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Abstract

Traditional viewpoints on the decision-making environment have been centered on a top-down perspective, guided by a hierarchy in which only one entity may establish desirable criteria and pursue them while considering others to be quiet agents in the system, merely obeying instructions and judgments taken. Many research studies and mathematical formulations have been developed with this centralized approach in mind. Although systems in the past were not entirely centralized in architecture, researchers assumed this simplification because systems were less complex and with fewer interdependent relationships. Therefore, considering a centralized mathematical formulation describing such systems' behavior was sufficient. However, today's complex systems, which have a very high degree of interconnectedness, exhibit distinct dynamics as subsystems are increasingly dependent on one another. Additionally, considering the free market environment, which encourages reserving government function to the privet sector, the number of decision-makers with distinguished interests and profiteering behavior that can affect the system has been increased. Therefore, two types of interdependencies can be realized in dealing with today's complex systems: (i) physical interdependency among the systems, and (ii) interdependency among decisions made by the decision-makers in the system.

Focusing on these two interdependency types, this research proposes algorithms incorporating mathematical formulations addressing decentralized decision-making environments considering interdependent complex systems. The game theory concept has been utilized to address the decentralized decision-making environment, while permanent

or temporal networks represent the interdependencies between the subsystems. This research has four research threads, each focusing on a decentralized decision-making environment where one can realize a kind of dependency among decision-makers and physical sub-systems. The first research thread, like the second, considers multiple decision-makers only interested in recovering their associated network post-disruption. However, the research addresses the problem of interdependent infrastructure network restoration where both decision-makers depend on each other. Therefore, the research develops an iterative algorithm on a game-theoretic basis, enabling to effectively search the solution space for possible Nash equilibria in the system. Also, we have illustrated that a lower bound could be discovered for each decision-maker's best response in every iteration of the algorithm, where using that can reduce the number of algorithm iterations and, subsequently, calculation time.

The second research thread proposes an adaptive algorithm using machine learning to integrate predictions about other decision-maker behaviors into interdependent network restoration planning under an imperfect information-sharing environment. The proposed algorithm considers a permanent network representing a dependent infrastructure network where each network has a separate decision-maker (i.e., utility) only interested in and benefiting from the recovery of its associated network post-disruption. As one decision-maker is dependent on the other and lacks complete information about the leader's restoration activities, a predictive model based on a machine learning approach was designed to predict the behavior of the leader decision-maker. The proposed algorithm provides a solution sufficiently close to the optimal solution showing the algorithm performs well in situations where the information sharing environment is incomplete.

The third research thread focuses on the impact of demand surge on critical supply chain networks due to disinformation propagation in the communities. In this research, we presented a framework that enables the modeling of these fluctuations and the evaluation of their impact on the adaptability of the supply chain. The objective was to identify any vulnerabilities or shortcomings within the supply chain by analyzing its flexibility in response to these variations.

The fourth research thread focuses on a semi-decentralized, three-stage algorithm based on game theory to resolve potential conflicts among multiple aircraft travelling in a 3D shared airspace. Considering the aircraft pilots as decision-makers, the main objective of this research was to allow pilots to pursue their interests and have the liberty to select their flight characteristics freely while maintaining a safe distance from others. Conflicts among aircraft were considered logical, and temporal interdependency relationships existed between them, considering their route and flight characteristics.

1. CHAPTER ONE: Introduction and motivation

1.1. Overview

Infrastructure networks are complex interconnected systems that encompass physical, digital, and social components, enabling the continuous flow and exchange of entities [1]. The interplay of these components, let's assume, having roles as either consumers or suppliers, creates a dynamic ecosystem of interactions and transaction where encompasses countless entities that participate in our everyday life. It is worth noting that the infrastructure networks are continuously evolving to become more interconnected, driven by the synergistic advantages gained from establishing highly interdependent systems. This increasing interconnectivity allows for enhanced collaboration, resource sharing, and optimized operations among the various components of the networks enabling new levels of efficiency, and resilience considering the complex systems [1–3]. Therefore, the vitality and stability of societies heavily rely on the proper functioning of these critical infrastructure networks. Their failure can significantly degrade the overall quality of life and compromise the security of communities, as disruptions can quickly propagate from one system to another, making them highly vulnerable. [1,2,4–9]. Hence, recognizing the significance of these intricate systems, the United States Cybersecurity and Infrastructure Security Agency (CISA) has declared 16 infrastructure sectors as critically important, as their failure would have severe consequences for the economy, health, and security of the society [10].

Furthermore, in reality, these infrastructure networks have generally been structured in a decentralized form, as several stakeholders exist with different natures and interests that can affect the system through their decisions. In other words, the decision-makers are interdependent, considering their available decision options and the potential payoffs they can receive from executing their decisions. In a decentralized decision-making environment, the design of critical infrastructure networks emphasizes distributed decision-making authority and autonomy among network participants. This approach allows individual components and subsystems within the network to make localized decisions based on their specific needs and conditions. Therefore, by solely focusing on maximizing the performance of an individual infrastructure network in isolated decision-making environment, the mutual benefits that arise from interdependency with other networks are disregarded, along with the fundamental reality of having interdependent decision-makers influencing the network [1,2,4,5]. The decentralized approach considers multiple decision-makers in the system, in the way that decisions taken by one decision-maker can influence others and vice versa. Thus, a decentralized decision-making approach provides a more realistic view of the problem, although it will come at the cost of having higher complexity [5,11–16].

In this research, we focused on the restoration of critical interdependent infrastructure networks, considering the decentralized decision-making approach. We considered multiple decision-makers who are expected to influence the decision-making environment, with their main objective being to satisfy their own interests and maximize their benefits. Additionally, we extensively investigated the impact of various variables, including information availability and disinformation propagation. Furthermore, we specifically emphasized the consideration of a temporal logical network that arises due to the decisions made by the decision-makers, considering the dynamic nature of time.

1.2. General Assumptions for Decentralized Model

Many researchers have approached the problem of restoring critical infrastructure network after disruption, assuming various problem definition and characteristics, illustrating cooperative or non-cooperative behavior among decision-makers, with various considerations of information availability [5,11–16]. In this regard, we have classified the restoration problem considering three different criteria, which are: (i) type of payoff function and decision-making constraints for decision-makers in the system, (ii) the decision-makers attitude regarding each other, and (iii) availability of information regarding the movements of the other decision-makers in the system. In models, algorithms, and formulation that we developed along this research, we have considered that the decision-makers are indifferent regarding each other's profit, although they focus only on their own interest. Therefore, finding the global optimal response to the restoration plan of a disrupted interdependent infrastructure network is not the case, when each decision-maker that could affect the environment following only its interest. As every decision-maker can affect and be affected by the decision-making environment, the decision-makers' decisions are also considered interdependent. The main source of this interdependency in decision-making is the physical interdependency relationships that exist between parent and child components of the interdependent network.

Furthermore, many solution approaches have been developed to deal with the complexities of having multiple decision-makers in the system. In this regard, many studies have used the bi-level or tri-level optimization, analyzing the problem of critical infrastructure network restoration after disruption addressing decentralized environments [11,15,17–25]. Note that high computation time is associated with this solution approach, especially when more than two decision-makers are involved in the decision-making environment [26,27].

The problem of decentralized decision-making has been investigated using various approaches, including the utilization of a Markov decision process (MDP) [28–31]. Åström [32] introduced the concept of partially observable MDP (POMDP) for centralized sequential decision-making, which is suitable for dealing with imperfect information. Building upon the POMDP framework, the decentralized partially observable MDP (DEC-POMDP) has been developed as a generalized version, specifically designed to address problems involving multiple decision-makers collaborating in an imperfect information environment [16]. However, research has demonstrated the considerable challenge in solving the DEC-POMDP problem involving multiple decision-makers, especially when compared to the single agent POMDP problem. It has been revealed that the finite-horizon DEC-POMDP problem is classified as exponential time complete (NEXP-complete) when there are only two decision-makers present in the system [33].

Moreover, game theory offers a well-established methodological approach for analyzing strategic interactions among multiple decision-makers who have conflicting interests. It has been widely employed in network-based research to explore the behaviors of decision-makers regarding to the network, allowing for efficient pursuit of their respective objectives [5,13,34–40]. Considering the interdependencies and strategic choices of decision-makers, game theory allows for a more comprehensive understanding of the dynamics and outcomes in the network. Therefore, we employed game theory as a valuable tool to address the presence of multiple decision-makers and their impact on the network.

The other major concept when analyzing the presence of multiple decision-makers in the system, is how decision-makers are informed regarding the options that the other decision-makers in the system do have. This involves the decision-makers' movements in the past, and their future. Having complete information allows the decision-makers to have a more accurate assessment regarding the other decision-makers' strategies, enabling them to have a better perspective over the outcomes of their potential actions [5,16,17,35]. This will enable decision-makers to devise effective strategies, have a better understanding of negotiation outcomes, and maximize their own benefits in a competitive or cooperative setting. In this research, we have focused on the level of information availability and its effect on the restoration of interdependent infrastructure networks. Also, we have presented a machine learning approach to predict the behavior of other decision-makers in the system. This allows us to adjust the restoration plan of each decision-maker based on the prediction, enhancing their ability to make informed decisions and optimize their individual strategies. In this regard we have also focused on the effect of disinformation on the critical supply chain network, providing a framework for anticipating the demand surge due to disinformation propagation in the communities.

Furthermore, the type of network structure plays an important role in determining the appropriate game-theoretic analysis required to accurately model and analyze the dynamics of strategic interactions among decision-makers, which impacting on the network. We have focused on three different categories of network in this research which are: (i) physical network, (ii) logical network, and (iii) logical temporal networks. In the large portion of this research, we have used physical networks for addressing the problem of critical interdependent infrastructure network restoration after disruption as the relationships existed between the components of the network can be physically realized (e.g., power or water networks). Also, we used critical supply chain network on which representing the logical transportation of goods between the different components, as the production and distribution processes are defined logically and can be changed considering different circumstances. Finally, we have also focused on the application of temporal networks in the context of air traffic control, where the path history of events in the airspace is critical for efficient and safe management of multiple aircraft in the airspace optimizing the air traffic flow.

1.3. The Structure of Dissertation

The remaining sections of the dissertation are structured into five additional chapters. Chapter 2 is dedicated to the development of a non-cooperative simultaneous game-theoretic algorithm to tackle the restoration of interdependent networks after disruptions. The algorithm considers a decentralized decision-making environment where multiple decision-makers operate with their own interests at stake. It is worth noting that these decision-makers are not treated as adversaries but rather as entities with potential conflicts of interest concerning the recovery of interdependent components, as they serve as key bargaining element in the process. As a result, we considered that the decision-makers have full information about each other's restoration strategies and their payoff.

In the third chapter, we have investigated the value of information, providing a methodology to anticipate the other decision-makes strategy regarding their restoration plan using machine learning, to better devise a plan for restoration of the components that are associated with the follower decision-maker. In this regard we have developed an adaptive algorithm using the notion of dictatorial game theory, integrating the prediction with the decision-making, addressing uncertainties regarding having an imperfect information sharing environment. we considered that the information is gradually updated while the decision-makers are restoring the disrupted interdependent network.

In Chapter 3, we extensively examine the value of information by proposing a methodology that utilizes machine learning to anticipate the restoration strategies of other decision-makers. This enables us to devise a more effective restoration plan for the components associated with follower decision-makers. To tackle uncertainties stemming from an imperfect information sharing environment, we developed an adaptive algorithm that integrates the prediction process with decision-making, considering the concept of dictatorial game theory. Furthermore, we consider the gradual updating of information as the decision-makers work towards restoring the disrupted interdependent network.

In Chapter 4, our effort was to provide a framework for modeling the demand surge of the critical goods and services due to disinformation discrimination in the community.

We also explored the impact of these demand variations on the agility of the critical supply chain networks, assessing factors like the speed of order fulfillment and the overall idle time within the supply chain. Examining these two criteria not only provides insights into the level of supply chain flexibility, but also helps identify potential weaknesses within the system.

In Chapter 5, we propose a semi-decentralized, three-stage algorithm based on game theory to resolve potential conflicts among multiple aircraft in a shared 3D airspace. The algorithm considers the sensitivity of deviation costs to the level of congestion in the airspace and offers valuable information to enhance coordination and decision-making for air traffic control and pilots in conflict scenarios.

In chapter 6, we have dedicated our attention to presenting comprehensive concluding remarks that encapsulate the key insights, contributions, and implications derived from the extensive research conducted.

2. CHAPTER TWO: GAME THEORETIC ALGORITHM FOR INTERDEPENDENT INFRASTRUCTURE NETWORK RESTORATION

2.1. Introduction

Infrastructure networks are described as connected cyber-physical-social systems that allow entities to flow and exchange among their components [1]. Countless entities can be realized in everyday life following this definition such that every component performs a role as a user or server asking for service or satisfying it. The well-being of the societies critically depends on many of these infrastructure networks, and failure in them could effectively degrade the life quality and security of the communities that rely upon them [1,2,4,5]. Thus, according to the United States Cybersecurity and Infrastructure Security Agency (CISA), 16 infrastructure sectors have been considered critical that their incapacitation would severely affect the economy, health, and security [10].

Human-made systems are becoming highly interdependent due to technological advances and the synergic gains obtained by connecting different systems [1–3]. Although high interdependency among systems could be considered a source of efficiency, these systems could be highly vulnerable to disruptions as failure could quickly propagate from one system to another [4,6–9]. That is, a component in a system could fail because another system's component was disrupted (e.g., disruption in a power network would disrupt the operation of a communication network).

Many studies have been conducted regarding critical interdependent infrastructure network vulnerability and resilience by considering centralized and decentralized approaches. Generally, one agent performs all analysis in the centralized approach and executes all decisions. In contrast, other agents are only supposed to follow the orders even if they do not benefit from them [41–48]. Note that although centralized control over the interdependent systems is a traditional assumption in this context [49,50], it cannot exist practically due to many stakeholders involved who have their own objectives to meet (e.g., infrastructure utilities, local and state governments). Therefore, most studies that consider a centralized approach do not satisfy the reality of these complex systems.

Moreover, the decentralized approach considers more than one agent in the decision-making environment, where one agent's decision can be affected by others and vice versa. This approach brings more features with realistic characteristics to the analysis, although it adds to the complexity of the problem. Many researchers have also investigated this approach considering various definitions and solution methods, demonstrating the cooperative or non-cooperative nature of decision-makers in the system [5,11–16].

Several different approaches have been proposed to address various types of decentralized decision-making. We considered some criteria to classify various decentralized environments that have been investigated regarding the restoration of interdependent infrastructure networks, which are: (i) type of payoff function for every decision-maker in the system, (ii) decision-maker intention regarding others in the system, and (iii) information availability.

In centralized bi-level or tri-level optimization, only one decision-maker is involved in the decision-making process, sequentially approaching the problem considering the conflicting objective function. In contrast, in a decentralized approach, more than one decision-maker with various objective functions is present in the system, presenting different attitudes regarding the others. Many studies have been conducted using bi-level or tri-level optimization, analyzing the problem of critical infrastructure network restoration after disruption in a centralized and decentralized environment, considering various assumptions while focusing on maximizing the resilience or minimizing the vulnerability of the network grid [11,15,18–25]. Note that high computation time is prevalent in this solution approach, especially when more than two decision-makers are involved in the decision-making environment [26,27]. Furthermore, Sharkey et al. [17] considered the value of sharing information among the cooperative decision-makers in the system until they reach an agreement on infrastructure restoration. However, the unlimited number of negotiations increases the algorithm's computational complexity for finding a solution in which all decision-makers agree [5]. Although the proposed algorithm provides excellent insight into the impact of sharing information among multiple decision-makers, the final solution may not converge to a satisfying solution concerning the objective function of the problem [5].

In this regard, game theory is a well-established concept, demonstrating a framework for analyzing the strategic interactions among various decision-makers with competing interests. The game theory concept has been applied in many network-related studies considering multiple decision-makers that strategically influence the network differently to more effectively serve their interest [5,13,34–40]. In recent years, game theory has been applied to the infrastructure restoration problem to better understand decision-makers' behavior and the impact of their actions on the restoration process, as noted below.

Chen and Zhu [40] developed a model based on the game theory where all the decision-makers have a single shared objective. Their objective is to maximize the algebraic connectivity of the global interdependent network. Note that the algebraic connectivity can be represented using the second smallest eigenvalue of the network Laplacian matrix. Furthermore, decision-makers rewire the interdependency relationships to form a new structure using the game theoretic approach considering the available resources. However, every decision-maker is limited and can only control specific variables of the network. In other words, although each decision-maker is responsible for restoring a particular portion of the network (decentralized environment), decision-makers show a complete cooperative nature in the restoration of the global network as both decision-makers follows the same interest. Note that this kind of relationship among the decision-makers in the system is rare, considering the actual situation where private companies' primary objective is to maximize their own profit.

Smith et al. [5] proposed a model based on sequential game theory to address a decentralized environment for restoring disrupted interdependent infrastructure networks where the decision-makers could have different payoff functions. The model has been designed in a turn-based order which makes the model less realistic as the decision are generally taken simultaneously in reality. Also, as information is gradually updated based on the decision-makers' turn, the one decision-maker who acts last can make a better decision as it has complete information about the actions taken by all other decision-makers.

Rangrazjeddi et al. [51] presented an adaptive algorithm based on dictatorial game theory. The system anticipates the leader's behavior using machine learning, updating the optimization input parameters, and preparing the optimal response for recovering an interdependent infrastructure network in which each decision-maker is solely accountable for and rewarded for its associated sub-network. The algorithm is efficient when there is only a dependency among sub-networks and when only the decisions of the follower decision-maker are dependent on the leader decision-maker.

In this research, we propose an algorithm for the decentralized restoration of interdependent infrastructure network in a non-cooperative environment where each decision-maker is only responsible and rewarded for restoring their assigned sub-network. Note that the decision-makers are not necessarily hostile regarding others, meaning that a lower payoff for one does not contribute to the higher payoff for the others. Therefore, the decision-makers are considered indifferent agents where the interdependency between the sub-networks is the primary source of conflict that can influence the payoff function of each decision-maker. Furthermore, we have assumed that the payoff of each decision-maker considering their selected strategy is known to every other decision-maker in the system. The key contributions of the proposed game theoretic algorithm are: (i) addressing the post-disruption restoration of the infrastructure network in a decentralized environment where the decision-makers can act simultaneously, (ii) searching the solution space, effectively finding multiple Nash equilibria for the problem enabling the decision-makers to select the best found Nash equilibrium, (iii) providing a lower-bound strategy solution at every iteration of the algorithm, (iv) providing analysis to calculate the cost of anarchy by representing the unmet demand cost difference between decentralized and centralized decision-making, and (v) providing analysis for assessing the computational time of the proposed algorithm.

The rest of this study is organized as follows. Section 2 reviews the appropriate literature. In Section 3, we describe the methodology of the proposed algorithm, starting with the problem statement and assumptions, and continued by describing the presented model and algorithm capable of finding the best response for the decision-makers in the network. We also describe the decision-maker's lower bound strategy in response to the other decision-maker's best strategy. In Section 4, results for realistic and three generated interdepends networks considering various disruption scenarios are provided, along with an analysis of cost of anarchy and algorithm computational time. Finally, the concluding remarks have been provided in Section 5.

2.2. Literature Review

In a centralized decision-making environment, decisions are assumed to be taken by a supervisory decision-maker or a small group, then communicated to the lower ranks of the system for execution. While in a decentralized decision-making environment, the decision-making process is distributed or performed by agents that could show different attitudes. The management of complex systems with network characteristics has primarily been addressed with a centralized approach [49,50]. However, a centralized management approach that dictates top-down decisions has become less practical, especially in environments where multiple interconnected networks with different applications managed by various decision-makers exist, and the functional status of one is relatively essential to another. Considering critical infrastructure networks, whose performance is critical regarding the functional societies, numerous studies have been conducted to reduce the vulnerability and improve the resilience of these interdependent networks in a centralized manner [41–48]. These models have neglected the fact that these naturally separated networks are generally managed by various entities with various attitudes regarding others that may even only follow their interests due to the conditions dictated by the business environment. Therefore, methodologies incorporating only a centralized approach are less practical, addressing the essential criteria governing the actual situation affected by the business environment.

Many studies consider a decentralized approach with various assumptions that govern how multiple decision-makers are involved in a system. Note that various criteria can affect the type of decentralized environment and subsequently affect the solution approach required to address them. Considering all decision-makers in the system, if each player shares the same payoff function (i.e., if one wins, all win), then they demonstrate a cooperative attitude concerning the others in the system. If each decision-maker follows a different objective, decision-makers can show indifferent or hostile attitudes toward each other. Such a hostile attitude leads the strictly competitive decentralized decision-makers toward zero-sum game where the loss of one decision-maker is the win for the other

[52,53]. The level of information availability and the ability to digest the information (rationality level) by decentralized decision-makers is the other factor that can influence the strategies they select regardless of their attitude [17,54,55]. A lack of information can cause decision-makers with a cooperative attitude to be less effective in contributing to the common cause. At the same time, such decision-makers with indifferent or hostile attitudes lack the agility to cope with the situation and change their strategy effectively in time.

Furthermore, with multiple decision-makers influencing the system, although it contributes to the practicality of the application, there exists an associated increase in the complexity of the methodology addressing the problem [4]. The decentralized Markov decision process (D-MDP) is one of the methodologies that has been utilized to optimize the joint reward function of the multiple decision-makers in the system with a cooperative attitude [28–31,33,56].

Some studies have used bi-level and tri-level optimization to analyze the vulnerability and resilience of infrastructure networks in a hostile decision-making environment, where a hostile force's objective is to attack the network components. In contrast, the other decision-makers are responsible for protecting and recovering the network's vulnerable and disrupted components [11,15,18–23]. Ouyang [23] proposed a tri-level optimization model to minimize the vulnerability of the network function by protecting the sensitive components. At the same time, the model's third level is responsible for providing redundant components to the network to minimize network performance loss due to the hostile attack. Ghorbani-Renani et al. [15] has used tri-level optimization to minimize the vulnerability and enhance the trajectory of restoration of an interdependent infrastructure network in a zero-sum competitive environment. However, bi- and tri-level optimization models are generally considered computationally costly, especially when more than two decision-makers are involved [57–59]. As these and many other network interdiction models developed to address the zero-sum environment, each decision-maker has a separate individual objective function that must be optimized in a non-cooperative manner. Furthermore, these models generally consider the attacker to have complete information on the defender's protection and recovery plan and not vice versa. Note that this type of information availability would tend to be rare in practice as decision-makers in a system have neither zero information nor complete information regarding their opponent.

Furthermore, Sharkey et al. [17] developed an iterative algorithm using a kind of bi-level mechanism, illustrating the value of information sharing where the decision-makers are willing to share their restoration plan with others. Although all decision-makers in the system have the same goal of restoring the network completely, each can control a portion of the network components using variables responsible for component functionality and restoration. This allows the decision-makers adjust their restoration plan, provided the other decision-makers plan regarding the interdependent components they control in the network in every iteration. In this study, the decision-makers exhibit a cooperative attitude as they share the same objective and exchange their restoration plan information. Although the amount of the unified objective function in each iteration improves relative to the previous iteration, the solution may not converge to optimality even after a high number of tedious and time-consuming algorithm reputations [5].

Game theory is a mathematical approach that studies how individuals or organizations make decisions and interact with one another in strategic situations. Subsequently, game theory has been used in many infrastructure network problems assuming decentralized decision-makers have various intentions affecting the network. Extensive research has been conducted on applying game theory to the defender-attacker problem considering infrastructure network security. As a result, numerous models and algorithms with diverse characteristics have been developed by researchers. Utilizing a Bayesian Stackelberg sequential game, Zeng et al. [60] have proposed an active defense approach considering the defender-attacker problem and introducing false information to the attacker. Using a mixed defense strategy, they assumed probabilities associated with adopting the defense strategy and the selection of different types of attackers. He et al. [61] proposed a discrete simultaneous and sequential game theoretic model addressing the defender-attacker problem for cyber-physical systems. They assumed a probability of a successful attack on and survival of network components. The model assumes that the attacker and defender have complete information on each other's strategy payoff. Chen et al. [62] explored an adversarial network of network operators and one attacker to capture a reliable design for an interdependent infrastructure network with the Internet of Things capabilities with limited resources. Under the assumption of a sequential game, subgame perfect Nash equilibrium was employed to address the problem. Also, Zhang et al. [63] used the concept of the sequential game for the defender-attacker problem in which the defender distributes defensive resources to the potential targets. Opting for the best strategy, the attacker allocates resources based on the target's value, adapting its strategy based on defense resources allocated by the defender. Considering simultaneous games, Feng et al. [64] proposed a Bayesian game-theoretic framework for reducing expected performance with limited defensive resources when multiple attack forms are imposed on the network. For different defense and attack strategies, they assess the expected loss of network performance and subsequently find the Bayesian Nash equilibrium solution where none of the players (defender or attacker) has the incentive to alter their strategy. Guan et al. [65] proposed a game theoretical model that addresses the defender-attacker problem while considering both parties' budget constraints. The study focuses on identifying the equilibrium point for resource allocation between the defender and the attacker. The Karush-Kuhn-Tucker conditions were employed as a solution approach to determine the equilibrium solutions for the defender-attacker problem, which is specifically designed for solving convex optimization problems.

Furthermore, game theory could be a valuable tool for devising plans to restore the infrastructure networks after disruption by considering the perspectives of the different decision-makers satisfying various intentions by affecting the network. In this regard, Belhaiza and Baroudi [66] proposed a noncooperative game-theoretic model for managing demand for the smart grid , ensuring that users and providers interact to maximize their payoff. The Nash equilibrium solution in this problem is the one which satisfies the primal conditions of the user and the dual conditions of the provider, though the methodology is only effective when the formulation used for decision-makers is continuous and the feasible area is convex. Lu et al. [67] proposed a game-theoretic equilibrium model that systematically investigates the mutual interest between infrastructure and communities, considering the same objective function illustrating a cooperative relationship. Equilibrium occurs when no community can reduce access cost to resources, while the interdependency constraints related to the service capacity and the functional status of the infrastructure are satisfied. Using cooperative game theory, Mohebbi et al. [68] developed a decentralized resource allocation model for restoring disrupted interdependent infrastructure networks. Using Shapley values, the model recognizes coalitional structures within the network and provides the restoration sequence of the disturbed components. Chen and Zhu [40], have developed an alternating play algorithm using a bi-level mechanism to maximize algebraic connectivity in an interdependent critical infrastructure network to improve performance. They assumed that every decision-maker has access to limited resources for adding or removing a link within its controlling network or the interdependent links that connect individual networks. Note that as the main objective is to maximize the connectivity of the global network, therefore similar to Sharkey et al. [17], decision-makers show a cooperative attitude. Also, the model has only focused on network connectivity while the flow of goods or services between the network supply and demand sectors has been neglected. More aligned with our research line, Smith et al. [5] developed a sequential game-theoretic algorithm to address the problem of interdependent infrastructure network restoration in a decentralized non-cooperative environment. Every decision-maker in this algorithm is only responsible and rewarded for restoring the components of the network that they have assigned. Although the algorithm provides an excellent insight and solution approach for addressing the post-disruption restoration of the infrastructure network, the algorithm has been designed in a turn-based order, making it less realistic and practical. As the decision-makers are gradually being informed about the actions made by the others, the turn-based approach gives decision-makers who act later better information to make the decision. This unparalleled update of information makes the algorithm more biased from the actual condition. The other concern regarding the developed algorithm is the computational time that could be very much sensitive calculating the Nash equilibrium due to the size of the problem and resources available for each decision-maker. Furthermore, as the algorithm had to calculate all possible strategy combinations and find the Nash equilibrium using the backward induction, the proposed algorithm had to enumerate all possibilities, which makes the algorithm highly expensive regarding time. Rangrazjeddi et al. [51] developed an innovative adaptive algorithm using a notion of dictatorial game theory. This algorithm uses machine learning to predict the leader's behavior and updates input parameters to generate the best possible response for restoring an interdependent infrastructure network. Each decision-maker is accountable for and rewarded for their specific sub-network. The algorithm is particularly effective in situations where there is only a dependency between sub-networks, where only the decisions of the follower decision-maker are impacted by the leader's decisions.

Summarizing the previous studies on the restoration of interdependent infrastructure networks shows an extensive focus on the zero-sum game concept where the loss of one decision-maker is a gain for others. However, considering the characteristics of a modern business environment, decision-makers are not necessarily adversaries but have conflicts of interest, meaning they are not regarded as natural enemies. Wanting to cooperate with other decision-makers and lacking information due to organizational hierarchy arrangements also lead decision-makers to act as individuals considering different payoff functions and reward systems. This results in a decentralized decision-making environment that can be addressed using a game-theoretic approach. Therefore, the investigations on game theoretic models and algorithms that only applies to zero-sum game applications neglect many real business applications. Likewise, decision-makers with a conflict of interest generally do not have a cooperative nature and act based on their interest. Therefore, studies investigating cooperative decentralized decision-making environments are suitable for certain societies and business environments where collective behaviors are more valued than individual ones. Furthermore, studies that use probability distributions can be validated, provided that probability distributions of the case study are known. Much of the literature related to game theory applications in infrastructure networks has focused on defender-attacker models and algorithms and less on network restoration. Restoration optimization models often take the form of mixed integer formulations where the feasible area in these problems is typically nonconvex and not continuous. Therefore, it is only possible to combine the primal and dual forms of the problem to find the Nash equilibrium if the optimization problem related to each decision-maker in the system is convex.

In this research, we propose a non-cooperative game-theoretic algorithm to address the post-disruption restoration of the infrastructure network where the decision-makers can act simultaneously. The decision-makers have different payoff functions representing their interest: minimizing the unmet demand of the sub-network with which they are associated. Therefore, the decision-makers are assumed to have indifferent attitudes, considering that their interests may not necessarily conflict in all situations. The primary basis for negotiation arises from the interdependence of the sub-networks, where the ability of one decision-maker to restore or maintain the functionality of their components is reliant on the actions of the other decision-maker. That is, the sub-networks are interconnected, and the decisions made by one decision-maker can significantly impact the other, creating opportunities for negotiation and compromise. To gain a comprehensive understanding of the difference between decentralized and centralized decision-making, we have analyzed the cost of anarchy in a system where multiple decision-makers have the liberty to pursue their own interests. Also, the algorithm’s computational cost for finding the first Nash equilibrium has been analyzed considering realistic and generated algorithm with various size of networks, interdependency, and disruption. The algorithm showed a strong performance in computation time regarding small and medium size networks. Furthermore, the model effectively searches the solution space to find other Nash equilibria, if there are any.

2.3. Methodology

This section formulates the interdependent infrastructure network restoration approach where different decision-makers are involved in the system. However, these decision-makers are rewarded only for restoring the part of the network allocated to them, and global restoration of the network is not their concern. Therefore, each decision-maker greedily follows its own interest regardless of others in the system developing their own different payoff functions, hence necessitating a game-theoretic structure.

We consider two interconnected sub-networks that have a interdependency relationship, where the functionality of the parent component in one sub-network is necessary for the functionality or recovery of the child component in the other sub-network.

Subsequently, the strategy selected by a decision-maker regarding parent component recovery time can affect the other decision-maker's payoff function and vice versa. Both decision-makers have parent components in their sub-network, therefore each can devise a strategy against the other decision-maker in a non-cooperative game-theoretic structure. These strategies simply seek the minimal recovery time of the parent components in every sub-network. As the parent components in neither sub-network have any predecessor requirements, the parent components' recovery time can determine the earliest time that its corresponding child component can be recovered or functional. Knowing the earliest time that a child component can be recovered, the decision-makers can find their best response using a centralized approach to maximize their payoff function.

Note that the main objective of every decision-maker, generally, is to prioritize the restoration of the allocated sub-network disrupted components to maximize the sub-network users' satisfaction by fulfilling their demands through the time horizon. Many studies that consider a centralized approach for infrastructure network restoration problems have defined their objective function similarly by considering the cumulative unmet demand in the network through the problem time horizon as the criterion to be minimized. However, they have bounded the objective function using constraints representing the availability of resources such as work crews and budget. In addition, these models and algorithms have added features to include more practicality by more effectively capturing the actual situation in a centralized decision-making environment.

To adapt these models to the developed algorithm, we need to insert a hard constraint representing the earliest time a child component can be recovered by the decision-maker responsible for the sub-network. In this study, we modified the time-based interdependent network design problem (TD-INDP) by Gonzalez et al. [46] for finding the decision-maker's best response given the strategy selected by the other decision-makers in the system. That is, the result of a given strategy is the time that the parent components of the other sub-networks are going to be functional.

Although this algorithm considers the decision-makers to be greedy and selfish, they are not enemies, suggesting that, although the decision-makers have separate payoff functions, these payoff functions are not at odds with each other (i.e., improving one payoff function does not necessary require the reduction of the other payoff function). Therefore, we assume that decision-makers have indifferent attitudes regarding each other.

The decision-makers in the system have two types of variables affecting their payoff function: (i) variables related to the independent disrupted components, and (ii) variables related to the interdependent disrupted components, which are the main source for decision-maker's bargaining and strategy conflict.

The more substantial the interdependent relationship between sub-networks, the higher the decision-maker's conflict of interest. Also, the best response of every decision-maker can be calculated assuming that the selected strategies by the other decision-makers are known using a modified version of TD-INDP model (MTD-INDP).

The general idea behind the algorithm is that if all the decision-makers select the best response against the selected strategy of the other decision-makers in the system, the provided solution converges to the Nash equilibrium where none of the decision-makers have any incentive to change their selected strategy. The following is a table related to the acronyms and abbreviations used in this study.

Table 1. Summary of acronyms, sets, parameters, variables, and algorithm terms.

Acronyms	Meaning
INDP	Interdependent network design problem
TD-INDP	time-dependent interdependent network design problem
MTD-INDP	Modified time-dependent interdependent network design problem
Index	**Meaning**
i, j	Index for the components in the networks, where $(i, j) \in N$
t	Index for the time in the recovery time horizon, where $t \in T$
l	Index for the type of the commodity, where $l \in L$
m	Index for the decision-maker, where $m \in M$
k	Index for the sub-network, where $k \in K$
r	Index for algorithm repetition
Sets	**Meaning**
T	Set of time periods in recovery time horizon
T'	Set of time periods where a disrupted child component cannot be recovered or functional, because its associate parent component is not functional in the other sub-network
N	Set of all components in the network
N'	Set of disrupted components in the network
M	Set of all decision-makers in the system
K	Set of all sub-networks
A	Set of links in the network
L	Set of commodities
R	Set of resources
S	Set of all possible strategy for a single decision-maker
s	Single strategy of a decision-maker defined as set of parent components time of recovery
s^*	Single best strategy of a decision-maker given the strategy selected by the others
O	Set of independent components in network, where $O \subset N$
P	Set of parent components in a network, where $P \subset N$
H	Set of child components in a network, where $H \subset N$

Parameters	Meaning
c_{ijlt}	Unit flow cost of commodity l through link (i,j) at time t
e_{it}	Recovery cost of component i at time t
M^{+}_{ilt}	Unit cost of component i excessive supply at time t for commodity l
M^{-}_{ilt}	Unit cost of component i unmet demand at time t for commodity l
b_{ilt}	Supply/demand of component i at time t for commodity l
u_{ijt}	Capacity of link (i,j) at time t
h_{irkt}	Required resource for recovery of component i at time t, from resource r
v_{rt}	Available resource r at time t
Variables	**Meaning**
δ^{+}_{ilt}	Continuous variable for excessive supply of component i at time t for commodity l
δ^{-}_{ilt}	Continuous variable unmet demand of component i at time t for commodity l
f_{it}	Binary variable for functionality of component i at time t
$\tilde{f}_{it}$	Binary variable for extent of recovery of component i at time t
x_{ijlt}	Continuous variable for flow on link (i,j) in network k and time t
Algorithm terms	**Meaning**
RO	Row candidate list
CO	Column candidate list
SP	Solution pool list
NE	Nash equilibria list

2.3.1. Best Response by Individual Decision-Makers

Mentioned previously, to find the best response for a decision-maker given the known strategies already selected by the other decision-makers in the system, we were inspired by the TD-INDP model, simplifying it by removing the constraints related to the components' interdependency relationship. Although these interdependency relationships are crucial in representing the key features of the interdependent network, the interdependency relationships cannot be addressed in a centralized manner as it has been addressed in the TD-INDP model due to having several decision-makers available in the system. However, our concern is to evaluate the decision-maker's best response strategy provided by the strategy selected by other decision-makers that is known or can be precisely assumed by the decision-maker. Furthermore, these selected strategies are the times the other decision-makers recover their parent components in their assigned sub-network. Therefore, these strategies imply that the decision-maker cannot recover its correspondent child components before the time that the parent component in the other sub-network is restored.

The objective function of the MTD-INDP model in Eq. (1) works as a payoff function for each decision-maker and consists of three key costs related to typical infrastructure networks: (i) flow cost of the commodity, (ii) recovery cost of each disrupted components, and (iii) unmet demand and excessive supply cost. Note that the main concern of this objective function is to minimize the unmet demand and excessive supply, as their associated terms are weighted heavily in Eq. (1) with M_{ilt}^{-} and M_{ilt}^{+}, respectively. However, the flow and components recovery costs are considered as the secondary objective that enhance the network recovery performance by giving restoration priority to the recovery of components which implies less cost to the system. We should note that the main objective of our model is to minimize unmet demand. To achieve this, the model places an emphasis on efficiently recovering disrupted components. Additionally, we prioritize minimizing unmet demand over providing excessive supply, as our primary concern is to satisfy the network demand. As such, we have developed a mechanism to prioritize unmet demand over excessive supply by considering $M_{ilt}^{+} < M_{ilt}^{-}$,.

$$\min Z = \sum_{t\in T}\left[\sum_{l\in L}\sum_{(i,j)\in A} c_{ijlt}x_{ijlt} + \sum_{i\in \acute{N}} e_{it}\tilde{f}_{it} + \sum_{l\in L}\sum_{(i,j)\in A}(M_{ilt}^{+}\delta_{ilt}^{+} + M_{ilt}^{-}\delta_{ilt}^{-})\right] \tag{1}$$

The logical constraint depicted in Eq. (2) maintains the flow balance at each node, regulating the inflow and outflow of commodities considering the supply, demand, or transshipment nature of the nodes. Eq. (3) is the other logical constraint enforcing that a disrupted node cannot be functional unless it has been restored in a previous period. Eqs. (4) and (5) enforce link capacity, while not letting any transshipment between nodes if either end node is not functional. Eq. (6) bounds the number of the components that can be restored in each time period according to available resources. Eq. (7) forces the model to keep disrupted components disrupted in the first time period. Finally, the nature of the decision variables is governed by Eqs. (8)-(10).

$$\sum_{j:(i,j)\in A} x_{ijlt} - \sum_{i:(i,j)\in A} x_{ijlt} = b_{ilt} - \delta^{+}_{ilt} + \delta^{-}_{ilt} \qquad \forall i \in N, \forall l \in L, \forall t \in T \tag{2}$$

$$f_{it} \le \sum_{t\in T} \tilde{f}_{it} \qquad \forall i \in N', \forall t \in T \tag{3}$$

$$\sum_{l\in L} x_{ijlt} \le u_{ijt} f_{it} \qquad \forall (i,j) \in A, \forall t \in T \tag{4}$$

$$\sum_{l\in L} x_{ijlt} \le u_{ijt} f_{jt} \qquad \forall (i,j) \in A, \forall t \in T \tag{5}$$

$$\sum_{i\in \acute{N}} h_{irt} \tilde{f}_{it} \le v_{rt} \qquad \forall r \in R, \forall t \in T \tag{6}$$

$$f_{i0} = 0 \qquad \forall i \in N' \tag{7}$$

$$x_{ijlt} \ge 0 \qquad \forall (i,j) \in A, \forall l \in L, \forall t \in T \tag{8}$$

$$\delta^{+}_{ilt}, \delta^{-}_{ilt}, \ge 0 \qquad \forall i \in N, \forall l \in L, \forall t \in T \tag{9}$$

$$f_{it}, \tilde{f}_{it} \in \{0,1\} \qquad \forall i \in N, \forall t \in T \tag{10}$$

The functions above only consider a single layer network with one decision-maker and without considering the interdependency relationships with any other networks. To evaluate the best response of the decision-maker subjected to the strategy selected by the other decision-maker, these interdependencies must be considered. The decision-maker should know the earliest time that an interdependent child component can be recovered or functional. That is, the information regarding the parent node recovery time is important for the decision-maker to prioritize the recovery sequence of the disrupted nodes in its sub-network. The recovery time of the parent components is considered as known input information, representing the recovery strategy of the other decision-maker in the system. This information must be provided to solve the MTD-INDP by considering interdependent relationships. Two major behaviors can be recognized considering interdependency relationships. First is that the disrupted child node cannot be recovered and subsequently be functional unless the corresponding parent node in the other sub-network is functional. Second is that the decision-maker can recover the child node regardless of the functionality status of it correspondent parent node, though it cannot be functional unless the parent node is functional.

$$\sum_{t\in T'} \tilde{f}_{it} = 0 \qquad \forall i \in N' \tag{11}$$

$$\sum_{t\in \acute{T}'} f_{it} = 0 \qquad \forall i \in N' \tag{12}$$

Using Eqs. (11) and (12), we can force the model to not recover (or make functional) a child node in periods where the parent nodes in the other sub-network have not yet been repaired. Subsequently, as this model can be solved optimally, the decision-maker's best response given the strategy selected by the other decision-maker regarding the recovery time of the parent components can be achieved. However, this conditional best response guarantees the best action for only one of the decision-makers in the systems. At the same time, it cannot find the best response for all the decision-makers in the system and subsequently calculate the Nash equilibrium as the solution for the entire problem. Therefore, to achieve the Nash equilibrium, we have developed an algorithm that uses the MTD-INDP model in a repetitive process to calculate the Nash equilibrium.

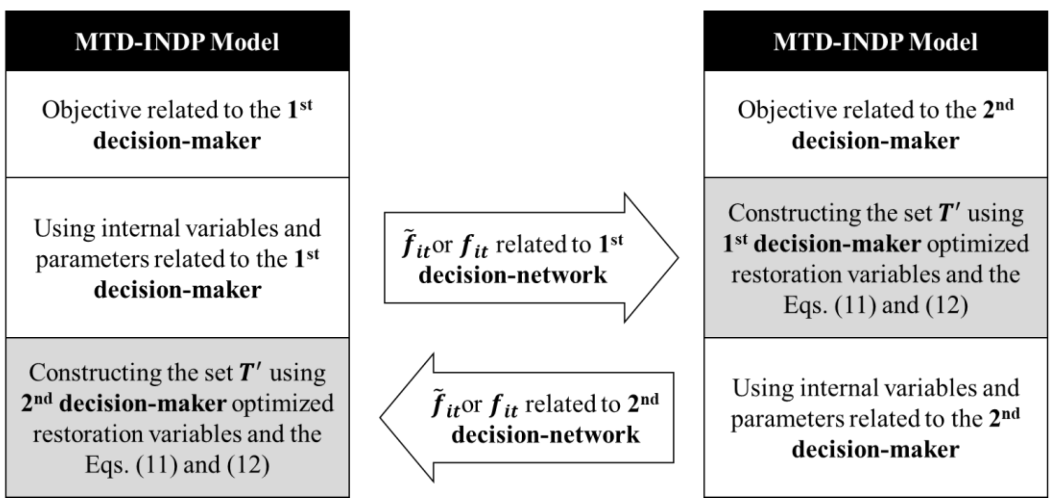

Figure 1: Schematic behavior of the algorithm

It worth noting that the information regarding each decision-maker's strategy related to its associated parent component will be transferred to other decision-maker in every iteration of the developed algorithm which we discuss in following sections. Figure 1 provides a schematic representation of the developed algorithm, depicting the process of transferring information between decision-makers responsible for restoring each interdependent sub-network. To give you a brief overview of how the developed algorithm works, it basically solves the MTD-INDP model for one decision-maker, to determine the best and most optimal response to the strategy that another decision-maker has chosen. This allows us to establish the recovery time of the parent components that are associated with that decision-maker. Once we know the recovery time for these parent components, we can then determine the earliest time that the relative child components in other sub-networks can be recovered or become functional. With these times, we can form a set T' that pertains to the child components of the other sub-network. This set is essential for providing the external input required in the construction of Eqs. (11) and (12) related to the other decision-maker.

2.3.2. Algorithm to Find the Nash Equilibrium

In non-cooperative game theory, there exists a Nash equilibrium when none of the active players does have any incentive to change their selected strategy. In other words, a Nash equilibrium is a solution when all decision-makers select the best response given the strategies selected by other decision-makers. Although the MTD-INDP model guarantees the best response for one decision-maker, the given strategies selected by the other decision-makers are unknown to be the best response to the best strategy selected by the other decision-maker. Therefore, finding a way to guarantee the best response for all the decision-makers while considering the strategy of the other decision-maker for their recovery plan enables us to achieve the Nash equilibrium.

Furthermore, related to the interdependency relationship between nodes, it is assumed that interdependent networks consist of three types of nodes: (i) independent nodes (O), (ii) parent nodes (P), and (iii) child nodes (H). Independent nodes have no relationship with the other sub-networks, and the decision-maker can determine their recovery without considering the interdependency relationship with other sub-networks. Parent nodes are also considered as an independent node where its recovery can affect the functionality of a child node in another sub-network. The child node, on the other hand, can be recovered by the decision-maker, but it cannot be functional until its associated parent node in the other sub-network is functional. Also, we denote S_m as the set of all possible strategy solutions related to decision-maker m where $m \in M$. Using MTD-INDP notation, s_m is a single strategy belong to S_m, representing the recovery sequence of the disrupted components in the sub-network related to the decision-maker m where: $s_m = \{\tilde{f}_{it} |\ i \in N', i \in P, t \in T, \tilde{f}_{it} \neq 0\}$. Assuming each decision-maker has sufficient resources to restore only one disrupted component in each period, the number of possible strategies in S_m would be: ${}^{|N'|}Pr_{|P|} = \frac{|N'|!}{(|N'|-|P|)!}$, calculated as the permutation (function Pr) of parent nodes from the total number of disrupted nodes in the sub-network. Therefore, although there is a finite number of strategies that can be selected by a decision-maker, this number can be quite high considering the size of disruption in the network, as well as the number of interdependency relationships among the sub-networks. The most straightforward approach to find the Nash equilibrium is to consider all strategy profiles among the decision-makers, provided they have feasible strategies, in the system and find the Nash equilibrium using the associated payoff that each decision-maker could have. Such an approach would result in exponential growth in problem size increasing the computational complexity searching the solution space for finding the Nash equilibrium. Considering the number of decision-makers in the system having the same network characteristics, the total number of strategy profile would be $\left({}^{|N'|}Pr_{|P|}\right)^{|M|}$, where $|M|$ is the total number of the decision-makers in the system. Using MTD-INDP model, evaluating the decision-makers' best response given the selected strategy by the other can dramatically reduce the problem size. Note that given the strategy forced by the other decision-maker, using the MTD-INDP model gives us the best response for the decision-maker, eliminates the need for investigating many of strategy profiles where their payoff is less than the best response since the best response dominates the other strategy profile given the forced circumstances implemented by the other decision-maker. Therefore, assume having two decision-makers in the system, the size of the problem would be reduced to $2 \times \left({}^{|N'|}Pr_{|P|}\right)$, which is a significant improvement in complexity. Mentioned previously, securing the best response for one of the decision-makers does not guarantee the best response for the others and subsequently achieving a Nash equilibrium. Denote s_1^* and s_2^* as the best response for the decision-makers 1 and 2, respectively. The following condition is required to be maintained if the strategy profile is a Nash equilibrium.

Lemma 1: Optimizing the MTD-INDP for decision-maker 2 given s_1, the strategy profile (s_1, s_2^*) is a Nash equilibrium if $s_1 = s_1^*$, when optimizing MTD-INDP for decision-maker 1 given the same s_2^*.

Proof: As $U_2(s_1, s_2^*) \geq U_2(s_1, s_2)$ and $U_1(s_1^*, s_2) \geq U_1(s_1, s_2)$, then using the MTD-INDP given s_2^*, if $s_1 = s_1^*$, then as $U_1(s_1^*, s_2^*) \geq U_1(s_1, s_2^*)$, and $U_2(s_1^*, s_2^*) \geq U_2(s_1^*, s_2)$, therefore strategy profile (s_1, s_2) is a Nash equilibrium as none of the decision-makers has the incentive to select a strategy with a better payoff value given the selected strategy by the other decision-maker.

That is, solving the MTD-INDP for the second player given the first player's strategy would be the best response for the second player. By fixing the second player strategy considering the best response and solving the MTD-INDP model for the first player, the best response for the first player is the result. Now if the first players' best strategy is equal to the strategy selected earlier, then we can conclude that the solution profile of both decision-makers calculated strategy is a Nash equilibrium. Using Lemma 1, we propose Algorithm 1 to find the Nash equilibrium for the interdependent network restoration problem. Assuming that there are two decision-makers in the system, we select one of the decision-makers in the system to start with (e.g., decision-maker 1). In the first stage, we optimize the recovery strategy for the first decision-maker assuming that no predecessor is required for any of the child components in the sub-network. That is, we assume that predecessor associated with sub-network's child components have been resolved before the time that these child components are required to be recovered in the ideal situation. Note that the provided solution in this stage can be considered the upper bound and maximum payoff for the first decision-maker because the second decision-maker shows submissive behavior assumed to serve the first decision-maker. We considered the evaluated best strategy of the first decision-maker $\hat{s}_1^*$ as the starting point for our algorithm. Note that naturally, it is in favor of the decision-makers not to deviate from this upper bound solution. Therefore, the Nash equilibrium is more likely to occur close to this ideal solution, making the algorithm to find the first Nash equilibrium faster with less computational effort.

Algorithm 1. Nash equilibria search algorithm.

Input initialization:

1. Optimize the recovery strategy for each network (N_1, N_2) regardless of any interdependencies using MTD-INDP model
2. Select the optimal solutions for the first network as candidates for starting point of the Nash equilibrium search algorithm $s_{1,0}$ and inserting them into the row candidate list (RO)
3. Fill up the solution pool (SP) list with optimal and non-optimal solutions of N_1 and N_2 for the purpose of jumping form Nash equilibrium, provide one has been found

Main loop:

4. While SP is not empty do:
5. $\quad r = r + 1$
6. $\quad$ **Performing the row operation for all $s_{1,r}$ in RO:**
7. $\quad\quad$ Find the best responses of $s_{2,r}^*$ for N_2 considering the candidate strategy of $s_{1,r}$ of N_1 using MTD-INDP model
8. $\quad\quad$ if $r > 1$, then:
9. $\quad\quad\quad$ if $s_{2,r}^* = s_{2,r-1}^*$, then:
10. $\quad\quad\quad\quad$ add $(s_{1,r}^*, s_{2,r}^*)$ to the Nash equilibria list (NE)
11. $\quad\quad\quad$ else: add $s_{2,r}^*$ into the column candidate list (CO)
12. $\quad\quad\quad$ remove $s_{1,r}^*$ from RO
13. $\quad\quad\quad$ if CO is empty, then: remove a solution from SP and insert it into CO
14. $\quad$ **Performing column operation for all $s_{2,r}$ in CO:**
15. $\quad\quad$ Find the best response of $s_{1,r}^*$ for N_1 considering the candidate strategy of $s_{2,r}$ of N_2 using MTD-INDP model
16. $\quad\quad$ if $s_{1,r}^* = s_{1,r-1}^*$, then:
17. $\quad\quad\quad$ add $(s_{1,r}^*, s_{2,r}^*)$ into the NE

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18. else: add $s^*_{1,r}$ into the RO
19. remove $s^*_{2,r}$ from CO
20. if RO is empty, then: remove a solution from SP and insert it into RO

21. **Output:** Nash equilibria list (NE)

Two types of operations have been defined in the main loop of the algorithm: (i) row operation, and (ii) column operation. Row operation refers to the situation where the second decision-maker optimizes their strategy for their assigned sub- network recovery given a fixed recovery strategy for the first decision-maker. The column operation is similar to the row operation except in this case the first decision-maker optimizes its sub-network recovery strategy given the second decision-maker's strategy. In each repetition, using the conclusion from Lemma 1, if the optimized strategy is the same as the strategy in the previous iteration considering each of the row or column operations, the calculated strategy is considered to be a Nash equilibrium. Note that in each operation, the MTD-INDP optimization problem could have multiple optimal solutions. Therefore, all these multiple optimal solutions are required to be considered in row or column candidate lists to be investigated for finding the Nash equilibrium, as mentioned in the algorithm. As the problem could have several Nash equilibria, the algorithm focuses on the same solution no matter how many time the row and column operation is performed. Therefore, to search the other parts of the solution space whenever a Nash equilibrium is found, we need to jump from a Nash equilibrium strategy to a non-Nash equilibrium strategy to renew the starting point of the algorithm. In this regard, a solution pool list has been considered in the algorithm consisting of the optimal and non-optimal solutions acquired during initialization process when optimizing the strategies for both decision-makers without considering the interdependent relationship. As the associated variables in relation to node recovery and functionality status are binary, several non-optimal solutions can be acquired while solving the problem to the optimality considering various optimality gaps. Such an optimality gap can be adjusted to decrease or increase the number of solutions in the solution pool list of the algorithm. Adjusting the optimality gap enables keeping only solutions considering their closeness to the optimal solution for a specific sub-network recovery strategy where the other decision-maker shows a submissive behavior regarding that sub-network. Using this solution pool, we consider another solution as a new starting point of the algorithm whenever a Nash equilibrium is found.

2.3.3. Lower Bound for Decision-maker Strategy

The algorithm is a repetitive algorithm searching for the Nash equilibria in the strategy profile set. While minimal iterations are sought, the iterations required to find all Nash equilibria could be time-consuming, especially when the number of disrupted components and the interdependency relationships between the sub-networks are large. As we can calculate the optimal strategy for each decision-maker assuming submissive behavior from other decision-makers, a fixed ideal upper bound can consistently be recognized for each decision-maker. In that case, none of the decision-makers can gain a better payoff using the MTD-INDP model. Note that the submissive behavior is regarded as the situation in which no predecessor recovery is required for the functionality of child components of the sub-network.

On the other hand, we believe that a lower bound strategy can always be recognized for each decision-maker payoff in each iteration of the algorithm. Therefore, even if the algorithm is unable to find any Nash equilibrium due to time limitations, we can always determine a strategy profile related to all decision-makers in the system as a sub-Nash equilibrium.

The proposed algorithm is a loop-based algorithm performing a back-and-forth procedure, optimizing one decision-maker's strategy provided the fixed and known strategy of the other decision-maker one at the time. In each procedure, we optimize the strategy once for each first and second decision-maker in the system by fixing the strategy for the other, assuming only two decision-makers in the system.

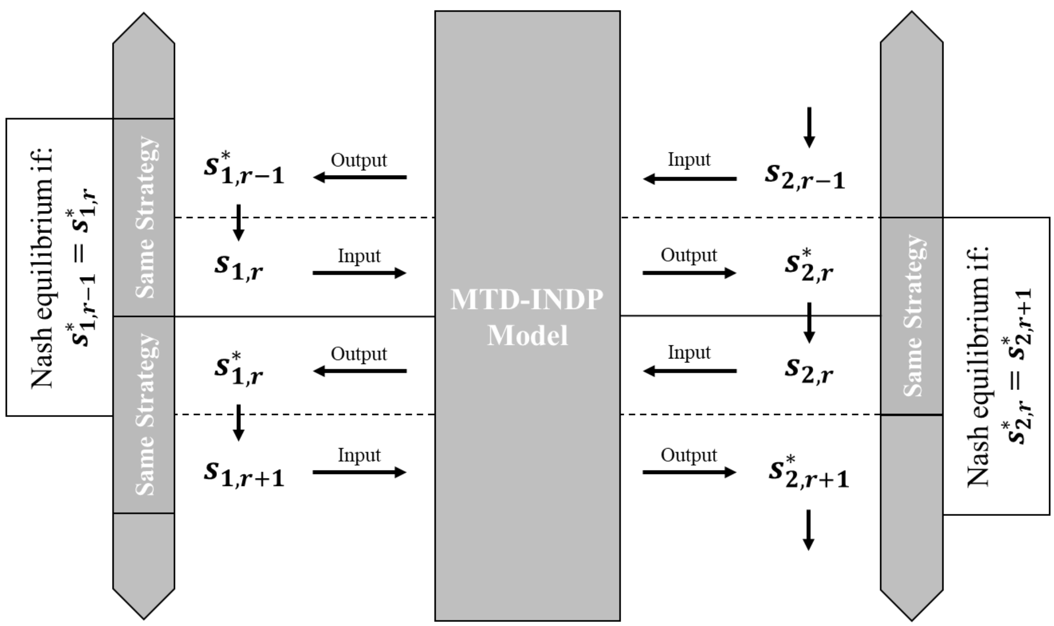


Figure 2: Schematic behavior of the algorithm

Considering $s_{1,r} = s^*_{1,r-1}$ as the fixed strategy of decision-maker 1 in algorithm repetition of r, $s^*_{-m,r}$ would be the decision-maker 2's best strategy of the algorithm repetition of r using the MTD-INDP model in the algorithm row operation. Subsequently, considering $s_{2,r} = s^*_{2,r}$ as the fixed strategy of decision-maker 2 in the same algorithm repetition r, we can calculate $s^*_{1,r}$ as the best strategy for the decision-maker using the MTD-INDP model in the column operation of the algorithm. Note that using the algorithm, $s_{1,r} = s^*_{1,r-1}$. Furthermore, using conclusion from Lemma 1, the Nash equilibrium can be found in the case that any of the following conditions, $s^*_{1,r-1} = s^*_{m,r}$ or $s^*_{2,r-1} = s^*_{2,r}$, occurs. The schematic behavior of the algorithm in selecting the strategies is illustrated in Figure 2.

Lemma 2: Using the algorithm and the MTD-INDP model to find the best response given the situation, we can consider $U_{1,r}\left(s_{1,r}, s^*_{2,r}\right)$ and $U_{2,r}\left(s^*_{1,r}, s_{2,r}\right)$ as the lower bound for the payoff function for decision-makers 1 and 2, respectively.

Proof: Considering the strategy profile of $\left(s_{1,r}, s^*_{2,r}\right)$ and $\left(s^*_{1,r}, s_{2,r}\right)$, we can realize that $s^*_{2,r}$ and $s^*_{1,r}$ are the best responses to the selected strategies $s_{1,r}$ and $s_{2,r}$ related to decision-makers 1 and 2, respectively, at the rth algorithm repetition. Therefore, we can recalculate $U_{1,r}$ and $U_{2,r}$ by assuming $s_{2,r} = s^*_{2,r}$ and $s_{1,r} = s^*_{1,r}$ as the fixed strategy of the decision-makers 2 and 1, respectively, while keeping their previous strategies. Note that we do not need to use MTD-INDP models for calculating the decision-makers' payoff as the strategies for both sides are known. Therefore, we can realize that $U_{1,r}\left(s_{1,r}, s_{2,r}\right)$ and $U_{2,r}\left(s_{1,r}, s_{2,r}\right)$ are the minimum payoff that decision-makers 1 and 2 can receive by keeping their previous strategy against the best selected strategy by decision-makers 2 and 1, respectively. Therefore, the following conditions in Eqs. (13)-(14) holds for decision-makers 1 and 2 payoff.

$$U_{1,r}\left(s_{1,r}^{*}, s_{2,r}\right) \geq U_{m,r}\left(s_{1,r}, s_{2,r}\right) \geq 0 \tag{13}$$

$$U_{2,r}\left(s_{1,r}, s_{2,r}^{*}\right) \geq U_{2,r}\left(s_{1,r}, s_{2,r}\right) \geq 0 \tag{14}$$

Therefore, $U_{1,r}\left(s_{1,r}, s_{2,r}\right)$ and $U_{2,r}\left(s_{1,r}, s_{2,r}\right)$ where $s_{2,r} = s_{2,r}^{*}$ and $s_{1,r} = s_{1,r}^{*}$ can always be the lower bound of the payoff for decision-makers 1 and 2, respectively.

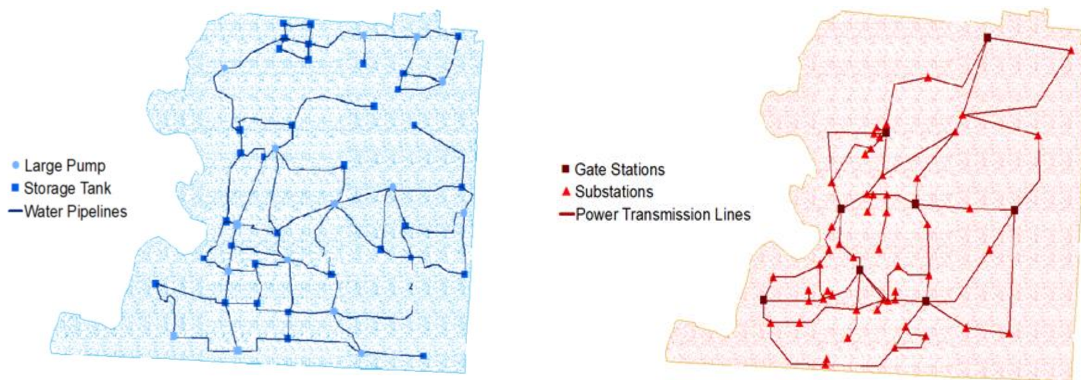
That is, after any of the row or column operations of the algorithm, at least we have one decision-maker with the best response to the selected strategy of the other decision-maker. Regardless of conclusion of Lemma 1, although we cannot guarantee that the selected strategy is not the best response to the best response of the other decision-makers, the selected strategy produces a lower bound for the first decision-makers payoff. Therefore, if the first decision-maker does not move from its selected strategy, this lower bound is the minimum payoff that the decision-maker can receive. As the algorithm searches for Nash equilibria, lower bounds for each decision-maker are determined. Tracing the lower bounds through algorithm repetitions, we can always consider the one that the decision-maker benefits from the most. Therefore, regardless of the Nash equilibrium, the decision-maker does not have any incentive to select strategies producing a payoff lower than the lower bound payoff.

Knowing that we can always have a lower bound in each algorithm repetition for both decision-makers, we also can conclude that the problem could have a minimum of one Nash equilibrium. When we calculate the best response for one decision-maker, we can conclude that Nash equilibrium has been found if the associated lower bound strategy is equal to the best response strategy. Otherwise, one decision-maker can always realize a strategy with a better payoff. In cases where that are no solutions with a better payoff than the lower bound, the lower bound would be the best response of the decision-maker. In the algorithm, we already know that the other decision-maker's strategy is the best response against the one decision-maker's lower bound strategy. Therefore, in the case of considering the lower bound strategy as the best response, both decision-makers are playing the best response strategy against each other. Subsequently, the strategy profile selected by both decision-makers would be Nash equilibrium. Furthermore, when there are multiple decision-makers involved, achieving the lower bound is only possible if the other decision-makers have already selected their best response strategy. This is because, in such cases, the selected strategy by the decision-maker can provide it with the minimum possible payoff as other decision-makers have no incentive to change their strategy. Also, in that case if there is no alternative available to the decision-maker, we can consider the lower bound as the Nash equilibrium.

2.4. Discussion and Results

To investigate the capability of the proposed algorithm in dealing with the various infrastructure networks, we have used two types of networks: one related to a realistic interdependent network and the other randomly generated. For a realistic network, we have applied our developed algorithm to the interdependent infrastructure network of Shelby

County, TN, USA [19,68,69], assuming two different decision-makers responsible for the recovery of each one of the water and power sub-networks, respectively. Figure 3 illustrates the Shelby County, TN water and power sub-networks, where each consisted of 49 and 60 components and 71 and 76 links, respectively.



(a) (b)

Figure 3. Shelby County, TN network topology for the (a) water and (b) power sub-networks (adapted from González et al., (2016)).

As mentioned before, we have considered each decision-maker to be associated with the recovery of only one of the sub-networks, being rewarded exclusively by minimizing the unmet demand of its assigned sub-network. We assume that the interdependent relationships between the sub-networks are the source of the conflict of interest between the decision-makers. Generating random disruption scenarios, we considered the different sizes of the disrupted component set by randomly selecting some interdependent relationships between the sub-networks and considering the components involved in those relationships as disrupted. Considering the Shelby County TN dataset, we have considered three different sizes of random disruption scenarios with 10, 20, and

30 interdependent disrupted relationships between the water and power sub-networks, respectively. As two components are involved in every interdependent relationship, parent and child, the total number of disrupted components in each designed disruption scenario would be 20, 40, and 60, respectively. We calculate the Nash equilibria strategies using the proposed algorithm. The recovery sequence for each decision-maker and its respective unmet demand related to each decision-maker strategy is then calculated.

Also, the algorithm's capability is tested against the randomly generated networks considering the different sizes of networks and the percent of connectivity between the sub-networks, checking the effect of each parameter on the time required for the algorithm to find the first Nash equilibrium of the problem. We designed a separate algorithm to generate a random network for each sub-network, considering the number of transshipment, supply, and demand components to be 40, 20, and 40 percent of all components in a sub-network, ensuring that every transshipment component degree is greater or equal to 2, so that we do not have any dead ends in the network not having any supply or demand. The parameters of the networks (e.g., flow cost and required supply and demand) have been randomly generated using normally distributed values whose parameters are found in Table 2 (with positive values only). To generate the random values for the network flow cost, we have used the normal distribution with the mean and standard deviation of 30 and 8, respectively for every different size of networks. Also, we have defined the amount of supply or demand as proportional to the number of components with supply or demand attributes in the correspondent network. Finally, considering the number of supply and demand components in the network, we set the maximum amount of supply or demand a network could generate, distributing them among the supply or demand components of the network using the normal distribution. We have set the mean automatically as the maximum supply or demand quotient based on the number of components in their category. Also, it was assumed that the standard deviation is a tenth of the mean value. Note that three levels of network size have been considered for the sub-networks: 20, 30, and 40 components per sub-network. As we have two sub-networks, the total number of components for the whole network is doubled. Three connectivity percent levels between the sub-networks have been considered for every scenario: 20%, 35%, and 50%.

Table 2: Normal distribution parameters (mean, standard deviation) for components' supply and demand.

	Water network components		Power network components	
Network size	Supply	Demand	Supply	Demand
40	(171, 17.1)	(167, 16.7)	(200, 20)	(166, 16.6)
60	(130, 13)	(77, 7.7)	(120, 12)	(111, 11,1)
80	(100, 10)	(67, 6.7)	(92, 9.2)	(77, 7.7)

2.4.1. Shelby County Examples

We have considered three various random disruption scenarios, considering different numbers of disrupted interdependent links between the sub-networks. Note that interdependent links consist of two components on each sub-network, providing a connection between the sub-networks. Therefore, applying the proposed algorithm, we searched for the Nash equilibria in the solution space belonging to each decision-maker responsible for recovering their sub-network. However, to find all Nash equilibria for the problem, it is required to search all the solution spaces of the problem, which could be very time-consuming and sometimes practically impossible considering the size of the sub-networks and the number of interdependent relationships. Note that theoretically, the solution space can be searched using the developed algorithm as the solution space is finite. Therefore, we are not bounding ourselves searching all solution space, but finding quite a handful of them would be generally enough to provide agreement between decision-makers.

To compare the performance of the strategy selected by each decision-maker, we need to compare them with the ideal strategy wherein neither of the decision-makers have independency limitations. That is, the ideal strategy can be defined as the strategy where neither of the decision-makers is required to wait for the other to act. Therefore, to find the ideal situation, which is not feasible, we need to relax the problem of minimizing unmet demand without considering the interdependency requirement. This reduces the problem to centralized optimization of one decision-maker objective function responsible for only one sub-network, using the MTD-INDP model without considering Eq. (11) and forces the precedence of the interdependent child components recovery in the sub-network. Therefore, we refer to the strategy resulting from this centralized optimization as relaxed, ideal strategy (RIS). Subsequently, as RIS is a relaxed centralized optimization, the decision-makers' recovery strategies in Eqs. (15)-(16) would always hold.

$$U_{RIS_1} \geq U_1(s_1^*, s_2^*) \geq U_1(s_1, s_2^*) \tag{15}$$
$$U_{RIS_2} \geq U_2(s_2^*, s_1^*) \geq U_2(s_2, s_1^*) \tag{16}$$

U_{RIS} is the decision-makers payoff, equivalent to the relaxed version of the MTD-INDP model objective function. The MTD-INDP model objective function calculates the cumulative cost of the network in the sense of unmet demand, excess supply, and recovery cost through the defined time horizon. Therefore, this cumulative dynamic cost must be calculated for both decision-makers to be an excellent baseline for the selected strategy assessment.

Figure 4 illustrates the cumulative unmet demand cost relative to each decision-maker responsible for their assigned sub-network through the time horizon for various defined scenarios of 20, 40, and 60 disrupted interdependent components in the whole network.

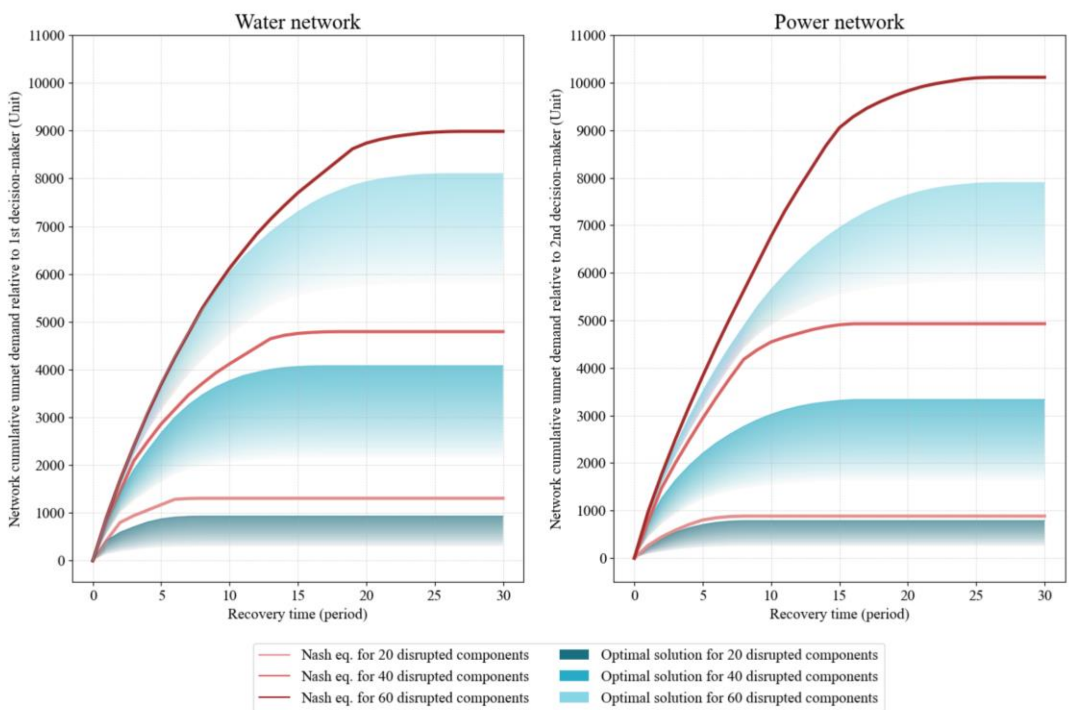


Figure 4: Cumulative unmet demand cost of each sub-network.

If we consider the Nash equilibrium strategy, a decision-maker may achieve a better payoff that is closer to its relaxed ideal strategy compared to the other decision-maker, indicating that one decision-maker has an advantage in bargaining power over the other.

The crucial factor is the importance level of the child and parent interdependent components for decision-makers as this affects their ability to reduce the unmet demand of their associated sub-networks. Suppose there is an instance where a parent component has low importance for a decision-maker. As a result, the decision-maker has little motivation to prioritize the early restoration of that particular component. Conversely, if the corresponding child component holds significant importance for the other decision-maker, then there may be a longer wait until the parent component is repaired, leading to a prolonged period of unmet demand. Therefore, implementing a direct or indirect approach to encourage the restoration of such parent components could potentially improve the overall performance of the network restoration plan. One possible approach to encourage such restoration efforts could involve offering rewards or imposing penalties by a third party who has the authority to regulate the relationship between decision-makers, with the aim of maximizing cooperation.

In addition, it is important to consider the technical aspect of identifying the Nash equilibrium, particularly in terms of the time it takes to find the first Nash equilibrium using the proposed algorithm, while taking into account the size of disruption and the level of conflict between decision-makers. Therefore, considering the defined disruption scenarios, the time to find the first point of agreement (first Nash equilibrium) between the decision-makers has been illustrated in Table 3. The algorithm was processed on a desktop computer that has an Intel Core i5 processor with 3.2 GHz and four cores and 8GB DDR3 memory. In addition, we used Python and the Gurobi package to implement the algorithm and optimize the MTD-INDP model, which calculates the best response for each decision-maker.

Table 3: Time (seconds) to find the first Nash equilibrium.

Number of disrupted interdependent components	Computation time (seconds)
20	12
40	181
60	9530

Table 3 shows that the required time to find the first Nash equilibrium increases as the number of disrupted interdependent components increases in the network. There are two reasons for this: (i) the combinatorial nature of the problem, as mentioned in the methodology, and (ii) the existence of several optimal solutions when we optimize the recovery plan of each decision-maker considering the strategy selected by the other decision-maker in every repetition of the algorithm. The calculation complexity is increased as we force the algorithm to find the best response for all equally optimal strategies selected by the other decision-maker in every repetition of the algorithm.

2.4.2. Calculating the Cost of Anarchy

In the event of a service outage in the infrastructure network, the restoration process can become difficult and costly due to conflicting interests and objectives among the involved parties. Individual decision-makers may prioritize their own interests over the broader goal of restoring the network, which can undermine the collective restoration efforts and increase the overall cost of repairing the infrastructure network. Consequently, the cost of restoration can be higher than it would have been if all parties had collaborated towards a shared objective. Therefore, in this section, we have attempted to find a way to compare the costs of centralized decision-making, assuming that all decision-makers are ideally collaborative and completely aligned, with the costs incurred by the network due to the presence of decentralized decision-making for infrastructure network restoration. Thus, we incorporated the interdependency relationships constraint inspired from the TD-INDP model as shown in Eqs. (17) and (18) to exclude other decision-makers from the process and address the issue in a centralized manner. To address the interdependency between sub-networks in centralized decision-making, we reformulated the problem by adding indices to the variables and associated parameters to account for the interdependent sub-networks. Note that the proposed algorithm requires only a single-layer network recovery optimization because decision-makers have control only over their associated sub-network components. Therefore, to calculate the effect of having a multi-layer network in a decentralized decision-making environment, we used Eqs. (11) and (12). However, to incorporate interdependency relationships in a centralized decision-making environment, we used Eqs. (17) and (18) with layer indices of k representing each sub-network instead of using Eqs. (11) and (12). Note that Eq. (17) is used when the recovery of component i in sub-network k logically depends on the functionality of the component j in sub-network $\tilde{k}$. On the other hand, Eq. (18) does not constrain the recovery of component i in sub-network k, but its functionality would depend on the functionality of the component j in sub-network $\tilde{k}$. This approach assumes that the centralized decision-maker possesses the resources of both other decision-makers.

$$\tilde{f}_{ikt} \leq f_{j\tilde{k}t} \qquad \forall i \in N'_k, \forall j \in N'_{\tilde{k}}, \forall k, \tilde{k} \in K, \forall t \in T \qquad (17)$$

$$f_{ikt} \leq f_{j\tilde{k}t} \qquad \forall i \in N'_k, \forall j \in N'_{\tilde{k}}, \forall k, \tilde{k} \in K, \forall t \in T \qquad (18)$$

By considering the interdependency constraint and having a single decision-maker in charge of all aspects of the interdependent network, any conflicting interests among decision-makers can be resolved, resulting in the most efficient solution for the restoration plan. Also, we can evaluate the cost of anarchy by examining the cost of unmet demand in a network if we compare the cost of a centralized decision-making restoration plan with that of a decentralized one which consider the costs incurred by both decision-makers. This cost of anarchy represents the additional unmet demand cost imposed on the network if each decision-maker is allowed to pursue their own interests.

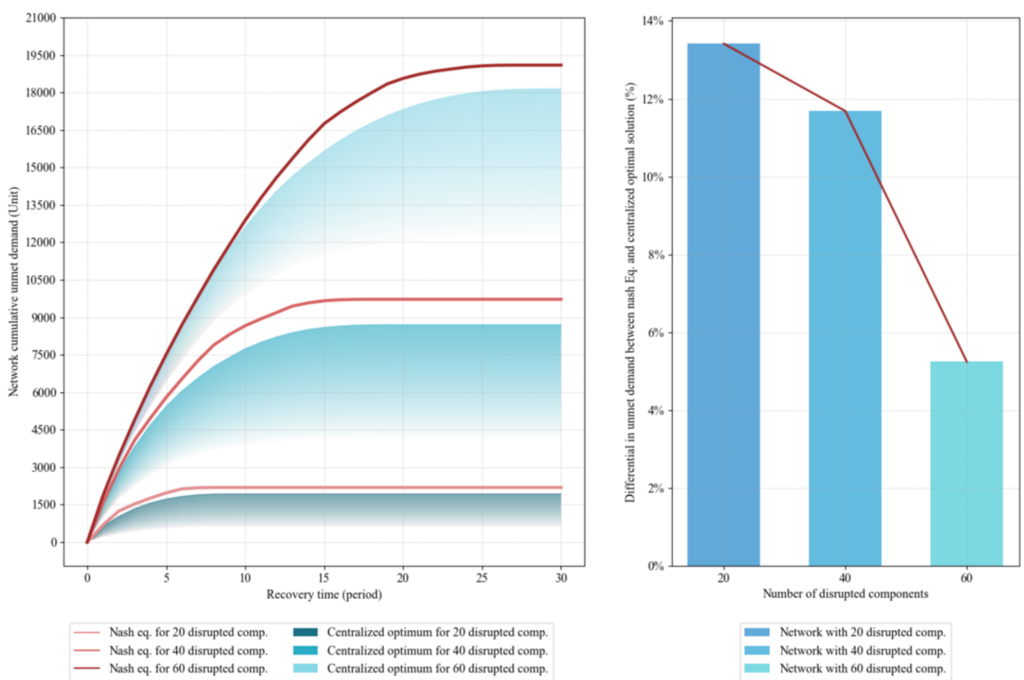


Figure 5: Cumulative unmet demand cost for decentralized vs. centralized decision-making.

To determine the cost of anarchy, we analyzed various scenarios of random disruptions, which we previously utilized in the Shelby County examples in section 4.1. By utilizing the MTD-INDP model, which incorporates interdependency constraints, we computed the total cost of unmet demand for each scenario. Additionally, we computed the total cost of unmet demand for the decision-makers' restoration strategies associated with their Nash equilibrium by summing up the cumulative costs incurred by each decision-maker. The results are shown in Figure 5.

Note that the total unmet demand of each sub-network when interdependency constraints were not considered is lower than the optimal solution of the centralized problem when interdependency constraints were considered. Naturally, the cost of unmet demand for centralized decision-making would be lower than the decentralized approach. However, as the size of the disruption increases, the proportion of the cost of anarchy decreases relative to the centralized cost of unmet demand. Therefore, we can conclude that the cost of decentralized decision-making grows closer to the centralized approach as the scale of disruption increases. This is due to the decreasing proportion of the cost of anarchy compared to the cost of unmet demand for centralized decision-making.

2.4.3. Generated Network Examples

In this section, we have designed an experiment to realize the effect of network size and interconnectivity between the sub-networks on the time required to find the first Nash equilibrium using the proposed algorithm. First, we defined the number of components in the network as an index for network size. Next, we considered the percentage of connected components between the sub-networks as an index for the interdependency of the network. The response for the experiment is the time spent to find the first Nash equilibrium using the proposed algorithm. To perform this experiment, we considered both low and high levels for both variables. 40 and 80 components (20 and 40 for each sub-network) have been considered for the low and high levels of the variable, representing the network size, respectively. Also, 20% and 50% have been considered for the experiment low and high levels of the variable representing the interdependency between the sub-networks. To realize if we have significant curvature in the response surface, we have considered two center points for the experiments with a network size of 60 components (30 for each sub-network) and 35%of interdependency. Augmenting the experiment, we performed two other experiment configurations with network component sizes of 40 and 80 while keeping the interconnectivity at 35%. Table 4 shows the summary of the experiment configuration.

Table 4: Experiment configuration summary.

Experimented point summary	Number of points	Factor A: Network size	Factor B: Interdependency
Corner Points	8	40 – 80	20% - 50%
Center Points	2	60	35%
Augmented Points	2	40 – 80	35%

Note in Table 4 that the experiment for corner points has been repeated twice for every corner, while the augmented points are located at the edge of the experiment by considering variable factor A and fixed factor B. Using the natural log transformation, a two-level factorial experiment design was conducted. The resulting ANOVA table is shown in Table 5.

Table 5: ANOVA summary.

Source	Sum of squares	DF	F stat	p-value
Model	**61.95**	**3**	**84.92**	**< 0.0001**
A: Network Size	32.27	1	132.72	< 0.0001
B: Interdependency	21.64	1	89.01	< 0.0001
AB	8.03	1	33.02	0.0007
Curvature	0.2339	1	0.9618	
Residual	**1.70**	**7**		
Lack of fit	0.6233	2	1.44	0.3198
Pure error	1.08	5		

The model and both factors (network size, interdependency) and their interaction are significant. On the other hand, the curvature is insignificant, meaning that the curvature term can be removed to have a simpler model without losing accuracy. Also, the lack of fit this low indicates that there is a 31.98% chance that it could occur due to noise. The predicted R^2 and adjusted R^2 are 0.935 and 0.961, respectively, indicating adequate precision and showing that the model can be used to navigate the designed space. Figure 6 illustrates the predicted response surface of the experimented space relative to the variation in network size and interdependency.

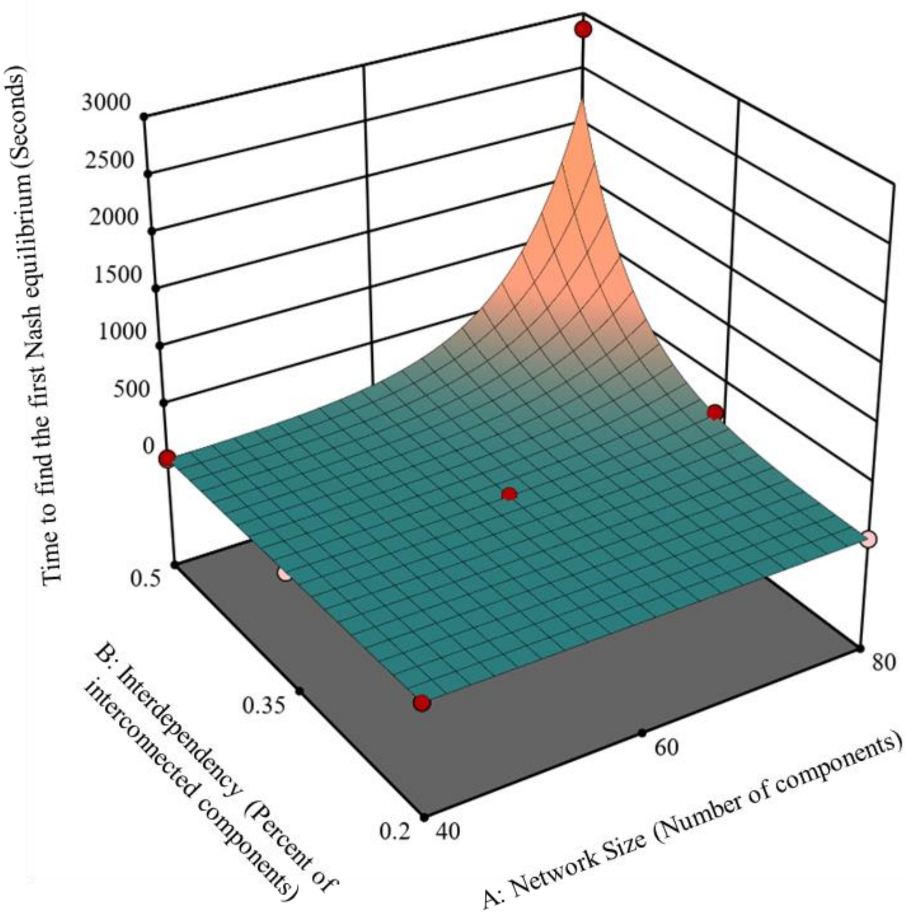


Figure 6: Predicted response surface of the time required to find the first Nash equilibrium.

According to Figure 6, the time required to find the first Nash equilibrium increases exponentially when the network size and interdependency between the sub-networks both tend to their maxima. This suggests a strong performance of the proposed model for small and medium size networks. Also, the calculation complexity could be low if the sub-networks of a network have medium to low interdependency relationships, regardless of the network size. Note that computational time is increased exponentially only by the existence of both factors at high levels. During our experiments, we found that the performance of our algorithm is impacted by the increasing number of multiple solutions with the same payoff value, which often occurs as the problem size grows. This results in a significant increase in computational time since the algorithm is designed to search for all these solutions in each iteration. Hence, we realized that the computational time required for larger interdependent networks could be minimized by modifying the algorithm based on this crucial insight. This important discovery has given us useful ideas to enhance the algorithm and minimize the computational time required for dealing with large-scale interdependent networks by utilizing more efficient search methods which we plan to explore these possibilities in our future research endeavors.

2.5. Conclusion

In this study, we have investigated the interdependent infrastructure network restoration after disruption from the perspective of having decentralized decision-makers in the system, each responsible for a sub-network and only getting rewarded for recovering their allocated components in the network. This scenario has been investigated under the assumption of having indifferent decision-makers considering none of them will get rewarded based on the success or failure of their opponents. As the interdependent parent and child components are the primary source of the bargain and conflict between the decision-makers, we have developed an iterative algorithm on a game-theoretic basis, enable to effectively search the solution space for possible Nash equilibria in the system. Also, we illustrated that a lower bound could be discovered for each decision-maker's best response in every iteration of the algorithm. To achieve a comprehensive understanding of the distinctions between decentralized and centralized decision-making, we performed an analysis on the cost of anarchy in a system where multiple decision-makers have the liberty in pursuing their individual interests. This in-depth investigation enabled us to gain a more understanding of the differences between these two approaches to decision-making. Finally, the algorithm's performance has been tested using several scenarios considering the actual interdependent network and randomly generated ones. The time required for the algorithm to find the Nash equilibrium has been considered an index for testing the algorithm's performance. Furthermore, we note that the network restoration plan's overall performance can be improved by incentivizing the restoration of parent components that might not be of significant importance to their respective decision-makers but hold great significance to other decision-makers due to their connected child components. The reason for this is that parent components deemed less important to an individual decision-maker are typically not restored as quickly, causing child components of higher importance to wait longer in the restoration queue. As a result, this leads to a higher level of unmet demand within the entire system. One potential avenue for future research involves exploring the effectiveness of incentivizing restoration efforts in a decentralized decision-making environment. Building on the findings of this study, it may be beneficial to investigate the feasibility of offering rewards or imposing penalties by a third-party entity that has the authority to regulate the relationship between decision-makers. By doing so, this approach could potentially increase cooperation and collaboration among decision-makers, thereby improving the overall effectiveness of the restoration process. Also, we can conclude that as the scale of disruption increases, the performance of a decentralized approach becomes more comparable to that of the centralized approach. This is because the proportion of the cost of anarchy decreases in relation to the cost of unmet demand for centralized decision-making. Our observations highlight the potential benefits of adopting a decentralized decision-making approach for managing disruptions, particularly in large-scale scenarios where the cost of anarchy can be relatively small compared to the centralized cost of unmet demand in the network. Regarding the methodology calculation time performance, we discovered that when both network size and interdependent relationship increased, the time required to find the first Nash equilibria grew exponentially. However, it is not the case for networks with low to medium size and interdependency relationships. The main reason worth mentioning is the combinatorial nature of these problems, which can be exacerbated by adding to the number of decision-makers in the system. Also, considering the behavior of the proposed algorithm, we realized that having multiple optimal solutions given the assumption imposed by the other decision-maker can cause higher computational complexities to the algorithm. Considering methodologies that can reduce the size of the considered network without losing the reality by removing parallel and unimportant transshipment components of the network would reduce the size of the network as well as the number of multiple optimal solutions in every iteration of the algorithm. Finding a way to reduce the available strategies for the decision-makes using the proposed lower bound as a valid inequality to the model seems to be very applicable for reducing the computation time and enhancing the model's performance, which we look to investigate in the future.

3. CHAPTER THREE: ADAPTIVE ALGORITHM FOR DEPENDENT INFRASTRUCTURE NETWORK RESTORATION IN AN IMPERFECT INFORMATION SHARING ENVIRONMENT

3.1. Introduction

Infrastructure networks are complex systems related to flow, movement, or exchange of entities such as electric power, water and wastewater, data and information, and critical goods and services, among many others [1]. Such infrastructure networks are often interdependent, implying that their integration and connection means that the performance of one network can affect others [6–9,69]. These infrastructure networks are regularly becoming more connected due to synergic efficiencies obtained by creating highly interdependent systems [1,2]. Therefore, maximizing the performance of an infrastructure network in isolation neglects the mutual benefits that each network gains from interdependency with other networks, as well as the basic reality of the connectedness of these networks [1,2,4,5].

The proper operation of these interdependent infrastructure networks is crucial for a functioning society, particularly from the perspective of economic productivity, public health, and national security, among others [4,70,71]. Therefore, the restoration of disrupted infrastructure networks has been an important area of study. The traditional approach in solving such problems is to follow a top-down, centralized process, which is more suitable for government strategy developers dictating a solution [49]. As a result, many models and algorithms have been developed to determine restoration schedules for disrupted networks from a centralized perspective, where a complex system is controlled by an overarching decision-maker [41–48]. However, in many real cases, the restoration of infrastructure networks is scheduled in decentralized manner, as different networks are controlled by different entities (e.g., utility companies), potentially autonomous and potentially with conflicting interests [5,12–14,16]. As such, game theory can be effective in describing the effect of the selection of different strategies by one decision-maker on those of another.

Further, oftentimes deterministic assumptions are made about various aspects of the restoration scheduling problem. Such assumptions may not be realistic as in many real decision environments, there exists uncertainty due to a lack of information. This is especially true in the time of disasters in which there is a lack of complete information with which to evaluate the magnitude of the events and resources available for decision-makers across the system of networks [5,72–75]. Without proper information, decision-makers cannot easily anticipate the preference of other decision-makers and their approach to the restoration problem. Generally, information regarding the functional status of various components in a disrupted system of networks is dynamically accumulated over time while the network is gradually recovered [76].

This paper proposes an iterative algorithm using ideas from (i) dictatorial game theory [77] to address the decentralized environment of restoring disrupted interdependent infrastructure networks and (ii) machine learning techniques to predict the restoration actions of other decision-makers under an imperfect information sharing environment. That is, the objective of the proposed algorithm for the follower player is to adapt to the network restoration movements of the leader player in the future when the leader is not willing to share or disclose their restoration schedule for interdependent components. The algorithm employs a modified version of the time-dependent interdependent network design problem (td-INDP) [78] to calculate the best response given the available prediction of the other decision-maker actions. The key attributes of the proposed algorithm are: (i) each network decision-maker (player) is considered to be self-interested, responsible only to minimize the unmet demand of their network, (ii) only one-way interdependency among networks is considered, so that the proposer player dictates their strategy selection to the responder player (representing a dictatorial game), and (iii) the responder player is not aware of the decisions that have been taken by the proposer player and needs to predict the proposer's actions using some features related to the network topology utilizing a machine learning model. Studies in this field have made contributions by considering various assumptions, such as showing cooperative nature among decision-makers where each decision-maker shares its plan with others until they converge into agreement [17], or by being informed gradually from other decision-makers actions without having any prediction of others action [5]. Although both studies illustrate the different perspectives of the problem, the utility decision-making often lies somewhere between contrasting assumptions of full information availability and no information sharing. Generally, neither players in the same decision-making environment have full access to the information of the status of the entire system, nor are they completely uninformed. In the proposed algorithm, we have considered a more realistic approach for predicting the moves of other decision-makers and then incorporating the prediction in the decision-making process as definite information gradually becomes available. Although machine learning is used in various studies, its application in the restoration scheduling of the disrupted interdependent infrastructure networks in a decentralized environment is unique in this paper.

The key contributions of the proposed algorithm are: (i) improving the ability of the responder player to be effectively and dynamically adaptive to the proposer player selected strategies, (ii) combining definite information and the predicted restoration schedule of the other decision-makers using a machine learning approach, and (iii) determining the best response strategy for the responder considering the accuracy of the prediction for the proposer's action. In simple terms, the best strategy response can be achieved for the responder if the proposer selected strategy can be predicted accurately. Therefore, the higher the accuracy of the prediction, the less costly strategy can be selected for the responder player. The remainder of this research is structured as follows. Section 2 provides an overview of the research literature and background focused on decentralized decision-making approaches. Section 3 presents the methodology employed to develop the presented algorithm. The performance and capability of the algorithm is explored in section 4. Section 5 provides the conclusion remarks of the study.

3.2. Literature review

Decentralized management has recently been applied to model the control of today's complex systems and infrastructure networks [5,16]. Researchers have studied the use of decentralized optimization in various practical domains such as path exploration systems for autonomous vehicles [79–81], air traffic management [82], resource allocation in networks [83], and sensor network management [84], among others. Generally, a decentralized optimization approach deals with problems of multiple objectives that each maximize the interests of a specific decision-maker [16,31,73,85]. This decentralization can be the result of either having variation among the goals of different decision-makers in a non-cooperating manner or due to the lack of clear information among them regarding their preference in a cooperative/semi-cooperative environment.

Furthermore, researchers have extensively employed deterministic and stochastic decentralized convex optimization to address decentralized decision problems. Tsitsiklis [86] and Tsitsiklis et al. [87] investigated how like-minded decisions can be made in a decentralized environment utilizing iterative optimization algorithms given delays in communication among decision-makers. Some researchers have focused on optimizing the sum of local functions, performing an averaging process for each decision-maker while descending stepwise along the local sub-gradient direction considering their local constraints [88–93]. The general drawbacks with this approach are slow convergence and lack of accuracy [94], therefore the alternating direction method of multipliers (ADMM) was used by Boyd et al. [95] to achieve linear convergence in exchange for higher computation time. Consequently, for improved computation time, linearized ADMM algorithms have been utilized by Ling et al. [94]. These methods emphasize the cooperative nature of the decision-makers to optimize a global objective function (i) that each understands only partially (ii) while their communication is also not perfect and occurs in time intervals. Note that the cooperative nature in a decision-making environment exists when only one objective function is optimized while each decision-maker in the system can control certain variables of the main problem. Therefore, considering the large size of the private market where these companies do their best to maximize their profit regardless of the others (non-cooperative nature), cooperative behavior becomes less practical in a free-market environment.

A Markov decision process (MDP) is another approach that has been utilized to investigate the problem of decentralized decision-making [28–31]. Using the concept of MDP, Åström [32] introduced the partially observable MDP (POMDP) for centralized sequential decision-making, which is appropriate in the imperfect information environment. In this regard, the decentralized partially observable MDP (DEC-POMDP) is the generalized version of the POMDP that accounts for problems with two or more decision-makers available in the system cooperating in an imperfect information environment [16]. Similar to decentralized convex optimization, these players in the system cooperate to optimize the joint reward function while each has its different observation function [31]. Unfortunately, studies showed the significant difficulty of solving the DEC-POMDP problem for multiple decision-makers compared to the single agent POMDP problem. It turned out that the DEC-POMDP problem finite-horizon is exponential time complete (NEXP-complete) when only two decision-makers are available in the system [33]. Therefore, considering the various forms of the MDP approach, the methodology is substantially time-intensive for problems with more than one decision-maker in the system.

Game theory represents a different approach that focuses on the behavior of different decision-makers in a system with a cooperative/non-cooperative nature that has gained popularity in solving problems with network characteristics [5,13,71,73,96].

Tosselli et al. [73] developed a repeated-negotiation game approach considering a conceptual enterprise model. The approach deals with two types of multi-layer project and resource decision-makers that are responsible for the scheduling of tasks and resources, respectively. Decision-makers maximize their objectives related to the project deliverables and resource utilization metrics having a client-server relationship. Their algorithm could be considerably time consuming as it uses repeated negotiations to converge to a Nash equilibrium or a close Nash equilibrium solution. Also, instead of predicting the other players' move, the algorithm considers a certain behavioral assumption which is altered according to a predefined probability in each iteration, and such an assumption could affect the practicality of the algorithm. Huang and Zhu [96] proposed an iterative algorithm based on the dynamic game framework to investigate the interactions between a stealthy attacker and a proactive defender related to the security of a cyber-physical system. Although this study does not provide a methodology for scheduling the recovery of a disrupted network, it presents a Bayesian game-theoretic algorithm for detecting the malevolent attacker of the cyber-physical system. The iterative algorithm assumes that it can observe all interactions the users/attackers can make in each iteration related to the system. Therefore, the defender of the system has access to the real-time information of all other users/attackers, which may not be the case in practice. Sharkey et al. [17] considered the cooperative nature among decision-makers, who find a resolution for their payoff function considering optimistic/pessimistic assumptions and share their plan for recovery with other decision-makers to update their assumptions and find new resolutions for their payoff function. However, the proposed framework assumes an unlimited number of negotiations between players related to their recovery plans, which is time consuming and may not to be converged in some cases since there is no upper bound for the number of negotiations [5]. Although several centralized approaches have been suggested by the researchers to address the problem of interdependent infrastructure network recovery, only a few works have used game-theoretic models to address the decentralized nature of decision-making in such networks. Smith et al. [5] developed an ad hoc sequential game-theoretic model to address the restoration problem of interdependent networks in a decentralized environment. They utilized the reduced version of the interdependent network design problem (INDP) model developed by González et al. [46] as the payoff function for the decision-makers of the system. The computational time of their algorithm could be sensitive to the availability of resources and the number of disrupted components related to each decision-maker in the system. This is because the algorithm must consider all available strategies in each period and keep track of the total cost through the decision path in the whole problem time horizon. Subsequently, the algorithm calculates the Nash equilibrium using backward induction, which can be time-consuming as all possible outcomes of strategy combinations are enumerated. Also, the assumption that decision-makers act in turn weakens the algorithm's practicality. Furthermore, considering only definite information about the actions of the other decision-makers is the other drawback of this research. Note that communication among decision-makers and the quality of the exchanged information is important in interdependent infrastructure network recovery problems [5,16,17,35]. Furthermore, Smith et al. [5] assumed decision-makers are unaware of the status of nodes on which they depend in the other networks controlled by other decision-makers, and they will be updated gradually when they are recovered. Therefore, although they do not have complete information about the other decision-makers, the information that is available to the decision-makers is considered definite. However, our proposed algorithm combines definite information and the best guesses of decision-makers in their decision-making process to achieve a better result. In addition, the proposed algorithm is an efficient framework for predicting future decision-maker behavior and devising an adaptive plan concerning the recovery of the disrupted interdependent infrastructure network. Therefore, with the aid of this method, an acceptable solution can be achieved efficiently following a disruptive event. We present the details of the proposed algorithm in the following section.

3.3. Methodology

It is important to determine the method for evaluating the benefits that each decision-maker gains relative to their objectives. Regardless of the number of decision-makers in the system, researchers have approached the network restoration problem from various angles. The main objectives generally drive the restoration of infrastructure networks is to restore the network components as quickly as possible so that user demand from the network is reinstated. In this regard, some have designed their objective function to impose a cost for every period in which user demand is not satisfied [15,42,44–46,97–99]. The other angles of this problem that have been generally illustrated in the constraints of these models are (i) network flow cost, (ii) interdependency, (iii) crew constraints, and (iv) resource and budget constraints, among others. In these models, to be efficient in any of these perspectives, some costs have been associated with them and included in the objective function [47]. They have considered different perspectives that exist in practical situations to mirror essential characteristics in real network disruption scenarios. The interdependent network adaptive recovery (INAR) algorithm proposed in this paper can adapt with the variety of these models predicting the proposer's network actions and implementing them in the responder network's optimization model to devise the best response action. In this research, we considered a modified, shortened version of the INDP model developed by González et al. [46] to calculate the best response given different predictions.

The algorithm is based on the continuous prediction of opponent player behavior, adaptive strategy optimization, and updating assumptions with new information. We assumed that each decision-maker has only full information about the network that they are responsible for, yet their awareness of the other networks is limited to that network's general topology and the importance of its components related to its supply-demand capabilities. Aggregating our assumption using opponent network features and considering the dependencies between the networks, we used a machine learning approach to predict the opponent player recovery schedule for subsequent periods. Therefore, the recovery process of the opponent's network can be approximated. As such, the responder can anticipate when to expect the dependency relationship to be resolved by the opponent player. Having a good guess for the opponent strategic recovery schedule enables the decision-makers to incorporate this indefinite information into their optimization models as parameters and constraint. Therefore, the higher the responder's prediction, the better the strategic reaction, and the closer it would represent the best and ideal response. We assumed that the responder could be informed of an interdependent component recovery as soon as the proposer recovers it, thus the deviation of the predicted recovery schedule from the actual can be realized as time passes. As the recovery process of disrupted infrastructure networks is developed over time, the predictions made at the beginning of the planning time horizon could be significantly biased from the opponent's actual strategic plan. Therefore, as the recovery process progress and information gradually aggregate, we need to renew the prediction to minimize the deviation between the predicted and actual plans.

Figure 7 provides a general overview of the proposed algorithm. Note that the algorithm breaks the problem into two parts of prediction and strategy optimization. Therefore, the first level predictions are inserted into a centralized recovery model as parameters and constraints to find the best reaction response.

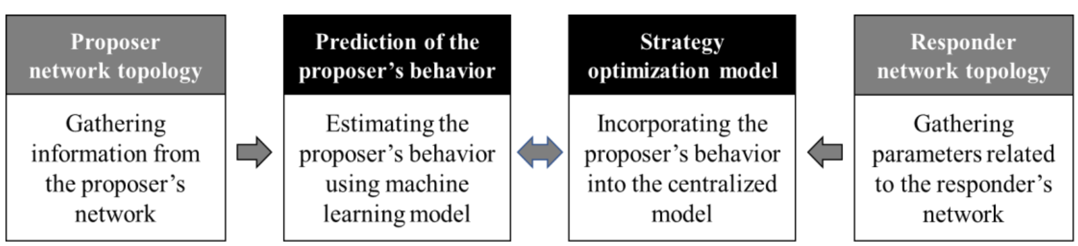


Figure 7. General overview of the proposed algorithm.

In the following three sub-sections, we describe (i) the prediction model characteristics, including the data preprocessing and the learning process of the prediction model, (ii) the optimization model responsible for finding the best response considering the available information, and (iii) the algorithm for updating the strategy, including the continuous strategy correction. Table 6 summarizes several acronyms used in our discussion, as well as the notation for sets, parameters, variables, and terms involved in Algorithms 1 and 2.

Table 6. Summary of acronyms, sets, parameters, variables.

Acronyms	Meaning
INDP	Interdependent network design problem
td-INDP	time-dependent interdependent network design problem
mtd-INDP	Modified time-dependent interdependent network design problem
INAR	The interdependent network adaptive recovery
PCA	Principal component analyses
Index	**Meaning**
$(i, j) \in N$	Index for the components in the networks
t	Index for the time in the recovery time horizon
l	Index for the type of the commodity
Sets	**Meaning**
T	Set of time periods in recovery time horizon
T'	Set of time periods where a disrupted interdependent component is predicted not be recovered in the proposer's network
T_a^*	Set of actual earliest recovery times for the interdependent components
T_p'	Set of predicted earliest recovery times for the interdependent components
N	Set of all components in the network
N'	Set of disrupted components in the network
$\widehat{N}$	Set of disrupted interdependent components in responder's network
N_I'	Set of all interdependent components in the network
A	Set of links in the network
L	Set of commodities
R	Set of resources
Parameters	**Meaning**
c_{ijlt}	Unit flow cost of commodity l through link (i, j) at time t
e_{it}	Recovery cost of component i at time t
M_{ilt}^+	Unit cost of component i excessive supply at time t for commodity l
M_{ilt}^-	Unit cost of component i unmet demand at time t for commodity l
b_{ilt}	Supply/demand of component i at time t for commodity l
u_{ijt}	Capacity of link (i, j) at time t
h_{irkt}	Required resource for recovery of component i at time t, from resource r
v_{rt}	Available resource r at time t
Variables	**Meaning**
δ_{ilt}^+	Excessive supply of component i at time t for commodity l
δ_{ilt}^-	Unmet demand of component i at time t for commodity l
f_{it}	Functionality of component i at time t
$\tilde{f}_{it}$	Extent of recovery of component i at time t

x_{ijlt}	Flow on link (i, j) in network k and time t
Algorithm terms	Meaning
i_t^*	Index of the component that is recovered in time t
t_i	Recovery time of the disrupted component i in the recovery time horizon
t_p	Current time of the algorithm
cn_i	Centrality metric for component i
rc_{it}	Rescaled centrality metrics for disrupted component i relative to the other disrupted components in time t
S_i^p	Minimum element in set T_p'
S_i^a	Minimum element in set T_a^*
cls_{it}	Actual recovery status of component i in time t

3.3.1. Prediction model

Adopting the Markov decision process concept that the best policy in a specific state is selected, the proposed algorithm predicts the component from the opponent network that is recovered in the next period. That is, the predictive model should classify whether the proposer recovers any specific disrupted node in the next period. The model checks every remaining disrupted component in each period and see if it is the next candidate for recovery. Hence, we employed the random forest ensemble model to cluster each component into two classes in each period: recovered and not recovered.

Beginning with Wollmer [100], who proposed an algorithm to find sensitive components of a network by removing a number of them to maximize the flow reduction in a network, numerous component importance measures have been developed to identify critical components in a network. To approximate each component's importance in the network, we used several network centrality measures as the predictors of our random forest model. We are motivated by the fact that, although decision-makers are not aware of the opponent player reward function and constraints, generally, they can assume that the components with higher importance are recovered earlier relative to others in the network.

So, the service flow through the network can be replenished faster and unmet demand costs could be reduced. Additionally, we determined the actual target classes using a *modified* version of td-INDP model, referred to as the mtd-INDP. In fact, when we find the solution of mtd-INDP model for the proposer's network recovery, then we can easily realize the class to which each node belongs at each iteration.

The following stages were used to prepare the data, preprocessing, and learning the random forest model: (i) perform data preparation considering various disruption scenarios, (ii) apply principal component analyses (PCA), and (iii) learn the random forest ensemble model using the prepared data and calculate the model accuracy. In the data preparation stage, we balanced the data using the under-sampling method [101] to reduce the size of the abundant class to improve the random forest model and more effectively distinguish the minority class. PCA was used to make the clustering process easier for the random forest model [102]. We used the random forest model for this study as it can provide accurate and efficient results for both prediction and clustering problems considering a variety of applications [103,104].

While the algorithm could be implemented with any number or selection of importance measures, we selected six network centralities due to their ubiquity in the literature: (i) degree, (ii) betweenness, (iii) closeness, (iv) Katz, (v) page-rank, and (vi) load. Naturally, different topological measures might be more useful than others depending on how importance is being defined [105]. However, note that it has been shown that in many instances topological measures are sufficient surrogates for different (and more difficult to estimate) flow-based measures [106]. Each of these centrality measures has been defined to represent the network components' importance from various perspectives. Degree centrality represents the connection of nodes to other nodes in the network. Betweenness centrality refers to the number of shortest paths in the networks that pass through each component. Closeness centrality considers the sum of the shortest path length between one node and all other nodes in the network. Katz centrality measures the number of immediate neighbors of a node and the other nodes that connect to the node through its immediate neighbors to compute the relative influence of that node in the network. Page-rank centrality is an extension of the Katz centrality, considering the advantage a node can receive connecting the network's critical nodes. The load centrality for a node is calculated considering the number of all shortest paths passing through the node, assuming each node sends a hypothetical unit of some materials to its neighbors.

Furthermore, in each period of recovery time horizon, these metrics are rescaled considering only the disrupted components and serve as the input variables for the classification model. Additionally, we considered the nominal demand/supply capacity of each component as another predictor, as such capacities may also provide insight on the importance of the component.

Algorithm 1. Data processing procedure.

Input: proposer's network relationships and parameters
Calculate the component actual recovery sequence using the mtd-INDP model (i.e., component actual recovery time in the set $\{t_i^*: t \in T,\ i \in N'\}$)
Calculate the component centrality metrics (i.e., component centrality in the set $\{cn_i: i \in N\}$)
for t in T **do**
    Rescale the centrality metrics (rc_{it}) for all $i \in N'$, for time t
    if $i = i_t^*$ **then**
        Set actual component class (cls_{it}) = 1
        Remove i from N'
    Else
        Set actual component class (cls_{it}) = 0

11.	**Output:** Rescaled centrality metrics corresponded to the actual recovery status of the component for every component in each period $\{(rc_{it}, cls_{it}): i \in N', t \in T\}$

The data processing algorithm (Algorithm 1) shows the data preparing process schematically, where T, N, and N' are the set of periods in the recovery time horizon, the set of all components in network, and the set of all disrupted components in the network, respectively (discussed in Table 7 in the context of the restoration optimization problem). The notation cn_i and rc_{it} represent the array of different centrality metrics calculated for each component and the array of rescaled component centralities considering only disrupted components, respectively. Note that whenever each component is restored, it is required to be removed from the set of disrupted components in each period. Therefore, the remaining component centrality metrics should again be rescaled to keep the summation domain of each centrality metric between [0, 1]. This way, we also keep updating the priority of each remaining disrupted component related to each centrality metric. Index i is associated with the components in the network, while i_t^* in Algorithm 1 is the component of the network that is candidate for recovery in specific period of t. The cluster to which a component belongs in a specific period is referred to as cls_{it}. Note that cls_{it} shows whether or not component i is recovered in time t.

After preparing data using the algorithm, PCA is performed on the data to find the predictors that capture the greatest variance. These principal components are added to the predictors for our random forest classification model. As we use a classification model categorizing the node's status individually, the random forest model can mistakenly categorize several components into the recovered class in each period. To address this concern, in case of having several candidate components to recover in a specific time, we will use the probability produced by the random forest model for each of those categorized components. We select the component with higher probability as the recovered component in each period, as we consider having the higher chance of correct classification.

3.3.2. Strategy optimization

After predicting opponent recovery behavior, these predictions are integrated into the strategy optimization. In this study, we were inspired by the td-INDP model developed by González et al. [46] which focused on the recovery of disrupted components in interdependent networks in a centralized manner and with full information about all networks. We have modified the td-INDP model to suit the proposed algorithm in this study. As the proposed algorithm assumes multiple players in the system in an imperfect information sharing environment, we need to address the interdependency perspective differently. Thus, the equations satisfying the interdependency relationships of the networks are no longer required for the proposed algorithm's assumptions (in the INAR algorithm, the interdependencies among networks are addressed by adding constraints representing the responder's prediction for when interdependent disrupted component is recovered in the proposer's network). The td-INDP also accounts for geographical constraints, focusing on minimizing the land preparation fixed cost, which we found deviant from the general practices in network recovery problems. Therefore, pruning the td-INDP model, we focused on the constraints responsible for minimizing the cost of unmet demand, excess supply, and network flow, considering a single-layered network environment.

The objective function of the mtd-INDP model in Eq. (19) consisted of three different terms representing the costs for (i) commodity flows represented by x_{ijlt}, (ii) component recovery represented by $\tilde{f}_{it}$, and (iii) unmet demand and excessive supply represented by δ_{ilt}^{-} and δ_{ilt}^{+}, respectively. Note that the $\tilde{f}_{it}$ is a binary variable showing if the disrupted component i is recovered in time t.

$$\min Z = \sum_{t\in T}\left[\sum_{l\in L}\sum_{(i,j)\in A} c_{ijlt}x_{ijlt} + \sum_{i\in \acute{N}} e_{it}\tilde{f}_{it} + \sum_{l\in L}\sum_{(i,j)\in A}(M_{ilt}^{+}\delta_{ilt}^{+} + M_{ilt}^{-}\delta_{ilt}^{-})\right] \tag{19}$$

Also, six groups of constraints have been considered for the model that govern the following: (i) the balance of goods/services flow through the network in Eq. (2), (ii) the activation mechanism for disrupted components in Eq. (3), (iii) the link capacities in Eqs. (4) and (5), (iv) the availability of resources in Eq. (6), and (v) enforcing disruption to the network for the first period in Eq. (7). The balance constraint in Eq. (2) governs that the commodity output and input outcomes be equal to the amount of consumption, unmet demand, and excessive supply. Constraint (3) is responsible for keeping the disrupted components non-functional until recovered. Constraints (4) and (5) prevent flow in link (i, j) if either node i or node j are not functional. The nature of the decision variables is described by Eqs. (8)-(10).

$$\sum_{j:(i,j)\in A} x_{ijlt} - \sum_{i:(i,j)\in A} x_{ijlt} = b_{ilt} - \delta_{ilt}^{+} + \delta_{ilt}^{-} \qquad \forall i \in N, \forall l \in L, \forall t \in T \tag{20}$$

$$f_{it} \leq \sum_{t \in T} \tilde{f}_{it} \qquad \forall i \in N', \forall t \in T \qquad (21)$$

$$\sum_{l \in L} x_{ijlt} \leq u_{ijt} f_{it} \qquad \forall (i,j) \in A, \forall t \in T \qquad (22)$$

$$\sum_{l \in L} x_{ijlt} \leq u_{ijt} f_{jt} \qquad \forall (i,j) \in A, \forall t \in T \qquad (23)$$

$$\sum_{i \in \acute{N}} h_{irt} \tilde{f}_{it} \leq v_{rt} \qquad \forall r \in R, \forall t \in T \qquad (24)$$

$$f_{i0} = 0 \qquad \forall i \in N' \qquad (25)$$

$$x_{ijlt} \geq 0 \qquad \forall (i,j) \in A, \forall l \in L, \forall t \in T \qquad (26)$$

$$\delta_{ilt}^{+}, \delta_{ilt}^{-}, \geq 0 \qquad \forall i \in N, \forall l \in L, \forall t \in T \qquad (27)$$

$$f_{it}, \tilde{f}_{it} \in \{0,1\} \qquad \forall i \in N, \forall t \in T \qquad (28)$$

To include the restoration predictions in the model, two approaches can be considered. In the first approach, it is assumed that the responder does not attempt to recover the interdependent disrupted components until its corresponding node in the proposer's network is fixed. This is governed by inserting Eq. (29) into the optimization model. Alternatively, in the second approach, the model recovers the disrupted node but it cannot be activated until its predecessor in the proposer's network are restored. To adopt this approach, Eq. (30) is instead inserted into the model.

$$\sum_{t \in \acute{T}} \tilde{f}_{it} = 0 \qquad \forall i \in N' \qquad (29)$$

$$\sum_{t \in \acute{T}} f_{it} = 0 \qquad \forall i \in N' \qquad (30)$$

3.3.3. Best response and strategy update

Using the random forest model predictions of the proposer 's behavior with recovered and not recovered classes for each node, we need to find the best response to the proposer 's behavior considering the mtd-INDP model. The best recovery schedule of the responder's disrupted nodes is found. Although we can make predictions at the primary stage of the recovery time horizon, as time advances, decision-makers acquire more information about the actions made by the opponent player. These pieces of information generally describe the interdependent components that have been recovered. Thus, the responder can simply realize the deviation between their prediction and actual actions made by the proposer. As this information updates gradually, the prediction can be renewed and subsequently the responder can more effectively adapt to the proposer’s decisions whenever they receive new information.

Gathering such new information could differ by the decision-making situation, the level of cooperation among decision-makers, and the cost of information. In this research, it is assumed that the responder player predicts the proposer’s behavior at the beginning of the time horizon and plans the recovery of the network on that prediction. Then, as the responder pursues the actual recovery plan over time, they will be informed whenever the proposer recovers an interdependent component necessary for functionality of a node in their network. Based on this assumption, three different situations may happen. The first is that the prior prediction is identical to the proposer’s actual recovery plan. The second is that an interdependent component is recovered sooner than expected. The third situation is that an interdependent component is not recovered by the expected time. Considering these situations, if the prediction is not identical with the actual plan, the proposer needs to consider the new condition, renew its prediction, and develop a new recovery plan for the rest of the remaining periods. The strategy updating algorithm (Algorithm) shows the strategy update procedure in detail.

Algorithm 2. Strategy updating algorithm.

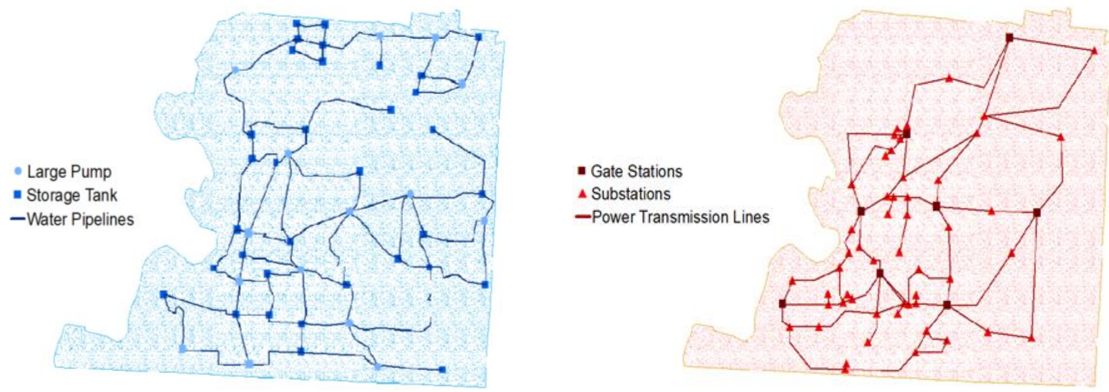
1. **Input:** proposer's and responder's network relationships and parameters
2. Setting the actual earliest recovery time set (T_a^*) for the interdependent components using mtd-INDP model on proposer's network: $T_a^* = \{t_i^* | t \in T, i \in N_I'\}$
3. Setting the predicted earliest recovery time (T_p') for interdependent components: $T_p' = \{t_i | t \in T, i \in N_I'\}$
4. Run mtd-INDP model to find the best recovery schedule set for the responder's network considering the prediction set T_p'
5. **While** $N_I' ! = \emptyset$ **do:**
6.   Select the minimum t_i from the set T_p': $S_i^p = min\ (T_p')$
7.   Select the minimum t_i^* from the set T_a^*: $S_i^a = min(T_a^*)$
8.   Set the algorithm time (t_p): $t_p = \{min\left(S_i^p, S_j^a\right) | i, j \in N_I'\}$
9.   **If** $S_i^p < S_j^a$
10.   **If** component i status is recovered
11.   Remove i from N_I': $i \setminus N_I'$
12.   Remove t_i^* from T_a^*: $t_i^* \setminus T_a^*$
13.   Renew the interdependent component recovery time prediction set T_p'
14.   Run mtd-INDP model to find the best recovery schedule set for the responder's network considering the prediction set T_p'
15.   **If** $S_i^p > S_j^a$
16.   Remove i from N_I': $i \setminus N_I'$
17.   Remove t_i^* from T_a^*: $t_i^* \setminus T_a^*$
18.   Renew the interdependent component recovery time prediction set T_p'
19.   Run mtd-INDP model to find the best recovery schedule set for the responder's network considering the prediction set T_p'
20. **Output:** Disrupted components recovery schedule set for the responder's network

Note that both proposer and responder are working concurrently to recover the disrupted nodes in their network. If the responder realizes a fault in the prediction in any period, it has the chance to revise their plan for only the remaining disrupted nodes that have not recovered yet. Therefore, although the INAR algorithm's solution is sub-optimal, renewing the predictions whenever we realize a conflict between the actual and prediction makes the solution stay close to the optimal as best as possible. Also, the algorithm terminated whenever no disrupted nodes remained in the disrupted interdependent component list (N_I').

Note that the running time of the prediction model is insignificant, as the prediction for the proposer's independent components can be acquired almost instantly. Also, as the mtd-INDP model is the simplified version of the td-INDP model and the interdependency relationships between the networks are defined using simple constraints, the time required to solve the model is insignificant. Therefore, running algorithm 2 should not be time consuming at all.

3.4. Results and discussion

To adequately explore the capability of the INAR algorithm, we applied this framework to four different case studies, one related to a realistic testbed and three randomly generated instances with different topological properties. The first case study is associated with the interdependent system in Shelby County, TN, USA [46,107,108]. To test the performance of our algorithm, as we have two decision-makers (each responsible for the restoration of a utility network) we focus our Shelby County case study on its water and power networks. The water and power networks have 49 and 60 nodes and 71 and 76 links, respectively. Figure 8 illustrate a general overview of Shelby County, TN water (a) and power (b) network.



(a) (b)

Figure 8. Shelby County, TN network topology for the (a) water and (b) power networks (adapted from González et al., (2016)).

The three remaining case studies correspond to: (i) Barabási-Albert network [109], (ii) Erdös-Renyi network [110] and (iii) Newman-Watts-Strogatz [111]. These three types were selected due to their popularity among the random network generation algorithms and for their varied structural properties. The Barabási-Albert algorithm generates a scale-free network where degree distribution of components follows the power law [109]. The Erdös-Renyi algorithm, one of the simplest random generation methods, produces networks with a binomial degree distribution [112]. The Newman-Watts-Strogatz network generation algorithm, derived from the Watts-Strogatz algorithm, produces networks with small-world properties [113]. The generated networks consisted of 120 nodes for each proposer and responder network, while the number of links varied based on the type of network generation algorithm. To keep the combinations of proposer and responder networks manageable, the pairs of interdependent proposer/responder networks are both generated by the same algorithm (e.g., a Barabási-Albert network paired with another Barabási-Albert network). Also, the interdependent nodes from both networks are selected and paired randomly. Note that the Barabási-Albert and Newman-Watts-Strogatz algorithms produce a connected network while the Erdös-Renyi network can create a disconnected algorithm based on the selected probability for link creation. To address this issue, the probability of link creation is increased incrementally so that only one connected component is produced. To generate these random networks, we use the networkx package in Python [114]. The number of links generated using the Barabási-Albert, Erdös-Renyi, and Newman-Watts-Strogatz algorithms are 273, 305, and 173 bidirectional links, respectively. The Shelby County networks, along with the three algorithm-generated networks, make up the four network types studied subsequently.

For each of the network types, 600 random disruption scenarios were generated, each with 40 disrupted components. Using the data processing algorithm (Algorithm 1), the data required for learning the random forest model were prepared for all four network types. Table 7 shows an example of prepared data for learning the random forest model. Columns 2 through 7 represent the rescaled value of the node's centrality measures. Columns 8 through 10 are the principal components that have been calculated considering the centrality measures along with the amount of demand or supply for the node. Note that we have only used three principal components as predictors for learning the random forest model.

Table 7. Illustration of variables and an example of prepared data.

Node	Betweenness	Page-rank	Katz	Closeness	Degree	Load	PC1	PC2	PC3	Cluster (cls)
78	0.012	0.013	0.015	0.021	0.012	0.003	418	0.031	0.02	1
38	0.012	0.013	0.018	0.023	0.012	0.002	0	0.061	-0.009	0
30	0.025	0.025	0.021	0.025	0.024	0.026	-112	0.081	0.013	0
44	0.019	0.025	0.018	0.023	0.024	0.015	-401	0.1	-0.021	1
23	0.025	0.035	0.028	0.027	0.036	0.025	-5	0.107	0.01	1

10	0.102	0.082	0.059	0.034	0.09	0.157	-522	0.219	0.137	0
102	0.014	0.014	0.019	0.025	0.013	0.004	492	0.092	-0.019	0
16	0.015	0.014	0.022	0.028	0.013	0.004	0	0.104	-0.035	1
53	0.013	0.015	0.016	0.023	0.013	0.002	-17	0.101	-0.039	0

We have considered the ideal situation, where the responder is able to observe the proposer's recovery plan beforehand as the comparison baseline. Furthermore, we evaluate the difference between (i) the solutions from the INAR algorithm with (ii) the ideal condition. As many works related to the network disruption have analyzed the network vulnerability by removing the network components based on centrality metrics [115–120], we compared the INAR algorithm solution to a restoration scheduling heuristic based on the ranking of the disrupted components according to individual centrality measures, as one might expect a decision-maker to choose important nodes to recover first.

Subsequently, we discuss the performance and validity of different parts of the INAR algorithm and describe its behavior. First, the performance of the random forest classification model for individual components in each period is described with different accuracy matrices. Then, we studied the INAR algorithm performance in predicting the recovery of the entire chain of the disrupted interdependent components in the proposer's network. Finally, we investigate how a better prediction can affect the network recovery cost.

3.4.1. Random forest classification performance

After completing the data preparation, we used the under-sampling method to balance the data by randomly deleting data from the majority class to the extent that the no information rate is less than roughly 52%, suggesting that the data do not suffer from imbalance issues that might lead to bias in the application of the machine learning techniques. Afterward, we use 20% of the data as the test data to validate the random forest model and calculate accuracy metrics. Table 8 shows several evaluation metrics illustrating the performance of the random forest model in classifying the restoration of each disrupted node in every period. The model accuracy for the four types of networks used ranges around 82-83% with low variability. As the no information rate for all different networks is around 51%, the random forest model operates at least 30% better than the random classification of disrupted component recovery status. As the training data were balanced with the proportion of each class being roughly 50%, the log-loss no information rate is roughly 0.69. The random forest model results in an improved log-loss accuracy ranging from 0.38 to 0.41. Also, kappa statistic of more than 0.60 shows a substantial strength of agreement between the actual and predicted classification [121].

Table 8. Random forest evaluation metrics.

Metrics	Network types			
	Shelby County	Barabási-Albert	Erdös-Renyi	Newman-Watts-Strogatz
Accuracy	0.82	0.83	0.83	0.83
No Information rate	0.51	0.51	0.52	0.51
Log-loss	0.40	0.38	0.39	0.41
Kappa statistic	0.65	0.66	0.66	0.66
Sensitivity	0.80	0.78	0.77	0.80
Specificity	0.85	0.88	0.89	0.86
Precision (Class 0)	0.84	0.86	0.86	0.84
Precision (Class 1)	0.81	0.81	0.81	0.82
F-Score (Class 0)	0.82	0.82	0.81	0.82
F-Score (Class 1)	0.83	0.84	0.84	0.84

The number of data required to achieve this level of accuracy has been determined using the elbow graph presented in Figure 9, where the classification model accuracy corresponds to the size of data used for learning purposes. Thus, we divided the data into batches of 1000 instances each and consider the learning data population to increase by a batch for every time that we learn the random forest model. The accuracy of the model for every learned model were recorded and plotted in Figure 9. Although a sharp error reduction can be observed with fewer than 10000 data records, the model's lack of accuracy appears at its minimum and approaching stability at more than 30000 data records.

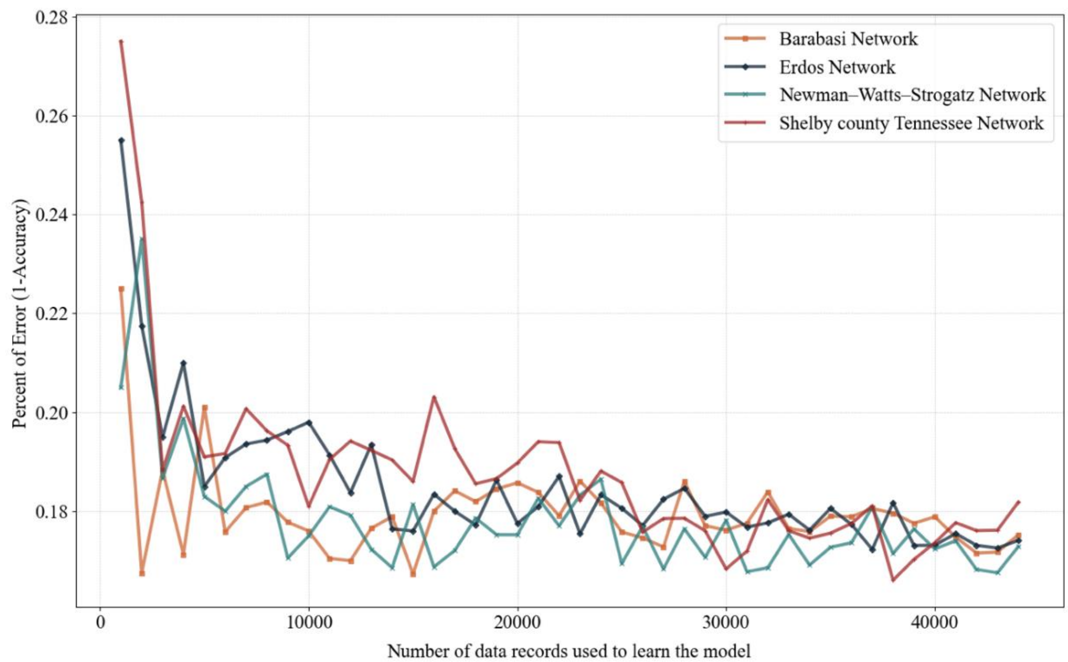


Figure 9. Random forest model accuracy elbow graph.

Furthermore, the behavior and the performance of the INAR algorithm relative to the structural changes in the network is required to be investigated. The main structural variables that have been considered in this part of the study are: (i) the network size, (ii) the number of the interdependency relationships, and (iii) the network type. Table 9 illustrates an experimental design configuration to explore the effects of these three variables. Note that the Shelby County network was not considered because its characteristics are fixed.

Table 9. Experiment design configuration.

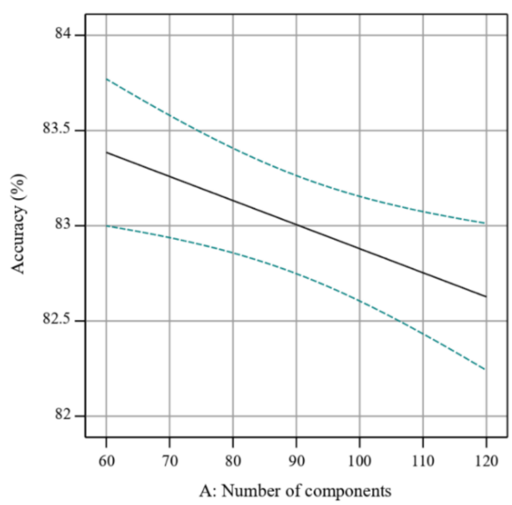
Factor	Levels
Number of nodes	2: 60/120
Number of interdependencies	2: 15/30
Network type	3: Barabási-Albert/Erdös-Renyi/Newman-Watts-Strogatz

Performing the full-factorial experiment, according to Table 9, the number of combinations is found by multiplying the numbers of levels for each factor, $2 \times 2 \times 3 = 12$. Considering one center point for each network type, the number of combinations increases to $12 + 3 = 15$. As we replicated each treatment twice, the number of observations analyzed is 30. As such, 30 interdependent networks representing various structures have been generated, and data were acquired according to data processing algorithm (Algorithm 1). With these experimental data, the random forest model was learned, and the model's accuracy was calculated for each repetition. After an initial full ANOVA, only the main effects were significant. The ANOVA was performed again only with the main effects, with results in Table 10.

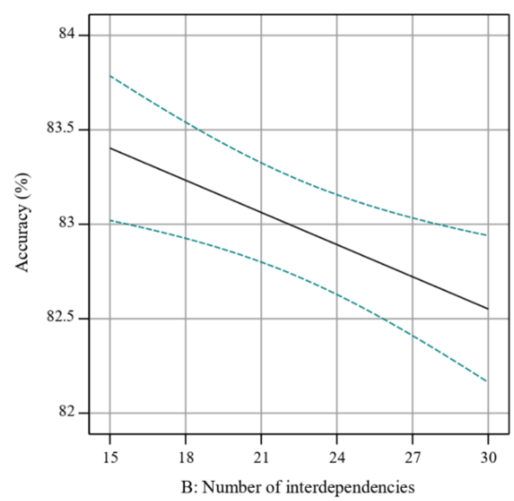
Table 10. ANOVA table for random forest model accuracy.

Source	Sum of squares	df	Mean square	F stat	p-value
Model	**25.25**	**4**	**6.31**	**13.51**	**< 0.0001**
Number of components	3.45	1	3.45	7.38	0.0118
Number of interdependencies	4.35	1	4.35	9.31	0.0053
Network type	17.45	2	8.73	18.67	< 0.0001
Residual	**11.69**	**25**	**0.4675**		
Lack of fit	4.19	10	0.4192	0.8389	0.6015
Pure error	7.50	15	0.4997		

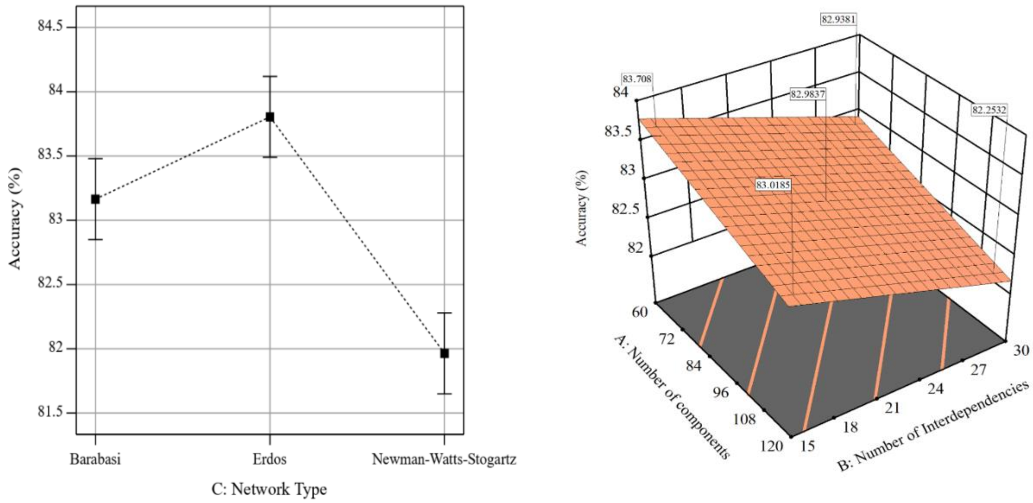
The R^2 is 0.5448 (adjusted R^2 of 0.6330) suggests a relationship between the controllable factors and the accuracy of the random forest model, although it explains only the 63% of the response total variance. This experiment's signal-to-noise ratio, comparing the range of the predicted values to the average prediction error, is 12.359, which is greater than 4, the rule of thumb that suggests an adequate signal [122]. Therefore, the experiment analysis confirms the significance of the main factors and the existence of a linear relationship between factors and the response. Figure 10 illustrates the effects of variation in the three factors. Although the accuracy of the model decreases with an increasing number of components and interdependencies in the network, the degradation of accuracy considering the experiment domain is around just 1%. This suggests that the variation in accuracy does not significantly affect the performance of the entire algorithm.



(a)



(b)



(c) (d)

Figure 10. Accuracy versus the (a) number of components, (b) number of interdependencies, (c) network type, as well as (d) the effect of number of components and number of interdependencies on accuracy considering the average over factor C.

3.4.2. Recovery time chain prediction

Mentioned previously, in each period, we use the random forest model to predict if each disrupted component is recovered or not in that period. Because the random forest model is not 100% accurate and each node is classified independent from the other components, more than one component can be predicted as recovered in each period. To rectify this problem, we rank the selected components based on the probability generated by the random forest model and select the one component with the highest probability as the component predicted to be restored in a specific period. That is, a short-list of the disrupted components is provided using the random forest model, and the one with the highest score among them is selected. So, we expect better accuracy in predicting the whole chain of disrupted interdependent components than the random forest model's independent predictions. The boxplots in Figure 11 show the accuracy of the prediction made by the INAR algorithm versus those made using rankings from individual centrality measures. To analyze the performance of the INAR algorithm, it is currently unrealistic to consider a massive number of disruption scenarios and to enumerate all possible combinations due to the computational effort required, mainly when dealing with the large number of disrupted components in an interdependent network. Therefore, we generated 30 random disruption scenarios, along with 40 disrupted components, including 15 and 10 interdependent components for the proposer's and responder's networks. Subsequently, we measured how different the predicted restoration and the actual restoration were in terms of the number of periods and lead the discussion using the mean and standard deviation of the random sample. The distributions of mean absolute error (found from the predictions of 15 disrupted components) are found in Figure 11. According to Figure 11, the INAR algorithm has the lowest mean absolute error of 3.49 (standard deviation of 0.9) for the Shelby County network. Similar results were found for the other network types. The performance of the INAR algorithm is substantially better than restoration decisions based on centrality measures across the network types. The use of centrality measures was substantially worse (and more variable) for Erdös-Renyi, which are networks whose connections are generated randomly. Centrality measures worked a bit better for Barabási-Albert networks, whose degree distribution follows a power law, suggesting only a small portion of the components has a high degree distribution. Thus, using centrality measures for decision-making naturally produces better performance.

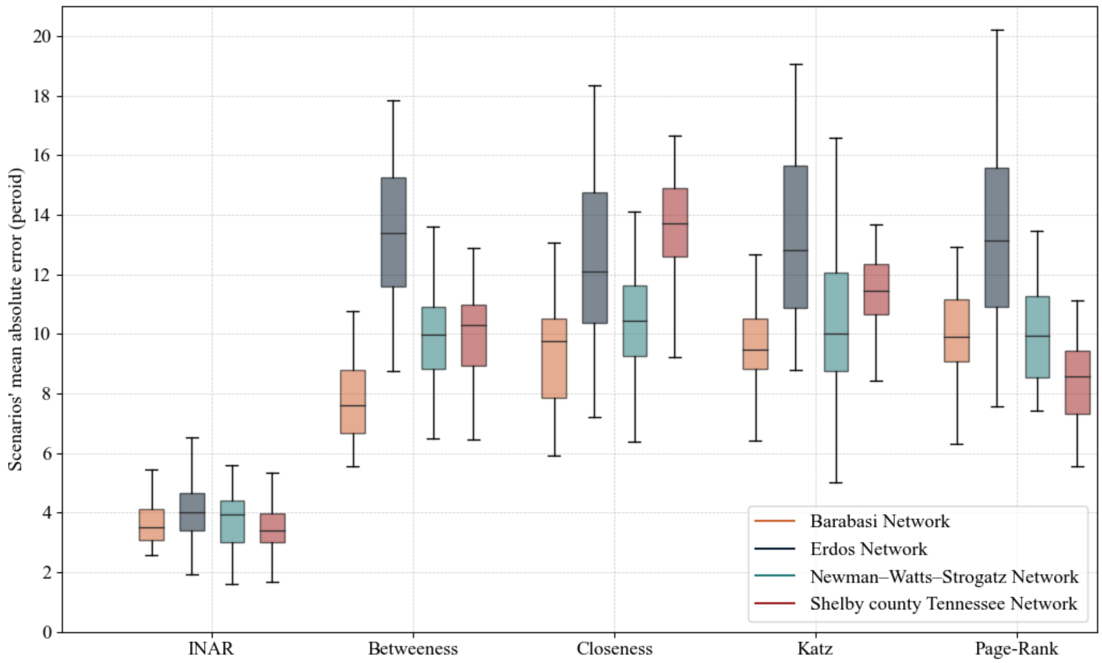


Figure 11. Mean absolute error for the disrupted interdependent component recovery period prediction.

Generally, it is easier to predict an incident happening in the near future relative to the distant future, which may be a function of a chain of uncertain events and decisions. That is, if we fail to predict the events happen in the near future correctly, the dependent incidents in the distant future are less likely to be predicted correctly. The INAR algorithm reflects this concept as we are dealing with a finite number of disrupted components that need to be recovered in time. As the INAR algorithm makes predictions period by period, the prediction made in later periods depends on the ones in earlier periods. Thus, as the time passes, we expect to lose accuracy in predicting restoration time. To address this concern, we consider a continuous reproduction process for if the responder realizes a deviation between prediction and the actual plan of the proposer. Therefore, the responder tries to keep the gap between their assumption and what actually happens as close as possible, so predictions made in the future are less important because they will likely be corrected in the algorithm process. In Figure 12, we considered the ascending order of restoration time for each disruption scenario and calculate the absolute error of actual and predicted time of recovery. Doing so shows the model prediction performance corresponding to the proposer's consecutive order of recovery.

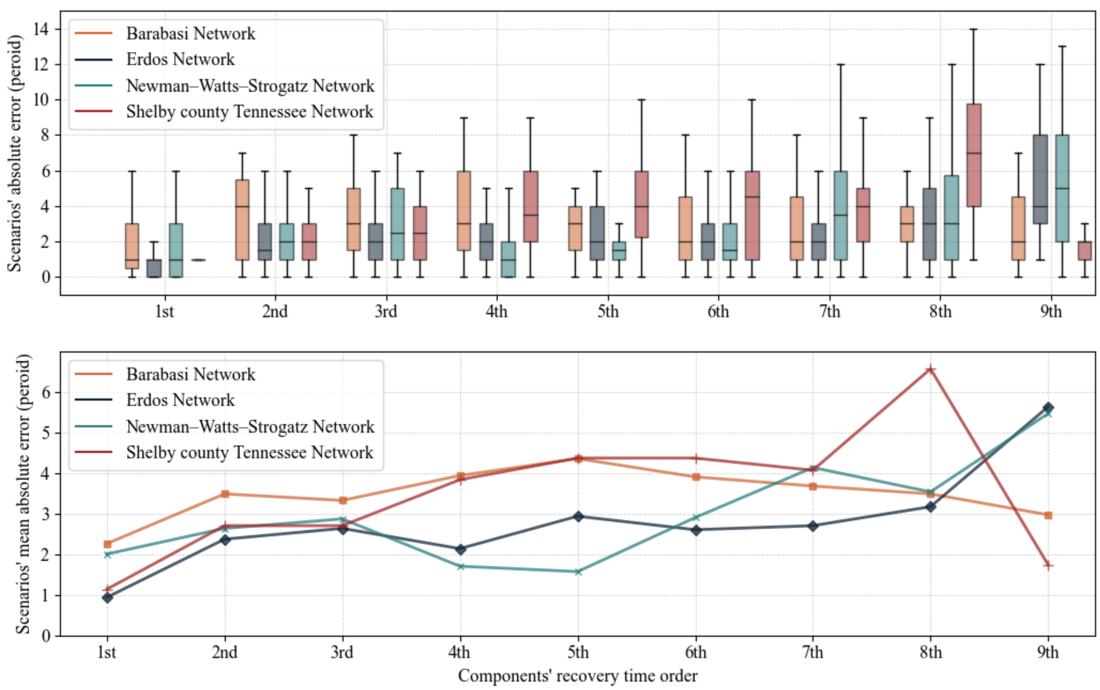


Figure 12. Prediction chain performance based on recovery time order.

It is evident in Figure 12 that the mean and standard deviation of prediction error generally increases as the component's recovery order increases. This indicates that the further in time an event happens, the less accuracy can be expected from the prediction model. Also, we can realize that the INAR algorithm has a better performance in predicting the near future with the mean absolute error of 2.25, 0.93, 2, and 1.13 for Barabási-Albert, Erdös-Renyi, Newman-Watts-Strogatz, and Shelby County network, respectively. Using a continuous reprediction mechanism whenever we find deviation between the actual and predicted plan, the INAR algorithm frequently sheds light on the near future and always puts the algorithm in position to predict the prior component that is recovered by the proposer.

3.4.3. Recovery cost

We have illustrated the performance and behavior of the prediction of the INAR algorithm in detail. In this section, we address how a lack of accuracy can affect the recovery cost of the responder's network. We considered the cost of the perfect condition, when the responder knows exactly how the opponent would behave, as the baseline for cost comparisons. Then we calculate the network unmet demand cost considering (i) the prediction made using INAR algorithm and (ii) by ranking disrupted component recovery time based on individual centrality metrics. To have a realistic point of view, Figure 13 shows the average unmet demand cost considering 20 different disruption scenarios during the network recovery time horizon for the Shelby County network.

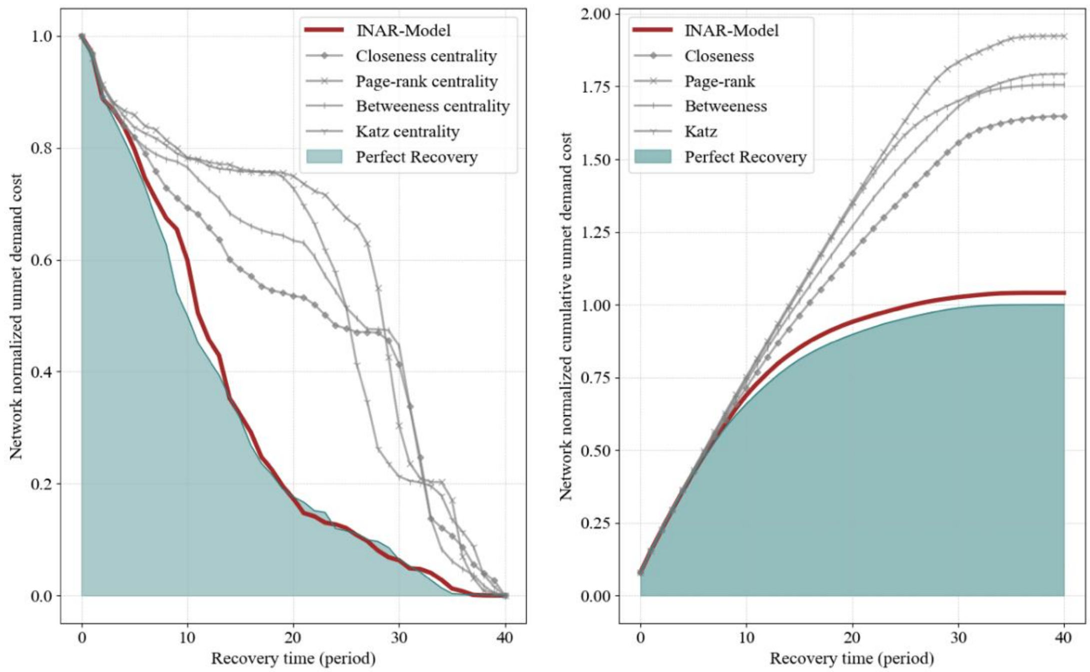


Figure 13. Network normalized unmet demand cost during the recovery process.

Illustrated in Figure 13, the recovery cost related to the INAR algorithm is closely aligned with the perfect recovery condition, suggesting that the network recovery solution provided by the INAR algorithm can successfully compete with the optimal solution. Furthermore, the same trend can be seen for other generated networks that represent different network structures in this study. Considering the perfect situation as the baseline, we calculate the percentage of the extra cost imposed due to the lack of accuracy in predictions made by utilizing the above-mentioned methods for various type of networks. Table 11 illustrates the optimal cumulative recovery cost as the baseline, and the amount of extra cost that imposed due to using different prediction algorithms. The cost deviation from the baseline optimal cumulative cost considering the INAR algorithm for generated networks is between 1% to 3%, while this amount for the Shelby County network is almost

4%. Evident from the Table 11, the INAR algorithm shows a better cost-performance than the other ranking methods considering the realistic Shelby County network. Therefore, Table 11 confirm the high performance of the INAR algorithm compared to the other prediction methods, especially for the Shelby County network, which is based on a real-world topology of actual interdependent infrastructure networks.

Table 11. Cumulative recovery cost of recovery.

		Percent of extra cost due to the lack of accuracy				
Network type	Baseline optimal cumulative cost	INAR algorithm	Betweennes s centrality ranking	Closeness centrality ranking	Page-rank centrality ranking	Katz centrality ranking
Shelby County	456,449,810	3.98%	48.24%	39.11%	59.29%	52.60%
Barabási-Albert	349,182,870	1.27%	3.52%	4.21%	5.13%	4.56%
Erdös-Renyi	426,206,130	2.76%	7.80%	8.92%	6.98%	8.47%
Newman-Watts-Strogatz	80,207,450	1.09%	7.11%	7.42%	6.98%	6.86%

Note from Table 12 that the running time of Algorithm 2 for the prediction and optimization process was insignificant relative to the time scale of the decision process and also similar across the selected network types.

Table 12. Computation time (seconds) for Algorithm 2 by network type.

Network type	Shelby County	Barabási-Albert	Erdös-Renyi	Newman-Watts-Strogatz
Computational time	52	54	47	39

3.5. Conclusion

The INAR algorithm is developed to address the concern of having multiple infrastructure decision-makers whose decisions affect (i) the performance of other infrastructures and thus (ii) the decisions of other infrastructure decision-makers. This is especially true when these decision-makers do not reveal their restoration plans to other players in the system. The INAR algorithm provides a framework for predicting future decision-maker behavior and devising an adaptive plan concerning the recovery of the disrupted interdependent infrastructure network. To do so, we have integrated machine learning with an analytical solution approach by defining a mechanism to include uncertain assumption (i.e., the best guesses of the intentions of other players) into the analytical model.

The main idea behind the INAR algorithm development objective is to use topological feature of the proposer's network, investigating and learning from disruption scenarios before they occur, so that a good solution can be achieved in a timely manner after a disruption. The proposed algorithm consists of two parts: (i) data preparation and learning using a random forest model and (ii) scheduling the recovery process. The prediction model showed good performance when predicting the recovery periods of each individual component in the proposed case studies, which include the system of interdependent water and power networks from Shelby County, TN, and three systems with different types of randomly generated networks: Barabási-Albert, Erdös-Renyi, and Newman-Watts-Strogatz.

Furthermore, we have studied the recovery prediction accuracy of the disrupted interdependent components complete sequence using the mean absolute error between actual and predicted plan. The results showed a high performance with an average deviation of around 4 periods from actual, while this amount for the near feature was around 2 periods. In this regard, we observed myopic behavior from the INAR algorithm prediction, which has been addressed using a continuous prediction loop to renew the predictions whenever a deviation was detected by the algorithm. Finally, we addressed the reflection of the prediction lack of accuracy objectively on the cost of the recovery. Using the INAR algorithm, unmet demand recovery costs approach the optimal solution recovery cost such that we can observe only a 4% cost deviation from the optimal solution for Shelby County case and less than 3% for the other generated networks. This implies a high efficiency of the proposed algorithm. The challenge for the future work is to enhance the overall performance of the algorithm, particularly when predicting the distant future events in the proposer's network. Deep learning models can help achieve better predictions and improve the algorithm overall performance. Furthermore, the proposed algorithm is suitable for real cases like hierarchical systems where the responder player does not have that much leverage over the proposer player in the system. Incorporating different levels of bargaining power for each player in the system could be another relevant future avenue of this research.

4. CHAPTER FOUR: FRAMEWORK FOR MEASURING THE IMPACTS OF WEAPONIZED DISINFORMATION ON SUPPLY CHAIN NETWORKS

4.1. Introduction

Maintaining secure and resilient supply chains is important for ensuring economic productivity, and in some cases, societal health and well-being. Such supply chains serve to safeguard the steady delivery of essential goods and services required for daily life by ensuring that items reach their intended destinations without interruption, protecting against potential hazards [123–125]. Natural disasters and the COVID-19 pandemic have recently affected supply chains severely, resulting in significant delivery disruptions and shortages of critical items, such as medical supplies [126–129] and critical minerals [130].

However, a different, perhaps over-the-horizon, hazard that could adversely impact supply chains comes from an adversary who seeks to attack such critical networks indirectly by altering the consumption behavior of human intermediaries who are influenced by weaponized disinformation distributed by the adversary. Due to the advances in new technologies allowing an easy access to and dissemination of information, efforts to spread misinformation have been on the rise. The well-publicized disinformation campaigns surrounding the recent US presidential elections and pandemic vaccine adoptions highlight how disinformation spreads regularly and quickly online and across media platforms [131–133]. In the case of supply chains, disinformation could compromise the availability of raw material or component parts by spreading false or misleading information among various actors in the system [134–136]. Bad actors can alter consumer assumptions regarding different characteristics of goods and services delivered by the supply chain network. Propagating false news about the shortage, quality, or credibility of a product being delivered could change consumption behavior and demand [137,138]. These behaviors can be realized as buying panic, hoarding, stockpiling, substitution of goods and services, or delayed purchasing [139–141]. Furthermore, disinformation can also pose a security threat by propagating false information about product authenticity and deceiving consumers to alter their demand behavior by buying or introducing substandard goods and services to the supply chain, which can compromise the safety of consumers [142]. The interconnectivity between different parts of a supply chain, combined with the widespread use of social media networks and online news sources, makes critical supply chains vulnerable to the dissemination of disinformation among communities related to the supply chain network [137,143].

Sudden changes in demand due to disinformation are critical because supply chains are not generally prepared to handle demand surges as they are adapted to the usual demand behavior of consumers [142,144–146]. On the other hand, being prepared all the time for high peak alterations in demand imposes significantly high costs as systems need to be designed with higher capacity, more warehouses, safety stock, and more capable transportation. Therefore, in most cases, overdesigning such preparation actions would be impractical as sudden changes occur perhaps infrequently in the time horizon while diminishing the profitability of the supply chains systems [147–151]. To design a supply chain that delivers critical goods and services, achieving a delicate balance between cost and vulnerability/resilience is crucial. While maximizing system performance is imperative for the supply chain to remain competitive, it is equally crucial to ensure that it can withstand disruptions and risks. Therefore, it is critical to intelligently allocate resources when designing a critical supply chain. This involves considering supply chain characteristics such as plant capacities, inventories, and transportation. By designing the supply chain in a smart way, we can maximize its performance while providing necessary flexibility and redundancy for the supply chain to withstand disruption, ensuring continuity of system operation. To evaluate the supply chain's responsiveness under different circumstances, it is necessary to understand the impact of demand alterations, which highlights the importance of having a framework for producing various demand alteration scenarios. This will enable us to identify potential weaknesses and vulnerabilities that may arise in the event of sudden variations in demand and take necessary steps to enhance the supply chain resilience.

To address the mentioned concerns, this study proposes a formulation for simulating the demand alteration pattern for products produced and distributed by a supply chain. The formulation considers various characteristics that represent real situations. The formulation addresses three main key consumer behavior: (i) the behavior of consumers reacting to disinformation, (ii) the magnitude of demand surge as consumers try to maximize their preparedness for the unwanted situation, and (iii) changes in demand norms after demand patterns return to normal. Additionally, we examine the potential impact of these demand fluctuations on the responsiveness of supply chain networks based on various measures. We employed a mixed integer optimization model for production fulfillment to minimize the time required to fulfill orders, which is an indicator of supply chain responsiveness. Additionally, we considered the risks associated with the various components of the supply chain. Section 2 reviews literature on supply chain demand alteration due to disinformation propagation in the communty. Section 3 proposes the methodology to provide a framework for anticipating the demand alteration due to disinformation and a method for assessing the weaknesses of the supply chain, with an illustrative example in Section 4. Concluding remarks are provided in Section 5.

4.2. Literature review

The dissemination of false information has emerged as a serious concern that can cause major disruptions in critical supply chains, resulting in a lack of vital goods and services [152,153]. With the widespread use of digital platforms and social media channels, disinformation can quickly spread and significantly influence consumer actions [154]. Such sudden changes in consumer behavior can trigger unexpected surges in demand, leaving supply chains unprepared to handle them. A notable example is the COVID-19 pandemic, which caused a high demand for medical supplies, disrupting supply chains and creating shortages of critical goods. Lockdowns, travel restrictions, and border closures have also led to delays and disruptions in the transportation of goods, further exacerbating the situations regarding the supply chain [155,156].

Therefore, the clear message of these catastrophes was the necessity of having supply chains with greater agility and resilience to better mitigate the impact of such crises in the future. With the possibility of disinformation causing significant consequences in supply chains, researchers have increasingly focused on exploring the ways supply chains can handle abrupt changes in demand triggered by disinformation campaigns. Xiao et al. [157] and Cao [158] have investigated a dual-channel supply chain when both market demands were disrupted due to false information that affected consumer demand behavior. The authors created models for pricing and production decisions, proposed a revenue sharing contract, and compared profits under normal and disrupted demand. They stressed that adjusting price and production quantity were the key factors in managing demand disruption and maximizing revenue. Feng [159] investigated the value of information sharing in supply chain management by using system dynamics, comparing the supply chain dynamics before and after efficient information sharing under market demand changes. Cao et al. [160] have used the Cournot competition model considering n Cournot competing retailers and one manufacturer under simultaneous production cost and demand fluctuation. The study examines the effects of the disruptions on revenue sharing contracts and investigates optimal strategies for different disruption levels under centralized and decentralized decision-making modes. Paul et al. [161] discussed a three-stage supply chain network with multiple manufacturing plants, distribution centers, and retailers proposing a predictive mitigation planning approach using a fuzzy inference system tool and a reactive mitigation plan using a quantitative model and an efficient heuristic. Chen et al. [162] proposed a supply chain disruption recovery strategy by changing the original product type to cope with the massive demand disruption caused by the COVID-19 pandemic. They developed a mixed integer linear programming model to maximize the total profit from product changes and to reduce the profit loss of the manufacturer due to late delivery and order cancellation. The model combines emergency procurement on the supply side, product changes by the manufacturer, and backorder price compensation on the demand side. Barman et al. [163] proposed the direct delivery channel as a recovery strategy to minimize the effects of disruptions in the food supply chain due to variations in demand caused by the COVID-19 pandemic. They optimized the recovery plan based on profit maximization criteria, demonstrating that the plan can withstand the reduction in demand and improve profit. The recovery plan involves the manufacturer appointing vendors and delivering products directly to customers through multiple delivery channels.

These studies have mainly addressed the problem of mitigating supply chain disruptions arising from demand surges, but they have not explored the characteristics of sudden demand variations due to the spread of critical false news or any catastrophic events. Therefore, given the potential impact of disinformation on supply chains, there has been growing interest in investigating the schematics of these sudden demand variations and how critical they can affect the supply chain networks responsible for delivering critical goods and services. In this regard, Petratos and Faccia [134] have examined the relationship between fake news, misinformation, and disinformation with supply chain disruptions, proposing blockchain strategies to mitigate and manage them. They have created a theoretical framework incorporating disinformation, which suggests how disinformation can exacerbate supply chain disruptions, particularly in cases where disinformation and fake news are intentionally disseminated. They suggest that blockchain technology can reduce vulnerability and improve the resilience of the supply chain. Chatterjee et al. [135] investigated the role of fake news and misinformation in supply chain disruption and its consequences on a firm's operational performance by developing a theoretical and conceptual model considering various hypotheses, which were tested by survey results. The conceptual model was validated using partial least squares structural equation modeling. They illustrated how disinformation can critically impact the supply chain and affect the firm's operational performance. Furthermore Wang et al. [138] have explored the important factors that could affect the acquisition and diffusion of misinformation concerning food safety. They used multivariate regression to examine the important factors and developed a structural equation model that shows the effect of each considered factor. Their results show that while exposure to online news reduces misinformation, social media websites and news outlets are the major sources of misinformation about food safety. Hossain et al. [136] have investigated the impact of disinformation disseminated by social media on supply chain performance due to demand alteration driven by the COVID-19 pandemic. They developed a theoretical model to study the relationship between fake news in social media and disruptions in a supply chain, testing their model with structural equation model applied to survey data. They have also used fuzzy set qualitative comparative analysis to identify various configurations of supply chain resilience that could reduce the supply chain disruption due to disinformation.

The studies mentioned above have primarily focused on gathering and analyzing survey data to test hypotheses and develop theoretical models that explore the impact of various variables and relationships on demand fluctuations resulting from the spread of disinformation within a system. The results show that consumer buying panic exacerbates due to dissemination of disinformation.

The impact of disinformation on other kinds of critical networks has also been studied. Waniek et al. [164] conducted a study examining how messages (e.g., “road closed ahead”) can dictate driving patterns keeping certain drivers off the road, demonstrating that explicit messages regarding physical actions can be leveraged to manipulate user interactions with transportation networks. Other work has showcased the susceptibility of the power grid to disinformation [165–167] . In particular, Jamalzadeh et al. [137] addressed the problem of disinformation campaigns affecting commodity consumption behavior and potentially causing shortages in electric power networks, integrating an epidemiological model of disinformation spread with a mixed-integer programming optimization model for infrastructure network performance. The proposed model objective is to maximize user demand satisfaction and minimize the shortages in network. Their susceptible-infectious-recovered model (SIR) to describe disinformation spread describes the membership of infrastructure users in different states: (i) susceptible to disinformation, (ii) infected by disinformation, or (iii) recovered or not influenced by disinformation. This analogy systematically illustrates the propagation and dismantling of disinformation among consumers of a system over time. It demonstrates how disinformation can lead to fluctuations in demand, reaching its peak within a specific time window. However, as the true information propagates, the demand gradually settles into a new norm, reflecting the impact of accurate information. Jamalzadeh et al. [137] have incorporated simplifying assumptions in their study, specifically by linking the dissemination of disinformation to the dynamics of time and frequent communication among users. While these assumptions offer a framework for objectively measuring the propagation of and resolution to disinformation, they do not explore the significant features and characteristics associated with the changes in demand over time resulting from disinformation.

Therefore, to realize the effects of demand alterations on a supply chain network, it is necessary to establish a systematic and logical framework for generating various scenarios of demand alterations, considering different disruption magnitude levels resulting from disinformation. By doing so, we can determine any possible shortcomings and weaknesses that could emerge because of abrupt variation in demand, enabling us to implement suitable actions aimed at strengthening the resilience of the supply chain. To address these concerns, this research introduces a framework that formulates the patterns of demand alterations in the supply chain resulting from the dissemination of disinformation. The framework takes into consideration multiple realistic characteristics to simulate real-world scenarios. Furthermore, we evaluate the potential effects of these demand fluctuations on the responsiveness of supply chain networks using various metrics.

4.3. Methodology

In this section, we propose a framework that enables us to anticipate changes in demand caused by the spread of disinformation among consumers of goods and services. The framework takes into account different stages where consumers exhibit varying patterns of demand behavior. It is important to note that the (i) dissemination of disinformation among consumers within a supply chain and (ii) its impact on consumer behavior are the two crucial factors underlying our proposed approach for understanding and predicting shifts in consumer demand. We test the demand alteration effect using a mixed-integer optimization model for production fulfillment developed by Soltanisehat et al. [129]. The aim of their optimization model is to minimize order fulfillment time, which serves as a reliable indicator of supply chain responsiveness. Furthermore, to examine the impact of various variables associated with sudden demand changes on supply chain responsiveness, we have conducted a factorial design experiment that analyzes the supply chain's performance in terms of delivery satisfaction (responsiveness) and changes in idle time (capacity utilization) in the production fulfillment process. Table 13 explains the set, parameter, and decision variable notation used in this paper.

Table 13: Summary of sets, parameters, and variables.

Demand alteration formulation terms	Meaning
t_0	Demand elevation starting time
t_1	Demand alteration peak
t_2	Demand degradation termination time
T_m	Demand elevation duration
T_n	Demand degradation duration
D_0	Demand before disinformation dissemination
D_1	Demand peak
D_l	Maximum demand increase
D_{norm}	New demand norm after demand degradation process
α_n	The proportion of maximum demand that accounts for the new demand norm
TDR	Total demand rate
Model sets	**Meaning**
N	Set of work centers indexed by i, j, or k for supplier, inspection, or operation
N_-	A virtual demand work center, $N_- \subseteq N$
$N_=$	Set of virtual supply work centers $N_= \subseteq N$
N_+	Set of actual supply work centers $N_+ \subseteq N$
M	Set of materials for raw material, sub material, and final product indexed by m, p, or q
M_-	Set of final products $M_- \subseteq M$
O	Set of orders indexed by o
T	Set of time indexed by t
A	Set of relations among work centers with their corresponding materials, such that $(m, i, p, j) \in A$ denote that material m in work center i is related to material p in work center j where $m, p \in M$ and $i, j \in N: i \neq j$
R_j	Set of risks corresponding to work center $i \in N$, which is indexed by r
Model parameters	**Meaning**

c_i	The capacity of work center $i \in N\{N_= \cup N_-\}, c_i \in \mathbb{Z}^*$
u^{mp}	Unit usage of material $m \in M$ for processing material $p \in M$ such that $(m, i, p, j) \in A$, $u^{mp} \in \mathbb{Z}^*$
s_o^m	Size of order $o \in O$ for final product $m \in M_-$, $s_o^m \in \mathbb{Z}^*$
w_{ot}^m	Importance weight of order $o \in O$ for final product $m \in M_-$ at time $t \in T$, $w_{ot}^m \in \mathbb{R}^+$
b_0^m	Initial inventory of product $m \in M$, $b_0^m \in \mathbb{Z}^*$
e_i	Process time of work center $i \in N$, $e_i \in \mathbb{R}^+$
Variables	Meaning
x_{ijt}^{mp}	Flow of material $m \in M$ transformed to material $p \in M$ at time $t \in T$ passing through work centers i and $j \in N$, respectively, such that $(m, i, p, j) \in A$, $x_{ijt}^{mp} \in \mathbb{Z}^*$
y_{it}^m	Amount of final product $m \in M_-$ reaching at virtual work center $i \in N_-$ at time $t \in T$, $y_{it}^m \in \mathbb{Z}^*$
v_t^m	Inventory of material $m \in M$ at $t \in T$, $v_t^m \in \mathbb{Z}^*$
z_{ot}^m	Equal to 1 if order o ∈ Oof final product $m \in M_-$ is met at time $t \in T$ otherwise equal to 0, $z_{ot}^m \in \{0, 1\}$

4.3.1. Demand Alteration due to Disinformation

Using the analogy of poison and antidotes, when disinformation (poison) spreads within a community, there is a probability that individuals who come across it will be influenced by its effect. Conversely, the dissemination of accurate information (antidote) to counter the disinformation generally lags behind, as the sudden spread of disinformation catches trustworthy sources off guard. Additionally, the time required for these reliable sources to influence the community, enlightening individuals about false information, may vary and encounter obstacles [168,169]. Therefore, a dynamic relationship exists between the propagation of disinformation and accurate information, depending on the level of connection between agents of each side and individuals within the communities. Equally important is how individuals perceive and consume both disinformation and information, as this reflects the resilience of the community towards them [170,171]. As such, several variables may influence the propagation of information and disinformation within a system. These variables include connectivity, system resistance, credibility of propagation sources, community demographics, and the presence of and the performance of agents responsible for countering disinformation [168]. These variables can have varying levels of importance assigned to them. Understanding the relationship between the propagation and breakdown of disinformation and its influence on consumer behavior and demand variation further adds complexity to the problem. Addressing these complications is important when adopting a bottom-up methodology to predict sudden demand variations caused by disinformation.

To address these concerns, three phases of demand alteration has been considered: (i) the behavior of consumers reacting to disinformation, (ii) the magnitude of demand surge as consumers try to maximize their preparedness for the unwanted situation, and (iii) changes in demand norms after everything returns to normal. Generally, consumers can exhibit various types of behavior when confronted with disinformation, demonstrating different levels of resistance to its influence. Initially, they may show resistance and not change their demand during the early stages of disinformation dissemination. However, over time, there could be a significant increase in demand as the impact of disinformation takes hold. Conversely, in other communities with different demographic and sociological characteristics, the opposite relationship can occur. Therefore, consumer demand could increase or decrease in any stages of disinformation propagation. Depending on the criticality of the disinformation, the dissemination rate due to a highly connected community network, and presence of any agents responsible countering disinformation, the magnitude of demand alteration could vary. Moreover, as consumers undergo a transition in their consumption patterns, their habits have the potential to shift in response to the spread of disinformation once the initial surge in demand has subsided. This underscores the dynamic nature of consumer responses to disinformation and its influence on their behavior.

Therefore, we have considered a three-stage demand alteration based on the aforementioned factors: (i) an initial demand increase caused by the propagation of disinformation, (ii) a subsequent decline in demand as the disinformation is debunked and individuals become aware of accurate information, and (iii) the establishment of new demand norms as the community adapts to a new consumption pattern and forms new habits. Figure 14 illustrates this three-stage process, with the horizontal axis representing the time (t) at which the demand alteration occurs and the vertical axis indicating the corresponding demand $D(t)$ from the supply chain over time. Term $T_m = (t_1 - t_0)$ is the considered to be the time that disinformation propagation is causing the increase in demand. We refer to this as the period demand elevation. Term $T_n = (t_2 - t_1)$ is considered to be the time that disinformation is revolve and correct information finds its way to the community, thus degrading demand $D(t)$. Demand is degraded, for $t > t_2$, until it settled on a new demand norm, D_{norm}.

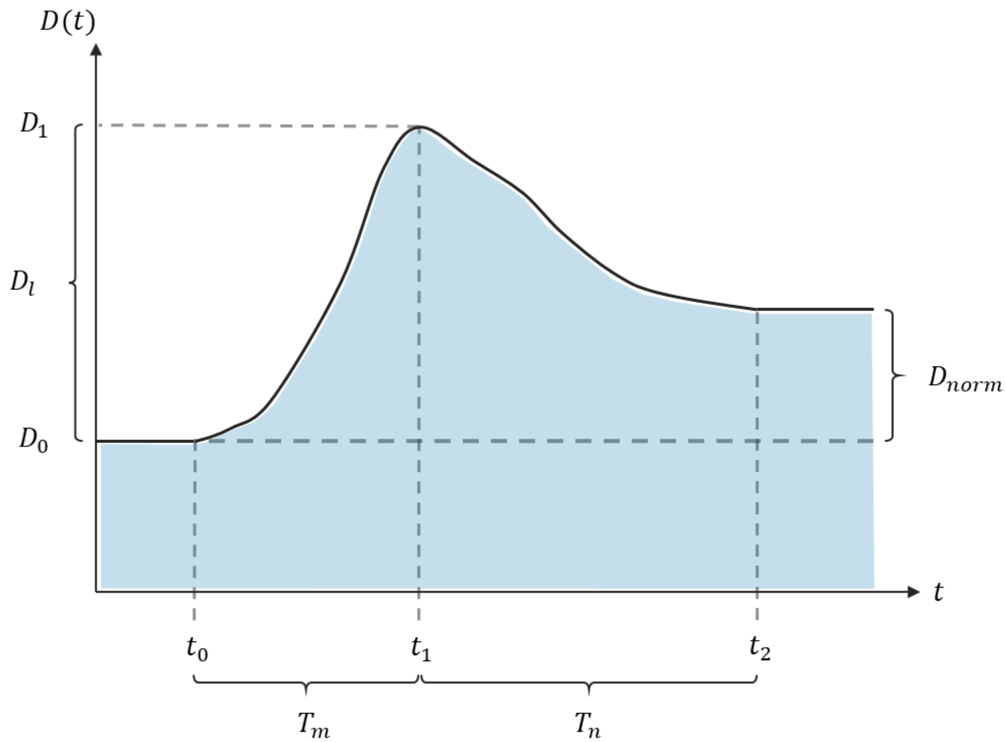


Figure 14: Demand alteration schematic overview.

We consider a steady demand (D_0) prior to disinformation propagation for $t < t_0$. The demand surges during the demand elevation period to its peak (D_1), considering the demand increase (D_l) from the demand level prior the elevation period during the time $t_0 < t < t_1$, where $D_l = (D_1 - D_0)$. As the new demand norm is created due to change in consuming habit, the demand from supply chain settled from its peak to the new demand norm level (D_{norm}). As such, we consider three different behaviors of demand elevation and degradation which are: (i) linear, (ii) exponential, and (iii) trigonometric, where each represent different consumer behavior resulting from disinformation spread and adoption.

The linear formulation is generally used when there is limited information available about the specific impact of different characteristics of disinformation on community demand behavior in the elevation or degradation stages. On the other hand, the exponential formulation is more appropriate in cases where users initially resist the spread of disinformation, resulting in a slow increase in demand that accelerates over time. Similarly, in the demand degradation process, there is a sudden decline in demand immediately after reaching its peak, followed by a gradual decrease. This formulation is suitable for situations where the effect of disinformation is rapid but short-lived. Additionally, we have proposed a trigonometric formulation for situations where there is a slow and gradual increase or decrease in demand during the initial and final phases of both elevation and degradation of demand alteration process. On the other hand, in the middle phases of both elevation and degradation process, a rapid change can be assumed to be experienced. The formulations are motivated by the resilience analysis of Song and Li [172]. Eqs. (31) and (32) represent the demand elevation and degradation respectively. Eq. (31) shows the demand formulation for demand elevation stage for time $t_0 < t < t_1$.

$$D_t(t) = \begin{cases} D_l\left(\dfrac{t - t_0}{T_m}\right) & (linear) \\ D_l\dfrac{\exp\left(-b\,\dfrac{T_m - t + t_0}{T_m}\right) - \exp(-b)}{1 - \exp(-b)} & (exponential) \\ 0.5D_l\left[1 - \left(\pi\dfrac{t - t_0}{T_m}\right)\right] & (trigonometric) \end{cases} \tag{31}$$

Eq. (32) shows the demand formulation for demand degradation stage for time $t_1 < t < t_2$.

$$D_t(t) = \begin{cases} D_l\left(1 - (1 - \alpha_n)\dfrac{t - t_1}{T_n}\right) & (linear) \\ D_l\left[(1 - \alpha_n)\left(\dfrac{\exp\left(-b\dfrac{t - t_1}{T_n}\right) - \exp(-b)}{1 - \exp(-b)}\right) + \alpha_n\right] & (exponential) \\ D_l\left[0.5(1 - \alpha_n)\left(1 + \pi\dfrac{t - t_1}{T_n}\right) + \alpha_n\right] & (trigonometric) \end{cases} \qquad (32)$$

Note that b is scale parameter in the exponential function representing the linearity of the demand alteration over time. In cases that the $b = 0$, the linear function can be used. Also, the larger the value of b, the higher the resistance of the community against disinformation dissemination in early time periods. In the degradation formulation, the new demand norm rate, α_n, describes the proportion of maximum demand that accounts for the new demand norm. As such, Eq. (33) provides the formulation for new norm demand (D_{norm}).

$$D_{norm} = \alpha_n D_l \qquad (33)$$

With variables D_l, T_m, T_n, and α_n, we can calculate the demand for any time t. Figure 15 illustrates the shape of the functions for both elevation and degradation stages of demand using all three types of functions: linear, exponential, and trigonometric.

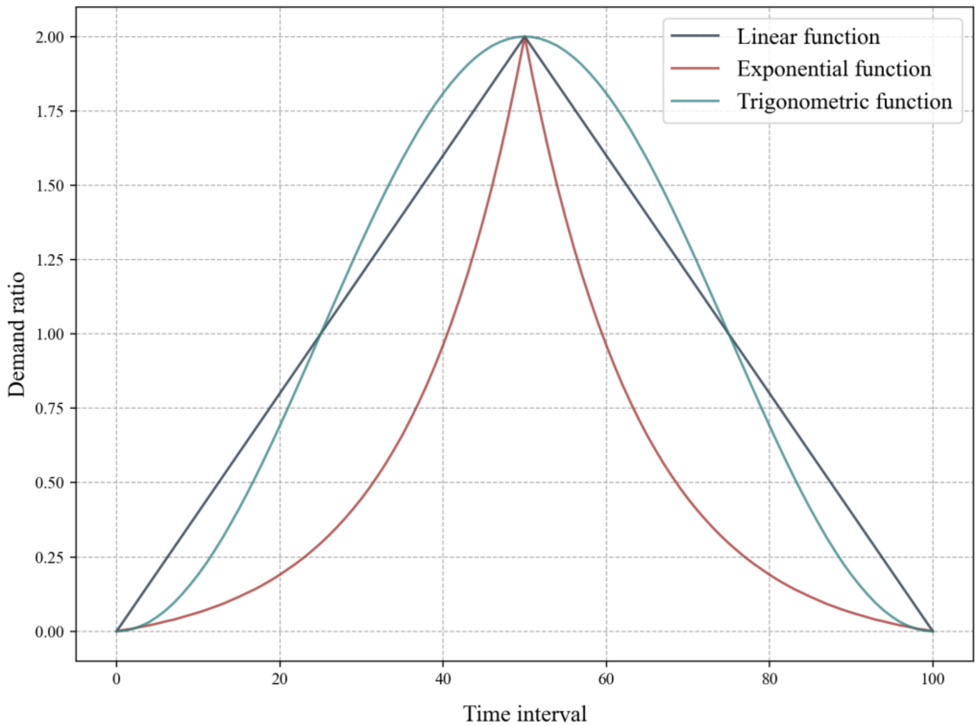


Figure 15: Developed formulation for both demand elevation and degradation stage.

The demand ratio in Figure 15 shows the times that the demand is greater than the normal demand prior to the disinformation propagation in the community.

With this approach, we can generate a scenario of demand alteration. Since predicting the duration of the elevation or degradation process is challenging, we explore randomly generated durations of these stages, as demand elevation and degradation can be regarded as an uncontrollable factor when generating a sample of scenarios to be used in our experiments. Exponential distribution is assumed to be used for determining values for T_m and T_n.

Furthermore, estimating the maximum demand increase D_l is generally challenging as it depends on various factors, including the duration of demand elevation and degradation, community behavior regarding disinformation dissemination, and the impact of disinformation on demand alteration. Mentioned previously, we randomly determine values for T_m and T_n, considering this randomness as an uncontrollable factor due to the subjective nature of disinformation influence on the community.

To address demand behavior related to demand alteration, we have introduced the three types of formulations. It is also important to assess the supply chain's responsiveness, as well as test its capability considering various demand magnitudes. Therefore, we have defined a ratio called Total Cumulative Demand (TDR) which represents the magnitude of demand alteration. TDR indicates the increase in demand compared to the normal condition (e.g., $TDR = 2$ signifies that the cumulative demand after disinformation propagation is twice as high as the normal condition). Therefore, for a continuous domain for time and Figure 14 in mind, we can formulate Eq. (**34**) to represent the total cumulative demand.

$$\frac{\int_{t_0}^{t_1} D^m(t)\, dt + \int_{t_1}^{t_2} D^n(t)\, dt}{(t_2 - t_0) D_0} = TDR \tag{34}$$

Replacing $D^m(t)$ and $D^n(t)$ with the formulations defined in Eqs. (31) (32)and (32), respectively, we can solve the Eq. (**34**) for D_l, which would be the demand peak after disinformation dissemination. In conclusion, having all three assumed input information of demand elevation/degradation, consumers demand behavior, and the demand peak, we can calculate the demand for the entirety of the time horizon.

4.3.2. Production Fulfilment Mixed Integer Optimization Model

In this section, we introduce a mixed-integer model that was developed by Soltanisehat et al. [129] and specifically designed to address the production fulfillment time and capable of examining the responsiveness of the supply chain.

In situations where there is a sudden and significant increase in demand, the primary objective is to fulfill as many orders as possible, given that the supply chain capacity is often insufficient. Therefore, decision-makers responsible for the supply chain typically prioritize orders based on their importance, rather than focusing on meeting specific delivery deadlines for each order. The objective of the model used in this research is to minimize the time required to fulfill each order, considering their respective importance levels. The model also considers factors such as resource availability, production capacity, and uncertain operational risks, which become particularly crucial in critical circumstances. The MILP model maximizes product delivery for each order by considering production system risks. Two interconnected networks were constructed: one based on material relations from the bill of material (BOM) and the production system structure, and another focused-on work center characteristics such as production time, capacity, and probabilistic risk values.

The demand information, whose formulation was provided in the previous subsection, is used model input. This information includes the order size and its priority, which can be determined by factors such as delivery deadlines, emerging priorities, or other important considerations. Note that, congruent with the optimization model, we have considered the demand for each period as the order size within that period, while utilizing time as a factor for determining order priority. In other words, we treat the demand for each period as an individual order and calculate its size using the formulation provided in the previous section. Additionally, we assign more importance to orders placed earlier.

The production system is represented by a directed network, denoted as $G = (N, L)$, where N represents the work centers connected by links in set L. This network incorporates material relations, work center relations, and combined material and work center relations derived from the BOM. The links within the network depict the flow of materials or assembled parts between different work centers, enabling the tracking of material flow and the propagation of risks throughout the system. Figure 16 illustrates the relationships among work centers and the progression of materials during production processes. As materials move through each work center, they undergo transformations, and a designated set of A, where $(m, i, p, j) \in A$, is employed to monitor these changes. The material flow within the network x_{ij}^{mp} and x_{jk}^{pq}, signifying the transformation of materials from one work center to another until the final product reaches the virtual demand node.

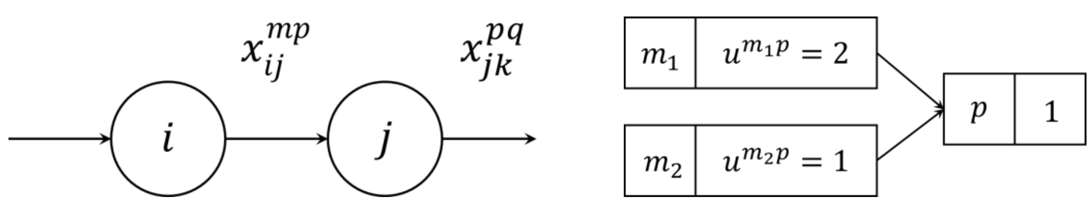


Figure 16: (a) work centers relations, and (b) material relations (adapted from Soltanisehat et al. [129]**)**

The optimization model consists of: (i) the objective function responsible for maximizing the order fulfilment as quickly as possible, considering the importance (priority) of each order, (ii) work center production capacity constraints, (iii) inventory balance constraints for every work center and the final virtual sink component that pulls demand from all other components, and (iv) the logical constraint that ensures each order to be satisfied only one in the time horizon. The objective function, represented in Eq. (35), aims to satisfy the orders in a timely manner, maximizing the sum of order weights multiplied by binary variable representing if an order is satisfied, forcing the model to fulfill the orders as fast as possible.

$$G = max \sum_{t \in T} \sum_{m \in M\_} \sum_{o \in o} w_{ot}^{m} z_{ot}^{m} \tag{35}$$

Constraint (36) restricts the outgoing flow which cannot exceed the capacity of the supplier and the work center. Constraint (**37**) is responsible to provide the inventory for the initial period at $t = 0$. Constraints (**38**) and (**39**) track the inventory of materials flowing through the work centers at each period in the time horizon. These equations consider the unit usage of the material and the processing time required for processing of the material at each work center. Constraint (**40**) guarantees the delivery of final product to the virtual demand node (final work center), while constraint (**41**) forces the model to satisfy the order considering the number of final products it consisted of. Constraint (**41**) also keeps track of the inventory for the final product reaching the virtual node in case that the minimum number of products has not been maintained to fulfil the order in time t.

$$\sum_{m:(m,i,p,j)\in A} \sum_{p:(m,i,p,j)\in A} \sum_{j:(m,i,p,j)\in A} x_{ijt}^{mp} \leq c_i \qquad \forall i \in N\{N_= \cup N_-\}: (m,i,p,j) \in A, \forall t \in T \tag{36}$$

$$v_{t-1}^m = b_0^m \qquad \forall m \in M, \forall t \in T: t = 1 \tag{37}$$

$$v_t^m = v_{t-1}^m - u^{mp} \sum_{q:(p,j,q,k)\in A} \sum_{k:(p,j,q,k)\in A} x_{jkt}^{pq} \qquad \forall j \in N\backslash\{N_-\}, \forall m \in M\backslash\{M_-\}: (m,i,p,j) \in A, \forall t \in T: (t - ej + r\tilde{}j) \leq 0 \tag{38}$$

$$v_t^m = \sum_{i:(m,p,i,j)\in A} x_{ij(t-e_j+r_i)}^{mp} + v_{t-1}^m - u^{mp} \sum_{q:(p,j,q,k)\in A} \sum_{k:(p,j,q,k)\in A} x_{jkt}^{pq} \qquad \forall j \in N\backslash\{N_-\}, \forall m \in M\backslash\{M_-\}: (m,i,p,j) \in A, \forall t \in T: (t - ej + r\tilde{}j) \geq 0 \tag{39}$$

$$\sum_{m:(m,i,p,j)\in A} \sum_{i:(m,i,p,j)\in A:m=p} x_{ijt}^{mp} = y_{it}^m \qquad \forall j \in N_-, \forall m \in M_-: m = p, \forall t \in T \tag{40}$$

$$v_t^m = y_{it}^m + v_{t-1}^m - \sum_{o\in O} s_0^m z_{ot}^m \qquad \forall i \in N_-, \forall m \in M_-, \forall t \in T \tag{41}$$

$$\sum_{t\in T} z_{ot}^m \leq 1 \qquad \forall m \in M_-, \forall o \in O \tag{42}$$

4.3.3. Experiment on Supply Chain Responsiveness

To gain insights into the effects of various variables on the responsiveness of the supply chain when confronted with sudden changes in demand, we conducted a factorial experiment designed to consider the different variables associated with demand alteration. The primary objective of this experiment was to evaluate the supply chain's responsiveness by analyzing critical aspects of customer satisfaction. We assessed how quickly orders could be fulfilled considering the surge in demand resulting from disinformation dissemination. Additionally, we aimed to assess the supply chain's flexibility in dealing with the challenging conditions of demand surges. Therefore, we focused on analyzing the capacity utilization of the supply chain, specifically examining changes in idle time during the production fulfillment process.

In this regard, we employed the center of gravity formula to evaluate the speed at which orders were stratified by the production fulfillment center. By calculating the center of gravity for the size of orders that had been satisfied, corresponding to the time of their fulfillment, we were able to assess the concentration of satisfied orders correspondent to the time. Comparing this criterion for both normal demand condition and demand situation after the disinformation dissemination, we can assess the responsiveness of the supply chain. Hence, the difference in the center of gravity concerning the time when orders are fulfilled under normal demand conditions compared to sudden demand surges is regarded as the response measure for this criterion. Furthermore, the center of gravity formula, also known as the weighted mean formula, calculates the center point or average location of a set of values based on their respective weights. In the context of assessing order satisfaction speed, the center of gravity formula can be expressed with Eq. (43). By applying this formula, we can determine the central point or average time at which orders are satisfied based on their sizes.

$$CG = \frac{\sum_{o \in O} s_0^m t}{\sum_{o \in O} s_0^m} \qquad \forall o \in O, t \in T: z_{ot}^m = 1 \qquad (43)$$

To assess the capacity utilization of the supply chain, we rely on Eq. (36) from the optimization model, which constrains the flow to be within the available capacity. By calculating the idle time of each work center at every point in time, we can effectively measure how efficiently the supply chain utilizes its resources throughout the production fulfillment process. By measuring the idle time of the work centers during different periods, we can gain valuable insights into the degree of capacity utilization and identify opportunities for enhancing efficiency. Hence, the difference in the work center total idle time when orders are fulfilled under normal demand conditions compared to sudden demand surges is regarded as the response measure for this criterion.

To calculate the total idle time of the work centers, we reformulate Eq. (36) to a summation of idle time across all work centers and throughout the time horizon. Using this formulation in Eq. (44), we can calculate the level of unused capacity considering alteration in demand, which also can be an indication for determining the flexibility of the supply chain. Comparing the system idle time before and after disinformation dissemination can inform about how the system can adapt itself to the new demand.

$$ID = \sum_{t \in T} \sum_{i:(m,i,p,j) \in A} \left(c_i - \sum_{m:(m,i,p,j) \in A} \sum_{p:(m,i,p,j) \in A} \sum_{j:(m,i,p,j) \in A} x_{ijt}^{mp} \right) \tag{44}$$

Mentioned previously, this experiment considered three different demand variables: (i) Total Cumulative Demand Rate (TDR), (ii) Level of New Demand Norm, and (iii) Consumer Demand Behavior. By combining different levels of these variables, various demand surge scenarios can be created. It is important to note that T_m and T_n are uncontrollable factors randomly generated using the exponential distribution, allowing for the creation of different demand surge scenarios using the same variable combination. The generated scenarios serve as inputs for the optimization model to determine the optimal response of the supply chain under different demand scenarios. The results of the optimization model are used to compute the desired responses using Eqs. (43) and (44).

With this data, we conducted a factorial design of experiment to identify the key variables that influence the responsiveness of the supply chain and its resource utilization.

4.3.4. More realistic view of critical supply chain

In this section we would like to discuss the other variables that can affect the supply chain network of critical goods, changing the resilience of the network considering the demand surge due to disinformation dissemination in the communities. In this regard we can define these variables as (i) internal and (ii) external variables. The internal variables are those that can be controlled by the decision-makers responsible for the supply chain network administration. These variables represent the decisions regarding the network components process capacity and the level of inventory that should be kept for safety. I worth noting that considering different circumstances, different variables can be defined representing different aspects of capacity and safe inventory variables. The external variables, on the other hand are those that are not in control of supply chain decision-makers, and generally depends on the behavior of the supply chain network consumers. These variables are generally related to the input and output of the supply chain such as the raw material supply, and final product demand. Figure {} shows a schematic overview of the internal and external variables effective in a supply chain.

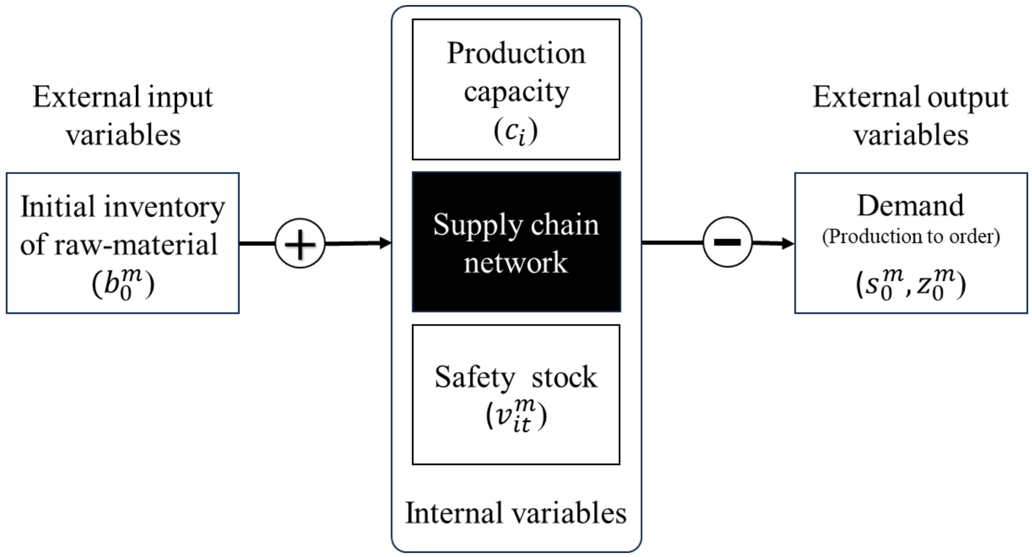


Figure 17: schematic overview of the internal and external variables

It is worth noting that we can always consider that the internal variables are generally affected by the external ones and vice versa. Considering the disinformation propagation in society, generally the external variables would be affected. Therefore, input supply and the output demand of the supply chain can be increased or decreased. Reacting to these changes, the supply chain changes its internal variable adapting to the new situation and maximizing its payoff. It is worth mentioning that after adjusting the internal variables some of the orders may be partially satisfied or not even satisfied, imposing higher costs to the supply chain. Also, dealing with high demand surges, the supply chain may decide to keep higher inventory to minimize the cost of unmet demand. However, keeping higher inventory for the final product, higher inventory cost is imposed to the supply chain. Therefore, considering any of these two policies, the decision-maker responsible for the supply chain administration should decide how to provide a balance between these two policies to minimize the total cost of the system. Therefore, we propose a different objective function for the production fulfilment mixed integer optimization model minimizing the fulfillment cost considering the mentioned policies which are: (i) unmet demand cost, and (ii) inventory cost. Eq. (45) is designed to address these two mentioned policies.

$$G = Min \sum_{t \in T} \sum_{m \in M_{-}} \sum_{o \in o} \delta_{tmo}^{-} + \sum_{i} \sum_{m} \sum_{t} v_{it}^{m} \tag{45}$$

Generally, in critical situations like disinformation propagation all consumers in the systems expect shortcomings, they try to minimize their unmet demand even if they know that they may not be able to receive their orders completely. Therefore, we propose a formulation to be added to the fulfilment mixed integer optimization model to allow partial fulfilment. Eq. [] is designed to address partial fulfilment in the supply chain.

$$H_{o}^{m} - \sum_{t \in T} \sum_{o \in O} z_{ot}^{m} = \delta_{tmo}^{-} \tag{46}$$

Note that z_{ot}^{m} in the formulation does not represent the orders, but the numbers of final product. However, H_{o}^{m} is considered as parameters which represent the number of final products m in order o. Considering these two changes in the model we have a basic model to calculate the best strategy for the supply chain given the circumstances.

Furthermore, supply chains are not only interdependent where the output of one supply chain serving as the input for another, but also controlled by different decision-makers pursuing their own interests through decentralized decision-making. These decision-makers operate autonomously, trying to maximize their profits and optimize their associated supply chain performance. Investigating the performance of interconnected supply chains without accounting for these decentralized decision-making dynamics would result in a biased understanding of the reality. Therefore, it is essential to consider the interdependency between supply chains and the self-interested behavior of decision-makers to gain a more comprehensive overview of the overall system dynamics. Addressing the mentioned concerns, we need to emphasize on decentralized decision-making models and algorithms finding the solutions where all decision-makers have the liberty to adjust their strategy using their available resources to maximize their benefit.

Note that the disinformation can be disseminated with different magnitudes, having local or global effect on demand alteration of one single supply chain or several of them. These demand alterations can be imposed using the presented formulations, considering different assumptions regarding the behavior of the consumers. Bearing in mind that components of a supply chain are interconnected, it is important to recognize that any changes in demand for one component can impact the entire system. As the demand increases for a specific component, this alteration in demand propagates throughout the supply chain, where every other connected component attempts to adapt itself to the new situation. Therefore, the speed and timing at which the demand alteration can propagate throughout the system must be anticipated and considered in the modeling. Although this dissertation does not focus investigation into addressing the specific concerns mentioned in this section, it recognizes the importance of these issues and acknowledges them as potential areas for future investigation. Understanding and tackling these concerns would contribute to broader knowledge in the field and could lead to valuable insights and advancements. By identifying these aspects as future avenues for research, this study opens the door for further exploration and invites researchers to pursue these important topics.

4.4. Results and Discussion

We conducted a factorial experiment design to systematically investigate the impact of multiple variables and their interactions on the responsiveness and resource utilization efficiency of the supply chain. By defining key variables and simultaneously varying them to create symmetric level combinations, while also considering random and uncontrollable factors, we can assess the individual and combined effects of these variables on the response variables. This analysis provides insights into how TDR, α_n, and consumer demand behavior influence the responsiveness and resource utilization of the supply chain, highlighting its strengths and weaknesses in adapting to sudden demand alterations caused by disinformation propagation. We consider levels of low and high for each independent numerical variable, and three levels for independent categorical variable that represent the type of consumers demand behavior, as shown in Table 14. As such, the experiment has $2 \times 2 \times 3 = 12$ corner points. Considering three experiment repetitions for every treatment we will have $3 \times 12 = 36$ runs for our experiment. To augment the experiment and check if any potential curvature or non-linearity can be detected considering the response variables, we use two center points concerning the numerical independent variables for each level of categorical variables, bringing the total experiment runs to $(2 \times 3) + 36 = 42$. We have considered two response variables related to the performance of the supply chain and three independent variables representing various characteristics of demand surge: (i) the difference in the center of gravity in relation to the time of order fulfillment under normal demand conditions versus sudden demand surges caused by disinformation propagation and (ii) the difference in work center total idle time.

Table 14: Experiment design summery

Factors		Factor types	Low levels	High levels
A	Total cumulative demand rate (TDR)	Numerical	1.5 (times the normal)	3 (time the normal)
B	New demand norm rate (α_n)	Numerical	0 (back to normal)	1 (stays at peak)
C	Consumer demand behavior	Categorical	Linear, Exponential, Trigonometric	

Note that we consider the TDR of 2.25 and α_n of 0.5 for the center point respectively, considering each categorical levels for consumer demand behavior. We have utilized the same case study employed by Soltanisehat et al. [129], which focused on the design specifications of a medical ventilator and its BOM, publicly accessible on the Gtech company website [173]. Additionally, realistic assumptions were made due to the limited availability of information concerning the production system.

After conducting the initial ANOVA analysis on the first target response variable, the difference in the center of gravity, we observed that only the (A) total cumulative demand rate, (B) new demand norm rate, and their (AB) interaction were found to be statistically significant. Table 15 provides an overview of the ANOVA results. The lack of fit is not significant relative to the pure error.

Table 15: ANOVA table for the difference in center of gravity

Source	Sum of squares	df	Mean square	F_0	p-value
Model	3235.39	3	1078.46	110.37	< 0.0001
A-Total cumulative demand rate	2922.67	1	2922.67	299.09	< 0.0001
B-New demand norm rate	115.57	1	115.57	11.83	0.0015
AB	192.60	1	192.60	19.71	< 0.0001

Residual	361.55	37	9.77		
Lack of fit	143.69	11	13.06	1.56	0.170
Pure error	217.86	26	8.38		
Cor Total	3596.94	40			

The ANOVA results for the difference in the work center total idle time is shown in Table 16. Diagnosing the initial model and looking at the Box-Cox plot we transformed the data using invers square root formulation as the corresponding Lambda was (-0.5). After transformation, we observed that only the (A) total cumulative demand rate, (C) Consumers' demand behavior, their interactions (AC), and the interaction exists between new demand norm rate and Consumers' demand behavior (BC) were found to be statistically significant. The lack of fit also appears to not be significant relative to the pure error.

Table 16: ANOVA table for the difference in work centers' total idle time.

Source	Sum of squares	df	Mean square	F_0	p-value
Model	4.368	8	0.546	148.86	< 0.0001
A-Total cumulative demand rate	3.981	1	3.981	1085.51	< 0.0001
B-New demand norm rate	0.011	1	0.011	2.93	0.0970
C- Consumers' demand behavior	0.173	2	0.086	23.55	< 0.0001
AC	0.214	2	0.107	29.12	< 0.0001
BC	0.028	2	0.014	3.87	0.0312
Residual	0.117	32	0.004		
Lack of Fit	0.037	6	0.006	2.01	0.1004
Pure Error	0.080	26	0.003		
Cor Total	4.485	40			

Fit statistics related to both defined target responses have been illustrated in the

Table 17. As predicted R^2 for both the target responses agree with the adjusted R^2 as their difference is less than 0.2. Also, adequate precision, which measures signal to the noise

ratio, is much greater than 4 (rule of thumb [122]) for both target responses, which implies that both models can be used to navigate the design space.

Table 17: Fit statistics of both target responses.

Statistics	Difference in center of gravity	Difference in total idle time.
R^2	0.899	0.973
Adjusted R^2	0.891	0.967
Predicted R^2	0.881	0.958
Adequate precision	23.552	31.824

Figure 18 displays the response surface depicting the variation in the center of gravity corresponding to the total cumulative demand rate and the new demand norm. Examining the ANOVA table reveals that the cumulative demand rate has a significant main effect on the response, which is also evident from the figure. However, consumers' demand behavior does not have a significant effect on the center of gravity of orders, indicating that regardless of the invulnerability of a group of users to the dissemination of disinformation, these slight changes in the form of sudden demand do not greatly impact consumer satisfaction.

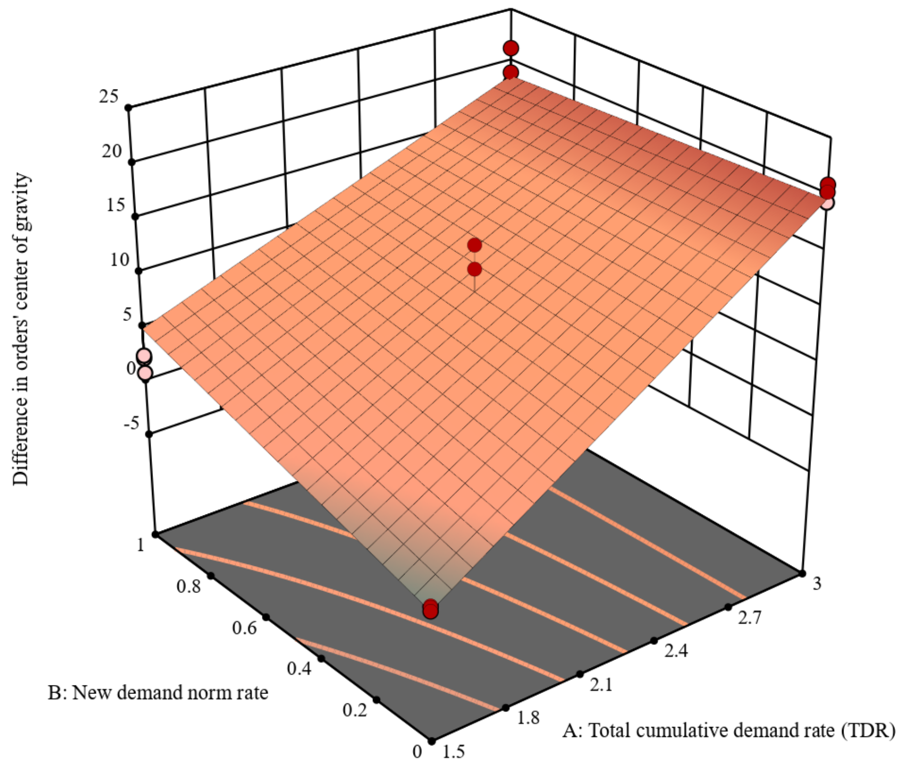


Figure 18: Response surface for the difference in orders' center of gravity

Notably, the response surface's lowest point corresponds to a total cumulative demand rate of only 1.5 times the normal condition, resulting in a center of gravity difference of approximately -3 days for order satisfaction. That is, despite the increased demand, the center of gravity for order fulfillment shifts to the left compared to the normal condition. This implies that, provided there is sufficient supply chain capacity, sudden changes in demand do not have a negative impact on system responsiveness but rather lead to enhanced performance as the higher number of orders can be satisfied in a same amount of time. These capacities can be maintained through various policies and strategies within the supply chain, such as maintaining additional inventories, providing a balanced supply chain network, and increasing the capacity of production and distribution work centers.

Regarding the second target response, the total cumulative demand rate overwhelmingly dominates as the most influential factor in determining work center utilization. This significance is evident from the ANOVA table, where the total cumulative demand rate overshadows the effects of other factors. This is also evident from Figure 19, which illustrates the response surface considering both the total cumulative demand rate and the rate of the new demand norm as numerical factors. It is worth noting that the rate of the new demand norm and its interaction with the total cumulative demand rate are not significant, particularly when the total cumulative demand rate is high. In such cases, the effect of the new demand norm becomes completely negligible.

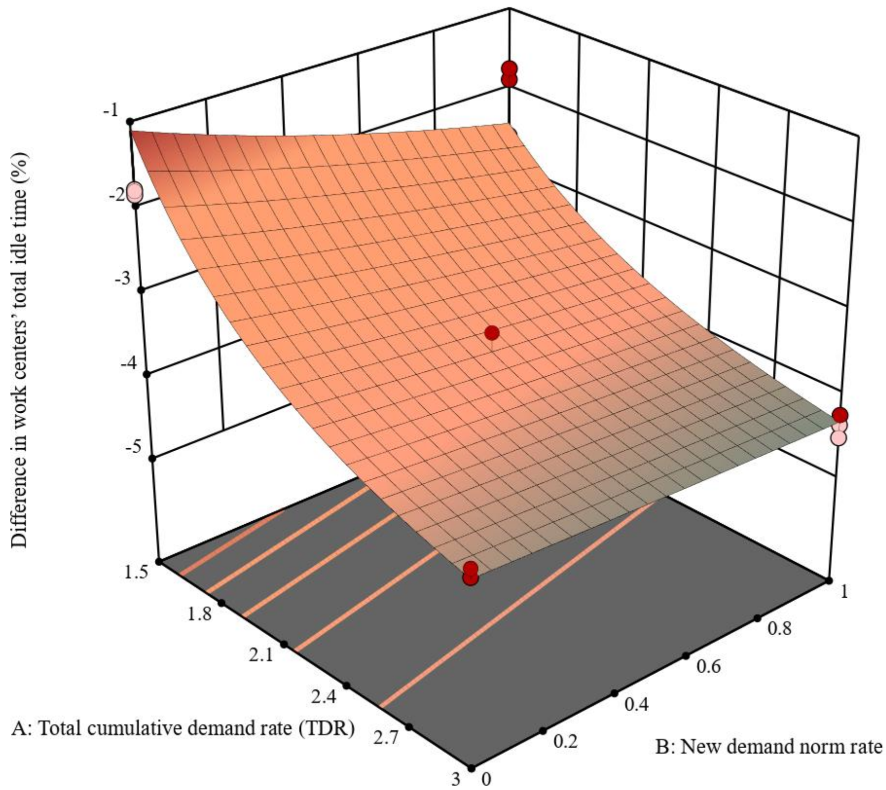
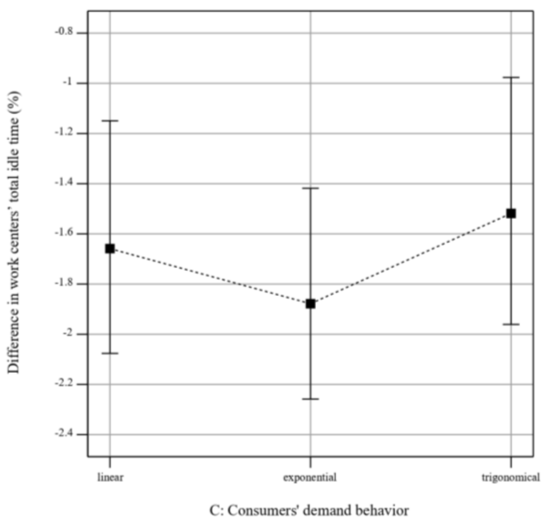


Figure 19: Response surface for the difference in work centers' total idle time

Figure 20(a) and (b) present the difference in system idle time corresponding to consumer demand behavior, specifically when the total cumulative demand rate is at its minimum (1.5) and maximum (3), respectively. Interestingly, there is a negligible difference in the total idle time of work centers across various factor levels of consumer demand behavior. However, when the total cumulative demand rate is high, exponential demand behavior noticeably exhibits lower capacity utilization, as demonstrated in Figure 20(b). This could be due to the nature of the exponential demand formulation, which reflects a gradual increase at the beginning to capture communities with higher levels of resilience against disinformation propagation. Consequently, the supply chain may have caught us off guard as it attempted to adjust to the initially moderate demand, requiring fewer resources and capacity during the early stages of the demand alteration. As a result, the capacity utilization remained relatively stable for an extended period after the onset of the demand alteration process.



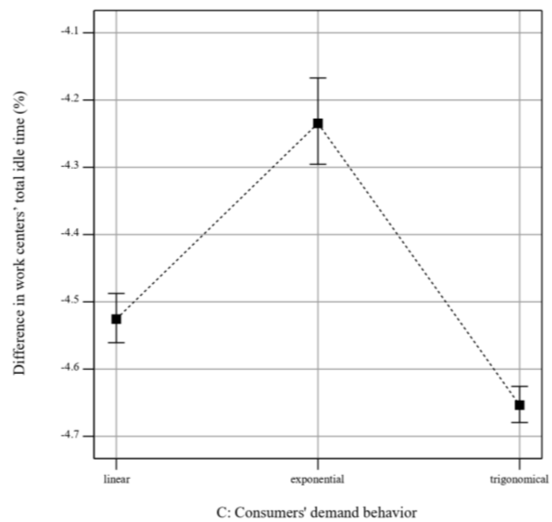


Figure 20: The difference in system idle time corresponding to consumers' demand behavior. (a) when the TDR is at its minimum, (b) when the TDR is at its maximum.

Note that by utilizing the second target response, we can immediately assess the level of flexibility of the supply chain. In our case, as significant increase in demand exceeds the system capacity, the decrease in work centers' total idle time was less than 5%. That is, the system's adaptability to the demand surge caused by disinformation propagation is limited. Therefore, the proposed framework can serve as a practical tool for evaluating the flexibility of the supply chain and critical networks.

4.5. Conclusion

This study highlights the critical concern of supply chain security in the face of disinformation. The rapid spread of false information within supply chain networks can lead to delays, disruptions, and security breaches. Our research emphasizes the importance of modeling and understanding the variations in demand behavior caused by disinformation and their impact on the responsiveness of supply chain networks. The analysis indicates that the magnitude of demand surge (cumulative demand rate) has a significant effect on how fast the consumers order (orders' center of gravity) can be satisfied. Also, it has a significant effect on the capacity utilization (work center total idle time) of the supply chain network. Note that consumer behavior after adopting disinformation and subsequently changing their demand behavior does not have an effect on consumer order satisfaction (order center of gravity) but does have an effect on supply chain capacity utilization. Furthermore, the new consumption norm, created after consumers experience the demand surge, has an effect only on supply chain responsiveness and not on the system capacity utilization (work center total idle time). Furthermore, the study suggests that as long as the supply chain has enough capacity, abrupt shifts in demand do not negatively affect the system's responsiveness but instead contribute to improved performance in consumer satisfaction. Regarding the utilization of supply chain capacity, lower capacity utilization can be observed when consumers' demand exhibits exponential behavior, representing their higher resilience to being influenced by disinformation during the early stages of dissemination. Additionally, we have demonstrated that by utilizing the presented framework and generating various demand scenarios, we can evaluate the adaptability and flexibility of any given supply chain network in coping with sudden changes in demand. This enables us to identify weaknesses within the supply chain and develop solutions to address them. The future avenue for this research could be focusing on different variables internal to the supply chain, examining how these variables can be changed to mitigate the shortcoming of the supply chain as the result of having demand surge. Also, we would like to approach the problem from the perspective of interconnected autonomous supply chains that can mutually impact each other, with each chain striving to protect its own interests considering having disinformation disseminated into the system.

5. CHAPTER FIVE: APPLIED GAME THEORY TO ENHANCE AIR TRAFFIC CONTROL IN 3D AIRSPACE

5.1. Introduction

Civil air transportation offers important advantages, including but not limited to speed, comfort, and efficiency over long distances [174,175]. Recent surveys show the growing popularity of air transportation in comparison to other modes of transportation [176–181]. In 2018, more than one billion passengers traveled in the US using civil air transportation [182]. As a result of this trend, the traffic volume in the airspace continues to increase, which subsequently increases the possibility of conflicts among aircraft [183]. Therefore, reliable decision-making procedures are necessary to enable the timely resolution of these conflicts in the airspace. In this regard, integration of innovative technological advances are essential to assist decision makers for maintaining a required level of safety, efficiency, and smooth traffic in the airspace [178,184–187]. From a conventional perspective, air traffic control (ATC) was a centralized operation responsible for providing safe and smooth airspace operations. Thereafter, new technologies have been developed to support ATC with the aim of establishing decentralized operation and control in the airspace as well as reducing the workload and stress of air traffic controllers [186,188–190]. Several government-led programs have led to the development of hardware and software technologies, such as instrumental landing systems, navigation systems, and data transfer technologies, for coping with the challenging decision procedures in the airspace [181,191,192]. The general program entitled "Next Generation Air Transportation

System” (NextGen) integrated all the programs and established infrastructure to accomplish the potentials associated with “free flight” (FF), a notion that includes (i) providing safe distance among aircraft, (ii) improving airspace capacity, and (iii) enhancing flight efficiency by improving predictability in the airspace [193,194]. Therefore, with the FF concept, pilots, particularly from commercial airlines, are involved in the decision-making procedures of “conflict detection and resolution” (CD&R) in a decentralized manner [191,195–198].

The problem of CD&R includes two major decision-making aspects: (i) detecting possible conflicts among aircraft flying in a shared airspace, and (ii) resolving those conflicts by providing a new set of maneuvers. Most studies have focused on the latter, in which presumed conflicts are resolved locally in the airspace without considering their future impacts [199]. However, due to the dynamic features of the problem, each solution may cause further conflicts in the corrected paths or other parts of the airspace. Considering the integration of both problems of CD&R is, clearly, very important regarding the efficiency in the airspace. Some studies approached this problem using agent-based simulation modeling to realize the conflict in the airspace, resolving it locally, using the level-*k* thinking approach considering the pilots' behaviors. Using the latter methodology, integrated with reinforcement learning for training the model, can provide a tool for pilots to adjust their flight behavior in different situations [200,201]. Such a methodology can be integrated with our proposed algorithm to examine its performance from a different, more simulation-based, perspective.

In addition, some researchers focused on the "tube-shape" airspace in which a limited number of routes between waypoints are considered in the form of a directed network that aircraft arrival time at each node is estimated for scheduling purposes [197]. However, this specific assumption deliberately leads to losing a large amount of the airspace capacity, and consequently a substantial loss of efficiency may occur, contradicting the FF concept [197]. Additionally, when resolving the conflicts, these studies considered a limited number of options for each party, where aircraft are not able to change variables such as trajectory and velocity concurrently, discussed further subsequently.

The business side of air transportation is driven by diverse profit-oriented variables controlled by several parties serving their own interests, where decisions taken by each significantly affect others. Due to this interdependency in the decision-making process, game theory is an appropriate method to address the problem of conflict resolution among multiple aircraft when each represent various airlines with various conflicting interests [82,202]. In this regard, Barker et al. [82] introduced an algorithm to address the CD&R problem considering 2D airspace using game theory to provide new sets of maneuvers for aircraft to avoid conflict and minimize aircraft deviation from the original path considering players conflicting interest. In this study, we propose a framework that extends the work by Barker et al. [82] to address the CD&R problem in the 3D airspace. In this research, both parts of the algorithm related to the CD&R problem are modified such that the algorithm can be applied in the airspace with various levels of altitudes. Also, pilots are assumed to compete with others to enhance their performance and efficiency concerning their interest in conflict situations to get a better flight pattern. In this algorithm, ATC is viewed as the supervisor and coordinator intervening in the decision-making process as a fair broker whenever more than one solution (Nash equilibrium) exists to choose the best solution as the final decision with the aim of minimize the system's total cost.

This study generally improves decision-making strategies with multiple players involved with various perspectives. The proposed algorithm enhances the efficiency of the decisions made by the air traffic controllers as they are the main components of air traffic management. The main contributions of this algorithm are: (i) enabling decision-making in a decentralized manner, (ii) providing a semi-cooperative algorithm based on game theory, (iii) assuming theoretically infinitely possible routes between an origin and destination pair, (iv) providing conflict-free solutions throughout the time horizon of the model, and (v) enabling the pilots to change trajectory and velocity concurrently, which produces options consisted of two actions with certain quantity.

5.2. Literature review

Before 1950, due to the limitations in technologies such as navigational instruments and communication devices, a rudimentary ATC system was only capable of providing limited information to pilots through radio communications, such as weather conditions in critical sectors of the airspace [185]. After the application of radar technology in civil air transportation, controllers were responsible for guiding airplanes using screens that depict nearby airspace sectors. Furthermore, flight strips containing information of aircraft entering a sector were passed among the controllers [185]. Given the significant increase in demand for air transportation, there emerged a need for finding an efficient solution to the conflict detection problem for an increasing number of aircraft [177,183,185,203]. In particular, the problem of CD&R involves both human and non-human aspects [177]. Air traffic controllers are the human component of the system that is charged with the important burden of managing and guiding aircraft in various circumstances. In other words, they are the decision-makers of the system [204,205]. The non-human components of ATC involve the various technologies (e.g., radars, satellites, communication systems) used by controllers to enhance their capabilities [184–186]. Key features of aircraft CD&R in a shared airspace include (i) the natural limitations associated with human mental capabilities to identify several interdependent sub-problems concurrently while considering a variety of perspectives related to the safety and efficiency of the system [205–208], (ii) the type and availability of technology being used by ATC and pilots to solve the conflicts [209], and (iii) the general environment of the problem, such as navigational characteristics and differences in the perspectives of each party involved.

Generally, the human plays a major role in ATC systems by making decisions given available technology to support the quality and performance of their decisions. In particular, ATC has been designed primarily as a centralized process, where all information regarding aircraft flying in the airspace is received and analyzed by the controller so that future conflicts are anticipated and corrected [186,188,190,210]. Accordingly, numerous methods have been developed based on a centralized perspective of ATC providing solutions for conflict resolution (CR). For example, Liu et al. [211] developed a stochastic optimization model, considering wind dynamics and centralized form of control in a 2D airspace. Also, Volpe Lovato et al. [174] developed a centralized fuzzy model manipulating the longitudinal speed of the aircraft without any deviation in trajectory to reduce the possibility of having conflicts. More recently, the application of reinforced learning in ATC has received attention. Tran et al [208] have developed a personalized digital assistant, utilizing artificial intelligence to keep controllers' work load manageable and improve the performance of the centralized system by incorporating controllers preferences. Brittain and Wei [206] have developed an actor-critic model combined with proximal policy optimization for having a centralized learning process while all aircraft using the shared learned model for conflict resolution. Tolstaya et al. [212] combined search-based motion planning and inverse reinforcement learning to estimate the reward function observing traffic controller actions when selecting trajectories for aircraft. Note that, although humans are often effective at handling this duty due to the ability to perform parallel computing and to incorporate various subjective and quantitative data, controllers can have difficulty in managing conflict situations. This is caused, among other reasons, by the high volumes of aircraft with conflicts in their flight plans, high workload and fatigue, various airline-specific criteria, safety characteristics, and the size and variety of the input data required to make decisions [213]. As a result, to maximize the quality and performance in the limited available time, controllers mostly act based on safety priorities. Therefore, especially in critical situations, some performance criteria is considered less relevant, and may be completely neglected [186,188,190,210,213–217]. Therefore, the centralized approach for controlling the airspace could be reliable but not as efficient according to the cost imposed to the airlines. Also, concentrating the whole decision-making process in one sub-system increases the system's vulnerability against disruption and decreases overall system robustness. In contrast, to overcome the weaknesses associated with a centralized approach in ATC, several researchers have investigated the possibility of a decentralized decision-making structure, by sharing the decisions (and responsibilities) among the diverse players involved [188–190,218]. Most of these studies have been done to deliver on the potential of free flight (FF) with the objective of maximizing the efficiency while maintaining the safety. It is assumed that, by giving more liberty to the pilots for choosing their flight characteristics, airlines would be able to follow their own profit variables related to their business plan, and consequently, the cumulative effect of each agent would effectively improve the overall efficiency of the system [219]. In this regard, Menon et al. [199] investigated the CR problem under a decentralized decision-making environment using an iterative multi-objective and multi-participant optimization model to minimize the travel time and fuel consumption. Furthermore, Wollkind et al. [220] proposed a multi-agent negotiation approach, emphasizing more on having agreement among agents rather than best response, using a monotonic concession protocol to select predefined maneuvers for each aircraft and resolve the detected conflict. Hill et al. [221] and Bellomi et al. [222] applied satisficing game theory focused on only cooperative games, providing cooperation among pilots to satisfy individual and group preferences. Note that the performance of these methods depends on the availability of appropriate infrastructure and hardware.

Using control theory, some studies have used aircraft control theory to anticipate the aircraft flight characteristics and behavior accurately. In this regard, Blight et al. [223], applying multivariable control theory for aircraft control law development, showcasing applications in lateral oscillation solution, longitudinal control law for stable and unstable airframes, and a high-agility fighter up to 90° angle of attack. Also, Liu et al. [224] have addressed the challenges in dynamics modeling, autopilot design, and field tuning for model-scale unmanned aerial vehicles with only two elevon control surfaces and they could solve the problem using nonlinear control algorithms. Furthermore, Julian et al. [225] have presented a comprehensive auto landing design using a cascaded control structure for a commercial aircraft model. They utilize classical loop shaping and robust control techniques to design individual control loops, ensuring safe aircraft landing under various scenarios, including strong crosswinds. Industry-grade verification and nonlinear Monte Carlo simulations are employed to assess the control system's performance and limitations in ensuring safe landings. These investigations generally enhance the control over the alteration of aircraft flight variables, such as flaps, ailerons, rudder, engine thrust, etc., to satisfy the pilot's desired handling of the aircraft and ensure aircraft safety. Therefore, the managerial aspect of air traffic control is not the primary focus of these investigations. Therefore, the managerial aspect of air traffic control is not the primary focus of these investigations. These studies are highly useful when used in a decision-making hierarchy. Once pilots have determined their desired trajectory at a higher level of decision-making, these control systems execute their decisions, adjusting all aircraft variables to achieve the desired outcome while minimizing the difference between the actual and desired states.

Therefore, the type of technology being used to support decision-making procedures is the other major factor in ATC, enabling us to determine the type of system required to maintain in order to enhance quality and performance [226]. Due to the high demand for air transportation (and its associated congestion), it is expected to use innovative technological advances to integrate and compute the necessary information to maintain adequate safety and efficiency levels, as well as to enable smooth air traffic [184–186]. Therefore, numerous government-led programs have been conducted to develop hardware and software technologies required for current and future demands [191,192]. Therefore, nowadays, this capability exists to involve players, such as pilots and airlines, in addition to air traffic controllers, to address the CD&R problem in a decentralized fashion [191,195–198]. On the other hand, by adding other independent decision-makers who are direct beneficent of the system, it is likely for every party to try bending the situation into their own favor regardless of others' interest [199]. Therefore, it is necessary to develop high performance models to guarantee the integrity of the decision-making procedures and serve the interests of various benefits of the system.

Therefore, in the concept of air traffic management, semi network-form games have been applied by Yildiz et al. [204] to address the confrontations happens in merging and landing procedures among the aircraft in the freeze horizon. Yildiz et al. [186] developed an algorithm using multi-stage games and reinforcement learning to address the CD&R problem in a 2D environment considering a layered tube-shape airspace. Likewise, Albaba et al. [2] and Musavi et al. [41] address the CD&R problem considering the 3D airspace using agent-based simulation for conflict detection and a level-*k* thinking game for conflict resolution. They have also used reinforcement learning for training the model providing a tool to investigate the pilots' behavior at the time of conflict. Using agent-based modeling, the authors generally focused on investigating the cumulative effect of pilot behavior regarding conflict resolution on the overall airspace. As conflict detection is based on simulation, it requires that the entirety of the simulation be completed so that all possible conflicts can be realized. Likewise, Pritchett and Genton [195] presented an iterative bargaining algorithm using game theory to address the CR problem enabling pilots to make negotiations in a decentralized decision framework. However, they only considered six continuous options for the pilots (e.g., left/right, up/down, faster/slower), but not simultaneous flight characteristic changes, such as a concomitant change in speed and trajectory. Also, future correction costs related to the aircraft altitude changes that are required when approaching the merging point have not been considered. The central assumption is that the algorithm converges if pilots honestly communicate their proposed maneuver cost among each other. Therefore, the iterative bargaining process can continue indefinitely without convergence while no measure (e.g., air traffic controllers) are anticipated to break the cycle.

In these kinds of problems, to obtain good solutions in a reasonable amount of time and at an affordable cost, the model should focus on the critical factors that affect the dynamics of the system. Thus, according to Table 18, the important factors related to the problem environment are: (i) the geometric properties of the problem that affect the navigation system required to produce solutions, and (ii) the effect of weather conditions on aircraft movements and the availability of various sectors in the airspace. With respect to general spatial dimension of the problem, studies can be distinguished by the type of rules and assumptions considered with respect to the geometry of the system. Sunil et al. [197] proposed four different categories for airspace structures: (i) an unbounded airspace structure where aircraft can move in any direction without any constraint, (ii) an altitude-layered structure where aircraft can move freely, (iii) a column-shape airspace structure, suitable for aircraft with vertical-fly abilities, and (iv) a tube-shape airspace structure, with predefined virtual highways on which airplanes should not deviate from.

Furthermore, additional dynamic characteristics need to be considered, to account for weather situations, military instructions, and airspace overload, among others. A few studies explore the CD&R problem considering weather conditions. Liu et al. [211] developed a stochastic algorithm for collision avoidance in a 2D airspace considering centralized control and random wind dynamics. Many studies have focused on finding the shortest route (between a fixed origin and destination) considering an inaccessible space in their path. Sutorius and Panagou [227] presented a decentralized control system to provide coordination among mobile agents considering both convex and non-convex obstacles in a 2D airspace. Furthermore, Ma et al. [81] developed an algorithm for a decentralized motion plan suitable for multiple UAVs with the aim of avoiding 3D polygonal obstacles. The algorithm addresses the CR problem considering the priority of the moving objects and produce collision-free trajectories. Balazs and Vasarhelyi [80] modeled coordinated self-driving autonomous drones in a decentralized structure in a 2D airspace with applications in package delivery. In this work, they use flocking algorithms to avoid agents from getting too close to each other, while letting them move circumspectly around each other and share information about the global target of their local neighbors.

These models are well designed for a low altitude environment where many obstacles (e.g., towers, buildings) can be identified, making drones and UAVs suitable for deliberately controlling their movement toward every direction. In general, there are six main features that can differentiate and categorize studies on guiding and controlling airplanes in airspace: (i) scope of the study (e.g., CR or CD&R), (ii) type of control (e.g., centralized or decentralized control), (iii) type of communication among multiple agents (e.g., cooperative, or non-cooperative), (iv) spatial dimension (e.g., 2D or 3D), (v) whether or not weather conditions were considered, and (vi) application (e.g., ATC or UAV). Table 18 outlines the features of some relevant studies in the field.

Table 18: Summary of previous research work on air traffic conflict resolution

No.	Authors	Title	Year	Method	Problem Type: - Conflict detection and resolution (CD&R) - Conflict resolution (CR) - Detect and avoid (DAA) - Not applicable (NA)	Type of Control: - Centralized (C) - Decentralized (D)	Type of Communication: - Cooperative (C) - Non-Cooperative (NC) - Not applicable (NA)	Spatial dimension: - 2 Dimension (2D) - 3 Dimension (3D)	Considering weather condition?	Application: - Air traffic control (ATC) - Unmanned air vehicle (UAV) - Unmanned air mobile (UAM)
1	Tran et al.	An intelligent interactive conflict solver incorporating air traffic controllers' preferences using reinforcement learning	2019	Reinforcement learning	CD&R	C	NA	2D	No	ATC
2	Brittain and Wei	Autonomous Air Traffic Controller: A Deep Multi-Agent Reinforcement Learning Approach	2019	Reinforcement learning	CD&R	C	NA	2D	No	ATC
3	Yang et al.	Multi-Agent Autonomous On-Demand Free Flight Operations in Urban Air Mobility	2019	Markov Decision Process, Monte Carlo tree search	NA	D	C	2D	No	UAM
4	Liu et al.	Aircraft Trajectory Optimization for Collision Avoidance Using Stochastic Optimal Control	2018	Stochastic optimal control	CR	C	NA	2D	Yes	ATC
5	Pritchett et al.	Negotiated Decentralized Aircraft Conflict Resolution	2018	Game theory	CR	D	C	3D	No	ATC
6	Balazs and Vasarhelyi	Coordinated Dense Aerial Traffic with Self-Driving Drones	2018	Agent base simulation model	DAA	D	C	2D	No	UAV
7	Ma et al.	3-D Decentralized Prioritized Motion Planning and Coordination for High-Density Operations of Micro Aerial Vehicles	2018	Multi agent	DAA	D	C	3D	No	UAV
8	Volpe Lovato et al.	A fuzzy modeling approach to optimize control and decision-making in conflict management in air traffic control	2018	Fuzzy model	CR	C	C	3D	No	ATC

No.	Authors	Title	Year	Method	Problem Type: - Conflict detection and resolution (CD&R) - Conflict resolution (CR) - Detect and avoid (DAA) - Not applicable (NA)	Type of Control: - Centralized (C) - Decentralized (D)	Type of Communication: - Cooperative (C) - Non-Cooperative (NC) - Not applicable (NA)	Spatial dimension: - 2 Dimension (2D) - 3 Dimension (3D)	Considering weather condition?	Application: - Air traffic control (ATC) - Unmanned air vehicle (UAV) - Unmanned air mobile (UAM)
9	Sutorius and Panagou	Decentralized Hybrid Control for Multi-Agent Motion Planning and Coordination in Polygonal Environments	2017	Defined algorithm	DAA	D	NA	2D	No	ATC
10	Sunil et al.	Analysis of Airspace Structure and Capacity for Decentralized Separation Using Fast-Time Simulations	2016	Simulation, Modified Voltage Potential (MVP)	NA	D	NC	3D	No	ATC
11	Yildiz et al.	Predicting Pilot Behavior in Medium-Scale Scenarios Using Game Theory and Reinforcement Learning	2014[1]	Multistage game, reinforced learning	CD&R	D	NC	2D	No	ATC
12	Hill et al.	A Multi-Agent System Architecture for Distributed Air Traffic Control	2005	Satisficing game theory, Simulation	CD&R	D	NA	2D	No	ATC
13	Wollkind et al.	Automated Conflict Resolution for Air Traffic Management Using Cooperative Multiagent Negotiation	2004	Cooperative, multi-agent negotiation techniques	CD&R	D	C	3D	No	ATC
14	Menon et al.	Optimal Strategies for Free-Flight Air Traffic Conflict Resolution	1999	Multicriteria optimization	CR	D	NC	3D	No	ATC

In this research, we have addressed the CD&R problem for manned commercial aircraft in the en route segment of the flight in the 3D spatial dimensions. Also, a non-cooperative game has been utilized to address the decentralized business environment where each pilot is eager to minimize its deviation from it predefined route and velocity.

5.3. Methodology

Game theory has been utilized in this study to address the CD&R problem among various airplanes traveling in the airspace and provide maneuvers to minimize deviation from their original plans. The proposed algorithm is defined in three stages: (i) predicting future aerial conflicts considering the aircraft flight plan, (ii) constructing the game by generating player options and calculating payoffs for their actions, and (iii) resolving the conflict and re-routing the aircraft. The algorithm repeated until no more conflicts can be found.

The main objective in conflict detection is to find situations where two hypothetical aircraft violate the regulatory safe distance from each other at a certain time [177,196,228,229]. As 3D layered airspace has been considered in this study, conflicts may happen in different altitude layers. Therefore, the aircraft planned route segments and waypoints are categorized based on the altitude layers where they lie. Afterward, the algorithm predicts the possibility of having conflicts in layer. Conflicts in each layer are discovered by utilizing the method inspired by Barker et al. [82] three-stage algorithm consisted of: (i) discovering candidates for conflict zones (CZ) provided the existence of crossing in aircraft route segments, (ii) checking the separation violation condition, considering time and velocity of the aircraft, and (iii) selecting conflicts that do not have a path history. The crossings in the original flight plans provided by the pilots are considered as the input of the algorithm, and waypoints determining the beginning and end of the route segments are extracted. The coordinates of the line crossing in the direction of any pair of route segments are calculated as:

$$x_c = \frac{m_i x_i - m_j x_j - y_i + y_j}{m_j - m_i} \text{ and} \tag{47}$$

$$y_c = \frac{m_j y_i - m_i y_j + m_i m_j x_j - m_i m_j x_i}{m_i - m_j}, \tag{48}$$

where $(x_c,\ y_c)$ are the crossing point coordinates, m_i and m_j are the headings or the slopes of the lines in the direction of route segments of the aircraft i and j, and $(x_i,\ y_i)$ and $(x_j,\ y_j)$ are the starting waypoint of the route segment belong to the involved aircraft. Our primary candidate for the CZ set is determined if the crossing point is located somewhere in segment routes pair.

The next filtering stage considers whether the aircraft flying in a candidate pair of route segments are going to be at the same place at the same time. Note that the position of a moving object is the function of velocity (v), heading (β), and time (t). Thus, the distance of two moving objects considering the Euclidean distance function can be rewritten again in the form of the objects' velocity, time, and heading, when the position coordinates of each follow as:

$$x_{it} = v_i \cos(\beta_i)\, t + x_{i0} \tag{49}$$

$$y_{it} = v_i \sin(\beta_i)\, t + y_{i0}, \tag{50}$$

therefore, the final distance equation is shown as:

$$d_t = \sqrt{(x_{it} - x_{jt})^2 + (y_{it} - y_{jt})^2}\,. \tag{51}$$

The objective now is to find the time that these two moving objects have the minimum distance with each other considering their velocity and heading. Therefore, we solved the derivative of Eq. (51) with respect to time to find the time at which these two moving objects are at their closest position from each other, as calculated by

$$t_{\min} = \frac{2\,[v_i\cos(\beta_i) - v_j\cos(\beta_j)] \times [x_{i0} - x_{j0}]}{2[v_i\cos(\beta_i) - v_j\cos(\beta_j)]^2 + 2[v_i\sin(\beta_i) - v_j\sin(\beta_j)]^2} + \frac{2[v_i\sin(\beta_i) - v_j\sin(\beta_j)] \times [y_{i0} - y_{j0}]}{2[v_i\cos(\beta_i) - v_j\cos(\beta_j)]^2 + 2[v_i\sin(\beta_i) - v_j\sin(\beta_j)]^2}\,. \tag{52}$$

If this minimum distance is less than the minimum regulatory distance determined by official requirements, we keep the point in our CZ set, otherwise we remove it from the set. The final stage considers the lineage of the events or path history of CZ occurrence. In this stage only CZs will remain in the set if any of the involved aircraft are not participating in any other CZs occurring earlier in time.

In this part of the algorithm, the minimum distance between two aircraft is calculated during their flight time, and a comparison is made to minimum regulatory separation distance. Additionally, the algorithm only considers independent conflicts that have no path history or no relationship with the outcome of the other conflicts in each iteration to resolve the conflicts that occur earlier and anticipate the conflicts that could potentially occur in the future given the decisions made in the past.

5.3.1. Game Construction

In the proposed algorithm, a two-player game is utilized to model conflicts with two aircraft involved. Each player (a pilot) maximizes their own cost function associated with the cost of its flight plan. However, they need to compromise to avoid conflicts, which will depend on the decisions made by other player. In the proposed model, we assume that pilots have diverse controllable variables, including aircraft trajectory (T) (e.g., left or right), altitude (A) (e.g., up or down), and velocity (V) (e.g., accelerate or decelerate) although each of these variables can remain without any change. The proposed model considers all possible combinations related to feasible values for T, A, and V, including concurrent changes in each or all of them.

The time that both aircraft should start executing the decisions related to the controllable factors is considered to be when the last aircraft enters into its corresponding pairs of route segments that conflict with the other. Thus, for the aircraft that enters later to the route segment, its location at the decision time would be the beginning point of the route segment. The coordinates of the other aircraft are calculated using Eqs. (3) and (4) considering the time the aircraft was cruising along its corresponding segment route before the latter arrived. Note that the trajectory is assumed to be changed discretely, meaning that the aircraft can change the headings considering a predefined degree to the left or right in every level of change. We considered the velocity to increase or decrease instantaneously at the decision time as a predefined fraction of the planned velocity that pilots defined initially at the route segment. Therefore, as the changes in variables are changed discretely, the proposed model can be easily generalized to account for additional options for each variable to a level that theoretically describes the continuous domain, although it would be subject to an increased computational complexity caused by the combinatorial nature of the problem.

Assuming that the number of options each pilot has for A, T, and V, are l, m, and n, respectively, the total number of options would be $l \times m \times n$. Although aircraft can alter T and V while also altering A, these combinations are dominated by those where only A is altered due to having a higher unnecessary deviation from the original route. In other words, if it is necessary for a pilot to change the aircraft altitude, according to

$$U_1(TAV, \text{any}) \ > \ U_1(A, \text{any}), \text{ and} \tag{53}$$

$$U_2(\text{any}, TAV) \ > \ U_2(\text{any}, A), \tag{54}$$

then changing the trajectory and velocity at the same time would only impose more unnecessary costs. As such, altering variables simultaneously, including a change in A, would always result in a worse utility function (U) than only altering A.

Therefore, the total number of options to be considered is $m \times n$ (the number of option combinations that a pilot has for T and V given the current altitude) plus $l - 1$ (the number of altitude levels except the current one). Using the same reasoning for both players, the size of the strategy set for both players would be the multiplication of each player's pure strategy, $[(m \times n) + (l - 1)]^2$.

Subsequently, if one aircraft decides to change the altitude, the other pilot does not have any incentive to deviate for its own original strategy. In short, if one aircraft changes its altitude, the best response of the other aircraft is to stay on its original path. According to

$$U_1(\text{unchanged}, A) < U_1(\text{changed}, A) \text{ and} \quad (55)$$
$$U_2(A, \text{unchanged}) < U_2(A, \text{changed}), \quad (56)$$

the utility for an aircraft deviating from its original path provided the other aircraft changing its altitude is always higher than when the first aircraft does not deviate. Thus, without loss of generality, the size of the strategy set could be further reduced to $(m \times n)^2 + 2(l - 1)$, on which we consider all strategy combinations in the current altitudes plus the number of strategies when one of the aircraft changes its altitude.

Also, since altering A for one level would be enough to resolve the conflict while the flight variables of the second aircraft remain unchanged, the first aircraft does not need to alter A for more than one step to resolve the conflict. Otherwise, the deviation cost would increase more than what is necessary. Therefore, as one level change in A dominates further changes in A, the number of l will not go beyond 3. Thus, the size of strategy set is further reduced to $[(m \times n)]^2 + 4$.

5.3.2. Cost Function

Designing a proper payoff function is a non-trivial task given the diversity in business plans and competitive strategies among different airlines. For example, low-cost carriers may value customer-service and responsiveness differently compared to legacy airlines. Additionally, companies may target customers with other concerns such as time, comfort, and reputation [230]. In addition, there are airline-specific aspects, including size, age, and the fleet type, that affect how costs and profits are calculated. However, although a variety of variables with different relationships can be incorporated to define the interests of an airline, the payoff functions in this industry are often related to aircraft fuel consumption, usually a product of aircraft travel distance, velocity, and change in altitude [231,232]. Additionally, since violating safe separation distances would greatly increase the probability of having a collision [233], the payoff function should penalize these instances. For this study, the proposed payoff function

$$U = \lambda\big((V_{ab}D_{ab} + V_{bc}D_{bc}) - V_{ac}D_{ac}\big) + (1-\lambda)L + I, \tag{57}$$

is used in the game-theoretic model to address the criteria that are generally affecting fuel consumption cost and penalizing possible collisions. Eq. (57) was inspired by the aircraft drag force formula [234] and the work of Barker et al. [82]. The formula has three major parts that are: (i) aircraft deviation in current level, (ii) aircraft deviation by changing the altitude, and (iii) safety violation index. The first two parts are contributing to more fuel consumption and subsequently to a higher flight cost. The third part ensures that the aircraft keep their distance from each other. Furthermore, λ is a binary variable that indicates if an aircraft changes or remains in the current altitude. V and D are aircraft velocity and traveled distance between origin and destination points, respectively that will be discussed later on. L is the cost relative to the changes in altitude. Finally, I serves as the penalization for safe distance violation, where, depending on if the safe distance among aircraft is violated or not, it would take a value of 0 or an infinite number, respectively. Thus, in the case of a violation in the minimum safe distance, the large number makes the related strategy too costly to be selected.

For the first term of the cost function, which considers aircraft deviations in trajectory and velocity (while maintaining a fixed altitude) it is assumed that each airplane deviates at way point (a) and turns back toward their destination way point (c) after passing the turning way point (b). Therefore, the objective is to calculate the additional costs incurred when deviating from the original flight plan. Accordingly, $V_{ab}D_{ab}$ and $V_{bc}D_{bc}$ are the multiplication of the aircraft velocity and travel distance from point (a) to (b) and (b) to (c) respectively. Likewise, $V_{ac}D_{ac}$ is the multiplication of the aircraft velocity and travel distance from point (a) to (c) representing the original plan cost Figure 21. We used the multiplication of aircraft velocity and travel distance because the drag force affected a moving object in a fluid has a direct relationship with the square of its velocity (V) [234].Therefore, as we are looking for the amount of the extra work (W) inflicted by deviation in the aircraft route and considering that the work formula in classical physics is equal to the multiplication of the force (F) and distance (X), logical equation $W = F \times X \ \propto \ V^2 \times X$ holds.

In our cost function, we simplified the concept in Eq. (57) to consider velocity raised only to the power 1.

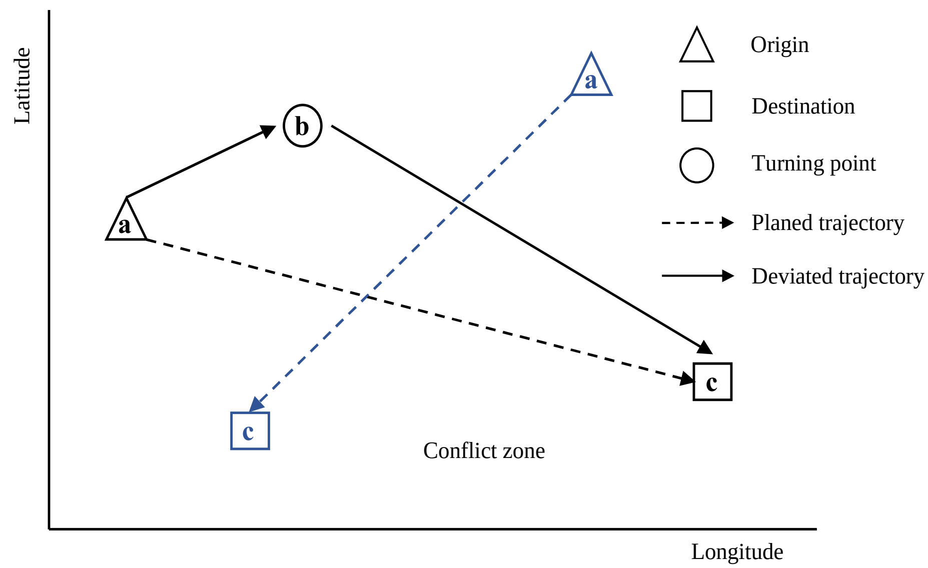


Figure 21. Aircraft trajectory deviation.

Although start and destination points are known, the turning way points are unknown and need to be calculated for both players. In other words, out of many possibilities for the turning point, we need to find the one which dominates the others and minimize the deviation cost for both players considering the strategy that they chose. Therefore, we assume that the selected turning way points are those that minimize the deviation costs while providing a safe separation distance. In addition, the locations of the optimal turning way points should guarantee that there is no conflict in any part of the traveled distance along deviation starting point to ending point. For this purpose, the infinite penalization for safe distance violation guarantees that a minimum safe separation distance is always maintained. Accordingly, the ant colony algorithm is used to minimize the cost function for both players by determining their turning points as described in detail by Barker et al.[82].

Furthermore, in the cases when pilots decide to change the altitude, instead of deviation horizontally, a fiscal cost is considered in the cost function for each layer of deviation in altitude. Accordingly, the binary value would be 0, only to consider the cost of changing the altitude. Note that as all aircraft should change altitude at the end of their journey to match up with the merging points (points where all aircraft are lined up for landing), a change in altitude to resolve the conflict may cause extra deviation costs. Therefore, a decision tree has been developed to anticipate the future cost of deviation concerning the altitude change in CZs. Figure 22 illustrates different parts of the decision tree, where A_0 and A_{merge} indicate initial aircraft altitude and destination merging point altitude, respectively. Similarly, C_a and C_d are ascending and descending costs for each layer of altitude change, respectively, while AC refers to the number of changed altitude layers.

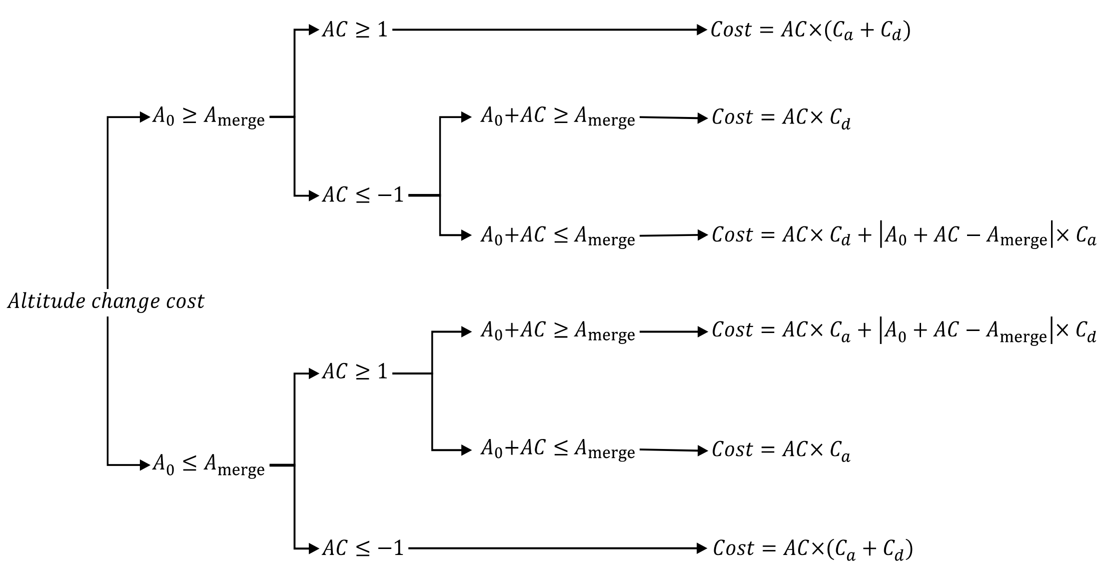


Figure 22. The decision tree for change of altitude deviation cost calculation.

5.3.3. Conflict Resolution

Once the game is constructed using the aforementioned payoffs for each pilot's deviation strategy, the resolution for the conflict is determined by finding the Nash equilibrium. Note that in non-cooperative games, a Nash equilibrium is associated with the strategy where no player can increase their utility function by moving to another strategy. Therefore, everyone in the game can maximize their own preference considering the decisions that the other players can make [235,236].

In the proposed game, there is a chance to find multiple Nash equilibria. In such cases, ATC intervenes and select from these preferred strategies, perhaps according to additional criteria. Without loss of generality, we selected the Nash equilibrium that has the minimum payoff aggregate of the involved players. Note that the ATC can consider any different criteria to automate selecting among the Nash equilibria strategies fully. In addition, human intervention can also be considered, making the algorithm semi-automatic or manual when it comes to selecting among various preferred strategies. Afterward, we assume that deviation strategies are incorporated into the airplanes flight plan, and the next iteration of the algorithm commences and continues until no independent conflicts are found. The algorithm restarts at each time interval set by the air traffic controller so that new aircraft entering and exiting the airspace are considered in the new iteration of the CD&R algorithm. The algorithm 1 pseudocode illustrates different part of CD&R problem in steps. Note that this algorithm should be repeated until no CZ can be found given the current state of the airspace.

Algorithm 3: CD&R problem pseudocode for internal steps of the algorithm.

Discovering candidates for conflict zone (CZ) in the airspace:

1) Setting the airspace conflict zone set $CZ = \{cz_k | \, cz_k = \left(m_i, n_j, t_k\right), i, j \in N, 1 \leq k \leq |CZ|\}$ where m and n are the route segment number of each aircraft in the set of all aircraft in the airspace (N), and t is the time that the two aircraft have their minimum distance (D_{min}) from each other
2) Check all aircraft route crossings using Eqs. (1) and (2)
3) Calculate D_{min} among the two aircraft considering each discovered route crossing using Eqs. (3) to (6)
4) If $D_{min} < D_{reg}$ where D_{reg} is the minimum regulatory separation distance, then:
   Consider (m_i, n_j, t_k) as candidate for cz_k
   Else:
   Remove (m_i, n_j, t_k) from the candidate list for cz_k

Check the events path history to find the independent conflict zones

5) For all elements in the CZ candidate list, do:
6) If any of the i or j in the candidate list for cz_k have been repeated in other conflict zones in the CZ set, then:
   Keep only the cz_k where has the minimum t_k in the candidate list for cz_k and remove the others
   Else:
   Consider (m_i, n_j, t_k) as candidate for cz_k
7) Consider CZ equal to the CZ candidate list

Conflict resolution: Game construction

8) For all elements in CZ, do:
9) Setting the set of strategy profiles $S = \{(s_i, s_j)| \, s_i = (T_i, V_i, A_i), s_j = (T_j, V_j, A_j), i, j \in N\}$ where T, V, and A are the various strategies for aircraft trajectory, velocity, and altitude considering the predefined levels for each of these flight characteristics.

Conflict resolution: Calculating the cost function for each strategy combination

10) If any of the A_i or A_j are going to be changed, then:
    Use the decision tree in Figure 22 to calculate altitude change cost (L)
    Use Eq. (11) to calculate the strategy payoff
    Else:
    Find the turning point for both aircraft using the ant colony algorithm considering the selected strategy
    Use Eq. (11) to calculate the strategy payoff

Conflict resolution: Nash equilibrium

11) Establish the set of Nash equilibrium (NE) strategy profiles $NE = \{[(s_i^*, s_j^*), (p_i, p_j)]| \, s_i^* = (T_i, V_i, A_i), s_j^* = (T_j, V_j, A_j), \; i, j \in A\}$, where p represents the payoff of the aircraft
12) If $|NE| = 1$ then:

Select the strategy in NE

Else:

Select the one strategy in NE that has the minimum cumulative payoff $\{(s_i^*, s_j^*) \mid (s_i^*, s_j^*) \in NE, (p_i + p_j) < (p_{-i} + p_{-j})\}$

13) Modify the aircraft flight characteristics (T, V, A) according to the selected strategy

Also, the actual monitored change of aircraft flight characteristics can be considered in the new iteration of the algorithm, and new conflicts in the system can be resolved continuously over time. Therefore, if pilots deviate from their planned trajectory, the algorithm can readdress the deviations in the new iteration of the algorithm and devise a new trajectory plan for the aircraft in airspace. Figure 23 provides more details regarding the main idea behind the algorithm.

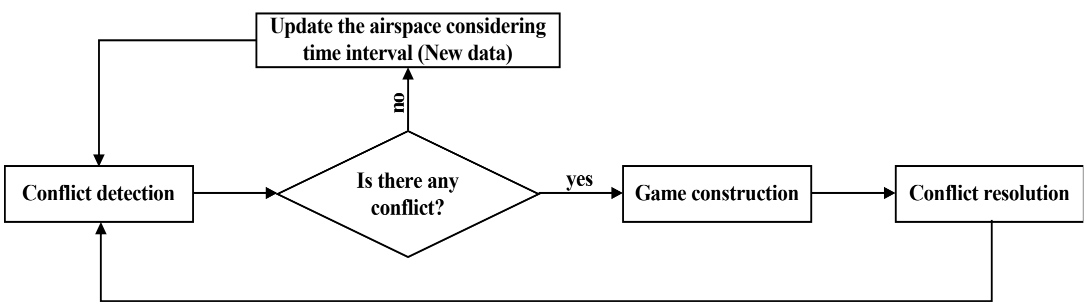


Figure 23. The general framework of the algorithm.

Concerning the characteristics of the proposed algorithm, the algorithm would theoretically be located at ATC land facilities considering the organizational structure hierarchy, security, information aggregation, and infrastructure availability required for the analysis and computation of aircraft flight paths in the airspace. Combining the intensive computational power of ATC land facilities with the satellite communication systems would result in a safe, accurate, and fast decision-making tool aiding the air traffic controllers and pilots navigating through the airspace. Note that the decision-making process and use of the proposed algorithm, is initiated by pilots determining and requesting a flight path from ATC. This flight path consists of the aircraft destination, route waypoints, velocity, and altitude in each route segment of the flight path. The request received by ATC is then processed and analyzed using the proposed algorithm, providing the pilots with a conflict-free flight path. Also, the communications between the pilot and ATC are performed using a fast and reliable satellite communication system. As for hardware infrastructure located on the aircraft, Automatic Dependent Surveillance-Broadcast (ADS–B) is a part FF program that can send the aircraft position to nearby aircraft and land facilities using satellite technology. Using ADS-B, any deviation between the confirmed flight plan and actual aircraft flight characteristics can be captured by the ATC land facility renewing the analysis using the algorithm updating aircraft flight path.

5.4. Results and Discussion

To better describe the application of the proposed algorithm, an illustrative example has been designed to assume an altitude-layered structure airspace consisting of five layers of altitude. In this example, we modeled 200 aircraft flying in direct paths between randomly generated origins and destinations. These paths follow the general desire of airlines for direct flight trajectories, so that the travel time and cost can be minimized. Also, we have represented the results for several conducted experiments to examine the algorithm behavior considering variations in the capacity of the airspace. The main aspects studied in these experiments were: (i) pilot variety of options, representing how much flexibility can affect the performance of the system, (ii) capacity of the airspace, showing how the algorithm could operate in dense airspace, and (iii) comparison between levels of cooperation among aircraft to examine the possibility of improving the system by maximizing the cooperation.

In this study, we analyzed the flexibility and liberty of pilots in strategy selection considering three different categories of flight characteristics, including: (i) three levels for velocity, trajectory and altitude, (ii) five levels of velocity and trajectory with three levels of altitude, and (iii) seven levels of velocity and trajectory with three levels of altitude. Note that by increasing the number of options for each variable, the pilots have more flexibility in order to select the strategies that are better suited with respect to their own interests. Likewise, as options available to pilots are discrete, the higher the number of options given for a particular variable, the closer this model would resemble a continuous domain for such variables, which enables pilots to choose from an infinite number of strategy options. Therefore, the algorithm better follows the free flight concept.

Additionally, the available capacity of the airspace varies, either by changing the number of airplanes in the sector or by increasing the required safety separation distance among aircraft. Increasing the number of aircraft means that the sector capacity must be shared among more aircraft. Also, increasing the minimum required distances means more space is required by aircraft to maneuver. Having lower capacity, generally, causes more potential conflicts in the airspace, giving harder times to the presented algorithm to resolve the conflicts. In this study, the effects of changing the airspace capacity have been investigated by considering changing the number of aircraft and required separation distance. Figure 24 illustrates solutions developed by the algorithm for a randomly generated case considering 200 aircraft and 100 aircraft merging points.

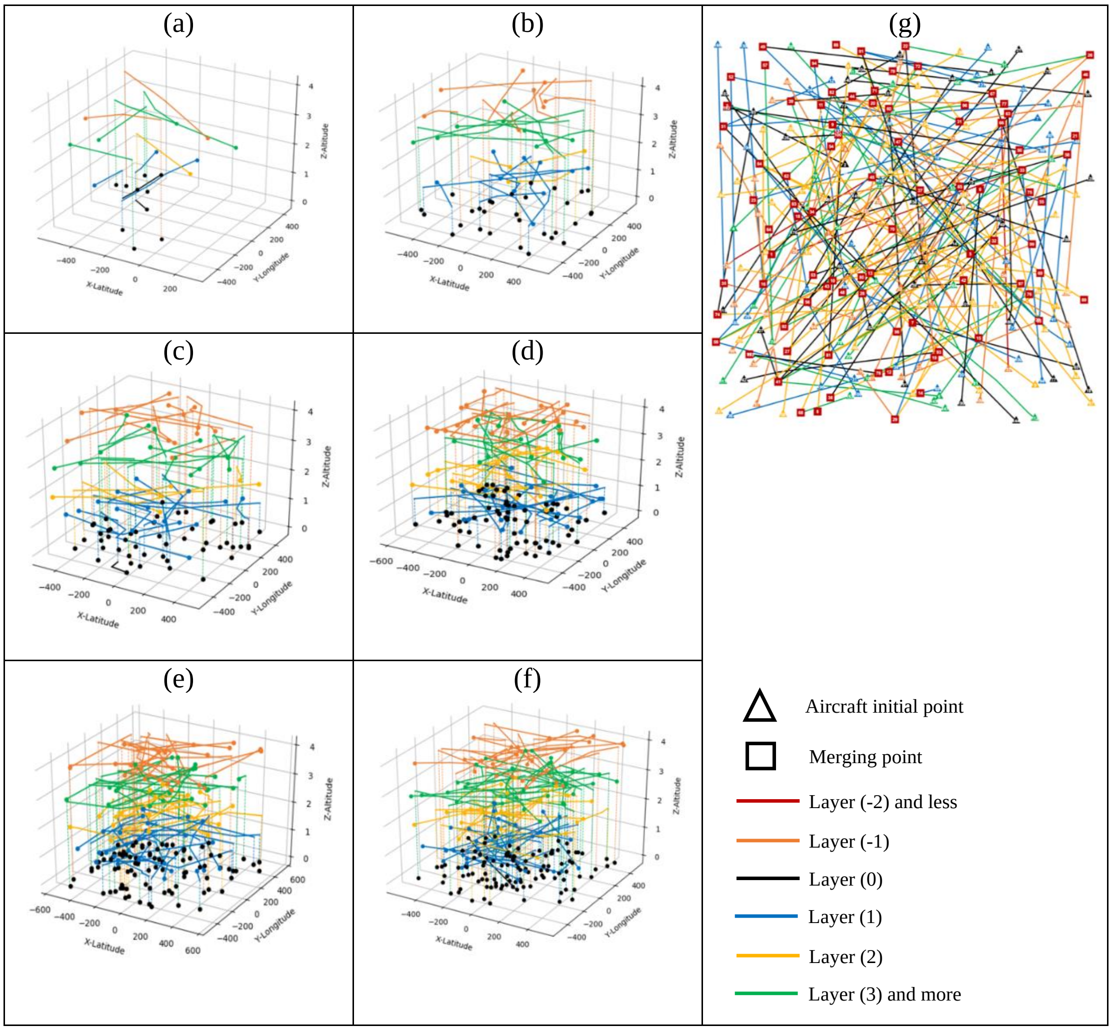


Figure 24. Algorithm solution for conflict resolution for (a) 10 aircraft, (b) 30 aircraft, (c) 50 aircraft, (d) 70 aircraft, (e) 90 aircraft, (f) 110 aircraft, and (g) 200 aircraft illustrated from top while colors shows the level of altitude

Note that the proposed algorithm solves the CR problem using a non-cooperative game by extracting Nash equilibrium. In case of two or more Nash equilibria, it is up to the ATC to select among them, building a kind of semi-cooperative, semi-decentralized instance on which pilots compete to satisfy their interest while the air traffic controller operates as a negotiator. In other words, among several Nash equilibria, the air traffic controller selects the strategy on which has a minimum cumulative cost to reduce the cost imposed on the airspace. Also, we have investigated the possibility of giving a centralized authority to the air traffic controller to make a comparison between the result of the developed semi-cooperative, semi-decentralized algorithm and cooperative, centralized approach. In the cooperative, centralized approach, regardless of the calculated Nash equilibria, the air traffic controller selects the strategy that imposes the least cumulative costs considering all available conflict-free solutions in the solution space.

Although the difference between semi-cooperative approach and centralized full cooperative approach is not significant, the total deviation cost for full cooperative approach is generally less than semi-cooperative approach. According to the experimental results depicted in Figure 25 and Figure 26, the effects of a full-cooperative approach are augmented by airspace capacity reduction and an increase in options for each variable. The general realization is that there is little difference in cooperative and semi-cooperative approaches unless we face high congestion lowering the available capacity of the airspace. Further, with congested airspace, the high number of strategies that pilots can choose could exacerbate the situation and increase the cost.

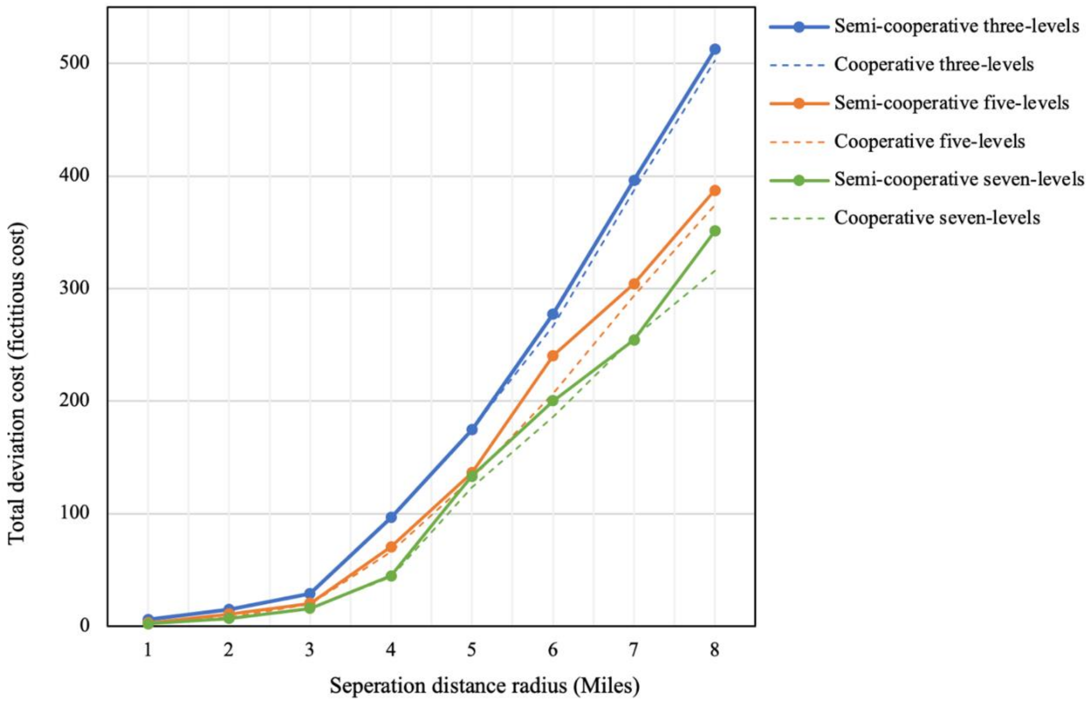


Figure 25. Variation in deviation cost relative to the separation distance radius.

According to Figure 25, the total deviation cost increases when the capacity of the airspace is reduced by increasing the safe separation distance. Also, a transition point related to the cost can be realized after three miles of separation distance, which means that there is somewhat of an exponential relationship between the minimum required safety and the total deviation cost. In other words, the higher the minimum separation distance, the higher the total deviation cost. Likewise, the more flexibility given to the pilots regarding the options they have the less total cost is expected, especially when airspace capacity is limited due to the high congestion. On the other hand, the effect diminishes when the number of options increases. On the other hand, the effect diminishes when the number of options increases. According to Figure 25, the curves are getting closer to each other when the number of levels is increased from 3 to 5 and then 7. Therefore, it can be inferred that having a finite limited number of variable options can appropriately approximate a continues domain for each variable.

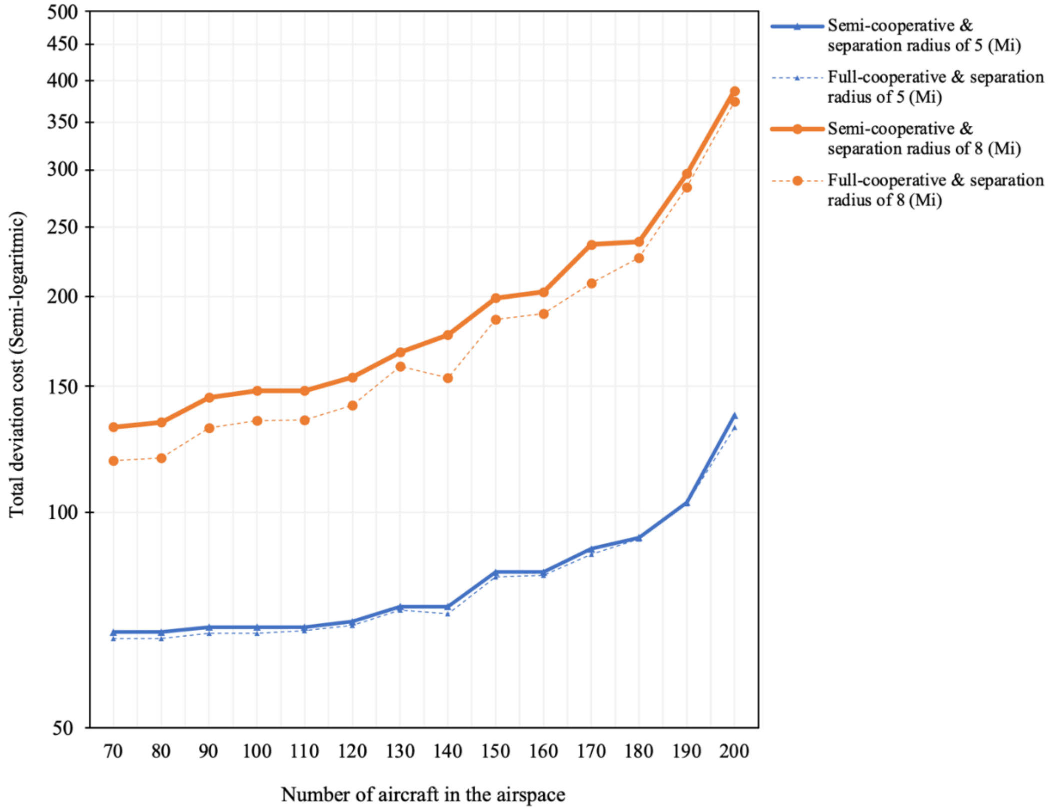


Figure 26. Variation in deviation cost relative to the number of aircraft.

Furthermore, in Figure 26, a significant gap is realized in total deviation cost when the minimum safety distance is increased from 5 to 8 nautical miles. Also, the situation exponentially exacerbates when the number of aircraft in shared airspace increases. Thus, one of the critical findings of this research is how the normal behavior of aircraft in the airspace can be jeopardized due to having unexpected congestion in the airspace.

Note that the computational complexity of the algorithm could be affected by the airspace capacity and the number of options that each aircraft could have regarding aircraft trajectory and velocity. Less capacity in the airspace either caused by more aircraft in the airspace or higher safety standard would generally produce more conflicts, and subsequently higher computational load would be imposed. Also, the more options the aircraft have, the more possibilities should be considered when resolving conflicts, and therefore the higher the computational complexity imposed by the algorithm. However, as each local CR problem has little complexity, the time required to solve each iteration is not significant. Table 19 demonstrate the algorithm computational time corresponding to the number aircraft the air space considering various minimum safety separation distance radii among aircraft.

Table 19: Algorithm computational time corresponding to the number of aircraft and separation reduction.

Number of aircraft	Separation (miles)			
	2	4	6	8
100	25 sec	73 sec	86 sec	148 sec
150	26 sec	73 sec	134 sec	194 sec
200	39 sec	159 sec	242 sec	356 sec

The algorithm can detect and resolve conflicts that could happen throughout the flight path. Therefore, any possible conflict can be realized almost as early as the aircraft enters the airspace. Subsequently, the time required by the algorithm to provide a conflict-free flight path is not significant in comparison to the computational times mentioned in Table 19. Also, computational times reported in Table 19 are recorded running the proposed algorithm using a desktop computer with limited computational abilities. Using faster computers with multi processors available at ATC land facilities can shorten the compound task of computing and information transmission between the aircraft and ATC negligible.

5.5. Conclusion

Due to the rapid advancement of technology and growth in air transportation demand, the problem of CD&R is of generalized interest from researchers, airlines, and federal agencies. In this study, a model based on game theory was presented to detect and resolve conflicts among aircraft flying in a shared airspace. In particular, the proposed model first detects possible conflicts and resolves them by providing a new set of conflict-free trajectories for the aircraft. In the proposed model, air traffic controllers supervise and coordinate the aircraft flying through a given airspace, giving some freedoms to pilots to select the flight characteristics that better serve their interests while maintaining mandatory safety standards.

To demonstrate the capability of the proposed model, we present a randomly generated example, imitating dense airspace by considering 200 aircraft in virtual airspace, where the effects of various factors, such as pilots' number of options, the capacity of the airspace, and cooperation level among the aircraft, were analyzed. The results showed that the deviation costs are highly sensitive to the variation in airspace capacity due to the congestion. In addition, a growing number of options leads to more flexibility and lower deviation costs. However, the case study also showed that as the number of decision options increase, the improvements in deviation cost reduce. With respect to the level of cooperation, the results show that the difference in deviation costs between semi- and full-cooperation approaches is not substantial, except for situations with highly congested airspaces. The presented algorithm has effectively reduced the complexity of the problem by decreasing the number of un-important option while maintaining the generality of the approach considering the dominated strategies for each player of the game. Also, the algorithm can be used to resolve the airspace conflicts in real-time while decreasing the number of cases that air traffic controllers are needed to intervene, and subsequently reduce their workload. The algorithm presented in this research can be used to resolve the conflicts related to various aerial transportation methods, especially conflict resolution for UAVs and drones. However, features needed to be added to the algorithm to consider the obstacles, such as buildings and towers, and avoid them along the planned trajectory. Note that the concept of using UAV and drones as a method of transportation, either for shipping goods and materials or carrying humans, is getting more significant. Therefore, such models and algorithms are required to resolve conflicts considering the interest of each aerial moving object in environments with obstacles.

6. CHAPTER SIX: CONCLUDING REMARKS

In this chapter we present comprehensive concluding remarks that synthesize the key insights, contributions, and implications derived from our research. This dissertation addresses the restoration planning of critical interdependent infrastructure networks utilizing a decentralized decision-making approach. We have explored various aspects of decentralized decision-making considering having multiple decision-makers in the system, pursuing their own interest by analyzing the value of information, and considering the application of game theory having different network structures. Our research has provided valuable insights into the complexities and potential solutions for managing interdependencies in these networks while we mainly focus on developing non-cooperating simultaneous game-theoretic algorithms. In Chapter 2 we addressed the restoration of critical interdependent infrastructure network after disruption considering interdependent decision-makers connected to each other by the physical interdependency exists between the sub-networks. Addressing the problem, we have developed an iterative algorithm based on game theory that offers a practical approach for searching Nash equilibria throughout the solution space, enabling effective decision-making in network restoration. Examining the cost of anarchy and the performance of the algorithm, the research has illustrated the benefits of decentralized decision-making, particularly in large-scale disruptions. Furthermore, in Chapter 3, our research has emphasized the value of information, utilizing machine learning and a notion of dictatorial game theory, in order to address the problem of critical interdependent infrastructure network restoration after disruption, in cases that one decision-maker restoration plan (follower) depends on the restoration plan of the other decision-maker (leader). The notable contribution of this research is considering the presence of uncertainty regarding the information available to the follower decision-maker regarding the leader decision-maker restoration plan. Therefore, utilizing the machine learning approaches, we have developed an adaptive approach to anticipate restoration strategies of the leader decision-maker, and integrate predictions into decision-making processes. The developed algorithm shows a strong performance in predicting the leader decision-maker restoration plan which can achieve a cost-efficient recovery outcome for follower decision-makers. It is worth noting that the developed model exhibits an insignificant difference in restoration cost resulted from the developed algorithm compared to the optimal solution. Focusing more on the effect of information and its availability on decision-making, in Chapter four we explored the impact of demand surge on critical supply chain networks due to disinformation propagation in the communities. In this research we provided a framework for modeling these variations and assessing their effect on supply chain flexibility to find the weaknesses of the supply chain. The analysis reveals that the magnitude of demand surge greatly affects the speed at which consumer orders are fulfilled and the capacity utilization of the supply chain network. Understanding these dynamics not only enhances our comprehension of supply chain operations but also allows for proactive measures to mitigate disruptions and improve overall performance. Furthermore, this dissertation focuses on different type of critical interdependent infrastructure in Chapter five which logical interdependencies between decision-makers must be removed to resolve conflicts between the aircraft route in an airspace. In this regard we have proposed a semi-decentralized, three-stage algorithm based on game theory to address aircraft conflicts in a shared 3D airspace. The developed game theory algorithm is designed to detect conflicts and provide conflict-free trajectories for the aircraft, with air traffic controllers supervising and coordinating the airspace. The results demonstrate the sensitivity of deviation costs to airspace capacity and the benefits of increasing pilots' options and cooperation levels. The presented algorithm reduces complexity, allows real-time conflict resolution, and has applications for UAVs and drones. Overall, this dissertation contributes to the body of knowledge in the field of interdependent infrastructure networks by offering insights, methodologies, and algorithms to address the complexities of decentralized decision-making. The research findings pave the way for more resilient, efficient, and coordinated management of critical infrastructure networks, thereby enhancing societal well-being, security, and sustainability.

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