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A STATISTICAL ANALYSIS OF THE PLACEMENT PROGRAM IN MATHEMATICS FOR FRESHMEN AT THE UNIVERSITY OF OKLAHOMA

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Statement of the Problem

The object of this research is to conduct a statistical analysis of the placement program for freshmen entering their initial college mathematics courses at the University of Oklahoma. The effectiveness of this program is to be compared with other possible placement practices. The data presented and the recommendations resulting from this study should prove of use to the Mathematics and Astronomy Department of the University of Oklahoma.

This study contains some data of secondary interest to placement practices in mathematics. Tables showing the different colleges of the University of Oklahoma and their requirements in mathematics, comparisons of freshmen and non-freshmen grades in elementary mathematics courses, and other related matters have been included because the investigator believes they will be of value in maintaining and improving instruction in mathematics.

Need for the Research

Starting the study of any area at the proper level of difficulty is important. Educational time cannot be used in today's culture to unnecessarily review previous work; the increasing horizons of knowledge demand efficiency of educational effort. Yet one cannot enter indiscriminately at an advanced level into a field of study, for many academic areas require an ordered, systematic conquering of concepts and techniques.

The placement of a student in a course which proves to be a review of previous experiences can result in boredom and a loss of interest by him in the entire field. Placement at too difficult a level is equally frustrating and unrewarding, and can generate an acute dislike for the subject. If the student thus placed fails, not only will he probably feel his time and effort have been wasted, but he still may lack the needed knowledge and skills to even repeat the course successfully. In addition, the psychological effects of failure are frequently detrimental and combined with the resulting grade point deficiency may cause lost opportunities and drop-outs by capable young people.

Either of the foregoing extremes in placement requiring competition in a course among students of vastly different backgrounds of competence can result in circumstances which prove to be a disservice not only to the individual, but to his classmates, his instructor, and the academic field under study. Educational institutions, most pointedly those of

Oklahoma, must make maximum utilization of available instructional facilities. The improperly-placed students consume the same physical facilities, the same teaching time, and the same instructional effort--in fact, often more of the latter--as the student capable of maximum profit from the course. This is a drain on our limited educational resources.

In the field of mathematics, the problem of correct placement takes on special urgency. Mathematics is a fabric consisting of various interwoven and dependent branches of study. The student's success, therefore, in a mathematics course is often dictated as much by his former studies in this field as by the new material presented in the course.

The placement of entering freshmen in mathematics is influenced, and properly so, by other major fields of study such as the physical sciences and engineering. Each of these fields requires understanding of many basic concepts in mathematics before one can enter certain specialized areas within the field. The freshman at the University of Oklahoma planning to pursue engineering who starts at a course level below Mathematical Analysis II (calculus) finds he cannot apply that credit on his engineering degree. He also finds he is further handicapped because his schedule of engineering courses may have to be delayed until he achieves the mathematical proficiency necessary to pursue them.

Other fields such as the natural sciences, social sciences, and business are putting increased emphasis on mathematical skills and concepts. Their sequences of courses are frequently based on the assumption that the advanced students will have attained the particular level of mathematical effectiveness necessary to understand and to enter into certain statistical studies and research areas.

Even with these needs and pressures, it would be unrealistic to force or even advise entry of a student in a course for which he is ill-equipped. All freshmen have had some work in mathematics previous to entry in college. The experience in mathematics results in a diversity of skills, knowledge, and even attitudes toward the subject. In some fields, psychology for example, a relatively homogeneous grouping of students with respect to knowledge of the subject might be found. Such homogeneity in the best selected beginning mathematics group will not be found. The wide range of courses available to the entering student, however, does permit some measure of grouping.

The following courses are offered by the mathematics department at the University of Oklahoma as logical choices for an initial course in college mathematics.

<u>1</u> <u>Remedial Mathematics</u>: one credit hour (but meets three hours per week). A review of beginning algebra. Offered only at night on campus or by the extension or correspondence divisions.

<u>2</u> <u>Intermediate Algebra</u>: three credit hours. Covers approximately one and one-half years of high school algebra.

<u>5</u> <u>College Algebra</u>: three credit hours. The standard college course of algebra.

<u>6</u> <u>Trigonometry</u>: three credit hours. An analytical development of plane trigonometry.

<u>21</u> <u>Mathematical Analysis I</u>: five credit hours. An integrated course in college algebra and analytic trigonometry with introduction to analytic geometry. Duplicates <u>5</u> and <u>6</u>.

<u>22</u> <u>Mathematical Analysis II</u>: five credit hours. An integrated calculus and analytical geometry course, starting a three-semester series.

The present admission policy at the University of Oklahoma does not require any specified units in any field of study. In view of this, it is not surprising that Mathematics <u>2</u>, intermediate algebra, had an enrollment of more than seven hundred students, most of whom were freshmen, in the fall semester of 1964. Yet from the same population, approximately one hundred freshmen were qualified to enroll immediately in Mathematics <u>22</u>, calculus. These two courses represent a wide diversity in the overt competencies of entering freshmen, and placement in the proper course, therefore, represents a major factor in determining the student's performance in class and his efficient use of the educational resources provided for him almost completely by the state of Oklahoma.

<u>Definition and Recognition of Proper Placement</u>

In order to conduct this research, the following question must be answered, "What is proper placement in first-year college mathematics and how is it recognized?"

First we realize that speaking of correctness in placement requires postulation of the existence of measurable variables related to an individual which, properly utilized, can aid in predicting his chances of success in a given course. A major part of this research will be the statistical examination of available data to test the validity of this assumption and to determine, if possible, the correct utilization of the variables for predicting success.

Proper placement is then the correct use of these predictive variables in the selection of that particular course among those available which will permit the maximal amount of success to be experienced by the student. But what is success in the educational process?

Ideally we could speak of a student achieving success in a course if all or any of the following occurs.

1. The student experiences a growth of his rational powers.[1]
2. He develops an interest and curiosity about the field of study and is motivated to continue work in that area.
3. He displays mastery of the material and is ready to study more advanced concepts in the field.
4. He has been challenged to put forth maximum efforts in constructive learning activities.
5. His time has been utilized in effective, significant ways during the course.

But these results are often nebulous and

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[1]We understand these to be the processes of recalling and imagining, classifying and generalizing, comparing and evaluating, analyzing and synthesizing, and deducing and inferring. The Central Purpose of American Education. Educational Policies Commission, NEA, Washington, D.C., 1961.

uncontrollable, sometimes unidentifiable, and if recognized and identified, difficult to measure. Moreover, when they can be measured, no absolute scale permits one to draw a line between a success and failure dependent on the amount of variation shown. Even if the foregoing five criteria represent proper outcomes from a course, they are not the well-defined common criteria which can be used to promote or judge educational efficiency.

We need, therefore, a criterion which is readily accessible, recognizable and acceptable to use in judging an individual's success in any course. We are, therefore, led to the academic grade a student receives in the course. That standard is understood and used at most educational institutions to determine the student's rate of achievement and progress toward a degree. In seeking educational efficiency both for the student and of available facilities, this is a pragmatic instrument for measuring progress. We will assume, therefore, that the most efficient, practical measure of a student's academic success is his final course grade.

One is cognizant both of the diversity and depth of feelings about such an assumption and the inherent difficulties involved. We are not claiming a grade is a completely accurate measure of a student's success in a course if success is defined by the long-range goals previously stated. We realize that mathematics grades may not reflect a uniform standard as they are issued each semester by a heterogeneous group of instructors. In some classes, only the truly outstanding students receive the grade of "A." In contrast, some instructors give many high grades and issue failing grades only in exceptional cases. Even the same instructor may be unable to grade students of similar ability and performance precisely the same in two different classes or during different years. We realize the freshmen mathematics courses are taught by many inexperienced teaching assistants whose very inexperience can affect the grades issued and the level of achievement reached by the students.

We maintain, however, that the nature of mathematics and its objective rather than subjective characteristic of testing permits grading practices more divorced from the instructor's individual feelings than possible in fields such as English, philosophy or education. Consequently, the following hypothesis will be tested.

> The correlations between the grades on a common final examination in one course and the predictive variables[2] for the group are significantly greater than the correlations between final grades as issued by the several instructors of the course and the same predictive variables.

Rejection of this hypothesis at the 5% level of significance would indicate that grades given by many instructors are more reliable than might be expected, considering the variety of philosophies of grading involved. It would

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[2]The predictive variables for the group are the measurements known on each entering freshman such as his scores on the American Testing Program or his high school grade point average.

indicate the individual instructor, involved for a period of time, can judge by his own testing instruments the performance of a student as skillfully or more so than can be done by the more standardized and validated, but highly impersonal, common examination.

Acceptance of this hypothesis will necessitate a closer look at the instructing, testing and grading policies followed in the mathematics courses under study. If examination of the common final finds it is a reasonably valid instrument for measuring the course objectives, then the teaching skills and objectives of the various instructors would have to be observed. The grading practices used should be evaluated and examined to see if factors other than basic course mastery were receiving heavy weight in determining the final grades. The data would have to be studied to determine if a few teachers were responsible for a disproportional amount of the variation. Finally, a further study would be needed to decide if the recent emphasis of the last two years on departmental supervision had varied the amount of correlation between success as we have defined it and the predictive variables being used.

We are also aware that a grade of "A" earned by different students in the same class under the same instructor may denote vastly different degrees of success on the part of the students. One such grade may reflect hard work by a student who succeeds in mastering the subject. The other may result from the effortless response of a student who took a course below his ability, thus he wasted educational facilities as well as his own time.

Similar extremes occur at every division of grading. Certainly every failing grade is not a sign of improper placement. Students placed correctly according to their background, ability, and curriculum may find their responses dominated by problems of college adjustment, work loads, health, or even by a dull, oppressive, unskilled teacher.

While acknowledging our lack of control over the above extraneous factors, we nevertheless accept grades as the best indicator of course success. We also believe that correct placement can measurably aid the student in achieving educational growth as measured by the "grade indicator." Our practices, therefore, in placement must be evaluated to determine their effectiveness and to justify their enforcement or change, and we will use the criterion of grades in this evaluation.

We do not expect correct use of the determined variables for placement to guarantee an absolutely accurate prediction of success for each individual concerned. Placement practices should, however, be based on the predictive variables of maximal value, accuracy, and usability for the largest number possible of the group for which it is formulated. The student's college advisor remains of primary importance; for his judgment, based on individual observation and data gleaned from the student's record as well as the suggested guidelines of a placement program, is invaluable and irreplaceable and must continue to dictate the final decisions in each course selection. A placement guide of proven value is just one tool to make the advisor's task easier.

The Placement Policies at the University of Oklahoma

Stating that a correct decision in placement in mathematics should be based on the individual student, his background of knowledge, his ability and his planned curriculum is easily done. Unfortunately, study of the high school transcript does not always result in an accurate assessment of the student's competence in a field, and an advisor, skilled in any of the many varied fields of study offered at the University, cannot be expected to place each of his many advisees at the correct level of mathematics, basing his decision solely on individual interviews.

The University College of the University of Oklahoma has recognized this problem and sends to all advisors of freshmen students a printed statement of placement policies in mathematics. This statement, formulated with the close cooperation of the mathematics department, is offered as a set of suggestions rather than as a set of requirements; rigid compliance of the conditions listed for entrance to each course is not checked or insisted upon.

The placement practices for mathematics at the University of Oklahoma have varied. When the University community was reasonably small, more individual supervision was possible than is presently provided. For some years, the instructors of the beginning courses tried to identify any misplaced students in their classes and advise those students of the necessary course adjustment during the first few weeks of every semester. Then the practice was adopted of administering a test (generally on basic algebra) to all beginning mathematics students at the end of two weeks of class work. On the basis of the student's performance on this examination, necessary course changes were counseled. Finally a mathematics placement examination (the O.U. Mathematics Test written and standardized by Dr. John Brixey and Professor Earl LaFon) was administered during registration to all entering students. The raw score of this examination, at times combined with the decile score of the Iowa High School Content Examination (IHSC) formed the main placement tool in mathematics at the University until 1960.[3]

During the 1960-61 school year, all Oklahoma colleges and universities adopted the American College Testing program in order to have a standard measure of academic ability and background for the entire state to use for comparison and research purposes. This battery of tests consists of an English Usage Test, a Mathematics Usage Test, a Social Studies Reading Test, and a Natural Sciences Reading Test. Each part[4]

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[3]Background data was obtained in interviews with Dr. John Brixey, Professor Earl LaFon, Don Patten, and Dr. C. E. Springer, all members of the mathematics department at the University of Oklahoma.

[4]Referred to hereafter as the ACT with subtests

requires approximately forty-five minutes of testing time and is usually administered to high school seniors prior to their acceptance at the University. The results of each subtest are reported both in form of a standard score varying from 1 to 36 and by a percentage ranking based on the population of all students participating in these ability and achievement measures. A composite standard score which is the average of the subtest scores and its comparative percentage rank are also computed. The stated purpose of this program is to provide college officials with a practical standard method of evaluating students to aid in decisions of admissions, sectioning in freshmen courses, scholarship and loan awards, advanced placement, and student counseling [8].

When this testing program started during the summer of 1960 at the University of Oklahoma, the placement guidelines for mathematics became dependent primarily on the ACTM. Intervals of values, partitioning the range of the ACT standard scores, were selected to determine membership in mathematics 1 (formerly called mathematics A), mathematics 2 and for mathematics 5 and above. Selection of the particular scores defining any one course classification was dependent on the necessary range of values which would maintain the customary percentage of students at that course level. Since that initial decision in classification procedures, there have

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indicated by ACTE (English), ACTM (Mathematics), ACTSS (Social Studies), ACTNS (Natural Sciences) and the composite score referred to by ACTC.

been only minor variations in the score divisions.

In 1962-63, it became necessary to formulate a standard policy to apply to the growing numbers of entering students who seemed prepared to start at the calculus level, mathematics 22. That policy states that if a student makes a top score on the ACTM and has taken the equivalent of four years of high school mathematics, he is permitted to take a departmental examination to test his readiness for mathematics 22. Originally this was given a few days after class work had begun; now it is taken during the pre-registration period.

In 1964 course prerequisites for mathematics 2, 5, 6, and 21 were also mentioned specifically on the Mathematics Placement Instructions[5] issued for the advisors by the University College. Still the main emphasis remained on the ACTM.

The ranges of ACTM scores which determine membership in certain mathematics courses were originally selected to maintain certain percentages of students at each level. Maintenance of those same ranges for classification purposes now overlooks the new set of norms established by the ACT program in 1962 which varied the meaning of any fixed standard score previously determined. Also ignored is the possibility of a change in mathematical competency on the part of high school students who have been exposed to renewed interest in science and mathematics resulting from space travel and to

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[5]Sample copies of these Placement Instructions are found in Appendix II.

invigorated courses in those areas affected by curriculum movements spawned by such groups as the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, and similar groups in science. Recent studies [30] undertaken by the Guidance Service at the University of Oklahoma indicate the freshmen class of 1965 when compared to freshmen groups of three or more years ago show increased competency as measured by all subtests of the ACT.

But primarily, the validity of any placement program should be demonstrated and there have been no studies evaluating the ACTM as a placement guideline for the particular mathematics courses offered at the University of Oklahoma. Studies of the role of the ACTM or ACTC [24,26,28,29] undertaken here for other purposes or at other universities for similar reasons do not validate the use of the ACTM placement guideline for courses offered by the Department of Mathematics and Astronomy. It must be demonstrated by statistical analyses that the standard scores of the ACTM alone or combined with the high school mathematics background, are correlated significantly with the students' performance in the mathematics courses of concern and distinguish significantly between successful membership in each. Moreover, the ACTM must be shown to be the maximal predictor of success among all usable and available predictive variables. Until this is done, it must be assumed our placement policy is based as much on whim as on reasoned judgement.

Survey of Literature

The difficulty of determining the correct initial placement of college freshmen in mathematics is not a unique problem of this university. As this project developed, it became desirable to investigate the practices of other colleges and universities and to study previously undertaken research relative to placement policies in mathematics.

An informal survey of the principal state universities of the United States was made and that investigation indicates most placement programs are combinations of two or more measurement instruments varying in type and emphasis. Thirty-five schools from twenty-nine states responded to letters of inquiry sent to their mathematics departments. Of this group, eight are using the ACT program and nine, the College Entrance Examination Boards (CEEB). Twenty-seven use locally designed tests or other standardized national examinations for placement decisions, advanced standings (with or without college credit), and exemption purposes. Twelve consider the high school grade point averages while eighteen checked the specific high school courses in mathematics taken by the student. A few emphasized individual interviews for decision purposes in placement, and choice of the major field of study played a role of diversified emphasis in most programs. Some colleges partially solve the problem by stringent entrance requirements in mathematics which help assure a certain level of mathematical maturity in the student qualified for admittance to their schools.

A copy of the letter of inquiry as well as a detailed summary of the responses received is included in Appendix III.

Thirteen schools indicated some research had been done or was in progress to validate their placement practices. Several institutions mentioned that justification for their present practices is based more on expert opinion and lack of complaints than on conclusions reached by statistical studies.

Lee W. Chatfield, the Director of the Junior Division at the University of Nebraska, provided the investigator with the results of a study made of their freshman class of 1958. In that study, the reliabilities of the high school rank in class, the high school grades, and achievement tests (the CEEB, ACT, NMSQT, and their Regents Examination) were compared as predictors of success in freshman chemistry, English, mathematics, and on first semester grade point averages. The conclusion was that the achievement tests are the least reliable of the three criteria.

Research at the University of Hawaii undertaken by George Fujita [31] compared the effectiveness of the Cooperative Mathematics Pre-Test for College Students (form Y) with the effectiveness of the high school rank, the entrance examination scores, and performance in high school mathematics courses as predictive variables for placement in mathematics. The Pre-Test proved significantly more effective than any other single variable; the high school mathematics grade was second. A combination of the Pre-Test score and the high school grades provided the best predictive combination.

Research at the University of Iowa [28] was undertaken to develop a function to use in classifying students in either intermediate algebra or college algebra courses. As a criterion variable, the opinions of the instructors about the mathematical competencies of the students in their classes were sought during the semester, after the mid-term testing period. From the predictive variables, i.e., the number of semesters of high school mathematics, the high school mathematics grade point average, the high school grade point average and the subtest and composite scores of the ACT, a function dependent on the number of semesters of mathematics, the mathematics grade point average, and the ACTM was developed for use in placement. By the criterion variable, this function misclassified twenty-three percent of this sample, thus was not accepted as being of sufficient reliability to remedy the original classification problem.

In 1955, Harold W. Linscheid [25] studied the freshmen mathematics program at the University of Oklahoma and analyzed available data on the freshmen class of 1952. The discriminant functions he formulated to provide the best practical separation between students properly placed in mathematics 1, 2, and 21 were based on the O.U. Mathematics Test raw scores and the decile rank of the Iowa High School Content Mathematics Subtest. The actual correlations found between the single variables and the criterion of the grade achieved in the first college mathematics course varied from .246 to .535 with the median .385. The coefficients of multiple correlation based on regression equations dependent on these variables ranged from .440 to .540.

The Guidance Service in conjunction with the University College has carried on several long range studies of different freshmen classes (notably the 1952, 1960, and 1962 groups) at the University of Oklahoma [27,30,32,33]. Research on the 1952 class indicated use of age, score on the Ohio State Psychological Examination, and the raw score on the O.U. Mathematics Test formed the best predictive combination for determining college grade point averages. In later studies on the 1960 freshman class with ACT test data included, the grade point average and the ACTC ranked as the two main predictors of first-year overall college grade-point averages. Although a noticeable rise in ACT scores was evident from 1960 to 1962, the predictive equations based on the 1960 study were validated again in a subsequent study. The ACTM subtest score was not found to be significant in formulating any predictive equation for grade point averages for any subset of students, whether the division was by grade point achievement or by field of study.

George Patterson [26] compared the predictive validity of the ACT and the Ohio State University Psychological Test (the OSUPT) and concluded the OSUPT was significantly better than the ACT as a predictor of college grade point averages at the 99% level of confidence. Multiple correlations using combinations of the OSUPT and the ACT subtest and composite scores did not show a significant increase in predictability over the OSUPT variable. Moreover, the low correlation noted between the ACTM subtest and college grade point averages prompted him to question its use as a placement tool in the mathematics area.

John Keihlbauch in his thesis [24] examined data on variables including the ACT subtest scores, the ACT composite score, the high school grade point average, and the number of first semester credit hours for a sample of four hundred freshmen students of 1960. He found the best correlations with freshmen grade point averages were .569 with the high school grade point average and .524 with the ACT Composite Scores. Lowest correlations were .399 with ACT Math scores and .281 with the number of credit hours attempted. His final predictive equation of maximal utility was based on the same variables used by the Guidance Bureau--high school grade-point averages and ACT Composite scores.

Dale Hassinger [23], working with a sample of students at Oklahoma State University, sought significant relationships between achievement in calculus and previous scholastic achievements. He concluded the best predictors of success in calculus were mathematics aptitude tests and high school grades in mathematics. A surprisingly low correlation (not significant at the 5% level) was found between achievement in calculus and the number of previous high school mathematics courses.

A brief study undertaken at Oklahoma State University by Royal H. Bowers and Dan Wesley [29] dealt directly with the value of the ACTM as a placement tool for freshmen entering their institution. As a result of their research, Oklahoma State University has discarded the ACTM and is now relying on the Cooperative Algebra Test--form Z--and the number of semesters of high school mathematics as placement tools in mathematics.

From the survey of the literature, it is evident that the high school grade point average is generally considered the best predictive variable of the college grade point average of freshmen [15]. In the field of mathematics, the high school mathematics grade point average and mathematical achievement tests concentrating on specific concepts and skills in algebra and trigonometry appear most commonly accepted as predictive tools. There is, however, considerable variation in placement policies, as cited in the study of the major state universities, and no one variable has yet been generally acclaimed as the reliable placement guideline for mathematics.

Selection and Description of Sample and Data

To examine the present placement policy at the University and determine some measures of its effectiveness, one must select a set of students which has been placed in certain mathematics courses by that policy. If any conclusions reached by the study are to be applicable, this group must be recent enough to be considered from the same population as our entering freshmen of the next few years. There must, however, have been a sufficient time lapse since the initial enrollment of the set of students to allow data to accumulate with respect to their success both in the mathematics courses and in college.

The 1962 freshman class fits the foregoing criteria. These students were placed in mathematics primarily on basis of their ACT mathematics score. The division points to determine the course level have remained constant since 1962.

This particular sample is being extensively studied in a project of the University Guidance Service and University College, a fact which permits ready accessibility to much data on these individuals. Most data are on IBM cards, a fact which allows immediate duplication and use. Moreover, many characteristics of this group, secondary to this study but of value in suggesting predictive relations, have been determined and are available for reference. The assumption that this group is representative of recent freshman classes has been validated by research of the Guidance Bureau [30,32].

This sample is also a subset of all Oklahoma freshmen college students of 1962 who are the subject of a long term research project being conducted under the direction of the Oklahoma State Regents for Higher Education [9]. This project provides a means of comparing the 1962 freshman University of Oklahoma student body with those of all other universities, colleges, and junior colleges scattered throughout Oklahoma.

A careful examination of our sample reveals that it is composed of two thousand two hundred fifty-four freshmen who enrolled the fall of 1962 in the University of Oklahoma. No student with any previous college course work in residence, night school, or by correspondence at any higher educational institute is included; neither are there any foreign students in this group. The set is partitioned into subsets of one thousand four hundred six males and eight hundred forty-six females, ninety-five percent of whom are from seventeen to nineteen years of age. Seventy-five percent of these students are residents of Oklahoma.

The data available on these individuals besides identifying means (I.D. number and name) specify sex, high school grade point average, the number of semesters of high school mathematics, the ACT English, Mathematics, Social Studies, Natural Science and Composite scores, plus college grades for courses taken during their first two years at the University. For a subset of students in Mathematics 2, their grades on a common final are also available.

In further work, there will be frequent mention of the available predictive variables. This is a reference to the following statistics known for each entering student: the standard scores of the ACTE, the ACTM, the ACTSS, the ACTNS, and the ACTC, the high school grade point average, the high school mathematics grade point average, and the number of semesters of high school mathematics. The course grades in college mathematics will be referred to as the criterion variable.

An exact description of the IBM card kept for each student in our sample is included in Appendix IV. Most computational work has been done by use of the 1410 computer at the University of Oklahoma Computer Laboratory. Appendix I contains a list of the nomenclature frequently used in this study as well as a list of the formulas and tests used in analyzing the data.

Outline of Approach

This study will be organized, in addition to the present section, into three additional chapters.

The second chapter will provide descriptions of the sample set and specific subsets of the sample in terms of the available predictive variables. These descriptions should give insights into variances between the sets of students who choose mathematics and those who do not, and between the sets of students placed at different course levels. Selection of the best single variables for placement purposes should then be possible.

The third chapter will be the development of discriminant functions, based on a combination of predictive variables, to use in classifying students into the courses they are capable of successfully undertaking. The effectiveness of the resulting functions as placement tools will be compared with the effectiveness of single variables in the same role.

Chapter four will be a summary of the information gained in this study. Based on that information, definite recommendations for the University of Oklahoma placement program in mathematics will be stated.

CHAPTER II

USE OF SINGLE PREDICTIVE VARIABLES

AS PLACEMENT GUIDELINES

Correlations of Variables with Course Grades and Common Examination Grades

In this study, the existence of measureable variables which are significantly correlated with the criterion variable of the course grade and which should be utilized in formulating a placement policy in mathematics was assumed. The search for these variables was limited, however, to those statistics known on each entering freshman for practical reasons (time, expense, and usability). The following information appeared relevant: high school grade point average, high school mathematics grade point average, the number of semesters of high school mathematics, and the standard scores on the ACTE, ACTM, ACTSS, ACTNS, and ACTC.

In selecting the course grade as the criterion variable, we also assumed the uncontrollable variables such as different instructors, diversified grading philosophies, and divergent teaching techniques have relatively insignificant roles in determining the final grade. Neither of these assumptions, however, is trivial nor self-evident.

We maintain the structure of the discipline of mathematics permits objective evaluation of student achievement and objective grading practices. Both of the foregoing procedures can be carried out quite independently of most subjective factors such as essay examinations and personality differences between teachers and students which could offset the learner's comprehension of the subject. Therefore, the following hypotheses were tested:

> $H_o\underline{1}$: The correlations between the predictive variables and the criterion variables of either course grade or final examination are not significantly different from 0.
>
> $H_o\underline{2}$: The correlations between the given predictive variables and the common examination grade in one multi-sectioned course are significantly greater than the correlations between the same predictive variables and the final course grades as issued by the several instructors of the course.

In testing these hypotheses, the sample consisted of two hundred eighty-three freshmen who took mathematics 2 the fall semester of 1962 and whose grades on a commonly administered and graded final examination were obtainable. This was not the entire set of mathematics 2 freshmen since two sections of the course pursued an experimental curriculum and complete data were not available on four other sections. Still the sample was large and represented fourteen different classes taught by ten different instructors.

The examination scores were available in terms of percentage scores; however, use of this finer division of scoring (rather than a division of A, B, C, D, or F) would tend to raise the correlation coefficients [7]. This would bias the comparison of these coefficients with the coefficients of correlation determined for the variables and course grade. Thus both criterion variables were recorded by a letter grade.

Table 1 summarizes the data, the hypotheses, and the necessary tests. The reader is again referred to Appendix I for a list of the nomenclature, formulas, and tests used throughout this paper.

Hypothesis 1 was rejected at the five percent level of significance. Thus these predictive variables are positively correlated to criterions of success in mathematics <u>2</u> and may prove of value in determining a placement program.

Hypothesis 2 was also rejected for each pair of correlations with the eight predictive variables at the five percent level of significance. Other studies have verified that correlations of given variables with a criterion such as a course grade which is an average of many tests tend to be equal or higher than those based on a single test score [15]. Thus the course grade is as valid a criterion as the more uniform, standardized examination which limits variances caused by different teaching philosophies, grading practices, etc. Course grades, therefore, remain the best available criterion of success for this study.

Specific Grades Which Identify Proper Placement

Since grades are the criterion of success to be used, those letter grades which will indicate a satisfactory measure

TABLE 1

COMPARISON OF THE COURSE GRADE AND THE COMMON EXAMINATION GRADE AS CRITERION VARIABLES

Mathematics 2, Fall, 1962
Sample Size: 283

Correlations r_{ij}	HSGPA r_{i1}	HSM GPA r_{i2}	SEM.HSM r_{i3}	ACTE r_{i4}	ACTM r_{i5}	ACTSS r_{i6}	ACTNS r_{i7}	ACTC r_{i8}
r_{1j} Course Grade	.449	.451	.266	.280	.346	.195	.241	.305
r_{2j} Exam Grade	.366	.389	.301	.244	.323	.150	.184	.250

N = 283 $r_{\underline{12}}$ = .872 (correlation between course and final grades)

$H_o\underline{1}$: $r_{ij} = 0$, $i = 1,2$; $j = 1,2,\ldots 8$

Level of significance: 5%
Student's t: two-tailed test

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}, \quad N-2 \text{ degrees of freedom}$$

H_o: rejected in every case

$H_o\underline{2}$: $r_{2j} > r_{1j}$, $j = 1,2,\ldots 8$

Level of significance: 5%
Student's t: one-tailed test

$$t = (r_{2j} - r_{1j})\sqrt{\frac{(N-3)(1+r_{\underline{12}})}{2(1-r^2_{\underline{12}} - r^2_{1j} - r^2_{2j} + 2r_{\underline{12}}r_{1j}r_{2j})}}$$

N - 3 degrees of freedom
H_o: rejected in each case

of educational progress and thus should identify correct placement must be decided upon.

A grade of F does not permit progress into the next course, thus students receiving failing grades will not be considered as properly placed. Grades of B and C are usually accepted as indicators of satisfactory progress. To aid in the decision about A and D grades, Table 2 presents data on all students of our sample who made grades of A or D the fall semester of 1962 and elected to take the next mathematics course the spring semester, 1963. The following hypothesis was tested.

> H_o3: The A subset of students and the D subset of students are from populations with equal proportions of students continuing into the next mathematics course level.

Table 3 summarizes the data of Table 2 to facilitate comparing the performance of these two sets of students in their subsequent classes of mathematics.

Hypothesis 3 was rejected as was anticipated for a satisfactory grade should encourage continuing work in mathematics and a failing or low grade suggests either review of previous work or changing to other fields of study.

The performance in the next course of the "A" students was remarkably consistent with previous work. Table 3 shows that approximately fifty-four percent repeated their A grade in the next course; ninety-six percent performed satisfactorily, completing the course with a grade of C or better. The majority of students receiving "A's" may be working at

TABLE 2

RECORD OF GRADES OF FRESHMEN EARNING A's OR D's IN THE FALL, 1962, ENROLLED IN THE NEXT MATHEMATICS COURSE, SPRING SEMESTER, 1963

Course	Grades in Following Course						Comparative Enrollments		Percentage Electing Second Course.
	A	B	C	D	F	W	Fall	Spring	
"A" Set									
$\underline{1}$		1	1				3	2	66.6
$\underline{2}$	11	5	4	1			42	21	50.0
$\underline{5}$	5	3					29	8	27.5
$\underline{21}$	17	13	1			1	42	32	76.2
$\underline{22}$	8	4		1			16	13	81.2
Total	41	26	6	2	0	1	132	76	57.6
"D" Set									
$\underline{1}$					3		9	3	33.3
$\underline{2}$		1	2	5	10	6	80	24	30.0
$\underline{5}$			2			1	10	3	30.0
$\underline{21}$				4		1	53	5	9.4
$\underline{22}$					2	1	12	3	25.0
Total	0	1	4	9	15	9	164	38	23.2

$H_o\underline{3}$: $P_A = P_D$ with P_A and P_D representing the proportions of the two populations enrolled in the next advanced course

$p_A = 76/132$ $\qquad$ $p_D = 38/164$

$N_A = 132$ $\qquad$ $N_D = 164$

Level of significance: 5%
z distribution: two-tailed test

$$z = \frac{p_A - p_D}{\sqrt{\dfrac{p(1 - p)(N_A + N_D)}{N_A N_D}}} \text{ with } p = \frac{N_A p_A + N_D p_D}{N_A + N_D}$$

H_o: rejected

TABLE 3

COMPARISON OF PERFORMANCE OF A AND D SUBSETS OF STUDENTS IN FOLLOWING MATHEMATICS COURSE

Performance in Following Course	'A' Subset		'D' Subset	
	Number	Percentage	Number	Percentage
Same Grade	41	53.9	9	23.7
Same Grade or Better	41	53.9	14	36.8
Satisfactory Grade of A, B, or C	73	96.1	5	13.2
Lower Grade or Dropping Course	35	46.1	24	63.2
Grade of D or F	2	2.6	24	63.2
Dropping Course	1	1.3	9	23.7
Unsatisfactory Grade or Dropping Course	3	3.9	33	86.8
Size of Sample	76		38	

their full potential; some, however, may have been capable of accelerating their educational progress by enrolling in the next advanced course. The investigator, therefore, will omit the subset of students receiving grades of A from the sample classified as properly placed.

In sharp contrast to the above performance, less than twenty-four percent of the D students repeated their grades of D, thirteen percent improved their work to C standard or above, but sixty-three percent received F's or withdrew from the higher course. Over eighty-six percent performed unsatisfactorily. Since a small percentage of D students choose to continue studies in mathematics (not counting repeaters of the initial course), the sample set is small. Still it is evident that while the D grade permits credit hours to be earned (without grade points), the data indicate that the student is not prepared to advance to the succeeding level of course work. This set of students, therefore, will also be excluded from the subset termed as properly placed mathematics students.

Thus, data from the set of students receiving B and C grades will determine the ranges of predictive variables designating proper placement. The other subsets of A, D, and F students will serve as comparison sets for the guidelines thus developed.

Course Requirements of the University in Mathematics

One thousand twenty-seven students in our sample did not take mathematics their freshman year. This large percentage of students avoiding mathematics prompted an investigation of possible cause.

The placement guidelines issued freshman advisors are usually blunt. "If the student's curriculum does not require additional mathematics, then do not give him any unless he especially wishes to go beyond his high school preparation."[6] The student, after completion of twenty-six hours of satisfactory college work under the auspices of the University College, enters one of the seven colleges or schools of the University. Table 4 summarizes the entrance requirements for mathematics in each of these divisions. The percentage of the total senior class of 1965-66 (of which our freshman sample of 1962 is an overlapping subset) enrolled in each college has been included to facilitate comparison of college enrollments.

College mathematics is an entrance requirement only for the College of Business Administration and the College of Engineering. The small percentages of students at the sophomore level and above enrolled in their initial college mathematics indicates few students select mathematics as an elective late in their college program. Naturally students electing specific fields within each college, such as mathematics teaching in the College of Education or science majors in Arts and Science realize their field requires mathematical

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[6] Appendix II, Mathematics Placement, 1-64-65.

TABLE 4

MATHEMATICS REQUIREMENTS OF THE COLLEGES AND SCHOOLS
FOR ENTRANCE OF STUDENTS FROM UNIVERSITY COLLEGE

College or School	Percentage of Seniors, 1965-66 (N = 2473)	Mathematics Requirements
College of Arts and Science	41%	at least two units of college preparatory mathematics in high school or college
College of Business Administration	18%	second year high school algebra and a score of 24 on the ACTM or mathematics 2 in college, also mathematics 5
College of Education	13%	none
College of Engineering	19%	three and one-half units of algebra, geometry, and trigonometry in high school or college and 22 (first calculus course), all with a C or better average
College of Fine Arts	5%	none
School of Nursing	1%	two units of high school mathematics or the college equivalent
College of Pharmacy	3%	second year high school algebra and a score of 24 on the ACTM or 2, the college equivalent

skills and elect mathematics their freshman year regardless of college requirements. Nevertheless, a high school mathematics background could fill the mathematics requirements for a majority of the 1965-66 seniors of the University of Oklahoma.

Comparison of Sets of Freshmen in Mathematics Courses with Non-math Freshmen

In our initial sample of two thousand two hundred fifty-four students, there were one thousand two hundred eighty-one individuals who completed a mathematics course during the four semesters between September, 1962 and January, 1964. Table 5 shows the exact distribution of these students in the various elementary courses. Excluded from this number are the few exceptional freshmen permitted to start at a more advanced level than 22 (calculus) and the students who, after enrollment, either officially dropped the course before the end of the free drop period or dropped it with a passing grade (WP).

Table 6 summarizes data about the predictive variables on three sets of students: (1) those who enrolled in mathematics their initial semester at Oklahoma University, (2) those enrolled in a course in mathematics their second semester here, and (3) those who did not take any college mathematics their first academic year. The total sample has decreased because of incomplete data on eighteen students and the high school grade point average is for academic subjects only, sophomore to senior years. The number of semesters of

TABLE 5

DISTRIBUTION OF FRESHMEN IN INITIAL COLLEGE MATHEMATICS COURSE

Semester	1	2	5	21	22	6 or 7	Totals
Fall, 1962	44	453	115	329	91	9	1041
Spring, 1963	11	111	41	17	--	2	182
Summer, 1963	--	2	2	--	--	-	4
Fall, 1963	--	34	17	2	--	1	54

TABLE 6

MEANS AND STANDARD DEVIATIONS OF PREDICTIVE VARIABLES ON ENTIRE SAMPLE SET

Predictive Variable	Initial Mathematics Fall, 1962			Initial Course Spring, 1963			Non-Mathematics Freshmen		
	$\overline{x}$	s	N	$\overline{x}$	s	N	$\overline{x}$	s	N
HSGPA	2.77	.73	1038	2.60	.77	182	2.69	.74	1016
HSM GPA	2.70	.89	1032	2.43	.88	179	2.33	.90	978
	2.77	.64	1032	2.51	.54	179	2.42	.49	978
SEM.HSM	5.19	1.50	1038	4.39	1.60	182	3.59	1.62	1016
ACTE	20.5	4.5	1038	20.4	5.3	182	20.2	4.7	1016
ACTM	23.5	5.9	1038	21.1	6.4	182	18.2	6.2	1016
ACTSS	22.2	5.4	1038	21.5	5.8	182	20.1	5.7	1016
ACTNS	22.8	5.7	1038	21.1	6.0	182	19.9	5.9	1016
ACTC	22.4	4.6	1038	21.2	5.2	182	19.7	4.8	1016

N = Sample Size

high school mathematics and the earned grade point average in mathematics cover also only grades ten through twelve inclusive. In every case, grades of A, B, C, D and F or WF (withdrawal while failing) have been interpreted numerically as 4, 3, 2, 1 and 0 respectively. The mean and standard deviation of the high school mathematics grade point average have been figured by two methods--the first is a straight average of each student's average in mathematics, the second weighs each average according to the number of semesters of mathematics taken by the student. The variation in sample size for the data on the high school mathematics grade point average indicates forty-seven students did not take any courses in mathematics.

The following hypothesis was formulated and tested for the data of Table 6.

> H_o4: The three samples with respect to each predictive variable are from populations of like means.

Table 7 summarizes the test conditions and results. Comparisons of the means of the high school mathematics grade point average were made only for the data obtained by the first computational method. Further work with this variable will follow this approach.

At the five percent level of significance, every possible pairing of the three samples with respect to the variables--the number of high school mathematics courses, the ACTM, the ACTNS, and the ACTC--shows positive differentiation between the three subsets of freshmen population.

TABLE 7

DETERMINATION OF SIGNIFICANT DIFFERENCES IN MEANS OF VARIABLES FOR THREE SAMPLES

H_o4: $\mu_{fi} = \mu_{si}$ f - set of fall students

$\mu_{fi} = \mu_{ni}$ s - set of spring students

$\mu_{si} = \mu_{ni}$ n - non-math freshmen

i - predictive variables 1,2,3, ... 8

Level of Significance: 5%

Student's t: two-tailed test

$$t = \frac{\bar{x}_k - \bar{x}_\ell}{\sqrt{\frac{s_k^2}{n_k} + \frac{s_\ell^2}{n_\ell}}}, \quad n_k + n_\ell - 2 \text{ degrees of freedom}$$

$k \in \{f,s,n\}$
$\ell \in \{f,s,n\}$
d.f. $\geq$ 1000

	HSGPA	HSM GPA	SEM.HSM	ACTE	ACTM	ACTSS	ACTNS	ACTC
$\mu_{fi} = \mu_{si}$	rej.	rej.	rej.	<u>acc.</u>	rej.	<u>acc.</u>	rej.	rej.
$\mu_{fi} = \mu_{ni}$	rej.	rej.	rej.	<u>acc.</u>	rej.	rej.	rej.	rej.
$\mu_{si} = \mu_{ni}$	<u>acc.</u>	<u>acc.</u>	rej.	<u>acc.</u>	rej.	rej.	rej.	rej.

rej. = rejected; <u>acc.</u> = accepted

All samples are from populations of like means only with respect to the ACTE variable. One notes the highest mean scores with respect to each variable are those of the subset choosing mathematics their initial semester in college. This could be a reflection of the students' natural interests and abilities or the result of the placement policy which tends to discourage those with low ACTM scores from taking mathematics courses by denying full credit in semester hours for remedial level work.

Comparison of Freshmen Vs. Other Mathematics Students

At this point, data comparing the performance of freshmen with all other students taking the same mathematics courses the fall of 1962 have been included. Table 8 presents the mean grades earned in each course by the set of freshmen students and by the set of students classified as sophomores or above. Table 9 shows the percentage of each set earning each grade in each course.

This second set of students is of mixed background with respect to mathematical experience. Some are taking their initial college mathematics; others are just reaching this level of work after successfully completing more remedial courses; many are repeating a course previously failed or dropped. They are a somewhat select group since they have shown the ability to survive at least one year of college work and have been admitted for further study.

Frequently the suggestion is made to delay taking

TABLE 8

MEAN GRADE EARNED BY FRESHMEN VS. NON-FRESHMEN IN ELEMENTARY MATHEMATICS, FALL, 1962

Classification		1	2	5	21	22	Total
Freshmen	$\bar{x}$	1.93	1.76	2.64	1.99	2.26	1.99
	n_i	44	453	114	329	91	1031
Non-Freshmen	$\bar{x}$	2.12	1.67	1.95	1.86	1.89	1.87
	n_i	25	153	247	87	140	652

TABLE 9

PERCENTAGES OF FRESHMEN (F) AND NON-FRESHMEN (U)
AT EACH GRADE LEVEL PER COURSE, FALL, 1962

Grade	1		2		5		21		22	
	F	U	F	U	F	U	F	U	F	U
A	6.8	8.0	9.3	7.8	25.4	10.9	12.8	12.6	17.6	12.1
B	22.7	20.0	19.0	17.0	32.5	19.0	23.1	20.7	26.4	17.1
C	38.6	56.0	32.2	31.4	28.1	36.0	31.3	24.1	31.9	37.9
D	20.5	8.0	17.7	22.2	8.8	21.9	16.1	25.3	13.2	13.6
F-WF	11.4	8.0	21.9	21.6	5.3	12.1	16.7	17.2	11.0	19.3
n_1	44	25	453	153	114	247	329	87	91	140

the more "difficult" course until after the initial year of adjustment in college. If maturity helps and if one learns by continued exposure to the same material, then the freshman set might be expected to have scored lower with respect to grades than this second group. Table 8 shows this occurred only in mathematics <u>1</u>.

The total average grade earned by freshmen in mathematics the fall of 1962 was 1.99; this could be compared to the average grade point of 2.02 earned in all subjects by the entire sample of freshmen of 1962 their initial semester in school [32].

Data on Freshmen in Mathematics, Fall, 1962

The primary interest of this research is not in determining those factors indicative of interest and aptitude in mathematics, but rather the determination of those factors among our available variables which will distinguish most effectively between degrees of mathematical skill and knowledge.

The most valid conclusions should result from data obtained on the subset of students enrolled in mathematics classes their initial semester at the University. Rapid decrease in total enrollment of the freshmen of 1962 is evident after this first semester, and small entries in the course subdivisions would invalidate use of some statistical tools. Moreover, choice of mathematics after the initial enrollment in college is strongly influenced by factors other than the recommendations of an advisor and departmental guidelines. Knowledge of course content, perusal of texts, advice of friends, requirements of a major field, success in other courses, and increased awareness of college expectations weigh heavily in selection of course work during subsequent enrollments.

Thus the subset of one thousand twenty-nine students enrolled in mathematics the fall of 1962 was selected for further study. The freshmen in mathematics 6 were excluded because of the small number in the subset.

This set of fall enrollees in mathematics was then partitioned by courses; the mean and standard deviation of each of the eight predictive variables were determined for each subset. Table 10 summarizes this data.

Examination of this table reveals the mean values of the variables increase as the course level becomes higher. Only between mathematics 5 and 21 are there counterexamples of this trend. These two courses, by designation, require like mathematical background and have identical ranges of ACTM scores as placement guidelines. In further work, grouping of the 5 and 21 samples together will be considered to simplify the data for use in placement functions.

The sharp steady increase of mean values for the course levels with respect to the ACTM and the number of semesters of high school mathematics probably is a reflection of bias resulting from the present placement policy. The reader is cautioned to remain cognizant of the inherent bias

TABLE 10

MEANS AND STANDARD DEVIATIONS OF PREDICTIVE VARIABLES FOR INITIAL MATHEMATICS ENROLLMENT, FALL, 1962

Predictive Variable		Mathematics 1	2	5	21	22	Total
HSGPA	$\bar{X}$	2.12	2.51	2.92	3.04	3.20	2.77
	s	.65	.72	.64	.63	.60	.73
HSM GPA	$\bar{X}$	*1.64	*2.35	*2.84	3.07	3.39	*2.70
	s	.72	.83	.74	.73	.65	.88
SEM.HSM	$\bar{X}$	2.89	4.60	5.46	5.90	6.34	5.19
	s	1.60	1.46	1.15	1.00	1.04	1.51
ACTE	$\bar{X}$	15.3	18.9	22.2	22.1	23.4	20.5
	s	4.4	4.3	3.8	3.6	3.6	4.5
ACTM	$\bar{X}$	11.2	19.6	26.0	27.4	31.4	23.5
	s	3.9	3.1	3.3	3.7	3.2	5.9
ACTSS	$\bar{X}$	16.4	19.8	24.3	24.4	26.7	22.2
	s	4.9	5.2	4.1	4.4	3.7	5.5
ACTNS	$\bar{X}$	15.9	20.0	23.9	25.8	27.4	22.7
	s	5.3	5.3	4.3	4.0	3.9	5.7
ACTC	$\bar{X}$	14.8	19.7	24.2	25.0	27.4	22.4
	s	3.5	3.5	2.9	3.1	2.9	4.6
N_1		44 *39	451 *449	114 *113	329	91	1029 *1021

The $\bar{X}$'s which are preceded by (*) refer to the N_1's which are preceded by (*).

in our sample as he examines the data and conclusions of this study. A correlation matrix was then generated between each predictive variable and the grade criterion. The "student's t" test was applied to determine which of the correlations differed significantly from 0. Table 11 presents both these data and the test conditions.

This table shows the correlation coefficients of the ACTM with our course grade criterion, while different from zero at the five percent level of significance at each course level, vary from .268 to .399, a low range of values. Moreover, the correlation of the number of semesters of high school mathematics is not different from zero for four of the five subsets. The high school grade point average and the high school mathematics grade point average have the highest correlations with grade criterion at every course level. This is in agreement with similar studies [15].

But the amount of correlation between a variable and the course grade does not necessarily indicate the power of the variable in classifying students in different courses according to their abilities and skills. Thus it is necessary to examine the statistics on the subset of properly placed students in our fall sample to determine which variables distinguish between the given courses with a minimum overlap in the ranges of values characteristic of each course level. Table 12 presents the mean and standard deviation statistics for each subdivision of this sample. The reader is reminded of the previous decision to denote the subset of students

TABLE 11

CORRELATIONS OF PREDICTIVE VARIABLES VS. COURSE GRADE IN INITIAL MATHEMATICS ENROLLMENT FALL, 1962

Predictive Variable	r_{ij} Mathematics 1	2	5	21	22
HSGPA	.492	.526	.544	.464	.545
HSM GPA	.499	.526	.500	.453	.481
SEM.HSM	.182	.263	.081	.008	.078
ACTE	.344	.319	.213	.220	.260
ACTM	.268	.399	.304	.373	.398
ACTSS	.274	.188	.167	.161	.381
ACTNS	.188	.262	.062	.242	.242
ACTC	.340	.339	.233	.309	.399
N	44	451	114	329	91

H_o5: $r_{ij} = 0$

$i \in \{1,2, \ldots 8\}$

$j \in \{1,2,5,21,22\}$

Level of significance: 5%

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$$, N - 2 degrees of freedom, two-tailed test

H_o: Accepted for r_{31}, r_{51}, r_{61}, r_{71}, r_{35}, r_{65}, r_{75}, $r_{3(21)}$, $r_{3(22)}$ and rejected in every other case.

TABLE 12

MEANS AND STANDARD DEVIATIONS OF PREDICTIVE VARIABLES FOR PROPERLY PLACED FRESHMEN IN MATHEMATICS, FALL, 1962

Predictive Variable		Mathematics 1	2	5	21	5-21	22
HSGPA	$\bar{x}$	2.23	2.67	2.81	3.10	3.02	3.24
	s	.67	.62	.57	.58	.59	.52
HSM GPA	$\bar{x}$	1.52	2.51	2.75	3.17	3.05	3.44
	s	.78	.70	.65	.69	.71	.54
SEM.HSM	$\bar{x}$	3.11	4.81	5.57	5.89	5.80	6.42
	s	1.65	1.29	1.13	1.06	1.09	.99
ACTE	$\bar{x}$	16.2	19.4	21.7	22.3	22.1	23.3
	s	4.2	4.0	3.7	3.3	3.5	3.7
ACTM	$\bar{x}$	11.9	20.3	25.6	27.8	27.1	31.6
	s	3.3	2.7	3.1	3.5	3.6	2.7
ACTSS	$\bar{x}$	17.5	19.9	24.0	24.5	24.4	27.1
	s	4.1	4.8	3.7	4.4	4.2	3.4
ACTNS	$\bar{x}$	17.0	20.5	23.3	26.2	25.4	27.5
	s	5.8	4.8	4.3	3.8	4.1	3.7
ACTC	$\bar{x}$	15.8	20.1	23.7	25.3	24.8	27.5
	s	3.0	3.0	2.6	2.9	2.9	2.6
N		27	232	69	179	248	53

earning B's and C's as properly placed.

In Table 13, the following hypothesis is tested.

H_o6: The subsets of the sample are from populations of equal means with respect to each predictive variable.

Table 13 indicates the high school grade point average in mathematics, the number of semesters of high school mathematics, the ACTM, the ACTNS, and the ACTC are most effective in distinguishing between courses by the mean statistic. The ACTE and the high school grade point average are the least effective considering every course division.

Using the grouping of the 5 and 21 students together and then comparing the means of subsets 1, 2, 5-21, and 22, there are differences at the five percent level of significance for every variable. A t-test applied to the means of the 5 and 5-21 subsets and to the 5-21 and 21 subsets shows eleven of the possible sixteen comparisons of means involve samples from populations of equal means at the five percent level of significance. Considering this and the like requirements of mathematics 5 and 21 with respect to background skills, data on these courses will be combined for all further analyses.

Illustrations 1 and 2 present graphical evidence of the amount of overlap in the range of each variable for the different courses according to the statistics computed on the properly placed subset of students. The mean, interquartile range, and an interval of two standard deviations centered about the mean have been indicated by |, ∿∿∿, and

TABLE 13

DETERMINATION OF SIGNIFICANT DIFFERENCES IN MEANS FOR PREDICTIVE VARIABLES

H_o6: $\mu_{ik} = \mu_{i\ell}$

$i \in \{1, 2, \ldots 8\}$

$(k,\ell) \in \{(1,2),(2,5),(5,21),(21,22),(2,5\text{-}21),(5\text{-}21,22)\}$

Level of significance: 5%

Student's t: two-tailed test

$$t = \frac{\bar{x}_k - \bar{x}_\ell}{\sqrt{\frac{s_k^2}{N_k} + \frac{s_\ell^2}{N_\ell}}}, \quad N_\ell - N_k - 2 \text{ degrees of freedom}$$

Predictive Variable	1-2	2-5	5-21	21-22	2-5,21	5,21-22
HSGPA	rej.	acc.	rej.	acc.	rej.	rej.
HSM GPA	rej.	rej.	rej.	rej.	rej.	rej.
SEM.HSM	rej.	rej.	rej.	rej.	rej.	rej.
ACTE	rej.	rej.	acc.	acc.	rej.	rej.
ACTM	rej.	rej.	rej.	rej.	rej.	rej.
ACTSS	rej.	rej.	acc.	rej.	rej.	rej.
ACTNS	rej.	rej.	rej.	rej.	rej.	rej.
ACTC	rej.	rej.	rej.	rej.	rej.	rej.

rej. = reject and acc. = accept

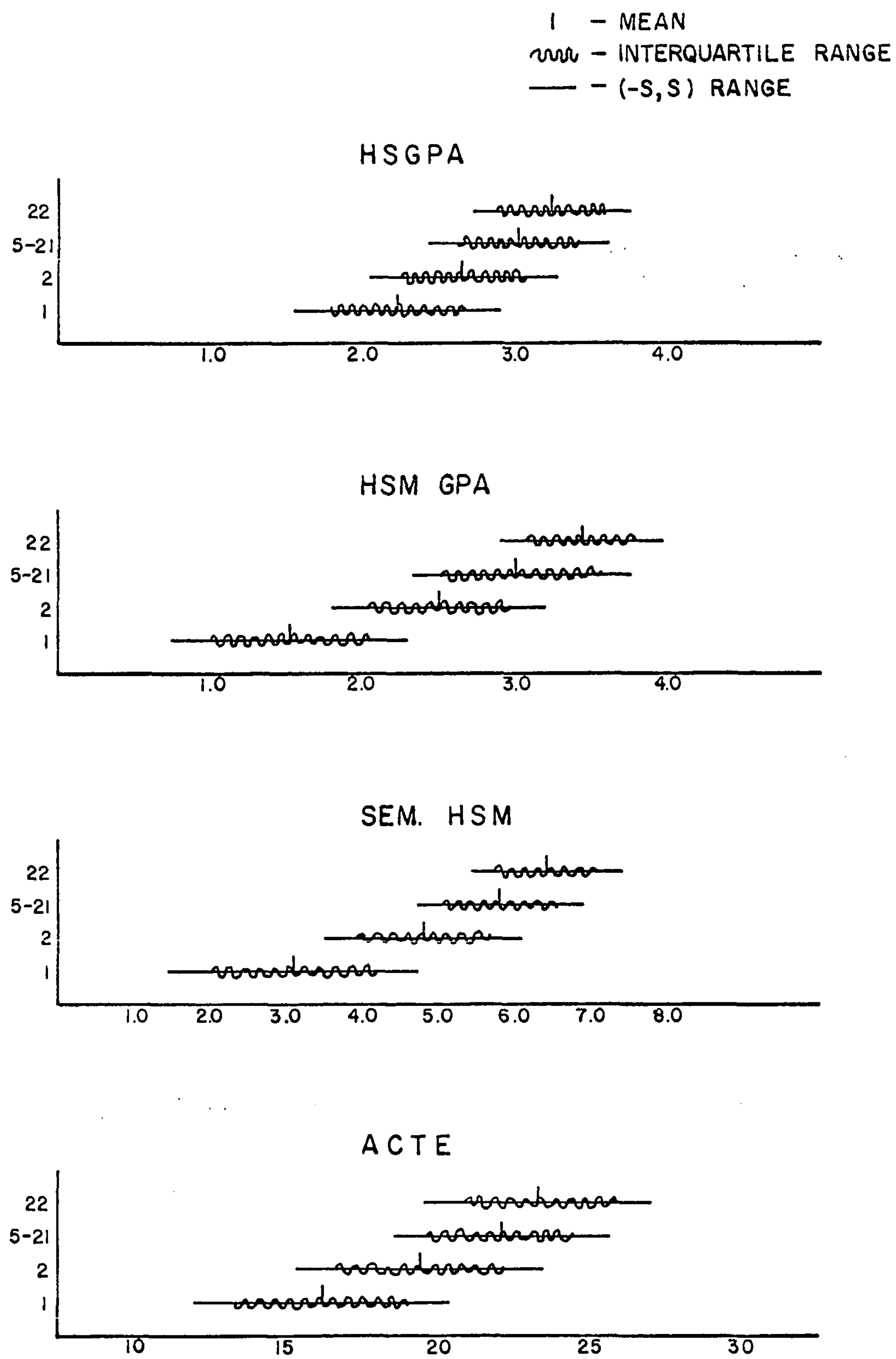

Figure 1.--Data on Ranges of Predictive Variables for Properly Placed Students in Mathematics
Fall, 1962

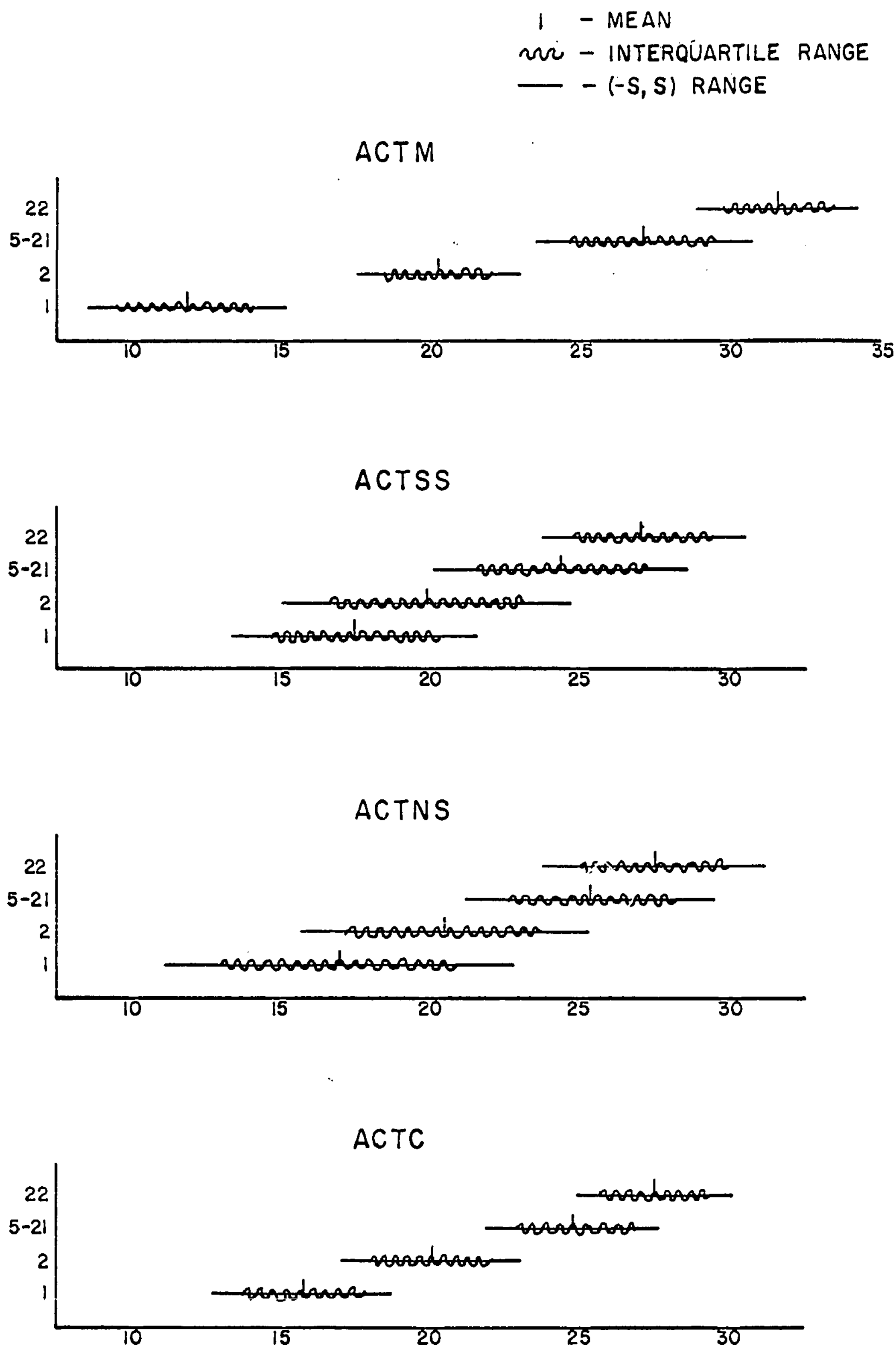

Figure 2.--Data on Ranges of Predictive Variables for Properly Placed Students in Mathematics Fall, 1962

___ respectively. Clearly the ACTM scores have a minimum amount of overlap in range of values for the different course levels. The original placement of the majority of this set by ACTM scores could account for this. The graphs do indicate that the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics are also partially effective in classifying the students into different courses, especially at the mathematics 1, 2, and 5-21 levels. Since a departmental test provides further screening between the 5-21 and 22 subsets, the lower effectiveness (rather the increased amount of overlap in ranges of values) at this level is of secondary importance.

Therefore, if a single variable among those available on entering freshmen is to determine placement guidelines, it should be one of the four mentioned above. The next division of this study will compare the effectiveness of each of these four variables in classifying the students in our fall sample who received grades of A, D, or F in their mathematics courses.

Comparison of the Effectiveness of Certain Predictive Variables in Placement

The ACTM, the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics appear to be the single variables of greatest value in distinguishing between degrees of mathematical skills and knowledge. Therefore, their effectiveness as placement guidelines in mathematics should be compared.

To do this, first the means and standard deviations of properly placed students in each course were utilized to determine the ranges of value for each variable dictating classification within each course. Then every freshman earning a grade of A, D, or F in mathematics during the 1962 fall semester was placed in a mathematics course by strict adherence to each guideline determined by one of the predictive variables. Table 14 summarizes the placement of this set by each of the four guidelines.

The suggested placement policy of the University of Oklahoma Mathematics Department shows a slight deviation in the ranges of the intervals of the ACTM scores for mathematics 1 and 2 compared to the ranges determined by use of the mean and standard deviation statistics. It also stipulates certain course pre-requisites for mathematics 5, 21 and 22. Therefore, classification of the A, D, and F subsets was done in compliance with the suggestions of the present placement program. Since the data on the students include only the number of semesters of mathematics taken by each student, and the transcripts contain such a variety of course titles, the exact nature of the high school mathematics taken by each student was impossible to ascertain. Therefore, three or more semesters of mathematics (from tenth grade up) have been considered as fulfilling the course pre-requisites for mathematics 5 and 21, and six or more semesters of high school mathematics comply with the requirements for mathematics 22. Table 15 summarizes placement by this combination of the

TABLE 14

SUGGESTED PLACEMENT OF FRESHMEN EARNING A, D, AND F GRADES BY SINGLE VARIABLES: FALL, 1962

Actual Course Completed	Grade	Suggested Placement by ACTM				Suggested Placement by ACTC				Suggested Placement by HSM GPA				Suggested Placement by SEM.HSM			
		Intervals of Classification				Intervals of Classification				Intervals of Classification				Intervals of Classification			
		0-16	17-23	24-29	30-36	0-17	18-22	23-26	27-36	0-2.04	2.05-2.73	2.74-3.27	3.28-4.00	0-4	5	6	7-8
		1	2	5-21	22	1	2	5-21	22	1	2	5-21	22	1	2	5-21	22
1	A	2	1			2	1					3		3			
	D	9				9				9				9			
	F	5				5				5				4	1		
2	A	2	34	6		5	16	17	4	5	2	7	28	15	4	21	2
	D	7	69	4		26	42	11	1	51	18	7	4	48	7	22	3
	F	31	66			41	39	16	1	74	13	7	3	65	9	19	4
5	A		3	18	8		3	14	12	1	1	11	16	9	2	16	2
	D		1	8	1		4	3	3	5	2	2	1	4	1	4	1
	F		2	4		1		4	1	3	3			1	2	3	
21	A		2	13	27		2	16	24		3	4	35	6	3	26	7
	D		12	32	9	2	13	26	12	9	17	13	14	4	5	39	5
	F		12	33	10	2	15	26	12	13	17	16	9	6	3	39	7
22	A			1	15			1	15			2	14	1		9	6
	D			3	9	1	2	3	6	2	1	4	5	1	1	7	3
	F		1	4	5		2	4	4	2	3	2	3	2		5	3

TABLE 15

SUGGESTED PLACEMENT OF FRESHMEN EARNING A, D, AND F GRADES BY PRESENT PLACEMENT POLICY: FALL, 1962

Actual Course Completed	Grade	Suggested Placement			
		1	2	5-21	22
1	A	2	1		
	D	9			
	F	5			
2	A	1	35	6	
	D	3	73	4	
	F	16	81		
5	A		3	22	4
	D		2	7	1
	F		2	4	
21	A		2	18	22
	D		12	33	8
	F		12	36	7
22	A			2	14
	D			5	7
	F		1	5	4
ACTM Classification Intervals		0-15	16-23	24-29	30-36
SEM.HSM Classification Intervals		-	-	≥ 3	≥ 6

ACTM and course requirements.

To compare these placement tables, the following assumptions were made.

1. A student will be considered "clearly misplaced" if by these guidelines: (a) he is placed in a lower course level than the one in which he earned an A, (b) he is placed in a higher course level than the one in which he earned a D or F grade.
2. "Improved placement" by a variable is defined in terms of possible reduction of the number of F grades. Therefore, a student is considered better placed if by adherence to the guidelines set by the variable, he has been placed in a lower level course than the one in which he earned a failing grade.
3. Combining these concepts, "probable misplacement" will include all students falling in category 1 as well as all F students placed in the same course by the variable as the one in which they failed. The students in <u>1</u> with failing grades will not be counted in this category since it is the most elementary mathematics course offered.

Table 16 compares the five guidelines with respect to the above concepts. By use of the z distribution and a one-tailed test at the five percent level of significance, the samples from populations of equal proportions with respect to each of the factors under consideration were determined. This information was utilized to rank each variable with

TABLE 16

COMPARISON OF EFFECTIVENESS OF CERTAIN VARIABLES AS PLACEMENT GUIDELINES

Sample: Set of 470 Freshmen Earning A's, D's, or F's, in Mathematics, Fall, 1962

Predictive Variable		Course					Total	
		1	2	5	21	22	No.	%
ACTM	CM	0	6	4	21	1	32	6.1
	IP	0	31	2	12	5	50	10.6
	PM	0	72	8	54	6	140	30.2
ACTC	CM	0	34	7	26	1	68	14.5
	IP	0	41	1	17	6	65	13.8
	PM	0	73	11	52	4	140	30.2
HSM GPA	CM	0	26	3	26	2	57	12.1
	IP	0	74	6	30	7	117	24.9
	PM	0	39	3	42	5	89	19.4
SEM.HSM	CM	1	63	12	21	10	107	22.8
	IP	0	65	3	9	7	84	17.9
	PM	1	72	15	60	12	160	34.0
ACTM & SEM.HSM	CM	0	5	4	17	2	28	6.0
	IP	0	16	2	12	6	36	7.7
	PM	0	86	8	53	5	152	32.8

CM - clear misplacement; IP - improved placement;
PM - probable misplacement

RANKING BY ABOVE INFORMATION

	ACTM	ACTC	HSM GPA	SEM.HSM	Present Policy
Minimizing CM	1	2	2	3	1
Maximizing IP	2	2	1	2	3
Minimizing PM	2	2	1	2	2

respect to its effectiveness in minimizing misplacement and maximizing the number of improved placements.

Table 16 indicates the high school mathematics grade point average might be as effective a guideline as the ACTM for placing students in the course at which they can successfully achieve as measured by grade criterion. Certainly its correlation with the grade criterion is higher and it places a higher percentage of the students making F in their present course at a lower course level, thus possibly reducing the number of failing grades.

To determine if the placement guidelines as dictated by the ACTM and the high school mathematics grade point average are equivalent, the following hypothesis was tested at the five percent level of significance:

> $H_o\underline{7}$: Placement of the sample of A, D, and F students in mathematics by the ACTM and the high school mathematics grade point average guidelines indicates these classification methods are similar.

The Chi-square method used to test this statement requires that each element be classified in precisely one category, thus the divisions used vary slightly from the previous ones. Table 17 summarizes this work.

Table 17 indicates these are different classification guidelines. Unless further investigation reveals definite reasons to choose one over the other, each has arguments in its favor for selection as the best single variable to determine placement in mathematics.

TABLE 17

COMPARISON OF THE ACTM AND THE HSM GPA AS CLASSIFICATION GUIDELINES

$i = 1,2$ $j = 1,2,3$ f_{ij}	(1)			(2)			$\sum_{j=1}^{3} f_{ij}$
	CM	IP	PP	PM	IP	PP	
ACTM	32	50	388	140	50	280	470
HSM GPA	57	117	296	89	117	264	470

CM - clear misplacement, IP - improved placement
PP - present placement, PM - probable misplacement

H_o7: The ACTM and the HSM GPA are similar classification methods using either divisions (1) or (2).

Level of Significance: 5%

Chi-square test

$$\chi^2 = \sum_{j=1}^{3} \frac{(f_{1j} - f_{2j})^2}{f_{1j} + f_{2j}}$$ with 2 degrees of freedom

f_{ij} = number of students in each category, $i \in \{1,2\}$, $j \in \{1,2,3\}$

(1) $\chi^2 = 41.78$

(2) $\chi^2 = 38.71$

H_o: rejected for both (1) and (2)

It appears a combination of several of these variables might prove still more effective as a guideline than any of the above variables. This possibility will be explored in Chapter III.

CHAPTER III

THE SEARCH FOR AN IMPROVED PLACEMENT POLICY

Selection of the Type of Function

The generation of a more accurate guideline to use in the placement of freshmen in their initial mathematics courses at the University of Oklahoma than is presently available will be explored in this chapter. This guideline will take the form of mathematical functions which will depend upon several combinations of the predictive variables. Again data available on the set of "properly placed" freshmen enrolled in mathematics the fall semester of 1962 will be utilized in determining these functions. The assumption is made that the entering student can be considered properly placed when a function determining placement puts him in the course taken by the group of well placed students that he is "most alike" in terms of measurable predictive variables.

The individual variables have defects as placement tools for none are highly correlated with the performance of the students as measured by the grades earned in any of these courses, and moreover, the ranges of scores for any one variable among successfully performing students in the different courses overlapped considerably. Thus a new function for placement must be correlated as highly as possible with the probable performance of the students as measured by the grade criterion, and its range of values in the different courses (as determined by students performing satisfactorily) should display a minimum amount of overlap as measured by the standard deviation of the variable in each course.

R. A. Fisher [14] originated a discriminant function which is suitable for classifying elements into two sets; this function is based on a combination of measurements available on all elements in the sets under study. The linear function is constructed so that it maximizes the ratio of the difference between means in the two groups to the standard deviation within the groups. This function when evaluated for the known measurements of an element determines the membership of that element in one of the given groups. It will result in a minimum amount of misplacement even when there is wide overlap in the ranges of the independent variables of measurement.

Such a discriminant function, based on the single variables of greatest value as placement tools, appears the logical choice for an accurate, practical placement guideline. Since classification is sought for four rather than two groups, it will be necessary to generate three discriminant functions. Each of these functions will dictate a boundary value between two sets. The first function will determine membership in either mathematics 1 and 2 or in mathematics 5-21 and 22, the second separating the students into mathematics 1 or 2, the last dividing the 5-21 and 22 set into either 5-21 or 22. Each of these functions when evaluated for the predictive variables known on an entering freshman can then be used to decide his correct course placement in mathematics.

An outline of the mathematical development of a discriminant function and a sample calculation of one such function is included in Appendix V.

Choice of the Independent Variables

In determining these functions, choice of the independent variables was of prime importance. One needed a reasonable number of predictive variables with the following characteristics: (1) high correlation of each variable with the grade criterion, (2) a minimum amount of intercorrelation of the variables within each group, (3) a significant difference in the means of the variables between the groups, (4) a minimum amount of overlap of the probable ranges of each variable between the specified groups.

Characteristics (1), (3) and (4) have already been examined with respect to the eight available measurements in the analysis of data undertaken in Chapter II. Table 11 indicated the high school grade point average and the high school mathematics grade point average correlated highest of all the variables with the grade criterion in each course. The high school mathematics grade point average, the number of semesters of high school mathematics, the ACTM, the ACTNS, and the ACTC had significant differences between the means for each pair of courses, as shown by the data in Tables 12 and 13. Illustrations 1 and 2 show the minimum overlap in ranges occurs with the ACTM and ACTC variables.

To determine characteristic (2)--the amount of intercorrelation between the variables--an eight by eight matrix of intercorrelation coefficients was generated for the available predictors. Table 18 presents these data.

Study of Table 18 shows that the highest intercorrelations are between elements of the following pairs: (HSGPA, HSM GPA), (ACTE,ACTC), (ACTSS,ACTC), and (ACTNS,ACTC). Therefore, inclusion as independent variables of both elements of any one pair would add relatively little in increased power to the function. The number of semesters of high school mathematics is the variable whose correlation with any of the other predictive variables is low. Thus use of it in a discriminant function with a combination of the other variables might add considerably to the function's discrimination power since new relationships would be considered.

Finding no one variable meeting all four of the desirable characteristics, the investigator has chosen the ACTM, the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics as the variables fulfilling the majority of the desirable qualities and are thus suitable as the independent variables of the discriminant functions. These are the same predictive variables which appeared most effective as single

TABLE 18

INTER-CORRELATION COEFFICIENTS OF THE PREDICTIVE VARIABLES FOR FRESHMEN, FALL SEMESTER, 1962 IN MATHEMATICS 1, 2, 5, 21, AND 22

		HSGPA	HSM GPA	SEM.HSM	ACTE	ACTM	ACTSS	ACTNS	ACTC
HSGPA	1	1.000	.596	.195	.399	-.114	.126	.135	.190
	2	1.000	.852	.063	.375	.257	.175	.246	.329
	5	1.000	.881	.079	.431	.251	.306	.223	.421
	21	1.000	.862	-.054	.297	.272	.146	.179	.275
	22	1.000	.802	-.055	.472	.454	.469	.431	.574
HSM GPA	1		1.000	.501	.009	.009	.065	.150	.128
	2		1.000	.125	.261	.261	.050	.130	.208
	5		1.000	.190	.275	.275	.232	.102	.295
	21		1.000	-.035	.293	.293	.063	.125	.211
	22		1.000	-.032	.503	.503	.404	.296	.487
SEM.HSM	1			1.000	-.100	-.127	-.003	-.125	-.099
	2			1.000	-.060	.206	-.045	.050	.022
	5			1.000	-.149	-.067	-.023	-.029	-.095
	21			1.000	-.137	-.029	-.092	-.068	-.106
	22			1.000	-.006	-.044	-.086	-.118	-.075
ACTE	1				1.000	.267	.609	.590	.785
	2				1.000	.342	.526	.547	.772
	5				1.000	.250	.447	.597	.800
	21				1.000	.412	.498	.464	.796
	22				1.000	.467	.527	.530	.807
ACTM	1					1.000	.461	.177	.592
	2					1.000	.295	.347	.543
	5					1.000	.235	.143	.502
	21					1.000	.413	.442	.713
	22					1.000	.454	.406	.713
ACTSS	1						1.000	.540	.825
	2						1.000	.689	.838
	5						1.000	.559	.765
	21						1.000	.645	.826
	22						1.000	.513	.805
ACTNS	1							1.000	.793
	2							1.000	.863
	5							1.000	.813
	21							1.000	.806
	22							1.000	.786
ACTC	1								1.000
	2								1.000
	5								1.000
	21								1.000
	22								1.000

Numbers of Students in Each Subset

1	44
2	451
5	114
21	329
22	91

predictors of proper placement.

Discriminant Functions of Four Variables

To classify students into four separate courses, three discriminant functions are necessary. They were determined by the analysis of data on the following subsets of properly placed freshmen: (1) the students in mathematics 1 and 2, (2) those in mathematics 2 and 5-21, and (3) those in mathematics 5-21 and 22. The predictive factors used were: (1) ACTM, (2) ACTC, (3) the high school mathematics grade point average, and (4) the number of semesters of high school mathematics. In further discussion each discriminant function will be referred to as α_{ij}, where i and j are natural numbers, i denoting the particular set of courses separated by the function as numbered above, and j indicating the number of independent variables. x_1, x_2, x_3, and x_4 will refer to the above listed variables respectively. Therefore, $\alpha_{32} = 16x_2 + 5x_3$ is a discriminant function with i = 3 indicating the courses referred to are mathematics 5-21 and 22, j = 2 indicating α_{32} is a function of two variables, and x_2 and x_3 representing the ACTC and the high school mathematics grade point average, respectively.

The reader is again referred to Appendix V where the mathematical development and the method of calculation of a discriminant function are discussed. By similar calculations the following discriminant functions were determined for the groups and variables previously described.[7]

$$\alpha_{14} = .003768x_1 + .000264x_2 + .006276x_3 + .002497x_4$$
$$\alpha_{24} = .001089x_1 + .000462x_2 + .001186x_3 + .001223x_4$$
$$\alpha_{34} = .001072x_1 + .000326x_2 + .001285x_3 + .001936x_4$$

Now we have three new variables α_{14}, α_{24}, and α_{34} dependent for their range of values on the variables x_i. Each function α_{14}, α_{24}, and α_{34} also determine a constant a_{i4} (i=1,2,3) which acts as a boundary point separating the ranges of the variable α_{i4} in the two courses of the i^{th} set. For the given functions, a_{14} = .086861, a_{24} = .044996, and a_{34} = .056697. A sample calculation of a_{i4} is also included in Appendix V.

Now we have sufficient tools to place an entering freshman in mathematics 1, 2, 5-21, or 22 by these discriminant functions α_{i4} (i=1,2,3). The following procedure is used.

(1) The student's scores on the ACTM and the ACTC, his high school mathematics grade point average, and the number of semester hours of high school mathematics taken must be known and used to evaluate the dependent variable α_{24}. This number will determine placement in either mathematics 2 and lower or in mathematics 5-21 and higher, dependent on its order relationship with a_{24} = .044996. Two alternatives result at this point.

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[7]Eight significant figures were carried in all IBM computer work.

(2) If α_{24} is greater than or equal to a_{24}, then the student's predictive measurements are used in evaluating α_{34}. If this number α_{34} is greater than or equal to $a_{34} = .056697$, then the student is placed in mathematics 22. If α_{34} is less than a_{34}, the suggested placement is either 5 or 21 dependent on his interests or purposed major.

(2') If α_{24} is less than a_{24}, the α_{14} function is evaluated for the student's x_i values. If this number α_{14} is less than $a_{14} = .086861$, placement in mathematics 1 is recommended. If α_{14} is greater than or equal to a_{14}, then placement in mathematics 2 is preferred.

Selection of equal unit coefficients for each variable in these functions would simplify manual computation of these placement numbers. It appears this would partially cancel the effectiveness of the equations since the variation of the coefficients shows the dependence of the functions on the specific classes under consideration.

It is logical, however, to use the computer as a tool here, making rapid accurate computation feasible for large numbers of students. In Appendix IV is included the one program necessary to both compute the placement constants on any individual student and give the suggested course placement according to the division points, a_{i4}.

Effectiveness of the Discriminant Functions of Four Variables

The question of the effectiveness of these functions in discriminating between membership in the different mathematics courses must be faced. By an analysis of variance table, the following hypothesis has been tested at the five percent level of significance.

> H_o8: Each pair of courses under consideration is homogeneous with respect to the variable α_{i4} related to that pair.

Table 19 summarizes the testing for mathematics 1 and 2 with α_{14}, for mathematics 2 and 5-21 with α_{24}, and for mathematics 5-21 and 22 with α_{34}.

Therefore, these functions α_{i4} are significantly distinguishing between membership in each pair of courses on the basis of "alikeness" with respect to a combination of the ACTM, the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics.

The effectiveness of these discriminant functions in placement must also be compared with the effectiveness of the single variables in this role. Desirable features of classification functions are: (1) high correlation of the function with the grade criterion in each course, (2) significant differences between the means of the variable for the courses under consideration, and (3) a minimum amount of overlap in the probable ranges of the function among the different courses. The effectiveness of the single variables

TABLE 19

ANALYSIS OF VARIANCE OF α_{i4}, (i=1,2,3)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Hypothesis
Between 1 and 2 Within 1 and 2	n_2 = 254 n_1 = 4	D = .00424233 C = .00043527	E = .00001670 G = .00010882	6.5152316	Rejected for α_{14}
Between 2 and 5-21 Within 2 and 5-21	n_2 = 475 n_1 = 4	D = .00120814 C = .00017496	E = .00000254 G = .00004374	17.196835	Rejected for α_{24}
Between 5-21 and 22 Within 5-21 and 22	n_2 = 296 n_1 = 4	D = .00119406 C = .00006226	E = .00000404 G = .00001557	3.8584944	Rejected for α_{34}

H_o8: Each pair of classes is homogeneous with respect to α_{i4}

Level of significance: 5%

Analysis of Variance - F test

$$F = \frac{N_1 \cdot N_2}{N_1 + N_2}(N_1 + N_2 - n_1 - 1)D$$

with n_1 and n_2 degrees of freedom

H_o: rejected in every case

n_1, number of variables = 4

$n_2 = N_1 + N_2 - 5$ where N_1 is number in the first set, and N_2 is number in the second set

$$d_i = \bar{x}_{2i} - \bar{x}_{1i}$$

$$D = \sum_{i=1}^{4} \alpha_{i4} d_i$$

$$C = D^2 \frac{N_1 N_2}{N_1 + N_2}$$

$E = D/n_2$

$G = C/n_1$

$F = G/E$

was tested with respect to each of the above characteristics in Chapter II. Examination of the discriminant function with respect to these features is now necessary.

Table 20 evaluates characteristic (1)--the amount of correlation the α_{14} variables show with the grade criteria for the set of all freshmen enrolled in mathematics the fall semester of 1962. Correlation coefficients for the grades earned in mathematics 2 and 5-21 subsets have been calculated against a pair of α_{14} since a combination of these functions determines membership (the upper and lower boundary points of the range of values for classification) in the course.

Comparing these correlations with those determined between the individual predictive variables and grades is difficult with the 5-21 grouping and the pair of coefficients involved in two of these courses. Noting the range of the correlation coefficients with the grade criterion is .268 to .399 for the ACTM, .453 to .526 for the high school mathematics grade point average, and .331 to .557 for these discriminant functions,[8] similar and better correlations appear evident for the high school mathematics grade point average and for the discriminant functions than for the ACTM.

To test characteristic (2)--significance difference between the means of α_{14} in the i^{th} set of courses--the means and standard deviations of α_{14} were calculated for each course with the sample of "properly placed" freshmen.

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[8]Table 11 of Chapter II.

TABLE 20

CORRELATIONS OF α_{i4} FUNCTIONS WITH GRADES IN MATHEMATICS FOR FRESHMEN, FALL, 1962

Course	No. of Students	Correlation	α_{i4}
1	44	.454	α_{14}
2	451	.557 .533	α_{14} α_{24}
5-21	443	.346 .331	α_{24} α_{34}
22	91	.461	α_{34}

Table 21 summarizes this data and tests the following hypothesis:

> H_o9: Each member of the pair of subsets (1,2), (2,5-21), and (5-21,22) is from a population of equal means with regard to the α_{14}, α_{24}, and α_{34} variables respectively.

Table 21 indicates the means of α_{14} are different for each course (as were the means of each of the four single variables selected as possible predictive tools), thus the second of the desirable features of a good placement function is satisfied by the α_{14}'s.

The third feature of a good placement variable is a minimum amount of overlap in the probable ranges of the variable for the different courses. Graphical evidence of the amount of overlap of the interquartile ranges and the standard deviation intervals is presented in Illustration 3. The mean, interquartile range, and an interval of two standard deviations centered about the mean are symbolized by |, ∿∿, and ___ respectively. Each value has been multiplied by 100 to simplify the graphing units.

This last table confirms, by comparison with Illustrations 1 and 2 of Chapter II, that the range of these discrimination functions are separated equally or more between pairs of courses, as determined by the number of standard deviations, than with any single predictive variable.

It appears the discriminant functions do have the desirable features of a placement function to an equal or better degree than any of the single variables which might

TABLE 21

MEANS AND STANDARD DEVIATIONS OF α_{14} FOR PROPERLY PLACED FRESHMEN IN MATHEMATICS, FALL, 1962

α_{14}		Mathematics Course			
		1	2	5-21	22
α_{14}	$\bar{x}$	.06628	.10952		
	s	.01189	.01309		
α_{24}	$\bar{x}$		.04025	.05174	
	s		.00453	.00523	
α_{34}	$\bar{x}$			.05234	.05970
	s			.00518	.00378
N		27	232	248	53

$H_o 9$: $u_{14} = u_{24}$ for α_{14}

$u_{24} = u_{(5-21)4}$ for α_{24}

$u_{(5-21)4} = u_{(22)4}$ for α_{34}

Level of significance: 5%

$$t = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}, \quad N_1 + N_2 - 2 \text{ degrees of freedom}$$

H_o: rejected in every case

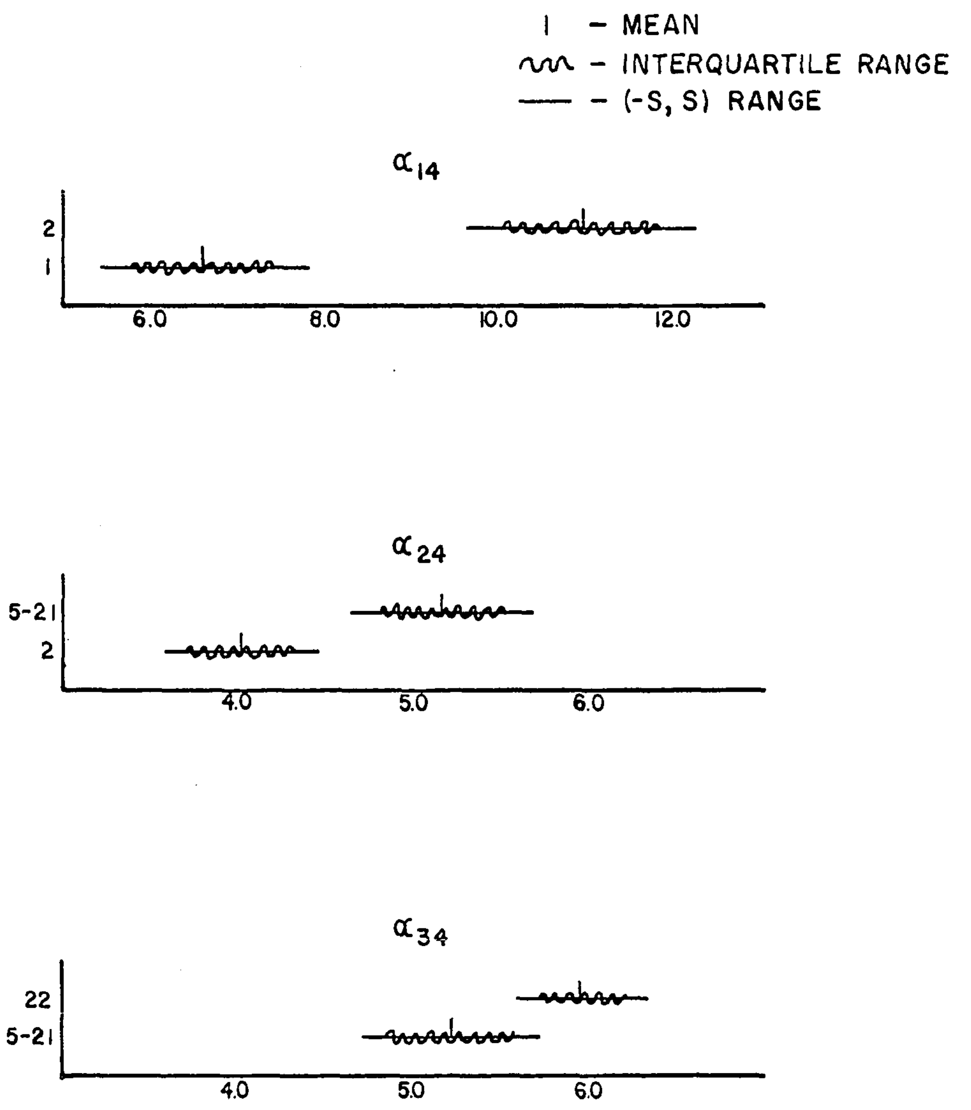

Figure 3.--Data on Ranges of Discriminant Functions for Properly Placed Students: Fall, 1962

be used as placement tools. Since the ACTM and the high school mathematics grade point average were the best single variables to use as placement guidelines for the A, D, and F sets of students of the fall semester, comparison of the α_{i4} with these variables in this role is of interest. Table 22 shows the proposed placement of the above set by strict adherence to the guidelines set by the α_{i4}'s, the discriminant functions. Table 23 summarizes the number of students falling in the categories of (1) clear misplacement, (2) improved placement, and (3) probable misplacement as judged by their actual performance in mathematics that semester.[9] Finally, Table 24 displays the necessary information to determine if the placement of this set is significantly different from the purposed placement of the same set by either the ACTM or the high school mathematics grade point average guidelines. This is done by testing the following hypothesis by a Chi-Square test at the five percent level of significance:

> H_o10: The proportions of freshmen in each placement classification as dictated by a single predictive variable are the same as the proportions determined by the discriminant functions α_{i4}.

Application of the Chi-Square test requires each element in the sets under consideration be classified in exactly one category. This condition is reflected in the chosen divisions in Table 24.

In Chapter II, we concluded the ACTM and the high

---

[9] The reader is referred to page 59 for definition of these terms.

TABLE 22

SUGGESTED PLACEMENT OF FRESHMEN EARNING A, D, AND F GRADES THE FALL SEMESTER, 1962 BY THE DISCRIMINANT FUNCTIONS, α_{14}, α_{24}, AND α_{34}

		Mathematics Course			
		1	2	5-21	22
1	A	2	1		
	D	9			
	F	5			
2	A		25	17	
	D	9	67	4	
	F	36	60	1	
5	A			25	4
	D		4	5	1
	F		2	4	
21	A			17	25
	D		7	41	5
	F		10	40	5
22	A			1	15
	D		1	4	7
	F		1	6	3
Boundary Values		a_{14} .086861		a_{24} .044996	a_{34} .056697

TABLE 23

SUMMARY OF THE PLACEMENT DATA OF TABLE 22

	1	2	5	21	22	Total No.
CM	0	5	1	10	1	17
IP	0	36	2	10	7	55
PM	0	65	5	50	3	123

CM - clear misplacement
IP - improved placement
PM - probable misplacement

TABLE 24

COMPARISON OF PLACEMENT OF FRESHMEN IN MATHEMATICS BY ACTM OR HIGH SCHOOL MATHEMATICS GRADE POINT AVERAGE WITH PLACEMENT BY DISCRIMINANT FUNCTIONS α_{i4}'s

i=1,2,3 j=1,2,3 f_{ij}	(1)			(2)			$\sum_{j=1}^{3} f_{ij}$
	CM	IP	PP	PM	IP	PP	
ACTM	32	50	388	140	50	280	470
HSM GPA	57	117	296	89	117	264	470
D.F. α_{ij}'s	17	55	398	127	55	288	470

D.F. - Discriminant Function
CM - Clear Misplacement
IP - Improved Placement
PP - Present Placement
PM - Probable Misplacement

H_o10: a. Placement by ACTM and by D.F. α_{ij}'s are equal by (1) and (2) classifications.

b. Placement by HSM GPA and by D.F. α_{ij}'s are equal by (1) and (2) classifications.

Level of Significance: 5%

$$\chi^2 = \frac{\sum_{i=1}^{3} (f_{1i} - f_{2i})^2}{f_{1i} + f_{2i}}$$ with 2 degrees of freedom

a(1) $\chi^2 = 4.96$ b(1) $\chi^2 = 61.49$

a(2) $\chi^2 = .98$ b(2) $\chi^2 = 30.08$

$\chi^2 = 5.99$ at the 5% level of significance

H_o: a. (1) and (2) accepted
b. (1) and (2) rejected

school mathematics grade point average were the best two single variables to use for placement purposes, yet they were different as placement tools (Table 17) at the five percent level of significance. Now we find the discriminant functions, whose method of construction maximizes the separation between the groups to insure minimum misplacement, and the ACTM dictate identical placements (at the ninety-five percent level of confidence) while placement by the high school mathematics grade point average is significantly different at the same level of confidence.

Therefore the ACTM variable, by its similarity in placement practices to the α_{14} functions, must be the most effective single variable for placement purposes. In evaluating this conclusion, one must remain cognizant of the inherent bias of the sample toward the ACTM because of the placement of the majority of the students in their first mathematics course by a policy primarily dependent on the ACTM scores.

It is important to note there has not been a significant amount of improvement in placement by use of the discriminant functions based on four variables over the placement dictated by the ACTM. Any set of discriminant functions based on a subset of the original four predictive variables can only be equal or less effective as placement guidelines than the α_{14}'s. Therefore, further pursuit of significant improvement in placement by using subsets of these variables to determine new discriminant functions is useless.

Chapter IV will summarize the conclusions of this study and present the investigator's recommendations for a placement policy in mathematics for freshmen at the University of Oklahoma.

CHAPTER IV

SUMMARY

This study was undertaken with the objectives of examining the present placement policy for entering freshmen of the Department of Mathematics and Astronomy of the University of Oklahoma, evaluating its efficiency, and comparing it with other possible placement programs. These alternative policies were formulated after study of both the particular courses offered by the department and the groups of students who had performed successfully in these courses. The 1962 freshman class composed of two thousand two hundred fifty-four students was utilized as our sample to provide data for analysis.

The following conclusions are of primary importance when the objectives of the research are considered.

(1) The only available predictive variables whose correlation coefficients with the course grade criterion differ from 0 at the five percent level of significance for each of the elementary mathematics courses (<u>1</u>, <u>2</u>, <u>5</u>, <u>21</u>, and <u>22</u>) are the high school grade point average, the high school mathematics grade point average, the ACTE and the ACTC. The coefficient of

the ACTM differs from 0 at the five percent level of significance in every course except 1. Even these significant coefficients, the highest of which is .545, account for little of the actual variation in grades.

(2) The means of the high school mathematics grade point average, the number of semesters of high school mathematics, the ACTM, the ACTNS, and the ACTC for the set of "properly placed" freshmen are different for each mathematics course under consideration at the 5% level of significance. These increase positively as the course level advances.

(3) The ACTM appears to be the best single variable for use as a placement guideline. This conclusion, however, is probably reflecting an inherent bias toward this variable resulting from its use as the primary guideline for the original placement of our sample in their mathematics courses.

(4) Discriminant functions dependent on the ACTM, the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics are of value in distinguishing membership in different mathematics courses at the five percent level of significance. However, they do not prove significantly better than the ACTM at this level as placement tools.

Of secondary importance to the objectives of this research, but of general interest and value are the following observations:

(1) The final grade is as valid a uniform criterion of success in a course as a common examination grade would be, even though the first is influenced by uncontrollable variables such as different instructors, diversity of teaching, testing, and grading philosophies, dissimilar composition of classes, etc.

(2) Students making grades of A in their initial mathematics course continue to perform well in succeeding courses of mathematics. Those earning D grades generally do not improve or even maintain that level of work in subsequent courses in mathematics.

(3) The students who choose to complete a mathematics course during their freshman year at this University are from a population whose mean scores with respect to all predictive variables except the ACTE are generally higher at the five percent level of significance than the mean scores representing the population of students not enrolled in any mathematics their initial year at the University of Oklahoma.

(4) The performance of freshmen in mathematics courses as judged by their earned grade point is not significantly different from their performance by the grade criterion in other college courses.

(5) Freshmen do equally well or better than all other students enrolled in the same elementary mathematics courses, with the exception of mathematics 1.

(6) Division of the range of the ACTM scores for placement in the different mathematics courses as dictated by the present placement policy appears satisfactory except between the 1 and 2 courses. Here the respective means and standard deviations dictate a more logical choice of a one to sixteen interval for mathematics 1 with the seventeen to twenty-three interval for 2.

As a result of this research, the investigator submits the following recommendations for study by the Mathematics and Astronomy Department:

(1) The discriminant functions developed in this study, based on the ACTM, the ACTC, the high school mathematics grade point average, and the number of semesters of high school mathematics should be used as a placement guideline in mathematics for some future freshmen class at this University. Although they have not been shown to be significantly better than the ACTM as a placement tool, they are equally efficient. Since they are less dependent on the single ACTM variable, use of these functions might nullify part of the bias inherent in this 1962 sample set. This might permit development of discriminant functions of greater value at some future time. The differences in ratios

of the coefficient variables at the different levels of course work do indicate changing roles and emphasis for the predictive variables in each function, a factor which must be ignored with use of a single variable. The apparently correct placement of all but one of the "A" students in our sample by these functions in contrast to the misplacement of nine "A" students by strict compliance with the ACTM guideline has influenced this recommendation, although no significant statistical improvement could be cited from this observation.

(2) To discriminate more accurately correct placement in mathematics 21 and 22, the test scores from the mathematics department Qualifying Examination should be utilized as a predictive variable in determining a suitable discriminant function which would also be dependent on the high school mathematics grade point average, the number of semesters of high school mathematics, and the ACTC score.

(3) Students making grades of D in any elementary mathematics course should be strongly advised not to continue in the next level of course work in mathematics unless they first repeat and improve their work at the original course level. The data on this aspect collected for the 1962 freshmen class should provide a convincing argument of this for some advisors.

(4) If the ACTM is used further as a placement tool, the score interval for membership in mathematics 1 should be changed to the one to sixteen interval. This should reduce part of the large number of F grades found in the records of mathematics 2.

(5) The objectives for each of the mathematics courses (1, 2, 5, 21, and 22) should be clearly formulated and included as part of the advisor's guidance sheets.

This study is not complete. The conclusions reached by analysis of the data of the 1962 freshmen class should be used and validated by application to a new freshmen class. The initial problem of determining the best placement guideline in mathematics is not solved; only an outline for action has been made. Further efforts will be necessary if an improved placement program is to result.

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APPENDIX I

NOMENCLATURE, FORMULAS AND TESTS

NOMENCLATURE

Listed below are the symbols and abbreviations with their referents used throughout this study.

<u>1</u>	Remedial Mathematics
<u>2</u>	Intermediate Algebra
<u>5</u>	College Algebra
<u>21</u>	Mathematical Analysis I
<u>22</u>	Mathematical Analysis II (calculus)
ACT	American College Testing Program
ACTE	English subtest of the ACT
ACTM	Mathematics subtest of the ACT
ACTSS	Social Studies subtest of the ACT
ACTNS	Natural Science subtest of the ACT
ACTC	Composite score of all ACT subtests
HSGPA	High School Grade Point Average
HSM GPA	High School Mathematics Grade Point Average
SEM.HSM	Number of Semesters of High School Mathematics
$\bar{x}$	Sample mean
$\bar{\mu}$	Population mean
s	Sample standard deviation
σ	Population standard deviation
r	Sample correlation coefficient
ρ	Population correlation coefficient
p	Sample proportion
P	Population proportion

Formulas and Tests

The following are the formulas and tests used in the computations and analysis of data in this dissertation.

(1) The mean $\bar{x}$ and the standard deviation s of a variable x in a set of size n are:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s = \sqrt{\frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}}$$

(2) The coefficient of correlation between two variables x_i and y_i in a set of size n is:

$$r_{xy} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

(3) Student's t distribution:

(a) Is $\rho_{xy} = \rho_{xz}$?

If n is the sample size and r_{xy}, r_{xz}, and r_{yz} are sample coefficients of correlation of variables x and y, x and z, and y and z respectively, then

$$t = (r_{xy} - r_{xz})\sqrt{\frac{(n-3)(1 + r_{yz})}{2(1 - r_{yz}^2 - r_{xy}^2 - r_{xz}^2 + 2r_{yz}r_{xy}r_{xz})}}$$

with n - 3 degrees of freedom.

(b) Is $\rho_{xy} = 0$?

If n is the size of the sample and r_{xy} is the sample coefficient of correlation of variables x and y, then

$$t = \frac{r_{xy}\sqrt{n - 2}}{\sqrt{1 - r^2}}$$

with n - 2 degrees of freedom

(c) Is $\mu_1 = \mu_2$?

If n_1 and n_2 are the sample sizes of sets A and B respectively and the means and standard deviations of A and B are $\bar{x}_1$, $\bar{x}_2$, s_1, and s_2, then

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

with $n_1 + n_2 - 2$ degrees of freedom

(4) z distribution:

Is $P_A = P_B$?

If the sample size of set A is n_A and the sample size of set B is n_B with p_A and p_B the sample proportions of sets A and B respectively, then

$$z = \frac{p_A - p_B}{\sqrt{\frac{p(1 - p)(n_A + n_B)}{n_A n_B}}} \quad \text{where } p = \frac{n_A p_A + n_B p_B}{n_A + n_B}$$

(5) χ^2 - Chi-square

Do the two classification criteria differ as judged by the proportion of the sample placed in each category? If a sample A is placed in k categories with frequencies f_{1j} and f_{2j}, $(j=1,2, \ldots k)$ dependent on classification methods 1 and 2, then

$$\chi^2 = \sum_{j=1}^{k} \frac{(f_{1j} - f_{2j})^2}{f_{1j} + f_{2j}}$$

with k - 1 degrees of freedom.

APPENDIX II

MATHEMATICS PLACEMENT INSTRUCTIONS

MATHEMATICS PLACEMENT INSTRUCTIONS

2-1959-60

Students whose curricula require additional math beyond that taken in high school will be required to begin their math at the level determined by the method given below. If the student's high-school record shows that he has had all of the math for the particular degree he seeks, no more math is required regardless of the placement test scores.*

Add the O.U. Math raw score to the I.H.S.C. Math decile and place students according to the following scale:

0-15 inclusive	Math A (Night classes or correspondence)
16-28 inclusive	Math 2
29-50 inclusive	(Eligible for courses above Math 2)

*Advisers in the College of Business Administration are to follow the special instructions for that college.

INSTRUCTIONS FOR PLACEMENT IN MATHEMATICS

Summer, 1960

Students whose curricula require additional math beyond that taken in high school will be required to begin their math at a level determined by the method given below. If the student's high school record shows that he has had all of the math for the particular degree he seeks, no more math is required regardless of the test scores.*

If the math score on the ACT score card is:

0 - 22	Math 1 (Remedial Mathematics). Formerly Math A
23 - 25	Math 2
26 and above	Eligible for courses above Math 2

*Advisers in the College of Business Administration are to follow the individual college instructions concerning math.

MATHEMATICS
1-62-63

Students whose curricula require additional math beyond that taken in high school will be required to begin their math at a level determined by the method given below. If the student's high school record shows that he has had all of the math for the particular degree he seeks, no more math is required regardless of the test scores.*

ACT Math score	0 - 15	Remedial Mathematics (Math 1; offered only in evening classes)
" " "	16 - 23	Mathematics 2 (Int. Alg.)
" " "	24 & above	Any required course above Math 2

*Advisors in the College of Business and the College of Pharmacy follow individual instructions concerning math.

PLACEMENT IN MATH 22

The student must have an ACT math score of at least 30, plus high school credit in college algebra and trigonometry, or a year course in analysis (which includes trigonometry).

At the second class meeting in Math 22, and for certain students in Math 21, a test will be given and any necessary changes will be made according to performance on the test.

MATHEMATICS PLACEMENT
1-64-65
2-64-65

Find out what mathematics the student had in high school

If student's curriculum does not require additional mathematics, then do not give him any unless he especially wishes to go beyond his high school preparation.

If student needs or desires more mathematics, then look at his ACT scores.

Mathematics 1: Remedial Mathematics

If student had no algebra in high school or if he feels he needs a review of high school algebra, he should be enrolled in Math 1--available in evening classes on the main O.U. campus through the extension division.

If the student has an ACT mathematics score of 15 or less, he must be enrolled in Mathematics 1 as above, irrespective of high school courses, providing he plans to take more mathematics.

Mathematics 2: Intermediate Algebra

If the student has had one year of algebra and feels he knows it and has a score of at least 16 on the ACT mathematics test, he may take Mathematics 2.

Mathematics 5 - College Algebra: or Mathematics 21 - Math Analysis I

If the student has had at least a year and a half of algebra and has a mathematics score of at least 24 on the ACT test he may enroll in Mathematics 5 or Mathematics 21. If he has less than 24, he must enroll in Mathematics 2 or Mathematics 1 before taking Mathematics 5 or Mathematics 21.

Mathematics 22: Mathematical Analysis II

If the student has had four years of mathematics in high school, including analytic trigonometry with good grades (A or B) and has a mathematics score of at least 30 on the ACT, he may enroll in Math 21, or, if he wishes, he may take the Math 22 Qualifying Exam given by the Department of Mathematics to determine whether or not he will be permitted to skip Math 21 and enroll in Math 22. This test will be given in Room 215, Buchanan Hall, at 9:00 a.m. and 2:00 p.m. during the regular enrollment period, Tuesday through Friday, September 8 through 11. An Advisor should not enroll a student in

Math 22 unless the student has either completed Math 21 (grade of C recommended) or has taken the Qualifying Exam and performed satisfactorily on it. If accepted he can receive ten hours of college credit upon satisfactory completion of Math 22 and by applying for advanced standing credit in Math 21.

Honors Mathematics 21

Especially able students who have had four years of college preparatory mathematics in high school with grades of A and B with more A's than B's and who are interested in honors work may enroll in Honors Mathematics 21 providing they present a slip like the attached, signed either by their high school principal and mathematics teacher or by a member of the O.U. Mathematics Department. It is harder to get into Honors Mathematics 21 than to skip Mathematics 21 and get into Mathematics 22. Honors 21 is designed for serious students who really want to know what makes Mathematics tick. Honors courses in Mathematics 22, 103, 104, 244, etc., are available for students who have completed Honors 21.

APPENDIX III

SURVEY OF PLACEMENT POLICIES

June 23, 1965

Dear Colleague:

We are attempting to evaluate and improve the placement of entering freshmen in their first university mathematics course at the University of Oklahoma. I am interested in learning what criteria are used at your University to aid in advising students in their choice of a first mathematics course. Furthermore, if any research has been undertaken to evaluate your placement policy, I would appreciate the opportunity to read it.

Any information you give me with respect to the above problem will be appreciated. A stamped, addressed envelope is enclosed.

Sincerely,

John W. Renner,
Professor of Education and
Chairman of Science,
University School

Encl.
JWR/mio

SURVEY OF PLACEMENT POLICIES IN STATE UNIVERSITIES

State and School	H.S. grades	H.S. courses	ACT	CEEB	Interview	Special Exams	Research
Arizona							
University of Arizona	X			X		if desired	none
Arkansas							
University of Arkansas						X	none
California							
University of California at Berkeley		X					----*
University of Southern California						if desired	none
Colorado							
University of Colorado				X			in progress
Georgia							
University of Georgia	X			X		for exemption	----
Hawaii							
University of Hawaii						X	yes
Illinois							
University of Illinois					X	X	none
Iowa							
Iowa State University	X	X				X	----
University of Iowa	X	X	X				in progress
Kansas							
University of Kansas	X	X				X	----
Kansas State University		X				X	none
Maine							
University of Maine				X	X		none
Maryland							
University of Maryland	X	X	X		X		----
Massachusetts							
University of Massachusetts	X	X				X	yes

SURVEY OF PLACEMENT POLICIES IN STATE UNIVERSITIES--Continued

State and School	H.S. grades	H.S. courses	ACT	CEEB	Interview	Special Exams	Research
Michigan							
University of Michigan		X		X		X	yes
Mississippi							
University of Mississippi	X	X	X			X	----
Missouri							
University of Missouri	X	X				X	in progress
Nebraska							
University of Nebraska	X	X	X	X		if lacks ACT & CEEB	yes
New Jersey							
Rutgers - State University					by major field		----
New Mexico							
University of New Mexico		X				X	yes
New Mexico State University			X				----
Ohio							
University of Ohio	X		X				----
Ohio State University						X	----
Oklahoma							
Oklahoma State University		X				X	in progress
South Carolina							
University of South Carolina				X		if desired	----
South Dakota							
University of South Dakota		X				X	----
Texas							
University of Texas						X	none
Texas A & M University				X		X	none
Utah							
University of Utah			X			X	yes

SURVEY OF PLACEMENT POLICIES IN STATE UNIVERSITIES--Continued

State and School	H.S. grades	H.S. courses	ACT	CEEB	Interview	Special Exams	Research
Vermont							
University of Vermont	X	X		X		X	yes
Washington							
University of Washington					by field of study	X	yes
West Virginia							
University of West Virginia	X	X				X	yes
Wisconsin							
University of Wisconsin	X				by field of study	X	----
Wyoming							
University of Wyoming	X					for some	none

*---- represents lack of information

APPENDIX IV

DATA CARDS AND PROGRAMS

Format of IBM Cards Kept on Individual Students

Card Column	
1	
2-6	I.D. number
7-30	name
31	sex
32-33	age
34-36	high school grade point average (HSGPA)
37-38	ACT English (ACTE)
39-40	ACT Mathematics (ACTM)
41-42	ACT Social Studies (ACTSS)
43-44	ACT Natural Sciences (ACTNS)
45-46	ACT Composite (ACTC)
47-49	first semester college grade point average
50-52	second semester college grade point average
53-55	second semester cumulative grade point average
56-57	first college mathematics course _1_, _2_, _5_, _6_, _21_, _22_
58	semester taken: 1 - fall, 1962; 2 - spring, 1963; 3 - summer, 1963; 4 - fall, 1963
59	mathematics course grade
60-61	section of course
62-64	high school mathematics grade point average (HSM GPA)
65	number of semesters of high school mathematics (SEM.HSM)
66	grade on common final in _2_

Fortran Program for Placing Students in Mathematics Courses by α_{14}'s

The following sets of numbers, representing the coefficients of the three discriminant functions, should be punched in (4E15.8) format.

α_{14}	.37683342E-02	.26414253E-03	.62760935E-02	.24966177E-02
α_{24}	.10894644E-02	.46159327E-03	.11857718E-02	.12227622E-02
α_{34}	.10724067E-02	.32573435E-03	.12850908E-02	.19362044E-02

A data card for each student must be punched with the following information: identification number, name, ACTM, ACTC, HSM GPA, SEM.HSM. The format could be (1X,I5,1X,A22,1X,F2.0,1X,F2.0,1X,F3.2,1X,F2.0).*

Program in Fortran, as Run on 1410 Machine

```
   DIMENSION X(4),A(3,4)
   READ 1, ((A(J,K),K=1,4),J=1,3)
1  FORMAT (4E15.8)
   READ 2, DD, PE, (X(I),I=1,4)
2  FORMAT (1X,I5,1X,A22,IX,F2.0,1X,F2.0,1X,F3.2,1X,F2.0)
   D = 0.0
   DO 3 K=1,4
3  D = D + A(1,K)*X(K)
   IF (.044996 - D)6,6,8
```

*If α_{34} was adjusted to be dependent also on the Mathematics Qualifying Exam, that data should be added and the necessary adjustments in α_{34} and the program should be made.

```
8  Q = 0.0
   DO 7 K = 1,4
7  Q = Q + A(2,K)*X(K)
   IF (.086861 - Q)9,9,10
9  M = 2
   GO TO 11
10 M = 1
   GO TO 11
6  Q = 0.0
   DO 12 K = 1,4
12 Q = Q + A(3,K)*X(K)
   IF (.056697 - Q)13,13,14
13 M = 22
   GO TO 11
14 M = 521
11 PRINT 4,DD,PE,M
4  FORMAT (1X,I5,1X,A22,I6)
   GO TO 5
   END
```

The printed results will list each student by identification number, name, and then the course suggestion--1, 2, 5-21, or 22.

APPENDIX V

THE DISCRIMINANT FUNCTION

The Discriminant Function

If elements belonging to several categories have been characterized by a set of measurements, then it should be possible to generate functions which will discriminate between membership in each of these categories. R. A. Fisher [14] established a method for determining a function when the problem is concerned with just two categories. In this study, his method has been adapted to classify students into four courses on the basis of known predictive measurements of properly placed members of the courses by simply considering two courses at a time, thus generating a set of three discriminant functions.

If there are p measurements $x_1, x_2, \ldots x_p$ known on each element of groups A and B, then we seek a linear function $\alpha = \sum_{i=1}^{p} c_i x_i$ which will distinguish more efficiently than any other linear function between groups A and B. To determine the coefficients c_i's of such a function, Fisher chose to maximize the ratio of the difference between the means of α in the groups to the standard deviation within the groups.

Let $d_i = \bar{x}_{2i} - \bar{x}_{1i}$ where $\bar{x}_{1i}$ and $\bar{x}_{2i}$ are the means of the i^{th} variable in groups A and B. The difference between the means of α in A and B would then be $D = \sum_{i=1}^{p} c_i d_i$. If S_{ij} is the sum of the squares and products of the deviations from the specified means, then the variance of α within the groups is proportional to $S = \sum_{i=1}^{p} \sum_{j=1}^{p} c_i c_j S_{ij}$.

To solve for c_i's which will maximize $\sqrt{D}/S$, one may differentiate the function D/S^2 with respect to each c_i and set each partial derivative equal to 0.

For $i \in \{1,2, \ldots p\}$,

$$\frac{\partial}{\partial c_i}\left(\frac{D^2}{S}\right) = \frac{S \cdot 2D \frac{\partial D}{\partial c_i} - D^2 \frac{\partial S}{\partial c_i}}{S^2} = \frac{D}{S^2}\left(2S \frac{\partial D}{\partial c_i} - D \frac{\partial S}{\partial c_i}\right)$$

$$\frac{\partial S}{\partial c_i} = \frac{\partial}{\partial c_i} \sum_{i=1}^{p} c_i d_i = d_i$$

$$\frac{\partial S}{\partial c_i} = \frac{\partial}{\partial c_i} \sum_{i=1}^{p} \sum_{j=1}^{p} c_i c_j S_{ij} = \sum_{j=1}^{p} c_j S_{ij}$$

Therefore, $\frac{\partial}{\partial c_i}\left(\frac{D^2}{S}\right) = 0$ implies $D \sum_{j=1}^{p} c_j S_{ij} = 2Sd_i$ for $i \in \{1,2, \ldots p\}$.

Now we have p linear equations with p unknowns where S, D, d_i and S_{ij} are known constants for our groups A and B. The c_i which satisfy the necessary conditions are unique only in their ratio to each other, not in magnitude, therefore solution for c_j's of the system of equations $\sum_{j=1}^{p} c_j S_{ij} = d_i$ where $i \in \{1,2, \ldots p\}$ satisfies the requirements of our problem.

After the necessary calculations to determine the c_i's for the function α, a boundary value a_{ip} dictating the division between the ranges of α in group A and group B must be determined. If the variances of α in groups A and B are not presumed equal, then the following procedure is necessary [14,21].

First, one evaluates α for $\overline{x}_{1i}$ and $\overline{x}_{2i}$, $i \in \{1,2,\ldots p\}$ to determine the mean values, $\overline{\alpha}_1$ and $\overline{\alpha}_2$ in groups A and B. Then it is necessary to find the sample standard deviations s_1 and s_2 of α for groups A and B by the usual methods. The least amount of errors in classification will result if

$$a_{ip} = \frac{s_1}{s_1 + s_2} (\overline{\alpha}_2 - \overline{\alpha}_1) + \overline{\alpha}_1.$$

If α is evaluated for an individual and equals k, and $k < a_{ip}$, the individual is placed in group A. If $k \geq a_{ip}$, placement in group B is recommended.

Sample Calculation of α_{14} and a_{14}

α_{14} is the function used to discriminate between membership in mathematics 1 and mathematics 2.

If $\alpha_{14} = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$, finding the coefficients c_i, $i \in \{1,2,3,4\}$ requires solving the system of 4 linear equations of 4 unknowns, $\sum_{j=1}^{4} c_j S_{ij} = d_i$ for $i \in \{1,2,3,4\}$. S_{ij} and d_i are determined by the known data x_1, x_2, x_3, x_4 (ACTM, ACTC, HSM GPA and SEM.HSM respectively) on each of the "properly placed" students in 1 and 2.

Table 25 shows the actual collection of data on

TABLE 25

COLLECTION OF DATA FOR α_{14}

k	1	2
N_k	27	232
Σx_{k1}	321.00000	4707.00000
$\bar{x}_{k1}$	11.88888	20.28879
Σx_{k2}	427.00000	4672.00000
$\bar{x}_{k2}$	15.81481	20.13793
Σx_{k3}	41.02000	581.65000
$\bar{x}_{k3}$	1.51925	2.50711
Σx_{k4}	84.00000	1116.00000
$\bar{x}_{k4}$	3.11111	4.81034
Σx_{k1}^2	4099.00000	97241.00000
Σx_{k2}^2	6989.00000	96198.00000
Σx_{k3}^2	78.16100	1572.90150
Σx_{k4}^2	332.00000	5752.00000
$\Sigma x_{k1}x_{k2}$	5172.00000	95681.00000
$\Sigma x_{k1}x_{k3}$	460.26000	11860.05000
$\Sigma x_{k1}x_{k4}$	959.00000	22807.00000
$\Sigma x_{k1}x_{k3}$	647.36000	11736.59000
$\Sigma x_{k2}x_{k4}$	1295.00000	22445.00000
$\Sigma x_{k3}x_{k4}$	142.97000	2800.15000

$d_1 = \bar{x}_{12} - \bar{x}_{11} = 8.39990$

$d_2 = \bar{x}_{22} - \bar{x}_{21} = 4.32311$

$d_3 = \bar{x}_{32} - \bar{x}_{31} = .98785$

$d_4 = \bar{x}_{42} - \bar{x}_{41} = 1.69923$

sets $\underline{1}$ and $\underline{2}$, as calculated by use of the 1410 computer, needed to set up these equations.

$$S_{ij} = \sum_{k=1,2} \Sigma x_{ki} x_{kj} - \sum_{k=1,2} \frac{\Sigma x_{ki} \Sigma x_{kj}}{N_k}$$

i.e.

$$S_{23} = (\Sigma x_{12} x_{13} + \Sigma x_{22} x_{23}) - \left(\frac{\Sigma x_{12} \Sigma x_{13}}{N_1} + \frac{\Sigma x_{22} \Sigma x_{23}}{N_2} \right)$$

Calculating each S_{ij}, using the fact $S_{ij} = S_{ji}$ and the data of Table 25, the following equations were set up.

$$2024.31800c_1 + 987.20400c_2 + 31.65200c_3 + 125.04100c_4 = 8.39990$$
$$987.20400c_1 + 2349.67000c_2 + 22.00000c_3 - 62.37500c_4 = 4.32311$$
$$31.65200c_1 + 22.00000c_2 + 130.48080c_3 + 17.56530c_4 = .98785$$
$$125.04100c_1 - 62.37500c_2 + 17.56530c_3 + 454.32190c_4 = 1.69923$$

Solutions of these equations for the c_i's determined

$$\alpha_{14} = .0037683342x_1 + .00026414253x_2 + .0062760935x_3 + .0024966177x_4.$$

To find the division point a_{14} separating the ranges of values of α_{14} of $\underline{1}$ and $\underline{2}$, the following calculations were necessary. The mean value of α in $\underline{1}$, $\bar{\alpha}_1 = \sum_{i=1}^{4} c_i \bar{x}_{1i}$ $= .06628$ and the standard deviation, $s_1 = .01189$. The mean value of α in $\underline{2}$, $\bar{\alpha}_2 = \sum_{i=1}^{4} c_i \bar{x}_{2i} = .10952$ and the standard deviation $s_2 = .01309$.

$$a_{14} = \bar{\alpha}_1 + \frac{s_1}{s_1 + s_2} (\bar{\alpha}_2 - \bar{\alpha}_1) = .06628 + \frac{.01189}{.02498} (.04324)$$
$$= .086861.$$