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A STUDY ON INTERACTION BETWEEN APTITUDES AND CONCRETE VS. SYMBOLIC TEACHING METHODS AS PRESENTED TO THIRD-GRADE STUDENTS IN MULTIPLICATION AND DIVISION

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A STUDY ON INTERACTION BETWEEN APTITUDES

AND CONCRETE VS. SYMBOLIC TEACHING METHODS

AS PRESENTED TO THIRD-GRADE STUDENTS

IN MULTIPLICATION AND DIVISION

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CHAPTER I

INTRODUCTION

The "sense realism" method of instruction had its beginning over 300 years ago. (7:199)[1] Review of the literature since that time has shown many educators repeating and strengthening the recommendations of the early instructional pioneers, especially in teaching arithmetic to young people.

Many educational reformers inferred that the concept of number, and the basic operations of counting and addition, can be truly learned and applied only if the student has contact with objects. There are few arguments with this statement in general. Disagreement lies in the extent of contact with objects and the length of time between contact and formation of the concept.

This study could be termed as part of a search for an optimal length in the chain between observation of objects and the concept of multiplication. The length of the chain is measured both in time and intermediate concepts. Assuming a child has developed the concepts of number, counting, and addition primarily from observations of objects, can he generalize these concepts to multiplication without formally returning to the objects; or will shortening the chain by almost daily exposing the child to the objects strengthen the development of the concept and skill of multiplication?

Possibly the most consistent idea appearing in the literature is the use of multiple senses in learning. If a child is allowed to touch, move, perhaps even to smell or

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[1] These bibliographical references mean the first number is the number assigned to the work in the bibliography and the second number is the page number.

taste objects, he will more easily develop the attendant concepts and skills. Here, too, there is little argument in general. The question is the degree to which multiple senses should be involved. Realizing all children are in contact daily with objects which they may manipulate as they choose, will it be beneficial to formally expose them to objects in the classroom when new concepts are to be learned? Will manipulation of objects utilizing multiple senses be more beneficial than observation of pictures and charts to strengthen the bond between the concrete and symbolic in learning multiplication?

A second, and equally important, question is: Do teaching methods and students interact in the statistical sense of interaction? Will a group of students do better given a first method of instruction than with a second method while a different group will excel when given the second method? E. P. Torrance found interaction in some areas:

> To me, by far the most exciting insight that has come from our research is that different kinds of children learn best when given opportunities to learn in ways best suited to their motivations and abilities. Whenever teachers change their ways of teaching in significant ways, a different group of learners become the stars or high achievers. (58:678)

Assuming Piaget (47:15-17) is correct concerning his theory on stages of development, a child in the concrete development stage who has previously learned the operations of arithmetic would probably benefit from the observation and manipulation of concrete objects in learning multiplication. The child in the formal operational stage may be able to generalize or transform his knowledge of addition to develop the concepts involved in multiplication from the definition.

In considering the mental age of children, a normal third-grade class may have a mental age range of about four to twelve years. If the stages of development apply to mental age, then the students on the lower end of the scale would possibly be in the preoperational stage and they would require a considerable amount of exposure to the concrete world while those at the upper end of the scale would be at or near the formal operational stage and these would be able to generalize their knowledge in addition to mastering multiplication without a return to the concrete objects. A test for interaction between mental age and the teaching procedures should shed some light on this supposition.

R. M. Gagné (3:25) implied that some students attempt to solve problems using a visual or spatial term while other students may apply a symbolic or verbal term. Students of these different characteristics may well benefit from instruction that stresses their methods of attack.

If different instructional methods are developed to take advantage of differences in students, a problem of equal importance arises--that of deciding which students are to be given the various types of instruction. Educators must be able to predict with some degree of certainty the best instructional procedures for each student, or the development of different teaching methods will be of little value.

Need for the Study

There is controversy at the present time concerning the virtues of concrete manipulation, activity-oriented instruction, and laboratory-type instruction. This author feels that much of the controversy is caused by a tendency on the part of some theoretists and educational researchers to overgeneralize.

The search for optimal teaching methods in mathematics may have to be done topic by topic. The fact that a concrete teaching method might prove superior in teaching second-grade addition and subtraction doesn't necessarily imply that an extension of the same teaching method will be superior in third-grade multiplication. One must also guard against overgeneralization of teaching methods. The manipulation of concrete objects in a set manner prescribed and led by the teacher will not necessarily lead to the same results on a given topic as will manipulation of the same objects in an uncontrolled "discovery" type of instruction.

The summary of the review of the research, page 28, indicates almost conclusively that teaching procedures which include manipulation of objects are generally beneficial to students in first and second grades. The summary is much less conclusive in studies done in the higher grades. This leaves the third grade on the boundary between these levels.

Moody, Abell, and Bausell (40) did a study in third-grade multiplication only. The instruction in their study covered multiplication from single-digit numbers through multi-digit numbers in a time period of only four weeks. The textbook used in the present study recommended about twelve weeks to cover this material; some of the other textbooks do not cover multi-digit multiplication until the fourth grade. This possibly accounts for the very low level achievement of all the groups in the Moody et. al. study. Dawson and Ruddel (17) did a study on fourth-grade division only. Their results indicate that the concrete method was superior in both computational achievement and problem solving. Three Cuisenaire studies (see page 23) were done on the third-grade level with diverse results.

The conflicting results in the above studies indicate that further testing is required in the area of third-grade multiplication. None of the above studies were designed to determine which students benefited most from the different types of instruction.

Statement of the Problem

The topic of this study is multiplication and division as presented to third-grade students. The teaching methods compared were Method S, a symbolic approach in which a textbook, blackboard and verbal instruction were the only methods of instruction; and Method M, manipulation of concrete objects in addition to the text, blackboard and verbal instruction. Chapter III, Methodology, contains a complete description of the teaching methods, and examples of lesson plans used in Method M are contained in Appendix A.

From the beginning, the study was intended to be school-oriented in order to determine if the teaching methods produced significantly different results when taught in a "typical" classroom by a teacher with no special training in mathematics, other than the planning sessions spent with the author. The independent variables tested for interaction, other than those resulting from the Torrance Creativity Test, were of the type readily available to most schools.

The Hypotheses Tested All Stated in the Null Form

Ho I: There is no difference between the two teaching methods in achievement in computational skills and application as measured by the post-tests in multiplication and division.

Ho II: No interaction exists between teaching methods and mental ability as measured by the Otis-Lennon Mental Ability Test. (45)

Ho III: No interaction exists between teaching methods and scholastic achievement scores as measured by the Stanford Achievement Test.[1] (54)

Ho IV: No interaction exists between teaching methods and various forms of creativity as measured by the Torrance Test of Creative Thinking.[2] (59)

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[1]There were eight separate tests in this battery. A test for interaction was performed for each test.

[2]There were four separate tests in this battery. A test for interaction was performed for each test.

CHAPTER II

REVIEW OF THE LITERATURE

Literature Related to the Teaching Methods

Kieren, in a review of manipulative activity learning, stated:

> It is obvious from reading the articles or the advertisements in any recent mathematics teachers' journal on this continent or across the Atlantic that the use of manipulative activities in the teaching and learning of mathematics is in vogue. . . . Nonetheless it is an understatement to say that research is needed into the role and effects of manipulative activity in mathematics. (38:228)

Moody et. al said:

> Piaget, for example, has often been interpreted as positing the superiority of activity learning: "To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation and as a consequence to understand the way the object is constructed."
>
> If activity-oriented instruction is more effective than more conventional forms of instruction, then it should result in either higher original learning, higher transfer, or higher retention of the instructional content. If none of these increments are effected, then those theoretical discussions which predicted, or are interpreted as predicting, increments, e.g., Piaget, Bruner and Dienes, are suspect. (40:208)

The reason for the skepticism of Moody and others is mainly attributable to the large gap between experimental evidence and the popularity of activity learning in both literature and usage in the schools. At the present time, commercial catalogs for school supplies contain many pages of manipulative materials and other aids that involve student participation. In workshops, the topics of activity-oriented materials, concrete manipulation and mathematical laboratories are very popular.

The December, 1971, issue of The Arithmetic Teacher was devoted almost entirely to the topic of mathematics laboratories. While the articles in this issue don't seem to be as skeptical as the statements by Kieren and Moody, they did recommend caution on the part of teachers to carefully evaluate methods of instruction and the devices used to teach the different topics.

On the other side of the gap, the research has been inconclusive. Only a few studies have shown the use of manipulative aids superior to all other methods; but, on the other hand, even fewer have shown that the use of manipulative aids obtains poorer results than other methods.

A Brief History

It is doubtful if one could designate a starting point for the theory of concrete manipulation. Activity learning was not of much consequence in formal education as handed down from early Greece and Rome. Roger Ascham (1515-1588) summed up the feeling of the teaching theory that prevailed from the classical Greeks to his time as "Learning teacheth more in one year than experience in twenty; and learning teacheth safely when experience maketh more miserable than wise." (7:185)

The type of instruction known as "sense realism" had its beginning in the seventeenth century. Brubacher considered Comenius (1592-1670) as the earliest realist and "The man who has most often been called the first modern educator . . . " Possibly Comenius' most outstanding achievement was that of writing the first textbook employing pictures as a teaching device.

Comenius looked upon methods of instruction as the most important area in all of education:

> As soon as we have succeeded in finding the proper method, it will be no harder to teach schoolboys, in any number desired, than with the help of the printing press to cover a thousand sheets daily with the neatest writing . . . (7:201)

Comenius laid down the general rule that everything should be taught through the senses: " . . . one should try to employ more than one sense at a time, for senses like hearing and seeing will then reinforce each other. Imagine trying to teach physics without recourse to sensory experience at all." (7:200)

Following Comenius in rapid succession were such educators as John Locke (1632-1704), the great English educator, and Johann Julius Hecker (1707-1768), the founder of the German "Realschule," both of whom prescribed " . . . pedagogy appealing to the senses through concrete objects in the child's environment . . . " (7:203)

While the men above opened the way to a theoretical position, it remained for Jean Jacques Rousseau (1712-1778) to formalize the theory of teaching through the senses. Rousseau also recommended such teaching procedures as discovery and freedom to inquire as the child sees fit. (7:204-205) These procedures, like activity learning, are popular today.

Johann Heinrich Pestalozzi (1746-1827) did much to put Rousseau's theory to practical use. Warren Colburn (1793-1833) wrote the textbook, First Lessons in Arithmetic on the Plan of Pestalozzi, that was popular in early America. Brubacher noted: "Colburn . . . based his book on the assumption that the concept of number is first acquired by observing sensible objects." (7:210) Florian Cajori gave this account of Colburn's first lesson:

> Instead of introducing the young pupil to the science of numbers, as did old Dilworth, by the question, "What is arithmetic?" and the answer, "Arithmetic is the art or science of computing by numbers, either whole or in fractions," he was initiated into this science by the following simple question: "How many thumbs have you on

> your right hand? How many on your left? How many on both together?" The idea was to begin with the concrete and known instead of the abstract and unknown, then proceed gradually by successive steps to subjects more difficult. (9:106-107)

Colburn's procedure of going from the concrete to the abstract would still be timely today but, also as today, his recommendations were not universally accepted, as pointed out by Daniel Adams:

> Instructors of academies and common schools have been so long attached to the old synthetic method of instruction, that, unhappily, many are still (1829) strongly opposed to the introduction of the valuable works of Colburn. (9:107)

There were many successors to Rousseau and Pestalozzi in the nineteenth and early twentieth centuries. Of all American educators, John Dewey (1859-1952) is generally considered to have made the greatest contribution to educational methods. (7:228) Dewey's "Problem Method" relied on the child's natural tendency for activity. He used activities both to clarify the child's understanding and to stimulate his interest. Dewey insisted:

> . . . there is no such thing as genuine knowledge and fruitful understanding except as the offspring of <u>doing</u>. The analysis and rearrangement of facts which is indispensable to the growth of knowledge and power of explanation and right classification cannot be attained purely mentally--just inside the head. Men have to <u>do</u> something to the things when they wish to <u>find</u> out something; they have to alter conditions. (18:321)

Dewey warned that activities should not be done for the mere sake of activity but they should be selected and directed to lead to the desired learning, an observation that is still timely. (19:86)

In 1902, E. H. Moore, in a speech as the retiring president of the American Mathematical Society, talked of the dangers of over-abstraction in all levels of mathematics. The editors of the first yearbook of the National Council of

Teachers of Mathematics felt Moore's talk was still timely in 1926 and published it under the title, "On the Foundations of Mathematics:"

> Would it not be possible for the children in the grades to be trained in power of observation and experiment and reflection and deduction so that always their mathematics should be directly connected with matters of thoroughly concrete character? The response is immediate that this is being done today in the kindergartens and in the better elementary schools. (41:45)

A Theory of Concrete Manipulation in Teaching Mathematics in Primary Schools

The use of manipulative material or concrete objects in today's education is extensive. Not many educators recommend, as did Rousseau and Dewey, that the student be sent to the farms or the factories to become involved in life, but rather that the objects be brought into the classroom. In some instances, realism is employed in the form of play stores or kitchens, etc., but often devices such as pegs, pebbles, or rods are designed to simulate the actual object.

As previously stated, much of the theory recommending concrete manipulation is centered around two ideas: (1) The concept of number is first acquired by observing many objects. (2) One should teach all things through the senses and employ more than one sense in the learning process.

Piaget used a merger of these two ideas as a foundation for teaching children. He asserted that a mathematical concept doesn't come from the objects but from the actions performed on the objects. He used the illustration of a child counting a set of pebbles. No matter how he arranges the pebbles when he counts them, he arrives at the same number. Piaget said: "He did not discover a property of pebbles; he discovered a property of the action of ordering. The pebbles had no order. It was his action which introduced a linear order or a cyclical order, or any kind of order." (47:12)

Piaget contributed a great amount to the literature of teaching procedures. Perhaps his best-known works are his formalizing of the stages of the child's development and his prescribing of material and teaching procedures appropriate for the various stages.

The first stage of the child's development that Piaget distinguished is the sensory-motor stage, lasting approximately two years. (12:15-17) In this stage, the child is restricted to direct interaction with his environment.

Lasting roughly to age seven, the preoperational stage comes next. It is during this period that symbolic representation takes meaning, mostly in the form of language but also in pictures.

During the next period, approximately seven to eleven, the child passes through the concrete operational stage. At this time, the child uses the concepts of conservation and invariance, etc., when various operations are performed.

Copeland stated that the term "concrete operational" is given to this stage

> . . . because the child is obtaining ideas from operations on such concrete objects as water, clay, etc. At the beginning of the concrete operational level, the ideas of a child are based on observation and experience with objects in the physical world, but he is beginning to generalize or break away from manipulations of objects as a way of "knowing." When these generalizations are complete and correct, the child is at the concrete-operational level. (12:16)

Copeland interpreted Piaget as saying that the stages are not fixed at age levels, but vary from person to person and also from concept to concept:

> The preoperational period is often described as lasting from two to seven years of age. However, this is only a rough guide . . . For some mathematical concepts, children do not leave the preoperational stage until nine or ten years of age. (12:15)

The fourth stage is the formal operational: " . . . he can now reason on hypotheses, and not only on objects. He constructs new operations, operations of propositional logic, and not simply the operations of classes, relations and numbers." (47:9-10) On the teaching of multiplication Piaget said:

> . . . the construction of additive operations and that of multiplicative operations are bound together. It is wrong to think of additive structures as being established first . . . these two structures develop through parallel stages, and in close mutual dependence (at about seven to eight years of age) they constitute a single operational organization . . . (48:195)

Piaget considered that a child reaches an understanding of the concept of multiplication when he is able to divide a set of m x n elements in m equal sets by developing an n to 1 correspondence. Copeland said:

> . . . it is the reversibility involved in multiplication and division . . . that is impossible for many children below the operational level of approximately seven years of age.
>
> Multiplication and division, because of their inverse relationship, must both be understood if either is to be understood. (12:113)

Copeland summarized Piaget's approach to teaching multiplication: " . . . concrete materials are manipulated by children in order to develop an understanding of the basic addition and multiplication facts." (12:118)

Bruner, who said that Piaget is " . . . unquestionably the most impressive figure in the field of cognitive development today," (8:7) expanded on the need for concrete examples:

> We have already remarked that by giving the child multiple embodiments of the same general idea expressed in a common notation we lead him to "empty" the concept of specific sensory properties until he is able to grasp its abstract properties. But surely this is not the best way of describing the child's increasing development of insight. The growth of such abstractions is important. But what struck us about the children as we observed them is that they not only understood the abstractions they had learned, but also had a store of concrete images that served to exemplify the abstractions. When they searched for a way to deal with new problems, the task was usually

> carried out not simply by abstract means but also by "matching up" images. (8:65)

Many other educators have expressed the need for manipulation of concrete objects as shown in the following quotes. Dienes said: "In an entirely fluid and objectless world, it is hard to imagine that the concept of natural number could arise . . ." (20:13) Dienes also stated:

> A visual aid, or any other single "aid," although no doubt better than no aid, merely links the symbolism of mathematics to a visual equivalent. The "picture" of the concept is not the concept itself . . . It is the realization of the common properties of a great many different such representations that constitutes the mathematical insight. (20:16-17)

The entire volume of the eighteenth yearbook of the National Council of Teachers of Mathematics in 1945 was devoted to "Multi-Sensory Aids in the Teaching of Mathematics." Edith Sifton stated in the opening chapter that " . . . children learn through other avenues than their eyes and ears--for example, their hands." (52:1) As an example, she cited teaching the concept of a remainder when dividing 11 by 3:

> Miss Smith could go over the work carefully at the board . . . She might even make a row of eleven marks on the board, cross them off three at a time . . . But suppose that, instead, she tells you to take some of the small log cabin blocks and asks you to show her how many threes can be made with eleven of them. Wouldn't you prefer to acquire division facts with your hands and eyes, as well as your head? Wouldn't you understand them better and remember them longer? (52:7)

In advocating the use of laboratories in mathematics classrooms, L. Grace Carrol stated that while multi-sensory aids were introduced by Rousseau, Huxley and others over a century ago, in the present-day high school

> . . . classical subjects which antedate Huxley, Spencer, Rousseau, and the activity program are all too commonly taught by the medieval methods of lecture, question, and answer. Except, perhaps, for the difference in dress and attitude of the students, a casual visitor might be

> unable to tell the difference between many 1944 classrooms in mathematics and their prototypes of the Middle Ages. (11:16)

The urge to use more manipulation of materials and activity-oriented instruction has been steady. In 1956, Fehr, in School Science and Mathematics, said:

> Laboratory procedures in which physical objects are counted, measured, subjected to formation, and built to scale are being employed in the modern classroom. All mathematical learning can ultimately be traced back to some sensory experience, even though at times the chain may be long and nebulous. (26:118)

In the twenty-first yearbook of the National Council of Teachers of Mathematics Van Engen stated:

> This concept of the part that actions, or manipulations, play in the development of concepts of the first order is of utmost importance to the teacher. Children and adolescents use manipulatory experiences to develop primitive concepts, and those concepts which are more nearly related to the action world of the child are the ones that are more easily developed. . . . Books, paper, pencils, blackboards, and the drill exercises which usually accompany these instructional tools are not sufficient except, possibly, for that relatively small percentage of pupils who are symbolically minded. Today's schoolrooms are barren of those small inexpensive objects which provide those opportunities for perceptual and manipulatory experiences for which the average child can abstract and generalize in order to take the first steps in formulating a concept. (62:86)

Max Beberman continued urging the use of manipulative materials: "We need to involve students in problems whose solutions require the use of manipulative processes. But the problems must make sense to the students, and the students must be allowed to invent the methods of solution." (1:37)

In the March, 1964, issue of The Arithmetic Teacher, W. J. Sanders wrote:

> The excitement of mathematics comes from discovering how it works, not in drilling on rules that someone tells you.
> Each child must actively work with the model himself. It must not be just a teacher demonstration. (50:165)

Robert Davis, noting that in 1965 the almost universal method of instruction in the American classroom consisted of teachers in front of the classroom with the students listening, talking, reading or writing, with an almost complete lack of physical apparatus, offered as an alternative:

> A greater use of physical material in mathematics classes, a greater diversity of types of experience in the child's day, the identification and early introduction of basic mathematics ideas and more emphasis on student originality and creativity within the school mathematics program . . . (16:355)

The January, 1970, and December, 1971, issues of The Arithmetic Teacher were devoted almost entirely to mathematics laboratories and manipulative material, and both repeated the request for more laboratories and more manipulation. There was a difference in the December, 1971, issue as compared to the earlier articles and books--that of a more cautious attitude. Robert E. Reys stated the reason for the caution:

> Many prominent mathematics educators have strongly urged greater use of manipulative materials in teaching mathematics. The rationale for this emphasis seems educationally sound. Unfortunately, research studies in this area have not been conclusive in either supporting or refuting the value of these aids. (49:552)

Vance and Kieren also warned that

> . . . haphazard or faddish use of the notions of "experience" and "activity" in mathematics may be undermining the achievement of the essential cognitive goals of the mathematics program. (61:585)

> The research indicates that students can learn mathematical ideas from laboratory settings. However, in maximizing achievement on cognitive variables, other meaningful instruction appears to work as well if not better. (61:588)

A Review of the Related Research

Before starting the review of the various research papers, a few terms will be defined to help clarify the material:

> The Active Concrete Method: A method of instruction that involves some object which is handled by the student. Any method of instruction that involves a multi-sensory impression, primarily touch and muscular action, in addition to sight.
>
> The Passive Concrete Method: Concrete objects are used but only in teacher demonstration.
>
> The Semi-Concrete Method: This method involves a representation of physical objects in the form of pictures, charts or cards.

The dividing line between methods is not distinct. The last two methods are closely related because the sense of sight is primarily involved. It is also difficult to differentiate between active concrete and semi-concrete because a set of cards with pictures of different numbers of elements would probably be considered semi-concrete; however, if the cards are moved about and overlapped to form single sets, a form of concrete manipulation is taking place.

The following studies compared concrete and semi-concrete methods with traditional methods in teaching arithmetic.

Howard--1947 (33): In teaching fractions to fifth- and sixth-grade students, three teaching methods were used--a symbolic approach that made use of extensive drills, a concrete method that involved the use of concrete teaching aids, and a third method that contained a combination of the first two. Howard's tests measured computation, application and understanding, but no significant differences were detected among the three methods on the post-test given immediately after the course of study. However, a test given after the summer vacation to measure retention showed the third, or combination, group scored higher.

Neureiter, Troisi--1952 (44): This study was similar to Howard's except only two teaching methods were employed to teach sixth-grade students fractions--a concrete method and an abstract, or symbolic, method. Two testing programs were given, one to evaluate computational ability and a second to test for understanding and application. No difference was detected between the groups in computational skills but the concrete material group was significantly superior in the application and understanding test.

Dawson, Ruddel--1955 (17): Concrete and abstract teaching methods were used to teach division on the fourth-grade level. The experiment ran one semester and the subjects were tested in computational achievement and the ability to solve problems involving application of division. In both tests, as well as in repeated tests to show retention, the group using concrete aids was significantly superior.

Gibb--1956 (28): Among the variables tested in this study on second-grade children involving instruction in subtraction was method of instruction. Gibb used three teaching methods--a concrete method involving small dolls and other objects; a semi-concrete method that used pictures of discs on cards; and an abstract symbolic method. The statistical analysis of the post-test indicated that a difference among the means existed at ($P = .01$). The order of preference was semi-concrete, concrete and abstract.

Swick--1959 (55): In a different type of study, the two methods compared were the "traditional" method in use in the Kingsport, Tennessee, school system in 1953-54, and an experimental method that included extensive use of multi-sensory aids. The grade levels were from two through five. After the students were given a test, the experiment started and the study was divided into two time periods. During the first period, the entire group was given the "traditional" method of instruction after which the same test was repeated and the progress recorded. During the second period, the multi-sensory method was used on all subjects. The same test was again given and the progress reported from the previous test. A statistical comparison was made on the amount of improvement made during the two instructional periods. The multi-sensory method had students scoring significantly higher in both computation and reasoning. No differences were detected between the attitude test scores of the two groups. Swick concluded that "The success of the program was revealed in better teaching attitude toward arithmetic and in continued use of multi-sensory aids." (55:ab)[1]

Jamison--1962 (35): This experiment was done on the seventh-grade level in which the students studied arithmetic bases other than base ten. Three teaching methods were used in the study--concrete, in which the students actively used a small variable-base abacus; passive concrete, in which the teacher demonstrated on the large variable-base abacus; and a third method in which the only teaching aid was a blackboard and chalk. The instructional time lasted five days and the statistical analysis of test scores measuring the students' understanding and computational skills showed no significant differences.

Schippert--1964 (51): For a period of one semester in the seventh grade, two teaching methods were employed using the S.M.S.G. text Mathematics for Junior High, Volume I, Part I. One method, a "laboratory method," had the students manipulate objects to help learn mathematical principles. A second method, the "abstract method," used only verbal and written descriptions to represent mathematical principles. The analysis of the tests in arithmetic skills indicated a significant difference favoring the laboratory group. No difference was detected in attitude change toward mathematics on the Dutton's scale.

Ekman--1966 (24): In instructing third-grade students on addition and subtraction algorithms, Ekman's methods were abstract, semi-concrete (using pictures) and concrete

[1]Indicates quote is taken from the dissertation abstract.

(manipulation of discs). He reported the concrete method students scoring higher than the first two in understanding at the 3.5 per cent level in the post-test and at the 15 per cent level after a four-week delay. In a "transfer scale," the students using the concrete and semi-concrete methods were superior to those using the abstract at the 8 per cent level on the post-test, and students under the concrete method were superior to the other students at the 4 per cent level after four weeks. On his "skills scale," Ekman noted that the concrete and semi-concrete methods were superior to the abstract initially at the 20 per cent level with no difference after the retention period. Ekman concluded that " . . . use of even the simplest and most inexpensive manipulative materials with third grade pupils is desired over pictures or algorithms only." (22:ab)

Trueblood--1967 (60): During this active versus passive concrete teaching procedures experiment, 7 fourth-grade teachers volunteered to teach 21 lessons about exponential notation and non-decimal bases. Teaching method one, T-1, allowed the students to manipulate the aid. Teaching method two, T-2, had the teacher demonstrating the aid while the student watched and offered suggestions. T-2 was marginally superior to T-1 in the post-test at ($P = .10$). No difference was noted in retention.

Toney--1968 (56): Two groups of fourth-grade students were instructed by active manipulation and by teacher demonstration for an entire semester. The students were given two tests, one to evaluate achievement in computation and one to evaluate understanding. No significant differences were noted between the means of the two groups in either test. However, in improvement the active manipulation method students scored higher in each test.

Weber--1969 (64): In teaching mathematical concepts to the beginning first-year student, two teaching methods were evaluated. Weber defined the methods as: (1) Reinforcement of mathematical concepts through the use of paper and pencil follow up. (2) Reinforcement of mathematical concepts through the use of manipulative and concrete material for follow-up activities. Post-tests were the Metropolitan Readiness Test and an author-made oral test of understanding. No significant differences were noted on the M.R.T., although a definite trend favored manipulation material. Children from the manipulative group scored significantly higher on the oral test of understanding. The Weber study was designed to detect interaction among a number of variables as will be reported on page 27.

Bisio--1970 (5): Bisio tested the comparative effectiveness of three methods of teaching addition and subtraction of like fractions to fifth-grade students. In the treatments, A involved no use of manipulative material; B used manipulative material demonstrated by the teacher; and C allowed the student and teacher to manipulate the material. No significant difference between means was detected in an analysis of variance test for the entire population. However, in a high socio-economic area the group receiving the passive instruction (B) was significantly higher in computational skills than Group A, non-use. There was no significant difference between A and C or B and C in the high socio-economic area. From this Bisio concluded:

> While the actual use of manipulative material appears to fit the need of most students, and is better than non-use, the passive use of material appears equally effective. Therefore, the passive use of manipulative material is an adequate exposure, with minimum cost of time and money, for the beginning student of fractions. (5:77)

Carmody--1970 (10): Concrete, semi-concrete and symbolic approaches were examined for their effect on the learning of number bases, properties of odd and even numbers and tests of divisibility. The testing system evaluated both computation and transfer. Statistical analysis of the test scores indicated statistical differences favoring the semi-concrete over symbolic on the computation test at ($P = .05$) and both the concrete and semi-concrete over the symbolic at (P = .01) in transfer. Carmody summarized the study:

> The experiment supported the use of concrete or semi-concrete material if the goal is transfer. The theory emphasized the importance of having specific behavioral objectives for the use of concrete aids and of recognizing the many factors that must be considered in deciding their use. (10:ab)

The necessity of distinguishing between the physical situation to which a mathematical concept is related and the media used for establishing the relationship was also indicated by the study.

Curry--1970 (15): "Clock arithmetic," both modulo 12 and modulo 8, was taught in this study conducted on a third-grade group of students. Three teaching procedures were evaluated--a concrete method, in which the students were given clock faces with movable hands; a semi-concrete passive method, in which the teacher demonstrated on pictures of clocks; and an abstract method. Computational and understanding tests were given. Curry reported in the results:

> A. The combined concrete and semi-concrete groups scored higher than the abstract group on all but the properties test taken without the use of aids.
> B. The concrete group scored higher than the semi-concrete group only on the properties test on which they were permitted to use clocks. (15:ab)

In conclusion, Curry said:

> A. Methods providing concrete materials or pictures resulted in greater computational skills and greater understanding of properties by third graders than did a verbal method . . .
> B. Providing concrete materials may foster better understanding of mathematical properties and principles but not greater computational skills than providing pictures. (15:ab)

Moody, Abell and Bausell--1971 (41): The instruction was presented to third-grade students for four weeks. Four teaching methods were employed to teach multiplication. The four methods were: (A) The activity-oriented treatment, which consisted of multiplication instruction starting each day with activities in which all subjects manipulated the material in each training session. No emphasis was placed upon memorization of basic facts or algorithms. (R) The rote treatment, which consisted of instruction in the multiplication unit of American Book Company's Developing Mathematics. Emphasis was placed upon memorization of basic multiplication facts and algorithms. (RW) The rote-word problem treatment, which consisted of the same multiplication instructions as R with the addition of practice in solving multiplication word problems. (C) The control group which received further instruction in addition only. The teaching procedures were evaluated by four tests on computation and transfer to applied problems immediately after the teaching period and after six weeks' time. The only significant difference shown was that the average of the three means of teaching methods A, R, and RW was greater than the mean of method C at ($P = .02$) in the computation test given immediately after the instruction.

Johnson--1970 (36): To evaluate the effects of teaching methods on the topics of number theory, measuring, rational numbers, and topics in geometry, the instruction was done on the seventh-grade level for an entire year. The methods of instruction were: (A) Text. Exclusive use of the textbook as the only mode of instruction. (B) Activity. Exclusive use of instructional modes other than textbooks. (C) Enrichment. A textbook-based mode which was augmented by enrichment activities from treatment B. Achievement tests in computation, application and understanding and a test on attitude toward mathematics were given to evaluate the teaching procedures. Johnson concluded that if activities become the dominant feature of instruction (method B), the achievement of the students appears inferior to the achievement of the students who have little or no activity. Johnson said no difference was noted in the achievement between A and C. No difference was detected in attitude toward mathematics among the three groups.

A large number of studies have been done evaluating the use of "Cuisenaire Rods." In this system, the rods are made in different lengths of one, two, three, etc., centimeters to represent the concept of number. Typical of the Cuisenaire studies are the following.

Brownell--1966 (6): Even though there were significant differences detected in this large study done in both England and Scotland in grades one through three, the results were inconclusive. In Scotland, the Cuisenaire method was superior to a traditional approach in "meaning abstraction." In England, the reverse was true; that is, the traditional was superior to the Cuisenaire method. No differences were detected in computation.

Crowder--1965 (14); Hollis--1965 (32); Fedor--1965 (25): These studies were done on the first-grade level and the outcomes varied. Crowder and Hollis' outcomes implied that the students using the Cuisenaire method scored higher than those students under the traditional method. Fedor indicated no significant difference and expressed a common criticism in using the Cuisenaire method for beginning students by noting:

1. Counting is a logical basis for beginning initial number experiences.
2. The use of color inhibited the initial study of rod relationships. (25:ab)

Fedor did, however, suggest that "Aspects of the Cuisenaire program should be incorporated into our present programs." (25:ab)

Nasca--1966 (42): Nasca conducted a study on the second-grade level. The students under the Cuisenaire method scored significantly higher than those under the traditional method in computational skills with no significant difference in understanding.

Lucow--1964 (39); Hays--1963 (30); Passy--1963 (46): All these men did studies on third-grade students. Hays found no signficant differences on a post-test in multiplication. Lucow's findings opposed this viewpoint. Students from the rural areas were superior when given the Cuisenaire method compared with a traditional approach. However, there were no detected differences for students from the city. Passy had the most significant results and concluded:

> 1. The group using Cuisenaire materials in a modified elementary mathematics program achieved significantly less in computational skill than either of the two comparison populations.
> 2. The group using Cuisenaire materials in a modified elementary mathematics program achieved significantly less in mathematical reasoning than either of the two comparison populations.
> 3. The group using Cuisenaire materials in a modified elementary mathematics program achieved significantly less than either of the two comparison populations at each level no matter what the manner of stratification. (46:ab)

Nelson--1964 (43): Nelson reviewed 50 experiments on the Cuisenaire method. The over-all consensus showed the Cuisenaire students superior in computation in the early grades. They were, however, significantly inferior in understanding and problem solving, and the superiority in computation disappeared in later school years.

Interaction Between Teaching Methods And Individual Differences (Aptitudes)

As far back as the fifth century B. C., Plato recognized the differences in individual abilities. He reasoned that the nature of man is made up of three components, appetite, spirit, and reason. The degree to which these components controlled one's life decided his best vocation and hence decided his method of education. (7:102)

It was not until the start of this century that much was done to identify individual differences. The works of Binet and others are well known and their tests were used, as

Cronbach said, to " . . . decide which pupils should be allowed to drop by the wayside or vegetate in an undemanding 'slow' classroom, and which should proceed briskly, be indoctrinated with high aspirations, and go on to higher education." (13:24)

Until recently, the only widely-used variable employed by schools to adapt for individual differences was the rate of instruction. Cronbach reported:

> This concept is distinctly arguable. Woodrow (1946), you may recall, spent twenty years compiling evidence that rate of learning is entirely inconsistent from one task to another and that there is no justification for identifying mental test scores with ability to learn. (13:25)

Cronbach called for a search for different teaching methods that interact with different abilities. He stated: "Aptitude information is not useful in adapting instruction unless that aptitude and treatment interact." (13:30)

Aptitudes and Interactions

General ability is the most often-used criterion in the selection of homogeneous classes and is the variable most frequently reported in research papers when teaching procedures are evaluated for interaction. Cronbach felt that this blanket approach has not been too successful because of the generally high correlation between general ability and achievement resulting from almost any type of instruction.

Gagné has suggested:

> The existence of individual differences, presumably relatively stable ones of the sort implied by the term "basic abilities," is a pervasive issue in mathematics teaching, as well as in mathematics itself. It is a widely held belief, for example, that some people tend to solve mathematical problems by thinking out the solutions in terms of visual and spatial models and then translating these into acceptable symbols, while other people think out solutions in terms of abstract symbols and symbol-operations. It is thought possible by some that problems may be solved, by

> some people, in terms of logical-verbal statements and then translated into symbolic form. With regard to these differences, it is probably significant that some of the best-defined and stable differences in basic abilities which have been measured by psychologists are spatial factors (or ability), verbal factors, and numerical factors. (3:25)

Gagné implies that varying teaching procedures should be designed to best take advantage of skills or abilities.

In 1959, Guilford developed a model of "The Structure of the Intellect," which implied that each mental factor is classified by three criteria: the kind of operation--evaluation, convergent thinking, divergent thinking, memory and cognition; the content--figural, symbolic, semantic, behavioral; the products involved when a given operation is applied to a certain content--units, classes, relations, systems, transformation, implications. The mental factors are then one of the ordered triplets from the 5 x 4 x 6 matrix. Guilford and others have developed tests to evaluate the main components and have isolated many factors by factor analysis. Guilford suggested that

> If education has the general objective of developing the intellects of students, it can be suggested that each intellectual factor provides a particular goal at which to aim. Defined by a certain combination of content, operation, and product, each goal ability then calls for certain kinds of practice in order to achieve improvement in it. This implies choice of curriculum and the choice or invention of teaching methods that will most likely accomplish the desired results. (21:478)

Cronbach suggested that variables other than intellectual be considered. He stated that "The most thoroughly documented at present involve attitudes having to do with confidence." (13:34) As an example he cited competition in the form of races or contests as being very effective for the constructively motivated child, while a defensive child may do best in a non-competitive situation.

Torrance has maintained that the creative child tends to react differently to various types of instruction than the unimaginative child. In general, he felt that the present school education appeals more to rote than to reason:

> In the main, current school curricula at all levels of education are designed to develop and make use of the kinds of thinking abilities reflected in traditional tests of intelligence. No one is suggesting that the development of these abilities be eliminated. It is only suggested that parallel treatment be given the creative thinking abilities . . . (57:68)

Torrance also reported a study in which a correlation of .53 existed between a creativity test score and scores that required understanding and application of previously-learned material. A second correlation of .03 existed between the creativity test score and scores of a test on memorized facts covering the same material. (58:149)

<u>Research on Interaction and Concrete Manipulative Teaching Methods</u>

Cronbach (13), Guilford (29), Behr (4), and Becker (2 and 3) have excellent reviews and bibliographies done on interaction with teaching methods other than those involving activities or manipulation of concrete material.

Schippert (51), Weber (64), and Bisio (5) designed their studies to detect interaction with socio-economic background. Schippert's study was done entirely in the inner city and did not offer a comparison with another area. But he concluded that

> The findings of the study indicated that an inner-city seventh grade student can demonstrate a greater understanding and facility in arithmetic skills if he is permitted to engage in the process of inquiry at a level of abstraction at which he can function effectively and at a pace suited to his abilities. (51:ab)

Weber's study allowed a comparison of socio-economic backgrounds and he concluded that "Interaction was not significant but the trend favored manipulative materials for low socioeconomic status children." (64:ab)

Bisio also compared the teaching methods and socio-economic backgrounds, as previously reported. He noted that students from the high socio-economic group performed higher at ($P = .01$) when they had been given the "teacher-demonstrated method" than those from the same area that used no concrete aids.

Johnson's study reported that low- and middle-ability students are apparently aided in learning some concepts in seventh-grade mathematics by the use of activities-oriented lessons. Johnson said: "Laboratory lessons in the study of geometry and measurement might be appropriate for low and middle ability students." (36:ab)

Summary of the Review of Research

Of the seven studies which compared some type of concrete method with a traditional method in first- through third-grade addition and subtraction, five studies showed the students given concrete methods scoring significantly higher on at least one of the post-tests. The other two studies reported no significant differences. None of the studies showed the traditional method superior. In Nelson's review of Cuisenaire studies, the Cuisenaire method yielded higher scores in computation in the early grades. The concensus of these studies is that use of concrete material is desirable for the first two grades or for early topics in arithmetic, such as addition and subtraction.

In the area of the present study, third- and fourth-grade multiplication and division, the results are not as conclusive. Of the six studies reported, the Dawson and Ruddel study in fourth-grade division found the concrete method superior, and Lucow's Cuisenaire study showed rural students who had the concrete method scored higher. Passy's findings strongly favored the symbolic approach. The other three studies revealed no significant differences. Another third-grade study, Curry's in modular arithmetic, showed the concrete methods superior.

Methods of teaching fractions in fifth and sixth grades were evaluated in three studies by Bisio, Howard, and

Neureiter and Troisi. Neureiter and Troisi found concrete methods superior in application while Howard's study reported the combination of methods was superior to either concrete or symbolic methods alone in retention.

Four studies were done in the seventh grade. Those of Carmody and Jamison involved non-decimal bases. Carmody's study showed students given concrete methods scored higher while Jamison's found no significant differences. Johnson and Schippert used laboratory methods. Johnson found both the textbook method and a combination of textbook and laboratory method superior to the laboratory method alone. Schippert's study showed the laboratory method superior to an abstract approach.

This summary seems to indicate that concrete aids in mathematics are desirable in the first and second grades. Their use for third and fourth grades is more questionable, and more studies are needed on this level. For grades five through seven, the cited studies have not shown concrete aids to be as valuable as in the earlier grades.

Three of the studies compared socio-economic backgrounds and some reported results in general intelligence levels. Otherwise, none of the studies tested for interaction.

CHAPTER III

METHODOLOGY

The teaching methods in this study were evaluated to determine their effects on third-grade students' learning of multiplication and division. The textbook, Elementary School Mathematics, Book 3, Robert E. Eicholz and Phares G. O'Daffer, Addison Wesley Company, 1968, was used in both teaching methods, and three chapters of the text were taught. In Chapter 4, multiplication was informally defined, the multiplication of single-digit whole numbers (zero through nine) was covered, and a number of topics such as the commutative, associative and distributive principles were discussed. Chapter 5 covered division of the numbers related to single-digit multiplication, and Chapter 8 involved multiplication of multi-digit numbers. In all three chapters, the application of the concepts was covered extensively in "story problems." With few exceptions, the lesson topics in the text were taught in the order given, one day for each topic.

The course of instruction lasted for twelve school weeks, starting during the second week of December, 1971, and ending in the second week of March, 1972. Forty-nine days were spent in instruction and eleven days were used for testing. The time included the Christmas vacation, the end of the first semester, and two days in which school was not held because of weather conditions.

Teaching Procedures

Method S--The Symbolic or Text Method

The teachers' edition of the textbook was followed generally in this method, giving one two-page lesson each day in the order covered in the book. A few of the lessons were deleted and some were given two days. Since the text makes use of pictures and charts, this method could be considered as semi-concrete. Other than those contained in the text, no pictures, charts or objects were used to represent equivalent sets.

When the blackboard was used, only mathematical symbols and numerals were written on the board. Since the second group, Group M, often took longer to be introduced to a given topic, each member of Group S was supplied a set of multiplication flash cards. These cards were used individually and in groups to help equalize the amount of time spent by the two groups on the topic.

Method M--The Manipulative Method

This teaching method was not considered a "laboratory method." In addition to following the same basic outline as the text and that used by Group S, an activity that involved student manipulation of concrete objects and devices was used by Group M to introduce or reinforce the daily topics. The instructions for the activities were written by the author, although some of the activities were patterned after those suggested by the teachers' supplement to the text.

The teacher was allowed to teach each class and topic in the manner best suited to her with respect to class control and individual discovery, etc., but with the understanding that in all instructional variables, such as time spent and class control, both groups would have as nearly as possible the same instruction except for the variable tested. The variable tested was the use or non-use of concrete manipulation to introduce and/or reinforce the various topics.

Rationale for the Selection of the Manipulative Activities

The first consideration in the selection of the manipulative devices was the availability and cost of the devices.

Since each child was to have easy accessibility to the devices, the variety of objects was kept to a small number. Secondly, the concept of number was treated only as the cardinal number of concrete sets except in a few cases in the textbook.[1] Therefore, none of the devices involved the concepts of equal or fractional length or weight. This eliminated such devices as the Cuisenaire rods and balance beams. Egg cartons and pegboards were used to satisfy these considerations.

A third consideration was the desire to have a device or scheme that promoted the idea of base and place. Three methods were considered. The often-used popsicle sticks and rubber bands in which sticks are put in groups of tens, ten tens, etc., was the first method considered; it was rejected because, while it does make use of the base, the concept of place is not used and the number of sticks needed becomes very large for multi-digit numbers. A base ten abacus was considered, one with ten beads or discs on each rod. The abacus was rejected in favor of the counting board (see page 37) because of cost, the versatility of the counting board, and the fact that the author had previously used the counting board for teaching operations in bases other than ten to high school students and college students in classes for elementary teachers.

Use of the Manipulative Devices

When the lesson plans for Group M were written, the primary consideration was whether the inclusion of kinesthetics and the sense of touch would improve the understanding of the concepts and lead to greater achievement. No attempt was made to design or write a course of study but only to augment the textbook.

Often the devices were used to simulate other concrete objects--pebbles were used to represent keys, pennies or

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[1]The textbook had a lesson on multiplication and the number line, page 88. Each class was given identical instruction as covered in the textbook.

shoes; pegs on a pegboard often represented fruit trees or basketball players. While it may be true that a more extensive variety of objects would have been more stimulating, the time, the cost and the over-riding principle of concrete manipulation limited the number of objects. The children seemed to have little trouble with the simulation and enjoyed the model building.

Another question that arose in the writing of the lesson plans was what proportion of the lesson time should be used for manipulative activities. It is the author's interpretation of the related literature that too much time is apt to be more harmful than too little (see the Johnson study, page 22). It is also the author's impression that the inclusion of concrete manipulation is of most benefit if the teacher helps the student connect the ideas of the manipulation to the symbolic representation and applications.

<u>The Egg Carton</u>

Each child was given a molded paper or plastic egg carton with separate holes or cells for each egg and a handful of uniformly-sized pebbles of the type used in gardens and lawn decorations. The carton was used in a number of ways to supplement the textbook, as in the following example.

The textbook introduced multiplication with a picture of three sets of keys, each set with four keys followed by the statements:

We see	3 sets of keys 4 keys in each set 12 keys in all
We think	3 fours are ////[1]
We write	3 x 4 = 12
We say	3 times 4 equals //// (23:84)

Group M simply added one step to this process. Prior

[1]The symbol //// is used throughout the textbook as a place holder, indicating an answer is wanted.

to opening their books, the students were told to place four pebbles in each of three cells of their cartons. Then the teacher would ask:

How many cells (holes) have pebbles?	<u>3</u>	
How many pebbles in each cell?	<u>4</u>	a
How many pebbles in all?	<u>12</u>	

giving the students time to count if necessary.

The teacher then wrote on the board:

We think	3 fours are 12
We write	$3 \times 4 = 12$
We say	3 times 4 equals 12

The students would then open their books and do similar problems starting with the pictures instead of the device. Sometimes the process was reversed and the students were given the problem in the book and asked to simulate this with the devices as in the following example.

The text contained a picture of five nickles, and the students were to find the number of cents for the five nickles. Here the students were asked to pretend that their pebbles were pennies and to place five pennies in each of five cells to simulate the value of the nickles. They were then asked how many cents in all and to write the equation.

A process like the one above was used on a number of occasions. It was called model building and later in the course the students were given more freedom to make their own models of the problems. This process was used primarily in the solution of application or "story" problems.

The egg carton was also used to teach the principles of multiplication. The commutative, or "order principle," was handled by placing three pebbles in each of five cells on one side of the carton. The students were asked if they could write the related statement, <u>5 threes are 15</u>, and the equation, <u>$5 \times 3 = 15$</u>. They were then told to move the

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[a]In places where the answer is underlined, the students were to supply the correct statement. Other statements are teacher supplied.

pebbles to the other side of the carton, placing an equal amount in each of three cells. They were asked the number of pebbles in each cell and asked to give the related statements and equations, 3 fives are 15 and $3 \times 5 = 15$.

The concept of division was also demonstrated using the the cartons. Twenty-four pebbles were placed, six in a cell, until all were used, or one at a time in each of four cells until they were gone, to demonstrate these two concepts of division. Both the teacher and the author felt that the use of the concrete object was very meaningful in teaching division.

The concept of remainders in division or, as the children expressed it, "not coming out even," evolved naturally even though it was not covered in the three chapters but appeared because a student miscounted. As the instruction progressed, the students were allowed to group the pebbles on their desks when the number of groups was small enough, generally five or less; this made the counting a little easier.

The Pegboard

Each student was supplied a small commercial pegboard, ten holes by ten holes, and a supply of pegs. As the textbook made extensive use of rectangular arrays of objects to teach a number of topics, the pegboard was used to simulate these arrays. The boards were made of white plastic so the students could write on them and erase.

As an example of the versatility of the pegboards, it was first established that three rows of five pegs represented the equation $3 \times 5 = 15$. To teach the commutative property, the students were asked to represent the equation, $2 \times 6 = 12$. They were shown an example of two rows of six pegs each, with the equation written below the example. The teacher then told the students to rotate their boards one quarter turn and demonstrated with her board. The students were then to write the proper equation represented on the pegboards, $6 \times 2 = 12$.

To demonstrate the distributive (addition-multiplication) property as found on pages 102-103 in the textbook, the pegboard was used as follows. The students were told to place three rows of seven pegs each and write the related equation, $3 \times 7 = 21$. Then they were told to move the last column of pegs one space to the right and write the related equation under each array, $\underline{3 \times 6 = 18}$ and $\underline{3 \times 1 = 3}$. They were asked:

How many pegs in all? $\underline{21}$
What is the sum of 18 and 3? $\underline{21}$

The teacher then put this arrangement on the board:

$18 + 3 = 21$ and $6 + 1 = 7$ so
$(3 \times 6) + (3 \times 1) = 3 \times (6 + 1)$

The students were asked to move the next to the last peg in each row forming the arrays 3×5 and 3×2, write the related equations, $\underline{3 \times 5 = 15}$ and $\underline{3 \times 2 = 6}$. The students then wrote equations in the same manner as those on the board:

$\underline{15 + 6 = 21}$ and $\underline{5 + 2 = 7}$ so
$\underline{(3 \times 5) + (3 \times 2) = 3 \times (5 + 2)}$

The students were told to continue the process of moving the columns of pegs one column at a time until all were in one group, writing the equations each time.

Throughout the instruction, the students were asked repeatedly to write the equation or symbolic representation of the physical model or conversely, to display a physical model representing the given symbolic statement or equation. This was done to develop and strengthen the bond between the symbolic and concrete.

The pegboards were used also in connection with the application or "story" problems. For example, early in the instruction, the textbook gave the problem:

Two basketball teams
Five basketball players on a team
How many players in all? (22:116)

Using pegs of different color for the two teams, the students were asked to make a model of the game. If, as Gagné has suggested (see page 25), some people solve problems in a spatial or physical way, this activity would develop the connection between the written statement and the physical representation.

In being given considerable freedom to develop their own models, which they seemed to enjoy, the students found that many problems were not quite so physical in nature:

A doubleheader baseball game
Each game nine innings
How many innings in all? (22:116)

A number of students came up with the idea of a scoreboard for this.

<u>The Counting Board</u>

The counting boards were made by the author from a piece of 3/8 inch plywood, 5 inches by 15 inches with four 3-inch holes evenly spaced across the board. The 3/8 inch piece was glued on a 5 inch by 15 inch piece of 3/16 inch fiberboard. Each student was supplied with a counting board painted flat white on which he could write.

The holes were labeled from right to left with the numbers 1, 10, 100, and 1,000. Prior to starting instruction in multiplication, the students in Group M practiced on these devices and learned to represent a number such as 23 by putting 2 pebbles in the "10" hole and 3 pebbles in the "1" hole. They learned to add 6 + 8 by placing 6 and then 8 pebbles in the "1" hole, then counting and removing 10 of the pebbles and carrying one to the "10" place. They also practiced subtraction.

The teacher mentioned that from the comments of the students the idea of borrowing in subtraction was made very meaningful in using the counting board and that she would surely use the device in the future. During this time, the students in Group S reviewed the symbolic operations of addition and subtraction.

The counting boards were used throughout the instruction on multiplication of one-digit numbers as repeated addition and division as repeated subtraction. Most of the students developed considerable skill on the counting boards. As with the other devices, the students were asked to do the operation on the counting board and then do it symbolically, often writing the equation on the counting board itself, to develop the bond between the symbolic and concrete operations.

Multiplication by 10 was developed by asking the students to count out 10 sets of 3 pebbles and place them in the "1" place and add. (Again stressing that this represented the statement, 10 threes, and the equational phrase, 10 x 3.) Upon adding, the students noticed the 3 pebbles in the "10" place and none in the "1" place. Similar work was done on problems such as 10 x 40, etc.

The multiplication of multi-digit numbers by single-digit numbers was done as follows. In multiplying 2 x 26, the students were asked to work in pairs. One student was asked to place two-26's on his counting board while his partner was to add two-6's and two-20's. As always, they were to write the equations, <u>2 x 26 = 52</u> and <u>2 x 6 + 2 x 20 = 52</u> therefore <u>2 x 26 = 2 x 6 + 2 x 20</u>. (23:232) Between themselves the students were then allowed to experiment with other combinations. Similar work was done for one by three- or four-digit numbers.

The last operation covered was two-digit by two-digit multiplication. The students first learned multiplication by multiples of 10, such as 20 x 3, by considering it as 2 x (10 x 3) or moving two-3's over to the "10" place. Multiplications such as 20 x 40 = 2 x 4 x 10 x 10 were represented by placing 2 groups of 4 in the 10 x 10 or "100" place. The problem, 24 x 3, was handled as 20 x 3 + 4 x 3. The final problem of 24 x 53 was done as 20 x 53 + 4 x 53.

Late in the instruction, the students were allowed considerably more freedom to manipulate the devices, after they developed the skills and the abilities to represent the problems on the devices. The students often worked in small groups. In one instance, they were encouraged to race the counting boards against the symbolic algorithm.

By design, most of the problems given on the counting boards used digits of five or less on at least one of the factors since this kept the counting and number of pebbles low and minimized the number of errors. However, examples were included that required "carrying" to the next place.

The Sample

The experiment was conducted in the third grade of the Northmoor Elementary School, Moore, Oklahoma. Two of three self-contained heterogeneous classes were selected by the principal to take part in the experiment. The 65 students in the two classes were pooled; each was assigned a number and the two groups were selected using the table of randomized number and procedure as described in Introduction to Statistical Analysis (21). Group M had 33 students assigned to it, and 32 students were assigned to Group S.

Moore, Oklahoma, is a rapidly-growing suburban city in northern Cleveland County between Oklahoma City and Norman. It is generally a middle-class community and none of the children in the groups appeared to be non-white.

The same teacher taught both the groups to eliminate between-teacher variance, while another teacher taught reading to the two classes during the time of the experiment. Although the teacher was a beginning teacher, she was highly recommended by the principal and the author feels her teaching was outstanding. She had taught multiplication during her student teaching in the school year 1970-71.

Supervision and Control

The author wrote supplemental lesson plans for Group M and the teacher used these to formulate her teaching plans. In addition to the author's meeting regularly with the teacher about once a week to discuss the lesson plans, the author and the principal frequently observed the teaching of both classes and the author went over the teaching procedures with the teacher during the next conference.

Measurement Instruments

Pretests

Each student was given the following battery of tests immediately prior to the experiment:

Otis-Lennon Mental Ability Test
Elementary Level 1, Form J
Harcourt, Brace and World, Inc., 1967, New York

Stanford Achievement Test
Primary Battery II
Harcourt, Brace and World, Inc., 1967, New York

Torrance Test of Creative Thinking
Figural Test Booklet A
January, 1970
Personnel Press Inc., Princeton, New Jersey

Each of these tests was given and scored by the methods prescribed in the test manual. The reliability and validity of these tests are published in the corresponding test manuals.

A pretest for competence in multiplication was also given (Test A, Appendix B). The test was an equivalent form of the post-tests but with fewer questions. The test consisted of 14 computation problems and 6 application, or "story," problems. The split half reliability derived for this test was .6219.

Before taking the multiplication pretest, the students were given a definition for multiplication, 4×3 is 4 threes added together or $3 + 3 + 3 + 3$ and $12 \div 4 = ////$ if $4 \times //// = 12$. No other instruction was given in multiplication or division prior to any of the pretests.

Post-Tests

The post-tests consisted of two parts (Tests B and C, Appendix B) of nearly equal difficulty which were given on successive days. Each part consisted of 12 computation problems and 8 application or "story" problems. The test parts were scored separately and by application, computation and total. A split-half reliability for these tests is given by the correlation between the two parts, as .8048 for the computation, .9061 for the application, and .8923 for the total.

The pretest and post-tests for competence in multiplication were written by the author using problems from the text, from other published textbooks, or from achievement tests in third-grade mathematics. The example tests and review exercises in these books were used to decide the number of each type of problem. It is assumed that this method has produced a valid test.

Statistical Procedures

Hypothesis I (see page 5) was examined using Student's t test for differences between the means and F test for differences between the variances of the two groups. Hypothesis I was rejected only if at least one of the following hypotheses was rejected:

Ho $\overline{X}$: There is no difference between the means of the scores in the achievement tests for computation and application.

Ho S^2: There is no difference between the variance of the scores in the achievement tests for computation and application.

If $\overline{X}_s$ and $\overline{X}_m$ were the mean scores of the two groups, and S_m and S_s were the estimates of standard deviation, then

$$t = \frac{\overline{X}_s - \overline{X}_m}{\sqrt{\frac{S_m^2}{N_m} + \frac{S_s^2}{N_s}}}$$ had a t distribution

with $N_m + N_s - 2$ degrees of freedom and Ho $\overline{X}$ was rejected only if $|t| > t\ (.975,\ N_m + N_s - 2)$. (31:320)

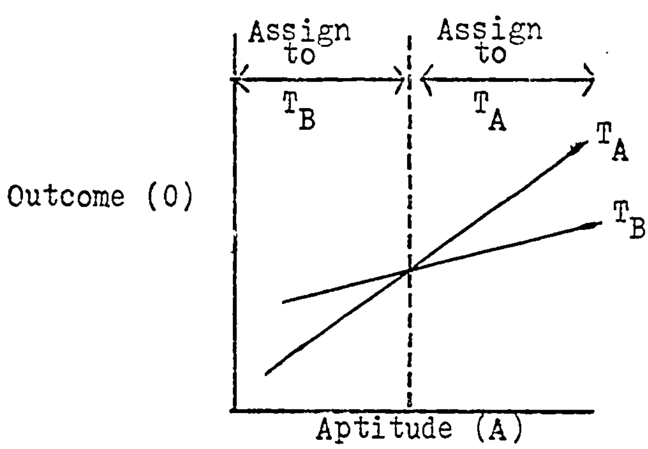
Similarly, the dispersion of each group was evaluated by an F test in which

$$F = \frac{S_s^{\ 2}}{S_m^{\ 2}}.$$

Ho S^2 was rejected only if $F > F\ (V_s,\ V_m,\ .975)$ or $F < F\ (V_s,\ V_m,\ .025)$. (21:102 and Table A-7c)

Hypotheses II, III, and IV pertaining to interaction (see page 5) were tested using the method suggested by J. P. Becker:

> . . . for adaptation of instruction to individuals to be profitable there must exist different instructional techniques (treatments) that teach for the same objectives or criterion and, moreover, there must exist aptitudes for which regressions of outcome scores on aptitude exhibit the pattern shown in figure 1 (T_A and T_B represent the regres-



[Figure 1]

> sion lines for treatments A and B respectively). In figure 1 we have an interaction effect represented (i.e., a crossover of the two regression lines). Each line corresponds to a treatment and each line represents the graph of a regression equation of the form $0 = a + bA$, where a is the intercept, b is the regression coefficient of the aptitude variable, and 0 and A represent the outcome and aptitude variables respectively. Here we should divide our groups of experimental

subjects into two subgroups, one of which would be given treatment A (those with "higher" aptitude scores) [i.e., higher than the crossover point] and the other to treatment B (those with "lower" aptitude scores). (3:22-23)

If $\hat{Y}_s = A_s + b_s X_a$ and $\hat{Y}_m = A_m + b_m X_a$ were the simple regression equations from aptitude A to the achievement scores for Group S and M respectively, the null hypothesis

Ho (Interaction): No interaction exists between the teaching methods and the scores on aptitude A

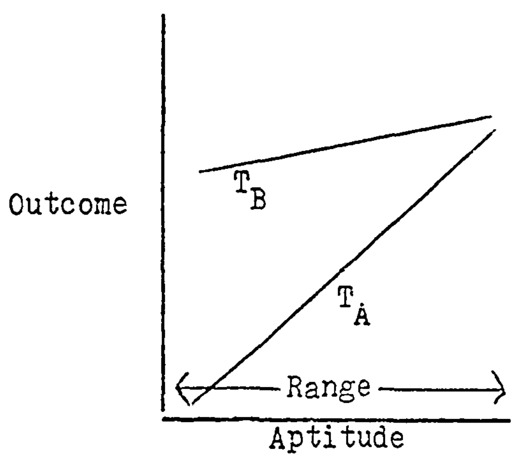
was rejected if and only if the following were satisfied:

1. The regression equations intersected within the range of the independent variable and

2. The hypothesis that $b_m = 0$ or $b_s = 0$ was rejected at $(P = .05)$ and

3. The hypothesis that $b_m = b_s$ was rejected at $(P = .05)$.

It is possible for interaction to exist in a strict statistical sense without meeting criterion 1 above (the regression equations intersect within the range of the independent variable). As an example:



The above indicates a significant difference in the slopes of the regression equations, but for each score in the range of aptitude a higher score would be predicted with T_B. It seems doubtful, however, that this condition could happen without the rejection of the hypothesis of equal means of the two distributions.

In a post hoc manner multiple regression equations were computed for each teaching method. If any null hypothesis for interaction was rejected, multiple regression scores were used to predict the scores of the groups, provided the multiple regression equation was significantly better than the best single predictor at ($P = .05$).

CHAPTER IV

ANALYSIS OF THE DATA AND RESULTS

Main Effects and Simple Regression Analysis

Scientific Subroutine MISR was used to analyze the data for the two teaching procedures. The program gives the information required to test the means and variances of each variable and the correlation and simple regression equations for each pair of variables. The description of Scientific Subroutine MISR is given in Appendix C, page 79.

Tables 1-M and 1-S are Zerox copies of the input matrices for Scientific Subroutine MISR taken from the computer printout.

Variable 1---S.A.T. Word Meaning
Variable 2---S.A.T. Paragraph Meaning
Variable 3---S.A.T. Science and Social Science
Variable 4---S.A.T. Spelling
Variable 5---S.A.T. Word Study
Variable 6---S.A.T. Language
Variable 7---S.A.T. Arithmetic Computation
Variable 8---S.A.T. Arithmetic Concepts
Variable 9---Otis-Lennon Mental Ability
Variable 10--Torrance Creative Fluency
Variable 11--Torrance Creative Flexibility
Variable 12--Torrance Creative Originality
Variable 13--Torrance Creative Elaboration[1]
Variable 14--Chronological Age[2]
Variable 15--Pretest in Multiplication and Division
Variable 16--Post-test Total
Variable 17--Post-test Computation
Variable 18--Post-test Application (Story Problems)
Variable 19--The Student's Number

The code score of 99 in the matrix indicates missing data.

---

[1]One-half the actual score.

[2]One-half the number of months.

TABLE 1-M

INPUT MATRIX FOR GROUP M

	VARIABLES																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	21.	33.	12.	12.	30.	21.	31.	15.	99.	22.	18.	19.	31.	53.	3.	28.	19.	9.	3.
2	25.	39.	20.	18.	41.	41.	35.	28.	54.	33.	23.	28.	27.	50.	11.	25.	12.	13.	4.
3	28.	45.	19.	15.	58.	43.	30.	23.	45.	31.	23.	38.	40.	54.	0.	19.	14.	5.	5.
4	28.	46.	26.	25.	55.	48.	33.	22.	61.	23.	14.	19.	43.	50.	7.	20.	13.	7.	8.
5	14.	11.	8.	5.	27.	10.	23.	11.	29.	37.	24.	51.	53.	48.	0.	10.	10.	0.	11.
6	15.	21.	11.	9.	21.	24.	37.	13.	33.	99.	99.	99.	99.	53.	3.	37.	23.	14.	13.
7	13.	20.	13.	8.	23.	39.	35.	12.	33.	28.	15.	26.	29.	57.	2.	29.	19.	10.	16.
8	20.	29.	20.	25.	45.	31.	33.	18.	44.	23.	99.	27.	37.	50.	0.	21.	17.	4.	18.
9	31.	54.	29.	27.	52.	48.	39.	31.	54.	21.	17.	24.	27.	55.	10.	37.	24.	13.	20.
10	5.	11.	24.	0.	15.	29.	12.	12.	42.	27.	12.	9.	22.	57.	99.	0.	0.	0.	22.
11	20.	37.	15.	19.	42.	41.	35.	21.	49.	29.	20.	18.	22.	53.	3.	24.	14.	10.	29.
12	13.	21.	10.	13.	26.	0.	22.	7.	99.	27.	15.	29.	20.	49.	0.	9.	5.	4.	30.
13	27.	48.	16.	24.	55.	49.	35.	33.	47.	40.	22.	43.	32.	50.	10.	31.	21.	10.	32.
14	26.	30.	21.	23.	51.	46.	37.	33.	59.	40.	23.	56.	41.	51.	99.	34.	22.	12.	34.
15	30.	60.	22.	27.	57.	61.	39.	37.	67.	37.	20.	41.	48.	52.	11.	35.	21.	14.	36.
16	13.	11.	21.	8.	27.	21.	35.	16.	45.	40.	21.	39.	31.	54.	6.	25.	21.	4.	37.
17	19.	24.	13.	7.	19.	20.	20.	10.	30.	31.	27.	29.	25.	55.	0.	6.	4.	2.	39.
18	26.	34.	16.	11.	30.	33.	27.	14.	31.	26.	20.	33.	38.	57.	4.	20.	15.	5.	40.
19	19.	38.	17.	19.	62.	20.	20.	26.	47.	40.	24.	62.	19.	57.	10.	18.	13.	5.	43.
20	24.	34.	18.	13.	37.	31.	35.	16.	53.	99.	99.	99.	99.	51.	1.	20.	17.	3.	44.
21	25.	37.	14.	18.	56.	32.	32.	18.	50.	40.	22.	37.	25.	50.	4.	27.	23.	4.	46.
22	25.	49.	17.	29.	54.	53.	37.	22.	51.	32.	20.	54.	32.	55.	5.	35.	22.	13.	49.
23	22.	33.	16.	4.	20.	21.	22.	21.	48.	40.	16.	15.	21.	51.	1.	17.	14.	3.	50.
24	31.	51.	28.	29.	56.	52.	37.	34.	63.	28.	19.	21.	24.	55.	5.	37.	22.	15.	52.
25	12.	14.	9.	3.	23.	16.	31.	17.	18.	99.	99.	99.	99.	49.	0.	9.	7.	2.	53.
26	25.	36.	22.	16.	25.	35.	26.	18.	99.	26.	16.	21.	52.	54.	1.	8.	6.	2.	56.
27	22.	27.	18.	12.	24.	37.	31.	17.	50.	34.	8.	10.	20.	53.	11.	35.	23.	12.	58.
28	22.	35.	14.	12.	37.	42.	14.	16.	41.	40.	19.	32.	40.	54.	3.	5.	4.	1.	60.
29	25.	43.	16.	18.	42.	39.	39.	28.	53.	26.	13.	42.	51.	51.	99.	19.	14.	5.	61.
30	22.	38.	15.	7.	20.	24.	27.	12.	30.	31.	19.	23.	16.	49.	1.	9.	8.	1.	62.

TABLE 1-S

INPUT MATRIX FOR GROUP S

	VARIABLES																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	22.	30.	11.	24.	35.	32.	22.	24.	38.	26.	23.	35.	20.	50.	4.	25.	20.	5.	1.
2	22.	30.	14.	16.	35.	37.	32.	18.	38.	19.	12.	16.	30.	50.	1.	10.	7.	3.	2.
3	23.	33.	15.	14.	46.	33.	28.	9.	31.	27.	18.	42.	38.	50.	0.	11.	9.	2.	6.
4	25.	45.	17.	11.	44.	25.	36.	20.	49.	99.	99.	99.	99.	51.	0.	29.	21.	8.	7.
5	27.	38.	22.	17.	41.	43.	37.	30.	45.	35.	25.	63.	32.	52.	8.	33.	21.	12.	9.
6	99.	99.	99.	5.	25.	29.	99.	99.	29.	33.	20.	28.	27.	51.	0.	12.	7.	5.	10.
7	26.	37.	18.	6.	36.	33.	8.	23.	43.	34.	13.	22.	51.	51.	0.	15.	12.	3.	12.
8	24.	29.	99.	99.	99.	26.	37.	99.	61.	25.	19.	21.	38.	52.	13.	30.	19.	11.	15.
9	18.	31.	11.	16.	45.	39.	30.	16.	44.	34.	17.	26.	34.	51.	4.	24.	18.	6.	17.
10	17.	26.	10.	11.	27.	36.	41.	36.	47.	28.	14.	19.	16.	56.	11.	34.	22.	12.	19.
11	31.	50.	21.	25.	61.	41.	36.	35.	64.	31.	25.	46.	24.	51.	9.	32.	20.	12.	24.
12	24.	27.	18.	6.	21.	30.	29.	32.	64.	21.	12.	19.	15.	51.	9.	17.	11.	6.	25.
13	25.	40.	18.	18.	42.	42.	42.	12.	53.	34.	23.	41.	30.	51.	10.	24.	16.	8.	27.
14	29.	45.	21.	8.	47.	54.	40.	24.	38.	40.	18.	33.	38.	53.	12.	32.	20.	12.	28.
15	18.	31.	12.	18.	21.	35.	31.	25.	39.	22.	18.	37.	28.	54.	11.	28.	19.	9.	31.
16	30.	49.	23.	25.	53.	28.	32.	37.	67.	34.	23.	42.	44.	53.	5.	35.	22.	13.	33.
17	15.	38.	14.	13.	30.	15.	28.	16.	47.	35.	27.	37.	40.	49.	3.	5.	5.	0.	35.
18	32.	54.	25.	30.	32.	49.	35.	34.	61.	22.	17.	23.	15.	49.	7.	34.	23.	11.	41.
19	18.	33.	12.	11.	27.	32.	28.	20.	33.	37.	24.	52.	31.	53.	7.	20.	17.	3.	42.
20	15.	29.	15.	8.	15.	18.	32.	15.	40.	33.	24.	32.	38.	54.	5.	19.	12.	7.	45.
21	21.	30.	10.	9.	30.	27.	29.	14.	50.	38.	20.	34.	18.	54.	6.	20.	14.	6.	47.
22	20.	9.	14.	8.	29.	29.	22.	8.	42.	33.	8.	36.	51.	54.	0.	13.	10.	3.	48.
23	26.	49.	27.	22.	56.	44.	35.	31.	61.	40.	19.	40.	30.	52.	9.	35.	23.	12.	51.
24	19.	31.	18.	17.	27.	32.	28.	8.	13.	28.	10.	13.	26.	58.	4.	19.	17.	2.	54.
25	22.	31.	14.	21.	27.	35.	35.	20.	54.	30.	21.	30.	42.	54.	3.	29.	22.	7.	55.
26	30.	45.	23.	28.	49.	56.	37.	32.	44.	31.	20.	59.	33.	51.	10.	32.	20.	12.	57.
27	12.	12.	19.	8.	21.	6.	30.	21.	39.	22.	14.	31.	65.	54.	4.	19.	14.	5.	59.
28	14.	20.	9.	7.	26.	29.	30.	8.	27.	35.	14.	31.	19.	55.	0.	16.	11.	5.	63.

Pretest Analysis

Analysis of the pretest data indicated that the two groups were well matched. Table 2 shows the ages of the children as well as means and ranges of each group on the pretests. The means of the groups were near the national mean in all areas except the S.A.T. Science and Social Science and Language scores, in which the means were near the thirtieth percentile, and the Arithmetic Computation scores, in which the means were near the seventieth percentile.

Table 3 gives the means and standard deviations for both groups for each of 15 scores and the t and F value for comparing the means and standard deviations. The hypotheses tested for the t and F tests were:

1. Ho $(\overline{X}_s = \overline{X}_m)$
2. Ho $(\sigma_s^2 = \sigma_m^2)$

In no case were these rejected at $(P = .10)$. The raw test scores of the S.A.T. and Otis-Lennon were used in the t and F tests as well as the regression analysis.

Table 4 contains correlation matrices of the independent variables in the regression. All of the S.A.T. scores and the Otis-Lennon score correlated positively with each other. The Torrance test scores were not consistently positively correlated with the other pretests or with each other.

Post-Test Analysis

Table 5 contains correlations between the independent variables and the post-test scores. The S.A.T. and Otis-Lennon scores were all significantly positive with the post-tests. None of the Torrance test scores correlated significantly with the post-tests. As would be expected, the S.A.T. scores in Arithmetic Computation and Arithmetic Concepts were the best predictors for the post-tests and reading ability was a better predictor for the post-test application problems than for computation.

TABLE 2

PRETEST MEANS AND RANGES

VARIABLE	MEAN		RANGE	
	Group		Group	
	S	M	S	M
1. S.A.T. Word Meaning[1]	50	50	16-96	1-94
2. S.A.T. Paragraph Meaning	44	44	1-96	2-99+
3. S.A.T. Science and Soc. Sci.	28	32	1-89	1-98
4. S.A.T. Spelling	54	54	11-99+	1-99
5. S.A.T. Word Study Skills	38	44	1-92	1-93
6. S.A.T. Language	28	28	1-94	1-98
7. S.A.T. Arithmetic Computation	70	66	2-95	6-89
8. S.A.T. Arithmetic Concepts	50	48	2-96	2-96
9. Otis-Lennon Mental Ability	48	49	1-96	1-96
10. Torrance Creative Fluency[2]	30.6	31.5	19-40	21-40
11. Torrance Creative Flexibility	18.4	18.8	8-27	8-27
12. Torrance Creative Originality	33.6	31.3	13-63	9-62
13. Torrance Creative Elaboration	64.6	64.1	30-130	32-106
14. Chronological Age	8 yr. 5 mo.	8 yr. 9 mo.	8 yr. 1 mo.- 9 yr. 8 mo.	8 yr. 0 mo.- 9 yr. 6 mo.
15. Pretest in Multiplication[3] of 20 Problems	5.50	4.14	0-13	0-11

[1]Lines 1 through 9 are in percentiles for school level--first one-third of grade three.

[2, 3]Lines 10 through 13 and Line 15 are in raw scores.

TABLE 3

PRETEST COMPARISONS USING RAW SCORES

Group	Mean	t-Value		N	Standard Deviation	F	
S	$\overline{X}_s$	$\dfrac{\overline{X}_s - \overline{X}_m}{\sqrt{\dfrac{S_s^2}{N_s} + \dfrac{S_m^2}{N_m}}}$		N_s	S_s	$\dfrac{S_s^2}{S_m^2}$	
M	$\overline{X}_m$			N_m	S_m		
Variable 1--S.A.T. Word Meaning							
Group	Mean	t	d.f.	N	S	F	d.f.
S	22.40	.509	55	27	5.50	.750	26, 29
M	21.60			30	6.35		
Variable 2--S.A.T. Paragraph Meaning							
S	34.15	.164	55	27	10.92	.702	26, 29
M	33.63			30	13.03		
Variable 3--S.A.T. Science and Social Science							
S	16.58	-.540	54	26	5.00	.889	25, 29
M	17.33			30	5.30		
Variable 4--S.A.T. Spelling							
S	14.89	-.150	55	27	7.33	.783	26, 29
M	15.20			30	8.28		
Variable 5--S.A.T. Word Study							
S	35.11	-.720	55	27	11.91	.639	26, 29
M	37.67			30	14.89		
Variable 6--S.A.T. Language							
S	33.39	-.055	56	28	10.83	.600	27, 29
M	33.57			30	13.98		
Variable 7--S.A.T. Arithmetic Computation							
S	31.48	.615	54	26	6.91	.864	25, 29
M	30.33			30	7.43		

TABLE 3--Continued

Group	Mean	t	d.f.	N	S	F	d.f.
Variable 8--S.A.T. Arithmetic Concepts							
S	21.85	.787	55	27	9.31	1.35	26, 29
M	20.03			30	8.00		
Variable 9--Otis-Lennon Mental Age							
S	45.04	-.121	53	28	12.72	1.16	26, 25
M	45.44			27	11.78		
Variable 10--Torrance Test for Fluency							
S	30.63	-.550	52	27	5.96	.853	26, 26
M	31.56			27	6.45		
Variable 11--Torrance Test for Flexibility							
S	18.44	-.320	52	27	5.02	1.33	26, 26
M	18.85			27	4.35		
Variable 12--Torrance Test for Originality							
S	33.63	.642	52	27	12.33	.783	26, 26
M	31.33			27	13.93		
Variable 13--Torrance Test for Elaboration							
S	32.33	.086	52	27	12.03	1.42	26, 26
M	32.07			27	10.09		
Variable 14--Age in Months							
S	104.58	-.438	56	28	4.28	.623	27, 29
M	105.14			30	5.42		
Variable 15--Pretest in Multiplication and Division							
S	5.54	1.265	53	28	4.18	1.00	27, 26
M	4.15			27	3.97		

TABLE 4

CORRELATION MATRICES FOR INDEPENDENT VARIABLES

Group M

Var.	2	3	4	5	6	7	8	9	10	11	12	13
1	.90	.50	.75	.69	.49	.71	.65	.65	-.05	-.16	.14	.27
2	1	.50	.80	.75	.75	.44	.74	.68	-.10	.08	.12	.14
3		1	.55	.44	.64	.27	.59	.74	-.26	-.22	.22	.04
4			1	.86	.72	.60	.75	.75	-.12	.13	.30	.21
5				1	.64	.44	.76	.69	.13	.34	.51	.22
6					1	.53	.74	.73	-.03	-.05	.05	.28
7						1	.55	.44	-.15	.01	.17	.20
8							1	.77	.18	-.15	.31	.21
9								1	.01	-.19	.03	.13
10									1	-.44	.51	-.06
11										1	.17	.11
12											1	.35
13												1

Group S

Var.	2	3	4	5	6	7	8	9	10	11	12	13
1	.77	.72	.58	.72	.70	.23	.57	.54	.08	.19	.27	-.17
2	1	.65	.62	.69	.58	.34	.54	.45	.25	.52	-.31	-.26
3		1	.47	.54	.41	.27	.54	.46	.07	.12	.27	.16
4			1	.53	.52	.32	.48	.41	-.17	.35	.32	-.21
5				1	.55	.26	.35	.41	.36	.29	.45	.02
6					1	.36	.37	.14	.17	.03	.26	-.39
7						1	.30	.30	.02	.31	.20	-.29
8							1	.67	-.13	.24	.19	-.22
9								1	-.03	.29	.11	-.05
10									1	.38	.45	.10
11										1	.14	.06
12											1	.13
13												1

TABLE 5

CORRELATION MATRICES OF INDEPENDENT VARIABLES AND POST-TESTS

Variable	Total	Computation	Applications
Group M			
1	.4984	.4672	.4644
2	.4645	.3864	.5027
3	.3091	.2504	.3440
4	.6216	.5437	.6347
5	.4965	.4783	.4442
6	.5678	.4664	.6228
7	.8158	.7923	.7210
8	.5809	.4958	.6108
9	.5570	.4923	.5551
10	.0449	.1240	-.0772
11	-.0112	.0678	-.1242
12	.1724	.2309	.0592
13	-.0385	-.0034	-.0832
Group S			
1	.5927	.4958	.6013
2	.5504	.5093	.5376
3	.5366	.4109	.5539
4	.5811	.6192	.4888
5	.4296	.4186	.4886
6	.5035	.4891	.5289
7	.5913	.4846	.6705
8	.7297	.6203	.7603
9	.5076	.4103	.5918
10	.0726	.0823	.1082
11	.2692	.2264	.2772
12	.2960	.2337	.2965
13	-.2053	-.1841	-.2034

Main Effects

The post-test data and the values of the t and F tests for testing $\overline{X}_m = \overline{X}_s$ and $\sigma_s^2 = \sigma_m^2$ are given in Table 6. None of the tests were significant at (P = .05).

TABLE 6

POST-TEST RESULTS

Group	Mean	t	d.f.	N	S	F	d.f.
			Total Score				
S	23.14	.580	56	28	8.81	.646	27, 30
M	21.63			30	10.96		
			Computation				
S	16.143	.765	56	28	5.448	.626	27, 29
M	14.900			30	6.885		
			Story Problems				
S	7.143	.405	56	28	3.875	1.020	27, 29
M	6.733			30	3.835		

Interaction

Tables 7-45 in Appendix D show the regression analysis of the independent variables to the post-tests. The points and the regression equations were plotted using the I.B.M. 1130 Plotter. Page 55 gives a description of the variables and the statistical tests used. The "crossover point," the F test for equal variances and the t test for equal regression coefficients were not computed for values unless at least one of the regression coefficients was significantly different from zero.

The null hypothesis, Ho ($b_i = 0$), was rejected at (P = .05) for all the S.A.T. scores and the mental age score from the Otis-Lennon for both groups on all post-tests. The null hypothesis, Ho ($b_i = 0$), was accepted at (P = .05) for the Torrance tests scores for both groups on all post-tests.

All tests for homogeneity of variance Ho ($S^2_{bm} = S^2_{bs}$) were accepted at (P = .05).

Null Ho ($b_s = b_m$) was accepted at ($P = .05$) with one exception. That exception indicated significant interaction between the teaching methods and Arithmetic Computation skills.

<u>Reported Values and Computational Procedures</u>
<u>For Interaction by Regression</u>

r---The correlation between the independent and dependent variables for the two groups.

a---The "Y intercept" for the regression equation of the two groups.

b---The regression coefficient or slope of each of the equations.

n---The number of pairs used in computation for each group.

S_b--The standard error of regression coefficient.

The test for $b_{yx} = 0$ from Hays is:

$$t = \frac{b_{yx}}{\frac{S_{y.x}}{S_x\sqrt{n-2}}} \qquad (31:505)$$

From the Scientific Subroutine MISR (see Appendix C):

$$S_{b_{ij}} = \sqrt{\frac{S_{jj} - r_{ij}^2 S_{jj}}{(n-2)\,S_{ii}}} \Rightarrow$$

$$S_{by.x} = \sqrt{\frac{S^2_y - r^2_{y.x} S^2_y}{(n-2)\,S^2_x}} = \sqrt{\frac{S_y^{\,2}\,(1 - r^2_{y.x})}{(n-2)\,S^2_x}}\,.$$

Since $\quad S_{y.x}^{\,2} = S_y^{\,2}\,(1 - r^2_{y.x})$

then $\quad S_{by.x} = \sqrt{\frac{S_{y.x}^{\,2}}{S^2_x\,(n-2)}} \quad$ and $\quad t = \frac{b_{ij}}{S_{b_{ij}}}$

as taken from MISR.

The hypothesis $b_m = b_s$ was tested by an F test in the analysis of covariance as reported in Snedecor and

Cochran. (53:432) However, to make use of the output of the Subroutine MISR and since $t^2 = F$ when only two treatments are used, a t test was used with pooled error term:

$$S_{pooled} = \sqrt{\frac{(N_s - 1)\, S^2_{bs} + (N_m - 1)\, S^2_{bm}}{N_1 + N_2 - 2}} \quad \text{and}$$

$$t = \frac{b_s - b_m}{\sqrt{S^2_{pooled} + S^2_{pooled}}} \; .$$

The assumption of homogeneous variance is necessary in this case and was tested using the F test.

Multiple Regression Analysis

The multiple regression analysis was done on the IBM System 360 using the Edstat Package "Regran." (63:295)

A stepwise regression indicated Variable 7, S.A.T. Arithmetic Computation, was the best single predictor for Group M's post-test score in computation. Variable 8, S.A.T. Arithmetic Concepts, was the best predictor for the post-test computation score for Group S. The addition of any other variables failed to significantly increase the predictive efficiency. Therefore, simple regression equations were used to predict future outcomes (see page 63).

Although no significant interaction was detected between any independent variable and the post-test total or applications, a stepwise multiple regression was done on these scores. Single predictors were again accepted for the post-test total using the same predictors as in the computation scores, i.e., Variable 7 to predict Group M and Variable 8 to predict Group S.

Multiple regression equations were significantly more efficient to predict the outcome of the post-test application problems. Variables 6 and 7, S.A.T. Language and Arithmetic Computation scores, proved to be the best pair of predictors for Group M. These two variables were significantly better than Variable 7 alone at ($P = .0042$). No other variable significantly increased the predicting efficiency in the presence of these two variables at ($P = .05$). The multiple regression equation to predict the application score is:

$$\hat{Y}_m = -8.8740 + .1713X_6 + .3177X_7 .$$

The multiple correlation for this equation is .8108.

For Group S, Variables 8 and 9, S.A.T. Arithmetic Concepts and Otis-Lennon Mental Age, were significantly better predictors than Variable 8 alone at ($P = .0038$). No other variable significantly increased the predicting efficiency in the presence of these two variables at ($P = .05$). The multiple regression equation is:

$$\hat{Y}_s = -6.9053 + .2658X_8 + .2596X_9 .$$

The multiple correlation for this equation is .8825.

The use of these multiple correlation equations should be approached with caution since their efficiency approaches the reliability of the post-tests. Following the suggestion of Dr. W. Alan Nicewander, Professor of Psychological Testing and Statistics, University of Oklahoma, the Lord-Nicholson estimation of the cross-validated $(\text{Rho})^2$ in the population was computed for both post-test computation and applications. The high positive values computed for Rho (ρ) indicate that the regression equations will be good predictors in any sample drawn from populations similar to those in the present study.

Assume a random regression model:

$$\text{Est } (\rho)^2 = 1 - \frac{N-1}{N-n-1} \cdot \frac{N-2}{N-n-2} \cdot \frac{N+1}{N} (1 - R^2)$$

where N = sample size, n = number of predictors.

Post-Test Computation

Group M: Variable 7 (S.A.T. Arithmetic Computation) was the best predictor with $R = .7460$, $N = 30$ and $n = 1$.

$$\rho^2 = 1 - \frac{29}{28} \cdot \frac{28}{27} \cdot \frac{31}{30} (1 - .7460^2) \quad \text{then}$$

$$\rho^2 = .50779 \text{ and } \rho = .7126$$

Group S: Variable 8 (S.A.T. Arithmetic Concepts) was the best predictor with $R = .6203$, $N = 26$ and $n = 1$.

$$\rho^2 = 1 - \frac{25}{24} \cdot \frac{24}{23} \cdot \frac{27}{26} (1 - .6203^2) \quad \text{then}$$

$$\rho^2 = .30556 \text{ and } \rho = .5528$$

Post-Test Applications

Group M: Variables 6 and 7 (S.A.T. Language and Arithmetic Computation) were used with $R = .8108$, $N = 30$ and $n = 2$.

$$\rho^2 = 1 - \frac{29}{27} \cdot \frac{28}{26} \cdot \frac{31}{30} (1 - .8108^2) \quad \text{then}$$

$$\rho^2 = .5905 \text{ and } \rho = .7684$$

Group S: Variables 8 and 9 (S.A.T. Arithmetic Concepts and Otis-Lennon Mental Age) were used with $R = .8825$, $N = 26$ and $n = 2$.

$$\rho^2 = 1 - \frac{25}{23} \cdot \frac{24}{22} \cdot \frac{27}{26} (1 - .8825^2) \quad \text{then}$$

$$\rho^2 = .7276 \text{ and } \rho = .8530$$

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Summary of the Findings

Four hypotheses, as listed on page 5, were tested.

Ho I: There is no difference between the two teaching methods in achievement in computational skills and application as measured by the post-tests in multiplication and division.

This hypothesis was accepted at (P = .05). The mean of the scores in Group S was higher but not enough to be statistically significant. The values for this test are found in Table 6, page 54.

Hypotheses II-IV were evaluated using the following procedures (see page 43):

If $\widehat{Y_s} = A_s + b_s X_a$ and $\widehat{Y_m} = A_m + b_m X_a$ are the simple regression equations from aptitude A to the achievement scores for Group S and M respectively, the null hypothesis

Ho (Interaction): No interaction exists between the teaching methods and the scores on aptitude A

was rejected if and only if the following were satisfied:

1. The regression equations intersected within the range of the independent variable and
2. The hypothesis that $b_m = 0$ or $b_s = 0$ was rejected at (P = .05) and
3. The hypothesis that $b_m = b_s$ was rejected at (P = .05).

Ho II: No interaction exists between teaching methods and mental ability as measured by the Otis-Lennon Mental Ability Test.

This hypothesis was accepted. The regression coefficients of the two groups were both significantly positive indicating this is a good predictor for achievement but the coefficients were not significantly different from each other (see Tables 31-33, pages 106-108).

Ho III: No interaction exists between teaching methods and scholastic achievement scores as measured by the Stanford Achievement Test.

The hypothesis was tested separately for each of the eight S.A.T. scores. All of the regression coefficients were significantly positive with the post-tests for both groups. The null hypothesis of no interaction was accepted in all but one instance. The null hypothesis was rejected for Variable 7, Arithmetic Computation, on the post-test for computation. The regression coefficients were significantly different and the point of intersection was in the range of the dependent variable (see Tables 7-30, pages 82-105).

Ho IV: No interaction exists between teaching methods and various forms of creativity as measured by the Torrance Test of Creative Thinking.

The null hypothesis was accepted for all four scores in the Torrance test with the post-tests for computation and application. The regression coefficient was not significantly different from zero in any test (see Tables 34-45, pages 109-120).

General Observations

The procedures as set forth in Chapter III were followed with respect to teaching methods. Both methods appeared to be acceptable for teaching the topics covered. The average score for computation was over 60 per cent for the two groups, and Group M had seven students with a score above

90 per cent, while Group S had five. The average score for applications was considerably lower than the computation score. The mean for each group was under 50 per cent with only one student scoring above 90 per cent.

This study, in conjunction with other studies on this grade level and topic, indicates that use of concrete teaching aids is not as valuable as it is for lower grades or for teaching addition and subtraction.

There are a number of possible explanations for this. The concrete applications of addition and subtraction are quite simple and obvious to a child. However, in multiplication the number of pegs or pebbles required for all but the simplest problems is quite large and the student possibly loses sight of the concept to be learned in the maze of objects. The significant interaction which indicated that only the better students benefited from manipulation seems to support this explanation.

Since the textbook made use of pictures and charts, Method S may be considered a combination of an abstract method and a semi-concrete method. Eight studies in the review of the literature compared active concrete methods to passive concrete or semi-concrete methods. This study and five of those reviewed indicated no significant differences. One study, Trueblood's, had the semi-concrete group marginally superior to the concrete group on one test. Ekman's study indicated that the concrete group was significantly higher than the mean of the semi-concrete and abstract groups. The active group in Curry's study scored higher than the passive group on a test that involved manipulation of objects on which the passive group had no experience.

This evidence seems to indicate that use of multisensory manipulation does not lead to better computational skills or understanding for the over-all sample when compared with the use of teacher demonstrations or semi-concrete devices in which the sense of sight is the primary sense stimulated.

In the summary of related research, page 28, it was stated that the evidence indicates concrete aids do benefit children in the lower levels in which the primary mathematics topics are addition and subtraction. Only two studies, those of Gibb and Curry, compared a concrete method to a passive or semi-concrete method. Neither of these two studies showed a significant difference; however, both showed the concrete group's mean score was lower than the other group's mean score on computation. It would seem that more research is needed in this area.

Significant Interaction

The significant interaction between computational ability of the students and teaching methods in this study is not consistent with general results of other studies. Johnson's study, and others, indicate the use of concrete aids to be of benefit for slower students. Data accompanying the Moody et. al. study revealed a smaller variance for the activities group which may be interpreted as possible interaction. In the present study, however, the symbolic group had the smaller variance. Although the outcome of this study is unusual, it is not unique; Lucow's study indicated the Cuisenaire method may be of more benefit to children of average or above average intelligence.

Possibly the interaction can be explained by further examination of the teaching methods. The textbook for the course stressed teaching for understanding: ". . . they [the students] learn much more quickly and effectively when the teaching method is based upon a conceptual approach that emphasizes the discovery and understanding of ideas." (23:V)

The lessons plans for the concrete aids were written as nearly as possible to fit the lessons presented in the book. To equalize the amount of time given for instruction, students in Group S spent more of their time in drills or completion of exercises presented in the book. It may be that the better students benefited more from the extra time spent in teaching for understanding by manipulative devices while those students with less ability were helped more with extra drill and memorizing. As mentioned before, the nature of multiplication tends to lead to large numbers and slow students may become lost in or frustrated with a large number of objects.

While a significant interaction between teaching methods and aptitudes in general is of interest, it is especially significant because the computational aptitude had the highest average correlation with the post-test score (.63).

Assuming different outcomes of this study are due to different teaching methods, then future classes of this type would be divided according to the regression analysis on S.A.T. Arithmetic Computation scores. The students above the crossover point, 33 (see Table 26, page 101), would be assigned Method M and those with 33 or less would be assigned Method S. Regression equations could be used to predict the outcome.

As an example, suppose this had been done for the students in the sample. Thirteen of the students in Group M had S.A.T. Computation scores greater than 33 and their average grade on the post-test was 19.3846. Eleven students in Group S had more than 33 on the S.A.T. Computation and would be assigned to Method M. Using the regression equation $\widehat{Y} = -7.3366 + .7339\ (X_7)$, the mean score of these 11 students on the post-test would be predicted at 20.0845. The predicted mean of the group assigned to Method M would be $\frac{11\ (20.0845 + 13\ (19.3846)}{24} \approx 19.77.$

Using the same procedures for predicting scores for the simulated Method S, 16 students in Group S had scores less than or equal to 33 on the S.A.T. Computation and their actual mean was 13.62 in the post-test.

Seventeen students in Group M should be assigned to Method S as they scored 33 or less in S.A.T. Computation. The multi-regression analysis has shown Variable 8, the S.A.T. Arithmetic Concepts score, to be the best single predictor for Method S. Using the mean score of the 17 students in Group M on Variable 8 and the regression equation $\hat{\overline{Y}} = 8.6381 + .3546\ (\overline{X}_8)$, the mean predicted for Group M if given Method S is 14.42. The over-all predicted mean of the group given Method S would be 14.0324.

Pooling the means of the two groups, a mean of 16.49 would be predicted. The actual pooled mean of the two groups was 15.31.

While the difference is small, the distribution of both groups is clustered around the score of 33. However, since the interest is in developing teaching methods to suit individual students, the following examples are cited. Three students had scores over 40 in the S.A.T. Arithmetic Computation, averaging 41. A score of 22.7 in the post-test would be predicted if these students were given Method M and a score of 19.5 in Method S. At the other end of the spectrum, an Arithmetic Computation score of 10 is about the mean for the lower one-fourth of the total sample and a post-test score of 0 would be predicted if the students were given Method M (a score actually received in this group) while a score of 8 would be predicted in Method S.

Teacher's Impression

The teacher felt there was a definite Hawthorne Effect at the start of instruction. The students in Group M were eager to use the teaching devices and those in Group S seemed jealous. However, these feelings subsided in time and seemed to disappear before the end of the course.

As the teacher was impressed with the use of aids, she felt she would use them again. She especially liked the counting boards and was anxious to use them for instruction in addition and subtraction.

Limitations

The experiment was conducted in a middle-class suburban area and all the students appeared to be white.

Both teaching methods were conducted by the same teacher, therefore the question of existence of interaction between teaching methods and teachers was not tested.

After the experiment, the students returned to their regular classes and teachers. Since further multiplication instruction was given, no attempt was made to evaluate retention.

The size of the sample was adequate to detect any difference between the means and variances of the two groups and to test for interaction by the simple regression equation. However, the sample was too small to accurately evaluate higher order interaction by multiple regression or to determine if the regression was non-linear.

The time spent learning to add and subtract on the counting boards prior to start of instruction on multiplication was not adequate. Therefore, part of the time intended for instruction was used to develop this skill.

The computational ability shown on the Stanford Achievement Test was not typical. The mean score of the two groups was in the national sixty-eighth percentile.

Recommendations

The only significant difference detected in this study was interaction between previous computational ability and computational skills developed by the teaching methods. This significance was accepted at ($P = .05$) but would have been rejected at ($P = .01$). The procedures should be repeated to check the significance and to remove the limitation of results.

However, since the crossover point for the interaction was above the mean, this would indicate the greatest difference between the methods is found at the lower levels. Therefore, this study indicates that third-grade students of low arithmetic ability will benefit more from the teaching procedures used in Method S than from working with manipulative devices to stress concepts. The incorporation of concrete exercises as done in this study does seem to benefit the better students in third grade and may be appropriate for instruction in fourth and fifth grades in classes of multiplication.

<u>Recommendations for a Replication of the Study</u>

1. The experiment should be started at the beginning of instruction in third grade and the teaching devices, especially the counting boards, should be used during the instruction on place value, addition and subtraction, as well as multiplication.

2. The size of the experiment should be increased to include at least three teachers, each teaching a class of each treatment. This would allow for testing variance of teachers and interaction between teachers and teaching methods.

3. The selection of a sample should include a wide range of socio-economic and racial backgrounds.

4. Some method of checking long-range effects should be incorporated.

5. A sample size of 100 or more in each group should be used and a multiple-regression analysis done to evaluate higher order interaction.

APPENDIX A

SAMPLE LESSON PLANS--METHOD M

Topic: One and Zero in Multiplication[1]
Pages 96-97

Concept: $3 \times 1 = 3$

Place one peg in the first position of each of three rows on your pegboards. What multiplication problem represents this display? 3×1 The answer is 3.

Turn your boards to make one row. What multiplication problem is represented now? 1×3 What is the answer? 3

Do problem 1, A and B on page 96 in your books ($9 \times 1 =$ //// and $1 \times 9 =$ ////). First, set up the problems on your pegboards and then work the multiplication problem.

Who can answer this question: The product of any number and one is____?

Concept: $2 \times 0 = 0$

Remove everything from your egg cartons.

Look in the first two cells of the carton. Write the multiplication problem. 2×0 What is the answer? 0

Look in all 12 cells and write the multiplication problem. 12×0 What is the answer? 0 If you looked in every cell of each carton of all the students in the room, your equation would be 396×0. What would the answer be? 0 What is the answer for $n \times 0$ for any number n? 0

Have the students do problems in their books for the rest of the period.

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[1]The lesson plans were written so the teacher could read the instructions to the students if she wished. The teacher and the author went over the lesson plans prior to presenting them to the class and the teacher generally presented them in her own words.

Topic: Grouping Principle for Multiplication (The Associative Property) Pages 100-101

Concept: $2 \times (3 \times 5)$

Have the students work in pairs to represent $2 \times (3 \times 5)$.

Each student place 5 pebbles in each of 3 cells of your egg carton. What multiplication equation is represented in your carton? $\underline{3 \times 5 = 15}$ Put all the pebbles in the 3 cells in just 1 cell.

Place your carton beside your partner's carton. What equation is represented? $\underline{2 \times 15}$ How many pebbles in both cartons? $\underline{30}$ Add if you have to.

We have just represented and solved $2 \times (3 \times 5)$. We put the parentheses around the numbers that we multiply first.

Concept: $(2 \times 3) \times 5$

Again, put 5 pebbles in each of 3 cells of your egg cartons. Place your cartons beside those of your partners. How many cells have pebbles in them, adding both cartons together? $\underline{6}$ Think: Two cartons of three cells each. What is the equation? $\underline{2 \times 3 = 6}$

If you have 6 cells with 5 pebbles each, what equation does this represent? $\underline{6 \times 5 = 30}$ You have solved $(2 \times 3) \times 5$. How did your answer compare with your answer in the first problem?

Open your books to page 100 and do the problems in No. 1.

Topic: Find the Missing Factors
Page 118

Do Review Problem 1 before starting in the book. With the pegboards arrange 3 rows of 4 pegs each. How many pegs do you have in all? 12 What is the equation? $3 \times 4 = 12$ Arrange 6 rows of 3 pegs each. How many pegs in all? 18 What is the equation? $6 \times 3 = 18$ Arrange 4 rows of 5 pegs each. How many pegs are there altogether? 20 What is the equation? $4 \times 5 = 20$

Now, find the missing factor $n \times 2 = 10$. Count out 10 pegs. Make rows of 2 pegs each until all 10 pegs are used. How many rows did you make? 5 What is the missing factor in $n \times 2 = 10$? 5

Count out 7 pegs. Place the same number of pegs in each row of 7 rows until all the pegs are used. How many pegs are in each row? 1 What is the missing factor in $7 \times n = 7$? 1

Let's find the missing factor in $6 \times n = 18$. Count out 18 pegs. Place the same number in each of 6 rows. (Note to teacher: You might demonstrate this first by putting 1 in each row, then 2, then 3.) What is the missing factor in $6 \times n = 18$? 3

(Note to teacher: Do the following using the egg cartons.) Count out 30 pebbles; put 5 in each cell of your egg cartons until all the pebbles are used up. How many holes have pebbles in them? 6 Find the missing factor in $n \times 5 = 30$. 6

Count out 12 pebbles. Put the same number of pebbles in each of 3 cells in the egg cartons. How many pebbles are in each of the cells? 4 Find the missing factor in $3 \times n = 12$. 4

See if you can find the solution to the following problem using the egg cartons: $4 \times n = 20$. 4 cells of 5 pebbles each. (Note to teacher: If some of the students solve the problem by 5 cells of 4 pebbles each, you can point out that 5 sets of 4 gives the equation $5 \times 4 = 20$. We know from the order principle on page 98 that 5 sets of 4 has the same number of elements as 4 sets of 5 or $5 \times 4 = 4 \times 5$.)

Do the problems on page 118 in the book and 119 if you have time.

<u>Topic:</u> <u>Introduction to Division</u>
<u>Pages 136-137</u>

Count out 12 pebbles. Place 4 in each cell of your egg cartons until all 12 pebbles are used up. How many cells have 4 pebbles in them? <u>3</u> We think 3 fours in 12, and write $12 \div 4 = 3$, and say 13 divided by 4 equals 3. (Note to teacher: Point out that $3 \times 4 = 12$, etc.)

Count out 20 pebbles. Place 5 pebbles in each cell of your egg cartons until all the pebbles are gone. How many cells have pebbles in them? <u>4</u> We think <u>4</u> fives in 20, write <u>$20 \div 5 = 4$</u> and we say <u>20 divided by 5 equals 4</u>.

Finish the problems on page 137 until the end of the period. You may use your egg cartons if you wish.

Different Ways to Think of Division
Pages 148-149

Concept: Three Ways to Represent $15 \div 3$

Arrange 15 pegs on the pegboard in 3 rows. How many pegs are in each row? 5 Circle each column of 3. How many 3's are in 15? 5

Put the representation of 15 on your counting boards, 1 in the "10" and 5 in the "1." Subtract 3 from this set. How many pebbles do you have left? 12 Write a subtraction equation for this, $15 - 3 = 12$. What do you have to do before you can subtract 3 more pebbles? Borrow 10 from the "10" place. Continue subtracting 3, writing the equation each time you do. $12 - 3 = 9$, $9 - 3 = 6$, $6 - 3 = 3$, $3 - 3 = 0$ How many 3's in 15? 5 Answer this equation: $15 \div 3 = 5$.

Count out the 15 pebbles again. Arrange them in the first 3 cells of an egg carton, placing the same amount of pebbles in each cell until all the pebbles are gone. How many pebbles are in each cell? 5 Write this as a missing factor multiplication problem, $n \times 3 = 15$, and as a division problem, $15 \div 3 = 5$.

Finish the exercises as time allows.

Topic: Multiplication by 10
Pages 210-211

Count out 10 sets of 2 pebbles each and place them in the "1" place on your counting boards. What multiplication problem is represented? 10 x 2 Now add.[1] What is the answer? 20 Write the equation. 10 x 2 = 20

Place ten-31's on your counting boards.[2] What multiplication problem does this represent? 10 x 31 Now add. What is the answer? 310 Write the equation. 10 x 31 = 310 (Repeat the process using 10 x 240.)

Put the answer to this problem, 10 x 397, on your counting boards without adding. What is the answer? 3,970

Do some of the problems in the book as time permits.

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[1]The children were familiar with the instruction "to add," which meant to count the number of pebbles in each place, starting with the "1" place. If there were more than 10 pebbles, the children would pick up 10 pebbles and carry 1 to the next place.

[2]To represent 31 on the counting board, 3 pebbles were put in the "10" place and 1 pebbles in the "1" place (see page 37).

Multiplication of One-Digit By Three- and Four-Digit Numbers
Pages 234-237

Multiply 123 x 4 on the counting boards by placing four 123's on the counting boards and adding. Count out the 10 one's and carry 1 to the "10." What is the answer? 492 Repeat by multiplying 4 x 3 and adding. Leave the 12 on the boards, then add four 20's and four 100's; count up the final answer. (Note to teacher: This will stress that four 123's is the same as 4 (3) + 4 (20) + 4 (100).)

Multiply 2,715 x 2 by placing 2,715 on the counting boards and 2,715 again. The answer is 5,430. Repeat by placing 2 x 5 = 10 on your counting boards. (Note to teacher: Let them do the multiplication in their heads and place only answer 10 on their counting boards.) Now place the answer to 2 x 10, or 20, on your counting boards. Next place the answer to 2 x 700, or 1,400, on your counting boards. (Note to teacher: They might place two 700's on their boards and add.) Now place the answer to 2 x 2,000 on your counting boards, 4,000. Add up the total on your counting boards. What is the answer? 5,430

(Note to teacher: Continue as in the book.)

APPENDIX B

PRETEST AND POST-TESTS

Test A--Pretest

Find the products:

(1.) 7 x 8 = ____ (2.) 3 x 9 = ____ (3.) 6 x 6 = ____

(4.) $\begin{array}{r} 47 \\ \underline{\times\ 4} \end{array}$ (5.) $\begin{array}{r} 36 \\ \underline{\times 12} \end{array}$

Find the missing factor:

(6.) $\square$ x 3 = 15

Find the quotient:

(7.) 24 ÷ 6 = ____

Solve these problems:

(8.) A football team scores 4 touchdowns. Each touchdown is 6 points. How many points were scored?________

(9.) You have 48 books stored in boxes. There are 6 books in a box. How many boxes are there?____________

(10.) A farmer has 8 dozen eggs. There are 12 eggs in a dozen. How many eggs does the farmer have?________

Find the product:

(11.) 5 x 4 = ____ (12.) 2 x 8 = ____

(13.) $\begin{array}{r} 21 \\ \underline{\times\ 5} \end{array}$ (14.) $\begin{array}{r} 438 \\ \underline{\times\ 3} \end{array}$ (15.) 78 x 100 = ____

<u>Find the missing factor</u>:

(16.) $6 \times \boxed{} = 42$

<u>Find the quotient</u>:

(17.) $63 \div 9 =$ ____

<u>Solve these problems</u>:

(18.) Each Cub Scout ate 5 marshmallows. There were 7 Cub Scouts. How many marshmallows were eaten?__________

(19.) If there are 7 rows of desks in your school room and there are 42 desks in all, how many desks are in each row?_____________

(20.) How much will it cost to buy 24 stamps if they cost 8 cents each?______________

Test B--Post-Test

Find the products:

(1.) $7 \times 9 =$ ____ (2.) $6 \times 8 =$ ____

(3.) $\begin{array}{r} 46 \\ \underline{\times\ 4} \end{array}$ (4.) $\begin{array}{r} 62 \\ \underline{\times\ 7} \end{array}$ (5.) $\begin{array}{r} 138 \\ \underline{\times\ 5} \end{array}$

(6.) $\begin{array}{r} 432 \\ \underline{\times\ 3} \end{array}$ (7.) $\begin{array}{r} 25 \\ \underline{\times 32} \end{array}$ (8.) $\begin{array}{r} 46 \\ \underline{\times 28} \end{array}$

Find the missing factor:

(9.) $\square \times 7 = 42$ (10.) $6 \times \square = 54$

Find the quotient:

(11.) $27 \div 3 =$ ____ (12.) $40 \div 8 =$ ____

Solve these problems.

(13.) A football team scores 6 touchdowns. Each touchdown is 6 points. How many points were scored?______

(14.) You have 45 books to store in boxes. If there are 9 books in a box, how many boxes are needed?_________

(15.) A farmer has 7 dozen eggs. There are 12 eggs to a dozen. How many eggs does the farmer have?__________

(16.) Each Cub Scout ate 9 marshmallows. There were 7 Cub Scouts. How many marshmallows were eaten?__________

(17.) If there are 6 rows of desks in your school room and 42 desks in all, how many desks are in each row?_____

(18.) How much will it cost to buy 28 stamps if each stamp costs 7 cents?____________

(19.) I have 45 cents and candy bars are 5 cents each. How many candy bars can I buy?_____________

(20.) 15 pencils are in each box. How many pencils do 24 boxes hold?______________

Test C--Post-Test

Find the products:

(1.) 8 x 6 = ____ (2.) 3 x 7 = ____

(3.) 59 x 3

(4.) 23 x 9

(5.) 243 x 7

(6.) 241 x 6

(7.) 36 x15

(8.) 64 x47

Find the missing factor:

(9.) $\square$ x 9 = 72 (10.) 5 x $\square$ = 40

Find the quotient:

(11.) 24 ÷ 4 = ____

(12.) 49 ÷ 7 = ____

Solve these problems:

(13.) Each baseball team has 9 players. There are 5 teams in the Little League. How many players in the league?

____

(14.) There are 30 students going on a field trip. In each car there will be 5 students. How many cars are needed for the trip?__________

(15.) Each girl in the Brownies has 8 badges. There are 6 girls in the Brownies. How many badges do they have altogether?__________

(16.) If each bag holds 14 apples and I have 8 bags, how many apples do I have?__________

(17.) If I have 63 books to store in boxes and there are 9 boxes, how many books will I put in each box?______

(18.) How much will it cost to buy 37 pencils if each pencil costs 6 cents?__________

(19.) I have 48 cents and apples cost 8 cents each. How many apples can I buy?__________

(20.) There are 12 eggs in a dozen. How many eggs are there in 25 dozen?__________

APPENDIX C

SCIENTIFIC SUBROUTINE MISR

This subroutine computes means, standard deviations, third and fourth moments, correlation coefficients, regression coefficients, and standard errors of regression coefficients when there is missing data. Effective sample sizes are also provided. Missing observations or certain values of the data can be skipped at the user's option. The computational steps are as follows:

1. Compute means:

$$\bar{x}_j = \frac{\sum_{\alpha=1}^{n} x_{\alpha j}}{n}$$

where $j = 1, 2, \ldots, m$ implies variables
n = number of nonmissing values for the jth variable

2. Compute sums of cross-products of deviations from means for complete sets of ith and jth variables:

$$S_{ij} = \sum_{\alpha=1}^{n'} (x_{\alpha i} - \bar{x}_i)(x_{\alpha j} - \bar{x}_j) - \frac{\sum_{\alpha=1}^{n'} (x_{\alpha i} - \bar{x}_i) \sum_{\alpha=1}^{n'} (x_{\alpha j} - \bar{x}_j)}{n'}$$

where $\bar{x}_i$, $\bar{x}_j$ = means of ith and jth variables computed as above
n' = number of sets where the ith and jth variables are both present

3. Compute product-moment correlations:

$$r_{ij} = \frac{S_{ij}}{\sqrt{S_{ii}}\sqrt{S_{jj}}}$$

4. Compute regression coefficients, intercepts, and standard errors of regression coefficients:

ith variable as independent and
jth variable as dependent:

Regression coefficient: $b_{ij} = \dfrac{S_{ij}}{S_{ii}}$

Intercept: $a_{ij} = \overline{X}_j' - b_{ij}\overline{x}_i'$

Standard error regression coefficient:[1]

$$S_{b_{ij}} = \frac{S_{jj} - r_{ij}^2 S_{jj}}{(n' - 2)\, S_{ii}}$$

. . . .

5. Compute standard deviations:

$$s_j = \sqrt{\frac{S_{jj}}{n - 1}}$$

where n = number of nonmissing values for jth variable (34:33-34)

---

[1]Careful reading of the program indicates that the values

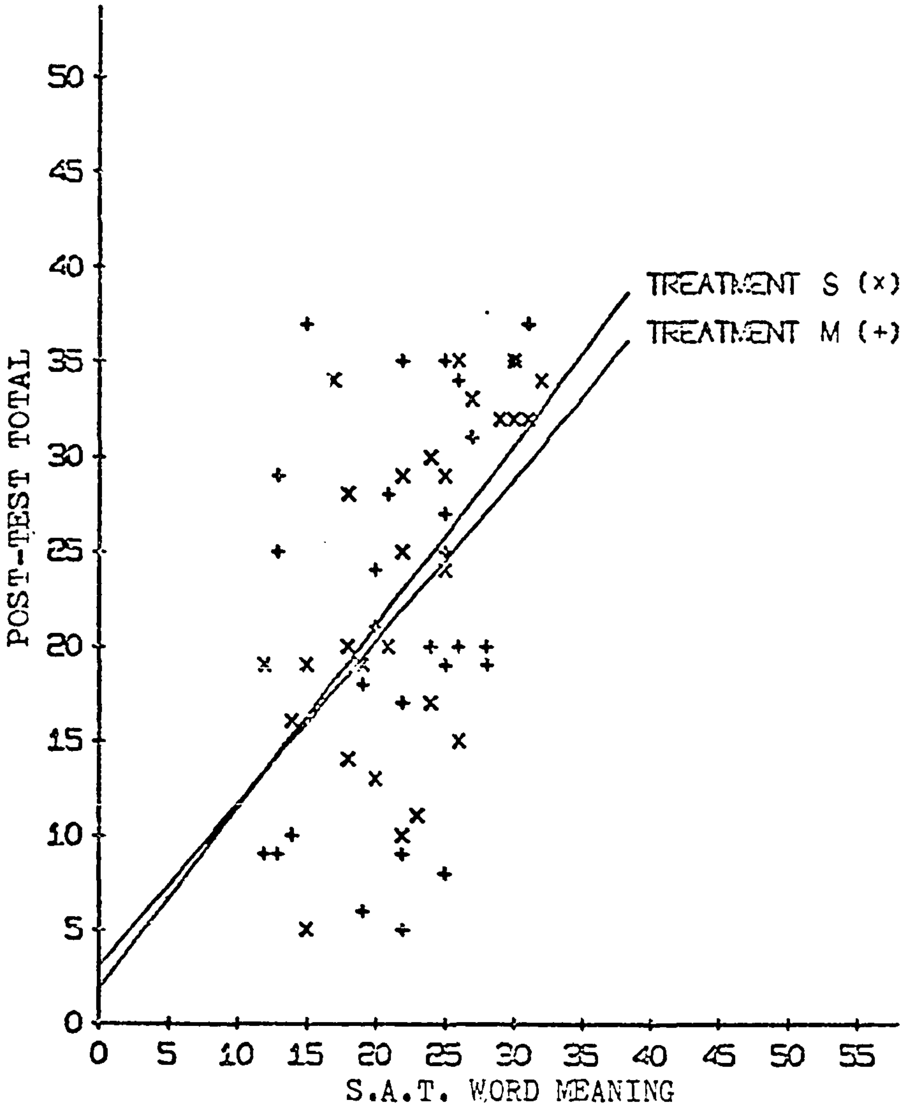
$$S_{b_{ij}} = \sqrt{\frac{S_{jj} - r_{ij}^2 S_{jj}}{(n' - 2)\, S_{ii}}}$$

APPENDIX D

REGRESSION ANALYSIS OF THE INDEPENDENT VARIABLES TO THE POST-TESTS

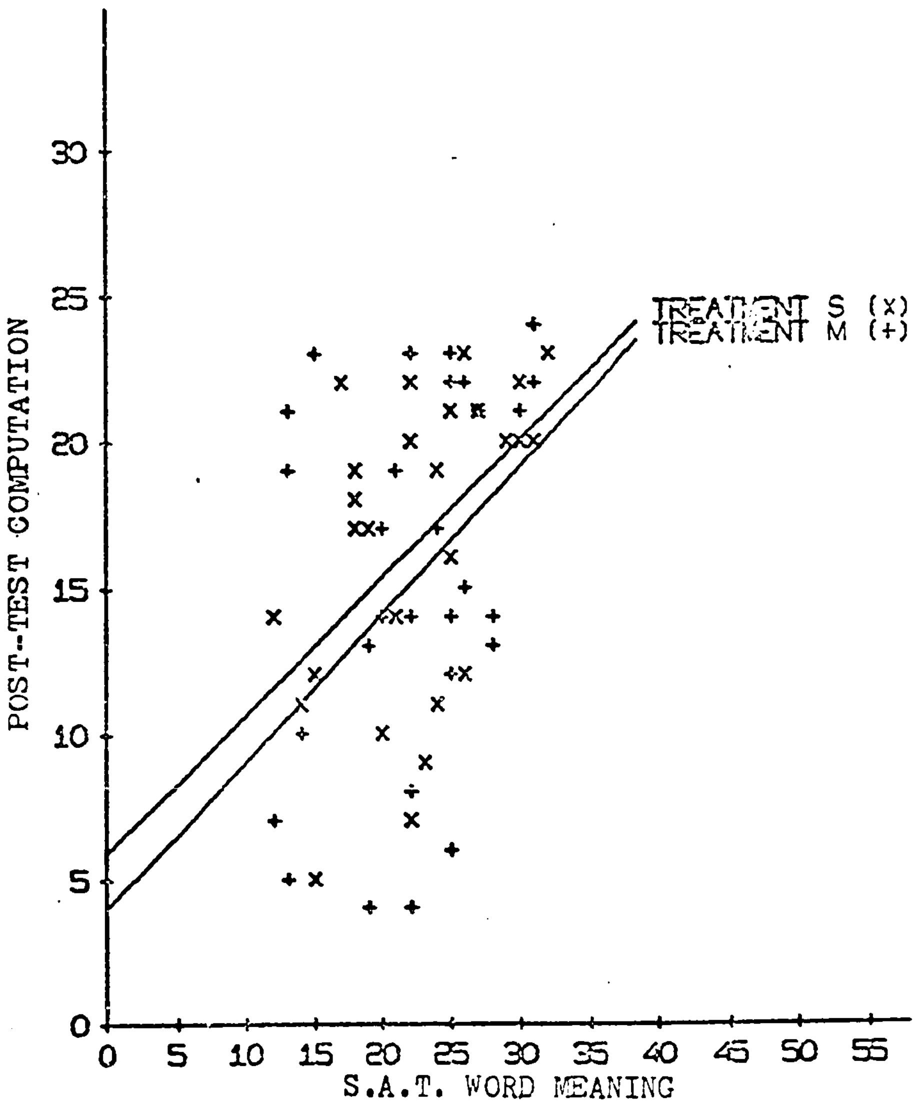
Tables 7-45

TABLE 7



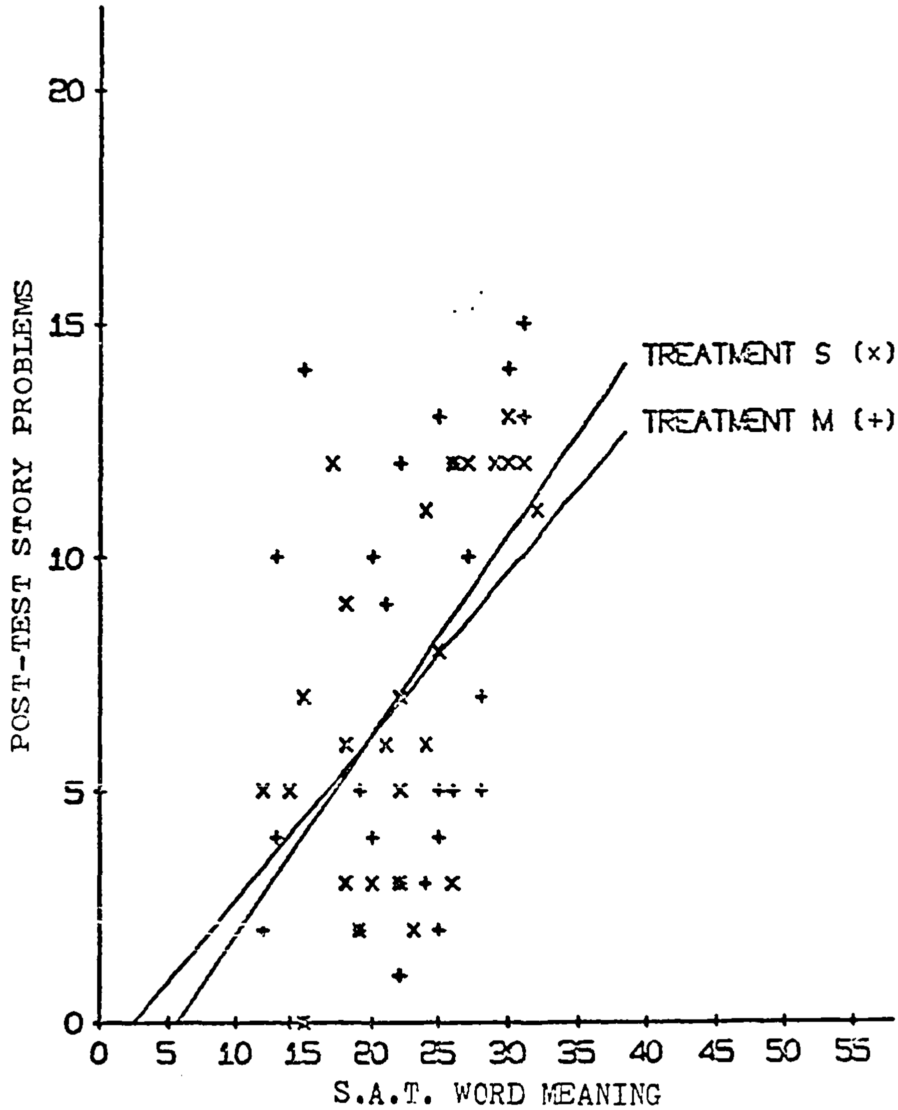
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5927	1.8714	.9578	27	.2603	3.6796	12.2	.848	
M	.4984	3.0690	.8595	30	.2825	3.0425			.2509

TABLE 8



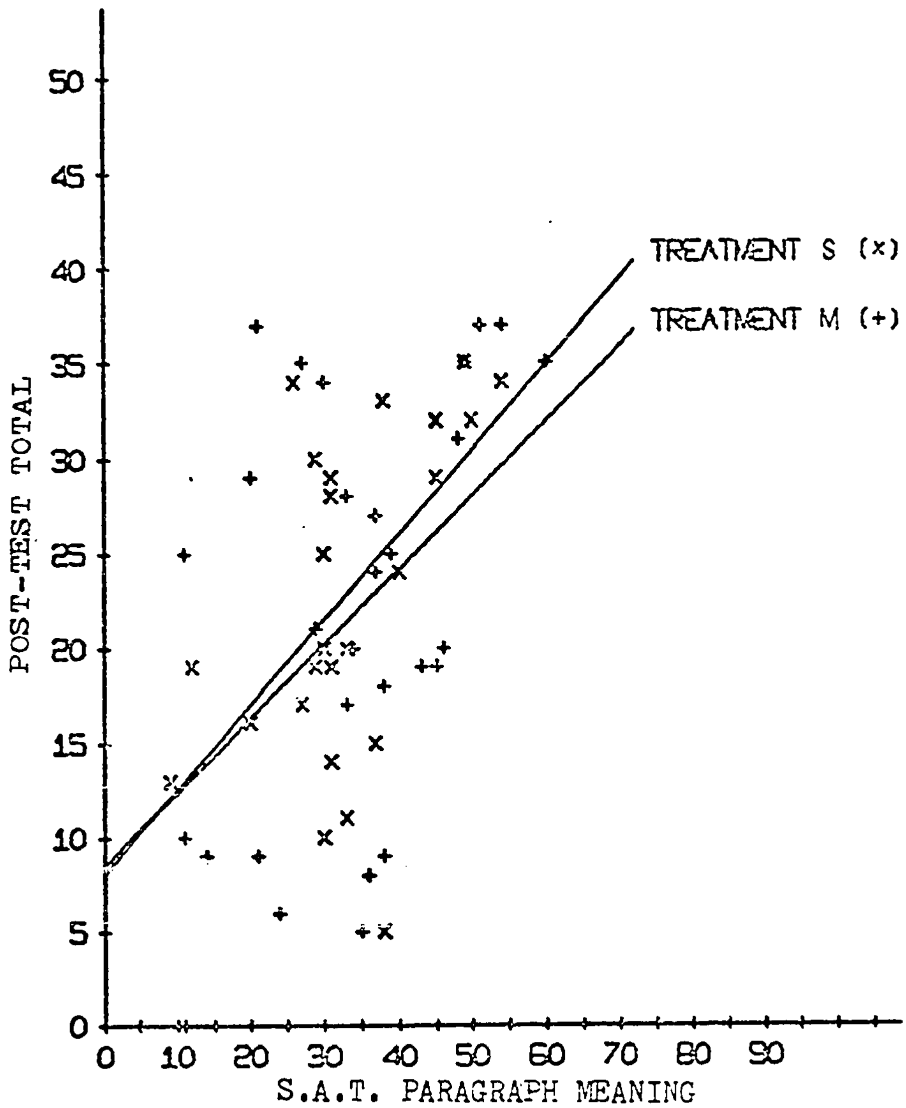
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4958	5.8919	.4726	27	.1656	2.8538	57.6	.837	
M	.4672	3.9672	.5061	30	.1810	2.7961			-.1339

TABLE 9



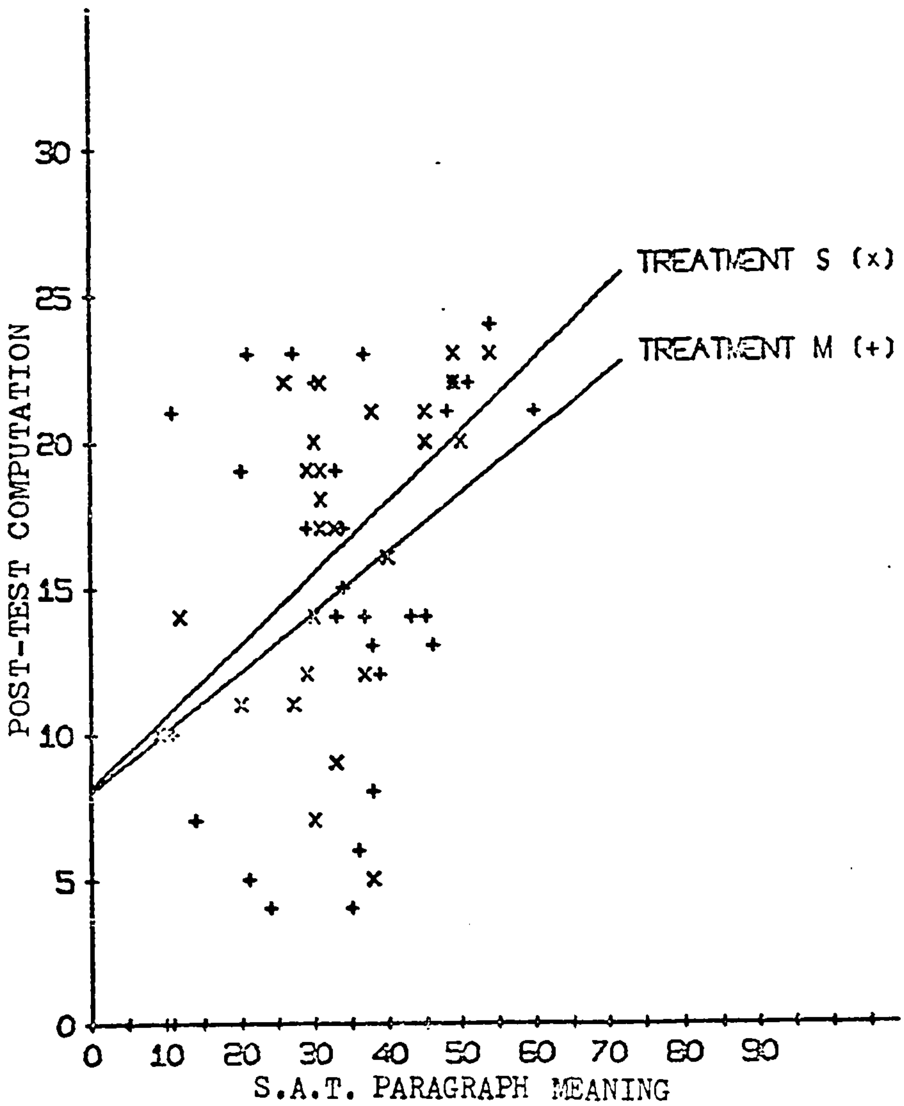
	r	a	b	N	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.6013	-2.3945	.4292	27	.1141	3.7616	19.7	.814	
M	.4644	- .8982	.3533	30	.1273	2.7533			.4338

TABLE 10



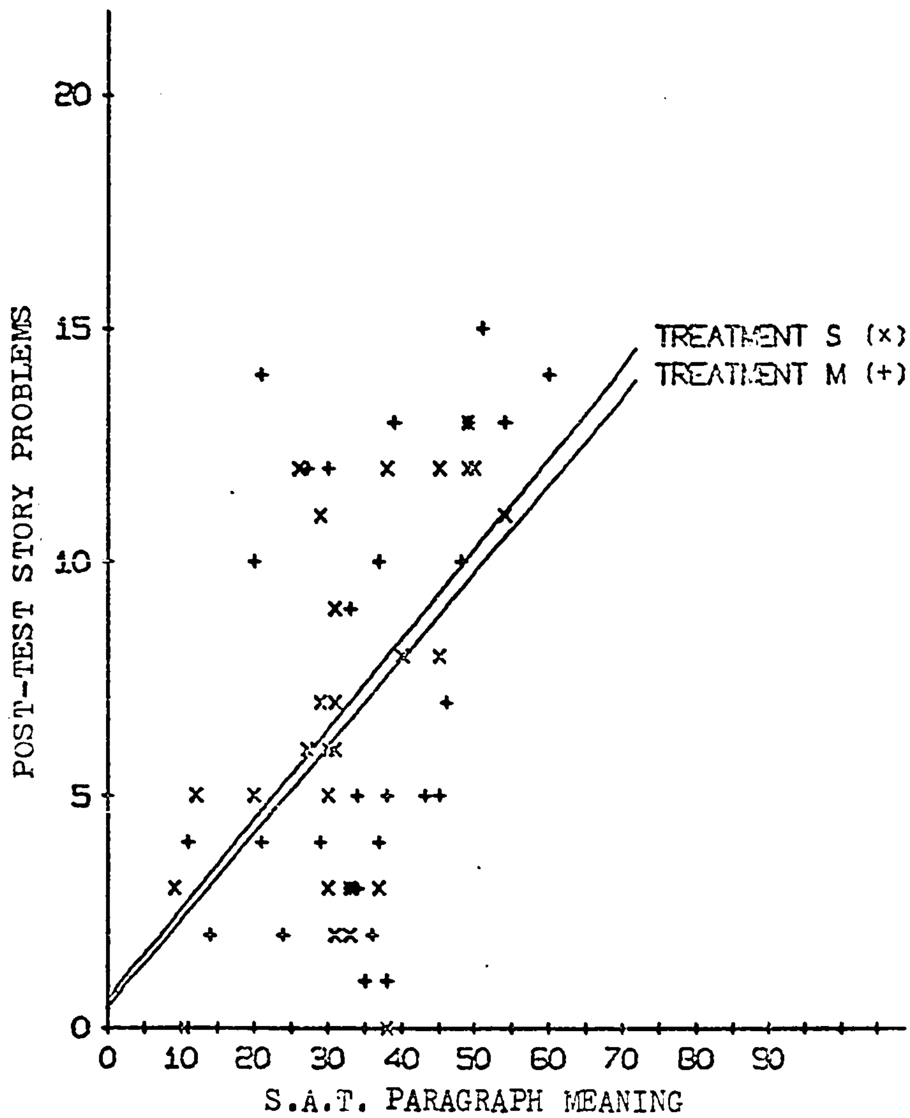
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5504	8.0335	.4480	27	.1359	3.2965	8.0	.934	
M	.4645	8.4904	.3908	30	.1408	2.7756			.2868

TABLE 11



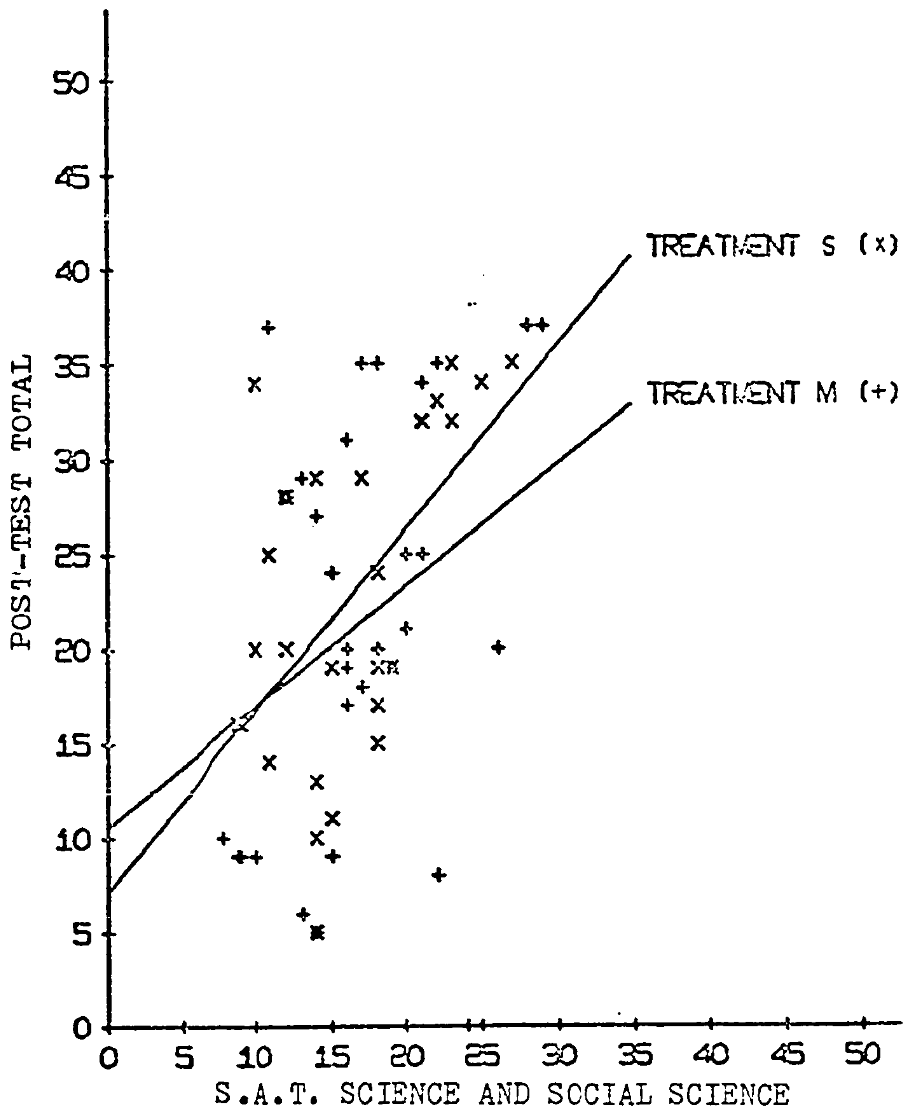
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5093	8.1293	.2466	27	.0827	2.9819	-2.3	.800	
M	.3864	8.0318	.2042	30	.0921	2.2171			.3361

TABLE 12



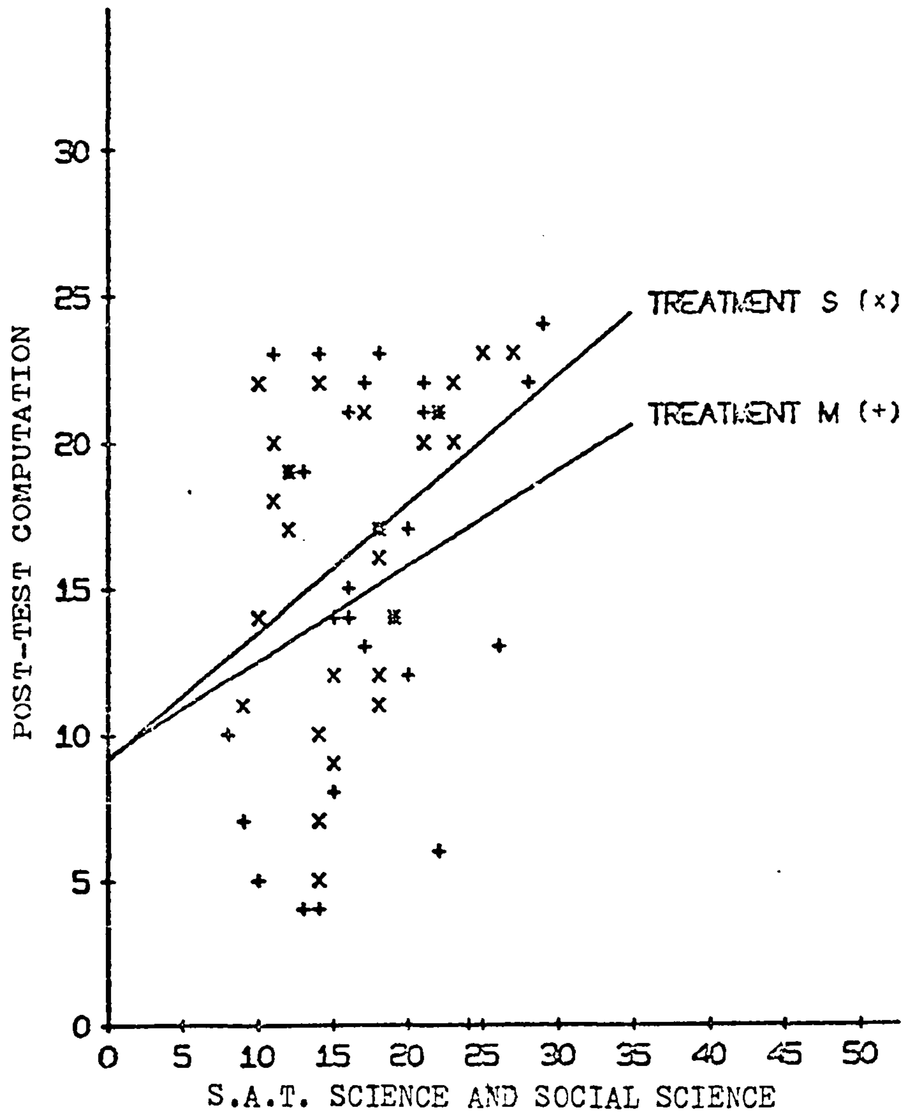
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5376	.6214	.1933	27	.0606	3.1898	-24.	1.000	
M	.5027	.4586	.1866	30	.0606	3.0792			.0769

TABLE 13



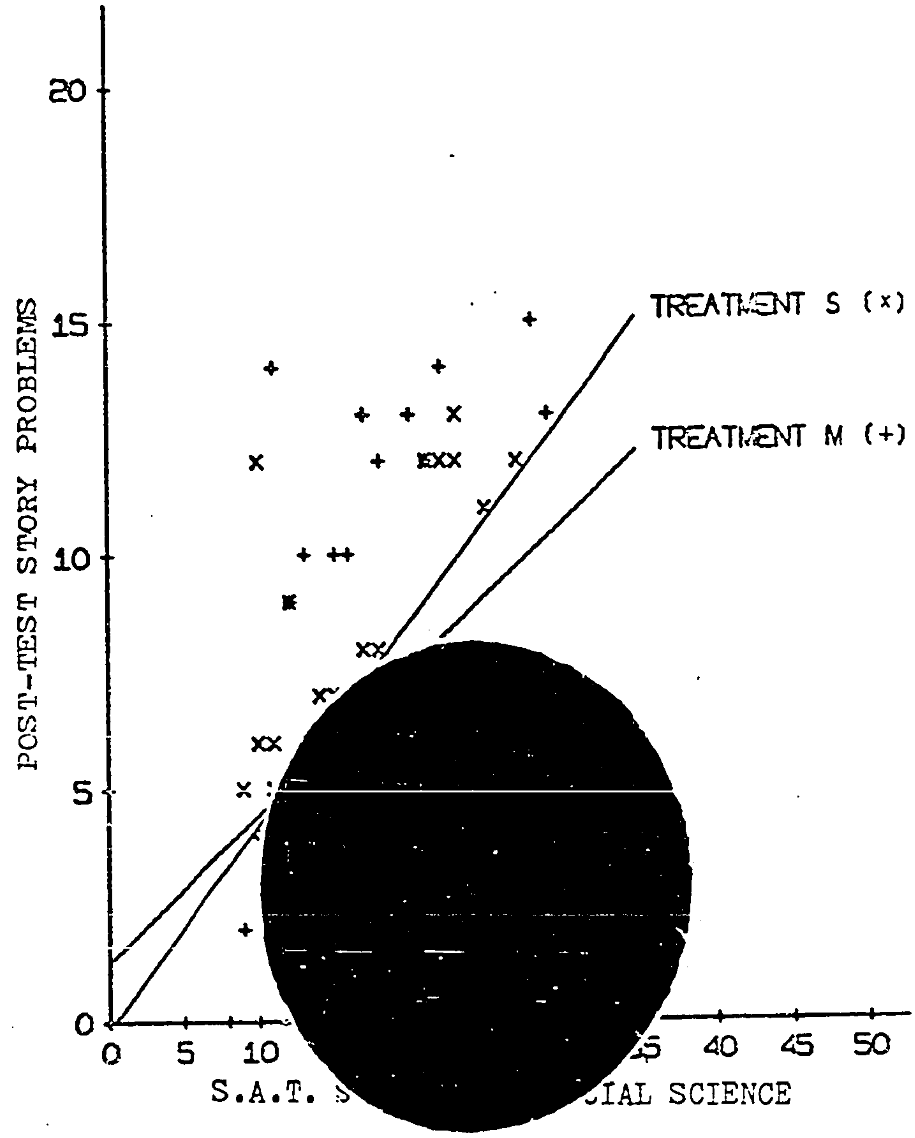
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5366	7.1240	.9624	26	.3089	3.1155	10.6	.6893	
M	.3091	10.5553	.6391	30	.3716	1.7198			.6327

TABLE 14



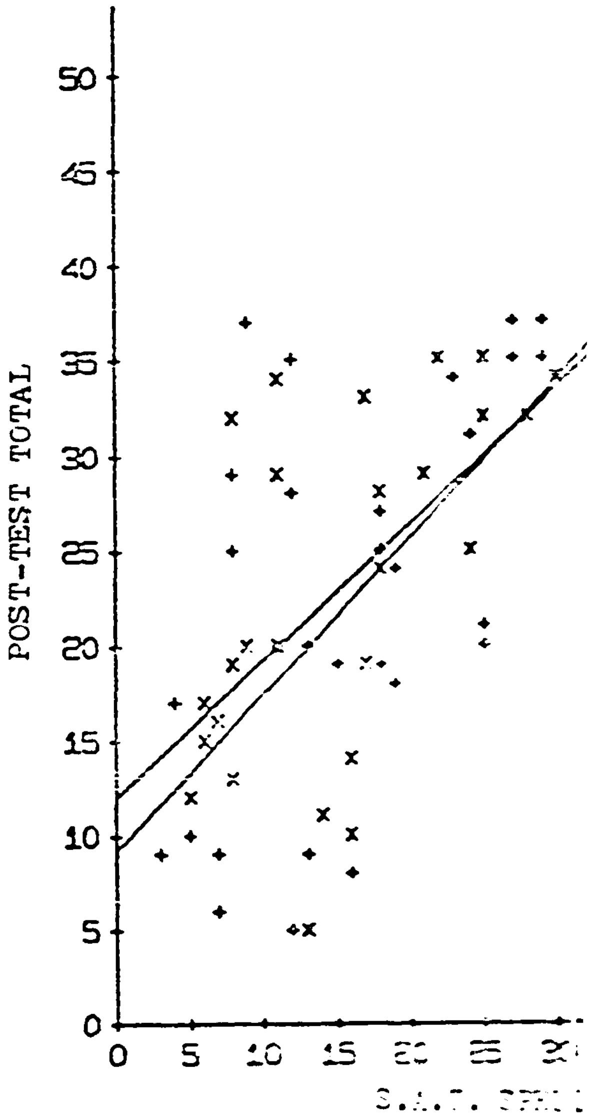
	r	a	b	N	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4109	9.1301	.4376	26	.1982	2.2078	1.1	.693	
M	.2504	9.2617	.3253	30	.2377	1.3685			.3541

TABLE 15



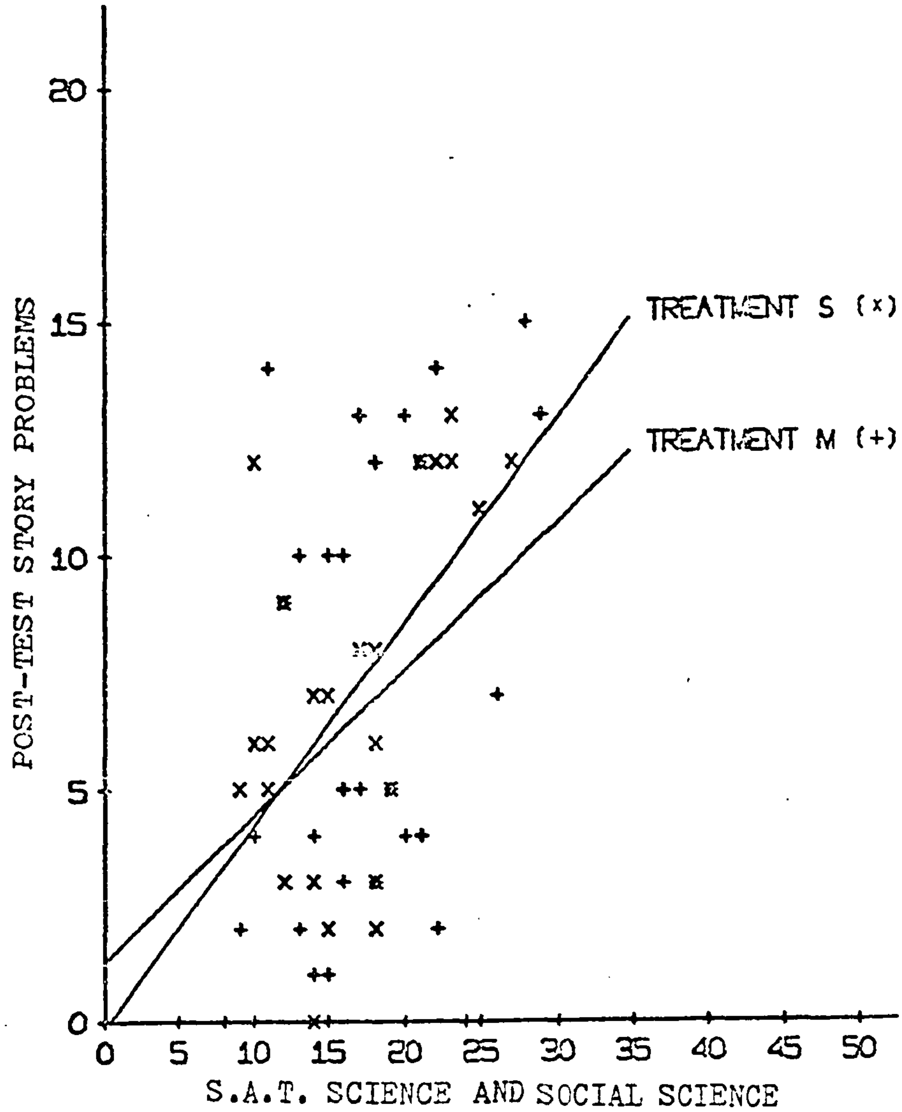
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5599	-.1408	.4354	26	.1136	3.8327	11.7	.492	
M	.3440	1.2936	.3138	30	.1619	1.9382			.5966

TABLE 16



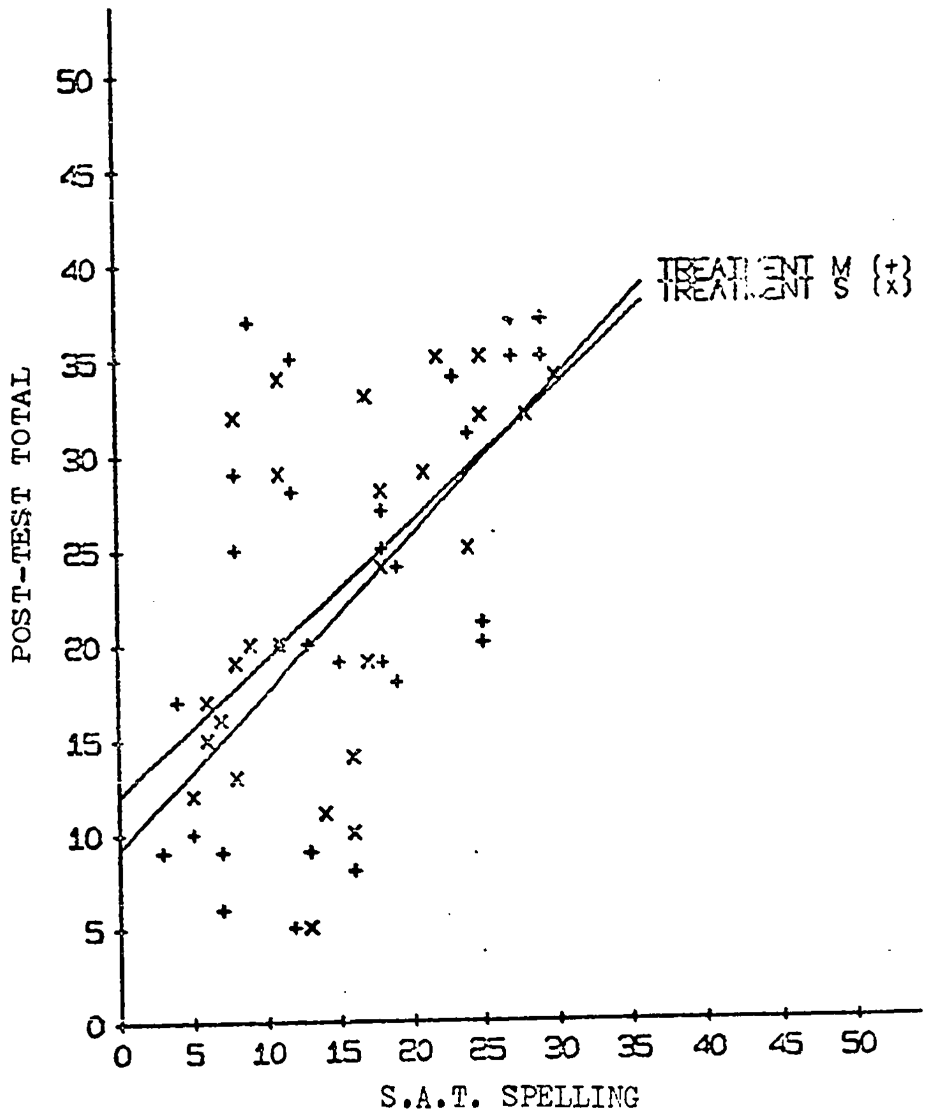
	r	a	b	N	[illegible]	[illegible]
S	.5811	12.003	[illegible]	[illegible]	[illegible]	[illegible]
M	.6216	9.1233	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 15



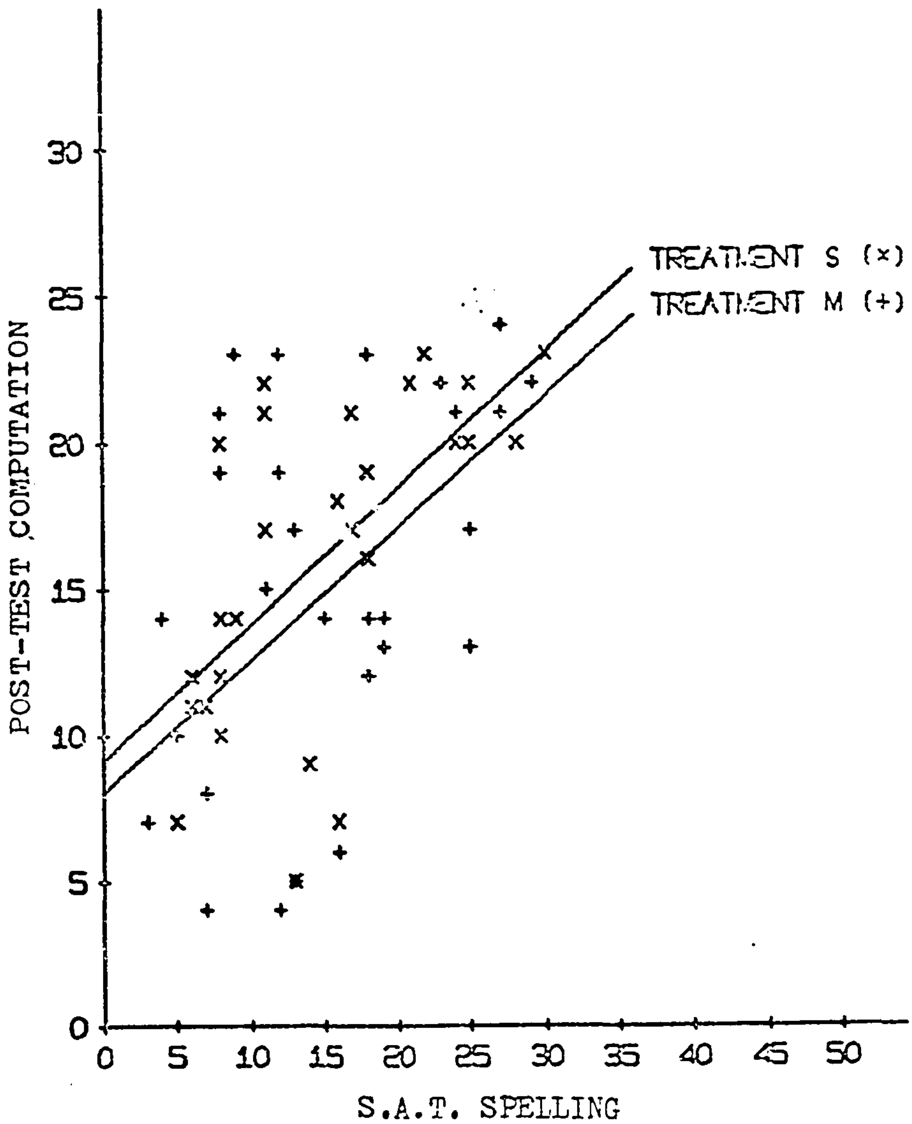
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5599	-.1408	.4354	26	.1136	3.8327	11.7	.492	
M	.3440	1.2936	.3138	30	.1619	1.9382			.5966

TABLE 16



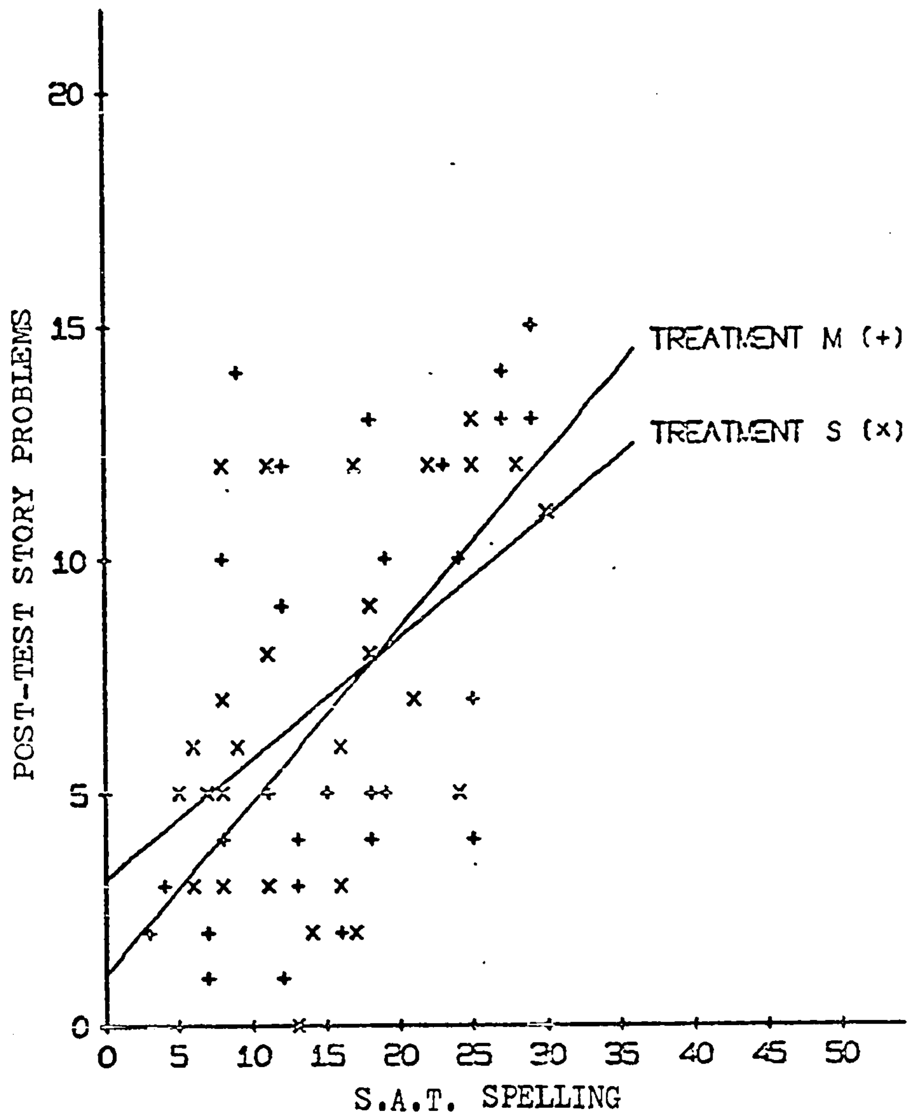
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5811	12.003	.7164	27	.2007	3.5695	27.0	1.049	
M	.6216	9.1233	.8230	30	.1960	4.1989			.3803

TABLE 17



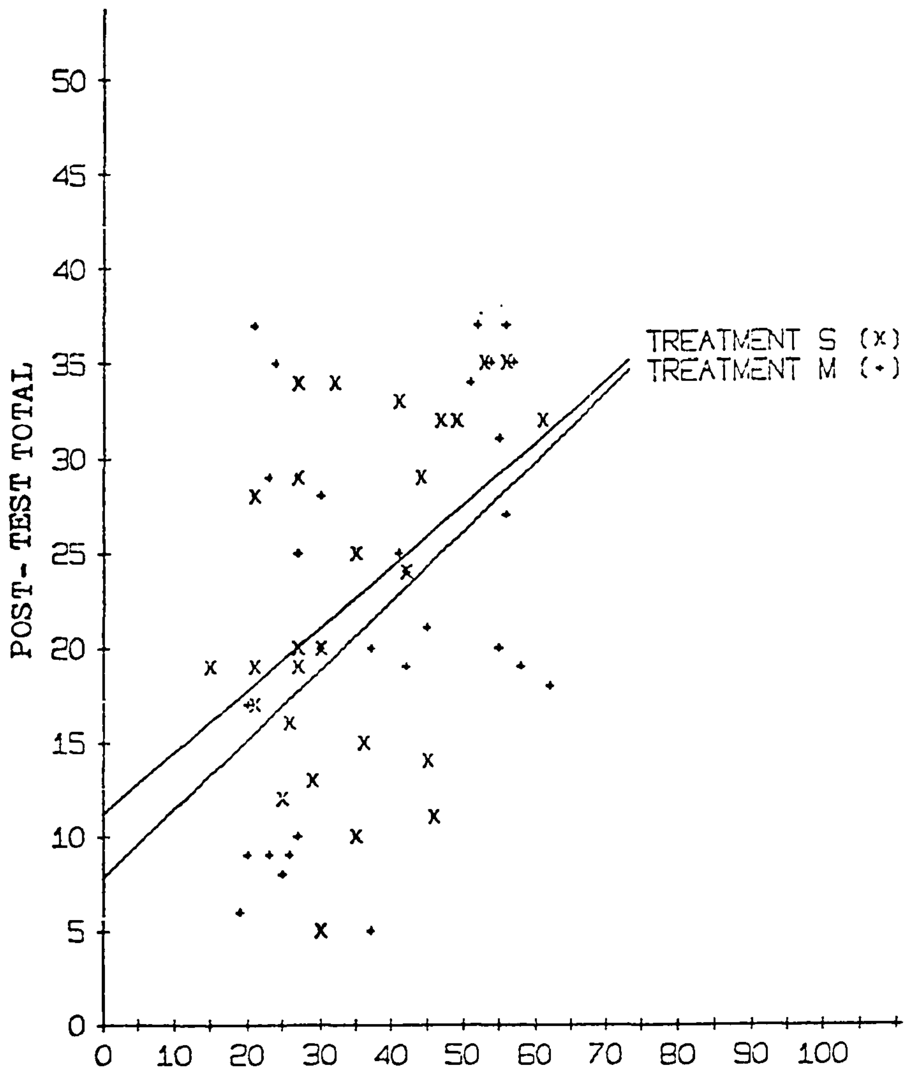
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	$\underline{S_b}$	t for $\underline{b = 0}$	X at Cross-$\underline{\text{over}}$	F for $\underline{S_s^2/S_m^2}$	t for $\underline{b_s=b_m}$
S	.6192	9.0953	.4662	27	.1182	3.9441	-76.9	.8033	
M	.5437	8.0253	.4523	30	.1319	3.4291			.0782

TABLE 18



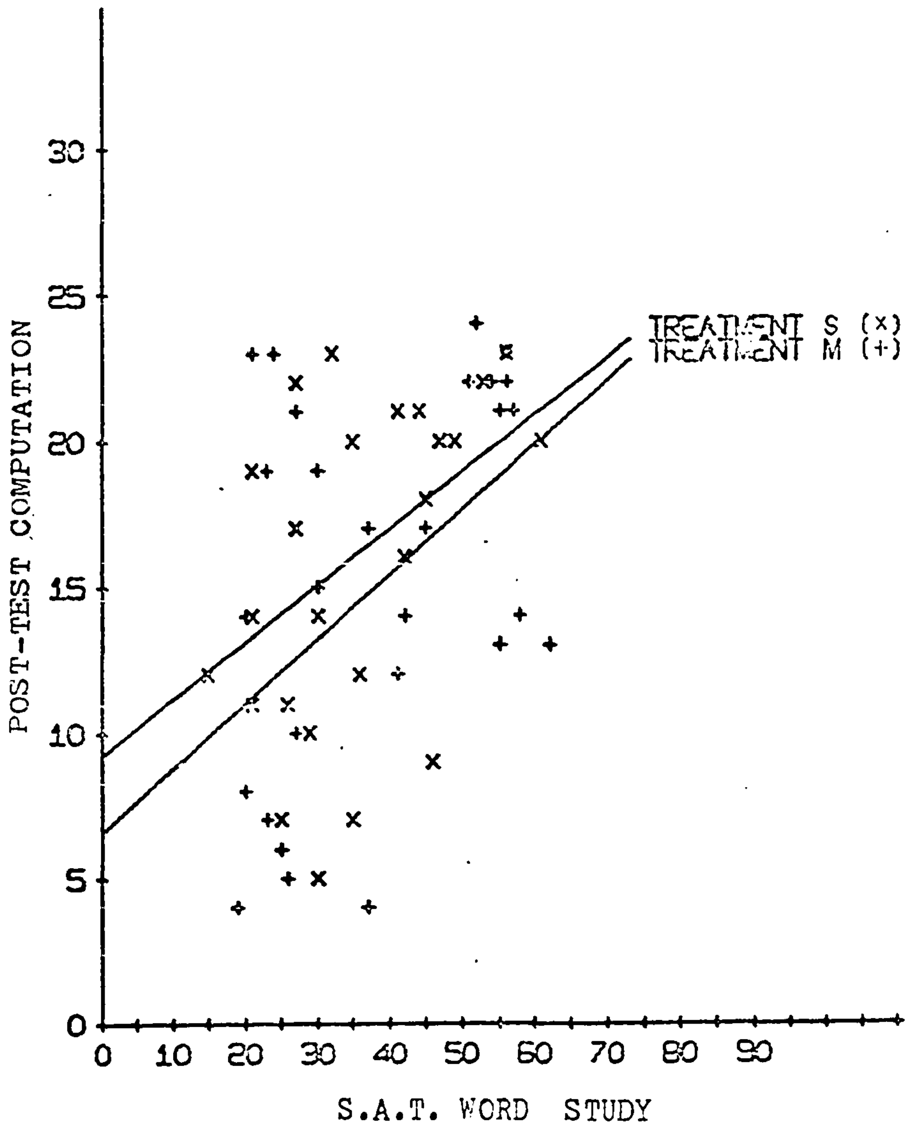
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4888	3.1571	.2581	27	.0921	2.8023	18.3	1.777	
M	.6347	1.0980	.3707	30	.0853	4.3458			.8996

TABLE 19



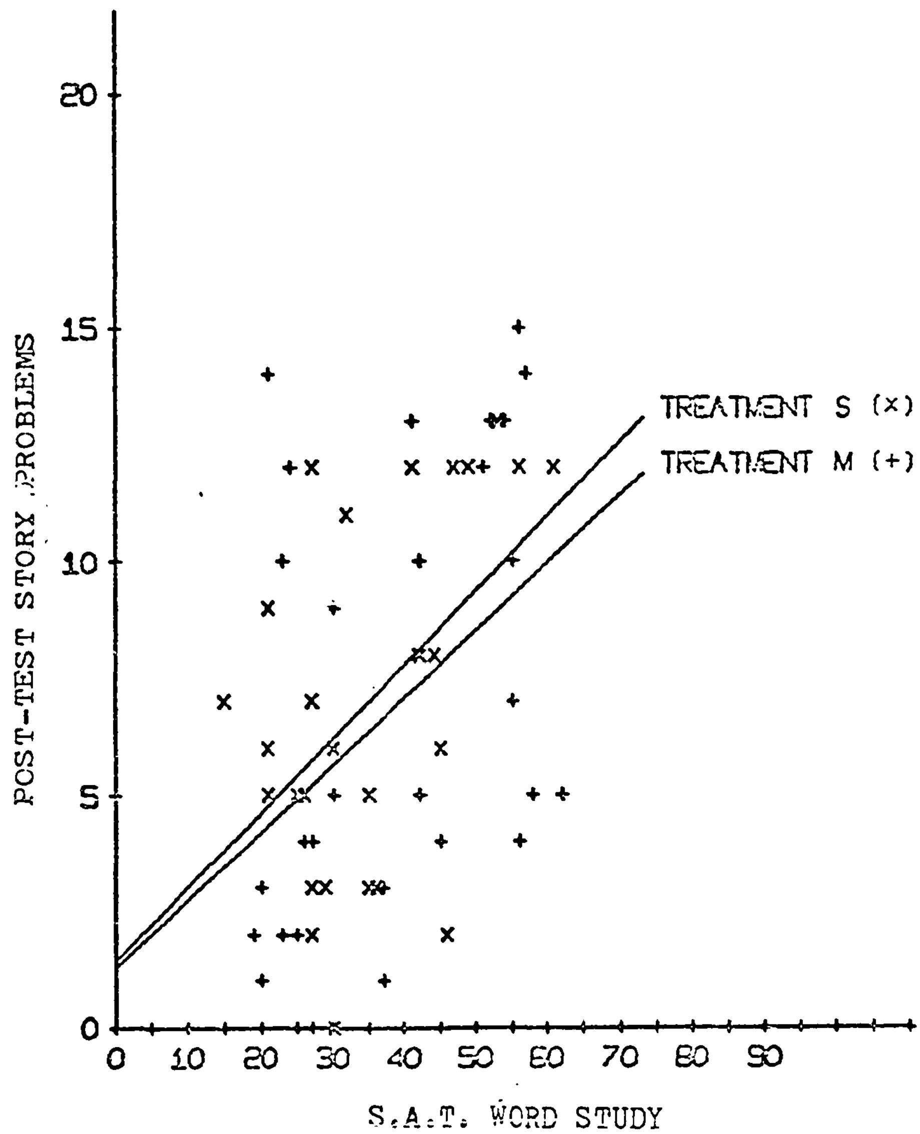
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4296	11.2157	.3261	27	.1371	2.3785	87.0	1.289	
M	.4965	7.8681	.3654	30	.1207	3.0273			-.2159

TABLE 20



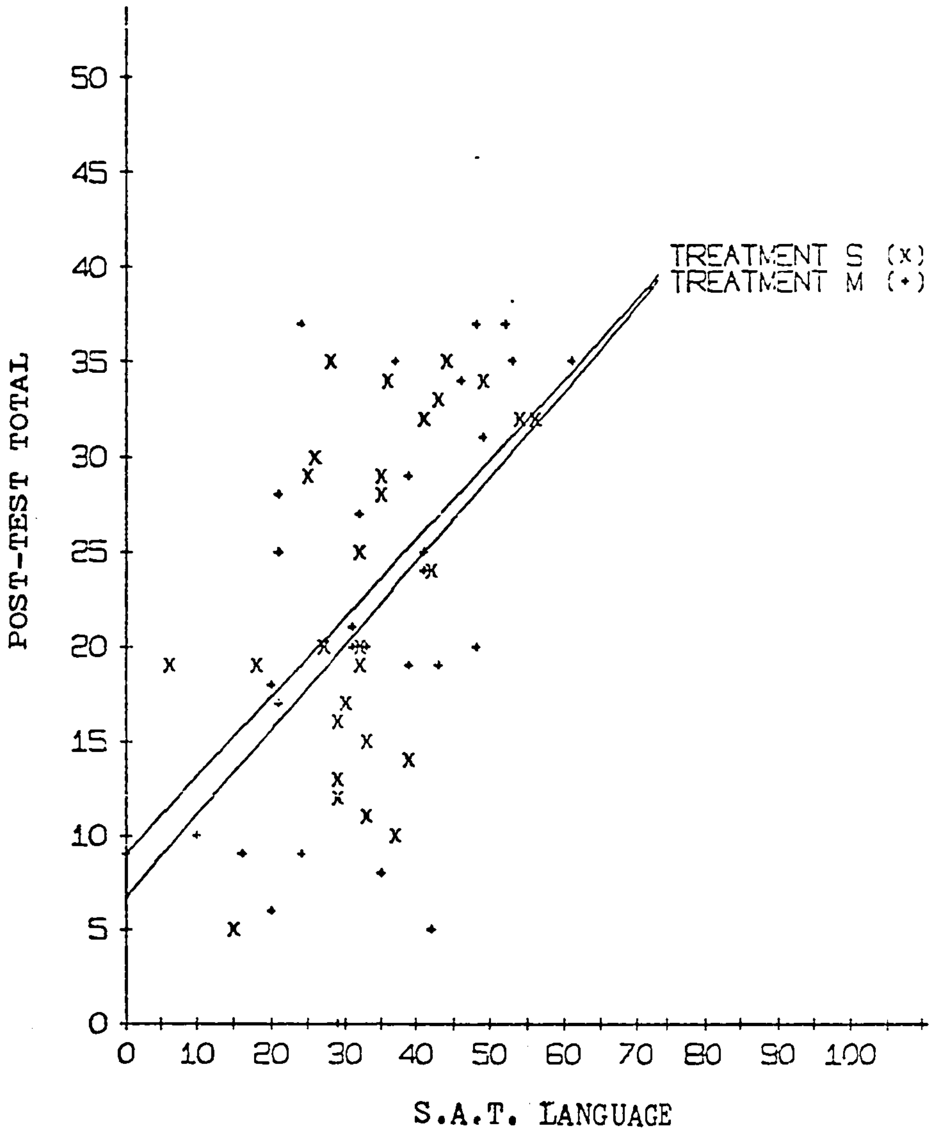
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4186	9.2228	.1941	27	.0842	2.3052	97.9	1.344	
M	.4783	6.5682	.2212	30	.0767	2.8839			-.2300

TABLE 21



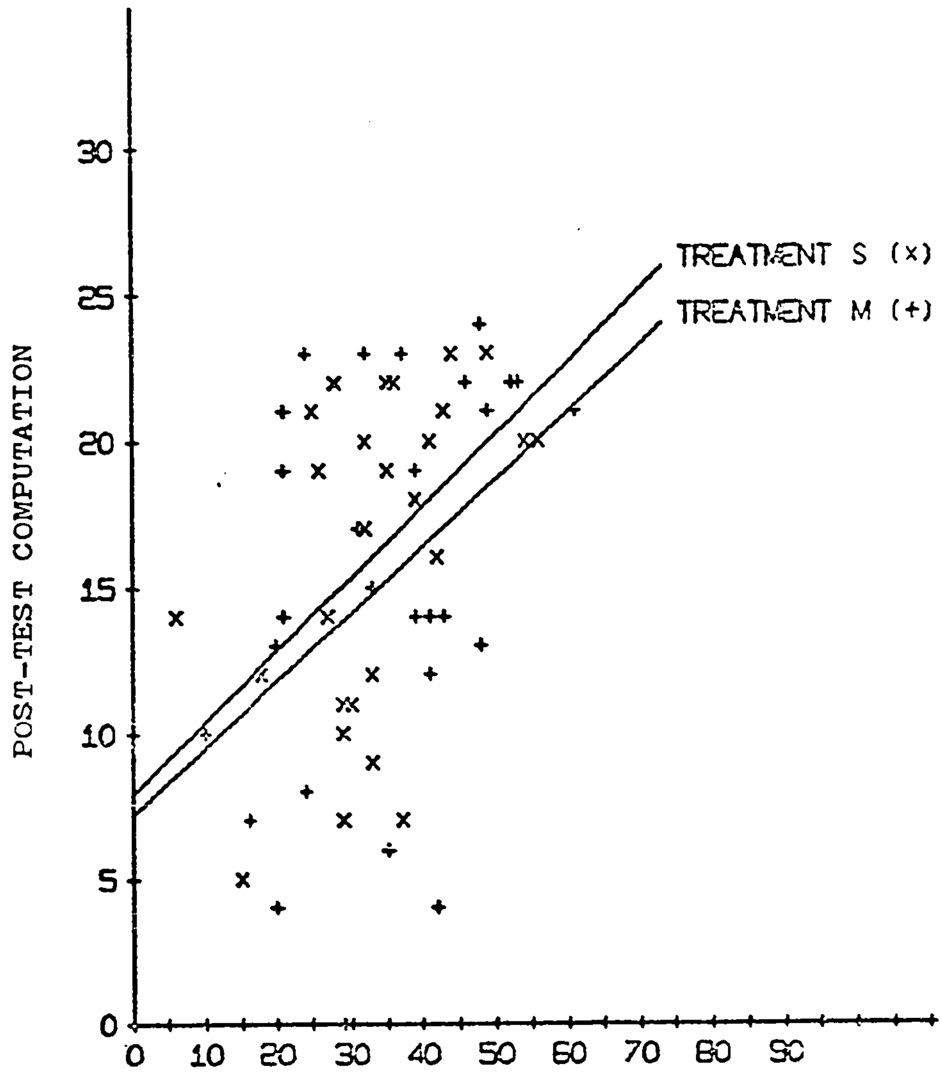
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4886	1.4221	.1589	27	.0567	2.8024	-8.3	1.066	
M	.4442	1.2999	.1442	30	.0550	2.6218			.1862

TABLE 22



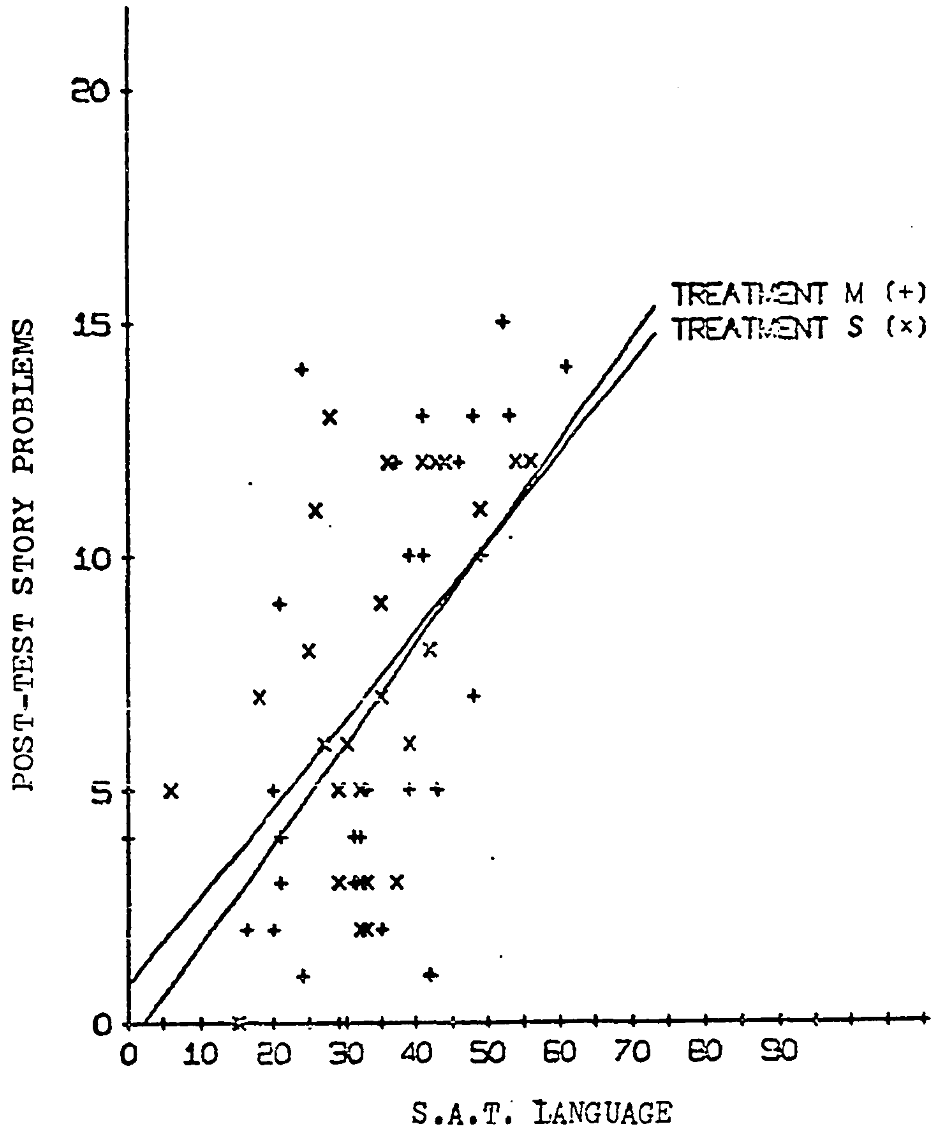
	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5035	8.9885	.4175	28	.1405	2.9715	83.7	1.331	
M	.5678	6.7038	.4448	30	.1219	3.6488			-.1472

TABLE 23



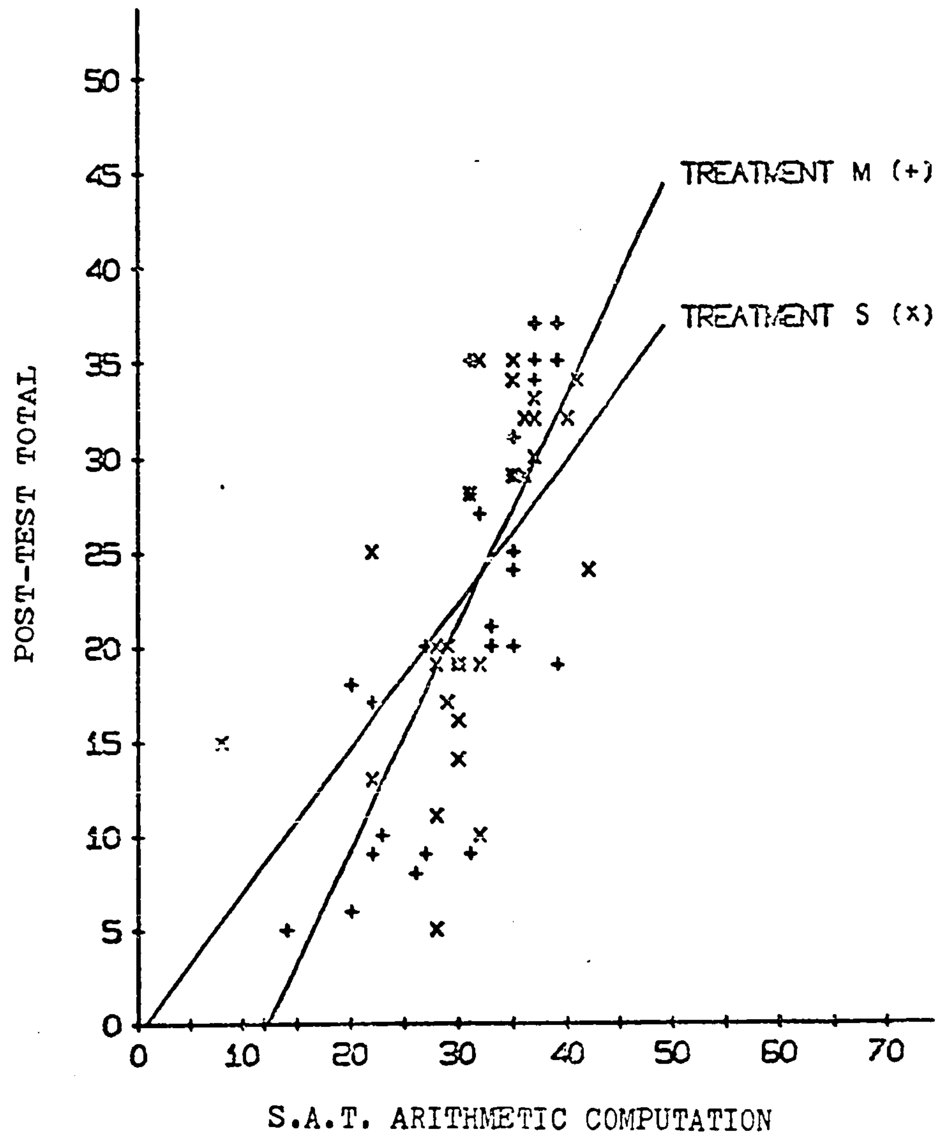
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4891	7.9289	.2460	28	.0860	2.8604	-44.4	1.104	
M	.4664	7.1952	.2295	30	.0823	2.7385			.1364

TABLE 24



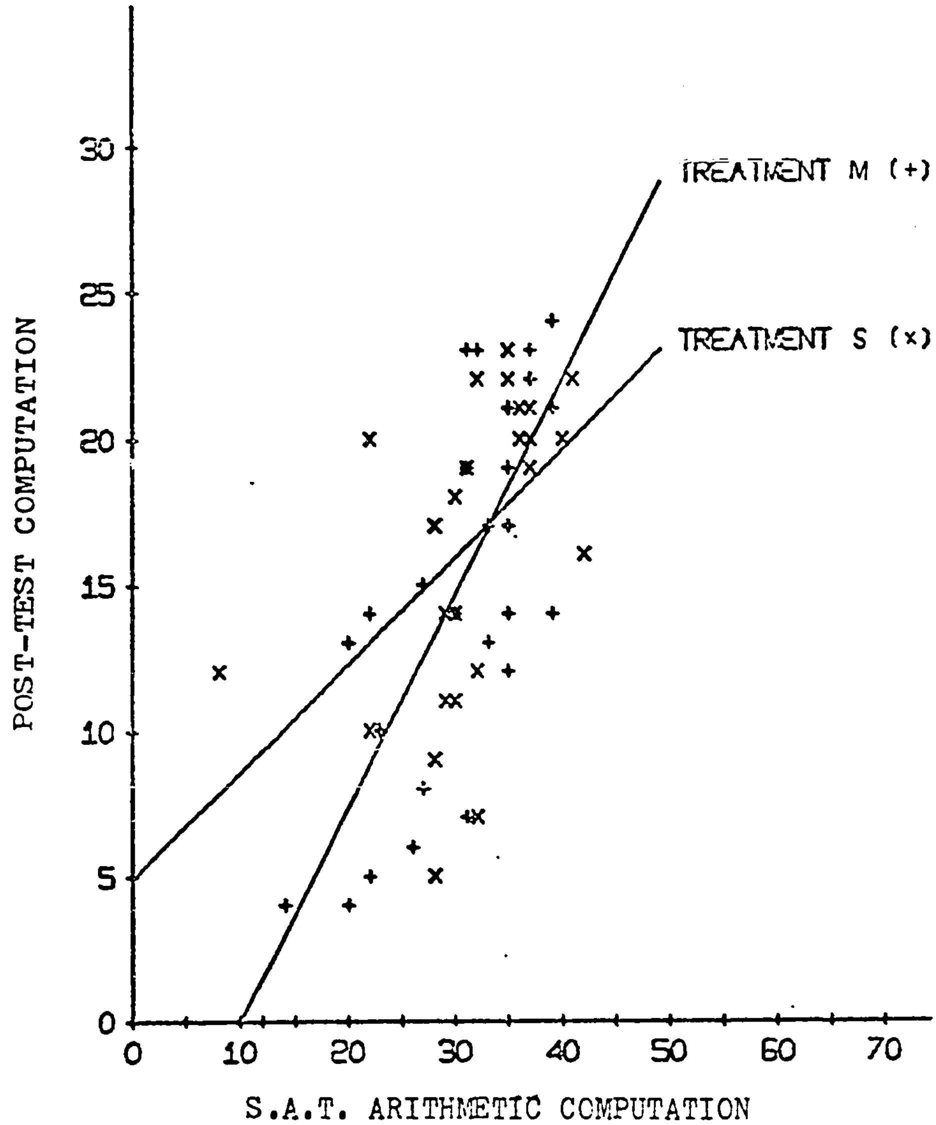
	r	a	b	N	S_b	t for $b = 0$	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5289	.8258	.1892	28	.0595	3.1798	50.6	1.346	
M	.6228	-.4914	.2152	30	.0511	4.2113			-.3271

TABLE 25



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5913	-.5972	.7601	27	.2074	3.6648	32.11	1.660	
M	.8158	-14.8125	1.2028	30	.1611	7.4661			-1.6975

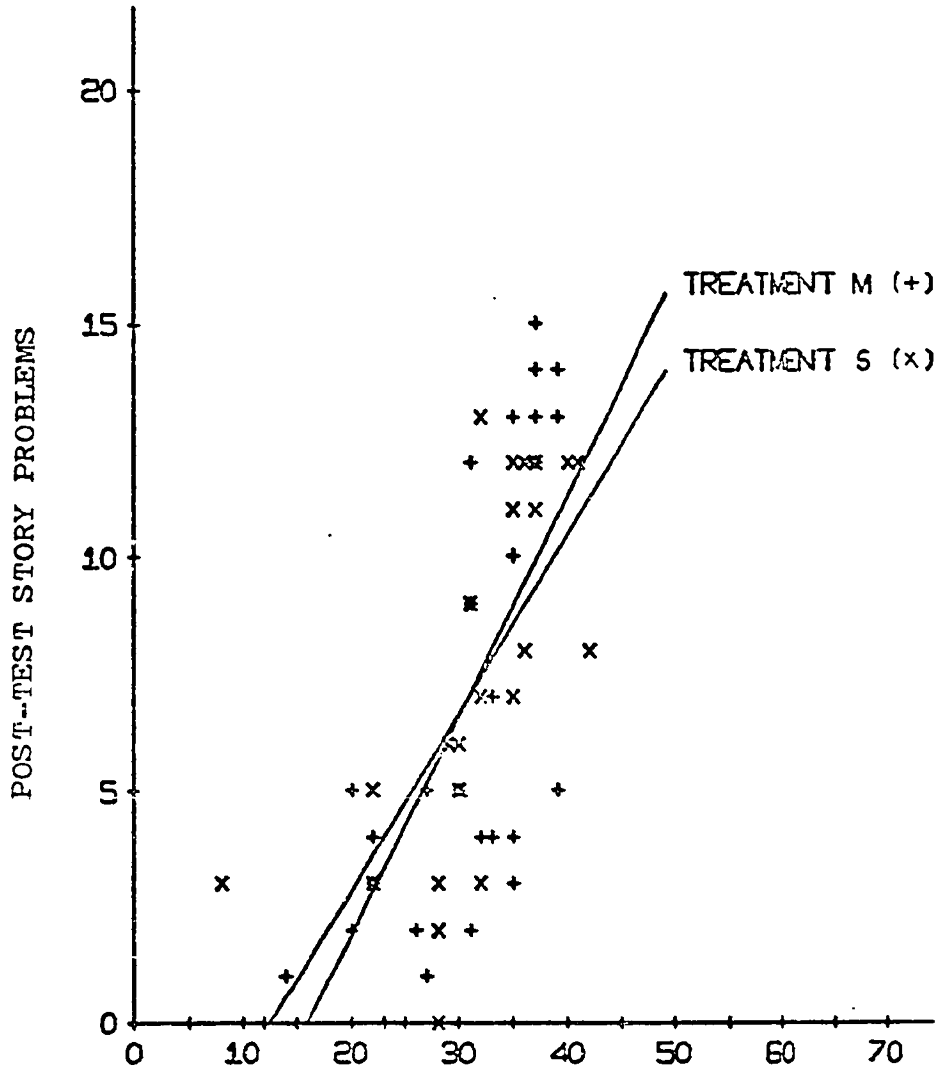
TABLE 26



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4846	4.9112	.3673	27	.1327	2.7679	33.40	1.543	
M	.7923	-7.3366	.7339	30	.1068	6.8717			-2.1656[a]

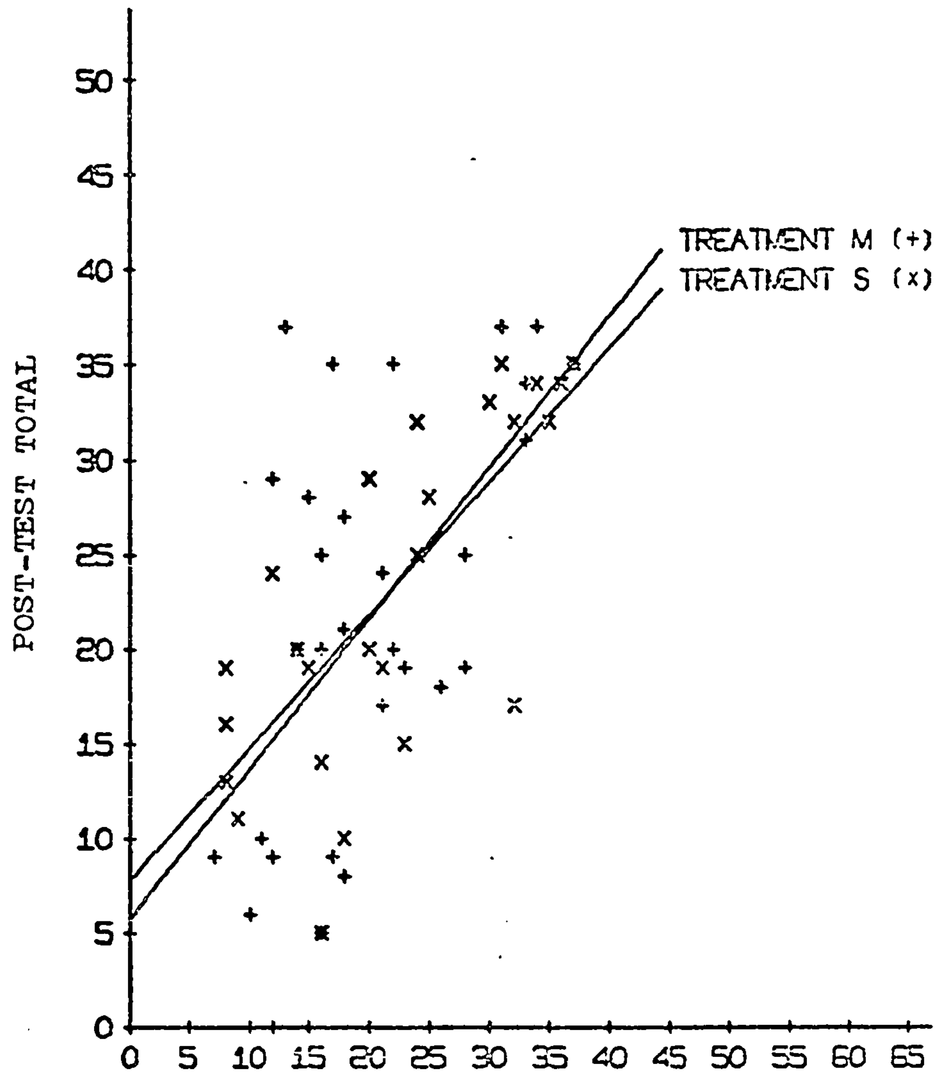
[a]Significant at (P = .05)

TABLE 27



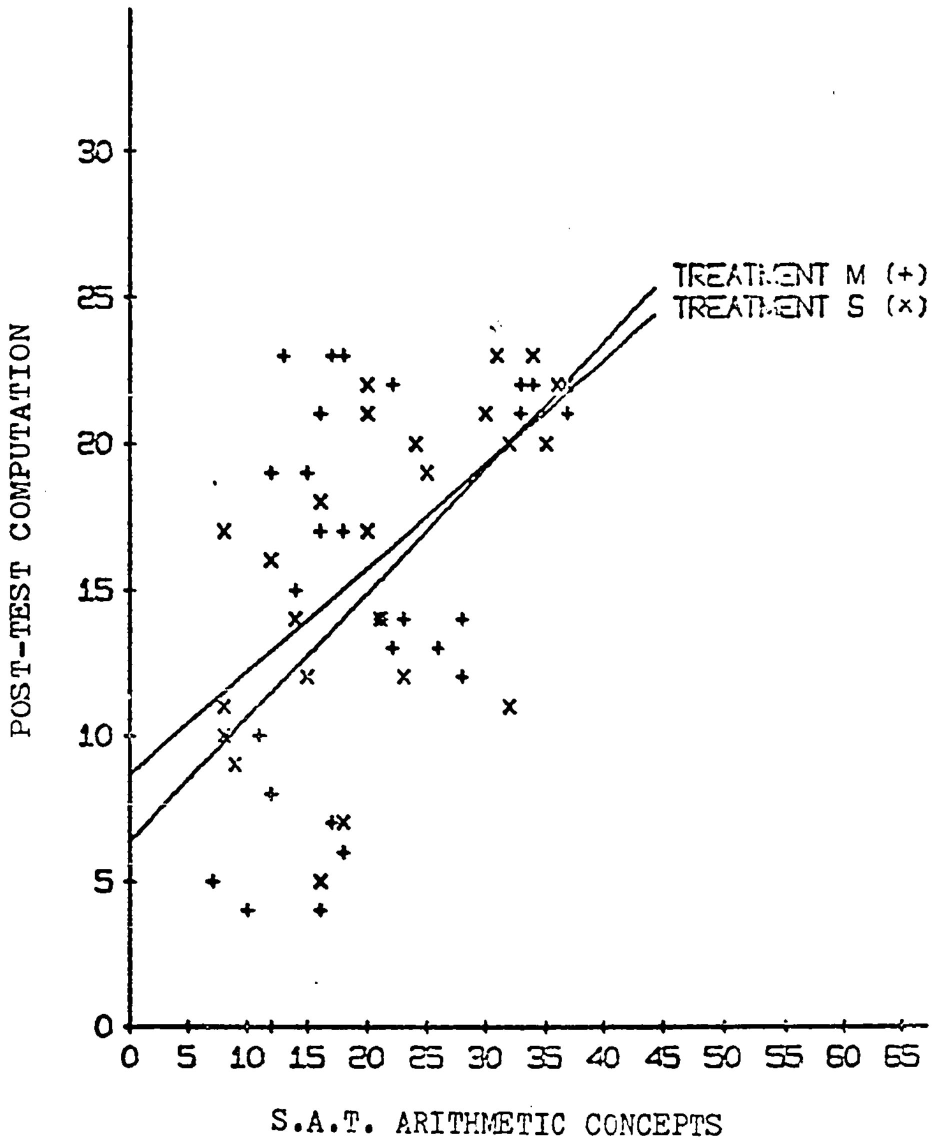
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.6705	-4.7627	.3807	27	.0843	4.5160	30.7	.986	
M	.7210	-7.4758	.4690	30	.0852	5.5046			-.7371

TABLE 28



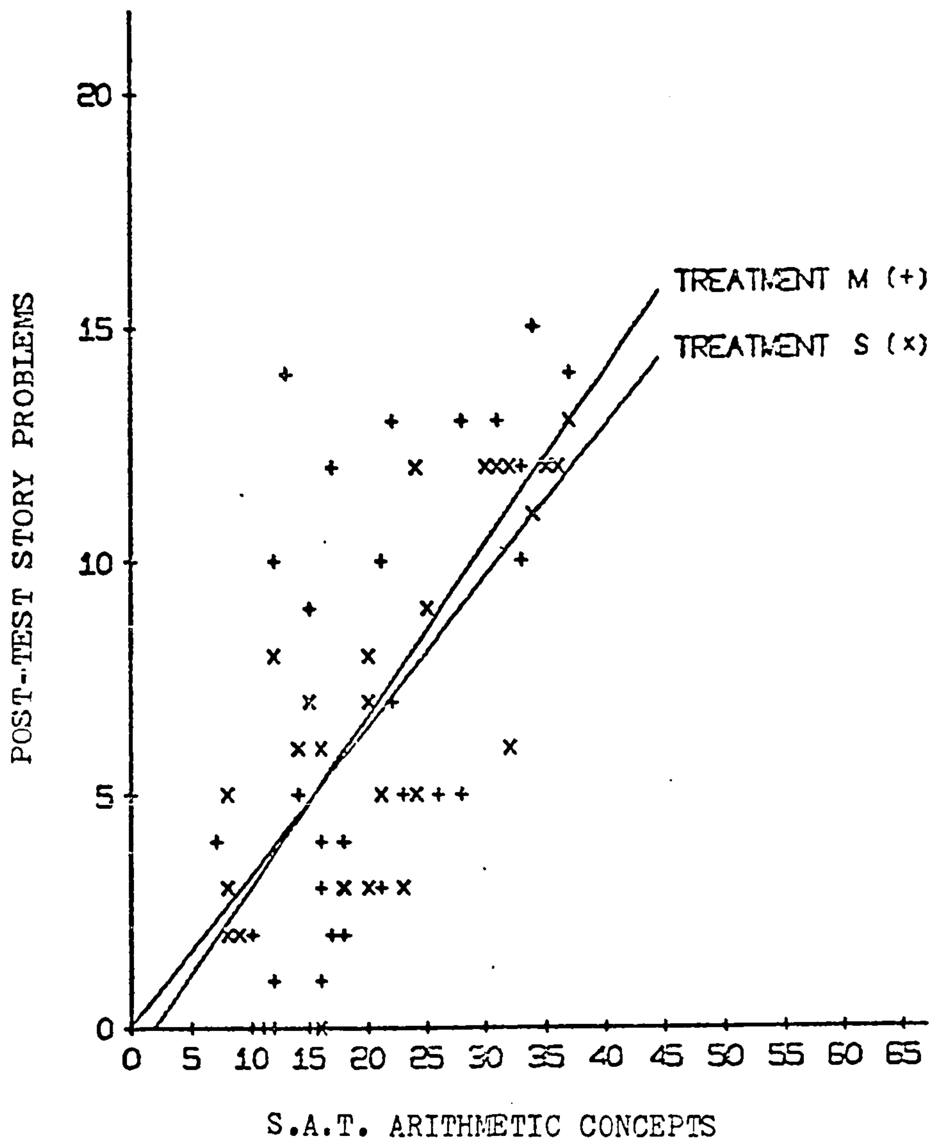
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.7297	7.7328	.7024	26	.1343	5.2308	21.8	.406	
M	.5806	5.6952	.7956	30	.2107	3.7759			-.3673

TABLE 29



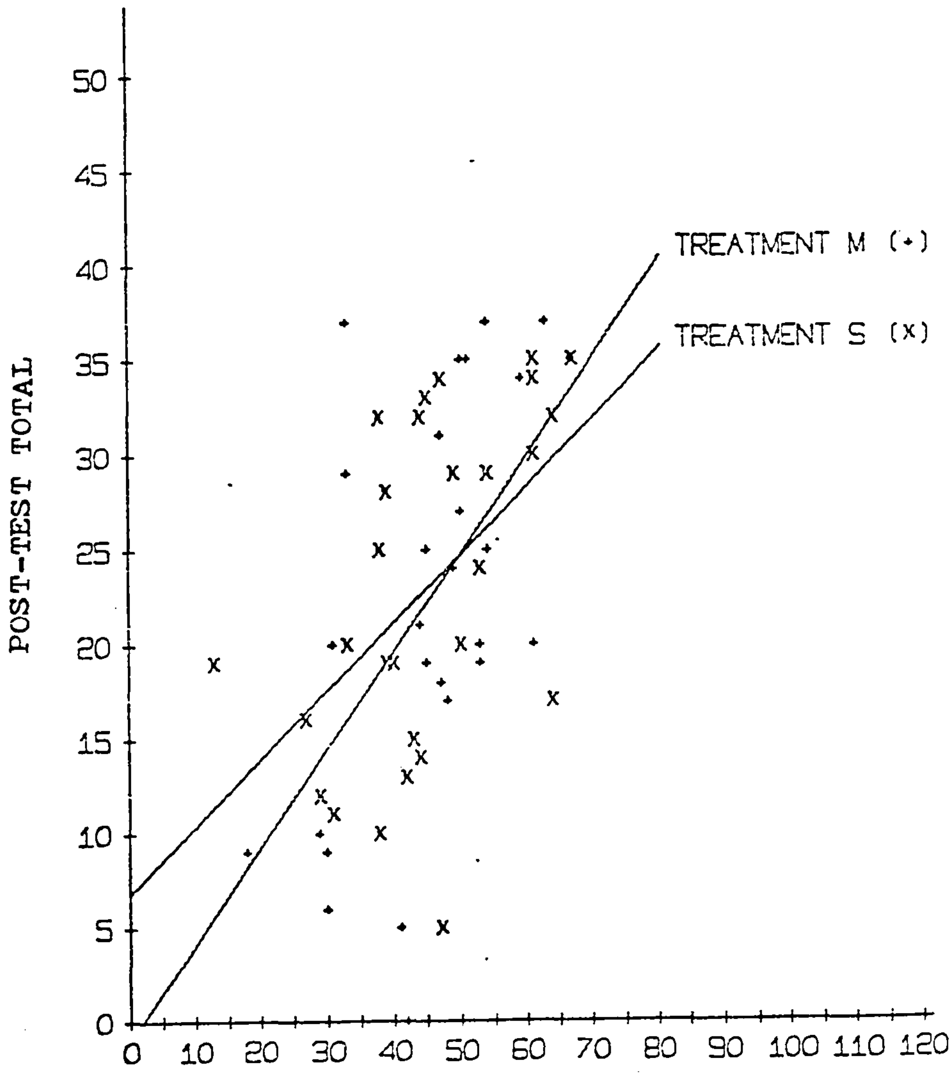
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.6203	8.6381	.3545	26	.0915	3.8743	33.5	.417	
M	.4958	6.3547	.4226	30	.1412	2.9929			-.3989

TABLE 30



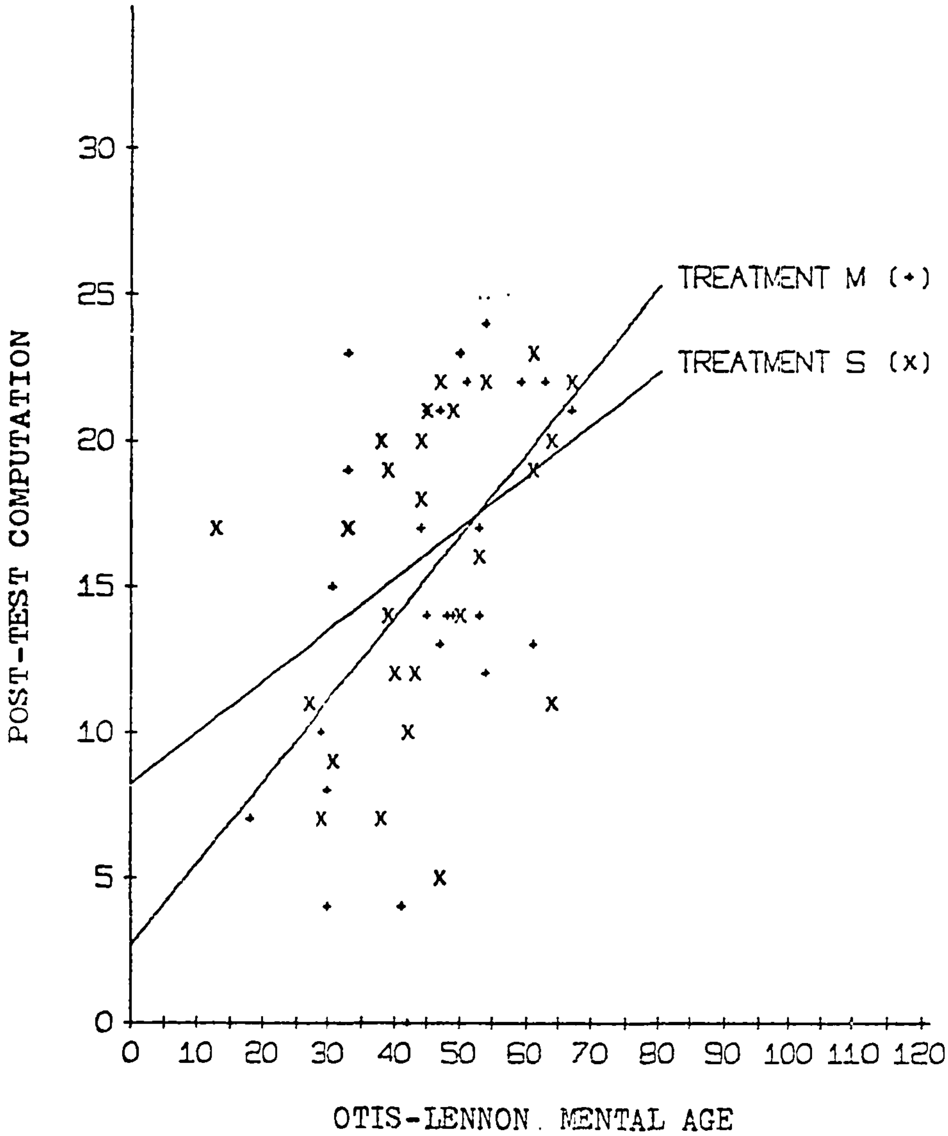
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.7603	.0686	.3208	26	.0559	5.7388	15.1	.3827	
M	.6108	-.6595	.3690	30	.0904	4.0818			-.4466

TABLE 31



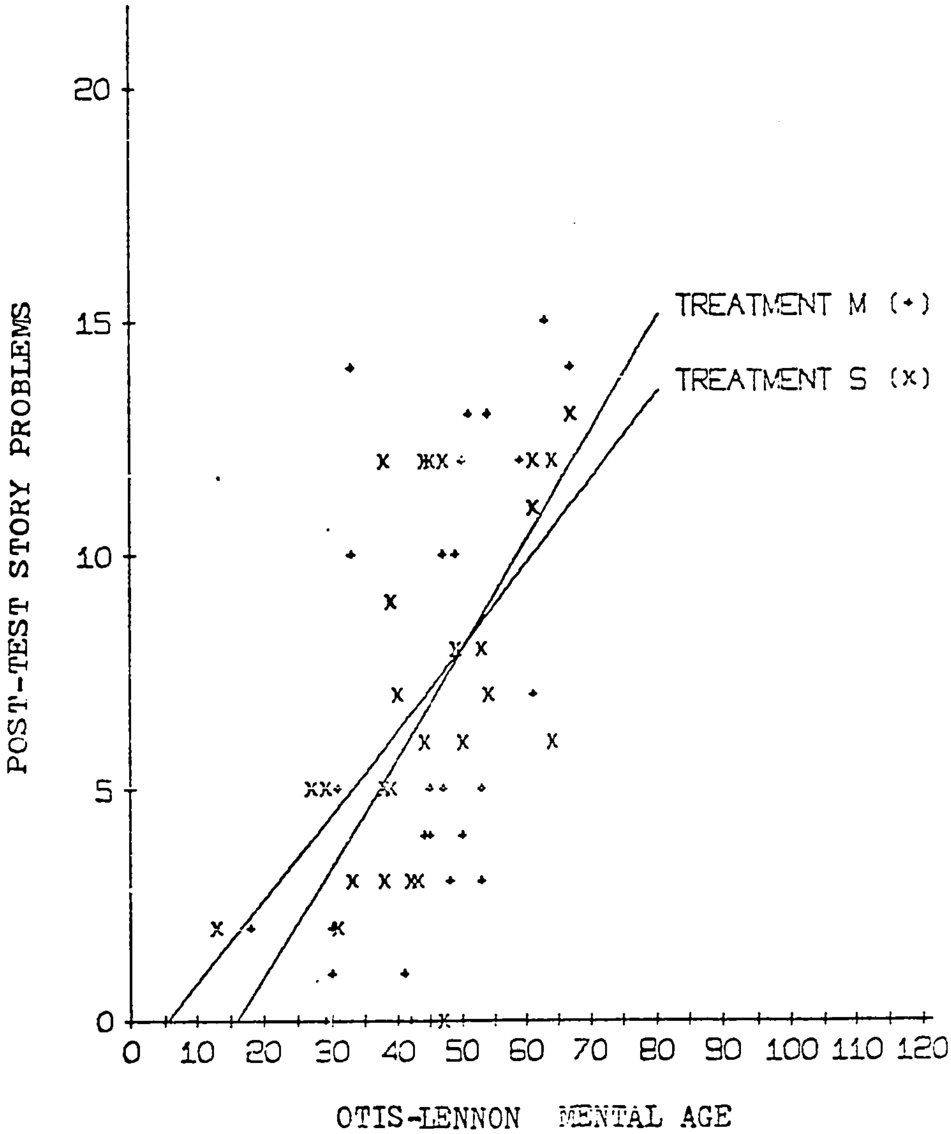
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5076	6.7927	.3583	28	.1193	3.0033	49.9	.664	
M	.5570	-1.0221	.5147	27	.1535	3.3530			-.8066

TABLE 32



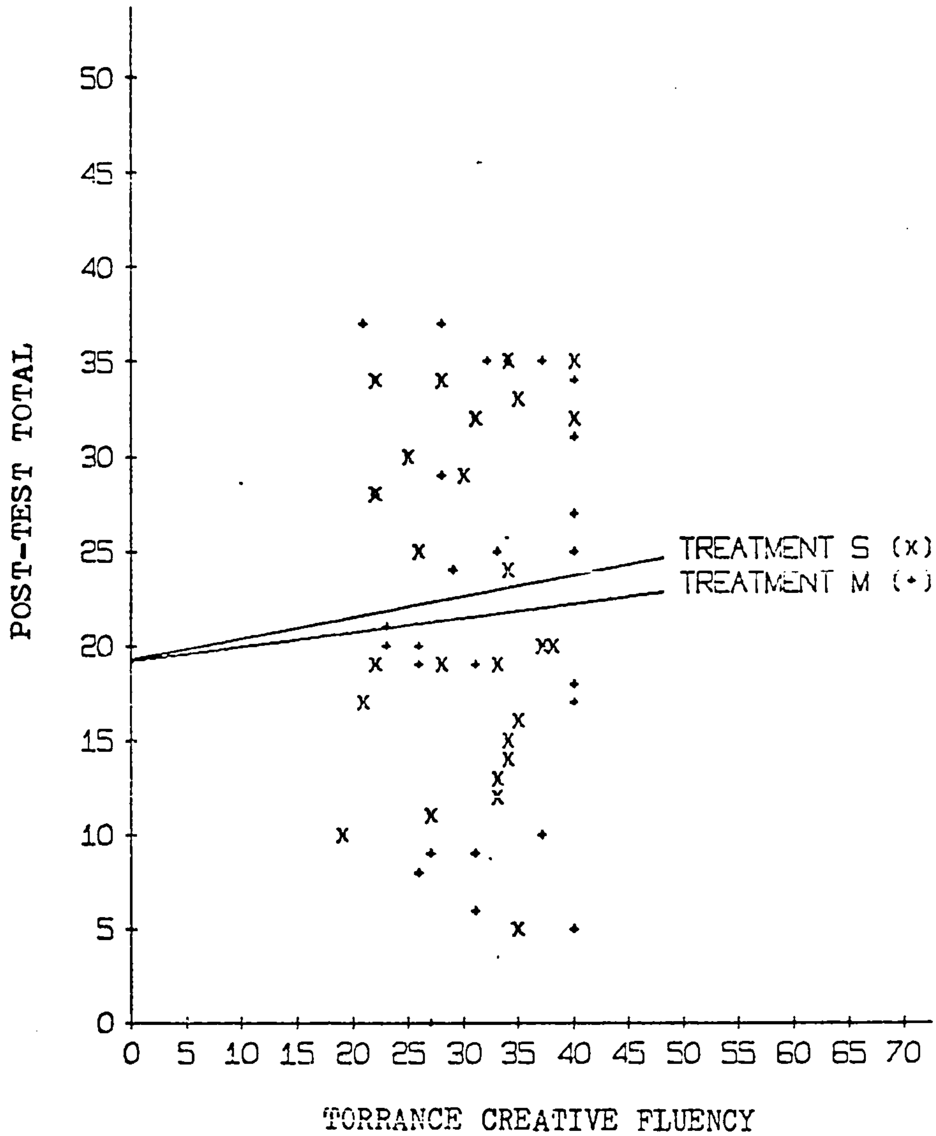
	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.4103	8.2313	.1715	28	.0766	2.2389	50.7	.591	
M	.4923	2.6918	.2806	27	.0992	2.8286			-.8730

TABLE 33



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for S_s^2/S_m^2	t for $b_s=b_m$
S	.5918	-.9747	.1802	28	.0481	3.7463	50.8	.465	
M	.5551	-3.7139	.2341	27	.0702	3.3347			-.6361

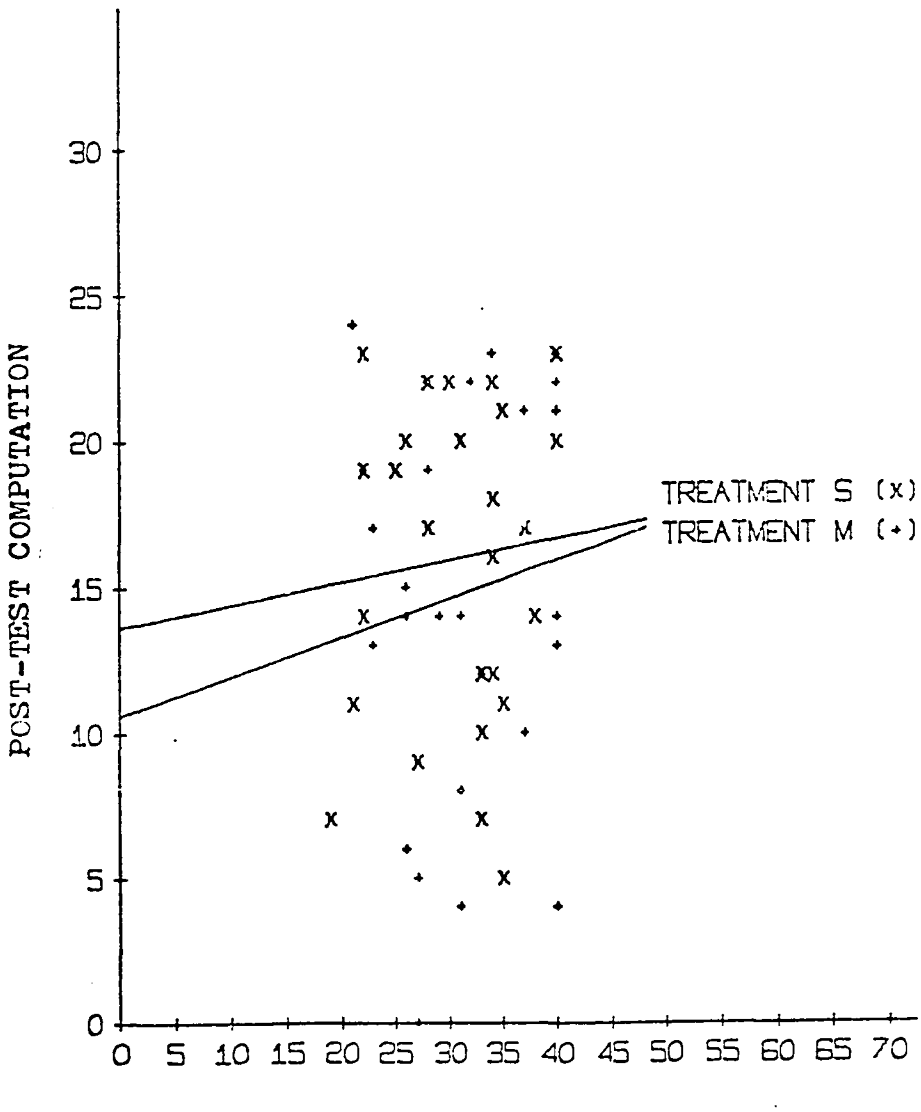
TABLE 34



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.0726	19.3190	.1106	27	.3038	.3605	-3.4		
M	.0449	19.1994	.0758	27	.3372	.2248			

[a, b]Reported only if b_s or b_m is significantly different from zero.

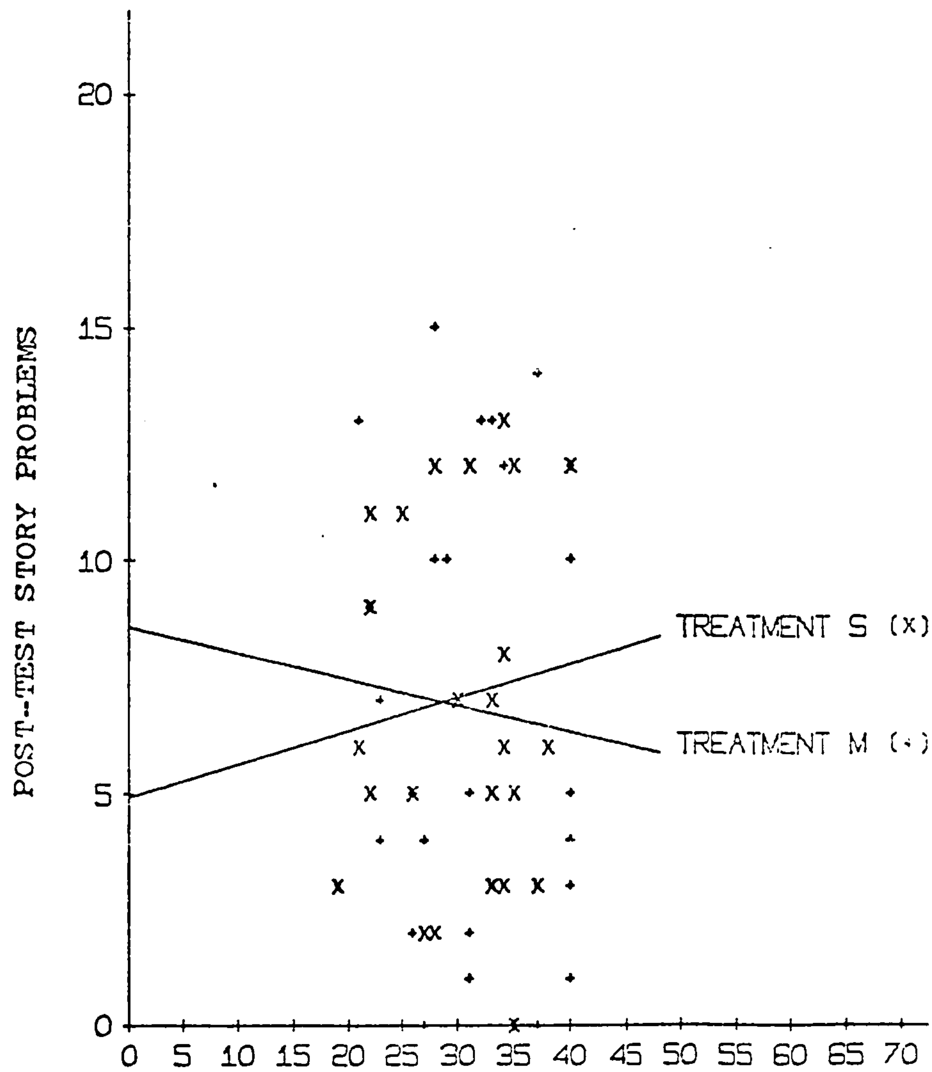
TABLE 35



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.0823	13.6505	.0755	27	.1829	.4127	52.8		
M	.1240	10.6242	.1328	27	.2126	.6246			

[a, b] Reported only if b_s or b_m is significantly different from zero.

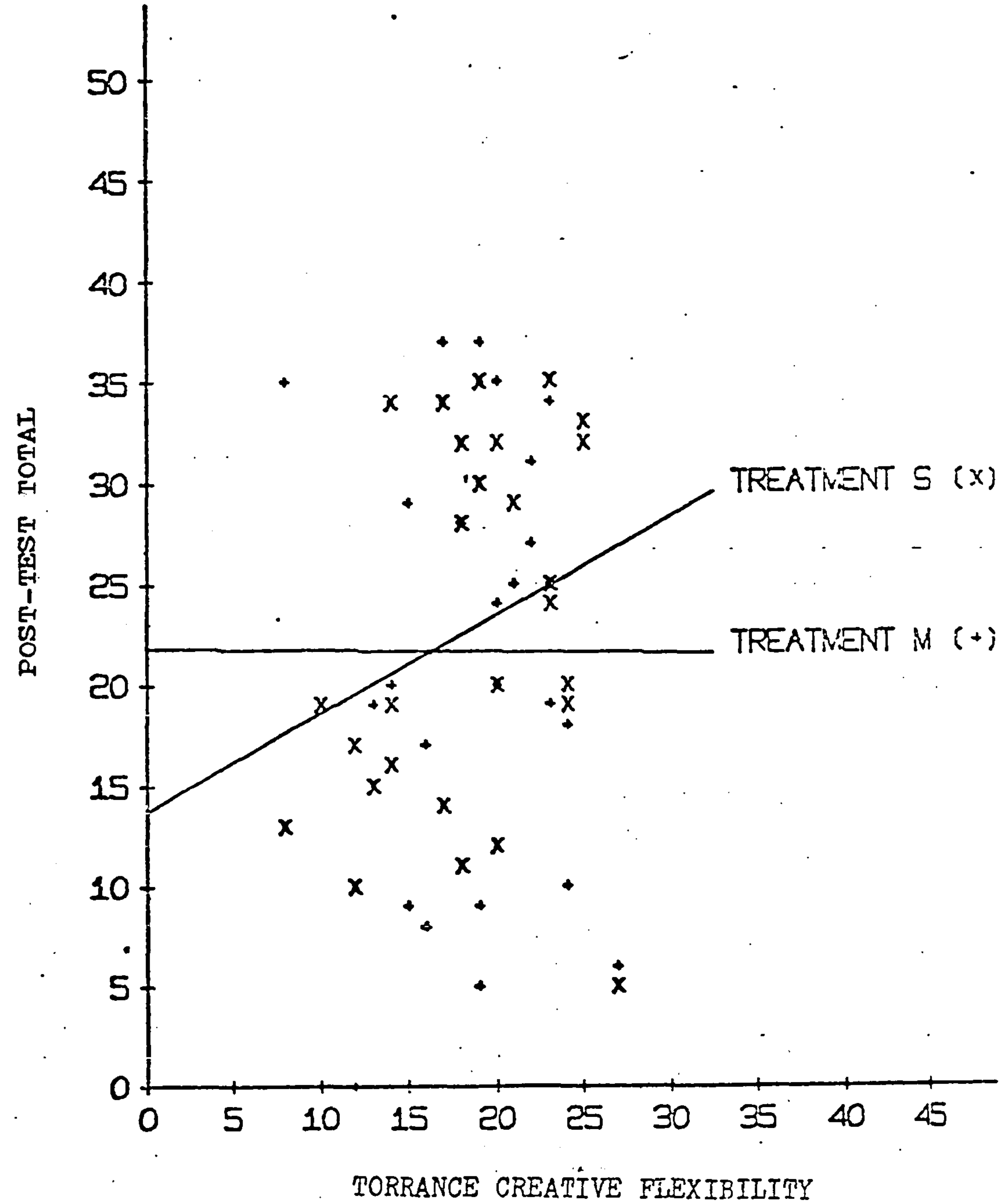
TABLE 36



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.1082	4.9156	.0717	27	.1317	.5441	28.4		
M	-.0772	8.5751	-.0570	27	.1470	-.3877			

[a, b]Reported only if b_s or b_m is significantly different from zero.

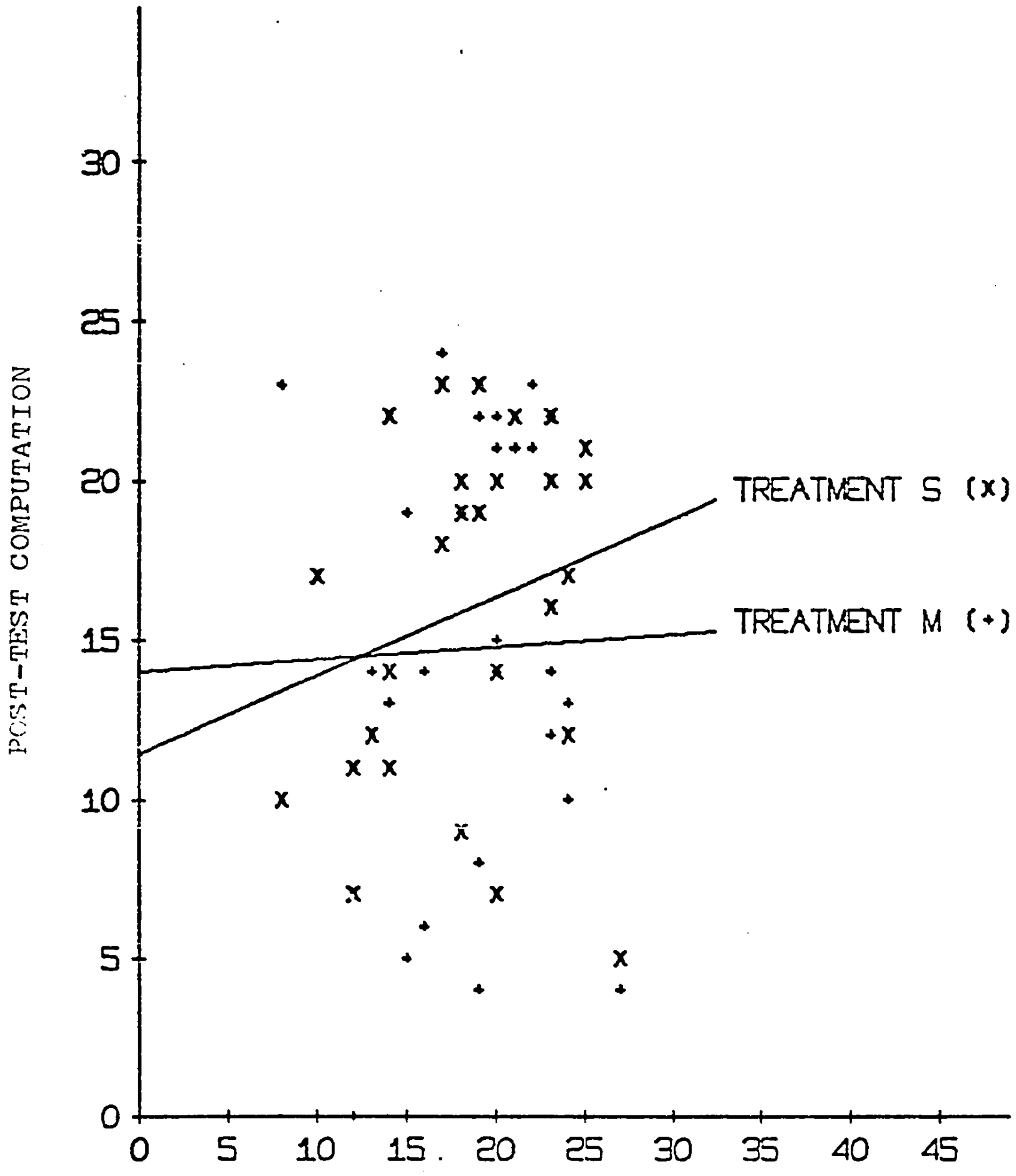
TABLE 37



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2692	13.7288	.4866	27	.3482	1.3974	16.246		
M	-.0029	21.7562	-.0075	26	.5210	-.1439			

a, b Reported only if b_s or b_m is significantly different from zero.

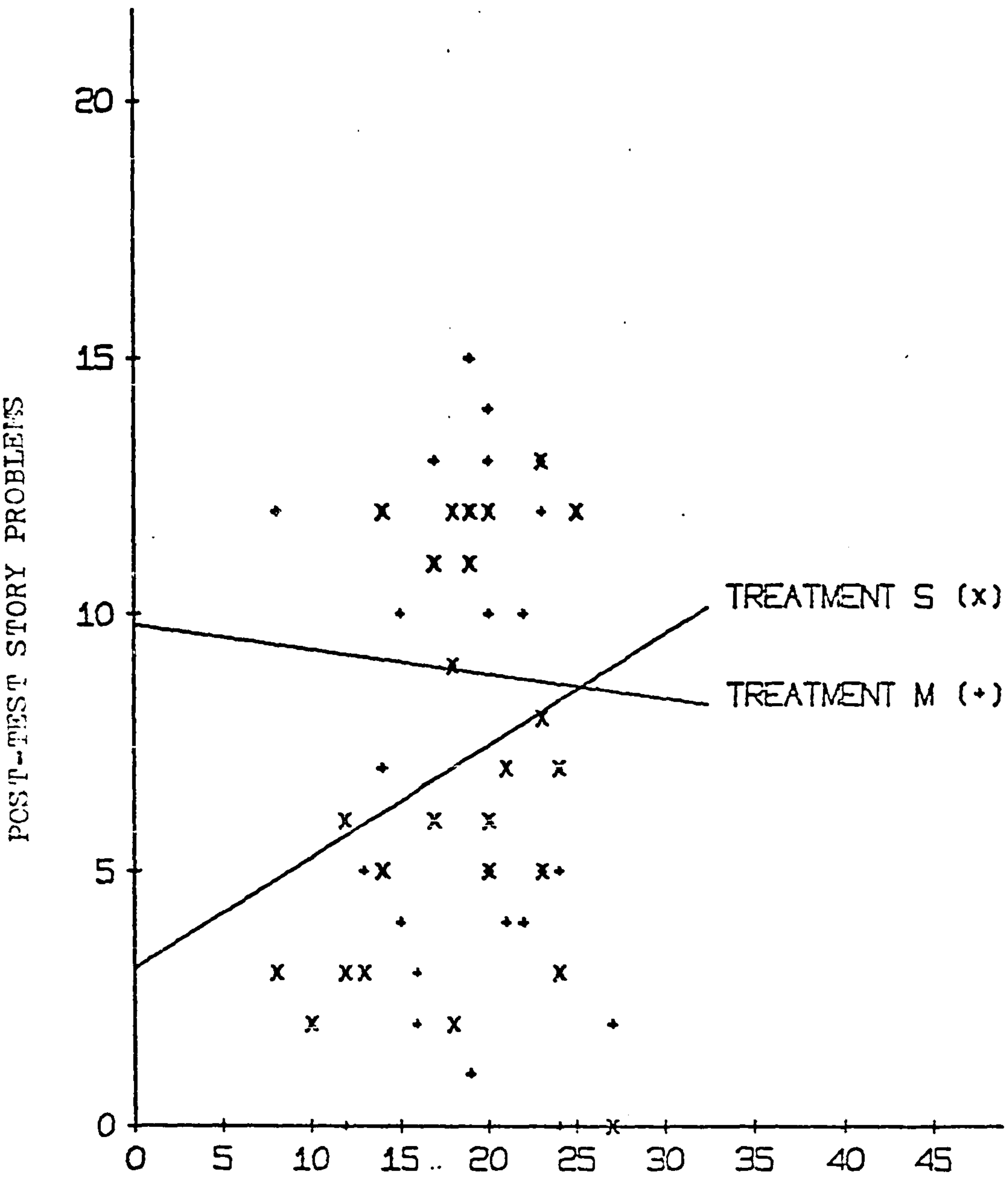
TABLE 38



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2264	11.4145	.2466	27	.2122	1.1621	12.40		
M	.0247	13.9774	.0400	26	.3299	.1212			

[a, b]Reported only if b_s or b_m is significantly different from zero.

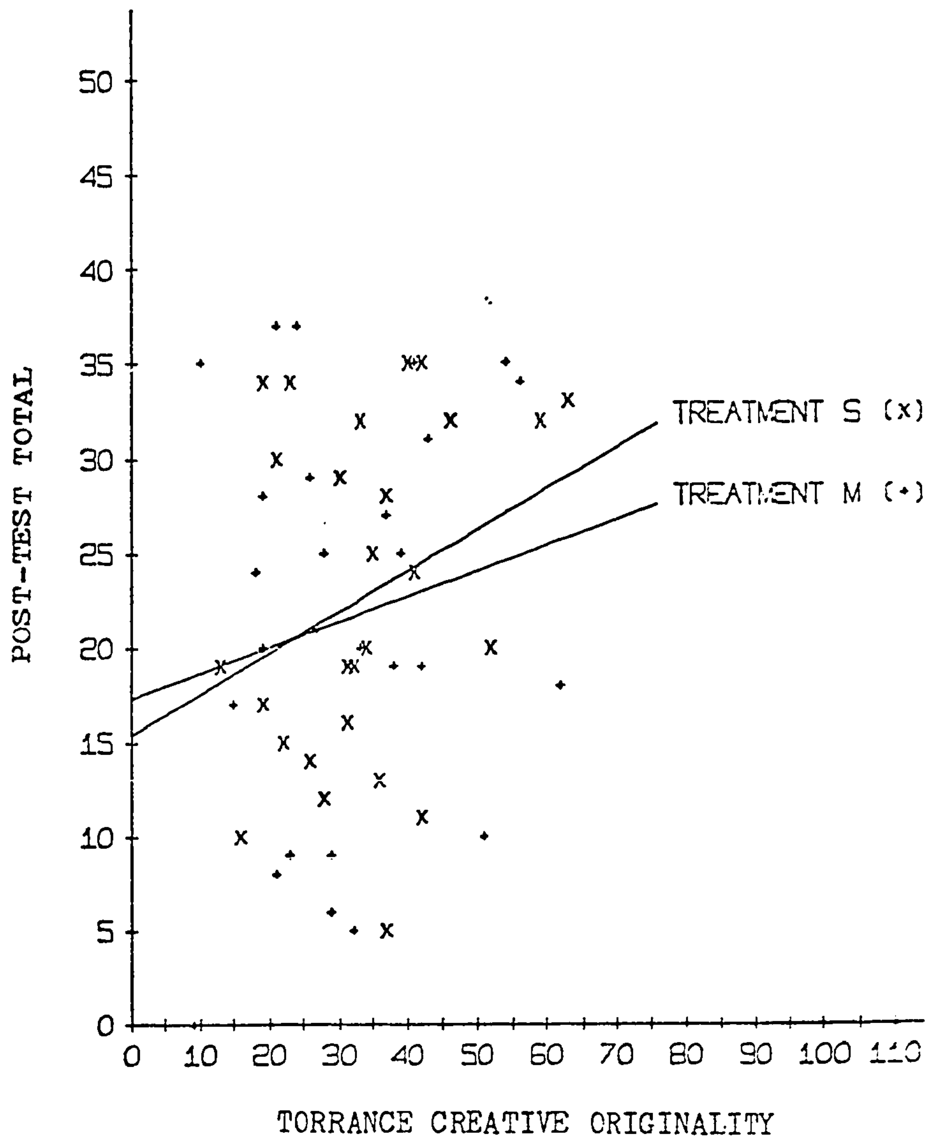
TABLE 39



	r	a	b	N	S_b	t for $b = 0$	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2772	3.0916	.2179	27	.1511	1.4420	17.66		
M	-.0428	7.7788	-.0474	26	.2259	.2098			

[a, b]Reported only if b_s or b_m is significantly different from zero.

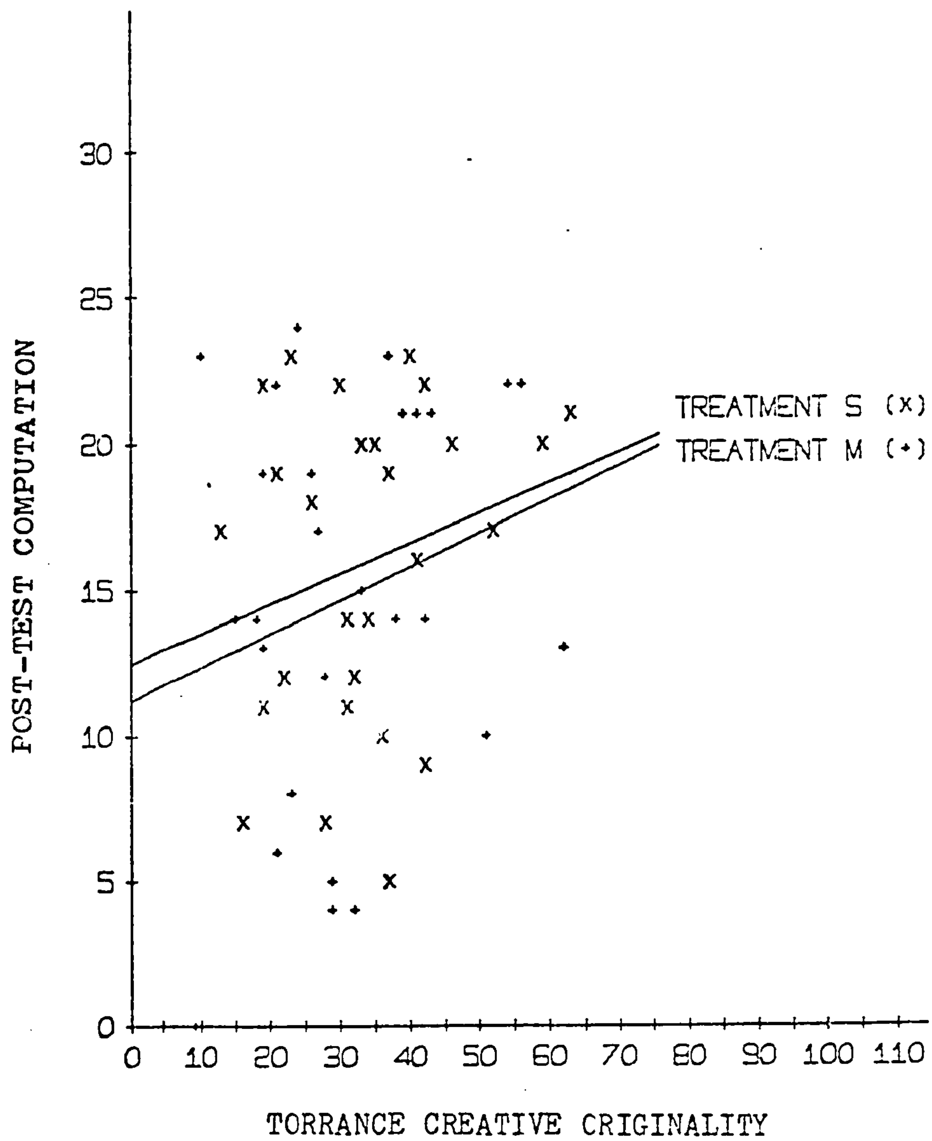
TABLE 40



	r	a	b	N	S_b	t for $b = 0$	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2960	15.3810	.2177	27	.1405	1.5494	23.91		
M	.1724	17.3671	.1349	27	.1541	.8740			

a, b Reported only if b_s or b_m is significantly different from zero.

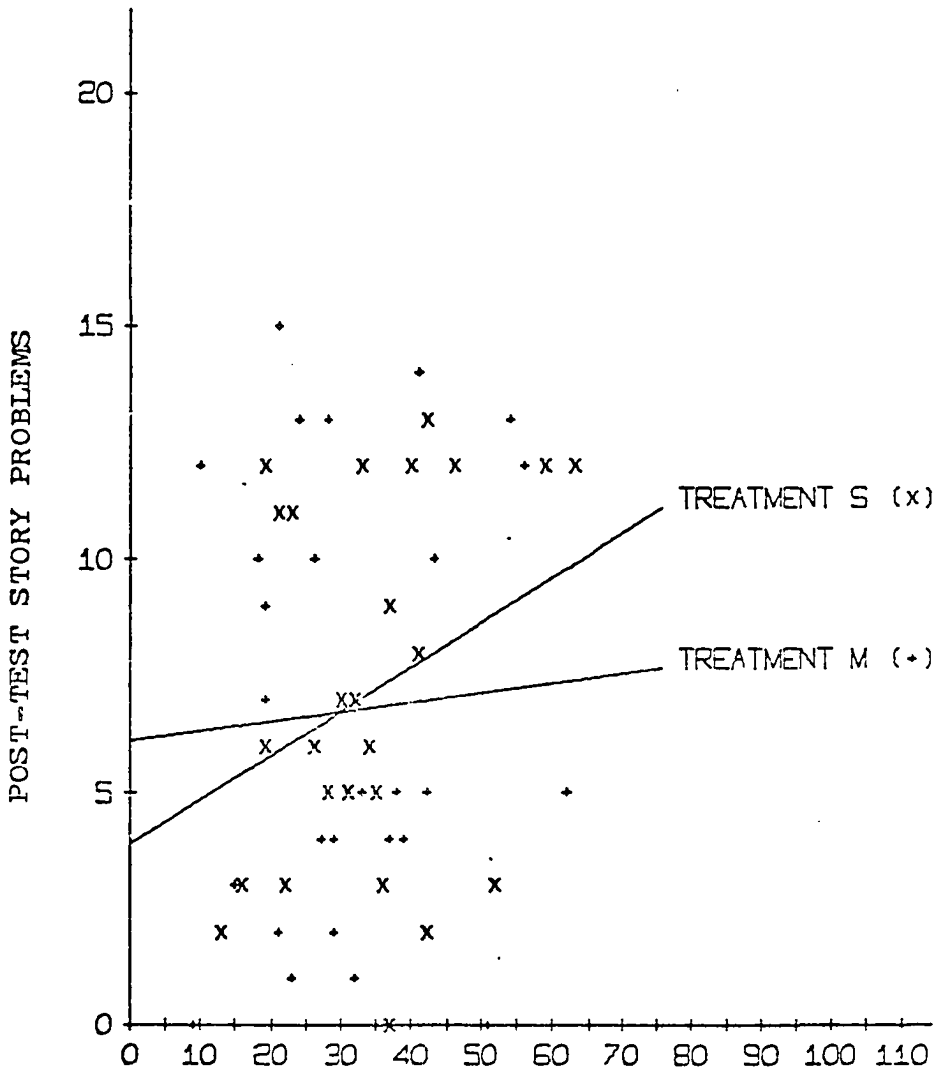
TABLE 41



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2337	12.4793	.1036	27	.0862	1.2018	114.5		
M	.2309	11.2235	.1146	27	.0966	1.1863			

[a, b]Reported only if b_s or b_m is significantly different from zero.

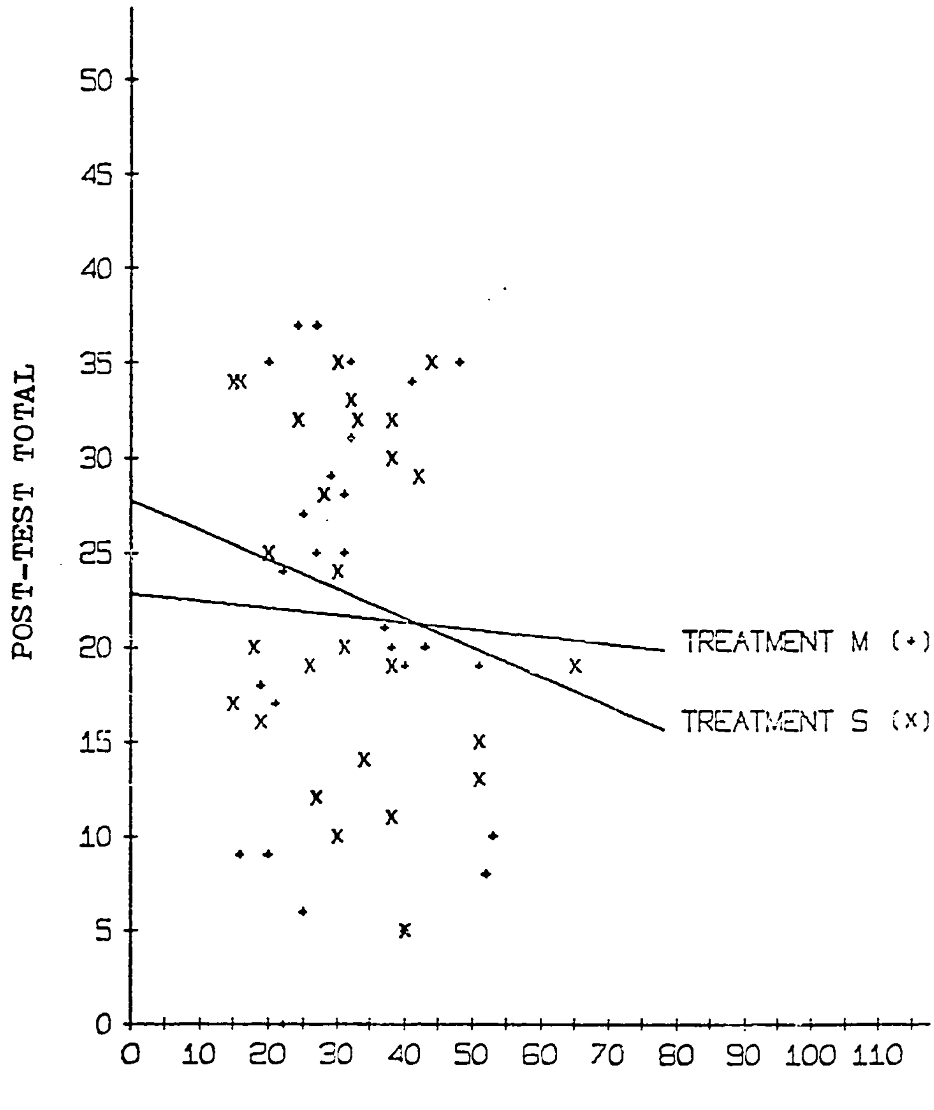
TABLE 42



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	.2965	3.9210	.0949	27	.0616	1.540	29.7		
M	.0592	6.1436	.0202	27	.0682	.2987			

[a, b]Reported only if b_s or b_m is significantly different from zero.

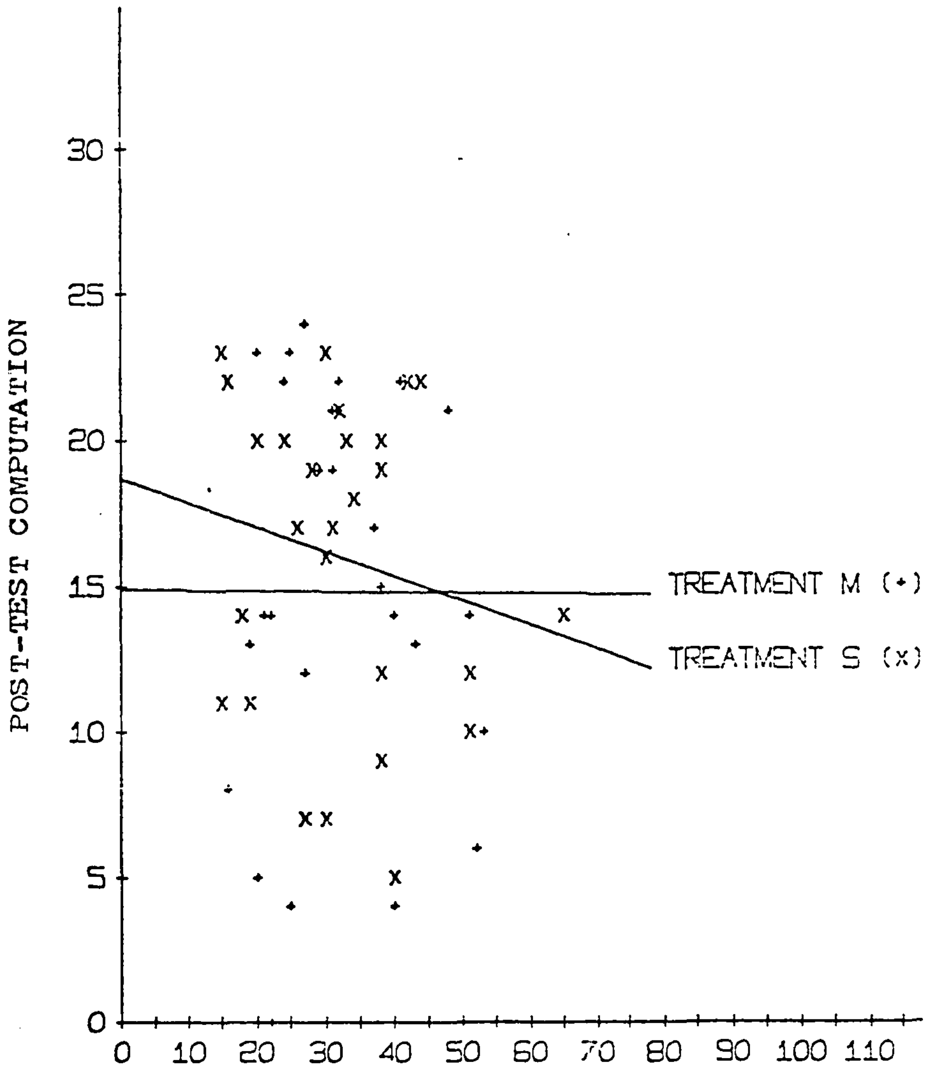
TABLE 43



	$\underline{r}$	$\underline{a}$	$\underline{b}$	$\underline{N}$	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	-.2053	27.7087	-.1548	27	.1476	1.0478	42.0		
M	-.0385	22.8234	-.0384	27	.1992	.1927			

[a, b]Reported only if b_s or b_m is significantly different from zero.

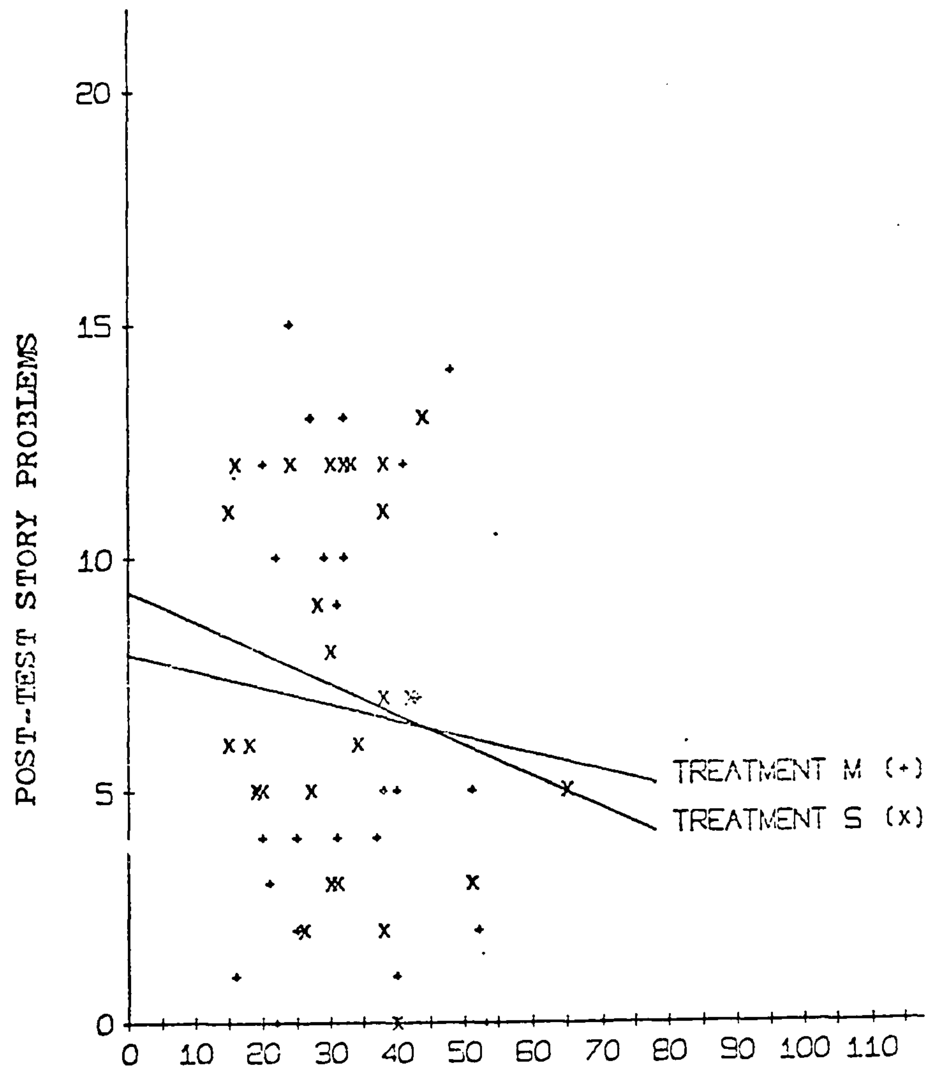
TABLE 44



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	-.1841	18.6674	-.0836	27	.0893	.9361	46.5		
M	-.0034	14.8833	-.0021	27	.1265	.0166			

[a, b]Reported only if b_s or b_m is significantly different from zero.

TABLE 45



	r	a	b	N	S_b	t for b = 0	X at Cross-over	F for[a] S_s^2/S_m^2	t for[b] $b_s=b_m$
S	-.2034	9.2984	-.0667	27	.0642	1.3084	44.2		
M	-.0832	7.9401	-.0362	27	.0868	-.4170			

[a, b]Reported only if b_s or b_m is significantly different from zero.

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