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STATISTICAL AND SPECTROPHOTOMETRIC IDENTIFICATION

OF BINARIES IN STAR CLUSTERS

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ABSTRACT

The incidence of duplicity and distribution of secondary masses in open clusters are investigated using statistical techniques on UBVRI photometry. Important results are: (1) identification of stars likely to be binaries in the Hyades, Praesepe, and the Pleiades, (2) the scatter about the main sequences of single stars for these clusters is compatible with that of a one-dimensional sequence within the limits of observational error, (3) the total scatter about the single star main sequences for the Hyades and the pleiades are analyzed and found to be incompatible with the secondary mass distribution function Abt and Levy (*Ap. J. Suppl.*, *00*, 000, 1976) have found for field stars, and (4) unresolved binaries in open clusters can produce an error in distance (as determined by main sequence fitting) of about 5 per cent.

The effects of unresolved duplicity on spectral features of solar-type stars are examined and it is found that the strengths of Na D lines of composites are strongly dependent on the mass ratio. The D lines of late-type Hyades stars are scanned using narrow band photometry. Results are: (1) the effects of duplicity on D line indices are not negligible (the analysis of these effects is independent of any knowledge of what features might be in the pass-bands of the photometry), (2) the effects of duplicity suggest themselves in an analysis of the scatter in a (B-V)-Index plot, and (3) there

is weak support for the earlier conclusion that the secondary mass distribution function of the Hyades is not compatible with Abt and Levy's field star results.

The technique of scanning D lines to analyze duplicity in open clusters looks promising and should be further studied.

CHAPTER I

INTRODUCTION

The research described in this thesis represents three lines of inquiry into various aspects of duplicity among stars. The first, discussed in Chapter III, seeks to determine the incidence of duplicity or multiplicity in several open clusters by analyzing UBVRI photometry of them. As a result of this analysis, the frequency of binaries (in clusters triple and quadruple stars do not last long and we can consider all multiples as binaries; see Chapter II, Section C) and the particular stars which are likely to be binaries whose components differ by less than about 3^m in brightness are identified. The second, described in Chapter IV, investigates the effects of unresolved duplicity on spectral features and derived atmospheric parameters by means of measured spectral properties and model atmosphere analysis. From second of these approaches, interesting consequences are found for elemental abundances, electron pressures and surface gravities if one unwittingly takes a double star for a single star and models an atmosphere for it. One result that is particularly interesting is that the strength of the Na D lines of the composite spectrum of two stars is strongly dependent on the ratio of the masses of the two components. This result leads to the third line of inquiry. It includes measurements I have made of D line strengths of late type dwarfs in the Hyades using narrow band photometry. These measurements are analyzed and the incidence of binaries with components differing by greater than about $2\overset{m}{.}5$ in brightness is discussed, and compared with the results of Chapter III.

I have tried to write this thesis so that the general reader will be able to follow. Chapter II contains background material for understanding the significance of the results and the reasonableness of the assumptions. This chapter is intended to be detailed only to the extent necessary for a good grasp of the purpose and results of the research. For more complete accounts of the subjects discussed, the interested reader may go to the references cited where he or she should have no trouble satisfying his or her curiosity. In Chapter IV, rather than burden the reader with a poor parroting of the excellent texts available, I have given a very cursory description of the model atmospheres approach and simply referred to those aforementioned texts. I have tried to be complete in my discussion of the actual research. It is my hope that the general reader will be able to see what assumptions were made, and understand what was done and how the conclusions were drawn.

CHAPTER II

BINARY STARS AND OPEN CLUSTERS

A. An Introduction to Binary Stars and their Importance to Astrophysics.

The study of binary stars is quite important to astrophysics; some of the things that can be determined from a knowledge of binaries are: the masses of stars, stellar radii, distances to stars, and limb darkening coefficients. All of these data are vital to the study of stellar physics. The incidence of duplicity among stars can provide important clues about star formation, the formation of planetary systems, galactic structure, and possible systematic errors in luminosity for real stars as opposed to model stars. For an elementary discussion of these and other interesting areas that a knowledge of binary stars pertains to, see Swihart(1); for a more advanced discussion, see Batten(2). It seems that the incidence of binaries may also have important implications for computer models of the evolution of open clusters (see Section C of this chapter).

A binary star is simply two stars in closed orbit about a common center of mass bound by their mutual gravitational attraction. One can specify the dynamical properties of most binary systems (there are some very interesting examples of binaries that exhibit odd behavior, such as apsidal motion, which require further specification) by seven parameters. These parameters are the orbital elements; they are: (see Figure 1)

P = the orbital period.

i = the inclination of the orbital plane to the tangent plane of the sky.

Ω = the position angle (measured from north through east) of the line of nodes joining the intersections of the orbital and tangent planes, and measured in the latter.

ω = the angle between the direction to the ascending node (at which the star crosses the tangent plane while receding from the observer) and that to the point of closest approach to the two stars (periastron) measured in the orbital plane, in the direction of orbital motion.

a = the semi-major axis of the orbit.

e = the eccentricity of the orbit.

T = the time at which the two stars pass through periastron.

(These definitions were taken from Batten, *op.cit.*) Sometimes, for convenience, other elements are used in place of these (see below). The number of elements that can be determined depends on the particular kind of binary under observation: visual, spectroscopic or eclipsing.

A visual binary is a system of two stars that can be resolved optically and are in orbit about one another. Astrometric binaries and common proper motion pairs are subclasses of the visual binaries. Astrometric binaries are stars that do not have visible companions but

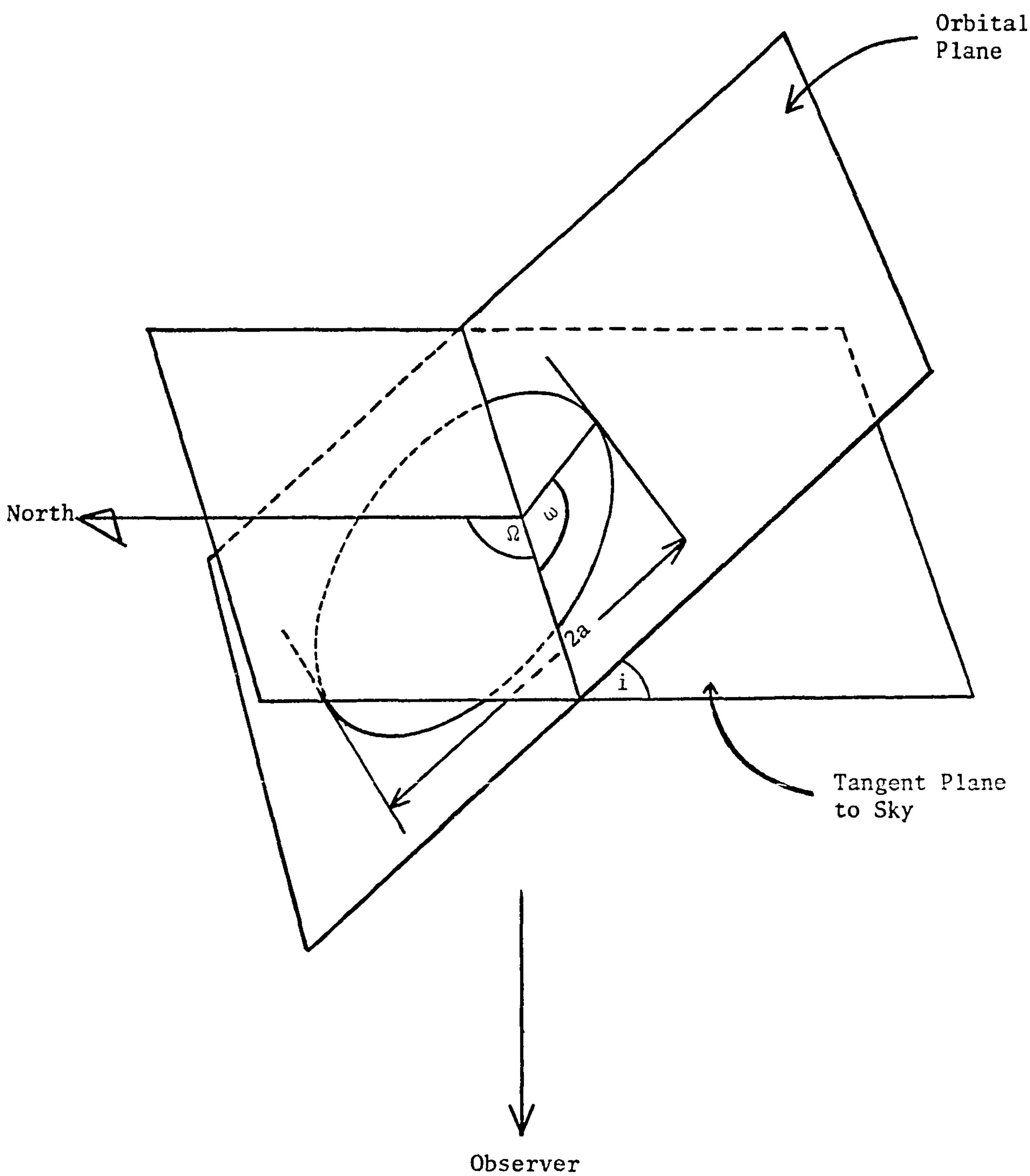

Figure 1.

can be seen in periodic motion relative to background stars; some of these are suspected of having planetary companions, these being the only indication of planetary systems other than our own. Common proper motion pairs are pairs of stars moving with respect to background stars at the same velocity but too widely spaced to show any presumed period over reasonable lengths of time. Visual binaries are detected mainly visually or sometimes by taking photographic plates of a suspected system at intervals (on the order of years) and carefully measuring the relative positions on the developed plates. The fact that visual binaries must be resolvable optically and show motion over reasonable time periods means that, astronomically speaking, they can't be too far away nor can their periods be too short or too long. Thus there are systems that would otherwise be visual binaries but that they have the misfortune of being, say, too far away. This and other selection effects to be discussed later lead to difficulties in obtaining good statistics, for example: incidence of binaries, mass spectrum of secondaries, mean angular momentum. From observations of a visual binary the elements P, $\pm i$, ω, e, and T can be determined and Ω to an ambiguity of $180°$ (unless one has radial velocity measurements one cannot distinguish the ascending node from the descending node). The apparent semi-major axis, a'', can be determined but a is unknown unless one also knows the system's distance. If radial velocity measurements are available, they can be used to scale the orbit and thus give the distance.

A spectroscopic binary is a binary system in which the two stars are close enough together and in the proper orientation to have radial velocities that vary enough so as to be detected by a spectroscope. Spectroscopic binaries aren't affected by a distance selection effect, but their discovery does depend on the inclination of the orbital plane; for example, a system with its orbital plane in the tangent plane of the sky would never be seen as a spectroscopic binary since it would have no radial velocity changes over its period. Spectroscopic binaries are further divided into two categories depending on whether only one spectrum is seen or both spectra are seen. The first case is denoted by "SB1" and the second, by "SB2". In as much as the orbital elements given above are not directly observable for a spectroscopic binary, for convenience, others are used. The elements of a spectroscopic binary are:

V_o = the radial velocity of the center of mass.

K_1 = half the total range of the radial velocity variation of the brighter star.

K_2 = half the total range of the radial velocity variation of the fainter star (this is only available for SB2's, of course).

The elements e, ω, and P can be determined from the velocity curve (the velocity of a component as a function of time). From Kepler's laws, $a \cdot \sin(i)$, $m_1\sin^3(i)$, and $m_2\sin^3(i)$ can be determined if both spectra are observable. If only one spectrum is observed one has instead: $a_1\sin(i)$, and $f(m) = m_2^3 \sin(i)/(m_1+m_2)^2$, the mass function.

An eclipsing binary is one in which the orbital inclination is sufficiently close to 90° so that the two components periodically intersect each other's cone of sight, resulting in an eclipse. If one observes such a system with a photometer or a series of photographic plates, one sees a periodic variation in the apparent magnitude, and, indeed, this is how such binaries are discovered. Thus, whether or not a binary is observed to be an eclipsing binary depends on its inclination.*

These three classes of binaries are by no means exclusive and there is little physical content in them. Indeed, Batten[(2)] has said, "The techniques used to observe these different kinds of binary are very different, and a classification based on them is very useful for distinguishing the astronomers who study binary systems. It has few other merits, however." For instance, if observed carefully enough with a spectroscope, an eclipsing binary would be a spectroscopic binary. The point that needs to be made here is that, in the discovery of each of these types, there are selection effects that keep one from obtaining a complete survey of binary stars as a run of all their physical variables. In certain cases, for particular orbital elements and types of stars, discovery can be extremely unlikely; whereas other situations lend themselves to ready discovery and observation. This is most unfortunate since statistics that might be derived from a complete survey could be very important to so many areas of astrophysics. It was my goal to get around some of

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*All eclipsing binaries are potential spectroscopic binaries.

these selection effects that prompted mu... of the research in this thesis.

B. The Incidence of Binary Systems.

The incidence of binaries has been a matter of much interest for some time now. Questions such as how does duplicity affects stellar evolution, is there any relation between an open cluster's mean rotation (per star) and its frequency of binaries and what is the mean angular momentum perstar in the galaxy are important to astrophysics and are related to the incidence of binaries. The answers to these questions have not been clear largely because of the difficulty in dealing with the various selection effects associated with each type of binary. One can fairly easily take account of the selection effect involving the inclination, but what is one to do with the fact that early stars have lines that aren't as sharp as the later type stars thus making discovery of spectroscopic binaries more difficult in early types? The situation is not hopeless, though; shortly, I will discuss a very important paper by Abt and Levy that takes account of selection effects and arrives at important conclusions.

Estimates for the duplicity rate among field stars have ranged from about 30 per cent to 70 per cent. This is indicative of the difficulty in estimating selection effects. It is difficult in the first place to obtain a random sample of stars; one obvious factor in selecting a group of stars to study is that bright stars are easier to see and consequently, the intrinsically faint stars that one observes will be on the average closer to us. Even among the bright stars, not all types of binaries will be equally easy to detect; for instance, visual binaries whose components are separated by less than an arcsecond and differ by more than 2^m will be more difficult to detect than those more widely spaced or equally bright.

One way around some of these problems is to select as the group of stars to be studied an open cluster (an open cluster is a cluster of about 10^2 stars weakly gravitationally bound to one another and is a few parsecs in diameter; an open cluster is also referred to as a galactic cluster). By doing so one would hope to get a reasonably complete sample of stars. The results of several studies on such clusters are given in Table 1 (from Batten). These studies were made as investigations of mean rotational velocity and duplicity in galactic clusters. Notice the wide disagreement between independent estimates. There are a couple of reasons for this: the already mentioned selection effects and the fact that in order to take into account the difficulty of observing faint stars one needs the van Rhijn or luminosity function (the distribution of stars as a function of luminosity or mass) which is uncertain for later type stars in galactic clusters. One is then in the awkward position of making statistical estimates based on already poor statistics.

In an effort to circumvent these and other similar problems, Abt and Levy(3) have selected as their sample 135 F3-G2 IV or V bright stars [for an elementary discussion of spectral types see Swihart(1)], that is, 135 roughly solar type stars. Quoting Abt and

Table I. Frequencies of Spectroscopic Binaries in Twelve Galactic Clusters and Associations [from Batten[(2)]]

Cluster	Percentage of Spectroscopic Binaries (known-suspected)	Investigators (see Batten)
α Persei	11.0-35.6[1]	Kraft (1967)
Coma	20.5-32.3	Kraft (1965)
Hyades	17.4-31.9[2]	Kraft (1965)
I.C. 4665	"many"	Chaffee and Abt (1967)
N.G.C. 6475	42.1-47.4	Abt *et al.* (1970)
Pleiades	12.3-24.6[3]	Anderson *et al.* (1966)
Praesepe	19.7-28.5[4]	Dickens *et al.* (1968)
Ursa Major	16.3	Geary and Abt (1970)
I Lacerta	16.9-47[5]	Blaauw and van Albada (1963)
I Orion	16.9-47[5]	Blaauw and van Albada (1963)
II Perseus	16.9-47[5]	Blaauw and van Albada (1963)
Sco- Cen	9.8-11.0	Slettebak (1968)

[1]Independent estimate by Petrie and Heard (1970) 6.5-16.8%.

[2]Independent estimate by Treanor gives 26%.

[3]Independent estimate by Abt *et al.*

[4]Independent estimate by Treanor from smaller sample, gives 8%.

[5]Binary frequency is mean for all three groups.

and Levy[3]:

> The present study is one of several that treat a specific section of the main sequence. It was deemed necessary because (1) previous studies gave very different binary frequencies for stars at the middle of the main sequence, namely 18 to 54 per cent spectroscopic duplicity, (2) those studies were based on published data that were subject to serious selection effects and allowance was usually not made for those effects, and (3) the studies often did not include all kinds of binaries, namely spectroscopic binaries, visual binaries, and common proper motion pairs. In addition, the apparent variation in duplicity along the main sequence from 72 per cent for B3V stars to 10 per cent for dwarf K and M stars does not include a factor for the increased difficulty in detecting spectroscopic binaries while progressing to low mass stars, and therefore does not necessarily represent a real difference.
>
> This study concentrates on the bright field stars of types F3-G2 IV or V. The range was selected for the following reasons: (1) to avoid the region of the Am and Ap stars, which terminate at F1 or F2; (2) to concentrate on stars with narrow lines so that accurate radial velocities can be determined; (3) to include the stars of solar type for comparison with the solar system; (4) to stop at G2 because later type dwarfs tend to be rate among the brighter stars. In addition, the selection of stars brighter than $V = 5\overset{m}{.}5$ includes stars that have been studied well by visual observers.

Abt and Levy took about twenty high dispersion spectra with the 2.1-m coude spectrograph of each of their program stars (save for a few, very well measured cases for which accurate radial velocity data were already available). From the measured radial velocities, they found twenty-five new spectroscopic binaries. In their sample there were already twenty-one known spectroscopic binaries, twenty-three visual binaries and twenty-five common proper motion pairs. They then analyzed the selection effects in their study under the following assumptions:

A. Random orientation of orbital axes, even within multiple systems. This seems reasonable because (1) the lack of evidence [Huang and Wade[4]] for preferred galactic distribution of eclipsing binaries, (2) a lack of dependence of inclinations in visual systems on galactic latitude [Finsen[5]], or (3) the evidence for random orientation of rotational axes of field Ap stars [Abt, Chaffee, and Suffolk[6]].

B. Failure to detect any SB1's with K_1 less than 2.0 km sec^{-1}.

C. Failure to detect any SB2's with K_1 less than 20.0 km sec^{-1}.

D. Failure to detect any VB's (visual binaries) with a less than 0".3 and ΔV greater than $0^m.4$.

E. Failure to measure half of the VB's with a greater than 0".3 and less than 1".0 and ΔV greater than $2^m.0$ and less than $3^m.5$ and essentially all of those with ΔV greater than $3^m.5$.

F. Failure to measure one third of the VB's or CPM's with a greater than 1" and V_2 greater than $10^m.0$ and less than $12^m.0$ or two thirds of those with V_2 greater than $12^m.0$ and less than $13^m.0^V$ or all of those with V_2 greater than $13^m.0$.

G. Failure to discover almost all CPM's with a greater than 75".

I refer the interested reader to Abt and Levy's paper for further comment on the reasonableness of these assumptions. Using A-G Abt and Levy estimate how many binaries have gone undetected. They divide binaries into categories of secondary mass (five different masses) and period (seven ranges) and estimate the incompleteness of their survey in each of the thirty-five categories. For instance:

> The primary stars of types F3-G2 have masses 1.5-1.0 $M_\odot$: the average is about 1.30 $M_\odot$. The double lined spectroscopic binaries have nearly equal (and known) secondary masses, but all systems with M_2 about 1.2 $M_\odot$ will not be detected because some of them have inclinations, i, (between the plane of the orbit and the plane of the sky) that are too small for the velocity amplitude $K_1 = 2 P^{-1} a_1(1-e^2)^{-\frac{1}{2}}\sin(i)$ to be greater than 20 km sec^{-1}. For random orientation of orbital planes, the probability of detecting an inclination such that $i_1 \le i < i_2$ is $\cos(i_1)-\cos(i_2)$. For a given small range in period, mean values of M_1 and $M_2(=1.2\ M_\odot)$, and a mean value of the eccentricity, we can compute the fraction of systems with K_1 greater than 20 km sec^{-1} relative to those with smaller values. We will identify the former group with the observed systems and compute the number of undiscovered systems in the latter group. For P about 10^{-2} years and $M_2 = 1.2\ M_\odot$ we observe six double lined systems with $\langle e \rangle = 0.13$; we compute that $K_1 < 20$ km sec^{-1} for $i < 21°$ and since $\cos 0° - \cos 21° = 0.065$, there will be $0.065 \times 6 \times (0.935)^{-1} = 0.42$ undetected binaries and a total of 6.42 binaries. These are the numbers given in the upper left box of Table 8.

Abt and Levy's Table 8 is reproduced as Table 2 in this thesis. In a similar fashion other estimates are made for most of the categories of undetected binaries, and Table 2 can be completed with only a small amount of extrapolation. Results from Table 2 are plotted in Figure 2.

Abt and Levy fit a curve to the left side of Figure 2 and find the number of secondaries with a mass m (in solar masses) to be proportional to $m^{0.35\pm0.1}$. (Since their abscissa is logarithmic, the distribution function for secondary masses is proportional to $m^{-0.65}$.) Using this fit, they estimate the number of undetected binaries in the few categories that couldn't be evaluated on the basis of the assumptions given earlier (this is the extrapolation mentioned above). They conclude from their study that binaries with periods less than 100 years or so were formed by the fission of a forming star and

Table 2. Summary of Frequencies of Various Secondary Masses

Period (Years)		$M_2(M_\odot)$					Total
		1.2	0.6	0.3	0.15	0.075	
10^{-3}-10^{-1}	Meas.	6	4	4	2	2	18
	Undet.	0.42	0.00	0.02	0.03	0.11	0.58
	Total	6.42	4.00	4.02	2.03	2.11	18.58
10^{-1}-1	Meas.	2	2	1	1	2	8
	Undet.	0.94	0.01	0.01	0.04	0.40	1.40
	Total	2.94	2.01	1.01	1.04	2.40	9.40
1-10	Meas.	3	6	5	3	0	17
	Undet.	---	0.17	0.50	1.78	---	---
	Total	---	6.17	5.50	4.78	---	---
10-10^2	Meas.	8	4	1			13
	Undet.	0.00	2.48	---			---
	Total	8.00	6.48	---			---
10^2-10^3	Meas.	4	5	4	1	0	14
	Undet.	0.00	0.4	6.8	---	---	---
	Total	4.00	5.4	10.8	---	---	---
10^3-10^5	Meas.	3	8	3	1	0	15
	Undet.	0.00	0.6	6.8	---	---	---
	Total	3.00	8.6	9.8	---	---	---
10^5	Meas.	1	2				3
	Undet.	---	---				---
	Total	---	---				---

that those with periods greater than about 100 years were formed by capture, giving as evidence that the two groups have markedly different secondary mass distributions and that of the latter is roughly the van Rhijn function. The results in Table 2 indicate that single stars are rare, and Abt and Levy conclude that most stars form as either double or quadruple systems, the final results, including the categories filled by extrapolation, are in Table 3 [from Abt and Levy[3]].

Table 3. Conclusions Regarding Total Multiplicity

	P<100 Years	P>100 Years
Stellar Companions	∿ 67%	∿ 72%
Non-Stellar Companions	∿ 33%	0-28%

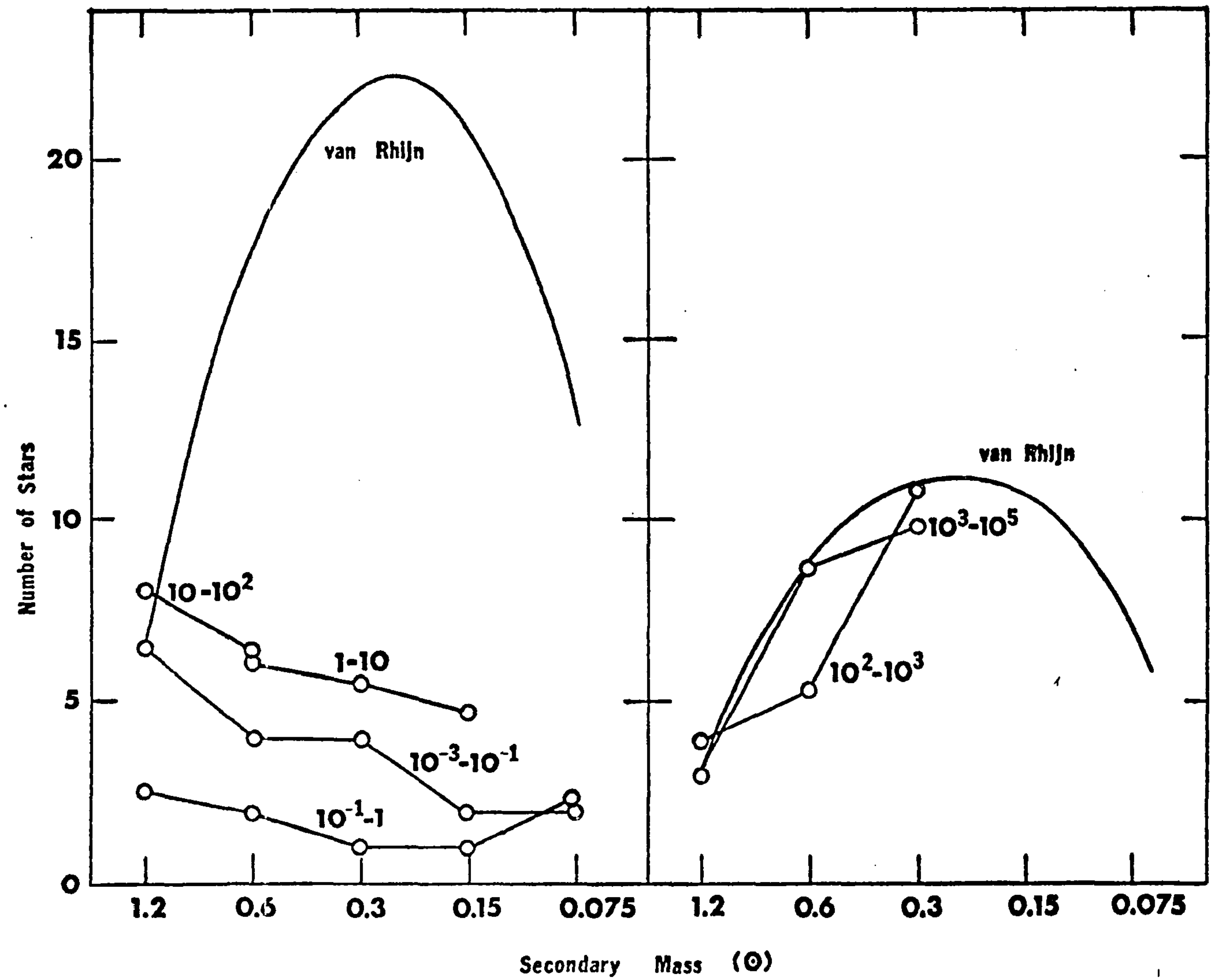

Figure 2. From Abt and Levy (3).

C. Open Clusters and their Dynamics.

The groups of stars studied in this thesis are open clusters. Open clusters have been interesting to astronomers for a long time; they have some nice properties: the stars in a galactic cluster were formed at the same time, out of the same stuff (some of the results of the research contained in Chapter III will lend weight to this assumption); the distance to the nearest one (the Hyades) may be determined by the moving cluster method, an independent astronomical yardstick; the distances to others may be measured by a method known as main sequence fitting.

Some comments about the main sequence and the Hertzsprung-Russell diagram are in order. If one does stellar evolution theory, one finds that the two most important parameters in determining a star's absolute magnitude, M_V, and color are its mass and age. If one restricts one's attention to a particular age group, one obtains a more or less one-dimensional sequence. If the particular age group in question is the group of stars that are still burning hydrogen in their cores, the resulting sequence is the main sequence. The one-dimensionality of this sequence for field stars of the appropriate age is vitiated to some extent by varied chemical compositions, but, as I have mentioned earlier, the chemical composition of an open cluster should be homogeneous; hence for <u>single</u> stars in a galactic cluster, one expects a one-dimensional sequence on a color-magnitude diagram (if the magnitude is M_V as opposed to the apparent magnitude, V, the color magnitude is referred to as a Hertzsprung-Russell or H-R diagram). The sequence one finds on H-R diagrams of open clusters aren't necessarily main sequences. The more massive stars evolve more rapidly than the lighter ones (this is another nice property of open clusters: their ages can be determined by the point on an H-R diagram where stars leave the main sequence). Even so, in as much as all the stars in an open cluster have a common age, the single star sequence is one-dimensional. For a very young cluster in which all the stars have just started their thermo-nuclear burning of hydrogen, one observes the zero age main sequence (ZAMS).

Because of these interesting properties, the open cluster has been a playground for astronomer and astrophysicist alike. Important information about the lives of stars has been gleaned from studies of these clusters. Open clusters have also attracted celestial mechanicians. They seek to describe the dynamical evolution of the clusters, hoping to predict lifetimes, binding energies, density distributions, the effects of multiplicity, and velocity dispersions. The galactic cluster represents a formidable problem for the theorist and "experimentalist" (Aarseth has called the modeling of clusters by computer, experimenting). The theorist is confronted with the worst kind of many body problem; it is intractable by perturbation and statistical methods alike. For systems like the solar system, dominated by one body, one may use perturbation theory; for systems like galaxies, one may use the Boltzmann equation; but for these clusters, neither approach works well. The experimentalist is more fortunate.

Since about 1960, people have been attempting to model gravitational interactions for about 10^2 particles on computers. Through persistence, clever approaches, and lots of computer time, they have achieved some success. Aarseth reviews this field in *Annual Review of Astronomy and Astrophysics,* 1975; here, I would like to focus attention on the recent work of Aarseth.[7,8]

Qualitatively, Aarseth's models come out as follows: using a truncated Maxwellian distribution for single stars and rather weak binding as initial conditions, the cluster first contracts and forms a core with a less dense halo around it; within this core, close encounters involving more than two stars can occur and, indeed, as a result of such interactions binaries may be formed and lighter members accelerated to escape velocities. Eventually, one or two heavy binaries form and dominate the custer's dynamics as an energy sink. A lighter member coming in from the halo in an eccentric orbit will have a close encounter with the heavy binary, the binary will become more tightly bound and give additional kinetic energy to the light halo star, possibly enough energy to escape the cluster altogether. This type of behavior leads to a characteristic mass-energy distribution that Pels, Oort, and Pels-Kluyver[9] believe they see in the Hyades; Pels *et al.* go on to speculate that the star, VB80, may be the dominate binary in the Hyades. The ultimate fate of an open cluster is for all its binary energy to be absorbed by the heavy binary leaving only the heavy binary. At this point, the models are still too unrealistic to describe real clusters; they start out with a distribution of single stars rather than binaries, for instance. Notably, the half lives predicted by Aarseth are off in some cases by an order of magnitude. There is one point of specific interest for this thesis: if a bound triple system forms in a cluster, it is unstable and the lightest member will eventually be expelled. Perhaps if such systems form and are disrupted frequently enough, as may be the case if one starts out with a distribution of binary stars as Abt and Levy imply would be more realistic, binaries would tend to evolve toward equal mass systems. There is at this time no way of treating such an initial distribution of binaries* so such a conclusion is highly speculative, but it will be interesting to see how this speculation fares in the next few chapters.

*See Chapter VI. Aarseth feels at this time there is no strong reason to suppose equal mass system would be retained preferentially.

CHAPTER III

STATISTICAL IDENTIFICATION OF BINARIES IN STAR CLUSTERS

A. Introduction

As I mentioned earlier, it is a consequence of stellar evolution theory that a star's position on a Hertzspring-Russell diagram is determined to first order by its mass and age. Therefore, if one constructs a color-magnitude diagram for a galactic cluster, one expects to see a one-dimensional sequence in as much as differences in distance, age and composition have been eliminated.

This is not quite the case as has been known since the classic work of Haffner and Heckmann[10]. They, and observers since, have noted that the scatter in a color-magnitude diagram for a galactic cluster is somewhat larger than allowed by the assumption of a one-dimensional sequence and the known observational error. Haffner and Heckmann attributed this to the presence of unresolved binaries and estimated the percentage of binaries. (People have even come to speak of a binary ridge.) Upton[11] and Taylor[12] assumed that the color-magnitude diagram for the Hyades is one-dimensional commensurate with observational error, eliminating stars systematically above a mean line as either suspected or known binaries.

This chapter extends this line of thought to a statistical method. To do so, one must know to what extent unresolved binaries contribute to the scatter in color-magnitude diagrams and how important other sources of scatter are. In the first place, what are the composite colors and magnitudes of various combinations of single stars? This question can be answered by considering two stars of magnitude V_1 and V_2 and of color (B_1-V_1) and (B_2-V_2) respectively. Then by definition

$$V_1-V_2 = -2.5 \log(F_{V1}/F_{V2}) \quad \text{(all logarithms are base 10)}$$

where F_{V1} and F_{V2} are the fluxes from each star in the passband of a V filter. (The photometry discussed on this chapter is UBVRI photometry, each letter representing a particular wide band filter from the ultraviolet to the infrared.) Then:

$$F_{V2} = F_{V1} 10^{(V_1-V_2)/2.5}$$

so if $\bar{V}$ is the composite magnitude and V_1 is the brighter of the two stars, then the binary is

$$\bar{V}-V_1 = -2.5 \log\left[\frac{F_{V1}+F_{V1}10^{(V_1-V_2)/2.5}}{F_{V1}}\right] = -2.5 \log[1+10^{(V_1-V_2)/2.5}] \tag{1}$$

brighter than the primary, [Sometimes M_V, the absolute magnitude (= apparent magnitude if the star were ten parsecs in distance from us), is given instead of the apparent magnitude V. In which case, $V_1-V_2 = M_{V1}-M_{V2}$.] One must also consider the change in color. Let us consider the color (B-V):

$$B_1 - B_2 = -\ 2.5\ \log(F_{B1}/F_{B2})$$

so

$$F_{B2} = F_{B1} 10^{(B_1-B_2)/2.5}$$

and

$$\bar{B} - B_1 = -\ 2.5\ \log[1+10^{(B_1-B_2)/2.5}]\ ;$$

using (1),

$$(\bar{B}-V)-(B_1-V_1) = -\ 2.5\ \log[\frac{1+10^{(B_1-B_2)/2.5}}{1+10^{(V_1-V_2)/2.5}}]\ ;$$

but

$$(B_1-B_2) = (B_1-V_1)-(B_2-V_2)+(V_1-V_2)\ .$$

So,

$$(\bar{B}-\bar{V})-(B_1-V_1) = -\ 2.5\ \log[\frac{1+10^{[(B_1-V_1)-(B_2-V_2)+(V_1-V_2)]/2.5}}{1+10^{(V_1-V_2)/2.5}}] \qquad (2)$$

where, again, $M_{v1}-M_{v2}$ may be substituted for V_1-V_2 if necessary. Equation (2) gives the difference in (B-V) color between the composite and the bright component. A similar agrument gives similar expressions for the (V-R) and (R-I) colors. Now, using the main sequence for a color-magnitude relationship, one can calculate the color and magnitude of a binary composed of two main sequence stars. Two stars of equal masses are $0^m.75$ brighter than a single star of the same mass and have the same color as the single star. One may calculate the runs of color and magnitude for a given primary as a function of secondary color. The results of such a calculation are shown for various primaries for (V-R) in Figure 3. Qualitatively similar results are obtained for (B-V) and (R-I), the tracks going further from and turning in a little more sharply to the main sequence for (B-V) and closer to and less sharply for (R-I) than for (V-R). However, since the main sequence increases in slope as one goes from (B-V) to (R-I), binaries stay about the same distance above the main sequence in each color-magnitude diagram. Thus, binaries do indeed add scatter if we assume a population of single stars to be present. Before Abt and Levy's paper (discussed in Chapter II) it was commonly thought that about half of all stars were in multiple systems leaving roughly two thirds of the systems single; if this were so, we might expect a sequence of single stars with binaries scattered above it. If Abt and Levy are correct in saying that single stars are rare, we would also expect the same sort of color-magnitude diagram since binaries composed of stars differing by more than 3^m would be roughly within the scatter of supposed single stars and the rest of the binaries would be distributed above the single star sequence. See Section D for a more rigorous discussion. So let us, as a temporary convenience, consider those binaries whose components differ by more than 3^m as single stars, and, of course, single stars with nonstellar companions are treated as single.

Other, competing sources of scatter include:

1. Range in Distance: For the Hyades the distance moduli (defined as $V-M_V$ whence by application of the inverse square law, one may readily obtain the distance) of Heckmann and Johnson[13] are used, the

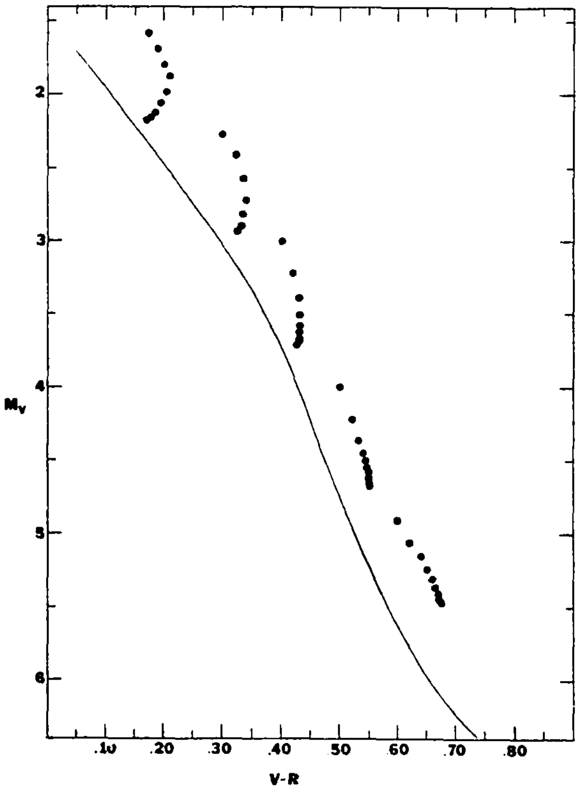

Figure 3. Positions of binary systems on a zero-age main sequence. The highest point on each track represents a system composed of two stars of equal mass. Succesive points on each track are binaries formed by keeping the brighter component fixed in mass and successively dropping the fainter one in mass by increments corresponding to $0^{m}.05$ in (V-R).

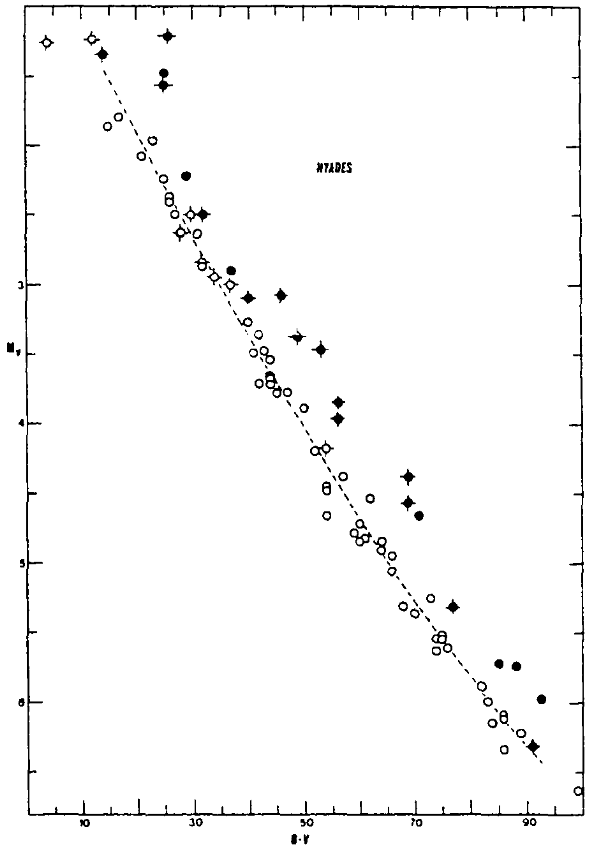

Figure 4. Color-magnitude diagram for the Hyades. The solid circles represent analyzed binaries; the crosses, observed binaries; and the open circles, single stars.

error quoted in these moduli are small compared to typical binary effects. For the Pleiades and Praesepe the contributions from this source of scatter are small (less than $0\overset{m}{.}1$). In all cases the errors are included in the total predicted error in Section D.

2. Rotation: For the Hyades and Praesepe rotation is small, and Dickens, Kraft and Krzemanski[14] have found for Praesepe that rotation is only weakly correlated with magnitude. Kraft and Wrubel[15] have found a correlation between color and magnitude for the Hyades, but in all cases the effect was less than $0\overset{m}{.}1$; indeed, for the majority of early type stars in the Hyades the effect is less than $0\overset{m}{.}05$. The Pleiades, however, contain stars earlier than those in Praesepe or the Hyades for which the rotation may be a problem. A rapidly rotating star seen pole on may be reddened enough to perhaps be identified as a binary by the technique described in this chapter. Even so, a greater problem for the method is the steepness of the main sequence in the region of these early Pleiades stars (see Section D). For either reason, the analysis is not so accurate for B stars in the Pleiades, and such stars are indicated in the results (Table 5). This problem does not significantly affect the statistical results.

3. Ap and Am Stars: These stars present a problem in that, even when they are known spectroscopic binaries, they are usually not brighter than other main sequence stars of the same color. They do not add additional scatter in the color system used; they have been left in the analysis for completeness even though the technique described will not work for them.

4. Blue Stragglers: In most clusters there are one or two of these and they will not be indicated by this analysis as binaries even if they are; however, their small number makes them statistically insignificant.

All this suggests a means of statistically distinguishing binaries from single stars in clusters provided that the components differ by less than about 3^m and that the photometry is sufficiently accurate. Binaries are systematically brighter than single stars and the scatter they produce is asymmetric. From this reasoning, the technique described in the next section evolved.

B. Technique

The problem at hand is that one is presented with photometric data of a galactic cluster consisting of color-magnitude diagrams in three different colors. These data represent a skewed distribution of points consisting of two classes of objects, one class being systematically above the other; one would like to use all data possible to resolve these two classes resulting in an unskewed distribution of single stars about a mean curve.

A computer program was written to perform a least squares fit of a polynomial of specified order to a given color-magnitude diagram, say (B-V), V, and to calculate the standard error of the fit. Data points having residuals more than a certain number, N, times the standard error were then eliminated and the polynomial refitted. This process was iterated until the data set converged or the points eliminated were evenly distributed above and below the polynomial.

N was determined by the following criteria. One would want to remove only points above the polynomial. Of course this is not possible by the means described because of the various sources of error. But, at least at the outset, one would not want to remove too many points from below the curve by using too small a value of N. Furthermore, a final unskewed distribution is desired so N should not be too large. It was found that by varying N, usually very noticeably, the value, N, that fit these criteria presented itself.

The order of the polynomial fit was also definitely determined. The computer was programmed to fit higher orders until no improvement in the fit was seen. The order of the polynomial used could be accurately estimated by eye.

The above process was repeated for two other color indices, (V-R) and (R-I), in the same manner with values of N appropriate to each index. Then the stars eliminated from all these color-magnitude diagrams were correlated. If a star was eliminated from two or more diagrams from above a mean curve, it was analyzed as a binary; if it was eliminated from only one, it was regarded as a single star. Stars below the mean curve are discussed in the next section.

C. Results

The results of this approach as applied to three galactic clusters are indicated in Figures 4-12 and Tables 4-8. Tables 9-11 list the stars used in the analysis. The photometry used is that of Mendoza[16]. Objects listed by Mendoza as other than main sequence stars were not included in the analysis. The final mean curves drawn

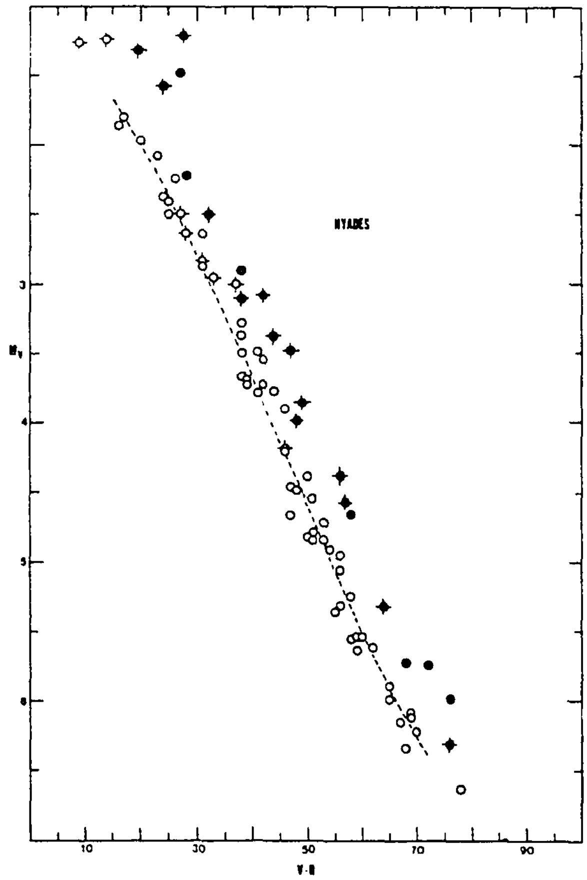

Figure 5. Color-magnitude diagram for the Hyades. Identification is the same as in Figure 4.

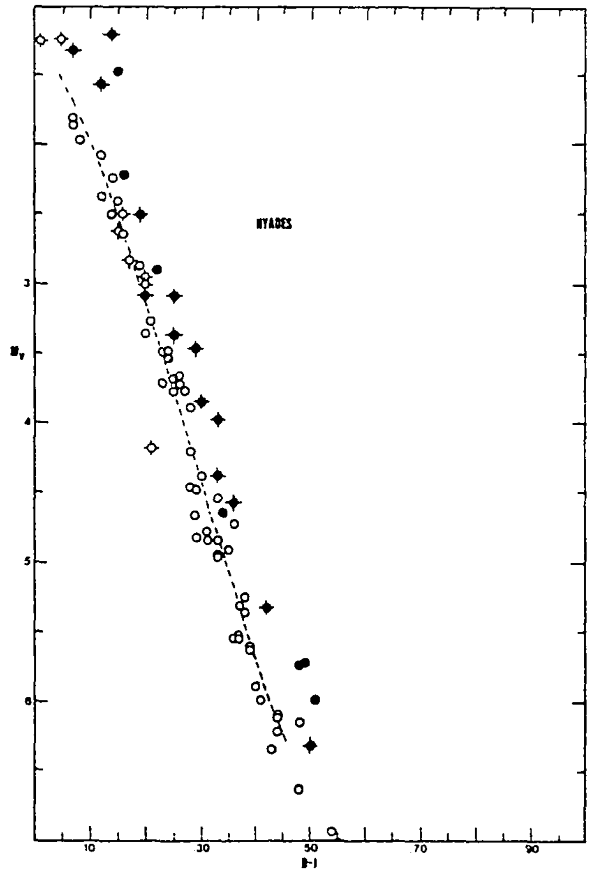

Figure 6. Color-magnitude diagram for the Hyades. Identification is the same as in Figure 4.

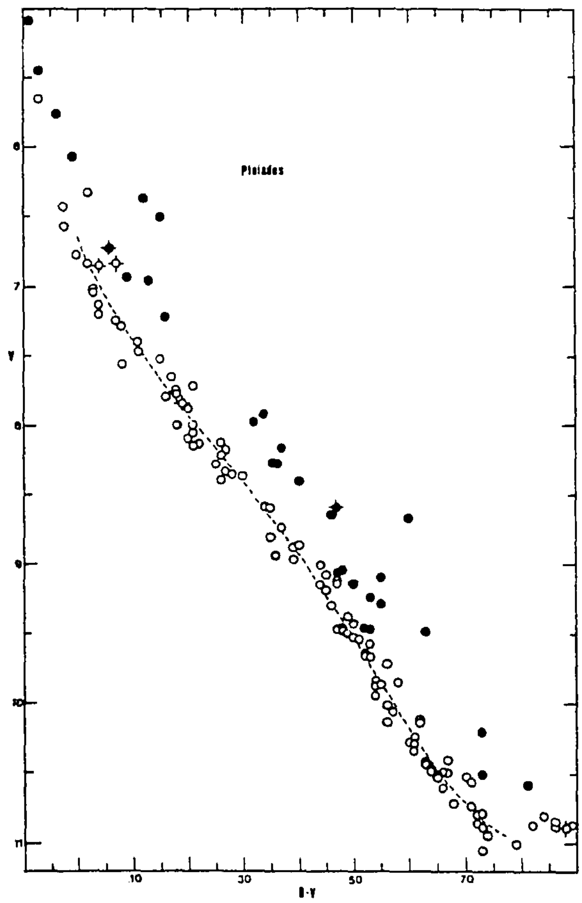

Figure 7. Color-magnitude diagram for the Pleiades. The solid circles represent analyzed binaries; the crosses, observed binaries; and the open circles, single stars.

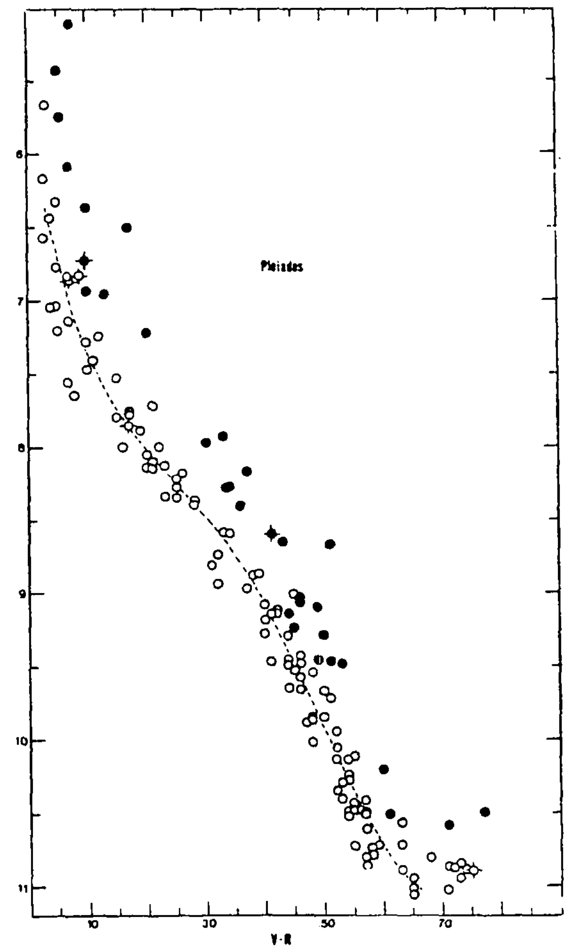

Figure 8. Color-magnitude diagram for the Pleiades. Identification is the same as in Figure 7.

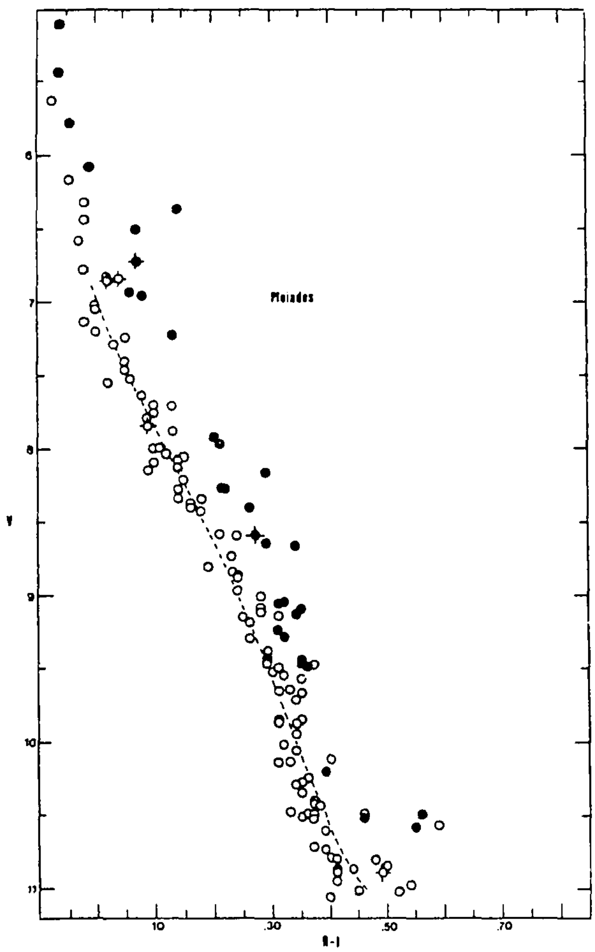

Figure 9. Color-magnitude diagram for the Pleiades. Identification is the same as in Figure 7.

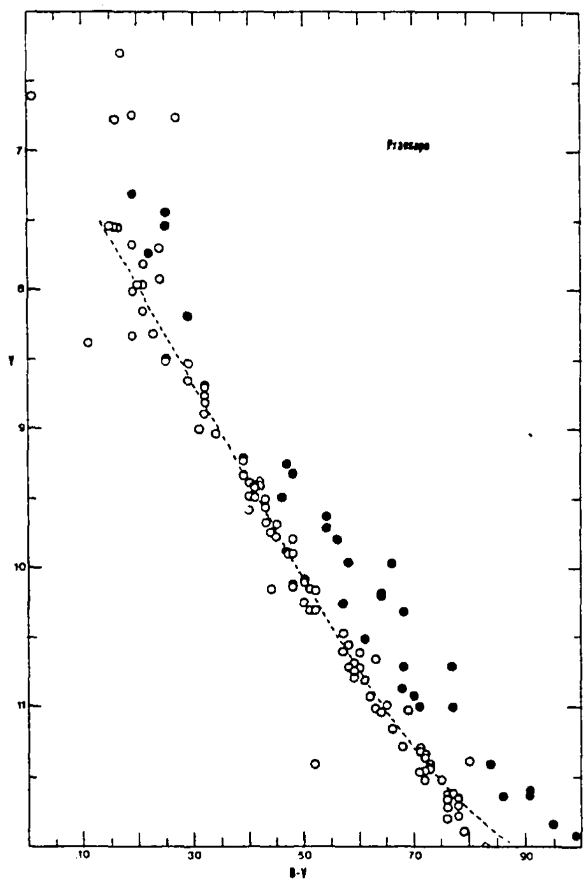

Figure 10. Color-magnitude diagram for Praesepe. The solid circles represent analyzed binaries; the open circles, single stars.

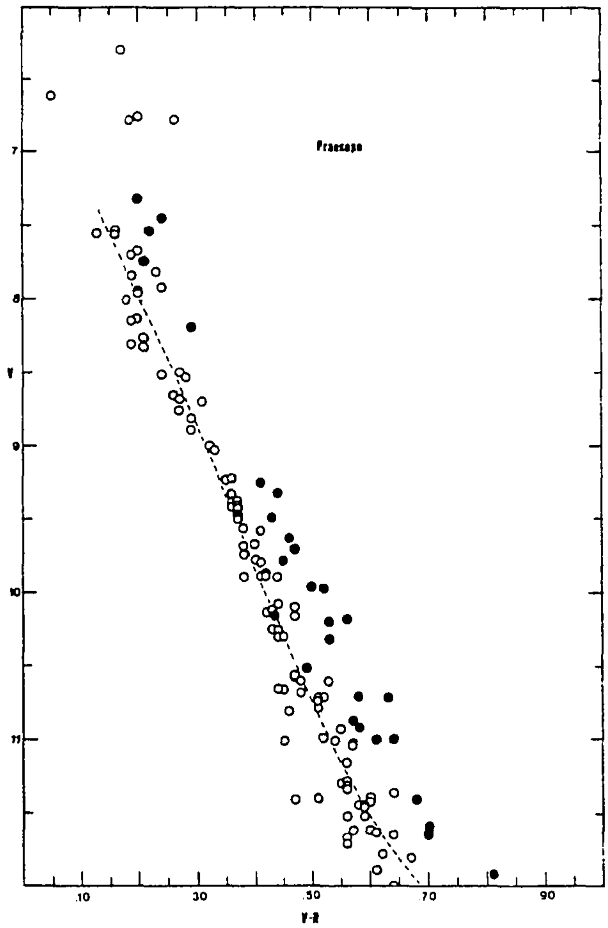

Figure 11. Color-magnitude diagram for Praesepe. Identification is the same as in Figure 10.

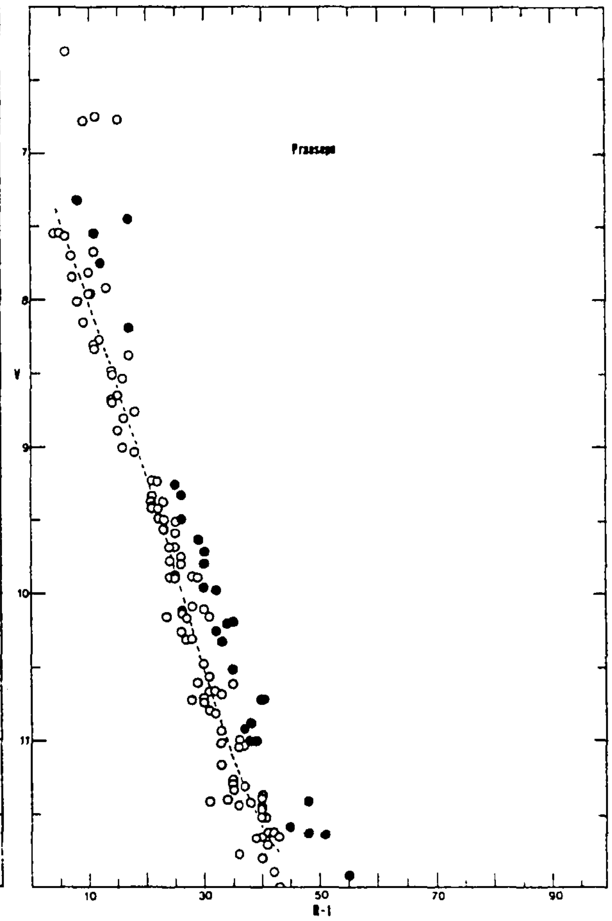

Figure 12. Color-magnitude diagram for Praesepe. Identification is the same as in Figure 10.

Table 4. Hyades Binaries*

VB		VB	
11 and 12	a,o	57	a,o
22	a,o	58	a,o
23	a,o	60	a,o
24	a	62	o
29	a,o	75	a,o
30	o	80	a,o
32	o	89	o
34	a,o	91	a
38	o(Am)	95	a,o
40	a,o	104	o
43	a	108	a,o
45	o(Am)	141	a
53	a	162	a
54	a	176	a
56	o	182	a

*a = analyzed, o = observed; numbers are those of van Bueren(17); observed binaries are from Upton(11)

Table 5. Pleiades Binaries*

HzII		HzII	
102	a	1431	o
157	a	1726	a
298 & 299	o	1762	a
333	a	1823	a(B8V)
338	a	1876	a
563	a,o(B6V)	1912	a
717	a	2181	a(B8p)
739	a	2500 & 2507	a,o
745	a	2866	a
801	o	Ts19	a
817	a(B8V)	Ts45 & 45x	a,o
948	a	Ts67	a
956	a	Ts78 & 78x	o
975	a	Ts127	a
1084	a	Ts149	a
1117	a	Ts156x	a
1266	a	Ts177	a
1338	a	Ts183x	a

*a = analyzed; o = observed. Numbers are those of Hertzsprung [HzII, Hertzspring[18] and Trumpler, Ts, Trumpler[19]]; observed binaries are from Abt *et al.*[20].

Table 6. Praesepe Binaries (Analyzed)*

VL	VL
237	934
265	947
363	967
441	993
508	1022
598	1033
744	1091
761	1142
828	1149
843	1220
847	1322
877	1416
921 (Am)	1421
925 (Am)	

*Numbers are those of Vanderlinden[21]

were those fitted to the set of data with analyzed binaries deleted, and the analyzed scatters were taken from this fit. The final mean curves were about $0^{m}.1$ below the curves fitted at the outset of the analysis; this figure varied at different parts of the curve. The rising of the mean curve by the presence of unresolved binaries should be taken into account when determining distance moduli by main sequence fitting, where it could cause a systematic error of about 5 per cent in distance.

There are a few points rather far below the mean curves. In the Hyades there was one, in Praesepe two, and in the Pleiades none that were eliminated from below the mean curves of more than one color-magnitude diagram. These points represent possibly extremal errors in magnitude or perhaps interloping field stars. Points eliminated from only a single color-magnitude diagram were interpreted as extremal errors in color index.

There is a selection effect in the faint end of Praesepe and the Pleiades. The photometry stops at a particular magnitude, so that a fitted curve will curl up at the faint end. This effect was compensated for by deleting points at the red end until only the entire width of the data was in the data set. Later, the points were reinserted and their distances from the mean curve measured and compared with the standard error; this involved only a small extrapolation of the fitted curve.

At the bright end of Praesepe there are evolved objects which are more than $0^{m}.75$ above the mean curve; these objects though analyzed as binaries are not listed as such in Table 6, there being no good evidence one way or the other for their duplicity (see also the discussion of the steepness effect in the next section).

The Hyades did not have the selection effect because the data represented a subset of a larger body of data: those Hyades with distance moduli quoted by Heckmann and Johnson[13].

D. Discussion

Taking the observational errors quoted by Mendoza[16] in color and magnitude, the slopes of the mean curves, and the errors introduced by assuming all stars in the particular cluster to be equidistant (in the Hyades, this was, instead, the error in the distance moduli), weighting them according to the number of stars appropriate to each magnitude range (errors were quoted for V less than $9^{m}.0$ and V greater than $9^{m}.0$), and combining them, one arrives at the predicted errors in Table 7. If the slope is large, errors in color index swamp other sources of error; the color index error is the limiting factor.

The comparison between predicted error and measured scatter in Table 7 indicates that the main sequence of each of these galactic clusters is a one-dimensional sequence within the limits of current photometric accuracy.

Table 4 compares analyzed and observed binaries in the Hyades. The comparison is quite good especially in the later type stars. There is, however, a problem as noted before for the Am and Ap stars (see also Figures 5-7). Tables 5 and 6 compare observed and analyzed binaries for the Pleiades and Praesepe. Some reasonable questions

Table 7. Predicted and Analyzed Scatter of Galactic Clusters; Errors are Indicated in Magnitudes of M_V or V

Cluster	B-V		V-R		R-I	
	P*	A	P	A	P	A
Hyades	0.14	0.13	0.17	0.21	0.21	0.24
Praesepe	0.15	0.13†	0.28	0.24	0.26	0.25
Pleiades	0.15	0.15	0.22	0.25	0.21	0.25

*P = predicted; A = analyzed.

†Three stars below the mean curve are deleted for the analysis of the error in the (B-V) magnitude relation for Praesepe.

Table 8. Frequencies* of Binaries

Cluster	Frequency
Hyades	0.26 (0.36, analyzed and observed)
Praesepe	0.20
Pleiades	0.22

*No. of analyzed binaries/no. of main-sequence objects.

at this point might be:

1. Are the results statistically significant; that is, could guessing do as well?

2. What is the likelihood that a single star has been analyzed as a binary?

3. Are the single stars really single or are they perhaps binaries whose components differ by more than 3^m and thus are unresolvable by this method?

The answers to these questions are:

1. About one-fourth of the sample of the Hyades used were well-known binaries (uncertain cases were not included). If one selected a subset of twenty-two stars at random from the Hyades, one would expect only about five or six of them to be known binaries. Using the technique in this chapter, thirteen of them are. Furthermore, some of the known binaries are Am and Ap stars for which the method will not work, and some may simply be binaries having components differing by more than 3^m. The results are significant.

2. Using the value of N, the number of standard deviations used in the computer analysis (this was usually about two), one can estimate that the probability that a single star would be analyzed as a binary is less than 0.01, if we assume that there is no asymmetric source of scatter for single stars. One also finds that the probability of detecting a binary with components differing by about 3^m is 0.5 and increases to near certainty for those differing by $2\overset{m}{.}5$.

3. This question brings up the assumption made in Section A about stars whose components differ greatly in magnitude. Although

I won't be able to give a complete answer to it (it will be examined from another point of view for the Hyades in Chapter V) some interesting results will follow. In the first place, the fact that the method worked at all tells us something about the way stars are scattered about the main sequence (the distributions of stars later than about F2 about the final fits are plotted as histograms in Figures 13-17): they must be scattered asymmetrically as assumed and as evident in the histograms. Let us calculate the distribution of stars about the main sequence using the results of Abt and Levy. Abt and Levy studied field stars; it is suspected that cluster dynamics play an important role in determining the secondary mass distribution in clusters and the luminosity function is uncertain for later type stars in open clusters. Nevertheless, let us assume Abt and Levy's results apply to open clusters and to spectral types from F2 to about K0 in clusters and see what the consequences are for the scatter around the main sequence.

Let us divide the secondary mass distribution into two parts: one, representing those binaries with periods less than 100 years, proportional to $m^{-2/3}$ (where m is in solar masses) and the other, representing the complement to the first, proportional to the van Rhijn function, both being normalized to the proportions indicated in Table 3. Consider the first part. Let ρ_m be the secondary mass distribution function; it is defined from m = 1.2 to m = 0.07 (those objects with m less than 0.07 are not stars). Then

$$\rho_m(m) = a \cdot m^{-2/3}$$

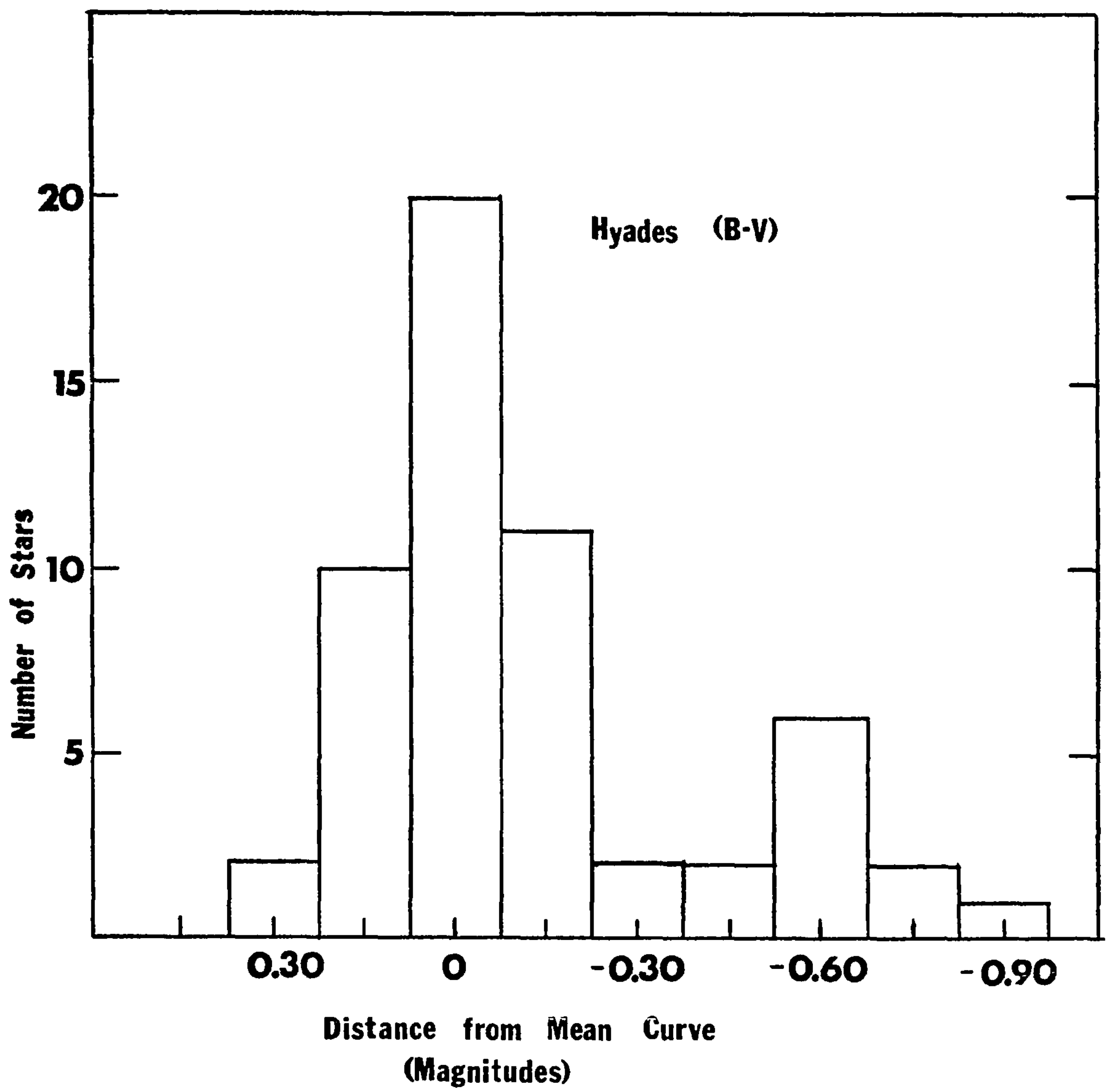

Figure 13. Histogram for Hyades stars later than about F2.

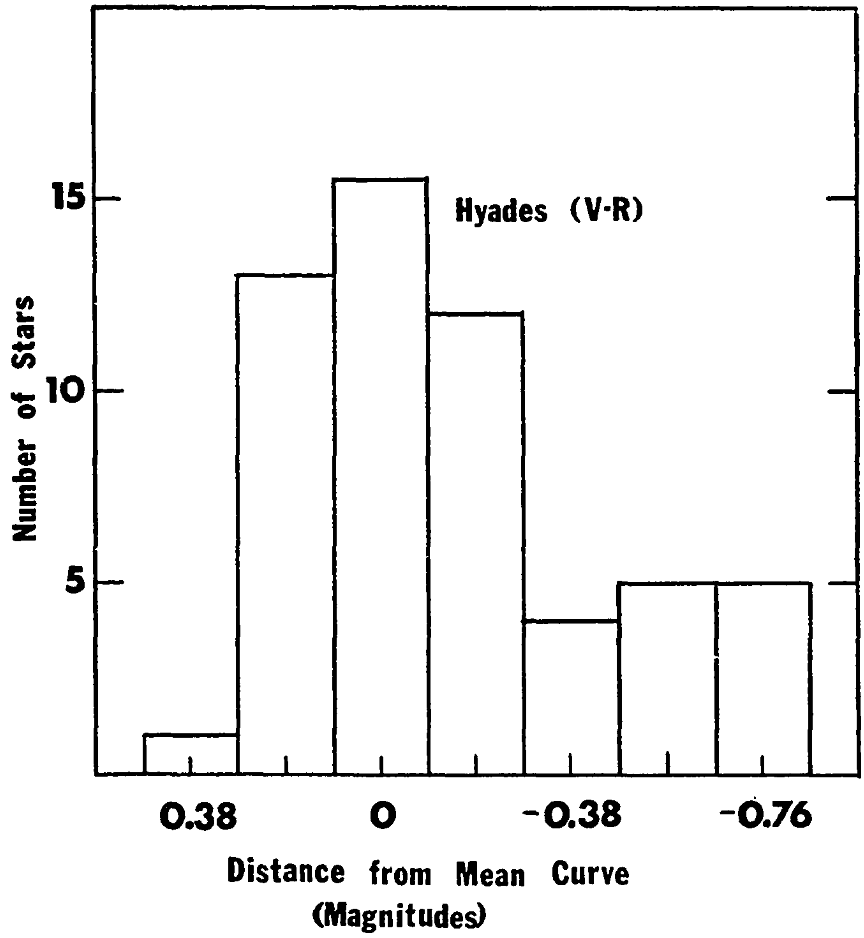

Figure 14. Histogram for Hyades stars later than about F2.

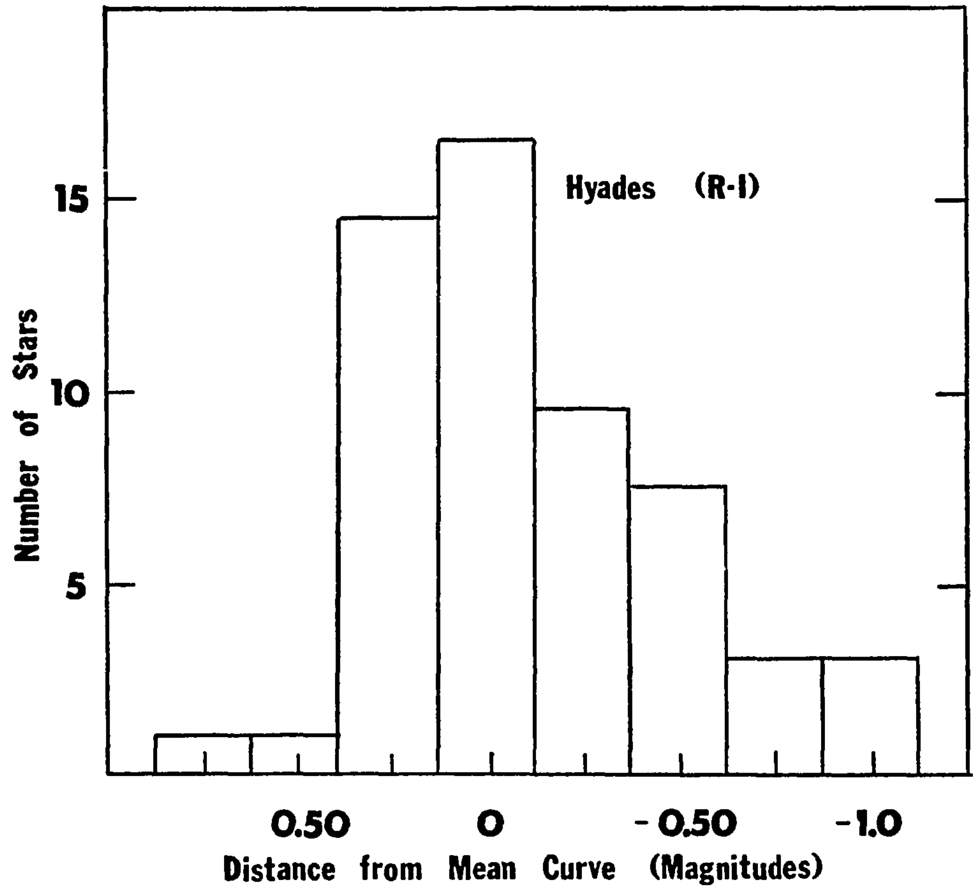

Figure 15. Histogram for Hyades stars later than about F2.

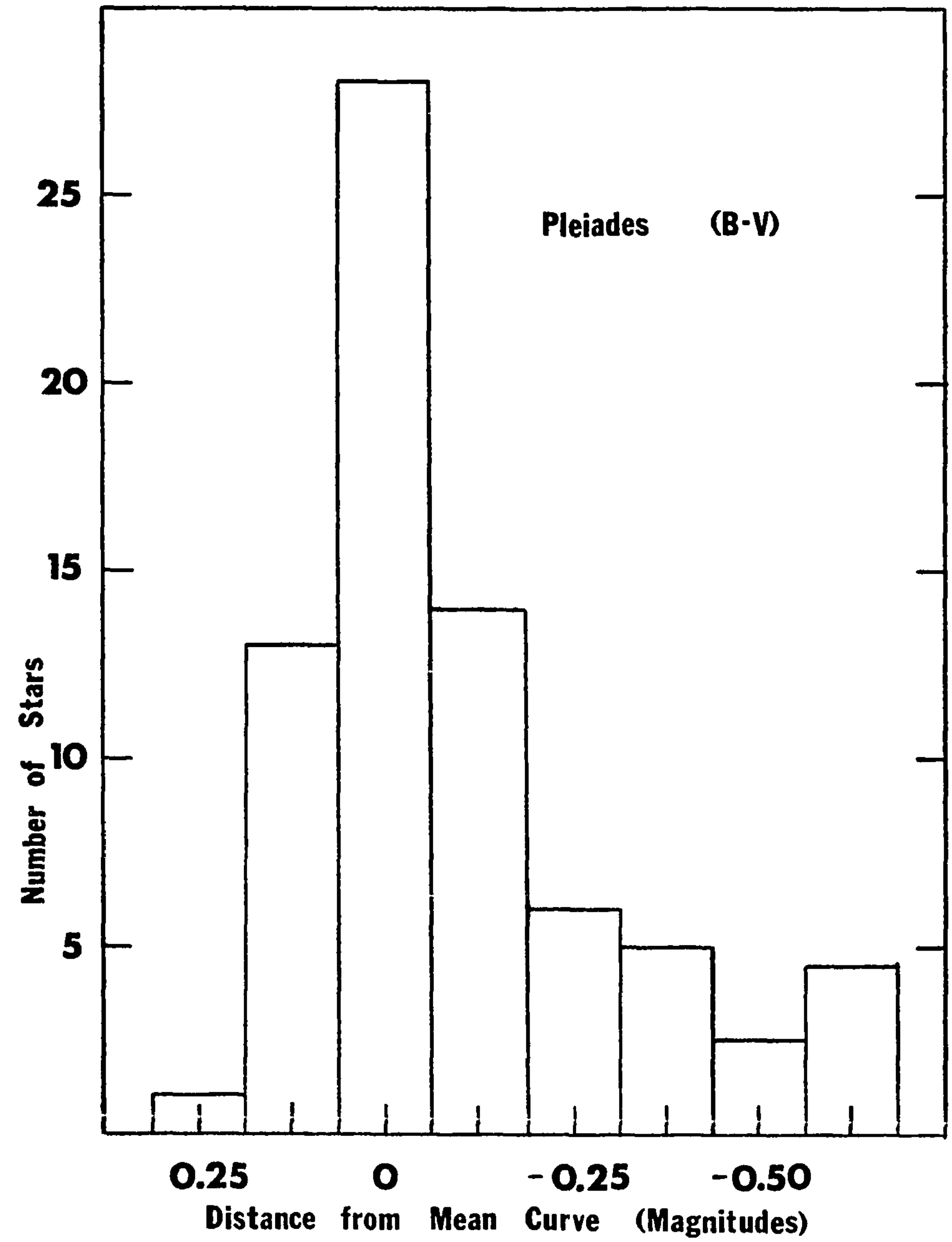

Figure 16. Histogram for Pleiades stars later than about F2.

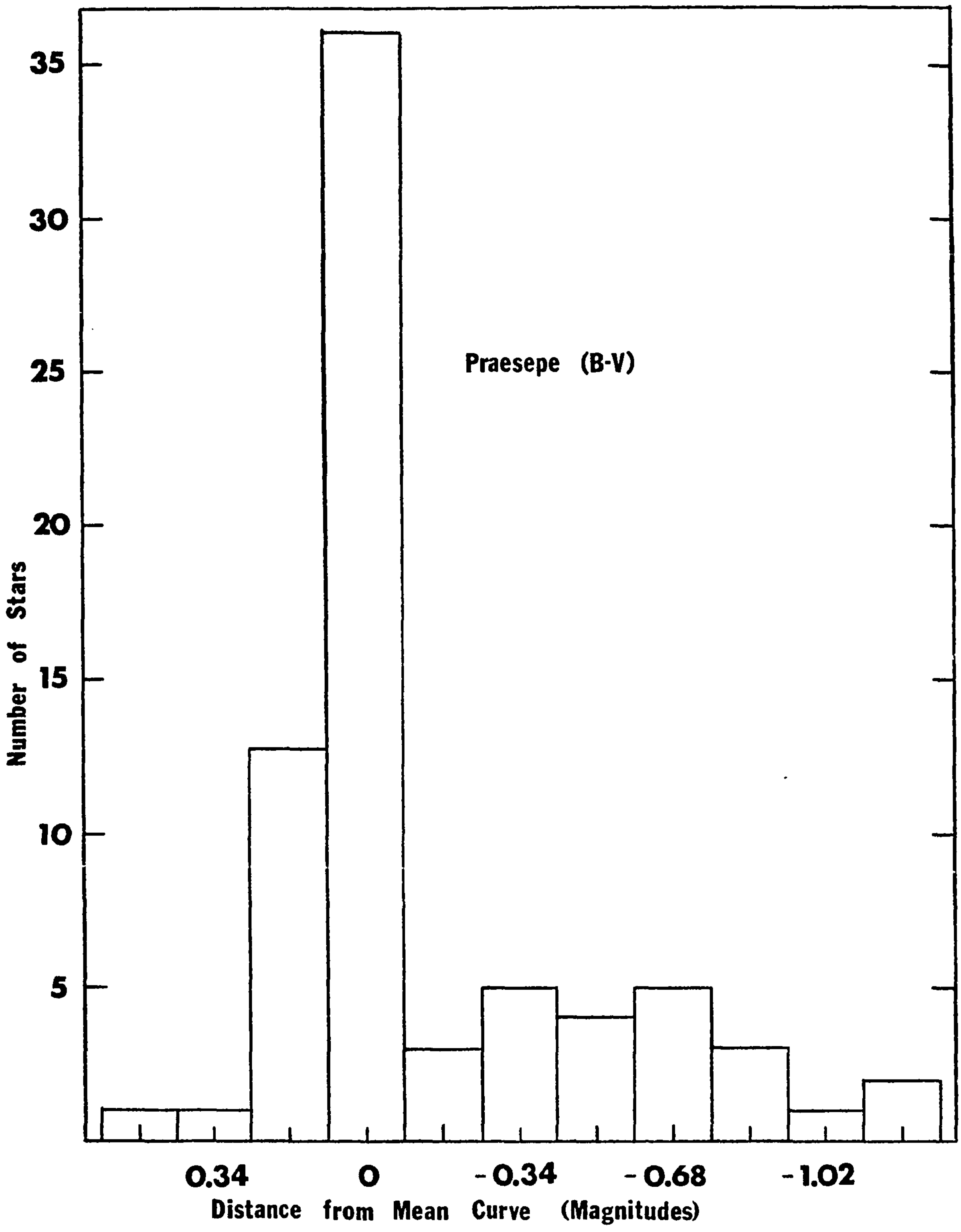

Figure 17. Histogram for Praesepe stars later than about F2.

and

$$a = 0.512.$$

Let $\rho(M_2)$ (where M_2 is the secondary magnitude) be the secondary magnitude distribution function, then

$$\rho(M_2) = \rho_m(m) \left|\frac{dm}{dM_2}\right| .$$

Now, using Harris, Strand and Worley's[22] empirical mass-luminosity relationship for bolometric magnitudes,

$$M_2 = 4.6\text{-}10.0 \log m \qquad (0<M_2<6.5)$$

$$M_2 = 5.2\text{-}6.9 \log m \qquad (6.5<M_2<11) \quad ,$$

we have

$$m = 10^{(4.6-M_2)/10}$$

$$m = 10^{(5.2-M_2)/6.9}$$

and we find:

$$\rho(M_2) = 0.118\cdot 10^{0.153-0.033M_2} \qquad (0<M_2<6.5)$$

$$\rho(M_2) = 0.171\cdot 10^{0.251-0.048M_2} \qquad (6.5<M_2<11) \quad .$$

We seek $\rho_\Delta(\Delta M)$ the distribution of the difference in magnitude ($\Delta M = M\text{-}M_1$ where M is the composite and M_1 is the primary bolometric magnitude) between the composite and the primary;

$$\rho_\Delta(\Delta M) = \rho(M_2) \left|\frac{dM_2}{d\Delta M}\right| .$$

as in Equation (1),

$$\Delta M = M-M_1 = -\ 2.5\ \log[1+10^{(M_1-M_2)/2.5}]$$

or

$$M_2 = M_1 - 2.5\ \log(10^{-\Delta M/2.5}-1);$$

and

$$\rho_\Delta(\Delta M) = \frac{0.118\cdot 10^{0.153-0.033[M_1-2.5\ \log(10^{-\Delta M/2.5}-1)]}}{1-10^{\Delta M/2.5}}$$

$$\rho_\Delta(\Delta M) = \frac{0.171\cdot 10^{0.251-0.048[M_1-2.5\ \log(10^{-\Delta M/2.5}-1)]}}{1-10^{\Delta M/2.5}} \quad (3)$$

for $0<M_2<6.5$ and $6.5<M_2<11$. Now we may evaluate (3) for various values of ΔM in intervals of $\delta\Delta M$ letting $M_1 = 3.8$ and calculate $\rho_\Delta(\Delta M)\delta\Delta M$ for each value. Using each value of M_2, a $M_{bol}-M_V$ relation (from *Basic Astronomical Data,* see reference 22) and the appropriate color-magnitude diagram (the Hyades diagrams were used for these calculations), we can calculate how much redder and brighter (in M_V) and hence how much above the main sequence each $\rho_\Delta(\Delta M)\delta\Delta M\cdot 100$ percentage of stars lies. We then construct the histograms in Figures 18-20 in which the derived distribution functions convoluted with a Gaussian of standard error appropriate to each color are shown.

Figures 18-20 represent only part of the secondary mass distribution for field stars. Missing from these histograms are the distributions that would be derived from the stars with non-stellar companions and those binaries whose secondaries are distributed as

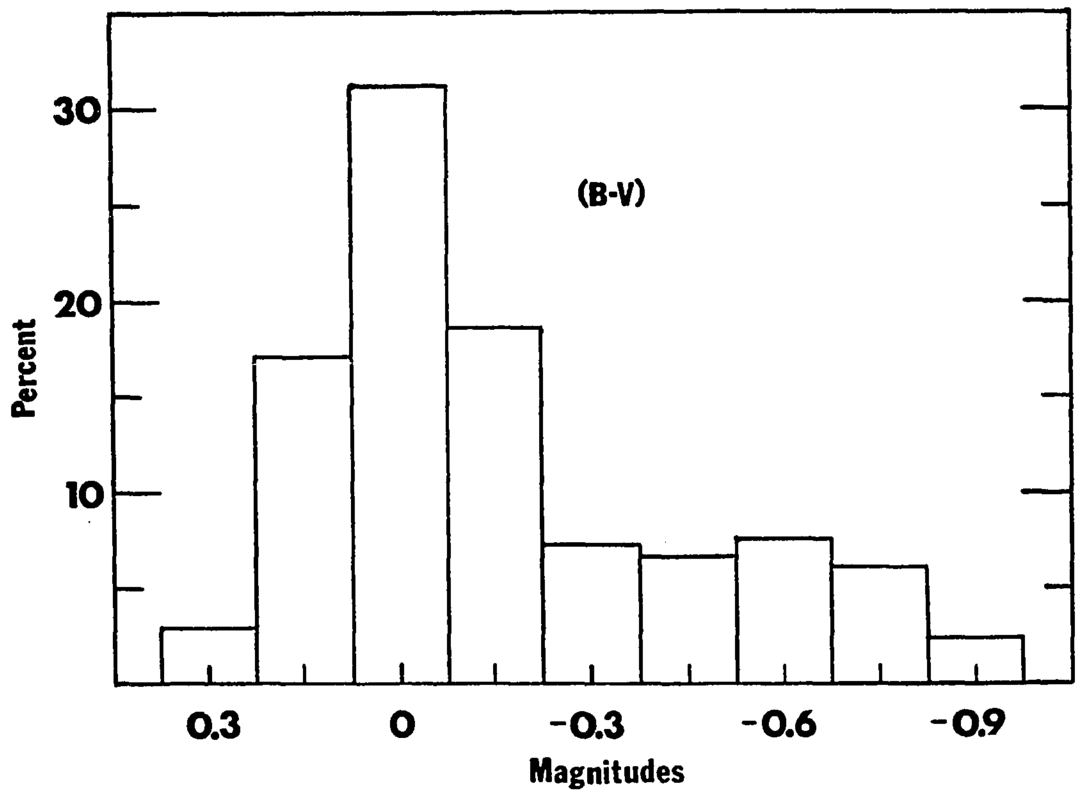

Figure 18. Calculated histogram of scatter about the main sequence derived from a secondary mass distribution function proportional to $m^{-2/3}$ and convoluted with a Gaussian of standard error 0.13 magnitudes. The zero of the abscissa is a main sequence taken from the final fit to the Hyades.

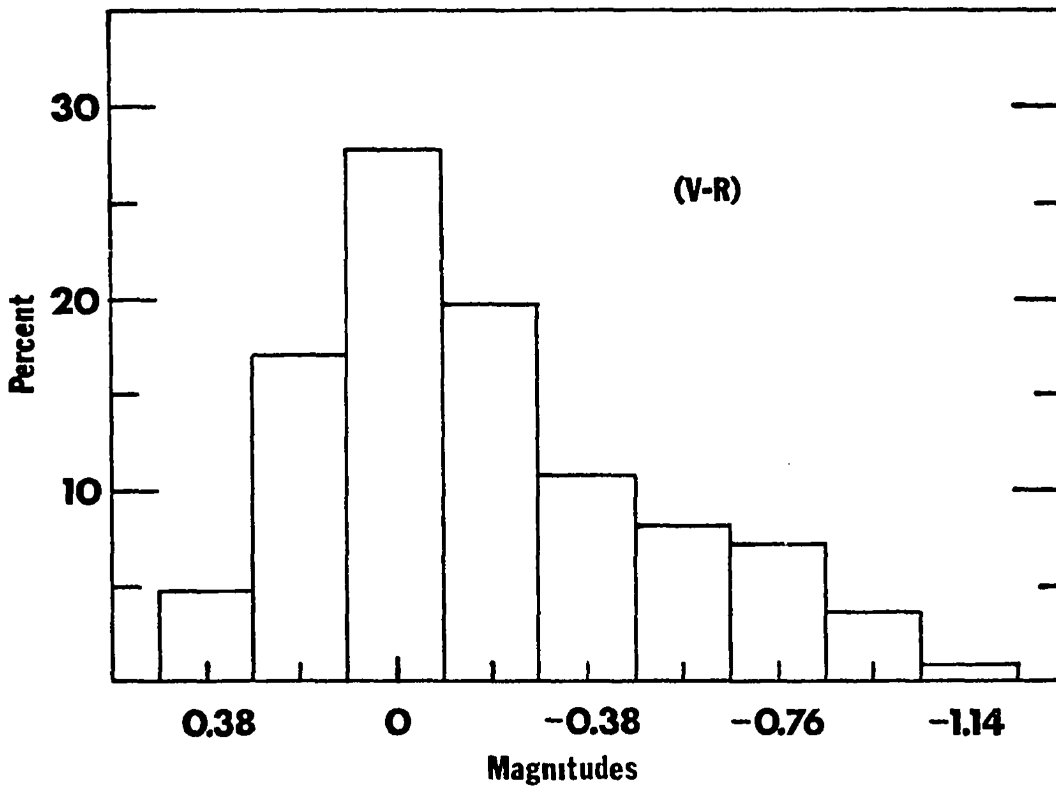

Figure 19. Calculated histogram of scatter about the main sequence derived from a secondary mass distribution function proportional to $m^{-2/3}$ and convoluted with a Gaussian of standard error 0.19 magnitudes. The zero of the abscissa is a main sequence taken from the final fit to the Hyades.

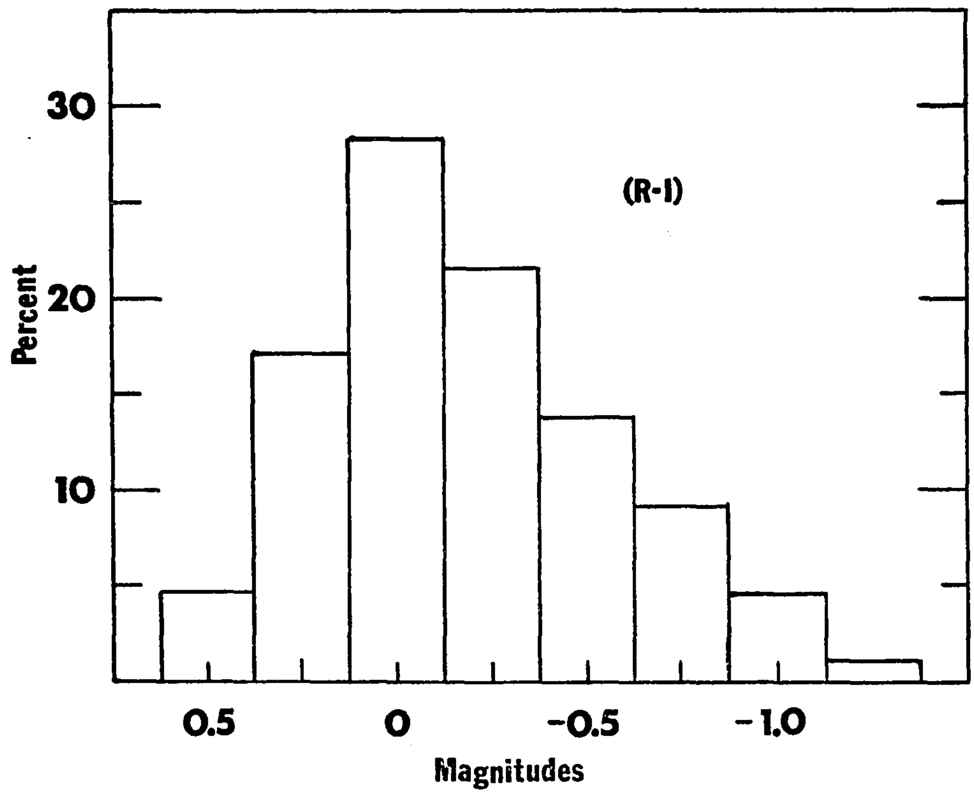

Figure 20. Calculated histogram of scatter about the main sequence derived from a secondary mass distribution function proportional to $m^{-2/3}$ and convoluted with a Gaussian of standard error 0.25 magnitudes. The zero of the abscissa is a main sequnce taken from the final fit to the Hyades.

the van Rhijn function. In the first case, we can see that, since stars with non-stellar companions of a given magnitude have the same colors as single stars of that magnitude, adding them could be done by simply adding a Gaussian of standard error appropriate to the color, whose area is about half that of the area under the distribution already calculated (see Table 3), centered on the main sequence for single stars and renormalizing. Doing this would increase the discrepancy already visible between the calculated histograms and those observed for the Hyades and the Pleiades. Notice, for example, the secondary peak in the observed (B-V) histograms as compared with the very slight secondary peak in Figure 18 which would be made even smaller by the addition of a Gaussian centered on the main sequence. The effect of adding the distribution of scatter about the main sequence due to the van Rhijn function distribution of secondary masses is very similar, adding a slightly skewed Gaussian of about equal weight (see Table 3) to the sum of the already discussed distributions with its peak centered about $0^{m}.05$ above the single-star main sequence. If we assume the luminosity function in clusters is the van Rhijn function [see Allen[23]], we find about 90 per cent of the stars are within $-0^{m}.25$ and 85 per cent are within $-0^{m}.1$ of the main sequence for single stars. This would further increase the discrepancy between the observed and calculated histograms. By comparing the observed and calculated histograms, it can be seen that the Hyades and the Pleiades secondary mass distributions are enhanced around secondary masses roughly equal to the primary mass (those objects around $0^{m}.6$ to $0^{m}.7$ above the mean curve) with respect to the secondary mass distribution for field stars, the Hyades more so than the Pleiades. It is also evident from the depth of the minimum between the two peaks in the histogram for the Hyades that the Hyades is probably deficient with respect to field stars in binaries whose secondaries are about 3^m fainter than their primaries (those objects about $0\overset{m}{.}4$ above the mean curve). Here, one might consider varying the relative weights of the van Rhijn and non-stellar companion distributions with respect to the $m^{-2/3}$ distribution. There is some justification for this since the weight factor for the non-stellar companion distribution involved an extrapolation of an empirical fit and since Aarseth (private communication) has said that, in clusters, if many binaries are present then dynamical binaries (those represented by the van Rhijn function) are unlikely. However, even by adjusting the relative weights of these distributions, one could not remove the discrepancy between the observed and calculated histograms for the Hyades and the Pleiades. For the Pleiades, one might demand that the calculated histogram agree with the observed frequency of equal mass component binaries but then the frequency of binaries whose components differ by about 3^m would be considerably deficient in the observed histogram with respect to the calculated histogram, the point being that the ratio of frequencies of the two types of binaries could never be in agreement with that derived from the calculated histogram no matter what weights are used. For the Hyades, the ratio of the two peaks in the observed (B-V) histogram can only be made more discrepant with respect to the calculated histogram by adding the other two distributions no matter what the weight.

Likewise, it would be impossible to force a fit on the minimum between the two peaks where those binaries whose components differ by about 3^m are. Praesepe is a borderline case; although there is an apparent secondary peak in its histogram it is not satistically significant. It is interesting that Praesepe should be different from the Hyades in not having a pronounced secondary peak since the similarity between the Hyades and Praesepe in other respects has been frequently noted, there even being speculation that the two clusters may have had a common origin. The secondary mass distribution function for Praesepe might be the same as that of field stars if the various uncertainties are considered.

Table 9 gives the frequencies of binaries for each cluster (the analyzed frequencies are for binaries whose components are less than 3^m fainter than the primary). Notice that Praesepe is again markedly different from the Hyades. Perhaps the individual cluster dynamics are the important factor in determining these differences.

Table 9. Hyades Stars which were Analyzed

VB	VB	VB	VB
11	38	63	93
12	39	64	94
15	40	65	95
16	42	67	97
17	43	68	99
20	45	69	101
21	46	73	102
22	47	75	103
23	48	77	104
24	49	78	108
25	50	79	141
26	51	80	162
27	52	81	173
29	53	82	174
30	54	83	175
31	55	84	176
32	56	85	178
33	57	87	180
34	58	88	181
35	59	89	182
36	60	91	183
37	62	92	

[See Table 4 for references]

Table 10. Pleiades Stars which were Analyzed.

HzII	HzII	HzII	HzII	Ts
25	563	1200	2027	26
102	605	1207	2147	29
120	627	1215	2172	37
152	652	1234	2181	39
153	697	1266	2195	42
157	708	1284	2220	45 & 45x
158	717	1309	2263	51x
164	727	1338	2278	61
173	739	1362	2289	67
232	745	1375	2341	76
233	761	1380	2345	78 & 78x
248	801	1384	2415	84
250	804	1397	2425	93
253	817	1407	2488	115
293	859	1425	2500 & 2507	127
298 & 299	923	1431	2506	137
303	948	1514	2644	142
314	956	1613	2786	144y
320	975	1726	2866	149
338	996	1762	3031	151x
344	1015	1766	3097	156x
405	1028	1776	3179	165
470	1084	1797	Tr60	177
476	1101	1823	Ts4	183x
489	1117	1856	Ts9	184
514	1122	1876	Ts19	184x
530	1132	1912	Ts23	185
531	1139	1924	Ts25	194
541	1182	1993	Ts25a	

(See Table 5 for references)

Table 11. Praesepe Stars which were Analyzed

VL	VL	VL	VL	VL
166	598	926	1044	1232
236	621	929	1049	1236
237	659	934	1054	1242
265	691	937	1057	1243
312	700	941	1070	1248
315	744	947	1071	1253
321	747	952	1077	1263
363	750	963	1078	1273
387	761	967	1086	1276
388	775	973	1090	1283
408	778	979	1091	1300
418	787	982	1092	1304
440	789	987	1098	1312
441	828	993	1100	1322
443	843	997	1113	1326
455	847	1003	1114	1345
460	867	1005	1142	1356
467	877	1008	1149	1361
483	889	1012	1150	1365
489	891	1013	1151	1399
498	900	1014	1152	1416
501	907	1022	1164	1421
508	908	1027	1180	1426
515	917	1028	1197	1452
545	921	1031	1205	1483
570	922	1033	1209	1517
588	925	1036	1220	1540
				1567

(See Table 6 for references)

CHAPTER IV

THE EFFECTS OF UNRESOLVED DUPLICITY ON SPECTRA OF SOLAR-TYPE STARS

A. Introduction

According to the results of Abt and Levy, duplicity among solar-type stars is considerably more common than previously supposed. Therefore it becomes of increased interest to consider the following question: what do the composite spectra of normal stars look like, and what errors occur when a spectroscopic analysis is applied on the mistaken assumption that one is dealing with a single star?

In this chapter, I discuss some selected features in the composite spectra of pairs of normal main sequence stars, the hotter star having roughly solar temperature. The body of this chapter is taken from Bettis and Branch[(24)]. To make the matter a little clearer, let me give a brief description of a model atmosphere calculation. [For an elementary treatment, see Swihart[(1)]; for more advanced theory see Mihalas[(25)] or Aller[(26)].] One starts with the equation of transfer, a first-order linear differential equation relating the specific intensity (the amount of energy, per unit frequency interval, passing through a unit area oriented normal to the beam, into a unit solid angle, in a unit time) and source function (the ratio of emmisivity to opacity at a particular frequency which will be in general a function of intensity) to the derivative of the specific intensity with respect to optical depth. One can find a formal solution to this equation, an integral equation in specific intensity. The model atmosphere problem is to give an accurate estimate of the run of physical variables with depth in the atmosphere and the emergent spectrum; in general this is very difficult and has not yet been done, but under certain assumptions it can be tractable. For instance: assume local thermodynamic equilibrium, hydrostatic equilibrium, radiative equilibrium, the atmosphere to be stratified into homogeneous plane parallel layers and steady state conditions throughout. Then the local distribution of atoms among their various states is determined by the local temperature and their distribution among various states of ionization is determined by the temperature and the electron density. From this, one may obtain the opacities as functions of temperature and electron density. The gas pressure is the total of the partial pressures; since the pressure due to ions is a function of electron density and temperature and the electron pressure is also a function of electron density and temperature, the gas pressure is a function of electron density and temperature. If one now assumes a temperature-optical depth relationship, the density structure of the atmosphere is determined by the equation of hydrostatic equilibrium. By integration of this equation, one derives step by step the pressure and electron density as a run of optical depth and from them, the opacities at all frequencies. One may then solve the radiative transfer equation by estimating the source function. From the solution of the equation of transfer, one can obtain the mean intensity (specific intensity averaged over solid angles) and the flux as a function of optical depth and then see if the condition of radiative equilibrium obtains. If not, assume a slightly different temperature-optical depth relationship and repeat the calculation until the desired accuracy is achieved. When the model atmosphere has been calculated, one can then proceed to calculate the emergent spectral lines by including the appropriate atomic data (oscillator strengths, excitation energies, ionization potentials, abundances, molecular weights, partition functions) and calculating how much radiation is absorbed out of the continuum at each depth.

B. The Sodium D Lines

Given the number of equivalent width measurements in the literature, the strengths of the Na D lines can be calculated without resorting to model atmospheres. Equivalent widths and B-V colors of composites were calculated using (1) a mean relation between the equivalent width of the $D_1 + D_2$ doublet and B-V colors (Figure 21), constructed from the data given by Tinsley[27] and Spinrad[28], (2) a mean relation between absolute visual magnitude, M_V, and B-V for nearby main sequence stars [Woolley *et al.*[29]], and (3) a relation between M_V and the monochromatic flux at λ5900, estimated from the absolute flux measurements of Whiteoak[30]. The composite equivalent widths were obtained by weighting the equivalent widths of the component stars according to the λ5900 flux.

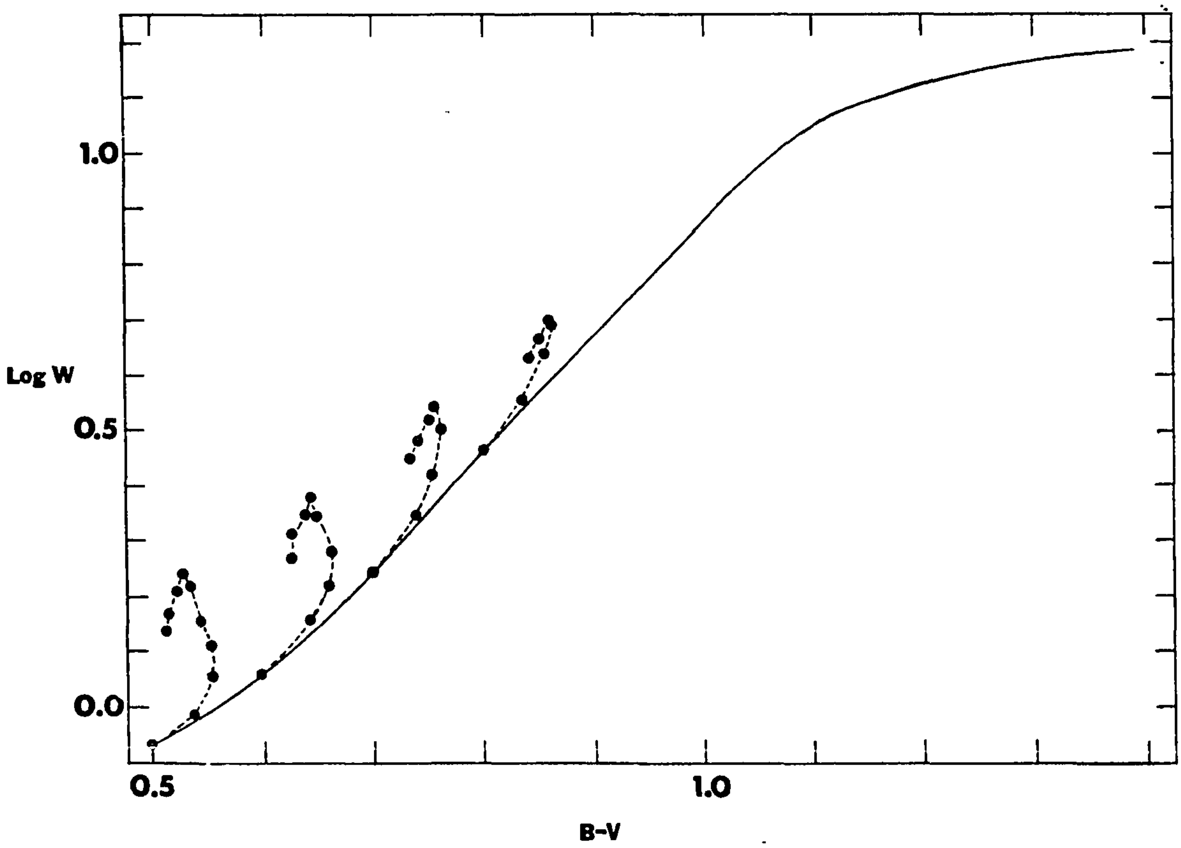

Figure 21. Mean relation between Na D_1+D_2 equivalent widths and (B-V) color index (full line) for main-sequence stars. Filled circles represent equivalent widths and color indices of composites consisting of a primary having $(B-V)=0^m.5$, $0^m.6$, $0^m.7$, or $0^m.8$ and secondaries having (B-V) incremented in steps of $0^m.1$ to $(B-V)=1^m.4$.

The composite equivalent widths, also shown in Figure 21, are in general significantly greater than those of single stars of the same color. In the most extreme case, the pair of stars having B-V = 0.5 and 1.1 forms a composite which has an equivalent width 90 per cent greater than the single star having the color, B-V = 0.53. A star having B-V = 0.5, combined with any star having B-V in the range 0.8 to 1.4, forms a composite which has D lines enhanced by more than 30 per cent. Judging from Figure 21, stars redder than B-V = 1.4 would also produce significant effects, but the problem would become complicated by TiO contamination of the D lines in the M stars. It must be mentioned that the relationship between equivalent width and color is not well determined for B-V greater than 1.0. After David Branch and I did the work from which this chapter is drawn, we received data from Oinas (private communication) that gave us a slightly lower curve for the mean B-V equivalent width relation. Using this curve, lower values for the composite equivalent widths were obtained but not significantly lower for our purposes. However, a theoretical relation based on model atmosphere calculations of D line equivalent widths [Cayrel de Strobel[31]] gives us a slightly higher curve, implying that the effects of duplicity have been underestimated. So Figure 21 represents a working compromise.

The actual effects of duplicity on equivalent widths measured on microphotometer tracings might be somewhat less drastic than implied by Figure 21. In the most extreme cases of D line enhancement, the major contribution of the cooler star is to produce broad, shallow wings on the composite profile (see Figure 22, based on model atmosphere calculations to be described in Section C). The unexpectedly shallow wings might go unnoticed, to some extent, by the measurer. Nevertheless, it seems clear that the D lines in spectra of main sequence stars should be used with extreme caution, if at all, for abundance work.

The effects of duplicity on narrow band photoelectric measurements of the D lines might be quite serious, since such measurements take full account of any line wings, however shallow, which fall within the passbands. This suggests the interesting possibility of spectrophotometric detection of binary stars in clusters, where chemical composition differences are presumably absent. This method would complement the approach in Chapter III by detecting those binaries whose components differ greatly in magnitude (more than about $2^{m}.5$); this idea is explored in Chapter V.

C. Lines of Neutral and Ionized Iron

Model atmospheres (log g = 4.0 i.e. main sequence stars) published by Carbon and Gingerich[32] have been used to calculate the profiles of the D lines shown in Figure 22, and the equivalent widths of Fe I and Fe II lines in composite spectra. For each composite a red color index, based on the monochromatic fluxes of the models at λ6095 and λ6994, was computed. A single model having the same red color index was then obtained by interpolation in the Carbon-Gingerich grid. In this way, composites consisting of a model having effective temperature 6000°K, combined with models of

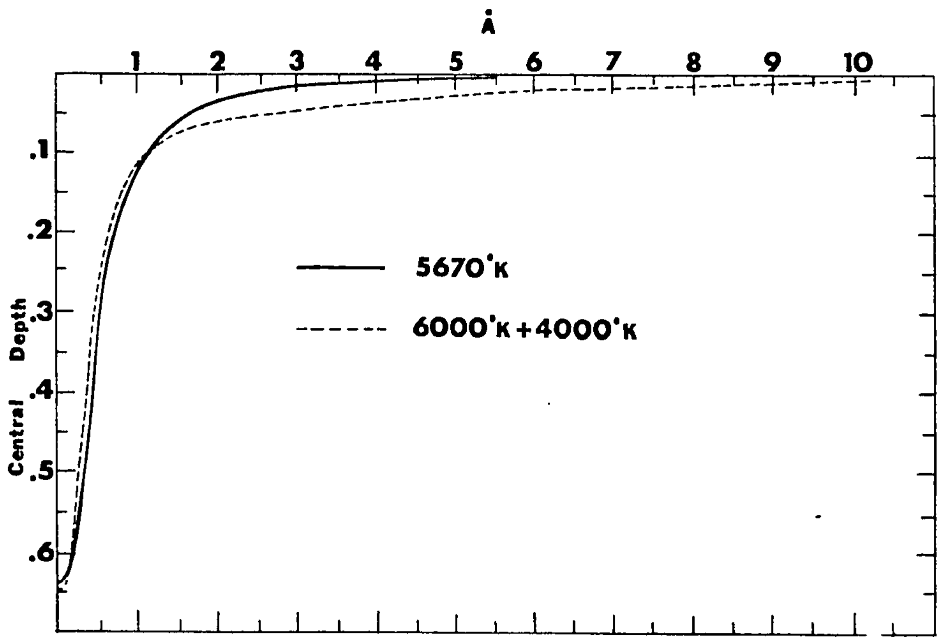

Figure 22. Calculated profiles of the Na D_2 line, based on model atmospheres having log g = 4.0.

5500, 5000, 4500, and 4000 °K were found to have "equivalent single models" of effective temperatures 5810, 5670, 5580, and 5670 °K, respectively.

In general, Fe I lines are weaker and Fe II lines are stronger in spectra of composites than in the spectra of the equivalent single models. Computed curves of growth for Fe I and Fe II, for the combination 6000 +4000 °K, are compared to the curves of growth for the equivalent single model in Figures 23 and 24. At λ5000 the horizontal separations of the composite and single star curves of growth (which increase with decreasing wavelength) are about 0.15 dex for Fe I and 0.10 dex for Fe II. Thus in a conventional abundance analysis of an unresolved 6000 + 4000 °K pair, treated as a 5670 °K single star, the iron-to-hydrogen ratio based on Fe I would be underestimated by 0.15 dex, and the electron pressure based on a comparison of Fe I and Fe II would be underestimated by 0.25 dex. The error in Fe/H would be barely significant for an individual star, being somewhat smaller than the acknowledged uncertainty of spectroscopic analysis, but the effects of duplicity will produce a slight tendency for field stars, on the average, to appear metal-deficient relative to the Sun. The electron-pressure error would result in underestimates of the surface gravity [being proportional to the electron pressure squared in late-type stars, see Swihart(1)] and mass by a factor of three! Unrealistically low masses have in fact, been derived by Oinas(33); however, it is unlikely that all of his troublesome stars have appropriate companions, and in any case Perrin, Cayrel de Strobel, and Cayrel(34) seem to have removed much of the

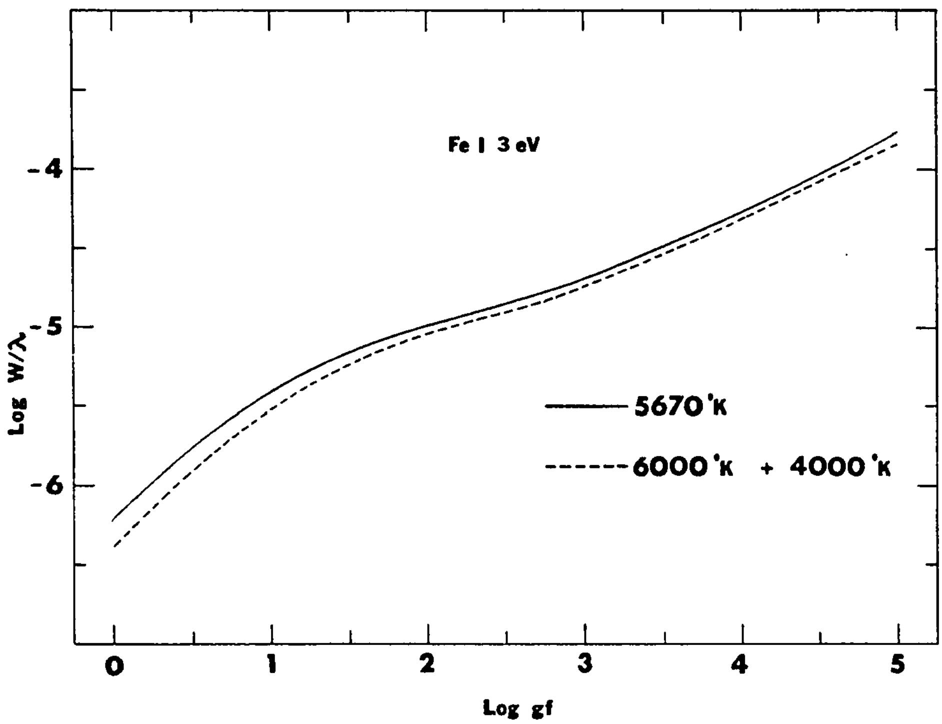

Figure 23. Curve of growth for Fe I; excitation potential 3eV, λ5900.

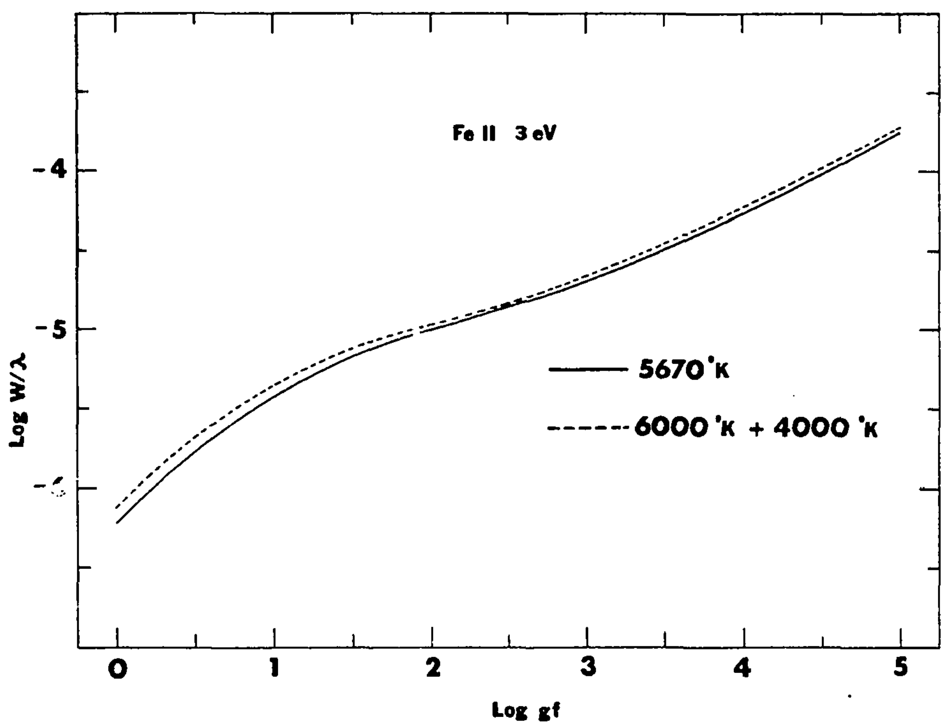

Figure 24. Curve of growth for Fe II; excitation potential 3 eV, λ5900.

difficulty. Still, when estimating the masses of late-type dwarfs on the basis of electron pressure, one should keep in mind the possible effects of unresolved duplicity.

CHAPTER V

D LINE INDICES AND DUPLICITY IN THE HYADES

A. Introduction

The work described in the previous chapter led me to believe that by observing D line strengths in the Hyades using narrow band photometry I might learn something about duplicity in that cluster. This chapter describes the results of observations that I made at Kitt Peak and their application to the problem of the secondary mass distribution in the Hyades.

To begin with, let us define an index, I, as $2F_b/(F_a+F_c)$, where F_b is the flux in some passband centered on the D lines and F_a and F_c are fluxes in passbands of the same width as the first centered on points in the continuum on either side of the D lines. Assuming there are no other features in any of the passbands (we will not need this assumption; I use it now to give the reader an idea of how the index is related to equivalent width) then $I = 1-W_\lambda/\Delta\lambda$, where W_λ is the equivalent width of the D lines and $\Delta\lambda$ is the width of the passband. So, a change ΔW_λ, in equivalent width, produces a change $\Delta I = -\Delta W_\lambda/\Delta\lambda$ in the index, and we expect D line enhancement due to duplicity to reduce the index of the composite below a mean (B-V)-Index relationship provided that I doesn't fall off too rapidly with (B-V). It turns out that, over the range of stars later than about G0, $\Delta\lambda = 30Å$ is a good passband width to use, taking in most of the wings of the D lines over that range yet not being so large as to be insensitive to changes in W_λ.

Filters were procured of the appropriate width and frequency. The centers for the continuum passbands were determined by looking at a solar spectral atlas and finding reasonably featureless regions of the passband width on either side of the D lines. The photon counting times through each filter for best efficiency and least error were calculated in the standard way [see Crawford[30]], giving a standard error in the index of 0.0026 for eighth magnitude stars and 0.0065 for tenth. This is the minimum error; things can be worse because of bad conditions, faculty equipment or poor observing practice. The stars observed, their (B-V) colors [from Mendoza[16]] and the measured indices are listed in Table 12 and the indices as a run of (B-V) are shown in Figure 25. When these indices are compared with similar indices Taylor[12] measured for some of the stars in Table 12, an estimate of the standard error in my measurements (given the error in Taylor's measurements) of 0.005 can be had, a value consistent with the calculated error averaged over the whole range of magnitudes of observed stars (from about $7\overset{m}{.}5$ to about $10\overset{m}{.}5$). This value will serve as an upper limit in the next section since there we are not concerned with the whole range from $7\overset{m}{.}5$ to $10\overset{m}{.}5$ but only those stars from $7\overset{m}{.}5$ to $8\overset{m}{.}5$, and the calculated value will serve as the lower limit.

Table 12. D Line Indices for Late-Type Hyades

VB	(B-V)	I	VB	(B-V)	I	VB	(B-V)	I
1	0.57	0.956	52	0.60	0.949	135	0.87	0.882
2	0.62	0.954	61	0.51	0.955	138	0.86	0.940
3	0.75	0.925	63	0.64	0.950	140	0.77	0.910
4	0.84	0.903	64	0.66	0.944	142	0.67	0.946
5	0.91	0.879	98	0.45	0.967	146	0.53	0.956
7	0.89	0.881	99	0.86	0.903	148	0.63	0.949
9	0.71	0.916	102	0.62	0.934	149	0.62	0.949
10	0.60	0.947	105	0.58	0.947	150	1.04	0.951
15	0.66	0.964	106	0.67	0.944	153	0.84	0.900
16	0.42	0.971	109	0.81	0.925	158	0.61	0.946
17	0.70	0.942	110	0.68	0.931	159	0.52	0.969
18	0.64	0.941	114	0.72	0.943	162	0.71	0.930
19	0.51	0.958	115	0.84	0.905	165	0.63	0.937
21	0.82	0.914	116	0.82	0.908	173	1.24	0.780
23	0.69	0.939	117	1.07	0.831	174	1.07	0.844
25	0.99	0.868	118	0.58	0.944	175	1.04	0.841
26	0.74	0.927	119	0.56	0.938	176	0.94	0.876
27	0.73	0.932	120	0.74	0.926	178	0.84	0.909
31	0.57	0.950	121	0.50	0.956	180	0.86	0.909
39	0.68	0.935	125	0.49	0.965	181	1.16	0.823
44	0.45	0.959	127	0.74	0.932	182	0.85	0.901
49	0.59	0.949				183	0.94	0.895

Numbers are those of van Bueren[17]; (B-V)'s are taken from Mendoza[16].

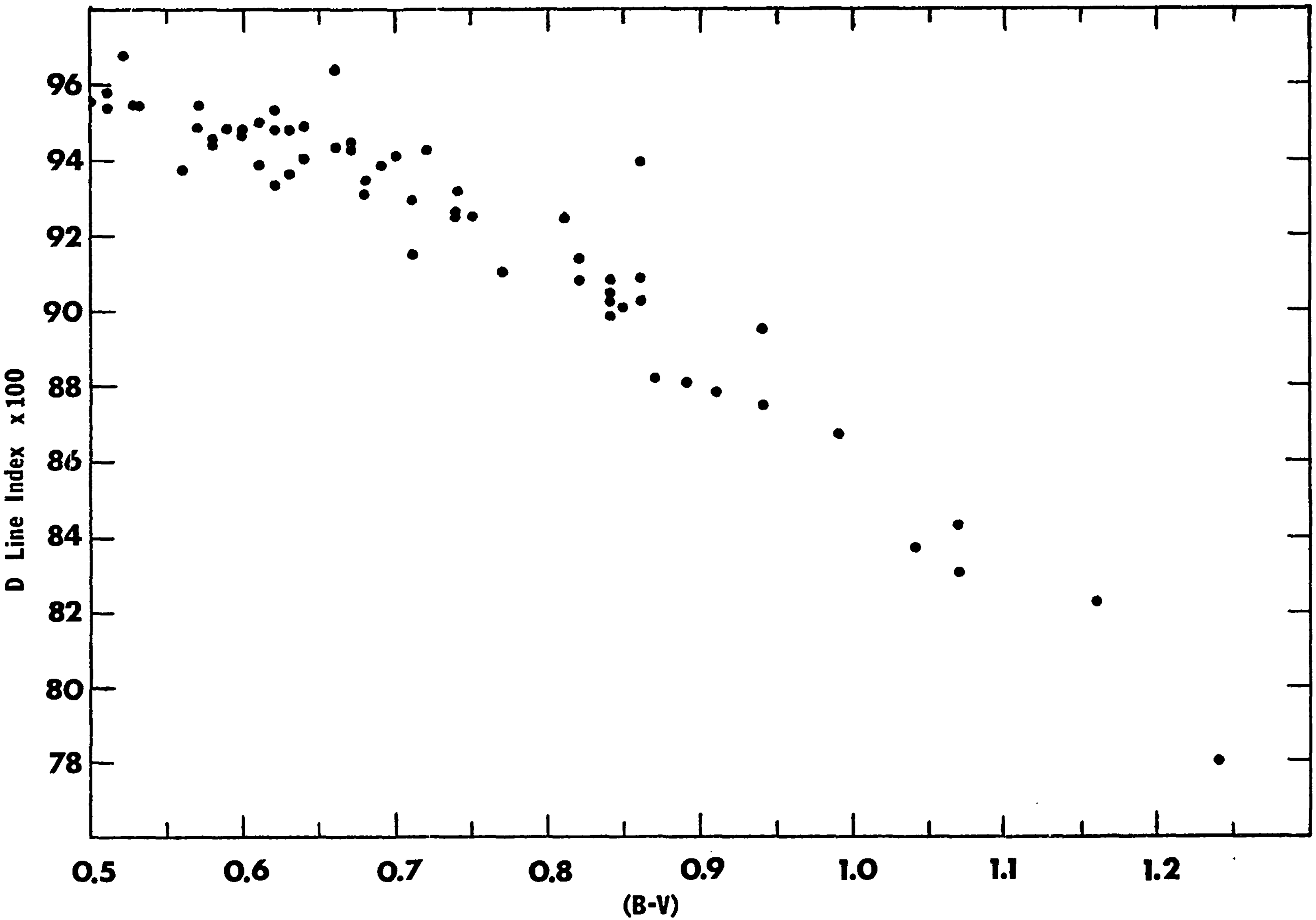

Figure 25. D Line Indices in the Hyades.

B. Duplicity and the D Line Index

The results of Chapter IV show us that the D line strength is increased markedly for a star with an appropriate companion over a single star with the same (B-V). How does the D line index behave under similar circumstances? I approached this question in two ways. The first was to take actual measured fluxes of stars and add them together (the continuum fluxes of two different stars had to be normalized according to their absolute magnitudes and the relation between absolute flux and flux at λ5900 mentioned in Chapter IV: usually these normalization corrections were small, the measured values being close to those that would have been obtained by absolute photometry) and compute the composite index. The second was to use a mean (B-V)-Index relation and add the indices together thus: $I_1/(1+1/a) + I_2/(1+a)$, where I_1 is the index of the first star, I_2 that of the second star and a is the ratio of the continuum flux of the first star to that of the second at λ5900. Both methods gave the same results; since the second was more convenient, I used it in the following calculations. Note that neither approach depends on a knowledge of the D lines or what other features might happen to be present in one of the passbands (the particular passbands used were centered on λ5826, λ5883 and λ5940 with half-widths of 28, 27, and 27 Angstroms). As a run of secondary (B-V), for a primary of (B-V) = 0.5, the composite index drops from 0.957 to 0.946 (when the secondary (B-V) ≈ 1.1: for secondary (B-V)'s from 0.8 to 1.1 the composite index was less than 0.948) and increases to 0.947 when the secondary (B-V) = 1.2. It should be remembered that the composites are also slightly redder than the primary though for the (B-V)-Index relation, this is not too important as the index does not vary sharply with (B-V). It is unfortunate that I don't have measured values for the indices of stars of (B-V) redder than 1.24 in that these values are necessary in calculating the scatter about the mean (B-V)-Index curve due to duplicity. I was able to extrapolate my mean curve to larger (B-V)'s by referring to Spinrad's[36] measurements of indices in M dwarfs and translating a mean relation to my system. Spinrad's measurements of a D line index show a great deal of scatter for M dwarfs so such a mean relation is not too well known.

Figure 28 shows the observed scatter about a mean curve (drawn by eye) for the (B-V)-Index relation from (B-V) = 0.5 to 0.8. This range was selected because in analyzing the effects of duplicity we need the run of secondary indices for faint stars, and as mentioned, past (B-V) = 1.2, these are not well known. So we can minimize the effects of this uncertainty by concentrating on the brighter primaries. The asymmetry in Figure 28 is slight and suggests the presence of binaries.

Let us now consider theoretical histograms of scatter much the way we did in Chapter III. Using Abt and Levy's results [considering a primary of (B-V) = 0.5 and renormalizing Equation (3) (Chapter III, Section D) accordingly], the composite indices as a run of secondary (B-V) and the reddening of the composite over the primary, we can calculate the distribution of indices. We can

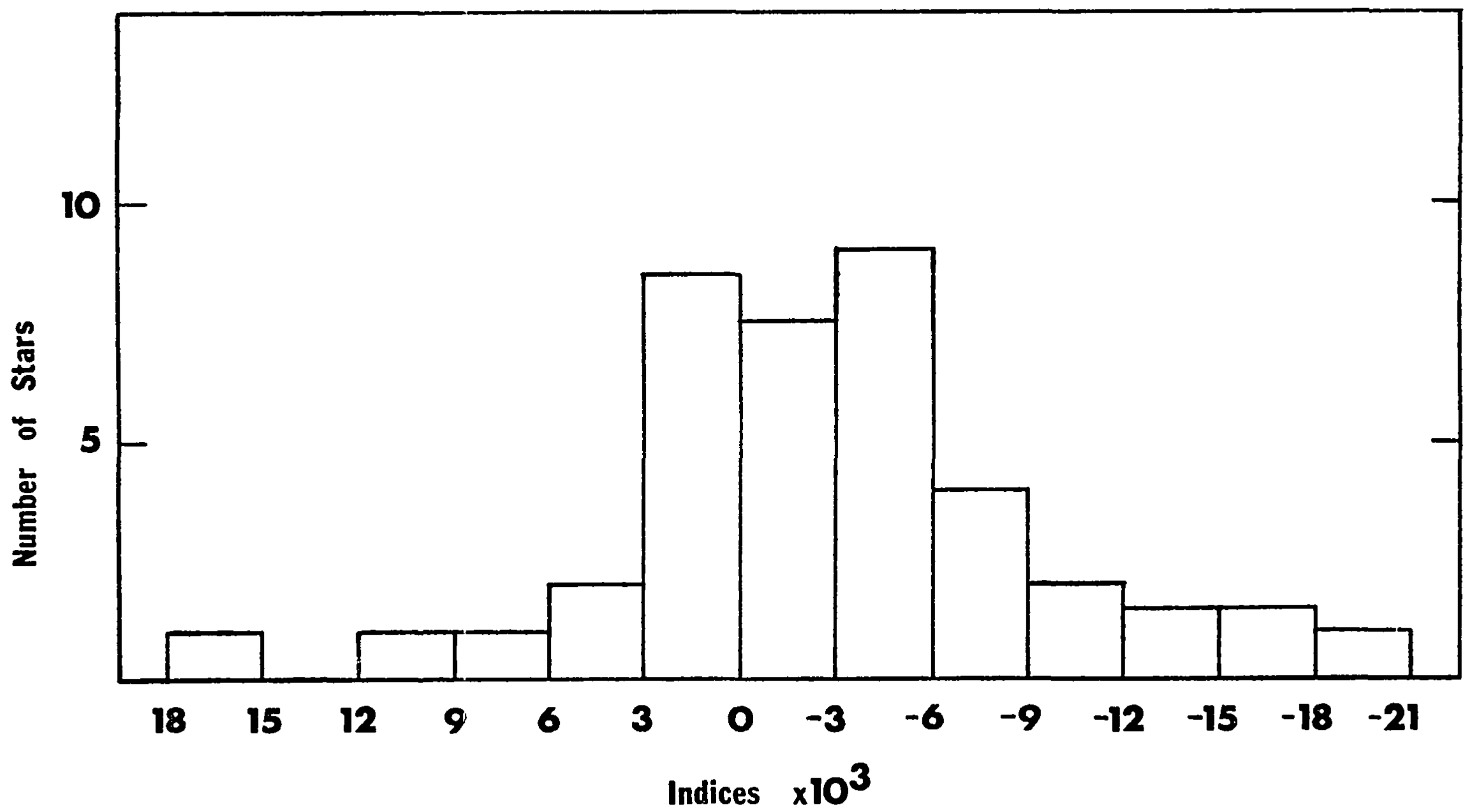

Figure 26. Histogram of scatter about a mean curve for Indices of Hyades stars from (B-V)= 0.5 to 0.8.

calculate the distribution of indices due to the van Rhijn function by using Allen's[23] tabulation of the van Rhijn function and we find that the distribution of indices due to this source is strongly peaked on the mean curve. We must now convolute these distributions with Gaussians of some standard error. What value of the standard error should be used? We know the lower limit is 0.0026 and the upper limit is 0.005, and we would like to use a value that would represent the average value of the standard error over the range (V = 7.5 to V = 8.5 roughly) of the observed histogram, Figure 26. I found that convolving the distributions with a Gaussian of too large a standard error results in a broad-peaked symmetric distribution in contrast to Figure 28. A standard error of 0.003 seemed to be a reasonable value to use, being a bit larger than the lower limit and not so large as to destroy the sharpness of the peak (and asymmetry) in both the distribution of indices derived from the secondary mass distribution function proportional to $m^{-2/3}$ and the distribution of indices due to the van Rhijn function. Figure 27 shows the distribution of indices due to the $m^{-2/3}$ secondary mass distribution and Figure 28 shows the distribution of indices due to the van Rhijn function and stars with non-stellar companions added to the distribution in Figure 27.

C. Discussion

As can be seen from comparing Figure 26 with Figure 27, the case for the lower mass secondaries being depleted with respect to the results of Abt and Levy is not clear though it may be compatible

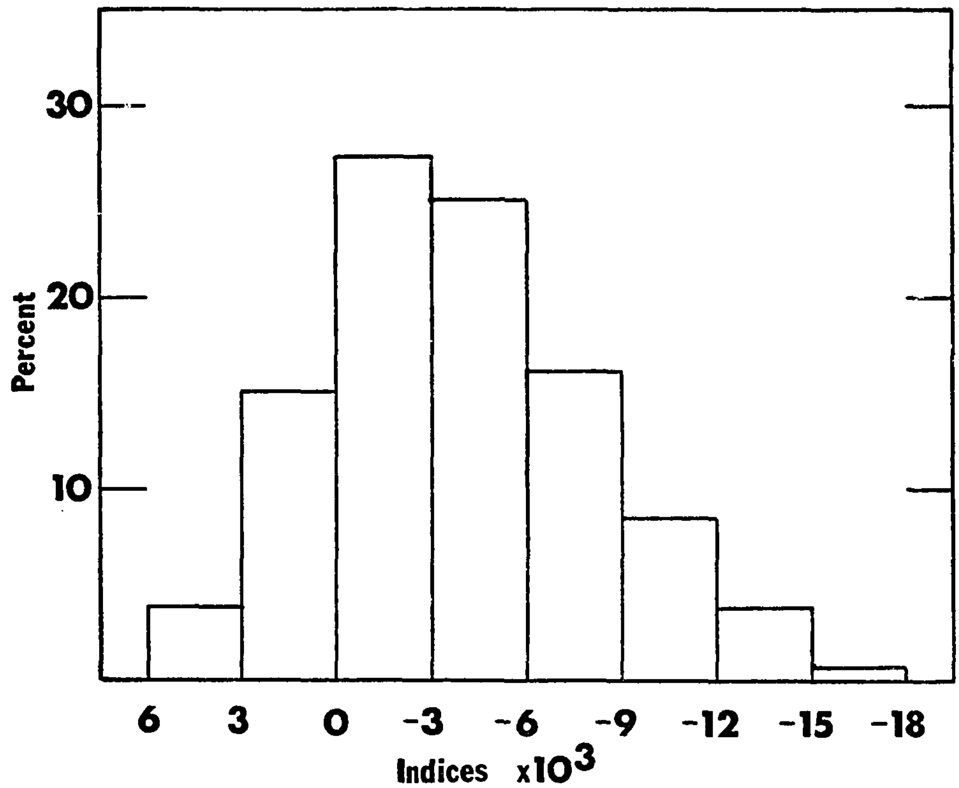

Figure 27. Calculated histogram of scatter about a mean (B-V)-Index relation for a secondary mass distribution proportional to $m^{-2/3}$ and convoluted with a Gaussian of standard error 0.003 indices.

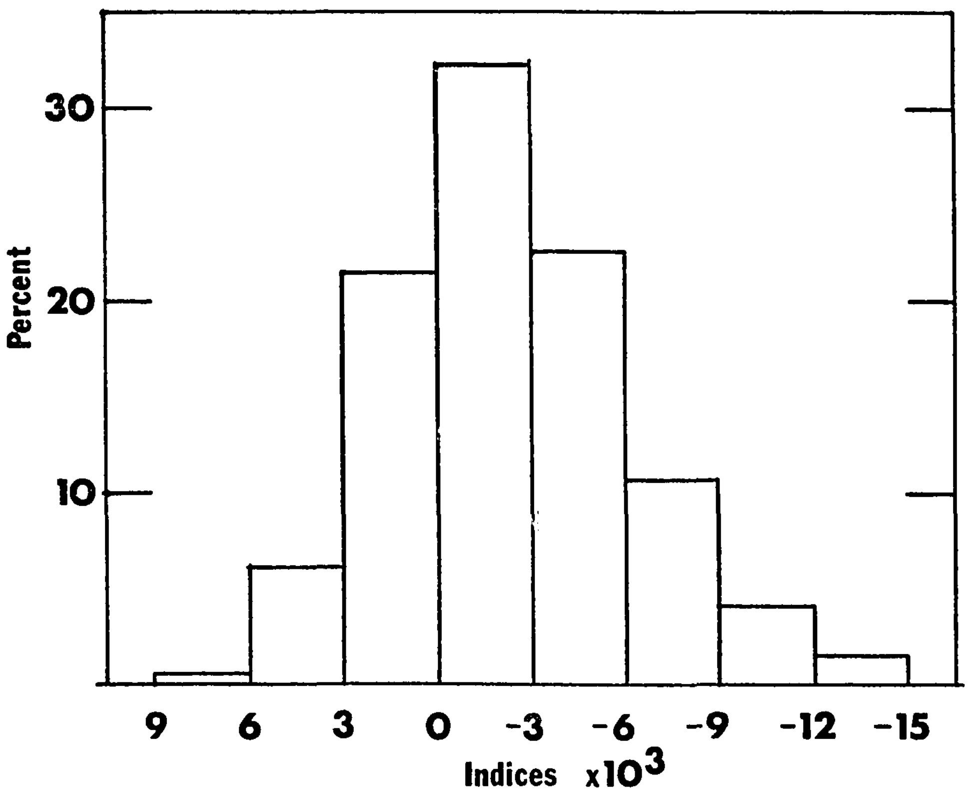

Figure 28. Calculated histogram representing the distribution of indices shown in Figure 27 added to a distribution derived from a secondary mass distribution proportional to the van Rhijn function and a distribution due to the stars with non-stellar companions, both being convolved with a Gaussian of standard error 0.003 indices.

The observed histogram looks a little depleted about the region from -0.006 to -0.012 which would indicate deficiencies in secondary frequency either from about $1^m.5$ to $2^m.5$ fainter or more than $3^m.5$ fainter than the primary, the latter value being the one compatible with the results of Chapter III. This is not a very strong conclusion because of the small number of stars in the observed histogram and because adding the distributions due to the van Rhijn and non-stellar companion secondary mass distributions can remove most of the discrepancy as can be seen in Figure 28. So the comparison is unfortunately strongly dependent on the questionable parts of the secondary mass distribution (see Chapter III, Section D). The interpretation of the histogram is also made uncertain because of the extrapolation involved in computing the composite indices and because of the uncertainty in the van Rhijn function for clusters, the former being the greater stumbling stone.

One may conclude, however, that the effects of duplicity on narrow band photometric measurements of the D lines are not negligible by the way the measured indices add together, those additions being independent of any spectroscopic interpretation. Certainly, one should not use a D line index as an indicator of sodium abundance unless the case under consideration rules out the possibility of duplicity playing a significant role. It is also safe to say that the effects of duplicity are present in the measured indices, the observed scatter being a bit larger than can be accounted for by a reasonable value for the standard error, no sodium abundance variation, and single star values of the D line index.

CHAPTER VI

SUMMARY

In this thesis, I have examined a few aspects of duplicity among stars. In Chapter III, I analyzed photometry of three galactic clusters and, in addition to identifying likely binaries, came to several conclusions: the single star main sequences of the three clusters are one-dimensional within the limits of observational error; the incidence of duplicity is higher in the Hyades than in the Pleiades or Praesepe; the scatter in the Hertzsprung-Russell diagram for the Hyades is not compatible with the scatter one would expect had the Hyades the same secondary mass distribution function as Abt and Levy have found for field stars; the scatter in the color-magnitude diagram for the Pleiades is also so incompatible. It is also clear that the secondary mass distribution function for the galactic clusters couldn't be just the van Rhijn function or the binary ridge could not be seen. (Remember, for the secondary mass distribution function proportional to the van Rhijn function, 90 per cent of the binaries are within $0\overset{m}{.}25$ and 85 per cent are within $0\overset{m}{.}1$ of the single star main sequence [for (B-V)], resulting in an almost Gaussian distribution around a line slightly brighter than the single star main sequence and consequently no binary ridge.) In Chapter IV, I discussed the effects of duplicity on spectral features of solar-type stars, and found interesting consequences for the model atmospheres approach. I also found a means of identifying binaries in star clusters using narrow band photometry of the D lines. In Chapter V, I tried that method on the Hyades. While I wasn't as successful as I would like to have been, nonetheless, several important conclusions followed: the D line indices add in such a way as to make duplicity a significant effect on a (B-V)-Index plot (this is independent of assumptions about what features are in the passbands used), hence sodium abundance determinations should not, in general, be based on D line indices; the effects of duplicity are suggested by the observer scatter in the (B-V)-Index relationship, that scatter being compatible with, but not offering strong support to, the earlier conclusion about the secondary mass distribution function in the Hyades.

I have written to Aarseth about this matter and have recently received a reply; in it, he says that binaries which are too close to have been formed dynamically are very likely to survive disruptive encounters, except by other close binaries. He also adds that he is not sure how effective such encounters are but hopes to investigate them when time permits. He points out that there is no preferential escape of lightest stars [*Astrophysical Letters 12*, 159 (1972)] though this conclusion may need to be modified when the effect of interstellar clouds is included in the computer simulations. He concludes that there is, at this time, no strong reason to suppose that binaries with equal mass components should be retained preferentially during the evolution of a cluster. This leaves open the question of why the secondary mass distribution for field stars should be different than that for the Hyades. It is especially interesting because it is commonly held that most stars are found in clusters and later escape to become field stars. I hope to further investigate the secondary mass distribution function for clusters, by scanning the D lines of other clusters and more observations of the Hyades (both D line and radial velocity observations).

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