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GRADUATE COLLEGE

UPSTREAM DETENTION METHODS AS A FLOOD CONTROL PRACTICE IN URBAN AREAS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the degree of

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BY

JOSEPH KWANG-CHAO CHENG

Norman, Oklahoma

1973

UPSTREAM DETENTION METHODS AS

A FLOOD CONTROL PRACTICE

IN URBAN AREAS

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DISSERTATION COMMITTEE

TO

HIS MOTHER DONGS

AND

HIS BROTHER SAMUEL

UPSTREAM DETENTION METHODS AS A FLOOD CONTROL PRACTICE IN URBAN AREAS

BY

JOSEPH KWANG-CHAO CHENG

MAJOR PROFESSOR: Dr. Jimmy F. Harp

ABSTRACT

The problem of urban area floodwater containment has become more and more serious as the process of urbanization continues. This research effort is an extensive study of the current graphical methods which have been most widely used. Hydrologic methods are not envisioned as candidate models, or equations, to be computerized. The basic partial differential equations of unsteady non-uniform flow are to be assembled and a digital computer assisted finite differences techniques developed. This eliminates the utilization of one of the classical approximate methods whereby storage-depth assumptions are made. An applicable, suitable, computer method is developed, and an application to urban areas is made.

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LIST OF SYMBOLS

A ---------- Area of Flow Cross Section

B ---------- Bottom Width of Channel

C ---------- Celerity of Gravity Wave

g ---------- The Acceleration of Gravity

h_f ---------- Head Loss

P ---------- Wetted Perimeter

Q ---------- Flow Rate of Discharge

R ---------- Hydraulic Radius

S_o ---------- Longitudinal Bed Slope

S_f ---------- Friction Slope

T ---------- Top Width of Channel

V ---------- Flow Velocity

X ---------- Distance

Y ---------- Depth of Flow

γ ---------- Specific Weight

ρ ---------- Density

I ---------- Inflow Rate

O ---------- Outflow Rate

S ---------- Storage

t ---------- Time

L ---------- Length of Reservoir

H ---------- The Head

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UPSTREAM DETENTION METHODS AS A FLOOD CONTROL PRACTICE IN URBAN AREAS

CHAPTER I

INTRODUCTION

Whenever heavy rainfall or melting snow provides more runoff than can be carried within the normal channels of existing streams, a flood results. The excess water overflows to adjacent areas, the valley lands, and invades developed areas. Then, the flood damage is relative to: transportation impediment, agricultural destruction, urban development inundation, and loss of human life and property. There is no known absolute method of controlling the rainfall itself apart from current weather modification efforts such as cloud seeding, etc. Nature alone controls the cycle from sea to sky to earth. Man's efforts are confined to attempts at guiding the water on that part of its course from earth back to the sea. The regulation of the waters that could cause floods present the flood-control problem.

Many structures are built to control water: waterways of proper depth and width control arteries of transportation; water discharged through water-wheels provides controlled power; water may be caught in storage basins and distributed for irrigation or water supply; and water

that is an actual or potential source of damage or danger to property or human life may be controlled to prevent floods. When this control of water involves improvement of reclamation of property not damaged in its present conditions, the control is termed "reclamation," and when it involves the prevention of flood damage is termed "flood control."

Flood problems are diverse in nature. Uncontrolled runoff results in erosion of land and contributes to sediment-deposition problems downstream. "Among the devices employed under the general concept of flood control, the storage, or detention, of excess flood waters in reservoirs designed for that purpose constitute one method of flood contro." Another is the diversion of excess flood waters into floodways specifically designed for that purpose. By contrast, levees, dikes, floodways and channel improvement merely serve to protect property from overflow but do not control floods; in fact, their presence tends to confine flood flow and thereby increase the height of flood stages. As population increases, the natural trend is for more people to crowd into the low valley lands, which are natural pathways for flood water. Subsequently, there is a greater demand for more extensive flood protection.

The function of a flood-control reservoir is to store a portion of the flood flow in such a way as to minimize the flood peak at the point to be protected. In an ideal case, the reservoir is situated immediately upstream from the protected area and is operated to "cut off" or alter the flood peak. This is accomplished by slowly discharging all reservoir inflow until the outflow reaches the safe capacity of the channel downstream. All flow above this rate is stored until inflow drops below the safe channel capacity and the stored water is released to recover storage

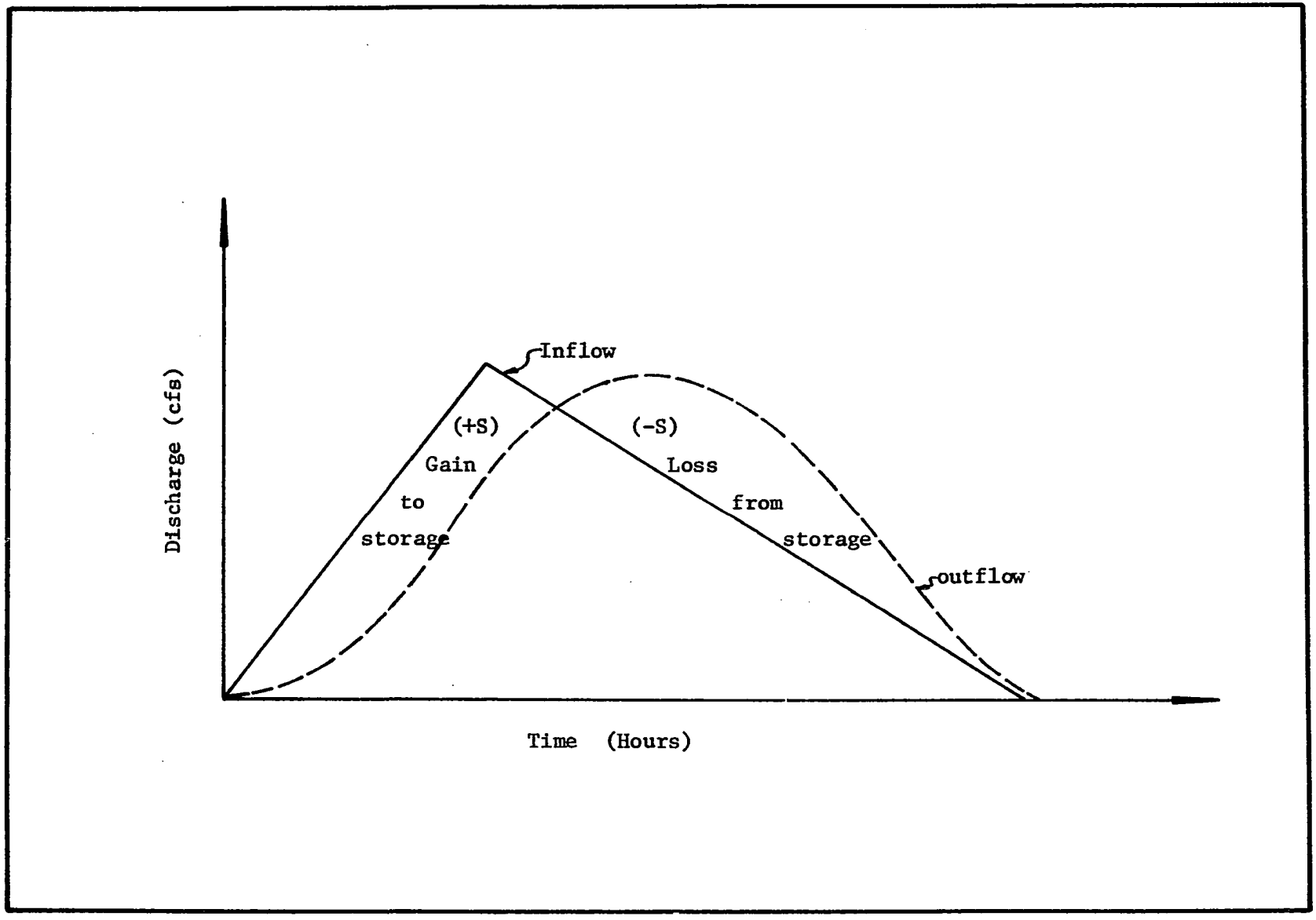


Fig. 1-1 Inflow and outflow Hydrograph Relationships

capacity for the next flood.

The common method of determining storage in a reach of natural channel is to use one of several storage equations in conjunction with obsered flows. Figure 1-1 shows an inflow and outflow hydrographs relationship. In the case of a proposed flood-control reservoir, it is necessary to determine the degree of flood protection offered. This requires that an operating plan for the reservoir be determined and that actual floods of various magnitudes be "routed" through the reservoir following the proposed operation schedule.

To solve the problem of hydrologic reservoir routing, the reservoir storage configuration must be known. A storage equation, "inflow minus outflow equals change in storage per unit time," becomes the basis of the routing. In routing a flood through a reach, it is necessary to know:

(1) Total inflow into reach

(2) Profile of water surface at any instant

(3) Storage under the profile

The storage equation is:

$$(I - O)\ \Delta t = \Delta S$$

in which I is the total inflow rate, O is the total outflow rate, ΔS is the change in volume in storage in the reach, Δt is the length of the time period.

Until recently, most of the water flood flow research has been aimed at determining the variables influencing runoff, and the relationships between the variable has been empirical or statistical because of the complexity of the problem and the lack of data involved. In recent

times, high speed digital computers have been used for both basic and applied research to flood flow problems. Most of the techniques for overland flow routing in streams are based, in various degrees, on the integration of the partial differential equations of motion. The equations of flood flow can be a powerful tool for the different areas of research. In this study, there are two fundamental partial differential equations. One is the conservation of mass or the continuity equation and the other is the momentum equation. These two partial differential equations from the basic mathematical models represent the various phenomena. These equations are not simple to solve by analytical methods, and numerical solutions prove to be too tedious for practical purposes. Simplified methods based primarily on the equation of conservation of mass have been widely used in the past. The digital computer has made it feasible to obtain numerical solutions for the complete system of equations of motion by the finite difference method.

Accordingly the flood problem, water in the stream channel system (see Fig. 1-2), has been observed to be unsteady non-uniform flow, etc., but the reservoir routing problem is different from the streamflow routing problem in its fundamental nature and by difference from local runoff entering along the length of the stream channel. This study will entail the use of partial differential equations to solve the problem of reservoir routing. The solution requires as input information:

(a) The reservoir geometric elements.

(b) The total time.

(c) The inflow hydrograph.

Programs for the solution of the partial differential equations of

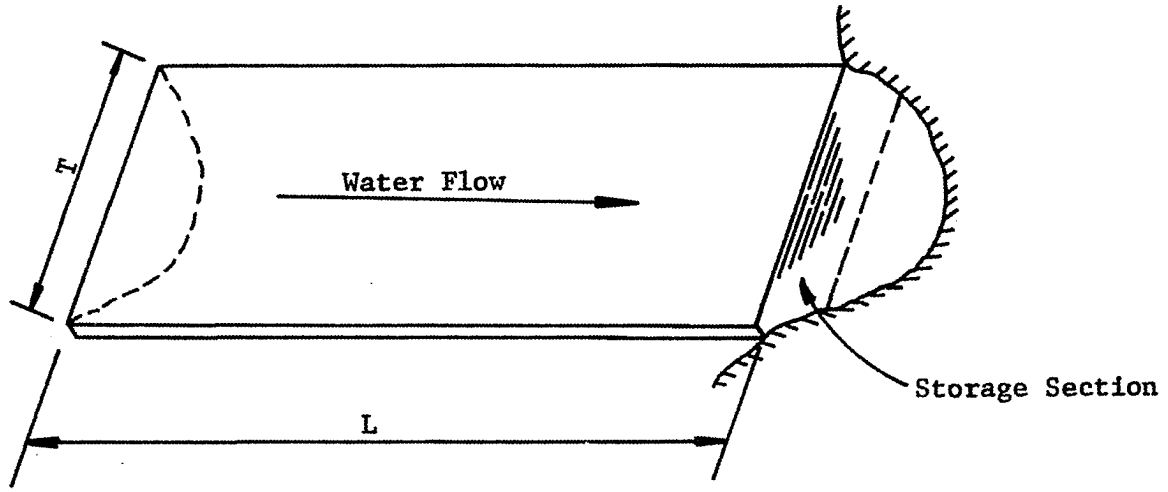


Fig. 1-2. Definition Sketch

the open channel flow have been written for solution by digital computers such as the IBM 360. The programs have been designed to utilize a variety of languages such as FORTRAN, MAD, ALGOL, etc.

The reservoir routing problem has been solved by several graphical methods which will be described in the literature review. The original purpose of this disseration was to present a solution similar to the ICES-HYDRO scheme (25) whereby a practical method is made available for the problem of urban area floodwater containment. However, due to circumstances beyond our control, a lesser goal was achieved. Specifically, this study will examine one aspect of this problem --- reservoir routing. Hopefully, the end result of this research will be the development of a practical, usuable technique for urban area floodwater containment.

The ICES-HYDRO (25) package solves the streamflow routing problem but not the reservoir routing problem solved here. This technique uses the ALGOL computer, which is not available at the University of Oklahoma computer laboratory.

CHAPTER II

REVIEW OF LITERATURE

The history of flood routing through reservoirs will be reviewed in this chapter. For many years most engineers have recognized the problem of water overflow and have expended considerable time and thought to its solution. Usually the investigators directed their efforts towards an analytical approach to the problem. A brief evaluation of the work that has been done in this field will be described.

In the past, most research has been done by two methods for optimum flood control. First, the unit hydrograph method along with a basic water balance relationship shows the water overflow as a linear system. Second, the flood routing method uses two complete partial differential equations for open channel flow, one being the continuity equation and the other, the momentum equation.

In order to study the storage influence on the outflow hydrograph, the differential equation containing the storage term (inflow minus outflow is the change of storage in a given time increment) is integrated over a cycle inflow function (an assumed hydrograph) and for outflow rating curve which is linear with respect to the reservoir level. This theory was completed by J. A. Seddon 1898 (1). He let the function $H = f(t)$ be a reservoir level hydrograph where the area of the reservoir is either constant or changing with the reservoir level, and where H is the reservoir

level and t is the time. The effect of storage on the amplitude of the crest flow and on phase shifting (the postponement of maximum Q) is derived analytically for one or more reservoirs.

In 1914 T. R. Running (2) used the storage equation and a graphical method for its integration:

$$P - Q = A \frac{dH}{dt} \tag{2-1}$$

where P = reservoir inflow in cfs

Q = reservoir outflow in cfs

A = area of rerservoir in square miles

H = depth or elevation of the reservoir area in feet

t = time

Since P = f(t), Q = f(H), and A = f(H), then $\frac{dH}{dt} = f(H,t)$. The family of curves of $\frac{dH}{dt} = f(t)$ for constant H can be plotted, when P, Q, and A are known. Using the starting point B_0 (H_0, t_0) (where B is the width of reservoir) and selecting the point B_1 on the line H_1 so that the area under B_0B_1 is $(H_1 - H_0)$, the point t_1 is obtained, etc. The function H = f(t) is thus derived.

H. A. Thomas 1917 (3) solved the storage equation by mass curves

$$P - Q = \frac{dW}{dt} \tag{2-2}$$

Given the mass curve of P, storage W = f(Q) computed form W = f(H) and Q = f(H). An interval Δt is selected as fixed, inside of which all changes are considered linear. On the storage W for each outflow Q, $1/2Q\Delta t$ is added and a new curve $W + 1/2Q\Delta t = f(Q)$ is plotted. If the accumulated

outflow is known for t_1 and Δt is added, the value of $\Sigma p_i \Delta t$ is determined for the center of the Δt range.

$$\Sigma P_i \Delta t - \Sigma Q_i \Delta t = W + 1/2 Q \Delta t \qquad (2\text{-}3)$$

and with the value from the curve $W + 1/2Q\Delta t = f(Q)$, the mean value of Q in the interval Δt is obtained. The outflow during the interval is $Q\Delta t$ and the new point of $\Sigma Q \Delta t$ becomes $\Sigma Q_i \Delta t + Q\Delta t$. In this manner, the integration is completed

where P = inflow of reservoir in cfs

Q = outflow of reservoir in cfs

W = storage volume

H = depth in reservoir in feet

t = time interval

R. E. Horton, 1918 (4) developed a storage equation of the form

$$P\Delta t = A\Delta H + [1/2(Q_1 + Q_2)]\ \Delta t \qquad (2\text{-}4)$$

His equation is discussed for integration, Q_1 and Q_2 at the beginning and at the end of Δt, A and H are area and height of reservoir surface respectively, Δt = time interval. For given Δt, P_1, A, and Q_1 and assumed ΔH, Q_2 can be determined so that ΔH and Q_2 can be obtained by successive approximations. In order to avoid this procedure, two functions are developed:

$$F_1 = A \frac{H_1}{\Delta t} + \frac{Q_1}{2} \qquad (2\text{-}5)$$

and

$$F_2 = A\frac{H_2}{\Delta t} + \frac{Q_2}{2} \qquad (2\text{-}6)$$

As Q_1 or Q_2 are functions of H_1 or H_2, $F_1 = f(H_1)$ and $F_2 = f(H_2)$, and two curves can be computed that may be easily plotted, $F_2 = P - F_1$, and for given F_1 and P_1, the value F_2 and $H_2(Q_2)$ can be obtained, and so on.

Where P = inflow discharge in cfs

Q = outflow discharge in cfs

F_1, F_2 = storage factors

Δt = time interval

J. C. Stevens, 1921 (5) used the integration of the storage differential equation

$$P - Q = \frac{dW}{dt} \qquad (2\text{-}7)$$

The function $W = f(Q)$ is replaced by $dW = f(Q)dQ$, wher $f(Q) = m$ is the shope of the storage function and may be considered practically constant in certain limits of Q. The trial-and-erro method is based on different Δt and corresponding m. The genral form of the storage differential equation for $Q = f(H)$ and $W = f(H)$ as power functions of H is developed, but not integrated because of the difficulty to fit the hydrograph $P = f(t)$ by a mathematical expression.

Where P = inflow discharge in cfs.

Q = outflow discharge in cfs

W = storage volume

H = depth in reservoir in feet

t = time interval

A trial-and-error method for flood routing by the use of the storage equation was developed by R. D. Goodrich, 1931 (6). He used

$$P - Q = \frac{dW}{dt} \tag{2-8}$$

for the selected time interval using the outflow storage factor equal to

$$P_1 + P_2 + W_1 - Q_1 = W_2 + Q_2 \tag{2-9}$$

so that W_2 and Q_2 are obtained by successive operations in tabulating the above values.

Where P = inflow discharge in cfs

Q = outflow discharge in cfs

W = storage volume

t = time interval

H. K. Barrows, 1933 (7), by using a graphical procedure for the determination of reservoir storage above the spillway level, computed the outflow from the reservoir. A simple short method, employing the mass curves of inflow and spillway discharge is used. As the starting point, storage W represents the increment of difference in level ΔH. Using the slope of the outflow mass curve for ΔH, a new point on the inflow mass curve is obtained for the new level $H + \Delta H$ and the computation process is repeated.

G. R. Clemens, 1945 (8) proposed a graphical method based on the storage equation. It is called "Reach Reservoir Method" and is used for flood routing in reservoirs and in valley storage. The curves

$$F_1 = W_1 + 1/2Q_1 \tag{2-10}$$

and

$$F_2 = W_2 - 1/2Q_2 \tag{2-11}$$

both equations are used as well as $Q = f(H)$ and $W = f(H)$, with known inflows. Curves of different scales can be plotted to increase the accuracy of the method. The time interval of the wave, the local inflow, and the valley storage effect are discussed. In the equation, F_1, F_2 = storage factors, Q = outflow discharge, W = storage volume and H = depth in reservoir.

The slide rule flood routing method was used by C. J. Posey 1935 (9). This method involved short, uniform time increments and required a separate set of scales for each reservoir having a different volume-depth or outflow-depth relation. Length along the slide and stock represent total volumes as "day-second-feet," the former bearing a simple scale with graduations defined by

$$F_2 = W_2 + Q_2 \frac{t}{2} \tag{2-12}$$

and the latter bearing two opposed scales with a common origin defined by $I = Pt$ and

$$F_1 = W_1 - Q_1 \frac{t}{2} \tag{2-13}$$

in which P = average inflow rate in cfs

t = length of time increment

W_1 = at beginning storage volume

W_2 = at end storage volume

Q_1 = at beginning outflow rate in cfs

Q_2 = at end outflow rate in cfs

The outflow rate is assumed to be a known function of the total storage, and the storage equation is of the form $(F_1 + F_2)$. Given the inflow and outflow rates at the beginning of a step, the outflow rate at the end of the step can be obtained directly by means of the slide rule or by means of a monograph in which F_1 and F_2 are storage factors.

The first attempt to complete the general partial differential equations of a solution suitable for flood control was begun by H. A. Thomas, 1937 (10). His was the first real significant contribution to this disciplinary area, "Hydraulics of Flood Movement in Rivers" using WPA funded research. His two partial differential equations were:
the energy equation

$$i - \frac{V^2}{K^2_\gamma 2\alpha} = \frac{\partial Y}{\partial X} + \frac{V}{g}\frac{\partial V}{\partial X} + \frac{1}{g}\frac{\partial V}{\partial t} \tag{2-14}$$

(1) (2) (3) (4) (5)

and the law of continuity:

$$a\frac{\partial V}{\partial X} + Vb\frac{\partial Y}{\partial X} = -b\frac{\partial Y}{\partial t} \qquad (2\text{-}15)$$

(6) (7) (8)

(1) Bed - slope term

(2) Hydraulic - friction term

(3) Depth - friction term

(4) Velocity - Head term

(5) Acceleration term

(6) Prism - storage term

(7) Wedge - storage term

(8) Rate - rise term

Thomas research paper (10) presents a systematic analysis of unsteady flow in reivers and of the approximate flood-routing methods that have been developed. The following are discussed: Review of laws of steady and unsteady flows, propagation of stable wave forms, difficulties of integration by exact methods and boundary conditions, use of hydraulic models for unsteady flow (which is recommended for accurate flood routing), and approximate methods of flood routing in uniform channels and in actual rivers. Three approximate methods are analyzed: First approximation, with a simple storage equation for each Δx and time interval Δt, based on the relationship of storage and outflow discharge (this method was found to be lacking in accuracy); second approximation, in which the slope of the reach is considered to be a straight line, with or without corrections for velocity head and acceleration term; and third approximation, employing two differential equations where the solution is very impractical and the

possibility of solving the equations by the finite difference approach is very difficult. H. A. Thomas did some sample computations discussing how the technique could be used in the field and acknowledged that the quantity of computation made its application quite difficult to apply.

A semigraphical method for integrating the storage differential equation was made by R. S. Goodridge, 1937 (11). For given inflow hydrograph, storage-elevation function, and outflow discharge-elevation function (rating curve), the outflow hydrograph and storage-time function are determined. The shortcut method described uses a process of direct integration without employing mass curves. It is based on the use of a selected time unit Δt (which is variable) for the equation $W = Q\Delta t$ and uses Δt as a constant (time required to fill a given volume by a given) discharge); Δt depends on the selected increments for storage and discharge in which W is storage volume, Q is outflow discharge, and Δt is time interval.

The function of a flood control reservoir is to reduce the height of flood peaks by temporarily storing part of the flood. The feasibility of the idea was exemplified in studies made by C. J. Posey and I. Fu-Te, 1940 (12). The principal problem in the functional design of a flood control reservoir is the determination of the relationship between the amount of storage and the corresponding reduction in the flood peak. They have been generalized and extended to apply to reservoirs with either orifice or weir-type outlets and valleys of a wide morphological configuration. Although the relationships derived can be used in the design of multiple-purpose reservoirs, the present discussion is restricted to reservoirs designed primarily for flood control.

H. A. Thomas, 1940 (13) made use of the two partial differential

equations for unsteady flow. A trial-and-error process of determining local inflow from given stage profiles for different times is given by using the stage-surface profile for a constant discharge as the reference base, from which the depth h is measured. By repeating the process, finite relations of $Q = f(t)$ for given stations and $A = f(x)$ for given times are determined. Having Δt and Δx for both families of curves, $\Sigma\Delta Q$ and ΔW are obtained, where $\Sigma\Delta Q$ is the difference between outflow and inflow and ΔW is storage of a reach Δx during the time, Δt. Local inflow

$$\Sigma P = \Delta W - \Sigma\Delta Q \qquad (2\text{-}16)$$

Solution of the trial-and-error process in the case of known local inflow is also given. Thomas (13) recognizes that the method is laborious but considers it justified.

D. Johnstone and W. P. Cross, 1949 (14) in "Elements of Applied Hydrology," Chapter 7, discusses the simple storage equation, flood routing through reservoirs and retading basins with storage as a function of discharge alone, an example of flood routing by use of a mass diagram and a storage-factor $(2W/\Delta t + Q)$ curve, along with discussion of flood routing in a stream where storage is used as a function of inflow and outflow. Derivation is given of the storage relationship for a reach and an example of the flood routing is shown. In which, W is storage volume. Q is the outflow rate, and t is the time interval used.

Most text books on hydrology such as Linsley, Kohler, and Paulhus, 1949, 1958 (15, 16) present a good summary on flood routing through reservoirs by using the continuity equation. It is the so-called "hydrology

equation, "written as

$$I - O = \frac{\Delta s}{\Delta t} \tag{2-17}$$

The term I represents the inflow discharge, O the outflow discharge, and $\Delta s/\Delta t$ the rate of change of storage. In words it simply states that the rate of change in storage in a system is equal to the difference between the inflow and the outflow.

The Hydrology Equation (2-17) may be rewritten as

$$S_1 + \int_{t_1}^{t_2} Idt = S_2 + \int_{t_1}^{t_2} Odt \quad \text{or} \tag{2-18}$$

$$\left(\frac{I_1 + I_2}{2}\right) - \left(\frac{O_1 + O_2}{2}\right) = \frac{\Delta s}{\Delta t} \tag{2-18a}$$

and transposing, we obtain

$$\int_{t_1}^{t_2} Idt - \int_{t_1}^{t_2} Odt = S_2 - S_1 \quad \text{or} \tag{2-19}$$

$$\frac{I_1 + I_2}{2}\Delta t - \frac{O_1 + O_2}{2}\Delta t = S_2 - S_1 \tag{2-19a}$$

In the latter form the equation says simply, the total quantity of inflow into a reach during a given period of time minus the total quantity of outflow from the reach furing the same period equals the change in the

volume of water stored in the reach.

Since neither I nor O can be expressed mathematically in terms of t, numerical integration is necessary for the solution of the hydrology equation. Selecting a time interval short enough that both I and O may be considered linear functions of t, the equation may be rewritten as:

$$(t_2 - t_1)\left(\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2}\right) = S_2 - S_1 \qquad (2\text{-}20)$$

By the terms of the problem, I_1 and I_2 are known and assuming that O_1 and S_1 are known, the equation has two unknown, O_2 and S_2. For a solution of the flood routing problem it is therefore necessary that another relation involving O and S be found. This relation may be obtained from the physical characteristics of the reach.

Flood routing by the hydrological method is based on the storage equation which K. E. Sorenson, 1949 (17) arranged in the form:

$$W_1 + 1/2(Q_1\Delta t) + [1/2(P_1 + P_2) - Q_1] = W_2 + 1/2(Q_2\Delta) \qquad (2\text{-}21)$$

The following curves are used by Sorenson (17) in his solution to the flood routing through reservoir problem: $W + 1/2(Q\Delta t) = f(H)$, $W = f(H)$, $Q = f(H)$ and $P = 1/2(P_1 + P_2) = f(t)$, $Q = f[W + 1/2(Q\Delta t)]$.

The functions $H = f(t)$ and $Q = f(t)$ are computed. A graphical method is used and a combined graphical and nomographic method in which P = inflow discharge in cfs, O = outflow discharge in cfs, W = storage volume, t = time interval, and H = depth in the reservoir in feet.

B. R. Gilcrest, 1950 (18) uses two appximation methods for flood

routing. The first approximation method is based on the neglect of the momentum equation and the second approximation method is based on the two differential equations.

These partial differential equations have been directed toward improving the techniques of solution using the method of characteristics. J. J. Stoker, E. Isaacson, and A. Troesch, 1953, 1954, 1956, 1958 (19, 20, 21, 22), respectively, did the research in this field with the aid of a digital computer. The last paper, (22), is actually a summary of the first three reports which were prepared for the U. S. Army Corps of Engineers, Ohio River Division. Those reports gave a complete discussion of the restrictions imposed by the characteristic directions, developed grids for solving the equations by finite differences method on the digital computer, and comparison with the actual data which was good in the case of a junction analysis and a reservoir operation. The method of characteristics is used to determine the time difference range for Δt when ΔX is selected and there is a good analysis of the mesh of points (Δt, ΔX) in the plane (t, X). Although the research efforts of Stoker, Isaacson and Troesch (19, 20, 21, 22) was directed specifically to stream routing, the principal ideas may be adapted to reservoir routing with some effort.

In Chow's Open-Channel Hydraulics, Part V, 1959 (23), unsteady flow is discussed in two chapters. Chapter 18 treats gradually varied unsteady flow, considering continuity of unsteady flow, dynamic equation, monoclinal rising wave, dynamic equation for uniformly progessive flow, wave propagation and solution of the unsteady surface flow. Chapter 20 treats flood routing, considering the method of characteristics, the method of diffusion analogy including principles and methods of hydrologic

routing, and a simple hydrologic method of routing.

Morgali and Linsley, 1965 (24) presented a method of synthesizing overland flow hydrographs by controlling the parameters. The hydrograph is constructed for a uniform rainfall on a flow plan of constant slope with uniform surface texture and a given length, and the effect of each parameter is isolated by varying it individually. These continuity and mementum equations are solved on a digital computer using the numerical procedure for boundary and initial conditions.

The HYDRO Program, 1966 (25) is a content-oriented computer language system which was first begun in the Civil Engineering Department, Massachusetts Institute of Technology and was completed later in the Department of Civil Engineering, Carnegie Institute of Technology. It was developed for the solution of hydraulic engineering problems and specifically for the stream flow routing. This system has two principle components: an ALGOL compiler and a procedure library. The general flood routing computation is based on the appximate methods of routing proposed by H. A. Thomas, 1937 (10) in "The Hydraulics of Flood Movements in Rivers," Carnegie Institute of Technology, Engineering Bulletin P.46-60, and is applicable to gradually varied unsteady flows in streamflow channels with no abrupt changes in the cross section. The method makes two basic approximations: (1) the storage in a reach (reach herein is defined as the channel length between two stations) is considered equal to the length of the reach times the average cross section area within the reach, assuming that the surface profile within the reach does not differ from a straight line by more than a negligible amount, and (2) the discharge rate at the downstream end of the reach is considered equal to the normal discharge

rate corresponding to the given stage at the end of the reach. The effects of abnormal surface slope in modifying the discharge are neglected.

Hartman, Ree, Schoof, and Blanchard, 1967 (26) developed a flood prevention program on Sugar Creek, a tributary to the Washita River in Oklahoma, where flood peaks were reduced by one-half. The recession parts of the after-treatment hydrographs were lengthened. The structures reduced the infiltration on the flood plane by reducing flood peaks. The recession flow was increased by the number of detention reservoirs.

The U. S. Army Corps of Engineers, Hydrologic Center, Davis, California, 1971 (27) have developed very recently a generalized computer program for the reservoir system analysis. This program was prepared for use in the CDC 6600 computer and is usable on other high speed computers if dimensions are changed to fit memory size. Using FORTRAN IV, it performs multipurpose routing of a reservoir system by any number of periods of uniform or varying length per year based on varying flow requirements at reservoirs, diversion, and downstream control points, and power peaking and energy requirements at reservoirs. Although it can accept any configuration of reservoirs, diversions, power plants, and control points and will accept system power demands that override individual power plant requirements, but it does not provide for channel routings. Therefore, a more fundamentally based formulation is required if one is to achieve a high degree of generality and flexibility.

A review of the literature relevant to the problem of reseroir routing has shown that most studies have dealt with practical methods and use a graphical technique. However, it was found that no research efforts

have been directed toward solving "flood routing through reservoirs". Therefore, it is the purpose of this study to provide development of a practical, usable technique for urban area floodwater containment.

Having reviewed the literature pertinent to this problem, Chapter III will present the theoretical work.

CHAPTER III

DEVELOPMENT OF PROCEDURE FOR MATHEMATICAL EQUATIONS

Part I. Basic Theory

In storage reservoirs for flood control, the inflow hydrograph and outflow-head relationships must be known (see Fig. 1-1). From the mathematical point of view, there are two fundamental partial differntial equations for open channel unsteady nonuniform flow. One is the continuity equation commonly called the law of conservation of mass, and the other one is the momentum equation or the law of conservation of momentum. These two equations are classified as nonlinear partial differential equations of the hyperbolic type.

Two Fundamental Partial Differntial Equations

A. Continuity Equation

In Figure 3-1, the ΔX is the length between the sections a-a and b-b. Letting X be the horizontal distance in feet in the same direction as the water flow, ρ is the water density, t is the time coordinate in seconds, A is the channel cross section area in square feet, Y is the flow depth in feet, E is the channel bottom elevation in feet, V is the average velocity in the same direction as the water flow in feet per second, B is the channel bottom width in feet, and Q is the volume flow rate entering

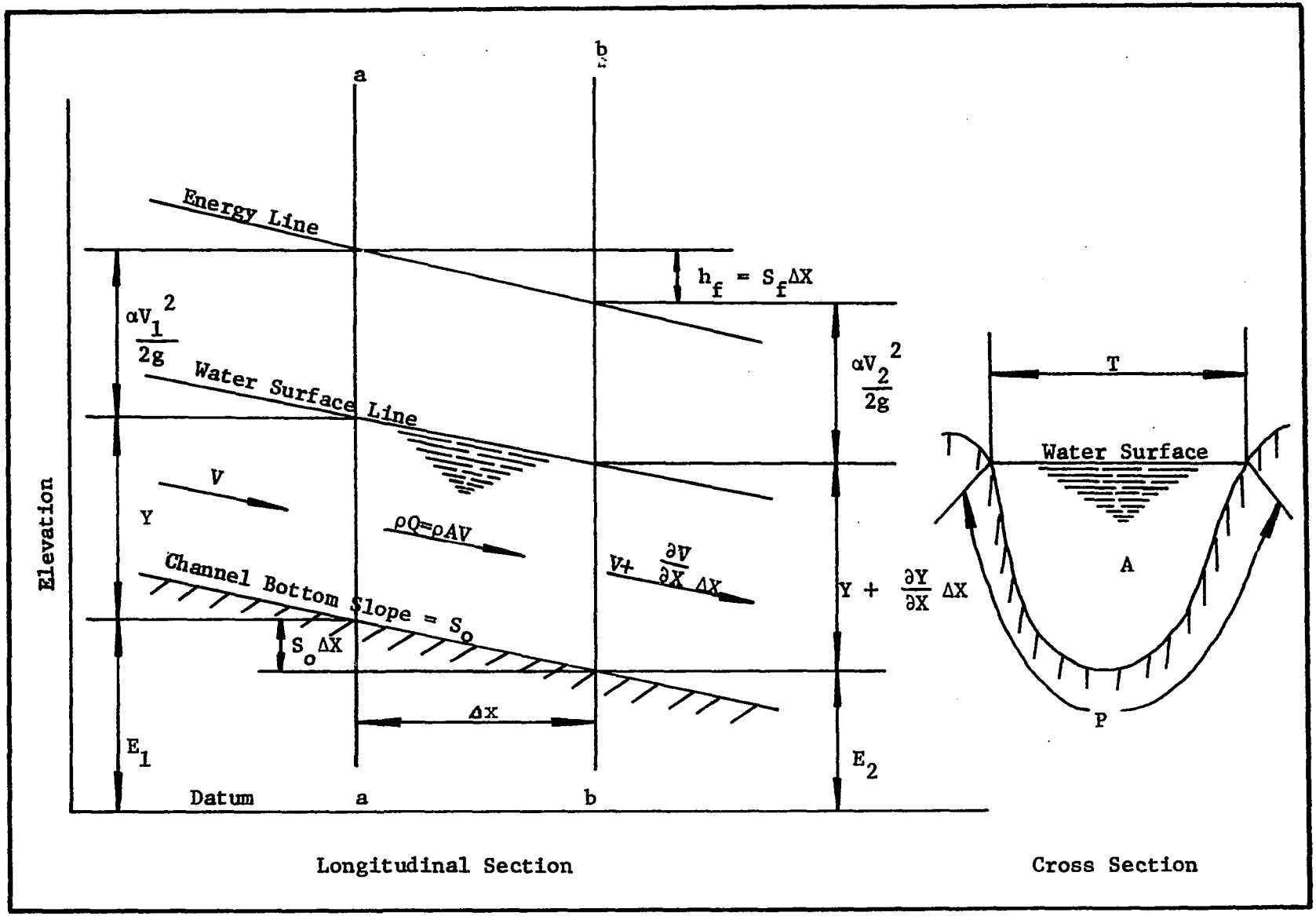

a
b
Energy Line
$h_f = S_f \Delta X$
$\frac{\alpha V_1^2}{2g}$
Water Surface Line
$\frac{\alpha V_2^2}{2g}$
V
Y
$\rho Q = \rho A V$
$V + \frac{\partial V}{\partial X} \Delta X$
Channel Bottom Slope = S_o
$S_o \Delta X$
$Y + \frac{\partial Y}{\partial X} \Delta X$
ΔX
E_1
E_2
Datum
a
b
Elevation
T
Water Surface
A
P
Longitudinal Section
Cross Section

Fig. 3-1

the channel through section a-a in cubic feet per second. This study assumes no lateral inflow.

The mass of water entering the channel reach during a time interval will be

$$\rho Q \Delta t \tag{3-1}$$

The volume outflow rate is

$$Q + \frac{\partial Q}{\partial X} \Delta X \tag{3-2}$$

The mass of water leaving the channel during the same time Δt, from expression (3-2) will be

$$\rho (Q + \frac{\partial Q}{\partial X} \Delta X) \Delta t \tag{3-3}$$

Combining the inflow and outflow from expressions (3-1) and (3-3) we have the following expression:

$$\rho Q \Delta t - \rho (Q + \frac{\partial Q}{\partial X} \Delta X) \Delta t \tag{3-4}$$

The change in storage is given by

$$\rho \frac{\partial A}{\partial t} \Delta X \Delta t \tag{3-5}$$

For the law of conservation of mass, inflow minus outflow equals the change in storage, so we equate the expressions (3-4) and (3-5)

$$\rho Q \Delta t - \rho (Q + \frac{\partial Q}{\partial X} \Delta X) \Delta t = \rho \frac{\partial A}{\partial t} \Delta X \Delta t \tag{3-6}$$

Dividing by $\rho\Delta t$ from equation (3-6)

$$Q - (Q + \frac{\partial Q}{\partial X} \Delta X) = \frac{\partial A}{\partial t} \Delta X \qquad (3-7)$$

Simplifying equation (3-7)

$$\frac{\partial Q}{\partial X} + \frac{\partial A}{\partial t} = 0 \qquad (3-8)$$

Equation (3-8) is the continuity equation and is the mathematical expression for the law of conservation of mass for open channel, non-uniform, unsteady flow. Since $Q = AV$, for a trapezoidal cross section the area $A = (B + ZY)Y$ and the top width $T = B + 2ZY$, so the continuity equation may be written as the following:

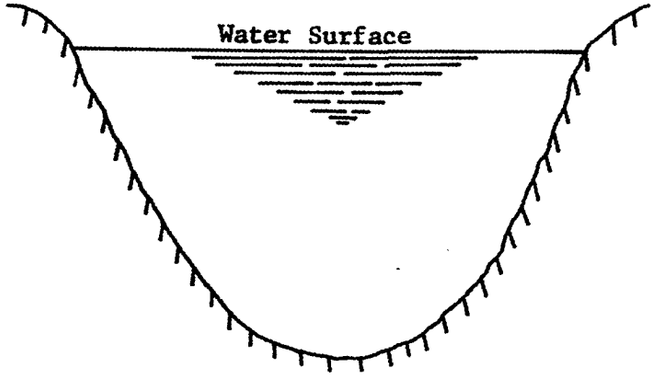
$$\frac{A}{T} \frac{\partial V}{\partial X} + V \frac{\partial Y}{\partial X} + \frac{\partial Y}{\partial t} = 0 \qquad (3-9a)$$

For a rectangular cross section, this expression is given by

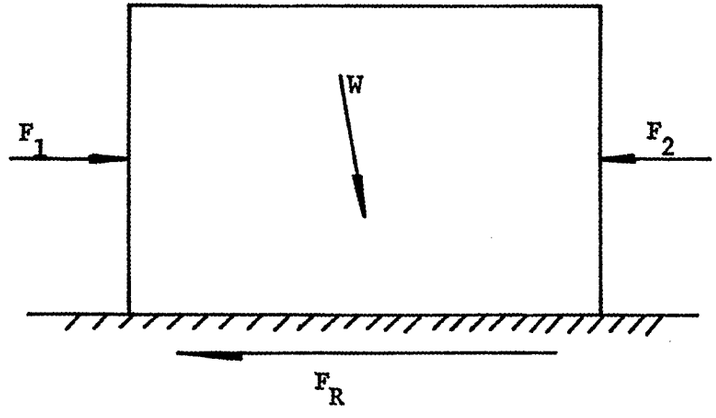
$$Y \frac{\partial V}{\partial X} + V \frac{\partial Y}{\partial X} + \frac{\partial Y}{\partial t} = 0 \qquad (3-9b)$$

B. Momentum Equation

The equation for conservation of momentum is given by Newton's second law of motion which states that total forces acting on an element are equal to the rate of mementum change. The forces acting on the element are shown in Figure 3-2. These forces are the result of the pressure, gravity and friction forces.



Cross Section



Forces Acting on the Flow Element

Fig. 3-2

Letting γ be the specific weight of the fluid ($\gamma = 62.4$ lb/ft^3 for water, g is the acceleration due to the gravity which denotes 32.2 ft per second per second. Also $\gamma = \rho g$, S_f refers to the friction slope in ft/ft, S_o is the channel longitudinal bottom slope in ft/ft, h_f is the head loss in ft, and the forces F_1 and F_2 represent the hydrostatic forces at the end faces of the element.

$$F_1 = \gamma YA - \gamma A \frac{\partial Y}{\partial X} \frac{\Delta X}{2} \tag{3-10}$$

$$F_2 = \gamma YA + \gamma A \frac{\partial Y}{\partial X} \frac{\Delta X}{2} \tag{3-11}$$

The water depth at section a-a is Y; at section b-b it is $Y + \frac{\partial Y}{\partial X}\Delta X$ The cross section area at section a-a is A; at section b-b, it is $A + \frac{\partial A}{\partial X}\Delta X$ The pressure force F_1 acts to the right and the pressure force F, acts to the left. It is assumed that the water depth at section b-b is greater than the section a-a water depth. So the resultant hydrostatic force is

$$F_p = -\rho g A \frac{\partial Y}{\partial X} \Delta X \tag{3-12}$$

The body force expression is equal to the weight of the fluid inside the element ($\rho g A \Delta X$) times the channel longitudinal bottom slope (S_o)

$$F_g = \rho g A \Delta X \cdot S_o \tag{3-13}$$

The head loss is equal to the friction slope (S_f) times the length of the element (ΔX)

$$h_f = S_f \Delta X \tag{3-14}$$

The energy losses caused by the boundary drag, turbulence, and modifications to the velocity distribution pattern produces an energy line with a slope S_f which results in a loss of head in the length ΔX. The resistance force is given by

$$F_R = \rho g A h_f \tag{3-15}$$

From equation (3-14) $h_f = S_f \Delta X$

$$F_R = \rho g A S_f \Delta X \tag{3-16}$$

Mass in the small element = $\rho A \Delta X$ and the acceleration = $a = \frac{dv}{dt}$. Rate of the momentum change of the fluid through the element is the change in time with respect to the momentum inside of the volume element.

$$\rho A \Delta X \frac{dv}{dt} \tag{3-17}$$

where $\frac{dv}{dt}$ is defined as

$$\frac{dv}{dt} = \frac{\partial v}{\partial t}\frac{dt}{dt} + \frac{\partial v}{\partial x}\frac{dx}{dt} = \frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x} \tag{3-18}$$

So the rate of the momentum change is given by

$$\rho A \Delta X \left(\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial X} \right) \tag{3-19}$$

Finally, using Newton's second law, F = Ma, and combining the equations of pressure, friction, gravitation, and the rate of the momentum change equations (3-12), (3-13), (3-16), and (3-19)

$$\rho g A \Delta X S_o - \rho g A \Delta X S_f - \rho g A \frac{\partial Y}{\partial X} \Delta X = \rho A \Delta X \left(\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial X} \right) \tag{3-20}$$

Simplifying equation (3-20)

$$\rho g A \frac{\partial Y}{\partial X} \Delta X + \rho A \Delta X \left(\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial X} \right) = \rho g A \Delta X S_o - \rho g A S_f \Delta X \tag{3-21}$$

Dividing equation (3-21) by $\rho A \Delta X$

$$g \frac{\partial Y}{\partial X} + \frac{\partial v}{\partial t} + V \frac{\partial v}{\partial X} = g S_o - g S_f \tag{3-22}$$

Simplifying equation (3-22)

$$g \frac{\partial Y}{\partial X} + \frac{\partial v}{\partial t} + V \frac{\partial v}{\partial X} = g \left(S_o - S_f \right) \tag{3-23}$$

Equation (3-23) is the momentum equation for open channel non-uniform, unsteady flow. But for this study, it is applied to a reservoir where friction slope is negligibly small, so let S_f equal zero, whereby equation (3-23) becomes:

$$g \frac{\partial Y}{\partial X} + \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial X} = g S_o \tag{3-24}$$

Part 2. Method of Solution

A strict closed form mathematical solution to the reservoir routing problem has been shown in the literature to be extremely complicated, difficult, and, as yet unsolved. However, various simplified methods have been developed for practical purposes. The approach dealt with in this problem is to use the finite difference technique related to the general method of characteristics. Now, it is the purpose of this research to seek a numerical methods type solution which is based on the solution to the set of differential equations of unsteady flow in reservoirs.

The equations governing the unsteady flow problem under consideration are treated in this study. Although a general solution is not available, the method of characteristics is used to transform the partial differential equations into particular total differential equations which are then solved by a first order finite difference technique. In order to obtain an orderly numerical solution on the digital computer, a method of specified time intervals is adopted.

The continuity and momentum equations (3-9a) and (3-24), form a pair of quasilinear hyperbolic partial differential equations in terms of two dependent variable, velocity (V) and depth (y), and two independent variable, distance (X) along the reservoir and time (t).

The slope of the characteristics curve is used to indicate the solutions of the following partial differential equations with independent variables distance (X) along the reservoir and time (t) and dependent variable flow depth (y) and flow velocity (v).

The continuity equation

$$V \frac{\partial y}{\partial X} + \frac{A}{T} \frac{\partial v}{\partial X} + \frac{\partial y}{\partial t} = 0 \qquad (3\text{-}25)$$

The momentum equation

$$g \frac{\partial y}{\partial X} + V\frac{\partial v}{\partial X} + \frac{\partial v}{\partial t} = g\ S_o \qquad (3\text{-}26)$$

The total changes in depth

$$\frac{\partial y}{\partial X} dx + \frac{\partial y}{\partial t} dt = dy \qquad (3\text{-}37)$$

The total change in velocity

$$\frac{\partial v}{\partial X} dx + \frac{\partial v}{\partial t} dt = dv \qquad (3\text{-}28)$$

In the above four equations, $\frac{\partial y}{\partial X}$ is the slope of water surface $\frac{\partial y}{\partial t}$ is the change of depth of flow with respect to time, $\frac{\partial v}{\partial X}$ is the change velocity with respect to distance, and $\frac{\partial v}{\partial t}$ is the change velocity with respect to time. These equations (3-25, 3-26, 3-27 and 3-28) are a set of nonhomogeneous linear equations in the four unknowns $\partial V/\partial X$, $\partial V/\partial t$, $\partial y/\partial X$ and $\partial y/\partial t$. These can be expressed by a single matrix equation as the following:

$$\begin{matrix} \partial y/\partial X & \partial y/\partial t & \partial V/\partial X & \partial v/\partial t \end{matrix}$$

$$\begin{bmatrix} V & 1 & A/T & 0 \\ g & 0 & V & 1 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix} \begin{bmatrix} \partial y/\partial X \\ \partial y/\partial t \\ \partial V/\partial X \\ \partial v/\partial t \end{bmatrix} = \begin{bmatrix} 0 \\ gS_o \\ dy \\ dv \end{bmatrix} \qquad (3\text{-}29)$$

The theory of linear algebraic equations show that if the determinant of the coefficient matrix vanishes, that is, if

$$\begin{vmatrix} V & 1 & A/T & 0 \\ g & 0 & V & 1 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{vmatrix} = 0 \qquad (3\text{-}30)$$

The equation (3-30) shows either an infity of solutions or no solution results and also show discontinuities to determine the characteristic direction (See Fig. 3-3).

Simplifying equation (3-30)

$$\left(\frac{dx}{dt}\right)^2 - 2V\left(\frac{dx}{dt}\right) + \left(V^2 - g\frac{A}{T}\right) = 0 \qquad (3\text{-}31)$$

$$\frac{dx}{dt} = V \pm \sqrt{\frac{gA}{T}} \qquad (3\text{-}32)$$

For a rectangular cross section, which could sometimes be used, this expression is given by

$$\frac{dx}{dt} = V \pm \sqrt{gy} \tag{3-32a}$$

$$\text{Let } C = \sqrt{\frac{gA}{T}} \text{ or } C = \sqrt{gy} \tag{3-33}$$

The characteristic curve

$$\left(\frac{dx}{dt}\right)_{\alpha} = V + C \tag{3-34}$$

$$\left(\frac{dx}{dt}\right)_{\beta} = V - C \tag{3-35}$$

Finite - Difference Equations

The method of the finite-difference technique is based on the determination of approximate solutions of the partial differential equations in a discrete net of points in the x-t plane. There are various procedures which can be used to solve approimate solutions. In general, the system computation normally adopted is used on a rectangular array of points in distance and time (X, t - plane) as shown in Fig. 3-3. When using a finite difference technique to solve a partial differential equation (when given initial and boundary conditions) a network of gird points is first established throughout the region of intersect occupied by the independent variables. For example, the distance coordinate x and the time t are independent variables, the respective grid spacings are ΔX and Δt, and the dependent variables are the flow depth y and velocity v.

One solves the characteristic equations by using the first order finite difference approximations. The subscripts are used to define the

location of the known and unknown quantity. These two partial differential equations (3-9) and (3-24a), the initial data, boundary conditions, and inflow hydrographs are given. These two equations have a uniquely determined solution (see Fig. 3-3) for the unknown quantities flow depth y and velocity v for all future times. The two unknowns y (x,t) and V(x,t) are advanced by a time increment Δt through using the partial differental equations.

To solve these two partial differential equations by using the first-order finite-difference method can combined the continuity and momentum equations together and multiplied by K to momentum equation. The K will make both equations dimensionally compatible.

$$\frac{A}{T}\frac{\partial v}{\partial x} + V\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} + Kg\frac{\partial y}{\partial x} + KV\frac{\partial v}{\partial x} + K\frac{\partial v}{\partial t} = KgSo \qquad (3\text{-}36)$$

$$K[\frac{\partial v}{\partial x}(V + \frac{A}{KT}) + \frac{\partial v}{\partial t}] + [\frac{\partial y}{\partial x}(V + Kg) + \frac{\partial y}{\partial t}] = KgSo \qquad (3\text{-}37)$$

Total derivative in velocity respect to time

$$\frac{dv}{dt} = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial t}\frac{dt}{dt} \qquad (3\text{-}38)$$

Total derivative in depth respect to time

$$\frac{dy}{dt} = \frac{\partial y}{\partial x}\frac{dx}{dt} + \frac{\partial y}{\partial t}\frac{dt}{dt} \qquad (3\text{-}39)$$

comparing equations (3-38) and (3-37) gives

$$\frac{dx}{dt} = V + \frac{A}{KT} \tag{3-40}$$

Comparing equations (3-39) and (3-37) gives

$$\frac{dx}{dt} = V + Kg \tag{3-41}$$

Solving for K from equations (3-40) and (3-41)

$$K = \sqrt{\frac{A}{gT}} \tag{3-42}$$

For a rectangular cross section, this expression is given by

$$K = \sqrt{\frac{y}{g}} \tag{3-42a}$$

Consider that initial conditions for velocity and depth are known at points L and R from Fig. 3-3. The two characteristic curves C+ and C-, passing through points L and R and intersect at point P where conditions are unknown.

Substituting K into the equation (3-37)

$$\frac{dy}{dt} + K\frac{dv}{dt} - KgSo = 0 \tag{3-43}$$

From equation (3-43) and Fig. 3-3 can write the following finite-difference equation

$$\frac{\Delta y}{\Delta t} + K \frac{\Delta v}{\Delta t} - KgSo = 0 \qquad (3-44)$$

or

$$\Delta y + K\Delta V - KgSo\Delta t = 0 \qquad (3-45)$$

From the characteristics curve can obtain the following four finite-difference equations

(1) The C+ Characteristic Curve

$$Y_P - Y_L + K\ (V_P - V_L) - KgSo\Delta t = 0 \qquad (3-46)$$

$$X_P - X_L = (V + C)\ (t_p - t_L) \qquad (3-47)$$

(2) The C- Characteristic curve

$$Y_P - Y_R - K\ (V_P - V_R) - KgSo\Delta t = 0 \qquad (3-48)$$

$$X_P - X_R = (V - C)\ (t_P - t_R) \qquad (3-49)$$

The above four equations have four unknowns V_p, Y_p, X_P and t_p. To solve for these unknowns, Let L be the length of the reservoir, N is the number of equal reaches.

$$\Delta X = X_P - X_L = X_P - X_R = L/N \tag{3-50}$$

and

$$\Delta t = t_P - t_L = t_P - t_R = \frac{\Delta X}{V+C} \tag{3-51}$$

Now, solving equations (3-46) and (3-48) are obtained for V_P and Y_P.

$$Y_P = \frac{1}{2}(Y_L + Y_R) + \frac{1}{2} K (V_L - V_R) + KgSo\Delta t \tag{3-52}$$

and

$$V_P = \frac{1}{2K}(Y_L - Y_R) + \frac{1}{2}(V_L + V_R) \tag{3-53}$$

The values of both Y_P and V_P are used to designate the interior points.

At the upstream end of the reservoir, the velocity V_{up} is determined by using the C_- Characteristic Curve based on the both continuity and momentum equations. The reservoir at fixed level, the velocity V_{up} is obtained directly from equation (3-48).

$$V_{up} = V_R + \frac{1}{K}(Y_P - Y_R) - g\ So\ \Delta t \tag{3-54}$$

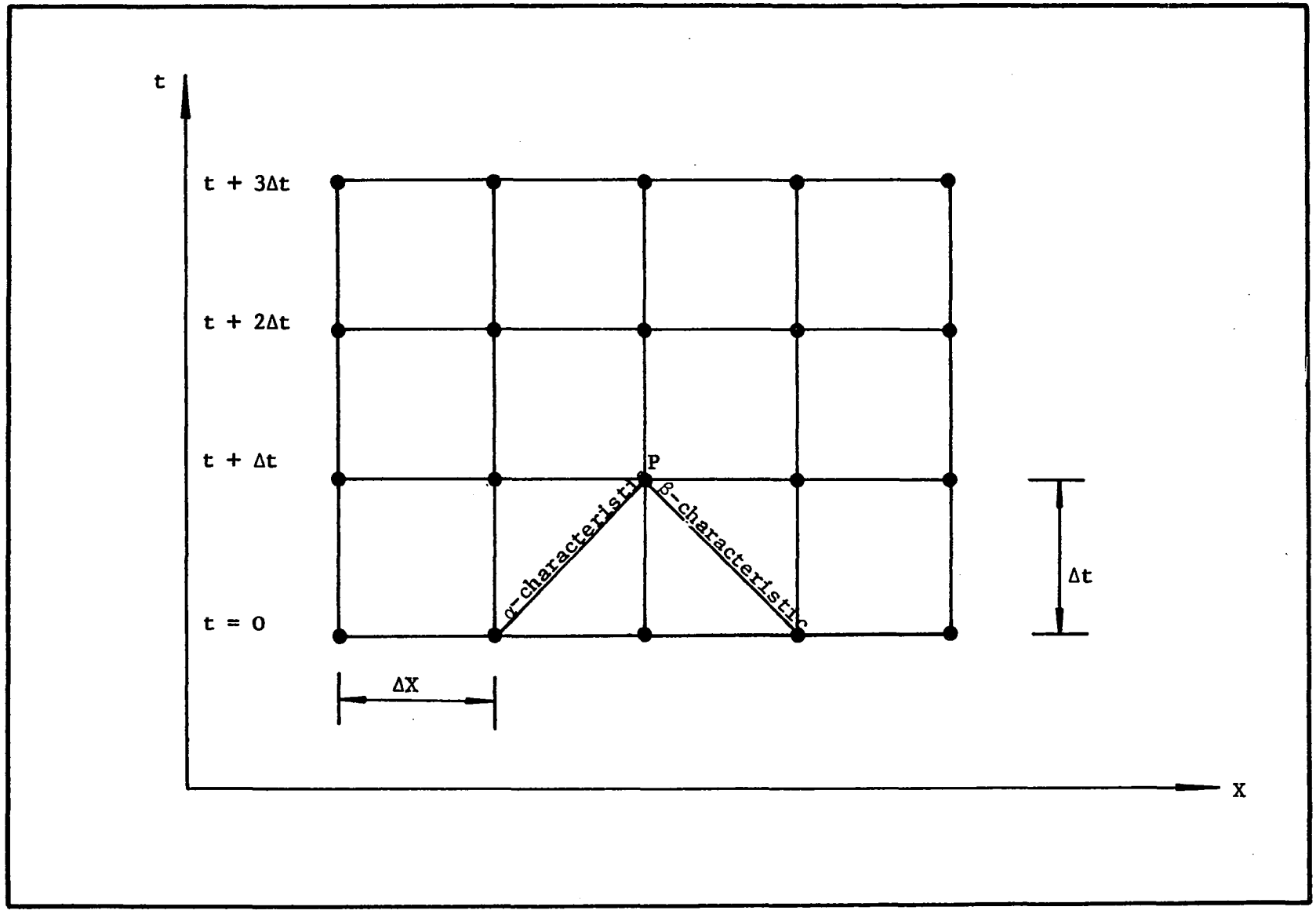


Fig. 3-3. Grid for Finite Difference Approximations

At the downstream end of the reservoir, the expression for velocity V_{DP} becomes

$$V_{DP} = \frac{C_w L_1 (H)^{3/2}}{AP} \tag{3-55}$$

Where: C_w = Coefficient of Discharge

L_1 = Length of crest

H = The head

A_p = The Area.

CHAPTER IV

DIGITAL COMPUTER SOLUTION

The computer solution discussed in this Chapter will provide a simplified solution to these partial differential equations solved in Chapter III. In solving these partial differential equations of unsteady flow, it is necessary to specify initial and boundary conditions.

Initial Conditions

The necessary initial conditions for the unsteady non-uniform flow in the reservoir are the inflow and outflow rate and area of water flow surface along the reservoir (see Fig. 4-1). They (initial conditions) are specified at fixed values of time (Δt) at various spatial locations.

Given the initial values of depth y and velocity V at time t=0 for a series of stations spaced ΔX along the reservoir, the values of y and V can be determined for the same stations at a time $t_1 = t_o + \Delta t$. By taking successive time increments the solution can be said to obtain in time from the initial conditions. A convergent solution requires that the selection of Δt with respect to ΔX be such that no point will lie outside of the area bounded by the characteristic curves. By the theory of characteristics, the maximum Δt can be determined by the following equation.

$$\Delta t_{(max)} = \frac{\Delta x}{V+C} \tag{4-1}$$

The term C in the above equation is used to represent the celerity of gravity wave, and equal to $\sqrt{gA/T}$.

Boundary Conitions

Boundary Conditions are conditions specified at fixed values of distance (X) at time (t) in various discharge versus time. The boundary conditions can be prescribed at the upstream end or left boundary (X=o, t=o), and at the downstream end or right boundary (X=L, t=o). These are all along the reservoir bottom.

Upstream Boundary Condition

The boundary condition at the upstream inlet of the reservoir is given by a discharge inflow hydrograph (see Fig. 4-2). A inflow hydrograph is a graph of discharge against time. The inflow hydrograph by the triangular method is used in the case where rainfall and runoff records are not available. The time to the peak flow and the time from peak flow to the end flow are computed by the following equations

$$T_p = \frac{1}{6} T \tag{4-2}$$

and

$$T_E = \frac{5}{6} T \tag{4-3}$$

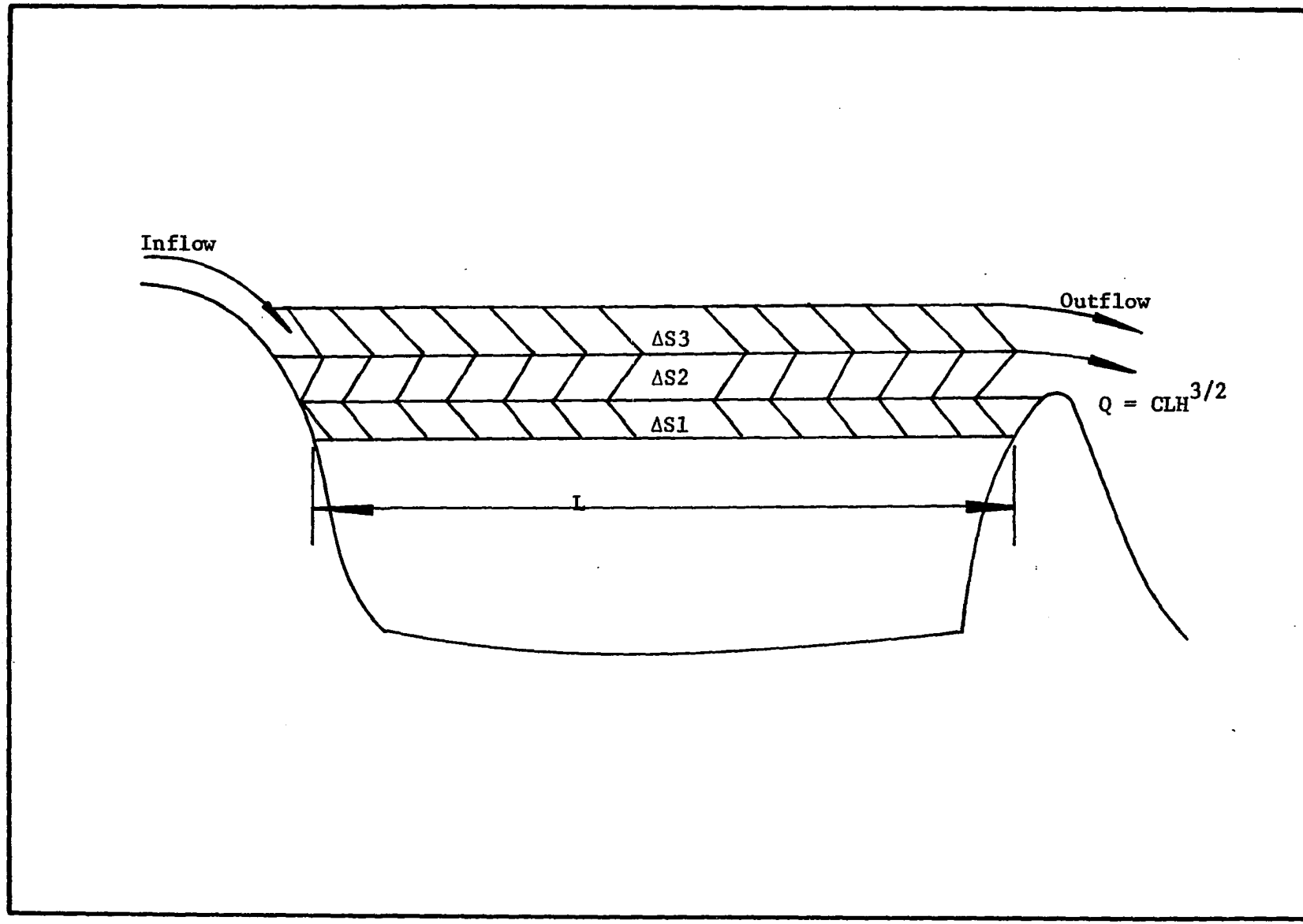


Fig. 4-1 Reservoir Longitudinal Section

in which: T_P = the time to the peak flow

T_E = the time from peak flow to the end flow

T = total time is given.

Downstream Boundary Condition

The downstream boundary condition is based on the principal spillway capacity and retarding storage amount. These are proportioned using the principal spillway hydrograph commonly called the outflow hydrograph. It is the safety valve for the flood control reservoir (See Fig. 4-3). The general formula for the free discharge of a spillway (23, page 362), 1959 is given by the following equation

$$Q = C_W L_1 (H)^{3/2} \qquad (4-4)$$

in which: Q = outflow rate in Cfs

C_W = Coefficient of discharge

L_1 = length of spillway in feet

H = the head in feet.

For the trapezoidal channel cross-section area is computed by the following equation

$$A = (B + ZY)\ Y \qquad (4-5)$$

in which: A = Cross section area in feet square

B = the channel bottom width in feet

Y = the channel depth in feet

Z = the side slope of channel

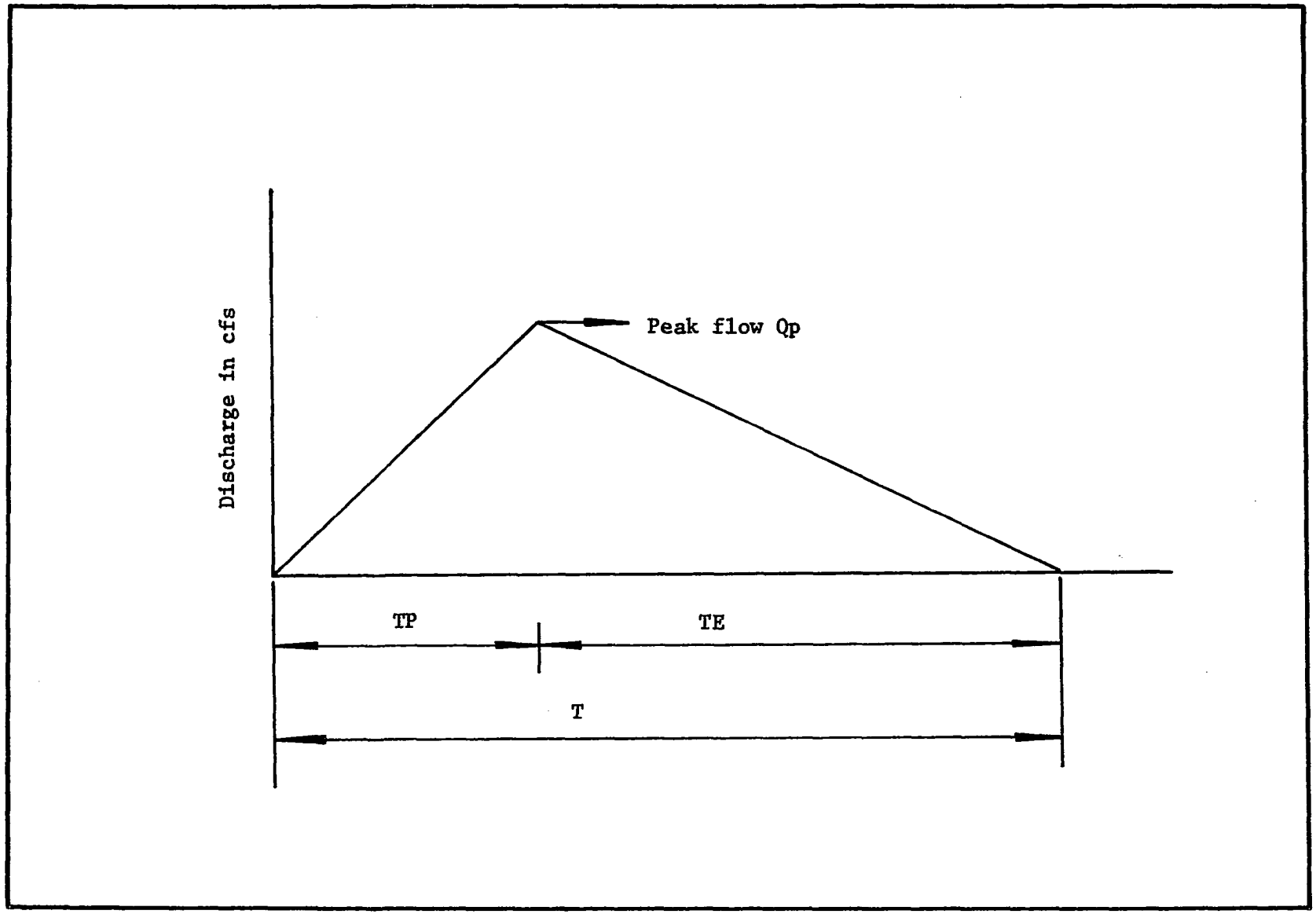


Fig. 4-2 Inflow Hydrograph into Reservoir

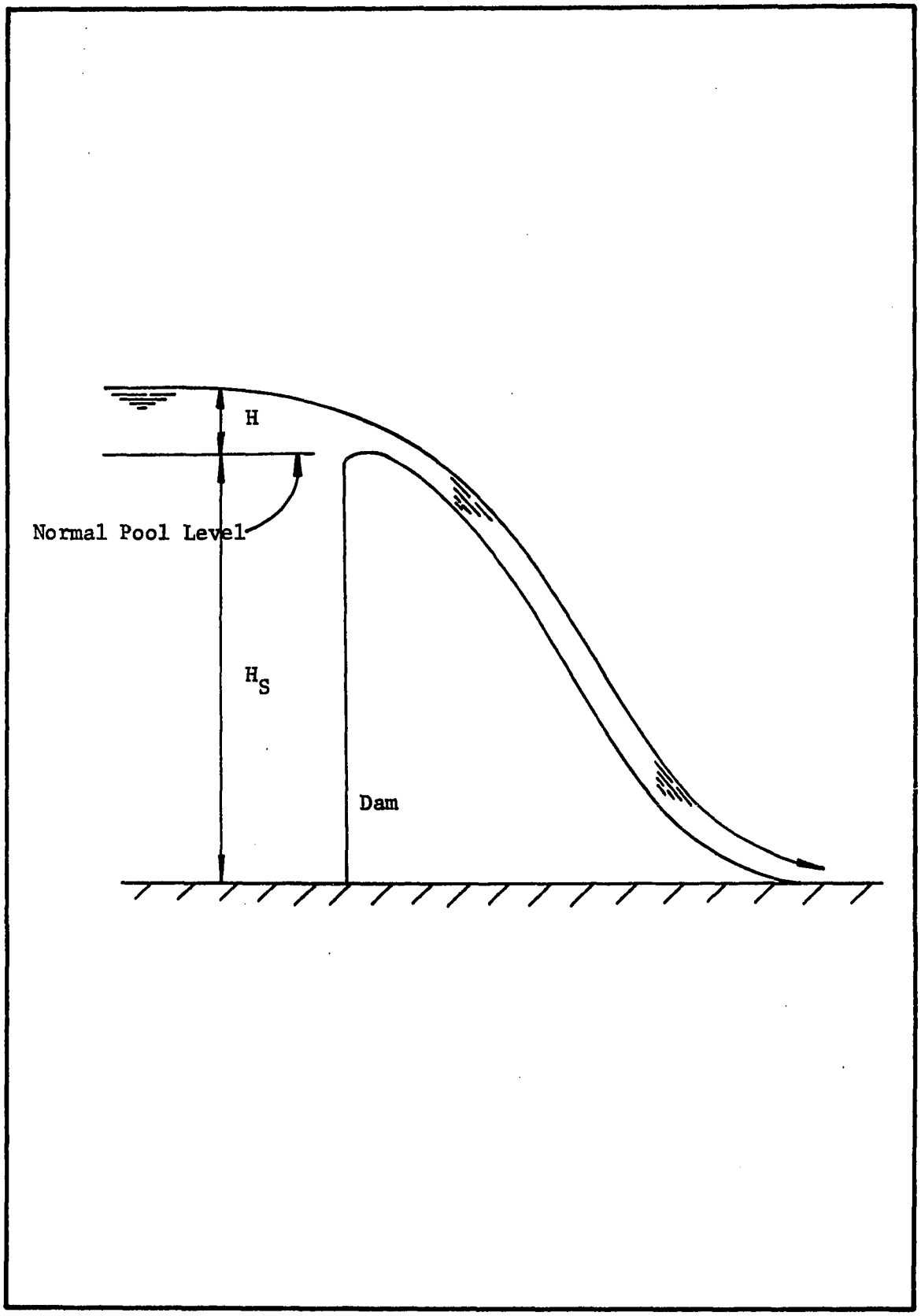


Fig. 4-3. Section of Spillway

Computer Programming

The computer program is based on the given initial and boundary conditions to compute the inflow and outflow rate. All programs were written in FORTRAN IV and were executed on the IBM System 360 Digital Computer.

Given typical assumed data for an example solution to be the following:

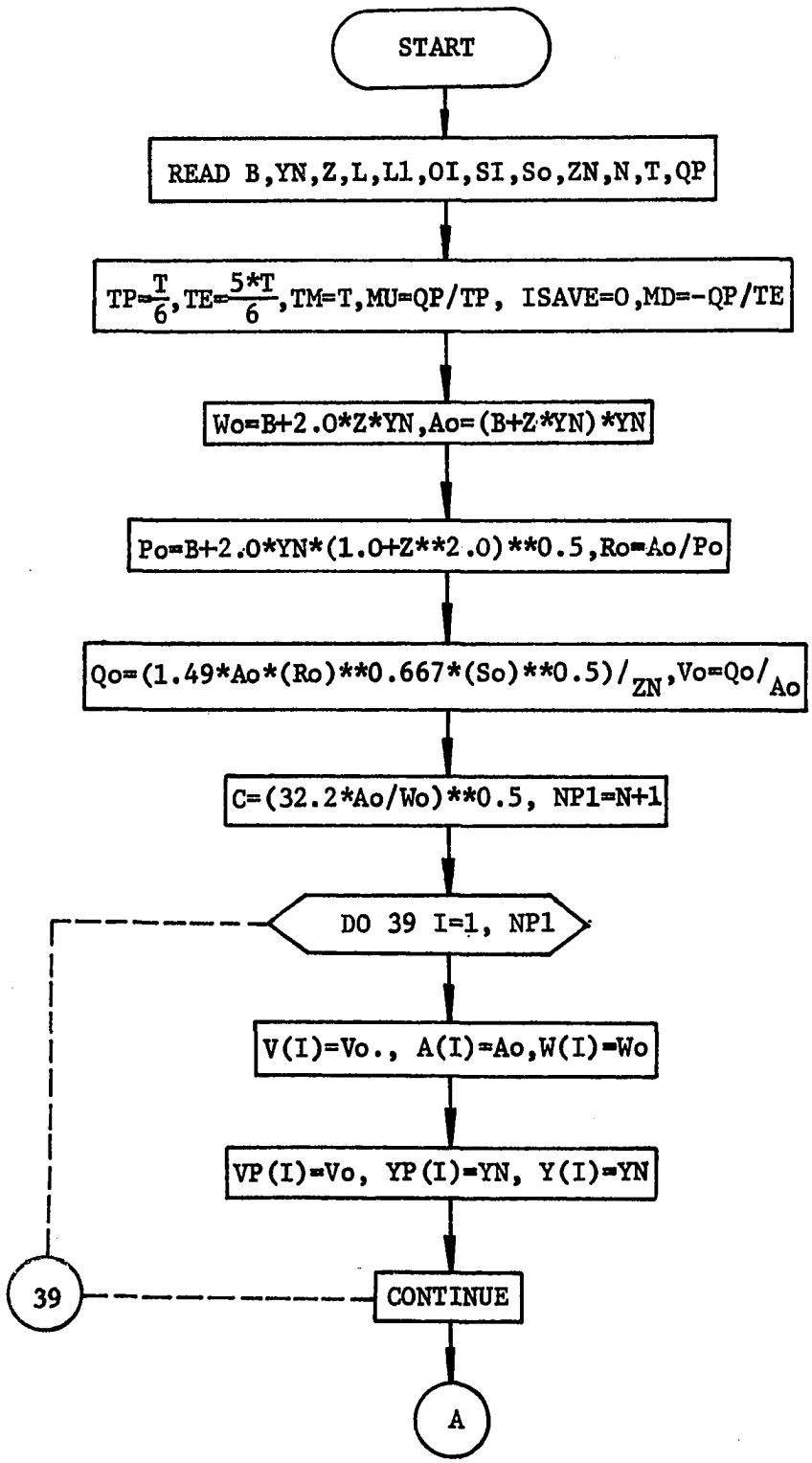
(1) Peak flow Q_p = 4200 Cfs

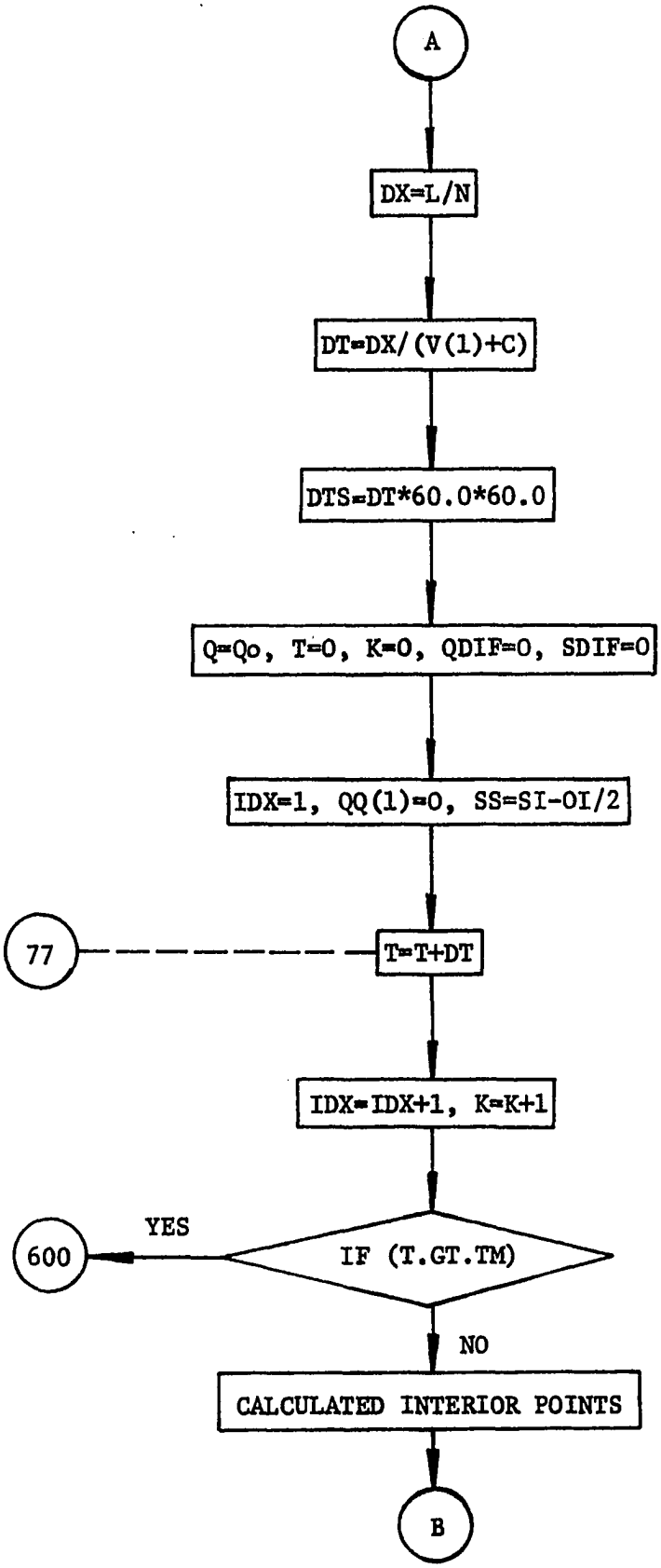
(2) Total time T = 36 hours

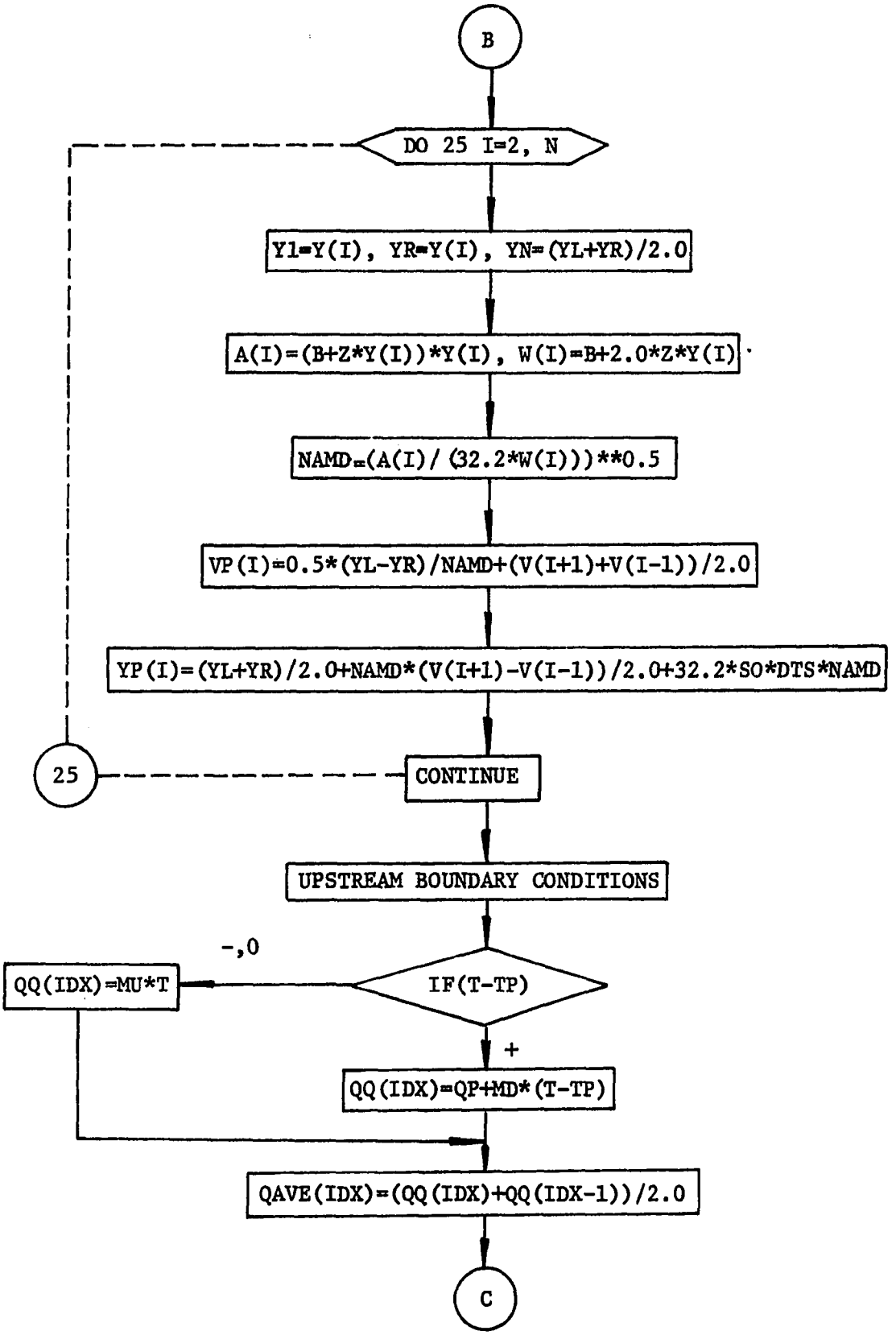
(3) The reservoir geometric elements are an assumed trapezoid channel, the bottom width of reservoir B=1000 feet, the depth of reservoir y = 5 feet, the side slope of reservoir Z = 0.25 the length of the reservoir L = 10,560 feet, the length of the Weir Crest Ll= 50 feet.

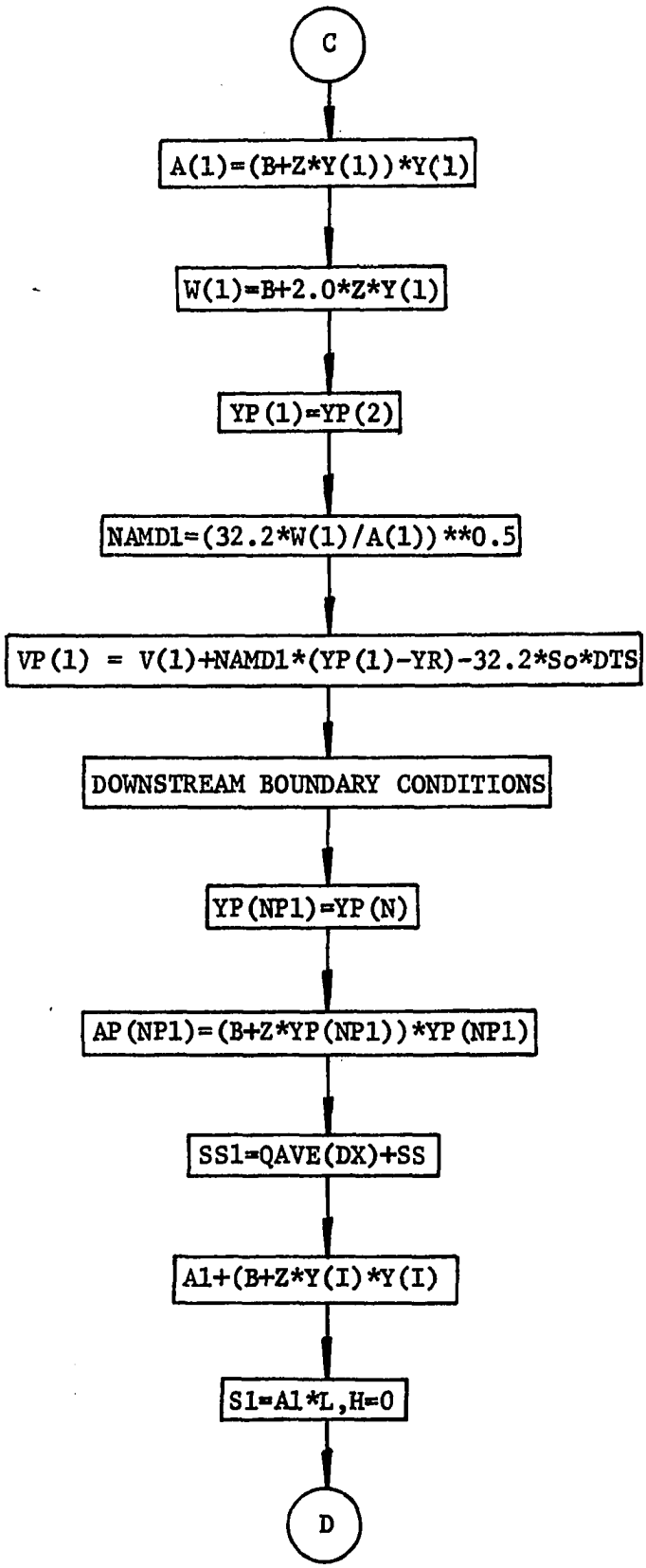
A flow chart for the hydraulic reservoir routing is given in Figure 4-4 and a computer printout is given in the Appendix A.

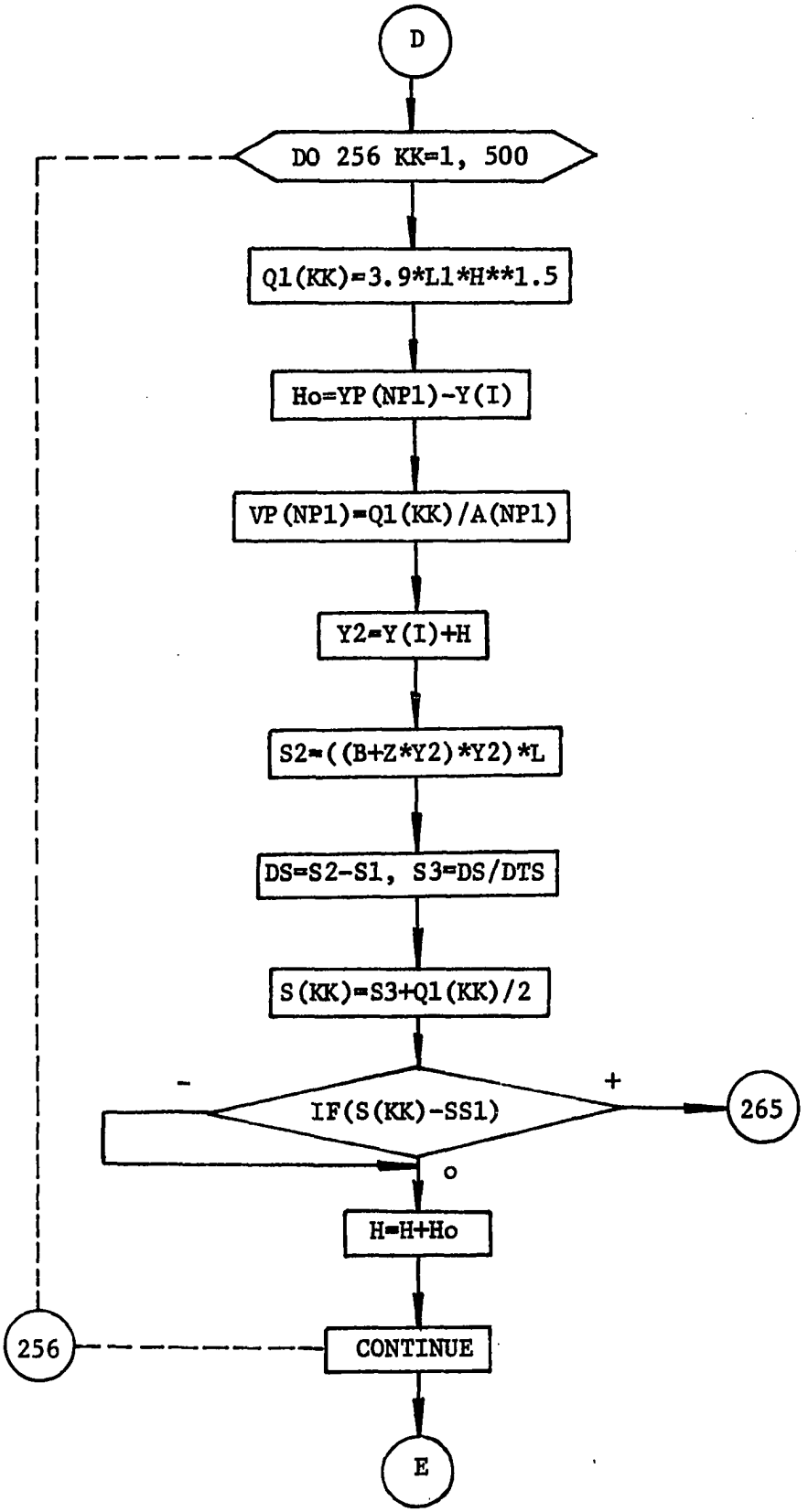
FLOW CHART FOR HYDRAULIC RESERVOIR ROUTING METHOD COMPUTATIONS




A
DX=L/N
DT=DX/(V(1)+C)
DTS=DT*60.0*60.0
Q=Qo, T=0, K=0, QDIF=0, SDIF=0
IDX=1, QQ(1)=0, SS=SI-OI/2
77
T=T+DT
IDX=IDX+1, K=K+1
YES
600
IF (T.GT.TM)
NO
CALCULATED INTERIOR POINTS
B


B
DO 25 I=2, N
Y1=Y(I), YR=Y(I), YN=(YL+YR)/2.0
A(I)=(B+Z*Y(I))*Y(I), W(I)=B+2.0*Z*Y(I)
NAMD=(A(I)/(32.2*W(I)))**0.5
VP(I)=0.5*(YL-YR)/NAMD+(V(I+1)+V(I-1))/2.0
YP(I)=(YL+YR)/2.0+NAMD*(V(I+1)-V(I-1))/2.0+32.2*SO*DTS*NAMD
25
CONTINUE
UPSTREAM BOUNDARY CONDITIONS
-,0
QQ(IDX)=MU*T
IF(T-TP)
+
QQ(IDX)=QP+MD*(T-TP)
QAVE(IDX)=(QQ(IDX)+QQ(IDX-1))/2.0
C


C
A(1)=(B+Z*Y(1))*Y(1)
W(1)=B+2.0*Z*Y(1)
YP(1)=YP(2)
NAMD1=(32.2*W(1)/A(1))**0.5
VP(1) = V(1)+NAMD1*(YP(1)-YR)-32.2*So*DTS
DOWNSTREAM BOUNDARY CONDITIONS
YP(NP1)=YP(N)
AP(NP1)=(B+Z*YP(NP1))*YP(NP1)
SS1=QAVE(DX)+SS
A1+(B+Z*Y(I)*Y(I)
S1=A1*L,H=0
D


D
DO 256 KK=1, 500
Q1(KK)=3.9*L1*H**1.5
Ho=YP(NP1)-Y(I)
VP(NP1)=Q1(KK)/A(NP1)
Y2=Y(I)+H
S2=((B+Z*Y2)*Y2)*L
DS=S2-S1, S3=DS/DTS
S(KK)=S3+Q1(KK)/2
IF(S(KK)-SS1)
-
+
265
o
H=H+Ho
256
CONTINUE
E

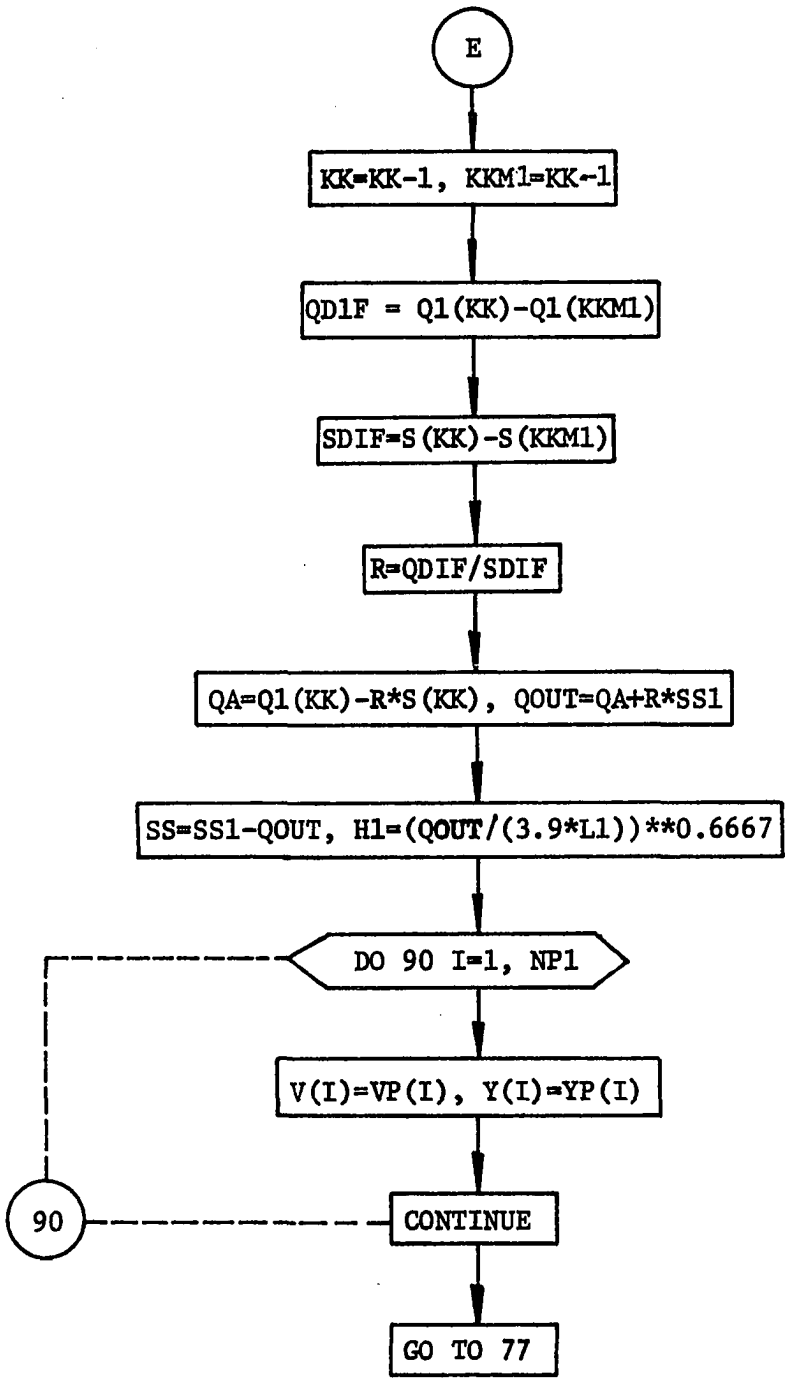


Fig. 4-4 Flow Chart

CHAPTER V

COMPARISON TO HYDROLOGIC RESERVOIR ROUTING

Reservoir routing methods entail the hydraulic and hydrologic routing. H. A. Thomas' (10), 1937 fundamental contribution to the literature on flood routing was found to be the use of continuity and momentum two partial diffential equations in hydraulic reservoir routing. Generally, hydrologic routing methods are formulations which are approximate in the sense that the basic continuity equation or conservation of mass differ from the momentum equation.

Since the function of reservoirs is to provide storage, their most important physical characteristic is storage capacity. The storage capacity of a reservoir of regular shape can be computed with the formulas for the volume of solids. The solution of the storage capacity in either hydraulic or hydrologic routing invariably requires the use of numerical analysis methods. The hydrologic reservoir routing are often called storage routing methods. Reservoirs have the characteristic that either storage is closely related to their outflow rate discharge. The storage discharge rate relation is used for repeatedly solving the continuity equation, each solution being a step in delineating the outflow hydroraph.

The basic point of initiation efforts to solve the hydrologic reservoir routing method is using the continuity equation or conservation

of mass. For a given time interval or routing period, the volume of inflow minus the volume of outflow equals to the change in volume of storage. The equation is often written in the following form:

$$I_{ave} - O_{ave} = \frac{\Delta S}{\Delta T} \tag{5-1}$$

in which: I_{ave} = the average rate of inflow during the time interval

O_{ave} = the average rate of outflow during the time interval

Δt = a time interval

ΔS = change in volume of storage during the time interval

The equation (5-1) inflow, outflow, and storage variable are expanded as the following

$$I_{ave} = \frac{I_1 + I_2}{2}$$

$$O_{ave} = \frac{O_1 + O_2}{2}$$

$$\Delta S = S_2 - S_1$$

$$\Delta t = t_2 - t_1$$

so that equation (5-1) change to

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t} \tag{5-2}$$

in which Δt = the time interval = $t_2 - t_1$

t_1 = the time of the beginning of the interval

t_2 = the time of the end of the interval

I_1 = inflow rate at time t_1

I_2 = inflow rate at time t_2

O_1 = outflow rate at time t_1

O_2 = outflow rate at time t_2

S_1 = storage rate at time t_1

S_2 = storage rate at time t_2

When routing using equation (5-1) the usual procedure is finding the outflow rate O_{ave}, with the equation (5-2) to find the outflow rate O_2. This means that two equations should be reset in some more convenient working form. Also, it is necessary to use the outflow rate to storage relationship in making a solution. Generally, it is using the storage-outflow relationship. Thus the unknowns in equation (5-2) are O_2 and S_2. Placing the knowns on the left side of the equation and the unknowns on the right side of the equation.

$$\frac{I_1 + I_2}{2} + \frac{A_1}{\Delta t} - \frac{O_1}{2} = \frac{S_2}{\Delta t} + \frac{O_2}{2} \qquad (5\text{-}3)$$

In the equation (5-3) $1/2(I_1 + I_2)$ are either taken from the mid-points of routing intervals of the inflow hydrographs or computed from the inflow tabulated at normal intervals. The S_1 and O_1 are knowns but the S_2 and O_2 are unknowns. It cannot be solved unless a second independent function is available such as a curve showing the relationship between outflow rate and discharge. Most of the earlier engineers used a graphical

method to solve the hydrologic reservoir routing. The purpose of this study is to describe hydrologic reservoir routing method solving by digital computers. The storage-indication method, which has been widely used for hydrologic reservoir routing, has inflow and outflow rate as the input and output. The method to be described here is suitable for practical purposes. This is base on computerization to solve the hyrologic reservoir routing method. It uses the following two assumptions:

(1). Develop the inflow hydrograph from the peak flow

(2). Select time interval (Δt)

The outflow discharge rate over the spillway (see Fig. 5-1) is a section of dam design to permit water to pass over the top of the Weir Crest. The spillway is the safety valve for the flood control reservoir (see Fig. 4-3). It must have the capacity to carry overflow discharge, and at the same time, keep the pool level below the maximum valve.

The discharge rate over a spillway can be computed by the experimental formula.

$$Q = C_w L1 H^{3/2} \tag{5-4}$$

in which Q is the outflow rate in cubic feet per second, C_w is the discharge coefficient, the C_w value varies from about 3.0 to 4.0, L1 is the length of the Weir Crest in feet, and H is the head above spillway in feet.

The computer program is based on the continuity equation to compute the inflow and outflow rate. All programs were written in FORTRAN IV and were executed on the IBM System 360 Digital Computer. A flow chart for

hydrologic reservoir routing is given in Fig. 5-2 and a computer printout is given in the Appendix B.

Given data as the same in the Chapter IV.

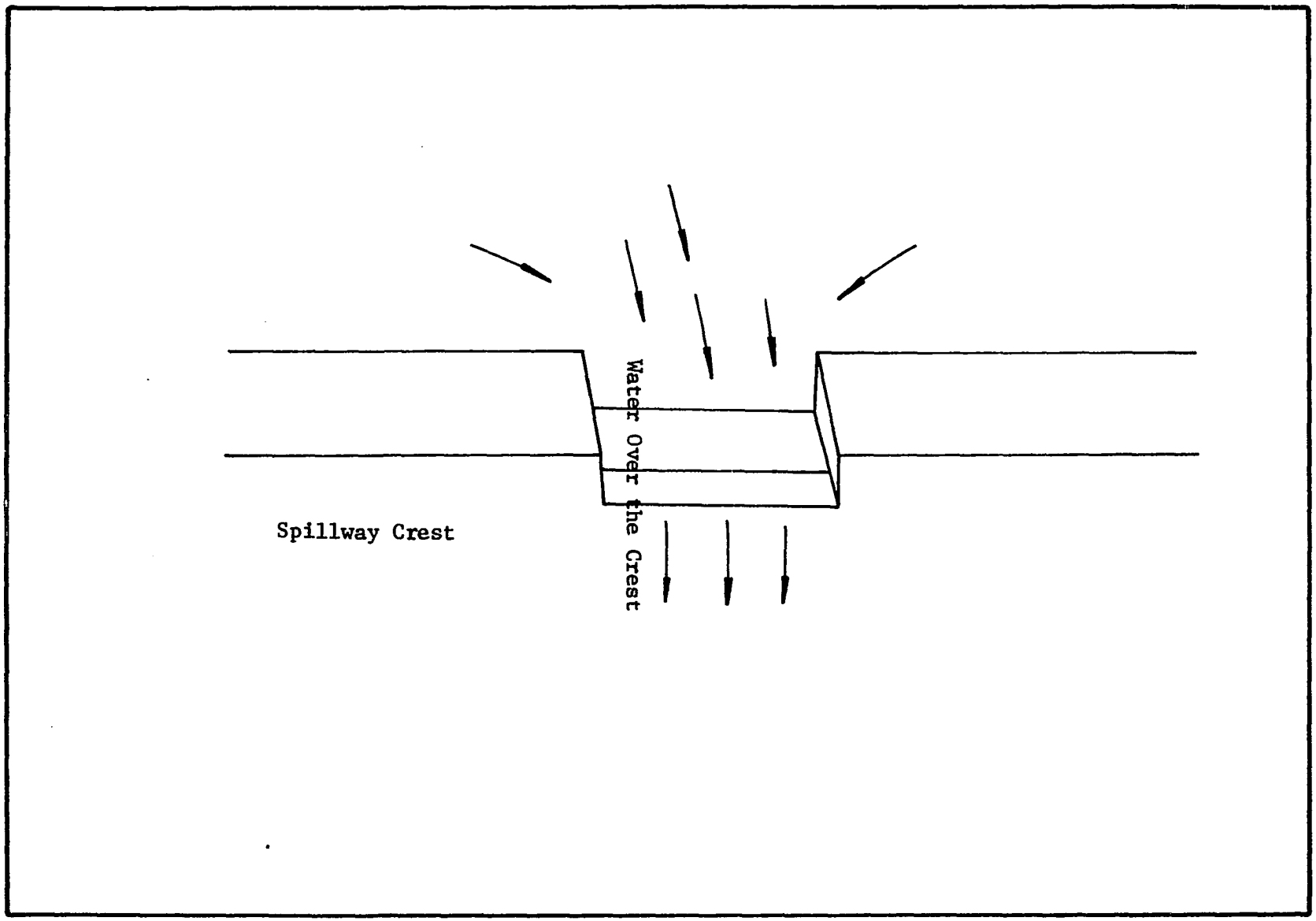
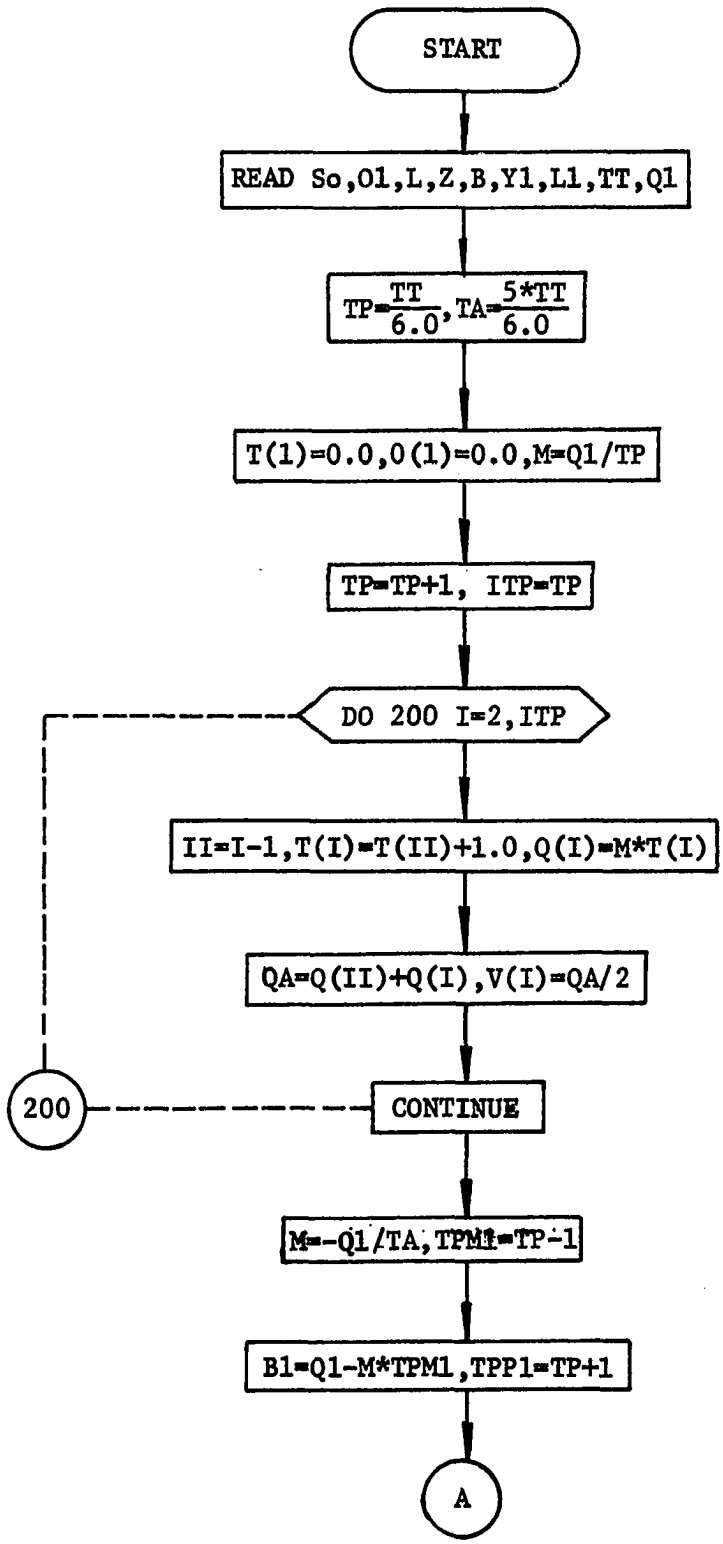
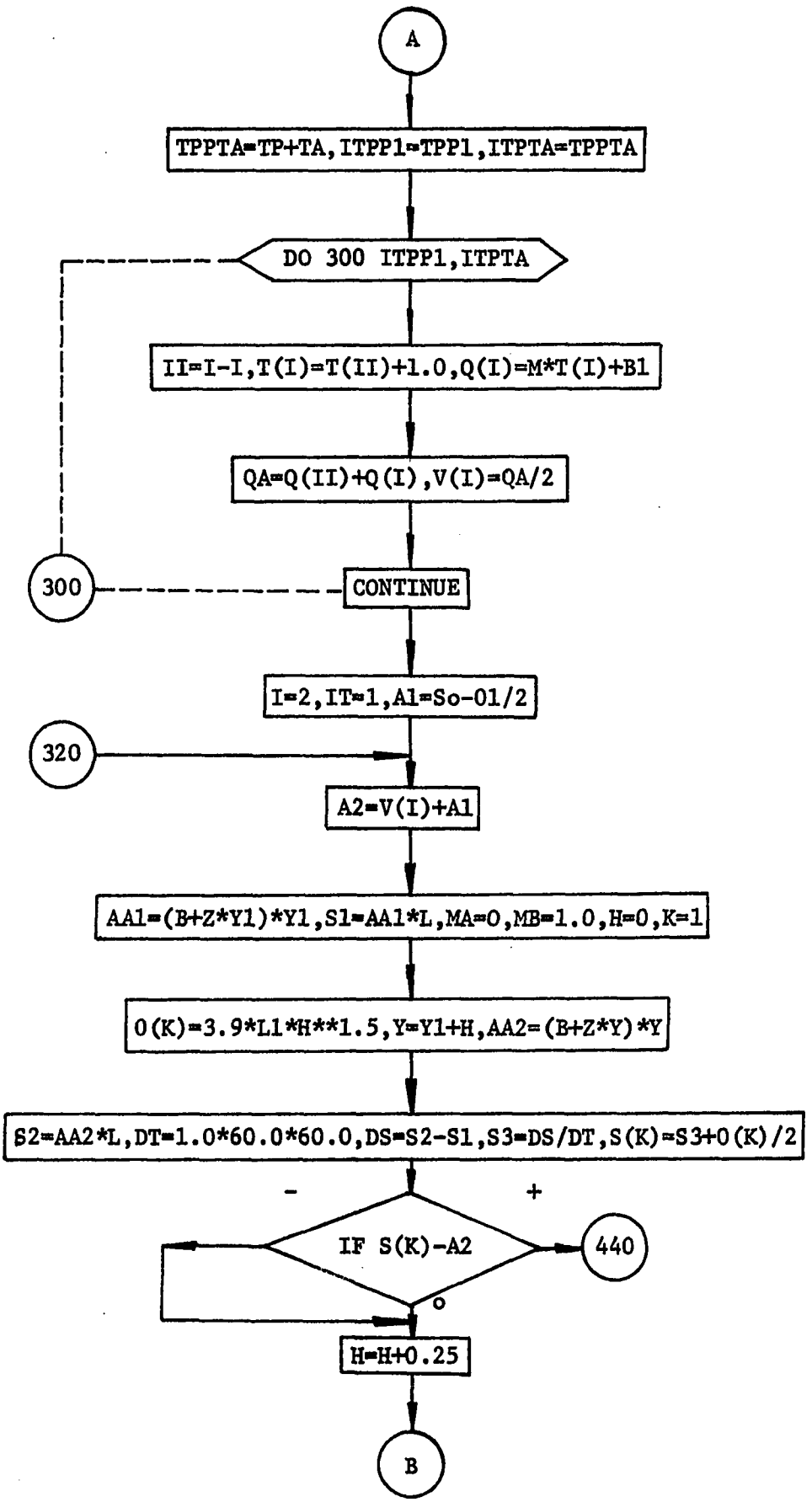


Fig. 5-1. Plan View of Spillway

FLOW CHART FOR HYDROLOGIC RESERVOIR ROUTING METHOD COMPUTATIONS




A
TPPTA=TP+TA,ITPP1=TPP1,ITPTA=TPPTA
DO 300 ITPP1,ITPTA
II=I-I,T(I)=T(II)+1.0,Q(I)=M*T(I)+B1
QA=Q(II)+Q(I),V(I)=QA/2
300
CONTINUE
I=2,IT=1,A1=So-01/2
320
A2=V(I)+A1
AA1=(B+Z*Y1)*Y1,S1=AA1*L,MA=0,MB=1.0,H=0,K=1
0(K)=3.9*L1*H**1.5,Y=Y1+H,AA2=(B+Z*Y)*Y
S2=AA2*L,DT=1.0*60.0*60.0,DS=S2-S1,S3=DS/DT,S(K)=S3+0(K)/2
-
+
IF S(K)-A2
440
o
H=H+0.25
B

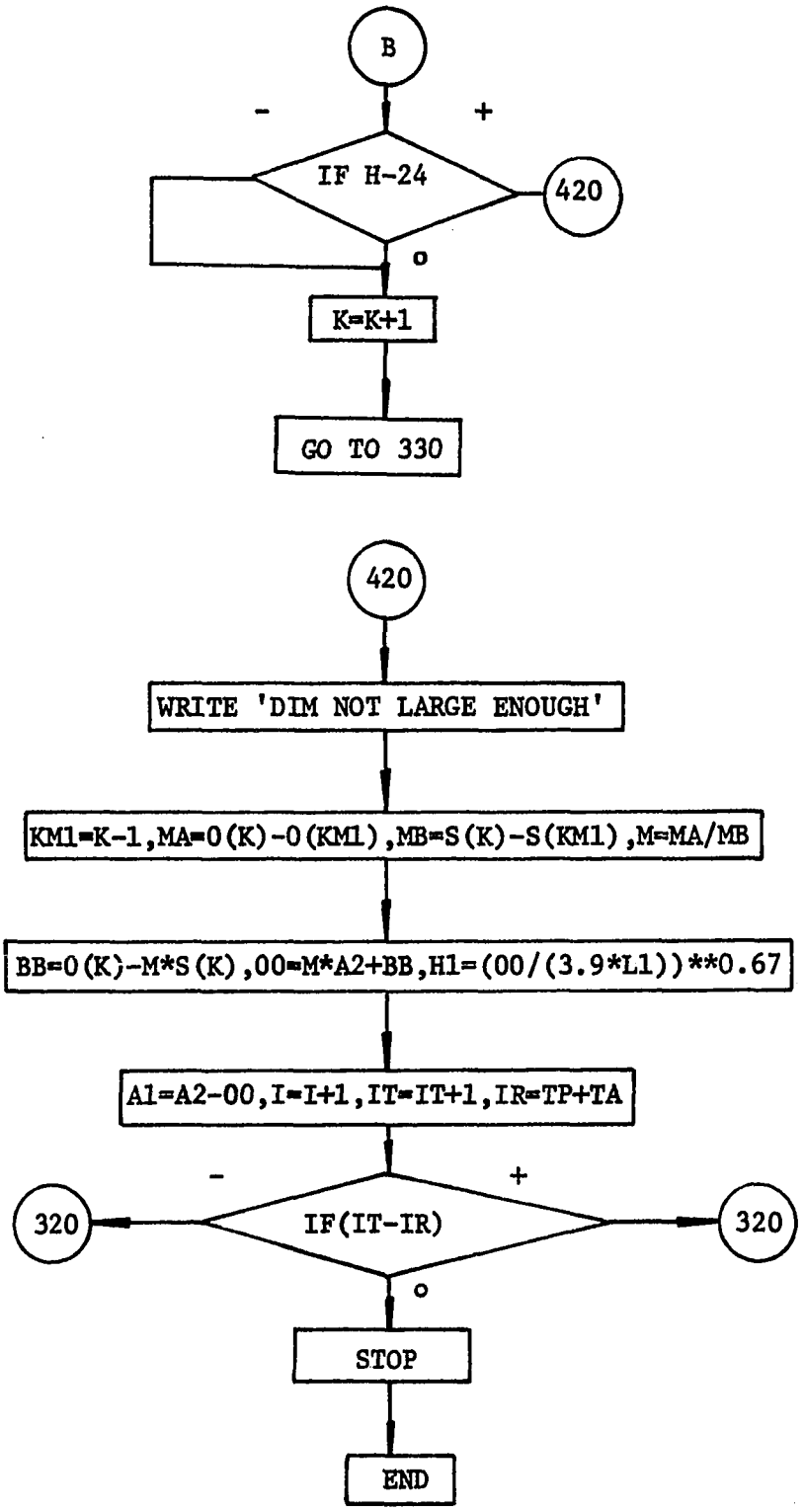


FIG. 5-2. FLOW CHART

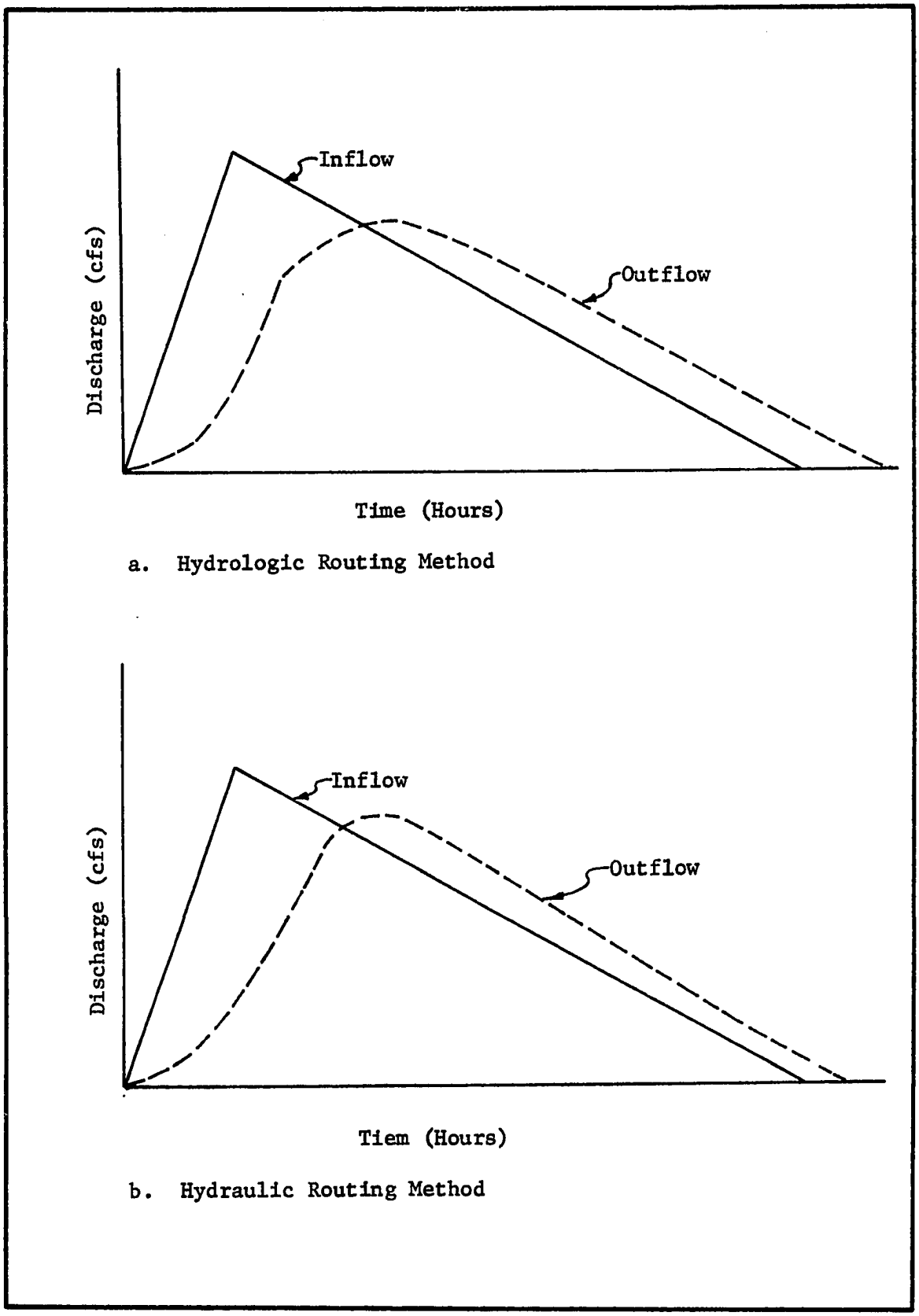
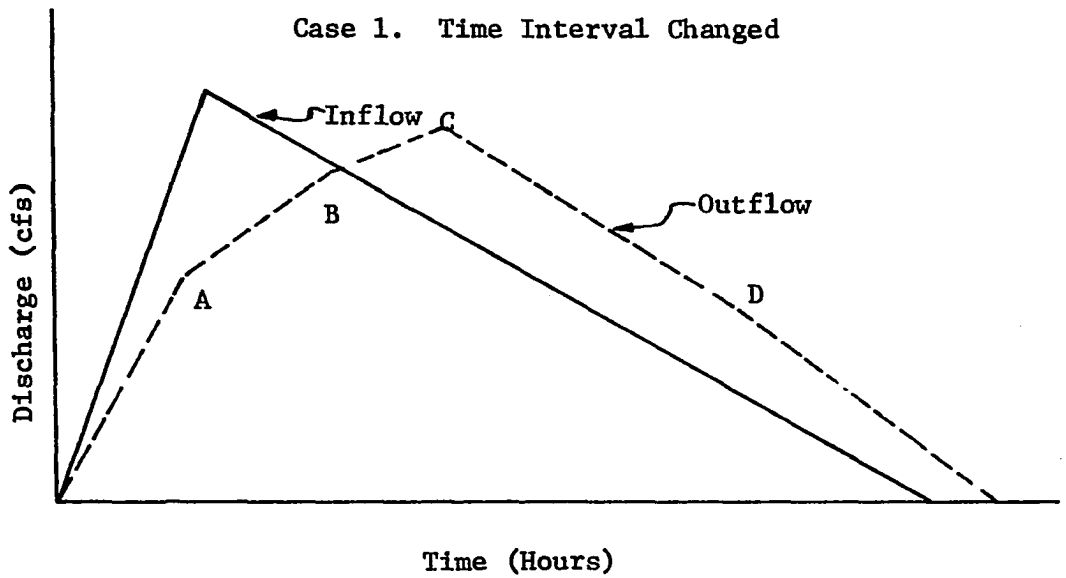
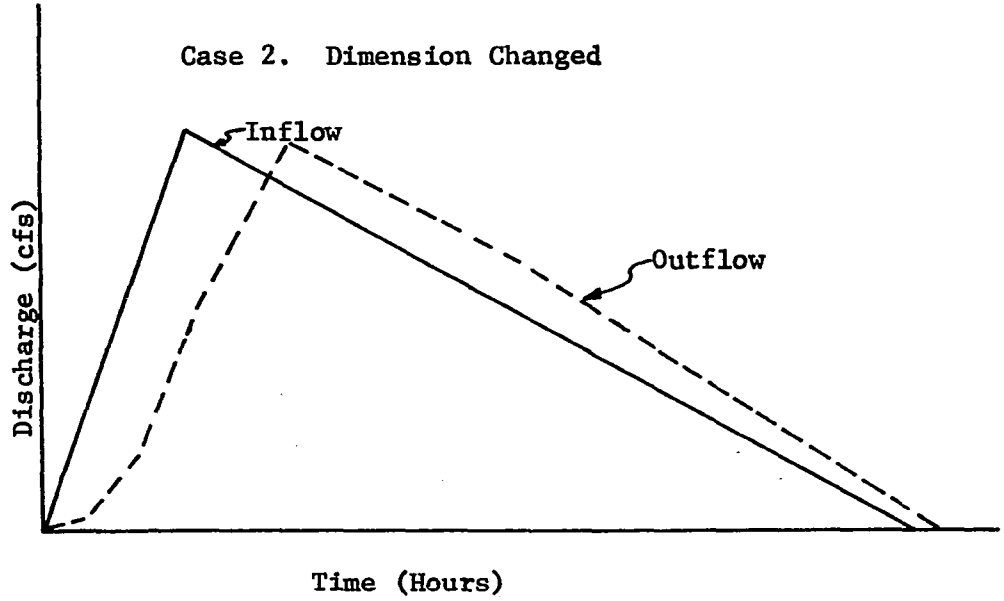


Fig. 5-3. Hydrologic and Hydraulic Routing Methods Inflow and Outflow Relationships



L = 10,560 ft.
L1 = 50 ft.
B = 1000 ft.
YN = 5 ft.
Δt = 5.7 Hrs.

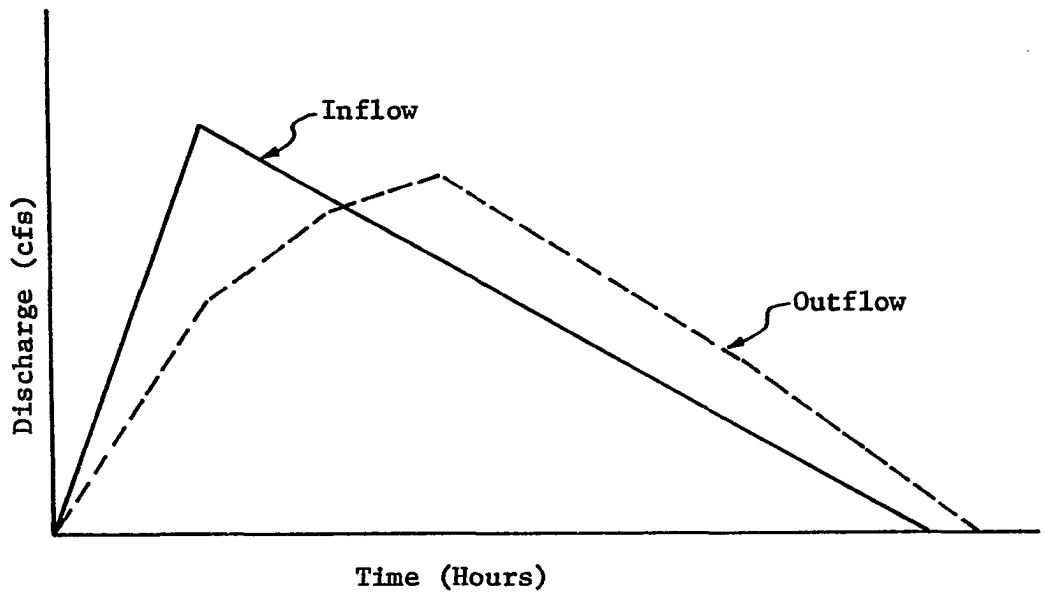
Fig. 5-4 Hydraulic Reservoir Routing Inflow and Outflow Relationships.



L = 7920 ft.
L1 = 45 ft.
B = 800 ft.
YN = 5 ft.
Δt = 1.9 Hours

Fig. 5-5 Hydraulic Reservoir Routing Inflow and Outflow Relationships

Case 3. Time Interval and Dimension Changed



L = 7920 ft.

L1 = 45 ft.

B = 800 ft.

YN = 5 ft.

Δt = 5.7 Hours

Fig. 5-6. Hydraulic Reservoir Routing Inflow and Outflow Relationships

The figure 5-3 shows differential outflow hydrographs. Because they have differential time interval, the hydrologic reservoir routing method, assumes Δt = 1 hour time interval, using the hydraulic reservoir routing method, the time interval is calculated by the formula $\Delta t = \Delta X/(VtC)$. The differential time interval gives differential outflow hydrograph shapes.

CHAPTER VI

DISCUSSION OF RESULTS

Although the basic issue is delaying or detaining water as it moves down well-defined reservoirs, several interesting results have appeared in the solutions to the problems under study.

Of primary interest is the similarity of the two methods illustrated for comparative purposes in Figure 5-3. The hydrologic reservoir routing method results (Fig. 5-3 (a)) shown in the upper portion of the illustration may be dependent on several factors for computing the resultant outflow hydrograph. These include the time interval, reservoir configuration, and outflow structure designate.

The other method, hydraulic reservoir routing, (Fig. 5-3 (b)) can be computed by using a time interval (Δt) which is a necessary input to both the continuity and momentum equations used herein. This method is readily adaptable to limited historical data of hydrologic events. The differences in the two outflow hydrograph shapes in Fig. 5-3 are due to inherent differences in the two methods. It should be noted that the peak discharge rates, the time lag intervals, and the total flow times are not significantly different for the two routing methods. At least not when the data is appropriately and carefully refined.

The hydraulic reservoir routing method receives major emphasis in

this research effort due to its mathematical appropriateness. Three specific cases using this method are discussed below.

In Case 1 (See Fig. 5-4), the time interval has been changed from two-hour increments to six-hour increments. The well-defined break points, A, B, C, D indicate a need for closer time spacing on an event of this particular duration. Increments of twelve and twenty-four hours are also computed by the method described herein. However, the limited number of points obtained using those segments did not adequately describe the outflow hydrograph. Therefore, the data and illustrations for increments greater than six-hours have been eliminated from this study.

In Case 2 (See Fig. 5-5), the reservoir dimension variables are changed to study the effect of reservoir size on the outflow hydrograph. The two-hour time increment is maintained, so that any resulting changes would be of direct consequence in the reservoir size. Interestingly, although the shape of the outflow hydrograph for the reservoir dimension and time intervals changes is quite different, the peak runoff rates are not affected. However, the reservoir characteristics are responsible for the delay of peak runoff rates as shown in Case 1 and Case 2. Once the reservoir is filled to capacity, the outflow rate will be approximately equal to the inflow rate. Depletion of stored water takes place at an almost constant rate due to the outflow and control characteristics of the reservoir. There appears to be no substantial change in depletion characteristics due to either time interval or reservoir dimension changes.

As a final exercise, the time interval and reservoir dimensions are both changed in Case 3 (see Fig. 5-6). The time interval is now in six-hour increments but the reservoir size retains the same dimensions as

in Case 2. The effective outflow hydrograph shape is similar to that of Case 2. The outflow hydrograph in Case 3 appears to be quite distinct with respect to time increments for a reservoir of this size. This seems to indicate that the time increments are adequate in this case. However, due to the duration of the event, the twelve and twenty-four hours time increment computations are deleted from this research.

It should be noted that if the event duration T is divided into time increments $\frac{T}{N}$, whereby N is an integer, that when N is smaller than about 3, unreliable results are obtained.

As to reservoir design characteristics, this research will permit a rapid evaluation of any assumed configuration dimensions with regard to hydrograph attenuation and other factors in flood control.

Although the graphical method has been widely used for the hydrologic reservoir routing in flood control treatment, it is subject to large possible errors due the use of long time intervals. In contrast, the method used in this research, the hydraulic reservoir routing method, is subject to fewer error because of two important factors. First, this method encompasses a specified time interval of any selected value. And mos importantly, whereas, the graphical method used for the hydrologic reservoir routing uses only the continuity equation, the hydraulic reservoir routing method incorporates the continuity equation as well as the momentum equation. Both equations are solved simultaneously by the finite-difference method.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Generally, every flood control problem in engineering hydraulics entails the prediction by either experimental or mathematical analytical methods one or more characteristics of flow. There are several types of predictions:

(1). By far the oldest is that of "engineering experience" gained in the field by each individual engineer.

(2). The engineering laboratory experimental method of studying each flood record by means of scale models. The early engineers usually used tables, charts, and coefficients for designing a model to solve the overflow control problems.

(3). The use of flood routing equations of hydrology are empirical and the methods of solution are equally crude trail-and-error procedures that depend on derived tables and graphs based on historical records.

(4). The utalization processes of mathematical analysis which are developing rapidly today.

(5). Digital computer solutions with numerical analysis methods and digital computer facilities now available can be utilized to solve most difficult and complex problems. At

least, they are no longer such an important item of concern. The input data for the computations are the inflow hydrograph, channel configuration, and outflow structure dimensions.

(6). The two approaches which have been used in this study to solve the flood routing problem in a reservoir. One is the hydrologic reservoir routing method based on the continuity equation only and a relationship between the inflow and outflow and storage and outflow. The other approach is called the hydraulic or hydromechanic routing, which utilizes more fully the basic hydrodynamic equations of continuity and momentum. A comparison of the two methods is shown in Fig. 5-3.

New techniques and procedures in numerical analysis methods are being developed for use with the electronic digital computers in many areas of study. This study is an attempt to solve reservoir flood routing problems by a better method than has heretofore been used. The results should be useful in the planning, design, construction and operation of flood control projects especially those in urban areas when intensified application is of current wide importance.

Unsteady nonuniform flow in reservoir is described by two partial differential equations. One is the continuity equation (3-9a) and the other is the momentum equation (3-24). These two equations were solved by the method of finite difference technique. The following conclusions are drawn.

(1). Any inflow flow hydrograph can be synthesized using appropricate numerical methods and mathematical models. For this study, use has been made of initial and configuration boundary conditions.

(2). The selection of an appropriate time increment (Δt) is one hour for hydrologic reservoir routing. A computed $\Delta t = \Delta x/(V+C)$ is used with hydraulic reservoir routing along with the finite difference technique to solve the equations of unsteady flow.

(3). The time increment (Δt) selection is a major influence on the shape of the routed outflow hydrograph.

(4). The inflow hydrograph is plotted as the discharge (Q) versus the time (t) for this study. Also the inflow hydrograph is assumed to be triangular. Any input hydrograph function could be assumed for use with this method. However, a triangular shape is not an unreasonable approxiation to natural occurring events.

(5). The method of finite differences technique provides an accurate assessment of the reservoir routing phenomenon.

Further research has to be done to refine the programming techniques to acieve more generality and flexibility. In this study, emphasis is given to the development of numerical methods rather than the actual programming.

The Integrated Civil Engineering System, so-called ICES, was developed and is being carried out at the M.I.T. Civil Engineering System

laboratory. This study is directed to developing a powerful computer based system which solves difficult problems. Recently, HYDRO has been added to solve "some" hydraulic and hydrologic problems. This subsystem is in the earlier stages of its development. HYDRO presents an opportunity for the research group who can contribute to its initial design and orientation. Therefore, it is hoped that this study can ultimately be added to the HYDRO capability, which was never completed, as a part of the ICES System.

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```
$JOB            000092768,KP=29,TIME=120
C      HYDRAULIC RESERVOIR ROUTING METHOD
C      SOLVED BY FINITE DIFFERENCES METHOD OF SPECIFIED TIME INTERVALS
C      FLOOD ROUTING THROUGH RESERVOIR INTO A TRAPEZOIDAL CHANNEL SECTION
C      QP=PEAK RATE OF INFLOW HYDROGRAPH IN CUBIC FEET PER SECOND
C      SI=INITIAL STORAGE IN CUBIC FEET
C      OI=INITIAL OUTFLOW IN CUBIC FEET PER SECOND
C      QI=OUTFLOW HYDROGRAPH IN CUBIC FEET PER SECOND
C      TP=TIME TO PEAK FLOW IN HOURS
C      TE=FROM PEAK FLOW TO THE END FLOW IN HOURS
C      L=LENGTH OF RESERVOIR IN FEET
C      L1=LENGTH OF WEIR CREST IN FEET
C      Z=SIDE SLOPE OF CHANNEL
C      YN=INITIAL NORMAL FLOW DEPTH IN FEET
C      B=CHANNEL BOTTOM WIDTH IN FEET
C      A=AREA IN SQUARE FEET
C      G=THE ACCELERATION DUE TO THE GRAVITY
C
C      ***************************************************************
C
       INTEGER N,K
       REAL L,L1,Z,B,MU,MD,NAMD,SDIF
       COMMON V(500),Y(500),VP(500),YP(500),A(500),W(500)
       COMMON QQ(500),QAVE(500),QI(500),S(500)
    35 READ(5,36) B,YN,Z,L,L1,OI,SI
    36 FORMAT(7F10.4)
   235 READ(5,236) S0,ZN
   236 FORMAT(2F10.6)
    37 READ(5,38) N
    38 FORMAT(I10)
   425 READ(5,426) T,QP
   426 FORMAT(2F10.4)
C
C      ***************************************************************
C
```

```
C       INITIAL CONDITIONS
        TP=T/6.0
        TE=(5.0*T)/6.0
        TM=T
        MU=QP/TP
        ISAVE=0
        MD=-QP/TE
        W0=B+2.0*Z*YN
        A0=(B+Z*YN)*YN
        P0=B+2.0*YN*(1.0+Z**2.0)**0.5
        R0=A0/P0
        Q0=1.49*A0*(R0)**0.667*(S0)**0.5/ZN
        V0=Q0/A0
        C=(32.2*A0/W0)**0.5
        NP1=N+I
        DO 39 I=1,NP1
        V(I)=V0
        A(I)=A0
        W(I)=W0
        VP(I)=V0
        YP(I)=YN
        Y(I)=YN
   39 CONTINUE
C
C       ***************************************************************************
C
        DX=L/N
        DT=DX/(V(1)+C)
        DTS=DT*60.0*60.0
        Q=Q0
        T=0
        K=0
        QDIF=0
        SDIF=1.0
        IDX=1
        QQ(1)=0
```

```
      SS=SI-OI/2
      WRITE(6,15) B,YN,Z,L,L1
   15 FORMAT(5F10.4)
      WRITE(6,215) SO,ZN
  215 FORMAT(2F10.6)
      WRITE(6,315) TP,TE,T,QP
  315 FORMAT(4F10.4)
      WRITE(6,65) QO,DX,DT,MU,MD
   65 FORMAT(5F10.4)
      WRITE(6,95)
   95 FORMAT(6X,'HOUR',10X,'HEAD',8X,'INFLOW',10X,'QAVE',7X,'OUTFLOW')
C
C     ******************************************************************
C
   77 T=T+DT
      IDX=IDX+1
      K=K+1
      IF(T.GT.TM) GO TO 600
C
C     ******************************************************************
C     CALCULATED INTERIOR POINTS
C
      DO 25 I=2,N
      YL=Y(I)
      YR=Y(I)
      YN=(YL+YR)/2.0
      A(I)=(B+Z*Y(I))*Y(I)
      W(I)=B+2.0*Z*Y(I)
      NAMD=(A(I)/(32.2*W(I)))**0.5
      VP(I)=0.5*(YL-YR)/NAMD+(V(I+1)+V(I-1))/2.0
      YP(I)=(YL+YR)/2.0+NAMD*(V(I+1)-V(I-1))/2.0+32.2*SO*DTS*NAMD
   25 CONTINUE
C
C     ******************************************************************
C     UPSTREAM BOUNDARY CONDITIONS
C
```

```
      IF(T-TP) 45,45,46
   45 QQ(IDX)=MU*T
      GO TO 47
   46 QQ(IDX)=QP+MD*(T-TP)
   47 QAVE(IDX)=(QQ(IDX)+QQ(IDX-1))/2.0
      A(1)=(B+Z*Y(1))*Y(1)
      W(1)=B+2.0*Z*Y(1)
      YP(1)=YP(2)
      NAMD1=(32.2*W(1)/A(1))**0.5
      VP(1)=V(1)+NAMD1*(YP(1)-YR)-32.2*S0*DTS
C
C     **********************************************************************
C     DOWNSTREAM BOUNDARY CONDITIONS
C
      YP(NP1)=YP(N)
      A(NP1)=(B+Z*YP(NP1))*YP(NP1)
      SS1=QAVE(IDX)+SS
      A1=(B+Z*Y(I))*Y(I)
      S1=A1*L
      H=0
      DO 256 KK=1,500
  245 Q1(KK)=3.9*L1*H**1.5
      H0=YP(NP1)-Y(I)
      VP(NP1)=Q1(KK)/A(NP1)
      Y2=Y(I)+H
      S2=((B+Z*Y2)*Y2)*L
      DS=S2-S1
      S3=DS/DTS
      S(KK)=S3+Q1(KK)/2
      IF(S(KK)-SS1) 255,255,265
  255 H=H+H0
  256 CONTINUE
  285 WRITE(6,295)
  295 FORMAT(1X,'DIM NOT LARGE ENOUGH')
      KK=KK-1
      KKM1=KK-1
```

```
      QDIF=Q1(KK)-Q1(KKM1)
      SDIF=S(KK)-S(KKM1)
  265 R=QDIF/SDIF
      QA=Q1(KK)-R*S(KK)
      QOUT=QA+R*SS1
      SS=SS1-QOUT
      H1=(QOUT/(3.9*L1))**0.6667
C
C     ***************************************************************
C
      DO 90 I=1,NP1
      V(I)=VP(I)
      Y(I)=YP(I)
   90 CONTINUE
      WRITE(6,85) T,H1,QQ(IDX),QAVE(IDX),QOUT
   85 FORMAT(5(F10.4,4X))
      GO TO 77
  600 STOP
      END

$EXEC
   1000.0000     5.0000     0.250010560.0000    50.0000
    0.000010   0.500000
      6.0000    30.0000     0.0000 4200.0000
    137.1932    24.0000     1.8886  700.0000 -140.0000
```

TABLE A

Operation Table For Hydraulic Reservoir Routing

HOUR	HEAD	INFLOW	QAVE	OUTFLOW
1.8886	0.8621	1321.9940	660.9973	156.0983
3.7771	1.8665	2643.9890	1982.9920	497.2393
5.6657	4.0616	3965.9840	3304.9860	1595.9910
7.5543	4.4829	3982.4040	3974.1930	1850.6650
9.4428	5.9798	3718.0050	3850.2030	2851.0830
11.3314	6.4247	3453.6060	3585.8040	3175.0930
13.2199	6.8043	3189.2070	3321.4060	3460.6000
15.1085	5.7048	2924.8080	3057.0070	2656.6850
16.9971	5.8988	2660.4100	2792.6070	2793.3080
18.8856	6.1594	2396.0130	2528.2100	2980.4620
20.7742	6.4733	2131.6160	2263.8140	3211.1760
22.6627	5.0627	1867.2190	1999.4170	2221.0200
24.5513	5.2214	1602.8220	1735.0200	2326.2500
26.4398	5.4237	1338.4250	1470.6230	2462.7790
28.3284	3.7753	1074.0280	1206.2260	1430.2570
30.2169	3.8732	809.6313	941.8298	1486.2890
32.1055	4.0237	545.2344	677.4329	1573.7480
33.9940	2.0858	280.8374	413.0359	587.3870
35.8826	2.1440	16.4414	148.6394	612.1418

```
$JOB            000092768,KP=29,TIME=120
C      HYDROLOGIC RESERVOIR ROUTING METHOD
C      FLOOD ROUTING THROUGH RESERVOIR INTO A TRAPEZOIDAL CHANNEL SECTION
C      TP=TIME TO PEAK FLOW IN HOURS
C      TA=FROM PEAK FLOW TO THE END FLOW IN HOURS
C      O1=PEAK RATE OF INFLOW HYDROGRAPH IN CUBIC FEET PER SECOND
C      S0=INITIAL STORAGE IN CUBIC FEET
C      O1=INITIAL OUTFLOW IN CUBIC FEET PER SECOND
C      L=LENGTH OF STORAGE REACH IN FEET
C      Z=SIDE SLOPE OF CHANNEL
C      B=CHANNEL BOTTOM WIDTH IN FEET
C      Y1=DEPTH FROM CHANNEL BOTTOM TO THE WEIR CREST IN FEET
C      L1=WIDTH OF WEIR CREST IN FEET
C      AA=AREA, H=CHANNEL HEAD IN FEET
C
C      ***************************************************************
C
C
       DIMENSION Q(100), T(100), V(100), S(100), O(100)
       REAL Q1,S0,O1,L,Z,B,Y1,L1,M
       READ(5,100) S0,O1,L,Z,B,Y1,L1
  100  FORMAT(7F10.4)
       READ(5,10) TT,Q1
   10  FORMAT(2F10.4)
       TP=TT/6.0
       TA=5*TT/6.0
       WRITE(6,125) B,Y1,Z,L,L1
  125  FORMAT(5F10.4)
       WRITE(6,135) TP,TA,TT,Q1
  135  FORMAT(4F10.4)
       T(1)=0.0
       Q(1)=0.0
C
C      ***************************************************************
C
```

```
      M=Q1/TP
      TP=TP+1
      ITP=TP
      DO 200 I=2,ITP
      II=I-1
      T(I)=T(II)+1.0
      Q(I)=M*T(I)
      QA=Q(II)+Q(I)
      V(I)=QA/2
  200 CONTINUE
C
C     ***************************************************************************
C
      M=-Q1/TA
      TPM1=TP-1
      B1=Q1-M*TPM1
      TPP1=TP+1
      TPPTA=TP+TA
      ITPP1=TPP1
      ITPTA=TPPTA
      DO 300 I=ITPP1,ITPTA
      II=I-1
      T(I)=T(II)+1.0
      Q(I)=M*T(I)+B1
      QA=Q(II)+Q(I)
      V(I)=QA/2
  300 CONTINUE
C
C     ***************************************************************************
C
      WRITE(6,310)
  310 FORMAT(3X,'HOUR',8X,'HEAD',8X,'INFLOW',8X,'QAVE',8X,'OUTFLOW')
      I=2
      IT=1
      A1=S0-Q1/2
  320 A2=V(I)+A1
```

```
C
C     ***************************************************************************
C
      AA1=(B+Z*Y1)*Y1
      S1=AA1*L
C
C     ***************************************************************************
      MA=0
      MB=1.0
      H=0
      K=1
  330 Q(K)=3.9*L1*H**1.5
      Y=Y1+H
      AA2=(B+Z*Y)*Y
      S2 = AA2 * L
      DT=1.0*60.0*60.0
      DS=S2-S1
      S3=DS/DT
      S(K)=S3+Q(K)/2
      IF(S(K)-A2) 400,400,440
  400 H=H+0.25
      IF(H-24) 410,410,420
  410 K=K+1
C
C     ***************************************************************************
C
      GO TO 330
  420 WRITE(6,430)
  430 FORMAT(1X,'DIM NOT LARGE ENOUGH')
      KM1=K-1
      MA=Q(K)-Q(KM1)
      MB=S(K)-S(KM1)
  440 M=MA/MB
      BB=Q(K)-M*S(K)
      QQ=M*A2+BB
      H1=(QQ/(3.9*L1))**0.67
```

```
      WRITE(6,500) IT,H1,Q(I),V(I),OO
  500 FORMAT(I5,4(4X,F10.4))
      A1=A2-OO
      I=I+1
      IT=IT+1
      IR=TP+TA
      IF(IT-IR) 320,600,320
  600 STOP
      END

$EXEC
  1000.0000     5.0000     0.250010560.0000    50.0000
     6.0000    30.0000    36.0000 4200.0000
```

TABLE B

Operation Table For Hydrologic Reservoir Routing

HOUR	HEAD	INFLOW	QAVE	OUTFLOW
1	0.2483	700.0000	350.0000	24.3750
2	0.4983	1400.0000	1050.0000	68.9429
3	1.2514	2100.0000	1750.0000	272.5205
4	1.7549	2800.0000	2450.0000	451.4312
5	2.7639	3500.0000	3150.0000	889.2698
6	3.5220	4200.0000	3850.0000	1276.8400
7	4.5340	4060.0000	4130.0000	1861.4580
8	5.2937	3920.0000	3990.0000	2345.7050
9	5.5471	3780.0000	3850.0000	2515.2340
10	6.0540	3640.0000	3710.0000	2865.9020
11	6.3075	3500.0000	3570.0000	3046.8740
12	6.3075	3360.0000	3430.0000	3046.8740
13	6.3075	3220.0000	3290.0000	3046.8740
14	6.5611	3080.0000	3150.0000	3231.5010
15	6.3075	2940.0000	3010.0000	3046.8740
16	6.3075	2800.0000	2870.0000	3046.8740
17	6.3075	2660.0000	2730.0000	3046.8740
18	6.0540	2520.0000	2590.0000	2865.9020
19	6.0540	2380.0000	2450.0000	2865.9020
20	5.8005	2240.0000	2310.0000	2688.6620
21	5.5471	2100.0000	2170.0000	2515.2340
22	5.5471	1960.0000	2030.0000	2515.2340
23	5.2937	1820.0000	1890.0000	2345.7050
24	5.0404	1680.0000	1750.0000	2180.1650
25	5.0404	1540.0000	1610.0000	2180.1650
26	4.7871	1400.0000	1470.0000	2018.7140
27	4.5340	1260.0000	1330.0000	1861.4580
28	4.2809	1120.0000	1190.0000	1708.5110
29	4.0278	980.0000	1050.0000	1559.9990
30	3.7749	840.0000	910.0000	1416.0590
31	3.7749	700.0000	770.0000	1416.0590
32	3.5220	560.0000	630.0000	1276.8400
33	3.2692	420.0000	490.0000	1142.5080
34	3.0165	280.0000	350.0000	1013.2490
35	2.7639	140.0000	210.0000	889.2698
36	2.5115	0.0000	70.0000	770.8049

TABLE C

Operation Table For Hydraulic Reservoir Routing (Case 1)

HOUR	HEAD	INFLOW	QAVE	OUTFLOW
5.6657	5.1732	3965.9840	1982.9920	2294.1660
11.3314	6.5034	3453.6060	3709.7940	3233.5870
16.9971	7.7675	2660.4100	3057.0070	4220.7810
22.6628	4.4969	1867.2150	2263.8120	1859.3440
28.3284	5.0627	1074.0190	1470.6170	2221.0000
33.9941	0.0000	280.8247	677.4221	0.0000

TABLE D

Operation Table For Hydraulic Reservoir Routing (Case 2)

HOUR	HEAD	INFLOW	QAVE	OUTFLOW
1.8888	0.8621	1322.1700	661.0854	140.4835
3.7776	2.8018	2644.3420	1983.2560	823.0205
5.6664	5.1664	3966.5130	3305.4250	2060.6680
7.5553	6.9141	3982.2630	3974.3860	3190.2290
9.4441	7.5375	3717.8290	3850.0440	3631.2310
11.3329	6.6409	3453.3940	3585.6110	3002.9890
13.2217	6.8142	3188.9600	3321.1750	3121.3120
15.1105	7.1319	2924.5260	3056.7420	3342.0960
16.9993	7.5051	2660.0930	2792.3080	3607.7990
18.8881	6.2878	2395.6600	2527.8760	2766.7010
20.7769	6.4676	2131.2270	2263.4430	2886.2180
22.6658	5.0530	1866.7940	1999.0100	1993.2000
24.5546	5.2127	1602.3600	1734.5770	2088.4230
26.4434	3.6114	1337.9270	1470.1440	1204.3570
28.3322	3.7292	1073.4940	1205.7100	1263.7330
30.2210	3.8656	809.0610	941.2776	1333.7250
32.1098	4.0293	544.6277	676.8442	1419.2910
33.9986	2.0839	280.1946	412.4111	527.9233
35.8874	2.1430	15.7617	147.9781	550.5513

TABLE E

Operation Table For Hydraulic Reservoir Routing (Case 3)

HOUR	HEAD	INFLOW	QAVE	OUTFLOW
5.6666	5.1733	3966.6140	1983.3060	2064.7590
11.3332	9.7819	3453.3540	3709.9820	5368.2960
16.9998	3.9770	2660.0320	3056.6930	1391.7420
22.6664	4.4508	1866.7100	2263.3710	1647.7150
28.3329	5.0409	1073.3890	1470.0500	1986.0170
33.9995	5.6925	280.0684	676.7290	2383.2860