

Cooperative effects in thin dielectric layers: Long-range Dicke superradiance

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The realization and control of collective quantum effects so far have predominantly focused on cold atomic ensembles. Quantum photonic platforms, with their engineered Green's functions and integration capability of advanced solid-state quantum emitters, provide opportunities to explore regimes of light-matter interaction beyond the scope of atomic systems. In this work, we demonstrate that embedding quantum emitters within a thin dielectric layer fundamentally alters their collective radiative behavior. The optical modes in the dielectric layer mediate long-range dipole-dipole interactions between emitters, enabling both total and directional superradiance between emitters separated by several wavelengths. Crucially, this mechanism supports Dicke superradiance even in parameter regimes where standard settings fail to support an interaction, unveiling a dimensionality-driven enhancement of cooperative effects. By bridging many-body quantum optics and photonic engineering, our work reveals a distinct interplay between surrounding dimensionality and collective quantum dynamics. Experimental realization of these predictions, readily achievable in solid-state quantum optics platforms, paves the way for scalable, directional quantum light sources and frontiers in many-body quantum optics.

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I. INTRODUCTION

The radiative behavior of a quantum emitter is fundamentally modified by the presence of other emitters, as first established by Dicke in 1954 [1]. In such ensembles, mutual coupling through the electromagnetic field can lead to *superradiance*—a collective burst of radiation that far exceeds the independent emission of isolated atoms [2]. Recent advances in experimental control of atomic arrays—via optical lattices and tweezers—have renewed interest in collective effects in one-dimensional (1D) waveguides and nanofibers [3–7]. Further, two-dimensional (2D) geometries offer tunable spatial arrangements that critically impact collective behaviors and directionality [8–10]. So far, superradiance has been observed in diverse systems, including atomic clouds [11–14], solid-state emitters [15–17], and cavity QED settings [18,19], affirming its universality across platforms.

While prior studies have primarily focused on emitters in zero-dimensional or homogeneous three-dimensional (3D) environments, the role of the surrounding photonic environment in tuning and controlling the collective phenomena has not been fully explored yet. Motivated by advances in

solid-state quantum emitters, such as quantum dots and rare-earth ions with radiation-limited linewidths [20,21], we explore the cooperative behavior of emitters embedded in a thin dielectric layer, as depicted in Fig. 1(a) (top). This configuration supports guided photonic modes, which qualitatively alter emitter-emitter interactions.

Unlike prior works, our platform addresses the role of the dimensionality of the surrounding medium. In particular, we find that these modes mediate long-range dipole-dipole interactions across distances spanning several wavelengths, giving rise to both *directional* and *total* superradiance. These effects persist even in the presence of spatial disorder, revealing a dimensionality-driven enhancement of collective emission. We quantify the dependence of superradiant behavior on emitter separation, spatial arrangement, and array size, and show that both total and directional superradiant emission can be engineered through a dielectric layer alone. Our results highlight a regime of collective emission governed not just by the emitter ensemble geometry but by the dimensionality and mode structure of the electromagnetic environment, offering a platform for scalable, robust, and highly directional quantum light sources. Furthermore, because of the modified interaction scaling, this platform allows us to study dissipative physics in the presence of long-range interactions, such as dissipative quantum many-body phenomena with long-range couplings, such as superluminal transport [22,23] and driven-dissipative phase transitions [24].

II. RESULTS

By integrating out photonic modes, a spin model can be derived that describes photon-mediated atom-atom

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interactions using the photonic Green's function [25–29]. In the Markovian limit, the joint density matrix of emitters $\hat{\rho}$ evolves as ($\hbar = 1$)

$$\dot{\hat{\rho}} = -i[\hat{H}_A + \hat{H}_F + \hat{H}_{\text{int}}, \hat{\rho}] + \mathcal{D}(\hat{\rho}), \quad (1)$$

where $\hat{H}_A$ describes the Hamiltonian of the free atom, $\hat{H}_F$ corresponds to the external drive, and $\hat{H}_{\text{int}}$ is the interaction as

$$\hat{H}_{\text{int}} = \sum_{n \neq m} J_{mn} \hat{\sigma}_{eg}^{(m)} \hat{\sigma}_{ge}^{(n)}. \quad (2)$$

The Lindblad terms, describing the dissipative dynamics, here due to the atom-atom interaction, have the following form:

$$\mathcal{D}(\hat{\rho}) = \sum_{m,n} \frac{\Gamma_{mn}}{2} (2\hat{\sigma}_{ge}^{(m)} \hat{\rho} \hat{\sigma}_{eg}^{(n)} - \{\hat{\sigma}_{eg}^{(n)} \hat{\sigma}_{ge}^{(m)}, \hat{\rho}\}). \quad (3)$$

In the above equations, $\hat{\sigma}_{eg}^{(m)} = (|e\rangle\langle g|)_m$ is the raising operator and $\hat{\sigma}_{ge}^{(m)} = (|g\rangle\langle e|)_m$ is the lowering operator of the m th atom. Furthermore, the coherent J_{mn} and dissipative Γ_{mn} couplings due to the interaction read as

$$J_{mn} - i\frac{\Gamma_{mn}}{2} = -\mu_0\omega_0^2 \vec{p}_m^* \cdot \mathbf{G}_E(r_m, r_n) \cdot \vec{p}_n, \quad (4)$$

where μ_0 is the vacuum permeability, $\mathbf{G}_E(r_m, r_n)$ is the photonic Green's function, which depends on the frequency, the position of the emitters (r_m, r_n), and the macroscopic properties of the surrounding medium (cf. Appendix A for more information), and $\vec{p}_m$ is the transition dipole moment of the m th atom.

In a homogeneous environment, referred to as 3D vacuum here, the spherical symmetry of the environment leads to a Green's function decaying as r^{-1} . In contrast, the reduced cylindrical symmetry of the slab geometry, here referred to as 2D vacuum, results in a decay of $r^{-0.5}$. This slower dropoff indicates that the photon-mediated atom-atom interactions can exist at larger separations.

Figure 1(a) (top) illustrates an array of quantum emitters with average spacing d , embedded in a dielectric layer of width W . Solid-state hosts typically have high refractive indices n , supporting multiple optical modes. Figure 1(a) (bottom) shows the effective refractive indices (n_{eff}) of the lowest-order transverse electric (TE) and transverse magnetic (TM) slab modes (purple and green) in a freestanding layer with $n = 3.5$, plotted versus W [30]. For thin layers, TE and TM modes have distinct n_{eff} , but they converge to n as W increases.

The extended-range interaction is reflected in the radiated power of two emitters. Figure 1(b) compares the total radiated power of two emitters in a homogeneous medium and a dielectric layer ($W = 200$ nm, $n = 3.5$) as a function of their separation d , normalized to the in-medium wavelength $\lambda = \lambda_0/n$. The power is normalized to twice that of a single emitter (black dashed line), with deviations indicating emitter-emitter interactions. Notably, the dielectric layer (red line) shows significant interactions at much larger separations

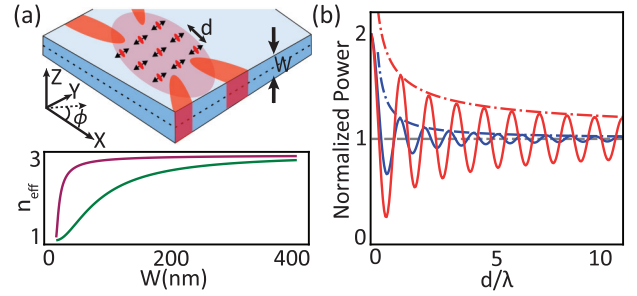


FIG. 1. Schematics. (a) Top: A schematic representing several quantum emitters, separated by d , in a dielectric layer. Bottom: The effective refractive index (n_{eff}) of the transverse electric (solid green line) and magnetic (solid purple line) modes of a dielectric layer with $n = 3.5$, as a function of the layer thickness W . (b) The total radiated power by two emitters, with the free-space wavelength of $\lambda_0 = 980$ nm, as a function of their separation in a homogeneous dielectric (blue) and a 200-nm-thick layer (red). The blue and red dash-dotted lines show the asymptotic behavior of r^{-1} and $r^{-0.5}$ for the homogeneous and thin-layer case, respectively. Atomic separation is normalized to the wavelength in dielectric, $\lambda = \lambda_0/n$.

compared to the homogeneous case (blue line), highlighting the longer-range coupling in the 2D vacuum. Additionally, the envelope of the radiated power in a homogeneous dielectric decays asymptotically as d^{-1} [blue dash-dotted line in Fig. 1(b)]. In contrast, the dielectric layer exhibits a slower decay of $d^{-0.5}$ (red dash-dotted line), consistent with the far-field behavior of the slab's Green's function (see Appendix A for the full analytical form). This slower falloff results in an interaction range that spans several wavelengths, substantially longer than in a homogeneous medium.

A. Effect of vacuum dimensionality

In terms of photon statistics, superradiance corresponds to situations where the emission of one photon enhances the likelihood of subsequent emissions, i.e., $g^{(2)}(0) \geq 1$. For an initial state with all atoms excited, this condition can be recast in terms of pairwise decay in Eq. (4) as

$$\sum_{m \neq n}^N (\Gamma'_{mn})^2 \geq N, \quad (5)$$

where N is the total number of atoms in the ensemble and $\Gamma'_{mn} = \Gamma_{mn}/\Gamma_{00}$ is the normalized decay rate, for Γ_{00} being the decay rate of an individual atom.

To derive general scaling behavior without delving into detailed numerical calculations, we assume ordered arrays with period d , and use the asymptotic forms of Γ'_{mn} , based on the far-field behavior of the Green's function as

$$\Gamma_{mn} \propto (R'_{mn})^{-\alpha}, \quad (6)$$

where $R'_{mn} = |\vec{r}_m - \vec{r}_n|/\lambda_0$ is the normalized separation between the pair (m, n) and $\alpha = 0.5$ and 1 , for the 2D and the 3D vacuum, respectively.

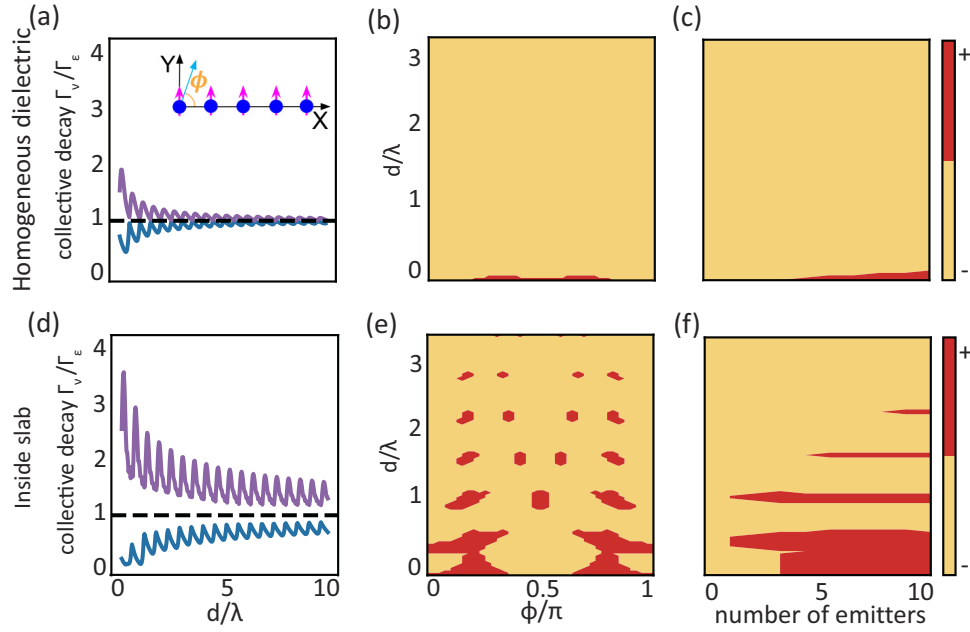


FIG. 2. Total and directional superradiance in a 1D array of Y -polarized emitters. The collective decay rate of five emitters in (a) a homogeneous dielectric and (d) a dielectric layer. The purple curve corresponds to the superradiant mode, while the blue curve corresponds to the subradiant one. Directional superradiance of a five-emitter 1D array in (b) a homogeneous dielectric and (e) a thin dielectric layer, as a function of atomic separation (d/λ) and the in-plane angle ϕ . The areas colored in red correspond to the regions where superradiance can be observed. The emergence of Dicke superradiance as a function of the number of emitters in a 1D array and their separations at $\phi = 0.22\pi$ inside (c) a homogeneous dielectric medium and (f) a dielectric layer. In all calculations, $n = 3.5$ and $W = 200$ nm.

For atoms arranged in a 1D chain, with normalized spacing $d' = d/\lambda_0$, we have

$$\sum_{m \neq n}^N (\Gamma'_{mn})^2 = \frac{1}{d'^{2\alpha}} \sum_{m \neq n}^N \frac{1}{|m - n|^{2\alpha}} \approx \frac{N}{d'^{2\alpha}} \sum_{n=1}^N \frac{1}{n^{2\alpha}} = \begin{cases} \frac{N}{d'^2}, & \alpha = 1, \\ \frac{N \ln N}{d'}, & \alpha = 0.5. \end{cases} \quad (7)$$

When combined with Eq. (5), we get the following condition for the maximum separation between emitters that the superradiance is still attainable:

$$d'_{\max} \propto \begin{cases} 1, & \alpha = 1, \\ \ln N, & \alpha = 0.5. \end{cases} \quad (8)$$

As can be seen, in a 3D vacuum, i.e., $\alpha = 1$, collective behaviors can only be obtained if atoms are closer than the wavelength, in agreement with previous results [31]. In the 2D vacuum, i.e., $\alpha = 0.5$, however, the superradiance can always be obtained for any separation, if the system size N increases accordingly.

Following similar analyses for a 2D lattice of emitters separated by d' , one arrives at the following critical spacing for obtaining the Dicke superradiance:

$$d'_{\max} \propto \begin{cases} \sqrt{\ln N}, & \alpha = 1, \\ \sqrt{N}, & \alpha = 0.5. \end{cases} \quad (9)$$

Unlike 1D chains, 2D arrays support Dicke superradiance in both 2D and 3D vacuum. However, for 2D vacuum emitters, for example, dielectric layer embedded emitters, collective

emission persists at significantly larger emitter separations, highlighting the enhanced interaction range compared to that in a homogeneous environment.

B. Collective decay

In Eq. (1), the unitary dynamics is determined via the first term on the right-hand side, while the second term describes collective dissipative dynamics as in Eq. (3). We define a decay matrix $[\Gamma_{mn}]$, with pair-wise-interaction-induced decays, Γ_{mn} between (m, n) atoms determined via Eq. (4). After diagonalizing this matrix, the dissipative dynamics can be rewritten as

$$\mathcal{D}(\hat{\mathcal{O}}) = \sum_v \frac{\Gamma_v}{2} (2\hat{L}_v \hat{\mathcal{O}} \hat{L}_v^\dagger - \{\hat{L}_v^\dagger \hat{L}_v, \hat{\mathcal{O}}\}). \quad (10)$$

Here, Γ_v is the decay rate associated with the v th collective mode and L_v is the corresponding jump operator that describes the decay of excited atoms to the ground state via this mode [8,25].

To explore interaction effects in emitter ensembles in 2D and 3D vacuum cases, we calculate the collective decay rates Γ_v from the dissipative matrix $[\Gamma_{mn}]$, whose elements are determined by the corresponding Green's functions $\mathbf{G}_E(r_m, r_n)$, in the dielectric layer and the homogeneous environment, respectively.

Figures 2(a) and 2(d) show the highest and lowest collective decay rates (purple and blue, respectively) for a 1D array of five emitters aligned along the x axis and polarized along the y axis [inset, Fig. 2(a)], as a function of normalized spacing d/λ , in a homogeneous medium and a dielectric

slab. Decay rates are normalized to the single-emitter value Γ_e (black dashed line). (See Fig. 5 in Appendix B for the case of Z-polarized emitters.) In the homogeneous dielectric, collective decay rates approach the single-emitter limit near $d \approx \lambda$, indicating the suppression of interactions at larger separations. In contrast, in the slab geometry, collective effects persist up to $d \approx 10\lambda$, reflecting the prolonged emitter-emitter interactions. This extended behavior sharply contrasts with free-space configurations, where superradiant effects are typically restricted to subwavelength arrays [10,31]. As shown in Eq. (8), while a 3D vacuum imposes a strict upper limit on emitter separation for collective behaviors to emerge, the 2D environment of the dielectric layer removes this constraint. In this case, superradiance remains possible at arbitrarily large separations, provided the system size increases as $N \propto e^{d'}$, as in Eq. (8).

C. Directional superradiance

To probe the onset of Dicke superradiance, we adopt an equivalent criterion based on the instantaneous photon emission rate at $t = 0$ [10]. This metric not only captures the collective enhancement of emission but also enables a directional analysis of photon statistics, resolving emission along a specific in-plane angle ϕ , depicted in Fig. 1(a). This approach provides deeper insight into directional superradiance and other emergent phenomena arising from the interplay between geometry and collective interactions.

The total photon emission rate in all directions is determined as

$$\gamma(t) = \sum_n \left[\Gamma_0 \langle \hat{\sigma}_{eg}^{(n)} \hat{\sigma}_{ge}^{(n)} \rangle(t) + \sum_{m \neq n} \Gamma_{mn} \langle \hat{\sigma}_{eg}^{(m)} \hat{\sigma}_{ge}^{(n)} \rangle(t) \right]. \quad (11)$$

An increase in the early-time photon emission rate, $\dot{\gamma}(0) \geq 0$, indicates photon bunching—a key signature of Dicke superradiance [10].

Similarly, the photon emission rate into a given $\mathbf{k}_f$ direction is given by

$$\gamma(t, \mathbf{k}_f) = \Gamma_0 \sum_n \left[\langle \hat{\sigma}_{eg}^{(n)} \hat{\sigma}_{ge}^{(n)} \rangle(t) + \sum_{m \neq n} e^{i\theta_{mn}} \langle \hat{\sigma}_{eg}^{(m)} \hat{\sigma}_{ge}^{(n)} \rangle(t) \right], \quad (12)$$

where $\theta_{mn} = \mathbf{k}_f \cdot \mathbf{r}_{mn}$.

By focusing on the directional instantaneous emission rate $\gamma(0, \phi)$ rather than the total emission, the aforementioned criterion can be extended to assess the emergence of superradiance along specific directions ϕ relative to the array axis. In particular, if $\dot{\gamma}(0, \mathbf{k}_f) \geq 0$, the emission is collectively enhanced in the $\mathbf{k}_f$ direction, signaling the onset of directional superradiance.

Figures 2(b) and 2(c) illustrate the emergence of directional superradiance in a 1D array as a function of normalized spacing d/λ , comparing a homogeneous dielectric and a thin slab. Red (yellow) regions denote directions ϕ where $\dot{\gamma}(0, \phi) \geq 0$ (≤ 0), indicating the presence (absence) of directional superradiance. In the homogeneous medium, superradiance is confined to small separations and limited angular ranges. In

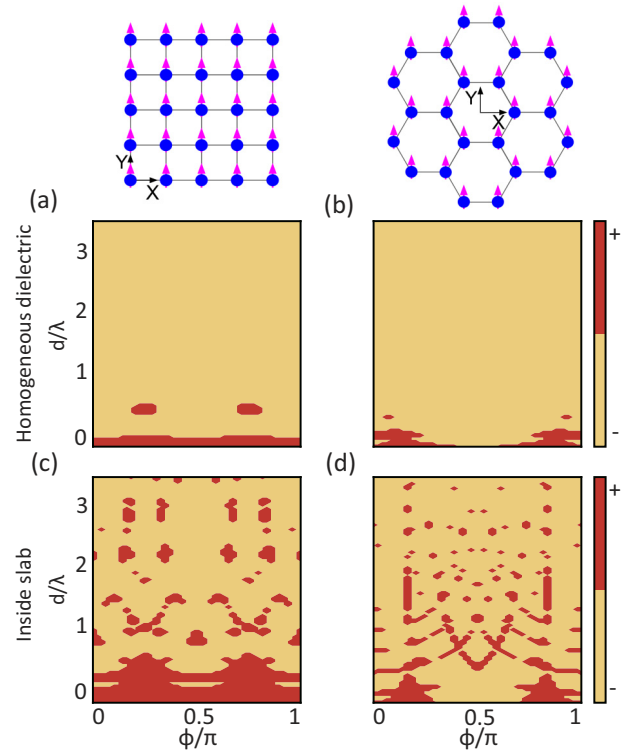


FIG. 3. Geometry effect on the total and directional superradiance in 2D arrays of Y-polarized emitters. Directional superradiance for Y-polarized dipoles arranged in a 2D geometry inside a homogeneous dielectric [panels (a) and (b)] and a thin layer [panels (c) and (d)], delimited by red regions. Emitters in panels (a) and (c) are arranged on a square lattice with 25 sites, while the emitters in panels (b) and (d) are arranged on a hexagonal lattice with 24 sites, as depicted by small schematics on the first row. Regardless of the lattice geometry, the attainable directional superradiance regions are extended in a thin layer compared to a homogeneous medium. In all calculations, $n = 3.5$ and $W = 200$ nm. Note that the hexagonal tiling will cover more area than the square one for the same lattice spacing.

contrast, the thin dielectric layer supports superradiant emission over a much broader range of directions and significantly larger emitter spacings.

To assess the impact of system size, Figs. 2(c) and 2(f) show the directional superradiance at $\phi = 0.22\pi$ as a function of system size N and spacing d . In the 3D vacuum, superradiance emerges only in large arrays with closely spaced emitters. In contrast, the 2D vacuum supports collective emission even in smaller arrays and at significantly larger separations, as indicated by the appearance of multiple red regions at larger d .

To explore the influence of array geometry on collective effects—particularly directional superradiance—we analyze Y-polarized emitters arranged in 2D square and hexagonal lattices, as shown in Fig. 3. The directional emission rate $\dot{\gamma}(0, \phi)$ is used to identify superradiant regions, with red areas indicating $\dot{\gamma}(0, \phi) \geq 0$. (See Fig. 6 in Appendix B for Z-polarized configurations.) Figures 3(a) and 3(c) compare square arrays of 25 emitters in a homogeneous dielectric and a 200-nm-thick dielectric layer, respectively. As in the 1D case, the dielectric

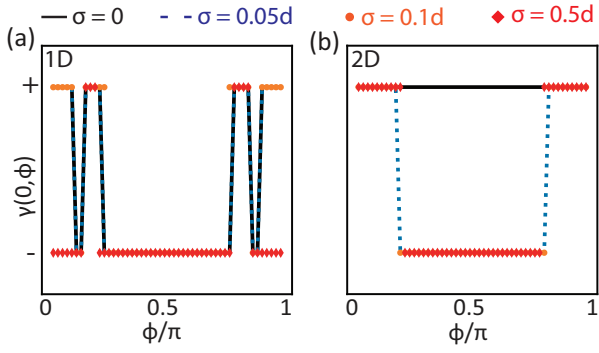


FIG. 4. Disorder effects on 1D and 2D Y -polarized emitters within a dielectric layer. Directional superradiance 25 emitters in (a) 1D linear and (b) 2D square arrangement, with $d = 0.54\lambda$. The black solid line, blue dashed line, orange dots, and red diamonds show the results for different disorders of $\sigma = 0, 0.05d, 0.1d$, and $0.5d$, respectively.

layer environment significantly extends the range and angular breadth of superradiance, enabling collective behavior at larger emitter separations.

In the homogeneous case, collective effects are confined to short emitter separations and narrow angular regions. In contrast, dielectric layer-embedded arrays exhibit enhanced superradiant behavior over significantly larger distances and broader angular sectors, with distinct directional features. These differences stem from geometry-dependent interference patterns inherent to 2D lattices, which modulate the collective response. The extended interaction range—originating from the slower spatial decay of the 2D vacuum Green’s function—enables Dicke superradiance to persist even in dilute arrays, well beyond the limits set by a 3D vacuum.

D. Robustness to disorder

To assess the robustness of collective effects under positional disorder, we examine how directional superradiance responds to random displacements of emitters from their ideal lattice sites. Disorder is introduced by shifting emitter positions in the XY plane using normally distributed random displacements with zero mean and standard deviation σ . The directional emission rate $\dot{\gamma}(0, \phi)$ is averaged over multiple random configurations to quantify the impact. Figures 4(a) and 4(b) show results for 25 Y -polarized emitters arranged in 1D and 2D square arrays, respectively, at various disorder values of $\sigma = (0, 0.05, 0.1, 0.5)d$, with $d = 0.54\lambda$ corresponding to the approximate threshold for omnidirectional superradiance in the ordered square array. Remarkably, even at high disorder, regions with $\dot{\gamma}(0, \phi) \geq 0$ persist, indicating the robustness of superradiance in this long-range interaction regime. (See Fig. 7 in Appendix B for results with Z -polarized emitters.)

III. CONCLUSION

We have demonstrated that Dicke superradiance extends far beyond the conventional near-field regime, persisting at significantly larger interatomic distances in one- and two-dimensional arrays embedded within dielectric layers, i.e.,

a 2D vacuum environment. This enhancement arises from the modified photon-mediated interactions encoded in the Green’s function, which extend the effective interaction range compared to homogeneous environments. By analyzing the collective decay eigenvalues and the early-time emission rate derivative $\dot{\gamma}(0)$, we reveal robust collective phenomena and rich directional superradiance patterns that emerge even in dilute emitter arrays.

Our results place current experimental platforms—such as optical tweezer arrays [21,32], optical lattices [33], and solid-state quantum emitters in 2D materials or bulk crystals—well within reach of observing these extended-range superradiant effects. Notably, state-of-the-art quantum dot systems exhibit the requisite coherence properties and low dephasing rates essential for realizing these collective behaviors [32,34]. Quantum dots are naturally embedded in high-index dielectric layers that can be released as freestanding through selective etching. Their planar geometry further allows local strain tuning of optical transitions [35], for instance, via phase-change materials. This work thus opens a pathway for exploring collective emission in scalable solid-state arrays, leveraging recent advances in fabrication and coherence control.

Looking ahead, our theoretical framework can be extended to more complex atomic-level structures and emitters coupled to tailored reservoirs, including nanophotonic environments [36–39]. The ability to engineer long-range dipolar interactions and dissipative dynamics in reduced dimensions paves the way to investigating exotic quantum phases [40,41], driven-dissipative phase transitions [42,43], and entanglement over unprecedented distances. Ultimately, these insights provide promising avenues toward photonic platforms for decoherence-resistant quantum states and quantum technologies through engineered light-matter interactions [44].

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A.J., R.T., and H.A. conceived the project and performed the theoretical studies, while A.K. helped with some numerical simulations. R.T., A.J., and H.A. wrote the manuscript and contributed to the discussion of the results.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DYADIC GREEN’S FUNCTION OF AN ELECTRIC DIPOLE IN STRATIFIED MEDIA

To find the Green’s function of the desired problem, we need to find the Green’s function of an electric dipole in a slab.

The formalism presented here follows the general approach developed for stratified media [45,46]. Here, we are interested in the simplified case of one slab containing the quantum emitter, as in Fig. 1, given as

$$(\nabla^2 + k_j^2)G(r, r') = \delta(x - x')\delta(y - y')\delta(z - z'), \quad (\text{A1})$$

where $k_j = \omega\sqrt{\mu_0\epsilon_0\epsilon_j}$ is the wave number of the j th layer with the relative permittivity of ϵ_j .

Due to the translational symmetry of the problem, the polarizations of the electromagnetic field are preserved quantities. Therefore, we decompose the Green's function to $\mathbf{G}_{0(\text{ref})}^{s(p)}(r, r')$, where the superscript s, p refers to two polarizations. In addition, subscripts (0, ref) refer to the Green's functions in the homogeneous medium and to the part due to the successive reflections from the slab interfaces.

The dyadic Green's function of the homogeneous medium reads as follows:

$$\mathbf{G}_0(r, r') = \frac{i}{8\pi^2 k_2^2} \iint_{-\infty}^{+\infty} dk_x dk_y \frac{e^{i(k_x(x-x') + k_y(y-y') + k_{z_2}|z-z'|)}}{k_{z_2}} \begin{bmatrix} k_2^2 - k_x^2 & -k_x k_y & \mp k_x k_{z_2} \\ -k_x k_y & k_2^2 - k_y^2 & \mp k_y k_{z_2} \\ \mp k_x k_{z_2} & \mp k_y k_{z_2} & k_2^2 - k_{z_2}^2 \end{bmatrix}.$$

After a bit of straightforward vector algebra, we can show that the polarization-decomposed parts are

$$\mathbf{G}_0^s(r, r') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} dk_x dk_y \frac{e^{i(k_x(x-x') + k_y(y-y') + k_{z_2}|z-z'|)}}{k_{z_2} k_\rho^2} \begin{bmatrix} k_y^2 & -k_x k_y & 0 \\ -k_x k_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$\mathbf{G}_0^p(r, r') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} dk_x dk_y \frac{e^{i(k_x(x-x') + k_y(y-y') + k_{z_2}|z-z'|)}}{k_2^2 k_\rho^2} \begin{bmatrix} k_x^2 k_{z_2} & k_x k_y k_{z_2} & \mp k_x k_\rho^2 \\ k_x k_y k_{z_2} & k_y^2 k_{z_2} & \mp k_y k_\rho^2 \\ \mp k_x k_\rho^2 & \mp k_y k_\rho^2 & \frac{k_\rho^4}{k_{z_2}} \end{bmatrix},$$

where $k_\rho = \sqrt{k_x^2 + k_y^2}$ is the in-plane transverse momentum, in the direction perpendicular to the layers.

With $r_{1,2}^s(k_x, k_y)$ and $r_{1,2}^p(k_x, k_y)$ being the Fresnel reflection coefficients for s and p polarization at the first and second interfaces, respectively, we can explicitly write the reflected Green's functions as

$$\mathbf{G}_{\text{ref}}^s(r, r') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} dk_x dk_y \frac{e^{i(k_x(x-x') + k_y(y-y') + k_{z_2}|z-z'|)}}{k_{z_2} k_\rho^2} \begin{bmatrix} k_y^2 & -k_x k_y & 0 \\ -k_x k_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times R_s^{(x,y,z)}(k_x, k_y), \quad (\text{A2})$$

where

$$R_s(k_x, k_y) = \Gamma_s(k_x, k_y) e^{ik_{z_2}(z-z_1)} (r_1^s(k_x, k_y) e^{ik_{z_2}l_1} + r_1^s(k_x, k_y) r_2^s(k_x, k_y) e^{ik_{z_2}(l_1+2l_2)}) + \Gamma_s(k_x, k_y) e^{ik_{z_2}(z_2-z)} (r_2^s(k_x, k_y) e^{ik_{z_2}l_2} + r_1^s(k_x, k_y) r_2^s(k_x, k_y) e^{ik_{z_2}(2l_1+l_2)}), \quad (\text{A3})$$

$$\Gamma_s(k_x, k_y) = \frac{1}{1 - e^{i2k_{z_2}l} r_1^s(k_x, k_y) r_2^s(k_x, k_y)}. \quad (\text{A4})$$

Similarly, for the p polarization with $r_{1,2}^p$, as the reflection coefficients from the first and second interfaces, we have

$$\mathbf{G}_{\text{ref}}^p(r, r') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} dk_x dk_y \frac{e^{i(k_x(x-x') + k_y(y-y') + k_{z_2}|z-z'|)}}{k_2^2 k_\rho^2} \begin{bmatrix} k_x^2 k_{z_2} & k_x k_y k_{z_2} & \mp k_x k_\rho^2 \\ k_x k_y k_{z_2} & k_y^2 k_{z_2} & \mp k_y k_\rho^2 \\ \mp k_x k_\rho^2 & \mp k_y k_\rho^2 & \frac{k_\rho^4}{k_{z_2}} \end{bmatrix} \times R_p^{(x,y,z)}(k_x, k_y), \quad (\text{A5})$$

where

$$R_p(k_x, k_y) = \Gamma_p(k_x, k_y) e^{ik_{z_2}(z-z_1)} (-r_1^p(k_x, k_y) e^{ik_{z_2}l_1} - r_1^p(k_x, k_y) r_2^p(k_x, k_y) e^{ik_{z_2}(l_1+2l_2)}) + \Gamma_p(k_x, k_y) e^{ik_{z_2}(z_2-z)} (r_2^p(k_x, k_y) e^{ik_{z_2}l_2} + r_1^p(k_x, k_y) r_2^p(k_x, k_y) e^{ik_{z_2}(2l_1+l_2)}), \quad (\text{A6})$$

$$\Gamma_p(k_x, k_y) = \frac{1}{1 - e^{i2k_{z_2}l} r_1^p(k_x, k_y) r_2^p(k_x, k_y)}. \quad (\text{A7})$$

As an aside, note that for both polarizations the poles of $\Gamma_{p,s}$ correspond to the slab modes. With the Green's functions at hand, we are now well positioned to address the interacting ensemble problem described by the generalized spin model in Eq. (1).

APPENDIX B: Z-POLARIZED EMITTERS

This Appendix provides complementary information about collective effects emerging in Z-polarized emitters.

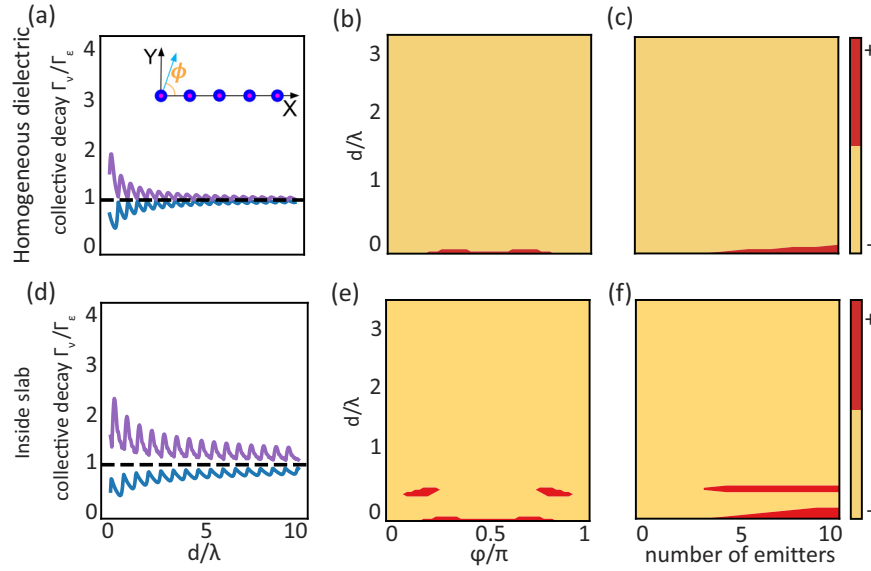


FIG. 5. Total and directional superradiance in a 1D array of Z-polarized emitters. The collective decay rate of five emitters in (a) a homogeneous dielectric and (d) a dielectric layer. The purple curve corresponds to the superradiant mode, while the blue curve corresponds to the subradiant one. Directional superradiance of a five-emitter 1D array in (b) a homogeneous dielectric and (e) a thin dielectric layer, as a function of atomic separation (d/λ) and the in-plane angle ϕ . The areas colored in red correspond to the regions where superradiance can be observed. The emergence of Dicke superradiance as a function of the number of emitters in a 1D array and their separations at $\phi = 0.22\pi$ inside (c) a homogeneous dielectric medium and (f) a dielectric layer. In all calculations, $n = 3.5$ and $W = 200$ nm.

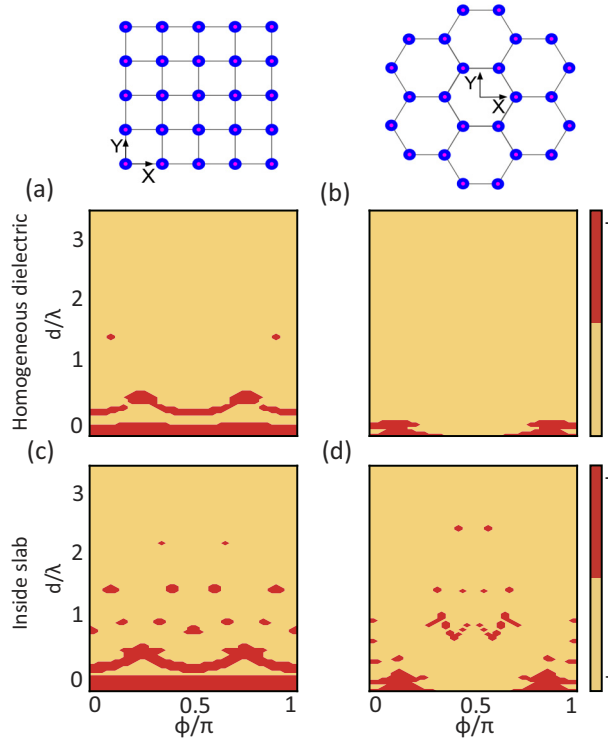


FIG. 6. Geometry effect on the total and directional superradiance in 2D arrays of Z-polarized emitters. Directional superradiance for Y-polarized dipoles arranged in a 2D geometry inside a homogeneous dielectric [panels (a) and (b)] and a thin layer [panels (c) and (d)], delimited by red regions. Emitters in panels (a) and (c) are arranged on a square lattice with 25 sites, while the emitters in panels (b) and (d) are arranged on a hexagonal lattice with 24 sites, as depicted by small schematics on the first row. Regardless of the lattice geometry, the attainable directional superradiance regions are extended in a thin layer compared to a homogeneous medium. In all calculations, $n = 3.5$ and $W = 200$ nm. Note that the hexagonal tiling will cover more area than the square one for the same lattice spacing.

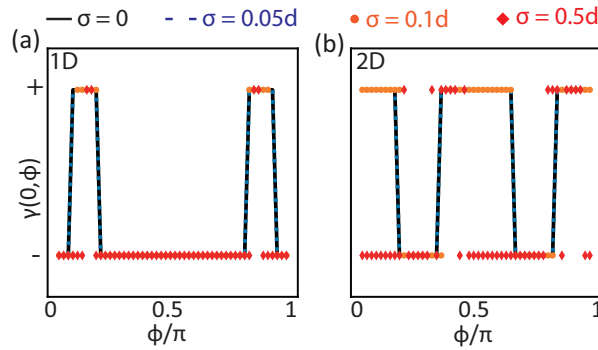


FIG. 7. Disorder effects on 1D and 2D Z-polarized emitters within a dielectric slab. Directional superradiance 25 emitters in (a) 1D linear and (b) 2D square arrangements, with $d = 0.54\lambda$. The black solid line, blue dashed line, orange dots, and red diamonds show the results for different disorders of $\sigma = 0, 0.05d, 0.1d$, and $0.5d$, respectively.

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