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STRUCTURAL DESIGN CRITERIA BY STATISTICAL METHODS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

JAMES EUGENE HAYES

Norman, Oklahoma

1965

STRUCTURAL DESIGN CRITERIA BY STATISTICAL METHODS

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PREFACE

Selection of a Dissertation Topic

One of the factors considered during the selection of a dissertation topic was that the topic must be one for which a need exists in the aero-space industry. One of the documents used to help determine this need was, "Important Research Problems in Advanced Flight Structures Design - 1960," (1) edited by Norris F. Dow in collaboration with the NASA Research Advisory Committee on Structural Design. The problem areas listed in this document (1) under the heading of Structural Design Criteria are:

1. To devise methods of assessing the performance of the structure in quantitative terms which will permit the establishment of rational design criteria. Such methods must take cognizance of the probabilistic nature of failure phenomena and the factors which influence them.
2. To devise practical procedures whereby the knowledge which exists regarding environment, loads, and structural performance, may be brought together to determine reliability; or alternatively, whereby design requirements may be identified for a stipulated reliability.

Mangurian (2) has pointed out the inadequacy of the old factor of safety method in light of the advancements in structural and aerodynamic knowledge.

One of the most recent publications indicating increasing interest in structural reliability (3) was done by Bert and Hyler concerning large solid-propellant rocket-motor cases.

Based on these reports and personal contact by the writer with the aero-space industry, a dissertation topic of "Structural Design Criteria by Statistical Methods" was selected.

Objective and Scope

The object of this dissertation is to establish a structural design criterion using statistical methods such that for any desired structural reliability an allowable design stress level can be calculated.

The scope of this dissertation is limited to vertically rising boost vehicles. These particular vehicles were selected for several reasons:

1. It was desirable to select a vehicle for which the loads were caused by a random process or a process approximately so, such as the atmospheric winds as opposed to an airplane whose loads are greatly influenced by the pilot, and thus are not random,
2. It was also desirable to select a vehicle which would be of prime interest to the aero-space industries located in Oklahoma. Two of the major aero-space companies located in Tulsa are

interested in large structures, such as boost vehicles, because they can make use of the Arkansas River Navigation Project, which will be completed in a few years, to ship large structures from Tulsa to the Gulf.

Acknowledgments

The writer would like to thank the many people who assisted in the preparation of this dissertation. In particular, I wish to thank the following:

1. Mr. Norman Sissenwine, of the Air Force Cambridge Research Laboratories, who assisted in the selection of the method for calculating loads;
2. Dr. Norman P. Hobbs, of AviDyne Research, Inc., who, with the permission of Mr. George Muller, Project Officer at Wright-Patterson Air Force Base, supplied a copy of his loading results on simulated flights;
3. Mr. James K. Boyer, of the Defense Metals Information Center; Mr. Glenn H. Beatty, of the Battelle Memorial Institute; Mr. Albert J. Belfour of the Technical Information Systems Division of Belfour Engineering Company; and Mr. Hugh Baker of The Dow Metal Products Company; who were of considerable assistance in obtaining mechanical properties data;

4. Messrs. Gary Greer, James McConnell, and Terry Middleton, of North American Aviation, Inc., Tulsa, who wrote the numerous computer programs required for this dissertation;
5. Mr. Keith Hunt of North American Aviation, Inc., Tulsa, who transcribed much of the mechanical properties data from reports to tabular sheets;
6. North American Aviation, Inc. for allowing me to work almost full time on this dissertation for seven months as well as furnishing the above mentioned computer programmers and several hours of 7094 computer time, plus a twelve month full time Fellowship beginning September, 1964;
7. The members of my Doctoral Committee for their suggestions, comments, criticisms, and guidance during the preparation of this dissertation.

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CHAPTER I

INTRODUCTION

Although increased emphasis has been placed on statistical concepts of structural safety, some of the early references were published over thirty years ago. Ebner pointed out in reference (4) that work by Küssner and Thalau (5), in 1932, suggested that the required strength of an aircraft for a single load application should be based on the expectance of failure, determined statistically from load measurements during flight. This idea was developed further by Küssner (6) in a paper published in 1935. Similar proposals for statistical investigations of the strength of aircraft were made in England by Pugsley and Fairthorne (7) in 1939. Also, in 1939, Weibull, of Sweden, developed a statistical theory of the strength of materials (8).

Practically all work in this area was halted during the early 1940's because of World War II. Since 1946, many publications have appeared on this topic; and because there are so many, only those that have a direct bearing on this dissertation will be listed. These works will be listed in later chapters where they apply.

CHAPTER II

STATISTICAL APPROACH

Introduction

The basic concept of how to attack this problem was to select the statistical distribution which best fits the loads data, select the statistical distribution which best fits the strength data, and combine these two distributions mathematically so that the structural probability of failure can be calculated.

Selection of the Statistical Distributions

It is, of course, not possible to investigate all distributions; and therefore, the few which are investigated should be selected with care.

The first distribution selected was the normal, or as it is sometimes called, the Gaussian distribution. This distribution was selected because more phenomena in the sciences fit the normal distribution than any other single distribution, hence the name "normal distribution." Also, several previous structural reliability investigators (3,9) have used the normal distribution.

The second distribution selected was the log-normal distribution with somewhat the same reasoning as was used to select the first distribution, namely, that the log-normal is considered the second most used distribution, usually being tried after the data prove to be non-normal, and a previous investigator (10) used this distribution in a structural reliability study.

It was decided that the third distribution which would be selected would be the one which gave the best fit to loads data based on the results of the literature search. Press (11), in his study of gust loads, found the double exponential distribution showed a good fit to gust loads when the maximum values were obtained from successive samples. Gumbel (12) gave mathematical proof that the double exponential distribution results if maximum values are selected from initial distributions of the exponential type. Since the normal and log-normal are both of the exponential type, it is felt that the possibility is very good that the initial distribution will be of the exponential type and the distribution of the maximum values, selected from the exponential type, will thus be the double exponential.

The fourth and last distribution was selected to give the best fit to the strength data based on a literature search. The search disclosed that Weibull (8) developed a statistical distribution especially for the breaking strength of materials. Weibull based his selection of this distribution, which bears his name, not only on the excellent fit of the theory to the test data, but also on philosophical reasoning.

Tests for Goodness of Fit

In order to mathematically determine the theoretical distribution which yields the best fit to the observed data, a Chi-Square test was attempted. It was found that in order to use the Chi-Square test, considerable lumping together of adjacent classes* at both tails of the distributions was necessary in order to have the calculated frequency be at least 1.0. This procedure defeated the purpose of accurately fitting a theoretical distribution to the observed data because of the greater importance of the tails of the distribution to a structural reliability study.

The most important part of the strength distribution is its weakest, and the importance of the fit between the theoretical distribution and the observed distribution decreases as the strength increases.

The reverse is true for the loads as the maximum load is the most important and minimum load is the least important from a structural reliability point of view. Thus it can be seen that the left hand (minimum) tail of the strength distribution and the right hand (maximum) tail of the load distribution are very important.

To be able to exactly weight this importance would require the answer to the complete structural reliability

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*The selection of the number of class intervals is based on the Sturges method (13). The number of class intervals is the integer nearest the value 1 + 3.3 ln (number of samples).

study in advance. Since this is not known, it was decided to use a conventional goodness-of-fit test (the Kolmogorov-Smirnov nonparametric test) for the entire distribution and to supplement this test with a test which places greater importance on the tails of the distributions. This new criterion or test is based on an intuitive modification of the Chi-Square test and will be called the Psi-Square test.

Similar to the Chi-Square test, a test was desired which was concerned with the discrepancy between the observed and the theoretical distribution. Also, here a relative discrepancy was desired, but unlike the Chi-Square test, the value was divided by the observed rather than the theoretical. This modification eliminated the possibility of having to lump adjacent classes together in order to have a frequency of 1.0. Next a method was needed for combining these relative discrepancies. One method would have been to sum all the relative discrepancies; however, these relative discrepancies had signs of plus and minus and their sum might have been very close to zero, even with large discrepancies, if the positive discrepancies approximately balanced the negative ones. This difficulty was overcome by using the same procedure as in the Chi-Square test, that of squaring the relative discrepancies. For the Chi-Square test the relative discrepancies were weighted because of the greater probability of scattor in the tails of the distributions. For the particular problem of structural reliability and the greater importance of the tails of the distributions, the relative discrepancies were not weighted for this test. Thus one arrives at the formula of the Psi-Square test:

$$\psi = \Sigma \left[\frac{f_o - f_t}{f_o}\right]^2$$

where f_o is the observed frequency and

f_t is the theoretical frequency.

One additional change from the conventional Chi-Square test was necessary. In the form used, both tails of the distribution were quite significant and it was desired that only the left hand tail (minimum) of the strength distribution and the right hand tail (maximum) of the load be significant.

Therefore the Psi-Square test was applied only to half of the classes, the lower half for the strength distributions and the upper half for the load distributions.

The preceding discussion is not a mathematical derivation, but rather a logical derivation of a method, to test the goodness of fit of the particular half of the distribution which is most significant for structural reliability. No probability can be assigned to the Psi-Square test, however, one is not necessary as the Kolmogorov-Smirnov (14) test will be used for this purpose. The Psi-Square test was merely a method of checking the Kolmogorov-Smirnov test to make sure that the theoretical distributions which gave the best fit to the entire observed data also gave the best fit when only the most significant half of the observed data was used. The Kolmogorov-Smirnov test is a nonparametric or "distribution-free" test which means that it is not necessary to know the statistical distribution of the population from which the sample was drawn.

In order to have as large a sample size as possible, nonparametric tests, such as the Kruskal-Wallis test (14), were used to determine if data from different sources were from the same population and could be combined.

Curve Fitting of Empirical Data

In certain areas of structural stability, analytical theory has not been able to adequately predict the results of buckling. In these areas, empirical methods have been developed. When only a few test results are available, the empirical design curve is generally constructed to approximate the test data by engineering judgment. However, when a larger number of data is available, it is desirable to use statistical methods to aid in the construction of the design curve. The following method was used to construct all empirical curves necessary in this dissertation.

A regression line was fitted to the test data by a statistical technique such that the sum of the squares of the values of the individual test data to the computed mean value were a minimum. This technique is known as the method of least squares.

In the method of least squares, a polynomial equation of the form,

$$y = C_0 + C_1x + C_2x^2 + \ldots C_nx^n,$$

where y is the dependent variable,

x is the independent variable, and

C_0, C_1, C_2, . . . C_n, are the computed coefficients,

was fitted to the test data. Several polynomial equations are available; the first degree equation is $y = C_0 + C_1x$: the second degree equation is $y = C_0 + C_1x + C_2x^2$; etc. Generally the higher the degree equation used the better the fit to the test data. It is possible to have an equation with the degree equal to one less than the number of test data and the equation will be a "perfect" fit since it will pass through every point. However, this is generally not the equation desired. For each increase in the degree of the equation, a degree of freedom is lost. The problem is then to decide when there is no longer a significant improvement in fit by going to the next higher degree polynomial equation. This is accomplished by an analysis of variance. The most powerful and widely used analysis of variance test is the F test, named in honor of R. A. Fisher, who originated the method. However, one of the assumptions associated with the statistical model of the F test is that the observations are independently drawn from normally distributed populations. Not wishing to restrict the analysis to normal distributions, nonparametric tests were investigated.

The Kruskal-Wallis test was selected because of its high power efficiency, 95.5 per cent, when compared with the F test, the most powerful parametric test (14).

CHAPTER III

LOADS

Selection of Method for Determining Loads

There are three basic methods for determining the loads on a vertically rising boost vehicle:

1. The synthetic wind profile,
2. The wind statistic matrices,
3. The statistical load survey.

The first of these methods, the synthetic wind profile, is the oldest being first described by Sissenwine (15) in 1954. Sissenwine described his method as the first of the generally recognized, synthetic wind profiles and a "guestimate" of the 99% wind for the windiest season and the windiest launch location. A similar type of follow-on study was performed by Williams and Bergst (16) of Lockheed Missile Systems Division in 1958. The Marshall Space Flight Center of the National Aeronautics and Space Administration has, within the last few years, published a number of reports on synthetic wind profiles based upon a wind speed and wind shear that is exceeded only a certain percentage of the time (17, 18, 19, 20, 21 and 22). One of the major reasons for the synthetic profile was to provide a method for handling the very large errors in the wind sounding collected in the years preceding 1954, which were less refined than today's AN/GMD-1 equipment. This was accomplished by averaging the shears of approximately the same portion of the same type of soundings, in which case the random error in this part of the synthetic profile would be $N^{-\frac{1}{2}}$ times the error in a single sounding, where N is the number of soundings used to obtain the synthetic profile.

Another reason for using the synthetic profiles in 1954 was that high speed computers had much less capacity and could not be made available for the long periods of time required for flying tentative designs through hundreds of soundings. Since these two arguments for the use of a synthetic wind profile are no longer valid, Sissenwine, who was one of the pioneers of synthetic profile, has now publicly declared he is against them (23).

The second of these methods, the wind statistic matrices, was pioneered by Bieber (24, 25) of Lockheed and Trembath (26) of Space Technology Laboratories. This method requires the following knowledge of the wind: a mean value, standard deviation, and correlation coefficients as a function of altitude for each of two orthogonal wind components and the cross correlation coefficients between the two components for the same and different altitudes.

Van Der Maas (27), also of Lockheed, conducted an extensive study into wind shear response of missiles and concluded that this procedure is entirely too complex and laborious to be used in the design of a missile. Another shortcoming of this method is that much subjectiveness is still required because estimates must be made in certain areas where little or no data are available.

The third method, the statistical load survey, was developed by Hobbs and his associates at AviDyne Research, Inc. (28). This approach involves the simulation on a computer of missile flights through a statistically adequate sample of wind soundings. The maximum load is determined for each simulated flight. This approach is completely straight forward. The only objection to this method is that it requires a large amount of computer time to simulate 200 flights, approximately 25 hours on a 1103A (or 704) computer or about 2.5 hours on a 7090 computer.

The writer attended the American Institute of Aeronautics and Astronautics and the American Meteorological Society Joint Meeting entitled "Meteorological Support for Aerospace Testing and Operation Meeting" held July 10-12, 1963, at Colorado State University, Fort Collins, Colorado, as well as the American Meteorological Society Meeting entitled, "Atmospheric Problems of Aerospace Vehicles" held March 2-6, 1964, at Atlantic City, New Jersey. Based on the papers presented, the discussions of the papers, and informal discussions with the other attendants, the writer is convinced that the statistical load survey method is the most accurate method known today.

Thus the statistical load survey was selected as the method for determining loads.

Determining the Loads

The wind soundings used in the statistical load survey for calculating vehicle loads were AN/GMD-1* meteorological balloon soundings taken by the United States Weather Bureau at seven locations within the United States:

1. Caribou, Maine,
2. Denver, Colorado,
3. Fort Worth, Texas,
4. International Falls, Minnesota,
5. Long Beach, California,
6. Montgomery, Alabama,
7. Seattle, Washington.

---

*The following brief description of the AN/GMD-1 equipment is taken from reference (29).

"Briefly, in the AN/GMD-1 system the elevation angle and azimuth of the balloon is computed from the temperature, pressure, and humidity data transmitted to the ground receiver by the radiosonde. Wind speeds are then obtained by calculating the horizontal distances traversed in a given time, utilizing the elevation angle and the tangent law to obtain the horizontal distance to the balloon, and the horizontal angle to determine azimuth. The use of the law of tangents is considered responsible for the major inaccuracy in AN/GMD-1 observed winds, since wind calculations are based on the formula that horizontal distance is equal to the altitude multiplied by the cotangent of the elevation angle. At low elevation angles, which correspond to maximum range and altitude, the cotangent value changes very rapidly for very small changes in the angle; therefore, any small elevation angle error due to hunting of the AN/GMD-1 antenna will result in erroneous wind calculations."

A study of the seasonal variations of winds indicated that the most severe winds occurred during the winter and soundings were taken for five winters in order to account for the variation of wind severity from year to year. Soundings were selected at approximately three day intervals because of the known persistency of the winds. Thus for the four winter months, beginning with December and ending with March, a sample of 40 per year or 200 for the five year period was obtained.

The computer program for calculating missile loads from the raw wind sounding balloon data described above is completely explained in "Digital Computer Programs Relating to Wind Loads on Vertically-Rising Vehicles" by Hobbs of AviDyne (30). This is a six-degree-of-freedom rigid body program and is considered quite adequate for calculating the loads encountered by fairly rigid vehicles.

The loads were calculated using the previously described procedure for five actual vehicles:

1. Atlas,
2. Dyna-Soar,
3. Minuteman,
4. Thor,
5. Titan.

Since the performance of these vehicles is classified, the loads cannot be identified with the vehicle. Therefore, the loads were identified by a random selection as vehicle B, C, D, E, and F. These five vehicles were flown on 200 simulated flights for each of the seven locations with the exception of Seattle, Washington, for vehicle C, and Montgomery, Alabama, for vehicle F, which were missing. This left 6,600 simulated flights for loads data which was considered quite adequate for this study.

Selection of Loads Distribution

In order to have as large a sample as possible to compare with the four selected statistical distributions, the Kruskal-Wallis (14) nonparametric analysis of variance test was applied to determine whether there was a significant difference in loads at the seven different locations studied.

Although the five different vehicles were flown on the same 200 simulated flights for each location, the resulting loads were quite different. This was due to the fact that the vehicles responded differently to different wind disturbances. Thus, one location which caused critical loads for one vehicle did not necessarily cause critical loads for another. Likewise two locations which caused loads that were not significantly different for one vehicle caused loads that were quite different for another vehicle. The results of the Kruskal-Wallis tests show that this was exactly what happened for the five vehicles and seven locations studied.

For vehicle B, there were four locations for which the loads were not significantly different: Fort Worth, Seattle, Long Beach, and Caribou. This was based on a significance level of 0.05 for the Kruskal-Wallis test. For vehicle C, there were only two locations for which there was not a significant difference in the resulting loads: Denver and International Falls. For vehicle D, three locations, Seattle, Denver, and Caribou were not significantly different. For vehicle E, three locations, Long Beach, Fort Worth, and Seattle were not significantly different. For vehicle F, three locations, Seattle, Long Beach, and Caribou were not significantly different.

In order to establish which of the four theoretical distributions gave the best fit to the observed loads, the results of the goodness-of-fit tests were examined for the five vehicle-combined locations (Figures 3-1, 3-9, 3-16, 3-24, 3-32) and the thirty-three separate vehicle-location combinations (Figures 3-2 through 3-8, 3-10 through 3-15, 3-17 through 3-23, 3-25 through 3-31, 3-33 through 3-38). Examination of the combined locations for all five vehicles shows that, according to the Kolmogorov-Smirnov test, the double exponential distribution gave the best fit for all five vehicles; and according to the Psi-Square test, the double exponential distribution gave the best fit to four of the five vehicles. For the fifth vehicle, which was vehicle E, the Weibull distribution gave the best fit for the Psi-Square test. By assigning the distribution which gave the best fit the number 1.0 and the distribution with the second best fit the number

2.0, etc., these assigned numbers were averaged to give an indication of the average rank of the various distributions. For the Kolmogorov-Smirnov test, the average of the ranks were:

Double Exponential	1.00,
Weibull	2.20,
Log-Normal	3.00,
Normal	3.80.

For the Psi-Square test, the average of the ranks were:

Double Exponential	1.60,
Weibull	2.00,
Log-Normal	3.00,
Normal	3.40.

The above average ranks were for the vehicle-combined locations. For the separate vehicle-location, the following average ranks occurred for the Kolmogorov-Smirnov test:

Weibull	1.64,
Double Exponential	2.09,
Log-Normal	2.76,
Normal	3.52,

and for the Psi-Square test:

Double Exponential	1.42,
Weibull	2.24,
Log-Normal	2.82,
Normal	3.52.

By combining the separate vehicle-locations and the vehicle-combined locations, the following ranks were obtained for the

Kolmogorov-Smirnov test:

Double Exponential	1.75,
Weibull	1.81,
Log-Normal	2.83,
Normal	3.60,

and for the Psi-Square test:

Double Exponential	1.48,
Weibull	2.17,
Log-Normal	2.88,
Normal	3.48.

These results showed the same order of ranking by both the Kolmogorov-Smirnov and Psi-Square tests. It should be pointed out that while the double exponential, log-normal, and normal distributions are two-parameter distributions; the Weibull is a three-parameter distribution and thus has one less degree of freedom. Therefore, the Weibull distribution is probably not as close a second choice as the above rankings indicate. Since the double exponential was the first choice as the theoretical distribution which gave the best fit to the observed loads, it was not necessary to investigate further the effects of the loss of a degree of freedom for the Weibull distribution.

Gumbel has shown in reference (12) that the double exponential distribution is the asymptotic distribution of the largest value where the initial distribution is an exponential type. This means that if the initial distribution of all the loads encountered during one flight is one of the exponential type, such as normal or log-normal, and the largest load is taken for each flight; then these largest loads have as an asymptote the double exponential distribution. Data were available for the initial distribution of vehicle D at Denver, Colorado, and vehicle E, at Long Beach, California. The Kolmogorov-Smirnov test was applied at a significance level of 0.05 and no significant difference was found for either the normal or log-normal distribution when compared to the observed load data. Thus it can be said that the initial distribution of the observed load is an exponential distribution and the distribution formed by selecting the largest values of observed loads should have the double exponentail distribution as an asymptote. This adds further justification for the selection of the double exponential distribution as the one which best represents the observed maximum loads.

It should be pointed out that a check for significant difference at the 0.05 level using the Kolmogorov-Smirnov test indicated no significant differences between the observed loads and all four theoretical distributions for a majority of the thirty-three separate vehicle-location combinations and the five vehicle-combined locations combinations. For vehicles B and D, all four theoretical distributions showed no significant difference without exception. For vehicle C, the normal distribution did not fit for Caribou, Maine, and Long Beach, California; and the log-normal distribution did not fit for

Long Beach, California. For vehicle E, at Denver, Colorado, the normal distribution did not fit; at Seattle, Washington, only the Weibull distribution fit; and for the combined locations for vehicle E, none of the distributions fit the observed loads. For vehicle F for the combined locations, the normal distribution showed a significant difference.

The distribution chosen to represent the loads was the double exponential; however, since the other distributions also fitted fairly well, the effect on the final structural reliability caused by using one of the other theoretical distributions will be investigated further in a later chapter.

Applied Bending Moment

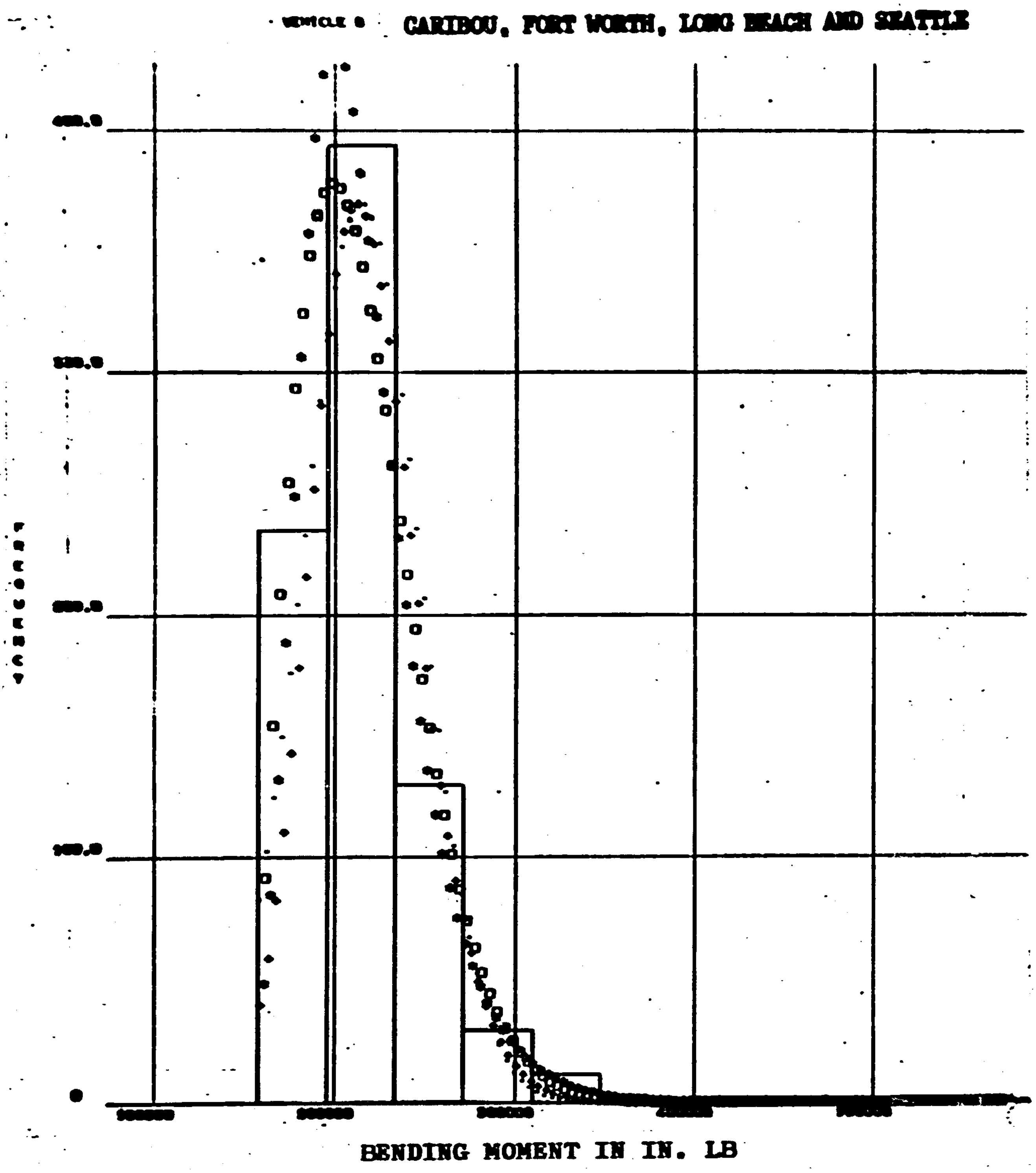
In the previous sections the loads were given in terms of an equivalent bending moment. It was called an equivalent bending moment because the effect of the axial load caused by the engine thrust was included in the bending moment. This is done by setting the compressive stress caused by the axial load (stress = load/cross-sectional area) equal to the classic bending formula for stress in the extreme fiber (stress = moment x distance from the neutral axis/moment of inertia) and solving for the moment,

$$\frac{P}{A} = \frac{Mc}{I}.$$

Solving for the moment gives:

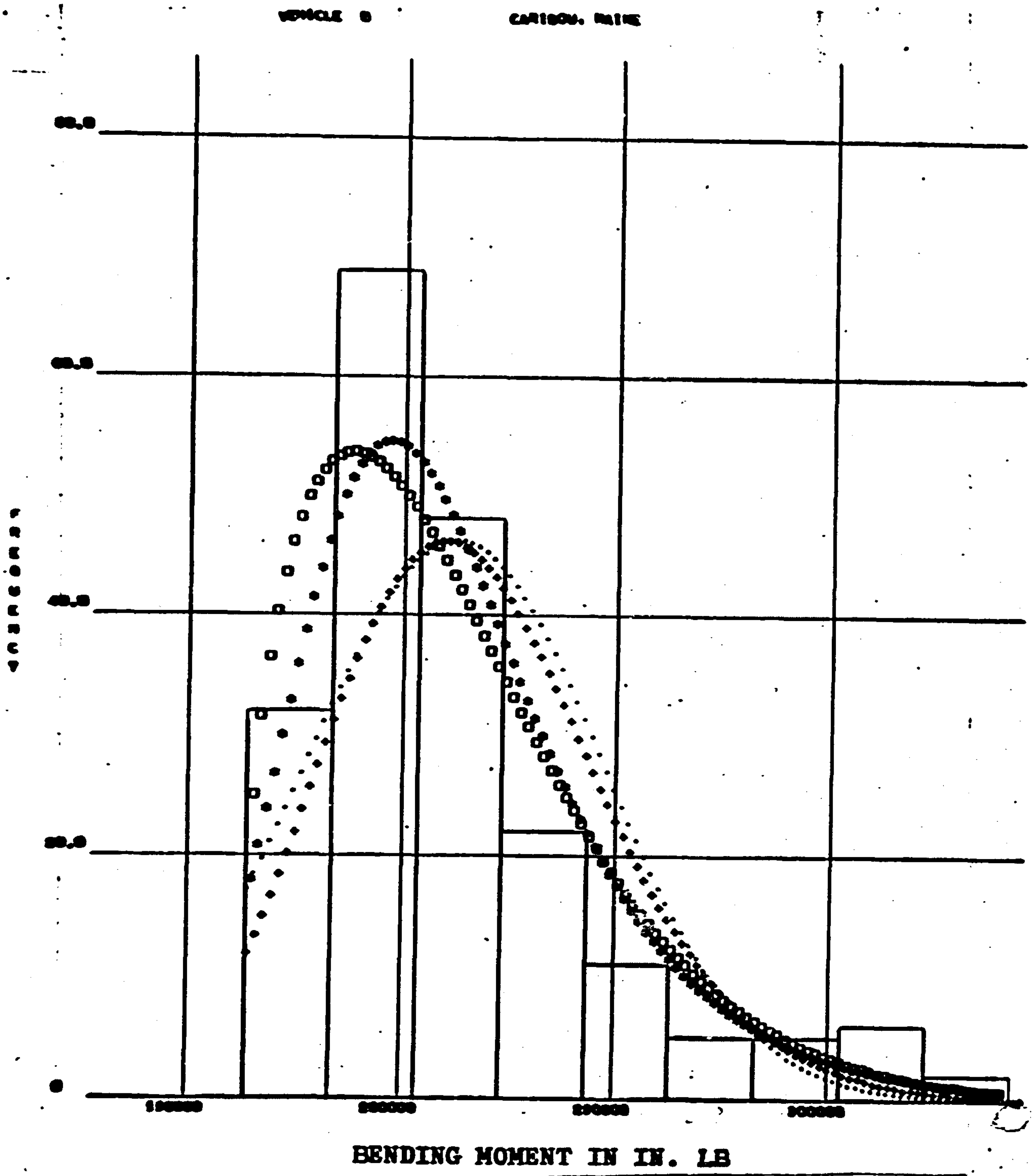
$$M = \frac{PI}{Ac}.$$

This fictitious bending moment was added to the real bending moment and the result was called an equivalent bending moment.



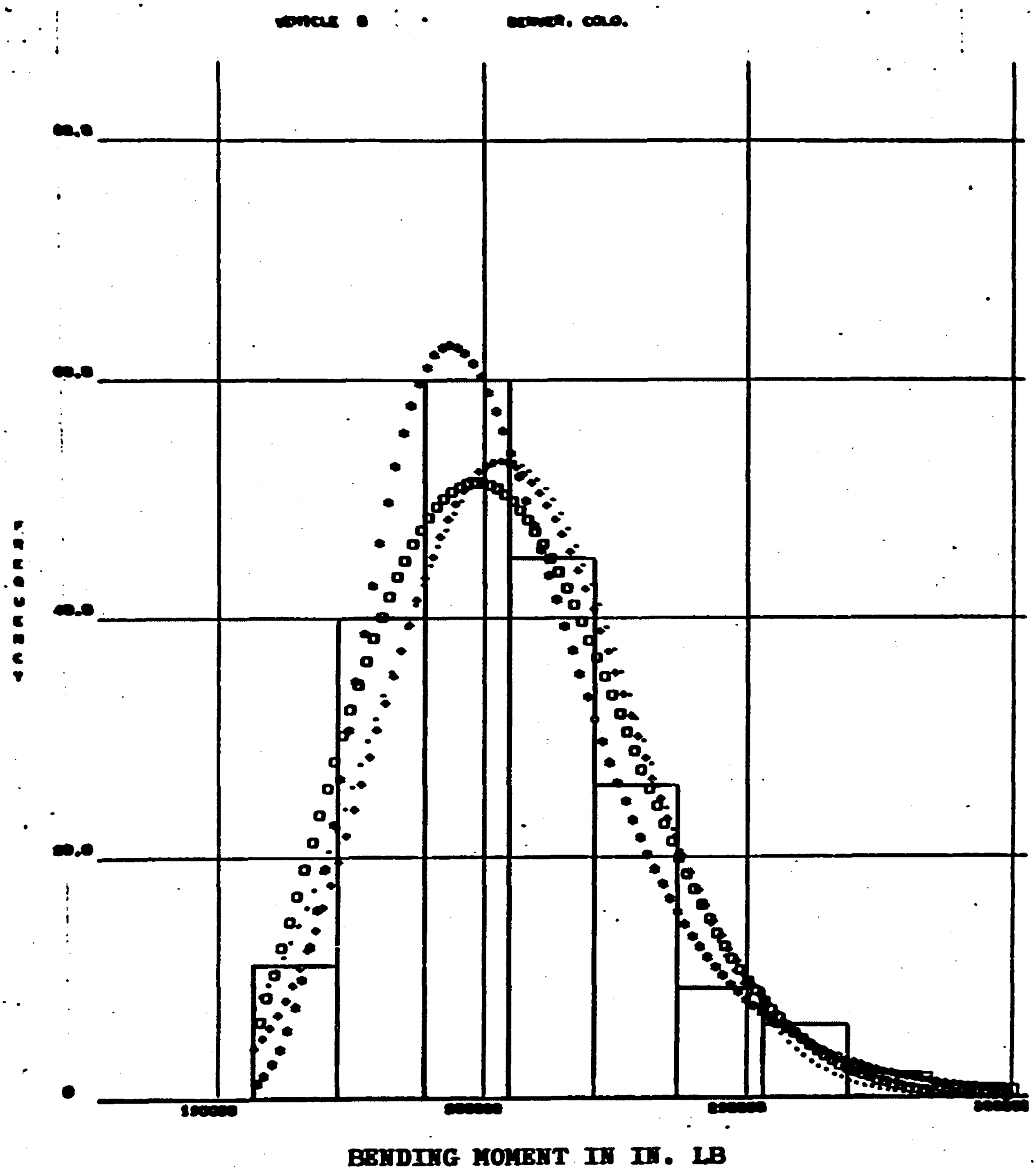
			PSI-SQ.	K-S
NO. OF SAMPLES = 800	•	NORMAL	1.00000	50.5198
NO. OF CLASS INTERVALS = 11	+	LOG NORMAL	1.00000	22.3036
MEAN = 2,138,300	*	DOUBLE EXPONENTIAL	.997804	14.5730
SIGMA = 328,786	□	WEIBULL	1.00000	33.1390

Figure 3-1



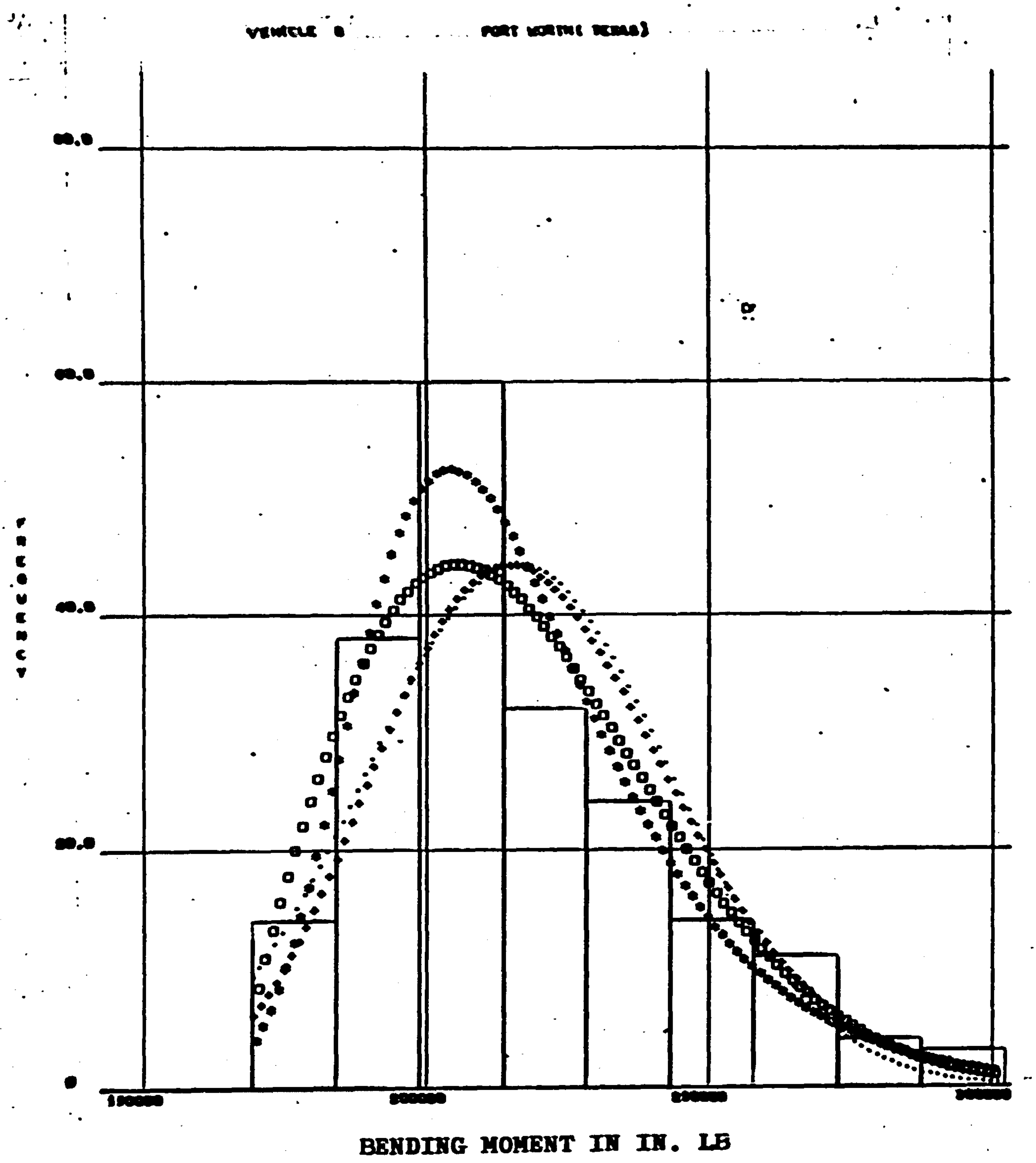
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	3.91154	23.4338
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	2.58537	16.9644
MEAN = 2,120,130	* DOUBLE EXPONENTIAL	1.41353	10.7908
SIGMA = 343,865	▫ WEIBULL	1.66676	11.5200

Figure 3-2



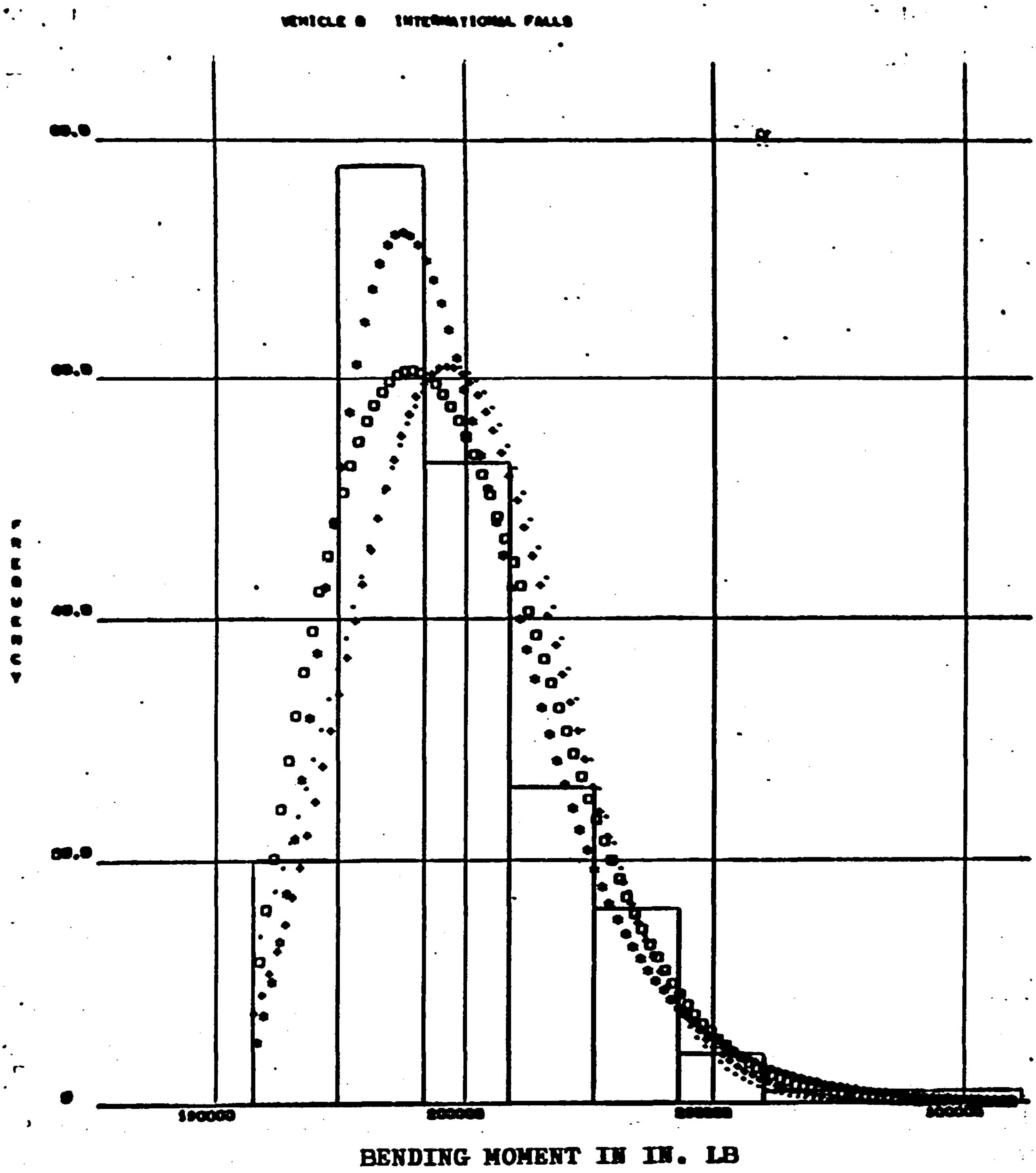
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.74837	9.41431
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.05158	4.73750
MEAN = 2,042,290	* DOUBLE EXPONENTIAL	.112646	4.69863
SIGMA = 241,212	□ WEIBULL	1.03053	6.30047

Figure 3-3



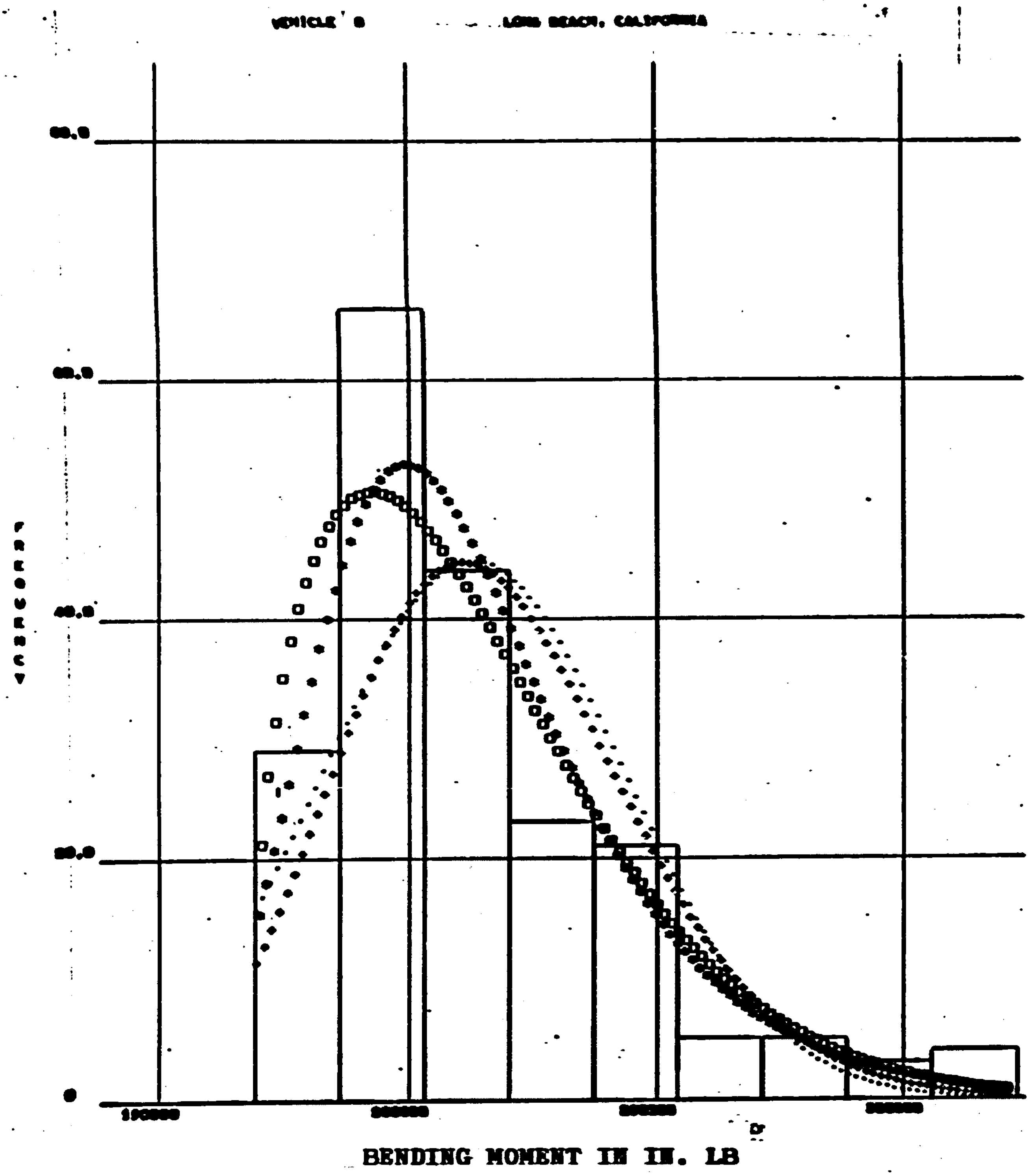
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.11056	19.9643
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.708904	15.4493
MEAN = 2,160,300	* DOUBLE EXPONENTIAL	.276646	6.35978
SIGMA = 267,569	□ WEIBULL	.353459	10.5505

Figure 3-4



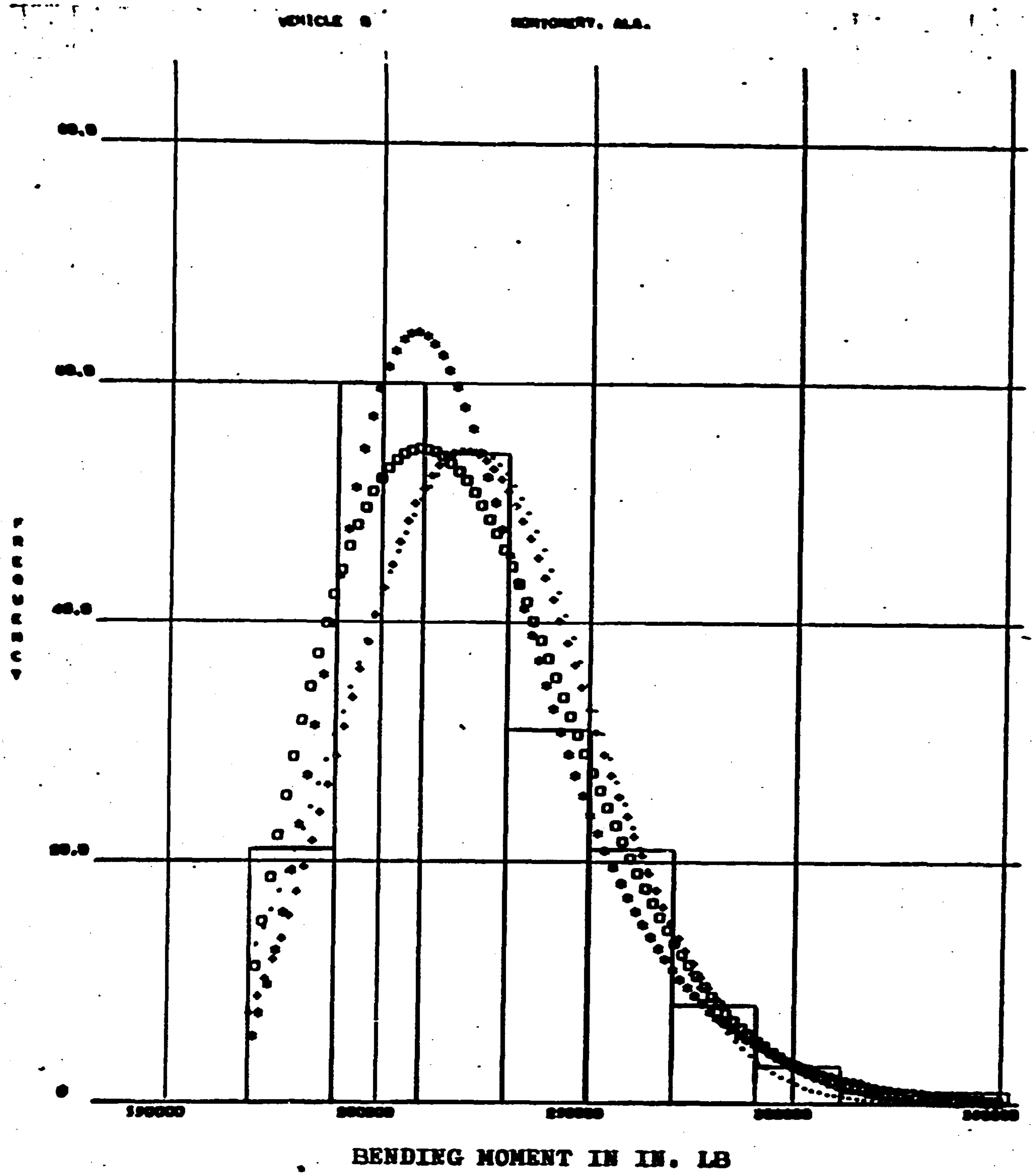
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	2.22418	18.4297
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.87162	15.6758
MEAN = 1,972,930	* DOUBLE EXPONENTIAL	1.53115	6.55586
SIGMA = 223,613	□ WEIBULL	1.69845	11.2085

Figure 3-5



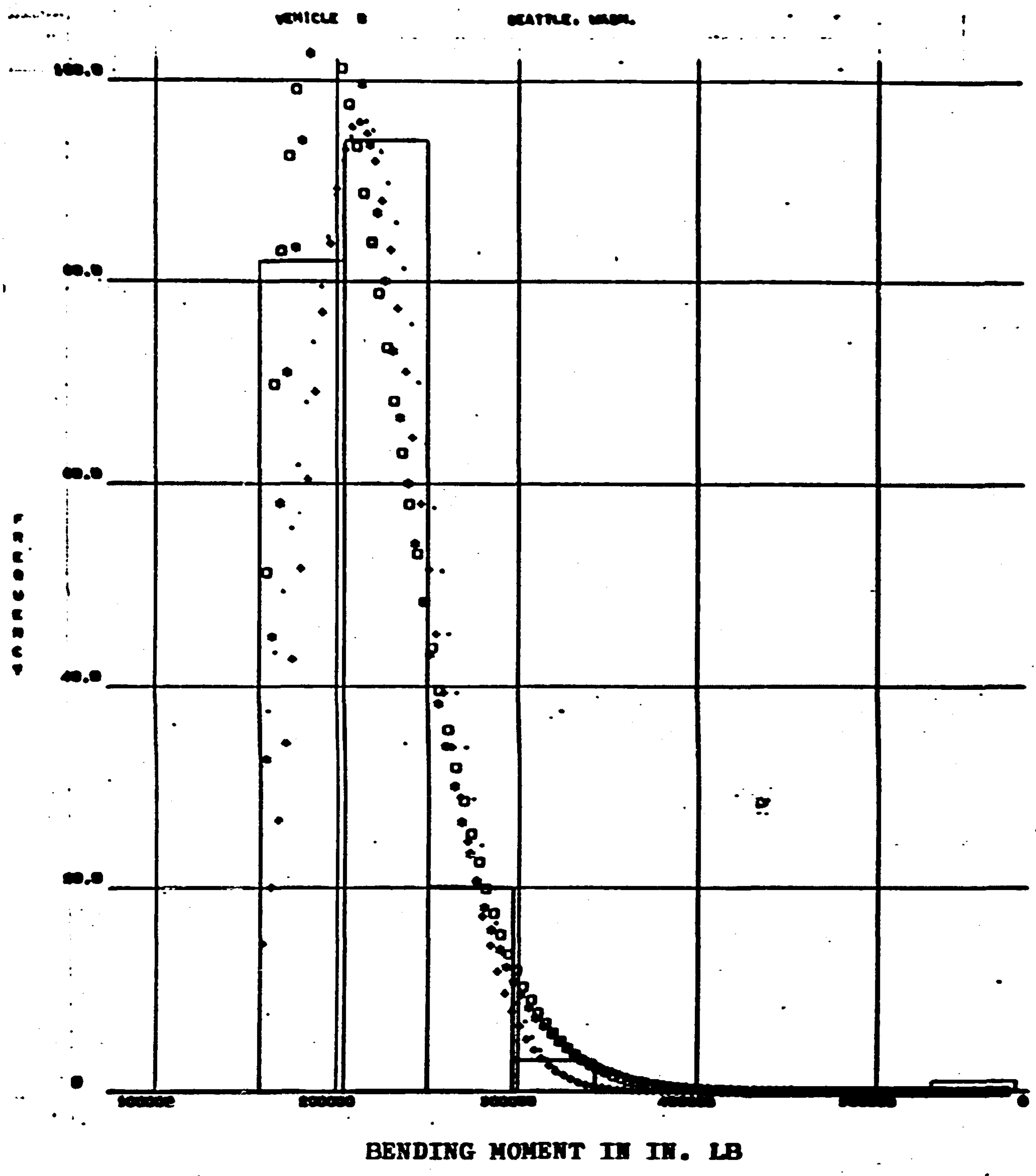
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	3.31858	20.5483
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	2.25620	17.6120
MEAN = 2,129,680	* DOUBLER EXPONENTIAL	1.34728	9.75369
SIGMA = 304,558	¤ WEIBULL	1.73214	9.62735

Figure 3-6



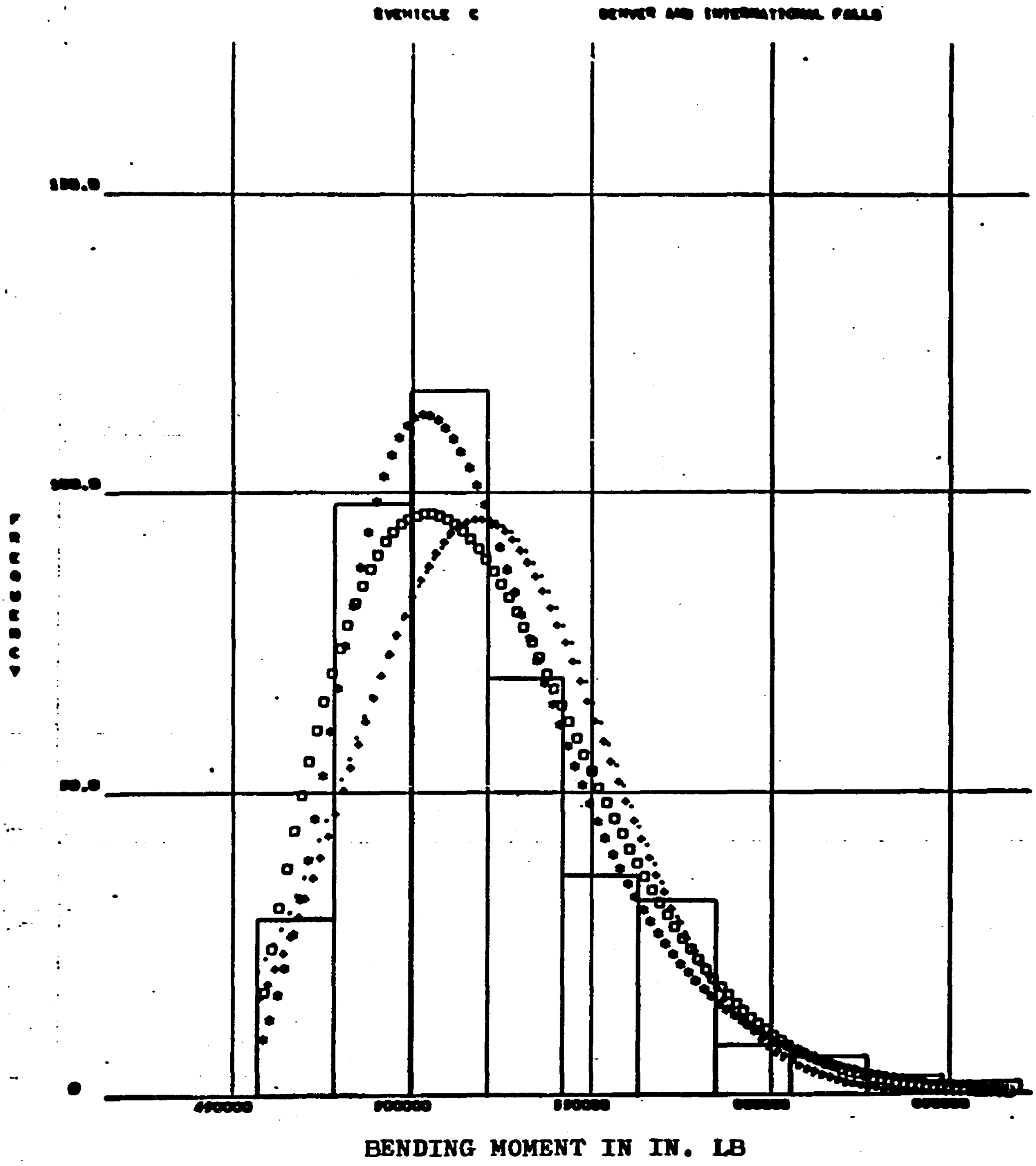
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.75340	11.9355
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.05495	8.26500
MEAN = 2,212,640	* DOUBLE EXPONENTIAL	.361232	4.28782
SIGMA = 298,301	▫ WEIBULL	.654169	7.08461

Figure 3-7



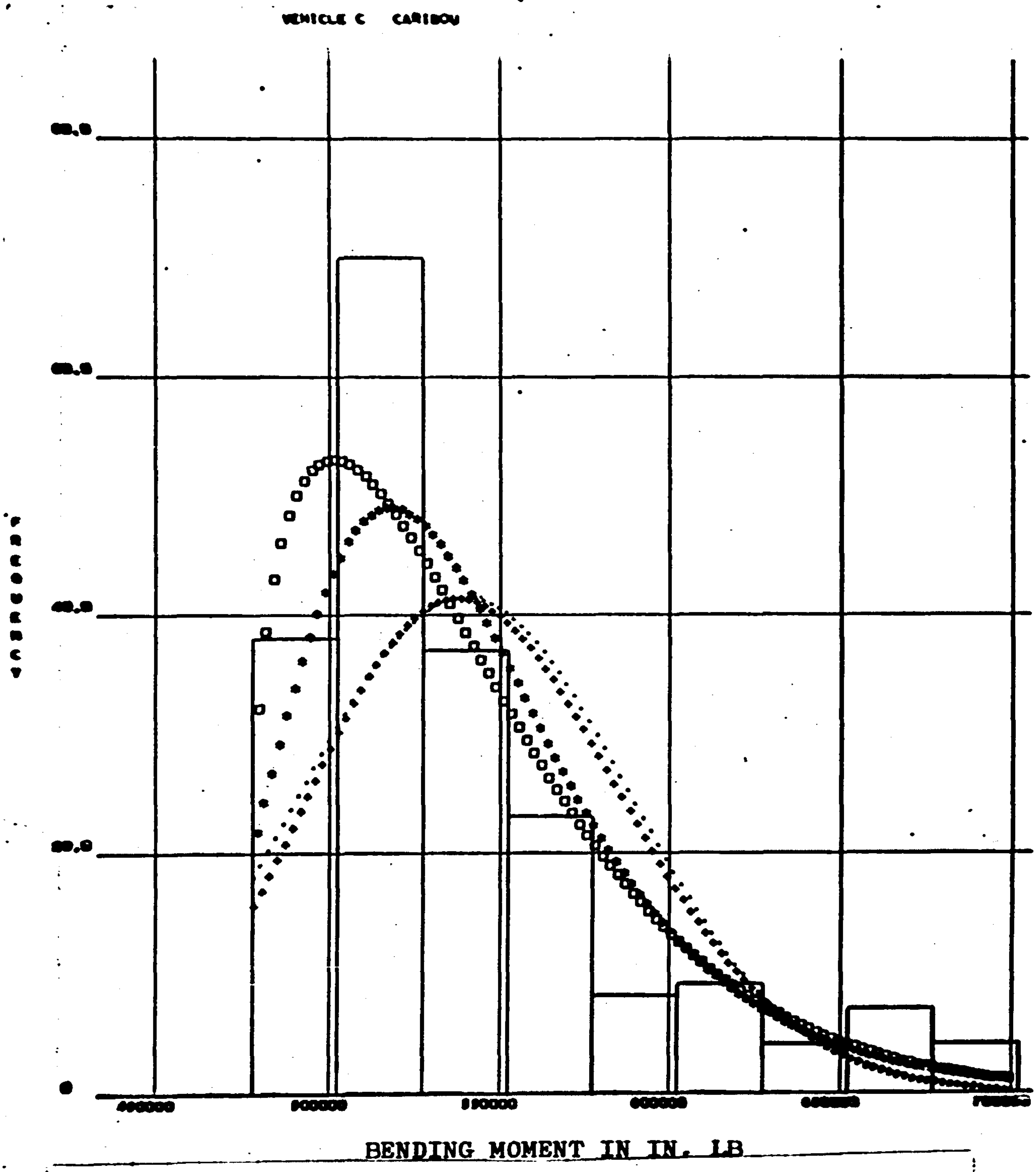
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	·NORMAL	.999997	11.4079
NO. OF CLASS INTERVALS = 9	+LOG NORMAL	1.00000	2.58022
MEAN = 2,143,100	*DOUBLE EXPONENTIAL	.994857	7.87863
SIGMA = 387,975	□WEIBULL	..999899	10.3609

Figure 3-8



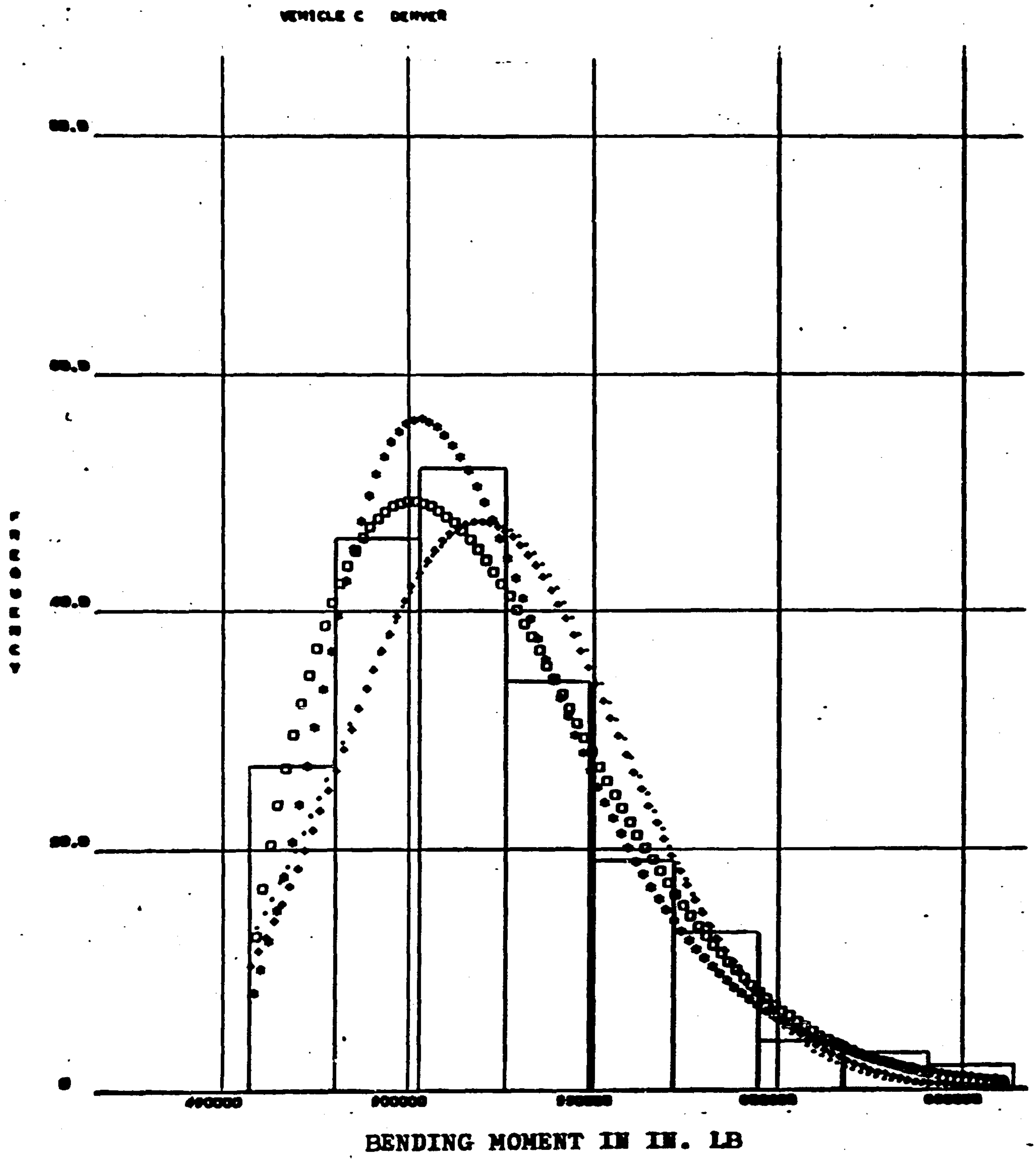
			PSI-SQ.	K-S
NO. OF SAMPLES = 400	.	NORMAL	2.11599	37.5913
NO. OF CLASS INTERVALS = 10	+	LOG NORMAL	1.86329	31.9793
MEAN = 5,190,670	*	DOUBLE EXPONENTIAL	.466362	9.32540
SIGMA = 355,046	▫	WEIBULL	1.14748	18.6196

Figure 3-9



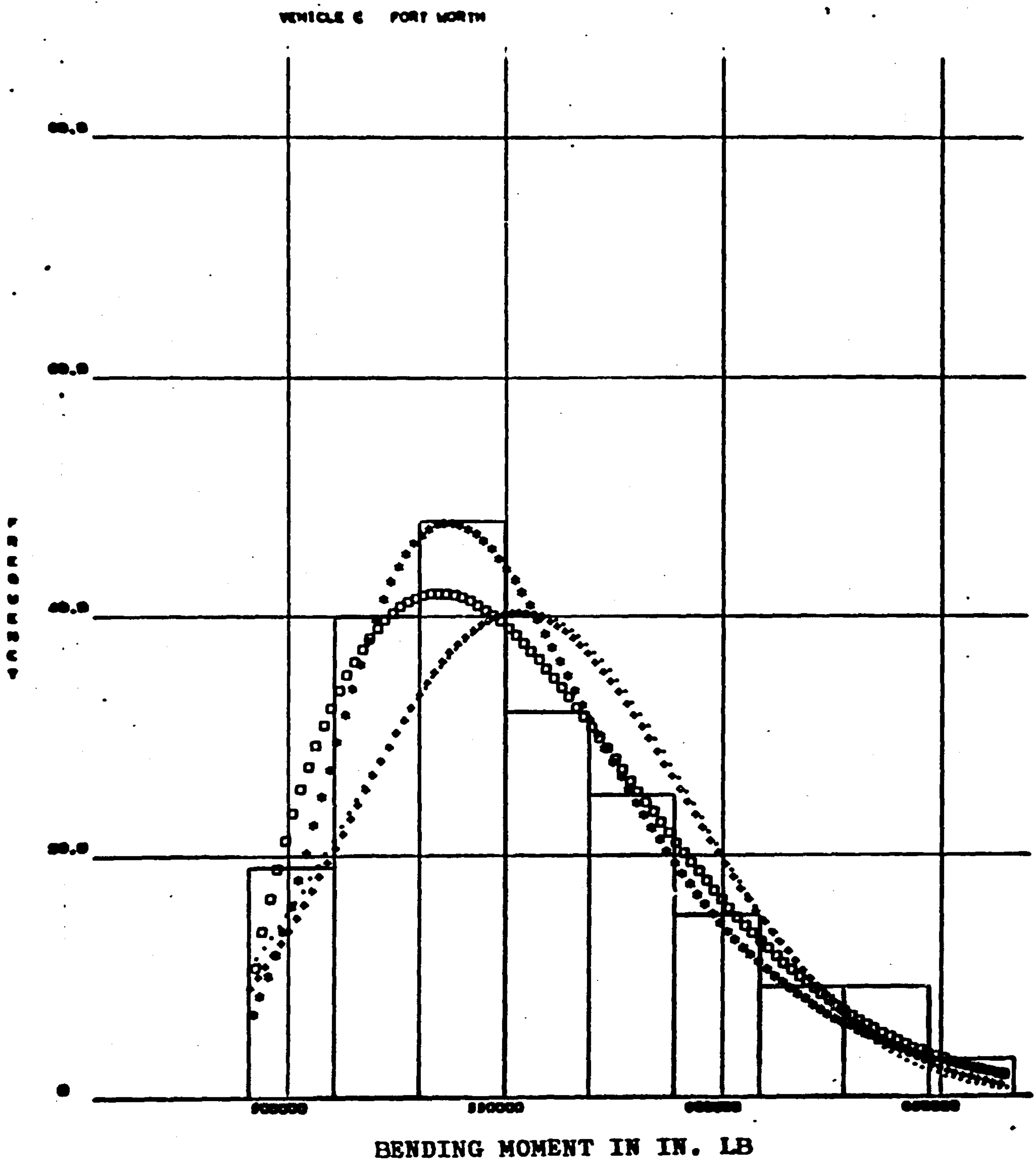
			PSI-SQ.	K-S
NO. OF SAMPLES = 200	·	NORMAL	5.68336	28.1320
NO. OF CLASS INTERVALS = 9	+	LOG NORMAL	4.53533	26.1138
MEAN = 5,396,200	*	DOUBLE EXPONENTIAL	2.18751	16.2020
SIGMA = 480,887	◻	WEIBULL	1.99126	10.5616

Figure 3-10



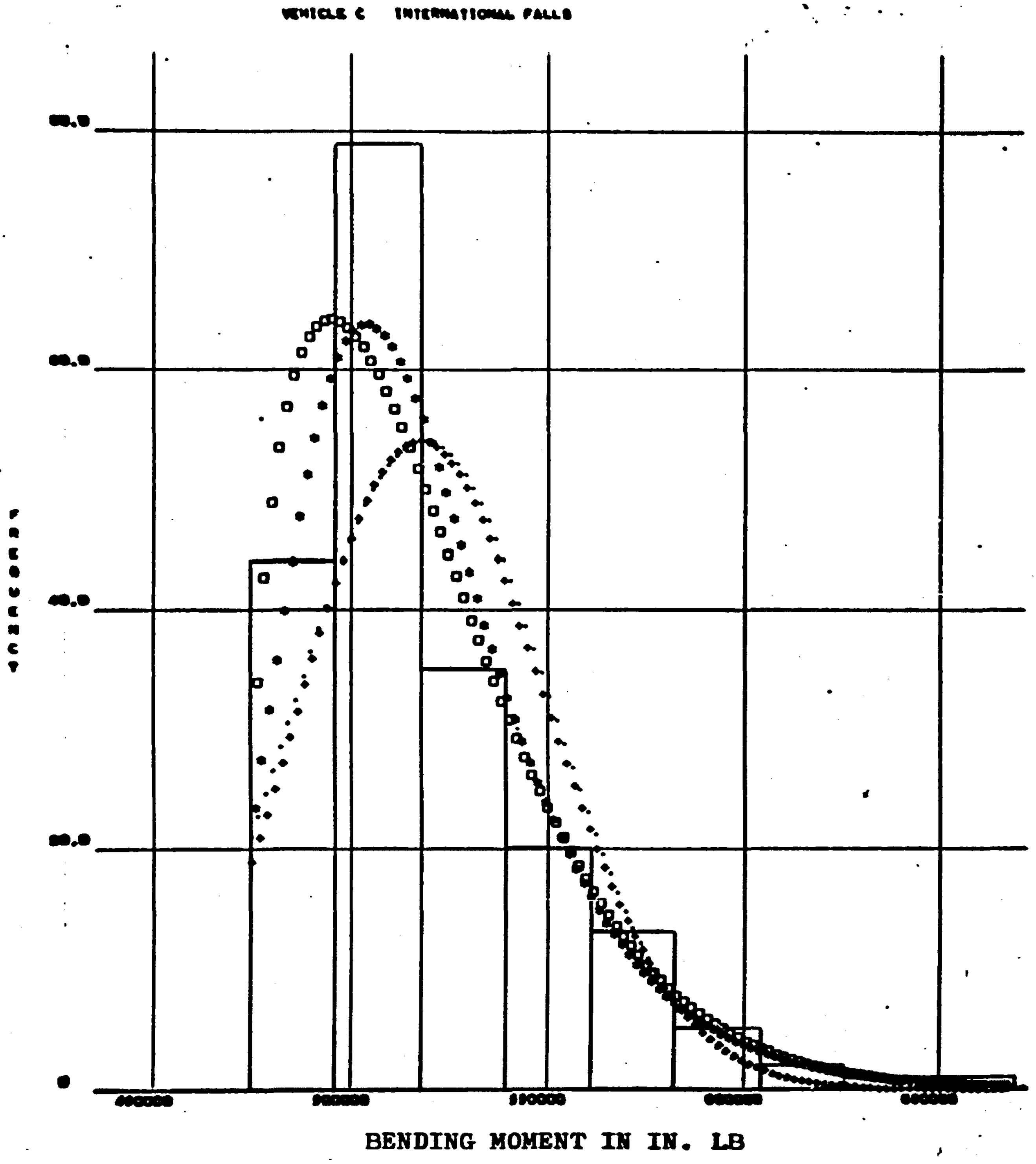
			PSI-SQ.	K-S
NO. OF SAMPLES = 200	·	NORMAL	1.51654	13.3419
NO. OF CLASS INTERVALS = 9	+	LOG NORMAL	1.23319	10.1843
MEAN = 5,200,520	*	DOUBLE EXPONENTIAL	.362832	3.49017
SIGMA = 386,910	◻	WEIBULL	.592106	3.23983

Figure 3-11



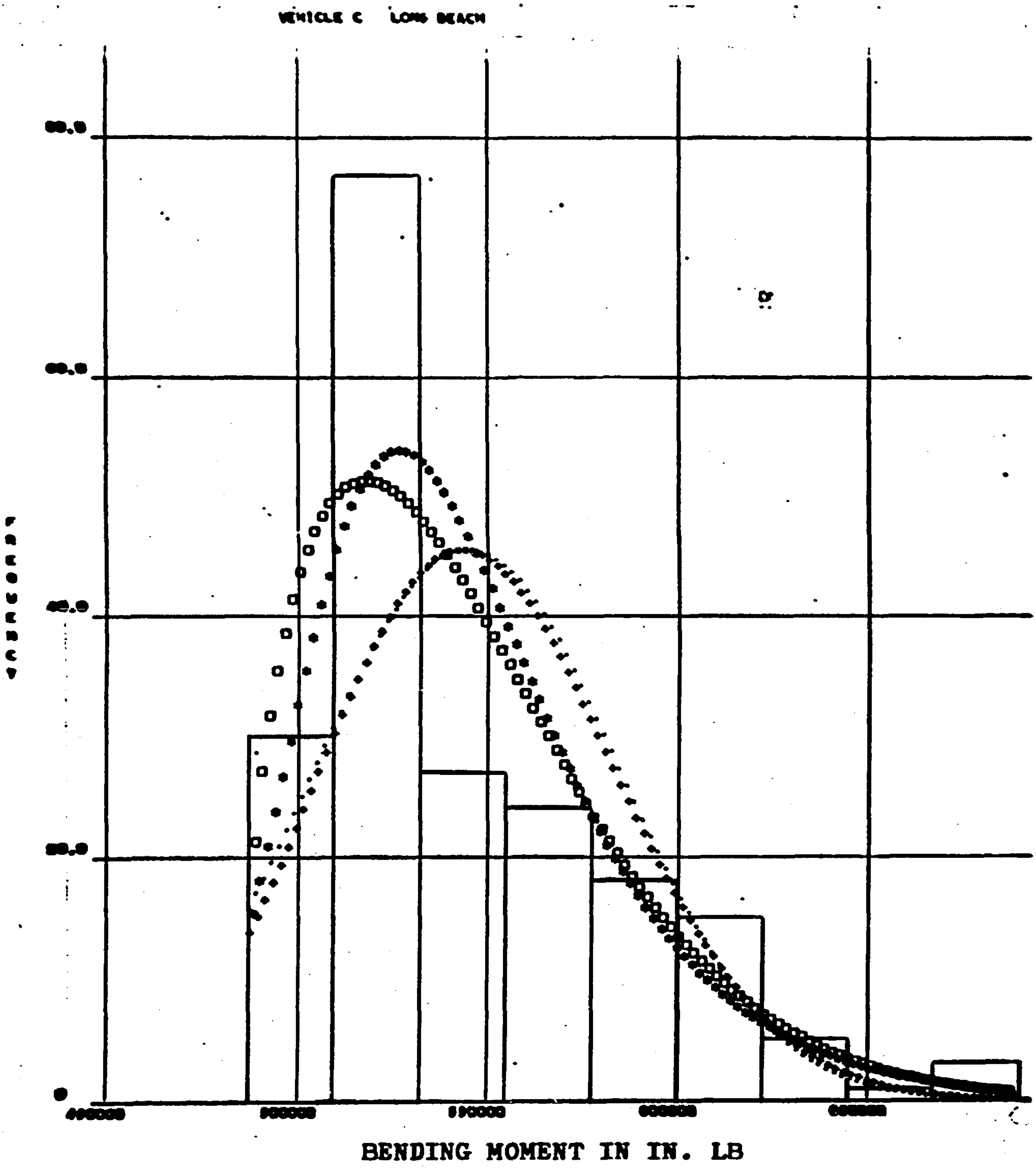
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	.928821	15.9990
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.717500	13.4869
MEAN = 5,540,010	* DOUBLE EXPONENTIAL	.307972	3.54378
SIGMA = 386,693	□ WEIBULL	.244966	4.33073

Figure 3-12



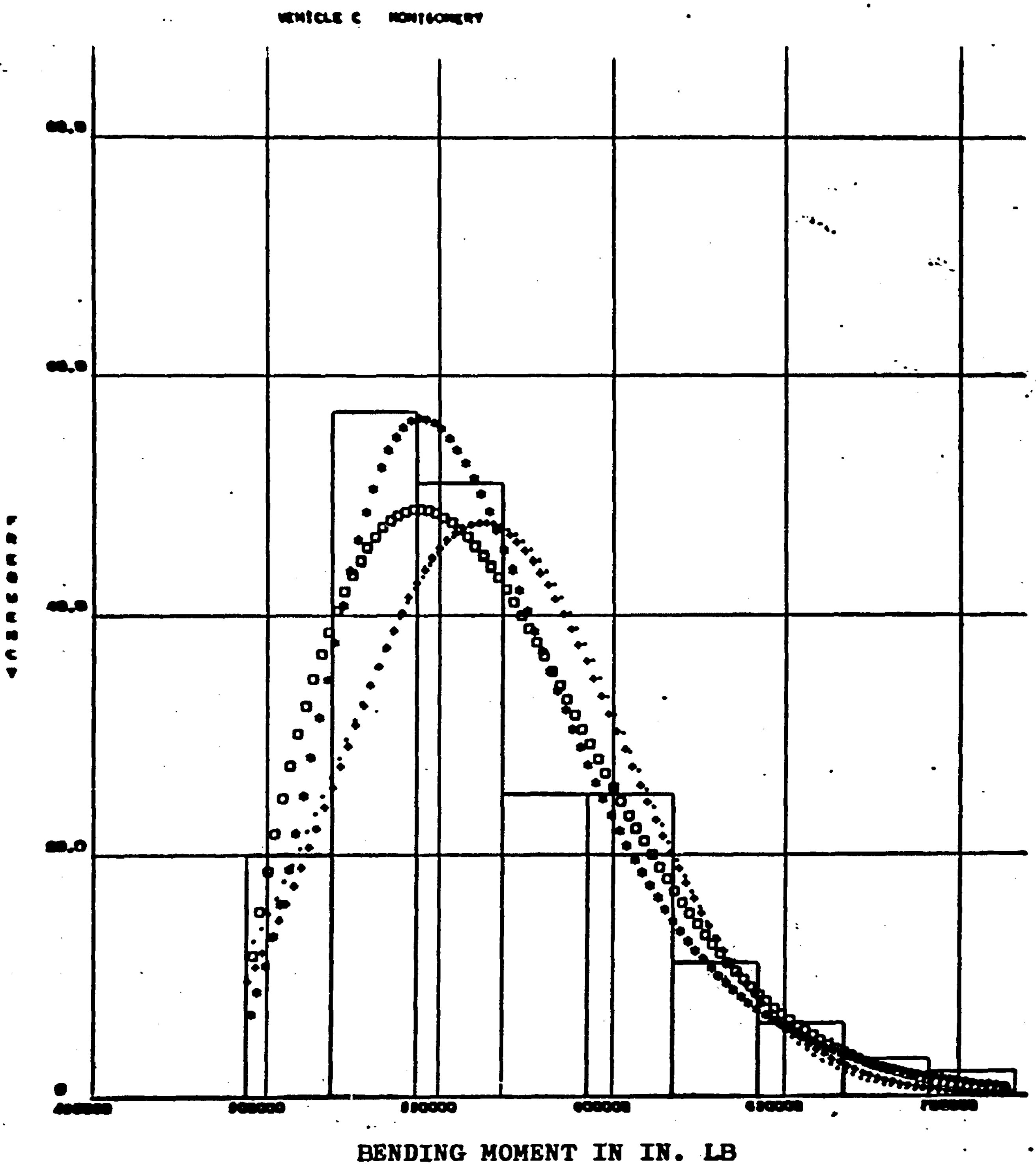
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	2.38390	24.6585
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	2.20514	22.2891
MEAN = 5,180,820	* DOUBLE EXPONENTIAL	.449189	10.6423
SIGMA = 320,709	▫ WEIBULL	.523142	11.6935

Figure 3-13



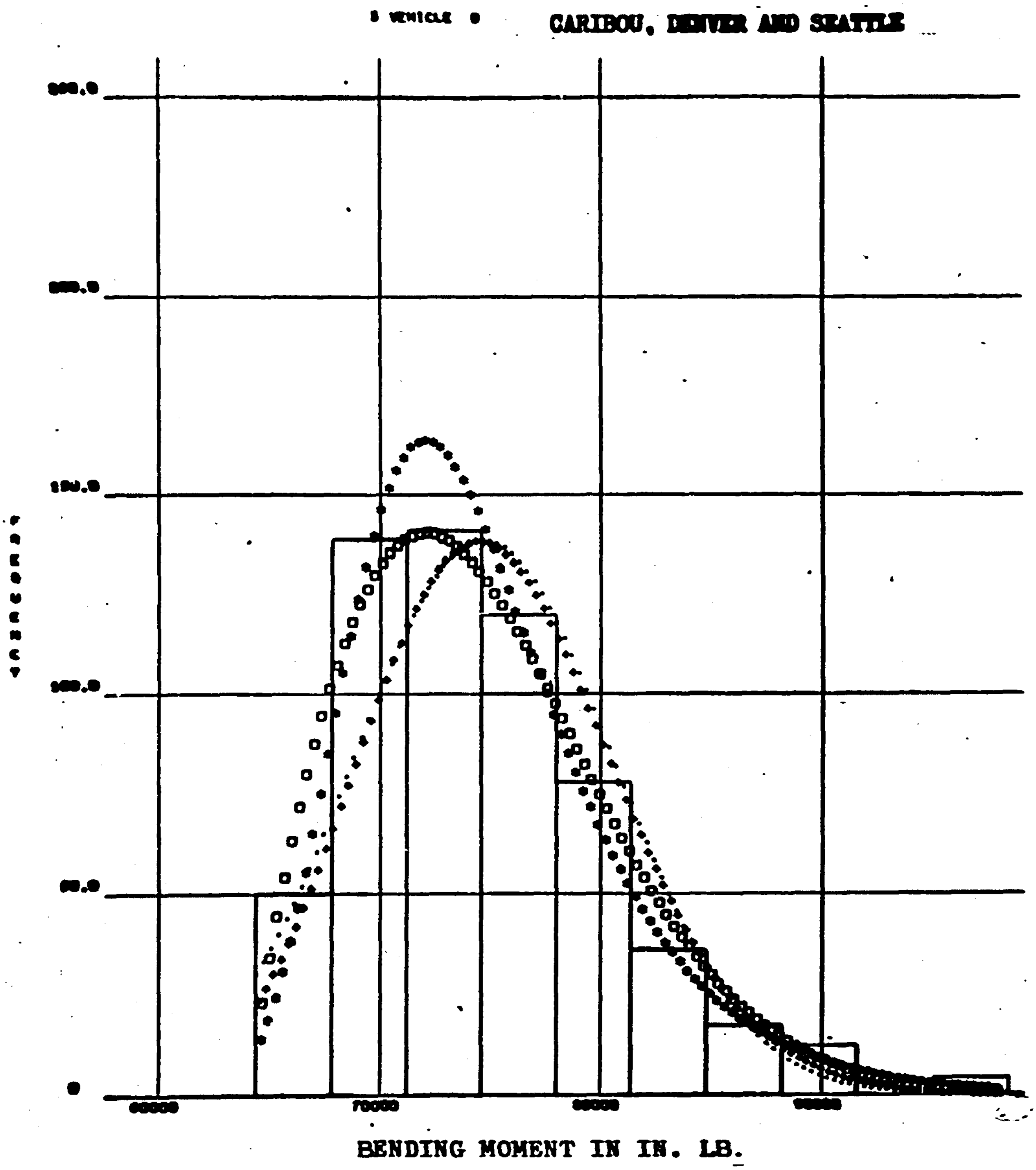
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.15333	31.5169
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.06447	29.9329
MEAN = 5,442,490	* DOUBLE EXPONENTIAL	2.03741	20.4872
SIGMA = 396,294	□ WEIBULL	2.45893	17.2505

Figure 3-14



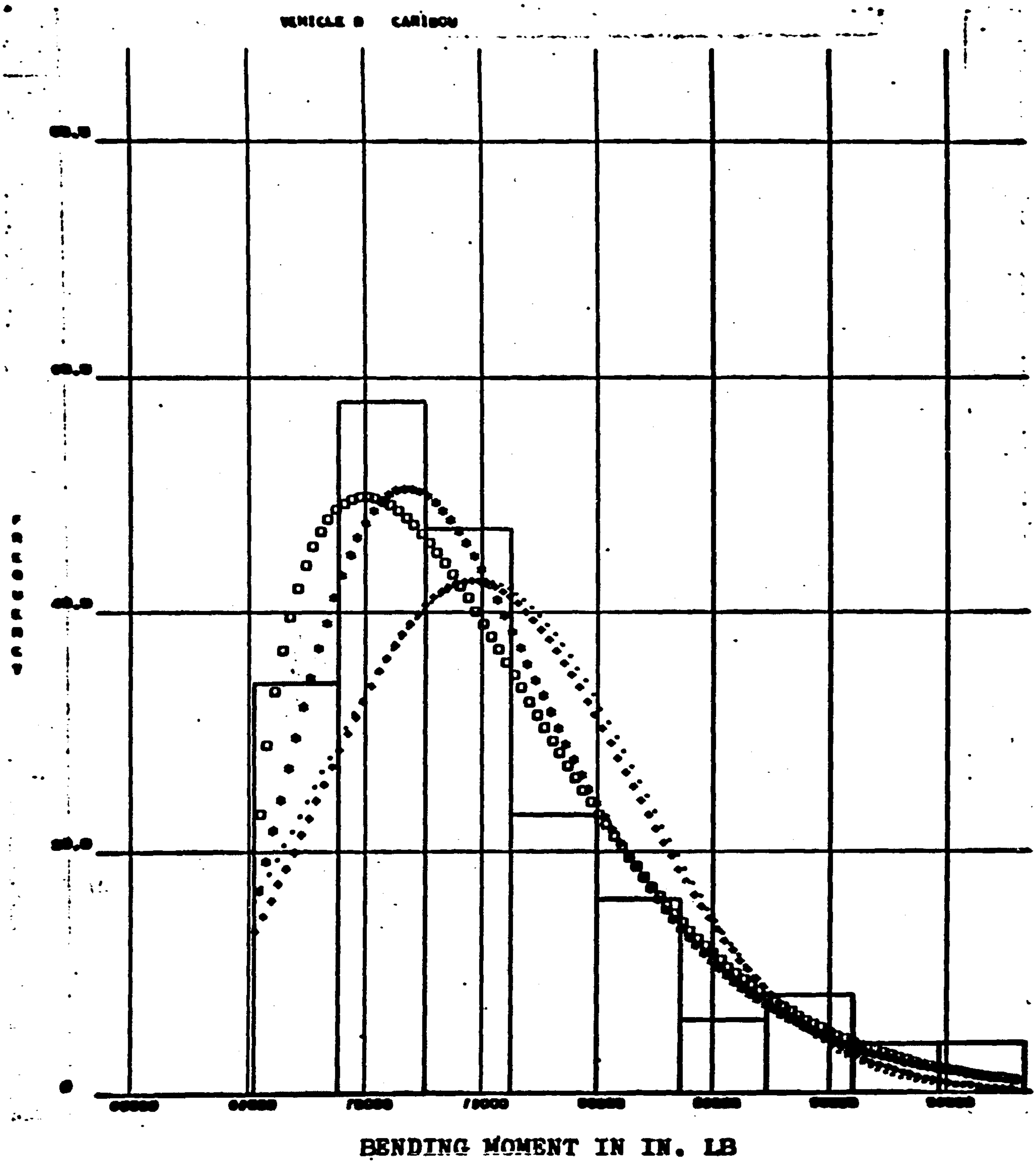
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.40549	18.7012
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.15482	15.5932
MEAN = 5,630,930	* DOUBLE EXPONENTIAL	.304129	6.76051
SIGMA = 414,501	□ WEIBULL	.439715	8.96307

Figure 3-15



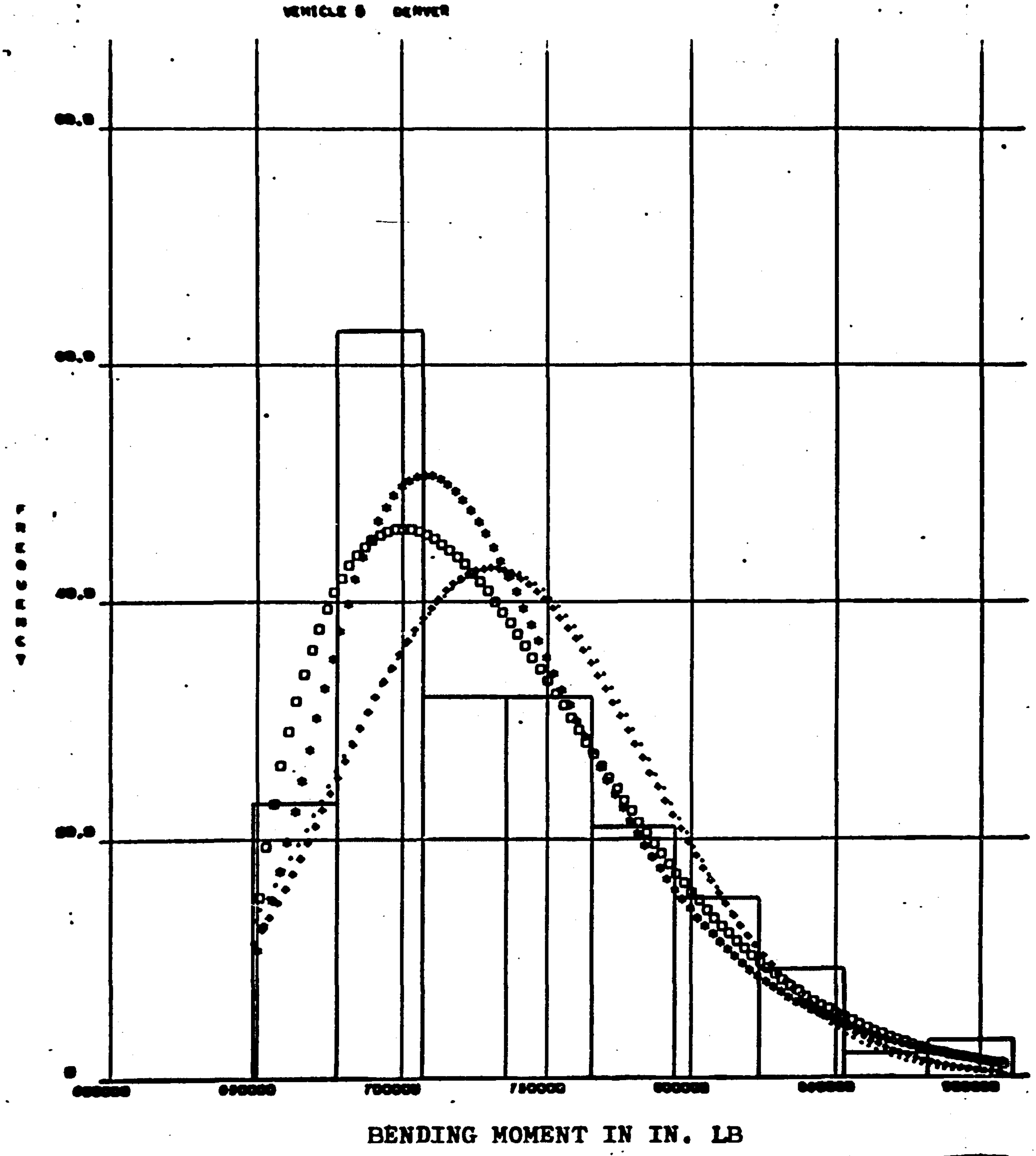
			PSI-SQ.	K-S
NO. OF SAMPLES = 600	·	NORMAL	1.84972	33.2808
NO. OF CLASS INTERVALS = 10	+	LOG NORMAL	1.40124	23.9946
MEAN = 7,465,130	*	DOUBLE EXPONENTIAL	.502403	8.83494
SIGMA = 588,040	□	WEIBULL	.727283	14.0107

Figure 3-16



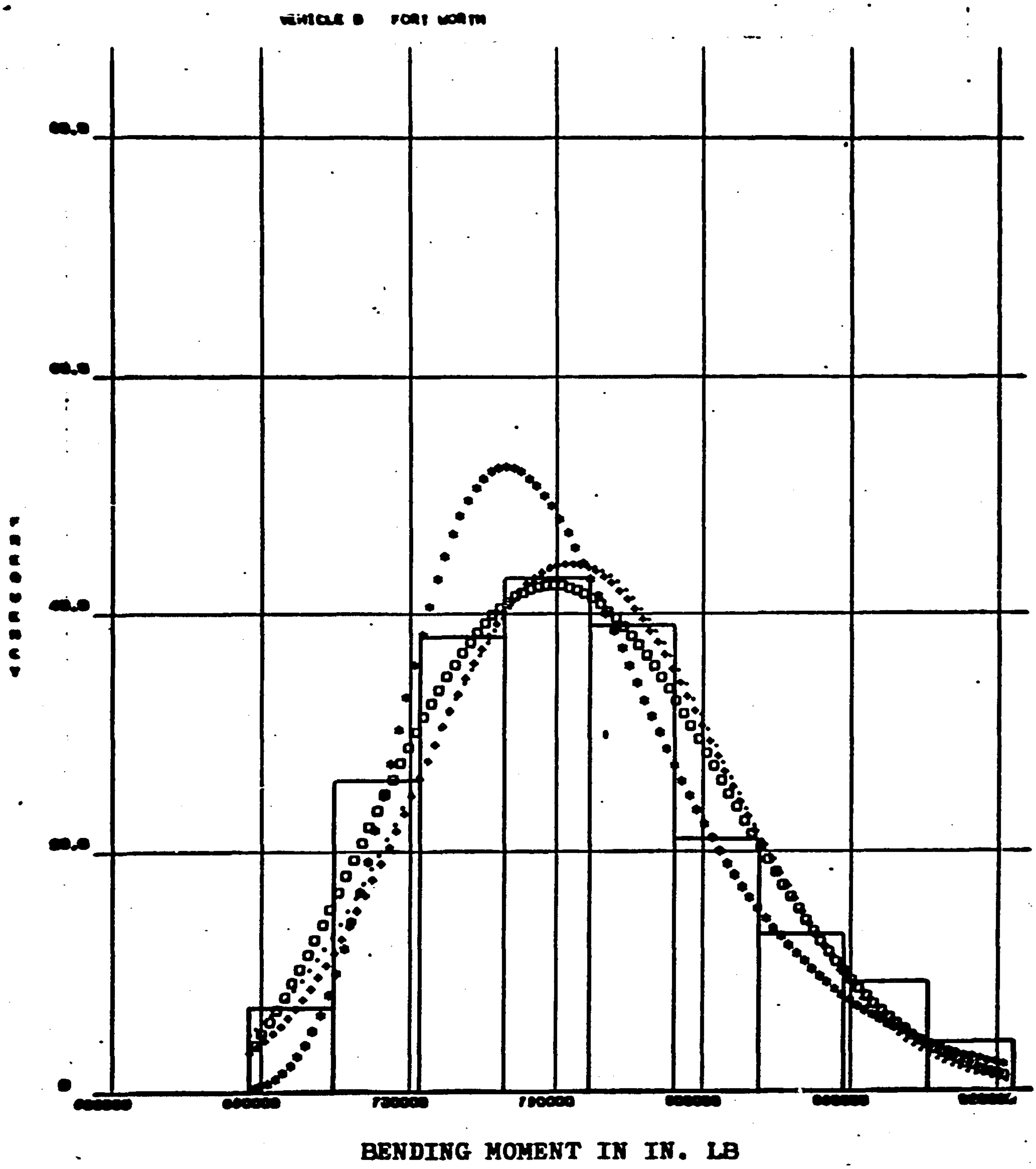
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	3.20082	23.1267
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	2.46654	19.0186
MEAN = 7,487,000	* DOUBLE EXPONENTIAL	1.12849	9.45326
SIGMA = 690,140	□ WEIBULL	1.21580	9.30659

Figure 3-17



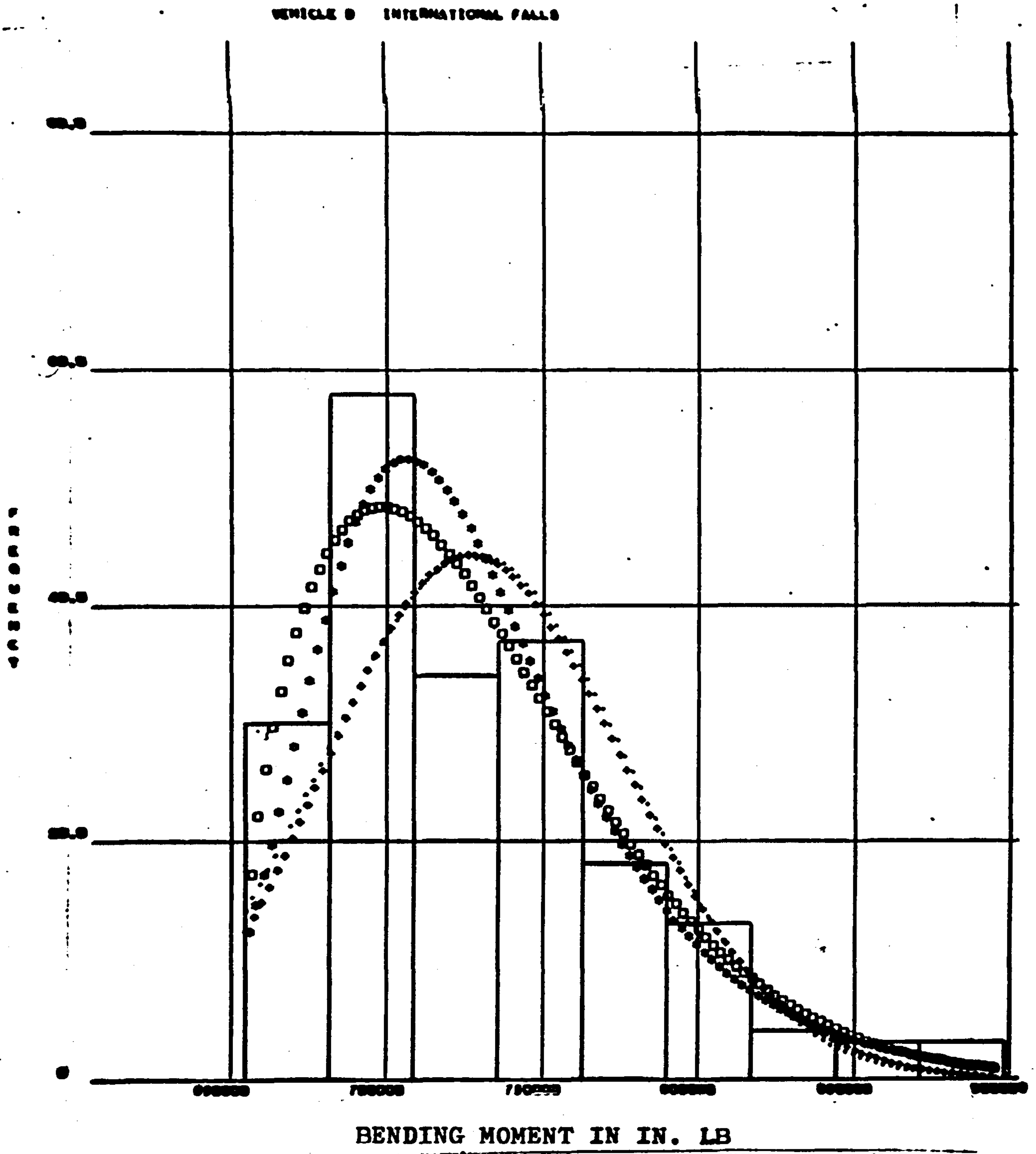
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	.924828	21.8521
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.792026	20.7541
MEAN = 7,319,890	* DOUBLE EXPONENTIAL	.705327	13.8941
SIGMA = 544,805	◻ WEIBULL	.853378	10.6145

Figure 3-18



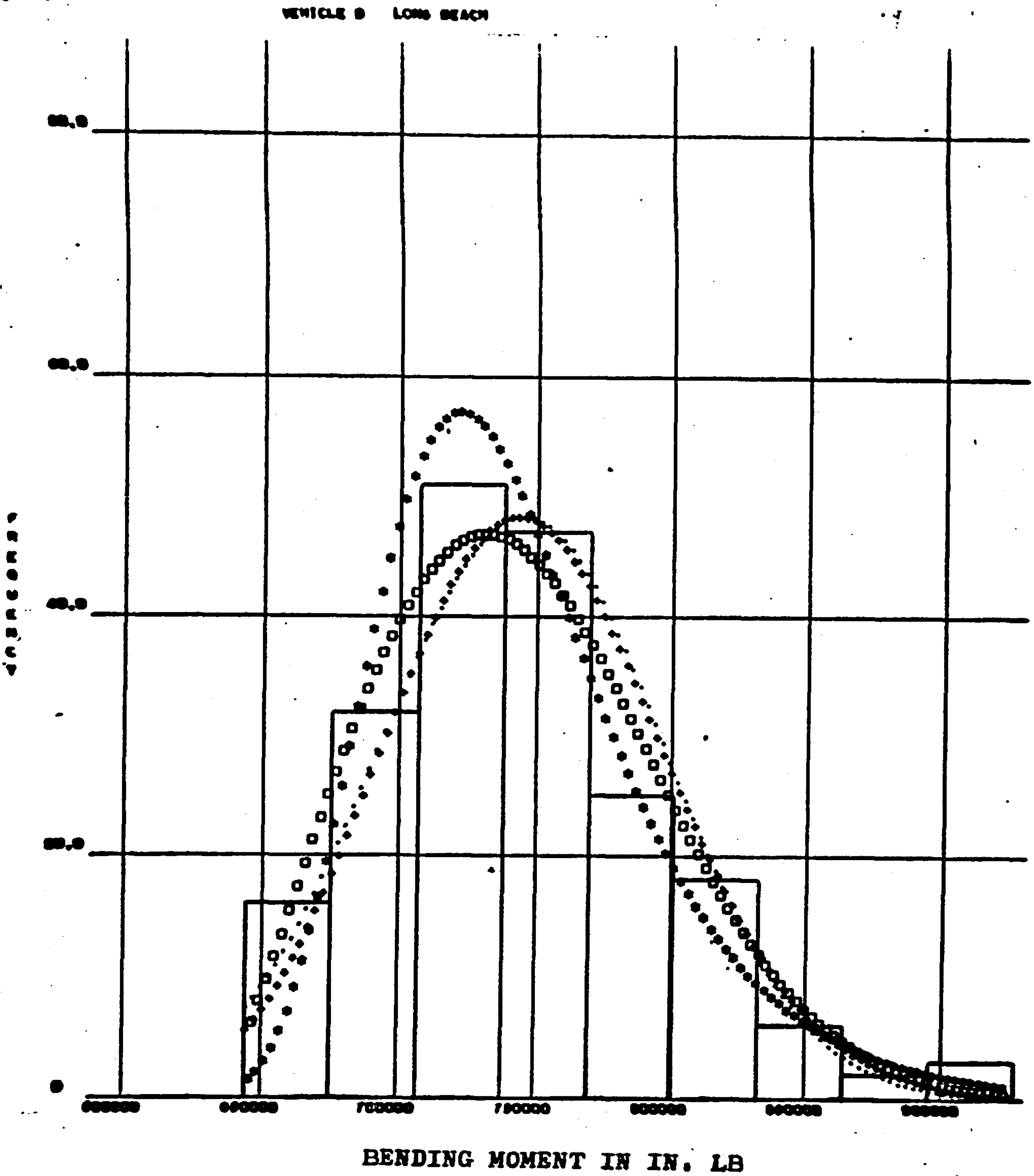
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	.636506	6.14773
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.466221	4.64700
MEAN = 7,558,820	* DOUBLE EXPONENTIAL	.248025	7.97785
SIGMA = 521,356	▫ WEIBULL	.375303	3.99807

Figure 3-19



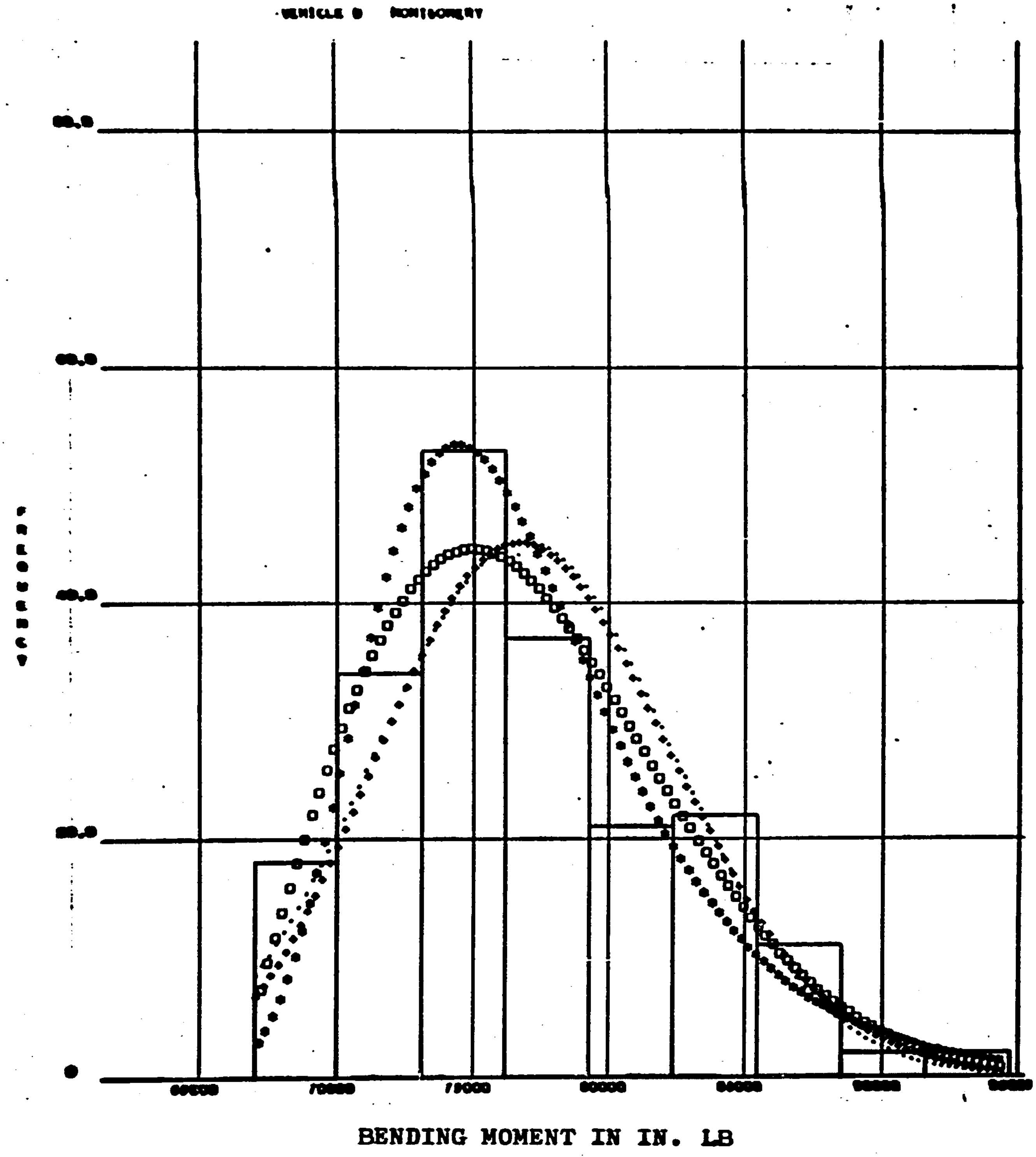
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.43666	19.0680
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.19583	17.8504
MEAN = 7,280,240	* DOUBLE EXPONENTIAL	.481194	9.68310
SIGMA = 489,485	◘ WEIBULL	.636746	6.59511

Figure 3-20



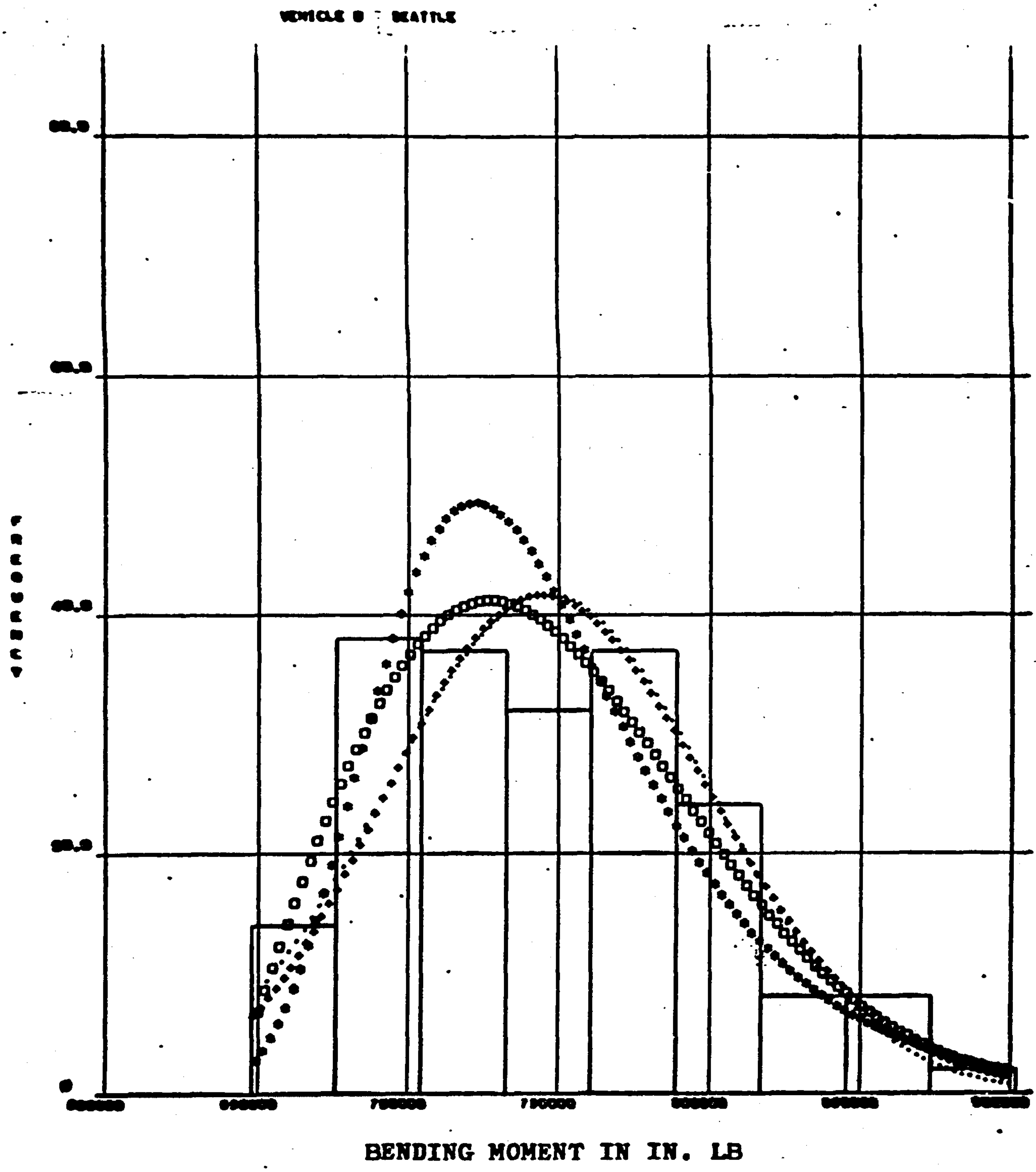
			PSI-SQ.	K-S
NO. OF SAMPLES = 200	•	NORMAL	1.00162	9.40047
NO. OF CLASS INTERVALS = 9	+	LOG NORMAL	.828293	6.88358
MEAN = 7,447,360	*	DOUBLE EXPONENTIAL	.660848	7.20503
SIGMA = 520,266	▫	WEIBULL	.868923	6.47437

Figure 3-21



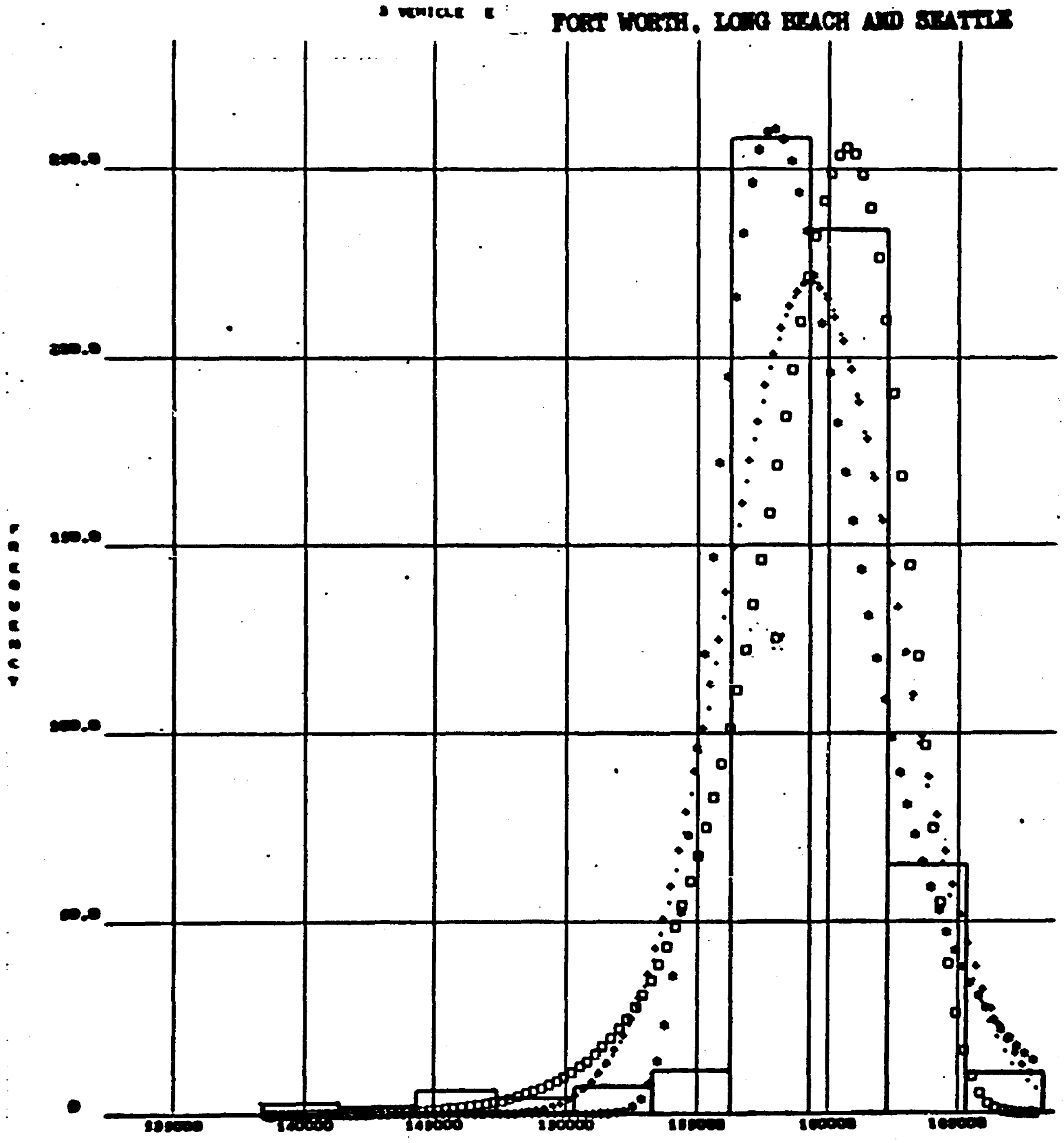
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	1.07100	14.9959
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.940547	12.4348
MEAN = 7,686,960	* DOUBLE EXPONENTIAL	.971979	8.40182
SIGMA = 545,593	□ WEIBULL	1.12098	6,49020

Figure 3-22



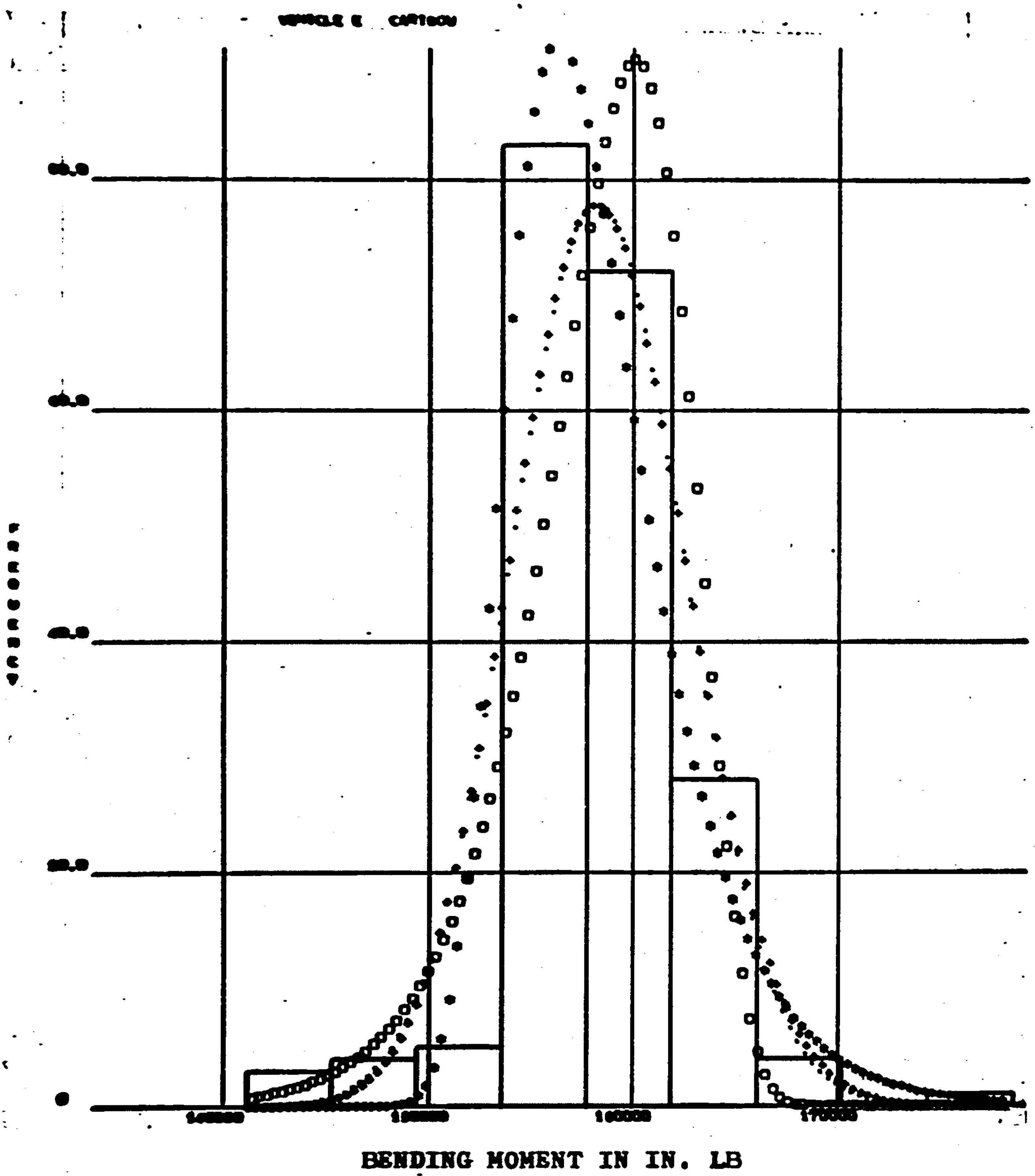
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	.523269	8.98978
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.370978	7.92358
MEAN = 7,461,060	* DOUBLE EXPONENTIAL	.396959	13.3950
SIGMA = 541,354	◻ WEIBULL	.401544	6,43485

Figure 3-23



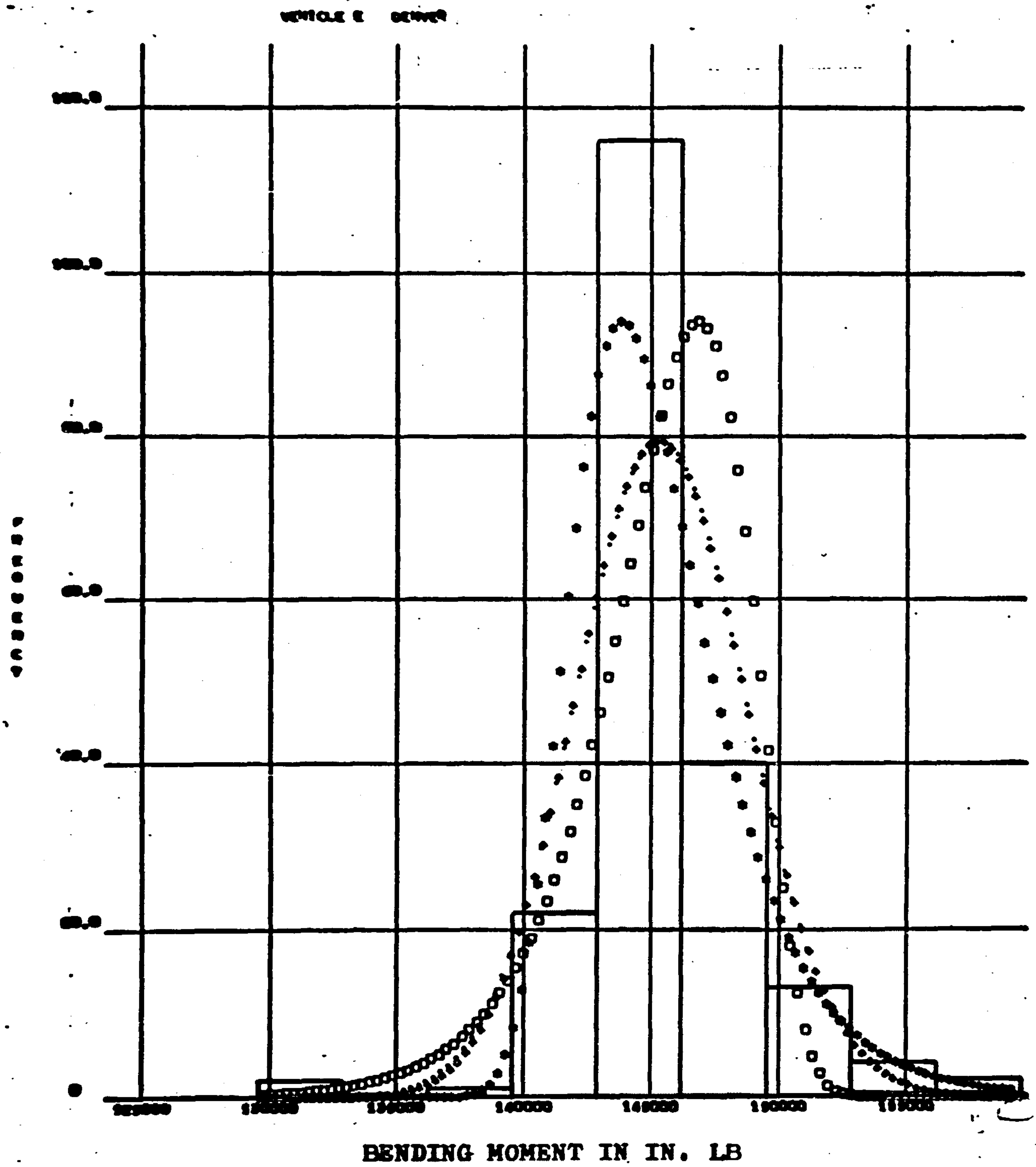
NO. OF SAMPLES = 600		PSI-SQ.	K-S
NO. OF CLASS INTERVALS = 10	· NORMAL	48.0594	72.8269
MEAN = 1,593,250	+ LOG NORMAL	52.5185	77.0425
SIGMA = 32,510	* DOUBLE EXPONENTIAL	56.9674	61.4444
	◘ WEIBULL	25.0196	62.3881

Figure 3-24



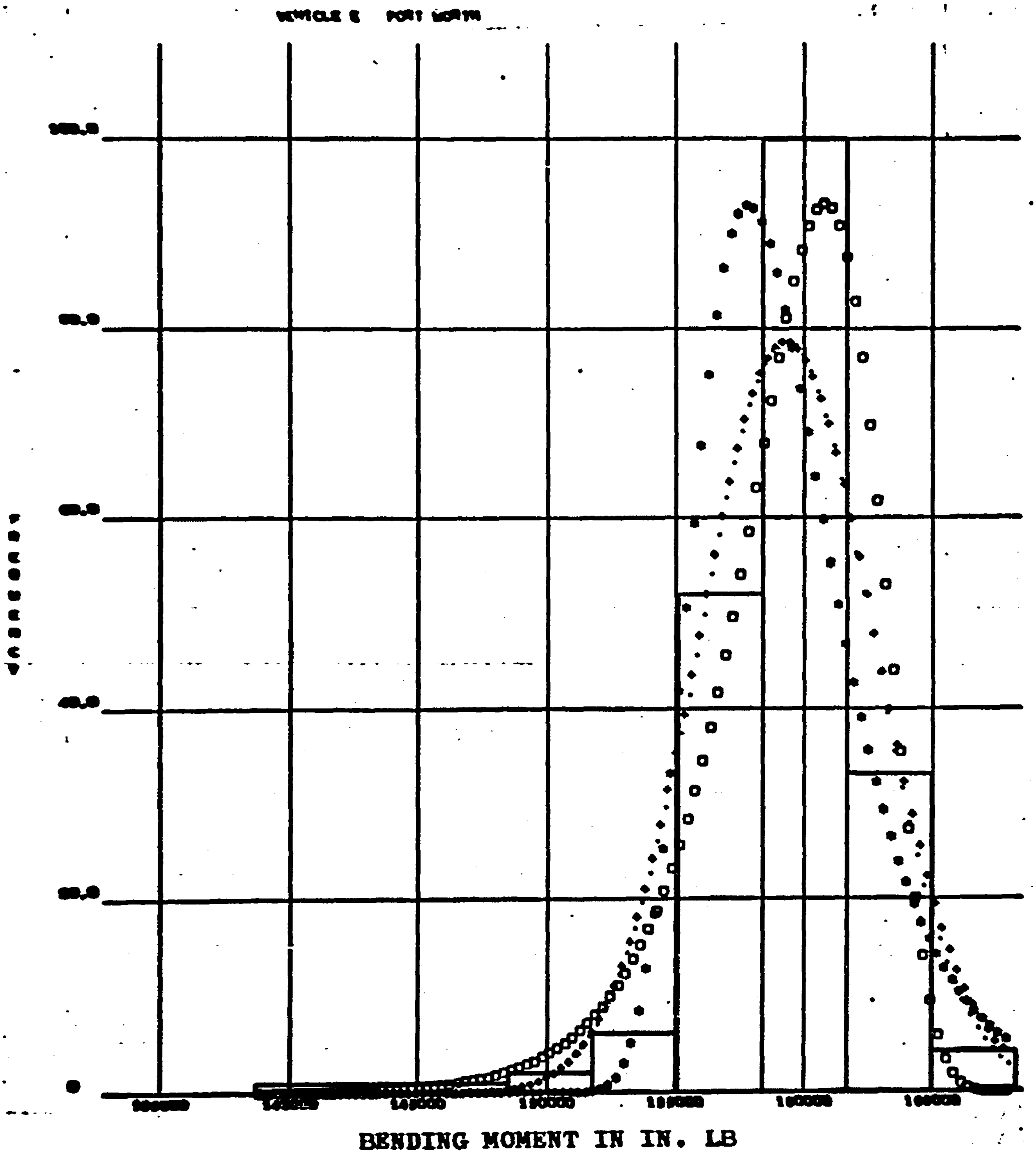
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	1.27277	15.5822
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.37772	15.8380
MEAN = 1,581,910	* DOUBLE EXPONENTIAL	.885224	9.45098
SIGMA = 42,683	¤ WEIBULL	1.79870	17.3045

Figure 3-25



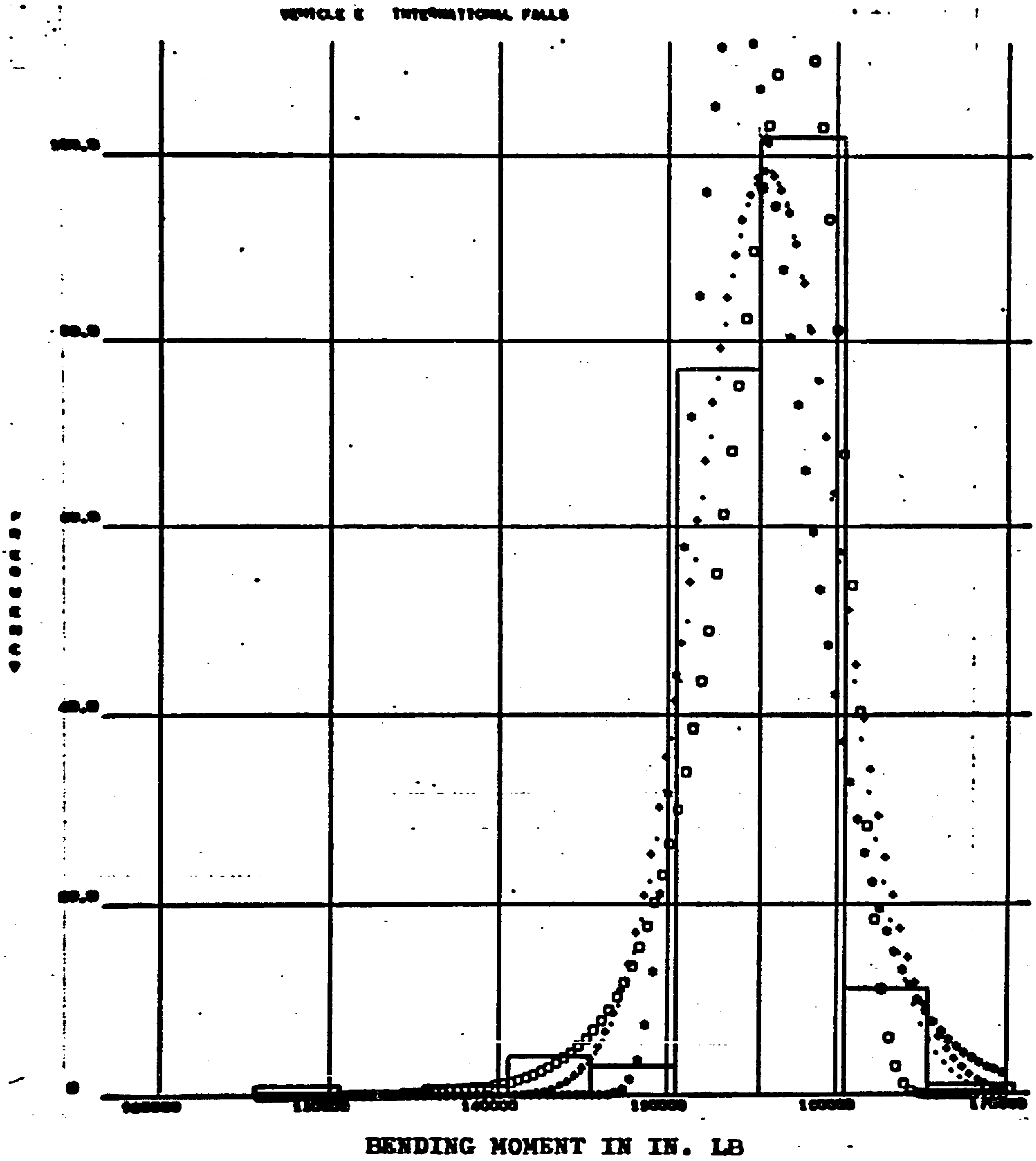
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	. NORMAL	1.58621	21.3737
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.46689	21.6201
MEAN = 1,453,040	* DOUBLE EXPONENTIAL	.256790	22.5306
SIGMA = 33,377	¤ WEIBULL	3.12533	31.8189

Figure 3-26



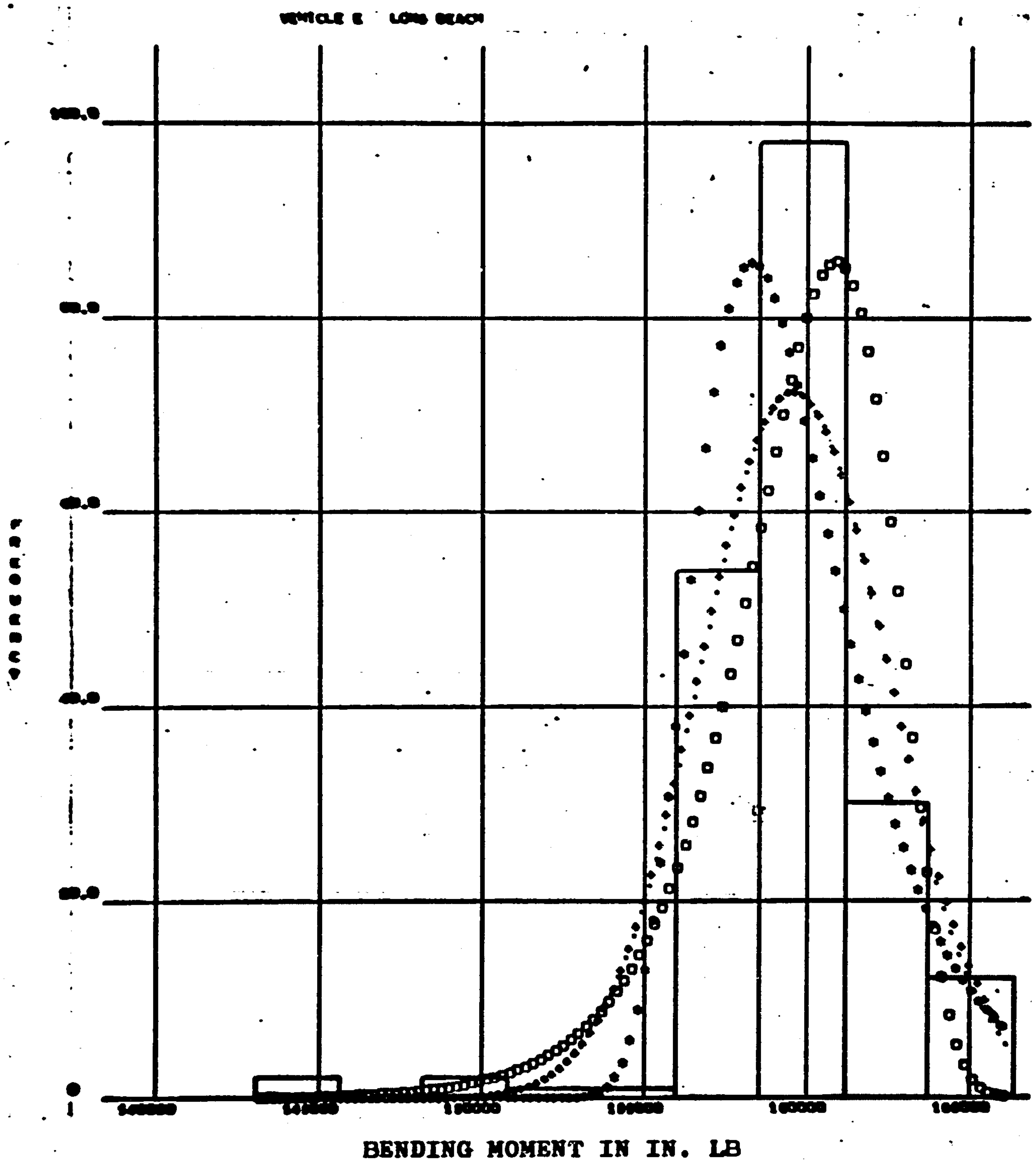
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	5.42057	15.3424
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	6.51395	16.6332
MEAN = 1,593,190	* DOUBLE EXPONENTIAL	2.73362	26.9773
SIGMA = 33,390	▫ WEIBULL	2.85826	12.2947

Figure 3-27



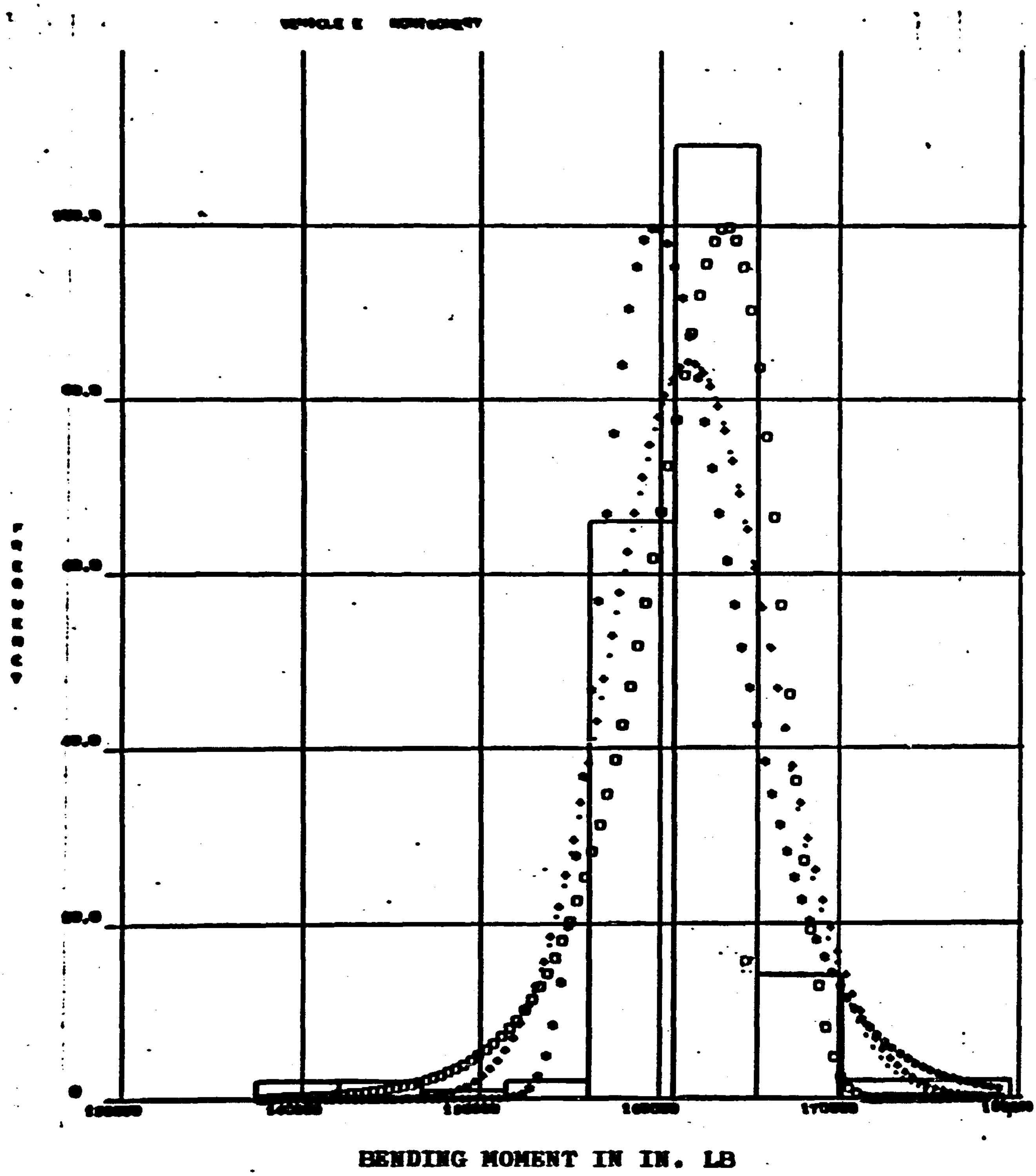
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	25.0011	13.2668
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	32.0567	15.4137
MEAN = 1,558,250	* DOUBLE EXPONENTIAL	14.9808	20.4093
SIGMA = 40,301	▫ WEIBULL	18.1596	10.4090

Figure 3-28



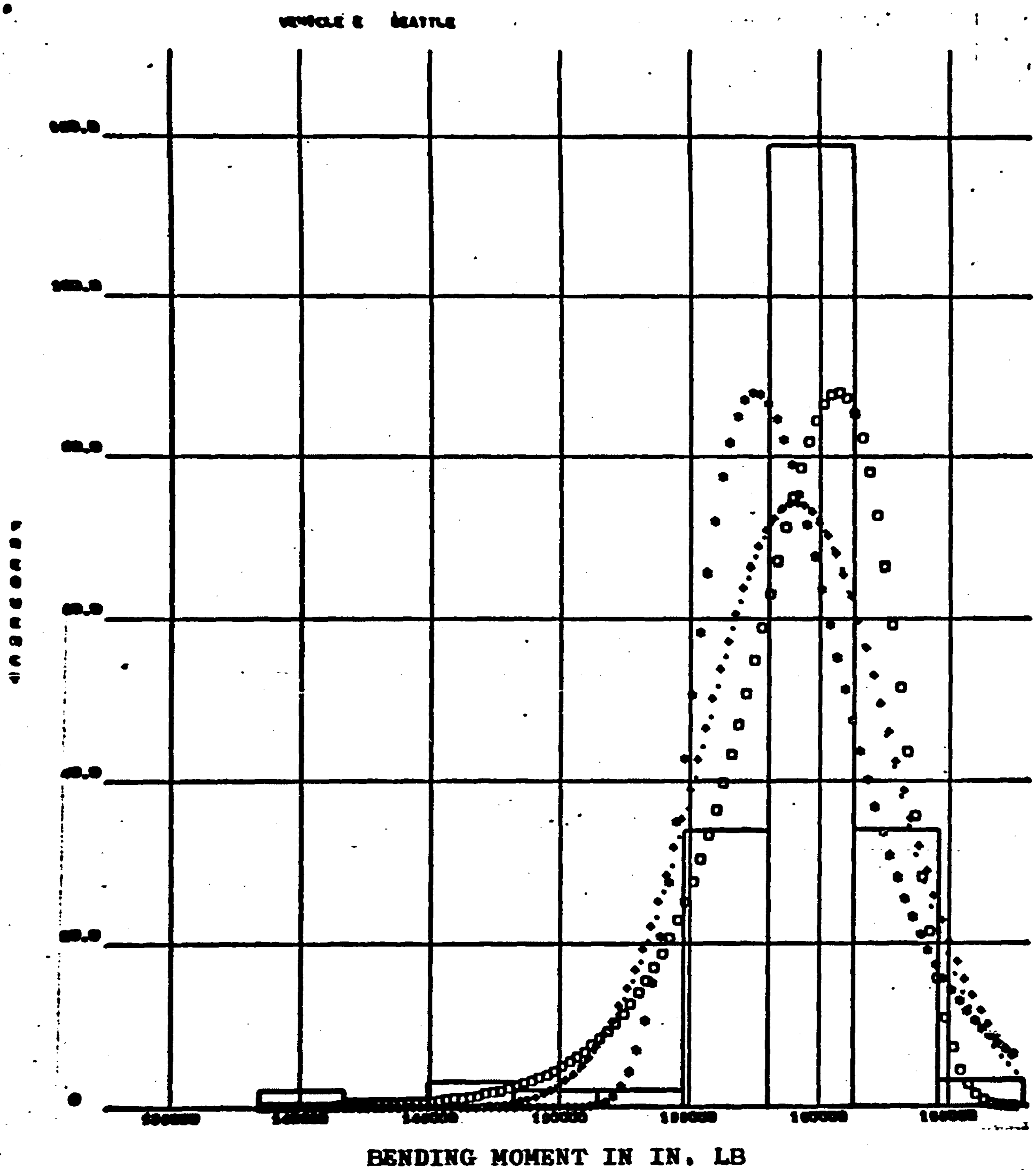
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	274.280	16,2807
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	294.408	16.4914
MEAN = 1,595,680	* DOUBLE EXPONENTIAL	111.076	22.1605
SIGMA = 28,502	◘ WEIBULL	173.704	22.1020

Figure 3-29



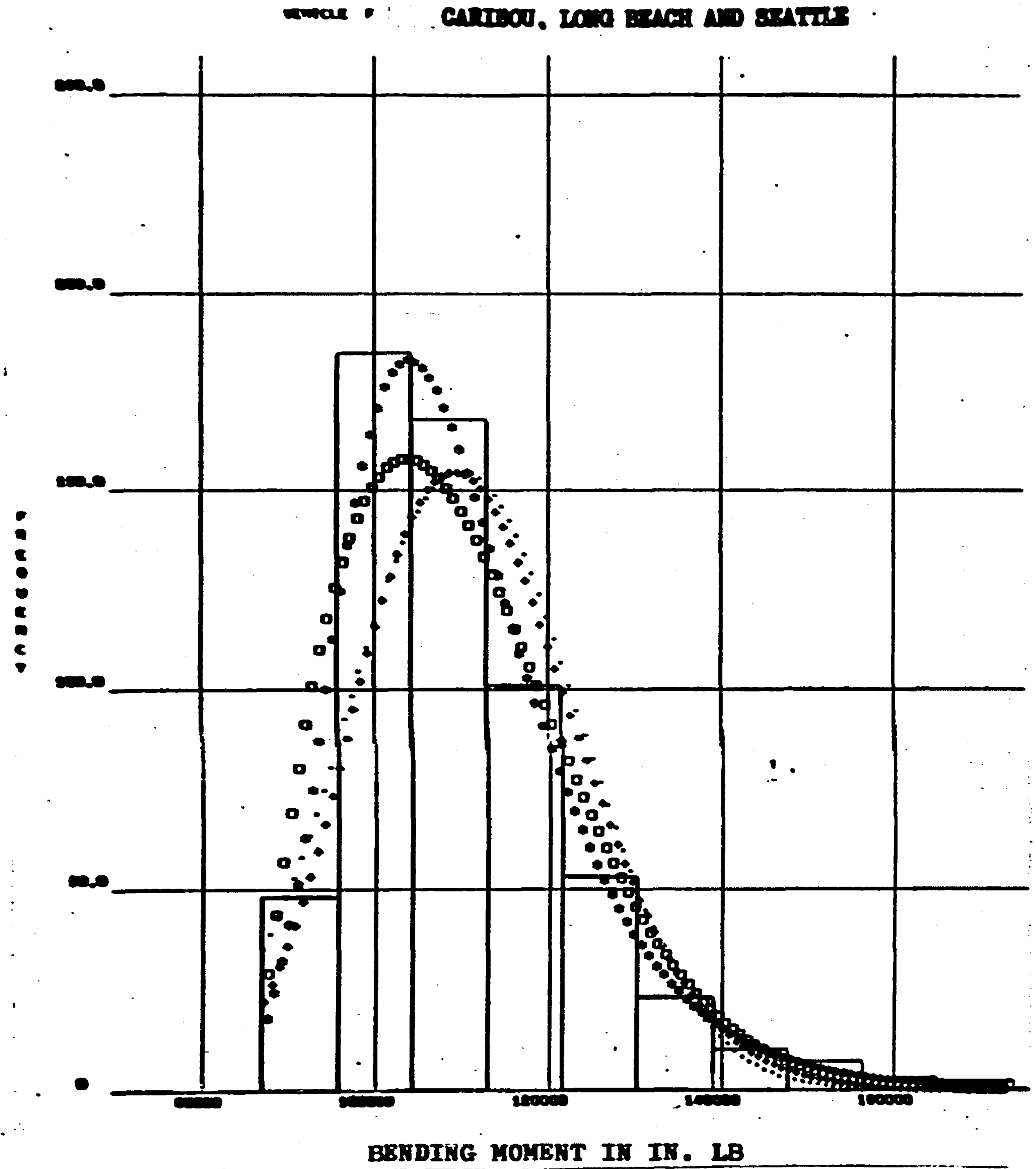
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	5.29865	20.8586
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	6.56401	21.6199
MEAN = 1,616,540	* DOUBLE EXPONENTIAL	6.79366	24.2365
SIGMA = 44,286	▫ WEIBULL	4.35652	18.5060

Figure 3-30



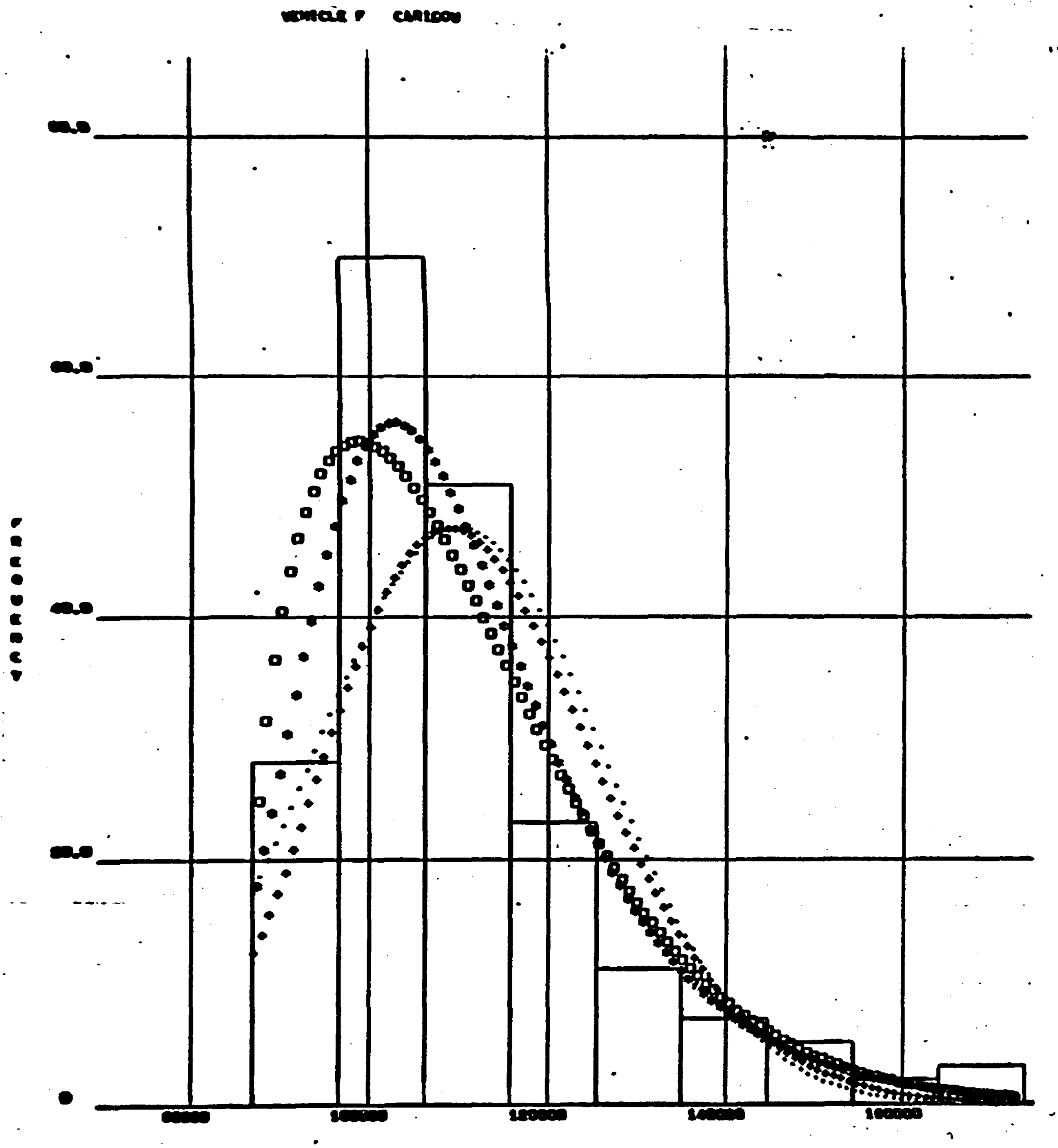
NO. OF SAMPLES = 200		PSI-SQ.	K-S
NO. OF CLASS INTERVALS = 9	• NORMAL	78.1393	32.7844
MEAN = 1,590,870	+ LOG NORMAL	89.4948	34.3200
SIGMA = 35,248	* DOUBLE EXPONENTIAL	39.0193	44.0980
	▫ WEIBULL	42.9057	19.8168

Figure 3-31



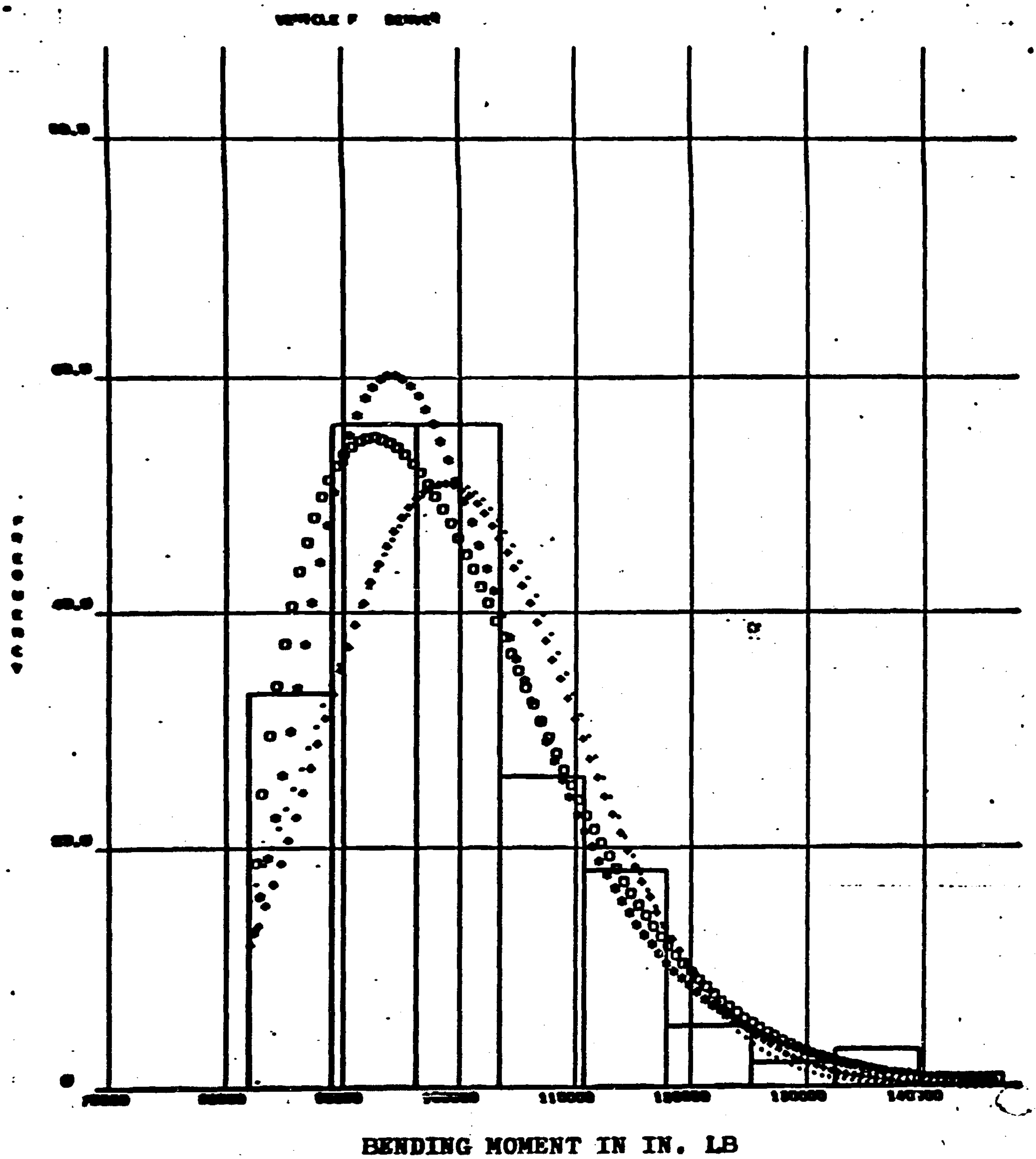
		PSI-SQ.	K-S
NO. OF SAMPLES = 600	· NORMAL	2.68156	49.6491
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	2.15663	33.8417
MEAN = 10,998,800	* DOUBLE EXPONENTIAL	.428260	15.8827
SIGMA = 1,341,330	◻ WEIBULL	1.37017	32.4478

Figure 3-32



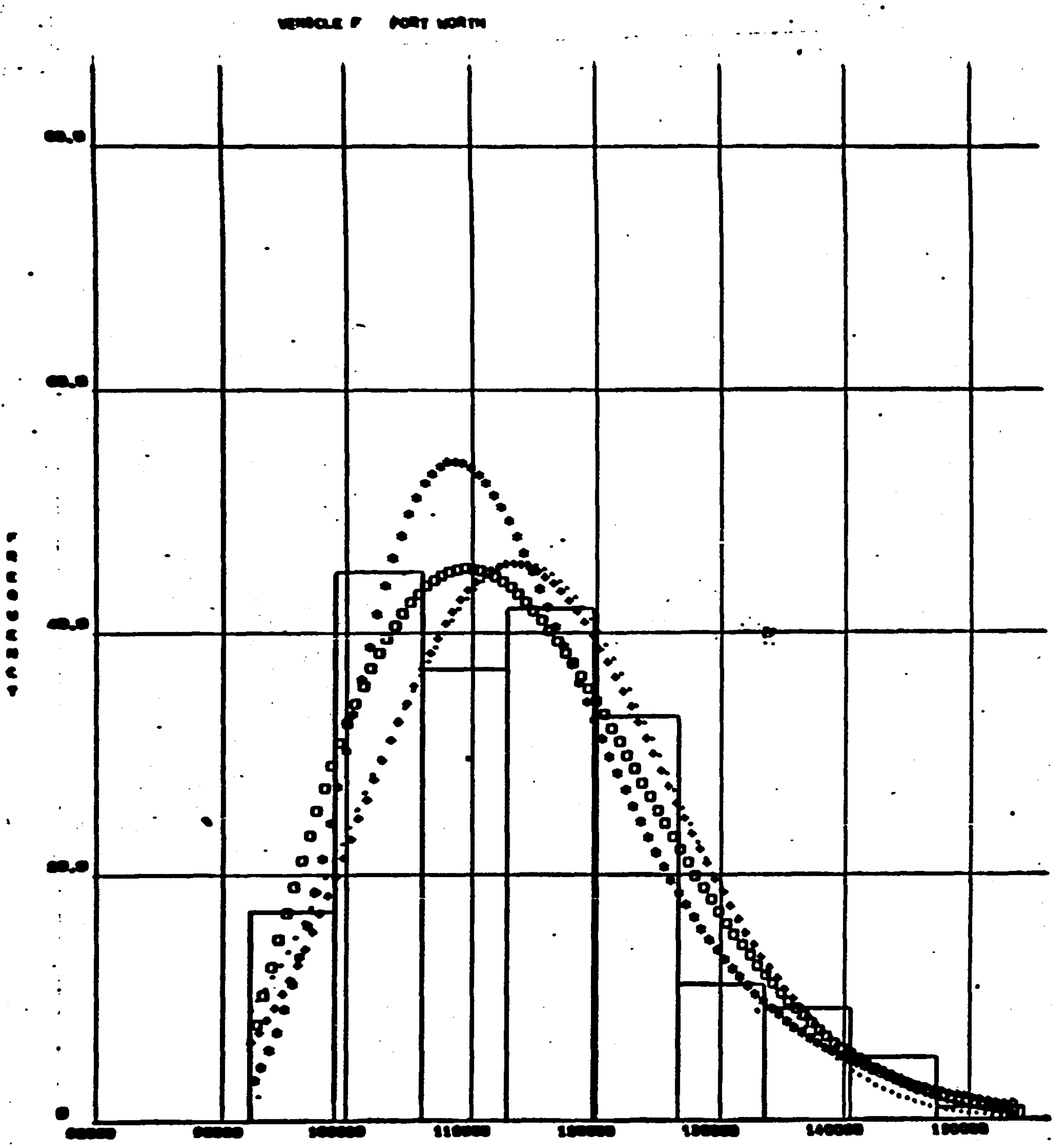
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	2.76293	20.8172
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.93651	14.1489
MEAN = 10,991,200	* DOUBLE EXPONENTIAL	.787201	11.4841
SIGMA = 1,623,190	◻ WEIBULL	.871230	15.8878

Figure 3-33



		PSI-SQ.	K-S
NO. OF SAMPLES = 200	. NORMAL	2.05409	15.5711
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.65292	10.7329
MEAN = 9,920,920	* DOUBLE EXPONENTIAL	.992708	3.51346
SIGMA = 1,131,250	▫ WEIBULL	1.61347	6.53421

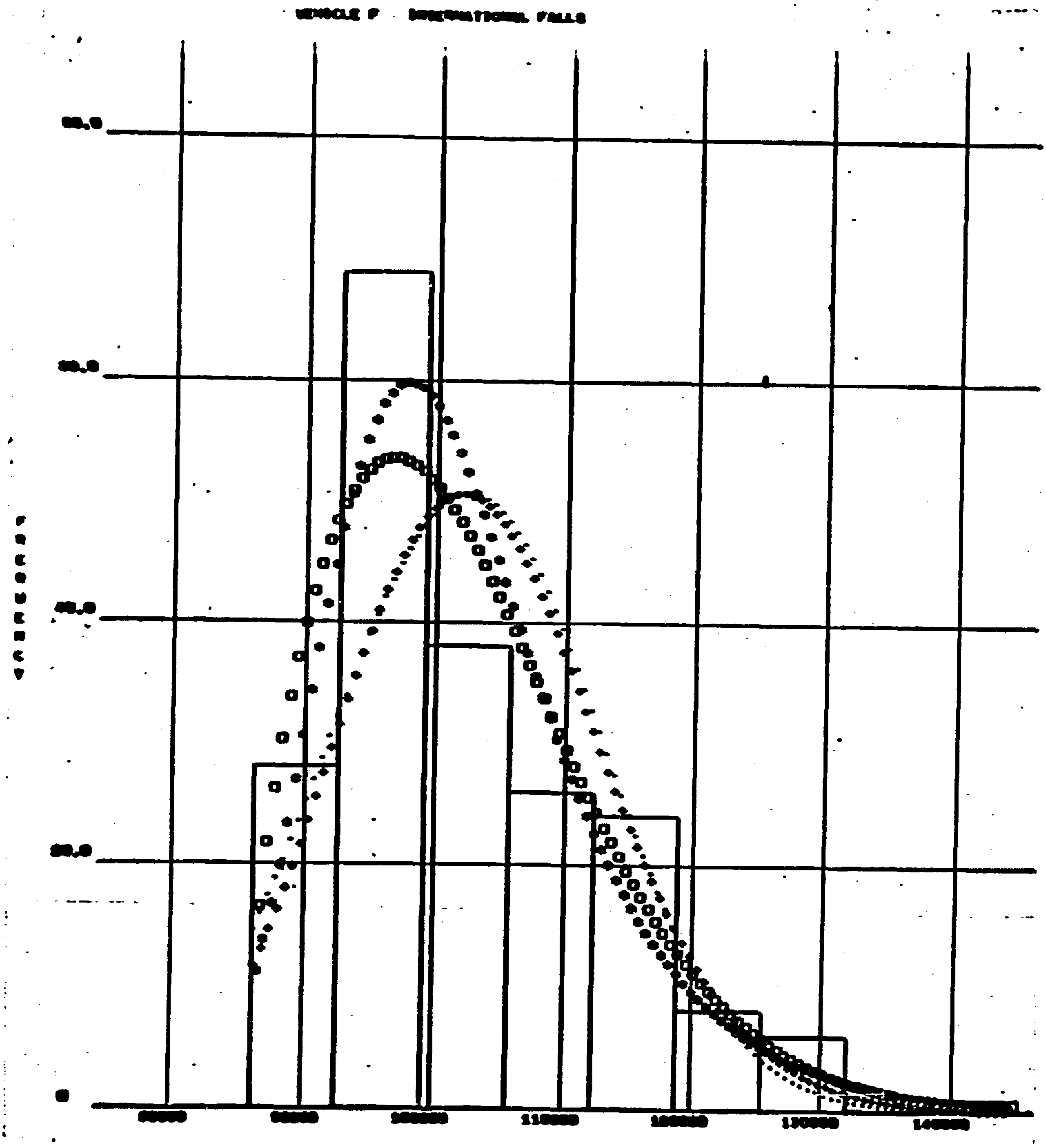
Figure 3-34



BENDING MOMENT IN IN. LB

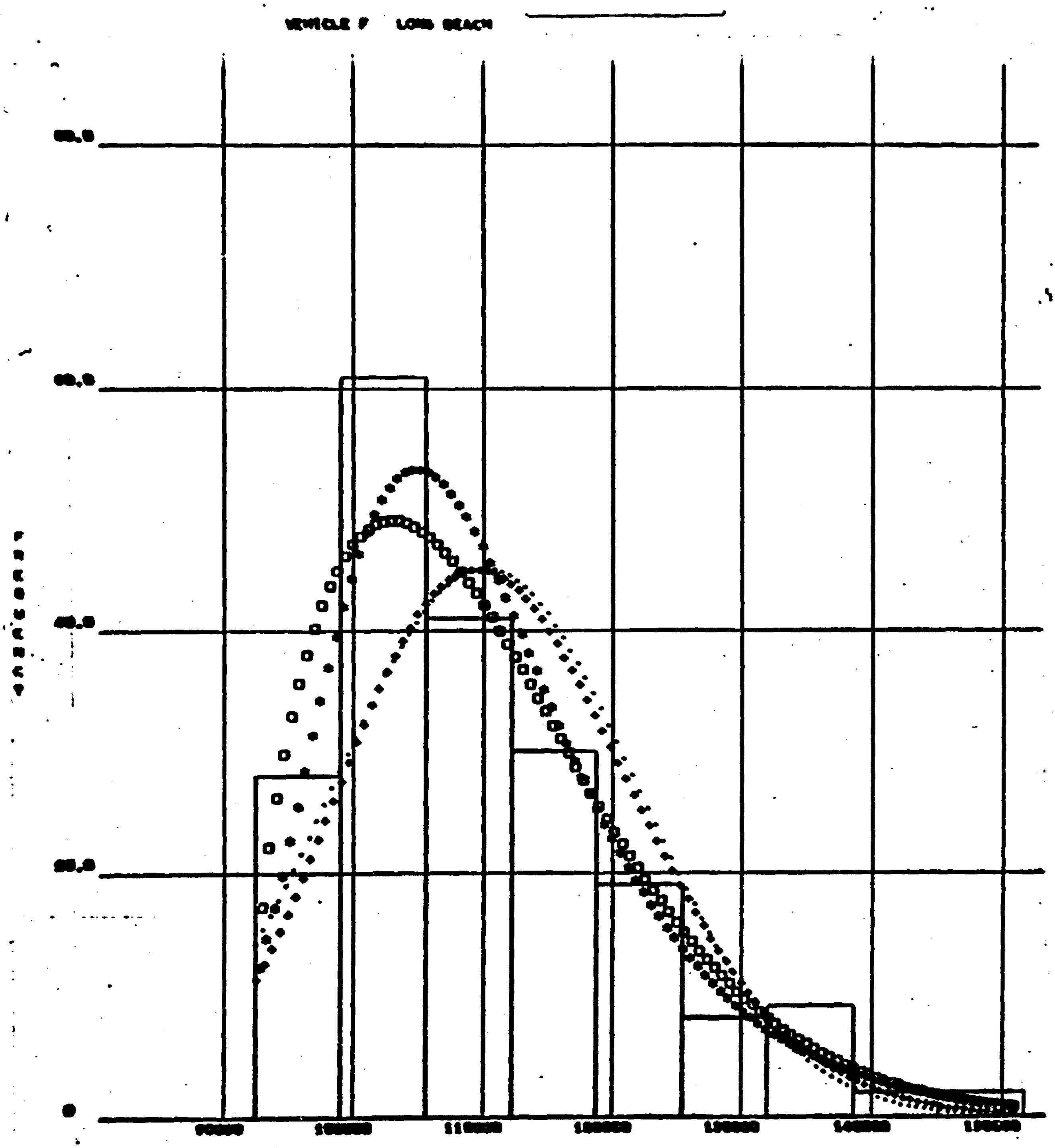
		PSI-SQ.	K-S
NO. OF SAMPLES = 200	. NORMAL	1.11609	11.1892
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.573220	10.3154
MEAN = 11,388,100	* DOUBLE EXPONENTIAL	.681514	8.14429
SIGMA = 1,206,820	◘ WEIBULL	.442139	5.13060

Figure 3-35



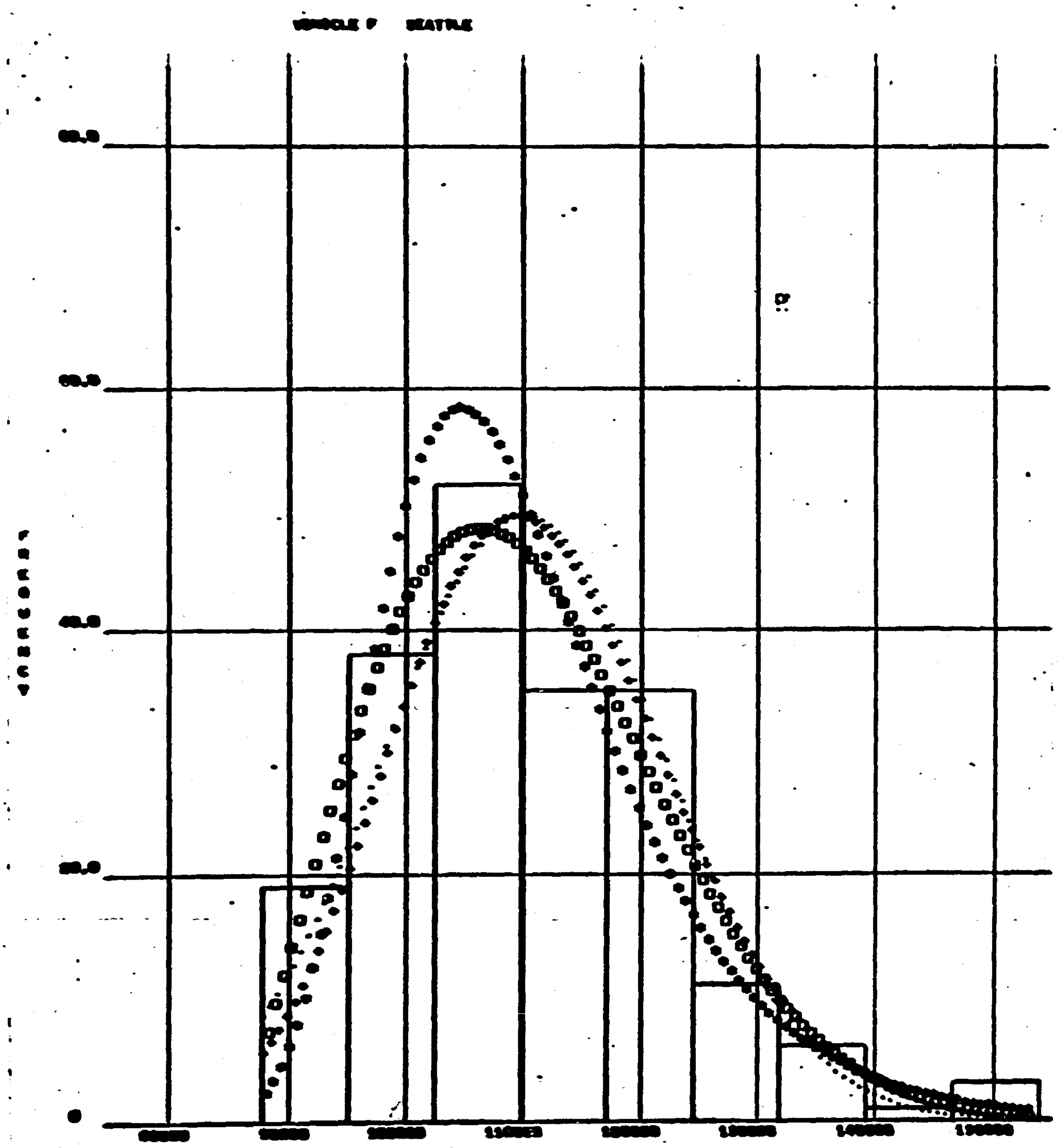
NO. OF SAMPLES = 200		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.32833	21.3512
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.11081	18.8107
MEAN = 10,235,000	* DOUBLE EXPONENTIAL	.365643	10.2846
SIGMA = 1,032,710	▫ WEIBULL	.470990	9.15972

Figure 3-36



		PSI-SQ.	K-S
NO. OF SAMPLES = 200	· NORMAL	1.72434	19.1831
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.20674	17.2016
MEAN = 11,009,600	* DOUBLE EXPONENTIAL	.469221	9.55506
SIGMA = 1,169,180	▫ WEIBULL	.612353	6.68886

Figure 3-37



BENDING MOMENT IN IN. LB

		PSI-SQ.	K-S
NO. OF SAMPLES = 200	• NORMAL	1.11582	10.4199
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	1.29140	6.20363
MEAN = 10,995,800	* DOUBLE EXPONENTIAL	2.99102	10.2632
SIGMA = 1,188,970	▫ WEIBULL	2.19584	3.65250

Figure 3-38

CHAPTER IV

STRENGTHS

Introduction

It is desirable to have as large a sample of structural strength data as possible. The results of tests on metal cylinders will yield a sample size of approximately one hundred to aid in the selection of a statistical distribution; however, Weibull (31) has shown that approximately one thousand observations may be necessary to distinguish between the log-normal and the Weibull distribution of strength properties. Thus it can be seen that on the basis of the cylinder test data alone it may not be possible to properly select the theoretical distribution.

It would be possible, of course, to test more cylinders, however the cost associated with constructing and testing a thousand metal cylinders makes this approach impractical. An alternate approach, which was chosen for this study, was to select the statistical distribution based on the mechanical properties of the structural material from which it is anticipated future boost vehicles will probably be constructed.

The assumption that the failure of a cylinder under a bending load has the same statistical distribution as the mechanical properties of the material from which the cylinder is constructed is not as far fetched as it might first appear. The structural shell of a boost vehicle is often pre-loaded in tension by applying an internal pressure. The buckling of the cylinder wall is delayed because the applied stress must first overcome the pretension due to the internal pressure before going into a state of compression. If the internal pressure is high enough, then the mode of failure changes from a buckling of the cylinder wall to a tensile failure on the opposite side of the cylinder. This type of failure is directly related to the ultimate tensile strength of the cylinder material. The ultimate tensile, tensile yield, and sometimes the compressive yield strengths are the mechanical properties data which are available in large quantities. It should be quite accurate to represent the failure of highly pressurized cylinders by the statistical distribution which best fits these mechanical properties data.

For nonpressurized and slightly pressurized cylinders a better choice would be the elastic modulus in place of the ultimate or yield strengths. Insufficient elastic modulus data at this time (less than two hundred) nullifies this approach. Therefore it is suggested, that until a more accurate approach can be found or more elastic modulus data can be obtained, that the same statistical distribution be used for both pressurized and nonpressurized cylinders. It is also suggested that this distribution be selected based on the currently available mechanical properties data.

Mechanical Properties Test Results

The three materials selected for investigation were aluminum, magnesium, and titanium.

Aluminum

Although numerous tests of mechanical properties have been conducted on aluminum alloys, the vast majority of these tests were conducted by the aluminum producers at their own expense. These producers are most cooperative in discussing the results of these tests in general terms, but they are reluctant to disclose specific test results as they consider this as proprietary information. A search of publications reveals only a very limited amount of test results of mechanical properties of the aluminum alloys. Thus the aluminum alloys were discarded as a possible source of a large number of test results of mechanical properties.

Magnesium

Efforts to obtain test results data from the magnesium producers proved more productive. As a part of the quality control program one of the magnesium producers conducted tests on the mechanical properties and was willing to supply a copy of the results. The tensile ultimate, the tensile yield, and sometimes the compression yield strengths are recorded in class intervals of one ksi. This procedure does not give the actual test result but only a one ksi range within which the result occurred. By assuming that all failures occurred at the mid point of the one ksi range, the mean and standard deviation of the entire sample can be calculated. For approximately symmetrical data (symmetrical with regard to the mean) this assumption should have little effect on the accuracy of the mean value; however, the standard deviation will be larger than actual. Fortunately the error in the standard deviation will result in conservative structural reliability calculations. Since the error in the analysis of the lumped data is considered small and in view of the large sample size, it was decided to use these data to help select the statistical distribution.

Titanium

A review of the published information on titanium alloy mechanical properties disclosed that the Department of Defense conducted a Titanium Alloy Sheet-Rolling Program which started in 1956 and ran for approximately five years. The major goal of this program was to accelerate the development of high strength, heat treated titanium sheet alloys for airframe and missile applications. As a result of this concentrated effort, a large number of tests were conducted on the mechanical properties of titanium sheet. The majority of the test results from this program were obtained from three sources: 1) The Defense Metals Information Center, 2) The Statistical Analysis Department of Battelle Memorial Institute, 3) The Technical Information Systems Division of Belfour Engineering Company.

Several months were spent transcribing the mechanical properties to IBM cards. After the first month, the literature search for mechanical properties was limited to four alloys:

1. 2.5 Al - 16 V,
2. 4 Al - 3 Mo - 1 V,
3. 6 Al - 4 V,
4. 3 Al - 13 V - 11 Cr.

These alloys were selected for two reasons: first, the results of the DOD Titanium Alloy Sheet-Rolling Program indicated these alloys showed the most promise for aircraft and missile applications, second, there were more test results of mechanical properties available for these four alloys than any other titanium alloys.

These alloys were subdivided by heat treatment, tension or compression loading, grain direction, gage, and producer. Using the Kruskal-Wallis test and a significance level of 0.05, a check was made for significant difference between producers for one thickness gage, one grain direction, one method of loading, one heat treatment, and one alloy. If the Kruskal-Wallis test shows no significant difference, the data from the titanium producers were combined; if not, then the producer with the largest sample size was used. The second step in this procedure was to compare the various gages and test for significant difference. In this case, however, if there was a significant difference the gage with mechanical properties data the farthest removed from the properties of the other gages was discarded. The gages were discarded one gage at a time until the Kruskal-Wallis test showed no significant difference in mechanical properties for the remaining gages. The third step was to compare the mechanical properties data for the two grain directions, longitudinal and transverse. If there was no significant difference, the two grain directions were combined; if not, both were kept separate. The final step in this procedure was to run the mechanical properties data on the goodness-of-fit computer program. All the titanium strength data finally used in this study were contained in references (32) through (47).

Selection of Strength Distribution

The results of the goodness-of-fit computer runs can be seen in Figures 4-1 through 4-26. A close examination of the results of magnesium alloy AZ 31B-H24 tested in the longitudinal grain direction shows these three sets of mechanical properties data (ultimate tensile strength, tensile yield strength, and compressive yield strength) are truncated at the guaranteed minimum value. A check with the magnesium producer disclosed that the following procedure was used in recording the results of the strength tests.

1. If a strength test resulted in a value below the guaranteed minimum, two additional tests were conducted on the same material. If both of these tests were above the guaranteed minimum value, the material was passed as acceptable but the test result below the guaranteed minimum was discarded.
2. If one of the additional tests was above the guaranteed minimum and one below, then two more tests were conducted.
3. If the two additional tests were both below the guaranteed minimum, the material was rejected and reprocessed.

In the third case, no error will result in this procedure because this material was not to be shipped to industry for use and therefore should not be considered as a part of the statistical distribution of mechanical properties. The first case does result in error as the material shipped can actually have some parts of it below the guaranteed minimum, and the test that records this was omitted. If in the second case, further testing results in rejection of the material, all test results above the guaranteed minimum are recorded and they should be discarded. This procedure results in a large number of test data just above the guaranteed minimum value and none just below it. Since this is not a true representation of the magnesium material which is shipped to industry, these three samples had to be discarded. This was regrettable as this was a large combined sample of 40,205 test results.

Examination of the remaining magnesium alloys and the titanium alloys disclosed nothing unusual, and these test results were accepted as typical.

In order to establish which of the four theoretical distributions gave the best fit to the observed strength test results, the same procedure used in establishing the loads distribution was used; namely, the distribution which gave the best fit was assigned the number 1.0, and the distribution which gave the second best fit the number 2.0, etc. These assigned numbers were averaged to give an indication of the average rank of the various distributions. For the magnesium strength data and the Kolmogorov-Smirnov test, the averages of the ranks were:

Normal	1.40,
Log-Normal	1.95,
Weibull	2.65,
Double Exponential	4.00.

For the magnesium strength data and the Psi-Square test, the averages of the ranks were:

Log-Normal	2.00,
Normal	2.20,
Double Exponential	2.65,
Weibull	3.15.

The strength data, unlike the loads data, were from different size samples and require some weighting. All the magnesium samples sizes were within 75 of 700, ranging from 638 to 775, except two samples 990 and 987. The samples close to 700 were not weighted, but the two larger samples were weighted by 1.5 since they were approximately 1.5 times the size of the other samples.

For the titanium strength data, a similar weighting was required. The titanium sample sizes also were grouped around 700, ranging from 619 to 776 with four exceptions: 974, 984, 1866, and 1874. The samples near 700 were not weighted, but the 974 and 984 samples were weighted by 1.5 and the 1866 and 1874 samples were weighted by 3.0. For the titanium strength data and the Kolmogorov-Smirnov test, the averages of the ranks were:

Normal	1.60,
Weibull	2.29,
Log-Normal	2.32,
Double Exponential	3.79.

For the titanium strength data and the Psi-Square test, the averages of the ranks were:

Normal	1.76,
Weibull	2.03,
Log-Normal	2.50,
Double Exponential	3.71.

Combining the magnesium and titanium results and weighting the titanium by 2.0 since the titanium sample size was approximately twice that of the magnesium (12,699 titanium to 6,881 magnesium), the Kolmogorov-Smirnov test gave the following averages of the ranks:

Normal	1.53,
Log-Normal	2.20,
Weibull	2.41,
Double Exponential	3.86.

For the Psi-Square test the following averages of the ranks were obtained:

Normal	1.91,
Log-Normal	2.33,
Weibull	2.40,
Double Exponential	3.36.

These results showed the same order of ranking by both the Kolmogorov-Smirnov and Psi-Square tests. The normal distribution gave the best fit to the observed strength data, with log-normal a second choice, Weibull a close third, and double exponential last.

A check for significant difference at the 0.05 level using the Kolmogorov-Smirnov test showed that although the Weibull distribution was the third choice of the distributions only twice out of the 23 sets of strength data was a significant difference indicated. The normal distribution showed a significant difference three times, the log-normal six times, and the double exponential distribution showed a significant difference 14 out of the 23 sets.

Thus the normal distribution was selected as the one which best representsd the observed strength test results.

Metal Cylinder Test Results

Non-Pressurized

A review of the published reports on bending tests conducted on metal cylinders disclosed that data are available for three materials: aluminum, steel, and brass. It is doubtful that brass will ever be used as a structural material for a missile. The number of test results on brass is small. For these two reasons, brass was discarded.

Of the remaining 125 aluminum and steel test cylinders, 25 were discarded because the L^2/rt ratio is less than 100,

where L = length of cylinder,

r = radius of cylinder,

t = skin thickness of cylinder.

This was done so that the cylinders analyzed will have structural parameters typical of present day missiles.

Following the procedure of Peterson (48) the aluminum and steel cylinder test results were plotted using as ordinate and abscissa the parameters obtained by small deflection theory (49). The cylinder-theory parameters used are:

$$\text{abscissa} = Z = (L^2/rt) \quad (1 - \mu^2)^{\frac{1}{2}},$$

and

$$\text{ordinate} = K = (\sigma t L^2)/(D \pi^2),$$

where μ = Poisson's ratio,

σ = critical compressive stress,

D = flexural stiffness = $Et^3/12\ (1 - \mu^2)$,

E = Young's modulus.

Plotted on log-log graph paper, theoretically the data should form a straight line with a 45 degree slope. The "least squares" straight line established by the IBM computer has a slope of 0.966, which indicates very good agreement with the theoretical slope of 1.000, note Figure 4-27.

Peterson (48) showed that in addition to the two cylinder-theory parameters, the buckling is also a function of the r/t ratio. This fact is also supported by Batdorf (49), Harris (50) and others. Since the buckling coefficient K is a function of both Z and r/t, in order to evaluate the effect of r/t it was necessary to cross-plot K as a function of r/t for a constant value of the geometrical parameter Z. This was accomplished by constructing a line parallel to the established straight line and passing through the test point. A value of Z equal to 1,000 was arbitrarily chosen, and a cross-plot of the projected values of K vs. r/t is shown in Figure 4-28. The idea of plotting K on a logarithmic scale and r/t on a linear scale was proposed by Harris (50) and Suer (51). Using the Kruskal-Wallis test and the curve fitting technique described in Chapter II, it was established that for both Figure 4-27 and 4-28 there was no significant improvement by using a third degree equation over the second degree equation. Checking the aluminum test results with the steel test results and using the Kruskal-Wallis test disclosed a significant difference at the 0.05 level between the aluminum and steel. Therefore, the steel test results were omitted. The aluminum data used were obtained from references (48), (52), (53), and (54).

Pressurized

The pressurized cylinders are analyzed following the procedure of Lo (55), Harris (50), and Suer (51). The test data are plotted in terms of two nondimensional parameters;

$$\bar{p} = (p/E)\ (r/t)^2, \text{ and}$$

$$\bar{\sigma} = (\sigma/E)\ (r/t),$$

where p = the internal pressure.

One difference discovered in the results of Lo (55), Harris (50), and Suer (51) and the present report, is that the previous investigators found that the scatter in test results was decreased when $\Delta\bar{\sigma}$ was used in place of $\bar{\sigma}$,

where $\Delta\bar{\sigma} = \bar{\sigma} - \bar{\sigma}_{p=o}$.

However, for this investigation less scatter was found by using $\bar{\sigma}$ as shown in Figures 4-29 and 4-30. Unlike the unpressurized data, the pressurized data are not affected by the r/t ratio. As in the unpressurized cylinder tests, a significant difference was found between the aluminum and steel test results and, therefore, the steel data were omitted. The test data on pressurized aluminum cylinders used in this report were obtained from reference (53).

Allowable Bending Moment of Cylinders

In the previous sections, the strength of circular cylinders has been investigated in terms of buckling or crippling stresses. The applied loads are given in terms of an equivalent bending moment. In order to compare the loads and strengths, they must be given in the same units. In this case, it was decided to convert the strength of the cylinders from buckling stresses to allowable bending moments. Using the classic bending equation, one obtains the bending moment in terms of the extreme compression fiber stress;

$$\sigma = M\, r/I,$$

where σ = critical buckling stress,

M = bending moment,

r = radius of the cylinder,

I = moment of inertia.

For circular sections with the wall thickness very small compared to the radius, the moment of inertia is given by:

$$I = \pi r^3 t.$$

Substituting into the classic bending equation one obtains:

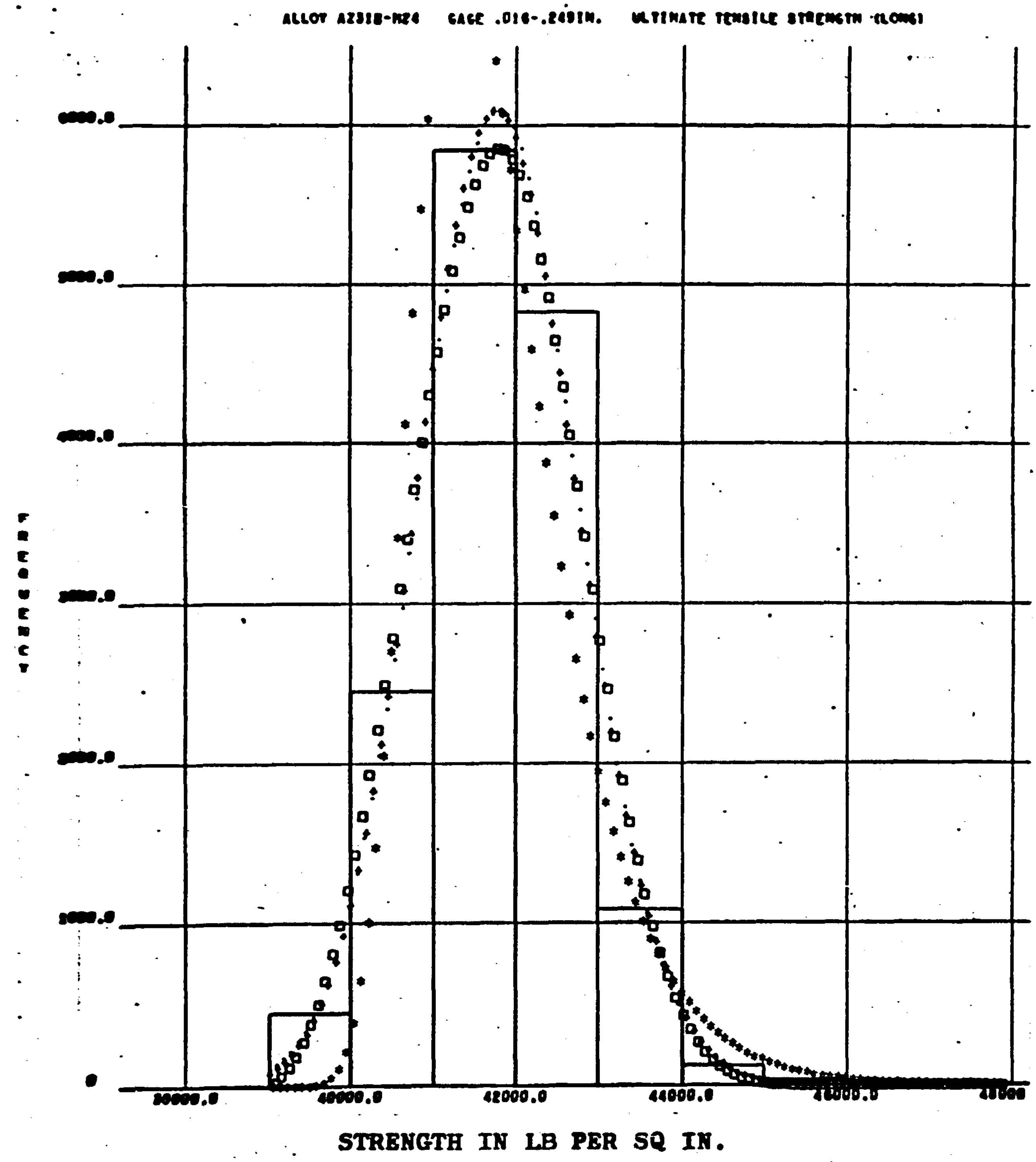
$$\sigma = M/\,(\pi r^2\, t).$$

The critical buckling stress is related to the buckling coefficient K, by the equation given in references (48) and (49):

$$\sigma = \frac{K \pi^2 \; E}{12\;(1 - \mu^2)} (t/L)^2.$$

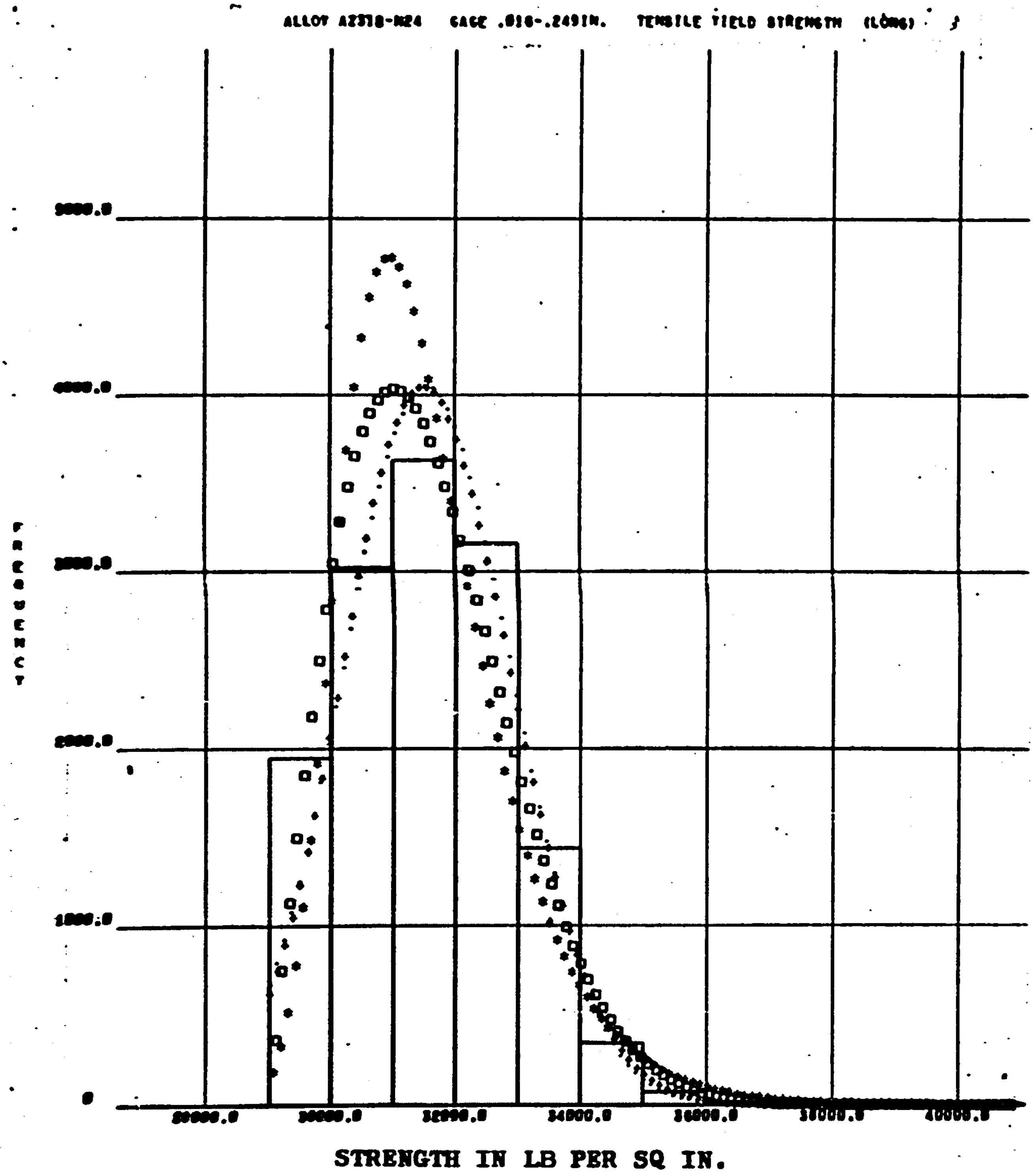
Combining with the above equation and solving for the bending moment, one obtains:

$$M = \frac{\pi^3 K E t^3 r^2}{12 (1 - \mu^2) L^2} .$$



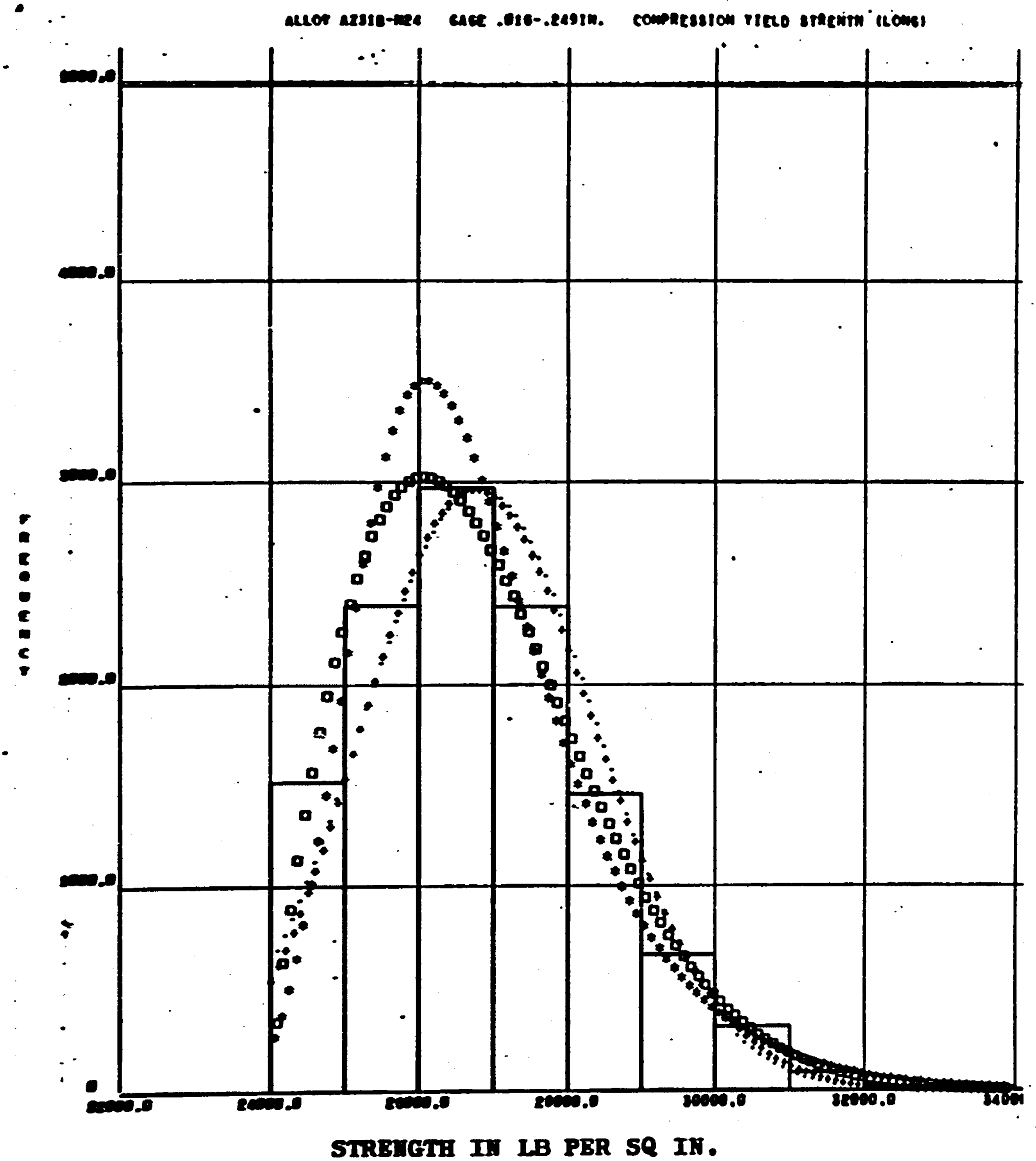
		PSI-SQ.	K-S
NO. OF SAMPLES = 14,807	· NORMAL	.086295	305.668
NO. OF CLASS INTERVALS = 9	+ LOG NORMAL	.078913	293.149
MEAN = 41,780.4	* DOUBLE EXPONENTIAL	.982877	994.137
SIGMA = 968.249	□ WEIBULL	.144042	363.745

Figure 4-1



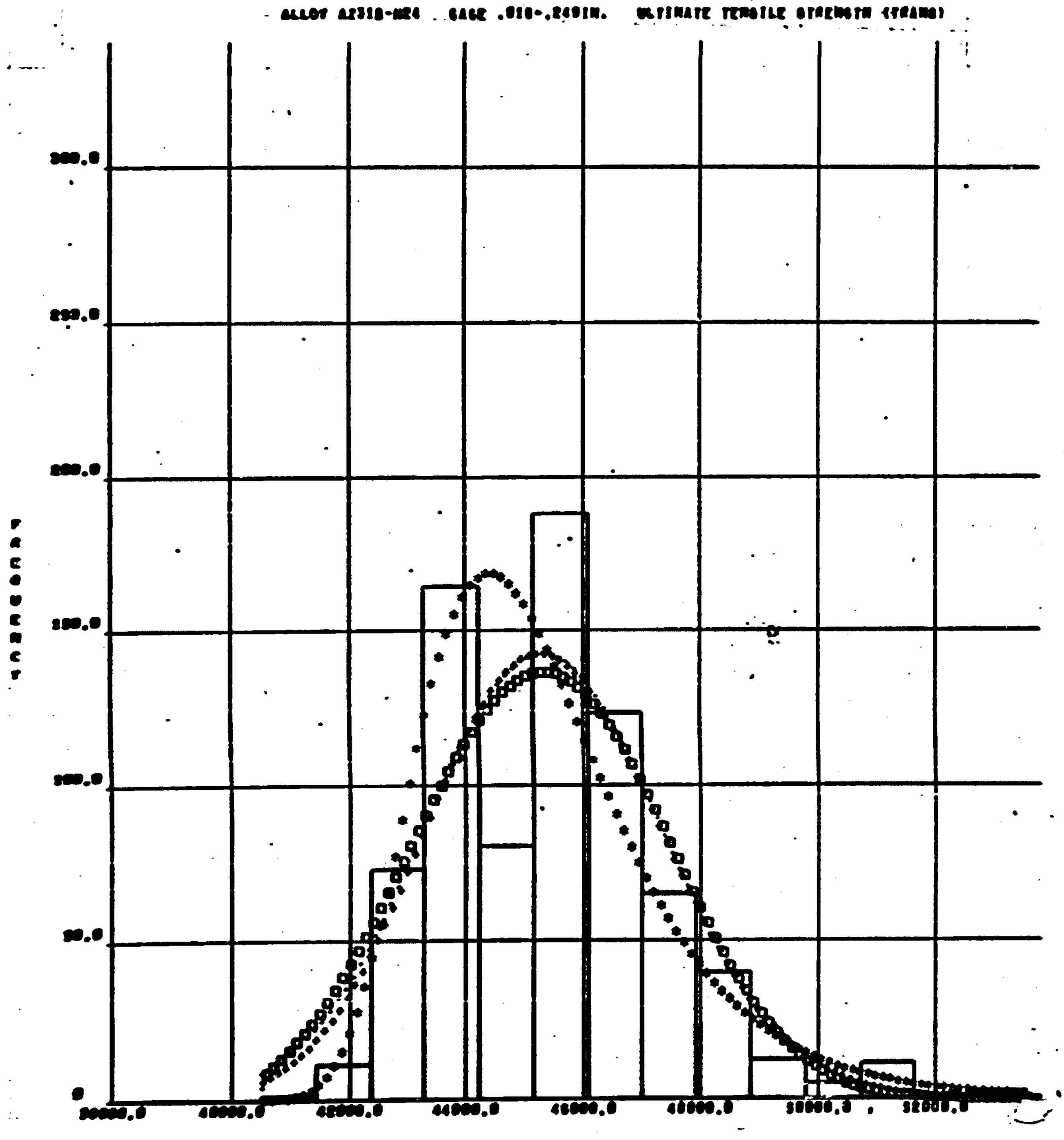
		PSI-SQ.	K-S
NO. OF SAMPLES = 13,587	· NORMAL	.115401	282.510
NO. OF CLASS INTERVALS = 12	+ LOG NORMAL	.110550	254.424
MEAN = 31,536.9	* DOUBLE EXPONENTIAL	.460121	895.887
SIGMA = 1,338.64	□ WEIBULL	.120176	462.358

Figure 4-2



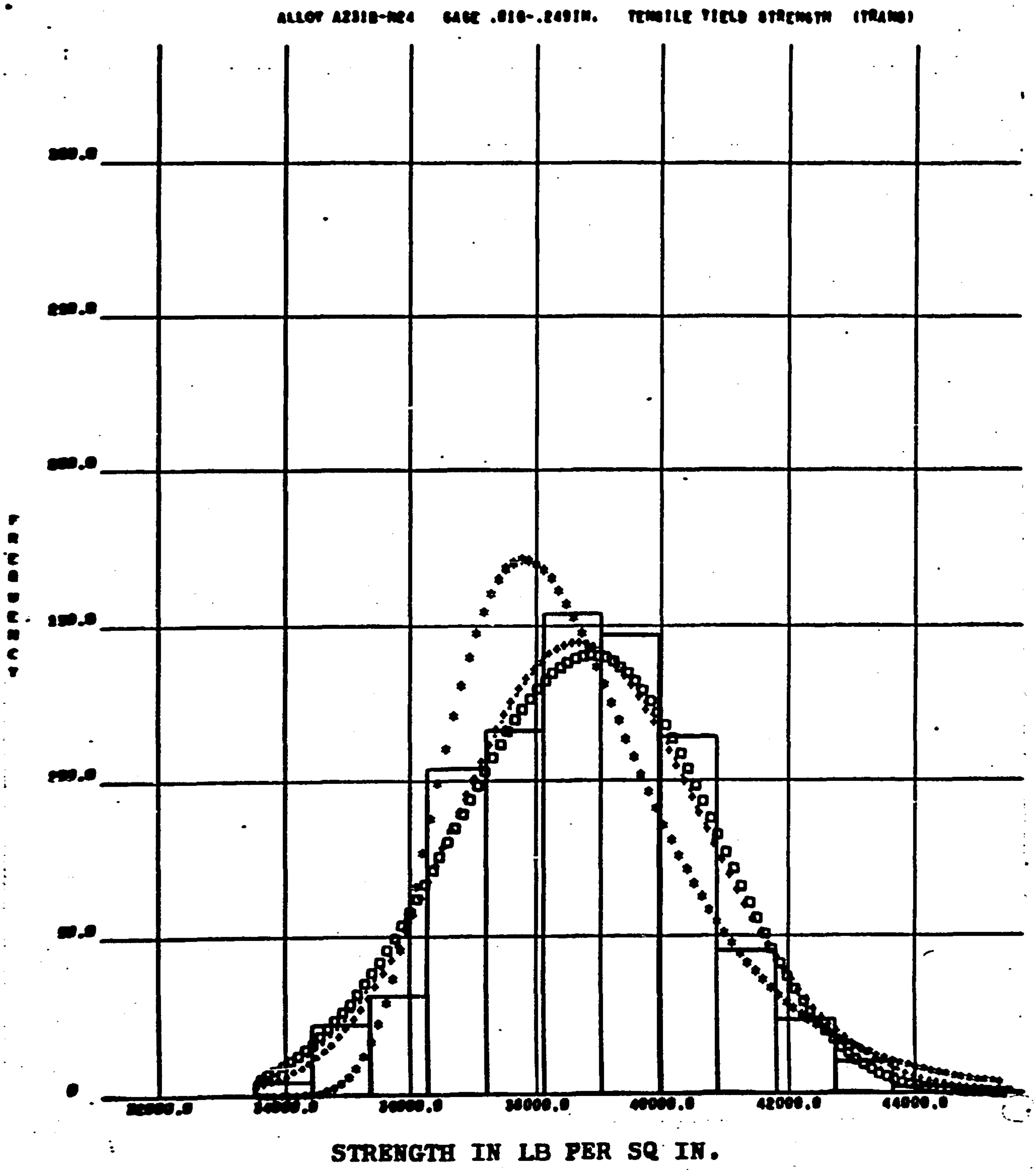
		PSI-SQ.	K-S
NO. OF SAMPLES = 11,811	• NORMAL	.154536	428.764
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.106596	260.541
MEAN= 26,814.2	* DOUBLE EXPONENTIAL	.220497	477.381
SIGMA = 1,588.92	◘ WEIBULL	.045438	209.291

Figure 4-3



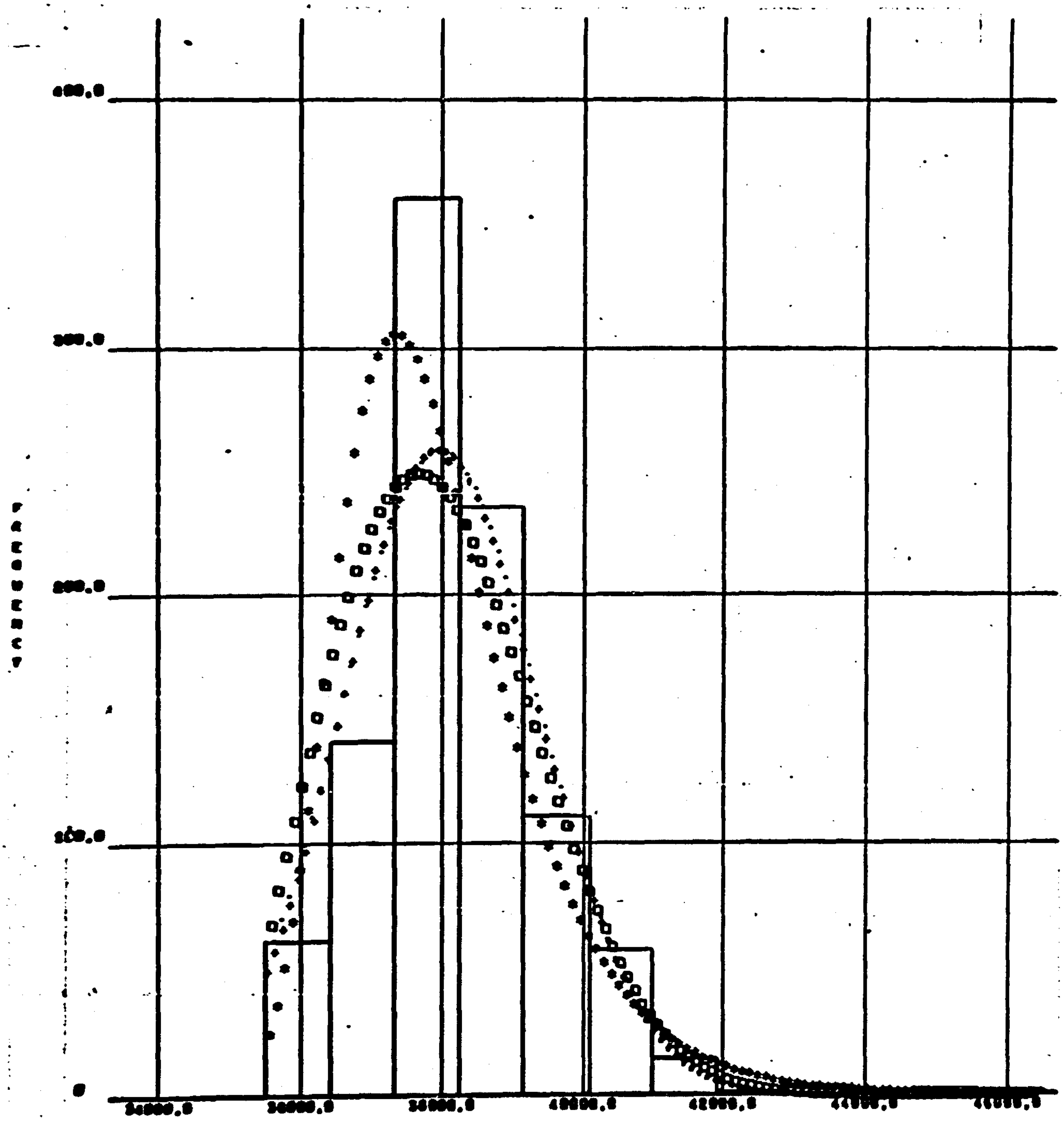
		PSI-SQ.	K-S
NO. OF SAMPLES = 775	· NORMAL	185.891	43.5615
NO. OF CLASS INTERVALS = 14	+ LOG NORMAL	141.177	39.6407
MEAN = 45,309	* DOUBLE EXPONENTIAL	1.62035	86.2321
SIGMA = 2,017.28	▫ WEIBULL	216.526	45.6433

Figure 4-4



		PSI-SQ.	K-S
NO. OF SAMPLES = 773	• NORMAL	1.60657	27.3743
NO. OF CLASS INTERVALS = 13	+ LOG NORMAL	1.03575	27.8806
MEAN = 38,688.9	* DOUBLE EXPONENTIAL	1.98456	64.7223
SIGMA = 1,965.51	◘ WEIBULL	2.62111	33.1365

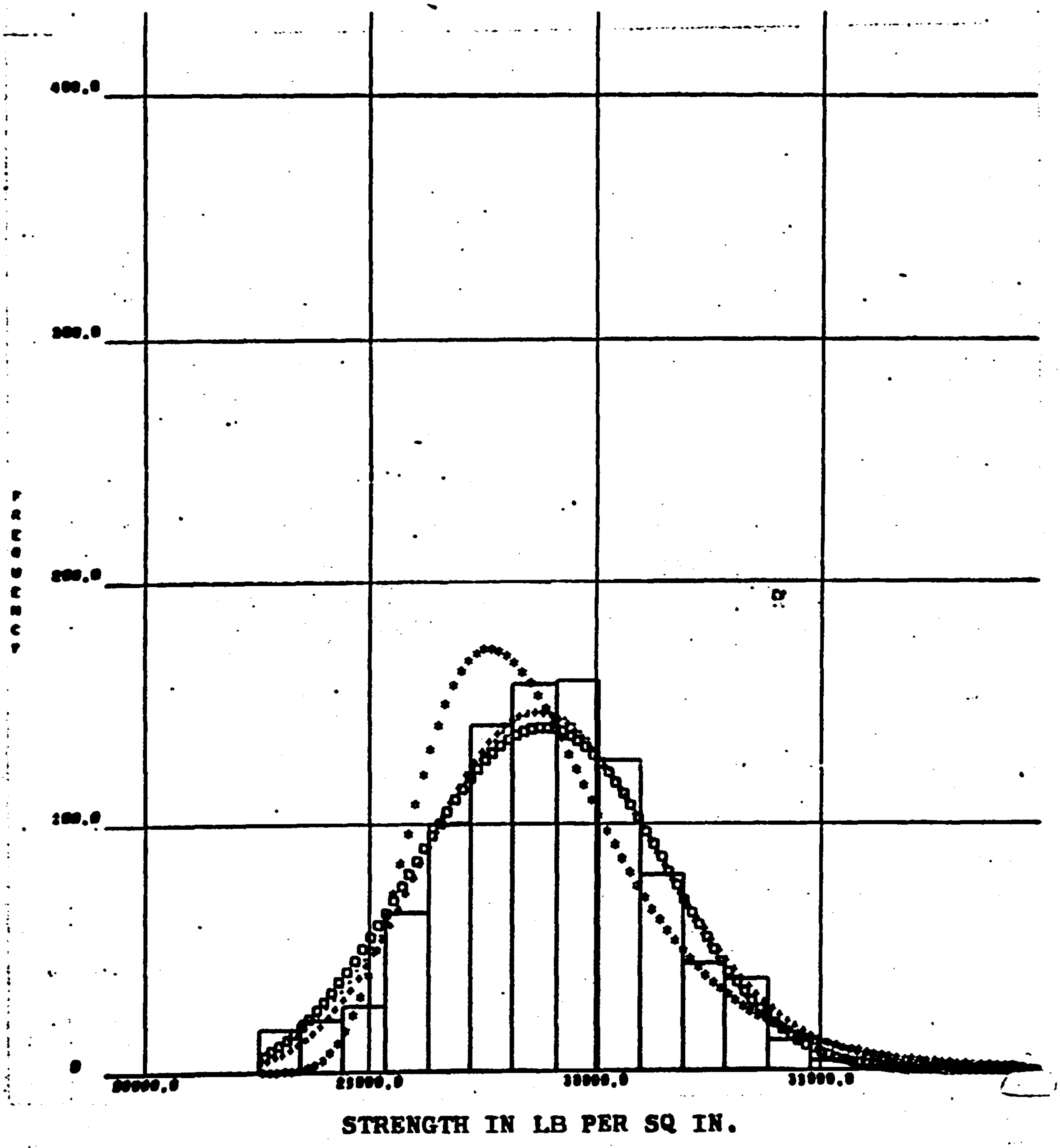
Figure 4-5



STRENGTH IN LB PER SQ IN.

		PSI-SQ.	K-S
NO. OF SAMPLES = 987	• NORMAL	.505176	116.583
NO. OF CLASS INTERVALS = 12	+ LOG NORMAL	.536727	119.465
MEAN = 37,975.2	* DOUBLE EXPONENTIAL	1.12414	156.493
SIGMA = 1,395.49	□ WEIBULL	1.18041	144.552

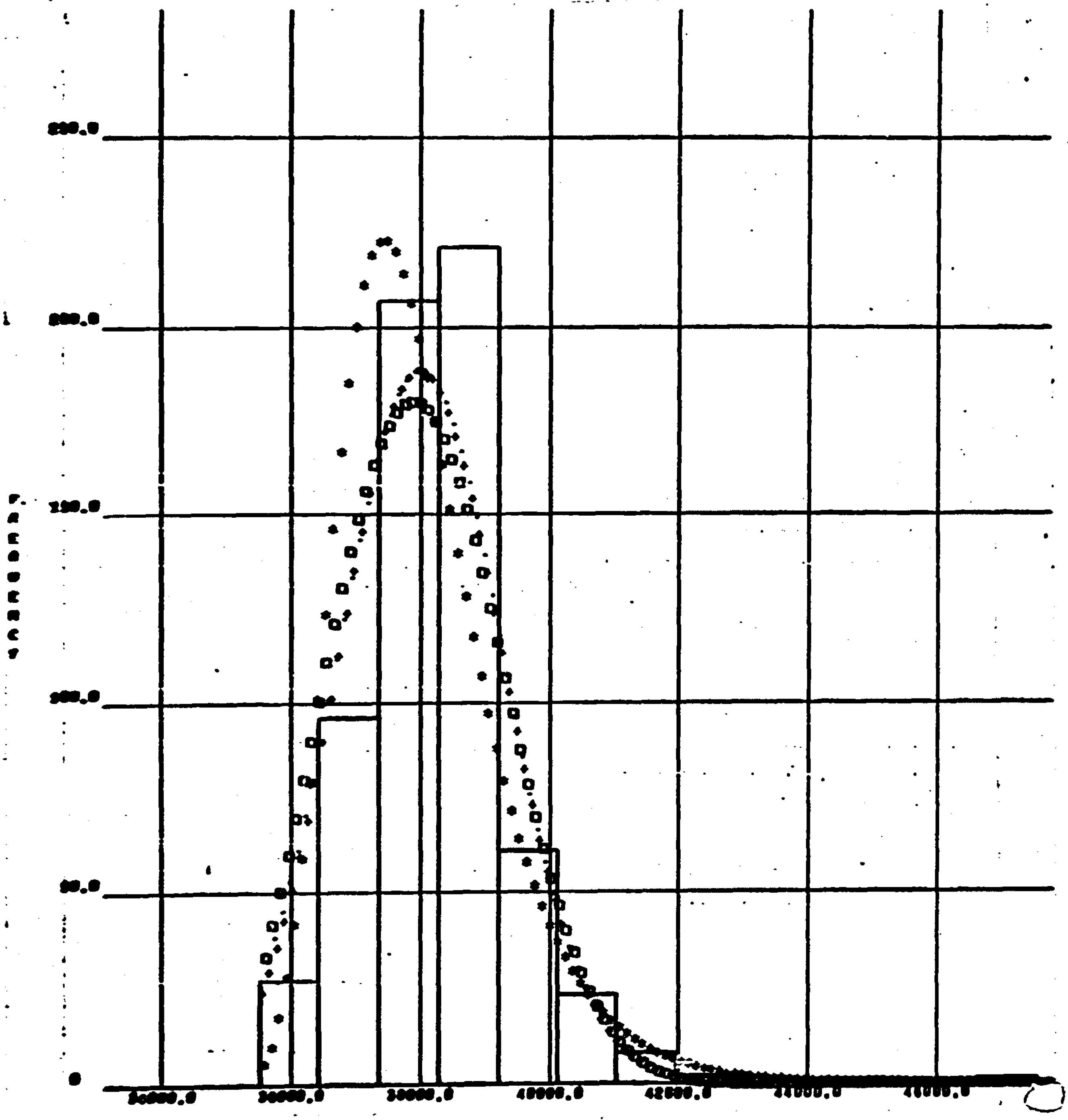
Figure 4-6



		PSI-SQ.	K-S
NO. OF SAMPLES = 990	· NORMAL	.704252	38.9547
NO. OF CLASS INTERVALS = 18	+ LOG NORMAL	1.00866	49.7439
MEAN = 28,791.9	* DOUBLE EXPONENTIAL	2.13091	90.1739
SIGMA = 2,557.06	□ WEIBULL	1.13040	47.9805

Figure 4-7

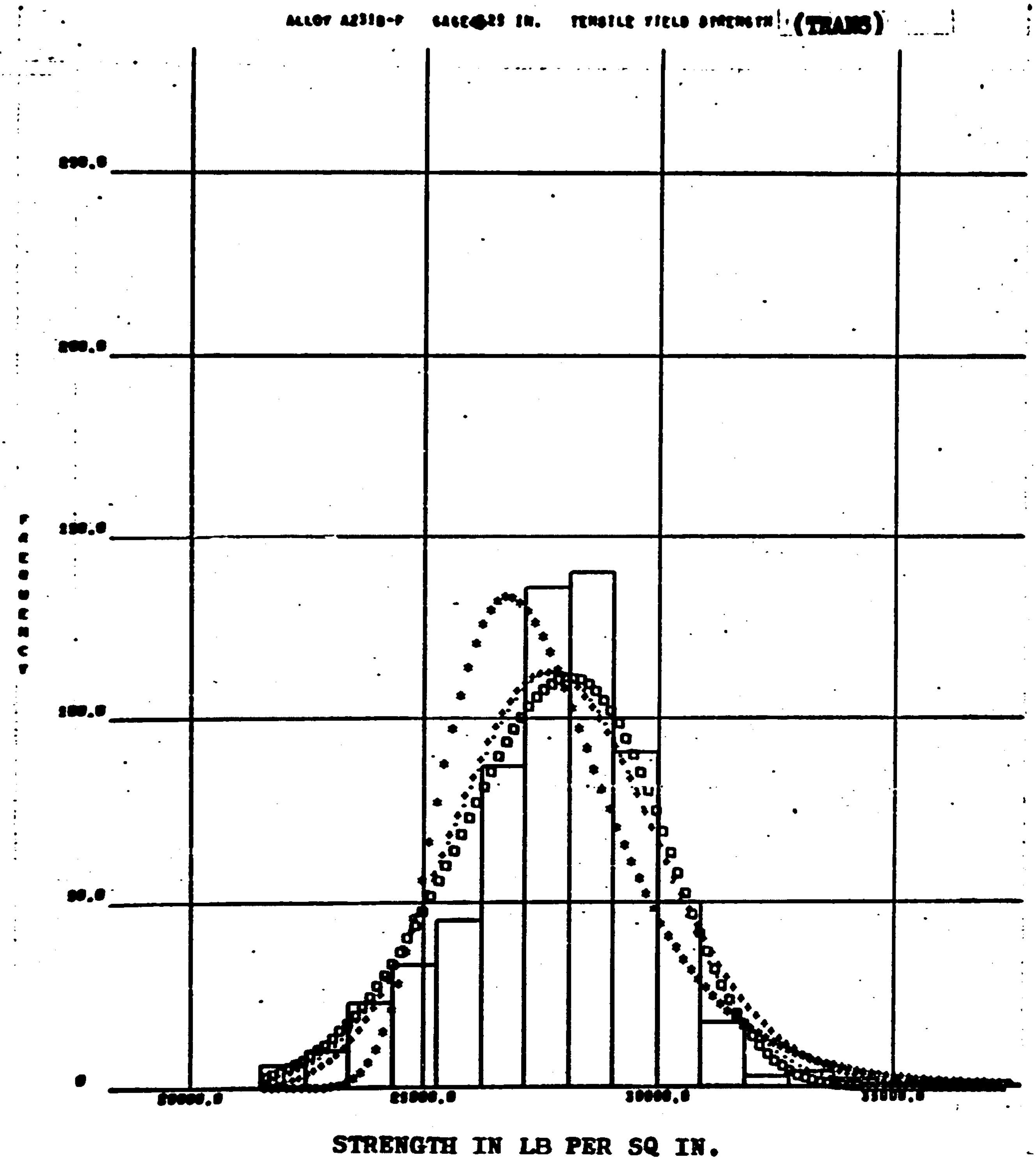
ALLOY [illegible] GAGE<.25 IN. ULTIMATE TENSILE STRENGTH (TRANS)



STRENGTH IN LB PER SQ IN.

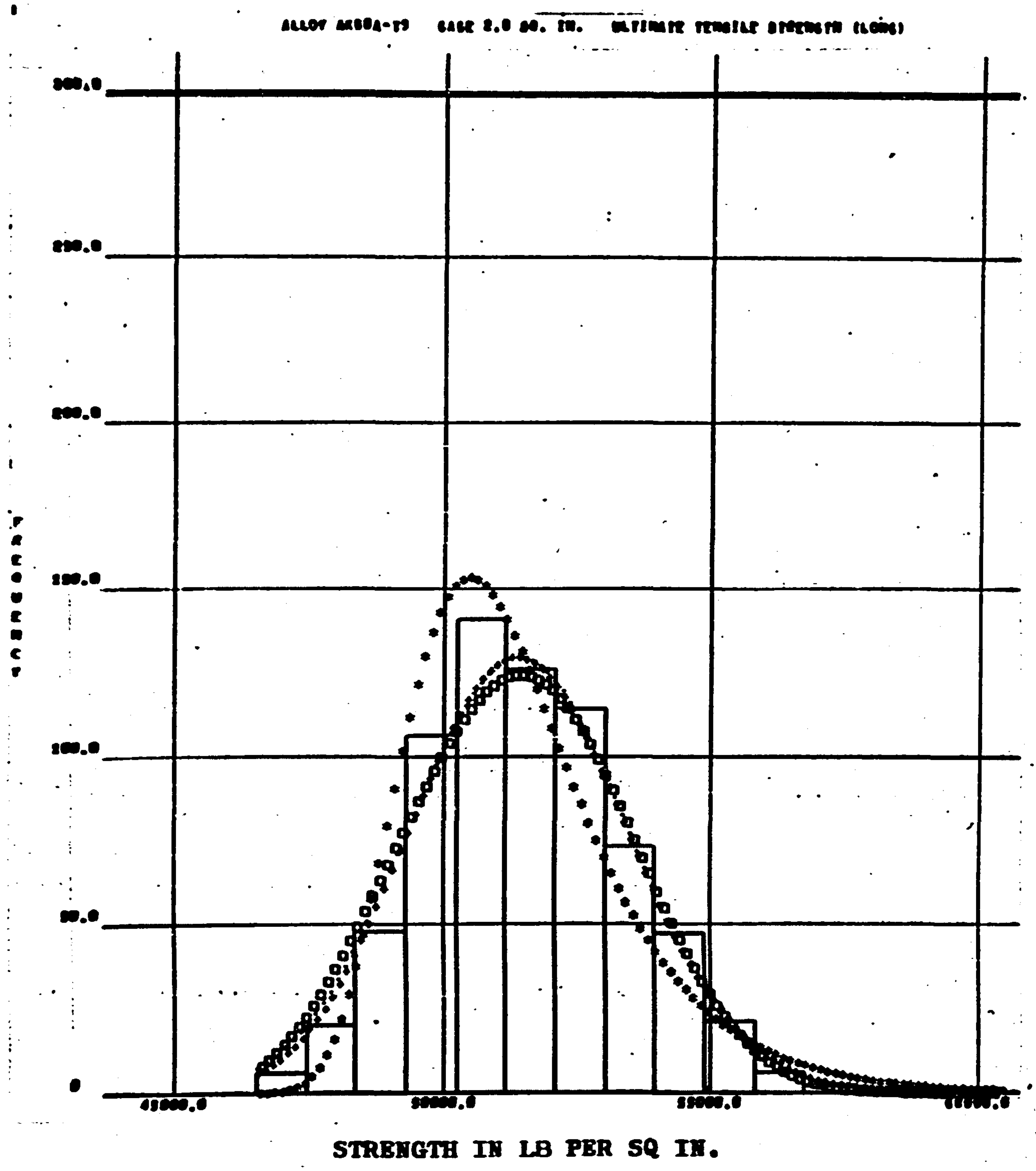
			PSI-SQ.	K-S
NO. OF SAMPLES = 647	.	NORMAL	1.27338	73.6498
NO. OF CLASS INTERVALS = 13	+	LOG NORMAL	1.24068	74.7680
MEAN = 37,994.6	*	DOUBLE EXPONENTIAL	1.15965	95.7935
SIGMA = 1,263.77	◻	WEIBULL	2.08048	84.5667

Figure 4-8



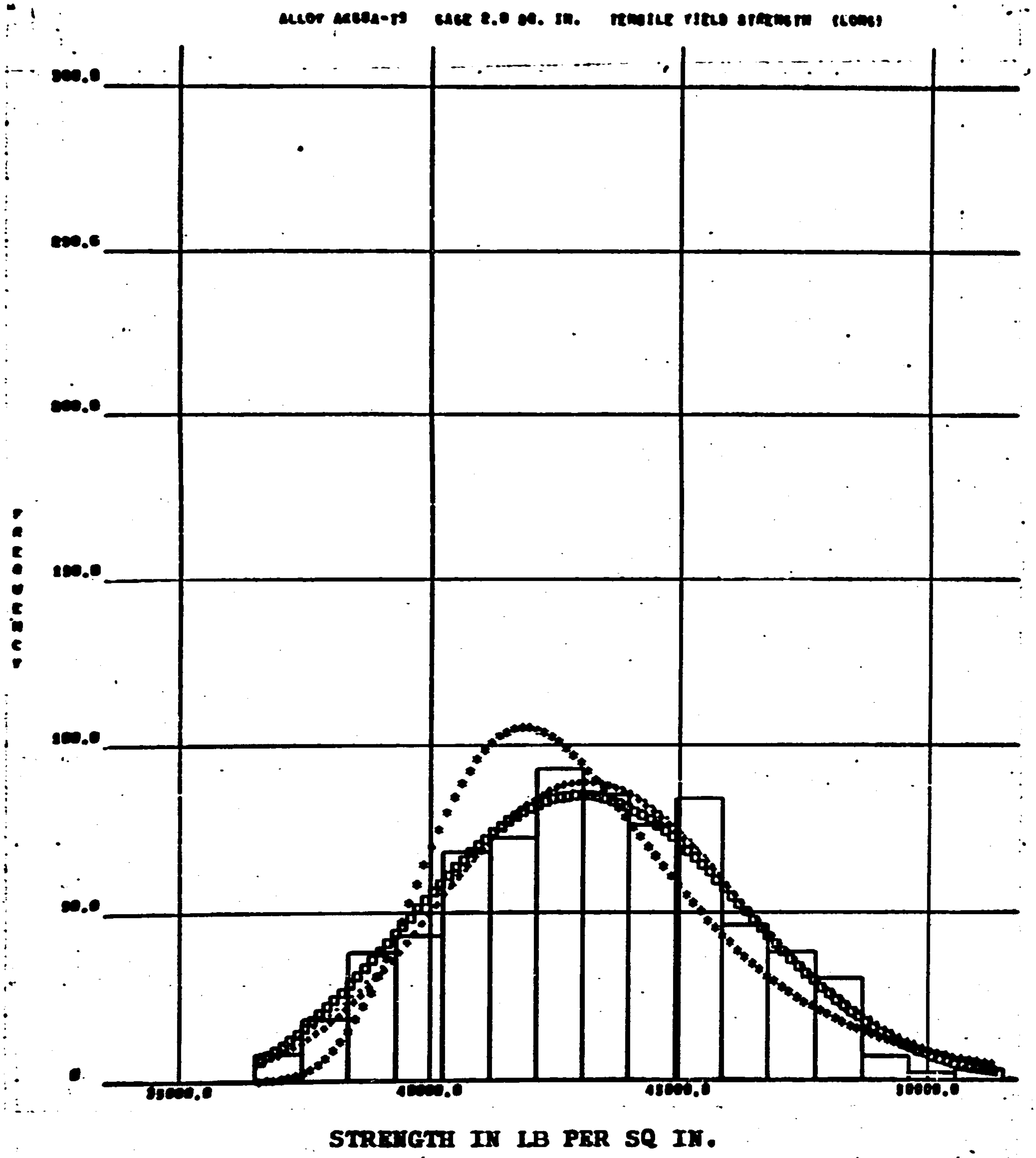
		PSI-SQ.	K-S
NO. OF SAMPLES = 648	· NORMAL	.730342	50.9437
NO. OF CLASS INTERVALS = 17	+ LOG NORMAL	1.21313	61.2288
MEAN = 27,729.9	* DOUBLE EXPONENTIAL	4.25681	88.8617
SIGMA = 2,155.73	◘ WEIBULL	.435940	41.6876

Figure 4-9



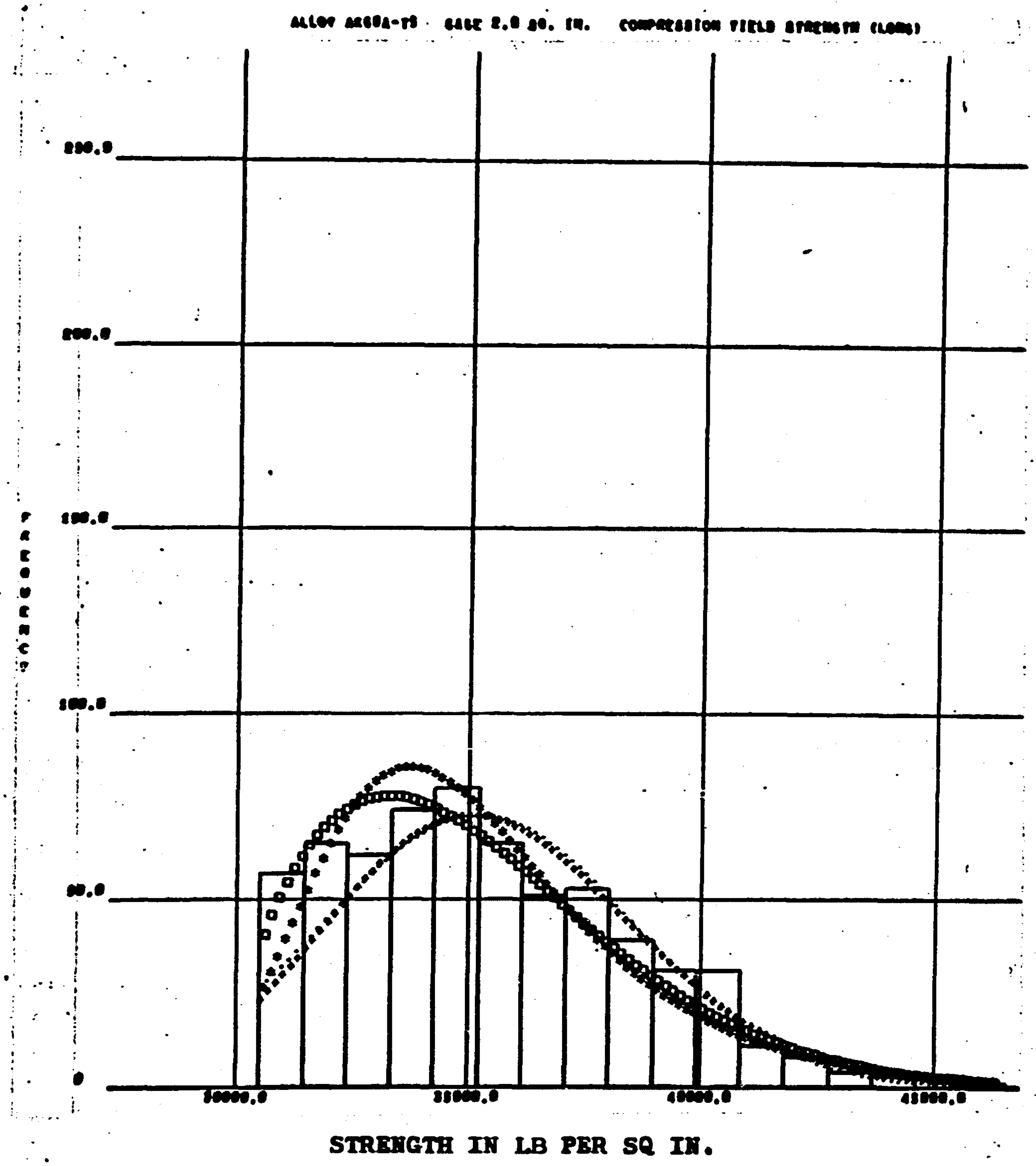
		PSI-SQ.	K-S
NO. OF SAMPLES = 712	• NORMAL	1.58239	33.7037
NO. OF CLASS INTERVALS = 15	+ LOG NORMAL	1.18533	32.6537
MEAN = 51,410.1	* DOUBLE EXPONENTIAL	1.09212	49.2152
SIGMA = 2,046.66	▫ WEIBULL	2.25489	39.4551

Figure 4-10



		PSI-SQ.	K-S
NO. OF SAMPLES = 711	. NORMAL	.121506	11.8195
NO. OF CLASS INTERVALS = 16	+ LOG NORMAL	.064748	16,2831
MEAN = 43,214.5	* DOUBLE EXPONENTIAL	1.50952	51.2336
SIGMA = 2,988.01	◻ WEIBULL	.126991	18.2075

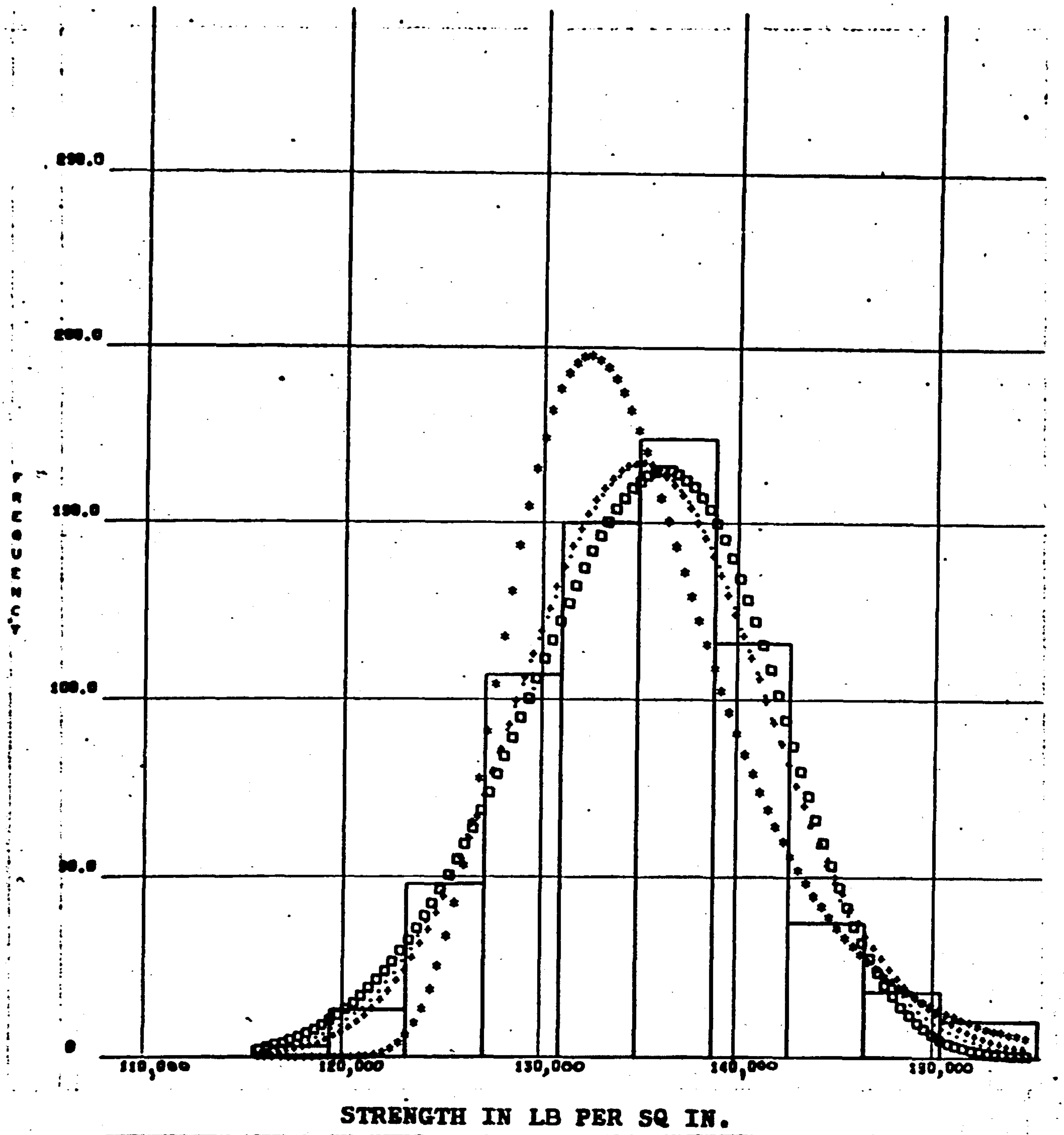
Figure 4-11



		PSI-SQ.	K-S
NO. OF SAMPLES = 638	· NORMAL	.330290	23.8004
NO. OF CLASS INTERVALS = 17	+ LOG NORMAL	.236408	18.3632
MEAN = 35,211.6	* DOUBLE EXPONENTIAL	.232679	32.6588
SIGMA = 3,300.47	▫ WEIBULL	.086246	30.0200

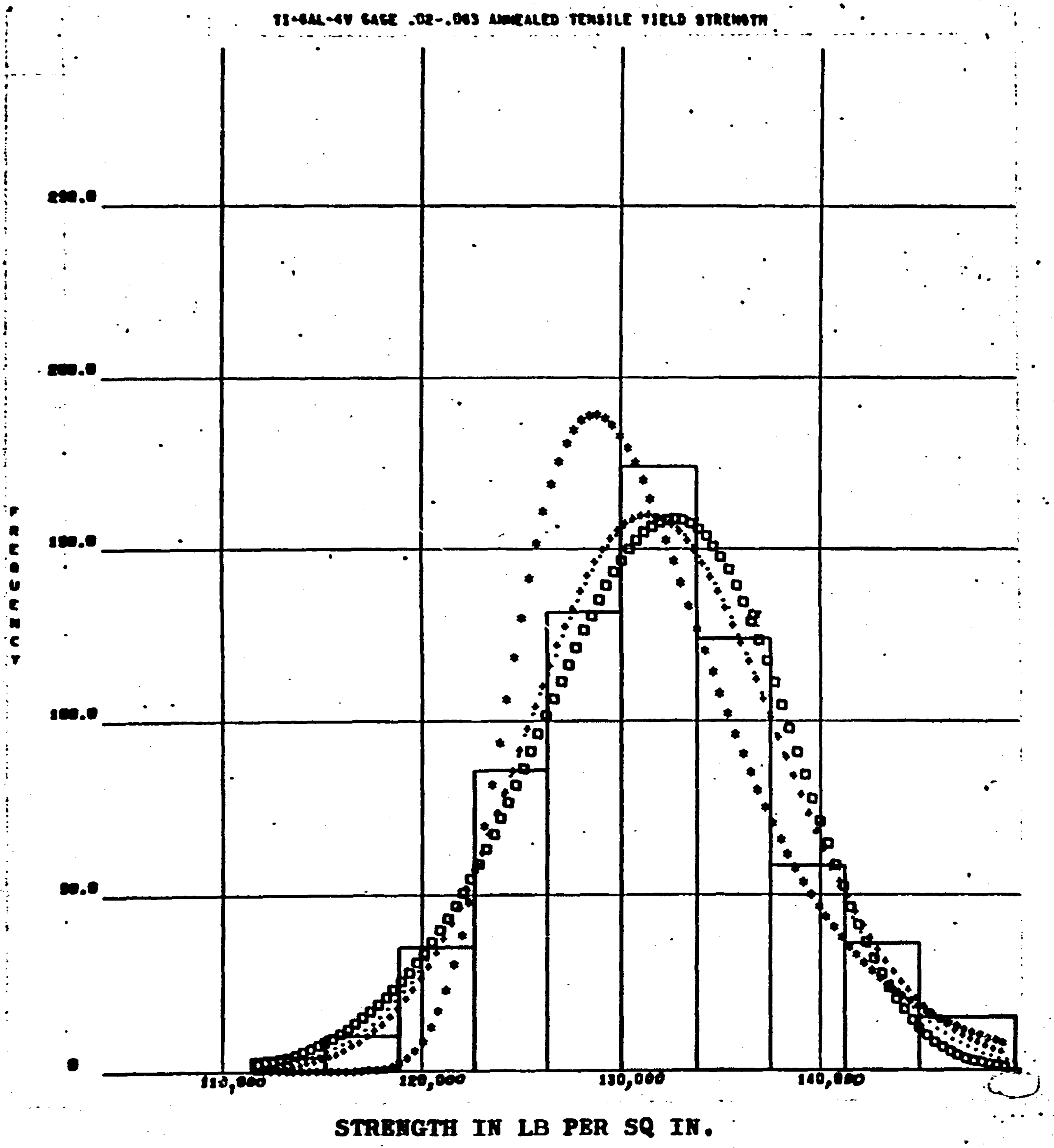
Figure 4-12

TI-6AL-4V GAGE .02-.063 ANNEALED ULTIMATE TENSILE STRENGTH



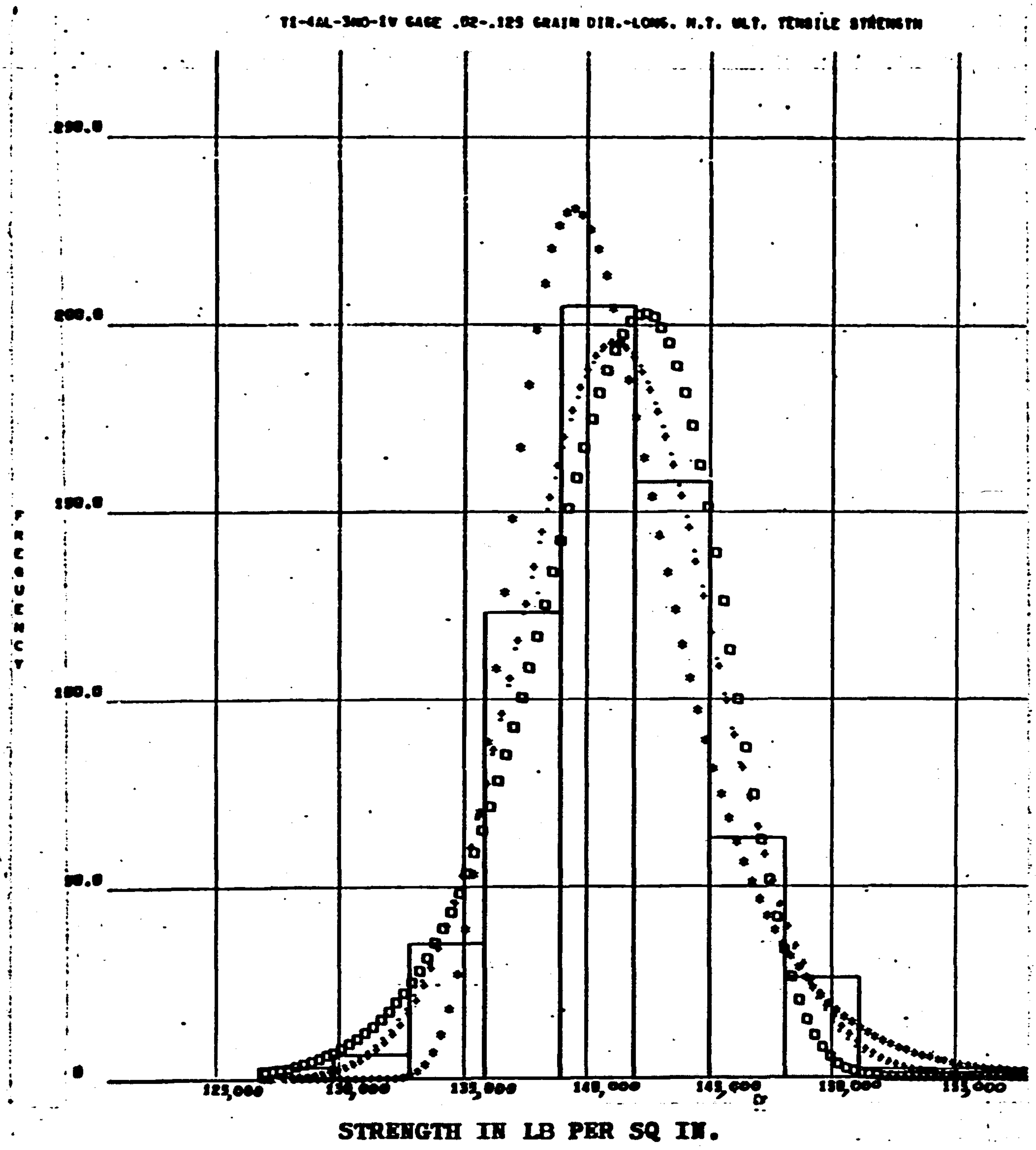
		PSI-SQ.	K-S
NO. OF SAMPLES = 676	• NORMAL	.075603	11.9647
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.044682	14.6273
MEAN = 135,101	* DOUBLE EXPONENTIAL	2.08670	55.5885
SIGMA = 6,266.32	▫ WEIBULL	.703328	16.8664

Figure 4-13



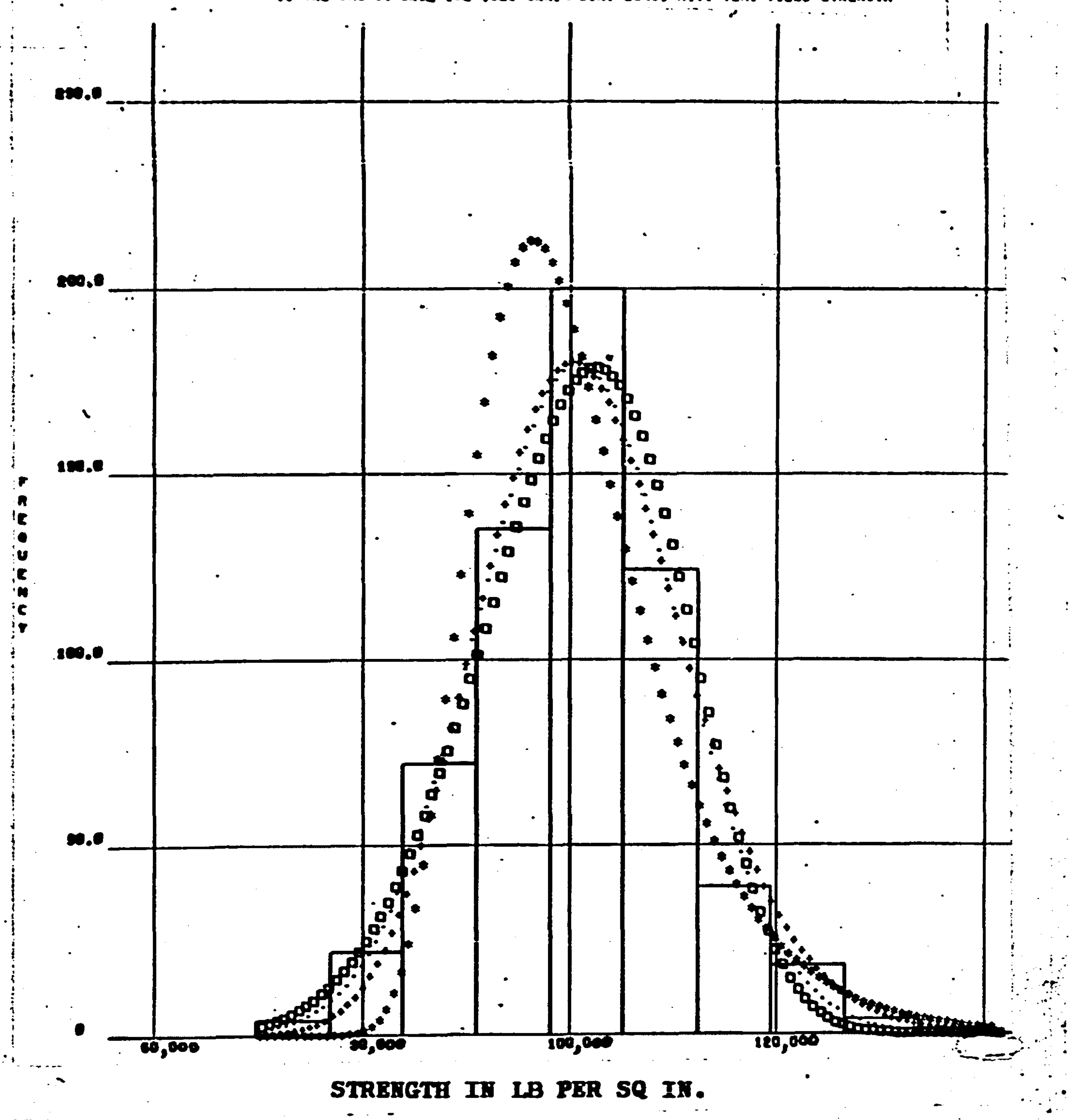
		PSI-SQ.	K-S
NO. OF SAMPLES = 674	· NORMAL	.160231	10.1985
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.324035	14.0329
MEAN = 131,512	* DOUBLE EXPONENTIAL	2.33977	49.1636
SIGMA = 6,275.07	□ WEIBULL	.276374	22.5985

Figure 4-14



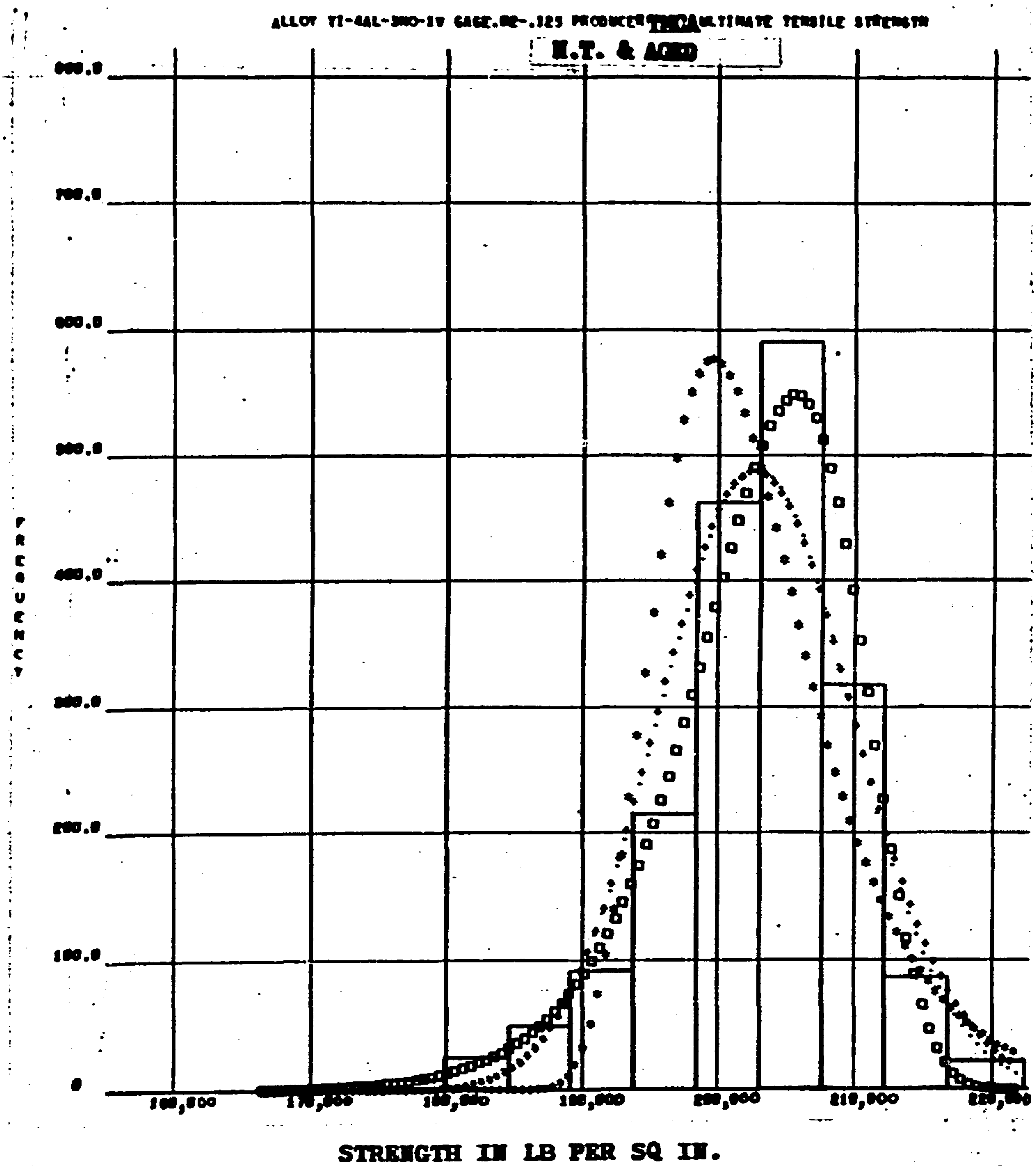
		PSI-SQ.	K-S
NO. OF SAMPLES = 623	• NORMAL	.742074	9.64042
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.759325	9.99186
MEAN = 141,149	* DOUBLE EXPONENTIAL	2.19000	32.9182
SIGMA = 3,881.41	◻ WEIBULL	1.70582	32.4121

Figure 4-15



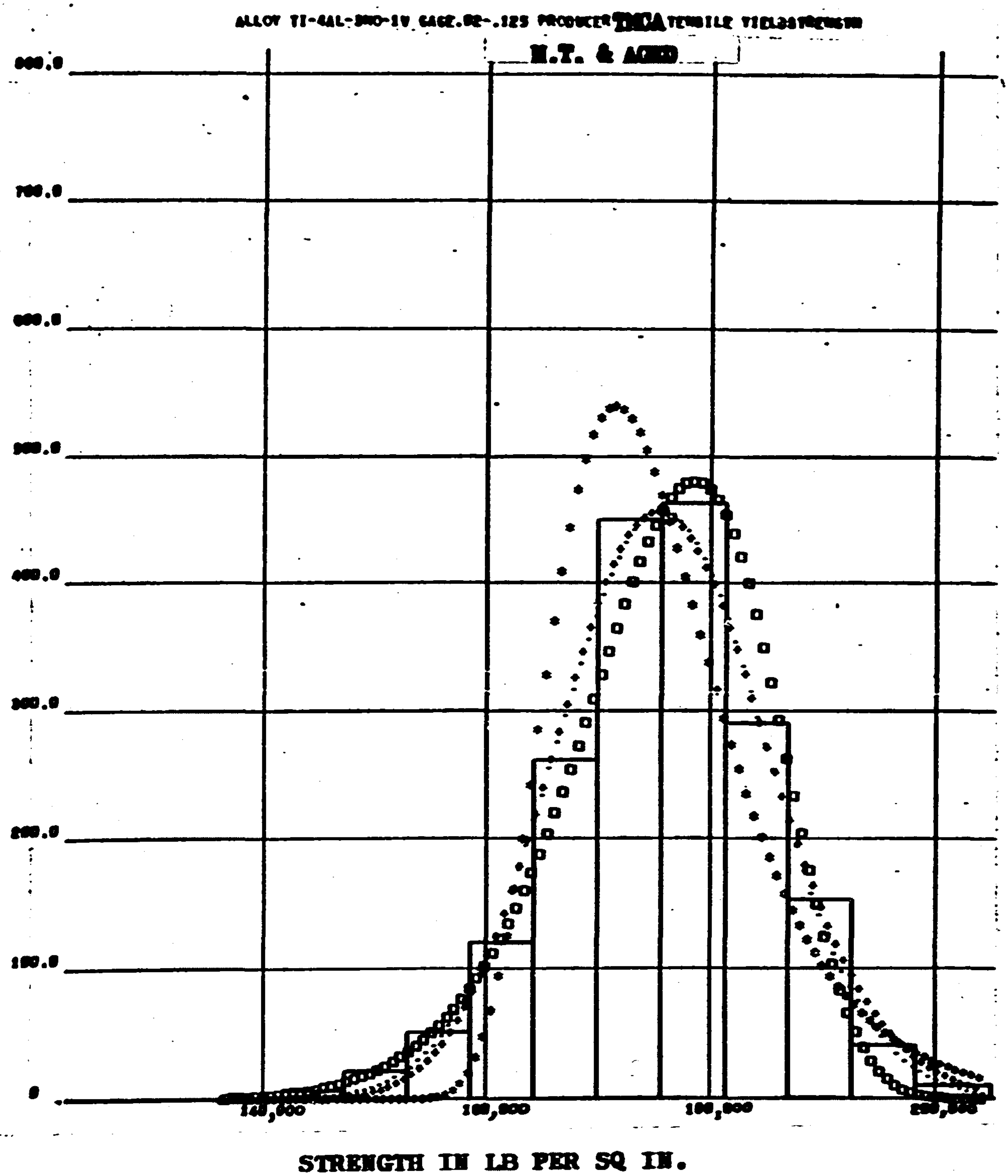
		PSI-SQ.	K-S
NO. OF SAMPLES = 619	· NORMAL	.022057	12.6084
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.349055	23.3718
MEAN = 100,709	* DOUBLE EXPONENTIAL	1.93613	48.7697
SIGMA = 9,758.13	□ WEIBULL	.306232	23.8621

Figure 4-16



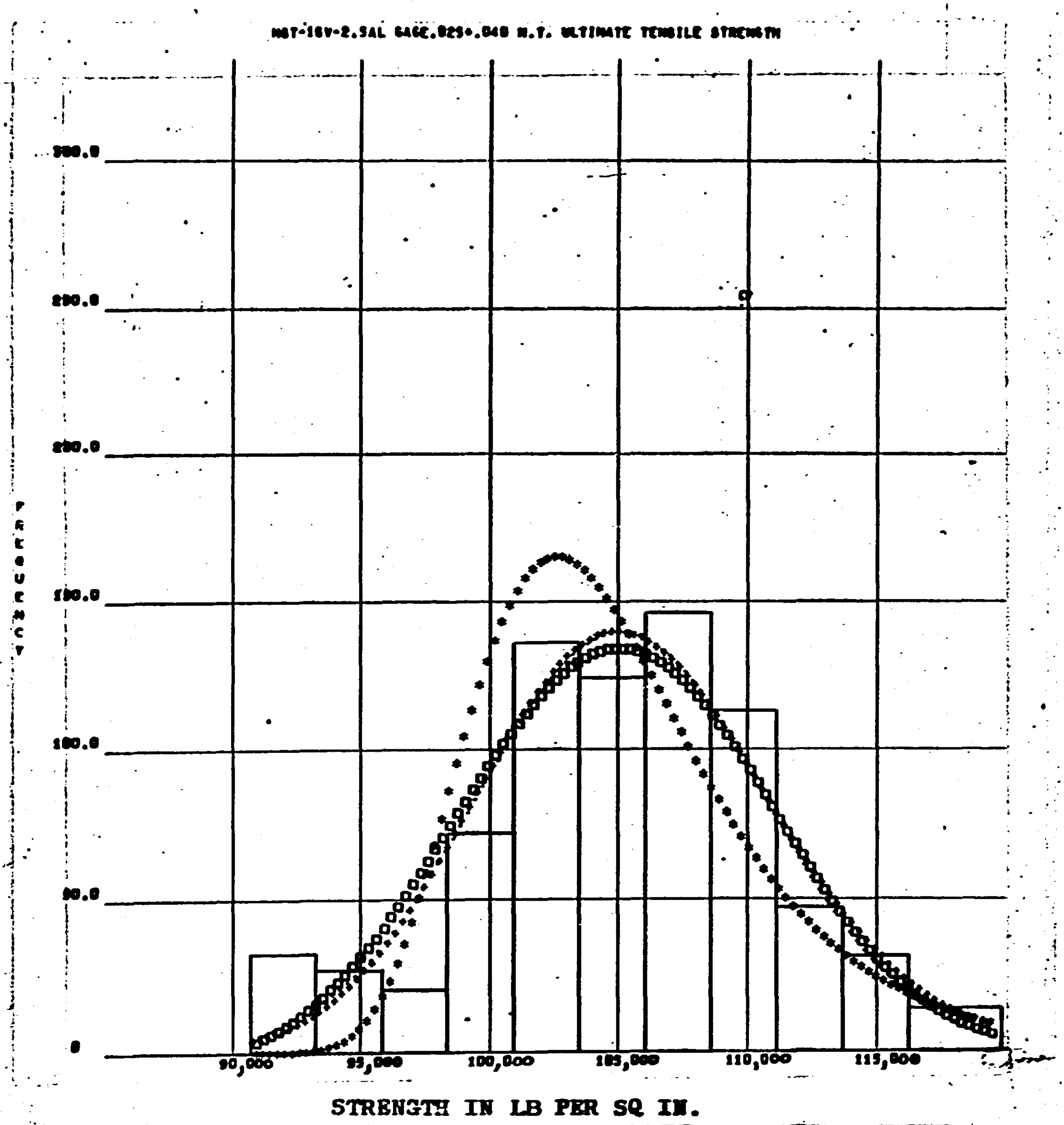
		PSI-SQ.	K-S
NO. OF SAMPLES = 1874	• NORMAL	3.17837	107.641
NO. OF CLASS INTERVALS = 12	+ LOG NORMAL	3.33262	121.754
MEAN = 202,667	* DOUBLE EXPONENTIAL	4.89806	238.737
SIGMA = 7,033.05	□ WEIBULL	.542391	60.5498

Figure 4-17



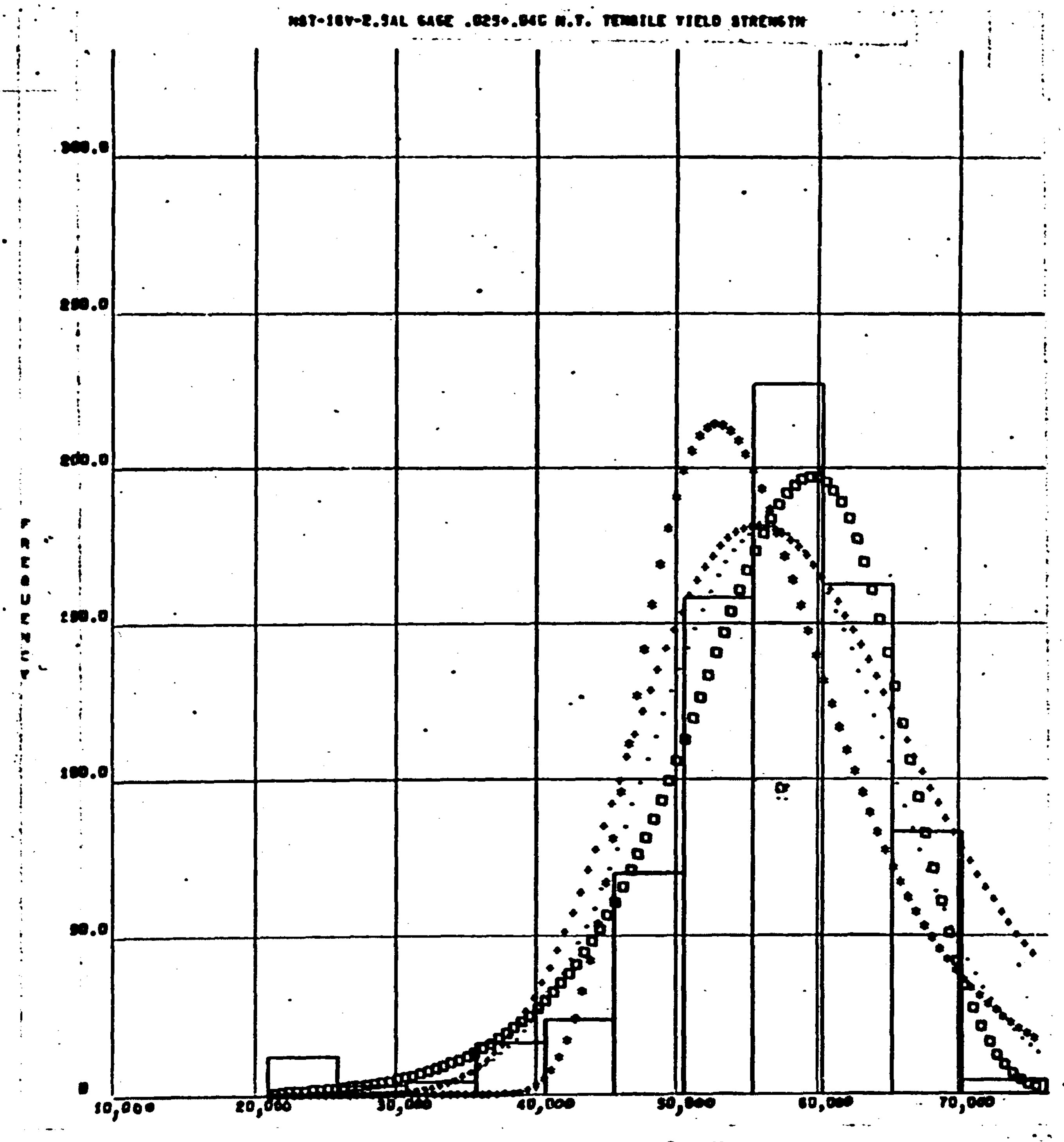
		PSI-SQ.	K-S
NO. OF SAMPLES = 1866	• NORMAL	1.37791	44.9629
NO. OF CLASS INTERVALS = 12	+ LOG NORMAL	2.02698	59.8750
MEAN = 175,587	* DOUBLE EXPONENTIAL	3.81061	156.091
SIGMA = 9,226.47	□ WEIBULL	1.59568	54.7335

Figure 4-18



		PSI-SQ.	K-S
NO. OF SAMPLES = 762	• NORMAL	2.86969	22.4602
NO. OF CLASS INTERVALS = 11	+ LOG NORMAL	3.33113	28.6614
MEAN = 105,078	* DOUBLE EXPONENTIAL	4.05014	74.2479
SIGMA = 5,571.75	□ WEIBULL	3.84445	30.3672

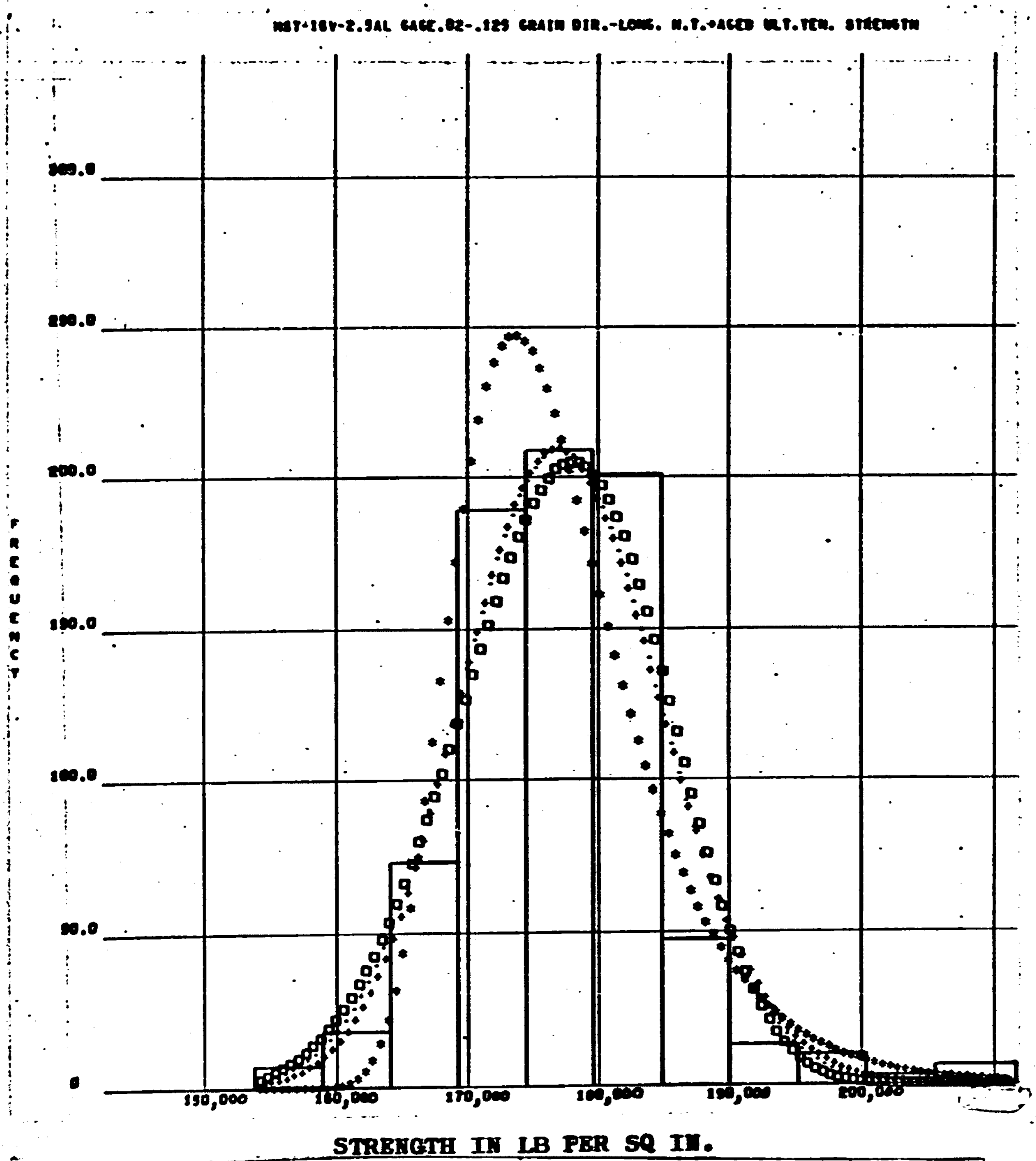
Figure 4-19



NO. OF SAMPLES = 762
NO. OF CLASS INTERVALS = 11
MEAN = 56,549
SIGMA = 8,282.77

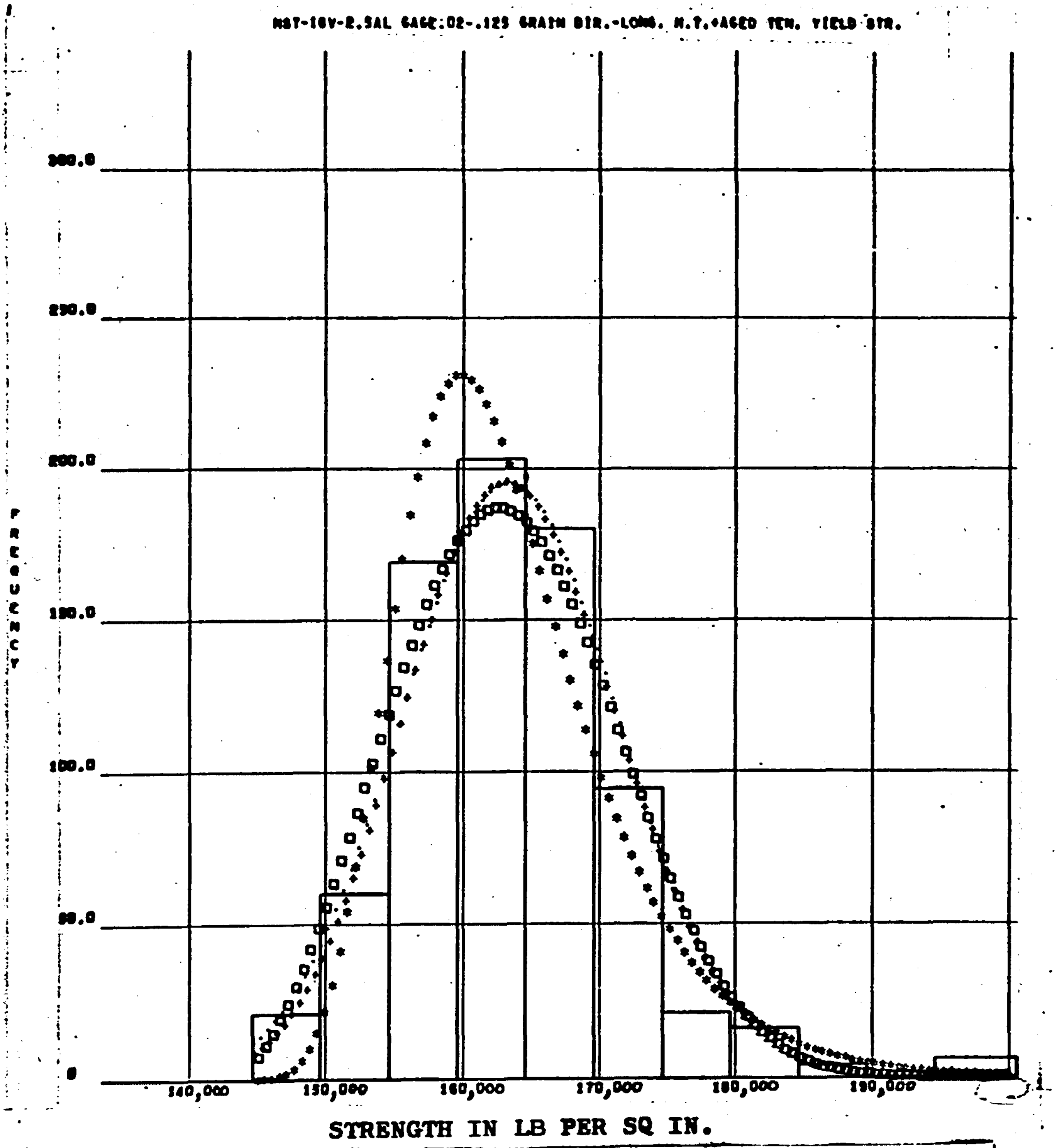
	PSI-SQ.	K-S
• NORMAL	2.97001	54.9481
+ LOG NORMAL	6.02194	91.6124
* DOUBLE EXPONENTIAL	4.11864	105.818
□ WEIBULL	2.94417	32.8541

Figure 4-20



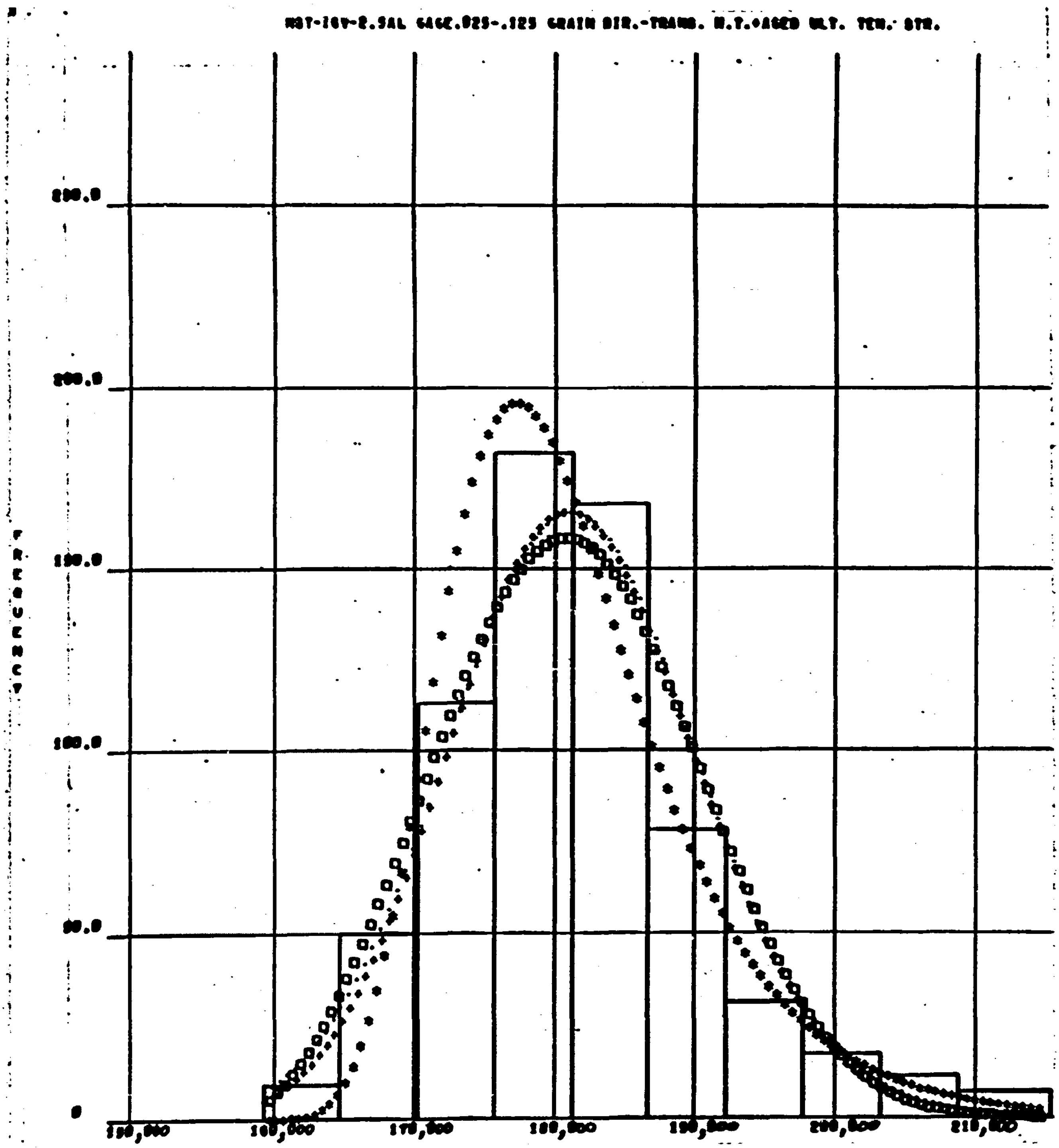
			PSI-SQ.	K-S
NO. OF SAMPLES = 776	·	NORMAL	.406268	40.8367
NO. OF CLASS INTERVALS = 11	+	LOG NORMAL	.399311	38.9741
MEAN = 176,989	*	DOUBLE EXPONENTIAL	1.57533	44.2024
SIGMA = 7,666.92	□	WEIBULL	.744153	42.9790

Figure 4-21



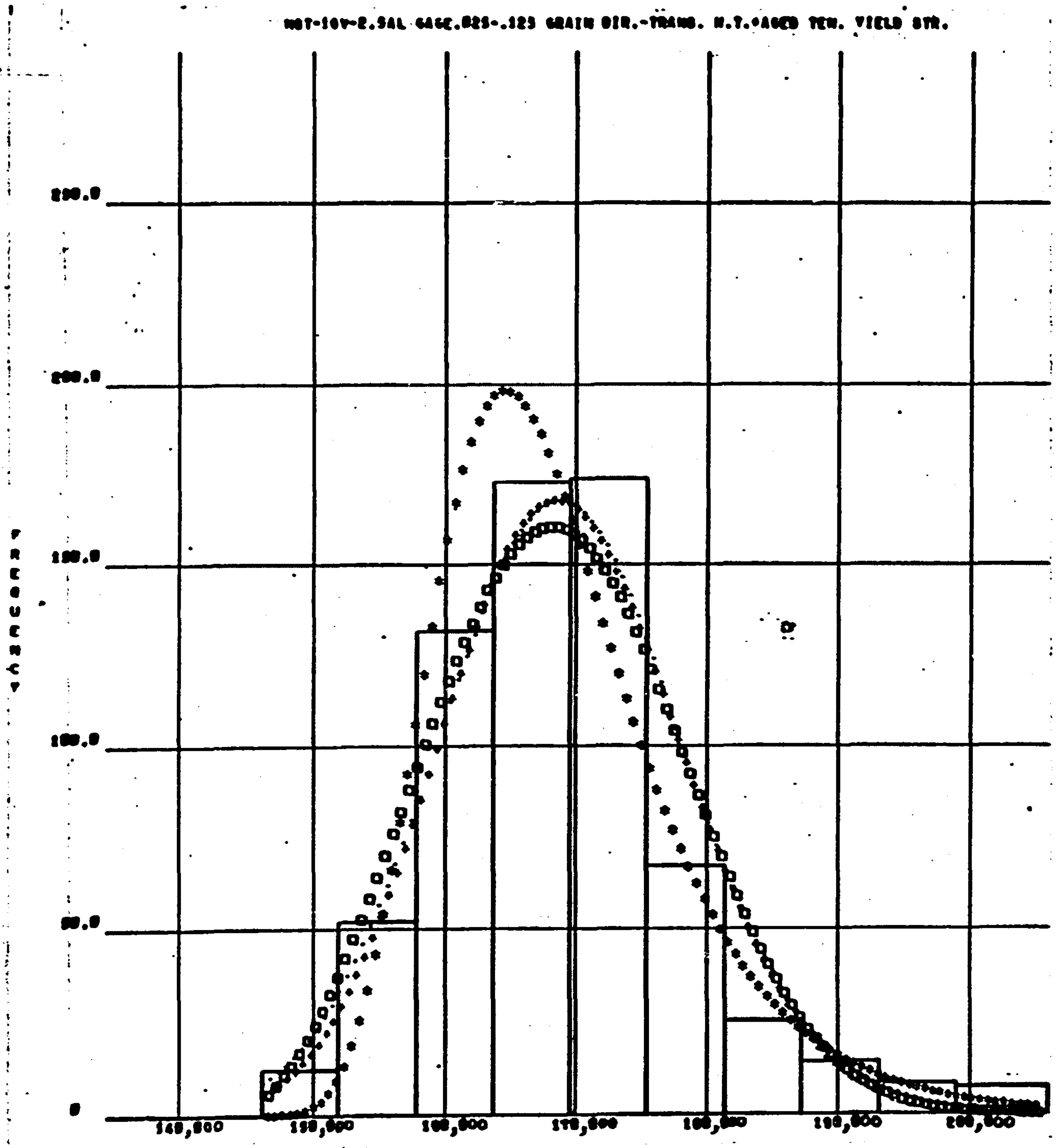
			PSI-SQ.	K-S
NO. OF SAMPLES = 774	•	NORMAL	.105347	25.3382
NO. OF CLASS INTERVALS = 11	+	LOG NORMAL	.070468	20.4748
MEAN = 163,358	*	DOUBLE EXPONENTIAL	.762691	36.0969
SIGMA = 7,906.03	□	WEIBULL	.210236	27.9501

Figure 4-22



		PSI-SQ.	K-S
NO. OF SAMPLES = 666	· NORMAL	.755307	31.8459
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.367643	26.5304
MEAN = 181,183	* DOUBLE EXPONENTIAL	.869624	28.2547
SIGMA = 8,823.19	□ WEIBULL	.716316	34.0031

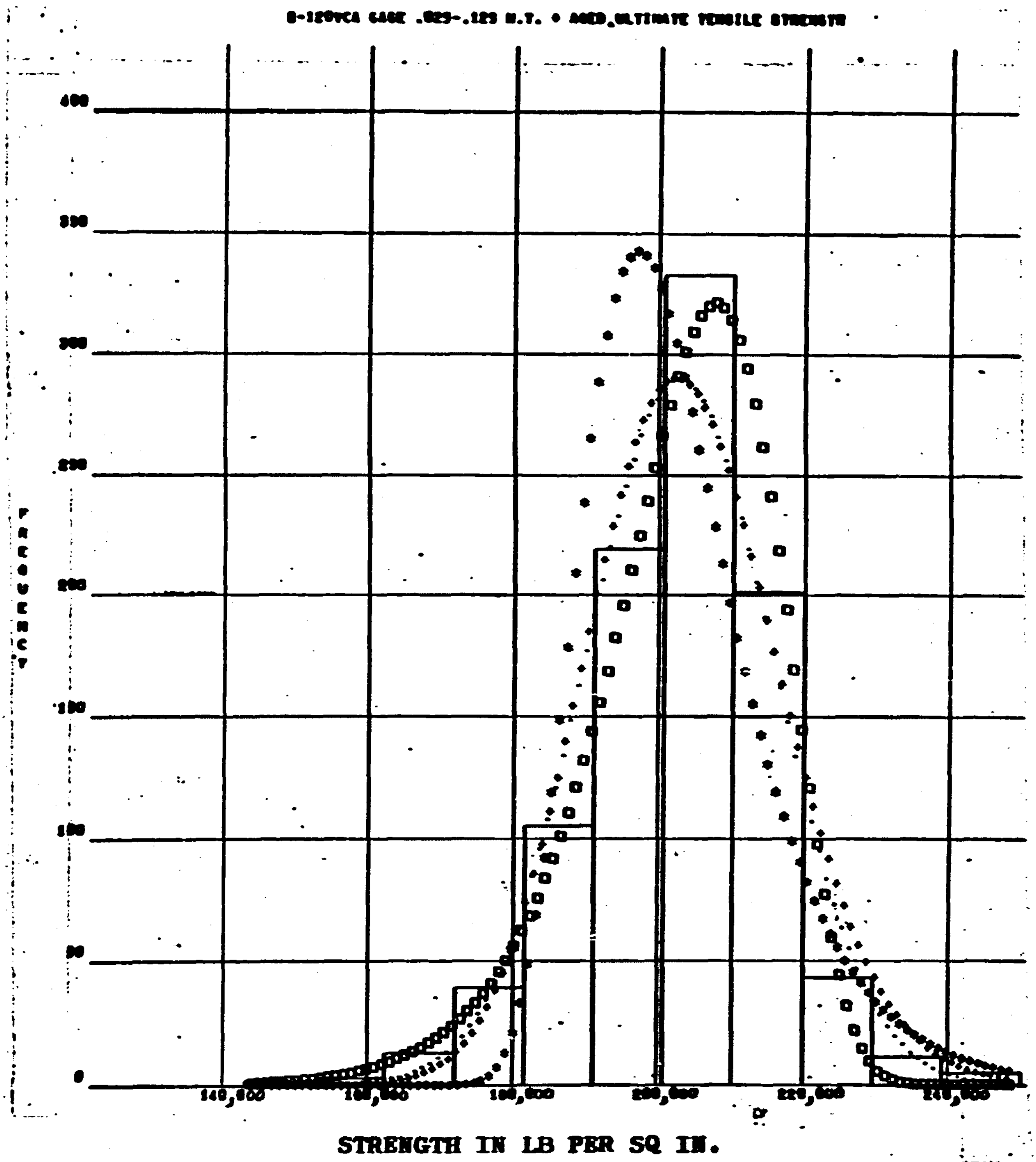
Figure 4-23



STRENGTH IN LB PER SQ IN.

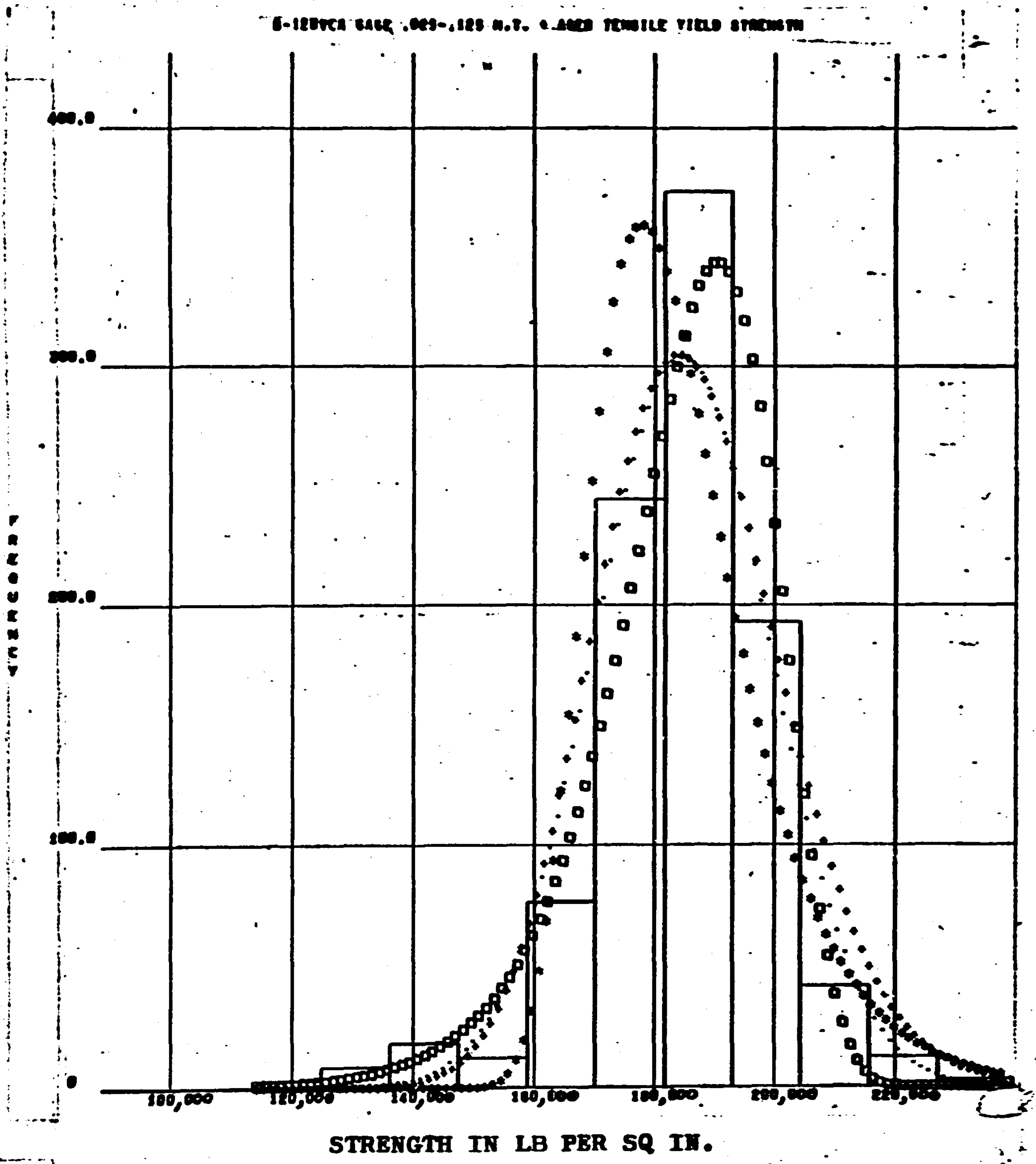
		PSI-SQ.	K-S
NO. OF SAMPLES = 664	• NORMAL	.345393	34.7974
NO. OF CLASS INTERVALS = 10	+ LOG NORMAL	.147924	29.8752
MEAN = 168,737	* DOUBLE EXPONENTIAL	.794080	33.8672
SIGMA = 9,276.65	□ WEIBULL	.367884	37.2035

Figure 4-24



			PSI-SQ.	K-S
NO. OF SAMPLES = 974	•	NORMAL	1.76566	44.1176
NO. OF CLASS INTERVALS = 11	+	LOG NORMAL	2.22940	57.4888
MEAN = 202,884	*	DOUBLE EXPONENTIAL	3.79378	107.881
SIGMA = 12,916.7	□	WEIBULL	.544842	35.2421

Figure 4-25



		PSI-SQ.	K-S
NO. OF SAMPLES = 984	· NORMAL	6.01016	52.6136
NO. OF CLASS INTERVALS = 11	+ LOG NORMAL	6.86038	71.3652
MEAN = 184,529	* DOUBLE EXPONENTIAL	3.91906	114.821
SIGMA = 14,660.8	□ WEIBULL	4.89295	54.9826

Figure 4-26

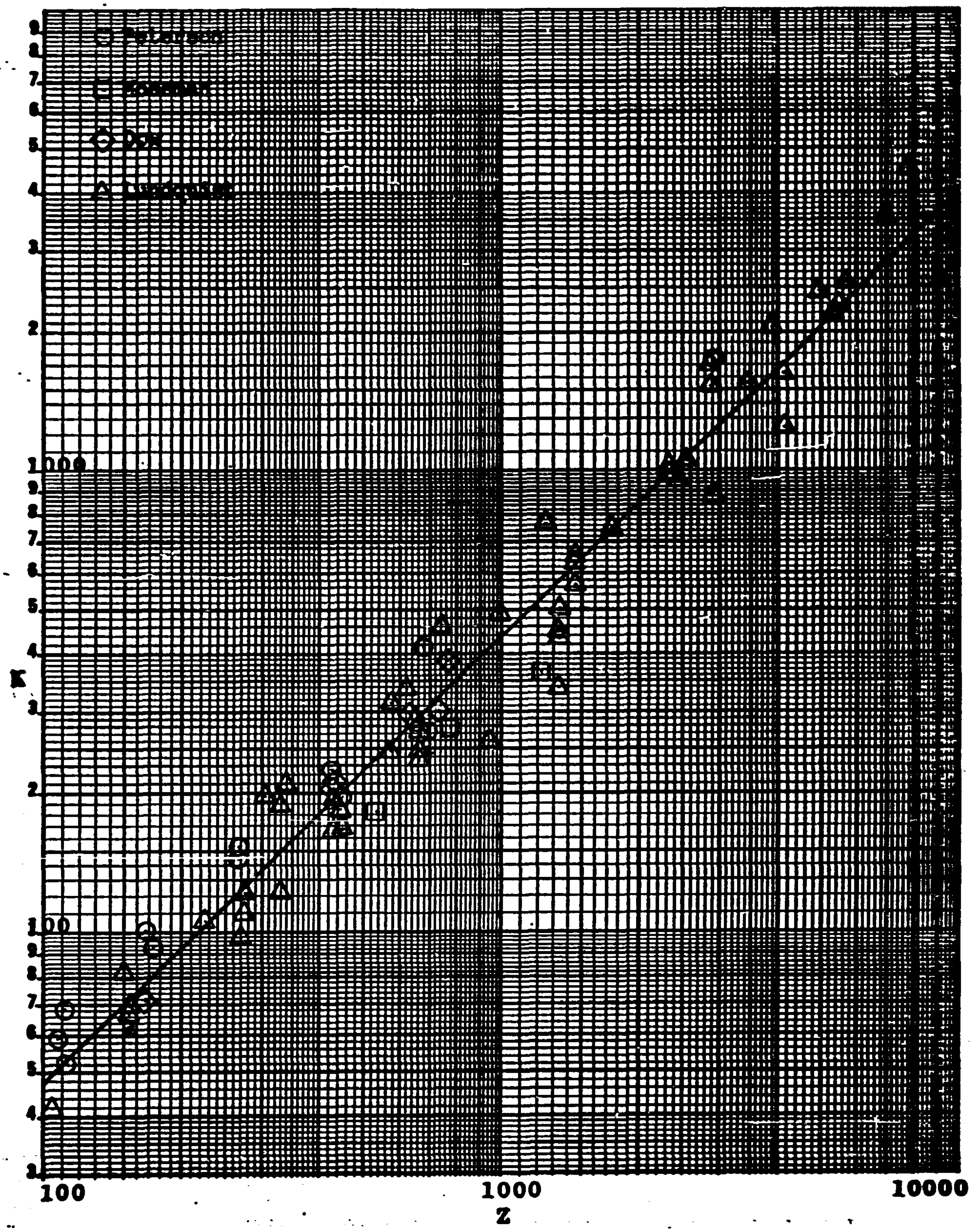


Figure 4-27

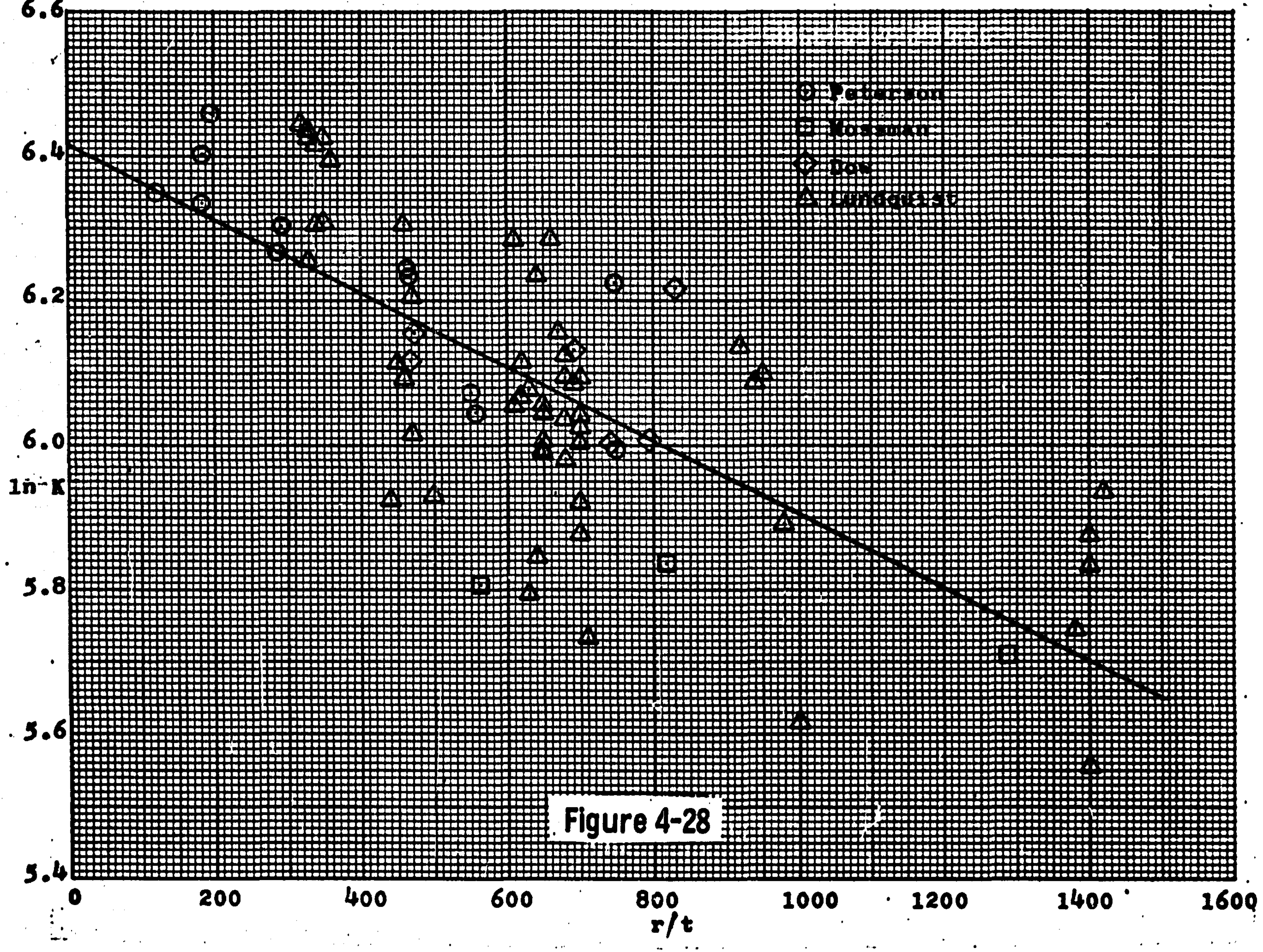

Peterson
Mossman
Dow
Lundquist
6.6
6.4
6.2
6.0
5.8
5.6
5.4
ln K
0
200
400
600
800
1000
1200
1400
1600
r/t

Figure 4-28

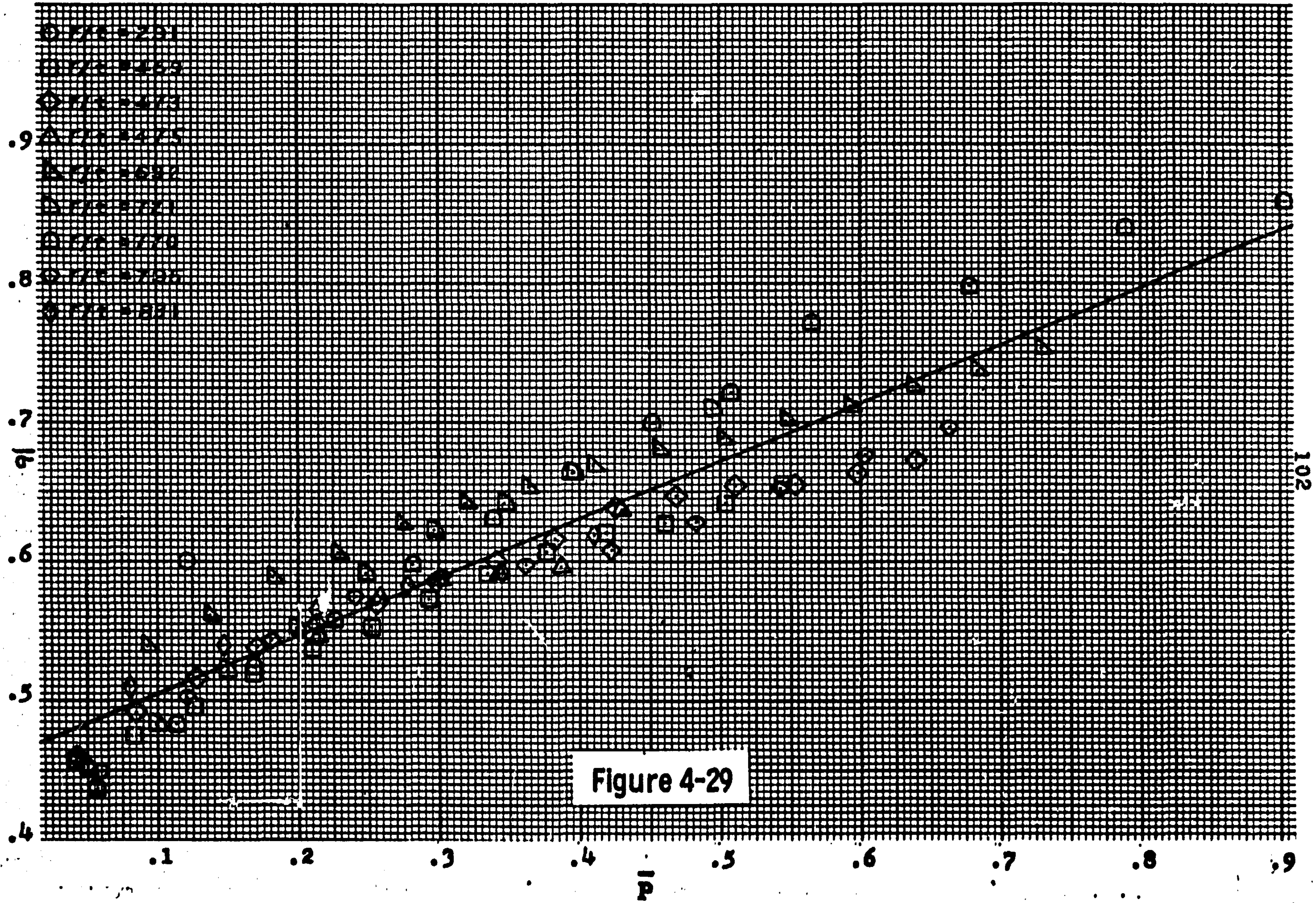


Figure 4-29

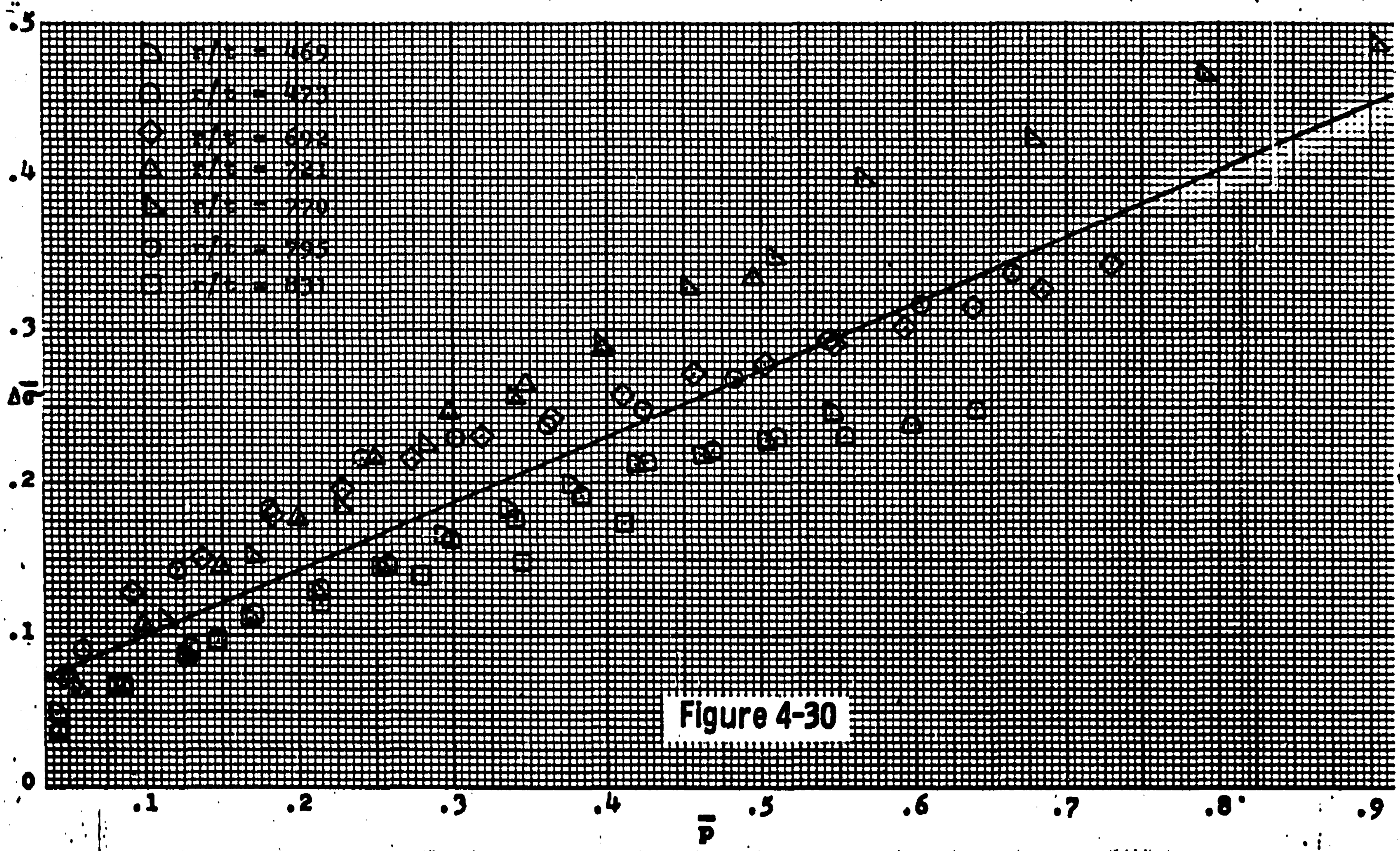


Figure 4-30

CHAPTER V

STRUCTURAL RELIABILITY

Introduction

Structural reliability will be defined here as the probability of the strength, as determined according to Chapter IV, being greater than the loads, as determined according to Chapter III. This probability will be calculated using two different methods. The first method for the calculation of structural reliability will use the double exponential distribution for the applied loads and the normal distribution for the strength. These two distributions gave the best fit to the loads and strengths data as shown in the previous chapters. Although these two distributions gave the best fit, a check for significant difference at the 0.05 level using the Kolmogorov-Smirnov test indicated no significant differences between the loads and strengths data and all four theoretical distributions for a majority of the data examined. Therefore, the calculation of structural reliability by different distributions will be used for the second method. For ease of calculation, the second method for the calculation of structural reliability will assume that both the applied loads and the vehicle strength are normally distributed.

Load Double Exponential and Strength Normal

Since both the double exponential and the normal distributions are of the exponential type difficulties are encountered when one attempts to integrate these distributions. In order to calculate the structural reliability using these distributions, the following numerical integration process was developed.

Referring to Figure 5-1, the probability of a strength occurring between X_1 and X_2 is equal to the probability of a strength occurring between negative infinity and X_2 minus the probability of a strength occurring between negative infinity and X_1. Loads less than X_1 will not cause failure. Loads between X_1 and X_2 will cause failure of some, and loads greater than X_2 will cause failure of all of the strengths between X_1 and X_2. The location between X_1 and X_2 for the loads causing failure is considered to be a second order magnitude factor and has been selected for convenience as half way between X_1 and X_2 and is called X_3. For this analysis all loads less than X_3 are considered not to cause failure, and all loads greater than X_3 are considered to cause failure of all strengths between X_1 and X_2. Since the load and strength distributions are considered independent, the probability that a structural failure will occur between X_1 and X_2 is the product of the probability that the strength will be between X_1 and X_2, and the probability that the load will be greater than X_3. The total probability of structural failure is the sum of all the products for X on the interval ($-\infty$, $+\infty$). In actual practice X is not taken to positive infinity, instead it is taken to a sufficiently large value so that the desired accuracy is obtained. This numerical integration process can be applied to any pair of distributions; the only limitation being that the probabilities of the distributions separately must be calculatable. An example of this numberical integration process is presented in the structural reliability example.

Load and Strength Both Normal

Given normal probability density functions of strength p_S and load p_L a third normal probability density of failure p_f will be defined by:

$$p_f = p_S - p_L$$

where $$p_S = \frac{1}{(2\pi)^{\frac{1}{2}} S_S} \exp\left(-\tfrac{1}{2}\left[(X_S - \bar{X}_S)/S_S\right]^2\right),$$

$$p_L = \frac{1}{(2\pi)^{\frac{1}{2}} S_L} \exp\left(-\tfrac{1}{2}\left[(X_L - \bar{X}_L)/S_L\right]^2\right),$$

and S is the sample standard deviation, X is the sample mean and the subscripts S and L refer to strength and load respectively.

Hoel (56) has shown that if p_S and p_L are normally and independently distributed then $p_f = p_S - p_L$ is normally distributed, and

$$\bar{X}_f = \bar{X}_S - \bar{X}_L$$

$$S_f^2 = S_s^2 + S_L^2 .$$

Therefore, the probability density of failure is:

$$p_f = \frac{1}{(2\pi)^{\frac{1}{2}} S_f} \exp\left(-\tfrac{1}{2}\left[(X_f - \overline{X}_f)/S_f\right]^2\right).$$

Failure is defined here as any condition where the load is greater than the strength, that is, when (S-L) is less than zero. Therefore, the probability of failure may be obtained by integrating over the negative range of p_f.

$$P_f = \frac{1}{(2\pi)^{\frac{1}{2}} S_f} \int_{-\infty}^{0} \exp\left(-\tfrac{1}{2}\left[(X_f - \overline{X}_f)/S_f\right]^2\right) dx$$

let $t = (X_f - \overline{X}_f)/S_f = X_f/S_f - \overline{X}_f/S_f$

$dt = dx/S_f \qquad dx = S_f \ dt$

at $X = 0 \qquad t = (0 - \overline{X}_f)/S_f = -\overline{X}_f/S_f$

at $X \rightarrow -\infty \qquad t \rightarrow \infty$

$$P_f = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{-\overline{X}_f/S_f} e^{-\frac{1}{2}t^2}\, dt$$

Structural Reliability Examples

First Example

For the first example the loads were taken for vehicle C at the combined locations of Denver and International Falls.

Thus

$$\bar{X}_L = 5{,}190{,}670 \text{ inch pounds},$$

$$S_L = 355{,}046 \text{ inch pounds},$$

The missile in this example is unpressurized and constructed of 2024-T3 aluminum alloy with

a radius equal to 100 inches,

a length between bulkheads of 100 inches,

and a skin thickness of 0.100 inches.

The problem is to solve for the structural reliability.

Solution

For 2024-T3 aluminum alloy, reference (57) gives a value of Young's Modulus of 10,700,000. In reference (58) a value of 0.33 is given for Poisson's Ratio. Following the procedure described in the previous chapter, the distribution of the buckling coefficient K is calculated for non-pressurized aluminum cylinders. The following results were obtained:

$$\bar{K} = 369.96,$$

$$S_K = 50.7157,$$

Using the allowable bending moment equation from the previous chapter,

$$M = \frac{\pi^3 \, K \, E \, t^3 \, r^2}{12 \, (1 - \mu^2) \, L^2}$$

one obtains an average and standard deviation of the moment by substituting in the average and standard deviation of the buckling coefficient.

Thus

$$\bar{X}_S = \frac{\pi^3 \bar{K} (10,700,000) (0.1)^3 (100)^2}{12 (1 - 0.33^2) (100)^2}$$

$$\bar{X}_S = 27,648 \bar{K},$$

and

$$S_S = 27,648 S_K.$$

Substituting in the values for $\bar{K}$ and S_K one obtains -

$$\bar{X}_S = 27,648 (369.96) = 10,228,700,$$

and

$$S_S = 27,648 (50.7157) = 1,402,200.$$

Assuming the loads are double exponentially distributed and the strengths are normally distributed, one obtains the structural reliability by the following numerical integration process.

The double exponential distribution, like the normal, is a two parameter distribution. The parameters are u and v*, and are related to the mean and standard deviation by the following equations:

$$u = \bar{X} - 0.450053S \text{ and } v = 0.779697S.$$

---

*In reference (11) this parameter is given as $1/\alpha$.

For vehicle C loads:

$u = 5,190,670 - 0.450053\ (355,046) = 5,030,880$

and $v = 0.779697\ (355,046) = 276,828$.

The parameter u in the double exponential is similar to $\bar{X}$ in the normal distribution in that when both distributions are transformed to their simpler forms, these two parameters are the location of the zero point on the abscissa of the probability density function. The parameter v is similar to S in that S is the unit division of the abscissa of the probability density function for the normal distribution and v is the unit division of the abscissa of the probability density function for the double exponential distribution.

For the numerical integration a common reference point and common strip widths must be established for the two distributions. All of the strip widths will be the same size except the first. This first strip will start at negative infinity and be as wide as possible and still maintain the desired accuracy. Examination of the cumulative probability table for the double exponential distribution, given in reference (11), shows that the point -2.5v, the probability is 0.999995 that the loads will be greater than this value. Thus, it would be conservative to say that all strengths less than this value fail, and the conservative error would be less than 0.000005. The point -2.5v will be taken as the reference point and the first strip width will be from negative infinity to this reference point. All other strip widths are taken as

0.1v.

For this numerical example the reference point is:

$$R.P. = u - 2.5v = 5{,}030{,}880 - 2.5\ (276{,}828) = 4{,}338{,}810.$$

In terms of the number of standard deviations from the mean for the strength:

$$t = (R.P. - \overline{X}_s)/S_s = (4{,}338{,}810 - 10{,}228{,}700)/1{,}402{,}200 = -4.20047.$$

The strip widths are:

$$S.W. = 0.1v = 0.1\ (276{,}828) = 27{,}683.$$

In terms of the standard deviations of the strength the width is:

$$S.W./S_s = 27{,}683/1{,}402{,}200 = 0.01974.$$

With the reference point and strip widths established, Table 5-1 is constructed to give the structural reliability. Column 1 is the number of standard deviations from the mean value of the strength data beginning with t and subtracting $S.W./S_s$ each time. This gives the locations of X_1 and X_2 shown in Figure 5-1. In this step by step procedure, for the first step, the first number in Column 1 is X_1 and the second number is X_2. In the second step the second number in Column 1 is X_1 and the third is X_2. Column 2 is determined from normal probability tables for the locations given in Column 1. Column 3 is the difference of consecutive probabilities given in Column 2 and thus represent the probability that a strength will occur between the X_1 and X_2 locations given in Column 1. In terms of load data the locations of X_1 and X_2 are given as -2.5v, -2.4v, -2.3v, -2.2v, etc. Thus, in terms of the loading values, the location of X_3 is given by -2.45v, -2.35v, -2.25v, -2.15v, etc. Column 4 gives the probability that the loads will be greater than the X_3's given above. These probabilities were obtained from the double exponential probability table contained in reference (11). Column 5 is the product of Column 3 and Column 4 and thus represents the probability that a strength will occur between X_1 and X_2, and simultaneously the load will be greater than X_3 and therefore result in a structural failure. The sum of Column 5 is the total probability of structural failure P_f.

Assuming both loads and strengths are normally distributed one obtains:

$$\bar{X}_f = \bar{X}_S - \bar{X}_L = 10{,}228{,}349 - 5{,}190{,}670 = 5{,}037{,}679$$

and

$$S_f^2 = S_s^2 + S_L^2 = (1{,}402{,}165)^2 + (355{,}046)^2$$

$$S_f = 1{,}446{,}418.$$

The structural reliability is equal to $1 - P_f$ and

$$P_f = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{-\bar{X}_f/S_f} e^{-\frac{1}{2}t^2}\, dt,$$

$$\text{and } -\bar{X}_f/S_f = -5{,}037{,}679 \;/\; 1{,}446{,}418 = -3.48287.$$

Using these limits and checking normal probability tables one obtains a structural reliability of 0.999748.

Other Examples

Missiles were also designed for the loads of vehicle B, vehicle D, vehicle E, and vehicle F. The missile radii were selected such that the wall thicknesses for all vehicle loads would be approximately the same. The resulting wall thicknesses for various structural reliability levels for both methods of calculation are shown in Figure 5-2. Figure 5-3 shows the percentage difference between the two methods of calculation. The maximum difference in wall thicknesses for the two methods of calculation is 7.6 per cent for vehicle B at a probability level of 0.999999. It should be noted that as the structural reliability level decreases the difference between the two methods of calculation decreases. It should also be noted that the assumption that both loads and strengths are normally distributed results in unconservative wall thickness when compared to the other method.

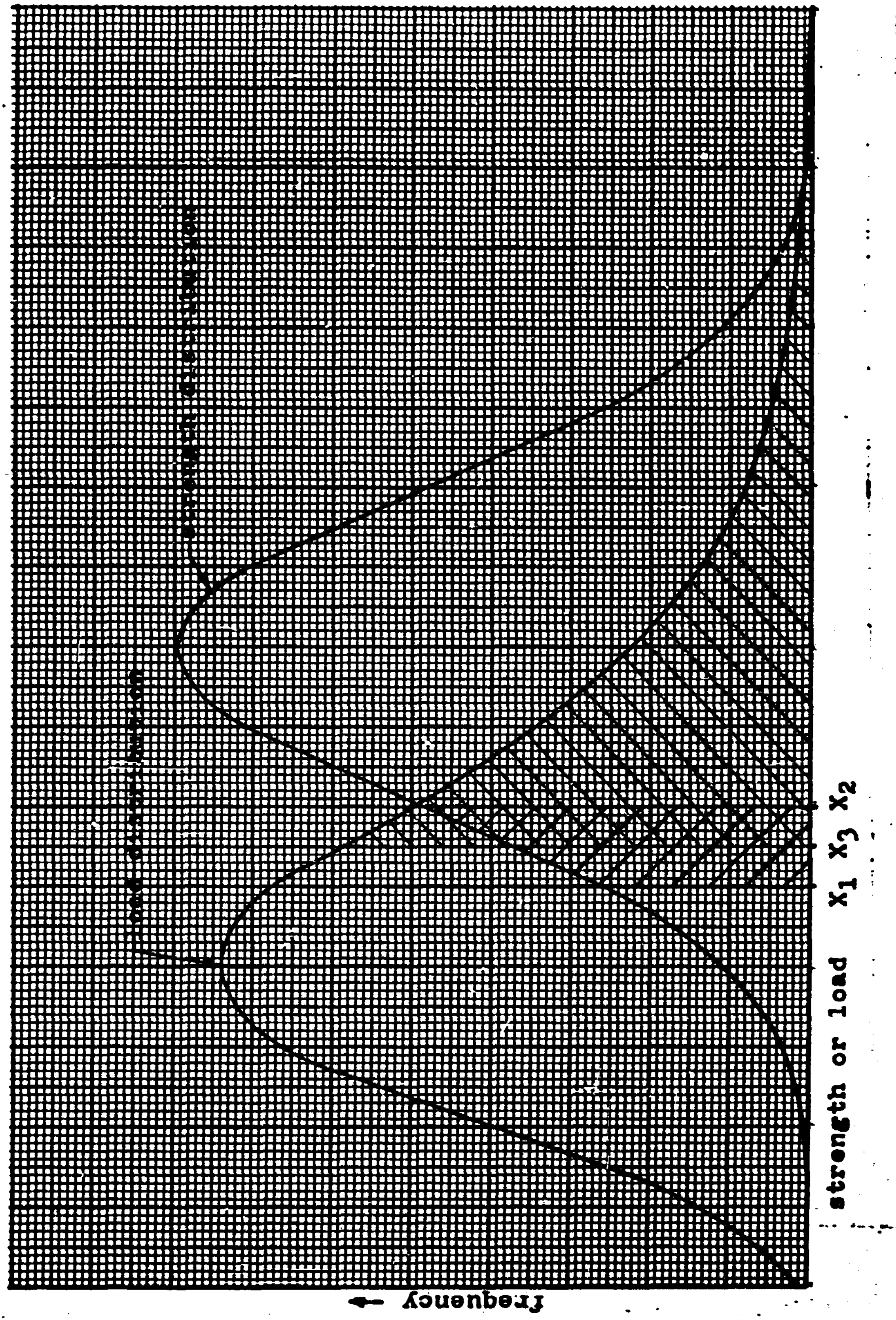
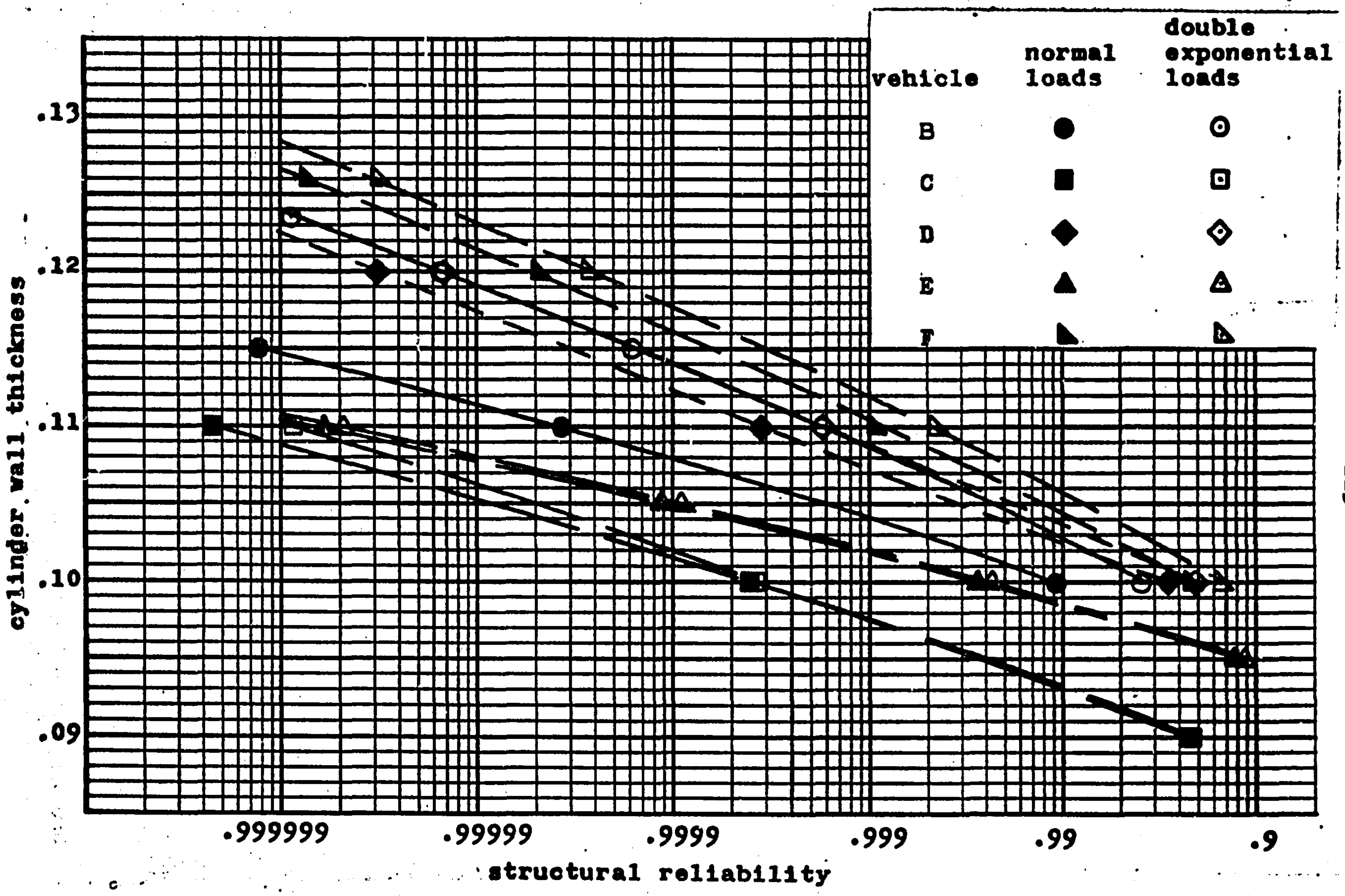


Figure 5-1



cylinder wall thickness
.13
.12
.11
.10
.09
.999999
.99999
.9999
.999
.99
.9
structural reliability
vehicle
normal loads
double exponential loads
B
C
D
E
F

Figure 5-2

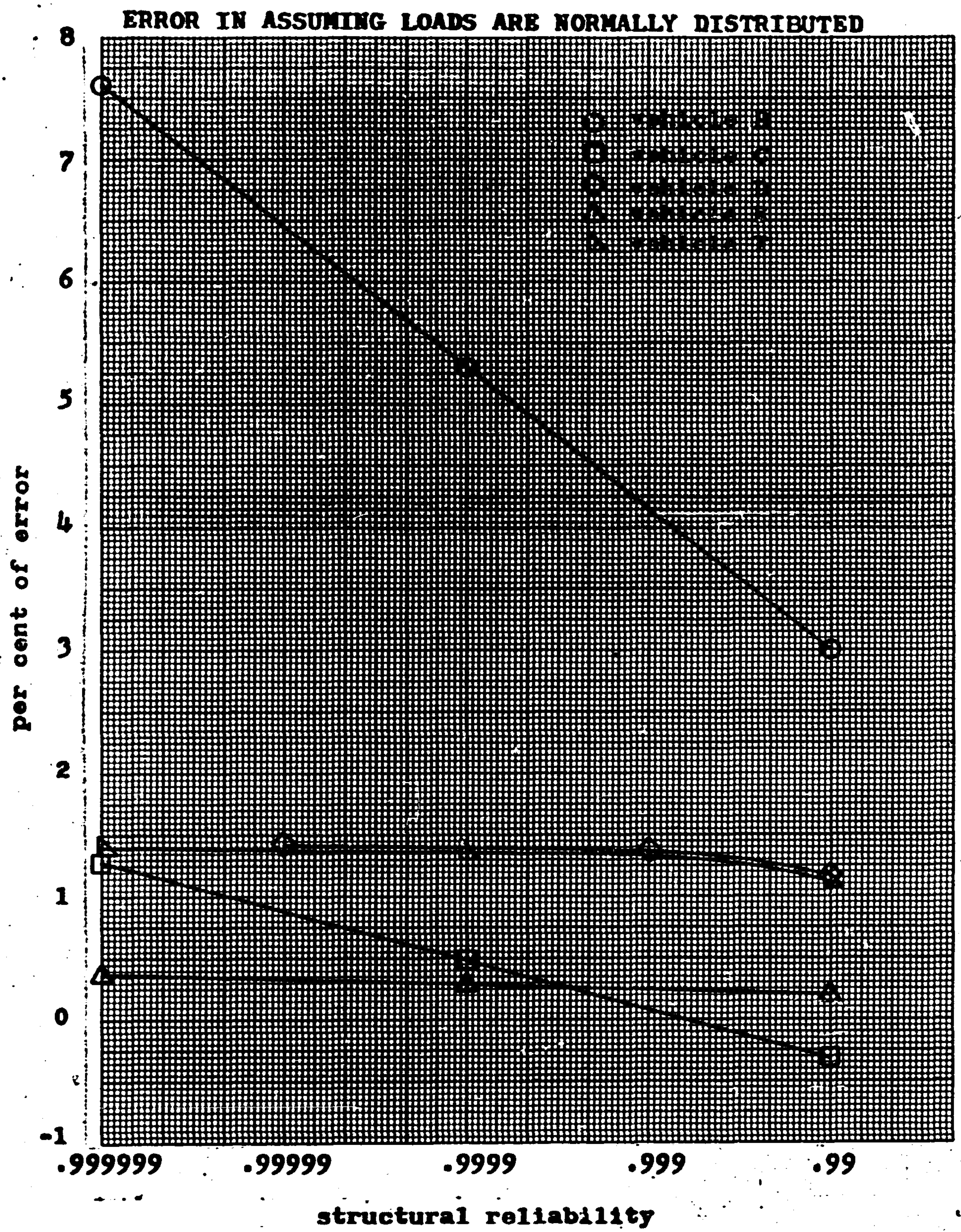

ERROR IN ASSUMING LOADS ARE NORMALLY DISTRIBUTED
per cent of error
8
7
6
5
4
3
2
1
0
-1
.999999
.99999
.9999
.999
.99
structural reliability

Figure 5-3

COL. 1.	COL. 2.	COL. 3.	COL. 4.	COL. 5
-0.	1.00000000	0.00001341	0.99999072	0.00001340
-4.20046997	0.99998659	0.00000121	0.99997206	0.00000121
-4.18072796	0.99998537	0.00000132	0.99992422	0.00000131
-4.16098595	0.99998406	0.00000143	0.99981308	0.00000142
-4.14124393	0.99998263	0.00000155	0.99957690	0.00000154
-4.12150192	0.99998108	0.00000168	0.99911390	0.00000168
-4.10175991	0.99997939	0.00000183	0.99827031	0.00000182
-4.08201790	0.99997757	0.00000198	0.99683182	0.00000197
-4.06227589	0.99997558	0.00000214	0.99452178	0.00000212
-4.04253387	0.99997345	0.00000232	0.99100842	0.00000230
-4.02279186	0.99997112	0.00000251	0.98592157	0.00000247
-4.00304985	0.99996861	0.00000271	0.97887763	0.00000265
-3.98330784	0.99996590	0.00000294	0.96950951	0.00000285
-3.96356583	0.99996296	0.00000318	0.95749743	0.00000304
-3.94382381	0.99995977	0.00000343	0.94259645	0.00000323
-3.92408180	0.99995634	0.00000371	0.92465728	0.00000343
-3.90433979	0.99995263	0.00000401	0.90363819	0.00000362
-3.88459778	0.99994862	0.00000433	0.87960758	0.00000381
-3.86485577	0.99994429	0.00000467	0.85273767	0.00000398
-3.84511375	0.99993962	0.00000504	0.82329119	0.00000415
-3.82537174	0.99993458	0.00000543	0.79160320	0.00000430
-3.80562973	0.99992914	0.00000586	0.75806031	0.00000443
-3.78588772	0.99992329	0.00000632	0.72307949	0.00000457
-3.76614571	0.99991697	0.00000679	0.68708813	0.00000466
-3.74640369	0.99991018	0.00000732	0.65050662	0.00000476
-3.72666168	0.99990285	0.00000788	0.61373400	0.00000484
-3.70691967	0.99989497	0.00000848	0.57713725	0.00000489
-3.68717766	0.99988649	0.00000911	0.54104380	0.00000492
-3.66743565	0.99987738	0.00000980	0.50573716	0.00000495
-3.64769363	0.99986757	0.00001054	0.47145532	0.00000496
-3.62795162	0.99985704	0.00001132	0.43839113	0.00000495
-3.60820961	0.99984572	0.00001215	0.40669438	0.00000494
-3.58846760	0.99983357	0.00001305	0.37647501	0.00000491
-3.56872559	0.99982052	0.00001399	0.34780706	0.00000487
-3.54898357	0.99980653	0.00001501	0.32073297	0.00000481
-3.52924156	0.99979152	0.00001609	0.29526801	0.00000475
-3.50949955	0.99977542	0.00001725	0.27140462	0.00000468
-3.48975754	0.99975818	0.00001848	0.24911650	0.00000460
-3.47001553	0.99973969	0.00001979	0.22836239	0.00000452
-3.45027351	0.99971990	0.00002118	0.20908936	0.00000443
-3.43053150	0.99969872	0.00002267	0.19123589	0.00000433
-3.41078949	0.99967605	0.00002424	0.17473433	0.00000423
-3.39104748	0.99965180	0.00002593	0.15951315	0.00000414
-3.37130547	0.99962588	0.00002772	0.14549865	0.00000403
-3.35156345	0.99959816	0.00002961	0.13261652	0.00000393

Table 5-1

-3.33182144	0.99956855	0.00003162	0.12079301	0.00000381
-3.31207943	0.99953693	0.00003376	0.10995583	0.00000371
-3.29233742	0.99950317	0.00003602	0.10003487	0.00000360
-3.27259541	0.99946715	0.00003843	0.09096275	0.00000349
-3.25285339	0.99942872	0.00004098	0.08267516	0.00000338
-3.23311138	0.99938774	0.00004368	0.07511114	0.00000328
-3.21336937	0.99934406	0.00004654	0.06821320	0.00000317
-3.19362736	0.99929752	0.00004958	0.06192736	0.00000307
-3.17388535	0.99924795	0.00005277	0.05620316	0.00000297
-3.15414333	0.99919517	0.00005617	0.05099361	0.00000286
-3.13440132	0.99913900	0.00005975	0.04625501	0.00000276
-3.11465931	0.99907925	0.00006354	0.04194698	0.00000266
-3.09491730	0.99901571	0.00006755	0.03803214	0.00000256
-3.07517529	0.99894816	0.00007177	0.03447606	0.00000247
-3.05543327	0.99887639	0.00007624	0.03124706	0.00000238
-3.03569126	0.99880014	0.00008095	0.02831603	0.00000229
-3.01594925	0.99871919	0.00008591	0.02565630	0.00000220
-2.99620724	0.99863328	0.00009115	0.02324338	0.00000212
-2.97646523	0.99854213	0.00009667	0.02105495	0.00000203
-2.95672321	0.99844546	0.00010248	0.01907054	0.00000195
-2.93698120	0.99834298	0.00010859	0.01727151	0.00000187
-2.91723919	0.99823438	0.00011504	0.01564083	0.00000180
-2.89749718	0.99811935	0.00012181	0.01416301	0.00000172
-2.87775517	0.99799754	0.00012893	0.01282390	0.00000165
-2.85801315	0.99786860	0.00013641	0.01161066	0.00000158
-2.83827114	0.99773219	0.00014428	0.01051158	0.00000151
-2.81852913	0.99758791	0.00015254	0.00951606	0.00000145
-2.79878712	0.99743538	0.00016120	0.00861439	0.00000139
-2.77904510	0.99727418	0.00017029	0.00779783	0.00000133
-2.75930309	0.99710388	0.00017983	0.00705840	0.00000127
-2.73956108	0.99692406	0.00018982	0.00638885	0.00000121
-2.71981907	0.99673424	0.00020029	0.00578263	0.00000115
-2.70007706	0.99653395	0.00021126	0.00523380	0.00000110
-2.68033504	0.99632269	0.00022274	0.00473692	0.00000105
-2.66059303	0.99609996	0.00023475	0.00428710	0.00000101
-2.64085102	0.99586521	0.00024731	0.00387992	0.00000095
-2.62110901	0.99561790	0.00026045	0.00351135	0.00000091
-2.60136700	0.99535745	0.00027417	0.00317774	0.00000086
-2.58162498	0.99508327	0.00028850	0.00287577	0.00000083
-2.56188297	0.99479477	0.00030348	0.00260247	0.00000079
-2.54214096	0.99449129	0.00031909	0.00235510	0.00000075
-2.52239895	0.99417220	0.00033539	0.00213122	0.00000071
-2.50265694	0.99383681	0.00035237	0.00192860	0.00000068
-2.48291492	0.99348444	0.00037008	0.00174523	0.00000064
-2.46317291	0.99311437	0.00038851	0.00157929	0.00000061
-2.44343090	0.99272586	0.00040771	0.00142911	0.00000058
-2.42368889	0.99231815	0.00042770	0.00129320	0.00000055
-2.40394688	0.99189045	0.00044848	0.00117020	0.00000052
-2.38420486	0.99146197	0.00047009	0.00105891	0.00000049
-2.36446285	0.99097188	0.00049256	0.00095818	0.00000047
-2.34472084	0.99047932	0.00051589	0.00086705	0.00000045

Table 5-1 (Continued)

-2.32497883	0.98996343	0.00054012	0.00078458	0.00000042
-2.30523682	0.98942330	0.00056527	0.00070994	0.00000039
-2.28549480	0.98885804	0.00059136	0.00064240	0.00000037
-2.26575279	0.98826668	0.00061841	0.00058129	0.00000036
-2.24601078	0.98764827	0.00064644	0.00052599	0.00000034
-2.22626877	0.98700183	0.00067548	0.00047595	0.00000032
-2.20652676	0.98632634	0.00070556	0.00043066	0.00000030
-2.18678474	0.98562078	0.00073668	0.00038968	0.00000028
-2.16704273	0.98488410	0.00076889	0.00035261	0.00000027
-2.14730072	0.98411521	0.00080217	0.00031907	0.00000025
-2.12755871	0.98331304	0.00083659	0.00028870	0.00000024
-2.10781670	0.98247645	0.00087213	0.00026124	0.00000022
-2.08807468	0.98160432	0.00090883	0.00023638	0.00000021
-2.06833267	0.98069549	0.00094671	0.00021389	0.00000020
-2.04859066	0.97974879	0.00098578	0.00019353	0.00000019
-2.02884865	0.97876301	0.00102606	0.00017512	0.00000018
-2.00910664	0.97773695	0.00106757	0.00015847	0.00000016
-1.98936462	0.97666938	0.00111033	0.00014338	0.00000016
-1.96962261	0.97555905	0.00115436	0.00012974	0.00000015
-1.94988060	0.97440469	0.00119966	0.00011740	0.00000013
-1.93013859	0.97320504	0.00124625	0.00010622	0.00000013
-1.91039658	0.97195879	0.00129414	0.00009612	0.00000012
-1.89065456	0.97066465	0.00134336	0.00008697	0.00000011
-1.87091255	0.96932129	0.00139389	0.00007869	0.00000010
-1.85117054	0.96792740	0.00144579	0.00007121	0.00000010
-1.83142853	0.96648160	0.00149900	0.00006443	0.00000009
-1.81168652	0.96498260	0.00155360	0.00005831	0.00000009
-1.79194450	0.96342900	0.00160954	0.00005275	0.00000008
-1.77220249	0.96181946	0.00166684	0.00004774	0.00000007
-1.75246048	0.96015262	0.00172551	0.00004320	0.00000007
-1.73271847	0.95842711	0.00178557	0.00003909	0.00000007
-1.71297646	0.95664154	0.00184696	0.00003536	0.00000006
-1.69323444	0.95479458	0.00190976	0.00003201	0.00000006
-1.67349243	0.95288482	0.00197392	0.00002895	0.00000005
-1.65375042	0.95091090	0.00203940	0.00002620	0.00000005
-1.63400841	0.94887149	0.00210627	0.00002371	0.00000004
-1.61426640	0.94676522	0.00217447	0.00002146	0.00000004
-1.59452438	0.94459075	0.00224403	0.00001942	0.00000004
-1.57478237	0.94234673	0.00231487	0.00001757	0.00000004
-1.55504036	0.94003186	0.00238703	0.00001590	0.00000004
-1.53529835	0.93764482	0.00246052	0.00001438	0.00000003
-1.51555634	0.93518431	0.00253522	0.00001301	0.00000003
-1.49581432	0.93264909	0.00261121	0.00001177	0.00000003
-1.47607231	0.93003788	0.00268842	0.00001065	0.00000002
-1.45633030	0.92734946	0.00276684	0.00000964	0.00000002
-1.43658829	0.92458262	0.00284641	0.00000873	0.00000002
-1.41684628	0.92173621	0.00292716	0.00000790	0.00000002
-1.39710426	0.91880905	0.00300905	0.00000715	0.00000001
-1.37736225	0.91580000	0.00309198	0.00000647	0.00000001
-1.35762024	0.91270802	0.00317594	0.00000586	0.00000001
-1.33787823	0.90953208	0.00326099	0.00000529	0.00000001

Table 5-1 (Continued)

-1.31813622	0.90627109	0.00334691	0.00000480	0.00000001
-1.29839420	0.90292418	0.00343381	0.00000434	0.00000001
-1.27865219	0.89949837	0.00352164	0.00000392	0.00000001
-1.25891018	0.89596873	0.00361025	0.00000355	0.00000001
-1.23916817	0.89235848	0.00369962	0.00000322	0.00000001
-1.21942616	0.88865886	0.00378978	0.00000291	0.00000001
-1.19968414	0.88486908	0.00388063	0.00000264	0.00000001
-1.17994213	0.88098845	0.00397208	0.00000238	0.00000001
-1.16020012	0.87701637	0.00406406	0.00000216	0.00000001
-1.14045811	0.87295231	0.00415663	0.00000195	0.00000001
-1.12071609	0.86879569	0.00424962	0.00000177	0.00000001
-1.10097408	0.86454687	0.00434302	0.00000159	0.00000000
-1.08123207	0.86020305	0.00443674	0.00000145	0.00000000
-1.06149006	0.85576631	0.00453059	0.00000131	0.00000000
-1.04174805	0.85123572	0.00462482	0.00000119	0.00000000
-1.02200603	0.84661090	0.00471899	0.00000107	0.00000000
-1.00226402	0.84189191	0.00481337	0.00000097	0.00000000
-0.98252202	0.83707855	0.00490765	0.00000088	0.00000000
-0.96278001	0.83217090	0.00500178	0.00000080	0.00000000
-0.94303801	0.82716912	0.00509571	0.00000073	0.00000000
-0.92329600	0.82207341	0.00518932	0.00000066	0.00000000
-0.90355400	0.81688389	0.00528292	0.00000060	0.00000000
-0.88381200	0.81160098	0.00537582	0.00000054	0.00000000
-0.86406999	0.80622515	0.00546840	0.00000049	0.00000000
-0.84432799	0.80075675	0.00556023	0.00000045	0.00000000
-0.82458598	0.79519652	0.00565159	0.00000040	0.00000000
-0.80484398	0.78954493	0.00574204	0.00000036	0.00000000
				SUM
				0.00027361

Table 5-1 (Continued)

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions were reached as a result of this study:

1). The distribution of maximum bending load on a vertically rising vehicle is best represented by the double exponential distribution,

2). The distribution of strengths is best represented by the normal distribution,

3). There is a significant variation in structural reliability for one of the five examples depending on which theoretical distributions are used in the calculations,

4). It is concluded that for structural reliability calculations it should be assumed that the loads are double exponentially distributed and strengths are normally distributed.

It is recommended that additional investigations be conducted on structural cylinders to determine a more accurate method of combining axial and bending stresses.

LIST OF REFERENCES

(1) Dow, Norris F., "Important Research Problems in Advanced Flight Structures Design - 1960," NASA TN D-518 (1960).

(2) Mangurian, George N., "The Aircraft Structural Factor of Safety," AGARD Report 154 (1957).

(3) Bert, C. W. and Hyler, W. S., "Design Considerations in Selecting Materials for Large Solid-Propellant Rocket-Motor Cases," DMIC Report 180 (1962).

(4) Ebner, H., "The Problem of Structural Safety with Particular Reference to Safety Requirements," AGARD Report 150 (1957).

(5) Küssner, H. G. and Thalau, K., "Die Entwicklung der Festigkeitvorschriften für Flugzeuge von den Anfängen der Flugtechnik bis zur Gegenwart," Luftfahrtforschung, Bd. 10 (1932), S. 1-54.

(6) Küssner, H. G., "Häufigkeitsbetrachtungen zur Ermittlung der Erforderlichen Festigkeit von Flugzeugen," Luftfahrtforschung, Bd. 12 (1935), S. 57-61.

(7) Pugsley, A. G. and Fairthorne, R. A., "Note on Airworthiness Statistics," ARC, London, R & M 2224 (1939).

(8) Weibull, W., "A Statistical Theory of the Strength of Materials," Ingeniors Vetenskaps Akadeniens, Handlingar Nr 151 (1939).

(9) Hilton, Harry H. and Feigen, Morris, "Minimum Weight Analysis Based on Structural Reliability," Journal of the Aerospace Sciences, Vol. 27, Number 9 (Sept., 1960), pp. 641-52.

(10) Freudenthal, A. M., "Safety, Reliability and Structural Design," Journal of the Structural Division, Proceeding of the ASCE, Vol. 87, No. ST 3 (March, 1961).

(11) Press, Harry, "The Application of the Statistical Theory of Extreme Values to Gust-Load Problems," NACA Report 991 (1949).

(12) Gumbel, E. J., Statistics of Extremes, New York: Columbia University Press (1960).

(13) Sturges, H. A., "The Choice of a Class Interval," Journal of the American Statistical Association, Vol. 21 (1926), pp. 65-66.

(14) Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences, New York: McGraw-Hill Book Company (1956).

(15) Sissenwine, Norman, "Wind Speed Profile, Wind Shear, and Gusts for Design of Guidance Systems for Vertical Rising Air Vehicles," Air Force Surveys in Geophysics, No. 57 (1954).

(16) Williams, J. J. and Bergst, G. L., Jr., "Design Wind Criteria for Air Force Missile Test Center," LMSD-2933, Lockheed Missile Systems Division (1958).

(17) Vaughan, William W., "Idealized Wind Profiles Applicable for the Study of Vehicle Performance Between 60 KM and 80 KM Altitude, Cape Canaveral (Atlantic Missile Range), Florida," NASA TN D-560 (1960).

(18) Smith, Orvel E., "A Reference Atmosphere for Patrick AFB, Florida, (Annual)," NASA TN D-595 (1961).

(19) Vaughan, William W., "Empirical Three - Sigma Wind Component Profiles 45 - Degree Missile Flight Azimuth Atlantic Missile Range Patrick AFB (Cape Canaveral), Florida," NASA TN D-606 (1961).

(20) Smith, J. W. and Vaughan, W. W., "Monthly and Annual Wind Distribution as a Function of Altitude for Patrick Air Force Base, Cape Canaveral, Florida," NASA TN D-610 (1961).

(21) Weaver, William L.; Swanson, Andrew G.; and Spurling, John F., "Statistical Wind Distribution Data for Use at NASA Wallops Station," NASA TN D-1249 (1962).

(22) Scoggins, James R. and Vaughan, William W., "Cape Canaveral Wind and Shear Data (1 through 80 KM) for use in Vehicle Design and Performance Studies," NASA TN D-1274 (1962).

(23) Sissenwine, N., "Panel Session Discussion," Proceedings of the National Symposium on Winds for Aerospace Vehicles Design, Vol. II, Air Force Surveys in Geophysics No. 140 (1962), p. 266.

(24) Bieber, R. E., "Prediction of Wind-Induced Missile Structural Loads," Proceedings of the National Symposium on Winds for Aerospace Vehicles Design, Vol. II, Air Force Surveys in Geophysics No. 140 (1962), pp. 11-30.

(25) Bieber, R. E., "Missile Structural Loads by Nonstationary Statistical Methods," Journal of the Aerospace Sciences, Vol. 28, No. 4 (April, 1961), pp. 284-94.

(26) Trembath, N. W., "Missile Control System Design Using Wind Statistics," Proceeding of the National Symposium on Winds for Aerospace Vehicles Design, Vol. II, Air Force Surveys in Geophysics No. 140 (1962), pp. 31-47.

(27) Van Der Maas, C. J., "Wind Shear Responses for Missile Systems," Lockheed Missile and Space Co., Report No. A003108A (March, 1962).

(28) Hobbs, Norman P., "Calculation of Wind Shear Loads on Missiles for Preliminary Design," presented at the AMS Atmospheric Problems of Aerospace Vehicles Conference held at Atlantic City, New Jersey (March, 1964).

(29) Dvoskin, Norman, and Sissenwine, Norman, "Evaluation of AN/GMD-2 Wind Shear Data for Development of Missile Design Criteria," Air Force Surveys in Geophysics No. 99 (April, 1958).

(30) Hobbs, Norman P., "Digital Computer Programs Relating to Wind Loads on Vertically Rising Vehicles," RTD-TDR-63-4004 (March, 1964).

(31) Weibull, W., Written discussion of paper titled, "Interpretation of Fatigue Test," ASTM Symposium on Statistical Aspects of Fatigue, ASTM Special Technical Publication No. 121 (1951).

(32) King, R. E., "Some Tension and Fatigue Data on Annealed 6AL-4V Titanium Alloy Sheets," Republic Aviation Corp., ERMR 4689 (1958).

(33) Anonymous, "Evaluation of Titanium Sheet Alloys," Grumman Aircraft Engineering Corp., ASTIA AD No. 248265 (Oct., 1960).

(34) Anonymous, "Properties of Titanium 6AL-4V Alloy Sheet Heat Treated to 160,000 psi Ultimate Tensile Strength," The Martin Co., Baltimore, ER 10120-6 (Feb. 6, 1959).

(35) Alesch, C. W. and Carpenter, S. R., "Titanium Development Program," Convair - SD, ASD TR 61-7-576 Final Technical Engineering Report (1961).

(36) Leach, A. E. and Clampett, H. A., Jr., "B120VCA Titanium Alloy Sheet Rolling Program," Crucible Steel Co. of America, Midwest Research Laboratory (March 31, 1961).

(37) McGee, W. M. and Matthews, B. R., "Determination of Design Data for Heat Treated Titanium Alloy Sheet," Lockheed-Georgia Co., ASD-TDR-62-335 (December, 1962).

(38) Anonymous, "Evaluation of New Titanium Sheet Alloys for Use in Airframe Construction," North American Aviation, Inc., Los Angeles, NA-57-184-10 through -14 (1960).

(39) Gunter, J. L., "Determination of Adaptability of Titanium Alloys," Boeing Airplane Co., Seattle, ASTIA No. AD-156058 (1958).

(40) Childs, J. K. and Lemcoe, M. M., "Determination of Materials Design Criteria for 6AL-4V Titanium Alloy at Room and Elevated Temperature," Southwest Research Institute, WADC TR 58-246, ASTIA No. AD-155863 (August, 1958).

(41) Peterson, J. J. and Young, S. G., "Evaluation of New High Strength Titanium Sheet Alloy," Chance Vought Corp., E1R-13121 (April, 1961).

(42) Beatty, Glenn H., Unpublished work sheets on the Titanium Sheet Rolling Program, data furnished by Crucible and Mallory-Sharon (1959).

(43) Beatty, Glenn H., Unpublished work sheets used to prepare DMIC Report 46J, "Statistical Analysis of Tensile Properties of Heat-Treated Ti-4AL-3MO-1V and Ti-2.5 Al-16V Sheet," (1961).

(44) Beatty, Glenn H., Unpublished work sheets used to prepare DMIC Report 46I, "Statistical Analysis of Tensile Properties of Heat Treated Ti-4AL-3MO-1V Sheet," (1960).

(45) Beatty, Glenn H., Unpublished work sheets on special TMCA data (April, 1960).

(46) Beatty, Glenn H., Unpublished work sheets on the Titanium Sheet Rolling Program, Battelle Memorial Inst., Project No. G-2610 (1961).

(47) Lumm, J. A. and Myers, D. E., "DOD High Strength Titanium Alloy Sheet Research Program," North American Aviation, Columbus Division, Report No. NA 57H-527 (September, 1962).

(48) Peterson, James P., "Bending Tests of Ring-Stiffened Circular Cylinders," NACA TN 3735 (1956).

(49) Batdorf, S. B.; Schildcrout, Murry; and Stein, Manuel, "Critical Stress of Thin-Walled Cylinders in Axial Compression," NACA Report 887 (1947).

(50) Harris, Leonard A.; Suer, Herbert S.; Skene, William T. and Benjamin, Roland J., "The Stability of Thin-Walled Unstiffened Circular Cylinders Under Axial Compression Including the Effects of Internal Pressure," Journal of the Aeronautical Sciences, Vol. 24, No. 8 (August, 1957), pp. 587-96.

(51) Suer, Herbert S.; Harris, Leonard A.; Skene, William T. and Benjamin, Roland J., "The Bending Stability of Thin-Walled Unstiffened Circular Cylinders Including the Effects of Internal Pressure," Journal of the Aeronautical Sciences, Vol. 25, No. 5 (May, 1958), pp. 281-87.

(52) Mossman, R. W. and Robinson, R. G., "Bending Tests of Metal Monocoque Fuselage Construction," NACA TN 357 (1930).

(53) Dow, Marvin B. and Peterson, James P., "Bending and Compression Tests of Pressurized Ring-Stiffened Cylinders," NASA TN D-360 (1960).

(54) Lundquist, E. E., "Strength Tests of Thin-Walled Duralumin Cylinders in Pure Bonding," NACA TN 479 (1933).

(55) Lo, Hsu; Crate, Harold and Schwartz, Edward B., "Buckling of Thin-Walled Cylinders Under Axial Compression and Internal Pressure," NACA Report 1027 (1949).

(56) Hoel, P. G., Introduction to Mathematical Statistics, New York: John Wiley and Sons (1954).

(57) Vosteen, Louis F., "Effect of Temperature on Dynamic Modulus of Elasticity of Some Structural Alloys," NACA TN 4348 (1958), p. 11.

(58) Smith, Robert W.; Hirschberg, Marvin H. and Manson, S.S., "Fatigue Behavior of Materials Under Strain Cycling in Low and Intermediate Life Range," NASA TN D-1574 (1963), p. 23.

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