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UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

MATHEMATICAL EMPOWERMENT: A CASE STUDY OF RELATIONAL CLASSROOM LEARNING

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

In partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

By

GLORIA NAN DUPREE
Norman, Oklahoma
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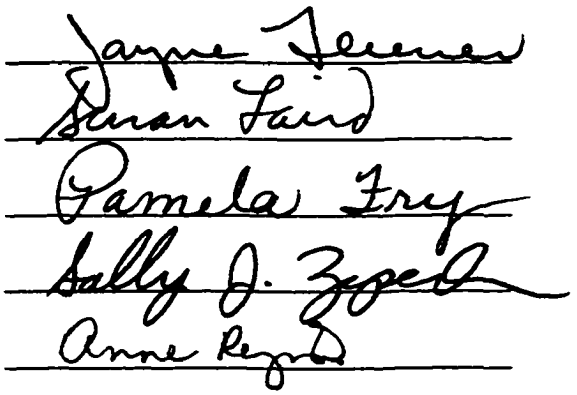
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MATHEMATICAL EMPOWERMENT:
A CASE STUDY OF RELATIONAL CLASSROOM LEARNING

A Dissertation APPROVED FOR THE
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND
ACADEMIC CURRICULUM

BY



DEDICATION

To my mother, Cleolua Draughon Dupree, who wanted to be a mathematics teacher but chose instead to be a mother and a kindergarten teacher, who helped me learn to love and appreciate the beauty of mathematics, who taught me how to live and love, and who has always believed in me.

To my son, James Gavin Donaldson, my daughter-in-law, Mary Susan Donaldson, my grandchildren, Taylor Lynn and Brandon James (B. J.), who have supported and encouraged me with their love to continue this endeavor.

To my sister, Donna Lyn Buckner, who has become my best friend and who has told me many times how proud she is of me and my accomplishments.

To the memory of my daddy, Morris Sheppard Dupree, who taught me how to live and walk in the truth and who always challenged me to be the best I could be.

To the memory of my daughter, Alicia Lynn Donaldson, who never had the opportunity to fulfill her dreams and who would be very proud of her momma.

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To my friend and mentor, M. Jayne Fleener, who chaired my committee, who believed in me even when I had doubts, who pushed me out of the nest when she knew I was ready to fly, and who shares with me the reality of a dream come true.

I love you all very much and am grateful for all of your love, encouragement, and praise. God has richly blessed me through you.

MATHEMATICAL EMPOWERMENT: A CASE STUDY OF RELATIONAL CLASSROOM LEARNING

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ABSTRACT

This is a study of the relationships that developed among the students in a small class of Mathematics for Critical Thinking. The organization of the class was based on the social constructivism of Vygotsky. There was an attempt to create an atmosphere that was sensitive and nurturing in which students could trust their classmates to be supportive of their efforts to solve mathematics problems. We worked together to establish the sociomathematical norms that would enhance the development of mathematical power within the students and promote the evolution of problem-solving skills.

Problems were selected as part of the evolution of the class to challenge the students, to reinforce the search for patterns, and to evoke questions and problem-posing from the students. Through interactions with each other and by writing their thought processes as they solved problems, the students were actively engaged in doing mathematics rather than watching mathematics being done. Our study of *Flatland* (Abbott, 1884/1994) afforded opportunities for the students to examine their relationships with mathematics and to recognize their own mathematical power.

Using the different ways of knowing presented by Belenky, et al. (1986) as a guide, it was determined that the students became less dependent on received knowledge and more competent and confident in their procedural and constructed areas of knowledge. The challenges of this class offered the students opportunities to engage in mathematics in a personal way that was empowering. They all left the class with new and positive attitudes towards mathematics and their abilities to actively engage in challenges and rewards that relationships with mathematics afford.

The findings indicate that the relationships within the classroom enhanced the development of viable relationships with mathematics. There were also indications that the fact that the students were all females may have contributed to the workable relationships with mathematics.

There are strong implications that mathematics can be made more accessible to more students by restructuring the mathematics curricula to accommodate the different ways that students approach the construction of their mathematical knowledge. My findings call for college and university mathematics departments to re-examine the traditional lecture method for disseminating mathematics to large numbers of students in a lecture hall and consider more opportunities for students to engage in mathematics at a personal level. This could revolutionize mathematics education in ways that would empower all students.

MATHEMATICAL EMPOWERMENT: A CASE STUDY OF RELATIONAL CLASSROOM LEARNING

CHAPTER 1

CLASSROOM PRACTICES AND GOALS

MATHEMATICAL POWER

> Mathematical power includes the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power. (NCTM, 1991)

If mathematical power is an important ability (NCTM, 1991; National Research Council, 1989), then how can teachers create a classroom environment which helps students develop mathematical power? What are the characteristics of a mathematics classroom that promotes mathematical power? Which instructional practices contribute to mathematical power? How do these effective practices vary for different students?

RELATIONAL MATHEMATICS CURRICULUM DEFINED

Palmer (1993) posits that when we know something really well, "we feel inwardly related to it: knowing it means that we have somehow entered into its life,

and it into ours" (p. 57). We can talk about a relational mathematics curriculum as one which allows students to engage themselves with mathematics in a way that encourages them to make mathematics a part of them as well enabling them to become part of mathematics.

Viewing the learner as an active participant in the process of building knowledge (Noddings, 1990; Steffe, 1990), the selection of problems becomes an important aspect of designing a relational mathematics curriculum. Recognizing the crucial role of problem-solving in the construction of mathematical knowledge (Confrey, 1991; NCTM, 1991; National Research Council, 1990), students should be encouraged to pose their own problems (Brown, 1986; Frankenstein, 1990; Morrow, 1996). Defining and identifying good problems are not easy. Characteristics of good problems include:

1. Open-ended with no solution or multiple solutions,
2. Solved using multiple approaches,
3. Exhibit certain patterns which lead to the solutions,
4. Lend themselves to the collaborative effort of several students,
5. Require multiple steps to reach a solution,
6. No ready or obvious answer,
7. May require students to seek assistance from sources outside the classroom,
8. Are interesting and challenging, and
9. Represent real-world situations.

What are some of the advantages of using problems with these characteristics? If students are to develop their own relationship with mathematics, including becoming good problem solvers and problem posers, they may need to become more mindful of their learning (Langer, 1997). Langer (1997) posits that rather than proceeding directly from the problem to the answer by following pre-learned rules and algorithms, a 'mindful' person remains open and flexible to optional perspectives. As an example of mindful problem solving, she tells of an entrepreneur who learned of a machine that was designed to spray crops, but failed because the spray froze and killed the plants. This man saw the possibilities created by the cause of the machine's failure and turned the idea into a snow-making machine. He was not hindered nor bound by the machine's intended purpose. "From a mindful perspective, one's response to a particular situation is not an attempt to make the best choice from among available options but to create options" (Langer, 1997, pp. 113-114). As 'mindful' learners, students may become better problem solvers when they are open to the possibilities presented by the problems rather than restricted by predetermined solutions.

Langer (1997) also speaks of the freedom we have to be mindful thinkers when we do not know all the 'rules.' Her example is that of a young student who had not been told that the sum of the angles of a triangle in a plane is 180°. The student consistently determined the sum of the angles of her triangle to be 183°. When the teacher said that it is impossible for a triangle to have more than 180°, the student asked why two lines on a globe that are both perpendicular to the equator intersect at the poles making a triangle with more than 180° in the sum of the angles? The teacher then explained that their triangles were in the plane. The student was free to be

mindful until the teacher imposed the rigid 'rules' of Euclidean geometry. Indeed the student was measuring on the carpet (not a plane) which may have accounted for the extra degrees. Inflexibility resulted in the teacher's missing an opportunity to let the students explore the geometry of curved surfaces.

Environmental characteristics, including opportunities to discuss problem solving efforts, may contribute to a more mindful approach to learning mathematics and therefore may contribute to a student's developing a relationship with mathematics. Morrow (1996) suggests that students who engage in detailed discussions of the processes of problem solving tend to become more astute in recognizing their own mistakes and thus become more confident in their abilities to solve problems. According to Morrow (1996), open discussions are more likely to occur when the problems are open-ended with multiple solutions. She says that such discussions are based on beliefs rather than doubts and provide students more freedom to express their ideas. Through this type of dialogue, students may develop techniques and strategies that may be applicable to future problems.

Learning to recognize and approach problems by exploring geometric and numeric patterns has been one strategy that seems to be particularly helpful, both as a problem solving technique as well as an organizer for understanding fundamental mathematical relationships (NCTM, 1991; National Research Council, 1990). Developing an awareness of patterns and relationships within mathematics may be an important aspect for students' developing their own relationships with mathematics. Similarly, generalizing and recognizing patterns may help students develop their own algorithms useful for future problem solving (Davis & Maher, 1990). Finally,

affectively, as students experience success in solving problems, an excitement and desire to tackle other problems may develop within the student (Confrey, 1991). They may be motivated to experiment with creating their own problems (Brown, 1986; Frankenstein, 1990; Morrow, 1996).

Thus a relational mathematics curriculum involves a careful selection of problems that promote thought and discussion and a classroom environment that encourages the development of relationships among students and the teacher that create a sense of openness. When the classroom is a learning community built on trust, students may be more comfortable discussing mathematics without fear of ridicule. In this safe environment the students may develop relationships with mathematics that empower them to become active participants in their journeys with mathematics. As active participants, willing to take risks, students may recognize their mathematical abilities to develop problem solving strategies and the importance of using available resources, including poeple, as one of these strategies. The ultimate goal of a relational mathematics curriculum is to promote the relationships which may empower students to become mathematically competent and confident.

A CRITIQUE OF TRADITIONAL MATHEMATICS INSTRUCTION

As we begin to study the effects of a relational mathematics curriculum, we need to examine traditional mathematics curriculum and the traditional way mathematics has been, and is being, taught to gain perspective on the possible need for change. What has led to the current or typical mathematics curriculum, and how does it fail to meet the needs of all students?

We begin our discussion by looking at the ways 'knowledge' has been defined.

Traditionally, western culture has perpetuated that ideas that learning is an individual, solitary activity, and that there is no validity in other ways of knowing (Code, 1991) while it has ignored women in defining and validating knowledge. 'Malestream' (Code, 1991, p. 12) thinking founded in the philosophies of men like Descartes and Aristotle has emphasized logic, reason, and the scientific method while ignoring insight, connections, and traditionally feminist traits as valid ways of knowing. According to these malestream perspectives, women owned no knowledge except that which they received from men. Knowledge was objective and defined as that which could be learned from books and was taught by men. Women have had to fight for the right to access this knowledge, while being discounted as knowledge makers (Code, 1991).

Because women have not been knowledge makers, very few women have been given opportunities to contribute to the body of mathematical knowledge. Some women mathematicians such as Hypatia (Greek, 370-415 A.D.), Maria Gaetana Agnesi (Italian, 1718-1799), and Emmy Noether (German, 1882-1935), were daughters of mathematicians, while others, including Emilie du Chatelet (French, 1706-1749) and Sophie Germain (French, 1776-1831), were of wealthy families and could afford the luxury of hiring people to run the household while they pursued their interest in mathematics (Perl, 1978). These early women mathematicians who did succeed and are recognized for their contributions did so because they struggled to make themselves heard, and in some cases, concealed their sex to be recognized as valid knowledge makers. While it seems that these women were afforded opportunities to investigate mathematics, the men considered that it was all right for

women to 'play' with mathematics, but they certainly did not think that women were smart enough to have discovered something worthwhile. For example, Sophie Germain (French, 1776-1831) submitted her work on analysis under the name of *M. LeBlanc* to insure that Lagrange would read it before dismissing it as the musings of a woman (Perl, 1978).

Our schools have been the perpetuators of this malestream thinking, especially in the teaching of mathematics. Consider the 'typical' college mathematics classroom. The classroom has a front--complete with a chalkboard, a podium with a lectern, and a teacher's desk. The students' desks (often bolted to the floor) are arranged in neat rows. There is always an authority figure--the teacher--who is, more often than not, male. He is the teller of knowledge, the authority on what is correct, and the judge, giver of grades.

Although there are many more women mathematics teachers now than in the past, schools are still operating under this traditional paradigm. Patriarchy is still the dominant theme with men's and women's responsibilities clearly delineated (Scering, 1997). In mathematics classrooms, many women teachers have assumed the masculine role of the keeper of knowledge in the classroom because they are held accountable for their students' performance on standardized tests (Scering, 1997). The teacher controls the mathematics taught and decides when a student has learned the material. If the goal is to "cover" the material, then that may become the driving force of the class, and the students may not learn the material before a new topic is presented.

The principal format of traditional mathematics instruction is the lecture (Confrey, 1990), with some questioning of or by the students. During this exchange

of questions and responses, the teacher often responds more to the males than the females (Leder, 1995). The teacher may answer questions and/or work problems from the previous assignment before introducing a new topic, working a few examples on the board, and making an assignment. The students may begin work on the new assignment during the remaining few minutes of class. Each day is very much the same as the previous one (Confrey, 1990).

The reports on the Third International Mathematics and Science Study (TIMSS) concur with this picture of American mathematics classroom teaching. "U. S. mathematics teachers' typical goal is to teach students how to do something, while Japanese teachers' goal is to help them understand mathematical concepts" (National Center for Education Statistics, 1997). The ineffectiveness of this type of instruction is reflected in the TIMSS data which places the United States twelfth graders below most of the other industrialized nations and below the international average in both mathematics literacy and advanced mathematics (National Center for Education Statistics, 1998; Schmidt, McKnight, & Raizen, 1997).

This traditional paradigm assumes that the students are the receivers of the mathematical knowledge. The students are expected to memorize and regurgitate the right answers at the right time. Those who do not follow the correct protocol are penalized and/or failed.

In these traditional mathematics classrooms, many girls and young women have taken a back seat to boys and men (Spender, 1986; Leder, 1995). Boys are expected to do well in mathematics. After all, they are the ones who will be the doctors, the engineers, and the scientists. Girls have been excused from excelling in

mathematics because they are deemed not as capable as the boys in these technical areas of study. Girls who are experiencing difficulties with the concepts are frequently told to try something easier, while the boys were challenged to try again; they could get it (Walden & Walkerdine, 1986). In many cases little has changed. The girls who have succeeded in mathematics have done so by adopting the aggressive tactics of the males, but even they have been discounted as being inferior to the boys (Scott-Hodgetts, 1986). This exclusion is not limited to the girls; I argue that there are many boys who are discounted mathematically. This exclusion may extend to include students of minority races, ethnicities, and cultures as well as those with less affluence.

While some students have mastered the art of learning mathematics in this type of mathematics class, many others have not. If we eliminate the stereotypical curriculum that separates the sexes and further discredits the abilities of girl students (Martin, 1991) in the relational mathematics class, then we may be able to establish a classroom environment that nurtures all students. Confrey (1990) points to evidence from research that direct instruction is inadequate for helping students develop long-term understanding.

Many feminists view the traditional and philosophical definitions of knowledge as oppressive to women. The essentialist defines knowledge as masculine, objective, linear, and logical. Thus, to the essentialist, feminine knowledge, which is subjective, nonlinear, and intuitive, cannot be counted as knowledge (Code, 1991). In a relational mathematics class where the problems promote thought, discussion, and ownership, and the classroom environment encourages discourse, we would discount this essentialism and recognize that there are different ways of knowing (Belenky, Clinchy,

Goldberger, & Tarule, 1986), that knowledge can be subjective as well as objective, and that we may learn from our interactions with other people and with ideas (Code, 1991). Skovsmose (1994) developed the notion 'mathemacy' to capture both the importance of mathematical literacy in our society and as a critique of the inequities and power relationships formed by mathematics and technical knowledge. He says that mathemacy calls for reflective knowing which in turn crosses the mathematical boundaries into other disciplines. Mathemacy may be the result of a relational mathematics curriculum which calls students to form a community of mathematics investigators.

Goals of a relational mathematics class would be to instill in all students confidence in their mathematical abilities (Bandura, 1986; Pajares, 1996; Pajares & Miller, 1994), feelings of self-worth, and a realization that they can do mathematics. Recognizing that "a fundamental belief in the students is more important than anything else" (Rich 1979, p. 66), we would work toward helping all students develop positive self-images (Bandura, 1986; Pajares, 1996; Pajares & Miller, 1994).

Will a departure from the traditional lecture format empower students to view themselves more positively as students of mathematics? There are other mathematics classroom environments that may encourage students to become more than just passive learners (NCTM, 1991). A relational mathematics class may solve some of these problems and help students develop confidence in their mathematical abilities. There are at least three academic relationships that students may develop in a relational mathematics classroom:

1. Relationships with the teacher,

2. Relationships with each other, and

3. Relationships with mathematics.

As we build a relational mathematics environment, we will consider these relationships and their contributions to the mathematical empowerment of students.

RELATIONSHIPS BETWEEN STUDENTS AND THE TEACHER IN A RELATIONAL MATHEMATICS CLASSROOM

How does a relationship between students and the teacher develop, especially if one goal of relational mathematics instruction is to combat the perception that the teacher is the holder of all knowledge? Several strategies suggest promise for helping students develop their relationships with mathematics which also may impact the relationships between the students and the teacher. Problem-centered learning, emphasizing both problem-solving and problem-posing student activity, as well as a focus on mathematical discourse (NCTM, 1991), may contribute to relationships in the classroom that are more student-focused and egalitarian. These approaches are consistent with Mezirow's (1991) theory of critical mathematics education which suggests three responsibilities for the teacher (of adults):

1. Encourage critical reflection,

2. Establish discourse, and

3. Direct students' actions for further learning.

As teachers encourage students to reflect on and discuss mathematics within the classroom (Noddings, 1990), opportunities arise for teachers to analyze what the students are learning about mathematics (Cobb, Wood, & Yackel, 1990), as well as encourage student ownership of their mathematics learning. By engaging students in

conversation about a particular problem, the teacher may be better able to understand the difficulties the students are experiencing, the strategies they have employed, and their analyses of the methods they have tried. Discourse within communities of learners similarly may encourage critical reflection and transformative learning.

When students are experiencing difficulties solving a particular problem, the teacher may be able to gain a perspective on the students' approaches to the problem by listening to the students' explanations of their solution processes. This may involve having the students explain their drawings, models, and/or procedures. Through explaining their perspectives on a particular problem, the students may be better able to modify their views so that the problems and solutions make sense (Confrey, 1990; Confrey, 1991). By encouraging the students to discuss their problem-solving strategies, the teacher may be liberating them to accept new challenges.

There may be times when a student thinks she/he is working a problem correctly but has either misunderstood a concept or made a careless mistake (Confrey, 1991; Noddings, 1990; Edwards, 1989). If the teacher enters into conversation with the student about the problem without specifically addressing the mistakes, the student may be able to discern her/his error(s). This may have the effect of building the student's confidence in her/his problem-solving skills. Dialogues such as this may "promote more self-reflection and a stronger approach to knowledge construction" (Confrey, 1991, p. 121).

A relational mathematics class is one which fosters an atmosphere where questioning and doubting are acceptable (NCTM, 1991). As a teacher in a relational mathematics class challenges students' answers and statements about mathematics to

encourage them to justify their responses, students may be encouraged to ask questions of themselves and others, to ask for reasons, and to challenge themselves and others (including the teacher) (Noddings, 1990; Cobb, Wood, & Yackel, 1990). By valuing the input of each student, the teacher is setting the stage for safe risk-taking. When students know that they can trust the teacher as a friend and colleague (Cobb, Wood, & Yackel, 1990), they may be more willing to discuss their mathematical ideas. As dialogue develops between the teacher and the students, all may engage in more self-reflection (Confrey, 1991), which may promote an increase in the students' understanding of themselves as learners, knowers, and doers of mathematics.

RELATIONSHIPS AMONG STUDENTS IN A RELATIONAL MATHEMATICS CLASSROOM

As we consider mathematical relationships among students in a relational mathematics classroom, we are drawn to Vygotsky's theory of social constructivism. Vygotsky posed that children learn in a social setting, and that they learn through their interactions with their teachers and their peers.

Research (Confrey, 1991; Steffe, 1991; Cobb, Wood, & Yackel, 1990) points toward the utilization of small groups for doing mathematics. In a small group setting, it is easier for all students to be engaged in mathematical activity (Cobb, Wood, & Yackel, 1990). In small groups students may be more willing to ask questions and to verbalize their misunderstandings. As the students establish the sociomathematical norms (Cobb, Wood, & Yackel, 1990) of their group and build a mathematical community, they may find that they are more mathematically capable than they once

thought.

In establishing a relational mathematics class, there should be opportunities for the students to be actively involved in the problem-solving efforts of a small group. As they engage in negotiation and cooperation by combining their efforts to solve the problems, a sense of camaraderie and responsibility to each other may develop. As students develop a sense of mathematical community (Noddings, 1990) through cooperating with other students to collectively solve problems, they may be able to assess their mathematical strengths and discover hidden talents within themselves. As they develop relationships with each other, reflection and discovery may lead to the establishment of viable relationships with mathematics.

STUDENTS' RELATIONSHIPS WITH MATHEMATICS

As we look at students' relationships with mathematics, let us return to Palmer's (1993) statement that when we know something really well, "we feel inwardly related to it: knowing it means that we have somehow entered into its life, and it into ours" (p. 57). We have discussed relationships between the teacher and the students and those among students in a relational mathematics classroom, but how can students enter into relationships with mathematics? If students are to 'enmesh' themselves in mathematics, then there may need to be more than relationships with teachers and other students. Maybe they must establish (or re-establish) relationships with themselves as students of mathematics.

One way of making the connections with their mathematical selves may be through writing (Countryman, 1980; Buerk, 1985, 1986, 1994a, 1994c, & 1995; Buerk & Szablewski, 1993). As students write solutions to problems rather than just

following the steps of some algorithm, they may develop a sense of their thought processes. If students are encouraged to jot down all their thoughts related to a problem, writing solutions may preclude the temptation to give up on a problem and just do nothing. Writing may also incorporate a commitment to remain connected to a problem for a longer period of time. Writing may encourage students to examine "how ... [they] feel about mathematics, how they solve problems, how they [construct and] process information, and how they view themselves as learners" (Standera, 1994, p. 25) of mathematics.

Writing may also give some students 'voices' (Belenky, et. al., 1986; Miller, 1990). There are students who are intimidated by mathematics to the point that they are silent in the mathematics classroom (Leder, 1995). Writing may give these students 'voices' while allowing them to remain non-vocal. This non-vocal mathematics communication may lead to the building of mathematical self-confidence for the students. This self-confidence may be indicative of students' growing relationships with mathematics and may lead to discussion of mathematics with their classmates.

Reading students' writings may provide the teacher an opportunity to determine if students are establishing effectual mathematical ideas. Responding to students' writings may promote in the students more accountability for assignments, a better understanding of problem solving as process, and heightened creativity in their solutions (Standera, 1994). This may foster in the students the notion that the learner is an active participant in the process of building mathematical knowledge (Noddings, 1990).

Through the use of writing, students may have the opportunity to be creative as they express their beliefs about mathematics through the use of metaphors (Fleener & Fry, 1994; Fleener, Dupree, & Craven, 1997). Students may also develop a sense of connectedness with mathematics that makes it more relational for them. They may even embrace mathematics as a source of power that exists within them rather than somewhere 'out there'. Ideally they would bask in the warmth of its radiance and beauty.

WHAT ARE THE QUESTIONS?

A relational mathematics class may have the effect of generating mathematical power for students. The following questions may lead to an understanding of some instructional practices that may foster mathematical power in students.

1. How do students experiencing a relational mathematics curriculum become more mathematically empowered, specifically, develop their ability to do mathematics and become more confident?
2. How are students' beliefs about themselves as students of mathematics affected by experiencing a relational mathematics curriculum?

In the next chapter we will examine some of the practices that have influenced this researcher to believe that the common lecture method of teaching mathematics may not empower all students, and that some students may learn mathematics more easily in a relational mathematics class. The relevant literature to this study includes the use of writing and the establishment of sociomathematical norms to enhance student-mathematics and student-student connections, as well as the literature on the impact of beliefs on mathematics learning. Framed within feminist and social constructivist

theories, this literature, along with a review of the literature on problem solving and writing in the mathematics classroom, will provide a basis for the design and focus of the study for understanding the potential of relational mathematics instruction.

CHAPTER 2

RELATED LITERATURE

Through my studies in recent years, I have become acutely aware of the inequities perpetuated by traditional mathematics. As a teacher I have been an agent in the distribution of these inequities. While I thought I was being sensitive to the needs of all my students, in reality I was blaming the students when they failed to learn. I thought that if they were not learning, there was something wrong with them, or they were not working hard enough. As I have looked at the literature and re-examined my teaching, I have come to realize that I was working in the traditional paradigm for teaching mathematics: If I worked enough examples and answered all the students' questions, if I gave them step-by-step algorithms for solving problems, and if they did all of their 'homework,' they would learn the mathematics and would perform well on the tests. The realization that there are different ways of knowing (Belenky, et. al., 1986; Code, 1991), that research points to social learning (John-Steiner, 1992; Lambdin, 1992; Schmittau, 1992; Taylor, 1992; Wilson, Teslow, & Taylor, 1992), and that beliefs affect learning (Bandura, 1986; Bullough & Baughman, 1995; Pajares & Miller, 1994; Pajares, 1996; Raymond & Santos, 1995) has brought me to the point of looking at relationships within the mathematics classroom that may lead to the mathematical empowerment of all students (NCTM, 1991).

As I began to consider the possibility of a relational mathematics curriculum to provide opportunities for students to develop within themselves mathematical power, I was informed by research literature from the areas of feminism, constructivism, problem solving/posing, writing/narrative, and beliefs. While none of these offered

explicit criteria for a relational mathematics curriculum, they did afford clues that guided me as I developed the curriculum and conducted my study.

RELATED FEMINIST LITERATURE

What can feminist literature contribute to the idea of a relational mathematics classroom? In the introduction we critiqued the traditional way that mathematics has been taught. The teacher lectures; the students are supposed to listen and learn so that they can reproduce correct answers on the tests. Belenky, et al. (1986) refer to this type of knowledge as received knowledge, and that the person who depends solely on received knowledge never learns to think for her/himself. Everything can be categorized as one of two extremes with no middle ground. Perry (1970) called this type of categorization dualism; something is either black or white, right or wrong, good or bad. This type of categorization makes life easy for the person who relies on received knowledge; decisions are easy because the rules are established by someone else.

According to Belenky, et al. (1986), as one begins to listen to one's own thoughts, one moves from received knowledge to subjective knowledge. Subjective knowledge comes from within when one begins to listen to one's own thoughts and is affirmed when others listen to these thoughts. Subjective knowledge leads to procedural knowledge.

With procedural knowledge, Belenky, et. al. (1986) found that women began to feel in control because they had figured out ways to obtain and communicate knowledge. The researchers acknowledged two categories of procedural knowledge--separate knowing and connected knowing.

In separate knowing the individual begins to think critically and to question everything. The women they (Belenky, et. al., 1986) classified as separate knowers were usually graduates of traditional liberal arts colleges, were extremely conventional, and frequently had been tomboys as children. These separate knowers believe in what Code (1991) refers to as the "autonomy of reason," which carries with it "the importance of detachment, impartiality, neutrality, and cognitive self-reliance" (p. 112). These women (Belenky, et. al., 1986) learned through explicit formal instruction and felt a desire to "teach" others how to reason.

Connected knowers believe that knowledge that is trustworthy comes from personal experience. They learn through conversations with others and making connections between their combined personal experiences. They are non-critical and non-judgmental. (Belenky, et. al., 1986). Connected knowers are reflected in Code's (1991) discussion of Annette Baier's notion of 'second personhood' (p. 74) to discount 'autonomy of reason' (p. 130-144) as the only way of knowing. This second personhood idea incorporates the interchangeability of the knower and the known when both are people. When people know each other, they interact with each other and share their stories; thus the knower and the known become indistinguishable because each knower is the known for the other person. By knowing people and sharing ideas about mathematics, students may gain an understanding of how others solve problems. Through working together for a common solution to a problem, students may develop a sense of community.

According to Belenky, et. al. (1986), constructed knowledge comes when one begins to combine all the other types of knowledge and creates her own knowledge.

She becomes the expert and is intimate with and passionate about her knowledge. These are the characteristics I want for my students. I want them to become the experts of their mathematical knowledge so that they may become passionate knowers and so intimate with their knowledge of mathematics that not only is it part of them, but they are part of it. (Palmer, 1993).

Through the writings of Buerk (1985, 1986, 1994a, 1994c, & 1995), Buerk & Szablewski (1993), and Countryman (1980), I met many women students through the recounting of their experiences with mathematics. There was Jackie (Buerk, 1993), who had taken calculus but never felt that she owned mathematics. Carolyn Werbel (Buerk, 1986) wrote that when she reread her journal for the class, Writing Seminar in Mathematics, she realized how much she had hated mathematics in the beginning of the class and how much she had learned. Her attitude toward mathematics was much more positive when she realized that she was thinking much more analytically as the semester progressed. There were Molly and Hilary (Countryman, 1980) who thought that their abilities to do mathematics centered on getting the right answers.

These students made me wonder about my teaching and the learning of my students. I began to realize the need for me to look at ways that I could change my approach to 'teaching' mathematics. The questions surfaced: Why were some of my students not learning mathematics? Was it because my classroom was a cold 'sterile' environment where students learned that mathematics was something to be observed and learned in a detached way? Did my students fear involvement with mathematics?

While the feminist literature elaborates on the oppression of women (and girls), I have begun to wonder if this oppression also extends to include men (and boys) as

well. As I thought about a relational mathematics curriculum, I realized that there are lessons to be learned from the sensitivity and compassion (Austin, 1996; Barnes, 1996; Becker, 1996; Laird, 1994/1995; 1995; Martin, 1992; Pollina 1995; Scering, 1997; Scott-Hodgetts, 1986) that abound in this literature. Some of these are summarized below.

"Instead of trying to change the way our female [and some male] students approach mathematics, ... we need to study the ways they do learn" (Pollina, 1995). Pollina (1995) points to Barbara McClintock's research in genetics and Jane Goodall's and Dian Fossey's "relational approach[es]" (p. 30) to studying primate behavior as examples of the sensitive ways to conduct research. Rather than using the scientific method in their research, Goodall and Fossey lived with the animals they researched, establishing personal relationships with them so that the animals were comfortable exhibiting their normal behavior. This had meaning for me because I met Jane Goodall, and when she talked about "her" chimps, there was love in her voice, and she spoke of them by name as one would of one's own family members. This kind of relationship within the mathematics classroom may create a comfortable environment for the exchange of mathematical ideas.

In our classrooms we also can be sensitive and nurturing while observing our students' approaches to problem solving. Pollina (1995) also offers suggestions she gleaned from studies of girls' schools and girls' classes. Relevant to this study are making connections with the real world, capitalizing on students' writing skills, carefully choosing metaphors and encouraging students to develop their own metaphors, promoting an atmosphere that results in active participation of all students,

and encouraging students to become the experts.

Martin (1992) in *The Schoolhome* speaks of developing a sense of community in the schools. She appropriately addresses the climate of the schoolhome by positing her model, "the three Cs of care, concern, and connection" (Martin, 1992, p. 39) as the dominant theme. Students should be encouraged to express love and concern for each other as well as for those outside the schoolhome. There should be mutual respect for each person as an individual. The tasks would be equally divided among the students with the unwritten message that there are no stereotypical tasks, and that everyone has a responsibility to make the environment a pleasant place to live. Students would be taught that the stereotypes of the past are not appropriate nor will they be tolerated in the schoolhome. The characteristics of Martin's (1992) schoolhome can be incorporated into any classroom and may create an environment in which all students work together to help each other solve problems and reinforce the notion that all students can learn mathematics.

Thinking about ways to encourage students to listen to each other in a sensitive and nurturing manner led me to consider a seminar format for a class of MATH 1473, Mathematics for Critical Thinking. This idea for using the seminar format was supported by the writings of Dorothy Buerk (1985; 1986; 1994a; 1994c; 1995; & 1996) and Buerk with her student, Jackie Szablewski (1993). She had taught a class entitled "Writing Seminar in Mathematics," in which she asked her students to think about mathematics, in groups and independently. The seminar format of the class encouraged the development of a mathematics community. An integral part of the class was journal writing, in which Buerk asked the students to write, not only about

mathematics, but also about their reactions to it. Every two weeks, she read the students' journals and responded to their writings with comments and questions designed to stimulate more thought. Through the journal writing the students were able to establish a personal relationship with Buerk and with mathematics. Code (1991) used an ecological model to describe a nurturing symbiotic relationship such as the one Buerk had with her students. By encouraging them to work in groups and by responding to their journal entries, Buerk was supporting their growth in interdependence as well as independence. However, in her research, Buerk did not explicitly examine the relationships the students may have established with each other or with her to ascertain any effects.

I approached the design of the class with caution, warned by an article by Wallace (1993/1994). She described her experiences in setting up an egalitarian format for a master's level course in education. She assumed that because the students were teachers who were working on a master's degree, that they would have a vested interest in the success of the class. That was not the case. She posits that her mistaken assumption led her to give the group total control of the syllabus, the class schedule, and the criteria for grades. The group decided that all would receive As. As the semester progressed, she realized that the students were not interested in learning; they only wanted a degree so their salary would increase. There were factions within the class, the students did very little work, and they seldom followed through on any of their ideas. Wallace was very discouraged that the students exhibited such apathy, but their grades were already established so there was no incentive to work, or even attend class.

From reading this article I realized that I needed to determine ways to allow students some freedom of choice in the mathematics curriculum while providing leadership in their selections. This led to the development of a curriculum that was comprehensive in its activities while allowing student selection of some topics. The curriculum was centered around a packet of materials that covered many topics in mathematics, but was not all-inclusive. The activities were selected because they encouraged students to think about and explore mathematics creatively.

I also was warned by Lather (1991) that students have a tendency to resist curriculum that may be liberating. Her study was based on research reports, journal entries, interviews, and her own writings, assembled over a three-year period. She draws no conclusions but quotes women who, when writing about a liberating curriculum, use words related to fear, dislike, hard work, new responsibility, and hesitance.

Laird (1994/1995; 1995) also prompted me to be cautious when designing a curriculum that would support mathematical learning for all students. She argues that "equal" education does not always mean that every student is treated the same; girls and women are still treated differently from boys and men in the classroom. My curriculum needed to be sensitive and nurturing so that the individual needs of each student would be met.

"Mathematics has a public image of an elegant, polished, finished product which obscures its human roots. It has a private life of human joy, challenge, reflection, puzzlement, intuition, struggle, and excitement" (Buerk & Szablewski, 1993, p. 151). If students are given the opportunities to experience some of the

private joys of mathematics and to realize that mathematics is not the finished product, but a process that is connected to our inner beings, they may become more "mindful" (Langer, 1997) learners. Langer (1997) says, "A mindful approach to any activity has three characteristics: the continuous creation of new categories; openness to new information; and an implicit awareness of more than one perspective" (p. 4).

CONSTRUCTIVISM AND SOCIOCULTURAL THEORY

Many mathematics educators support the theory that students construct their mathematical knowledge based on their experiences (Cobb, 1994). As I searched the literature to make sense of constructivist theories and to investigate ways to make mathematics more meaningful for my students, the sociocultural theory of Vygotsky (John-Steiner, 1992; Schmittau, 1992) seemed to make the most sense to me. I decided that further study of Vygotsky's theory might open doors to developing a mathematical environment which affords students the opportunity to investigate mathematical ideas within the context of a social setting. This prompts the question: What advantages does this view of mathematics education have over the traditional mathematics classroom? I discovered studies that involved elementary and secondary students as well as preservice and inservice teachers, but none investigated the relationships that college undergraduate students might develop in a sociomathematical classroom.

Vygotsky was a Russian psychologist during the early part of the twentieth century, a contemporary of Piaget. The results of his research have only recently been translated into English. The focus of his studies was the difference between the way children develop knowledge when working with a teacher or a more capable student

and when working alone. This developed into his Zone of Proximal Development (ZPD) Theory which he defined as the "distance between the actual developmental level as determined through independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." (Vygotsky, 1934/1978, p. 86). He determined through his studies that a student solves problems more easily when he is able to discuss his ideas with someone else who can ask appropriate questions to lead the her/him to the solutions.

Schmittau (1992) contrasts this Vygotskian theory that cognitive development is the result of social interaction with that of the Piagetian theory that cognitive development is a natural process as one matures. Vygotskians purport that through dialogic interaction with others, an individual thinks about the concepts and discusses them. As a result, the concepts become more formalized for the individual (John-Steiner, 1992). John-Steiner (1992) also points to multiforms of reasoning that an individual uses in processing information and developing concepts in a social setting.

Wilson, Teslow, and Taylor (1992) proffer the following considerations in developing a mathematics curriculum that fosters student interest. They suggest that we move from a framework based on individual cognition to one based on a social mathematical environment.. According to them students are often taught how to manipulate symbols to determine an answer to a problem without really understanding why the algorithm works; these kinds of exercises should be replaced with mathematical activities relate to the real world. They point to dialogue as important in promoting student understanding and contend that it should be extended beyond

communication between individuals to include communication with technology. These authors also suggest that Vygotsky's ZPD for a student can be spanned through interaction with the teacher and one's peers. This may have the effect of making the student more independent and in control of developing her/his knowledge.

The goals of the research of Cobb, Wood, and Yackel (1990) were to analyze how young children learn in a constructivist environment, how teachers can help students communicate effectively about mathematics, how teachers can learn from promoting this kind of environment, and how they, as researchers, could help teachers create a constructavist environment. The research was conducted with a second grade teacher who was comfortable with the constructivist philosophy. The teacher learned that she had to be flexible and that it was not enough to tell the students what was wrong and what they should do to correct their mistakes. Often her "show and tell" solutions to the students' problems confused them. The researchers determined that teachers and students work together to establish a social environment in which to construct mathematics, which is consistent with Vygotsky's sociocultural theory. The researchers now see their roles as assistants helping teachers "develop forms of pedagogical practice that improve the quality of their students' mathematical education, not to spread a particular philosophical doctrine" (p. 145). This prompted me to re-examine the structure of the curriculum for my study. I needed to be flexible in allowing time and space for negotiation among the students to make their mathematical development productive.

Several researchers (Buerk, 1994a; Bock, 1994; Keikin & Zaslavsky, 1997; Lambdin, 1992; NCTM, 1991) suggested that when students are working

cooperatively in a sociomathematical group, they may become more active learners of mathematics. While Bock (1994) writes of his own experience in the use of cooperative groups in which he established the guidelines for the functioning of the groups, he does report that the students were actively involved in doing mathematics and that they preferred doing mathematics to watching someone else do it. Noddings (1992) cautions that in the establishment of cooperative groups, "we must somehow ensure that the community is a mathematical community" (p. 17). This suggests the importance of curriculum and the responsibility of the teachers to help students establish the sociomathematical norms for viable functioning of the groups.

Cobb (1994) contends that constructivist and sociocultural perspectives of knowledge development actually complement each other. Considering the research of others, he theorizes that the acquisition of mathematical knowledge is actually the product of individual construction and sociocultural interaction (p. 13). While engaged in solving mathematics problems within the context of a sociomathematical group, students may be able to capitalize on the interactions within the group to formulate their own ideas. Through conversation and reflection, students may be able to organize their thoughts to make meaningful contributions to the knowledge base of the group.

Lambdin (1993) indicates that when students work together in a cooperative effort to solve problems, their abilities to solve other problems improved more than when they attempted the problems alone. She based her conclusions on a study she conducted with two preservice elementary teachers working together to solve a mathematics problem. By analyzing the audiotape of their conversation as they

worked the problem, she ascertained how they established the sociomathematical norms and division of tasks as they worked. She determined that their individual roles complemented each other as they collaboratively solved the problem.

Wells and Wells (1992) address the importance of collaborative talk to the development of "literate thinking" (p. 70). Recognizing the importance of the individual's contribution to the emerging knowledge of a collaborative group may enable students to become active participants in the construction of their own knowledge. As members of a group, students also may develop a sense of responsibility for ensuring that all members of the group are given opportunities to contribute to collective body of mathematics information. If knowledge building is at least partially the result of interactions with others, then each individual in a group should have the opportunity to formulate and discuss her/his ideas.

If the mathematics classroom environment is to be one in which students develop mathematical relationships with each other, then there must be opportunities for collaboration and the establishment of sociomathematical norms (Yackel & Cobb, 1996). Yackel and Cobb (1996) describe sociomathematical norms as "normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom" (p. 461). They posit that as a representative of the mathematics community, the teacher has a responsibility to help students develop a sense of what constitutes these characteristics. By acting as a guide the teacher can lead students through mathematical discussions that enable students to differentiate among solutions and develop an understanding of acceptable explanations and justifications of solutions

within the established sociomathematical norms (King, 1994).

In order for a sociomathematical community to evolve, students must be able to trust each other and the teacher (Breitborde, 1996). She theorizes that the members (including the teacher) of a class must prove themselves trustworthy by listening and valuing others' comments. The teacher may have the responsibility to help students establish these social norms; because they lack experience, they may not realize the importance of everyone's having an opportunity to speak. Breitborde (1996), reporting her experiences, writes of creating a community within a multiculture education class. As part of the orientation she used questions designed to help the students understand that we are all minorities sometimes. Through this process, a sense of trust developed among the members of the class that fostered an interest in learning about the many different cultures represented in the class. This trust also resulted in the students' pride in sharing the histories of their families with the class. In a mathematics class a sense of trust may be critical in the establishment of an environment that fosters a willingness among the students to share their private struggles with mathematics.

Trowell's (1994) research with college students in a problem-solving mathematics class concurs that students with the leadership of the teacher can establish sociomathematical norms that contribute to the development of an intellectual community. She observed through videotapes a problem-solving class of university freshmen in which there were no lectures by the teacher. The focus of her research was the means the students used to negotiate the sociomathematical norms of their class environment and the effects on their learning. She determined that the students

began to see the importance of sharing procedures even though there might be computational errors, listening reflectively to each other, and making sense of the mathematics. She also determined that the teacher has a role to ensure that the students establish the norms. Although her research was related to mine, she did not explicitly address the relationships that may develop within a sociomathematical community of learners.

This search of the literature is not exhaustive, but I think it represents the body of knowledge of the impact of establishing a sociomathematical community within the mathematics classroom. We have learned that students appear to acquire more mathematics knowledge in a social environment and that they incorporate this into the ways they think about mathematics. We now must look at the relationships created within these sociomathematical communities and how they impact students in their construction and understanding of mathematics. Looking at the mathematics classroom as a community may change the ways we do and think about mathematics. By creating a nurturing community of mathematics learners using a relational mathematics curriculum, we may open doors for the mathematical empowerment of students.

PROBLEM SOLVING/POSING

The primary goal of the National Council of Teachers of Mathematics has been to foster mathematics education to meet the needs of all students. Through the collaborative efforts of teachers, mathematicians, mathematics educators, and parents, NCTM published its *Curriculum and Evaluation Standards for School Mathematics* in 1989. The goals of this document were to state NCTM's position on mathematics

education to the nation and to provide guidelines for the implementation of a mathematics curriculum that would actively engage students in doing mathematics. To emphasize the importance of mathematical literacy, they posited five goals of mathematics instruction for all students:

1. That they learn to value mathematics,
2. That they become confident in their ability to do mathematics,
3. That they become mathematical problem solvers,
4. That they learn to communicate mathematically, and
5. That they learn to reason mathematically (NCTM, 1989).

With this in mind I searched the literature to find what others had to say about these goals.

Confrey (1990), writing about constructivism, claims that "mathematics is a language of human action," (p. 109) and that reflection on the process of the mathematics is essential for the individual to connect the language (mathematics) with her/his world. Her study was designed to examine the practices of a teacher who incorporated his constructivist beliefs into his teaching in an effort to create a model for moving away from the traditional lecture method of teaching mathematics. The students were high school women participating in a summer mathematics program. The focus was on the interactions between the teacher and the students. Five one and one-half hour sessions were videotaped and two students were interviewed. The teacher designed activities around the content of the lessons to actively engage the students in discussing the problems among themselves and reflecting on the processes. Confrey concluded that this study indicates that there are other ways of guiding

students through mathematics that may be more effective than the traditional lecture method.

Maybe the mathematics content is a contributing factor in perpetuating the traditional lecture method. In searching for mathematics problems that would afford more opportunities for students to investigate mathematics, I was informed by the literature that supports nonroutine problems (Brown, 1986; Buerk, 1994a; Clarke, 1997; Duckworth, 1996; Frankenstein, 1990; Lambdin, 1992; NCTM, 1989 & 1991) as a means of actively involving all students in learning to think mathematically (NCTM, 1991; Schoenfeld, 1992).

Often students have ideas about problems that are different from what the teacher intended. If we can capitalize on these ideas we may open doors that promote the NCTM's goals. Duckworth (1996) speaks of "the having of wonderful ideas" (p. 1). She points to open-ended questions as a means of promoting the having of wonderful ideas. She says "that of all the virtues related to intellectual functioning, the most passive is the virtue of knowing the right answer. Knowing the right answer requires no decisions, carries no risks, and makes no demands. It is automatic. It is thoughtless" (p. 64). If we want our students to learn to make good decisions, become risk-takers, and become more mindful in their learning (Langer, 1997), then maybe we need to provide more opportunities through carefully selected problems for them to develop these characteristics.

The value of not knowing lies in the quest to discover as much as one can about a problem rather than trudge mindlessly toward the 'right answer.' This type of problem solving also requires that students (and teachers) develop courage, insight,

and confidence to become risk-takers and mindful learners. This may not happen if the problem-solving rules are already in place.

Brown (1986) proposes that we move from posing problems (with definite solutions) to suggesting situations in an attempt to encourage students to start asking questions. He says that this strategy allows students to move in two directions--"from situations to posing and from posing to de-posing" (p. 210). He contends that another key to students' developing good problem-solving skills is helping them learn to neutralize a problem to facilitate their efforts to "re-pose" (p. 210) it.

Brown (1986) describes an event that occurred while he was team teaching with a colleague. The problem was,

"$x^2 + y^2 = z^2$ What are some answers?" (p. 207).

As students began responding with Pythagorean triples, all integers, a student posed the triple, 1, 1, $\sqrt{2}$. Others caught on to the student's humor and began suggesting others containing radicals. Brown and his colleague jokingly reprimanded the students and explained that they were looking for rational solutions. In their later discussions they realized that their question carried many implicit connotations and that the students were actually being creative. Brown and his colleague decided that maybe they needed to open more opportunities for students to explore mathematics creatively.

Brown (1986) is critical of those who would avoid selecting problems they perceived as being beyond the students' capabilities. He asks the question, "[W]hat happens if in the creation of a problem from a situation, a student defines a problem

that we know is beyond his/her ability to handle?" (p. 212) He answers that it may not be necessary for the student to solve his own problem because the value of the exercise is the posing itself. He also argues that creativity and intelligence may not be connected, and we cannot always know which problems students are able to solve. The important thing is to give the students an opportunity to try.

Frankenstein's (1990) research involved creating a "critical mathematical literacy curriculum" (p. 336) that would involve students in "confront(ing) various race and gender issues while simultaneously learning basic mathematics." (p. 338) She selected problems that incorporated cultural, political, social and economic issues such as Mandela (p. 339), Castro (p. 340), electric rates (p. 344), welfare (p. 345), and capitalism (p. 345) to draw her students into thinking about mathematics. She reported that there was resistance from her students in the beginning. They complained that all the words and discussion confused them about the mathematics. She attributes their difficulties to the comfort of the traditional mathematics to which they were accustomed. She used their frustrations to show them how much mathematics they really knew. The implications of her research are applicable to empowering all students. We may need to work more diligently to help students understand that mathematics is intertwined in all that we do.

Skovsmose (1994) coined the word "mathemacy" to parallel the word "literacy." He proposes, "If mathemacy has a role to play in critical education, similar to but not identical with the role of literacy, then mathemacy must be seen as being composed of different competencies, a mathematical, a technological and a reflective competence. But especially: Reflective knowing has to be developed to provide

mathemacy with an element of empowerment" (p. 117). This echoes the work of Frankenstein (1990) in her attempt to bring the world into her mathematics classroom.

The major goal of the study of Silver and Cai (1996) was to determine the problem-posing skills of students in middle school. Their subjects were approximately 500 sixth- and seventh-grade students who were given a test with one problem-posing task and eight open-ended problem-solving tasks. To minimize the effect of the order, the problem-posing task appeared as the second question on one-third of the tests, fifth on another third, and eighth on the remaining third. In the problem-posing task, the students were given a situation and asked to write three possible questions that could be answered from the information given. The researchers found that 80% of the students were able to write at least one mathematically sound question, and almost 60% of them wrote three questions. Approximately 25 of the questions were not questions or were not mathematically sound.

In the report of this study, Silver and Cai (1996) indicated a high correlation between students' abilities to solve problems and their abilities to pose mathematically sound problems. This supports the importance Brown (1986) and Frankenstein (1990) place on providing opportunities for students to pose problems.

BELIEFS

I believe that students come to mathematics classes with a set of beliefs, preconceived ideas of what constitutes mathematics. They may view mathematics as computation, algebra, geometry, and all those course names that abound. Their beliefs about mathematics and their mathematical abilities may affect their success or failure with mathematics. My interest in students' beliefs about their efficacy in mathematics

prompted me to search the literature on beliefs to learn what I could that would inform me as I sought to impact my students' perceptions of themselves as students of mathematics? Since my study involved students who planned to teach young children, I was also interested in what the research said about the effect of teachers' beliefs had on their teaching of mathematics.

Research (Bandura, 1986; Bornholt, Goodnow, & Cooney, 1994; Pajares & Miller, 1994; Taylor, 1992) supports my belief that an individual's actions are determined by her/his beliefs about her/his abilities. Bandura (1986) differentiated between self-efficacy and self-concept. Self-efficacy (or inefficacy) involves the beliefs a person has about her/his capabilities of completing a single task; self-esteem is one's measure of her/his self-worth; and self-concept, which encompasses both self-efficacy and self-esteem, is the collective belief about oneself based on experiences and the evaluations of others. While these beliefs are interrelated, they may not be affected by changes in one of the others. For example, a student may be unable to solve a mathematics problem (efficacy) without changing her/his self-concept or self-esteem.

Bandura (1986) theorized that one's belief about her/his abilities is the greatest determining factor in her/his behavior. Pajares and Miller (1994) based their study of 350 undergraduate mathematics students partly on Bandura's theories of self efficacy. In their study they also examined the following characteristics of the students: perceived usefulness of mathematics; mathematics anxiety; math self-concept; prior experience with mathematics, and mathematics performance. The students were unpaid volunteers and responded to questionnaires and problem-solving tests that were all administered within one class period. The researchers used path analysis

techniques to determine the effects of each measure on mathematics performance.

Pajares and Miller (1994) confirmed Bandura's theory that self-efficacy had the greatest effect on performance. They also found that mathematics anxiety was also directly linked to self-efficacy. They suggested that classroom teachers may better understand how to help their students if they assess the students' self-efficacies.

Newman and Tuckman (1997) tested Bandura's (1986) proposal that when students are successful, their self-efficacy and performance improve. They conducted their study with 60 undergraduate teacher education students. The students' task was to write test questions on the class material. The class was divided into two groups with one group coached on how to write good multiple choice questions. The results showed no significant impact of self-efficacy on performance but did show that both self-efficacy and performance experienced parallel improvement in the experimental group.

The purpose of the study of Bornholt, Goodnow, and Cooney (1994) was to determine the effects of gender on students' perceptions of their performance on tests in mathematics and English. The subjects were 116 Australian students in Years 7 through 10. The results of their study indicated that in mathematics the boys were more likely to overestimate their performance than girls. The boys appeared to have a higher self-efficacy than the girls, indicating that girls' performance on a mathematics test is more predictable from their self-efficacy than that of boys. They attributed part of this discrepancy to the pervading notion that boys are better than girls in mathematics.

Ma and Kishor (1997) conducted a meta-analysis to determine the relationships

between students' attitudes toward mathematics and their achievement in mathematics across gender, grade, ethnicity, sample selection, sample size, and time. They searched the literature from major journals to gather the data that met their prescribed criteria. They found that random samples proved more valuable in assessing the relationship between mathematics attitude and achievement and that samples of 300 were adequate for analyzing the relationships. They determined that the relationship between attitude and achievement in mathematics was not affected by gender but was affected by grade level and ethnicity. They said that the results of their study would probably be more meaningful at the secondary level, and that among the ethnic groups, the Asians exhibited the most significant results. They did not determine any reliable evidence of the effect of any interaction among gender, grade, and ethnicity.

My conjecture that students' beliefs affect the way they learn and do mathematics is supported by this research. In establishing healthy relationships with mathematics, it may become necessary for students to examine their beliefs about themselves as students of mathematics.

Since my subjects aspired to be elementary or early childhood teachers, it became important for me to examine the literature concerning teachers' beliefs. We learn from research that teachers' beliefs (Anderson & Piazza, 1996; Brosnan, Edwards, & Erickson, 1996; Brown, 1992; Cabello & Burnstein, 1995; Clark, Moss, Goering, Herter, Lamar, Leonard, Robbins, Russell, Templin, & Wascha, 1996; Copeland, Birmingham, DeMeulle, D'Emidio-Caston, & Natal, 1994; Fennema & Franke, 1992; Mahlios & Maxson, 1995; Manouchehri, 1997; McDermott, Gormley, Rothengerg, & Hammer, 1995; McLeod, 1992; Orton, 1996; Pajares, 1993; Pajares &

Bengston, 1995; Richardson, Anders, Tidwell, & Lloyd, 1991; Rodriguez, 1993; Thompson, 1992) affect the methods teachers employ and the classroom environments they create.

Pajares (1993) asserts that pre-service teachers come into the field of education with preconceived notions of how to teach. He contends that teacher educators have a responsibility to challenge the beliefs of pre-service teachers as a means of helping them identify and clearly define their beliefs. Before people can change their beliefs, they need a clear understanding of what those beliefs are. Pajares does not propose that teacher educators try to force pre-service teachers to change their beliefs but to challenge them to examine their beliefs. The pre-service teachers may choose to change their beliefs once they are cognizant of what they believe.

The study of Brosnan, Edwards, and Erickson (1996) was designed to examine the changes in teachers' beliefs and practices as they implemented the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991). The subjects were four teachers from a middle school with 62 teachers and 845 students. The project lasted two years that included a year of planning, intensive professional development in the summer, and a year of implementation. The data was gathered through interviews, observations, seminars, and surveys. The researchers noted the following changes in the project's teachers' beliefs and practices from the first to the second year: beliefs about mathematics and mathematics teaching and learning; organization of the classroom; the use of technology; and the use of time. The teachers reported that their classes became less lecture-oriented with more small

group work, greater use of manipulatives, and increased involvement of students in discussions and explanations.

Anderson and Piazza (1996) conducted a study of preservice teachers in a constructivist classroom. Their goal was to assess changes in their beliefs about the way mathematics should be taught. The major differences in this class were the use of small groups and the incorporation of manipulatives. Their data came from the journals of 50 students randomly selected from a group of 154. The students were classified into one of four different categories based on their acceptance of constructivism: those strongly committed; those in agreement with some aspects; those who appeared to be noncommittal; and those who were antagonistic. They categorized the students' writings into ten classes. The largest number of students felt they benefited from the use of manipulatives and small groups. Very few mention student discourse as being important. The students did express a decrease in their own mathematics anxiety, an increase in their understanding of mathematics, and more enjoyment and confidence in their abilities to do mathematics. The researchers did conclude that the preservice teachers did experience a change in their beliefs after experiencing a constructivist curriculum. The research of Vacc and Bright (1999) supports this evidence that the program of study for preservice teachers may change their beliefs about the ways mathematics should be taught.

However, in a case study of six elementary teachers, Raymond (1997) reports inconsistencies between the teachers' beliefs about the teaching and learning of mathematics and their practices in the classroom. She said that in all cases the beliefs were less traditional than the teaching practices. This indicates that changes in the

methods of teaching methods will not happen over night.

IMPLICATIONS FOR MY STUDY

The purpose of establishing a relational mathematics classroom is to provide opportunities for students to develop mathematical power (NCTM, 1991) and to find their mathematical voices (Koch, 1996; Miller, 1990). We have examined the different ways of knowing from the feminist literature, the social aspects of constructivist thought, the importance of problem solving and posing, and the impact of beliefs on learning and teaching mathematics. Throughout all of this, dialogue and writing have played an important part. Based on the literature in this chapter, I have attempted to create a relational mathematics environment in which students may find their mathematical voices and develop mathematical power. The methods I used are discussed in the next chapter.

CHAPTER 3

METHODOLOGY

Langer (1997) posits that rather than proceeding directly from the problem to the answer, a "mindful" person remains open and flexible to optional perspectives. As an example of Mindful" problem solving, she tells of an entrepreneur who learned of a machine designed to spray crops but failed because it froze them. This man saw the possibilities created by the machine's failure and turned it into a snow-making machine. He was not blinded by the machine's intended purpose.

She speaks of the freedom we have to be "mindful" thinkers when we do not know all the "rules." Her example is that of a young student who had not been told that the sum of the angles of a triangle in a plane is 180°. The student consistently gets 183° when measuring the angles of her triangle. When the teacher says that number is impossible, the student asks, "Then why do the lines on a globe that are perpendicular to the equator intersect at the poles making a triangle with more than 180° in the sum of the angles?" The teacher then explains that their triangles were in the plane. The student was free to be "mindful" until the teacher imposed the rigid "rules." The teacher missed an opportunity to let the students explore the geometry of curved surfaces. Indeed the student was measuring on the carpet; this accounted for the extra degrees.

In light of this I selected mathematics problems and exercises for a college level mathematics class that would foster "mindful" learning. (See Appendix D.) One technique that may enable students to become good problem solvers is selecting problems that incorporate the use of patterns (NCTM, 1991). Observing patterns in

nature such as the Fibonacci sequence may prompt students to look for number patterns that may lead to the solutions of problems. In designing the curriculum for this class I also considered problems that would involve multiple techniques and would foster the cooperation of two or more students in pursuit of the solution. Relationships among the students and me that could lead to positive relationships with mathematics became the focus for the selection of problems. As the semester progressed, some problems were discarded and others resequenced to meet the needs of the evolving learning process and student interests.

CONTEXT

MATH 1473, Mathematics for Critical Thinking, is a general education course that fulfills the mathematics requirements for most majors in Arts and Sciences and Education. Traditionally this course enrolls approximately 300 students in four large sections with eight recitation sections in the fall semester and 225 students in three large sections with six recitation sections in the spring semester. The sections are taught using the lecture format with a standard textbook. The purpose of the recitation sections is to provide an opportunity for students to receive help and to have their questions answered.

The prerequisite for taking MATH 1473 is passing a placement examination or successfully completing a remedial non-credit mathematics course. Approximately ten to twenty-five percent of the students who enroll in MATH 1473 fail to complete the course with a passing grade.

THE PILOT CLASS

In the fall of 1996 a pilot section of MATH 1473 was implemented. The

concept of mathematics problem solving in a smaller classroom context was presented to a larger section, and fourteen students volunteered to participate. In this section of MATH 1473 the students were introduced to problem-solving. Unlike traditional mathematics classrooms where the students sit in rows, the students and the teacher sat around a table in a seminar format (Buerk, 1985; 1986; 1994a; 1994b; 1994c; & 1995). Rather than working alone, the students worked in small groups and reported their solutions to the class.

In the small groups, everyone was able to participate, and each student contributed to the solutions of the problems. As the semester progressed, the students became more comfortable talking with each other about mathematics. They were eager to discuss the assignment problems and often were engaged in mathematical discussions with each other as they entered the classroom. Although the mathematics learning environment was more relaxed and less structured than traditional mathematics classes, the students worked with a sense of purpose. This type of mathematics class was something most of them had never experienced. Students who had never felt comfortable asking questions in a mathematics class were willing to offer suggestions that might lead to the solution of a problem.

The problems were designed and selected to incorporate multiple steps and were often open-ended with multiple solutions. The purpose was to encourage the students to engage in dialogue and to examine their own strategies (NCTM, 1991). Were there patterns? Had they observed these patterns in other problems, or were they new? What new questions arose from the solution to a problem? Through the use of manipulatives (NCTM, 1991) and discourse (Keikin & Zaslavsky, 1997;

NCTM, 1991) the students were able to reach a consensus, and each felt that she/he contributed to problem solutions.

Writing (Buerk, 1985, 1986, 1994a, 1994b, 1994c, & 1995; Buerk & Szablewski, 1993; & Countryman, 1980) was an integral part of the format of the pilot class. Through the use of journals the students discussed their solutions to problems, recounting a step-by-step discussion of their process, including the tactics they tried that did not work. They included explanations of why they thought particular tactics did not work while others did. An integral part of their journal entries was a discussion of their reactions to the problems. What was it like to struggle with a problem and finally arrive at a solution? How did their successes make them feel about themselves as doers of mathematics? Their responses to these questions led me to believe that they were growing more comfortable in their relationships with mathematics.

RELATIONAL MATHEMATICS CURRICULUM

As the semester progressed with this pilot section of MATH 1473, it became clear that the students were becoming more comfortable with mathematics. They expressed excitement at their successes and were able to explain their solutions to the class. Some commented that for the first time in a long time, they actually liked mathematics. They were developing relationships--with me, with each other, and with mathematics.

In reflecting on my experiences with this class, the students' comments in class, and their final papers, it became apparent that this class may have made a difference in the ways that these students perceived themselves and their abilities to think about, to

know, and to do mathematics. In light of a relational mathematics curriculum, these reflections prompted the following questions.

1. How do students experiencing a relational mathematics curriculum become more mathematically empowered, specifically, develop their ability to do mathematics and become more confident?
2. How are students' beliefs about themselves as students of mathematics affected by experiencing a relational mathematics curriculum?

With these questions in mind I decided that teaching another section of Math 1473 in the spring semester of 1997 might clarify some of the questions that arose during the semester with the pilot group. Did a class based on a relational mathematics curriculum really make a difference in students' understanding of mathematics and their views of themselves as students of mathematics? Would another group of students have similar experiences and reactions to a relational mathematics curriculum?

SELECTION OF PARTICIPANTS

In the pilot class, five of the fourteen students were elementary or early childhood education majors. In their final essays, these students expressed attitudes (Bandura, 1986; Bullough & Baughman, 1995; Pajares & Miller, 1994; Pajares, 1996) of being much more confident in their abilities to do mathematics as a result of being in the experimental class. Since MATH 1473 is a prerequisite for EDMA 3052, Introduction to Mathematics Education, I was interested in how other students who were majoring in elementary or early childhood education would respond to a relational mathematics curriculum. Would they have similar experiences? Would the

experiences in a relational mathematics class change their perceptions of the ways mathematics should be taught?

The request for volunteers to participate in a second experimental section of MATH 1473 was presented to the two large sections. To qualify for selection to participate, the students

1. Should be planning to major in early childhood or elementary education.
2. Must be willing to participate in this study.

DESCRIPTION OF PARTICIPANTS

The participants in this study were eleven young women who volunteered from the two larger sections of MATH 1473 to be part of a second experimental section. (See Appendix G for the human subjects' letters and approval forms.) Of these students, all had taken two years of algebra and one year of geometry in high school, an admissions requirement to the University of Oklahoma. Three had taken college algebra, and one had taken calculus in both high school and college. Seven of the students were given permission to take the class because they passed the mathematics department's placement test. Of these seven students, three were required to take Math 0123, a remedial non-credit mathematics class before taking Math 1473. One of these three took Math 0123 twice and another, three times, before passing the final examination required for permission to take mathematics classes for credit.

Two of the students were early childhood education majors, while seven students were majoring in elementary education. One of the elementary education majors was minoring in early childhood education, one was minoring in Spanish, and

one was seeking a mathematics endorsement and potentially a Spanish endorsement. One student was majoring in speech pathology with a minor in elementary education, and one student was undecided but interested in working with young children. Because of their interest in elementary education and their willingness to participate, these last two students were included.

CLASS ORGANIZATION

The class met two days a week (Monday and Wednesday evenings) for 1 hour 40 minutes (6:00-7:40 p.m.). There were 30 class sessions beginning January 13, 1997, and ending April 30, 1997. (See Appendix A for syllabus.) Such a detailed syllabus is not consistent with a constructivist philosophy and is not representative of the way the class was conducted. Because this was an experimental class of Mathematics for Critical Thinking, the Mathematics Department requested a syllabus. This syllabus, which was given to the students at the beginning of the semester, represents the order that evolved in the pilot class. There are some similarities between the syllabus and the order of topics, but as in the pilot class, the topics evolved as the semester progressed to meet the mathematical needs and input of the students.

As in the previous class, a major theme of the class was the search for patterns as a means of solving problems. The pattern themes incorporated number patterns (Pascal's triangle, primes, Pythagorean triples, Fibonacci numbers), resulting algebraic patterns, and geometric patterns (points, intersecting lines, regions of a circle, building with cubes). The purpose of using patterns to model mathematical situations was to help the students develop a variety of tools for solving problems.

The class utilized a seminar format with the students and the instructor (me) sitting around a large table for discussion. To establish a sense of camaraderie, I requested that we all call each other by our first names. I wanted them to view me as a learner and colleague as well as a teacher and leader of discussions.

Another important component of this class was the emphasis on mathematical discourse. Building on my experiences with the previous class, the students were asked to divide into groups of two or three for the class assignments. The composition of the groups was determined by student choice. The mathematics activities were designed to involve students in cooperative learning and group work. By having an opportunity to work through their trials and errors in a semi-private setting, the intent was that students would learn that it is acceptable to be unable to solve a problem quickly. They may also have learned through working together and by listening to what others in the group said about the problem to think of other ways to approach the problems. In small groups the students may have been less likely to fear criticism if their profferings did not work for a particular problem.

Writing as a component of discourse was an important aspect of the students' experiences in this class. (Buerk, 1985, 1986, 1994a, 1994b, 1994c, & 1995; Buerk & Szablewski, 1993; & Countryman, 1980) can provide a vehicle for students to examine their approaches to solving a particular problem. By organizing their steps, they can more easily identify the strategies that did or did not work. Following each class the students were asked to write as a journal entry their group's solution to the problem. In this journal entry, they were asked to include a discussion of the group's strategies along with any difficulties they encountered, a synopsis of what they learned, and their

reactions to the lesson. How did the lesson fit in with previous lessons? What did the lesson contribute to their understanding of mathematics and of themselves as learners of mathematics?

Another organizational strategy was to utilize a journey metaphor to ground the students' experiences with mathematics. To journey is to "[t]ravel from one place to another" (Morris, 1976). In our journeys we sometimes go directly to the point of our destination, and sometimes we take side trips for sightseeing, relaxation, or rejuvenation. Occasionally we become lost and must retrace our path, choose an alternate route, or ask for directions. We meet interesting people who enrich our lives by sharing stories of their journeys. The learning of mathematics is like a journey: we begin in one place (that of not knowing some mathematical concept) and travel to another (one where we know and understand the concept). The previous experimental class explored metaphors as a powerful means of describing mathematics learning experiences, but the journey metaphor was not explicitly utilized to help frame their experiences. The need for common language with which to describe experiences was evident, and the choice of the journey metaphor also fits with the narrative and the historically connected focus of this class. (See Appendix B for a description of my journey.)

The students kept journals recounting their mathematical journeys in this class. In addition to the class assignment entries, they were sometimes given individual problems to work outside class. For these problems they were permitted to work together or ask other people. They were asked to submit a solution, to describe their approaches, with results, and to discuss their reactions to the problem. These

reactions should have included their first thoughts after reading the problem, descriptions of their struggles as they moved toward a solution, and their feelings when they finally determined a solution. In other journal entries the students responded to questions designed to promote thought about their mathematical histories and journeys. (See Appendix C.) The purpose of these questions was to enable the students to assess their progress with mathematics prior to and through participation in this class.

The students also submitted several papers during the semester. In the first paper the students recounted their mathematical histories detailing what they knew about mathematics prior to the beginning of this class, their attitudes toward mathematics, and their experiences with mathematics that informed their attitudes. They were encouraged to be candid and specific without using the names of former teachers. I did not want to know who these teachers were, and I wanted the students to focus on themselves and their relationships with mathematics.

The second paper was a report on the history of a woman mathematician. Since the class was composed of only women students, it seemed appropriate for them to have an opportunity to examine the struggles of some of the women who have contributed to the body of mathematical knowledge. The third paper was a fantasy putting their woman mathematician in *Flatland* (Abbott, 1998/1994) which we had read and discussed. The fourth paper was a critique of the film "Donald in Mathmagic Land" (The Walt Disney Company, 1959).

Their final paper was a recounting of their mathematical journeys, including their experiences in this class. (See Appendix E for the final examination questions.)

The questions for this paper were designed to enable the students to reflect on their growth in mathematics and changes in their perceptions of themselves as students of mathematics, and to think about how they will address future encounters with mathematics.

DATA COLLECTION

When we look at constructivist research, we realize that research settings, the participants, and the researcher are all in a process of flux. The dynamics are constantly changing (Guba & Lincoln, 1989). Thus we considered the changes in the students' statements about their beliefs (Bandura, 1986; Bullough & Baughman, 1995; Pajares & Miller, 1994; Pajares, 1996) about themselves as students of mathematics throughout the semester. Their journal entries, their three major papers, and audiotapes of class discussions, as well as my journal entries, were utilized as an on-going record of student development in the course. Audiotapes of class discussions and my journal entries also provided sources of data.

In addition to these data sources, there was an audiotaped follow-up meeting of the class in the fall. The purpose of this meeting was to allow the students to share with each other any postscripts they wished to add to the previous class discussions, and to share their reflections on the class and any on-going experiences with mathematics. I wrote individual letters to the students giving my assessment of their progress in the class. (See Appendix J for the letters to Colleen, Fran, Jill, Katie, and Linda.) They were given time to read their letters and an opportunity to offer corrections and additional comments about my analysis and conclusions. There were also questions that I asked them to discuss (See Appendix I for the questions.) during

this follow-up meeting.

DATA ANALYSIS

Constant comparative methods of multiple data sources, i. e., the students' journal entries, audiotapes of some class discussions, the instructor's journal entries, and the students' papers, were utilized and organized according to the research questions. As the semester progressed, and I compared my notes with the students' journal entries, papers, and mathematical discourse in class, it appeared that they were building a community in which everyone had an active role.

The first class meeting was the contracting stage, in which I led the students in a discussion of the characteristics of a classroom in which learning is a priority and students feel comfortable making contributions to the class discussions. As we discussed the expectations that I had listed in the syllabus (See Appendix A.), the students contributed the following additions.

1. Be sensitive in offering constructive criticism.
2. Appreciate the opinions of others.
3. Everyone should participate in the discussions.
4. Exhibit a positive attitude. (Leave your troubles outside the door.)
5. Don't be afraid to ask questions.
6. Work toward bringing out the best in each other.
7. Be willing to discuss your work with others. (My Journal Entry (JE) 1.)

The negotiations continued throughout the semester as they worked on the class problems in their groups and decided which person performed which task and which

solution suggestions merited consideration.

As the semester progressed, and I compared my class notes with the students' journal entries, it became apparent that the students shared a common experience, as evidenced by the parallels among their comments in class and their journal entries. There were also indications of changes in their attitudes about mathematics and, more importantly, growth in their understanding of mathematics. The excitement they displayed when they recognized a pattern or solved a problem perpetuated a positive climate in the classroom.

The components of my research that support the trustworthiness (Guba & Lincoln, 1989) of the data include prolonged engagement, persistent observation, peer debriefing, negative case analysis, progressive subjectivity, and member checks (pp. 237-239). By actively engaging the students in a continuous cycle of problem solving, analysis of their actions, and discussion of the surrounding activities, I was able to observe the students over a long period of time. Throughout the semester I discussed the class events and activities with mathematics educators and others as a means of determining the progress of the group.

> With this class, as with the pilot class,
>
> I envisioned an opportunity for me to share my love of mathematics with some college students who might not share my enthusiasm. My hope was to create a curriculum and an environment that would enable students to develop healthy relationships with mathematics. I also hoped that the events of the semester as together we explored different avenues of mathematics might change their perceptions of mathematics and their relationships with it (My Mathematical

Journey, Appendix B).

Rather than ask them directly if, or how, their views of mathematics were changing, I asked them to describe their ideal mathematics class or to discuss the differences between this class and other mathematics classes. As I read their essays I looked for clues that their ideas were, or were not, changing. I discussed these essays with others to determine possible changes that would influence their perceptions.

During the semester there were times when the problems seemed too difficult for the students. Since their problem-solving skills were evolving, I wanted the problems to be challenging but not impossible for their skill levels. By re-evaluating the difficulties they experienced, I was able to continue selecting problems that both helped build their problem-solving skills and boost their self-confidence.

As students worked in their groups, I observed and listened to assess their progress. Occasionally it became necessary for me to intervene with relevant questions to redirect their approach to the solution of a problem.

The next chapter is an account of our journey through Mathematics for Critical Thinking. During the time we spent together, relationships evolved. We became friends in search of solutions to mathematical conundrums. As a result some of us developed new relationships with mathematics.

CHAPTER 4

MATHEMATICS FOR CRITICAL THINKING

In this chapter we will look at the students, the mathematics, our interactions with mathematics and each other, and the changes that occurred as the students moved toward mathematical empowerment. The following questions will guide us as we examine this class of Mathematics for Critical Thinking.

1. How do students experiencing a relational mathematics curriculum become more mathematically empowered, specifically, develop their ability to do mathematics and become more confident?
2. How are students' beliefs about themselves as students of mathematics affected by experiencing a relational mathematics curriculum?

As we consider these questions, we will use a journey metaphor to move through the semester, looking at the characteristics of the students as they entered the class, some of their encounters with mathematics and how they worked together to solve problems, and changes they experienced in their confidence and abilities to do mathematics. We will see that as we worked together to solve problems, the students began to recognize patterns and build on these patterns to solve other problems. By the end of our journey most of the students had become better problem solvers and expressed a greater confidence in their mathematical abilities than when they began the semester.

PREPARATION FOR THE JOURNEY: Getting Acquainted

This is the account of a mathematical journey. When we began this journey, I was not sure where we would go, by what means we would travel, or whether our paths would diverge or converge. However, there was a beginning with a focus that would lead us through some rough terrain. We would be challenged by problems that would require that we struggle, combine our resources, and work to ensure that all would accomplish the tasks encountered. The problems selected were consistent with Doll's (1993) criteria that he called the four Rs--richness, recursion, relations, and rigor.

The richness of our journey was provided by our search for patterns that would enable us to solve the problems we encountered. The search for patterns had a built-in recursive component that allowed us to revisit some patterns in situations that seemed to have no connections. Some of the patterns we encountered were related to Pascal's triangle, Fibonacci numbers, and figurate numbers, for example, triangular, square, and pentagonal numbers. This promoted thought and discussion of the mathematical relations that abound in our world. The rigor of the problems was such that the students were compelled to think and discuss possible approaches to the solutions.

On this journey I took along with me the students of a small class of Mathematics for Critical Thinking. As we wound our way through various mathematical pathways, relationships were created, refined, and polished. Our journey is the story of how our relationships with each other impacted our relationships with mathematics, sometimes in surprising ways. Our journey is reflected through our

journal entries, class discussions, and the students' papers.

Our mathematical journeys merged in mid-January, and as we embarked together, we held open discussions of the responsibilities we had to each other. I had listed some of my expectations for the class in the syllabus (Appendix A), and as we discussed these, the students expressed their needs for being comfortable in a mathematics class. To make the journey successful for each of us, we determined that we should all work towards making the environment nurturing and comfortable for everyone. Each of the students wanted to be free to discuss solutions to problems without fear of ridicule. To ensure that the journey would be beneficial to all, we agreed that we each had a responsibility to attend all sessions and be prepared to participate in the discussions.

It was somewhat disappointing that even though we had emphasized the importance of attendance and participation, the average daily attendance was 9.4 of 11 students or 85.7 percent. Three of the students attended less than 80 percent of the classes while four attended more than 90 percent, including one student with perfect attendance. There were only 5 days of 29 when everyone was in attendance. This affected our efforts to maintain continuity in our discussions.

As a preface to our journey, we introduced ourselves and gave an overview of our mathematics histories and goals. While the students' experiences were varied, there were some similarities. All of the students were elementary or early childhood education majors except Abby and Sue. Abby was minoring in early childhood education, and Sue was interested in working with children in some capacity. All but

Katie had experienced difficulties with mathematics at some point in their lives. For most these students, their difficult experiences marked the beginning of their dislike of mathematics.

It was interesting that all of the students had credited both their good and bad experiences to teachers. The good experiences were punctuated by teachers who knew how to explain mathematics and took extra time to work with the students. The bad experiences were attributed to teachers who had taught straight from the book, who allowed only one way to solve problems, or who had embarrassed the students in front of their peers for not understanding mathematics. These are the words of some of the students.

Katie: My ninth grade geometry teacher was so organized, easy to follow, understanding and personable that she is the reason why I decided to be a teacher (Katie, JE 1).

Linda: I was very lucky, however, because my teacher ... was willing to spend as much time with a student as was needed to help them understand. The most wonderful thing that she did for me was to show me different ways of working the same problem (Linda, JE 1).

Colleen: At that point my teachers weren't really concerned with my math skills as much as they were with my reading and writing skills. Maybe that was the beginning, I don't honestly know when math became such a difficult subject for me (Colleen, JE 1).

Fran: I got exactly what I needed in a class, credit, and teacher: patience,

> thought, and understanding. ... Then life threw in another mathmatic (sic) obstacle as I entered my algebra two classroom. This teacher was not so patient. At this point I had developed a good base and was confident in my skills. That all changed the moment these five words were spoken in my direction, "Are you ignorant or something?" How any teacher could be so cruel, as to humiliate a student in front of all their peers, got all over me. So, there I was shot back to zero (Fran, JE 1).

Others expressed similar sentiments.

Even though all of the students had taken geometry and two years of algebra in high school to fulfill the university's admission requirements, three of them were placed in the non-credit remedial classes as their first college mathematics class. Of these, two had taken the remedial class more than once before passing the test for placement into Mathematics for Critical Thinking.

THE TRAVELERS: Participant Descriptions

The descriptions of these mathematical tourists come from both the students' and my journal entries, class discussions, the students' papers, a post-class meeting held the following fall, and some telephone interviews. I have selected five of the eleven students as representative of the group to provide detailed accounts of their responses. Among the students there were several who had many common characteristics and experiences; they are represented by Jill and Fran. I selected the other three, Colleen, Linda, and Katie, because of the ways their unique characteristics impacted the class.

Jill

Jill represents those students who came to class most of the time, actively contributed to the class discussions, and viewed mathematics from a practical stance. To her mathematics was associated with stockbrokers, accountants, balancing checkbooks, flying planes, averaging grades, and calculating distances (Jill, JE 1). In her first essay describing her mathematical history, she told of "time tests" in multiplication and how she practiced every night using flash cards and a calculator to prepare for these tests. She was convinced that mathematics learning came about through "practice and patience" (Jill, Final Examination (FE)). Later in the semester when we were discussing the students' earliest memories, Jill mentioned this time period. Again when asked in a journal to recount a time when she felt most comfortable, she gave the same response (Jill, JE 15).

Throughout the semester, Jill worked diligently on the assignments and actively participated in the class discussions, but always checked her thoughts with some of her classmates or me before making a commitment to her own ideas. This was a reflection of her earlier quest for a perfect paper on the multiplication facts. While she subscribed to the social aspects of learning mathematics, Jill still placed the responsibility for learning mathematics on the teacher. In her essay on the ideal mathematics classroom she wrote, "It is the students responsibility to learn mathematics, but it is the teachers job to make sure they are responsible by educating the students and creating a stable education for them" (Jill, JE 19).

Another illustration of Jill's practicality was her response to the class

assignment to prepare and teach a lesson to her classmates. Jill's lesson dealt with writing checks and balancing a checkbook. She had made and stapled together sample checks and a check register. She did not actually teach the lesson; she just told us what she would do when she was the teacher. When asked on the final examination, "What was the most significant topic or lesson in this class?", Jill cited this lesson plan. She told again, in detail, what she would "tell" her students to do. She commented,

> I found it to be very beneficial because it really applies to the work that I will be doing as a teacher someday. By creating my own lesson, I felt that I had accomplished a small step of the road to becoming a teacher. ... I am a creative person and I wanted to create a lesson that would be fun but also benefit my students (Jill, FE).

Jill left the class at the end of the semester with the same preconceived ideas about what it means to learn and teach mathematics that she had at the beginning of the semester: Mathematics must be practical; the teacher must tell the students what to learn; and through much practice the students will master mathematics.

"Jill said that she had always liked mathematics, but it has not always been easy. She credited her limited success to perseverance and hard work. Her ideas of mathematics revolve around applications that involve computation" (My JE 1). "Without math, it would be impossible to balance checkbooks, fly planes, average grades. ... [M]ath can determine how many billions of miles a star in the sky is from earth" (Jill, JE 1). "I hope that she will change her perceptions of mathematics and learn to appreciate the other aspects of mathematics" (My JE 1).

Fran

Fran represents the few students who were vocal about their dislike of mathematics and their inabilities to learn mathematics. "Tonight Fran was outspoken in her dislike of mathematics, saying she had no confidence in her abilities to do mathematics. She attributed her great distaste to many unpleasant experiences with high school mathematics. She expressed much frustration at what she saw as an inability to grasp mathematical concepts" (My JE 1.) Fran also blamed a teacher who had asked her if she were ignorant (Fran, JE 1). "I hope Fran will learn mathematics in a way that will make a difference in her perception of mathematics and herself as a student of mathematics" (My JE 1).

In spite of her lack of confidence in her mathematical abilities, Fran did express excitement about being in this class. As the semester progressed, she frequently expressed her elation and surprise when she 'discovered' some mathematical tidbit. For example, when we were counting the squares on a geoboard, Fran recognized the algebraic pattern for determining the number of squares in a rectangle of given dimensions. She practically shouted out her discovery (My JE 14).

When working with others on a problem, Fran assumed a leadership role. She never hesitated to try her ideas and never expressed embarrassment when her suggestions sometimes failed. She expressed much creativity in her approaches to solving problems. Fran had fun with mathematics and told many interesting stories about the people in her life with whom she shared her assignments. She told how she gave one of the problems to the ten-year-olds on her sister's soccer team. Some of

them got out their calculators and worked on it. She said that they had fun working together to solve the problem (My JE 3).

Linda

Linda is in a category by herself. She had taken four years of precalculus mathematics but, unlike Katie, had not taken calculus. For Linda,

> Math used to be a subject that was just like all the others to me. It came easily. It was what some people would call an 'easy A.' I never had to study, and I always did very well. Then came algebra. Everything changed after that. I still made A's in all my math classes, but it wasn't easy any more. I had to really work at it. I studied at home, and I had to get extra help from my math teachers (Linda, JE 1).

In this journal entry, Linda writes of the support she had from her teachers. In spite of her difficulties, she never lost confidence in her mathematical abilities. "I guess I never had any 'break through' moments, but I feel that my accomplishment is knowing that even though something is not easy for me, I can still succeed" (Linda, JE 1).

Linda's experiences with mathematics have been good, and although she has had some difficulties, she likes mathematics. In her history (Linda, JE 1) she depicts herself as a hard worker who is willing to spend as much time as she needs to ensure her success. I expect her to work hard and anticipate that she will discover that she does have mathematical abilities" (My JE 1).

Linda was unique in that although she was reticent, when she did speak she expressed herself quite well. When working in groups, Linda was a follower. Even

though she knew more mathematics than some of the students she worked with, she was hesitant to offer her thoughts unless asked a question. She seemed to need to know the solution to a problem before offering her suggestions.

In her journal entries, Linda was thorough in her discussions of the assigned problems and readily expressed her frustrations with her inabilities to solve a particular problem. "Now I can honestly say that I'm beginning to get frustrated" (Linda, JE 3). "This is a really frustrating problem" (Linda, JE 4). "I have been trying to figure out this problem for about 45 minutes. I am very, very frustrated. I feel like throwing these stupid pennies across the room" (Linda, JE 11). In spite of all her frustrations, Linda never quit. Many times just a few clues would prompt her to move in the direction of a solution to a problem.

Colleen

I chose to introduce Colleen because she struggled more than the others. The first weekend of the semester one of her sorority sisters died in an accident, and Colleen missed two classes to attend the wake and services. This tragedy made it difficult for her to concentrate when she was in class. Later in the semester she had severe dental problems that interrupted her efforts to learn. Through all of this she made progress; she began to participate more in class, and her confidence in her abilities to solve problems began to grow. She still has much to do in her journey, but she has a good start.

My first impressions of Colleen are recorded in my journal.

When Colleen entered the room tonight, she sat down and lowered her eyes,

> avoiding eye contact. I see her as shy and vulnerable, unsure if she really wants to be here. She said that she got really nervous and scared just thinking about taking a mathematics course. She was afraid that she would be made to look ignorant and would be embarrassed (My JE 1).

Colleen had attended a Spanish-speaking elementary school, and when she transferred to regular school, she had to relearn everything in English. She said that her teachers worked more on her reading than mathematics. In her first journal entry she stated that her biggest fear was being called on in class because many times she had been embarrassed in class for not knowing the answer to a question. "Math to me is something I have always struggled with my whole life, but it is something that totally amazes me" (Colleen, JE 1). "There is hesitance, but I hope enough interest to draw her into mathematics" (My JE 1).

Katie

Katie really likes mathematics and wants to teach it in elementary school. I was interested in her interactions with the other students because she knew much more mathematics than the other students and had a confidence in her abilities that the others lacked. "Katie is a veteran of mathematics, having taken calculus in high school and college. She had already taken one semester of mathematics for elementary teachers and wanted an elementary mathematics endorsement" (My JE 1). For Katie, mathematics "has become second nature" (Katie, JE 1). "I wonder if Katie will be challenged enough in this class. It is important for this class to be a meaningful experience for her as well as for the others" (My JE 1).

As the semester progressed I observed that Katie watched and listened to her classmates as they solved problems. Even when she knew how to solve a particular problem, she let her partners experience the struggle of assuming ownership of mathematics. Only when asked specific questions did she offer suggestions. She was kind, sensitive and patient with the others and never made any of them feel that they could not solve a problem. She seemed to revel in their successes as well as her own. For these reasons, Katie was a valuable member of the class.

EVENINGS AROUND THE CAMPFIRE: Building A Relational Mathematics Environment

The class was arranged in a seminar format with everyone seated around a large table. Everyone could see each other, and I sat with them to convey the message that we were all in this journey together. When the students worked on in-class assignments, they worked in groups of two, three or four where they were, or they moved to another table with their group.

The first evening of our journey was a time of getting to know each other. I thought that it was important for us to establish a rapport with each other that would facilitate the learning of mathematics. Some of the students knew each other from other classes and their sororities; this enabled us to build on existing relationships. In order for us to establish a comfortable environment that would promote mathematics learning, we agreed that we needed some rules. It was important for these rules to be flexible to accommodate unexpected events. Building on my expectations listed in the syllabus, the students proposed the following additions.

1. Be sensitive in offering constructive criticism.
2. Appreciate the opinions of others.
3. Everyone should participate in the discussions.
4. Exhibit a positive attitude. (Leave your troubles outside the door.)
5. Don't be afraid to ask questions.
6. Work toward bringing out the best in each other.
7. Be willing to discuss your work with others. (My JE 1.)

In the beginning of our journey some students were like Colleen, who sat quietly, listening to the others in her group as they discussed the solutions to problems. Colleen was shy, insecure, and not really sure she wanted to be in a mathematics class. As the semester progressed, she began to loosen up a little and to ask questions in class. In her final paper she expressed the freedom she felt when she was working with her classmates. "I was always too scared or not allowed to share problems with my classmates. It also makes me feel good when I can show another person how to do something that I had understood" (Colleen, FE). This indicates that Colleen had developed a relationship with the members of the class that allowed her to grow mathematically. She further states, "[T]his class has made me not only a more confident and better math student, but a better all around student and person. ... I also realized I like sharing my thoughts and ideas with other people whereas before this class I didn't say a word to anyone unless they talked to me first or asked me a question" (Colleen, FE). There were other students who would echo these same sentiments.

Fran represents the students who were quite vocal about their dislike of mathematics. In spite of her distaste for mathematics, she began to realize that she could learn mathematics and that learning gave her a good feeling. After the first few weeks, her entrance into the classroom was almost a production; she had to tell the class how she had shared the problem with someone else. It was good to see her develop an enthusiastic and positive relationship with mathematics.

Fran was an active participant in class discussions and group assignments, often assuming the role of leader. She valued the support and encouragement of her classmates. In her final paper she wrote, "[I]t is important that they know their responsibilities. Encouragement to fellow class mates to keep on trying [and] compassion to those who might need a little extra attention ..." (Fran, FE). In her paper on *Flatland*, Fran also shows that she values cooperation in the classroom by having four girl students learn to form a square so they could go to school as a boy.

Even though Linda was quiet, she was actively involved in the group activities of the class. She acknowledged the importance of developing mathematical relationships with others in the classroom when she wrote, "[K]nowing everybody makes me feel more confident and not afraid to speak out. Also, hearing each others ideas helps me a lot. Someone else might work a problem in a way that I would never think about. The method that they use might be easier for me to understand. It also gives me an idea of how other people think." (Linda, FE). She also expressed the importance of being able to share her ideas without fear of ridicule.

In her final paper, Jill spoke of the importance of classroom and group

discussions to the learning of mathematics. However she felt that it was the teacher's responsibility to make sure that the discussions moved in a predetermined direction. "It is their [the teacher's] job to motivate the students and encourage their studies and participation. ... But as a student it is vital to pay attention and be corapritive (sic) with the teacher" (Jill, FE). Jill still carried the ideas that the teacher was supposed to tell, and the students were supposed to learn. I do not think she realized the value of her own thinking and problem solving skills.

Because of her solid background in mathematics, I wondered how Katie would interact with others in a small group setting. Often students who have had less than pleasant experiences with mathematics are intimidated by those they perceive as smarter than they. I was concerned that she would take over and solve all the problems quickly and then tell the others what they should have done. This was not the case. Rather than working the problems for the other students, she observed them as they worked. When asked questions, she offered hints and suggestions rather than answers. By allowing them the opportunity to struggle until they solved a problem rather than giving them a quick answer, she helped them renew their faith in themselves as students of mathematics. Katie's positive attitude and willingness to help others made a difference in the climate of the classroom. Her manner and patience were helpful rather than threatening to the others in the class.

While the relationships that developed within the classroom were varied, the students indicated that these relationships had a positive impact on their understanding and their relationships with mathematics. Their increasing abilities to recognize

patterns and solve problems support this theory.

THE JOURNEY: Developing Mathematical Empowerment

In our mathematical journey, the students and I were embarking on an adventure in which there were no lectures. The assignment problems (See Appendix B for examples.) were non-routine and rigorous problems which often had an unspecified number of solutions. The problems frequently required collaboration among the travelers to reach a conclusion. The general theme of the journey was a search for patterns in the problems to determine if there were clues that would lead to a solution. The students were encouraged to work together both in and out of class and to discuss the problems with people who were not in the class.

In the beginning of our journey, several of the students were apprehensive about their abilities to contribute to the discussions, but as we all got to know each other better, they loosened up and began offering their ideas. Throughout the journey, the students were asked to write detailed discussions of solutions to problems. As our journey progressed their journal entries became more detailed and explicit. They were also asked to respond to journal prompts that were designed to keep them focused on their mathematical journeys.

To encourage the students to see their relationships with mathematics in preparation for this journey, it was important for them to look at their past histories with mathematics. For their first journal entry they were asked to describe their journeys from the beginning of their memories of mathematics. All of the students wrote about having good experiences with mathematics in elementary school. For

most of them their relationships with mathematics took a turn for the worse as they entered middle (or junior high) school. Some noted teaching methods, including teaching straight from the book and not allowing different approaches to problem solving, as the reasons for the changes. Others related experiences in the classroom that made them believe that they were dumb and incapable of doing mathematics.

Throughout our journey as we all became more comfortable talking with each other about mathematics, several themes began to emerge. The vehicles for our journey were our journal writings (private reflections), our conversations about mathematics and the learning of mathematics (evenings around the campfire), and the strategies and techniques of problem solving we utilized (on the trail). The nourishment for our growth in understanding mathematics came from the relationships which developed--relationships with each other and, ultimately, our relationships with mathematics.

ON THE TRAIL: Mathematical Adventures

Our pursuit of mathematics led us down many paths. The problems were selected to provide opportunities for the students to ruminate, conjecture, and explore possible approaches to the solutions. Some of the problems were given as class assignments, while others were individual assignments. For the individual assignments, the students were asked to write detailed accounts of the steps they took, including approaches that did not work. In their accounts they also discussed their feelings as they worked through their problems. The purpose of these assignments was to encourage them to think about all that goes into solving problems rather than

simply arriving at an 'answer.' The following is an accounting of some of the mathematics problems we encountered and the students' responses and reactions to them.

I also wanted the students to experience success with mathematics, so the first in-class mathematical activity was an exercise with letter patterns. The students were asked to work in pairs to complete the following letter pattern at least five different ways.



The class was quiet in the beginning, but as the students began to share ideas, an excitement began to build. As I listened, I could tell that all the students were participating in the discussions. It was exciting for me to listen as they explained how they devised their patterns to each other. Later in the class several commented that that was the first time in quite a while that they had enjoyed a mathematics class. (My JE 1).

When we shared our patterns, each student was able to share one that was different from the others. It was interesting, but not astounding, that all of their patterns were based on the number of letters in each group above or below the line. No one had considered that the geometrical shapes of the letters could also determine a pattern.

One of the take-home problems was my version of the following car tag problem.

Car tag problem:

> The license plate on my car contains five different digits. Although my brother installed it upside-down, it still shows a five-digit number. The only thing is, the new number is 63,783 more than the old number. Find the original license plate number (Lambdin, 1992).

My version of the problem reads as follows.

> I had to buy a new car tag and asked my neighbor to install it. He put it on upside-down. If the tag has 5 numbers, the new number is smaller than the correct number, and the difference between the two numbers is 76,203, what should my tag number be?

Some of the students had difficulty visualizing the transformation that occurs when a car tag is installed upside-down. Some of their comments as they entered the classroom are recounted below.

Annie: My mom said you couldn't install a car tag upside-down.

Fran: I gave it to my sister and her ball team. They pulled out their calculators to see what the numbers looked like upside-down. They had a blast with it!

Linda: It took me a while to realize I had to rotate it instead of flipping it over.

Wendy: I had paper all over my room (Audiotape 3).

All of the students had begun the problem by listing the digits that are the same (0, 1, and 8) when rotated. Later they realized that "6" became "9" when it was rotated. Some tried "2" and "5" but decided rotating one did not produce the other. Some tried "3" until they realized they should rotate instead of turning the tag over. I was

pleased that they thought the problem was interesting enough to share it with others and to work until they had solved it.

Another problem that the students were given as a take-home individual assignment was a version of the locker problem.

> One thousand students have lined up in a very long hall with 1000 closed lockers. One by one the students run through the hall and perform the following ritual: The first student opens every locker. The second student goes to every second locker and closes it. The third student goes to every third locker and changes its state. If it is open, the student closes it; if it is closed, the student opens it. In a similar manner the fourth, firth, sixth, ... students change the state of every fourth, fifth, sixth, ... locker. After all 1000 students have passed down the hall, which lockers are open? (NCTM, 1989)

The students seemed to have more problems with the locker problem than with the car tag problem. Very few had solved it. As we discussed possible approaches, I suggested that they use coins to represent the lockers. Working in pairs with coins, they soon solved the problem. We then looked at the divisors of numbers to determine why the open lockers had numbers that were perfect squares. They were amazed at their abilities to determine that perfect squares have an odd number of divisors while other numbers have an even number of divisors. Thus the lockers with perfect square numbers will be changed an odd number of times leaving them open, while the others will be changed an even number of times and will be left closed (My

JE 7).

There were some problems that the students were unable to solve on their own or in class working together. One example was the "Hefty Hippos" (See Appendix B.), which proved to require algebra skills that many of them lacked. However, there was value in their struggles with the problem in that it led us on a side trip examining the algebra necessary to solve the problem. We spent parts of two sessions discussing the "Hefty Hippos." The first evening, they all expressed frustration at their inabilities to solve the problem. Some of them did not understand what was meant by "weights of all the pairs." Rather than work the problem for them, I led them through a discussion that helped them determine how many hippos there were and other clues that would help them continue working on it. I was somewhat disappointed that with all the work we had done, no one had been able to complete the solution, so in the following class, we continued to work together until the problem was solved.

As the semester progressed the students became more comfortable discussing mathematics. This was evidenced by their animated chatter about the assignments as they entered the classroom, their active participation in the class assignments, and their journal entries. Their journal entries included with their solutions to the problems, their approaches and their feelings as they worked. The stigma of being unable to 'do math' seemed to be lifting and the students were excited and ready to share their attempts at problem solving. They acted as if they really enjoyed coming to class. When asked to reflect on how the course had changed their views of mathematics, Colleen said in a journal entry,

> I found myself getting much less frustrated than I normally would because I can ask my classmates things I don't understand and I can add my input to those who don't understand something I might. I have found that math is now a fun and exciting subject that I enjoy learning and a class I look forward to. I no longer think of it as my greatest weakness in school (Colleen, JE 20).

Fran echoed Colleen with, "I learned math is still fun ..." (Fran, JE 20). Katie's response had a different character.

> I honestly thought this class would be easier than it is only because I have worked throught (sic) business calculus, but the work really has challenged me and that is what attracts me to math (Katie, JE 20).

I had been concerned that Katie would not be challenged. The responses to this prompt informed me that the students were developing mathematical power with problems that were challenging.

The lesson with tangrams began with step-by-step verbal instructions. (See Appendix F.) As we folded and cut out the shapes, the students were able to answer the questions. The hard part was putting the tangram pieces back together to form the square. We also used the pieces to make other shapes. Some of the students found that they needed smaller pieces to fit some pre-drawn outlines. They were very resourceful in downsizing while maintaining the proportions (My JE 20).

Their journal assignment for the next class was to reflect on the class exercise with tangrams and what they had learned. Some excerpts from their journal entries follow.

Colleen: I learned another new aspect of learning mathematics. I never understood all the geometry terms until we cut out all of the shapes in class. By doing those hands on activities and not always using only a textbook really helped my understanding of geometry. The step by step procedures we went through in class also really helped me understand. I learned a new way of figuring out patterns for geometric figures as well. I really liked trying to put the shapes back together and building new things. The relaxed atmosphere of the class also eased the tension and worries I had about geometry (Colleen, JE 20).

Katie: This class has really taught me so much more about mathematics and how to use it in the classroom. I have discovered so many different ways to solve a problem and I value that. I have also enjoyed working in pairs or groups because that is how I have been able to understand how others think (Katie, JE 20).

Jill: I learned that I remember more about Geometry than I thought. I realized how things you learn long ago can come back to you by simply reviewing. The funny thing is, that I understand it better now than it did when I first learned it. As you explained the lesson I listened and could imagine myself back in my geometry class in high school. I had fun cutting out the shapes and I enjoyed putting the puzzles together. I also learned that I am most definitely a visual learner. When I was younger, I use to love puzzles. I was impressed with myself for being able to figure out the tangrams so easily. I learned that maybe I don't hate geometry now as much as I did in high school (Jill, JE 20).

The students really had fun with tangrams, and their journal reflections indicate that the activity may have renewed their confidence in their abilities to understand and remember some things they learned in geometry.

THE WOMEN OF FLATLAND

Near the middle of our journey, we visited some women mathematicians. Consistent with the pilot group, the students were asked to choose a mathematician of their sex to research, and the students on this journey were all women. The characteristics and names of these mathematicians will be revealed later as the students' essays are discussed.

We continued our journey with a visit to *Flatland* (Abbot, 1884/1994), a novel about mathematics written by Edwin A. Abbott (1838-1926). The teller of the story of Flatland is a square who shares with the reader how his visit with a sphere opens his eyes to the possibilities of other dimensions. Flatland is a two-dimensional world in which a man's shape determines his social status. The lowest ranking men are very narrow isosceles triangles; next in order are the equilateral triangles, whose sons will be squares; the squares will father regular pentagons, who will father regular hexagons, etc. The highest ranking men are the priests who are circles, or regular polygons with so many sides that they look like circles.

All women are line segments, and as such, have no status. Because their sharp ends are hazardous to the safety of men since all movement is within the plane, the women are required by law to hum constantly and keep their rear ends in motion when they are out in public. They supposedly, according to Abbott, have no brains and thus

no memories of their actions. This means that there is no reason to think that they can learn or should go to school. This stimulated much discussion and many conjectures about the prejudices and injustices that abound in *Flatland*.

Initially, the students had difficulty understanding *Flatland*. They asked questions such as "How do they watch a movie?" and "How do they eat?" They struggled with the notion of everything being flat and of the people sliding around on the surface of a plane.

At one time in Flatland, a pentagon began experimenting with colors. He painted everything he owned different colors for identification, including himself. Each of his five sides was a different color. The fad caught on and soon everyone was colored except women and the priests--neither of whom had sides. The women were line segments and the priests were circles. The introduction of color eliminated the need to teach recognition by feeling in the schools. This disturbed some of the aristocracy because they had learned that recognition by feeling had replaced recognition by sight earlier. They thought the schools were becoming easier and easier. There was a revolutionary group which was pushing a universal color bill which would standardize the colors and require everyone including women and priests to paint themselves for identification purposes. What follows is an example of some of the discussions we had concerning *Flatland*.

Fran: I thought we would go to the color part. At first when I was reading about the color bill I thought that it was where the women were painted, and the priests were painting the front of the house red, but they kept talking about another

color bill, and I thought it had already been passed, so I don't understand what the color bill is.

Me: Does anybody want to comment on the color bill? Does anyone have any thoughts about it?

Unidentified student 1: My understanding is I thought they threw it out.

Me: They did. Now why did they throw it out?

Unidentified student 1: Because I thought it was because the workers, the isosceles triangles, they just kind of like which was who was fair to be colored (sic) and who wasn't. And everyone wanted to have a color and they it just got too crazy and so they just threw it all out. Well, that was my understanding.

Unidentified student 2: Well, what about when they got the priests and the women mixed up 'cause they were like colored the same. Is that why they kind of threw it out?

Me: That may be part of it.

Unidentified student 3: That gave status to the women that they were thought of as priests that they did not actually have or deserve or whatever.

Unidentified student 4: Was what they were saying on the priests and the women was that if a woman was seen from the side she would be half red and half green and the priests were also half red and half green but they were circles, so if you saw a circle you might think it was a woman or the other way around?

Me: Exactly. Now, what is another possibility with color? Well, what if you weren't looking at the priests exactly that way? What if you were looking at them from

a different direction?

Unidentified student 5: Then you could tell the difference. Weren't they like painting themselves colors that they were not supposed to be painting?

Me: Ahhhhhh! So, people were not always what they seemed to be. Sue?

Sue: That just infuriated me. I mean, it's like they are so snotty. They are so stuck on classes and I mean, I know that is kind of like our world today, but it even seems worse. I mean I guess, in America we aren't so bad anymore. (Audiotape--March 4)

This is only part of this discussion but gives some of the flavor of our discussions. The students were quite animated in their comments and very offended at the way women were treated. This also illustrates the comfort the students felt discussing *Flatland* with each other and with me.

While the students focused their questions and discussions on the political and social implications of Flatland, we also had some healthy discussions on the mathematics presented. When Square is teaching his grandson, Hexagon, that the area of a square 3 inches on a side is 3^2 square inches, Hexagon asked his grandfather what 3^3 means. Square replied that it had no meaning in his world which is only two-dimensional. This led to a discussion of dimensions, and during the next class period we constructed a tesseract, a fourth-dimensional cube (Simonson, 1984).

The discussion of Lineland prompted a discussion of the properties of numbers, including the density and completeness of the number line and the sizes of infinity. We also examined the geometry of the surface of a sphere where the sum of

the angles of a triangle is greater than 180°, and the shortest distance between two points is not a straight line segment on a flat map, but the arc of a great circle of a sphere.

As we read and discussed *Flatland*, I asked the students to think about how their women mathematicians would carry out their work if they were to be transported into Flatland. When we had completed our study of *Flatland*, their assignment was to write a fantasy placing their woman mathematician in Flatland. The purposes of this exercise were to help the students examine the influences of mathematics on their lives, to enable them to assess their relationships with mathematics, and to release their imaginations. I hoped that the students would leave this class with a knowledge base and a freedom in mathematics that they may not have experienced before. This included turning loose their imaginations and letting their creative spirits free, because that is the way mathematics is done. The fantasies the students wrote about their mathematicians in *Flatland* revealed much information about the students and their progress on their journeys.

Colleen visited the mathematician, Mary Fairfax Somerville (Scotland, 1780-1872). While she did not write a fantasy with Mary in Flatland, she did discuss some of the similarities between Mary's life and lives of the women of Flatland. Colleen wrote of the expectations of Mary's family that she would forget her interests in mathematics and be content to be just a wife and mother. Unlike the women of Flatland, Mary never forsook her love of mathematics and became well known as a writer of mathematics textbooks.

Colleen wrote,

> I have many thoughts about how the women [of Flatland] could have broken away from the rule in Flatland. Maybe they did have memories but were told so many times that they didn't [have memories] so they believed that (Colleen, *Flatland* essay).

She may have been thinking of herself when she wrote this. Colleen had had many struggles in coming to the point of saying that she could do mathematics, and that she felt good about herself as a student of mathematics. She also wrote, "I think Mary Somerville would have been an interesting figure if she had lived in the fictional Flatland, and she probably would have succeeded because of her hard work and dedication" (Colleen, *Flatland* essay.) In her journal entries and final examination paper, Colleen comments that she, too, can succeed in mathematics if she is dedicated to working hard. "All in all I don't have anything standing in my way anymore. I have proved to myself that I can do things I put my mind to doing" (Colleen, FE) was part of her response to the final exam question, "How has your view of yourself as a student of mathematics evolved?" In our follow-up meeting, Colleen confirmed that Mathematics for Critical Thinking was the best experience she had had with mathematics. However she was having difficulty with the first course of Mathematics for the Elementary Teacher. It was taught in a traditional manner, and she said she had trouble following the lectures.

Katie had taken more mathematics courses than the other students. Her confidence in her abilities to understand mathematics and her enthusiasm to share her

excitement with others is reflected in her fantasy, "Evelyn Boyd Granville [United States of America, 1924-Present] vs. Flatland." She depicts Evelyn as a lower class woman in Flatland who realizes her ability to "think logically and remember." (Katie, *Flatland* essay). Evelyn works very hard to learn all she can, and although only men can attend the university, she works out an elaborate scheme to get into the university. Once admitted, she proves that women can learn by graduating at the top of her class.

Evelyn's conviction that all people, both men and women, regardless of class, have a right to an education leads her to work with the lower class. Katie expresses her desire to teach and her excitement when observing others learn through Evelyn's work with her students. She recounts a dream that Evelyn had about expanding her horizons by becoming three dimensional. Evelyn immediately shares her dream and is met with skepticism.

> She told the others of her dream and they immediately thought her insane. They all wanted proof. They wanted to see her become a sphere. She couldn't remember how this had happened, but she did know that it happened in her sleep while her mind was being stretched to its fullest potential. She began to relax her mind and stretch her thoughts back to the images of her dream and felt the nausea grow deep in her stomach again. The low class people watched her slowly disappear and then all they had of her was her voice. They could hear her voice all around them, not coming from one area. She was instructing them about how to relax their minds and stretch their thoughts to achieve the three-dimensional form. Stretch!! Slowly, they began to pop up off of the

> surface like boiling water and they squealed with delight as they floated freely through the air. How exciting that knowledge had set them free! They revealed their beautiful spherical shapes to Evelyn and asked how they were going to influence others (Katie, *Flatland* essay).

In her essay, Katie explains that some of the more affluent people of Flatland refused to believe Evelyn's theory that people could change when they were willing to stretch their minds, but those who did follow her were able to experience the joy of floating in the third dimension. Katie's excitement about learning and sharing mathematics had a positive effect on the students on this journey. Because she encouraged the others and observed with pleasure their excitement in their discoveries, the other students were willing to try their own ideas for solving problems without fear of ridicule. This mirrors her statements,

> She [Evelyn] explained that there was much more in the world to be learned and that this was just the tip of the iceberg. She charged the others to a life of growth and knowledge (Katie, *Flatland* essay).

Katie also saw herself as a learner. She was interested in learning mathematics and how people learn mathematics. She worked well with the others, always making sure that those in her group had opportunities to offer their suggestion. I frequently noticed that she was quietly observing her classmates as they worked on the solution to a problem. Her ability to work with those with less experience in mathematics is reflected in her characterization of Evelyn: "She also attributed her success to the network of friends around her. She enjoyed interacting with other people so much

that it excited her to pass on her findings to them" (Katie, *Flatland* essay).

> In my evaluation letter to Katie, I wrote (See Appendix J.)
>
> Your paper, "Evelyn Boyd Granville vs. *Flatland*," seemed to be a reaffirmation of your success in mathematics, and yet a statement that one person can effect changes to make education accessible to all students. Your final paper indicates there have been changes in your philosophy of teaching mathematics. You seemed to have discovered the power of writing in mathematics, of analyzing mistakes or misunderstandings, using manipulatives, and helping students develop a sense of community in the mathematics classroom (My follow-up letter to Katie).

In the follow-up meeting with the students, Katie affirmed my assessment of her beliefs about mathematics education with the statement, "I value my experiences in this class and the package of materials. I have changed so much, and I will carry what I've learned with me when I start teaching" (Audiotape, December 4, 1997).

Although Jill remained practical throughout the semester, she did reveal some creativity in her fantasy about Sophie Germain (France, 1776-1831). She tells of Sophie's interest in mathematics from an early age and how Sophie bent herself into a pentagon so that she could go to school as a boy. Under the pseudonym of M. LeBlanc, she continued to study mathematics and secretly submit mathematical papers to her professor. The professor [Joseph-Louis Lagrange, 1736-1813] was intrigued by these papers and curious about the author. Sophie met with Lagrange disguised as the pentagon, M. LeBlanc. Out of curiosity, she asked Lagrange if he thought a woman

could have written the papers. He naturally said it was impossible. To his surprise Sophie then straightened herself out to prove she was a woman. As a result of her accomplishments, Sophie opened doors for other women in Flatland to achieve recognition for their accomplishments.

Jill expresses in her essay a belief that women can overcome obstacles and achieve greatness by making significant contributions and social changes. Her practical side points to hard work and perseverance of the individual in accomplishing one's goals. Just as Sophie worked under hardship and the handicap of being a woman in a male-dominated society, Jill confirms her previously stated belief that she can achieve her goals if she works hard enough and refuses to quit.

In her fantasy, Linda depicts Ada Byron Lovelace (England, 1815-1852) as a spherical visitor to Flatland. Ada had left town to avoid embarrassment over her gambling problems. As Ada moves closer to Flatland, Linda describes how the scenery begins to lean and becomes flatter until everything is only two dimensional. Linda's characterization of Ada is as a gregarious visitor to Flatland with a desire to change the political climate that pervades Flatland and prevents women from making positive contributions to society.

Linda may have been reflecting the changes in her relationship with mathematics that she expressed in her final paper.

> One of the most important things that I am going to take with me from this class in an open mind. This class really made me look at things from a different perspective. I realize now that when you open up your mind, get creative, and

use your imagination, the possibilities are endless. Reading *Flatland* made me understand that more than anything.

When I first began *Flatland*, it blew my mind. All I kept thinking was that all of this was impossible. However, when we had to write our own story, I realized that I had to make it possible. I let my imagination run wild. It was great! I started creating my own story with *Flatland*, and everything began to make sense. I could picture it all in my head. It turned out to be one of the best assignments that we had (Linda, FE).

When Ada enters Flatland a woman invites her to dinner. Ada questions her about the education system and learns that girls and women were not educated. Ada convinces the women that they can learn and then organizes them in a march on Flatland Headquarters where the priests had gathered. The women threaten a revolution if the priests do not listen to Ada. Ada then speaks to them about computers and asks them if they know what a computer is. They know about computers, but no one knows how to program one. When Ada tells them that she can, the priests begin to listen, and Ada is able to convince them that it is important for women to be educated.

Linda's account of Ada in Flatland illustrates Linda's belief that women students can achieve if they are given the opportunities (Linda, FE).

That is when they finally began to take women and their ability to learn and teach seriously. From then on things in Flatland changed for the better. Women were given more and more rights everyday. Their opinions were taken

seriously, and when they had something to say people listened up (Linda, *Flatland* Essay).

Linda also tells how the women of Flatland tell their daughters how a strong leader such as Ada can give women "the courage to stand up for themselves making it possible for women to be treated as equals" (Linda, *Flatland* paper). In a telephone conversation I had with Linda in the fall, she expressed renewed confidence in her abilities. She specifically mentioned the *Flatland* assignment as one of the contributing factors.

Fran depicts a television show host Cord the Cube interviewing Sonya Kovalevskaya (Russia, 1850-1891), a native of Flatland. Sonya's life was pretty typical of a girl growing up in Flatland; she married a pentagon selected by her father and had one daughter. Following Sonya's abduction by a group of rebels who began to teach her basic skills to support the revolutionaries, she realized how smart she really was and why the men did not want girls and women educated. After the rebels helped Sonya find her daughter, she began to teach the girls and women of Flatland. In order to move freely around Flatland, Sonya taught the younger girls how to hook themselves together and move in tandem as a square. The women were soon able to take over Flatland.

Fran may have been telling the story of her rise from feeling ignorant of mathematics to a belief in her own mathematical abilities. "I feel as if the most significant lesson I got from this math class is that everyone can learn math" (Fran, FE) was Fran's first sentence in her response to the question, "What was the most

significant topic or lesson in this class?" on the final examination. Fran's affirmation of her confidence in the power of people working together to solve problems is also reflected in her final examination.

> The class we are in this semester was like an open book. We were able to compile our knowledge to solve problems, we were able to talk as if what we had to say might be important, and we were able to ask questions of the teacher and not be shamed (Fran, FE).

The impact of *Flatland* on these students was observable in many ways. The students seemed more comfortable discussing mathematics and issues that affect the teaching of mathematics. Their willingness to tackle difficult problems and their creativity in solving problems improved dramatically. They also began to bring interesting problems to class for the others to solve.

PRIVATE REFLECTIONS: Students' Evolving Beliefs about Mathematics and Themselves as Students of Mathematics

Writing was a critical element of the course. Through journal entries students were asked to write detailed solutions to problems, discuss the approaches they tried that did not work, assess the reasons these approaches did not work, analyze the approaches that did work, and discuss their reactions and feelings as they worked through a problem. I wanted them to think about the processes of problem solving rather than memorizing and reproducing an algorithm.

Writing in a mathematics class was a new experience to these students. It was not always easy. In the beginning some of them found it very difficult, but as the

semester progressed, they became more comfortable and articulate. Sometimes when working a problem, they would reach a solution but find it difficult to figure out how they reached it. As Linda said, "Figuring out a way to explain what I had just done was often much more difficult than actually working the problem itself." (Linda, FE)

After about six weeks of class I asked the students to address the issue of writing mathematics. The following comments come from an audiotape (February 10) of a discussion about the impact of writing on their understanding of mathematics.

Helps keep up with the steps.

Helps me keep my thoughts organized.

Forces you to explain it to yourself.

Helps you reflect on what you did.

Makes you observe more.

Provides source for questions.

It's not always easy to explain what I did.

Because they had struggled with the writing of mathematical solutions to problems, I wanted them to address the issue of writing as part of their final examination. The question "How has writing impacted your learning of mathematics?" was part of a series of questions intended to help them reflect on their experiences with mathematics during this class. The comments of these students may be biased because they may have been writing what they thought I wanted to read.

Fran: I am a smart student ... being able to write down my thoughts and emotions regarding a problem has helped mold this new attitude of respect for

mathematics. Writing seems pointless in a math class, but experiencing the actual thought of it has changed my thoughts on certain problems (Fran, FE).

Katie: It has been so helpful to write out my experiences and thought processes as I attempt to solve the assigned problems, because I can later look back and see the different methods I attempted. I have never been put in this position before in a math class and I think that it has been very important to the learning process (Katie, FE).

Jill not only writes about what writing mathematics has done for her, but also the reactions of her friends.

Jill: Writing about mathematics has most definitely impacted my learning and understanding math. By writing I have written things I would have never thought of otherwise. I have learned to express my thoughts on paper. Writing has also helped me organize my thoughts of mathematics. I have never had to keep a journal in math class before and when I tell people I have to write a math paper they laugh and ask why. They do not understand the importance of math and the view that I see it. By writing about math it allows me to think beyond my imagination. I am able to express my thoughts beyond numbers. Writing about math has impacted my learning math in a positive way by improving my answers and decisions (Jill, FE).

Linda expresses in detail how writing made her think differently about solving problems.

Linda: I believe that part of what has helped me understand math better has been the

> writing, because it has helped me to understand myself. Writing everything down has forced me to look at how my thought process works. In order to write down how I worked a problem, I have to be able to completely think it through and understand what I am doing. ... I think that I have benefited from the writing in this class, because I have really had to think about each and every step to working problems. In doing so, I am now better able to understand myself and how my mind works when it comes to math (Linda, FE).

Colleen did not specifically address the issue of writing in her final paper, but like the others, the quality of her solution discussions improved steadily throughout the semester. From these comments it appears that writing not only enabled the students to actively involve themselves with mathematics but allowed them better to see themselves as capable students of mathematics.

END OF THE TRAIL: Questions and Answers?

We have defined a relational mathematics curriculum as one which allows students to engage themselves with mathematics in a way that encourages them to make mathematics a part of them as well enabling them to become part of mathematics. We posited that in a relational mathematics class, the students are active participants in solving problems. By working together in a sociomathematical community where trust and respect develop among the members, they cultivate relationships with each other that are sensitive and nurturing. Through these relationships with each other and with writing as a vehicle for expressing their

understanding of mathematics, they develop relationships with mathematics that empower them to accept mathematical challenges with confidence. We saw that the students experienced a relational mathematics curriculum through their search for patterns, their interactions with each other and mathematics as they solved problems, their studies of women mathematicians, and their visits to *Flatland*. Let us now consider the questions we sought to answer.

1. How do students experiencing a relational mathematics curriculum become more mathematically empowered, specifically, develop their abilities to do mathematics and become more confident?
2. How are students' beliefs about themselves as students of mathematics affected by experiencing a relational mathematics curriculum?

According to research (Bandura, 1986; Pajares & Miller, 1994), students' beliefs about their abilities determine their performance on given tasks. Subscribing to this, then the two questions are interrelated, and it is appropriate that we consider them together. As we look back at the individuals who represented the class, we see the changes in their abilities to solve mathematics problems as well as their confidence in, and beliefs about, themselves as students of mathematics.

Colleen began Mathematics for Critical Thinking with a much greater mathematics handicap than most of the other students. As we worked together to solve problems, she began to relax and enter into the discussions. She confirmed this comfort and trust in her classmates in her final examination. Colleen's solution to the

assignments became more detailed as the semester progressed. Her confidence in her mathematical abilities was demonstrated in her class participation and comments on the value of the class. "[T]his class has made me not only a more confident and better math student, but a better all around student and person" (Colleen, FE). By the end of the semester, Colleen was beginning to emerge as a student of mathematics. She still has a long way to go in her development as a mathematics student, but maybe this class was the boost she needed to continue her journey with mathematics.

Fran really hated mathematics when the semester began. In recent years, she had had nothing but unpleasant experiences with mathematics (Fran, JE 1), and she was quite vocal about her feelings. As the semester progressed, she frequently expressed how her attitude had changed and how much better she liked mathematics when she could work with others. She also began to believe that she could do mathematics. "I am a smart student ... being able to write down my thoughts and emotions regarding a problem has helped mold this new attitude of respect for mathematics" (Fran, FE). At the follow-up meeting, Fran commented that even though she was having difficulties with Mathematics for the Elementary Teacher, there were several others from our class who worked with her on the assignments. Later in a chance meeting, Fran told me she had done well the preceding semester and was really enjoying the second semester of Mathematics for the Elementary Teacher. She said the class was more like ours, and some of the exercises were even the same. She said she no longer disliked mathematics because most of her recent experiences had been good.

For Linda, Mathematics for Critical opened her mind to new possibilities for creativity in mathematics (Linda, FE). Her relationship with mathematics had been dependent on hard work and helpful teachers (Linda, JE 1). In this class she learned how working with others and writing her solutions could enhance this relationship. Because she was unable to attend the follow-up meeting, I had a long telephone conversation with her. She was attending a different university and was taking their mathematics for elementary teachers. She spoke of the benefits she had gained from the experiences in our class--working with others, searching for patterns, and writing solutions. While Linda had always succeeded in mathematics through hard work, she now believes that mathematics is much more accessible through her own creativity. "I need to be able to look at the same thing from different perspectives just as different students in a classroom would be doing" (Linda, FE). I believe that her relationships with mathematics will continue to grow.

I do not think that Jill's beliefs about herself as a student of mathematics were affected by experiencing a relational mathematics curriculum. She was the only one who had perfect attendance, and she worked well with the others, but throughout the semester, she never expressed her own ideas without checking with someone else first. Her confidence was in hard work. "Today I am still benefiting from my hard work in the third grade" (Jill, FE). As evidenced by her lesson plan on check writing and her final examination, she still believed that if students worked hard enough and did what the teacher told them to do, they could learn to do mathematicas. This was the same view that she brought with her when the class began.

Katie's experiences with the relational mathematics curriculum were different from those of the other students because she was already a competent and confident mathematics student. However she did come to value the relationships she observed and the ways the curriculum enhanced the development of those relationships. When asked in the follow-up meeting what the most valuable aspect of the class was for her, she said the materials packet and learning how to actively engage students in doing mathematics (Audiotape, December 4, 1997).

For most of the students, the relational mathematics curriculum provided opportunities for them to struggle with mathematics problems, work with others in their struggles, and experience the success of arriving at solutions to the problems. All of this appeared to contribute to an increased confidence in themselves as students of mathematics.

According to Perry's (1970) scheme, humans progress through a series of nine developmental stages in their life's education. He posits that we begin thinking in a very simple dualistic manner where everything is considered in extreme terms, and we gradually begin to question authority. As we move on this continuum, our dualism becomes more complex as it gives way to multiplicity and relativistic thought. If we continue to move in this direction, we make a commitment to self and grow to believe in our own abilities to make wise decisions. Because there is a continuum, when we are not ready to move to the next level, there may be no recourse but to retreat. We may react negatively, or we may be carried along without ever taking a stand or having ideas of our own. However, if we continue forward on Perry's track, the

authority moves from being external to the individual to being internal.

In contrast the work of Belenky, et al. (1986) purports an interaction among the various learning stages which they call received knowledge, subjective knowledge, separate procedural knowledge, connected procedural knowledge, and constructed knowledge. These are related to the steps of Perry's (1970) scheme, but rather than putting the stages on a continuum, these researchers posit that when individual is in one of the stages, she may remain connected to any of the others. When she is able to construct her own knowledge and become the expert, she is able to pull knowledge from all of the stages and use it to support her beliefs. In many ways this model of Belenky, et al. (1986) than Perry's (1970) scheme because it brings all of the stages together and allows the individual to pull knowledge from the whole.

If we look at the students through Belenky's, et al. (1986) model, we find that most of them were developing in several stages of knowledge. At the end of the semester they still were rather dependent on received knowledge, but through their interactions with each other and with me, their struggles to make sense of mathematics made them more confident and more competent. By Perry's (1970) standards most of the students were not very far along his continuum, but using Belenky's, et al. (1986) model, we can see that they all were making progress in several areas of knowledge.

When the semester began, Colleen was very dependent on received knowledge, but she was very doubtful that she could succeed in a mathematics class. Her experiences had compounded her beliefs that she would never "be good at math" (Colleen, JE 1). If we examine the experiences of Colleen in this class, we see that by

Belenky's, et al. (1986) model, she began to develop her procedural knowledge by questioning herself and others about mathematics. Evidence of her constructing knowledge comes from the end of the semester when she was able to contribute to the discussions of mathematics and explain her solutions. In Perry's (1970) scheme, Colleen has made little progress, but by Belenky's, et al. (1986) model, she has experienced growth in all knowledge areas.

Fran made considerable progress in all areas of knowledge. While still dependent on received knowledge, through her development of pattern recognition, she experienced tremendous growth in her connected procedural knowledge. With all of this growth came expansion of her constructed knowledge. She now believes in herself and her mathematical abilities.

Jill experienced growth in all areas as well, but she still relies primarily on received knowledge and does not give herself credit for her mathematical abilities to construct knowledge that is reliable. While she was in class working on the solutions to problems with others, she exhibited insight and the ability to make connections. However she still depends on those she perceives as authorities for confirmation of her ideas. What she sees as creative is really a reconstruction of something someone else has shown her.

Linda was really awakened to her mathematical abilities. She had a strong mathematics background but never saw herself as creative mathematically. She had depended strongly on received knowledge but underneath she had developed substantially in procedural and constructed knowledge. While her separate procedural

knowledge was more developed than her connected procedural knowledge at the beginning of the semester, she was able to capitalize on her strengths and experiences in the class to build stronger knowledge bases in all areas. Linda left the class with a sense of confidence and understanding of her mathematical abilities that she had not known existed.

Katie entered the class with the strongest mathematics background and abilities. By Belenky's, et al. (1986) model, her separate procedural knowledge was highly developed. She had the confidence that she could meet any mathematical challenge and master it. Prior to this class she was unaware of the power of connected procedural knowledge. Because of her strong mathematics knowledge base, I was concerned that she would not be challenged by the mathematics in this class. She was challenged, not only through the problems, but through her interactions with the others in the class. Through her experiences Katie developed an understanding of the ways she and others construct knowledge through challenging problems and interactions with each other. All of this combined to strengthen her constructed knowledge.

Experiencing a relational mathematics curriculum was different from other mathematics classes the students had experienced. Maybe the effects are more long termed than immediate, and as they continue their education, parts of this phase of their education will be interwoven in the individuals they become. Most of these students plan to be elementary teachers. Maybe their experiences in Mathematics for Critical Thinking will challenge them to allow their students opportunities to experience mathematics in a personal way.

CHAPTER 5

JOURNEY'S END, OR BEGINNING?

The organization of the class was based on the social constructivism of Vygotsky (Cobb, Wood, & Yackel, 1990; Schmittau, 1992; Wilson, Teslow, & Taylor, 1992). I attempted to create an atmosphere that was sensitive and nurturing (Breitborde, 1996; Code, 1991; Martin, 1992) in which students could trust their classmates to be supportive of their efforts to solve mathematics problems. We worked together to establish the sociomathematical norms (King, 1994; Yackel & Cobb, 1996) that would enhance the development of mathematical power within the students and promote the evolution of problem-solving skills (NCTM, 1989, 1991; Trowell, 1994).

Problems were selected as part of the evolution of the class to challenge the students (Brown, 1986), to reinforce the search for patterns, and to evoke questions and problem-posing from the students (Frankenstein, 1990; Silver & Cai, 1996). The students were actively engaged in doing mathematics rather than watching mathematics being done (NCTM, 1989, 1991) through interactions with each other and by writing their thought processes as they solved problems (Buerk, 1986; Buerk & Szablewski, 1993; Countryman, 1980).

REUNION: Follow-up Meeting

In the fall following our journey, we held a reunion to reflect on the events of our journey with mathematics. I prepared evaluations of each student's progress and asked them to comment on my report. (See Appendix J for my evaluations of Fran, Jill, Linda, Colleen, and Katie.) They each agreed that my assessment was accurate,

and that it was beneficial to them to read my thoughts.

In our discussion I asked them to reflect on the most important aspects of the course. Most of them concurred that the greatest impact came from the friendships that they formed and the confidence in their mathematical abilities that developed within them. Katie said that the most important aspect of the course was the packet of materials because she would use much of the material when she began teaching.

Some of the students expressed difficulties with the mathematics class they were currently taking. According to them the instructor was very traditional in the teaching methods used and allowed for no flexibility in the solutions of problems. This led me to wonder if these students had really learned to believe in themselves. According to Perry's (1970) scheme, as we move along the continuum from dualism to commitment to self, the authority moves from being external to the individual to being internal. Mathematically these students may still have doubts about their own abilities, and in Perry's scheme may still be in or have retreated to dualism. However, if we look at the students through the eyes of Belenky, et al. (1986), we see that they experienced growth in several areas of knowledge. Their dependence on received knowledge is still strong, but they were more subjective and questioning than they had been. Their abilities in procedural knowledge when they were able to collaborate steadily improved throughout the semester, and they were actually constructing some mathematical knowledge that they could own. Even though they were expressing some self-doubts, they still felt confident about their accomplishments.

WHERE DO I GO FROM HERE?

As I reflect on my research questions:

1. How do students experiencing a relational mathematics curriculum become more mathematically empowered, specifically, develop their ability to do mathematics and become more confident?
2. How are students' beliefs about themselves as students of mathematics affected by experiencing a relational mathematics curriculum?

and the events of the study, I am convinced that the students' understanding of mathematics evolved both in terms of repeating patterns and processes as well as historical and relational meanings and connections. In our search for patterns we found new ones and revisited old ones. We found patterns from Pascal's triangle in the "Twelve Days of Christmas" and in the number of paths on a city map with perpendicular streets. We found Fibonacci numbers in nature, art, and architecture. Through our study of women mathematicians, we were able to connect history with mathematics. The students became quite adept at recognizing familiar patterns even when they were disguised or hiding in some new context.

In their writings the students found that they understood the problem solutions better than they had before because they had to rethink what they had done to solve the problem. As Linda said, "In order to write down how I worked a problem, I have to be able to completely think it through and understand what I am doing" (Linda, FE). As the semester progressed, they became more detailed and accurate in their written solutions and more articulate in their class discussions of the solutions.

Similarly, the results of my study suggest that their experiences in the relational setting increased their confidence and valuing of mathematics. Near the end of the semester, the students were more willing to take risks in their approaches to solving

problems. They began to recognize familiar patterns and to ask new questions. For example, when counting the squares on a geoboard, someone asked if the squares could have diagonals of other squares for sides. This says to me that the events of this class may have changed the ways these students perceive mathematics and themselves as students of mathematics.

ALL GIRLS

Experiencing mathematics in a sex-segregated class also seems to have had some unanticipated results. Throughout the semester I was conscious of the fact that all of the participants in the class were women but cautiously avoided mentioning the situation to the students. I did not want them to think that I was making concessions or selecting easier problems. I actually chose problems that would stretch them to their mathematical limits and gave as little assistance as was feasible. They accepted the challenges and surprised themselves with their accomplishments. I did, however, want them to reflect on this aspect of the class and asked the following question on the final examination.

> Compare and contrast this class with those mathematics classes in which there were males. How did being in an all-female class make a difference in the way you perceive yourself as a student of mathematics?

All of the students thought that the all-female class made a difference for them.

Colleen: The class being all girls was a good thing because it was a little less intimidating especially in the beginning. ... I never imagined I would be able to talk to the girls in the class like I do now (Colleen, FE).

Jill: By being an all girl class, I felt more comfortable. I looked at everyone in the

class as equals. I believe that by being an all-female class helped our discussions and made communications easier. ... As a student of mathematics I felt more confident and outspoken in this class. I think that by being an all girl class that we felt closer and that we were all equal in our thoughts as well as our actions. I really enjoyed this class and I am glad I was able to have the opportunity to be part of an all-girls class (Jill, FE).

Annie: This class being one with all females even the teacher I believed helped me perceive myself as a better math student (Annie, FE).

Lucy: Being in an all female class this semester has definitely gave (sic) me more confidence to speak up and try harder. I perceive myself as a more successful mathematical student than ever before (Lucy, FE).

Fran: As far as perceiving myself as a mathematics student differently, I have to say yes. This may sound stupid coming from me, but without there being males in the classroom there was no need for anything other than math. I could sit there and soak up all the information I could hold without any feeling of shame or embarrassment. I did not need that extra primping time or that time during class to make "goo-goo" eyes at some boy. I felt comfortable being myself. Females tend to put up a front when they first meet people and they even do this with other girls. Simply having no males shortened that time considerably and we could get on to real things (Fran, FE).

For these and others, being in an all-female class may have contributed to their advancement and perceptions of themselves as students of mathematics. These findings were similar to the reports of Campbell (1995) on several summer

mathematics programs for girls and Morrow's (1996) report on SummerMath at Mount Holyoke College. These programs are designed to provide opportunities for girls to experience mathematics in an active constructivist setting. They both reported that in these programs, the girls leave with new perspectives on mathematics and more confidence in their mathematical abilities.

Some of my students also wrote of their friendships with the other students in the class.

Annie: I even became friends with some of the girls in the class and I hope to keep the friendship past this class (Annie, FE).

Abby: I know that we all learned a lot from each other and also made a lot of new friends (Abby, FE).

While Fran was very positive about her experiences in the class, she did share one observation that was unique among the students' responses.

Fran: Being in an all girl class was a different experience. There are not a lot of math classes that are entirely female. The first thing that pops into mind first is that I never realized until this class that girls had so many excuses for anything and everything. I have given my fair share but I have never heard so many different ones in such a short period of time. Also by being an all female class we tended to gossip a lot more than a normal class. That could have also been because the class was so small and personalized. Anyhow I did notice some petty differences I wanted to share (Fran, FE).

Linda's response to the question was very explicit. She expressed her lack of fear to speak up as the major factor contributing to her growth as a mathematics student.

Linda: This class has been different in a lot of respects, but one of the biggest differences is that I have never taken a class in which there were only females. I think that this factor has been very beneficial. I believe that it made everyone more comfortable and willing to speak out. I know that I definitely felt a lot more confident. I was not afraid that I would say the wrong thing and be embarrassed. I feel that this relaxed atmosphere is what made such a difference for me this semester. It is very difficult to learn when you are afraid to ask questions. In this class I felt comfortable asking questions, giving my opinions, and I just felt good about myself as a whole. This class gave me the chance to really experience math for the first time. It was not just about getting the right answer. It was about understanding the process to get to the right answer (Linda, FE).

Linda was unable to attend the reunion because she had transferred to another school. In a follow-up phone call with her, she reiterated these sentiments and spoke excitedly about the elementary education mathematics course she was taking. She felt that she was enjoying it so much because of the confidence she had gained in Mathematics for Critical Thinking.

Of all the responses, Katie's was the most detailed. She wrote of a specific incident and the way that it affected her.

Katie: The utopia created in our classroom was due to the fact that we were all female. Studies have shown that males tend to get favored in the general classroom. I have always found this fact interesting because I never felt discriminated before in my past classes. However, I did notice a definite

difference in the structure if this particular class. There was a connection, a bond, that probably would not have existed had there been a male in the room. I certainly didn't view us as a group of feminists, but we had a security among us that we were all equals. It seems strange, but I had never seriously thought about this issue until it appeared on the prompt for this paper. Reflecting on the disposition of the classroom I can now see the role that gender played in our education. I don't know how I had been so blind to it before, but there was a definite unity feeling in our classroom. I attended co-ed schools throughout my life with the exceptance of my first year in high school when I attended a parochial girls' school. I am a member and live in a sorority house. I guess I have always taken the presence of women for granted, but I have always felt safe when I'm around my friends. On the other hand, I did notice a change in the mood of the class when Sue would bring her male friend. He almost posed a threat. I found myself hurrying faster through my work because I didn't want him to finish before me. I felt like we had to prove ourselves or something to him. I wanted him to see that I was smart and competent. I never had feelings for the man; I never even knew his name. I have a serious boyfriend, so I knew that emotional feelings had nothing to do with it. I felt competition and I believe that my classmates did too. It was interesting to watch their reactions change when he was there. They would be more articulate in their participation, some would laugh more, others were very quiet. Unmistakably, there was a difference when a male was in the room. I must anticipate this mood should I teach in a co-ed school in the future

(Katie, FE).

When I read Katie's response I was surprised that I had not made those observations. After reflecting on her comments and the two class periods to which she referred, I realized that there was a difference in the milieu of the class. The students were noisier those two nights, and they did seem to have more difficulty concentrating on their work. It may have been due to the distraction of a stranger in the classroom or to the fact that he was male. Reflecting on some recent studies of girls in both single-sex and coeducational mathematics classes (Brody, Fuller, Gosetti, Moscato, Nagel, Pace, & Schmuck, 1998; Rennie & Parker, 1997; Seitsinger, Barboza, & Hird, 1998), there may be some evidence here for the implementation of all-female mathematics classes for some students. In those studies, the authors indicated that the differences may have been due to the more nurturing environment or a consturctivist approach to mathematics in the girls classes.

SIGNIFICANCE OF THE STUDY

Most of the problems were presented with few, if any, instructions. Consistent with a constructivist approach (Cobb, 1994; Confrey, 1990), I did not want the students to be locked into one particular approach. This made them very uncomfortable, and most expressed frustration at an inability to even begin working. They found that making tables for specific cases often led to observable patterns. They also learned that manipulatives such as coins for the open and closed doors of the locker problem could often provide the clues they needed. They listened to each other and combined their ideas. Through the process of problem solving, they developed relationships with each other and with mathematics that proved beneficial to

a better understanding of the mathematics and themselves as students of mathematics. This is reflective of Perry's (1970) report that for the majority of the students in his study, "the most important support seemed to derive from a special realization of community. This was the realization that in the very risks, separateness and individuality of working out their Commitments, they were in the same boat not only with each other but with their instructors as well" (p. 213).

The significance of this study may be both for providing a clearer understanding of how students approach and develop their relationships with mathematics and for offering guidelines for developing classes, especially for adult learners, where mathematical empowerment rather than mathematics as a filter or barrier becomes the norm. Beyond reporting how this relational experience affected these students, changes in how mathematics is taught and, perhaps, what mathematics is considered to be important, may offer support for a radical reinterpretation of school mathematics.

QUESTIONS

The majority of the students taking Mathematics for Critical Thinking are fulfilling a university graduation requirement and will not take another mathematics class. Approximately ten to twenty-five percent of these students do not complete the course with a passing grade. The course is usually taught using the traditional lecture method in a large auditorium. Maybe it would be appropriate to conduct a study to determine if smaller classes with a constructivist approach would reduce the number of failures.

All but two of the students in my study were elementary or early childhood

education majors. Their problem-solving abilities steadily improved throughout the semester, and they indicated orally and in writing that they viewed mathematics and themselves as students of mathematics in a more positive way. It would be interesting to track these students for the first three to five years of teaching through interviews and observations to determine if this course has had any impact on their organization of mathematics classes.

IMPLICATIONS

The traditional lecture method of teaching mathematics to the masses in colleges and universities has done a disservice to many students who might otherwise choose to embrace mathematics rather than avoid it. By keeping mathematics on the board and in the textbooks, teachers and professors have presented mathematics as something to be worshipped and feared. Only the ones who seem to survive this system by passing the tests are allowed the pleasure of the company of mathematics. This is supported by the fact that large numbers of freshmen and sophomore college students turn their backs on mathematics and avoid it like the plague. One reason for this avoidance is reflected in the high failure rates in freshman and sophomore level mathematics classes.

This study speaks to the need for different approaches to make mathematics accessible to more students. When we look at the students in this section of Mathematics for Critical Thinking and their successful experiences with a relational mathematics curriculum, we must conclude that something made a difference for these students. From Colleen who had had few positive experiences with mathematics prior to this class to Katie who had been successful in the traditional mathematics classes,

the challenges of this class offered the students opportunities to engage in mathematics in a personal way that was empowering. They all left the class with new and positive attitudes towards mathematics and their abilities to actively engage in challenges and rewards that relationships with mathematics afford.

Considering my research I challenge college and university mathematics departments to consider restructuring their mathematics curricula to accommodate students' different ways of knowing. If we really want students become empowered mathematically, then we must afford them opportunities to explore mathematics, to develop their reasoning abilities, to learn to communicate mathematically, and to learn to make connections between mathematics and other areas of their lives. Through the development of relationships with mathematics, students may become more confident and competent students of mathematics. As Palmer (1993) would say, when we know mathematics really well, "we feel inwardly related to it: knowing it, [mathematics], means that we have somehow entered into its life, and it into ours" (p. 57). Mathematicians have this relationship with mathematics, and at varying levels, we can help our students develop this kind of relationship with mathematics.

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APPENDIX A

MATHEMATICS and CRITICAL THINKING
MaTH 1473--Section 902
SYLLABUS--Spring, 1997
(REVISED)

Instructor: Gloria Nan Dupree
Phone: 521-6436 (Work)
359-1019 (Home)
email: gdupree@ossm.edu

Office Hours: By Appointment.

Participation in this class requires that you agree to be a subject for my dissertation research. There will be periodic writing assignments, in addition to weekly journal entries. Some of the class discussions may be audiotaped.

Philosophy: It is my belief that every person can learn mathematics!

There are many areas in our day-to-day living in which we encounter mathematics. Some of these are obvious, while others are more obscure. I hope that through your experiences in this class, you will become more aware of the mathematics displayed in your surroundings, and maybe even develop a greater appreciation for mathematics, while having a little fun along the way. **The success of this class and each member depends on everyone. You have a responsibility to attend all class sessions. When you are absent, you deprive the class of your valuable contributions.**

Texts: Abbott, Edwin A. (1963). *Flatland: A Romance of Many Dimensions*. (5th Ed.). New York: Harper Collins.

Burger, Dionys. (1965). *Sphereland: A Fantasy About Curved Spaces and an Expanding Universe*. New York: Dover.

(NOTE: These books come bound together for about $10.00)

Selected exercises. (May be purchased at King Kopy for about $15.00).

Expectations: You will be expected to

1. attend class unless there is an emergency, in which case, please call me to explain the circumstances.
2. arrive to class on time.
3. come to class prepared to enter into the class discussions.
4. participate in class activities.

5. keep a journal with a minimum of 2 one-page entries per week.
6. turn in all assignments on time.

Grades:

Requirement	Percentage Points
First paper	10
Attendance (-1 point for each absence)	20
Participation in discussions	10
Assignments on time (-1 point for each tardy)	20
Second paper	15
Final paper	25
TOTAL	**100**

Schedule:

MONDAY, JANUARY 13
Discussion of group dynamics and group establishment of rules for the class.
Counting squares.
WEDNESDAY, JANUARY 15
Counting rectangles.
Letter patterns.
Pascal's Triangle and related patterns.
MONDAY, JANUARY 20
HOLIDAY (Martin Luther King Day)
WEDNESDAY, JANUARY 22
Journal entry 1 due: Complete "Mathematics is ... because ..." in at least 250 words (approx. 1 page doubled spaced). Recount the mathematical experiences in your life that have influenced your response.
The 12 Days of Christmas and more patterns.
Figurate numbers.
MONDAY, JANUARY 27
Journal entry 2 due: Car tag problem
Circles, segments, and intersections
WEDNESDAY, JANUARY 29
Chinese checkers and dominoes
MONDAY, FEBRUARY 3
Journal entry 3 due: Diffy and p35, #D
Magic squares
WEDNESDAY, FEBRUARY 5
Tetraminoes and pentaminoes
MONDAY, FEBRUARY 10
Palindromes
Journal entry 4 due: Hefty hippos
WEDNESDAY, FEBRUARY 12
One hundred dollar words.
MONDAY, FEBRUARY 17

Journal entry 5 due: Candy problem.
Problems related to the candy problem

WEDNESDAY, FEBRUARY 19

Family birthday problem
Orange juice problem

MONDAY, FEBRUARY 24

Journal entry 6 due: Report on your woman mathematician.
Flatland, pp. 1-40.
Oral reports
Word puzzles

WEDNESDAY, FEBRUARY 26

Flatland, pp. 41-88.
Cutting the moldy cheese
Drawing solids

MONDAY, MARCH 3

Journal entry 7 due: Write a summary of what you have read in *Flatland.*
Flatland, pp. 89-120.
Discussion of *Flatland.*

WEDNESDAY, MARCH 5

Sphereland pp. 1-26.
Probability with Skittles

MONDAY, MARCH 10

Spring Break

WEDNESDAY, MARCH 12

Spring Break

MONDAY, MARCH 17

Patterns on billiard tables

WEDNESDAY, MARCH 19

Patterns on billiard tables (continued)

MONDAY, MARCH 24

Journal entry 8 due: Paper on *Flatland* and *Sphereland.*
Geodesic domes

WEDNESDAY, MARCH 26

Geodesic domes (continued)

MONDAY, MARCH 31

Journal entry 9 due: What does it mean to you to know mathematics?
Tangrams

WEDNESDAY, APRIL 2

Tiling a plane with regular polygons
Fibonacci numbers

MONDAY, APRIL 7

Journal entry 10 due: Prepare to discuss Fibonacci numbers in nature and the Golden Ratio
"Donald in Mathemagic Land" video.

Projects on tiling a plane due
Tessellations

WEDNESDAY, APRIL 9

Describe your reactions to "Donald in Mathemagic Land".
Geometry-Aestheometry (Art).

MONDAY, APRIL 14

Journal entry 11 due: Project on curve stitching due

WEDNESDAY, APRIL 16

Paradoxes and oxymorons
Three-bean salad

MONDAY, APRIL 21

Journal entry 12 due: Write up two examples of paradoxes to share with the class. Write your solutions to the Three-bean salad problems.
Shortest distances.
Graphs.
Who's guarding the museum?

WEDNESDAY, APRIL 23

General discussion of the course and your final paper.
Pythagorean triples
Fractions
Fractals

MONDAY, APRIL 28

Logic and truth tables
Alice in Puzzle Land.
The Lady or the Tiger?

WEDNESDAY, APRIL 30

Final paper due.

MAY 5-9 FINAL EXAMS

ADDITIONAL TOPICS

Geometry of angles
Continued fractions and the Euclidean algorithm.
Prime numbers and the sieve of Eratosthenes.
Perfect, abundant, and deficient numbers.

#-#

Any student in this course who has a disability that may prevent him or her from fully demonstrating his or her abilities should contact me personally as soon as possible so we can discuss accommodations necessary to ensure full participation and facilitate your educational opportunities.

APPENDIX B

MY MATHEMATICAL JOURNEY

"Mathematics has a public image of an elegant, polished, finished product which obscures its human roots. It has a private life of human joy, challenge, reflection, puzzlement, intuition, struggle, and excitement" (Buerk, 1993, p. 151). This is the beginning of Dorothy Buerk's article "Getting Beneath the Mask, Moving Out of Silence" which she wrote with her student, Jackie Szablewski. Together they paint a vivid picture of Jackie's journey and struggle to make her private thoughts about mathematics public.

After reading this article, I began to muse about my mathematical journey. Where HAD I been mathematically, what had brought me to my current mathematical place, and why did this article trouble me so? Where do I go from here?

I am sure that my mathematical journey began before my earliest memories of mathematics, and maybe I only remember those incidences which were somewhat troublesome. In elementary school, each year we took 'Mrs. Heard's' tests; they must have been some kind of standardized tests. I remember that after the tests in the third grade, my teacher told me that numbers could have more than two decimal places. My only previous experience with decimals had been in the context of money, so I 'knew' that a number could not have more than two decimal places. In my naiveté' I had 'corrected' the test. In numbers with three decimal places, the decimal points became commas; in others with more than three decimal places, I just 'moved' the decimal point and added a dollar sign. All of my computations were correct, and I still had a high score. In the fourth grade, I (along with most of the class) was paddled for

failing an arithmetic test.

Junior high school was punctuated by Miss Jernigan's speed tests on computation. I frequently had to stay after school because I did not have a perfect paper. I knew the facts but could not write the answers as quickly as she called out the problems. In spite of this I DID like mathematics. Maybe it was the challenge; I was very competitive. As an eighth grader, I was elated when they told us that in high school we could choose our subjects. Naively I thought that now, at last, I could take math all day long and not have to 'mess around' with the boring stuff like history and English. Was I in for a surprise!

As much as I liked mathematics, I only took two years of algebra and geometry. There were other courses such as Latin, chemistry, physics, and biology that filled my schedule. In college, after scoring third highest of 500 freshmen on the mathematics placement test, I changed my major from speech therapy to mathematics. I was successful in mathematics, always near, or at the top of the class. I just never thought of math as something that I was not supposed to be able to do. I knew that other girls did not have the same feelings that I had about mathematics, but there were also boys who could not do mathematics either. I never considered the possible differences; I just thought that people who could not understand mathematics were not trying hard enough. I Jilld these thoughts with me as I began teaching.

In the early years of my teaching career, I became the AUTHORITY IN MY CLASSROOM, the teller of the facts, the keeper of the answers, the giver of grades. There was an invisible barrier between my students and me. I was convinced that if students were not learning, then THEY must be doing something wrong. It could not

be my fault! I was an excellent teacher! So I blamed them. It would be many years before I realized in order for students to experience mathematics in a personal way, there must be an environment in the classroom that promotes relationships--among the students, with the teacher, and with mathematics.

As I have reflected on who I am as a mathematics educator, I have become acutely aware of the lessons of my students. I remember Mike, who after 2 weeks of studying absolute values, asked me while we were reviewing for a test, "What's that '1 times 1' ($|x|$) stuff?" Mike has taught me that students do not always learn what I think they are learning. They may only glean bits and pieces of what transpires in the classroom. Several hours later when they attempt their assignments, they may be missing critical pieces of information, become frustrated, and just give up. This can develop into a vicious cycle from which escape is difficult at best, and for some students almost impossible. By listening and discussing mathematics with my students and providing opportunities for their active involvement, I may enable them to develop new strategies for experiencing mathematics.

Larry taught me a similar lesson. I have always been interested in the history of mathematics and try to incorporate relevant 'tidbits' to interest students in learning mathematics. Larry sat in the back of the classroom and watched the girls on the tennis courts. I know that girls' tennis matches were far more interesting that studying algebra at 2 o'clock in the afternoon, but Larry may have been listening more than I thought. One day I mentioned something about Aristotle; the next day before class, Larry asked me, "That stuff you said yesterday, did you say that Eric Estrada did it first?" Larry WAS hearing some of what transpired in class. I should have recognized

it then and made a greater effort to bring Larry back into focus on mathematics.

John could not understand why I insisted that he use algebra to solve "those dreaded word problems." He could solve the problems; he just could not work them using the methods that I insisted he use. On one 'word problem' test, John wrote, "Mission Impossible." John taught me that there is a place for creativity in mathematics problem solving; I now search for problems that encourage students to stretch their imaginations. I also provide opportunities for students to incorporate their creativity with mathematics.

I taught Peggy in an 8th grade algebra class; she was one of the silent students who did well but never drew attention to herself. At their 10-year reunion I was surprised to learn that Peggy was teaching mathematics, and that I was the one who had inspired her. From Peggy, I learned that I never know how I may be influencing someone else.

My association with The College Board's Advanced Placement Program since 1983 has enabled me to establish a network of outstanding mathematicians and mathematics educators across the nation. There have been opportunities for me to grow in my understanding of calculus and effective teaching strategies. The advances in technology and the calculus reform movement have opened doors for developing curriculum that is relevant, interesting, and challenging.

Through my active involvement with the Alabama Council of Teachers of Mathematics, Oklahoma Council of Teachers of Mathematics, and Central Oklahoma Association of Teachers of Mathematics, I have had opportunities to help organize and coordinate monthly and annual mathematics conferences. I have been able to share my

experiences with technology in the classroom with hundreds of teachers across the state of Oklahoma and nation, but more importantly, I have learned from the participants' sharing their experiences.

Because I was a state finalist for the 1983 Presidential Awards for Excellence in Science and Mathematics Teaching, I was asked to teach in the first Rickover Science Institute (Now Research Science Institute). Working with Admiral Hyman G. Rickover, Father of the Nuclear Navy, was one of my most enriching experiences. He and his staff introduced the faculty and students to many famous people including astronauts, scientists, and politicians. Through working with exemplary teachers to develop a curriculum from scratch, I learned how resourceful teachers can be. It was also exciting to see high school students involved in mentorships with some of the leading researchers in the country.

My experiences with The Johns Hopkins University's Center for the Advancement of Academically Talented Youth (CTY) was an adventure. CTY was begun by Julian Stanley for 12 to 15 year-olds to get a jump-start on mathematics. Until 1988, the precalculus instructors were former CTY students. These students had served as teaching assistants for a couple of years while waiting for the instructors, who were also CTY prodigies to graduate from college and find full-time vocations. Because the program had begun to lose some of its credibility with high school mathematics teachers in the northeastern states, the program directors decided to hire some high school teachers to remedy this situation. I did not realize this until the first staff meeting. My friend Martha (who retired from teaching the following year) and I were the only people in the room who were over the age of 22. The third teacher,

Bob, had not arrived. We were met with hostility from these young instructors because many of them had had unpleasant experiences with what they considered incompetent high school mathematics teachers. By the end of the first three-week session, they had accepted us and acknowledged our competence. I learned from these young instructor and the students that even intelligent students approach the study of mathematics with different styles, and that intelligence is not a measure of someone's readiness to develop an understanding of mathematics. I learned that these precocious youth need understanding, encouragement, and time instead of criticism.

The most valuable aspects of this program were the late-night sessions in the basement of our boarding house where we adults gathered to play cards or share mathematics teaching strategies. In addition to precalculus, there were teachers in other disciplines as well: etymology, anthropology, and paleontology, as well as Advanced Placement chemistry, physics, and biology. The intellectual stimulation of this group of professionals was like a juicy ripe peach. The juice gets on everything, but it does not matter because it is so delicious. One of the topics frequently discussed was how similar these classes of very bright students were to our regular classes, and even to general math classes. We came to the conclusion that any class of students will stratify themselves into three groups--those who go above and beyond what is expected, those who do just enough to satisfy the requirements, and those who get lost early in the year and soon perceive themselves as dumb. I have a big responsibility to the students in the latter group because I believe they can learn.

In the summer of 1990, as the result of a National Endowment for the Humanities grant proposal, I was privileged to be one of fifteen high school teachers

selected to spend five weeks studying "Great Theorems of Mathematics in Historical Context." The teachers were from various disciplines and from all across the country plus one teacher from Guam. We research manuscripts in the mathematicians native languages and shared our findings with each other. This experience opened my eyes to the struggles of the early mathematicians, especially the women, who were thwarted at every turn in their efforts to be taken seriously.

In 1989 when I was hired as the first mathematics professor for the Oklahoma School of Science and Mathematics (OSSM), little did I know what opportunities lay in wait for me. OSSM, truly an awesome environment in which to work, is a state residential school for high school juniors and seniors gifted in mathematics and/or science. Not only do we have a group of highly specialized professionals, but almost every region of the world is represented in our faculty. I have been privileged to learn different approaches for teaching and learning mathematics from my colleagues from China and Poland

In the 1970s Marva Collins founded the Westside Preparatory School in Chicago for young black children for whom the public schools have given up hope. She was concerned that young black children were not afforded the same opportunities or challenged as much as other children in the Chicago schools. The basic philosophy of her school is that every child can learn, and someone must believe in these children, and help them develop respect for themselves, their abilities, and for others. Oklahoma's Great Expectations is a program for elementary schools that is modeled after Marva Collins' school. It was my privilege in the summer of 1995 to conduct two five-week sessions in mathematics for elementary teachers whose schools

were committed to the Great Expectations program. I learned that elementary teachers really want support from the mathematics community that will help them feel more comfortable with mathematics in the classroom.

Reflecting as I read Buerk's and Szablewski's (1993) article, I saw much of myself in Jackie--not in the 'math phobic', but in her statement, "I was speaking of myself as if I was someone else; not only that, I was speaking of myself as if I was male" (p. 155). It smacked me right in the face! I realized that in my study of mathematics and in my teaching, I had behaved as a male: competitive and authoritarian. How could I reveal my struggles to my students? Wouldn't I lose my credibility? Oh, sure, I have tried various techniques to encourage my students to think about mathematics--study groups, cooperative learning situations, journaling, projects, etc., but I always seem to come back to the same format--I tell; you learn; you tell; I grade. Time seems to be a critical factor; I really want my students to appreciate mathematics and their abilities to think about it, but the administration always seemed to have other ideas--high scores on SAT, ACT, and Advanced Placement Exams. I must continue to work toward establishing a classroom environment that encourages students to explore mathematics and their relationships with it and that promotes experimentation and discussion about mathematics.

Through the years my experiences with students and colleagues have convinced me that there are many variables that influence the events of the classroom. I have come to realize that there are many students who in an attempt to avoid mathematics have terminated their mathematical journeys. There are others like Jackie, Dorothy Buerk's student, who had taken calculus in high school and had been

successful in the course. However, Jackie never felt as if she owned the mathematics; it always belonged to someone else. Mathematics, in its private world, is as subjective as any of the humanities, and it is only when we remove the 'public mask' that we can see its inner beauty. Dorothy Buerk said, "... to bring the private world of mathematics into the classroom and really listen and share in the intellectual struggles of our students takes courage" (Buerk & Szablewski, 1993, p. 163).

My goal as a teacher is to empower my students to remove their mathematical 'masks,' and to instill in them the abilities to recognize their own personal mathematical strengths and to develop their own strategies for incorporating mathematics into their journeys.

I now realize that learning mathematics is a cooperative effort that involves me as a facilitator and the students as active participants. Palmer (1993) in To Know as We are Known: Education as a Spiritual Journey posits that when we know something really well, "we feel inwardly related to it; knowing it means that we have somehow entered into its life, and it into ours" (p. 57).

As I considered possibilities for the focus of my research, I reflected on the writings of some of my current students. These students are bright, but their mathematics backgrounds are limited. As we study the algebra of functions, I use writing as a means of helping them make connections with what they already know and establish new relationships with mathematics. Through creating their own function signatures and using other metaphors, they are sometimes better able to describe their relationships with mathematics and develop their own self-assessments. In one class I asked them to choose a plant that best described the way they learn

mathematics. Some of their responses follow.

1. I believe that the mahogany tree personifies my feelings toward math. Once I get a concept, I'm strong in it, and I almost never forget how to do it. But right now, it feels like my mahogany forest is being cut down faster than it can grow back. Things are coming at me so fast that they can't take root and grow. My math skills could be valuable and treasured in the future, but only if they are given the time to grow strong.
2. Rubarb (sic)--because it's really not that good, but if you fix it the right way I kind of like [it].
3. Venus fly trap because once I've got something, I've got it.
4. African violet--Because math is hard for me and African violets are hard to grow.
5. Math is like a grass, because there is way too much of it. Sometimes I really like math & grass, but they are usually just annoying things I deal with every day.
6. Spider plant. Has a base where everything is connected, then innumerable offshoots. Some are harder to follow than others, some seem completely unrelated, but they all go back to the base.
7. An oak tree. The oak grows with nurturing and becomes a strong harbor to other creatures. Of course, poison oak grows around and on oak trees, but once you get over the itch and the rash and the cold, pink calamine lotion, they everything is OK.
8. I compare math to the grapevine, like the entangling mass of vines, math can

seem impossible with no purpose, but its rewards at the end are well worth it, like the grapes.

This type of communication provides a vehicle for students to examine their relationships with mathematics and enable me to understand how to help each student develop a good working relationship with mathematics. I have also found it beneficial to ask the students to write an essay at the beginning of each course telling me what they know about mathematics and an essay at the end of the course detailing what they have learned about mathematics and themselves as students of mathematics. This enables them to evaluate themselves and their progress.

In 1996 when I requested and was granted permission to teach a small section of Mathematics for Critical Thinking (MATH 1473), I envisioned an opportunity for me to share my love of mathematics with some college students who might not share my enthusiasm. My hope was to create a curriculum and an environment that would enable students to develop healthy relationships with mathematics. I also hoped that the events of the semester as together we explored different avenues of mathematics might change their perceptions of mathematics and their relationships with it.

My major struggles came, not with the students, but with the request of the mathematics department for a detailed syllabus. I obligingly turned in the syllabus (both fall and spring semesters). Because this syllabus was incongruent with constructivism and the relational focus of the class, it was partially abandoned except as a check list for topics. I resolved the conflicts I had with the syllabus by choosing topics that seemed appropriate for the needs of the students as we journeyed through the semester. The syllabus found in APPENDIX A is actually a revision that reflects

the sequence of topics in the spring semester class of 1997. It bears little semblance to the copies the students received at the beginning of each semester.

This is only part of my mathematical journey. There will be more students, more mathematics, and more opportunities to share my love and intimacy with mathematics. Dorothy Buerk said, "True, to bring the private world of mathematics into the classroom and really listen and share in the intellectual struggles of our students takes courage" (Buerk & Szablewski, 1993, p. 163), and this I must do.

APPENDIX C

Questions for Journal Entries[1]

1. Describe a mathematics class that stands out in your memory.

2. What was your most exciting experience with mathematics?

3. What was your most disturbing experience with mathematics?

4. What was your most difficult experience with mathematics?

5. What was your most surprising experience with mathematics?

6. What does it mean to think about mathematics? Have your opinions changed?

7. What does it mean to know mathematics? Have your opinions changed?

8. What does it mean to do mathematics? Have your opinions changed?

9. How did you view yourself as a student of mathematics before this class?

10. How do you view yourself as a student of mathematics now that you have been in this class for almost a semester?

11. What is the teacher's role in your learning of mathematics?

12. What is your role in your learning of mathematics?

13. How does writing make a difference in your learning of mathematics?

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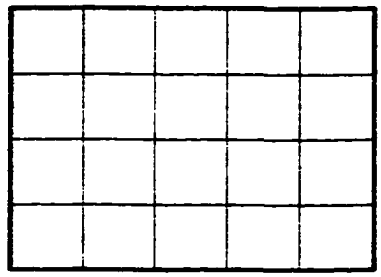
[1] These questions were addressed by the students throughout the semester in their journal entries and their essays. The questions were not always explicitly stated, but the students' responses sometimes emerged in their creative writings.

APPENDIX D

SAMPLE PROBLEMS

How many paths are there from A to B if one can only move to the right or down along the lines?

A



B

Every year the members of the loyal order of Pygmy Hippopotami march in the Independence Day parade. When they tried on their uniforms this year, they decided they had better lose a little weight before the Fourth. a motion was introduced, amendments were debated, and the hippos resolved to meet once a week at a gym. After the workout they'd weigh themselves and retire to their clubhouse to watch a ball game. The hippo who had lost the least number of pounds that week would have to pay for beer and pizza.

One morning they forgot to weigh themselves before leaving the gym. They found a scale in a nearby warehouse, but it started at 300 pounds, more than any of them weighed. Undaunted, they weighed all the possible pairs of hippos. At the clubhouse, they thought, it would be a simple matter to calculate the individual weights. Here are the weights of all the pairs: 361, 364, 376, 377, 380, 389, 392, 393, 396, 398, 408, 411, 414, 426, 430.

The question: How much did each hippo weigh?

You can assume that all the weights are whole numbers. If you look at them carefully, there is a relatively easy way to solve this puzzle (Stover, 1990).

What is the maximum number of pieces of pie with 1 cut? With 2 cuts? With 3 cuts? With 4 cuts? With n cuts?

What is the maximum number of segments created when 2 points on a circle are connected? When 3 are connected? When 4 are connected? When n are connected?

Complete the following letter pattern at least five different ways.



I had to buy a new car tag and asked my neighbor to install it. He put it on upside-down. If the tag has 5 numbers, the new number is smaller than the correct number, and the difference between the two numbers is 76,203, what should my tag number be?

One thousand students have lined up in a very long hall with 1000 closed lockers. One by one the students run through the hall and perform the following ritual: The first student opens every locker. The second student goes to every second locker and closes it. The third student goes to every third locker and changes its state. If it is open, the student closes it; if it is closed, the student opens it. In a similar manner the fourth, firth, sixth, ... students change the state of every fourth, fifth, sixth, ... locker. After all 1000 students have passed down the hall, which lockers are open? (NCTM, 1989)

APPENDIX E

MATH 1473
MATHEMATICS AND CRITICAL THINKING
Spring, 1997
Gloria Nan Dupree
FINAL EXAMINATION QUESTIONS

1. What was the most significant topic or lesson in this class? Write a complete discussion of this topic as if you were teaching it to someone else.

2. Recreate your mathematical journey. Discuss what you learned about yourself as a student of mathematics as a result of being in this class? Consider the following prompts.
   Describe your most (exciting, disturbing, difficult, surprising) experience with mathematics.
   What does it mean to (think about, to know, to do) mathematics?
   How have your opinions changed?
   How has your view of yourself as a student of mathematics evolved?
   How has writing impacted your learning of mathematics?

3. Compare and contrast the methods used in this class with the methods of your previous mathematics classes. What is the (teacher's, student's) role in the learning of mathematics?

4. Compare and contrast this class with those mathematics classes in which there were males. How did being in an all-female class make a difference in the way you perceive yourself as a student of mathematics?

APPENDIX F

TANGRAM INSTRUCTIONS (VERBAL)

1. Take your 8 ½ in. by 11 in. sheet of paper, and fold the upper right-hand corner to the long edge of your paper making a triangle. What are the properties of this triangle? What shape is created at the bottom of your paper? With this corner folded over, in what shape is the paper now?

2. Cut the bottom rectangle off, discard it, and open the sheet of paper. What shape do you have now?

3. Cut the square along the fold. What kinds of triangles are these?

4. Take one of the triangles and fold it in half. Now cut along the fold. What kinds of triangles are these? What relationship do they have with the remaining triangle?

5. Take the remaining large triangle, and locate the midpoint of the hypotenuse. Do not fold the triangle in half. Now take the vertex of the right angle, and fold it to the midpoint of the hypotenuse. Cut along the crease. What kind of triangle do you have? What relationship does it have with the other triangles? What is the shape of the remaining piece?

6. Hold the isosceles trapezoid with the short base at the top. Fold one of the upper vertices to the center of the long base. Cut along the crease. What are the two shapes created?

7. Lay aside the parallelogram. Take the remaining trapezoid, and fold the two sides so that they overlap and create a square. Cut off these triangles. What kind of triangles are they? What relationship do they have with the other triangles?

8. You now have seven (7) tangram pieces. Put them back together to form the original square. (Remember that you threw away the rectangle that you cut off first.)

APPENDIX G

UNIVERSITY OF OKLAHOMA

AGREEMENT TO PARTICIPATE

TITLE: "What Can We Learn by Examining Students' Mathematical Narratives? How Are these Narratives Influenced by Different Types of activities in the Mathematics Classroom?"

INVESTIGATOR: Gloria Nan Dupree

I, _____, hereby agree to participate as a volunteer in the above named research project, which has been fully explained to me.

I understand that I am free to refuse to participate in any procedure or to refuse to answer any question at any time without prejudice to me. I further understand that I am free to withdraw my consent and to withdraw from the research project at any time without prejudice to me.

DATE: January ____, 1997

STUDENT SIGNATURE:_____

APPENDIX H

MATH 1473 QUESTIONNAIRE

Name_____ S. S. # ____ - ____ - ____

Date of Birth__/____/_____Age _____

High School _____

Population of Home Town:

- Less than 10,000
- 10,000 - 20,000
- 20,000 - 30,000
- 30,000 - 50,000
- 50,000 - 100,000
- More than 100,000.

Mathematics Courses Taken in High School:

(OPTIONAL: Grades.)

- _____General or Business Math
- _____Algebra I
- _____Algebra II
- _____Geometry
- _____Algebra III, Precalculus, Math
- _____Analysis, Advanced Math, or
- _____Trigonometry
- Calculus
  - _____1 Semester
  - _____2 Semesters
  - _____AP (AB or BC)
  - _____AP Exam Score
  - _____Not AP

In College (name):

- College Algebra
- College Precalculus or Trigonometry
- Calculus I
- Calculus II
- Calculus III
- Calculus IV
- Others (Please list.)

Major _____ Minor_____

APPENDIX I

QUESTIONS FOR CONSIDERATION ON THURSDAY, DECEMBER 4, 1997

These questions may be used as prompts, and they may lead to other questions that you think need to be addressed. Please feel free to respond as you wish. I am still in the learning mode, and you have much to share with me.

1. Now that you have completed MATH 1473, what were the most important aspects of the class?

2. What improvements would you suggest?

3. What impact has the class has on you as an individual and on the mathematics you are currently doing?

4. How has your relationship with mathematics been affected as a result of your experiences in the experimental section of MATH 1473?

APPENDIX J

My evaluations of Jill, Colleen, Fran, Linda, and Katie follow.

November 30, 1997

Dear Katie,

You came into MATH 1473 with a solid background in mathematics. Your positive attitude and willingness to help others made a difference in the climate of the classroom. Often students who have had less than pleasant experiences with mathematics are intimidated by those they perceive as smarter than they. You have a manner and patience that was helpful rather than threatening to the others in the class.

In the beginning of the semester, I wondered if you would be challenged enough. As the semester progressed I realized through reading your journal entries and my observations that you were observing the other students as they worked on the problems. By allowing them the opportunity to struggle until they solved a problem rather than giving them a quick answer, you helped them renew their faith in themselves as doers and knowers of mathematics. You also seemed to benefit from your observations.

Your paper, "Evelyn Boyd Granville vs. *Flatland*," seemed to be a reaffirmation of your success in mathematics, and yet a statement that one person can effect changes to make education accessible to all students. Your final paper indicates there have been changes in your philosophy of teaching mathematics. You seemed to have discovered the power of writing in mathematics, of analyzing mistakes or misunderstandings, using manipulatives, and helping students develop a sense of community in the mathematics classroom. You may have even changed your views on single-sex mathematics classes, as indicated by your astute observations of the class when Shannon's boyfriend visited our class. You have the qualities for being an excellent teacher (mentor) of mathematics.

Thank you for all that you taught me.

Gloria

November 30, 1997

Dear Colleen,

As I read your journal entries, your papers, and your final, and as I reflected on my observations of you in class, several things come to mind. I first saw you as shy, insecure, and not really sure you wanted to be in a mathematics class. You sat with your head down and didn't seem to want to make eye contact. With your history with mathematics, that was not surprising. When your friend was killed, I was afraid that you might not be able to pull everything together.

As the semester progressed, you began to loosen up a little. In your journal entries, you seemed willing to express your frustrations, and you began to ask questions in class.

In your paper on *Flatland*, your comment on the women, "Maybe they did have memories but were told so many times that they didn't so they believed that." seemed to reflect your experiences with mathematics. I do not think you believed that you could learn and do mathematics, much less think about it. In your paper, you also comment on Mary Somerville in *Flatland*, "...she probably would have succeeded because of her hard work and dedication." I think you may have begun to believe this about yourself. You indicated in your final paper how beneficial this exercise had been for you.

I believe you can do mathematics. You have come a long way in one semester, but your journey is not complete. You seem to realize the importance of determining the reasons for incorrect answers, so that mistakes or misunderstandings can be remedied. No one knows or understands all of mathematics, and it is important to know that we can admit that we do not understand something, if we are willing to find the answers.

You seem to understand the importance of building community in the classroom. Your comments in your final paper validate the freedom you seemed to feel when you were working with your classmates. I observed a happier and more outgoing student as the semester progressed. Reread your final paper as well as your notes. As you said, "I have proved to myself that I can do things I put my mind to doing."

Thank you for all you taught me.

Gloria

December 4, 1997

Dear Jill,

Although you have had some difficulties with mathematics, you seem to feel somewhat successful because of your diligence and hard work. At the beginning of the semester, I sensed that you may have had a narrow view of mathematics as being mostly computational. You seem to feel most comfortable with the computational aspects of mathematics. You also commented that mathematics gives you a "sense of freedom."

In your essay on *Flatland*, you creatively had Sophie change herself into a pentagon so that she could gain acceptance and recognition for her work in mathematics. It also seemed important to you for her to resume her identity and work to promote women everywhere.

You seem to recognize the importance of asking questions and pursuing a mathematics problem until you have solved it. In your final paper you acknowledge the value of working in small groups. You speak of establishing "relationship[s] of friendship as well as trust." I thought your analogy of mathematics as "another language" was on target.

You seem more confident in your abilities to do mathematics and commented in your final paper, "After being in this class I actually like math again." This is important for you to remember. You also expressed the importance to you of writing in mathematics as a way to promote more thinking about mathematics. You also seemed to flourish in the all-girls setting.

Thank you for all that you taught me.

Gloria

December 4, 1997

Dear Fran,

At the beginning of the semester, I detected that you have let a few distasteful experiences with mathematics affect your relationships with it. As the semester progressed you seemed to find new meaning in mathematics. You seemed so excited when you had figured out the solution to a problem. I liked the way you shared problems with your sister and her friends.

In your paper on *Flatland*, you had four young women join themselves together to form a square so that they could enter the realm of the men so that they could get an education. This indicates that you value small groups for solving mathematical problems.

In your final paper, you indicate a change in your beliefs about the importance of mathematics for everyone. You also seem to believe that you can do mathematics, and that this has had a positive impact on your attitude towards mathematics. "I am a smart student and this class has proven to me that if I try hard enough if there is an answer I can find it." You also seem to value writing as a way to learn mathematics. "Anything that comes from the writings is going to be positive either for them or even another student."

You also seemed concerned about the amount of socialization and indicated that it might have been due to the fact that there were no guys. I appreciate your candor, but I also need to know if it interfered with your ability to concentrate and learn mathematics. Maybe this is something we need to discuss as a group.

Thank you for all that you taught me.

Gloria

December 4, 1997

Dear Linda,

Prior to taking MATH 1473, you seemed to have had good experiences with mathematics even though some topics were difficult. You kept really good journal notes. You have been very reflective of your experiences with mathematics in this class. "... I need to be more open-minded, imaginative, and creative." As the semester progressed, I caught glimpses of all of this in you.

Your paper on *Flatland* indicates that you believe that one person can effect changes in the way it has always been. You may also believe that people working together can solve problems more easily and efficiently than one person working alone.

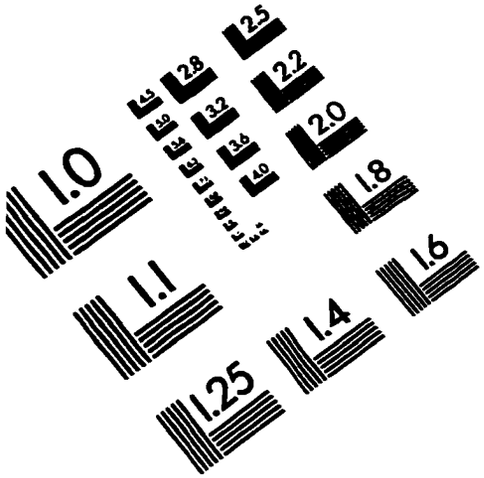
Your final paper indicates a positive change in your attitude towards mathematics. You state that "math is not all bad. Actually, some things are quite fun." You also addressed the positive impact that writing the journal entries had on your learning, "because I have really had to think about math and every step to working problems. In doing so, I am now better able to understand myself and how my mind works when it comes to math."

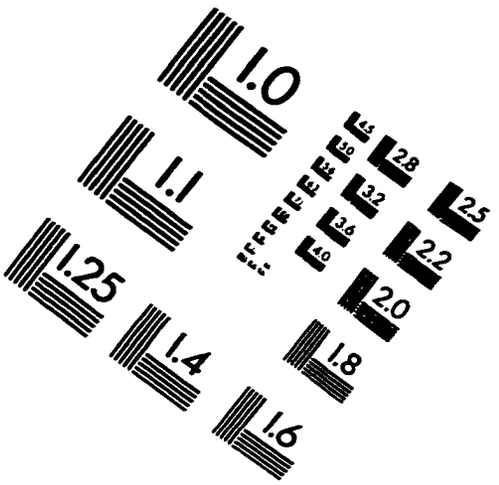
You also stressed the importance of working in small groups, of having students actively involved in doing mathematics, and of giving students the opportunity to correctly construct their own solutions to problems. In our telephone conversation, you stated that you had already used much of the mathematics that you had learned, and that you really felt more confident in your abilities to do mathematics.

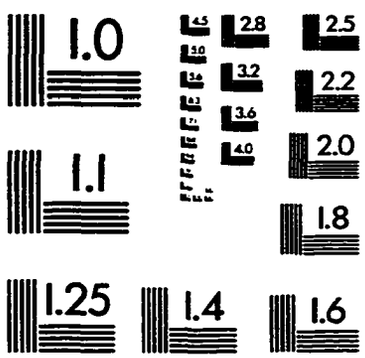
Thank you for all that you taught me.

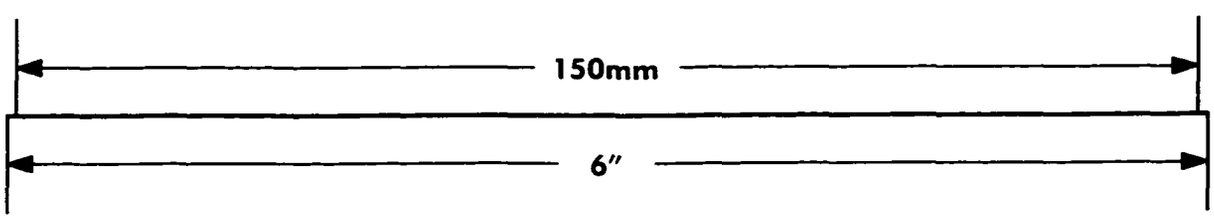
Gloria

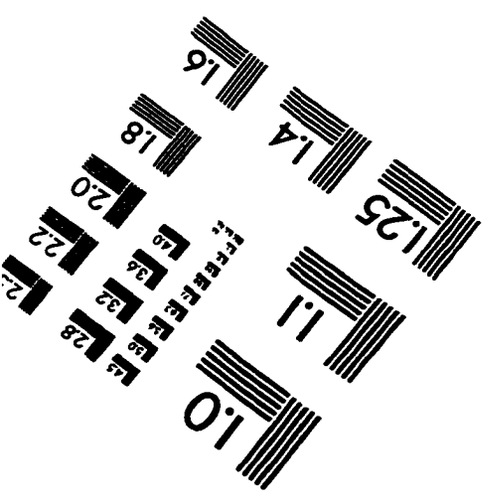
IMAGE EVALUATION
TEST TARGET (QA-3)











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