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Modular Open-Source Software for Item Factor Analysis

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Abstract

This article introduces an item factor analysis (IFA) module for `OpenMx`, a free, open-source, and modular statistical modeling package that runs within the R programming environment on GNU/Linux, Mac OS X, and Microsoft Windows. The IFA module offers a novel model specification language that is well suited to programmatic generation and manipulation of models. Modular organization of the source code facilitates the easy addition of item models, item parameter estimation algorithms, optimizers, test scoring algorithms, and fit diagnostics all within an integrated framework. Three short example scripts are presented for fitting item parameters, latent distribution parameters, and a multiple group model. The availability of both IFA and structural equation modeling in the same software is a step toward the unification of these two methodologies.

Keywords

item factor analysis, item response theory, modular, open-source

Introduction

When a test consisting of items is administered, it is desirable to understand what those items measure. Furthermore, if a test is to be summarized as a single score, the items should assess a single latent dimension. This is especially important for

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computer adaptive testing because each examinee may receive a different set of items intended to measure the same latent ability. These issues have received attention in the literature since at least the 1950s. However, it was the publication of the marginal maximum likelihood algorithm (MML; Bock & Aitkin, 1981) and the increasing availability of computing power that opened the way for efficient analysis of practical problems grounded with a sound theoretical footing. With demonstrations of the flexibility and sensitivity of MML for modeling tests and questionnaires (e.g., Bock, Gibbons, & Muraki, 1988), full-information item factor analysis (IFA) has grown in popularity.

A wide variety of software is available for estimation of IFA models. Examples include ConQuest (M. L. Wu, Adams, Wilson, & Haldane, 2007), EQSIRT (E. J. C. Wu & Bentler, 2012), flexMIRT (Cai, 2012), IRTPRO (Cai, Thissen, & duToit, 2011), and `mirt` (Chalmers, 2012). In light of the many excellent choices already available, it may be surprising that there is an opportunity for new IFA software. Even so, the present article introduces the availability of IFA software that is substantially different from what is currently available.

Why New Software for IFA?

Modular design is an important software attribute that has been neglected by current IFA software. Modular software is written such that each section of code operates independently and is accessed via a well-defined interface. A benefit of modularity is that many programmers can work on the code simultaneously as long as each module maintains the expected behavior of its interface. Our new IFA software is itself a module within `OpenMx` (Boker et al., 2011), a free and open-source software originally designed for structural equation modeling (SEM). `OpenMx` runs inside the R statistical programming environment (R Core Team, 2014) in heterogeneous computing environments. Similar to the `OpenMx` style of model specification for SEM, the IFA module offers a novel model specification language.

Open-Source

`OpenMx` is open-source; hence, the source code is available for everybody to view, modify, and use. To help organize a community around the project, the `OpenMx` team maintains a website (http://openmx.psyc.virginia.edu) that hosts binary and source versions of the software and several forms of tutorials and reference documentation. Help with `OpenMx` is available on the website from discussion forums and a community-maintained Wiki.

`OpenMx` is not the only open-source software for IFA. The R package mirt is also open-source. In fulfillment of some of the claimed benefits of open-source software, a friendly exchange of code and ideas has taken place between `OpenMx` and `mirt` (Pritikin, 2014). The `OpenMx` team welcomes contributors and encourages cross-pollination between implementations. Of course, open-source

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software is peer-reviewed. In many software engineering projects, peer-review has led to more reliable and higher quality software in comparison with a closed-source approach (Aberdour, 2007).

Heterogeneous Computing Environments

`OpenMx` runs on a variety of operating systems including GNU/Linux, Mac OS X, and Microsoft Windows. `OpenMx` scripts that are written on one operating system can be used on other operating systems without modification. This platform-independence is useful in today's heterogeneous computing environments, where each researcher on a team may have a different preferred computing platform. Moreover, since `OpenMx` is free, no license obligations hinder the use of `OpenMx` on computer clusters or exotic supercomputers.

Model Specification

While IFA software has grown more capable, model specification generally follows a structure that has changed very little since the 1980s. Specification of IFA models is generally accomplished with a serially ordered script written in a domain specific language unique to a particular software package. One of the key innovations in the design of `OpenMx` was the recognition that models can be specified directly in `R`, leveraging the fact that `R` is interactive and offers a full programming language. One may use `OpenMx` without changing one's conception of model building, engaging in programming minimally. However, consider the challenge of specifying nominal item models for testlets.

The nominal model (e.g., Thissen, Cai, & Bock, 2010) can be defined as

$$\mathbf{a} = T_a \alpha$$
$$\mathbf{c} = T_c \boldsymbol{\gamma}$$
$$\Pr(\text{pick} = k | s, \theta, a_k, c_k) = C \frac{1}{1 + \exp(-(s\theta a_k + c_k))}$$

where the kth entry of a and c are the result of multiplying two vectors of free parameters α and γ by fixed matrices T_a and T_c, respectively; a_0 and c_0 are fixed to 0 for identification; and C is a normalizing factor such that $\sum_k \Pr(\text{pick} = k) = 1$. For locally dependent items that share a common stimulus, Thissen, Steinberg, and Mooney (1989) suggested fitting a nominal item model to their sum-score. Orthogonal polynomial contrasts were used for the T_a and T_c matrices. To avoid overfitting, a proportion of the α_k and γ_k parameters were fixed to zero and the remainder estimated as free parameters, typically obtaining satisfactory fit with a less than full-rank model.

The specification of such nominal items is somewhat labor intensive. The transformation matrices, T_a and T_c, depend on the number of outcomes and the proportion of free parameters may be controlled by freeing or fixing individual parameters. A

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function is given in Appendix A that takes as arguments a dataset, the item name, the number of factors, the proportion of full-rank of α, and proportion of full-rank of γ. This is a simpler interface because, conceptually, these are the form of the parameters that a user wants to control and the user has direct control of them. Granted, there is nothing that precludes an analyst from writing a similar function for any IFA software package. However, this is often regarded as extra work and is rarely, if ever, done.

Automation of the specification of nominal testlet items is one example of the use of programming to ease a modeling task. One of the main contributions of `OpenMx` has been to encourage treatment of statistical models and their components as things that can be generated and manipulated within a programming environment. The `OpenMx` IFA module also uses this style of model specification. Not all users will prefer to exploit such a low-level user interface. Our goal is to extend the range of utility in both directions, toward both expert and novice users. Sophisticated users are empowered to build nonstandard models or to simplify application of routine analyses by developing higher-level interfaces. For example, the `metaSEM` package (Cheung, 2014) simplifies the specification of `OpenMx` models for meta-analysis. A similar package could simplify the specification of nominal testlets to make the technique simpler for nonspecialists to put into practice.

Modular Components

`OpenMx` is written using modular programming techniques in the `R` and `C++` languages with the intent that it will be maintained and extended by members of the research community. The core programming team helps and encourages statistical and quantitative researchers to add their research projects to the larger `OpenMx` framework. We believe that nurturing a sustainable community around `OpenMx` is essential to the project's long-term prosperity (Nakakoji, Yamamoto, Nishinaka, Kishida, & Ye, 2002). To work on part of `OpenMx`, one does not need to understand the inner workings of all other modules; it is only necessary to understand and adhere to the interface of the thing that one wishes to customize or extend. Although `OpenMx` originally focused on SEM, the scope of `OpenMx` is often stretched in new directions, expanding its scope (e.g., state-space modeling, Hunter, 2014). The IFA module is another instance of this happening. In creation of the IFA module, economy of effort was realized by leveraging the existing model specification and data handling methods already built into `OpenMx`. The IFA module, in turn, adds a new set of interfaces that can be customized and extended. Those who practice basic research on IFA are invited to incorporate their research projects into the `OpenMx` IFA module.

Item Models

The IFA module of `OpenMx` relies on the RPF package for response probability functions (Pritikin, Chalmers, Weeks, Cai, & Houts, 2014). Three functions are

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Table 1. Interface for Response Probability Functions.

Method
Descriptive
Count of nonestimated parameters
Count of estimated parameters
Default upper and lower bounds (if any)
Imperative
Plausible random parameters
Response probability function
First and second derivatives
Estimable parameters with respect to the log likelihood
Outcome probabilities with respect to examinee ability

currently available: the 4PL dichotomous response model (Loken & Rulison, 2010), graded response model (Samejima, 1969), and nominal response model (Thissen et al., 2010). Of course, the 1PL, 2PL, 3PL, partial credit model (Masters, 1982), and generalized partial credit model (Muraki, 1992) can be constructed as restrictions of the available models. All models use the logistic metric and a multidimensional parameterization.

The community is invited to expand the list of available models. All that is required to add a new response probability function is to contribute computer code to implement the RPF interface (see Table 1). Beyond this modular interface, no additional computer code is required to add a new type of item model. From the point of view of the rest of the software suite, a new item model is indistinguishable from any of the three original item models. Item estimation and fit statistics would work seamlessly with a new item model just as they do with the item models that are currently implemented.

Item Parameter Estimation

Item parameters are optimized using MML (Bock & Aitkin, 1981) with an equal interval quadrature. For a convenient performance boost, automatic detection of two-tier covariance structure is implemented (Cai, 2010a). Bayesian priors of any functional form can be placed on any parameter. As per usual SEM practice, a free parameter can be fixed to a constant or equated with other free parameters. Precision of estimates (e.g., standard errors) can be assessed by the direct method (Oakes, 1999), covariance of the row-wise gradients, Richardson extrapolation (Jamshidian & Jennrich, 2000), Supplemented EM (Meng & Rubin, 1991; Tian, Cai, Thissen, & Xin, 2013), sandwich-type covariance (Louis, 1982; Yuan, Cheng, & Patton, 2013), likelihood-based confidence intervals, and bootstrap. Accuracy of parameter recovery and standard errors is indistinguishable from accuracy reported by Cai, Yang, and Hansen (2011). However, users are encouraged to verify our accuracy claims for

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themselves. A number of ready-to-run simulation studies are included with OpenMx, including both simulation studies from Cai, Yang, et al. (2011).

Optimizers

IFA is closely related to factor analysis (Kamata & Bauer, 2008; Takane & De Leeuw, 1987). However, IFA makes the simplifying assumption of conditional independence (Equation 1) to achieve higher performance. The Hessian of the completed data model is block diagonal (Cai, 2010b). OpenMx takes full advantage with a block-wise matrix inversion routine and Newton–Raphson optimizer. To demonstrate and exercise this code, a test in the OpenMx test suite routinely fits a model with 1,536 2PL items. This test completes in 17 seconds on a developer laptop, less than the time it takes to invert a $3{,}072 \times 3{,}072$ matrix once on the same hardware. MML offers remarkable efficiency but at the cost of flexibility. To compute likelihood-based confidence intervals, a general nonlinear optimizer is required.

OpenMx currently offers two nonlinear optimizers: NPSOL (Gill, Murray, Saunders, & Wright, 1986) and CSOLNP (Zahery, Gillespie, & Neale, 2014). Since optimizers use a modular interface, a nonlinear optimizer can be substituted for the Newton–Raphson optimizer in the M-step of MML or can optimize an IFA model directly without MML. The nonlinear optimizers are much slower than Newton–Raphson, but can optimize a much broader class of problems. The ability to switch optimizers can also simplify debugging. For example, if Newton–Raphson cannot solve a problem then it is a simple matter to check whether NPSOL can solve it. NPSOL tries to optimize the fit function without the use of analytic derivatives. If NPSOL cannot solve it either then, most likely, there is a mistake in the problem specification.

Test Scores

Examinee scores can be obtained with expected a posteriori (EAP; Bock & Mislevy, 1982), maximum a posteriori (MAP), and sum-score EAP (Thissen, Pommerich, Billeaud, & Williams, 1995) methods. However, the user interface is different than what many traditional IFA packages offer (e.g., menu options or buttons). EAP family scores do not involve optimization whereas MAP scores do. Therefore, EAP family scores are available from the RPF package, while MAP scores involve running an OpenMx optimization function. This seemingly minor difference is of great philosophical importance to the OpenMx design team. OpenMx is intended to accurately reflect the underlying mathematical process. We believe that closely following the math is the best way to avoid designing ourselves into a corner. Our users expect their data analysis scripts to continue working year after year, even as we add features and evolve the software. If we design ourselves into a corner then we will be forced to break backward compatibility and users will be

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forced to update their scripts. This means that some of the OpenMx IFA interfaces are not as streamlined as most other IFA software in popular use. One could regard OpenMx as a middle layer to support a point-and-click style IFA user interface for analysts who are satisfied with fitting an occasional single factor 2PL model. On the other hand, we believe sophisticated users will appreciate the power and consistency of the OpenMx approach when building intricate or nonstandard models.

Fit Diagnostics

OpenMx offers popular model fit statistics for IFA models including CFI, TLI, RMSEA, AIC, and BIC, consistent with treatment of structural equation models. In addition, the RPF package includes IFA specific diagnostic tests. These include a test of the assumption of conditional independence (Chen & Thissen, 1997), a sum-score-based test of item fit (Kang & Chen, 2008; Orlando & Thissen, 2000), and a full-information multinomial fit test (Bartholomew & Tzamourani, 1999). These tests are optimized for two-tier covariance structure. Also available are Rasch residual-based fit statistics (Wright & Masters, 1982). Consistent with our theme of modularity, each diagnostic procedure is an independent R function. Easy access to single tests make it straightforward to verify the uniformity of the null distribution or compute a bootstrap p value.

Example Scripts

Before we present the scripts, a brief review of IFA models will be helpful. The conditional likelihood of response x_{ij} to item j from person i with item parameters ξ_j and latent ability θ_i is

$$L(x_i|\xi,\theta_i) = \prod_j \Pr(\text{pick} = x_{ij}|\xi_j,\theta_i). \quad (1)$$

One implication of Equation 1 is that items are assumed to be conditionally independent given the latent ability θ. That is, the outcome of one item does not have any influence on another item after controlling for ξ and θ_i. The unconditional likelihood is obtained by integrating over the latent distribution θ_i,

$$L(x_i|\xi) = \int L(x_i|\xi,\theta_i)L(\theta_i)\,\mathrm{d}\theta_i. \quad (2)$$

With an assumption that examinees are independently and identically distributed, we can sum the individual log likelihoods,

$$\Gamma = \sum_i \log L(x_i|\xi). \quad (3)$$

One important observation about Equation 3 is that there are two kinds of parameters. There are item parameters ξ and latent distribution parameters θ of the

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participants. For didactic purposes, we will start with a model for item parameters ξ and neglect the latent distribution, assuming that the latent distribution is standard multivariate normal. Once there is some experience with item models, we will fix item parameters and focus on estimating latent distribution parameters. Finally, an example will be given of a model that estimates both item and latent distribution parameters simultaneously.

Readers familiar with prior versions of `OpenMx` know that only one strategy was available to find the maximum likelihood estimate and obtain an estimate of the information matrix. Many more options are available for IFA models. To accommodate the additional choices, the user can attach an explicit compute plan to a model. The compute plan is flexible way to communicate to `OpenMx` which operations to perform.

Item Parameters

Suppose you regularly administer the PANAS (Watson, Clark, & Tellegen, 1988), but instead of scoring participants by adding up the item scores, you want to try IFA. Here is how you might do it. Without loss of generality, we will only consider the positive affect part of the scale.

```
library(OpenMx) # load the OpenMx and RPF packages into R
library(rpf)

spec <-list() # spec will map item models to data columns
  # rpf.grm creates a graded response item model
spec[1:10] <- rpf.grm(outcomes = 5)
data <- read.csv('demo1.csv')
  # Coerce data to ordered factors
data <- mxFactor(data, levels = 1:5)

  # make starting values matrix
startingValues <- matrix(c(1, seq(1, -1, length.out = 4)),
                         ncol = length(spec), nrow = 5)
  # make an mxMatrix object to hold the free item parameters
imat <- mxMatrix(name = 'item', values = startingValues, free = TRUE,
                 dimnames = list(names(rpf.rparam(spec[[1]])), colnames(data)))
rownames(imat)[1] <- 'posAff'
  # Labels create equality constraints
imat$labels[1,] <- 'slope'

  # Give instructions on how to optimize the model
computePlan <-mxComputeSequence(steps = list(
    mxComputeEM(expectation = 'expectation', predict='scores',
      mstep=mxComputeNewtonRaphson()),
    mxComputeOnce(from = 'fitfunction', what= 'information', how= 'meat'),
    mxComputeStandardError()))
  # Construct an mxModel object called 'panas1' containing data, expectation,
  # fit function, and compute plan.
```

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```
29  panas1 <- mxModel ( model = 'panas1', imat,
30              mxData ( observed=data, type='raw'),
31              mxExpectationBA81 ( ItemSpec=spec ),
32              mxFitFunctionML ( ), computePlan )
33  panas1 <-mxRun (panas1, silent=TRUE)   # run the model
34  summary (panas1)                       # print a summary of the results
```

The item parameters are stored in the item matrix with 1 item per column (Line 15). Regardless of item model, the first rows contain the factor loadings. The exact parameterization of the graded response model used (Lines 6 and 13) is given in the RPF package (Pritikin et al., 2014). We use traditional parameter names from the item model except for the first row where we override the name with the name of our factor (Line 17). An equality constraint is placed on the parameters in the first row of the item matrix by giving them a label of slope (Line 19). This causes the estimation of a Rasch model instead of unconstrained 2PL items. The compute plan (Line 22) tells `OpenMx` what we want to do with the model. The default compute plan is to optimize the parameters of a model using a nonlinear optimizer. This is usually suitable for structural equation models, but for IFA models, it is much more efficient to use the Expectation–Maximization algorithm (Dempster, Laird, & Rubin, 1977). We must request this explicitly with `mxComputeEM` (Line 24). The information matrix is approximated by the inverse of the covariance of the row-wise first derivatives (Line 25). This estimate is called meat because it also forms the inside of a sandwich covariance matrix (White, 1994). The last of step the compute plan, `mxComputeStandardError` (Line 26), summarizes the information matrix as standard errors.

Latent Distribution Parameters

Suppose an enterprising researcher has administered the PANAS to a large sample from the general population, fit an item model to these data, and published item parameters. You are testing an experimental intervention that should increase positive affect and wish to check whether your sample has significantly more positive affect than the general population mean.

```
35    # Here we are going to reuse the item parameters from our first
36    # example. Instead of estimating them, we are going to fix
37    # them and assume that they are the true population parameters.
38  panas1$item$free [, ] <- FALSE
39
40    # set up the matrices to hold our latent free parameters
41  m.mat <- mxMatrix ( name = 'mean', nrow = 1, ncol = 1, values = 0, free = TRUE )
42  rownames ( m. mat) <- 'posAff'
43  cov.mat <- mxMatrix(name= 'cov', nrow = 1, ncol = 1, values = diag ( 1 ), free=TRUE)
44  dimnames (cov.mat) <- list ( 'posAff', 'posAff' )
45
46    # build the item part of the model
```

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```
47  data <- read.csv ( ' demo2 .csv ' )
48    # Coerce data to ordered factors
49  data <- mxFactor ( data, levels = 1:5 )
50  panasModel <- mxModel (model =panas1, m.mat, cov.mat,
51                         mxData ( observed=data, type='raw'), name='panas')
52
53    # build the latent distribution part of the model
54  latentModel <- mxModel ( model = ' latent ',
55    mxDataDynamic ( type = ' cov ', expectation='panas.expectation'),
56    mxExpectationNormal ( covariance= ' panas.cov ', means= 'panas.mean'),
57    mxFitFunctionML ( ) )
58
59  computePlan <- mxComputeSequence ( steps = list(
60    mxComputeEM ( expectation = ' panas.expectation ', predict = ' scores',
61      mstep=mxComputeGradientDescent(fitfunction= ' latent.fitfunction ' ) ),
62    mxComputeConfidenceInterval ( ) ) )
63
64    # build the container model
65  e1Model <- mxModel(model= 'experiment1', panasModel, latentModel,
66    mxFitFunctionMultigroup ( ' panas.fitfunction ' ),
67    mxCI ( ' panas.mean ' ),
68    computePlan )
69  e1Model <- mxRun (e1Model, silent=TRUE)  # run the model
70  summary(e1Model)                         # print a summary of the results
```

If you want to run this code on your own computer, note that this example is not self-contained but depends on the previous example. Inside the M-step (Line 61), the latent distribution of an IFA model is independent of item parameters (Cai, 2010b). The discrete masses accumulated in the quadrature are converted to a multivariate normal distribution by `mxDataDynamic` (Line 55). The model multivariate normal distribution is fit against these dynamic data by `mxExpectationNormal` (Line 56). Since the latent model does not have any constraints, the optimizer will find a model distribution that is essentially identical to the data distribution. However, this is not generally the case. Within the M-step, we can fit an arbitrary structural equation model to the data covariance and mean vector. In fact, `mxExpectationNormal` is exactly the same function that we could use to specify a SEM. The compute plan (Line 59) tells `OpenMx` what we want to do with the model. It is necessary to specify an explicit compute plan because there are more choices to make than can be accommodated by the default plan. `mxComputeGradientDescent` invokes a nonlinear optimizer (Line 61). Our use of `mxFitFunctionMultigroup` is not strictly necessary (Line 66). This fit function is used because it generalizes to more than 1 group (as in Appendix B). We request a likelihood-based confidence interval around the mean (using `mxCI`) to obtain a more robust test of our hypothesis than we could expect from a Wald test (Line 67).

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Item and Latent Distribution Parameters

The reader may be aware that other IFA software is more succinct when it comes to model specification. Only a single line of R code is required to fit our first example with mirt. The benefits of the OpenMx approach become apparent in the specification of more intricate models. Multiple group models often involve one group anchored to a standard normal distribution with latent distribution parameters of the other groups estimated relative to the anchoring group. An example is given in Appendix B that is approximately a combination of our first two examples. The specification is lengthy, but the user has control over every detail. As few as another 10 to 20 lines of R code are required to turn this example into a Monte Carlo study to predict statistical power at a range of candidate sample sizes (e.g., Kelly et al., in press).

Discussion

OpenMx is an open-source statistical modeling software package with IFA and SEM modules that runs in the R statistical computing environment. The IFA module builds on a strength of the OpenMx design wherein a full programming language is used for model specification. The IFA engine (Bock & Aitkin, 1981) has been carefully tuned to perform well on multicore hardware (Levon, 2014; Valgrind Developers, 2014) and can fit a wide variety of multidimensional IFA models. Once an IFA model is fit, a selection of factor scores, fit statistics, and diagnostic tests are available.

Projects like OpenMx are never fully completed and the IFA module is no exception. For higher-dimensional models, the Metropolis–Hastings Robbins–Monro algorithm (Cai, 2010b) would make a useful addition, since Bock and Aitkin (1981) become impractically slow as the number of dimensions increase (the curse of dimensionality). Once we can efficiently estimate models with more factors then it would be a straightforward extension to allow RAM (McArdle & McDonald, 1984) or LISREL (Jöreskog & Van Thillo, 1972) notation to model the mean and covariance matrix. The assumption that the latent distribution is multivariate normal could be relaxed using splines (Woods, 2006) or empirical histograms (Woods, 2007). A limited-information goodness of fit test (Cai & Hansen, 2013) would be helpful since the full-information test performs poorly as the number of items increase. Scripts to automate exploration of differential item function and test linking would ease these common tasks.

The current article only briefly covers the many features and facilities of OpenMx. To learn more, obtain a free download of the software, and participate in the forums please go to http://openmx.psyc.virginia.edu (Boker et al., 2011).

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Conclusions

OpenMx, a freely available open-source statistical software package, is now capable of estimating IFA models via MML. However, datasets often include other indicators measured on an interval or ratio scale. For example, in an educational context, students are often recorded with an age and a socioeconomic score. Educators want to understand how school performance is influenced by demographic factors in a general SEM setting. There is some work in this direction (e.g., Adams, Wilson, & Wang, 1997). However, at the time of writing, there is still no consensus on how to best combine IFA and SEM approaches without resorting to Monte Carlo methods (Cai, 2013). The availability of an IFA algorithm within capable, open-source SEM software is a step toward the unification of IFA and SEM. We hope OpenMx will spur research in to a more satisfactory reconciliation of these two methodologies.

Appendix A

Nominal Item Model, Testlet Construction

```
71  library (OpenMx) # load the OpenMx and RPF packages into R
72  library ( rpf )
73  nominalTestlet <- function ( data, name, factors, aRank, cRank) {
74    lev <- length ( levels ( data [ [ name ] ] ) ) # find the number of outcomes
75    spec1 <- rpf.nrm ( lev, factors ) # create a nominal item model
76    nthresh <- spec1$outcomes - 1
77
78    # determine which parameters are free
79    free <- rep ( FALSE, spec1$factors + 2 * nthresh)
80    free[1] <- TRUE
81    base <- spec1$factors+1
82    if (nthresh * aRank >= 1) {
83       free [ ( 1 + base ):(base + nthresh * aRank - 1)] <- TRUE
84    }
85    base <- base + nthresh
86    if (nthresh * cRank >= 1) {
87       free [ base:(base + nthresh * cRank - 1 ) ] <- TRUE
88    }
89
90    # suggest feasible starting values
91    sv <- free * c(rep(1,spec1$factors), rep(1, nthresh), rep(0, nthresh))
92    sv[ c(1,spec1$factors+1) ] <- 1
93
94    list ( spec1, free=free, startingValues=sv) # return results
95  }
96
97  # create some demonstration data and items
98  data <- data.frame ( fruitfly=mxFactor ( sample.int (5, 10, replace=TRUE), 1:5),
99                       lincoln=mxFactor ( sample.int ( 6, 10, replace=TRUE), 1:6) )
```

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```
# What the user would do to create a nominal testlet once they had data
t1 <- nominalTestlet(data, 'fruitfly', 1, .1, .5)
t2 <- nominalTestlet(data, 'lincoln', 1, .5, .3)
```

Appendix B

A Two Group Model, Estimating Item and Latent Distribution Parameters

```
library(OpenMx) # load the OpenMx and RPF packages into R
library(rpf)
spec <- list()    # spec will map item models to data columns
  # rpf.grm creates a graded response item model
spec[1:10] <- rpf.grm(outcomes = 5)

g1Data <- read.csv('demo1.csv')
  # Coerce data to ordered factors
g1Data <- mxFactor(g1Data, levels = 1:5)

  # make starting values matrix
startingValues <- matrix(c(1, seq(1, -1, length.out = 4)),
                         ncol = length(spec), nrow = 5)
  # make an mxMatrix object to hold the free item parameters
imat <- mxMatrix(name = 'item', values = startingValues, free = TRUE,
       dimnames = list(names(rpf.rparam(spec[[1]])), colnames(g1Data)))
rownames(imat)[1] <- 'posAff'
  # label item parameters to equate them across groups
for(rx in 1:nrow(imat)) for(cx in 1:ncol(imat)) {
  imat$labels[rx, cx] <- paste('I', rx, ',', cx, sep='')
}

  # The latent distribution of this group is fixed to standard normal.
anchor <- mxModel(model='anchor', imat,
                  mxData(observed = g1Data, type = 'raw'),
                  mxExpectationBA81(ItemSpec = spec), mxFitFunctionML())

  # set up the matrices to hold our latent free parameters
m.mat <- mxMatrix(name = 'mean', nrow = 1, ncol = 1, values = 0, free = TRUE)
rownames(m.mat) <- 'posAff'
cov.mat <- mxMatrix(name = 'cov', nrow = 1, ncol = 1, values = diag(1), free = TRUE)
dimnames(cov.mat) <- list('posAff', 'posAff')

  # build the item part of the 2nd group
g2Data <- read.csv('demo2.csv')
  # Coerce data to ordered factors
g2Data <- mxFactor(g2Data, levels = 1:5)
panasModel <- mxModel(model = 'panas2', m.mat, cov.mat, imat,
    mxData(observed = g2Data, type = 'raw'),
    mxExpectationBA81(ItemSpec=spec), mxFitFunctionML())

```

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```
144     # build the latent distribution part of the 2nd group
145   latentModel <- mxModel ( model = ' latent ',
146       mxDataDynamic ( type = ' cov ', expectation = ' panas2 . expectation ' ),
147       mxExpectationNormal ( covariance = ' panas2.cov ', means = ' panas2.mean ' ),
148       mxFitFunctionML ( ) )
149
150     # build the container model
151   groups <- c ( ' anchor ', ' panas2 ' )
152   mstepPlan <- list (
153       mxComputeNewtonRaphson ( freeSet = paste ( groups, ' item ', sep = ' .' ) ),
154       mxComputeGradientDescent ( fitfunction = ' latent . fitfunction ',
155                                  freeSet = c ( ' panas2 . mean ', ' panas2 .cov ' ) ) )
156   cModel <- mxModel ( model = ' container ', anchor, panasModel, latentModel,
157       mxFitFunctionMultigroup ( c ( ' anchor.fitfunction ', ' panas2.fitfunction ' ) ),
158       mxComputeEM ( expectation = paste ( groups, ' expectation ', sep = ' . '),
159         predict = ' scores ', mstep = mxComputeSequence ( steps =mstepPlan ) ) )
160   cModel <- mxRun ( c Model )       # run the model
161   summary ( cModel )                # print a summary of the results
```

This example uses the same data as the first two examples. However, instead of using only the first data set to estimate item parameters (Line 30), both data sets combined influence item parameters. In the `mstepPlan` (Line 152), `freeSet` is used to partition the free parameters into item parameters and latent distribution parameters (Lines 153 and 155). These two sets of parameters are optimized separately every E-M cycle.

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