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INVESTIGATING THE PROBLEM WITH PROBLEMS:

PRESERVICE MATHEMATICS TEACHERS' UNDERSTANDING OF PROBLEMS AND PROBLEM SOLVING

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Abstract

Problem solving is a key component of mathematics education. Waves of educational reform have focused on the problem-solving aspect within the K-12 educational setting. Yet, the teacher plays a critical role in establishing the problem-solving environment of the classroom. Therefore, preservice secondary mathematics teachers (PSMTs) must examine their understanding of problem solving and tasks that engage their future students in problem solving. This research project aimed to explore and interpret the meanings that PSMTs associate with problem-solving tasks. By examining their experiences with these tasks and the connections they make, this study sought to understand how PSMTs determine the problem-solving criteria they will apply in their future classrooms and investigate the cognitive structures underlying their decision-making within the context of mathematics education. This study centered around questions involving PSMTs' understanding of problems, problem solving, and their criteria for a worthwhile task. This study was a case study design with qualitative data collection within a first mathematics methods course. Participants were three PSMTs enrolled in their first mathematics methods course. Data collection methods involved interviews and analysis of problem-solving tasks and assignments. Common themes for understanding problems were solvable and thinking/sense-making. Problem solving is centralized on uncomfortableness and connecting prior knowledge with new information. The themes involving criteria for worthwhile tasks were mathematical connections, collaboration, and student engagement. Based on the findings of this study, there is a need for mathematics educational institutions to create opportunities for PSMTs to evaluate tasks and refine their criteria for worthwhile tasks. Both mathematics methods courses and content courses should have activities to edit tasks to meet the criteria for worthwhile tasks.

Chapter 1: What's the Problem with Problems

Problem solving is a critical skill that is developed through active engagement in a problem-solving situation. Throughout the history of mathematics education research, there has been a tug-of-war between a focus on basic mathematical skills or a critical mind through problem solving. Current mathematical education research has shifted to shaping mathematical content within a problem-solving lens by establishing process standards to focus on the skill of problem solving. This chapter will focus on the mathematical educational progression throughout history to current mathematical educational trends of problem solving.

History of Problem Solving in Mathematics Education Research

Over the last 120 years, mathematics education research has fluctuated through various ideologies, schooling structures, and goals based on influences from collegiate level professors to policymakers to military involvement to community investors. These influences have shifted the theoretical mindset based on the individuals in power's intended goal. Within the last 40 years, the focus has shifted to an attempt to balance problem solving and fluency of mathematical skills. From the National Council of Teachers of Mathematics' (NCTM) reform of introducing a new goal of problem-solving skills in 1980s to No Child Left Behind in the early 2000s to the rewriting of mathematical standards with the push of Common Core, the focus on mathematics education has pendulum swung between a need basic skills or ability to problem solve.

The Back to the Basics era, 1970s, focused on just the basic skills for everyday life. Kline (1973) urged for curriculum to "be broad rather than deep. It should be a truly liberal arts education wherein students not only get to know what a subject is about but also what role it plays in our culture and society" (p. 145). However, it limited the problem-solving component within mathematics education. Students were spending too much time "on routine problems by applying a known procedure and not enough time working on nonroutine, unstructured problems that required them to learn how to think, reason, and solve problems using disciplinary practice" (Tekkumru-Kisa, et al., 2020, p. 609).

By the late eighties, the National Council of Teachers of Mathematics (NCTM) (1989) introduced standards calling for changes in content and pedagogy. The change focused on integrating real-world connections through problem-solving experiences. Through the NCTM initiative, the goal was for learners to be mathematically literate, lifelong learners, have opportunity for all, and be informed citizens. To be mathematically literate requires that "in-school experiences reflect to some extent those of today's workspace" (NCTM, 1989, p. 4). Lifelong learners need the ability to adept and problem solve. Therefore, schools were urged to create spaces "so that students can create, accommodate to changed conditions, and actively create new knowledge over time" (NCTM, 1989, p. 4). To create such spaces, fundamental changes to how students were taught, and the tools used to teach students were needed (Remillard, 1999).

In response to this push from NCTM through the Curriculum and Evaluation Standards of School of Mathematics, four curricula for elementary mathematics were developed starting in the late 1980s. These curricula differed in their approach to integrating problem solving as NCTM intended. Yet, each emphasized the need for problem solving and the nature of creativity, real-world, and how multifaceted problem solving is.

However, "pure" mathematics enthusiasts were still calling for routine based curriculum. There was still a push for skills to develop through traditional algorithms (Herrera & Owens, 2001). Therefore, there was a struggle between an emphasis on nonroutine problem solving and routine skill based. This shift started math wars between traditional mathematics education and reform mathematics education. Since there was a disconnect between the two sides of the math war, the battle ground came by means of curriculum resources, the textbook. Both sides chose to solicit school districts to adopt their textbook based on the opposing perspective (Herrera & Owens, 2001).

The math wars caused a split of educational focus for student improvement. This split allowed for the federal government to begin playing a bigger role in education, focused on student improvement. The 2000s became a time of increased political influence on educational reform. No Child Left Behind (NCLB) was signed by President Bush in 2001. President Bush stated that NCLB was needed to "compete with education systems in China and India. If we fail to give our students the skills necessary to compete in the world of the 21st century, the jobs will go elsewhere" (2006). NCLB had two main focuses: standardized testing requirements to increase proficiency levels and high qualified teacher certifications. If those two focuses were not met, sanctions were in place to force change, including replacement of staff (Hursh, 2007). The driving inspiration "for this reform is the notion that publicizing detailed information on school-specific performance and linking that 'high-stakes' test performance to the possibility of meaningful sanctions can improve the focus and productivity of public schools" (Dee & Jacob, 2011, p. 418). The idea that having specific standardized measurements for all schools would give society a focal point for additional reforms per school. Schools were required to report on teacher quality and standardized test scores. Through these reports, NCLB aimed to provide parents with educational knowledge (Darling-Hammond, 2006).

The Common Core Initiative (2010) increased political influence in education. The Common Core Initiative became a result of a push for a national curriculum. This initiative "developed these standards as a state-led effort to establish consensus on expectations for student knowledge and skills that should be developed in Grades K-12" (Porter et al., 2011, p. 103). The development of the standards was in collaboration with the National Governors Association Center, Council of Chief State School Officers which represented the top K-12 education officials from all US states and territories, and two major teacher unions (National Education Association and the American Federation of Teachers). The US Department of Education (USDE) was not involved in creating the standards. However, the USDE facilitated the Race to the Top competition. The Race to the Top competition involved creating standardized assessments of the Common Core standards. Two companies created standardized assessments, SMARTER Balanced Assessment Coalition and Partnership for Assessment of Readiness for College and Careers (PARCC). As the standards were written, states were given the option to adopt the Common Core standards or create their own standards that would align. Forty-six states initially chose to adopt the Common Core standards. However, many states later opted to create their own standards, including the state this research study was conducted. States who opted out of the implementation of Common Core believed having a federal curriculum would take rights away from the state. These states created standards that meet the rigorous level of academic knowledge and aligned with a college and career readiness level. Whether states adopted Common Core standards or created their own standards, every state's standards included at least one processing standard that focuses on problem solving (Common Core, 2010; XXXXX, 2022).

The pendulum has again shifted to a call for problem solving within mathematics. NCTM (2014) has integrated problem solving as an essential component within the teaching practices. One of the effective mathematics teaching practices developed by NCTM is to "implement tasks that promote reasoning and problem solving" (NCTM, 2014, p. 17). Problem solving aligns with "mathematics instruction as a means to encourage high-level student thinking and reasoning, and hence maximize student learning" (Kartel et al., 2020, p. 86).

The Common Core standards (2010) and the standards the state this study was conducted (XXXXX, 2022) integrated standards for mathematical practices that go beyond the immediate mathematical content. Both Common Core and this study state's mathematical standards start with how problem solving is essential to a mathematics learning environment. Common Core's (2010) first mathematical practice focuses on problem solving by stating students need to "make sense of problems and persevere in solving them" (p. 6). The study state's standards (XXXXX, 2022) states in their guiding principles that "an effective mathematics program focuses on problem solving" (p. 4), therefore, one of their Mathematical Actions and Process standards is to "develop strategies for problem solving" (p. 8). In both standards, learning in mathematics is described as "an active process, in which each student builds his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers and other adults, and themselves" (NCTM, 2014, p. 9). Mathematics allows for the world to be connected through collaborative problem solving.

Why is problem solving important?

Though processing standards highlight problem-solving skills, not all mathematics classrooms focus on problem solving. Issues arise with only procedure-based and rote memorization teaching (Jonsson et al., 2014; Lithner, 2008) where teachers focus on procedures only in their classrooms through mathematical examples that only require the learner to demonstrate their ability to provide memorized steps (Kapur, 2016). Resources, especially textbooks, have lacked the alignment of problem-solving tasks and have produced examples that follow procedures to solve through step-by-step guidance for the learner and teacher (Cai, et al., 2016; Jäder et al., 2020). For teachers, this lack of problem solving in mathematics classrooms could result from a lack of personal experience with problem-solving tasks or not having sufficient supporting resources in choosing problem-solving tasks. With a lack of this personal experience, one may not have a belief in the crucial need for problem solving. When studying in-service teacher (IST), it was found that "Without a core belief that learning mathematics is a process that best occurs through solving genuine problems, matched with skills in planning and delivering problem-based lessons, ISTs often maintain the status quo by using a traditional approach to instruction" (King, 2019, p. 169). Thus, for preservice teachers (PSTs) to develop the ability to create problem-solving centered classrooms, they must have experiences with problem-solving tasks that confront the beliefs they developed through their K-12 mathematical experiences and highlight the critical need for problem solving in mathematics classrooms.

Yet, PSTs are often challenged to engage in problem solving and value problem solving and the skills that are developed. PSTs tend to solve nonroutine problems using personally chosen strategies (Gözde, 2020). The strategies used by the PSTs aligned with their abilities to solve the task. With a limited set of strategies, the PSTs were unable to complete both tasks, showing a need for more experiences with various strategies in an educational setting. Further, PSTs could not create additional problems to pose based on a mathematical problem (Rosli et al., 2015). PSTs' personal experiences with problem-solving tasks frame their extended use and creation of problem-solving tasks.

Personal experiences and belief also create a mindset of how a person views problem-solving tasks. When asked to create metaphors for problem solving, PSTs' metaphors aligned to their beliefs on the intention of the task (Son & Lee, 2020). For example, one metaphor was "problem solving is like a difficult riddle" (Son & Lee, 2020, p. 136). This metaphor was classified as having a mindset that problem solving is about the product; therefore, the task must result in a final product to be complete. Some metaphors provoked emotions from the participants, like "Problem solving is like the stench of failure because I've never been good at math and I always feel like I am failing at it" (Son & Lee, 2020, p. 138). When problem solving provokes a negative emotion, learners may avoid problem-solving tasks until they can overcome the negative emotion. To help combat the negative emotion, a teacher may scaffold tasks for learners to focus on overcoming the negative emotion to build problem-solving skills.

Supporting problem solving in the classroom

To assist students in improving their problem-solving skills, teachers have been reaching out to various avenues to seek mathematical tasks to implement in the classroom. One of the avenues is the classroom textbook. The textbook gives multiple types of mathematical scenarios for students to build their mathematical skills. However, not all mathematical scenario support building problem-solving skills. A study was conducted to compare the top two elementary school textbooks in the United States and in China (Cai, et al., 2016). The findings showed that textbooks in both countries are less likely to have higher-level problem-solving tasks than lower-level ones.

Since textbooks do not provide the needed levels of problem-solving tasks, teachers have begun to search social media outlets, like Pinterest and TPT formerly known as Teachers Pay Teachers, to supplement tasks. Yet, these outlets are not vetted for accuracy or cognitive demand. Teachers are more likely to choose a task to implement based on the amount of pins the task originally received on social media versus tasks with a significant mathematical content connection (Sawyer, et al., 2019). If veteran teachers are choosing based on arbitrary criteria like the number of pins received, how might PSTs choose tasks? Before this question can be investigated, one must seek to understand what PSTs see as a problem and a problem-solving task. The basis of choosing a mathematical task is found in one's understanding of a problem and problem solving. One's understanding paves the way for how one chooses a task that aligns with their understanding and current need for the classroom.

Research Questions

Historical events mentioned above have changed focal points on mathematics educational research. Throughout the changes, resources have been created to support the changes during a specific time of educational reform. Educational research has highlighted a focus on how educational resources either use or do not use problem-solving tasks (Ayalon & Hershkowitz, 2018; Cai et al., 2016; Santos-Trigo, 2014) and how teachers use the resources to facilitate the classroom experience (D'Ambrosio, 2003; Lester & Cai, 2016; Lubienski, 2000; Saadati et al., 2019). Additional research has been conducted with PSTs on their abilities to problem solve, using Pólya's levels of problem-solving process (Gözde, 2020; Rosli et al., 2015; Son & Lee, 2020). Yet, literature is limited in how PSTs define problem solving to use problem-solving tasks in their future classroom. This research study aims to fill that gap in the literature by asking the following questions:

1. What prior knowledge and understandings do preservice secondary mathematics teachers (PSMTs) have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?

a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

Study Organization

In this study, I begin by delving into the existing literature that explores the interplay between constructivism, authentic learning, and problem-solving. Subsequently, I delve into the key features of problem-solving, drawing insights from research that focuses on problem-solving skills and processes. Next, I examine the presence or absence of problem-solving in the classroom setting. In the subsequent section, I define problems within this context and outline potential criteria for problem-solving tasks. Finally, I review the literature on how teachers' beliefs and experiences shape their perception of problem-solving tasks and influence their decision-making process when selecting tasks. My review of the literature will demonstrate a lack of research concerning PSMTs and problem solving. In doing so, I identify a gap in the literature regarding PSMTs and their choice of criteria and influential factors.

After reviewing the literature, I describe the methods used to address the identified gap including the research design of this case study. This case study used semi-structured interviews to understand participants views on problems and problem solving and think-aloud protocols as PSMTs analyzed tasks. Next, I will organize my findings by research question topic of problem, problem solving, and criteria. Each participants' experiences will be described in chronological order. Finally, I discuss the significance of the findings by means of current literature. I will conclude with implications to current mathematical educational settings and future potential research.

Chapter 2: Review of the Literature on Problems and Problem Solving

For this literature review, I will first lay a foundation of constructivist theory and demonstrate why a constructivist lens is appropriate for this research project. Using the constructivism lens, problems, tasks, and problem solving will be defined. Next, I will discuss research that focuses on teachers' understanding of problems and problem solving, including how personal experience shapes one's understanding and usage of problem-solving tasks. Then I will identify a gap in the literature with PSMTs and their understanding of problems and problem-solving tasks. This gap will lead to my research questions:

1. What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?
    a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

A Constructivist Theoretical Framework

Each person experiences the beauty of the world through their own lens, formed through their experiences and learning. That beauty is demonstrated through patterns and connections. "Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us" (Schoenfeld, 2016, p. 2). This quote from Schoenfeld captures the beauty of mathematics. It gives people a way to see the world through the connections that are revealed through mathematics. Mathematics educators use problems, tasks, and problem solving to support students in connecting mathematics, the world, and the mind. By engaging in authentic mathematical problem-solving tasks, students construct their own understanding which allows for the patterns and connections that make mathematics beautiful to emerge.

Constructivism asserts that individuals construct meaning from their own experience and use that meaning to make decisions in their realities. In other words, “people construct knowledge based on their beliefs and experiences in situations” (Schunk, 2020, p 315). Learning requires the learner to actively engage with themselves and the environment. “From the constructivist point of view, learning is considered as the process of mental construction whereby the individual adds new information onto a constructed understanding and knowledge” (Bozkurt, 2017, p. 210). Through lived experiences, meaning is altered to fit the constraints of the situation, which in turn changes one's reality.

There are three general perspectives of constructivism: exogenous, endogenous, and dialectical (Schunk, 2020). The exogenous perspective is based solely on the environment and recreation through models, teaching, and experiences. This perspective follows an input and output model of learning specific traits from environment for recreation. The endogenous perspective is based solely on the individual's prior experiences. This perspective follows the pattern of Piaget's cognitive development in that learning occurs through the development of cognitive mental structures. Dialectical perspective is "the outcome of mental contradictions that results from the interactions with the environment" (Schunk, 2020, p. 317). This creates a balance between the environment and the individual.

Constructivism Key Players

Piaget, von Glasersfeld, and Vygotsky were three prominent influencers within constructivism. Piaget formed the theoretical framework describing stages of development for children's cognition. The stages of development establish the flow of a child's learning by building on the previous stage. Piaget's constructivist stages align with the endogenous perspective of constructivism. Von Glasersfeld highlights the environment and how it aligns with the individual. Vygotsky integrated one's learning with the environment through the interaction between the social world and oneself. Both Vygotsky and von Glasersfeld demonstrate the dialectical view of constructivism through the relationship between the environment and the individual. However, Vygotsky includes the social aspects of others within the environment. Through a constructivist lens established by these theorists, students learn through interaction and thoughts between the individual and the social environment. These theories suggest that mathematics teachers, like any learners, construct their understanding of problem solving through their personal experiences and the influences of the world around them. Below, I will discuss each of the seminal theorists in constructivism and discuss the implications on their work for mathematics teachers' understanding of problem solving.

Piaget. Piaget is known for formalizing four stages of cognitive development: sensorimotor, preoperational, concrete operational, and formal operational (Dimitriadis & Kamberelis, 2006; Miller, 2016; Mousley, 2015; Piaget, 1964; Schunk, 2020). At the sensorimotor stage, a child processes the world through immediate actions with objects. The sensorimotor stage is typically between the ages of birth to two years old. The preoperational stage focuses on beginning to understand future events. For example, a child is unable to distinguish reality and fiction. They start to develop language to communicate thought processes. The preoperational stage is typically between the ages of two and seven years. The concrete operational step increases abstract thinking, language acquisition, and the idea of reversibility. The concrete operational stage usually happens between ages seven- and eleven years old. The final stage is the formal operational stage. At this stage, the learner increases reasoning skills and the ability to think abstractly in multiple aspects. This stage progresses from age eleven through adulthood. Though the stages have varying age ranges, the progression through the stages is not limited to age (Dimitriadis & Kamberelis, 2006; Schunk, 2020).

For learning to occur, a learner evaluates new information through current schemas to either be added to, adapted, or rejected from the existing schemas. A schema is a way of understanding or knowledge (Miller, 2016; Piaget, 1964). An initial point of disequilibrium starts the learning process, disrupting the current knowledge. Then, new knowledge is either assimilated or accommodated. Assimilation is the process of accepting new knowledge within the existing structure. Accommodation is changing systems to fit new knowledge. Therefore, learning happens between the movements from disequilibrium to equilibrium through assimilation and accommodation of further information (Dimitriadis & Kamberelis, 2006; Miller, 2016; Schunk, 2020). “The logical structure is reached only through internal equilibration, by self-regulation, and the external reinforcement of seeing that the balance did not suffice to establish this logical structure” (Piaget, 1964, p. 183). Piaget's theory focuses on the individual construction of knowledge. For mathematics teachers to learn how to develop problem solving in students, they must first engage in problem solving themselves: the individual learner to understand what problem solving is. Yet, the individual understanding of problem solving is not sufficient for teachers. Teachers must learn how other people solve problems. To learn how others solve problems, the students can be treated as part of the environment created by the teacher, and the teacher learns by observing and interacting with that environment. There is not a single method of problem solving that works equally well for all learners. This leads to Von

Glaserfeld's stance that there is no single objective truth since each individual person experiences an environment through the lens of their own experiences.

Von Glasersfeld. Piaget laid the foundation for von Glasersfeld's notion of radical constructivism. Radical constructivism "breaks with convention and develops a theory of knowledge in which knowledge does not reflect an 'objective' ontological reality, but exclusively an ordering and organization of a world constituted by our experience" (von Glasersfeld, 1984, p. 21). Through the lens of our experiences, knowledge is gained by placing structures in order by the individual. This organization is an active endeavor by the individual to construct the world through what we perceive (Walshe, 2020). Radical constructivism has two main principles:

(a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;

(b) Cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld 1984, pp. 22–23)

Radical constructivism is an evolving, adapting, and isolated learning experience. When presented with new knowledge, students evaluate current schemas to see where the new knowledge may fit. This evaluation can cause changes in schemas to be able to adopt further information when needed. Radical constructivism focuses on the individual mind, where students must see how knowledge connects to personal experiences to create new learning pathways. The teacher facilitates educational situations for students to confirm or reconstruct schemas. The students verbalize their thinking to connect concepts. Through this cognitive feedback loop, learning occurs. Piaget focuses on the developmental stages of a person and its effects on learning. Von Glasersfeld differs in that individual experiences play a part in the development of a person's learning.

Vygotsky. Vygotsky emphasized "socially meaningful activity as an important influence on human consciousness" (Schunk, 2020, p. 331). He believed a person's knowledge was constructed through the social interaction between two or more people and the environment. Through social interaction, a learner can develop skills to self-regulate. Therefore, language is the crucial element and tool needed to construct knowledge since a social aspect is necessary (Vygotsky, 1998). Social constructivism is a theory of learning that focuses on the collective experiences of a group through communication (Ernest, 1996; Mousley, 2015). The discussion starts with a question posed, whether implicitly or explicitly, by one involved in the environment. Through the conversation, learners can share experiences that allow all other learners to see the topic of the discussion from many perspectives to create new knowledge. "The social constructivist model of the world is that of a socially constructed world that creates (and is constrained by) the shared experience of the underlying physical reality" (Ernest, 1996, p. 343).

Vygotsky is best known for the development of a concept called the zone of proximal development (ZPD) (Dimitriadis & Kamberelis, 2006; Miller, 2016; Mousley, 2015; Schunk, 2020; Vygotsky, 1998). The ZPD is the zone that a learner progresses through based on what the learner can do on their own and what they can do with the scaffolding of others. "Learners bring their own understandings to social interactions and construct meanings by integrating those understandings with their experiences in the context" (Schunk, 2020, p. 333). By processing learning through individual experiences and working in a social setting, a learner can learn from other people's experiences to increase cognitive development. "We assist each child through demonstration, leading questions, and by introducing elements of the task's solution" (Vygotsky, 1987, p. 209). Through a social constructivist lens, when learning how to teach problem solving, mathematics teachers need space and places with peers and more knowledgeable others to discuss what problem solving is and how to do it.

Epistemological Beliefs

Social constructivism aligns to my epistemological beliefs about how people learn (Ernest, 1996; Mousley, 2015). Radical constructivism focuses on the individual mind processing current information within an environment that challenges their existing schemas—this individual time allows students to reconstruct a current schema or start with a new one. Social constructivism engages a community around learning to challenge personal understanding with potential new knowledge and meaning. The learner adapts their thought process through interactions with the community of learning. "Knowledge is co-constructed through interactions with others. Through the exchange of ideas with peers and the teacher, students develop shared meanings that allow group members to communicate effectively with each other" (Wheatley, 1991, p. 19). By including the social aspect of learning, students share their thinking with others who can elaborate, correct, or disprove the thinking. Through discussion, one can form beliefs about subjects and challenge new ones. The information gathered through social time is reflected upon during an individual time to process the new information.

Authentic Learning

Constructivism details how a person learns, yet it does not describe how the environment is established for the most conducive learning. Authentic learning is derived from social constructivism, by which learning is constructed through a collaborative space of shared information and challenging knowledge built by individuals and the environment (Vygotsky, 1987). Authentic learning consists of four aspects, which are construction of knowledge (Lui et al., 2017), real-world connections (Darling-Hammond et al., 2021), inquiry-learning (Chatterjee et al., 2009; Kuhlthau et al., 2015), and student-centered learning (Manyukhina & Wyse, 2019; Moses et al., 2020).

Authentic learning subsumes constructivism ideas as the first component of authentic learning, construction of knowledge. Construction of knowledge is the foundation of constructivist theory (Fox, 2001; Piaget, 1964; Vygotsky, 1987). People construct their knowledge through interactions with the environment by grappling with how new information gained from these interactions either connects to or is disconnected from their previous understanding (Lui et al., 2017). This process creates space for students to assess their prior knowledge to shape and reshape their learning structure. Reshaping the learning structure allows the knowledge to transform from short term to long term memory to be accessed later in new situations, creating new structures in the brain (Lui et al., 2017). For students to engage in learning, students need to first recall their current knowledge to engage with the learning environment. Students then explore the learning activity to challenge or support their previous knowledge. The other aspects of authentic learning (real-world connections, inquiry-learning, and student-centered) describe the environment that must be developed to support construction of knowledge.

Connections within academic content allows a student to create meaningful alignments of the content and the real-world (Darling-Hammond et al., 2021). When students understand the alignment of course content with the real-world, students can transfer skills and knowledge to other areas both within and outside the school setting (Osher et al., 2020). When students see a reason for their knowledge and skills outside of the school, they can visualize themselves applying their knowledge in various ways (Koomen et al., 2018; Singer et al., 2020). This visualization also allows the student to understand the content's importance for potential future endeavors throughout their lives (Beier et al., 2018). Integrating real-world connections within the problem-solving task allows the students to connect their content to a real-world situation and align to a future problem-solving task. If a student is connected to their learning, they are more likely to persevere through the solving process of learning. By understanding how real-world situations require problem-solving skills to solve, PSMTs can experience mathematical tasks to identify spaces for future students to connect their learning.

Inquiry-based learning, the next aspect of authentic learning, is a cyclical process by which the learner is actively seeking answers to questions posed by themselves or others (Chatterjee et al., 2009; Kuhlthau et al., 2015). This process requires the student to think through the information and questions being posed. As the student thinks through what they understand about the information they have, the student can collaborate with peers to create discourse to discover potential answers to the questions posed. As information and potential answers are presented, the students reflect on how these items align with their understanding to make sense of what they know or do not know (Kranzfedler et al., 2019; Ligozat et al., 2017). Using a question, or questions, and collaborative learning environment, students can challenge their own understanding of a concept and make sense within the learning structure. By continuing to question one’s own understanding and experiences within a social context of learning, one can challenge their own thinking to either help solidify or adapt understanding. For PSMTs, questioning their thinking and strategies of problem-solving tasks help shape better understanding of how and when to use problem-solving tasks within their future classroom.

The last component is student-centered learning. Placing the student at the center of the learning allows the students to make connections to themselves and the world around them. The students can develop agency which will drive their ability to become independent learners (Lee & Hannafin, 2016; Manyukhina & Wyse, 2019; Moses et al., 2020; Reeve & Shin, 2020). The learning environment lays the foundation for the necessary scaffolding of learning for the student and the inquiry process (De Backer et al., 2016; Mariage et al., 2019; Schwartz et al., 2021). The real-world connections establish a cultural nature for learning by which the students can connect to personal experiences both inside and outside of school (Hammond, 2015; Kelly et al., 2021; Colter & Ulatowsky, 2017). This allows students to engage more authentically in problem solving.

Problem Solving

Posing a worthwhile authentic task does not mean a learner will be engaged in solving. A problem-solving process is needed to provide access to pathways between the learner and the problem. This process is a cyclical flow between the learner's understanding, the environment, the task, and new information. How these elements flow together creates the problem-solving process.

Definitions

Throughout the years of mathematics educational reform, problem solving has varied in its definition. Problem solving has become a complex term with multiple variation of definitions. Some definitions carry similarities, while other have contrasts. Within the last few decades, research still has not clearly defined the concept of problem solving. Mamona-Downs & Downs (2013) defined problem solving as "engagement on any mathematical task that is not judged procedural, or the student does not have an initial overall idea how to proceed in solving the task" (p.139). Some researchers (Lester, 1994; Lester & Kehle, 2003; Mayer, 1998; Schoenfeld, 2016) have described problem solving as an art form. Within the problem-solving process, a picture is developed by "the socially constructed and contextually situated nature of concept development and problem-solving ability" (Son & Lee, 2020, p. 147).

The definitions of problem solving mentioned above have similarities and differences. One similarity is the notion of having multiple approaches (English & Gainsburg, 2015; Liljedahl et al., 2016). Multiple approaches allow learners to work through the problem in a sensible way. One learner can approach a problem using one method, while another learner chooses a different approach. Each learner can connect with the problem in a way that makes sense to them to start the problem-solving process.

Collaboration among peers, the problem, and the environment allow the learner to make connections beyond their current understanding. This collaborative space lays a foundation for a layer of sense making for learners. "Through the exchange of ideas with peers and the teacher, students develop shared meanings that allow group members to communicate effectively with each other" (Wheatley, 1991, p. 19). Collaboration gives way to different perspectives of the problem in which students can broaden their understanding and methods for a problem.

Problem solving leads to the use of numerous strategies and various heurisms to solve the problem. Heuristics/heurisms are strategies used to process a problem, mental capacities to assist in solving a problem. They are "effective approaches to problem-solving" (Liljedahl et al., 2016, p2). However, heuristics are "personal entities that are dependent on the solver's prior knowledge as well as their understanding of the problem at hand" (Liljedahl et al., 2016, p15) and therefore highly individualized.

Another similar aspect of the definitions of problem solving is that problem solving requires a high level of thinking of the solver (Lester & Kehle, 2003; Son & Lee, 2020). Problem solving is nonroutine, with higher order thinking skills involved in solving (Gözde, 2020; Lester & Kehle, 2003; Liljedahl et al., 2016). Nonroutine moves away from procedural steps to solve a problem. Higher order thinking skills goes beyond just memorization and gives a learner a chance "to actualize the knowledge they have into innovative and independent learners" (Nur et al., 2020, p. 332). Higher order thinking skills can include critical thinking, synthesizing, reflecting, and applying to a new situation.

While there have been debates within the mathematics educational realm as to whether problem solving is a mathematical concept itself or just a method for teaching a specific mathematical concept, what can be agreed upon is the cyclical nature of problem solving. Solvers move from strategies to processing thinking to actions, and reflections which may spawn additional strategies (Carlson & Bloom, 2005; Collet & Bruder, 2008; Da Ponte, 2007; Pólya, 1957; Schoenfeld, 2014b). Liljedhal (2021) states that problem solving "requires students to get stuck and then to think, to experiment, to try and fail, and to apply their knowledge in novel ways in order to get unstuck" (p. 20). This cyclical process requires a learner to identify potential methods to solve, try the chosen method, reflect on the usage of the method, and begin again if needed.

Because of the similarities that exist in the definitions of problem solving in the literature, for this study, problem solving will be defined as the active and collaborative process of using various strategies and heurisms through a cycle of hypotheses, action, and reflection to obtain a solution to a situation that cannot be solved procedurally.

Problem-solving skills and processes

Problem-solving skills are a fundamental part of mathematics (Pratiwi & Widjajanti, 2020). One of the key contributors to defining problem-solving skills is Pólya, whose research emphasized strategies and phases of the problem-solving process (1957). Pólya began his research on problem solving through heuristic and the process of solving a problem. Pólya defined four phases of problem solving. Pólya's stages consist of understanding the problem, developing a plan, carrying out the plan, and looking back to check validity (Pólya, 1957). Additional research has confirmed the validity of Pólya’s problem-solving process (Carlson & Bloom, 2005; Collet & Bruder, 2008; Da Ponte, 2007; Schoenfeld, 2014b).

Researchers have used different terms to describe the solving process, but the fundamental components are the same as Pólya asserted. For example, in Carlson and Bloom's (2005) research study involving research mathematicians and Ph.D. candidates, the terms used to describe each phase of the process were orientating, planning, executing, and checking. These terms emerged from patterns of data related to how the participants were progressing through the problem-solving process. The first phase for both Pólya and Carlson & Bloom required the participants to read the problem for understanding. The participants understood that understanding the problem and what it was asking would direct the next step. The second phase of Carson and Bloom’s framework was producing a strategy to solve the problem. Like in Pólya’s second phase, developing a plan, the participants could develop multiple approaches and pathways to solve the problem based on their understanding of the problem and their background knowledge. The third phase, executing (Carlson & Bloom) or carrying out the plan (Pólya), involved the participants using their strategies to solve the problem. The act of putting their approach into action allowed the participants to demonstrate their understanding of the problem and their mathematical knowledge. The final phase was returning to the problem to check the soundness of the solution. Participants would return to a previous phase if the answer did not make sense. This cyclical cycle of problem solving opened the process of self-reflection within each phase. Pólya's last phase aligned with Carlson and Bloom's final stage by reflecting on the soundness of the validity of their solution. As each decision is made and executed by a solver, reflection takes place on if it was the appropriate choice for the situation. This creates the cyclical processing of problem solving in which both Pólya and Carlson & Bloom exhibited in their research. These four stages give a basis for the beginning of the general problem-solving cycle. Through the cycle of problem solving, one can actively interact with the task by using different solving heurisms to complete and reflect on the mathematical situation. Whether the problem-solving process is successful, however, depends on factors influencing one's ability to make hypotheses, conjectures, and discoveries. In the next section, factors that influence one's ability to successfully solve problems are discussed.

Factors that Influence Problem Solving

Alan Schoenfeld, a professor at the University of California Berkley, has spent decades focusing on research involving problem solving using design experiments, which are "about improving both theory and practice" (2014a, p. 404). Schoenfeld described experiments designed to investigate three aspects of metacognition learners engage in as they solve problems (2014a). First, someone has a "theory" about learning and the design of learning come together. Secondarily, that person develops an intervention based on their understanding of the theory. Thirdly, the users implement and reflect upon the intervention to refine both the theory and intervention. He created a course on problem solving which focuses on supporting these metacognitive strategies. During and after the course, he engaged in a continual cycle of reflections and refinement. After engaging in this work, he concluded a transformation is needed within mathematics to support problem solving. Experiences in mathematics classes need to extend beyond exercises that only require memorization and procedures to problems that work with patterns through discovery and exploration to hypothesize conjectures (Schoenfeld, 2016). Additionally, through this research, he arrived at a theory which describes the factors influencing whether a person can problem solve successfully.

Schoenfeld (2014a) stated, there are four main reasons a solver succeeds or fails with mathematical problem solving:

a. The domain-specific knowledge and resources available to the solver
b. The solver's ability to access to productive "heuristic" strategies for making progress on challenging problems in that domain
c. The solver's monitoring and self-regulation abilities (aspects of metacognition)
d. The solver's belief systems regarding mathematics and one's sense of self as a thinker in general and a doer of that domain (p. 405)

One needs to have some mathematical knowledge to understand a problem. Resources, like textbooks, collaboration, previous experiences, etc., can help a solver process their understanding of the task and possible heurisms to use to solve the task. Just knowing a strategy to solve does not mean one has a "productive" strategy to use. Though mathematics allows for different strategies, the strategies that would be most beneficial for a given task depends on specific characteristics of the task. For example, to solve 16 x 19, a student may use a strategy by rounding 19 to 20 and then multiplying 16 and 20. Since the student rounded up from 19 to 20, the student would have to subtract 16 from their product to get the correct answer. This same strategy would not be as beneficial for the problem 17 x 13. Since both of those numbers are too far from having a 0 in the ones place, this strategy does significantly ease the pathway to finding the solution. Thus, having multiples strategies to choose from allows the solver to have a higher potential for being successful in solving the task.

If a solver has access to multiple strategies for solving a task, it becomes extremely important for the solver to monitor their performance through their process of solving to consider whether their current strategy is proving successful or if they need to attempt to use a different strategy. Monitoring is important aspect of metacognition, which is thinking about how you think (Flavell, 1976).

Metacognition

Mathematics is about learning patterns (Cai & Nie, 2007; Masingila et al., 2017), developing thinking skills from concrete to abstract (Gözde, 2018; Nur et al., 2020), inquiry-based learning (Saadati et al., 2019), persistency in solving (Carlson & Bloom, 2005), decision-making skills (Gözde, 2018), justifying reasoning (Henningsen & Stein, 1997), and developing higher-order thinking skills (Cai & Nie, 2007; Nur et al., 2020; Ünveren-Bilgiç & Argün, 2018). “The purpose of teaching problem-solving in the classroom is to develop students' problem-solving skills, help them acquire ways of thinking, form habits of persistence, and build their confidence in dealing with unfamiliar situations” (Cai & Nie, 2007, p. 471). Problem solving also strengthens students' metacognitive skills to reflect on their learning (Carlson & Bloom, 2005; Orgoványi-Gajdos, 2016). As students learn to self-reflect, they can self-guide their knowledge in several ways in the mathematics classroom and the real world.

Metacognition has been divided into two areas: metacognitive knowledge and metacognitive behaviors (Bannert & Mengelkamp, 2008). Metacognitive knowledge is related to a solver’s understanding of what, how, when, and why they can or should engage in metacognitive behaviors. Metacognitive behaviors are the actions people take when engaging in metacognition and align with the individual monitoring of their own learning. Several educational researchers focus on metacognition behaviors solvers engage in while problem solving (Bannert & Mengelkamp, 2008; Brown, 1987; Desoete, 2007; Efklides, 2001; Nelson & Naren, 1994). By continuously reasoning, checking, and verifying one's thinking, the solver can move towards a solution.

Problem-Solver beliefs

The way in which one engages in metacognitive behaviors relates to one's belief system of how they think about and do mathematics. If one's belief about mathematics is procedural, their lens for metacognition processes the steps taken to solve (Boaler et al., 2022; Boaler, et al. 2020; Carlson & Bloom, 2005). If one's belief of mathematics is problem solving, their lens focuses on the connections between strategies, their knowledge basis, and the problem (Boaler et al., 2022; Boaler, et al. 2020; Carlson & Bloom, 2005). Additionally, if the solver does not believe they will be successful, they may lack the motivation to identify the strategies or mathematical knowledge they could use to succeed with the task. If one believes they can be successful on the given task, they are motivated to work through the challenges to use or discover new strategies to persevere through the task. Having a belief that one can persevere through the challenges of thinking and doing mathematics gives the learner a foundation to continue working through the problem-solving task. Pólya's cycle of problem solving gives the problem solver specific mini cycles of trying and verifying as a perseverance tool within the problem-solving task, as mentioned by Carlson & Bloom (2005). Problem solving is a process. One's thinking of the process establishes layers of understanding and use for solvers.

Problem Solving in the classroom

National standards have emphasized problem solving and how the curriculum should be centralized around activities that highlight problem solving (Lubienski, 2000; NCTM, 2000). A teacher must organize the curriculum around problem-solving activities to promote the importance of problem solving in that classroom (Santos-Trigo, 2014). Teachers can use problem solving to support students' development of specific strategies. For example, consider the previous multiplication task of 16 x 19. When one student or group of students uses the rounding method to solve 16 X19, the teacher can ask other students to examine to solution and discuss why it works. The teacher can then present a problem, like 17 X 13, for which the strategy is less helpful. Students can justify why this strategy works well for 16 X 19 but not 17 X 13. Teachers' beliefs and expectations of problem solving therefore play an essential role in how problem solving is implemented in the class.

Problem solving is defined as the action of "engaging students in challenging tasks that involve active meaning making, and support meaningful learning" (Kartel et al., 2020, p. 86). The interaction between the learner, the task, and the environment creates the space for learning to happen. This constructivist mindset gives the learner room to piece together knowledge, through their pre-existing schema and new information. Therefore, the learning environments allow learners to construct their mathematical understanding. With the environment, a social dimension enters the learning process. Problem-solving learning environments "provide opportunities for children to share their ideas with peers, both in small groups and within the society of the classroom" (Wheatley, 1991, p. 12). Within the learning environment, the learner connects to the tasks by reading the task through the lens of their own experiences. The experiences frame the task in the context needed for the learner to make sense of the task and proceed on the problem-solving process. Thus, a problem-solving classroom aligns with the tenets of authentic learning.

Benefits of Problem Solving for Students

Problem solving is for all mathematics students. Students from elementary and lower math levels need to have the opportunity to develop problem-solving skills that can be built on throughout all levels of mathematics (Boero & Dapueto, 2007). Using problem solving in a mathematics classroom, students can formulate multiple methods to generalize ways to solve future problems. Teaching using problem solving allows students to approach tasks using their current knowledge to build future knowledge (D'Ambrosio, 2003). Problem solving should allow students to generalize their conceptual understanding; yet many students focus on the specific situation to solve a problem. The mathematics classroom should be a place for students to encounter various problems to develop mathematical and real-world skills. However, most mathematics classrooms focus on lower achievement level standards-based instruction to focus on the types of problems typically seen on high stakes standardized testing (Cai & Nie, 2007). This is particularly true of schooling experienced by lower socioeconomic status (SES) students. By seventh grade, lower SES students prefer lower cognitive demand mathematics activities, while higher SES students were found to most likely to embrace activities which required higher order thinking skills (Lubienski, 2000).

Lower SES students asked for specific steps or explanation from the teacher to solve the problem more often and were unable to think through open-end problems or how to connect their understanding of mathematical concepts. The lower SES students repeatedly expressed their previous experiences of solving routine style problems only and having confidence in solving through provided set steps or processes. Thus, Lubienski (2000) attributed these characteristics of the lower SES students to be a result of their previous mathematical experiences. Lubienski concluded that, to increase achievement scores of lower SES students, there is a need to develop problem-solving skills and significant persistence in students of lower SES through open-ended questions. Engaging lower SES students in problem solving and modeling would establish a stronger foundation with problems and allow students to persevere through the problem-solving cycle.

Choosing Appropriate Problems

The constructive process of learning and the environment in which learning takes place lay the foundation for a teacher to determine a problematic task to be used for learning. To engage students in problem solving, chosen tasks must be both authentic to students and require problem solving. To choose high quality task then, teachers must understand what makes a worthwhile problem. Choosing a high-quality mathematical task starts with how one defines a problem within a mathematical context.

Problem is a term used in a range of situations from when something goes wrong, from "Houston, we have a problem," to when a mathematical situation to be solved is presented in a mathematics classroom. Karl Duncker (1945), a German psychologist, defined a problem as "when a living creature has a goal but does not know how this goal is to be reached" (p.1). Schoenfeld (1983) differentiated between a problem and an exercise: "A problem is only a problem (as mathematicians use the word) if you don't know how to go about solving it" (p. 41). In comparison, an exercise is a problem that "can be solved comfortably by routine or familiar procedures" (p. 41). According to Schoenfeld, a problem can take various skills and processes, while the solver can solve an exercise using steps already known by the solver. In other words, an exercise is a routine instance that a learner has seen or completed before.

A problem has also been defined as “a mathematical task that a person tries to solve and for which that person does not have a straight and known way to solve” (Felmer & Perdomo-Diaz, 2016, p. 290). Like Schoenfeld’s definition, Felmer and Perdomo-Diaz align that a key component of a problem is a sense of unknown within the solving element. A problem is a non-routine situation in which a learner must use a deep understanding and connection to mathematical concepts to meet a goal.

For problem solving to be effective, it must lead to new learning. Gözde (2020) states that a “problem is a situation that can be solved by analyzing and synthesizing previously acquired information” (p. 215). Therefore, for high quality problems “the solution strategy is not immediately discerned...the mathematics resources learnt earlier are thus necessary but not sufficient to solve the problems” (Leong et al., 2016, p. 312). The connection to a student’s prior understanding of mathematical concepts gives students a basis to go deeper into the concept in various ways. Like the components of an authentic learning structure, new knowledge challenges the student’s personal connection and prior knowledge for learning to occur. As the student questions existing knowledge, the student goes farther in discovery of the content they are learning.

Each of the definitions of problems described above shares the idea that a problem cannot be immediately solved with prior skills. However, a problem uses those prior skills to connect on the solver’s understanding of the mathematical situation. Therefore, this paper defines the term “problem” as a mathematical task that cannot be solved by routines alone, yet those routines can be a starting place for the solving process. For a problem, there is not a clear pathway to a potential solution. The knowledge needed to establish a full solution may not exist yet for the solver but can be developed through the process of solving the problem (Herbst, 2006).

This act of accessing known understanding to connect unknown or new information aligns to a problem's non-routine nature: solver must align information to find gaps in understanding and searching for ways to help fill the knowledge gap. The solver can make conjectures to determine the reasonableness of their thought process and potential solutions. These processes are part of problem solving. Yet not all routine problems will engage learners to develop new mathematical understandings. A mathematical task engages learners in the problem-solving process in ways that are likely to lead to new mathematical understandings.

Tasks

Henningsen and Stein (1997) define a task as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill" (p. 528). By focusing on the intention, the task can be used to define both a problem and an exercise. Other definitions of a task include "a segment of a classroom activity devoted to the development and assessment of a disciplinary idea and/or a practice" (Tekkumru-Kisa et al. 2020, p. 607) and a learning activity with the goal of developing "students' understanding of mathematical concepts by engaging them with mathematical processes" (Choy, 2016, p. 423). Each of the definitions mention the notion of developing a mathematical concept. There is also a similarity in that each definition calls for active participation from the learner; the learner can develop their knowledge of the mathematical concept by engaging in the task. Therefore, this study will define a task as an activity that intends to develop the learner's understanding of a mathematical concept through active learning.

Recalling Schoenfeld's distinction between a problem and an exercise, if the task's intention is to build the learner's fluency skills, an exercise will be appropriate for the chosen task. On the other end, if the intention is to build problem-solving skills with justifications and exploration, a problem would be a better fit for the teacher to choose. Both a problem and an exercise have relevant spaces in the classroom and can be presented as a task. A task, whether actualized as a problem or an exercise, can vary in categories based on the cognitive demand, design of the task, and the intention of the teacher.

Levels of Cognitive Demand. The process of engaging learners in problem solving begins "by [the teacher] identifying the key mathematical ideas to be weaved into the task and provide opportunities for students to do mathematics" (Choy, 2016, p. 423). However, not all tasks provide opportunities to do mathematics or problem solving. A task can be divided into two main categories: low cognitive demand and high cognitive demand (Stein et al., 1996). When a task is a low cognitive demand, it "requires students to memorize or reproduce facts or to perform relatively routine procedures without making connections to the underlying mathematical ideas" (Wilhelm, 2014, p. 638). For a low cognitive demanding task, a student must memorize the specific procedures and reproduce the steps in their sequential order to arrive at the desired answer. Based on that criterion, a low cognitive demanding task is equivalent to an exercise. An exercise and a low cognitive demanding task require a student to know specific steps and reproduce those steps on similar mathematical tasks.

There is a place for low cognitive demanding tasks in the classroom. One of the national mathematical actions and processes standards focuses on the connections made between mathematical concepts “to produce a coherent whole” (NCTM, 2000, p.65). Two of the Common Core process standards are “attend to precision” and “look and make use of structure” (Common Core State Standards Initiative, 2010, p.7-8). Low cognitive demanding tasks allow students to develop procedural fluency to see how the whole structure functions.

A high cognitive demanding task, however, "requires students to make connections to underlying mathematical ideas" (Wilhelm, 2014, p. 638). These connections develop as a student determines how to engage with the task and what strategies to use to solve the task. A high cognitive demanding task “allows students to engage in active inquiry and exploration or encourage students to use procedures in ways that are meaningfully connected with concepts or understanding” (NCTM, 2014, p. 19). For a task to have high cognitive demand, the task needs to be open so that there is no specific process or strategy that a solver must take to solve the task. Open tasks give the opportunity for students to transition from tasks that only require procedural memorization to solve to tasks that require students to make connections and explore potential solution(s) and pathways to reaching understanding (Boaler et al., 2022). "Multiple-solution tasks offer students the opportunity to solve problems in many different ways, in turn encouraging three hallmarks of mathematical creativity in school: fluency, flexibility, and novelty" (Levenson, 2013, p. 270). Therefore, there is a connection between a high cognitive demand task, which requires multiple possible solution strategies, and a problem, which requires that students cannot identify a routine solution strategy. “Tasks that ask students to perform a memorized procedure in a routine manner led to one type of opportunity for student thinking: tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking” (Stein & Smith, 1998, p. 269).

The intention of the task will change the level of cognitive demand. If the goal is for the students to develop fluency in procedures, the cognitive load would be low as the students are just asked to follow the procedures. If the intention is for students to critically analyze or make meaningful correlations, the task would be a high demand task. Thus, teachers must be able to identify and be motivated to choose high cognitive demand tasks to use in their classrooms if students are to develop problem solving skills.

Table 1

Researchers and their criteria for high-quality tasks

Researcher	Criteria
Smith and Stein (1998)	Level of cognitive demand
Arbaugh and Brown (2005)	Level of cognitive demand, process to solve, the end result
Boaler et al. (2022)	Open tasks: multiple pathways to potential solutions
Levenson (2013)	Communication, multiple pathways to potential solutions, use of illustrations, manipulatives, and real-world connections
Son & Lee (2020)	Problem Solving as an art: analysis, application, interpretation, exploration, and reflection

This research study defines a "worthwhile" task as a task that meets the following criteria: contains a mathematical connection; requires a high level of cognitive demand; involves a process of analysis, application, exploration, and reflection; provides opportunities for communication; uses manipulatives, representations, and/or real-world connections; and allows for multiple pathways and solutions.

Textbooks as a Source of Problem-Solving Tasks

Mathematics curriculum has historically been driven by mathematical textbooks (Remillard, 2005; Remillard, 2009; Usiskin, 2010). However, textbooks often reduced problem solving to "procedural algorithms" (English & Gainsburg, 2015, p. 316). Two research studies (Ayalon & Hershkowitz, 2018; Cai et al., 2016) focused on tasks within textbooks and how textbook tasks are categorized.

Both in the US and abroad, mathematics education reforms have called for more focus on problem solving, yet analysis of textbooks continue to document a lack problem solving tasks. In an international comparative study looking at the most common Chinese and US textbooks for elementary schools (Cai et al, 2016), the textbooks of both counties were found to have few tasks that engaged students in problem solving. Cai et al.'s (2016) looked at the two most popular elementary textbooks in both countries (China's textbooks: published by PEP and Beijing Normal University; USA's textbooks: *Everyday Mathematics* and *Investigations in Number, Data, and Space*). The authors first counted the number of tasks per grade level and then categorized problem-solving tasks by content area, presentation of information within tasks, and the type of tasks. The authors began their task analysis by identifying problem-solving tasks within the textbooks. Once the problem-solving tasks were identified, the author employed a framework that focused on what the task requires the student to do. The framework for the tasks included five categories: 1) the task is a replicate of a problem presented earlier; 2) the task is a slight variation of an earlier problem; 3) the task asked for additional information based on an example; 4) the task requires students to pose questions based on the information given, and 5) the task does not have constraints. For category one, the task is the exact same problems at the beginning of the section but with different numbers. Category two adjusts the problem by altering one aspect of the task to not be an exact replica. Category three extends the task by using the example but asking the student to answer scaffolding questions. The next category presents information to the learner but requires the learner to pose additional questions about the information beyond the question given. The final category presents a task without specific instructions given within the task. This category gives the student free range to pose questions to be solved based on the information given.

Analysis of the textbooks demonstrated that all four textbooks had low problem-solving tasks per grade level. For example, within the grade 1 Chinese textbook published by PEP, there are 527 problems; however, there is only 3.61% classified as problem-solving tasks. *Investigations in Number, Data, and Space* have 490 problems in the grade 1 curriculum, yet 0% of the problems were classified as problem-solving tasks. The tasks for the US fell within the low-end task types for replicas or slight variations. The tasks contained in the Chinese textbooks were categorized as tasks involving additional questions based on the sample or the given information. Both research studies demonstrate that though there have been strives toward more problem-solving tasks, there is still a need for more.

Ayalon and Hershkowitz's study (2018) asked seventeen 7th-grade teachers to choose which tasks within a standard textbook the teachers believed encouraged argumentation. According to Ayalon and Hershkowitz (2018), argumentation was defined as a social activity in which students are "building claims, providing evidence to support the claims, and evaluating this evidence to judge the validity of the claims" (p. 164). Within the textbook, the researchers identified a total of seventy-two tasks. Of the seventy-two tasks, the participants only chose twenty-one tasks that they believed encouraged the students to provide claims and justify their thinking. The researchers, in partnership with the authors of the chosen curriculum (*Integrated* Mathematics), categorized the complete seventy-two tasks into three dimensions: 1) making claims/justifications and proving demonstration; 2) making claims only; 3) other types. The researchers placed twenty-four of the seventy-two tasks into the first dimension, 33% of the total tasks. The participants also categorized the twenty-one tasks the participants chose into the exact three dimensions. Of the participant-chosen tasks, the participants felt that 76% of the tasks only engaged the students in making claims and demonstrating their knowledge. Based on the discrepancy in the classification between the researchers and participants, the curriculum user understanding of task and problem solving may not align with that of curriculum developers. Curriculum writers can write tasks intended for problem solving; yet it does not meet what teachers expect of problem-solving tasks. With this lack of connection or understanding, teachers could seek problem-solving tasks elsewhere. Yet not all outlets for "resources" have been verified by mathematical experts, like teachers, to be aligned with problem solving. Depending on the location the teachers seek tasks, their search may or may not be productive.

Teachers influence on tasks

Within most mathematics classrooms, students look to the teacher for guidance and support in learning. Therefore, there is a need to look at teachers' beliefs and personal confidence in problem-solving skills (Carlson & Bloom, 2005). "Teachers' instructional practices and beliefs may also be influenced by national culture, school orientation, and mathematics curriculum" (Saadati et al., 2019, p. 1010). How teachers teach and choose instructional materials for their classrooms is based on the teachers' personal experiences and understanding of the content. Teachers are constructing their understanding through the lens of their personal experiences.

Research has examined teachers' self-efficacy related to problem solving and teaching problem solving (Saadati et al., 2019). They found the higher the teacher's self-efficacy in using problem-solving strategies, the higher the likelihood a teacher will use similar techniques within the classroom. Watson (2019) stated that "teachers may profess beliefs in reform-oriented student-centered ambitious teaching, yet their teaching is largely traditional and teacher-centered" (p. 6). Thus, teachers may state a particular belief within their self-efficacy, yet their actions and decisions do not support their words.

Teacher Beliefs

Choosing the right tasks requires teachers to consider the characteristics of the classroom and the students. The proper task "may influence not only the mathematical content that is learned, but also how students experience mathematics" (Levenson, 2013, p. 270). The intention, not only of mathematics education but education in general, drives implementation within the classroom. Is the intent to continue the status quo of a working and elite class (Anyon, 1980)? Or perhaps, could the intent to foster thinkers who gain understanding from experiences and reflections (Raymond, 2018)?

Anyon's research (1980) demonstrated how the intention of the task may be influenced by teachers' perceptions of their students. Her study shed light on the hidden curriculum tied to social class through five types of elementary schools. Two schools were working-class schools. These schools focused their lessons on steps and procedures through rote behavior. These working-class schools left little choice for the students to make decisions on what to do; they were expected to do exactly what the teacher showed them and nothing more. The next school was a middle-class school whose focus was getting the right answer and good grades. The lessons gave some choice to the learner but had to be neat and orderly. The next school was an affluent professional school. The school allowed for activities to have a creative element that should be completed independently. Negotiation was allowed in this setting. The final school, executive elite, were required to use "analytical intellectual powers" to think logically by how everything fits together in the big picture (Anyon, 1980, p. 83). These five types of schools display a hidden curriculum to keep the status quo of society by teaching students to remain in their social class. Therefore, based on the intention, the learner is either part of the working class to just follow orders, the middle class with tasks intended to limit thinking skills, an affluent group with a creative element, or part of the elite class with tasks focusing on big picture problem solving (Anyon, 1980).

Teachers' beliefs about the purposes of mathematics education also serve to shape the tasks they used in their classroom (Raymond, 2018). During the Space Race, many teachers believed the purpose of education was to help prepare creative and critical thinkers needed to win the race to space, and problem solving became more of a focus in mathematics classrooms. The Back to the Basic era shifted the climate of mathematics education toward a need to solidify students' ability to use basic procedures. Mindsets constantly change. One common mindset is the belief that learners should develop skills that transcend the mathematics classroom and connects one's learning through interactions with experiences and personal reflection. This mindset relates to teachers' beliefs about the goal of mathematics. Teachers with this mindset may believe "By constructing their own understandings of mathematics and connecting those understandings, students are empowered and develop creativity and agency that can improve their lives regardless of the endeavors they choose to pursue in the future" (Raymond, 2018, p. 8). For the students to develop agency, learning must be constructed to foster independent thought through collaborative elements that help establish skills for deeper thinking. Because the learner connects with the task through active engagement, the intention can also shift due to the student's intentions and knowledge (Raymond, 2018).

What may be considered problematic for one student, may not be problematic for another student. The problematic nature of a task can also vary from environment to environment or from a time in one's life to another. Yet, "in order to identify potential problematic situations the teacher must focus on the students' understandings" (Wheatley, 1991, p. 15). Teachers need to understand how students work on problems and how problems can be the instigator of interactions between the students and the learning environment (Herbst, 2006).

Despite the different intentions within the classroom, the goals for each are what drive the choices for mathematical tasks. Therefore, the “right” task may vary depending on multiple variables, including the teacher, the students, school policies and views, and even the views of parents and other community members as well as the environment in which the learning is taking place (Masingila et al., 2017). So, how might a teacher determine if a task aligns to key ideas and provides opportunities to learn aligned with the chosen goals?

In addition to deciding intention or goal of the task, “teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them” (NCTM, 2000, p. 19). Each of these decisions will influence the environment of learning. The decisions a teacher makes will reflect on their own beliefs and experiences.

Several studies centralized on the process of decision making and identified two types of decision making (Clark, 1986; Evans & Stanovich, 2013; Johnson-Laird, 2006; Shavelson & Stern, 1981). Type 1 is unconscious and intuitive decisions. These types of decisions are the choices being made through immediate reactions to a situation or environment. Type 2 is the conscious deliberative decisions made through thoughtful thinking, “conscious reasoning uses working memory to construct mental models and assess possibilities prior to making a decision and taking action” (Watson, 2019, p. 4).

Watson (2019) emphasized how routine plays a role in decision making. The routines within a classroom give way to the expectations on how it should go and allow space to change based on actions from the students from the class to adjust. Deliberated decisions are made to establish the classroom routines, yet intuitive decisions are made based on the breaking of the routines. If a classroom experience does not go according to the expectations from the deliberate decisions that were made, a teacher's ability to adapt are the intuitive decisions that come from personal experiences in the learning environment. Veteran teachers have more experience with both types of decision-making styles, whereas PSTs must develop their intuitive decision-making skills.

A teacher's level or length of experience in teaching plays a part in a teacher's decision-making abilities and expectations about behaviors within the classroom. These abilities and expectations are factors in the routines teachers use to make immediate decisions within a lesson (Fogarty et al., 1983; Parker & Gehrke, 1986; Peterson & Comeaux, 1987; Veenman, 1984). Expert teachers flow through three phases of decision-making, "preactive, interactive, and postactive" (Westerman, 1991, p. 298). The preactive phase focuses on planning what the teacher expects the students to do during the lesson. The interactive phase is during the actual teaching of the lesson where the teacher is making immediate decisions based on the students. The postactive time is after the lesson has concluded and the teacher is reflecting on the teaching experience. The expert teachers flow more naturally between these three phases, whereas the novice teachers have to take deliberate time to separate each phase.

Decision making within the classroom requires an interaction between the teacher, the learner, the problem, and the context. This interaction aligns with transactional theory. According to Rosenblatt's (1978) transactional theory, meaning of a text (in this instance a problem) is made by the interaction between all components of the environment (people, place, context) and the text. Each component plays a part in the construction of meaning for the learner and decisions the teacher makes. As the learner reads the problem, the learner considers their previous understanding, the context in which the problem is presented, the way the teacher introduces the problem, and how the problem emerges within the collaborative space.

Transactional theory claims there exist two stances from which the learner who is experiencing the problem might approach the problem: purely aesthetic to purely efferent (Rosenblatt, 1978). For the aesthetic stance, the learner looks at all components of learning when considering potential connections to the problem to see how it affects them. These components include the learner's previous understanding, current environment situations, and the problem itself. "In aesthetic reading, the reader's attention is centered directly on what he is living through during his relationship with that particular text" (Rosenblatt, 1978, p. 25). Since the aesthetic stance places an emphasis on the current situation of the learner, how a learner interacts with the problem can be adjusted as their understanding and situation changes. The efferent stance is goal-orientated (Rosenblatt, 1978). This stance is centralized on key information within the text to solve without connecting the information to the learner's current situation of previous experiences. The learner can move between these two stances across texts or within a single text. Depending on the learner, the learner can focus on a desired outcome (efferent) or seeing how the text is important to them (aesthetic).

Teachers make decisions on which resource to use for developing mathematical learning and understanding. According to Remillard (2013), there are three lenses for interactions with resources: visualize the resource as a tool, an artifact, or an object for design work. The first lens, resources as a tool, is when teachers who use resources as a tool for design work are looking for a tool to guide learning. Both the tool and the teacher hold authority and "are active participants in the instructional design process" (Remillard, 2013, p. 926). The second lens, resources as artifacts, is when teachers focused on how the task is "transformed through their use" (Remillard, 2013, p. 926). The interconnectedness between the resource, the teacher, the learning environment, and the learner creates a space for the resource to be transformed to enhance the design work. The final lens transitions to the results of the design process. "For teachers who collaboratively adapt or generate resources as an object or product of their design work expands the notion of what it means to teach and where and when teaching occurs" (Remillard, 2013, p. 926).

How teachers view resources determine their relationship with the curricular element. "Teacher must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies – all in the pursuit of their instructional goals" (Brown M., 2009, p. 18). Each element of the instructional design requires the teacher to decide how, when, or even if the tool of instruction is to be used. There are three ways of "understanding the interaction between teachers and curriculum artifacts: (a) curriculum materials play an important role in affording and constraining teachers' actions; (b) teachers notice and use such artifacts differently given their experience, intentions, and abilities; (c) 'teaching by design' is not so much a conscious choice as an inevitable reality" (Brown M., 2009, p. 19). The relationship between the curriculum and the teacher is a dynamic flow of experience and understanding, of themselves as the teacher and learner, the classroom setting, and how each influences the other. "Ultimately, the teacher-tool relationship involves bi-directional influences; how curriculum artifacts, through their affordances and constraints, influence teachers, and how teachers, through their perceptions and decisions mobilize curriculum artifacts" (Brown M., 2009, p. 23). Through the lens of experience and support, teachers use previous knowledge to utilize tools to enact curriculum. M. Brown (2009) used the term, "*pedagogical design capacity*" which he defined "as a teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" (p. 29). The higher a teacher's pedagogical design capacity, the higher capability "to deconstruct curriculum materials, recognize their essential elements, and reconstruct them in order to suit their needs" (Brown M., 2009, p. 31).

Teachers' criteria for choosing tasks

Tekkumru-Kisa et al. (2020) revisited over 30 years of research to identify if (and how) problem solving and mathematical tasks have been defined and changed. The authors refer to what is involved in a problem-solving task, why problem solving is essential to revisit, and what some curricula within mathematics use for tasks. Tekkumru-Kisa et al. summarized four characteristics of any task: a) the design of the task lays a foundation for student thinking; b) the teacher set-up helps determine the student thinking; c) students are the prominent thinkers within the task; d) students take ownership of their thinking (2020).

The first characteristic, laying a foundation, focuses on how the task is written, i.e., the explicit or inexplicit instructions. Teachers focus on the "feasibility" of the task. This characteristic connects to "the amount of time that could be devoted, the sequence in which lessons may occur, and the physical resources and materials required" (Raymond, 2021, p. 514). Each of those design elements create the space for students to begin thinking.

The second aspect creates a basis for what the teacher expects from the students regarding the task. The developmental appropriateness of a task allowed the teachers to reflect on their individual students to align the task to the desired outcome (Raymond, 2021). Teachers align goals to standards and curriculum. Teachers make deliberate decisions regarding the initial learning environment to meet goals for the interaction between the students and the standards and curriculum. If the goal is for students to explore a new mathematical concept, the teacher could create an environment that allows for freedom to explore through various methods. If the goal is for students to practice mathematical fluency, the teacher could create a more structured setting to develop fluency.

The third characteristic emphasizes the student's actual thinking process as they engage in the task. Tekkumru-Kisa et al. use frameworks from Doyle (1988) and Stein & Smith (1998) to describe the thinking processes of students. Doyle's framework consists of only two levels of cognitive demand (low and high). Low cognitive demand focuses on memorization or "search-and-match", that a learner is searching only for the keywords to identify the algorithm to be used (Doyle, 1988, p. 170). According to Doyle (1988), a high cognitive demanding task focuses on comprehension and flexibility of knowledge application. Students making connections with themselves, a concept or another disciple emphasized the relationship between the task and the student.

The final aspect of a task is summarizing the student's thinking and learning by connecting with the task and the mathematical concept. Identifying how each part of the task connects to mathematical understanding and student learning gave the teacher an avenue to grow the student's problem-solving abilities. Many agree that teachers should sharpen problem-solving skills in the mathematics classroom (NCTM, 2014; Osana et al., 2006; Sapkota, 2022). To support the improvement of problem-solving skills, an appropriate task needs to be chosen.

Lappan and Phillips (1998) developed ten criteria for a “worthwhile” problem. First, the task needs to have helpful mathematics interwoven within it. Without mathematics, the task lacks the central part of a problem. For a task to even begin to be considered for learners, the task must have mathematical representation that learners can connect to prior knowledge or personal experience. Representation can be multifaceted in terms of being visual, physical, contextual, verbal, or symbolic (NCTM, 2014).

Secondly, the task should have multiple entry points to use various strategies. Therefore, the task needs to allow the solver to enter the task at various levels. Along with multiple entry points, the task should allow for either multiple solutions or stances to solve for justification. If there is only one entry point and one approach, the task is no longer a task but an exercise. A task can allow for a low floor access point for students who are struggling with the mathematical concept, as well as extend the mathematical thinking of students who have grasped the concepted by extending the task to a high ceiling aspect through students using creativity to find multiple solutions (Boaler, 2022).

The third criterion lets students justify their reasoning for their solution in several ways. This justification is considered as mathematical discourse. Students need “opportunities to share ideas and clarify understandings, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things through other perspectives” (NCTM, 2014, p. 29). This means a student can choose their preference of displaying their thinking, whether to draw, write, verbalize, use manipulatives, or other media to justify their thinking and solution. The act of justifying one’s reasoning is only a part of discourse. This act of discourse allows learners to use and connect mathematical ideas (NCTM, 2014).

The fourth criterion is the collaborative nature of the task; students' engagement in the task through discussions and discourse. A collaborative task allows students to verbally process their understanding or get feedback on their problem-solving strategies. This also allows an opportunity for students to learn new heurisms to help solve future tasks from their peers. This aspect connects to the collaborative nature of discourse, hearing others' thinking to compare, contrast, question, and verify with one's own thinking (NCTM, 2014). During the collaborative environment, there are responsibilities from the learner and the teacher in the classroom. Hufferd-Ackles and colleagues developed a framework for "Levels of the Math-Talk Learning Community" (2004, p. 87). The authors defined a "math-talk learning community" as a community where both "the teacher and the students use discourse to support the mathematical learning of all" (2004, p. 82). The framework shifted the focus on whose was doing the mathematical thinking by the student being the key participant in questioning, answering, explaining, and taking responsibility for their learning.

The next criterion focuses on the level of thinking of the students. If a task does not aim to engage students in high cognitive demand thinking, the task is no longer a problem to be solved. The task should be just out of the reach of the solver to aim to cause the problem-solving process to begin. This criterion connects to the idea of Vygotsky's Zone of Proximal development, in that the solver should not be able to solve the task easily, but through scaffolding or connections to strategies, the solver can proceed (Dimitriadis & Kamberelis, 2006; Miller, 2016; Mousley, 2015; Schunk, 2020).

The sixth and seventh criteria are how the task allows the students to continue developing conceptual understanding and connections to other mathematical ideas. Like the first criterion of mathematics is at the center learning, the task needs to allow the students to see how mathematical patterns connect to each other and their prior knowledge. The sixth and seventh criteria align with the extension of learning, whereas the first criterion focuses on the understanding from past mathematical experiences.

The next criterion looks at the use of mathematics and the appropriate application of the skills needed to do mathematics. “Students must have opportunities actually to be involved in doing mathematics – to explore interesting mathematical situations, to look for patterns, to make conjectures, to look for evidence to support their conjectures, and to make logical arguments for their conjectures” (Lappan & Friel, 1993, p. 524). The act of “doing” mathematics allows the student to grasp the flexibility and flow of problem solving within mathematics. The ability to extend their skills to other content bridges a student’s learning abilities and problem-solving skills.

The ninth criterion provides the space for the students to practice the skills they have acquired. This criterion helps to align “that procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving” (NCTM 2014, p. 42). Students need the time and space to be able to try different strategies and ideas to solve a task. Through trying, students can refine their thinking to use for current and future tasks. This allows for students to try strategies or mathematical understanding in other situations.

The final criterion is how the teacher assesses students' learning and knows where they are struggling. The teacher is in a supportive role throughout the task. This allows the teacher to ask clarifying questions to understand the students’ current level of understanding to help them process potential next steps to solve. “Essential mathematics teaching relies on questions that encourage students to explain and reflect on their thinking” (NCTM, 2014, p. 35). The types of questions the teacher asks the student are determined by what information is needed to support the student (NCTM, 2014). If a student needs help recalling information, a teacher would ask gathering questions: What is the formula for finding volume of a rectangular prism?. These types of questions focus the student on answering a key element that will assist them in their task, without the teacher giving the information. Probing style questions are centered around the student's need to justify their thinking. Questions in this style ask the student to show or explain their work. The third style of questioning allows for making the invisible visible. When students are working with a mathematical concept, it is important for connections to mathematical ideas and structures to become visible for them. This question type asks students to discuss their mathematical journey to the connections they see. The final type of questions reflects the students' understanding. By justifying their reasoning, students can create an argument for validity of their understanding and mathematical work (NCTM, 2014). Each of these question styles (gathering information, probing thinking, making the mathematical visible, encouraging reflection and justification) give the teacher evidence of the students' thinking in order to guide or support the student in their mathematical journey.

Lappan and Phillips's criteria align with Tekkumru-Kisa et al.'s four characteristics of a task. Lappan and Phillips's criteria of integration of mathematics, multiple pathways to a potential solution, and justification of solutions fall within the characteristic of how the design of the task lays a foundation for student thinking. How the task is structured determines the environment for students to engage with the task. The way the teacher supports student thinking, Tekkumru-Kisa et al.'s second characteristic, aligns with the conceptual development of mathematical connections, the creation of space for students to grapple with the mathematical ideas. The teacher should know the goal for the task and direct students to be the main thinkers within it. The next characteristic of Tekkumru-Kisa et al., the student is the main thinker, relates to Lappan and Phillips's criteria of the level of cognitive load and practicing the mathematical skills necessary work through the task. The final characteristic, students taking ownership, aligns with students actively making decisions and having discourse with collaboration with peers.

Table 2

Comparison of Researchers' criteria of high-quality tasks

Tekkumru-Kisa's Criteria	Lappan & Phillips Criteria
Design of task lays foundation for student thinking	Integration of mathematic; multiple pathways to a potential solution; and justification of solutions
Teacher set-ups for student thinking	Conceptual understanding; development of mathematical connections; creation of space for students to think; provide a supportive role for students
Students as prominent thinkers	Level of cognitive load; practice and apply mathematical skills
Students' ownership of thinking	Students actively making decisions; discourse and collaboration with peers

Teachers' perceptions of the cognitive demand of tasks

Teachers' beliefs about their abilities to understand and teach mathematics and their views on curriculum tools influence how they choose tasks with the instructional design process. "No other decision that teachers make has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics" (Lappan & Friel, 1993, p. 524). In choosing a task, the teacher must first consider the students and the classroom norms (Smith

& Stein, 1998). This consideration must include the students' prior experiences, individual experiences, and social and cultural norms. There are four categories of tasks: "memorization, procedures without connections to concepts or meaning, procedures with connections to concepts or meaning, and doing mathematics" (Smith & Stein, 1998, p. 345). The level of cognitive demand increases across these categories. For example, memorization requires low cognitive demand because the students must reproduce a previously memorized fact or formula. The second level, procedures without connections, requires students to be able to produce steps in various exercises without considering how those exercises and methods connect. The third level is where students make connections to understand why they are doing specific procedures. Finally, the highest level of cognitive demand is doing mathematics. This level requires analyzing, using multiple strategies, exploration, and nonroutine algorithms to solve.

A study by Arbaugh and Brown (2005) examined mathematical tasks and how those tasks influence a teacher's decision to use them in the classroom. The authors provided twenty middle school mathematics tasks to the nine teacher participants to sort into categories of the teachers' choosing. First, the teachers described their thought processes for choosing the categories. Then, after explaining their categories, the authors asked the participants to resort their tasks into dissimilar categories than they had created before. The authors had the teachers do the two sorts twice during the study, one at the beginning and one at the end. In between the task sorts, the teachers provided the authors with all the tasks they completed in their high school mathematics course. The teachers also attended study group meetings with the participants and the authors to be analyzed for the level of cognitive demand. These sessions concentrated on the teachers' thought processes for mathematical tasks, understanding of the level of cognitive demand, and use of tasks in the classroom.

The findings of this study centered on the teachers' increased awareness of cognitive levels of tasks from the initial task sort to the final one. The initial task sorting resulted in five groupings: mathematical concept, process the student takes to solve the task, the result of the task, characteristics of the task, and others. These groupings focused on actions the learners were/would be doing. At the end of the study, the participants changed the task sort into three categories: level of cognitive demand, the process the students take to solve the task, and the result. The final groupings centralized on the notion of the learner bring an "active thinker" (Arbaugh & Brown, 2005, p. 515) where the learner is increasing the cognitive load and using problem-solving skills. The authors also noted the increase in the usage of higher cognitive demanding tasks by the teachers in the classroom. With the increase of higher cognitive demanding tasks, students engage in problem-solving tasks.

PSMTs and Problem Solving

PSMTs have been the focus of multiple research studies studying problem solving (Gözde, 2020; Masingila et al., 2017; Rosli et al., 2015; Son & Lee, 2020; Thanheiser, 2015). Some of these research studies focused on the PSMTs' problem-solving skills (Gözde, 2020; Rosli et al., 2015; Son & Lee, 2020). Other studies highlight how tasks are designed to support PSMTs (Thanheiser, 2015).

Teachers' performance on nonroutine problems has been the subject of multiple research studies. For example, Gözde (2020) used a 0-5 scale rubric for performance on two nonroutine problems. The lowest level of the rubric (0) was used if a PSTs left the problem blank or had a wrong answer with wrong reasoning. Solutions categorized as Level 1 showed the beginning of a correct strategy or a correct answer but incorrect or missing reasoning. Level 2 focused on the mentioning of the correct strategy, but the strategy was not applied, and a correct answer was given without reasoning. At this level, the participant did not show use of the strategy or how the correct answer was determined. Level 3 was for a correct strategy that was used incorrectly and arrived at the wrong answer. Level 4 is like level 3, except that the misconception was at the misevaluation of data. This misconception caused the solution to focus on the wrong aspect of the given data. Therefore, the participant used the wrong number within their problem solving but arrived at an answer using the wrong number. Level 5, the highest level, used a correct and complete strategy and answer.

This framework was used to analyze the problem-solving work of the PSTs on two nonroutine problems to investigate how the difficulty level of the problem affected the participants' work by considering the participants' successes or failures during each part of Pólya's problem-solving process. The first non-routine problem was at a lower difficulty level compared to the second problem. The researcher found that 72% of the participants reached level 5 on the first problem, yet only 18% on the second problem. The researcher noted that those using the strategy of working backward could solve the second problem, whereas those who used a traditional method of an equation were unable to answer it. When looking at both problems using Pólya's problem-solving steps, most participants understood the problem (94% for problem 1, 96% for problem 2). However, the participants struggled with the reflective stage (62% for problem 1, and 96% for problem 2). This representation demonstrates the participants' ability to understand the problem and what is being asked to solve, yet through the act of problem solving, the participant was unable to reflect on the correct usage of a plan or strategy. Considering the performance of the stages of problem solving and the chosen heurisms, the performance level of the participants was low. This study reinforces the need to continue developing problem solving, both the action and the reflection of personal heurisms, in the preservice education curriculum.

The development of problem-solving skills can be supported in teacher preparation programs. Three instructors at the same university who taught various levels of mathematics method courses conducted a self-study with PSMTs to reflect on problem-solving aspects within their curriculum (Masingila et al., 2017). For these instructors, the course followed the same general structure of the instructor beginning by presenting a mathematical task to the class, followed by a collaborative discussion of the task and small group time for PSMTs to work on the tasks, and ending with a whole class discussion on the strategies used to solve the task. During the time the PSMTs worked on the task, the instructor would walk between groups to discuss and question the strategies the group was implementing with the task. Through reflections and observation notes, the researchers determined that the instructors had three main methods for supporting the PSMTs' success: clear mathematical goals throughout the methods courses, choosing appropriate tasks to facilitate, and scaffolding for the PSMTs through questioning. There was more success with the PSMTs connecting problem-solving aspects with the instructor's support through questioning. During both the whole class discussion and small group time, the instructor focused on asking questions to have the PSMTs explain their chosen methods and if the method was successful or not. This result can serve as a model for using purposeful questions to support PSMTs' engagement of problem solving.

However, PSMTs must be able to do more than solve problems themselves, they must be able to develop an understanding of what problems are and how to teach problem-solving skills. One research study examined how preservice elementary teachers conceptualize problem solving through metaphors (Son & Lee, 2020). The researchers first asked the participants to create and explain a metaphor representing their problem-solving thoughts. Next, the participants solved a task and had a follow-up interview to see if there were connections between the metaphor and the strategies used to solve the task. The preservice elementary teachers' metaphors were categorized into end-product, processes, and emotions. The end-product metaphors concentrated on arriving at a solution or the end of the task. A process metaphor focused on the path the solver takes throughout the task. The emotional metaphors surrounded a negative emotion involving problem solving. There was an alignment between the participants' association with problem solving and their performance on the task when incorporating the task performance. The more positive the metaphor and concept of problem solving the higher performance on the task. If a participant described problem solving as an end-product, they had a higher computational skill. Since the metaphors created by the participants described their belief of mathematical task, the metaphors correlate to their belief of problem solving. For example, if the problem solving is meant to reach an end goal, a teacher will choose a task that has a clear end goal for students to arrive at. Teachers with a belief of problem solving as a positive idea has the mentality of connecting tasks and mathematical context.

Choosing a task requires a teacher to make a decision about the problems they pose to students. Thus, teachers need to be able to pose problems as well as solve them. Problem posing is the act of posing additional problems based on the current task. Asking additional questions about the problem can take place within any stage of problem solving. In a mixed methods study, researchers investigated how middle school PSMTs use their problem-solving skills to solve problems and use the given problem to recreate additional problems to pose (problem posing) to others (Rosli et al., 2015). The researchers of this study used Silver's (1994) problem-posing model, which inserts problem reformulation in a cyclical cycle after one solves a problem. The participants were divided into two groups. The first group was given a three-part task to solve and reflect on their potential problems. The second group was asked to pose additional problems before solving the problem. The participants' solutions were classified using a rubric that consisted of understanding the problem, heurisms used to solve the problem, and coherence of the solution. Group A was able to pose higher cognitive problems after experiencing the problem. Group B posed lower cognitive problems before encountering the problem. This result shows how experience with problem solving increases the extension of problem posing. PSMTs' experiences with problem solving and problem posing increases the chance that they will incorporate these experiences into their future teaching.

Each of the studies mentioned above connects PSTs' performances on problem-solving tasks and their level of understanding with the concept and the problem-solving process. Son and Lee (2020) used metaphors to connect how PSTs see problem solving and how they solve a task. Rosli et al. (2015) expanded beyond the problem-solving process to making connections to problem posing for future classrooms. With the assorted studies that focus on PSTs' problem-solving skills, there is a lack of research on how PSMTs develop beliefs about what problems are worth solving and how to determine if a problem is worthwhile. Therefore, research is needed that asks those questions of PSMTs.

Gap in the Literature

Problems and problem solving in mathematics education research have been a focus on mathematical reforms since The Committee of Ten (United State Bureau of Education, 1893), which requested a course specifically for "a greater number of exercises in simple calculation and in the solution of concrete problems" (p105). Different reforms focused on the purpose of mathematics in the educational realm. There has been a shift with a current focus on developing and deepening problem-solving skills (NCTM, 2014). One way of deepening problem-solving skills has been by choosing mathematical tasks. Various research focused on what criteria make a task worthwhile to be used in the classroom (Choy, 2016; Henningsen & Stein, 1997; Lester & Cai, 2016; Levenson, 2013; Masingila et al., 2017; Smith & Stein, 1998; Tekkumru-Kisa et al., 2020). With the assorted studies that focus on PSTs' problem-solving skills (Gözde, 2020; Masingila et al., 2017; Rosli et al., 2015; Son & Lee, 2020; Thanheiser, 2015), there is a lack of research on the development of PSMTs' beliefs about problems and problem solving, and criteria that determines if a problem worth using.

This study sought to understand how individuals constructed the meaning of problems and problem solving and how it affected their criteria for choosing tasks for their classroom by utilizing a social constructivist mindset. By examining the thinking of individual PSMTs through the social environment of the same mathematics methods courses, I identified potential interrelations between the individual and the collective to understand how PSMTs chose problem-solving tasks for their future students to complete in a mathematics classroom. Social constructivism centralizes individual experiences interlaced with social connections to build understanding; therefore, it is important to acknowledge those experiences in the understanding process of problem solving. While utilizing the characteristics of social constructivism to inform my work, this study sought to investigate the following research questions:

1. What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?

a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

Constructivism "suggests that each one's way of making sense of the world is as valid and worthy of respect as any other" (Crotty, 1998, p. 58). Each person sees the world differently and deserves to be recognized for their meaning through their lived experiences. Since the individual cognitive process is something to be recognized and respected, my research focused on how PSMTs make decisions on problem-solving tasks under the constraints of their experiences with problem-solving applications. Social constructivist theory allowed me to answer research questions derived from the idea of the individual cognitive process of decision making withing the social context of a mathematics methods course. Approaching this research from a constructivist perspective allowed me to investigate PSMTs' definitions of problems and problem solving and how they understand a problem-solving task as worthwhile. Some examples of the types of questions that shed light on the PSMTs' perspectives are "what is problem solving?" or "how does your definition of problem solving align with your criteria of a worthwhile task?". These questions allow individuals to explain their problem-solving process to connect their reality of choosing problem-solving tasks for their classroom.

Chapter 3: Methodology and Study Design

This case study is designed to investigate PSMTs' understandings and values related to problem solving. Case study "is an in-depth description and analysis of a bounded system (Merriam & Tisdell, 2016, p. 40). Creswell & Poth defined case study as "a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving *multiple sources of information* (e.g., observations, interviews, audiovisual material, and documents and reports)" (2018, p. 96, emphasis in original). A bounded system is "as a single entity, a unit" that is being studied (Merriam & Tisdell, 2016, p. 40). The bounded system for this study is the first mathematics education method course of a state flagship university. This research project aimed to understand and interpret PSMTs' meanings associated with problem-solving tasks. By investigating the experiences PSMTs have with problem-solving tasks and making connections to their experiences, I sought to uncover how PSMTs decide what problem-solving criteria to use for their future classrooms. This research project investigated the cognitive structures of decision-making for the PSTs within the mathematics education environment.

Research Design

This research study investigates PSMTs' understanding of problems and problem solving. Additionally, this research sought to identify characteristics PSMTs use to determine what problem-solving tasks to implement in the classroom. With a focus on problem-solving in mathematics classrooms, it is essential to look at criteria for what constitutes a worthwhile problem and why those criteria are crucial. If one's goal is for students to reach a deeper understanding of a mathematical concept, a teacher will have a lens for choosing specific tasks over others and making decisions to create a task to support a higher cognitive level. The goal of including significant opportunities for students to engage in high cognitive demand tasks in mathematics classes led to the following research questions for this study:

1. What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?
    a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

Focusing the research questions on the participants understanding before and during the beginning semester of their teacher preparation program gives insight into influences that define their understanding and how their understanding does or does not change throughout their first methods course. By using this focus, I wanted to capture the PSMTs' perspectives through their first methods course to shed light on how they interpreted different experiences, and which experiences were meaningful to their understanding of problems and problem solving.

Participants

This study aimed to understand how PSMTs' understand problems, problem solving, and the criteria they use to assess the worthwhileness of tasks. Purposeful sampling was used to gather participants by establishing specific criteria for participants to meet. Purposeful sampling uses specific criteria to select participants of a research study, in the case that criteria must allow the researcher to learn from PSMTs entering the first mathematics methods course at a state flagship university in a southern plains state. Therefore, the sampling criteria required the participant to be enrolled in the first mathematics methods course during the time of this study. The reason for binding this study to the first mathematics methods course was to understand the beginning mindset of the PSMTs. By understanding the participants' perspectives and experiences when entering the beginning of the degree-specific courses, one can structure courses to build on problem-solving knowledge.

The goal was to recruit at least three participants. Triangulation calls for at least two to three aspects used to investigate (Merriam & Tisdell, 2016). By having at least three participants, this study can cross-check experiences of understanding. Data sources, discussed later, support triangulation within this research study. Recruitment letters were given to students enrolled in the targeted methods course during the first class (see Appendix A for the recruitment letter and Appendix B for the consent form). Time for questions and answers was given during the delivery of the recruitment letters. Follow up questions were answered via email or phone calls.

Three participants volunteered to participate in this research study. The pseudonyms of Kiley, Ashley, and Hilary were given to the participants to secure their identities. All participants were female secondary mathematics undergraduate students in their early 20s. No concern was raised with having all female participants as most of the education community in the state this research was completed is female. At the time of this study, all participants were enrolled in the first mathematics methods course. A description of the methods course will be given in the setting portions of this literature. Kiley initially was declared a mathematics major. She moved to education because of her desire to want to help students understand mathematics. Kiley enjoys working out mathematical problems and exercises. Ashley also had previously declared different majors at the state flagship university. She originally was an engineering major. Ashley is full of energy. Ashley is expressive in the way she talks. As her excitement for a subject develops, her face lights up and her hand movements become more robust. Hilary had completed the previous years of her undergraduate studies at a different college in the Southern Plains state. She is a former volleyball player and used her athlete skills at her previous college to receive a scholarship. Hilary prefers items to be organized and structured. She describes herself as a math person verses a reading person. Her belief in herself is lacking at times when she is asked to describe her understanding.

Setting

The setting for this study was a state flagship university in the Southern Plains state. The university had an undergraduate population of approximately 28,000 students. It offers over 170 different program areas. With the college for education, bachelor's degrees are offered in early childhood, elementary, language arts, mathematics, social studies, science, and world languages education. Graduate degrees are offered in eleven different areas, including all bachelor areas of studies, biomedical education, reading specialist, and curriculum and instruction.

Mathematics Methods Course

The mathematical methods course, Developing Problem Solving Environments for Secondary Mathematics Learning, that stood as the setting for this study was the first secondary mathematics methods course. This course is the first of three total methods courses for secondary mathematics undergraduate students. This mathematical methods course centers around problem solving through the content lens of grades 5-8. The second course is Fundamental Concepts of Secondary Math Learning. This course continues the building of problem solving and curriculum through algebraic reasoning. The third course is Teaching and Learning of Mathematics Reasoning and Proof which centralizes on reasoning and proof by means of secondary mathematics topics involved with geometry and trigonometry. The third course aligns with a field experience for undergraduate students to be in secondary mathematics classroom within the Southern Plains state. As students continue in the method courses, the students engage in mathematical learning experiences that can challenge or align with their previous understanding to extend to the potential of their future classroom.

This case study is bounded by the first course in the mathematics methods sequence during the fall semester. It is important to inquire about the students' beginning understanding of problems and problem solving. The focus of this first course is developing a problem-solving environment within a secondary mathematics classroom using content levels of grades 5^{th}-8th. Throughout the semester, students engaged in problem-solving tasks and assignments. PSMTs worked in small groups to solve problem-solving tasks to reflect on potential correlation between the task and readings.

One assignment had students write portions of lessons to include problem-solving tasks to engage their future learners. The lessons were structured in a 5E format (Bybee, 2014; Bybee, 2015). The first E in the lesson is the engage. This portion of the lesson is to establish the students' engagement and activate their prior knowledge. The second E is the explore. This section is for the students to actively explore the concept for the lesson. This can be done individually or in small groups. The third E is the explain. This portion is designed for discussions with peers and facilitators to validate thinking and formalize their understanding of the topic. The fourth E is extend. During this section, students are using their new or clarified understanding of the topic to be applied to other scenarios. The final E is evaluate. This portion is to evaluate the students' understandings of the topic.

A required reading for this course is *Building Thinking Classrooms in Mathematics* (Liljedahl, 2021). This book is centered on 14 principles to transfer a mathematics classroom into a thinking classroom. The 14 principles are (Liljedahl, 2021, p. 14):

1) What types of tasks we use
2) How we form collaborative groups
3) Where students work
4) How we arrange the furniture
5) How we answer questions
6) When, where, and how tasks are given
7) What homework looks like
8) How we foster student autonomy
9) How we use hints and extensions
10) How we consolidate a lesson
11) How students take notes
12) How we choose to evaluate
13) How we use formative assessment
14) How we grade

Data Sources

Three sources of data were collected for this study. These sources included interviews with each participant at the beginning, middle, and end of the course, task analysis activities at the end of each interview, and a document analysis of participants' chosen course assignments. Three 45-60-minute interviews and three 30-minute task analyses were held. Each participant opted to complete the interview and task analysis during the same allotted time.

Interviews

I conducted three 45-60-minute semi-structured interviews with each participant. The interviews were audio recorded and transcribed verbatim via Otter.AI (Artificial Intelligence) for comparison within the data analysis portion of this study. The transcripts were reviewed for accuracy. Any identifiers in the transcripts were removed for anonymity of participants and specific settings. Interviews were semi-structured to allow for follow-up questions to further investigate participants' responses when appropriate.

Interview questions were open-ended, focusing on three main topics: personal understanding of problems and problem solving, experiences of problems and problem solving, and criteria for a worthwhile problem-solving task. Personal experiences with problems and problem solving help shape a personal understanding of problems and problem solving (Gözde, 2020; Son & Lee, 2020). Teachers use those experiences and understanding to lay the foundation for what criteria is important when choosing a problem-solving task (Carlson & Bloom, 2005; King, 2019; Rosli et al., 2015).

Interview one was intended to be completed at the beginning of the mathematics methods course. The first part of interview one was designed to solicit the participants’ understandings of problems and problem solving. The first set of questions informed my first research question: What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program? (See Appendix C). The goal was to understand the participants' thoughts and experiences surrounding problems and problem solving as they entered their mathematics methods courses. The questions in this interview (See Appendix C for interview questions) investigated participants’ prior understanding and experiences of problem solving to establish a basis for the participant's connection to problem-solving criteria.

The second set of questions in interview one focused on the participants' criteria for a worthwhile task and their rationale. Questions focused on the criteria for a worthwhile task were: "What criteria make a task a "worthwhile" problem or task? Why is that important?"; and How does that criterion promote problem solving?". These questions aligned with the sub-question of the first research question (What prior knowledge and understandings do PSMTs have about a problem and problem solving as they enter their teacher preparation program?) by gaining insight into the participant's belief of problem-solving tasks and how to distinguish their criteria for a task to be worth using. Hence, interview one was used to address fully the first research question.

The second and third interviews were spaced out during the participants' enrollment in the mathematics methods course. The second interview was completed halfway through the course, while the final interviews for Kiley and Ashley were completed during finals week of the course. Hilary had a schedule conflict and had to reschedule her final interview for after finals week. The spacing of the interviews was designed for the participants to progress through the mathematics methods course through social interactions in learning and individual reflections within their own learning.

During the second and third interviews, the participants were asked if any changes had occurred to their understanding of problems and problem solving and their criteria for a worthwhile task as they engaged in the first mathematics methods course. If the participants had adjusted their ideas, I asked what had influenced the change. By asking about changes in the participants' ideas, I sought to find the influences for change. A complete list of interview questions for Interview 2 (Appendix D) and Interview 3 (Appendix E) are included.

Task Analysis Activity

The initial task analysis activity was completed after the first interview questions. This activity consisted of four pre-algebra tasks where the participants completed a "think aloud" about the task and justify if the task aligned with their criteria for a problem-solving task or not (Bannert & Mengelkamp, 2008). In the state in which this research was conducted, the state standards identify four strands of mathematical standards. They are Numbers & Operations, Algebraic Reasoning & Algebra, Geometry & Measurement, and Data & Probability (XXXXX Education, 2022). The course in which the participants were enrolled used the mathematical strands through the middle school level mathematics content. Each of the four tasks were chosen from a different mathematical strand for participants to focus on the pedagogical situation and not compare mathematical tasks from the same strand. Having different mathematical strands for the tasks allowed the participant to focus on their criteria of a “worthwhile” task, rather than comparing tasks to each other. Each task will be discussed in terms of their mathematical strand and alignment to problem-solving criteria below.

A think aloud is a strategy used to gather insight into participants' thoughts and feelings of a task. When the participants were presented with a task, they were asked to share their initial thoughts and feelings about the task. After the participants gave their initial thoughts, follow-up questions were asked to focus on their criteria of a worthwhile problem-solving task. The purpose of the participants engaging in a think aloud was to hear what the participant was thinking about while looking at the individual tasks.

Four pre-algebra tasks were chosen by using a rubric created by combining aspects of Smith & Stein's (1998) reflection on levels of cognitive, Son & Lee's (2020) analysis framework, and Levenson's (2013) task analysis rubric. Smith & Stein's (1998) reflection on cognitive levels divided tasks into low and high cognitive demanding tasks. Tasks within the low category focused on memorization or producing procedures without connections to a mathematical concept or routine exercises. Higher cognitive tasks make procedures connecting to a mathematical concept and/or "doing" mathematics. Son & Lee (2020) identified five areas for tasks by being a form of art, an analysis, an application of understanding, interpretation of information, exploration of a concept, or reflection of understanding. Levenson's (2013) framework used specific criteria within the tasks. For example, a task should require communication from the learner, identifiable with multiple pathways to solve, and the ability to use various manipulatives, illustrations, or real-world context for a learner to connect with math concepts. See Appendix F for a formalized rubric. The rubric highlighted the components that foster high order thinking through problem solving.

Tasks. The preselected tasks matched pre-algebra content because the methods course the participants were enrolled in focused on middle school mathematics. Therefore, focusing on pre-algebra allowed the tasks to be mathematically accessible to participants. Each task was aligned to a different strand and standard within the state standards for the state in which the study was conducted. Though there is no one task that meets all criteria on the rubric, each element was at least met one time. A variety of tasks are needed to meet each component of the rubric. All tasks can be found in Appendix G. As a former pre-algebra teacher in the state, my content knowledge facilitated my choice of tasks. See Appendix H for completed rubrics for each task.

Task 1, Diving Distance, meets the state pre-algebra standard, "Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects" (XXXXX Education, 2022, p. 31). In this task, the learner is asked to use their knowledge of functions to establish a representation for a drive from one local city to another and the drive back. The learner was given information regarding the speed and time of the drives. The task asked for the learner to determine the distance of a portion of the drive based on the information given. The task allowed the learner to analyze and interpret the information to apply previous knowledge and strategies to the context. Reflection on their solution and the context created a space for communication to understand their pathway of solving and their potential solution.

Using the task selection rubric, this task met the criteria of procedures with connections, problem solving as an art, use of real-world connections, and multiple pathways. This task did not meet the criteria on the rubric for doing mathematics. However, the task met the requirements for procedures with connections, level 3 of Smith & Stein's (1998) levels of cognitive demands because the learner uses procedures to connect the mathematics of distance with speed and time. Problem solving as an art requires students to understand the task to apply their prior knowledge and strategies. Once known information is applied, the student can interpret the strategy based on its effectiveness and reflect on its reasonableness. Real-world connections are met through locations the participants are familiar with in personal experiences.

Task 2, Algebraic Rectangle, meets the pre-algebra standard, "Develop and apply the properties of integer exponents, including (with $a \neq 0$), to generate equivalent numerical and algebraic expressions" (XXXXX Education, 2022, p. 31). Task 2 asks the learner to find at least 5 potential areas of a rectangle with a perimeter of $4x + 24$. This task requires the learner to understand perimeter and area of rectangles, multiplication and simplifying expressions including rule of integers.

This task met multiple criteria on the task selection rubric including multiple pathways and solutions, procedures with connections, and visual representations. The task allowed the learner to connect areas of rectangles with exponential properties with multiplication. The learner can start the task using different strategies to arrive at different solutions. This task did not meet the criteria on the rubric for real-world context and communication. The task can be extended for the learner to explain why or justify their reasoning; however, the written task did not use a real-world connection or ask for justification. Communication could be added to this task if the task asked for the learner to justify their reasoning.

Task 3, Drinking Glass Task, meets the pre-algebra standard, "Develop and use the formulas and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for pi (T). Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate units (e.g.,)" (XXXXX Education, 2022, p. 32). Task 2 has two parts for the learner to solve. The first portion incorporates a cylinder's volume range for the learner to find potential dimensions. The second portion of the task uses knowledge of the volume of cylinders and cubes to determine how many ice cubes can be added to the drink.

This task aligned with the criterion of doing mathematics, allowing for multiple pathways and solutions and attaching to real-world context. The learner must apply their understanding of volume to compare volumes within the glass. An extension of volume was given with the task's second part when asked about the addition of the ice cubes in the cyclical volume. The learner is asked to use a real-world situation to solve the task. This requires the learner to extend beyond the procedures of volume to doing mathematics by critically processing their understanding and information given to use strategies to solve the task.

Task 4, Locker Likelihood, meets the pre-algebra standard, "Determine how samples are chosen (randomness) to draw and support conclusions about generalizing a sample to a population, including identifying limitations and biases" (XXXXX Education, 2022, p. 32). Task

4 asks the learner to use knowledge of probability to determine likelihood cases for lockers being searched based on the population percentage between male and female students. Once the learner finds the probability of the scenarios, the learners must identify a scenario they believe is most fair and justify their reasoning.

Using the task alignment rubric, this task met the criterion of connections with procedures problem solving as an art in the instance of analysis, application, interpretation, reflection, and communication. The learner uses their understanding of probability within a situation to apply a procedure. The learner is asked to reflect on their solutions. This task also meets the criterion of communication to justify which situation they believe is fairer.

Table 3

Task criteria rubric

Criteria	Task			
	1	2	3	4
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)	Met	Met	Met	Met
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Procedures with connections*	Met	Met	Not Met	Not Met
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Doing Mathematics*	Not Met	Not Met	Met	Met
Problem Solving as an art (Son & Lee, 2020)	Met	Met	Met	Met

-Involves analysis, application, interpretation, exploration, and reflection				
Communication (Lappan & Phillips, 1998; Levenson, 2013)	Met	Not Met	Met	Met
Use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013)	Met	Met	Met	Met
Multiple pathways to solution(s) (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	Met	Met	Met	Met
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	Not Met	Met	Met	Met

Every task did meet the criteria of math content, task as an art, use of resources, and multiple pathways. Each task aligns to a mathematical standard. It is important for mathematical content to be a large portion of the focus of the task. For a task to be an art, the task involves questioning strategies that could be used, applying the strategies chosen, and analyzing/reflection how the strategy was used. Using resources allows for both relevant and use connections to understand and solve the task. These tasks either allow for real-world connections or use of physical or visual manipulatives to connect understanding and application to the tasks. The tasks allow learners to use multiple pathways to conclude. The learners can start from their understanding of the task's content to start problem-solving it.

The two criteria that are not met for all tasks are communication and multiple solutions. Task 2 does not explicitly ask for the learner to discuss or justify their reasoning. Though this task does not meet the criteria for communication, the task could be altered by facilitator. Task 1 does not have multiple solutions. There is only one correct answer.

By focusing on only pre-algebra tasks, the content was consistent across a specific grade-level. The pre-algebra tasks allowed for participants' accessibility to focus on pedagogical thinking versus the participants' conceptual understanding. The methods course the participants were enrolled in also focused on middle school (6^{th} through 8^{th} grade) content. The task choices aligned with aspects they may encounter in the methods course.

Participants' Chosen Tasks. Participants were asked to bring a task they believed was worthwhile to the second interview. With the task, the participant was asked to explain why they thought it was a worthwhile problem-solving task and how it aligned with their definition of a problem. After the participants justified their reasons for their task selection, follow up questions were shaped based off the research-chosen task rubric. A complete list of questions is included in Interview Protocol 2 (Appendix D). The tasks were added to the final task analysis activity to be completed at the end of the semester. The final task analysis activity the two participant-chosen tasks from the other participants' second interview. Participants did not know which participant had chosen which task. Similar interview questions to the first task analysis were asked for the final task analysis (Appendix E).

The task analysis activity was completed at the beginning and end of the course. See Interview 2 Protocol (Appendix D) and Interview 3 Protocol (Appendix E) for a list of think aloud questions. Follow-up questions were used to determine the participants' justification for their task analysis.

Document Analysis

At the end of the second interview, participants were asked to bring to the last interview a course assignment they believed connected to problem solving and their criteria for selecting a task. This data collection was completed at the end of the semester. This portion of the interview began with the participant explaining the connection between the chosen assignment and problem solving. Participants were asked questions like "How does this assignment align with your understanding of problem solving?". These questions explored the participant's justification for bringing the assignment and their belief in how it aligned with problem solving. In addition to questions that align with the participant's criteria for problem solving, the researcher asked questions that align with the task analysis rubric, such as, "Do you believe this allows for multiple solutions? Why or why not?"—these styles of questions aided in understanding the criteria participants used for the task analysis. A complete list of questions is included in Interview 3 Protocol (Appendix E).

Subjectivity Statement

As a former middle school mathematics teacher, I have been in the position of choosing types of tasks and problems I would give my students. Upon reflection, there were common considerations I would make when selecting tasks. For example, I would think about what content knowledge my students already have, the purpose of the task or problem, and the content knowledge being taught. Being a former mathematics teacher can be both a disadvantage and an advantage in my research study. Having a professional and educational background like my participants, I can establish a foundation of trust sooner. Since this research has a short timeline, I was able build trust quickly to dive into the PSMTs' perspectives on problem-solving criteria for tasks chosen for a classroom. However, as a disadvantage, PSMTs may feel a sense of "knowing" connection, limiting their interview responses. I needed to consciously focus on engaging the participants in continued elaborations on their responses to counter short responses caused by on participants relying on my previous experience.

My post-secondary education has been in the same department as the university in this study. As a Ph.D. candidate, I have spent time in master's and doctoral programs focusing on what research says is a worthwhile problem-solving task. Within my educational background, I have taken mathematics education classes focused on problem-centered learning. Because of those graduate classes, I developed interview questions that were semi-structured. Because the graduate classes impacted my view of problem-centered tasks, as a researcher, I needed to be disciplined in distancing my biases and preferences with problem-solving to understand others' understanding and criteria for a specific task.

I was a mathematics curriculum specialist, coaching mathematics teachers and writing lessons. Within my career, I worked with mathematics teachers in 24 rural schools across the state in which this study was conducted. Coaching aspects of my job required me to converse with teachers about why they picked specific strategies/activities for their classroom. Within those conversations, I learned how the teachers believe those strategies/activities align with student-centered criteria to build on students' prior knowledge through meaningful questioning, substantive discussions, and real-world connections. However, I wonder what knowledge of problem solving and problem-solving tasks PSMTs bring to their programs and how those change in one mathematics methods course.

As a former Algebra 1 and Geometry teacher in the state in which the study took place, I have my notions of problem solving and the types of tasks that encourage problem-solving skills. I put precautions in place to uphold an ethical standard. I had an outside researcher interview me using this study's interview questions to gather my thoughts on the critical aspects of this study. My interview allowed me to bracket my experiences to establish a clear mind when interviewing the participants. Bracketing is when “investigators set aside their experiences, as much as possible, to take a fresh perspective” (Creswell & Poth, 2018, p. 78). I identified my understanding of the research topics to set aside that understanding to allow my participants' experiences to become precedent in the research.

Data Analysis

Inductive analysis was used for this study. Inductive analysis uses rounds of coding to create codes, categories, and themes by looking for patterns in the data (LeCompte et al., 1993; Morse, 1994; Thomas, 2003). I began coding by using open coding. Open coding is an initial coding process used during beginning data collection where potential themes or patterns emerge (Merriam & Tisdell, 2016). Open coding allowed meaning to emerge with connections to ideas through the participants' whole interviews and the task-sorting activities. This round of coding was done via hand-written notes during the interviews.

I transcribed the audio recordings of the interviews verbatim using Otter A.I. The interview transcripts were anonymized to protect the participants' identities. The data was open-coded using line-by-line coding. Line-by-line coding allowed the researcher to read each data line for meanings to create codes. For example, Kiley stated, “I think our problem like anything that can be answered whether that’s one specific answer or many different answers.” This line was coded under Problem definition as “to be answered with one or more answers”. Hilary stated, “I think when I see a problem, it’s something it has to be solved”. This line was coded using a similar code to the line from Kiley as” to be answered”. After line-by-line coding, I compiled all the open codes to transfer to a spreadsheet, each tab representing a different participant. Participants were contacted for clarification of data and confirmation of codes. This confirmation process is called member checking, in which the researcher requests feedback from the participants (Merriam & Tisdell, 2016). Member checking is important because the participants need to see their experience through the interpretation the research is making of the data (Merriam & Tisdell, 2016). Upon participants' confirmation of codes, I analyzed categories and themes by synthesizing codes into groups. For example, the lines provided above by participants 1 and 3 were coded under the same theme as “to be answered”. The grouping of themes allowed for meaning making to occur (Merriam & Tisdell, 2016). Finalized themes were used to conclude the study for implications on education.

The participant chosen tasks were coded using the same format as the interviews. I first coded the descriptions the participants used to align their understanding of problem, problem solving, and criteria for a task. I also used follow-up questions regarding the research-chosen task rubric to determine if the participant believed task met or did not meet the criterion used as a basis for a worthwhile problem-solving task within this study. The participant chosen tasks were used during a portion of the third interview for participants to analyze other participant tasks. This part of the task analysis was included in third interview transcript for coding.

The document analysis followed a similar analysis pattern as the participant chosen task. Participant brought the assignment during the final interview. The document analysis was intended for participants to align their understanding of a problem and problem solving for the end of the study to be represented through the document they provided. The document was coded first using line-by-line coding with the description and connection given by the participant who provided the items. Like the task analysis, the participants were asked to depict how the items they chose aligned to their understanding of problem, problem solving, and criteria for a task. I then, like the participants' chosen task, asked follow-up questions that connected to the research-chosen task rubric to determine if the assignment met or did not meet this study's criteria for a worthwhile task.

Trustworthiness

Triangulation was implemented to ensure trustworthiness in this study. Triangulation is the act of comparing and cross-checking data collected (Merriam & Tisdell, 2016). Three participants were included in this study creating multiple voices. Data was collected using multiple means of interviews, task analysis and document analysis. Through the data analysis process, coding and theme checking were validated by the participants to confirm the accuracy of the themes. Participants confirmed or contradicted the codes and theme in person. At the end of the final interview, I presented the current coding system with participants for clarification on understanding. This negotiation regarding codes became part of my analysis because the participants' responses led me to refine my codes and my subsequent analysis. Kiley stated how she was interested in the progression of her understanding through her learning. Seeing it through the alignment of codes gave a depiction of her growth. A clarifying question as to the steps of analysis were addressed by means of my rubric. Seeing the rubric to understand how I determined tasks helped her understand how I analyzed the task she provided during interview two. Ashley did not have any conflicts with the codes and analysis. This instance affirmed my coding strategies. Hilary asked for clarification as I coded an aspect of her understanding of problem solving as "decoding". Hilary stated, "When you're presented with a problem, in order to solve it, you have to take into consideration what you know and then the new material you've learned. And kind of combine the two to create a new method or a way of navigating how to get a solution." I had coded this line as both "use of prior knowledge" and "decoding understanding". I explained I used the code "decoding" to describe the strategy of when a solver is present with a problem and decodes the information in terms of their previous knowledge and the new information they know. After the given justification, Hilary expanded on her thinking. She explained that she was placing the emphasis on the connection between prior knowledge and new information. Therefore, I removed the code for "decoding" to edit code to "connection". In this instance, using member checks altered my analysis to more closely match the code with the participant's intent.

Conclusion

This study aimed to understand PSMTs' definitions and influences of problems and problem solving and what criteria they have for worthwhile tasks to promote problem solving. Using a constructivist lens, I asked:

1. What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?
    a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

To answer these questions, I engaged in interviews, task analysis, and reflections on coursework during their first mathematics methods course. The participants were recruited through a state flagship university's initial secondary mathematics methods course. Written consent was obtained from the participants as the interviews were conducted. The Otter AI (Artificial

Intelligence) recording device was used to record and transcribe the interviews. I secured all recordings and documents on an external drive, to maximize the security of the data. The recordings were deleted after the completion of the verbatim transcription. Transcripts were shared with participants to secure the accuracy and clarity of codes, categories, and themes.

Chapter 4: Findings

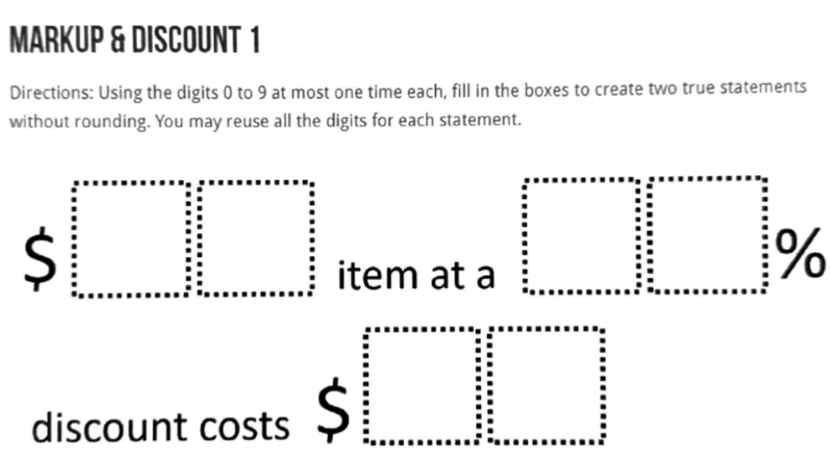
Each participant had a unique mathematics education background and, therefore, their understanding of problems and problem solving and how to implement problem-solving tasks in a secondary mathematics classroom unfolded in unique ways. As the participants progressed through the methods course, class experiences altered their understanding to reflect a social constructivist view in which teachers develop collaborative experiences to support students in learning new ideas. This chapter describes the participant chosen tasks, interviews, task analysis, and document analysis. After the review, each individual participant's story and connections to their understanding are presented.

Participant Tasks

Kiley brought the task titled, Markup & Discount 1 (Figure 1). The task asks the learner to use the digits 0 to 9 only one time each to fill in the blank spaces to create a true statement about the initial cost of an item with a discounted percentage for the final cost.

Figure 1

Kiley's Task



Ashley brought the task titled, Ancient Egyptians and Fractions (See Figure 2). This task gives a brief history on how Ancient Egyptians used unit fractions to write other fractions. The learner is asked to write 5/8 and 8/9 like the Ancient Egyptians would. The final aspect of this task asks the learner to use subtraction, instead of addition, to discover another way to write 5/8.

Figure 2

Ashley's Task

Ancient Egyptians only used unit fractions (½, ⅓, ¼, etc.)
For ⅔, they would write ⅓ + ⅓. How might they write ⅝?

How would they write 8/9?
What if we used subtraction? What is another way they could write ⅝?

Hilary brought the task titled, Find the odd man out (Figure 3). This task lists four algebraic expressions and asks the learner to find which one does not belong with the other three expressions.

Figure 3

Hilary's Task

Find the odd man out

$$\frac{16}{8}x^2 - 2$$

$$\frac{36}{18}x^2 - 2$$

$$-4x^2 - 2$$

$$2x^2 - 2$$

All three tasks met the criteria for mathematical content, use of manipulatives, illustrations or real-world, and multiple pathways. Kiley's task connects real-world situations with discounts and costs of shopping. Ashley's task, though it does not specify, allow for manipulatives or illustrations for fractions. Hilary's task could use graphical illustrations to represent the functions. All the tasks allow the learner to use different strategies to come to a solution. Each task also allows for the learner to analyze, interpret and apply strategies to solve. Kiley's and Ashley's tasks met the criteria for procedures with connections and multiple solutions. Ashley's task is the only task that explicitly has questions that lead to communication. However, all tasks would allow for communication with learners through a collaborative discourse setting. Hilary's task is the only task that only has one solution. The following table summarizes how each task was evaluated according to the rubric for this study.

Table 4

Summary of Participants' Tasks

Criteria	Kiley	Ashley	Hilary
Connection to Mathematical Content	Met	Met	Met
High Cognitive Demand *Procedures with connections*	Met	Met	X
High Cognitive Demand: *Doing Mathematics*	X	X	X
Problem Solving as an art	Met	Met	Met
Communication	Met	Met	Met
Use of manipulatives, illustrations, real-world context	Met	Met	Met
Multiple pathways to solution(s)	Met	Met	Met

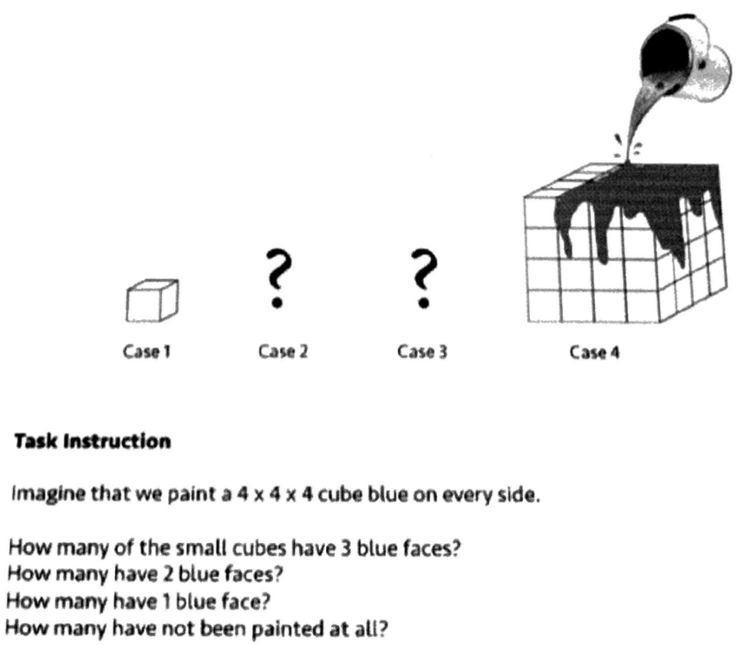
More than one solution	Met	Met	X

Participant Assignments

Kiley brought an assignment from the mathematics methods course that is a task involving a 4x4x4 cube and paint (Figure 4). The task asked the learner to consider a 4x4x4 cube with blue paint on every side. The learner is asked four questions about how many small cubes within the 4x4x4 cube have specific sides painted or not painted.

Figure 4

Kiley's Assignment



Ashley brought an assignment involving three candy bars to find measurements and their volumes (Figure 5). The learner is asked to measure three different candy bars to determine the volume of the bars. Once the learner finds each of the volumes, they must decide which candy bar they would choose to eat and why.

Figure 5

Ashley's Assignment

Candy Bar Volume

Candy Bar	Height	Length	Width	Volume
Twix				
Snickers				
Milyway				

Using the provided ruler, measure the height, length, and width of three candy bars. Keep in mind, you only get to eat one at the end of the lesson so choose wisely!

Which candy bar did you choose to eat and why? (can you think about this mathematically?)

Hilary brought an assignment from her Precalculus class (figure 6). The assignment was a quiz question that Hilary struggled with during the semester. The question focused on finding the value of the sine of an angle given its cosine without using a right triangle. Hilary's ability to overcome struggles she faced in this course throughout the semester is why she brought this assignment.

Figure 6

Hilary's Assignment

Question 2

Let θ be an angle in Quadrant 1. If $\cos\theta = \frac{5}{7}$, what is the exact value of $\sin\theta$?

Do not use a right triangle to answer this question. Use information from Worksheet 2 or 3.

For the participant assignments, only two criteria for this research study were met, connection to mathematical content and multiple pathways. Kiley's and Ashley's assignments allow different strategies to approach the task to implement and reflect on. Their assignments allow for multiple pathways to find a solution. Manipulatives, illustrations and real-world context are used for the learner to solve. Hilary's assignment lacked multiple of the criteria for this research study. Besides the criteria all three assignments met, Hilary's assignment focused on procedures with connections by the learner having specific examples to return to for assistance. The following table shows how each assignment was evaluated according to the rubric for this study.

Table 5

Summary of Participants' Assignments

Criteria	Kiley	Ashley	Hilary
Connection to Mathematical Content	Met	Met	Met
High Cognitive Demand *Procedures with connections*	X	X	Met
High Cognitive Demand: *Doing Mathematics*	Met	Met	X
Problem Solving as an art	Met	Met	X
Communication	Met	Met	X
Use of manipulatives, illustrations, real-world context	Met	Met	X
Multiple pathways to solution(s)	Met	Met	Met
More than one solution	X	X	X

Each participant was asked to justify why they chose their task and connect their task to their criteria for a worthwhile task. Also, each participant was asked to evaluate the tasks of other participants. Their evaluations will be shared in the participants' descriptions later in this chapter.

Kiley

Kiley decided to major in mathematics education because of a desire to help others. In interview one, she said, "I wanted to help serve people and help them, like, learn math." Upon entering the university, she originally chose to major in mathematics because she had always liked math. However, she had a desire to help others enjoy mathematics as well. She could not envision how majoring in mathematics would help people like mathematics. She stated, "I was thinking how I wanted to use my knowledge and math to help other people…but I didn't think it was my place to do that for myself." She decided to make the transition to math education to help people learn mathematics and how mathematics can be applied to a variety of sciences.

Many of her previous math teachers also inspired her. She said her teachers gave her a reason "to try and look to want to be a teacher." Kiley's face lit up as she described the impact of one specific teacher in high school. This teacher stood out to her because of the way the teacher used different manipulatives and activities in AP Calculus to make math fun for everyone. She compared her AP Calculus teacher to her other math teachers in middle school and high school through their different teaching styles. Kiley said, "Many teachers, even in middle school and high school will give a lecture style lesson, but she [AP Calculus teacher] was always using, like, manipulatives. This teacher really inspired [her] to try and be a teacher who makes math fun for people who don't like it."

Interview one

For the first interview, Kiley's understanding of problems was “anything that can be answered.” Kiley separated the idea of problems and equations by describing “problems that helped create a more learning and thinking environment, instead of just following the procedures to find what is X if you have X plus two is five.” She mentioned that growing up, her preference in her math classes was the “plug-and-chug equations.”

Kiley’s initial influence on understanding problems was personal experience with problems. In her personal learning, she expressed how she enjoyed equations that could just be plugged with numbers to solve. She said, “throughout high school, I loved whenever they gave me an equation, and I just plugged into an equation.” However, looking back on her personal experience, she realized she “wasn’t really thinking anything beyond [plugging into the equation].” Parts of her experiences in high school were aligned with these equation-style mathematical tasks that just required her to plug in numerical values to solve.

In contrast, in college, she felt she needed to think more about the process for what she was solving. She stated, “The student might have to come up with the procedure without it being specified in the problem or something.” She mentioned that she would have to develop a process to solve problems that she believed aligned with the creative side of problems. She said, “I realized I was just plugging into an equation. I wasn't really thinking anything beyond that. So, I think what makes a problem good is invoking thought.”

Reflecting on her experiences in college, Kiley came to believe that “there are definitely different types of problems.” Kiley said, “there are the ones [problems] that can be interpreted many different ways, and I think these ones are the problems that helped create a sort of problem like a more of a learning and thinking environment.” Kiley felt that problems allow for a creative aspect as a learner interprets the problem to determine the pathway for solving. The learner’s experience and known strategies are what help choose a potential pathway to solve a problem. She said, “if there's some like creative aspect, so the student might have to come up with this procedure without it being specified in the problem or something. I think that makes a problem good.” Kiley described her understanding to problem solving as aligning to “the whole process like reading the problem, interpreting it or decoding what it’s saying. And then answering this question in your own way.” She continued to tie one’s prior knowledge into the problem-solving process, “most of problem-solving is thinking about what prior knowledge do you have that can help you solve a new problem.” As she continued to elaborate on the aspects of problem solving, she kept coming back to prior knowledge. She stated, “usually when you’re given a problem or when a student is given a problem, it will reflect back to something they’ve learned.” This prior knowledge could be “a definition, or just like some procedure…but it’s going to show it in a new light.” Kiley gave a problem-solving example she experienced in her Differential Equations class. She mentioned the problem was “kind of procedural, however the problem gave you the information…but you still had to interpret what each number meant.” The process of problem solving involves active thinking. Kiley stated “I was actively thinking through the problem. And since it took me a while, I can say that I was problem solving.”

When asked about her criteria for choosing a worthwhile task, Kiley started with the idea that a worthwhile task met curricular goals and expectations. She said, “I think at some point it has got to meet the curriculum, because it’s totally your job to teach people this curriculum.” Though she perceives a responsibility of the teacher to teach the curriculum, Kiley did state that “there are so many different ways to approach teaching this curriculum.” The different approaches would be based on the classroom setting and the learners. Her second criterion was collaboration “between students, I feel like every problem could be collaborative to an extent like just by going over your answers." She did, however, say she was not sure of her understanding and felt "like I would have to read more about like how to make a problem collaborative."

The third criterion she mentioned was higher order thinking, "I want it to invoke higher order thinking." She connected this criterion to reading from one of her course textbooks, *Building Thinking Classrooms in Mathematics* (Lijedahl, 2021). Kiley said, [the author] suggests "it might be better to start with a non-curricular task that is engaging and then relate that to these tasks, by use of like manipulatives or visual tasks." Kiley's final criterion was to engage all students. "I think it is really important to have ... for the problem to be engaging for all students." She mentioned that engaging all students can be difficult at times because:

> I know many times students, when they develop their identity, might say 'Oh, I hate math I don't want to do anything the teacher says.' To try and make math interesting for them, you might want to use a problem that doesn't feel like math in the beginning.

Kiley emphasized that a problem-solving task does not always have to be "really difficult" but is a balance between discomfort and building confidence. She believes "that can definitely lead people, like, feeling discouraged to keep continuing." She added, "every once in a while, it's good to have a difficult one. But as long as the students are thinking and working together, I think that's a really good environment for problem solving."

Task Analysis One

Task One. When given the task, Driving Distance (Appendix G), Kiley read the task out loud and verbally started to process the task to solve it. She stated that, "I see velocity, average velocity. I see average time. I can multiply and like, her total distance." Throughout her initial processing of identifying the information given in the task, she mentioned that "if I were an 8th grader, I'm not sure if I would make the connection in my head." After her think-aloud, she was asked to consider her understanding of problem solving with the task. Kiley emphasized that she "really liked this one because it doesn't give you a set procedure." The lack of specific procedures and equations to use to solve opened opportunities to use different methods and "different questions that can be asked from this problem."

Kiley went through each of her criteria to discuss whether she believed the task met her criteria or not. Kiley's first criteria was that it met curricular expectations. For task one, Kiley first stated that the task met curricular expectations "for sure". However, she also then redefined her idea of curriculum by saying "I think I might have gone too vague when I said curriculum because I feel like every task will be curricula unless it says read this for fun." Therefore, she readjusted when she meant, to clarify that she believes a task should have a "connection to a mathematical context." She connected this task to solving distance.

Kiley's second criteria was that tasks should have a collaborative element. She said, "I would definitely want to collaborate with students." She identified a lack of hands-on with her understanding of use of manipulatives, "it's not exactly manipulatives, like especially lower-level math with fraction strips and stuff". She ranked the collaborative level as not "super collaborative, but I still think it builds conversations."

Kiley's third criterion was higher order thinking. She said this task did meet criteria three because "you can make your own procedure, which I think is quality of higher order, when students can make their own way to solve this problem."

Kiley's final criteria was engaging all students. Kiley assumed that the students who engage with this task would live near Texas based on the locations mentioned in the task. She thought "it's engaging in like the cultural sense because it gives something that people they can relate to."

Task Two. The second task, Algebraic Rectangle (Appendix G), brought out excitement from Kiley. She stated she loved these types of tasks because "I could pick this up for days because obviously there's no right answer", the personal connection to determine a process and use that process to try solving. Kiley believed "it's really good for curriculum." She believed the criterion of meeting curricular expectations was met because she felt "like it hits on so many different parts, like so many different curriculums." When asked to expand what she meant by "different curriculums", she said "different contexts". For Kiley, different contexts represent aspects of mathematics content that can be highlighted.

Criteria two was collaborative. Kiley claimed this task was "very collaborative" because students could "brainstorm together and maybe even come up with a fifth one." Since the task is open-ended, she could see how answers could extend thinking, and conversations could deepen reasoning and understanding. This connects to her third criterion, higher order thinking.

In terms of higher order thinking, Kiley stated "I think it's pretty good." She supported that thought by saying she liked this task because "it asks for five different values." She appreciated that this task was not "wordy" because "sometimes higher order is associated with more words on the page." She does not believe more words equates to higher order thinking.

Kiley's final criteria was engaging for all learners. She said that this task is engaging "because it offers different answers for one problem." She described how the engagement for this task is different than task one, Driving Distance. She said the task one “interpretation of engaging was more cultural, but this one doesn’t really have that much cultural.” She did believe that even though it does not have a cultural connection, it is engaging because of the different ways a learner can approach the task to find various solutions.

Task Three. Task three, Drinking Glass Task (Appendix G), made Kiley pause after reading. Her facial expression showed her thinking of what she just read. She followed her pause with “this one’s difficult.” This task would be “one of those problems where I would spend a lot of time actively thinking.” She made this connection to one of her third criteria, higher order thinking, by saying, “this one is probably more higher-level thinking than the other two.” She was intrigued by the task giving a range of volume, instead of a specific number the student is given “a choice of where to start in the problem.”

She continued her thinking as she reflected on the collaborative aspect of the task, criterion two. She believed this task “really shows the value of working with your peers” based on the difficulty level. Kiley also felt the task met her criterion of meeting curricular expectations because “it’s definitely seems to be talking about the volumes of geometric shapes.” For her final criterion, engaging all learners, she had to pause to think. Tying to cultural aspects of engagement, she mentioned how the unit of measurement is different between America and other countries, “like imagine a little British kid being like I know how much that is. But unfortunately, here we don’t use those measurements.” Kiley said there is connection to the 3D object in this task to a student is “obviously drinking water and sodas.”

Task Four. For Task Four, Locker Likelihood (Appendix G), Kiley’s think-aloud started by her verbally solving the task. Through her verbal solution, she determined this task connected to percentages, which meet her first criterion concerning meeting curriculum expectations.

For criteria two, collaborative, she said, “it’s not super collaborative.” The collaboration piece would only come because “people might get stuck and want to ask their friend.” Kiley’s criteria three was met by connecting higher-order thinking to the fluidity between fractions and percentages. For the final criteria, engaging all students, Kiley turned the task to personal experience of high school locker search by thinking “students may ask themselves ‘what if my locker gets searched next year?’.” However, Kiley did not mention connections to mathematical content to engage the learner in ways to approach the task.

Interview Two

Interview two started with me reviewing her understanding of problems from the first interview which was “problems that helped create a more learning and thinking environment, instead of just following the procedures.” Kiley expanded her understanding to state that interpretation sets the foundation of learning. The problem is not the whole thinking environment, it is only a portion of it. Kiley stated, “I had the misconception that you can give a student a problem and they will learn from that problem alone.” She continued to include the teacher/facilitator as a factor to the learning environment by giving feedback and asking questions of the learner. She equated her understanding changed because of her engagement in the mathematics methods course. She asserted that readings within a course required book, *Building Thinking Classrooms in Mathematics* (Liljedahl, 2021), expanded her thinking to include other aspects of the learning environment like the teacher.

During the second interview, Kiley separated problem solving between individual problem solving and collaborative problem solving. Her description of individual problem solving aligned with the definition of problem solving she gave in interview one, which was a process of actively thinking by reading, interpreting, and decoding a task by considering prior knowledge to reach a potential solution. However, she defined collaborative problem solving as a setting in which students were able to discuss with others to include the group's previous knowledge and the decoding/interpretations unique to each individual: "Instead of just thinking about your own prior knowledge, you would maybe discuss everyone's previous knowledge and like the diversity of thoughts within your group in order to come to a solution." Kiley altered her understanding to include a collaborative aspect because of her time in the mathematics methods course. She said, "Whenever I was first thinking of problem solving, I was simply thinking about like an individual completing the problem items by themselves. But now, with the context of the class (mathematics methods course), I think more about how students work together to solve." She restated that "it's just because of exposure to a collaborative setting that makes me think that way." Different backgrounds and prior knowledge gathered from everyone benefit the whole group when solving the problem.

Experiences in the mathematics methods course influenced her change in thinking. Kiley experienced working in groups firsthand in class. She mentioned the course book, *Building Thinking Classrooms in Mathematics* (Lijedahl, 2021), in which the seating of the classroom "affects collaborative learning and how collaborative learning affects like makes the learning more effective for the students." Collaboration fills gaps in learning by working with more people's prior knowledge than just one's own previous understanding; "it completed the puzzle."

Kiley did not change her criteria for a problem-solving task during the mathematics method course. She asked for verification that she mentioned curriculum in the first interview. "So, I talked about curriculum?" After confirmation that she did mention curriculum in the first interview, Kiley said, "I think that covers at least what I can think of now."

Kiley's Task. Kiley aligned her task (Figure 1) with her understanding of problems because the task allows the learner to enter the task through multiple pathways. Kiley aligned this task to their understanding of collaborative problem solving. Kiley described how a group setting allows learners to use each person's previous knowledge to help see gap in understanding for the task and support each other to find a solution. This understanding "supports students creating their own model for how to solve a problem." Kiley chose this task because of her experience in the mathematics methods course working with ratios and proportions. She mentioned doing open middle tasks in the mathematics methods course. She described open middle tasks as tasks where "the middle of the process is completely open" leaving the learner options of where to begin the task.

Kiley believed this task met all her problem-solving task criteria. Her first criterion was that the task met "curriculum expectations." This task aligns with the mathematical concepts of ratios and proportions which she believed "is emphasized in seventh grade math, but it can also be discussed in sixth and eighth grade." The second criterion was that the task allowed for collaboration. Kiley commented that "collaborative aspects kind of goes into how the teacher presents it." Her experience in the mathematics methods course with these types of tasks was in a collaborative setting. In these settings, the instructor structured collaboration so that people "were spitting out their ideas because they didn't feel like there was a punishment or any consequences of having the wrong answer."

Kiley's third criterion was that the task should engage students in higher order thinking. Kiley felt this criterion was met based on "being able to come up with your own method to solving a task." The final criterion was engaging all students. Kiley also believed this task met the final criteria. She mentioned accommodations for students with physical disabilities and different cultural aspects of English language learners. She stated, “like with any problem, the teacher has to be thinking of these things ahead of time and able to make accommodations.” She mentioned accommodations for students with disabilities and English language learners because of conversations in the methods class concerning these topics.

After Kiley aligned her task to her criteria, she was asked follow-up questions to see if she believed her task aligned with this research study’s criteria, focusing on multiple solutions and pathways, various representations, and allows for mathematical communication. Kiley stated the different mathematical representations were present in this task because “students think about how to present a percent as a fraction or a decimal when they’re thinking about taking the percent of something.” This fluidity between the representations of the percentage would be present “in the process of solving the answer, they’re gonna go, subconsciously, through these different representations.”

Kiley connected the requirement that task should engage students in mathematical communication criteria to the discussions students would engage in when working in groups. She suggested the possibility that the discussion within the groups “sparks that idea of ratio and proportions.” Yet, she felt that there could be “better problems for promoting mathematical discourse.”

Interview Three

For the final interview, Kiley’s understanding of problems expanded again to state that some problems can only have one answer but can have multiple pathways to arrive at an answer. She stated, “sometimes, like a math problem only has like one final answer and that’s okay. It doesn’t make it any less valid.” She emphasized, “one of the more important things is those multiple pathways being available." Kiley felt that this expansion of her understanding was due to being exposed to different problem styles in her mathematics methods course. She described how the professor of the mathematics methods course exposed the students to many different types of problems. As she engaged in the tasks, she felt conflicted because some problems had only one answer, but she thought it should have had multiple solutions. Then, in the class, she had an experience working on a particular task that changed her thinking. She said, "I was like, wow, this is a great task, and there's only one answer. I was kind of conflicted because I had believed it had to have more than one answer." This internal conflict led to Kiley expanding her understanding that a problem could have either one or multiple solutions.

Kiley stated her understanding of problem solving had not changed since the last interview. She again emphasized collaboration is "really important" and stated, "I'm glad I added that last time." She realized that her understanding is based on her experiences and stated, "I don't think I've been exposed to anything since the last meeting that would change my mind." However, she described a specific activity in the mathematics methods course concerning probability that caused her to reflect on her understanding of problem solving and its relation to prior knowledge. Small groups were given two dice and asked to find probability of different combinations of dice displays. During this activity, she mentioned "we were all thinking, and everyone had different backgrounds." She said she could identify her peers that had taken a previous course called Discrete Math in which students had exposure to similar tasks. Kiley mentioned that those who had taken the Discrete math course approach the task differently that the ones who had not: "People who had taken that course probably had kind of a first step, because they had practiced these tasks compared to other who didn't. It was interesting because without that knowledge, the prior knowledge of this class, people approach the same task completely differently." Thus, although she elected not to change her definition explicitly, she did indicate new understanding of the relationship between prior-knowledge and problem solving.

During the final interview, Kiley kept the same criterion which were meeting curricular goals, collaboration, higher order thinking, and engaging all learners. When asked if she would add or change any of her criteria, she mentioned, "if anything I might add but at the moment, I can't think of anything."

Kiley's Assignment. Kiley brought her assignment (Figure 4) because she believed "it's engaging to students because it's like a puzzle." She described the experience in the mathematics methods course with the use of manipulatives for a physical representation: "I think that it especially helped everyone feel more confident or able to attack this problem." Kiley stated that this task aligned with her first criteria of a good problem, meeting curricular expectations, because it included examining 3D objects. Her second criterion, collaboration, was met because "it was more hands on and [students are] able to work with a partner or table group." Kiley felt the third criteria, higher order thinking, was met by the follow up questions embedded within the task: "it would not have been as higher order thinking if they had only done case one which is a one by one." The final criterion, engaging all students, was met by using manipulatives, which Kiley felt "helps people who might need a physical or visual representation and can't picture in their minds."

Ashley's Task. When Kiley was presented with Ashley's Task (Figure 2), her initial thoughts on this task was "I think this can be really cool, especially if you have manipulatives so that students can check what they're thinking in their heads." Because this task does not give specific procedures, Kiley stated "it allows the reader to use their imagination." When considering her criteria for worthwhile tasks, Kiley felt the task met the first criteria, the task should meet curriculum expectations, by using fractions, addition, and subtraction. Kiley felt her second criteria, collaboration, was met by the use of manipulatives, which allowed for multiple solution pathways: "I think if it used manipulatives, it would definitely foster collaboration since there are so many different ways they can compare and contrast what they did." Kiley also felt her next criteria, higher order thinking, was met "due to the openness of the problem and the ability to be as creative as you want." Since this task allows students to be creative, Kiley believed her final criteria, engaging all learners, was also met.

Hilary's Task. As Kiley began to examine Hilary's task (Figure 3), she aligned it to experiences from her mathematics methods course with tasks that she referred to as "which one doesn't belong." She then began identifying aspects for each function that was the same. For example, she mentioned the y-intercepts were the same. In thinking about her first criteria of a worthwhile task, meets curriculum expectations, Kiley aligned the task to concepts of equivalent fractions and functions.

Collaboration, the second criterion Kiley chose for a worthwhile task, is, according to Kiley, met as students "think to themselves first and then discuss with their group mates" what they notice about the functions. However, Kiley felt her third criterion, higher order thinking, was not met. Her reasoning for saying no was "there's one targeted answer and the way to do it is kind of procedural." Kiley believed that this task met her final criteria, engaging all students, because the strategy of "find the odd man out, I feel like its engaging" for students to start thinking of what they know.

Summary of findings related to Kiley

Initially, Kiley expressed her understanding of problems as exercises to be solved. Her personal educational experiences led her to prefer the plug-and-chug equations. However, her college coursework evolved her understanding of problems. She recognized problems as creating space for learning and thinking. She no longer felt problems were just about finding a solution. She began to see value in problems that encouraged multiple approaches and interpretations. Her understanding of problem solving involves a process of interpreting and decoding the task to solve. The process includes individual and collaborative elements. She came to highly value collaboration in problem solving. For Kiley, collaboration allows different perspectives and prior knowledge enrich the learning space.

At the beginning of the study, Kiley identified four criteria for a worthwhile task. The first criterion is that a task should meet curriculum expectations but allows for flexibility and exploration. This connects to her understanding of problems and problem solving by using prior knowledge to find an entry point to explore the new situation of the task. Kiley's second criterion focuses on collaborative potential of a task. Kiley felt the task should encourage peer collaboration and discussion. Kiley began to connect collaboration to both problems and problem solving. Through collaboration, a person can hear of other people's understandings and see new perspectives to understanding the task.

The third criterion was incorporating higher order thinking. Problem solving, according to Kiley, requires a person to actively think about the problem throughout the whole process of reading, interpreting, decoding, and answering the task. Kiley's final criterion, engaging all students, makes the task interesting to the students and accessible to all students, with potential for differentiation and accommodations.

Kiley's understanding of problems, problem solving, and her criteria can be seen through her analysis of tasks and her assignment choice. The task that Kiley brought, Markup & Discount 1 (Figure 1), encourages students to actively think of numbers for a discount scenario. Kiley believed the task connects to the curriculum and allows for higher order thinking through the flexibility of numbers. Collaboration with peers allows for multiple prior knowledge and experiences to be used to interpret and answer the task.

Kiley's assignment meets all her understanding by using multiple elements in the task to connect to curriculum and allow for creative thinking on decoding and interpreting the task. Kiley believed collaboration allows for different entry points to be used to solve the task. Visual aspects of the task engage different students through various learning styles. The task has multiple questions to align with understanding of the content. Each question of the assignment highlights different aspects of 3D shapes. This aligns with higher order thinking for the learner to critically think about their understanding and apply the appropriate knowledge to the situation.

Kiley's experiences reflect a growing understanding of problems and problem solving. She emphasizes the importance of creating a supportive and engaging learning environment where students can actively participate, collaborate, and develop a deeper understanding of mathematical concepts.

Ashley

Ashley thought from a young age that she would be a teacher. She "realized that since I was like seven years old and could write on a whiteboard that I was, like, 'I'm going to be a teacher someday'." However, she did not immediately seek a career in education. As a freshman in college, salary served as the largest deterrent for entering education. Instead, she initially chose to pursue an engineering degree because of the potential income range: "engineering made more money." However, she realized she wanted to be in a classroom with students to invoke a love for math. She stated, "I just want to teach, I just want to be with the kids, and I don't know, show I love math." She emphasized, "I talked a lot and I was like, none of the engineering students are chatty like this." This realization helped guide her to mathematics education for her major.

Religion, family, and community played a role in her decision to become a teacher as well. Ashley described her large family as having love and support for each other. Her mother was a particularly important influence in her decision to change her major to education. Her mother was a teacher and her biggest role model, "a big important piece." Ashley said, "looking at her [mom] as like, she's lived the most fulfilling life." Through prayer and discussions with the family, she made the final decision to become a secondary mathematics educator.

Interview One

Ashley was initially challenged when considering the difference between a problem and an equation. When asked what makes a good mathematical problem, her initial response was to connect problems with equations to be solved. Ashley described equations by stating "you don't expand on that topic per se, you just find what works and get the job done." After the initial response, she stated that a problem is something more than an equation. She started by stating, "first what came to mind was like, just an equation like that's a problem that you need to solve." Then she stated, "a problem is kind of open-ended…something they can work through." It is bigger than just a procedure and "more applicable to life." There is a process to think and work through.

Her understanding of problems developed from her personal experiences in classes. Her previous mathematics classes focused on “the equation portion.” There was no collaborative aspect when growing up outside of asking a peer for help on solving the equation that was assigned. She mentioned that she was one of the students who would ask a peer to describe the process they took to get the answer because she did not wish to just know the answer. She would say be like, “don’t tell me the answer, tell me how to get to the answer.” In college, she learned that a learning experience is more than just solving. It is about a thinking environment. To have a thinking environment, she connected the need for a problem to be open-ended, require a thinking process, and have real-world connections.

For interview one, Ashley described her understanding of problem solving through a description of the environment: “I have like this image in my head of just students in like groups of three or something in a classroom, and they’ve got a big poster board or whiteboard or something to write on and they’re talking and they’re interacting.” She believed the environment of the classroom is collaborative, “working in small groups to have the freedom to try different methods and approaches”, so that “they can just interact and work through together and not have to worry about the pressure of like getting it right or wrong.” Ashley stated that “in the problem-solving process, it’s finding that right answer and using your mistakes to make a masterpiece at the end.”

Ashley used the term “mental discomfort” to discuss how one knows they are engaged in problem solving. She believed, “when you feel that mental discomfort and that little tug that you’re like, wait a minute, I don’t know exactly what’s happening. You’re kind of forced to sit back and look at the situation as a whole and then find little pieces that you want to pull out and like really analyze." Therefore, "that mental discomfort kind of clues me in that I'm problem solving right now."

Ashley had three criteria for a problem-solving task. Criteria one is that the task must address the goal of the lesson: "the lesson should probably be involved, that is a handy thing". The second criterion is connecting to other topics. Ashley said connecting to other topics means to "highlight certain points that'll be carried on…really highlighting those bigger topics and making sure those are prevalent in the task at hand." She continued to add that "if you can't build on it and expand on it, it was like a lot of time spent for nothing." The final criterion was hands on. She stated that "I think just building such a fun bright, peppy hands-on kind of classroom, students are going to focus better and really get something out of it when they're touching and engaging rather than just sitting and looking." Ashley's definition of a hands-on approach also incorporated collaboration as an aspect of problem solving: "the hands-on part is kind of working with other students using manipulatives, writing on a board, anything that kind of helps your brain filter all of its crazy thoughts into writing and something that can be conferred with other people."

Ashley's criteria were influenced heavily by her own personal experiences and her observations of her mother's teaching practices. The personal need to make connections of the topics lead to her need to "keep the knowledge I previously had and then expand upon it." Her mother integrated collaborative hands-on approaches in her classes. Ashley stated that her mother would encourage people to move and use manipulatives, so "they have hands on activity to keep them engaged."

Task Analysis One

Task one. When Ashley was given task one, Driving Distance, she immediately commented "all the sixes are crazy, kind of fun." This task brought back memories of her middle school days with "solving these back in 8^{th} grade." She mentioned that "it's real-world problem, which is nice." She also appreciated the structure of the sentences of the task, "broke it up into various pieces, different parts."

Ashley's first criteria of a worthwhile task was that it addressed the goals of the lesson. When asked if this task aligned with her first criteria, she went back to her middle school days to "remember that we did like distance equals rate times time." She remembered that this concept was "a huge lesson for me back in the day." She ended her thoughts with "I'd say the lesson is well incorporated in this task."

Criteria two was connecting to other topics. She connected the distance formula to her physics class and her current math classes: "I definitely still use the distinct distance equals rate times time, even if it's a bit more done up now." Ashley's final criteria for a worthwhile task was that it allowed students to be hands-on. To address this criterion, Ashley described the classroom environment that students would be in when solving this task. For Ashley, the environment involved "standing in groups of three or so with big sticky notes." By using large writing surfaces, Ashley visualized students "just working in groups to talk through it and everyone kind of applies what they know and what they see."

Task Two. For task two, Ashley immediately stated "I like the five possible answers." She expanded to connect her liking of the task to the mathematics methods course; "it's like there are multiple ways to teach and the importance of fostering student learning." She immediately began to analyze the task using her chosen criteria without being prompted. For her first criteria, incorporating lesson goals, she said that "perimeter and area has always been a pretty big lesson."

Ashley felt connecting to other topics, criteria two, was met because she saw connections to trigonometry and triangles. She stated that "if a student really learns this [perimeter and area] now they can definitely further that knowledge and also apply that knowledge with later concepts." Ashley's final criteria was that the task allowed students to take on a hands-on approach. She believed this criterion could be met if students use manipulatives and gave the example of using square inch tiles. Students "can definitely solve for the five different values by moving those [inch tiles] around, manipulating them."

Task Three. For Ashley, she honed into the structure of task three like she did in task one. Since task three asks two questions, she said "I do like the presentation of it" in terms that there was a visual and have multiple questions within the task. She enjoyed how the task mentioned adding ice cubes because the "addition of the ice cube is definitely real world there."

She felt this task met her first criteria, incorporating lesson goals, because "this kind of builds off of perimeter and area, then volume." Like task one, Ashley initially aligned her second criteria, connecting topics, to physics. "I don't know if I use volume a ton. I think we did a little bit with it in physics." However, she also described the connection "back to perimeter and area and those easier or different algebraic tasks."

Ashley decided her final criteria, allowing students to use a hands-on approach, was not met. She felt "like this is one of ones where it's easier to solve when you just have the equation." She did think that the task could be altered to "make it hands on because I'm all about the fun." However, when asked how she might alter to meet her criteria, she stated that it may be difficult because “you’re going to have that hands-on demonstration or something, would have to measure out all the dimensions.” Though the hands-on aspect would be difficult, she did believe this task could be collaborative by allowing students to “build off of each other’s knowledge and use peers to help them build confidence.”

Task Four. After reading the task, Locker Likelihood (Appendix G), Ashley reflected on her personal experiences of tasks that required justification of thinking, “these always terrified me as a student.” Yet, she understands that “if you build the right classroom and the right encouragement”, students would be encouraged to justify their answers by explain to their peers how they got their answer.

Ashley felt her first criteria, incorporating the goals of the lesson, was met. She mentioned the math concept of probability, since the task’s mathematical concept is probability.

For her criteria, connecting to other topics, she highlighted that this task is real-world that “could be applied later on in life and learning.” Though she mentioned a future life application, she did think “probability is usually its own lesson and not really taken further.” She did not believe this specific task was hands-on, criteria three. Ashley could not envision how the task would allow students to use manipulative she usually associated with probability, “like with the cards and with the dice, those are definitely manipulatives you can use for probability.” Thus, she was unable to determine hand-on approaches students might take.

Interview Two

I began interview two reminding Ashley of the definitions of problem and problem solving she had developed in interview one. Ashley’s definition of problems was open-ended that requires a process to think through and is more than just a procedure. Ashley chose to add to her definition of problems by stating that that a problem can have multiple pathways or multiple solutions, which "keeps it open and it gives the kids like, a lot of avenues to thinking and it's not just one set thing." Ashley believed that having multiple pathways can help eliminate the notion that mathematics was "the mimicking and just like memorize the process and then dump it out later." This addition to her definition was influenced by readings and experiences in the mathematics methods course. She described how *Building Thinking Classrooms in Mathematics* (Liljedahl, 2021) discusses how mimicking and just using a procedure does not mean someone is learning. Having problems that use multiple pathways allows a learner to identify those pathways and leads to learning. Ashley said, "Giving them a problem where there's so many different ways to get an answer, they can kind of sit and work together and think through it or sit on their own to work through it." Ashley described the easiness of reading the book and examples given within the book.

Ashley did not change her understanding of problem solving during interview two. She stated that "I was happy with that one" when I restated her definition from interview one. She continued, aligning this definition to the thinking she used to choose the task that she brought to interview two: "I liked what I said. I think that really fits this problem, works for it. So, I'm gonna stay there."

Ashley mentioned that she had engaged in problem solving within her calculus III class. During the calculus III class, Ashley had the opportunity to work in a collaborative environment with her peers and to "go up to the board and like write things out and like the process that we took to get there." The professor also required several of her peers to display their process on the board so students could see the different methods to get to a solution. Reflecting on this experience, Ashley realized "that's what we've been talking about in [the mathematics methods class].” Thus, the experience of solving problems collaboratively supported and reinforced this aspect of her initial definition and her confidence in it.

When asked if her criteria have changed (incorporating lesson goals, connections to other topics, and hands-on approaches), Ashley stated “I don’t think my criterion has changed.” Her initial understanding involved a collaborative environment to have the freedom to try different approaches for seeking a solution. She focused her attention on the second criteria for a problem she had posed, that the problem needed to make connects to other aspects of student knowledge. She reiterated that it is important for learning to be connected. She stated that in her experience with previous mathematics content classes where she was just given a worksheet to complete, she would like to know “how do these apply? How do they work together? I’m always a little frustrated.” This frustration led to her understanding of there is “kind of applicability of it like being able to apply it to real life for future lessons or previous lessons, even could be the why.” This emphasis was influenced by recent tutoring experiences Ashley had. Ashley tutored a high school student between the first and second interview. She mentioned wanting to explain the why behind to the math topic to the student she was tutoring.

Ashley’s Task. Ashley chose her task (Figure 2) because she was drawn to the task by the unit fraction and the connection to “everything here in college.” She liked the fact that “fractions could be applied later.” When solving this task for her own enjoyment, she realized “there wasn’t a specific way to find 5/8.” She ended her reasoning for picking this task with “I liked how it was not necessarily complicated, but there’s a lot of thinking in developing.”

Ashley aligned this task with her first criteria, incorporating lesson goals, by describing where she would place this task within a lesson: “I thought this was kind of a fun activity to do after you learn about fractions.” The second criteria, connections to other topics, was met starting with vocabulary of “unit fraction.” Ashley said, “I’ve definitely seen it pop up a lot in physics and then calculus II.” She also tied fractions into the real-world through the “use of them in recipes, you do them in analysis, and all sorts of sports stats.” For Ashley’s final criteria, allows students to engage in hands-on approaches, she described the environment of the classroom for group setting and “have them all stand up and get on whiteboard.” She mentioned the use of manipulatives like “fraction strips or magnets or there’s a pie chart.”

Ashley was asked if the criteria for this research study were met with this task. Ashly felt this task allows for multiple solitons “because you can write 5/8 in a lot of different ways.” Since there are multiple solutions, Ashley believed there are multiple pathways: “As long as they use the unit fraction, they can do whatever the heck they want to get to an end goal.”

Ashly had difficulty in determining of the task incorporated multiple representations. She mentioned “it doesn’t explicitly say that, but I guess if we wanted to think a little deeper on it, it could.” She made connections between fractions and percentages and decimals. Ashley did connect the multiple representation criteria to the manipulatives like the fraction strips. Ashley felt structuring this task in a collaborative environment would allow it to meet the criterion of engaging students in mathematical communication: “I think when they’re all standing up and working together, they’re gonna have to talk to each other. They’re gonna have to show what they’re thinking.”

Interview Three

For interview three, Ashley did not change or add to her understanding of problems. She stated, “I honestly kind of like what I said, I’m not gonna lie. I think I am gonna stick with it.”

She also stated that though the mathematics methods course was ending, she understands her thinking can change if she is exposed to more opportunities for different situations.

However, when discussing problem solving, Ashley added emphasis to the need for collaboration, which she initially included under her criteria for a hands-on approach. She also integrated collaboration and her original idea of the students being allowed the freedom to make mistakes. She stated: “I like the collaborative piece and then like the no pressure. I think it’s really great to a learning environment to be able to just kind of think things through instead of getting caught up with right or wrong.” Ashley reflected on her courses and stated that she believed she has not engaged in problem solving recently because “teachers have been cramming to get things done.” She stated, “in my calc class, we had some like discussion going on, which was nice and collaborative. But I don’t know if it checked every box, per se.”

The final interview did not bring change to Ashley’s criteria for a problem-solving task. She stated, “these are my criteria. Yes, from day one. I was sold on these guys.” When asked if she would add or change any, she mentioned that she likes “to keep it short and simple three criteria.”

Ashley’s Assignment. Ashley chose this assignment (Figure 5) because of the use of candy bars to pique the interest of students. She mentioned a personal experience in her elementary science class that used Oreos to visualize the phases of the moon to make it “tangible”. She related tangibility to her criteria of allowing hands-on approaches. Using physical tools and candy bars allows the student to “see how volume applies to a lot of different things in your life.” Ashley mentioned collaborative aspects of group discussion concerning the measurement of the candy bars and how each group could get a slightly different estimation per group. Ashley’s criterion of incorporating lesson goals was met through addressing solving or volume. Additionally, Ashley felt this task connects to other topics, her second criteria, by using specific items of 3D shapes.

Kiley's Task. When Ashley analyzed Kiley's task (Figure 1), she stated that this task was "super approachable." Ashley felt her first criteria of incorporating lesson goals was met with the integration of "decimals and rounding, and percentage." For the second criteria, connections to other topics, she believed this task connected topics by "bringing in the money aspect." She added she "liked the money and how the decimals can connect to the percentages like it's in the real world." Her third and final criterion was that the task allowed for hands-on approaches. Ashley gave "a middle thumb" for this criterion because she believed this task could be written on vertical whiteboards, but "I guess it's just writing again." Her understanding of hands on is "manipulatives and something you can physically like hold and turn it around."

Hilary's Task. When presented with Hilary's task (Figure 3), Ashley immediately said "how fun, it's like a little spy." She followed that statement with "I found the odd man out, so I feel very intelligent right now." Ashley agreed that criteria one, incorporates lesson goals, was met using "fractions, some equations making, some identifying numbers and coefficients." Her second criteria, connecting to other topics, was met by "transferring over to a graph…and looking at the different graphs" and extend other concepts involving different times of algebraic functions. Ashley did not think her final criteria, allowing hands-on approaches, was met because she did not think it allowed for students to use manipulatives.

Summary of findings related to Ashley

Ashley first connected problems with equations to be solved. Yet, during the first interview, her understanding evolved to be open-ended to allow for multiple pathways and solutions. Problem solving focused on the learning environment being a space of active thinking and collaborative in nature. The learning environment creates an opportunity for a task to provide mental discomfort for the learner to dive deeper into analyzing the task.

Ashley's held true to her original three criteria through all her interviews. First, the task should align to the mathematical goal of the lesson. Ashley insisted that the open-ended aspect of problems still must connect to mathematical goals of the lesson. Secondly, Ashley felt the task needs to have connections to other concepts, whether they were mathematical, a different educational context, or a real-world situation. The final criterion focuses on a hands-on element for the task. Although Ashley sometimes indicated that this element could involve manipulatives, group work, or other active learning strategies, she sometimes decided a task did not meet this criterion based on a lack of manipulatives only.

Ashley's understanding of problems, problem solving, and worthwhile tasks was demonstrated through her analysis of tasks and assignments. Her task, Ancient Egyptians and Fractions, aligned with her understanding of problems by being open-ended and have multiple pathways and solutions to solve. By being open-ended and having multiple pathways, the task allows for collaboration among peers to have freedom to explore different methods to solve problems. Ashley's assignment connects to real-world connections using real-life materials of candy bars. She felt conversations of measurements and outcomes allow the learning environment to be a safe place to explore.

Ashley demonstrated a strong commitment to an engaging learning environment with freedom to try different strategies for learning. Her evolving understanding focused on creativity within the learning experience for learners to work through their mental discomfort by connecting mathematical content, personal understanding, group prior knowledge, and hands-on elements to the task. Thus, despite not explicitly changing her criteria for problem solving, her understanding of problems and problem solving evolved.

Hilary

Hilary stated she had always enjoyed doing math: "I just like doing math because I've always wanted to be a teacher." Math gave her peace of mind because "math was always simple because it's just the same all the time." She found peace in that, because she described herself as "a type A person so everything when it like lines up perfectly makes me happy." She liked the routine of patterns and understanding of numbers. Hilary states she was influenced by a high school teacher who gave her the encouragement and support she needed when she was struggling with concepts. She described her teacher's style as a "natural way" that made her feel as if "I already knew what we were learning, this concept like she made it very natural." She described this same teacher as "Google" in the way that teacher "could just spit out an answer."

This teacher also played a support role outside of the classroom during a tough personal time for Hilary. This teacher "cared beyond just in the classroom." Hilary described her observations of her teachers as "they provide curriculum, but they also provide emotional support. And I really want to do that too." Therefore, Hilary was motivated to become an educator by the desire to be that type of support and role model for other students. Her love of mathematics, combined with her admiration for these role models led her to make the decision to be a secondary mathematics educator.

Interview One

Hilary stated, "when I see a problem, it's something that has to be solved." Hilary described "plug-and-chug equations" as "I can just plug it in there to try to cut down on option." However, more than something that must be solved, she expanded to say, its "an obstacle that you to have to tackle, but your solution can further explain like why there is an obstacle in the first place." When approaching the obstacle to be solved, Hilary acknowledged the need for past knowledge to navigate to a solution: "When I think of a problem, I think of something that I have to use like past knowledge to navigate and get through." She tied this understanding to her Pre-Calculus class, "when you're presented with the problem, you have to apply what you know and then use the new stuff that's on there to contribute to which you already know."

Hilary connected her understanding of problem solving with her understanding of problems: "When you're presented with a problem, in order to solve it, you have to take into consideration what you know and then the new material you've learned." Through this combination of previous knowledge and new information, Hilary says, "the two (previous knowledge and new information) create a new method or way of navigating how to get a solution." Hilary mentioned that problem solving requires a "level of uncomfort(ableness) to learn something." This uncomfortableness occurs "when you're learning to be able to apply new material" to new situations.

Hilary quickly developed her understanding of problem solving through her mathematics methods course. She described how, through class discussions of the 5E model of lessons, she came to align the explore portion of the lesson with a problem-solving task. She explained that during the explore portion, students can "expand one student's way of thinking versus another and maybe they can discover that way of thinking is easier, or it makes more sense."

Hillary's understanding of problem solving was also influenced by problem solving experiences in her Precalculus class. She said, "I have to take what I already know and plug it in and see how it were to work out." This process gives Hilary a feeling of satisfaction: "once you're able to problem solve with new materials, it's like, a level of satisfaction…of learning something new and being able to apply it."

Hilary described several criteria for a problem-solving task. The first criterion she described was the use of previous knowledge: "I like using previous knowledge. That has to be the criteria of it, like there has to be some form of knowledge that they can apply." Once previous knowledge has been identified, Hilary mentioned there must also be a way for the learner to communicate their thinking: "So that way, when I come in, I'm able to see where they went wrong, or where they went right." She describes this criterion by means of working out a problem and answers questions. The third criterion that Hilary mentioned was that the problem should be accessible, which, for Hilary, meant that the learner should be able to try to solve the problem without teacher assistance: "I would want them to try to figure it out on their own first, by themselves first." Later in the interview, she revised this criterion to include a collaborative element by allowing for a group of peers to try to solve without teacher assistance. She described learners "can use their peers and stuff to understand, to bounce off ideas and thinking".

Hilary's next criterion focused on the teacher's ability to facilitate and scaffold learning, The teacher's role "would definitely be to kind of introduce them and steer them in the right direction." The final criterion Hilary described was the use of discovery learning. She connected this criterion to the course textbook (Lijedahl, 2021). She mentioned that her mathematics methods course focused on a "thinking classroom and if you were to try to control a way a person thinks that they're not going to retain anything that you're talking about." She believes "that it's important for the teacher to initiate and then let the learners have the freedom to explore." Her final criterion of discovery connects to the role of the teacher and establishing a space for students to explore.

Task Analysis One

Task One. When given task one, Driving Distance, Hilary immediately started the process of solving the task. She tied this task to her personal experiences of driving to Dallas by saying, “I was like sixty miles per hour driving in Texas is kind of crazy, like people down there drive really fast. So, she would probably get a thumbs down from a lot of drivers.” Thus, Hilary made a cultural connection to the problem.

Before Hilary discussed her criteria with the task, she reflected that a learner would first “have to figure out what the task wants you do” to start determining what the obstacle within the problem was there. Hilary’s first criteria involved use of previous knowledge. She believes this task meets her first criteria “because that information of rate and speed” is previous knowledge. The second criterion was the ability for students to communicate their thinking. She stated this criterion was met because one would “visually see where they’re putting these numbers and what they’re doing to them, to keep it organized.”

Hilary’s third criterion was that the task should be accessible to students without teacher assistance. In analyzing this task, she referred to the collaborative aspect of working with peers to clarify misunderstanding of the problem. Hilary’s fourth criterion was that the teacher could facilitate and scaffold the task. Hilary again referred to students working in groups to justify how this task met criteria four. She said “just like in the groups, when they’re talking about it, and I hear something along the lines of like let’s add the two speeds together and then just add the time to the speed, then I can sit down say well. ‘why do you think that is?’.” Hilary’s final criterion was the use of discovery learning. Hilary believed this criterion was met because “they’re able to identify there’s a speed and there’s also a time and then what all you can do when you’re given the speed and time.” This explanation contradicts her original understanding of discovery.

Task Two. Hilary read task two and said, “That’s a lot.” From her initial thought, she believed “this one would kind of be definitely a good one for a group to figure out.” Though she felt the task was challenging, she acknowledged that her first criterion, using previous knowledge, was met because students would need to use “what you know about the perimeter to find the area.” At this point, Hilary expressed this task would be a “big challenge with five possible values of the area.” She said she “definitely could use a group” to solve this task.

Hilary felt her second criteria, communicating student thinking, was also met. She described how she would visualize her future classroom “in groups of four working together with four individual pieces of paper, but then there’s also one in the middle to show how they came to the conclusion.”

In evaluating her third criterion, accessible without teacher assistance, Hilary described her understand that the placement of the task in the lesson would influence the accessibility of the task, stating: “I think that this task definitely would just be like an introduction to the lesson.” She would want “to see where they would all start with a problem like this.”

Hilary stated that the teacher facilitation, criterion four, would primarily be met by “making sure that there’s a conversation happening in their groups.” Hilary was unable to determine if her final criterion, discovery, was met, stating it was “a maybe”. She felt she needed to spend significant time considering this criterion “because I mean what was throwing me for a loop is like the five possible values for x.” She believed that “one value would be enough for them to feel that satisfaction” of being able to solve the problem. Though Hilary stated she would need more time to consider her final criterion, her explanation contradicts her understanding of discovery by focusing on the number of solutions, instead of the exploration factor of the task.

Task Three. Like task two, Hilary's initial comment about task three is "this is a lot." She thought this task "would be another group one" for students to find a solution. She thought this task would be most appropriately used if it were to "apply to a test or towards the end of the unit."

Yet, she determined that her first criteria, previous knowledge, was met by recalling "the shape of the cylinder, discusses volume." However, for her second criterion, communicating thinking, Hilary envisioned how she would have solved the task. She determined this criterion was met because she "would need to see a picture just to have them get their numbers organized" to show work in solving. Hilary believed her third criterion, the task is accessible without teacher assistance, was also met by having the students be in group "because there's a lot of information when you first look at it, to kind of take it one by one." Thus, she equated accessibility with collaboration.

For teacher facilitation, criteria four, Hilary considered the amount of information the teacher could provide, but was challenged in considering how a teacher might scaffold the task, stating there was "not a lot of information you can give to them without giving them a direct route on how they should go about it." She imagined that for this task "students asked you a question, and you respond with a question." Therefore, she believed the task only partly met the criterion. For Hilary's final criteria, discovery learning, she stated, "I think a lot of discovery" is here because of connecting all the pieces of the problem being applied. Thus, Hilary made a connection between discovery learning and mathematical connections.

Task Four. Hilary's stated "I do like the last question" when she turned her attention to the fourth and final task. Her reason for liking that question was how it met her first criteria, the use of previous knowledge to solve the problem. She immediately began to consider her first criteria, previous knowledge, to understand the terms used in the task. For criteria two, communicating thinking, she reflected on her personal experience of solving probability problems in school, where she would “tap some buttons on my calculator and they’d [teachers] be like, show your work.” She did believe criteria two would be met by the students showing the “math part” of solving, but just using the calculator to solve the task contradicts her explanation of discovery.

For criteria three, accessible without teacher assistance, Hilary once again believed it was met by students working in groups. She described a classroom setting where different groups solve different parts of the task, “then all together as a group they have to decide which scenario would be more fair.” Since the group setting would be prominent, she stated teacher facilitation, criteria four, would be met by focusing on classroom management aspect: “I don’t think there’s a lot of facilitating, rather than just like the regular inside voices, please.” Hilary also decided that the final criterion, discovery learning, was met because the student “have to reason why their scenario is more fair, they had to pick one of those and argue why.”

Interview Two

During interview two, Hilary added to her understanding of problems by stating “there could be, like, multiple solutions” but it depends on the problem. The problem determines what solutions work: “If there are only two solutions, they’re the only ones that could work.” The largest influence for this change in understanding is personal experiences in engaging with problems during the mathematics methods course. During the course, Hilary explored problems which allowed students to use different methods to solve problems.

During interview two, Hilary was asked if her understanding of problem solving had changed. She stated no, and did not add understanding to her definition from the first interview. She mentioned again that she had engaged in problem solving within her Precalculus class by connecting her prior knowledge of triangles into new situations with Heron's formula. With the merging of prior knowledge and new learning, Hilary mentioned her learning in the mathematics methods course with her understanding of constructivism and the building of knowledge. She connected how she writes the lesson for the mathematics method course is built to use the notion of constructivism.

For interview two, Hilary stated her criteria had not changed. Her criteria were use of previous knowledge, communicating thinking, accessible without teacher assistance, teacher facilitation, and discovery learning. When asked if she could add or edit anything, Hilary stated "I think I am good."

Hilary's Task. For Hilary's task (Figure 3), she imagined it would be placed in the engage section of the lesson "just to get it going." She envisioned a class discussion by "first starting off with, what do you notice and what do you wonder." Hilary aligned this task with her first criteria, previous knowledge, by use of equations and functions, "so they're able to recognize the x and know that means if you plug something into x, you're get an output." The second criterion, communicating thinking, would be met through "the discussion portion for the viewer to state 'what do you notice'." Hilary felt the next criterion, accessible without teacher assistance, was met by the student simplifying the individual problems of the task. Hilary believed teacher facilitation, her fourth criterion, was met by the teacher asking, "questions like 'what is something that they all have in common?' because that is the main question to think about." Hilary also felt her final criteria, discovery learning, was met because she could imagine that students “discovered that 36 over 18 and 16 over 8 both equal 2, to be able to see multiple ways of this problem.” Her explanation for the final criterion contradicts her understanding for discovery based on her task being met by solving the equivalent fractions.

Hilary was asked if her task met the criteria for this research study. Hilary believed her task did not meet the criteria for multiple solutions. She stated that “there is only one solution to this task.” However, she did believe this task allowed for multiple pathways to the solution, another criteria of this research study. Hilary stated that, depending on where the student decides to start the task, the students could arrive at the answer through different avenues. Hilary also felt her task incorporated multiple representations. She described that the equations “can also visually show a graph.” Finally, she felt mathematical communication was present by talking “about these equations, you’re able to use vocabulary…like shifts of slope and what that all means.”

Interview Three

During interview three, Hilary did not change her understanding of problems, which was to understand why an obstacle was there. When asked if she would add, alter, or change her understanding, Hilary said, “Nope.” She did not choose to expand on her answer. Hilary also said that her understanding of problem solving had not changed. Problem solving to Hilary was a combination of previous knowledge and new information that brings a level of discomfort. She discussed engaging in problem solving during the week before her final in her Precalculus class. Throughout the review for the final, Hilary said, “I was able to visually see how we were just applying new information to the original stuff.” She brought the mental discomfort back up when she stated, “uncomfortableness is when you get frustrated when it doesn’t come right away, and then you have to work on it.”

Hilary initially said that she would not change her criterion for a problem. However, she then added that teacher demonstration could be added if necessary. She stated that a "demonstration can be added in certain parts." She described an example of when a teacher could provide a worked-out example for students to help connect prior knowledge to new information. She believes that "when you're instructing them, it can be just flat out like saying like, this is what's happening, but it still would be applying to your previous knowledge." This understanding contradicts her criterion of discovery.

Hilary's Assignment. Hilary stated her assignment (Figure 6) aligned with her first criteria of building on previous knowledge by using her prior understanding of trigonometric functions to "know that cosine of theta is the adjacent over the hypotenuse." She explained how she was able to communicate her thinking by using the given information to "then work it all out." For criterion three, accessible without teacher assistance, Hilary referred to times "we obviously have a problem like this before in class." Hilary felt teacher facilitation, criteria four, was met by class examples and the instructions used to solve the problem. Hilary stated that final criteria, discovery by the student, was met by "it could be like the algebra to get sine by itself and then discover to plug in the cosine."

Kiley's Task. When presented with Kiley's task (Figure 1), Hilary immediately recognized this type of task from the mathematics methods course. She expressed her experience with this type of task as "a doozy". She reminisced that the class "was all stuck" and needed "a lot of clarification." One of the struggles for Hilary was not a recognizing a direct procedure she could use to solve, "instead of saying like one plus one equals two, you have to think of what plus one equals this." When analyzing her first criterion, previous knowledge, Hilary stated her criteria was met because the task had "dividing and multiplying and stuff like that and would get a little adding into the mix too." The second criterion Hilary chose was mathematical communication. Hilary believed this criterion was met because students "would definitely need a scrap piece of paper for this, to mark up each scenario they come up with." Hilary also determined that the next criteria, accessible without teacher assistance, was met because students would be engaging in "a lot of plug and chug, because that is always the first attempt with students" allowing students to have potential starting place.

After students attempted the task without teacher assistance, Hilary believes "if a teacher were to facilitate this, they may have to drop a hint." Therefore, the fourth criteria, teacher facilitation, would be met by the teacher giving hints for next steps, which connects to her additional understanding of a teacher needed to demonstrate. The final criterion Hilary asserted was discovery learning. She felt this criterion was also met because "the strategy for problems like these is all unique depending on what student is solving it." She ended her explanation by stating, "there could be multiple different ways to solve this problem."

Ashley's Task. Hilary first noticed that the example in Ashley's task (Figure 2) involved 1/3s but then the first question changed to 1/8. She started to reason through equivalent fractions and giving examples "to look at how 2/8 equals ¼." As she was reasoning through the task, she mentioned "there's many ways that I would go, but I also can see people going different routes."

Hilary's first criterion was the use of previous knowledge. She believed this criterion was met by the integration of the example given, which illustrates the connection to an understanding of addition. The second criterion was mathematical communication, which Hilary felt was also met because students would need "to write out their work and new information" to understand the process to solve. Hilary noted that students can try different options for fractions, which she believes meant that the task met the third criterion, accessible without teacher assistance. For the next criterion, teacher facilitation, Hilary believed it was met by stating that the teacher could "share" additional examples. Hilary believes the final criterion, discovery learning, would also be met because "they [students] can discover the equivalences like 2/8 is equivalent to ¼."

Summary of findings related to Hilary

Hilary's understanding of problems started by being an obstacle that requires previous knowledge and new information. She extended her understanding to the number of solutions for one problem can have more than one if the problem calls for it. Her understanding of problems connects to problem solving, which she said was the process of using previous knowledge and connecting it to new information. Hilary believed there would be a point of uncomfortableness where the previous knowledge and new information connected. This uncomfortableness allows the learner to explore other ways of thinking that could be completing new ways of thinking, helping the problem make more sense, or allowing for easy access to understanding.

Hilary's criteria aligned with her understanding of problems and problem solving. Her first criteria, usage of prior knowledge, requires students to access what they already know about the topic to start the process of working through the discomfort of problem solving. Communicating thinking, the second criterion, allows for observation of the learner's thinking to see the process for solving the task. She contradicted herself for criterion two when evaluating a task by equate this criterion with showing work of solving the task. Her third criterion, student try without teacher assistance, gives the student, either individually or with a group, the opportunity to use different strategies to work through the discomfort of connecting prior knowledge and new information. The teacher is responsible for facilitation, the fourth criterion Hilary chose. She believed tasks should allow teachers to provide appropriate scaffolding and guidance for the students, such as providing structures, hints, clarifying questions, or even examples as needed. This could allow for new ways of thinking for the learner. The final criterion was discovery learning. The student needs to discover their own solutions and strategies for solving. As Hilary analyzed tasks, her explanation of discovery focused mainly on just solving the task using procedures.

Hilary’s connections between these criteria and problem solving are evident in her choice of task and assignment. Hilary’s Task, Find the odd man out, emphasized the application of prior knowledge with new information. Students would need to communicate their thinking to find multiple pathways to the solution. Since there are multiple pathways for Hilary’s Task, the students can explore different options to find which strategy works best for them. Teacher facilitation would then be able to be used to help students to clarify as needed.

Hilary’s assignment also connects to her personal experience as a learner working through her problem-solving process. She displayed her connection to prior knowledge and new information through her communication of her thought process of solving the problem. She found new strategies to solve through teacher facilitation of clarifying questions and scaffolding.

Hilary focuses on the expectations of both the learner and the teacher in a learning environment. The learner is expected to access their previous understanding of the mathematical topic to enter the moment of uncomfortableness to apply new information. The teacher’s role is to provide scaffolding and guidance when the uncomfortableness overwhelms the learner. The interaction between the learner, the task, and the teacher allows the learner to discover their own solutions or strategies to support the task.

Collective Understanding

All three participants connected the need for a task to connect to a mathematical content and previous knowledge. Each participant expressed the need for a learner to understand what they already know to apply to a new situation. Kiley specificslly aligned her criteria of aligning to curriculum as the connection to the mathematical content. Ashley's criteria of addressing the goals of a lesson focuses on the intent of the mathematical content. Hilary's connection to mathematical content aligns with the relationship she describes with applying previous mathematics knowledge with new mathematical information.

Collaboration was a large emphasis for all participants all throughout the semester. Kiley specifically mentioned collaboration for her second criteria. During the second interview, she expanded her understanding of problem solving to have an individual and a collaborative aspect. Ashley addressed collaboration when she described her thinking environment by working in groups to have the freedom to try different strategies. When discussing her criteria of hands-on approaches, she included aspects of collaboration among groups and using manipulatives and resources to explore the task. Hilary added a collaboration aspect to her criteria, accessible without teacher assistance. Her initial explanation of her criteria only included the individual elements. However, before the end of the first interview, she changed her understanding to include the collaborative aspects of working with peers to try to solve a task.

Hilary added the option for a teacher to demonstrate example of problems. She connected this to connecting prior knowledge from examples a teacher did in previous classes. This contradicts her criteria for discovery by establish a procedural aspect to the task. When analyzing tasks, she had difficulty determining if her criteria was met or not when the task asked for multiple solutions or did not have a direct procedure.

All participants mentioned how both the mathematics method course and other content courses shaped their understanding. Kiley mentioned how the mathematics method course broadened her understanding concerning higher-order thinking and the learning environment. She mentioned that higher-order thinking involves active thinking through open-ended tasks. The learning environment includes the teacher, the learner, their peers, the task and the general environment. Kiley believed the aspects of the learning environment plays a part in the collaboration of learning. Kiley connected the content course, Differential Equations, to actively thinking about what you know and what you need to work through a strategy to solve. She also mentioned Discrete Math course when reflected on her experience of working through a task in the mathematics methods course. She reflected on how those who had previous taken the Discrete Math course had appropriate strategies to use for the worthwhile task in the methods course.

For Ashley, the mathematics method course aligned to Kiley’s notion of a learning environment. Ashley mentioned how the learning environment establishes a foundation for the collaboration aspects, that is important to Ashley. When mentioning her Calculus course, Ashley focused on the collaboration element as well. She mentioned how the professor allowed students to show their strategy for solving on the board for everyone to see. This gave Ashley the sense that she could learn from her peers to see what strategies work well for her and to try new strategies.

Hilary’s experiences in her Precalculus course represented her struggle for working through mathematical problems. She explained how the problems she had in her Precalculus class connected prior knowledge to new situations. However, she did reflect on how she struggled first with learning the content because she did not understand the procedure.

Relearning throughout the semester gave her the confidence to know how to solve similar problems. When analyzing tasks, Hilary mentioned the mathematics method course and the 5E lessons for that class. She described where tasks would fit in a lesson plan. Like Kiley and Ashley, Hilary mentioned a thinking environment in terms of the teacher is the one to establish the environment for the student to have freedom to explore.

Summary

This chapter presented each participant's understanding of problems, problem solving, and their criteria for a problem-solving task. As the participants attended the mathematics methods course, their experiences mostly deepened their initial understanding. Both mathematics methods course readings and engagement in problem solving within mathematics classes influenced the participants' understanding.

The common themes of their understanding of problems were the need for a solution and required thinking to make sense of the scenario. Kiley's initial understanding of problems surround around learning and the thinking environment by a problem being something to be answered with one or more solutions. Through the semester, she emphasized how the problem is only part of the thinking environment which includes the learner, teachers, and peers and can have multiple pathways to arrive at a solution. Ashley's initial understanding of problems was an open-ended situation to be solved and is connected to a larger context and real-world. Her experiences during the semester brought emphasis on the need for a problem to be open-ended that could have multiple pathways to multiple solutions. Hilary's initial understanding of problems was a situation that needs a solution to make sense of an obstacle. Throughout the semester, Hilary connected that the situation could have multiple solutions to explain the existence of the obstacle.

Participants understanding of problem solving is centralized on two themes: participants felt problems should prompt a level of uncomfortableness for solvers and require solvers to combine prior knowledge and new information. Kiley began the semester thinking of problem solving as a process of reading, decoding, determining appropriate prior knowledge to work through a moment of uncomfortableness to answer in the learner's own way. Her experiences throughout the semester caused a reconsideration of the balance between the individual learner and the collective collaboration of those within the learning environment. Ashley's understanding of problem solving started by describing the learning environment being collaborative with interactive materials to allow freedom to try different strategies to engage prior knowledge in connection with new information. Throughout the semester, she emphasized the collaborative element to allow freedom among those in the learning environment to learn from the uncomfortableness of potential mistakes. Hilary was consistent in her thoughts on problem solving as the point of uncomfortableness between past knowledge and new information.

A teacher chooses a task based on their criteria for a problem-solving task. All three participants had criteria that connected to alignment with lesson/curriculum, collaboration, and student engagement. See Table 6 for summary of all criteria and alignment with each participant, as described above. The next chapter will expand on the themes and connections to literature.

Table 6

Summary of all participants' criteria

Criteria	Kiley	Ashley	Hilary
Align with lesson/curriculum	Yes	Yes	Yes (as part of lesson placement)
Collaboration	Yes, increasingly	Yes (as part of "hands-on")	Yes (as part of accessibility)
Higher order thinking	Yes	No	No
Student engagement	Yes	Yes (as part of "hands-on)	Yes (as part of accessibility)
Connections to (prior) knowledge within/beyond math	No	Yes	Yes
Communicate thinking	No	No	Yes
Accessibility	No	No	Yes
Teacher facilitation	No	No	Yes
Discovery learning	No	No	Yes

Chapter 5: Discussions and Implications

This study aimed to understand how PSMTs construct their meanings of problems and problem solving and how those meanings connect to their criteria for a problem-solving task. This chapter will focus on the themes regarding problems and problem solving, and criteria of a worthwhile task. Themes will be discussed within connection to the research question topic (problem, problem solving, or criteria for a problem-solving task) they correlate to. Implications for practice will be highlighted. Limitations and extensions of this study will also be presented. Finally. Recommendation for supporting PSMTs' understanding of problems and problem solving will be provided. The research questions for this study were:

1. What prior knowledge and understandings do PSMTs have about problems and problem solving as they enter their teacher preparation program?
    a. What criteria do PSMTs use at the beginning of their first methods course for describing a problem-solving task?
2. How does PSMTs' understanding change during their first methods course?
    a. What criteria do PSMTs use during their first methods course for describing a problem-solving task?

In this chapter, I will discuss the themes surrounding problem and problem solving. Throughout this study, the participants did not change their formal definitions of a problem. However, their understandings were deepened and enriched. Next, specific commonalities surrounding criteria for worthwhile tasks will be connected to the participants and research. Discussion of one participant, Hilary, will be outlined based on the contradictions she presented throughout her analysis of tasks and assignments. Implications of the findings will be presented linking to the educational spaces. Limitations to this research study will be presented and described why these limitations are issues. Future research will be laid out to help further the body of literature involving PSMTs and their understanding of problems, problem solving, and worthwhile tasks.

Understanding of Problems

Extant literature defined a problem as a mathematical task that cannot be solved by routines alone. However, researchers have used different characteristics to describe a problem (Gözde, 2020; Felmer & Perdomo-Diaz, 2016; Herbst, 2006; Long et al., 2016; Schoenfeld, 1983). Likewise, the three participants described their understanding of problems in different ways. Common themes between all three participants were (a) problems were something to be solved and (b) problems required a thought process to connect content and scenario. Each participant indicated the need for a solution to a problem. Additionally, for participants, the thinking environment goes beyond just the mathematical scenario of the problem to make sense of a problem.

Solvable

One theme surrounding problems is the need to be solved. This researcher recognizes that there are potential problems in the universe that currently would be classified as unsolvable. However, each participant expressed that a problem is solvable. Kiley aligned problems with “anything that can be solved” but goes beyond just a “plug-and-chug equations”. Ashley equated problems to something more than just a procedure and something you “memorize the process and then dump it out later”. A problem can have one or multiple solutions, but the number of solutions is determined by the problem. A problem establishes a scenario for a student to navigate a path towards a solution (Felmer & Perdomo-Diaz, 2016). The path to the solution can vary based on the scenario and the student actively thinking about potential solutions (Chatterjee et al., 2009; Kuhlthau et al., 2015). Hilary's understanding of a problem was "something that has to be solved" but more than that the obstacle within a scenario.

Thinking and Sense Making

The second theme concerning problems is a problem is something to work through one's thinking to make sense of a scenario. Hilary visualized a problem as an obstacle that the student needs to make sense of. To make sense of a problem, the learning environment comes into play (Herbst, 2006; Wheatley, 1991). Kiley aligned a problem with the thinking environment. She mentioned that the scenario of the mathematical task is only a portion of the problem. The learning environment is an interaction with the student, the teacher, the problem, and the surrounding environment. Ashley aligned to this thinking concerning the learning environment through the connections between the different "avenues of thinking and it's not just one set thing" by using the collective thinking of the setting to "work together and think through it". The relationship between the elements of the learning environment helps construct understanding and sense making (Schunk, 2020; Vygotsky, 1987). The problem's scenario starts the thinking process for a student to reflect on what they know and don't know (Ligozat et al, 2017).

Expanding Understandings of Problems

Throughout the semester during which this study took place, the participants expanded or changed their understanding of a problem in unique, individual ways that reflected their personal experience within the social group of the mathematics education majors enrolled in the methods course, their experiences in mathematics content courses, and their personal reflections. The design of classroom activities and assigned readings in the mathematics method course and personal experience in the mathematics method courses were particularly important influences over the change in participants' understanding, whether in a positive or negative way. Kiley and Ashley were positively impacted by their personal experiences with their courses. Kiley mentioned her connection to the Discrete Math course in how she sees problems and approaches to solve tasks. Ashley expressed how Calculus II used problems in a collaborative setting to work towards a solution. Hilary had a negative impact with her Precalculus class. She struggled with keeping her justification of problems to align with her criteria.

Kiley transitioned from seeing problems as exercises with answers to viewing problems as learning opportunities that promote diverse approaches and collaboration. Ashley initially saw problems as equations to be solved. However, she later embraced problems as open-ended challenges that foster critical thinking, collaboration, and deeper analysis. Hilary first thought of problems as obstacles that require application of prior knowledge. Her understanding changed to exploratory opportunities that could have multiple solutions depending on the content.

Understanding of problem solving

Extant literature described problem solving as the active and collaborative process through the lens of different strategies to hypothesize, take action, and reflect in order to find a solution (Carlson & Bloom, 2005; Collet & Bruder, 2008; Liljedahl, 2021; Pólya, 1957; Mamona-Down & Downs, 2013; Remillard, 1991; Schoenfeld, 2014b). When defining problem solving, the most common theme among participants focused on the uncomfortableness that happens at a point in which prior knowledge and new information meet.

Uncomfortableness

To solve a problem, there is a moment of unknown and mental discomfort. This instance of uncomfortableness triggers a thinking process to begin (Liljedahl, 2021). Hilary described the discomfort as “when you get frustrated when it doesn’t come right away, and then you have to work on it”. Ashley described the point of uncomfortableness by being “forced to sit back and look at the situation as a whole and then find little pieces that you want to pull out and really analyze”. The moment of uncomfortableness can lead to a need for additional information or scaffolding from a more knowledgeable other. This notion connects to Vygotsky’s idea of ZPD (Miller, 2006; Schunk, 2020; Vygotsky, 1998). ZPD is built by the students as they move through zones of known and unknown. When a student moves to a zone of unknown, there is a need for the learning environment to support the student.

Connection of prior knowledge and new information

Problem solving starts with a student understanding what they already know. This recollection of prior knowledge aligns with the beginning of constructing one’s knowledge (Lui et al, 2017). It is important for students to begin with what they already know to support a pathway to an answer. Kiley highlighted the need to think “about what prior knowledge do you have that can help you solve a new problem”. Once a student understands what they already know, they can identify what they do not know to make sense of the problem. Hilary mentioned the need to consider “what you know and the new materials you’ve learned” to start making sense of the problem. A social aspect of the learning environment gives the student more opportunities to help fill the gap of unknown knowledge (Wheatley, 1991; Vygotsky, 1998). Kiley connected to the social aspects through “the diversity of thoughts within your group” can help you arrive at a potential solution. The learning environment allows students to discuss with peers their process for solving the problem and reflect on new knowledge or strategy used to solve (Lappan & Phillips, 1998; NCTM, 2014).

Participants changing views of problem solving

When asked to define or describe problem solving, Kiley and Hilary initially focused on the individual components of problem solving. Kiley mentioned the process of solving by reading, decoding, interpreting. In her view, as a person progresses through the process of solving, ideas are being generated to use prior knowledge to provoke new ideas and solutions. Ashley focused her understanding of problem solving on the learning environment. For her, problem solving happens in an environment that is collaborative, exploratory, and allows for freedom to try strategies. Hilary pinpointed that she viewed the connection between prior knowledge and new information as the point of uncomfortableness. This point for the individual is the opportunity to apply new information using various strategies.

During the second and third interviews, the participants reiterated some of their specific criteria for problem solving but also expanded their definitions to include collaborative elements. They expressed that collaboration allows for the extension of prior knowledge from multiple people to create space for new strategies to try. For each participant, this deeper understanding was influenced by their mathematics methods course and personal experiences in mathematics content classes.

Criteria for a worthwhile task

This study used a rubric (See Appendix F) to choose worthwhile tasks. The rubric included connection to mathematical content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020), high cognitive demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998), problem solving as an art (Son & Lee, 2020), communication (Lappan & Phillips, 1998; Levenson, 2013), use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013), multiple pathways to solution(s) (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013), and more than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013). The participants' criteria focused on three themes: connections to mathematical concepts, collaboration, and student engagement. The participants' theme of connections to mathematical concepts aligns with the researcher's rubric for the criteria of connection to mathematical content. The next theme, collaboration aligned to the researcher's rubric through the communication criterion. The final theme of student engagement connects to multiple criteria of the researcher's rubric: high cognitive demand, problem solving as an art, and use of manipulatives. Though the participants did not have the criteria of multiple pathways to solution(s) or more than one solution, the participants did mention these items during their explanations of their understanding of problems.

Participants Understanding of Worthwhile Tasks

Each participant was asked about their criteria for a worthwhile task. All three participants had criteria that connected mathematical content, a collaborative element, and student engagement.

Connections to mathematical concepts

As Ashley said, "The lesson should probably be involved, that is a handy thing." Lappan and Phillips's (1998) first criteria for a problem-solving task focused on connections to mathematical concepts. This case study supports the primacy of that idea. All three participants identified the need for the problem to have mathematical connections. Kiley expressing the need to meet the curriculum within the mathematical context. Ashley had two criteria that connected with this theme, incorporate the lesson and connect to other topics. Connections lay the groundwork for learning, whether the connection confirms or challenges a topic. Therefore, these participants believed a problem-solving task needs a connection to a mathematical concept. For example, Kiley suggested a task needed to meet curricular expectations. The probelm should begin with a connection to mathematical content whether it be process or content specific. For Kiley, the connection to mathematical content should still allow for the learner to have the flexibility to explore the content in their own unique way. Likewise, Ashley included the criterion that a problem was to incorporate lesson goals. There must be a reason to include the problem within the lesson. This can be content specific or exploratory problem to try new strategies for problem solving. Hilary's criterion related to mathematics content was the use of previous knowledge. Identifying previous knowledge that is useful for the problem gives a necessary foundation for the learner to start the problem-solving process. Hilary's focus on the connection to prior knowledge demonstrated her perceived need for a problem-solving task be placed into a learning environment at the moment a student needs a mathematical connection between previous knowledge and new concepts (Lappan & Philips, 1998). This timing can support the student to identify potential patterns in their mathematical understanding to extend to new ideas.

Collaboration

Social constructivism requires a collaborative social interaction to build on one's knowledge set (Wheatley, 1991; Vygotsky, 1998). It is about interacting with others to compare, contrast, question, and/or verify one's understanding (NCTM, 2014). The learning environment of the teacher, student, peers, and the problem-solving task create a space for collaborative elements. Yet, it does require the initial question posing to begin the interaction. The collaborative element brings a collective thinking to the learning environment.

Collaboration became a stronger emphasis for the participants throughout the semester. Participants believed collaboration allows for discussion among those within the learning environment to expand personal understanding to include understanding of the whole and new ways to apply understanding. The participants described their learning environments start with the initiation of a problem-solving task for the group to discuss their understandings of the mathematical concepts so that individual students can question how the understandings connect to their personal understanding. Kiley believed collaboration builds conversations within those engaged in learning to work together towards a solution. Kiley's criterion of collaboration showcased the importance of diversity of the group's understanding of the problem. Hilary's criterion requires tasks to be accessible without teacher assistance but allows for collective thinking to "bounce ideas off of each other" to determine what strategies are appropriate for solving. The collaborative nature is about creating discourse among peers concerning a problem (NCTM, 2014). It is important to note that Hilary's criterion was originally that a problem should be accessible without teacher assistance. She later included that a group of peers could be used to ensure that a problem was accessible. For Hilary, accessibility without teacher assistance includes working with peers to expand personal understanding. Likewise, Ashley's criterion requiring hands-on experiences equated to using manipulatives in a group or working with peers on erasable surfaces on the problem. By allowing the environment to be fundamental about group work, the learners have the opportunity to work together by using mental understanding and physical or visual elements to solve a problem.

Student Engagement

An authentic learning experience contains an element of inquiry-based exploration (Chatterjee et al, 2009; Kuhlthau et al, 2015) and student-centered learning (Manyukhina & Wyse, 2019; Moses et al., 2019). Student engagement involves the learner being active in their thinking process by being engaged in questioning, analyzing, and exploring strategies to solve (Hufferd-Ackles, 2004; Tekkumri-Kisa et al, 2020). This starts with the learner processing what they understand about the problem in terms of their prior knowledge. Though Hilary was the only participant that explicitly had a criteria connecting prior knowledge, all participants understood the important role prior knowledge plays when solving problem-solving tasks. All participants mentioned this need of activating prior knowledge is part of problem solving.

Once prior knowledge has been established, students activate strategies to use with the problem. The strategies can vary by student. Kiley mentioned multiple times the importance of using one's own choice of strategy or model to try solving. Like Pólya's (1957) and Carlson & Bloom's (2005) phrases for problem solving, the cyclical process requires one to explore know strategies with a problem to see what can work. The problem-solving process gives way to freedom to engage with the problem. Like Ashley's understanding of freedom to try new strategies without the fear of being wrong. Hilary's idea of initiation of a problem, which could come from the teacher or something else in the learning environment, starts the students having the freedom to explore the problem.

While exploring the problem, students can use strategies that include physical and visual supports. Ashley's criterion of hands on was about using manipulatives. She believed that "students are going to focus better and really get something out of it when they're touching and engaging rather than just sitting and looking." Therefore, physical and visual supports give a way for the learner to be engaged with the problem.

Hilary's contradictions

Hilary presented multiple contradictions to her understanding of problems and problem solving. As she analyzed tasks, she stated the task met her criteria, but her justification did not align with her criteria. This was apparent with two of her criteria, communicating thinking and discovery. Multiple times when addressing the criteria of communicating thinking, she would describe the learner as just showing their work or showing "the math part." She mentioned when she analyzed Task 4 (Appendix G) for her criteria of discovery, that the learner could give a "reason why their scenario is more fair…and argue why." This understanding aligned with her criteria of communicating thinking.

When discussing her criteria of discovery, she equated this criterion with multiple solutions or just plugging into the problem to solve. This connects to her desire for teacher demonstrations, that she mentioned in her third interview, and preference of procedures. She mentioned her struggle with problems in both her Precalculus course and the mathematics methods course because of the lack of procedure to solve. During her explanation of her assignment from her Precalculus course (Figure 6), she expressed how she struggled with problems similar to this one earlier in the semester and needed teacher assistance to know the procedure to solve. Kiley's task (Figure 1) gave Hilary pause when analyzing because Hilary's desire for a procedure. For Ashley's task (Figure 2), Hilary mentioned this task could use additional examples given by the teacher.

Implications

Implications of this case study focus on the support needed in the educational realm of the method courses. It is crucial for students to regularly engage with problem-solving tasks to hone their problem-solving skills that are necessary for future educational experiences. The personal experiences whether in the mathematics method course or a mathematics content course, like Calculus, influenced the participants' understandings. PSMTs must first question their understanding of problems and problem solving as a learner before they can explore how their understanding connects to their beliefs as a teacher. As PSMTs engage in problem-solving tasks, it is important to reevaluate one's understandings and beliefs. For example, what a PSMT believes about themselves at the beginning of the first mathematics methods course could change by the end of the third mathematics methods course. Not only does one need to reflect their understanding of problems and problem solving, PSMTs need to engage in problem solving. Both mathematical methods courses and content courses can create a learning environment for PSMTs to explore problem-solving tasks. By means of active engagement, PSMTs can experience problem solving as a learner and reflect on the problem-solving task as an educator.

Engaging in problem solving hones problem-solving skills. Yet, choosing an appropriate problem-solving task is a skill needed by mathematical educators. PSMTs need time to consider their criteria for problem-solving tasks. Readings in the mathematics methods course help shape the PSMTs' understandings of important criteria for a task. However, PSMTs need space to verbalize their own criteria. Instructors for mathematics methods courses at any university can have the PSMTs collectively create a class list of criteria. After course readings, PSMTs need the opportunity to revisit the class list to reevaluate the criteria. After PSMTs engage in problem solving, they need the opportunity to critique the experience with the class criteria. This critique would be important to complete both in the methods course and after problem-solving experiences in a mathematical content course.

As PSMTs gain an understanding and ability to critique mathematical tasks, they also need to gain experience with altering mathematical tasks. The participants in this study mentioned that when a task did not meet their criteria the task could be changed. However, with the lack of experience in altering tasks, the participants were not comfortable with making edits. Therefore, PSMTs need support and opportunities to take a mathematical task they believe is lacking in areas and adjust the task to increase the areas of their specific criteria.

Limitations of study

The first limitation of this study centralized around the timing of case. This study's case focused on the PSMTs' understanding prior and during the first methods course. It is important to acknowledge PSMTs' understanding at the beginning of their journey through the method courses. However, limiting the study to just the first methods course restricts the case to a course that has an underlying theme of problem solving. Since the course centralizes on problem solving, the participants' mindset was focused on the idea of problem solving. Understanding is not limited to one specific course. Therefore, it would be important to investigate more context the participants experience. For example, interview questions could be added to explicitly ask about potential connections to mathematical content courses. Two participants of this study mentioned the content classes they were enrolled in during the methods course. Additional questions to see if there are connections to content and methods course would be needed.

A second limitation is the bounding of the case. This case study was only at one major state university. This limitation restrains the understanding to just how the one university structured their mathematics methods course. This is a limitation because it creates a silo of structures for learning with only one initial methods course. Because of this limitation, this study could extend to another major university in the same state. Other universities could structure their methods courses in a different way. This would allow the data to broaden to methods courses that do not focus on problem solving as the main theme of the course. More research would need to be done to analyze the different objectives of the courses to see how the different universities aligned.

A final limitation for this study was the individualization of the data collection. The personal experience of each participant is important. However, understanding can happen in a collective environment. A group interview could be added to the data collection portion of this study. The participants would be able to share their experiences within the course with their peers.

Future research

This case study focused on PSMTs in one mathematics methods course at one major university in a southern plains state. To further this body of research, a longitudinal study could be designed to follow participants through all the mathematics methods courses at this university. This is important for multiple reasons. First, other mathematics methods courses focus on different content levels. The first mathematics methods course focuses on middle school mathematics content. The following mathematics methods courses focus on algebraic content and then geometric content. Second, a longitudinal study would allow for a collective understanding of how the participants build on their understanding of problem-solving tasks from course to course. Thirdly, the participants would be able to implement their understanding in an educational setting with on-level learners because the mathematics methods courses eventually have field experiences involved. Data could be collected based on implementation of giving a problem-solving task within a secondary mathematics classroom.

Another potential study could be a multiple case study to include other major universities in the southern plains state. The focal point of this study was the PSMTs' understanding of problems, problem solving, and their criteria for problem-solving tasks during their first methods course. Therefore, a need to align the method courses at each major university would need to be completed to identify the timing of the study.

Conclusion

This research aimed to explore how PSMTs understand problems and problem-solving, and how these understandings influence their criteria for selecting problem-solving tasks. The PSMTs experiences revealed that problems are scenarios that are solvable to make sense of a situation. Problem solving is the need to engage in a moment of uncomfortableness that ignites prior knowledge with new information. Worthwhile tasks are a part of the learning environment that sets the foundation for problem solving through a collaborative network for solving. How PSMTs see problem solving and worthwhile tasks is influenced by personal experiences with problem solving by means of the mathematics methods and content courses. This study was just a glimpse at these participants' educational journey to their own classroom with future mathematics students exploring the wonders of mathematics.

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Appendices

Appendix A: Recruitment Letter

If you are a secondary mathematics education major entering your first math methods course, you are invited to participate in research about experience with problem-solving tasks and problems, and your personal criteria for justification of a worthwhile task or problem.

If you agree to participate, you will be asked about your thoughts and experiences with problem-solving within your mathematical educational experience. You will be asked to participate in three 45-60-minute interviews, three task analysis sessions, and share class assignments aligning with problem-solving.

Please contact Amber Stokes (amberstokes@ou.edu) for more information.

The University of Oklahoma is an equal opportunity institution.

Appendix B: Participant Consent Form

Consent to Participate in Research

University of Oklahoma

You are invited to participate in research about experience with problem-solving tasks and problems, and your personal criteria for justification of a worthwhile task or problem.

If you agree to participate, you will be asked about your thoughts and experiences with problem-solving within your mathematical educational experience. You will be asked to participate in three 45-60-minute interviews, three task analysis sessions, and share class assignments aligning with problem-solving. You will be audio recorded. There is no compensation for your participation.

There is a risk of accidental data release if we collect your data using audio recordings. If this occurred, your identity and the statements you made could become known to people who are not on the research team. To minimize this risk, the researchers will transfer data to, and store your data on, a secure platform approved by the University's Information Technology Office.

You will be asked to transfer documents, images, or other personal records to the researchers. There is a risk that someone who is not part of the research team may gain access to this data during the transfer process. We will give you information about transferring data to the researchers to minimize this risk. When we receive your data, we will transfer the data to, and store your data on, a secure platform approved by the University's Information Technology Office.

Your participation is voluntary and your responses will be anonymous.

I will be asking some questions to find out how you want me to report your ideas. You can refuse any that you do not like without any penalty.

Do you agree for data records to include identifiable information? ___Yes ___ No

Do you agree to be quoted directly, without the use of your name? ___ Yes ___ No

Do you agree to have your name reported with quoted material? ___Yes ___ No

Do you agree for your data to be archived for scholarly and public access? ___Yes ___ No

Do you consent to audio recording? ___ Yes___ No

Your audio records may be used in University research reports unless you tell me not to do this.

May I contact you to gather additional data or recruit you for new research? ___Yes ___ No

Even if you choose to participate now, you may stop participating at any time and for any reason.

If you have questions about this research, please contact:

Amber Stokes (amberstokes@ou.edu) and Kate Raymond (kate.m.raymond@ou.edu).

You can also contact the University of Oklahoma – Norman Campus Institutional Review Board at 405-325-8110 or irb@ou.edu with questions, concerns, or complaints about your rights as a research participant or if you don't want to talk to the researcher.

You will be given a copy of this document for your records.

I agree to participate in this research.

____ ______________________

Signature of Participant Date

____ ____________

Signature of Researcher Obtaining Consent Date

Appendix C: Interview 1 Protocol

Thank you for your time and willingness to participate. I am interested in your definition of problem-solving and your criteria for decision-making for problem-solving tasks. Particularly, I am trying to understand how secondary preservice teachers define problem-solving and align personal criteria to choosing problem-solving tasks. My research questions are:

1. What prior knowledge and understandings do preservice secondary mathematics teachers have about a problem and problem solving as they enter their teacher preparation program?
    a. What criteria do preservice secondary mathematics teachers use at the beginning of their first methods course for describing a problem-solving task?
2. How does their understanding change during their first methods course?
    a. What criteria do preservice secondary mathematics teachers use during their first methods course for describing a problem-solving task?

Do you have any questions before we begin?

Background information:

1. Are you a secondary mathematics undergraduate student currently enrolled in EDMA 4233, Developing Problem Solving Environments for Secondary Mathematics Learning?
    a. Is this your first mathematics education course?
    b. What other educational courses have you taken?
2. Why did you choose to be a secondary mathematics undergraduate student?

a. What influenced your decision to be a secondary mathematics undergraduate student?

Definition and influencers of problem solving:

1. How would you explain your understanding of a problem in mathematics?
    a. What has influenced that understanding you have of a problem?
2. How would you explain your understanding of problem solving in mathematics?
    a. What has influenced your definition of problem solving?
3. What are examples of problem-solving tasks you have experienced as a learner?
    a. How do you know you are engaged in problem solving?
    b. How does the task align with your definition of problem solving?
    c. How does or does not align with your definition of a problem?
4. What are examples of problem-solving tasks you would use in your future classroom?

Criteria for Worthwhile Tasks:

1. What criteria make a task a "worthwhile" problem or task?
    a. Why is that important?
    b. What influenced your criterion of a “worthwhile” problem or task?
2. How does your criterion promote problem-solving?

Task analysis:

I am going to give you a task to read. I will ask you to read the task aloud. As you read this document, I want you to consider your definition of problem-solving and your criteria for a worthwhile task. If at any time you want to pause in your reading to comment on what I've asked you to consider, please do so. Please also share any other thoughts you might have as you read it. If you fall quiet as you read, I might ask you what you are thinking to prompt you to think aloud again. Do you have any questions?

After they finish reading

1. Can you tell me what you thought of that task?
2. I asked you to consider your previous mentioned understanding of problem-solving and your criteria for a worthwhile task. Can you tell me your thoughts about that now that you've finished reading?
3. Using your criteria for a "worthwhile" task, do you think this task aligns with your criteria?
    a. Why or why not?

Appendix D: Interview 2 Protocol

Thank you for your continued participation in this research study. As a reminder, I am interested in your definition of problem-solving and your criteria for decision-making for problem-solving tasks.

Do you have any questions before we begin?

Definition and influencers of problem-solving:

1. From our last interview, you stated that your understanding of a problem was ________, correct?
    a. Has your understanding changed? If so, how?
    b. If it changed, what influenced the change in your understanding?
2. From our last interview, you stated that your understanding of problem solving was ______, correct?
    a. Has your understanding changed? If so, how?
    b. If it changed, what influenced the change in your understanding?
3. Have you engaged in problem-solving tasks recently that align with your thinking?
    a. If so, what was the task?
    b. How does the task align with your thinking?

Criteria for Worthwhile Task:

1. From our last interview, you stated the following as your criteria for a worthwhile task _________, correct?
    a. Have your criteria changed? If so, how?
    b. If it changed, what influenced the change in your criteria?

c. If it changed, how does your new criteria promote problem-solving?

Participant Chosen Task Think Aloud:

Tell me about the task you chose to bring.

After participant describes the task

1. How does this task align with your criteria of a worthwhile task?
2. Do you believe this task allows for multiple solutions? Why or why not?
3. Do you believe this task allows for multiple pathways to a potential solution? Why or why not?
4. Do you believe this task allows for various representations? Why or why not?
5. Do you believe this task allows for mathematical communication? why or why not?

Appendix E: Interview 3 Protocol

Thank you for your continued participation in this research study. As a reminder, I am interested in your definition of problem-solving and your criteria for decision-making for problem-solving tasks.

Do you have any questions before we begin?

Definition and influencers of problem-solving:

1. From our last interview, you stated that your understanding of a problem was ________, correct?
    a. Has your understanding changed? If so, how?
    b. If it changed, what influenced the change in your understanding?
2. From our last interview, you stated that your understanding of problem solving was ______, correct?
    a. Has your understanding changed? If so, how?
    b. If it changed, what influenced the change in your understanding?
3. Have you engaged in problem-solving tasks recently that align with your thinking?
    a. If so, what was the task?
    b. How does the task align with your thinking?

Criteria for Worthwhile Task:

1. From our last interview, you stated the following as your criteria for a worthwhile task _________, correct?
    d. Have your criteria changed? If so, how?
    e. If it changed, what influenced the change in your criteria?

f. If it changed, how does your new criteria promote problem-solving?

Task analysis:

I am going to give you a task to read. I will ask you to read the task aloud. As you read this document, I want you to consider your definition of problem-solving and your criteria for a worthwhile task. If at any time you want to pause in your reading to comment on what I've asked you to consider, please do so. Please also share any other thoughts you might have as you read it. If you fall quiet as you read, I might ask you what you are thinking to prompt you to think aloud again. Do you have any questions?

After they finish reading

1. Can you tell me what you thought of that task?
2. I asked you to consider your definition of problem-solving and your criteria for a worthwhile task. Can you tell me your thoughts about that now that you've finished reading?
3. Using your criteria for a "worthwhile" task, do you think this task aligns with your criteria?
    a. Why or why not?

Document Analysis:

Tell me about the course assignment(s) you chose to bring.

After participant describes the assignment(s)

1. How does this assignment align with your criteria of a worthwhile task?
2. Does this assignment demonstrate your understanding of problem-solving?

3. Do you believe this assignment allows for multiple solutions? Why or why not?
4. Do you believe this assignment allows for multiple pathways to a potential solution? Why or why not?
5. Do you believe this assignment allows for various representations? Why or why not?
6. Do you believe this assignment allows for mathematical communication? why or why not?

Appendix F: Task Selecting Rubric

Characteristics	Meet	Does Not Meet	Justification
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)			
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Procedures with connections*			
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Doing Mathematics*			
Problem Solving as an art (Son & Lee, 2020) Involves analysis, application, interpretation, exploration, and reflection			
Communication (Lappan & Phillips, 1998; Levenson, 2013)			
Use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013)			

Multiple pathways to solution(s) (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)			
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)			

Appendix G: Tasks

Task 1: Driving Distance

DRIVING DISTANCE

Hilary drove from her house to Dallas and back. On the way to Dallas, she drove at an average speed of 60 miles per hour. On the way back, she drove at an average speed of 66 miles per hour. She spent a total of 6 hours and 6 minutes driving. How far is the drive from her house to Dallas?

Task 2: Algebraic Rectangle

Algebraic Rectangle

The perimeter of a rectangle is $4x+24$.
What are five possible values for the area of the rectangle in terms of x?

Task 3: Drinking Glass Task

Drinking Glass Task

- A drinking glass, in the shape of a cylinder, has a volume between 390 and 460 cubic centimeters. The radius and height of the drinking glass, when measured in centimeters, are integers. What could the dimensions of the glass be?

- James poured cubic 355 cubic centimeters of water into this drinking glass. He then realized the water was not quite cold and wanted to add ice cubes. The ice cubes were 2.5 centimeters on each side. How many ice cubes can James add?

Task 4: Locker Likelihood

Locker Likelihood

In a high school of 1000 students, ten lockers were searched. All the searched lockers belonged to male students. What was the likelihood that the lockers were chosen at random if the school population was 50% male? What is the school was 70% male? 90% male?

Which scenario do you think more fair? Why?

Appendix H: Task Rubrics Completed

Task 1 Rubric

Characteristics	Meet	Does Not Meet	Justification
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)	X		PA.A.2.4 Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects.
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Procedures with connections*	X		Students use prior knowledge of functions to make connections to multiple components of the task.
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998)		X	

Doing Mathematics			
Problem Solving as an art (Son & Lee, 2020) -Involves analysis, application, interpretation, exploration, and reflection	X		Students must understand the task to apply known information and strategies, interpret if the strategy is appropriate and reflect on reasonableness.
Communication (Lappan & Phillips, 1998; Levenson, 2013)	X		Students can discuss strategies and methods to solve.
Use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013)	X		Task connects to real-world. Students have a personal connection with driving to Dallas. Can use illustrations, graphs, or equations.
Multiple pathways to solution(s) (Boaler et	X		The task is open for students to approach

al., 2022; Lappan & Phillips, 1998; Levenson, 2013)			solving using varies ways.
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)		X	There is only one solution for this task.

Task 2 Rubric

Characteristics	Meet	Does Not Meet	Justification
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)	X		PA.N.1.1 Develop and apply the properties of integer exponents, including $a^0 = 1$(with $a \neq 0$), to generate equivalent numerical and algebraic expressions.
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Procedures with connections*	X		Connects procedures for perimeter and expressions to use varies methods to find multiple solutions.
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998)		X	Asks students to connect procedures to tasks involving perimeter and expressions.

Doing Mathematics			
Problem Solving as an art (Son & Lee, 2020) -Involves analysis, application, interpretation, exploration, and reflection	X		Students explore multiple strategies to find multiple reasonable solutions.
Communication (Lappan & Phillips, 1998; Levenson, 2013)		X	As written, it does not call for communication. However, it could be written to justify reasoning.
Use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013)	X		Students can use diagrams and manipulatives. Lacks real-world connection, as written.
Multiple pathways to solution(s) (Boaler et	X		Varies methods could be used to solve.

al., 2022; Lappan & Phillips, 1998; Levenson, 2013)			
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	X		Task states there is more than one solution.

Task 3 Rubric

Characteristics	Meet	Does Not Meet	Justification
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)	X		PA.GM.2.4 Develop and use the formulas $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for pi (T). Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate units (e.g., cm^3).
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998)		X	No clear procedure, only math context.

Procedures with connections			
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Doing Mathematics*	X		Extension of task to add elements.
Problem Solving as an art (Son & Lee, 2020) -Involves analysis, application, interpretation, exploration, and reflection	X		Students must determine strategies that can support. Exploration of dimensions. Reflection on reasonableness with both questions.
Communication (Lappan & Phillips, 1998; Levenson, 2013)	X		Justification to verify solution.
Use of manipulatives, illustrations, real-world context (Lappan &	X		Real-world context, illustrations could be used.

Phillips, 1998; Levenson, 2013)			
Multiple pathways to solution(s) (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	X		Different ways to approach problems.
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	X		More than one possible solution for glass based on volume.

Task 4 Rubric

Characteristics	Meet	Does Not Meet	Justification
Connection to Mathematical Content (Henningsen & Stein, 1997; Tekkumru-Kisa et al., 2020)	X		PA.D.2.2 Determine how samples are chosen (randomness) to draw and support conclusions about generalizing a sample to a population, including identifying limitations and biases.
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998) *Procedures with connections*		X	Goes beyond procedures with connections to justification
High Cognitive Demand: (Arbaugh & Brown, 2005; Smith & Stein, 1998)	X		Students were not given procedures. Justify reasoning with multiple scenarios

Doing Mathematics			
Problem Solving as an art (Son & Lee, 2020) -Involves analysis, application, interpretation, exploration, and reflection	X		Students must explore multiple possibilities to reflect on fairness.
Communication (Lappan & Phillips, 1998; Levenson, 2013)	X		Students are asked to justify their reasoning.
Use of manipulatives, illustrations, real-world context (Lappan & Phillips, 1998; Levenson, 2013)	X		Real-world connection to school environment.
Multiple pathways to solution(s) (Boaler et al., 2022; Lappan &	X		Students can state at various parts of the first question.

Phillips, 1998; Levenson, 2013)			
More than one solution (Boaler et al., 2022; Lappan & Phillips, 1998; Levenson, 2013)	X		Asking for students to reason and justify.