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Scaling Torsional Drilling Vibrations: A Simulation-Based Comparison of Downscale and Upscale Drill Strings Under Varying Torque Conditions

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Abstract: Torsional vibrations pose a serious challenge in drilling operations and can lead to effects such as stick-slip phenomena, tool wear, and reduced drilling efficiency. While previous research has been conducted on torsional vibrations, there is a notable gap in comparative studies that assess the scalability of downscale models to real-world drilling conditions. This study fills this gap by systematically comparing torsional vibrations in downscale and upscale drill strings under different torque conditions at three different depths, shedding light on scaling effects in drilling vibrations. Numerical simulation was carried out taking into account non-linear interactions, damping effects, and torque variations. The laboratory set-up was for a well length of 15 m and was geometrically scaled to represent an upscale well of 450 m. Certain operational parameters such as rotation speed, torque, density, and friction coefficients were modified to keep realistic dynamic behavior, and all models were run at an identical speed of rotation to enforce consistency. The results show that both the upscale and downscale models exhibited stick-slip behavior, but differences in vibration intensity and stabilization trends point out how scaling affects torsional dynamics. Notably, the upscale bit first faced higher torsional oscillation than the set rotation speed after overcoming stick-slip before stabilizing, whereas the downscale bit went through prolonged stick-slip instability before synchronization. This study enhances the understanding of scaling effects in torsional drilling vibrations, offering a foundation for optimizing experimental setups and improving predictive modeling in drilling operations.

Keywords: upscale; downscale; stick slip; torque; numerical simulation

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1. Introduction

Torsional vibrations are very concerning for drilling operations, especially in deep wells where drill string length, speed of rotation, weight, flow rate, and torque conditions can result in undesirable effects such as stick-slip phenomena, tool wear and decreased drilling efficiency [1–4]. Understanding and mitigating torsional vibrations is critical for improving the reliability and performance of drilling operations, particularly in challenging environments [5,6]. While extensive work has been done in torsional vibration studies, mainly in large-scale operations [7,8], and drill string behavior in a downscaled experimental setup [2], a significant gap exists in direct comparative studies that evaluate the scalability of downscale models to real-world drilling conditions.

Various methods have been adopted in studying torsional drilling vibrations, such as analytical, numerical, and experimental approaches. Christoforou and Yigit [9] put forward a lumped parameter model that included axial as well as torsional and lateral vibrations. They showed that feedback control can be quite effective in suppressing stick-slip vibrations. Ahmadian et al. [10] compared 3-mode (axial, torsional, and lateral) and 2-mode coupled models; the results pointed toward the absence of torsional coupling, which produced greater lateral impacts. Such vibrational stability models that consider bit–rock interaction have been studied in great detail for both finite-dimensional [11–16] and distributed systems [17,18]. Other studies [19–23] have carried out comparative analysis for controller design aiming to suppress torsional vibrations, which proved that adjustments in the parameter of weight on bit (WOB), top-drive torque, and rotational speed (RPM) can be equally used as mitigation strategies. Controller design for drilling torsional vibrations has been addressed for finite-dimensional models [24–28]. Attenuation of axial–torsional drilling vibrations was studied in [3,29] for lumped-parameter models, in [30] for distributed parameter models, and in [31] for a partial differential equation–ordinary differential equation (PDE-ODE) coupled model. Experimental studies on small-scale drilling setups have provided valuable insights into drilling vibrations. Westermann et al. [32] and Patil et al. [33,34] developed small vertical rigs to investigate vibration phenomena, while Kapitaniak et al. [35] designed an experimental rig replicating undesired drill string behaviors such as stick-slip, whirling, and buckling. Mihajlovic [36] studied friction-induced vibrations using a DC motor, rotating discs, and a low-stiffness string. Recent efforts by Marquez and Teodoriu [37] initiated an experimental setup that enabled the investigation of torsional behavior in different wellbore sections with the downscaling ratio set as 1:40. Khulief and Al-Sulaiman [38] and Patil et al. [34] considered in more detail torsional wave propagation by means of their experimental rigs, focusing on the effects of various operational parameters, such as WOB and RPM. Additionally, Melakhessou et al. [39] examined geometric nonlinear interactions and mechanical friction effects in drill string dynamics under compressive loads. A complete look at drill string vibrations shows how important non-linear dynamics are in knowing what a system does. To gain a better understanding, researchers have used non-linear analysis tools. Leine [40] applied bifurcation theory to look at a coupled torsional–lateral model of the drill string, and to compare results against experimental field data. Similarly, Divenyi et al. [41] studied axial–torsional coupling dynamics by nonlinear analysis. Their parametric study using bifurcation theory showed that it could be a useful tool for assessing how the system behaves under different operating conditions. In a related study, Liao et al. [42] conducted a parametric assessment of drill string motion to enhance understanding of dynamic responses. Additional work in this area was provided by Gupta and Wahi [43], who examined the stability of a lumped axial–torsional model. It was found in that work that under certain parameter values, vibrations can coexist with periodic solutions (period-1 and period-2). This behavior was demonstrated through numerical simulations. In the work carried out by Bakhtiari-Nejadz and Hosseinzadeh [44], drill string stability was defined, with the main focus being on the importance of proportional damping coefficients as well as rock intrinsic energy in defining stable drilling conditions. From their findings, it can be concluded that rock intrinsic energy is an important parameter concerning bit–rock interaction introduced by Detournay et al. [45]. System damping was found to have little influence on the minimum rotational speed for stable drilling. However, intrinsic rock energy raised a downside deviation of the stability boundary and accommodated lowering the maximum permissible WOB. Another key finding concerns the common assumption of constant axial velocity at the top of the drill string. Research suggests that this simplification could significantly influence stability predictions and should be carefully reconsidered in dynamic modeling [46].

In spite of the valuable insights from these studies, the challenge of translating results from downscale experiments and their applicability to full-scale drilling operations remains unresolved. Scaling effects introduce discrepancies in material properties, geometric constraints, and applied forces, potentially limiting the applicability of downscale findings to real-world scenarios [47,48]. While some studies have examined torsional vibrations across different scales, no comprehensive comparative work directly assesses how scaling influences the dynamics of vibration reported. This study will address this gap in a systematic comparison between torsional vibrations of downscale and upscale drill strings under various torque conditions. In this study, we propose to look at torsional vibration intensities, stick-slip phenomena, and dynamic responses for different depths of the string, which would further help in understanding how scaling affects drilling vibrations. By relating downscale results to upscale results, this work sheds light on how experimental design can be better optimized and predictive modeling enhanced for drilling operations.

2. Numerical Simulation

The simulator used in this study is called Wellscan v3.8 and it gives a physics-based representation of drill string vibrations. This shows how the drill bit and formation work together in a nonlinear way, including damping effects, and allows for real-time simulation of torque and rotational speed variations. The calibration of model parameters was based on empirical data from previous experimental works [48] and field data [49] so as to accurately represent down-hole conditions. The downscale model was based on the laboratory small-scale setup introduced by [48], which had a total length of 15 m. By applying a geometrical scaling factor of 1:30 for the total length and bit diameter [49], the simulation produced an upscaled well with a length of 450 m. Table 1 presents a detailed comparison of the key parameters used for both the upscale and downscale drill strings. This scaling allows for a realistic replication of the drill string's dynamics on a larger scale, while maintaining the proportional relationships between the components and forces involved. The bit diameter was equally scaled, with the upscale bit measuring 5.625 inches in diameter, while the downscale bit diameter was proportionally reduced to 0.20 inches. While geometric scaling principles for length and diameter were adhered to, certain operational parameters like RPM, torque, density, and static and dynamic friction did not strictly follow the 1:30 scaling factor. This slight deviation was quite necessary due to the inevitable physical limitations and practical constraints of the downscale setup. It was impossible to scale down these parameters in a linear manner because doing so would have resulted in certain values that would be beyond the realistic operational range of the downscale system and would, therefore, render it and the model inaccurate. The precise values for the RPM, torque, density, and friction coefficients were thus chosen independently for both setups so that the dynamic response of the upscale drill string under torsional loads could be accurately 'modeled' by the downscale setup. Both strings were subjected to the same top RPM. The upscale string was subjected to torques of 500 kgfm, 1500 kgfm, and 4500 kgfm at distinct phases, reflecting the forces typically experienced during actual drilling operations. The downscale string, however, experienced significantly lower torques, scaled down to 0.01 kgfm, 0.03 kgfm and 0.09 kgfm.

To carry out the comparison, the drilling dynamics module within the simulator was utilized to evaluate the torsional vibration signatures for both the upscale and downscale drill strings. The simulator operates using two primary torque and drag models: the soft-string model and the stiff-string model. The soft-string model, as the name suggests, assumes continuous contact with the wellbore, simplifying the analysis by neglecting the stiffness and bending moment effects. However, the stiff-string model (Abis) provides a more accurate representation of the drill string behavior accounting for stiffness and is particularly useful in wells with complex trajectories. The model applied the Abis stiff string model due to its capability to deliver a more realistic assessment of downhole configurations, making it ideal for examining the torsional dynamics in tortuous well paths.

Table 1. Key parameters comparison.

Parameters	Upscale Values	Downscale Values
Length	450 m	15 m
Bit diameter	5.625 inches	0.20 inches
Rotation speed (RPM)	0–150 RPM	0–150 RPM
Torque	500 kgfm	0.01 kgfm
	1500 kgfm	0.03 kgfm
	4500 kgfm	0.09 kgfm
Density	1.2 SG	0.001 SG
Static/Dynamic friction	0.2	0.01
Depth of observation	0 m (bit)	0 m (bit)
	100 m	3.3 m
	300 m	10 m

2.1. Downscale Setup

The downscale string functions as a laboratory experimental rig, providing a valuable platform for investigating drill-string dynamics at a lower scale [32]. This approach offers a unique alternative to full-scale testing by simulating downhole conditions in a controlled, repeatable environment. The key advantages of a downscale laboratory setup include its economic feasibility, allowing multiple tests without the high costs associated with full-scale operations. Moreover, the ease of replicating downhole conditions enables researchers to conduct experiments with high precision and consistency. Additionally, the setup allows for comprehensive data collection, with easy access to both surface and downhole parameter measurements, offering detailed insights into the dynamics of the drill string [48]. Figure 1 presents the torsional vibration setup for the downscale string, designed to replicate the key components of a full-scale drilling operation. The setup included a bit, drill collar, heavy weight drill pipe (HWDP), and drill pipe, all scaled down to mimic real-world drilling scenarios while allowing for a controlled and detailed analysis of torsional vibrations.

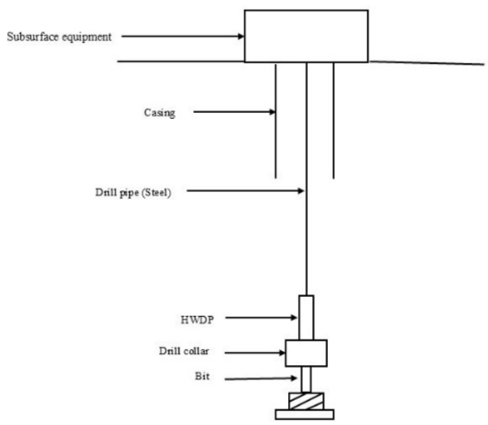

Figure 1. Experimental setup for torsional vibration for downscale string.

The downscale drill string was modeled with a total length of 15 m, closely replicating the configuration of the experimental setup. The bit diameter was 0.20 inches, and the string included the same components used in the experimental arrangement, ensuring consistency for accurate comparison. Table 2 provides a detailed breakdown of the string's components, which consist of the drill bit, drill collar, heavy weight drill pipe (HWDP), and standard drill pipe.

Table 2. Modeled downscale string.

Type	Length (m)	OD (inch)	ID (inch)	Gauge (inch)	Total Length (m)	Mass (kg)	Total Mass (kg)
Bit	0.03	-	-	0.20	0.03	0.001	0.001
Drill collar	0.15	0.25	0.20	-	0.18	0.01	0.011
HWDP	5.32	0.50	0.15	-	5.50	0.20	0.211
Drill pipe	9.50	0.12	0.10	-	15.00	0.15	0.361

2.2. Upscale String

Similarly, an upscale drill string was modeled with a total length of 450 m and a bit diameter of 5.625 inches, designed to accurately represent the configuration of a typical drill string used in oil and gas field operations. This model was chosen to simulate real-world drilling conditions, ensuring that the upscale string mirrored the physical dimensions and behavior of strings deployed in actual wellbores. The upscale string comprised the same configuration as the downscale. Table 3 presents a detailed breakdown of each component, including their outer diameter, inner diameter, mass, and length. These comprehensive data allow for a clear understanding of the string's structural composition, highlighting the precise specifications and design considerations essential for optimal drilling performance in the field.

Table 3. Modeled upscale string.

	Length (m)	OD (inch)	ID (inch)	Gauge (inch)	Total Length (m)	Mass (kg)	Total Mass (kg)
Bit	0.5	-	-	5 5/8	0.5	28.16	28.16
DC × 8	72	4 1/2	2 1/8	-	73	4506.18	4534.34
HWDP × 10	111	3 1/2	2 1/16	-	184	3170.08	7704.42
DP × 28	266	3 1/2	2 5/8	-	450	6416.2	14,120.6

2.3. Simulation Matrix

The upscale and downscale simulations were conducted over 80 s, during which the top RPM was varied. This strategic approach enables comprehensive monitoring of torsional vibrations at three specific depths along the drill string, providing insights into how these vibrations propagate under differing torque conditions, as outlined in Table 1. By focusing on these key depths, the performance of the drill string in critical zones can be analyzed, enhancing understanding of the torsional dynamics involved. The decision to use steel as the material for each setup was pivotal. Steel's inherent properties, such as its high tensile strength and durability, ensure that every component can withstand the extreme stresses, vibrations, and potential corrosive effects encountered in the challenging environments of drilling operations. This material choice not only contributes to the structural integrity of the drill string but also plays a vital role in minimizing fatigue and failure under cyclic loading conditions. Additionally, the careful selection of essential parameters such as RPM, torque, density, and static and dynamic friction coefficients is crucial for accurately modeling the behavior of drill strings. These parameters were carefully chosen to align with the specific characteristics of each string configuration, thereby enhancing the fidelity of the simulations. By rigorously calibrating these variables, the simulations capture the complex interactions and responses within the drill string during operational conditions. This not only leads to more reliable results regarding torsional behaviors but also provides deeper insights into optimizing drilling parameters.

3. Results and Discussions

3.1. Stick Slip Vibration Effect on the Bit

The stick-slip effect on the bit in both the upscale and downscale models was analyzed over an 80-s duration at varying torque values, as shown in Figure 2. In this simulation, the upscale bit operated with a torque of 500 kgfm, while the downscale bit functioned at a significantly lower torque of 0.01 kgfm. Both bits were run under the same top RPM to compare their behavior under identical operating conditions. From the graph, it is evident that the upscale bit initially rotated at a higher RPM than the top RPM, but it soon experienced stick-slip oscillations. After the first 5 s, the bit overcame the initial sticking phase and exhibited torsional oscillations, attempting to synchronize with the top RPM. As the top RPM increased, the upscale bit's rotation gradually stabilized and became more easily associated with the top RPM over time to allow the severity of oscillations to reduce. On the other hand, the downscale bit had somewhat dissimilar behavior. At the start of the run, an apparent sticking at 0 RPM was observed. This probably means some level of instability at the start-up. However, after this happened, the downscale bit also experienced a moment of stick-slip. Eventually, quite similar to the upscale bit, the downscale bit overcame the stick-slip effect and began to rotate in sync with the top RPM as the system stabilized.

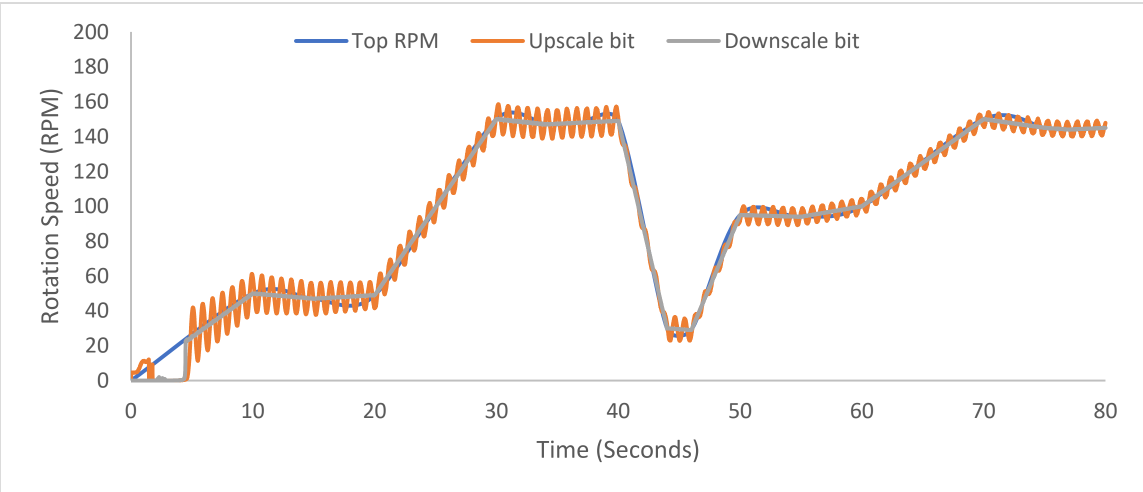

Figure 2. Upscale and downscale bit RPM at torque values of 500 kgfm and 0.01 kgfm.

The torque values for both setups were increased to evaluate their response under higher torque conditions. Specifically, the torque on the upscale bit was raised to 1500 kgfm, while the downscale bit's torque was increased to 0.04 kgfm. Figure 3 illustrates the behavior of both bits at these elevated torque levels. As observed, both bits initially exhibited similar dynamics to their performance under the lower torque values with stick slip at the onset. However, the stick-slip periods became more pronounced for both the upscale and downscale bits, indicating that higher torque amplified torsional oscillations (especially for the upscale bit). Despite the extended stick-slip phases, both bits eventually synchronized with the top RPM over time, stabilizing their rotational speeds in alignment with the system's overall rotation.

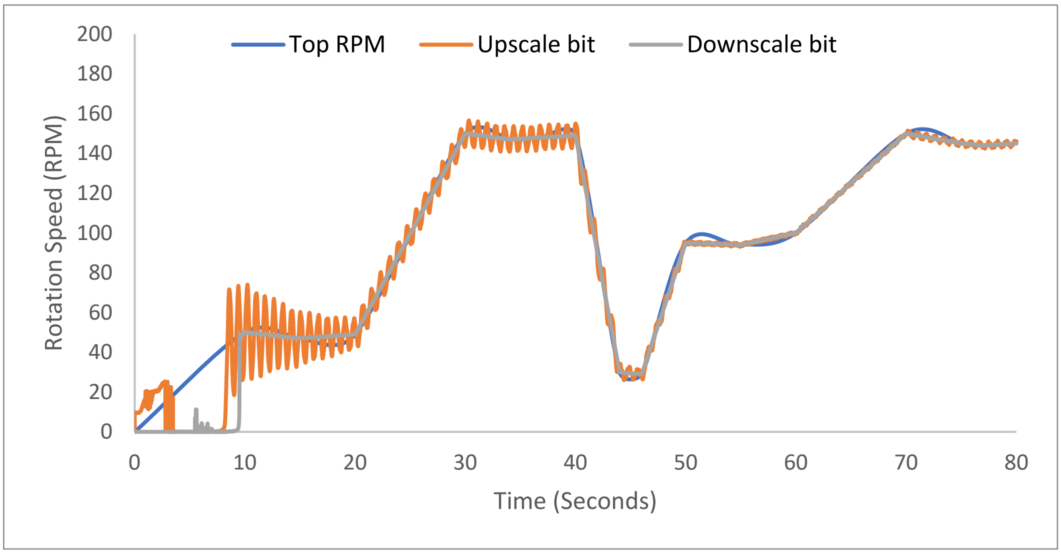

Figure 3. Upscale and downscale bit RPM at torque values of 1500 kgfm and 0.04 kgfm.

The torque on both setups was further increased to assess their behavior under more extreme conditions. Specifically, the upscale torque was raised to 4500 kgfm, while the downscale torque was increased to 0.09 kgfm. Figure 4 illustrates the effect of these higher torque values on both bits. It is evident from the figure that the stick-slip period became significantly longer, with both bits experiencing more prolonged sticking phases before resuming rotation. This behavior emphasizes that higher torque amplifies the intensity and duration of torsional vibrations, making it increasingly difficult for the bits to achieve smooth, continuous rotation. This further highlights the critical role torque plays in influencing stick-slip severity, requiring more effort and precise control to stabilize the system at higher loads.

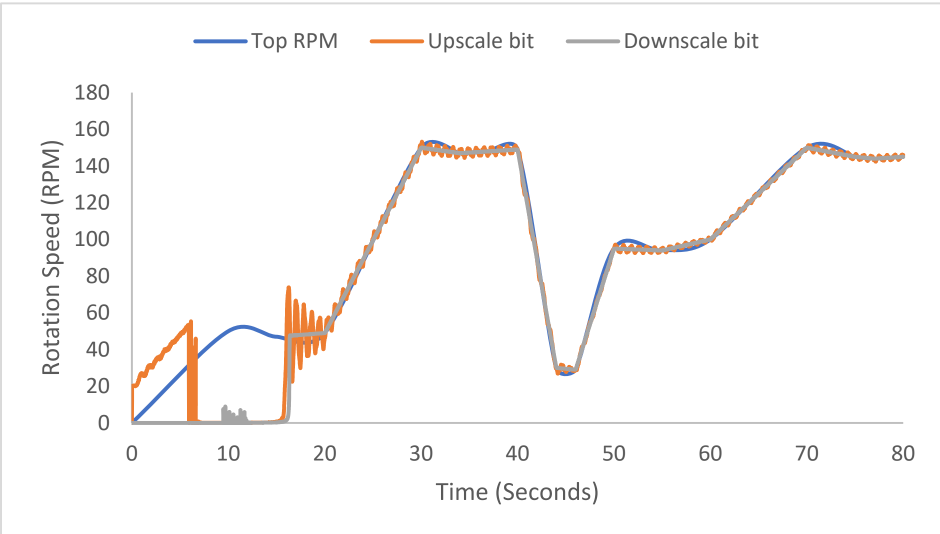

Figure 4. Upscale and downscale bit RPM at torque values of 4500 kgfm and 0.09 kgfm.

3.2. Stick Slip Behavior at 100 m (Upscale) and 3.3 m (Downscale) Depth

This analysis helps to understand the propagation of torsional forces through the drill string and their effects on the rotational behavior at different scales. Figures 5–7 present comparative results for both the upscale and downscale drill string models under different torque conditions, showcasing the emergence and behavior of stick-slip oscillations.

A torque of 500 kgfm was applied to the upscale string, and 0.01 kgfm to the downscale string (Figure 5). From the onset, it can be seen that both strings closely mimicked the top RPM, indicating that the rotational energy was effectively transmitted through the drill string without significant disruption. However, as time progressed, the upscale string started to exhibit stick-slip behavior, where periods of sticking (when the rotation momentarily stops) alternate with slipping (sudden rotational jumps). Interestingly, the stick-slip effect was more severe in the downscale string, which reflects its higher sensitivity to torque variations due to its scaled-down geometry and reduced mass. Both strings entered the stick-slip phase simultaneously, suggesting that the onset of torsional oscillations in both models was influenced by a common factor—likely the transmission of top-end rotational fluctuations. However, after approximately 5 s, both the upscale and downscale strings managed to overcome the stick-slip effect, gradually stabilizing and synchronizing with the top RPM. This indicates that the systems could reach a steady rotational state after overcoming initial torsional disturbances.

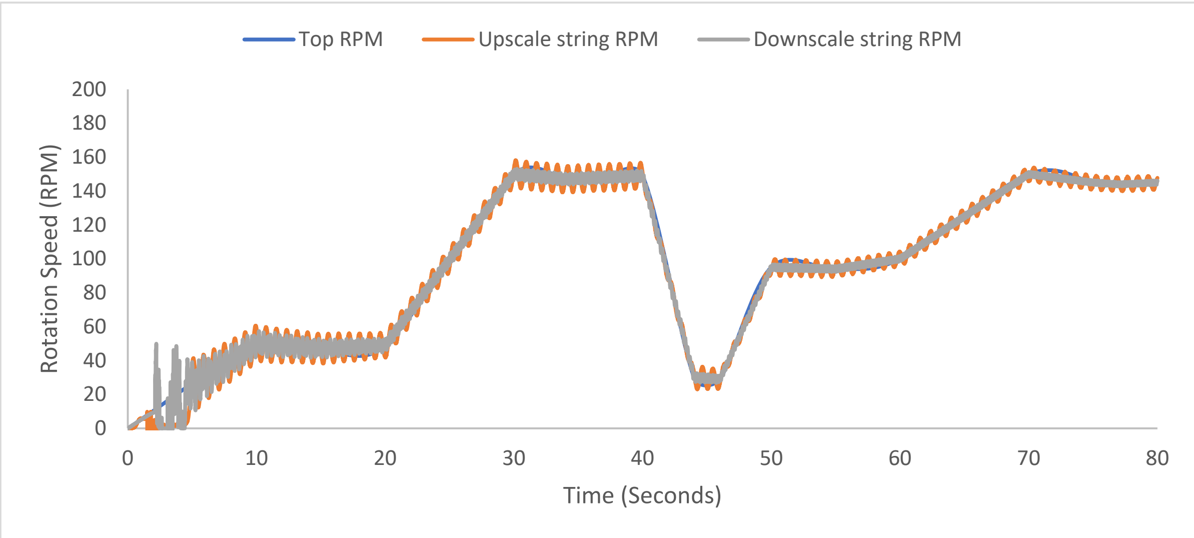

Figure 5. Upscale and downscale string RPM at torque values of 500 kgfm and 0.01 kgfm at 100 m (upscale) and 3.3 m (downscale).

The applied torque was increased to 1500 kgfm for the upscale string and 0.04 kgfm for the downscale string (Figure 6). As observed in the first case, both strings initially mirrored the top RPM's rotation without noticeable torsional instability. However, after a brief period, both strings entered a stick-slip phase, but the severity of the oscillations was much greater than in the first scenario. This suggests that the magnitude of the applied torque has a direct influence on the intensity and persistence of stick-slip behavior. The downscale string continued to exhibit more severe stick-slip oscillations compared to the upscale string. This outcome aligns with the expectation that smaller-scale systems, due to their reduced inertia and stiffness, are more prone to sudden rotational disturbances. Moreover, increased torque levels amplify the system's vulnerability to torsional vibrations, further validating the importance of optimized torque management during drilling operations to minimize mechanical wear and instability.

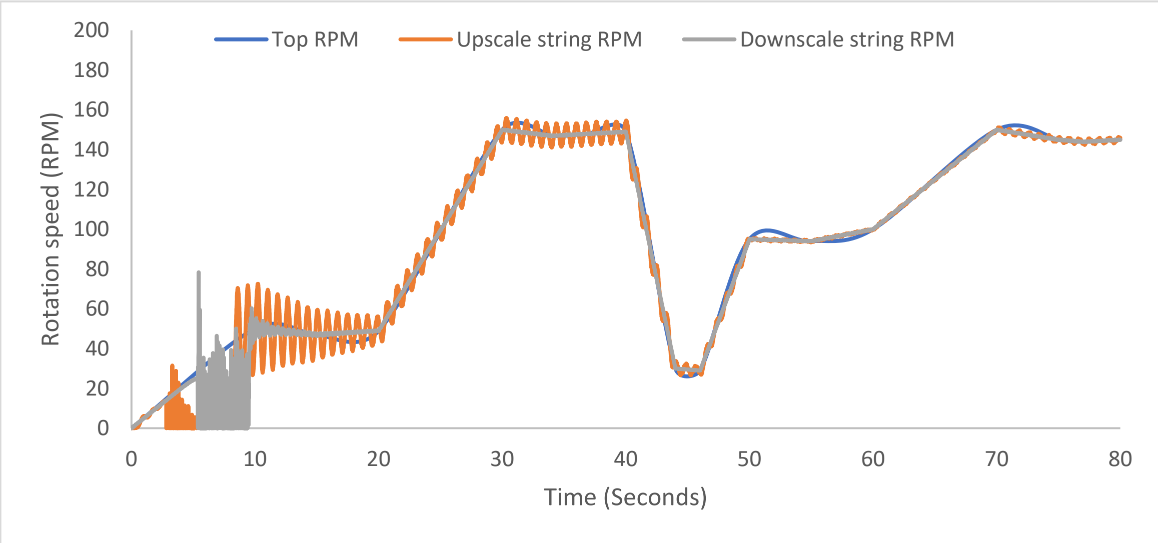

Figure 6. Upscale and downscale string RPM at torque values of 1500 kgfm and 0.04 kgfm at 100 m (upscale) and 3.3 m (downscale).

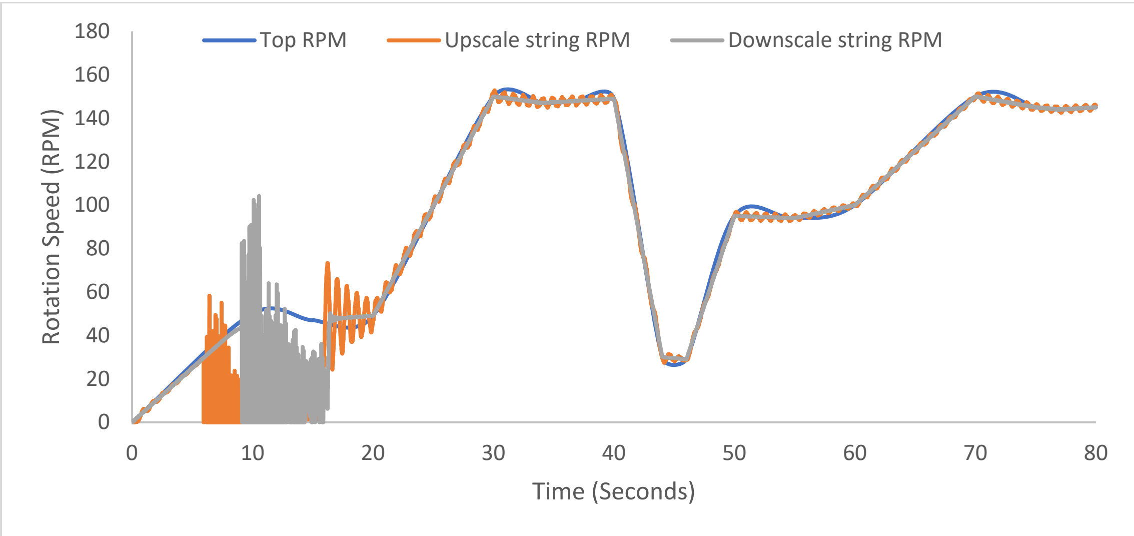

Figure 7. Upscale and downscale string RPM at torque values of 4500 kgfm and 0.09 kgfm at 100 m (upscale) and 3.3 m (downscale).

Torque was increased to 4500 kgfm for the upscale string and 0.09 kgfm for the downscale string, as shown in Figure 7. As in the previous tests, both strings maintained synchronization with the top RPM for a longer period before experiencing stick-slip oscillations. This implies that at elevated levels of torque, there could be a transient delay in the development of torsional vibration in the drill string until stick slip sets in. Even though both strings underwent stick slip, the downscale string had a more drastic stick-slip phenomenon owing to its smaller dimensions and reduced structural rigidity. This shows how scaling can affect the dynamic response of drill strings, with smaller systems being more vulnerable to torsional instabilities. Nonetheless, it is important to note that both strings stabilized and resumed rotation in time with the top RPM, showing that the system achieved the steady-state condition after it overcame transient instabilities.

3.3. Stick Slip Behavior at 300 m (Upscale) and 10 m (Downscale) Depth

This phase of the investigation focused on the behavior of the drill strings with an upscale string operating at 300 m and a downscale string operating at 10 m. Thus, the subsequent comparisons give detailed information regarding the influence of depth and system size on the transmission of torsional forces and the stability of the rotational system. Results from these simulations are illustrated in Figures 8–10.

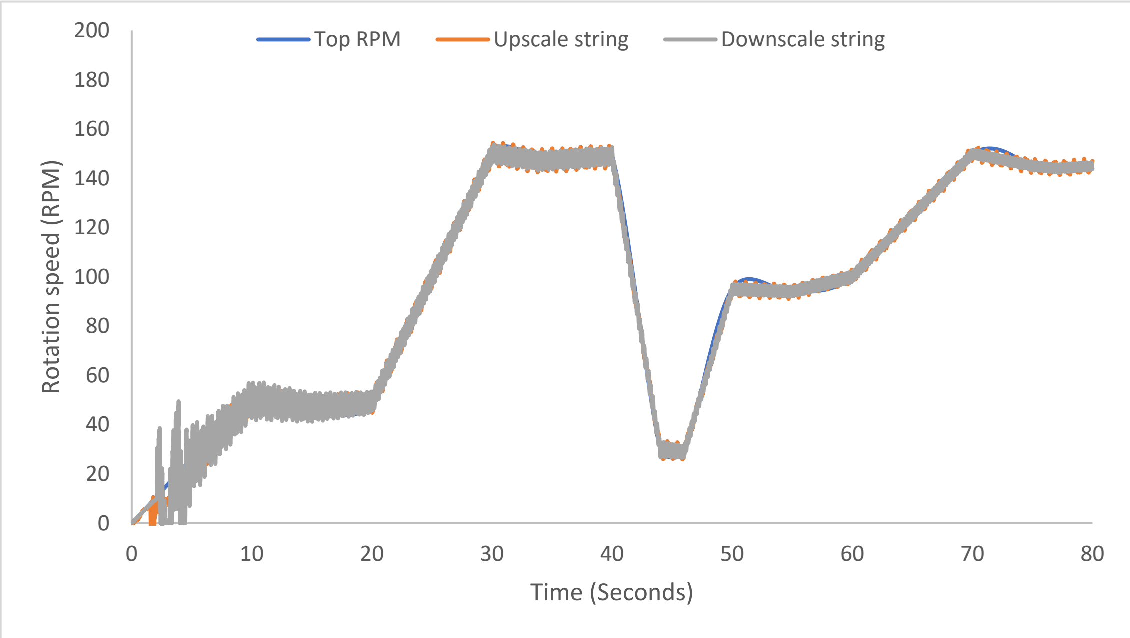

Figure 8. Upscale and downscale string RPM at torque values of 500 kgfm and 0.01 kgfm at 300 m (upscale) and 10 m (downscale).

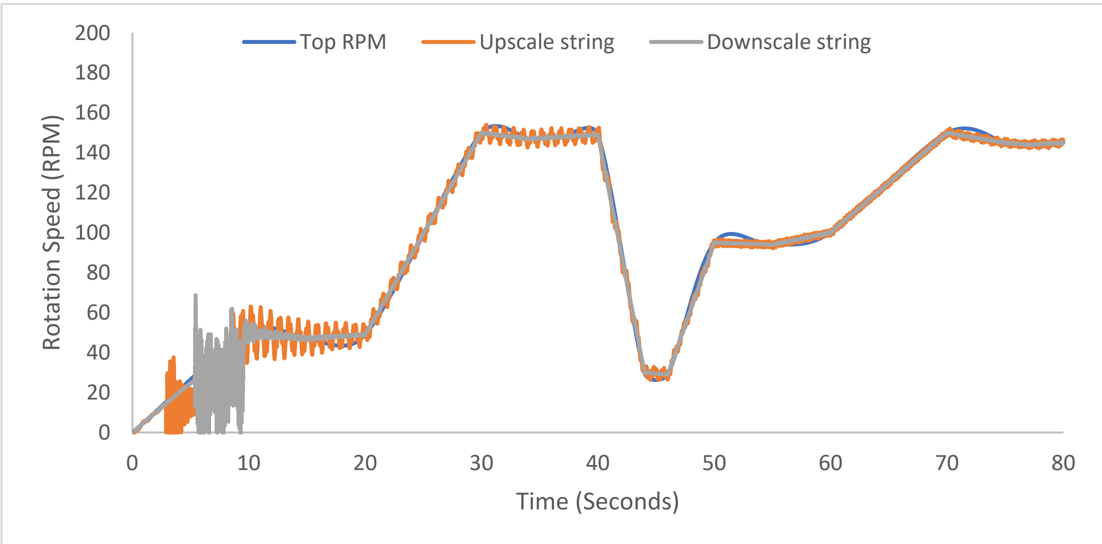

Figure 9. Upscale and downscale string RPM at torque values of 1500 kgfm and 0.04 kgfm at 300 m (upscale) and 10 m (downscale).

In the first scenario, a torque of 500 kgfm was applied to the upscale string, while 0.04 kgfm was applied to the downscale string (Figure 8). At this torque level, both strings started by mimicking the top RPM. However, it is evident from the results that the stick-slip effect was more pronounced in the downscale string compared to the upscale string. This aligns with previous findings, where smaller, lighter strings are more sensitive to rotational instabilities due to their reduced inertia and stiffness. Despite the downscale string's higher stick-slip severity, both systems managed to overcome the torsional oscillations within a short period, stabilizing their rotations and synchronizing with the top RPM. This outcome reflects the capacity of both systems to self-correct and reach a steady rotational state after passing through an initial phase of torsional instability.

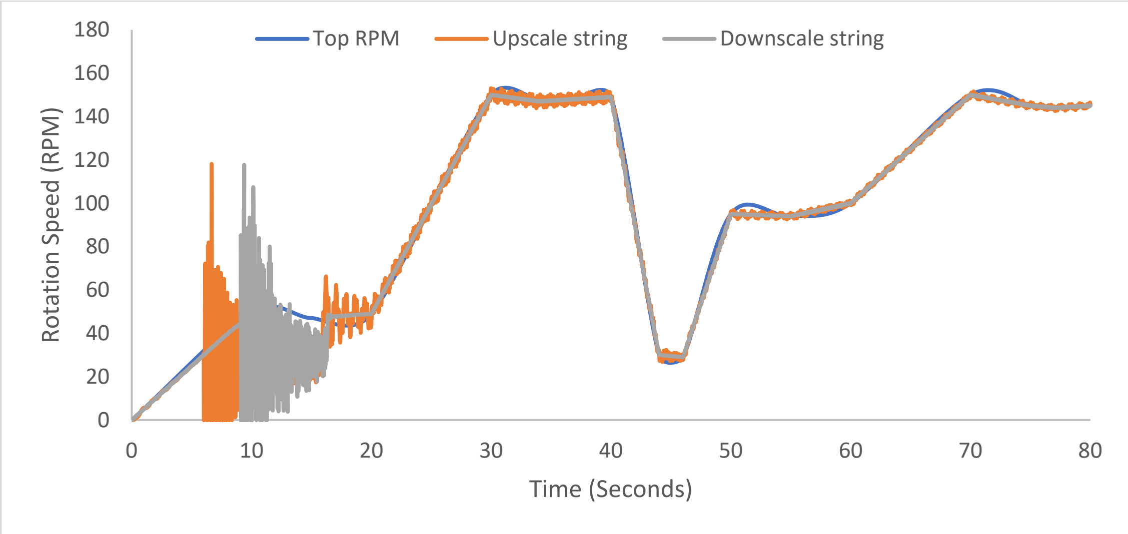

Figure 10. Upscale and downscale string RPM at torque values of 4500 kgfm and 0.09 kgfm at 300 m (upscale) and 10 m (downscale).

In the second scenario, the torque was increased to 1500 kgfm for the upscale string while the torque for the downscale string remained at 0.04 kgfm (Figure 9). As expected, both strings initially mirrored the top RPM's rotation. However, the upscale string entered the stick-slip phase earlier than the downscale string. This observation suggests that at higher torque levels, even the upscale string, despite its larger size and greater structural stability, became prone to torsional disturbances. Although the downscale string took longer to enter the stick-slip phase, it once again exhibited greater severity of oscillations compared to the upscale string. This behavior is consistent with the trend observed in previous tests, where smaller-scale systems experience more pronounced instabilities. The higher stick-slip severity could indicate that the smaller system has more difficulty dissipating the torsional energy efficiently, causing more abrupt rotational jumps.

In the final scenario, the torque was raised to 4500 kgfm for the upscale string and 0.09 kgfm for the downscale string (Figure 10). At this heightened torque level, both strings again started by rotating in sync with the top RPM, but they eventually entered the stick-slip phase. A key observation at this torque level is that the upscale string now entered the stick-slip phase earlier than the downscale string. This reversal in behavior suggests that the upscale string's stability is more easily compromised under extreme torque conditions. Additionally, the severity of stick-slip oscillations for both strings increased at this higher torque. The larger rotational disturbances observed indicate that the transmission of excessive torque introduces significant challenges for maintaining smooth rotational behavior, even for the more robust upscale string. Both strings underwent severe torsional vibrations at one point, but over time, they stabilized and rotated in sync with the top RPM. This shows the inherent resilience of the system, demonstrating that both models, despite initial instability, can recover and resume stable rotation.

4. Conclusions

This study compares the torsional stick-slip behavior between upscale and downscale drill strings at three different depths under different torque conditions. Results obtained show that both systems initially followed the top RPM, then eventually transitioned into the stick-slip phase, signaling the onset of torsional instability. The severity of stick-slip vibrations was consistently greater in the downscale system, attributed to its reduced inertia and increased sensitivity to rotational disturbances. A major point noticed in this study is that when torque is applied, the upscale drill string showed larger torsional oscillation than the low-scale drill string right after overcoming the occurrences of sticks and slips. This behavior can be attributed to the higher value of upscale string inertia in proper consideration, naturally leading to a delayed and more abrupt type of onset of instability. At elevated torque levels, the stored kinetic energy in the upscale system is greater, making the upscale system more prone to rapid destabilization when the critical threshold is exceeded. These results emphasize the need for accurate torque control, especially in full-scale drilling operations where the management of rotational inertia becomes imperative. Real-time monitoring and adaptive control approaches need to be installed to detect the phenomena of stick slips at their early onset and suppress severe oscillations well before they can damage the integrity of the drill string.

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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

BHA	Bottomhole assembly
FEM	Finite element assembly
PDE-ODE	Partial differential equation-ordinary differential equation
DC	Drill collar
HWDP	Heavy weight drill pipe
DP	Drill pipe
RPM	Rotation speed
WOB	Weight on bit

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