they generally did not incorporate real-time predictions of lithology and compressional wave velocity at the bit, which could help in mitigation of sensor offset and improve calculation of uniaxial compressive strength and confined compressive strength at each depth interval. In this study, we introduced

an approach that used machine learning to predict compressional wave velocity and lithology at each depth (Sharma et al., 2023a, b; Burak et al. 2024). These predictions allowed for uniaxial compressive strength and confined compressive strength calculations at the bit through established correlations, which we then incorporated into an ROP prediction model. This method ensured a data-driven, adaptive approach that reflects formation properties at the bit, representing an advancement in ROP modeling and its application in diverse drilling conditions.

2. Field overview

The dataset used for this analysis comes from a field located on the Norwegian Continental Shelf. A total of five well datasets (Well A, Well B, Well C, Well D, and Well E) were used, including both drilling and Logging while drilling (LWD) data. For predicting compressional wave velocity and lithology at the bit, four well datasets (Well A, Well C, Well D, and Well E) were used for model training, while the Well B dataset was reserved for testing/validation as a blind test. Additionally, the 8-½" section of the Well B dataset (2440m–2898m MD) was used for hybrid ROP modeling.

3. Methodology

In this study, a PDC bit ROP model was presented, adopted from Kerkar et al. (2014), which incorporated elements like bit hydraulics, bit wear and blade count, bit profile, operational parameters, and rock strength for a more realistic representation of drilling ROP. The ROP model was represented as:

$$ROP = \left[\frac{A \times WOB^{b} \times \mathrm{RPM}^{c} \times \cos(\mathrm{SR})}{CCS^{d} \times \text{bit diamater} \times \mathrm{Tan}(\mathrm{BR})}\right] \times W_f \times \mathrm{NOB}_f \times \mathrm{hydraulics}_f \quad (1)$$

Where CCS is confined compressive strength, $\mathrm{hydraulics}_f$ is bit hydraulics efficiency coefficient, NOB_f is PDC number of blades function, W_f is bit wear factor, BR and SR are back and side rake angles of PDC bit and A, b, c and d are constants obtained through iteration. Calculating CCS at each depth interval was important for precise ROP calculations in real-time across varying depth intervals. To compute CCS in real-time at the bit, it was necessary to estimate the compressional wave velocity and lithology type at each depth interval. Rock strength was determined using empirical correlations based on compressional wave velocity or travel time data and lithology type, usually derived from laboratory experiments. However, these correlations varied considerably with different lithologies. Additionally, field-recorded compressional wave velocity data presented challenges as it was gathered through sonic logs, which are often not run for top holes. Moreover, compressional wave velocity data obtained through LWD was recorded 20–30 m above the bit, called as sensor offset. Consequently, even if LWD compressional wave velocity data was available, it might not accurately reflect conditions at the bit, making real-time CCS calculations at the bit infeasible. For example, if the sonic sensor was positioned 30 m above the bit and drilling ROP was 10 m per hour, it would take 3 h for sensor to reach the drilling depth due to this offset. Therefore, predicting sonic log values and lithology type in real-time was needed to mitigate sensor offset challenges and achieve a reliable rock strength estimate at the bit for each depth interval. Machine learning algorithms were used to predict lithology and compressional wave velocity at the bit. The predicted compressional wave velocity and lithology data were used to estimate rock strength at the bit, as outlined in subsequent sections. Also, the

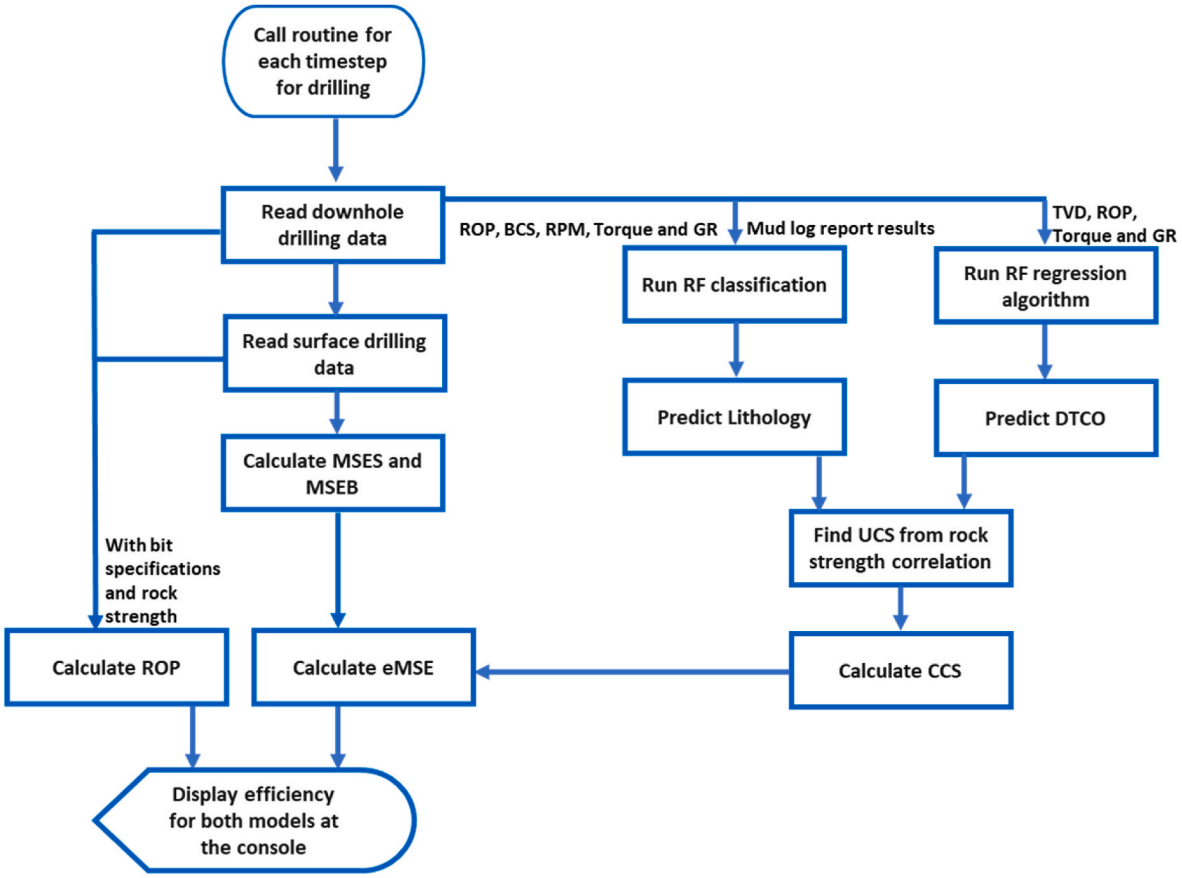


Fig. 4. Steps involved in ROP modeling.

same methodology to calculate rock strength has been followed by (Burak, 2025; Burak et al. 2025a, 2025b) to estimate borehole stability in real-time. The ROP modeling using Equation (1) was conducted in six steps, as shown in Fig. 4 and detailed in sections 3.1 to 3.6.

3.1. Predict compressional wave velocity (V_p) at the bit

In a prior study Sharma et al. (2023a) conducted in the same field, machine learning (ML) models were explored for real-time prediction of compressional wave velocity (*Vp*) at the bit using drilling and logging-while-drilling (LWD) data. Four well datasets were used to develop the ML model, while one dataset (Well B) was reserved for testing/validation through a blind test. The aim was to improve the accuracy and efficiency of travel time predictions during drilling operations. Various regression models were evaluated based on error metrics, including Multi-Linear Regression (MLR), K-Nearest Neighbor (KNN), Support Vector Machines (SVM), and Random Forest (RF).

Input parameters for the regression models were selected through feature engineering to ensure relevance to real-time applications. These included true vertical depth, rate of penetration, torque, and gamma ray. The input parameters for the algorithm were selected with careful consideration of their proximity to the bit. Since the methodology proposed in this study focuses on real-time implementation, it was crucial that the algorithm relies on data originating directly from the bit. Unlike previous studies such as Olayiwola and Sanuade (2021), which used only wireline log data to predict compressional and shear wave velocity for post-drilling analysis, our approach integrates both drilling and near-bit log parameters to predict compressional wave velocity at the bit in real time.

The 8-½" reservoir section of Well B was selected for the prediction of compressional wave velocity and lithology. The RF regression model, developed using the four well datasets, demonstrated superior performance when tested on the 8-½" section of Well B, achieving an R^2 value of 93%. The robustness of RF lies in its ability to handle noisy and missing data, making it ideal for real-time drilling applications. This predicted compressional wave velocity was subsequently utilized in this study to calculate CCS at each depth interval.

3.2. Prediction of lithology at the bit

In a prior study conducted in the same field by Burak et al. (2024), multiple ML algorithms were developed and tested to predict lithology in real-time using a combination of petrophysical and drilling parameters. Four well datasets were used for model training, while the Well B dataset was reserved for testing/validation as part of a blind test.

Among the methodologies developed, the combination of manual labeling with the Random Forest classification method proved to be the most effective, achieving an accuracy rate of 96% for the 8-½" reservoir section of Well B. The manual labeling approach involved using lithology data from mud log reports to train the model, which provided accurate lithology data in the dataset for classification.

This approach, distinct from conventional clustering-classification techniques, employed ML algorithms such as KNN, RF, SVM, Multi-Layer Perceptron, and Deep Learning without relying on clustering. The RF model's accuracy and ability to generalize to unseen data demonstrated its effectiveness for lithology prediction compared to traditional petrophysical methods and other ML-based approaches. The same RF classification model was applied in this study to ensure lithology predictions at the bit for each depth interval, which were critical for real – time lithological identification and rock strength estimation.

3.3. Calculate UCS and CCS

Following the prediction of compressional wave velocity and lithology at the bit, the next step involved calculation of UCS using the predicted compressional wave velocity and lithology type data. The ML algorithms discussed in Sections 3.1 and 3.2 provided the real-time data necessary for estimating rock strength at the bit (Hareland and Nygaard, 2007).

$$UCS = \left(\frac{1}{t_1 \times (\Delta sonic\ travel\ time - t_2)}\right) + t_3 \tag{2}$$

The constants in the UCS equation were derived from the developed UCS correlation and varied based on the lithology encountered at each depth interval. Once lithology was predicted, the algorithm selects the appropriate constants for that lithology, as outlined in Table 1 below.

CCS to be used in the ROP calculation as shown in Equation (1), is calculated based on correlation mentioned in Hareland and Nygaard (2007) as shown below.

$$CCS = UCS\left(1 + a_s(BHP - PP)^{b_s}\right) \tag{3}$$

Where BHP is bottom hole pressure, PP is pore pressure, and a_s and b_s are constants values assigned for different types of lithology.

3.4. Calculation of bit wear factor

The bit wear factor, W_f, served as a wear function to incorporate the effect of bit wear into the ROP models. W_f was calculated using Equation (4), where ΔBG represented the IADC dull grade bit wear state, reported when the bit was pulled out to the surface and valued between 0 and 8. In this field of study, the reported cumulative bit wear (ΔBG) for the 8-½" section was 2. A new or "green" bit, with 0 cumulative bit wear, had W_f equal to 1, while a fully worn bit had a W_f value of 0. In this equation, ΔBG was calculated as a function of normalized WOB, RPM, CCS, and formation abrasiveness (ABR) for in-depth (d_{in}) to out-depth (d_{out}) of the bit run. All four parameters were normalized to their maximum values within the equation. The constant C_a referred to as the bit wear coefficient, was determined through iterative calculations using field drilling data, with the final bit condition recorded upon retrieval, reported as 2 on the scale of 0–8. After the lithology was determined using ML, the formation abrasiveness was assigned for different rock types as mentioned in Table 2. ABR represents the coarseness of the rock and is linked to the quartz mineral content within it. Higher rock abrasivity leads to an increased rate of bit wear.

$$W_f = 1 - \left(\frac{\Delta BG}{8}\right) \tag{4}$$

$$\Delta BG = C_a \sum_{i=d_{in}}^{d_{out}} normalized(WOB_i \times RPM_i \times CCS_i \times ABR_i) \tag{5}$$

3.5. Calculate number of blades factor

The function representing the number of blades (NOB) for the PDC bit was defined as follows:

$$NOB_f = \frac{RPM^{1.02-0.02\times NOB}}{RPM^{0.92}} \tag{6}$$

According to Equation (6), a five-blade bit produced a value of 1 for NOB_f across all RPM values. This NOB_f term rewarded the ROP if the NOB was less than 5 and penalized the ROP when the NOB exceeded 5. With an NOB of exactly 5, NOB_f remained equal to 1 regardless of RPM

Table 1
Experimental constants for different lithology type to compute UCS at the bit (Hareland and Nygaard, 2007).

Lithology	t_1	t_2	t_3
Sandstone	0.0011	50	3.42
Shale	0.0013	50	−2.66
Carbonate	0.0012	50	0.22

Table 2
Abrasiveness coefficient of different rock types.

Lithology Type	Abrasiveness constant
Shale	0.11
Sandstone	0.85
Carbonate	0.57

changes, having no impact on ROP. In simple term, it was observed that NOB_f increased as the NOB decreased, resulting in a higher ROP. However, bits with fewer NOB generally exhibited lower wear resistance, leading to faster bit wear. In this study, NOB for this 8-½" bit is 7.

3.6. Calculation of bit hydraulics factor

Drilling fluids play an important role in drilling operations, with efficient cuttings removal being a key factor influencing overall drilling performance. To advance to Phase II of drilling, it is essential to model the bit hydraulic effect on the overall ROP. Drilling fluids supplied energy at the bit to enhance drilling efficiency, represented as:

$$P_{bit} = \frac{156 \times Drilling\ fluid\ density \times Flowrate^2}{Total\ flow\ area} \tag{7}$$

$$HSI = \frac{\frac{\text{Flowrate} \times P_{bit}}{1714}}{\frac{\pi}{4} \times bit\ diameter^2} \tag{8}$$

where P_{bit} is the pressure loss at the bit, and the total flow area was calculated based on the number of nozzles. In this study, calculations for the 8-½" PDC bit used 5 nozzles of 12/32″ and 2 nozzles of 13/32". The function representing the bit hydraulics for the PDC bit was defined as:

$$hydraulics_f = a_1 \times \left(\frac{\text{HSI} \times \frac{\text{JSA}}{2 \times \text{bit diameter}}}{(clean\ \text{ROP})^{c_1}} \right)^{b_1} \tag{9}$$

where a_1, b_1 and c_1 were coefficients determined through iterative calculations using field drilling data. JSA, the junk slot area, had a value of 25 in^2 for the 8-½" bit size used. Clean ROP was calculated by removing all the constants, bit wear factor, NOB factor and bit hydraulics factor from Equation (1).

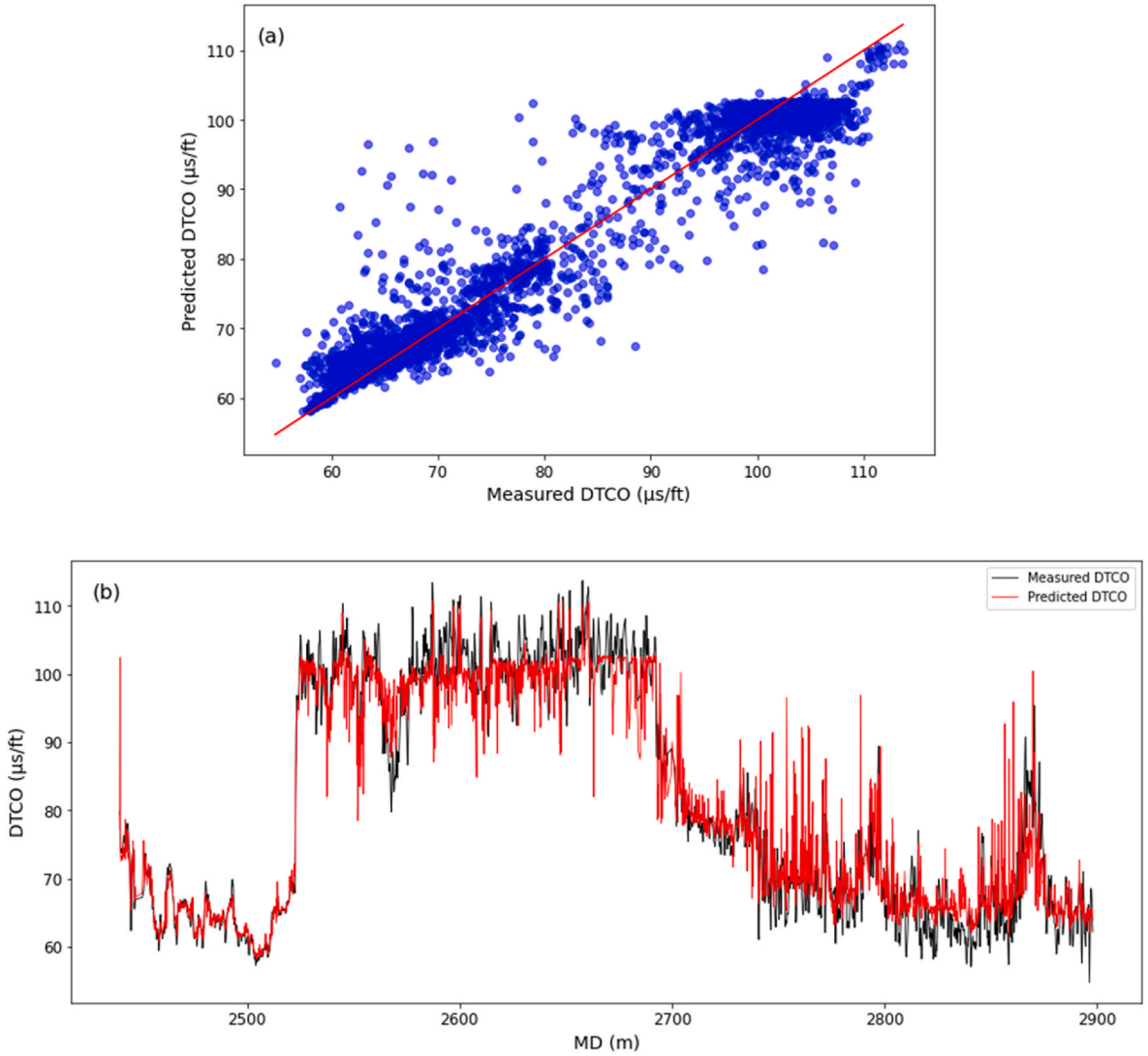


Fig. 5. (a) Scatterplot, (b) Line plot of predicted and measured compressional wave velocity at the bit using Random Forest regression. Four well data were used to train the model and Random Forest regression was used to predict the DTCO for 8-½" well section (R^2 = 93%). The input parameters were drilling parameters and gamma ray.

4. Results

4.1. Compressional wave velocity prediction results

The predicted versus observed compressional wave velocity data for the Random Forest regression model is shown in Fig. 5. The results indicate that the model demonstrates high prediction accuracy, with a close alignment between the predicted and observed data at each depth interval. The RF regression model achieved an R^2 value of 93% for predicting compressional wave velocity.

4.2. Lithology prediction results

A depth-by-depth comparison of the predicted lithologies obtained from the classification algorithms developed with the blind test well data (discussed in section 3.2) is showed in Fig. 6. The RF classification algorithm achieved an accuracy of 96%. The RF classification was not able to detect two thin layers of Carbonate formation around the measured depths of 2545m and 2555m, otherwise RF classification prediction quality was comparatively high when compared to KNN and Deep learning classification methods.

4.3. Rock strength calculation results

Fig. 7 displays several parameters: WOB, RPM, UCS, CCS, Mechanical Specific Energy (MSE) and enhanced MSE (eMSE) plotted against depth. The real-time WOB, RPM, ROP, and MSE data were recorded during drilling, while UCS and CCS values were derived using Equations (2) and (3) based on predicted compressional wave velocity and predicted lithology at the bit. MSE equation use only drilling parameters as shown below.

$$MSE = \frac{4 \times WOB}{D_{bit}^2 \times \pi} + \frac{480 \times RPM \times Torque}{D_{bit}^2 \times ROP} \tag{10}$$

where D_{bit} is bit diameter. To calculate MSE on a universal scale and comparable for a particular lithology type, MSE value needs to be normalized with the rock strength value as shown below. When MSE value is normalized with CCS value, it is called as enhanced MSE (*eMSE*) and eMSE has been plotted along with other drilling parameters, calculated UCS and calculated CCS.

$$eMSE = \frac{\text{MSE}}{CCS} \tag{11}$$

4.4. Hybrid ROP modeling results

Fig. 8 shows the validation results for the ROP model developed in the previous Section 3, applied to a 8-½" bit section from the Well-B drilled in the Norwegian continental shelf. This verification used drilling data specifically from this section for validation purposes. Machine learning algorithms outlined in Sections 3.1 and 3.2 provided the essential data for estimating rock strength properties at the bit for each depth interval, enabling a precise ROP calculation using Equation (1). The process, detailed in Sections 3.1 to 3.6, allowed for a comparison between measured and calculated ROP, highlihting the effectiveness of the hybrid ROP model. The comparison in the plot demonstrates alignment between model predictions and actual measurements, underscoring the model's potential for real-time applications in challenging drilling environments. The model was able to capture most of the variations as indicated in the measured ROP.

4.5. Sensitivity analyses

Sensitivity analyses were performed to understand the developed ROP model and examine the effects of various parameters influencing ROP. This analysis evaluated the impact of operational, rock strength,

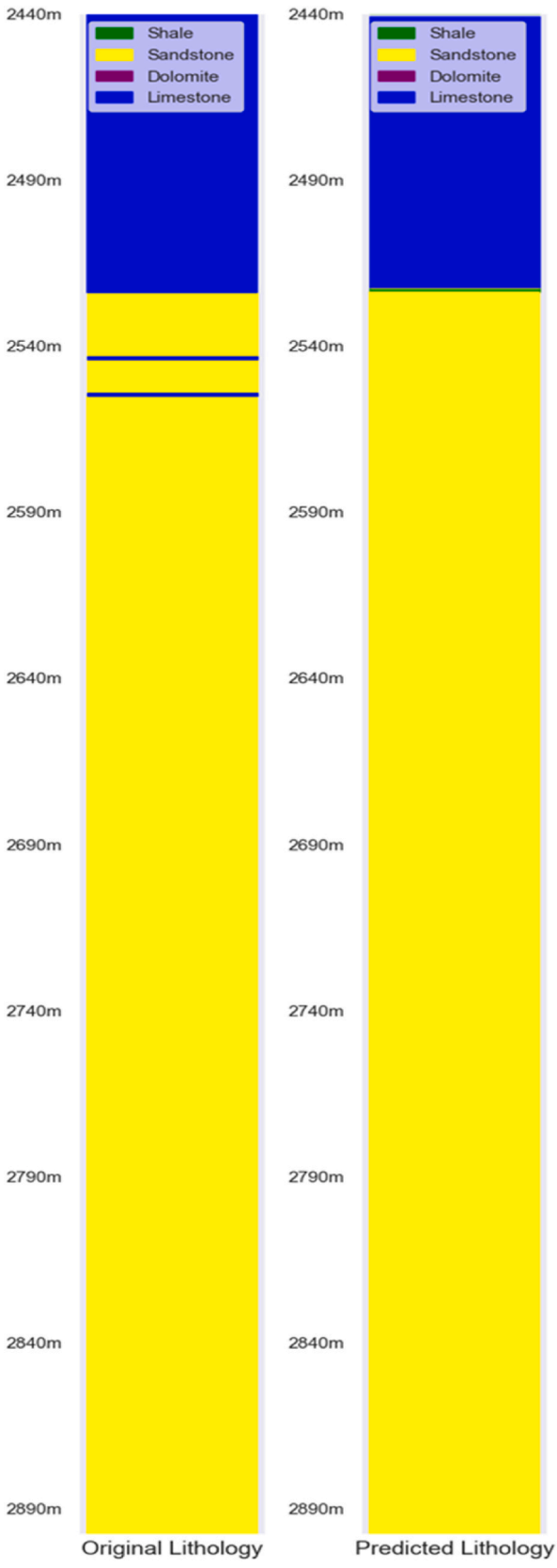


Fig. 6. Lithology classification from manually labeled data and Gamma ray for the 8-½" section of the well B (blind testing). The left column shows the original lithofacies of the 8-½" section of the Well B, and the right column represents results for predicted lithology using Random Forest classification with manual labeling.

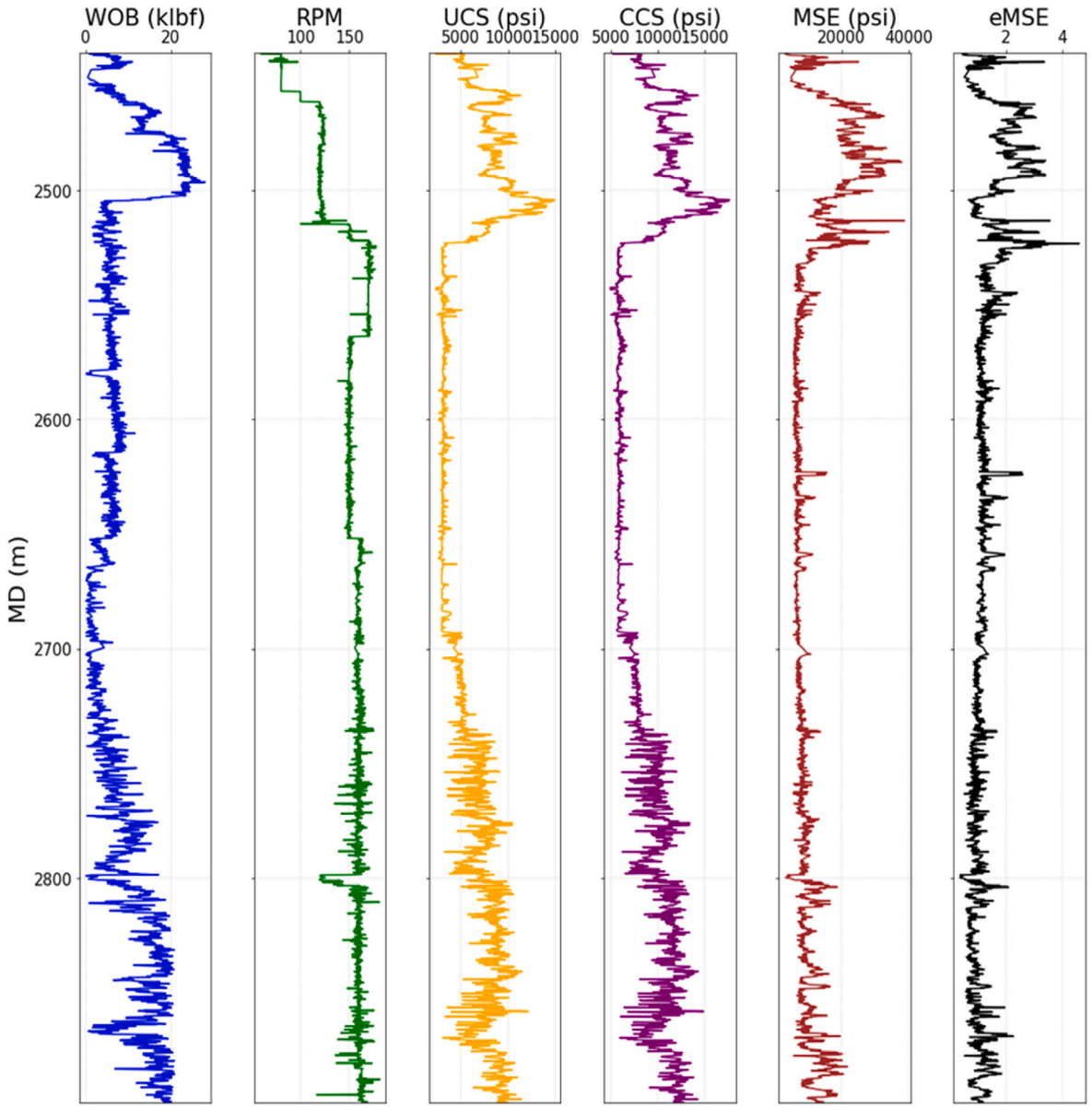


Fig. 7. (Left to Right) WOB, RPM, UCS, CCS, MSE and eMSE plots with depth for 8-½" section of Well B drilled in Norwegian continental shelf. The WOB and RPM were recorded while drilling and MSE was calculated using drilling parameters. The UCS and CCS were calculated using previously developed correlations by using predicted compressional wave velocity data at the bit. eMSE is calculated by normalizing the MSE value with CCS value for each depth interval.

and bit design parameters, including WOB, RPM, drilling fluid density, flowrate, bit hydraulic horsepower, CCS, back rake angle, side rake angle, bit diameter, Junk Slot Area, and nozzle size. Each parameter was normalized against a baseline set of values, as shown in Table 3, to facilitate direct comparison and create a star plot visualization of their effects. Establishing a base case for operating, rock strength, and bit design parameters (as shown in Table 3) is essential, as it allows us to assess the effect of changing one parameter on ROP while maintaining other parameters same. Normalizing the parameter values according to the base values aligns all plots at a common point, ensuring consistency in comparisons.

Fig. 9 illustrates the sensitivity analysis for operational and rock strength parameters. In the analysis, each parameter was varied individually while keeping all other parameters constant to observe the specific impact on ROP. For example, to analyze the effect of WOB, only the WOB value was modified, with all other parameters kept unchanged. Fig. 9 shows both WOB and RPM have a positive correlation with ROP, meaning that as these parameters increase, the ROP increases. This relationship shows that higher WOB increases the depth of cut, enhancing rock penetration. Similarly, a higher RPM speeds up rock removal by the drill bit, further boosting penetration rates. Flowrate also positively affects ROP by improving bottom-hole cleaning efficiency, which reduces the chance of cuttings accumulating under the bit and enhances drilling efficiency. In contrast, drilling fluid density shows an inverse relationship with ROP, as drilling fluid density increases, ROP decreases. This is due to the chip hold-down effect, where the higher hydrostatic pressure from a heavier drilling fluid density holds rock cuttings against the formation, making it harder for the drill bit to remove them. Hydraulic Horsepower (HHP) has a moderately positive effect on ROP. By enhancing the removal of cuttings and preventing bit balling, higher HHP support sustained ROP, especially in demanding drilling conditions. This parameter is crucial in high-energy drilling environments where efficient cuttings removal is necessary to maintain ROP. Overall, these trends highlight the complex, interdependent roles of operational parameters in optimizing drilling efficiency. CCS, representing rock strength, was a crucial parameter in the analysis. Higher

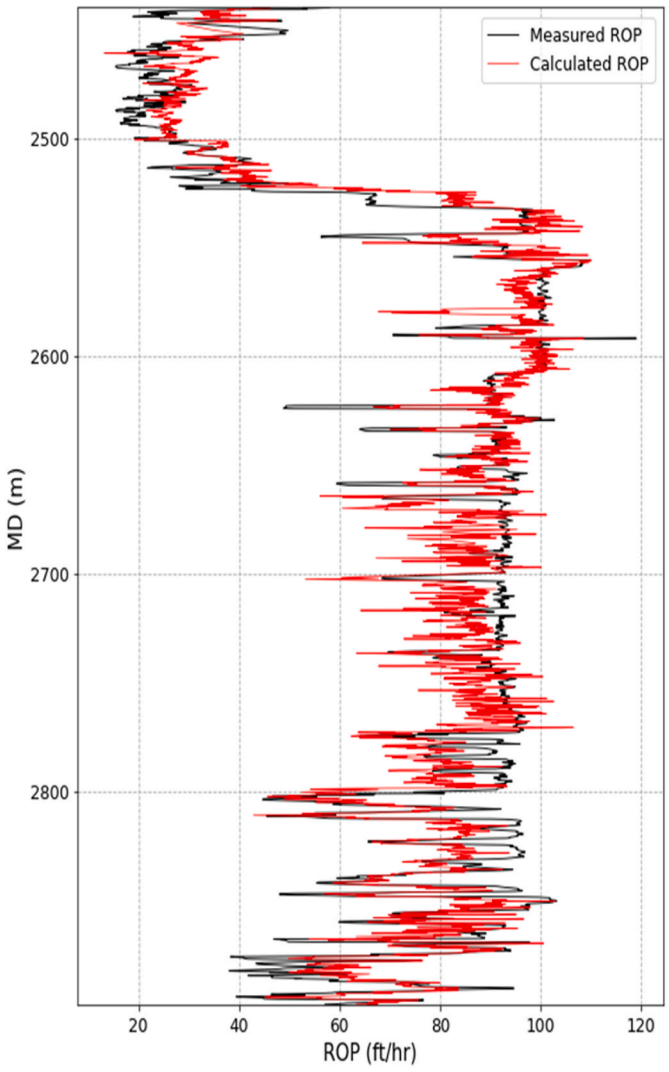


Fig. 8. Comparison of calculated ROP model from measured ROP with depths.

Table 3
Base case values for operational, rock strength and bit design parameters used in the sensitivity analysis.

Parameter	Base case value
WOB	5.2 klbf
RPM	150
ROP	90 ft/h
Drilling fluid density	9.5 ppg
Flowrate	464 GPM
Hydraulic horsepower	70 hp
Confined compressive strength	5846 psi
Back rake angle	20 deg
Side rake angle	15 deg
Bit diameter	8-½ inch
Junk slot area	25 $inch^2$
Nozzle size	$2 \times 13/32''$ and $5 \times 12/32''$

CCS values indicated more resistant formations, leading to a reduced ROP, as shown in the comparative analysis in Fig. 9. In real-time drilling, adjustments to other parameters like WOB and RPM can help mitigate the effects of high CCS, maintaining operational efficiency and ROP.

In Fig. 10, the sensitivity analysis highlights the influence of bit design parameters on ROP, underscoring the importance of optimized bit design in maximizing drilling efficiency. The PDC bit design parameters analyzed include back rake angle, side rake angle, bit diameter, junk slot area (JSA), and nozzle size. An increase in the BR angle, which makes the cutter orientation less aggressive, leads to a decrease in ROP, as higher resistance to rock chipping reduces penetration efficiency. This aligns with the behavior expected in harder formations, where a more aggressive BR angle would be needed to maximize ROP. SR angle, on the other hand, shows minimal impact within standard ranges, serving more to aid cutting lift than to drive significant changes in ROP. Bit diameter also plays a critical role, with larger diameters distributing the WOB across a wider surface, thus reducing the penetration rate. Conversely, increasing the JSA, which enhances the open face volume of the bit, improves bottom hole cleaning efficiency by aiding cuttings removal. This results in an increase in ROP by reducing obstructions at the bit face. Finally, nozzle size exhibits an inverse relationship with ROP. Smaller nozzles increase jet impact force, enhancing cuttings transport and bit cleaning effectiveness, ultimately supporting higher ROP. This trend is consistent with hydraulic optimization practices where nozzle size is tailored to achieve a balance between impact force for cuttings removal and hydraulic energy for efficient drilling. Overall, these findings emphasize the interdependency between bit design and operational efficiency, highlighting that an optimal combination of these parameters can substantially enhance ROP, particularly in variable formation conditions.

As described in Equation (10), the axial and torsional components contribute to the Mechanical Specific Energy (MSE) equation. Fig. 11 shows the sensitivity analysis of the drilling parameters. In this analysis, each parameter varied independently while keeping all other parameters constant to isolate its specific impact on MSE. For instance, when analyzing the effect of RPM, only the RPM value was adjusted while maintaining the other parameters unchanged. The results indicate that the axial component represented by WOB, axial component in MSE equation, has a negligible contribution to the MSE calculation, as evident in Fig. 11. Conversely, an increase in torque and RPM leads to an increase in MSE. On the other hand, increases in bit diameter and ROP result in a decrease in MSE. This analysis highlights the varying degrees of influence each parameter has on MSE during drilling operations.

4.6. Discussion

The hybrid ROP model developed in this study provides a comprehensive approach by incorporating both machine learning and analytical methods, addressing critical gaps in traditional models. Unlike conventional ROP models, which often rely on empirical relationships or limited operational parameters, this model includes predictive rock strength for each depth. The study incorporated data from five wells in the Norwegian Continental Shelf, with four wells (Well A, C, D, and E) used for training and one well (Well B) reserved for blind testing. The Random Forest regression model effectively predicted compressional wave velocity, achieving an R^2 of 93%, and the Random Forest classification model predicted lithology with 96% accuracy, ensuring precise uniaxial compressive strength and confined compressive strength calculations. These enhancements allow the ROP model to adapt to real-time changes in rock characteristics, mitigating the challenges posed by sensor offset issues in traditional downhole measurements.

Additionally, the sensitivity analysis shows the complex interactions among operational, rock strength, and bit design parameters, emphasizing the significant impact of WOB, RPM, rock strength and bit hydraulics on ROP. The inverse relationship between drilling fluid density and ROP was particularly evident, as the chip hold-down effect from higher drilling fluid density inhibits the removal of rock cuttings. The role of hydraulic horsepower in maintaining efficient bottom-hole cleaning was shown through star-plot, supporting sustained ROP in high-energy drilling environments. The bit design sensitivity analysis revealed that parameters like back rake angle, side rake angle, bit diameter, junk slot area, and nozzle size play pivotal roles in drilling efficiency, with optimal configurations required to minimize resistance and maximize penetration in varying formations.

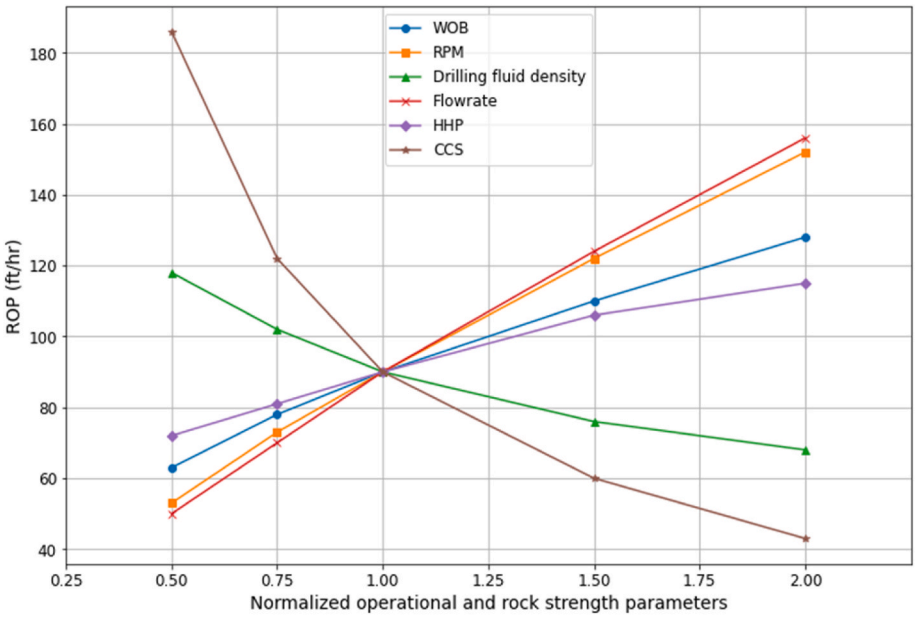


Fig. 9. Sensitivity analysis of operational parameters for PDC bit using ROP model.

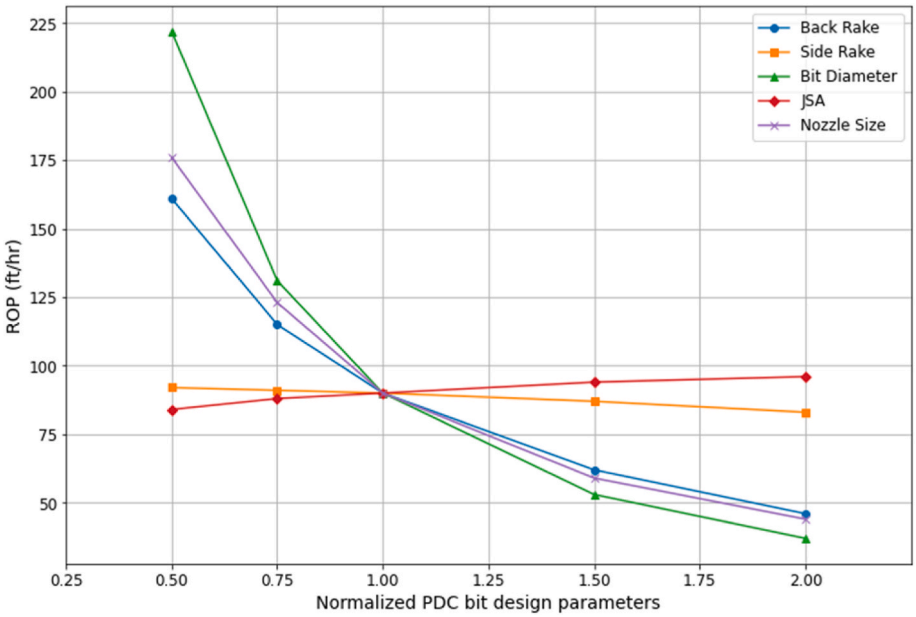


Fig. 10. Sensitivity analysis of PDC bit parameters using ROP model.

Unlike previous studies, our approach integrates both drilling and near-bit log parameters to predict compressional wave velocity at the bit in real time, effectively mitigating the challenges posed by LWD sensor offset. This sensor offset provided an opportunity to apply machine learning to predict compressional wave velocity directly at the bit. Additionally, in the context of the ROP equation, no previous studies have used the confined compressive strength values calculated from both predicted lithology and predicted compressional wave velocity, a novel aspect of our methodology. This integration enabled a more precise, real-time estimation of rock strength, which is not performed in earlier ROP models.

These findings indicate the importance of integrated modeling approaches in drilling optimization. The hybrid model not only aligns closely with measured ROP values but also adapts to dynamic downhole conditions, making it valuable for real-time drilling applications across diverse formations. However, a key limitation of the developed workflow is its geological dependence. While it proves robust in predicting compressional wave velocity and lithology within the Norwegian Continental Shelf region, its accuracy may be compromised in different geological settings. This challenge is inherent to all machine learning models, as they are location specific. Nevertheless, by re-training the model with data from new wells in different regions, its accuracy can be significantly improved. While the same methodology can be applied, incorporating near field data is essential to maintain reliability across varied geological environments.

4.7. Conclusions

In this study, a hybrid ROP model that integrates analytical and machine learning approaches, addressing the limitations of traditional

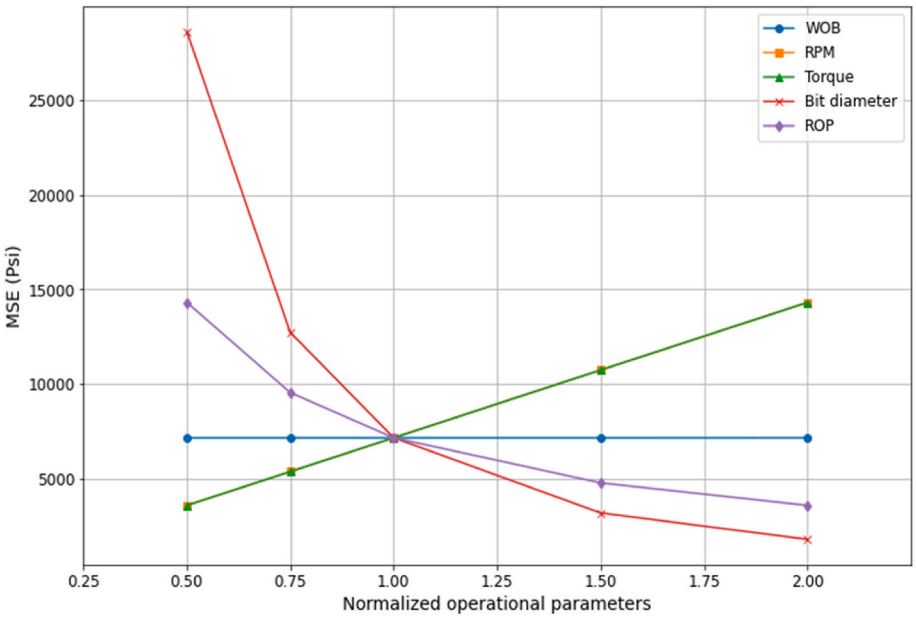


Fig. 11. Sensitivity analysis of drilling parameters using MSE calculation.

ROP models, was developed. By incorporating real-time predictions of compressional wave velocity and lithology, the model enabled rock strength estimations at the bit for each depth interval, leading to more precise and reliable ROP calculations. Model validation with the field data demonstrated close alignment between measured and predicted ROP values, confirming the model's accuracy and robustness. The sensitivity analysis highlighted the importance of optimizing operational and bit design parameters, such as WOB, RPM, drilling fluid density, hydraulic horsepower, and bit geometry, to maximize ROP and improve drilling efficiency. These findings highlight the model's potential for real-time applications in complex and challenging drilling environments, offering practical benefits in reducing operational costs and improving performance. Future research should focus on extending the model to diverse geological settings and incorporating additional data from various lithologies and well trajectories and re-train the model to further enhance its robustness. Such advancements would further enhance the predictive capabilities and versatility of the hybrid ROP model, making it a tool for drilling optimization across different drilling conditions.

CRediT authorship contribution statement

Ashutosh Sharma: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Tunc Burak:** Writing – review & editing, Visualization, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Runar Nygaard:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Espen Hoel:** Writing – review & editing, Supervision. **Tron Kristiansen:** Writing – review & editing, Supervision. **Morten Welmer:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Runar Nygaard reports financial support was provided by Aker BP ASA. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

The authors would like to thank AkerBP ASA and Sekal AS for their financial and technical support for this project.

Nomenclature

ABR	Abrasiveness constant
ΔBG	Cumulative bit wear
BR	Back rake angle, *deg*
CCS	Confined compressive strength, *psi*
D_b	Bit diameter, *inches*
d_{in}	Bit run-in depth, *m*
d_{out}	Bit pulled out of hole depth, *m*
DOC	Depth of cut, *ft/rev*
DL:	Deep learning
DTCO	Compressional slowness or sonic trave time, *μs/ft*
eMSE	Enhanced mechanical specific energy, *unitless*
FR	Drilling fluid Flow rate, *gpm*
G	Bit coefficient
GR	Gamma ray, *API*
HHP	Hydraulic horsepower, *hp*
HSI	Hydraulic horsepower per square inch, *hp/psi*
$hydralics_f$:	Bit hydraulics function
IADC	International association of drilling contractors
JSA	Junk slot area, *inches*
KNN	k-nearest neighbors
LWD	Logging while drilling
MD	Measured depth, *m*
ML:	Machine learning
MLR	Multi linear regression
MLP	Multi-layer perceptron
MSE	Mechanical specific energy, *psi*
MW	Drilling fluid density, *ppg*
NOB	Number of blades
NOB_f	Number of blades function
P_{bit}	Pressure losses at the bit, *psi*
PDC	Polycrystalline diamond compact
RF	Random Forest
ROP	Rate of penetration, *ft/hour*
RPM	Revolution per minute, *RPM*

SR	Side rake angle, *degrees*
SVM	Support vector machine
TOB	Torque on bit, *Klbf-ft*
TVD	True vertical depth, *m*
UCS:	Uniaxial/unconfined compressive strength, *psi*
V_p:	Compressional wave velocity, *ft/sec*
W_f	bit wear factor
WOB	Weight on bit, *Klbf*
YM	Young's modulus

Data availability

Data will be made available on request.

References

Akhtarmanesh, S., Atashnezhad, A., Hareland, G., Al Dushaishi, M., 2021. ROP model for PDC bits in geothermal drilling. In: 55th U.S. Rock Mechanics/Geomechanics Symposium 2021.

Anemangely, M., Ramezanzadeh, A., Tokhmechi, B., Molaghab, A., Mohammadian, A., 2018. Drilling rate prediction from petrophysical logs and mud logging data using an optimized multilayer perceptron neural network. J. Geophys. Eng. 15, 1146–1159. https://doi.org/10.1088/1742-2140/aaac5d.

Bingham, M.Grant, 1965. A new approach to Interpreting– rock drillability LK. https://Ou.on.Worldcat.Org/Oclc/8276505.

Bourgoyne, A.T., Young, F.S., 1974. A multiple regression approach to optimal drilling and abnormal pressure detection. Soc. Petrol. Eng. J. 14, 371–384. https://doi.org/10.2118/4238-PA.

Burak, T., 2018. Application of Artificial Neural Networks to Predict the Downhole Inclination in Directionally Drilled Geothermal Wells. Middle East Technical University.

Burak, T., 2025. Real-Time Analysis of Borehole Integrity Through Model Based and Data Driven Methods. University of Oklahoma. Dissertation. Available from ProQuest Dissertations & Theses Global. (3168167725). Retrieved from. https://login.ezproxy.lib.ou.edu/login?qurl=https%3A%2F%2Fwww.proquest.com%2Fdissertations-theses%2Freal-time-analysis-borehole-integrity-through%2Fdocview%2F3168167725%2Fse-2%3Faccountid%3D12964.

Burak, T., Akin, S., 2021. Estimation of downhole inclination in directionally drilled geothermal Wells. In: World Geothermal Congress 2020+1.

Burak, T., Sharma, A., Hoel, E., Kristiansen, T.G., Welmer, M., Nygaard, R., 2024. Real-time lithology prediction at the bit using machine learning. Geosciences 14, 250. https://doi.org/10.3390/geosciences14100250.

Burak, T., Sharma, A., Hoel, E., Kristiansen, T.G., Welmer, M., Hellvik, S., Nygaard, R., 2025a. Real Time Borehole Stability Analysis at the Bit with Time Dependent Effects.

Burak, T., Sharma, A., Hoel, E., Kristiansen, T.G., Welmer, M., Hellvik, S., Nygaard, R., 2025b. Real-time borehole stability analysis at the bit using analytical and machine learning approaches. In: Paper Presented at the 59th U.S. Rock Mechanics/Geomechanics Symposium. June 2025.

Gan, C., Cao, W.H., et al., 2019. Prediction of drilling rate of penetration (ROP) using hybrid support vector regression: a case study on the shennongjia area, Central China. J. Petrol. Sci. Eng. 181. https://doi.org/10.1016/j.petrol.2019.106200.

Hareland, G., Nygaard, R., 2007. Calculating unconfined rock strength from drilling data. Proceedings of the 1st Canada-US Rock Mechanics Symposium - Rock Mechanics Meeting Society's Challenges and Demands 2, 1717–1723. https://doi.org/10.1201/noe0415444019-c216.

Hareland, G., Rampersad, P.R., 1994. Drag - bit model including wear. In: Proceedings of the SPE Latin American and Caribbean Petroleum Engineering Conference, pp. 657–667. https://doi.org/10.2118/26957-ms, 2.

Hegde, C., Soares, C., Gray, K., 2018. Rate of penetration (ROP) modeling using hybrid models: deterministic and machine learning. In: SPE/AAPG/SEG Unconventional Resources Technology Conference 2018. https://doi.org/10.15530/urtec-2018-2896522. URTC 2018.

Jiao, S., Li, W., Li, Z., Gai, J., Zou, L., Su, Y., 2024. Hybrid physics-machine learning models for predicting rate of penetration in the Halahatang oil field, Tarim Basin. Sci. Rep. 14. https://doi.org/10.1038/s41598-024-56640-y.

Kerkar, P.B., Hareland, G., Fonseca, E.R., Hackbarth, C.J., 2014. Estimation of rock compressive strength using downhole weight-on-bit and drilling models. In: All Days. https://doi.org/10.2523/IPTC-17447-MS.

Li, C., Cheng, C., 2020. Prediction and Optimization of Rate of Penetration Using a Hybrid Artificial Intelligence Method Based on an Improved Genetic Algorithm and Artificial Neural Network.

Ma, Z., Hsu, M., Hu, H., Wang, T., Zhou, X., Li, G., Tian, S., 2024. Hybrid strategies for interpretability of rate of penetration prediction: automated machine learning and SHAP interpretation. In: 58th U.S. Rock Mechanics/Geomechanics Symposium. https://doi.org/10.56952/ARMA-2024-0315.

Mazen, A.Z., Mujtaba, I.M., Hassanpour, A., Rahmanian, N., 2020. Mathematical modelling of performance and wear prediction of PDC drill bits: impact of bit profile, bit hydraulic, and rock strength. J. Petrol. Sci. Eng. 188. https://doi.org/10.1016/j.petrol.2019.106849.

Motahhari, H.R., Hareland, G., James, J.A., 2010. Improved drilling efficiency technique using integrated PDM and PDC bit parameters. J. Can. Petrol. Technol. 49, 45–52. https://doi.org/10.2118/141651-PA.

Olayiwola, T., Sanuade, O.A., 2021. A data-driven approach to predict compressional and shear wave velocities in reservoir rocks. Petroleum 7, 199–208. https://doi.org/10.1016/j.petlm.2020.07.008.

Rashidi, B., Hareland, G., Nygaard, R., 2008. Real-time drill bit wear prediction by combining rock energy and drilling strength concepts. In: All Days. https://doi.org/10.2118/117109-MS.

Ren, C., Huang, W., Gao, D., 2023. Predicting Rate of Penetration of Horizontal Drilling by Combining Physical Model with Machine Learning Method in the China Jimusar Oil Field.

Sharma, A., 2024. Incorporating Rock Behavior in Real-Time Drilling Analysis. University of Oklahoma. Dissertation. Available from ProQuest Dissertations & Theses Global. (3142749857). Retrieved from. https://login.ezproxy.lib.ou.edu/login?qurl=https%3A%2F%2Fwww.proquest.com%2Fdissertations-theses%2Fincorporating-rock-behavior-real-time-drilling%2Fdocview%2F3142749857%2Fse-2%3Faccountid%3D12964.

Sharma, A., Al Dushaishi, M., Nygaard, R., 2021. Fixed bit rotary drilling failure criteria effect on drilling vibration. In: Paper Presented at the 55th U.S. Rock Mechanics/Geomechanics Symposium. Virtual, June 2021.

Sharma, A., Burak, T., Nygaard, R., Hoel, E., Kristiansen, T., Hellvik, S., Welmer, M., 2023a. Projecting petrophysical logs at the bit through multi-well data analysis with machine learning. In: Day 3 Thu. https://doi.org/10.2118/215587-MS. September 07, 2023.

Sharma, A., Burak, T., Nygaard, R., Hellvik, S., Hoel, E., Welmer, M., 2023b. Projection of logging while drilling data at the bit by implementing supervised machine learning algorithm. In: Society of Petroleum Engineers - SPE Oklahoma City Oil and Gas Symposium 2023. OKOG 2023. https://doi.org/10.2118/213070-MS.

Sharma, A., Al Dushaishi, M.F., Nygaard, R., 2024. Evaluating PDC bit-rock interaction models to investigate torsional vibrations in geothermal drilling. Geothermics 122, 103060. https://doi.org/10.1016/j.geothermics.2024.103060.

Warren, T.M., 1987. Penetration-rate performance of roller-cone bits. SPE Drill. Eng. 2, 9–18. https://doi.org/10.2118/13259-PA.

Zhou, F., Fan, H., Liu, Y., Zhang, H., Ji, R., 2023. Hybrid model of machine learning method and empirical method for rate of penetration prediction based on data similarity. Appl. Sci. 13. https://doi.org/10.3390/app13105870.