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NOVEL CONFORMAL MAPPING MODEL FOR MINIATURIZATION OF MICROSTRIP
PATCH ANTENNAS WITH MAGNETO-DIELECTRIC MATERIALS

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PATCH ANTENNAS WITH MAGNETO-DIELECTRIC MATERIALS

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Abstract

Magneto-dielectric (MD) materials have enabled significant miniaturization while theoretically improving bandwidth and efficiency of microstrip patch antennas (MPA), however computationally efficient design and performance models for MD are needed to enable rapid development. Conformal mapping (CM) provides an efficient algorithm which translates an orthogonal basis to another by rotating and scaling each point while the angle between them remains unaltered. This is an important methodology that can transform the potential electric and magnetic fields into a linear grid to simplify integration while preserving gradients and angle of incidence at boundary conditions. This research investigates utilization of CM on microstrip structures to evaluate the effective relative permeability, μ_{ref} , for more robust modeling and analysis when MD media is incorporated. The methods are experimentally verified to microstrip structures fabricated on Rogers Corporation's MAGTREX 555 (MX), a commercially off the shelf (CoTS) MD substrate. The two-port microstrip transmission-lines (TL), microstrip low pass filter (LPF), and probe fed microstrip patch antenna (MPA) were simulated with these methods and compared to the measurement data. Additional validation of novel CM methods was performed by parametric simulation in HFSS to evaluate a larger range of material parameters and strip widths.

Chapter 1

Introduction

1.1 Overview

Pursuit of miniaturized electronic components has come a long way, but there is room for further reduction of the size, weight, power consumption, and cost (SWaP-c) of antennas [1], [2]. Conventionally, the size of a dipole perfect electric conductor (PEC) radiator in free-space is on the order of a half wavelength, $\lambda/2$. Wavelength is inversely proportional to the resonant frequency, meaning low frequency antennas must be very large and decrease in size as frequency increases. Many industries including communications, medical, aerospace, and defense are heavily driving the efforts to optimize antenna SWaP-c [1]. The communications and medical industries are continuously seeking novel methods to decrease the size and weight of antennas while increasing bandwidth and efficiency for mobile, flexible wearables, radio frequency identification (RFID), and implantable applications [3], [4]. The aerospace and defense industries have a vested interest in miniaturized antennas for communication systems, enemy detection and ranging, and remote instrumentation monitoring [5]. These applications aim to reduce size and maximize performance, while reducing costs [1]. While each industry has differing motivations, they all share the desire to find antenna miniaturization solutions that increase efficiency.

To bypass the requirement for antenna miniaturization, antenna sizes could be reduced by utilizing higher operational frequencies [1], [6]. Caveats to that approach are the legal limitations on spectrum utilization and increased cost of integration with existing back-end electronics [1], [6].

The International Telecommunications Union (ITU) governs and designate international utilization of shared frequency bands [1], [6], [7]. Additionally, each government further regulates frequency allocations [1], [6], [7]. Figure 1.1 demonstrates allocations contributing to the crowded frequency spectrum which with the proliferation of technology has increased the number of radio frequency interference (RFI) sources [1], [6], [7]. Additional non-allocated RFI sources include wideband systems, unlicensed spectral use, or unregulated noise sources [6]. The spectral space becomes further polluted with increased numbers of RFI sources, which to prevent antenna performance degradation drives the need for increases isolation to maintain signal-to-interference-plus-noise ratio (SINR) requirements [1], [6], [7]. These limitations isolate each industry to finding solutions within their designated frequency band while managing the trade space to optimize SWaP-c.

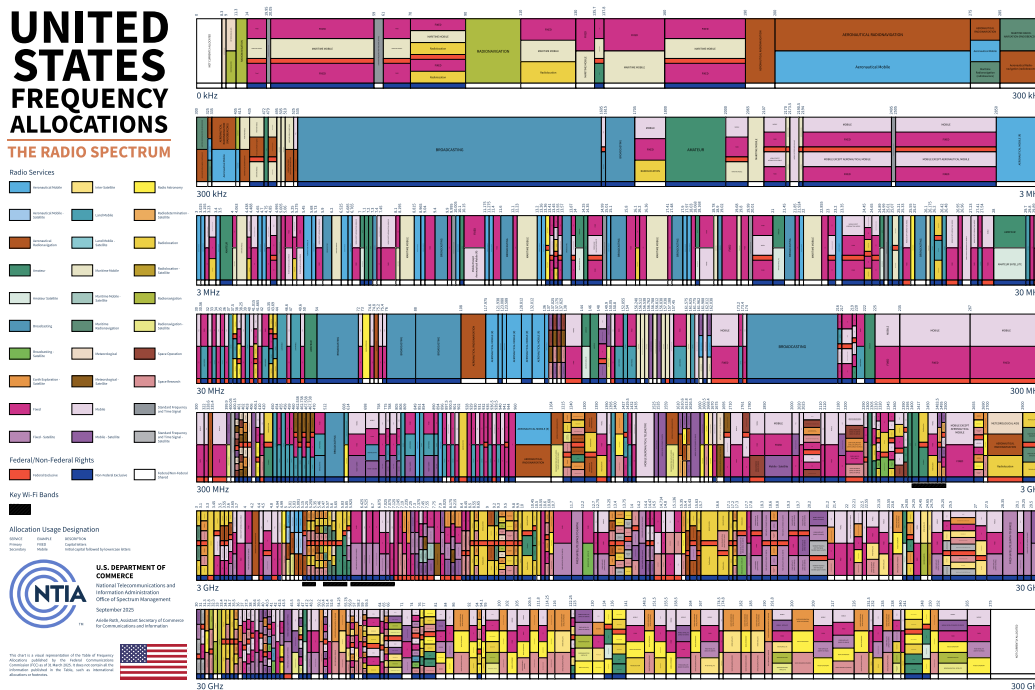


Figure 1.1: Frequency spectrum allocation chart demonstrating the competing use of frequency bands that result in spectral crowding [7].

To break free of the traditional bounds that limit antenna miniaturization, the fundamental physics that drive antenna performance must be further explored and a standardized Figure of Merit (FoM) developed that quantifies performance and comparison between traditional and modern antennas.

1.2 Motivation

MDs are composites of a dielectric material with magnetic inclusions providing a high refractive index with low dielectric and magnetic loss tangents which make these materials promising for antenna improvements in miniaturization, radiation efficiency, and bandwidth [3], [5], [8], [9], [10], [11]. Realizing the benefits of MD materials in antenna design has been limited by the difficulty in producing the desired intrinsic material characteristics, the high costs of establishing well controlled manufacturing processes, and the lack of models to accurately facilitate design or predict performance metrics [3], [12], [13], [14], [15]. The electromagnetic (EM) industry requires models with versatility to account for modern material properties to enable accelerated design and analysis of MD antennas.

There is an ever increasing demand for more efficient yet smaller antennas. The bounds of antenna size, quality factor (Q-factor), and efficiency have been highly investigated for well behaved passive electrically small antennas. The demand for antennas to be further miniaturized and reach greater efficiency has led to the pursuit of the absolute limits of these parameters [13], [16], [17]. The upper and lower bounds of the Q-factor have been asserted by multiple authors, but are limited by assumptions applicable only to passive antennas. The quest for an antenna that is both miniaturized and has frequency agility has further increased the complexity of antenna design. As antenna complexity increases, the efficiency and Q-factor may no longer be an appropriate gauge of behavior over an antenna's tuning range. If this is found to be so, a new way to quantify performance of these antennas must be formulated and standardized as a FoM.

As technology advances to utilizing much higher frequencies, GigaHertz (GHz) and above, it becomes more apparent that lossless models no longer approximate the effects on a signal as it propagates through space, along a TL, or within materials. Linear and time invariant (LTI) assumptions impose fundamental bounds of antenna miniaturization, which break down in cases utilizing lossy materials [12]. Engineering the next generation of technology will require ingenuity in manipulating lossy materials to break past the formerly asserted bounds. These attributes drive the necessity of models that accurately account for material properties and have the versatility to

accelerate rapid design and analysis of radio frequency (RF) circuits and antennas [13].

Engineers have explored many techniques to overcome size and performance limitations of MPAs. Topological methods achieve a reduced form-factor by modifying current paths by introducing defects, meandering margins, or periodic fractional length arms, however they often suffer from reduced SINR and be poor far-field radiators [12], [18]. Reactive loads offer a hybrid approach to introduce capacitance or inductance to shift an antenna's resonant frequency through incorporated resonant structures, cavities, or reactive lumped-elements [12], [18], [19], [20]. These approaches lead to improved bandwidth, but can also have adverse effects on the Q-factor and 3 dB bandwidth related to additional loss mechanisms [18]. Engineered media, such as dielectrics and MDs, have altered material properties which introduce reactance, the imaginary component of impedance, to load the antenna volume and shift the resonant frequency [1], [3], [14], [18], [19], [20], [21].

It is essential to understand the relation between fundamental material parameters and observable properties for selection of the best modeling technique. Common simplified forms of equations used to evaluate EM parameters of materials no longer hold true due to the frequent use of mixed media and MD materials. The most robust modeling techniques are full wave solutions, however these are considerably limited by simulation times to evaluate complex structures. To facilitate accelerated development while exploiting novel media, the EMs engineering community needs models that promote both rapid design and performance analysis while capturing increased structural and media complexities.

Exploration into utilizing MD materials for antenna miniaturization qualifies the engineering community's need for models that account for expanded material properties and help to drive capitalization of this design space. This thesis presents a CM algorithm for modeling microstrip structures such as microstrip TLs, or MPAs which investigates two proposed methods to find the effective relative permeability. Together, effective relative permittivity and permeability are used as isotropic effective parameters to evaluate microstrip and multi-layered microstrip structures incorporating MD media. The effective values found by CM can be fed into other modeling techniques to more accurately characterize the behavior of a microstrip TL or MPA. These novel

CM techniques are the first known CM models to account for MD material parameters in modeling multi-layered microstrip structures.

Also proposed in this work is the introductory work to develop a non-linear TL method to accurately predict the MPA resonant frequency. These models are key contributions to the engineering community that will accelerate antenna design by decreasing reliance on time intensive methods such as iterative full-wave simulation or prototyping to tune a MPA's frequency response.

The following section provides a review of important, but lesser utilized properties of MD materials. The review will be followed by the objectives and organization of this thesis.

Review of Material Properties

The foundation of intrinsic material properties has been introduced in context of exploitation for miniaturization of RF structures and antennas. This section is going to dive deeper to discuss the impact of manipulating each material property and how that effects extrinsic, or measurable, properties. It is essential to understand the relation between fundamental material parameters and observable properties for selection of the best design and modeling techniques.

Intrinsic parameters are material characteristics such as permittivity, permeability, and characteristic impedance that are independent of geometry, but cannot be measured directly. Permittivity, ε , and permeability, μ , are descriptive of how easily a field can travel through a medium related to electric and magnetic fields, respectively. When operating under LTI assumptions, the absolute permittivity and absolute permeability represent a product between free space and relative values, $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$, where ε_0 is the permittivity of free space in Farad per meter, ε_r is the relative permittivity of the medium, μ_0 is the permeability of free space in Henry per meter, and μ_r is the relative permeability of the medium. Most often the relative value, which is relative to the medium, is of most interest as it is the portion that can be used to engineer materials to desired properties [18].

Altering the relative permittivity, ε_r , or relative permeability, μ_r , of a medium has an inverse effect on the wave velocity and therefore the wavelength of an EM wave traveling through that

medium [18]. This inverse relationship between the constitutive parameters and wavelength is well demonstrated by $v = \frac{1}{\sqrt{\mu\varepsilon}}$ and $\lambda = \frac{v}{f} = \frac{1}{f\sqrt{\mu\varepsilon}}$, where λ is the wavelength in the medium given in meters, v is the propagation velocity of a wave within the medium in meters per second, and f is the frequency in Hertz of the EM wave traveling within a medium with absolute permittivity, ε , and absolute permeability μ [18]. Effective relative values of the constitutive parameters can be utilized to approximate a singular value to represent mixed media as single isotropic medium. Derivations for more accurate estimation of effective values for these relative constitutive parameters based on microstrip geometry and material layers will be covered in Chapter 4.

Engineered materials can be anisotropic due to fabrication methods used to combine dielectric and magnetic materials into a composite form. The permeability of an anisotropic material is represented by a tensor that describes the material properties in each cartesian direction is given in matrix form by

$$[\mu] = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \quad (1.1)$$

[5]. Equation 1.1 simplifies under a $\hat{z}$ biased condition to

$$[\mu] = \begin{bmatrix} \mu & jk & 0 \\ -jk & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}, \quad (1.2)$$

where $j = \sqrt{-1}$ and $k = \mu_0 \frac{w w_m}{w_0^2 - w^2}$, and the precession frequency is $w_0 = \mu_0 \gamma H_0$, where $w_m = \mu_0 \gamma M_s$, $\gamma = 1.759 \times 10^{11} [C/kg]$ is the gyromagnetic ratio, $H_0 = j E_0 \sqrt{\frac{\varepsilon_0 \varepsilon_r}{\mu_0 \mu_r}}$ is an applied field bias, and M_s is the saturation magnetization of the magnetic moment, a constant specific ferrite components [5], [22], [23]. Permittivity also has a tensor form similar to Equation 1.1 [5].

Relative permittivity and relative permeability are complex values expressed as $\varepsilon_r = \varepsilon' - j\varepsilon''$ and $\mu_r = \mu' - j\mu''$, respectively. The real component of each is notated with a single apostrophe referred to as "prime," and the imaginary component notation is two apostrophes called "double

prime.” The real component of relative permittivity, ε' , is also referred to as the dielectric constant or k in some literature. The imaginary components of relative permittivity and relative permeability are representative of energy loss in the form of stored energy or conductive losses. This stored energy can be expressed as a dimensionless ratio of imaginary to real components called the loss tangent given by $\tan \delta_E = \frac{\varepsilon''}{\varepsilon'}$ and $\tan \delta_M = \frac{\mu''}{\mu'}$ for electrical and magnetic losses, respectively. These notations can also be combined to represent the complex permeability and permittivity as

$$\varepsilon_r = \varepsilon'(1 - j\tan\delta) \quad (1.3)$$

[24]. The dielectric loss tangent is sometimes referred to as the dissipation factor which is also reciprocal of the Q-factor. The Q-factor is discussed in more depth in Section 3.1 [1].

The imaginary part of permeability and permittivity is related to conductivity, σ , by

$$\varepsilon'' = \frac{w\varepsilon' \tan \delta_E - \sigma}{w} \quad \mu'' = \frac{w\mu' \tan \delta_M - \sigma}{w}. \quad (1.4)$$

The characteristic impedance, also referred to as intrinsic or wave impedance, of a material is given by the ratio of the transverse, or orthogonal, components of the electric and magnetic fields,

$$Z_0 \hat{z} = \frac{E\hat{x}}{H\hat{y}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \quad (1.5)$$

[25]. Any impedance discontinuity will create a boundary condition which causes energy to be reflected, transmitted, and must consider refraction or conditions which induce currents. When boundary conditions are present, the product of the ratio of impedances in each medium to the impedance of free space is used and referred to as the refractive index, which in a vacuum is $\eta_0 = 120\pi \approx 377\Omega$. For MD materials, if the relative components of epsilon and mu are nearly equal, the refractive index should remain well matched to the impedance of free space. The preferred notation for refractive index is η , rather than the boundary condition notation of Equation 1.5.

It is essential to understand the relation between fundamental material parameters and observable

properties for selection of the best modeling technique. Common simplified forms of equations used to evaluate EM parameters of materials no longer hold true due to the frequent use of mixed media and MD materials. The most robust modeling techniques are full wave solutions, however each has considerable limitations. To facilitate accelerated development while exploiting novel media, the EMs engineering community needs models that promote both rapid design and performance analysis while capturing increased structural and media complexities.

1.3 Objectives and Scope

The key objective of this thesis is to explore the behavior of MD materials, evaluate related material limitations, and to contribute to advancement of microstrip models that accommodate both modern MD and traditional media. The theoretical development of two novel techniques utilizing CM algorithms to estimate effective relative permeability are presented in detail. The CM technique will be utilized to estimate the effective values for relative permittivity and relative permeability by preserving the electric and magnetic field paths.

An improved TL model accounting for non-linear effects of MD material properties to aid in rapid design or modeling MPAs will be presented as an Appendix of this thesis. This non-linear TL model accurately models microstrip TLs and MPA configurations with traditional and novel MD materials. Evidence will be presented that this novel CM model is adaptable to differing configurations and accounts for the effects of both structural dimensions and MD material properties.

The major contributions of this thesis include the following:

1. An improved conformal mapping (CM) algorithm which expands the application to microstrip structures with incorporated magneto-dielectric (MD) media. The presented methods are experimentally verified for use with magneto-dielectric (MD) substrates in two-port and microstrip patch antennas (MPA)s. These methods have increased accuracy over other CM methods for evaluation of the effective relative permittivity and effective relative permeability

of microstrip transmission lines (TL) and microstrip patch antennas (MPA). The algorithms are formatted for application to microstrip structures with flush superstrate, gapped superstrate, or without a superstrate.

2. Parameters found by these CM techniques are suited for use in conjunction with other modeling techniques, such as Bernhard and Tousignant's algorithm to find the natural resonant frequency (f_r) of a MPA or inputs for a TL model [26].
3. Improved approximation of a MD microstrip's characteristic impedance.

1.4 Thesis Organization

To systematically explore these objectives, this work is organized as follows. Chapter 2 provides a thorough review of the unique properties of magnetic materials used to create MD media. Mathematical models related to properties of magnetic materials is covered in Section 2.3.4 where the non-linear (NL) effects of MD materials becomes apparent. Section 2.4 explores methods to measure and characterize the intrinsic properties of MD materials. This chapter will be concluded by an introduction to Magtrex 555 (MX), a commercial, off-the-shelf (CoTS) planar MD material introduced by Rogers Corporation for antenna miniaturization applications.

Chapter 3 first introduces metrics used by the antenna community to quantify the performance of antennas. Section 3.1 on performance metrics leads into Section 3.1.1 to quantify metrics and miniaturization achieved with MD materials. A review of typical MPA design models is covered in Section 3.2. The traditional equations used to design and model MPAs are modified as necessary to account for the effects of adding magnetic properties.

Chapter 4 introduces CM techniques and discusses the applicability to modern MPA design in Section 4.1. A review of historical and modern microstrip TL conformal mapping techniques is given in Section 4.2 which leads into qualifying the need for a CM method to find the effective relative permeability in Section 4.4. Section 4.4.2 presents the derivation of a novel method to use CM techniques to find the effective relative permeability of a microstrip structure applicable to

mixed media and MD material microstrip configurations, TL or MPA. Section 4.5 gives a detailed procedure for CM techniques in application to TLs and MPAs then gives two methods to further evaluate a MPA's frequency behavior.

Chapter 5 thoroughly analyzes the presented CM technique by comparison to results from prior CM works, other techniques commonly used to estimate the effective relative permeability, and demonstrating the increased accuracy in predicted frequency when applied to a TL model for MPAs. Chapter 6 CM will experimentally validate the CM method to prototype measurements of microstrip structures fabricated on MX substrate including: 1) Two-port microstrip 50Ω TLs of various lengths 2) A fifth-order LPF implemented as a two-port microstrip structure 3) A probe fed 400 MHz MPA from [27]. Concluding Chapter 6, measurements of these aforementioned structures will be presented and compared to the predictions from CM methods.

Parametric simulation data will be utilized to verify the presented CM method to a larger span of microstrip widths and constitutive material parameters. Chapter 8 will draw conclusions from the research performed herein. Future work needed to continue this line of research will be outlined and pertinent disclosures will also be stated.

Chapter 2

Magneto-dielectric (MD) Materials

Chapter 1 introduced the influences that drive the need for antenna miniaturization. This Chapter will introduce material properties important to EM applications, classification of magnetic materials, and discuss macroscopic and microstructural perspectives that lead to specific magnetic behaviors. This Chapter will be concluded with challenges in developing realizable MD material characteristics and obtaining accurate measurements.

The information in this chapter will provide a foundation to understanding the physics, trade-offs, and limitations of structural and material parameters in realizing ideal MD materials for RF and microwave (MW) applications. Magnetic materials are any material with a relative permeability greater than one [28]. General properties of a magnetic material will first be introduced in Section 2.1. The five classifications that describe magnetic materials at the macroscopic level are discussed in Section 2.2 [28]. Sections 2.3 and 2.3.4 will explore microstructural mechanisms and models used to understand net observable magnetic behaviors. Measurement of material properties is covered in Section 2.4. Chapter 2 is concluded by introduction to a CoTS MD medium marketed for antenna miniaturization.

Chapter 3 will thoroughly explore the increased complexities of MD MPA design. CM techniques will be introduced in Chapter 4 and develop new methods for MD applications. Introduction to the presented CM methods will be followed by analysis, verification, and validation.

2.1 Characterization of Magnetic Material Properties

Many material properties have a similar quantity to describe the electrical and magnetic properties, such as permittivity and permeability. Magnetic materials have additional unique qualities that must be considered when these media are incorporated into RF and MW structure design. Some of these unique properties are observable at macroscopic scale or as a bulk property, but the micro-structural properties are responsible for the manifestation and will be discussed in greater depth in later sections.

Molecular Polarization

Magnetic molecules are often represented by magnetic dipoles, where the net molecular charge is slightly negative on one side and opposite side is slightly positive. These polarized molecules can be disorganized within a material resulting in zero net charge, but when aligned or polarized these molecules contribute to a net material charge. There are variations of molecular polarization used to classify magnetic materials, however that will be discussed in context of magnetic material classification in Section 2.2.

Temperature Dependence

The Curie temperature, T_c , is the minimum temperature where the particles within a ferromagnetic material become free moving and disorganized or depolarized in structure. Han describes this phenomenon as when permanent magnetism changes to induced magnetism [19]. Most applications and operational conditions do not have a concern of exceeding the Curie temperature, however high power utilization of MD material or use in high temperature environments are special applications that would be sensitive to excursions above the Curie temperature. MD materials utilize a suspending matrix giving an additional metric that can be engineered to increase a material's Curie temperature. Similarly, the Néel temperature is the temperature above which antiferromagnetic material loses polarization order becoming a paramagnetic material [19].

Frequency Dispersion

Permeability is defined as a complex-valued material property, $\mu = \mu' - j\mu''$, but in practice this value is also frequency dispersive [9]. Frequency dispersion occurs when the phase speed of a wave depends on the frequency. When an alternating magnetic field of high frequency is applied to substance, certain magnetic resonance effects appear, and they are important loss mechanisms of magnetic materials for MW applications. Figure 2.1 is a representation of complex permeability and permittivity across frequency [28]. The term permeability spectrum is used to describe this representation of complex permeability versus frequency [9]. The real permeability, μ' , is fairly constant across frequency until the permeability breakdown region where the magnitude rapidly decreases resembling a LPF's frequency response [9]. Where the complex permeability is no longer near the static permeability is referred to as the permeability breakdown region and when μ'' reaches a maximum is described as the cutoff frequency [9], [28]. The ferromagnetic resonance (FMR) occurs after the cutoff frequency when again μ' decreases rapidly and μ'' demonstrates a second resonant peak [9]. This frequency dependent characteristic is referred to as frequency dispersion [9], [28].

The thickness of a structure and its averaged static permeability are proportional to and are limiting to the logarithm of the reflection coefficient integrated over a wavelength from zero to infinity [29]. A composite material's resonant frequency depends on the uniformity of particle distribution, shape, and orientation; therefore is dependent on the composition and technology of the material fabrication processes [19]. The dispersive properties of a material can also be heavily influenced by environmental factors such as temperature and humidity [19]. Material degradation over time or from environmental conditions contribute to the mechanism of altered frequency dispersion. These altered frequency dispersion properties of a material are primarily shifted due to phase velocity and attenuation within the medium.

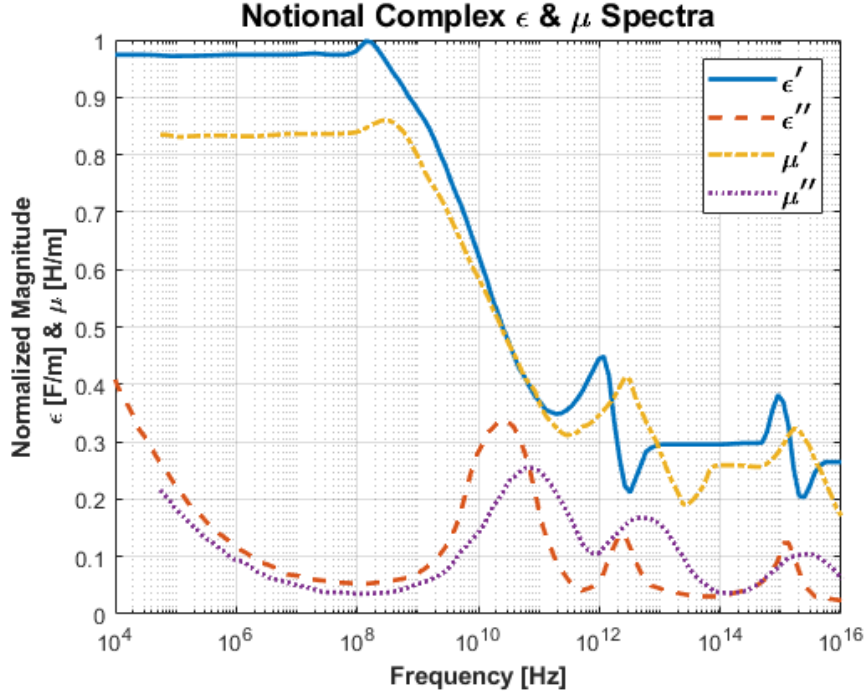


Figure 2.1: Notional example of the permittivity and permeability spectra. Permeability spectrum has a slightly broader appearance that lags the permittivity response due to hysteresis [28]

Magnetic Susceptibility

Magnetic susceptibility defines a medium's relative permeability, $\mu_r = 1 + \chi$. As the defining term of a complex and frequency dependent μ_r , the definition of χ must also be complex and frequency dependent

$$\begin{aligned}\chi'(w) &= \mu'(w) - 1 \\ \chi''(w) &= \mu''(w),\end{aligned}\tag{2.1}$$

where χ' and χ'' are the real and imaginary parts of magnetic susceptibility, respectively [30]. Magnetic susceptibility is a unitless quantity that represents the ratio of magnetism to magnetic field strength, both of which have units of Amperes per meter $[A/m]$ which cancel to give a unitless value.

Hesitivity

Hesitivity is utilized by [10] to characterize families of magnetic materials by the maximum Debye and Lorentz magnetic conductivities. To complete the analogy from the relationship of electric conductivity to ε'' to the relationship of magnetic conductivity and μ'' , Heaviside defined magnetic conductivity as

$$\sigma_m = w\mu_0\mu'' \quad (2.2)$$

[10]. Yousefi et al. formed an analytical function for hesitivity, their term for maximum magnetic conductivity for a single Debye material

$$h_{mDebye} = 2\sigma_m(w_0) = w_0\mu_0(\mu_{DC} - 1) \quad (2.3)$$

and a single Lorentzian material,

$$h_{mLorentz} = \sigma_m(w_0) = \frac{w_0\mu_0(\mu_{DC} - 1)}{\alpha}, \quad (2.4)$$

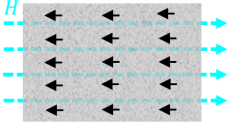
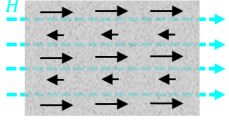
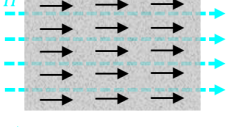
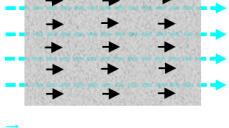
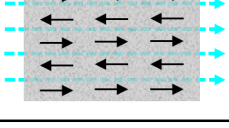
where α is a Lorentzian damping factor [10]. It is of interest that the hesitivity formulas both present with the Snoek relation, which confirms that hesitivity is another method of characterizing families of magnetic materials [10].

2.2 Classification of Magnetic Materials

Magnetic materials are classified by their observable properties. The properties of each class is outlined in the bulleted list below and summarized in Table 2.1.

- **Diamagnetic materials** oppose an applied magnetic field with a small, negative, and linear response that is essentially temperature independent. Most non-ferrous materials are classified as diamagnetic with susceptibility on the order of $-1 \times 10^{-9} < \chi < -1 \times 10^{-4}$ [28], [31].

Table 2.1: Classification of magnetic materials and relation to the magnetic susceptibility [31].

Classification	Magnetic Susceptibility	Net Response
Diamagnetic	Negative and small	
Ferrimagnetic	Positive and large	
Ferromagnetic	Positive and large	
Paramagnetic	Positive and small	
Antiferromagnetic	Positive and small	

- **Ferrimagnetic materials** contain two sub-lattices, which respond to an applied field in antiparallel directions with unequal magnitudes resulting in a NL net positive response. Ferrimagnetic materials such as spinel ferrites like magnetite (Fe_3O_4) are generally described by their permeability, which spans a large range [28], [31].
- **Ferromagnetic materials** behave similarly to ferrimagnetics, but additionally have already magnetized local domains within the lattice. This NL response is referred to as hysteresis where under an applied field the local domains will align until saturation is achieved. Ferromagnetic materials are temperature dependent, becoming paramagnetic above the Currie temperature (T_c) [28], [31]. These materials tend to be polycrystalline alloys with relative permeability that often exceed 100 [28]. Examples of ferromagnetic materials include elemental α -Fe, Ni, Co, and their

alloys [28].

- **Paramagnetic materials** respond to an applied field with a small, positive, and linear response resulting from the random alignment of magnetic moments at the atomic scale often resulting from thermal agitation. These materials may be ferromagnetic, ferrimagnetic, or antiferromagnetic below the Curie or Néel temperatures (T_C or T_N) becoming paramagnetic once exceeded [28], [31]. The susceptibility of paramagnetic materials have typical values between unity and ten, but can be much greater [28], [31].
- **Antiferromagnetic materials** contain two sub-lattices that respond to an applied magnetic field in antiparallel directions with magnitudes that largely cancel [28], [31]. Canted antiferromagnetic materials have the same behavior with the exception that they are tilted at equal and opposing angles rather than parallel. These materials are temperature dependent becoming paramagnetic above the Néel temperature, T_N [28]. Most antiferromagnetic materials are rock salts such as NiO, FeO, and CoO having a small net positive susceptibility in the range of $1 \times 10^{-5} < \chi < 1 \times 10^{-4}$ [28], [31].

2.2.1 Ferromagnetic Materials

There are two main categories of ferromagnetic materials: magnetic ferrite ceramics and ferrite polymer composites [9]. Magnetic ferrite ceramics are often ferrimagnetic ionic compounds containing iron oxides and are generally easily magnetizable [9]. Advantages of these ferrite ceramics include moderate to high magnetization, high electrical and thermal resistivity, high Currie temperature, good mechanical rigidity and strength, excellent corrosion resistance, and are low cost to manufacture [9].

Ferrites have crystalline structures which are described as spinel, garnet, or hexagonal [9]. Ferrite polymer composites or MD materials are artificially created with an electrically insulating matrix containing homogeneously dispersed ferrite particles [9]. Advantages include the ability to fabricate complex morphologies with reduced material weight, size, and eddy current losses as well

as high production rates at low cost with good uniformity and reproducibility [9]. These materials also offer an additional degree of freedom in tailoring the magnetic permeability and frequency dispersion characteristics by selecting the size, shape, distribution, density, and orientation of the ferrite particles [9]. Examples of ferrites used in creating MD materials include nickel, cobalt, manganese, chromium, and iron [32].

The low cost, reproducibility, and uniform structure of ferrites as inclusions to engineer MD materials make them a desirable material for many use cases. Koledintseva et al. suggests a thin layer of ferrites to attenuate common-mode currents on TLs like edge-coupled microstrip lines, power cords, and coaxial cables [33]. MD materials can conversely increase the radiation efficiency of an antenna because the ferrite material effectively excludes stored energy, or the imaginary component of energy, within the antenna volume [11].

2.3 Magnetic Microstructure

Magnetic materials are classified by the net magnetic responses to an applied field which was discussed in Section 2.2 and summarized in Table 2.1. For most of the magnetic materials, the energy dissipation is associated with FMR resonance [9], [30]. The FMR is often interpreted as a boundary above which the magnetic losses become too large for a device to operate [34]. This concept impresses how significant the FMR is to the operation of RF structures and circuits.

2.3.1 Ferromagnetic Resonance (FMR)

Contributing to FMR, the domain wall and gyromagnetic spin resonances each have differing underlying mechanisms related to the microstructure of the magnetic material. Domain wall resonance relates to low frequency resonance, while gyromagnetic spin rotation relates to high frequency resonance, therefore, it is common to treat these as a superposition problem and define permeability as

$$\mu = 1 + \chi_{spin} + \chi_{dw}, \quad (2.5)$$

where χ represents the magnetic susceptibility of spin and domain wall motion, respectively [30], [35].

Domain Wall Resonance

The parallel components of the DC and AC magnetic fields interact constructively and destructively as the amplitude of the AC signal cycles between the same or opposite direction to the DC field [9].

The Lorentz equation for domain wall motion is described by the magnetic susceptibility as

$$\chi_{dw} = \frac{\chi_{dw,0}}{1 - \frac{f}{f_{dw,r}}^2 + j \frac{f}{f_{dw,D}}}, \quad (2.6)$$

where $\chi_{dw,0}$ is the static susceptibility of the domain wall motion, $f_{dw,r}$ is the resonant frequency given by

$$f_{dw,r} = \frac{1}{2\pi} \sqrt{\frac{\alpha}{m_w}}, \quad (2.7)$$

$f_{dw,D}$ is the relaxation frequency given by

$$f_{dw,D} = \frac{\alpha}{2\pi\beta}, \quad (2.8)$$

m_w is the effective mass of the domain wall, β is the viscous damping coefficient, and α is the restoring constant [9]. The domain wall unit area is the basis of defining m_w , β , and α . In most cases the relaxation frequency is much greater than the resonant frequency ($f_{dw,D} \gg f_{dw,r}$) so the domain wall magnetic susceptibility becomes a Debye-type function

$$\chi_{dw} = \frac{\chi_{dw,0}}{1 + j(\frac{f}{f_{dw,D}})} \quad (2.9)$$

[9].

Gyromagnetic Spin Resonance

The dipole moment of unpaired electron spins is the main contributor to the magnetic properties of ferromagnetic materials [9]. The Landau-Lifshitz-Gilbert (LLG) equation describes the change of the dipole moment (torque) in the presence of a DC magnetic field with the orthogonal component of an AC magnetic field as

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H} + \frac{\alpha}{|\vec{M}|} \vec{M} \times \frac{\partial \vec{M}}{\partial t}, \quad (2.10)$$

where $\gamma = 1.76 \times 10^7 [s^{-1} Oe^{-1}]$ is the gyromagnetic ratio in centimeter-gram-second units and α is the damping coefficient [9]. Li [9] gives the natural resonant frequency and static susceptibility of a polycrystalline material with uniaxial anisotropy as

$$\omega_{spin}^{res} = \gamma(H_A) \quad (2.11)$$

and

$$\chi_{spin,0} = \frac{8\pi M_s}{3H_A} \quad (2.12)$$

Tsutaoka et. al. [35] found that there is also a demagnetizing field with a magnitude much greater than the anisotropic field ($H_d \gg H_A$)

$$\omega_{spin}^{res} = \gamma(H_A + H_d) \approx \gamma H_d, \quad (2.13)$$

where γ is gyromagnetic ratio, H_a , is the magnetic anisotropy field (small for Ni-Zn ferrite), and H_d is the demagnetizing field [35]. This demagnetizing field is caused by the pore size and spatial distribution of pores [9]. The saturation magnetization, M_s is proportional to the resonant spin frequency of the ferrite composites [35].

$$\chi_{spin} = \frac{4\pi M_s}{H_A + H_d} \approx \frac{4\pi M_s}{H_d} \quad (2.14)$$

[35]

$$\chi_{spin} \omega_{spin}^{res} \propto M_s \quad (2.15)$$

which represents Snoek's law. [35]. This shows that Snoek's relation applies to the complex permeability spectra with both temperature variation and ferrite content variation [35].

The Kittel equations for gyromagnetic spin resonance is an adjacent method that has also been utilized by Xu et al. to extract the complex permeability parameters from thin-films [36].

2.3.2 Particle Size

[37] found that microstructural magnetic models describe alterations from expected EM observations for composite MD materials in two cases: 1) When the incident field exceeds the critical strength of the ordered internal anisotropic magnetic fields. 2) When the particle absorption spectra overlaps with the incident waveforms frequency content, also noting the thresholds vary with shape, size, and orientation of the magnetic particles [37]. Han also noted that the nanoscale particle size can decrease losses and eliminate the FMR [19]. This assumption can be utilized up to frequencies where the wavelength becomes sensitive to the anisotropy or the particles appear electrically large [19].

The relative permeability of MD materials is a function of particle size and layer thickness which is found by

$$\mu_{ref} \left(\frac{\delta}{D} \right) = \frac{\mu_B \left(1 + \frac{\delta}{D} \right)}{1 + \mu_B \frac{\delta}{D}}, \quad (2.16)$$

where D is the average particle size and δ is the average thickness of the non-magnetic, or dielectric, resin layer [35], [38]. [35] refers to $\frac{\delta}{D}$ as the gap parameter. Van overcame variance in grain shape by taking the mean linear intercept as a measure of the grain size, D , because it gives an unequivocal definition of D , independent of the grain shape [38].

Particle distribution in a matrix is ideally uniform, but realistically there are some effects due to particle clustering. [9] extended the Bruggeman formula to consider both particle-size and cluster-size distributions which, corrects the overestimation of the traditional Bruggeman formula.

For commercially available MD materials, the grain size, shape, and clustering effects may be considered proprietary and not readily available.

Common practice is to use superposition of the spin and domain wall magnetic susceptibilities to characterize magnetic materials [9]. Domain wall motion causes low frequency resonances while the gyromagnetic spin rotation induces the high frequency resonances [9], [30]. Any unknown parameters can then be extracted by curve-fitting to Equation 2.5.

Conversely, some materials exhibit fast relaxation properties from spin-orbit coupling contributing to reduction in losses without actual reduction in the $\tan \delta$ as exhibited in $CoFe_2O_4$ [39].

2.3.3 Magnetization

Magnetization of a metal is a function of two mechanisms, molecular polarization and hysteresis. These phenomena are utilized at direct current (DC) to create a permanent magnet. When the conditions of applied fields relate to EM waves, the polarized particles experience constantly changing applied magnetic fields which relate magnetic susceptibility to the applied reactive power. This manifests as decreased μ' with increasing spin susceptibility with increasing temperatures in sintered ferrites, while Ni-Zn ferrite composites maintain a nearly constant spin susceptibility with increased temperature up to the Currie temperature or cutoff frequency for permeability [35]. The underlying mechanisms relating to polarization and hysteresis are discussed in greater detail in the following subsections.

Polarization

Section 2.1 introduced the concept of magnetic materials having a molecular polarization. Particles cannot polarize or magnetize instantaneously when an electric or magnetic field is applied, therefore is a function of time [19]. Polarization is a function of convolution between the past applied field with the time-dependent susceptibility both quantities either electric or magnetic. A Fourier transform of the time-dependent polarization results in a frequency domain representation that is

more easily evaluated

$$P(\omega) = \varepsilon_0 \cdot \chi_m(\omega) \cdot H(\omega), \quad (2.17)$$

where P is the particle polarization and χ_m is the magnetic susceptibility [19]. The magnetization depends on polarization of the material which is only related to the past magnetic field, therefore has a causal relationship which must satisfy Kramers-Kronig constraints which describe the relationship of the real part of susceptibility to the imaginary part and vice versa [19].

Hysteresis

DC-hysteresis and low frequency resonances are attributed to the domain wall movement while alternating current (AC)-hysteresis and high frequency resonances are due to gyromagnetic spin resonances [9], [35], [38]. However, it is well established that coercive force, H_c , and hence the dissipation, scales as $H_c \propto \frac{1}{D}$ [38]. This latter relation has been the basis for the traditional practice, to reduce H_c , and suppress the effect of DC-hysteresis by employing ferrites composed of large grains [38]. In clear contrast to the AC-hysteresis data presented here; thus suggesting that a clear distinction should be made between DC- and AC-hysteresis [38]. Boundary conditions give us the relation for the demagnetization field in the $\hat{z}$ direction is

$$M_z \hat{z} = -H_z \hat{z} \quad (2.18)$$

[40].

The area enclosed in the hysteresis loop can be approximated by $\approx 4\mu_0 M_S \times H_C$ which represents the energy loss in a complete hysteresis cycle of magnetization and demagnetization [38]. Energy loss relates to power loss and therefore brings the question of conservation of energy. Hysteresis is not only dependent on current state, it depends on past inputs in form of magnetic flux and magnetic fields. The relation of polarization to hysteresis shows that the time-domain form of complex magnetic susceptibility represents the phase lag between B and H due to losses. Frequency domain representation of the hysteresis curve is required to meet the Kramers-Kronig constraints

for causality, but has not been well explored. The susceptibility could be modeled as a transfer function, but would have to be modified by a material specific memory term.

2.3.4 Modeling Magnetic Materials

Frequency Dispersion Models

Application of the general Lorentzian type dispersion law, based on classical physics of a damped harmonic oscillator, to the complex permeability

$$\mu(w) = 1 + \frac{(\mu_s - 1) w_{res}^2}{w_{res}^2 - w^2 + j\beta w w_{res}}, \quad (2.19)$$

where μ_s is the static or DC permeability, w_{res} is the angular ferromagnetic resonant frequency, and β is an empirical damping factor [10], [15]. This general form is the formula from which both Snoek's and Archer's Laws are derived [15]. This is a less common general form of the more often utilized modified Lorentzian form [15].

Magnetic Conductivity

Yousefi et al. had also identified an important function of permeability which was given in the frequency dispersive form as

$$f(\mu) = \frac{\mu''(\omega)}{|\mu_r(\omega) - 1|^2} = \frac{\mu''(\omega)}{(\mu'(\omega) - 1)^2 + \mu''(\omega)^2}, \quad (2.20)$$

which can be transformed to the form of a circle with the center at $\left(0, \frac{1}{2f(\mu)}\right)$ with a radius of $\frac{1}{2f(\mu)}$ [10]. When combined with either the Debye or Lorentzian form of hesitivity results in a single definition as

$$f(\mu) = \frac{\omega \mu_0}{h_m} \quad (2.21)$$

[10]. The author concluded that an electrically small antenna's radiation efficiency is primarily influenced by a single material parameter, the peak value of the Heavyside magnetic conductivity [10]. This finding defines a family of magnetic materials and is also proportional to the more well-known Snoek's product, $\omega_0\mu_0(\mu_{DC} - 1)$, used to guide MD material selection for antenna design by selecting the material with the highest μ'' in the desired frequency band [10].

This function of permeability relates to the instantaneous magnetic loss tangent previously introduced in Section 1.2 [10]. Yousefi et. al give a more accurate representation as

$$\tan \delta_M = \frac{\mu''}{|\mu'|^2 + |\mu''|^2} \quad (2.22)$$

where they demonstrated high efficiencies at both very high values of μ'' and with a very low magnetic loss tangent [10]. This formulation allow either a low loss magnetic material where $\mu'' \ll \mu'$ or a perfect magnetic conductor (PMC) with $\mu' \mu''$ can be utilized to maximize radiation efficiency [10].

Snoek's Law

Snoek's Law is most commonly used for bulk magnetic materials and demonstrates the inverse relationship between permeability and resonant frequency. [15]. Snoek's Law gives

$$(\mu_s - 1)\omega_{res} = \frac{4\pi}{3}\gamma 4\pi M_s \quad (2.23)$$

where μ_s is the static permeability, $4\pi M_s$ is the saturation magnetization, and $\gamma \approx 3 \text{ GHz/kOe}$ [15]. Note that the left side of Snoek's Law also appears in the hesitivity Equations 2.3 and 2.4, which indicates that both characterize families of magnetic materials [10]. Snoek's Law relates the reciprocal relationship of static permeability and angular resonant frequency to the saturation magnetization, which is determined by the material composition.

Archer's Law

Two special cases, ferromagnetic thin-films and hexagonal ferrites, more closely follow Archer's Law,

$$(\mu_s - 1)\omega_{res}^2 = c(\gamma 4\pi M_s)^2, \quad (2.24)$$

which permits a higher static permeability compared to Equation 2.23 at the same resonant frequency [15]. Ferromagnetic thin films with in-plane magnetic isotropy have a $c \approx 4\pi^2$ [15]. Thin-film materials can be combined in layers to create a bulk laminate for use as a microstrip substrate or superstrate [15]. Bulk laminates use an alternate value, $c = 4\pi^2\rho$, in Equation 2.24, where ρ is the volume fraction of magnetic to laminate components [15].

Hexagonal ferrites exhibit

$$c = \frac{4\pi^2}{3 \left(1 + \frac{H_\theta + H_\phi}{4\pi M_s}\right)}, \quad (2.25)$$

where H_θ and H_ϕ are out-of-plane and in-plane anisotropy, respectively [15]. The term on the right of Equation 2.24 is material specific, but conventionally converted to linear frequency and referred to as Archer's constant [15].

Modeling Dispersive Magnetic Susceptibility

Under LTI assumptions, realistic materials must obey causality, linearity, passivity, and energy conservation, therefore they must meet the Kramers-Krönig relations [15]. Two classic analytical functions that meet these requirements for ferromagnetic and ferrimagnetic materials are the Lorentz function

$$\chi_{Lorentz} = \frac{\chi_{DC}}{1 + j \frac{w}{w_0} \alpha - \left(\frac{w}{w_0}\right)^2} \quad (2.26)$$

and the Debye function,

$$\chi_{Debye} = \frac{\chi_{DC}}{1 + j \frac{w}{w_0}} \quad (2.27)$$

where χ_{DC} is the DC susceptibility, w_0 is the resonant frequency (or relaxation frequency), and α is the Lorentzian damping factor [10]. Ferromagnetic materials and composites tend to follow the Debye dispersion function as will the majority of MD materials presented in this work [21].

Several works have expanded on literature defining a magnetic susceptibility in terms of domain wall motion and gyromagnetic spin by introducing relative density and average grain size to describe the material's microstructure at a fixed angular frequency such as Equation 2.16 [30].

Assuming MD material obeying the modified Lorentzian (low-frequency approximation) type dispersion,

$$\mu = \mu(\omega) = \mu_s + \frac{A\omega^2}{\omega_0^2 - \omega^2 + j\omega\Gamma} \quad (2.28)$$

where μ_s is the static permeability, A is the amplitude factor ($0 < A < 1$), ω_0 is the undamped angular frequency of the medium (zeroth pole pair), and Γ is the loss factor [15]. To model complex permeability the general Lorentzian type dispersion law from Equation 2.28 is expanded given by

$$\mu(\omega) = 1 + \frac{(\mu_s - 1)\omega_{res}^2}{\omega_{res}^2 - \omega^2 + j\beta\omega\omega_{res}}, \quad (2.29)$$

where μ_s is the static permeability, ω_{res} is the angular FMR, and β is an empirical damping factor [15].

Using a differing approach, Wallace et al. solved the LLG equation to develop a formula for intrinsic permeability given by

$$\mu(f) = \mu_0 \left[1 + \frac{(4\pi M_s)^2}{(4\pi M_s)H_s - (\frac{f}{\gamma})^2 + j\alpha(4\pi M_s)\frac{f}{\gamma}} \right], \quad (2.30)$$

which varies with the frequency of an applied AC magnetic fields [41].

2.4 Measurement Techniques

Resonant measurement methods extract EM parameters at one or more discrete frequencies [9].

Resonator or resonant perturbation methods have been the predominant method for quantifying

low-loss materials, often narrow-band, but are not well suited to for lossy MD materials [9].

Non-resonant measurement methods which obtain general properties over a frequency range [9]. The Nicolson-Ross-Weir (NRW) method developed in the 1970's is a broadband non-resonant measurement technique that measures s-parameters with a vector network analyzer (VNA) [9]. NRW calculates the transmission and reflection coefficients from the s-parameters and closed-form transmission line expressions to extract complex permittivity and permeability [9]. This method has the flexibility to be used with most any electromagnetic wave TL: coaxial cable, planar transmission lines, waveguides (hollow and dielectric), and free space, which is why it has been the standard of measuring homogenous and isotropic materials with low conductivity for over 40 years [9].

The NRW method does have potential for several sources of error in measurements. The measurements can be sensitive to sample's position in relation to the calibration plane and the sensitivity to position increases as frequency increases [9]. There have been several attempts at finding position insensitive methods, but each at the cost of increasingly complicated algorithms and computational time [9]. Li proposed a simple, explicit, non-iterative, single measurement method for precisely determining the sample position within the test fixture. This method calculates the phase angles that validate the surface locations through a minimum of two scattering parameter (s-parameter)s at nonresonant frequencies [9]. After finding the sample position, the conventional NRW method is used to extract the complex permittivity and permeability.

Magneto-Dielectric Characterization

Material characterization is difficult with MD materials due to difficulty in differentiating the frequency-dependent characteristics of ϵ' and ϵ'' from μ' and μ'' [12], [19]. Permittivity and permeability are inversely coupled, essentially introducing a relative permeability to the substrate increases the inductive effect, which reduces the capacitive effects effectively decreasing the relative permittivity [1]. Saini et al. found that $\epsilon_{r_{ef}}$ is a function of patch width normalized by substrate height, W/h , and μ_r , with a linear dependence on ϵ_r [1]. This is not reciprocal according to [1] where they find $\mu_{r_{ef}}$ is still a function of W/h , μ_r , and ϵ_r , but the dependency on μ_r appears

more similar to a root function curve than linear. Frequency dispersion and Snoek's Limit also demonstrate this frequency dependence of μ_r . Thin ferromagnetic films work well as microstrip antenna substrates because of they allow a higher static permeability with the same resonant frequency as shown by Equation 2.23 [15].

Weir provides a single loss tangent for a lossy MD material as

$$\tan \delta = \frac{\delta_r''}{\delta_r'}, \quad (2.31)$$

where $\delta_r' = \mu_r' \varepsilon_r' - \mu_r'' \varepsilon_r''$ and $\delta_r'' = \mu_r' \varepsilon_r'' + \mu_r'' \varepsilon_r'$ such that when $\mu_r'' = 0$, $\tan \delta = \varepsilon_r'' / \varepsilon_r'$, and [42]. This formula is insightful in demonstrating that the imaginary minimum of either the relative permittivity or permeability for a material could be exploited for decoupling the constitutive parameters. The MD material attenuation in linear units, Nepers per unit length, can be determine by

$$\alpha = \frac{\pi \sqrt{2\delta_r'}}{\lambda_0} \left[\sqrt{1 + \tan^2 \delta} - 1 \right]^{1/2} \quad (2.32)$$

per Weir [42].

Electrical Characterization

Material characterization methods evaluate a material or a structure's behavior across frequency in response to an applied electrical stimulus. Typically these techniques are used to collect extrinsic performance or intrinsic material parameters. There are many material characterization methods with varying accuracy. This section will provide a broad overview of these methods.

Scattering Parameters (S-parameters)

S-parameters provide important information to characterize a network when standard circuit analysis cannot be easily performed. S-parameters of a network are measured or calculated across frequency and represented in matrix form. The subscript convention for two-port networks says the s_{ij} -parameter describes the network being excited by port j and measured at port i . For a two-port

network the s-parameters appear as

$$[s] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}. \quad (2.33)$$

A reciprocal two-port network has $s_{12} = s_{21}$ which indicates that the input and output ports are interchangeable. Reciprocal networks must be passive, meaning that a passive network cannot contain any active devices, ferrites, or plasmas.

Coaxial Air-line Method

Roger's suggests taking coaxial airline measurements then the NRW iterative method from Section 2.4 to extract the complex permittivity and permeability [43]. These methods are generally valid up to 1 GHz [19].

Cavity Perturbation Method

Han presents a novel cavity perturbation method (CPM) to extract electrical and magnetic properties with a substrate integrated waveguide (SIW) cavity resonator [19]. The material is placed in the waveguide at a location of maximum electric field magnitude to extract the complex permittivity [19]. The device constructed by Han is valid up to 4 GHz, but due to the large sample size could not isolate the permittivity from permeability at higher frequencies [19].

Split Post Dielectric Resonator

Requires two morphologies designed at two different frequencies to extract parameters, using 500 MegaHertz (MHz) for permeability extraction and 1 GHz for permittivity extraction [31]. This method may be prone to introducing additional sources of error.

2.5 Commercial Magneto-Dielectric Products

MAGTREX 555 (MX) is a MD substrate that is commercially available from Rogers Corporation. The MX substrate has a relative permittivity and relative permeability of 6.5 ± 0.5 and 6.0 ± 0.2 , respectively, in the X-Y directions [44], [45]. The wavelength within a material is found by

$$\lambda = \frac{c}{f \sqrt{\mu_r \epsilon_r}}, \quad (2.34)$$

or simplified to $\lambda/6.24$ for MX [44], [45]. Using the approximately matched relative permittivity and permeability, a microstrip design should theoretically remain impedance matched to free space and benefit by size reduction to one-sixth the size of the equivalent free space structure. This is a desirable impedance characteristic for planar printed MPAs because it alleviates the problems from the image phase destruction [12]. MX's miniaturization factor is supposed to be comparable to a very high permittivity substrate of $\epsilon_r = 32$ with a relative permittivity equal to unity [46].

MX datasheet has material properties from 10 MHz to 2 GHz, where ϵ_r is fairly constant at 6.5. The real part of relative permeability, μ'_r , is near 6 up to 600 MHz then increases to 6.5 at 715 MHz before declining to 1 near 2 GHz [20]. The magnetic losses increase to 0.1 at 600 MHz and continually increase due to the relationship of real and imaginary components. The loss tangent is found by $\tan \delta_m = \mu''/\mu'$ so the magnetic loss tangent increases while the real component continues to decrease [20].

2.6 Conclusion for Magnetic Materials

As technology continues to advance, the need for smaller form factor microwave circuits is ever increasing. Common miniaturization techniques result in decreased bandwidth and efficiency. Miniaturization while retaining bandwidth and efficiency of the full-size circuit requires novel approaches and inclusion of magnetic materials at the design phase of the microwave circuit design.

This chapter explored unique characteristics, classification, and models of the underlying

mechanisms for magnetic materials. Chapter 3 will introduce typical antenna performance metrics, design equations, and measurement techniques. These metrics will be expanded to include MD magnetic material parameters, but there remains room for improved methods for MD MPAs. CM techniques will be derived for historical and novel CM methods in Chapter 4. Chapters 5, 6, and 7 will evaluate the presented CM methods with in depth analysis, experimental verification, and validation by simulation, respectively. This work will be followed by conclusions and future work in Chapter 8.

Chapter 3

Microstrip Patch Antennas (MPA)

To break the bounds of miniaturization, the fundamental physics that drive material and antenna performance must first be understood [12]. As demonstrated in Chapter 2, lossy and magnetic materials greatly increase the complexity of material modeling methods. This chapter begins with an introduction to characterize and quantify antenna performance. The fundamentals lead into a brief summary of planar MPA design considerations with emphasis on use with MD materials. Miniaturization approaches are presented and discussed followed by a review of novel miniaturization techniques. This chapter will be concluded with challenges in developing realizable MD material characteristics. The information in this Chapter will provide a foundation to understanding the physics, trade-offs, and limitations of structural and material parameters throughout the increased complexity of design introduced by incorporating MD media. This is followed by the development of more robust modeling techniques developed in Chapter 4.

3.1 Antenna Performance

Antenna performance parameters are generally measures of the antenna's ability to receive few or many frequencies, efficiency of power transfer, beam directionality, and beam width. As the structure of an antenna becomes increasingly complex, each of these parameters requires more careful consideration to prevent adverse affects to the overall performance. A fundamental understanding of how an antenna functions is imperative to making the right design choices.

Bandwidth

Fractional bandwidth, the percentage of the center frequency, is most often utilized to describe bandwidth of narrow-band antennas [18]. Alternatively, broadband or ultrawideband (UWB) antennas more often use a ratio of the upper to the lower frequency to describe their bandwidth [18]. The bandwidth of an electrically small antenna is limited by radiation resistance, proportionality, and increased energy storage as reactance to reduced antenna volume per wavelength [8], [15]. Hansen et al. used a zero order analysis to derived the bandwidth of a MD material antenna to be,

$$BW = \frac{96 \frac{h}{\lambda_0} \sqrt{\frac{\mu_r}{\varepsilon_r}}}{\sqrt{2}(4 + 17\sqrt{\varepsilon_r \mu_r})}, \quad (3.1)$$

where h is the height, ε_r is the relative permittivity, and μ_r is the relative permeability all in relation to the MD material substrate [8]. Evaluating Equation 3.1 shows that increasing the relative permittivity and relative permeability together has a greater impact on bandwidth than increasing the relative permittivity alone [8]. It is also noted that there is a limit at which the mu-epsilon product in the denominator will become large enough to negate the bandwidth improvement gained by increasing both the relative permittivity and relative permeability [27].

A large mu can improve bandwidth even if epsilon is near one [8]. Similarly, a substrate with relative permeability and unitary relative permittivity reduces the resonant length by $\sqrt{\mu_r}$, but the smaller version maintains the same bandwidth as a permeability-only substrate [8]. Mishra et al. found the width of the patch most greatly effected the bandwidth, but length also effected bandwidth without significantly impacting the operational frequency [47].

$$BW = \frac{1}{\sqrt{2}} \left(240 \frac{h}{W} G_r^{patch} \sqrt{\frac{\mu_r}{\varepsilon_r}} + \tan \delta_E + \tan \delta_M \right), \quad (3.2)$$

where the radiation conductance is given by

$$G_r^{patch} = \frac{4}{377\pi^2} \iint_0^\pi \left[\frac{\sin \frac{k_0 \cos \theta}{2}}{\cos \theta} \cos \left(\frac{k_0 \sin \theta \sin \phi}{2k_g} \right) \right]^2 \times (\sin \theta)^3 d\theta d\phi, \quad (3.3)$$

$k_g = k_0 \sqrt{\mu_r \varepsilon_r}$, k_0 is the free space propagation constant, and θ and ϕ are the elevation and azimuth angles of the antenna radiation pattern [3], [5].

Efficiency

Radiation efficiency of an antenna is also an important parameter. The efficiency of an antenna is the product of the radiation efficiency and the match efficiency or described as a power ratio,

$$\eta_a = \eta_{rad} \eta_m = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_{loss}} \left[1 - |\Gamma|^2 \right], \quad (3.4)$$

where P_{rad} is the radiated energy, P_{in} is the input power, R_{rad} is the radiation resistance, R_{loss} is the resistive losses in the network, and Γ is the reflection coefficient [48], [49]. Niamien et al. derived the radiation efficiency of a MD MPA from this power ratio to be

$$\eta_{rad}^{patch} = \frac{1}{1 + \frac{\tan \delta_E + \tan \delta_M}{240 G_r^{patch}} \frac{W}{h} \sqrt{\frac{\varepsilon_r}{\mu_r}}}, \quad (3.5)$$

where G_r^{patch} is given by Equation 3.3 [3].

The key to characterizing antenna efficiency is to separately characterize the radiation resistance and loss resistance of the antenna, such as with the Wheeler cap method [9]. The radiation efficiency can be difficult to interpret due to the radiation and loss conductance being in parallel [10], [50]. An interesting contribution from Yousefi et al. found the efficiency of a permeable MD dipole antenna to be

$$\eta_a = \frac{1}{1 + \frac{\mu''(w)}{(\mu'(w)-1)^2 + (\mu''(w))^2} \frac{6}{\left(\frac{\rho}{l}\right)^2 (kl)^3} + \frac{3}{4} \frac{\varepsilon''(w)}{(kl)}} \quad (3.6)$$

which considers magnetic conductor losses from ε'' and maintains the μ'' term for evaluating high $\tan \delta_m$ MD materials [10], [50]. The work of [10] used Equation 3.6 to highlight the missing terms from loss tangent calculations. This demonstrated that very low or very high magnetic loss tangent materials can yield high radiation efficiency, contrary to most literature [10].

MD material increase in efficiency is a direct result of decreasing conductance losses by a factor of $\sqrt{\frac{\epsilon_r}{\mu_r}}$ [3], [8]. Radiation conductance, the inverse of resistance, for an epsilon-mu product 1 to 10 is,

$$G_{rad} \cong \frac{1}{40\sqrt{\epsilon\mu} + 170\epsilon\mu} \quad (3.7)$$

[8]. Hansen found when the patch width divided by wavelength is small and the epsilon-mu product is large, $\epsilon\mu > 3$, the radiation conductance is well approximated by,

$$G_{rad} \cong \frac{1}{360\epsilon\mu} \quad (3.8)$$

[8]. Saini et al. finds that the ohmic conductivity must decrease inversely to μ_r for the radiation conductance to remain constant, but the result yields increased efficiency [1]. Namien et al. found an expression for the total loss of a patch antenna on a MD substrate

$$P_{MD} = P_D + P_M = \frac{1}{2} |E_0|^2 \frac{Wd(\tan \delta_e + \tan \delta_m)}{240} \sqrt{\frac{\epsilon_r}{\mu_r}}, \quad (3.9)$$

where E_0 is the electric field amplitude at the radiating edge, W is the patch width, and d is the substrate height [3].

Realized Gain

Directivity is related to the radiating lobes as a measure of how much of an antenna's power is directed in a given direction normalized to an isotropic radiator. Measure of the main radiating lobe at half power is called the 3 dB beam width. When related to signal processing, the peak power is more useful as a difference between the main and first side lobe or related to the angle of any grating lobes. The realized gain of the main lobe is found by,

$$G_{RG} = \eta_{rad}\eta_a D, \quad (3.10)$$

where η_{rad} is the radiation efficiency, η_a is the efficiency of the antenna as a load, and D is the directivity.

Quality Factor (Q-factor)

The most important parameters to determine an antenna's characteristics are the Q-factor and transmission efficiency [16]. The Q-factor is a dimensionless ratio quantifying the energy stored versus the power lost defined as

$$Q = \frac{2\omega W_{stored}}{P_{rad}}, \quad (3.11)$$

where ω is the center or operating frequency in radians per second, W_{stored} is the stored electric or magnetic energy, P_{rad} is the power radiated by the antenna, and the factor of two accounts for the resistive matching network which theoretically would double the stored energy [16], [51], [52]. Equation 3.11 easily shows that a large reactive field in the near zone would result in a large Q-factor. This would also imply high currents and high resistive loss which would cause a narrow bandwidth [51]. The Q-factor also has a reciprocal relationship with the bandwidth and loss tangent [1]. Saini et al. found that for a voltage standing wave ratio (VSWR) = 2, the bandwidth is $\frac{1}{\sqrt{2Q}}$ [1].

The radiation Q-factor can be easily found as a function of loss tangent,

$$Q_r = Q_0 + \tan \delta_E + \tan \delta_M, \quad (3.12)$$

where Q_0 is the unloaded Q-factor [15]. Unfortunately, the reactive field energy within the sphere is hard to calculate to find an accurate Q-factor. Collin used a novel approach of calculating the far-field energy flow velocity and subtracting that from the total energy density in the field to find the stored reactive energy in the near field [51]. Collin states that Equation 3.11 represents the maximum achievable Q-factor value because it also encompasses the inherent physical limitation of antenna size due to the energy stored within a sphere of radius, $r = \lambda/2\pi$ [51], [52]. This physical limitation to antenna size creates an inversely proportional relationship between the Q-factor and bandwidth [16].

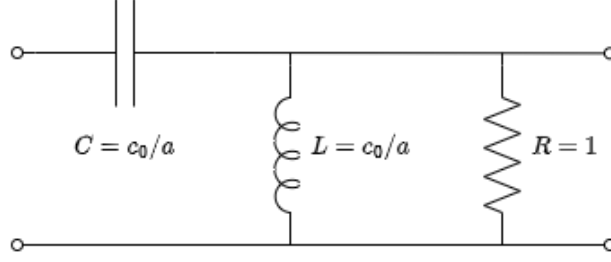


Figure 3.1: Equivalent circuit model used to derive the Q-factor of an electrically small antenna [16], [52] .

McLean derived the Q-factor of an electrically small antenna [52]. Electrically small antennas are commonly described as any antenna that would be enclosed in a sphere of the radius $a = 1/k$, where $k = \frac{2\pi}{\lambda}$ is the wave number of the resultant EM fields [52]. To do so [52] indirectly found the NL quantity for the non-propagating stored energy by taking the difference between the total electric-energy density and the propagating electric-energy density $W'_e = \frac{4\pi\eta}{3w} \left[\frac{1}{k^3 a^3} + \frac{1}{ka} \right]$ [52]. For a spherical TM_{01} spherical mode [52] finds

$$Q = \frac{2wW'_e}{P_{rad}} = \frac{1}{k^3 a^3} + \frac{1}{ka}, \quad (3.13)$$

where $P_{rad} = \frac{8\pi}{3}\eta$ is the radiated power and the factor of two comes from the concept that the electric and magnetic energy densities must be equal so the total energy density can be found by computing a single energy density value therefore $W'_{total} = W'_e + W'_m = 2W'_e = 2W'_m$ [52].

McLean also gives the equivalent circuit model shown in Figure 3.1 first used by Chu to show that Equation 3.13 can also be found using the circuit model. He continues to demonstrate that the exact solution agrees with Chu and Wheeler for values of $ka < 0.8$, but increasingly differs above this range [52]. The impact of this finding is that the lowest achievable radiation Q-factor is greater than previously thought, which conversely decreases the maximum achievable bandwidth [52].

Best investigated this concept further by assessing multiple antenna geometries with similar effective volumes and self-resonate frequencies demonstrating that the Q-factor for a self-resonating antenna is most dominantly affected by the antenna's height and effective volume in relation to the resonating wavelength [53]. Best Concluded that the Q-factor was most limited by the height

and effective volume of the antenna, but it must be noted that these parameters will also have the opposite effect on the bandwidth [53]. This study still falls short of the effects of an active antenna would have on the Q-factor.

Gustafsson et al. utilize a different approach by designing a current optimization problem with both the Q-factor and antenna efficiency, η_a [54]. This approach requires method of moments and linear algebra methods to solve, but provide optimization of multiple parameters to quantify an antenna [54]. The authors of [54] found that the optimization still required prioritization of either Q-factor or efficiency, but that a relatively small sacrifice in Q-factor could result in a large increase of efficiency. Further work using this methodology is warranted and could potentially be extrapolated to include active antenna design [54].

Figure of Merit

Interpretation of antenna efficiency is apparent, but is limited to describing inherent loss without relating efficiency to other important parameters. FoM is utilized to describe antenna behavior by the relationship between efficiency, fractional bandwidth, and antenna volume which is defined as

$$\text{FoM} = \frac{\eta_a B_{frac}}{v_a}, \quad (3.14)$$

where η_a is the antenna efficiency and v_a is the antenna's physical resonator volume [55].

Wallace et al. describes using a graphical representation for FoM in the form of the bounded area on a LOG-LOG scale with 1% to 100% of absolute value of the reflection coefficient on the vertical axis and frequencies as bounding quantities of the reflection spectrum as shown in Figure 3.2 [41]. This form of FoM was preferred over utilizing bandwidth as to not exclude the possibility of bimodal or multi-modal curves during optimization [41].

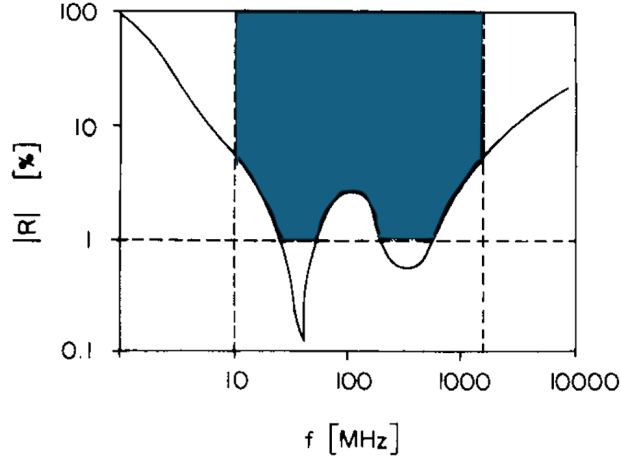


Figure 3.2: Graphical representation for the Figure of Merit (FoM) of an antenna as recommended by [41].

Non-Linearity

LTI assumptions apply strict bounds a system to simplify Maxwell's equations to buy frequency independence and reciprocity [13]. LTI limitations guarantee the system performance remains in a flat region of performance, which makes the system stable, predictable, and simplifies modeling. These assumptions include that a bounded input will always result in a bounded output, all poles of the system lie in the left-hand side of the complex plane, and the natural response approaches zero as time approaches infinity. Traditional methods have restricted antenna design to homogeneous passive materials that remain within LTI regions of operation. NL and/or time-variant methods must be explored in order to reach the absolute bounds of antenna size, Q-factor, and efficiency.

Active antennas achieve frequency, polarization, or pattern agility by utilizing semiconductor devices to tune or reconfigure the antenna [56]. The non-LTI behavior of these active antennas is due to the nonlinear current versus voltage (I-V) characteristic of the semiconductors, such as a varactor or Positive-Intrinsic-Negative (PIN) diodes [56].

NL distortion is not isolated to active antennas. Hysteresis from ferrites in MD materials and junctions of dissimilar metals, to include metal–oxide–metal junctions from oxidation, can also cause passive inter-modulation distortion (PIM) [56]. Steel and aluminum are prone to oxidation and therefore are avoided for sensitive RF designs [56]. PIM is a mixing between time-varying

elements with passive NL elements causing distortion and harmonics to be generated within the passband as transmit power levels increase [56]. PIM has presented long-standing issues in wireless communications [57].

Blosser demonstrated this increase of undesirable power losses for a MD material MPA when used as a high-power transmitter when compared to a patch designed at same operational frequency on Rogers MX and R03006 substrates, respectively [57]. The same MX MPA was found to behave linearly as a receiver due to the significantly decreased power of a received signal [27].

Qin and Wang also noted nonreciprocal behavior attributed to a time-varying capacitance adding a forward-traveling modulating wave that modulates the input frequency producing harmonic and third-order intermodulation (IM3) output terms that mix to demodulate the tones upon receive [58]

3.1.1 Quantifying MD MPAs

MD materials are composites of a dielectric material with ceramics, ferrimagnetic materials, or ferrimagnetic compounds [5]. MD materials used for antenna miniaturization are defined as low loss materials having both relative permittivity, ϵ_r , and relative permeability, μ_r , greater than unity and exhibit dielectric and magnetic properties that are not found to occur in natural materials [1], [8]. Careful selection of the values for relative permittivity and relative permeability will miniaturize the footprint of an antenna [1].

Inclusion of magnetic materials suspended in a dielectric matrix are often referred to as MD materials in literature. The MD materials used for antenna miniaturization are further defined as low loss materials having both relative permittivity, ϵ_r , and relative permeability, μ_r , greater than unity and exhibit dielectric and magnetic properties that do not occur in natural materials [1], [8]. Incorporation of MD materials in a microstrip configuration requires careful selection of the values for relative permittivity and relative permeability to miniaturize the structure's footprint, as well as understanding of how these properties are coupled [1].

MD material can be designed to fit a specific application by offering two degrees of freedom in varying material properties [1]. Additionally, the structural and electromagnetic properties

can be further customized to an application through deliberate selection of the raw materials and synthesis processes [1]. Compounding magnetic material particles, or nano-ferrite materials, with dielectrics through material synthesis has been a promising solution to realizing high-bandwidth planar antenna miniaturization [1], [12]. MD materials demonstrate increased dielectric strength at high frequencies resulting in decreased or even negligible eddy current losses [1]. Selection of the ferrite inclusion may be based on the desired properties: soft ferrites are good conductors of magnetic fields while hard ferrites can make permanent magnets [1]. These compounded MD materials have an inherent anisotropy and nonreciprocal properties which permit variable frequency tuning and antenna polarization diversity [14]. External biasing of these MD material substrates can force them into a cutoff state to beam steer, change radiation pattern shape, and control the antenna's effective electrical size [14].

By matching μ_r and ε_r , the MD material intrinsic characteristic impedance, $Z_c = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$, is matched to the intrinsic impedance of free-space, while the refractive index, $\eta = \sqrt{\varepsilon_r \mu_r}$, can be increased [18]. Selection of moderate values of relative permittivity and permeability ensure that the mu-epsilon product in the denominator of Equation 3.1 is not so large that it negates the overall bandwidth improvement seen in the numerator [1], [8], [20], [27]. MD materials are inherently anisotropic so the miniaturization factor given in Equation 3.17, $\eta = \sqrt{\varepsilon_r \mu_r}$, should be represented as a cross-product so bandwidth becomes,

$$BW = \frac{96 \frac{h}{\lambda_0} \sqrt{\frac{\mu_r}{\varepsilon_r}}}{\sqrt{2(4 + 17\sqrt{\varepsilon_r \times \mu_r})}}. \quad (3.15)$$

MD materials can result in antenna miniaturization with improved radiation efficiency, impedance matching, and bandwidth when the material characteristics are thoroughly evaluated and compensated for during the design process [18].

The most significant limitation of using MD materials is creating these material characteristics without introducing significant loss [18]. Efficiency of MD material antennas improve with either an increase in radiation conductance or a decrease in ohmic conductivity [1]. To keep the radiation

conductance constant, the ohmic conductivity must decrease as μ_r increases resulting in higher efficiency [1]. In ferrite materials, both dielectric and magnetic losses co-exist so overall loss is reduced by a factor of $\sqrt{\frac{\epsilon_r}{\mu_r}}$ [1].

There has been little study of the hysteresis effects of incorporated ferrites, the non-LTI behavior of these materials, or how to model these behaviors for MD materials [27]. Blosser expected that high power applications may invoke loss through hysteresis and PIM effects, which was supported by their findings [27]. If PIM is present on an antenna, it is radiated with the transmitted signal or interferes with the received signal [59].

Miniaturization Factor

The miniaturization factor or area reduction of a microstrip antenna is defined as

$$\eta = \sqrt{\epsilon_r \times \mu_r}, \quad (3.16)$$

when anisotropic relative permittivity and relative permeability are represented as tensors or in an isotropic medium Equation 3.16 simplifies to

$$\eta = \sqrt{\epsilon_r \mu_r}. \quad (3.17)$$

Equations 3.16 and 3.17 show that relative permittivity and/or relative permeability must increase largely to have a significant influence on the miniaturization factor [1]. Saini et al. achieved a miniaturization factor of 3.55 with a MD material substrate (only 1.95 with dielectric only substrate) which equated to a 44% reduction in patch area without degradation in the bandwidth (increased -10 dB RL bandwidth) [1]. Alternatively, [20] utilizes a ratio of the free space fundamental frequency and the miniaturized operational frequency, $m = f_m / f_0$, and physical size reduction to quantify miniaturization [20].

Patch antennas on a ground plane backed MD material substrate utilizes the permeability of a high-magnetic-field zone to achieve miniaturization [20]. If the $\mu_r \gg \epsilon_r$ and $\mu_r \gg 1$, a

half-wavelength MPA maintains its impedance bandwidth [15].

3.2 Design Considerations

Planar microstrip antennas are highly preferred in mobile systems due to ease of fabrication, compact size, low profile form factor, and being conformal [1], [5], [12], [60]. These attributes provide ease of fabrication and integration minimizing overall production costs for applications such as Global Positioning System (GPS), RFID, and wireless networking (Wi-Fi) [1], [5], [12], [60]. The benefits of planar MPAs include conformal shape, reconcilability, decreased noise, and an increased front-to-back ratio because the ground plane shields underlying electronics from the antenna's fields [5], [12], [60]. Conversely, a planar antenna's performance is limited due to the patch's surface currents opposing the image theory currents resulting in canceled fields [12]. This destructively coupled image from antenna currents increases stored energy and as a consequence decreases efficiency and narrows the bandwidth [12]. The goal with microstrip patch antennas is to achieve the smaller form factor without sacrificing performance characteristics such as bandwidth and efficiency. This section will cover the material and dimensional considerations evaluated for optimal MPA design.

Microstrip patch antenna MPA design is not a new concept, but traditional formulas do not account for relative permeability greater than unity, therefore are not fit to design MPAs utilizing novel materials. This section will summarize formulas found in literature either derived or found by empirical methods for patch antenna designs and when necessary modified to account for relative permeability.

Design and Fabrication Initial Considerations

A microstrip line is structurally a conductive strip with a width, W , and thickness, t , suspended above or supported by a substrate at a height, h , above a conductive ground plane as shown in Figure 3.4 [61]. Typically the conductive strip is supported by a substrate with a dielectric constant

or relative permittivity, ϵ_r . With the introduction of MD media into microstrip structures, the relative permeability, μ_r , must also be considered as previously discussed in Chapter 2.

When a wave propagates along a microstrip structure, the EM fields are extending through two media, the dielectric substrate and air above [62]. Because the fields encounter multiple media, the structure itself is inhomogeneous and cannot support true TEM wave propagation [62]. To approximate the effects of mixed media and fringing fields, effective parameters are used to approximate the equivalent properties that represent a microstrip embedded within an single isotropic medium. The effective relative permittivity, ϵ_{ref} , relates to how a material interacts with an electric field. Similarly, the magnetic field requires an effective value for the relative permeability, μ_{ref} . Together the effective material parameters with effective microstrip width and length account for differing properties of the substrate in addition to effects of the finite distance between the microstrip and the ground plane. This section will cover general design considerations for MPAs. Section 3.3 will dive into specific design methods and equations.

Mode of Propagation

The dominant transverse magnetic (TM) mode of the guided wave behaves like a TEM mode, so the term quasi-TEM is used. The propagation velocity of the microstrip structure depends not only on the constitutive properties, but also the physical microstrip dimensions and frequency [62], [63]. For microstrip structures, both impedance and phase velocity vary with frequency, but occurs at a very slow rate per octave so for design purposes is assumed constant [63].

A MPA can be classified as a distributed line or patch resonator which contains at least one oscillating EM field [62]. These typically have a dominant TM_{010} mode as demonstrated in Figure 3.4.

Substrate

The selection of an appropriate substrate material and it's height is paramount to achieving the desired operational characteristics for a patch antenna. The authors of [64] found that as the relative

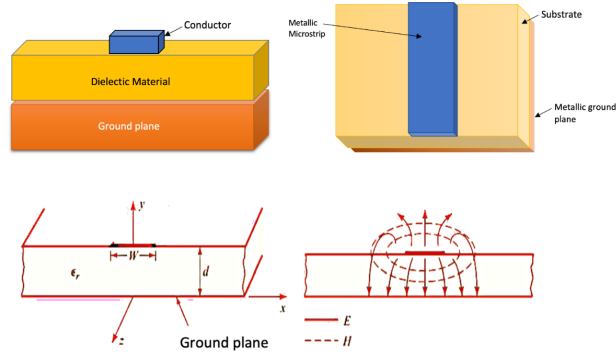


Figure 3.3: Electric and magnetic field distribution of microstrip with TM_{010} mode of propagation [61].

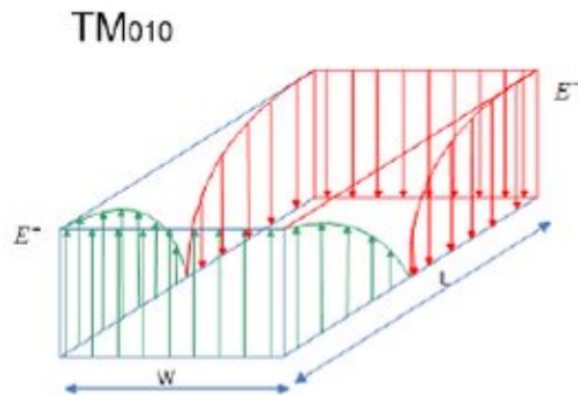


Figure 3.4: Rectangular patch antenna electric field distribution in TM_{010} .

permeability, ε_r , of the dielectric substrate increased, that patch dimensions decreased, but has repercussions of decreased gain and bandwidth. The trade-off during the design phase is to either optimize gain and bandwidth or to minimize the design dimensions. For the majority of applications the former is most desired, however with miniaturization of electronics, the foreseeable future will have more emphasis on minimization of the antenna footprint or volume.

Conductor

A PEC is often used in simulations to estimate the behavior of the design to cut down on simulation time. This is done by setting the conductivity, σ , to zero making the simulation much less calculation intensive. This is a good method in evaluating the radiation pattern morphology, but over-estimates other performance parameters. Input impedance would likely be the parameter most significantly impacted by the neglected conductivity losses. Realistic and cost-effective conductors are typically metals like copper, brass, or aluminum, but precious metals have specialized use cases as well.

Manufacturing Tolerances

The miniaturization effect from MD materials also reduces manufacturing tolerances, sometimes becoming nearly diminutive tolerances to achieve the desired radiating characteristics.

Patch Dimensions

There is no known closed-form analytical solution for the electromagnetic fields of a patch antenna because approximations used for the dipole are no longer valid for this much more complex structure. Instead the cavity and transmission line models have been developed to approximate the microstrip patch structure [65], [66]. The MPA fields are approximated by these models which allows design equations for design parameters such as length and width to obtain the desired operational frequency.

The morphology of a microstrip also introduces fringe fields and edge effects that affect the relative permittivity and permeability. This is due to the limited height, h , of a MPA causing the presence of fringe electric fields, $\vec{E}$ which lie outside the patch volume [1]. Figure 3.6 demonstrates

fringing of the electric field, which is accounted for by using an effective width of the microstrip conductor [61]. Effective values are utilized to represent the apparent parameters to account for the effects of mixed media and fringe fields for relative permittivity ε_r , relative permeability μ_r , length L , and width W [1]. There have been many formulas developed to estimate the effective relative permittivity, ε_{ref} , but effective relative permeability, μ_{ref} , has traditionally not been considered.

A MPA is typically on the order of half a wavelength to the fundamental resonant frequency in length, or when terminated may operate as a quarter wavelength resonator [62]. Length is approximately given by

$$L_{MPA} = \frac{c_0}{2f_r \sqrt{\mu_{ref} \varepsilon_{ref}}}, \quad (3.18)$$

where $c_0 = 2.9979 \times 10^8 m/s$ is the velocity of light in free space, f_r is the resonant frequency, ε_{ref} is the effective relative permittivity, and μ_{ref} is the effective relative permeability. The physical length of the patch varies slightly based on which design model is utilized. The most popular design models include the cavity model and the TL, which will be covered more thoroughly in Section 3.3.1 and Section 3.3.2, respectively.

The width of the MPA is often selected to maximize the bandwidth, but care taken to avoid widths that may excite higher order modes, particularly transverse electric (TE) modes. It is common for the width to exceed the length of the MPA, however to suppress higher order modes Blosser found it optimal to use a width bounded by $L/2 < W < L$ [27].

Finite Ground Plane Dimensions

The size of a finite ground plane and substrate height can increase the fringe fields present in a microstrip patch antenna design. Saini et al. extended the ground plane by nearly six times the substrate height, $h = 0.2$ cm, in x and y dimensions from the patch dimensions to approximate an infinite ground plane to mitigate the fringe field effects caused by a finite ground plane [1]. The limited height between the conductors also contributes to fringe fields, which increase proportionally to height [1].

Feed Method

The desired feed point of a patch antenna depends on the desired fundamental mode. The TM_{010} fundamental mode, shown above in Figure 3.4, or the desired polarization [32]. To feed the intended TM fundamental mode with linear polarization without exciting higher order modes, the feed should be placed along the midline of the patch's width. Along the length the midpoint is a null in the MPA fields, which acts as a short with 0Ω impedance, but the desired mode cannot be excited from a null in that mode. A TM_{010} half-wavelength MPA has a null, or short, at the center and the length to the ends of the patch are $\lambda/4$ from the center null, therefore these ends act as an electrical open. The signal will be partially reflected if the antenna feed is not impedance matched, with the short having zero impedance and the open has an impedance of $Z_{open} = 120\pi\sqrt{\mu_{ref}/\epsilon_{ref}}$. The usual convention is to feed at approximately $\lambda/3$ from the short with a standard 50Ω method, however this assumes the impedance at the radiating slot to be equivalent to free space, 377Ω , and places the feed near the patch's 50 to 100Ω location. With incorporation of MD media, the feed location must be more precisely calculated. Calculation methods will be discussed in Section 3.2.

Feed methods can be divided into contact and non-contact feed methods. Common contact feed methods include probe fed with an SMA connector, an inset microstrip feed, or coplanar waveguide. Non-contact methods are typically coupled through proximity or an aperture in the ground plane. If a configuration's design requires a specific feed location, an impedance match can be still be made by designing a matching network as long as the matching network is more broadband than the radiator it feeds.

*Feed Location

The feed method must be optimized for maximum power transfer while minimizing the reflected power, generally $VSWR \leq 2$. Impedance matching has been well documented for dielectric patch antennas, however MD media introduce greater complexity in identifying the optimal location for a matched feed.

The resulting reflection coefficient at the patch antenna feed's boundary condition is calculated

by

$$\Gamma = \frac{Z_{feed} - Z_a}{Z_{feed} + Z_a}, \quad (3.19)$$

where Z_{feed} is the impedance of the coaxial cable and Z_a is the impedance of free space. The resulting VSWR is found by

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}. \quad (3.20)$$

A $VSWR < 2$ is desired. The required antenna impedance can be solved for by solving for Γ from Equation 3.20, then equating it to Equation 3.19

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1} = \left| \frac{Z_1 - Z_2}{Z_1 + Z_2} \right| \quad (3.21)$$

Plugging in the upper limit of 2 for VSWR and known feed impedance, the antenna impedance is bounded by

$$\frac{Z_{feed}}{2} \leq Z_a \leq 2Z_{feed}. \quad (3.22)$$

For a conventional 50Ω feed, the value of Z_a must be between 25Ω and 100Ω to have a VSWR of two or less. Maximum radiated power, P_{rad} , occurs at $VSWR = 2$, so $Z_a = 2Z_{feed}$ is the most desired feed location. Due to diminishing tolerances, $VSWR < 2$ may be utilized in practice. The phase angle in degrees at the feed point is given by,

$$\Theta_{feed} = \arccos \frac{VSWR \cdot Z_{feed}}{\eta_{open}}, \quad (3.23)$$

where $\eta_{open} = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$ is the open circuit refractive index. Relative permeability in many early design equations is assumed as unity to simplify the calculations, but with MD materials it must be included. The length of a patch is on the order of half a wavelength for a wave confined within the substrate media and the wavelength is found by

$$\lambda_g = \frac{v}{f} = \frac{c_0}{f \sqrt{\mu_r \epsilon_r}} \quad (3.24)$$

The distance along the length edge from the open is found by

$$d_{feed}^{open} = L_{ef} \frac{\Theta_{feed}^{\circ}}{180^{\circ}}, \quad (3.25)$$

where $L_{ef} \approx \frac{\lambda_{ef}}{2}$ is approximated by the effective wavelength. The distance from the open is preferred by certain models, but leads to confusion in computer-aided design (CAD) models or during fabrication if the feed location is measured from a non-physical point. Additional confusion may arise if the physical patch's free edge is assumed to be equivalent to the theoretical open. A MPA has the short at the center along the length so the distance to the desired feed location is more universally defined by

$$d_{feed}^{short} = \frac{L_{ef}}{2} - d_{feed}^{open}. \quad (3.26)$$

3.2.1 Patch Design Remarks

As an antennas structure gets increasingly complex, each parameter requires more careful consideration during the design process. Each design specification can adversely affect the overall performance of the antenna. A fundamental understanding of how an antenna functions from the material parameters to the physical dimensions is imperative to making the right design choices.

3.3 Microstrip Patch Antenna (MPA) Modeling Methods and Limitations

Before the development of computational electromagnetics (CEM) analysis methods, the design, analysis, and optimization of electromagnetic structures was done through iterative experimentation [67]. CEM algorithms provide fundamental insight into EM problems by giving visual representation and optimization capabilities that couldn't be achieved without these computational methods [67].

Manual calculation methods use approximations for dipoles to derive closed-form solutions. Those approximations are no longer valid for more complex structures which have no closed-form analytical solution for the complex EM fields. With increasing computational capabilities, CEMs

naturally became a sought after capability. Quick computational solvers began in the 1970's with the identification of Surface Integral Equations (SIE) functionally being a convolution problem, which sparked use of uniform discretizations and the fast Fourier Transform (FFT) [68]. From there many algorithms have been developed to speed up simulation time and improve accuracy for complex structures.

To address the need for modeling microstrip TLs and MPAs, many approximation methods have been developed. The MPA fields are approximated by these models, which use the desired operational frequency, material properties, and structural dimensions to find design parameters such as length and width for a MPA [65], [66].

The morphology of a microstrip also introduces fringe fields and edge effects that affect the relative permittivity, ϵ_r , and relative permeability μ_r . This is due to the limited height, h , and asymmetry of a MPA creating fringe electric fields, $\vec{E}$ which lie outside the patch volume [1]. Figure 3.4 demonstrates fringing of the electric field, which is accounted for by using an effective width of the microstrip conductor [61]. Effective values are utilized to represent the apparent parameters to account for the effects of mixed media and fringe fields for relative permittivity ϵ_r , relative permeability μ_r , length L , and width W [1]. There have been many formulas developed to estimate the effective relative permittivity, ϵ_{ref} , but effective relative permeability, μ_{ref} , has traditionally been assumed to equal unity and not been considered in design or modeling.

3.3.1 Cavity Model

Rectangular MPAs resemble a cavity loaded with a substrate material. The pseudo-cavity is defined as a PEC patch and ground-plane with PMC bounding the vertical sides to define a lossy resonant cavity [69]. Microstrip patch antennas propagate in a quasi-TEM mode or truly TM mode and this model assumes the net fields are a superposition of various resonant modes as demonstrated in Figure 3.4 [69]. This quasi-TEM mode causes both impedance and phase velocity to vary with frequency, however this occurs at a very slow rate per octave and is generally assumed constant for design purposes [63].

A major advantage of using the cavity models is the design equations consider the propagating modes along the height, length, and width of the cavity. Figure 3.5 demonstrates the field amplitudes along each dimension of the MPA cavity as well as fringing fields [69], [70]. The resonant frequency of the cavity is found by

$$f_{r_{mnp}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \cdot \sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2}, \quad (3.27)$$

where m , n , and p represent the mode number in directions of the height, length, or width, respectively [70]. The dominant mode of a MPA is the mode with the lowest-order resonant frequency. If dimensional constraints meet $W < L$, $h \ll L$, and $h \ll W$, then the order of modes in ascending order are $L > W > h$, with $f_{r_{010}}$ being the dominant mode. Balanis demonstrates that the optimal width is $\frac{L}{2} < W < L$ in order to suppress the second-order modes along the length and width of the patch [27], [70]. Equation 3.27 can be solved to find each higher-order mode and guide dimensional adjustments to meet design requirements.

A major shortcoming of cavity models is that it is the least accurate of common models and it requires an experimentally determined equivalent feed width to obtain adequate correspondence between predictions and experiments [65], [70]. Additionally, traditional cavity models do not account for modern materials that have a relative permeability greater than one. Blosser modified the cavity model to include relative permeability, but experimental verification demonstrated more error than expected [27].

Cavity Model Design Equations

Blosser utilized the cavity model to design a MPA on MD substrate and found more iterations of HFSS were necessary to optimize feed placement and operating frequency when compared to when designing a dielectric substrate material, RO3006 [57]. A common approach to estimating effective relative permittivity are to average the substrate material parameters with that of free space

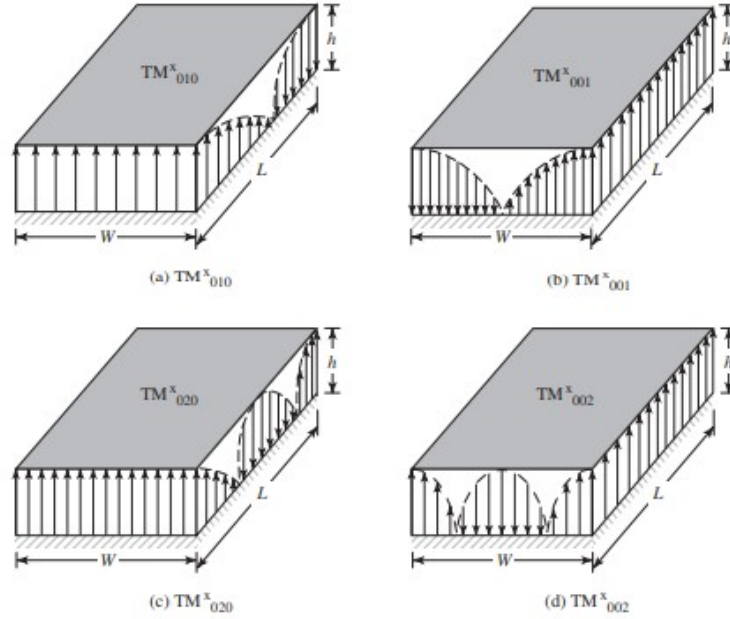


Figure 3.5: Microstrip patch antenna modes of operation. (a) TM_{010} (b) TM_{001} (c) TM_{020} (d) TM_{020} [70].

in calculations for cavity dimensions, which has the form

$$W = \frac{c_o}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}}. \quad (3.28)$$

The square root portion represents a common rough estimate of effective relative permittivity if the substrate's ϵ_r were averaged with that of the free space superstrate. Introduction of relative permeability for both the substrate and superstrate regions, Equation 3.28 becomes,

$$W = \frac{c_o}{2f_r} \sqrt{\frac{2}{\epsilon_1\mu_1 + \epsilon_2\mu_2}} = \frac{c_o}{2f_r\sqrt{\epsilon_{ref}\mu_{ref}}}. \quad (3.29)$$

The averaging approach of Equation 3.29 is an improvement, but lacks the ability to weight the substrate and superstrates to account for field density within each material as should be done when approximating ϵ_{ref} and μ_{ref} . The width is commonly greater in dimension than the width to increase bandwidth of the MPA. However, depending on application suppression of second-order modes leads to a width that is constrained by the length, $\frac{L}{2} < W < L$ [70].

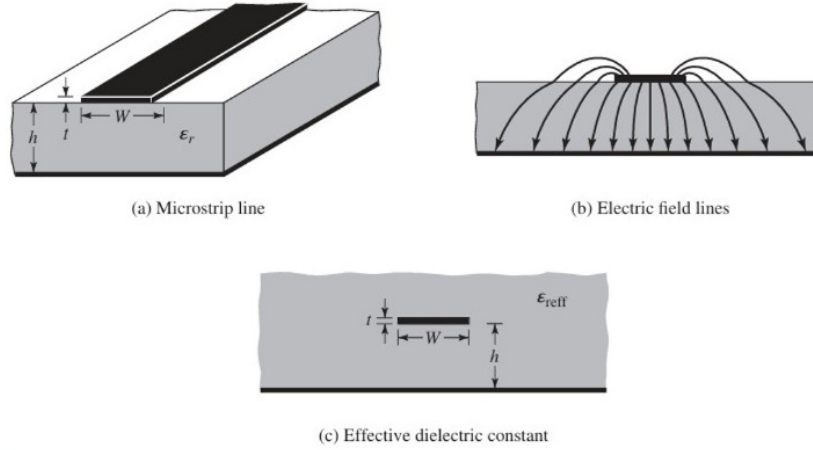


Figure 3.6: The transmission line (TL) model for a microstrip structures utilizes the physical dimensions and estimated fields to find effective parameters representing a homogeneous structure [70].

3.3.2 Transmission Line (TL) Models

This section will review TL models seen in literature. This will lead into the derivation of a well-known TL model for patch antennas which will be analyzed and further developed to account for modern MD materials in Appendix A. Telegraphers equations, sometimes referred to as the Heavyside equations, are the TL analog of Maxwell's equations as the solution to the second-order differential Helmholtz's equations.

The traditional TL model uses the physical parameters of a microstrip line and takes into account the fringing fields due to anisotropic field paths to estimate effective parameters of length, width, and constitutive parameters to represent the TL surrounded by an equivalent homogeneous material, which has a closed-form solution. This concept is well depicted in Figure 3.6.

The TL model assumes the dielectric constant, or relative permittivity, remains constant at low frequencies, termed static permittivity, and at intermittent frequencies increases monotonically until it approaches the value accepted as the dielectric constant [70].

TL Design Equations

For a MPA, the patch length represents the radiating edge length, while the width represents the distance between two radiating slots [1]. Traditionally, the resonant length, L , of a half wavelength

patch antenna is given by

$$L = \frac{c_0}{2f_r \sqrt{\epsilon_r \mu_r}}, \quad (3.30)$$

where $c = 2.9979 \times 10^8 m/s$ is the velocity of light in free space, f_r is the resonant frequency, ϵ_r and μ_r are the relative permittivity and permeability, respectively, of the substrate [1]. The effective length of the patch is greater than the patch's actual length due to fringing fields and found by,

$$L_{ef} = (L + 2\Delta L) = \frac{c_0}{2f_r \sqrt{\epsilon_{r_{ef}} \mu_{r_{ef}}}}, \quad (3.31)$$

where $L = \lambda/2$ for dominate mode TM_{010} and ΔL is the additional perceived length to each end of the patch [1].

To prevent simultaneous radiation of higher order modes, Blosser notes that the ratio of $W/L < 1$ is desirable, yet not so small as to decrease the bandwidth [27]. Conversely, Mishra et al. finds that $W/L = 1.5$ is optimal for maximizing a patch's bandwidth including multiple radiating modes, however it is unclear if the authors matched the 50Ω feed as patch dimensions (and operating frequency) were altered [47]. Patch width most significantly affects the bandwidth, but can also be affected by patch length with little impact on the resonant frequency [47].

Characteristic Admittance

Characteristic admittance, the inverse of impedance, of a microstrip patch is described by

$$Y_0 = \frac{\lambda}{2\eta\mu h}, \quad (3.32)$$

where η is the characteristic impedance of free-space, $\eta = 120\pi$, and h is substrate height [1], [8]. The characteristic admittance is important in terms of impedance matching to ensure maximum power is transmitted. Evaluating the effects each selected material may have on the characteristic admittance must be carefully considered. Equation 3.32 demonstrates the characteristic admittance is a function of μ , but not ϵ .

Equivalent Matrix Methods

ABCD analysis is a common method used to evaluate N-port networks. A simple microstrip TL can be represented by a two-port network and the quantities found by voltage or current ratios under open- or short-circuit conditions. A and C can be found when a source is placed on port 1 and port 2 remains an open circuit causing $I_2 = 0$. Subsequently, B and D can be found when a source is placed on port 1 and port 2 is short-circuited causing $V_2 = 0$. Figure 3.7 shows the ABCD representation of a two-port network under these conditions. The matrix representation of a two-port network is found by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left. \frac{V_1}{V_2} \right|_{I_2=0} & \left. \frac{V_1}{-I_2} \right|_{V_2=0} \\ \left. \frac{I_1}{-V_2} \right|_{I_2=0} & \left. \frac{I_1}{I_2} \right|_{V_2=0} \end{bmatrix}. \quad (3.33)$$

A two-port network is considered reciprocal if $AD - BC = 1$. Two-port networks can be represented by cascading $ABCD$ -matrices which are simply multiplied. This property makes $ABCD$ -parameters a good method for cascading the individual $ABCD$ -matrix of each cascaded lumped element or differential length of TL. A length of microstrip is represented by the TL $ABCD$ -matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_0 \sinh \gamma l \\ Y_0 \sinh \gamma l & \cosh \gamma l \end{bmatrix}, \quad (3.34)$$

where l is a unit length of the TL, j is the imaginary number representing $\sqrt{-1}$, and $\gamma = \alpha + j\beta$ is the complex propagation constant [71]. The $ABCD$ matrix can be further simplified under lossless

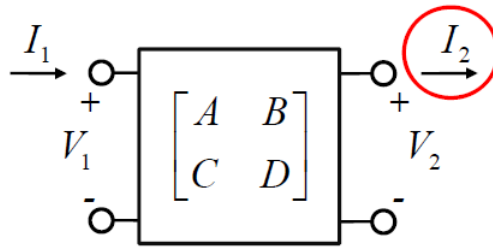


Figure 3.7: Diagram of a two-port network with defined current direction for deriving $ABCD$ -parameters.

assumptions to

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{lossless}} = \begin{bmatrix} \cos \beta l & jZ_0 \sin \beta l \\ jY_0 \sin \beta l & \cos \beta l \end{bmatrix}, \quad (3.35)$$

where β is the phase constant approximated by $\beta_{\text{lossless}} = 2\pi f \sqrt{\varepsilon\mu}$ or for high frequencies $\beta_{\text{lossless}} = \frac{2\pi f}{v_p}$, where v_p is the phase velocity in the medium.

The $ABCD$ method is not the best matrix format under certain conditions, such as a splitter, but these matrices can easily be converted to another matrix method that simplifies the calculation such as Z, Y, G, H , or s-parameters. Figure 3.7 demonstrates the voltage and current definitions of a two-port network of $ABCD$ -parameters. The red circle in Figure 3.7 is to differentiate between the defined current direction differentiating $ABCD$ -parameters from the similarly derived A -parameters.

Lumped Element Circuits

The lumped port transmission line model is a good first-order approximation for TL structures. The lumped element TL is reduced to a single element, most commonly a resistance, R , and inductance, L , in series with a reactance, G , and capacitance, C , in shunt configuration as seen below in Figure 3.8. The ohmic resistance is due to finite conductivity of the conductive layers [72]. The conductance, reciprocal of resistance, occurs in shunt to account for material dielectric and magnetic loss tangents [72]. The inductance and capacitance are related to self-inductance and the proximity of conductors, respectively [72]. To simplify the lumped element transmission line model for simulation, the lossless material assumptions reduce the number of terms by setting $R = 0$ and $G = 0$. The lossless assumption results in only the lumped series inductance and shunt capacitance terms remaining.

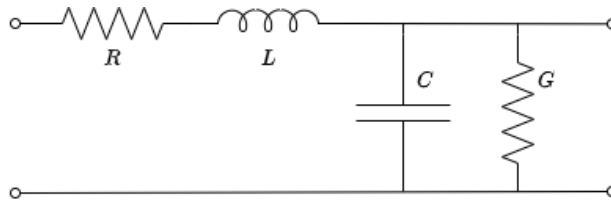


Figure 3.8: Equivalent circuit model for a lumped element transmission line (TL).

R is resistance in ohms is the resistance due to finite conductivity of the conductor, L is inductance in Henrys is the self-inductance (internal), G is shunt conductance in Siemens (loss tangent or dielectric loss in material), and C is capacitance in Farads is the capacitance due to the proximity of conductors [72]. Total self-inductance includes both internal and external inductance, but the external mutual inductance from coupling is generally very small and ignored [72].

Distributed Parameter Transmission Lines (TL)

Distributed parameter TL methodology is very similar to the lumped element approach from Section 3.3.2, but the values are normalized so that it can be scaled per unit length along the TL. The distributed parameter TL model does this by dividing a length of TL into infinitesimally small sections and depicts the resistance (R), inductance (L), reactance (G), and capacitance (C) each found per unit length of the TL.

The lossless TL model is much easier to evaluate compared to a distributed lossy TL model, but becomes less accurate as frequency and the length of line increases. Competitively, the lossy TL model covered in Section 3.3.2 accounts for material losses which make the approximations more accurate, but the equations are also much more difficult to evaluate compared to a lossless model. These TL methodologies can also be applied to more complex structures to develop an equivalent circuit model

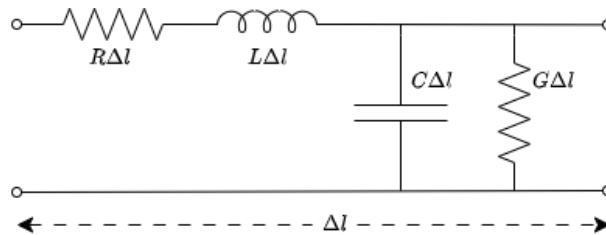


Figure 3.9: Equivalent circuit of a differential segment of transmission line (TL).

Lossless Transmission-Lines (TL)

Lossless TLs are a simplified way to demonstrate the phase changes of a wave as it travels through a TL without real impedance losses. This tool is great for finding the impedance at the port or

boundary to quantify the reflection or transmission coefficients or to design a matching network. A system of equations can be used to solve for the inductance and capacitance using two lossless TL equations with unknown values of inductance and capacitance, L and C , respectively. Figure 3.10 demonstrates this simplified model. Equations 3.36 thru 3.41 show how to solve for these values through substitution of equations.

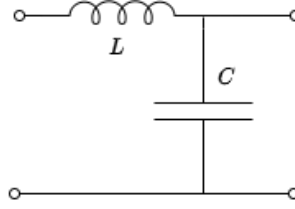


Figure 3.10: Equivalent circuit for a the lossless transmission line (TL) model.

It is known that in a lossless TL the characteristic impedance, Z_0 , is found by,

$$Z_0 = \sqrt{\frac{L}{C}}. \quad (3.36)$$

By convention it is standard for systems to be designed with a characteristic impedance, Z_0 , of 50Ω to match with other system components. By plugging Z_0 into Equation 3.36, L can be found as a function of capacitance,

$$L = Z_0^2 C. \quad (3.37)$$

The phase constant, β , of a medium determines many characteristics of a transmission line. This variable is present in formulas to determine phase velocity, wavelength, and the imaginary component of the complex propagation constant. The phase constant of this lossless transmission line example is found by,

$$\beta = \omega \sqrt{LC}, \quad (3.38)$$

where ω is the frequency in radians per second or is equivalent to $2\pi f$ in Hertz. The value of L from equation 3.37 can be substituted into Equation 3.38,

$$\beta = \omega \sqrt{Z_0^2 C^2} = \omega Z_0 C, \quad (3.39)$$

and solve for the remaining unknown,

$$C = \frac{\beta}{\omega Z_0} \left[\frac{F}{m} \right]. \quad (3.40)$$

Plugging the result of Equation 3.40 back into Equation 3.37,

$$L = Z_0^2 C \left[\frac{H}{m} \right]. \quad (3.41)$$

The inductance and capacitance in Equations 3.40 and 3.41 are both found per unit of length, Δl , so the number of cascaded segments and the length of each is very important.

The overall length of a TL is conventionally found in terms of wavelengths, λ . The wavelength within a medium is a function of the phase constant, β , which is given by,

$$\lambda_g = \frac{2\pi}{\beta} [m]. \quad (3.42)$$

The $\lambda/10$ convention can be utilized as the maximum segment length for linear approximations. The total length of the TL must be divided into a minimum of 10 cascaded sections of a series impedance and a shunt capacitance to meet the $\lambda/10$ convention requirement. The length of each segment is found to by,

$$l_s = \frac{l_{tot}}{N} \left[\frac{m}{\Delta l} \right], \quad (3.43)$$

where l_{tot} is the total length of the TL in wavelengths and N is the number of segments. For example a 2λ long TL would require 20 segments to meet the $\lambda/10$ convention and the length of each segment found by $l_s = \frac{2\lambda}{20} \left[\frac{m}{\Delta l} \right]$. The values found by Equations 3.40 , 3.41, 3.43 represent a small section of lossless TL as series inductance and shunt capacitance as represented in Figure 3.9.

Lossy Transmission Line

More realistically than the ideal TLs in Section 3.3.2, the equivalent circuit model of a lossy TL will still have a series inductance and a shunt capacitance per unit length but also introduces a complex

propagation constant given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}, \quad (3.44)$$

where α is the attenuation constant in linear units of Nepers per meter, and β is the phase constant in linear units of radians per meter. Convention is to convert α into decibels, $\alpha[\frac{dB}{m}] = 8.686 \cdot \alpha[\frac{Np}{m}]$ [25]. The lossless approximation for β cannot be used any longer at high frequencies, so the approximation $\beta_{lossless} = \omega\sqrt{\varepsilon\mu}$ becomes $\beta = \omega/v_{phase}$ [25].

Parasitic Capacitive Coupling

Parasitic capacitive coupling is traditionally modeled by either a series or parallel inductor-capacitor (LC) resonant circuit [73]. For a MD material the LC components represent the coupled relationship between permittivity and permeability [1]. The MD material MPA characteristics indicate the parallel LC model is representative of the internal coupling mechanism by demonstrating inductive coupling that becomes capacitive after the resonant frequency [27], [73]. The preferred parallel LC circuit model is generally a first-order resonant circuit with effective equivalent values for an inductor and capacitor that represent the overall structures distributed behavior [73]. The first-order approach does not capture the higher-order resonant peaks that are necessary to precisely model high-frequency common-mode and differential-mode currents [73]. To quantitatively evaluate the parasitic capacitive coupling, literature proposes using a device figure of merit [73].

3.3.3 Microstrip Patch Antenna (MPA) Transmission Line (TL) Models

The permittivity of a mixed medium MPA is represented by an effective relative permittivity value representing the equivalent homogeneous material filling the structure that would give the same performance. The effective relative permittivity is found by

$$\varepsilon_{ref} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \left(1 + 12\frac{h}{W}\right)^{-1/2} \quad (3.45)$$

for a microstrip of $\frac{W}{h} > 1$ [70]. Traditionally, materials for microstrip structures had a unitary permeability, which significantly simplified design formulas. Methods of calculating effective relative permeability have not been thoroughly studied compared to those for its counterpart, permittivity. The magnetic fields form a semicircle around the microstrip, so a very rough estimation of the effective relative permeability, μ_{ref} , is to take the average of the substrate and superstrate,

$$\mu_{ref} = \frac{\mu_{r1} + \mu_{r2}}{2}, \quad (3.46)$$

where the subscripts one or two represent the substrate and superstrate material parameters, respectively. This averaging method does not capture field ratios nor dynamic effects related to field density.

Radiation efficiency is predominantly determined by the width of an MPA, therefore a practical width of an efficient radiator is

$$W = \frac{c_0}{2f_r} \sqrt{\frac{2}{\varepsilon_r + 1}} = \frac{c_0}{2f_r} \sqrt{\frac{2}{\varepsilon_{r1}\mu_{r1} + \varepsilon_{r2}\mu_{r2}}} = \frac{c_0}{2f_r \sqrt{\varepsilon_{ref}\mu_{ref}}}, \quad (3.47)$$

where the approximation increases in accuracy from left to right and the final iteration being most representative of the mixed media properties of a microstrip structure [70].

Traditionally, the physical resonant length, L , of a half-wave MPA is given by Equation 3.29

$$L = \frac{c}{2f_r \sqrt{\varepsilon_r}} = \frac{c}{2f_r \sqrt{\varepsilon_r \mu_r}}, \quad (3.48)$$

where $c = 2.9979 \times 10^8 \text{ m/s}$ is the velocity of light in free space, f_r is the resonant frequency, ε_r is the relative permittivity, and modified to include μ_r which is the relative permeability [1]. The effective length of the patch is greater than the patch's actual length due to fringing fields and is found by,

$$L_{ef} = (L + 2\Delta L) = \frac{c_0}{2f_r \sqrt{\varepsilon_{ref}\mu_{ref}}}, \quad (3.49)$$

where $L = \lambda/2$ for the dominant mode TM_{010} and ΔL is the additional perceived total length

found by

$$\Delta L = 0.412 \cdot h \frac{(\epsilon_{ref} + 0.3) \left(\frac{W}{h} + 0.264\right)}{(\epsilon_{ref} - 0.258) \left(\frac{W}{h} + 0.8\right)} \quad (3.50)$$

[1], [70]. The dominant mode frequency is often calculated with actual length and permittivity per [70], but should utilize the effective values, especially when utilizing MD materials. Modifying the equation given by [70] to include effective relative permeability, the dominant mode frequency is found by

$$(f_{rc})_{010} = \frac{c_0}{2L_{ef}\sqrt{\epsilon_{ref}\mu_{ref}}} = \frac{c_0}{2(L + 2\Delta L)\sqrt{\epsilon_{ref}\mu_{ref}}}. \quad (3.51)$$

3.3.4 Pues and Van de Capelle TL Model

Due to the nature of MPAs, they are well represented by admittance parameters, or Y -parameters. The general form of admittance is $Y = G + jB$, where G is conductance and B is susceptance. Y -parameters are usually in matrix form to represent a shunt component or an n -port network, where n represents the number of ports. The Y -parameters of multiple shunt components can be added in series making this model ideal for representing a small segment of TL. Pues and Van de Capelle use this method in their improved model individually deriving characteristic admittance, self admittance, and mutual admittance which are used to calculate the three-port network Y -parameters and from that calculate the input admittance, Y_{in} [65].

Traditional TL models are depicted as a two-port networks, but coaxial shunt fed MPAs have a third port representing the feed point [65]. The typical three-port TL network shown in Figure 3.11 represents an excited slot with an infinite length and width and a height equal to the thickness of the substrate [65]. Pues and van de Capelle identified three shortcomings of this traditional model:

1. Inaccurate for modeling narrow patches
2. Mutual coupling between radiating slots is neglected
3. Radiation conductance of side slots is neglected [65].

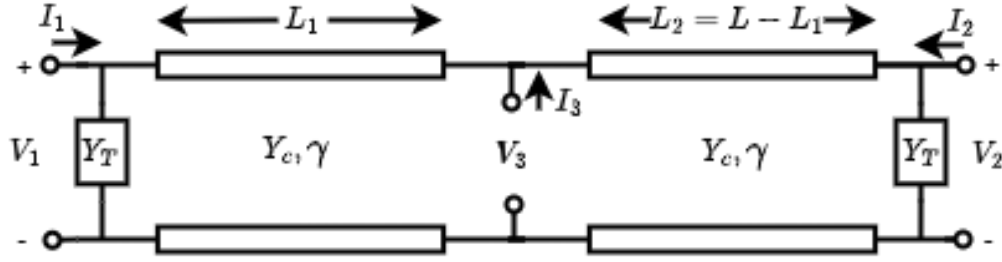


Figure 3.11: A generic three-port equivalent circuit of a probe fed microstrip patch antenna (MPA) represented as the admittance of a transverse electric (TE) excited slot of infinite length and width equal to the substrate height [65].

To address the shortcomings stated above, Pues and van de Capelle proposed a modified TL model shown in Figure 3.12 that accounts for self-admittance, Y_s , as a shunt component, mutual-admittance (radiative), Y_m , as a voltage dependent current source placed in shunt, along with the distributed characteristic admittance, Y_c , of the conductors and substrate [65]. The complex propagation constant, γ , is utilized to account for all of the losses from copper and the substrate [65].

The admittance matrix representing the three-port TL model shown in Figure 3.11 is

$$[Y] = \begin{bmatrix} Y_s + Y_c \coth(\gamma L_1) & -Y_m & -Y_c \operatorname{csch}(\gamma L_1) \\ -Y_m & Y_s + Y_c \coth(\gamma L_2) & -Y_c \operatorname{csch}(\gamma L_2) \\ -Y_c \operatorname{csch}(\gamma L_1) & -Y_c \operatorname{csch}(\gamma L_2) & Y_c \coth(\gamma L_1) + Y_c \coth(\gamma L_2) \end{bmatrix}, \quad (3.52)$$

where $\coth$ and csch are the complex trigonometric functions representing hyperbolic cotangent and hyperbolic cosecant, respectively, and $L_1 = L - L_2$ are the lengths of the patch as depicted in Figure 3.12 [65]. The input admittance derived from Equation 3.52 is found to be

$$Y_{in} = 2Y_c \left[\frac{Y_c^2 + Y_s^2 + Y_m^2 + 2Y_s Y_c \coth(\gamma L) - 2Y_m Y_c \operatorname{csch}(\gamma L)}{(Y_c^2 + Y_s^2 + Y_m^2) \coth(\gamma L) + (Y_c^2 + Y_s^2 + Y_m^2) \cosh(2\gamma \Delta) \operatorname{csch}(\gamma L) + 2Y_s Y_c} \right], \quad (3.53)$$

where $\cosh$ is the complex hyperbolic sine function and the remaining variables align with Equation 3.52.

Line Parameters

The Pues and Van de Capelle TL model utilized the planar waveguide model formulas from [74] to find the characteristic impedance, propagation constant, and attenuation constant

$$Z_c = \frac{\eta_0}{\sqrt{\epsilon_{ref}}} \frac{h}{W_{ef}} \quad (3.54)$$

$$\beta = k_0 \sqrt{\epsilon_{ref}} \quad (3.55)$$

$$\alpha = 0.5\beta \tan \delta_E, \quad (3.56)$$

where η_0 is the wave impedance in free-space, ϵ_{ref} is the effective relative permittivity of the substrate, h is the substrate height (thickness), W_{ef} is the effective width of a patch, $k_0 = \frac{2\pi}{\lambda_0}$ which is the free-space wave number, and $\tan \delta_E = \frac{\epsilon_r''}{\epsilon_r'}$ is the substrate's dielectric loss tangent.

Pues and Van de Capelle note neglecting surface waves and frequency dependence of Δl are acceptable if $k_o h \sqrt{\epsilon_r} \leq 0.3$, thus giving self susceptance approximations

$$G_s = \frac{1}{\pi \eta_o} \left[\left(w Si(w) + \frac{\sin(w)}{w} + \cos(w) - 2 \right) \left(1 - \frac{s^2}{24} \right) + \frac{s^2}{12} \left(\frac{1}{3} + \frac{\cos(w)}{w^2} - \frac{\sin(w)}{w^3} \right) \right] \quad (3.57)$$

$$B_s = Y_c \tan(\beta \Delta l), \quad (3.58)$$

where $w = k_o W_e$, W_e is the TE radiating slot length, $s = k_o \Delta l$, Δl is the MPA's open-end length extension, and Si is the sine integral of a sinc function, $\sin(w)/w$ or $\text{sinc}(w)$ [65].

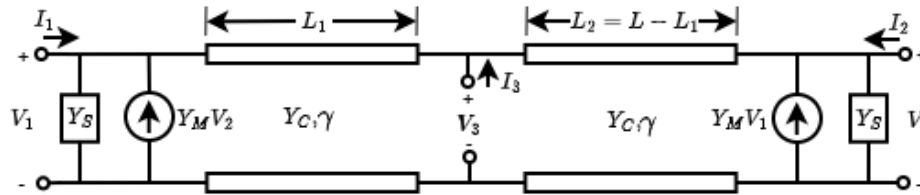


Figure 3.12: Three-port equivalent circuit representation of Pues and Van de Capelle's improved probe fed MPA TL model [65].

The mutual admittance functions require correction factors, general form given as

$$G_m = G_s F_g K_g \quad (3.59)$$

$$B_m = B_s F_b K_b, \quad (3.60)$$

where $F_g = g_m/g_s$ and $F_b = b_m/b_s$ are coupling functions to express the per-unit-length ratio between mutual and self-admittance and correction factors K_g and K_b . For $s \leq 1$ the following coupling functions are accurate

$$F_g = J_o(l) + \frac{s^2}{24 - s^2} J_2(l) \quad (3.61)$$

$$F_b = \frac{\pi}{2} \frac{Y_o(l) + \frac{s^2}{24 - s^2} Y_2(l)}{\ln(\frac{s}{2}) + C^e - \frac{3}{2} + \frac{s^2/12}{24 - s^2}}, \quad (3.62)$$

where $l = k_o L_e$ is the normlized center distance between the slots, J_i and K_i are the i th-order Bessel functions of the first and second kind, respectively, and $C^e \approx 0.57721$ is the Euler–Mascheroni constant. Approximating the correction factor $K_g = 1$ works well for parameters within the ranges $w \geq 0.1$, $l \leq 3.2$, and $s \leq 1$. A curvefit formula was derived for the correction factor $K_b = 1 - e^{-0.21w}$

In the case that the MPA is series fed by a microstrip feed, the length L_1 goes to zero, $L_1 \rightarrow 0$, and $I_2 = I_3 = 0$, the main slot facing the feed line can be reduced by a factor $r = 1 - \frac{W_m}{W_e}$, where W_m is the width of the microstrip feed line and W_e is the effective slot width.

Limitations

The Pues and van de Capelle model succeeds at accounting for inaccuracies of previous models. Their TL model was more accurate for narrow patches, accounted for mutual coupling between radiating slots, and incorporated the conductance of side slots [65]. There is room for further improvement on the Pues and van de Capelle model. It does not model magnetic or MD materials.

Pues and Van de Capelle model has an inherent limitation of neglecting surface waves and

the frequency dependence of ΔL in self-susceptance formulas, which should be accounted for when $\beta \cdot h_{sub} > 0.3$ [65]. Frequency dispersion is also unaccounted for in this model. Dispersion can significantly vary the effective relative values of the constitutive parameters and with greater magnitude variation for frequencies above the material's breakdown region. Essentially this means that the phase speed within the material changes across frequency and could result in altered resonant behavior of the MPA.

Linear And Time Invariant (LTI)

LTI assumptions simplify Maxwell's equations to buy frequency independence and reciprocity at the cost of strict LTI bounds [13]. These LTI limitations guarantee the system performance remains in the flat region of operation which is more predictable and easily modeled. LTI systems are strictly bound to remain stable and within a predictable region of performance. These assumptions include that a bounded input will always result in a bounded output, all poles of the system lie in the left-hand side of the complex plane, and the natural response approaches zero as time approaches infinity. Traditional methods have constrained antenna design to passive materials that remain within LTI regions of operation.

3.3.5 Modeling Remarks

Modernized Modeling Methods for Magneto-Dielectrics

This thesis will derive equations that account for relative permeability using LTI assumptions, because the analytically derived models traditionally assumed a relative permeability of free space, $\mu_r = 1$. An infinite ground plane must be assumed for this derivation in order to find a closed form solution since none exists for microstrip with a finite ground plane.

Appendix A will present an adaption to the Pues and Van de Capelle TL Model to utilize effective relative permeability and introduce a NL component to capture the realistic non-linear effects for a MPAs on MD substrate.

3.4 MPA Concluding Remarks

As technology advances to utilizing higher and higher frequencies, GHz and above, it becomes more apparent that the lossless models no longer approximate the effects on a signal as it propagates along a TL or within materials. Engineering the next generation of technology will require ingenuity and utilizing the advantages of lossy materials to break performance and miniaturization bounds. These attributes drive the need for improved and more versatile MPA design models.

Chapter 3 built upon the fundamental material properties of MD materials discussed in Chapter 2 with the introduction of antenna performance parameters, miniaturization methods, and modeling for design and analysis of MPAs. MD material with the advantages of a high refractive index, matched constitutive parameters, and low dielectric and magnetic loss tangents are promising for antenna improvements in both miniaturization and radiation efficiency [9].

The antenna community has need of adapted techniques to enable design and modeling of MD antennas. MD MPAs have typically required computationally intensive simulation and often iterative prototyping to achieve the desired performance characteristics. There is a need for simplified yet accurate design formulas. Before these simplified models can be developed, improved estimation methods for effective material parameters for mixed-media microstrip structures must first be refined and matured. Chapter 4 will introduce novel CM methods for approximating the effective relative permeability for microstrip and MPA configurations with or without a superstrate. The methods derived in Chapter 4 will be used to assess microstrip performance followed by experimental verification and validation by simulation.

Chapter 4

Microstrip Conformal Mapping (CM) Algorithms

Chapters 2 and 3 introduced fundamental material properties, quantification of MD antenna performance, and MPA design models under LTI assumptions. It is evident throughout Chapters 2 and 3 that the antenna community has need of improved and computationally efficient models for MD MPAs and other RF circuitry. This chapter will review historical CM derivations and present novel CM techniques for finding the effective relative permeability of microstrip structures.

4.1 Motivation for use of Conformal Mapping Methods

Microstrip is a popular TL method for RF and MW devices and MPAs due to the planar structure and highly desirable SWaP-c. The design of a microstrip TL or MPA is highly dependent on material parameters, physical structural dimensions, and the desired frequencies for transmission or emission. Many methods have been developed to estimate the effective relative permittivity, ϵ_{ref} , related to dielectric material substrates to aid in design and modeling these structures. However there is a lack of methods that account for MD materials, which requires consideration of the magnetic component's relative permeability.

Chapter 2 demonstrated the complex non-linear behaviors of magnetic materials and how the realization of desired bulk material properties is difficult to achieve. MD materials further complicate the design when incorporated into microstrip or MPA design as demonstrated in Chapter 3. With the industry in pursuit of miniaturization of RF devices as well as development of millimeter wave communications, there is an unmet need for accurate methods to estimate the effective relative

permeability for design or modeling of a microstrip structures with MD materials or mixed media. This need can be realized through modified CM methods.

CM analysis has been a technique utilized to simplify the assessment of complex geometries, such as microstrip structures, for nearly a century and the process is well documented [75], [76], [77]. CM preserves the magnitude and direction of angles between intersecting arcs from one orthogonal basis to an alternate orthogonal basis [75]. In simpler terms, this means that curves at a point are rotated and scaled by CM, but that the angle between them remains unaltered. Boundary conditions, partial differential equations, diffusion, vibration, and harmonic functions satisfying Laplace's equation for potential fields are often transformed through CM techniques to simplify their evaluation [75].

Sychev identifies that the CM techniques maintain relevance today because CM offers computationally efficient algorithms that can easily incorporate complex microstrip design synthesis, multivariate analysis, and optimization [77]. CM techniques have also been combined with other numerical methods for EM analysis such as combining with Finite Element Method (FEM) for an approximate full-wave solution or combining multiple simple polygonal regions to obtain exact analytical solutions through integration [77], [78]. Perhaps most importantly, the image plane adds visual clarity by distorting physical characteristics in favor of visualizing linear electric fields while maintaining incident angles at boundary conditions [77].

This chapter will first review the history of CM methodology for microstrip structures and re-derive foundational formulas in modern notation inclusive of developments that expanded applicability of CM analysis. The review will be followed by derivation of two novel CM techniques that enable incorporation of MD media into design and performance assessment. The proposed improvements include a modified characteristic impedance formula, CM boundary condition approximation for magnetic fields, and two novel methods for determining the effective relative permeability for multi-layered microstrip structures with MD materials. This chapter will conclude by presenting the CM algorithms combining preexisting formulas and presented methods for application to MD microstrip structures. These methods will be evaluated for accuracy in modeling two-port microstrip

TLs and MPAs through experimental verification and simulated design validation in Chapter 6 and Chapter 7, respectively.

4.2 Origins of Conformal Mapping Methods

Derivations and proofs used in microstrip CM analysis is well documented in publications which span many decades. The CM derivations by Wheeler yield important formulas to design and evaluate a basic microstrip configuration [79], [80]. Section 4.2 will review these foundational works in development of microstrip CM methods and variability in notation across publications will be standardized. Section 4.3 will incorporate superstrate layers to the CM algorithm. Svacina first introduced a CM method for evaluation of microstrip with a superstrate layer, which was later improved and generalized by Bernhard and Tousignant [26], [81]. Section 4.4 introduces application of CM to find the effective relative permeability for ferrite substrate application [82], [83]. This topic will lead to the proposal of two novel methods for estimation of effective relative permeability for application to MD multi-layered microstrip structures. The contributions from each author will be analyzed, retained formulas identified, and summarized in presentation of a novel CM algorithm for MD microstrip structures.

Wheeler's Conformal Mapping Analysis

Wheeler used the Schwarz-Christoffel transformation to develop his CM technique [75], [80]. The Schwarz-Christoffel transformation is a formalized method of two sequential CM transformations that transform complex geometries into a format for simplified evaluation [75]. The Schwarz-Christoffel transformation in CM text books consisting of a transform from the z -plane of complex spatial coordinates to the complex T -plane of co-linear coordinates, then to the w -plane of complex flux-potential coordinates also referred to as the image plane [75]. The desired axes for the w -plane are first selected to obtain a convenient symmetry, then assumptions to achieve this symmetry are exerted onto the physical spatial z -plane model [75]. Functions are developed in the form,

$\mathbf{w} = \mathbf{w}(z)$, which are used to map an existing physical configuration to the model or image plane [75].

The nomenclature in Wheeler's publications deviate greatly from modern convention, instead using z -plane, w -plane, and z' -plane to refer to the formal notation of z -plane, T -plane, and w -plane, respectively [75], [79], [80]. The deviation from modern convention also applies to variables used by Wheeler, most notably half strip width, a , and substrate height, b , which are conventionally denoted by $w/2$ and h , respectively [79], [80]. This review of Wheeler's contributions is standardized to conventional notation for consistency with later developments.

Wheeler's motivation to use the CM technique was to find very close approximations to relate electrical and structural properties of microstrip structures in terms of exponential and hyperbolic functions rather than the more difficult exact solutions as elliptic functions [79], [80]. The conductive strip width normalized by the distance from the ground plane being the primary structural property and characteristic impedance the primary electrical parameter both of which influence performance of the microstrip [79], [80]. The characteristic impedance experienced by a traveling wave is a function of both permittivity and permeability of the effective microstrip structure [79], [80]. Wheeler begins his derivation with a configuration with a known closed-form solution for EM-fields, a paired TL of PEC strips in free space, later introducing a dielectric sheet between the conductive strips [79], [80]. A cross-sectional quadrant of these paired strips is representative of a microstrip first by image theory and further reduced by symmetry. The half-microstrip cross-section problem is reduced through CM to a problem resembling parallel conductors with a mixed dielectric medium between these conductors of unlimited extent [79], [80]. Boundary conditions are preserved in CM through the transformation from one orthogonal basis to another by maintaining the angle of refraction at all material boundaries [79].

The microstrip CM was developed under several critical assumptions. The microstrip conductors are very thin PEC and are separated by a substrate that is homogeneous, isotropic, and non-dispersive [80]. TEM must be the primary mode of propagation and this assumption holds as long as the width of the strip is less than half a wavelength within the substrate [79]. Assuming TEM propagation

has great importance because it allows second-order effects of surface waves and dispersion to be approximately non-existent and ignored [79], [84]. This strip width limitation relates to suppressing higher-order modes by keeping the operational frequency far below the structure's cutoff frequency [79].

The derivation begins as a pair of very thin parallel PEC conductive strips of width $2a$ and separated by a substrate of height $2b$ [80]. To align with various nomenclatures across publications, the microstrip's width and height from the ground plane will be notated as w and h , respectively, from this point forward. This pair of strips is used to define the wave impedance in the closed-form of a balanced paired TL [80]. To better resemble microstrip line, a sheet of material is added between the balanced pair and the material's dielectric constant, ϵ_r , added to the wave impedance equation discussed later in more detail [79]. Image theory is utilized place a ground plane at the point of symmetry between the conductive strips [80]. The ground plane transforms the balanced pair TL into a microstrip configuration where a conductive strip lies over an infinite ground plane with a substrate between, which in this form has half the original wave impedance [80]. The microstrip cross-section's symmetry allows truncation where only the right-hand quadrant remains to define the spatial z -plane [79], [80]. Invoking symmetry both simplifies the CM analysis and doubles the wave impedance back to the original value of the balanced pair [79], [80].

Wheeler defined key structures and boundaries on the complex z -plane for which he developed relations to both the T -plane and w -plane [80]. The technique used by Wheeler fits the Riemann mapping theorem condition of mapping three boundary points based on like features [75]. The

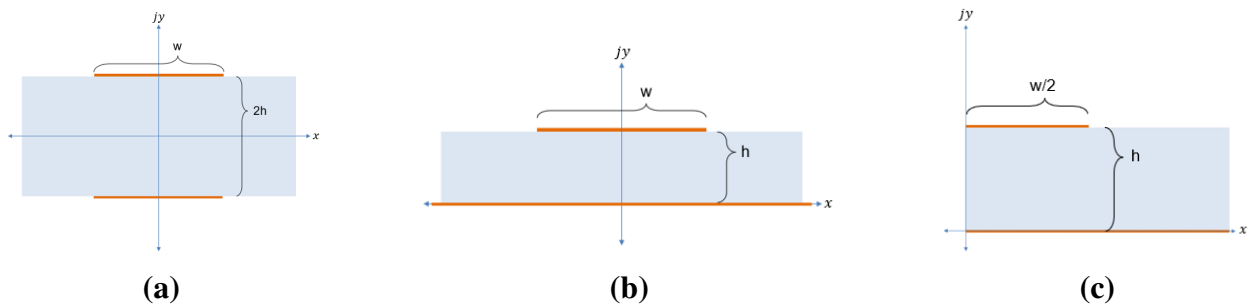


Figure 4.1: Wheeler's cross-section approximations in the physical z -plane [79]. (a) Pair of conductive strips separated by a substrate. (b) Image theory transformed to microstrip. (c) A single quadrant from symmetry to be used in conformal mapping.

z -plane is carefully mapped to the T -plane to enforce geometry to ensure gradients on the w -plane will be rational functions [80]. This is accomplished by limiting changes in direction to only those that occur while tracing a continuous conductor boundary. The gradient is given as

$$\begin{aligned}\frac{\delta(z)}{\delta(T)} &= 1 + \frac{d^2 - c^2}{T^2 - d^2} \\ z(T) &= jh + T - \ln \frac{1 + T/d}{1 - T/d} \\ &= j\pi + T - 2 \tanh^{-1} \frac{T}{d},\end{aligned}\tag{4.1}$$

where c is the reversal point at the boundary of the conductive strip's outward face to the inner conductor-dielectric boundary, d is the reversal at the conductive strip's termination along the conductor-dielectric boundary [80]. Both points c and d exist on the co-linear boundaries of the T -plane, where $h = \pi$ equates the substrate height to the angle of a half-circle corresponding to the aforementioned direction reversals along the conductor boundaries [80]. He next defines the gradient between the T and w -planes

$$\begin{aligned}\frac{\delta(w)}{\delta(T)} &= \frac{2d'}{(d')^2 - T^2} \\ w(T) &= jh + \ln \frac{1 + T/d'}{1 - T/d'} \\ &= j\pi + 2 \tanh^{-1} \frac{d'}{T} \\ T(w) &= d' \tanh \frac{1}{2}w,\end{aligned}\tag{4.2}$$

where w is a point that exists on the w -plane, T is the corresponding location of w on the co-linear T -plane, d' is to account for the effects of second-order spatial harmonics that distort the rightmost boundary, but under the initial assumptions may be approximated as $d' \approx d$ [79]. From Equations 4.1 and 4.2 the relation between the z and w -planes is

$$z(w) = j\pi + d' \tanh \frac{1}{2}w - 2 \tanh^{-1} \left(\frac{d'}{d} \tanh \frac{1}{2}w \right),\tag{4.3}$$

where d and d' are assumed to be nearly equal and $j\pi$ is the height in the complex w -plane from the T -plane's angle of π radians [80].

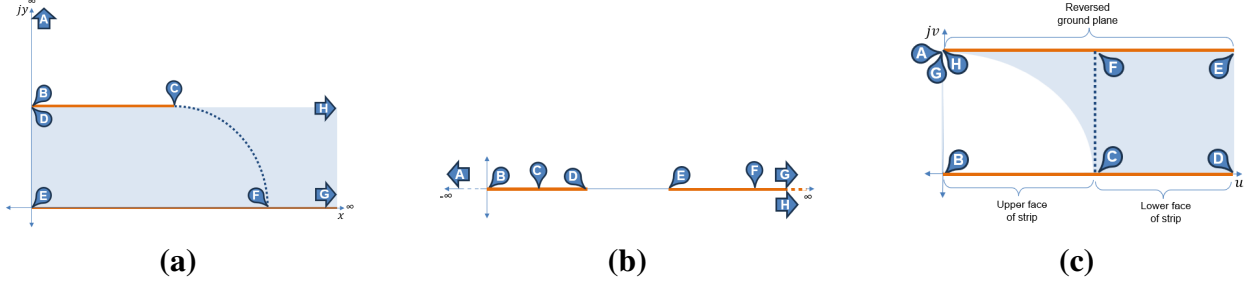


Figure 4.2: Wheeler's conformal mapping planes with key points labeled [79]. (a) z -plane (b) T -plane (c) w -plane

The flux line, the dashed line from point c to f in Figure 4.2, represents the boundary between the electric field lines which terminate on the inner or the outer faces of the conductive strip [80]. From this, the flux ratio is defined by the electric flux terminating on the outter conductor boundary in relation to the total electric flux [80]. Figure 4.2 shows the regions separated by the flux line where in the z -plane the flux terminating on the inner conductor represents the fields bound within the substrate which when translated to the w -plane becomes the right half-plane. Likewise, the fringe fields are those flux lines terminating on the outter surface of the strip which translate to the left half of the image plane. Figure 4.3 highlights the fringe field regions

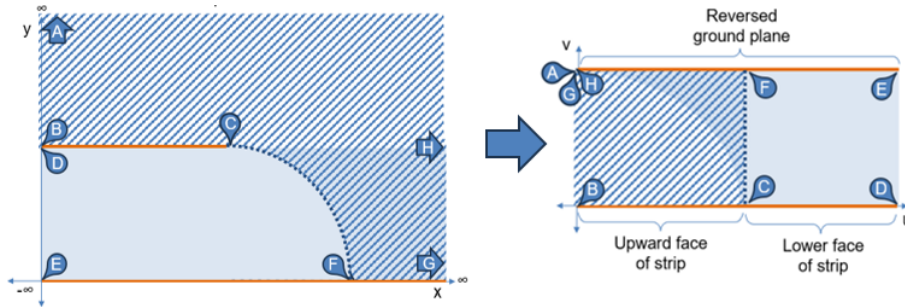


Figure 4.3: Visual aid to demonstrate the transformation from (left) the z -plane to (right) the w -plane. The hatch shading represents the region outside the equipotential line containing the fringe electric fields. The smooth fill represents the substrate filling regions. When transformed to the w -plane, the substrate boundary becomes curved, while the flux line becomes linear [79].

Figure 4.4 demonstrates how the electric and magnetic fields, because orthogonal fields are maintained, are transformed through Wheeler's conformal mapping. Later publications use Fig-

ure 4.4b, but reverse the conductor voltages, which also reverses the field directions so the w -plane can be evaluated like a parallel plate capacitor. By convention, the remainder of this work will depict the w -plane as an analogous mixed media capacitor.

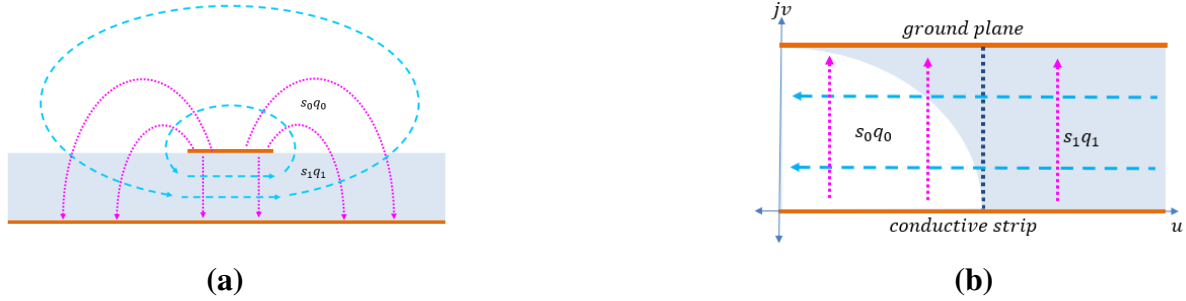


Figure 4.4: Transformation of $\vec{E}$ -fields and $\vec{H}$ -fields from (a) the z -plane to (b) the actual fields in the w -plane. The direction of fields in (b) are conventionally reversed to resemble a mixed media capacitor.

Effective Width

To calculate the filling fraction, q , on the w -plane the effective width, w_{ef} , must first be found. The effective width of the conductive strip compensates for fringing electric fields and can be found from the flux ratio for wide to very wide strips by

$$\begin{aligned}
 w_{ef} &= w + 2\Delta w \\
 &= w + \frac{2h}{\pi} \ln \left[2\pi e \left(\frac{h}{2w} + 0.92 \right) \right],
 \end{aligned} \tag{4.4}$$

and $2\pi e = 17.09$ can be substituted for ease of calculation [80]. The value of Δw rapidly reaches a limit which for very wide strips is $\Delta w < \frac{h}{\pi} \ln 4$ [79]. When related to the total effective width, this is not a true limit, but the maximum fringing electric fields that can be accounted for without exciting surface waves or higher order modes [79].

Filling Fractions

Wheeler utilizes the flux ratio to find the filling fraction of the substrate material, which will be step-wise translated into the standard notation. The filling fraction q is a w -plane ratio defined by

the area of a specific medium divided by the total area of the w -plane and given by

$$\begin{aligned} q &= 1 - \frac{a'}{g'} \left(1 - \frac{s'}{a'} \right) \\ &= 1 - \frac{b}{\pi a} \ln \frac{2\pi a}{b} \left(1 - \frac{s'}{a'} \right), \end{aligned} \quad (4.5)$$

where $\frac{a'}{g'}$ is the flux ratio, $\frac{a}{b} = \frac{w_{ef}}{2h}$ is the shape ratio in standard notation, and $\frac{s'}{a'}$ is the area to the left of the flux line in the w -plane that is occupied by substrate also called the area fraction [80], [79]. Wheeler demonstrates the maximum of the area fraction is well estimated by the area outside an ellipse inscribed in a rectangular quadrant, approximated by $\frac{s'}{a'} < \left(1 - \frac{\pi h w_{ef}}{8} \right)$ [79]. The maximum of $\frac{s'}{a'}$ is reached near the square condition when $h = \frac{w_{ef}}{2}$, so the maximum is given by $\max(\frac{s'}{a'}) = \left(1 - \frac{\pi}{4} \right)$ [79]. Converting to standard notation for the shape variables and using the maximum area fraction, the filling fraction from Equation 4.5 becomes

$$q = 1 - \frac{h}{2w_{ef}} \ln \left(\frac{\pi w_{ef}}{h} - 1 \right), \quad (4.6)$$

where h is the substrate height and $\frac{w_{ef}}{2}$ is half the effective width that represents the conductive strip in the microstrip half-plane [79], [81].

To simplify visualization of the w -plane's mixed-material filling ratios, the filling fraction, or area with three bounds at the curved material boundary, flux line, and conductor is approximated, the area of Figure 4.2c is redrawn as a rectangle which approximates the total area of that material

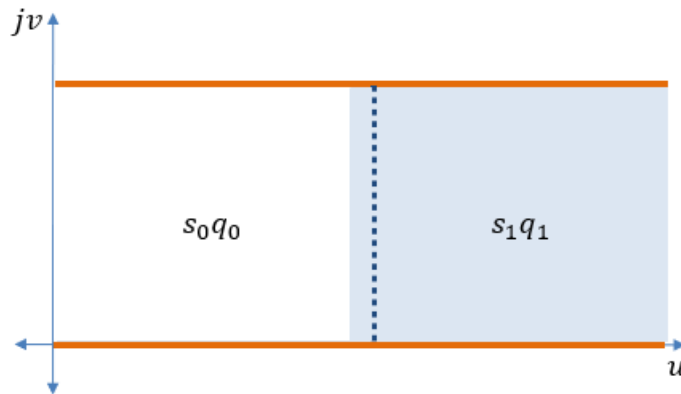


Figure 4.5: Wheeler's approximated material boundaries in the w -plane [79].

as demonstrated in Figure 4.5 [79].

Wave Resistance

Wheeler asserts that the wave resistance at the flux line is the same as the resistance of a balanced pair of strips [79], [80]. The flux line on the z -plane is a circle with center at $w/2$ and radius equal to the height [80]. The flux-bisection boundary joins the potential boundaries at right angles in the w -plane and the width is divided proportionally to the flux ratio while the substrate height remains equivalent to that of the z -plane, $h = \pi$ [80]. From Equation 4.1 he finds $d = w + 1 + \ln \left(2\pi \left(\frac{w}{2\pi} + 0.94 \right) \right)$ used to find the ratio for $\frac{w}{2h} > 0.90$, very wide strips

$$\frac{R_0}{R} = \frac{w}{2h} + \frac{1}{\pi} \ln \left[2\pi e \left(\frac{w}{2h} + 0.92 \right) \right], \quad (4.7)$$

where $e=2.71828$ is Euler's number and 0.92 is used instead of 0.94 to correct the relation for $R/R_0 = \frac{1}{2}$ [80]. The formula is then modified to account for a dielectric substrate

$$\frac{R_0}{R} = \frac{\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\varepsilon_r + 1}{2\pi\varepsilon_r} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\varepsilon_r - 1}{2\pi\varepsilon_r^2} \ln \frac{e\pi^2}{16}}{\sqrt{\frac{1}{\varepsilon_r}}}, \quad (4.8)$$

where ε_r is the dielectric constant and the additional terms interpolate between the upper and lower bounds of the effective filling fraction [79]. Equation 4.8 is accurate for wide strips, $\frac{w}{h} > 1$, with a relative error less than 1% [79].

Wheeler's Continuous Formulas

In continued works, Wheeler went on to formulate expressions that bridge the discontinuity between the formulas derived for narrow and wide strip widths [84]. The caveat to utilizing these continuous formulas is the reduced accuracy as the formulas reach the extrema of strip widths [84]. The increased error limits utility of the continuous formulas for effective width and characteristic impedance in application to MPAs, therefore the more precise formulas for wide strips from earlier

works will be retained [79], [80], [84].

Wheeler's later approach to CM no longer requires a filling fraction for evaluation, but he gives

$$q = \frac{1}{2} \left(1 + \frac{1}{\sqrt{1 + \frac{10h}{w}}} \right), \quad (4.9)$$

an empirical formula found by Schneider as a good approximation of the substrate filling fraction [84]. Equation 4.9 is notable because it works well across all shape ratios without the discontinuity between narrow and wide strip widths, although like the other formulas given, Equation 4.9 also lacks some of the shape peculiar at each extrema [84].

4.3 Developments in Conformal Mapping Techniques

Wheeler's CM methods were limited to simple substrate supported microstrip structures, but laid the foundation which has been extended to additional microstrip cases.

4.3.1 Layered Microstrip Filling Fractions

Svacina applied Wheeler's filling fractions and material boundary approximation to a microstrip with a flush superstrate as shown in Figure 4.6 [81]. Svacina retained Equations 4.4, 4.6, and 4.8 to derive a value for the superstrate filling fraction and height of the superstrate, h_2 [81]. The height of the superstrate on the w -plane is analogous to the voltage at that point between conductors of a parallel plate capacitor

$$v_e = \frac{2h}{\pi} \arctan \left[\frac{\pi}{\frac{\pi w_{ef}}{2h} - 2} \left(\frac{h_2}{h} - 1 \right) \right], \quad (4.10)$$

where w_{ef} is found by Wheeler's method in Equation 4.4 [81]. Svacina then used Equations 4.6 and 4.10 to determine the filling fractions for each material

$$\begin{aligned}
q_1 &= \frac{A_{s_1}}{A_{total}} = 1 - \frac{1}{2} \frac{\ln \left(\frac{\pi w_{ef}}{h} - 1 \right)}{\frac{w_{ef}}{h}} \\
q_2 &= 1 - q_1 - \frac{A_{s_0}}{A_{total}} = 1 - \frac{1}{2} \frac{h - v_e}{w_{ef}} \ln \left[\frac{\pi w_{ef}}{h} \frac{\cos \left(\frac{v_e \pi}{2h} \right)}{\pi \left(\frac{h_2}{h} - \frac{1}{2} \right) + \frac{v_e \pi}{2h}} + \sin \left(\frac{v_e \pi}{2h} \right) \right] \\
q_0 &= \frac{A_{s_0}}{A_{total}} = 1 - q_1 - q_2,
\end{aligned} \tag{4.11}$$

where A is the ratio of each medium to the total area in the w -plane, w_{ef} uses Wheeler's Equation 4.4 for very wide strips, subscript one represents the substrate, subscript two represents the superstrate, and free space is assumed to fill the region above the superstrate [81].

4.3.2 Finding Effective Relative Permittivity with CM

Svacina applied the filling fractions found by in Equation 4.11, approximated material boundaries, and effective field theory to derive an equation for the effective relative permittivity

$$\begin{aligned}
\varepsilon_{r_{ef}} &= \varepsilon_{r_1} q_1 + \varepsilon_{r_2} q_2 \parallel \varepsilon_{r_0} q_0 \\
&= \varepsilon_{r_1} q_1 + \frac{\varepsilon_{r_0} \varepsilon_{r_2} (q_0 + q_2)^2}{\varepsilon_{r_0} q_0 + \varepsilon_{r_2} q_2} \\
&= \varepsilon_{r_1} q_1 + \frac{\varepsilon_{r_0} \varepsilon_{r_2} [(1 - q_1 - q_2) + q_2]^2}{\varepsilon_{r_0} (1 - q_1 - q_2) + \varepsilon_{r_2} q_2} \\
&= \varepsilon_{r_1} q_1 + \frac{\varepsilon_{r_2} (1 - q_1)^2}{\varepsilon_{r_2} (1 - q_1 - q_2) + q_2},
\end{aligned} \tag{4.12}$$

where ε_{r_0} and q_0 are free space quantities and $q_0 = 1 - q_1 - q_2$ [81].

Corrected Filling Fractions

Bernhard and Tousignant found q_2 from Equation 4.11 to overestimate the flush superstrate filling fraction since q_2 does not go to zero without a flush superstrate when $h_2 \rightarrow h_1$ [26]. To compensate for this limitation [26] introduces an additional filling fraction, q_4 [26]. The value of q_4 is equal

to half the value of q_2 in the limit, $\lim_{h_2 \rightarrow h}$ [26]. To give more generalizable filling fractions, the uppermost region is assigned the subscript three instead of assuming free space [26]. Figure 4.6 shows a comparison of each of the CM planes for comparison of between both methods [26], [81].

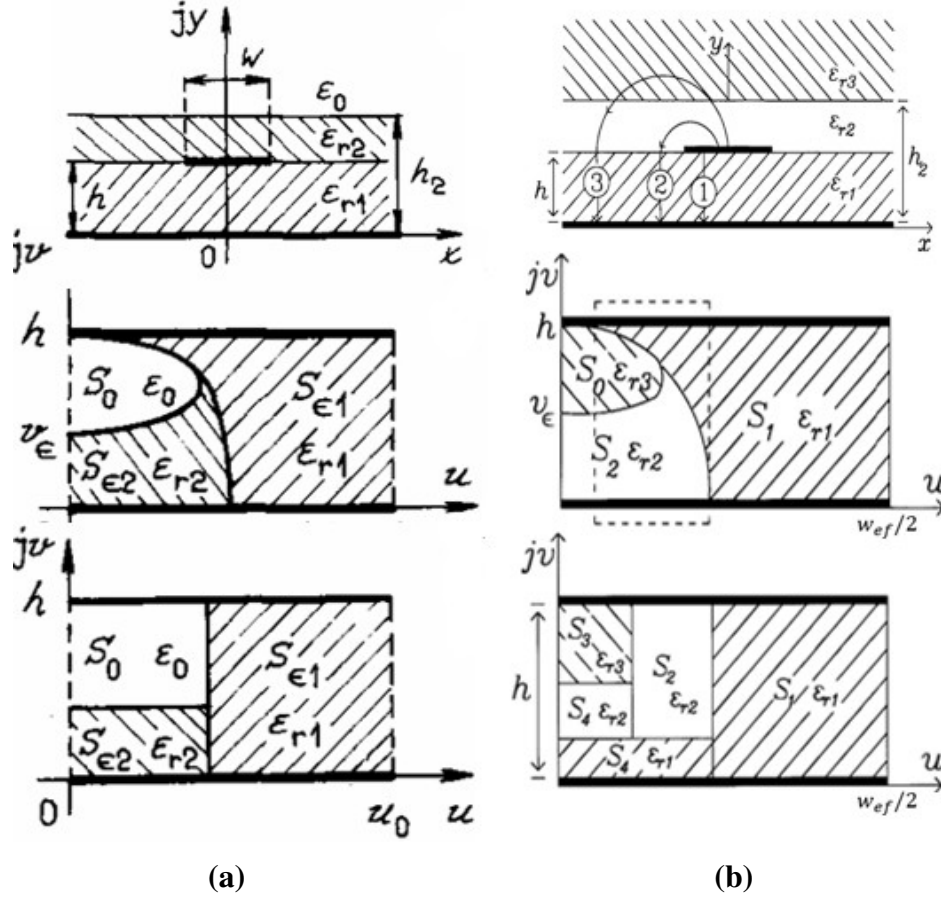


Figure 4.6: Comparison between CM transform planes for layered microstrip configurations as published by (a) Svacina [81] and (b) Bernhard and Tousignant [26]. Both demonstrate comparable configurations. (Top) z -plane, (Middle) w -plane, and (Bottom) w -plane with approximated material boundaries.

The corrected and generalized form of the filling fractions are given by

$$\begin{aligned}
 q_1 &= 1 - \frac{h}{2w_{ef}} \ln \left(\frac{\pi w_{ef}}{h} - 1 \right) - q_4 \\
 q_2 &= 1 - q_1 - q_3 - 2q_4 \\
 q_3 &= \frac{1}{2} \frac{h - v_e}{w_{ef}} \ln \left[\frac{\pi w_{ef}}{h} \frac{\cos \left(\frac{v_e \pi}{2h} \right)}{\pi \left(\frac{h_2}{h} - \frac{1}{2} \right) + \frac{v_e \pi}{2h}} + \sin \left(\frac{v_e \pi}{2h} \right) \right] \\
 q_4 &= \frac{h}{2w_{ef}} \ln \left(\frac{\pi}{2} - \frac{h}{2w_{ef}} \right)
 \end{aligned} \tag{4.13}$$

[26].

Preserving Electric Flux Paths

Bernhard and Tousignant additionally noted that the electric flux path was not well preserved by Svacina's approximations, so they utilized the q_4 filling fraction to approximate the material boundaries in a way that preserves the electric flux path seen in Figure 4.6 [26], [81]. Their newly derived expression to calculate the effective relative permittivity in a more generalized form is given by

$$\begin{aligned}
\varepsilon_{ref} &= \varepsilon_{r1} q_1 + \varepsilon_{r2} q_2 \left| \varepsilon_{r1} q_4 + \varepsilon_{r3} q_3 \right| \varepsilon_{r2} q_4 \left| \varepsilon_{r1} q_4 \right| \\
&= \varepsilon_{r1} q_1 + \frac{\varepsilon_{r1} \varepsilon_{r2} q_2 q_4}{\varepsilon_{r1} q_4 + \varepsilon_{r2} q_2} + \frac{\varepsilon_{r1} \varepsilon_{r2} \varepsilon_{r3} q_3 q_4^2}{\varepsilon_{r1} \varepsilon_{r2} q_4^2 + \varepsilon_{r1} \varepsilon_{r3} q_3 q_4 + \varepsilon_{r2} \varepsilon_{r3} q_3 q_4} \\
&= \varepsilon_{r1} q_1 + \varepsilon_{r1} (1 - q_1)^2 \\
&\quad \times \frac{\varepsilon_{r2}^2 q_2 q_3 + \varepsilon_{r2} \varepsilon_{r3} (q_2 q_4 + (q_3 + q_4)^2)}{\varepsilon_{r2}^2 q_2 q_3 q_4 + \varepsilon_{r1} (\varepsilon_{r2} q_3 + \varepsilon_{r3} q_4) (1 - q_1 - q_4)^2 + \varepsilon_{r2} \varepsilon_{r3} q_4 (q_2 q_4 + (q_3 + q_4)^2)},
\end{aligned} \tag{4.14}$$

which can be applied to many general cases of microstrip configurations including substrate only, flush superstrates, gapped superstrates, and layered flush superstrates [26].

4.3.3 Improved Formula for Effective Width

Bernhard and Tousignant derived a new formula for the effective width of a microstrip conductor with flush or gapped superstrates which requires an iterative sequence to find a more accurate value for the effective width of the microstrip conductor in the presence of flush or gapped superstrates [26]. The effective width derived by [26] can be found by

$$w_{ef} = \sqrt{\frac{\varepsilon'_r}{\varepsilon_{ref}}} \left[w + 0.882h + 0.164 \frac{h (\varepsilon'_r - 1)}{(\varepsilon'_r)^2} \right] + \sqrt{\frac{\varepsilon'_r}{\varepsilon_{ref}}} \frac{h (\varepsilon'_r - 1)}{\pi \varepsilon'_r} \left[\ln \left(\frac{w}{2h} + 0.94 \right) + 1.451 \right], \tag{4.15}$$

where the first iteration approximates $\varepsilon'_r \approx \varepsilon_{r_1}$ and $\varepsilon_{r_{ef}} \approx \varepsilon'_r$ [26]. The first iteration of w_{ef} is used for the initial calculation of Equations 4.10, 4.13, and 4.14 [26].

The second iteration recalculates w_{ef} from Equation 4.15 using a ratio between an ideal single substrate microstrip structure that would mimic the multi-superstrate structure under consideration and the first iteration result for $\varepsilon_{r_{ef}}$ [26]. The ideal single substrate microstrip structure would have a substrate with a relative permittivity, ε'_r , found by

$$\varepsilon'_r = \frac{2\varepsilon_{r_{ef}} - 1 + A}{1 + A}, \quad (4.16)$$

where $\varepsilon_{r_{ef}}$ is the initial result from Equation 4.14 and $A = \sqrt{\left(1 + \frac{10h}{w_{ef}}\right)}$ [26]. The second iteration of Equation 4.15 is followed by re-evaluating Equations 4.10, 4.13, and 4.14 [26].

4.4 CM Methods for Effective Relative Permeability

Several authors have worked to extend the conformal mapping technique to work well with MD materials. Pucel and Masse applied the duality of Maxwell's equations, $\varepsilon_r \propto \frac{1}{\mu_r}$, and Wheeler's work to find an expression for effective relative permeability, $\mu_{r_{ef}}$ [82]. Yeo and Lancaster revisited this concept and from Wheeler's later works found an expression without the discontinuity of Pucel and Masse [82], [83], [84]. Methods given by [82] and [83] were significant improvements over taking a rough average of relative permeability, but were developed in application to very simple microstrip configurations without consideration of preserving the magnetic flux paths. There still remains room for improved methods to be developed.

Wheeler utilized the speed ratio to extract the effective permittivity from the velocity of a wave in the effective media [79], [80]. This speed ratio was defined as

$$\frac{R}{R_0} = \frac{1}{\sqrt{\varepsilon_r}}, \quad (4.17)$$

where R is the resistance of the microstrip configuration scaled by the material's constitutive

parameters and R_0 the characteristic resistance a balanced pair of strips in free-space [79], [80]. [83] used the inverse of Wheeler's speed ratio squared to solve for the effective relative permittivity given by

$$\begin{aligned}\varepsilon_{r_{ef}}(\varepsilon_r) &= \left[\frac{R_0(1)}{R_c(\varepsilon_r)} \right]^2 \\ &= \frac{\varepsilon_r + 1}{2} \left[\frac{\ln 1 + 2a^2 + a\sqrt{4a^2 + \pi^2}}{\ln 1 + 2a^2b + a\sqrt{a^2b^2 + c\pi^2}} \right]^2,\end{aligned}\quad (4.18)$$

where

$$a = \frac{4h}{w} \quad b = \frac{14 + \frac{8}{\varepsilon_r}}{11} \quad c = \frac{1 + \frac{1}{\varepsilon_r}}{2} \quad (4.19)$$

[84]. [83]. Yeo next invoked reciprocity by substituting $\mu_{r_{ef}}(\mu_r) \rightarrow \frac{1}{\varepsilon_{r_{ef}}(\frac{1}{\varepsilon_r})}$ to find

$$\mu_{r_{ef}}(\mu_r) = \frac{2\mu_r}{\mu_r + 1} \left[\frac{\ln 1 + 2a^2b' + a\sqrt{a^2b'^2 + c'\pi^2}}{\ln 1 + 2a^2 + a\sqrt{4a^2 + \pi^2}} \right]^2, \quad (4.20)$$

where

$$b' = \frac{14 + 8\mu_r}{11} \quad c' = \frac{1 + \mu_r}{2}, \quad (4.21)$$

and a is given in Equation 4.19. Yeo's method is derived from Wheeler's less accurate, but continuous resistance equation introducing greater error for very wide strip widths and not applicable to superstrate configurations [84], [83].

Saini et al. found an analytical solution for the effective relative permeability of a MPA given by

$$\mu_{r_{ef}} = \left[\left(\frac{\mu_r + 1}{2\mu_r} + \frac{(1 - \mu_r)f_1}{2\mu_r} \right) \times f_2 \right]^{-1}, \quad (4.22)$$

where

$$f_1(W/h, \mu_r, \varepsilon_r) = (10u\mu_r\varepsilon_r^2)^{-0.001u(\mu_r+0.2\varepsilon_r)} \quad u = \frac{W}{h} \quad f_2(\varepsilon_r) = 0.98 - 0.08e^{-0.157\varepsilon_r}, \quad (4.23)$$

however is limited in accuracy for substrates with high values of μ_r [1]. [1] gave Equation 4.22 in the inverse form, but when solving for μ_{ref} results in decreasing μ_{ref} as μ_r of the substrate increases suggesting an algebraic error in the published formula [1].

4.4.1 Hargraves Dual Method

Bernhard and Tousignant found a highly accurate general expression to find the effective relative permittivity applicable to a microstrip with a flush or a gapped superstrate given by Equation 4.14 [26]. According to methods by [40] and [82], reciprocity can be invoked and applied directly a ε_{ref} formula by simply swapping in $\frac{1}{\mu_r}$ for ε_r when relative permeability is not unitary. Application of reciprocity to Bernhard and Tousignant's formula for ε_{ref} will be identified as the Hargraves Dual Method for effective relative permeability. The dual formula is given by

$$\mu_{ref}^{dual} = \left[\frac{q_1}{\mu_{r1}} + \frac{(1 - q_1)^2}{\mu_{r1}} \times \frac{\frac{q_2 q_3}{\mu_{r2}^2} + \frac{q_2 q_4 + (q_3 + q_4)^2}{\mu_{r2} \mu_{r3}}}{\frac{q_2 q_3 q_4}{\mu_{r2}^2} + \frac{(\frac{q_3}{\mu_{r2}} + \frac{q_4}{\mu_{r3}})(1 - q_1 - q_4)^2}{\mu_{r1}} + \frac{q_4(q_2 q_4 + (q_3 + q_4)^2)}{\mu_{r2} \mu_{r3}}} \right]^{-1}. \quad (4.24)$$

[26]. Bernhard and Tousignant's method approximated material boundaries for conservation of the electric field paths, which are partially confined within the substrate only [26].

The predominant magnetic flux paths of the microstrip would be normal to the electric fields, therefore horizontally oriented in the image plane seen in Figure 4.9b. Comparing Figure 4.9b to Figure 4.9c, it is evident that using Bernhard and Tousignant's approximated boundaries does not well represent or preserve the magnetic flux paths [26]. Visual assessment of Figure 4.9c clearly indicates that Equation 4.24 derived from Equation 4.14 is not sufficient to accurately model the structure's effective relative permeability. There remains a need for a more accurate CM model to estimate the effective relative permeability.

4.4.2 Approximating Magnetic Material Boundaries

Time-Harmonic EM Fields

CM must preserve the magnitude and angle at each point from one orthogonal plane to another orthogonal plane to satisfy Laplace's equation for potential fields, applied by Wheeler in the electrostatics case [75], [79], [80]. Wheeler used this concept to map the electric field vectors and the orthogonal equipotential boundaries onto a conformal plane [80]. The Hargraves CM method assumes the electric and magnetic fields are approximately orthogonal under quasi-TEM conditions as shown in Figure 4.4. This assumption means that the magnetic field vector can be substituted for Wheeler's equipotential boundary to approximate the effective relative permeability for microstrip structures with MD materials.

To extend Wheeler's electrostatic CM technique into a time-harmonic EM case, the CM technique is required to satisfy Helmholtz equations [75]. Wheeler's analysis of this CM method has already proven to satisfy the Laplace requirements for gradient fields, a special case of Helmholtz equations, therefore this requirement has already been satisfied [75], [79], [80]. Combined with careful consideration to preserve magnetic field paths, the proposed Hargraves CM Method will give a generalized formula to find the effective relative permeability of a microstrip structure with or without layered superstrate media.

Preserving the magnetic flux paths is essential to achieve accuracy in the calculation for effective relative permeability. The approximated material boundaries must be estimated in the direction of the magnetic flux vector, similarly to Bernhard and Tousignant's boundary approximations in the direction of the electric flux vector [26]. The material boundaries will be approximated to preserve the magnetic flux path through each material boundary.

EM Boundary Conditions

Section 4.2 demonstrated that the w -plane of a microstrip structure is distorted, resulting in curvilinear material boundaries in favor of linearized electric and magnetic fields. Interaction of the fields

at material boundaries can be decomposed into the perpendicular and tangential field components relative to the normal vector.

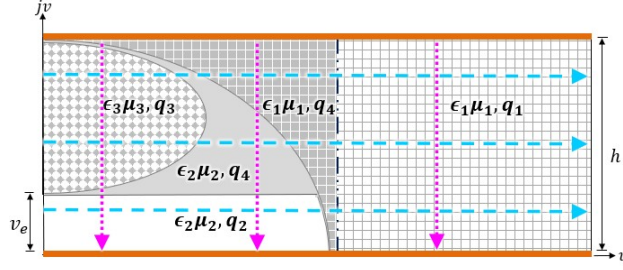


Figure 4.7: The Conformal Mapping (CM) image plane is with curvilinear material boundaries. The shaded regions represent the additional filling of the substrate and flush superstrate affected by the correction term, q_4 .

Ampere's and Faraday's laws are used to derive the boundary condition for the tangential, or parallel component of the magnetic field, $\vec{H}_{\parallel}$, to the surface plane and is found by

$$\hat{n} \times (\vec{H}_{1\parallel} - \vec{H}_{2\parallel}) = \vec{J}_s, \quad (4.25)$$

where $\vec{J}_s$ is the surface current. If medium two is non-conductive, Equation 4.25 is simplified by setting $\vec{J}_s = 0$, such is the case of dielectric and MD interfaces.

The perpendicular component of magnetic field boundary conditions are derived from the integral form of Gauss' Law. This derivation guides the requirements for magnetic field components at normal incidence, $\mu \vec{H}_{\perp}$, to the material interface which is described as

$$\hat{n} \cdot (\mu_1 \vec{H}_{1\perp} - \mu_2 \vec{H}_{2\perp}) = 0, \quad (4.26)$$

where $\hat{n}$ is the unit vector normal to the boundary and points from the surface of medium two into medium one. This condition enforces continuity of the perpendicular component of the magnetic field across the boundary, but the ratio of transmission to reflection is influenced by the impedance match or mismatch. If the second medium is non-conductive, the perpendicular component given by Equation 4.26 must be continuous across the boundary.

Magnetic fields are more vulnerable than electric fields to destructive interference from reflec-

tions at unmatched impedance boundaries. This vulnerability is due to the reflected magnetic field experiencing a 180-degree phase shift to flip the direction of the propagating vector also called the Poynting vector. Reflected magnetic fields experience a destructive phase interference between the incident and reflected wave. If equal in magnitude such as complete reflection, the reflected field can cause complete wave destruction.

The angle of incidence at the boundary of each medium is also important to evaluate for phenomena such as the Brewster and Critical angles which together define the conditions for reflection and transmission governed by Fresnel's equations. The critical angle is more significantly related to angle of incidence causing a total reflection, which as described above would result in complete destructive interference. The Brewster angle relates to the polarization of the incident and reflected waves. Layered and anisotropic media further complicate these mechanisms, often resulting in phasing which oscillates between regions of constructive and destructive interference.

Evaluation of the generalized CM with curvilinear material boundaries shown in Figure 4.7 makes it apparent that the tangential magnetic field meets the requirement to be discontinuous along the conductive PEC surfaces, leaving only the perpendicular field component to consider at non-conductive material boundaries adjacent to the PEC upper and lower margins. The paraboloid contour of medium three creates a primarily parallel or grazing angle of interaction for the majority of the medium. It seems intuitive that medium two and medium three should both be conserved by adding the regions in parallel, but only when the height of medium two is large. When medium two is thin, it primarily fills the region adjacent to the discontinuity along the conductors. This analogy is generally the case for all material boundaries in the CM of a microstrip with layered superstrate. Only the series components of the magnetic fields at each material boundary is a significant contributor.

As frequency increases or for very high constitutive parameters, the wavelength will decrease until the material boundaries become electrically large requiring the tangential component to be considered when approximating each material boundary. Once the primary tangential and perpendicular magnetic field contributors are defined, they drive the orientation for material boundary

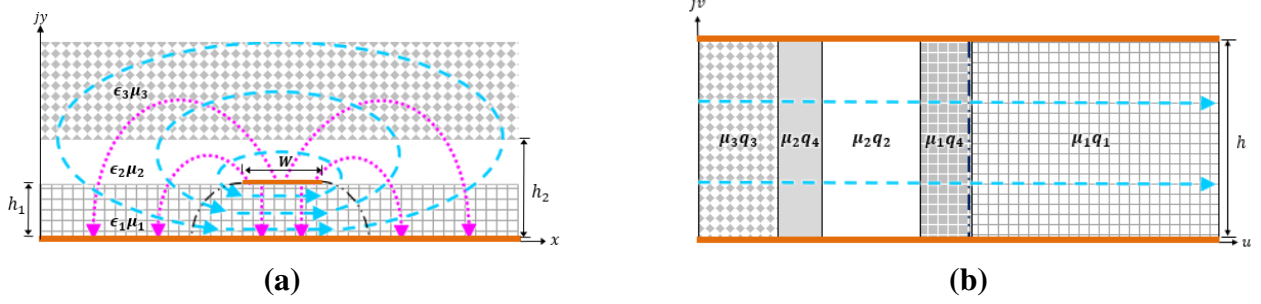


Figure 4.8: The Hargraves CM Method for magnetic fields. (a) Microstrip cross section with quasi-TEM fields. (b) CM material boundary approximation conserving the predominant magnetic field paths.

approximation. Integration along the magnetic field lines add in series at perpendicular material boundaries and added in parallel at parallel non-conductive material boundaries. For the purposes of the frequencies and material parameters explored in this thesis, the predominate component of the magnetic flux density is the perpendicular portion which is continuous across non-conductive boundaries. Integration along the magnetic fields for a microstrip with or without layered superstrate media is conducted by adding the contribution of each medium in series as depicted in Figure 4.9d.

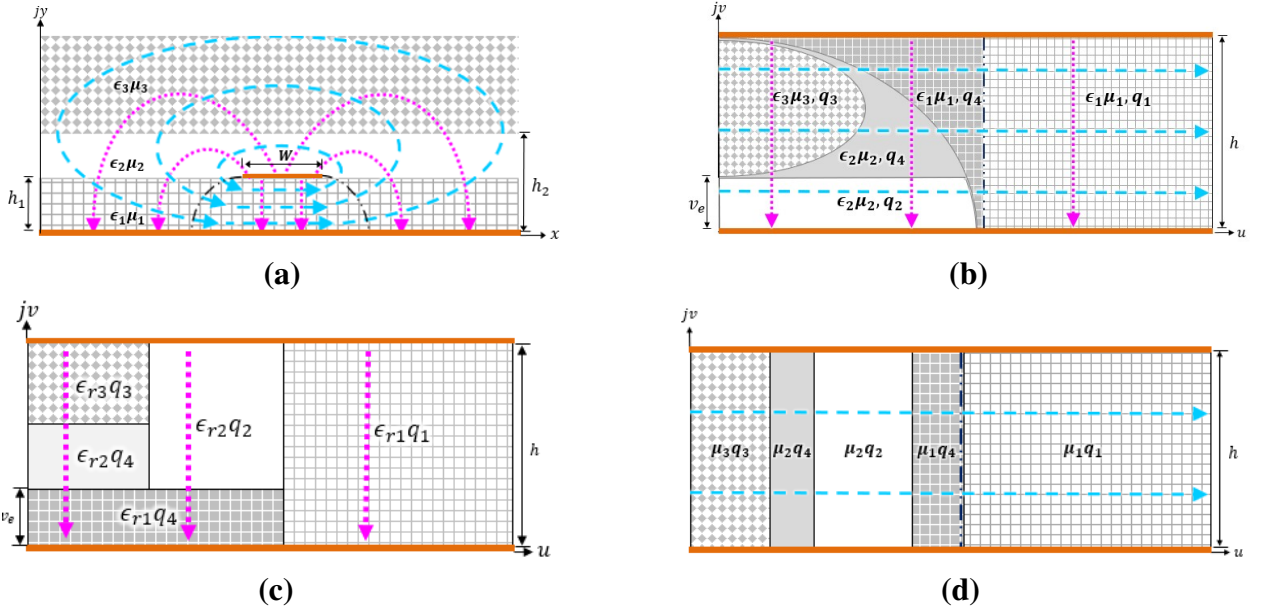


Figure 4.9: Summary of the presented CM planes. (a) Predominant $\vec{E}$ -fields and $\vec{H}$ -fields of a microstrip with a layered superstrate (b) Svacina's CM plane [81] (c) Bernhard and Tousignant's approximated material boundaries to preserve electric flux paths [26] (d) Hargraves CM Method to approximate material boundaries while conserving the magnetic flux paths.

4.4.3 Derivation of the Hargraves CM Method

Material boundaries are approximated as vertical bounds perpendicular to the magnetic field angle of incidence. The material boundaries were approximated to conserve the magnetic flux paths where each medium is in series as shown in Figure 4.9d.

To find the effective relative permeability, the newly approximated material boundaries from Figure 4.9d are utilized with Equation 4.13's filling fractions [26]. The variable s can represent each sub- or superstrate medium to derive and simplify the formula prior to substituting in the appropriate constitutive value. The s is later swapped for the relative permeability and utilizes the reciprocal relationship $\varepsilon_r \rightarrow \frac{1}{\mu_r}$. The effective relative permeability is found by the relation of the magnetic field vector to each material boundary. Boundaries parallel to the magnetic flux line add in parallel and the material boundaries normal to the magnetic flux lines add in series to find,

$$s_{ref} = s_{r3}q_3 + s_{r2}q_2 + s_{r2}q_4 + s_{r1}q_1 + s_{r1}q_4, \quad (4.27)$$

where s is a temporary variable representing the constitutive parameter, q the material filling fraction, and subscripts represent each layer as seen in the z -plane.

The inverse relationship of effective relative permeability to permittivity is evident once $s = \frac{1}{\mu_r}$ is substituted into Equation 4.27 resulting in

$$\begin{aligned} \frac{1}{\mu_{ref}} &= \frac{q_3}{\mu_{r3}} + \frac{q_4}{\mu_{r2}} + \frac{q_2}{\mu_{r2}} + \frac{q_4}{\mu_{r1}} + \frac{q_1}{\mu_{r1}} \\ &= \frac{q_3}{\mu_{r3}} + \frac{q_2 + q_4}{\mu_{r2}} + \frac{q_1 + q_4}{\mu_{r1}}. \end{aligned} \quad (4.28)$$

The form given by Equation 4.28 must be inverted to solve for the effective relative permeability.

The resulting equation due to inverting to solve for μ_{ref} resembles the results found by adding each

region in parallel

$$\begin{aligned}
\mu_{ref}^{CM} &= \left[\frac{q_3}{\mu_{r3}} + \frac{(q_2 + q_4)}{\mu_{r2}} + \frac{(q_1 + q_4)}{\mu_{r1}} \right]^{-1} \\
&= \frac{1}{\left(\frac{q_3}{\mu_{r3}} + \frac{q_2 + q_4}{\mu_{r2}} + \frac{q_1 + q_4}{\mu_{r1}} \right)} \\
&= \frac{\mu_{r1}\mu_{r2}\mu_{r3}}{\mu_{r1}\mu_{r2}q_3 + \mu_{r1}\mu_{r3}(q_2 + q_4) + \mu_{r2}\mu_{r3}(q_1 + q_4)}.
\end{aligned} \tag{4.29}$$

Equation 4.29 represents the novel Hargraves CM Method for calculation of μ_{ref} .

The Hargraves Dual and Hargraves CM Methods given by Equations 4.24 and 4.29, respectively, will be further investigated for model validation of MD microstrip configurations with or without layered superstrates in Chapter 5.

4.4.4 Modified Characteristic Impedance

Recall Wheeler's Equation 4.8 for wide conductors, $\frac{w}{h} > 1$, with a dielectric substrate for the half microstrip plane can be written as

$$\frac{R}{R_0} = \frac{\sqrt{\frac{1}{\varepsilon_r}}}{\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\varepsilon_r - 1}{2\pi\varepsilon_r} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\varepsilon_r - 1}{2\pi\varepsilon_r^2} \ln \frac{e\pi^2}{16}},$$

where ε_r is the dielectric constant and $e = 2.71828$ is Euler's number [79]. Now recall that Wheeler invoked two symmetry plane assumptions in Section 4.2 describing the initial steps of this CM derivation [80]. The first symmetry assumption halved the characteristic impedance by using the image plane, then doubled the characteristic impedance by retaining only the right half of the microstrip cross section seen in Figure 4.1 [79], [80]. To reverse the second symmetry assumption for the twin line derivation, the impedance should be divided by a factor of two.

To account for effective relative permeability of MD materials, the numerator becomes $\sqrt{\frac{\mu_{ref}}{\varepsilon_{ref}}}$ to have the form of free-space impedance. Yeo initially proposed this for Wheeler's impedance formula, however did not account for Wheeler's symmetry assumptions [79], [80]. The corrected

impedance formula is given by

$$Z_0 = \frac{\eta_0 \sqrt{\frac{\mu_{r_{ef}}}{\varepsilon_{r_{ef}}}}}{2 \left(\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\varepsilon_r - 1}{2\pi\varepsilon_r} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\varepsilon_r - 1}{2\pi\varepsilon_r^2} \ln \frac{e\pi^2}{16} \right)} \quad (4.30)$$

[83].

4.5 Complete CM Procedure

The CM procedure for a microstrip configuration has been well established. This thesis proposes a method to approximate the material boundaries while preserving the magnetic flux paths. This methodology could be replicated to approximate more complex structures. To utilize the proposed effective relative permeability calculation for a simple microstrip or a microstrip with a superstrate that is flush or air-gapped, the user must simply utilize the following mathematical procedure without need to repeat the CM procedure given previously. Once the effective constitutive parameters have been determined, the user can use these values in design, optimization, or analysis of microstrip structures or MPA by utilizing the CM output values as input values to their modeling option of choice. Chapters 5, 6, and 7 will explore results from the CM procedure.

1. The initial step of this procedure is to approximate the effective width of the microstrip using Equation 4.15,

$$w_{ef} = \sqrt{\frac{\varepsilon'_r}{\varepsilon_{r_{ef}}}} \left[w + 0.882h + 0.164 \frac{h(\varepsilon'_r - 1)}{(\varepsilon'_r)^2} \right] + \sqrt{\frac{\varepsilon'_r}{\varepsilon_{r_{ef}}}} \frac{h(\varepsilon'_r - 1)}{\pi\varepsilon'_r} \left[\ln \left(\frac{w}{2h} + 0.94 \right) + 1.451 \right],$$

where $\varepsilon'_r = \varepsilon_{r_{ef}} = \varepsilon_{r_1}$ [26].

2. The flush or gapped superstrate fills the region indicated by material two in the z -plane. The z -plane height is transformed to a w -plane value, v_e , by the same method utilized in

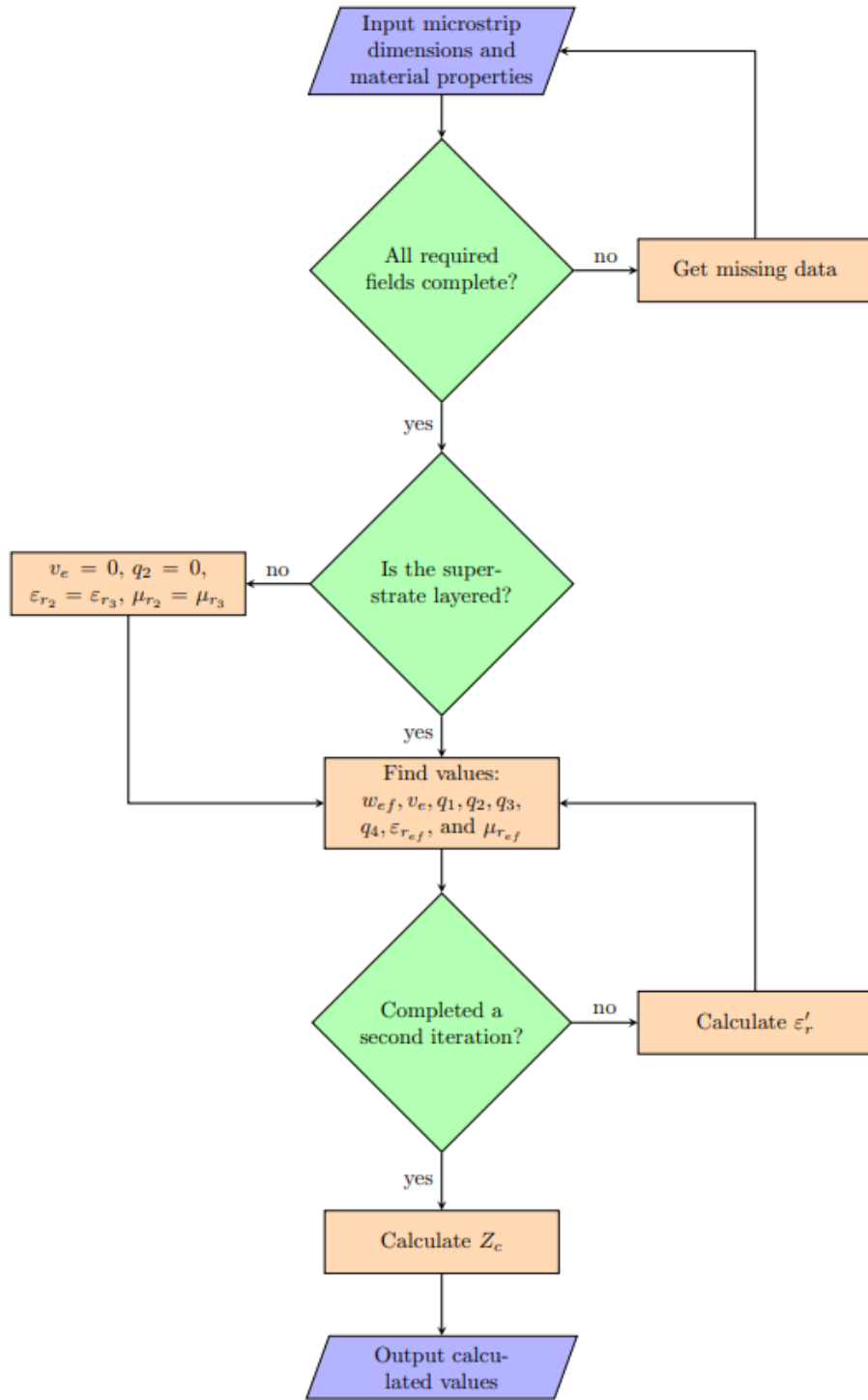


Figure 4.10: A block diagram representing the presented Conformal Mapping (CM) algorithm.

Equation 4.10 by both [26] and [81]

$$v_e = \frac{2h}{\pi} \arctan \left[\frac{\pi}{\frac{\pi w_{ef}}{2h} - 2} \left(\frac{h_2}{h} - 1 \right) \right].$$

3. The values w_{ef} and v_e are used to find the filling fractions per Bernhard and Tousignant's method given by Equations 4.13 [26].

$$\begin{aligned} q_1 &= 1 - \frac{h}{2w_{ef}} \ln \left(\frac{\pi w_{ef}}{h} - 1 \right) - q_4 \\ q_2 &= 1 - q_1 - q_3 - 2q_4 \\ q_3 &= \frac{1}{2} \frac{h - v_e}{w_{ef}} \ln \left[\frac{\pi w_{ef}}{h} \frac{\cos \left(\frac{v_e \pi}{2h} \right)}{\pi \left(\frac{h_2}{h} - \frac{1}{2} \right) + \frac{v_e \pi}{2h}} + \sin \left(\frac{v_e \pi}{2h} \right) \right] \\ q_4 &= \frac{h}{2w_{ef}} \ln \left(\frac{\pi}{2} - \frac{h}{2w_{ef}} \right) \end{aligned}$$

The filling fractions for each material described by the subscript, while subscript four is designated to indicate the curved shaded regions shown in Figure 4.9b that is being approximated. When no superstrate is present, $h_2 = h$ and the other parameters for materials in region two should be updated to reflect the configuration values of $v_e = 0$, $q_2 = 0$, $\varepsilon_{r_2} = \varepsilon_{r_3}$, and $\mu_{r_2} = \mu_{r_3}$ since region two is non-existent.

4. The effective relative permittivity is given by Equation 4.14

$$\begin{aligned} \varepsilon_{r_{ef}} &= \varepsilon_{r_1} q_1 + \varepsilon_{r_1} (1 - q_1)^2 \\ &\times \frac{\varepsilon_{r_2}^2 q_2 q_3 + \varepsilon_{r_2} \varepsilon_{r_3} (q_2 q_4 + (q_3 + q_4)^2)}{\varepsilon_{r_2}^2 q_2 q_3 q_4 + \varepsilon_{r_1} (\varepsilon_{r_2} q_3 + \varepsilon_{r_3} q_4) (1 - q_1 - q_4)^2 + \varepsilon_{r_2} \varepsilon_{r_3} q_4 (q_2 q_4 + (q_3 + q_4)^2)}. \end{aligned}$$

The effective relative permittivity preserves the electric flux paths to represent the mixed media microstrip as a single homogeneous medium [26].

5. Calculate the effective relative permeability with either the Hargraves Dual or CM Method.

Hargraves Dual Method given by Equation 4.24

$$\mu_{r_{ef}}^{dual} = \left[\frac{q_1}{\mu_{r_1}} + \frac{(1-q_1)^2}{\mu_{r_1}} \times \frac{\frac{q_2 q_3}{\mu_{r_2}^2} + \frac{q_2 q_4 + (q_3 + q_4)^2}{\mu_{r_2} \mu_{r_3}}}{\frac{q_2 q_3 q_4}{\mu_{r_2}^2} + \frac{(\frac{q_3}{\mu_{r_2}} + \frac{q_4}{\mu_{r_3}})(1-q_1-q_4)^2}{\mu_{r_1}} + \frac{q_4(q_2 q_4 + (q_3 + q_4)^2)}{\mu_{r_2} \mu_{r_3}}} \right]^{-1}.$$

Hargraves CM Method given by Equation 4.29

$$\mu_{r_{ef}}^{cm} = \frac{\mu_{r_1} \mu_{r_2} \mu_{r_3}}{\mu_{r_1} \mu_{r_2} q_3 + \mu_{r_1} \mu_{r_3} (q_2 + q_4) + \mu_{r_2} \mu_{r_3} (q_1 + q_4)}.$$

6. A second iteration to find w_{ef} using Equation 4.15

$$w_{ef} = \sqrt{\frac{\varepsilon'_r}{\varepsilon_{r_{ef}}}} \left[w + 0.882h + 0.164 \frac{h(\varepsilon'_r - 1)}{(\varepsilon'_r)^2} \right] + \sqrt{\frac{\varepsilon'_r}{\varepsilon_{r_{ef}}}} \frac{h(\varepsilon'_r - 1)}{\pi \varepsilon'_r} \left[\ln \left(\frac{w}{2h} + 0.94 \right) + 1.451 \right],$$

where $\varepsilon_{r_{ef}}$ was found in step 4 above and ε'_r is found by Equation 4.16 [26]. This step is followed by repeating steps 2, 3, 4, and 5 as introduced by Bernhard and Tousignant [26].

7. The proposed characteristic impedance of the microstrip is given by Equation 4.30

$$Z_0 = \frac{\eta_0 \sqrt{\frac{\mu_{r_{ef}}}{\varepsilon_{r_{ef}}}}}{2 \left(\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\varepsilon_r - 1}{2\pi \varepsilon_r} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\varepsilon_r - 1}{2\pi \varepsilon_r^2} \ln \frac{e\pi^2}{16} \right)},$$

where $\eta_0 = 120\pi \approx 377 \Omega$ is the resistance to free-space and $e = 2.71828$ is Euler's number.

4.6 Conformal Mapping Conclusion

Chapter 4 presented two novel methods for utilizing CM methods to estimate the effective relative permeability of microstrip structures. These methods build upon prior CM methods to expand the application to microstrip structures with MD media. Because this work is built upon prior derivations, these have been provided in Chapter 4. Chapter 5 will analyze and compare prior methods with those presented here. Analysis will be followed by experimental verification in

Chapter 6 and validated through parametric simulation in Chapter 7.

Chapter 5

Comparison and Analysis of CM Methods

Chapters 1 through 3 established the need for design and analysis methods for MD microstrip structures. Chapter 4 presented two novel CM techniques to approximate the effective relative permeability of MD microstrip structures.

This chapter will compare each advancement in CM techniques to Wheeler's original analytical solutions. These methods will further be analyzed to evaluate the effect on filling fractions with increased relative constitutive parameters. Additionally, the effective parameters found by each of these methods will be compared and analyzed.

5.1 Continuity With & Without Superstrates

It is important to know the bounds of geometry that may limit a formula's validity. To verify that the μ_{ref} calculation is generally applicable to microstrip configurations without a superstrate, the equation must maintain accuracy at the limit when the height of the flush superstrate goes to the height of the substrate, $\lim_{h_2 \rightarrow h}$. This situation is captured by a standard microstrip as the simplest case of a microstrip trace on a single substrate with free-space above the microstrip as seen in Figure 5.1a. When $h_2 = h$ the filling fraction $q_2 \rightarrow 0$ and the conformal map is simplified to the form derived by Wheeler shown in Figure 4.4b and 5.1b [79]. The approximated material bounds that introduce the q_4 filling fraction remain applicable.

This case only has two materials, μ_{r1} and μ_{r3} , respectively, because the area of q_2 has diminished to zero the filling fraction q_4 is added to the remaining media occupying q_1 and q_3 regions. Using

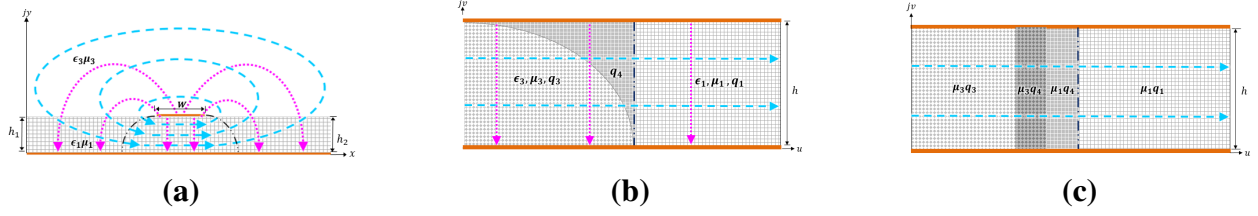


Figure 5.1: Simple microstrip physical and CM representations. (a) Physical cross-section of a microstrip. (b) CM of a simple microstrip. (c) CM with approximated media boundaries for preservation of the magnetic flux path.

the filling fractions given by [26] the effective relative permeability is found by

$$\frac{1}{\mu_{r_{ef}}} = \frac{q_3}{\mu_{r_3}} + \frac{q_4}{\mu_{r_3}} + \frac{q_1}{\mu_{r_1}} + \frac{q_4}{\mu_{r_1}}, \quad (5.1)$$

which is next inverted to solve for $\mu_{r_{ef}}$ the equation simplifies to

$$\mu_{r_{ef}} = \frac{\mu_{r_1} \mu_{r_3}}{\mu_{r_1} (q_3 + q_4) + \mu_{r_3} (q_1 + q_4)}. \quad (5.2)$$

Figures 5.1b and 5.1c demonstrate that when $h_2 = h$, the value of $q_2 \rightarrow 0$ and the left-hand of the w -plane with approximated material boundaries is filled with only q_3 and the q_4 portions of each medium. Therefore in the CM domain this condition can be compensated for by setting $\mu_{r_2} = \mu_{r_3}$. Starting with Equation 4.29 and substituting these values

$$\begin{aligned} \mu_{r_{ef}} &= \frac{\mu_{r_1} \mu_{r_2} \mu_{r_3}}{\mu_{r_1} \mu_{r_2} q_3 + \mu_{r_1} \mu_{r_3} (q_2 + q_4) + \mu_{r_2} \mu_{r_3} (q_1 + q_4)} \\ &= \frac{\mu_{r_1} \mu_{r_3}^2}{\mu_{r_1} \mu_{r_3} (q_3 + q_4) + \mu_{r_3}^2 (q_1 + q_4)} \\ &= \frac{\mu_{r_1} \mu_{r_3}}{\mu_{r_1} (q_3 + q_4) + \mu_{r_3} (q_1 + q_4)} \end{aligned}$$

demonstrates that this method simplifies to match Equation 5.2.

Equation 4.29 remains valid for simple substrate only microstrip configurations by setting q_2 to zero and μ_{r_2} to the value of μ_{r_3} as shown in the CM flowchart given in Figure A.1. Figure 5.2 demonstrates that the solution utilizing Equation 4.29 is equal to that obtained using Equation 5.2 across a wide range of normalized strip widths and various substrate relative permeability values.

Equation 4.29 gives a general equation to find the effective relative permeability of microstrip structures configured to have only a substrate, a flush superstrate, or a gapped superstrate.

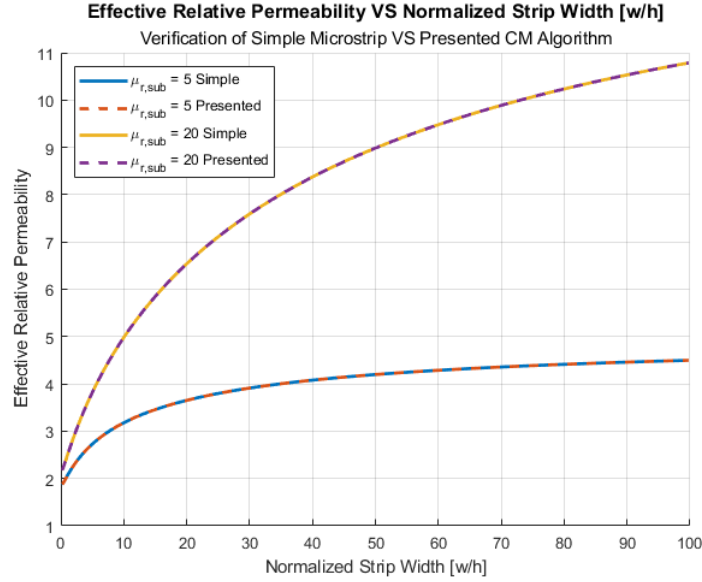


Figure 5.2: The value of μ_{ref} is equivalent for a simple microstrip when using Equation 5.2 or the presented method with Equation 4.29.

5.2 Alternate Derivation of the Hargraves CM Method

The Hargraves CM Method given by Equation 4.29 works well for moderate values of relative permeability and at microwave frequencies. The microstructure of the sub- and superstrate media will eventually become electrically large features with increasing frequency. Both higher frequency models and very thick flush superstrates, $h_2 \gg h_1$, are likely to require material boundary re-approximation to include the parallel component of q_2 . A model with both parallel and perpendicular material boundary contributions may become rather complex to compute for which the dual form of the CM plane may simplify the calculation without requiring inversion to obtain a general form equation.

Figure 5.3b and 5.3c are the dual configurations of Figures 4.9b and 4.9d, respectively, utilizing the CM with preserved $\vec{H}$ -fields paths. This is the dual configuration of the microstrip CM plane, where $\mu_{ref} = \frac{1}{\mu_{ref}^d}$ and the superscript d indicates that duality has been invoked. The effective

relative permeability derived from Figure 5.3bb is

$$\begin{aligned}
 \mu_{ref} &= \frac{1}{\mu_{ref}^d} = \frac{q_3}{\mu_{r3}} \left\| \frac{q_4}{\mu_{r2}} \right\| \left\| \frac{q_2}{\mu_{r2}} \right\| \left\| \frac{q_4}{\mu_{r1}} \right\| \left\| \frac{q_1}{\mu_{r1}} \right\| \\
 &= \frac{q_3}{\mu_{r3}} \left\| \frac{(q_2 + q_4)}{\mu_{r2}} \right\| \left\| \frac{(q_1 + q_4)}{\mu_{r1}} \right\| \\
 &= \frac{\mu_{r1} \mu_{r2} \mu_{r3}}{\mu_{r1} \mu_{r2} q_3 + \mu_{r1} \mu_{r3} (q_2 + q_4) + \mu_{r2} \mu_{r3} (q_1 + q_4)}.
 \end{aligned} \tag{5.3}$$

Equation 5.3a was derived from Figure 5.3c with boundaries approximated by preserving the magnetic flux paths, therefore reciprocity can be applied directly without inverting the formula.

Mathematically, Equation 5.2 could have been found from Equation 4.28 by utilizing the inverse function to swap the series and parallel operations. It is apparent that this method for a simple parallel or series configuration leads to the same result, but has potential to simplify the calculation of more complex mixed media conditions or a combination of both multilayered sub- and superstrates.

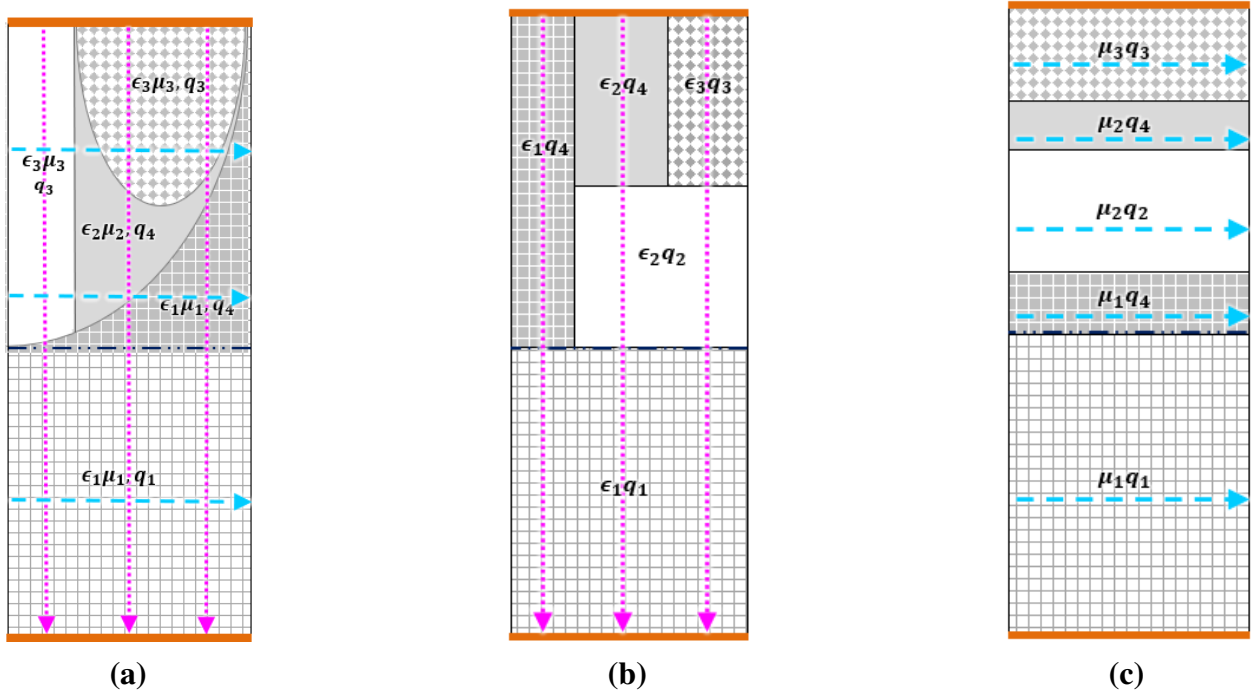


Figure 5.3: (a) $\vec{E}$ -fields and $\vec{H}$ -fields of a the dual conformal map (CM) of a microstrip with a superstrate. (b) Dual conformal map (CM) with approximated material boundaries to find ϵ_{ref}^d . (c) Approximated material boundaries of the dual configuration for deriving the μ_{ref}^d .

5.3 Comparison of Conformal Mapping Methods

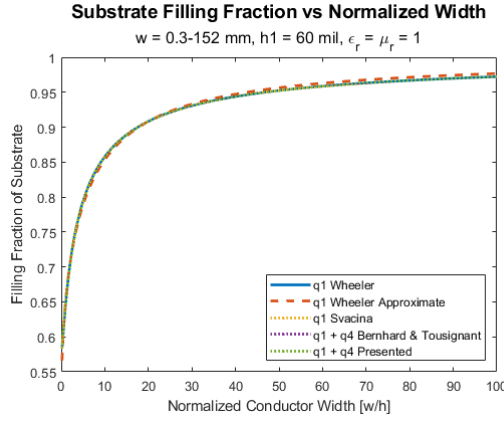
Comparison with prior models is important for verification and validation of a novel method that extends applicability to more complex situations. Doing so ensures that the foundation of theory remains intact while the new development is extending the application.

Agreement Between Filling Fractions

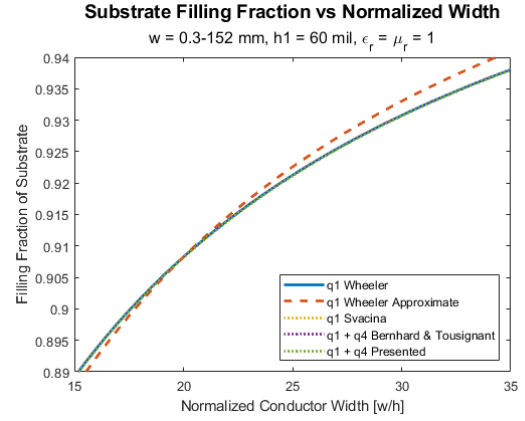
The filling fractions, q , represent the area of each medium with respect to the total area in the w -plane, or image plane. Wheeler gave two methods for determining the filling ratio of the substrate, q_1 in Equations 4.6 and 4.9 where the first was a derived analytical solution for wide microstrip conductors and the second an empirical solution that works fairly well for narrow and wide microstrip conductors [79], [80], [84]. Svacina introduced a superstrate and a respective filling fraction, q_2 [81]. Bernhard and Tousignant generalized Svacina's method to be applicable for flush and gapped superstrates by adding a correction term, q_4 , to rectify the overestimation of q_2 so that it goes to zero when no superstrate is present [26], [81]. The presented method and that of [26] are evaluated by adding q_4 to both q_1 and q_2 [26]. This method captures the entire filling area without overestimation of the substrate or the flush superstrate [26]. The curves of Figures 5.4 and 5.5 represent filling fractions found by different methods versus the normalized strip width for the substrate and superstrate, respectively, of a simple microstrip.

As demonstrated by [26], the substrate filling ratio is equivalent to $A_1 = q_1 + q_4$, where A_1 is the total substrate filling fraction. The filling fraction q_1 represents the substrate left of the flux line in the half plane cross-section z -plane or right of the flux line in the conformal mapping image or w -plane [80]. Bernhard and Tousignant derived q_4 to account for the portion of the fringing electric fields from the upper conductor surface that interact with both q_1 and q_2 [26], [79].

Filling fractions are mainly dependent on the microstrip structure influenced by the conductor width and substrate height with only a second-order dependence on the constitutive parameters that influence the effective width [84]. Figure 5.11 demonstrates the total filling fraction of each region



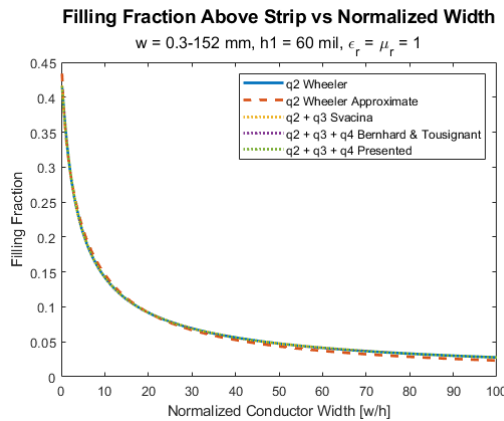
(a)



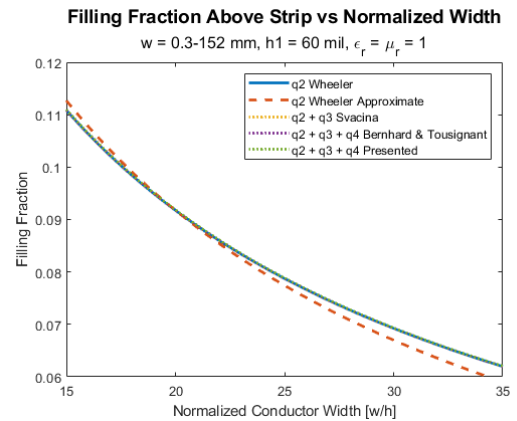
(b)

Figure 5.4: Substrate filling fraction versus the normalized microstrip width as published in [26], [79], [81], [84], and this thesis. (a) Over a large range of normalized microstrip width. (b) Zoomed view.

and how each is affected by the thickness of a flush superstrate when the substrate and superstrate are matched. The substrate total fractional area seen in Figure 5.4 is unchanged by the flush superstrate thickness when $\epsilon_r = \mu_r = 1$ for all media. Introduction of constitutive parameters greater than one demonstrates the influence of constitutive parameters on the substrate filling fraction as well as a slight influence by thickness of the flush matched superstrate. These effects have diminishing additional impact once the superstrate thickness exceeds the substrate height. This aligns with radiative near field effects that attenuate as higher order functions of distance for the excited EM



(a)



(b)

Figure 5.5: Superstrate total filling fraction versus the normalized microstrip width as published in [26], [79], [81], [84], and presented in this thesis. (a) Over a large range of normalized microstrip width. (b) Zoomed view.

fields.

Figure 5.11a demonstrates that the substrate filling fraction, q_1 , is independent of the superstrate dimensions. Figure 5.11b demonstrates that q_4 is also unaffected by thickness or presence of a superstrate. The solid lines in Figure 5.11b represent only free-space, therefore q_2 is zero for all values of conductor width. The filling ratio of q_3 for the dashed and dotted lines have an initial rise, peaking near $\frac{w}{h} = 2.5$ for a thin flush superstrate, $h_2 = 1.5h$, and near $\frac{w}{h} = 17$ for a thick flush superstrate, $h_2 = 5h$. The thick superstrate filling fractions q_2 and q_3 become nearly equivalent as the normalized width surpasses 50.

Comparing Relative Constitutive Parameter Methods

Chapter 4 introduced CM methods found in the literature for finding the effective relative permittivity and permeability of microstrip structures. Table 5.1 summarizes the equations used to find each parameter for comparison between the different methods.

Figures 5.6 and 5.7 demonstrate the effective relative permittivity and permeability found by each CM method as the substrate's relative parameters increase and for various normalized strip widths, $W/h = 5, 20, 50$. The blue lines represent the results for ε_{ref} using the average for substrate and free space, Yeo's method given in Equation 4.18, and Bernhard and Tousignant's method given in Equation 4.6b [26], [83]. The presented method utilizes Bernhard and Tousignant's equation, however slight variations in preceding calculations given in Section 4.5 warrant demonstrating that the results align [26]. Shown in red-orange are different methods for finding the μ_{ref} by Yeo's Equation 4.20, Pucel and Masse's inverse method applied to Bernhard and Tousignant's Equation A.1 results in Equation 5.3, and the presented Equation 4.29 [26], [82], [83].

Figure 5.7 demonstrates the alignment of the different methods for finding ε_{ref} that converge well for values less than 10 and $w/h = 50$. Interestingly, the values for ε_{ref} appear to increase linearly while most apparent in $w/h = 5$, each μ_{ref} method resembles the hyperbolic tangent function appearing to have a maximum attainable value. These methods for determining μ_{ref} do appear to begin converging as the normalized strip width increases, however assessing over a larger

Table 5.1: CM methods for finding effective relative constitutive parameters.

Methods and related formulas	
Averaged	
Eq. 4.14	$\varepsilon_{ref} = \varepsilon_{r1} q_1 + \varepsilon_{r1} (1 - q_1)^2$ $\times \frac{\varepsilon_{r2}^2 q_2 q_3 + \varepsilon_{r2} \varepsilon_{r3} (q_2 q_4 + (q_3 + q_4)^2)}{\varepsilon_{r2}^2 q_2 q_3 q_4 + \varepsilon_{r1} (\varepsilon_{r2} q_3 + \varepsilon_{r3} q_4) (1 - q_1 - q_4)^2 + \varepsilon_{r2} \varepsilon_{r3} q_4 (q_2 q_4 + (q_3 + q_4)^2)}$
Eq A.1	$\mu_{ref} = \frac{\mu_{r1} + \mu_{r2}}{2}$
Yeo	
Eq. 4.18	$\varepsilon_{ref} = \frac{\varepsilon_{r1} + 1}{2} \left[\frac{\ln 1 + 2a^2 + a\sqrt{4a^2 + \pi^2}}{\ln 1 + 2a^2 b + a\sqrt{a^2 b^2 + c\pi^2}} \right]^2$
Eq. 4.20	$\mu_{ref} = \frac{2\mu_{r1}}{\mu_{r1} + 1} \left[\frac{\ln 1 + 2a^2 b' + a\sqrt{a^2 b'^2 + c'\pi^2}}{\ln 1 + 2a^2 + a\sqrt{4a^2 + \pi^2}} \right]^2$
Hargraves CM	
Eq. 4.14	$\varepsilon_{ref} = \varepsilon_{r1} q_1 + \varepsilon_{r1} (1 - q_1)^2$ $\times \frac{\varepsilon_{r2}^2 q_2 q_3 + \varepsilon_{r2} \varepsilon_{r3} (q_2 q_4 + (q_3 + q_4)^2)}{\varepsilon_{r2}^2 q_2 q_3 q_4 + \varepsilon_{r1} (\varepsilon_{r2} q_3 + \varepsilon_{r3} q_4) (1 - q_1 - q_4)^2 + \varepsilon_{r2} \varepsilon_{r3} q_4 (q_2 q_4 + (q_3 + q_4)^2)}$
Eq. 4.29	$\mu_{ref} = \frac{\mu_{r1} \mu_{r2} \mu_{r3}}{\mu_{r1} \mu_{r2} q_3 + \mu_{r1} \mu_{r3} (q_2 + q_4) + \mu_{r2} \mu_{r3} (q_1 + q_4)}$
Hargraves Dual	
Eq. 4.14	$\varepsilon_{ref} = \varepsilon_{r1} q_1 + \varepsilon_{r1} (1 - q_1)^2$ $\times \frac{\varepsilon_{r2}^2 q_2 q_3 + \varepsilon_{r2} \varepsilon_{r3} (q_2 q_4 + (q_3 + q_4)^2)}{\varepsilon_{r2}^2 q_2 q_3 q_4 + \varepsilon_{r1} (\varepsilon_{r2} q_3 + \varepsilon_{r3} q_4) (1 - q_1 - q_4)^2 + \varepsilon_{r2} \varepsilon_{r3} q_4 (q_2 q_4 + (q_3 + q_4)^2)}$
Eq. 4.24	$\mu_{ref} = \left[\frac{q_1}{\mu_{r1}} + \frac{(1 - q_1)^2}{\mu_{r1}} \times \frac{\frac{q_2 q_3}{\mu_{r2}^2} + \frac{q_2 q_4 + (q_3 + q_4)^2}{\mu_{r2} \mu_{r3}}}{\frac{q_2 q_3 q_4}{\mu_{r2}^2} + \frac{(\frac{q_3}{\mu_{r2}} + \frac{q_4}{\mu_{r3}})(1 - q_1 - q_4)^2}{\mu_{r1}} + \frac{q_4 (q_2 q_4 + (q_3 + q_4)^2)}{\mu_{r2} \mu_{r3}}} \right]^{-1}$

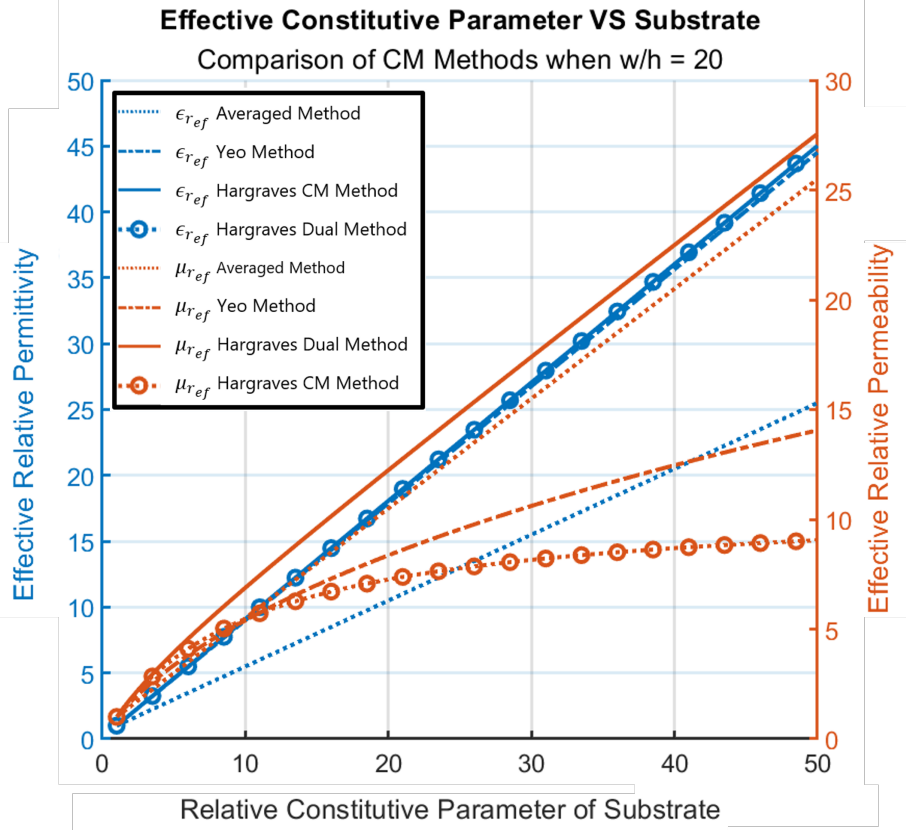


Figure 5.6: Comparison of CM for $w/h = 20$ over a large range of substrate constitutive parameters.

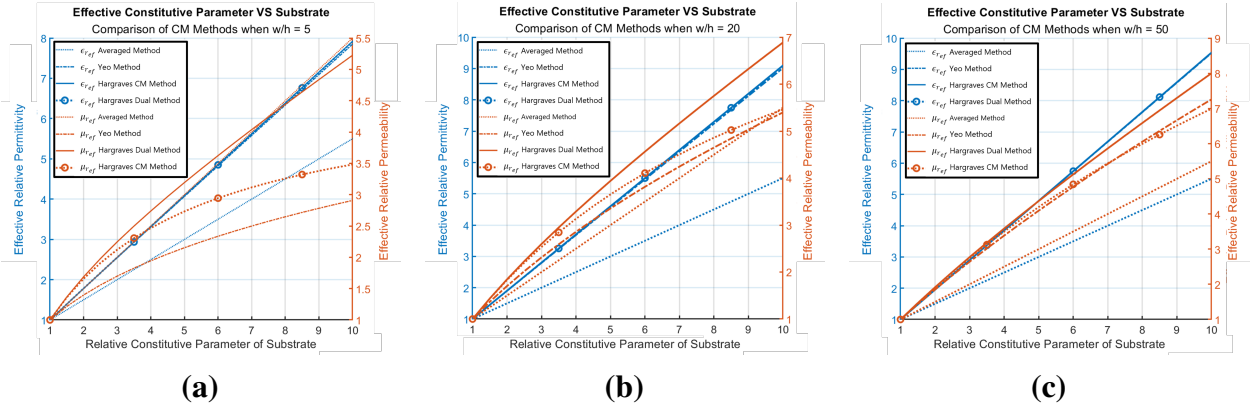


Figure 5.7: Comparison of CM method results for ϵ_{ref} and μ_{ref} versus substrate relative values at $w/h = 5, 20, 50$.

range in Figure 5.6 demonstrates this is only true for lower values of substrate relative permeability.

Effective relative permeability increases in a hyperbolic tangent fashion with increased normalized strip width because the rate at which the width of the strip increases is not proportional to the rate at which the effective magnetic field line lengths increases within the substrate. The the

majority of the electric field lines are enclosed within the substrate between the conductive strip and the ground plane and the fringing fields are relatively unchanged by the substrate alone. [3] has also noted that the effective relative permeability increases much slower than the substrate's relative permeability when compared to the effective relative permittivity versus the substrate relative permittivity [3].

Microstrip Patch Antenna (MPA) Operational Frequency

Many empirical formulas curve fit to increase the accuracy of modeling the effective relative permittivity, ε_{ref} , have been published in literature, but very little work has been conducted in efforts to extend these concepts to modeling the relative effective permeability, μ_{ref} , as discussed in Section 4.4. Comparison between a free-space MPA, a dielectric MPA, and a MD MPA with resonant frequency at $400MHz$ helps to give insight to the relative miniaturization that can be achieved through use of material properties. The length of a MPA is approximately given by Equation 3.18

$$L_{MPA} = \frac{c_0}{2f_0\sqrt{\mu_{ref}\varepsilon_{ref}}}.$$

A $400MHz$ free-space patch with constitutive parameters $\varepsilon_r = 1$ and $\mu_r = 1$ would have a length of $37.474cm$. Percent length reduction is found by

$$\%_{reduction} = \frac{|L_0 - L_{red}|}{L_0}, \quad (5.4)$$

where L_0 is the free-space length and L_{red} is the length of the reduced length of the miniaturized MPA. Another metric is to use the miniaturization factor given in Equation 3.16 as $\eta = \sqrt{\varepsilon_r \times \mu_r}$, but a more accurate estimate is given by using the effective relative constitutive parameters as

$$\eta = \sqrt{\varepsilon_{ref} \times \mu_{ref}}. \quad (5.5)$$

A dielectric gives miniaturization, but suffers from degraded bandwidth. MD materials have

been shown to give a greater extent of miniaturization than a dielectric material alone. Utilizing Blosser's patch parameters and the MX manufacturer's datasheet, a 400MHz has a length of 74.6cm and width of 37.3cm on 60mil or 1.524mm with the substrate $\varepsilon_r = 6.5$ and $\mu_r = 6.0$ [27], [44].

Figure 5.8 demonstrates a plot of the The equivalent patch for a dielectric with $\varepsilon_r = 6.5$ and $\mu_r = 1$ has $\varepsilon_{r_{ef}} = 6.0214$ when using the presented CM would have a length of 15.272cm for a miniaturization factor of 2.45 or length reduction of 59.2%. With MX the patch length is further reduced to 7.382cm for a miniaturization factor of 5.08 or a reduction in length of 80.3% using the effective parameters $\varepsilon_{r_{ef}} = 6.0214$ and $\mu_{r_{ef}} = 4.2796$ as found by the presented CM method.

For comparison of the dielectric and MD MPAs, Blosser's patch dimensions were utilized and only the material parameters were altered to evaluate the effect on the MPA's resonant frequency [27]. Using $f = \frac{c_0}{2L}$, a metal patch of equivalent dimensions surrounded by only free-space would have a fundamental resonant frequency near 2.0093GHz but due to fringing fields from proximity to a ground plane. The Kirschening method formulas used with CM are covered in Section A.1. This method finds a resonant frequency of 1.9368GHz , however this method only gives the structural resonant frequency.

To plot the impedance versus frequency the TL algorithm from Appendix A.0.1 is utilized using the CM values for characteristic impedance and effective relative constitutive parameters as input parameters. The TL method results find the resonant frequency at 1.9378GHz for a percent difference between the CM and TL methods is 0.05%. Altering the material properties only to a dielectric only versus the same with MX materials, the CM patch resonant frequencies are reduced to 803.22MHz or 51.7% calculation and 388.28MHz or 80%, respectively, when compared to a free-space microstrip patch. Figure 5.8 shows the impedance plotted across frequency for these two designs when first the effective relative constitutive parameters are found by the presented CM and used as the isotropic relative constitutive parameters input to the linear PUES and Capelle TL model described in Appendix A.0.1 [65]. The linear TL model results give a fundamental resonant frequency at 387.8MHz for the MX patch. Without the additional reduction from relative

permeability the dielectric patch has a fundamental resonant frequency at 803.59 MHz with the linear TL model.

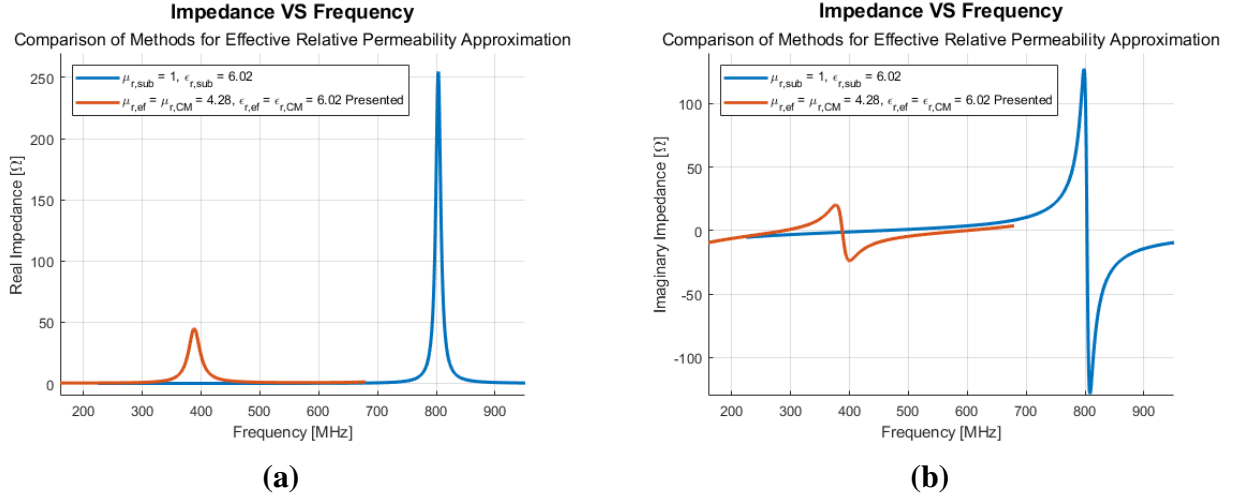


Figure 5.8: Comparison of dielectric and MD MPA with length 74.6cm. (a) Real impedance versus frequency. (b) Imaginary impedance versus frequency.

It is well documented in literature that adding effective relative permeability to the substrate increases the miniaturization of a MPAs when compared to a dielectric only substrate. There are many analytical and empirical methods for increased accuracy of modeling with effective relative permittivity, but methods used to approximate the effective relative permeability are generally more primitive. The two most common methods applied to initial prototype design are directly use the substrate's relative permeability from the datasheet or to using the Averaged Method to take the mean of the relative permeability of free space and the substrate. These methods are generally followed by rigorous full-wave simulations to tune the design. Figure 5.9 demonstrates the results of these methods and compares them to the presented method.

Section 4.4 presented some alternative methods for assessing the effective relative permeability. Yeo presented a method derived from Wheeler's continuous formula published in 1977, but Wheeler's later formula lacks accuracy in the extrema of small and very wide normalized strip widths [83], [84]. Yeo's method given in Equation 4.20 is also limited as it is not easily modified to account for microstrip structures with superstrates as is needed in more complex microstrip designs [83]. The Hargraves Dual Method utilized the duality of permittivity to permeability by

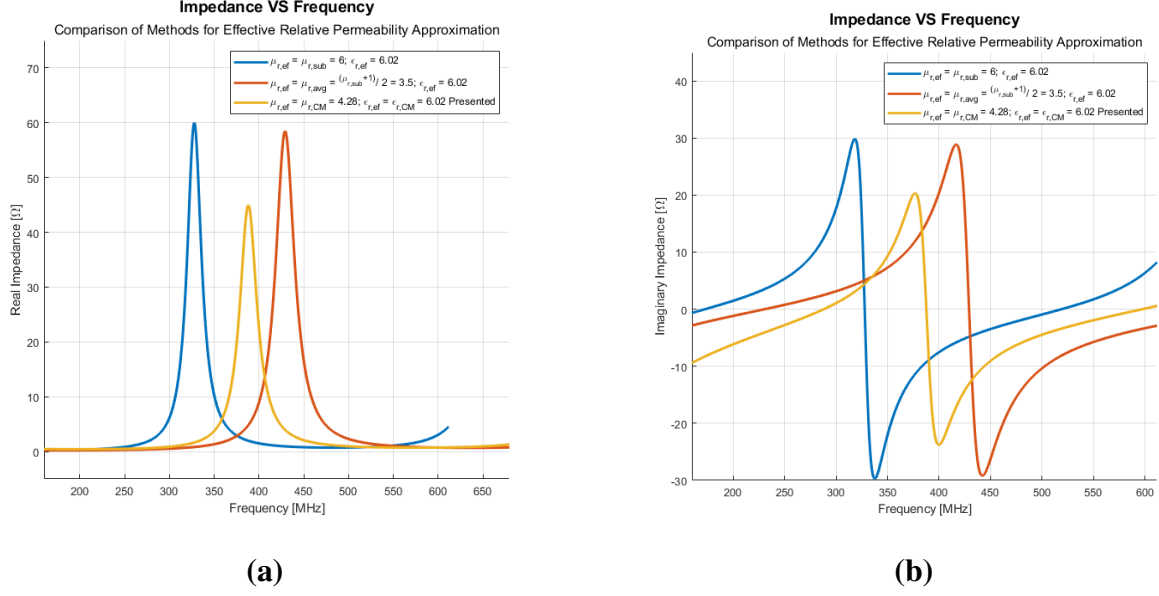


Figure 5.9: Comparison of MD MPA with length 74.6cm using common methods to approximate $\mu_{r,ef}$. (a) Real impedance versus frequency. (b) Imaginary impedance versus frequency.

swapping $\epsilon_r \rightarrow \frac{1}{\mu_r}$ [82]. The Hargraves Dual Method concept can be applied to each of the CM methods given in Equations 4.12 and 4.14 [26], [81], [82]. The Hargraves CM Method was derived by approximating boundary conditions along the magnetic flux paths. These four methods of approximating $\mu_{r,ef}$ are plotted for comparison in Figure 5.10.

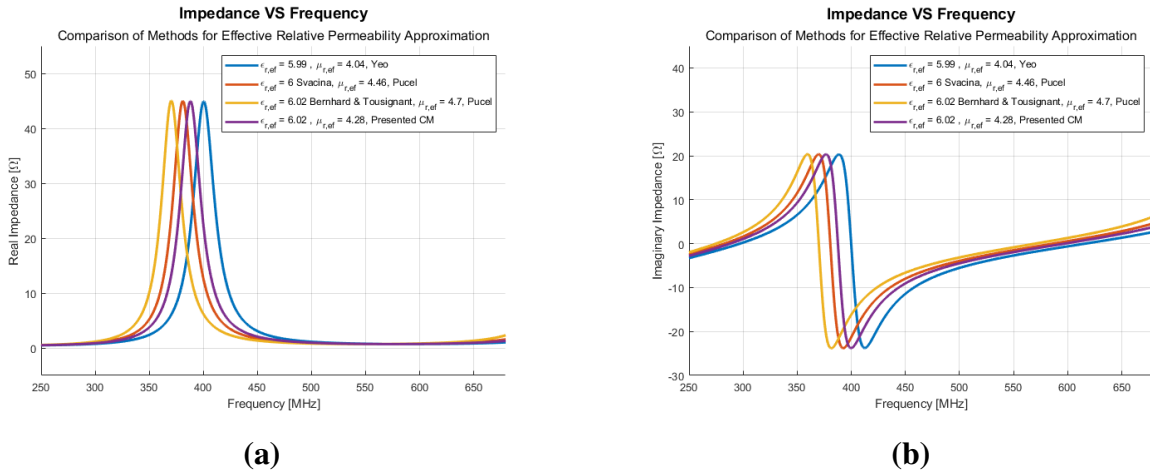
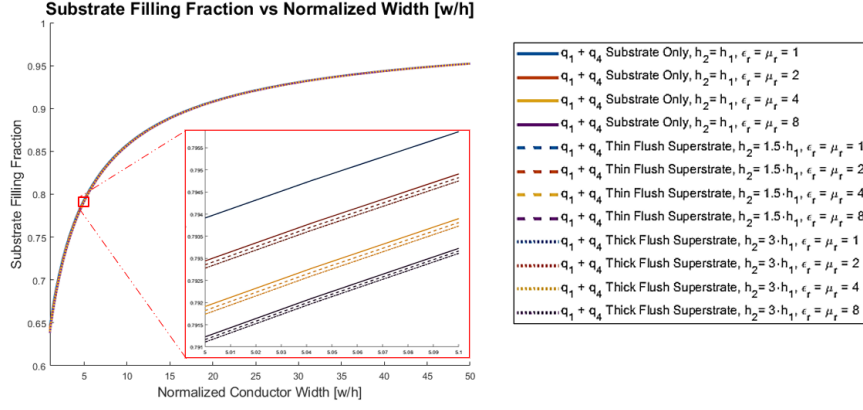


Figure 5.10: Comparison of MD MPA with length 74.6cm using CM methods to approximate $\mu_{r,ef}$. (a) Real impedance versus frequency. (b) Imaginary impedance versus frequency.

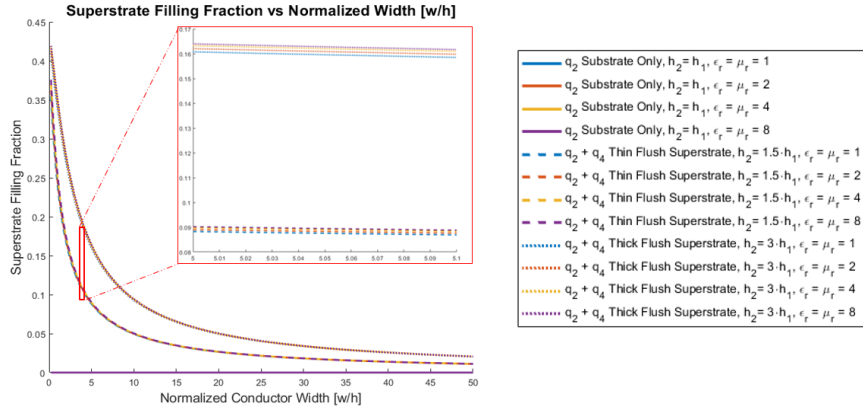
5.4 Effective Parameter Analysis

Filling Fractions and Effective Width

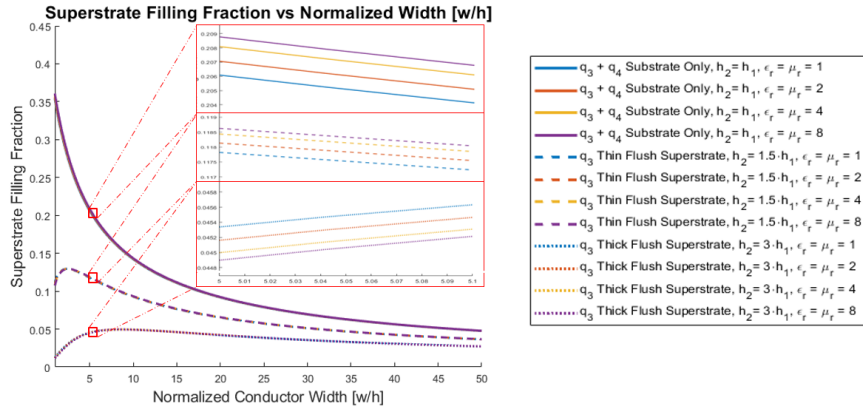
The fringing fields cause the conductive strip or patch to effectively be larger than the physical dimensions. Microstrip structures experience fringing fields which are accounted for by the effective width found by Equation 4.4. . Figure 5.13 demonstrates that flush superstrates of various thicknesses do not significantly effect the fringing fields.



(a)



(b)



(c)

Figure 5.11: Comparison of various superstrate thicknesses and matched constitutive parameters to demonstrate the effect on filling fractions. (a) Substrate filling fraction $A_1 = q_1 + q_4$. (b) Flush superstrate filling fraction $A_2 = q_2 + q_4$. (c) Gapped substrate filling fraction $A_3 = q_3$.

The structural dimensions have a significant impact on filling fractions as shown above. The normalized strip width also has an impact on the maximum effective relative constitutive parameters that can be reached. Figure 5.12 demonstrates that the normalized width has a rapid increase in effective relative permittivity occurs from $w/h = 1$ to $w/h = 15$ due to the increase in fringing fields as the conductor's upper surface area increases towards an asymptote where fringing fields no longer increase with the normalized width. This effect is more pronounced as the relative permittivity of the substrate is increased. The effective relative permeability also appears to ascend towards an asymptote, however at a much lower slope, or rate of change. Also of note is that the maximum effective value for permeability is lesser for the same initial substrate value than the maximum for permittivity.

5.4.1 Limits of Effective Material Properties

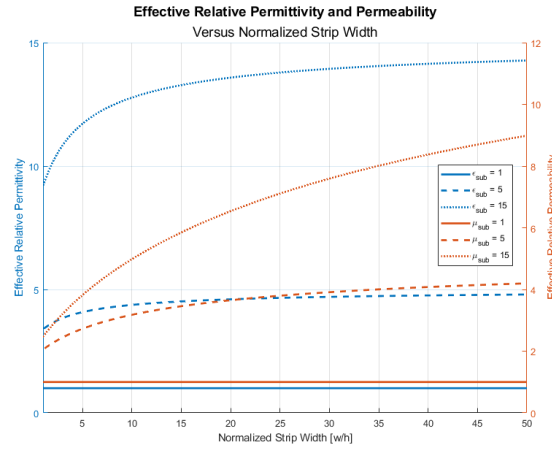


Figure 5.12: Evaluation of trends in how the normalized strip width affects the effective constitutive parameters for low, moderate, and high starting values.

By swapping the x -axis for the substrate's relative constitutive values in Figure 5.14, it is easily visualized that increasing the relative permittivity of the substrate for a given normalized width results in a linearly increasing effective relative permittivity and that the structural normalized width has most impact at lower values. Conversely, the effective relative permeability increases logarithmically with increased relative permeability of the substrate. Both the normalized width and the relative permittivity of the substrate appear to reach a maximum for effective relative

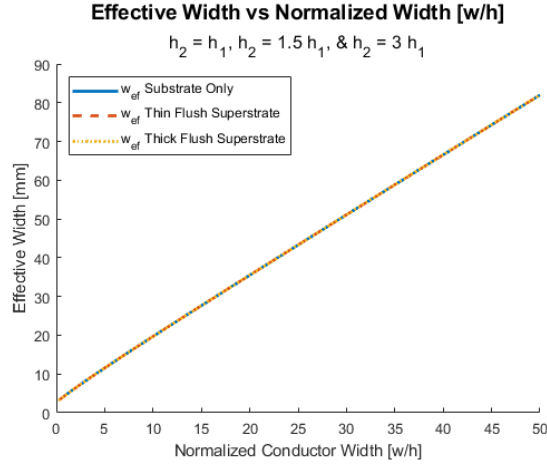


Figure 5.13: Demonstration that microstrip effective width is not significantly affected by superstrate thickness.

permeability that can be reached. This finding is not surprising as permeability and permittivity have a dual relationship, but can be a significant challenge for achieving a $\mu_{ref} > \epsilon_{ref}$ to realize increased radiation efficiency and bandwidth [3].

Figure 5.14 demonstrates that the effective relative permittivity increases linearly with the substrate permittivity independent of the relative permeability, but the slope or rate of change for effective relative permeability highly depends on the width of the conductive strip. The effective relative permeability resembles a logarithmic or hyperbolic tangent function approaching an asymptote, The normalized width of the strip influences the initial slope of μ_{ref} and the value of the perceived asymptote or maximum realizable value.

The effect of normalized strip width on the calculated values is demonstrated in Figure 5.14 for normalized widths of 1, 25, and 100. For moderately wide and very wide microstrips, both ϵ_{ref} and μ_{ref} appear to increase approximately linearly when the substrate values are between one and two. The increase in estimated μ_{ref} appears to be reaching an upper bound in the transition from wide to very wide strips. This is a secondary effect related to the magnetic flux path increasing in length with increased normalized strip width. The electric flux paths always have a confined region below the strip. The surface are confined under the strip and above the strip for fringing fields continues to increase in proportion to the increased normalized strip width.

For narrow strips, the substrate constitutive parameters have the least effect on effective relative

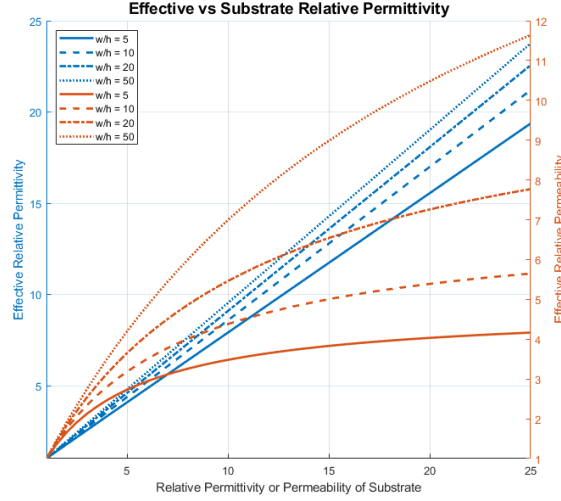


Figure 5.14: Demonstration that the effective relative permeability maximum value is a function of relative permeability and normalized strip width.

values. Effective relative permittivity increases more rapidly with increasing conductor width, but the difference from moderately wide to very wide strips shows a diminishing effect. [20] also demonstrated with increased thickness of a MD material at the base of a monopole had diminishing return on miniaturization attributing it to the maximum effect at the peak of the magnetic fields. [20] The structure of a MPA has the electric and magnetic field maxima at the radiating slots, however the electric fields are reduced as thickness of the substrate increases as well as the fringing fields increase with the strip width. Normalized width, w/h , experiences these effects simultaneously

Effective relative permeability increases in a hyperbolic tangent fashion with increased normalized strip width because the rate at which the width of the strip increases is not proportional to the rate at which the effective magnetic field line lengths increases within the substrate. The majority of the electric field lines are enclosed within the substrate between the conductive strip and the ground plane and the fringing electric fields increase with the normalized strip width, but not at the same rate.

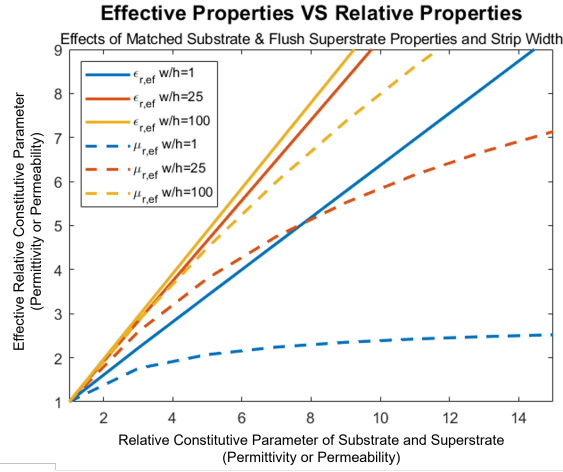


Figure 5.15: Effective relative permittivity and effective relative permeability for narrow, wide, and very wide microstrip patch antennas with matched substrate and superstrate properties of equal thickness.

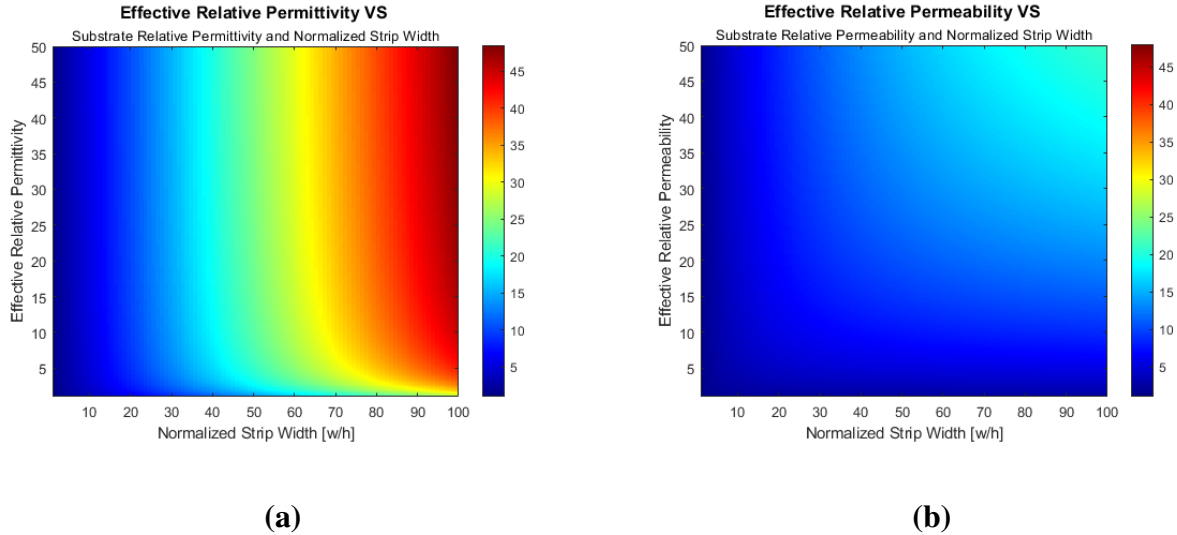


Figure 5.16: Heat maps with like scales demonstrating the difficulty in matching effective constitutive parameters. (a) Effective relative permittivity. (b) Effective relative permeability.

5.4.2 Effect of Limits on Bandwidth

Figure 5.11 also demonstrates that structural dimensions of the microstrip have the most significant impact on the effective relative parameters. This relationship has not been demonstrated in the literature reviewed for this work. Relating this phenomenon back to Equations 3.1 and 3.15, the bandwidth of a microstrip patch antenna is more accurately defined with $\varepsilon_{r_{ef}}$ and $\mu_{r_{ef}}$ as functions of normalized strip width.

$$BW\left(\frac{W}{h}\right) = \frac{96 \frac{h}{\lambda_0} \sqrt{\frac{\mu_{r_{ef}}}{\varepsilon_{r_{ef}}}}}{\sqrt{2} \left(4 + 17 \sqrt{\varepsilon_{r_{ef}} \times \mu_{r_{ef}}}\right)},$$

where h is the substrate height, λ_0 is the free space wavelength, $\mu_{r_{ef}}(\frac{W}{h})$ is the effective relative permeability as a function of normalized width, $\varepsilon_{r_{ef}}(\frac{W}{h})$ is the effective relative permittivity as a function of normalized width, and the cross product in the denominator relates to inclusion of anisotropic materials.

Bandwidth and radiation efficiency benefit when $\mu_r > \varepsilon_r$, but the rigor required to ensure that $\mu_{r_{ef}} > \varepsilon_{r_{ef}}$ is difficult to achieve when the rate of increase for effective relative permittivity outpaces that of effective relative permeability with increasing normalized strip width [3], [85]. There is a fundamental limitation on how much $\mu_{r_{ef}}$ can be increased which is dependent on the normalized width. For a normalized width $\frac{W}{h} = 1$, the $\mu_{r_{ef}}$ appears to be nearing an asymptote near $\mu_{r_{ef}} = 2.5$ with a substrate $\mu_r = 15$. By increasing the normalized width to $\frac{W}{h} = 25$, the same substrate has a $\mu_{r_{ef}} = 7$. concept of utilizing matched material parameters to increase the radiation efficiency has been over simplified in literature. Similarly, the $\varepsilon_{r_{ef}}$ increases with substrate values, but in a linear fashion. These values also relate to limitations to the miniaturization factor given in Equation 3.16 as $\eta = \sqrt{\varepsilon_r \times \mu_r}$, which for a microstrip is better captured by

$$\eta_{ms}\left(\frac{W}{h}\right) = \sqrt{\varepsilon_{r_{ef}} \times \mu_{r_{ef}}}. \quad (5.6)$$

Effective relative permeability increases in a hyperbolic tangent fashion with increased normal-

ized strip width because the rate at which the width of the strip increases is not proportional to the rate at which the effective magnetic field line lengths increases within the substrate. The majority of the electric field lines are enclosed within the substrate between the conductive strip and the ground plane and the fringing electric fields increase with the normalized strip width, but not at the same rate.

5.5 Conformal Mapping Remarks

The engineering community needs computationally efficient and more accurate methods to accelerate design and fabrication of MD MPAs. This chapter evaluated the methods presented in Chapter 4 , which align with Wheeler's methods while extending application to layered microstrip configurations and enabling new methods for evaluation for MD microstrip structures.

While this Section 5.4.2 demonstrated that the filling fractions are not significantly altered by material parameters, antenna engineers must stay cognizant that the guided wavelength could still be significantly altered. Decreased guided wavelength will introduce unintended propagating modes and undesired surface currents.

Chapter 6

Experimental Validation of Presented Conformal Mapping Methods

Although there are benefits to using MD materials as substrates in RF microstrip designs, there are also potential consequences. How does the ferromagnetic loading of the substrate affect the magnetic fields on the microstrip trace? Does introducing ferromagnetic materials introduce effects of hysteresis? If so what effect does it have on the operational frequency? This chapter will explore the answer to these questions by designing, fabricating, and analyzing the scattering parameters of a 50Ω transmission line, a microstrip LPF, and a microstrip patch antenna on a magneto-dielectric substrate.

The MX was characterized to ensure good understanding of the material's frequency response in order to predict the behavior when used to design an antenna. The characterization of the material was carried out by evaluating the material across frequency in various configurations, a TL, LPF, and microstrip patch antenna.

Realization of desired MD material parameters in a lab setting is wrought with challenges. Chapter 2 discussed the many obstacles to reliably achieving uniform materials prior to scaling production, which hinders large scale commercial availability of MD materials. MX was introduced in Section 2.5 as one of the few CoTS MD materials available. Chapter 4's CM methods will be utilized in Chapter 6 to design and fabricate microstrip structures on MX substrate for experimental validation of the CM techniques presented.

Chapter 6 will provide experimental validation of CM methods. The microstrip structures from simple to complex include three lengths of 50Ω TLs, a fifth-order LPF filter, and Blosser's probe fed

400 MHz MPA [27]. Section 6.1 will cover design of the two-port microstrip structures, followed by fabrication and measurement. Validation for two-port structures will be discussed in Section 6.3. Validation to MPAs is covered in Section 6.4.

MAGTREX 555 (MX)

Section 2.5 introduced MAGTREX 555 (MX), a CoTS MD material made available by Rogers Corporation's in 2018 for antenna miniaturization applications.

Figure 6.1 shows the condensed relative permeability and permittivity spectrum for the MX material. The real part of relative permeability, μ'_r , has three notable transitions in trend notated as f_1 , f_2 , and f_3 . The static permeability begins to rise at $f_1 = 499$ MHz, peaks at $f_2 = 715$ MHz, and reaches unit at $f_3 = 3.9$ GHz [44]. The cutoff frequency of MX occurs at $f_t = 2.8$ GHz, which is defined at the peak of imaginary relative permeability [44].

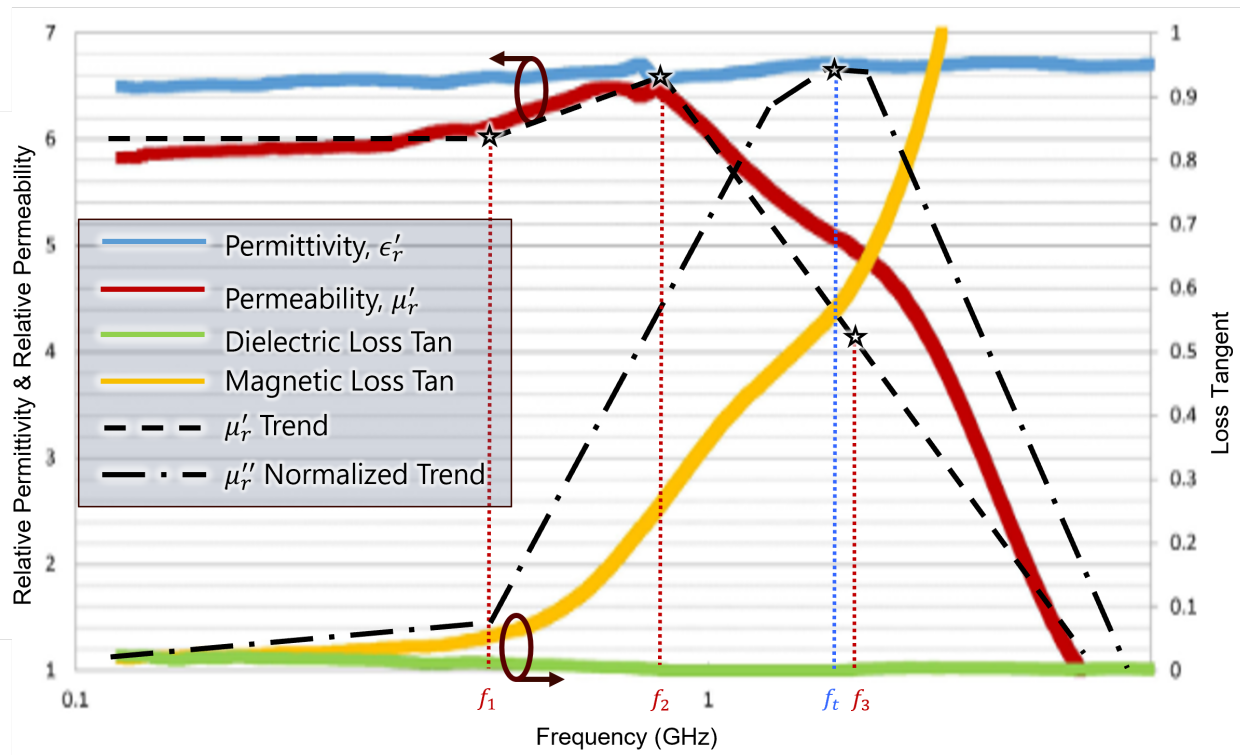


Figure 6.1: Condensed MX datasheet permeability and permittivity spectra provided by Rogers Corporation [44].

6.1 Magneto-Dielectric Two-Port Microstrip Designs

Magneto-Dielectric Transmission Line (TL) Design

The simplest microstrip structure to design and prototype for characterization of material properties is a two-port TL. The width and height of a TL can be optimized to the desired characteristic impedance. Three lengths of TL were designed at 50Ω to match SMA edge launch connectors that were to be soldered to the board for measurements of the two-port s-parameters.

Initially ADS's LineCalc was used to guide design for the 50Ω microstrip line width with three lengths representing different phase lengths at 450 MHz. Unfortunately the version of LineCalc and MLIN available provided a field for the substrate's relative permeability, but entering a value in the field did not alter the resultant design dimensions. This was apparent because the MX datasheet values for $\epsilon_{r_{x,y}} = 6.5$ and $\mu_r = 6$ should have produced a miniaturization factor of $\approx 1/6$ per Equation 3.17. Therefore, alternative design methods had to be used.

An iterative simulation approach was highly favored over iterative fabrication due to the limited quantity of MX on-hand. This iterative design approach utilized several methods before the final design was reached. The simplest structure, the 50Ω TLs, were approximated for length and width by writing a MathWorks' MATLAB script utilizing modified microstrip impedance equations from Hong [62]. These dimensions were then used to perform a parametric analysis by full-wave simulation in Ansys' HFSS for a 60 mil thick MX substrate to find the optimal width, $w = 4.976$ mm. Table 6.1 lists the finalized design dimensions and Figure 6.2 demonstrates the HFSS model for the 45 mm TL used for tuning the TL width. The fabrication and measurement processes for these TLs and LPF will be discussed in parallel.

Magneto-Dielectric Low-Pass Filter (LPF) Design

Microstrip filters are often desired for the beneficial SWaP-c from the small form factor, planar structure, and ease of fabrication. A simple stepped-impedance L-C ladder-type LPF design was utilized because these are well documented for traditional materials [62]. This filter was designed

Table 6.1: 50 Ω MAGTREX 555 (MX) transmission line (TL) dimensions by designed and realized.

	Trace width [mm]	Trace length [mm]	Thickness [mil]
TL design 1	4.98	45	60
TL design 2	4.98	60	60
TL design 3	4.98	81.7	60

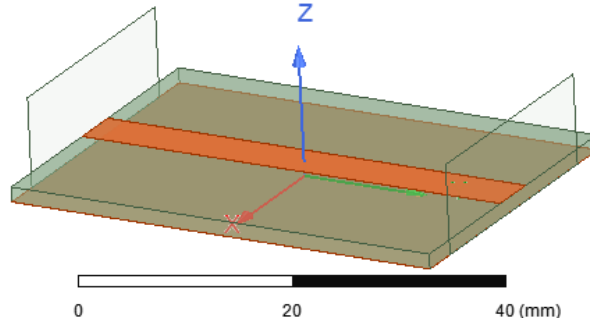


Figure 6.2: HFSS model of MX TL utilized to optimize the strip width to 50 Ω .

on MX to evaluate if the MD substrate introduces any unexpected behavior to the performance of the LPF. The LPF performance expectations were to comply with the following specifications:

- Chebyshev LPF with a pass-band ripple of 0.25 dB
- Cutoff frequency, $f_c = 450$ MHz
- At least 35 dB of rejection at 900 MHz

The minimum order to achieve the desired attenuation in the stop-band of a generalized LPF is found by

$$n \geq \frac{L_A + L_R + 6}{20 \cdot \log(S + \sqrt{S^2 - 1})}, \quad (6.1)$$

where L_A is the stop-band insertion loss in dB, L_R is the pass-band return loss in dB, and S is the selectivity found by $S = \frac{\omega_s}{\omega_p} \geq 1$, where ω_s is the stop-band angular frequency and ω_p is the pass-band angular frequency [71]. The insertion loss was not given; therefore must be calculated from s-parameters from the relationship to the return loss which was given. This relationship is

found by

$$L_{A_{linear}} = |s_{21}(jw)|^2 = 1 - |s_{11}(jw)|^2 = 1 - L_{R_{linear}}, \quad (6.2)$$

where s_{11} and s_{21} are s-parameters in linear units and $L_{R_{linear}} = |s_{11}(jw)|^2$. The return loss was a given requirement, $L_{R_{dB}} > 35$ dB at 900 MHz, which must be converted to linear units before use in Equation 6.2 resulting in $L_{R_{linear}} = 10^{(-L_{R_{dB}}/10)} = 3.16 \times 10^4$. Equation 6.2 can now be calculated giving $L_{A_{linear}} = 0.9997$. $L_{A_{linear}}$ must be converted back into dB for use in Equation 6.1, found by $L_{A_{dB}} = -10 \log L_{A_{linear}} = 1.37 \times 10^3$ dB. With the values of $L_{R_{dB}}$, $L_{A_{dB}}$, and S known, the filter order, n can be found by Equation 6.1 giving $n = 3.584$. The result for n is always round up to the next integer, so $n = 4$ indicates that the filter must be at minimum a fourth order filter to achieve 35 dB of suppression at 900 MHz. It is well known that an even valued order for a Chebyshev filter would require a matching network, so a fifth order design was selected.

Textbook procedures for finding the Chebyshev coefficients for a normalized fifth order LPF prototype was used [62]. The coefficients for a 0.25 dB pass-band ripple were not given in look-up tables so they were calculated with Equations 6.3-6.7 and results shown in Table 6.2 [62]. The formula to find the filter coefficients are given as

$$g_0 = 1.0 \quad (6.3)$$

$$g_1 = \frac{1}{\gamma} \sin \frac{\pi}{2n} \quad (6.4)$$

$$g_i = \frac{1}{g_{i-1}} \frac{4 \sin \frac{(2i-1)\pi}{2n} \sin \frac{(2i-3)\pi}{2n}}{\gamma^2 + \sin^2 \left(\frac{(i-1)\pi}{n} \right)}, i = 2, 3, \dots n \quad (6.5)$$

$$g_{n+1} = 1.0 \quad (6.6)$$

where

$$\gamma = \sinh \frac{\beta}{2n} \quad \beta = \ln \left(\coth \frac{L_{A_r}}{17.37} \right) \quad (6.7)$$

and L_{A_r} is the pass-band ripple in dB [62].

Table 6.2: Calculated Chebyshev prototype coefficients

Order n	Coefficients						
	g_0	g_1	g_2	g_3	g_4	g_5	g_6
Fifth	1.000	1.4145	1.3180	2.2414	1.3180	1.4145	1.000

Scaling LPF Coefficients

Next, the coefficients must be both impedance and frequency scaled. Coefficients g_1 to g_5 in a stepped-impedance L-C ladder-type LPF represent the normalized values of each series inductor or shunt capacitor. Starting with a series inductor, the new scaled values are found by

$$L'_r = \frac{R_0 L_r}{\omega_c}, \quad (6.8)$$

where L'_r is the scaled inductor value, R_0 the equivalent impedance of the system, L_r is the g_r prototype coefficient, and ω_c the cutoff frequency in radians per second [62]. Equation 6.8 was used to find L'_1 , L'_3 , and L'_5 . The shunt capacitor values are given by

$$C'_r = \frac{C_r}{R_0 \omega_c}, \quad (6.9)$$

where C'_r is the scaled capacitor value and C_r is g_r the prototype coefficient [62].

Keysight's ADS was used to model an ideal lumped element LPF with the calculated prototype values. The ideal lumped element LPF ADS model is shown below in Figure 6.3 followed by the frequency response in Figure 6.4.

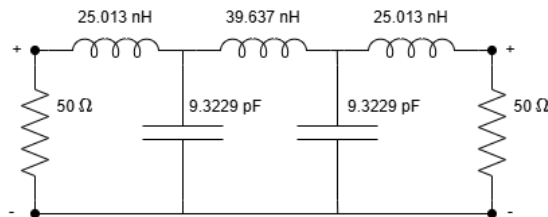


Figure 6.3: Ideal fifth-order Chebyshev LPF equivalent circuit model that meets the design requirements.

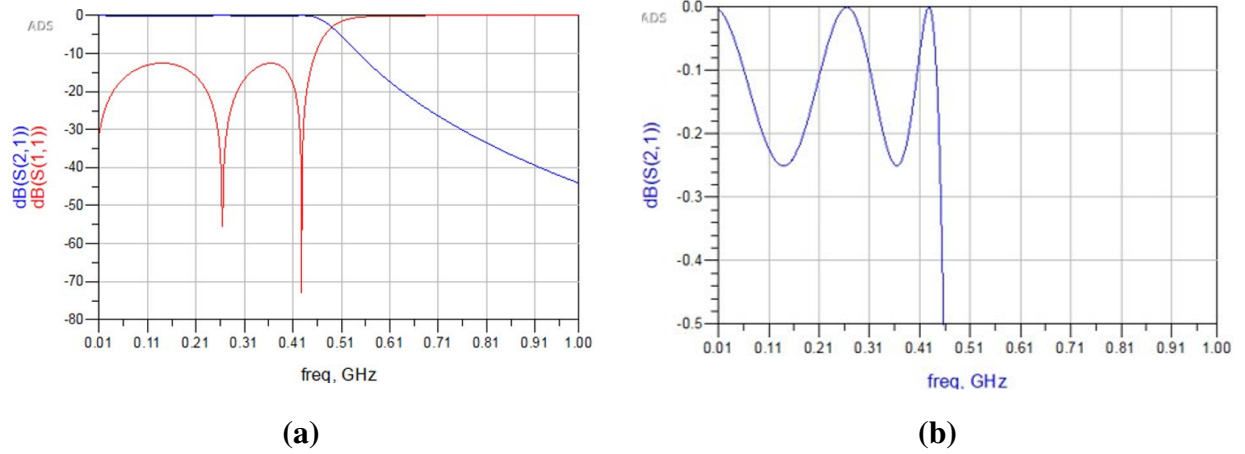


Figure 6.4: ADS Frequency response of ideal lumped element LPF. (a) Pass-band demonstrates a cutoff frequency at 450 MHz with 35 dB of rejection at 900 MHz. (b) Enlarged to appreciate the 0.25 dB ripple in the s_{21} pass-band.

To transform the lumped element LPF design into a microstrip design on MX, the desired microstrip lengths and widths were calculated. ADS LineCalc was used to calculate the line trace width of each segment of microstrip to achieve the desired impedances, but the calculations were noted to not take into account the specified relative permeability, μ_r , as noted in Section 6.1. The LineCalc effective permittivity and width were used as a starting points to Hong's formulas for calculating the line length of each segment to achieve the desired inductance or capacitance. All design dimensions and impedance values are presented below in Table 6.3 [62].

Table 6.3: Initial dimensions for Fifth-order Chebyshev microstrip design on MX substrate.

	L_1	C_2	L_3	C_4	L_5
Resistance, R_n	120 Ω	20 Ω	120 Ω	20 Ω	120 Ω
Width, W	0.188 mm	8.332 mm	0.188 mm	8.332 mm	0.188 mm
Reactance, X_n	25.013 nH	9.323 pF	29.637 nH	9.323 pF	25.013 nH
Length, L	24.978 mm	18.151 mm	39.582 mm	18.151 mm	24.978 mm

LPF Simulation

Once the design parameters were found, the microstrip schematic could be simulated for verification of expected results. The design was simulated in ADS then tuned to the desired cutoff frequency as seen in Figure 6.5a and Figure 6.5b to better account for the relative permeability, μ_r , that was neglected in the LineCalc calculations and with line widths that are realizable within the etching tolerances of available milling machines.

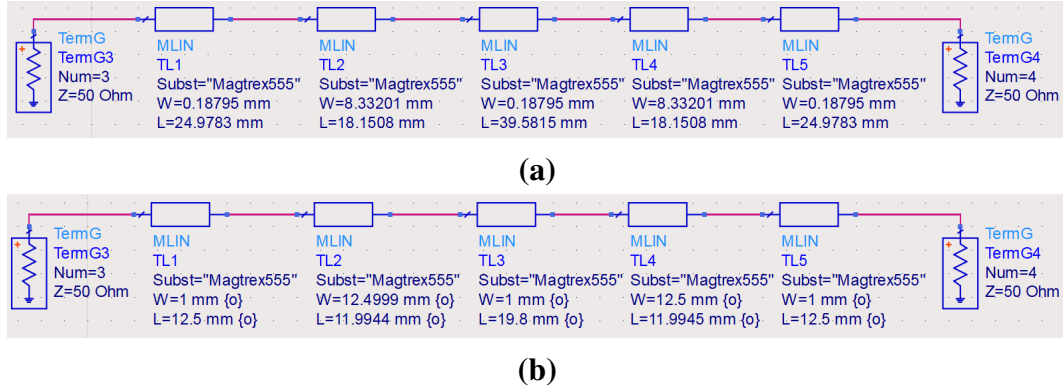


Figure 6.5: Schematics for microstrip LPF on MX. (a) Initial model of LPF from TX Line length calculations. (b) Tuned and optimized model of LPF filter.

Table 6.4: Tuned dimensions for Fifth-order Chebyshev microstrip design on MX substrate.

LPF ideal, ADS	L_1	C_2	L_3	C_4	L_5
Width, W [mm]	1.0	12.5	1.0	12.5	1.0
Length, L [mm]	12.5	12.0	19.8	12.0	12.5
LPF tuned, HFSS	L_1	C_2	L_3	C_4	L_5
Width, W [mm]	0.76	18.36	0.76	18.36	0.76
Length, L [mm]	13.9	10.0	23.9	10.0	13.9

After verifying the expected results from Figure 6.5b, the full three-dimensional (3D) model was designed in HFSS for full-wave simulation to verify frequency response prior to fabrication. The HFSS board layout is shown in Figure 6.6 demonstrating the 50 Ω transmission line then narrowing and widening for the sections representing L_1 , C_2 , L_3 , C_4 , L_5 before terminating in another 50 Ω transmission line for SMA edge-launch connectors. HFSS allows for customization of

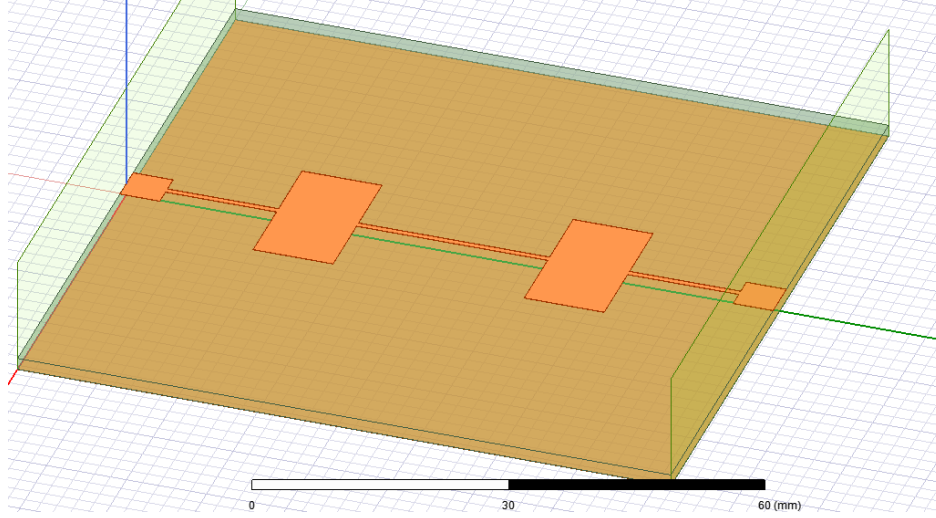


Figure 6.6: Model designed in HFSS for simulation purposes.

material properties, which for this MX substrate includes a relative permeability, μ_r , of 6.0 and an anisotropic relative permittivity, ϵ_r , of 6.5 in x and y and 5.3 in z [44]. The ground plane and traces were set to the 1 ounce copper thickness of $35 \mu m$. From the HFSS simulation results, electric and magnetic fields were plotted as vector fields in Figure 6.7a and Figure 6.7b, respectively.

The s-parameters from the ideal LPF simulation in ADS and the simulated results in HFSS were exported and graphed together in MATLAB for comparison. Figure 6.8 demonstrates that the HFSS better captures the effects of the lossy MX substrate and there is a notable shift of the cutoff frequency by approximately 50 MHz. This shift can be attributed to the more accurate calculation of effective constitutive parameters within a full-wave simulation compared to the approximation methods used in ADS.

6.2 MX Fabrication and Measurement

MX Fabrication

The microstrip TL and LPF design files were exported from HFSS and transferred to a milling machine for fabrication. These microstrip designs were fabricated on the MX substrate by a mechanical etch milling machine. The milling surface was first leveled and the board secured to the vacuum surface of the machine. A machining bit was used to rub away the desired sections of

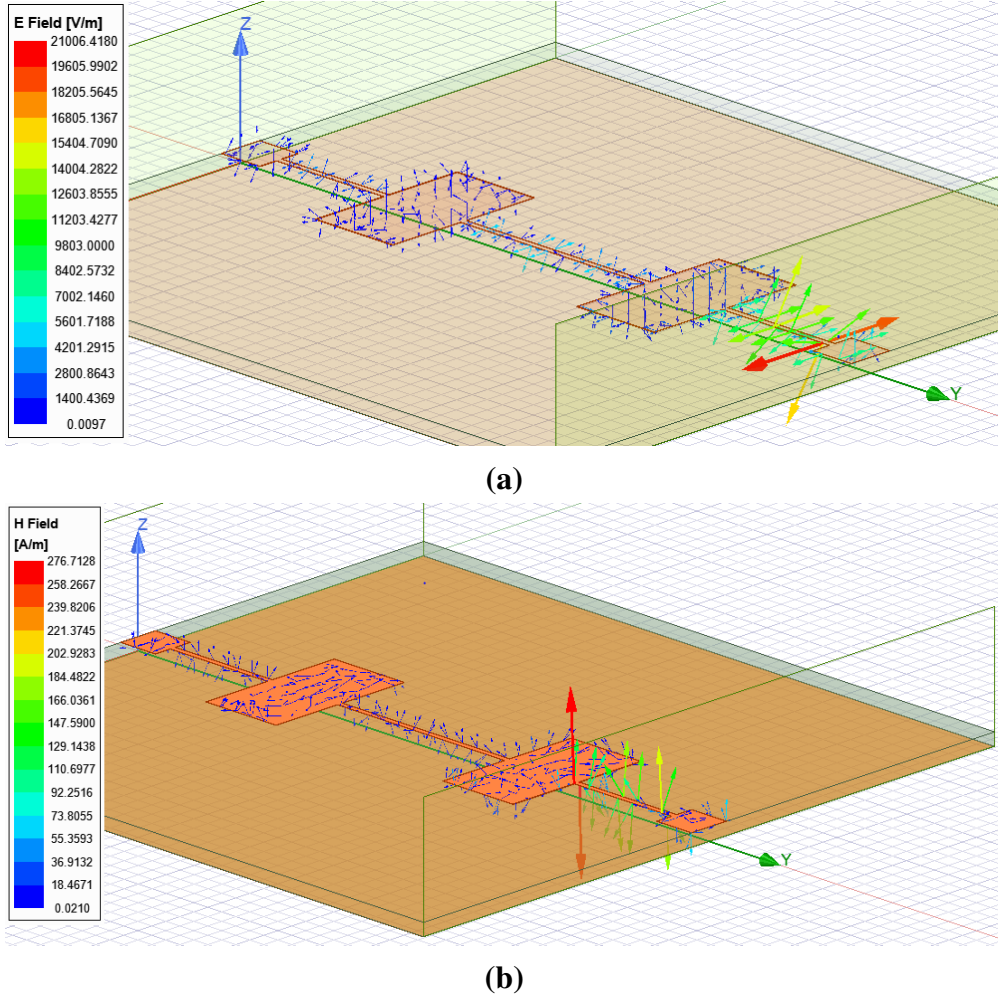


Figure 6.7: HFSS simulation of the microstrip filter's electric and magnetic fields. (a) Vector electric fields. (b) Vector magnetic fields.

the top layer of copper while leaving the underlying substrate. The TLs and LPF were fabricated simultaneously from the same board of MX. Once completed, a hand operated lever cutter was used to separate the four sections of microstrip and trim to length. Edge launch SMA connectors were soldered to the boards for evaluation by a network analyzer. The connectors were modified by removing the upper support arms with wire cutters due to near proximity to the edges of the $50\ \Omega$ traces. The fabricated LPF is displayed in Figure 6.9.

Table 6.5 demonstrates how the tolerances of a fabrication method can impact the realized microstrip board. The overall structural size was miniaturized by the MX material parameters, however it is also important to remember the fabrication method's dimensional tolerances are also

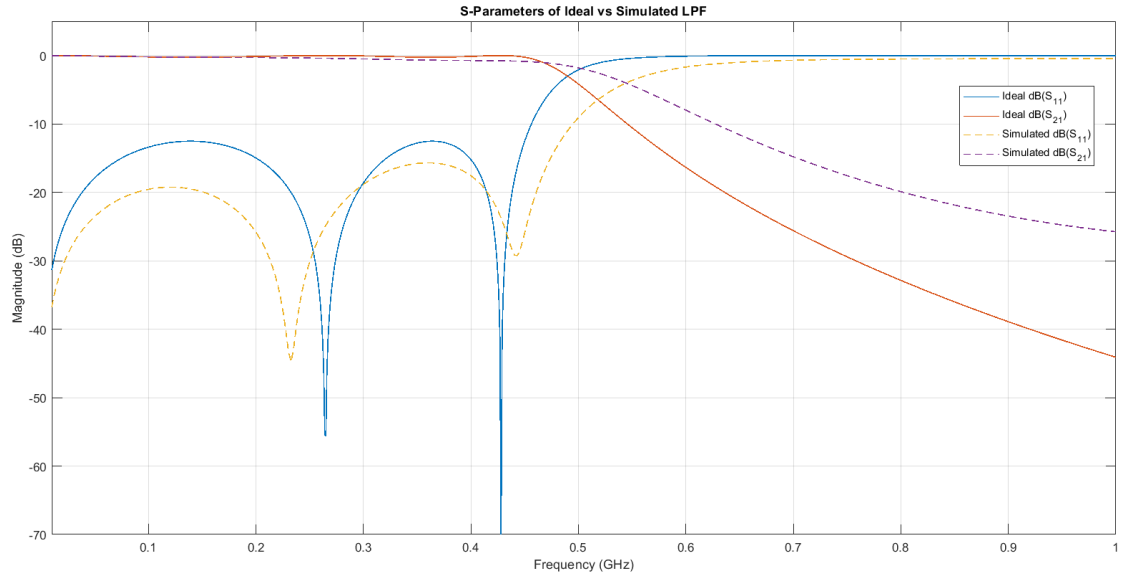


Figure 6.8: S-parameter frequency response comparing the HFSS simulation versus the ideal lumped element LPF.

scaled. These diminishing tolerances become more critical to design requirements with greater miniaturization.

Table 6.5: MX dimensions as designed versus fabricated for $50\ \Omega$ TL and 5th order LPF.

	Trace width [mm] Design / Actual	Trace length [mm] Design / Actual	Ground width [mm] Actual	Ground length [mm] Actual	Thickness [mil] Design / Actual
TL design 1	4.98 / 4.99	45 / 44.2	34.2	44.2	60 / Avg 63.0
TL design 2	4.98 / 4.98	60 / 58.48	32.40	58.48	60 / Avg 64.5
TL design 3	4.98 / 5.00	81.7 / 80.95	32.10	80.95	60 / Avg 62.0
5th order LPF	4.98 / 4.84	81.7 / 80.95	44.4	80.95	60 / Avg 63.0
	L1 [mm] Design / Actual	C2 [mm] Design / Actual	L3 [mm] Design / Actual	C4 [mm] Design / Actual	L5 [mm] Design / Actual
LPF width	0.76 / 0.75	18.36 / 18.30	0.76 / 0.75	18.36 / 18.28	0.76 / 0.85
LPF length	13.9 / 13.97	10.0 / 10.00	23.9 / 23.82	10.0 / 10.02	13.9 / 13.97

Measurement Methods

The four fabricated 2-port networks, three TLs and a LPF, were evaluated by measurement of **s-parameters!** (**s-parameters!**) on an Agilent Technologies N5225A MW programmable network



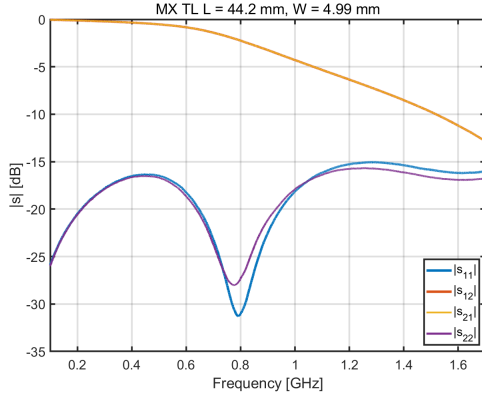
Figure 6.9: Fabricated two-port microstrip structures on MX substrate. (Top) 45 mm TL (Middle) 60 mm TL (Bottom) 81.7 mm TL (d) 5th order LPF

analyzer (PNA) after completion of a calibration procedure.

The fabricated TLs were evaluated from 100 MHz to 5 GHz at a sampling frequency of near 400 kHz. The fabricated microstrip filter was evaluated over the frequency range from 10 MHz to 1.5 GHz on a network analyzer. The s-parameter data was saved to a portable USB drive then compared to HFSS simulation results in MATLAB.

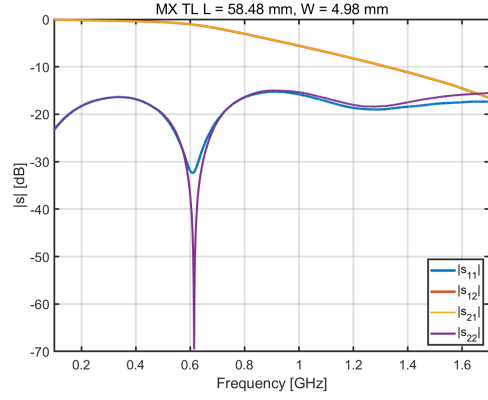
The third TL seen in Figure 6.10c demonstrates an abnormality in the s_{22} parameter. This is most likely a secondary unintended effect related to the SMA connector, but removal of the upper supporting pins, verification of solder contact with pin and TL, and network analyzer connector troubleshooting did not resolve this result. There may be material defect from the manufacturing

Amplitude of Measured S-parameters VS Frequency



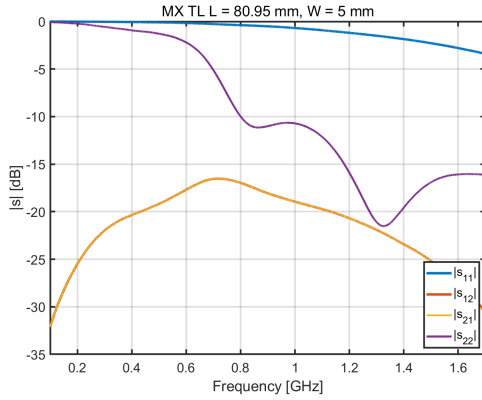
(a)

Amplitude of Measured S-parameters VS Frequency



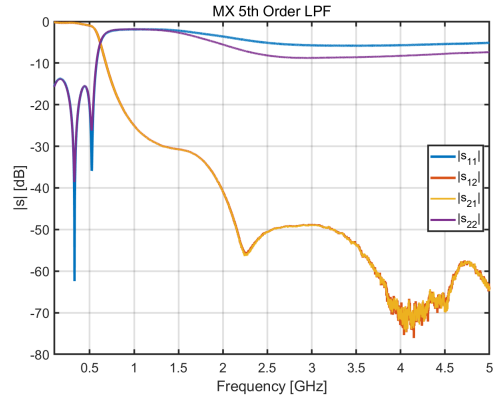
(b)

Amplitude of Measured S-parameters VS Frequency



(c)

Amplitude of Measured S-parameters VS Frequency



(d)

Figure 6.10: Two-port s-parameter measurements for the fabricated microstrip structures on MX substrate (a) 45 mm TL. (b) 60 mm TL. (c) 81.7 mm TL. (d) 5th order LPF.

or fabrication process. Ideally non-destructive methods for evaluation of an occult defect would be utilized, but these methods were not accessible. The data collected from this defective board was excluded from further data analysis as this outlier does not align with the predicted results nor results from the other fabricated microstrip structures.

The s-parameters of the fabricated LPF demonstrated a cutoff frequency shift, increasing by approximately 20 MHz, when compared to the HFSS simulation using an isotropic $\epsilon_r = 6.0$ as demonstrated in Figure 6.11. The 450 MHz cutoff frequency is very near the end of the MX recommended operational frequency range, likely near the FMR cutoff frequency. All the two-port measurements in Figure 6.10 indicate the MX material demonstrates inherent LPF qualities. These are due to frequency dispersive properties of both the relative permittivity and permeability at low

frequencies up to the first FMR which also represents the FMR cutoff frequency as discussed in Chapter 2.

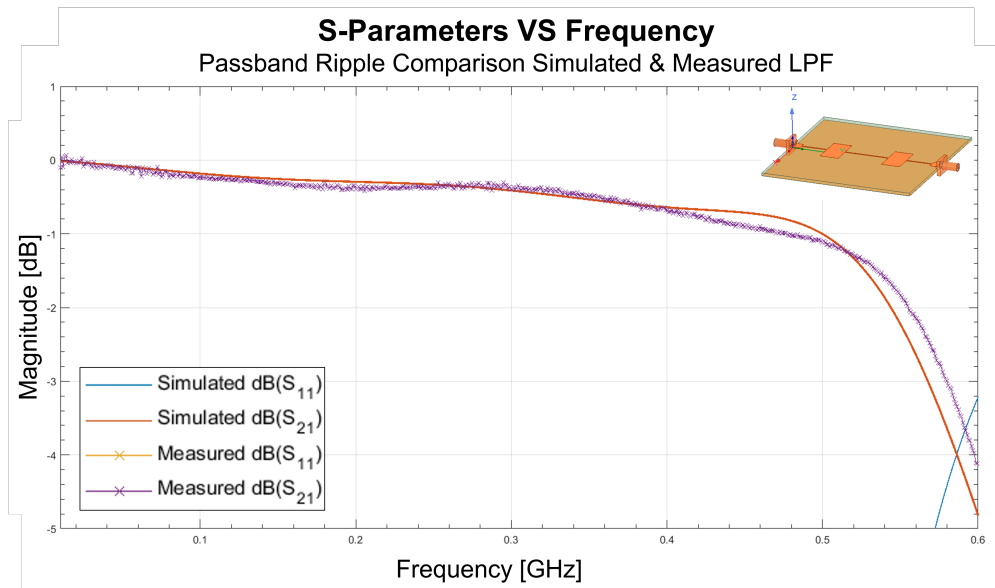
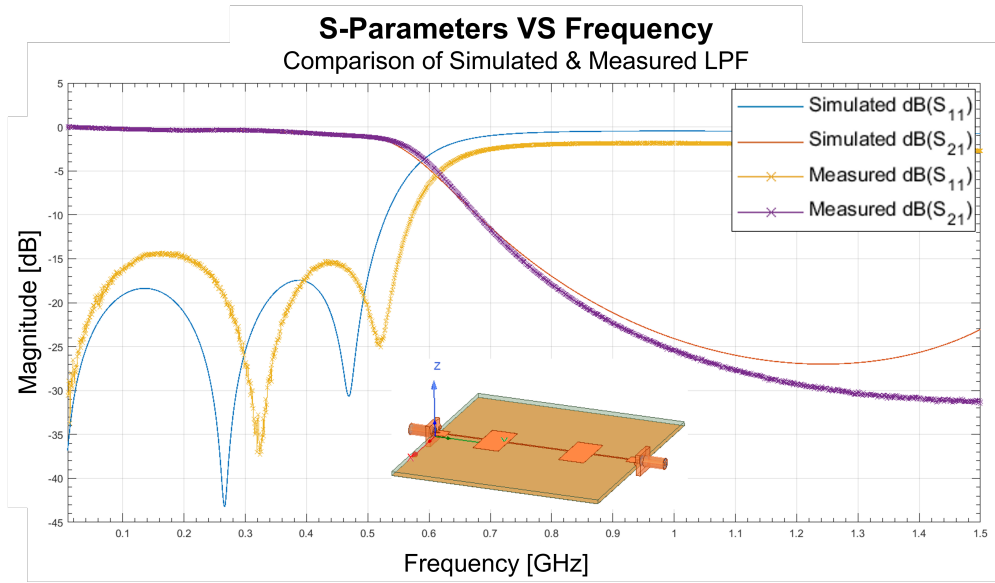


Figure 6.11: LPF frequency response as measured versus HFSS simulation. (a) LPF frequency response. (b) Zoomed to appreciate passband ripple.

6.3 Two-Port Network Validation

These fabricated microstrip structures were utilized for experimental validation of the Hargraves CM and Hargraves Dual Methods presented in Chapter 4. The two-port structures were evaluated with the CM methods to find the effective relative permittivity and permeability. These values were directly applied to standard two-port $ABCD$ -parameter formulas for a length of TL given in Equation 3.34 [71].

The measured data includes the SMA connectors soldered to each port. In order to simulate SMA connectors, they were modeled in HFSS and simulated at the same frequency range and number of samples that will be used for the CM procedure. The range from 100 MHz to 1.7 GHz was determined to be sufficiently above the peak permeability for MX per the datasheet and 2001 samples in this range was used. Section 2.1 introduced the permeability spectra where μ_r value is somewhat constant and near the static permeability value until the first peak of μ'_r , which begins the permeability breakdown region. The FMR occurs when μ'_r again rises and falls coinciding with the second μ''_r peak. The MX datasheet reports the permeability breakdown region and cutoff frequency occur at 715 MHz and 2.8 GHz, respectively [44].

Once the results were found, the complex s-parameters were exported from HFSS, read into MATLAB, then converted to $ABCD$ -parameters. The s-parameters for a SMA connector are slightly asymmetric, so the the far connector was represented as the reciprocal $ABCD$ -matrix to the initial SMA connector. $ABCD$ -parameters can be multiplied in series to find the parameters of the structure

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total_{TL}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{SMA_1} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{CM_{TL}} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{SMA_2}. \quad (6.10)$$

Figure 6.12 demarcates the microstrip edge planes with a red dashed line. The dashed lines divide the regions represented by each $ABCD$ -matrix. The network analyzer has a load impedance $Z_L = 50 \Omega$ calibrated to open, short, and through standards. As described above, all of the two-port

microstrip structures were designed to have $50\ \Omega$ terminating load for a good impedance match to SMA edge launch connectors. The input impedance convention for all two-port measurements utilize the convention $Z_L = 50\ \Omega$, which is maintained throughout experimental validation unless otherwise noted.



Figure 6.12: Cascaded $ABCD$ -matrices overlaid on a microstrip TL. The region for each matrix is divided by a red dashed line.

Recall that the 5th order LPF had LC sections as well as the $50\ \Omega$ strip for impedance match and soldering SMA edge launch connectors. The LPF board was modeled similarly by using the presented CM method individually for each section of different widths, placing those values into the TL equations, converting to $ABCD$ -parameters, then multiplying the matrices in series. The LPF microstrip board without connectors has seven segments of different widths and with the SMA connectors totals nine $ABCD$ matrices.

S_{21} Phase Angle

The primary effect a MD substrate will have is altering the phase velocity of the guided wave along the length of TL. To assess this attribute, the phase angle of s_{21} , or $\angle S_{21}$, gives the phase change versus frequency of the s-parameter measured at port 2 when excited from port 1. Typically the phase difference of a signal is used to calculate group delay. However, this metric is being utilized to evaluate how the propagation phase is affected by the MD substrate. The measured values are plotted for comparison to CM methods and CM with cascaded SMA connectors in Figures 6.13 and 6.14, respectively.

Initial evaluation of Figure 6.13 shows the effective relative permeability is well approximated by the Averaged and Hargraves Dual Methods. This assessment is limited because CM algorithms only account for the behavior of the microstrip structure without the effect of SMA connectors. To

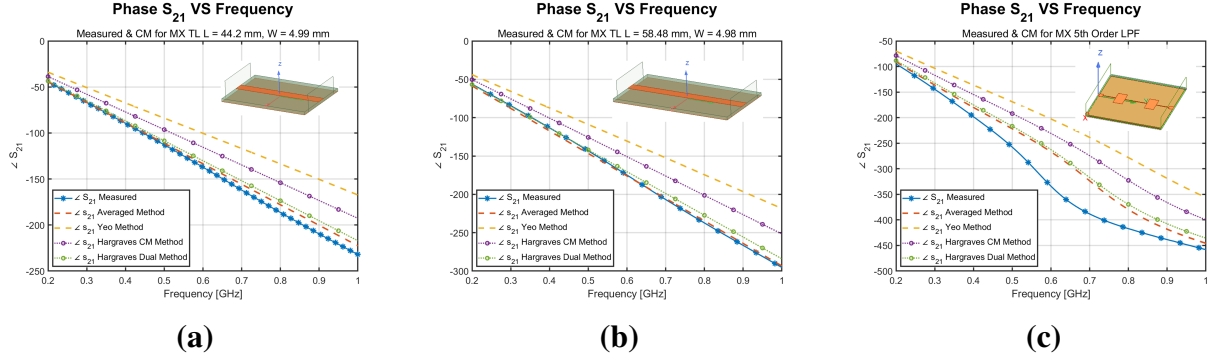


Figure 6.13: Comparison of $\angle S_{21}$ across frequency from fabricated MX measurement and CM algorithms. (a) 45 mm TL. (b) 60 mm TL. (c) 5th order LPF.

better capture the frequency behavior of the entire structure, the SMA ports were cascaded with the CM results then reevaluated in Figure 6.14.

Figure 6.14 demonstrates better alignment of the $\angle S_{21}$ results to measurement. When comparing like structures, microstrip with series SMA connectors, it becomes apparent that the Hargraves CM Method is the best method for assessing the phase angle, $\angle S_{21}$, of 50 Ω TLs.

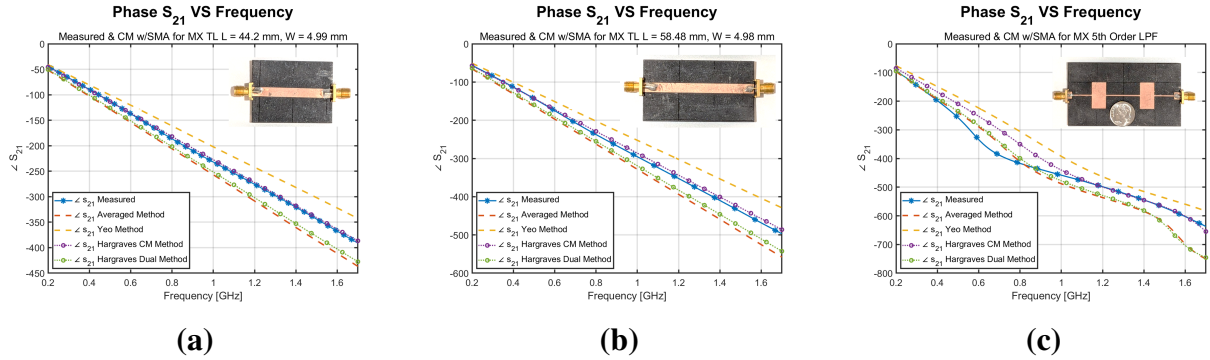


Figure 6.14: Comparison of $\angle S_{21}$ across frequency from fabricated MX measurement and CM algorithms with cascaded SMA ports as given in Equation 6.10. (a) 45 mm TL. (b) 60 mm TL. (c) 5th order LPF.

As expected, more error is noted in analysis of the LPF shown in Figure 6.14c. The average $\angle S_{21}$ error of the Averaged, Yeo, Hargraves CM, and Hargraves Dual Methods were 10.5 %, 9.9 %, 1.2 %, and 9.2 %, respectively. These values are in agreement that the Hargraves CM Method is the best CM algorithm for assessing the electrical length of these microstrip structures.

The LPF measurement demonstrates a significant decrease in the phase velocity near the permeability breakdown region occurring at 715 MHz. This is where both ϵ'_r and μ'_r both experience

a sharp decrease in value affecting the phase constant found by $\beta = k_0 \sqrt{\epsilon_{ref} \mu_{ref}}$, while the lossy terms, ϵ_r'' and μ_r'' both peak increasing stored energy loss, $\alpha = \frac{q_1 k_0 \sqrt{\epsilon_{ref} \mu_{ref}}}{2} (\tan \delta_E + \tan \delta_M)$, which is dominated by the loss terms. These additional loss mechanisms were not significant in measurements from simple TL structures. However, by design the LPF has an increased rejection per decade above 450 MHz allowing the mechanism to become dominant on measurements. This concept will be important in evaluation of MPAs as they functionally act as a spatial filter, but generally have a single pole. The MPA analysis will follow in Section 6.4

Application of these CM methods to the LPF does capture the altered phase change above the respective cutoff frequency, however it does appear shifted in all of the CM algorithms. MD materials have complex loss mechanisms that were not included in these CM algorithms. Inclusion of frequency dispersion for a well modeled material with consistent permeability and permittivity spectra would better capture the spectral behavior and worth the added computational cost. Computational efficiency was prioritized in the CM algorithms to provide proof of concept for microstrip design and general characterization.

Amplitude of S_{11}

The magnitude of s_{11} gives the amplitude of signal reflected at port 1 when excited from port 1. The plots of Figure 6.15 do demonstrate a null near the measured frequency for both lengths of TL. The 45 mm TL demonstrated a $|s_{11}|$ minimum at $f = 790$ MHz with the Yeo and Hargraves CM Methods found the null at 856 MHz (8.4% error) and 700 MHz (11.5% error), respectively. Similarly, the 60 mm TL demonstrated the $|s_{11}|$ null at $f = 610$ MHz with the Yeo and Hargraves CM Methods finding 674 MHz (10.6% error) and 539 MHz (11.6% error), respectively.

The LPF's $|s_{11}|$ was not well approximated. This result is not surprising because the LPF was evaluated as nine cascaded two-port s-parameter matrices. However, the MATLAB function *cascadesparams()* converts each to an *ABCD*-matrix assuming a default 50 Ω impedance. The characteristic impedance terminating each differing width of microstrip line along the LPF are mismatched. To capture this mechanism, the reflection coefficient would need to be calculated at

each transition using

$$\Gamma = \left| \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right|, \quad (6.11)$$

where η is the material refractive index and the subscript is the order of interaction. To utilize the reflection coefficient in a cascaded network, the response across frequency must be found then converted to s-parameters.

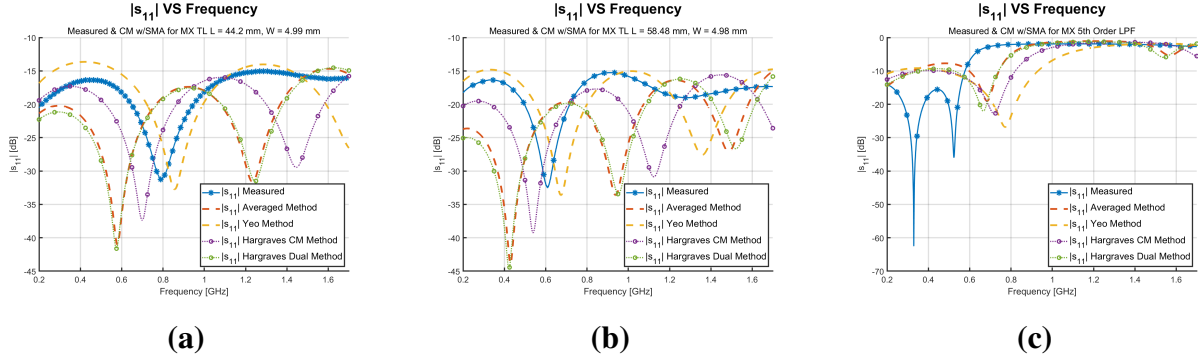


Figure 6.15: Comparison of $|s_{11}|$ as found by CM algorithm, CM with SMA ports, and MX measurements of fabricated board (a) for 45 mm TL (b) for 60 mm TL (c) 5th order LPF

Impedance of Two-port TLs

Figures 6.15a and 6.15b indicate the inherent loss of the MD substrate is not entirely captured by the CM model. The initial loss at 200 MHz from measurement are 10 dB less than what was captured in the CM model, which is an entire order of magnitude less than predicted. The characteristic impedance of a microstrip calculated by Pozar's method does not capture the effect of permeability from the MD substrate. The characteristic impedance of a microstrip depends on both the structural dimensions and the material parameters. Additionally, the TL equations are better suited to structures with true TEM and closed form solutions to Helmholtz equations. The effective dimensions and constitutive parameters attempt to capture the results as a quasi-TEM mode of propagation, but lack the effects of a finite ground plane, finite thickness of conductors, and potential mutual effects along the open edges of the TL.

Recall Equation 4.30 where

$$Z_c = \frac{\eta_0 \sqrt{\frac{\mu_{ref}}{\epsilon_{ref}}}}{2 \left(\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\epsilon_{ref} + 1}{2\pi\epsilon_{ref}} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\epsilon_{ref} - 1}{2\pi\epsilon_{ref}^2} \ln \frac{e\pi^2}{16} \right)},$$

where $\eta_0 = 120\pi \approx 377 \Omega$ is the resistance to free-space and $e = 2.71828$ is Euler's number. Wheeler developed the initial analytical formulation of the characteristic impedance at the flux boundary and Yeo introduced the permeability term [79], [80], [83]. Wheeler's derivation was based on the twin line assumptions where the impedance halved with the image plane to represent a microstrip structure, therefore the formula above introduced a factor of one-half. Wheeler's derivation was along the flux boundary which in the image plane, divides the two half-planes. This line within the image domain bounds the fringing fields to the left half image plane and the substrate confined fields to the right half of the image plane. The characteristic impedance formula should utilize both effective relative permittivity and permeability throughout the equation to capture fringing field effects.

When the relative permeability and relative permittivity found by CM are used with the modified formula for Z_c to calculate the microstrip line behavior, the TL analysis results improved. Figure 6.16 demonstrates the phase error for Z_{in} is a challenge to capture for the fabricated MX TLs, but has increasing accuracy using characteristic impedance found by Equation 4.30 compared to Wheeler's standard formula, while the LPF results were only slightly shifted.

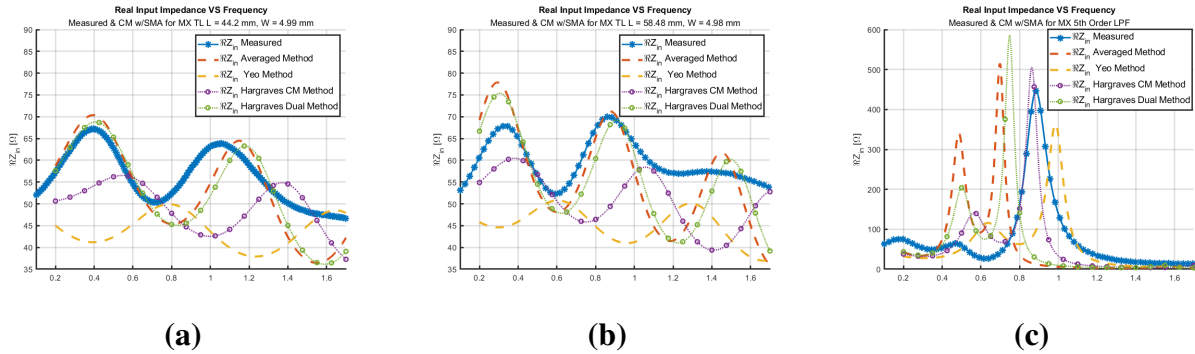


Figure 6.16: Comparison of Z_{in} as found by CM with SMA ports and measurement. (a) $\Re Z_{in}$ for 45 mm TL. (b) $\Re Z_{in}$ for 60 mm TL. (c) $\Re Z_{in}$ for 5th order LPF.

6.4 MPA Validation

Blosser fabricated and measured a MPA on MX substrate, dimensions provided in Table 6.6 [27]. Figure 6.17 shows Blosser's measured results versus those obtained with an HFSS full-wave simulation and the presented CM method utilizing the modified Yeo method for characteristic impedance. Both the measured and HFSS simulations included an SMA port at the feed location, the CM here is presented without the SMA port. The input impedance with the CM method is slightly reduced from the measured and simulated values. The SMA probe feed method would be a series input with the lengths of MPA on either side being in parallel. The CM was normalized to $50\ \Omega$, but does not account for the coaxial TL effects. To include the SMA port effects, the effect of the coaxial pin increasing the inductance at the feed point should be reflected in the $\Im Z_{in}$.

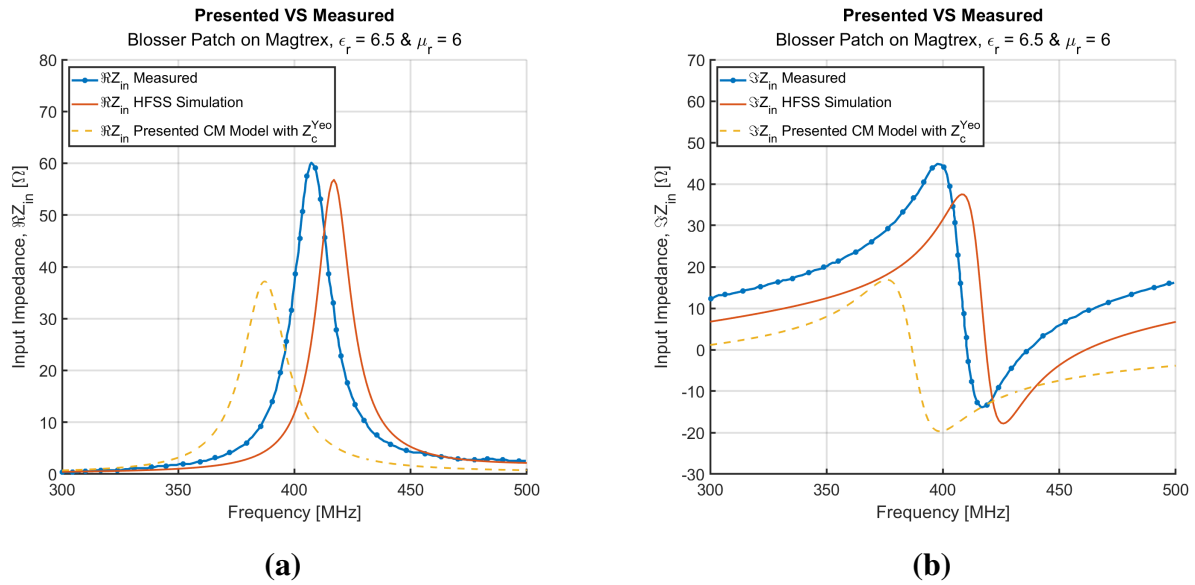


Figure 6.17: Blosser's MPA on MX substrate Z_{in} as measured versus combination of the presented CM and Pues and Van de Capelle's TL method [65].

Table 6.6: Dimensions of Blosser's MPA on MX substrate [27].

	L	W	h	$feed$	f_0
Design	74.6 mm	37.3 mm	60 mil	24 mm	400 MHz
Actual	74.61 mm	37.42 mm	62 mil	23.3 mm	410 MHz

Overestimation of the starting relative permittivity and permeability of the substrate could cause the frequency shift to the left that is seen in Figure 6.1. The frequency shift noted in Figure 6.1 for Z_{in} could relate to material parameter variation because realization of the MD properties at fabrication is difficult, as covered in Chapter 2. The relative permeability can be affected by the particle size, shape, settling, and particle clumping, therefore likely has a larger tolerance margin than relative permittivity. Looking at the MX datasheet provided by Rogers Corporation, Figure 6.1 shows the real part of the relative permeability begins to rise at 499 MHz, peaks at 715 MHz, then rapidly falls. Furthermore, the relative permeability shown on Figure 6.1 appears to be slightly less than 6 before the value begins increasing at 499 MHz. Considering likely fabrication tolerances and the MX datasheet indicates a slightly lower relative permeability when graphically presented than the value printed for reference. Between reported values being slightly lower and expected manufacturing tolerances, this frequency shift is not surprising.

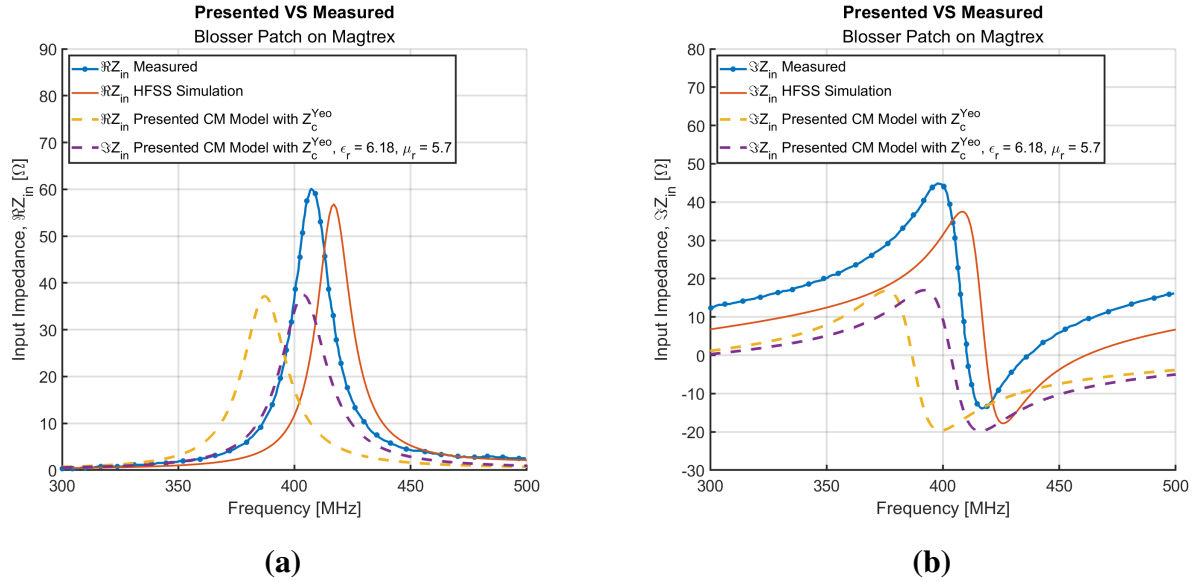


Figure 6.18: Experimental verification of Conformal Mapping (CM) parameters to model input impedance of probe fed microstrip patch antennas (MPA).

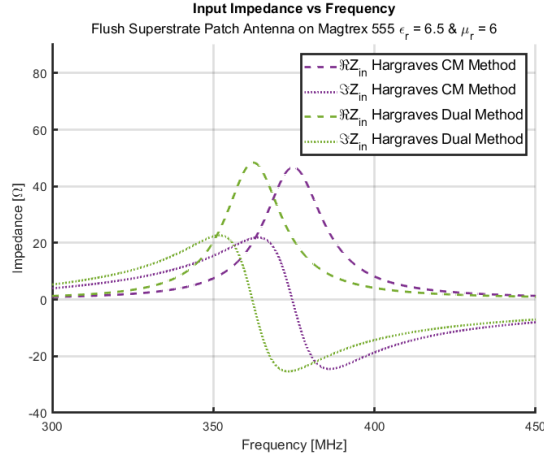


Figure 6.19: MX MPA with flush superstrate.

6.5 Experimental Validation Remarks

The presented CM methods were experimentally validated to microstrip structures fabricated on MX, a CoTS MD substrate material. The data obtained from two lengths of microstrip TL, a microstrip 5th order LC LPF, and a MPA support validation of these Hargraves CM and Hargraves Dual Methods for MD materials of moderate and closely matched constitutive values.

There is not a wide enough range of material parameters available to generally validate these methods. To obtain a wide range of material parameters, a parametric simulation will be utilized to identify the bounds for which this CM method is valid.

Chapter 7

Verification of CM by Simulation

Chapters 4 presented a novel method for calculating the effective relative permeability of a microstrip structure by CM. This method expands upon prior CM techniques which together expand the application to microstrip structures incorporating MD media. This work was experimentally validated in Chapter 6 for MD materials that have closely matched and moderate valued constitutive parameters. To find the bounds of validity of the presented CM methods, a parametric simulation will be utilized.

7.1 Simulation Methods

Access to MD materials with high values of relative permittivity, relative permeability, or both are not widely available for the fabrication of prototypes. Full-wave simulation can provide robust data for comparison of the presented CM algorithm. A parametric analysis will provide solutions for narrow to very wide microstrip structures with permutations of constitutive parameters and configured as both one-port and two-port networks.

Chapter 6 validated the CM methods to experimental measurements of two-port TLs and single-port probe fed MPAs. These same structures will be simulated for verification. Microstrip TL simulation in HFSS allows excitation by wave ports, removing any external effects introduced experimentally by material tolerances, fabrication tolerances, and SMA connectors. The two-port model only considers the effects of the ground plane, ideal material parameters, and conductive strip for direct comparison to the presented CM technique. This will result in a robust evaluation of

the CM algorithm and define the bounds for which the method is valid.

Simulated TLs and MPAs

The HFSS simulations utilized an infinite PEC ground plane to exclude the edge effects. This method also helped to minimize the simulation time because only the region above the ground plane required simulation. The radiation box was sized from microstrip, it was a quarter wavelength in height from the microstrip plane and width of the strip plus a quarter wavelength on each side. The lowest frequency simulated always has the longest wavelength, so the 200 MHz wavelength was used.

One of the primary benefits of using a simulation for comparison is removal of the SMA port effects from the overall behavior. The two-port network used a wave-port defined at the board termination as six times the strip width and ten times the substrate height. The MPA could not be completely decoupled from the SMA because it depends on the center pin to feed the patch, but the infinite ground plane shields the patch from any SMA field interactions.

The microstrip TLs were designed utilizing local variables so the dimensions would scale automatically when parametric analysis was used. The length and height were set constant and the width altered to give a range of normalized widths, $0.5 < W/h < 110$. A custom MD material was created and defined global variables for relative permittivity and permeability. The only realistic MD material characteristics known for the utilized frequency range are from MX, therefore the electric and magnetic loss tangents of MX were used. The constitutive values included 1 to 16 by steps of 5, for 16 combinations parametric analysis used nine widths ($1 \leq W \leq 165$ mm), and four values of constitutive values, defined to simulate the various dimensions in all permutations.

The CM and simulation frequency sweep and number of samples were matched, both swept from 200 MHz to 600 MHz with 1001 data points. When a greater frequency range was utilized on the CM analysis, an additional fifty percent in range and sampling rate were appended to the range matching the simulation, half to the beginning and remainder to end of frequency range. The interpolation basis was increased and the minimum converged passes increased from one to two.

The analysis section was also utilized to converge on parameters specified: characteristic impedance, the propagation phase, and phase angle of s_{21} . Additionally, a port only solution was requested with the minimum port mesh refined for characteristic impedance accuracy.

7.2 Verification for Two-Port Networks

Two-port Characteristic Impedance, Z_0

A parametric HFSS simulation was created to capture a range of constitutive parameters and normalized strip widths. The relative permittivity and relative permeability were simulated from 1 to 16 in increments of 5. The permutations of constitutive parameters resulted in sixteen paired values. The width had nine values that ranged from 1 to 165 mm and height remained constant at 1.524 mm. This resulted in normalized widths from 0.7 to 108.

The large parametric dataset from HFSS was condensed to a heatmap representation shown in Figure 7.1 by using the mean of the real portion of characteristic impedance across frequency. The features of Figure 7.1 have x -axis values representing the normalized strip width, W/h , y -axis values represent the paired ϵ_{ref} and μ_{ref} values, and the hue represents the magnitude of characteristic impedance, Z_0 . The constitutive values were sorted to show the ascending magnitude of z_0 with increasing μ_{ref} and decreasing ϵ_{ref} .

Recall Wheeler's formula for characteristic impedance given in Equation 4.8 given by

$$R = \frac{\eta_0 \sqrt{\frac{1}{\epsilon_r}}}{\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\epsilon_r + 1}{2\pi\epsilon_r} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\epsilon_r - 1}{2\pi\epsilon_r^2} \ln \frac{e\pi^2}{16}},$$

where $e = 2.71828$ is Euler's number [79]. It is apparent that Equation 4.8 is a function of relative permittivity, but does not change with varying relative permeability. As expected, this trait is demonstrated in Figure 7.2 with the averaged relative permeability CM method and the Hargraves CM method. The Hargraves Dual method yields the same results as the Hargraves CM method because they both utilized Bernhard and Tousignant's method for relative permittivity [26].

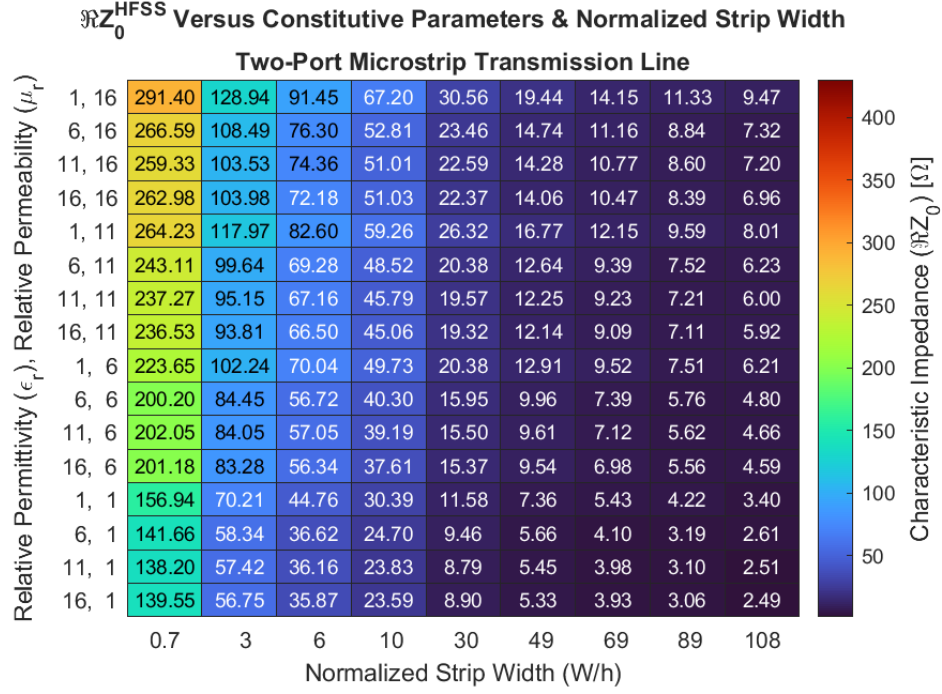


Figure 7.1: HFSS Parametric simulation data condensed to show the magnitude of mean real characteristic impedance versus the relative constitutive values and normalized strip width. The color scale is to the maximum characteristic impedance found by any method for easy comparison.

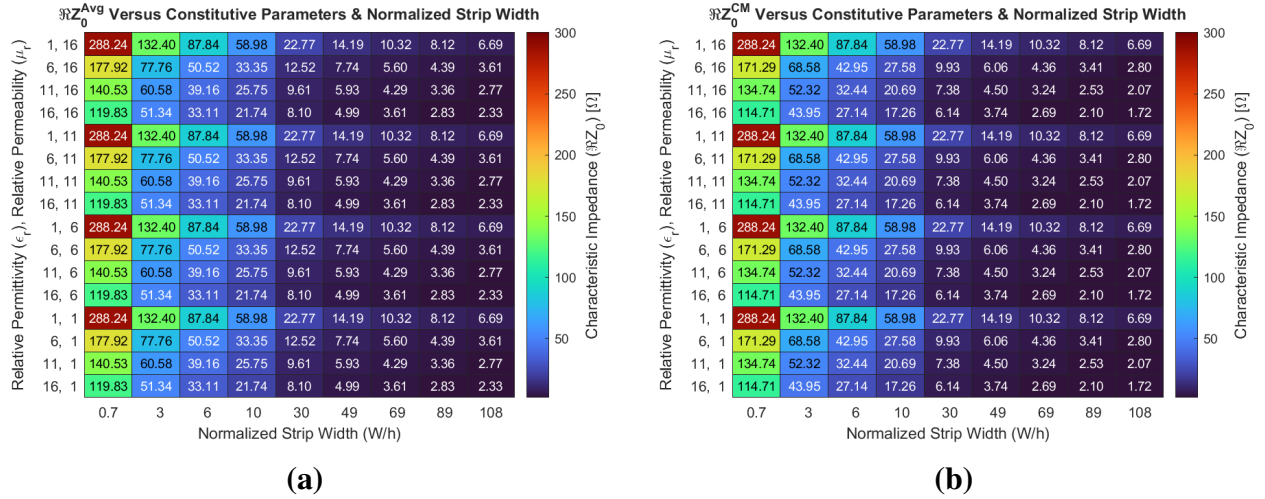


Figure 7.2: Condensed parametric CM demonstrating the magnitude of characteristic impedance found by Wheeler's method when incorporating MD materials [79]. (a) CM with averaged relative permeability. (b) Hargraves CM method.

Recall Equation 4.30 where

$$Z_c = \frac{\eta_0 \sqrt{\frac{\mu_{ref}}{\epsilon_{ref}}}}{2 \left(\frac{w}{2h} + \frac{1}{\pi} \ln 4 + \frac{\epsilon_{ref} + 1}{2\pi\epsilon_{ref}} \ln \left[\frac{\pi e}{2} \left(\frac{w}{2h} + 0.94 \right) \right] + \frac{\epsilon_{ref} - 1}{2\pi\epsilon_{ref}^2} \ln \frac{e\pi^2}{16} \right)},$$

was presented by Yeo and corrected by Hargraves. calculated from inputs for the effective parameters calculated from the averaged relative permeability, Yeo's, Hargraves' CM, and Hargraves' Dual methods is shown below in Figure 7.3.

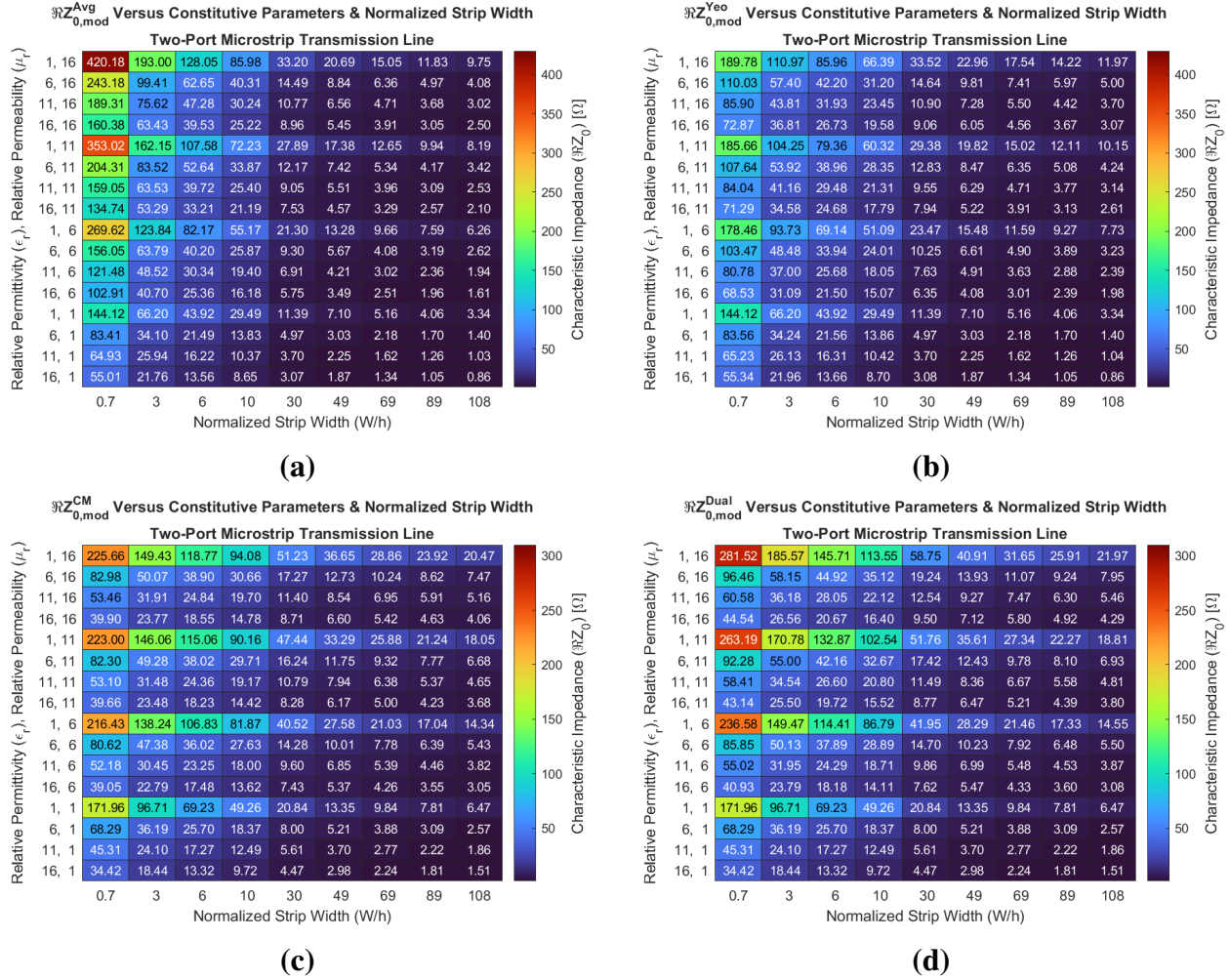


Figure 7.3: Condensed parametric CM demonstrating the magnitude of characteristic impedance found by the modified characteristic impedance from Equation 4.30. (a) CM with averaged relative permeability. (b) Yeo's CM method [83]. (c) Hargraves CM method. (d) Hargraves Dual method.

Comparison of each subfigure from Figure 7.3 to the HFSS simulation data in Figure 7.1 reveals improved trends. Generally, all of the methods shown in Figure 7.3 agree with simulation giving the highest magnitude when the substrate has maximum relative permeability and unitary relative permittivity. They also demonstrate decreased magnitude with both increasing relative permittivity and decreasing relative permeability. The general trend that the characteristic impedance decreases as the normalized strip width increases. Figure 7.3a demonstrates the maximum magnitude, which

was found by the averaged relative permeability method.

The percent error of each method was calculated by

$$\%error = \frac{|Z_0^{HFSS} - Z_0^{calc}|}{Z_0^{HFSS}} \times 100, \quad (7.1)$$

where Z_0^{calc} is the value found by each CM method. Rearranging the paired constitutive parameters into a gray code reveals regions of increased error and demarcates the most accurate region of each method. The percent error of each method is shown in Figure 7.4d. Regions of accuracy and increased error become grouped by rearranging the paired constitutive parameters into a gray code where each adjacent row has only one parameter changed. The numerator of the modified Z_0 formula has the greatest impact with the $\sqrt{\mu_{r_{ef}}/\epsilon_{r_{ef}}}$ term, which has well demarcated high accuracy with the Haragraves Dual method when relative permittivity is unity for normalized strip width near 1.

Figure 7.5 demonstrates the input impedance of each method when the port impedance is equivalent to the characteristic impedance. The simulation input impedance is near the characteristic impedance, but demonstrates some variability from dispersive mechanisms. Slight differences between the convergence settings for z -parameters and Z_c may also contribute, but was a necessary trade for computational efficiency for an already large parametric simulation.

Two-port Propagation Phase Angle, β

Validation to simulation has the unique opportunity to validate directly to the complex propagation parameters for increased confidence. The HFSS simulation was set to converge on the imaginary portion of complex propagation constant which allows direct comparison of the propagation phase, β , between full-wave simulation results and the TL calculation using CM results. Figure 7.6 demonstrates the β values for various normalized strip widths.

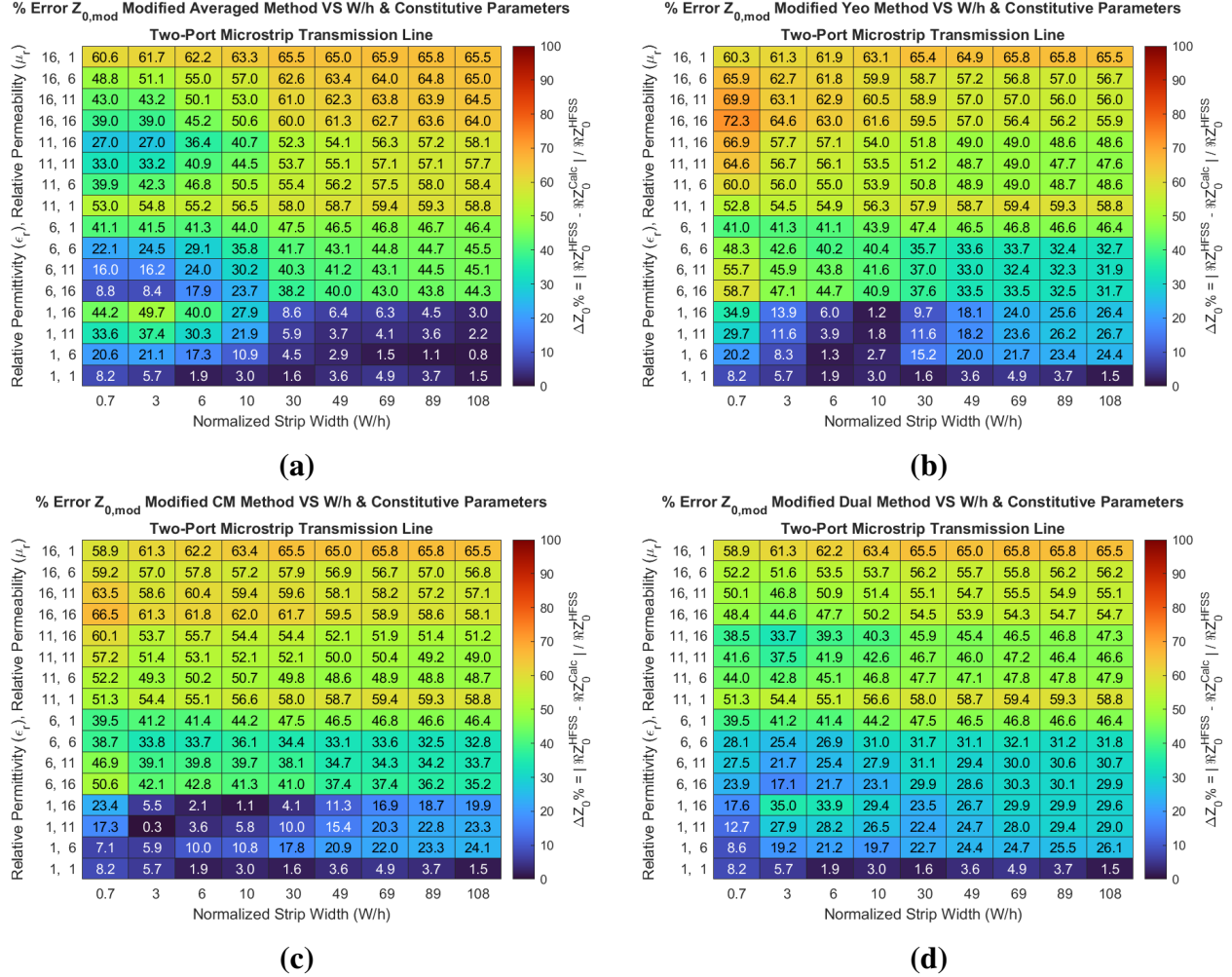
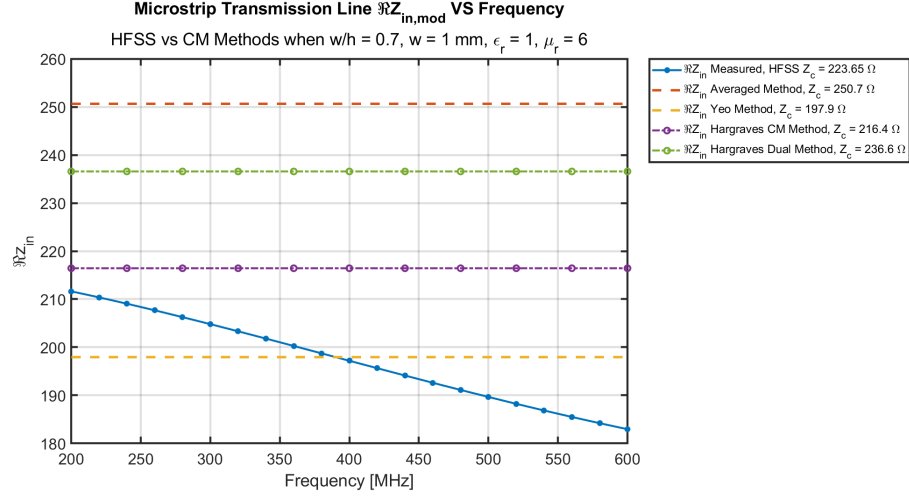


Figure 7.4: Condensed parametric CM demonstrating the percent error of the magnitude of characteristic impedance found by the modified Yeo characteristic impedance from Equation 4.30 compared to simulation. (a) CM with averaged relative permeability. (b) Yeo's CM method [83]. (c) Hargraves CM method. (d) Hargraves Dual method. Each cell summarizes a plot as the mean percent error. Figure 7.5 is two representative graphs summarized here.

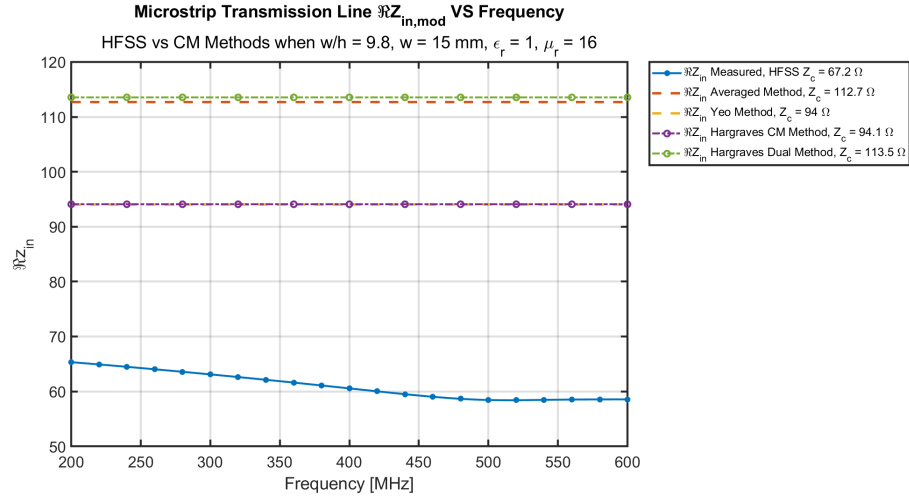
7.3 Microstrip Patch Antenna (MPA) Verification

Microstrip Patch Antenna Input Impedance, Z_{in}

Early models for microstrip and MPAs assumed unity relative permeability, therefore only considered relative permittivity in the curve fitting solutions. It is important for models to be extended to account for inclusion of MD materials, but ideally it maintains the accuracy of dielectric only materials. Figure 7.7 demonstrates the agreement of input impedance for high dielectric materials



(a)

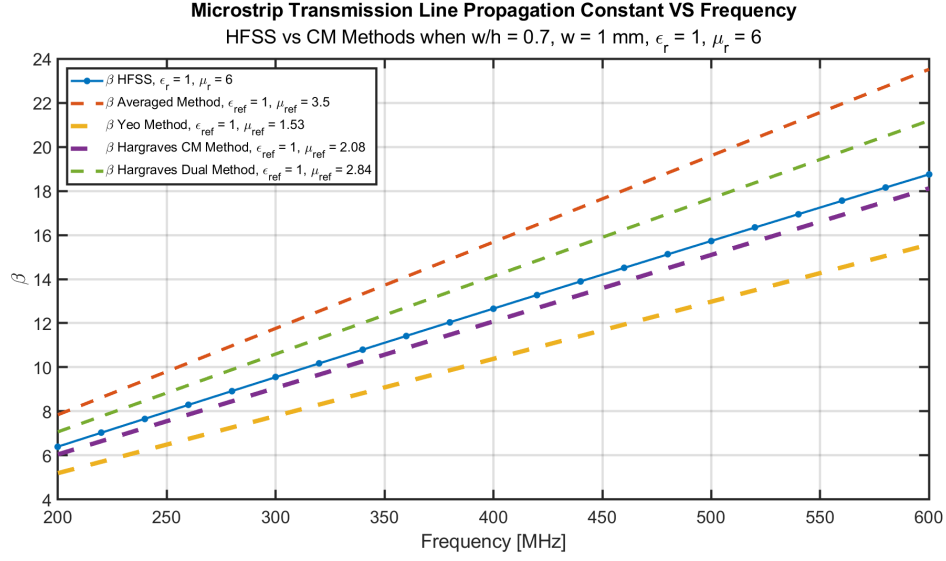


(b)

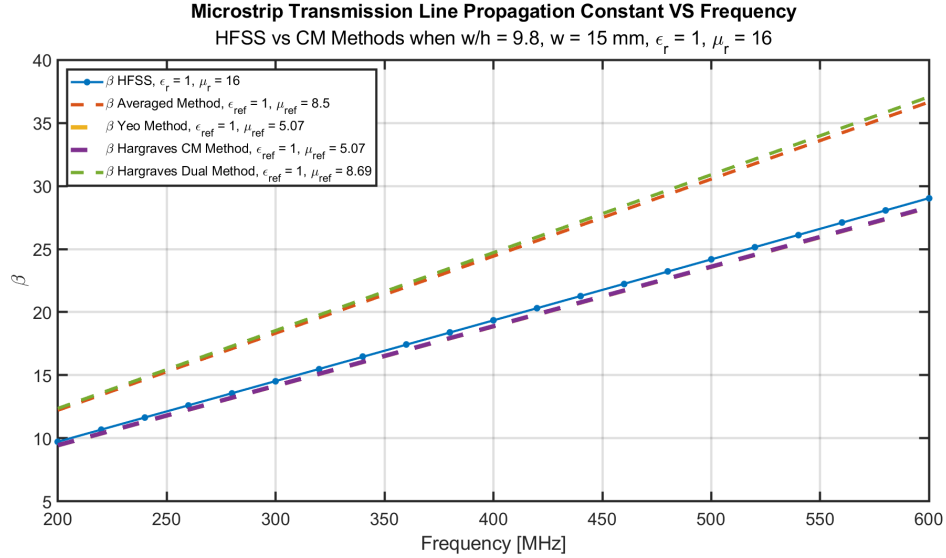
Figure 7.5: Input impedance versus frequency from simulation and each CM method. (a) Microstrip with $W/h = 0.7$, $\mu_{ref} = 6$, and $\epsilon_{ref} = 1$. (b) Microstrip with $W/h = 9.8$, $\mu_{ref} = 16$, and $\epsilon_{ref} = 1$.

modeled by different CM models with the results from simulation in HFSS. The imaginary portion is noted to have increased inductance, but this is a known effect related to the inner conductor of the probe feed method electrically appearing like an inductor.

The reverse case is a high relative permeability substrate with the relative permittivity of free space. This case was modeled for agreement with simulation and to observe the widened bandwidth effects reported in literature. As discussed thoroughly, MD media tend to have well behaved electrical characteristics while the relative permeability is not as well behaved due to size, shape,



(a)



(b)

Figure 7.6: Input impedance versus frequency from simulation and each CM method. (a) Microstrip with $W/h = 0.7$, $\mu_{ref} = 6$, and $\epsilon_{ref} = 1$. (b) Microstrip with $W/h = 9.8$, $\mu_{ref} = 16$, and $\epsilon_{ref} = 1$.

and distribution of inclusions. Hysteretic effects further complicate the behavior of magnetic inclusions. Figure 7.8 shows a slight frequency shift in frequency on the simulation and a much higher peak impedance when compared to the dielectric patch.

The input impedance characteristics when computed from terminating the line with the characteristic impedance, Z_c , calculated by the modified Yeo method captures the shape and peak of the

The decision to utilize static dimensions was made to remove variation from the microstrip that may influence the effective relative constitutive parameters of the microstrip structure. Figure 7.9 demonstrates the CM results in combination with standard TL methods for MPAs follows the expected envelope for the primary mode of operation, but does not capture higher order modes along the length or width of the patch nor does it capture any inter-modulation, or PIM, effects that could be seen with MD materials when used as high power transmitters.

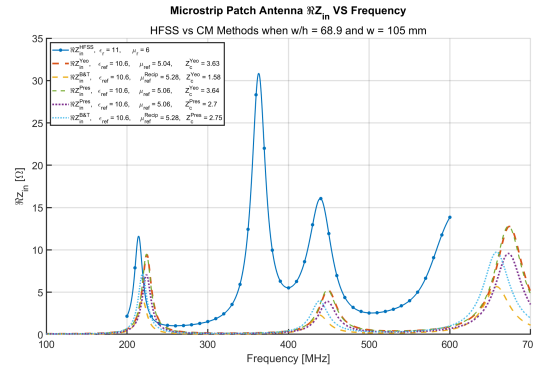


Figure 7.9: Input impedance from simulation versus CM methods. The material parameters allowed excitation of higher order modes that the CM methods did not capture.

7.4 Simulation Remarks

The MPA presented a unique challenge to isolating the fundamental mode when increasing material parameters. The probe feed method limited the geometries available for defining the propagating waves polarity. However, the data aligned fairly well without including the SMA connectors. There remains much opportunity for future work to improve the characteristic impedance calculations by curve fitting data to material and structural parameters.

Chapter 8

Conclusion Future Work

8.1 Summary of Chapters

The drive of biomedical, communications, aerospace, and defense industries to innovate, optimize, and achieve maximum antenna miniaturization was discussed in Chapter 1. Use of MD media for antenna design enables these industries to pursue the absolute bounds of antenna performance previously confined to LTI assumptions, while realizing the desired miniaturization. The trade space between performance and miniaturization is further complicated by difficulty achieving desired MD material characteristics and lack of computationally sparing modeling techniques for both antenna design and antenna performance analysis. The work in this thesis was motivated by the lack of computationally efficient methods for design and basic characterization of MPAs.

To have a thorough understanding of benefits and consequences of introducing magnetic inclusions into antenna design, Chapter 2 thoroughly explored the properties of magnetic materials and consequences when used in EM applications. This was followed in Chapter 3 by review of antenna performance metrics and identified the need of revised methods or a FoM that permits direct comparison for informed design trade studies.

Chapter 3 also reviewed the most frequently utilized simple modeling techniques for design and characterization of MPAs in order of ascending complexity. Each model was modified to include the relative permeability for accommodation of MD when applicable, however these modifications have been documented in literature to require iterative full-wave simulations to achieve desired

operational characteristics. This motivated pursuit of a more accurate simple computational model that would enable more rapid MPA design and prototyping. A well documented analytical model that could help to achieve this goal is microstrip CM models.

The history and derivation of prior supporting work in CM of microstrip structures and MPAs was covered in Chapter 4. This method analytically converts a microstrip right-hand plane into a format with linear electrical and magnetic fields for simplified integration while maintaining boundary conditions and orthogonal angles at each point between the domains. To expand the method to applications with MD media, two novel methods utilized CM algorithms to find the effective relative permeability of a microstrip structure.

The results from the presented method in conjunction with prior CM methods give parameters that approximate the mixed media as an isotropic medium surrounding the microstrip. These results were then applied to other modeling techniques to characterize composite microstrip structures to identify the fundamental resonant frequency of a MPA or utilizing TL theory find the operational frequency and define characteristics of propagation for two-port TLs and probe fed MPAs.

The presented CM methods which characterize microstrip structures by finding the effective relative permeability and characteristic impedance were experimentally verified then validated through simulation in Chapters 6 and 7, respectively.

8.2 Contributions to Industry

The presented CM methods will enable rapid design and characterization of microstrip structures and MPAs. The methods have been verified and validated for the incorporation of MD media, the each method of CM has a optimal region for application to both TLs and MPAs.

8.3 Future work

More analysis is required to curve fit for a characteristic impedance correction factor that accurately scales with the normalized width and material parameters. Once the impedance formula is improved,

there may be a distinct method that stands out for overall MD microstrip modeling.

With the current capabilities, the Hargraves CM method appears to best approximate propagation phase along a two-port microstrip. The data for MPAs is not as clearly defined due to excitation and difficulty isolating higher order modes. Each method has a clear ideal region identified by low error percentage in Figure 7.4.

The formula for μ_{ref} only accounting for material parameters and filling fractions, but does not account for increasing significance of capacitance as the effect of parallel conductive plates becomes a dominant mechanism. This mechanism decreases the effective relative permeability, but curve fitting a correction factor for β or Z_0 is an area for future study.

8.4 Disclosures

No conflicts of interest were present while conducting this graduate research.

Appendix A

Transmission Line Models

Accounting for Permeability

For a microstrip line, the magnetic field intensity, $\vec{H}$, is approximately half in the substrate and half in the superstrate. The effective relative permeability has historically been approximated by the average relative permeability of the superstrate and substrate,

$$\mu_{ref} = \frac{\mu_{r,super} + \mu_{r,sub}}{2}. \quad (\text{A.1})$$

This thesis presented CM methods for better approximating the effective relative permeability with the Hargraves CM and Hargraves Dual formulas given in Equations 4.29 and 4.24, respectively. These formulas enable better models of microstrip structures, both TLs and MPAs.

A.0.1 Two-Port Transmission Line (TL) Methods

Microstrip is a popular type of TL for propagation of a quasi-TEM signal and can be represented as a 2-port network. Equation 3.34 gave the 2-port $ABCD$ -parameters for a length of TL as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma L) & \frac{1}{Y_c} \sinh(\gamma L) \\ Y_c \sinh(\gamma L) & \cosh(\gamma L) \end{bmatrix},$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant and L is the physical TL length [71]. Use of the complex propagation constant requires hyperbolic trigonometry functions rather than the common form with only the phase constant.

MD media have high loss, therefore it is important to includes attenuation for both the media and conductor losses. Pozar gives microstrip attenuation for a dielectric medium as

$$\alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan \delta}{2 \sqrt{\varepsilon_e} (\varepsilon_r - 1)}, \quad (\text{A.2})$$

which is obtained by multiplying $\alpha = \frac{k_0 \sqrt{\varepsilon_e} \tan \delta_E}{2}$ by a filling fraction, $\frac{\varepsilon_r (\varepsilon_e - 1)}{\varepsilon_e (\varepsilon_r - 1)}$ [71]. The filling fraction of each medium was found above so the loss in the medium will be calculated from that value while including the relative permeability and magnetic loss tangent in the form of

$$\alpha_n = \frac{q_n k_0 \sqrt{\varepsilon_{r_n} \mu_{r_n}} (\tan \delta_{E_n} + \tan \delta_{M_n})}{2}, \quad (\text{A.3})$$

where n is the subscript assigned to each medium in the CM procedure. The conductive losses are found by

$$\alpha_c = \frac{R_c}{Z_0 W}, \quad (\text{A.4})$$

where $R_s = \sqrt{\frac{j\omega\mu_0}{2\sigma}}$ is the conductor's surface resistivity [71]. The conductive losses typically dominate the loss mechanism for a dielectric microstrip, however lossy media such as MD materials are an exception [71]. The total attenuation will be the sum $\alpha_{total} = \alpha_c + \alpha_n$ and may have more than two terms for multi-media microstrip structures. The frequency range can be set by the user or calculated from a center frequency value.

The resultant 2-port $ABCD$ -parameters are ideal for addition of SMA ports by multiplying in series, $\text{SMA}_1^{ABCD} \times \text{TL}^{ABCD} \times \text{SMA}_2^{ABCD}$, which can be compared directly to measured data. The $ABCD$ -parameters are also easily converted to z -parameters or using a termination impedance to s -parameters.

A.1 Resonant Frequency from Conformal Mapping Results

With a better approximation of effective constitutive parameters for microstrip structures, a more accurate frequency prediction follows. The next sections present common methods for finding the resonant or operational frequency of MPAs with slight modifications to enable application to MD MPAs. The flowchart in Figure A.1 allows the user to either calculate the structural frequency with Equation A.7 or response across frequency from the inverse of Equation A.13.

A.1.1 Structural Resonant Frequency

Bernhard and Tousignant described the w_{ef} calculation as an iterative approach from which the second iteration result is used to approximate the open end effects along the length of a MPA to estimate the resonant frequency. This method evaluates the natural resonant behavior of the MPA structure as a radiator independent of the response to excitation or feed location. Kirschning et al. gave the empirical formulas for determining the length extension to each radiating slot at the ends of the MPA [86].

$$\begin{aligned}
 \chi_1 &= 0.434907 \cdot \frac{\varepsilon_{r_{ef}}^{0.81} + 0.26}{\varepsilon_{r_{ef}}^{0.81} - 0.189} \cdot \frac{(w/h)^{0.8544} + 0.236}{(w/h)^{0.8544} + 0.87} \\
 \chi_2 &= 1 + \frac{(w/h)^{0.371}}{2.358 \cdot \varepsilon'_r + 1} \\
 \chi_3 &= 1 + \frac{0.5274 \cdot \arctan \left[0.084 (w/h)^{1.9413/\chi_2} \right]}{\varepsilon_{r_{ef}}^{0.9236}} \\
 \chi_4 &= 1 + 0.0377 \cdot \arctan \left[0.067 (w/h)^{1.456} \right] \cdot \left[6 - 5e^{0.036 \cdot (1 - \varepsilon'_r)} \right] \\
 \chi_5 &= 1 - 0.218 \cdot e^{-7.5(w/h)}
 \end{aligned} \tag{A.5}$$

$$\Delta L = h (\chi_1 \chi_3 \chi_5) / \chi_4, \tag{A.6}$$

where $\varepsilon_{r_{ef}}$ is the value found by Equation 4.14, ε'_r is found by Equation 4.16, w is the microstrip conductor width, h is the substrate height, and ΔL is the single-sided length extension due to edge

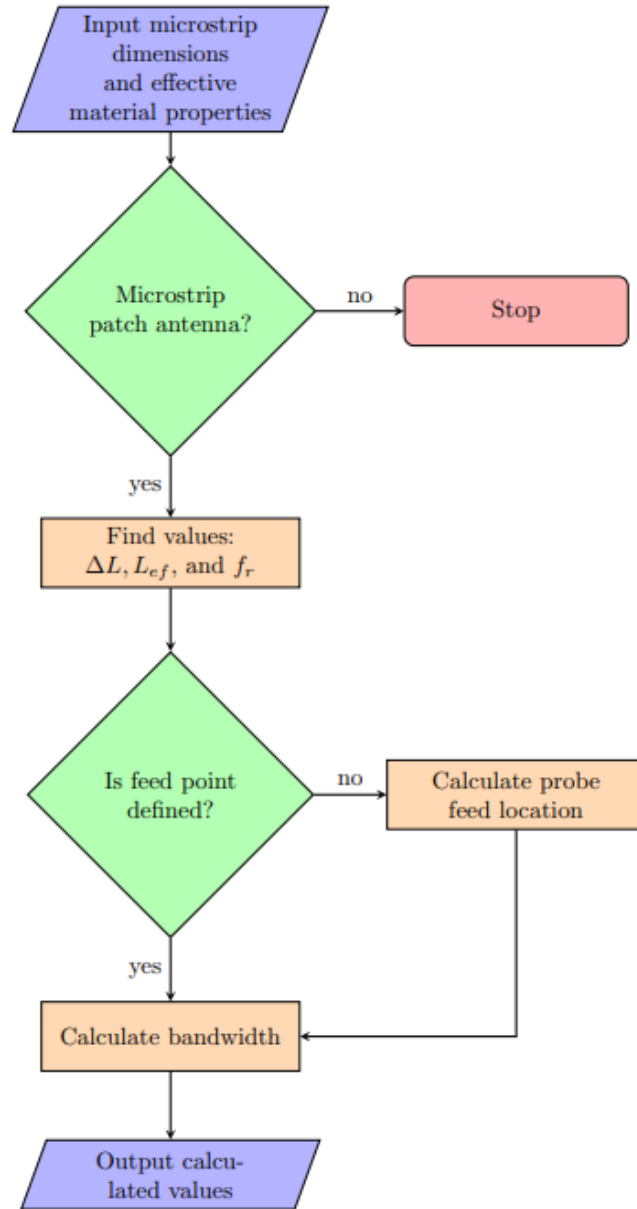


Figure A.1: A block diagram of the presented algorithm adapted from Bernhard and Tousignant to find the resonant frequency of MPAs with flush or gapped superstrates [26].

effects and fringing fields [26], [86]. The rectangular MPA's resonant frequency was found next by

$$f_r = \frac{c_0}{2(L + 2\Delta L)\sqrt{\mu_{r_{ef}}\epsilon_{r_{ef}}}} = \frac{c_0}{2L_{ef}\sqrt{\mu_{r_{ef}}\epsilon_{r_{ef}}}}, \quad (\text{A.7})$$

where c_0 is the speed of light in a vacuum and L is the physical length of the MPA [26].

The addition of steps to calculate the feed location if not given and the bandwidth are helpful for designers, but not required to find the resonant frequency by CM. With these values already calculated, the output from the flow chart in Figure A.1 can be directly fed into a TL model for determination of the impedance or reflection loss versus frequency plots. This will be demonstrated next in Section A.0.1 utilizing the Pues and Van de Capelle TL model [65].

A.1.2 MPA Operational Frequency with Pues and Van de Capelle TL Model

MPAs are also TL structures, however other models can account for the feed location and capture additional mechanisms that are disregarded for two-port TLs. The Pues and Van de Capelle method introduced in Section A.1.2 was utilized to find the resonant frequency using resulting values from the CM procedure presented above in Section 4.5 [65]. The frequency range over which impedance was calculated was based on the design frequency of the patch being modeled or by the calculated resonant frequency found by Equation A.7 and notated as f_{cm} . The formula used to formulate the frequency range was determined by

$$f_{min} = 0.4f_{cm} \quad (\text{A.8})$$

$$f_{max} = 1.7f_{cm} \quad (\text{A.9})$$

$$f_{step} = f_{max}/1000, \quad (\text{A.10})$$

where the lowest frequency is given by f_{min} and is increased in increments of f_{step} until the maximum frequency, f_{max} , is reached. This method allowed the fundamental frequency to be isolated from any harmonic frequencies.

The Pues and Van de Capelle transmission line method captures some material and conductive losses, but the losses of MD materials are better captured by also utilizing the medium's electric and magnetic loss tangents, $\tan \delta_E$ and $\tan \delta_M$, respectively. This was accounted for by modifying Equations 3.55 and 3.56 to

$$\beta = k_0 \sqrt{\mu_{r_{ef}} \epsilon_{r_{ef}}} \quad (\text{A.11})$$

and

$$\alpha = 0.5\beta(\tan \delta_E + \tan \delta_M), \quad (\text{A.12})$$

where k_0 is the frequency dependent term of the complex propagation constant, $\gamma = \alpha + j\beta$ [65].

The characteristic impedance was given by Equation 4.30 so the characteristic admittance is easily found by $Y_c = 1/Z_c$. The self admittance, Y_s , and mutual admittance, Y_m , were found by formulas from [65] and discussed in Section A.1.2.

To find the input impedance of a MPA, the feed location must first be determined. Pues and Van de Capelle demonstrate a shunt feed method, commonly referred to as a probe feed or SMA feed, for their TL equivalent circuit model [65]. The feed locations used in verification of the CM algorithm were given by publication or calculated per Section 3.2. The length of patch from radiating edge to feed is referred to as L_1 or $L_{1,ef} = L_1 + \Delta L$ and L_2 and $L_{2,ef}$ can be found by $L_2 = L - L_1$ and $L_{2,ef} = L_2 + \Delta L$, respectively.

The input admittance and impedance was found by Pues and Van de Capelle's methods. Recall the 3-port network, where the feed is defined at port 3, with an admittance matrix given by Equation 3.52

$$[Y] = \begin{bmatrix} Y_s + Y_c \coth(\gamma L_1) & -Y_m & -Y_c \operatorname{csch}(\gamma L_1) \\ -Y_m & Y_s + Y_c \coth(\gamma L_2) & -Y_c \operatorname{csch}(\gamma L_2) \\ -Y_c \operatorname{csch}(\gamma L_1) & -Y_c \operatorname{csch}(\gamma L_2) & Y_c \coth(\gamma L_1) + Y_c \coth(\gamma L_2) \end{bmatrix},$$

and input admittance at the feed by Equation 3.53

$$Y_{in}^{MPA} = 2Y_c \left[\frac{Y_c^2 + Y_s^2 + Y_m^2 + 2Y_s Y_c \coth(\gamma L) - 2Y_m Y_c \operatorname{csch}(\gamma L)}{(Y_c^2 + Y_s^2 + Y_m^2) \coth(\gamma L) + (Y_c^2 + Y_s^2 + Y_m^2) \cosh(2\gamma \Delta) \operatorname{csch}(\gamma L) + 2Y_s Y_c} \right]. \quad (\text{A.13})$$

$Z_{in}^{MPA} = 1/Y_{in}^{MPA}$ was calculated using these impedance equations substituting in the effective lengths for each frequency in the range calculated above. The real and imaginary impedance were plotted versus frequency and the maximum of the real impedance used to find the linear TL resonant frequency.

A.2 Modernizing the TL model

With the popularity of MD materials, the Pues and Van de Capelle TL model is a good model that could be modified to capture the non-linear effects induced by high power excitation. Blosser identified this mechanism with his MX patch antenna [27], [57].

A.2.1 Modeling Non-Linear Behavior

Varactor Diodes

Blosser noted NL characteristics of a patch antenna on MX, a MD material, with high power applications [27]. Diodes are common components with NL properties, which will be added to the TL model to capture the NL behavior of a MD material. First, a diode will be incorporated into the TL model as a shunt lumped element placed in reverse bias. A reverse bias diode will capture the NL behavior of the MD material without introducing high currents that would be present if placed in forward bias.

Reverse Bias Diode Admittance

The small signal admittance of a diode, Y_d , is given by,

$$Y_d = G_d + jB_d, \quad (\text{A.14})$$

where

G_d = conductance of the diode and

B_d = susceptance of the diode.

The conductance is found by $G_d = \frac{1}{R_d}$, however in reverse bias the high resistance causes the conductance to approach zero [87]. The reverse bias susceptance of a diode is functionally that of a capacitor with voltage dependence [87]. The susceptance between the patch and the ground plane can be found by,

$$B_d = \omega C = \omega \frac{\varepsilon A_{eff}}{W_{dep}}, \quad (\text{A.15})$$

where

C = capacitance

$\varepsilon = \varepsilon_0 \varepsilon_{reff}$ = the absolute permittivity

A_{eff} = effective area of the patch ($L_{eff} * W_{eff}$) in square meters and

W_{dep} = depletion width [87].

The depletion region width, W_{dep} , is much greater in reverse bias and introduces a voltage dependence. The depletion region width is found by

$$W_{dep} = \sqrt{\frac{2\varepsilon(\beta - V_A)}{I_0}}, \quad (\text{A.16})$$

where

β = propagation phase constant

V_A = DC bias voltage

I_0 = reverse bias current [87]. Equation A.16 demonstrates that the depletion region width increases

as substrate height increases.

The DC bias voltage, $V_A = 0$, for this transmission-line model of a microstrip patch antenna. This relationship is important in applications such as frequency agile antennas, but for a passive substrate material there is no DC voltage biasing the radiating slots of a patch antenna.

The capacitance of a varactor diode is a function of the DC bias voltage which can be visually demonstrated by a $\frac{1}{C_{dep}^2}$ versus V_A plot [87]. DC bias voltage could be used to tune the center frequency effectively increasing the bandwidth of the antenna.

A.2.2 Deriving Y-parameters

The microstrip patch is a three-port network of components in shunt. Y-parameters are good for modeling shunt components since they add in parallel. Pues and Van de Capelle modeled the characteristic admittance, Y_c , and self-susceptance, Y_s , as a shunt admittance and represents the mutual coupling as a voltage dependent current source [65].

$$\begin{aligned}
Y_{11} &= \left. \frac{I_1}{V_1} \right|_{V_2=V_3=0} = Y_s + Y_c \coth(\gamma l_1) \\
Y_{12} &= \left. \frac{I_1}{V_2} \right|_{V_1=V_3=0} = -Y_m \\
Y_{13} &= \left. \frac{I_1}{V_3} \right|_{V_1=V_2=0} = \frac{-Y_D}{1 + \frac{(\gamma l_2)}{(\gamma l_1)}} \\
Y_{21} &= \left. \frac{I_2}{V_1} \right|_{V_2=V_3=0} = -Y_m \\
Y_{22} &= \left. \frac{I_2}{V_2} \right|_{V_1=V_3=0} = Y_s + Y_c \coth(\gamma l_2) \\
Y_{23} &= \left. \frac{I_2}{V_3} \right|_{V_1=V_2=0} = \frac{-Y_D}{1 + \frac{(\gamma l_1)}{(\gamma l_2)}} \\
Y_{31} &= \left. \frac{I_3}{V_1} \right|_{V_2=V_3=0} = -Y_c + \coth(\gamma l_1) \\
Y_{32} &= \left. \frac{I_3}{V_2} \right|_{V_1=V_3=0} = -Y_c + \coth(\gamma l_2) \\
Y_{33} &= \left. \frac{I_3}{V_3} \right|_{V_1=V_2=0} = Y_D + Y_c(\coth(\gamma l_1) + \coth(\gamma l_2))
\end{aligned}$$

A.2.3 Nonlinear TL Model Remarks

The presented non-linear TL method places a lumped element diode in reverse bias to introduce non-linear behavior. MD materials are likened to semiconductor materials, therefore may be captured by a single lumped element in shunt at the feed or at each radiating edge.

A more robust method would be to capture the behavior per unit length of a distributed transmission line, however this is also more complex to capture as a wave propagates down the length of the MPA.

The concept has been considered, but verification and validation was outside the scope of this thesis. Time-domain methods are recommended for validation of any nonlinear models because nonlinear phenomenon are favored by time-domain representation [40].

Appendix B

Acronym List

3D	three-dimensional
AC	alternting current
ADS	Advanced Design Systems
CAD	computer-aided design
CEM	computational electromagnetics
CM	Conformal Mapping
CoTS	commercial, off-the-shelf
CPM	cavity perturbation method
DC	direct current
EM	electromagnetic
FEM	Finite Element Method
FFT	fast Fourier Transform
FMR	ferromagnetic resonance
FoM	Figure of Merit

GHz	GigaHertz
GPS	Global Positioning System
HFSS	High Frequency Structure Simulator
IM3	third-order intermodulation
I-V	current versus voltage
ITU	International Telecommunications Union
LC	inductor-capacitor
LLG	Landau-Lifshitz-Gilbert
LPF	low-pass filter
LTI	linear and time invariant
MD	magneto-dielectric
MHz	MegaHertz
MPA	microstrip patch antenna
MW	microwave
MX	MAGTREX 555
NL	non-linear
NRW	Nicolson-Ross-Weir
PEC	perfect electric conductor
PIM	passive inter-modulation distortion
PIN	Positive-Intrinsic-Negative

PMC	perfect magnetic conductor
PNA	programmable network analyzer
Q-factor	quality factor
RF	radio frequency
RFI	radio frequency interference
RFID	radio frequency identification
s-parameter	scattering parameter
SIE	Surface Integral Equations
SINR	signal-to-interference-plus-noise ratio
SIW	substrate integrated waveguide
SMA	SubMiniature version A
SWaP-c	size, weight, power consumption, and cost
TEM	transverse electromagnetic
TE	transverse electric
TL	transmission-line
TM	transverse magnetic
UWB	ultrawideband
VNA	vector network analyzer
VSWR	voltage standing wave ratio
Wi-Fi	wireless networking

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