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**UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE**

**MAGNETO-OPTICAL PROPERTIES
OF InSb BASED QUANTUM WELLS**

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

**In partial fulfillment of the requirement for the
degree of**

Doctor of Philosophy

By

**GITI A. KHODAPARAST
Norman, Oklahoma
2001**

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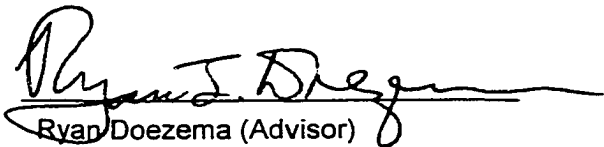
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**MAGNETO-OPTICAL PROPERTIES
OF InSb BASED QUANTUM WELLS**

**A DISSERTATION APPROVED FOR THE
DEPARTMENT OF PHYSICS AND ASTRONOMY**

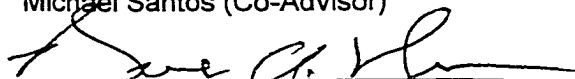
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Chapter 1

Introduction

The special properties of III-V narrow-gap semiconductors make them the natural candidates for studying fundamental physics as well as device applications. Of the binary III-V systems, InSb has the smallest effective mass ($0.014m_0$), the highest intrinsic electron mobility at room temperature, the largest g factor (-51), and the most non-parabolic band structure. These characteristics make InSb one of the best candidates for high-speed devices, infrared detectors, and spin-polarized electronic devices (spintronics).

Both basic research and industrial applications require a remotely-doped InSb quantum well. In 1991, some experiments by Saker et al.[1] demonstrated that quantum confinement in InSb wells could be provided using $\text{Al}_x\text{In}_{1-x}\text{Sb}$ as the barrier materials. In early 1995, InSb quantum wells with $\text{Al}_x\text{In}_{1-x}\text{Sb}$ barriers were grown in the Molecular Beam Epitaxy (MBE) laboratory of the University of Oklahoma.

The motivation of this work is to study the band structure and spin properties of the InSb quantum wells experimentally. These efforts resulted in many new observations such as spin resolved cyclotron resonance and zero field spin splitting in InSb quantum wells.

Theoretically, the band structure can be described by Kane ($k\cdot p$) or Luttinger models [2, 3]. Band non-parabolicity is reflected in an energy dependent effective mass and g factor, which can be probed by cyclotron resonance and spin resonance experiments respectively. Part of this work will address CR in InSb quantum wells and both experimental and theoretical aspects will be covered. The non-parabolicity allows us to observe well-resolved spin-split resonances in high mobility InSb quantum well samples. Some features of the spin resolved resonances are unexpected and will be discussed in Chapter 4.

Recently the study of spin-orbit coupling and spin splitting in semiconductors, especially narrow gap materials has attracted much theoretical and experimental attention. In asymmetric heterostructures, as first formulated by Bychkov and Rashba, spin splitting can occur even in the absence of an external magnetic field. This effect has been proposed as the basis for a type of transistor proposed by Datta and Das in 1990 [4].

Most of the experimental evidence of Rashba zero field spin splitting in heterostructures has been restricted to the observation of beating in Shubnikov-de Hass (SdH) oscillations. Using far infrared spectroscopy, electron spin resonance first was observed in InSb inversion layers by Därr [5]. In this work we also used far-infrared spectroscopy to probe the spin splitting in InSb symmetric and asymmetric quantum wells. Our experimental observations allow us to determine the zero field splitting and the Rashba spin-splitting parameter α . Our results provide a convincing confirmation of the spin-orbit coupling picture of Bychkov-Rashba [6, 7]. The details will be covered in Chapter 5.

Chapter 2

Theoretical foundations

The purpose of this chapter is to discuss some aspects of non-parabolicity in the band structure of narrow gap semiconductors, numerical methods to calculate the sub-bands in doped and undoped quantum wells, and finally the band offset determination in $\text{InSb}/\text{Al}_x\text{In}_{1-x}\text{Sb}$ quantum wells. [8].

Theoretically, the non-parabolicity in the band structure can be described by Kane (k.p) or Luttinger models [2, 3]. The band non-parabolicity is reflected in an energy dependent effective mass and g factor, which can be probed by cyclotron resonance and spin resonance experiments respectively. The bound state calculations is necessary to probe the physics of two-dimensional electrons in narrow-gap InSb as well as device applications. This work will address some techniques and the results of the bound state calculations for InSb quantum wells(QW).

2.1 Introduction

The III-V binary compounds such as InSb with a zinc-blende lattice structure, have 8 outer shell electrons per unit cell which make the chemical bonding and contribute to the electronic properties. The outer most electrons hybridize and make tetrahedral bonds between an In atom and its four Sb nearest neighbors which

quantum mechanically is described by sp^3 -hybrid wave functions. The chemical bonds plus the symmetry of the lattice structure can be used to investigate the band structure [9]. A typical band structure near the band edges of the direct band gap is shown in Fig. 2.1.

InSb is a direct gap semiconductor, with strong non-parabolic band structure. The band gap is about 0.236 eV at 4K with a $0.014m_0$ band edge effective mass. Non-parabolicity in the band structure implies an energy dependent effective mass and g-factor. In addition, in InSb/ $Al_xIn_{1-x}Sb$ heterostructures, as a result of lattice mismatch, the QW is under strain, which causes a strain dependent band gap and splitting of the heavy-hole and the light-hole bands (see Fig. 2.2) [8]

Theoretically, the non-parabolicity in the band structure can be described by the Kane model and will be discussed in the next section [9, 10]. Experimentally, cyclotron and spin resonance are the most direct tools to probe the non-parabolicity and spin properties of semiconductors, which are the topics of Chapter 4 and 5.

2.2 Kane model (k.p)

The band structure of a solid (the dependence of energy on wave vector) for different bands is defined by the Schrödinger equation. For an electron in a periodic potential in the absence of the spin-orbit interaction, the Schrödinger equation is given by [9, 10, 11]:

$$\left[\frac{-\hbar^2}{2m_0}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E(\mathbf{k})\psi(\mathbf{r}) \quad (2.1)$$

$$V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R}) \quad (2.2)$$

$$\mathbf{R} = n_1\mathbf{e}_1 + n_2\mathbf{e}_2 + n_3\mathbf{e}_3 \quad (2.3)$$

$\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are the lattice vectors and n_1, n_2, n_3 are integers.

e_1, e_2, e_3 are the lattice vectors and n_1, n_2, n_3 are integers.

Both $\psi(\mathbf{r})$ and $\psi(\mathbf{r} + \mathbf{R})$ are solutions of the Hamiltonian which suggests the Hamiltonian is invariant under lattice vector translation $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{R}$.

From the Bloch theorem, the solution of the above Hamiltonian can be written as:

$$\psi_{nk}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} u_{nk}(\mathbf{r}) \quad (2.4)$$

where $u_{nk}(\mathbf{r})$ is a periodic function, n is the band index and $\mathbf{k}$ is the wave vector. By substituting (2.4) in (2.1) and using $\frac{\hbar}{i} \nabla = \mathbf{p}$ the Schrödinger equation will have the following form [11]:

$$\left[\frac{p^2}{2m_0} + \frac{\hbar}{m_0} \mathbf{k} \cdot \mathbf{p} + V(\mathbf{r}) \right] u_{nk}(\mathbf{r}) = \left[E_n(\mathbf{k}) - \frac{\hbar^2 k^2}{2m_0} \right] u_{nk}(\mathbf{r}) \quad (2.5)$$

This is the $\mathbf{k} \cdot \mathbf{p}$ Hamiltonian in the absence of the spin-orbit interaction and results in a 2-band model which includes conduction band (CB) and valence band (VB). Adding spin-orbit interaction removes the degeneracy in the VB resulting in heavy-hole (HH), light-hole (LH) and the split-off (SO) bands. The $\mathbf{k} \cdot \mathbf{p}$ Hamiltonian with the spin-orbit interaction is given by [11]:

$$\left[\frac{p^2}{2m_0} + \frac{\hbar}{m_0} \mathbf{k} \cdot \mathbf{p} + V(\mathbf{r}) + \frac{\hbar}{4m_0^2 c^2} [\nabla V \times \mathbf{p}] \cdot \boldsymbol{\sigma} + \frac{\hbar^2}{4m_0^2 c^2} \nabla V \times \mathbf{k} \cdot \boldsymbol{\sigma} \right] u_{nk}(\mathbf{r}) = E' u_{nk}(\mathbf{r}) \quad (2.6)$$

$$E' = \left[E_n(\mathbf{k}) - \frac{\hbar^2 k^2}{2m_0} \right] \quad (2.7)$$

The choice of the basis functions for spin up and down are :

$$|iS \uparrow\rangle, \left| \frac{X-iY}{\sqrt{2}} \uparrow \right\rangle, |Z \uparrow\rangle, \left| -\frac{X+iY}{\sqrt{2}} \uparrow \right\rangle.$$

and

$$|iS \downarrow\rangle, \left| -\frac{X+iY}{\sqrt{2}} \downarrow \right\rangle, |Z \downarrow\rangle, \left| \frac{X-iY}{\sqrt{2}} \downarrow \right\rangle.$$

To derive the eigenvalue equation is not the main point of this work, but rather the result is important. The dispersion relations are doubly degenerate and are the solutions of : (assuming the bottom of CB as zero point energy [9]):

$$E' = -E_g \quad (2.8)$$

$$E'(E' + E_g)(E' + E_g + \Delta) - k^2 P^2 (E' + E_g + \frac{2}{3}\Delta) = 0 \quad (2.9)$$

The parameter P is a measure of the coupling between the conduction band and the valence band which is given by:

$$P = -i \frac{\hbar}{m_0} \langle S | p_z | Z \rangle$$

Δ is the spin-orbit split-off energy, and E_g is the band-gap. At $k \approx 0$, there will be three solutions to this equation,

$$E' = 0, E' = -E_g, E' = -E_g - \Delta$$

In the vicinity of $k = 0$, we can consider a small term ($\varepsilon(k^2) \ll E_g$ and Δ) and add to the solutions above. Using the following solutions in the vicinity of $k = 0$:

$$E' = 0 + \varepsilon(k^2)$$

$$E' = -E_g + \varepsilon(k^2)$$

$$E' = -\Delta - E_g + \varepsilon(k^2)$$

the CB, HH, LH, SO dispersion relations will have the following forms (assuming the top of VB as zero energy[11]):

$$E_{CB} = E_g + \frac{\hbar^2 k^2}{2m_0} + \frac{k^2 P^2 (E_g + 2\Delta/3)}{E_g(E_g + \Delta)} \quad (2.10)$$

$$E_{HH} = \frac{\hbar^2 k^2}{2m_0} \quad (2.11)$$

$$E_{LH} = \frac{\hbar^2 k^2}{2m_0} - \frac{2k^2 P^2}{3E_g} \quad (2.12)$$

$$E_{SO} = -\Delta + \frac{\hbar^2 k^2}{2m_0} - \frac{k^2 P^2}{3(E_g + \Delta)} \quad (2.13)$$

The Kane model does not include the effect of higher bands, and results in an incorrect effective mass for heavy holes. The model is sufficient to describe the effective mass of electrons and the light holes. From (2.9) the energy dependent effective mass for light particles can be written as:

$$\frac{1}{m^*(E')} = \frac{2P^2}{3} \left[\frac{2}{E' + E_g} + \frac{1}{E' + E_g + \Delta} \right] \quad (2.14)$$

In narrow gap materials like InSb, the energy E' is not normally negligible compared to E_g and Δ which implies the non-parabolicity corrections are important. The result of the Kane model for the effective mass will be used in the following sections in order to define the sub-band energies in doped and undoped QW.

2.3 Sub-bands for doped QW:

2.3.1 Self-Consistent calculations :

The main part of the computer simulations [12, 13] in this section is solving Schrödinger and Poisson equations for a doped $\text{Al}_x\text{In}_{1-x}\text{Sb}/\text{InSb}$ Square Quantum Well(SQW) to determine the sub-band energies for conduction and valence bands. Because of the small effective mass and large dielectric constant of InSb, the exchange potential has a small contribution and the Hartree potential is the dominant term which should be calculated self consistently. For a square quantum well, the Schrödinger equation inside the well is given by:

$$\left[\frac{\partial}{\partial z} \left(\frac{-\hbar^2}{2m_1^*(E_i, z)} \right) \frac{\partial}{\partial z} - E_i + V_1(z) + \frac{\hbar^2 K_t^2}{2m_1^*(E_i, z)} \right] \Psi_{1i,kt}(z) = 0 \quad (2.15)$$

where i is a sub-band index and K_t is the transverse wave vector. Similarly for outside of the well:

$$\left[\frac{\partial}{\partial z} \left(\frac{-\hbar^2}{2m_2^*(E_i, z)} \right) \frac{\partial}{\partial z} - E_i + V_2(z) + \frac{\hbar^2 K_t^2}{2m_2^*(E_i, z)} \right] \Psi_{2i,kt}(z) = 0 \quad (2.16)$$

If we define the bottom of the quantum well as zero energy, then $V_1(z) = V_H(z)$ the Hartree or electrostatic potential, and $V_2(z) = (E_{g_{Barrier}} - E_{g_{Well}}) \times Q$, where Q is the offset. The offset has been defined experimentally for InSb QW and is about 62% [8]. In the Schrödinger equations $m_1^*(E_i, z)$ and $m_2^*(E_i, z)$ are the energy dependent effective mass in the well and the barrier respectively.

The Hartree or electrostatic potential is given by:

$$\nabla^2 V_H(z) = \frac{-n(z)e^2}{\kappa \epsilon_0} \quad (2.17)$$

ϵ_0 is vacuum dielectric constant and κ is the material dielectric constant (17.7 for *InSb*). The distribution of electrons is obtained by:

$$n(z) = \sum_{i=1}^N \left(\sum_{K_t=0}^{K_{Fi}} |\psi_{i,K_t}(z)|^2 \right) \quad (2.18)$$

where N is the number of occupied sub-bands. The summation can be written as:

$$\sum_{K_t=0}^{K_{Fi}} |\psi_{i,K_t}(z)|^2 = \frac{\widehat{2}^{Spinfactor}}{(2\pi)^2} \int_0^{K_{Fi}} d^2 K_t |\psi_{i,K_t}(z)|^2 = \frac{1}{\pi} \int_0^{K_{Fi}} K_t dK_t |\psi_{i,K_t}(z)|^2 \quad (2.19)$$

The final form for the Poisson equation is given by:

$$\nabla^2 V_H(z) = \frac{-e^2}{\kappa \epsilon_0} \sum_{i=1}^N \frac{1}{\pi} \int_0^{K_{Fi}} K_t dK_t |\psi_{i,K_t}(z)|^2 \quad (2.20)$$

where K_{Fi} is the Fermi wave vector for different sub-bands. In a self-consistent procedure, a set of wave functions, assuming $V_H(z) = 0$, will be generated. The wave function will be used in Eq.(2.20) to calculate $V_H(z)$ (see appendix F) which

itself will be substituted back into the Schrödinger equation in order to generate a new set of wave functions. This procedure will be continued until variation in the potential converges to a small value. Fig. 5.2 shows a doped QW profile .

The result of this calculation can be used to define the doping level for single and double subband occupancies. Samples 669 ($n = 3.8 \times 10^{11} \text{cm}^{-2}$, $250 \text{\AA}$ QW) and 671 ($n = 5.2 \times 10^{11} \text{cm}^{-2}$, $250 \text{\AA}$ QW) are the examples of the samples with one and two occupied subbands respectively which were predicted by the self consistent calculations.

2.4 Numerical Techniques

2.4.1 Solving Schrödinger equation

In this work, the *Runge – Kutta* [13] method of fourth order was used to solve the Schrödinger equation numerically. To use this method, the spatial dimension is divided into evenly spaced points. For simplicity, in the entire calculation, the width of the quantum well is scaled to unity, and the Schrödinger equation is divided by $(\frac{\hbar^2}{2m \cdot W^2})$ where W is width of the well [13].

Using the *Runge – Kutta* method, the wave function and the first derivative of the wave function on each grid point can be determined as following :

$$\begin{aligned} \psi(z+h) = & \left[1 + \frac{h^2 A(z)}{6} + \frac{h^2 A(z+h/2)}{3} + \frac{h^4 A(z)A(z+h/2)}{24} \right] \psi(z) \\ & + \left[1 + \frac{h^2 A(z+h/2)}{6} \right] h \psi'(z) \end{aligned} \quad (2.21)$$

$$\begin{aligned} \psi'(z+h) = & \left[1 + \frac{h^2 A(z+h/2)}{3} + \frac{h^2 A(z+h)}{6} + \frac{h^4 A(z+h)A(z+h/2)}{24} \right] \psi'(z) \\ & + \left[A(z) + 4A(z+h/2) + A(z+h) + \frac{h^2 A(z)A(z+h/2)}{2} + \right. \\ & \left. \frac{h^2 A(z+h)A(z+h/2)}{2} \right] \frac{h}{6} \psi(z) \end{aligned} \quad (2.22)$$

Where $A(z) = \frac{V(z)-E_{i,K_t}}{(\frac{\hbar^2}{2m^*W^2})} + k_t^2 W^2$ is a unit less variable and h is the grid size. The *Runge – Kutta* method needs an initial guess for the WF and the first derivative of the WF.

2.4.2 Shooting Method

In a quantum well system such as $\text{Al}_x\text{In}_{1-x}\text{Sb}/\text{InSb}$, the bound states can be found by propagating the WF from a grid point in the barrier and the well regions to a matching point(*i.e.* edge of the well or the region that the potential is flat which usually works better). Using Eq.(2.21) and Eq.(2.22), the wave function and the first derivative of wave function can be determined on each grid point. The bound states are those for which the Wronskian:

$$Wr(z) = \psi_l(z)\psi'_r(z) - \psi_r(z)\psi'_l(z) \quad (2.23)$$

at the matching point goes to zero. Indices l and r are correspond to the solutions from the left and right sides of the matching point.

The result of the numerical calculations for the doped and undoped QW has been used to identify optical transitions [14, 8, 15] and also in the design of the sample structures. In the following sections, the Bastard formalism [9] will be discussed, and the results will be compared with the numerical method in the limit $V_H(z) \rightarrow 0$.

2.5 Sub-bands in undoped QW (Bastard formalism):

2.5.1 Subband energies ($K_t = 0$)

Consider a square QW of narrow gap material with the Hamiltonian given by[9]:

$$\left[p_z \frac{1}{2m(E, z)} p_z + V_s(z) \right] f(z) = E f(z) \quad (2.24)$$

The function $f(z)$ and $\frac{1}{m(E, z)} \frac{df(z)}{dz}$ are continuous at the well-barrier interface. $m(E, z)$ is the energy dependent effective mass and in the presence of band bending has an explicit position dependence. The solutions of the square QW are well known with the dispersion relation for both odd and even solutions in the well :

$$\cos(k_w L) - \frac{1}{2} \left[-\xi + \frac{1}{\xi} \right] \sin(k_w L) = 0 \quad (2.25)$$

$$\xi = (m_b(E) K_w) / (m_w(E) K_b) \quad (2.26)$$

$$\frac{1}{m_w} = \frac{2P^2}{3} \left[\frac{2}{E + E_{gw}} + \frac{1}{E + E_{gw} + \Delta_w} \right] \quad (2.27)$$

$$\frac{1}{m_b} = \frac{2P^2}{3} \left[\frac{2}{E - V_s + E_{gb}} + \frac{1}{E + E_{gb} + \Delta_b} \right] \quad (2.28)$$

The Kane model gives the wave vectors in the well and barrier for electrons and light holes :

$$K_w^2 = \frac{E(E + E_{gw})(E + E_{gw} + \Delta_w)}{\hbar^2 P^2 (E + E_{gw} + \frac{2\Delta_w}{3})} \quad (2.29)$$

$$K_b^2 = \frac{(E - V_s)(E - V_s + E_{gb})(E - V_s + E_{gb} + \Delta_b)}{\hbar^2 P^2 (E - V_s + E_{gb} + \frac{2\Delta_b}{3})} \quad (2.30)$$

For heavy holes, the dispersion relation is parabolic and the wave vectors for the well and barrier are given by:

$$k_w = \sqrt{-2m_{hhw}(E_{gw} + E)/\hbar} \quad (2.31)$$

$$k_b = \sqrt{-2m_{hbb}(E_{gw} + E + V_p)/\hbar} \quad (2.32)$$

The heavy holes effective mass is related to the Luttinger parameters that will be discussed later. The bound states are the solutions of equation (2.25), and can be calculated using any numerical method. $V_s(z)$ and $V_p(z)$ are defined in Fig. 2.4.

2.5.2 Subbands energies for $K_t \neq 0$

The wave vectors in the well and barrier for $K_t \neq 0$ have the following forms [16]:

$$K_w^2 = \frac{E(E + E_{gw})(E + E_{gw} + \Delta_w)}{\hbar^2 P^2 (E + E_{gw} + \frac{2\Delta_w}{3})} - K_t^2 \quad (2.33)$$

$$K_b^2 = K_t^2 - \frac{(E - v_s)(E - v_s + E_{gb})(E - v_s + E_{gb} + \Delta_b)}{\hbar^2 P^2 (E - v_s + E_{gb} + \frac{2\Delta_b}{3})} \quad (2.34)$$

The bound states are the solutions of the following dispersion relation.

$$\cos(k_w L) + \frac{1}{2} \left[\frac{K_b m_w}{m_b K_w} - \frac{K_w m_b}{m_w K_b} - \frac{K_t^2}{K_w K_b} \frac{(\eta_w - \eta_b)^2}{m_w m_b} \right] \sin(k_w L) = 0 \quad (2.35)$$

where

$$\frac{1}{\eta_w} = \frac{2P^2}{3} \left[\frac{1}{E + E_{gw}} - \frac{1}{E + E_{gw} + \Delta_w} \right] \quad (2.36)$$

$$\frac{1}{\eta_b} = \frac{2P^2}{3} \left[\frac{1}{E - V_s + E_{gb}} - \frac{1}{E + E_{gb} + \Delta_b} \right] \quad (2.37)$$

In the limit when $V_H=0$ the shooting method and Bastard formalism should give almost identical results which has been plotted in Fig. 2.5.

2.5.3 Luttinger model & band edge mass

This model describes the valence bands more accurately than the Kane model and includes the warping of the energy surface which can not be ignored in InSb. The details of this model can be find in many references[17, 3] but the result is rather important. The valence band dispersion relation in this model is given by:

$$E(\mathbf{k}) = \frac{\gamma_1 \hbar^2 k^2}{2m_0} \pm \frac{\hbar^2}{m_0} \left[\gamma_2^2 k^4 + 3(\gamma_3^2 - \gamma_2^2)(k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2) \right]^{1/2} \quad (2.38)$$

In this expression γ 's are standard Luttinger's parameters which can be found in Landolt-Börnstein [35]. This dispersion relation can be used to define the heavy and light hole effective masses with the (+) sign for light holes and (-) for heavy holes.

As it was mentioned earlier, the Kane model does not give the correct effective mass for heavy holes [18] these should be defined using Luttinger model. The dispersion relation for heavy and light holes using Luttinger's valence band Hamiltonian for $k \parallel [001]$ from the above equation can be written as :

$$E_{\pm} = \frac{\hbar^2}{2m_0}(\gamma_1 \pm 2\gamma_2)k^2 \quad (2.39)$$

Using the dispersion relation, band edge effective masses for the heavy and light holes are:

$$m_{hh}^* = \frac{m_0}{(\gamma_1 - 2\gamma_2)} \quad (2.40)$$

$$m_{lh}^* = \frac{m_0}{(\gamma_1 + 2\gamma_2)} \quad (2.41)$$

where γ_1 and γ_2 are Luttinger parameters.

Cyclotron masses can be related to the Luttinger parameters and for $B \parallel [001]$ are given by:

$$m_{hh}^* = \frac{m_0}{\gamma_1 - \{\gamma_2^2 + \frac{3}{4}(\gamma_2 + \gamma_3)^2\}^{\frac{1}{2}}} \quad (2.42)$$

$$m_{lh}^* = \frac{m_0}{\gamma_1 + \{\gamma_2^2 + \frac{3}{4}(\gamma_2 + \gamma_3)^2\}^{\frac{1}{2}}} \quad (2.43)$$

Only when the constant energy surfaces are spherical($\gamma_2=\gamma_3$), is the band edge effective mass and cyclotron mass are the same. It is important to distinguish between the cyclotron mass and the band edge effective mass in the systems (such as InSb) with the warping in the energy surface.

2.6 Determination of band offset

Excitonic transitions in parabolic quantum wells (PQW) can be used to determine the band offset [8, 19, 20]. Parabolic quantum wells can be constructed by alternate

growth of undoped and thin layers of InSb and $\text{Al}_x\text{In}_{1-x}\text{Sb}$. In PQW the spacing between the sub-band energies depend on the curvature of the well, and for an infinite well with parabolic band structure the conduction band offset is given by:

$$\frac{\Delta E_c}{\Delta E_{hh}} = \left[\frac{Q_c m_{hh}}{(1 - Q_c) m_e} \right]^{\frac{1}{2}} \quad (2.44)$$

where Q_c is the offset in the conduction band. Unlike the infinite well, for a finite PQW the ratio has a weak width dependence. The sub-band energies can be calculated assuming a smooth parabolic potential well, or a series of square wells. These methods will be discussed in the following section.

2.6.1 Sub-bands energy of PQW

While the parabolic quantum well is constructed from a series of square quantum wells, the shooting method described earlier can be used to determine the sub-bands energies. In order to propagate the wave function and the first derivative (using the *Runge – Kutta* method), the alternative layers of InSb and $\text{Al}_x\text{In}_{1-x}\text{Sb}$ should be divided into equally spaced grid points. The functions that are propagated from the two edges of the parabolic well to a point in the middle can be used to calculate the zeros of the Wronskian as described earlier .

In a similar procedure, instead of a series of square wells one can define a parabolic potential and solve the Schrödinger equation numerically. In a parabolic potential assumption, the change in the bandgap and effective mass at each grid point should be considered. The numerical results from the two different methods are in a good agreement, which suggest accounting for the digital composition of the parabolic well is not necessary [21]. These series of calculations have been used to define the band offset of $\text{InSb}/\text{Al}_x\text{In}_{1-x}\text{Sb}$ for the first time. Fig.2.6 shows an example of the optical transition in which the theoretical model have been used to

assign the transitions for undoped PQW.

As in $GaAs/Al_xGa_{1-x}As$ parabolic wells, the forbidden $\Delta n = 2$ excitonic transitions are observed which allows one to determine sub-band spacing for the hole and electron bands.

In order to define ΔCB_{1-3} , the difference of the $HH3 - CB3$ and $HH3 - CB1$ transitions or the difference of the $HH1 - CB3$ and $HH1 - CB1$ transitions can be used. Similarly for the heavy holes, ΔHH_{1-3} can be extracted from $HH3 - CB1$ and $HH1 - CB1$ transitions. For wells as wide as 1000 Å, the ΔCB_{2-4} and ΔHH_{2-4} spacing can be obtained as well.

Calculated $\frac{\Delta CB_{1-3}}{\Delta HH_{1-3}}$ which is almost independent of the well thickness for different band offset values is plotted in Fig. 2.7. The crossing of experimentally observed ratio with the theoretical curves determines the offset. We find a conduction band offset ratio of 0.62 ± 0.04 for Al concentrations in the range 2%-12%. The offset value is almost independent of Al concentration which suggests a lack of strain dependence in $InSb/Al_xIn_{1-x}Sb$ system for the range of Al concentrations.

2.7 Interaction of 2DES with magnetic field

2.7.1 Integer Quantum Hall Effect

In order to understand the oscillatory behavior of the resistivity at high magnetic field in 2DEG, which was observed by K. von. Klitzing in 1980 [22], the Landau levels and the effect of impurities should be introduced. In the classical case the motion of an electron in a uniform magnetic field follows a closed circular orbit and quantum mechanically the energy of an electron in such a closed orbit can have quantized values. The energy levels of the two dimensional motion of electrons are called Landau levels [23, 24].

Consider electrons in a uniform magnetic field in the z direction which the Hamiltonian is given by [13]:

$$H = \frac{1}{2m}(\nabla + e\mathbf{A})^2 + V(z) \quad (2.45)$$

For B in the z direction, $\mathbf{A}$ in symmetric gauge is given by:

$$\mathbf{A} = -\mathbf{B}/2(y, -x, 0) \quad (2.46)$$

Using the following transformation:

$$\mathbf{A} = \mathbf{A} + \nabla\Lambda \quad (2.47)$$

$$\Lambda = -\frac{\mathbf{B} \times \mathbf{r}}{2} \quad (2.48)$$

will result the Landau gauge:

$$\mathbf{A} = -B(y, 0, 0)$$

and the Hamiltonian can be written as :

$$H = \frac{1}{2m}(\nabla_x - eBy)^2 + \frac{1}{2m}\nabla_y^2 + \frac{1}{2m}\nabla_z^2 + V(z) \quad (2.49)$$

The Hamiltonian commutes with ∇_x therefore the wave function can be written as:

$$\Psi = e^{ikx} f(z) \Phi(y) \quad (2.50)$$

The Schrödinger equation can be written as:

$$\nabla_y^2 \Phi(y) f(z) + \left[\frac{2mE}{\hbar^2} - \frac{m^2 \omega_c^2}{\hbar^2} (y - y_0)^2 \right] \Phi(y) f(z) - [\nabla_z^2 f(z) + V(z) f(z)] \Phi(y) = 0 \quad (2.51)$$

where $\omega_c = \frac{eB}{m}$ and $y_0 = \frac{\hbar k_x}{eB}$. The above Schrödinger equation is the equation for a shifted harmonic oscillator with the eigen-solution:

$$E = (n + 1/2)\hbar\omega_c + E_z \quad (2.52)$$

The first term is corresponding to energies due the electronic motion in the plane(xy plane) perpendicular to the magnetic field (Landau levels) and E_z is the solution along z axis.

If we add electron spin to the picture, the above equation can be written as:

$$E = E_z + (n + 1/2)\hbar\omega_c \pm \frac{1}{2}g\mu B \quad (2.53)$$

Classically, cyclotron resonance is a resonance absorption by the charge carriers which are rotating about the magnetic field with frequency ω_c in a plane perpendicular to the field in the presence of a circular polarized light. Quantum mechanically, the transitions between the spin conserved Landau levels are called cyclotron resonances and transitions within one Landau level between two spin states are called spin resonances. Fig. 2.8 shows a simple diagram of these transitions in the absence of non-parabolicity.

In an actual system, the samples have impurities and defects (in one electron picture) [23, 24]. Most of the energy dissipation occurs when electrons are scattered by collisions with the impurity atoms or defects. The quantum state associated with the impurity states are more stable. For example, if an impurity atom gives up an electron it would be more likely to find an electron near the impurity atom. Therefore; a sharp energy level that makes up a Landau level in a pure material, in the presence of impurities will spread out into a band containing many distinct energy levels. The states near the bottom of each Landau level are localized states

and the electrons trapped in these states would never leave that region and cannot carry current.

Near the center of each band there are spatially extended states which are spread out over the sample. Electrons in these states are mobile and can carry current. Each Landau level can be occupied by a large number of electrons with the same energy. Which the populations of the levels change with the strength of the magnetic field. The size of the circular orbits have a radius $l = (\frac{\hbar}{eB})^{1/2}$ (magnetic length) that decreases with increasing magnetic field, resulting in an increasing degeneracy of the Landau levels [23, 24]. Each Landau level can have

$$D = \frac{1}{2\pi l^2} = \frac{eB}{h}$$

electrons per unit sample area, which D is the degeneracy. The number of occupied Landau levels (filling factor) can be defined by:

$$\nu = N/D$$

where N is the number of electrons per sample unit area. As shown in Fig. 2.9 [23] at very high magnetic field all electrons can lie in the lowest Landau level ($\nu < 1$), and by reducing the magnetic field the spacing and the degeneracy of the Landau levels decreases. There will be a magnetic field for which the lowest Landau level is exactly filled at $\nu = 1$.

For a smaller magnetic field the two lowest ($\nu = 2$) Landau levels will be exactly filled and so on. For any integer value i there is a magnetic field

$$B_i = \frac{N\hbar}{ie} \tag{2.54}$$

for which the i th lowest Landau levels are completely filled and all higher levels are empty. This effect is an introduction to integer quantum Hall effect (IQHE).

The vanishing of ρ_{xx} [23, 24] implies electron transport without dissipation in the energy. Dissipation exists if the electrons can scatter into an empty energy level. At an integer filling factor the empty levels are at higher energies and at low temperatures cannot be reached.

The zeros of ρ_{xx} correspond to the flat regions (plateaus) in the Hall resistance. The value of the Hall resistance in terms of the filling factor is given by:

$$R_H = \rho_{xy} = \frac{B}{Ne} = \frac{hD}{Ne^2} = \frac{h}{e^2\nu}$$

Obviously R_H for an integer filling factor is independent of magnetic field and just depends on the fundamental constants. The Hall resistance is $\approx 25.5 \text{ K}\Omega$ at $\nu = 1$.

To understand the plateaus in the Hall resistance we should return to the defects and impurities picture. The electrons trapped by impurities or defects cannot participate in the current flow. Small changes of the magnetic field from the B field at an integer filling factor changes the number of the trapped electrons but not the the number of extended states. This implies $\rho_{xx} = 0$ and no change in R_H (plateaus on R_H trace). A larger magnetic field changes the number of trapped electrons as well as the number of occupied levels.

The integer quantum Hall effect is a powerful tool to study fundamental physics as well as to characterize the samples. In a SdH trace the ρ_{xx} minima correspond to the integer filling factors, which along with the Eq.(2.54), can be used to determine the electron density. In our samples, the SdH traces are not identical in different orientations of which suggests an asymmetric behavior in electron conductivity in different directions of the samples. In the Van der Pauw geometry, the average of ρ_{xx} at $B = 0$ can be used in Eq.(2.54) to evaluate the resistivity and then the

mobility.

The Hall trace can be used to evaluate the density as well. The slope of the Hall trace at low magnetic field is proportional to the density. The middle of each plateau is at the same magnetic field of the SdH minima. The best method to define the filling factors is to look at the values of ρ_{xy} . At $\nu = 1$, the Hall resistance or $\rho_{xy} = 25.5K\Omega$, and for $\nu = 2$, $\rho_{xy} = 12.25K\Omega$, and so on. In the samples with good qualities the deviations in ρ_{xy} are less than 1%. Fig. 2.10 is a representative of many InSb QW which demonstrating the integer quantum Hall effect at 4.2 K.

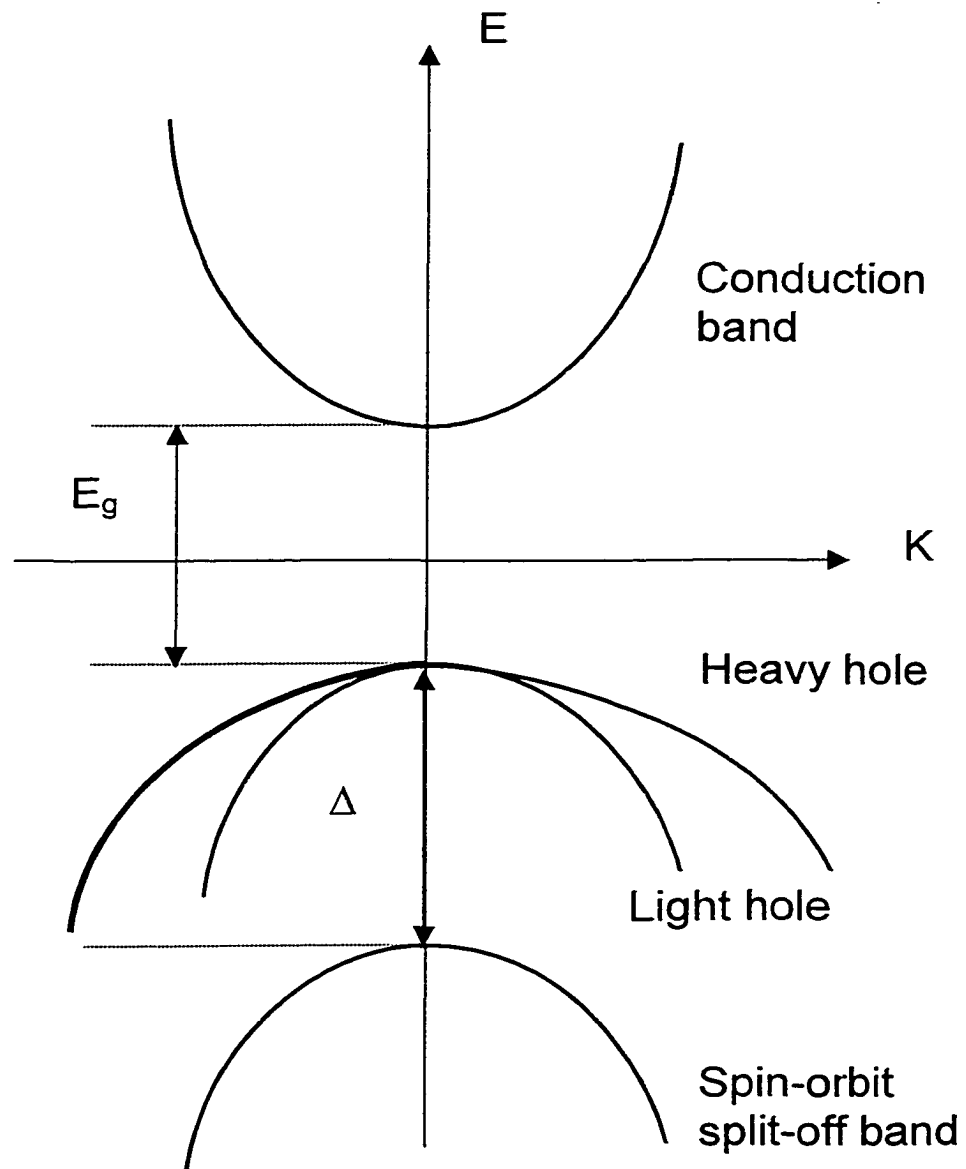


Figure 2.1: The band structure near the band edges showing the conduction, heavy-hole, light-hole, and spin-orbit bands

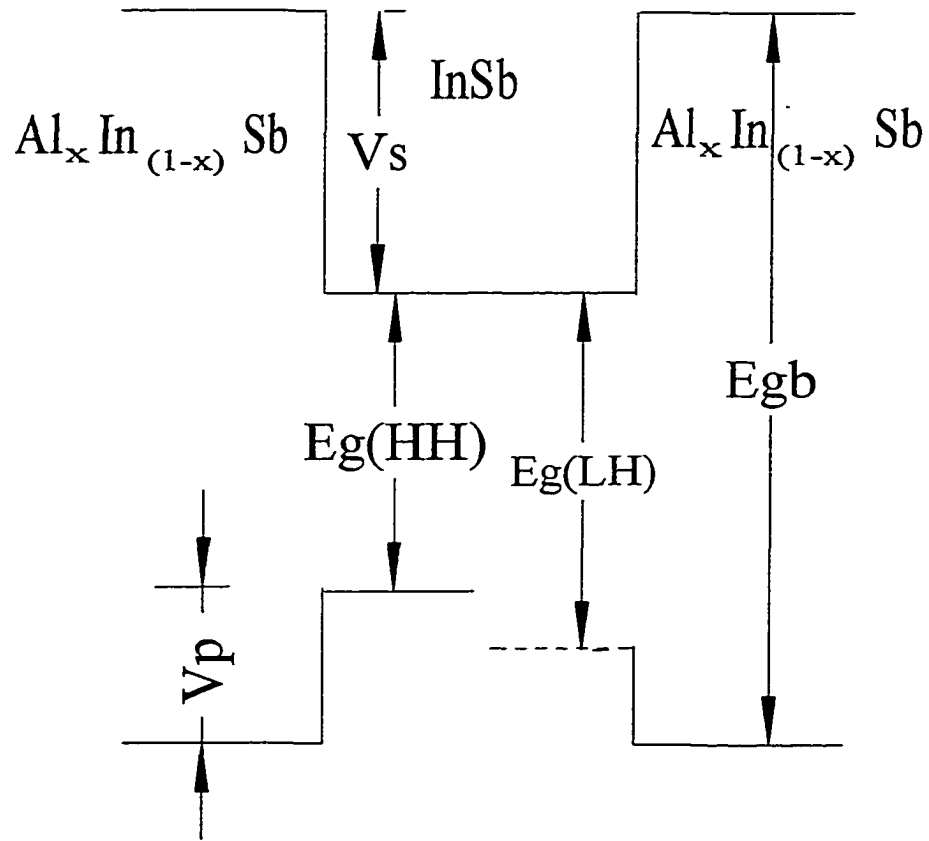


Figure 2.2: $\text{InSb}/\text{Al}_x\text{In}_{1-x}\text{Sb}$ band alignment. The well is under strain which splits the heavy hole and the light hole bands.

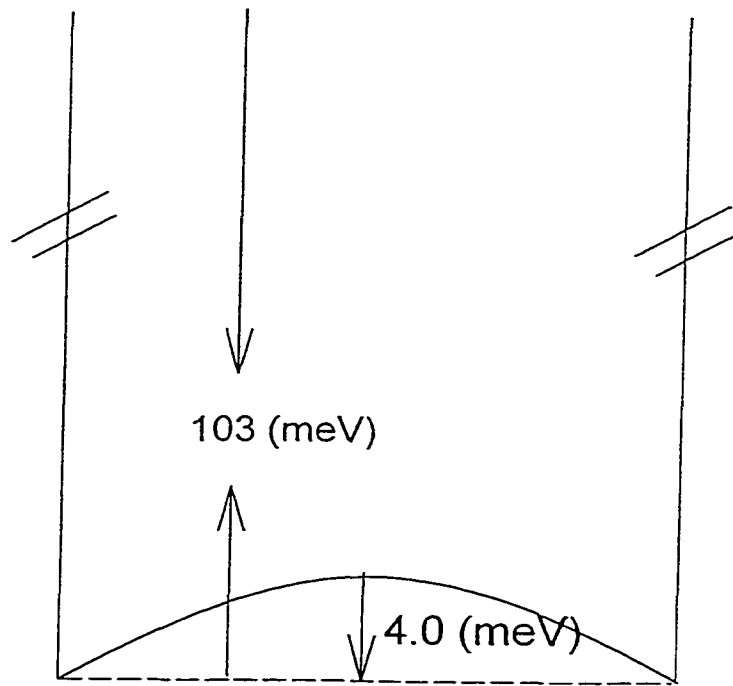


Figure 2.3: Self consistent calculated InSb quantum well profile for 9% Al concentration, 250Å well width and the density $n = 2 \times 10^{11} \text{cm}^{-2}$

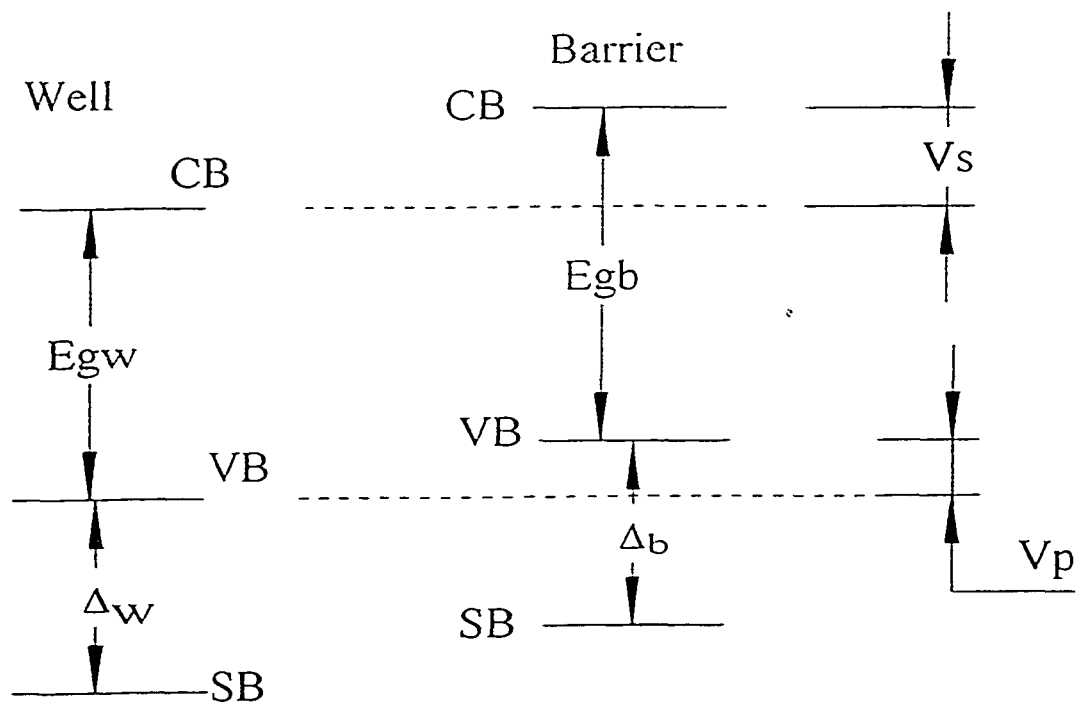


Figure 2.4: Relationship between V_s , V_p and the band parameters

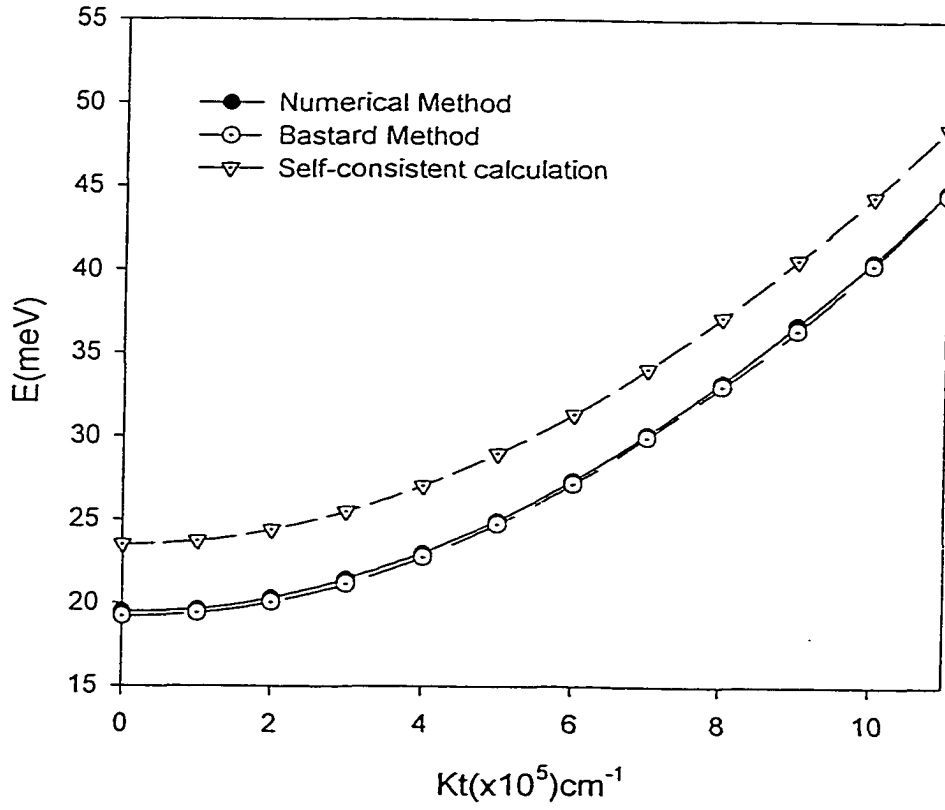


Figure 2.5: Subband energies using Bastard formalism, the numerical methods in the limit $V_H \rightarrow 0$, and the self consistent calculation. The upper curve shows the shift of the sub-bands due to the electron-electron interaction in a 250Å quantum well for electron concentration $n = 2 \times 10^{11} \text{cm}^{-2}$. The Bastard method and the numerical method without e-e interaction overlap.

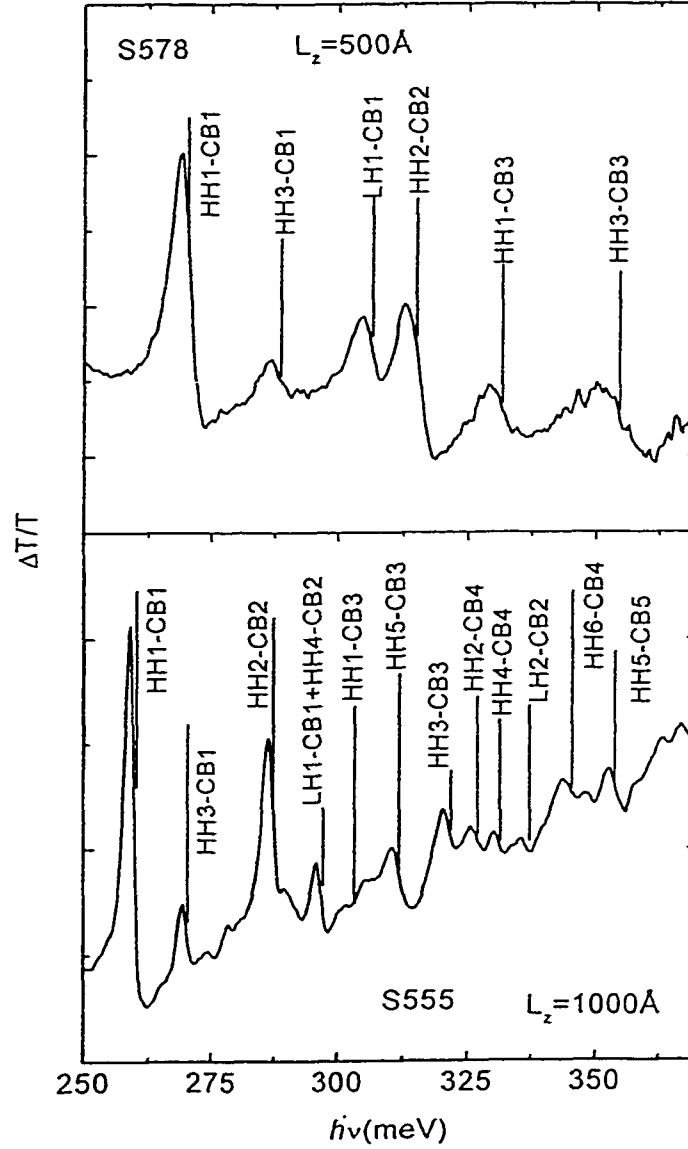


Figure 2.6: Differential transmission spectra of parabolic InSb MQW samples with 500 and 1000 Å wells. The transmission difference, $T(4.2K) - T(30K)$ relative to the average transmission T is plotted vs photon energy.

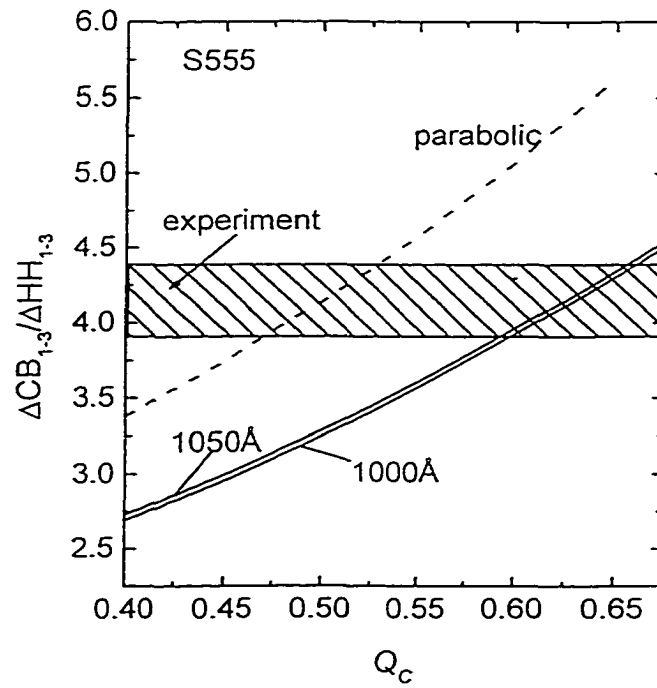
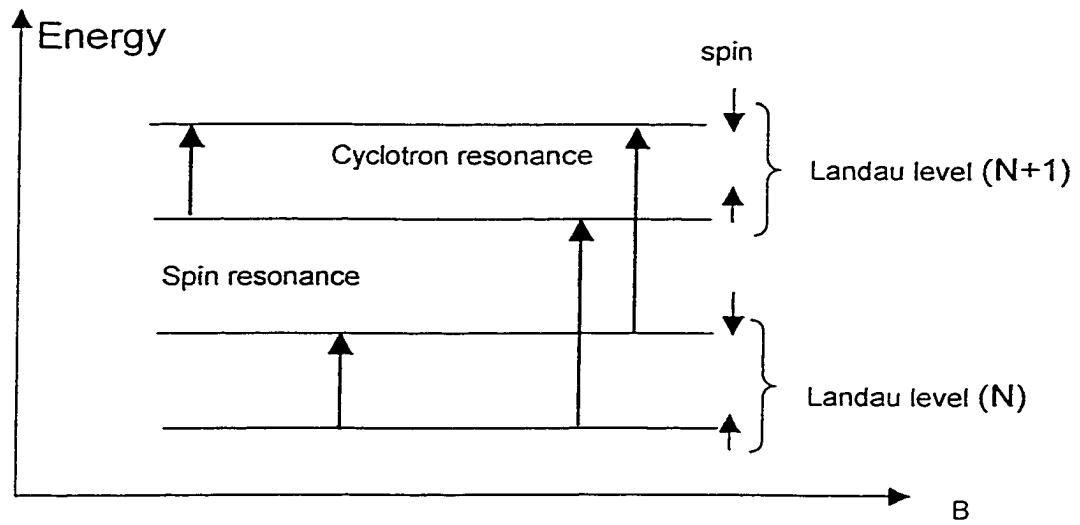


Figure 2.7: Calculated ratio of the ground-state to second-excited-state spacing for the conduction-band to that of the heavy-hole band vs conduction-band offset ratio. Coincidence with the experimentally observed ratio determines the offset. The upper curved labeled as parabolic shows the theoretical calculations in the parabolic regime.



Cyclotron resonance ($\Delta n = 1, \Delta s = 0$)

Spin resonance (weak) ($\Delta n = 0, \Delta s = -1$)

Figure 2.8: A simple picture of cyclotron resonance and spin resonance.

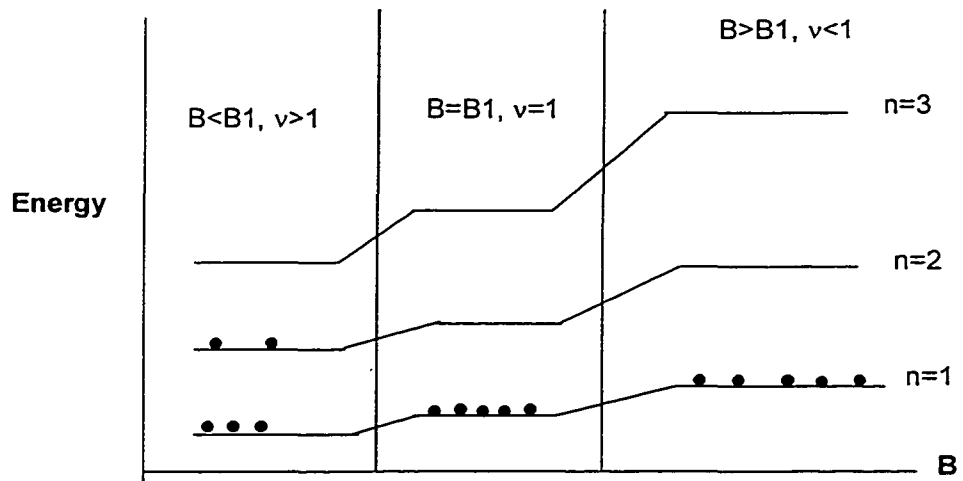


Figure 2.9: Three lowest Landau levels for a five-electron system. At $B=B_1$ the lowest Landau level is exactly filled. Eventually for smaller B field some electrons will be forced to go to higher levels resulting in reducing the degeneracy of the system [23].

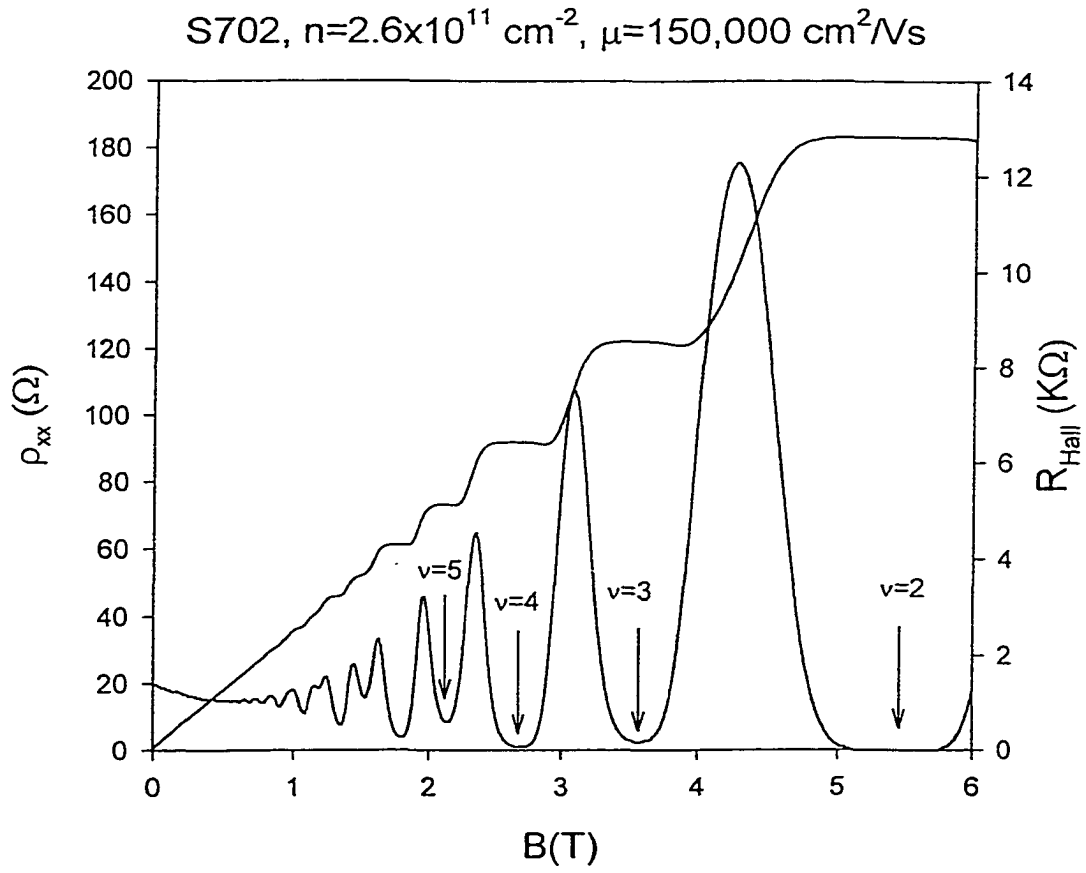


Figure 2.10: Plot of the Hall resistance and ρ_{xx} as a function of B at 4.2K for an InSb QW.

Chapter 3

Experimental Techniques

The purpose of this chapter is to discuss the InSb quantum well(QW) structure, the sample preparation, and some of the experimental aspects of transport, transmission, and photo-conductivity measurements which have been used to study the fundamental properties of InSb QW.

3.1 The quantum well structure

The samples used in the magneto-optical experiments of this work are single InSb ($\text{Al}_x\text{In}_{1-x}\text{Sb}$ barrier material) MBE-grown QW grown on GaAs (001), where remote doping has been provided with Si delta doped layers in the barrier material [25, 26]. Delta doping is ideal for remotely doping structures leading to the low ionized scattering and providing high mobility electrons inside the well. Fig. 3.1 shows an example of the layer sequence for our InSb QW structures grown by MBE.

Because of the lattice mismatch between InSb and $\text{Al}_x\text{In}_{1-x}\text{Sb}$ (about 0.5% for the alloy composition with $x = 0.09$) the QW is under strain. The lattice mismatch causes a strain dependent band gap with a larger light hole to conduction gap compared to the heavy hole gap (see Fig. 2.2). The strain effect in InSb QW system is well characterized [8].

In this work, the remotely-doped QW with densities in the range $(0.95\text{-}2.8) \times 10^{11} \text{ cm}^{-2}$ and mobilities in the range $(70,000 - 150,000) \text{ cm}^2/\text{Vsec}$ have been used. The delta doped layers can be either on one side or on both sides of the QW, resulting in asymmetric and symmetric wells respectively. An additional Si doping layer must be placed near the surface to prevent depletion of the quantum well by the surface states.

Fig. 3.2 shows, when the doping layers are on both sides of the wells, electrons feel no net external field and the electron probability density will be symmetric in the well. When the remotely doped layer is on one side of the well as in Fig. 3.2(b), the electrons experience a non zero net electric field along the growth direction which moves the electron probability density toward the edge of the well (toward the positive ions).

An asymmetric QW can be constructed with different Al concentrations in the barriers on either side of the well or by applying a gate voltage. Using a gated sample has the advantage of controlling the density and the asymmetric profile of the well. In 2D heterostructures, an asymmetry of the confining potential lifts the spin degeneracy even in the absence of an external magnetic field. This effect has been proposed as the basis for a type of transistor by Datta and Das in 1990 [4]. In this work far-infrared electron spin resonance (ESR) has been used to probe this zero-field spin splitting in asymmetric InSb QWs this will be discussed in Chapter 5.

3.2 Sample preparation

3.2.1 Ohmic contacts

In order to measure the electron density and study photo-conductivity effects such as ESR, ohmic contacts were made on the the samples. These are created by annealing four small In contacts on the corners of a sample typically 5 mm square. We use only clean pieces of In which are cut by a clean razor blade. A clean soldering iron which has been used only for In, can be used to put a small amount of In on the four corners of the samples.

The annealing temperature is in the 200–230° C range in a mixture of H₂(20%)-N₂(80%) (to avoid oxidation) for about 5 minutes. Watching the temperature during the anealing procedure is necessary because excessive heat will damage the samples. We remove the sample from annealing chamber,when the temperature drops at least to 60° C and then wire up the sample using 99.99% Au wires.

3.2.2 Wegding the samples

For the optical measurements, in order to decrease Fabry-Perot interference the back of the samples should be wedged. A South Bay Technology inc. (model 155) hand lapping fixture was used with a mounting block with $\approx 2^\circ$ angle. The mounting block is held firmly against a precision center slide. A threaded knob with graduations of 500 microns per revolution can position the sample relative to the bottom of the lapping fixture and controls the amount of material which should be removed.

We mount the sample on a small piece of clean glass using melted white wax after cleaning by Methanol. A small amount of Silicon Carbide powder (600Grit/14.5 Micron) mixed with water on a piece of flat glass can be used to wedge the back

of the samples. Smooth motion of the hand lapping fixture on the water-Silicon Carbide mixture will wedge the sample in a few minutes.

In order to remove the sample, the mounting block can be heated (not above 100° C) and the sample can be cleaned by boiling in Trichloroethylene for 2 minutes and in Acetone and Methanol for 2-5 minutes. We use filtered N_2 to dry the sample's surface.

3.2.3 Van der Pauw technique

The most common geometries for electronic transport measurements are the Hall and the Van der Pauw [27]. The Van der Pauw geometry is more convenient for routine measurements. In this technique the contacts are symmetric on the four corners of a square sample as in Fig. 3.3, and the resistivity of the material is given by:

$$\frac{\rho}{d} = \frac{\pi}{\ln 2} \frac{(R_{AB,CD} + R_{BC,DA})}{2} f \left(\frac{R_{AB,CD}}{R_{BC,DA}} \right) \quad (3.1)$$

Where d is the conducting thickness of the sample and $R_{AB,CD} = \frac{V_D - V_C}{I_{A-B}}$ with similar expression for $R_{BC,DA}$. The current I is typically in the order of 1 μA . The function f in this model is almost equal to 1 if the sample is a homogeneous square with $R_{AB,CD} = R_{BC,DA}$. The density can be obtained from the slope of the Hall resistance vs. the magnetic field, and the mobility is given by:

$$\mu_H = \frac{1}{\rho n}$$

where $n = N/d$ is the 2-D density. The samples are not usually homogeneous and the resistivity is not identical in different directions of the samples which implies $f \neq 1$. The density measured by this method varies in different pieces of a wafer by at least 10% .

Contact resistance of the InSb QW samples is typically a few 100 Ω at room temperature. It increases by 20-30% at 77K, and by not more than 50% at 4.2K. A two point measurement can check the ohmic behavior of the contacts. This is done by applying current between two contacts and measuring the voltage between the same two contacts. A non-linear IV suggests contact problems which can be caused by unclean annealing environment or dirty tools. Sometimes if the *In* contacts do not diffuse deep enough into the conductive region (*i.e.* the quantum wells) a large contact resistance and a non-linear IV behavior can be observed. The ohmic contacts can be checked through observation of current-voltage characteristics in four-point measurements as well.

3.3 Experimental setup

The experimental setup of the magneto-optical experiments of this work consists of an Edinburgh CO_2 pumped FIR laser model 100 (see Appendix A for the laser operation), which provides FIR radiation with wavelengths in the 40-1200 μm range. Fig. 3.4 shows a schematic of the experimental setup in which the FIR laser beam can be directed inside a vacuum-can/light-pipe assembly to the sample in the light pipe. The assembly is located at the center of a super-conducting magnet.

The measurements are at liquid Helium temperature (4.2 K) and a Ge:Ga doped photoconductor or a bolometer detector can be used to collect the transmission signals from the samples. We monitor a small portion of the FIR laser beam as a reference signal (using a polyethylene beam splitter) by a room temperature pyroelectric detector. The size of the reference signal depends on the laser-line intensity and the wavelength.

3.3.1 Superconducting magnet

A Cryomagnetics Inc. superconducting (NbTi) magnet which is located inside a cryogenic dewar provide the 7.7 Tesla of magnetic field. The maximum field can be increased by pumping on the liquid helium. The magnet power supply is connected to a programmer which controls the maximum field, the sweep direction, and the sweep rate. In order to measure the field, voltage across a shunt (on the back of the power supply) is measured and converted to strength of the magnetic field (field to current ratio 969.3 Gauss/Ampere). The following considerations are very important to operate the magnet properly:

- The power supply should be turned off only when the magnetic field is zero.
- The direction of the magnetic field can be changed using the switch box located in the far infrared lab only when the power supply is off.
- The sweep rate should not exceed 0.1 T per second otherwise quenching of the magnet can occur. The rate can be changed using a sensitive potentiometer which allows a sweep rate as low as 0.01 Amp. per second.
- At the low sweep rate (less than 0.05 A/sec) the programmer does not change the sweep direction. It is important first to set the sweep rate at a larger value (not greater than 0.1 A/sec) and then adjust it to a lower rate (sometimes the programmer should be switched to the pause mode temporarily).
- The operation of the magnet especially at high field requires sufficient amount of liquid Helium, otherwise quenching of the magnet can occur.
- The magnet leads are vapor cooled which the holes on the top of the leads should not be covered during the normal operation.
- We dry the magnet before cooling down using nitrogen gas.
- The magnet leads carry high current and good electrical isolation is required.

3.3.2 Cool down the system

We pre-cool the dewar and the magnet using liquid nitrogen the night before the experiment after pumping on the cryogenic dewar at least up to 10^{-5} Torr. In our system, pre-cooling requires at least 30 liters of liquid Nitrogen which should be transferred at a slow rate.

The magnet resistance is about $600\ \Omega$ at room temperature and $70\text{--}77\Omega$ at 77K. A few hours before transferring liquid helium the liquid nitrogen should be removed completely by pressurizing the dewar with Helium gas (The pressure should not exceed 2 PSI). We monitor the resistance of the magnet and a $100\ \Omega$ Carbon resistor ($\approx 130\Omega$ at 77K and $1.3K\Omega$ at 4K) which is located at the bottom of the dewar or at the end of a long tube inside the dewar. The Magnet and the Carbon resistor resistances should be $80\text{--}90\ \Omega$ and $110\text{--}120\ \Omega$ respectively after removing the liquid nitrogen.

The sample holder requires pre-cooling as well. We first pump on the vacuum-can/light-pipe assembly with a diffusion pump at least down to 10^{-4} Torr, and then add a small amount of exchange gas (helium) and finally put the vacuum can inside a nitrogen dewar for at least minutes before inserting it in the magnet dewar. For a long term experiment, especially at high magnetic field, we transfer at least 20 cm on the Helium sensor gauge. Driving the magnet at high magnetic field with less than 5 cm helium level is not recommended.

In our system, initial transfer usually requires 30-35 liters of liquid helium in order to transfer 20 cm (measurement is from the bottom of the magnet) helium and after that 15-20 liters are sufficient to reach to the same level. The liquid helium transfer should be performed at a slow rate and might takes at least 1 hour for the initial transfer and 15-20 minutes after that.

3.3.3 Warm up

To warm up the magnet faster to room temperature, nitrogen gas can be used after all the liquid helium is boiled off and the magnet resistance increases to a value above 100Ω .

Our vacuum-can can be warmed up using a heat gun, it is important to avoid hot air around the Indium seal between the detector and the vacuum can!! After warming up the can for a few minutes, we pump out the helium gas and then, in order to make the warm up cycle faster, add a small amount of nitrogen gas (only when the temperature inside the vacuum can is above 77 K). After adding nitrogen gas it is recommended to wait at least 30 min before taking the light pipe out of the vacuum-can to avoid any water condensation.

3.3.4 The detector

A *Ge : Ga* Infrared Laboratory (serial no. 2754) photo-conductor detector was used in the magneto-optical experiment of this work. The detector operating temperature is in the range 1.5-4.2K with the spectral range of $40\ \mu\text{m}$ - $120\ \mu\text{m}$. The detector unit consists of a Winston cone and a cold amplifier which is placed close to the detector. The detector unit is sealed to the vacuum can using Indium wire. Before making the Indium seal, the surface of the flange on the vacuum can and the flange on the detector should be cleaned using Methanol. We use a small amount of vacuum grease on the Indium wire before putting the wire in the Indium groove. The eight 4-40 screws which should be tightened in a star and symmetric pattern make the seal between the detector unit and the vacuum can. The assembly should be checked for any leak using a leak detector.

The pre-amplifier control box (LN6) is supplied by two internal 9 volts batteries

and a 15 volt battery for the bias. The life time of the batteries is usually 100 hours. DC bias during normal operation is adjusted by turning the bias adjust 10-turn potentiometer when the bias switch is on. The detector's wire are Constantan (30 or smaller) which are connected to a 6 pin connector on the vacuum can. See Appendix (D) for the detector's wire codes.

3.4 Transport measurements

In order to measure density, mobility, and study photo-conductivity effects such as ESR, ohmic contacts were made on the samples. The ordinary low field Hall effect can give basic information about the sample characteristics but more precise information can be achieved in the quantum Hall regime. A lock-in amplifier can be used to generate a small AC current (about $1\mu\text{A}$ in our case) with a small oscillation frequency i.e. 17 Hz (to reduce the noise in the signal) by putting a large resistor (i.e. a $2\text{M}\Omega$ plus a $1\text{K}\Omega$ resistor) in series with the sample.

In the Van der Pauw geometry, as shown in Fig. 3.3, applying current in the AC direction and measuring the voltage between BD will result in transverse resistivity (ρ_{xy}). In a similar fashion driving current in AB direction (or any other longitudinal direction) and measuring the voltage between the opposite side (CD) the longitudinal resistivity (ρ_{xx}) can be determined. both (ρ_{xy}) and (ρ_{xx}) have oscillatory behavior in high magnetic field which leads to many important results (See the Chapter 2). Fig. 3.5 shows an example of the resistivity measurements in the quantum Hall regime for an InSb QW. Using the concepts described in this chapter and Chapter 2, the density and the mobility of our sample can be measured.

3.5 Magneto-transmission experiments

The magneto-transmission experiments in this work were performed by using the FIR laser in the presence of magnetic field under Faraday or tilted configurations as in Fig. 3.6. In the Faraday configuration the direction of the laser beam is parallel to the direction of the applied magnetic field and perpendicular to the surface of the sample. In the tilted configuration the sample normal is tilted with respect to the common direction of laser beam and magnetic field.

The magnetic field is provided by a superconducting magnet, and the experiments were carried out at 4.2K. According to the profile of the magnet, the field in z direction is uniform in ± 0.5 cm from the center of the field. It is important that the position of the sample stays in this range.

In order to utilize lock-in amplification, the laser beam is chopped by a mechanical chopper. The best response frequency for the room temperature pyroelectric detector (provided by the Edinburgh company) is about 150 Hz which we used for the Ge:Ga photo-conductor detector in cyclotron resonance measurement as well.

As was mentioned earlier a small, portion of the chopped laser beam using a polyethylene beam splitter can be monitored by the pyroelectric detector and the rest can be directed inside the Brass light pipe (see Fig. 3.7). If no polarization of the beam is required, the sample would be the only object between the laser beam and the detector (the light pipe and the Ge:Ga doped detector are sealed by a wedged polyethylene window)

For a fixed laser line while sweeping the magnetic field, the sample's transmission is monitored by the detector. An amplifier next to the detector amplifies the signal and the output signal is fed into a lock-in amplifier and then recorded by a Lab View computer program.

We used a linear polarizer (Graseby-Specac) tilted 45° respect to the optical axis of x-cut $\lambda/4$ Quartz wave plate (supplier Sawyer crystal system) in order to make circularly polarized light [28]. The surface of the Quartz can be polished and coated with NiCr anti-reflection layer[29].

In order to eliminate the high scattered light which might cause multiple reflection, the sample, the quartz, and the linear polarizer holders can be made out of Eccosorb (supplier Emerson-Cuming) which is a castable resin and absorbs FIR radiation (see appendix B for more details). It is also very important to eliminate any stray light which might leak around the sample and reach the detector. We observed CR in both magneto-transmission and photo-conductivity measurements over a wide range of wave-lengths $119\mu\text{m}$ - $46\mu\text{m}$ in a wide range of magnetic field 1.5-6.5 Tesla.

3.6 Photo-conductivity measurements

Both CR and ESR can be observed in photo-conductivity (double modulation) measurements via the change of the longitudinal resistance ($\Delta\rho_{xx}$) due to absorption of the laser light. In this method, the sample itself works as a detector and the signal to noise ratio is better compared to the transmission experiments. Fig. 3.8 shows a simplified diagram of the photo-conductivity measurements.

In this method an AC current about $1\mu\text{A}$ ($f_1 \approx 130\text{ Hz}$) is applied to the sample and at the same time the laser light which is chopped at a small frequency ($f_2 \approx 13\text{ Hz}$ to provide enough response time and at the same time not a multiple of 60 Hz) is directed onto the sample. In this method the value of ρ_{xx} is changed by the laser radiation by a small amount $\Delta\rho_{xx}$. Two lock-in amplifiers are required to measure the change in ρ_{xx} . The first lock-in is locked to the AC current frequency

with small time constant ($1-10 \text{ ms} \ll 1/f_2$) and measures a signal proportional to ρ_{xx} this contains an oscillatory component with the frequency f_2 proportional to $\Delta\rho_{xx}$ which is then measured by the second lock-in set to f_2 with a time constant 3-10 sec to reduce the noise. $\Delta\rho_{xx}$ reflects the periodicity of the Shubnikov-de Hass oscillations and has two components. One can be attributed to [30] a non-resonant heating of the sample ($\frac{dR}{dT}$ which T is the temperature) and the other to a resonant structure such as ESR or CR to be discussed in the Chapters 4 and 5.

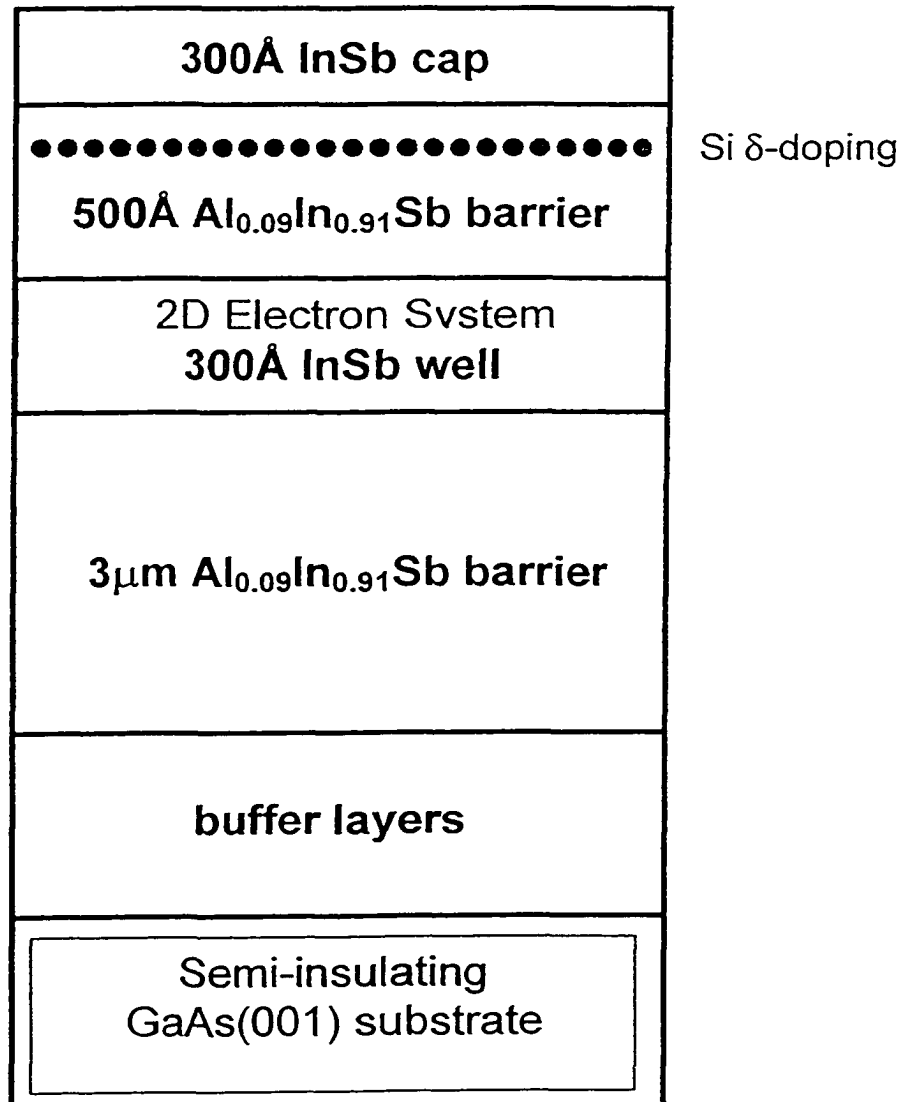


Figure 3.1: A Typical layer sequence for the remotely doped quantum well structure. In this case, the delta doped layer is on one side of the well resulting in an asymmetry well.

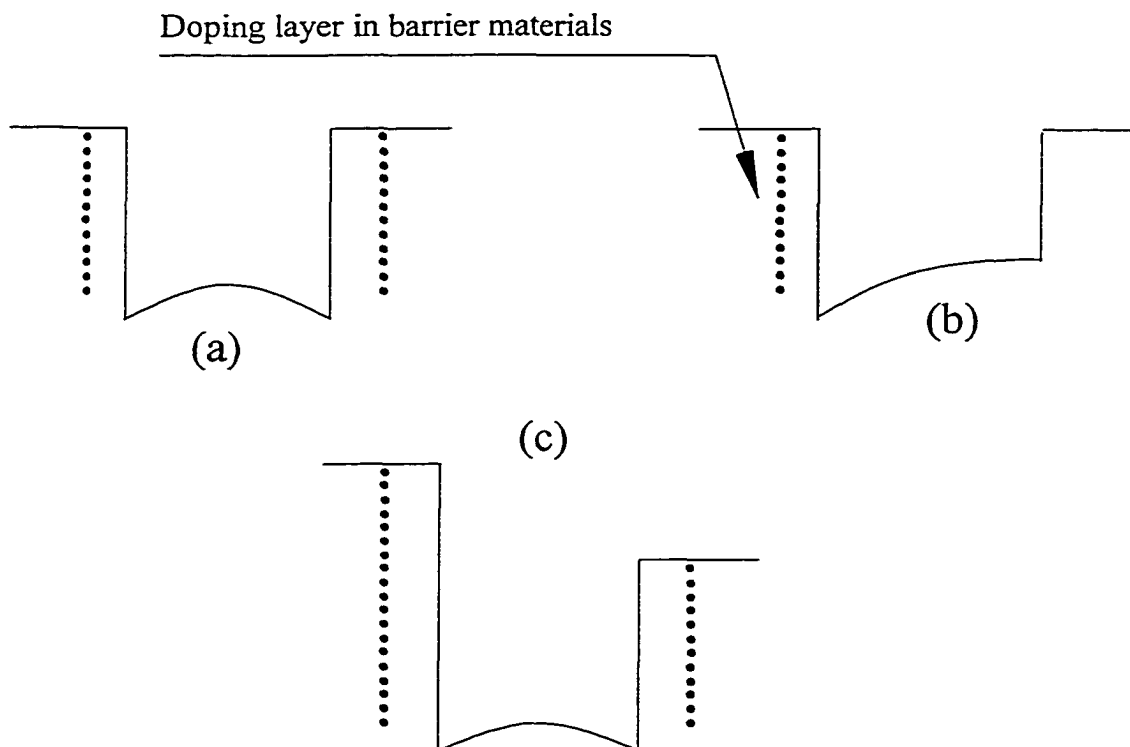


Figure 3.2: a) Doping on both sides of the well resulting in a symmetric quantum well. b) An example of an asymmetric quantum well in which the doping is on one side of the well. c) Asymmetry in quantum well achieved by difference in Al concentration in the barriers.

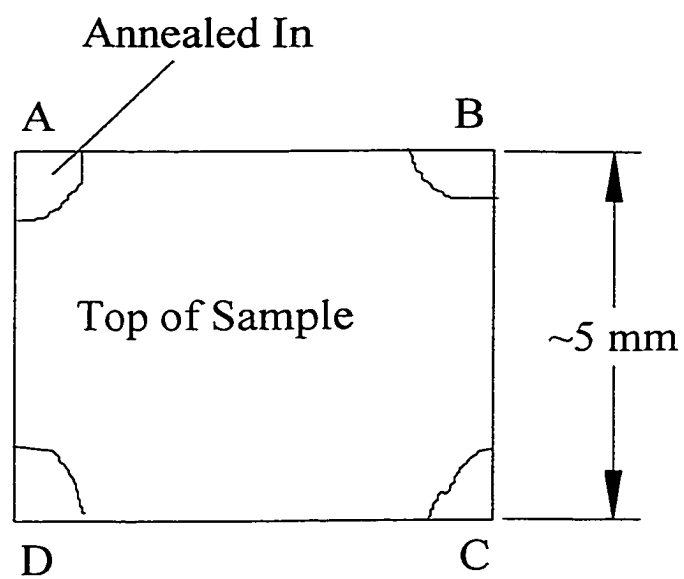


Figure 3.3: The Van der Pauw geometry for the electronic transport measurements.

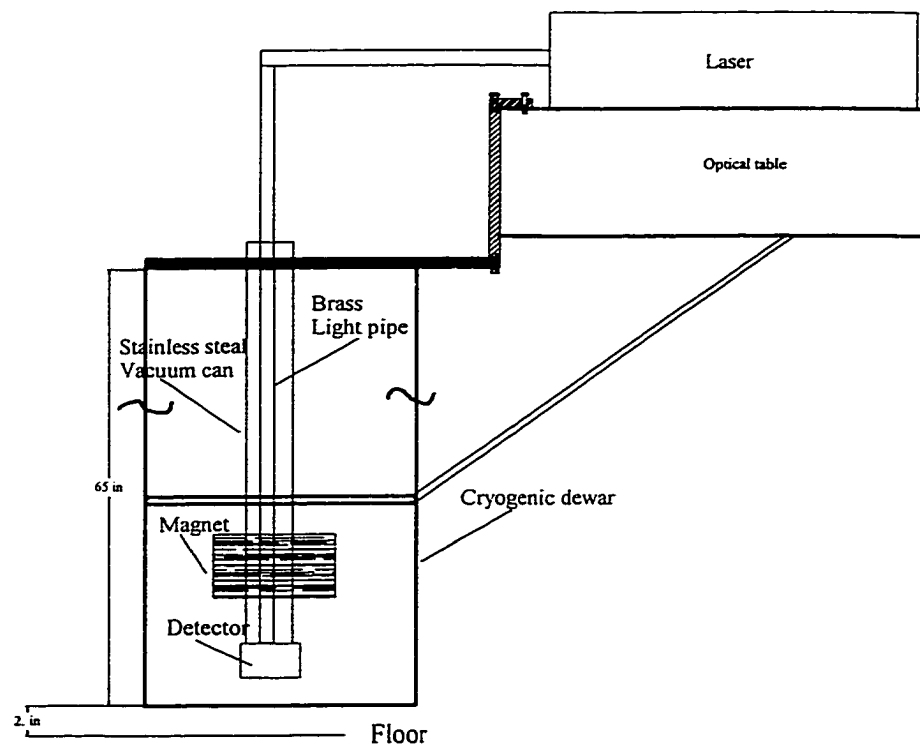


Figure 3.4: Schematic of the experimental setup. The far infrared laser beam can be directed inside a vacuum-can/light-pipe assembly for magneto-optical measurements using a superconducting magnet.

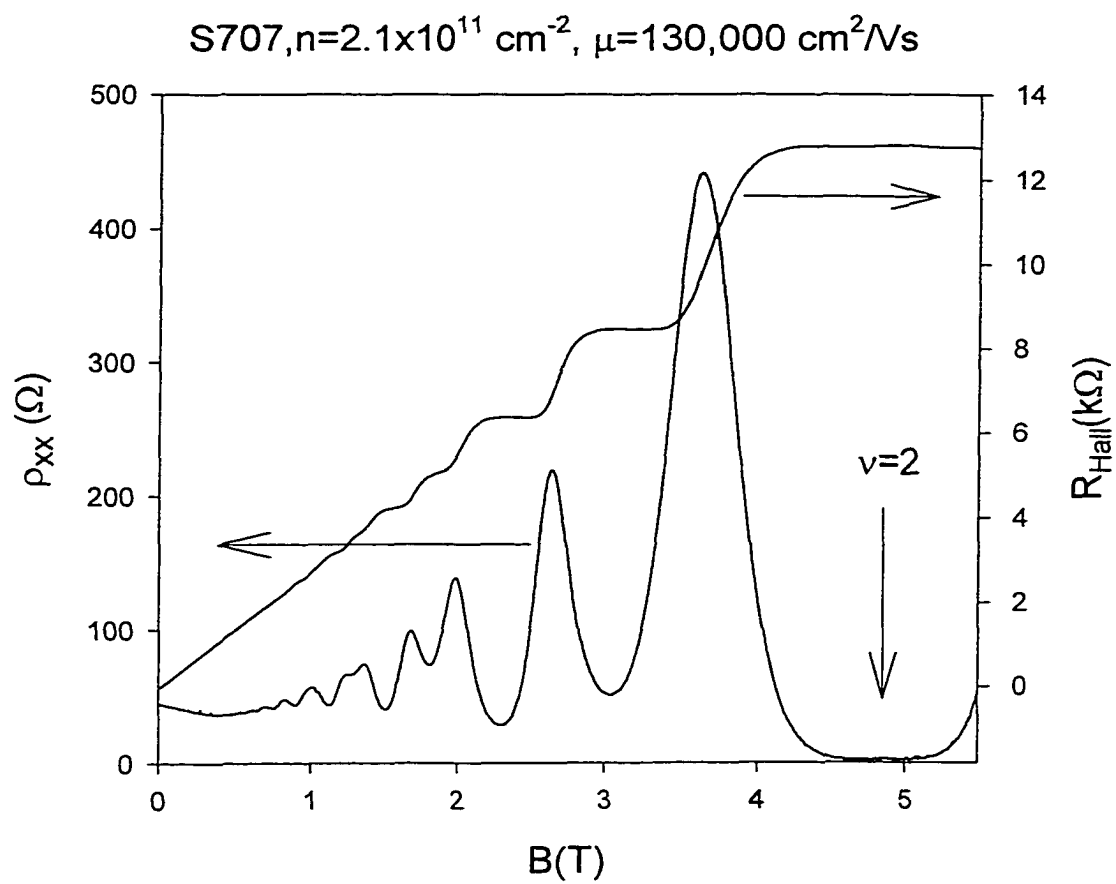


Figure 3.5: Plot of the Hall resistance and the longitudinal resistivity as a function of B at 4.2K demonstrating the integer quantum Hall effect in an InSb single QW.

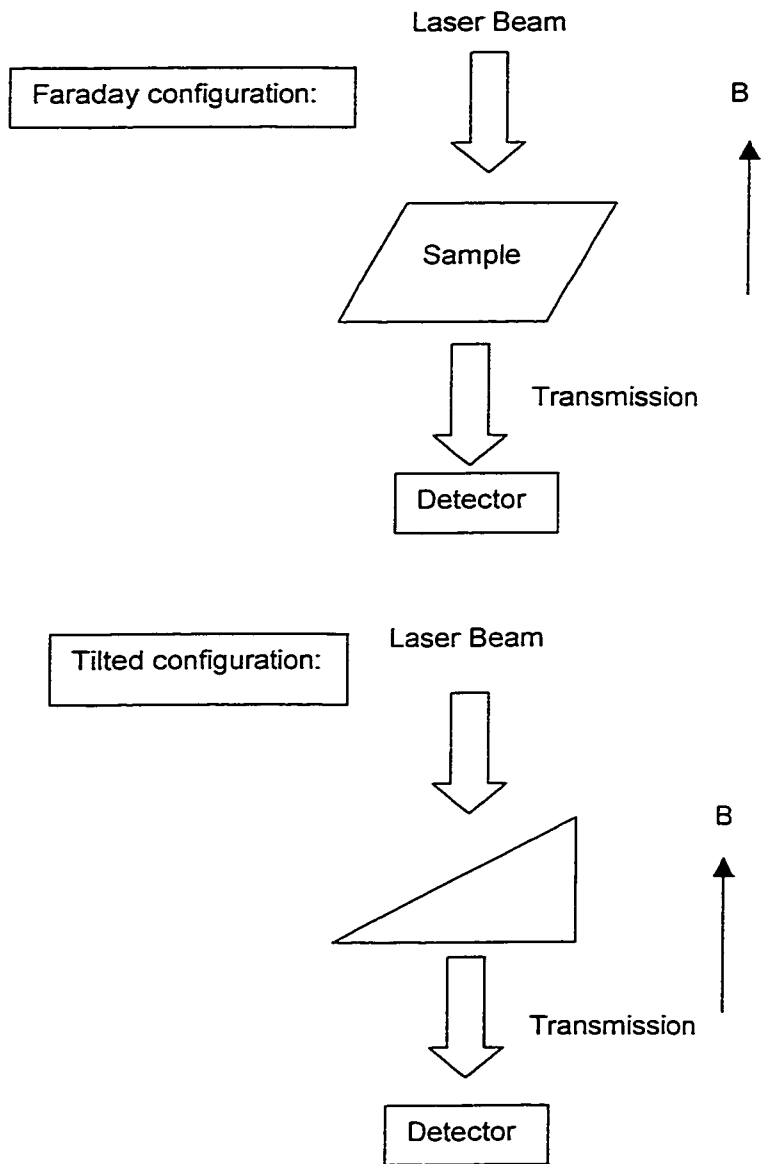


Figure 3.6: Simple schematic of the Faraday and the tilted configurations in the magneto-transmission experiments.

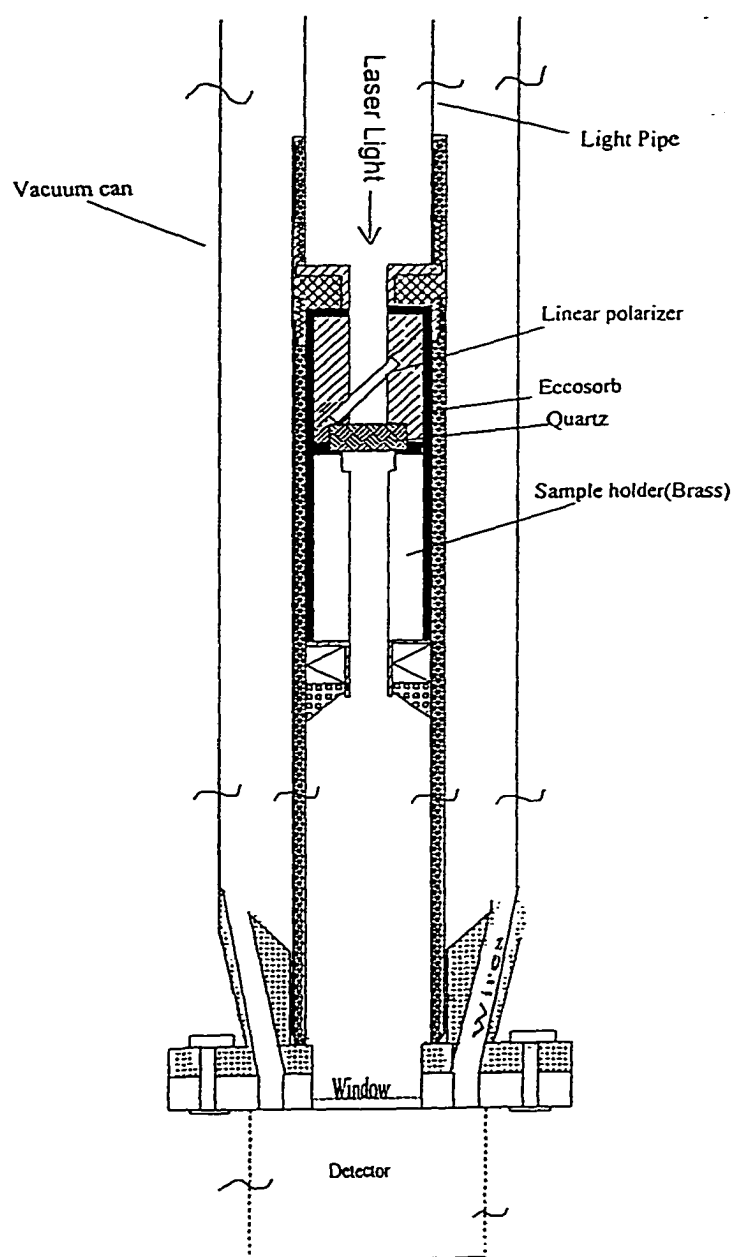


Figure 3.7: Part of vacuum-can/light-pipe assembly. The light pipe is made out of Brass with OD of 0.625 in. The vacuum can is stainless steal with OD of 1.5 in. In order to eliminate the light multi-reflection the sample holder can be surrounded by Eccosorb.

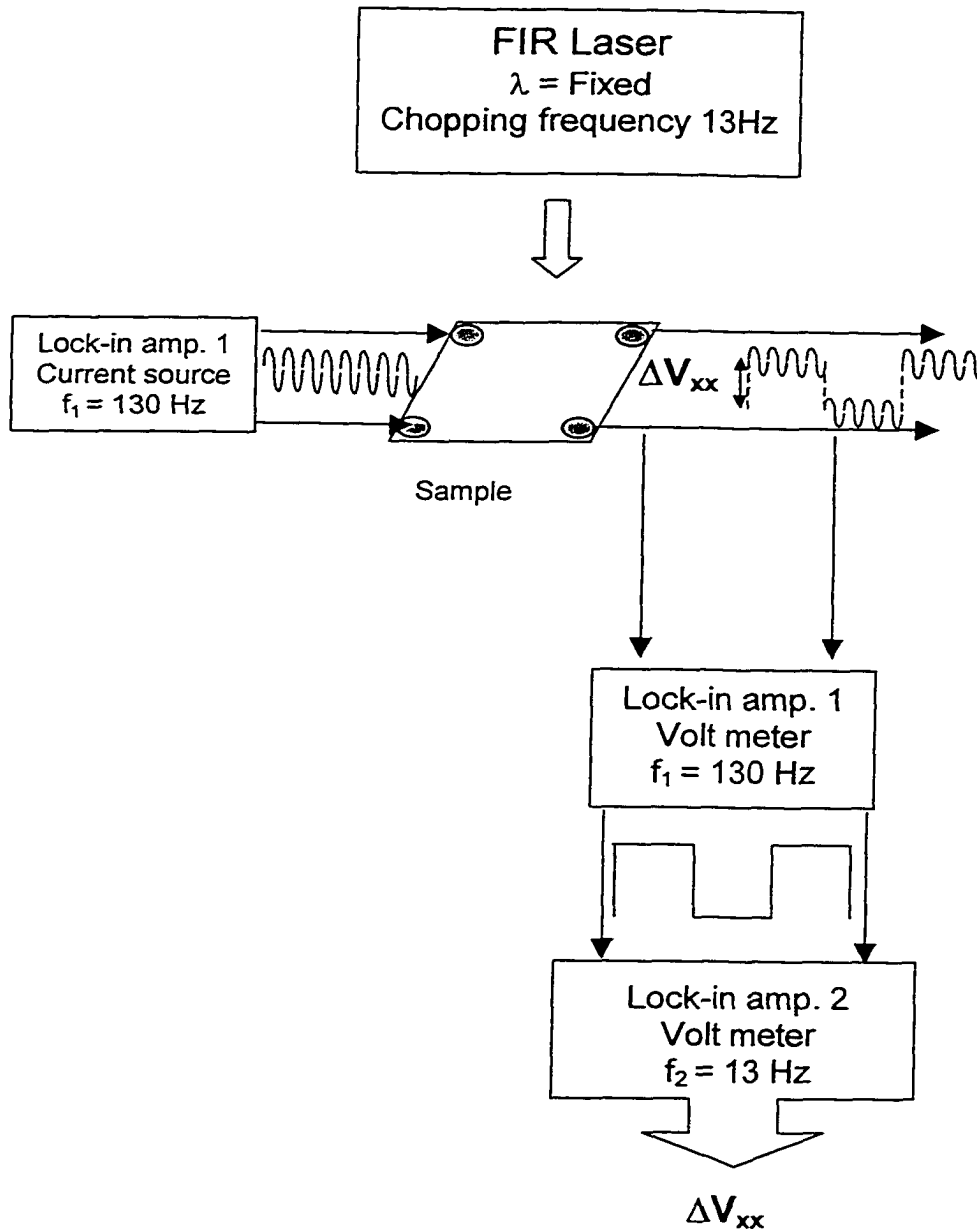


Figure 3.8: A diagram which describes photo-conductivity experiment. ΔV_{xx} is the change of V_{xx} by the laser beam.

Chapter 4

Cyclotron Resonance

The band structure of InSb, one the best examples of a narrow-gap semiconductor, has been studied extensively experimentally and theoretically. It was first recognized by Kane [2] theoretically that as a result of the small band gap, the conduction and the light hole band structures of this material have a non-parabolic energy wave-vector relation. InSb has the smallest effective mass ($0.014m_0$), the highest intrinsic electron mobility at room temperature, the largest g factor (~ 51), and the most non-parabolic band structure of all III-V semiconductor compounds.

Our experiments were focused on InSb/ $\text{Al}_x\text{In}_{1-x}\text{Sb}$ MBE-grown quantum wells (QW) samples with symmetric and asymmetric confinement potential with only one sub-band occupied. We measure CR in both magneto-transmission and photo-conductivity measurements and observed for the first time in InSb QWs the spin-split resolved cyclotron resonance. Our observed data can be fitted by a single electron picture predicted by the Kohn theorem [31] along with the modified Pidgeon and Brown model for the effective g-factor [32].

According to Kohn's theorem the external electric field couples only with the center-of-mass motion of electrons and is unaffected by internal forces, which is equivalent having a single harmonic oscillator with cyclotron frequency ω_c . In

an inhomogeneous or non-parabolic system, the Kohn theorem breaks down. We observed some unexpected features in the spin-split resolved cyclotron resonance which could be a result of deviation from the Kohn theorem.

In this chapter, the CR selection rules, the energy spectrum in the external magnetic field, and the CR experimental results will be discussed.

4.1 Selection rules

In Chapter 2, the Hamiltonian and the energy spectrum of a two dimensional electron system (2DES) in the presence of a perpendicular magnetic field was introduced. Now if we add the interaction with the electromagnetic (EM) radiation the total electronic Hamiltonian can be written as [33]:

$$H_{tot} = \frac{1}{2m_0} (\mathbf{p} + e\mathbf{A} + e\mathbf{A}')^2 \quad (4.1)$$

where $\mathbf{A}$ is the magnetic vector potential and $\mathbf{A}'$ is the radiation field potential (classical E&M approximation). Neglecting terms of order A'^2 for the EM radiation interaction with the charged particles we have:

$$H_R = e\mathbf{v} \cdot \boldsymbol{\xi} A' \quad (4.2)$$

$$\mathbf{v} = \mathbf{P}/m_0 = (\mathbf{p} + e\mathbf{A})/m_0 \quad (4.3)$$

$$\boldsymbol{\xi} \cdot \mathbf{P} = \xi_+ P_- + \xi_- P_+ + \xi_z P_z \quad (4.4)$$

with $\boldsymbol{\xi}$ as the directional unit vector (polarization) where $\xi_{\pm} = \frac{(\xi_x \pm i\xi_y)}{\sqrt{2}}$.

Using the property of the ladder operators, we can write:

$$\xi_- P_+ |n, m_l, K_z, m_s\rangle \propto |n+1, m_{l+1}, K_z, m_s\rangle$$

The only non-vanishing matrix element $\langle n+1 | H_R | n \rangle$ for a left circular polarized light (ξ_-) has

$$\Delta n = +1, \Delta m_l = +1, \Delta m_s = 0$$

and similarly for a right circular polarized light (ξ_+)

$$\Delta n = -1, \Delta m_l = -1, \Delta m_s = 0$$

Therefore; cyclotron resonance active (CRA) transitions are spin conserved transitions with $\Delta n = +1$ for a left circularly polarized light (ξ_-).

The interaction of the EM radiation with the spin angular momentum can be included by adding the following term to the Hamiltonian above:

$$H_s = \frac{e}{m_0} \mathbf{B} \cdot \mathbf{s} = \frac{eB}{m_0} \eta \cdot \mathbf{s} \quad (4.5)$$

where η is a unit vector along $\mathbf{B}$. Using an argument similar to that for the CR selection rules, the spin resonance selection rules for materials like InSb with a negative g-factor for CR inactive mode (CRI) (right circularly polarized light) are:

$$\Delta n = 0, \Delta m_l = 0, \Delta m_s = -1$$

The selection rules break down in non-parabolic systems [33].

4.2 Magnetic field energy dispersion

In the presence of an external magnetic field perpendicular to the plane of the two dimensional electron system, the motion of electrons in the x-y plane determines the transverse part of the energy without the field :

$$E = \frac{\hbar^2 K_t^2}{2m^*}$$

In a magnetic field, this energy is quantized into Landau levels and is no longer free electron like. By adding the electron spins to the system, the total energy will be:

$$E = E_z + (n + 1/2)\hbar\omega_c \pm (1/2)g^*\mu_B \quad (4.6)$$

with E_z corresponds to the subband energies due to the potential along the growth axis.

In the calculations discussed in Chapter 2, the value of $\frac{\hbar^2 K_z^2}{2m^*}$ can be replaced by $(n + 1/2)\hbar\omega_c$ and a set of sub-band energies for a given Landau level index at different magnetic fields can be calculated numerically. In InSb QWs, the energy dependent g-factor can be described by a Pidgeon-Brown [34, 32] model (modified) to include the non-parabolicity.

The Pidgeon-Brown model is a modified version of the Luttinger and Kohn model for the magnetic energy levels of the conduction and valence bands in 3 dimensional systems. The coupling between the conduction and the valence band, the effect of higher bands, the non-parabolicity, and the warping of the conduction and heavy-hole bands are included. The details of this method can be found in Ref. [34]

The result of this method has been summarized in two sets of 4×4 Hamiltonians for spin-up and spin-down. There will be 4 eigen-values for each spin orientation corresponding to the conduction band (CB), heavy hole (HH), light hole(LH) and spin-orbit split(SO) bands for a given value of magnetic field and Landau level index. The difference between the spin-up and spin-down Landau levels can be used to determine the energy dependent g-factor :

$$\Delta E = g^*\mu_B B$$

The parameters required in this model are standard band structure parameters

which can be found in Landolt-Bornstein [35]. In order to use this method for a 2 dimensional system, the ground state sub-band energies in the CB and VB can be added to the energy gap. The only fitting parameter in this model is the momentum matrix element which in this case depends on the effective band gap [32]. In our strained InSb/Al_xIn_{1-x}Sb QW system the effective band gap also changes somewhat with the Al concentration [8]. Fig. 4.1 shows the calculated g-factor for different Landau levels for a QW with 300Å width.

At B=0, there is a reduction in g-factor from -51 (the value for the bulk InSb) to -48 using the modified Pidgeon-Brown model due in nearly equal parts to the change in the band gap from both the strain effect in our InSb QW and from the sub-band quantization. For our 300Å InSb/Al_xIn_{1-x}Sb QWs with 9% alloy concentration the bandgap changes from 236 meV to $\approx$ 248 meV and the first conduction subband energy is about 15 meV.

4.2.1 Fan diagram

A fan diagram is an energy spectrum versus magnetic field plot for different Landau levels and is given by:

$$E(B, n) = E_z + (n + 1/2)\hbar\omega_c \pm \frac{1}{2}g^*\mu B \quad (4.7)$$

The non-parabolicity through an energy dependent mass is included in the ω_c term, and in the g^* -factor using the Pidgeon-Brown model. This energy spectrum is a single electron approximation based on the Kohn theorem. Fig. 4.2 shows an example of a fan diagram for sample S203 which is a 300Å InSb QW with density $n = 1.0 \times 10^{11} \text{ cm}^{-2}$.

The Born approximation [36, 12, 13](see appendix E) can be used to include the broadening of the Landau levels in the calculation of the fan diagram. In this

approximation the density of states can be written as:

$$DOS(E, E_{i,n,\sigma}) = N_l(1 - (\frac{E - E_{i,n,\sigma}}{\Gamma})^2)\Theta(\Gamma - |E - E_{i,n,\sigma}|) \quad (4.8)$$

$$E_{i,n,\sigma} = E_{i,\sigma} + \hbar\omega_c(n + 1/2) \quad (4.9)$$

where $N_l = \frac{3}{8\pi\Gamma l_B^2}$, Γ is a width parameter arising from scattering (≈ 1 meV for our system and in this approximation independent of magnetic field), i the sub-band index, σ spin index, n the Landau level index, and $\Theta(\Gamma - |E - E_{i,n,\sigma}|)$ is the step function. The density of states versus energy in the Born approximation is plotted in Fig. 4.3.

From the density of states, the two dimensional electron density can be calculated which will be used in the self consistent calculations discussed in chapter 2. Fig. 4.4 shows an example of the fan diagram in the frame work of the Born approximation. Both the fan diagram using just a single electron model(see Fig. 4.2) and the fan diagram using the Born approximation can explain our cyclotron resonance transition positions.

4.2.2 CR with circularly polarized light

We used a linear polarizer tilted 45° with respect to the optical axis of a x-cut quartz plate located very close to the sample [28] (see Chapter 3 for the details) to produce circularly polarized light. We measured transmission by sweeping the field with two different polarities of the magnet, reverse (B pointed in the -z direction) and forward (B pointed in the +z direction). We observed CR only in the reverse polarity at $119 \mu\text{m}$ which suggests a perfect circular polarization at the wave-length for the corresponding quartz thickness. The thickness of the quartz plates (d) for different wavelengths (λ) can be defined using the following relation:

$$d = \frac{\lambda}{4\Delta n}$$

where Δn is the difference between the ordinary and extra-ordinary index of refraction in quartz for which we used the experimental data of Ref [28]. Fig. 4.5 shows the CR trace of sample S285 in the circularly polarized configuration.

This result suggests that the electron cyclotron resonance can be observed in the reverse and the hole cyclotron resonance in the forward polarity of our magnet if the light is circularly polarized. This effect is important to distinguish between the CR experiment in an electron system or hole system. For most of our experiments we used unpolarized light.

4.2.3 Cyclotron effective mass

From the Landau level spectrum and the definition of the CR transition, the cyclotron effective mass is given by:

$$m^*(B) = \frac{\hbar e B}{E_{n+1} - E_n} \quad (4.10)$$

where E_{n+1} and E_n are the Landau level energies of $n+1$ and n states respectively with the same spin at field B . As in Fig. 4.2 the CR transitions can occur between a filled or partially filled state and an empty or partially empty state by absorbing the light.

In all of our samples, the predicted resonance positions and the experimental data are in a good agreement. Fig. 4.2 and Fig. 4.10 are the plot of the measured and predicted CR transitions in a wide range of magnetic fields.

For S203 with the density $n = 1 \times 10^{11} \text{cm}^{-2}$, the effective mass at 1.7 T is about $0.0188m_e$ and at 2.95 T is about $0.0194m_e$ which shows an increase in the effective mass with increasing energy (see Fig. 4.6).

Fig. 4.7 are the CR traces for sample S499 at $119\mu\text{m}$ and $96.5\mu\text{m}$ with the density $n = 2.1 \times 10^{11} \text{cm}^{-2}$. As a result of the non-parabolicity, experimentally we

can observe a small shift in the CR resonances positions in S499 compared to S203 due to the different electron densities.

4.2.4 Spin resolved cyclotron resonance

From the fan diagram in the last section one can see the non-parabolicity in the conduction band of InSb. The non-parabolicity makes the energy difference of Landau level (n-1) and (n) larger than that of Landau level (n) and (n+1) around odd-integer filling factor which would result in two different spin conserved CR transitions (Δm^* splitting). Unlike for the InAs system [32, 37] we have not observed Δm^* splitting in the InSb QW using a laser source and we will investigate this splitting using a FTIR system in future.

Because of the energy dependence of the g-factor, the Zeeman splitting of the nth Landau level is larger than that of the (n+1)th Landau level in the vicinity of even filling factors. In this case the two spin-conserved CR transitions between adjacent pairs of Landau levels happen at two different frequencies (Δg^* splitting). See Fig. 4.8 for a simple picture of Δm^* and Δg^* splittings.

Spin resolved CR first was observed by M.J. Yang et al. [32] and J.Scriba et al. [37] in InAs QW with AlSb as the barrier material. InAs is another narrow-gap semiconductor with band edge g-factor ≈ -15 and the band edge effective mass $\approx 0.023m_0$. The first experimental group included the strain and the penetration of the wave function into the barriers as well as non-parabolicity in order to get the best fit to their experimental data. The second group used a simplified Kane model which just includes the non-parabolicity in the band structure and were able to achieve a satisfactory fit to their experimental data. Both groups observed Δm^* and Δg^* splittings with the expected intensity patterns from the non-parabolicity

of band structure.

We observed spin resolved CR only in S203, S344, and S377 doped QWs which all show an unexpected amplitude pattern at $70.6\mu\text{m}$. S203 is an asymmetric $300\text{\AA}$ QW with 7% alloy concentration. The density of the sample is $\approx 1.0 \times 10^{11}\text{cm}^{-2}$ with the mobility of about $100,000\text{ cm}^2/\text{Vs}$. S344 and S377 both are symmetric $300\text{\AA}$ with 9% alloy concentration. S344 has a density $\approx 2.3 \times 10^{11}\text{cm}^{-2}$ and a mobility of about $90,000\text{ cm}^2/\text{Vs}$. S377 is a high mobility sample $\mu \approx 150,000\text{cm}^2/\text{Vs}$ with a density of $\approx 2.5 \times 10^{11}\text{cm}^{-2}$.

Fig. 4.2 and Fig. 4.9 show the fan diagram and the CR trace at $70.6\mu\text{m}$ for S203. The spin resolved CR (Δg^* splitting) is well resolved but with an unexpected amplitude pattern.

From the non-parabolicity picture and the oscillation of the Fermi energy which are plotted in the fan diagram for S203, the intensity of the first resonance at the lower magnetic field should be stronger than the second resonance. The first transition is from a completely filled state and the second one from a partially filled state. The reversed intensity pattern is the same for S344 and S377. The models discussed earlier can predict the positions of the CR transitions in all of our InSb samples over a wide range of magnetic fields, but not the peak intensity.

In an inhomogeneous system in which the translational invariance is broken (e.g. by electron-impurity scattering, band nonparabolicity or electron-phonon interaction) Kohn's theorem is not valid. The deviation from the Kohn's theorem could be responsible for the unexpected intensity pattern at $70.6\mu\text{m}$, but a better understanding of the mechanisms in our InSb QWs requires a detailed study using a FTIR spectrometer and a wide band detector like a Bolometer.

A similar reversed order of strength in the spin resolved CR have been observed

in high density GaAs QW[38]. These observation are explained by adding the electron-phonon interaction in the picture. In our case the LO-phonon frequency is around $40\mu\text{m}$ and the spin resolved CR occurs at $70.6\mu\text{m}$ which suggest the electron-phonon interaction can not be responsible for the reversed feature in the observed resonances intensities in InSb QWs.

4.2.5 CR in the tilted experiment

In order to observe electron spin resonance (ESR) in our InSb QW, we tilted the sample with respect to the direction of the applied external field and the incident laser radiation (see chapter 5 for the details on ESR). At the same time, we examined the CR signals in a tilted configuration in which the Landau quantization depends only on the magnetic field normal to the 2DES ($B\cos(\theta)$), with θ the tilt angle but the electron spins respond to the total B-field. From our experimental data the CR peak positions in the tilted configuration are scaled by the factor ($B\cos(\theta)$), and the energy spectrum can be written as:

$$E(B, n) = E_z + (n + 1/2)\hbar \frac{eB\cos\theta}{m^*} \pm \frac{1}{2}g^*\mu B$$

A summary of the experimental data of S209 in the tilted configuration in a wide range of magnetic fields can be found in Fig. 4.10.

4.2.6 Density and mobility

Cyclotron resonance traces can be used to determine the density and the mobility of the samples in the frame work of the Drude model. Using a matrix method described in Ref.[39] and Ref.[40], the complex index of refraction and the transmittance can be calculated and are given as:

$$t_{\pm} = \frac{2}{[(1 + n_0)\cos\Theta_{\pm} + i(n_{\pm} + n_0/n_{\pm})\sin\Theta_{\pm}]} \quad (4.11)$$

$$T(B) = \frac{n_0(|t_+|^2 + |t_-|^2)}{2} \quad (4.12)$$

$$n_{\pm} = [\epsilon - i\sigma_{\pm}(B)/\omega\epsilon_0]^{1/2} \quad (4.13)$$

$$\sigma_{\pm} = \frac{n_s e^2}{zm_{CR}^*} \frac{1}{1/\tau_{CR} + i(\omega \pm eB/m_{CR}^*)} \quad (4.14)$$

The $\pm$ sign refers to the right and left circularly polarized light, ϵ is the dc dielectric constant, $n_{\pm}$ is the complex index of refraction, $\Theta_{\pm} = (\omega/c)n_{\pm}z$, z is the thickness of the 2DES, $n_0 = \sqrt{\epsilon} = 4.2$ for InSb, and $\sigma(B)$ is Drude conductivity. The density n_s can be extracted from the transport measurements or fitting to the CR curves, the CR effective mass m_{CR}^* can be defined from the position of the cyclotron resonance using:

$$\omega_c = eB/m^*$$

The last fitting parameter in this model is the CR lifetime, τ_{CR} , which is related to the mobility as following:

$$\mu_{CR} = e\tau_{CR}/m^* \quad (4.15)$$

The value of τ_{CR} is usually in the order of few pico-seconds in our InSb QWs. Fig. 4.7 shows the fittings to CR traces of sample S499 using the discussed model. The half width half maximum (HWHM) of the CR line is inversely proportional to the mobility. For the samples with spin resolved CR two separated fitting will be added to fit the experimental data. For example in Fig. 4.9, the fitting on the first and the second cyclotron resonance traces result in $n_1 = 0.34$, $\tau_{CR} = 1.64ps$ and $n_2 = 0.63$, $\tau_{CR} = 1.9ps$ respectively.

4.2.7 Landau level broadening

Cyclotron resonance can be used to study scattering phenomena by measuring the scattering life time τ_{CR} from the line width of CR (Γ_{CR})[41]:

$$\tau_{CR} = \hbar/\Gamma_{CR}$$

The CR line width reflects the Landau level broadening which can be caused by different scattering processes such as electron scattering by impurities, acoustic phonons, and electrons themselves.

τ_{CR} must be long enough to observe CR, the electrons should be able to travel at least $1/2\pi$ of a revolution before a scattering event.

$$\tau_{CR} > \frac{1}{\omega_c} \Rightarrow \omega_c \tau_{CR} = \mu B > 1$$

where ω_c is the cyclotron resonance frequency and μ is the mobility of the electrons. Sharp resonances correspond to high mobility samples and longer CR life time. In our InSb QWs the CR life times are on the order of few pico-seconds.

4.2.8 Photo-conductivity

We observe CR in $InSb/Al_xIn_{1-x}Sb$ in photo-conductivity measurements as well as magneto-transmission measurements. The observation of CR in photo-conductivity measurements depends on the position of the resonance in the $\Delta\rho_{xx}$ trace [42, 43]. In photo-conductivity measurements, two effects are possible. One is the heating of the sample at the resonance position and the second mechanism is the resistance change due to the redistribution of carriers between Landau levels. When the Fermi level lies between one Landau level that is completely filled and the next higher one which is completely empty then $\rho_{xx} = 0$. In the presence of FIR radiation, the photoexcited carriers in the higher Landau level and the empty states in the

lower Landau level which are left behind, change the picture and increase scattering which results in a strong increase of ρ_{xx} .

A relatively weaker change of resistance can occur when the Fermi level is in the middle of a Landau level which is partially empty. If the cyclotron resonance transition lies in the region of $\rho_{xx} = 0$ (plateaus) the absorption cannot be observed.

Fig. 4.11 shows the CR traces from photo-conductivity and magneto transmission for sample S206 at $119\ \mu\text{m}$. The strong CR signal in the photo-conductivity trace can be explained by the positions of the resonances in SdH traces and the argument above. The peak positions are identical from magneto-optics and photo-conductivity measurements (see Fig. 4.12).

We also observed the spin resolved cyclotron resonance in S377 using the photo-conductivity measurements with the same reversed intensity pattern. Fig. 4.13 shows the CR trace of the spin resolved cyclotron at $70.6\ \mu\text{m}$. The sample was tilted 30° .

4.3 Summary

Our CR experimental results are in good agreement with our theoretical model. The values of the effective mass show the expected non-parabolicity behavior. We observed spin resolved CR in the high mobility samples with a rather unexpected amplitude pattern at $70.6\ \mu\text{m}$ which might be as a result of deviation from the Kohn theorem. In an inhomogeneous system in which the translational invariance is broken (e.g. by electron-impurity scattering, band nonparabolicity or electron-phonon interaction) Kohn's theorem is not valid. The simple model can predict the CR transition positions very well. More experiments using FTIR are required to understand the spin resolved CR in InSb.

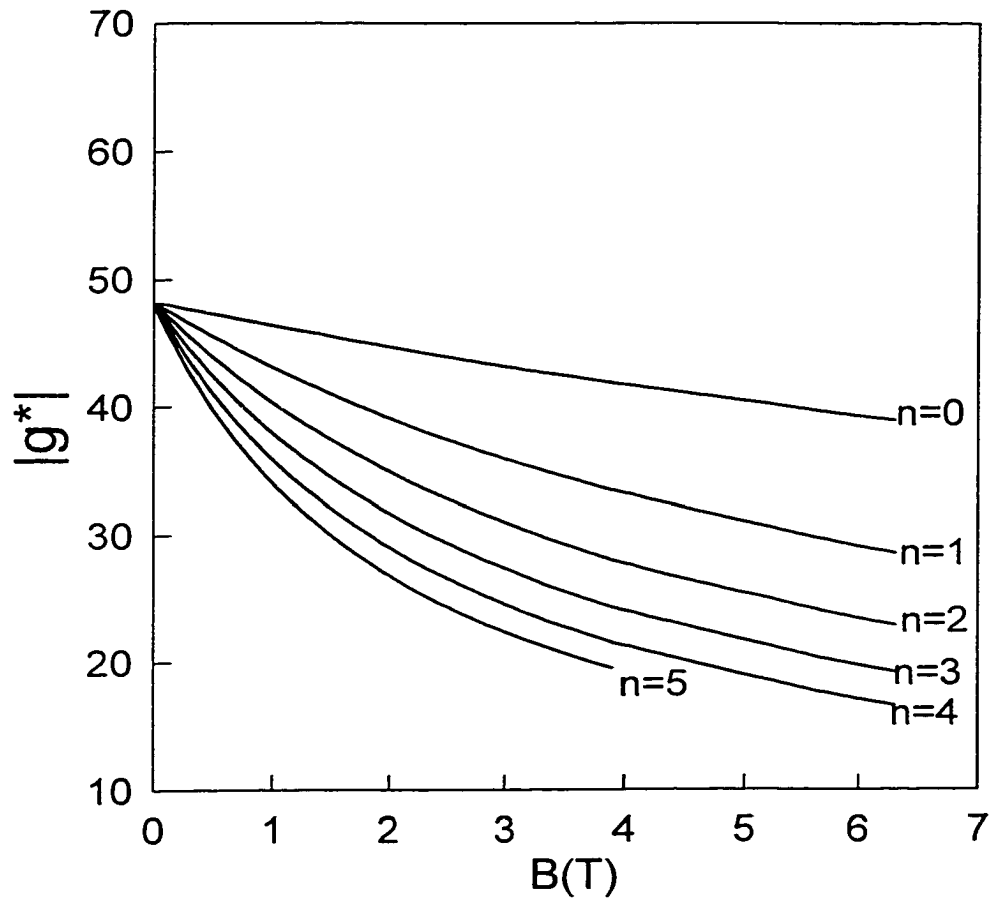


Figure 4.1: The effective g-factor for different Landau level index is calculated for a 300Å InSb QW with 9% Al concentration using the modified Pidgeon-Brown model which shows the effect of non-parabolicity in the g-factor.

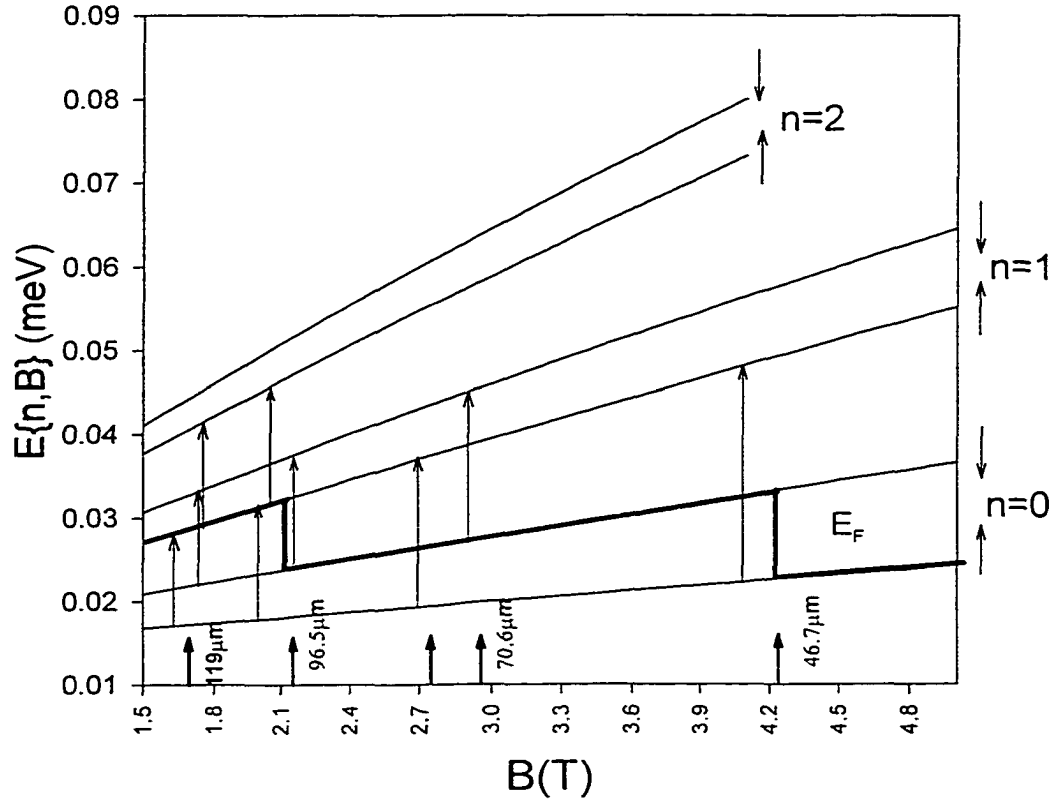


Figure 4.2: Partial fan diagram for S203 with density $n = 1.0 \times 10^{11} \text{cm}^{-2}$. The experimental CR transitions are shown by arrows at the bottom and the theoretical predictions on the top. We observed spin resolved CR transitions with a reversed intensity pattern only at $70.6 \mu\text{m}$.

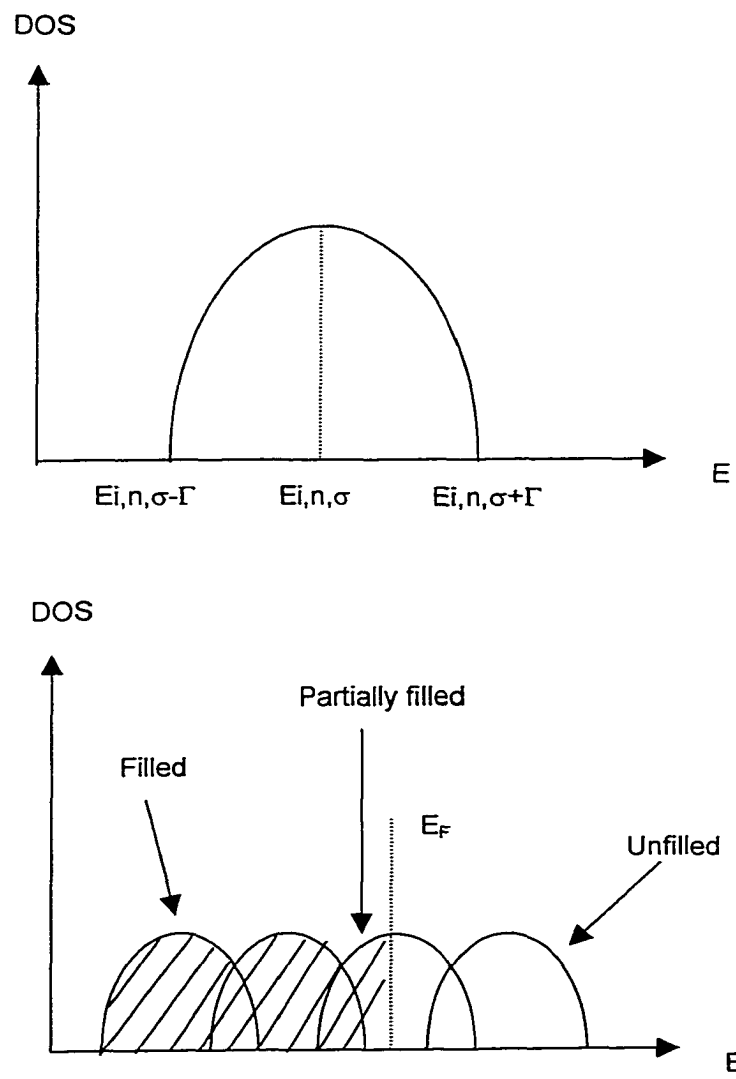


Figure 4.3: The density of states in the Born approximation versus energy

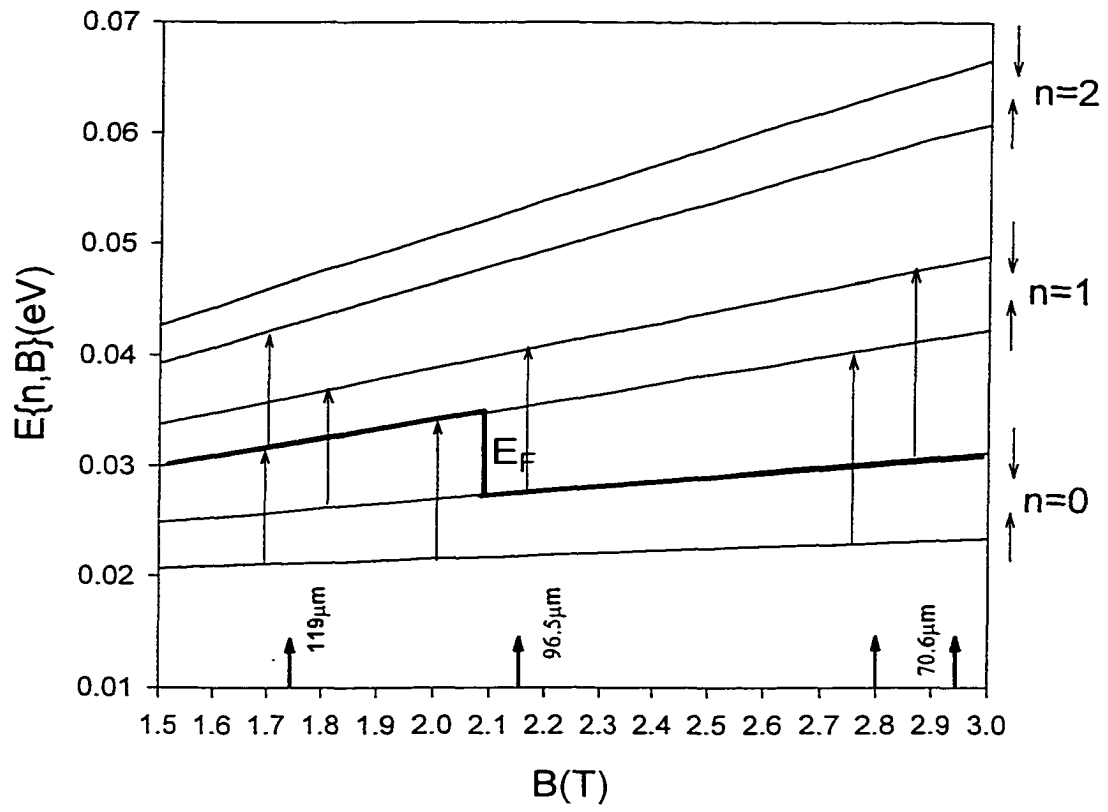


Figure 4.4: Fan diagram for S203 using the Born approximation which can describe our experimental CR transitions as well as the single particle model.

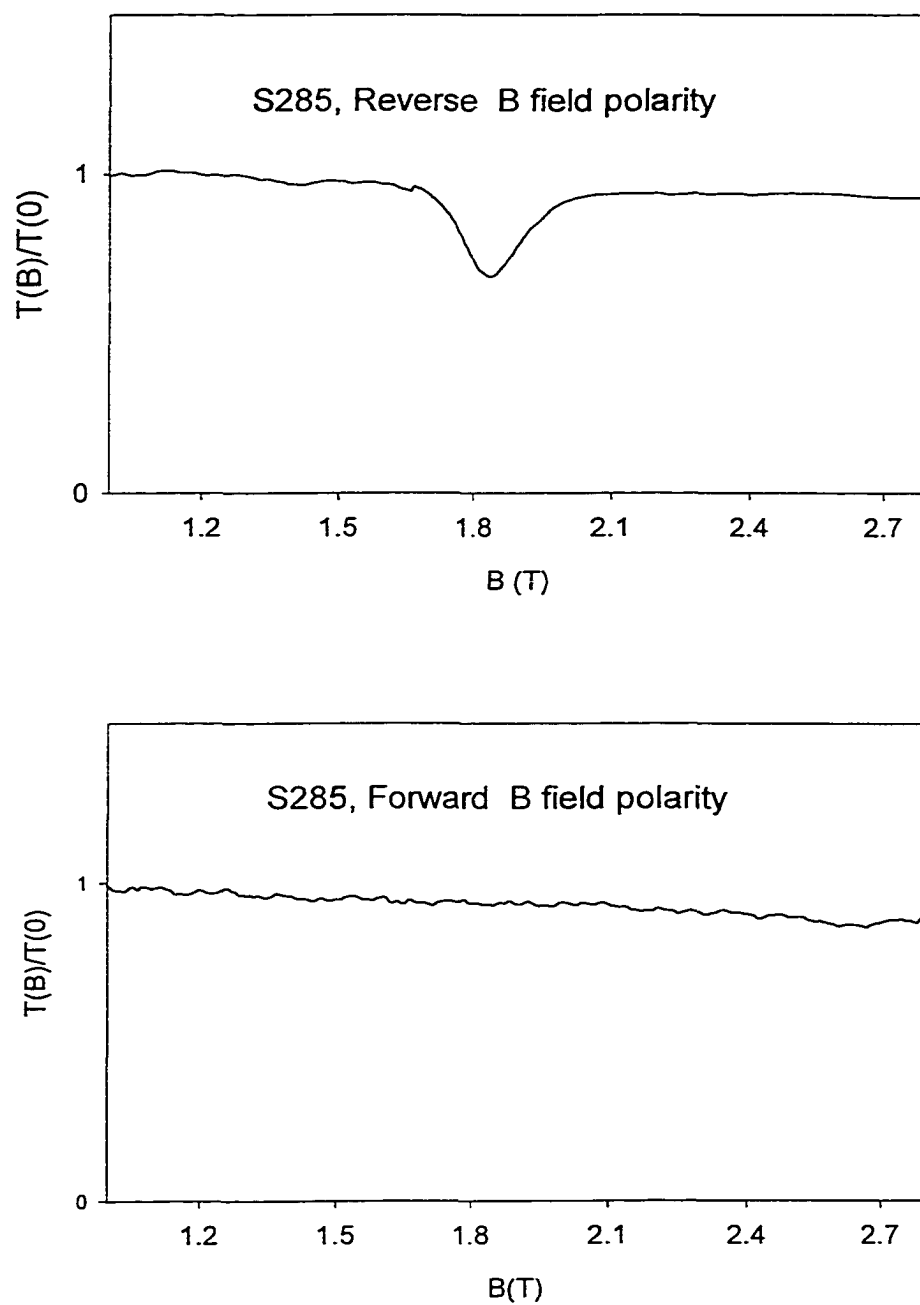


Figure 4.5: Cyclotron resonance with circularly polarized light for S285. The reverse polarity of the magnet is the only polarity that the electron CR of the $119\mu\text{m}$ line can be observed which suggests the opposite magnet polarity for the hole CR.

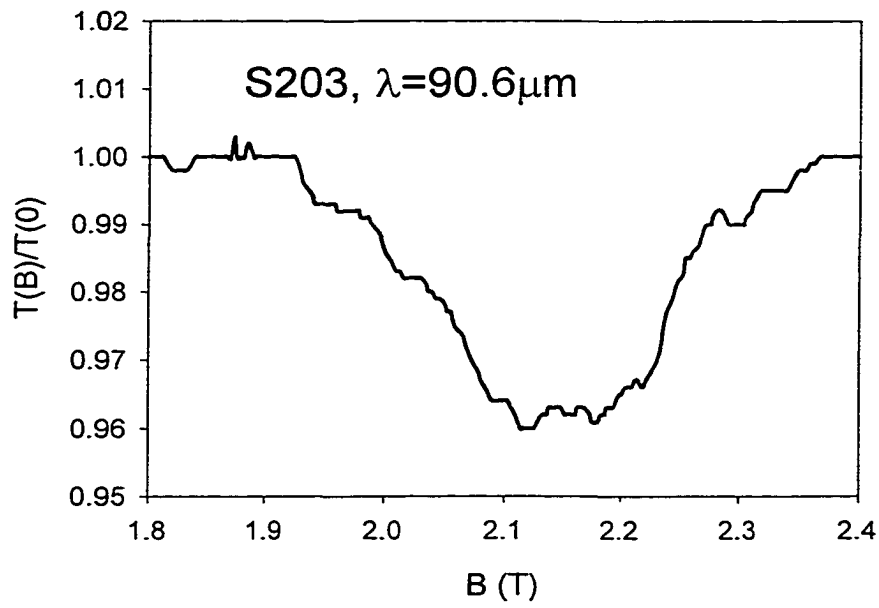
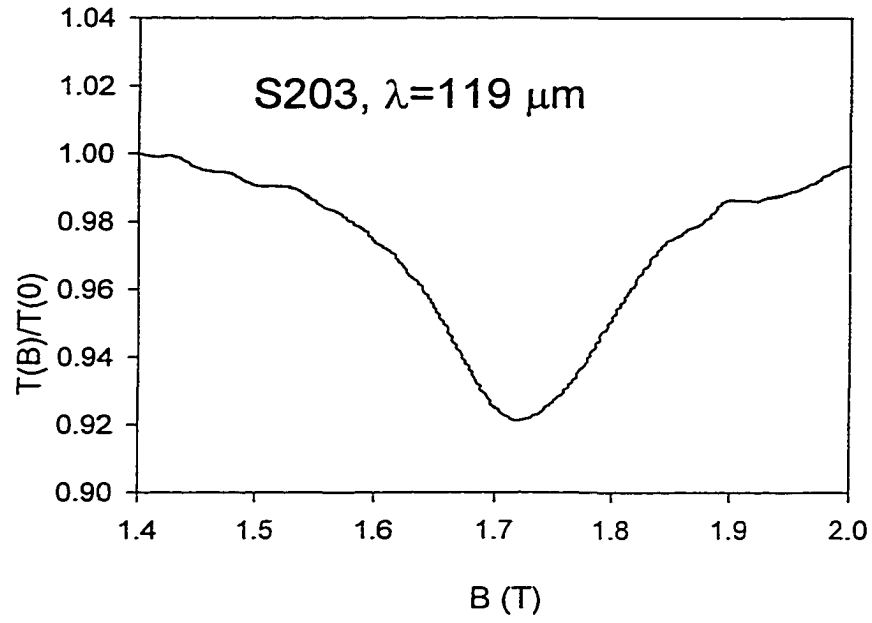


Figure 4.6: Cyclotron traces of S203 at $119 \mu\text{m}$ and $96.5 \mu\text{m}$ lines.

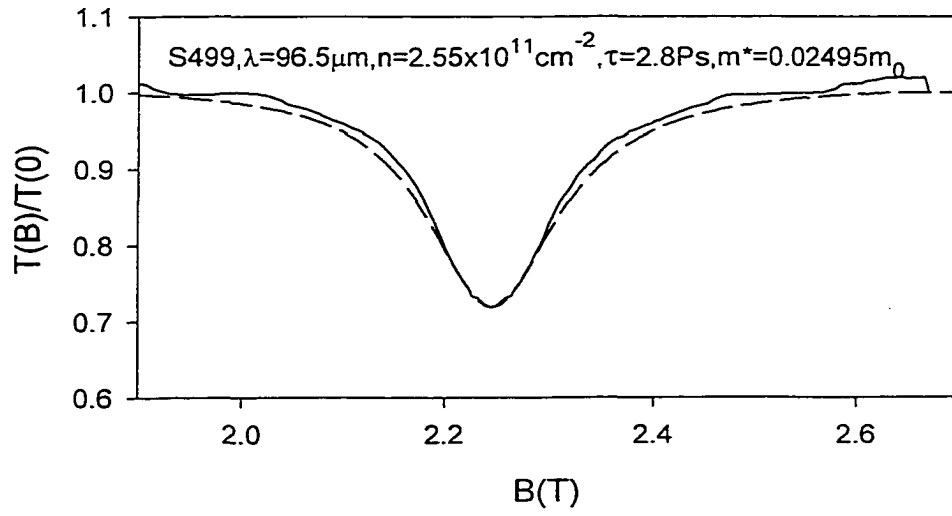
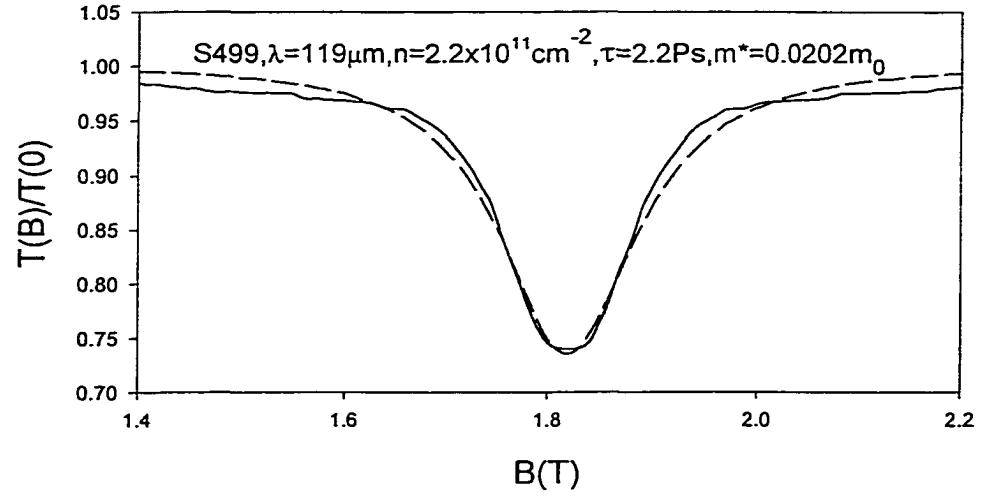
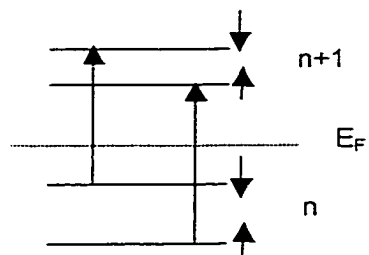
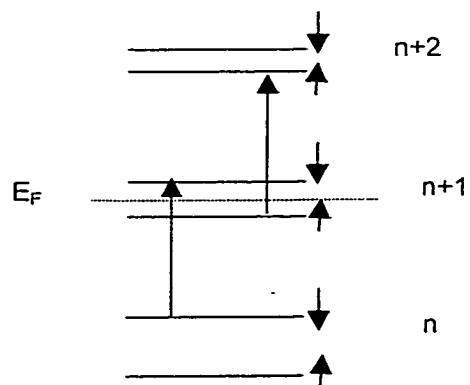


Figure 4.7: Cyclotron resonance of S499 with the 119 μm and 96.5 μm lines.



Δg^* splitting



Δm^* splitting

Figure 4.8: A simple picture of Δm^* and Δg^* splittings

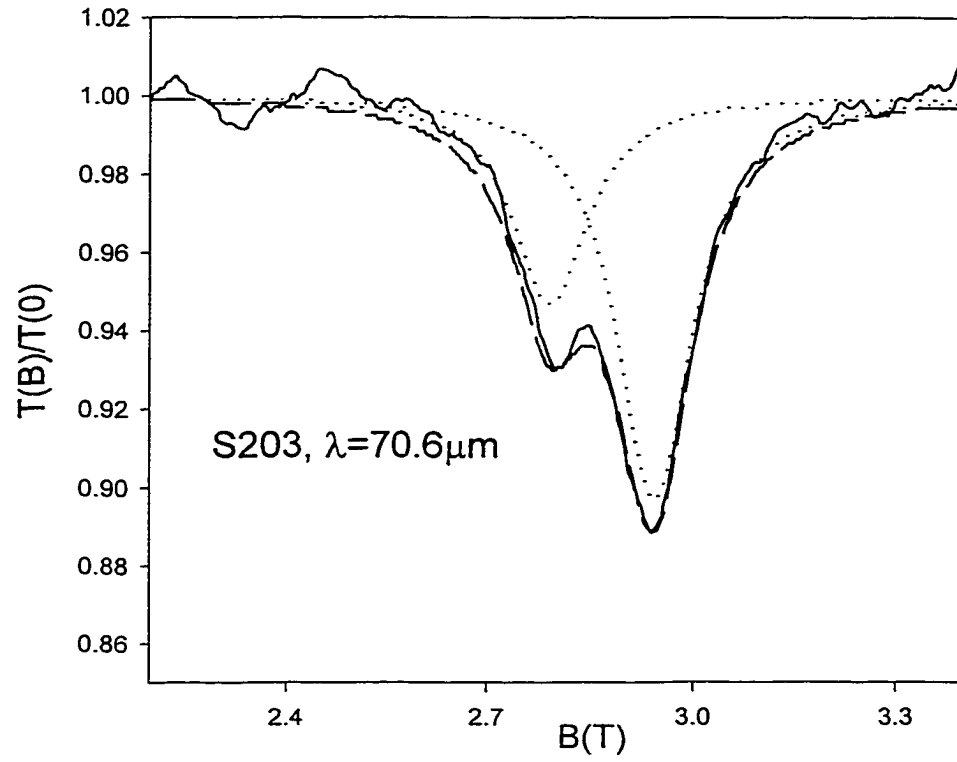


Figure 4.9: Spin resolved cyclotron resonance of S203 at $70.6\mu\text{m}$. From the simple Fermi factor picture, the intensity of the first resonance should be stronger than the second resonance.

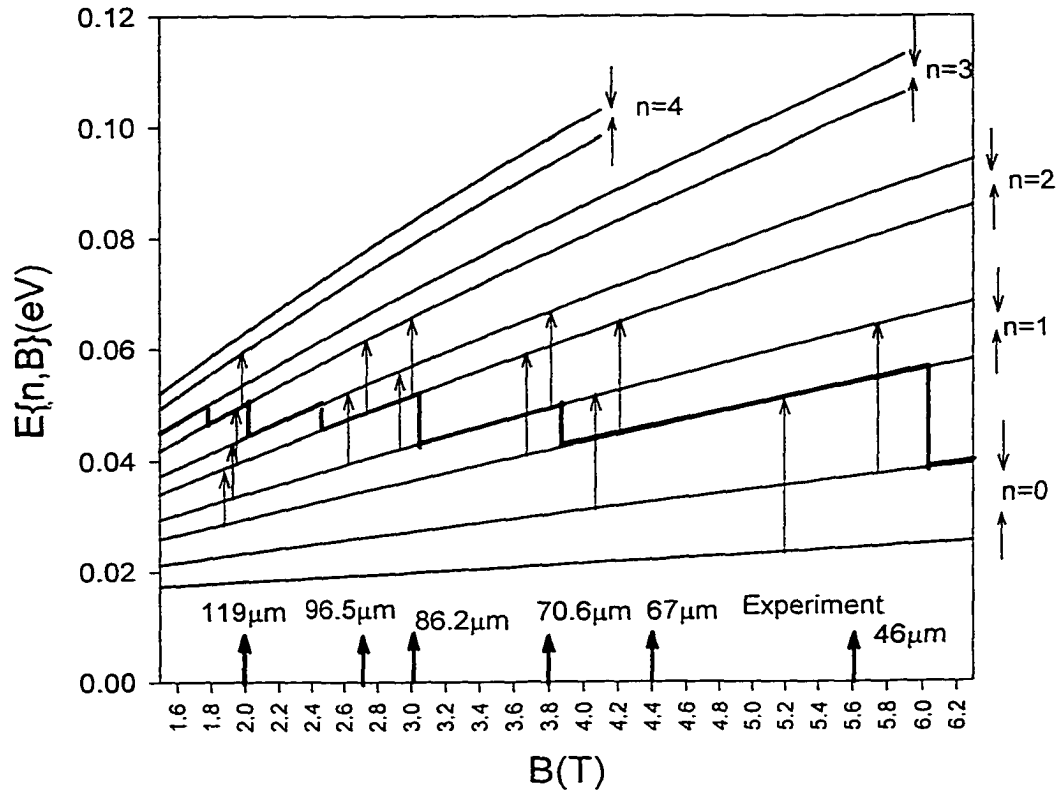


Figure 4.10: Fan diagram for S209 in tilted configuration with the observed CR transitions in a wide range of magnetic field. The electron density is $\approx 2.5 \times 10^{11} \text{ cm}^{-2}$ and the tilt angle is 30° .

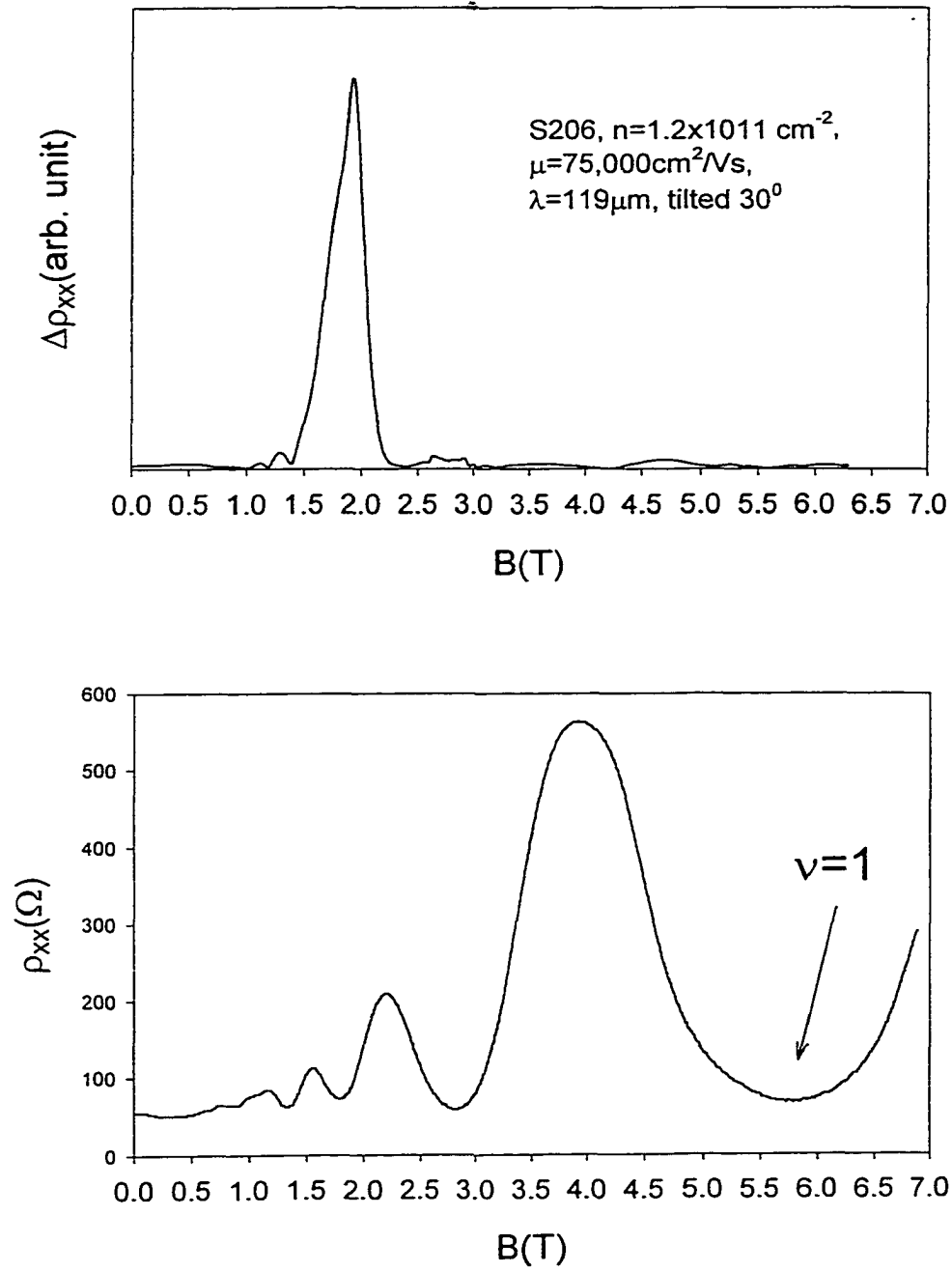


Figure 4.11: Cyclotron resonance of S206 at $119 \mu\text{m}$ from photo-conductivity measurements and the SdH trace of the sample. The resonance appears at the B field where $\rho_{xx} \neq 0$.

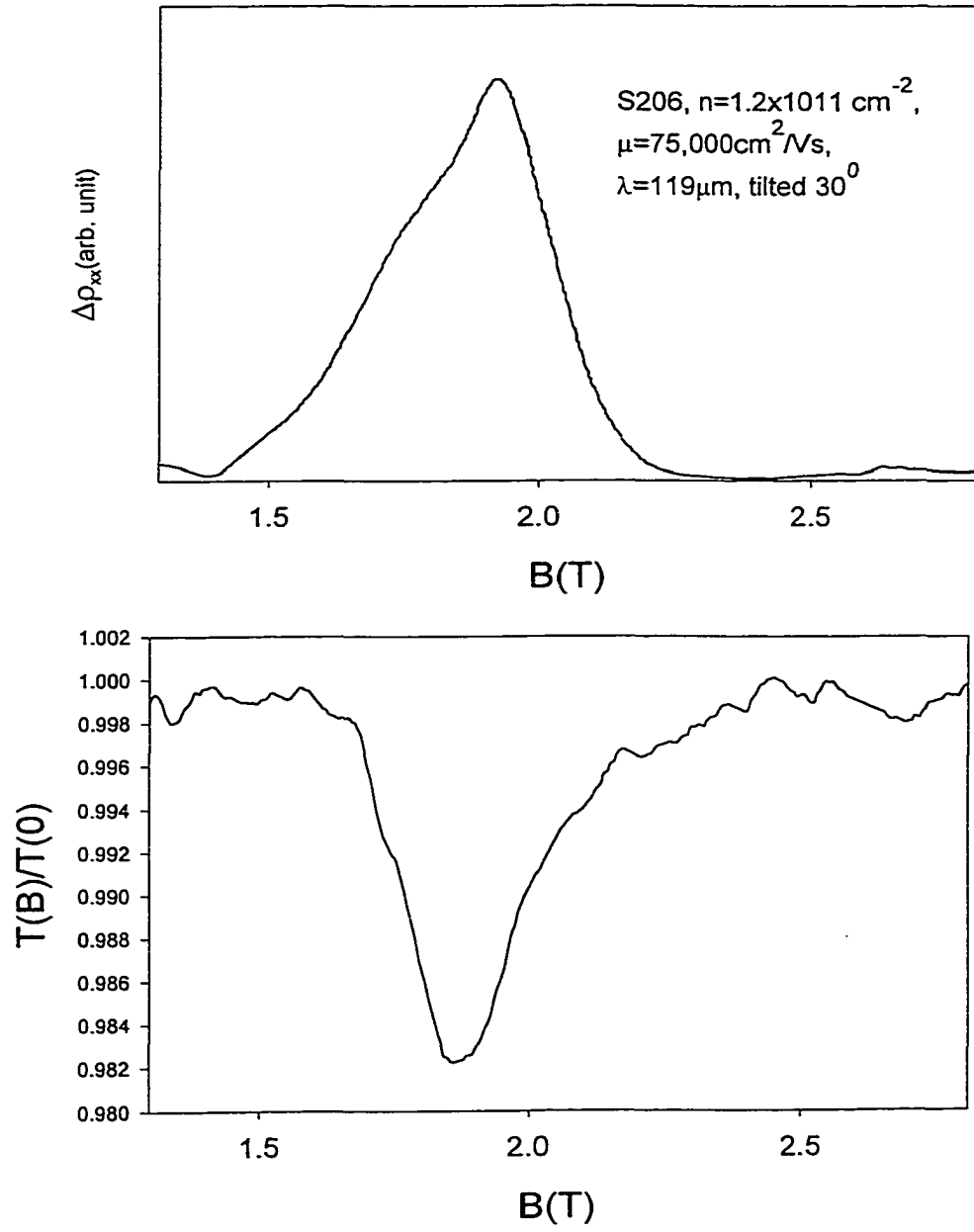


Figure 4.12: Cyclotron resonance of S206 at $119 \mu\text{m}$ from photo-conductivity and magneto-transmission experiments with identical resonance positions .

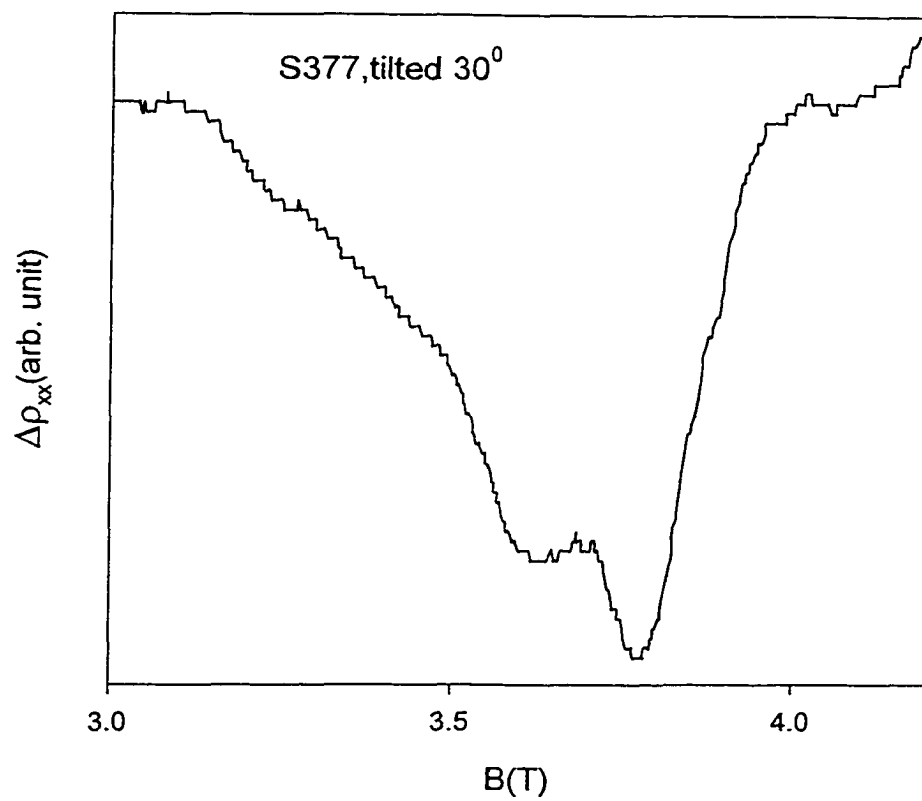


Figure 4.13: Spin resolved cyclotron resonance in S377 at $70.6\ \mu\text{m}$ shows the same reversed intensity pattern in photo-conductivity measurements. The sample was tilted 30° .

Chapter 5

Electron Spin Resonance

Recently, the study of spin-orbit coupling and spin splitting in semiconductors, especially narrow gap materials, has attracted much theoretical and experimental attention. In asymmetric heterostructures, as first formulated by Bychkov and Rashba[6, 7], spin splitting can occur even in the absence of an external magnetic field. This effect has been proposed as the basis for a type of transistor proposed by Datta and Das in 1990 [4].

Most of the experimental evidence for Rashba zero field spin splitting in heterostructures has been restricted to the observation of beating in Shubnikov-de Haas (SdH) oscillations [44-48]. In our InSb QWs, electron concentrations and electron mobilities are both low compared to GaAs and InAs systems, which might make beating patterns not observable .

2D electron spin resonance (ESR) using far-infrared spectroscopy in an inversion layer on InSb was first observed by Därr et al. [5]. In GaAs QWs, a millimeter-wave photo-conductivity technique has been used to observe ESR[30].

Resonant optical measurements and photo-conductivity techniques are the only methods to determine the bare effective g factor. While transport measurements are influenced by electron-electron interaction, and the measured g factor can be

enhanced by the exchange interaction.

In InSb quantum wells, we observe ESR in both magneto-transmission and photo-conductivity measurements using a far-infrared laser. Our results provide a convincing confirmation of the spin-orbit coupling picture of the Bychkov-Rashba model [6, 7].

5.1 Background:

In a 2 dimensional electron system(2DES) without a magnetic field, the motion of electrons perpendicular to the 2DES is quantized, resulting in energy sub-bands. In the presence of a magnetic field, the sub-bands split into Landau levels (see chapter 3 for more details), which themselves are split into levels with spin up and down. The energy spectrum of a 2DES in a perpendicular magnetic field has the following form:

$$E_{n\pm} = E_z + (n + \frac{1}{2})\frac{\hbar eB}{m^*} \pm \frac{1}{2}g^*\mu_B B \quad (5.1)$$

where E_z is the sub-band energy for the z direction, n is the Landau level index, g^* and m^* are the effective g-factor and effective mass.

If there were no spin-orbit(SO) interaction, an electron in a magnetic field would have two independent motions, one associated with the orbital and one with the spin degree of freedom [7]. The orbital motion is cyclotron rotation with the frequency given by:

$$\omega_c = \frac{eB}{m^*} \quad (5.2)$$

and with the magnetic length as a spatial scale corresponding to cyclotron motion given by:

$$l_B = (\frac{\hbar}{eB})^{1/2} \quad (5.3)$$

The frequency of the spin transition is determined by:

$$\omega_s = \frac{g^* \mu_B B}{\hbar} \quad (5.4)$$

The electric-dipole interaction associated with a transition between quantum states under an a.c. electric field is $H_{e.d.} = eEl_B$, which is typically much larger than the magnetic dipole interaction, $H_{m.d.} = g^* e \hbar B / m_0$. While the intensity of CR is stronger than magnetic resonance, the spin-orbit interaction leads to coupling between the orbital and the spin motions which will increase electric excitation of electron spin resonance[7].

Recently R. Meisels et al. [49] have done calculations based on an 8-band model and **k.p** theory for an $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ triangular potential which show that ESR is a magnetic dipole transition (MDT) with bigger transition matrix element than for electric-dipole transitions (EDT). Their theoretical model explains their ESR experimental results well.

5.2 Narrow gap semiconductors and spin resonance

In narrow gap semiconductors (NGS) with large spin-orbit coupling, the interaction of the conduction and the valence bands is strong, which results in mixing of the electronic energy states. Therefore the spin up and down states of the Landau levels intermix with each other and as a result the wave function for a given Landau quantum number and spin $\pm$ contains both spin-up and down as well as harmonic oscillator states of quantum numbers n and $n \pm 1$ this allows transitions between two spin states with or without a change of orbital state. The complete expressions for the wave functions explicitly calculated by Zawadzki and can be found in Ref. [33, 50].

The matrix element for EDSR transitions in NGS materials is not small, which makes the observation of ESR possible. In a simpler form, from the result of a calculation by Zawadzki [50] in the parabolic region of electron energies, $E_n^\pm \ll E_g$ the spin resonance matrix element can be written as:

$$\langle \Psi_n^- | H_R | \Psi_n^+ \rangle = -\frac{eA'_0}{4\sqrt{2}m^*} \hbar k_z \frac{|g^*| \mu_B B}{E_g} \quad (5.5)$$

Here $\Psi_n^\pm$ are the spin up and down wave functions for a given Landau level n and A'_0 is the vector potential of the radiation field. The smallness of the band gap E_g along with a small effective mass m^* and a large $|g^*|$ factor enhances the spin resonance intensity.

In InSb quantum wells, we observe ESR in both magneto-transmission and photo-conductivity measurements over a wide range of magnetic field and the Landau level index. The width of the ESR transitions is very narrow (30-40 mT) which suggests a long relaxation time ($\approx 5 \times 10^{-10}$ seconds, ≈ 100 times longer than cyclotron resonance life time). For device applications a long relaxation time is ideal in order to achieve larger scattering length (on the order of $10 \mu\text{m}$ in InSb). The experimental results for our InSb symmetric QWs can be explained by the modified Pidgeon-Brown model without zero field spin splitting. (see chapter 4 for the details). The ESR data from asymmetric QW samples show a different behavior which will be presented in the following sections.

5.3 Zero field spin splitting

The spin-orbit interaction couples the electron spin to the electron motion and occurs strongly in narrow gap semiconductors. In two-dimensional electron systems with an asymmetric confinement potential, the moving electrons feel an effective magnetic field which is proportional to the vector product of the electrons in-plane

velocity and the electric field which exists because of the asymmetric confinement potential.

The effective magnetic field $\mathbf{B}_{\text{eff}} = (\mathbf{v} \times \mathbf{E}/c)$ in Fig. 5.2 removes the spin degeneracy and results in zero field spin splitting. The interaction of the electron spins with the effective magnetic field has opened a new area of research in which both the electron charge and spins can be used.

The interaction of the electron spins with the effective B field can make the spin precess. If we start at $t = 0$ with a spin state in the x direction S_x , later the expectation value of S_x will have the following form:

$$\langle S_x(t) \rangle \propto \cos \omega_s t$$

In the system with the zero field spin splitting, the $\hbar\omega_s$ (the energy difference between spin-up and spin-down) is proportional to the spin-orbit coupling parameter α . In a material with large α such as InSb, a faster precession can be achieved which is important in device applications. For example, in a normal field-effect transistor (FET) a gate voltage controls the density of the electrons and therefore the current. In a spin polarized transistor (SPT) in which a magnetic source injects spin polarized electrons and a magnetic drain detects the electrons with the same spin polarization. The gate voltage controls the electron density as well as the α (therefore the precession rate) by changing the asymmetry profile of the confinement potential.

The advantage of a SPT over FET is the transconductance. Fig. 5.1 shows schematically the source to drain current I_{SD} vs the gate voltage (V_G) for FET and SPT. From the comparison between the slopes in the I_{SD} vs the gate voltage (V_G), one can see that a smaller change in the (V_G) is required to change the I_{SD} in SPT

compared to a FET.

5.3.1 Relativistic picture

In the absence of any external field the dispersion relation in a two-band model is given in a simplified two-band Kane model by[51]:

$$E = \sqrt{\left(\frac{E_g}{2}\right)^2 + \frac{E_g P^2}{2m_0^*}}, \quad (5.6)$$

similar to the form of the relativistic relation for electrons in vacuum:

$$E = [p^2 c^2 + m_0^2 c^4]^{1/2}, \quad (5.7)$$

with $2m_0 c^2$ replaced by E_g and m_0 by m_0^* . The velocity u in this model is by analogy: $c = (2m_0 c^2 / 2m_0)^{1/2}$ can be replaced by $u = (E_g / 2m_0^*)^{1/2}$. For InSb u is about $1.2 \times 10^8 \text{ cm/s}$. For a density $n = 1 \times 10^{11} \text{ cm}^{-2}$ the effective B field is about 0.3 T which results in a zero field spin splitting of a few meV using $g^* = -51$. We will examine this value using our electron spin resonance experimental results later in this chapter.

5.3.2 The Rashba model

In the Rashba model [6, 7], the asymmetry of the confining potential leads in first order to an additional term in the Hamiltonian for a 2DES as :

$$H_R = \alpha(\vec{\sigma} \times \mathbf{k}) \cdot \mathbf{e}_z$$

where α is the spin-orbit coupling constant, $\vec{\sigma}$ are the Pauli spin matrices and $\mathbf{e}_z$ is the unit vector along the growth direction. The spin-orbit coupling parameter in the Rashba term is proportional to the average effective electric field [52]:

$$\langle E \rangle = \left\langle -\frac{1}{e} \frac{d}{dz} (E_c + V) \right\rangle \quad (5.8)$$

Here E_c is the conduction-band-edge profile, V is the space charge or applied electrostatic potential energy, and z is the growth direction. In the samples with symmetric structure $\langle E \rangle = 0$ (see Fig. 5.2).

According to the calculations by E.A. Silva *etal.* [52] the band-edge profile E_c and the electrostatic potential V do not play the same role.

The motion of electrons in the plane of a 2DES can be represented by $e^{ik_t \cdot r_t}$. Due to the lifting of the spin degeneracy, the energy dispersion in the Rashba formalism is split in two branches [48]:

$$E = \frac{\hbar^2 k_t^2}{2m^*} \pm \alpha |k_t| \quad (5.9)$$

As in Fig. 5.3, for a given energy E , the spin-up and spin-down states have different k values.

The density of states $D_{\pm}(E)$ can be calculated as a function of electron energy for the electrons with spin-down(-) and spin-up(+) as:

$$D_{\pm}(E) = \frac{m^*}{2\pi\hbar^2} \left[1 \mp \frac{1}{\sqrt{1 + (2(E - E_z^{sub})\hbar^2)/(\alpha^2 m^*)}} \right] \quad (5.10)$$

This expression for the density of states suggests two different populations for spin up and down electrons this can be used to extract the parameter α . By taking the differences in carrier concentration $\Delta n = n_- - n_+$ of the spin sub-bands:

$$\Delta n = \int_0^{E_F} D_- dE - \int_0^{E_F} D_+ dE \quad (5.11)$$

$$\Rightarrow \alpha = \frac{\Delta n \hbar^2}{m^*} \sqrt{\frac{\pi}{2(n - \Delta n)}} \quad (5.12)$$

Experimentally, the parameter α has been obtained by observing beating of SdH oscillations presumably resulting from different populations of electron spin states at zero magnetic field. This beating pattern has been studied in a number of heterostructures to obtain a measure of the parameter α [45, 46, 47, 48].

In 1999, Brosig et al. [53] presented experiments on high-quality InAs-AlSb quantum wells. In spite of a perfectly clean signal with SdH oscillations up to filling factors of 200 resolved, no indication of beating was observed. This experiment and a more recent experiment and calculations[54] have questioned the interpretation of the beating in SdH oscillations.

We use far-infrared spin resonance as a direct method to determine the spin splitting. In InSb spin resonance occurs in an accessible range of magnetic fields, and because of its large effective g-factor and predicted large α is a well-suited material for spin based devices. The experimental results of this work give the α value and zero field spin splitting in asymmetric InSb QWs.

In the presence of a magnetic field $B\|e_z$ the energy spectrum is given by [6, 55]:

$$E_s^\pm = \hbar\omega_c \varepsilon_s^\pm \quad (5.13)$$

$$\varepsilon_0 = \delta \quad (5.14)$$

$$\varepsilon_s^\pm = s \pm \sqrt{\delta^2 + \gamma^2 s}, s \geq 1 \quad (5.15)$$

$$\omega_c = \frac{eB}{m^*}, \gamma = 2\sqrt{\Delta/\hbar\omega_c}, \delta = \frac{1}{2} - \beta, \beta = \frac{m^*}{2m_s}, m_s = \frac{2m_0}{g}$$

with $\Delta = \frac{m^* \alpha^2}{2\hbar^2}$, m_0 the free electron mass, and g as the g-factor (the sign of the g-factor should be considered).

In the terms above, s is an integer but not the same as the Landau level index.

The spin splitting in this model for a given integer n value (when the g -factor is negative) is given by:

$$\Delta_s(n) = E_n^+ - E_{n+1}^- \quad (5.16)$$

The splitting between spin-up and spin-down in the energy spectrum of the Rashba model can be constructed as follows:

$$\Delta_s(n) = E_n^+ - E_{n+1}^- \quad (5.17)$$

$$\Delta_s(n=0) = \hbar\omega_c\delta - \hbar\omega_c[1 - \sqrt{\delta^2 + \gamma^2}] \quad (5.18)$$

$$\Delta_s(n) = \hbar\omega_c[n + \sqrt{\delta^2 + \gamma^2 n}] - \hbar\omega_c[n + 1 - \sqrt{\delta^2 + \gamma^2(n+1)}], n \geq 1 \quad (5.19)$$

At $B=0$:

$$\Delta_s(B=0) = E_-(k_F) - E_+(k_F) = 2\alpha k_F$$

In the limit $\gamma^2 s \ll \delta^2$:

$$E_s^\pm \approx \hbar\omega_c[s \pm \delta(1 + \frac{\gamma^2 s}{2\delta^2})] \quad (5.20)$$

and $\Delta_s(n) = E_n^+ - E_{n+1}^-$ will have the following form:

$$\delta\Delta_s(n) \approx \frac{4\Delta(n+1/2)}{\delta} \quad (5.21)$$

The above relation has a linear dependence on $(n+1/2)$ which we use below to extract the value of α and zero field spin splitting experimentally. For high Landau level indices, it has been shown [59, 60] the $\delta\Delta_s(n)$ can be written as:

$$\delta\Delta_s(n) \approx [(\hbar\omega_c - g^*\mu_B B)^2 + \Delta_0^2]^{1/2} \quad (5.22)$$

$$\Delta_0 = 2\alpha k_F \quad (5.23)$$

Thus by fitting experimental data at low field (high Landau level indices) with Δ_0 as the only fitting parameter, the zero field spin splitting can be extracted.

5.3.3 Spin resonance experiment

Our samples are symmetric and asymmetric InSb single QW of widths $300\text{\AA}$ and $200\text{\AA}$ with $\text{Al}_{0.09}\text{In}_{0.91}\text{Sb}$ barrier materials which are δ -doped with Si. The Si δ -layers are either located on one side of the QW or on both sides of the QW resulting in asymmetric and symmetric confinement potentials respectively.

The δ -doped layers are typically located $700\text{\AA}$ from the QW center within the barrier. The barrier layers on the substrate and the surface side of the QW are $3\mu\text{m}$ and $1600\text{\AA}$ thick respectively. Therefore the samples doped symmetric on both sides of the well are structurally symmetric within about $1700\text{\AA}$ from the QW center. This distance is larger than the surface depletion length which implies the shape and the symmetry of the wells would be determined only by the well/barrier mismatch and the mismatch in the doping layers.

The electron concentrations range from $(0.95 \rightarrow 2.9) \times 10^{11}\text{cm}^{-2}$ where only the first sub-band is occupied. The mobilities of the samples range from 70,000 to $150,000\text{ cm}^2/V\text{sec}$ [25, 26].

We observe the electron spin resonance (ESR)[56] in magneto-transmission of far-infrared laser radiation or by observing the laser-induced change in magneto-transport (photo-conductivity measurements) [30] at 4.2K. As for InSb inversion layers [5], the ESR is strongest in the tilted configuration in which the sample normal is tilted with respect to the B field which also coincides with the direction of the laser beam. In our experiments no ESR was observed below 30° or above 50° . The experimental techniques of magneto-transmission and photo-conductivity

effect can be found in Chapter 3.

In the tilted configuration, the spin splitting and ESR respond to the total B but the Landau level quantization depends on the component of the B field perpendicular to the 2D layer. The ESR is only observable at odd filling factors when the Fermi level is located between spin-split Landau levels within the same level index.

Fig. 5.4 shows the observed ESR in both transmission and photo-conductivity for a symmetric QW. The ESR signals are very small and narrow, 30-40 mT, similar to ESR in GaAs [30, 57, 58]. The ESR signals from the magneto-transmission experiments have Lorentzian line shape but the photo-conductivity signals which appear in the back ground of SdH traces, are not pure absorption lines and reflect a conductivity mechanism which is not well understood.

By using samples with different electron concentrations and by varying tilt angle and laser frequency, we could observe the ESR over a wide range of magnetic field (0.6-6.2 T) and Landau level index ($n = 0 \rightarrow n = 9$). Fig. 5.5 shows the photo-conductivity and SdH trace of sample S644 where the ESR is observable at $B \approx 3.4$ T for $184\mu\text{m}$ laser line. At low magnetic field we observed ESR using the photo-conductivity method. Fig. 5.6 shows the traces for samples S499 ($n = 2.0 \times 10^{11}\text{cm}^{-2}$, symmetric) and S356 ($n = 1.3 \times 10^{11}\text{cm}^{-2}$, asymmetric) at two different wave-lengths. Both samples were tilted 45° .

Also for sample S356 as is shown in Fig. 5.7, the shift in the ESR for two different wavelengths is observable.

Figure 5.8 shows SdH data for the asymmetric structure (S707) with the largest number of low- B oscillations of the four asymmetric samples that we have studied. No beating is seen in the trace and only three oscillations are evident before

spin splitting is resolved (the sample is tilted 30° , we did not observe any beating even when the sample was not tilted). In the same sample we observe ESR using photoconductivity measurements at low B-field which is plotted in Fig. 5.8b. It is unclear whether beating would be observed if the 2DES had less disorder or a much higher density.

In order to study the spin properties of our InSb QW, we used a series of symmetric samples for which the observed ESR can be fit using the modified Pidgeon-Brown[32, 34] model, without Rashba zero-field spin splitting. Only one fitting parameter (P the momentum matrix element) is required to fit the experimental data. All the other band parameters are their accepted values [35] (P itself is within the expected range).

The ESR signals of asymmetric samples show different behavior, most obviously at low B fields. In Fig. 5.9 the observed spin splitting $\Delta_s(B)$ of symmetric and asymmetric samples are plotted. The difference between the measured $\Delta_s(B)$ in a given asymmetric sample and the $\Delta_s(B)$ value predicted for a symmetric sample at the same Landau level index n will result in $\delta\Delta_s(n)$ which is easily shown for $B \gg B_{min}$ can be written as:

$$\delta\Delta_s(n) \approx \frac{8m^*m_0\alpha^2(n+1/2)}{[\hbar^2(2m_0 - m^*g^*)]} \quad (5.24)$$

where B_{min} is given by:

$$B_{min} = \frac{32n(m^*m_0\alpha)^2}{[e\hbar^3(2m_0 - m^*g^*)^2]}$$

where e is the electron charge. We find $B_{min} \approx 0.01T$ in our case.

These differences are plotted in Fig. 5.10 versus $(n+1/2)$ with n determined from the SdH measurement in each sample. The linear dependence was predicted by Eq. 5.21 without taking non-parabolicity into account.

We estimated the effect of nonparabolicity by examining the multiplier for $\alpha^2(n+1/2)$ in Eq. 5.24 . From our band structure calculation the multiplier remains in the range $(0.072 \pm 0.04)m_0/\hbar^2$ for the 200 Å wells and $(0.06 \pm 0.05)m_0/\hbar^2$ for 300Å wells from 0.75T to 6T which suggest that non-parabolicity does not change the linearity with $(n+1/2)$ of the parabolic Eq. 5.24 significantly.

The slope of the plots in Fig. 5.10 can be used to extract the α values. We find the α values of 1.6×10^{-9} , 1.3×10^{-9} , 1.6×10^{-9} , 1.5×10^{-9} eV cm for samples S285, S360, S356, and S707 respectively.

Equation 5.24 is only applicable to the case when B field is perpendicular to the quantum well, our experiment conducted under a tilted configuration which as shown by Pfeffer et al. [59] the effect of parallel component cannot be ignored and as a result the value of α might be smaller than presented here.

Using these α values, the zero-field spin splitting can be calculated from the sample densities using Eq. 5.23. For these four samples the zero field spin splitting is in the range of 2.9-3.6 meV. The values of $\delta\Delta_s(n)$ versus $n+1/2$ for the symmetric samples are plotted in Fig. 5.10 which show very small deviations from zero.

The deviations in the apparent g-factors are far beyond the InSb bulk g-factor in the low field region which is plotted in Fig. 5.11. The labels show the Landau level index to which the data points are belong (defined from SdH measurements). As one can see the deviation is larger at low magnetic fields.

The α values measured in this work are among the largest reported as would be expected for a material like InSb with a large bulk g-factor (g-factor is a measure of spin-orbit coupling strength). Recently, in gated InAs samples [61] α values ranging from 2×10^{-9} to 4×10^{-9} eV cm have been measured which suggest that even larger α values in InSb QW can be achieved.

Our results provide valuable confirmation of the Bychkov-Rashba picture . This work can be expanded to gated samples and to samples where the asymmetry is due to two different Al concentrations in the barrier materials.

5.3.4 Summary

We observe ESR using FIR laser spectroscopy in symmetric and asymmetric InSb QW over a wide range of magnetic field and the Landau level index. The behavior of the asymmetric wells at low magnetic fields with g-factors far in excess of the bulk g-factor of InSb is due to spin splitting at zero magnetic field. Asymmetry-induced shifts in the spin resonance at high fields depend on the Landau level index as predicted by the Bychkov-Rashba model.

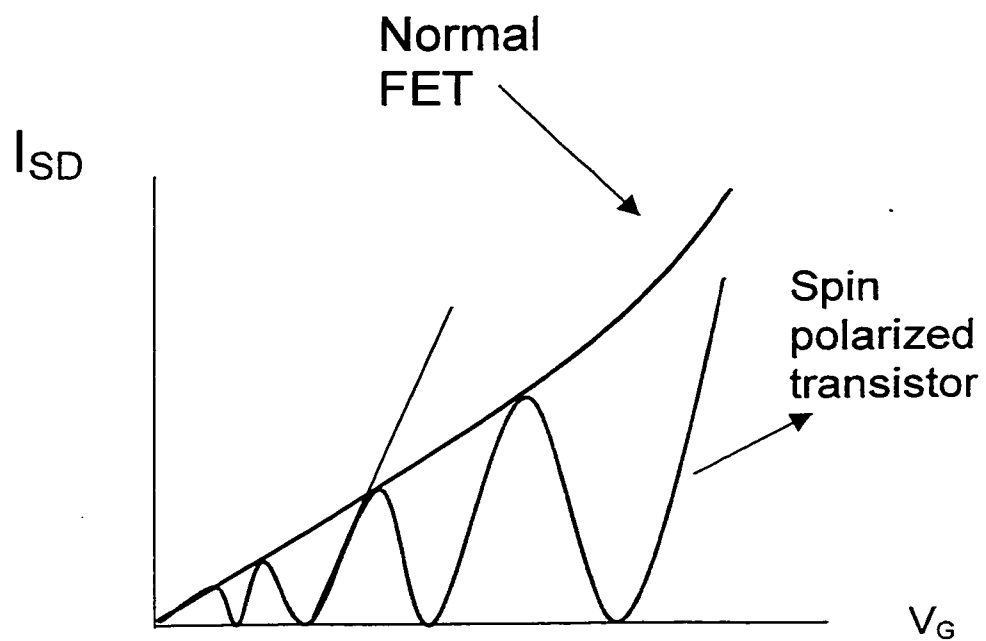
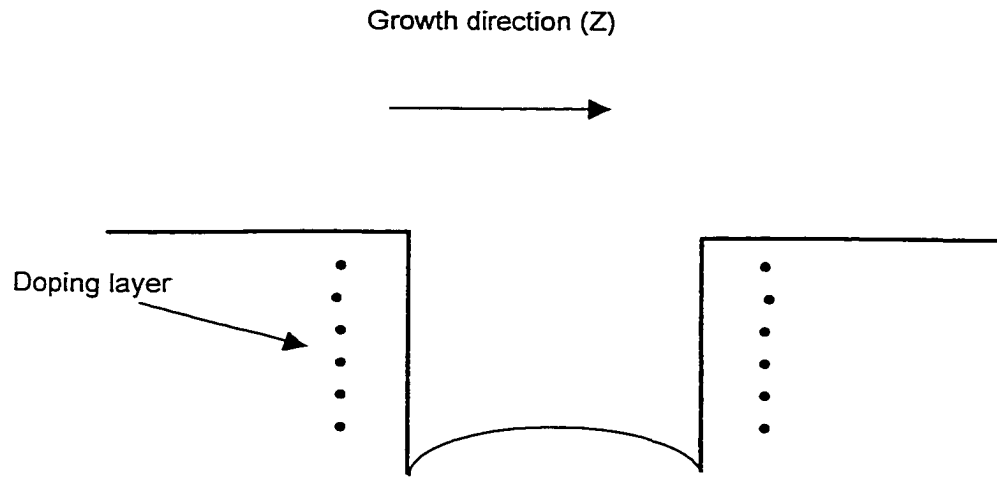


Figure 5.1: Source to drain current in a normal FET compared to a spin polarized transistor versus gate voltage.

Symmetric quantum well



Asymmetric quantum well

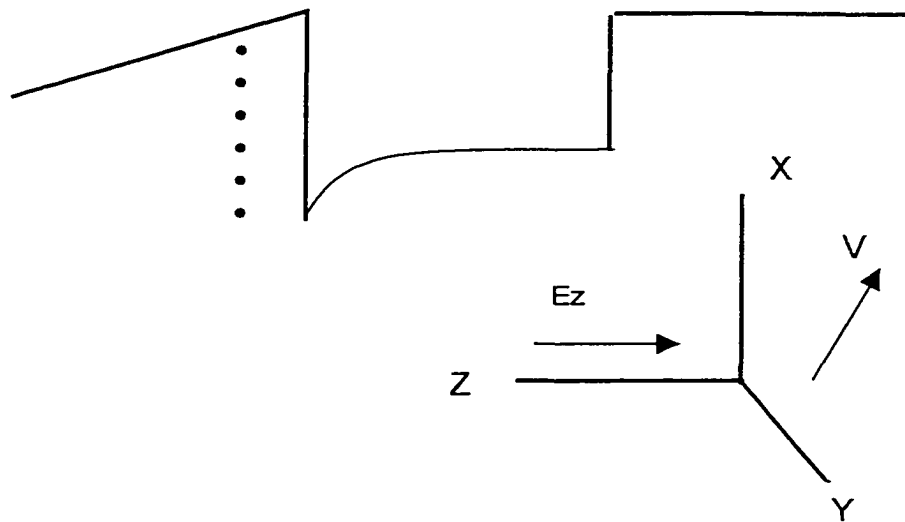


Figure 5.2: Quantum well profiles for symmetric and asymmetric wells. In an asymmetric well there is a non-zero net electric field which in the relativistic limit results in an effective magnetic field.

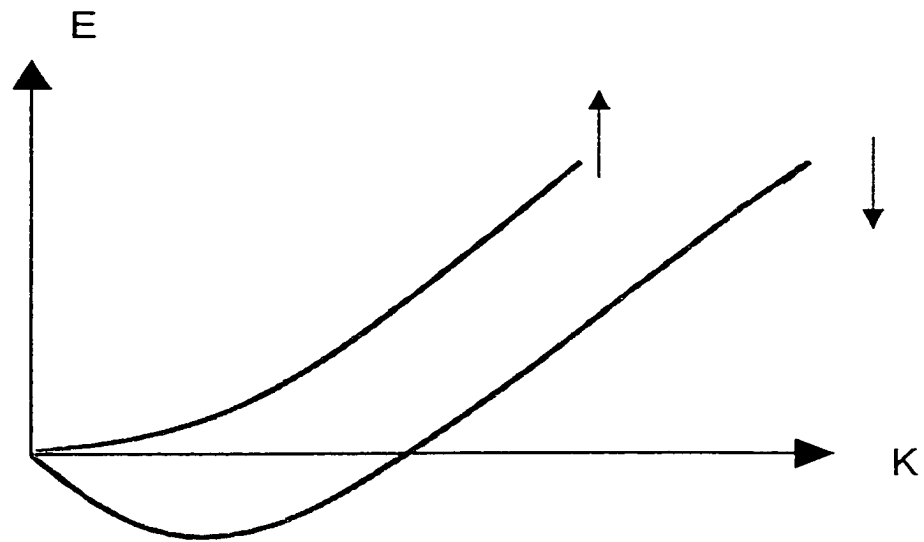


Figure 5.3: The dispersion relation with the zero field spin splitting

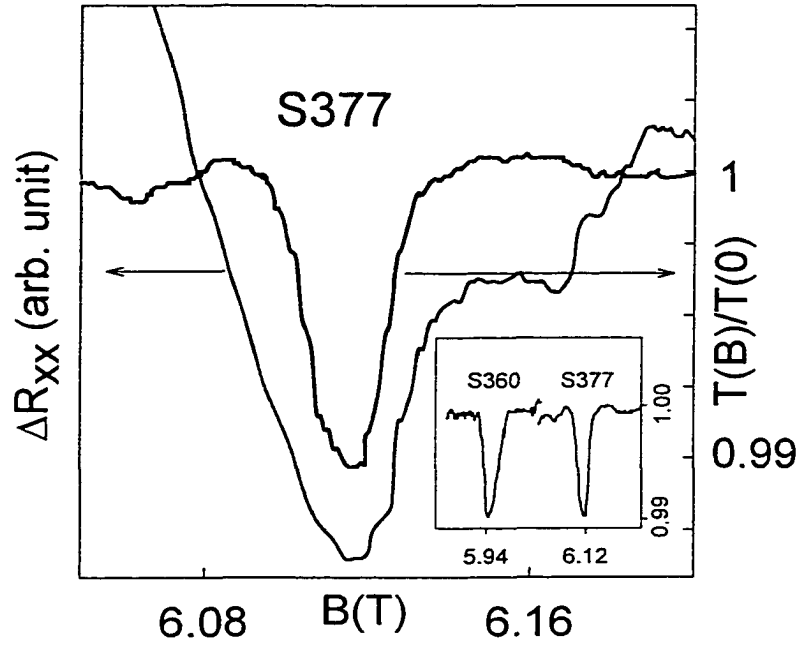


Figure 5.4: Electron spin resonance in S377 a symmetric quantum well for $\lambda = 119\mu\text{m}$, the sample is tilted 45° . ESR can be observed in both the magneto-transmission and the photo-conductivity measurements. The inset shows the experimental shift of ESR in an asymmetric sample S360 compared to S377.

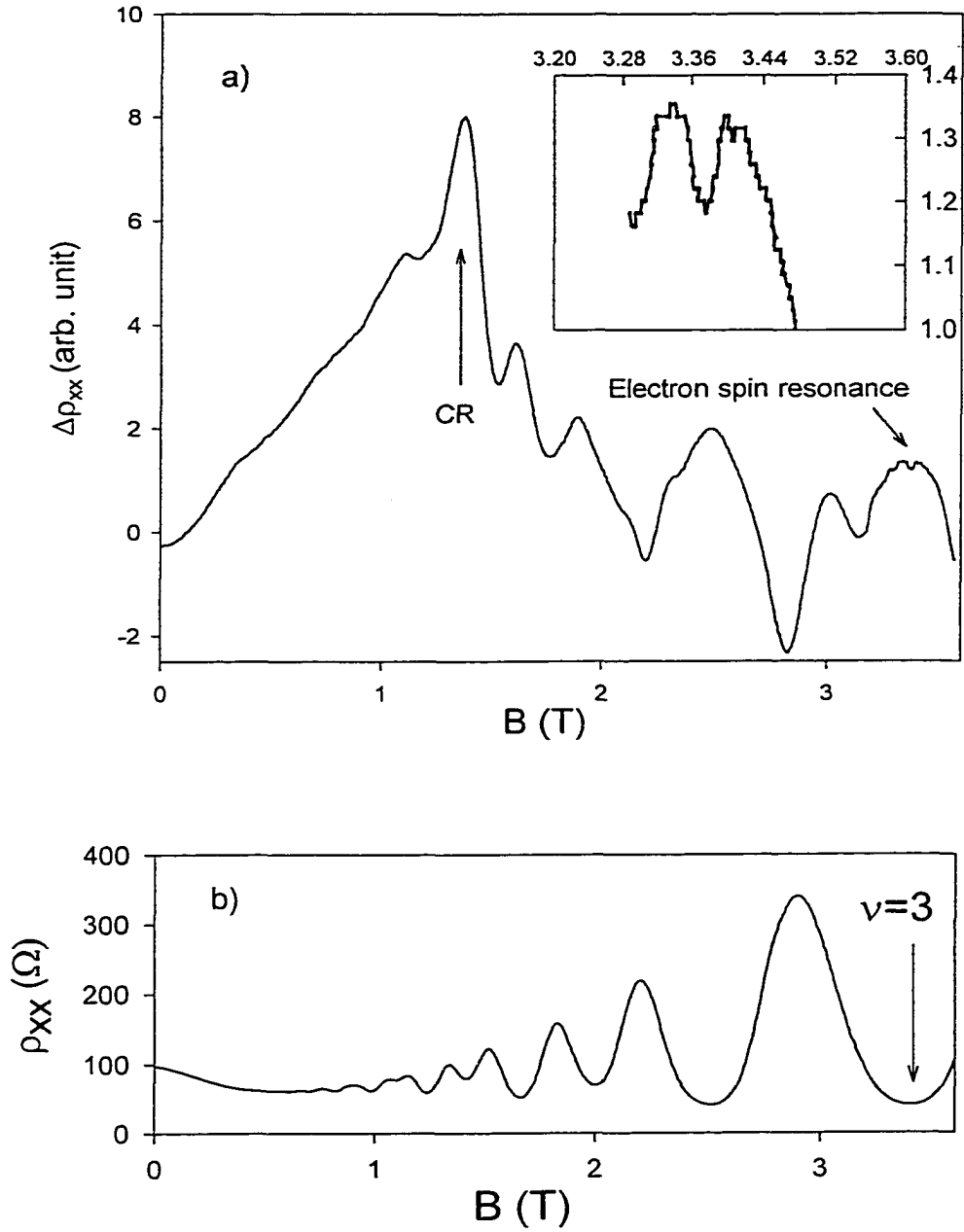


Figure 5.5: a) Electron spin resonance in S644 a symmetric quantum well at 184 μm using photo-conductivity measurements. The sample was tilted 30°. b) SdH trace of the same sample with the same tilt angle. The maxima at $\Delta\rho_{xx}$ is corresponding to the minima at ρ_{xx} while the photo-conductivity has a larger effect at the $\rho_{xx} = 0$.

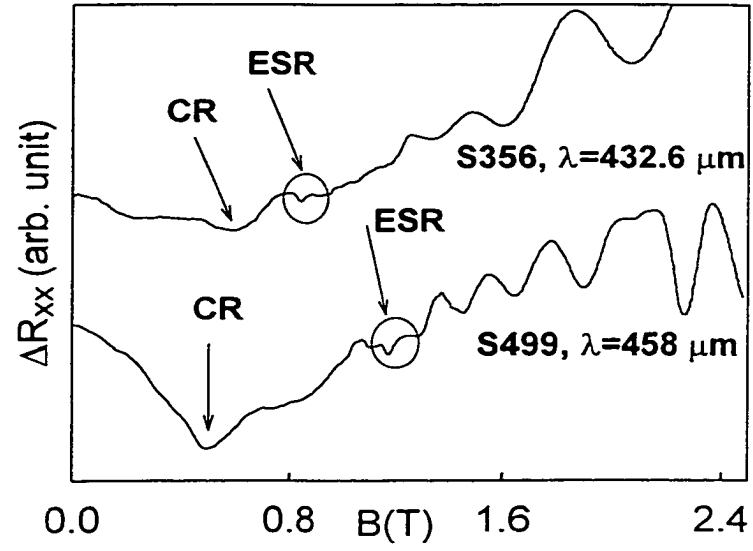


Figure 5.6: ESR at low magnetic field using photo-conductivity measurements in an asymmetric sample(S356) and a symmetric sample (S499) which both are tilted 45° .

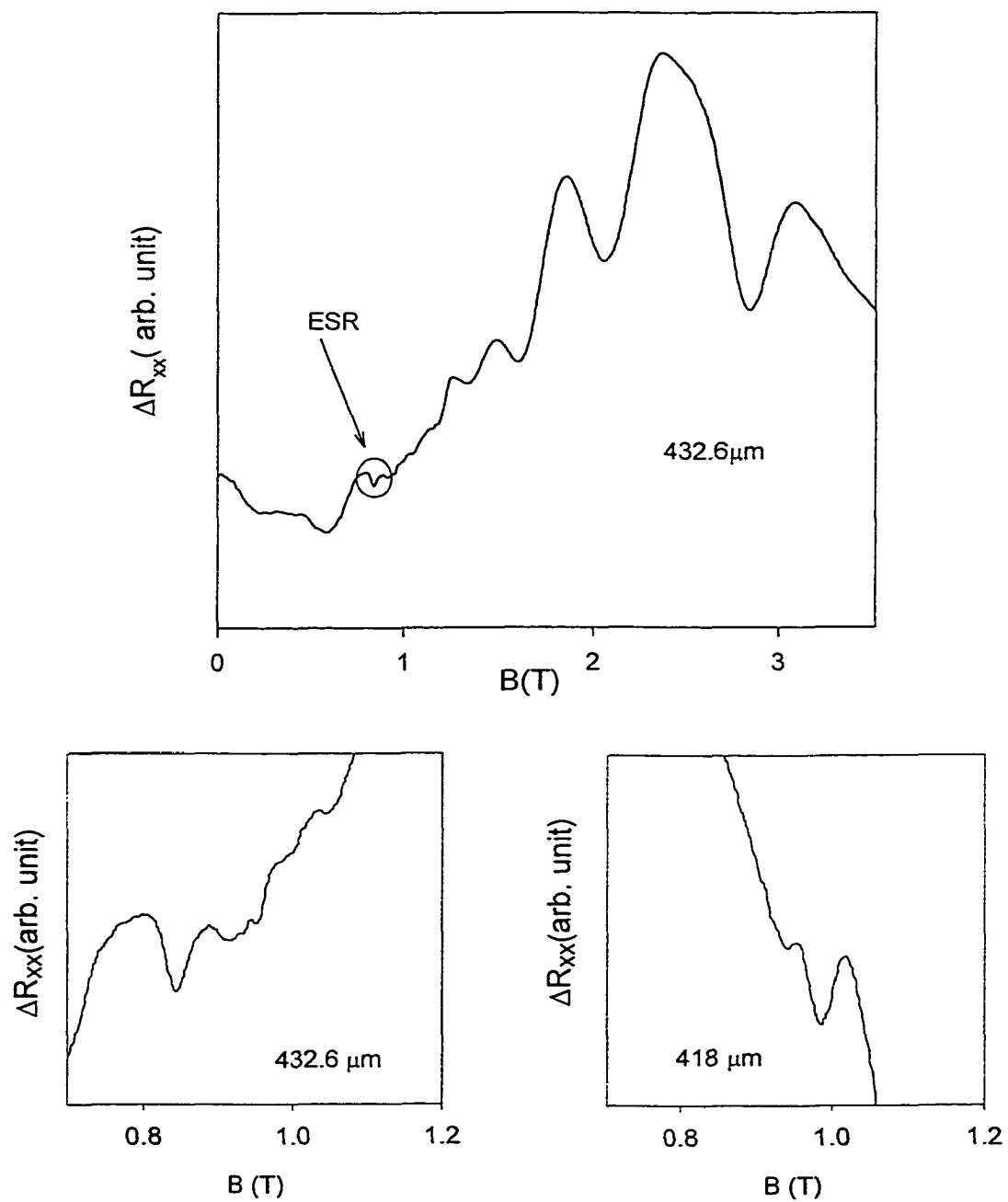


Figure 5.7: ESR at two different wave lengths for S356 which is tilted 45°.

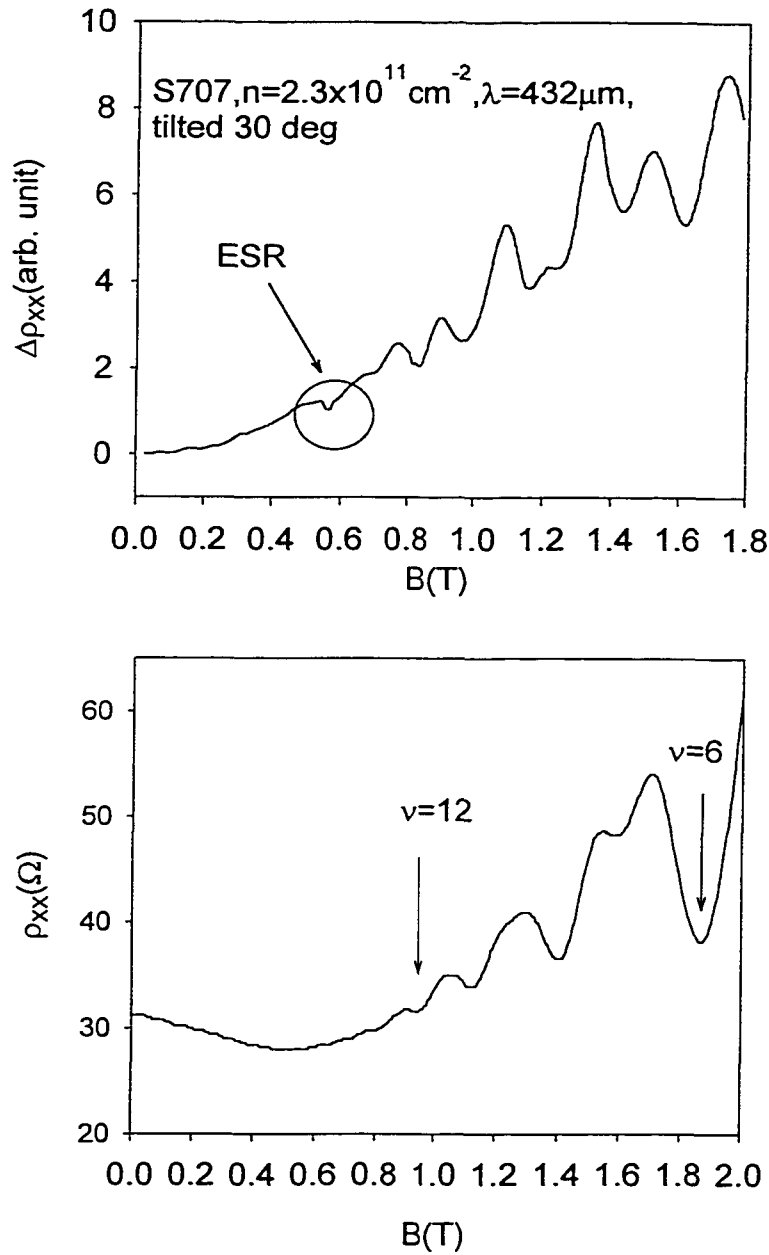


Figure 5.8: a) Shubnikov de Haas oscillations in magnetoresistance versus applied magnetic field. Several spin-split Landau level filling factors are indicated. b) Electron spin resonance at low magnetic field, which results in an apparent g -factor of ≈ -80

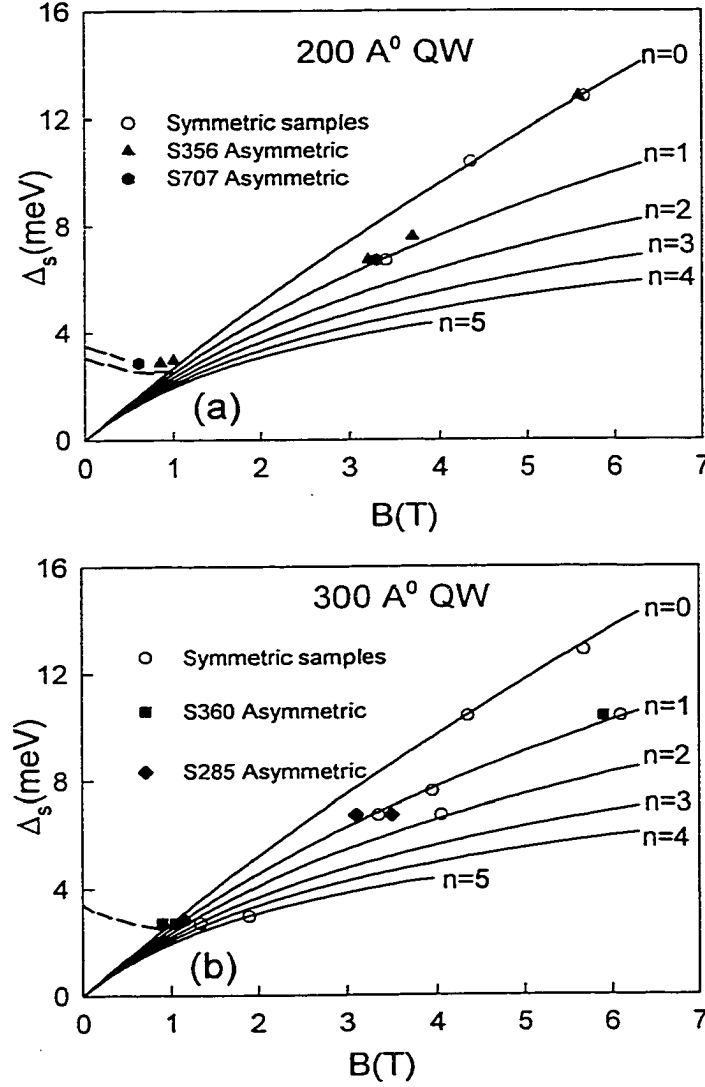


Figure 5.9: Measured spin splitting for symmetric and asymmetric samples. The solid curves are a single-parameter fit to the data of the symmetric samples. The dashed curves give the low-field prediction for zero field spin splitting using the experimental α values.

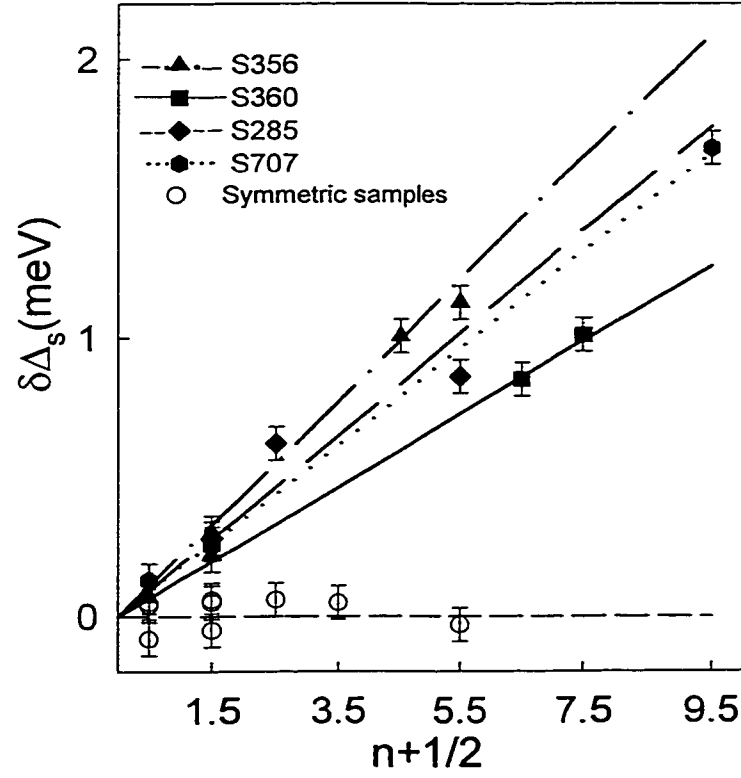


Figure 5.10: High-field spin-splitting shifts for 4 asymmetric samples. Differences between the measured spin splitting in asymmetric samples and the model for symmetric samples is plotted against $n+1/2$. The open circles are symmetric samples which show very small deviations from zero.

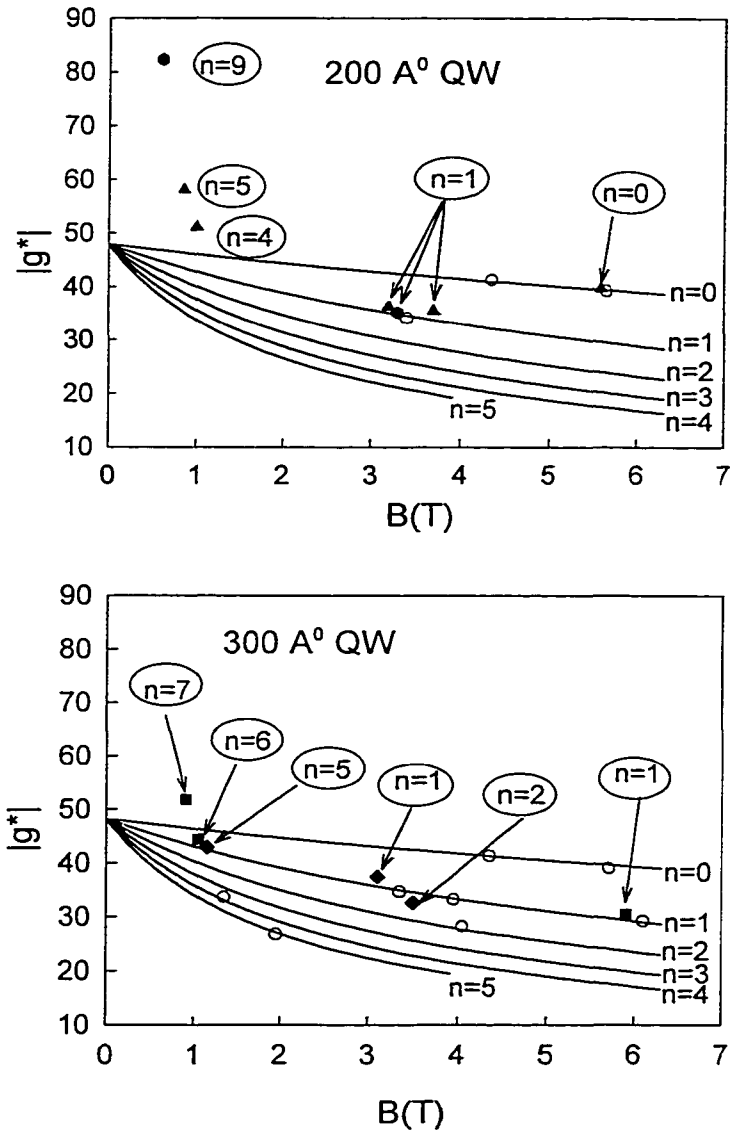


Figure 5.11: g-factor for symmetric and asymmetric samples. The solid curves are a single-parameter fit to the data of the symmetric samples. The deviations in g-factors are far beyond the InSb g-factor in the low field region.

Summary and future work

The goal of this work was to study the band structure and spin properties of the InSb quantum wells experimentally. Many new observations resulted such as spin resolved cyclotron resonance and zero field spin splitting in InSb quantum wells.

Our cyclotron resonance experimental results are in good agreement with our theoretical model. The values of the effective mass show the expected nonparabolicity behavior. We observed spin resolved cyclotron resonance in the high mobility samples with a rather unexpected amplitude pattern at $70.6\text{ }\mu\text{m}$ which might be a result of deviation from the Kohn theorem. More experiments using FTIR are required to understand the spin resolved cyclotron resonance in InSb.

We observe electron spin resonance using FIR laser spectroscopy in symmetric and asymmetric InSb quantum wells over a wide range of magnetic field and the Landau level index. The behavior of the asymmetric wells at low magnetic fields with g-factors far in excess of the bulk g-factor of InSb is due to spin splitting at zero magnetic field. Asymmetry-induced shifts in the spin resonance at high fields depend on the Landau level index as predicted by the Bychkov-Rashba model. In an extension of this work, we plan to compare samples where the asymmetry in the confinement potential is due to differing Al concentrations in the barriers on either

side of the quantum well to samples with asymmetric doping which were studied in this work.

The α values measured in this work ($1.5 \times 10^{-9} \text{eVcm}$) are among the largest reported as would be expected for a material like InSb with a large bulk g-factor. Recently, in gated InAs samples [61] α values ranging from 2×10^{-9} to 4×10^{-9} eV cm have been measured which suggest that we can achieve even larger α in InSb quantum wells. We are extending our spin resonance studies to gated samples. These should give us the ability to study the spin resonance in the absence of any applied magnetic field.

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Appendix A

The Laser operation

The following sections are a review on the laser operation which should be followed carefully. Reading the manual along with the instructions is necessary.

A.0.5 CO_2 laser:

The CO_2 laser is a CW laser which can generate an output power over 50 Watts on the strongest line with wavelengths in the range 9-11 μm . The size of the beam is about 7.5 mm which is vertically polarized.

The recommended gas mixture is 7% Carbon Dioxide, 18% Nitrogen(Oxygen Free) and 75% Helium 99.999% pure. Each mixture cylinder usually lasts for 60 hours and our supplier is the Air Gas company.

Following steps are a summary of the CO_2 laser operation:

- Check the level of the antifreeze-water mixture in the Chiller before each run.
- Be sure all the valves connected to the laser including the needle valve on the laser, are closed.
- Turn on the pump and let it warm up for few minutes.
- Open the main valve of the gas mixture, open the outlet valve and set the gas pressure to 15 Psi(using the pressure control valve on the cylinder regulator).

- Open the main valve on the pump and pump on the hose (be sure the pressure gauge on the pump shows zero).

- Open the valve which is located after the hose and before the needle valve (double check that the needle valve is closed). The gauge should still show zero.

- Open the needle valve slowly until the CO_2 pressure gauge reaches to a value between 10 $\rightarrow$ 15 mbar.

- **never pump on the CO_2 laser without gas flowing in the laser.**

- Turn on the chiller and let the temperature reaches to the set temperature (10-15 deg C). Be sure the heater's red light on the chiller occasionally flashes.

- Turn on the CO_2 laser power meter.

- **Avoid the CO_2 laser aperture at any time.**

- After achieving the operating temperature, turn the current control knob to its maximum position and switch the power supply on. At this point the interlock light should be on. After about 10 seconds the ready light will be on and the "HV ON" switch should be depressed. At this point the laser current should be about 30mA and can be adjusted later depending on the wavelength (see the laser manual for the calibration table).

- Depending on the wave length, the power meter should show few values which can be optimized by increasing the gas pressure and by adjusting the current according to the calibration table of the manual. **The operating pressure should never exceed 35mbar (The needle valve is very sensitive, try to adjust the pressure very slowly).**

The CO_2 laser has 1 hour warm up cycle and any adjustment to gain more laser power should be made after the warm up cycle (especially changing the grating position). **Do not direct the CO_2 laser beam into the FIR laser section**

unless FIR laser section is filled with the gas, otherwise over-heating of the optical parts will damage the O-rings and will cause a leak. For the CO_2 laser lines power, see the manual. The measured values might be slightly different from the recorded values.

A.0.6 FIR laser

The FIR laser covers the wavelengths between 40 μm and 1200 μm with the maximum output power of about 150mW. The size of the beam is about 11mm and the most commonly used active materials for the FIR lasers are : CH_3OH (Methanol), CD_3OD (D_4 Methanol, supplier OU chemistry department), CH_3OD (D_1 Methanol), $HCOOH$ (Formic acid, 98-100% extra pure) all of the chemicals are toxic (see the manual for a complete list).

The following steps are a summary of FIR laser operation, but in order to operate the laser one should read the manual as well!

- Be sure all the valves connected to the FIR laser are closed.
- Turn the FIR laser pump and let it warm up for a few minutes.
- Open the main valve on the pump and pump on the hose connected to the laser.
- Open the valve between the hose and the gauge, and pump up to the pump base pressure.
- Open the valve labeled FIRL gas fill and pump on the laser for a while and then open the valve labeled "flow through" and pump to the base pressure.
- Open the toggle and the needle valves, this way the manifold up to the valve on the bottle would be evacuated.
- If the liquid in the bottle is new and the air inside the bottle has not been

evacuated, freeze the liquid using liquid nitrogen and then open the valve of the bottle. Pump on the bottle to the pump base pressure. The liquid should be kept frozen during the pumping cycle. Close the bottle's valve and let the liquid warm up. Repeat the freezing procedure at least two times.

- At this point, open the valve on the bottle and fill the laser with gas. The optimum operating pressure depends on the wavelength and can be adjusted using the needle valve .

- The CO_2 beam can be directed in to the FIR laser by moving the mirror on the CO_2 side to CO_2 beam internal (avoid the beam aperture).

- The FIR laser has at least 30 minutes warm up cycle. During this time no adjustment should be performed.

- Depending on the intensity of the laser line a signal can be detected on an oscilloscope using a pyro-electric detector. The size of the signal can be optimized by adjusting the grating, the CO_2 or FIR gas pressure, CO_2 pump frequency, and the FIR laser cavity length.

- Usually, for the weak lines start from a lower pressure FIR gas (about 0.1 mbar) and after detecting the signal, adjust the pressure to gain the maximum signal.

The following table gives a list of FIR gas pressure for the most commonly used wave lengths which can be used as a guideline. For the other specifications of the laser lines, see the laser manual.

Wave length (μm)	FIR gas pressure (mbar)
513	0.08
458	0.2-0.1
432	0.4-0.6
418	0.2-0.1
393	0.2-0.1
290	0.2-0.4
253	0.2-0.4
184	0.2-0.4
163	≤ 0.1
119	0.2-0.1
103	0.6
96.5	0.4-0.6
70.6	0.08-0.1
67.5	0.4-0.6
46.7	0.2

A.1 Switch off

There are many important steps which should be followed carefully in order to switch off the lasers and will be discussed in the following sections.

A.1.1 The CO_2 laser:

- Depress the "H.V. OFF" switch located on the CO_2 power supply.
- **Turn the chiller off immediately, otherwise water condensation on the grating can occur.**
- Turn the key to the "power off" position.
- Close the CO_2 needle valve.
- Close the rest of the valves in the line between the CO_2 pump and the CO_2 laser and turn off the pump .
- Close the main valve on the CO_2 gas cylinder.

A.1.2 The FIR laser:

- Close the valve on the liquid's bottle.
- Open the needle valve all the way.
- Keep pumping on the FIR laser up to the pump base pressure which might take more than 1 hour.

This step is extremely important, otherwise condensation of the laser active materials will damage the optics.

- Close all the valves connected to the laser and the pump.
- It is not necessary to turn off the pump.

A.1.3 Maintenance:

- Check the level of antifreeze-water mixture every week.
- Check the batteries in the pyro-electric detector and the amplifier Box.
- Change the alumina beads and the oil of the pumps regularly. (For the CO_2 laser pump after at least 50 hours of the laser run)
- Check the CO_2 laser power on a specific wave length after the warm up cycle and fine tuning of the laser. If the laser power is significantly different from the recorded values most likely the problem is related to the optics.
- Remove the laser cover. Using a mirror check the Brewster windows and the grating.
- If the Brewster windows are dirty follow the steps in the manual to remove them. The ZnSe Brewster windows are very soft and should be handle with care.
- In case of minor contamination The following procedure can be used to clean the windows:
 - 1- Use Acetic acid (optically graded, supplier Fisher scientific) and make it

dilute to 35% concentration. Leave the Brewster window(s) for 3-5 minutes in the acid.

2- Put the Brewster window(s) on lens tissues and rinse with Acetone. Wipe the window gently by lens tissues. If the windows are too dirty leave them in Acetone over night. Repeat this step using Alcohol.

If the above procedure is not sufficient to clean the windows, they should be polished. Contact II-VI (two-six) company in order to polish the windows.

In order to optimize the output power, adjustment of the CO_2 output coupler mirror would be necessary after mounting the Brewster windows. Follow the instruction in the manual using a special hex key screw driver from the laser tool box. Avoid the laser beam and wear a safety glass.

The alignment of the output coupler mirror is very critical and one should be extremely careful not to scratch the surface of the mirror. Usually changing the Brewster windows changes the calibration of the grating.

In case the power lost is not due to the windows, contact the company for more information.

A.2 Laser stability

One of the most important steps to gain the best signal to noise ratio is fine tuning of the laser lines. Any fluctuation in the CO_2 or FIR lasers gas pressure would effect the laser stability. Monitoring the gas pressure during the experiment, especially for the FIR laser is very important.

Another important source of the laser fluctuation is the variation in the CO_2 laser cavity length which can be tuned by a piezo ceramic (PZT) attached to the

CO_2 output coupler. The tuning of the PZT can be obtained using the “PUMP FREQUENCY” knob on the hand control unit or by using the Edinburgh ALS1 laser Stabilizer (see the Stabilizer manual). Tuning of the FIR laser length (using $\pm$ keys on the laser hand control) improves the laser line power and the stability as well. See the manual for more details.

Appendix B

The samples

List of the samples for transport, cyclotron resonance and spin resonance experiments:

Sample	Well Width	Al Concentration
Number	Å	%
S203	300	7
S206	300	7
S209	300	11
S285	300	9
S341	300	9
S344	300	9
S346	300	9
S356	200	9
S360	300	9
S372	300	9
S377	300	9
S387	300	5
S391	300	5

Sample	Well Width	Al Concentration
Number	Å	%
S499	300	9
S550	300	9
S577	300	9
S617	300	9
S644	300	9
S702	200	9
S704	200	9
S707	200	9
S710	300	9
S716	300	9
S717	300	9
S724	300	9

The samples with the numbers between S679-S725 which are not listed above were used only for transport measurements. The following symmetric(S) and asymmetric(A) samples were used in the spin resonance experiments. We also observed spin resonance in S203(A) and S391(S) with 7% and 5% Al concentrations respectively. The changes in the densities listed in the following table for a specific sample are due to different cool downs.

Sample Number	Sample Density ($\times 10^{11} \text{cm}^{-2}$)	Wavelength+ Tilt μm
S285(A)	1.85	184,432, 30°
	1.7	184,50°
S356(A)	1.3	96.5,30°
	1.6	163,45°
	1.2	184,50°
	1.3	432,418,45°
S360(S)	2.5	119,45°
	2.6	458,30°
	2.2	458,45°
S372(S)	2.8	184,50°
S377(S)	2.5	119,45°
	2.9	418,50°
S499(S)	2.0	458 45°
S644(S)	2.1	184 30°
S704(S)	1.1	184,50°
	0.95	119,30°
S707(A)	2.3	184,432,30°
S710(S)	0.95	119,30°
S716(S)	1.4	96.5,30°
	1.4	163,50°

Appendix C

Eccosorb

C.1 Eccosorb

Eccosorb (Series 110, Emerson & Cuming Inc.) is a castable resin (toxic) and is supplied as a two component system X and Y (100 part X / 12 part Y).

It is recommended to follow the instruction given below (See the Emerson& Cuming instruction sheet as well) :

It is important to wear mask, gloves and safety glasses all the time, even during machining.

1-Mix part X in its shipping container to a uniform consistency before removing any material.

2-If crystals appear in Part Y, gently heat to 150° F until crystals go into solution.

3- Weigh out the desired amounts of both parts.

4- Heat part X to about 150° F. This will reduce its viscosity.

5- Blend part X and part Y thoroughly and remove air by vacuum evacuation.

6- Pour in a mold (preferably pre-heat to about 150° F) and heat up the mixture for 1 hour at 300° F (165° F for 12 hours or 200° F for 4 hours or 250° F for 2 hours).

Appendix D

Wire Codes

D.1 The Ge:Ga doped detector

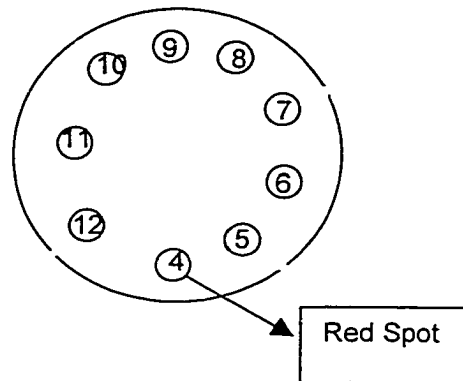
The amplifier box (Ln6) is connected to the top of the vacuum-can by two shielded cables. The pins on the connector of the amplifier (labeled by letters A $\rightarrow$ F) are connected to the Fisher connector (labeled by numbers) on the the Vacuum can as following:

- Pin A $\rightarrow$ Bias (pin 6 , green wire of the 2nd cable).
- Pin B $\rightarrow$ Drain (pin 5 , black wire of the 1st cable).
- Pin C $\rightarrow$ Gain (pin 10, white wire of the 1st cable).
- Pin D $\rightarrow$ Source2 (pin 12, green wire of the 1st cable).
- Pin E $\rightarrow$ Source1 (pin 4 , red wire of the 1st cable).
- Pin F $\rightarrow$ Feedback (pin 8 , white wire from the 2nd cable).

D.2 The helium level sensor

The Fisher connector on the top of the dewar is connected to the helium level sensor and to the resistor on the bottom of the dewar:

- Pin 4 $\rightarrow$ Red wire.
- Pin 6 $\rightarrow$ Blue wire.



-Pin 8 → Black wire.

-Pin 10 & 11 connected to the 100Ω resistor.

Appendix E

2D density of states in a magnetic field

This section is a brief note on the 2D density of states in a magnetic field [12, 13, 36] from private communications with Dr. Bruce Mason. The purpose of this calculation is to define the 2D electron density in a magnetic field which can be used in the self consistent calculations in chapter (2) to define the electron-electron interaction potential. The potential can be used in the Schrödinger equation to find the eigen states in the growth direction (z direction) which will be used to calculate the fan diagram (see chapter(4)). Assumptions:

- a) **B** field is perpendicular to the 2D electron gas (2DEG).
- b) Analytic density of states using the Born approximation.

Described energy eigen states can be written as:

$$E_{i,n,\sigma} = E_{i,\sigma} + \hbar\omega_c(n + 1/2) \quad (\text{E.1})$$

$$E_{i,\sigma} = E_i \pm 1/2g^*\mu B \quad (\text{E.2})$$

where i is the subband index, σ is spin index, n is the landau level index, and g^* is the energy dependent g-factor described in chapter (4)). Assuming that the per-

pendicular magnetic field does not change the Schrödinger equation in z direction, we can write:

$$-\frac{\hbar^2}{2m^*} \frac{d^2 f_i(z)}{dz^2} + V(z)f_i(z) = E_i f_i(z) \quad (\text{E.3})$$

where m^* is the energy dependent effective mass in chapter(2) and $V(z)$ is the potential in the quantum well along z direction.

In the Born approximation each Landau level is of the form:

$$DOS(E, E_{i,n,\sigma}) = N_l(1 - ((E - E_{i,n,\sigma})/\Gamma)^2)\Theta(\Gamma - |E - E_{i,n,\sigma}|) \quad (\text{E.4})$$

where Γ is a width parameter arising from scattering and Θ is a step function. The density of the state in this picture is plotted in Fig. E.1.

In order to calculate N_l we can write:

$$\int DOS(E, E_{i,n})dE = \frac{1}{2\pi l_b^2} \Rightarrow N_l = \frac{3}{8\pi\Gamma l_b^2}$$

where $l_b = (\frac{\hbar}{eB})^{1/2}$ The 2D electron density is given by:

$$\begin{aligned} N_{2D} &= \int_0^{E_F} DOS(E)dE \\ DOS(E) &= \sum_{i,n,\sigma} DOS(E, E_{i,n,\sigma}) \\ &= N_l \sum_{i,n,\sigma} \int_0^{E_F} (1 - ((E - E_{i,n,\sigma})/\Gamma)^2)\Theta(\Gamma - |E - E_{i,n,\sigma}|)dE \\ N_{2D} &= N_l \sum_{i,n,\sigma} \left[\frac{4\Gamma}{3}\Theta(E_F - (E_{i,n,\sigma} + \Gamma)) + ((E_F - E_{i,n,\sigma}) - \frac{(E_F - E_{i,n,\sigma})^3}{3\Gamma^2} + \right. \\ &\quad \left. \frac{2\Gamma}{3})\Theta(\Gamma - |E_F - E_{i,n,\sigma}|) \right] \end{aligned}$$

From the definition of filling factor, number of filled Landau level is given by:

$$\nu = \sum_{i,n,\sigma} [\Theta(E_F - (E_{i,n,\sigma} + \Gamma)) + (1/2 + \frac{3(E_F - E_{i,n,\sigma})}{4\Gamma} - \frac{1}{4}(\frac{E_F - E_{i,n,\sigma}}{\Gamma})^3)\Theta(\Gamma - |E_F - E_{i,n,\sigma}|)]$$

In order to define the Fermi energy one can search for the root of the following expression[13]:

$$F(E) - \nu = 0$$

where $\nu = \frac{N_{2D}h}{eB}$ and $F(E)$ is given by:

$$F(E) = \sum_{i,n,\sigma} [\Theta(E - (E_{i,n,\sigma} + \Gamma)) + (1/2 + \frac{(E - E_{i,n,\sigma})}{4\Gamma}(3 - ((E - E_{i,n,\sigma})/\Gamma)^2))\Theta(\Gamma - |E - E_{i,n,\sigma}|)]$$

For any value of E , $F(E)$ can have 3 different values:

$$E_{i,n,\sigma} < E - \Gamma \rightarrow F(E) = 1$$

$$E_{i,n,\sigma} > E + \Gamma \rightarrow F(E) = 0$$

$$E - \Gamma < E_{i,n,\sigma} < E + \Gamma \rightarrow F(E) = (1/2 + \frac{E - E_{i,n,\sigma}}{4\Gamma}(3 - \frac{(E - E_{i,n,\sigma})^2}{\Gamma^2}))$$

The value of the Fermi energy can be used to define the 2D electron density and therefore the potential in the quantum well.

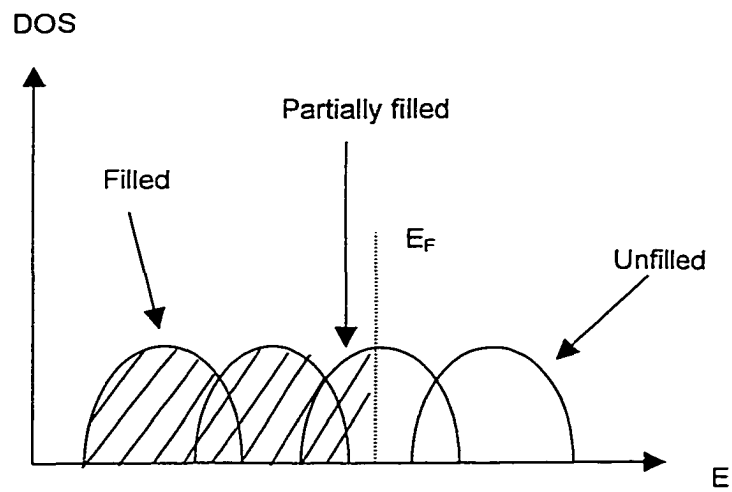
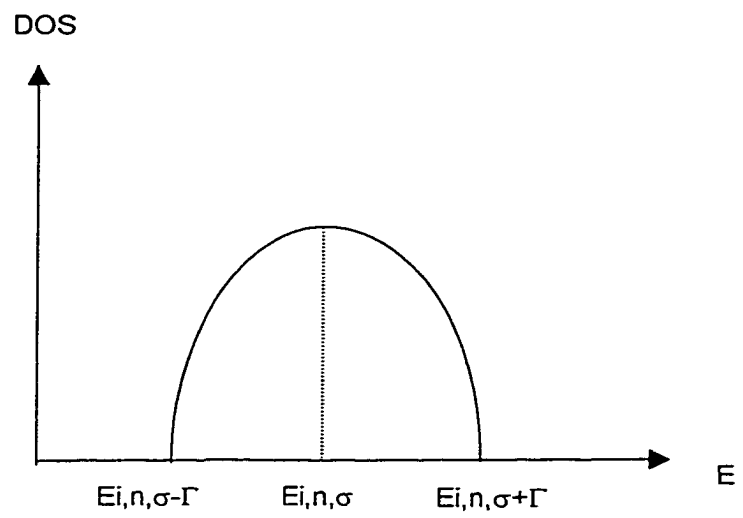


Figure E.1: The density of states in the Born approximation.

Appendix F

Solving the Poisson equation

Suppose we want to solve a two-point boundary conditions problem which is given by[62]:

$$\frac{d^2y}{dz^2} = F(z) \quad (\text{F.1})$$

with two boundary conditions $y(z_0) = f_0$ and $y(z_{M-1}) = f_{M-1}$, where f_0 and f_{M-1} are known and M is the number of grid points in z direction. At any point z_i , the above equation can be written as a finite-difference equation:

$$\frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta z^2} = F(z_i) \quad (\text{F.2})$$

This equation is equivalent to a system of M-2 linear equations and unknowns. As an example, if M=6 we can write:

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \Delta z^2 F(z_1) - y_0 \\ \Delta z^2 F(z_2) \\ \Delta z^2 F(z_3) \\ \Delta z^2 F(z_4) - y_5 \end{bmatrix} \quad (\text{F.3})$$

For any arbitrary large M, the structure of Eq.(F.3) remains the same, and such a matrix is called tri-diagonal. Gauss elimination can be used to solve a tri-diagonal system of equations. In our case, the same procedure can be used to calculate the

Hartree potential described in chapter(2) which is given by:

$$\nabla^2 V_H(z) = \frac{-e^2}{\kappa \epsilon_0} \sum_{i=1}^N \frac{1}{\pi} \int_0^{K_{Fi}} K_t dK_t |\psi_{i,K_t}(z)|^2$$

The right hand side of the equation is just a function of z , which can be calculated at each grid point. At the edges of the well $V_H(z) \rightarrow 0$ which would be the boundary conditions in this case .