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THE UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE

THE POROMECHANICS OF NATURALLY FRACTURED
ROCK FORMATIONS: A FINITE ELEMENT APPROACH

A DISSERTATION
SUBMITTED TO THE GRADUATE FACULTY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF
DOCTOR OF PHILOSOPHY

BY
RAJESH S. NAIR
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THE POROMECHANICS OF NATURALLY FRACTURED ROCK FORMATIONS: A FINITE ELEMENT APPROACH

A DISSERTATION APPROVED FOR THE
SCHOOL OF CIVIL ENGINEERING
AND ENVIRONMENTAL SCIENCE

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Dedicated to my family, friends, colleagues and teachers

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Abstract

Naturally fractured rock formations form important subsurface flow systems with a high degree of local heterogeneity. A basic differentiating characteristic of such formations is that a small volume of the medium (i.e., the fractures) transmits a large portion of the flow through it. The displacement of oil and gas from fractured reservoirs, estimated to contain a substantial amount of reserves is of importance to the oil and gas industry. In order to accurately predict production from such reservoirs, it is essential to understand fluid flow behavior in such systems. On the other hand, estimates of losses close to a billion dollars due to borehole instability, necessitate that drilling operations carried out in fractured rock formations be preceded by an accurate assessment of well-bore stability. This in turn requires an in-depth understanding of fully-coupled rock deformation and fluid flow. In the case of fractured rock formations, the presence of fractures increases the overall compliance of the rock-fluid system - a factor which may cause tensile effective stresses near the borehole, thereby affecting its stability during the initial drilling phase. Further, various field studies relating to reservoir compaction and subsidence problems (such as the Wilmington oil field, California; the Groningen gas field, Netherlands and; the Ekofisk field, North Sea) highlight the importance of accurately predicting flow and deformations. In addition, simulation of oil and gas recovery from fractured rock formations, reinjection of drill cuttings and reservoir compaction require fully-coupled multiphase flow and rock deformation, under isothermal as well as non-isothermal conditions, to be accurately modeled.

In this dissertation, an attempt has been made to develop a theoretically consistent model for fully-coupled multiphase flow in a fractured rock formation under non-isothermal conditions. The model has been systematically developed following a dual-porosity poromechanics approach wherein a fractured porous medium is envisaged as being composed of two distinct but overlapping media. The first medium represents the matrix and void spaces corresponding to matrix pores (primary porosity) whereas the second medium represents the fractures (secondary porosity), and a complementary solid part. Thus, the fluid flow and solid domains are both represented by two distinct but overlapping continua in this model with Barenblatt's original concept of two fluid pressures at a point and Aifantis's extension of Biot's poroelastic theory serving as basis. The intensity of interaction between the two media is controlled by fluid mass exchange rates, assumed to be proportional to the permeability of the primary medium and a quasi-steady pressure differential between the two media. Thermo-hydro-mechanical coupling has been incorporated by adopting the "single-temperature" approach wherein a single representative thermodynamic continuum is assumed to be sufficient to describe the temperature dependent response of a fractured formation. Further, the black-oil and limited compositional models have been incorporated to simulate fully-coupled oil and gas flow in a fractured formation under isothermal and non-isothermal conditions, respectively.

The mathematical models have been cast into finite element form and verified against analytical solutions for one-dimensional consolidation and an inclined wellbore in a fully saturated single-porosity formation subjected to pore pressure and thermal gradients. The displacements, pressures in the two media and the temperature (in the non-isothermal case) are the primary unknowns in the finite element model. The saturations and capillary pressures are the secondary unknowns obtained from saturation-capillary pressure-temperature (in the non-isothermal case) relations. Also, the relative

permeabilities, formation volume factors, gas-solubility ratios and oil volatility ratios are auxiliary unknowns estimated from relations involving the saturations and individual phase pressures. The resulting system of non-linear equations is solved using a direct solver and the stability is checked within each time-step. The finite element model has been extensively applied to the problem of an inclined wellbore and the sensitivity analyses carried out in this dissertation focus on the effect of thermal loading, heat transport by conduction and convection, secondary medium (representing the fractures) characteristics and, phase saturations on the pore pressures and effective stress distributions near the wellbore.

Land subsidence and fluctuation of water-table levels in the vicinity of a vertical well penetrating an aquifer have been studied employing a three-dimensional finite element model based on the mathematical equations developed herein. Two cases, one in which the aquifer is assumed to be fully saturated and, the other in which the aquifer is assumed to be unsaturated, are considered. The two-phase dual-porosity poroelastic model developed incorporating the black-oil model, has been modified in order to be applied to the unsaturated case. The effect of compliance and permeability of the secondary medium on the vertical displacements and water-table levels has been studied.

Chapter 1

Introduction

1.1 General Comments

The physicochemical environment of geologic systems is host to various coupled hydraulic, mechanical, thermal and chemical processes taking place at various rates, dependent on the nature and strength of the sources or driving energies available to the systems. Scientific interest in these coupled physicochemical processes have been aimed at understanding the hydraulic, mechanical, thermal and chemical response to technogenic perturbations such as drilling in chemically active (or inactive) formations, thermal and/or hydraulic fracturing of oil reservoirs, oil recovery by steam flooding, reinjection of drilling wastes into shallow formations, withdrawal of groundwater from aquifers and nuclear waste disposal (Sherwood, 1993; Clarke, 1949; Perkins and Krech, 1968; Jankovic, 1993; Olivella et al., 1994; Crookston et al., 1979; Coats et al., 1974; Kim and Parizek, 1999; Moschovidis et al., 1993; Scott et al., 1998; Noorishad and Tsang, 1996, Zaman et al, 2000).

Traditionally, geologic systems have been viewed as porous media composed of a deformable skeleton and one or more fluids saturating the pore space. An accurate assessment of the aforementioned coupled phenomena in porous media, requires a formulation of each with correct consideration of the major contributing factors (Noorishad and Tsang, 1996). To this end, phenomenological laws based on Onsager's theory have

been combined with a description of the porous medium (wherein the kinematics and deformations are referred to that of the skeleton), thereby leading to poromechanics models (De Groot, 1952; Coussy, 1995; Abousleiman, 2003).

In the case of fractured formations, the simulation of various coupled physicochemical processes poses a challenge, due in large part to the complex fracture geometries, irregular fracture spacing and lack of information regarding matrix-fracture interaction pertaining to fluid, heat and ion exchange. In addition, the presence of fractures weakens the system elastically and induces a “local preference” for flow and diffusion paths (Aifantis, 1977; Berryman and Wang, 2000). The discrete element, discrete fracture, single continuum and dual continua approaches are currently used for simulation of naturally fractured porous media. The discrete element method is a discontinuum modeling approach for simulating the behavior of fractured rock masses. The method was initially used to analyze the stability of fractured rock slopes (Cundall, 1971) and later extended to soils by using two-dimensional discs and three-dimensional spheres (Strack and Cundall, 1978). The medium is represented as an assemblage of blocks, discs or spheres interacting through contacts. The contact forces may be shear or normal forces and, the interaction between the discrete elements governed by simple force-displacement laws. Contacts can be formed or broken (when the normal force reduces to zero) and the contact forces are updated in each calculation cycle of an explicit time-finite difference scheme. Thallak et al. (1991a) incorporated the steady-state pore pressure response into this model to simulate hydraulic fracturing in granular media and, later applied the same model to borehole mechanics (Thallak et al., 1991b). However, most of the discrete element models developed do not account for the change in pore pressure due to change in the pore volumes. The discrete fracture approach, on the other hand, accounts for flow through the fracture network assuming negligible matrix permeability. This approach focuses on the flow through each individual fracture accounting for the location,

orientation, aperture variations and permeability. It is applicable when the fractures provide the essential porosity and permeability, thereby rendering the description of the fracture network crucial to reservoir simulations. However, this approach assumes that all fractures in the rock formation can be located exactly and, an explicit knowledge of fracture characteristics exists (Shapiro and Andersson, 1983). In both these approaches, accounting for thermal gradients, multiphase flow and rock-fluid interaction is a challenging task. Hence, an alternative continuum based conceptualization of the medium is required, especially in the case of a densely fractured rock formation.

In the single continuum approach, the fracture system is represented by an effective permeability tensor which takes into account the anisotropy arising from the fracture distribution and flow coupling between the matrix and fractures. However, this approach is applicable when the fractures are discrete or disconnected (Nakashima et al., 2001). In the dual porosity/dual permeability approach, two co-located, distinct continua are assumed to exist, representing the fractures and the matrix. The approach takes into account the flow in the fractures as well as the matrix and, the matrix-fracture interaction. The dual porosity/dual permeability approach is applicable when the fractures are well developed to form a flow network thereby providing the essential permeability, in contrast to the matrix which provides the essential porosity (Ganzer, 2002; Hsieh, 2002). Instead of dealing with the intricacies of individual fractures and pore spaces, average characteristics and responses of the medium are hypothesized to exist for both the fractures and the porous matrix at each point in the domain. Dual-porosity models provide a convenient and “least-damaging” approach to characterizing the coupled thermo-hydro-mechanical response and multiphase flow, especially in case of densely fractured rock formations (Abousleiman, 2003). In this dissertation, the coupled physical processes (i.e., thermal, hydraulic and mechanical) in naturally fractured media will be addressed following the dual-porosity approach.

1.2 Background

Some examples of both ancient and modern technogenic perturbations of geologic systems are - building of the first reservoir on the river Nile 5000 years ago, the first oil well drilled south of Baku, Azerbaijan in 1848, early hydraulic fracturing (then known as “pressure parting”) of formations reported by Yuster and Calhoun (1945) and, development of nuclear waste repositories in the Yucca mountains (www.hydrocomp.com/journal4.htm; www.ee-environment.net/countries/aze.shtml; Clark, 1949; www.ymmp.gov). Understanding the implications of especially the early projects would have been hindered by a lack of knowledge about the coupled phenomena that would have been induced or altered. Biot’s theory of poroelasticity serves as a cornerstone for recent scientific assessment of the coupled physical processes. Aifantis’s extension of Biot’s theory provides a basis for simulation of the coupled processes in naturally fissured formations. Although considerable scientific literature exists pertaining to simulation of single-phase fluid flow under isothermal conditions in a fractured porous medium, the coupled deformation-flow processes under non-isothermal conditions in a fractured porous medium saturated with a single or multiple fluids have received little attention. The attempt in this dissertation will be towards a mathematically rigorous formulation of a model to simulate fully-coupled, two-phase fluid flow and heat transport in a fractured porous medium. The parameters governing the response of such a medium to technogenic perturbations will be analyzed carrying out a sensitivity analysis.

1.3 Objectives

The objectives of this dissertation are:

1. Develop a dual-porosity model incorporating fully coupled flow in both, the primary medium (representing the matrix and pores) and the secondary medium (representing the fractures and a complimentary solid).

2. Develop a finite element model based on the aforementioned mathematical formulation in order to simulate fully coupled flow in a naturally fractured formation.
3. Incorporate the black oil model into the dual-porosity formulation in order to consider two-phase (oil and gas flow) in a fractured porous medium.
4. Develop a dual-porosity porothermoelastic model taking into account fully coupled fluid flow and heat transport in both media. Based on the mathematical formulations a finite element model will be developed to facilitate simulation of coupled thermal, hydraulic and mechanical processes in a fractured porous medium.
5. Incorporate the limited compositional model into the dual-porosity porothermoelastic model in order to simulate fully-coupled non-isothermal flow of oil and gas in a fractured formation.
6. Validate the aforementioned models against relevant analytical solutions.
7. Carry out a sensitivity analysis to identify the parameters affecting the thermo-hydro-mechanical response of a fractured formation subjected to perturbations.

1.4 Literature Review

1.4.1 Fully-Coupled Fluid Flow in Porous Media

The theory of isotropic linear poroelasticity introduced by Biot (1941) has been the basis for modern scientific analysis of the coupled diffusion-deformation phenomenon which characterizes the response of fluid saturated porous media. Work by Biot and Willis (1957), Geertsma (1957), Nur and Byerlee (1971) and Rice and Cleary (1976) have provided a clear understanding of the physical meanings of the Hookean, micromechanical and bulk engineering constants inherent in this theory. Based on the generalized Hooke's law, the isotropic theory was extended to an anisotropic one by Biot (1955). A practical

model for laboratory measurement has been developed wherein the material coefficients of Biot's anisotropic poroelasticity have been reinterpreted under assumptions of micro-homogeneity and micro-isotropy, following micromechanical considerations (Cui, 1995; Cheng, 1997).

An alternative approach (in contrast to Biot's macroscale theory), adopted while dealing with the mechanics of a deformable porous medium, is based on the theory of mixtures. In mixture theories, the porous medium is represented by spatially superposed interacting media. Pioneering work by Truesdell (1962), Atkin and Craine (1976) and, Bowen (1982) have been the basis for later work by Coussy et al. (1998) wherein a Eulerian description of the deformation and kinematics of the solid and fluid constituents was facilitated by application of an averaging procedure (spatial convolution method) at the microscale. Further, a link between the two approaches was established by Coussy et al. (1998) by reformulating the macroscale equations from mixture theories in terms of measurable quantities involved in Biot's theories.

Biot's isotropic and anisotropic theories have been the basis for assessment of stress and pore pressure distributions, in the vicinity of an inclined borehole and a cylinder subjected to loading conditions encountered in the laboratory (Cheng, 1998; Abousleiman and Cui, 1998; Abousleiman and Kanj, 2002; Kanj et al., 2002; Ekbote et al., 2002). Investigations of groundwater level fluctuations while pumping from an aquifer (Verrijt, 1969; Kim and Parizek, 1997; Wang, 2000), consolidation settlement of bridge approach foundation (Zaman et al., 1991), modeling of porous noise control materials (Kang et al., 1999), modeling three-dimensional fracture propagation under a specified wellbore pressure (Lam and Cleary, 1988), bone poroelasticity (Cowin, 1999), study of wave propagation in poroelastic media (Degrande et al., 1998), rheological characterization of fibroblastic biological collagenic gels (Naili et al., 1998), analysis of mine collapses (Grgic et al., 2002), micromechanical analysis of brittle materials (Kondo et

al., 2002) and, quasi-static and dynamic rock characterization (Scott and Abousleiman, 2002a and 2002b) are some of the diverse areas where Biot's theories have found recent application. The work carried out by Biot (1956), Coussy (1989) and Abousleiman et al. (1993) pertain to extending Biot's poroelastic theory to poroviscoelasticity and poroelastoplasticity. A more comprehensive list of publications in this regard may be found in Coussy (1995).

1.4.2 Coupled Thermo-Hydro-Mechanical Processes in Porous Media

The development of geothermal energy projects and nuclear waste repositories has led to an increasing interest in the coupled thermal, hydraulic and mechanical processes in geologic media. In the petroleum industry, in situ extraction of oil by steam injection has necessitated an understanding of fluid injection and fracture propagation processes under non-isothermal conditions (Agar et al., 1983). Laboratory measurements of thermal conductivity, permeability and rock deformations under high temperatures and pressures (van Buskirk, 1985; Voegele and Brace, 1985; Brodsky et al., 1985) have been pioneered by the early works of Griggs et al. (1960) and Paterson (1970). Laboratory experiments carried out subjecting rock samples to elevated temperatures and pressures have highlighted the impact of coupled thermo-hydro-mechanical processes on the strength and modes of failure (Althaus et al., 1994). The coupled thermal, hydraulic and mechanical processes have been accounted for by extensions of Biot's isothermal theory that incorporate thermal expansion of both the pore fluid and solid skeleton (Schiffman, 1971; Brownell et al., 1977; Bear and Corapcioglu, 1981; Palciauskas and Domenico, 1982). Booker and Savvidou (1984, 1985) presented the analytical solutions for consolidation of a medium comprised of incompressible constituents around a spherical and point heat source. Later, the governing equations for fluid-saturated, porous, linear thermoelastic media were developed and specialized to one-dimensional deformation. Analytical solu-

tions for a porothermoelastic half space subjected to constant temperature and heat flux boundary conditions, as well as for drained and impermeable fluid boundary conditions were obtained (McTigue, 1986). Kurashige (1989) obtained analytical solutions for the problem of hot (or cold) water injection from a spherical cavity into an infinite porous medium. Also, numerical solutions accounting for convective heat flow in the absence of conduction were presented. Axisymmetric analytical solutions for pore pressure and temperature around a heated wellbore (with a constant heat flux boundary condition) in a poroelastic medium were developed by McTigue (1990). The correspondence between thermoelastic variables and material parameters and their poroelastic counterpart was elucidated by Norris (1991). The issue of estimating effective poroelastic parameters for an inhomogeneous medium was translated into the analogous problem for a thermoelastic medium (Norris, 1991). Employing the Crank-Nicholson implicit scheme, Kurashige (1992) and Kodashima and Kurashige (1996) provided solutions for thermally induced stresses in a poroelastic cylinder and hollow sphere, respectively. Wang and Papamichos (1994) provided analytical solutions for coupled fluid flow and heat diffusion from an injection well into a two-dimensional, plane strain poroelastic medium under non-hydrostatic loading. A porothermoelastic model with thermo-osmosis and thermal-filtration along with solutions for consolidation of one-dimensional soil column subjected to a temperature boundary condition was presented by Zhou et al. (1998). The aforementioned theoretical development and analytical solutions have culminated in the recent work of Abousleiman and Ekbote (2002) wherein analytical solutions for the problem of an inclined borehole subjected to a three dimensional in-situ state of stress in a transversely isotropic medium, have been obtained.

1.4.3 Fluid Flow in Naturally Fractured Porous Media

Fluid flow in naturally fractured porous media is often modeled by the dual-porosity approach which is based on Barenblatt's original idea of representing the fluid domain

by two overlapping continua (*"If the system of fissures is well developed, the motion of the liquid can be investigated by the following method. Unlike the classical seepage theory, for each point in space, not one liquid pressure but two, p_1 and p_2 are introduced. The pressure p_1 represents the average pressure of the liquid in the fissures in the neighborhood of the given point, while the pressure p_2 is the average pressure of the liquid in the pores in the neighborhood of the given point"* - Barenblatt et al. (1960)). In Barenblatt's model, the interporosity flow between the matrix and fractures was characterized by a transfer function expressed in terms of the quasi-steady pressure difference between the matrix and fractures, matrix permeability and a fracture spacing parameter reflecting the geometry of the matrix blocks.

Warren and Root (1963) introduced this concept to the petroleum industry when they presented a model for well transient testing wherein, the matrix rock was represented as homogeneous, isotropic, rectangular blocks onto which a network of continuous, uniform, orthogonal fractures was superimposed. The matrix blocks were assumed to act as fluid sources for the fractures that provided conduits for fluid flow toward the well. However, fluid flow between the matrix blocks was neglected. Odeh (1964) attempted to remove assumptions of orthogonal fractures in his model which was otherwise similar to that developed by Warren and Root (1963). Subsequently, a "layer cake" model, wherein thin highly permeable layers representing the fractures alternated with thin layers of lower permeability representing the matrix, was developed by Kazemi (1969). Kazemi (1969) also investigated the validity of assuming the quasi-steady state assumption for matrix-fracture flow in his work. A model incorporating flow in both matrix and fractures was developed to simulate single-phase, slightly compressible fluid flow through fractured porous media by Duguid and Lee (1977). The fractured porous medium was treated as an elastic incompressible solid containing two different porosities. Two sets of governing equations for flow - one for each type of porosity, were formulated. These two sets of

equations were coupled by incorporating interaction between fluid in the primary pores and fractures. A finite-element procedure based on the Galerkin method was used to solve this system of equations. The model was then applied to the problem of transient flow of water in a confined leaky aquifer.

1.4.4 Multiphase Flow in Fractured Porous Media

One of the earliest models for multiphase flow in fractured formations was developed by Mattax and Kyte (1962) to predict imbibition oil recovery from matrix blocks in water-drive reservoirs. Yamamoto (1971) developed a compositional oil-gas simulator accounting for interphase mass transfer between oil and gas as well as interporosity flow between a given matrix block and surrounding fractures. Braester (1972) developed a model in which the flow between matrix and fractures was expressed in terms of the potential gradient in the fractures, capillary pressure difference between the matrix and fractures and, the density difference between the liquid phases. Kazemi (1976) developed a three-dimensional two-phase (oil and water) flow simulator for fracture reservoirs. The equations developed in his work were two-phase extensions of the single-phase flow equations used in the model proposed by Warren and Root (1963). The model developed by Rossen (1977) accounted only for flow in the fracture system and considered the rock matrix blocks as sources. Interporosity flow was modeled using semi-implicit source terms which were functions of rock matrix and fluid properties, with fracture saturations and pressures defining the boundary conditions. In the early 80's, a sophisticated three-dimensional, three-phase model for simulating water, oil and gas flow in a naturally fractured reservoir was developed by Thomas et al. (1983). The reservoir was assumed to be comprised of a continuous fracture system filled with discontinuous matrix blocks. Most of the flow in the reservoir was assumed to be through the fracture system. Local transfer of fluids between the fractures and matrix blocks was also accounted for. However, fluid transfer between the matrix blocks was neglected.

This limitation was overcome in the dual-porosity, dual-permeability models developed by Gilman (1986) and Wu and Pruess (1988). Later, Choi et al. (1997) proposed a dual-porosity/dual-permeability model wherein Forchheimer's equation was used to describe non-Darcian flow through the fractures. Darcy's law was assumed to hold for the flow in the matrix. The behavior of the model for different values of matrix permeability, fracture width and fracture spacing was highlighted.

1.4.5 Dual-Porosity Poroelasticity

The aforementioned models for single and multiple-phase flow in fractured formations either treated the formation as a rigid body or ignored the coupling between fluid flow and deformation. Aifantis (1977) was the first to combine Barenblatt's concept with Biot's theory of linear isotropic poroelasticity thereby providing a viable means to overcome this limitation. In Aifantis' theory, quasi-static constitutive equations linearly relate the strains to the stress and pore pressures in both the matrix pores as well as the fissures. The constitutive equations are then combined with the equilibrium equation. Time-dependent quasi-static fluid flow is incorporated by combining the continuity equations with Darcy's law (for fluid flow in both, the matrix as well as fractures) (Wilson and Aifantis, 1982). Analytical solutions for the problem of one-dimensional consolidation and an axisymmetric solution for the borehole problem were provided by Wilson and Aifantis (1982). Based on Galerkin's version of the weighted residual method, a finite element model was developed by Khaled et al. (1984) to obtain numerical solutions of Aifantis's equations of dual-porosity poroelasticity. Numerical solutions for the problems of one-dimensional consolidation, two-dimensional layer and two-dimensional half-space (refer to Khaled et al., 1984, for a detailed description of the problems) were presented to illustrate the method and assess the impact of dual-porosity parameters on the response of the fractured porous medium.

Berryman and Wang (1995) attempted to identify the coefficients in the govern-

ing equations for a dual-porosity medium. Acknowledging the fact that some of the parameters may not be readily measured in the laboratory, they attempted to clarify the significance of these parameters on the dual-porosity response of a fractured porous medium. Their procedure was essentially an extension of the approach employed by Biot and Willis (1957) for a single-porosity medium. Analyzing the response of a fractured rock sample subjected to a series of drained and undrained (albeit gedanken) tests, the parameters relating the change in volumetric strains to change in matrix and fracture pore pressures were identified. Their work provided an insight into the difficulty in parametrization of a dual-porosity medium and the choice of time-scales to be considered while deducing Skempton-like or drained parameters.

Tuncay and Corapcioglu (1995) presented a formulation similar to that of Wilson and Aifantis (1982) and derived an effective stress principle for dual-porosity media based on the volume averaging concept. Later, analytical solutions for the problem of one-dimensional consolidation of a dual-porosity medium were presented by Lewallen and Wang (1998) wherein the approach outlined by Berryman and Wang (1995) was followed. Callari and Federico (2000) carried out one-dimensional consolidation experiments on artificially structured clayey soils. The soils were modeled as dual-porosity media and, based on a formulation identical to that of Wilson and Aifantis (1982), a finite element model was developed. The numerical results from the finite element calculations were then compared with the experimental results and a satisfactory agreement between the two was observed.

1.4.6 Multiphase Poroelasticity

A model for two-phase fluid flow in a deforming porous medium was developed by Li et al. (1990). The flow-deformation coupling was taken into account on basis of Biot's theory. The effect of matrix and fluid compressibilities, interphase mass exchange and capillarity were also considered. The viscosities were assumed to be functions of the respective fluid

pressure and temperature. The relative permeabilities and fluid compressibilities were assumed to be dependent on the saturations. The solid displacements, pressure of the wetting phase and saturation of the wetting phase were the primary unknowns in the Galerkin-based finite element equations. The model was applied to the problem of one-dimensional consolidation of a poroelastic medium saturated with two fluids. Next, numerical calculations for the stress, fluid pressures and saturation distributions during injection of water into an oil formation were carried out. Li and Zienkiewicz (1992) adopted the same approach as Li et al. (1990) and discussed the direct solution method as well as a staggered solution technique for the finite element equations. Li and Fan (1997) presented a finite element model for analysis of transient deformation and seepage in unsaturated soils. An elasto-plastic constitutive model was used to describe the non-linear mechanical behavior of the soil. The model was applied to the problem of a flexible footing resting on a strip of unsaturated soil and, contours of the saturation and pore water pressure were obtained. The hydro-mechanical behavior of unsaturated soils was also studied by Klubertanz et al. (1997) who developed a fully coupled two-phase (liquid and gas) finite element model. The model was used to simulate Liakopoulos' (1965) laboratory experiments of vertical drainage of water through a sand column. The finite element calculations for water outflow and water pressure were compared with the experimental results. A satisfactory agreement between the two was observed after 20 minutes. Prior to this time, the experimental results did not agree very well with the finite element calculations. However, employing a single-phase finite element model yielded a better match for the initial stages of the experiment (Klubertanz, et al., 1997). It was concluded that since the soil was saturated at the beginning of the experiment the air phase would have been discontinuous (if at all present) and hence a two-phase model was not applicable to model the initial stages of the experiment. A similar conclusion for the same problem was drawn by Gawin et al. (1997).

1.4.7 Multiphase Dual-Porosity Poroelasticity

Lewis and Ghafouri (1997) developed a three-dimensional finite element model for multiphase (oil, water and gas) flow in a deforming fractured porous medium. The Galerkin-based finite element method was used to discretize the governing equations in the space and time domains. The fractured porous media was visualized as two overlapping continua. The first one represented the deforming porous matrix and the second represented non-deforming fractures, i.e., the compressibility of the fractured porous medium was assumed to be not affected by that of the fracture network. The solid displacements and fluid pressures were treated as the primary unknowns in the finite element model. The solution for the primary unknowns for each time level was obtained in an iterative fashion with the coefficient matrices associated with the non-linear system of equations being updated during each iteration. Nair et al. (2000 and 2003b) developed a finite element model for two-phase (oil and gas) flow in a dual-porosity poroelastic medium. In this model, the solid and fluid domains were both represented by two distinct and overlapping continua thus overcoming the theoretical inconsistency of representing the solid domain with a single continuum (Masters et al., 2000). The model accounted for fully coupled two-phase flow in both continua. The solid displacements and pore pressures were the primary variables in the finite element model. The non-linear set of equations was solved by a direct method to obtain the solutions for the primary variables at each time level. Since the coefficient matrices in the non-linear system of equations are dependent on the unknowns, an iterative procedure was employed within each time-step. At each iteration level, the nonlinear coefficients were updated using the values of the unknowns calculated in the previous iteration. The stability of the solution was monitored using a convergence criterion (Lewis and Ghafouri, 1997; Nair et al., 2000). The finite element calculations for the problem of one-dimensional consolidation and an inclined borehole in a saturated single-porosity poroelastic medium were compared with

the analytical solutions (Cui et al., 1997; Ekbote et al., 2002). An excellent agreement between the finite element and analytical solutions was observed. The model was then used to investigate the effect of initial phase saturations and dual-porosity parameters on the stress and pore pressure distributions in the vicinity of the inclined borehole.

1.4.8 Coupled Thermo-Hydro-Mechanical Processes in Fractured Porous Media

Masters et al. (2000) extended the dual-porosity poroelastic model developed by Ghafouri and Lewis (1996) to incorporate thermal effects. As in the model developed by Ghafouri and Lewis (1996), the fluid domain was represented by two continua, representing the deformable porous matrix and the non-deformable fracture network. A single thermodynamic continuum was assumed to be representative of the matrix and fracture network. The governing equations were discretized in the space and time domains by the Galerkin-based finite element method. The finite element model was then applied to the problem of non-isothermal consolidation of a one-dimensional column. An earlier model for heat extraction from hot dry rocks was proposed by Aifantis and Beskos (1980) wherein the “double temperature” approach analogous to Aifantis’s (1977) multiporosity approach was proposed. The thermal state of the system was characterized by two temperature fields corresponding to the solid and fluid temperatures. The solid deformation was ignored and two energy balance equations (one for the solid and the other for the fluid) were introduced with a term corresponding to the fluid-solid thermal interaction. However, Aifantis and Beskos (1980) did not consider fluid flows in the matrix and fractures separately. Also, distinct porosities characterizing the storage capacities of the matrix and fractures were not considered. Hence, in the author’s view it is questionable as to whether their model can be viewed as a non-isothermal dual-porosity model in the classical sense (as opposed to the comment made by Masters et al., 2000). Indeed, the final energy equations of Aifantis and Beskos (1980) have only a *form similar* to that

of fluid flow equations for a dual-porosity medium of Aifantis (1977) and must not be interpreted to be equations of a non-isothermal dual-porosity model.

The “single temperature” approach of Masters et al. (2000) was adopted by Nair et al. (2002 and 2003a) who developed a dual-porosity porothermoelastic model on lines of the model developed by Nair et al. (2000 and 2003b). Heat transport by conduction as well as convection was considered in this model. The model was verified against the analytical solutions (Abousleiman and Ekbote, 2002) for an inclined borehole subjected to pore pressure and temperature boundary conditions. The effects of convection, dual porosity parameters and thermal loading on the pore pressures and stresses were subsequently analyzed.

1.4.9 Fully-Coupled Non-Isothermal Multiphase Flow in Porous Media

To the author’s knowledge, the issue of non-isothermal multiphase flow in a deforming, fractured porous medium has not yet been addressed. However, non-isothermal multiphase flow in a deforming porous medium has received attention due to active consideration of nuclear waste disposal schemes, oil recovery techniques involving steam injection and thermal insulation of buildings. Thomas and He (1995) presented the theoretical formulation for the analysis of coupled heat, moisture and air flow applicable to deforming unsaturated soils. The unsaturated soil was viewed as a three “phase” system comprising of a solid non-linear elastic skeleton (solid phase), pore water (liquid phase) and a mixture of water vapor and dry air (gas phase). Heat transport by conduction and convection as well as latent heat flow in the soil water vapor were considered in the model. The governing differential equations were solved using the Galerkin-based finite element method for spatial discretization and a finite-difference based time-stepping scheme. A plane strain analysis of heating of a montmorillonite clay block was carried out using the finite element model and the calculations for pore pressure, water saturation and

temperature distributions were compared with experimental results for the same problem. Gawin and Schrefler (1996) developed a finite element model for fully coupled heat, water and gas flow in a deforming porous medium, based on the work of Gawin et al. (1995). The gas phase was modeled as an ideal gas comprising of dry air and water vapor. Phase changes of water, i.e. evaporation and condensation, heat transfer through conduction as well as convection and, latent heat transfer were accounted for in this model. The Galerkin-based finite element method was used to discretize the governing equations in space, and the time-domain was discretized by a fully implicit backward difference scheme. The gas pressure, capillary pressure, temperature and displacements were the primary unknowns in the finite element model. The experimental results obtained by Liakopoulos (1965) for isothermal drainage of water from a vertical column of sand were used to partially validate the model. The model was then applied to the problem of drying of a concrete wall subjected to a rapid variation of capillary pressure and uniform external loading. Pao et al. (2001) incorporated thermal effects into the model developed by Lewis and Schrefler (1987) and developed a finite element model for fully coupled, non-isothermal, three-phase (oil, water and gas) flow in a deforming porous medium. The mechanisms of heat transport considered in this model were conduction and convection. A finite element spatial discretization of the governing equations was carried out using Galerkin's method. Discretization in the time domain was carried out using a variable- θ scheme (Pao et al., 2001). The displacements, pore pressures and temperature were the primary unknowns in the finite element model whose solutions were obtained using a relaxation based iterative method. The model was first applied to the problem of a reservoir composed of Berea sandstone, in which an injector and a producer (wells) were located. Water at a temperature higher than that of the formation was injected and the resulting pore pressure and temperature profiles were analyzed. The effect of convective heat transport was analyzed by comparing the results with

those obtained by consideration of conductive heat transport alone. Next, the model was applied to the non-isothermal Mandel problem and non-isothermal one-dimensional consolidation of a partially saturated soil column. The finite element calculations of the displacements and pore pressures for the non-isothermal Mandel problem were identical to the isothermal case. Pao et al. (2001) tended to concur with Masters et al. (2000) view that the displacements and pore pressure are insensitive to temperature effects. It must be emphasized that this is not in agreement with either the results obtained from the finite element calculations (Nair et al., 2002) or analytical solutions (Abousleiman and Ekbote, 2002), for the inclined borehole problem, wherein a significant impact of thermal loading was observed.

1.4.10 Summary of Literature Review

On basis of the literature review conducted, it can be pointed out that fully coupled two-phase flow (isothermal and non-isothermal) in fractured porous media has received little attention. Difficulty in the mathematical formulation and understanding of the phenomena of heat and mass transport in fractured porous media has hampered modeling even single-phase, non-isothermal flow in such media. The numerical complexity introduced by convective terms which couple the fluid flow and energy equations further complicate the problem. The issue of whether to use one or two thermodynamic continua, while modeling non-isothermal flow in fractured porous media, needs to be rationalized further. Also, formulations for two-phase non-isothermal flow in fractured porous media have not appeared in literature to date. In view of these facts, the goal of this dissertation is set to development of a theoretically consistent dual-porosity model, facilitating accurate consideration of single and two-phase flows, under isothermal as well as non-isothermal conditions. The development of a finite element code based on the mathematical formulations is carried out with an aim to provide scientists and engineers a tool to handle complex geometries and boundary conditions. As a final note to

this section, the author would like to mention that the books by Zienkiewicz and Taylor (1989 and 2000), Desai and Abel (1987), Bathe (1996) and Reddy (1993), provide an adequate explanation of the basics, as well as illustrate the flexibility, of the finite element method.

1.5 Dissertation Outline

In Chapter 2, a dual-porosity poroelastic model has been developed based on Barenblatt's original concept of two fluid pressures at a point and Aifantis's extension of Biot's poroelastic theory. The fluid and solid domains are both represented by two distinct but overlapping continua representing the matrix and pores (primary medium) and, the fractures and a complementary solid (secondary medium). Separate effective stress and constitutive laws are implemented for both continua, in order to facilitate a quantification of the individual deformations while analyzing the response of a fractured porous medium to perturbations. A finite element model has been developed based on the mathematical formulations and verified against analytical solutions for the problems of one-dimensional consolidation and an inclined borehole in a fully saturated poroelastic medium.

In Chapter 3, a dual-porosity porothermoelastic model has been developed which accounts for thermo-hydro-mechanical coupling in both, the primary as well as the secondary medium. The model relies on the "single temperature" approach, i.e., a single representative thermodynamics continuum is assumed to be sufficient to describe the temperature dependent response of a fissured porous medium. The model accounts for heat transport through conduction as well as convection. Based on the mathematical formulation, a finite element model has been developed and verified against analytical solutions of an inclined wellbore in a fully saturated single-porosity medium subjected to pore pressure and thermal gradients. Further, a sensitivity analysis has been carried

out to study the impact of thermal loading and, the effect of convection and secondary medium characteristics on the stress, pore pressure and temperature profiles in the vicinity of a wellbore drilled in a fissured porous medium.

In Chapter 4, the dual-porosity poroelastic model presented in Chapter 2, has been extended to consider the simultaneous flow of a two-phase fluid system, comprised of oil and gas. The black-oil model, applicable to low-volatile oil systems, is incorporated into the formulation in Chapter 2 and, differential equations governing deformations of the primary and secondary media coupled with two-phase (i.e., oil and gas) flow are obtained. In the finite element model developed subsequently, the solid deformations and fluid pressures are treated as the primary unknowns while, the saturations are the secondary unknowns obtained from the capillary-saturation relations. The model is then applied to the problem of an inclined wellbore in a fractured porous medium saturated with oil and gas, and a sensitivity analysis is carried out to identify the effect of initial phase saturations and secondary medium characteristics.

In Chapter 5, the mathematical formulations in Chapters 3 and 4 are extended to formulate fully-coupled non-isothermal oil-gas flow in fractured porous media. The limited compositional model accounting for volatility of the oil has been incorporated into the formulation. The fluid and solid domains are represented by two distinct overlapping continua as in Chapters 2-4 and the “single temperature” approach in Chapter 3 has been adopted. The displacements, pressures (in the oil and gas phases) and temperature are the primary unknowns in the finite element model developed based on the mathematical formulation. The saturations and capillary pressures are the secondary unknowns to be obtained from capillary pressure-saturation-temperature relations. The case of an inclined wellbore in a fractured porous medium saturated with oil and gas and, a temperature gradient between the drilling and formation fluids has been analyzed. The parameters influencing the spatial and temporal distributions of pore pressure, stresses

and temperature in the vicinity of the wellbore have been identified by carrying out a sensitivity analysis.

In Chapter 6, the land subsidence and reduction in water table levels in the vicinity of a vertical discharging well penetrating an aquifer has been analyzed. Two cases, one in which the aquifer is assumed to be fully saturated and, the other in which the aquifer is assumed to be unsaturated, are considered. The two-phase dual-porosity poroelastic model developed in Chapter 4 has been modified in order to be applied to the latter case. The effect of fracture density and permeability on the vertical displacements and water table levels have been highlighted using a three-dimensional finite element model.

Finally, Chapter 7 includes a summary of this dissertation and recommendations for future work.

Chapter 2

Dual-Porosity Poroelasticity and the Finite Element Method

2.1 Introduction

A basic differentiating characteristic of a fissured formation is that a rather small volume of the medium (i.e., the fractures) transmits a large portion of the flow through it (Duguid and Lee, 1977). Such formations exhibit local heterogeneities - a “local preference” for flow or diffusion; primary and secondary porosities (corresponding to the pore spaces and fractures, respectively) and; discrete fractions of varying compressibilities. The concept of fluid flow through fissured media was crystallized by Barenblatt’s phenomenological model which adopted two fluid pressures at every point in space (Barenblatt et al., 1960). This has been the basis for all dual-porosity, dual-permeability models that have developed since and applied to modeling fractured porous media. Aifantis’s incorporation of Biot’s theory of poroelasticity within the dual-porosity concept is the earliest model in which the quasi-static constitutive equations linearly relate the strains to the stress and pore pressures in both, the pores spaces and fractures (Aifantis, 1977; Wilson and Aifantis, 1982).

A majority of dual-porosity poroelastic models based on Aifantis’s seminal formulation either ignore fracture deformations or, fail to differentiate between the deformations of the matrix and fractures. In other words, either the matrix blocks alone are attributed

elastic properties or, lumped parameters are used to characterize the elastic response, thus rendering the contributions of either media to the overall deformation less perspicuous. With this backdrop, the need for a model in which the displacement fields of both media are considered separately, within the framework of separate constitutive and effective stress laws, is discernible. To the author's knowledge, a mathematically rigorous derivation of such a model is not available in literature. In the following sections, a dual-porosity poroelastic model is systematically developed wherein (i) the underlying assumptions; (ii) constituent volumes, associated porosities and their variations in terms of the field quantities and; (iii) constitutive, conduction and effective stress laws are woven into the fabric of the continuity and equilibrium equations governing the response of a fluid saturated fissured porous medium. This serves as a precursor for the development of finite element equations as well as further extensions to incorporate thermal effects and two-phase flow.

2.2 Dual-Porosity Approach

A fluid saturated, fissured porous medium subjected to perturbations experiences changes in pore pressures within the matrix pores and the fractures as well as simultaneous changes in matrix and fracture volumes. The changes in pore pressures affect the equilibrium of the fissured system with the reciprocal consequence that volumetric changes of the solid constituents exert an influence on the fluid flux. In addition, pressure differentials between the matrix and fractures due to their respective flow and mechanical parameters cause interporosity flow between the matrix pores and the fractures. These retroactive and simultaneous changes in the state of the contrariant solid and fluid constituents of the fissured medium coupled with interporosity flow are referred to as dual-porosity poroelasticity.

The dual-porosity model elucidated herein envisages a fissured medium as being

composed of two distinct but overlapping media. The first medium is comprised of the matrix and void spaces corresponding to matrix pores (primary porosity). The second medium comprises of voids representing the fractures (secondary porosity), and a complementary solid part. Thus the fluid flow and solid domains are both represented by two distinct but overlapping continua. This approach helps to overcome the theoretical inconsistency inherent in many dual-porosity models, of representing the solid domain with just one continuum, as well as identify the individual contributions to the overall volumetric change of the system.

2.3 Assumptions

The assumptions relating to the physical and mathematical aspects of the model are listed as follows:

1. A fractured porous medium is visualized as being composed of two distinct but overlapping media. The first medium, referred to as the primary medium, represents the matrix and the pore spaces (primary porosity). The second medium referred to as the secondary medium represents the fractures (secondary porosity) and a complimentary solid part. The fracture network is assumed to be sufficiently well developed so that at each point in space two fluid pressures can be introduced (Barenblatt et al., 1960).
2. The analysis in this study is related to the macroscopic level, i.e., the Representative Elemental Volume (REV; Bear, 1972) concept is applicable.
3. The fluid pressures and deformations in both media are considered separately. At each point the changes in total stresses related to each media are assumed equal and the overall deformations are obtained by lumping the strains in each media. The deformations themselves are assumed to be small, i.e., the fractured porous

medium is assumed to be a linear poroelastic material.

4. The fractured porous medium is assumed to be homogeneous and isotropic.
5. In both, the primary and secondary media, the flow is assumed to be laminar so that Darcy's law is valid. In addition, the permeability and fluid viscosity in both media are assumed to be constant.
6. Both, the primary and secondary media are assumed to be fully saturated. The fluid flow in each continuum is considered to be independent of the flow in the other and is linked only by a quasi-steady mass exchange term representing interporosity flow. The mass exchange term is assumed to be proportional to the permeability of the primary medium and quasi-steady pressure difference between the two media.

2.4 Governing Equations

This section aims at developing a complete set of macroscale governing equations that describe the behavior of a fully saturated fractured porous medium under isothermal conditions. This set comprises of the equilibrium and fluid mass conservation equations for both the primary and secondary media. When combined with the constitutive, effective stress and conduction laws, this set describes the load-deformation behavior and, how the change in fluid pressures affect the quasi-static equilibrium along with a simultaneous conservation of fluid mass which is influenced by the deformation of the fractured medium.

2.4.1 Definitions

In the following derivations, the superscripts I and II always refer to the primary medium (rock matrix) and secondary medium (representing fractures). It is assumed that a Representative Elementary Volume (REV) concept herein applies. The domain

is envisioned as composed of two distinct but overlapping media - the first medium is comprised of void spaces corresponding to the matrix pores and the complementary solid part while, the second medium comprises of voids representing the fractures and a complementary solid part. Thus the total volume of the medium is

$$V = V_s^I + V_p^I = V_s^{II} + V_p^{II} \quad (2.1)$$

where V represents the total volume, V_s^I , V_s^{II} refer to the solid volumes in the two media and, V_p^I , V_p^{II} refer to the void spaces occupied by the fluids in the two media.

The porosities are defined for each of the media as

$$\phi^{(\eta)} = \frac{V_p^\eta}{V} \quad (2.2)$$

Computing the derivatives of primary and secondary porosities with respect to time, gives

$$\frac{D\phi^\eta}{Dt} = \frac{1}{V} \left(\frac{DV_p^\eta}{Dt} - \phi^\eta \frac{DV}{Dt} \right) \quad (2.3)$$

where $\frac{D}{Dt}$ is the total material derivative given as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} \quad (2.4)$$

where u_i is the solid velocity.

2.4.2 Solid Mass Conservation

The solid mass conservation equations for the primary and secondary media can be expressed as follows

$$\frac{\partial (1 - \phi^\eta) \rho_s^\eta}{\partial t} + \frac{\partial (1 - \phi^\eta) \rho_s^\eta u_i^\eta}{\partial x_i} = 0 \quad (2.5)$$

Expanding the derivatives we get,

$$\left\{ \begin{array}{l} \frac{\partial \rho_s^\eta}{\partial t} - \phi^\eta \frac{\partial \rho_s^\eta}{\partial t} - \rho_s^\eta \frac{\partial \phi^\eta}{\partial t} + u_i^\eta \frac{\partial \rho_s^\eta}{\partial x_i} \\ + \rho_s^\eta \frac{\partial u_i^\eta}{\partial x_i} - \phi^\eta u_i^\eta \frac{\partial \rho_s^\eta}{\partial x_i} - \rho_s^\eta u_i^\eta \frac{\partial \phi^\eta}{\partial x_i} \\ - \phi^\eta \rho_s^\eta \frac{\partial u_i^\eta}{\partial x_i} = 0 \end{array} \right. \quad (2.6)$$

Grouping terms appropriately so as to use equation 2.4, we get

$$(1 - \phi^\eta) \frac{D\rho_s^\eta}{Dt} + \rho_s^\eta (1 - \phi^\eta) \frac{\partial \rho_s^\eta}{\partial x_i} - \rho_s^\eta \frac{D\phi^\eta}{Dt} = 0 \quad (2.7)$$

which gives

$$\frac{D\phi^\eta}{Dt} = \frac{(1 - \phi^\eta)}{\rho_s^\eta} \frac{D\rho_s^\eta}{Dt} + (1 - \phi^\eta) \frac{\partial u_i^\eta}{\partial x_i} \quad (2.8)$$

In equations (2.8)-(2.11), ρ_s^η are the solid densities of the two media, i.e., matrix and fractures and, u_i^η are the respective solid velocities which may be related to the overall solid velocity by superimposing the strains from the two media.

2.4.3 Volumetric Variations

A micromechanical approach has been adopted to elicit the volumetric variations of the components of the model. The pore volumes are assumed to be a function of mean stress, $\bar{\sigma}$ and pore pressure P^η , i.e.,

$$V_p^\eta = V_p^\eta(\bar{\sigma}, P^\eta) \quad (2.9)$$

Hence,

$$\frac{dV_p^\eta}{V_p^\eta} = \frac{1}{V_p^\eta} \frac{\partial V_p^\eta}{\partial \bar{\sigma}} d\bar{\sigma} + \frac{1}{V_p^\eta} \frac{\partial V_p^\eta}{\partial P^\eta} dP^\eta \quad (2.10)$$

The pore volume variations can hence be expressed as

$$\frac{dV_p^\eta}{V_p^\eta} = -\frac{dP^\eta}{K_s^\eta} + \frac{\alpha^\eta}{\phi^\eta K^\eta} (d\bar{\sigma} + dP^\eta) \quad (2.11)$$

where α^η are the equivalent Biot's effective stress parameters for the respective media.

Also, using equations 2.11, the solid volume variations can be expressed in terms of variations of the solid densities as

$$\begin{aligned} \frac{1}{\rho_s^\eta} \frac{d\rho_s^\eta}{dt} &= -\frac{1}{V_s^\eta} \frac{dV_s^\eta}{dt} = \\ \frac{1}{(1 - \phi^\eta)} &\left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \frac{dP^\eta}{dt} - (1 - \alpha^\eta) u_{i,i}^\eta \right] \end{aligned} \quad (2.12)$$

The derivations of equations 2.11 and 2.12 are provided in Appendix A.

2.4.4 Fluid Mass Conservation

The mass conservation equation for the fluid in both media can be written as

$$\frac{\partial}{\partial t} (\phi^\eta \rho_f) + \frac{\partial}{\partial x_i} (\phi^\eta \rho_f U_i^\eta) + (-1)^\eta \Gamma = 0 \quad (2.13)$$

The Darcy velocities for fluid in both media are defined as

$$v_i^\eta = \phi^\eta (U_i^\eta - u_i^\eta) \quad (2.14)$$

In Equations 2.13 and 2.14, U_i^η are the intrinsic fluid velocities in the two media; ρ_f is the fluid density and Γ is the rate of fluid mass transferred between the two media.

From equations 2.13 and 2.14 we have,

$$\frac{\partial}{\partial t} (\phi^\eta \rho_f) + \frac{\partial}{\partial x_i} (\phi^\eta \rho_f u_i^\eta + \rho_f v_i^\eta) + (-1)^\eta \Gamma = 0 \quad (2.15)$$

Expanding derivatives, we have

$$\begin{cases} \rho_f \frac{\partial \phi^\eta}{\partial t} + \phi^\eta \frac{\partial \rho_f}{\partial t} + \rho_f u_i^\eta \phi_{,i}^\eta + \phi^\eta u_i^\eta (\rho_f)_{,i} \\ + \phi^\eta \rho_f u_{i,i}^\eta + \rho_f v_{i,i}^\eta + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{cases} \quad (2.16)$$

Grouping terms appropriately and using equations 2.4 and 2.16 we have,

$$\begin{cases} \rho_f \frac{D\phi^\eta}{Dt} + \phi^\eta \frac{D\rho_f}{Dt} + \phi^\eta \rho_f u_{i,i}^\eta + \\ \rho_f v_{i,i}^\eta + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{cases} \quad (2.17)$$

From equations 2.8, 2.12 and 2.17 we have,

$$\begin{cases} \rho_f \frac{(\alpha^\eta - \phi^\eta) DP^\eta}{K_s^\eta Dt} + \phi^\eta \frac{D\rho_f}{Dt} + \rho_f \alpha^\eta u_{i,i}^\eta + \\ \rho_f v_{i,i}^\eta + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{cases} \quad (2.18)$$

Now,

$$\frac{1}{\rho_f} \frac{D\rho_f}{Dt} = \frac{1}{K_f} \frac{DP^\eta}{Dt} \quad (2.19)$$

and invoking Darcy's law,

$$v_i^\eta = -\frac{k^\eta}{\mu_f} (P^\eta)_{,i} \quad (2.20)$$

where k^η are the permeabilities of the two media and μ_f is the fluid viscosity. Thus using equations 2.18-2.20, letting $\frac{D}{Dt} = \frac{\partial}{\partial t}$ and grouping terms we get the final form of the mass conservation for fluid in both media as,

$$\left\{ \begin{array}{l} \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} \\ + \alpha^\eta \rho_f u_{i,i}^\eta - \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{i,i} - \\ + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{array} \right. \quad (2.21)$$

2.4.5 Equilibrium Equation

The equilibrium equations are the stress balance equations obtained from standard considerations of static equilibrium. These may be expressed as

$$\sigma_{ij,i} = -F_j \quad (2.22)$$

where, F_j the vector of body force. The relationships between changes in total stresses σ_{ij} and intergranular stresses σ'_{ij} are expressed in terms of separate effective stress laws for both the primary and secondary media as follows

$$\sigma_{ij}^\eta = (\sigma'_{ij})^\eta - \alpha^\eta P^\eta \delta_{ij} \quad (2.23)$$

where,

$$(\varepsilon_{ij}^\eta) = C_{ijkl}^\eta (\sigma'_{kl})^\eta \quad (2.24)$$

In equations 2.23 and 2.24, $(\sigma'_{ij})^\eta$ represent the effective stresses, α^η are the equivalent Biot's effective stress parameters for the respective media, and δ_{ij} is the *Kronecker delta*. After considering requirements of local stress equilibrium and lumping the strains (Nair et al. 2000), we have,

$$\begin{aligned} \sigma_{ij}^I &= \sigma_{ij}^{II} = \sigma_{ij} \\ \varepsilon_{ij} &= \varepsilon_{ij}^I + \varepsilon_{ij}^{II} \end{aligned} \quad (2.25)$$

Using equations 2.22-2.25 the equilibrium equation may thus be written as

$$\begin{cases} (D_{ijkl}^{I,II} \varepsilon_{kl} - D_{ijkl}^{I,II} C_{klmn}^I \alpha^I \delta_{mn} P^I - \\ D_{ijkl}^{I,II} C_{klmn}^{II} \alpha^{II} \delta_{mn} P^{II})_{,i} + F_j = 0 \end{cases} \quad (2.26)$$

where $D_{ijkl}^{I,II}$ is the elasticity tensor for the whole solid domain, C_{ijkl}^I and C_{ijkl}^{II} are the compliance tensors for the primary and secondary media, ε_{ij} is the total body strain and,

$$D_{ijkl}^{I,II} (C_{klmn}^I + C_{klmn}^{II}) = \frac{1}{2} (\delta_{im} \delta_{jn} + \delta_{in} \delta_{jm}) \quad (2.27)$$

2.4.6 Contributions of Individual Deformations

Equations 2.23 and 2.24 capture the essence of the double effective stress laws and separate constitutive equations for both, the primary and secondary media. In addition, this aspect of the model facilitates quantification of the individual deformations in each media and their contribution to the overall deformation. The usefulness of this approach would be evident when the secondary medium is used to delineate the fractures. The significant impact of fracture deformation due to the higher compressibility of the fractures, as compared to the matrix, on the load-deformation response of the fissured medium can then be assessed.

This section aims to quantify the volumetric strain manifested in each of the two media in terms of the total volumetric strain. This may be done by expressing the volumetric strains of the individual media as a proportion of the total body strain. Equation 2.24 gives,

$$\varepsilon_{ij}^I = C_{ijkl}^I (\sigma_{kl}^I)', \quad \varepsilon_{ij}^{II} = C_{ijkl}^{II} (\sigma_{kl}^{II})' \quad (2.28)$$

Equations 2.25 and 2.28 give,

$$\varepsilon_{ij} = (C_{ijkl}^I + C_{ijkl}^{II}) \sigma_{kl} + C_{ijkl}^I \alpha^I P^I \delta_{kl} + C_{ijkl}^{II} \alpha^{II} P^{II} \delta_{kl} \quad (2.29)$$

Hence,

$$\sigma_{ij} = D_{ijkl}^{I,II} (\varepsilon_{kl} - C_{klmn}^I \alpha^I P^I \delta_{mn} - C_{klmn}^{II} \alpha^{II} P^{II} \delta_{mn}) \quad (2.30)$$

Equations 2.23, 2.28 and 2.30 result in,

$$\varepsilon_{ij}^I = C_{ijkl}^I \left[D_{klmn}^{I,II} \left(\varepsilon_{mn} - \frac{C_{mnpq}^I \alpha^I P^I \delta_{pq}}{C_{mnpq}^{II} \alpha^{II} P^{II} \delta_{pq}} \right) + \alpha^I P^I \delta_{kl} \right] \quad (2.31)$$

Thus,

$$\varepsilon_{kk}^I = C_{kkmm}^I \left[\frac{D_{mmnn}^{I,II} (\varepsilon_{nn} - C_{nnpp}^I \alpha^I P^I \delta_{pp})}{-C_{nnpp}^{II} \alpha^{II} P^{II} \delta_{pp}} + \alpha^I P^I \delta_{mm} \right] \quad (2.32)$$

Taking derivatives with respect to time gives

$$\dot{\varepsilon}_{kk}^I = C_{kkmm}^I \left[\frac{D_{mmnn}^{I,II} (\dot{\varepsilon}_{nn} - C_{nnpp}^I \alpha^I \dot{P}^I \delta_{pp})}{C_{nnpp}^{II} \alpha^{II} \dot{P}^{II} \delta_{pp}} - \alpha^I \dot{P}^I \delta_{mm} \right] \quad (2.33)$$

where the overdot signifies the differentiation with respect to time.

If the pressure changes, $\dot{P}^I$ and $\dot{P}^{II}$ are set to zero, the variation of fluid content depends solely on the solid displacement rates $u_{i,i}$ ($= \dot{\varepsilon}_{ii}$). Since $\dot{\varepsilon}_{kk}^I$ is the volumetric strain under conditions of no change in pore pressures, the relation between $\dot{\varepsilon}_{kk}^I$ and $\dot{\varepsilon}_{kk}$ can be obtained by setting $\dot{P}^I = \dot{P}^{II} = 0$ in equation 2.33 as

$$\dot{\varepsilon}_{kk}^I = C_{kkmm}^I [D_{mmnn}^{I,II} \dot{\varepsilon}_{nn}] \quad (2.34)$$

Similarly,

$$\dot{\varepsilon}_{kk}^{II} = C_{kkmm}^{II} [D_{mmnn}^{I,II} \dot{\varepsilon}_{nn}] \quad (2.35)$$

The mathematical development of the dual-porosity model has been completed. The model, based on double effective stress and separate constitutive laws for each overlapping medium facilitates assessing the impact of individual deformations. The next section focuses on the development of a finite element dual-porosity poroelastic model.

2.5 The Finite Element Method in Dual-Porosity Poroelasticity

The process of subdividing all systems into their components or ‘elements’ and then rebuilding the original system from such components is a natural way engineers, scientists and analysts in various fields proceed (Zienkiewicz and Taylor, 1989). In this approach, known as *going from part to whole*, the solution to a problem for the entire system is obtained by combining the solutions for each constituent component (Desai and Abel, 1987). Indeed, viewing systems in isolation from their environments, breaking them down into their simplest component parts and using the behavior of these parts to predict the unfolding future of the system is one of the cornerstones of classical or “Newtonian” physics (Marshall and Zohar, 1997). The finite element method is an embodiment of these ideas and was developed on a physical basis for analysis of problems in structural mechanics. However with the advances in computing technology, the method has been applied to solutions of many classes of continuum problems. Today, the finite element method is viewed as a general discretization procedure of continuum problems posed by mathematically defined statements (Zienkiewicz and Taylor, 1989; Bathe, 1996).

2.5.1 Preliminaries

The finite element method solves the mathematical model consisting of differential equations and boundary conditions idealizing the physical problem. Formally, we seek an unknown function ϑ such that it satisfies a differential equation set

$$\mathbf{A}(\vartheta) = \mathbf{0} \tag{2.36}$$

in a domain Ω , together with certain boundary conditions

$$\mathbf{B}(\vartheta) = \mathbf{0} \tag{2.37}$$

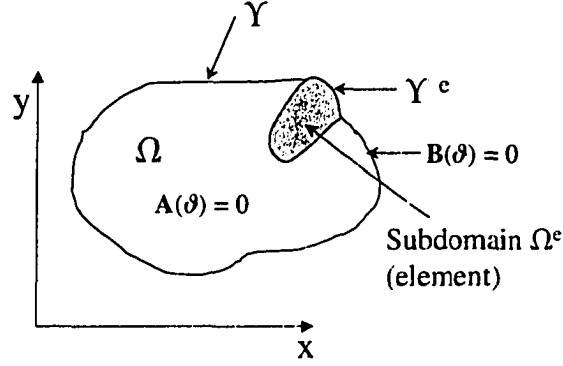


Figure 2.1: Problem domain Ω and boundary Υ (Zienkiewicz and Taylor, 1989)

on the boundary Υ of the domain (see figure 2.1). The condition that equation 2.36 is satisfied over the whole domain and equation 2.37 is satisfied over the boundary requires

$$\int_{\Omega} \mathbf{p}^T \mathbf{A}(\boldsymbol{\vartheta}) d\Omega + \int_{\Upsilon} \mathbf{q}^T \mathbf{B}(\boldsymbol{\vartheta}) d\Upsilon = 0 \quad (2.38)$$

where $\mathbf{p}$ and $\mathbf{q}$ are a set of arbitrary functions. In the finite element procedure, the solution is sought in the approximate form given by

$$\boldsymbol{\vartheta} \approx \hat{\boldsymbol{\vartheta}} \quad (2.39)$$

The integral statement, equation 2.38 allows the approximation in equation 2.39 to be made if we use a finite set of prescribed functions, $\mathbf{a}$ and $\mathbf{b}$ in place of $\mathbf{p}$ and $\mathbf{q}$ respectively (Zienkiewicz and Taylor, 1989), i.e.,

$$\mathbf{p} = \mathbf{a}; \quad \mathbf{q} = \mathbf{b} \quad (2.40)$$

From equations 2.38 and 2.40, we have

$$\int_{\Omega} \mathbf{a}^T \mathbf{A}(\hat{\boldsymbol{\vartheta}}) d\Omega + \int_{\Upsilon} \mathbf{b}^T \mathbf{B}(\hat{\boldsymbol{\vartheta}}) d\Upsilon = 0 \quad (2.41)$$

In equation 2.41 $\mathbf{A}(\hat{\boldsymbol{\vartheta}})$ and $\mathbf{B}(\hat{\boldsymbol{\vartheta}})$ represent the residual error due to introduction of the approximation into the differential equations and boundary conditions. Hence, equation 2.41 is a weighted integral of the residual and the approximation is called the *method of weighted residuals* (Zienkiewicz and Taylor, 1989). The relevance of the weighted residual in the finite element method will be demonstrated in the next section wherein the finite element form of the governing equations will be derived.

2.5.2 Finite Element Equations

Equations 2.21 and 2.26 form the governing differential equations comprising the set $\mathbf{A}(\boldsymbol{\vartheta})$ with $\boldsymbol{\vartheta} = \mathbf{u}, P^\eta$ and the associated boundary conditions comprise $\mathbf{B}(\boldsymbol{\vartheta})$. Equation 2.21 can be written as

$$\bar{A} \equiv \begin{cases} \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} \\ + \alpha^\eta \rho_f u_{i,i}^\eta - \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{i,i} - \\ + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{cases} \quad (2.42)$$

with the prescribed pore pressures and the condition for continuity of flow across the boundary of the domain forming the boundary conditions as in

$$\bar{B} \equiv -q - \mathbf{n}^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} = 0 \quad (2.43)$$

where $\mathbf{n}$ is the unit normal vector and q is the outflow rate per unit area of the boundary surface (Lewis and Schrefler, 1987).

$$P^\eta = (P^\eta)_{boundary} \quad (2.44)$$

As in equation 2.41 we have

$$\int_{\Omega} \mathbf{a}^T \bar{A} d\Omega + \int_{\Gamma} \mathbf{b}^T \bar{B} d\Upsilon = 0 \quad (2.45)$$

From equations 2.42 and 2.45 we get

$$\begin{aligned}
& \int_{\Omega} \mathbf{a}^T \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} d\Omega + \\
& \int_{\Omega} \mathbf{a}^T \alpha^\eta \rho_f u_{i,i}^\eta d\Omega - \int_{\Omega} \mathbf{a}^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{i,i} d\Omega - \\
& \int_{\Omega} \mathbf{a}^T v_i^\eta (\rho_f)_{,i} d\Omega + (-1)^\eta \int_{\Omega} \mathbf{a}^T \Gamma d\Omega - \\
& \int_{\Upsilon} \mathbf{b}^T \mathbf{n}^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} d\Upsilon - \int_{\Upsilon} \mathbf{b}^T q d\Upsilon = 0
\end{aligned} \tag{2.46}$$

The Green's theorem may now be employed to reduce the required order of continuity for $\boldsymbol{\vartheta}$ ($= \mathbf{u}, P^\eta$) necessitated by the appearance of second order derivatives, i.e., the term containing $(P^\eta)_{i,i}$ as in equation 2.46.

$$\begin{aligned}
& \int_{\Omega} \mathbf{a}^T \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} d\Omega + \\
& \int_{\Omega} \mathbf{a}^T \alpha^\eta \rho_f u_{i,i}^\eta d\Omega + \int_{\Omega} (\nabla \mathbf{a})^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} d\Omega - \\
& \int_{\Upsilon} \mathbf{a}^T \mathbf{n}^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} d\Upsilon + \int_{\Omega} \mathbf{a}^T v_i^\eta (\rho_f)_{,i} d\Omega + \\
& (-1)^\eta \int_{\Omega} \mathbf{a}^T \Gamma d\Omega - \int_{\Upsilon} \mathbf{b}^T \mathbf{n}^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} d\Upsilon - \\
& \int_{\Upsilon} \mathbf{b}^T q d\Upsilon = 0
\end{aligned} \tag{2.47}$$

Subsequently, letting $\mathbf{a} = -\mathbf{b}$ without any loss of generality as both $\mathbf{a}$ and $\mathbf{b}$ are arbitrary and cancelling some of the the boundary terms in equation 2.47 we have,

$$\begin{aligned}
& \int_{\Omega} \mathbf{a}^T \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} d\Omega + \\
& \int_{\Omega} \mathbf{a}^T \alpha^\eta \rho_f u_{i,i}^\eta d\Omega + \int_{\Omega} (\nabla \mathbf{a})^T \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{,i} d\Omega \\
& + \int_{\Omega} \mathbf{a}^T v_i^\eta (\rho_f)_{,i} d\Omega + (-1)^\eta \int_{\Omega} \mathbf{a}^T \Gamma d\Omega + \\
& \int_{\Upsilon} \mathbf{a}^T q d\Upsilon = 0
\end{aligned} \tag{2.48}$$

Equation 2.48 is the weak form of the weighted residual statement, equation 2.46. The next step in the development of the finite element equations is the discretization of the continuum into elements and expressing the approximations of the primary variables $\mathbf{u}$ and P^η within an element in terms of their nodal values. Thus we have

$$P^{\eta*} \simeq \mathbf{N} P^\eta; \mathbf{u}^* \simeq \mathbf{N} \bar{\mathbf{u}}; \boldsymbol{\varepsilon} \simeq \mathbf{B} \bar{\mathbf{u}} \tag{2.49}$$

where the superscript (*) denotes the finite element approximation and $\mathbf{N}$ represents the elemental shape functions (Zienkiewicz and Taylor, 1989). $\bar{\mathbf{u}}$ and $\mathbf{P}^\eta$ refer to the nodal displacements and pressures respectively and, the (-) will be dropped herewith. Also, the strains of both media, i.e. $u_{i,i}^\eta$, can be expressed in terms of the total body strain following equation 2.35 as (Nair et al. 2000)

$$\mathbf{m}^T \mathbf{B} \frac{\partial \mathbf{u}^I}{\partial t} = \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} \frac{\partial \mathbf{u}}{\partial t} \quad (2.50)$$

where the vector $\mathbf{m}$ is $[1, 1, 1, 0, 0, 0]^T$ and $\mathbf{B}$ is the strain displacement matrix.

Substitution of the approximate quantities (equation 2.49) into equation 2.48 and using equation 2.50 gives

$$\begin{aligned} & \int_{V_e} \mathbf{a}^T \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \mathbf{N} dV_e \frac{d\mathbf{P}^\eta}{dt} + \\ & \int_{V_e} \mathbf{a}^T \alpha^\eta \rho_f \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV_e \frac{d\mathbf{u}}{dt} + \\ & \int_{V_e} (\nabla \mathbf{a})^T \rho_f \frac{k^\eta}{\mu_f} \nabla \mathbf{N} dV_e \mathbf{P}^\eta + \\ & + \int_{\Omega} \mathbf{a}^T \eta_i^\eta (\rho_f)_{,i} d\Omega + \int_{\Gamma} \mathbf{a}^T q d\Upsilon + \\ & (-1)^\eta \int_{V_e} \mathbf{a}^T \Gamma dV_e = 0 \end{aligned} \quad (2.51)$$

where V_e is the elemental volume and the subscript will be dropped herewith. The form of the function $\mathbf{a}$ needs to be specified and as in the *Galerkin method* (Bubnov-Galerkin), $\mathbf{a}$ is chosen to be identical to $\mathbf{N}$. Also, neglecting the spatial variation of the fluid density, and assuming (Warren and Root, 1963)

$$\Gamma = \rho_f \frac{k^I}{\mu_f} \Psi (P^I - P^{II}) \quad (2.52)$$

(where Ψ is a transfer coefficient) the finite element form of the fluid mass conservation

equation is obtained as

$$\begin{aligned}
& \int \mathbf{N}^T \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \mathbf{N} dV \frac{d\mathbf{P}^\eta}{dt} + \\
& \int \mathbf{N}^T \alpha^\eta \rho_f \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \frac{d\mathbf{u}}{dt} + \\
& \int (\nabla \mathbf{N})^T \rho_f \frac{k^\eta}{\mu_f} \nabla \mathbf{N} dV \mathbf{P}^\eta + \\
& (-1)^\eta \int \mathbf{N}^T \rho_f \frac{k^I}{\mu_f} \Psi \mathbf{N} dV (\mathbf{P}^I - \mathbf{P}^{II}) + \mathbf{Q}_f^\eta = 0
\end{aligned} \tag{2.53}$$

The equilibrium equation, based on the principle of virtual work, can be written as

$$\int_v \mathbf{B}^T \partial \boldsymbol{\sigma} dV - \partial \mathbf{f} = 0 \tag{2.54}$$

where $\partial \mathbf{f}$ is an incremental vector of body forces. Using equations 2.26 and 2.54 and differentiation with respect to time, we have,

$$\begin{aligned}
& \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{B} dV \frac{d\mathbf{u}}{dt} - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \mathbf{m} \mathbf{N} dV \frac{d\mathbf{P}^I}{dt} \\
& - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \mathbf{m} \mathbf{N} dV \frac{d\mathbf{P}^{II}}{dt} = \frac{d\mathbf{f}}{dt}
\end{aligned} \tag{2.55}$$

Equations 2.53 and 2.55 are the finite element equations for the coupled isothermal flow-deformation response for a fully saturated fractured porous medium. They can be expressed as

$$\mathbf{K} \frac{d\mathbf{u}}{dt} + \mathbf{L}_{fl}^I \frac{d\mathbf{P}^I}{dt} + \mathbf{L}_{fl}^{II} \frac{d\mathbf{P}^{II}}{dt} - \frac{d\mathbf{f}}{dt} = 0 \tag{2.56}$$

$$\begin{cases} \mathbf{K}_f^\eta \frac{d\mathbf{u}}{dt} + (\mathbf{H}_D^\eta + \mathbf{H}_{tr}) \times \mathbf{P}^\eta \\ -\mathbf{H}_{tr} \mathbf{P}^\eta + \mathbf{S}^\eta \frac{d\mathbf{P}^\eta}{dt} = -\mathbf{Q}^\eta \end{cases} \tag{2.57}$$

Equations 2.56 and 2.57 now represent a set of differential equations in time which may be represented in a compact form as

$$\begin{aligned}
& \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{H}_D^I + \mathbf{H}_{tr} & -\mathbf{H}_{tr} \\ 0 & -\mathbf{H}_{tr} & \mathbf{H}_D^{II} + \mathbf{H}_{tr} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \end{Bmatrix} + \\
& \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fl}^I & \mathbf{L}_{fl}^{II} \\ \mathbf{K}_f^I & \mathbf{S}^I & 0 \\ \mathbf{K}_f^{II} & 0 & \mathbf{S}^{II} \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \end{Bmatrix} = \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}^I \\ -\mathbf{Q}^{II} \end{Bmatrix}
\end{aligned} \tag{2.58}$$

Equation 2.58 need to be discretized in the time-domain in order to obtain the values of $\mathbf{u}$, and $\mathbf{P}^\eta$. The discretization in time is carried out using a finite difference scheme as follows

$$\begin{cases} \left(\frac{d\mathbf{u}}{dt} \right)^{t+\Delta t} = \frac{1}{\Delta t} (\mathbf{u}^{t+\Delta t} - \mathbf{u}^t) \\ \left(\frac{d\mathbf{P}^\eta}{dt} \right)^{t+\Delta t} = \frac{1}{\Delta t} [(\mathbf{P}^\eta)^{t+\Delta t} - (\mathbf{P}^\eta)^t] \end{cases} \quad (2.59)$$

From equations 2.58 and 2.59, we get

$$\begin{aligned} & \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fl}^I & \mathbf{L}_{fl}^{II} \\ \mathbf{K}_f^I & \mathbf{H}_{Dtr}^I \Delta t + \mathbf{S}^I & -\mathbf{H}_{tr} \Delta t \\ \mathbf{K}_f^{II} & -\mathbf{H}_{tr} \Delta t & \mathbf{H}_{Dtr}^{II} \Delta t + \mathbf{S}^{II} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \end{Bmatrix}^{t+\Delta t} \\ &= \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fl}^I & \mathbf{L}_{fl}^{II} \\ \mathbf{K}_f^I & \mathbf{S}^I & \mathbf{0} \\ \mathbf{K}_f^{II} & \mathbf{0} & \mathbf{S}^{II} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \end{Bmatrix}^t + \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}^I \\ -\mathbf{Q}^{II} \end{Bmatrix}^{t+\Delta t} \end{aligned} \quad (2.60)$$

where,

$$\mathbf{H}_{Dtr}^\eta = \mathbf{H}_D^\eta + \mathbf{H}_{tr} \quad (2.61)$$

The sub-matrices in equation 2.60 are listed in Appendix A.

2.5.3 Model Verification

The model developed has been verified against analytical solutions assuming a single-porosity medium saturated with a single fluid for the following cases:

1. One-dimensional column consolidation
2. An inclined wellbore subjected to a three-dimensional in-situ state of stress

Verification With One-Dimensional Consolidation

The finite element results are compared with the analytical solution for one-dimensional consolidation of a column of soil saturated with a slightly compressible fluid. The input parameters in Table 2.1 are representative of a homogeneous single-porosity medium

Parameter	Units	Value
Elastic Modulus (E)	MPa	6.0
Poisson's Ratio (ν)	-	0.2
Porosity (ϕ^I)	-	0.375
Permeability (k^I)	m ²	1.02×10^{-15}
Grain Bulk Modulus (K_s)	MPa	10^7
Fluid Bulk Modulus (K_f)	MPa	3.3×10^3
Fluid Viscosity (μ)	MPa.s	10^{-9}
Applied Vertical Stress (σ_z)	MPa	1.0

Table 2.1: Parameters for verification (one-dimensional consolidation)

(depth = 1.0m) saturated with an incompressible fluid. 20 four-noded elements with a total of 42 nodes were used in the finite element layout. Comparison with the analytical solution for such a problem serves as a partial validation of the proposed model. Figures 2.2 and 2.3 show a comparison between the analytical and finite element results of the temporal displacement at the top of the column and, pore pressure distribution along the column for different times respectively. An excellent agreement with the analytical solution is evident in both figures.

Verification With Inclined Wellbore Solutions

Figures 2.4 and 2.5, schematically depict an inclined wellbore and corresponding far-field stress components, respectively. As shown in figure 2.4, the Cartesian coordinate system ($x' y' z'$) coincides with the principal axes of the in-situ compressive stresses, ($S_{x'}$, $S_{y'}$ and $S_{z'}$) with the initial in-situ pore pressure denoted by p_o . The local coordinate system (xyz) pertaining to the wellbore is obtained by rotating x' by φ_x and z' by φ_z as shown. The far field boundary conditions as shown in figure 2.5 are

$$\sigma_{xx} = S_x, \quad \sigma_{yy} = S_y, \quad \sigma_{zz} = S_z, \quad \tau_{xy} = S_{xy}, \quad \tau_{yz} = S_{yz}, \quad \tau_{xz} = S_{xz} \quad (2.62)$$

with the initial pore pressure being

$$P^I = P^{I(0)} \quad (2.63)$$

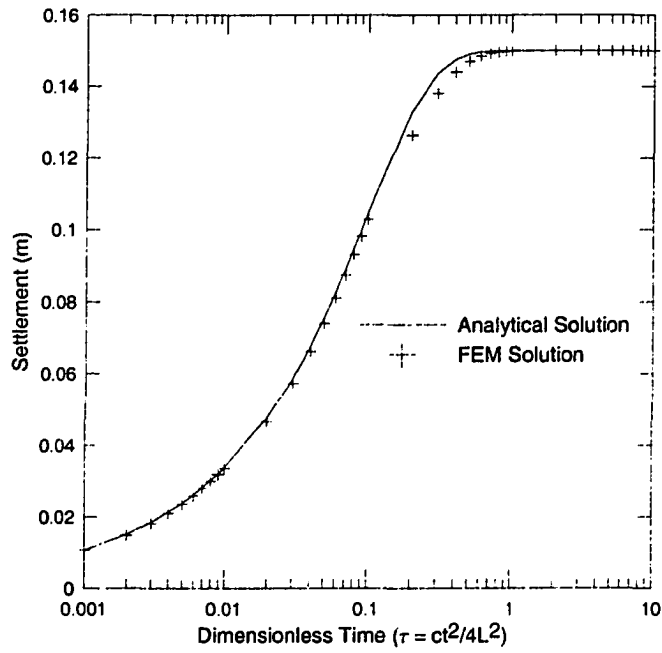


Figure 2.2: Verification of temporal settlement of the top layer for one-dimensional consolidation

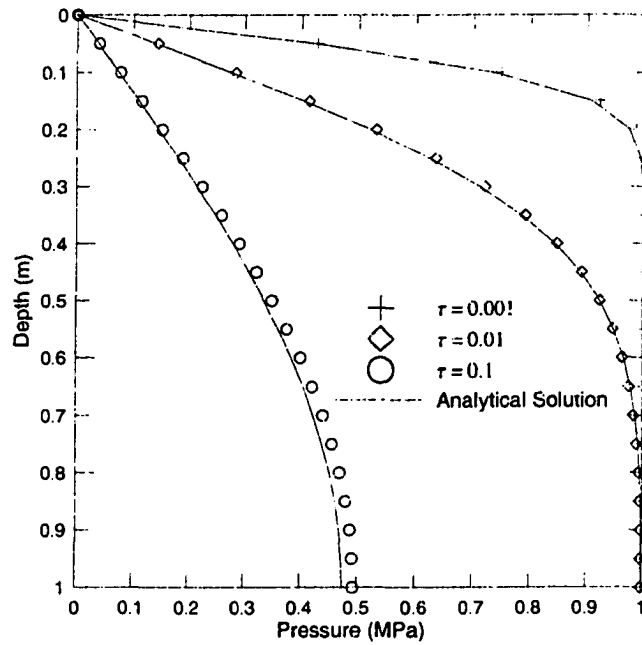


Figure 2.3: Verification of pore pressure distribution for one-dimensional consolidation (τ - Dimensionless Time)

Parameter	Units	Value
Elastic Modulus (E)	GPa	20.6
Poisson's Ratio (ν)	-	0.189
Porosity (ϕ^I)	-	0.02
Permeability (k^I)	m ²	10 ⁻¹⁹
Grain Bulk Modulus (K_s)	GPa	48.21
Fluid Bulk Modulus (K_f)	MPa	419.2
Fluid Viscosity (μ)	MPa.s	10 ⁻⁹

Table 2.2: Parameters for verification (inclined wellbore problem)

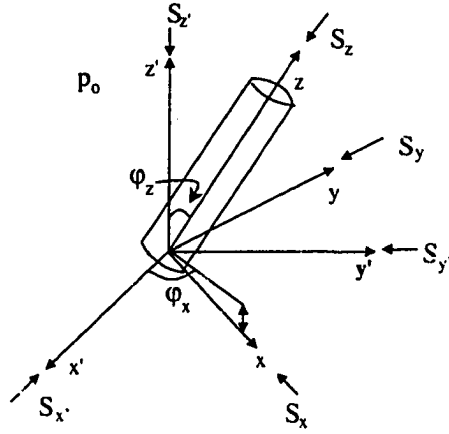


Figure 2.4: Schematic of an inclined borehole

where the superscript “(0)” refers to the initial condition. In the present analysis, the wellbore radius is chosen as 0.1 m with $\varphi_x = 0^\circ$ and $\varphi_z = 70^\circ$. The boundary stresses and pore pressures are $S_x = 25.5$ MPa, $S_y = 20.0$ MPa, $S_z = 28.5$ MPa, $S_{xz} = 1.3$ MPa, $S_{xy} = S_{yz} = 0.0$ MPa, and $P^{I(0)} = 10.0$ MPa. The medium was discretized by 352 four-noded isoparametric elements with the material parameters as in Table 2.2.

Also, a generalized plane strain approach is adopted with the condition (Cui, 1995)

$$u_i = u_i(x, y) \quad (2.64)$$

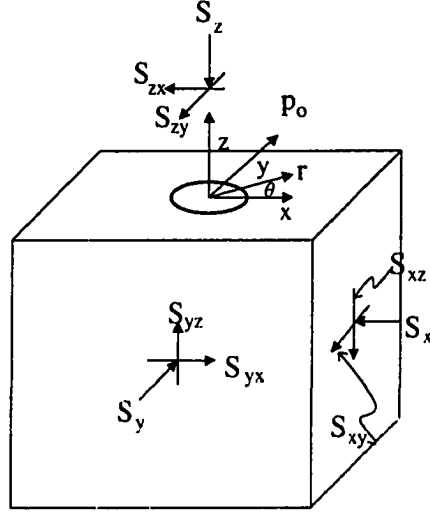


Figure 2.5: Anisotropic far-field stresses

The strain-displacement matrix, $\mathbf{B}$, can be expressed as

$$\mathbf{B} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & \frac{\partial}{\partial x} \end{bmatrix} \mathbf{N} \quad (2.65)$$

A detailed explanation of the analytical solution and the spatiotemporal variation of the pore pressure and effective radial stress for the inclined wellbore problem can be found in literature (Cui et al., 1997, Ekbote et al., 2002). Figures 2.6-2.8 show a comparison between the analytical solutions and finite element calculations along the radial

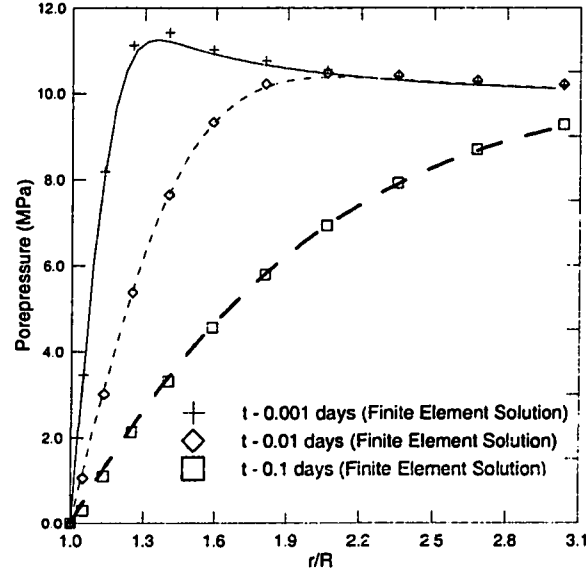


Figure 2.6: Verification of pore pressure distribution (inclined wellbore problem).

direction ($\theta = 87.63^\circ$)¹. The poroelastic effects manifest at the peak of the pore pressure distribution (Figure 2.6) and the corresponding tensile region (Figure 2.7) around the borehole wall. These early time responses are consistent with the cases reported in literature (Cui et al., 1997, Ekbote et al., 2002). The finite element computations have also been recently verified with analytical solutions for an inclined borehole in a dual-porosity medium saturated with a single fluid (Abousleiman and Nguyen, 2003).

The verification results presented in this section were obtained by successive refinement of the mesh density and an appropriate choice of the initial time-step. The effect of the size of the smallest element and time-steps on the stability of the solution is discussed in the next section.

2.5.4 Oscillations and Stability

A process is considered unstable if small disturbances lead to large consequences with the progression of time. In the context of numerical procedures such as the finite element

¹In Figures 2.6-2.8, the solid and dotted lines are from the analytical solutions (Ekbote et al., 2002)

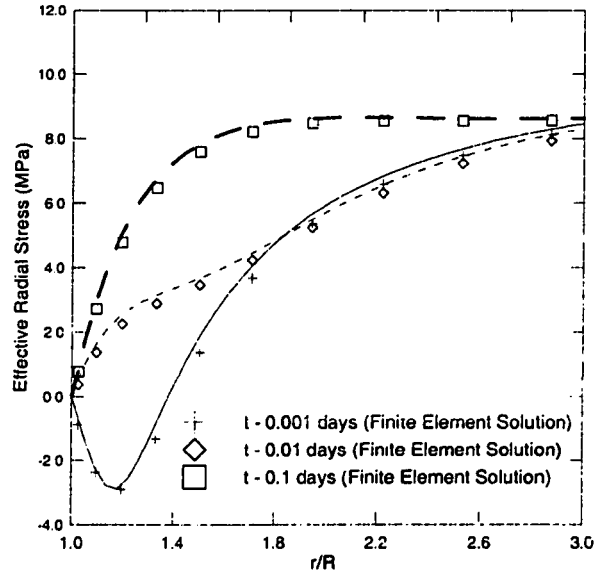


Figure 2.7: Verification of effective radial stress distribution (inclined wellbore problem).

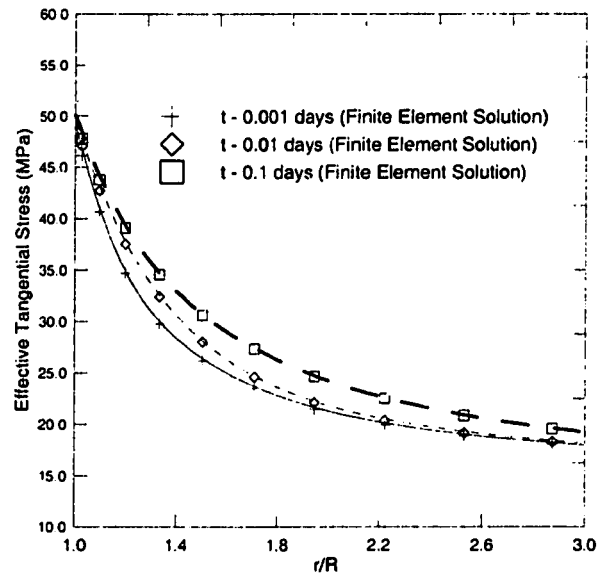


Figure 2.8: Verification of effective tangential stress distribution (inclined wellbore problem).

method, the “disturbances” may be due to truncation errors or deviations from the exact solution at a given instant of time due to the finite element approximations (Vermeer and Verruijt, 1979). Although the backward difference time-stepping scheme employed in the earlier sections is unconditionally stable, the finite element solutions may exhibit numerical oscillations if the time-step is smaller than a critical value (Thomas and Zhou, 1997). Vermeer and Verruijt (1979) analyzed the system of equations for one-dimensional consolidation using a one-dimensional two-noded element. They showed that a necessary and sufficient condition to prevent oscillations in a one-dimensional consolidation scenario is

$$\Delta t \geq \frac{1}{6} \frac{(\Delta h)^2}{c'} \quad (2.66)$$

where Δt is the time-step chosen, Δh is the length of the smallest element and c' is given by

$$c' = \frac{k/\mu}{1/K + \phi/K_f} \quad (2.67)$$

Equation 2.66 implies that the mesh density and material parameters influence the earliest instant of time for which stable solutions can be obtained. From a physical point of view it can be argued that since consolidation is a surface phenomena at earlier times, a reasonable finite element approximation to the solutions can be expected only after the pore pressures in the first row of nodes is affected by the progress of the process. Thomas and Zhou (1997) developed a similar criterion for the two-dimensional heat diffusion problem. For two-dimensional four and eight-noded elements they obtained the lower limit for the minimum time-step as

$$\Delta t \geq \frac{1}{2} \frac{L^2 C_v}{\lambda} \quad (2.68)$$

and

$$\Delta t \geq \frac{1}{20} \frac{L^2 C_v}{\lambda} \quad (2.69)$$

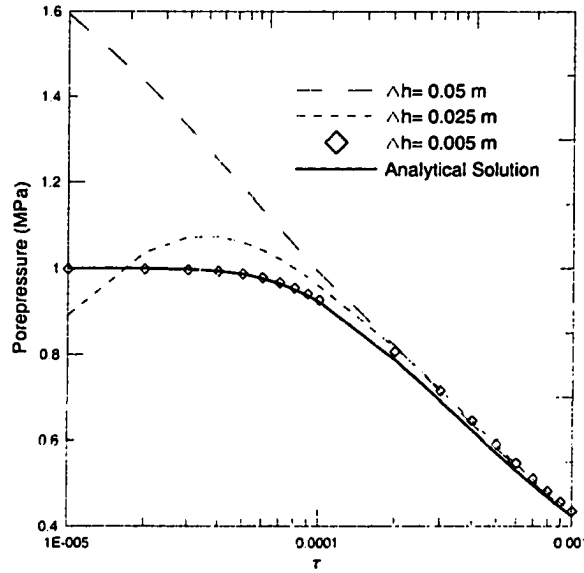


Figure 2.9: Impact of violating the mesh density criterion (one-dimensional consolidation, parameters from Table 2.2; τ - Characteristic Time)

respectively, where, C_v is the heat capacity, λ is the coefficient of thermal conductivity and, L is the smallest of the elemental dimensions along the direction of conduction.

The data used in the verification of the one-dimensional consolidation problem was chosen to illustrate the importance of the time-step and mesh density criteria as in equations 2.66 and 2.68. For a one-dimensional consolidation problem modeled with two-dimensional four noded elements, the smallest mesh dimension is given by

$$\Delta h \leq (2c' \Delta t)^{0.5} \quad (2.70)$$

For the data used in the verification exercise we have from equation 2.70, $\Delta h \leq 0.006m$. Figure 2.9 shows the significant difference between the analytical and finite element solutions at the beginning of the consolidation process when this criterion is violated. For the mesh having $\Delta h = 0.005m$ close to the draining surface, the agreement between the finite element and analytical solutions is excellent even at very short times.

For the mesh having the largest elements ($\Delta h = 0.05m$) close to the draining surface,

we have the criterion, $\Delta t \geq 368s$ and the maximum deviation from the analytical solution when this is violated. It must be noted however that for $\tau > 0.0007(t = 413s)$, the differences between the analytical and finite element calculations are insignificant even if the smallest element is larger than that demanded by equation 2.70. This would suggest that for a one-dimensional consolidation scenario, the early numerical oscillations created due to the time-step/mesh-density criteria being violated die down for $t > \Delta t$ suggested by equation 2.70. Hence this criterion assumes significance only when short time solutions are desired.

2.6 Dual-Porosity Parameters

The material parameters in the single-phase, dual-porosity model developed herein are the compliance tensors for the two media, C_{ijkl}^η , elasticity tensor for the porous medium, $D_{ijkl}^{I,II}$, porosities of the two media, ϕ^η , intrinsic permeabilities of the two media, k^η and the fluid transfer coefficient, Ψ . Of these, ϕ^η and $D_{ijkl}^{I,II}$ (for an isotropic dual-porosity medium) may be estimated by thin section analysis and drained triaxial tests on fractured rock samples (Berryman, 2002). Similarly, C_{ijkl}^I and k^I (for an isotropic dual-porosity medium) can be estimated by drained triaxial tests and permeability measurements on an intact rock specimen. Several researchers have proposed analytical estimates of Ψ based on assumed fracture geometries and orientations. A comprehensive list of the same has been compiled by Abousleiman and Nguyen (2003). However, a description of accurate laboratory and field measurements of Ψ has not appeared in literature to date. Also, the significance of C_{ijkl}^{II} and k^{II} may be realized only through “*thought experiments*” (Berryman and Wang, 1995; Berryman, 2002). These dual-porosity parameters may be estimated in course of back analysis such as the history-matching process carried out in reservoir simulation (Mattax and Dalton, 1977) and offset well reviews during an integrated borehole stability analysis (van Oort et al., 1997).

2.7 Summary

In this chapter a dual-porosity model characterized by equations 2.21 and 2.26 has been developed to analyze the response of a fractured porous medium fully saturated with a single fluid, under isothermal conditions, to perturbations. Barenblatt's original concept of two fluid pressures at a point and Aifantis's extension of Biot's poroelastic theory were used within the framework of the separate and overlapping technique. Though this technique is inherent in all dual-porosity models, representation of both the fluid and solid domains by two distinct but overlapping continua marks a significant deviation from classical dual-porosity models. A viable quantification (equations 2.34 and 2.35) of the impact of individual deformations results as a consequence of implementing double effective stress and separate constitutive laws in the model.

Numerical methods, especially the finite element method, have been the dominant tool for application of the dual-porosity poroelastic theory to problems in geomechanics. A finite element model has been developed, based on the mathematical formulation presented herein. The model has been verified against analytical solutions for the problems of one-dimensional consolidation and an inclined borehole in a fully saturated poroelastic medium. The issue of numerical oscillations arising due to small time-steps has been investigated in detail. For a one-dimensional consolidation scenario, the analysis carried out indicates that violation of the mesh density criterion leads to significant errors at the beginning of the consolidation process. For the purpose of organization, a sensitivity analysis involving variation of the dual-porosity parameters will be presented at a later stage. In the next chapter, the mathematical model presented will be extended to include thermal effects. Also, a finite element dual-porosity porothermoelastic model will be developed and verified against analytical solutions.

Chapter 3

Dual-Porosity Porothermoelasticity

3.1 Introduction

Traditionally, problems in the realm of geomechanics involving an interaction between fluid flow, heat transfer and, resultant deformations and stresses, are treated using a partially decoupled approach wherein, the coupled thermo-hydro-mechanical processes are argued to degenerate into situations where merely thermoelastic or poroelastic effects are manifest. There is however a wide range of problems that involve a strong coupling between transport of heat, pressurization and motion of interstitial fluid, and deformation of the porous matrix. (Demirdžić and Muzaferija, 1994; McTigue, 1986). While the simultaneous occurrences of skeleton deformation and pore pressure change and, their interactive influences on each other has received cognizance, it is also widely acknowledged that thermal loading of a porous medium serves to enhance these effects. This is so because the thermal expansion of both, the solid skeleton and pore fluid may lead to additional deformation. Also, a temperature change may significantly influence the pore pressure response, since the pore volume change is constrained by the lower expansion coefficients of the solid constituents as compared to the pore fluid. On the other hand, even in an initially isothermal environment, processes such as hydraulic pressurization inside a borehole may induce heat flow, the so-called Joule-Thompson effect (Wang and Papamichos, 1994). Hence a detailed knowledge of hydromechanical,

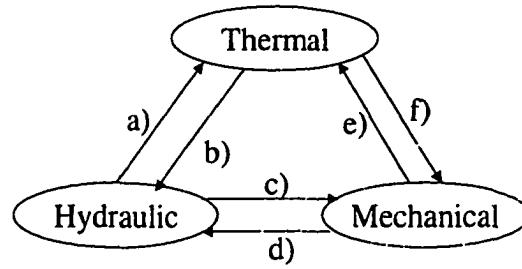


Figure 3.1: Porothermoelastic processes

hydrothermal and thermomechanical processes and their interdependence is necessary to describe a fully coupled behavior of fluid saturated media in the presence of fluid flow. Figure 3.1 illustrates the various coupled processes constituting the porothermoelastic response of a fluid saturated porous medium:

- a) Heat transfer by convection (due to pore fluid flow)
- b) Influence of rate of thermal expansion or contraction of fluid and solid constituents on fluid flux
- c) Modification of effective stress due to change in pore pressure
- d) Influence of volumetric strain rate on fluid flux
- e) Reversible change in energy due to solid elastic deformation
- f) Modification of total strain due to thermal expansion or contraction

In the case of fractured formations, sudden non-uniform temperature changes may increase the fracture density at both micro- and macro-scales. In a nuclear waste repository, thermally induced microcracks may form potential pathways for fluid flow and cause a reduction in strength (Wang et al., 1989; Althaus et al. 1994). In addition,

the mechanisms of heat transport in the matrix and fractures may be different. In the case of the rock matrix with its interconnected grain structures, heat transport via conduction may be dominant, whereas in the case of fractures acting as conduits for fluid flow, the dominant mode may be intuitively reasoned to be convection. Since the thermal conductivity of the matrix blocks is significantly higher than that of the fractures, the heat transfer process between the matrix and fractures is significantly different from that of interporosity flow. Due to the small volume of the fractures and the more rapid dissipation of temperatures facilitated by the matrix, the dual-porosity response of a fractured formation in the context of heat flow is likely to be masked. Hence, although heat flow by conduction is analogous to fluid flow, the paradoxical phenomena of mass and heat flow has attracted little attention as compared to isothermal modeling of a fractured formation (Masters et al., 2000). The dominant influence of the matrix blocks, as far as heat transport is concerned, renders the extension of the multi-porosity approach (Aifantis, 1977) to the multi-thermodynamics continua model (Aifantis and Beskos, 1980) questionable. Also, due to the difficulty in experimental determination of a heat transfer coefficient in a fissured medium, a single-temperature approach appears to be more viable as compared to the 'double-temperature' approach proposed by Aifantis and Beskos (1980).

In this chapter, the mathematical and finite element formulations for coupled heat transfer and pressure-deformation changes in a linearly elastic dual-porosity medium are presented. The fluid flow and solid domains are both represented by two distinct but overlapping continua with separate effective stress and constitutive laws for both, as in Chapter 2. The intensity of interaction between the primary and secondary media is controlled by fluid mass exchange rates which are associated with the pressure difference and matrix permeability. Following the work of Masters et al. (2000), heat transfer via conduction and convection is incorporated within the framework of a single representa-

tive thermodynamics continuum. This renders the concept of energy transfer between the two media as redundant and also negates the necessity of a heat transfer coefficient. Also, lumped parameters for thermal expansion and conductivity are assumed to be representative of the fissured porous medium as a whole. However, the convective heat fluxes in the primary and secondary media are considered separately. Based on the mathematical formulation, a finite element model has been developed wherein the solid deformations, fluid pressures and temperature are the primary unknowns. The resulting nonlinear system of equations in the finite element model is solved using a direct solver. The model presented herein has been verified against relevant analytical solutions (Nair et al. 2002).

Further, the dual-porosity porothermoelastic model is used to analyze the impact of thermal loading, heat transport by convection and dual-porosity parameters on the pressures (in both media) and stresses in a fully saturated fissured formation. Attention has also been focused on the effect of dominance of one mode of heat transport over the other.

3.2 Assumptions

The assumptions made during the development of the mathematical model are formally listed in addition to those in Chapter 2.

1. Temperature equilibrium between the fluid and solid constituents is reached quickly in comparison to the heat transport by conduction and convection.
2. The heat flux is linearly related to the temperature gradient alone and, the pressure gradient term representing a phenomenon analogous to the Dufour effect is neglected. Also, the fluid flux is proportional to the pressure gradient alone and, the conjugate term proportional to the temperature gradient analogous to Soret diffusion is ignored (McTigue, 1986).

3. Only one thermodynamics continuum is assumed to be representative of the primary and secondary media.
4. The coefficients of thermal expansion and conductivity are averaged over the representative elemental volume (REV) and are representative of the fissured porous medium as a whole.
5. The kinetic energies of the fluid and solid constituents are neglected and the potential energies are assumed to be constant in time.
6. The elastic properties and fluid viscosity are assumed to be independent of temperature.

3.3 Mathematical Formulation

The equations for equilibrium, mass conservation of the fluid in both the primary and secondary media, and energy balance constitute a complete set of governing equations describing the macroscale behavior of a fractured porous medium under non-isothermal conditions. This set will be developed taking into account heat transport and fluid flow through the primary and secondary pore spaces. The simultaneous and retroactive deformations of the fractured porous medium and changes in fluid pressures and temperature are described by incorporating the constitutive, effective stress and conduction laws for fluid flow and heat transport into the aforementioned set of equations.

3.3.1 Volumetric Variations

As in Chapter 2, the superscripts I and II always refer to the primary and secondary media, respectively. Also, the definitions of the constituent volumes, associated porosities and the equation relating change in porosities to the change in solid densities and volumetric strain rates (equation 2.8) are as in Chapter 2. A micromechanical approach

has been adopted to elicit the volumetric variations of the components of the model. The pore volumes are assumed to be a function of mean stress, $\bar{\sigma}$, pore pressure P^η and temperature T , i.e.,

$$V_p^\eta = V_p^\eta(\bar{\sigma}, P^\eta, T) \quad (3.1)$$

Hence,

$$\frac{dV_p^\eta}{V_p^\eta} = \frac{1}{V_p^\eta} \frac{\partial V_p^\eta}{\partial \bar{\sigma}} d\bar{\sigma} + \frac{1}{V_p^\eta} \frac{\partial V_p^\eta}{\partial P^\eta} dP^\eta + \frac{1}{V_p^\eta} \frac{\partial V_p^\eta}{\partial T} dT \quad (3.2)$$

The pore volume variations can hence be expressed as

$$\begin{aligned} \frac{dV_p^\eta}{V_p^\eta} = & -\frac{dP^\eta}{K_s^\eta} + \frac{\alpha^\eta}{\phi^\eta K^\eta} (d\bar{\sigma} + dP^\eta) \\ & + \frac{1}{\phi^\eta} (\beta - (1 - \phi^\eta)\beta_s^\eta) dT \end{aligned} \quad (3.3)$$

where α^η are as in Chapter 2; β and β_s^η are the bulk and solid thermal expansion coefficients respectively, given by

$$\frac{1}{V} \frac{\partial V}{\partial T} = \beta; \quad \frac{1}{V_s^\eta} \frac{\partial V_s^\eta}{\partial T} = \beta_s^\eta \quad (3.4)$$

Also, using equations 3.3 and 3.4, the solid volume variations can be expressed in terms of variations of the solid densities as

$$\begin{aligned} \frac{1}{\rho_s^\eta} \frac{d\rho_s^\eta}{dt} = & -\frac{1}{V_s^\eta} \frac{dV_s^\eta}{dt} \\ = & \frac{1}{(1 - \phi^\eta)} \left[\begin{aligned} & \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \frac{dP^\eta}{dt} \\ & - (1 - \alpha^\eta) u^\eta + \\ & \left(\frac{(1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta}{(1 - \phi^\eta)\beta_s^\eta} \right) \frac{dT}{dt} \end{aligned} \right] \end{aligned} \quad (3.5)$$

The derivations of equations 3.3 and 3.5 are provided in Appendix A.

3.3.2 Fluid Mass Conservation

The mass conservation equation for the fluid in both media can be written as

$$\frac{\partial \phi^\eta \rho_f}{\partial t} + \frac{\partial \phi^\eta \rho_f U_i^\eta}{\partial x_i} + (-1)^\eta \Gamma = 0 \quad (3.6)$$

The Darcy velocities for the fluid in both media are defined as

$$v_i^\eta = \phi^\eta (U_i^\eta - u_i^\eta) \quad (3.7)$$

In Equations 3.6 and 3.7, the superscript $\eta = I, II$ stands for the primary and secondary media, respectively. u_i^η are the respective solid velocities; U_i^η are the intrinsic fluid velocities in the two media; ϕ^η are the porosities; ρ_f is the fluid density and Γ is the rate of fluid mass transferred between the two media.

From equations 3.6 and 3.7 we have,

$$\frac{\partial}{\partial t} (\phi^\eta \rho_f) + \frac{\partial}{\partial x_i} (\phi^\eta \rho_f u_i^\eta + \rho_f v_i^\eta) + (-1)^\eta \Gamma = 0 \quad (3.8)$$

Expanding derivatives, we have

$$\left\{ \begin{array}{l} \rho_f \frac{\partial \phi^\eta}{\partial t} + \phi^\eta \frac{\partial \rho_f}{\partial t} + \rho_f u_i^\eta \phi_{,i}^\eta + \phi^\eta u_i^\eta (\rho_f)_{,i} + \\ + \phi^\eta \rho_f u_{i,i}^\eta + \rho_f v_{i,i}^\eta + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{array} \right. \quad (3.9)$$

Grouping terms appropriately and using equations 2.4 and 3.9 we have,

$$\left\{ \begin{array}{l} \rho_f \frac{D\phi^\eta}{Dt} + \phi^\eta \frac{D\rho_f}{Dt} + \phi^\eta \rho_f u_{i,i}^\eta + \\ \rho_f v_{i,i}^\eta + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{array} \right. \quad (3.10)$$

From equations 2.43, 3.5 and 3.9 we have,

$$\left\{ \begin{array}{l} \rho_f \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \frac{DP^\eta}{Dt} + \phi^\eta \frac{D\rho_f}{Dt} + \rho_f \alpha^\eta u_{i,i}^\eta + \\ \rho_f v_{i,i}^\eta + \rho_f ((1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta) \frac{DT}{Dt} \\ + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{array} \right. \quad (3.11)$$

Now,

$$\frac{1}{\rho_f} \frac{D\rho_f}{Dt} = \frac{1}{K_f} \frac{DP^\eta}{Dt} - \beta_f \frac{DT}{Dt} \quad (3.12)$$

and invoking Darcy's law,

$$v_i^\eta = -\frac{k^\eta}{\mu_f} (P^\eta)_{,i} \quad (3.13)$$

where k^η are the permeabilities of the two media and μ_f is the fluid viscosity. Thus using equations 3.11 - 3.13, letting $\frac{D}{Dt} = \frac{\partial}{\partial t}$ and grouping terms we get the final form of the mass conservation for fluid in both media as,

$$\left\{ \begin{array}{l} \frac{\partial P^\eta}{\partial t} \left\{ \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \right\} + \\ \frac{\partial T}{\partial t} \left\{ \rho_f \left(\begin{array}{l} (1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta \\ - \phi^\eta\beta_f \end{array} \right) \right\} \\ + \alpha^\eta \rho_f u_{i,i}^\eta - \rho_f \frac{k^\eta}{\mu_f} (P^\eta)_{i,i} - \\ + v_i^\eta (\rho_f)_{,i} + (-1)^\eta \Gamma = 0 \end{array} \right\} \quad (3.14)$$

where $(\cdot)$ denotes derivative with respect to time.

3.3.3 Equilibrium Equation

The relationships between changes in total stresses σ_{ij} and intergranular stresses σ'_{ij} extended to dual-porosity are as follows

$$\sigma_{ij}^\eta = (\sigma'_{ij}^\eta)' - \alpha^\eta P^\eta \delta_{ij} \quad (3.15)$$

Also,

$$(\varepsilon_{ij}^\eta)' = C_{ijkl}^\eta (\sigma_{kl}^\eta)' + \frac{\beta^\eta}{3} T \delta_{ij} \quad (3.16)$$

where

$$(\beta^\eta) = \delta_{ij} C_{ijkl}^\eta D_{klmn}^{I,II} \delta_{mn} \frac{\beta}{3} \quad (3.17)$$

In equations 3.15 and 3.16, $(\sigma'_{ij}^\eta)'$, α^η and δ_{ij} are as defined in Chapter 2 and β is the bulk thermal expansion coefficient. After considering requirements of local stress equilibrium and lumping the strains (Nair et al. 2000), the equilibrium equation may be written as

$$\left\{ \begin{array}{l} (D_{ijkl}^{I,II} \varepsilon_{kl} - D_{ijkl}^{I,II} C_{klmn}^I \alpha^I \delta_{mn} P^I - D_{ijkl}^{I,II} C_{klmn}^{II} \alpha^{II} \delta_{mn} P^{II} - \\ D_{ijkl}^{I,II} \frac{\beta}{3} T \delta_{kl})_{,j} + F_i = 0 \end{array} \right. \quad (3.18)$$

where $D_{ijkl}^{I,II}$, C_{ijkl}^I , C_{ijkl}^{II} , ε_{kl} and F_i are as in Chapter 2. The volumetric strain rate manifested in each of the two media needs to be expressed in terms of the total volumetric strain rate. This may be done by as in Chapter 2, i.e.,

$$\dot{\varepsilon}_{kk}^\eta = C_{kkmm}^\eta D_{mmnn}^{I,II} \dot{\varepsilon}_{nn} \quad (3.19)$$

3.3.4 Energy Balance

The macroscopic energy balance equations are developed by averaging the microscopic equations for a point in the fluid and solid continua over an REV of the dual-porosity porothermoelastic medium (Bird et al. 1960, Bear and Corapcioglu 1981). Neglecting the rate of internal energy increase by compression and viscous dissipation, the microscopic equation for the rate of gain of internal energy per unit volume for the fluid may be expressed as (Bird et al., 1960)

$$\frac{\partial (\rho_f^* \hat{U})}{\partial t} = - \frac{\partial (\rho_f^* U_i^* \hat{U})}{\partial x_i} - \nabla \cdot (\mathbf{q}^{f*}) \quad (3.20)$$

where $\hat{U}$ is the internal energy per unit mass, $\mathbf{q}^{f*}$ is the heat flux vector and all symbols with an asterisk denotes microscopic values. Expanding equation 3.20 we have,

$$\rho_f^* \frac{\partial \hat{U}}{\partial t} + \hat{U} \frac{\partial (\rho_f^*)}{\partial t} = - \hat{U} \frac{\partial (\rho_f^* U_i^*)}{\partial x_i} - \rho_f^* U_i^* \frac{\partial \hat{U}}{\partial x_i} - \nabla \cdot (\mathbf{q}^{f*}) \quad (3.21)$$

Combined with the mass conservation equation at the microscopic level expressed as

$$\frac{\partial (\rho_f^*)}{\partial t} = - \frac{\partial (\rho_f^* U_i^*)}{\partial x_i} \quad (3.22)$$

equation 3.21 becomes,

$$\rho_f^* \frac{d_f \hat{U}}{dt} = - \nabla \cdot (\mathbf{q}^{f*}) \quad (3.23)$$

where

$$\frac{d_f}{dt} = \frac{\partial}{\partial t} + U_i^* \frac{\partial}{\partial x_i} \quad (3.24)$$

Assuming $d\hat{U} \approx C_{vf}dT^{f*}$ in order to express equation 3.23 more conveniently in terms of fluid temperature, we have

$$\rho_f^* C_{vf} \frac{dT^{f*}}{dt} = -\nabla \cdot (\mathbf{q}^{f*}) \quad (3.25)$$

From equations 3.24 and 3.25 we have,

$$\rho_f^* C_{vf} \left(\frac{\partial T^{f*}}{\partial t} + U_i^* \frac{\partial T^{f*}}{\partial x_i} \right) = -\nabla \cdot (\mathbf{q}^{f*}) \quad (3.26)$$

Using equations 3.22 and 3.26 we have,

$$\frac{\partial (\rho_f^* C_{vf} T^{f*})}{\partial t} = -\frac{\partial (\rho_f^* C_{vf} U_i^* T^{f*})}{\partial x_i} - \nabla \cdot (\mathbf{q}^{f*}) \quad (3.27)$$

Equation 3.27 may now be averaged over each of the fluid continua and added to obtain the macroscopic ,

$$\frac{\partial [(\phi^I + \phi^{II}) \rho_f C_{vf} T]}{\partial t} = -\frac{\partial (\phi^I \rho_f C_{vf} U_i^I T^I + \phi^{II} \rho_f C_{vf} U_i^{II} T^{II})}{\partial x_i} - \nabla \cdot [(\phi^I + \phi^{II}) \mathbf{q}^f] \quad (3.28)$$

Using the macroscopic mass conservation equations for the fluid in both media (equation 2.13), expressing the intrinsic fluid velocities U_i^η in terms of the Darcy velocities (after neglecting the solid velocities) as in equation 2.14 and invoking Fourier's heat conduction law as in

$$\mathbf{q}^f = -\lambda_f \nabla T^f \quad (3.29)$$

we have

$$\begin{cases} [(\phi^I + \phi^{II}) \rho_f C_{vf}] \frac{\partial T^f}{\partial t} + [\rho_f C_{vf} (\mathbf{v}^I + \mathbf{v}^{II})] \cdot \nabla T^f - \\ \nabla^T [(\phi^I + \phi^{II}) \lambda_f] \cdot \nabla T^f = 0 \end{cases} \quad (3.30)$$

where λ_f in equation 3.29 is the coefficient of thermal conductivity for the fluid and T^f is the averaged temperature for the fluid. Similarly, for the solid we have (Bear and Corapcioglu, 1981; Nowacki, 1975)

$$\begin{cases} [(1 - \phi^I - \phi^{II}) \rho_s C_{vs}] \frac{\partial T^s}{\partial t} + [(1 - \phi^I - \phi^{II}) \rho_s C_{vs} \mathbf{u}^s] \cdot \nabla T^s - \\ \nabla^T [(1 - \phi^I - \phi^{II}) \lambda_s] \cdot \nabla T^s + (1 - \phi^I - \phi^{II}) 3K\beta T^s \frac{\partial \varepsilon_{kk}}{\partial t} = 0 \end{cases} \quad (3.31)$$

In equation 3.31, C_{vs} is the specific heat capacities of the solid, $\mathbf{u}^s$ is the solid velocity vector, λ_s is the coefficient of thermal conductivity for the solid and T^s is the averaged temperature for the solid. Also, the last term in the above equation corresponds to the reversible change in energy due to solid elastic deformation. Now, we assume $T^f = T^s = T$, i.e., the averaged temperatures for the fluid, solid and the whole porous medium, are the same. This assumption is based on the one that the solid grains are small compared to the fluid channels and that temperature equilibration between the fluid and solid constituents is quicker than the transport processes occurring in the whole domain (Bear and Corapcioglu, 1981). Also, the solid velocity in equation 3.31 may be assumed to be negligible compared to the fluid velocities in equation 3.30. Incorporating these assumptions and adding equations 3.30 and 3.31, the thermal energy conservation equation for the medium as a whole is obtained as

$$\left\{ \begin{array}{l} \left[\begin{array}{l} (\phi^I + \phi^{II})\rho_f C_{vf} + \\ (1 - \phi^I - \phi^{II})\rho_s C_{vs} \end{array} \right] \frac{\partial T}{\partial t} + \\ \left[\rho_f C_{vf} (v_i^I + v_i^{II}) \right] \cdot \nabla T - \\ \nabla T \left[\begin{array}{l} (\phi^I + \phi^{II})\lambda_f + \\ (1 - \phi^I - \phi^{II})\lambda_s \end{array} \right] \cdot \nabla T + \\ (1 - \phi^I - \phi^{II}) 3K\beta T \frac{\partial \varepsilon_{kk}}{\partial t} = 0 \end{array} \right. \quad (3.32)$$

It must be noted that equations 3.30 and 3.31, used to obtain the final form of the macroscopic energy balance equation, reflect the fact that a single thermodynamic continuum was assumed to be representative of both the primary and secondary media. Hence, unlike as in the fluid mass conservation equations (where we had mass transfer terms representing interporosity flow), terms corresponding to energy transfer between the two media are absent.

3.4 The Finite Element Method in Dual-Porosity Porothermoelasticity

3.4.1 Finite Element Equations

The finite element equations are developed with the displacements, pore pressures and temperature as the primary variables, i.e., $\mathbf{u}$, P^I , P^{II} and T . These are expressed in terms of approximated nodal quantities as

$$P^{\eta*} \simeq \mathbf{N}\mathbf{P}^\eta; \quad T \simeq \mathbf{N}\mathbf{T}; \quad \mathbf{u}^* \simeq \mathbf{N}\bar{\mathbf{u}}; \quad (3.33)$$

where the superscript (*) denotes the finite element approximation and $\mathbf{N}$ represents the shape functions. $\bar{\mathbf{u}}$ refers to the nodal displacements and the (-) will be dropped herewith. Also, the strains of both media, i.e. $u_{i,i}^\eta$, can be expressed in terms of the total body strain as in equation 2.50

Substitution of the approximate quantities (equation 3.33) into equation 3.14, using equation 2.50 and application of the Galerkin's principle and Green's theorem gives

$$\begin{aligned} & \int_V \mathbf{N}^T \rho_f \left[\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} + \frac{\phi^\eta}{K_f} \right] \mathbf{N} dV \frac{d\mathbf{P}^\eta}{dt} + \\ & \int_V \mathbf{N}^T \rho_f ((1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta - \phi^\eta\beta_f) \mathbf{N} dV \frac{d\mathbf{T}}{dt} \\ & + \int_V \mathbf{N}^T \alpha^\eta \rho_f \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \frac{d\mathbf{u}}{dt} + \int_V (\nabla \mathbf{N})^T \rho_f \frac{k^\eta}{\mu_f} \nabla \mathbf{N} dV \mathbf{P}^\eta \\ & + (-1)^\eta \int \mathbf{N}^T \rho_f \frac{k^I}{\mu_f} \Psi \mathbf{N} dV (\mathbf{P}_o^I - \mathbf{P}_o^{II}) + \mathbf{Q}_g^\eta = 0 \end{aligned} \quad (3.34)$$

The finite element equation for momentum equilibrium based on equations 2.54 and 3.18 is

$$\begin{aligned} & \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{B} dV \frac{d\mathbf{u}}{dt} - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \mathbf{m} \mathbf{N} dV \frac{d\mathbf{P}^I}{dt} \\ & - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \mathbf{m} \mathbf{N} dV \frac{d\mathbf{P}^{II}}{dt} \\ & - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \frac{\beta}{3} \mathbf{m} \mathbf{N} dV \frac{d\mathbf{T}}{dt} = \frac{d\mathbf{f}}{dt} \end{aligned} \quad (3.35)$$

Similarly, the finite element equation for the energy balance can be obtained from 3.32

as

$$\begin{aligned}
& \int_V \mathbf{N}^T \begin{bmatrix} (\phi^I + \phi^{II}) \rho_f C_{vf} + \\ (1 - \phi^I - \phi^{II}) \rho_s C_{vs} \end{bmatrix} \mathbf{N} dV \frac{dT}{dt} + \\
& \int_V (\nabla \mathbf{N})^T [\rho_f C_{vf} (\mathbf{v}^I + \mathbf{v}^{II})] \mathbf{N} dV \mathbf{T} + \\
& \int_V (\nabla \mathbf{N})^T \begin{bmatrix} (\phi^I + \phi^{II}) \lambda_f + \\ (1 - \phi^I - \phi^{II}) \lambda_s \end{bmatrix} \nabla \mathbf{N} dV \mathbf{T} + \\
& \int_V \mathbf{N}^T (1 - \phi^I - \phi^{II}) 3K\beta T \mathbf{m}^T \mathbf{B} dV \frac{d\mathbf{u}}{dt} + \mathbf{Q}_t = 0
\end{aligned} \tag{3.36}$$

The final form of the finite element governing equations are:

Equilibrium

$$\mathbf{K} \frac{d\mathbf{u}}{dt} + \mathbf{L}_{fI}^I \frac{d\mathbf{P}^I}{dt} + \mathbf{L}_{fII}^{II} \frac{d\mathbf{P}^{II}}{dt} + \mathbf{L}_t \frac{dT}{dt} - \frac{d\mathbf{f}}{dt} = 0 \tag{3.37}$$

Continuity

$$\begin{cases} \mathbf{K}_f^\eta \frac{d\mathbf{u}}{dt} + (\mathbf{H}_D^\eta + \mathbf{H}_{tr}) \times \mathbf{P}^\eta \\ -\mathbf{H}_{tr} \mathbf{P}^\eta + \mathbf{S}^\eta \frac{d\mathbf{P}^\eta}{dt} + \mathbf{S}_t^\eta \frac{dT}{dt} = -\mathbf{Q}^\eta \end{cases} \tag{3.38}$$

Energy Balance

$$\mathbf{T}_s \frac{dT}{dt} + (\mathbf{C}_t + \mathbf{H}_{tF}) \times \mathbf{T} + \mathbf{K}_t \frac{d\mathbf{u}}{dt} = -\mathbf{Q}_t \tag{3.39}$$

Equations 3.37-3.39 can be represented in a compact form as

$$\begin{aligned}
& \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{H}_D^I + \mathbf{H}_{tr} & -\mathbf{H}_{tr} & 0 \\ 0 & -\mathbf{H}_{tr} & \mathbf{H}_D^{II} + \mathbf{H}_{tr} & 0 \\ 0 & 0 & 0 & \mathbf{C}_t + \mathbf{H}_{tF} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \\ \mathbf{T} \end{Bmatrix} + \\
& \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fI}^I & \mathbf{L}_{fII}^{II} & \mathbf{L}_t \\ \mathbf{K}_f^I & \mathbf{S}_f^I & 0 & \mathbf{S}_t^I \\ \mathbf{K}_f^{II} & 0 & \mathbf{S}_f^{II} & \mathbf{S}_t^{II} \\ \mathbf{K}_t & 0 & 0 & \mathbf{T}_s \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \\ \mathbf{T} \end{Bmatrix} = \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}^I \\ -\mathbf{Q}^{II} \\ -\mathbf{Q}_t \end{Bmatrix}
\end{aligned} \tag{3.40}$$

The backward difference time-stepping scheme in Chapter 2 is employed to discretize

equation 3.40 in the time domain as follows:

$$\begin{cases} \left(\frac{d\mathbf{u}}{dt} \right)^{t+\Delta t} = \frac{1}{\Delta t} (\mathbf{u}^{t+\Delta t} - \mathbf{u}^t) \\ \left(\frac{d\mathbf{P}^\eta}{dt} \right)^{t+\Delta t} = \frac{1}{\Delta t} [(\mathbf{P}^\eta)^{t+\Delta t} - (\mathbf{P}^\eta)^t] \\ \left(\frac{d\mathbf{T}}{dt} \right)^{t+\Delta t} = \frac{1}{\Delta t} [(\mathbf{T})^{t+\Delta t} - (\mathbf{T})^t] \end{cases} \quad (3.41)$$

From equations 3.40 and 3.41, we get

$$\begin{aligned} & \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fl}^I & \mathbf{L}_{fl}^{II} & \mathbf{L}_t \\ \mathbf{K}_f^I & \mathbf{H}_{Dtr}^I \Delta t + \mathbf{S}^I & -\mathbf{H}_{tr} \Delta t & \mathbf{S}_t^I \Delta t \\ \mathbf{K}_f^{II} & -\mathbf{H}_{tr} \Delta t & \mathbf{H}_{Dtr}^{II} \Delta t + \mathbf{S}^{II} & \mathbf{S}_t^{II} \Delta t \\ \mathbf{K}_t & \mathbf{0} & \mathbf{0} & \mathbf{C}_{tF} \Delta t + \mathbf{T}_s \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \\ \mathbf{T} \end{Bmatrix}^{t+\Delta t} \\ &= \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{L}_{fl}^I & \mathbf{L}_{fl}^{II} & \mathbf{L}_t \\ \mathbf{K}_f^I & \mathbf{S}^I & \mathbf{0} & \mathbf{S}_t^I \\ \mathbf{K}_f^{II} & \mathbf{0} & \mathbf{S}^{II} & \mathbf{S}_t^{II} \\ \mathbf{K}_t & \mathbf{0} & \mathbf{0} & \mathbf{T}_s \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}^I \\ \mathbf{P}^{II} \\ \mathbf{T} \end{Bmatrix}^t + \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}^I \\ -\mathbf{Q}^{II} \\ -\mathbf{Q}_t \end{Bmatrix}^{t+\Delta t} \end{aligned} \quad (3.42)$$

where,

$$\begin{aligned} \mathbf{H}_{Dtr}^\eta &= \mathbf{H}_D^\eta + \mathbf{H}_{tr} \\ \mathbf{C}_{tF} &= \mathbf{C}_t + \mathbf{H}_{tF} \end{aligned} \quad (3.43)$$

The sub-matrices in equation 3.42 are listed in Appendix A.

3.4.2 Model Verification With Inclined Wellbore Solutions

The model developed has been verified against analytical solutions of an inclined wellbore subjected to “a three-dimensional in-situ” state of stress in a single-porosity medium, saturated with a single fluid under non-isothermal conditions. The generalized plane strain approach described in Chapter 2 has been adopted. The far field boundary stresses and initial conditions are available in literature (Abousleiman and Ekbote, 2002; Nair et al., 2002). The input parameters for the model listed in Table 3.1 have been selected to approximate a homogeneous, isotropic, single-porosity medium which was discretized as in

Chapter 2. Figures 3.2-3.5 show the comparison between the finite element calculations and analytical solutions (Abousleiman and Ekbote, 2002) for three different times¹. In order to compare the finite element and analytical solutions, the convective terms in the energy equation were neglected. In addition, the following approximations have been made to represent the heat capacity and thermal conductivity for the porous medium as a whole (Bear and Corapcioglu, 1981)

$$\rho C_v = (\phi^I + \phi^{II})\rho_f C_{vf} + (1 - \phi^I - \phi^{II})\rho_s C_{vs} \quad (3.44)$$

$$\lambda = (\phi^I + \phi^{II})\lambda_f + (1 - \phi^I - \phi^{II})\lambda_s \quad (3.45)$$

where, $\rho = (\phi^I + \phi^{II})\rho_f + (1 - \phi^I - \phi^{II})\rho_s$. Also, the difference in the expansion coefficients β and β_s reflecting the difference in the thermal response of the bulk porous medium and that of the solid constituents alone, has been neglected. Thus, with the assumption $\beta = \beta_s$, the coefficient of $\frac{\partial T}{\partial t}$ in equation 3.14 becomes (McTigue, 1986; Abousleiman and Ekbote, 2002),

$$- [\alpha\beta + \phi(\beta_f - \beta)] \quad (3.46)$$

The simplifications in equations 3.44-3.46 will be used henceforth in this chapter. The spatial distribution of porepressure, temperature and effective radial stresses are in a direction along that of the minimum horizontal stress in the local wellbore coordinate system.

3.5 Parametric Analysis

A parametric study of an inclined wellbore in a dual-porosity medium under non-isothermal conditions has been carried out to analyze

¹In Figures 3.2-3.5, the solid and dotted lines are from the analytical solutions (Abousleiman and Ekbote, 2002)

Table 3.1: Parameters for verification (inclined borehole, non-isothermal case)

Parameter	Units	Value
Elastic Modulus (E)	GPa	9.474
Poisson's Ratio (ν)	-	0.24
Matrix Porosity ($\phi^{(1)}$)	-	0.14
Matrix Perm. ($k^{(1)}$)	md	5×10^{-5}
Grain Bulk Modulus (K_s)	GPa	10.041
Fluid Bulk Modulus (K_f)	MPa	4335
Fluid Viscosity (μ)	MPa.s	10^{-9}
Thermal Conductivity (λ)	W/m/ $^{\circ}$ C	2.65
Thermal Expansion Coefficient (β)	/ $^{\circ}$ C	1.8×10^{-5}
Formation heat cap. C_v	J/kg/ $^{\circ}$ C	837
Fluid heat cap. (C_{vf})	J/kg/ $^{\circ}$ C	4186
Bulk density (ρ)	kg/m 3	1980
Drilling fluid temperature	$^{\circ}$ C	50
Mudweight	MPa	12

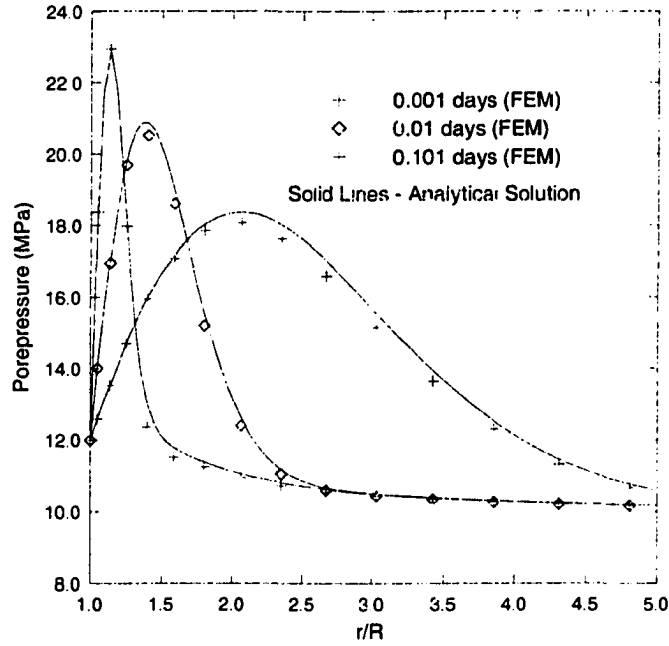


Figure 3.2: Verification of pore pressure distribution (inclined borehole, porothermoelastic solution).

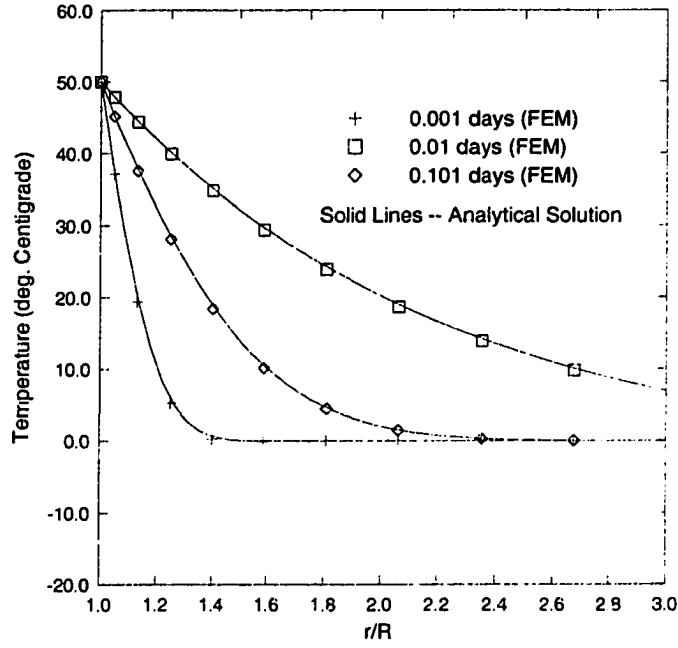


Figure 3.3: Verification of temperature distribution (inclined borehole, porothermoelastic solution).

- a) The impact of thermal loading
- b) The effect of heat transport via convection and
- c) The effect of dual-porosity parameters

In order to carry out the dual-porosity porothermoelastic modeling of a fractured formation, the secondary medium was characterized with the following parameters - stiffness (K_n and K_{sh}) and fracture spacing (s) governing the compliance of the secondary medium; secondary permeability (k^{II}) and; a secondary porosity (ϕ^{II}). Formally, C_{ijkl}^{II} was chosen to be

$$C_{ijkl}^{II} = \delta_{il}\delta_{jk} \left(\frac{1}{K_n s} - \frac{1}{K_{sh} s} \right) + \frac{\delta_{ik}\delta_{jl}}{K_{sh} s} \quad (3.47)$$

where K_n and K_{sh} are the normal and shear stiffness of the fractures and in the transfer coefficient influencing the interporosity flow in equation 2.52 is given by (Warren and

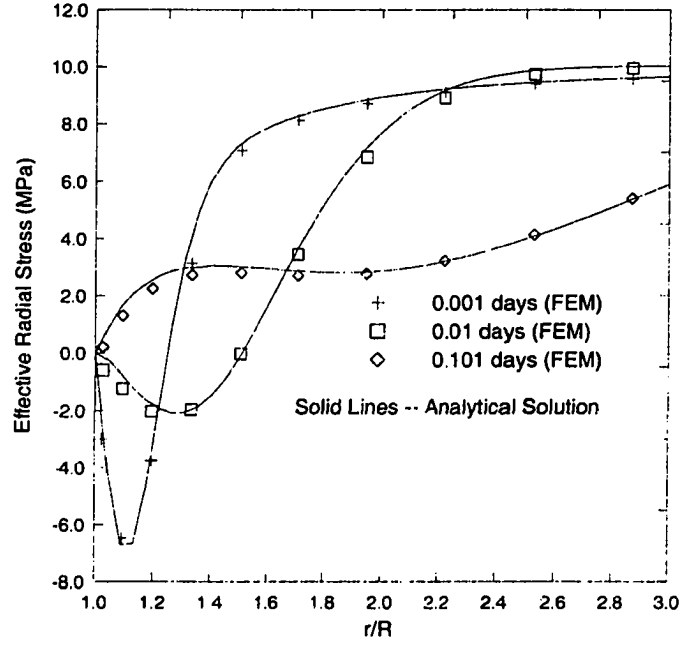


Figure 3.4: Verification of effective radial stress distribution (inclined borehole, porothermoelastic solution).

Root, 1963),

$$\Psi = \frac{60}{s^2} \quad (3.48)$$

3.5.1 Impact of thermal loading

The effect of temperature gradients on the pore pressure and stress distribution can be assessed by comparing results for an inclined borehole under both isothermal and non-isothermal conditions. The input parameters for the primary medium representing the matrix are as in Table 3.2 (Gulf of Mexico Shale). The secondary permeability and porosity are set to values 10 times greater and lower than the respective values for the primary medium (Wilson & Aifantis 1982), and the stiffness of the secondary medium is 10 GPa/m (i.e., $K_n = K_{sh} = 10.0$ GPa/m).

The impact of the thermal loading on the pressure and stresses can be gaged from Figure 3.6 where the pressures in both media are significantly higher for the non-isothermal

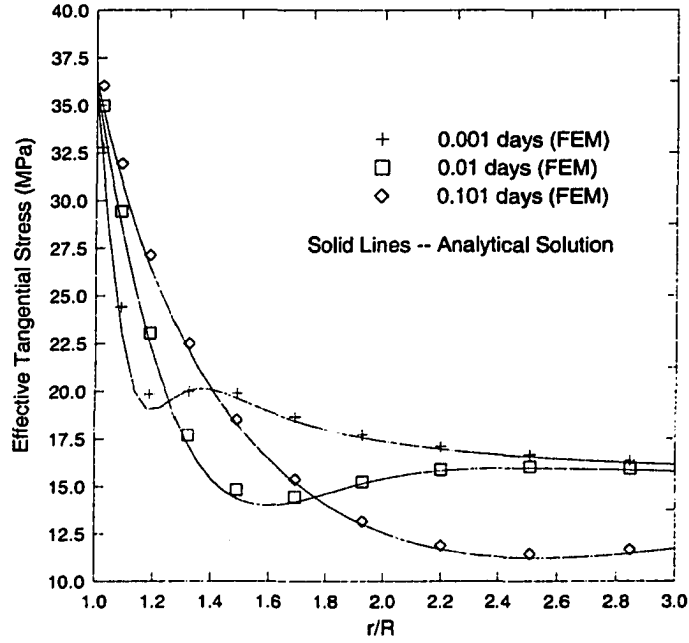


Figure 3.5: Verification of effective tangential stress distribution (inclined borehole, porothermoelastic solution).

Table 3.2: Input parameters (Gulf of Mexico Shale).

Parameter	Units	Value
Elastic Modulus (E)	GPa	1.853
Poisson's Ratio (ν)	-	0.22
Matrix Porosity ($\phi^{(1)}$)	-	0.14
Matrix Perm. ($k^{(1)}$)	md	1×10^{-4}
Grain Bulk Modulus (K_s)	GPa	27.5
Fluid Bulk Modulus (K_f)	MPa	1733
Fluid Viscosity (μ)	MPa.s	10^{-9}
Thermal Conductivity (λ)	W/m/ $^{\circ}$ C	3.0
Thermal Expansion Coefficient (β)	/ $^{\circ}$ C	1.8×10^{-5}
Formation heat capacity ρC_v	MJ/kg/ $^{\circ}$ C	1.7
Fluid heat capacity ($\rho_f C_{vf}$)	MJ/kg/ $^{\circ}$ C	4.186
Bulk density (ρ)	kg/m ³	1980
Drilling fluid temperature	$^{\circ}$ C	50
Mudweight	MPa	12

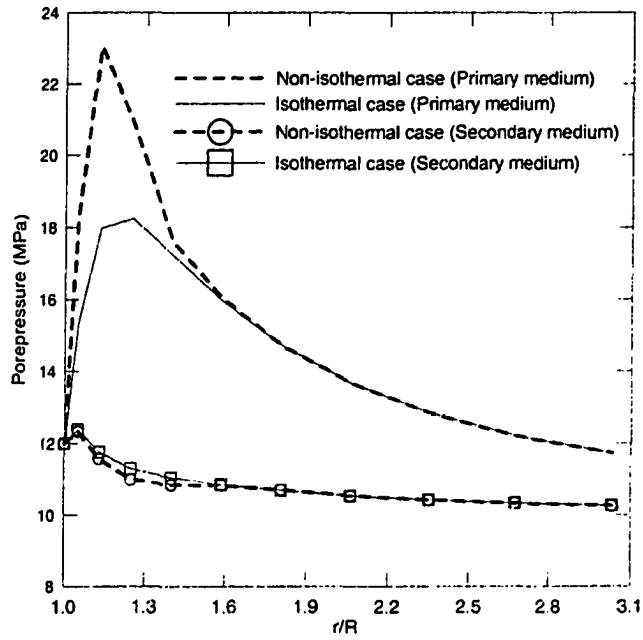


Figure 3.6: Effect of thermal loading on porepressures (dual-porosity, porothermoelasticity, $t = 0.001$ days, Gulf of Mexico Shale)

case. Correspondingly, the effective radial stress close to the borehole for the non-isothermal case is more tensile as seen in Figure 3.7. Also, the effective tangential stress is less compressive for the non-isothermal case as seen in Figure 3.8. Comparisons for the single-porosity case using analytical solutions for the stress and pore pressure profiles have been carried out by Abousleiman and Ekbote (2002). It must be pointed out that in the aforementioned analyses, only heat transfer through conduction was considered. The pore pressure and stress profiles in the isothermal and non-isothermal cases are significantly different as opposed to the conclusions drawn by Masters et al (2000).

3.5.2 Effect of heat transport via convection

The effect of heat transport by convection varies with the type of rock and hence this effect has been analyzed herein using data for Charcoal granite, Ruhr sandstone and Weber sandstone as in Tables 3.3-3.5 (Garcia and Natale, 1999). The results in this

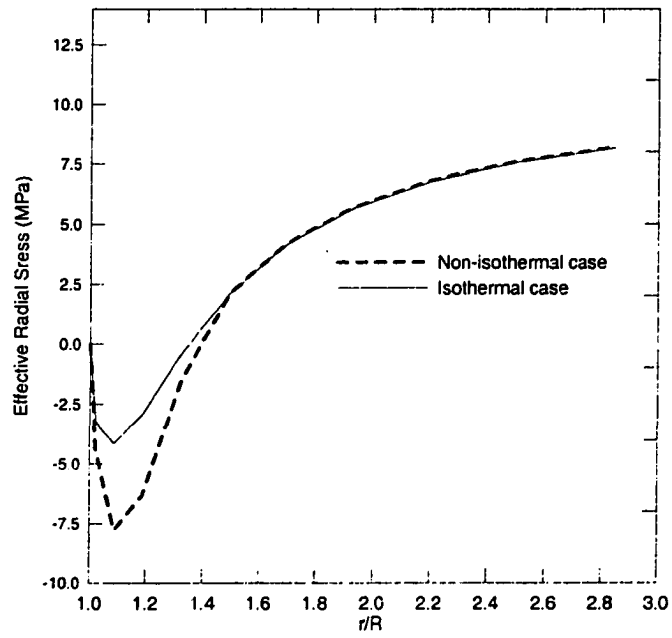


Figure 3.7: Effect of thermal loading on effective radial stress (dual-porosity, porothermoelasticity, $t = 0.001$ days, Gulf of Mexico Shale)

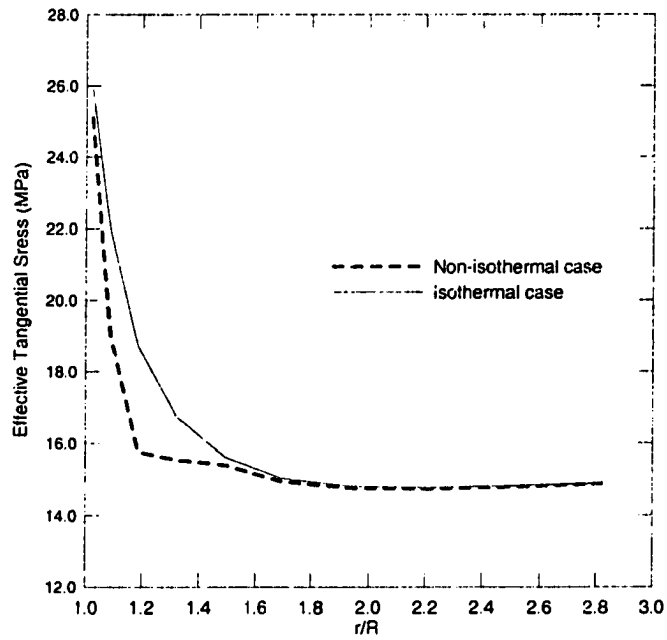


Figure 3.8: Effect of thermal loading on effective tangential stress (dual-porosity, porothermoelasticity, $t = 0.001$ days, Gulf of Mexico Shale)

section have been obtained by considering heat transport by conduction and convection. In order to analyze the convective effect, corresponding results obtained by neglecting convective terms in the model have also been included. Figures 3.9, 3.10, 3.11 and 3.12 show the spatial and temporal variations of pore pressure, temperature, effective radial stress and effective tangential stress for an inclined wellbore in a single-porosity medium of Charcoal granite. Due to the low permeability of Charcoal granite, the convective effects are negligible at all times and the heat transport occurs primarily via conduction.

Table 3.3: Input parameters (Charcoal Granite).

Parameter	Units	Value
Elastic Modulus (E)	GPa	48.26
Poisson's Ratio (ν)	-	0.27
Matrix Porosity ($\phi^{(1)}$)	-	0.02
Matrix Permeability ($k^{(1)}$)	md	1×10^{-4}
Grain Bulk Modulus (K_s)	GPa	45
Fluid Bulk Modulus (K_f)	MPa	3300
Fluid Viscosity (μ)	MPa.s	10^{-9}
Thermal Conductivity (λ)	W/m/ $^{\circ}$ C	3.0
Thermal Expansion Coefficient (β)	/ $^{\circ}$ C	2.4×10^{-5}
Formation Heat Capacity ρC_v	MJ/kg/ $^{\circ}$ C	2.63
Fluid Heat Capacity ($\rho_f C_{vf}$)	MJ/kg/ $^{\circ}$ C	4.186
Bulk Density (ρ)	kg/m ³	2568
Drilling Fluid Temperature	$^{\circ}$ C	50
Mudweight	MPa	12

In contrast, Ruhr sandstone has a larger fluid diffusivity and the convective effects are noticeable. As seen in Figure 3.13, the pore pressure profiles for all three times are those for the steady state. At the initial stages (0.001 days), the heat transfer is mainly through conduction as can be seen in Figure 3.14. However, as time progresses, the temperature and compressive effective radial and tangential stress distributions shown in Figures 3.14-3.16 exhibit an increase in magnitude when the convective terms in the model are included.

In the case of Weber sandstone, the fluid diffusivity is even higher. As in case of

Table 3.4: Input parameters (Ruhr sandstone).

Parameter	Units	Value
Elastic Modulus (E)	GPa	29.12
Poisson's Ratio (ν)	-	0.12
Matrix Porosity ($\phi^{(1)}$)	-	0.02
Matrix Permeability ($k^{(1)}$)	md	0.2
Grain Bulk Modulus (K_s)	GPa	36.0
Fluid Bulk Modulus (K_f)	MPa	3300
Fluid Viscosity (μ)	MPa.s	10^{-9}
Thermal Conductivity (λ)	W/m/ $^{\circ}$ C	3.0
Thermal Expansion Coefficient (β)	/ $^{\circ}$ C	3.0×10^{-5}
Formation Heat Capacity ρC_v	MJ/kg/ $^{\circ}$ C	3.02
Fluid Heat Capacity ($\rho_f C_{vf}$)	MJ/kg/ $^{\circ}$ C	4.186
Bulk Density (ρ)	kg/m ³	2960
Drilling Fluid Temperature	$^{\circ}$ C	50
Mudweight	MPa	12

Table 3.5: Input parameters (Weber sandstone).

Parameter	Units	Value
Elastic Modulus (E)	GPa	27.6
Poisson's Ratio (ν)	-	0.15
Matrix Porosity ($\phi^{(1)}$)	-	0.02
Matrix Permeability ($k^{(1)}$)	md	1.0
Grain Bulk Modulus (K_s)	GPa	36
Fluid Bulk Modulus (K_f)	MPa	3300
Fluid Viscosity (μ)	MPa.s	10^{-9}
Thermal Conductivity (λ)	W/m/ $^{\circ}$ C	3.0
Thermal Expansion Coefficient (β)	/ $^{\circ}$ C	3.0×10^{-5}
Formation Heat Capacity ρC_v	MJ/kg/ $^{\circ}$ C	2.32
Fluid Heat Capacity ($\rho_f C_{vf}$)	MJ/kg/ $^{\circ}$ C	4.186
Bulk Density (ρ)	kg/m ³	2128
Drilling Fluid Temperature	$^{\circ}$ C	50
Mudweight	MPa	12

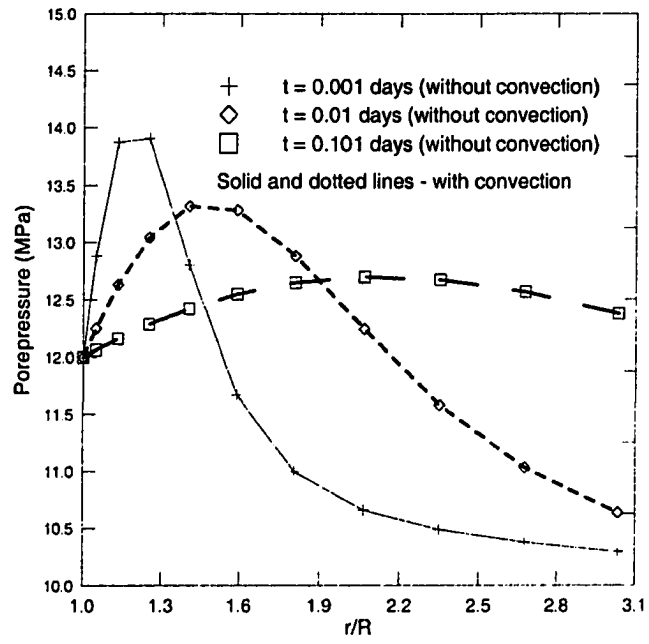


Figure 3.9: Spatiotemporal variation of porepressure (Charcoal Granite)

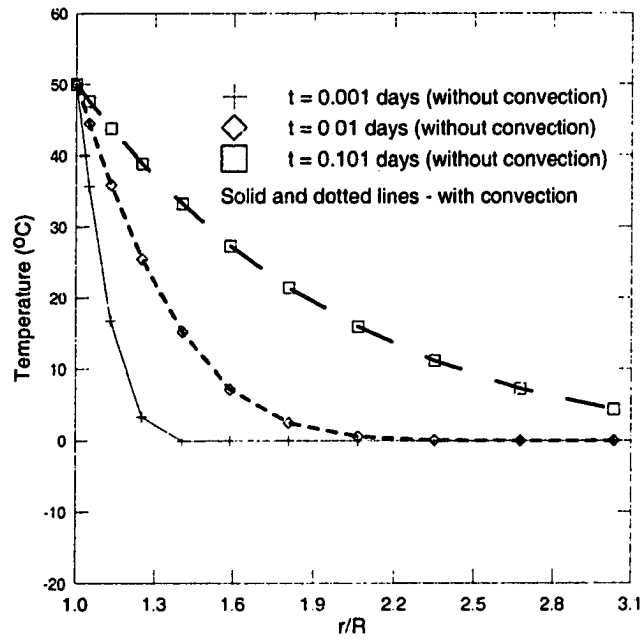


Figure 3.10: Spatiotemporal variation of temperature (Charcoal Granite)

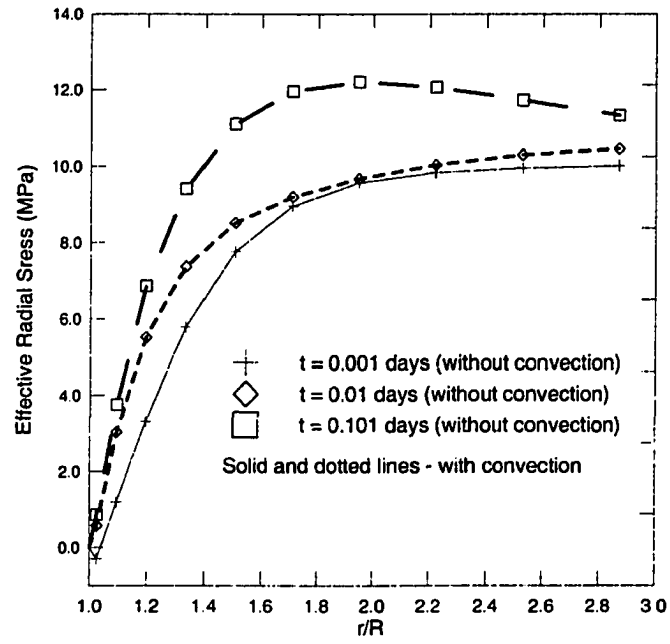


Figure 3.11: Spatiotemporal variation of effective radial stress (Charcoal Granite)

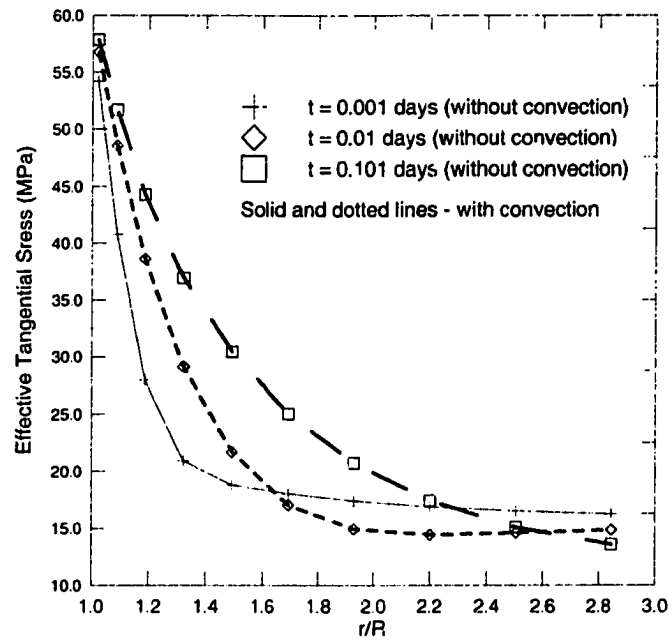


Figure 3.12: Spatiotemporal variation of effective tangential stress (Charcoal Granite)

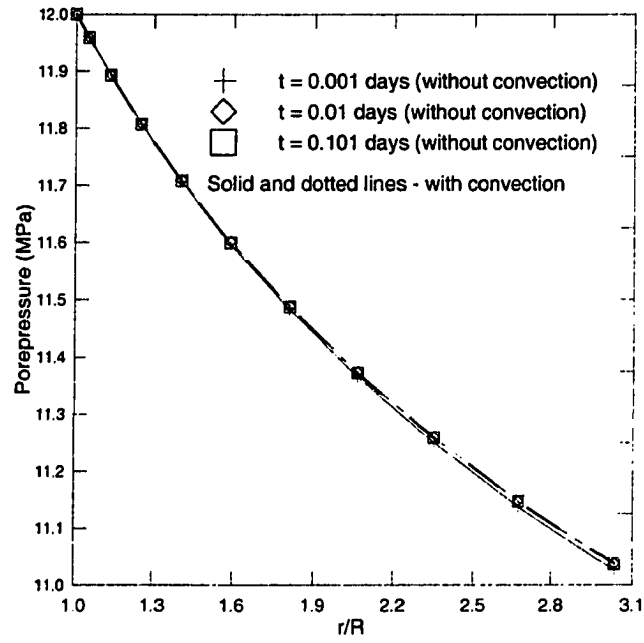


Figure 3.13: Spatial variation of porepressure (Ruhr sandstone)

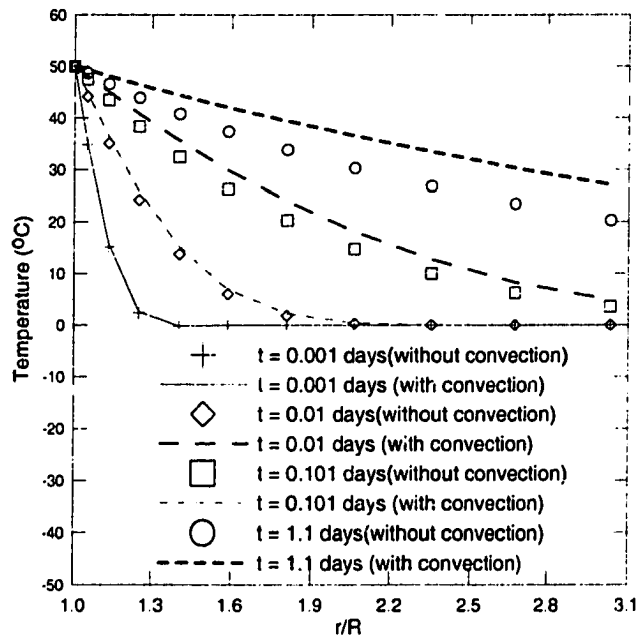


Figure 3.14: Spatiotemporal variation of temperature (Ruhr sandstone)

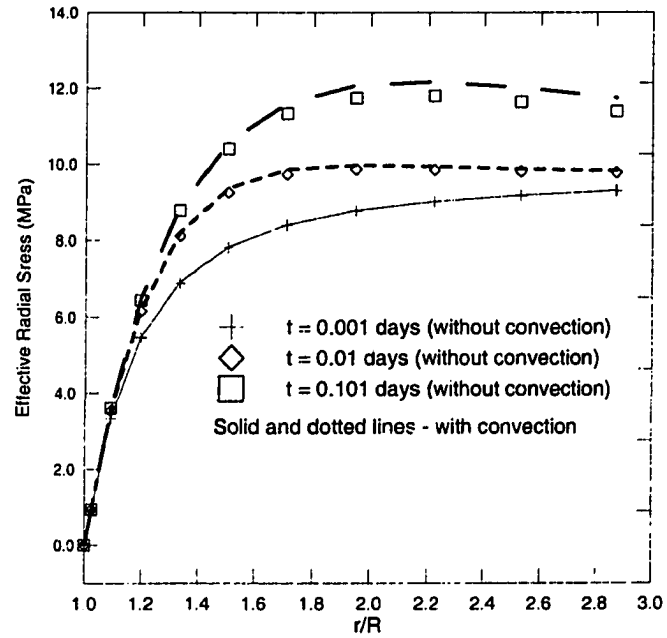


Figure 3.15: Spatiotemporal variation of effective radial stress (Ruhr sandstone)

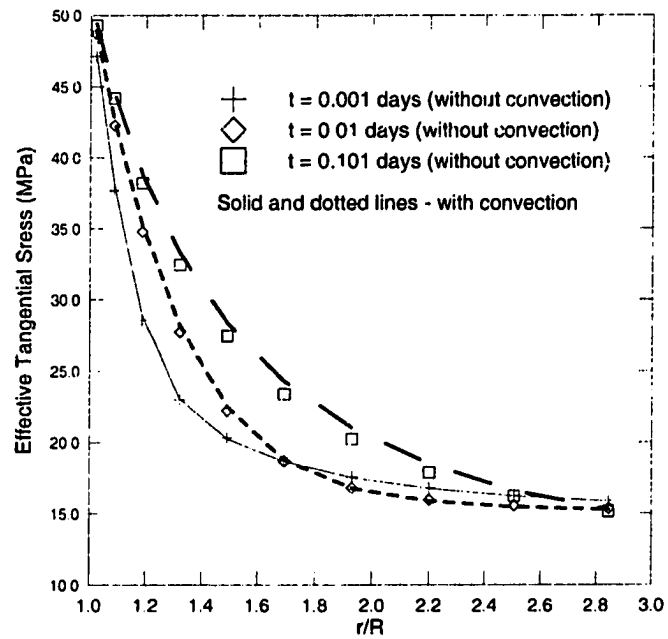


Figure 3.16: Spatiotemporal variation of effective tangential stress (Ruhr sandstone)

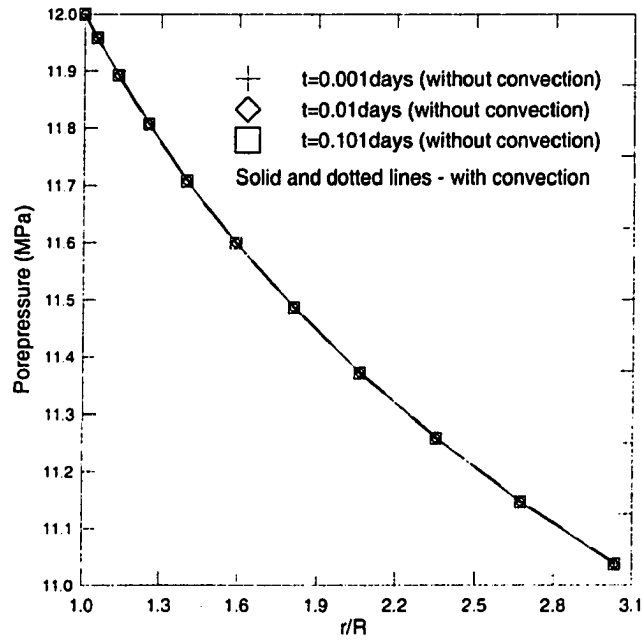


Figure 3.17: Spatial variation of porepressure (Weber sandstone)

Ruhr sandstone, the pore pressure exhibits no temporal variation due to the large fluid diffusivity (see Figure 3.17). However, the temperature profiles for 0.01, 0.101 and 1.1 days show a progressive change in trend when convective terms are included. Specifically, for 0.01 days, the trend of the temperature profiles are similar and they differ only in the magnitude at different locations along the radial direction due to convection. However at 0.101 days, a “convective front” characterized by a zone of higher temperature develops. In other words, due to convective effects, the temperature curve bulges upward and outward into the formation thus marking a significant change in the trend of the profile. Also, this “bulge”, i.e., zone of high temperature extends further into the formation at 1.1 days, an effect which is not visible when convection is ignored (see Figure 3.18). Correspondingly, as time progresses, the effective radial and tangential stresses become more compressive when convective terms are included (see Figure 3.19).

The effect of heat transport by convection varies with the ratio of thermal to fluid

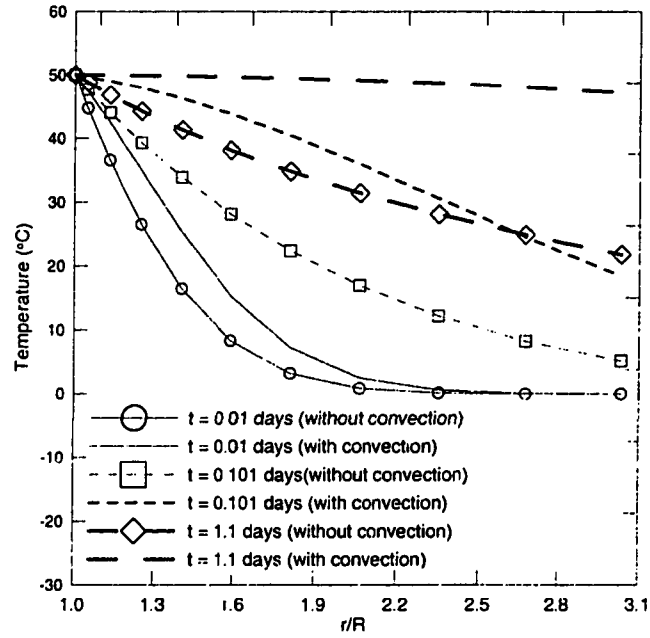


Figure 3.18: Convective effect on spatiotemporal variation of temperature (Weber sandstone)

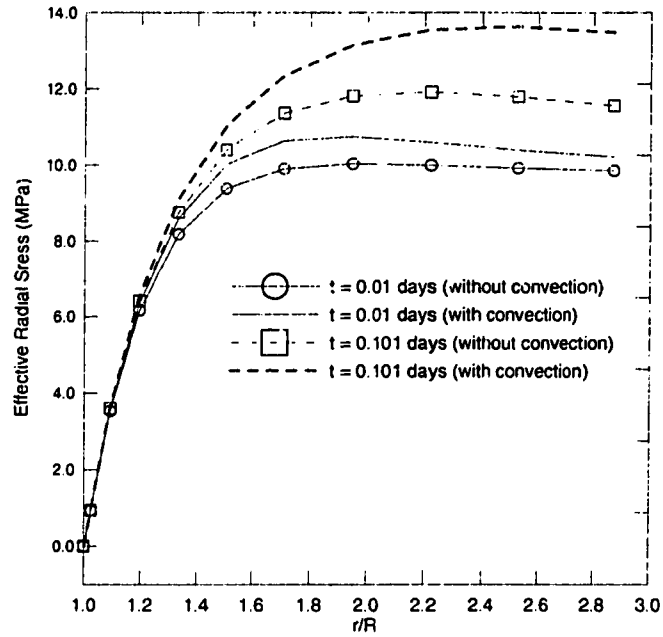


Figure 3.19. Convective effect on spatiotemporal variation of effective radial stress (Weber sandstone)

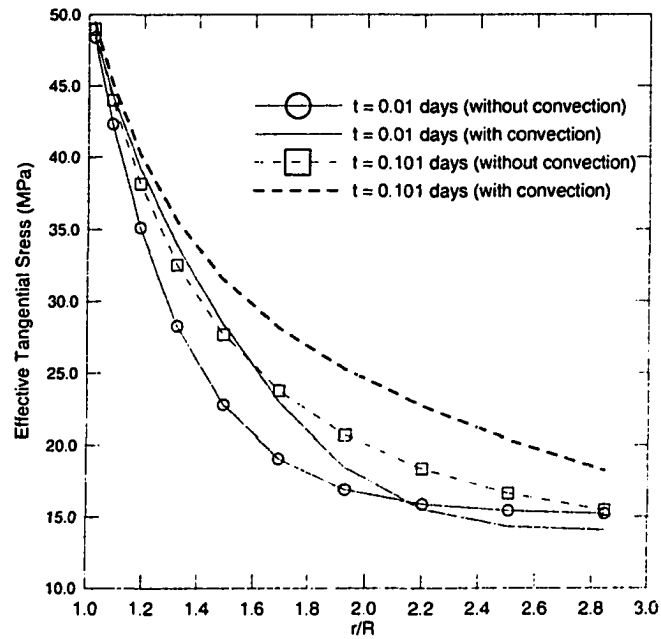


Figure 3.20: Convective effect on spatiotemporal variation of effective radial stress (Weber sandstone)

diffusivities (see Table 3.6). When the thermal diffusivity is of the same order (or higher) as the fluid diffusivity, the heat transport occurs primarily via conduction. For formations with a lower ratio of thermal to fluid diffusivity (as in case of Ruhr and Weber sandstone), heat transport by convection has a greater impact on the temperature and stress profiles. On the other hand, when convective effects are negligible, the temperature profiles are more or less independent of the fluid pressures and stresses. (Aifantis and Beskos 1980, Kurashige 1992, Kodashima and Kurashige 1996, Nair et al. 2002).

Table 3.6: Ratio of thermal to fluid diffusivities.

Rock Type	$C(m^2/s)$	$\frac{\lambda}{\rho C_v}(m^2/s)$	$\frac{\lambda}{\rho C_v}/C$
Gulf of Mexico Shale	1.79×10^{-7}	1.598×10^{-6}	8.91
Charcoal Granite	7.6×10^{-6}	1.14×10^{-6}	0.15
Ruhr sandstone	5.1×10^{-3}	9.9×10^{-7}	1.95×10^{-4}
Weber sandstone	1.978×10^{-2}	1.293×10^{-6}	6.54×10^{-5}

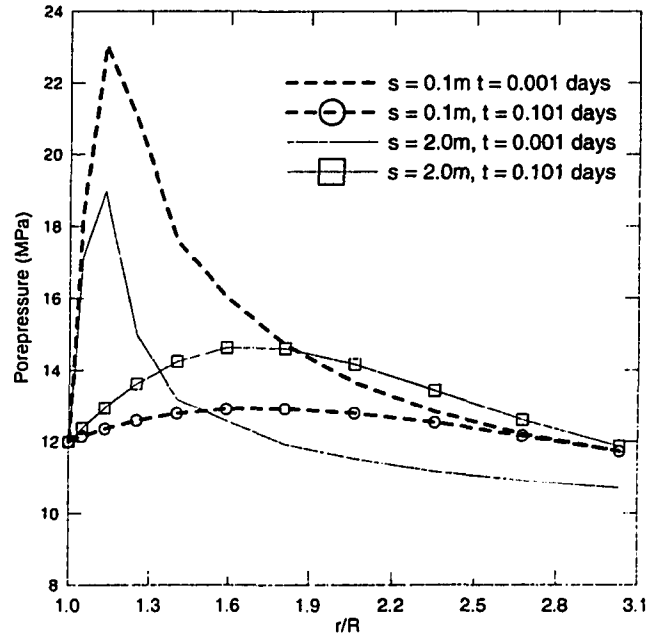


Figure 3.21: Effect of fracture spacing (s) on porepressure (Gulf of Mexico Shale)

3.5.3 Effect of dual-porosity parameters

The dual-porosity response of the medium as shown in Figures 3.21 and 3.22 is evident as the fracture spacing is decreased up to $s = 0.1m$. For a smaller fracture spacing, the overall compliance of the medium is higher resulting in an increased initial pore pressure in the primary medium. As pressure dissipates faster in a medium with a higher fracture density (i.e., smaller fracture spacing), the pressure in the primary medium drops rapidly as time progresses. This dual-porosity characteristic can be observed in Figure 3.21 which shows the pressure in the primary (matrix) medium for 0.001 and 0.1 days. Correspondingly, the effective radial stress in a dual-porosity medium with smaller fracture spacing attains higher tensile values initially (0.001 days). As the pressure in the whole system begins to dissipate (due to the higher permeable secondary medium, i.e., fractures), the effective radial stress becomes more compressive (see Figure 3.22).

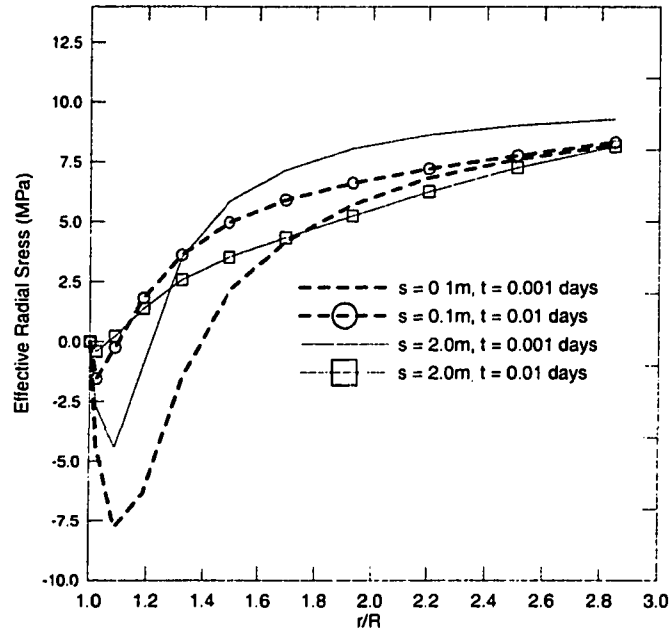


Figure 3.22: Effect of fracture spacing on effective radial stress (Gulf of Mexico Shale)

3.6 Summary

A dual-porosity porothermoelastic finite element model has been developed which accounts for thermo-hydro-mechanical coupling in both, the primary as well as the secondary medium. Heat transport through conduction as well as convection has been incorporated into the model. The model assumes that a single representative thermodynamics continuum is sufficient to describe the temperature dependent response of a fissured porous medium. The model has been used to analyze the response of an inclined wellbore in a fully saturated dual-porosity medium subjected to pore pressure and thermal gradients. From the numerical examples it is concluded that the thermal loading induces a significant increase in pore pressure and, tensile effective radial stresses at short time intervals in the vicinity of the wellbore thereby undermining its stability during the initial drilling phase.

The effect of heat transport by pore fluid flow was analyzed for different thermal to

fluid diffusivity ratios. In all cases, the temperature, pore pressure and stress profiles were compared with corresponding results obtained by neglecting the convective effect. It was observed that the heat transport by pore fluid flow is characterized by a zone of high temperature which progressively extends into the formation. This effect is particularly significant for formations with smaller values of the aforementioned ratio (i.e., Ruhr and Weber sandstone). The pore pressure profiles in both these cases did not exhibit a temporal variation. On the other hand, for larger thermal to fluid diffusivity ratios, the heat transport is mainly through conduction. In addition, it is to be noted that, the density of fractures governs the overall compliance of the medium thereby influencing the magnitude of stress and pore pressures near the borehole wall.

In the next chapter, the mathematical formulation presented in Chapter 2 will be extended to incorporate fully coupled isothermal oil and gas flow through a fractured porous medium. Also, a finite element dual-porosity poroelastic model for two-phase flow will be developed based on the mathematical formulations.

Chapter 4

Two-Phase Dual-Porosity Poroelasticity

4.1 Introduction

The formalism presented in Chapter 2 provides an adequate basis for analysis of either intact or fractured porous media saturated with a single fluid. It however lacks the flexibility to model formations such as oil reservoirs or coastal aquifers. An oil reservoir, for instance, almost always contains several fluids (oil, water and gas) in its pores, all of which may exchange mass, momentum and energy during the reservoir's productive life. Moreover, the development process of an oil field itself may involve the influx of a fluid tending to displace the oil or gas. Other instances where multiple fluids may coexist in a porous medium are, when water of a different quality is introduced into an aquifer by artificial recharge methods, and, during disposal of sewage, radioactive wastes or drill-cuttings (resulting from borehole drilling operations) into shallow or deep formations. The presence of two or more fluids, with different compressibilities, in the interstices of a porous medium modifies the rock-fluid(s) interaction, important for engineering applications such as stability analysis of boreholes or unsaturated soil slopes.

In general, two types of flow are possible when two fluids in motion occupy a porous medium:

1. *Miscible displacement* - The two fluids are completely soluble in each other, with the interfacial tension between them being zero. A distinct fluid-fluid interface does not exist. This type of flow is also referred to as hydrodynamic dispersion.
2. *Immiscible displacement* - In this case we have a simultaneous flow of two or more fluids or phases (oil, water or gas) in the porous medium. The interfacial tension is non-zero with a distinct interface separating the two phases within each pore. A pressure difference exists across this interface at every point on it, termed as the capillary pressure.

One of the best known cases of miscible displacement is that of the intrusion of sea water into coastal aquifers. In this case, a transition zone develops due to hydrodynamic dispersion and across this zone, the composition of the fluid varies from one fluid to the other. Another example is that of a secondary oil recovery process called solvent drive, wherein a fluid miscible with the oil is injected into the formation to displace it. A mixed zone is created between the two fluids behind which we have a region occupied entirely by the displaced fluid. Liquid propane or butane gas are examples of the injected fluid in this case (Bear, 1972).

Immiscible displacements, i.e., simultaneous flow of two or more fluids occur commonly in practically all oil reservoirs during various production stages. In the water drive mechanism, water completely or partially displaces oil and gas by intruding into oil-producing regions along its boundaries. Simultaneous flow may also occur when gas, air or water is injected into an oil reservoir in order to increase its production. A special case of immiscible flow of two liquids is that of unsaturated flow of water through soil. In this case, the air, which partly fills the void space, is assumed to be practically immobile (Bear, 1972).

When two immiscible fluids coexist in a porous medium, one phase will in general wet the solid. The saturation of a fluid is defined as the fraction of the pore space filled

by the fluid phase under consideration. The three general types of saturation regimes possible with respect to any one phase are (Scheidegger, 1974):

1. *Complete Saturation Regime* - The porous medium is completely saturated with one fluid, as seen in Chapters 2 and 3.
2. *Pendular Regime* - The porous medium has the lowest possible saturation with one phase, which occurs in the form of pendular bodies throughout the porous medium. These bodies are isolated and no pressure can be transmitted from one to another. Also, there is no possibility of flow for this phase.
3. *Funicular Regime* - The porous medium exhibits an intermediate saturation with both phases, i.e., the pendular bodies expand with the addition of the corresponding fluid so that they begin to merge. A continuous network of both phases results with simultaneous flow of both phases occurring within the tortuous pore spaces.

In this chapter, the simultaneous flow of a two-phase fluid system, comprising of oil and gas is considered. In other words, the saturation regimes are implicitly assumed to be either pendular or funicular. Biot's self-similar theory is extended to describe the rock-fluid(s) interaction for a multiphase system. The change in fluid phase is accounted for within the flexibility of the black-oil model, explained in detail in the following section. The macroscopic parameters such as the interfacial tension, capillary pressure and relative permeabilities which allow one to rationalize the microscopic fluid-fluid interaction have been explained. The governing equations and finite element solutions for fully coupled oil-gas flow in a fractured porous medium are then presented in the subsequent sections. The formulation presented in Chapter 2 is extended to obtain differential equations governing deformations of the primary and secondary media coupled with two-phase (i.e., oil and gas) flow. Specifically, the black-oil model, applicable

to low-volatility oil systems, is used within the framework of the dual-porosity concept established in Chapter 2. The mathematical formulation is then cast into a finite element form wherein the solid deformations and fluid pressures are treated as the primary unknowns while, the saturations are the secondary unknowns obtained from the capillary-saturation relations. The nonlinear system of equations in the finite element model is solved using a direct solver and the stability is checked at each time level. The model is then applied to the problem of an inclined wellbore in a fractured porous medium saturated with oil and gas. A parametric analysis is carried out to identify influential parameters governing the spatial and temporal distributions of pore pressure and stresses in the vicinity of the wellbore.

4.2 Preliminaries

4.2.1 Black-Oil Model

The “black-oil” model is a standard tool used primarily by petroleum reservoir engineers to describe the hydrocarbon (oil-gas) equilibrium of low-volatility oil systems. In the oil-gas system, within the framework of the black-oil model, two components are considered - the “oil” component is the residual liquid left after differential vaporization while the “gas” component is the remaining fluid. It must be noted here that though the black-oil model can account for pressure-dependent solubility of the gas in oil, it cannot take into account the changes in oil or gas compositions. In other words, the fluid system is modeled by identifying the components (“pseudo”-components, since the compositional changes in oil and gas are not considered) in accordance with the segregation which occurs at standard conditions of pressure and temperature (Peaceman, 1977; Allen et al., 1988; Mattax and Dalton, 1990).

In order to distinguish clearly between the individual phases and components, upper-case subscripts are used to identify the components, while lower-case subscripts are used

for the respective phases. A sample of fluid (both oil and gas) at reservoir conditions may contain oil and gas components of mass W_O and W_G , respectively. Hence the volume occupied by the oil and gas components at standard conditions, i.e., V_{OS} and V_{GS} are

$$V_{OS} = \frac{W_O}{\rho_{OS}} \quad ; \quad V_{GS} = \frac{W_G}{\rho_{GS}} \quad (4.1)$$

where ρ_{OS} and ρ_{GS} are the densities of the oil and gas components at standard conditions. To incorporate the pressure-dependent solubility of the gas in oil, the gas-oil solubility ratio R_s is defined as the volume of gas dissolved at a given reservoir pressure in a unit volume of oil, but *measured* at standard conditions. Hence,

$$R_s = \frac{(V_{GS})_{diss}}{V_{OS}} \quad (4.2)$$

From equations 4.1 and 4.2, we have

$$R_s = \frac{(W_G)_{diss} \rho_{OS}}{W_O \rho_{GS}} \quad (4.3)$$

The pressure-dependent changes in the fluid densities are accounted for by using formation-volume factors for oil and gas. In the case of the oil phase, the volume at reservoir conditions is larger than V_{OS} , because of the gas dissolved in the oil under reservoir pressure. The formation volume factor for oil, B_o is defined as the ratio of the volume of oil and the gas dissolved in it (at reservoir conditions) to the volume of the oil component measured at standard conditions. Hence,

$$B_o = \frac{V_o}{V_{OS}} \quad (4.4)$$

Also,

$$V_o = \frac{(W_O + (W_G)_{diss})}{\rho_o} \quad (4.5)$$

where ρ_o is the density of oil at reservoir conditions. Hence from equations 4.1, 4.4 and 4.5, we have

$$B_o = \frac{(W_O + (W_G)_{diss}) \rho_{OS}}{W_O \rho_o} \quad (4.6)$$

The formation volume factor for gas, B_g , is the ratio of the volume of gas not dissolved in the oil, measured at reservoir conditions to the volume of the same gas at standard conditions. Hence,

$$B_g = \frac{V_g}{V_{GS}} \quad (4.7)$$

Since,

$$V_g = \frac{W_G}{\rho_g} \quad (4.8)$$

where ρ_g is the density of gas at reservoir conditions. Thus, we have from equations 4.1, 4.7 and 4.8

$$B_g = \frac{\rho_{GS}}{\rho_g} \quad (4.9)$$

The mass fractions of the oil and gas components in the oil and gas phases can now be determined in terms of the densities, formation volume factors and gas-oil solubility ratio - all of which are functions of the pressures. It must be pointed out that, so far, the following set of rules have been implicitly followed. Firstly, the gas phase is allowed to dissolve in the oil phase, i.e., the oil phase is comprised of a gas and oil component as can be seen in equation 4.5. Secondly, the oil is not allowed to evaporate into the gas phase. Thus the gas phase is made up of the gas component entirely as reflected by equation 4.8. The mass fraction of the gas and oil components *in the oil phase*, i.e., w_{Go} and w_{Oo} , respectively, can be expressed as

$$w_{Go} = \frac{(W_G)_{diss}}{(W_O + (W_G)_{diss})} \quad ; \quad w_{Oo} = \frac{W_O}{(W_O + (W_G)_{diss})} \quad (4.10)$$

From equations 4.3, 4.6 and 4.10, we have

$$w_{Go} = \frac{R_s \rho_{GS}}{\rho_o B_o} \quad ; \quad w_{Oo} = \frac{\rho_{OS}}{\rho_o B_o} \quad (4.11)$$

Finally, since the oil is not allowed to evaporate into the gas phase, the mass fraction of the oil component in the gas phase is zero and that of the gas component is unity. Thus

$$w_{Gg} = 1 \quad ; \quad w_{Og} = 0 \quad (4.12)$$

The subscript *diss.* in the above equations refer to the gas dissolved in the oil. Also ρ_{os} and ρ_{gs} will be used to denote the densities of oil and gas under standard conditions henceforth in this chapter.

4.2.2 Interfacial Tension and Capillary Pressure

Interfacial Tension

When a liquid is in contact with another liquid, gas or a solid, the difference between the inward attraction of the molecules in the interior of each phase and those at the contact surface, gives rise to a free *interfacial energy*. This in turn manifests as interfacial tension, since a surface possessing free energy tends to contract. Figure 4.1 shows oil and gas in contact with a solid, where θ is the angle between the oil-gas interface and the solid. Requirements of equilibrium leads to Young's equation, i.e.,

$$\chi_{GO} \cos \theta = \chi_{SG} - \chi_{SO} \quad (4.13)$$

where χ_{ik} is the interfacial tension, defined as the amount of work required to separate a unit area of substance *i* from substance *k*. θ is also called as the contact angle and the left-hand side of equation 4.13 is called the adhesion tension determining which fluid (oil or gas) preferentially wets the solid. As in figure 4.1, since $\theta < 90^\circ$, oil is said to wet the solid, i.e., adhere to it and spread over it. Hence in an oil-gas system, oil is the wetting phase and gas is the non-wetting phase. The interfacial tension and wettability may be different when an oil-gas interface is receding or advancing on a solid, a phenomenon called hysteresis (Bear, 1972).

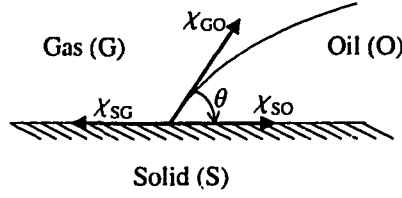


Figure 4.1: Interfacial Tensions (Bear, 1972)

Capillary Pressure

When the saturation regimes are pendular or funicular, the two fluids are in contact in the interstitial spaces of the porous medium. At any given relative saturation, a certain pressure differential exists across the interface separating them and depends on the interface curvature. This difference in pressure, P_c , is called the capillary pressure, i.e.,

$$P_c = P_{nw} - P_w \quad (4.14)$$

where P_{nw} and P_w are the pressures in the non-wetting (gas) and wetting (oil) phases respectively. The capillary pressure is the tendency of the porous medium to suck in the wetting phase and to repel the non-wetting phase (Bear, 1972). The capillary pressure is dependent on the interfacial tension, size of the solid grains (governing the contact angles), geometry of the pore space, nature of the solids and liquids and the degree of saturation, all of which are statistically averaged over the REV of the porous medium. Capillary pressure, like the contact angle and interfacial tension, exhibits the phenomenon of hysteresis. The geometry of the void space is extremely irregular and complex in natural porous media (intact or fractured) and hence cannot be described analytically. This renders the definition of the geometrical shape of the fluid-fluid interface (satisfying the requirement that a minimum energy be consumed in forming the interface) by analytical means nearly impossible. The most practical means to characterize the inter-

acting motion of each phase and evolution of their respective pressures is to correlate the capillary pressures and phase saturations, i.e.,

$$P_c = P_c(S_w) \quad (4.15)$$

where S_w is the saturation of the wetting phase. This correlation may be thought of as representative of all the aforementioned factors that affect the distribution of each phase and may be obtained experimentally. A detailed description of the laboratory methods for determining the relationship in equation 4.15 may be found in Bear (1972) and Scheiddeger (1974).

4.2.3 Relative Permeabilities

The extension of the equations of motion of a single fluid, to that of simultaneous flow of two or more fluids in a porous medium, is based on the concept of relative permeability. It is also, not surprisingly, intricately linked to the extension of Darcy's law for multiphase flow. The most common assumption for two-phase flow is that Darcy's law holds for both the wetting and non-wetting phases (Bear, 1972), i.e.,

$$v_i^w = -\frac{k_w}{\mu_w} P_{w,i} \quad ; \quad v_i^{nw} = -\frac{k_{nw}}{\mu_{nw}} P_{nw,i} \quad (4.16)$$

where μ_w and μ_{nw} are the viscosities of the wetting and non-wetting phases respectively. k_w and k_{nw} are the effective permeabilities of the porous medium to the two phases. In the case of two-phase flow, the permeability of the porous medium with respect to either phase would be reduced due to the simultaneous presence of the other in the pore space. Thus, the effective permeabilities (when the porous medium is saturated with two fluids) depend not only on the permeability, k , of the medium to a *single* fluid completely saturating it, but also on the degree to which each fluid phase blocks the flow of the other. The most obvious phenomenological postulate allowing for this dependence is to stipulate that the effective permeability of each phase increases with its respective

saturation, i.e., (Allen et al., 1988; Bear, 1972)

$$k_w = k k_{rw}(S_w) \quad ; \quad k_{nw} = k k_{rnw}(S_w) \quad (4.17)$$

where the functions k_{rw} and k_{rnw} are called the relative permeabilities of the wetting and non-wetting phases obeying the bounds $0 < k_{rw}, k_{rnw} < 1$. The relative permeability model is an approximation in terms of macroscopic variables which attempts to account for the complex interdependence between the flow of the fluid phases occurring at the microscopic level. The relative permeabilities exhibit the phenomenon of hysteresis, i.e., a dependence on the saturation history of the system. In addition, they may also vary as functions of the fluid composition, pressure or temperature. Hence, the assumption that the relative permeabilities are single-valued functions of the saturations alone, must be regarded as a working approximation (Scheidegger, 1974). Details of the measurement of relative permeabilities in a laboratory environment may be found in Scheidegger (1974).

4.2.4 Effective Stress

As in Chapter 2 (equation 2.24), the constitutive laws governing the deformation of the solid constituent of the solid-fluid(s) system may be introduced through the concept of the effective stress σ' , i.e.,

$$\sigma_{ij} = \sigma'_{ij} - \alpha \bar{P} \delta_{ij} \quad (4.18)$$

where $\bar{P}$ is an average pressure of the mixture of the wetting and non-wetting fluid phases. Generally, this pressure is postulated as a weighted average pressure of the fluid phases given by (Li and Zienkiewicz, 1990)

$$\bar{P} = \sum_{\iota=w,nw} \pi_{\iota} P_{\iota} \quad (4.19)$$

where the weighting parameter π_{ι} is a function of the saturations as $\pi_{\iota} = \pi_{\iota}(S_{\iota})$ and P_{ι} is the pressure in the ι -th phase. In this work, it is assumed that each phase is in contact

with the solid grains to some extent and the weighting function π_i can be defined as (Li and Zienkiewicz, 1990)

$$\pi_i = S_i \quad ; \quad i = w, nw \quad (4.20)$$

Thus from equations 4.19 and 4.20 we have,

$$\bar{P} = S_w P_w + S_{nw} P_{nw} \quad (4.21)$$

The volume averaged pore pressure in equation 4.21 may be used in equation 4.18 to obtain

$$\sigma_{ij} = \sigma'_{ij} - \alpha (S_w P_w + S_{nw} P_{nw}) \delta_{ij} \quad (4.22)$$

Equation 4.22 thus, relates the total and intergranular stresses in a porous medium saturated with two fluids.

4.3 Assumptions

In addition to the assumptions listed in Chapter 2, the following assumptions are made during the formulation of fully-coupled oil-gas flow through a fractured porous medium. It must be noted that the assumption that the fractured porous medium is fully saturated by a single fluid has been relaxed herein.

1. Both, the primary and secondary media are fully saturated by a low volatile oil and gas under isothermal conditions, i.e., for both media,

$$S_g + S_o = 1$$

2. The influence of the dissolved gas on the viscosity of the oil phase is neglected, i.e., the oil and gas viscosities are assumed to be constant.
3. Darcy's law, originally used to describe the flow of a single fluid through a porous medium, may be extended to describe the simultaneous flow of oil and gas through both media.

4. The relative permeabilities of oil and gas (k_{ro} and k_{rg} , respectively) are functions of the respective saturations only, i.e., $k_{ro} = k_{ro}(S_o)$; $k_{rg} = k_{rg}(S_g)$.
5. The capillary pressure is a function of the saturation of the oil phase, i.e., $P_c = P_c(S_o)$.
6. Each phase is assumed to be in contact with the solid constituent to some extent and the volume averaged pore pressure is assumed to be the sum of the pressures in the individual phases weighted with their respective saturations.
7. The solubility of the gas in oil is assumed to be dependent on the pressure in the gas phase alone, i.e., $R_s = R_s(P_g)$.

4.4 Mathematical Formulation

The basic concepts and assumptions made in the earlier sections are incorporated into the governing equations for fully-coupled oil-gas flow in a fractured porous medium. As in Chapters 2 and 3, the fractured porous medium is visualized as a dual-porosity medium. The separate and overlapping concept has been used wherein each medium saturated with oil and gas is assumed to pervade the entire domain, thus forming two distinct but overlapping continua. The intensity of interactions between the two media are controlled by mass exchange terms for each fluid phase. These are, in turn, dependent on the pressure difference between the two media, of the individual phases and, the matrix permeability to the respective phase.

4.4.1 Definitions

The definitions for the capillary pressures, total averaged pore pressures and parameters within the black-oil model presented in this section are based on equations 4.1-4.15. The saturations and capillary pressures for the oil and gas phases for the primary and

secondary media are given as:

$$S_g^\eta + S_o^\eta = 1 \quad (4.23)$$

$$P_c^\eta = P_g^\eta - P_o^\eta \quad (4.24)$$

where S_o^η and S_g^η are the oil and gas saturations in both media; P_o^η , P_g^η and P_c^η are the oil, gas and capillary pressures in both media and the superscripts $\eta = I, II$ always refer to the primary and secondary media, respectively, as in Chapters 2 and 3. The weighted average pressures P^η for both media can be expressed as:

$$P^\eta = S_g^\eta P_g^\eta + S_o^\eta P_o^\eta \quad (4.25)$$

In view of the black-oil model, the mass fractions of the oil component in the gas and oil phases based on equations 4.11 and 4.12, i.e., w_{Og}^η and w_{Oo}^η respectively, are,

$$w_{Og}^\eta = 0; \quad w_{Oo}^\eta = \frac{\rho_{os}^\eta}{\rho_o^\eta B_o^\eta} \quad (4.26)$$

where B_o^η are the formation volume factors for oil, and ρ_{os}^η are the densities of oil under standard conditions. Also, the mass fractions of the gas component in the gas and oil phases, i.e., w_{Gg}^η and w_{Go}^η respectively, are

$$w_{Gg}^\eta = 1; \quad w_{Go}^\eta = \frac{R_s^\eta \rho_{gs}^\eta}{\rho_o^\eta B_o^\eta} \quad (4.27)$$

and

$$\rho_g^\eta = \frac{\rho_{gs}^\eta}{B_g^\eta}; \quad \rho_o^\eta = \frac{\rho_{os}^\eta + R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \quad (4.28)$$

where B_g^η are the formation volume factors for gas; ρ_{gs}^η are the densities of gas under standard conditions and; R_s^η are the gas-oil solubility ratios.

4.4.2 Mass Conservation for Oil Component

Since the black-oil model allows for a pressure dependent solubility of gas in oil, the mass of the oil and gas phases are no longer conserved. However, the total mass of the oil and gas components in each phase is still conserved. Hence, the mass conservation relations for oil-gas flow are expressed for the components rather than for the phases.

The definitions of the constituent volumes and associated porosities are as in Chapter 2 (see equations 2.1-2.3). Also, the equations for rate of change of porosities, pore and solid volumetric strains remain the same as in equations 2.8, 2.11 and 2.12 respectively. The mass conservation equation for oil in the primary and secondary media can be written as

$$\begin{aligned} \frac{\partial}{\partial t} (\phi^\eta S_g^\eta \rho_g^\eta w_{og}^\eta) + \frac{\partial}{\partial x_i} (\phi^\eta S_g^\eta \rho_g^\eta U_i^{\eta g} w_{og}^\eta) + \\ \frac{\partial}{\partial t} (\phi^\eta S_o^\eta \rho_o^\eta w_{oo}^\eta) + \frac{\partial}{\partial x_i} (\phi^\eta S_o^\eta \rho_o^\eta U_i^{\eta o} w_{oo}^\eta) - \Gamma_o = 0 \end{aligned} \quad (4.29)$$

where $U_i^{\eta o}$, $U_i^{\eta g}$ are the intrinsic oil and gas velocities and ρ_o^η are the oil densities at reservoir conditions. The Darcy velocities of oil $v_i^{\eta o}$ are given by

$$v_i^{\eta o} = \phi^\eta S_o^\eta (U_i^{\eta o} - u_i^\eta) \quad (4.30)$$

Therefore, equations 4.26, 4.28-4.30 give

$$\frac{\partial}{\partial t} \left(\phi^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \right) + \frac{\partial}{\partial x_i} \left(\phi^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} u_i^\eta + \frac{\rho_{os}^\eta}{B_o^\eta} v_i^{\eta o} \right) - \Gamma_o = 0 \quad (4.31)$$

Expanding derivatives, grouping terms and using equations 2.4, 2.8 and 2.12 yield

$$\begin{aligned} S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \frac{DP^\eta}{Dt} + \phi^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{DS_o^\eta}{Dt} + \\ \phi^\eta S_o^\eta \rho_{os}^\eta \frac{D}{Dt} \left(\frac{1}{B_o^\eta} \right) + \alpha^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} u_{i,i}^\eta + \\ \frac{\rho_{os}^\eta}{B_o^\eta} v_{i,i}^{\eta o} + v_i^{\eta o} \rho_{os}^\eta \left(\frac{1}{B_o^\eta} \right)_{,i} - \Gamma_o = 0 \end{aligned} \quad (4.32)$$

Also, using equations 4.23-4.25 and invoking Darcy's law we have,

$$\begin{cases} \frac{\partial}{\partial t} \left(\frac{1}{B_o^\eta} \right) = - \frac{1}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \frac{\partial P_o^\eta}{\partial t} \\ \frac{\partial P^\eta}{\partial t} = \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \frac{\partial P_o^\eta}{\partial t} + \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \frac{\partial P_g^\eta}{\partial t} \\ v_i^{\eta o} = - \frac{k^\eta k_{ro}^\eta}{\mu_o^\eta} P_{o,i}^\eta \\ \frac{\partial S_o^\eta}{\partial t} = \frac{\partial S_o^\eta}{\partial P_c^\eta} \left(\frac{\partial P_o^\eta}{\partial t} - \frac{\partial P_g^\eta}{\partial t} \right) \end{cases} \quad (4.33)$$

where, k^η are the absolute permeabilities and k_{ro}^η , μ_o^η the relative oil permeabilities and oil viscosities. Using equations 4.32 and 4.33 and letting $\frac{D}{Dt} \simeq \frac{\partial}{\partial t}$ the final form of the governing equations for oil flow in the matrix and fractures can be written as

$$\begin{aligned} & \frac{\partial P_o^\eta}{\partial t} \left[S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) - \phi^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} - \right. \\ & \left. \frac{\phi^\eta \rho_{os}^\eta S_o^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \right] + \\ & \frac{\partial P_g^\eta}{\partial t} \left[S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) + \phi^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} \right] + \\ & \alpha^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} u_{i,i}^\eta - \frac{\rho_{os}^\eta}{B_o^\eta} \frac{k^\eta k_{ro}^\eta}{\mu_o^\eta} (P_o^\eta)_{i,i} + v_i^{\eta o} \rho_{os}^\eta \left(\frac{1}{B_o^\eta} \right)_{,i} - \Gamma_o = 0 \end{aligned} \quad (4.34)$$

4.4.3 Mass Conservation for Gas Component

The mass conservation equation for gas in the primary and secondary media can be written as

$$\begin{aligned} & \frac{\partial}{\partial t} (\phi^\eta S_g^\eta \rho_g^\eta w_{Gg}^\eta) + \frac{\partial}{\partial x_i} (\phi^\eta S_g^\eta \rho_g^\eta U_i^{\eta g} w_{Gg}^\eta) + \\ & \frac{\partial}{\partial t} (\phi^\eta S_o^\eta \rho_o^\eta w_{Go}^\eta) + \frac{\partial}{\partial x_i} (\phi^\eta S_o^\eta \rho_o^\eta U_i^{\eta o} w_{Go}^\eta) - \Gamma_g = 0 \end{aligned} \quad (4.35)$$

where again $U_i^{\eta o}$, $U_i^{\eta g}$ are the intrinsic oil and gas velocities and; ρ_g^η are the gas densities at reservoir conditions. The Darcy velocities, $v_i^{\eta g}$, are

$$v_i^{\eta g} = \phi^\eta S_g^\eta (U_i^{\eta g} - u_i^\eta) \quad (4.36)$$

From equations 4.27, 4.28, 4.35 and 4.36, we get

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\phi^\eta S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \right) + \frac{\partial}{\partial x_i} \left(\frac{\rho_{gs}^\eta}{B_g^\eta} v_i^{\eta g} + \phi^\eta S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} u_i^\eta \right) + \\ & \frac{\partial}{\partial t} \left(\phi^\eta S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) + \frac{\partial}{\partial x_i} \left(\frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} v_i^{\eta o} + \phi^\eta S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} u_i^\eta \right) \\ & - \Gamma_g = 0 \end{aligned} \quad (4.37)$$

As in the derivation of the continuity equation for oil, derivatives are expanded while the relationships in equations. 2.4, 2.8 and 2.12 are employed. As a result, we get

$$\begin{aligned} & \phi^\eta S_g^\eta \rho_{gs}^\eta \frac{D}{Dt} \left(\frac{1}{B_g^\eta} \right) + \\ & \left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \frac{DP^\eta}{Dt} \\ & + \phi^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{DS_g^\eta}{Dt} + \phi^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{DS_o^\eta}{Dt} + \\ & \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{DR_s^\eta}{Dt} + \phi^\eta S_o^\eta R_s^\eta \rho_{gs}^\eta \frac{D}{Dt} \left(\frac{1}{B_o^\eta} \right) + \\ & \left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \alpha^\eta u_{i,i}^\eta + \rho_{gs}^\eta \frac{\partial}{\partial x_i} \left(\frac{v_i^{\eta g}}{B_g^\eta} \right) + \\ & \rho_{gs}^\eta \frac{\partial}{\partial x_i} \left(\frac{R_s^\eta v_i^{\eta o}}{B_o^\eta} \right) - \Gamma_g = 0 \end{aligned} \quad (4.38)$$

Also, using equations 4.23-4.25 and invoking Darcy's law we have,

$$\left\{ \begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{B_o^\eta} \right) &= - \frac{1}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \frac{\partial P_o^\eta}{\partial t} \\ \frac{\partial}{\partial t} \left(\frac{1}{B_g^\eta} \right) &= - \frac{1}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} \frac{\partial P_g^\eta}{\partial t} \\ \frac{\partial S_o^\eta}{\partial t} &= - \frac{\partial S_g^\eta}{\partial t} = \frac{\partial S_o^\eta}{\partial P_c^\eta} \left(\frac{\partial P_o^\eta}{\partial t} - \frac{\partial P_g^\eta}{\partial t} \right) \\ \frac{\partial P^\eta}{\partial t} &= \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \frac{\partial P_o^\eta}{\partial t} + \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \frac{\partial P_g^\eta}{\partial t} \\ v_i^{\eta o} &= - \frac{k^\eta k_{ro}^\eta}{\mu_o^\eta} P_{o,i}^\eta, \quad v_i^{\eta g} = - \frac{k^\eta k_{rg}^\eta}{\mu_g^\eta} P_{g,i}^\eta \\ \frac{\partial R_s^\eta}{\partial t} &= \frac{\partial R_s^\eta}{\partial P_g^\eta} \frac{\partial P_g^\eta}{\partial t} \end{aligned} \right. \quad (4.39)$$

wherein, k_{rg}^η , μ_g^η are relative gas permeabilities and gas viscosities, respectively. From equations 4.38 and 4.39, letting $\frac{D}{Dt} \simeq \frac{\partial}{\partial t}$ and grouping terms, the final form of the

governing equation for gas flow can be expressed as

$$\begin{aligned}
& \frac{\partial P_o^\eta}{\partial t} \left[\left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \right. \\
& \quad \left. - \phi^\eta \frac{R_s^\eta S_o^\eta \rho_{gs}^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} + \phi^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} - \phi^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} \right] + \\
& \frac{\partial P_g^\eta}{\partial t} \left[\left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \right. \\
& \quad \left. - \frac{\phi^\eta S_g^\eta \rho_{gs}^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} - \phi^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} + \right. \\
& \quad \left. \phi^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} + \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial P_g^\eta} \right] + \\
& \left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \alpha^\eta u_{i,i}^\eta - \rho_{gs}^\eta \frac{\partial}{\partial x_i} \left(\frac{k^\eta k_{rg}^\eta}{B_g^\eta \mu_g^\eta} P_{g,i}^\eta \right) \\
& - \rho_{gs}^\eta \frac{\partial}{\partial x_i} \left(\frac{R_s^\eta k^\eta k_{ro}^\eta}{B_o^\eta \mu_o^\eta} P_{o,i}^\eta \right) - \Gamma_g = 0
\end{aligned} \tag{4.40}$$

4.4.4 Equilibrium Equation

As in Chapter 2, the equilibrium equation for a dual-porosity medium under isothermal conditions may be written as

$$\begin{aligned}
& (D_{ijkl}^{I,II} \varepsilon_{kl} - D_{ijkl}^{I,II} C_{klmn}^I \alpha^I P^I \delta_{mn} - \\
& D_{ijkl}^{I,II} C_{klmn}^{II} \alpha^{II} P^{II} \delta_{mn})_{,j} + F_i = 0
\end{aligned} \tag{4.41}$$

where $D_{ijkl}^{I,II}$, C_{ijkl}^I , C_{ijkl}^{II} and F_i are as described in Chapter 2. Following equation 4.25, we have

$$\begin{aligned}
& (D_{ijkl}^{I,II} \varepsilon_{kl} - D_{ijkl}^{I,II} C_{klmn}^I \alpha^I (S_g^I P_g^I + S_o^I P_o^I) \delta_{mn} - \\
& D_{ijkl}^{I,II} C_{klmn}^{II} \alpha^{II} (S_g^{II} P_g^{II} + S_o^{II} P_o^{II}) \delta_{mn})_{,j} + F_i = 0
\end{aligned} \tag{4.42}$$

The volumetric strain manifested in each of the two media may be expressed as a proportion of the total body strain as in equations 2.34 and 2.35.

4.5 Finite Element Equations

The finite element equations are developed on similar lines as in Chapters 2 and 3. The displacements and, oil and gas pressures, i.e., $\mathbf{u}$, P_o^I , P_o^{II} , P_g^I and P_g^{II} are the primary

variables. These are expressed in terms of approximated nodal quantities as

$$P_o^{\eta*} \simeq \mathbf{N} \mathbf{P}_o^\eta; P_g^{\eta*} \simeq \mathbf{N} \mathbf{P}_g^\eta; \mathbf{u}^* \simeq \mathbf{N} \bar{\mathbf{u}} \quad (4.43)$$

where again, the superscript (*) denotes the finite element approximation, $\bar{\mathbf{u}}$ refers to the nodal displacements and $\mathbf{N}$ represents the shape functions. Also, the (-) will be dropped herewith.. The strains of both media, i.e. $u_{i,i}^\eta$, can be expressed in terms of the total body strain following equation 2.50. Substitution of the approximate quantities (equation 4.43) into equations 4.34 and 4.40, using equation 2.50 and application of the Galerkin's principle and Green's theorem gives the finite element equations for oil phase as

$$\begin{aligned} & \int_V \mathbf{N}^T \left[S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) - \right. \\ & \left. \phi^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} - \frac{\phi^\eta \rho_{os}^\eta S_o^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \right] \mathbf{N} dV \frac{\partial \mathbf{P}_o^\eta}{\partial t} + \\ & \int_V \mathbf{N}^T \left[S_g^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_g^\eta + P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) + \right. \\ & \left. \phi^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} \right] \mathbf{N} dV \frac{\partial \mathbf{P}_g^\eta}{\partial t} + \\ & \int_V \mathbf{N}^T \alpha^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \frac{\partial \mathbf{u}}{\partial t} + \\ & \int_V (\nabla \mathbf{N})^T \frac{\rho_{os}^\eta}{B_o^\eta} \frac{k^\eta k_{ro}^\eta}{\mu_o^\eta} \nabla \mathbf{N} dV P_o^\eta + \\ & + (-1)^\eta \int \mathbf{N}^T \frac{\rho_{os}^I}{B_o^I} \frac{k^I k_{ro}^I}{\mu_o^I} \Psi \mathbf{N} dV (\mathbf{P}_o^I - \mathbf{P}_o^{II}) + \mathbf{Q}_g^\eta = 0 \end{aligned} \quad (4.44)$$

and for gas phase as

$$\begin{aligned}
& \int_V \mathbf{N}^T \left[\left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \right. \\
& \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) + \phi^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} - \\
& \left. \phi^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} \right] \mathbf{N} dV \frac{\partial \mathbf{P}_o^\eta}{\partial t} + \\
& \int_V \mathbf{N}^T \left[\left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \right. \\
& \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) - \frac{\phi^\eta S_g^\eta \rho_{gs}^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} - \\
& \phi^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} + \phi^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{\partial S_o^\eta}{\partial P_c^\eta} + \\
& \left. \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial P_g^\eta} \right] \mathbf{N} dV \frac{\partial \mathbf{P}_g^\eta}{\partial t} + \\
& \int_V \mathbf{N}^T \left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \frac{\partial \mathbf{u}}{\partial t} - \\
& \int_V (\nabla \mathbf{N})^T \rho_{gs}^\eta \frac{k^\eta k_{rg}^\eta}{B_g^\eta \mu_g^\eta} \nabla \mathbf{N} dV \mathbf{P}_g^\eta - \\
& \int_V (\nabla \mathbf{N})^T \rho_{gs}^\eta \frac{R_s^\eta k^\eta k_{ro}^\eta}{B_o^\eta \mu_o^\eta} \nabla \mathbf{N} dV \mathbf{P}_o^\eta \\
& + (-1)^\eta \int \mathbf{N}^T \frac{\rho_{gs}^I}{B_g^I} \frac{k^I k_{rg}^I}{\mu_g^I} \Psi \mathbf{N} dV (\mathbf{P}_g^I - \mathbf{P}_g^{II}) \\
& + (-1)^\eta \int \mathbf{N}^T \frac{R_s^I \rho_{gs}^I}{B_o^I} \frac{k^I k_{ro}^I}{\mu_o^I} \Psi \mathbf{N} dV (\mathbf{P}_o^I - \mathbf{P}_o^{II}) + \mathbf{Q}_g^\eta = 0
\end{aligned} \tag{4.45}$$

The finite element equilibrium equation can be expressed using equations 2.54, 4.42 and, the third and fourth expressions from equation 4.39 as

$$\begin{aligned}
& \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{B} dV \frac{\partial \mathbf{u}}{\partial t} - \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \\
& \left(S_o^I + P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \mathbf{m} \mathbf{N} dV \frac{\partial \mathbf{P}_o^I}{\partial t} \\
& - \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \left(S_g^I - P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \mathbf{m} \mathbf{N} dV \frac{\partial \mathbf{P}_g^I}{\partial t} \\
& - \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \left(S_o^{II} + P_c^{II} \frac{\partial S_o^{II}}{\partial P_c^{II}} \right) \mathbf{m} \mathbf{N} dV \frac{\partial \mathbf{P}_o^{II}}{\partial t} \\
& - \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \left(S_g^{II} - P_c^{II} \frac{\partial S_o^{II}}{\partial P_c^{II}} \right) \mathbf{m} \mathbf{N} dV \frac{\partial \mathbf{P}_g^{II}}{\partial t} = \frac{\partial \mathbf{f}}{\partial t}
\end{aligned} \tag{4.46}$$

Equations 4.44-4.46 are the finite element equations for fully-coupled oil-gas flow in a dual-porosity medium under isothermal conditions. They can be expressed in a more convenient form as

Equilibrium:

$$\mathbf{K}\dot{\mathbf{u}} + \mathbf{O}_c^I \dot{\mathbf{P}}_o^I + \mathbf{O}_c^{II} \dot{\mathbf{P}}_o^{II} + \mathbf{G}_c^I \dot{\mathbf{P}}_g^I + \mathbf{G}_c^{II} \dot{\mathbf{P}}_g^{II} - \dot{\mathbf{f}} = 0 \quad (4.47)$$

Oil in the primary medium:

$$\mathbf{K}_o^I \dot{\mathbf{u}} + (\mathbf{O}_D^I + \mathbf{O}_T) \mathbf{P}_o^I - \mathbf{O}_T \mathbf{P}_o^{II} + \mathbf{O}_o^I \dot{\mathbf{P}}_o^I + \mathbf{O}_g^I \dot{\mathbf{P}}_g^I = -\mathbf{Q}_o^I \quad (4.48)$$

Gas in the primary medium:

$$\begin{aligned} \mathbf{K}_g^I \dot{\mathbf{u}} + (\mathbf{G}_{dD}^I + \mathbf{G}_{dT}) \mathbf{P}_o^I + (\mathbf{G}_D^I + \mathbf{G}_T) \mathbf{P}_g^I - \mathbf{G}_{dT} \mathbf{P}_o^{II} \\ - \mathbf{G}_T \mathbf{P}_g^{II} + \mathbf{G}_o^I \dot{\mathbf{P}}_o^I + \mathbf{G}_g^I \dot{\mathbf{P}}_g^I = -\mathbf{Q}_g^I \end{aligned} \quad (4.49)$$

Oil in the secondary medium:

$$\mathbf{K}_o^{II} \dot{\mathbf{u}} + (\mathbf{O}_D^{II} + \mathbf{O}_T) \mathbf{P}_o^{II} - \mathbf{O}_T \mathbf{P}_o^I + \mathbf{O}_o^{II} \dot{\mathbf{P}}_o^{II} + \mathbf{O}_g^{II} \dot{\mathbf{P}}_g^{II} = -\mathbf{Q}_o^{II} \quad (4.50)$$

Gas in the secondary medium:

$$\begin{aligned} \mathbf{K}_g^{II} \dot{\mathbf{u}} + (\mathbf{G}_{dD}^{II} + \mathbf{G}_{dT}) \mathbf{P}_o^{II} + (\mathbf{G}_D^{II} + \mathbf{G}_T) \mathbf{P}_g^{II} - \mathbf{G}_{dT} \mathbf{P}_o^I \\ - \mathbf{G}_T \mathbf{P}_g^I + \mathbf{G}_o^{II} \dot{\mathbf{P}}_o^{II} + \mathbf{G}_g^{II} \dot{\mathbf{P}}_g^{II} = -\mathbf{Q}_g^{II} \end{aligned} \quad (4.51)$$

where the overdot signifies differentiation with respect to time. Equations 4.47-4.51 can be expressed in a compact form as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{O}_D^I + \mathbf{O}_T & 0 & -\mathbf{O}_T & 0 \\ 0 & \mathbf{G}_{dD}^I + \mathbf{G}_{dT} & \mathbf{G}_D^I + \mathbf{G}_T & -\mathbf{G}_{dT} & -\mathbf{G}_T \\ 0 & -\mathbf{O}_T & 0 & \mathbf{O}_D^{II} + \mathbf{O}_T & 0 \\ 0 & -\mathbf{G}_{dT} & -\mathbf{G}_T & \mathbf{G}_{dD}^{II} + \mathbf{G}_{dT} & \mathbf{G}_D^{II} + \mathbf{G}_T \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}_o^I \\ \mathbf{P}_g^I \\ \mathbf{P}_o^{II} \\ \mathbf{P}_g^{II} \end{Bmatrix} + \quad (4.52)$$

$$\begin{bmatrix} \mathbf{K} & \mathbf{O}_c^I & \mathbf{G}_c^I & \mathbf{O}_c^{II} & \mathbf{G}_c^{II} \\ \mathbf{K}_o^I & \mathbf{O}_o^I & \mathbf{O}_o^I & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_g^I & \mathbf{G}_o^I & \mathbf{G}_g^I & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_o^{II} & \mathbf{0} & \mathbf{0} & \mathbf{O}_o^{II} & \mathbf{O}_g^{II} \\ \mathbf{K}_g^{II} & \mathbf{0} & \mathbf{0} & \mathbf{G}_o^{II} & \mathbf{G}_g^{II} \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}_o^I \\ \mathbf{P}_g^I \\ \mathbf{P}_o^{II} \\ \mathbf{P}_g^{II} \end{Bmatrix} = \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}_o^I \\ -\mathbf{Q}_g^I \\ -\mathbf{Q}_o^{II} \\ -\mathbf{Q}_g^{II} \end{Bmatrix}$$

The sub-matrices in equation 4.52 are listed in the Appendix. Since equation 4.52 contains terms such as saturations and capillary pressures which are dependent on the primary variables, it represents a set of non-linear equations.

4.5.1 Discretization in the Time Domain

The discretization in time is carried out using a finite difference scheme as in Chapters 2 and 3:

$$\begin{cases} \dot{\mathbf{u}}^{t+\Delta t} = \frac{1}{\Delta t} (\mathbf{u}^{t+\Delta t} - \mathbf{u}^t) \\ (\dot{\mathbf{P}}_o^\eta)^{t+\Delta t} = \frac{1}{\Delta t} [(\mathbf{P}_o^\eta)^{t+\Delta t} - (\mathbf{P}_o^\eta)^t] \\ (\dot{\mathbf{P}}_g^\eta)^{t+\Delta t} = \frac{1}{\Delta t} [(\mathbf{P}_g^\eta)^{t+\Delta t} - (\mathbf{P}_g^\eta)^t] \end{cases} \quad (4.53)$$

From equations 4.52 and 4.53, we get

$$\begin{aligned} & \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{O}_c^I & \mathbf{G}_c^I & \mathbf{O}_c^{II} & \mathbf{G}_c^{II} \\ \mathbf{K}_o^I & \mathbf{O}_{dT}^I \Delta t + \mathbf{O}_o^I & \mathbf{O}_g^I & -\mathbf{O}_T^I \Delta t & \mathbf{0} \\ \mathbf{K}_g^I & \mathbf{G}_{dT}^I \Delta t + \mathbf{G}_o^I & \mathbf{G}_g^I & -\mathbf{G}_T^I \Delta t & -\mathbf{G}_T^I \Delta t \\ \mathbf{K}_o^{II} & -\mathbf{O}_T^I \Delta t & \mathbf{0} & \mathbf{O}_{dT}^{II} \Delta t + \mathbf{O}_o^{II} & \mathbf{O}_g^{II} \\ \mathbf{K}_g^{II} & -\mathbf{G}_T^I \Delta t & -\mathbf{G}_T^I \Delta t & \mathbf{G}_{dT}^{II} \Delta t + \mathbf{G}_o^{II} & \mathbf{G}_{dT}^{II} \Delta t + \mathbf{G}_g^{II} \end{bmatrix} \\ & \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}_o^I \\ \mathbf{P}_g^I \\ \mathbf{P}_o^{II} \\ \mathbf{P}_g^{II} \end{Bmatrix}^{t+\Delta t} \\ & = \frac{1}{\Delta t} \begin{bmatrix} \mathbf{K} & \mathbf{O}_c^I & \mathbf{G}_c^I & \mathbf{O}_c^{II} & \mathbf{G}_c^{II} \\ \mathbf{K}_o^I & \mathbf{O}_o^I & \mathbf{O}_g^I & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_g^I & \mathbf{G}_o^I & \mathbf{G}_g^I & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_o^{II} & \mathbf{0} & \mathbf{0} & \mathbf{O}_o^{II} & \mathbf{O}_g^{II} \\ \mathbf{K}_g^{II} & \mathbf{0} & \mathbf{0} & \mathbf{G}_o^{II} & \mathbf{G}_g^{II} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{P}_o^I \\ \mathbf{P}_g^I \\ \mathbf{P}_o^{II} \\ \mathbf{P}_g^{II} \end{Bmatrix}^t + \begin{Bmatrix} \frac{d\mathbf{f}}{dt} \\ -\mathbf{Q}_o^I \\ -\mathbf{Q}_g^I \\ -\mathbf{Q}_o^{II} \\ -\mathbf{Q}_g^{II} \end{Bmatrix}^{t+\Delta t} \end{aligned} \quad (4.54)$$

where

$$\begin{aligned} G_{dT}^\eta &= G_{dD}^\eta + G_{dT} \\ G_{DT}^\eta &= G_D^\eta + G_T \end{aligned} \quad (4.55)$$

4.5.2 Evaluation of Coefficients

The coefficients of the nonlinear equations 4.47-4.51 account for relative permeabilities, rock and fluid compressibilities, capillary pressures and gas solubilities. In the present study, solutions to equation 4.54 (for every time-step) are obtained using a direct solver. Since the coefficients are dependent on the unknowns an iterative procedure is employed within each time-step. At each iteration level, the nonlinear coefficients are updated using the values of the unknowns calculated in the previous iteration. The stability of the solution is monitored using the convergence criterion

$$\left\| \frac{\bar{\Upsilon}_i^{k+1} - \bar{\Upsilon}_i^k}{\bar{\Upsilon}_i^k} \right\| \leq \epsilon \quad (4.56)$$

where $\bar{\Upsilon}_i$ are the unknowns, at node i ; k and $(k + 1)$ are the old and new iteration levels, respectively; and, ϵ is the convergence limit which is chosen as 0.05 in the present study (Nair et al., 2002; Ghafouri and Lewis, 1997).

4.6 Application to Inclined Wellbore Geometry

In order to identify critical parameters in the dual-porosity modeling with generalized plane strain idealization, a parametric study of an inclined wellbore is carried out for the following cases: (a) single-porosity, two-phase, (b) dual-porosity, single-phase and (c) dual-porosity, two-phase.

The effect of phase saturation, dual-porosity parameters and interaction between the two media are highlighted in the first two cases, whereas the combined effect is examined in the last case.

Parameter	Units	Value
Elastic Modulus (E)	GPa	3.55
Poisson's Ratio (ν)	-	0.2
Porosity (ϕ^I)	-	0.01
Permeability (k^I)	m ²	10 ⁻¹⁸
Grain Bulk Modulus (K_s)	GPa	10 ⁷
Fluid Viscosity (μ)	MPa.s	10 ⁻⁹

Table 4.1: Parameters for the single-porosity, two-phase model (inclined wellbore problem)

4.6.1 Single-Porosity, Two-Phase Modeling

In this case, the formation was assumed to be a single-porosity medium saturated with oil and gas. Figures 4.2 and 4.3 show the spatial variations of the oil pressure and effective radial stress for $\theta = 87.63^\circ$ and $t = 78$ sec. for different initial oil saturations. The input parameters are listed in Table 4.1.

Relations between relative permeability and saturation, capillary pressure and saturation, formation volume factors and pressure and, gas solubility ratio and pressure are based on a linear least square analysis of data from literature (Lewis and Ghafouri, 1997). The peak in the oil pressure curve (Figure 4.2) and tensile stresses (Figure 4.3) around the borehole decrease for a lower value of initial oil saturation ($S_{oi} = 0.65$). This is expected since a larger initial gas saturation increases the overall fluid compressibility thereby leading to a decrease in the poroelastic effect.

4.6.2 Dual-Porosity, Single-Phase Modeling

In order to model a fractured formation using the dual-porosity concept, the following fracture characteristics were associated with the secondary medium as in Chapter 3 - stiffness K_n and fracture spacing s (governing the compliance of the secondary medium); secondary porosity ϕ^{II} (same as in equation 2.2, representing the storativity of the fractures); and, secondary permeability k^{II} . The formation was assumed to be a dual-porosity medium saturated with a single fluid with the material parameters as listed in

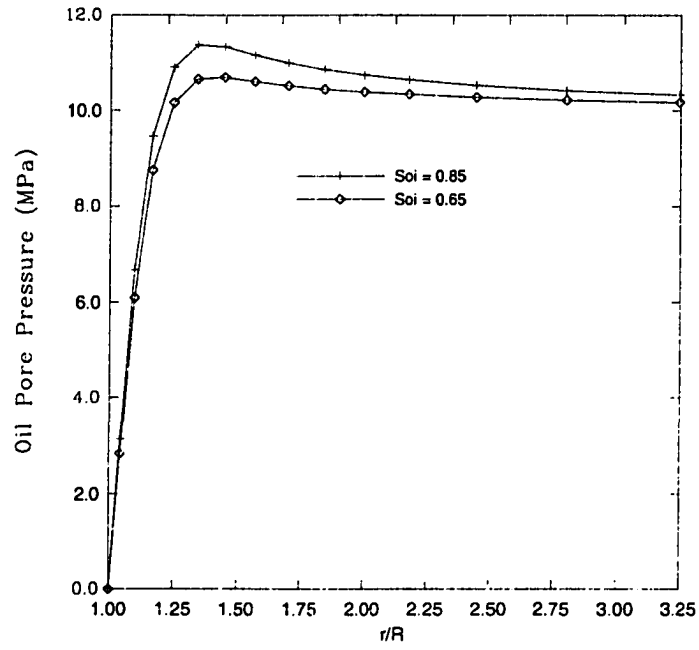


Figure 4.2: Oil pressure distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

Table 4.2.

The parameters for the secondary medium are selected such that the secondary (fracture) permeability is 10 times the primary (matrix) permeability and, the primary porosity (representing storage capacity of the matrix) is 10 times the secondary porosity (Wilson and Aifantis, 1982). The pore pressure and effective radial stress distributions for different fracture spacings are shown in Figures 4.4 and 4.5, respectively, for $\theta = 87.63^\circ$ and $t = 78$ sec. Both figures indicate that, for large fracture spacings, the dual-porosity model behaves similarly as the single-porosity model. As the fracture spacing becomes smaller (e.g. $s = 0.25$ m), the flow in fractures becomes more dominant. The decrease in tensile stresses around the borehole for smaller fracture spacing is due to the dissipation of pressures in the highly permeable fractures. The matrix-fracture interaction is characterized by the fluid transferred between the two media.

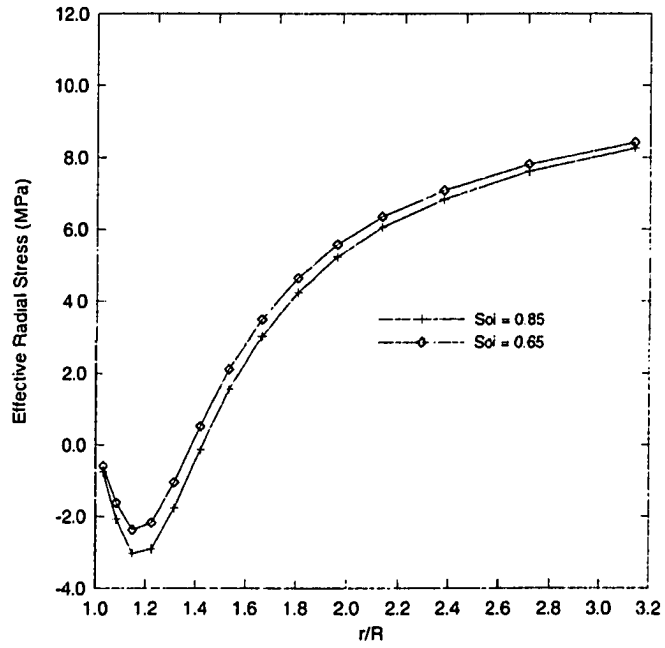


Figure 4.3: Effective radial stress distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

Parameter	Units	Value
Elastic Modulus (E)	GPa	20.6
Poisson's Ratio (ν)	-	0.189
Matrix (Primary) Porosity (ϕ^I)	-	0.02
Matrix (Primary) Permeability (k^I)	m^2	10^{-19}
Grain Bulk Modulus (K_s)	GPa	48.21
Fluid Viscosity (μ)	MPa.s	10^{-9}
Fracture (Secondary) Permeability (k^{II})	m^2	10^{-18}
Fracture Stiffness (K_n)	GPa/m	48.21
Fracture Spacing (s)	m	.5, 2.5, 10
Fracture (Secondary) Porosity (ϕ^{II})	-	0.002

Table 4.2: Parameters for dual-porosity, single-phase model (inclined wellbore problem)

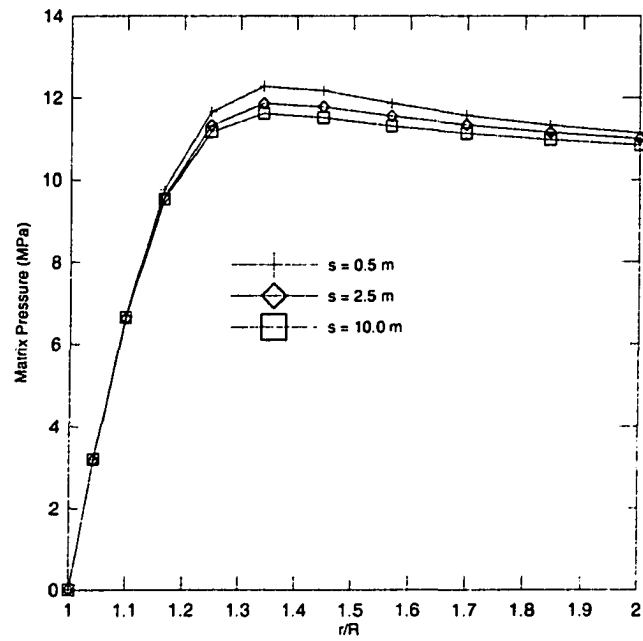


Figure 4.4: Primary medium (Matrix) pore pressure distributions for different fracture spacings (dual-porosity, single-phase; s - fracture spacing)

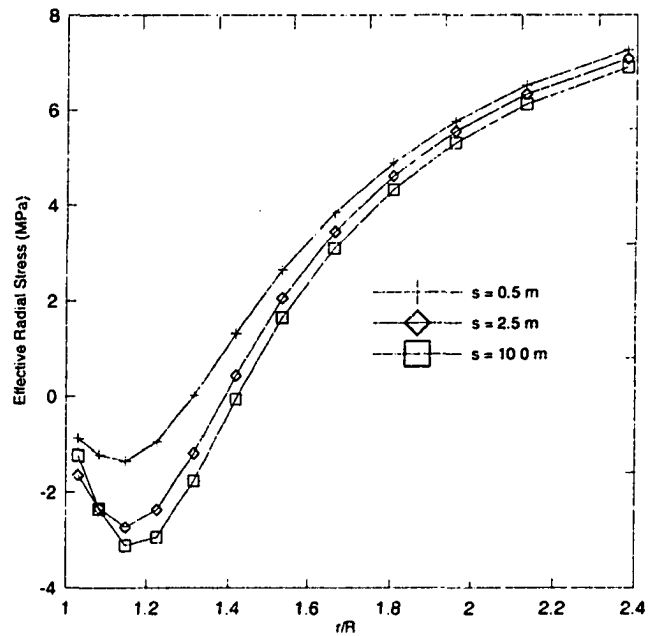


Figure 4.5: Effective radial stress distribution for different fracture spacings (dual-porosity, single-phase; s - fracture spacing)

Parameter	Units	Value
Elastic Modulus (E)	GPa	3.55
Poisson's Ratio (ν)	-	0.2
Matrix (Primary) Porosity (ϕ^I)	-	0.01
Matrix (Primary) Permeability (k^I)	m ²	10^{-18}
Grain Bulk Modulus (K_s)	GPa	10^7
Fluid Viscosity (μ)	MPa.s	10^{-9}
Fracture (Secondary) Permeability (k^{II})	m ²	10^{-17}
Fracture Stiffness (K_n)	GPa/m	20.0
Fracture Spacing (s)	m	.1, 1, 10^7
Fracture (Secondary) Porosity (ϕ^{II})	-	0.002

Table 4.3: Parameters for dual-porosity, two-phase model (inclined wellbore problem)

4.6.3 Dual-Porosity, Two-Phase Modeling

The dual-porosity two-phase modeling of the inclined wellbore problem has been carried out by assuming an initial oil saturation of 0.85 in both the primary and secondary media. The input parameters are as listed in Table 4.3.

Further, for simplicity, the relationships for capillary pressure-saturation, relative permeability-saturation and others, used in the single-porosity two-phase model, were adopted for both media. Figure 4.6 shows the spatial variations in oil pressure in the primary medium at $t = 15.6$ sec. The initial fluid transfer from the secondary medium to the primary medium leads to higher pressures in the latter. This effect is more pronounced for smaller fracture spacings ($s = 0.25$ m). As in the single-porosity, two-phase case, the oil pressure in the primary medium near the borehole ($r/R \simeq 1.15$) is lower for a lower initial oil saturation of 0.65. Figure 4.7 shows the spatial distribution of the effective radial stress for $t = 78$ sec. As expected, the tensile stresses around the borehole are smaller for lower values of initial oil saturation. This is consistent with the result in the single-porosity, two-phase case. Also, for the same value of initial oil saturation, the tensile stresses around the borehole decrease as the fracture spacing decreases, which is consistent with the result in the dual-porosity, single-phase case.

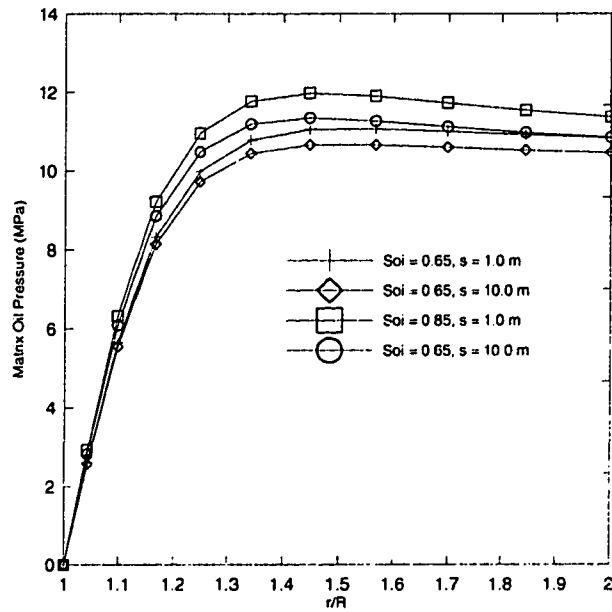


Figure 4.6: Primary medium (Matrix) oil pressure distribution for different fracture spacings and initial oil saturation, $S_{oi} = 0.85$ (dual-porosity, two-phase; s - fracture spacing)

It should be emphasized that the demonstrated results imply the coupled phenomena where stress variations are controlled by the double effective stress laws.

4.7 Discussions

The effects of the interaction between the primary and secondary media are clearly seen in Figure 4.8, which shows the temporal variations of pressures in both media at different locations ($r/R = 1.3, 2.0$, where R is the wellbore radius and r is the radial distance from the wellbore). In this case the medium is assumed to be saturated with a single fluid. At each location, the initial pressure in the secondary medium is greater than the pressure in the primary medium. This pressure gradient causes flow into the primary medium and an increases in pressure (Lewallen and Wang, 1998). When the pressures in the two media dissipate, the curves cross each other and inter-porosity flow is negligible.

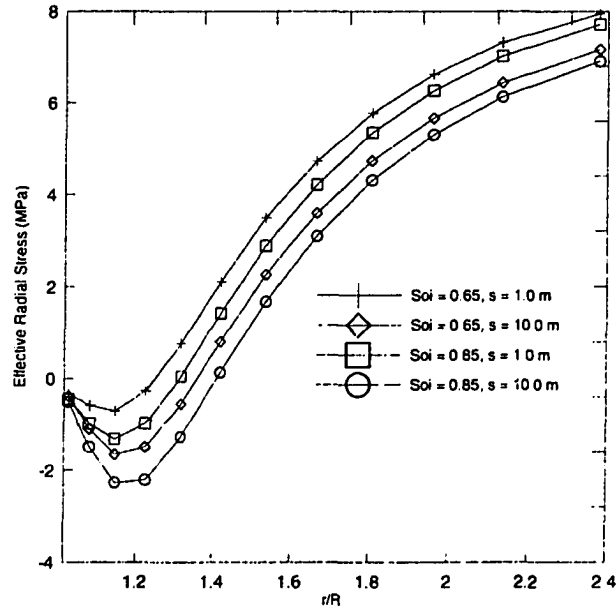


Figure 4.7: Effective radial stress distribution for different fracture spacings and initial oil saturations (dual-porosity, two-phase; S_{oi} -- initial oil saturation; s - fracture spacing)

For a larger distance from the borehole, the crossover time is more, since the fracture flowpath to the borehole is longer.

Figure 4.10 shows the effect of compliance of the secondary medium on the temporal variation of pressures in both media. Closely spaced fractures increase the compressibility of the secondary medium. This manifests itself as an increase in the initial pressure in the secondary medium and fluid transfer into the primary medium. The pressure buildup in the primary medium is thus higher for smaller fracture spacing ($s = 0.1$ m) and the result is consistent with those in Figures 4.4 and 4.6.

The effect of permeability ratios are investigated by varying the secondary (fracture) permeability. The pore pressure dissipates faster in a more permeable fracture system (i.e., secondary medium), resulting in lower pressures in both media (as seen in Figure 4.10) and a shorter crossover time.

Figure 4.11 shows the temporal variation of the effective radial stress near the well-

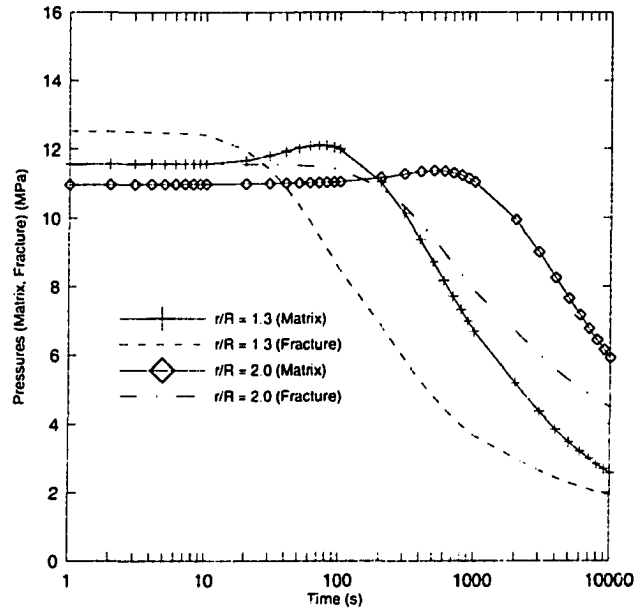


Figure 4.8: Temporal variation of primary (matrix) pore pressure at different locations ($s = 0.25$ m)

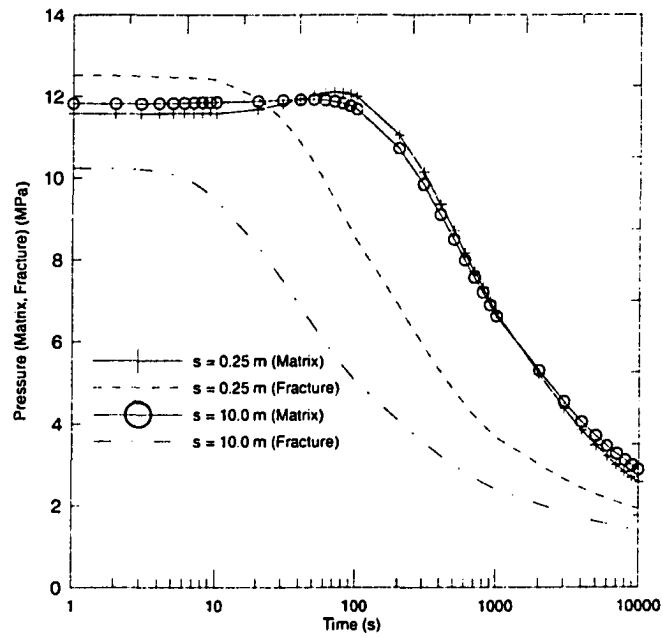


Figure 4.9: Effect of fracture spacing on temporal variations in primary (matrix) and secondary medium (fracture system) pressures ($r/R = 1.3$)

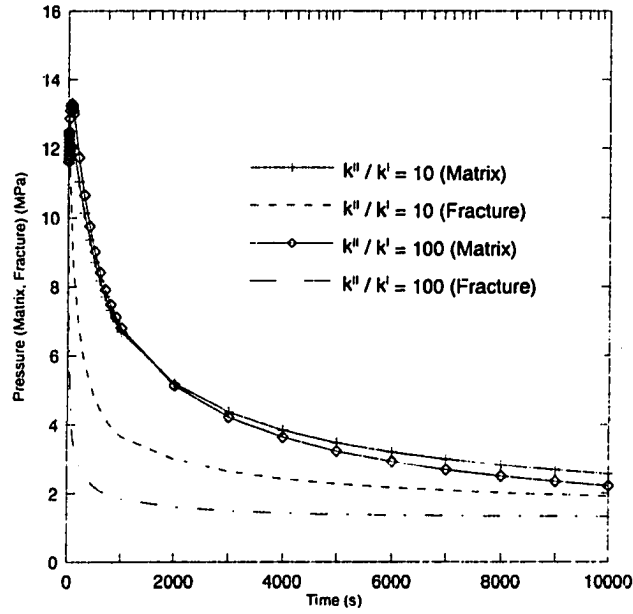


Figure 4.10: Effect of permeability ratio on the temporal distribution of primary and secondary medium pressures (fracture spacing, $s = 0.25$ m, $r/R = 1.3$)

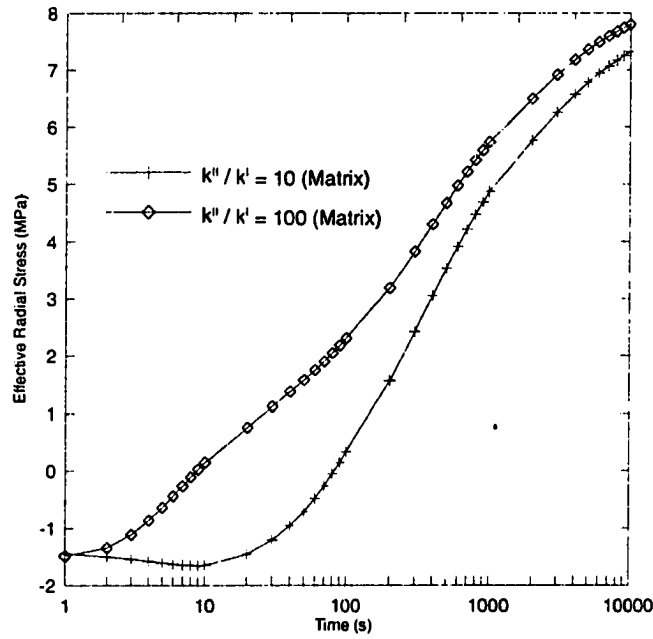


Figure 4.11: Effect of permeability ratio on the temporal distribution of effective radial stress (fracture spacing, $s = 0.25$ m, $r/R = 1.3$)

bore. The poroelastic effects, characterized by tensile stresses near the wellbore, expectably diminish more quickly for a secondary medium having a higher permeability (representing a highly permeable fracture system).

4.8 Summary

A dual-porosity poroelastic model incorporating oil and gas flow has been developed. As in Chapters 2 and 3, the model relies on the “separate and “overlapping” technique. More specifically, separate equations characterizing the responses of the primary and secondary media account for the contribution of fully coupled deformations in both media. In an oil-gas system, the saturations of the individual phases play an important role in determining the effective fluid compressibility. This is so because, the compressibilities of oil and gas differ by orders of magnitude. The results obtained using the single-porosity two-phase and dual-porosity two phase models, respectively, indicate that the fluid compressibilities (and hence the phase saturations) affect the magnitude of stresses and pore pressures perturbations near the borehole wall. Assuming the formation to be saturated with a single incompressible fluid leads to an overestimation of the tensile stresses and pore pressures developed at the borehole wall. A consideration of the mechanisms of two-phase flow is hence realistic since formations are generally saturated with more than one fluid of different compressibilities.

Fractured formations may be modeled using the dual-porosity concept by assigning fracture-like characteristics to the secondary medium. The initial response is governed by the fluid transfer between the two media. In the present study, the fluid transfer between both media is assumed to be a function of quasi-steady pressure differentials. The results obtained from the dual-porosity single-phase model show that the fluid transfer at early times is affected by the compliance of the secondary medium. Neglecting these effects could lead to an inaccurate estimation of near wellbore pressures and stresses.

Chapter 5

Two-Phase Dual-Porosity Porothermoelasticity

5.1 Introduction

The mathematical formulation presented in Chapter 4 was developed on basis of the assumption that the temperature remains constant and uniform throughout the fractured porous medium. However, for a fairly important class of problems, such as oil recovery by thermal methods, transport of non-aqueous phase liquids (NAPL) in groundwater systems, drying processes and building insulations, such an assumption may be inappropriate. In the case of thermal oil recovery techniques, heat is transferred to oil to reduce its viscosity, especially when reservoirs contain heavy, viscous oil. Other temperature-related effects include the distillation of light hydrocarbon fractions which results in interphase mass transfer and hence compositional considerations. In the area of building insulation, water vapor condensation in a porous insulation releases the latent heat of vaporization, thereby affecting its thermal performance (Allen et al., 1988; Vafai and Chen, 1989). With respect to groundwater systems, it is to be noted that the partially saturated zone of soil moisture near the earth's surface plays a critical role in partitioning precipitation into surface runoff, evapotranspiration and groundwater recharge (Milly, 1982).

The black-oil model described in Chapter 4 is not suitable for dealing with the afore-

mentioned problems. This is so because in case of oil-gas systems with volatile oil under high temperatures, the gas phase may contain a significant amount of vaporized oil. To account for these effects, fully compositional models have been developed which track the individual components (such as carbon dioxide, methane etc.) of the hydrocarbon system. However, these models are computationally expensive to employ, since the computing requirements are proportional to the number of components in the hydrocarbon system (Cook et al., 1974). Some of the advantages offered by the fully compositional model may be achieved by adopting the scheme proposed by Cook et al. (1974) wherein the effect of oil volatility is included within the framework of the black-oil model. The limited compositional model thus developed can account for relatively more complex compositional effects as compared to the black oil model and is computationally less intensive as compared to the fully compositional model (Peaceman, 1977).

In this chapter, the mathematical formulations in Chapters 3 and 4 are extended to formulate fully-coupled non-isothermal oil-gas flow in a fractured porous media. Specifically, the black oil model described in Chapter 4 has been extended to the limited compositional model in order to account for thermal effects. The volatility of the oil has been incorporated into the formulation. Thus, the gas phase is comprised of a gas and oil component unlike as in Chapter 4, where the gas phase was assumed to be made up of the gas component entirely. The oil phase, as in Chapter 4 has both, the oil component and a dissolved gas component. In addition, the viscosities of the oil and gas phases, formation volume factors for oil and gas, gas-oil solubility ratio, oil volatility ratio and the saturations are assumed to be dependent on the temperature. As in Chapters 2-4, the fluid and solid domains are represented by two distinct overlapping continua within the framework of the dual-porosity concept. As in Chapter 3, a single thermodynamic continuum is assumed to be representative of the fractured porous medium. The finite element method is employed to solve the non-linear set of differential equations, wherein

the displacements, pressures (in the oil and gas phases) and temperature are the primary unknowns. The saturations and capillary pressures are the secondary unknowns to be obtained from capillary-saturation-temperature relations. As in Chapter 4, the nonlinear system of equations in the finite element model is solved using a direct solver with iterations employed within each time-step to check the stability. The model is then applied to the problem of an inclined wellbore in a fractured porous medium saturated with oil and gas and a temperature gradient between the drilling and formation fluids. A parametric analysis is carried out to identify influential parameters governing the spatial and temporal distributions of pore pressure, stresses and temperature in the vicinity of the wellbore.

5.2 Limited Compositional Model

The limited compositional model is an extension of the black-oil model, wherein we permit solubility of the gas in oil phase and vaporization of the oil into the gas phase. Thus the oil component exists in both the gas and oil phases. As in Chapter 4, a sample of fluid (both oil and gas) at reservoir conditions is assumed to contain oil and gas components of mass W_O and W_G , respectively. The volatility of oil in the gas is expressed in terms of the oil volatility ratio defined as

$$R_v = \frac{(V_{OS})^{ev.}}{V_{GS}} \quad (5.1)$$

where V_{OS} and V_{GS} are as defined in Chapter 4. Also, the formation volume factor for gas is given by

$$B_g = \frac{V_g}{V_{GS}} = \frac{\rho_{GS}}{\rho_g} \quad (5.2)$$

where

$$V_g = \frac{(W_G)^{free} + (W_O)^{ev.}}{\rho_g} \quad (5.3)$$

The mass fractions of oil and gas components in the gas phase, i.e., w_{Gg} and w_{Og} , respectively, are

$$w_{Gg} = \frac{(W_G)^{free}}{(W_G)^{free} + (W_O)^{ev.}} \quad ; \quad w_{Og} = \frac{(W_O)^{ev.}}{(W_G)^{free} + (W_O)^{ev.}} \quad (5.4)$$

Hence from equations 5.2-5.4, we have,

$$w_{Gg} = \frac{\rho_{GS}}{\rho_g B_g} \quad ; \quad w_{Og} = \frac{\rho_{OS} R_v}{\rho_g B_g} \quad (5.5)$$

The mass fractions of oil and gas components in the oil phase are as in the black-oil model, i.e., equation 4.11. The notations to distinguish between the phases and components are as in Section 4.2.1. The superscripts *ev.* refers to the oil in the gas phase and *free* refers to the gas not dissolved in the oil, commonly referred to as “free gas”. Also ρ_{os} and ρ_{gs} will be used to denote the densities of oil and gas under standard conditions henceforth in this chapter.

5.3 Assumptions

The following assumptions are made in addition to those in Chapters 3 and 4 in order to formulate the equations for fully-coupled oil and gas flow under non-isothermal conditions in a fractured porous medium. It must be pointed out that the assumptions of a low-volatility oil has been relaxed in order to facilitate the evaporation oil into the gas phase.

1. The temperature equilibration between the fluid constituents, i.e., oil and gas, and the solid takes place quickly as compared to the heat transport by conduction or convection. Thus the oil and gas components in both, the oil and gas phase have the same temperature as the solid constituents, even though the spatiotemporal variation in temperature itself is accounted for.
2. The relative permeabilities of oil and gas (k_{ro} and k_{rg} , respectively) are functions of the respective saturations only, i.e., $k_{ro} = k_{ro}(S_o, T)$; $k_{rg} = k_{rg}(S_g, T)$.

3. The saturation of the oil and gas phases are functions of the capillary pressure and temperature, i.e., $S_o = S_o(P_c, T)$; $S_g = S_g(P_c, T)$.
4. The gas-oil solubility ratio and oil volatility ratio are functions of the pressure in the gas and oil phases and the temperature, i.e., $R_s = R_s(P_g, T)$; $R_v = R_v(P_o, T)$.

5.4 Mathematical Formulation

As in Chapter 4, the framework of the dual-porosity concept with two distinct overlapping continua for the solid and fluid domains is retained. A single thermodynamic continuum is assumed to be representative of the fractured porous medium. The limited compositional model is woven into this framework in order to develop the equations governing fully-coupled oil-gas in a dual-porosity medium under non-isothermal conditions.

5.4.1 Definitions

The mass fractions of the oil component in the oil and gas phases in both the primary and secondary media are based on equations 4.11 and 5.5 i.e., w_{Og}^η and w_{Oo}^η are given as

$$w_{Og}^\eta = \frac{\rho_{os}^\eta R_v^\eta}{\rho_g^\eta B_g^\eta} ; w_{Oo}^\eta = \frac{\rho_{os}^\eta}{\rho_o^\eta B_o^\eta} \quad (5.6)$$

where B_o^η , B_g^η are the formation volume factors for oil and gas as in Chapter 4, R_v^η is the volatility ratio of oil and ρ_{os}^η is the density of oil under standard conditions. Similarly, from equations 4.11 and 5.5 the mass fractions of the gas component in the oil and gas phases for both media, i.e., w_{Gg}^η and w_{Go}^η , respectively, are

$$w_{Gg}^\eta = \frac{\rho_{gs}^\eta}{\rho_g^\eta B_g^\eta} ; w_{Go}^\eta = \frac{R_s^\eta \rho_{gs}^\eta}{\rho_o^\eta B_o^\eta} \quad (5.7)$$

5.4.2 Mass Conservation for Oil Component

The mass conservation equation for oil in a dual-porosity medium under non-isothermal conditions may be obtained using equations 4.29, 4.30 and 5.6 as,

$$\left\{ \begin{aligned} & \frac{\partial}{\partial t} \left(\phi^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \right) + \frac{\partial}{\partial t} \left(\phi^\eta S_g^\eta \frac{\rho_{os}^\eta R_v^\eta}{B_g^\eta} \right) + \\ & \frac{\partial}{\partial x_i} \left(\phi^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} u_i^\eta + \frac{\rho_{os}^\eta}{B_o^\eta} v_i^{\eta o} \right) \\ & + \frac{\partial}{\partial x_i} \left(\phi^\eta S_g^\eta \frac{\rho_{os}^\eta R_v^\eta}{B_g^\eta} u_i^\eta + \frac{\rho_{os}^\eta R_v^\eta}{B_g^\eta} v_i^{\eta g} \right) - \Gamma_o = 0 \end{aligned} \right. \quad (5.8)$$

Expanding derivatives, and incorporating equations 2.4, 2.8 and 2.12 into 5.8, we have,

$$\left\{ \begin{aligned} & \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \frac{DP^\eta}{Dt} + \phi^\eta \rho_{os}^\eta \left(\frac{1}{B_o^\eta} - \frac{R_v^\eta}{B_g^\eta} \right) \frac{DS_o^\eta}{Dt} \\ & + \phi^\eta S_o^\eta \rho_{os}^\eta \frac{D}{Dt} \left(\frac{1}{B_o^\eta} \right) + \phi^\eta S_g^\eta \rho_{os}^\eta R_v^\eta \frac{D}{Dt} \left(\frac{1}{B_g^\eta} \right) \\ & + \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta \alpha^\eta u_{i,i}^\eta + \frac{\rho_{os}^\eta}{B_o^\eta} v_{i,i}^{\eta o} + \\ & \frac{\rho_{os}^\eta R_v^\eta}{B_g^\eta} v_{i,i}^{\eta g} + \phi^\eta \frac{\rho_{os}^\eta S_g^\eta}{B_g^\eta} \frac{DR_v^\eta}{Dt} \\ & + \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta ((1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta) \frac{DT}{Dt} \\ & + v_i^{\eta o} \rho_{os}^\eta \left(\frac{1}{B_o^\eta} \right)_{,i} + \rho_{os}^\eta v_i^{\eta g} \left(\frac{R_v^\eta}{B_g^\eta} \right)_{,i} \\ & - \Gamma_o = 0 \end{aligned} \right. \quad (5.9)$$

Now,

$$\left\{ \begin{aligned} & \frac{\partial}{\partial t} \left(\frac{1}{B_o^\eta} \right) = - \frac{1}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \frac{\partial P_o^\eta}{\partial t} - \frac{1}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial T} \frac{\partial T}{\partial t} \\ & \frac{\partial}{\partial t} \left(\frac{1}{B_g^\eta} \right) = - \frac{1}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} \frac{\partial P_g^\eta}{\partial t} - \frac{1}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial T} \frac{\partial T}{\partial t} \\ & \frac{\partial S_o^\eta}{\partial t} = - \frac{\partial S_o^\eta}{\partial P_o^\eta} \frac{\partial P_o^\eta}{\partial t} = \frac{\partial S_o^\eta}{\partial P_c^\eta} \left(\frac{\partial P_g^\eta}{\partial t} - \frac{\partial P_o^\eta}{\partial t} \right) + \frac{\partial S_o^\eta}{\partial T} \frac{\partial T}{\partial t} \\ & \frac{\partial P^\eta}{\partial t} = \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \frac{\partial P_o^\eta}{\partial t} + \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \frac{\partial P_g^\eta}{\partial t} \\ & \quad - P_c^\eta \frac{\partial S_o^\eta}{\partial T} \frac{\partial T}{\partial t} \\ & v_i^{\eta o} = - \frac{k^\eta k_{ro}^\eta}{\mu_o^\eta} P_{o,i}^\eta ; v_i^{\eta g} = - \frac{k^\eta k_{rg}^\eta}{\mu_g^\eta} P_{g,i}^\eta \\ & \frac{\partial R_v^\eta}{\partial t} = \frac{\partial R_v^\eta}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial R_v^\eta}{\partial P_o^\eta} \frac{\partial P_o^\eta}{\partial t} \\ & \frac{\partial R_s^\eta}{\partial t} = \frac{\partial R_s^\eta}{\partial P_g^\eta} \frac{\partial P_g^\eta}{\partial t} + \frac{\partial R_s^\eta}{\partial P_g^\eta} \frac{\partial P_g^\eta}{\partial t} \end{aligned} \right. \quad (5.10)$$

From equations 5.9, 5.10, letting $\frac{D}{Dt} \approx \frac{\partial}{\partial t}$ and grouping terms we get the final form of the governing equation for oil flow as

$$\begin{aligned}
 & \left\{ \begin{aligned}
 & \dot{P}_o^\eta \left\{ \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) - \right. \\
 & \quad \left. \phi^\eta \rho_{os}^\eta \left(\frac{1}{B_o^\eta} - \frac{R_v^\eta}{B_g^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} - \frac{\phi^\eta \rho_{os}^\eta S_o^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \right. \\
 & \quad \left. + \phi^\eta \frac{\rho_{os}^\eta S_g^\eta}{B_g^\eta} \frac{\partial R_v^\eta}{\partial P_o^\eta} \right\} \\
 & \dot{P}_g^\eta \left\{ \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta \frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) - \right. \\
 & \quad \left. \frac{\phi^\eta \rho_{os}^\eta S_g^\eta R_v^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} + \phi^\eta \rho_{os}^\eta \left(\frac{1}{B_o^\eta} - \frac{R_v^\eta}{B_g^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} \right\} + \\
 & \dot{T} \left\{ \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta \left(\frac{(1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta}{K_s^\eta} P_c^\eta \frac{\partial S_o^\eta}{\partial T} \right) \right. \\
 & \quad \left. - \frac{\phi^\eta \rho_{os}^\eta S_o^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial T} - \frac{\phi^\eta \rho_{os}^\eta S_g^\eta R_v^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial T} \right. \\
 & \quad \left. + \phi^\eta \frac{\rho_{os}^\eta S_g^\eta}{B_g^\eta} \frac{\partial R_v^\eta}{\partial T} + \phi^\eta \rho_{os}^\eta \left(\frac{1}{B_o^\eta} - \frac{R_v^\eta}{B_g^\eta} \right) \frac{\partial S_o^\eta}{\partial T} \right\} + \\
 & \alpha^\eta \left(S_g^\eta \frac{R_v^\eta}{B_g^\eta} + \frac{S_o^\eta}{B_o^\eta} \right) \rho_{os}^\eta u_{i,i}^\eta - \frac{\rho_{os}^\eta k^\eta k_{ro}^\eta}{B_o^\eta \mu_o^\eta} (P_o^\eta)_{i,i} - \\
 & \frac{\rho_{os}^\eta R_v^\eta}{B_g^\eta} \frac{k^\eta k_{rg}^\eta}{\mu_g^\eta} (P_g^\eta)_{i,i} + v_i^{\eta o} \rho_{os}^\eta \left(\frac{1}{B_o^\eta} \right)_{,i} \\
 & + \rho_{os}^\eta v_i^{\eta g} \left(\frac{R_v^\eta}{B_g^\eta} \right)_{,i} - \Gamma_o = 0
 \end{aligned} \right\} + \quad (5.11)
 \end{aligned}$$

where $(\cdot)$ denotes derivative with respect to time.

5.4.3 Mass Conservation for Gas Component

Using equations 4.29, 4.30 and 5.7 the mass conservation equation for gas in a dual-porosity medium under non-isothermal conditions may be written as,

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left(\phi^\eta S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} \right) + \frac{\partial}{\partial x_i} \left(\frac{\rho_{gs}^\eta}{B_g^\eta} v_i^{\eta g} + \phi^\eta S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} u_i^\eta \right) + \\
 & \frac{\partial}{\partial t} \left(\phi^\eta S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) + \frac{\partial}{\partial x_i} \left(\frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} v_i^{\eta o} + \phi^\eta S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} u_i^\eta \right) - \Gamma_g = 0 \quad (5.12)
 \end{aligned}$$

Expanding derivatives, and incorporating equations 2.4, 2.8 and 2.12 into 5.12, we have,

$$\left\{ \begin{aligned} & \phi^\eta S_g^\eta \rho_{gs}^\eta \frac{D}{Dt} \left(\frac{1}{B_g^\eta} \right) + \left[\left(S_g^\eta \frac{\rho_{gs}^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \right] \frac{DP^\eta}{Dt} \\ & + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{DS_o^\eta}{Dt} + \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{DR_s^\eta}{Dt} + \\ & \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) ((1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta) \frac{DT^\eta}{Dt} \\ & + \phi^\eta S_o^\eta R_s^\eta \rho_{gs}^\eta \frac{D}{Dt} \left(\frac{1}{B_o^\eta} \right) + \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \alpha^\eta u_{i,i}^\eta \\ & + \rho_{gs}^\eta \left(\frac{v_i^{\eta g}}{B_g^\eta} \right)_{,i} + \rho_{gs}^\eta \left(\frac{R_s^\eta v_i^{\eta o}}{B_o^\eta} \right)_{,i} + \\ & \rho_{gs}^\eta v_i^{\eta g} \left(\frac{1}{B_g^{(2)}} \right)_{,i} + \rho_{gs}^\eta v_i^{\eta o} \left(\frac{R_s^\eta}{B_o^{(2)}} \right)_{,i} - \Gamma_g = 0 \end{aligned} \right. \quad (5.13)$$

From equations 5.10, 5.13, letting $\frac{D}{Dt} \approx \frac{\partial}{\partial t}$ and grouping terms we get the final form of the governing equation for gas flow in the matrix as

$$\left\{ \begin{aligned} & \dot{P}_o^\eta \left[\begin{aligned} & \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \\ & + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} - \frac{\phi^\eta S_o^\eta R_s^\eta \rho_{gs}^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_c^\eta} \end{aligned} \right] + \\ & \dot{P}_g^\eta \left[\begin{aligned} & \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \\ & - \frac{\phi^\eta S_g^\eta \rho_{gs}^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} + \\ & \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial P_g^\eta} \end{aligned} \right] + \\ & \dot{T}^\eta \left[\begin{aligned} & \left(S_o^\eta \frac{R_s^\eta}{B_o^\eta} + \frac{S_g^\eta}{B_g^\eta} \right) \rho_{gs}^\eta \left(\frac{(1 - \alpha^\eta)\beta^\eta - (1 - \phi^\eta)\beta_s^\eta}{K_s^\eta} P_c^\eta \frac{\partial S_o^\eta}{\partial T} \right) - \\ & \frac{\phi^\eta \rho_{gs}^\eta S_g^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial T^\eta} - \frac{\phi^\eta \rho_{gs}^\eta S_o^\eta R_s^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial T^\eta} \\ & + \phi^\eta \frac{\rho_{gs}^\eta S_o^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial T^\eta} + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial T} \end{aligned} \right] \\ & + \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \alpha^\eta u_{i,i}^\eta - \rho_{gs}^\eta \left(\frac{k^\eta k_{rg}^\eta}{B_g^\eta \mu_g^\eta} P_{g,i}^\eta \right)_{,i} \\ & - \rho_{gs}^\eta \left(\frac{R_s^\eta k^\eta k_{ro}^\eta}{B_o^\eta \mu_o^\eta} P_{o,i}^\eta \right)_{,i} + \rho_{gs}^\eta u_i^{\eta g} \left(\frac{1}{B_g^{(2)}} \right)_{,i} + \rho_{gs}^\eta u_i^{\eta o} \left(\frac{R_s^\eta}{B_o^{(2)}} \right)_{,i} \\ & - \Gamma_g = 0 \end{aligned} \right. \quad (5.14)$$

5.4.4 Equilibrium Equation

Following equations 3.18 and 4.25 the equilibrium equation for a dual-porosity medium saturated with oil and gas under non-isothermal conditions may be written as

$$\begin{aligned} & (D_{ijkl}^{I,II} \epsilon_{kl} - D_{ijkl}^{I,II} C_{klmn}^I \alpha^I (S_g^I P_g^I + S_o^I P_o^I) \delta_{mn} - \\ & D_{ijkl}^{I,II} C_{klmn}^{II} \alpha^{II} (S_g^{II} P_g^{II} + S_o^{II} P_o^{II}) \delta_{mn} \\ & - D_{ijkl}^{I,II} \frac{\beta}{3} T \delta_{kl})_{,j} + F_i = 0 \end{aligned} \quad (5.15)$$

The expressions for the volumetric strains of the individual media developed in Chapter 2 remain unchanged.

5.4.5 Energy Balance

The macroscopic energy balance equations for a dual-porosity medium saturated by oil and gas may be expressed as

$$\left\{ \begin{aligned} & [(\phi^I S_g^I \rho_g^I + \phi^{II} S_g^{II} \rho_g^{II}) C_{vg} + (\phi^I S_o^I \rho_o^I + \phi^{II} S_o^{II} \rho_o^{II}) C_{vo}] \frac{\partial T}{\partial t} + \\ & [(\rho_g^I v_g^I + \rho_g^{II} v_g^{II}) C_{vg} + (\rho_o^I v_o^I + \rho_o^{II} v_o^{II}) C_{vo}] \cdot \nabla T - \\ & \nabla^T [(\phi^I S_g^I + \phi^{II} S_g^{II}) \lambda_g + (\phi^I S_o^I + \phi^{II} S_o^{II}) \lambda_o] \cdot \nabla T = 0 \end{aligned} \right. \quad (5.16)$$

for the fluids (oil and gas) and equation 3.31 for the solid. In equations 5.16 C_{vg} and C_{vo} are the heat capacities of gas and oil respectively; λ_g and λ_o are the coefficients of thermal conductivity for gas and oil; ρ_g^η and ρ_o^η are as in equation 4.28 and; v_g^η , v_o^η , S_g^η and S_o^η as described earlier. The thermal energy conservation equation for the dual-porosity medium saturated with oil and gas is obtained by adding equations 3.31 and 5.16.

$$\left\{ \begin{aligned} & \left[\begin{aligned} & (\phi^I S_g^I \rho_g^I + \phi^{II} S_g^{II} \rho_g^{II}) C_{vg} + \\ & (\phi^I S_o^I \rho_o^I + \phi^{II} S_o^{II} \rho_o^{II}) C_{vo} + \\ & (1 - \phi^I - \phi^{II}) \rho_s C_{vs} \end{aligned} \right] \frac{\partial T}{\partial t} \\ & + \left(\begin{aligned} & (\rho_g^I v_g^I + \rho_g^{II} v_g^{II}) C_{vg} + \\ & (\rho_o^I v_o^I + \rho_o^{II} v_o^{II}) C_{vo} \end{aligned} \right) \cdot \nabla T - \\ & \nabla^T \left[\begin{aligned} & (\phi^I S_g^I + \phi^{II} S_g^{II}) \lambda_g + \\ & (\phi^I S_o^I + \phi^{II} S_o^{II}) \lambda_o + \\ & (1 - \phi^I - \phi^{II}) \lambda_s \end{aligned} \right] \cdot \nabla T + \\ & (1 - \phi^I - \phi^{II}) 3K\beta T \frac{\partial \epsilon_{kk}}{\partial t} = 0 \end{aligned} \right. \quad (5.17)$$

It must be noted that the solid velocity term has been neglected in the final form of the energy conservation equation (equation 5.17).

5.5 Finite Element Equations

As in chapters 3 and 4, the finite element equations are developed with displacements, oil and gas pressures, and temperature, i.e., $\mathbf{u}$, P_o^I , P_o^{II} , P_g^I , P_g^{II} and T as the primary variables. These are expressed in terms of approximated nodal quantities as

$$P_o^{\eta*} \simeq \mathbf{N}\mathbf{P}_o^\eta; P_g^{\eta*} \simeq \mathbf{N}\mathbf{P}_g^\eta; \mathbf{u}^* \simeq \mathbf{N}\bar{\mathbf{u}}; T \simeq \mathbf{N}\mathbf{T}; \quad (5.18)$$

where the superscript (*) denotes the finite element approximation. The strains in both media may be expressed as in equation 2.50. Substitution of the approximate quantities (equation 5.18) into equations 5.11 and 5.14, using equation 2.50 and application of the Galerkin's principle and Green's theorem gives the finite element equations for oil phase in a dual-porosity medium under non-isothermal conditions as

$$\begin{aligned}
& \int_V \mathbf{N}^T \left[\left(S_g^{\eta} \frac{R_v^{\eta}}{B_g^{\eta}} + \frac{S_o^{\eta}}{B_o^{\eta}} \right) \rho_{os}^{\eta} \frac{(\alpha^{\eta} - \phi^{\eta})}{K_s^{\eta}} \left(S_g^{\eta} + P_c^{\eta} \frac{\partial S_o^{\eta}}{\partial P_c^{\eta}} \right) - \right. \\
& \quad \phi^{\eta} \rho_{os}^{\eta} \left(\frac{1}{B_o^{\eta}} - \frac{R_v^{\eta}}{B_g^{\eta}} \right) \frac{\partial S_o^{\eta}}{\partial P_c^{\eta}} - \frac{\phi^{\eta} \rho_{os}^{\eta} S_o^{\eta} \partial B_o^{\eta}}{(B_o^{\eta})^2 \partial P_o^{\eta}} + \\
& \quad \left. \phi^{\eta} \frac{\rho_{os}^{\eta} S_o^{\eta} \partial R_v^{\eta}}{B_g^{\eta} \partial P_o^{\eta}} \right] \mathbf{N} dV \dot{\mathbf{P}}_o^{\eta} + \\
& \int_V \mathbf{N}^T \left[\left(S_g^{\eta} \frac{R_v^{\eta}}{B_g^{\eta}} + \frac{S_o^{\eta}}{B_o^{\eta}} \right) \rho_{os}^{\eta} \frac{(\alpha^{\eta} - \phi^{\eta})}{K_s^{\eta}} \left(S_g^{\eta} - P_c^{\eta} \frac{\partial S_g^{\eta}}{\partial P_c^{\eta}} \right) - \right. \\
& \quad \left. \frac{\phi^{\eta} \rho_{os}^{\eta} S_o^{\eta} R_v^{\eta} \partial B_g^{\eta}}{(B_g^{\eta})^2 \partial P_g^{\eta}} + \phi^{\eta} \rho_{os}^{\eta} \left(\frac{1}{B_o^{\eta}} - \frac{R_v^{\eta}}{B_g^{\eta}} \right) \frac{\partial S_o^{\eta}}{\partial P_c^{\eta}} \right] \mathbf{N} dV \dot{\mathbf{P}}_g^{\eta} + \\
& \int_V \mathbf{N}^T \left[\left(S_g^{\eta} \frac{R_v^{\eta}}{B_g^{\eta}} + \frac{S_o^{\eta}}{B_o^{\eta}} \right) \rho_{os}^{\eta} \left(\frac{(1 - \alpha^{\eta}) \beta^{\eta} - (1 - \phi^{\eta}) \beta_s^{\eta}}{K_s^{\eta} \frac{\partial S_g^{\eta}}{\partial T}} \right) - \right. \\
& \quad \left. - \frac{\phi^{\eta} \rho_{os}^{\eta} S_o^{\eta} \partial B_o^{\eta}}{(B_o^{\eta})^2 \partial T} - \frac{\phi^{\eta} \rho_{os}^{\eta} S_g^{\eta} R_v^{\eta} \partial B_g^{\eta}}{(B_g^{\eta})^2 \partial T} + \right. \\
& \quad \left. \phi^{\eta} \frac{\rho_{os}^{\eta} S_g^{\eta} \partial R_v^{\eta}}{B_g^{\eta} \partial T} + \phi^{\eta} \rho_{os}^{\eta} \left(\frac{1}{B_o^{\eta}} - \frac{R_v^{\eta}}{B_g^{\eta}} \right) \frac{\partial S_o^{\eta}}{\partial T} \right] \mathbf{N} dV \dot{\mathbf{T}} + \\
& \int_V \mathbf{N}^T \alpha^{\eta} \left(S_g^{\eta} \frac{R_v^{\eta}}{B_g^{\eta}} + \frac{S_o^{\eta}}{B_o^{\eta}} \right) \rho_{os}^{\eta} \mathbf{m}^T \mathbf{C}^{\eta} \mathbf{D}^{I,I'} \mathbf{B} dV \dot{\mathbf{U}} - \\
& \int_V (\nabla \mathbf{N})^T \frac{\rho_{os}^{\eta} k^{\eta} k_{\tau o}^{\eta}}{B_o^{\eta} \mu_o^{\eta}} \nabla \mathbf{N} dV \mathbf{P}_o^{\eta} - \\
& \int_V (\nabla \mathbf{N})^T \frac{\rho_{os}^{\eta} R_v^{\eta} k^{\eta} k_{\tau g}^{\eta}}{B_g^{\eta} \mu_g^{\eta}} \nabla \mathbf{N} dV \mathbf{P}_g^{\eta} + \\
& + (-1)^{\eta} \int \mathbf{N}^T \rho_{os}^I \frac{k^I k_{\tau o}^I}{B_o^I \mu_o^I} \Psi \mathbf{N} dV (\mathbf{P}_o^I - \mathbf{P}_o^{\eta}) \\
& + (-1)^{\eta} \int \mathbf{N}^T \frac{R_v^I \rho_{gs}^I k^I k_{\tau g}^I}{B_g^I \mu_g^I} \Psi \mathbf{N} dV (\mathbf{P}_g^I - \mathbf{P}_g^{\eta}) + \mathbf{Q}_o^{\eta} = 0
\end{aligned} \tag{5.19}$$

and for gas phase as

$$\begin{aligned}
& \int_V \mathbf{N}^T \left[\rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) \right. \\
& + \phi^\eta \rho_{gs}^\eta \left(\frac{1}{B_g^\eta} - \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} - \frac{\phi^\eta S_o^\eta R_s^\eta \rho_{gs}^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial P_o^\eta} \left. \right] \mathbf{N} dV \dot{\mathbf{P}}_o^\eta + \\
& \int_V \mathbf{N}^T \left[\rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \left(\frac{(\alpha^\eta - \phi^\eta)}{K_s^\eta} \right) \left(S_g^\eta - P_c^\eta \frac{\partial S_g^\eta}{\partial P_c^\eta} \right) \right. \\
& - \frac{\phi^\eta S_o^\eta \rho_{gs}^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial P_g^\eta} + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial P_c^\eta} + \\
& \left. \phi^\eta S_o^\eta \frac{\rho_{gs}^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial P_g^\eta} \right] \mathbf{N} dV \dot{\mathbf{P}}_g^\eta + \\
& \int_V \mathbf{N}^T \left[\left(S_o^\eta \frac{R_s^\eta}{B_o^\eta} + \frac{S_g^\eta}{B_g^\eta} \right) \rho_{gs}^\eta \left(\frac{(1 - \alpha^\eta) \beta^\eta - (1 - \phi^\eta) \beta_s^\eta}{K_s^\eta} P_c^\eta \frac{\partial S_o^\eta}{\partial T} \right) \right. \\
& - \frac{\phi^\eta \rho_{gs}^\eta S_g^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial T^\eta} - \frac{\phi^\eta \rho_{gs}^\eta S_o^\eta R_s^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial T^\eta} + \\
& \left. \phi^\eta \frac{\rho_{gs}^\eta S_o^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial T} + \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial T} \right] \mathbf{N} dV \dot{\mathbf{T}} + \\
& \int_V \mathbf{N}^T \rho_{gs}^\eta \left(\frac{S_g^\eta}{B_g^\eta} + S_o^\eta \frac{R_s^\eta}{B_o^\eta} \right) \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \dot{\mathbf{u}} - \\
& \int_V (\nabla \mathbf{N})^T \rho_{gs}^\eta \frac{R_s^\eta k^\eta k_{ro}^\eta}{B_o^\eta \mu_o^\eta} \nabla \mathbf{N} dV \mathbf{P}_o^\eta - \\
& \int_V (\nabla \mathbf{N})^T \rho_{gs}^\eta \frac{k^\eta k_{rg}^\eta}{B_g^\eta \mu_g^\eta} \nabla \mathbf{N} dV \mathbf{P}_g^\eta + \\
& + (-1)^\eta \int \mathbf{N}^T \frac{\rho_{gs}^I}{B_g^I} \frac{k^I k_{rg}^I}{\mu_g^I} \Psi \mathbf{N} dV (\mathbf{P}_g^I - \mathbf{P}_g^{II}) \\
& + (-1)^\eta \int \mathbf{N}^T \frac{R_s^I \rho_{gs}^I}{B_o^I} \frac{k^I k_{ro}^I}{\mu_o^I} \Psi \mathbf{N} dV (\mathbf{P}_o^I - \mathbf{P}_o^{II}) + \mathbf{Q}_g^\eta = 0
\end{aligned} \tag{5.20}$$

The finite element equilibrium equation can be expressed using equations 3.35 and the third and fourth expressions from equation 4.39 as

$$\begin{aligned}
& \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{B} dV \dot{\mathbf{u}} - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \\
& \left(S_o^I + P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \mathbf{mN} dV \dot{\mathbf{P}}_o^I \\
& - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^I \alpha^I \left(S_g^I - P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \mathbf{mN} dV \dot{\mathbf{P}}_g^I \\
& - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \left(S_o^{II} + P_c^{II} \frac{\partial S_o^{II}}{\partial P_c^{II}} \right) \mathbf{mN} dV \dot{\mathbf{P}}_o^{II} \\
& - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^{II} \alpha^{II} \left(S_g^{II} - P_c^{II} \frac{\partial S_o^{II}}{\partial P_c^{II}} \right) \mathbf{mN} dV \dot{\mathbf{P}}_g^{II} \\
& - \int_v \mathbf{B}^T \mathbf{D}^{I,II} \frac{\beta}{3} \mathbf{mN} dV \dot{\mathbf{T}} = \dot{\mathbf{f}}
\end{aligned} \tag{5.21}$$

Similarly, the finite element equation for the energy balance can be obtained from 5.17 as

$$\begin{aligned}
& \int_v \mathbf{N}^T \left[\begin{array}{l} (\phi^I S_g^I \rho_g^I + \phi^{II} S_g^{II} \rho_g^{II}) C_{vg} + \\ (\phi^I S_o^I \rho_o^I + \phi^{II} S_o^{II} \rho_o^{II}) C_{vo} + \\ (1 - \phi^I - \phi^{II}) \rho_s C_{vs} \end{array} \right] \mathbf{N} dV \dot{\mathbf{T}} + \\
& \int_v (\nabla \mathbf{N})^T \left(\begin{array}{l} (\rho_g^I v_g^I + \rho_g^{II} v_g^{II}) C_{vg} + \\ (\rho_o^I v_o^I + \rho_o^{II} v_o^{II}) C_{vo} \end{array} \right) \mathbf{N} dV \mathbf{T} + \\
& \int_v (\nabla \mathbf{N})^T \left[\begin{array}{l} (\phi^I S_g^I + \phi^{II} S_g^{II}) \lambda_g + \\ (\phi^I S_o^I + \phi^{II} S_o^{II}) \lambda_o + \\ (1 - \phi^I - \phi^{II}) \lambda_s \end{array} \right] \mathbf{N} dV \mathbf{T} + \\
& \int_v \mathbf{N}^T (1 - \phi^I - \phi^{II}) 3K\beta T \mathbf{m}^T \mathbf{B} dV \dot{\mathbf{u}} + \mathbf{Q}_t = 0
\end{aligned} \tag{5.22}$$

Equations 5.19-5.22 are the finite element equations for fully-coupled oil-gas flow in a dual-porosity medium under non-isothermal conditions. They can be expressed in a more convenient form as

Equilibrium:

$$\mathbf{K} \dot{\mathbf{u}} + \mathbf{O}_c^I \dot{\mathbf{P}}_o^I + \mathbf{O}_c^{II} \dot{\mathbf{P}}_o^{II} + \mathbf{G}_c^I \dot{\mathbf{P}}_g^I + \mathbf{G}_c^{II} \dot{\mathbf{P}}_g^{II} + \mathbf{L}_t \dot{\mathbf{T}} - \dot{\mathbf{f}} = 0 \tag{5.23}$$

Oil in the primary medium:

$$\begin{aligned}
& \bar{\mathbf{K}}_o^I \dot{\mathbf{u}} + (\mathbf{O}_D^I + \mathbf{O}_T) \mathbf{P}_o^I + (\bar{\mathbf{O}}_{eD}^I + \bar{\mathbf{O}}_{eT}) \mathbf{P}_g^I - \mathbf{O}_T \mathbf{P}_o^{II} \\
& - \bar{\mathbf{O}}_{eT} \mathbf{P}_g^{II} + \bar{\mathbf{O}}_o^I \dot{\mathbf{P}}_o^I + \bar{\mathbf{O}}_g^I \dot{\mathbf{P}}_g^I + \bar{\mathbf{S}}_{ot}^I \dot{\mathbf{T}} = -\bar{\mathbf{Q}}_o^I
\end{aligned} \tag{5.24}$$

Gas in the primary medium:

$$\begin{aligned} & K_g^I \dot{u} + (G_{dD}^I + G_{dT}^I) P_o^I + (G_D^I + G_T^I) P_g^I - G_{dT} P_o^{II} \\ & - G_T P_g^{II} + G_o^I \dot{P}_o^I + G_g^I \dot{P}_g^I + \bar{S}_{gt}^I \dot{T} = -\bar{Q}_g^I \end{aligned} \quad (5.25)$$

Oil in the secondary medium:

$$\begin{aligned} & \bar{K}_o^{II} \dot{u} + (O_D^{II} + O_T) P_o^{II} + (\bar{O}_{eD}^{II} + \bar{O}_{eT}) P_g^{II} - O_T P_o^I \\ & - \bar{O}_{eT} P_g^I + \bar{O}_o^{II} \dot{P}_o^{II} + \bar{O}_g^{II} \dot{P}_g^{II} + \bar{S}_{ot}^{II} \dot{T} = -\bar{Q}_o^{II} \end{aligned} \quad (5.26)$$

Gas in the secondary medium:

$$\begin{aligned} & K_g^{II} \dot{u} + (G_{dD}^{II} + G_{dT}^{II}) P_o^{II} + (G_D^{II} + G_T^{II}) P_g^{II} - G_{dT} P_o^I \\ & - G_T P_g^I + G_o^{II} \dot{P}_o^{II} + G_g^{II} \dot{P}_g^{II} + \bar{S}_{gt}^{II} \dot{T} = -Q_g^{II} \end{aligned} \quad (5.27)$$

Energy Balance:

$$\bar{T}_s \dot{T} + (\bar{C}_t + \bar{H}_{tF}) T + K_t \dot{u} = -\bar{Q}_t \quad (5.28)$$

Equations 5.23-5.28 can be expressed in a compact form as

$$\begin{aligned} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & O_D^I + O_T & \bar{O}_{eD}^I + \bar{O}_{eT} & -O_T & -\bar{O}_{eT} & 0 \\ 0 & G_{dD}^I + G_{dT}^I & G_D^I + G_T^I & -G_{dT} & -G_T & 0 \\ 0 & -O_T & -\bar{O}_{eT} & O_D^{II} + O_T & \bar{O}_{eD}^{II} + \bar{O}_{eT} & 0 \\ 0 & -G_{dT} & -G_T & G_{dD}^{II} + G_{dT}^{II} & G_D^{II} + G_T^{II} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{C}_t + \bar{H}_{tF} \end{bmatrix} \\ & \begin{bmatrix} u \\ P_o^I \\ P_g^I \\ P_o^{II} \\ P_g^{II} \\ T \end{bmatrix} + \quad (5.29) \\ & \begin{bmatrix} K & O_c^I & G_c^I & O_c^{II} & G_c^{II} & L_t \\ \bar{K}_o^I & \bar{O}_o^I & \bar{O}_g^I & 0 & 0 & \bar{S}_{ot}^I \\ K_g^I & G_o^I & G_g^I & 0 & 0 & \bar{S}_{gt}^I \\ \bar{K}_o^{II} & 0 & 0 & \bar{O}_o^{II} & \bar{O}_g^{II} & \bar{S}_{ot}^{II} \\ K_g^{II} & 0 & 0 & G_o^{II} & G_g^{II} & \bar{S}_{gt}^{II} \\ K_t & 0 & 0 & 0 & 0 & \bar{T}_s \end{bmatrix} \frac{d}{dt} \begin{bmatrix} u \\ P_o^I \\ P_g^I \\ P_o^{II} \\ P_g^{II} \\ T \end{bmatrix} = \begin{bmatrix} \frac{df}{dt} \\ -\bar{Q}_o^I \\ -\bar{Q}_g^I \\ -\bar{Q}_o^{II} \\ -\bar{Q}_g^{II} \\ -\bar{Q}_t \end{bmatrix} \end{aligned}$$

Parameter	Units	Value
Elastic Modulus (E)	GPa	3.55
Poisson's Ratio (ν)	-	0.2
Porosity (ϕ)	-	0.01
Permeability (k)	m ²	10^{-18}
Grain Bulk Modulus (K_s)	GPa	10^7
Thermal Expansion Coefficient(β)	/°C	1.8×10^{-5}
Solid heat cap. C_{vs}	J/kg/°C	582
Oil heat cap. C_{vo}	J/kg/°C	2093
Gas heat cap. C_{vg}	J/kg/°C	1000
Thermal Conductivity of solid (λ_s)	W/m/°C	2.65
Thermal Conductivity of oil (λ_o)	W/m/°C	1.3
Thermal Conductivity of gas (λ_g)	W/m/°C	0.3
Mudweight	MPa	12
Temperature of drilling fluid	°C	50

Table 5.1: Parameters for the single-porosity, two-phase model (inclined wellbore problem)

Equations 5.23-5.28 represent a set of partial differential equations governing fully-coupled oil-gas flow and heat transfer in a dual-porosity medium. Since these equations contain terms dependent on the primary variables, they represent a set of non-linear equations. The discretization in the time-domain and the solution scheme are carried out as in Chapter 4 and will not be reiterated herein.

5.6 Application to Inclined Wellbore Geometry

A parametric study of an inclined wellbore is carried out in order to study the effect of phase saturations and dual-porosity parameters for the following cases: (a) single-porosity, two-phase and (b) dual-porosity, two-phase.

5.6.1 Single-Porosity, Two-Phase Modeling

Figures 5.1 and 5.2 show the spatial variations of the oil pressure and effective radial stress for different initial oil saturations. The input parameters are listed in Table 5.1.

The relations between (i) relative permeability and saturation (ii) capillary pressure

and saturation (iii) formation volume factors and pressure, temperature and (iv) oil, gas viscosities and temperature pressure are based on data from literature (Lewis and Ghafouri, 1997; Zeigler, 1987; Pao et al., 2001). These may be expressed as follows

$$\begin{aligned}
k_{ro} &= -1.818S_g + 1.0 \quad ; \quad k_{rg} = 0.6S_g \\
B_g &= 0.1(T + 273)/(273P_g) \\
\mu_o &= 0.25T^{-3.46} \text{ MPa-s} \quad ; T > 23.89^\circ\text{C} \\
&= 4.2 \times 10^{-6} \text{ MPa-s} \quad ; T \leq 23.89^\circ\text{C} \\
\mu_g &= 1.5 \times 10^{-12} \text{ MPa-s} \\
S_g &= 1 - 0.028/(P_c^{0.456}) \quad ; 0.0005 \leq P_c \leq 0.0025 \\
&= 0.575 \quad ; P_c > 0.0025 \\
&= 0.0 \quad ; P_c < 0.0005
\end{aligned} \tag{5.30}$$

As in the isothermal case seen in Chapter 4 (Figures 4.2 and 4.2), the peak in the oil pressure curve (Figure 5.1) in the vicinity of the borehole decrease for a lower value of initial oil saturation ($S_{oi} = 0.5$). Correspondingly, the effective radial and tangential stresses (shown in Figures 5.2 and 5.3) become more compressive for a lower value of initial oil saturation. Due to the low porosity, low heat capacity of gas, and lower thermal conductivities of the oil and gas as compared to the solid constituent, the initial oil saturation does not affect the temperature distribution as seen in Figure 5.4. This may also be attributed to the fact that the thermal diffusivity of the porous medium as a whole ($\sim 1.6 \times 10^{-6} \text{ m}^2/\text{s}$) is higher than the fluid diffusivity ($\sim 1.7 \times 10^{-7} \text{ m}^2/\text{s}$). Hence as shown in Chapter 4, heat transport by convection is negligible.

5.6.2 Dual-Porosity, Two-Phase Modeling

As in Chapters 3 and 4, the following fracture characteristics were associated with the secondary medium as in Chapter 3 - stiffness K_n and fracture spacing s (governing the compliance of the secondary medium); secondary porosity ϕ'' (same as in equation 2.2, representing the storativity of the fractures); and, secondary permeability k'' . The formation was assumed to be a dual-porosity medium saturated with oil and gas. The

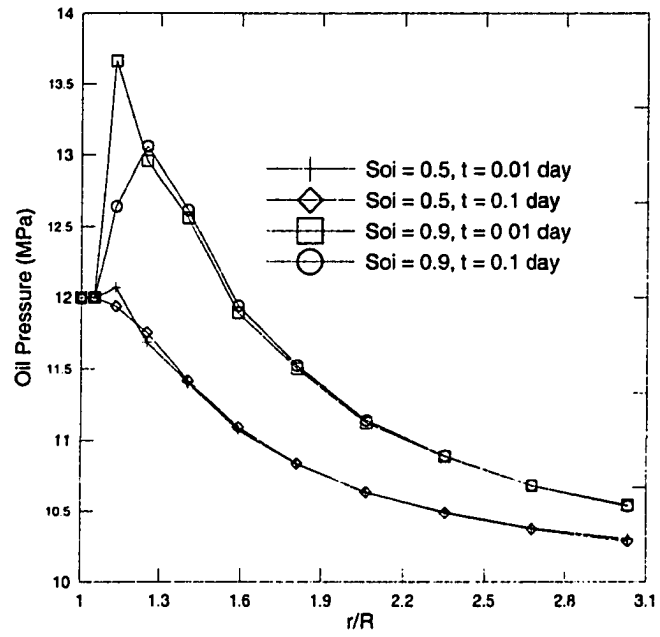


Figure 5.1: Oil pressure distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

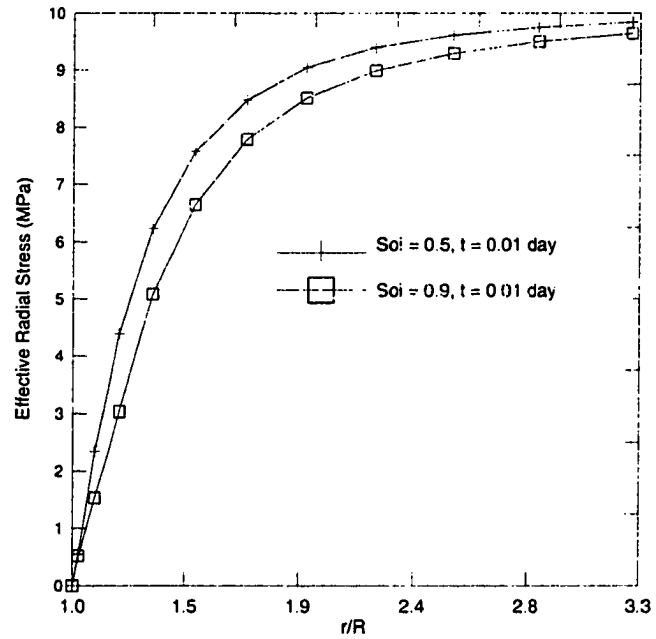


Figure 5.2: Effective radial stress distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

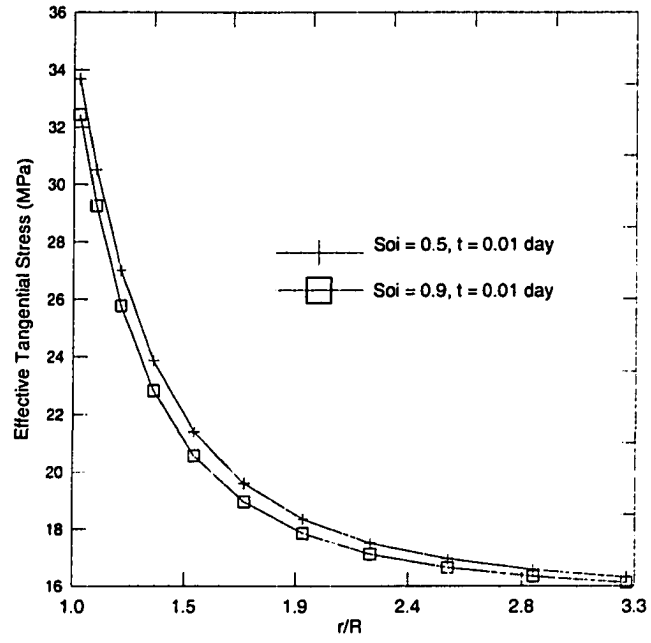


Figure 5.3: Effective tangential stress distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

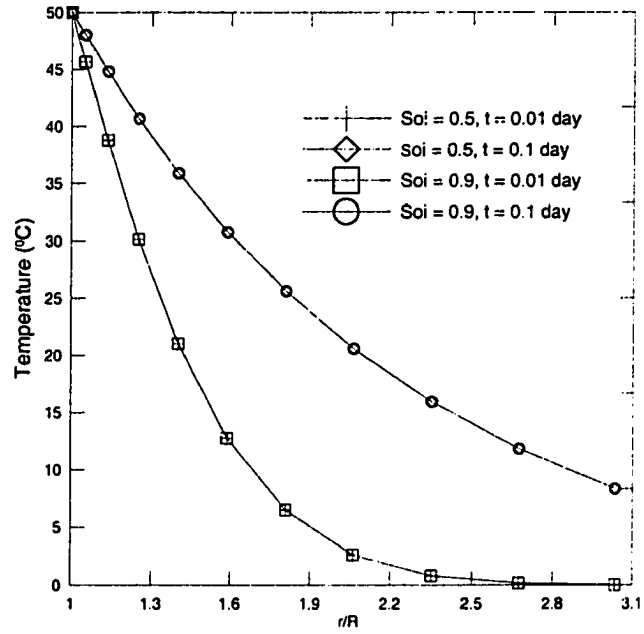


Figure 5.4: Temperature distribution for different initial oil saturations (single-porosity, two-phase; S_{oi} — initial oil saturation)

Parameter	Units	Value
Fracture (Secondary) Permeability (k'')	m^2	10^{-17}
Fracture Stiffness (K_n)	GPa/m	20.0
Fracture Spacing (s)	m	.1, .25, .5
Fracture (Secondary) Porosity (ϕ'')	-	0.002

Table 5.2: Parameters for dual-porosity, single-phase model (inclined wellbore problem)

material properties of the primary medium are as in Table 5.1 whereas the secondary medium parameters are listed in Table 5.2.

A smaller fracture spacing, increases the overall compliance of the dual-porosity medium thereby resulting in a higher initial oil pressure in the primary medium. This may be observed in Figure 5.5, wherein the peak of the oil pressure curve in the primary medium (in the vicinity of the borehole) is highest for a fracture spacing of 0.1m . This results in tensile effective radial stresses around the borehole, the magnitude of which reduces with an increase in fracture spacing as seen in Figure 5.6. Also, the tangential stresses for a smaller fracture spacing are less compressive as seen in Figure 5.7. The temperature distribution, on the other hand, is likely to be unaffected by the secondary medium parameters. As seen in equation 5.22, the porosity of the secondary medium affects the time rate of change in temperature and the heat transport by conduction. On the other hand, the fluid diffusivity of the secondary medium affects the heat transport by convection. Due to the low fluid diffusivity and storage capacity of the secondary medium (see Table 5.2), the secondary medium parameters do not affect the temperature distribution as seen in Figure 5.8.

5.7 Summary

A dual-porosity model incorporating fully-coupled non-isothermal flow of oil and gas, has been developed based on the “separate” and “overlapping” technique. The model accounts for the solubility of gas in the oil as well as evaporation of oil into the gas phase.

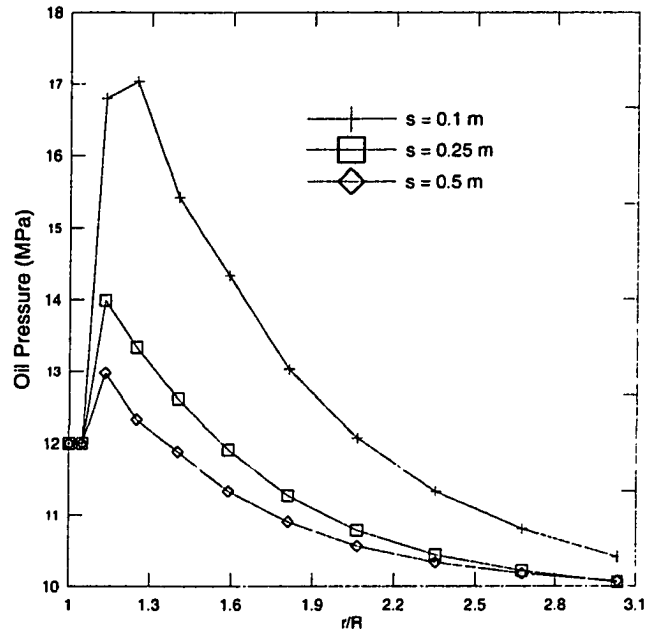


Figure 5.5: Primary medium (Matrix) oil pressure distribution for different fracture spacings and initial oil saturation, $S_{oi} = 0.5$ (dual-porosity, two-phase; s - fracture spacing, $t = 0.01$ day)

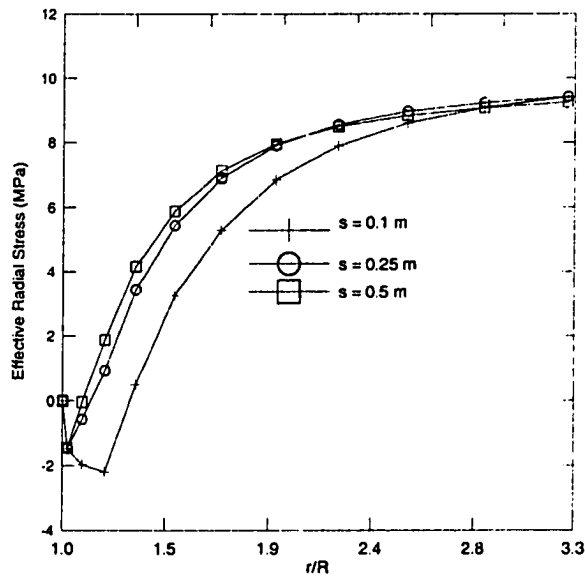


Figure 5.6: Effective radial stress distribution for different fracture spacings and initial oil saturation, $S_{oi} = 0.5$ (dual-porosity, two-phase; s - fracture spacing, $t = 0.01$ day)

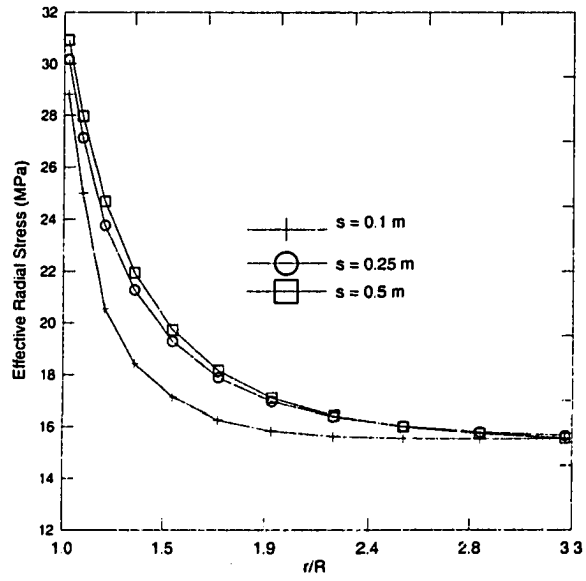


Figure 5.7: Effective tangential stress distribution for different fracture spacings and initial oil saturation, $S_{oi} = 0.5$ (dual-porosity, two-phase; s - fracture spacing, $t = 0.01$ day)

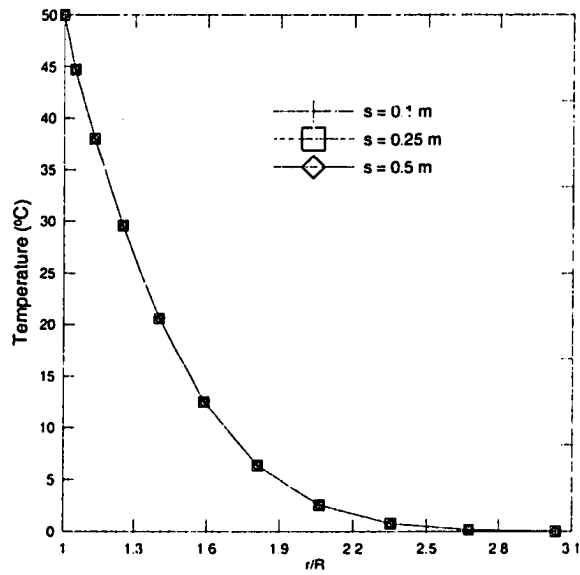


Figure 5.8: Temperature distribution for different fracture spacings and initial oil saturation, $S_{oi} = 0.5$ (dual-porosity, two-phase; s - fracture spacing, $t = 0.01$ day)

As in Chapter 4, the pore pressure and effective stress distributions in the vicinity of the borehole at short times are affected by the initial oil saturation. In the case of fractured formations, the compliance of the secondary medium, governed by the fracture density affects the pore pressure and effective stress distributions. For a medium with low fluid diffusivities, the initial oil saturation and secondary medium parameters do not affect the temperature distributions.

The models presented in Chapters 2-5, traverse the complexity associated with simulating a naturally fractured porous medium saturated with single or multiple fluids, under isothermal or non-isothermal conditions. These models will be applied to the problem of subsidence of aquifers in the following chapter.

Chapter 6

Model Application

Land deformation due to withdrawal of groundwater (or other fluid(s)) from porous media such as soils and sedimentary rocks is a well-known hydrogeomechanical phenomenon (Najjar and Zaman, 1993). When a subsurface water system is pumped, water levels in the vicinity of the well decline, creating a cone of depression around the well. Thus the pore water pressure in the region of withdrawal decreases as the extraction of groundwater takes place. This leads to an increase in the effective stress acting on the solid skeleton and subsequent compaction of the aquifer. Thus, groundwater extraction acts as a hydraulic stress, intrinsically inducing a fully-coupled hydro-mechanical interaction between fluid flow and the deformation of the solid skeleton (Kim and Parizek, 1999). This fully-coupled flow-deformation phenomenon may be better explained by Biot's theory of poroelasticity, in which a simultaneous solution is sought for the change in pore pressure and both, vertical and horizontal displacements.

In the case of fissured subsurface systems, the compliance and permeability of the fractures and, inter-porosity flow is likely to affect the rate of groundwater discharge and subsidence. The models developed in Chapters 2 and 4 facilitate a mathematically rigorous and realistic treatment of fully-coupled groundwater flow and solid deformation due to groundwater withdrawal from a fully or partially saturated subsurface system that may be either intact or fissured. In this chapter, the process of three-dimensional

consolidation due to groundwater withdrawal from an aquifer system under isothermal conditions is analyzed. The responses of both, a fully saturated as well as unsaturated aquifer system are considered. In addition, the dual-porosity model has been applied to study the effect of fracture compliance and permeability on surface subsidence in both cases.

6.1 Aquifer Drawdown

As per the definition adopted by the Water Quality Control Commission on February 9, 1999 (<http://www.cdphe.state.co.us/ap/wqdef14.pdf>), the term “aquifer” refers to a formation, group of formations, or part of a formation containing sufficient saturated permeable material that could yield a sufficient quantity of water that may be extracted and applied to a beneficial use. In the present section, the following cases of an aquifer system, subjected to groundwater extraction, are simulated using the models developed in Chapters 2 and 4:

- a) Single-porosity, fully saturated
- b) Dual-porosity, fully saturated
- c) Single-porosity, unsaturated
- d) Dual-porosity, unsaturated

6.1.1 Unsaturated Flow

Unsaturated flow refers to the flow of water through a porous medium at saturation less than 100%, with stagnant air filling that portion of the void space not occupied by water. Due to its importance in soil sciences and analyses of infiltration, drainage of agricultural lands, irrigation, slope stability etc., unsaturated flow has been treated as a separate subject by soil physicists and hydrologists. However, unsaturated (air-water)

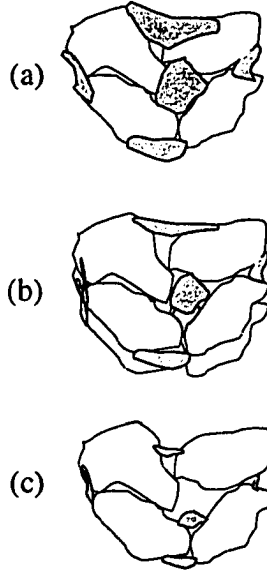


Figure 6.1: Possible water saturation states [(a) - funicular; (c) -pendular]

flow may be treated as a special case of simultaneous flow of two fluids (water and air) with air assumed to be stagnant. Thus an unsaturated system may be considered to be an air-water system, with water as the wetting phase and air as the non-wetting phase, along with the assumption that only water is in motion. In a drainage process, as water saturation is reduced, air enters the pores and forms air-water interfaces. Figure 6.1 describe the water distribution in the pore space in the funicular to the pendular stages developed during drainage.

The discussion on interfacial tension and capillary pressure in Section 4.2.2 is applicable to unsaturated flow with oil and gas replaced by water and air, respectively. Assuming air pressure to be a datum (i.e., $P_{nw} = 0$), equation 4.14 is modified as (Bear, 1972)

$$P_c = -P_w \quad (6.1)$$

where P_w is the water pressure adjacent to the interface. When equation 6.1 is averaged

over many such microscopic interfaces present in a representative elementary volume (REV) of the porous medium, P_w denotes the water pressure in a macroscopic (i.e., averaged over the REV) sense. The capillary pressure, P_c , in equation 6.1, is also referred to as suction or tension and is assumed to be a function of the water saturation alone. The capillary pressure head, h_c , is defined as

$$h_c = \frac{P_c}{\gamma_w} \quad (6.2)$$

where γ_w is the specific weight of water. The curves representing the relation between the capillary pressure (or the capillary pressure head, h_c) are called retention curves and describe how water is retained by capillary forces against gravity. The retention curves exhibit the phenomena of hysteresis due to change in contact angles during drainage and imbibition cycles and, compaction or consolidation (Bear, 1972).

The discussion on relative permeability in Section 4.2.3 is also applicable to unsaturated flow. As the water saturation reduces (during drainage), the larger pores drain first and subsequent flow takes place through the smaller pores. This reduction in the cross-sectional availability available for flow along with an increase in tortuosity of flow paths causes a rapid reduction in the effective permeability as the water saturation reduces. In unsaturated flow, only the relative permeability to water (wetting fluid), k_{rw} is relevant and is assumed to be a function of the capillary pressure head alone. The relative permeability to water also exhibits the phenomenon of hysteresis.

6.1.2 Problem Description (Kim and Parizek, 1999)

A system of vertical drainage wells fully penetrate an unconfined aquifer beneath a landfill site. The water table is initially located at a height of 21.43m above the bottom of the aquifer. Drainage wells are placed at regular intervals of 420.0m in the x direction and 280.0m in the y direction as shown in Figure 6.2, in order to lower the water table elevation below the base of the landfill. The water level in the wells is suddenly lowered

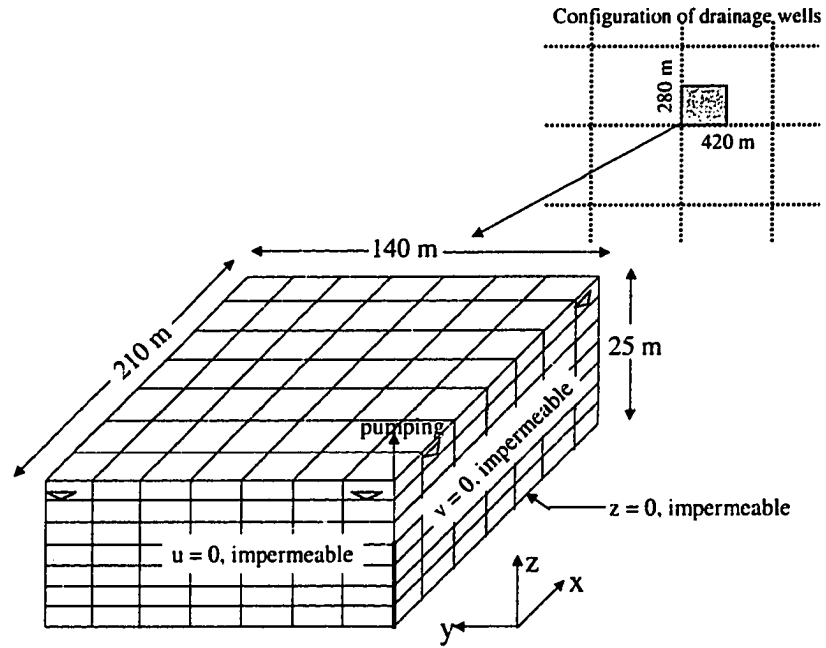


Figure 6.2: Problem geometry and finite element mesh

to 10m and maintained thereafter by groundwater pumping. The aquifer system is discretized into 343 ($7 \times 7 \times 7$) elements with 512 ($8 \times 8 \times 8$) nodes as shown in Figure 6.2.

The material properties of the aquifer are representative of unconsolidated sandy and silty (loam) materials and are as in Table 6.1 (Kim and Parizek, 1999)

Initial and Boundary Conditions

The initial hydraulic head is equal to 21.43m everywhere, prior to the start of pumping. The well is located at $x = y = 0.0m$ and a constant hydraulic head boundary condition of 10m is applied for $0.0m \leq z \leq 10.0m$. The top surface is permeable and is free to move horizontally and vertically, whereas the bottom surface is impermeable and fixed in the vertical direction. On the vertical planes at $x = 0.0m$ and $x = 210.0m$,

Table 6.1: Material properties of the aquifer system

Parameter	Units	Value
Elastic Modulus (E)	MPa	14.7
Poisson's Ratio (ν)	-	0.25
Porosity (ϕ)	-	0.25
Permeability (k)	m ²	3.72×10^{-12}
Grain Bulk Modulus (K_s)	MPa	10^7
Fluid Bulk Modulus (K_f)	MPa	2000
Fluid Viscosity (μ)	MPa.s	10^{-9}

the displacement in the y direction is constrained. On the vertical planes at $y = 0.0m$ and $y = 140.0m$, the displacement in the x direction is constrained. All four vertical boundaries are impermeable.

6.1.3 Numerical Results

The aquifer system has been discretized using three-dimensional 8-noded brick elements for which the strain displacement matrix may be expressed as

$$\mathbf{B} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \mathbf{N} \quad (6.3)$$

where $\mathbf{N}$ represent the elemental shape functions.

Single-Porosity, Fully Saturated Case

The spatial distributions of the vertical displacement at the top (i.e., $z = 25.0m$) and the hydraulic head are shown in Figures 6.3 and 6.4 respectively. For this case, the

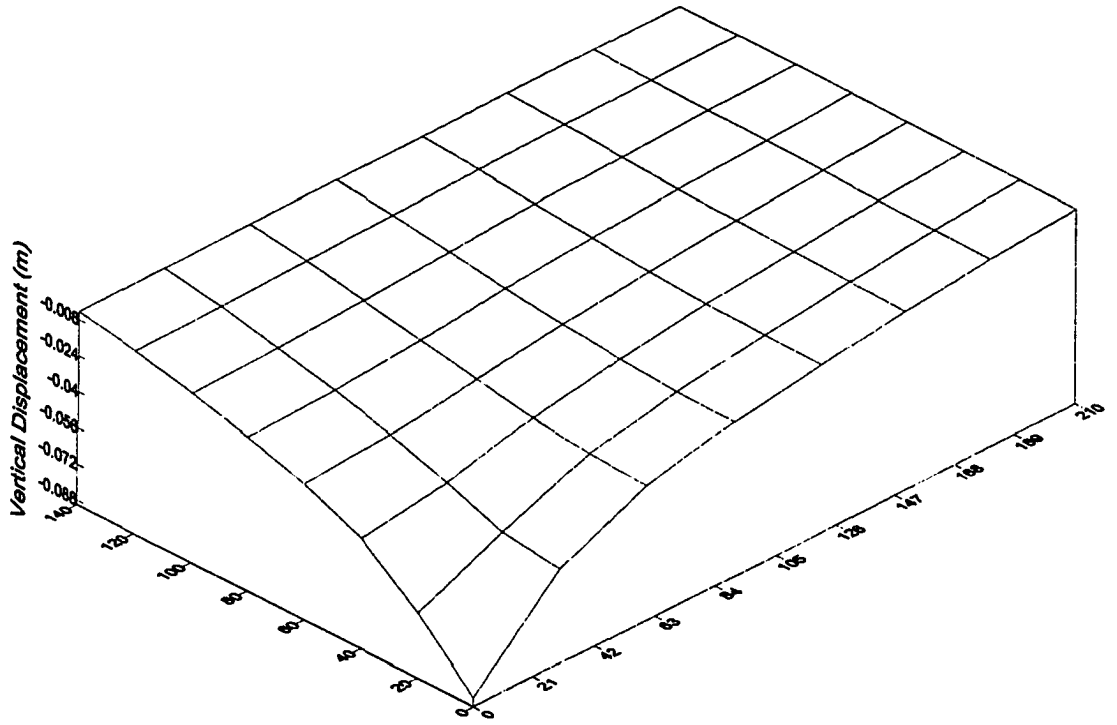


Figure 6.3: Spatial distribution of the vertical displacement at the surface after $t = 70$ days (single-porosity, fully-saturated)

material properties are as in Table 6.1. As seen in Figure 6.3, the vertical displacement is maximum directly above the discharging well. The hydraulic head distribution reflects the zone of depression around the well when it is discharging. At the well ($x = 0.0m$, $y = 0.0m$), the hydraulic head is equal to $10.0m$ which is the applied boundary condition. The vertical displacement reduces away from the well and correspondingly, the hydraulic head approaches the initial value of $21.43m$.

Figure 6.5 shows the temporal distribution of the vertical displacement at $x = 120.0m$, $y = 80.0m$ and $z = 25.0m$. The vertical displacement consistently decreases until it reaches the steady state value. The same trend is observed in the temporal distribution of the water table elevation as seen from the bottom of the aquifer at $x = 120.0m$ and $y = 80.0m$, in Figure 6.6.

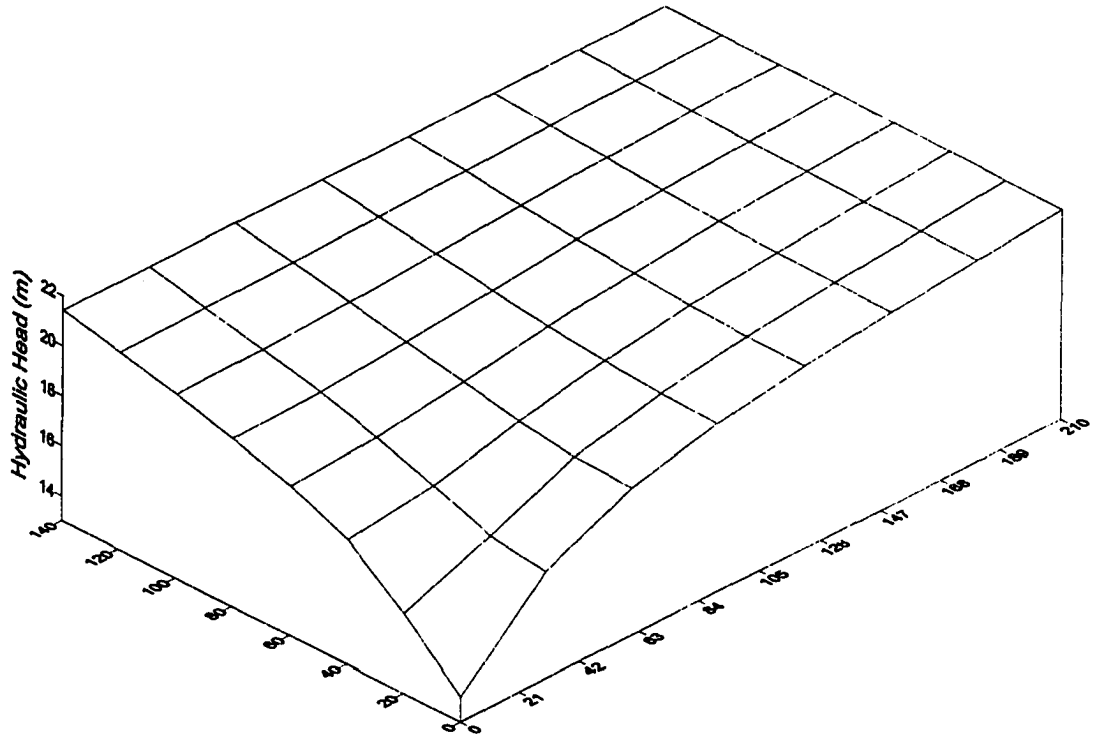


Figure 6.4: Spatial distribution of the hydraulic head after $t = 70$ days (single-porosity, fully-saturated)

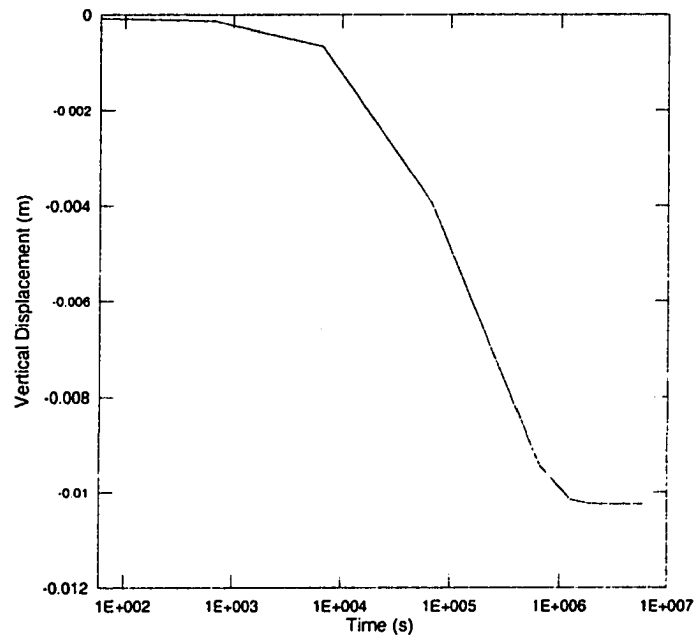


Figure 6.5: Temporal distribution of surface displacement at $x = 120.0m$ and $y = 80.0m$ (single-porosity, fully saturated)

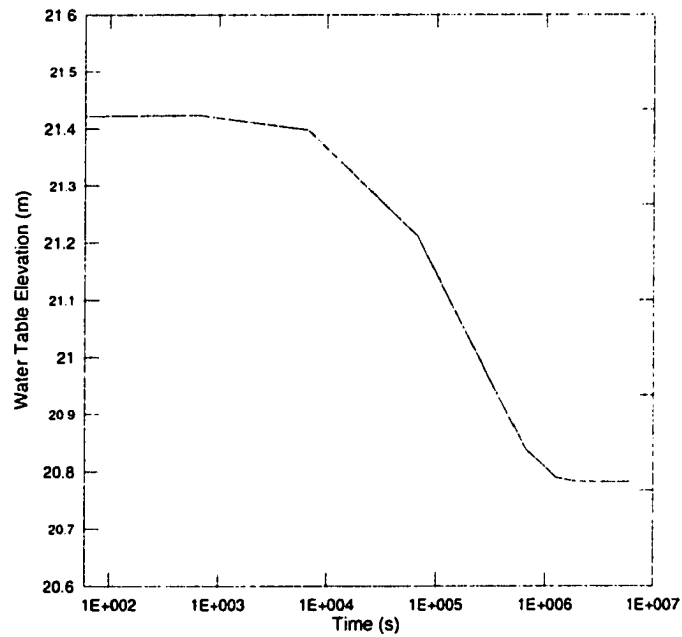


Figure 6.6: Temporal distribution of water table elevation at $x = 120.0m$ and $y = 80.0m$ (single-porosity, fully saturated)

Table 6.2: Characteristics of the secondary medium (representing the fractures) for the aquifer system

Parameter	Units	Value
Fracture Stiffness (K_n)	GPa/m	0.98
Fracture Spacing (s)	m	0.1, 0.5, 1.0
Fracture (Secondary) Porosity (ϕ^{II})	-	0.025
Fracture (Secondary) Permeability (k^{II})	m ²	3.72×10^{-11} , 3.72×10^{-10}

Dual-Porosity, Fully Saturated Case

The dual-porosity modeling of the aquifer system was carried by associating the following parameters with the secondary medium - stiffness K_n and fracture spacing s (governing the compliance of the secondary medium); secondary porosity ϕ^{II} representing the storativity of the fractures); and, secondary permeability k^{II} . The values for these parameters are as in Table 6.2.

The relevant material parameters for the primary medium are as in Table 6.1. Figures 6.7 and 6.8 show the spatial variation in the vertical displacement at the top and the hydraulic head for a spacing of $s = 0.1m$. As in the single-porosity case, the vertical displacement is maximum directly above the discharging well. Also, a zone of depression (of lower water pressure) is formed in the vicinity of the well.

Figure 6.9 shows the temporal variation in the vertical displacement at $x = 120.0m$, $y = 80.0m$ and $z = 25.0m$ for different values of the fracture spacing and ratios of the permeabilities of the primary and secondary media. As the fracture spacing is decreased up to $0.1m$, the overall compliance of the medium reduces which is reflected in a larger vertical displacement for $s = 0.1m$. When the permeability of the secondary medium is increased, it facilitates a faster dissipation of the pressures. Hence the vertical displacement reaches its final steady-state value faster for $k^{II}/k^I = 100$. As, seen in Figure 6.10, the permeability of the secondary medium has a greater effect on the water table elevation from the bottom of the aquifer. When the spacing is reduced the reduction in water table elevation from the bottom of the aquifer is greater. On the other hand, the

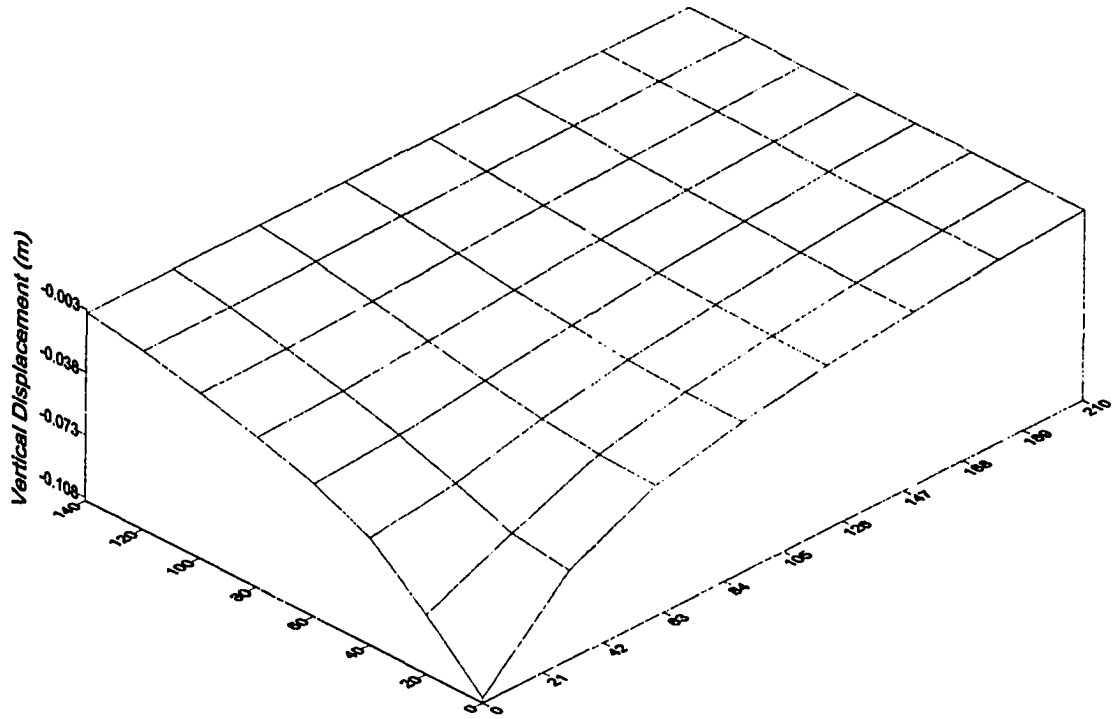


Figure 6.7: Spatial distribution of the vertical displacement at the surface after $t = 77$ days (dual-porosity, fully saturated, $s = 0.1m$)

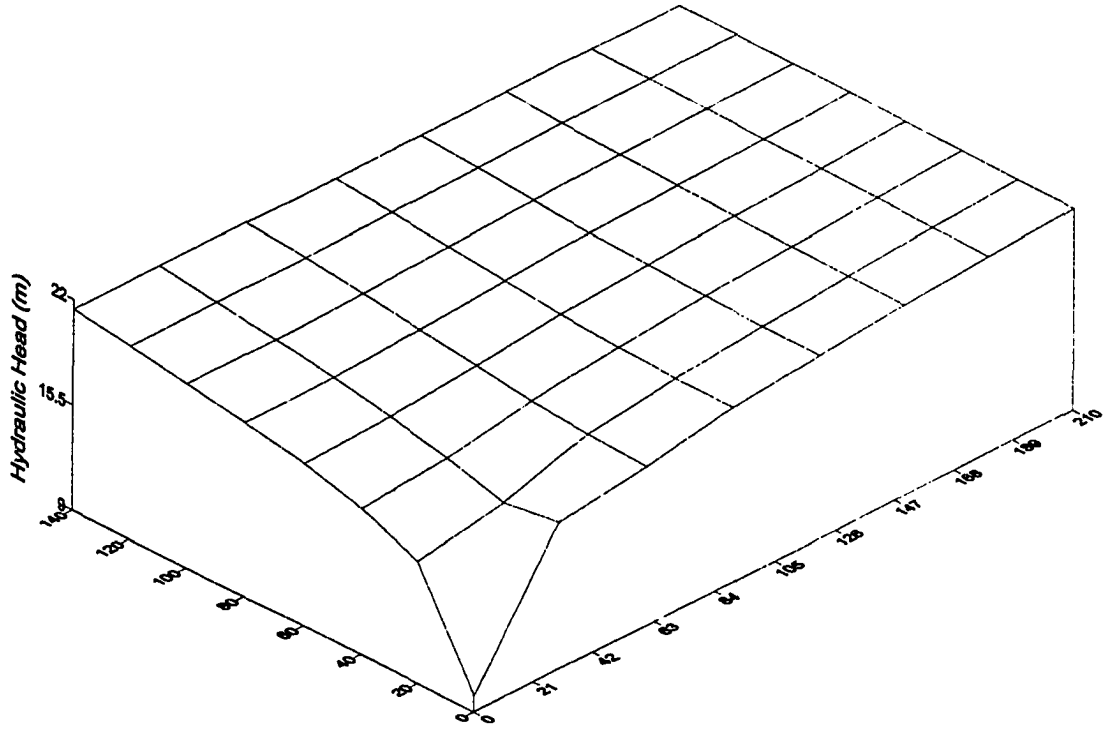


Figure 6.8: Spatial distribution of the hydraulic head after $t = 77$ days (dual-porosity, fully saturated, $s = 0.1m$)

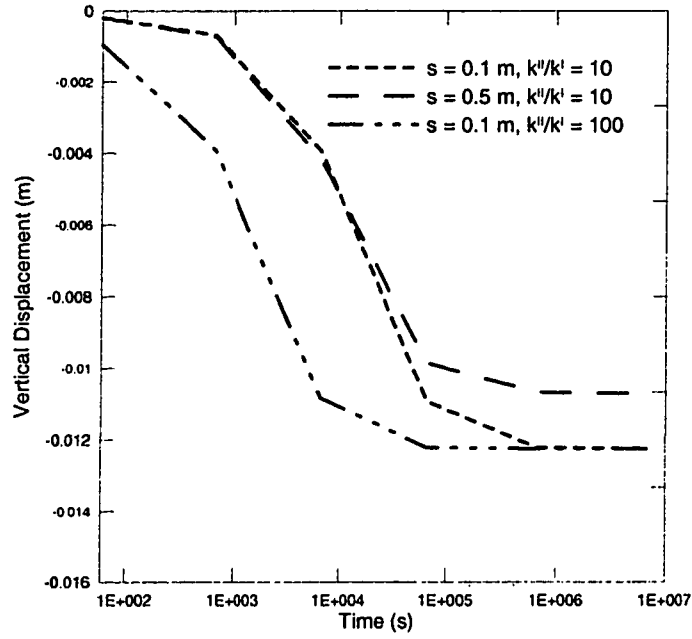


Figure 6.9: Temporal distribution of surface displacement at $x = 120.0m$ and $y = 80.0m$ (dual-porosity, fully saturated)

reduction in the water table elevation is faster when the permeability of the secondary medium is higher.

Single-Porosity, Unsaturated Case

For the unsaturated case, the following relationship between the water saturation (S_w) and hydraulic head (h) has been adopted (Kim and Parizek, 1999)

$$S_w(h) = S_{wr} + \frac{(1 - S_{wr})}{(1 + 0.5h^2)} \quad (6.4)$$

where S_{wr} ($= 0.05$, in the present example), is the residual water saturation. The following relationship between the relative permeability of water (k_{rw}) and hydraulic head has also been adopted (Kim and Parizek, 1999)

$$k_{rw} = \frac{1}{(1 + 0.5h^2)^2} \quad (6.5)$$

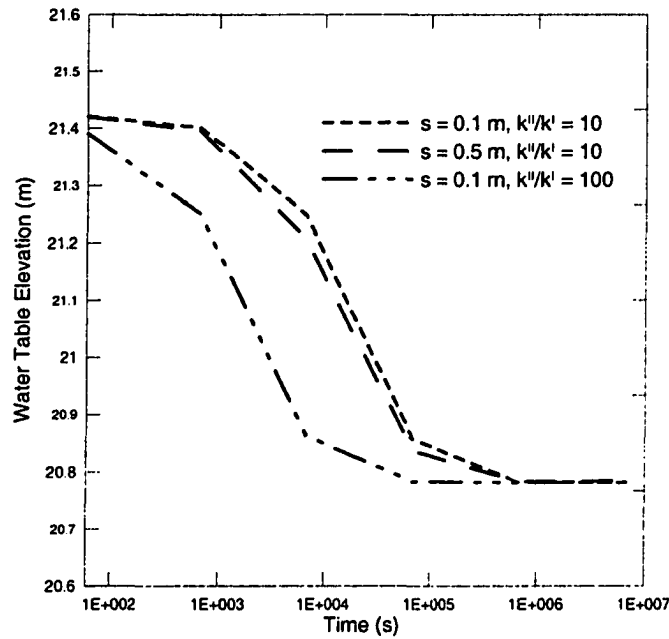


Figure 6.10: Temporal distribution of water table elevation at $x = 120.0m$ and $y = 80.0m$ (dual-porosity, fully saturated)

Figures 6.11 and 6.12 show the spatial variations in the vertical displacement at the top surface and the hydraulic head. As in the earlier cases, the vertical displacement is maximum directly above the well and a zone of low water pressure is formed in the vicinity of the discharging well.

Figure 6.13 shows the temporal variation in vertical displacement at $x = 120.0m$, $y = 80.0m$ and $z = 25.0m$ whereas Figure 6.14 shows the temporal variation in the water table elevation from the bottom of the aquifer at $x = 120.0m$ and $y = 80.0m$. The trend in the distributions is the same as in the single-porosity, fully saturated case.

Dual-Porosity, Unsaturated Case

For the dual-porosity, unsaturated case, the relationships in equations 6.4 and 6.5 were adopted for both, the primary and secondary media. The parameters chosen for the secondary medium are as given in Table 6.2. Figures 6.15 and 6.16 show the spatial

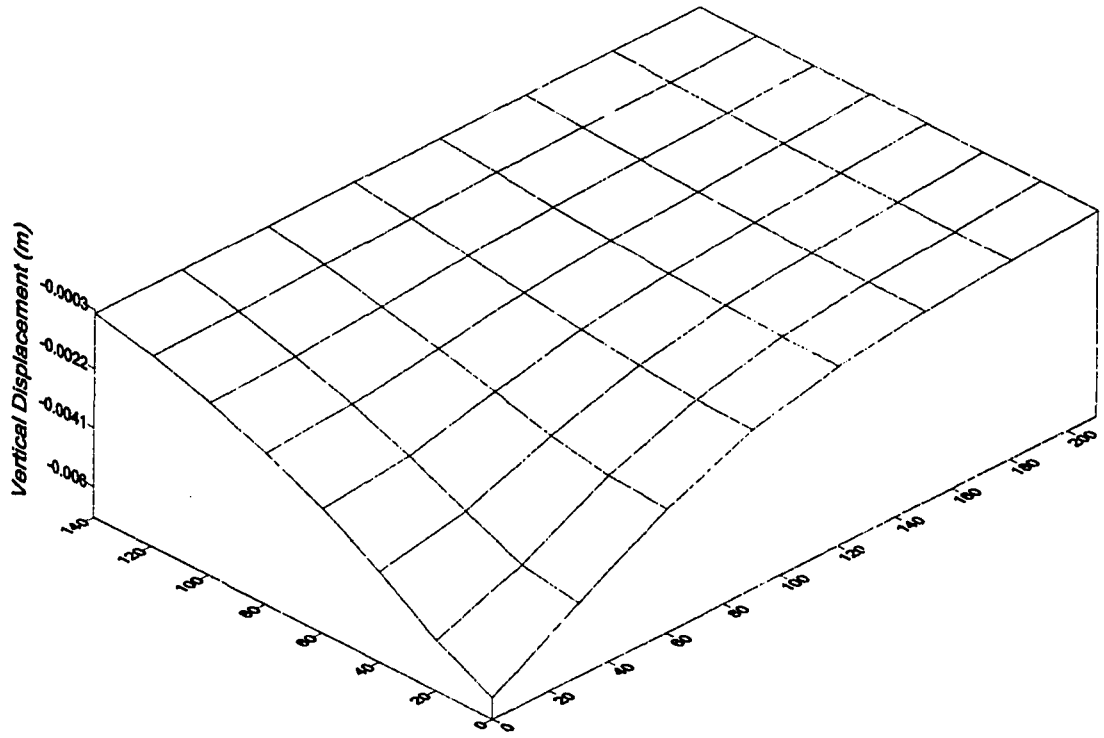


Figure 6.11: Spatial distribution of the vertical displacement at the surface after $t = 77$ days (single-porosity, unsaturated)

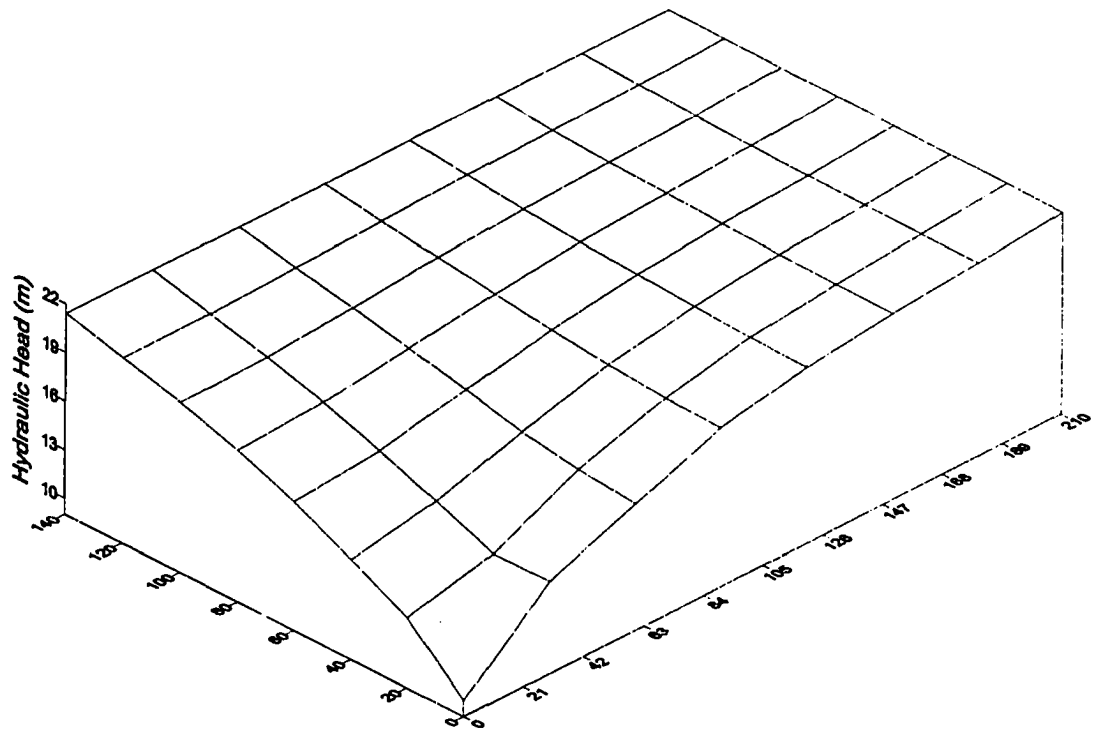


Figure 6.12: Spatial distribution of the hydraulic head after $t = 77$ days (single-porosity, unsaturated)

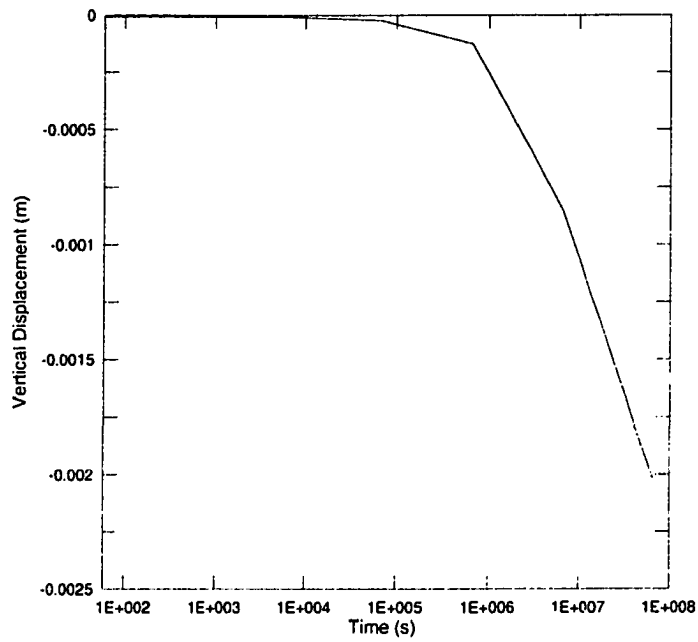


Figure 6.13: Temporal distribution of surface displacement at $x = 120.0m$ and $y = 80.0m$ (single-porosity, unsaturated)

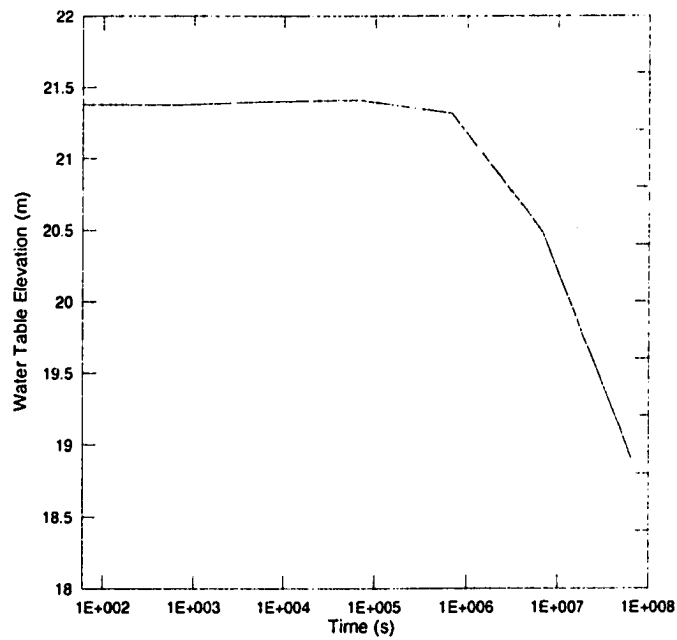


Figure 6.14: Temporal distribution of water table elevation at $x = 120.0m$ and $y = 80.0m$ (single-porosity, unsaturated)

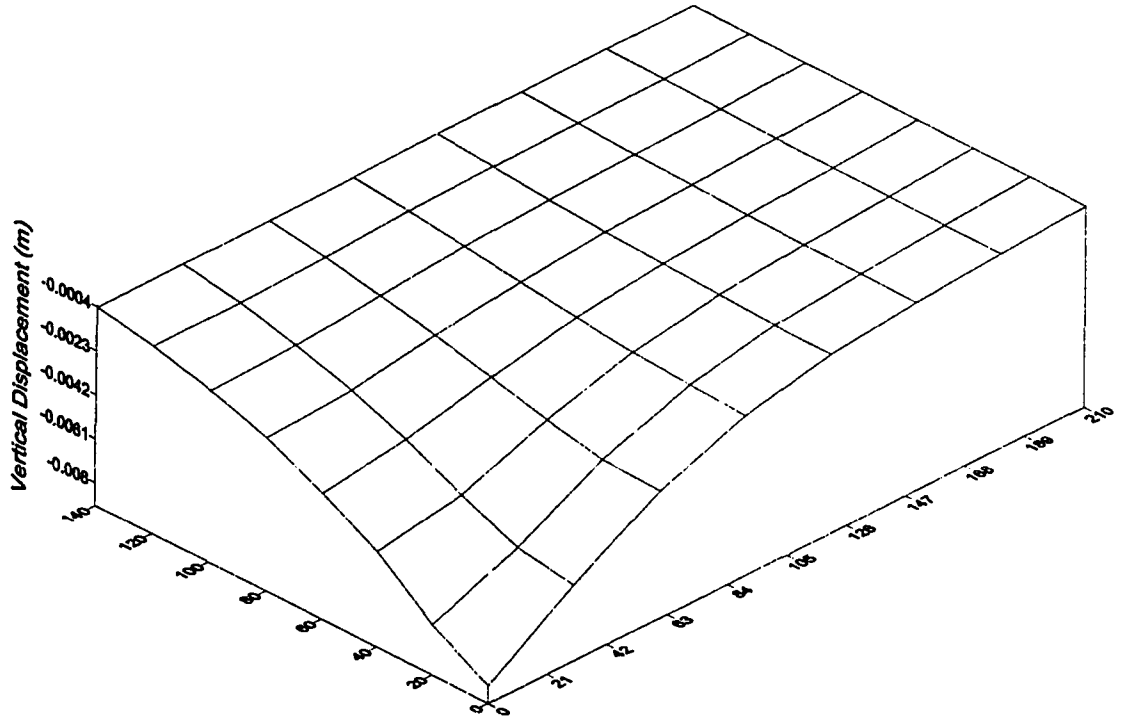


Figure 6.15: Spatial distribution of the vertical displacement at the surface after $t = 7.7$ days (dual-porosity, unsaturated, $s = 0.1m$)

variations in the vertical displacement of the top surface for $s = 0.1m$ and $s = 0.5m$. In both cases, the vertical displacement directly above the well is greatest. For $s = 0.1m$, the vertical displacements are greater since the overall compliance of the fractured medium is higher for a smaller fracture spacing.

Figures 6.17 and 6.18 show the spatial variations in the hydraulic head for the same cases (i.e., $s = 0.1m$ and $s = 0.5m$). A zone of depression corresponding to low water pressures in the vicinity of the discharging well is formed in both cases. Away from the well, the hydraulic head approaches the far-field value of $21.43m$.

Figure 6.19 shows the temporal variation in the vertical displacement at $x = 120.0m$, $y = 80.0m$ and $z = 25.0m$ for different values of the fracture spacing ($s = 0.1m$ and $0.5m$) and permeability ratios of the primary and secondary media. The vertical displacement

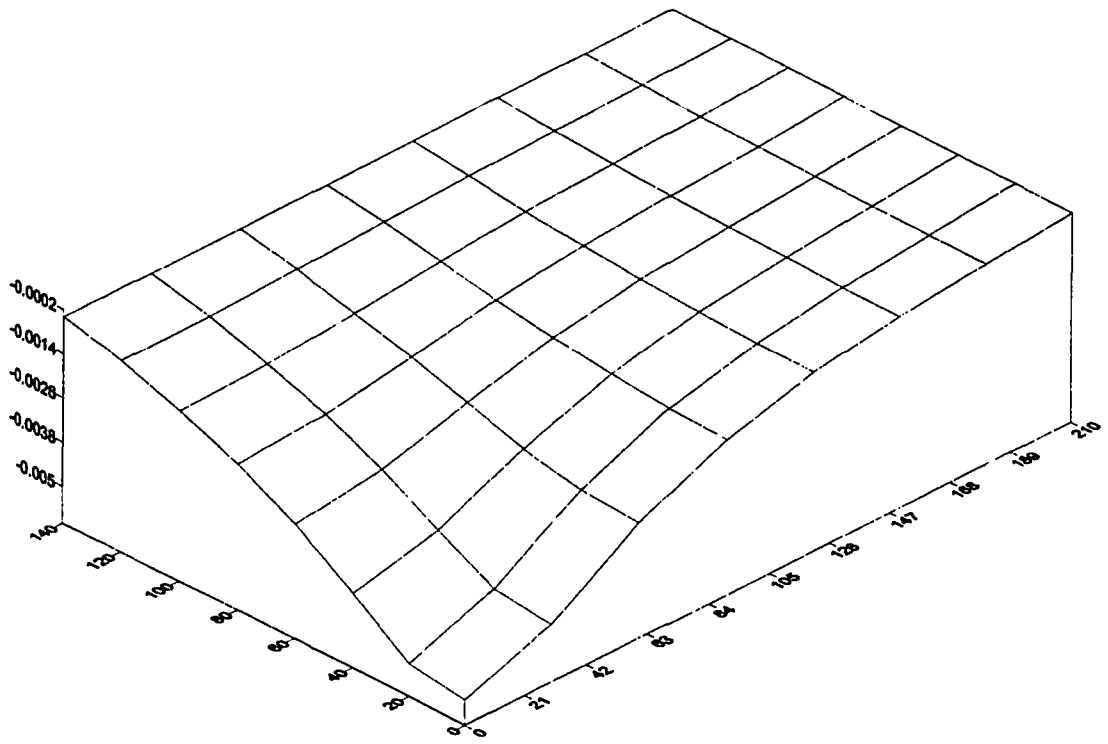


Figure 6.16: Spatial distribution of the vertical displacement at the surface after $t = 7.7$ days (dual-porosity, fully-saturated, $s = 0.5m$)

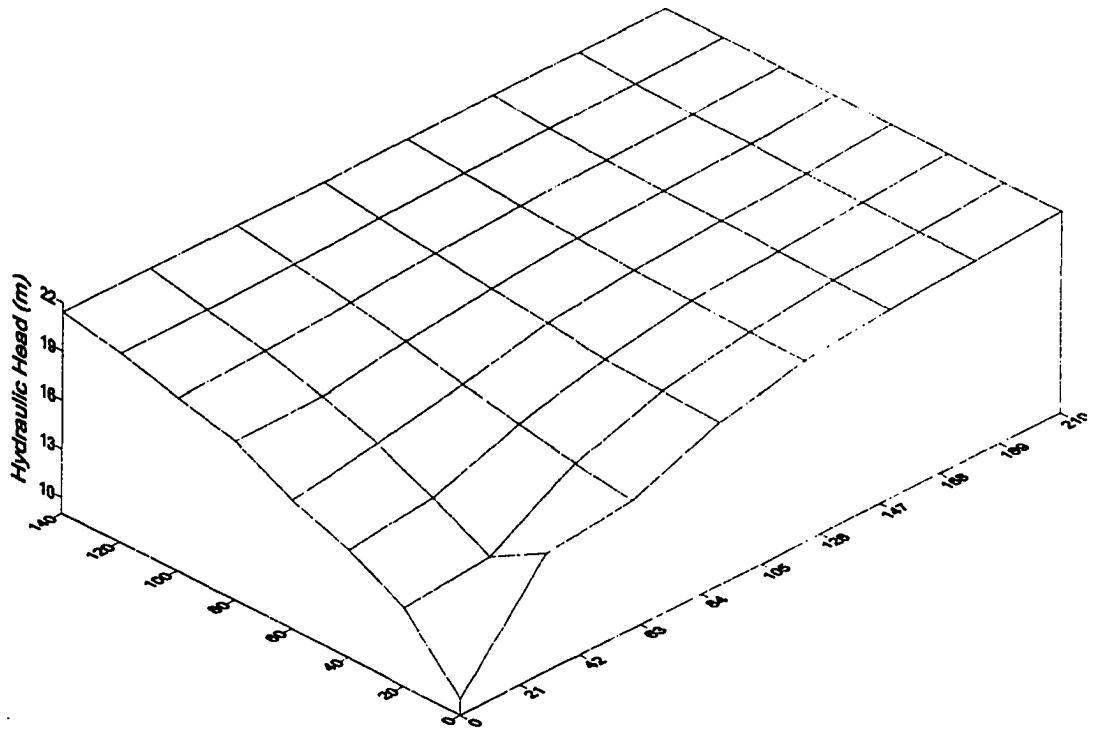


Figure 6.17: Spatial distribution of the hydraulic head after $t = 7.7$ days (dual-porosity, unsaturated, $s = 0.1m$)

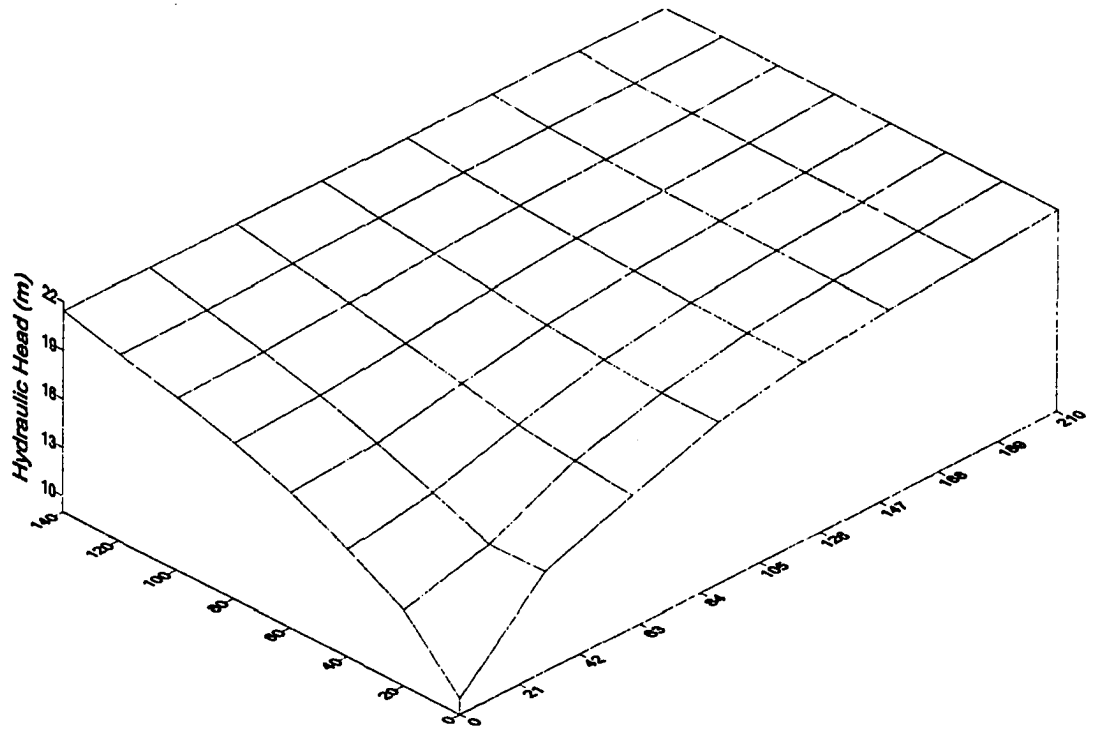


Figure 6.18: Spatial distribution of the hydraulic head after $t = 7.7$ days (dual-porosity, unsaturated, $s = 0.5m$)

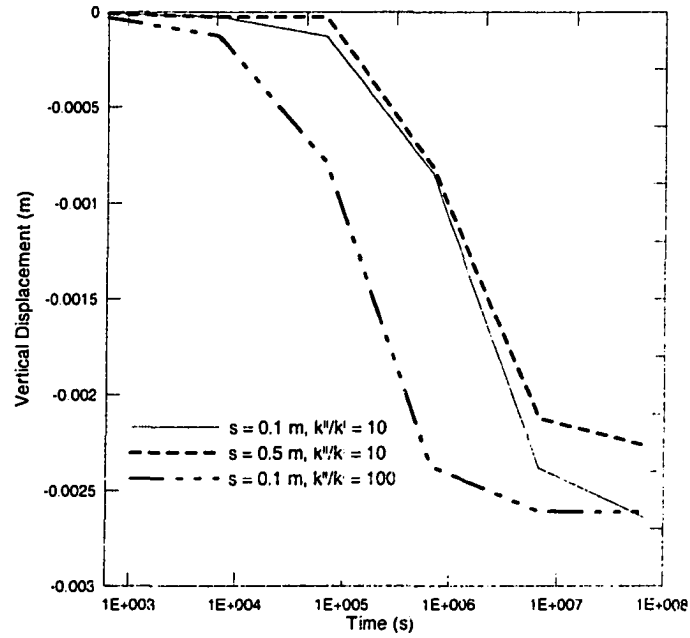


Figure 6.19: Temporal distribution of surface displacement at $x = 120.0m$ and $y = 80.0m$ (dual-porosity, unsaturated)

for a smaller fracture spacing is greater since the overall compliance of the medium is higher. A faster dissipation of the pressure is facilitated by a more permeable fracture system (i.e., secondary medium) and hence, the rate of consolidation is greater. Hence as seen in Figure 6.19 the vertical displacement for the case when $k^{II}/k^I = 100$ is higher and it reaches the steady-state value faster. Figure 6.20 shows the temporal variation in the groundwater table elevation from the bottom of the aquifer at $x = 120.0m$ and $y = 80.0m$. As seen, the permeability of the secondary medium affects the water table elevation by facilitating a faster drainage (and hence dissipation of the water pressure) and, the water table elevation reaches its steady-state value faster when $k^{II}/k^I = 100$. These results are consistent with the dual-porosity fully saturated case.

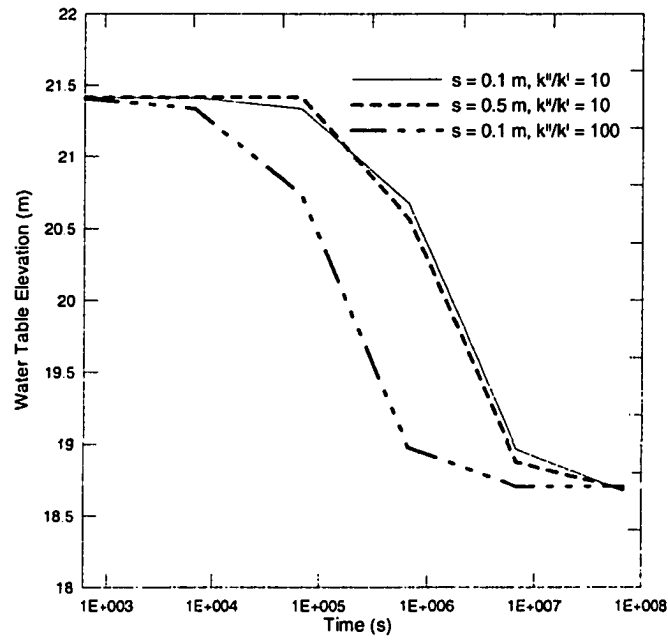


Figure 6.20: Temporal distribution of water table elevation at $x = 120.0\text{m}$ and $y = 80.0\text{m}$ (dual-porosity, unsaturated)

6.2 Summary

The two-phase (oil and gas) dual-porosity model developed in Chapter 4 was modified to analyze land subsidence and reduction in water table levels in the vicinity of a vertical discharging well penetrating an aquifer for both the fully saturated and unsaturated cases. The impact of characteristics of the secondary medium representing the fractures was analyzed by varying the fracture spacing and fracture permeability. It was observed that the fracture spacing affects the overall compliance of the system thereby increasing the magnitude of the vertical displacement at the surface. The effect of fracture permeability is to increase the magnitude of the vertical displacements and the rate at which the steady state values for the water table levels and vertical displacements are attained.

Chapter 7

Summary, Conclusions and Recommendations

7.1 Summary and Conclusions

A dual-porosity model has been developed to analyze the response of a fluid saturated fractured porous medium to perturbations. A fractured porous medium is visualized as being composed of two distinct but overlapping media. The first medium (primary medium) is comprised of the matrix and void spaces corresponding to matrix pores (primary porosity). The second medium (secondary medium) is comprised of voids representing the fractures (secondary porosity), and a complementary solid part. Thus, Barenblatt's original concept of two fluid pressures at a point and Aifantis's extension of Biot's poroelastic theory have been used within the framework of the separate and overlapping technique. Separate continuity equations along with effective stress and constitutive laws are implemented for both the primary and secondary media. The intensity of interaction between the two media are controlled by mass exchange rates which are assumed to be dependent on the quasi-static pressure difference between the two media and the permeability of the primary medium. Based on the mathematical formulations, a finite element model has been developed which has been verified against analytical solutions for the problems of one-dimensional consolidation and an inclined borehole in a fully saturated poroelastic medium.

A dual-porosity porothermoelastic finite element model has been developed which accounts for thermo-hydro-mechanical coupling in both the primary as well as the secondary media. The model accounts for heat transport through conduction as well as convection. A single representative thermodynamics continuum is assumed to be sufficient to describe the temperature dependent response of a fissured porous medium. The model has been used to analyze the response of an inclined wellbore in a fully saturated dual-porosity medium subjected to pore pressure and thermal gradients. The numerical examples show that the thermal loading induces a significant increase in pore pressure and tensile effective radial stresses at short time intervals in the vicinity of the wellbore, thereby undermining its stability during the initial drilling phase. Also, heat transport by convection is characterized by a zone of high temperature which progressively extends into the formation. Convective effects are particularly significant for formations with low values of the ratio of thermal to fluid diffusivities. For larger thermal to fluid diffusivity ratios, the heat transport is mainly through conduction, and convective effects may be ignored. In addition, it is observed that, the density of fractures governs the overall compliance of the medium thereby influencing the magnitude of stress and pore pressures near the borehole wall.

A dual-porosity poroelastic model incorporating oil and gas flow has been developed. Pressure-dependent change in fluid phase, is accounted for by incorporating the black-oil model into the continuity equations for oil and gas in the two media. The governing equations are cast into a finite element form wherein the displacement and pressures in the oil and gas phases are the primary unknowns. The saturations are the secondary unknowns obtained from capillary-saturation relations. The non-linear system of equations is solved by a direct solver and the stability is checked at each time step. In an oil-gas system, the saturations of the individual phases play an important role in determining the effective fluid compressibility. This is so because the compressibilities of

oil and gas differ by orders of magnitude. The results obtained by applying the single-porosity two-phase and dual-porosity two-phase models to the problem of an inclined borehole, indicate that the fluid compressibilities (and hence the phase saturations) affect the magnitude of stresses and pore pressures perturbations near the borehole wall. Assuming the formation to be saturated with a single incompressible fluid leads to an overestimation of the tensile stresses and pore pressures developed at the borehole wall. A consideration of the mechanisms of two-phase flow is hence, realistic since formations are generally saturated with more than one fluid of different compressibilities.

A dual-porosity model incorporating fully-coupled non-isothermal flow of oil and gas has been developed based on the “separate and overlapping” technique. The solubility of gas in the oil as well as evaporation of oil into the gas phase has been accounted for by incorporating a limited compositional model into the continuity equations for both media. A finite element model has been developed based on the mathematical formulations and applied to the problem of an inclined borehole drilled under non-isothermal conditions in a fractured porous medium saturated with oil and gas. The numerical results show that, the pore pressure and effective stress distributions in the vicinity of the borehole at short times are affected by the initial oil saturation. In the case of fractured formations, the compliance of the secondary medium, governed by the fracture density, affects the pore pressure and effective stress distributions. For a medium with low fluid diffusivities, the initial oil saturation and secondary medium parameters do not affect the temperature distributions.

The two-phase (oil and gas) dual-porosity model has been modified to analyze land subsidence and reduction in water table levels in the vicinity of a vertical discharging well penetrating an aquifer for both the fully saturated and unsaturated cases. The impact of characteristics of the secondary medium representing the fractures has been analyzed by varying the fracture spacing and fracture permeability. It is observed that

the fracture spacing affects the overall compliance of the system thereby increasing the magnitude of the vertical displacement at the surface. Further, the rate at which the steady state values for the water table levels and vertical displacements are attained is higher when the secondary medium has a higher permeability.

7.2 Recommendation for Future Work

The dual-porosity models need to be validated against results from laboratory experiments and field case studies. During operations such as drilling of boreholes in fractured formations, the critical mudweight (borehole pressure) for fracturing, collapse and spalling failure modes (Cui et al, 1999) needs to be assessed. The time required for finite element calculations of safe windows for mudweight for different borehole orientations and depths, warrant the development of analytical solutions for the governing equations developed herein. Analytical solutions, particularly for the cylinder geometry need to be developed in order to assess secondary medium parameters (possibly by a “history matching” process as in reservoir simulation studies).

Non-linear poromechanics has not been addressed in this dissertation. It is envisaged that a non-linear elastic or elastoplastic constitutive model could be applicable for the secondary medium representing the fractures.

A majority of drilling operations for oil and gas are carried out in chemically active formations such as shale which may be fissured in addition to being saturated with more than one fluid. Hence a model accounting for non-isothermal chemical interaction between the fissured formation and the saturating (and invading) fluid(s) needs to be developed.

The dual-porosity model presented in this dissertation needs to be extended to incorporate wave propagation effects. Theoretical work in this regard would help in the dynamic characterization of fractured rock, an area where only limited work has been

carried out.

The finite element method has been the dominant tool for obtaining numerical solutions to various problems in geomechanics. However, other numerical techniques such as the element-free Galerkin method (EFGM) have been recently applied to problems of heat conduction, wave propagation and two-phase flow in deforming porous media. Since no predefined element connectives are required for meshless methods such as the EFGM, the time-consuming meshing and remeshing (for refinement of numerical calculations) processes can be avoided. Hence, it is suggested that the governing equations for dual-porosity poromechanics, developed in this dissertation, be cast into a form amenable for application of meshless methods such as the EFGM. The Gauss Quadrature approach may also be explored as an alternative to obtain the solutions to the governing (Santosh and Karlin, 2002).

Nomenclature

Vectors and Tensors

C_{ijkl}^η	- Compliance tensors
$D_{ijkl}^{I,II}$	- Elasticity tensor for the porous medium
F_j	- Body forces
U_i^η	- Intrinsic fluid velocities
$U_i^{\eta\pi}$	- Intrinsic velocities of phase π
u_i^η	- Solid velocities
v_i^η	- Darcy velocities
$v_i^{\eta\pi}$	- Darcy velocities of phase π
δ_{ij}	- Kronecker delta
ε_{ij}	- Total body strain tensor
ε_{ij}^η	- Strain tensors
σ_{ij}	- Total stress tensor
$(\sigma'_{ij})^\eta$	- Effective stress tensors

Material Parameters

Single Phase

C_v	- Specific heat of the porous medium
C_{vf}	- Specific heat of the fluid
C_{vs}	- Specific heat of the solid
K^η	- Bulk moduli
K_f	- Fluid bulk modulus
K_s^η	- Solid grain bulk moduli
α^η	- Equivalent Biot's effective stress parameters
β	- Bulk thermal expansion coefficients
β_s^η	- Solid thermal expansion coefficients
β_f^η	- Fluid thermal expansion coefficients

- λ - Coefficient of thermal conductivity of the porous medium
- λ_f - Coefficient of thermal conductivity of the fluid
- λ_s - Coefficient of thermal conductivity of the solid
- μ_f - Fluid viscosity
- ρ_f - Fluid density
- ρ_s^η - Solid densities
- k^η - Intrinsic permeabilities
- ϕ^η - Porosities
- Ψ - Coefficient of fluid transfer

Two-Phase

- B_π^η - Formation volume factors for phase π
- $C_{v\pi}$ - Specific heat of phase π
- $k_{r\pi}^\eta$ - Relative permeabilities of phase π
- R_s^η - Gas-oil solubility ratios
- R_v^η - Oil volatility ratios
- λ_π - Coefficient of thermal conductivity of phase π
- ρ_π^η - Densities of phase π under reservoir conditions
- $\rho_{\pi s}^\eta$ - Densities of phase π under standard conditions
- μ_π - Viscosity of phase π

Variables

- Δh - Element length (one dimensional element)
- P^η - Averaged pore pressures
- P_π^η - Pressures in phase π
- P_c^η - Capillary pressures
- S_π - Saturation of phase π
- t - Time
- Δt - Time-step
- T - Temperature
- V - Total volume
- V_p^η - Pore volumes
- V_s^η - Solid volumes
- $V_{\Pi s}$ - Volume occupied by component Π at standard conditions
- $w_{\Pi\pi}^\eta$ - Mass fractions of the Π component in the π phase
- W_Π - Mass of component Π
- x_i - Cartesian coordinates

- $\bar{\sigma}$ - Mean stress
- Γ - Rate of fluid exchange between the two media
- Γ_{π} - Rate of exchange of phase π between the two media
- Ω - Domain under consideration
- Υ - Boundary of the domain

Superscripts and Subscripts

Superscripts

- $\eta, \bar{\eta}$ - $\begin{cases} I & \text{- primary medium representing the matrix} \\ II & \text{- secondary medium representing the fractures} \end{cases}$
- o - Oil
- g - Gas
- w - Wetting phase
- nw - Non-wetting phase
- $\mathbf{T}$ - Transpose of a matrix

Subscripts

- i, j, k, l, m, n, p, q - Indices as per Einstein's convention
- f - Fluid
- s - Solid
- o - Oil
- g - Gas
- w - Wetting phase
- nw - Non-wetting phase

Finite Element Notations

- $\mathbf{B}$ - Strain displacement matrix
- $\mathbf{m}^T$ - Row matrix, $[1 \ 1 \ 1 \ 0 \ 0 \ 0]$
- $\mathbf{N}$ - Shape function matrix
- $\mathbf{P}_{\pi}^{\eta}$ - Nodal phase pressures
- $\mathbf{T}$ - Nodal temperatures
- $\mathbf{u}$ - Nodal displacements

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Appendix A

Finite Element Submatrices

$$\mathbf{K} = \int_V \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{B} dV \quad (\text{A.1})$$

$$\mathbf{L}_f^\eta = - \int_V \alpha^\eta \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^\eta \mathbf{m} \mathbf{N} dV \quad (\text{A.2})$$

$$\mathbf{L}_t = - \int_V \alpha^\eta \mathbf{B}^T \mathbf{D}^{I,II} \frac{\beta}{3} \mathbf{m} \mathbf{N} dV \quad (\text{A.3})$$

$$\mathbf{K}_f^\eta = \int_V \mathbf{N}^T \alpha^\eta \rho_f \mathbf{m}^T \mathbf{B} dV \quad (\text{A.4})$$

$$\mathbf{H}_D^\eta = - \int_V \nabla \mathbf{N}^T \frac{k^\eta}{\mu} \rho_f \nabla \mathbf{N} dV \quad (\text{A.5})$$

$$\mathbf{H}_{tr} = - \int_V \mathbf{N}^T \rho_f \frac{k^{(1)}}{\mu} \Psi \mathbf{N} dV \quad (\text{A.6})$$

$$\mathbf{S}^I = \int_V \mathbf{N}^T \rho_f \left(\frac{(\alpha^I - \phi^I)}{K_s} + \frac{\phi^I}{K_f} \right) \mathbf{N} dV \quad (\text{A.7})$$

$$\mathbf{S}^{II} = \int_V \mathbf{N}^T \rho_f \left(\frac{(\alpha^{II} - \phi^{II})}{K_{ns}} + \frac{\phi^{II}}{K_f} \right) \mathbf{N} dV \quad (\text{A.8})$$

$$\mathbf{S}_t^\eta = \int_V \mathbf{N}^T \rho_f ((1 - \alpha^\eta) \beta^\eta - (1 - \phi^\eta) \beta_s^\eta - \phi^\eta \beta_f) \mathbf{N} dV \quad (\text{A.9})$$

$$\mathbf{T}_s = \int_V \mathbf{N}^T \rho C_v \mathbf{N} dV \quad (\text{A.10})$$

$$\mathbf{C}_t = - \int_V (\nabla \mathbf{N})^T [\rho_f C_{vf} (\mathbf{v}^I + \mathbf{v}^{II})] \mathbf{N} dV \quad (\text{A.11})$$

$$\mathbf{H}_{tF} = \int_V (\nabla \mathbf{N})^T \lambda \nabla \mathbf{N} dV \quad (\text{A.12})$$

$$\mathbf{K}_t = \int_V \mathbf{N}^T (1 - \phi^I - \phi^{II}) 3K \beta \mathbf{m}^T \mathbf{B} dV \quad (\text{A.13})$$

$$\mathbf{O}_c^\eta = - \int_V \alpha^\eta \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^\eta \mathbf{m} \mathbf{N} \left(S_o^\eta + P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) dV \quad (\text{A.14})$$

$$\mathbf{G}_c^\eta = - \int_V \alpha^\eta \mathbf{B}^T \mathbf{D}^{I,II} \mathbf{C}^\eta \mathbf{m} \mathbf{N} \left(S_g^\eta - P_c^\eta \frac{\partial S_o^\eta}{\partial P_c^\eta} \right) dV \quad (\text{A.15})$$

$$\mathbf{K}_o^\eta = \int_V \mathbf{N}^T \alpha^\eta S_o^\eta \frac{\rho_{os}^\eta}{B_o^\eta} \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \quad (\text{A.16})$$

$$\mathbf{O}_D^\eta = - \int_V \nabla \mathbf{N}^T \frac{k^\eta k_{ro}^\eta}{\mu_o^I} \frac{\rho_{os}^\eta}{B_o^\eta} \nabla \mathbf{N} dV \quad (\text{A.17})$$

$$\mathbf{O}_T = - \int_V \mathbf{N}^T \frac{k^I k_{ro}^I}{\mu_o^I} \frac{\rho_{os}^I}{B_o^I} \Psi \mathbf{N} dV \quad (\text{A.18})$$

$$\begin{aligned} \mathbf{O}_o^I = & - \int_V \mathbf{N}^T \phi^I S_o^I \frac{\rho_{os}^I}{(B_o^I)^2} \frac{\partial B_o^I}{\partial P_o^I} \mathbf{N} dV - \\ & \int_V \mathbf{N}^T \phi^I \frac{\rho_{os}^I}{B_o^I} \frac{\partial S_o^I}{\partial P_c^I} \mathbf{N} dV + \int_V \mathbf{N}^T S_o^I \frac{\rho_{os}^I}{B_o^I} \left(S_o^I + P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \\ & \left(\frac{(\alpha^I - \phi^I)}{K_s} \right) \mathbf{N} dV \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} \mathbf{O}_g^I = & \int_V \mathbf{N}^T \phi^I \frac{\rho_{os}^I}{B_o^I} \frac{\partial S_o^I}{\partial P_c^I} \mathbf{N} dV + \\ & \int_V \mathbf{N}^T S_o^I \frac{\rho_{os}^I}{B_o^I} \left(S_g^I - P_c^I \frac{\partial S_o^I}{\partial P_c^I} \right) \left(\frac{(\alpha^I - \phi^I)}{K_s} \right) \mathbf{N} dV \end{aligned} \quad (\text{A.20})$$

$$\mathbf{K}_g^\eta = \int_V \mathbf{N}^T \rho_{gs}^\eta \alpha^\eta \left(\frac{S_g^\eta}{B_g^\eta} + \frac{R_s^\eta S_o^\eta}{B_o^\eta} \right) \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \quad (\text{A.21})$$

$$\mathbf{G}_{dD}^\eta = - \int_V \nabla \mathbf{N}^T \frac{R_s^\eta \rho_{gs}^\eta}{B_o^\eta} \frac{k^\eta k_{ro}^\eta}{\mu_o} \nabla \mathbf{N} dV \quad (\text{A.22})$$

$$\mathbf{G}_D^\eta = - \int_V \nabla \mathbf{N}^T \frac{\rho_{gs}^\eta}{B_g^\eta} \frac{k^\eta k_{rg}^\eta}{\mu_g} \nabla \mathbf{N} dV \quad (\text{A.23})$$

$$\mathbf{G}_T = - \int_V \mathbf{N}^T \frac{\rho_{gs}^I}{B_g^I} \frac{k^I k_{rg}^I}{\mu_g^I} \Psi \mathbf{N} dV \quad (\text{A.24})$$

$$\begin{aligned}
& \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_n^g}{R_n^g} - \frac{B_n^o}{1} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{B_n^g}{R_n^g} \frac{\partial T}{\partial S_n^g} \\
& - \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial T}{\partial B_n^g} \frac{(B_n^g)_z}{S_n^g R_n^g} \\
& - \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial T}{\partial B_n^g} \frac{(B_n^g)_z}{S_n^g} \\
& \left(\frac{\partial T}{\partial S_n^g} \frac{K_n^s}{P_n^s} - \frac{(\alpha_n - \phi_n)}{1} \right) \\
& = \int_V N^T \left(S_n^g \frac{B_n^g}{R_n^g} + \frac{B_n^o}{S_n^g} \right) \rho_{,n}^{os}
\end{aligned}
\tag{A.33}$$

$$\begin{aligned}
& \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_n^g \frac{B_n^g}{R_n^g} + \frac{B_n^o}{S_n^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_n^g}{1} - \frac{R_n^g}{\partial S_n^g} \right) \frac{\partial P_n^g}{\partial P_n^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_n^g}{\partial B_n^g} \frac{(B_n^g)_z}{R_n^g} \\
& = \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_n^g \frac{B_n^g}{R_n^g} + \frac{B_n^o}{S_n^g} \right) \left(\frac{\partial P_n^g}{\partial S_n^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_n^g}{1} - \frac{R_n^g}{\partial S_n^g} \right) \frac{\partial P_n^g}{\partial P_n^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_n^g}{\partial B_n^g} \frac{(B_n^g)_z}{R_n^g}
\end{aligned}
\tag{A.32}$$

$$\begin{aligned}
& \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_n^g \frac{B_n^g}{R_n^g} + \frac{B_n^o}{S_n^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_n^g}{\partial P_n^g} \frac{\partial P_n^g}{\partial S_n^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_n^g}{1} - \frac{R_n^g}{\partial S_n^g} \right) \frac{\partial P_n^g}{\partial P_n^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_n^g}{\partial B_n^g} \frac{(B_n^g)_z}{R_n^g} \\
& = \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_n^g \frac{B_n^g}{R_n^g} + \frac{B_n^o}{S_n^g} \right) \left(\frac{\partial P_n^g}{\partial S_n^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_n^g}{1} - \frac{R_n^g}{\partial S_n^g} \right) \frac{\partial P_n^g}{\partial P_n^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_n^g}{\partial B_n^g} \frac{(B_n^g)_z}{R_n^g}
\end{aligned}
\tag{A.31}$$

$$\begin{aligned}
& \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_{n,II}^g \frac{B_{n,II}^g}{R_{n,II}^g} + \frac{B_{n,II}^o}{S_{n,II}^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_{n,II}^g}{\partial P_{n,II}^g} \frac{\partial P_{n,II}^g}{\partial S_{n,II}^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_{n,II}^g}{1} - \frac{R_{n,II}^g}{\partial S_{n,II}^g} \right) \frac{\partial P_{n,II}^g}{\partial P_{n,II}^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_{n,II}^g}{\partial B_{n,II}^g} \frac{(B_{n,II}^g)_z}{R_{n,II}^g} \\
& = \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(S_{n,II}^g \frac{B_{n,II}^g}{R_{n,II}^g} + \frac{B_{n,II}^o}{S_{n,II}^g} \right) \left(\frac{\partial P_{n,II}^g}{\partial S_{n,II}^g} \right) \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \left(\frac{B_{n,II}^g}{1} - \frac{R_{n,II}^g}{\partial S_{n,II}^g} \right) \frac{\partial P_{n,II}^g}{\partial P_{n,II}^g} \\
& + \int_V N^T \phi_{,n} \rho_{,n}^{os} \frac{\partial P_{n,II}^g}{\partial B_{n,II}^g} \frac{(B_{n,II}^g)_z}{R_{n,II}^g}
\end{aligned}
\tag{A.30}$$

$$\bar{\mathbf{K}}_o^\eta = \int_V \mathbf{N}^T \rho_{os}^\eta \alpha^\eta \left(\frac{S_o^\eta}{B_o^\eta} + \frac{R_v^\eta S_g^\eta}{B_g^\eta} \right) \mathbf{m}^T \mathbf{C}^\eta \mathbf{D}^{I,II} \mathbf{B} dV \quad (\text{A.34})$$

$$\bar{\mathbf{O}}_{eD}^\eta = - \int_V \nabla \mathbf{N}^T \frac{R_v^\eta \rho_{os}^\eta}{B_g^\eta} \frac{k^\eta k_{rg}^\eta}{\mu_g} \nabla \mathbf{N} dV \quad (\text{A.35})$$

$$\bar{\mathbf{O}}_{eT} = - \int_V \mathbf{N}^T R_v^I \frac{\rho_{os}^I}{B_g^I} \frac{k^I k_{rg}^I}{\mu_g^I} \Psi \mathbf{N} dV \quad (\text{A.36})$$

$$\begin{aligned} \bar{\mathbf{S}}_{gt}^\eta = & \int_V \mathbf{N}^T \left(S_o^\eta \frac{R_s^\eta}{B_o^\eta} + \frac{S_g^\eta}{B_g^\eta} \right) \rho_{gs}^\eta \\ & \left(\frac{(1 - \alpha^\eta) \beta^\eta - (1 - \phi^\eta) \beta_s^\eta}{K_s^\eta} P_c^\eta \frac{\partial S_o^\eta}{\partial T} \right) \mathbf{N} dV \\ & - \int_V \mathbf{N}^T \frac{\phi^\eta \rho_{gs}^\eta S_g^\eta}{(B_g^\eta)^2} \frac{\partial B_g^\eta}{\partial T^\eta} \mathbf{N} dV \\ & - \int_V \mathbf{N}^T \frac{\phi^\eta \rho_{gs}^\eta S_o^\eta R_s^\eta}{(B_o^\eta)^2} \frac{\partial B_o^\eta}{\partial T^\eta} \mathbf{N} dV + \\ & \int_V \mathbf{N}^T \phi^\eta \frac{\rho_{gs}^\eta S_o^\eta}{B_o^\eta} \frac{\partial R_s^\eta}{\partial T} \mathbf{N} dV + \\ & \int_V \mathbf{N}^T \phi^\eta \rho_{gs}^\eta \left(\frac{-1}{B_g^\eta} + \frac{R_s^\eta}{B_o^\eta} \right) \frac{\partial S_o^\eta}{\partial T} \mathbf{N} dV \end{aligned} \quad (\text{A.37})$$

$$\bar{\mathbf{T}}_s = \int_V \mathbf{N}^T \left[\begin{array}{c} (\phi^I S_g^I \rho_g^I + \phi^{II} S_g^{II} \rho_g^{II}) C_{vg} + \\ (\phi^I S_o^I \rho_o^I + \phi^{II} S_o^{II} \rho_o^{II}) C_{vo} + \\ (1 - \phi^I - \phi^{II}) \rho_s C_{vs} \end{array} \right] \mathbf{N} dV \quad (\text{A.38})$$

$$\bar{\mathbf{C}}_t = - \int_V (\nabla \mathbf{N})^T \left(\begin{array}{c} (\rho_g^I v_g^I + \rho_g^{II} v_g^{II}) C_{vg} + \\ (\rho_o^I v_o^I + \rho_o^{II} v_o^{II}) C_{vo} \end{array} \right) \mathbf{N} dV \quad (\text{A.39})$$

$$\bar{\mathbf{H}}_{tF} = \int_V (\nabla \mathbf{N})^T \left[\begin{array}{c} (\phi^I S_g^I + \phi^{II} S_g^{II}) \lambda_g + \\ (\phi^I S_o^I + \phi^{II} S_o^{II}) \lambda_o + \\ (1 - \phi^I - \phi^{II}) \lambda_s \end{array} \right] \nabla \mathbf{N} dV \quad (\text{A.40})$$