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By

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ANTIFORCE ELECTRON DYNAMICAL WAVES

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CHAPTER I

INTRODUCTION

Although the passage of luminous pulses between charged electrodes has been recognized since the work of Hauksbee in 1705, the systematic study of such phenomena in the laboratory has received sporadic attention. The general features of the pulses are well known but detailed calculations of their structure have proven to be very difficult. A large portion of the difficulty can be traced to the lack of experimental data regarding the physical characteristics of the moving luminous fronts. Obtaining such data requires very sophisticated measurement techniques and, until the theory has been advanced somewhat, it is not clear what data is required for the construction of theoretical models. The high speeds of the luminous pulse together with an unanimous report of the absence of a doppler shift supports the conclusion that the pulses are due to an electron fluid motion.

A model based on shock theory has been largely successful in predicting the front velocity for cases where the electron motion is in the direction of the force due to the applied electric field. However, the shock wave theory predicts a minimum velocity for these waves for the so called proforce case. The minimum wave velocity predicted by a strong shock does not apply to the antiforce case. Here proforce and antiforce are terms applied to waves which propagate in directions along and opposed to the force on electrons due to the applied electric field.

The present work extends the fluid dynamic model to the antiforce case and it is shown that velocities which are below the minimum predicted by shock theory can occur. The antiforce wave is driven by electron pressure and the existence of either preionization or photoionization is not required to explain the observed waves. The present model predicts fast waves to be accompanied by quite high electron temperatures and that the wave front will be considerably thicker than that seen in proforce waves. As in the proforce case, the wave front is succeeded by a quasi-neutral region where the electrons and ions gradually come to thermal equilibrium. The results indicate that a one-dimensional time-independent fluid model can be successfully applied to the problem of luminous fronts for both pro- and antiforce waves even though a true one-dimensional antiforce wave probably does not exist. Available data is matched by the results of the present theory and requirements for future experiments are presented.

CHAPTER II

BACKGROUND

According to the electron fluid dynamic theory of breakdown waves in gas discharges, anyone who has observed a lightning stroke has seen a breakdown wave. However, the first such observation in the laboratory was probably that of Hauksbee in 1705 [1]. In a study of gas propagation with rotating mirrors, Wheatstone was able to report luminous front velocities in excess of 10^5 meter/sec but his apparatus was inadequate to provide quantitative measurements [2]. Thomson used improved apparatus but was unable to solve the problem of synchronization of pulse initiation with observation of front passage [3]. Forced to rely on hit or miss observation, Thomson was still able to estimate a speed of around 10^8 meter/sec. Thomson's waves invariably moved from the anode to the cathode. Thomson further asserted that no doppler shift of the emitted light could be observed and thus concluded that the mass motion of the emitting species was negligible. Fowler has concluded that Thomson's observations were of a return stroke, passing through a previously ionized region, rather than a true breakdown wave [4]. Fowler gives credit for the first laboratory observation of a breakdown wave to Beams, who was initially looking at lags in spectral line emission in discharges, but soon began to investigate the breakdown process itself [5]. Working with rotating mirrors, Beams noted that waves always traveled from the ungrounded to the grounded electrode with speeds up to 4×10^7 meter/sec [6]. Snoddy, *et al.*, also observed waves from electrodes of both polarities with velocities up to 10^8 meter/sec [7]. They attempted to measure supporting currents and reported values up to 940 amp. The pressure dependence of such waves was also investigated. Later work by Snoddy and coworkers reported a linear dependence of velocity with applied field [8]. None of the experimenters report observations of doppler shifts in the emitted light.

Working with plasma shock tubes, Fowler and Hood reported waves with speeds of between 10^6 meter/sec and 4×10^6 meter/sec in argon and hydrogen [9]. They reported wave speeds which were linear with applied field. The wave speed was independent of pressure at low pressures, decreasing with pressures at pressures above 0.5 Torr.

The first attempt to make quantitative measurements of wave structures in a breakdown tube is due to Haberstich, who measured velocities for both pro- and antiforce waves in helium and argon [10]. Haberstich used photomultipliers to determine wave velocities. By use of voltage probes, he demonstrated the fact that the luminous front was accompanied by electron charge. By using microwave interferometer techniques, estimates of electron densities were obtained. The velocities measured by Haberstich range from 10^6 meter/sec to 1.4×10^7 meter/sec in helium and argon.

The most recent measurements reported are by Blais and Fowler [11]. Also working with helium, they measured wave velocities from 10^6 meter/sec to 5×10^7 meter/sec. The wave velocities were generally linear with the ratio of field to pressure for both proforce waves and antiforce waves. They also were able to estimate the temperature of the luminous region by optical means from observation of intensities of different spectral lines. Electron densities were estimated from absolute optical intensity measurements. Peak densities for anti-force waves were around 4×10^{14} electrons/meter^3, around a factor of 100 less than observed by Haberstich. The difference in observed densities between Blais and Fowler's work and that of Haberstich is attributed to the different resolution time of their measurement techniques, since Haberstich used microwave probes.

The state of the theory of breakdown waves has been very slow to advance, probably due in large part to the lack of quantitative experimental data. In reporting his investigation of lightning, Schonland gave an estimate of the velocity of leader strokes based on the kinetic energy gained by an electron between collisions from an externally applied electric field [12]. The concept of the electron motion being the determining factor in the speed of the lightning stroke is due to Simpson, who pointed out that the general features of lightning could be explained by assuming the presence of an electric field of sufficient intensity to produce ionization and that the relative mobility of the electrons and positive ions is such that the positive ions could be assumed at rest while the electrons rapidly passed away from the point of ionization [13]. The estimate by Schonland of the speed was greater than 10^5 meter/sec which was of the proper order of magnitude of speeds observed in breakdown waves, but predicted the speed to depend upon the square root of the applied field as opposed to the observed linear dependence.

The initial attempt to apply the concepts of fluid dynamics to the breakdown waves was by Paxton and Fowler [14]. The model was that of an electron shock wave propagating into a neutral gas, assuming ionization to be produced by either electron impact or photoionization. Writing the fluid equations for a gas componsed of electrons, ions, and neutrals; applying shock conditions at the interface with the neutral gas; and assuming that the velocities of the ions and neutrals were constant, they were able to predict wave speeds in rough agreement with observations. The work was important in demonstrating that a one-dimensional, time-independent solution of the fluid equations could be applied to the processes occuring in breakdown tubes and pointing out that electron pressure could be a sufficient driving force for the propagation of the wave. The pressure of the electrons was supplied by the high temperature attained in the electric field imposed on the gas.

The promise of the Paxton and Fowler theory led Shelton and Fowler to a more careful examination of the fluid dynamics of the three-component gas [15]. They found that, even though the heavy ion velocities were quite small, the momentum and energy changes of the heavy ions and neutrals were of comparable magnitude to those of the electrons and thus that they could not be neglected. However, they were able to produce a set of equations involving the electrons alone in which the heavy particle velocity could validly be considered constant and were then able to decouple the equations.

Using a one-dimensional time-independent frame of reference, Shelton and Fowler obtained expressions for collisional transfer of momentum and energy between the electrons and neutrals and were able to derive a solution for a strong shock propagating into a neutral gas for the proforce case (force on the electrons due to electric field is in the direction of wave motion). The Shelton-Fowler solution predicts a minimum velocity for propagation of the shock waves which is above observed velocities but is quite successful in predicting the general features of waves in the velocity regimes where it applies. The antiforce waves are not predicted by the Shelton-Fowler theory, although they point out that their equations may possess a weak shock solution. Prior to the present work, the only quantitative theory for antiforce waves is due to Albright and Tidman [16] and Klingbeil, Tidman and Fernsler [17]. The latter authors base their theory on the contention by Leob that photoionization is required to explain antiforce waves [18]. However, the presence of antiforce waves in an atomic gas cannot be due to self induced photoionization because of energy considerations and a theory based on photoionization cannot be considered satisfactory for all cases. The present effort demonstrates that electron pressure is adequate to produce the propagating force for antiforce waves in all cases and that a weak shock solution is available for all wave speeds observed.

CHAPTER III

FLUID MODEL

The basis for this investigation of antiforce waves in breakdown tubes will be the three-component fluid equations as adopted by Fowler [19]. These equations are the statements of conversation of mass, momentum, and energy and are written in Eulerian form. These equations are easily derived by considering quantities within control volumes and equating their time rate of change with flux through the surface of the control volume. When written for particle densities, these equations are as follows:

The equations for mass continuity are:

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} nv = \beta n \ ,$$

$$\frac{\partial}{\partial t} N_i + \frac{\partial}{\partial x} N_i V_i = \beta n \ , \qquad (1)$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} NV = - \beta n .$$

The problem is assumed to be one-dimensional with the x axis taken in the direction of wave propagation and t is, of course, time. The electron density in electrons per unit volume is n, the electron velocity v, ion density and velocity N_i and V_i and the neutral particle density and velocity N and V. The ionization is assumed due to electron collisions with neutrals and β represents the ionization frequency. β is given by $\langle\sigma_i VN\rangle$ where σ_i is the ionization cross section and the brackets represent a suitable average over the electron and neutral distributions. More will be said about β after the remainder of the equations is presented. A constant mass term has been factored out of each of the above equations.

The equations for conservation of momentum (Newton's second law) are:

$$\frac{\partial}{\partial t}(nmv) + \frac{\partial}{\partial x}(nmv^2+P_e) = -enE-\Delta_e(mv) + \Delta_i(mv)$$

$$\frac{\partial}{\partial t}(N_iM_iV_i+NMV) + \frac{\partial}{\partial x}(M_iN_iV_i^2+MNV^2+P_i+P) = eN_iE + \Delta_e(mv)-\Delta_i(mv) \tag{2}$$

The forces are assumed due to an external electric field E with no magnetic field present. The electron mass is m, ion mass M_i, neutral mass M, electron pressure P_e, ion pressure P_i and neutral pressure P. The terms $\Delta_e(mv)$ and $\Delta_i(mv)$ are the respective elastic and inelastic momentum transfer terms for electron-neutral collisions. e represents the charge (positive) of an electron. The heavy particle equations have been combined.

The equations for conservation of energy are:

$$\frac{\partial}{\partial t}\left(\frac{nmv^2}{2} + W_e\right) + \frac{\partial}{\partial x}\left[\frac{nmv^3}{2} + (P_e+W_e)v+q_e\right] = -envE - \Delta_e(\tfrac{1}{2}mv^2)+\Delta_i(\tfrac{1}{2}mv^2),$$

$$\frac{\partial}{\partial t}\left(\frac{N_iM_iV_i^2}{2} + \frac{NMV^2}{2} + W_i+W\right) + \frac{\partial}{\partial x}\left[\frac{NM_iV_i^3}{2} + \frac{NMV^3}{2} + (P_i+W_i)V_i+(P+W)V+q_i+q\right]$$

$$= eN_iV_iE + \Delta_e(\tfrac{1}{2}mv^2)-\Delta_i(\tfrac{1}{2}mv^2) \quad . \tag{3}$$

Again, the equations for ions and neutrals have been combined. The internal energy densities (energy/volume) of the electrons, ions, and neutrals are given by W_e, W_i, W. The electron, ion, and neutral heat conduction terms are q_e, q_i, and q. The elastic and inelastic collisional energy transfer terms are given by $\Delta_e(\frac{1}{2}mv^2)$ and $\Delta_i(\frac{1}{2}mv^2)$ where, as before, only collisions between electrons and neutrals are considered.

Now the terms $\Delta_e(mv)$ and $\Delta_e(\frac{1}{2}mv^2)$ can be calculated exactly from the kinetic theory of collisions using a technique due to Fowler [20].

Consider a particle of mass M_1 and velocity $\bar{V}_1$ which collides elastically with a particle of mass M_2 and velocity $\bar{V}_2$. The bar denotes vector quantities.

The velocity $\bar{G}$ of the center of mass of the system of two particles is given by:

$$\bar{G} = \frac{M_1\bar{V}_1 + M_2\bar{V}_2}{M_1 + M_2} . \tag{4}$$

The relative velocity $\bar{g}$ of the two particles is:

$$\bar{g} = \bar{V}_1 - \bar{V}_2 .$$

In the center of mass system the velocity of particle 1 is $\bar{U}_1$, that of particle 2 is $\bar{U}_2$. Conservation of momentum in the center of mass system is expressed by:

$$M_1\bar{U}_1 + M_2\bar{U}_2 = 0, \qquad \bar{G} = 0 \tag{5}$$

Now, since the relative velocity is independent of the frame of reference,

$$\bar{g} = \bar{U}_1 - \bar{U}_2 . \tag{6}$$

Thus,

$$\bar{U}_1 = \frac{M_2\bar{g}}{M_1+M_2} , \qquad \bar{U}_2 = -\frac{M_1\bar{g}}{M_1+M_2} . \tag{7}$$

Suppose the collection of particles of type one have an equilibrium temperature T_1, those of type two an equilibrium temperature T_2. Define M^* and $\bar{G}^*$:

$$\begin{aligned} M^* &= \frac{T_2}{T_1} M_1 , \\ \bar{G}^* &= \frac{M^*\bar{V}_1+M_2\bar{V}_2}{M^*+M_2} . \end{aligned} \tag{8}$$

Then $\bar{V}_1$ and $\bar{V}_2$ can be written in terms of $\bar{G}^*$ and $\bar{g}$.

$$\begin{aligned} \bar{V}_1 &= \bar{G}^* + \frac{M_2\bar{g}}{M^*+M_2} , \\ \bar{V}_2 &= \bar{G}^* - \frac{M^*\bar{g}}{M^*+M_2} . \end{aligned} \tag{9}$$

Now,

$$(M_1+M_2)\bar{G} = M_1(\bar{g}+\bar{V}_2) + M_2\bar{V}_2 ,$$

or

$$\bar{G} = \bar{G}^* + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}\,\bar{g} . \tag{10}$$

Now the change in kinetic energy of particle one due to the collision is given by:

$$\Delta E = \tfrac{1}{2}M_1(V_1^2-V_1'^2) ,$$

where the prime denotes the value of the quantity following the collision. This can be written as

$$\Delta E = \tfrac{1}{2}M_1\left[\bar{G} + \frac{M_2}{M_1+M_2}\,\bar{g}\right]^2 - \tfrac{1}{2}M_1\left[\bar{G} + \frac{M_2}{M_1+M_2}\,\bar{g}'\right]^2 \quad , \text{ since } \bar{G} \text{ is constant.}$$

Expanding:

$$\Delta E = \frac{M_1M_2}{M_1+M_2}\,\bar{G}\cdot(\bar{g}-\bar{g}') + \frac{\tfrac{1}{2}M_1M_2}{M_1+M_2}\,(g^2-g'^2) \ ,$$

or

$$\Delta E = \frac{M_1M_2\bar{G}}{M_1+M_2}\cdot(\bar{g}-\bar{g}') \quad , \tag{11}$$

since $g = g'$ for elastic collisions.

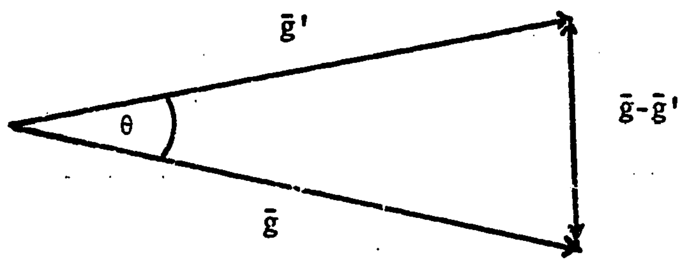

Figure 1. Relative velocity relation in center of mass system.

Then

$$\Delta E = \frac{M_1M_2}{M_1+M_2}\left[\bar{G}^*\cdot(\bar{g}-\bar{g}') + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}\,\bar{g}\cdot(\bar{g}-\bar{g}')\right] \ ,$$

or

$$\Delta E = \frac{M_1M_2}{M_1+M_2}\left[\bar{G}^*\cdot(\bar{g}-\bar{g}') + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}\,2g^2\sin^2\frac{\theta}{2}\right], \tag{12}$$

where θ is the angle between $\bar{g}$ and $\bar{g}'$. Now the expression for ΔE must be averaged over the distributions of particles N_1 and N_2:

$$\langle\Delta E\rangle = \iint dN_1 dN_2 \Delta E \; g \int I(\theta)\, d\Omega_{cm}$$

where dN and dN represent the density of particles of type one and two and $\int I(\theta)d\Omega_{cm}$ is the collision cross section. The term $g\int I(\theta)d\Omega_{cm}$ represents the collision frequency, and Ω_{cm} is the solid angle in the center of mass system. It should be noted that <ΔE> does not have the same dimensions as ΔE, and in fact is not the same quantity. If the distributions are considered to follow the Maxwell-Boltzmann form, in the center of mass system:

$$dN_1dN_2 = N_1N_2\left(\frac{M_1}{2\pi kT_1}\right)^{3/2}\left(\frac{M_2}{2\pi kT_2}\right)^{3/2} e^{-\frac{M_1(\bar{V}_1-\bar{W}_1)^2}{2kT_1}} e^{-\frac{M_2(\bar{V}_2-\bar{W}_2)^2}{2kT_2}}$$

$$\cdot d\bar{V}_1 d\bar{V}_2 \quad ,$$

where k is the Boltzmann constant, and N_1, N_2 are the number of particles/volume of type one and two. The particles are assumed to have non zero average velocities W_1 and W_2.

This can be written as:

$$dN_1dN_2 = N_1N_2\left(\frac{M^*}{2\pi kT_2}\right)^{3/2}\left(\frac{M_2}{2\pi kT_2}\right)^{3/2} e^{-\frac{1}{2kT_2}[M^*(\bar{V}_1-\bar{W}_1)^2+M_2(\bar{V}_2-\bar{W}_2)^2]}$$

$$\cdot d\bar{V}_1 d\bar{V}_2 \ .$$

The transformation from V_1, V_2 to G*, g as integrating variables has the Jacobian:

$$\left|\frac{\partial(G^*,g)}{\partial(V_1,V_2)}\right| = \left\|\begin{matrix} \frac{M^*}{M^*+M_2} & \frac{M_2}{M^*+M_2} \\ 1 & -1 \end{matrix}\right\| = 1 \ .$$

So, in terms of these variables:

$$<\Delta E> = \frac{M_1M_2}{M_1+M_2} N_1N_2[\frac{M^*M_2}{(2kT_2)^2}]^{3/2}\iiint [e^{-\frac{M^*}{2kT_2}(\bar{G}^*+ \frac{M_2\bar{g}}{M_1^*+M_2} - \bar{W}_1)^2}]$$
$$\cdot[e^{-\frac{M_2}{2kT_2}(\bar{G}^*- \frac{M^*\bar{g}}{M_1+M_2} - \bar{W}_2)^2}]g[\bar{G}^*\cdot(\bar{g}-\bar{g}')+ \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)} 2g^2\sin^2\frac{\theta}{2}]$$
$$\cdot d\bar{G}^*d\bar{g}\ I(\theta)d\Omega_{cm} \ . \tag{13}$$

Now consider the integral over the solid angle $d\Omega_{cm}$. The term $2g^2\sin^2\theta/2$ can be written as $g^2(1-\cos\theta)$. If the polar axis is taken to be along $\bar{g}$, the components of $(\bar{g}-\bar{g}')$ are given by:

$$(\bar{g}-\bar{g}')_z = g(1-\cos\theta),$$
$$(\bar{g}-\bar{g}')_y = g\sin\theta\sin\phi \ ,$$
$$(\bar{g}-\bar{g}')_x = g\sin\theta\cos\phi \ .$$

Then the scalar product $\bar{G}^*\cdot(\bar{g}-\bar{g}')$ is given by:

$$G^*\cdot(g-g') = G_x^* g \sin\theta\cos\phi + G_y^* g\sin\theta\sin\phi + G_z^* g\ (1-\cos\theta) \ .$$

When this is integrated over $d\Omega_{cm}$, the first two terms yield zero. Since the polar axis was taken along $\bar{g}$, the last term is equivalent to $\bar{G}^*\cdot\bar{g}(1-\cos\theta)$.

Writing $M_R \equiv M_1M_2/(M_1+M_2)$, the expression for $<\Delta E>$ is:

$$<\Delta E> = M_RN_1N_2[\frac{M_2M^*}{(2kT_2)^2}]^{3/2}\iint [e^{-\frac{M^*}{2kT_2}(\bar{G}^*+ \frac{M_2\bar{g}}{M^*+M_2} -\bar{W}_1)^2}]$$
$$\cdot[e^{-\frac{M_2}{2kT_2}(G^*- \frac{M^*\bar{g}}{M_1^*+M_2} -\bar{W}_2)^2}]g[\bar{G}^*\cdot\bar{g}+ \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)} g^2]$$
$$\cdot d\bar{G}^*d\bar{g}\int 2\pi(1-\cos\theta)I(\theta)\sin\theta d\theta \ . \tag{14}$$

Now one further change in variables is in order:

$$\bar{s} = \bar{g} - (\bar{W}_1-\bar{W}_2) = \bar{g} - \bar{W} ,$$

$$\bar{t} = \bar{G}^* - \frac{M^*\bar{W}_1 + M_2\bar{W}_2}{M^* + M_2} .$$

The Jacobian of this transformation is unity. The utility of this change in variables is to simplify the exponential factors, at the possible expense of more complicated terms for g^2 and $\bar{G}^*\cdot\bar{g}$. In terms of $\bar{s}$ and $\bar{t}$, the exponential terms become:

$$M^*(\bar{G}^*+\bar{g}\,\frac{M_2}{M^*+M_2} -\bar{W}_1)^2 + M_2(\bar{G}^*-\bar{g}\,\frac{M^*}{M^*+M_2} -\bar{W}_2)^2$$

$$= M^*(\bar{t}+ \frac{M_2}{M^*+M_2}\,\bar{s})^2 + M_2(\bar{t}- \frac{M^*}{M^*+M_2}\,\bar{s})^2,$$

$$= (M^*+M_2)t^2 + \frac{M^*M_2}{M^*+M_2}\,s^2 .$$

The factors g^2 and $\bar{G}^*\cdot\bar{g}$ are:

$$g^2 = s^2 + 2\bar{s}\cdot(\bar{W}_1-\bar{W}_2) + (\bar{W}_1-\bar{W}_2)^2 ,$$

$$\bar{G}^*\cdot\bar{g} = \bar{s}\cdot\bar{t} + \bar{s}\cdot(\frac{M^*\bar{W}_1 + M_2\bar{W}_2}{M^*+M_2}) + (\bar{W}_1+\bar{W}_2)\cdot(\frac{M^*\bar{W}_1 + M_2\bar{W}_2}{M^*+M_2}) .$$

Finally, <ΔE> is:

$$\langle\Delta E\rangle = M_R N_1 N_2 [\frac{M_2M^*}{(2\pi kT_2)^2}]^{3/2} \iint e^{-\frac{M^*+M_2}{2kT_2}t^2}\, e^{-\frac{M^*M_2}{2kT_2}s^2}$$

$$\cdot\sqrt{s^2+2\bar{s}\cdot\bar{W}+\bar{W}^2}\,[\bar{s}\cdot\bar{t}+\bar{s}\cdot(\frac{M^*\bar{W}_1 + M_2\bar{W}_2}{M^* + M_2}) +\bar{W}\cdot(\frac{M^*\bar{W}_1 + M_2\bar{W}_2}{M^* + M_2})$$

$$+ \bar{t}\cdot\bar{W} + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}\,(s^2+2\bar{s}\cdot\bar{W}+W^2)]d\bar{s}d\bar{t}\int 2\pi(1-\cos\theta)I(\theta)\sin\theta d\theta.$$

where $\bar{W}_1 - \bar{W}_2$ has been written as $\bar{W}$.

Consider the integration over t. The terms $\bar{s}\cdot\bar{t} = s_x t_x + s_y t_y + s_z t_z$ and $\bar{t}\cdot\bar{W} = t_x W_x + t_y W_y + t_z W_z$ integrate to zero since they are all of the form:

$$\int_{-\infty}^{\infty} e^{-\frac{M^* + M_2}{2kT_2} t_x^2} t_x dt_x = 0 \ .$$

After performing the t integration:

$$\langle\Delta E\rangle = M_R N_1 N_2 \left[\frac{M^* M_2}{2kT_2(M^*+M_2)}\right]^{3/2} \int e^{-\frac{M^*M_2}{M^*+M_2}\frac{s^2}{2kT_2}} \sqrt{s^2+2\bar{s}\cdot\bar{W}+\bar{W}^2}$$

$$\cdot\left[\bar{s}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right)+\bar{W}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right) + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}(s^2+2\bar{s}\cdot\bar{W}+\bar{W}^2)\right]$$

$$\cdot d\bar{s}\int 2\pi(1-\cos\theta)I(\theta)\sin\theta d\theta \ .$$

Experimental data for elastic collision cross sections show that for a wide range of energy, $I(\theta)$ is proportional to $1/g$ [21]. If we write $I(\theta) = I_o(\theta)/g$, then

$$\int 2\pi(1-\cos\theta)I(\theta)\sin\theta d\theta = \frac{1}{g}\int 2\pi(1-\cos\theta)I_o(\theta)\sin\theta d\theta \equiv \frac{\sigma_o}{g} \ .$$

Now the expression for $\langle\Delta E\rangle$ is easily integrated:

$$\langle\Delta E\rangle = \sigma_o M_R N_1 N_2 \left[\frac{M^* M_2}{2kT_2(M^*+M_2)}\right]^{3/2} \int e^{-\frac{M^*M_2}{M^*+M_2}\frac{s^2}{2kT_2}}$$

$$\cdot\left[\bar{s}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right)+\bar{W}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right) + \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)}\right.$$

$$\left.\cdot(s^2+2\bar{s}\cdot\bar{W}+W^2)\right]d\bar{s} \ . \qquad (17)$$

Using the same argument as for the integration over t, the terms $\bar{s}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right)$ and $2\bar{s}\cdot\bar{W}$ integrate to zero. Performing the integration:

$$\langle\Delta E\rangle = \sigma_o M_R N_1 N_2 \left[\bar{W}\cdot\left(\frac{M^*\bar{W}_1+M_2\bar{W}_2}{M^*+M_2}\right) + W^2 \frac{M_2(M_1-M^*)}{(M_1+M_2)(M^*+M_2)} + \frac{3M_2(M_1-M^*)}{(M_1+M_2)M^*M_2} kT_2\right] .$$

Substituting $M^* = \frac{M_1}{T_1} T$, this simplifies to:

$$\langle\Delta E\rangle = \sigma_o \frac{M_R N_1 N_2}{M_1+M_2} \left[(\bar{W}_1-\bar{W}_2)\cdot(M_1\bar{W}_1+M_2\bar{W}_2)+3k(T_1-T_2)\right] . \tag{18}$$

Now for the case of collisions of electrons with neutrals, M_1 = mass of electron, M_2 = mass of neutral, and M_R/M_1+M_2 is very nearly equal to M_1/M_2^2. If the speed of the electron is greater than that of the neutrals ($W_1>W_2$) and the electron temperature is much greater than the neutral gas, ($T_1>T_2$), $\langle\Delta E\rangle$ can be approximated by:

$$\langle\Delta E\rangle \simeq \sigma_o N_1 N_2 \left\{M_1\bar{W}_2\cdot(\bar{W}_1-\bar{W}_2) + 3\frac{M_1}{M_2} kT_1\right\} . \tag{19}$$

This is the desired result for the elastic collisional energy transfer term. Setting $\langle\Delta E\rangle = \Delta_e(\frac{1}{2}mv^2)$, we have:

$$\Delta_e(\tfrac{1}{2}mv^2) = n\sigma_o N[mV(v-V)+3\frac{m}{M} kT_e] . \tag{20}$$

$\sigma_o N$ has the dimensions of 1/sec and represents a collision frequency. Writing $K_1 \equiv \sigma_o N$,

$$\Delta_e(\tfrac{1}{2}mv^2) = n\ K_1[mV(v-V)+3\frac{m}{M} kT_e] . \tag{21}$$

A similar treatment of the collisional elastic momentum transfer term has the result:

$$\Delta_e(mv) = nmK_1(v-V) \quad . \tag{22}$$

The expressions for $\Delta_i(mv)$ and $\Delta_i(\frac{1}{2}mv^2)$ are not at all easily derived from collisional calculations. Shelton was able to derive a form for $\Delta_i(mv)$ by demanding that the equation for momentum conservation remain invariant under a Galilean frame transformation [22]. This must be true since Newton's second law holds under Galilean transformations. Shelton found that this principle of invariance demanded that $\Delta_i(mv)$ have the form:

$$\Delta_i(mv) = \beta nmV \quad . \tag{23}$$

Applying the same technique to the energy conservation equations, an expression found for $\Delta_i^e(mv^2/2)$ was given as:

$$\Delta_i^e(\tfrac{1}{2}mv^2) = \tfrac{1}{2}\beta nmV^2 - \beta ne\phi_i \tag{24}$$

where $e\phi_i$ is the energy required to ionize a neutral atom. This term is on less firm foundation than that for $\Delta_i(mv)$ since the energy equation is a scalar equation and thus the addition of any scalar quantity would not be affected by a frame change. The argument in support of equation (24) is that ionization of a neutral creates an electron which would be expected to possess the original kinetic energy it possessed prior to ionization, that is, $\frac{1}{2}mV^2$. Ionization thus should add an energy/volume of $\frac{1}{2}\beta nmV^2$ to the electron gas. However, the ionization process requires an amount of energy $e\phi_i$ for each electron-ion pair creation. This energy represents a loss to the electron gas so that the net energy/volume change for the electrons should be given by equation (24), retaining the reservation that other terms may be neglected by this argument but hopefully are not of importance. The term $\Delta_i^n(\frac{1}{2}mv^2)$ for the neutrals is assumed to be $\frac{1}{2}\beta nmV^2$, corresponding to the loss of the kinetic energy of the created electron.

In addition to the one-dimensional form for the description of the three component gas of electrons, ions, and neutrals, a solution which is independent of time is assumed to exist. That is to say, in the frame which moves with the wave front, the densities and velocities of the electrons, ions, and neutrals have no dependence on time. Since no doppler shift is detected in the wave front, the heavy particles have small or zero velocity in the laboratory. Thus, if the wave front has a laboratory velocity V, the heavy particles will have a velocity -V in the wave frame. In the wave frame, equations (1) are now:

$$\begin{aligned} \frac{\partial}{\partial x} nv &= \beta n \ , \\ \frac{\partial}{\partial x} N_i V_i &= \beta n \ , \\ \frac{\partial}{\partial x} NV &= -\beta n \ . \end{aligned} \tag{25}$$

Equations (2) are:

$$\begin{aligned} &\frac{\partial}{\partial x} (nmv^2+nkT_e) = -enE - K_1 nm(v-V) + \beta nmV \ , \\ &\frac{\partial}{\partial x} [N_i M_i V_i^2+NMV^2+N_i kT_i+NkT] = eN_i E + K_1 nm(v-V) - \beta nmV \ , \end{aligned} \tag{26}$$

and equations (3):

$$\frac{\partial}{\partial x}\left(\frac{nmv^3}{2}+\frac{5}{2}nvkT_e+q_e\right) = -envE - K_1nmV(v-V) + \tfrac{1}{2}\beta nmV^2-\beta ne\phi_i \ ,$$

$$\frac{\partial}{\partial x}\left(\frac{N_iM_iV_i^3}{2}+\frac{NMV^3}{2}+\frac{5}{2}N_iV_ikT_i+\frac{5}{2}NVkT+q_i+q\right) = eN_iV_iE$$

$$+ K_1nmV(v-V) - \tfrac{1}{2}\beta nmV^2 \quad . \qquad (27)$$

The term $3\frac{m}{M}kT_en$ has been neglected in equation (27) and it has been assumed that the electrons, ions, and neutrals behave like perfect gases so their internal energy densities are given by $\frac{3}{2}nkT_e$, $\frac{3}{2}N_ikT_i$, and $\frac{3}{2}NkT$, respectively. From equation (25):

$$\frac{\partial}{\partial x}(nv-N_iV_i) = 0,$$

or

$$N_iV_i - nv = \text{constant.}$$

Since the gas in front of the wave front is neutral, the one-dimensional time-independent case has:

$$N_iV_i - nv = 0 \ . \qquad (28)$$

This is the zero current condition since $e(N_iV_i-nv)$ represents a current. Thus, for a stationary plane wave there can be no current. Now in the stationary frame with no external magnetic field, the only non-zero member of Maxwell's equations is Poisson's equation, which becomes:

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_0}(N_i-n) \quad . \qquad (29)$$

ε_o is the permittivity in the MKS system. By summing equations (26), and using equation (29), an integrable expression results. Performing the integration:

$$nmv^2 + M_iN_iV_i^2 + MNV^2 + nkT_e + N_ikT_i + NkT = \frac{\varepsilon_o}{2} E^2 + C \ ,$$

where C is a constant to be determined by conditions ahead of the wave. Now ahead of the wave n and N_i are equal to zero, V equals V_o, E equals E_o, and N equals N_o. Then:

$$nmv^2 + N_iM_iV_i^2 + NMV^2 - MN_oV_o^2 + nkT_e + N_ikT_i + NkT - N_okT_o = \frac{\varepsilon_o}{2}(E^2-E_o^2) \ . \tag{30}$$

Similarly, by summing and integrating equations (27), making use of equations (25) and (28):

$$nmv^3 + N_iM_iV_i^3 + NMV^3 + 5nvkT_e + 5N_iV_ikT_i + 5NVkT$$
$$+ 2(q_e + q_i + q) + 2nve\phi_i = N_oMV_o^3 + 5N_oV_okT_o \ . \tag{31}$$

From the last pair of equations (28)

$$N_iV_i + NV = N_oV_o \ ,$$

and

$$M = M_i + m \ .$$

It is reasonable to assume that the neutrals and ions have essentially the same temperature since they have almost the same mass and should stay in thermal equilibrium. Making use of this assumption and the relations between the particles, equations (30) and (31) can be put in the form:

$$nm(v-V_o)v + nkT_e - \frac{\varepsilon_o}{2}(E^2-E_o^2) = nMv(V_i-V_o) - N_ok(T-T_o) \quad , \qquad (32)$$

$$nmv(v^2-V_o^2) + 5nvkT_e + 2nve\phi_i = -2nMvV_o(V_i-V_o) - 5N_okV_o(T-T_o)$$
$$-2(q_e+q_i+q) \quad . \qquad (33)$$

Equation (32) is the global momentum equation and equation (33) the global energy equation for the three component gas composed of electrons, ions, and neutrals. The left sides of these equations are functions of electron quantities alone while the right sides contain the terms V_i-V_o and $T-T_o$ which must be evaluated if the equations are to be useful in determining electron quantities. The energy equation also contains the terms $q_e + q_i + q$ which are heat conduction elements. It is customary to assume that the heat conduction is negligible over the time span of the wave passage. It will be seen later that this may be a questionable assumption for antiforce waves but it will be used for the time being. The absence of a doppler shift indicates that neither the ions nor the neutrals have appreciable motion in the laboratory and, based on this, the velocity of the ions and neutrals have both been assumed to be equal to V_o except where their velocity difference occurs. It will be seen that it is this slight difference in velocity, V_i-V_o, between the ions and neutrals which will allow a solution for the antiforce waves.

Before investigating the term V_i-V_o, the temperature difference term, $T-T_o$, will be discussed. The source of the temperature term, $T-T_o$, must be collisions of neutrals with electrons since the electron-ion collisions are so infrequent as to be negligible. The energy transfer rate for electron-neutral collisions has been calculated and is given by equation (18). If the neutrals are assumed to be at rest, $W_2 = 0$, and the electron temperature is much larger than the neutral, $T_e \gg T_o$:

$$\langle \Delta E \rangle = \sigma_o N_o n \frac{2\,mM}{(M+m)^2} [m \frac{v^2}{2} + \frac{3kT_e}{2}] \quad .$$

Now

$$\langle \Delta E \rangle = \frac{d}{dt} (\frac{3}{2} kT) \quad .$$

If

$$\frac{mv^2}{2} \approx \frac{3}{2} kT_e ,$$

$$\frac{d}{dt} (\frac{3}{2} kT) \approx \frac{m}{M} 4 \; K_1 n \frac{3}{2} kT_e \quad ,$$

where K_1 has been written for $N_o \sigma_o$. Writing the Eulerian form of the time derivative:

$$N_o \frac{\partial}{\partial t} (\frac{3}{2} kT) + \frac{\partial}{\partial x}(N_o V_o \frac{3}{2} kT) = 4 \; K_1 \frac{m}{M} (\frac{3}{2} nkT_e) \quad .$$

Then $T - T_o$ is of the order $4K_1 \frac{n \; T_e}{N_o V_o} \frac{m}{M} x$ since $\frac{\partial}{\partial t} \frac{3}{2} kT = 0$ in the stationary frame. For helium, m/M is of the order 10^{-4} and is even smaller for gases of higher atomic weight, n/N_o is quite small, and V_o is large. Thus for values of x expected for waves of interest (around 0.1 meter), the term $T - T_o$ should be negligible (on the order of 10^{-7} °K).

To calculate the expression for $V_i - V_o$, consider heavy ions moving in the presence of neutrals with an external field acting on the ions. From the above discussion of collision phenomena, the equation for conservation of ion momentum in a stationary frame can be written as:

$$\frac{d}{dx}\,(N_iM_iV_i^2) = eN_iE - K_iN_iM_i(V_i-V_o) \quad . \tag{34}$$

Here K_i represents the collision frequency of ions with neutrals and V_o is the neutral particle velocity as above. If the field E can be considered to be constant over a region of integration and N_i is considered constant over the same region:

$$V_i\,\frac{dV_i}{dx} + \frac{K_i}{2}\,V_i = \frac{e\,E}{2M_i} + \frac{K_iV_o}{2} \quad . \tag{35}$$

This can be integrated by writing:

$$y = V_i - \frac{\dfrac{e\,E}{2M_i} + \dfrac{K_iV_o}{2}}{\dfrac{K_i}{2}} \,,$$

and

$$[y+(\frac{eE}{M_iK_i} + V_o)]\,\frac{dy}{dx} + \frac{K_i}{2}\,y = 0 \ .$$

Integrating:

$$y + (\frac{eE}{M_iK_i} + V_o)\,\ell n\ y = -\frac{K_i}{2}\,x + (\frac{eE}{M_1K_1} + V_o)\ \ell nC \ ,$$

where C is to be evaluated. The solution can be written:

$$y = C\ \exp\left|\frac{-\dfrac{K_i}{2}\,x}{\dfrac{eE}{M_iK_i}+V_o}\right| \quad \exp\left|\frac{-y}{\dfrac{eE}{M_iK_i}+V_o}\right| \quad ,$$

or

$$V_i - \left(\frac{e}{M_iK_i} E + V_o\right) = C \exp\left[1 - \frac{V_i}{V_o + \frac{eE}{M_iK_i}}\right] \exp\left[\frac{-\frac{K_i x}{2}}{\frac{eE}{M_iK_i} + V_o}\right] .$$

To evaluate C, set $x = 0$, $V_o = V_i$. Then:

$$-\frac{eE}{M_iK_i} = C \exp\left[\frac{\frac{eE}{M_iK_i}}{\frac{eE}{M_iK_i} + V_o}\right] ,$$

and

$$V_i - V_o = \frac{eE}{M_iK_i} \left[1 - \exp^{\left(\frac{V_o - V_i}{\frac{eE}{M_iK_i} + V_o}\right)} \exp^{\left(\frac{-K_i \frac{x}{2}}{\frac{eE}{M_iK_i} + V_o}\right)}\right] . \qquad (36)$$

The equation for ion mobility is $V_i - V_o = bE$ where b is the mobility constant. If b* is defined as $b^* = e/M_iK_i$, equation (36) has the form of a mobility equation with a correction factor. Since $V_i - V_o$ is assumed to be small, a first approximation for its value is:

$$V_i - V_o = b^*E .$$

Substituting this into the exponential factors,

$$V_i - V_o = b^*E\left[1 - \exp^{\frac{V_o - V_i}{V_i}} e^{\frac{-K_i x}{2V_i}}\right] .$$

The first exponent is very nearly equal to unity and V_i can be replaced by V_o without much error in the second exponent. Doing this:

$$V_i - V_o \simeq b^*E\left(1 - \exp^{-\frac{K_i x}{2V_o}}\right) ,$$

which is the desired expression for $V_i - V_o$.

The system of equations which will be taken to represent the action of the electrons for a stationary wave solution is now:

$$\frac{\partial}{\partial x} nv = \beta n \ , \tag{38}$$

$$\frac{\partial}{\partial x} [nmv^3 + nkT_e] = -enE - K_1 nm(v-V_o) + \beta nmV_o \ , \tag{39}$$

$$\frac{\partial}{\partial x} [nmv(v^2-V_o^2) + 5nvkT_e + nve\phi_i] = -2nevE - 2K_1 nmV_o(v-V_o) \ , \tag{40}$$

$$nm(v-V_o)v + nkT_e - \frac{\varepsilon_o}{2}[E^2-E_o^2] = -Mnvb^*E(1-\exp^{-K_i x/2V_o}) \ , \tag{41}$$

$$nmv(v^2-V_o^2) + 5nvkT_e + 2nve\phi_i = -2MnvV_o b^*E(1-\exp^{-K_i x/2V_o}) \ . \tag{42}$$

The final equation needed is Poisson's equation. Using equation (28), Poisson's equation is:

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_o} n(\frac{v}{V_i} - 1) \ ,$$

or

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_o} n(\frac{v}{V_o + b^*E(1-\exp^{-K_i x/2V_o})} - 1) \ . \tag{43}$$

CHAPTER IV

ANALYSIS OF FLUID EQUATIONS

In principle, one merely needs to solve equations (38) through (43) of Chapter III in order to arrive at a complete representation of all the quantities which characterize the electrons, ions, and neutrals as a function of position behind the wave front. The model is that of an infinite plane wave which, in the laboratory, is traveling in the positive x direction with speed V_o. Remembering that the heavy neutrals are assumed to be at rest in the laboratory because of the absence of observed doppler shift in the emitted radiation, the neutrals are being swept into the wave at a velocity V_o. In the frame where the wave front is considered to be stationary at $x = 0$ the neutrals thus have a velocity $-V_o$ and the stationary wave extends from $x = 0$ to $x = -\infty$. The plane $x = 0$ divides the neutral gas in front of the wave from the three component gas composed of the electrons, ions, and neutrals behind the wave. The initial conditions on the neutral side are easily stated. There are no ions or electrons so n and $N_i = 0$. The velocity of the neutrals is, from the above discussion, given by $-V_o$, and the electric field is E_o for the antiforce case which is being considered, since the force on the electrons from the external field is in a direction to drive them away from the wave front, that is, in the negative x direction. On the negative side of $x = 0$, the initial conditions require some thought. Obviously, in this model, the line x = 0 must represent a shock front. However, whether the shock is a strong discontinuity where densities and temperatures change discontinuously or a weak discontinuity where the derivatives of these quantities change discontinuously but the quantities themselves are continuous, must be decided.

Shelton, in analyzing the proforce waves, choose a strong discontinuity solution [15]. Assuming that the electrons and ions were formed discontinuously at x = 0, the shock conditions for the global momentum and energy equations were found to be:

$$n_o[v_o(v_o-V_o)+\frac{k(T_e)_o}{m}] = 0 ,$$

and

$$n_o\, v_o[v_o^2-V_o^2+5\,\frac{k(T_e)_o}{m}+\frac{2e\phi_i}{m}] = 0.$$

The subscript o denotes that the quantity has the value assumed at $x = 0$. For the strong shock, $n_o \neq 0$ and it is possible to solve for v_o in terms of V_o and $e\phi_i$. Specifically,

$$v_o = \frac{5}{8}V_o \pm \frac{(9V_o^2 + 16\,\frac{2e\phi_i}{m})^{\frac{1}{2}}}{8},$$

and

$$\frac{k(T_e)_o}{m} = v_o(V_o-v_o) \quad .$$

Since $(T_e)_o$ must be greater than or equal to zero, the absolute magnitude of v_o must be less than that of V_o and the negative sign must be chosen in the equation for v_o. Also, $|v_o|$ must be greater than zero. From this, Shelton found that the strong discontinuity imposed a lower limit on the velocity V_o. For helium, this number is 2.9×10^6 meter/sec. Velocities well below this value have been observed and the immediate reaction from this might be to question the formulation. However, one notes that $n_o = 0$ will also satisfy the global equations at the discontinuity. This condition along with $(\partial n/\partial x)_o \neq 0$ would constitute a weak discontinuity at $x = 0$, and could represent another potential solution which does not appear to demand a minimum wave velocity. Another argument which can be advanced in support of the weak discontinuity rather than a strong discontinuity is to consider the acoustic speed of a disturbance in an electron gas. This is given by $(\frac{5}{3}\frac{kT_e}{m})^{\frac{1}{2}}$ and it does not require a very large temperature to attain quite large speeds. Blais reports measurement of temperatures ranging from 1.5×10^5 °K to around 4×10^5 °K which correspond to sound speeds of 4.5×10^6 meter/sec and 7.4×10^6 meter/sec [11]. These are at the lower limit of the range of observed wave speeds which implies that for electron temperatures in excess of 10^6 °K, the observed wave speeds do not have very large mach numbers and it may be reasonable to expect weak discontinuity rather than strong discontinuity. In any event, the assumption will be made that a weak discontinuity solution exists and to some extent the results will be relied upon to justify the assumption.

This has immediate consequences with regard to the initial value of the electron velocity. From equation (38) of Chapter III:

$$\frac{\partial}{\partial x} nv = \beta n \quad .$$

Differentiating:

$$n \frac{\partial v}{\partial x} + v \frac{\partial n}{\partial x} = \beta n \ . \tag{44}$$

Now, setting $n_o = 0$ at $x = 0$ in (44) we have:

$$v_o (\frac{\partial n}{\partial x})_o = 0 \quad . \tag{45}$$

The assumption that $(\partial n/\partial x)_o \neq 0$ demands that $v_o = 0$ because $v_o \neq 0$ leads to a singularity at $x = 0$. Thus, the production equation gives the initial conditions at $x = 0$: $n_o = 0$, $v_o = 0$, $(\partial n/\partial x)_o \neq 0$.

Now Poisson's equation, (43) of Chapter III, gives

$$(\frac{\partial E}{\partial x})_o = \frac{e}{\varepsilon_o} n_o [\frac{v_o}{V_o + b^*E(1-e^o)} -1] = 0 \quad . \tag{46}$$

The initial conditions for the field are then, $(E)_{x=0} = E_o$, $(\frac{\partial E}{\partial x})_o = 0$.

The electric field has an initial value of E_o at the wave front and an initial slope of zero. As one proceeds into the wave structure, n attains a non-zero value and the electrons possess some velocity. Since the gas in front of the wave front is neutral, the electrons cannot cross $x = 0$ in the positive direction. From the zero current condition, $v = N_i V_i/n$, so that v/V_i is positive. Now

$$V_i = V_o + b^*E(1-e^{-K_i x/2V_o}) \quad .$$

The factor $[1-e^{-(K_i x/2V_o)}]$ is always less than or equal to unity, b^* has a maximum value of around 1 meter /sec-volt [23], and for waves with velocities of 10^7 meter/sec, E_o is of the order of 10^5 volt/meter. Thus $V_i - V_o$ is of the order of 10^5 meter/sec at the maximum and will be much less than this for large x. Even though V_i-V_o is reasonably large, setting $V_i = V_o$ represents a 1 per cent error at most and usually the error will be much smaller. Thus v has the same sign as V_o, the velocity of the neutrals. Since the neutrals have a negative velocity in the wave frame, the electron fluid has a general motion away from the wave front, with an initial speed of zero.

From this, it is a good approximation to write Poisson's equation as:

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_o} n(\frac{v}{V_o} -1) \ . \tag{47}$$

Now $\partial E/\partial x$ has an initial value of zero, and as n and v assume non-zero values, $\partial E/\partial x$ is negative until the point where $v/V_o = 1$. Since passing through the wave corresponds to proceeding in the negative x direction this implies that the magnitude of the field increases in the interior of the wave until the point where the electrons and the ions have the same velocity. As the electrons acquire velocity in excess of the ions, the field decreases in magnitude. Ultimately the field must go to zero since a conductor in equilibrium cannot support a field and it is assumed that the ions and electrons must ultimately come into an equilibrium state. This behavior of the field is due to the vastly larger mobility of the electrons versus the ions. The electrons respond to the external field almost instantly while the ions, due to their large mass, are much slower to react.

In this model, the electric field has some maximum value which is larger than the initial value E_o. This has consequences regarding the validity of the form of the momentum and energy conservation equations which have been derived. The global momentum equation:

$$nm(v-V_o)v + nkT_e = \frac{\varepsilon_o}{2}(E^2-E_o^2) - Mnvb^*E[1-e^{-(K_i x/2V_o)}] \quad , \quad (48)$$

and the global energy equation:

$$nmv(v^2-V_o^2) + 5nvkT_e + 2nve\phi_i = -2MnvV_o b^*E[1-e^{-(K_i x/2V_o)}] \quad , \quad (49)$$

should be valid throughout the wave. If the term $b^*E[1-e^{-(K_i x/2V_o)}]$ is eliminated by multiplying equation (48) by $-2V_o$ and summing with equation (49), the result is:

$$nmv(v-V_o)^2 + nkT_e(5v-2V_o) + 2nve\phi_i = -\varepsilon_o V_o(E^2-E_o^2) \quad . \quad (50)$$

Now this equation must hold where $v = V_o$, and $E = E_1$:

$$3n_1k(T_e)_1V_o + 2n_1V_oe\phi_i = -\varepsilon_o V_o(E_1^2-E_o^2) \quad .$$

Dividing by V_o,

$$3(nkT_e)_1 = -2n_1e\phi_i - \varepsilon_o(E_1^2-E_o^2) \quad . \quad (51)$$

Now E_1 is $>E_o$ and $ne\phi_i$ is >0. Thus equation (51) gives a negative temperature at the point at which the field reaches its maximum value. A negative temperature is clearly not allowed so there must be some fault in the equations which lead to that conclusion. When v is set equal to V_o and E set equal to E_1 in the momentum equation (48):

$$(nkT_e)_1 = \frac{\varepsilon_o}{2}(E_1^2-E_o^2) - Mn_1V_ob^*E_1[1-e^{-(K_i x_1/2V_o)}] \quad . \quad (52)$$

The first term on the right is positive since $E_1>E_o$. E_1 is positive since E_o is positive, V_o is negative and b* is positive, so the right side of equation (52) is positive and the difficulty with a negative T_e does not appear. Examining equation (49) at this point:

$$5n_1V_ok(T_e)_1 + 2n_1V_oe\phi_i = - 2Mn_1V_o^2b^*E_1[1-e^{-(K_ix_1/2V_o)}] . \qquad (53)$$

Dividing by V_o:

$$n_1(kT_e)_1 = - \frac{2}{5}\, n_1e\phi_i - \frac{2}{5}\, Mn_1V_ob^*E_1[1-e^{-(K_ix_1/2V_o)}] . \qquad (54)$$

This is not compatible with equation (52). Equation (52) was essentially derived from Newton's second law and there is no reason to suspect that any forces of consequence have been overlooked. The technique of demanding that the inelastic momentum transfer term remain invariant under a Galilian frame transformation should properly account for that term since the quantities in question are vectors. The energy equation, on the other hand, is a scalar equation so the frame invariance is not so useful since scalars will automatically satisfy that requirement. The possibility certainly exists that the argument for the form of the inelastic energy transfer operator has failed to include some term which is of consequence. The other possibility is that the heat conduction terms which have been assumed to be small may indeed not be small at all, particularly for the electrons. According to Fourier's law of heat conduction,

$$q = - K_T dT/dx \quad , \qquad (55)$$

where q is the heat conduction, K_T is the coefficient of thermal conduction, and T is the temperature. For the ions and neutrals, their large mass and essentially constant velocity would argue that this term would indeed be small since one would expect dT/dx to be small. The same may not necessarily be true for the electrons. The difficulty in including this term lies in the unknown nature of K_T. An estimate can be derived from kinetic theory but the electron mean-free path is involved and an accurate determination of K_T involves a knowledge of the solution of the very equations with which we are concerned. This situation could perhaps be handled by successive iterations of the system of equations. However, this is not only unrewardingly tedious, but offers little physical insight into the nature of the solution. Fortunately, it is not necessary to resort to this technique, since there are sufficient equations available to effect a solution without utilizing the energy equations. Properly speaking, the energy equations should be redundant to the momentum equations in any event and it should not matter which form is used. Thus, rather than attempt to determine the correct form of the equations for conservation of energy, the momentum conservation equations will be assumed to be fundamental.

The system of equations to be solved will thus be taken to be:

$$\frac{\partial}{\partial x} nv = \beta n \,, \tag{56}$$

$$\frac{\partial}{\partial x} [nmv(v-V_o)+nkT_e] = - enE - K_1 nm(v-V_o) \quad , \tag{57}$$

$$nmv(v-V_o) + nkT_e = \frac{\varepsilon_o}{2} (E^2-E_o^2) - Mnvb^*E[1-e^{-(K_i x/2V_o)}] , \tag{58}$$

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon_o} (\frac{v}{V_o} -1) \quad . \tag{59}$$

Before proceeding further with an analysis of these equations, it is convenient to put them in non-dimensional form. This is achieved by introducing the following variables:

$\psi = v/V_o$, the reduced electron velocity.

$\eta = E/E_o$, the reduced electric field.

$\nu = 2e(\phi_i/\varepsilon_o E_o^2)n$, the reduced electron density.

$\xi = \dfrac{eE_o}{mV_o^2}\, x$, the reduced position variable.

$\theta = \dfrac{kT_e}{2e\phi_i}$, the reduced electron temperature.

Also define:

$j \equiv \nu\psi$, the reduced electron current.

$\alpha \equiv \dfrac{2e\phi_i}{mV_o^2}$, the ratio of the ionization potential to the kinetic energy of the electron traveling at wave velocity.

$\omega \equiv \dfrac{E_o}{V_o}\, b^*$, the ratio of the difference of the ion and neutral velocities to the neutral velocity.

$\Omega = 3\,\dfrac{M_i}{m}\,\omega.$

$\kappa = \dfrac{m}{e}\,\dfrac{V_o}{E_o}\,K_1$.

$\mu = \dfrac{K}{K_1}\,\beta.$

Remembering that $b^* = e/M_iK_i$ and $M_i \approx M$, $K_i x/2V_o$ becomes $3\xi/2\Omega$ and the system of equations become:

$$\frac{\partial}{\partial \xi}\, j = \frac{\mu j}{\psi}\,, \qquad (60)$$

$$\frac{\partial}{\partial\xi}[j(\psi-1)+\frac{\alpha\theta j}{\psi}] = -\frac{j\eta}{\psi} - \frac{\kappa j(\psi-1)}{\psi} \quad , \tag{61}$$

$$j(\psi-1) + \frac{\alpha\theta j}{\psi} = \frac{\alpha}{2}(\eta^2-1) - \frac{\Omega}{3} j\eta[1-e^{-(3\xi/2\Omega)}] \quad , \tag{62}$$

$$\frac{\partial\eta}{\partial\xi} = \frac{j}{\alpha}(\frac{\psi-1}{\psi}) \quad . \tag{63}$$

For the sake of completeness, the non-dimensional energy equations are:

$$\frac{\partial}{\partial\xi}[j(\psi^2-1)+5\alpha\theta j+\alpha j-\chi\frac{d\theta}{d\xi}] = -\frac{2j\eta}{\psi} - 2\kappa j\frac{(\psi-1)}{\psi} \quad , \tag{64}$$

$$j(\psi^2-1) + 5\alpha\theta j + \alpha j = -2\frac{\Omega}{3} j\eta[1-e^{-(3\xi/2\Omega)}] + \chi\frac{d\theta}{d\xi} \quad , \tag{65}$$

where χ represents the thermal heat conductivity in the reduced units.

In the reduced system, ξ goes from zero to negative infinity from the front to the rear of the wave, since x goes from zero to minus infinity. Since V_o is negative and E_o is positive as is K_1, κ is intrinsically negative as are μ, ω, and Ω.

One thing remains to be done prior to solving these equations and that is to discuss the form of β. β represents the ionization frequency for collisions of electrons with neutrals. Smith has determined the ionization cross section as a function of energy for argon, helium, and neon [24]. His expression for the fit to the data was given by:

$$\varepsilon = 3.383(\frac{V_o}{V_a})^{\frac{1}{2}}(1-e^{-5.4\frac{V_o}{V_a}})^{\frac{1}{2}}[1-e^{-(V_a-V_o/2.28\ V_o)}] \quad ,$$

where ε is the probability of ionization per unit path at unit pressure, V_o is the ionization potential of the gas, and V_a is the applied voltage. Fowler has shown that for a wide range of energy the cross section corresponding to this expression can be closely approximated by:

$$\sigma_i = \sigma_o \frac{v_i^2}{v^2} (1 - \frac{v^2}{v_i^2}) \quad ,$$

where v_i is the velocity of the electron which corresponds to the ionization energy and σ_o is 4.6×10^{-17} for helium [25]. Using this cross section and a Maxwell-Boltzmann distribution which included the relative velocity between the electron and the neutral, Shelton derived the ionization frequency in terms of incomplete gamma functions [22]. β is a function of the electron temperature, θ, and the ionization potential of the gas. From their definition, it is seen that μ/κ is the ratio of the ionization frequency to the collision frequency. Setting $\mu/\kappa = \mu_o$, Fowler has plotted μ_o as a function of θ and this is reproduced here as Fig. 2 [20]. Figure 3 is a plot of $\mu_o\theta$ versus θ, which is also useful. For the purpose of calculation it is sometimes convenient to approximate μ_o as a power function of θ. For $\theta > 1.5$, μ_o will be taken as constant = 0.2. For $0.46 < \theta < 1.5$, μ_o will be given by $\mu_o = 0.13\theta$. For $0.1 < \theta < 0.46$, μ_o will be taken as $\mu_o = 0.48\theta^{2.68}$ and for $0 < \theta < .1$, $\mu_o = 6.1 \times 10^{-3}\theta^7$.

It is now in order to look at the general nature of the problem and how a solution may be attained.

The model which is now assumed to represent the situation in a discharge tube is that of a stationary, infinite plane wave with all

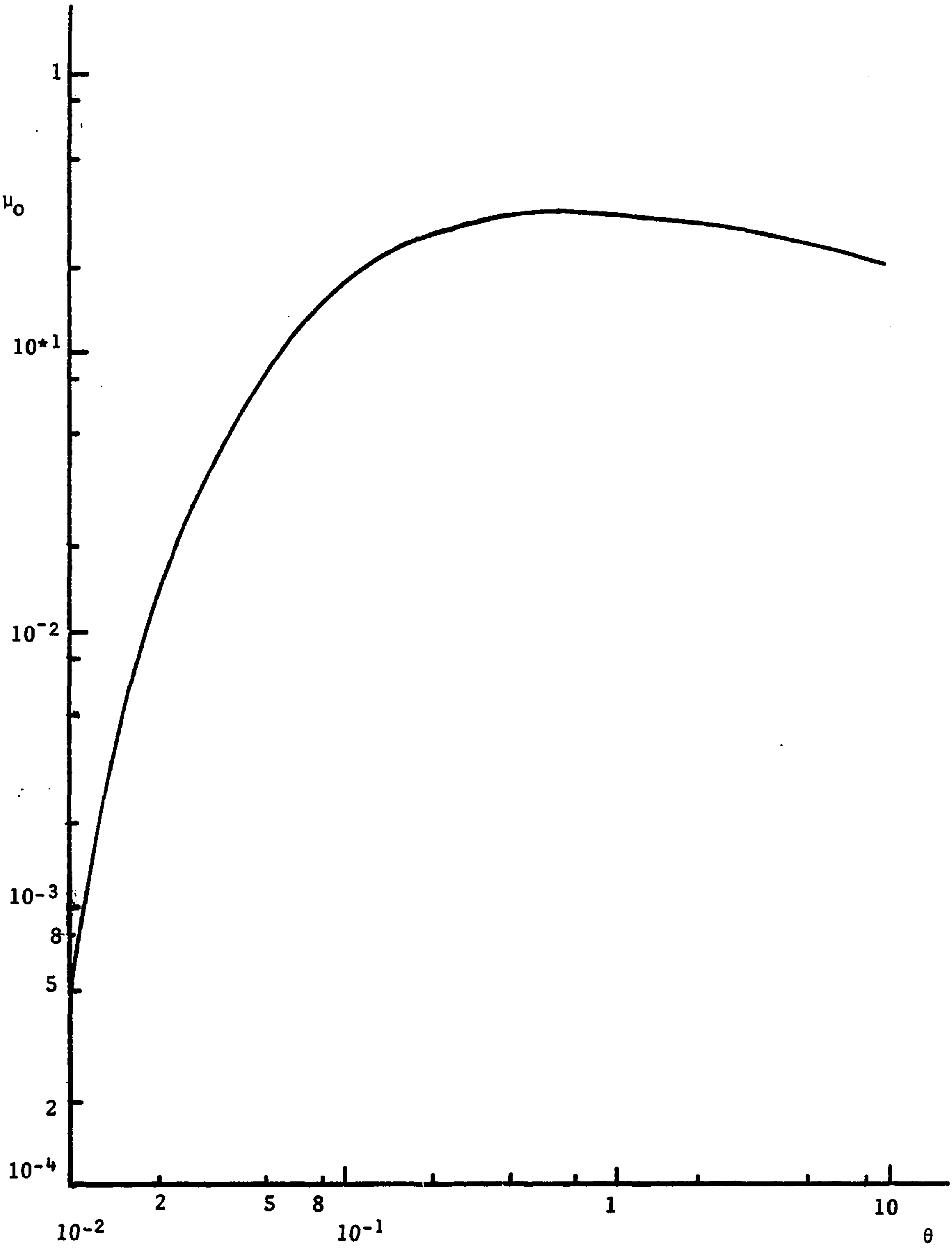

Figure 2. μ_o vs. θ

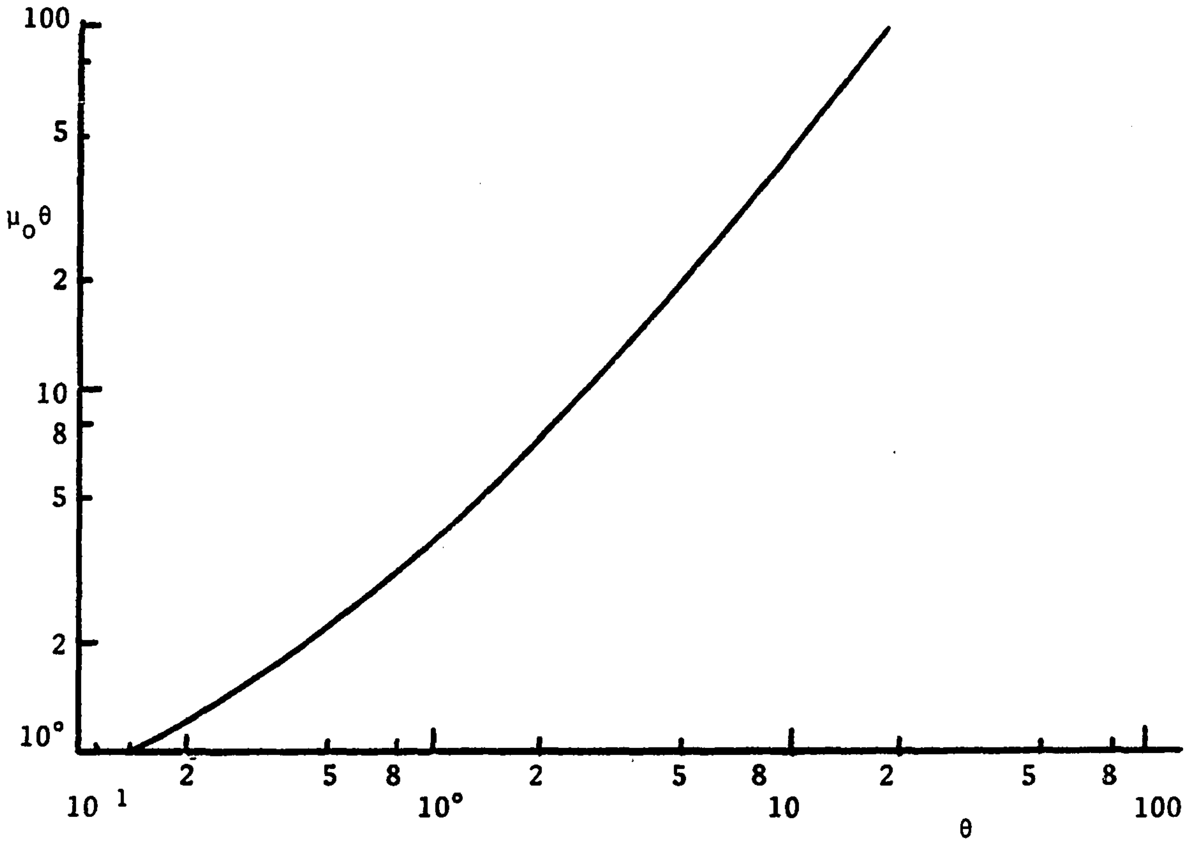

Figure 3. A. θ vs $\mu_o\theta$

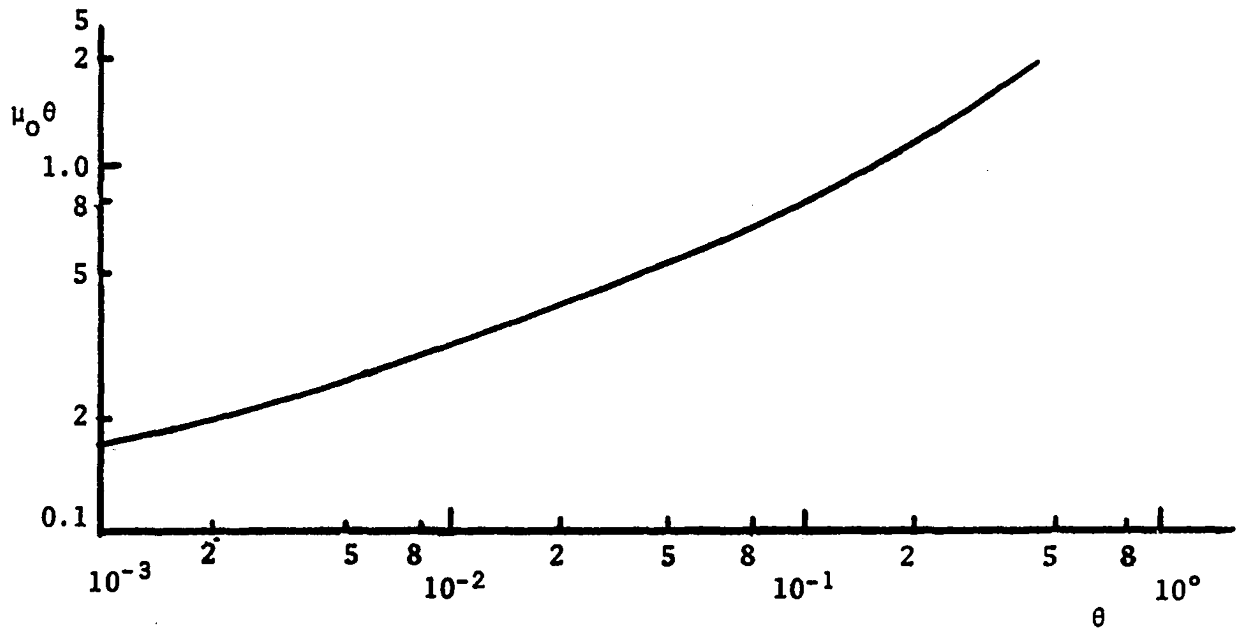

Figure 3. B. θ vs. $\mu_o\theta$

quantities (such as electron density, electron temperature, and velocity, and the corresponding quantities for the ions and neutrals, as well as the electric field) functions of position alone. The actual motion of the wave in the laboratory is governed by the actions of the electrons; and the ion and neutral motions are very small. The direction of the electric field is such as to cause the average drift velocity of the electrons to be away from the wave front. The electron fluid pressure is assumed to be large enough to provide the driving force to cause the wave to move down the tube with the observed velocities. This implies that the electron temperature must be great enough to provide enough ionization near the front of the wave to sustain its motion despite the net electron drift away from the wave front. Thus, waves with large speeds would be expected to be accompanied by large electron temperatures, while slower waves would have correspondingly lower temperatures.

The electrons start with zero velocity at the wave front and, due to the acceleration of the electric field, are accelerated so as to achieve velocity in the negative ξ direction. Since V_o is negative, $\psi = v/V_o$ will thus have an initial value of zero and as one proceeds into the wave in the negative ξ direction, ψ will increase. In addition to the pressure and the force of the electric field acting on the electrons, there is a velocity dependent retarding force due to the collisions with the neutral atoms of the gas. Eventually the velocity of the electrons will become large enough so that the force due to the electric field will balance the retardation due to collisions and the electron will experience no further non-pressure dependent acceleration.

In the reduced nondimensional form, the electric field acceleration is given by the term $-j\eta/\psi$; the retarding collision term, $-\kappa j(\psi-1/\psi)$. The condition that the electron have no net non-pressure force acting on it is thus that:

$$-\frac{j\eta}{\psi} - \kappa j \frac{(\psi-1)}{\psi} = 0 \quad . \tag{66}$$

When equation (66) is satisfied, the relation between ψ and η is given by:

$$\psi = 1 - \frac{\eta}{\kappa} \quad . \tag{67}$$

This condition establishes a boundary condition for ψ and η, for, if η increases, ψ will also increase until the retarding collisional force again balances the field. On the other hand, if η decreases, the retarding force will cause ψ to decrease again until the field is balanced. Thus, the relation (67) sets the ultimate upper limit for ψ for given η and κ.

The qualitative behavior of the field can be seen from Poisson's equation. As previously demonstrated, the initial conditions $\psi_o = 0$ and $v_o = 0$ (keeping in mind that the subscript o denotes conditions at the front of the wave) demand that $(\frac{\partial\eta}{\partial\xi})_o$ also be equal to zero. Thus η starts with the initial value $\eta_o = 1$ with a zero slope. Repeating Poisson's equation:

$$\frac{\partial\eta}{\partial\xi} = \frac{j}{\alpha}\left(\frac{\psi-1}{\psi}\right) \quad , \tag{63}$$

the quantities j and ψ determine the structure of η for a given ξ. Now α is a constant for a given wave speed V_o, and is positive. j is the product of ψ and ν, and ν represents the electron density, which must be non negative. From the above discussion, ψ starts with a value of zero at the wave front and acquires positive values as one proceeds into the wave structure. The assumption of a weak shock solution implies that ψ is continuous and thus ψ is less than one when it attains an initial non zero value. The sign of $\partial\eta/\partial\xi$ is thus negative when it attains an initial non zero value. Now ξ is negative into the wave so a negative $\partial\eta/\partial\xi$ means that η is increasing as a function of position away from the wave front into the wave. Since the sign of $\partial\eta/\partial\xi$ is determined by $(\psi-1)$, η must continue to increase until the point at which $\psi = 1$. Beyond $\psi = 1$, $\partial\eta/\partial\xi$ will be positive so that η decreases and the point $\psi = 1$ must correspond to a maximum for η.

Initial conditions have been established for ψ, ν and η, but so far nothing has been said about θ. At first sight it would appear that θ_o is arbitrary, since equation (62) is automatically satisfied at $\xi = 0$ by the conditions $\nu_o = 0$, $\eta_o = 1$; and θ_o has no influence. Since the electrons were attached to the neutrals prior to ionization, it might be reasonable to assign the electron temperature as that corresponding to the temperature of the ions. It is not completely obvious whether the temperature corresponding to the average neutral velocity should be assigned - which would lead to a factor of m/M_i times the temperature of the ion for the electron temperature - or, whether the temperature corresponding to the average neutral energy should be assigned, in which case the electron and ion temperature should be the same.

The question of the initial electron temperature will be deferred until the solution of the wave structure has been more advanced. Once solutions for ψ, j, and η have been found, θ should be determined without the necessity of specifying its initial value.

CHAPTER V

SOLUTION TO FLUID EQUATIONS

The equations which must be solved in order to characterize the wave structure are, from Chapter IV:

$$\frac{\partial j}{\partial \xi} = \frac{\mu j}{\psi} \quad , \tag{68}$$

$$\frac{\partial \eta}{\partial \xi} = \frac{j}{\alpha} \left(\frac{\psi-1}{\psi}\right) \quad , \tag{69}$$

$$\frac{\partial}{\partial \xi} \left[j(\psi-1) + \frac{\alpha\theta j}{\psi}\right] = - \frac{j\eta}{\psi} - \kappa(\psi-1) \frac{j}{\psi} \quad , \tag{70}$$

$$j(\psi-1) + \frac{\alpha\phi j}{\psi} = \frac{\alpha}{2} (\eta^2-1) - \frac{\Omega}{3} j\eta(1-e^{-\frac{3}{2}\frac{\xi}{\Omega}}) \quad . \tag{71}$$

This is a set of four equations in the four unknowns j, η, ψ, and θ with κ a constant and $\mu_o = \mu/\kappa$ a known function of θ and κ. Of course the value of κ is unknown and it is one of the tasks to determine its value since κ determines the relation between the initial applied field and the laboratory value of the wave speed. In principle all that is necessary is to solve the coupled equations, but, in practice, this has not been feasible to do in a direct fashion. Instead, it has proved necessary to resort to an approximation procedure which may not be an accurate representation of the wave structure in every detail but should provide answers of sufficient accuracy to test the model and to compare with the experimental data which is available.

As the initial step, it is convenient to express the equations as functions of η and ψ rather than ξ, if possible. By dividing equation (68) by equation (69):

$$\frac{\partial j}{\partial \eta} = \frac{\mu\alpha}{\psi-1} \quad . \tag{72}$$

Dividing equation (70) by equation (69):

$$\frac{\partial}{\partial \eta}\left[j(\psi-1) + \frac{\alpha\theta j}{\psi}\right] = -\frac{\alpha\eta}{\psi-1} - \kappa\alpha \ . \tag{73}$$

Now consider the behavior of η as a function of ψ. η starts with an initial value of 1 at $\psi = 0$, and since j is always positive, η increases as ψ increases toward 1, and has a maximum at $\psi = 1$. η then decreases with further increase of ψ until the line $\psi = 1 - \eta/\kappa$ is reached. As shown in Chapter IV, the line $\psi = 1 - \eta/\kappa$ must constitute a limiting path for the wave in the (ψ,η) plane and the relation $\psi = 1-\eta/\kappa$ must be satisfied by the ultimate solution, as the electrons come to equilibrium. Thus, in the (ψ,η) plane, the solution path must have the general features of Fig. 4. To the right of $\psi = 1$, η must decrease from its maximum value η_1, and ψ must ultimately meet the line $\psi = 1-\eta/\kappa$.

For the model of the gas as an infinite plane conductor, the field η must decrease to zero as the ions and electrons come to equilibrium and the electron and ion densities become equal with zero net charge density. The electrons are slower to react to the retarding force of collisions than to the effect of the electric field because of the finite time required for the collision process and the large electron mobility. For this reason, the solution path would be expected to approach the $\psi = 1-\eta/\kappa$ line from the right as η decreases and it is reasonable to expect the $\psi = 1-\eta/\kappa$ line to be intersected by the solution path at the point $\eta = 0$, $\psi = 1$.

For proforce waves, it has been shown that the solution to the fluid equations is not a strong function of the solution path in the (ψ,η) plane, as long as the path represents a valid solution to the equations [26]. Rather than attempt an interative step-by-step numerical solution to the equations, it will be assumed that the antiforce situation is also relatively insensitive to the path in the (ψ,η) plane and an analytical function will be sought. A single analytical function has not been found which can be taken to represent a solution path over the entire range of ψ and η for the wave. Instead, the expedient of dividing the (ψ,η) plane into different regions with separate paths for each region, and smoothly joining the paths where the regions intersect, will be adopted. Even with this great simplification, the solution is tedious and to some extent inexact. The result is useful, however,

In the (ψ,η) plane, the path for $0\leq\psi\leq 1$ will be called path I. Calling η_1 the maximum value of η at $\psi = 1$, the path between $1\leq\psi\leq 1 - \eta/\kappa$ will be called path II. As previously stated, path I has zero slope at $\psi = 0$ and at $\psi = 1$. Path II, in order to match smoothly, must also have zero slope at $\psi = 1$. Path II will be taken to intersect $\psi = 1 - \eta/\kappa$ at $\eta = 0$, $\psi = 1$.

The equation

$$\eta = 1-(\eta_1-1)(2\psi^3-3\psi^2) \tag{74}$$

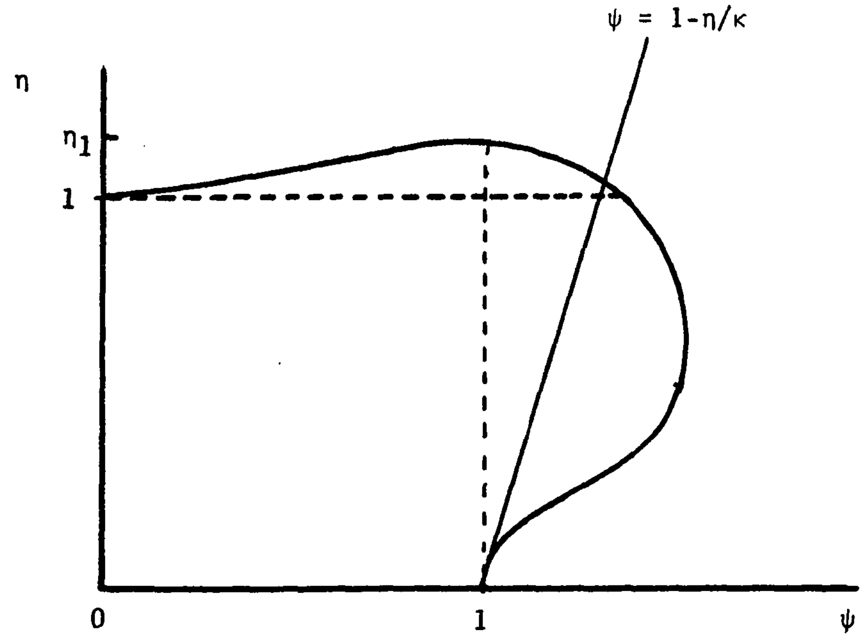

Figure 4. Solution path in (ψ,η) plane

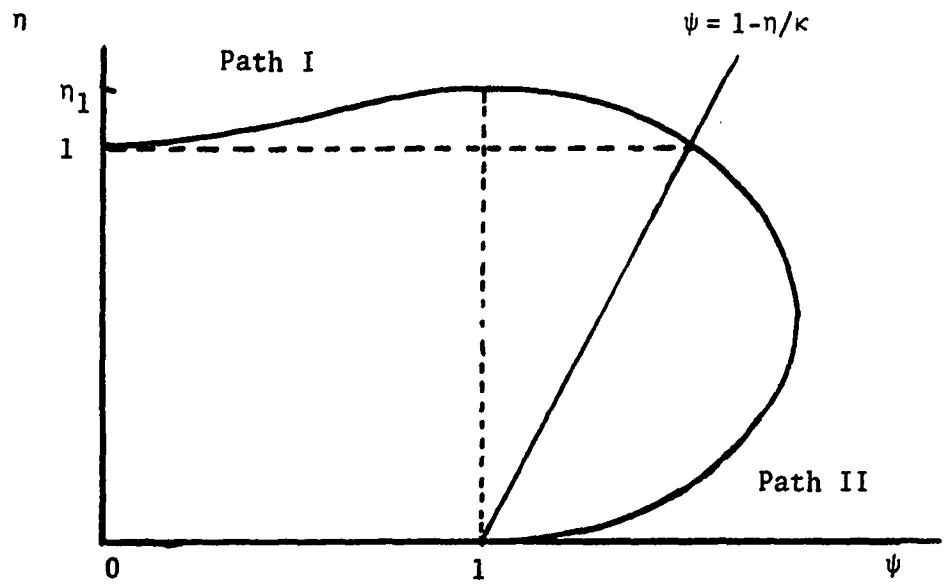

Figure 5. Assumed solution path.

has the essential properties of $\eta_o = 1$ and $\eta = \eta_1$ when $\psi = 1$ with $d\eta/d\psi = 0$ at both $\psi = 0$ and $\psi = 1$. This relation has the further desirable property of allowing equation (73) to be integrated. Equation (74) will be used as path I. Path II will be taken to be the ellipse given by equation (75) which connects $(\eta_1,1)$ with $(0,1)$.

$$\left(\frac{\eta - \frac{\eta_1}{2}}{\frac{\eta_1}{2}} \right)^2 + \frac{4\kappa^2}{\eta_1^2}(\eta_1-1)(\psi-1)^2 = 1. \tag{75}$$

This also has the advantage of being integrable and gives a zero slope for η at $\eta = 0$, $\psi = 1$.

The solution to the fluid equations along paths I and II is, in principle, now reduced to determination of η_1 and κ since equation (72) gives a relation between j and η; equation (69) gives the η and ξ relation and θ can be determined from either equation (70) or equation (71). E_o and κ determine the value of α. However, μ is a function of θ and equation (69) really is a relation between j, θ, and η. Without knowledge of the initial value of θ, it is not possible to determine which portion of the μ_o versus θ relation is appropriate to use. As a first estimate, it will be assumed that along the solution path, θ is large enough that μ can be taken as constant, where $\mu_o = \mu/\kappa = 0.2$ or $\mu = 0.2\kappa$.

Proceeding with the integration along path I, equation (74) gives $d\eta$.

$$d\eta = -\ 6(\eta_1-1)\psi(\psi-1)d\psi\ .$$

Substituting into equation (73):

$$d[j(\psi-1)+ \frac{\alpha\theta j}{\psi}] = 6\alpha(\eta_1-1)\eta\psi d\psi - \kappa\alpha d\eta\ ,$$

or

$$d[j(\psi-1)-\frac{\alpha\theta j}{\psi}] = 6\alpha(\eta_1-1)\psi[1-(\eta_1-1)(2\psi^3-3\psi^2)]d\psi-\kappa\alpha d\eta \ .$$

Integrating:

$$j(\psi-1)+\frac{\alpha\theta j}{\psi} = 6\alpha(\eta_1-1)[\frac{\psi^2}{2}-(\eta_1-1)\psi^4(\frac{2}{5}\psi-\frac{3}{4})]-\kappa\alpha\eta+C \ , \tag{76}$$

where C is a constant determined by the boundary conditions $\eta = 1$, $j = 0$ when $\psi = 0$. The resulting expression is:

$$j(\psi-1)+\frac{\alpha\theta j}{\psi} = 6\alpha(\eta_1-1)[\frac{\psi^2}{2}-(\eta_1-1)\psi^4(\frac{2}{5}\psi-\frac{3}{4})]-\kappa\alpha(\eta_1-1) \ . \tag{77}$$

Equation (72) is now:

$$dj = -6\mu\alpha(\eta_1-1)\psi d\psi \ .$$

With constant μ this can be integrated:

$$j = -3\mu\alpha(\eta_1-1)\psi^2 \ , \tag{78}$$

since j is zero when ψ is zero. With (78), equation (68) becomes:

$$2\psi = \mu d\xi \ .$$

Since ξ is 0 at the front of the wave where $\psi = 0$,

$$\mu\xi = 2\psi \ . \tag{79}$$

Now at the point $\psi = 1$, $\eta = \eta_1$, the right side of equation (77) is:

$$6\alpha(\eta_1-1)[\frac{1}{2}+\frac{7}{20}(\eta_1-1)]-\kappa\alpha(\eta_1-1) \ .$$

This must be equal to the right side of equation (71) at the same location:

$$6\alpha(\eta_1-1)[\frac{1}{2}+\frac{7}{20}(\eta_1-1)]-\kappa\alpha(\eta_1-1) = \frac{\alpha}{2}(\eta_1^2-1)-\frac{\Omega}{3}j_1\eta_1(1-e^{-\frac{3}{2}\frac{\xi_1}{\Omega}}).$$

Now

$$\xi_1 = \frac{2}{\mu} \ ,$$

and

$$j_1 = -3\mu\alpha(\eta_1-1) \ .$$

Experimentally, $|\kappa|$ is of the order of magnitude of unity. For helium, Ω is of the order of $200/\kappa$ and $[1-e^{-(3/2 \ \ \xi_1/\Omega)}]$ can be replaced by $3/\Omega\mu$ without appreciable error. Then:

$$6\alpha(\eta_1-1)[\frac{1}{2} + \frac{7}{20}(\eta_1-1)]-\kappa\alpha(\eta_1-1) = \frac{\alpha}{2}(\eta_1^2-1) + 3\alpha\eta_1(\eta_1-1) \ . \tag{80}$$

One solution to equation (80) is $\eta_1 = 1$. This is not useful since it merely expresses the fact that one solution is no wave at all, but a completely constant situation. The solution of interest gives a relation between κ and η_1 as:

$$\eta_1 = \frac{0.4-\kappa}{1.4} \ . \tag{81}$$

The condition $\eta_1>1$ imposes a minimum value for the absolute magnitude of κ,

$$|\kappa|>1 \ . \tag{82}$$

Now from equations (77) and (81), the value of θ at $\psi = 1$ can be determined in terms of η_1:

$$\theta_1 = -\frac{1}{6\alpha\mu}[7\eta_1+1]$$

or

$$\theta_1\mu_o = \frac{1}{6\alpha}(\frac{7\eta_1+1}{1.4\eta_1-0.4}) \ . \tag{83}$$

The value of θ_1 is thus determined by α and η_1. Remembering that α is small for fast waves and large for slow waves, it is encouraging to see that θ_1 has the qualitative features expected by the model of electron pressure supplying the impetus for wave motion.

Proceeding with the solution along path II, equation (75) gives:

$$(\psi-1) = -\frac{\eta_1}{2\kappa(\eta_1-1)^{\frac{1}{2}}}\left[1-\left(\frac{2\eta-\eta_1}{\eta_1}\right)^2\right]^{\frac{1}{2}} .$$

Substituting this into equation (73):

$$d\left[j(\psi-1)+\frac{\alpha\theta j}{\psi}\right] = \frac{\alpha\, 2\kappa(\eta_1-1)^{\frac{1}{2}}\eta d\eta}{\eta_1\left[1-\left(\frac{2\eta-\eta_1}{\eta_1}\right)^2\right]^{\frac{1}{2}}} - \kappa\alpha d\eta .$$

Integrating:

$$j(\psi-1)+\frac{\alpha\theta j}{\psi} = -\alpha\frac{\eta_1}{2}\kappa(\eta_1-1)^{\frac{1}{2}}\left[1-\left(\frac{2\eta-\eta_1}{\eta_1}\right)^2\right]^{\frac{1}{2}}+\eta_1\alpha\kappa(\eta_1-1)$$

$$\cdot\sin^{-1}\left(\frac{2\eta-\eta_1}{\eta_1}\right) - \kappa\alpha\eta + C ,$$

where C is again determined by the boundary conditions. Evaluating C at $\eta = \eta_1$ at $\psi = 1$ and rewriting, this becomes:

$$j(\psi-1)+\frac{\alpha\theta j}{\psi} = -\kappa\alpha(\eta-\eta_1) +\alpha\theta_1 j_1 + \alpha\kappa^2(\eta_1-1)(\psi-1)$$

$$+\frac{\eta_1}{2}\alpha\kappa(\eta_1-1)^{\frac{1}{2}}\sin^{-1}\left[\frac{2\kappa(\eta_1-1)}{\eta_1}(\psi-1)\right] . \qquad (84)$$

Now along path II,

$$d\eta = \frac{2\kappa^2(\eta_1-1)(\psi-1)d\psi}{\eta_1[1-\frac{4\kappa^2}{\eta_1{}^2}(\eta_1-1)(\psi-1)^2]^{\frac{1}{2}}} \quad .$$

Substituting into equation (72):

$$dj = -\mu\alpha\frac{2\kappa^2}{\eta_1}\frac{(\eta_1-1)\ d\psi}{[1-\frac{4\kappa^2}{\eta_1{}^2}(\eta_1-1)(\psi-1)^2]^{\frac{1}{2}}} \quad .$$

Integrating and imposing $\eta = \eta_1$ when $\psi = 1$:

$$j = j_1 - \mu\alpha\kappa(\eta_1-1)^{\frac{1}{2}}\sin^{-1}[\frac{2\kappa}{\eta_1}(\eta_1-1)^{\frac{1}{2}}(\psi-1)] \quad ,$$

or, using

$$j_1 = -3\mu\alpha(\eta_1-1) \quad ,$$

$$j = -\mu\alpha(\eta_1-1)\{3+\frac{\kappa}{(\eta_1-1)^{\frac{1}{2}}}\sin^{-1}[\frac{2\kappa}{\eta_1}(\eta_1-1)^{\frac{1}{2}}(\psi-1)]\} \quad . \qquad (85)$$

Now it should be possible to equate equation (84) and equation (71) at the point $\eta = 0$, $\psi = 1$ and obtain a solution for κ by use of equation (81), which gives the relation between η_1 and κ. However, there are difficulties. Equation (71) was derived under the assumption that η remains constant over the region of application. Along path II, η is decreasing from its peak value of η_1 towards zero and equation (71) thus cannot be valid at the point $\eta = 0$, $\psi = 1$.

Even if equation (71) were valid at the point $\eta = 0$, $\psi = 1$, its use requires a knowledge of the value of ξ at that point. From equation (68):

$$\mu d\xi = \frac{\psi dj}{j} .$$

Along path II

$$\mu d\xi = \frac{2\kappa^2}{\eta_1} \frac{\psi d\psi}{[1-\frac{4\kappa^2}{\eta_1{}^2}(\eta_1-1)(\psi-1)^2]^{\frac{1}{2}}[3+\kappa(\eta_1-1)\sin^{-1}\frac{2\kappa}{\eta_1}(\eta_1-1)^{\frac{1}{2}}(\psi-1)]} . \tag{86}$$

Attempts to integrate this expression analytically have been unsuccessful.

It is necessary to seek another relation to replace equation (71) if a solution is to be found. The electron energy equation will be reexamined to determine if the reasons for disregarding it are still valid near the point $\eta = 0$, $\psi = 1$.

The electron energy equation was previously discarded because of the presence of the unknown quantity representing the heat conduction. Now the magnitude of the heat conduction term, even though unknown, is a function of the temperature and should be small when the temperature is small or slowly varying. It has been shown that for the proforce case, the heat conduction term does not play a strong role, at least for a strong shock. Experiment indicates that, in the so-called quasi-neutral region where ψ approaches one and η approaches zero, the proforce and antiforce waves are very similar in behavior [11]. The temperature becomes small in this region for the proforce case and it is reasonable to assume that the heat conduction term may be small and can be ignored. If this is the case, the differential form of the electron energy conservation equation may be written:

$$\frac{d}{d\xi}\left[j(\psi^2-1)+5\alpha\theta j+j\alpha\right] = -\frac{2j\eta}{\psi} - 2\kappa(\psi-1)\frac{j}{\psi}\ . \tag{87}$$

Since ψ is very close to unity, it can be replaced by one without appreciable error, and equation (87) is:

$$\frac{d}{d\xi}\left[2j(\psi-1)+5\alpha j\theta+j\alpha\right] = -2j\eta - 2\kappa(\psi-1)j\ . \tag{88}$$

The differential momentum equation, with the same approximation is:

$$\frac{d}{d\xi}\left[j(\psi-1)+\alpha\theta j\right] = -j\eta - \kappa(\psi-1)j\ . \tag{89}$$

Multiplying (89) by 2 and subtracting from (88):

$$\frac{d}{d\xi}\left[3\alpha\theta j+\alpha j\right] = 0\ ,$$

or

$$j(1+3\theta) = \text{constant.} \tag{90}$$

Now even though the formulation for V_i-V_o does not apply in this region, the term which represents this quantity appears in both the global momentum and energy equations in the same manner. Thus, with the same assumptions regarding ψ and the heat conduction term these can be written:

$$j(\psi-1) + \alpha\theta j - \frac{\alpha}{2}(\eta^2-1) = -\Delta V$$

$$2j(\psi-1) + 5\alpha\theta j + \alpha j = -2\Delta V$$

where ΔV represents the unknown ion drift term. Eliminating ΔV:

$$3\alpha\theta j + \alpha j = \alpha(1-\eta^2)\ .$$

As η approaches zero:

$$3\alpha\theta j + \alpha j = \alpha$$

or

$$j = \frac{1}{1+3\theta} \tag{91}$$

which is in accord with equation (90) with the constant equal to one. Thus, as in the proforce case, in the quasi-neutral region j approaches unity as θ approaches zero. Since ψ is approximately one here, this implies that ν is one. Remembering the definition of ν as

$$\nu = \frac{2e\phi_i}{\varepsilon_o E_o^2} n ,$$

the result is the same as the proforce case in that the energy originally in the field is expended in production of the electron-ion pairs.

Now what about the assumption that the heat conduction is small? With $\psi \simeq 1$, equation (67) is:

$$\frac{dj}{d\xi} = \mu j \quad .$$

Now, dj is given by equation (91):

$$dj = - \frac{3d\theta}{(1+3\theta)^2}$$

and

$$\frac{dj}{j} = - \frac{3d\theta}{(1+3\theta)} \quad .$$

Since θ is small compared to unity in this region:

$$\frac{dj}{j} \simeq - 3d\theta ,$$

and

$$\frac{d\theta}{d\xi} = - \frac{\mu}{3} \quad .$$

The heat conduction term is proportional to $d\theta/d\xi$ and since μ is small when θ is small, the neglect of the heat conduction term at least appears reasonable. The resultant solution neglecting the heat conduction for the quasi-neutral region gives a satisfactory fit to the data presently available.

Now, from equation (84), when $\eta = 0$, $\psi = 1$ (using the subscript 2 to denote values at that point:

$$\alpha\theta_2 j_2 = \alpha\theta_1 j_1 + \kappa\alpha\eta_1 - \frac{\pi\alpha}{2}\eta_1\kappa(\eta_1 - 1)^{\frac{1}{2}} \quad . \tag{93}$$

From equation (91):

$$\alpha\theta_2 j_2 = \alpha[\frac{1-j_2}{3}] \quad . \tag{94}$$

Equation (85) gives:

$$j_2 = - \mu\alpha(\eta_1 - 1)[3 - \frac{\pi\kappa}{(\eta_1 - 1)^{\frac{1}{2}}}] \quad . \tag{95}$$

Now,

$$\alpha\theta_1 j_1 = - \kappa\alpha(\eta_1 - 1) + 6\alpha(\eta_1 - 1)[\frac{1}{2} + \frac{7}{20}(\eta_1 - 1)] \quad ,$$

so,

$$\frac{1 + \mu\alpha(\eta_1 - 1)[3 - \frac{\pi\kappa}{(\eta_1 - 1)^{\frac{1}{2}}}]}{3} = \kappa[1 - \eta_1(\eta_1 - 1)^{\frac{1}{2}}\frac{\pi}{2}] + 6(\eta_1 - 1) \cdot [\frac{1}{2} + \frac{7}{20}(\eta_1 - 1)] \quad . \tag{96}$$

Equation (96), together with equation (81), gives values for κ in terms of μ and α.

Figure 6 gives a plot of κ versus α and Fig. 7 gives θ_2 versus α. For α greater than around 0.5, it is found that the assumption of a constant μ is no longer valid since θ takes on values in the region where μ is a function of θ. The curves for $\alpha>0.5$ were fit by an iterative procedure by requiring consistency between an assumed value of μ and the value of μ implied by θ_2. This procedure is not expected to give valid results since the equation for j depends upon an integration which was performed assuming μ was constant. The value of κ is seen to be a weak function of α and thus, so is η_1. η_1 decreases with increasing α as does θ_2. This is the expected behavior since increasing α corresponds to slower waves and decreasing η and θ correspond to lower field and temperatures.

With this, it is possible to calculate the wave structure as a function of position for a given initial field. As an example of the technique, solutions will be exhibited for the case of small α which corresponds to a fast wave.

Figures 8 through 13 give the results for $\alpha = 0.086$ which corresponds to a wave with a speed of around 10^7 meter/sec in helium. For this value of α, κ is found to be -1.247 and η_1 is 1.1764, assuming $\mu = 0.2\kappa$, where $\mu_0 = 0.2$ and $\mu = \mu_0\kappa$.

The equations for the solution for $\alpha = 0.086$ are:

Path I:

$$\eta = 1-(0.1764)\psi^2(2\psi-3) \ ,$$

$$\psi = - 0.125\ \xi,$$

$$j = 0.114\ \psi^2,$$

$$\theta = -\frac{j\eta\xi}{2} - 0.043(\eta^2-1) + \frac{j\psi(1-\psi)}{0.086} \ .$$

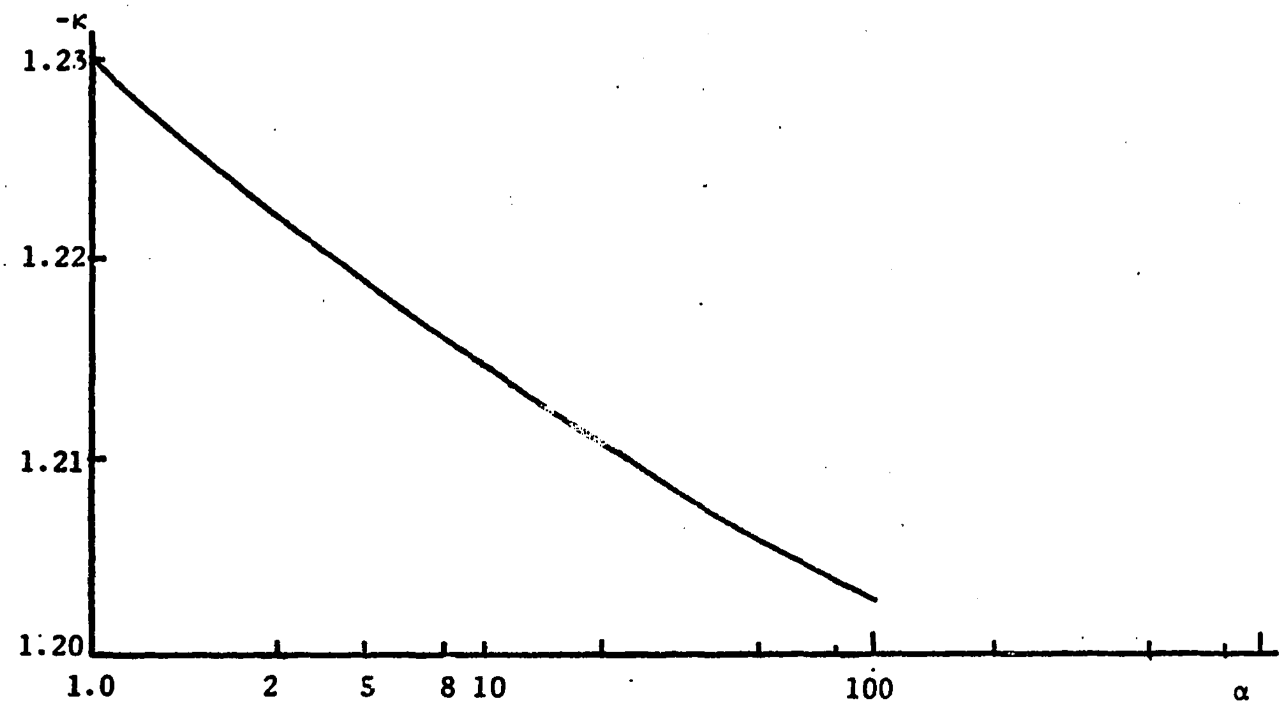

Figure 6. A. -κ vs. α

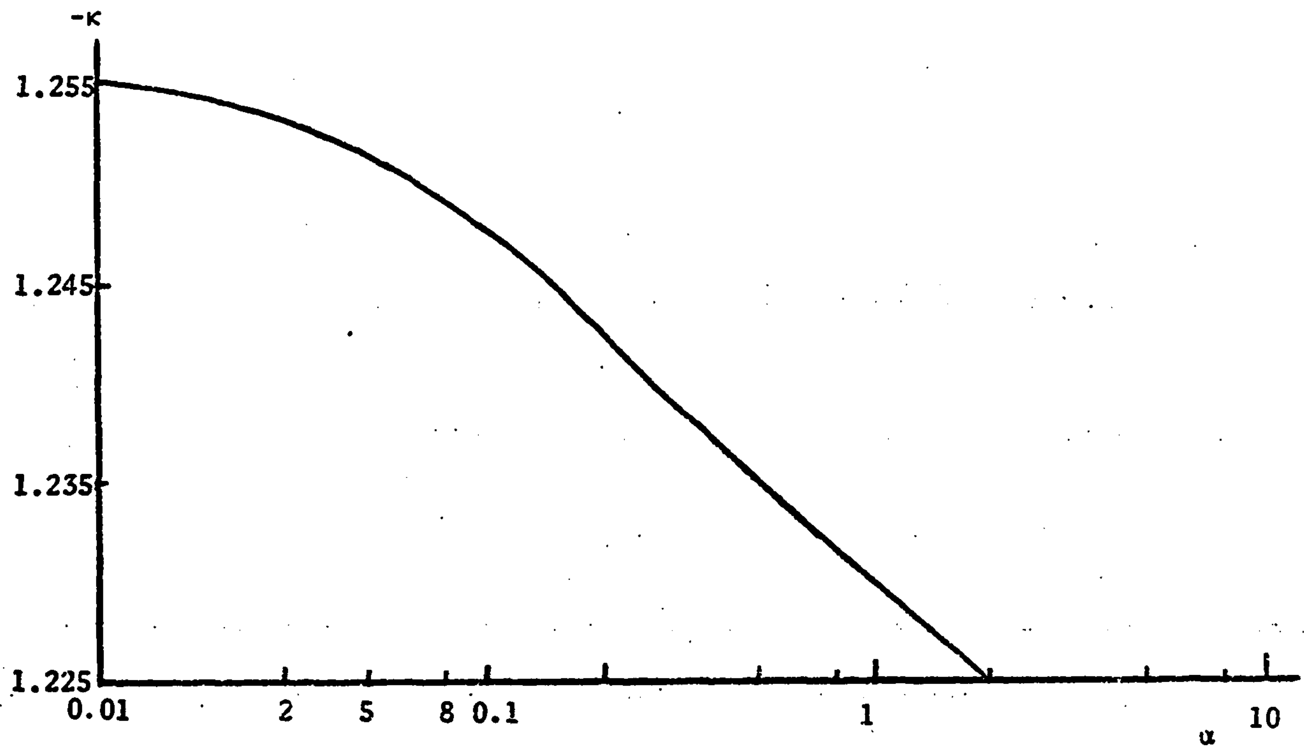

Figure 6. B. -κ vs. α

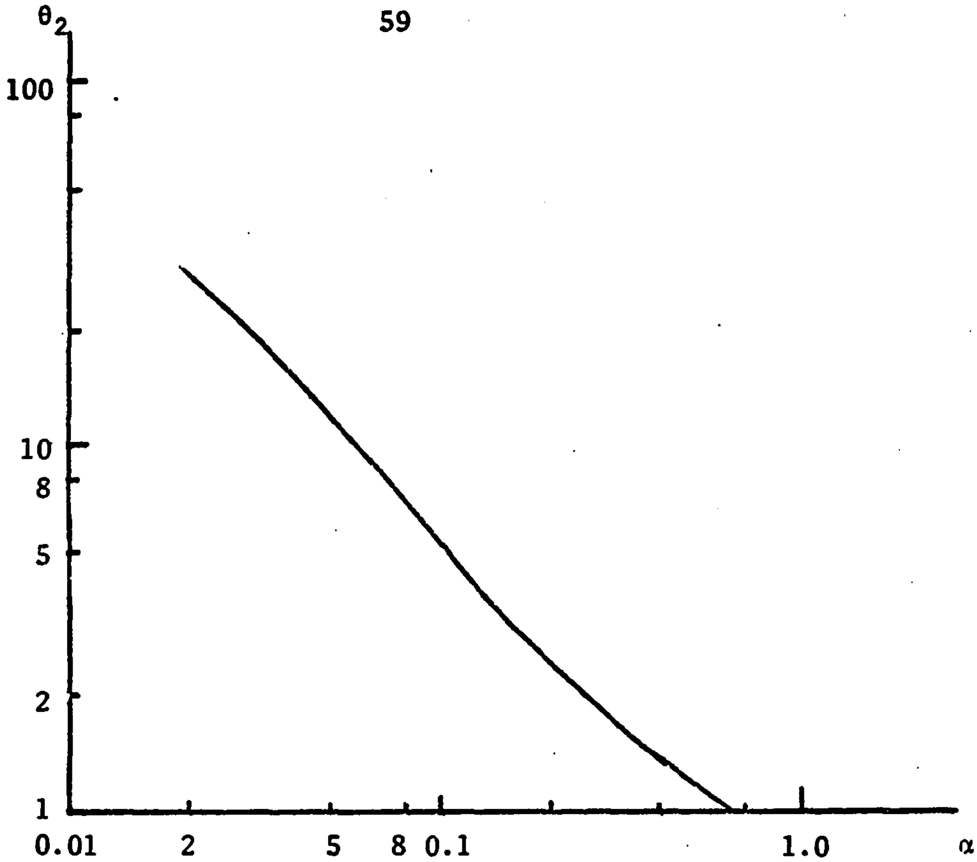

Figure 7. A. θ_2 vs. α

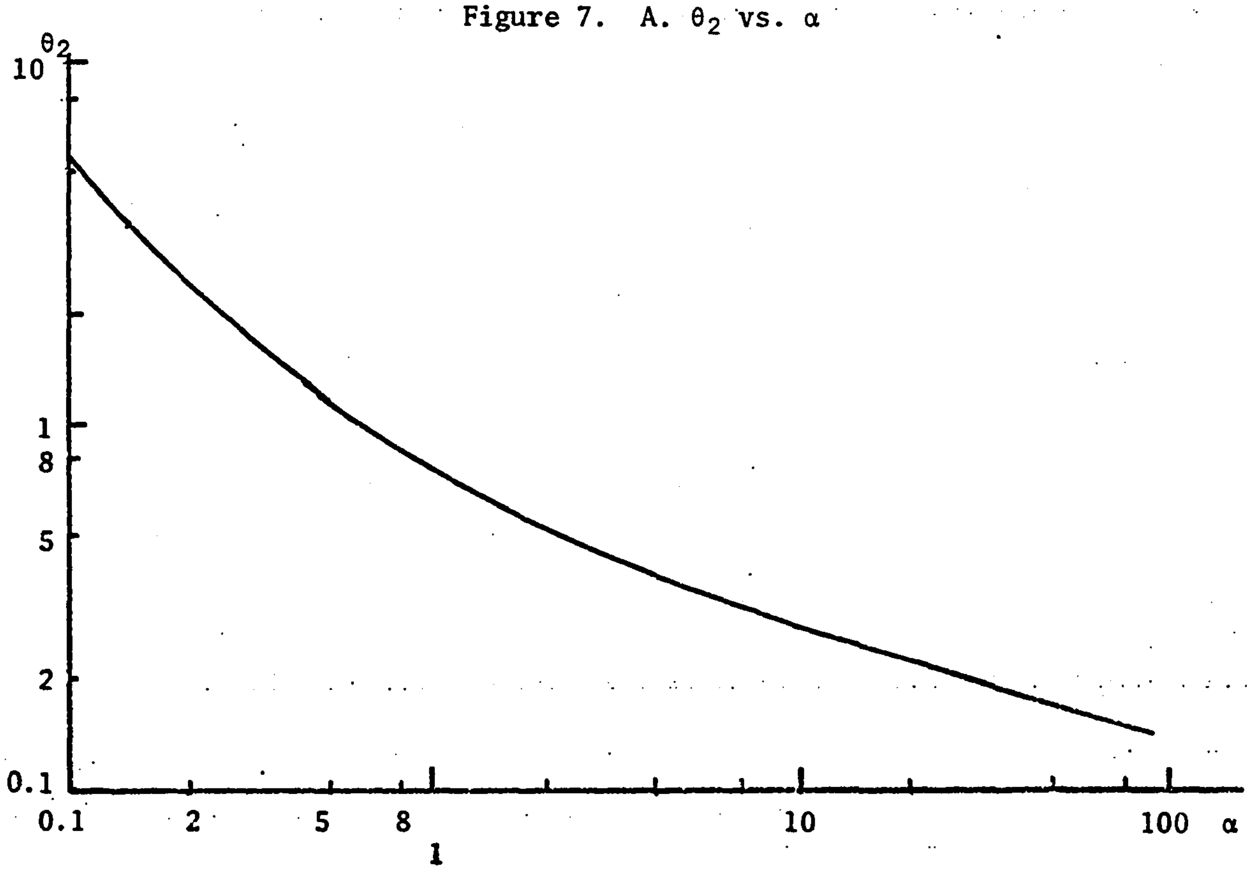

Figure 7. B. θ_2 vs. α

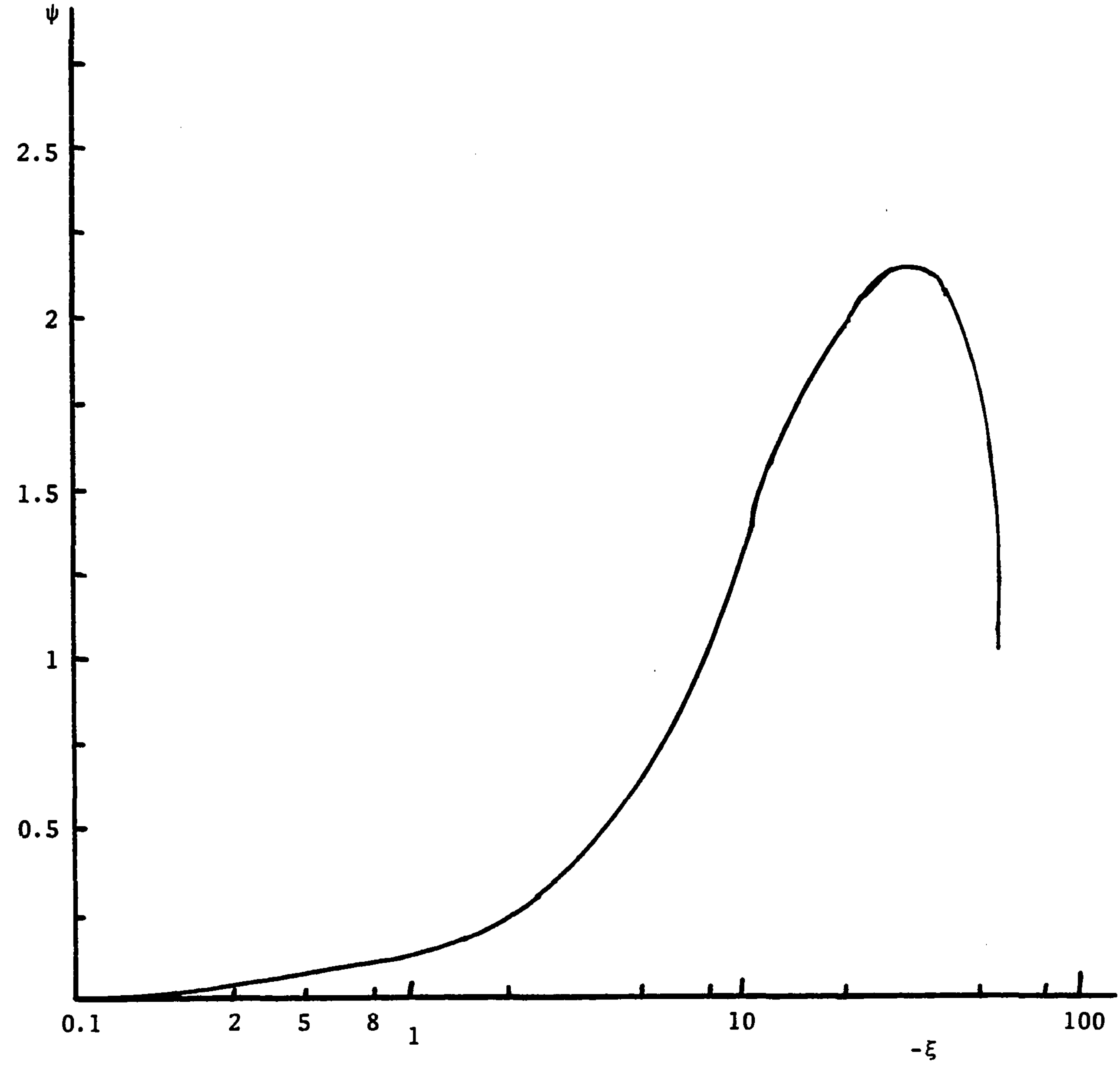

Figure 8. ψ vs. -ξ

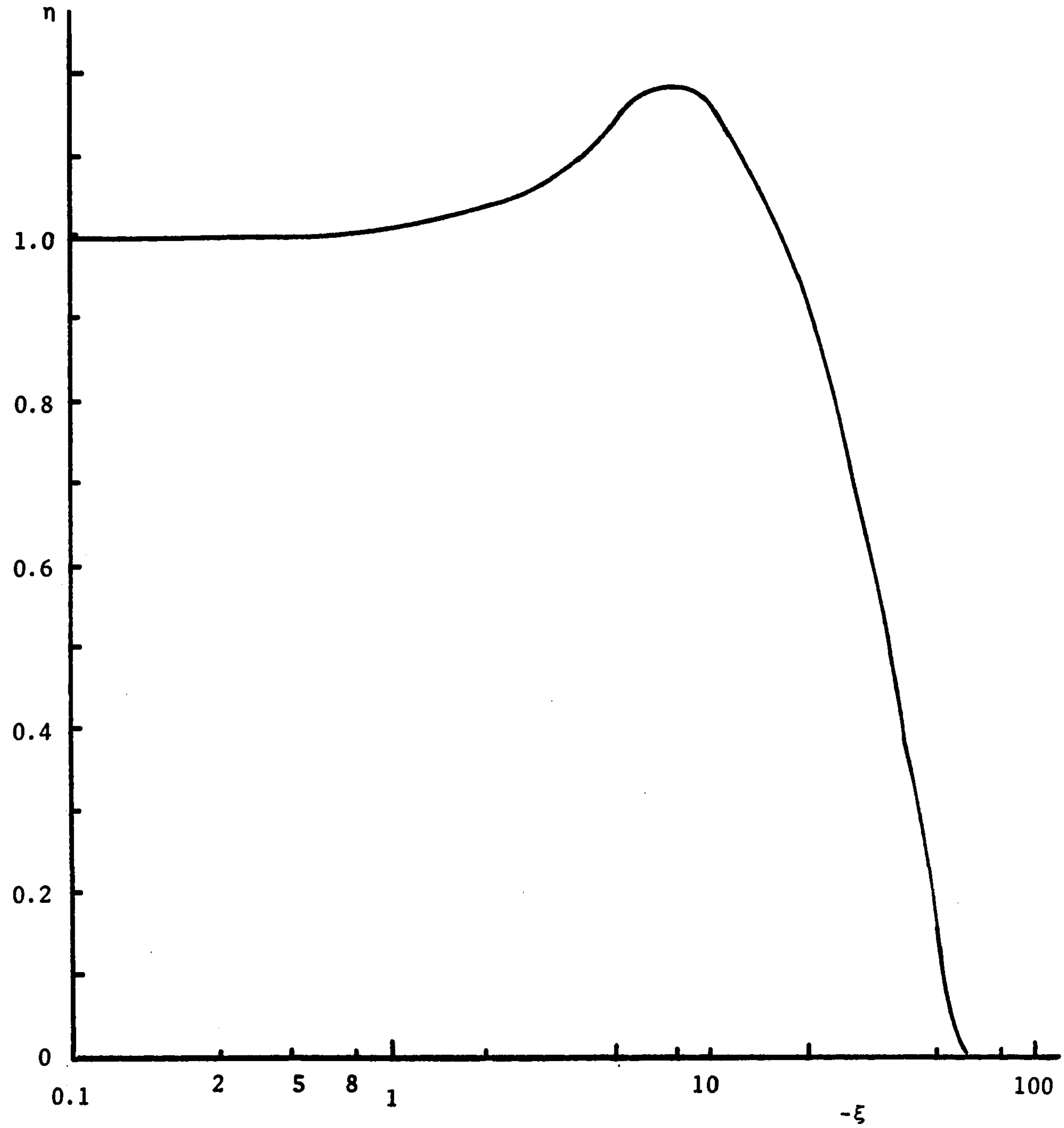

Figure 9. η vs. -ξ

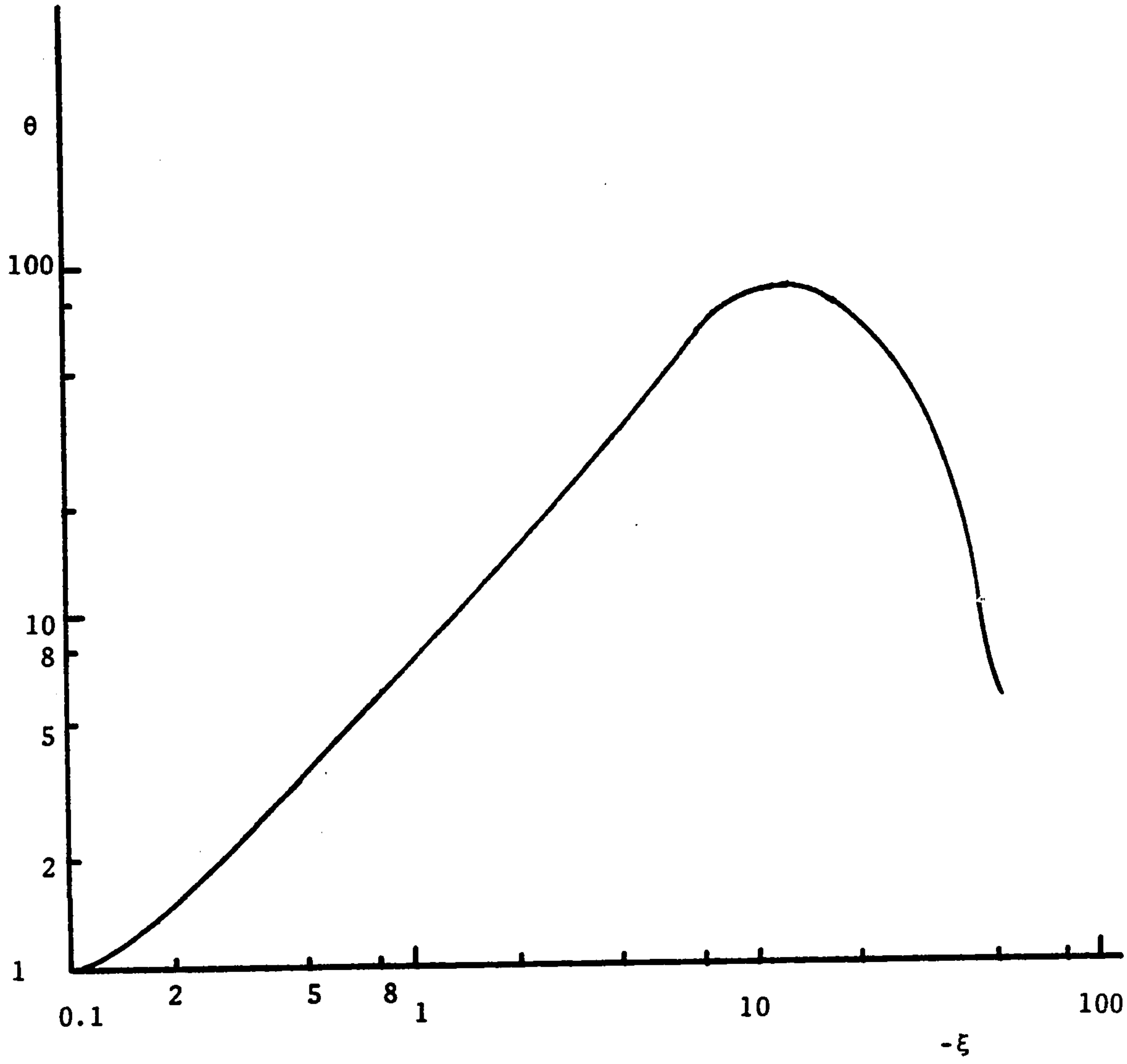

Figure 10. θ vs. -ξ for sheath region.

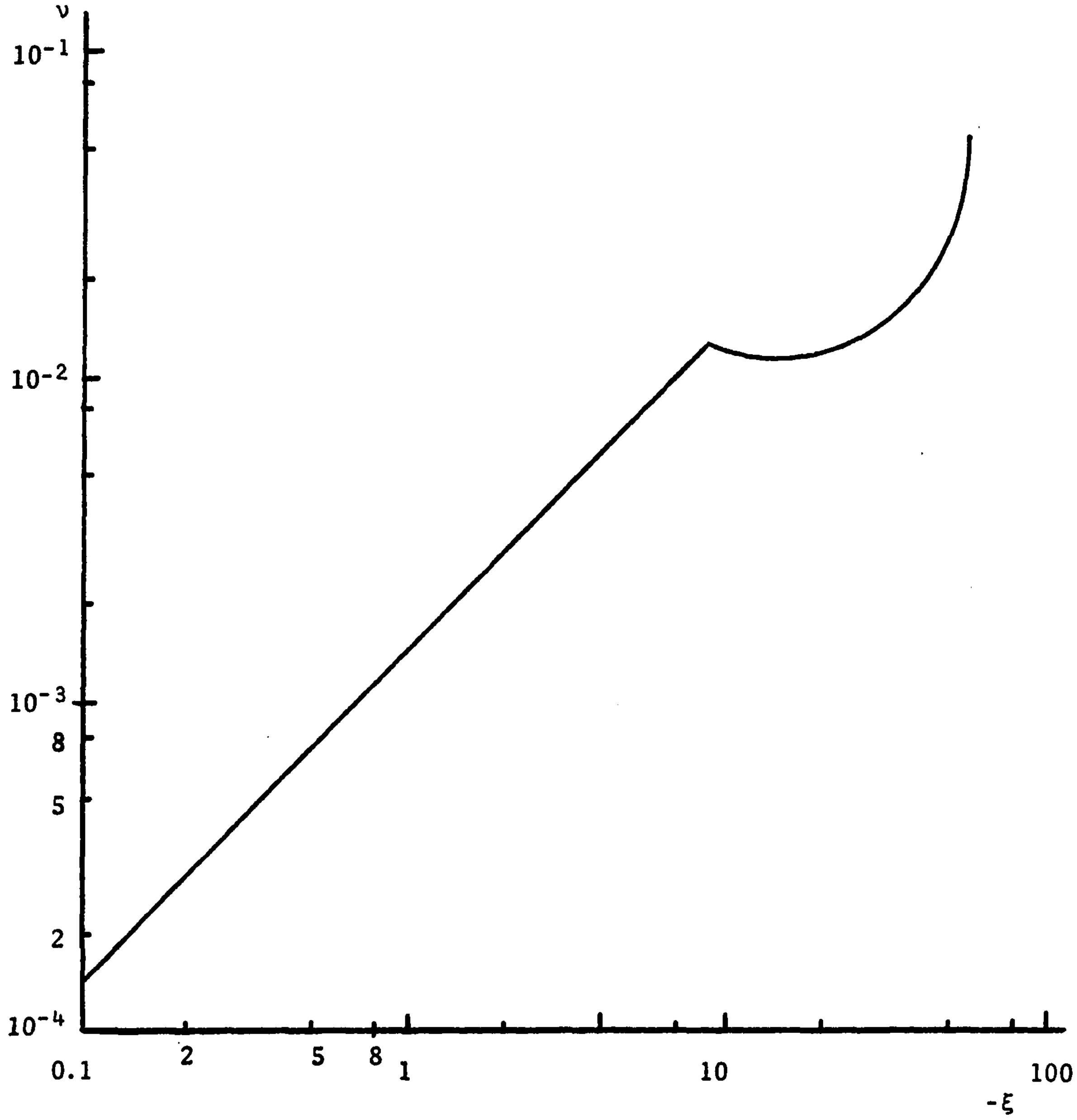

Figure 11. ν vs. -ξ for sheath region.

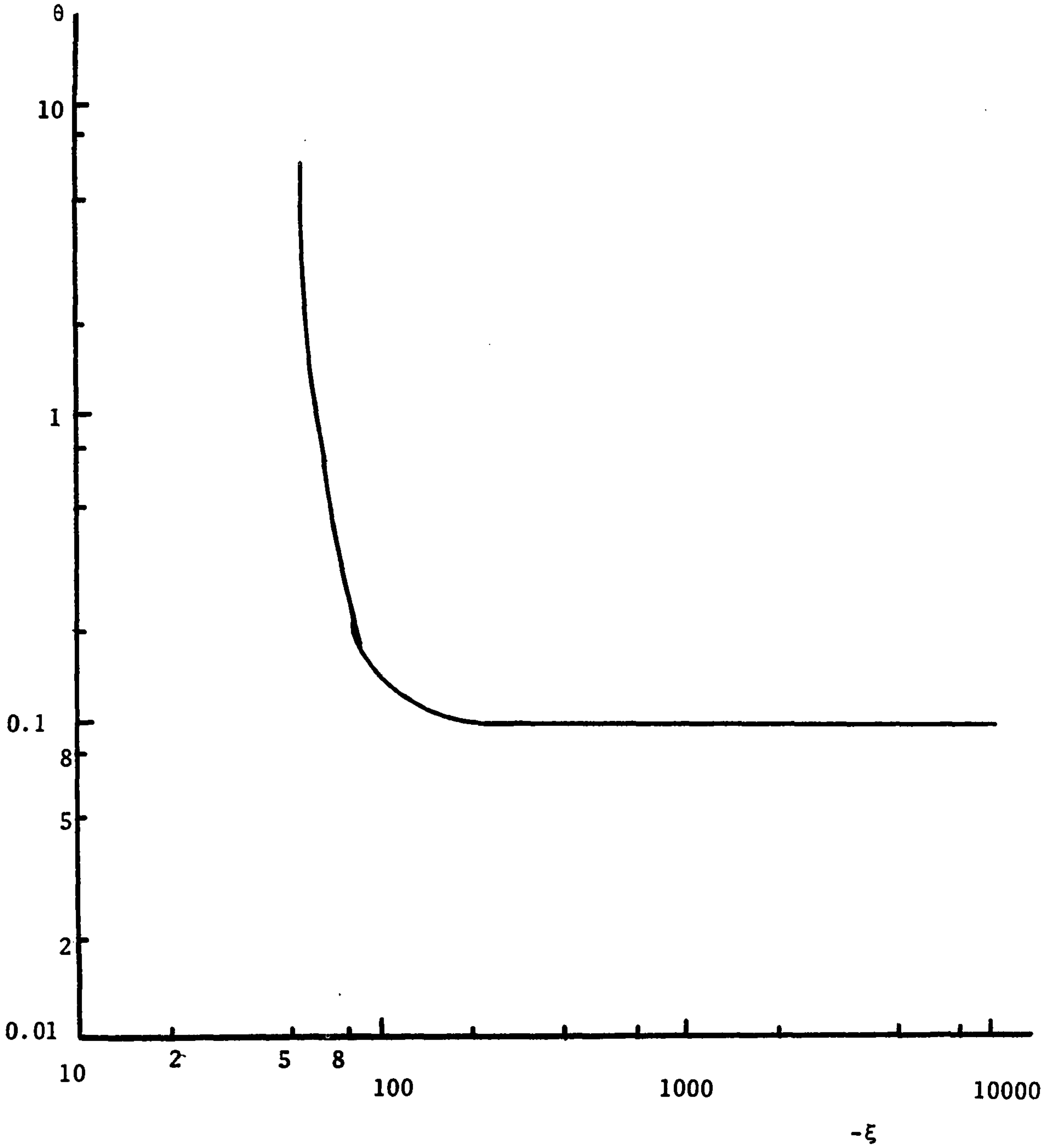

Figure 12. θ vs. $-\xi$ for quasi-neutral region.

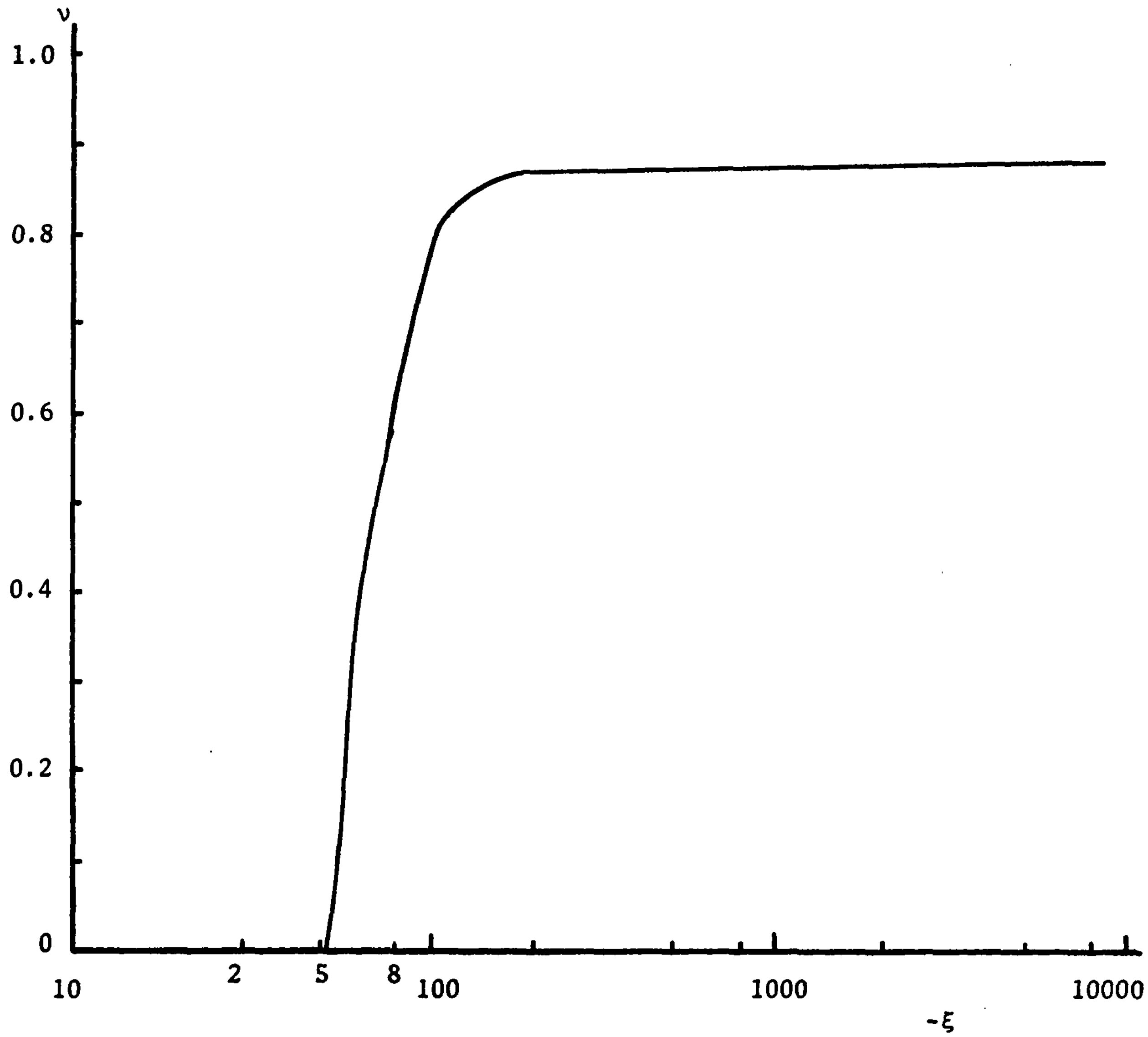

Figure 13. ν vs. -ξ for quasi-neutral region.

Path II:

$$\eta = 0.588\ (1+ \sqrt{1-0.793(\psi-1)^2},\ \eta > \frac{\eta_1}{2}$$

$$\eta = 0.588\ (1- \sqrt{1-0.793(\psi-1)^2},\ \eta < \frac{\eta_1}{2}$$

$$j = 1.134 \times 10^{-2} + \{1.123 \times 10^{-2} \sin^{-1}[0.8904(\psi-1)]\}\ ,$$

$$\theta = \frac{\psi}{j}\ [0.814 - 1.25(1.177-\eta) + 0.31 \sin^{-1} 0.8904(\psi-1)$$

$$+ 0.274(\psi-1) - j(\psi-1)(11.62)]\ .$$

Now along path II, ξ is given by equation (86), which cannot be integrated. An approximation for ξ can be obtained by neglecting the arc sine term and setting:

$$\xi \approx \frac{1}{\mu}\ [2+ \frac{2\kappa^2}{3\eta_1}\int \frac{\psi d\psi}{[1- \frac{4\kappa^2}{\eta_1}\ (\eta_1-1)(\psi-1)^2]^{\frac{1}{2}}}]\ .$$

This will give an ξ which is larger than the true value, but over most of the path, the neglect of the arc sine term is not too bad since ψ-1 is small for much of the path. The result for ξ is:

$$\xi = -[8.02 + 17.82\ \left\{1\mp\sqrt{1-0.793(\psi-1)^2}\right\}$$

$$+ 3.97 \sin^{-1} 0.8904(\psi-1)].$$

The positive sign of the radical applies for the lower portion of the path.

To obtain the structure of the wave for large α, an iterative technique is necessary since μ cannot be considered constant. When $\alpha = 10$, for instance, the expression for θ_1 from equation (83) gives a value

$$\theta_1 = 0.62 ,$$

for $\kappa = -1.247$, $\eta_1 = 1.764$, and $\mu_o = 0.2$. Thus, θ is in the region where μ is a function of θ and the assumption of a constant μ in the integration for j and ψ is not valid. A method for arriving at a solution for this case would be to determine the structure of the wave under the assumption of constant μ, and, using the values of θ, obtain μ as a function of position, ξ. These values of μ could then be used to determine a second approximation for j and so on.

However, there is another difficulty when α is large. When the equation for electron production is written

$$\frac{dj}{d\xi} = \mu \frac{j}{\psi}$$

and μ is considered to be strictly due to thermal collisions, θ cannot be allowed to go to zero at the front of the wave, since then μ would also be zero and there would be a singularity in the solution at that point. Actually μ is a function of applied field as well as the temperature. If μ is written as:

$$\mu = \mu_T + \mu_F \eta^2 ,$$

where μ_F is constant and μ_T is the temperature dependent part of μ, then for fast waves, μ_T is approximately constant. Now η_1 has been shown to be of order 1, so that over path I, the term $\mu_F \eta^2$ is also relatively constant. For large temperatures, μ_T is expected to be large compared to μ_F. Thus, for small α, the solution obtained from assuming,

$$\mu = \mu_T ,$$

where μ_T is constant, should be valid. For large α, on the other hand, μ_T becomes quite small over much of the solution path and the term $\mu_F \eta^2$ must also be considered.

Unfortunately, the value of μ_F depends upon the detailed theory of electron mobility and ionization frequency in a strong electric field. This theory is not presently available, so it is not known at what temperature the field dependent portion of μ becomes comparable to the temperature dependent term.

CHAPTER VI

DISCUSSION OF RESULTS

The solution presented in Chapter V cannot be expected to represent the actual situation in a discharge tube for a number of reasons. In the first place, laboratory waves occur in three dimensions rather than in the idealized one-dimensional form of the model. The one dimensional zero current condition may not hold for three-dimensional waves, although the divergence of the current must be zero. For cylindrical symmetry, this condition becomes:

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_r}{\partial r} + \frac{i_r}{r} = 0 \; , \qquad (97)$$

where x is along the axis of the tube, r is the radius and i_x and i_r are the axial and radial components of the current. If condition (97) is to be fulfilled on the axis of the tube, i_r must go to zero as r goes to zero. Then, on the axis of the tube (97) becomes:

$$\frac{\partial i_r}{\partial x} + \frac{\partial i_x}{\partial x} = 0$$

and, since there is no current in front of the wave, the axial current must be zero on the tube's axis. Thus since $i_x = i_r = 0$, the zero current condition does indeed hold true on the axis of the discharge tube. Since most observers report that breakdown waves have either a planar appearing front or have their leading edge on the axis of the tube, the one dimensional model is probably not a bad representation.

The second difficulty with the solution of Chapter V is the technique of patching together solution paths which represent the assumed relationship between electron fluid velocity and electric field. The necessity for utilizing this technique is the lack of a proper theory of electron mobility in a microscopic sense. That is, the detailed nature of the dependence of the electron velocity on applied field is not available from theory. Thus, one is forced to resort to depicting a generalized behavior in terms of satisfying Poisson's equation and the balance of macroscopic forces acting on the electrons, with the criteria in mind that the equations should be capable of being integrated along the solution path.

With these reservations in mind, the implications of the solutions can be discussed. In order to facilitate the inspection of the results, a knowledge of the conversion factors between the non-dimensional parameters and the usual laboratory quantities will be useful. For the case of $\alpha = 0.086$, V_o is equal to 10^7 meter/sec. For helium, which has an ionization potential $e\phi_i = 24.58$ electron volts, K_1 is equal to 2.41×10^9 sec^{-1}/P, where P is pressure in millimeters of mercury [4].

Now V_o is opposed to E_o in direction for the antiforce wave. Thus, the value of $K = -1.247$ gives a value for E_o/P as:

$$\frac{E_o}{P} = \frac{m}{e}\frac{V_o}{\kappa}\frac{K_1}{P} = 1.1 \times 10^5 \frac{\text{Volts}}{\text{Meter-mm-Hg.}}$$

Thus $\eta = 1$ corresponds to a field of 1.1×10^5 volts/meter for a pressure of 1 mm Hg. For helium, $\theta = 1$ corresponds to a temperature of 3.80×10^5 °K. Also for helium the field $E_0 = 1.1 \times 10^5$ volts/meter implies that $\nu = 1$ corresponds to an electron density of 1.36×10^{16} electrons/meter^3. Finally, $\xi = -1$ corresponds to a value of -5.11×10^{-3} meters for x. Since x represents distance from the wave front, a negative x represents position behind the front.

Using these values, it is seen from Chapter V that as one proceeds into the wave, the field increases until it reaches a maximum value of 1.294×10^5 volts/meter and then decreases toward zero. The field attains this maximum value at a position 4.092×10^{-2} meters behind the wave front. Thus, the field peaks very quickly and then gradually decreases over a large distance.

The electron temperature increases very rapidly away from the wave front until it reaches a peak value of around 3.17×10^7 °K. The temperature has its peak value at a distance of 5.4×10^{-2} meters behind the wave front.

The electron fluid reaches a peak velocity of 2.123×10^7 meters/sec at a distance of 0.1886 meters behind the wave front.

The antiforce wave thus reaches quite high temperatures and electric fields close to the wave front. The electrons then cool slowly and as they cool, continue to cause ionization. For the very fast wave, the electrons cool over such a large distance behind the wave front that it is doubtful that the state of thermal equilibrium is ever approached in tubes of laboratory dimensions.

The weak solution for the antiforce waves does not predict a minimum wave velocity as opposed to the strong discontinuity theory of the proforce waves. An argument can be made against the existence of the strong discontinuity for the antiforce wave. The electric field drives the electrons away from the wave front for antiforce waves and there should thus be a net heat conduction away from the wave front. But a strong discontinuity requires an elevation in temperature across the shock interface and, since there are no electrons ahead of the shock front, the temperature rise is required to be supplied by heat conduction from the wave behind the front. There is thus a contradiction in the electron motion and the temperature behavior for a strong discontinuity for the antiforce waves. The weak discontinuity solution, on the other hand, predicts a temperature rise as one proceeds into the wave, which one would expect if the electron motion were away from the wave front. It is thus reasonable to suppose that only the weak discontinuity solution is applicable to antiforce waves.

It was mentioned earlier that the energy equations could not be used due to their incompatibility with the momentum equations. This was attributed to the failure to properly include terms such as the heat conduction. Now that a solution for θ is available, the value of $d\theta/d\xi$ can be estimated, even though it must be done graphically. When $\xi = -0.4$, $d\theta/d\xi$ has a value of -7.5, and when $\xi = -20$, $d\theta/d\xi$ has a value of around 2.7. The magnitude of the coefficient of thermal conduction can be estimated from kinetic theory and is given by:

$$K_T = \frac{2}{3\sqrt{\pi}} \frac{nm}{T} I ,$$

where I is a function of temperature [20]. For $\xi = -0.4$, T has a value of around 10^6 °K and the value of I is approximately 10^{18} meter^4 sec^{-3}. When $\xi = -20$, $T = 2.3 \times 10^7$ °K and I is around 2×10^{18} meter^4 sec^{-3}. Now $dT/dx = 7.44 \times 10^7 \, d\theta/d\xi$, and putting in the values for n at these values of ξ, it is found that at $\xi = -0.4$, the thermal conduction term has a magnitude of 291 Joules/(meter^2-sec), and at $\xi = -20$ the thermal term has a magnitude of 190 Joules/(meter^2-sec). These values must be compared with the flux of kinetic energy, $\frac{1}{2}nmv^3$ and enthalpy, $\frac{5}{2}$ vnkT at these points. At $\xi = -0.4$, $\frac{1}{2}nmv^3 = 8.62$ Joules/(meter^2-sec), and $\frac{5}{2}$ nvkT = 132 Joules/(meter^2-sec) and at $\xi = -20$, $\frac{1}{2}nmv^3 = 3 \times 10^5$ Joules/(meter^2-sec), and $\frac{5}{2}$ nvkT = 1.05×10^5 Joules/(meter^2-sec). Thus, it is entirely reasonable to assume that the heat conduction term may be important near the wave front but becomes negligible compared to the other terms in the energy equation as one proceeds into the wave.

CHAPTER VII

CONCLUSION

A theory has been presented which applies the concepts of fluid dynamics to the motion of electrons and ions in an initially strong electric field. The solution has been assumed to be in the form of a weak discontinuity at the junction between the electron wave and the neutral gas in front of the wave. For a fast antiforce wave, the field is nevertheless seen to peak at a short distance behind the wave front. The electron velocity and temperature also attain peak values close to the wave front. As in the strong discontinuity proforce solution, the antiforce wave exhibits a sheath region between the wave front and the quasi-neutral region. In the quasi-neutral region the electrons and ions have the same velocities and densities, but the electrons still exhibit a large temperature. The sheath region is much thicker for the antiforce waves than for the proforce waves and the peak temperature is much higher for the antiforce wave than for the proforce wave with the same velocity.

The predicted velocities for the antiforce waves are in quite good agreement with the data of Blais and Fowler [11], especially in view of the approximations involved in obtaining the solution. The electron densities reported by the same authors agree reasonably well with the value of 1.36×10^{15} electrons/meter^3 which the theory predicts

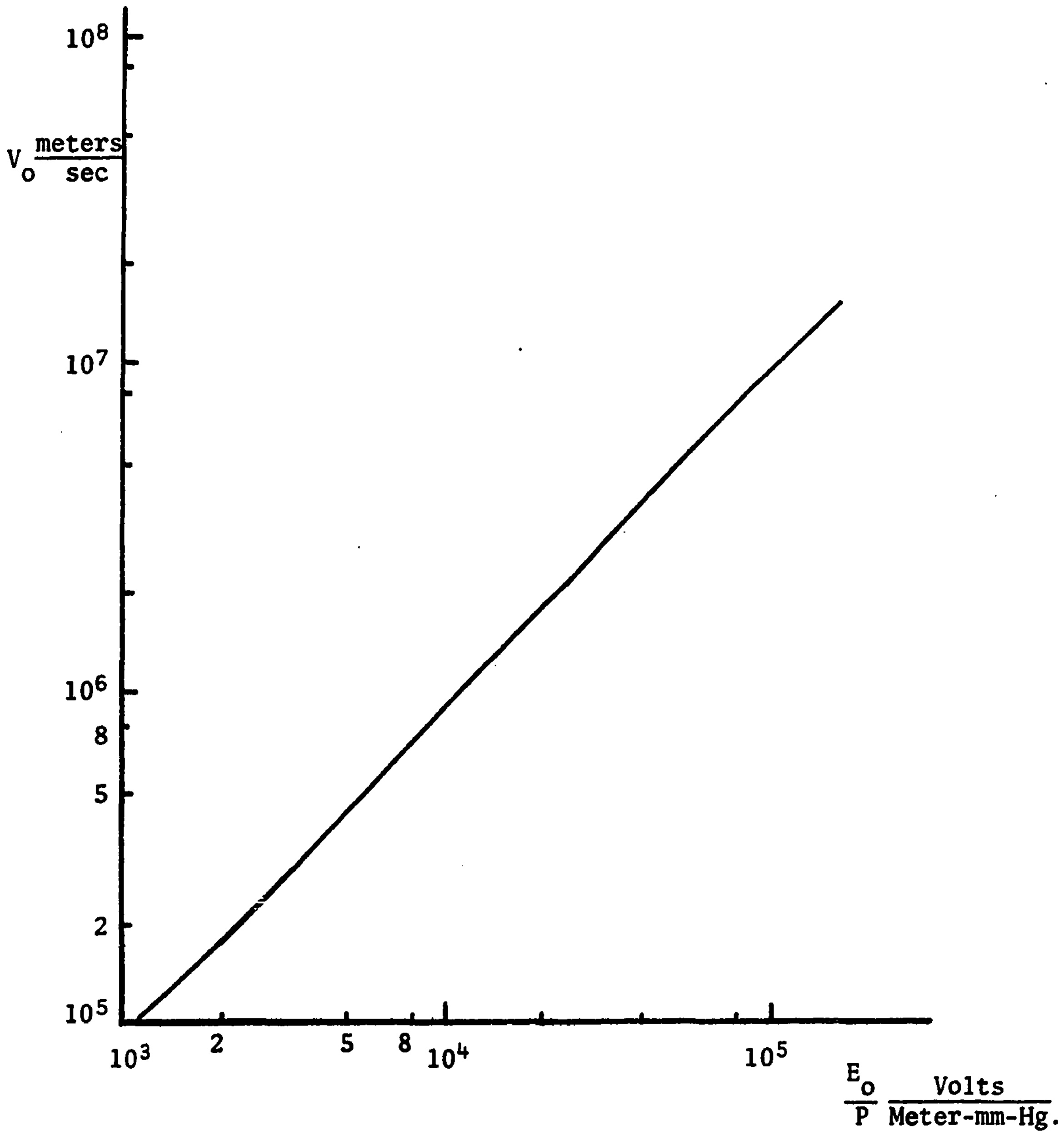

Figure 14. Wave velocity vs. applied field.

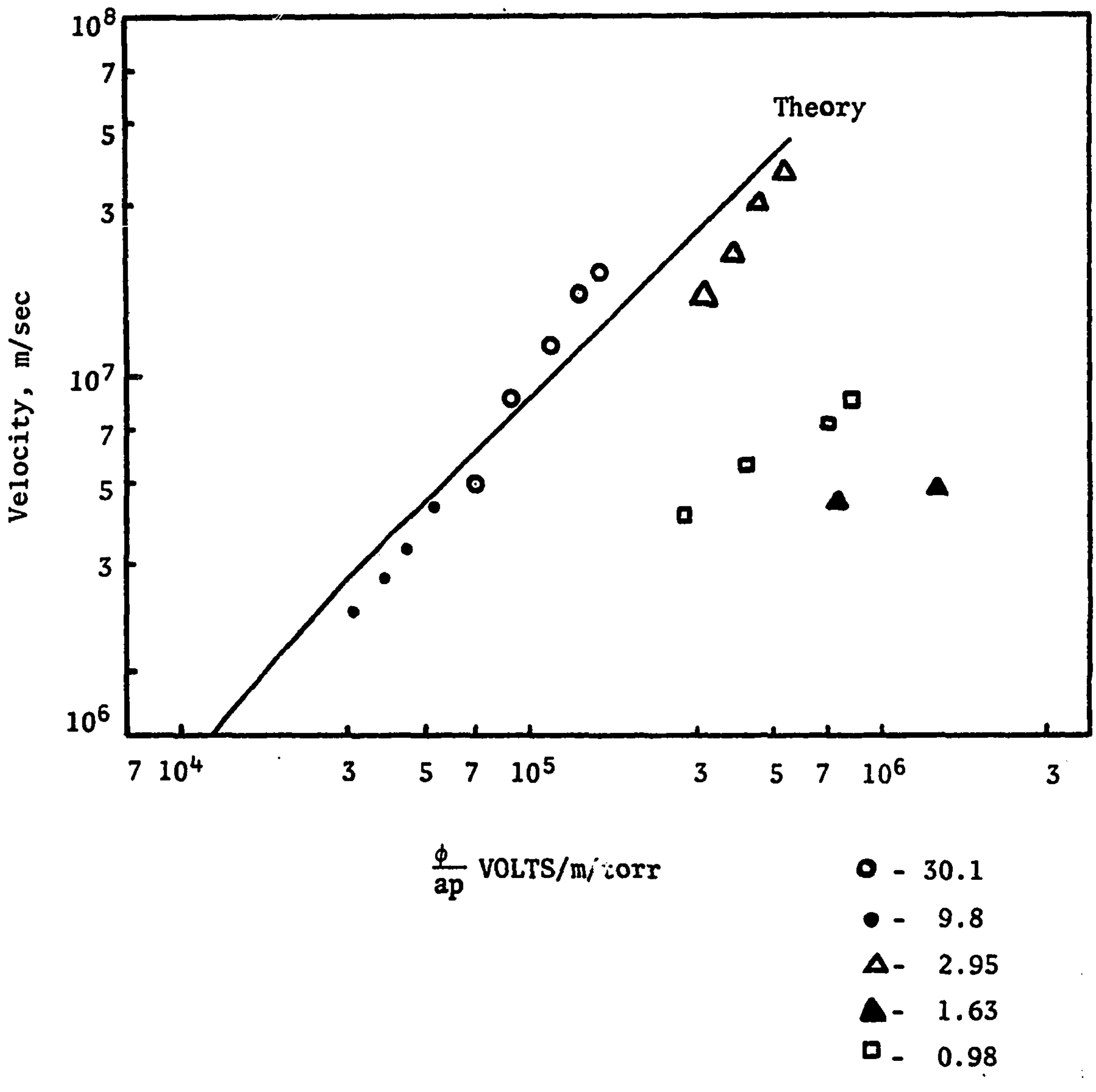

Figure 15. Comparison of predicted velocity with experiment. Data points by Blais and Fowler [11]. Solid line is theoretical prediction. Upper right legend is pressure in Torr.

over most of the quasi-neutral region for the conditions of their experiment. The observed temperatures, however, are far lower than those predicted by the theory. In addition, the theory predicts that the temperature will be higher for a fast wave than for a slow wave, which is in complete variance with the results of Fig. 6 of Blais and Fowler's article.

The difficulty may lie in the fact that the measured temperature is derived from optical data and, as mentioned by Blais and Fowler, depends upon the response of the atoms to excitation, a process which requires a finite amount of time. The slower wave may provide more time for excitation at a given location than the fast wave and thus the measured temperatures may not be representative of the electron temperatures. Since the antiforce waves exhibit a sheath structure similar to the fast proforce waves, the resolving time of the instruments making the measurements determines whether the sheath structure is observed. Since the sheath thickness depends upon the wave velocity as well, it is probable that the temperature reported for the fast waves represent measurements in the quasi-neutral region rather than the sheath. On the other hand, the slower waves may permit temperatures of the sheath to be observed.

The model requires that the electron temperature provide the driving force to allow the wave to proceed in opposition to the electric field. The concept that fast waves require higher temperature than slow waves is thus central to the model and the tentative conclusion is that the measured temperatures do not represent the true electron temperatures.

The difficulty of obtaining the detailed structure of the waves experimentally makes it all the more important to possess a theory which can assist in planning experiments and subsequent analysis of the data. This dissertation does not purport to provide a complete or highly accurate theory but it is hoped that the techniques of analysis presented may be beneficial to future work which others may undertake. The theory of ionization frequency in the presence of a strong electric field is required for proper extension of the results to the case of very slow waves. However, the results presented should serve as a guide as to the general nature of the solution. At the present time, the effort of this thesis, combined with the pro-force solution of Shelton and Fowler, constitutes the only theory for electron and ion mobilities in a strong electric field.

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