74-6960

KONTOGIANES, John T., 1941-
THE EFFECTS ON ACHIEVEMENT, RETENTION, AND ATTITUDE OF AN INDIVIDUALIZED INSTRUCTIONAL PROGRAM IN MATHEMATICS FOR PROSPECTIVE ELEMENTARY SCHOOL TEACHERS.

The University of Oklahoma, Ph.D., 1973
Education, theory and practice

University Microfilms, A XEROX Company, Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

THE EFFECTS ON ACHIEVEMENT, RETENTION, AND ATTITUDE OF AN INDIVIDUALIZED INSTRUCTIONAL PROGRAM IN MATHEMATICS FOR PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

JOHN T. KONTOGIANES

Norman, Oklahoma

1973

THE EFFECTS ON ACHIEVEMENT, RETENTION, AND ATTITUDE

OF AN INDIVIDUALIZED INSTRUCTIONAL PROGRAM IN

MATHEMATICS FOR PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

APPROVED BY

Thomas J. Hill

Eunice Lewis

Harold V. Huneke

W. Alan Dvarnander

Herbert R. Hengst

DISSERTATION COMMITTEE

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the members of the dissertation committee for their assistance during this study. They were: Dr. Thomas Hill, Chairman, Dr. Herbert Hengst, Dr. Harold Huneke, Professor Eunice Lewis, and Dr. Alan Nicewander.

A special debt of gratitude is owed to Dr. Hill, Dr. Huneke, and Mr. Robert Wendling, who served on the committee to evaluate materials used in the experimental learning model.

TABLE OF CONTENTS

Chapter

LIST OF TABLES

CHAPTER I

INTRODUCTION

Background of the Problem

More students are attending college than ever before. They represent many more levels of academic ability and academic preparation than in previous years. In addition, they bring to the campus attitudes and values from many more cultural backgrounds than before. Many are in college because of parental or societal pressure. Students are pursuing a wide variety of occupational goals, many of which require lengthy academic preparation. As a result of the recommendations of various committees and study groups, such as the Commission on Mathematics of the College Entrance Examination Board and the Committee on the Undergraduate Program in Mathematics, much new and varied content has been added to the college mathematics curriculum. This academic, economic, and social heterogeneity on the college campus is reflected in student attitudes and achievement.

Present educational practices and policies continue to produce the same normal achievement distribution in classes that seemingly has persisted always. Instead of providing students with opportunities to learn in ways individually suited to their own abilities and backgrounds, instructors hold learning time and instructional material constant for all students, thereby promoting an achievement distribution corresponding to the normal distribution of the students' abilities and backgrounds. A fundamental task in education should be to develop strategies of instruction which will take into account individual differences in such a way as to promote, rather than inhibit, the fullest development of every individual learner.

One such instructional strategy can be derived primarily from the work of John Carroll. Briefly, the model proposed by Carroll indicates that if students are normally distributed with respect to aptitude for some subject and all students are given exactly the same instruction in terms of amount and quality of instruction and learning time allowed, then achievement, measured upon completion of the instructional sequence, will be normally distributed. Under such conditions the correlation between aptitude and achievement will be relatively high. On the other hand, if students are normally distributed with respect to aptitude, but the kind and quality of instruction and learning time allowed are made appropriate to the characteristics and needs of each learner, the majority of students will achieve subject mastery. The correlation between aptitude and achievement should approach zero. (Carroll, 1963)

Statement of the Problem

This study investigates the effects of incorporating behavioral objectives in an instructional strategy based on the model for mastery learning of John Carroll. Two instructional methods for teaching a unit on sets and set operations and their application to non-metric geometry in a mathematics course for elementary education majors are compared for differences of effects on immediate knowledge acquisition, retention, and attitude towards mathematics. One method of instruction is characterized by the use of behavioral objectives, independent study, and formative evaluation tests, while the other method consists of the traditional classroom lecture method.

The problem, consequently, is manifested in three objectives. The first is to design an instructional strategy which can be used as an alternative or supplement to existing traditional methods of teaching. The second is to compare the outcomes of the experimental strategy with that of the traditional lecture approach to the subject matter. The third objective is to assess the amount and type of individualized instruction taking place during the course of the experiment and to evaluate the attitude of the subjects toward this individualization.

Need for the Study

Increased college enrollments have resulted in heterogeneous student populations. Students of widely varying abilities and backgrounds are taking many of the same courses, are attending the same lectures, and are expected to progress at the same rate. As a result, the distribution of students' achievement and attitude mirrors the distribution of their abilities and backgrounds. Methods of instruction must be revised and re-directed to problems of the individual if mastery of courses is to be achieved and if favorable attitudes are to be attained by more than a few.

Experimenters continue to explore the effects of such learning activities as team teaching, programmed instruction, behavioral objectives and various methods of individualizing instruction. This research has had an effect on improving instruction at the elementary and secondary levels. However, except for isolated cases, this research has not created much impact at the college level. Perhaps this lack of application is attributable to the complexity of the strategies themselves and the contrasting simplicity and convenience of the lecture method for presenting content.

Many of the experimental studies employ research designs and instructional strategies which yield significant effects due to the complex interaction of the components of the strategy. The presence of too many independent variables often makes it difficult, if not impossible, to determine which components of the instructional strategy caused the increase in the measurement of a specific dependent variable such as achievement, retention, or attitude. In addition, the complex nature of the strategies makes their widespread adoption prohibitive. This study attempts to incorporate contemporary trends in education in a teaching model which will provide an alternative to or a supplement to traditional college mathematics instruction. By employing intact groups and concentrating on the improvement of oral and written communication between the teacher and student, this study attempts to provide an instructional strategy which is oriented toward individual student mastery and can be adopted in the conventional classroom without extensive modification.

A unique aspect of the study is the use of a unit on geometry as well as sets and set operations as the content for mastery. The use of a mathematically naive set of subjects, prospective elementary school teachers, should aid in the elimination of the effect of previous mathematics achievement. Consequently, the results should be more tenable.

In addition to student mastery of a curriculum, another important goal in education is to develop learning skills in self-directed study. The knowledge explosion demands that the educated continue to learn. Such skills of independent study become even more important to the student after his graduation as he maintains competence in his profession. Unfortunately, for many learning ceases with graduation. Strategies of instruction are needed which provide the student with the satisfaction of being able to define an area of interest and proceed to study it and master it independently. In this experiment, the instructional content was the same for all students. The study investigates a strategy which enables a student to master a unit of mathematics content largely through directed independent study.

Terms Used in the Study

The term "behavioral objective" refers to a statement of an objective for learning. Such a statement describes the ways in which students are to be changed by their interaction with the process and materials of instruction. The behavioral objective should be expressed in terms of desired student behavior with respect to subject matter. The objectives must communicate precisely what the teacher expects the learner to be able to do after completing a unit of instruction. In accordance with the cognitive theory of Benjamin S. Bloom, objectives must be defined so that they are not open to multiple interpretations. This involves translating the verbs that are open to inference into action verbs that entail direct observations. The time to be used and the level of mastery must be specified for the formative evaluation test for each set of objectives.

In this study, the phrase "formative evaluation test" refers to any test used as a diagnostic tool for the purpose of improving the students' learning of the subject matter. In formative evaluation, the teacher strives to develop the kinds of evidence that will be the most useful in helping the student overcome his difficulties. The assumption is that the users of the formative evaluation will find ways of relating the results of the evaluation to the learning and instructional goals they regard as important and worthwhile.

The term "summative evaluation" is used to indicate the type of evaluation used at the end of the course of instruction. The essential characteristic of the summative evaluation is that a judgment is made about the student with regard to the effectiveness of learning or instruction, after the learning or instruction has taken place. A student is said to have achieved a "level of mastery" if he makes a score of ninety per cent or higher on any formative evaluation test or at a level of eighty per cent or higher on the summative evaluation test.

The Research Questions

The purpose of individualizing instruction stems from the assumption that education should have as its central purpose the meeting of the needs of students as learners. In any program of individualized instruction, if each student is allowed to progress through the curriculum at his own rate and to reach objectives by means of tasks assigned on the basis of individual abilities, the subject matter will be mastered.

This study investigates some of the basic assumptions growing out of the idea of individualized instruction. Behavioral objectives are used to form the nucleus of a learning strategy to be used in a mathematics course for elementary education majors. The strategy employs, in addition to behavioral objectives, several features designed to raise the level of expectation of the student and to provide individualized direction and opportunity for study. The most significant aspect of the learning strategy is that each member of the experimental group is allowed to proceed at his own pace. Emphasis is placed on making the student responsible for his learning and providing him with the freedom to choose activities which best meet his learning needs.

A statistical investigation will be made of the following questions:

1. Will students learning under the experimental strategy perform better on achievement and retention tests than students receiving the traditional lecture method of instruction?

2. Will students in the experimental group exhibit a better attitude toward mathematics than students receiving the traditional lecture method of instruction?

3. Will the students learning under the experimental strategy experience a growth in favorable attitudes toward mathematics?

Although they will not be tested statistically, the following two questions will also be investigated:

1. What are the attitudes of the students of the experimental group toward the learning strategy itself?

2. What are some of the problems associated with the implementation of the learning strategy?

CHAPTER II

RELATED LITERATURE AND HYPOTHESES

Theoretical Framework

The idea of universal learning for mastery originated in the 1920's with two major attempts to produce such an instructional strategy. One was the Winnetka Plan of Carleton Washburne and his associates (Washburne, 1922); the other was an approach developed by Henry C. Morrison (1926) at the University of Chicago Laboratory School. The two approaches were similar with respect to several features. Both defined mastery in terms of particular educational objectives each student was expected to achieve. Instruction was organized into well-defined learning units. Each unit consisted of a collection of learning materials systematically arranged to teach the desired unit objectives. Mastery of a unit was required of students before proceeding to the next unit. Ungraded diagnostic progress tests were administered at the completion of each unit to provide feedback to both the teacher and the student on the adequacy of the student's learning.

Supplementary or corrective instruction was given to those students needing it, as indicated by their performance on the diagnostic tests. Morrison used a variety of corrective procedures - reteaching, tutoring, restructuring the original learning activities, and redirecting student study habits. Time was used as a variable in individualizing instruction. Under the Winnetka plan student learning was self-paced; that is, achievement was fixed per subject matter units and time was varied to fit the individual capacities of the learners. Each student was allowed all the time he needed to master a unit. Under the Morrison plan, each student was allowed the time the instructor required to bring all or nearly all students to unit mastery.

Interest in this type of instruction dwindled, perhaps because of the lack of technology required to sustain a successful strategy. The idea emerged again in the late 1950's and early 1960's in conjunction with programmed instruction. (Block, 1971) A basic idea underlying programmed instruction was that the learning of any behavior, no matter how complex, rested upon the learning of a sequence of less complex component behaviors. Theoretically, by breaking a complex behavior down into a chain of component behaviors and by insuring student mastery of each link in the chain, it would be possible for any student to master even the most complex skill.

A useful model for individualizing instruction did not appear, however, until John Carroll's article, "A Model of School Learning," appeared in 1963. (Block, 1971) The model was a conceptual paradigm outlining the major factors influencing student success in school learning and indicated how these factors interacted. The most innovative feature of Carroll's model was its definition of aptitude, not as an index of the level to which a student could learn a particular subject, but as a measurement of the amount of time required to learn a task to a given level under ideal conditions. (Carroll, 1963)

In its simplest form, his model proposed that if each student were allowed the time he needed to learn to some given level, and he spent that required learning time, then he could be expected to attain the level. However, if the student were not allowed enough time, then the degree to which he could be expected to learn was a function of the ratio of the time actually spent in learning to the time needed:

$$\text{Degree of Learning} = f\left(\frac{\text{Time actually spent}}{\text{Time needed}}\right).$$

The numerator of this fraction was set equal to the smallest of the following three quantities: (1) opportunity - the time allowed for learning, (2) perseverance - the amount of time the learner is willing to engage actively in learning, and (3) aptitude - the amount of time needed to learn, increased by whatever amount necessary in view of poor quality of instruction and lack of ability to understand less than optimal instruction. This last quantity (time needed to learn after adjustment for quality of instruction and ability to understand instruction) was also the denominator of the fraction. (Carroll, 1963)

Quality of instruction meant the extent to which the presentation, explanation, and arrangement of the elements of the learning task approached the optimum for each learner. This variable applied not only to the performance of a teacher but also to the characteristics of textbooks, workbooks, films, etc. If the quality of instruction is anything less than optimal, it is possible that the learner will need more time to learn the task than he would otherwise need. Some learners will be more handicapped by poor instruction than others. The extent of this handicap was thought to be a function of the learner's ability to understand instruction. Hence, in the learning model, the amount of additional time needed by the learner in view of less than optimal instruction is inversely related to his ability to understand instruction. (Carroll, 1963)

Benjamin S. Bloom transformed this conceptual model into an effective working model for mastery learning. Bloom argued that if students were normally distributed with respect to aptitude for a subject and if they were provided uniform instruction in terms of quality and learning time, then achievement at the completion of the learning task would be normally distributed. However, if students were normally distributed on aptitude but each learner received optimal quality of instruction and the learning time he required, then a majority of students could be expected to attain a level of mastery. (Bloom, 1968)

James H. Block, in his book Mastery Learning: Theory and Practice, provides an excellent starting point for reviewing the literature pertaining to the recent research on the variables involved in the mastery learning situation described by Carroll and Bloom. The book is divided into two parts. Part One discusses the model of Carroll, its augmentation by Bloom, and possible theoretical, as well as some practical, implications. Part Two consists of an annotated bibliography of major research on mastery learning. It is especially useful since it contains reviews of research studies which were never published. (Block, 1971)

Review of Experimental Research

The review of literature which follows attempts to discuss some of the major results of studies which pertain to relevant elements of the working model of school learning as developed by Carroll and by Bloom and his associates. The review includes abstracts from pertinent experiments on the use of behavioral objectives and their incorporation into various learning strategies. While nearly all of the studies deal with more than one aspect of the learning model, they will be discussed under the following categories: (1) the effect of the use of behavioral objectives on achievement and/or retention, (2) the use of time as a variable element of learning, (3) the use of small groups and tutorials, and (4) the effect of individualized instructional strategies on student attitudes.

Henry H. Walbesser and Theodore A. Eisenberg have reviewed recent research on behavioral objectives and on learning hierarchies. Their paper is presented as a Mathematics Education Report by the ERIC Information Analysis Center for Science, Mathematics and Environmental Education. A unique feature of this review is the tabulation of research hypotheses into supporting and non-supporting categories arranged by general research hypotheses. When this information is used in conjunction with the paper's extensive bibliography, the reader can structure research in the area of behavioral objectives very quickly. This document also contains reviews of research studies which have not been published and others to which the reader would have very little access. (Walbesser, 1972)

Behavioral Objectives - Effect on Achievement and Retention

James Cook investigated the question, "If a group of students is informed of the behavioral objectives and the learning hierarchy of a unit of instruction and another group of students receiving the same unit of instruction is not so informed, then will there be differences in effect on learning and retention?" Eighty-eight elementary education majors in a four year college were blocked on ability levels and randomly assigned to four treatments. While receiving different information about the behavioral objectives and the hierarchical learning sequence, all four groups received the same set of self-instructional text material covering a unit of mathematics instruction. After the completion of the instruction unit, achievement post-tests were administered immediately to compare the degree of learning and after two weeks to compare the amount of retention. The results of the study do not substantiate the thesis that informing students of behavioral objectives and/or learning hierarchy can enhance their performance on an immediate achievement test. However, the study does suggest that giving students statements and examples of the behavioral objectives is an instrumental method that will result in resistance to forgetting. (Cook, 1969)

George Nix compared the effects of individualized instruction and group-oriented instruction to determine their effect on student achievement, student reaction, and teacher effort. Pre-test data obtained from his eighth grade subjects, as well as sex, age, intelligence, and overall achievement were used to divide the sample into subgroups. Nix found that not all students achieved more from individualized instruction in general mathematics than from group-oriented instruction. Students of average mathematics ability, students of below average intelligence quotients, and boys achieved significantly more under individualized instruction than the corresponding subgroups under group-oriented instruction. Also, students developed a more favorable attitude toward school and mathematics, were more satisfied with their classmates, and fared better with their teachers when taught general mathematics under the individualized method. (Nix, 1970)

Charles Doty examined the relative effectiveness of two teaching strategies used in a unit of an industrial arts course for seventh grade male students. Both experimental and control groups received the same classroom instruction, but the experimental group received, in addition, a list of behavioral objectives prior to instruction. The results indicated that students who had prior knowledge of educational objectives for the unit scored significantly higher on the posttest for achievement. (Doty, 1968)

Dorris Boardman, however, achieved a contradictory result in her study of behavioral objectives. The subjects of her study were enrolled in a course in remedial chemistry. They were provided with a list of specific behavioral objectives prior to instruction, but the analysis of achievement test results showed no significant differences between the experimental and control groups. (Boardman, 1970)

In an unpublished study, Kenneth Collins investigated the effectiveness of Bloom's mastery learning strategy for the teaching of freshman college mathematics. The mastery students were not only given a list of the objectives covered in each unit, class session and assignment, but also were given five to ten minutes to solve a problem based on the objectives covered in the preceding session. Both groups used the same textbook, received the same assignments, covered the same material in class, and took the same unit examinations. In the modern algebra classes, seventy-five per cent of the experimental group achieved the mastery criterion of an A or B grade. In the calculus classes, the achievement at this level was sixty-five per cent. In the control groups the mastery level was achieved by only thirty per cent and forty per cent for the algebra and calculus classes, respectively. (Block, 1970)

In an unpublished doctoral dissertation, Ronald McBride studied the effects of incorporating diagnostic unit progress tests in the traditional method. The subjects were mathematics majors enrolled in a beginning calculus course. The progress tests, given only to the experimental group, were closely related to the list of unit objectives which all subjects were provided. Ample opportunity for individual help from the classroom instructor was provided members of both the experimental group and the control group. If a student fell below mastery level on a progress test a recommendation was given to him to complete a retest over the same unit objectives, after first completing the additional work suggested on the diagnostic sheet. The use of any additional aid, however, was optional. The study showed that members of the experimental group achieved significantly better scores than the members of the control group. The members of the experimental group sought much more help from the classroom instructor outside of scheduled class sessions. Also, the students of the experimental group indicated a strong appreciation of the experimental method. (McBride, 1971)

Wheatley, Kane and Kulm at the 51st Annual meeting of the National Council of Teachers of Mathematics reported on a study investigating the effect on achievement and attitude of using objectives, formative evaluation and recommendations in a number systems course for elementary education majors at Purdue University. Four instructors taught six sections of the course each semester. During the fall semester two classes received formative evaluations, two received formative evaluations with recommendations and two classes acted as control. In the spring two classes received objectives and formative evaluations with recommendations; two received objectives and formative evaluations and two received objectives only. No attempt was made to individualize instruction with respect to time. (Wheatley, 1973)

An analysis of covariance, using a pre-test and SAT scores as covariates, showed no effect of giving objectives to students, or of using weekly formative evaluations on either achievement or attitudes, with one exception. On the first hour examination the classes receiving objectives, formative evaluations and recommendations scored higher than the other groups. The authors speculated that if a completely random design had been possible together with provisions for individual rates of learning the results may have been different.

Robert Olsen departed from the tradition of focusing upon individuals as the experimental unit. In an unpublished doctoral dissertation, he investigated class effects of behavioral objectives. Eight classes received instruction in ninth grade physical science with stated behavioral objectives and six classes received the same instruction without knowledge of the objectives. Olsen reported mean scores of classes with knowledge of behavioral objectives significantly higher than the classes with no knowledge of behavioral objectives on an immediate posttest and on a retention test. (Olsen, 1971)

Time as a Learning Variable

John Yeager and Mary Ann Kissel examined the relationship between various learning-rate measures and selected student characteristics. Their study hypothesized that there would be a significant positive relationship between the student's initial state of readiness to learn and the number of days he required to master a given task. Their results confirmed this hypothesis. Data were collected in connection with an individually prescribed instruction program for eight samples of student performance. Each sample was taken from one of four mathematics units - addition, subtraction, multiplication, and division - at one of two criterion levels of achievement. The student's pretest score, the number of skills to be mastered in the unit, and the student's age were highly predictive of the time the student needed to learn. Intelligence quotient had little predictive power, supporting earlier findings that it has little effect on progress in a program where the student proceeds at his own rate and is capable of mastery at some time. (Block, 1971)

William Wright investigated the relationship between subject matter mastery and time for subtests of several of the 1964 Stanford Achievement Test batteries. For each subtest a mastery level was defined according to the results of the battery's first administration as the score corresponding to the eightieth percentile. The results indicated that: (1) a large percentage of students eventually attained the predefined mastery level, (2) some students reached mastery faster than others, and (3) the time it took for a majority of students to reach mastery varied for the different subject matter subtests. (Block, 1971)

Henry Smith and Merrill Eaton found that retention is independent of the original learning speed. They examined this relationship using six different types of learning materials varying from nonsense syllables to poetry, in each of three different degrees or conditions of learning - partial learning, complete learning, and overlearning. The study indicated that the rate at which a student learns does not affect his retention of the material he has learned, and that the student's rate of learning will vary for different types of tasks. (Block, 1971)

Douglas Sjogren specifically tested the proposition that the degree of learning is a function of the ratio of the time spent to the time needed to learn. A sample of two hundred adults participated in each of three different learning programs. The subjects were randomly assigned so that each subject might learn each program under a different time condition. Two of these conditions allowed the subject to proceed at or near his own rate, but the third condition gave the subject a fixed amount of time to learn the program from the time he had taken under the other two conditions. The ratio of the time spent to the time needed was then calculated and related to scores on achievement tests over the program studied under the fixed time condition and to scores on an aptitude measure. There was a significant positive relationship between the ratio of time spent to the time needed and the learning measure - the achievement tests and the aptitude scores. In addition, a measure of general intelligence was found to be highly related to the ratio. From this, Sjogren hypothesized that the ratio of time needed to time spent might be the equivalent of an aptitude measure. (Sjogren, 1967)

Hogwon Kim investigated the relation between aptitudes and both learning rates and achievement levels. He was interested in the hypotheses that learning rate can be predicted by relevant aptitudes and that different aptitudes must be used to predict learning rates on different learning tasks. Learning tasks involving beginning German, simple statistics, and logical reasoning were taught to mastery levels for samples of about fifty fifth and sixth graders. For each learning task, the learning rates and levels of achievement at several time periods in the learning were correlated to their Primary Mental Abilities Test and their Otis Quick Scoring Mental Ability Test scores. Measures of final achievement correlated highly with measures of achievement at the end of each time period, suggesting that learning rates and achievement levels were interchangeable in this type of learning situation. Measures of verbal ability and general intelligence gave moderate correlations with learning rates in each of the learning tasks, suggesting that ability to understand instruction affects learning rate. (Block, 1971)

Small Group and Tutorial Learning

Ronald G. Taylor investigated the use of a tutorial program for freshman engineering students. Two matched samples were used to compare the effect of tutorial assistance in the area of mathematics, physics, and English. Taylor found that the students that were tutored achieved significantly better than those that did not receive such assistance. The analysis indicated that the student who was more frequently tutored was also more likely to benefit from the tutorial help. The study also pointed out that the tutorial program was beneficial for students who had less than a 2.00 cumulative grade point average. (Taylor, 1969)

Ursula C. Schwerin investigated the effectiveness of small structured study groups in improving the academic achievement of one hundred freshman enrolled in a course in dental hygiene. Four sections of the course were randomly selected, with two serving as experimental groups and two as control. Within the two experimental groups ten study groups of five subjects each were formed. Each group consisted of one high, three average, and one low achiever. The students were categorized according to prior performance. The control groups received a traditional individual study approach while the experimental groups studied together. Schwerin found that the experimental group performed significantly better on a test of academic achievement. (Schwerin, 1970)

Ronald McBride used small student groups as an activity to supplement classroom methods in order to maximize the student's opportunity to master a first course in calculus. The attendance at such small group sessions, primarily used for clarification or further study of material missed on formative evaluation tests, was not mandatory. The sessions were held as required. A total of twelve sessions were held over an eight week instructional period. The average number in attendance was seven. Of the twenty-nine members of the experimental group, only three did not attend any sessions. Eighteen members of the experimental group sought individual assistance from the instructor. These students sought tutorial assistance on forty-five separate occasions, with an average tutoring session lasting approximately thirteen minutes. The instructor spent a total of ten hours in tutoring sessions over the eight week period. McBride felt that the study sessions and tutorials helped to increase the perseverance of some members of the experimental group. (McBride, 1969)

Individualized Learning Strategies - Effect on Attitudes

George Nix studied reactions to a method of individualized instruction in mathematics. The subjects involved were eighth grade students engaged in the learning of general mathematics. Nix used a student questionnaire for the analysis. He discovered that students developed a more favorable attitude toward school and mathematics, felt that they had learned more mathematics under the individualized instruction method than previously, and seemed to be better satisfied with the relationship between themselves and their classmates and also their teacher. (Nix, 1970)

McBride (1971) found that students of his experimental group indicated a strong appreciation of the experimental method, which was characterized by diagnostic progress-tests and individual tutorial sessions. Every member of the experimental group reacted positively to the methods for teaching analytic geometry and calculus, and each member desired that the strategy be extended to the remainder of the course. The members of the experimental group sought much more help from the classroom instructor outside of scheduled class sessions. This contradicts Bloom's results in mastery learning which indicated that students at the secondary or higher education level do not seek this type of help frequently. (Bloom, 1968) Although no attendance records were kept, McBride stated that class attendance by members of the experimental group was much higher than by members of the control group.

Arthur Hendrickson found no significant differences in achievement between groups using a mathematics laboratory, an enrichment problem approach, or a conventional approach, though all gained. Attitude also improved, although only one significant difference was found, favoring the conventional group. (Hendrickson, 1970)

Ivan Lach conducted a study on the use of programmed workbooks to provide for partially individualized mathematics instruction for junior high school students. He found that for pupils matched for sex and intelligence quotient, achievement and attitude were generally higher for those using programmed workbooks than for those having teacher-led work on sample exercises. (Lach, 1970)

John Urban designed an undergraduate mathematics course incorporating team teaching, individualized instruction, and team supervision at the University of Pittsburgh which was compared to the traditional lecture and lecture-discussion methods for effects on achievement and attitude. In the experimental group, the students made decisions concerning the objectives they would study and their rates of study based on their interests, learning characteristics, and ambitions in the course. The results of a final examination indicated that students in the experimental model had significantly higher achievement than did students in the other two groups. An attitude scale indicated that students in the experimental group exhibited a more favorable attitude toward mathematics than did students in the large-lecture group. However, no significant difference was found between the responses of the students in the experimental and the conventional groups on the scale. Results of a course evaluation instrument indicated that the students in the experimental teaching model exhibited a significantly more favorable attitude toward the method of instruction employed in their course than did the students utilizing the other two methods. (Urban, 1972)

Raymond Schmelter investigated attitude change of elementary teachers in an in-service mathematics education program. He found that there were significant gains in attitude toward mathematics and toward specified instructional techniques, including individualized instruction. (Schmelter, 1970)

In a paper read at the American Educational Research Association Annual Meeting of 1968, Philip Tiemann reported his investigation of student preferences toward the specificity of statements of objectives. Eight videotaped lectures were presented to two groups of college students. One group was also provided with a set of general objectives for the lectures and a second group was given a set of behavioral objectives for the same lectures. Tiemann's findings show the students with a greater preference for more objectives, greater use of more specific objectives, and a more favorable attitude toward the presentation of lessons with specific objectives. (Walbesser, 1972)

Pagni, Sharman, and Randolph at the 51st Annual Meeting of the National Council of Teachers of Mathematics reported on their evaluation of a mathematics laboratory curriculum project in a California junior high school. Some of the components of the mathematics laboratory were electronic calculators, tapes, individualized instruction packets for skill development, and flow charting devices for problems in logic. Over a period of two academic years, the mathematics laboratory approach was evaluated by comparing the progress of a selected group of low achieving students with a similar group of students at another school on a pretest-posttest basis. Questionnaires on student self-concept and student attitude toward mathematics were administered also. The results indicated that the laboratory approach had positive effects on students' self-concepts and achievement. (Pagni, 1973)

Hypotheses to be Tested

The literature offers support for the hypothesis that knowledge of the objectives of instruction in behavioral terms affects performance on an immediate achievement test and on a retention test. There are studies which report the contrary, but the direction of the effect, in general, appears to be positive. Perhaps the reason for the lack of clarity is that the objectives, in many experiments, are presented only once to the learners, usually at the beginning of the instructional period.

The instructional strategy of this study is based on behavioral instructional objectives, but in addition, employs many of the elements for mastery recommended by Bloom. The experimental method employs student self-pacing as a key component, formative evaluation tests, small group sessions, and tutorials. All of these features are designed to raise the level of expectation of the student and to provide individualized direction and opportunity for study. Results are obtained by comparing the experimental group with a control group taught by the conventional lecture method. In the null form, the hypotheses to be tested are:

> H_1: There is no significant difference in achievement between the control and experimental groups.

H_2: There is no significant difference in retention between the control and experimental groups.

H_3: There is no significant difference in attitude toward mathematics between the control and experimental groups.

H_4: There is no significant change in attitude of the experimental group due to the experimental strategy.

CHAPTER III

THE EXPERIMENT

The Experimental Learning Strategy

A primary concern in the selection of a teaching model was to place the student in an environment suggested by the mastery learning model developed by Carroll and Bloom and the extensive approaches to learning found in the review of the literature. A strategy was desired which would improve student self-confidence and maximize learning opportunities primarily through more effective written and oral communication between teacher and student and among students themselves. Specifically, a strategy was chosen which would accomplish the following:

1) Provide the student with specific behavioral objectives, including levels of competency for demonstrating content mastery.

2) Provide the student with justification of the content to be mastered in terms of the relevance and usefulness of the concepts.

3) Allow the student to proceed through the instructional period at a speed commensurate with his ability and other demands upon his time.

4) Insure that the student proceed to new material only after demonstrating mastery of that which preceded.

5) Provide the student with a means for self-evaluation of progress toward mastery of lesson objectives and a means for diagnosing his difficulties.

6) Permit repeated testing, immediate grading, and tutoring as a means of encouragement and increasing perseverance.

7) Provide the student with learning sources in addition to lectures and the textbook.

With the preceding objectives as a guide, a learning packet was prepared for part of the course Arithmetic for Elementary Teachers. The packet dealt with material on sets, set operations and non-metric geometry. The instructional unit was designed to extend over a period of four weeks beginning with the third class meeting of the spring semester 1973. This is equivalent to twelve fifty minute class periods. The first two class meetings were spent in collecting data and pre-testing. The instructional content was divided into two major units. The first treated the material on sets and set operations, and the second unit dealt with the topics in geometry. The units were further divided into two lessons each.

The students were informed in a preface to the learning packet that an attempt was being made to individualize instruction in order to bring every learner to a pre-determined level of mastery. They were told that they would be able to progress through the material at their own rates, and that there would be ample opportunity for them to consult with the instructor and with each other about the lessons. They were also informed that a final examination on the material contained in the learning packet would determine part of their grade for the entire course and that this necessitated that they read the entire packet very carefully.

Introduction to Modern Mathematics, by Dora McFarland and Eunice M. Lewis, served as the textbook for the course. It was the primary source of information for the students. Chapter Two provided the material for sets and Chapter Five, together with Section Six of Chapter Ten, contained the geometric concepts. In addition, a few supplementary topics such as convexity and one-to-one correspondence were included in the learning packet. Specific reading and problem assignments from the textbook were provided with each lesson of the learning packet.

In case the textbook did not provide sufficient explanation for the students to meet the lesson objectives, or in case they wished to pursue a concept in more detail or from a different point of view, a list of reference books was provided the students. These books were located in the same building in which the class was to meet. In addition, the students were told that the instructor had several additional volumes which he would lend them.

Primarily for motivational purposes, each unit of the learning packet was preceded by an introduction. The intent was to give the students some information about the topic which would make it more relevant or which would give them examples of where and how the concepts being studied might be applied For example, in the introduction to the unit on sets the following statement appears: "Number meaning and operations on numbers, such as addition, are presently taught in terms of sets."

The lessons of the learning packet consisted of two parts. The first part set forth the objectives of the lessons, stated in behavioral terms.

These were written by the experimenter in the style recommended by Robert F. Mager in his book, Preparing Instructional Objectives. Also in accordance with Mager, the students were told how much time to spend taking the formative evaluation test corresponding to each lesson and a mastery level was set at ninety per cent. The second part of the lesson contained reading and problem assignments and any related supplementary material not found in the text.

Four lectures, distributed evenly over a four week interval, were given to the experimental group during the instructional period. The students were informed of the dates and content of the lectures in advance. The lectures dealt with the major topics of the material in the learning packet and also with procedural matters concerning the use of the packet. These were given at the regularly scheduled class time at the appointed place for the class meetings. Attendance was not required.

Table 1: Schedule of Experimental Group Lectures.

Lecture	Topics	Date
1	Sets - terminology and symbols, cardinal numbers, one-to-one correspondence	1/19/73
2	Set operations- complement, difference, union, intersection	1/26/73
3	Sets of Points, separation of line and plane	1/31/73
4	Figures in the plane, angles, convexity	2/7/73

The twenty-one students of the experimental group were divided into three sub-groups of seven members each. The assignment was made on the basis of scores made on the achievement test given during the second class meeting. The intent of the experimenter was to assign students of various abilities and backgrounds to each sub-group in order to provide the students with more opportunity for interaction during small-group meetings. The students were told that those who completed the learning packet sooner than others would be asked to continue participating in small-group sessions as tutors.

The groups were asked to establish two meeting times per week, allowing approximately one-half hour for each session. Two groups chose to meet during the regularly scheduled class meeting time and the third group chose to meet during the afternoon. The students were told that the instructor would meet with each group at every meeting. The students were informed that the purpose of the small groups was to provide each student with a relaxed atmosphere in which to solve problems, ask questions about content, or to take formative evaluation tests. Office hours were established so that every student had the opportunity to meet with the instructor for individual tutoring.

The formative evaluation examinations were referred to as lesson self-tests in the learning packet. The students were told to request a self-test when they thought they had mastered the objectives of a particular lesson. After the test had been taken and scored, then it could be used as a diagnostic tool. The tests were not intended to be used as study guides, as were the behavioral objectives and lesson assignments.

It was explained that the emphasis in the course was on progressing to a new lesson only if the preceding lesson had been mastered. Therefore, it was expected that each student retest on the objectives of a lesson until he had demonstrated mastery at the required level of competency. Upon successful completion of the learning packet, the students could then take the final summative evaluation examination.

The Control Group Strategy

The instructional procedure for the control group was similar to that used in many college mathematics classes. The period began with a discussion of the homework assignments, usually involving the teacher or student volunteers working problems requested by members of the class. Attention would then be directed to the new lesson. In addition to lecturing, the instructor made an attempt to discuss new topics and to involve the students as much as possible. Most of the activity, however, was teacher-oriented. The class period was concluded with a reading and problem assignment.

The content for the control group was identical to that presented in the experimental learning packet. At the beginning of the semester the control group was given the same reading and problem assignments as the experimental group. However, these were arranged in the form of twelve class periods of fifty minutes each. The entire period of instruction extended over the first five weeks of the spring semester 1973. The additional time was used for data collection and testing.

The supplementary material contained in the learning packet was given to the control group as a separate handout. The behavioral objectives, upon which the learning packet was based, were given to the control group orally by the instructor as the concepts were encountered during the course of study. No attempt was made to present these with any particular emphasis other than that which would be used to introduce any topic in the traditional lecture approach.

The instructor did encourage the students to seek his help during office hours, which were the same as those established for the experimental group. The students were also told that the instructor would be glad to recommend additional texts for further emphasis or enrichment. No attempt was made to make use of small group study sessions.

Selection of the Sample

It was not possible at the University of Oklahoma, site of the study, to select students by methods of random sampling. During the spring semester 1973, the Mathematics Department established four sections of Mathematics 2213, Arithmetic for Elementary Teachers. Enrollment procedures at the University of Oklahoma permit students to select any section, with the only restriction being the availability of space in the section chosen at the time of enrollment. The names of the instructors of the four sections were not known by the students in advance. Two sections of the course were assigned to the experimenter, a morning section meeting at eight-thirty and an afternoon section meeting at twelve-thirty. The classes were scheduled to meet on a Monday-Wednesday-Friday basis throughout the fifteen week semester and carried three hours of college credit. By a toss of a coin, the morning section was designated the experimental group and the afternoon section the control group.

By the end of the second week of classes, the enrollment had stabilized at twenty-one for both the experimental and control groups. Each section was composed of one male and twenty females. Most of the forty-two students participating in the experiment were sophomores or upper classmen enrolled in the College of Education. The majority were interested in careers in either elementary or special education. Very few had taken a course in college mathematics, but nearly all had successfully completed at least two years of high school mathematics.

Instruments for Evaluation, Measurement, and Analysis

The implementation of the study required the development of formative evaluation tests for each lesson of the learning packet and summative evaluation tests to test the hypotheses of the study concerned with achievement and retention. The formative evaluation tests were developed by the experimenter in strict adherence to the behavioral objectives of the lesson. These were designed to require approximately fifteen to twenty minutes for completion. Each objective of the lesson was tested implicitly, if not explicitly. Two parallel forms of each formative test were developed in order to implement the retest aspect of the strategy. The formative tests, excluding the parallel forms, were reviewed along with the learning packet by a professional mathematics educator for appropriateness of form and content.

In creating the achievement test, the guiding criterion was a high correlation of the test items with the behavioral objectives of the instructional units. An attempt was made to include test items which reflected several levels of difficulty in order to provide separation and to increase validity. An objective multiple-choice test was decided upon in order to insure uniformity of grading. Fifty test items were submitted for an evaluation by a panel of three members of the instructional staff of the Department of Mathematics who were familiar with the course and its content. The panel was instructed to evaluate each item by awarding it a rating of three, two or one in the direction of appropriate to non-appropriate. Any item not receiving a composite score of five was discarded. Two of the fifty items did not meet the pre-established criterion. The resultant multiple-choice examination contained forty-eight questions with five possible responses for each question.

The retention examination was a parallel form of the achievement test, the only changes being a rearrangement of the sequence of items and replacement of the mathematical variables, such as sets and their elements. The content and objectives tested by each question remained the same. The retention test was not submitted for an evaluation of content validity.

Although a content evaluation had been made, the Kuder-Richardson Formula No. 20 was applied to the results of an item analysis of the achievement test in order to obtain an estimate of its reliability. This coefficient would confirm the dependability of the test and its relative freedom from errors of measurement. The item analysis yields a difficulty measure for each item of the test. Difficulty in this context is defined as the proportion or percentage of those responding to an item who answered it correctly. The formula for Kuder-Richardson No. 20 is

$$r_{tt} = \frac{k}{k-1}\left[1 - \frac{\sum p(1-p)}{s^2}\right]$$

where k is the number of items on the test, s^2 is the variance of the test and $\sum p(1-p)$ is the sum of difficulty measures for all test items.

Table 2: Components for Computing Reliability Coefficient of Achievement Test according to Kuder-Richardson No. 20

r_{tt}	k	s^2	p(1-P)
.85	48	35.36	6.10

The computational procedure was obtained from Basic Statistical Methods by N. M. Downie and R. W. Heath. The authors state the reliability coefficients of well-made standardized tests tend to be .90 or above. Hence since Kuder-Richardson No. 20 yields a conservative estimate, we may assume that the coefficient of .85 indicates that the achievement test is a reliable measure of the instructional content.

The Mathematics Attitude Scale devised by Lewis R. Aiken was selected as the instrument for measuring attitudes. This scale is constructed by Likert's method of summated ratings. Versions of the scale have been used at several levels of education. Consistent with the findings of other investigations, tests of the scale made by Aiken show that the reliability and validity of this scale vary somewhat with grade level, with the scale being somewhat more valid in high school and college. (Aiken, 1972) Investigations using the scale on a female college population attested to a reliability coefficient of .94 for test-retest. In addition, a test of independence between the scores on the attitude scale and scores on four items designed to measure attitudes toward academic subjects in general suggested that attitudes specific to mathematics were being measured. (Aiken, 1961) The scale appears as Appendix D to the study.

Secondary data consisting of student characteristics, opinions, and attendance were collected. An accurate log of the amount of time each student spent in lectures and tutorials was kept for both the experimental and control groups. In addition, a log was kept of the amount of time spent by students of the experimental group in small group sessions. A questionnaire based on that used by Ronald McBride (1971) was given to members of the experimental group in order to summarize their reactions toward the learning strategy and also to draw inferences about their reactions to the various aspects of the experiment. The Student Questionnaire appears as Appendix E to the study.

Method of Analysis

The analysis of the achievement and retention of the two groups was based on the data obtained from the achievement test given as a pre-test and posttest and from the retention test which was administered after a period of four weeks following the end of instruction. The analysis of covariance was particularly appropriate for comparing the achievement and retention results since the experiment, of necessity, involved the use of intact groups. Because of the nature of the experiment it was impractical to assign the two different teaching methods randomly to members of both classes. The alternative was to assign the two methods randomly to the two different sections. Although this design has serious defects, by means of the analysis of covariance it is possible to statistically control one or more extraneous sources of variation believed to affect the dependent variable. (Kirk, 1968)

It was assumed by the experimenter that previous achievement in mathematics, specifically with respect to the content of the instructional unit, presented the greatest potential for biasing the evaluation of achievement and retention. Consequently, the analysis of covariance was used with the pre-test of achievement as the concomitant variable.

The procedure for the analysis was that described by Roger E. Kirk in his book _Experimental Design: Procedures for the Behavioral Sciences_. The procedure used is designated as the completely randomized analysis of covariance with two treatments (CRAC-2) design with one concomitant variable. The model for this design involves several assumptions. They are the following:

1) The experimental errors are independent both within treatments and between treatments.

2) The experimental errors are normally distributed within each treatment population.

3) The variance due to experimental error within each treatment is homogeneous.

4) Population within-group regression coefficients are homogeneous. (Kirk, 1968)

The Mathematics Attitude Scale of Lewis R. Aiken was administered to the experimental and control groups both at the beginning and at the end of the instructional period. The range of the scale extends from zero to eighty in the direction of unfavorable to favorable attitudes toward mathematics. Because of this extensive range and because of the equal probability of the occurance of any score, interval scaling was assumed so that the analysis of covariance could be used to compare attitudes of the experimental and control groups toward mathematics. The analysis of covariance was used in order to remove statistically the effect of previous attitudes toward mathematics on present attitude toward the subject.

A non-parametric test, the Wilcoxon Matched-Pairs Signed Ranks Test, was used to compare the attitude of the experimental group before and after the instructional period. This test utilizes information about the relative magnitude as well as the direction of the differences within pairs. The test gives more weight to a pair which shows a large difference between the two observed scores than to a pair which shows a small difference. (Siegel, 1956)

In this study the Mathematics Attitude Scale was administered as both a pre-test and as a posttest and each subject in the experimental group was matched with himself. The data for the Wilcoxon test were a set of twenty-one paired scores. The difference between each pair was calculated. The absolute values of the differences were then ranked and the sign of the difference was attached to each rank. Under the null hypothesis the sum of the positive ranks will tend to equal the sum of the negative ranks. If a marked difference between the sums is observed, this constitutes evidence for the rejection of the hypothesis that the two sets of measurements are from the same population.

CHAPTER IV

PRESENTATION AND ANALYSIS OF THE DATA

Introduction

Primary data was collected by means of the achievement and retention tests, and the Mathematics Attitude Scale. The analysis of covariance was applied to this data to compare the effects of the experimental learning strategy and the traditional lecture approach on student knowledge acquisition, retention, and attitude toward mathematics. The Wilcoxon Matched-Pairs Signed-Ranks test was used to investigate the change in attitude of the experimental learning group.

Secondary, descriptive data was collected by means of an attendance log and a questionnaire concerning the experimental method. This data was examined to draw inferences and conclusions concerning the strengths and weaknesses of the strategy and student reaction toward it.

Achievement

The summative evaluation test (Appendix B) was administered to both the experimental and control group as a pre-test and upon completion of the content of instruction, using the pre-test scores as the covariate in the analysis of covariance. The test was constructed to compare the two groups with respect to the following behavior:

1) Ability to remember or recall definitions and notation

2) Use of operations

3) Mastery of Concepts

4) Ability to interpret symbolic data and put data into symbols.

5) Ability to analyze problems and determine the operations which may be applied toward the solution.

The examination questions were written in accordance with the specific lesson objectives, written in behavioral performance terms, for each of the instructional units.

Because of the high reliability of the covariate and the precautions taken to equate the administrative aspects of the study for both groups, it was assumed by the experimenter that the assumption of independence of errors had been met. In general for groups of equal size, tests of significance in the analysis of covariance are robust with respect to violation of the assumptions of normality and homogeneity of residual variance. Before the application of the analysis of covariance, however, an investigation has to be conducted to verify that the assumption of homogeneity of within-treatment regression coefficients is tenable. Kirk recommends that a numerically large level of significance (α = .10 or .25) should be used for this test in order to avoid the error of accepting the null hypothesis when it is false. (Kirk, 1968) Essentially, the test investigates the ratio of variation of the two within-group regression coefficients to the variation of the individual observations around the unpooled within-group regression lines. The computation formulas are given in Appendix H. The results of this test are summarized in Table 3.

Table 3: Test for Homogeneity of Regression Coefficients for the Achievement Test.

Source	Sums of Squares	df	Mean Square	F
Groups	54.93	1	54.93	2.78
Residuals	749.37	38	19.72	
Total	804.30	39		

The test was conducted at the significance level of $\alpha = .10$. The computation yielded an F-ratio of 2.78 with 1 and 38 degrees of freedom. This ratio is less than the tabled value of $F(.10; 1,40)$ which is 2.84. Hence, the assumption of homogeneity of within-group regression coefficients was satisfied. The analysis of covariance was then applied to the data in order to compare the two groups with respect to achievement. Table 4 contains the elementary statistical data for the achievement test given as a pre-test and as a post-test. Table 5 summarizes the analysis of covariance for the achievement test. The examination was composed of forty-eight questions, each having a scoring value of one point.

Table 4: Elementary Statistics for the Achievement Test.

	Mean	Std. Dev.	Max	Min	Range
Pre-Test					
Experimental	13.95	8.20	30	5	25
Control	17.19	10.12	41	3	38
Posttest					
Experimental	40.29	4.10	47	31	16
Control	36.67	5.95	47	28	19

Table 5: Analysis of Covariance for the Achievement Test.

Source	Sum of Squares	df	Mean Square	F
Between Groups	227.55	1	227.55	13.75
Within Groups	645.35	39	16.55	
Total	872.90	40		

It is interesting to note that the gain score of the experimental group was much higher than that of the control group. The mean score of the control group was higher on the pre-test than that of the experimental group, but the mean score of the control group on the posttest was lower than that of the experimental group. The F-ratio obtained from the analysis of covariance was significant beyond the .01 level. Therefore, it was concluded that the use of the experimental strategy did result in a higher level of achievement by the experimental group over the control group for the instructional period. Table 6 contains the distribution of scores for both groups according to percentage of total points received. Eighty percent was the pre-determined criterion for mastery.

Table 6: Distribution for the Achievement Test

Percent	Range	Grade	Control Grp.	Experim. Grp.
90-100	43-48	A	6	7
80-89	38-42	B	2	10
70-79	33-37	C	7	3
60-69	29-32	D	5	1
0-59	0-28	F	1	0

The effect of the strategy on achievement is emphasized by the fact that all but four of the experimental group achieved the pre-determined level of mastery. Only eight members of the control group attained this level. The effect of the strategy was to skew the distribution in the direction of high achievement. Consistent with the findings of other experiments in mastery learning, the correlation between the pre-test and the posttest for the experimental group was only .372. The same correlation for the control group was .759.

Retention

A parallel form of the achievement test was administered to all members of the experimental and control groups four weeks after the completion of the instructional period. The analysis of covariance was applied to the data, using the pre-test achievement scores as the concomitant variate. Table 7 summarizes the results of the test for homogeneity of regression coefficients for the retention test.

Table 7: Test for Homogeneity of Regression Coefficients for the Retention Test.

Source	Sum of Squares	df	Mean Square	F
Groups	46.55	1	46.55	1.90
Residuals	929.67	38	24.46	
Total	976.22			

The test was conducted at the significance level of $\alpha = .10$. The computation yielded an F-ratio of 1.90 with 1 and 38 degrees of freedom. This ratio is less than the tabled value of F(.10; 1, 40), which is 2.84. Hence, the assumption of homogeneity of within-group regression coefficients was satisfied. Table 8 contains the elementary statistical data for the retention test for both experimental and control groups. Table 9 summarizes the analysis of covariance for the retention test.

Table 8: Elementary Statistics for the Retention Test.

	Mean	Std. Dev.	Max	Min	Range
Experimental	38.62	5.12	46	28	18
Control	34.38	7.45	48	20	28

The data reveals decreases from the achievement mean scores by both groups, with that of the control group slightly greater. The experimental group retention test mean was 1.67 less than their achievement mean score. The control group mean decreased by 2.29.

Table 9: Analysis of Covariance for the Retention Test

Source	Sum of Squares	df	Mean Square	F
Between Groups	326.21	1	326.21	13.03
Within Groups	976.22	39	25.03	
Total	1302.42	40		

The F-ratio obtained from the analysis of covariance was significant beyond the .01 level. It was concluded that the use of the experimental strategy resulted in a significantly higher performance on the retention test by the experimental group. Table 10 contains the distribution of scores for both groups according to the percentage of total points received. Mastery, as for the achievement test, is represented by any score greater than or equal to eighty per cent.

Table 10: Distribution for the Retention Test

Per Cent	Range	Control Group	Exp. Group
90 - 100	43 - 48	3	5
80 - 89	38 - 42	5	8
70 - 79	33 - 37	3	4
60 - 69	29 - 32	6	3
0 - 59	0 - 28	4	1

This table reveals that thirteen students of the experimental group were still performing at the mastery level of eighty per cent, while only eight of the control group were maintaining the same level of mastery. Perhaps even more significant is the fact that ten members of the control group had fallen below the seventy per cent level, while only four members of the experimental group failed to surpass this level. As in the case of the achievement test, the effect of the learning strategy was to skew retention in the direction of a high level of performance.

Attitude

The Mathematics Attitude Scale of Lewis R. Aiken was administered to both experimental and control groups before and after the instructional period. The analysis of covariance was applied to the data to compare the attitude of both groups toward mathematics. The composite score of the scale ranges from zero to eighty in the direction of an unfavorable to favorable attitude toward mathematics. Aiken has shown that the test is a reliable and valid predictor of success in mathematics for college women when used in conjunction with a test of mathematical ability. (Aiken, 1963) Table 11 summarizes the results of the test for homogeneity of regression coefficients for the attitude test.

Table 11: Test for Homogeneity of Regression Coefficients for the Attitude Test.

Source	Sum of Squares	df	Mean Square	F
Groups	200.35	1	200.35	.82
Residuals	9313.26	38	245.08	
Total	9513.61	39		

The test was conducted at the significance level of $\alpha = .10$. The computation yielded an F-ratio of .82 with 1 and 38 degrees of freedom. This ratio is less than the tabled value of F(.10; 1, 40), which is 2.84. Hence, the assumption of homogeneity of within-group regression coefficients was satisfied. Table 12 contains the elementary statistical data for the Mathematics Attitude Scale for both the experimental and control groups. Table 13 summarizes the analysis of covariance for the attitude test.

The elementary statistics reveal that the control group began the instructional period with a considerably more favorable attitude toward mathematics than the experimental group. A score of 40 on the Mathematics Attitude Scale represents an "average" attitude. This would presumably correspond to a person who was either uncertain of his attitude toward mathematics or relatively indifferent toward the subject. A score of 40 would also be expected of a person who had achieved a few successes along

Table 12: Elementary Statistics for the Mathematics Attitude Scale

	Mean	Std. Dev.	Max	Min	Range
Pre-Test					
Experimental	31.86	18.68	67	11	56
Control	44.23	16.52	66	11	55
Posttest					
Experimental	38.19	20.58	79	3	76
Control	48.57	14.47	68	14	54

with an approximately equal number of negative experiences with mathematics. The pre-test control mean was 44.23, more than four points above the "average" score of 40, while the experimental group measured 31.86, more than eight points below.

The data also indicates that both groups gained in mean score from pre-test to posttest. The control group mean rose to 48.57, an increase of more than four points on the scale, while the experimental group mean rose to 38.19, almost seven points. The analysis of covariance was applied to the data to determine if there was a significant difference in attitude toward mathematics between the two groups.

The F-ratio of .012 with 1 and 39 degrees of freedom shown in Table 13 did not prove to be a statistically significant value. A value this small may be expected to occur by chance alone more than twenty-five times out of one hundred. Therefore, it was concluded that there was no significant

Table 13: Analysis of Covariance for the Mathematics Attitude Scale

Source	Sum of Squares	df	Mean Square	F
Between Groups	1.02	1	1.02	.012
Within Groups	3351.13	39	85.93	
Total	3352.15	40		

difference in attitude toward mathematics between the experimental and control groups.

The Wilcoxon Matched-Pairs, Signed-Ranks Test was applied to the data to investigate the effect of the learning strategy on the attitude of the experimental group. The same test was also applied to the control group data. If the attitude change in both groups proved to be statistically significant, then it would not follow logically that the change in the experimental group could be attributed to the learning strategy. Table 14 summarizes the analysis of the data for the Wilcoxon Matched-Pairs, Signed-Ranks Test for both the control and experimental groups.

Table 14: Analysis of Attitude Change of Experimental Group by Wilcoxon Matched-Pairs Signed-Ranks Text

	Sum Neg. Ranks	Sum Pos Ranks	N	T
Experimental	-37.0	194.0	21	37
Control	-66.0	165.0	21	66

The F-ratio of 37 computed for the experimental group is statistically significant beyond the .01 level for a two-tailed test. The ratio of 66 computed for the control group, however, is not significant. It is reasonable to assume that the increase in attitude of the control group, shown in Table 12, was due primarily to the combined effects of the instructor and the material being learned. Since the same instructor taught both groups and the content of instruction was held constant, it was concluded that the statistically significant increase in attitude by the experimental group was attributable primarily to the experimental learning strategy.

Discussion of Small Groups and Tutorials

Attendance at lectures was taken for the control group and the experimental group, and for the latter a log was kept of attendance at small-group sessions. Also, a record was kept of the amount of time each individual in both sections spent with the instructor in individual tutoring sessions. Attendance was not required of either class, and no penalty was incurred by the student if his attendance was infrequent. The average attendance at the twelve lectures for the control group was eighteen. For the four lectures which were presented to the experimental group, the average attendance was nineteen. The total enrollment was twenty-one for each group. Table 15 summarizes the attendance at small-group sessions. for each sub-group of the experimental class. Table 16 contains the summary of individual tutoring sessions for members of both the experimental and control groups.

The small-group meetings were scheduled for a maximum of thirty minutes,

Table 15: Summary of Small - Group Sessions for Members of the Experimental Group

Sub-Group	I	II	III
Number of Students Assigned	7	7	7
Number of Meetings Held (One-half hour)	8	8	8
Average Meeting Attendance	5	4	4
Number of Students Not Attending any Session	1	0	0
Total Instructor Time Used (Hours)	4	4	4

and were of an informal nature. The problem sessions were directed toward clarifying lesson objectives and working problems suggested in the lesson study guide. The instructor usually began each session by asking if the students had any questions on the objectives for the lesson or problems they would like to discuss. Students would then identify areas of difficulty by referring to specific homework questions or behavioral objectives. The instructor attempted to incorporate student suggestions in his solution of a problem or discussion of an objective, and urged students to interact with each other.

Initially, the students were inclined to ignore the objectives after they had been read. By the third or fourth meeting however, the instructor had impressed upon them the importance of using the objectives as learning aids. The students were shown how to identify areas of difficulty by relating the results of the formative evaluation tests to the lesson objectives corresponding to items missed. Gradually the students began to see the importance of the objectives as guides for study and self-improvement. After this initial period of adjustment, the lesson objectives facilitated communication.

During the first few sessions, the activities of the meeting served every student's needs. However, as the semester progressed, student differences began to appear. Some students, who were progressing faster than the rest, saw little need for attending the small-group meetings. The majority continued to attend and take an interest in the questions of others. They were encouraged by the instructor to assist the slower students with their problems. By the end of the instructional period, the activities of the small-group sessions were varied. Some students were taking and correcting formative tests while others were asking questions and solving problems. The teacher was helping students, and students were interacting with each other.

In several cases, however, a student indicated thorough confusion either by his performance on the formative evaluation test for the lesson or by his response to questions asked in the small-group meetings. It was evident that he needed a considerable amount of help. The student was asked to visit the instructor individually so that he could be given the necessary assistance.

Table 16: Summary of Individual Assistance Given by the Instructor

	Experimental	Control
Total Number of Student Visits	11	7
Average time for tutoring sessions (minutes)	15	25
Total seeking such assistance	7	4
Total instructor time used (hours)	3	3

It is evident from the table that the average time spent per student in a tutoring session is less for the experimental group than for the control group. It can be argued that the areas of difficulty for students of the experimental group could be diagnosed much more easily and quickly in view of the structure of the experimental learning strategy. In general, students of the experimental group would come to the tutoring session with a problem already formulated or with a question in mind. Most of the students came to discuss their inadequate performance on a formative evaluation test and to retest on the lesson objectives. Students in the control group came for a review of what had been covered in class. More time was spend in identifying their problem areas than was spent for the students in the experimental group.

The proper administration of the formative evaluation tests was, to a great extent, a function of the students' desire to apply the strategy correctly. From the beginning of the instructional period, the practice of mastering one unit lesson to a pre-determined criterion level before progressing to the succeeding lesson was emphasized. The students were instructed to request a formative test from the instructor when they felt that they were ready to exhibit mastery of the lesson objectives. The instructor gave the tests to the students upon the completion of a lecture, during a small-group meeting, or in his office. The students were urged to take the test in the presence of the instructor so that he could assist them in making a diagnosis of their learning difficulty immediately. More often than not, the students preferred to take the tests independently, and discuss the results with the instructor at a later time.

The self-pacing feature of the experimental strategy resulted in the students' completing the material in the learning packet at different times. Table 17 categorizes, by week of completion, the time it took the students of the experimental group to complete the learning packet and take the final summative evaluation test.

Table 17: Order of Completion of Instruction by Experimental Group

Week of Instruction	1	2	3	4	5	6	7	8
Number of Experimental Group Taking Summative Evaluation Test	0	0	2	4	9	2	3	1

The achievement test was administered to the control group at the end of the fifth week. After the same period of time, six members of the experimental group had not completed the instructional packet. It is interesting to note that of these six students, only one did not perform at the mastery level of eighty per cent. Most of these students referred to their inability to discipline themselves in correct study habits as the primary reason for their not being able to complete the material at a faster pace. As the semester progressed, it became necessary for the instructor to monitor the activity of these students more closely.

The Student Questionnaire

The student questionnaire was a very useful instrument in gathering information about student reaction to specific aspects of the learning strategy. Such student-oriented information would not have been revealed if the students had not been provided the opportunity to respond to the instructional strategy directly. The questionnaire, consisting of a total of eleven questions, was given to each student to complete after he had taken the final summative evaluation test. Students were asked to respond to the questionnaire anonymously. They were told that their responses would assist the instructor in evaluating the experimental strategy with respect to its effectiveness in improving achievement and attitude toward mathematics. The questionnaire is included as Appendix E.

The first question asked the students to rate the components of the instructional strategy as being very effective, moderately effective, or having little or no effect on their mastery of the content of the course. No further explanation was given. Table 18 summarizes the ratings given by the students to the various components of the strategy. Each component was rated with a one, a two, or a three in the direction of decreasing effectiveness.

Table 18: Student Ratings of the Components of the Experimental Learning Strategy

Ratings	1	2	3
Lesson objectives	14	5	2
Outside readings	4	5	12
Home Assignments	16	5	0
Large group lectures	9	10	2
Small group sessions	11	9	1
Individual tutoring sessions with the instructor	7	10	4
Formative Evaluation System	21	0	0

The table reveals unanimous agreement among the students concerning the effectiveness of the formative evaluation system. In the second question, the students were asked to state which was more valuable to them, the go-at-your-own-pace feature or the formative evaluation system. Fourteen students preferred the formative evaluation system and seven chose the self-pacing feature as more valuable for them. From a learning theory point of view, it might be concluded that the reinforcement alone, provided by the formative evaluation tests, was sufficient to result in greater achievement on the part of the experimental group.

It is interesting to note the favorable rating awarded to the large group lectures. In question three, the students were asked to choose the form of communication which was most valuable to them - classroom lectures, small group sessions, or individual tutorials. Ten students preferred the small group sessions, six students chose the large group lectures, and five considered the individual tutoring sessions with the instructor as most valuable. In attempting to meet the needs of individual students, one cannot deny that some students thrive on conventional methods of instruction. In many cases, students have no desire to change this approach to learning. Through the use of the lectures it was possible to include this aspect of traditional college teaching procedures as a part of the experimental teaching method.

The next seven items of the questionnaire were questions which could be answered affirmatively or negatively. The student responses are summarized in Table 19. Space was provided for students to react to the questions at some length if desired. Several students commented rather candidly on what they felt were the strengths and weaknesses of the strategy. A few felt that the homework assignments should have been used as instruments for evaluation. Many students commented on the effectiveness of the learning packet as a whole. Others felt that the time spent in small-group problem sessions should have been longer. One individual observed that some students were not spending enough time doing the problem assignments for themselves outside of class. Several students indicated that the use of tests as tools of learning helped them to overcome their previous fears of tests. All students felt that there was ample opportunity to interact with the instructor and with the other students.

The final item on the Student Questionnaire asked each student to recommend any additional components which, in his opinion, would make the strategy more effective. Very few recommendations were made. One response showing considerable insight recommended that the course be expanded to a

Table 19: Responses to Dichotomous Items of Student Questionnaire on Experimental Strategy

Question	Yes	No
Do you feel that this method of instruction helps to alleviate the problem of teaching students with widely divergent backgrounds?	21	0
Do you feel that you have greater mastery of the material than you would have in a conventional lecture approach?	19	2
Do you think this method will result in a longer retention of the material?	16	5
If you had the opportunity, would you take a course taught entirely by this self-paced method?	19	2
Was your effort in this course more than in other of your college courses?	12	9
Would you eliminate any of the components of the strategy because of ineffectiveness or any detrimental effect?	21	0
Did you have enough opportunity to interact with other students and with your instructor?	21	0

six credit hour course combining methodology with content and that the learning strategy incorporate the teaching of arithmetic in some way. One student suggested that the list of objectives for each lesson be accompanied with more extensive definitions of terms. Many students used the space allotted to this question to make favorable comments on the learning strategy as a whole.

It is evident from student responses to the questionnaire and student behavior throughout the course of instruction that the students maintained a favorable attitude toward the purpose for which the strategy was designed and the manner in which it was applied. Having clearly stated objectives for each lesson and being able to retest until they could demonstrate mastery of these objectives gave the students a feeling that they were directly responsible for their attainment in the course. The self-pacing feature added to this feeling. For a few, the burden of responsibility led to procrastination. The majority, however, accepted the challenge with a favorable attitude.

CHAPTER V

SUMMARY, FINDINGS, AND CONCLUSIONS

Summary of the Study

This study was designed to investigate the effects on student achievement, retention, and attitude of an individualized instructional strategy incorporating behavioral objectives with a formative evaluation system, and was conducted at the University of Oklahoma during the first five weeks of the spring semester of the 1972-1973 academic year. It involved two classes of education majors enrolled in Mathematics 2213, Arithmetic for Elementary School Teachers. The effects of the learning strategy were investigated by means of a statistical comparison of the strategy and a traditional lecture approach for teaching a unit on sets, set operations, and their application to non-metric geometry.

The experimental learning strategy contained several features which were designed to improve student self-confidence and to facilitate learning by improving oral and written communication between teacher and student and among students themselves. Each student was allowed to progress through the course at a pace commensurate with his ability and previous mathematical experiences. This self-pacing aspect of the strategy was intended to maximize learning for all students in the experimental group. The content was divided into two major units of two lessons each. Mastery at the eighty per cent level was required of the student for each lesson before proceeding to the next lesson. Students in the experimental group were provided with the opportunity to attend lectures, participate in small group problem sessions, and to visit with the instructor for individual tutoring.

Members of both groups were administered an achievement test, a retention test, and a mathematics attitude test in order to compare the effects of the two strategies on the corresponding dependent variables. Secondary data, including attendance logs and students' responses to a questionnaire concerning the experimental strategy were collected in order to assess various components of the strategy and to summarize student reaction toward the strategy.

Findings

Stated in the null form, the hypotheses submitted to statistical analysis were the following:

H_1: There is no significant difference in achievement between the control and experimental groups.

H_2: There is no significant difference in retention between the control and experimental groups.

H_3: There is no significant difference in attitude toward mathematics between control and experimental groups.

H_4: There is no significant change in attitude of the experimental group toward mathematics due to the experimental strategy.

Secondary data, including attendance information and student responses to the student questionnaire concerning the experimental strategy were used to answer the following questions:

1. What are the attitudes of the students of the experimental group toward the learning strategy itself?

2. What are some of the problems associated with the implementation of the learning strategy?

The analysis of the results of the achievement test leads to the rejection of the null form of the first hypothesis. Members of the experimental group did achieve significantly better scores than the members of the control group. Seventeen of the twenty-one members of the experimental group performed at the pre-determined mastery level of eighty per cent. Only eight of the twenty-one members of the control group achieved at the same level or higher.

The analysis of results for the retention test leads to the rejection of the null form of the second hypothesis. Four weeks after the completion of the instructional period, members of the experimental group did retain more of the content than did students of the control group, as evidenced by the scores on the retention test. Thirteen members of the experimental group performed at a level of eighty per cent or higher, while only eight members of the control group attained the same level of performance.

The analysis of scores obtained on the Mathematics Attitude Scale leads to the acceptance of the null form of the third hypothesis. Statistically, there was no difference between the attitudes of members of the experimental and control groups toward mathematics. The attitude scale, administered as a pre-test, revealed that the control group began the instructional period with a much more favorable attitude toward mathematics than the experimental group. Although the attitude of the experimental group did improve under the learning strategy, the change was not great enough to raise the attitude of the experimental group to a significantly higher level than that of the control group, even though the attitude of the latter group was adjusted in the analysis of covariance.

The analysis of the change in attitude toward mathematics of the experimental group leads to the rejection of the null form of the fourth hypothesis. The students did experience a significant change in attitude toward the subject as a result of the experimental learning strategy.

The analysis of the secondary data reveals that members of the experimental group formed an overall favorable attitude toward the learning strategy. Although not all components of the strategy were equally satisfactory to the students, there was no recommendation to alter the strategy by deleting any specific part. In the opinion of the students, it was the formative evaluation system that contributed most to their success. No student seemed disturbed by the fact that he had achieved at least ninety per cent on an examination but had not received credit for it. All the students of the experimental group accepted the diagnostic purpose of the formative tests and used them accordingly.

An interesting outcome of the student questionnaire was the affirmation of the importance of the lecture for many students. This is confirmed by the relatively large attendance at the lectures. It may be argued that this affinity for the lecture is merely the result of many years of conditioning. It may be argued also that most students, when confronted with several sources for learning, will not substitute an alternative for the lecture, but will supplement the lecture with other learning activities.

Two problems arose during the preparation and implementation of the experimental strategy that are worthy of comment. The problem that appears to be the most crucial is that of instructor time spent. A considerable amount of time must be spent to develop the learning packet which serves as the basis of instruction. This includes the organization of content, the preparation of the behavioral objectives for each lesson, the preparation and validation of formative and summative tests, and the collection of supplementary sources. Also, time must be spent in preparing and giving lectures, meeting in small group sessions and in individual tutorials. This burden can be alleviated by adding staff members to conduct small group sessions, to proctor tests, and to meet with students individually. Learning packets are available commercially, but these may prove to be inadequate unless they fit the specific learning context in which they are to be used.

The second problem arises as a result of the inability of many students to pace themselves. A few students in the experimental group commented that they let their work pile up and fell behind other students. Their lack of progress became evident to them in the small-group sessions. This reaction is to be expected since, for most of the students, this was the first time that they had been given any responsibility to regulate their own learning rate. Careful monitoring of the students' progress should be made if the optimum learning rate for each student is to be achieved.

Conclusions

The general implication from this study is that experimental strategies based on the model for school learning of Carroll and Bloom can be used in the classroom effectively. An instructional strategy can be designed which results in higher achievement and retention and an increase in positive attitudes toward mathematics. This can be accomplished in part, as in the case of this particular study, by improving verbal communication between student and teacher.

Having clearly stated objectives for each lesson and being able to take advantage of a formative evaluation system help to elevate a student's expectation and self-confidence. Being able to regulate his own learning rate helps to give each student a feeling that he is more directly responsible for his attainment in the course. This situation can result in more favorable attitudes toward the subject matter and an increase in achievement motivation.

On the other hand, some students may find the responsibility of directing their own learning somewhat disturbing. If through conventional methods college students are conditioned to a passive role in the learning process, they may find independent study more difficult and less meaningful. It then becomes the instructor's task to monitor the student's progress closely and to help him apply the strategy to his own particular needs in order to promote self-reliance and to enhance the student's opportunity for academic success.

The learning model of this study promotes student interaction. It encourages students to cooperate with each other in pursuit of solutions to their problems and fosters an environment in which students and teacher work together informally. There is no pressure to discuss a specific topic or cover a particular amount of material. The student can introduce a problem and discuss it with the instructor or consult with other students.

No elaborate educational hardware was used in this study. The instructional materials were developed by the experimenter. The learning model can be augmented easily by the inclusion of tapes, films, programmed texts, and other audio-visual equipment in order to provide the learner with additional sources of information. The intent was to use the learning model with an intact group in a realistic setting in order to lend credence to the ease with which the strategy may be implemented. Once the learning packet has been prepared, the only major change in conventional classroom procedure that need be made to implement the learning model is to provide the opportunity for members of the class to work together in small groups. The successful implementation of the learning model is largely a function of the desire of the instructor to provide for the individual differences of the learners.

The most valuable component of the experimental strategy from the students' point of view was the system of formative evaluation tests. These tests not only aid the student in diagnosing his areas of difficulty, but also provide the instructor with important information concerning the pacing of student learning. The tests yield information which can be used to alter instruction or to review ideas with which students are having difficulty.

When the subject matter is sequential in a course, as in mathematics, poor learning of the early units is likely to result in poor learning of all ensuing ones. A formative evaluation system insures that students master the pre-requisites for succeeding material. For students who have achieved mastery or near mastery of a unit of learning, the results of the formative evaluation test act as a reward. This reinforcement, repeated over several lessons, increases the probability of the student's continuing to invest the appropriate effort and interest in the subject.

Recommendations for Further Study

No attempt was made in this experiment to isolate the effect of any specific learning source. Even though the form of the strategy was a relatively simple one, it is not possible to state exactly which component of the learning model produced what effect. For example, the effect and value of offering lectures as one of many possible learning sources is difficult to assess. Further research is needed to determine the effect on learning of the individual components of the learning model.

In this study, no analysis of the data was made to investigate the effect of the learning model on students of varying ability levels. The study could be replicated by blocking on high, medium, and low mathematical ability levels in order to determine the effect of the learning model on each type of student. Also, more research is needed to determine which components of the learning model work best for students of various ability levels.

In order to become even more individualized, the experimental strategy could be altered to provide the student with more choices during the learning period. The student could be provided with a choice of learning objectives and also levels of mastery. Perhaps in this manner, an investigation could be made as to the suitability of the learning model for developing the student's independent learning skills.

In order to increase its validity, the experiment should be replicated with the same type of student over an entire semester's work. In addition, the experiment could be altered to include a test of the effect of the learning model on transfer of learning. This could be accomplished quite easily with the unit on geometry by extending the summative evaluation test to cover concepts in three dimensional space.

Since the successful implementation of the learning model depends, in part, on time available and desire of the instructor to meet the academic needs of his students, a study investigating the effects of the learning model with different kinds of instructors would yield valuable information concerning the compatibility of instructor and strategy. Perhaps studies are needed in which instructors are taught how to use behavioral objectives and formative evaluation systems effectively.

Finally, the experiment must be replicated not only in mathematics but also in other areas of education with a broad cross section of students. Only in this manner will the model gain extensive external validity. The latter is necessary if the goals of individualized instruction and mastery learning are to be achieved in college teaching.

Bibliography

Aiken, Lewis R. "Research on Attitudes Toward Mathematics," The Arithmetic Teacher, Vol. 19, No. 3, March, 1972.

Aiken, Lewis R. and Dreyer, R.M. "The Effect of Attitudes on Performance in Mathematics," Journal of Educational Psychology, Vol. 52, 1961, 19-24.

Block, James H. Mastery Learning: Theory and Practice. New York: Holt, Rinehart and Winston, Inc., 1971.

Bloom, Benjamin S., ed. Taxonomy of Educational Objectives: Classification of Educational Goals, Cognitive Domain. New York: McKay, 1956.

Bloom, Benjamin S. "Learning for Mastery." UCLA CSEIP Evaluation Comment. Los Angeles: Center for the Study of Evaluation of Instructional Programs, Vol. 1, No. 2, 1968.

Bloom, Benjamin S.; Hastings, J. Thomas; and Madaus, George F. Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw - Hill Book Co., 1971.

Boardman, Dorris E. "The Effects of Students' Advanced Knowledge of Behavioral Objectives on their Achievement in Remedial Chemistry." Doctoral dissertation, UCLA, 1970, DAI, Vol. 31, 3286 - A, 1970.

Carroll, John B. "A Model of School Learning," Teachers College Record, Vol. 64, May, 1963, 723 - 733.

Cook, James Marvin. "Learning and Retention by Informing Students of Behavioral Objectives and their Place in the Hierarchical Learning Sequence." Doctoral dissertation, University of Maryland, 1969, DAI, Vol. 31, 1112 - A, 1970.

Doty, Charles R. "The Effect of Practice and Prior Knowledge of Educational Objectives on Performance." Doctoral dissertation, Ohio State University, 1968, DAI, Vol. 29, 3035 - A, 1968.

Downie, N.M. and Heath, R.W. Basic Statistical Methods. New York: Harper and Row, 1965.

Hendrickson, Arthur Dean. "A Study of the Relative Effectiveness of Three Methods of Teaching Mathematics to Prospective Elementary School Teachers." Doctoral dissertation, University of Minnesota, 1969, DAI, Vol. 31, 1117 - A, 1970.

Kirk, Roger E. Experimental Design: Procedures for the Behavioral Sciences. Belmont, California: Wadsworth Publishing Company, 1968.

Lach, Ivan John. "Report of a Study on the Use of Programmed Workbooks to Provide for Partially Individualized Mathematics Instruction in the Junior High." Mathematics Teacher, Vol. 63, 512 - 515, October, 1970.

Mager, Robert F. Preparing Instructional Objectives. Belmont, California: Fearon Publishers, 1962.

McBride, Ronald L. "A Learning Strategy for Calculus and Analytic Geometry." Doctoral dissertation, The University of Oklahoma, 1971.

Morrison, H.C. The Practice of Teaching in the Secondary Schools. Chicago: University of Chicago Press, 1926.

Nix, George Carol. "An Experimental Study of Individualized Instruction in General Mathematics," Doctoral dissertation, Auburn University, Auburn Alabama, 1970, DAI, Vol. 30, 3368 - A, 1970.

Olson, Robert C. "A Comparative Study of the Effect of Behavioral Objectives on Class Performance and Retention in Physical Science." Doctoral dissertation, University of Maryland, 1971. DAI, Vol. 33, 224 - A, 1972.

Pagni, David L.; Sharman, Fred; and Randolph, John. "The Effects of a Mathematics Laboratory Approach on Students' Self Concepts, Attitudes, and Achievement," Research Reporting Sections, NCTM, 51st Annual Meeting. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1973.

Schmelter, Raymond Charles. "A Study of Attitude Change of Elementary Teachers in an In-Service Mathematics Education Program." Doctoral dissertation, The University of Wisconsin, 1969. DAI, Vol. 31, 667 - A, 1970.

Schwerin, Ursula C. "The Effect of Group Study on Individual Academic Achievement and Individual Study Orientation in Two-Year Higher Education." Doctoral dissertation, New York University, 1970, DAI, Vol. 31, 3303 - A, 1970.

Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw - Hill Book Company, 1956.

Sjogren, Douglas D. "Achievement as a Function of Study Time." American Educational Research Journal, Vol. 4, 337 - 344.

Taylor, Ronald G. "Tutorial Services and Academic Success." The Journal of Educational Research. Vol. 62, January 1969, 195 - 197.

University of Oklahoma, Bulletin of the College of Arts and Sciences. Norman, Oklahoma: The University of Oklahoma Press, 1970.

Urban, John David. "The Design and Evaluation of an Undergraduate Mathematics Course Incorporating Team Teaching, Individualized Instruction, and Team Supervision," University of Pittsburgh, 1971, DAI, Vol. 32, 6301 - A, 1972.

Walbesser, Henry H. and Eisenberg, Theodore A. A Review of Research on Behavioral Objectives and Learning Hierarchies. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1972.

Washburn, C.W. "Educational Measurements as a Key to Individualizing Instruction and Promotions," Journal of Educational Research, Vol. 5, 1922, 195 - 206.

Wheatley, Grayson H.; Kane, Robert B.; and Kulm, Gerald. "Objectives and Formative Evaluation as Factors in Mathematics Achievement and Attitudes of College Students in a Number Systems Course," Research Reporting Sections, NCTM, 51st Annual Meeting. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1973.

APPENDIX A

LEARNING PACKET FOR MATH 2213

This is a learning packet to be used for the first two units of the course dealing with sets and geometry. It is designed so that the concepts are presented as clearly and as well organized as possible. It is your instructor's belief that, if each student's individual differences are considered he can master the content of the packet to a pre-determined level, regardless of his previous mathematical experiences.

You will be able to progress through the material at your own rate. There will be ample opportunity for you to consult with your instructor and with each other about the lessons. A final examination on the material contained in this packet will determine part of your grade for this course. Therefore, it is important that you read the entire packet carefully.

Suggested Learning Sources

Textbook: McFarland and Lewis, Introduction to Modern Mathematics

The textbook will serve as a primary source of information to assist you in mastering the objectives of each lesson. Chapter Two provides the material for sets and Chapter Five, together with section six of Chapter Ten, is devoted to geometric concepts. Specific reading assignments will be provided with each lesson.

Reference Books: If you find that the textbook does not provide sufficient explanation to enable you to meet the lesson objectives, if you would like to pursue a concept in more detail, or if you would like to examine a concept from another point of view, the reference books will be helpful.

These books, listed below, are located in the Mathematics Library located in the Physical Sciences Center, second floor. Your instructor has several additional volumes in his office, 927 Physical Science Center. If you desire assistance, your instructor will be glad to recommend a book to fit your particular need.

Author	Title
Charles F. Brumfiel, Eugene F. Krause	Elementary Mathematics for Teachers
Jack E. Forbes, Robert E. Eicholz	Mathematics for Elementary Teachers
Anne E. Kenyon	Modern Elementary Mathematics
Merlin M. Ohmer, et. al.	Elementary Contemporary Mathematics
Merlin M. Ohmer	Elementary Geometry for Teachers
John A. Peterson, Joseph Hashisaki	Theory of Arithmetic

Ruric E. Wheeler	Modern Mathematics, An Elementary Approach
John E. Young, Grace A. Bush	Geometry for Elementary Teachers

Unit Objectives: The unit objectives will be stated in behavioral terms. The objectives will state precisely what you must do in order to gain and exhibit mastery at a level of eighty per cent.

Unit Study Guide: The study guide will contain new material, or will suppplement material found in the textbook. The reading and problem assignments will also be given here.

Large-group sessions: There will be four lectures given to the class as a whole. The lectures will be of a formal nature and will deal with the content of the units as well as with procedural matters concerning the use of the packet. These will be given at the regularly scheduled class time at the appointed place. Following is the schedule:

Lecture		
1	Sets - terminology & symbols, cardinal numbers, one-to-one correspondence	Friday, Jan. 19
2	Set operations - unary and binary	Friday, Jan. 26
3	Sets of points, separation of line and plane	Wednesday, Jan. 31
4	Figures in the plane	Wednesday, Feb. 7

Small - group sessions: The class will be divided into smaller groups and each group will establish its own meeting time. Each group will allow for two small-group meetings per week, with each meeting lasting approximately one-half hour. The instructor will meet with each group at every meeting. The members of the groups may request specific lessons or objectives to be discussed during these meetings. The purpose of the small group is to provide each student with a more relaxed atmosphere in which to discuss some of the problems he is having with the material. Attendance is not mandatory.

Individual tutoring: Office hours will be established so that you may consult with the instructor for individual help and tutoring. You are encouraged to take advantage of these tutoring sessions.

Testing: The material in this packet consists of two units, one on sets, and the other on geometry. Each unit is divided into two lessons. At the end of each lesson your instructor will provide you with a self-test which is based on the objectives presented at the beginning of the lesson. You should take the self-test when you feel that you can do well. If you do not score at least ninety percent on this test you must see your instructor for additional instruction and another self-test.

Upon completing both units, you will take an examination over the content of this learning packet. Your performance on this exam will count as part of your grade for the course.

Introduction to Sets

The notion of set or collection is probably as primitive as the notion of number. Actually, the two are not unrelated. When a child hears the word "two" he thinks of a set or sets consisting of two objects with which he is familiar. The idea of collecting certain objects into a single whole seems to be quite natural. It has been found that the set concept is so simple and helpful that it is being used in the elementary schools - as early as kindergarten in some places - as a foundation on which to build ideas of mathematics.

In the latter part of the nineteenth century, the German mathematician George Cantor (1845-1918) proposed the first formal treatment of sets as mathematical entities. The theory and notation of sets gained popularity among twentieth century mathematics because the notion of set represents the one concept which unifies most of the branches of mathematics. Algebra, geometry, logic, probability, etc. can all be built on a set theory foundation. Nearly every branch of mathematics can be considered as a study of sets of objects of one kind or another.

There are two aspects to the study of sets. One is set theory, which is generally taught in advanced courses in mathematics. The other is set language. Set language is being used extensively in mathematics programs at both elementary and secondary levels. The notions of sets and set language can be used to great advantage in clarifying and unifying many mathematical concepts. For example, number meanings and operations on numbers such as addition, are presently taught in terms of sets. An understanding of the concepts and language of sets is an important step in the orderly and meaningful development of mathematical ideas.

In this unit we discuss only the very basic concepts of set theory and introduce a set language which will be used to develop the number system, its operations, and properties. This language will also be used to examine some topics in geometry. With this training you will see that there is much in mathematics that can be made clear and concise, simple and easy to understand, interesting and, at times, even intriguing.

Unit I - Sets

Lesson 1 - Set language; subsets, one-to-one correspondence; cardinal and ordinal numbers; trichotomy property.

Objectives:

1. You should be able to select the well-defined sets from a list of sets.
2. You should be able to use the symbols ϵ and $\subset$ correctly in mathematical sentences. Example: $2 \epsilon \{2,3,4\}$; $\{2\} \subset \{2,3,4\}$
3. You should be able to represent sets using: a. a rule or description, b. roster notation, and c. set-builder notation.
4. For a given set, A, you should be able to:
   a. Tell whether an element is in A or not in A.
   b. Determine if A is finite or infinite.
   c. Determine the cardinal number of A, if A is finite.
   d. Determine the proper and improper subsets of A.
5. For two given sets, A and B, you should be able to:
   a. Determine if the two sets are equal.
   b. Determine if the sets are equivalent.
   c. Determine if one of the sets is a proper or improper subset of the other.
6. You should be able to identify a number as being either a cardinal number or an ordinal number, depending upon its use in a sentence.

Time limit on the self-test for this lesson: unrestricted

Mastery level: 90%

Study Guide

Unit I, Lesson 1

Textbook: Read pp. 7 - 17 and pp. 29 - 35

Problem Assignment:
Ex. 2.1, p. 8: 1,2,4
Ex. 2.2, p. 10: 1,3,4
Ex. 2.3, p. 14: 2,4,6,7,8
Ex. 2.4, p. 17: 1,2
Ex. 2.6, p. 28: 1,2,3
Ex. 2.8, p. 34: 1,2,7

Supplementary Material

1. Finite Sets and Infinite Sets

   a. A set, A, is finite if there is a counting number, "a", such that there exists a one-to-one correspondence between the elements of A and the elements of the set {1,2,3...a} . If so, "a" is called the cardinal number of A. N(A) = a.

   Example:
   $$\begin{array}{cccccc} \{2, & 4, & 6, & 8, & 10, & 12\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{1, & 2, & 3, & 4, & 5, & 6\} \end{array}$$

   Hence, N(A) = 6

   b. An infinite set is one which is not finite.

   Example: The counting numbers, {1,2,3,4...}

2. The cardinal number of the empty set is 0. $N(\phi) = N\{\ \} = 0$.

Unit I, Lesson 1

Self - Test

1. Circle the letters which correspond to <u>well-defined</u> sets.

   a. The set of thin people in Norman

   b. The three planets in our solar system that are closest to the sun

   c. The set of women presidents of the U.S.

   d. The most difficult course at Oklahoma University

   e. The five best coaches in college football

2. Let A = $\{1,2,3,4\}$ B = $\{a,b,c,d\}$ C = $\{2,4\}$ D = $\{a,2,4\}$

   Referring to the sets above, place a circle around the <u>true</u> statements below:

   a. $2 \subset C$ c. $\{b\} \subset B$

   b. $\{2\} \in A$ d. $4 \in D$

3. Complete the following mathematical statements:

   a. N $\{a,b,c\}$ = ______________

   b. N $\{\phi\}$ = ______________

   c. N $\{(a,b), (c,d)\}$ = ______________

4. Let A = $\{o,1,2\}$ B = $\{a,b\}$ C = $\{x,w,y,z\}$ D = $\{\square, \bigstar, \circ\}$ E = $\{m,n\}$

   Referring to the sets above, make the following statements <u>True</u> by naming four different sets, one in each blank.

   a. _________ and _________ are equivalent.

   b. _________ and _________ are equivalent.

5. Let A = $\{a,b, (1,2),3,\phi\}$ C = $\{a,b\}$ B = $\{a,b,1,2\}$

   Referring to the sets above, for each of the following encircle T for true or F for false.

   a. T F $1 \in A$ b. T F $(1,2) \subset A$ c. T F $C \subset A$

   d. T F $\phi \subset A$ e. T F $3 \subset A$

6. $\mathcal{U} = \{1,2,3,4...\}$ $A = \{x \in \mathcal{U} : 8 \leq x \leq 21\}$

   a. Is A finite or infinite?______________

   b. If A is finite, then N(A) = ______________.

   c. Express A using roster notation

      A = ______________________________________

7. Consider the sentence, "Turn to page 23 and work the next 5 problems."

   a. The ordinal number is ________.

   b. The cardinal number is________.

8. Let $A = \{a,b,c,d,e,f\}$ $B = \{c,d,f\}$ $C = \{a,b,e\}$

   Which of the following is True?

   a. $B \subset C$

   b. $C \subset B$

   c. $B \subset A$

   d. $A \subset C$

Answer sheet for Unit I, Lesson 1, Self-Test

Score 2 points for each correct answer.

Mastery Level - 36

1. b, c are well-defined sets

2. c and d are true

3. a. 3 b. 1 c. 2

4. a. A and D b. B and E

5. a. F b. F c. T d. T e. F

6. a. Finite b. 14 c. A = {8,9,10...21}

7. a. 23 b. 5

8. c is true

Unit I - Sets

Lesson 2 - Set operations

Objectives:

1. For a given set A, you should be able to determine the complement of A, denoted by A'

2. For two given sets, A and B, you should be able to:

   a. Find their intersection, $A \cap B$

   b. Find their union, $A \cup B$

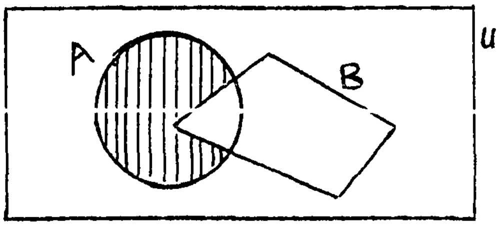
   c. Find the complement of one set with respect to the other set, A-B, B-A.

3. You should be able to draw Venn diagrams to represent various operations and relations on sets. Some of these are:

   $A \cap B$, $A - B$, $A \subset B$, $A \cap B'$, $(A \cup B)'$

4. Given a Venn diagram of two or more sets, you should be able to describe the diagram with a mathematical expression involving set relations and operations.

   Example:



A - B

5. For two or more given sets, you should be able to determine the elements in or the relationships between various complex sets formed from the given sets by means of set operations.

   Example: Find the elements in $(A \cup B - A \cap C)$, for the given sets A , B and C.

Time limit on the self-test for this lesson: unrestricted

Mastery level: 90%

Study Guide

Unit I, Lesson 2

Textbook: Read pp. 18 - 24

Problem Assignment: Ex. 2.5, p. 23: 1,2,3,4,5,6,7,
8,9,11,13,14,15

Ex. 2.8, p. 34: 5,6

Supplementary Material

1. The operation of complementation

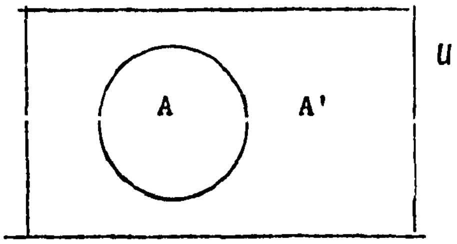
If $\mathcal{U}$ is a universal set and A is a subset of $\mathcal{U}$, then the complement of A, denoted by A', consists of every element in $\mathcal{U}$ that is not contained in A.

Example: $\mathcal{U} = \{1,2,3,4,5...\}$

$A = \{1,3,5,7...\}$

$A' = \{2,4,6,8...\}$

In terms of a Venn diagram, the complement of a set A is shown as follows:

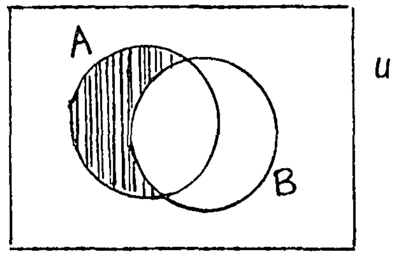


2. The complement of a set B with respect to another set A, denoted A - B, is the set of all elements that are contained in A but not in B. $A - B = \{x: x \in A \text{ and } x \notin B\}$.

Example: $A = \{2,6,7,8,9\}$ $B = \{1,2,3,6\}$

$A - B = \{7,8,9\}$

Venn diagram showing A - B:



3. Intersect and Intersection

We say that any two sets have an intersection, but not every two sets intersect. The intersection may be the empty set. Two sets, however, intersect only if they meet or overlap. Hence, if two sets have an empty intersection, then we say that they do not intersect or do not meet. If two sets do not intersect, they still have an intersection - the empty set.

Unit I, Lesson 2

Self - Test

1. Let $U = \{0,1,2,3\ldots\}$ $A = \{0,2,4,6\ldots\}$ $B = \{1,3,5,7\ldots\}$

   $C = \{0,1,2,3\}$ $D = \{3,6,9\}$

   Complete the following:

   a. $A \cap D =$

   b. $B \cap A' =$

   c. $C \cup D =$

   d. $(A \cup B)' =$

   e. $B \cup D =$

2. In the following encircle T for True or F for False.

   Let A, B and C represent any sets.

   a. T F $(A - B) \subset A$

   b. T F $(A \cap B) \subset (A \cup B \cup C)$

   c. T F $(A \cup B) \subset (A \cap C)$

   d. T F $(A - B) - C = A - (B - C)$

   e. T F $A \cup (B \cup C) = A \cup (B \cap C)$

3. Complete the following statements:

   a. If A is a subset of B and B is a subset of A then__________________.

   b. If A is a subset of B then $A \cap B =$ ______________.

   c. $A \cap \phi =$ _______________.

   d. $A \cup \phi =$ _______________.

4. If A and B are sets such that $N(A) = 8$, $N(B) = 6$ and $N(A \cap B) = 3$, then which of the following are true?

   a. $N(A \cup B) = 14$

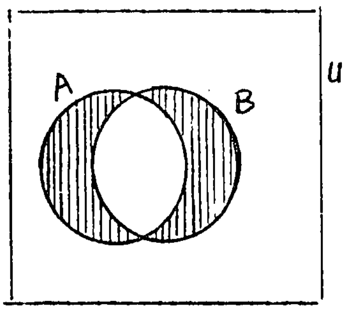
   b. $N(A - B) = 2$

   c. $N(A) + N(B) = 14$

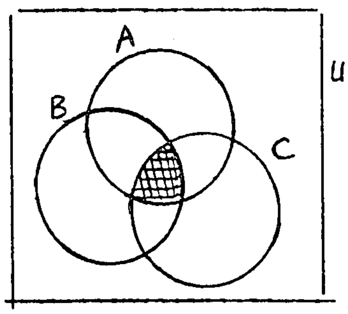
   d. $N(A \cup B) = 11$

   e. $N(B-A) = -2$

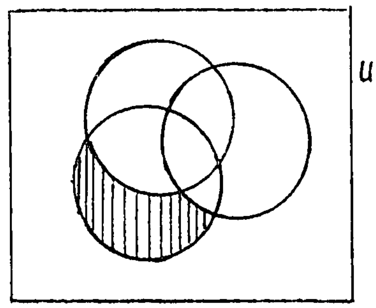
5. Describe each of the Venn Diagrams with a mathematical expression.



a.__________________



b.__________________



c.__________________

6. a. Draw a Venn Diagram showing $(A \cup B) \cap C$.

b. Draw a Venn Diagram showing $A \cap B'$.

Answer sheet for Unit I, Lesson 2, Self-test

Score 2 points for each correct answer.

Mastery Level - 36

1. a. {6} c. {0,1,2,3,6,9} e. {1,3,5,6,7,9,11...}

   b. B d. ϕ

2. a. T c. F e. F

   b. T d. F

3. a. A = B c. ϕ , A

   b. A

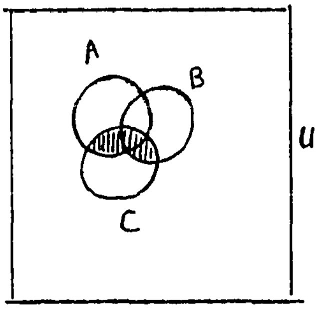
4. c and d are True

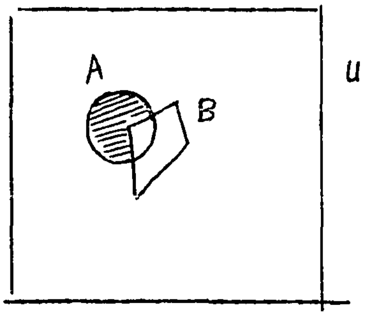
5. a. $(A \cup B) - (A \cap B)$

   b. $A \cap B \cap C$

   c. $C - (A \cup B)$

6.

a. 

b. 

Introduction to Non-metric Geometry

You may recall high-school geometry as a course which dealt primarily with lines, triangles, axioms, and proofs, with little relationship to the real world. However, long before you studied geometry, you were introduced to geometric ideas. Even in pre-school years children can identify basic "shapes" such as squares, circles, and triangles and can understand the difference between "straight" and "curved" lines.

Often, children are exposed to geometry incidentally in the classroom. For example, by coloring pictures in an art class, children gain an intuitive idea of the separation of a plane surface into regions to be colored differently. Children develop the notion of congruence when they attempt to fit pieces correctly in a jig-saw puzzle. Spatial relationships are not always pointed out in the classroom but they do exist, and students develop many intuitive geometric ideas simply by being exposed to them.

The word "geometry" means "earth measurement," which implies an intimate relationship between geometry and the world around us. Geometric figures are abstractions from our observations of this world. For example, we derive the notion of a sphere from such objects as an apple, a basketball, a scoop of ice cream, etc. The study of geometric figures and of their properties and relationships is known as geometry.

In this unit we shall use the language of sets to examine some concepts in non-geometric geometry. By the term "non-metric" we mean that part of geometry which does not have to do with measurement. We consider space (an abstraction of ordinary every-day three dimensional space) to be a set of points. Intuitively, a point represents and is represented by a position or location in space. We shall consider some of the basic properties of space and its subsets, some of which are of fundamental importance in geometry. Such subsets are lines and planes. In addition, we shall study the notion of symmetry in the plane.

By using the language of sets, precise definitions of geometric figures can be given, so that they can be identified accurately. Children can readily grasp non-metric notions such as straight lines and closed curves as long as they are presented clearly. Studies in education have shown that geometry can and should be included in the elementary school curriculum. Children like to work in "visual geometry," and they enjoy discovering geometric properties through the manipulation of geometric models.

As a result of your training in this unit, you will be able to appreciate the use of the language of sets as a facilitator in the study of geometry. You will be exposed to geometric concepts which, when mastered, will enable you to guide grade-school students in their own development of geometric concepts.

<u>Unit II</u> - Geometry

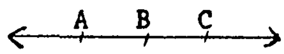
<u>Lesson 1</u> - Sets of points; union and intersection; betweenness and separation

<u>Objectives</u>:

1. You should be able to describe the intersection of

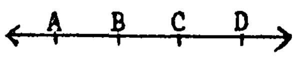
   a. two lines
   b. two rays
   c. a line and a plane
   d. two angles
   e. two line segments
   f. two planes

2. Given a line in which several points have been identified and given any two subsets of the line, you should be able to identify the union or intersection of the subsets.

   Example: Given the line m 

   $\overrightarrow{AB} \cap \overrightarrow{CB} = ?$ $\qquad$ $\overline{AB} \cup \overline{BC} = ?$

3. Given the description of a set of points, you should be able to identify two sets of points whose union or intersection matches the description.

   Example: Given the line m 

   Identify two subsets of m whose intersection is ϕ .

4. You will be given a figure in which several subsets of a plane, such as rays, lines, angles, etc., have been identified. Using this figure you should be able to:

   a. Find the intersection of any two subsets.
   b. Find the union of any two subsets.
   c. Given the description of a set of points, identify two subsets of the plane whose union or intersection matches the description.

d. Specify the location of a point with respect to two half-lines or two half-planes.

Time limit on the self-test for this lesson: unrestricted

Mastery level: 90%

Study Guide

Unit II, Lesson 1

Textbook: Read pp. 70 - 81

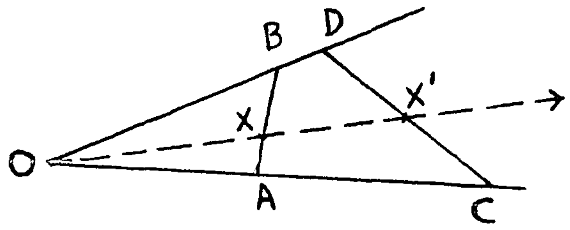
Problem Assignment: Ex. 5.3, p. 73: 1,4,5
Ex. 5.4, p. 76: 1,3,4,5,6,7
Ex. 5.5, p. 80: 1,3,4,5,6
Ex. 10.6, p. 191: 1,2,3,4,5,6

Supplementary Material

1. The symbol " $\parallel$ " is used to mean "parallel." For example, $\overline{AB} \parallel \overline{PQ}$ means the segments $\overline{AB}$ and $\overline{PQ}$ are parallel to each other.

2. One-to-one correspondence between two line segments

Suppose $\overline{AB}$ and $\overline{CD}$ are two segments as pictured below.

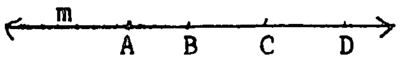


We wish to establish a one-to-one correspondence between the set of points of $\overline{AB}$ and the set of points of $\overline{CD}$. For each point X of $\overline{AB}$, let X' be the point of $\overline{CD}$ on the ray $\overrightarrow{OX}$. For each point of $\overline{AB}$ there is exactly one such ray and on each such ray containing a point of $\overline{AB}$ there is exactly one point of $\overline{CD}$. Furthermore, each point of $\overline{CD}$ is on one such ray. Hence, by the use of these rays through O, we have a one-to-one correspondence between the set of points of $\overline{AB}$ and the set of points of $\overline{CD}$.

Unit II, Lesson 1

Self-Test

1. Consider the line



Complete the following:

a. $\overline{AB} \cap \overline{BC} =$ ________

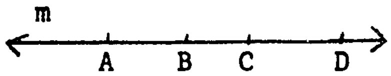
c. $\overline{AB} \cup \overline{CB} =$ ________

b. $\overrightarrow{BA} \cap \overleftrightarrow{DC} =$ ________

d. $m = \overrightarrow{AB} \cup$ ________

e. $\overline{AB} \cap \overline{CD} =$ ________

2. Consider the line



Using this figure:

a. Name two different line segments whose intersection is $\overline{BC}$.

________ and ________

b. Name two different rays whose intersection is $\overline{BC}$.

________ and ________

c. Name two different rays whose union is line m.

________ and ________

d. Name two line segments whose intersection is $\{C\}$.

________ and ________

e. Name two rays whose intersection is the empty set, ϕ .

________ and ________

3. Circle T for True or F for False

T F a. The union of two rays is always an angle.

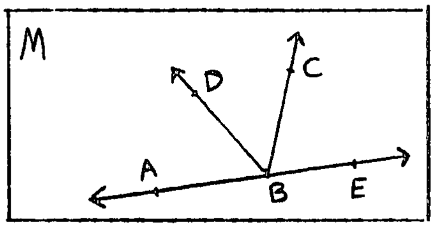
T F b. Two angles may intersect in three distinct points.

T F c. If two lines have an empty intersection then they are parallel.

T F d. Two planes may intersect in a line segment.

T F e. The union of two angles is a triangle.

4. Consider the figure below with plane $\mathcal{M}$, <ABC, <ABD, <DBC, etc.



Complete the following:

a. $\angle DBA \cap \angle CBE$ = ______________

b. $\angle DBC \cap \angle CBA$ = ______________

c. (Interior of $\angle CBA$) $\cap$ (Interior of $\angle CBE$) = ______________

d. $\overrightarrow{BD} \cup \overrightarrow{BC}$ = ______________

e. $\{D\} \cap \angle CBA$ = ______________

Answer sheet for Unit II, Lesson 1, Self-test

Score 2 points for each correct answer.

Mastery level - 36

1. a. {B} c. $\overline{AC}$ e. ϕ

   b. $\overrightarrow{BA}$ d. $\overrightarrow{BA}$ or $\overrightarrow{CA}$ or $\overrightarrow{DC}$, etc.

2. a. $\overline{AC}$ and $\overline{BD}$ c. $\overrightarrow{AB}$ and $\overrightarrow{CA}$ e. $\overline{BA}$ and $\overline{CD}$

   b. $\overrightarrow{BD}$ and $\overrightarrow{CB}$ d. $\overline{AC}$ and $\overline{CD}$

3. a. F b. T c. F d. F e. F

4. a. {B} c. empty set, ϕ e. empty set, ϕ

   b. $\overrightarrow{BC}$ d. <DBC

Unit II - Geometry

Lesson 2 - Curves in the plane, union and intersection of curves, regions, convex figures.

Objectives:

1. You should be able to determine the intersection or union of
   a. two plane curves
   b. the regions of two simple closed curves
2. Given a simple closed curve and a point, you should be able to determine if the point lies in the exterior, the interior, or on the path of the curve.
3. Given the representations of several curves, you should be able to select the curves which are:
   a. simple and closed
   b. not simple and closed
   c. simple, and not closed
   d. neither simple nor closed
4. Given a figure which contains a simple closed curve and other subsets of the plane such as lines, angles, etc., you should be able to determine:
   a. The intersection or union of any subset and the curve
   b. The intersection or union of any subset and the interior of the curve
   c. The intersection or union of any subset and the exterior of the curve
   d. The intersection or union of any subset and the region of the curve
5. Given a set of figures in the plane, you should be able to select those that are convex.

Study Guide

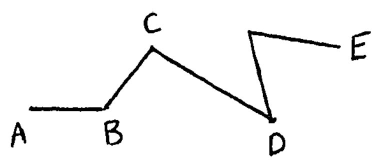
Unit II, Lesson 2

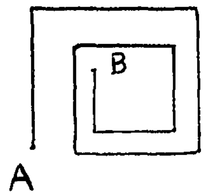
Textbook: Read pp. 82 - 83

Problem Assignment: Exercise 5.6, page 83: 1,2,3,4,5,6

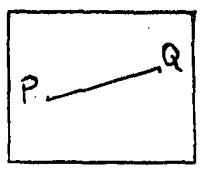
Supplementary Material

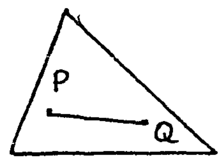
1. A polygonal path (broken-line path) is a union of segments such that each has an endpoint in common with the following one and there are no other intersections. Examples:



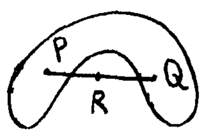


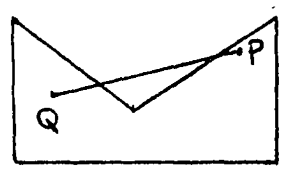
2. A set of points, A, is called convex, if for every two points P and Q of A, the entire segment $\overline{PQ}$ lies entirely in A. Some examples are:





To show that a set, say B, is not convex, you have to show that there are two points P and Q, both belonging to B, such that the segment PQ does not lie entirely in B. Examples:



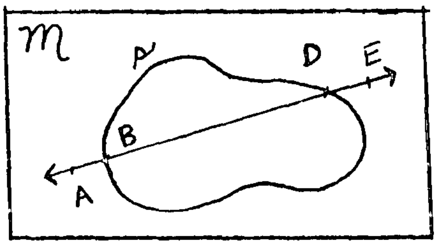


Are line segments convex? Is a set consisting of only one point convex?

Unit II, Lesson 2

Self - Test

1. Consider the figure below:



Complete the following:

a. $m \cap$ (Exterior s) = ________________

b. $m \cap s$ = ________________

c. $m \cap$ (Region s) = ________________

d. $\overrightarrow{DE} \cap$ (Interior s) = ________________

e. $\overrightarrow{BA} \cap$ (Region s) = ________________

2.

a b c d e f g h

Referring to the figures above:

a. List the simple, closed curves ____________________

b. List the non-simple, closed curves ________________

c. List the simple, non-closed curves ________________

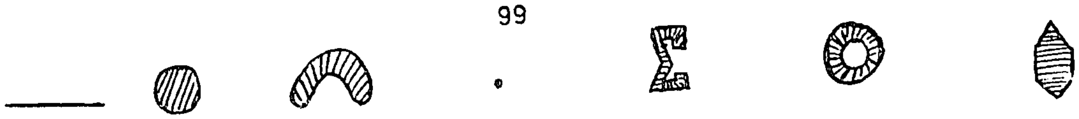
d. List the non-simple, non-closed curves ________________

3. a. The figure  contains ________ simple closed curves.

b. Draw two simple closed curves whose interiors intersect in three different regions.

c. Draw two regions whose intersection is a point.

d. Draw two angles whose intersection is exactly three points.

4. 

a. b. c. d. e. f. g.

List the convex sets, referring to the sets above.

____

5. Circle T for True and F for False

T F a. The union of two convex sets is convex.

T F b. If s is a curve, then $s \cap$ (interior of s) $= \phi$.

T F c. An angle is a simple closed curve.

T F d. A simple closed curve separates the plane in which it is contained.

T F e. The set of points in the plane bounded by two parallel lines is a convex set.

Answer sheet for Unit II, Lesson 2, Self-test

Score 2 points for each correct answer.

Mastery Level - 36

1. a. (Half-line on A side of B) $\cup$ (Half-line on E side of D)

   b. {B,D}

   c. $\overline{BD}$

   d. empty set, ϕ

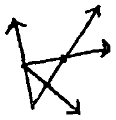
   e. {B}

2. a. b,e

   b. a,f

   c. c,g

   d. d,h

3. a. 3 b.  c.  d. 

4. a,b,d,g

5. a. F b. T c. F d. T e. T

APPENDIX B

EVALUATION TEST

Use the following sets to answer questions 1,2, and 3

$U = \{0,1,2,3...\}$ $A = \{0,2,4,6...\}$ $B = \{1,3,5,7...\}$ $D = \{2,5,6,8\}$

$E = \{3,5\}$ $F = \{2,4,6\}$ $G = \{1,4,7\}$

1. $A \cap D$ =

   a. $\{2,6\}$ c. $\{2,6,8\}$ e. $\{2,4,6\}$

   b. $\{0,2,4,6...\}$ d. $\{2,5,6,8\}$

2. $F \cup D$ =

   a. $\{2,6\}$ c. $\{4,5\}$ e. E

   b. $\{2,4,5,6,8\}$ d. ϕ

3. $(E \cap G)'$ =

   a. ϕ c. U e. $\{3,5\}$

   b. $\{1,3,4,5,7\}$ d. B

4. $N\{\phi, \{\phi,\phi\}\}$ =

   a. 0 b. 1 c. 2 d. 3 e. 4

5. $U = \{0,1,2,3...\}$ $A = \{x \in U: 7 \leq x \leq 23\}$

   N(A) =

   a. 2 b. 16 c. 17 d. 18 e. 23

6. Two sets, A and B, are said to be disjoint if

   a. $A \cap B = B$ c. $A \cup B = \phi$ e. $A \cap B = A'$

   b. $A \cup B = U$ d. $A \cap B = \phi$

7. A set of 2 elements has how many subsets?

   a. 2 b. 4 c. 5 d. 1 e. 3

8. Consider the following sets:

   1. The first two positive even integers.
   2. The students enrolled in Math 2213
   3. The ten most attractive women in Norman
   4. The three best bridge players in Maine
   5. The ten tallest students at O.U.

   Which of the above sets are well-defined?

   a. 1,3,4 c. 2,4,5 e. 1,2,5

   b. 1,2,3 d. 1,4,5

9. The symbol $A \not\subset B$ means

   a. A and B are disjoint

   b. B is not a subset of A

   c. $A \cup B = \phi$

   d. A is not a subset of B

   e. B is an improper subset of A

Use the following sets to answer questions 10 and 11.

$A = \{a,b,c,d,e\}$ $B = \{\{a,b\},\{b,c,d\},\{a,d,e\}\}$ $C = \{a,b, ,f\}$

10. Which of the following is true?

   a. $A \subset B$ c. $A \cap B = \phi$ e. $A - B = B$

   b. $B - A = A$ d. $A \cup B = A$

11. $A - C =$

   a. $\{a,b,c,d,e\}$ c. $B \cup C$ e. $\{c,d\}$

   b. $B \cap A$ d. $\{c\}$

12. Let $A = \{a,d,e,g\}$ $B = \{1,2,3,4\}$ Which is true?

   a. $A = B$ c. $A \subset B$ e. $B \subset A$

   b. $A \cap B \neq \phi$ d. $N(A) = N(B)$

13. Let $A = \{a,b,c,d\}$ $B = \{a,b,e,g\}$

Which of the following is true?

a. $N(A) = 5$ c. $N(B) = 3$ e. $N(A \cup B) = 6$

b. $N(A \cup B) = 8$ d. $N(A \cap B) = 1$

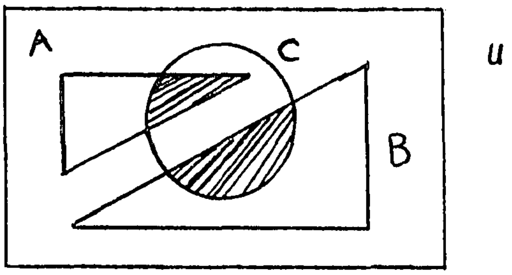
14. If A and B are sets and $N(A) = 15$, $N(B) = 19$, and $N(A \cap B) = 3$, then which statement below is correct?

a. $N(A \cup B) = 34$ c. $N(B - A) = 4$

b. $N(A \cup B) = 28$ d. $N(A \cup B) = 31$

e. all of the above are incorrect.

15. Consider the diagram:



Which mathematical expression below identifies the shaded region?

a. $(A \cup B) \cap C'$ c. $A \cap B \cap C$ e. $(A \cup B) - C$

b. $(A \cup B) \cap C$ d. $(B \cap A) \cup C$

16. Let $U = \{0,1,2,3,4...\}$ $A = \{x \in U:$ x is divisible by 3$\}$

A can also be expressed as:

a. $\{3\}$ c. $\{0,1,2,3...\}$ e. ϕ

b. $\{0,3,6,9...\}$ d. $\{3,6,9\}$

17. Let $U = \{0,1,2,3...\}$ $A = \{7,8,9,10,11,12,13,14,15,16,17,18,19,20,21\}$

A can also be expressed as:

a. $A = \{x \in U: 7 < x \leq 21\}$ d. $A = \{x \in U: 6 < x < 21\}$

b. $A = \{x \in U: 6 \leq x \leq 21\}$ e. $A = \{x \in U: 6 < x < 22\}$

c. $A = \{x \in U: 7 < x < 22\}$

Use the following sets for problems 18 - 21.

$\mathcal{U}$ = {j,k,l,m,n,o,p,q,r,s}

A = {j,l,n,o,p} B = {j,m,n,o,q}

C = {j,m,o} D = {k,m,q}

18. Which of the following pairs of sets are disjoint?

    a. A and B c. A and C e. C and D

    b. A and D d. B and C

19. Which of the following sets is equal to $(A \cap B) \cup D'$?

    a. $A \cup B$ c. $\mathcal{U}$ e. $C \cap D$

    b. $A \cap B$ d. D'

20. The cardinal number of $\mathcal{U}$ is

    a. 0 c. 9 e. greater than 7

    b. less than 10 d. equal to $N(\mathcal{U} \cap \phi)$

21. Which of the following statements is <u>false</u>?

    a. A is equvalent to B

    b. C is a proper subset of B

    c. $(A \cup B)' = \{k,r,s\}$

    d. Any two equivalent sets are equal

    e. $\phi \subset B$

22. A = {2,4,6,8} B = {4,6} C = { } D = {3,6,8}

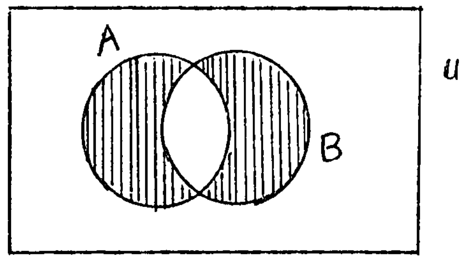
    E = {0,1}

    Which of the following statements is <u>True</u>?

    a. $3 \subset D$ c. $C \subset A$ e. $\{4\} \subset D$

    b. $0 \in C$ d. $4 \in E$

23. Consider the Venn Diagram:



Which of the following expressions correctly identifies the shaded portion of the diagram above?

a. $A \cap B$

b. $A \cup B$

c. $B - A$

d. $A - B$

e. $(A \cup B) - (A \cap B)$

24. The textbook has 250 pages. The assignment is on page 53. Referring to the sentences above, which of the following is correct.

a. 250 is ordinal and 53 is cardinal

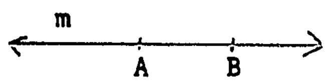
b. Both numbers are cardinal

c. Both numbers are ordinal

d. 250 is cardinal and 53 is ordinal

e. Neither number is ordinal

25. Consider the line



The line segment AB contains :

a. exactly two points

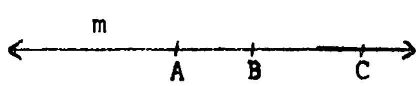
b. one point only

c. infinitely many points

d. finitely many points, but more than 2

e. exactly three points

Use the figure below in problems 26 and 27.



26. Which of the following statements is <u>False</u>?

a. $\overline{AB} \cap \overrightarrow{BC} = \phi$

b. $B \in \overline{BC}$

c. $\overline{AB} \subset \overrightarrow{CA}$

d. $\overline{AB} \cup \overline{BC} = \overline{AC}$

e. $\overline{BC} \subset \overline{AC}$

27. Which of the following statements is True?

a. $\{C\} \subset \overline{AB}$

b. $\overline{AB} \cup \overline{BC} = \phi$

c. $\overrightarrow{AB} \cup \overrightarrow{BC} = m$

d. $\overrightarrow{BA} \cup \overrightarrow{CB} = m$

e. $\overrightarrow{AB} \cup \overrightarrow{CB} = m$

28. Complete the following so that the statement is False

The intersection of two rays can be

a. a point

b. the empty set

c. exactly two points

d. a ray

e. a line segment

29. Complete the following so that the statement is true .

The union of two rays can be

a. a ray

b. a plane

c. a line segment

d. a point

e. the empty set

30. An angle is the union of

a. two line segments

b. two rays

c. two lines

d. two regions

e. a point and two line segments

31. Select the incorrect response to the following statement:

The intersection of two angles may be

a. one point

b. three points

c. two points

d. a line segment

e. a line

32. 

The sets of points shown above are examples of

a. concave sets

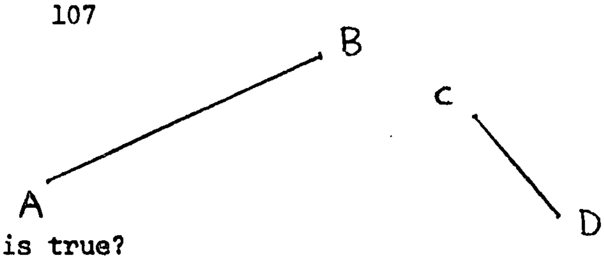
b. regions in the plane

c. simple closed curves

d. convex sets

e. none of the above

33. Consider the line segments



Which of the following is <u>true</u>?

a. $\overline{AB} \cup \overline{CD} = <ABC$

b. $\overline{AB}$ can be placed in one-to-one correspondence with $\overline{CD}$

c. $\overline{AB}$ is parallel to $\overline{CD}$

d. $\overline{AB} \cap \overline{CD} \neq \phi$

e. $\overline{AB}$ contains more points than $\overline{CD}$

Use the following figures to answer questions 34 - 37.

a b c d e f

34. The simple closed curves are

a. a and f

b. c and d

c. b and d

d. a and e

e. c and e

35. The non-simple, closed curves are

a. a and b

b. a and e

c. b and d

d. a and f

e. c and d

36. The non-simple curves are

a. a,b, and f

b. c,d, and e

c. a,b, and d

d. c,d, and e

e. none of the above

37. The non-simple, non-closed curves

a. a and f

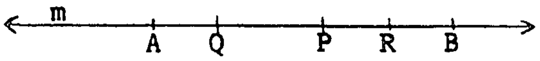
b. b only

c. f only

d. b and c

e. a and f

Use the following figure to answer questions 38 - 41.



38. Point P separates m into

    a. Two line segments

    b. Two half-lines

    c. Two rays with a common endpoint

    d. Two lines

    e. none of the above

39. Line m is the union of rays

    a. $\overrightarrow{QA}$ and $\overrightarrow{RB}$

    b. $\overrightarrow{PQ}$ and $\overrightarrow{RP}$

    c. $\overrightarrow{RP}$ and $\overrightarrow{AQ}$

    d. $\overrightarrow{AQ}$ and $\overrightarrow{RB}$

    e. $\overrightarrow{PR}$ and $\overline{QA}$

40. $\overline{AP}$ is the intersection of

    a. $\overrightarrow{AP}$ and $\overrightarrow{PQ}$

    b. $\overline{AR}$ and $\overline{QP}$

    c. $\overline{AR}$ and $\overline{PR}$

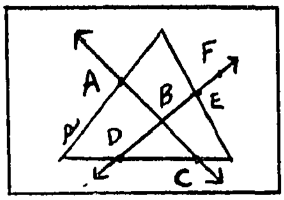
    d. $\overrightarrow{PQ}$ and $\overrightarrow{PB}$

    e. $\overline{AP}$ and $\overline{QP}$

41. $\overrightarrow{PR} \cap \overrightarrow{PA} =$

    a. $\overline{QP}$ b. {Q}   c. $\overline{PR}$ d. {R}   e. {P}

Use the figure below to answer questions 42 - 45.



42. $<ABF \cap <DBC =$

a. $\overline{AB}$

b. $\overline{DB}$

c. $\{D,B,A,C\}$

d. $\{B\}$

e. ϕ

43. (Exterior s) $\cap \overrightarrow{DB} =$

a. ϕ

b. Half-line on F side of E

c. $\overrightarrow{EF}$

d. Exterior s

e. $\overline{EF}$

44. $\overleftrightarrow{DF} \cap$ region of s =

a. $\{D,B\}$

b. $\{D,B,E\}$

c. $\overline{DE}$

d. $\{D\}$

e. ϕ

45. $\overline{AB} \cap \overline{BC} \cap$ (Interior s) =

a. ϕ

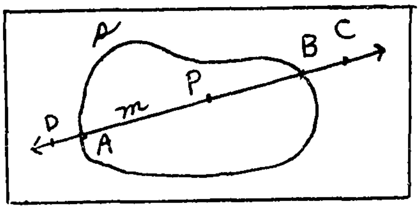
b. $\overline{AC}$

c. $\{A,B,C\}$

d. $\{B\}$

d. $\overline{BC}$

Use the following figure to answer questions 46-48.



46. $\overrightarrow{AP} \cap s =$

a. $\overline{PB}$ b. $\overline{AB}$ c. $\{B\}$ d. $\overrightarrow{AB}$ e. $\{A,B\}$

47. $\overrightarrow{BC} \cap$ (Interior s) =

a. $\overline{BC}$ b. ϕ c. $\{B\}$ d. $\{P\}$ e. $\overline{AB} - \{A,B\}$

48. (line m) $\cap$ (region of s) =

a. $\{A,P,B\}$ b. $\{A,B\}$ c. $\{D,C\}$ d. $\overline{AB}$ e. $\overline{DM}$

APPENDIX C

RETENTION TEST

Use the following sets to answer questions 1, 2, and 3.

$\mathcal{U} = \{0,1,2,3\ldots\}$ $A = \{0,1,3,5,7\ldots\}$ $B = \{2,4,6,8\ldots\}$

$D = \{3,7,9\}$ $E = \{3,5\}$ $F = \{2,4,6\}$ $G = \{2,5,7\}$

1. $A \cap D =$

   a. $\{3,5\}$ c. $\{0,5,7\}$ e. $\{3,9\}$

   b. $\{3,7,9\}$ d. $\{0,3,4\}$

2. $F \cup G =$

   a. $\{2,4,6,7\}$ c. $\{2\}$ e. $\{4,5,6,7\}$

   b. $\{2,4,5,6,7\}$ d. E

3. $(E \cup F)' =$

   a. $\mathcal{U}$ c. ϕ e. G

   b. $\{2,3,4,5,6\}$ d. $\{0,1,7,8,9\ldots\}$

4. $\mathcal{U} = \{0,1,2,3,4\ldots\}$ $A = \{x \in \mathcal{U}: 13 \leq x < 21\}$

   $N(A) =$

   a. 8 b. 9 c. 7 d. 10 e. 21

5. Let $A = \{a,d,e,g\}$ $B = \{5,6,7,8\}$

   Select the only true statement below:

   a. $A = B$ c. $A \subset B$ e. $B \subset A$

   b. $A \cup B = \phi$ d. $N(A) = N(B)$

6. The symbol $A \not\subset B$ means:

   a. A and B are disjoint
   c. $A \cup B = \phi$
   b. B is not a subset of A
   d. B is an improper subset of A
   e. A is not a subset of B

7. Let $A = \{a,b,c,d\}$ $B = \{a,b,q,e,g\}$

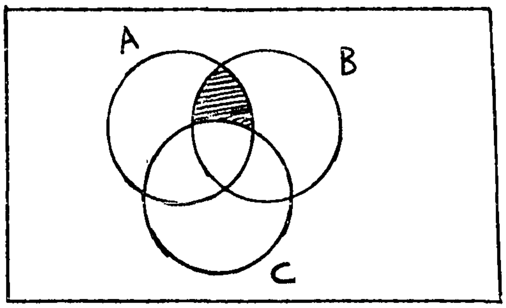
   Which of the following is true?

   a. $N(A) = 5$
   c. $N(B) = 3$
   e. $N(A \cup B) = 7$
   b. $N(A \cup B) = 8$
   d. $N(A \cap B) = 1$

8. If A and B are sets and $N(A) = 14$, $N(B) = 21$ and $N(A \cap B) = 5$, then which statement below is correct?

   a. $N(A \cup B) = 30$
   c. $N(A-B) = 16$
   e. $N(A) + N(B) = 30$
   b. $N(A \cup B) = 35$
   d. $N(B - A) = 9$

9. Consider the diagram:



   Which mathematical expression below identifies the shaded region?

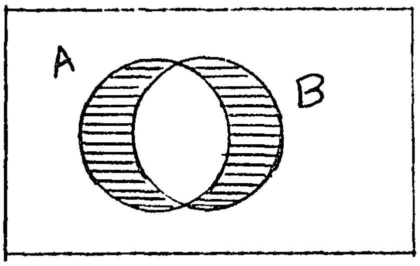
   a. $A \cap B \cap C$
   c. $(A - B) \cap C'$
   e. $(B - C) \cap A$
   b. $A \cap B \cup C$
   d. $(B - A) \cup C$

10. $A = \{2,4,6,8\}$ $B = \{4,6\}$ $C = \phi$ $D = \{3,6,8\}$ $E = \{0,1\}$

   Which of the following statements is True?

   a. $3 \subset D$
   c. $C \subset A$
   e. $\{4\} \subset D$
   b. $0 \in C$
   d. $4 \in E$

11. Consider the Venn Diagram:



Which of the following expressions correctly identifies the shaded portion of the diagram above?

a. $A \cap B$

b. $A \cup B$

c. $B - A$

d. $A - B$

e. $(A \cup B) - (A \cap B)$

Use the following sets to answer 12 and 13.

$A = \{a,b,c,d,e\}$ $B = \{(a,b), (a,c), (c,d)\}$ $C = \{a,b,e,f\}$

12. Which of the following is true?

a. $A \subset B$

b. $A \cap B = \phi$

c. $A - B = A$

d. $B - A = A$

e. $C \subset A$

13. $C - A =$

a. $\{c,e,f\}$

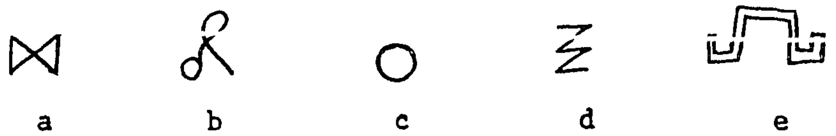
b. $\{c,d,e\}$

c. $\{a,b,e\}$

d. $\{f\}$

e. ϕ

Use the following figures to answer questions 14 - 17.



14. The simple closed curves are

a. a and f

b. c and d

c. b and d

d. a and e

e. c and e

15. The non-simple closed curves are

a. a and b

b. a and e

c. b and d

d. a and f

e. c and d

16. The non-simple curves are

a. a,b, and f

b. c,d, and e

c. a,b, and d

d. c,d, and e

e. none of the above

17. The non-simple, non-closed curves are:

a. a and f

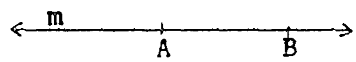
b. b only

c. f only

d. b and c

e. a and f

18. Consider the line



The line segment AB contains:

a. exactly two points

b. one point only

c. exactly three points

d. infinitely many points

e. finitely many points, but more than 2

Use the following figure to answer questions 19 - 21.

19. Line m is the union of rays

a. $\overrightarrow{QA}$ and $\overrightarrow{RB}$

b. $\overrightarrow{PQ}$ and $\overrightarrow{RP}$

c. $\overrightarrow{RP}$ and $\overrightarrow{AQ}$

d. $\overrightarrow{AQ}$ and $\overrightarrow{RB}$

e. $\overrightarrow{PR}$ and $\overrightarrow{QA}$

20. AP is the intersection of

a. $\overrightarrow{AP}$ and $\overrightarrow{PQ}$

b. $\overrightarrow{AR}$ and $\overrightarrow{QP}$

c. $\overline{AR}$ and $\overline{PR}$

d. $\overrightarrow{PQ}$ and $\overrightarrow{PB}$

e. $\overline{AP}$ and $\overline{QP}$

21. $\overrightarrow{QR} \cap \overrightarrow{PB} =$

a. $\overline{PR}$

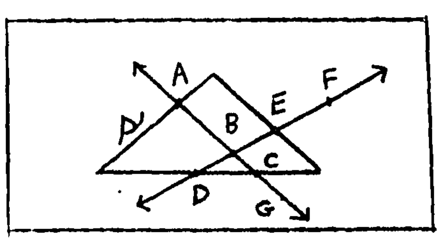
b. $\overrightarrow{QP}$

c. $\overline{RB}$

d. $\overline{PB}$

e. $\overrightarrow{PR}$

Use the figure below to answer questions 22 - 24.



22. $\angle ABF \cap \angle ABD =$

a. $\overrightarrow{AB}$ b. $\overline{AB}$ c. $\overrightarrow{BA}$ d. $\{B\}$ e. $\overrightarrow{DF}$

23. (Exterior s) $\cap$ BC =

a. ϕ b. $\overline{CG}$ c. Half-line on G side of C d. $\{G\}$

e. $\overrightarrow{CG}$

24. $\overline{AB} \cap \overline{BC} \cap \overline{DC} =$

a. ϕ b. $\{B,C\}$ c. $\{B,C,D\}$ d. $\{B\}$ e. $\angle ACD$

25. $\overleftrightarrow{DF} \cap$ Region of s =

a. $\{D,B\}$ b. $\{D,B,E\}$ c. $\overline{DE}$ d. $\{D\}$ e. ϕ

26. $N\ \{\phi,\{\phi\},0,\{0\}\} =$

a. 0 b. 1 c. 4 d. 3 e. 2

27. Two sets, A and B, are said to be disjoint if

a. $A \cap B = B$ c. $A \cup B = \phi$ e. $A \cap B = A'$

b. $A \cup B = U$ d. $A \cap B = \phi$

28. A set of 3 elements has how many subsets?

a. 3 b. 6 c. 8 d. 2 e. 0

29. Consider the following sets:

1. The set of all positive numbers less than 3.
2. The five best football players at O.U.
3. The ten tallest structures in Oklahoma.
4. The set of women presidents of the United State
5. The students enrolled in Math 2213.

Which of the above sets are well-defined?

a. 3,4,&5 c. 2,4,&5 e. 1,3,4,&5

b. 1,2,3,&4 d. 1,2,&3

Use the following sets to answer questions 30 and 31.

$A = \{a,b,c,d,e\}$ $B = \{\{a,b\}, \{b,c,d\}, \{a,d,e\}\}$

$C = \{a,b,e,f\}$

30. Which of the following is true?

a. $A \subset B$ c. $A \cap B = \phi$ e. $A - B = B$

b. $B - A = A$ d. $A \cup B = A$

31. $A - C =$

a. $\{a,b,c,d,e\}$ c. $B \cup C$ e. $\{c,d\}$

b. $B \cap A$ d. $\{c\}$

32. Let $\mathcal{U} = \{0,1,2,3,4...\}$ $A = \{x \in U\colon\ x \text{ is divisible by } 5\}$

A can also be expressed as:

a. $\{5\}$ c. $\{0,1,2,3...\}$ e. ϕ

b. $\{0,5,10,15...\}$ d. $\{5,10,15\}$

33. Let $\mathcal{U}=\{0,1,2,3...\}$ $A = \{11,12,13,14,15,16,17\}$

A can also be expressed as:

a. $A = \{x \in \mathcal{U}\colon\ 11 < x < 17\}$ c. $A = \{x \in \mathcal{U}\colon\ 11 < x < 18\}$

b. $A = \{x \in \mathcal{U}\colon\ \{10 < x < 18\}$ d. $A = \{x \in \mathcal{U}\colon\ 10 \leq x \leq 17\}$

e. $A = \{x \in \mathcal{U}\colon\ 11 \leq x \leq 18\}$

34. In the command, "Turn to page 33 and work any 5 problems"

a. 33 is cardinal and 5 is ordinal

b. 33 and 5 are both cardinal

c. 33 and 5 are both ordinal

d. Neither number is cardinal

d. 33 is ordinal and 5 is cardinal

Consider the following sets when answering questions 35 and 36.

$\mathcal{U} = \{j,k,l,m,n,o,p,q,r,s\}$

$A = \{j,l,n,o,p\}$ $B = \{j,m,n,o,q\}$

$C = \{j,m,o\}$ $D = \{k,m,q\}$

35. Which of the following pairs of sets are disjoint?

a. A and B c. A and C e. C and D

b. A and D d. B and C

36. Which of the following statements is false?

a. A is equivalent to B c. $(A \cup B)' = \{k,r,s\}$

b. C is a proper subset of B d. $\phi \subset B$

e. Any two equivalent sets are equal

37. 

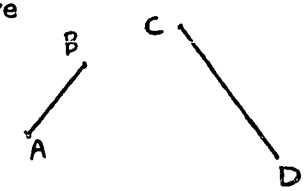
The sets of points shown above are examples of

a. concave sets c. convex sets

b. regions in the plane d. simple closed curves

e. none of the above

38. Consider the line segments



Which of the following is false?

a. $\overline{AB} \cap \overline{CD} = \phi$

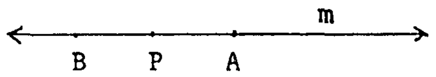
b. $\overline{AB}$ cannot be placed in one-to-one correspondence with $\overline{CD}$

c. $\overline{AB}$ is not parallel to $\overline{CD}$

d. $\overline{AB}$ contains infinitely many points

e. $\overline{AB} \cup \overline{CD}$ is a well-defined set

Use the following figure to answer questions 39 - 42



39. Point p separates m into:

a. two line segments c. two rays e. none of these

b. two half-lines d. the plane

40. $\overline{BP} \cup \overrightarrow{BA} =$

a. $\overline{BA}$ b. $\overrightarrow{AB}$ c. $\overrightarrow{BA}$ d. m e. $\overrightarrow{PA}$

41. Which of the following statements if <u>False</u>?

a. $\overline{AB} \subset \overrightarrow{BP} =$ c. $\overrightarrow{AB} \cap \overrightarrow{PA} = \overline{AB}$ e. $\{B\} \cap \overline{PA} = \phi$

b. $\{B\} \quad \overline{BP}$ d. $\overline{AP} \cup \overrightarrow{PB} = \overline{AB}$

42. Which of the following statements is True?

a. $P \subset \overline{AB}$ c. $\overline{PB} \cup \overline{PA} = \overrightarrow{BA}$ e. $\overrightarrow{AP} \cup \overrightarrow{PB} = \overrightarrow{PB}$

b. $\{P\} \subset \overline{AB}$ d. $\overrightarrow{BP} \cap \overrightarrow{PA} = m$

43. Complete the following so that the resulting statement is <u>false</u>:

The <u>intersection</u> of two rays can be

a. a point c. exactly two points e. a line segment

b. the empty set d. a ray

44. Complete the following so that the resulting statement is <u>true</u>:

The <u>union</u> of two rays can be

a. a ray c. a line segment e. the empty set

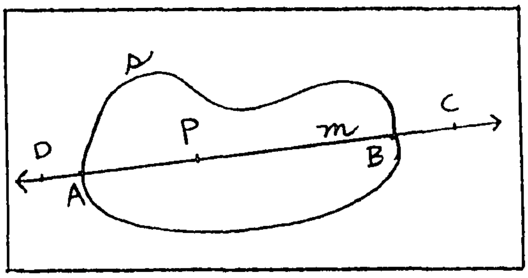
b. a plane d. a point

45. An angle is the <u>union</u> of

a. two line segments c. two lines e. two simple closed curves

b. two rays d. two regions

Use the following figure to answer questions 46, 47 and 48



46. $\overrightarrow{AP} \cap s =$

a. $\overline{PA}$ b. $\overline{AB}$ c. {A,B,P} d. {A,B} e. ϕ

47. $\overrightarrow{BC} \cap$ (Interior of s) =

a. $\overline{BC}$ b. ϕ c. {P} d. {B} e. $\overline{AB}$ - {A,B}

48. (line m $\cap$ (region of s) =

a. {D,C} b. Interior of s c. $\overline{AB}$ d. $\overline{DP}$ e. {A,P,B}

APPENDIX D

MATHEMATICS ATTITUDE SCALE

Directions: Please write your name in the upper right-hand corner. Each of the statements on this opinionnaire expresses a feeling or attitude toward mathematics. You are to indicate, on a five-point scale, the extent of agreement between the attitude expressed in each statement and your own personal feeling. The five points are:

SD - Strongly Disagree D - Disagree U - Undecided

A - Agree SA - Strongly Agree

Draw a circle around the letter giving the best indication of how closely you agree or disagree with the attitude expressed in each statement.

1. I am always under a terrible strain in a mathematics class.	SD	D	U	A	SA
2. I do not like mathematics, it scares me to take it.	SD	D	U	A	SA
3. Mathematics is very interesting to me, and I enjoy arithmetic and mathematics courses.	SD	D	U	A	SA
4. Mathematics makes me feel secure and at the same time it is stimulating.	SD	D	U	A	SA
5. Mathematics is fascinating and fun.	SD	D	U	A	SA
6. My mind goes blank and I am unable to think clearly when working mathematics.	SD	D	U	A	SA
7. I feel a sense of insecurity when attempting mathematics.	SD	D	U	A	SA
8. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD	D	U	A	SA
9. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA

10.	The feeling that I have toward mathematics is a good feeling.	SD	D	U	A	SA
11.	Mathematics is something that I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word mathematics, I have a feeling of dislike.	SD	D	U	A	SA
13.	I approach mathematics with a feeling of hesitation, resulting from a fear of not being able to do mathematics.	SD	D	U	A	SA
14.	I really like mathematics.	SD	D	U	A	SA
15.	Mathematics is a course in school that I have always enjoyed studying.	SD	D	U	A	SA
16.	It makes me nervous to even think about having to do a mathematics problem.	SD	D	U	A	SA
17.	I have never liked mathematics, and it is my most dreaded subject.	SD	D	U	A	SA
18.	I am happier in a mathematics class than in any other class.	SD	D	U	A	SA
19.	I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
20.	I feel a definite positive reaction toward mathematics; it's enjoyable.	SD	D	U	A	SA

APPENDIX E

STUDENT EVALUATION OF THE INSTRUCTIONAL STRATEGY

The following questions are concerned with the instructional strategy which was used during the first five weeks of this course, Math 2213. In order to evaluate the strategy with respect to its effectiveness in improving achievement and attitude, I would appreciate your honest responses to the questions below.

1. Consider the various components of the strategy listed below. Rate the components with a 1 - very effective, 2 - moderately effective, or 3 - little or no effect.
   a. List of objectives for each learning unit ____________
   b. List of outside readings ____________
   c. Homework assignments ____________
   d. Large lecture groups ____________
   e. Small problem sessions ____________
   f. Individual tutoring sessions with instructor ____________
   g. The formative evaluation system ____________
2. Which of the following features of the instructional strategy was most valuable to you? (Place an X beside your choice)
   a. The go-at-your-own-pace feature ____________
   b. The unit - mastery aspect which requires 90% mastery in order to advance to the next unit. ____________

3. Place an X beside the form of communication which was most valuable to you?

   a. Classroom lectures ____________________

   b. Small problem sessions ____________________

   c. Individual tutorials ____________________

Answer Yes or No to Questions 4 - 9. Comment briefly on each one, if you wish.

4. Do you feel that this method of instruction helps to alleviate the problem of teaching students with widely divergent backgrounds?

5. Do you feel that you have greater mastery of the course content than you would have in a conventional lecture approach?

6. Do you think this method will result in a longer retention of the material?

7. If you had the opportunity, would you take a course taught entirely by this self-paced method?

8. Was your effort in this course more than in other of your college courses?

9. Would you eliminate any of the components of the strategy because of ineffectiveness or any detrimental effect? Please enumerate and briefly explain.

10. Did you have enough opportunity to interact with other students and with your instructor?

11. What additional methods or components would you add to make the strategy more effective? (If any)

APPENDIX F

SAMPLE RAW SCORES FOR ACHIEVEMENT AND RETENTION TESTS

	Experimental			Control		
	Achievement		Retention	Achievement		Retention
n	Pre	Post		Pre	Post	
1	30	43	46	41	46	48
2	28	45	44	37	47	46
3	27	45	44	30	45	42
4	25	44	44	27	36	32
5	23	38	40	23	39	40
6	20	41	38	22	45	44
7	14	41	40	21	43	40
8	14	40	42	21	35	34
9	14	39	30	17	34	34
10	13	31	28	17	34	26
11	11	33	37	14	38	32
12	11	47	34	13	32	30
13	10	45	42	12	32	30
14	9	41	32	11	29	26
15	8	41	40	11	33	32
16	8	36	42	10	32	30
17	6	36	32	9	43	40
18	6	38	44	8	33	24
19	6	43	40	7	28	34
20	5	41	36	7	36	38
21	5	38	36	3	30	20

APPENDIX G

SAMPLE RAW SCORES FOR ATTITUDE SCALE

Experimental				Control	
n	Pre	Post		Pre	Post
1	67	79		66	59
2	46	45		65	66
3	52	55		51	57
4	59	71		35	37
5	28	31		60	61
6	12	21		61	53
7	23	56		50	57
8	49	46		53	56
9	24	25		24	50
10	17	16		37	54
11	25	32		27	35
12	12	21		57	68
13	11	21		38	36
14	65	76		62	61
15	34	51		51	58
16	49	39		58	55
17	22	36		21	28
18	22	21		11	35
19	11	23		33	28
20	11	3		47	52
21	30	34		22	14

APPENDIX H

COMPUTATIONAL SYMBOLS AND FORMULAS FOR THE ANALYSIS OF COVARIANCE

The procedure followed is that found in Experimental Design: Procedures for the Behavioral Sciences, by Roger E. Kirk. The following symbols are used:

N - total number of observations

n - number of subjects per treatment group

k - number of treatment groups

X - a covariate measure score

Y - a criterion measure score

Computational Symbols

$$\sum_{1}^{k} \sum_{1}^{n} X_{ij} = \sum_{i}^{N} BS_x \qquad \sum_{1}^{k} \sum_{1}^{n} Y_{ij} = \sum_{1}^{N} BS_y$$

$$\sum_{1}^{k} \sum_{1}^{n} X_{ij}^2 = [BS_x] \qquad \sum_{1}^{k} \sum_{1}^{n} Y_{ij}^2 = [BS_Y]$$

$$\sum_{1}^{k} \frac{\left(\sum_{1}^{n} X_i\right)^2}{n} = [B_x] \qquad \sum_{1}^{k} \frac{\left(\sum_{1}^{n} Y_i\right)^2}{n} = [B_y]$$

$$\frac{\left(\sum_{1}^{N} BS_x\right)^2}{N} = [X] \qquad \frac{\left(\sum_{1}^{N} BS_y\right)^2}{N} = [Y]$$

$$\frac{\left(\sum_{1}^{N} BS_x\right)\left(\sum_{1}^{N} BS_y\right)}{N} = [XY] \qquad \sum_{1}^{N} (BS_y)(BS_x) = [BS_{xy}]$$

$$\sum_{1}^{k} \frac{\left(\sum_{1}^{n} B_x\right)\left(\sum_{1}^{n} B_y\right)}{n} = [B_{xy}]$$

Computational Formulas

$$T_{yy} = [BS_y] - [Y] \qquad T_{xx} = [BS_x] - [X]$$

$$B_{yy} = [B_y] - [Y] \qquad B_{xx} = [B_x] - [X]$$

$$S_{yy} = [BS_y] - [B_y] \qquad S_{xx} = [BS_x] - [B_x]$$

$$T_{xy} = [BS_{xy}] - [XY] \qquad T_{adj} = T_{yy} - \frac{(T_{xy})^2}{T_{xx}}$$

$$B_{xy} = [B_{xy}] - [XY] \qquad S_{adj} = S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$$

$$S_{xy} = [BS_{xy}] - [B_{xy}] \qquad B_{adj} = T_{adj} - S_{adj}$$

The F ratio to determine homogeneity of the regression coefficients is given by

$$F = \frac{S_2/(k-1)}{S_1/k(n-2)}$$

where

$$S_2 = \sum_{1}^{k} \frac{(S_{xyi})^2}{S_{xxj}} - \frac{(S_{xy})^2}{S_{xx}} \quad \text{and} \quad S_1 = S_{yy} - \sum_{1}^{k} \frac{(S_{xyj})^2}{S_{xxj}} \quad .$$

In the above, $S_{xyj} = [BS_{xyj}] - [B_{xyj}]$ and $S_{xxj} = [BS_{xj}] - [B_{xj}]$.