Plate Wave Propagation in Transversely Isotropic Materials

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ABSTRACT: In this work, the modes of plate wave propagation in a fiber reinforced composite (assumed to be transversely isotropic) were investigated. The waves were modeled as harmonic plane waves propagating in the plane of an infinite plate whose bounding surfaces were assumed to be stress free. A numerical analysis procedure was developed to calculate the dispersion relationships for plate waves propagating in arbitrary directions in the plate. Particle displacements and stress distributions were calculated for several important propagation modes. Possible applications to nondestructive testing of composites are discussed.

INTRODUCTION

THE PROPAGATION OF guided elastic waves in plate structures has attracted a great deal of research interest since the pioneering work of Lord Rayleigh in 1889 [1]. Based upon Rayleigh's mathematical formulation, Lamb [2] extensively investigated the properties of these waves and today they are known as Rayleigh-Lamb waves. However, the mathematical complexity of the formulation as well as the fact that these waves can only be propagated at ultrasonic frequencies resulted in little advancement in knowledge of plate wave characteristics until recently. Through the work of modern investigators [3–6], the precise nature of the Rayleigh-Lamb spectrum was fully revealed. While the bulk of research work in this area has been devoted to isotropic media, there has been limited attention paid to anisotropic materials. The mathematical basis for plate wave propagation in anisotropic media was first presented by Ekstein [7] in 1945. Mindlin, motivated by an interest in the vibration of quartz piezoelectric transducers, used the Ekstein formulation to examine plate waves in crystals of monoclinic symmetry along a symmetry axis [8]. More recently, Green and co-workers [9–12] have examined the propagation of the bending and extensional waves in

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transversely isotropic media. Of particular note in Green's research is the investigation of propagation in non-symmetry directions and the limiting behavior for bending and extensional waves for short and long wavelengths. Green has also shown how this research might be extended to laminated composites in an initial study of a three-layer, cross-ply laminate. However, the algebraic complexity of the problem has thus far prevented a complete study of the Ekstein spectra for transversely isotropic or generally anisotropic media. The present work is motivated by these earlier studies as well as the need to nondestructively characterize the mechanical response of structural composites. The directional dependence of material properties in these materials as well as other effects such as bending-stretching coupling make the nondestructive characterization of these materials a challenging task indeed. Much of the research in this area thus far has been based on bulk wave propagation measurements to characterize elastic stiffnesses [13–16]. However, this approach requires multiple incidence angles in order to be useful. Thus, specimens must either be sectioned [8–10] or the experimental geometry adjusted [11,12], a cumbersome process, to be effective. An attractive alternative is the use of guided waves. While not yet widely utilized, several investigators have recognized the potential of guided waves for characterizing the laminated plate structures typical of most composites. Henneke [17] used longitudinal transducers to generate and sense both longitudinal and flexural waves in composites. In this way, both stiffnesses and flexural moduli could be experimentally obtained. Researchers at the Naval Surface Weapons Laboratory [18] have used guided wave propagation in various directions (including nonsymmetry directions) to characterize unidirectionally reinforced composites. Although they analyzed the results using a simplified, two-dimensional theory, good agreement between theory and experiment was obtained. These works clearly indicate the need for a more complete understanding of plate wave propagation in anisotropic materials in general and composites in particular. It is hoped that, with the development of accurate mathematical models for plate wave propagation in composites, suitable methods for experimentally characterizing material properties (particularly elastic moduli in the plane of reinforcement) will be devised.

THEORY

The equations of motion for an elastic continuum are given by (in the absence of body forces)

$$C_{ijk\ell} U_{k,j\ell} = \varrho \ddot{U}_i \tag{1}$$

For a transversely isotropic medium the $C_{ijk\ell}$ matrix may be simplified to a 6×6 array of constants using symmetry arguments and the standard index substitution

$$11 \rightarrow 1,\ 22 \rightarrow 2,\ 33 \rightarrow 3,\ 23 \rightarrow 4,\ 31 \rightarrow 5,\ 12 \rightarrow 6$$

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to yield

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{C_{11} - C_{12}}{2} \end{bmatrix} \tag{2}$$

where the z-axis is assumed to be the symmetry axis (fiber axis) as shown in Figure 1. In this case we may write the equations of motion in operator notation as (19)

$$L_{ij}[u_j] = 0 \tag{3}$$

where

$$\begin{aligned} L_{11} &= c_{11}\frac{\partial^2}{\partial x^2} + \frac{1}{2}(c_{11} - c_{12})\frac{\partial^2}{\partial y^2} + c_{44}\frac{\partial^2}{\partial x^2} - \varrho\frac{\partial^2}{\partial t^2} \\ L_{12} &= L_{21} = \frac{1}{2}(c_{11} + c_{12})\frac{\partial^2}{\partial x \partial y} \\ L_{13} &= L_{31} = (c_{13} + c_{44})\frac{\partial^2}{\partial x \partial z} \\ L_{22} &= \frac{1}{2}(c_{11} - c_{12})\frac{\partial^2}{\partial x^2} + c_{11}\frac{\partial^2}{\partial y^2} + c_{44}\frac{\partial^2}{\partial z^2} - \varrho\frac{\partial^2}{\partial t^2} \\ L_{23} &= L_{32} = (c_{13} + c_{44})\frac{\partial^2}{\partial y \partial z} \end{aligned} \tag{4}$$

and

$$L_{33} = c_{44}\frac{\partial^2}{\partial x^2} + c_{44}\frac{\partial^2}{\partial y^2} + c_{33}\frac{\partial^2}{\partial z^2} - \varrho\frac{\partial^2}{\partial t^2}$$

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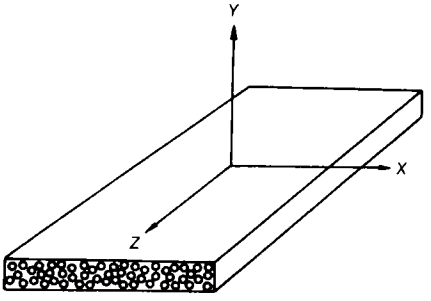


Figure 1. *Coordinate system.*

One of the major simplifications associated with wave propagation in transversely isotropic media is that the equation of motion may be reduced to yield a single relatively simple partial differential equation (the product of a second order differential operator and fourth order differential operator) for each of the displacement components (u,v,w) given by

$$\text{Det}\,(L_{ij}) = 0 \tag{5}$$

where

$$\begin{aligned}\text{Det}\,(L_{ij}) = &\left[c_{44}\frac{\partial^2}{\partial x^2} + \frac{1}{2}(c_{11} - c_{12})\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \varrho\frac{\partial^2}{\partial t^2}\right] \\ &\times \left[c_{44}c_{33}\frac{\partial^2}{\partial z^2} + \{c_{44}^2 + c_{11}c_{33} - (c_{13} + c_{44})^2\}\right. \\ &\times \frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + c_{44}c_{11}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \\ &- \varrho(c_{33} + c_{44})\frac{\partial^4}{\partial z^2\partial t^2} - \varrho(c_{11} + c_{44}) \\ &\left.\times \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \varrho\frac{\partial^4}{\partial t^4}\right] = 0\end{aligned} \tag{6}$$

Following the approach for isotropic media, for wave propagation in an anisotropic plate we assume a solution of the form

$$\underset{\sim}{u}(\underset{\sim}{x},t) = A_0\underset{\sim}{\alpha}h(y)\, e^{i(kx\cos\theta + kz\sin\theta - \omega t)}$$

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where

k = wave number
A_0 = amplitude
ω = frequency
$\underset{\sim}{\alpha}$ = polarization vector

i.e., a plane wave propagating in the plane of the plate at an angle θ with respect to the x-axis. Inserting this solution into Equation (6) we obtain two differential equations (one second order, one fourth order) for $h(y)$ which must be satisfied for wave propagation, i.e.:

$$\left[C_{44}\{-k^2 \sin^2\theta\}h(y) + \frac{1}{2}(C_{11} - C_{12})\{-k^2\cos^2\theta h(y) + h''(y)\} + \varrho\omega^2 h(y)\right]$$

$$\left[c_{44}c_{33}k^4 \sin^4\theta h(y) + \{c_{44}^2 + c_{11}c_{33} - (c_{13} + c_{44})^2\}\right.$$

$$\{k^4 \sin^2\theta \cos^2\theta h(y) - k^2 \sin^2\theta h''(y)\}$$

$$+ c_{44}c_{11}\{k^2\cos^2\theta - 2k\cos\theta h'(y) + h''''(y)\} - (c_{33} + c_{44})\varrho\omega^2 k^2 \sin^2\theta h(y)$$

$$\left. - \varrho\omega^2(c_{11} + c_{44})\{k^2\cos^2\theta h(y) - h''(y)\} + \varrho^2\omega^2 h(y)\right] = 0 \tag{7}$$

The solutions to these equations are of the form $h(y) = e^{i\beta y}$ which yield three possible values for β^2 (or six values for β), i.e.,

$$\beta_1^2 = \frac{\varrho\omega^2 - c_{44}k^2\sin^2\theta - \frac{1}{2}(c_{11} - c_{12})k^2\cos^2\theta}{\frac{1}{2}(c_{11} - c_{12})} \tag{8}$$

$$\beta_2^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{9}$$

$$\beta_3^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

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where

$$a = c_{11}c_{44}$$

$$b = [c_{44}^2 + c_{11}c - (c_{13} + c_{44})^2][-k^2 \sin^2\theta]$$
$$+ 2c_{44}c_{11}(-k^2\cos^2\theta) + \varrho\omega^2(c_{11} + c_{44})$$

$$c = c_{33}c_{44}k^4 \sin^4\theta + [c_{44}^2 + c_{11}c_{33} + (c_{13} + c_{44})^2]k^4 \sin^2\theta \cos^2\theta$$
$$+ c_{44}c_{11}k^4\cos^4\theta - (c_{33}c_{44})\varrho\omega^2k^2\sin^2\theta + \varrho\omega^2(c_{11} + c_{44})k^2\cos^2\theta + \varrho^2\omega^4$$

In practice the β^i can be pure real, zero, or pure imaginary, giving rise to different regions of the frequency spectrum.

Once the possible values for β^i have been calculated, the associated polarization vectors ($\underset{\sim}{\alpha}^i$) can then be determined by substituting the potential solution

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = A_0^i \begin{bmatrix} \alpha_1^i \\ \alpha_2^i \\ \alpha_3^i \end{bmatrix} e^{i\beta^i y} e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \tag{11}$$

into the equations of motion [Equation (3)].

One may consider the problem to a modified eigenvalue problem when the βs represent the eigenvalues and the αs represent the eigenvectors (see for example, Green [20] for a more complete discussion of wave propagation in anisotropic media). Thus, the general form of the solution is given by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^{6} A_0^i \begin{bmatrix} \alpha_1^i \\ \alpha_2^i \\ \alpha_3^i \end{bmatrix} e^{i\beta^i y} e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \tag{12}$$

The task then is reduced to finding a solution which satisfies the boundary conditions for the problem, which are given by

$$\sigma_{yy}\big|_{y=\pm b} = 0 \tag{13}$$

$$\sigma_{xy}\big|_{y=\pm b} = 0 \tag{14}$$

$$\sigma_{yz}\big|_{y=\pm b} = 0 \tag{15}$$

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For a transversely isotropic medium, these equations become

$$\sigma_{yy}\Big|_{y=\pm h} = \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}\Bigg|_{y=\pm h}$$

$$= \left\{ \frac{(c_{11} - c_{12})}{2} \right\} \sum_{i=1}^{6} A_0^i \, (\alpha_1^i \beta^i + a_2^i k \cos\theta) \, e^{\pm i\beta^i h} e^{i\psi} = 0 \tag{16}$$

where $\psi = kxdn\theta + kz \sin\theta + \omega t$

$$\sigma_{yy}\Big|_{y=\pm h} = \left\{ c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} + c_{13} \frac{\partial w}{\partial z} \right\}\Bigg|_{y=\pm h} = 0$$

$$\sum_{i=1}^{6} (c_{12}\alpha_1^i k \cos\theta + c_{22}\alpha_2^i \beta^i + c_{13}\alpha_3^i k \sin\theta)$$

$$\times \, e^{\pm i\beta^i h} e^{i\psi} = 0 \tag{17}$$

and

$$\sigma_{yz}\Big|_{y=\pm h} = c_{44} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)\Bigg|_{y=\pm h}$$

$$c_{44} \sum_{i=1}^{6} A_0^i (\alpha_2^i k \sin\theta + \alpha_3^i \beta^i) e^{\pm i\beta^i} e^{i\psi} = 0 \tag{18}$$

This set of equations has a solution for the six A_0^i when the determinant of the coefficient matrix is zero. It should be noted that this approach is essentially the same as that employed by Green and co-workers [9–12] except that all possible types of motion (both symmetric and antisymmetric modes) are included.

In order to develop a solution to this problem a numerical approach was used. The decision to obtain a numerical rather than a closed form is due to the algebraic complexity of the problem. The solution method relies upon the observation that, for each value of the wave number k (assumed to be real), there are multiple values of ω (associated with the various modes of propagation and also assumed to be real), hence β^i, which satisfy the boundary conditions for the problem. Since a numerical approach was used, it was first necessary to verify the accuracy of approach. This was achieved by comparing these results to the results of Mindlin [4] for isotropic media. Good agreement was obtained. It should be noted that complex wavenumbers are possible. However, they would be

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associated with solutions which decayed exponentially with propagation distance. Given the large intrinsic attenuation (principally associated with scattering from the fibers) for wave propagation in composites, it is felt that these solutions would not be of large practical importance. The approach to solving this problem was:

1. Fix wavenumber k.
2. Begin search for possible from $\omega = 0$. For each pair of values of ω and k, calculate wave propagation parameters ξ^i and associated polarizations α^i.
3. Find the determinant of the coefficient matrix.
4. Check for bracketing of possible root by comparing the sign of the determinant with that obtained for the previous value of ω. If there was no sign change, increment ω and repeat the process. Otherwise:
5. Find root of determinant (as function of ω with k fixed) using regula falsi method.
6. Increment ω and repeat process until desired number of roots are found for given k.
7. Increment k and repeat procedure (from Step 2) until desired maximum wavenumber is reached.

In this way, frequency spectra for any desired propagation direction could be constructed. Once the modes were identified, the nature of the motion could be further explored with particle displacement profiles and stress distributions being determined for several of the modes. The symmetry of the mode shapes were determined in two ways:

1. Direct calculation from the A_is using the general solution, i.e., $A_1 = A_2$ corresponds to a symmetric solution and $A_1 = -A_2$ corresponds to an antisymmetric, etc.
2. Forcing the solutions to be either symmetric or antisymmetric (this reduces the boundary constraints for the problem from 6 to 3) and comparing results with the general solution. Both methods were found to yield identical results for the propagation direction investigated.

RESULTS AND DISCUSSION

Using the approach presented in the previous section and experimental data of Kriz and Stinchcomb [16] as shown in Table 1, plate wave propagation for arbitrary directions in transversely isotropic media was investigated. Results analogous to those for isotropic media were generated for plate waves propagating in a symmetry direction (at 0° and 90° with respect to the reinforcing fibers) as well as in arbitrary directions in the plate. It is found that for 0° propagation, the boundary conditions can be satisfied by either pure mode SH wave propagation or by coupled P-SV propagation. For 0° propagation, the two shear roots to the wave equation are distinct. However, due to decoupling of P-SV and SH waves, this results in only a slight difference from the isotropic case in that there are more branches to the frequency spectra to be considered. Figures 2 and 3 present the frequency spectra [normalized frequency ($\bar{\omega} = \omega b/\sqrt{c_{66}/\varrho}$) vs

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Table 1. Material properties.

c_{33}	$\|6\|$ GPa
c_{11}	c_{22} = 14.5 GPa
c_{23}	c_{13} = 6.5 GPa
c_{66}	3.63 GPa
c_{44}	c_{55} = 7.10 GPa
ϱ	1.61 × 10^{-5}N-s²/cm²

wavenumber ($\bar{k} = bk$)] for the 0° direction for the symmetric and antisymmetric modes respectively. SH and P-SV modes are combined in the two plots.

The cases of 90° propagation as well as several other propagation directions, 15°, 30°, 45°, 60° and 75°, were investigated in this study and found to be quite different. For the 90° case, the two shear roots to the wave equation are degenerate since this is the axis of transverse isotropy for the medium. For an arbitrary direction in a transversely isotropic medium, the three roots of the wave equation correspond to one pure shear mode as well as two modes whose particle displacements have components both parallel and perpendicular to the direction of propagation and are known as quasi-longitudinal (QL) or quasitransverse (QT) waves depending on the dominant motion. In this study we have found that SH type plate waves cannot be propagated nor can waves analagous to the coupled P-SV mode in isotropic media. For all directions other than 0°, plate waves in transversely isotropic media require the participation of all three modes of wave propagation in the propagation direction in order to satisfy the traction free boundary conditions (even for 90°). One reason for this is the nature of reflection at a free surface in anisotropic media. Redwood [21] has shown how the reflection and interference of longitudinal (D) and shear (T) waves in isotropic media may be combined to produce the symmetric (or antisymmetric) displacements associated with plate waves [see Figure 4(a)]. Note: Snell's Law requires both sets of waves to have the same projected velocity along the boundary, hence in the propagation direction. For isotropic media, the incidence of a longitudinal (or SV shear) wave requires the generation of a longitudinal and shear wave to satisfy the traction-free boundary conditions at any angle of incidence other than normal incidence hence the need for P-SV coupling. SH waves only require a reflected SH wave and, therefore, can propagate alone. For anisotropic media, in general, the boundary conditions require three waves to be generated. The exception to this would be the 0° case where the incident shear (SH) or coupled longitudinal and shear (SV) waves would be pure modes (assuming a random fiber distribution) and only one (SH) or two (P-SV) reflected waves would be needed as for the isotropic case [Figure 4(b)]. Despite the symmetry of the 90° case, three reflected waves are required as illustrated in Figure 4(c). Even though the projections of the velocities of the waves incident upon the boundary correspond to that of a pure mode longitudinal wave, neither of the incident waves themselves are pure mode. Since they propagate at some angle with respect to the reinforcing fibers,

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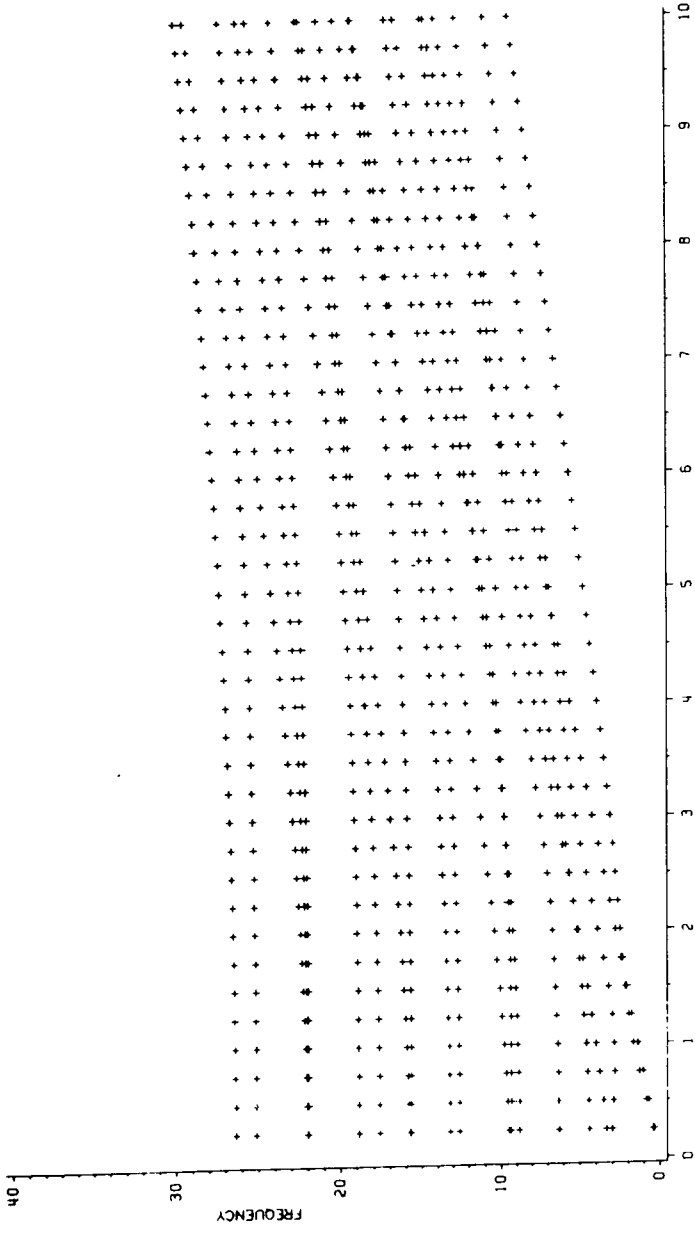


Figure 2. *Frequency spectra-symmetric modes (perpendicular to fibers).*

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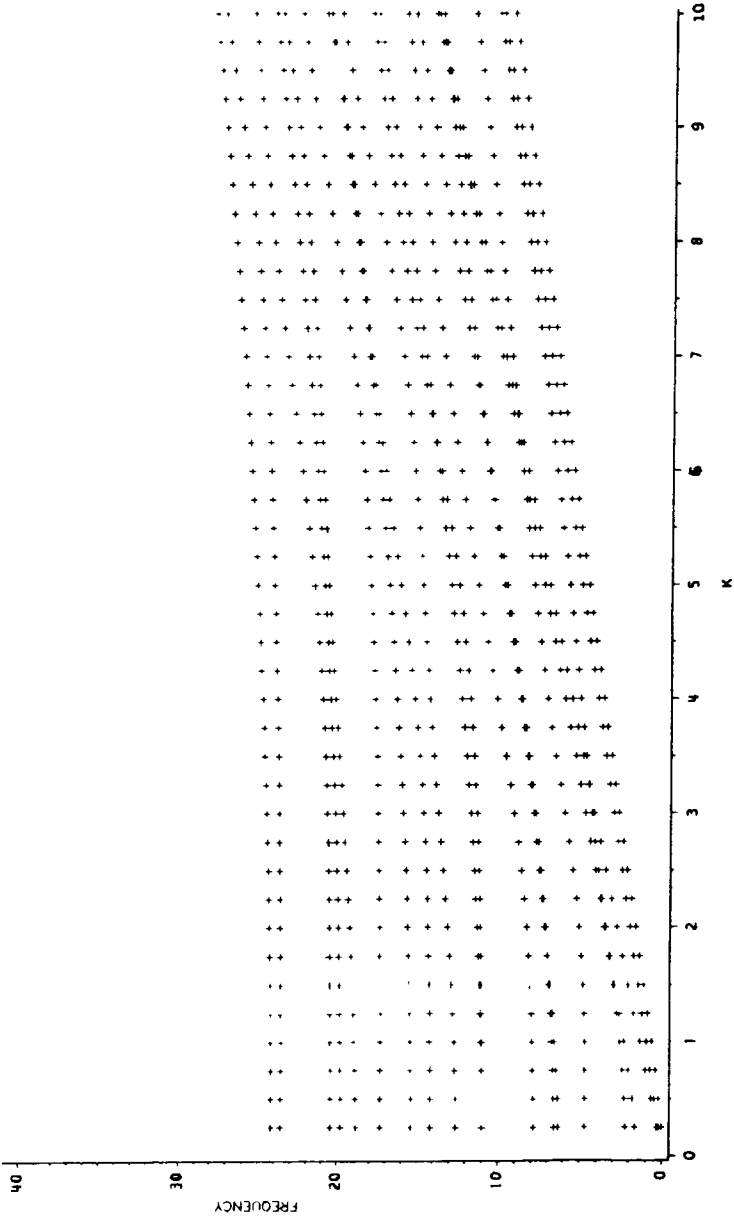


Figure 3. *Frequency spectra-antisymmetric modes (perpendicular to fibers).*

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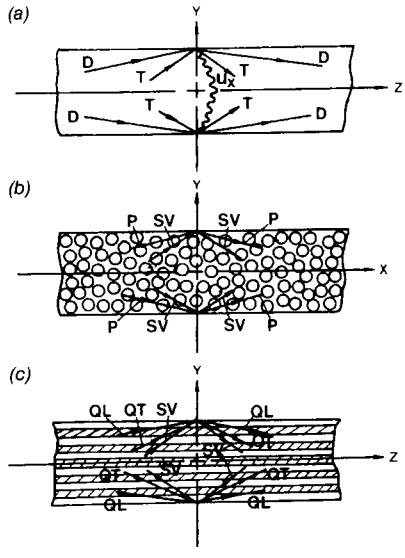


Figure 4. *(a) Reflection of longitudinal and shear waves at free surface in isotropic medium and how they may be combined to produce a plate wave (after Redwood [20]), (b) reflection at a free surface (0° case) and (c) reflection at a free surface (90° case).*

they are quasilongitudinal or quasitransverse in nature. Hence, the increased complexity of the reflection problem at the interface.

In our research, we found that the symmetry of the particle displacements was similar to that found for isotropic media, i.e., "symmetric" modes with u, w symmetric and v antisymmetric with respect to the midplane or "antisymmetric" modes with u and w antisymmetric and v symmetric. No other displacement patterns were found.

The most important finding in our research was the existence of one nondispersive mode of propagation for every direction studied, analagous to the zero order SH mode of propagation was associated with

$$\beta^5 = -\beta^6 \cong 0$$

corresponding to a velocity of propagation close to that of one of the bulk wave propagation velocities for this direction. (Note: $\beta^{5,6} = 0$ corresponds to a quasilongitudinal wave, not quasitransverse as might be expected.) This is illustrated in Figures 5–6 where the frequency spectra for symmetric and antisymmetric propagation modes are presented for each of the directions, other than 0° studied here. The solid line in Figure 5 illustrates this nondispersive mode. It should be noted that the motion, unlike SH waves, requires contributions from the other 2 waves to propagate. Thus, even though $\beta^5 = \beta^6 \cong 0$, the remaining β_i are nonzero and the particle displacements do vary through the thickness of the plate. This is in contrast to zero order SH waves. It should be noted that SH waves

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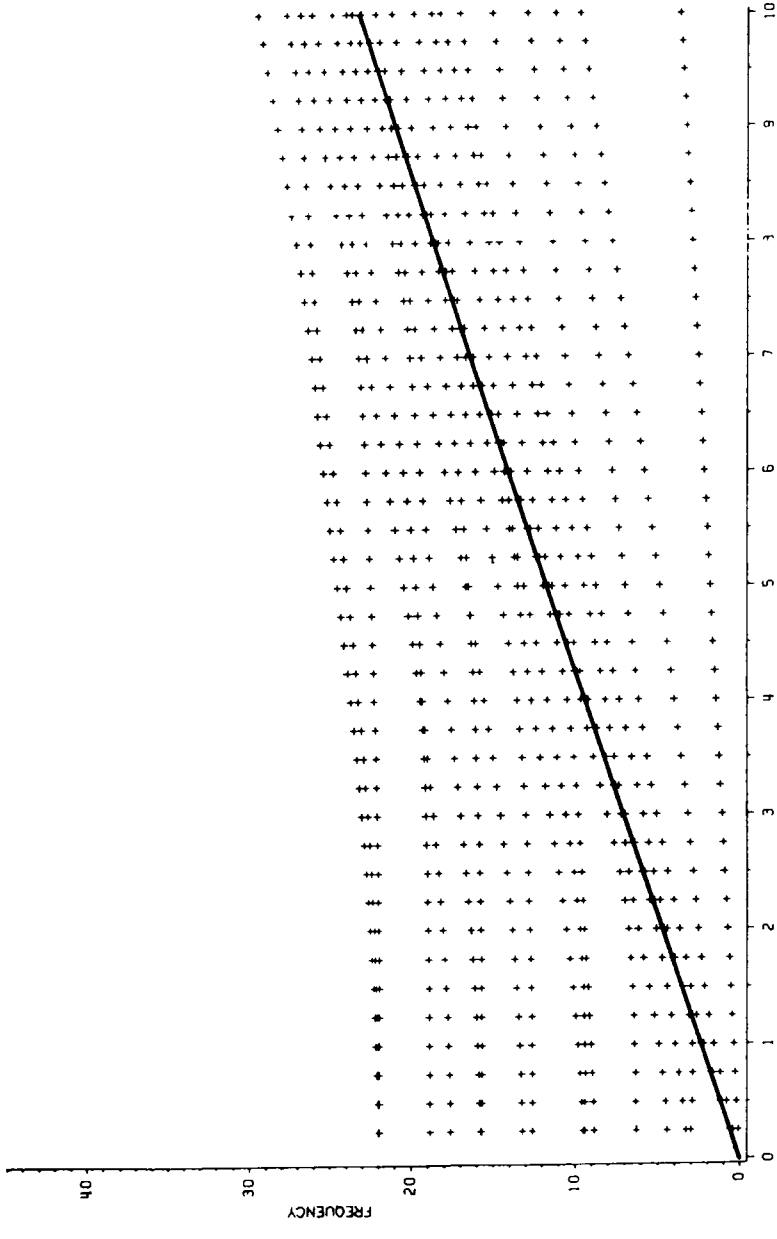


Figure 5a. *Frequency spectra as function of propagation direction (symmetric modes) at 15°.*

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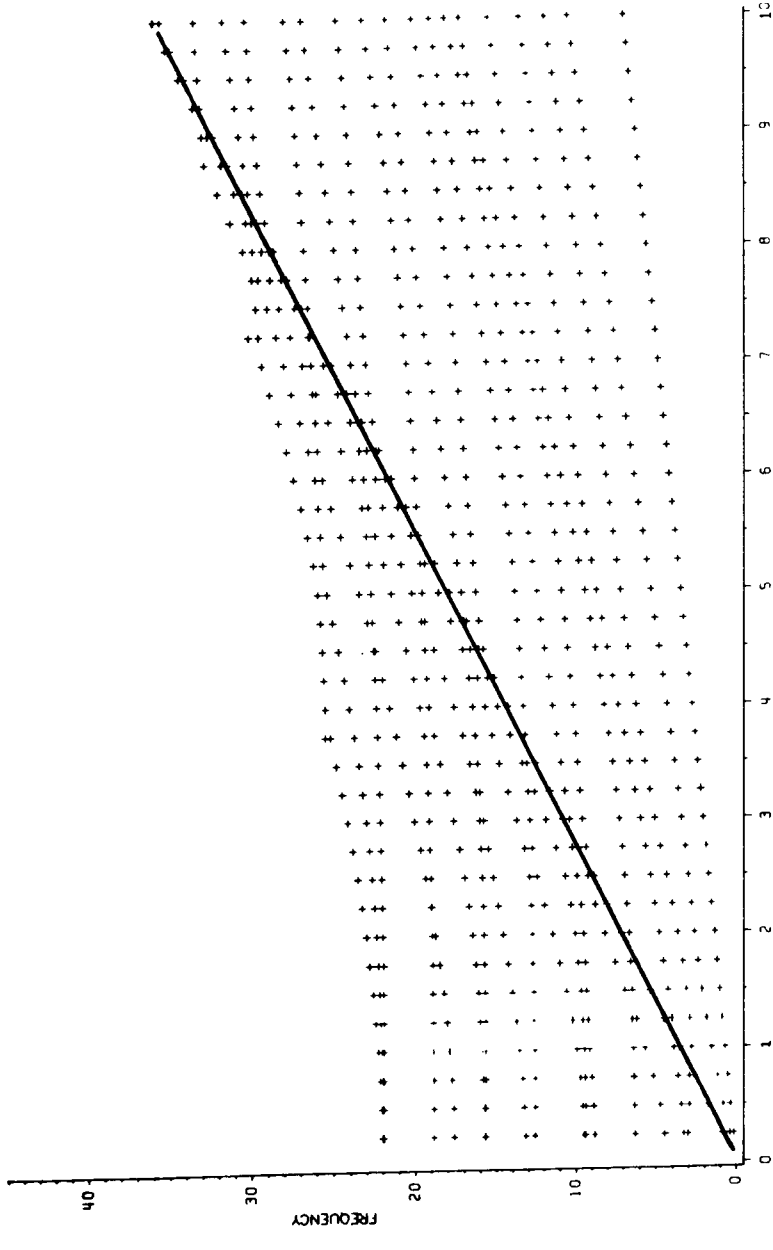


Figure 5b. *Frequency spectra as function of propagation direction (symmetric modes) at 30°.*

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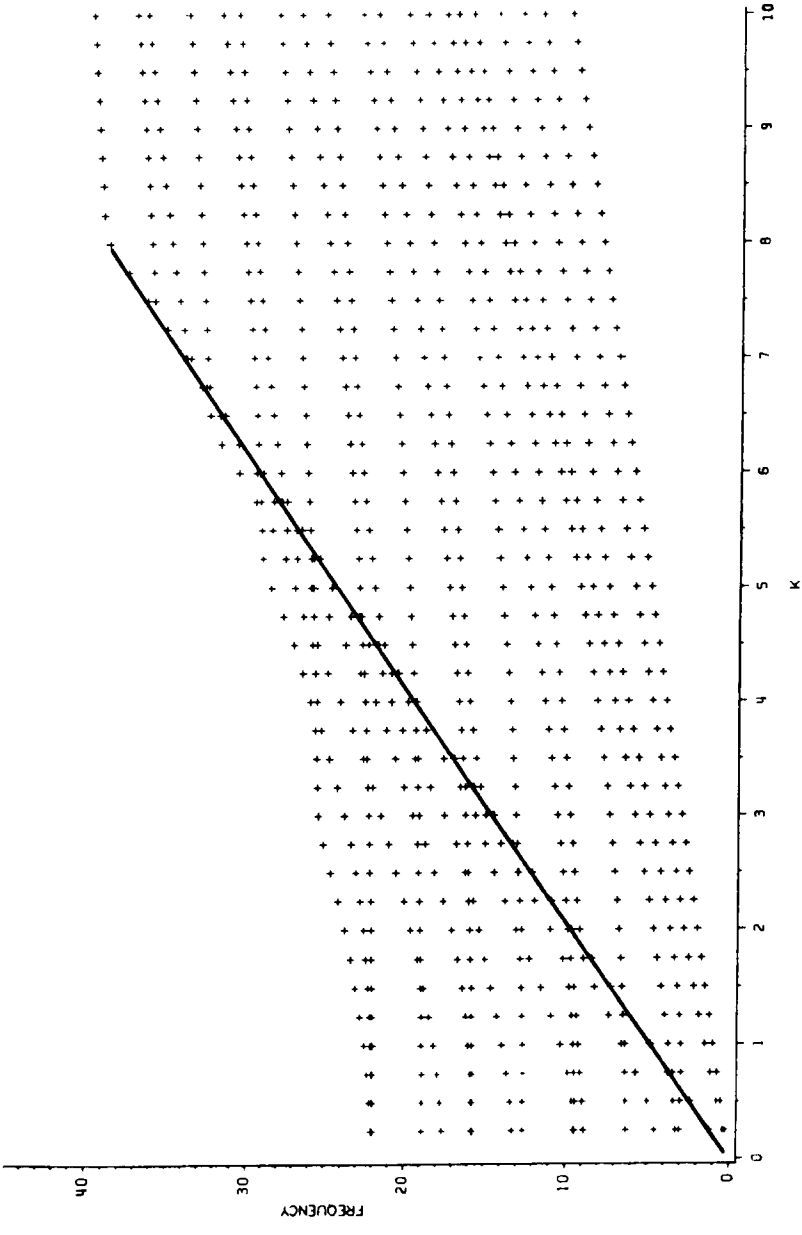


Figure 5c. *Frequency spectra as function of propagation direction (symmetric modes) at 45°.*

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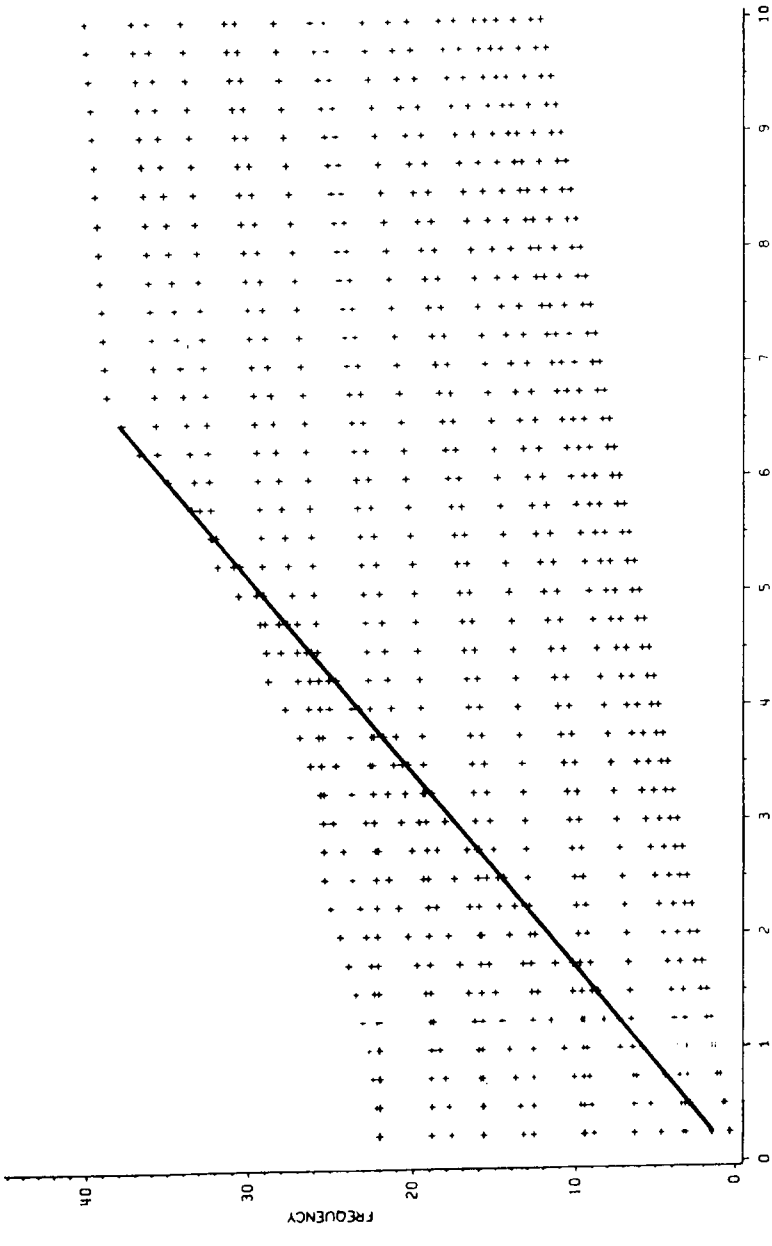


Figure 5d. *Frequency spectra as function of propagation direction (symmetric modes) at 60°.*

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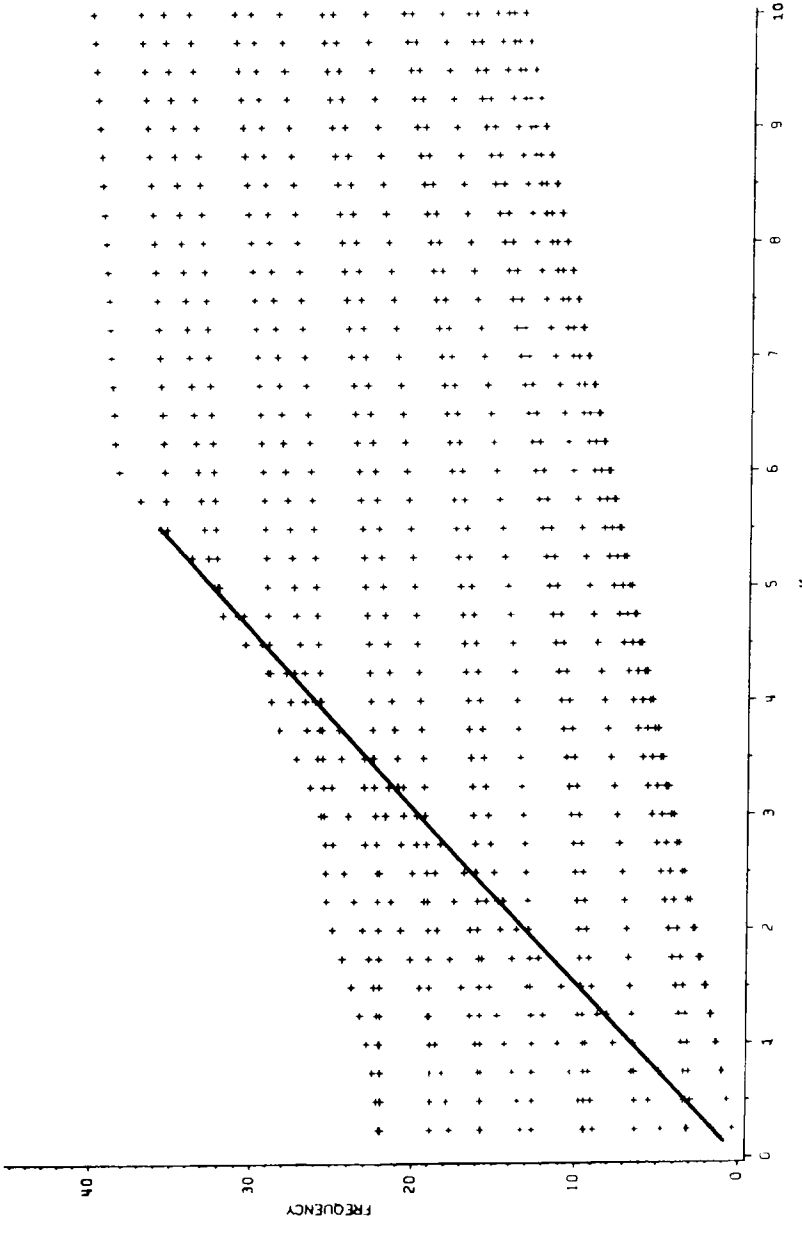


Figure 5e. *Frequency spectra as function of propagation direction (symmetric modes) at 75°.*

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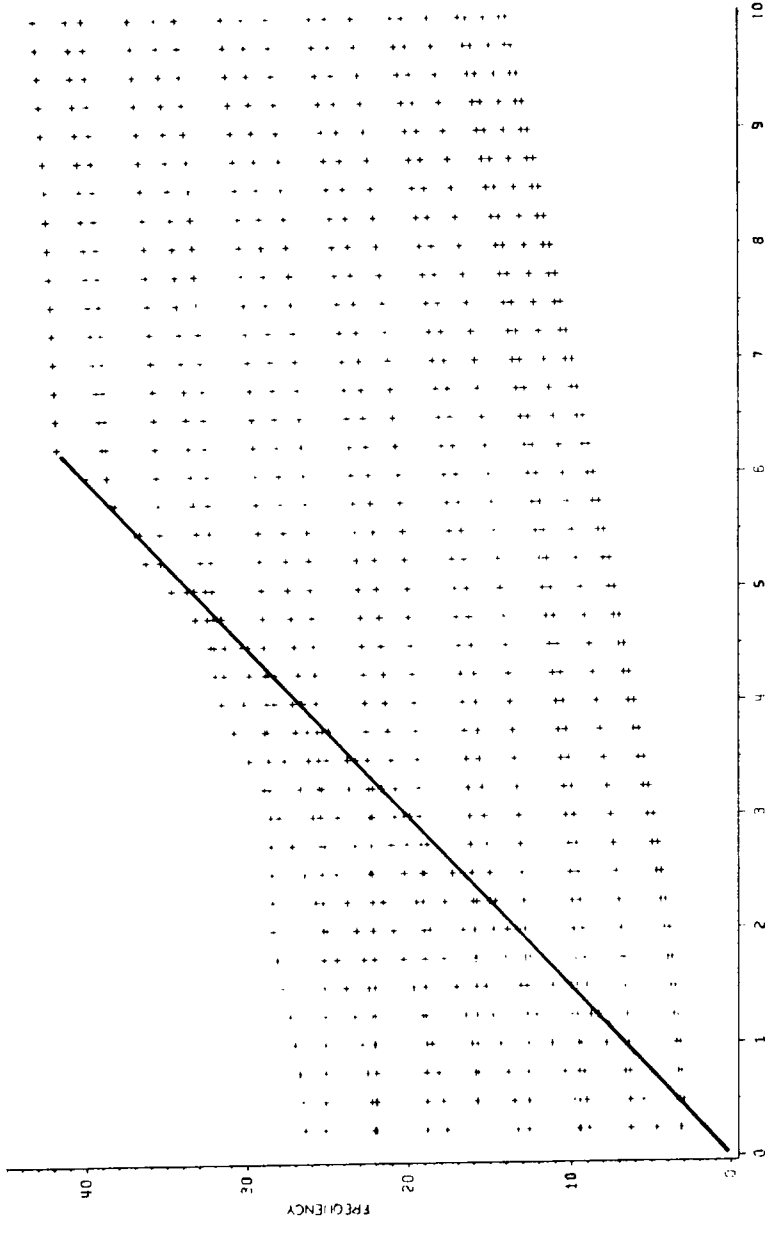


Figure 5f. *Frequency spectra as function of propagation direction (symmetric modes) at 90°.*

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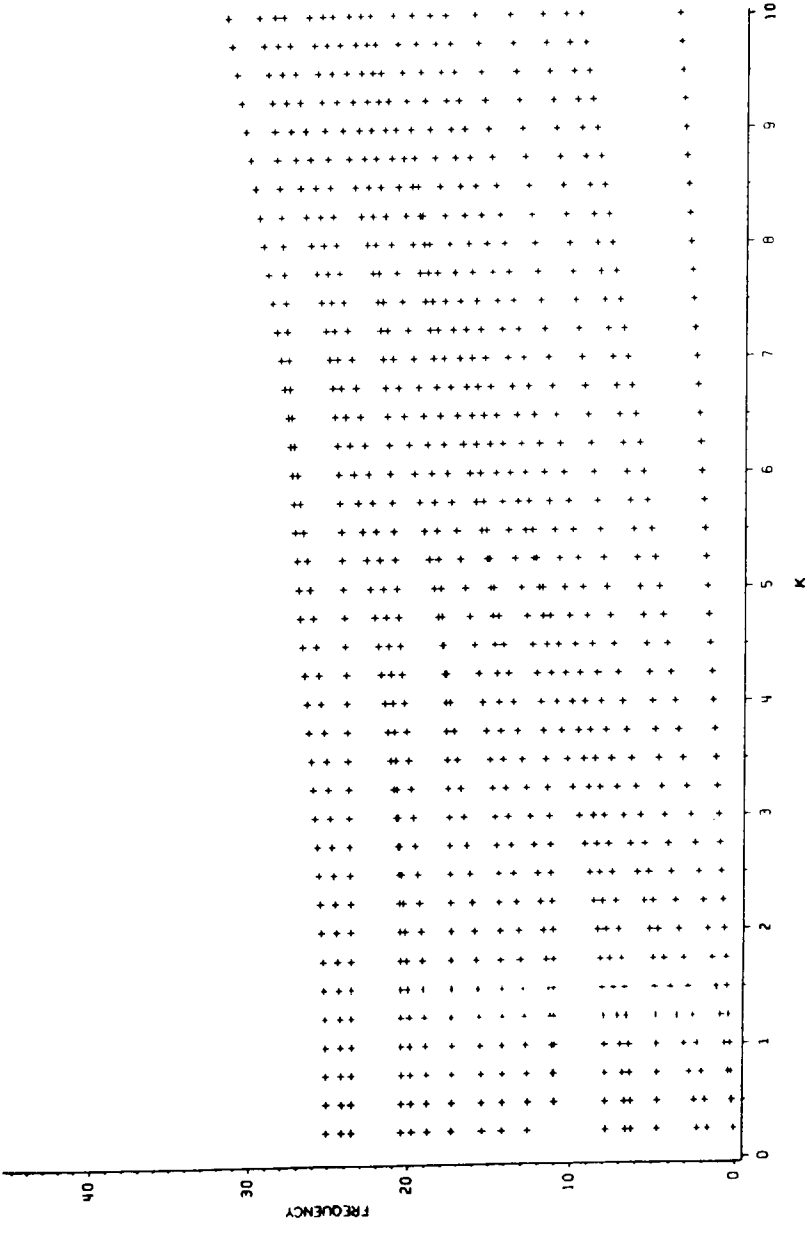


Figure 6a. *Frequency spectra as function of propagation direction (antisymmetric modes) at 15°.*

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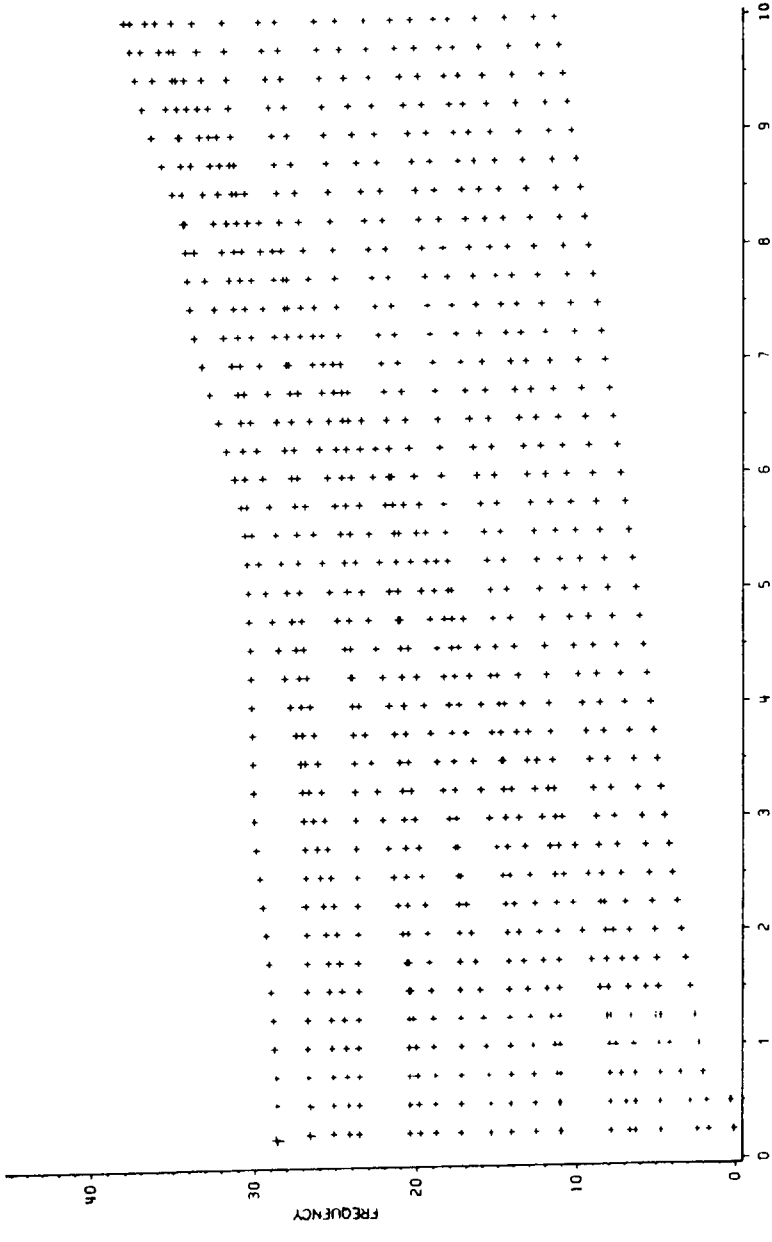


Figure 6b. *Frequency spectra as function of propagation direction (antisymmetric modes) at 30°.*

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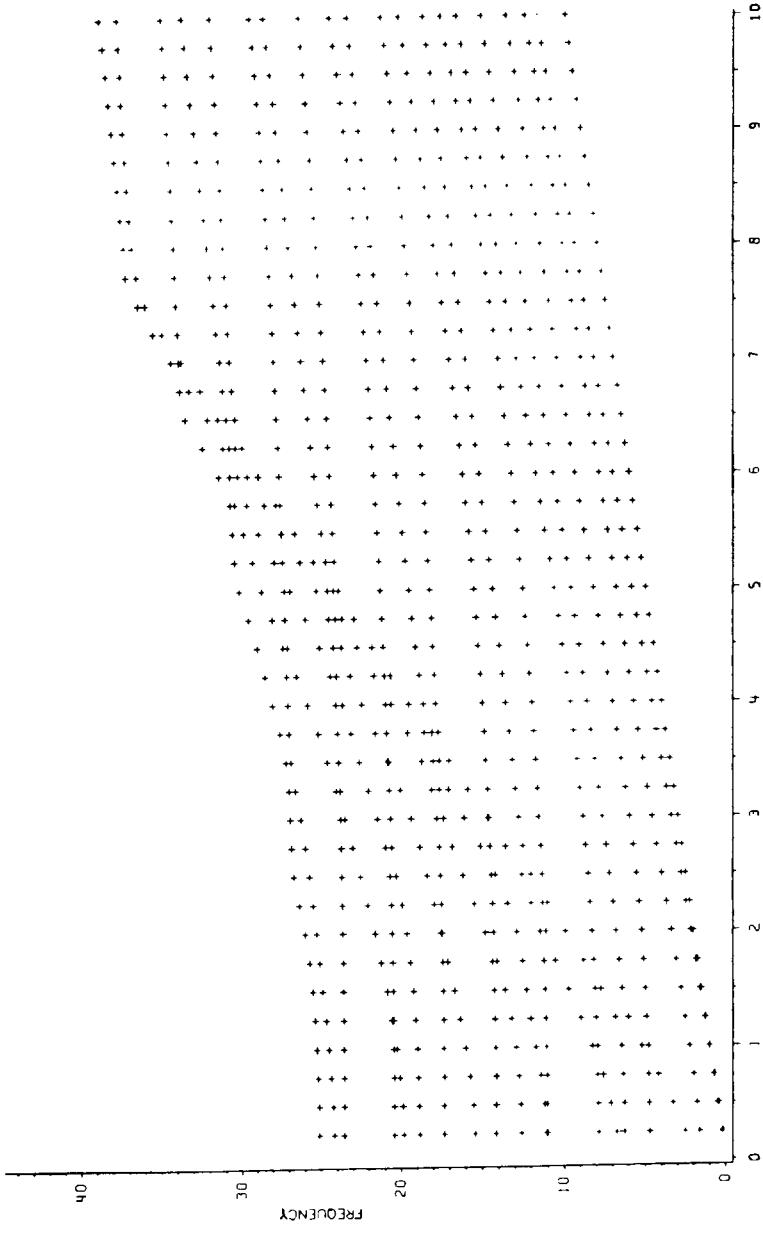


Figure 6c. *Frequency spectra as function of propagation direction (antisymmetric modes) at 45°.*

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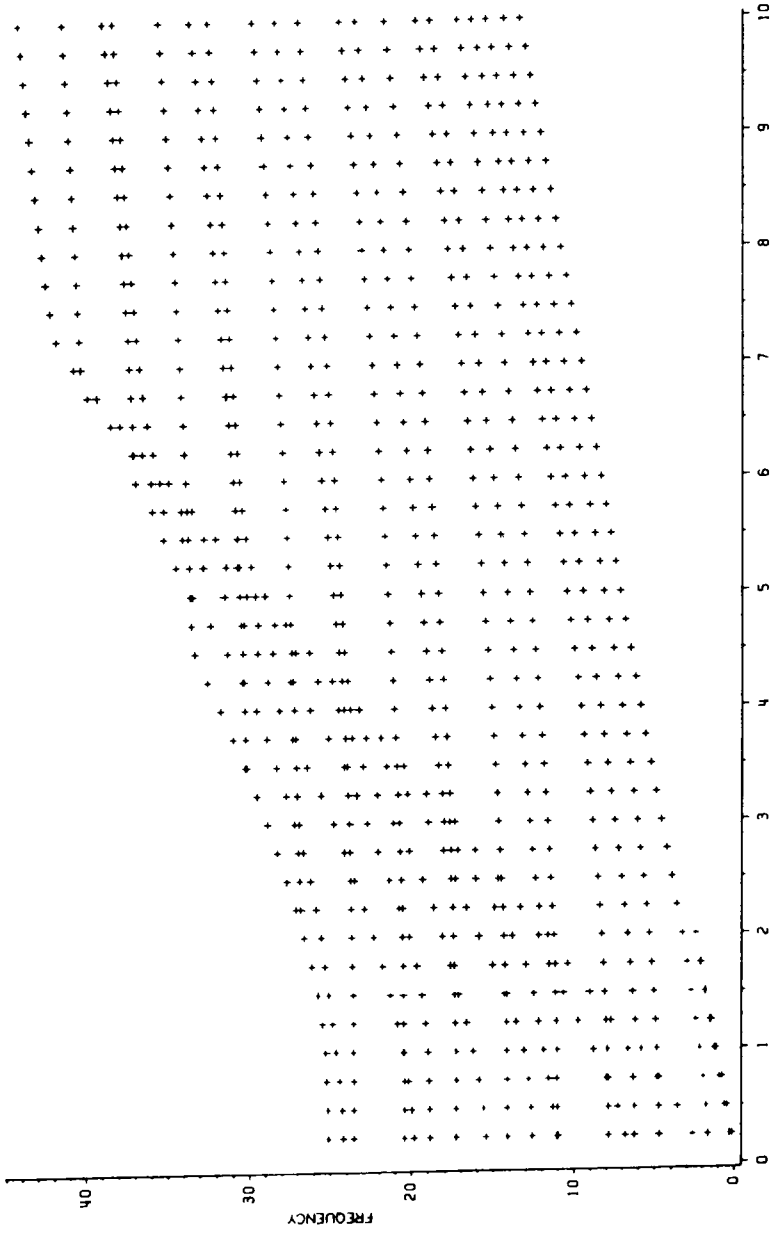


Figure 6d. *Frequency spectra as function of propagation direction (antisymmetric modes) at 60°.*

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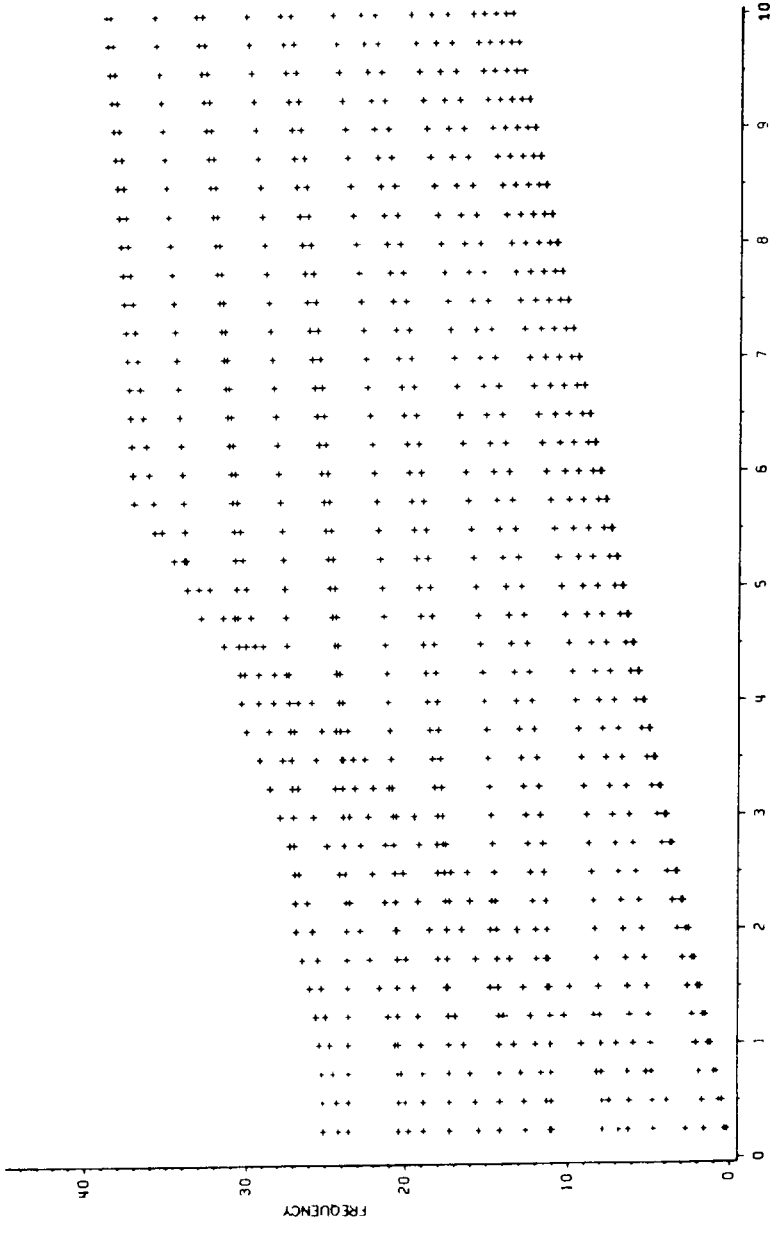


Figure 6e. *Frequency spectra as function of propagation direction (antisymmetric modes) at 75°.*

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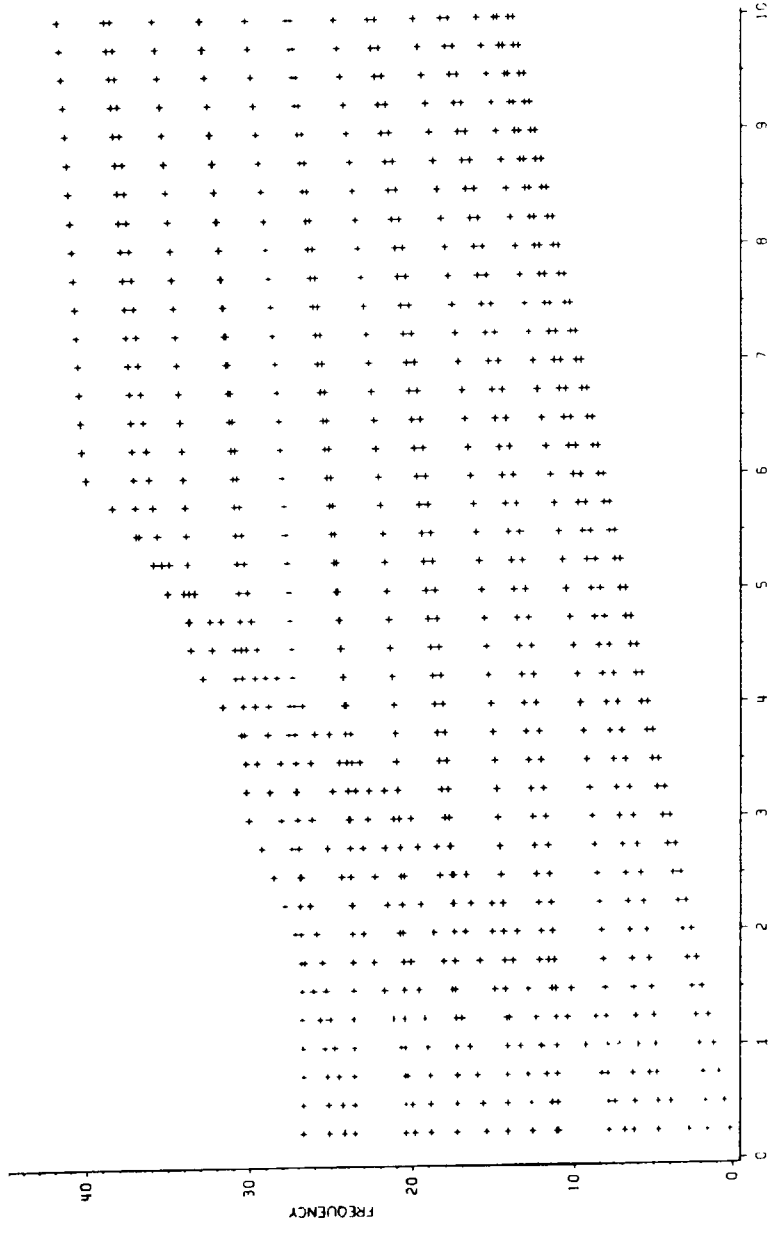


Figure 6f. *Frequency spectra as function of propagation direction (antisymmetric modes) at 90°.*

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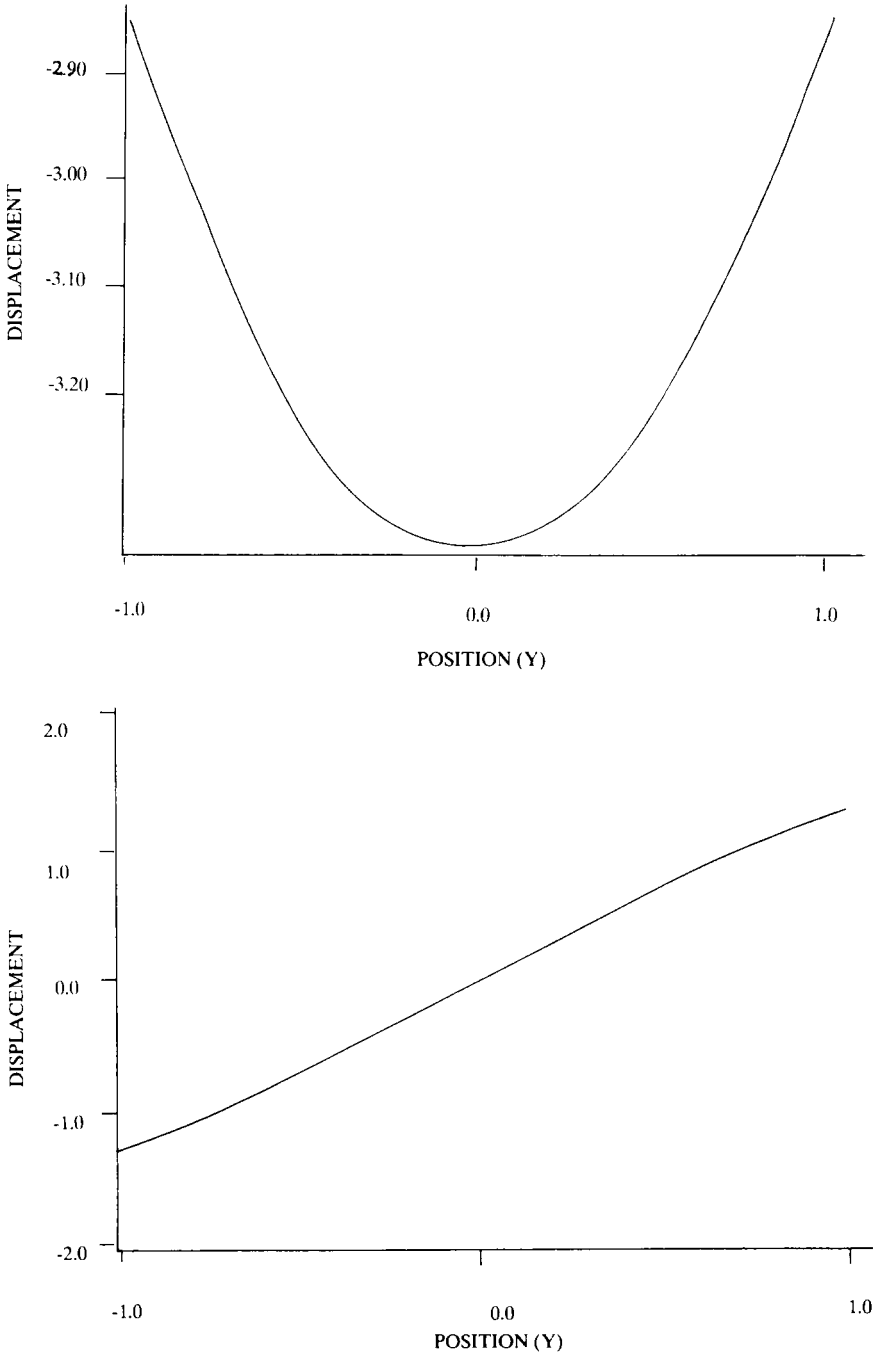


Figure 7. *Particle displacements-symmetric (45°).*

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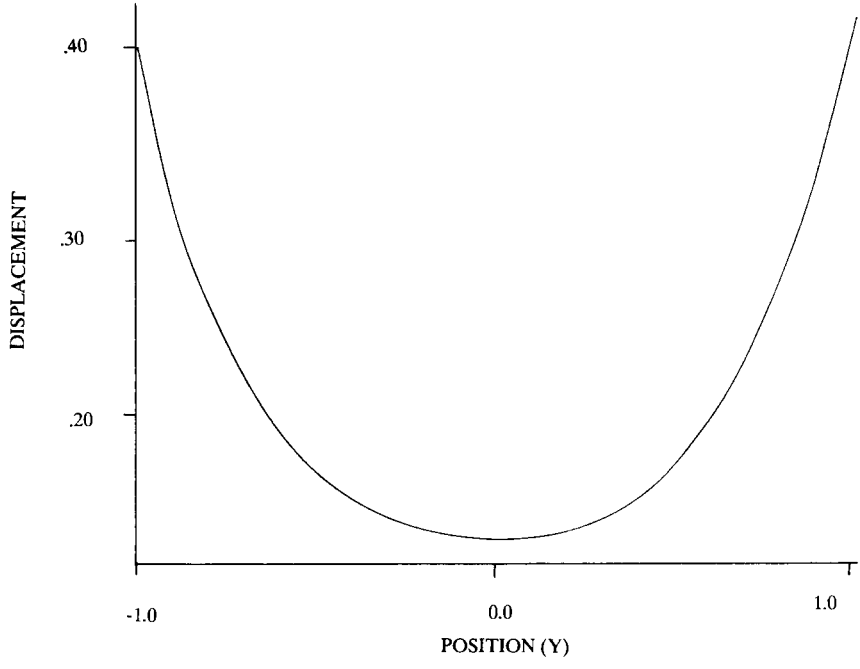


Figure 7 (continued). *Particle displacements-symmetric (45°).*

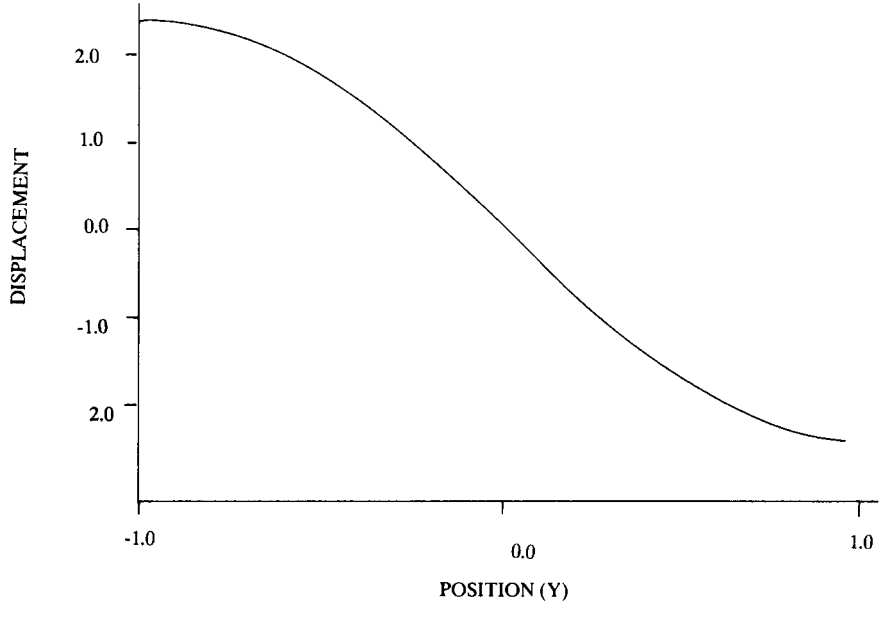


Figure 8. *Particle displacements-antisymmetric (45°).*

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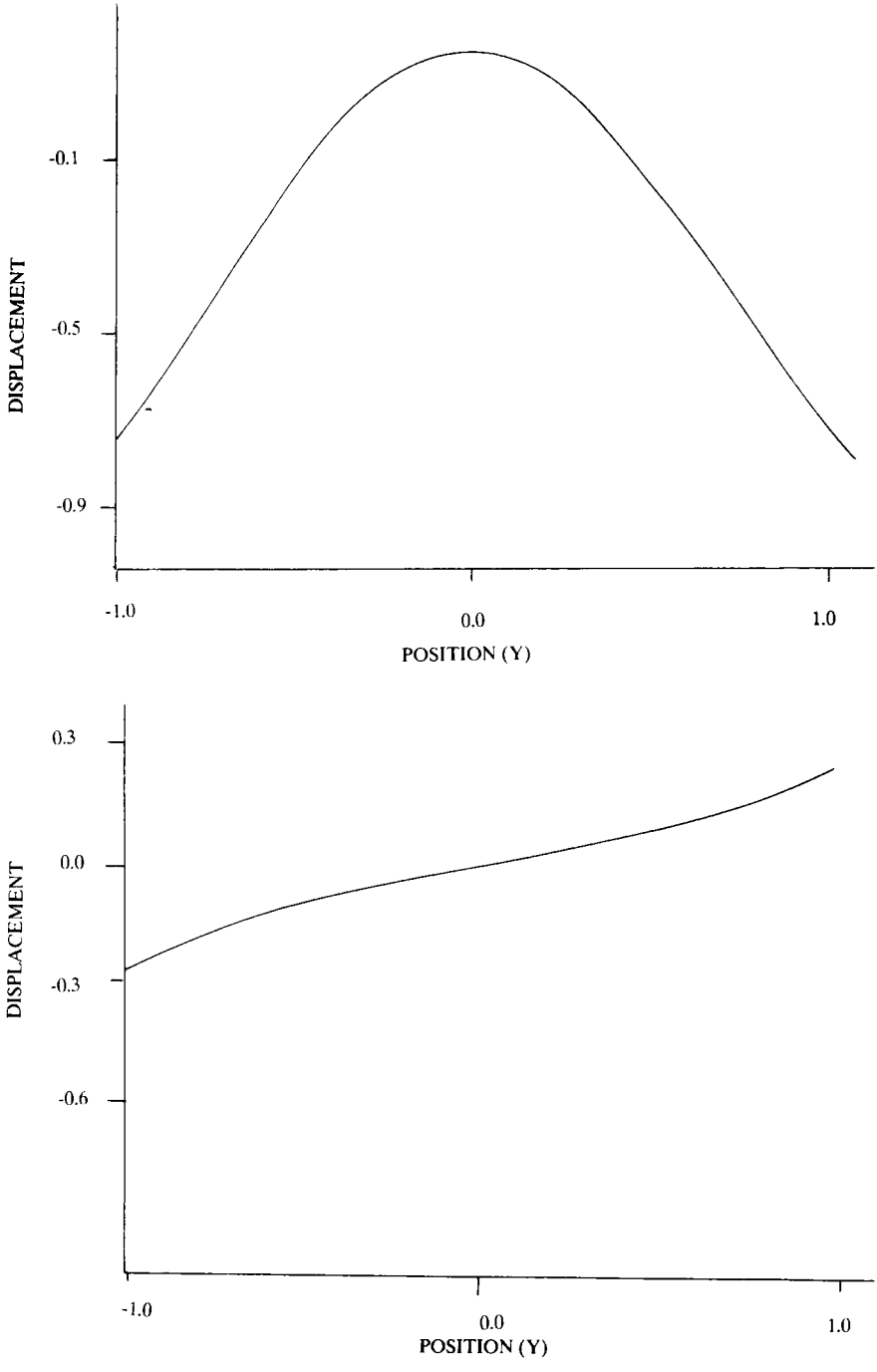


Figure 8 (continued). *Particle displacements-antisymmetric (45°).*

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are a family of modes ($\beta^i = n\pi/2, n = 1, 2, 3, \ldots$) whose velocities approach the shear velocity asymptotically as the wavenumber approaches infinity. No such family of waves could be identified for the directions under consideration here.

Typical particle displacements and stress distributions are illustrated in Figures 7–8 for the lowest order symmetric and antisymmetric modes which propagate at 45° with respect to the reinforcing fibers (for $k = 1.0$). The in-plane displacements (u,w) are found to be roughly parabolic with v being basically linear for the lowest "symmetric" case while the reverse is true for the lowest antisymmetric mode with parabolic v and an approximately linear variation. The stresses were found to exhibit similar behavior.

CONCLUSIONS

In this work, plate wave propagation in transversely isotropic media has been investigated. The propagation media was modeled as an infinite composite plate with unidirectional fiber reinforcement. A numerical algorithm was developed to characterize the various modes of propagation (dispersion relations, mode shapes, etc.) possible. Results were obtained for propagation in both material symmetry (0°, 90° w.r.t. reinforcing fibers) and nonsymmetry directions. For the symmetry directions, the results were quite similar to those obtained for isotropic media. For nonsymmetry directions, however, some interesting differences emerge, most notably the coupling of all three propagation modes. The approach taken in this study is readily adaptable to arbitrary propagation directions in generally anisotropic media. It is expected that, with suitable modifications, this approach to plate wave propagation could be used for the nondestructive characterization of laminated composites.

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