

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

PHOTOGRAMMETRIC STUDIES OF A KANSAS TORNADO
AND A HAWAIIAN TORNADIC-WATERSPOUT

A THESIS

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

MASTER OF SCIENCE

IN METEOROLOGY

BY

RANDAL ALLAN ZIPSER

Norman, Oklahoma

1976

LD
4321
.8t
cop.1

PHOTOGRAMMETRIC STUDIES OF A KANSAS TORNADO

AND A HAWAIIAN TORNADIC-WATERSPOUT

A THESIS

APPROVED FOR THE DEPARTMENT OF METEOROLOGY

I would first like to express gratitude to my committee chairman, Dr. Joseph Golden, who made this study at the National Severe Storms Laboratory (NSSL) possible. Drs. Jay Fein and Jeff Kimpel of the University of Oklahoma are also to be commended for their assistance during this research. An extended appreciation, however, goes to all individuals who contributed photographs, movies, and information to this research effort. Eyewitness photographers are listed in Appendix A. Lt. Steve Tegtmeler of the USAF, and Mr. Don Rostenwald of the Salina-Salina County Civil Defense, are also acknowledged for contributing material used in support of this research.

Thanks are also due to the many NSSL research staff members who contributed ideas through stimulating discussions. A special word of thanks, in particular, goes to Mr. Rodger Brown, who devoted many hours to the formulation of the derivations in Appendix C. In addition, I greatly appreciated the field-surveying assistance of Mr. Dan Furell of NSSL.

I would especially like to thank Dr. Thomas Schroeder of the University of Hawaii, Mr. Kerry Wilson and Mr. Arne O. [redacted] Observatory at Hilo, for enthusiastically [redacted]

Further appreciation goes to Mrs. [redacted] typing the manuscript. Ms. Jennifer Farris [redacted] also to be thanked for drafting the figures [redacted] illustrations, respectively.

ACKNOWLEDGMENTS

I would first like to express gratitude to my committee chairman, Dr. Joseph Golden, who made this study at the National Severe Storms Laboratory (NSSL) possible. Drs. Jay Fein and Jeff Kimpel of the University of Oklahoma are also to be commended for their assistance during this research. An extended appreciation, however, goes to all individuals who contributed photographs, movies, and information to this research effort. Eyewitness photographers are listed in Appendix A. Lt. Steve Tegtmeier of the USAF, and Mr. Don Rectenwald of the Salina-Saline County Civil Defense, are also acknowledged for contributing material used in support of this research.

Thanks are also due to the many NSSL research staff members who contributed ideas through stimulating discussions. A special word of thanks, in particular, goes to Mr. Rodger Brown, who devoted many hours to the formulation of the derivations in Appendix C. In addition, I greatly appreciated the field-surveying assistance of Mr. Dan Purcell of NSSL.

I would especially like to thank Dr. Thomas Schroeder of the University of Hawaii, Mr. Kerry Wilson and Mr. Arne Oustring, both of the Cloud Physics Observatory at Hilo, for enthusiastically supporting the Kailua-Kona study.

Further appreciation goes to Mrs. Estis Wynn for her diligent work in typing the manuscript. Ms. Jennifer Farris and Mr. Charles Clark of NSSL are also to be thanked for drafting the figures and readying the photographic illustrations, respectively.

A very special thanks are due to my parents, Lester and Helga Zipser, who not only have provided the financial support for much of my graduate education, but also have encouraged me to pursue the field of meteorology as a career.

This research was conducted entirely at the National Severe Storms Laboratory in association with graduate study at the University of Oklahoma, and was partially supported by an Interagency Agreement with the AEC under contract AT(49-25)-1004.

The greatest net wind speeds derived were 39 m s^{-1} in the Kansas case and 36 m s^{-1} for the Hawaiian vortex. Both of these maxima occurred below 70 m. Maximum apparent diameters were 115 m for the Salina and 99 m for the Salina-Kansas vortices.

Synoptic-scale and near-vortex environment descriptions are included for each vortex case. Photographic scaling procedures are also outlined.

ABSTRACT

Single-camera terrestrial photogrammetry was used to investigate the Salina, Kansas tornado of 25 September 1973 and the Kailua-Kona, Hawaii tornadic-waterspout of 28 January 1971. Carefully controlled and scaled 16 mm movie loops were used to derive the apparent vertical, horizontal, and net windspeed distributions for three types of air flow tracers associated with these vortices. In addition, dimensional variations of the Salina tornado and its attendant cloud base structure were synthesized from still photographs to provide insight into the vortex's evolutionary stages.

The greatest net windspeeds derived were 69 ms^{-1} in the Kansas case and 56 ms^{-1} for the Hawaiian vortex. Both of these maxima occurred below 70 m. Maximum apparent diameters were 318 m for the Salina and 99 m for the Kailua-Kona vortices.

Synoptic-scale and near-vortex environment descriptions are included for each vortex case. Photographic scaling procedures are also outlined.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	iii
ABSTRACT.	v
LIST OF TABLES.	vii
LIST OF ILLUSTRATIONS	viii
LIST OF SYMBOLS	x
Chapter	
I. INTRODUCTION	1
II. SYNOPTIC AND MESOSCALE STUDIES	5
III. PHOTOGRAPHIC DATA ANALYSIS	12
IV. PHOTOGRAMMETRIC ANALYSIS RESULTS	17
V. CONCLUDING REMARKS AND SUGGESTIONS FOR FURTHER RESEARCH .	36
BIBLIOGRAPHY.	39
APPENDIX A.	45
APPENDIX B.	46
APPENDIX C.	49

LIST OF TABLES

Table	Page
1. Apparent Looping Effect in the Decaying Salina Tornado	29
2. Summary of Photogrammetric Results for the Salina Tornado System.	32
3. Summary of Photogrammetric Results for the Kailua-Kona Tornadic-Waterspout	35
4. List of Photographers	45
5. Angular Distortion Error.	47
6. Regional Map of Kansas Synoptic Surface Network Stations Showing Wichita Under Attack, 2130 GMT, 25 September 1973.	50
7. Wichita Radar Echo Sequence of the Salina Tornado's Parent Thunderstorm.	58
8. Surface Analysis, 1200 GMT, 24 January 1971.	59
9. 500 mb Analysis, 0000 GMT, 25 January 1971.	59
10. Temperature/Dew Point Soundings for Hilo, Hawaii, 0000 GMT, 24 January 1971.	59
11. Net Velocity Trajectories for Cloud Tags Surrounding the Salina Tornado	60
12. Net Cloud Tag Velocities Within the Salina Tornado's Wall Cloud	60
13. Vertical Velocities for a Cloudless Tag Prior to Primary Salina Tornado Formation	61
14. Net Cloud Tag Velocities Associated with Debris Surrounding the Primary Salina Tornado.	61
15. Net Velocity Trajectories for a Perturbation and Debris Aggregates During the Salina Tornado's Striking Stage.	62
16. Net Velocity Trajectories for Debris Aggregates During the Salina Tornado's Mature Stage	62

LIST OF ILLUSTRATIONS

Figure		Page
1.	Surface Analysis, 0000 GMT, 26 September 1973.	55
2.	Surface/Upper Air Composite Synoptic Analysis, 1200 GMT, 25 September 1973, 0000 GMT, 26 September 1973 . . .	55
3.	ATS-3 Satellite Photographs for (a) 2003 GMT, 25 September 1973, 0000 GMT, 26 September 1973	56
4.	500 mb Analysis, 0000 GMT, 26 September 1973	56
5.	Temperature/Dew Point Soundings for (a) Topeka, and (b) Dodge City, Kansas, 0000 GMT, 26 September 1973. . . .	57
6.	Regional Map of Kansas Synoptic Surface Network Stations Showing Wichita Radar Echoes, 2230 GMT, 25 September 1973.	58
7.	Wichita Radar Echo Sequence of the Salina Tornado's Parent Thunderstorm.	58
8.	Surface Analysis, 1200 GMT, 28 January 1971.	59
9.	500 mb Analysis, 0000 GMT, 29 January 1971	59
10.	Temperature/Dew Point Sounding for Hilo, Hawaii, 0000 GMT, 29 January 1971.	59
11.	Net Velocity Trajectories for Cloud Tags Surrounding the Salina Tornado	60
12.	Net Cloud Tag Velocities Within the Salina Tornado's Wall Cloud	60
13.	Vertical Velocities for a Cloudbase Tag Prior to Primary Salina Tornado Formation	61
14.	Net Cloud Tag Velocities Associated with Funnel Succeeding the Primary Salina Tornado.	61
15.	Net Velocity Trajectories for a Perturbation and Debris Aggregates During the Salina Tornado's Shrinking Stage.	62
16.	Net Velocity Trajectories for Debris Aggregates During the Salina Tornado's Mature Stage	62

Figure		Page
17.	Net Velocity Trajectories for Debris Aggregates During the Salina Tornado's Decay Stage.	62
18.	Dimensions of the Salina Tornado and Wall Cloud Throughout Life Cycles	63
19.	Stretched Vortex During the Salina Tornado's Decay Stage.	64
20.	Scaled Diagram of Stereophotographic Site Geometry for Stretched Salina Vortex	64
21.	Sequential Axis and Vortex Center Positions of Decaying Salina Tornado	64
22.	Sequential Radius-vs.-Height Position Plots for Debris Eddies Within the Decaying Salina Tornado's Debris Cloud Relative to Its Center.	65
23.	Sequential Radius-vs.-Height Position Plots for Secondary Vortices Prior to the Primary Salina Tornado Relative to Its Center	65
24.	Program ARCLEN Output: (a) Near Side Element, (b) Far Side Element	66
25.	Net Solid-Debris Velocity Trajectories as the Kailua-Kona Vortex Makes Landfall and Travels Inland	67
A-1(a).	Map of Salina Area Photographers	68
A-1(b).	Map of McPherson County Area Photographers	68
A-2.	Map of Kona Coastline Showing Tornadic-Waterspout Path	68
B-1(a).	Illustration of the Tangent Law.	69
B-1(b).	Illustration of Azimuthal Angular Distortion of Linear Distance in the Image Focal Plane	69
C-1.	Geometry of Three-Dimensional Arc Length Travelled by a Vortex Tracer Element in a Horizontal Circular Orbit.	70
C-2.	Program ARCLEN Listings.	71
C-3.	Schematic Interpretation of Negative Arc Length.	72
C-4.	Correction of Tangential Velocity for a Distant Observer	72

LIST OF SYMBOLS

a_w	average vertical acceleration
D	linear distance increment measured across the image focal plane
d_1	horizontal distance between the apparent tornado center and one of the two surveyed landmarks in the effective image plane
f	focal length of camera lens
f'	effective focal length
P	wind pressure force
P_F	farthest point on a circular vortex from the observer
P_N	nearest point on a circular vortex to the observer
PP	principal point (azimuth) in the effective image plane
PP'	principal point (azimuth) in the image focal plane
R	distance (range) from the camera site to the tornado center in actual space
R_s	distance (range) from the camera site to the tornado center on a scaled map
r	turning radius; apparent turning radius (used interchangeably with x')
$\bar{r}$	mean radius
S	arc length travelled by a vortex tracer element in a circular orbit
TH	true horizon line
t	time
V_c, T_c	apparent vortex or tornado center (azimuth)
v	tangential velocity; apparent horizontal velocity component when vortex is moving
v_n	apparent net velocity
v_{tr}	translation speed of vortex

w	apparent vertical velocity component
$x', x_{()}$	horizontal distance of a tracer element from the vortex center in the effective image plane (used interchangeable with r)
$z, z_{()}$	vertical height, apparent vertical distance to the true horizon line
$\bar{z}$	mean height
$\alpha, \alpha_{()}$	angular displacement of a tracer element in its orbit around a circular vortex measured counterclockwise from the camera's principal axis
β	angle between the observer's line-of-sight to a tracer element or landmark in the effective image plane and the principal axis; angular separation between two surveyed landmarks when determining the scale ratio
β_c	critical angle for tangential velocity correction
γ	angle between the observer's line-of-sight to a tracer element on a circular vortex and its radius
ΔT_c	angular separation between the azimuth of one of the two surveyed landmarks and the azimuth of the apparent tornado center
ΔX	horizontal distance in actual space between two surveyed landmarks normalized to the range of the tornado's center
Δx	horizontal distance between two surveyed landmarks measured across the effective image plane
$\zeta_{()}$	elevation angle of a surveyed landmark
λ	apparent horizontal distance estimate between two surveyed landmarks measured across the effective image plane when determining the true horizon
ξ	horizontal distance of a landmark in the effective image plane from the principal point azimuth

PHOTOGRAMMETRIC STUDIES OF A KANSAS TORNADO
AND A HAWAIIAN TORNADIC-WATERSPOUT

CHAPTER I

INTRODUCTION

Photogrammetry is the science of obtaining reliable measurements by means of photography (Thompson, 1966). In addition to obtaining high quality photography, however, one should be able to interpret accurately what is seen on the photographic image. Such photointerpretation skill can aid in relocating the physical position of the photographer. Once the camera site is relocated, one may obtain by means independent of additional photography certain data which are necessary for deriving the numerical relationship between length measured across the photograph and the actual linear distance it represents.

The type of photogrammetry to be employed in the present application is known as analytical photogrammetry because our primary objective is to derive tornado windspeeds from mathematical computations involving measurements made from photographs as input data. Moreover, each film image analyzed was taken by one camera located on or near the ground. This is referred to as single-camera terrestrial photogrammetry (Thompson, 1966). Additional information on the subject of photogrammetry may be found in several excellent general texts (Sharp, 1951; Hallert, 1960; Moffitt, 1967; Crone, 1968; Williams, 1969; Wolf, 1974).

The primary objectives of this study are: (a) to derive and physically interpret the vertical and horizontal velocity fields of three types of visible motion tracers associated with the Salina, Kansas tornado of 25 September 1973 and the Kailua-Kona, Hawaii tornado of 28 January 1971, (b) to develop a new methodology for improving apparent tangential velocity underestimates, and (c) to briefly summarize the synoptic-scale forcing conditions for both tornado events.

The Present State of Mesometeorological Photogrammetry

After Hoecker (1960a) pioneered the use of conventional terrestrial photogrammetry techniques as a means for studying windspeeds in a filmed tornado, little additional work on tornado structure and windspeed distributions has appeared in the published literature (Fujita, 1960a; Hoecker, 1960b, 1961; Goldman and Fujita, 1963; Goldman, 1965) due to a lack of suitable movies. However, the 1970's have seen a resurgence of work on this subject (Fujita et al., 1970; Golden, 1973; Golden and Davies-Jones, 1975; Golden and Purcell, 1974, 1975; Agee et al., 1975; Blechman, 1975; Church, 1975). This recent increased interest in the photogrammetric approach to the study of tornadoes probably has developed for the following reasons:

- (1) A greater awareness by the general public of proper antecedent conditions for tornado development due to improved warnings, resulting in an increase of eyewitness tornado photography.
- (2) A public better informed of the needs of the meteorological research community to have such photography available for study.
- (3) Studies by civil engineers of damage inflicted upon wind engineered structures have led to an alternate approach for obtaining vortex windspeed estimates near the ground and comparison of results with photogrammetry (Segner, 1960; Mehta et al., 1971; Wen, 1975; Yeh, 1975; Sadeh, 1975).
- (4) Simplicity of field-survey data acquisition and subsequent laboratory analysis procedures.

Although Hoecker (1960a) was one of the first to attempt to quantify tornado windspeed estimates with photogrammetry, most earlier tornado windspeed estimates were based solely on structural damage observations. Perhaps the first of such estimates was that of Groat (1899), who, using the wind pressure-velocity relation $P = 0.005 v^2$, derived a 60 ms^{-1} velocity estimate for the New Richmond, Wisconsin tornado of 12 June 1899. Even more remarkably, Kimball (1901) cites the results of an earlier photogrammetric computation by Bigelow (1898-99) for the Cottage City, Massachusetts waterspout of 19 August 1896. Results include funnel diameter, outward radial velocity, rotational velocity, and upward vertical velocity at twelve levels. Closer examination of Bigelow's results shows the outer Rankine vortex criterion $v_r = \text{constant}$ (same for all z) to hold approximately. Marshall (1932) observed damage to three different objects following the Washington, Kansas tornado of 4 July 1932 and computed wind velocities of 94 ms^{-1} , 57 ms^{-1} , and 65 ms^{-1} , respectively, based on assumed maximum wind pressures against these objects. However, it was Van Tassel (1955) who first performed terrestrial photogrammetric measurements, using scaled photographs of the Scottsbluff, Nebraska tornado of 27 June 1955. Unfortunately, Van Tassel computed only the horizontal and vertical dimensions of the tornado in this manner. He analytically computed a surface windspeed of 216 ms^{-1} using aerially-surveyed elliptical ground markings (cycloids). Hoecker's (1960a) analysis of the Dallas tornado of 2 April 1957 differs principally from Van Tassel's in that the maximum tangential speed (76 ms^{-1}) and upward speed (67 ms^{-1}) were derived directly by tracking debris, dust parcels, and cloud tags in scaled movies of that tornado, similar to the procedures used in the study reported here.

More recently, Golden (1973, 1974a) has used aerial photogrammetric techniques to track the rotation of clearly recognizable spray vortex features

around Florida Keys' waterspouts, and has derived tangential windspeed profiles by applying techniques developed earlier by both Fujita (1960a) and Hoecker (1960a). Furthermore, Golden and Davies-Jones (1975) have reported a photogrammetrically-derived, horizontal windspeed maximum of at least 80 ms^{-1} at 90 m above ground level (AGL) and 200 m from the axis of the Union City, Oklahoma tornado of 24 May 1973. Blechman (1975) has recently applied photogrammetric techniques to the motion study of individual vortices in the multi-synoptic scale features leading to the outbreak of severe weather on 25 September 1973 which included the well-documented Salina tornado. The following analysis by Agee *et al.* (1975) of the Parker, Indiana tornado of 3 April 1974, has yielded a maximum rotational velocity of 50 ms^{-1} for a secondary vortex about the center of its parent vortex (or $v \geq 75 \text{ ms}^{-1}$ relative to the ground). Most significant, though, are the recent findings of Golden and Purcell (1975). Their photogrammetric measurements of the Great Bend, Kansas tornado of 30 August 1974, indicate that the cyclostrophic flow assumption and hydrostatic approximation are probably not valid for the lowest 100 m of the vortex column in that case. If future tornado studies substantiate these results, then previous attempts by several investigators (Glaser, 1960; Fujita, 1960a; Dergarabedian and Fendell, 1970, 1971, 1975) to estimate maximum tangential windspeeds using the cyclostrophic wind equation, hydrostatic approximation, and Rankine-combined vortex model from still photograph measurements may be in serious doubt.

Figure 2 is a composite analysis of key synoptic-scale features at the surface and aloft over the central U.S. on 25 September 1973. Although the first tornado of the long-track "Salina series"¹ was sighted at 2205 GMT, tornadoes were still occurring two hours later, when some of the features in Fig. 2 are valid. A southwest-northeast jet axis at 200 mb was located over northwest Kansas at 1200 GMT. Twelve hours later the 200 mb jet axis had a similar orientation but shifted slightly eastward. In addition, 200 mb jet streaks were located (but not shown) over southern New Mexico and eastern Nebraska at 0000 GMT. The latter isotach maximum was probably still associated

¹This refers to the longer-lived and considerably more devastating of two distinct tornado families, having a total damage path length of 254 km. The other family, which may be referred to as the "Bennington series," travelled for 116 km, but will not be dealt with in this study.

CHAPTER II

SYNOPTIC AND MESOSCALE STUDIES

Tornado at Salina, Kansas--25 September 1973

The purpose of this section is to acquaint the reader with the major synoptic scale features leading to the outbreak of severe weather on 25 September 1973 which included the well-documented Salina tornado. The following synoptic ingredients principally contributed to the formation of severe activity: a large-amplitude, mid-level long wave trough situated over the western U.S., and a rapidly intensifying surface low over the south-central U.S. The surface low appeared initially as a weak (1004 mb) frontal wave over eastern New Mexico at 1200 GMT on 25 September, but had elongated northeastward into a broad area of low pressure by 0000 GMT, 26 September (Fig. 1).

Figure 2 is a composite analysis of key synoptic-scale features at the surface and aloft over the central U.S. on 25 September 1973. Although the first tornado of the long-track "Salina series"¹ was sighted at 2205 GMT, tornadoes were still occurring two hours later, when some of the features in Fig. 2 are valid. A southwest-northeast jet axis at 200 mb was located over northwest Kansas at 1200 GMT. Twelve hours later the 200 mb jet axis had a similar orientation but shifted slightly eastward. In addition, 200 mb jet streaks were located (but not shown) over southern New Mexico and eastern Nebraska at 0000 GMT. The latter isotach maximum was probably still associated

¹This refers to the longer-lived and considerably more devastating of two distinct tornado families, having a total damage path length of 254 km. The other family, which may be referred to as the "Bennington series," travelled for 116 km, but will not be dealt with in this study.

with tornadic activity in northeastern Kansas. Lee and Galway (1956) have established a correlation between tornado occurrence and the proximity of the 200 mb jet. At the 850 mb level, a well-defined trough had moved from central Arizona at 1200 GMT into eastern New Mexico at 0000 GMT. Furthermore, the 850 mb trough over the eastern Texas Panhandle at 1200 GMT could be discerned over extreme southwestern Kansas by 0000 GMT, marking the region of strong low-level convergence and positive vorticity. A relatively weak, quasi-stationary cold front was analyzed at the surface across northern Oklahoma at 1200 GMT. By 0000 GMT, the frontal system had moved rapidly northward as a warm front and was situated over central Kansas to the east of a deepening low pressure center in southeastern Colorado. The rapid northward advancement of this warm front (arrow A) through southern Kansas is evident from the ATS-3 satellite photograph in Fig. 3(a). Severe thunderstorms were also occurring in southwestern Kansas by 0000 GMT.

Figure 4 indicates the advection of colder air into central Kansas at 500 mb by 0000 GMT. Only neutral or weak positive vorticity advection was indicated at 500 mb over and upstream of the Kansas region at both 1200 GMT, 25 September, and 0000 GMT, 26 September.

Upper-air temperature/dew point soundings for Dodge City and Topeka, Kansas for 0000 GMT are presented in Fig. 5. Neither sounding resembled the classical Type I tornado precedent sounding common to the Great Plains region (Fawbush et al., 1951; Fawbush and Miller, 1954; Beebe, 1955, 1958; Wills, 1969; Darkow and Fowler, 1971; Maddox, 1973), since tornadic storms were already in existence nearby (see Fig. 1). Nevertheless, both soundings indicated unstable airmass characteristics conducive to tornadic thunderstorm development. Vertical wind profiles at both these stations indicated the dominance of the major upper trough to the west, with rapid veering through the

lower moist layer, characteristic of the synoptic "Type B" pattern (Miller, 1972). Strong low-level winds above the surface frontal inversion in Fig. 5(a) suggest that the area along the warm front south of Topeka would be favored for tornadic thunderstorm development (Miller, 1972). The tornadic cell northeast of Salina at 0000 GMT (see Fig. 1) was located near the intersection of the axis of maximum surface winds and the warm front.

Surface moisture was abundant over all of Kansas, especially outside the thunderstorm areas, by 0000 GMT. Even north of the front (see Fig. 1), dew point temperatures were above the critical value (12.8°C) for severe storm occurrence (Fawbush *et al.*, 1951; Miller, 1972). Two hours prior to initial thunderstorm development, surface dew point temperatures were generally increasing and pressures were rapidly falling over southwestern Kansas. The first radar echoes rapidly developed 87 km west of Wichita (ICT) at 2013 GMT. These cells apparently formed along the surface front within a region of enhanced moisture convergence, apparently associated with the cloudiness band over central Kansas marked B in Fig. 3(a). By 2100 GMT, however, the cells had developed explosively into the cumulonimbus complexes labelled A and B in Fig. 3(b). Complex A began to produce the Salina family of tornadoes only one hour later.

Radar echoes for thunderstorms A and B in Fig. 3(b) are shown at 2230 GMT in Fig. 6. Most of the tornadoes that occurred during the five hour outbreak beginning at 2200 GMT were apparently associated with these two parent thunderstorms. The Bennington series tornadoes, which lasted from 2220 GMT 25 September, to 0040 GMT, 26 September, were probably spawned from cell B (Fig. 6) as it travelled northeastward. Cell A also moved northeastward, but its tornadoes (Salina series) followed a different, parallel track to the east of the Bennington series path.

The time history of the intense echo of cell A (Fig. 6) is shown in Fig. 7 for the entire tornadic portion of the storm's existence. Located near Little River, Kansas at 2155 GMT, the cell spawned its first funnel only ten minutes later. It then continued northeastward toward Salina (SLN) at 2242 GMT, affecting Salina briefly with large hail. Subsequently, a tornado passed to the southeast of Salina by only 1.6 km. However a mobile home court on Salina's southeastern outskirts was demolished at 2315 GMT. From Salina, the tornadic thunderstorm complex moved northeastward over New Cambria and Niles to Longford, which was struck by a tornado at 0010 GMT. A four-block wide vortex struck Clay Center at 0100 GMT. The last Kansas tornadoes associated with the immense double-cored storm complex after 0112 GMT struck Garfield Center at 0115, Greenleaf at 0152, and Hanover at 0215 GMT (U.S. Dept. of Commerce, NOAA, 1973; Weigel, 1974). Between 2242 and 0112 GMT, the echo was moving from 223° at 45 km hr^{-1} . From 0112 to 0202 GMT, however, the main core (on the right) was travelling from 185° at 50 km hr^{-1} (Fig. 7).

The total number of tornadoes in the Salina series is unknown since a subsequent aerial damage survey was never conducted. Although the apparent continuity of the damage swath has led several investigators to document the Salina event as one long-lived tornado (U.S. Dept. of Commerce, NOAA, 1973; Ostby and Pearson, 1974; Weigel, 1974), new photographic evidence confirms the existence of many discrete tornadoes within a series.

Only photographic data of the tornado's passage through the southeastern Salina area were analyzed here. However, additional photography of earlier tornadoes in the series was also obtained (see Appendix A) and will be analyzed in the future.

the major resort of Kailua-Kona. A few similar cases of waterspouts passing

The Kailua-Kona, Hawaii Tornadic-Waterspout--28 January 1971

Although daily weather in the Hawaiian Islands is dominated by north-easterly trade winds, synoptic-scale disturbances of mid-latitude origin occasionally interact with Hawaii's island topography to produce a variety of severe weather regimes. However, in contrast to Kansas, damaging tornado vortices are a relatively rare occurrence in Hawaii (Schroeder, 1975).

Hawaii has a seasonal severe weather maximum as does Kansas, although it occurs in a different time of the year. The maximum frequency of tornadoes is in January for Hawaii (Schroeder, 1975) and in May for Kansas (Pautz, 1969). When the frequency of frontal passages is combined with the frequency of surface lows for individual months, the maximum occurrence of surface disturbances is in January at all the Hawaiian Islands (Worthley, 1967).

Two major types of synoptic-scale cyclonic storms affect the Hawaiian Islands. These are the (1) Kona storms (Simpson, 1952), and (2) the familiar cyclones of the middle latitudes. In addition, a variety of subtropical cyclones and tropical disturbances influences Hawaii's surface weather from time to time (Ramage, 1962; Worthley, 1967).

The tornado outbreak associated with an occluded frontal system that passed through the Hawaiian Islands on 27-28 January 1971 was probably the most extensive and destructive in the recorded weather history of Hawaii (Schroeder, 1975). As many as six waterspouts may have struck Hawaii Island's (western) Kona coast. The largest and most extensively photographed vortex swept inland from Kailua Bay at 2040 GMT (1040 HST) as a large cyclonic waterspout. It appeared to maintain a continuous funnel from cloudbase to ground as it travelled 1.6 km inland through the downtown business and hotel district of the major resort of Kailua-Kona. A few similar cases of waterspouts passing

over islands or narrow land masses as tornadoes have been already documented (Varney, 1926; Macky, 1953; Dinwiddie, 1959; Golden, 1969).

Of 11 Hawaiian tornado reports from 1963-1974, four occurred with the 27-28 January 1971 frontal passage. In addition, 6 of these 11 tornadoes occurred during the calendar year 1971, which probably was a record year for reported Hawaiian funnels (Schroeder, 1975).

The Kailua-Kona vortex was spawned within a broad band of pre-frontal showers and thunderstorms associated with a deep occluding surface low (988 mb) centered to the north of the Hawaiian Islands (Fig. 8). Twenty-four hours after the analysis shown in Fig. 8, the surface low center had moved rapidly northeastward and had fully occluded.

Figure 9 shows the 500 mb geopotential field over the North Pacific Ocean at 0000 GMT, 29 January 1971, just 3 hours after the Kailua-Kona vortex had occurred. Notice that a major trough axis at that level was located to the west of the Hawaiian Islands, over the region of strong cold advection at the surface (see Fig. 8).

A proximity sounding for the Kailua-Kona vortex taken at nearby Hilo, Hawaii (see Fig. 8) approximately 3 hours after the event appears in Fig. 10. Except for the stable layer between 700 and 665 mb, the sounding was characteristic of the Type II tornado airmass common to the Gulf coast of the continental U.S. (Fawbush and Miller, 1954). In addition, the sounding possessed several other properties found to exist frequently on Hawaiian funnel-cloud days (Price and Sasaki, 1963). The deep superadiabatic layer from 630 to 590 mb may have resulted from frontal layer lifting of a preexistent trade inversion.

In summary, the sounding in Fig. 10 is dissimilar to the "typical" trade wind sounding (Price and Sasaki, 1963) associated with Hawaii's

prevailing weather regime. Large vertical shears in the winds aloft suggest that the Kailua-Kona tornadic-waterspout probably formed in association with a major mid-latitude baroclinic disturbance. Consequently the Kailua-Kona tornadic-waterspout possessed many environmental and vortex characteristics of a land tornado despite its birth over water.

During this study, air motion tracers were tracked across a carefully controlled, projected image (drawing screen) and velocities were computed using a derived scale ratio. The scale ratio was determined from a site survey of foreground landmarks appearing in the film.

A certain minimum of information is necessary before any tornado photogrammetry analysis can be performed (Morgan, 1974). Specifically, this includes: (a) the location of the camera during filming, (b) camera framing rate, and (c) damage track of the tornado. Nonessential but useful information consists of the vertical and horizontal angular orientation of the camera during filming, camera focal length, and location of the optical axis of the camera on the exposed film (principal point). Saunders (1963) has noted some applications of photogrammetric measurements in meteorological investigations. Morgan (1974) has outlined a specific digitized approach to tornado photographic data analysis. More recently Helle (1975) has developed a scaling procedure for ground-based single-camera photogrammetry for a tilted horizon. The analysis procedures employed in the present study will now be reviewed.

Scale Ratio Determination

(1) The exact location of the camera position can be located precisely on a scaled, up-to-date topographic map during a photographic site survey.

(2) From the camera site, azimuth and elevation angles of conspicuous permanent landmarks on the film horizon may be determined using a transit.

(3) After determining the true azimuths of any two such landmarks in the film, the true azimuth of the tornado center (T_c) from the camera site at any given instant in time may be linearly interpolated (or extrapolated) from the landmark azimuths using

CHAPTER III

PHOTOGRAPHIC DATA ANALYSIS

During this study, air motion tracers were tracked across a carefully controlled, projected image (drawing screen) and velocities were computed using a derived scale ratio. The scale ratio was determined from a site survey of the two landmarks on the image plane, and Δx is the total horizontal distance (cm) measured across the image between the two landmarks. Thus, ΔT is the angular separation between the known azimuth of the one landmark and the desired azimuth of T_c .

A certain minimum of information is necessary before any tornado photogrammetry analysis can be performed (Morgan, 1974). Specifically, this includes: (a) the location of the camera during filming, (b) camera framing rate, and (c) damage track of the tornado. Nonessential but useful information then found by measuring on the topographic map the straight-line distance between the camera site and the point where the line-of-sight through T_c computed in the previous step intersects the known tornado path. (4) The range (in cm) from the camera site to the tornado (R) is then found by measuring on the topographic map the straight-line distance between the camera site and the point where the line-of-sight through T_c computed in the previous step intersects the known tornado path. (5) The tornado range, R , determined above may then be transformed into actual distance R (in meters) using the scale of the 7.5-minute series topographic map. Morgan (1974) has outlined a specific digitized approach to tornado photogrammetric measurements in meteorological investigations. More recently Holle (1975) has developed a scaling procedure for ground-based single-camera photogrammetry for a tilted horizon. The analysis procedures employed in the present study will now be reviewed.

Scale Ratio Determination

(1) The exact location of the camera position can be located precisely on a scaled, up-to-date topographic map during a photographic site survey.

(2) From the camera site, azimuth and elevation angles of conspicuous permanent landmarks on the film horizon may be determined using a transit. It is recommended that the true horizon line be drawn first.

(3) After determining the true azimuths of any two such landmarks in the film, the true azimuth of the tornado center (T_c) from the camera site at any given instant in time may be linearly interpolated (or extrapolated) from the landmark azimuths using

$$\Delta T_c = \frac{\beta d_1}{\Delta x} \quad (3.1)$$

where β is the total angular separation (degrees) between the two landmarks, d_1 is the horizontal distance (cm) between the apparent tornado center and one of the two landmarks on the image plane, and Δx is the total horizontal² distance (cm) measured across the image between the two landmarks. Thus, ΔT_c is the angular separation between the known azimuth of the one landmark and the desired azimuth of T_c .

(4) The range (in cm) from the camera site to the tornado (R_s) is then found by measuring on the topographic map the straight-line distance between the camera site and the point where the line-of-sight through T_c computed in the previous step intersects the known tornado path.

(5) The tornado range, R_s , determined above may then be transformed into actual distance R (in meters) using the scale of the 7.5-minute series topographic map:

$$R = \frac{(1000 \text{ m})(R_s \text{ cm})}{(4.15 \text{ cm})} \quad (3.2)$$

(6) The true horizontal distance ΔX between the two landmarks at the range of the tornado's center T_c (see Appendix B) is given by

$$\Delta X = R \tan \beta \quad (3.3)$$

²It is recommended that the true horizon line be drawn first.

(7) The distance scale ratio for objects in the image plane (upon which the tracers were tracked) is then written as the ratio

$$\frac{\Delta x}{\Delta X} \quad (3.4)$$

True (Lens) Horizon Determination Procedure

(1) Locate two surveyed landmarks on the image plane and measure the apparent horizontal distance between them (cm) across this plane. Define this distance as λ .

(2) Compute β as the angular separation (i.e., azimuthal difference) between the two landmarks.

(3) Convert the elevation angles (in minutes) of the tops of each of these landmarks to degrees. Designate each of these angles as ζ_1 and ζ_2 .

(4) The apparent vertical distances z_1 and z_2 (cm) downward when $\zeta_1, \zeta_2 > 0$ (upward when $\zeta_1, \zeta_2 < 0$) from the tops of each of these landmarks to the level of the true horizon line can be computed as follows:

$$z_1 = \frac{\lambda \zeta_1}{\beta}, \quad z_2 = \frac{\lambda \zeta_2}{\beta} \quad (3.5)$$

In other words, the true horizon line will lie a distance z_1, z_2 (cm) in the apparent vertical below (when $\zeta_1, \zeta_2 > 0$) the tops of the two landmarks.

(5) Measure the distances z_1 and z_2 computed from Eq. (3.5) beneath the tops of each of these landmarks and mark these points on the image plane.

(6) A straight line drawn through the two points marked on the image plane in the previous step is the true horizon line (TH) or lens horizon (Saunders, 1963). The true horizon was the basis for all horizontal references used in the photogrammetric computations of this study.

Error Analysis

A word should be said about possible windspeed errors arising from both the photogrammetric data reduction and physical makeup of the tracers themselves. A subjective (nonstatistical) estimate of the largest possible net error in any one photogrammetric measurement here was placed at 15%.

However, particular windspeeds were probably more accurate since any questionable results were retracked carefully several times to minimize random errors.

The fifteen-percent error (in the worst case) comes from three possible apparent sources: tracking (7%), measurement (5%), and angular distortion (3%). Tracking errors might arise from positioning the tracer's point on the drawing screen. Measurement errors result from measuring with a finer accuracy than the measuring device allows; this also affects scale ratio accuracy. Causes of angular distortion are discussed in Appendix B.

In addition to the apparent windspeed errors cited above, physical factors involving debris form (solid or aggregate) may also result in lower velocities. The Salina tornado's debris tracers consisted of clumps or aggregates of debris, whereas those of the Kailua-Kona vortex were solid, flat objects. In the latter case, each solid mass of debris would be subject to both lift and drag forces resulting from pressure exerted on it by the flow. Even if such a wind-borne mass were tumbling, it would still be subject to an average drag force even though its average lift force would be zero (Beeth and Hobbs, 1975). The above considerations, however, would clearly not apply to a volume of space containing debris, such as the Salina aggregates. Accordingly, the Kailua-Kona vortex's windspeed estimates of the true air speed should be slightly poorer than the Salina tornado's. However, the smaller a solid debris particle is, the better it approximates, as well as responds to

changes in, the true air motion. This, together with a considerably greater tracking confidence for the Hawaiian case, should more than compensate for such physical error effects. In either case, this type of error would be difficult to quantify for any given debris tracer. Hence, its contribution to the net error estimate quoted above was neglected in this study.

CHAPTER IV Introduction

Hoecker (1960a) photogrammetrically derived the radial-vertical distribution of tangential and vertical wind components above the ground near the Dallas tornado from 16 mm newsfilm. He did this by compositing in one profile the derived speeds of three types of airflow tracers—solid debris particles, cloud and dust parcels, and funnel-surface perturbations—over a 19 minute time period. However, he found that such a temporal compositing led to an unrealistic distortion in the radial-vertical speed distribution patterns.

Hoecker did not specifically take into account that using different types of tracers together may in itself contribute to windspeed estimation error. There are three reasons for this. First, solid-debris particle velocities are more dependent on inherent tracer properties (e.g., mass, shape, exposed surface area, etc.) and less on meteorological parameter changes within the local environment traversed; the opposite is true for cloud tags. Second, dust parcels (aggregates) may rapidly change shape and optical density characteristics as a result of radial shears in tangential velocity, making accurate tracking of apparent centroids difficult. Lastly, since funnel-wall perturbations are probably the result of interior funnel (core) processes (Ward, 1972; Davies-Jones and Keasler, 1974), their apparent motion may be primarily unaffected by exterior airflow characteristics.³

³A distinction between perturbation wave speed and particle speed of the other two tracer types should always be made.

For the reasons outlined above, windspeed results are presented in this study separately for each tracer type. Temporal compositing is over

CHAPTER IV

11 seconds or less. Unless otherwise stated, the windspeeds quoted here are not velocities relative to the ground. They are the resultant sum

of vertical, tangential, radial, and translational velocity components. Excep-

Introduction

Hoecker (1960a) photogrammetrically derived the radial-vertical distribution of tangential and vertical wind components above the ground near the Dallas tornado from 16 mm newsfilm. He did this by compositing in one profile the derived speeds of three types of airflow tracers—solid debris particles, cloud and dust parcels, and funnel-surface perturbations—over a 19 minute time period. However, he found that such a temporal compositing led to an unrealistic distortion in the radial-vertical speed distribution patterns.

Hoecker did not specifically take into account that using different types of tracers together may in itself contribute to windspeed estimation error. There are three reasons for this. First, solid-debris particle velocities are more dependent on inherent tracer properties (e.g., mass, shape, exposed surface area, etc.) and less on meteorological parameter changes within the local environment traversed; the opposite is true for cloud tags. Second, dust parcels (aggregates) may rapidly change shape and optical density characteristics as a result of radial shears in tangential velocity, making accurate tracking of apparent centroids difficult. Lastly, since funnel-wall perturbations are probably the result of interior funnel (core) processes (Ward, 1972; Davies-Jones and Kessler, 1974), their apparent motion may be primarily unaffected by exterior airflow characteristics.³

³ A distinction between perturbation wave speed and particle speed of the other two tracer types should always be made.

Defined by Fujita (1960a).

For the reasons outlined above, windspeed results are presented in this study separately for each tracer type. Temporal compositing is over 11 seconds or less. Unless otherwise stated, the windspeeds quoted here are net velocities relative to the ground. Such windspeeds are the resultant sum of vertical, tangential, radial, and translational velocity components. Exceptions to this frame of reference will be noted when multivortex structures, funnel perturbations, and "wall cloud"⁴ motions are considered.

Salina Tornado

Cloud Tags

Meteorologists have always been interested in cloud tags (Hoecker, 1960a) around tornadoes primarily because they provide information about the circulation of the tornado. In the Salina tornado case, cloud tags were probably the most visible and trackable of the tracer types.

Figure 11 exemplifies the net velocity distribution for several large, persistent cloud tags which were rotating around the lower edge of the south side of the Salina tornado's wall cloud (see Fig. A-1, Appendix A). Motions were predominantly horizontal, and net cloud tag velocities were a relatively uniform $30\text{--}45\text{ ms}^{-1}$. The apparent movement of the tornado itself was 10 ms^{-1} towards the northeast, while the maximum net velocity measured (shown in Fig. 11) among the peripheral cloud tags was 49 ms^{-1} in the same general direction. Early in the Salina tornado's life history (Fig. 11), such peripheral cloud tags were located about 349 m above the true horizon (ATH).

Composited trajectories with net velocities plotted for cloud elements revolving around the periphery of the fully-developed wall cloud above the Salina tornado were also studied (Fig. 12). Some cloud tracers (A and B) were

⁴Defined by Fujita (1960a). Turbulent condensate matter formed midway

pendant from the far (southeast) side of the wall cloud base and the rest (C, D, and E) appeared as bright spots on the near edge of the wall cloud (see Fig. A-1, Appendix A). Translation of the wall cloud was removed from all net velocities in Fig. 12 so that they would better represent the tornado cyclone's rotational windspeed distribution. Figure 12 clearly reveals a general helical-upward flow in the wall cloud, with a maximum rising speed of 45 ms^{-1} indicated in tag E. Similar helical motion has also been observed in waterspouts (Golden, 1973, 1974a), dust devils (Sinclair, 1973) and even tornadoes (Golden and Purcell, 1975; Fujita, 1975). Horizontal cloud tag trajectory A in Fig. 12 contained $12\text{--}30 \text{ ms}^{-1}$ velocities. These were comparable to the $5\text{--}26 \text{ ms}^{-1}$ tangential speeds observed for cloud tags around the shrinking Dallas tornado (Hoecker, 1960a, b). In addition, vertical velocities of $5\text{--}45 \text{ ms}^{-1}$ out to a ringlike bulges which travelled upward along the funnel walls of the dissipating South Platte Valley tornadoes on 7 June 1953. $6\text{--}22 \text{ ms}^{-1}$ for tags out to 457 m from the shrinking Dallas tornado.

Vertical velocities of a rapidly ascending tag at cloudbase, above a short-lived, abortive funnel prior to formation of the primary Salina tornado, were near the apparent funnel edges where horizontal motions are given in Fig. 13. This cloud pendant showed an average upward acceleration of 17.5 ms^{-2} for the first 67 m of its trajectory, with a smaller acceleration thereafter. The average upward acceleration for its entire ascent was 9.5 ms^{-2} at $z = 337 \text{ m}$ AGL, which is nearly the magnitude of gravitational acceleration. Such acceleration would, of course, indicate that the turbulent cloudbase layer above the forming funnel was significantly nonhydrostatic. In fact, the vertical pressure gradient near this cloud tag would have been nearly proportional to twice gravitational acceleration.

Rapidly-ascending cloud tags were also associated with the second (cyclonically) away from the left funnel edge. Finally, it should be noted that funnel translation was eliminated from the perturbation's net motion in primary Salina tornado (Fig. 14). Turbulent condensate matter formed midway

below the cloudbase ahead of the vortex and ascended with trajectories parallel to the slope of the left funnel edge. Net velocities for cloud tracers in Fig. 14 were generally $20\text{--}40\text{ ms}^{-1}$, with no obvious cloud tag accelerations.

However, the vertical velocity component of the tag labelled A increased from 13 ms^{-1} to 28 ms^{-1} within a 39 m rise, reflecting an average vertical acceleration of 0.8 g . This result also tends to raise serious doubts about the hydrostatic approximation around the upper funnel region of some of the Salina-series tornadoes.

Funnel Perturbations

Moving protuberances on a tornado funnel were first documented from visual observation by Reber (1954). The funnel perturbations consisted of ringlike bulges which travelled upward along the funnel walls of the dissipating South Platte Valley tornadoes on 7 June 1953.

The Salina tornado exhibited similar funnel perturbations during portions of both its organizing and shrinking stages. Unfortunately, most trackable perturbations were near the apparent funnel edges where horizontal motions were unresolvable in the image plane. This was also the case with perturbation A (Fig. 15), which was located near the left edge of the funnel during partial vortex breakdown.

Although perturbation A's apparent tangential components were significantly underestimated, its vertical components were among the strongest observed in perturbations: $w = 34\text{ ms}^{-1}$ at $z = 212\text{ m}$ and $r = 37\text{ m}$. Net velocities in the trajectory A varied essentially from $10\text{--}35\text{ ms}^{-1}$. As expected, the trajectory showed a sequential increase in tangential velocity as the perturbation moved (cyclonically) away from the left funnel edge. Finally, it should be noted that funnel translation was eliminated from the perturbation's net motion in

Fig. 15. Trajectories B and C denote debris elements, and will be treated later.

Dust and Debris Aggregates

Unlike higher-level tracers, visual resolution for windspeed tracers within the lower Salina vortex boundary was reduced as a result of the large apparent camera-to-tornado range and intervening ground objects. The tornado's closest approach to photographer A (see Appendix A) was 3.7 km. Furthermore, foreground trees 9 m tall appearing in his film obscured the lowest 50-75 m of the debris cloud.

Nevertheless, above this obscured layer photographic evidence indicated strong radial asymmetry in the debris distribution below 150 m when the (south) side of the funnel, however, the aggregate at $r = 129$ m and $r = 102$ m funnel was in contact with the ground. This radial asymmetry appeared to be indicated the greatest upward speed measured: 31 ms^{-1} . Hence, the highest greatest in the image planes of photographers A and B, who were on opposite sides of the damage track (see Appendix A). Photographer A observed the funnel consistently offset to the right of the debris cloud's geometrical center. Locally dense concentrations of debris (aggregates) were observed to travel quasi-horizontally outward in advance of the tornado; at the same time, strong upward motion was observed in the trailing dust cloud which was confined to a smaller radius (see Fig. 16).

One notable exception to the asymmetric debris distribution of the mature stage (Fig. 16) was observed later when the tornado funnel lifted momentarily. With the vortex tip suspended aloft, dust and debris surrounded the lower half of the funnel as an axially symmetric sheath which appeared to disperse upward and outward. Hoecker (1960b) noted a similar behavior in the dust cloud of the Dallas tornado. Kuo (1971) has attributed this to diminished frictional effects on the boundary layer rotation, which would restore cyclostrophic balance.

Stronger windspeeds due to the additive effect of translation in the right-front quadrant are probably responsible for the debris distribution asymmetry observed in Fig. 16 (Brown, 1976). Several other recent investigations have also found observational evidence that horizontal and vertical flow asymmetries may be relatively common in both waterspouts (Golden, 1973, 1974a, b) and tornadoes (Golden and Purcell, 1975; Golden and Davies-Jones, 1975; Blechman, 1975).

Net velocities for all aggregates left of the tornado in Fig. 16 varied somewhat ($21\text{--}65\text{ ms}^{-1}$), with typical speeds ranging from 38 to 47 ms^{-1} near the left funnel edge. Higher windspeeds appeared to be concentrated between $z = 105\text{--}125\text{ m}$ and were apparently northeast of the funnel. On the right (south) side of the funnel, however, the aggregate at $\bar{z} = 129\text{ m}$ and $r = 102\text{ m}$ indicated the greatest upward speed measured: 31 ms^{-1} . Hence, the highest velocities in Fig. 16 were evidently associated with dense debris aggregate trajectories at comparatively large radii from the funnel. These, of course, may not have necessarily been the highest speeds existing around the vortex; there may have been higher speeds where tracers were not tracked or were absent. Lastly, it was not possible to compute apparent turning radii for the aggregates tracked in Fig. 16 because each aggregate appeared to lose its identity while spiralling apparently outward and upward from the tornado center.

Reexamination of Fig. 15 reveals that debris aggregate trajectory C contained the peak net velocity (relative to the ground) measured for any of the tracked Salina tracers: 69 ms^{-1} at $z = 67\text{ m}$ and $r = 114\text{ m}$. Similar to the 65 ms^{-1} windspeed maximum in Fig. 16, this maximum appeared to be predominantly horizontal ($v = 68\text{ ms}^{-1}$) and occur at a large apparent radius in the right-front quadrant of the debris cloud, northeast of the funnel. Also apparently

debris objects photographed at close range, while those available in the Salina

northeast of the funnel but having a smaller apparent turning radius ($r = 58$ m) was aggregate B. In contrast to other right-front quadrant aggregates tracked (see Fig. 16), this aggregate possessed an unusually strong rising motion component ($w = 24 \text{ ms}^{-1}$ at $z = 94$ m). Indspeed (64 ms^{-1}) was found at a larger

radial Debris aggregate trajectories near the surface during the early decay stage of the Salina tornado are given in Fig. 17. Denser dust vortices (eddies) were also embedded within the debris cloud during this stage. Trajectories for such vortices were not analyzed in Fig. 17, but rather will appear in later figures. Aggregates A, B, and C, however, are possibly exceptions:

Size and Shape Characteristics
whether or not they were associated with dust vortices at lower heights could not be determined.

A scaled time-sequence analysis of certain size and height features characteristic of the Salina tornado and its parent cloudbase structure is shown in Fig. 18. The photograph sequence (Weigel, 1974) is especially appropriate for photogrammetric study because the entire life cycle of the tornado was captured on film. More importantly, the sequence may indicate that family-vortex began to dissipate could not be confirmed. In fact, there is some indication in the limited tracer samples available that windspeeds may have been somewhat higher during tornado decay (Fig. 17) than during maturity (Fig. 16). Trajectories A, B, and C in Fig. 17, having some net windspeeds under 20 ms^{-1} , were located on the northwest side of the vortex, where rotational speeds were diminished by the northeastward translation. Nonetheless, Oklahoma tornado (Moller et al., 1974; Brown, 1976).

secondary dust vortices could have possibly also accounted for these apparent slower speeds, as well as for the stronger ones in those trajectories.

Brilliant subcloud contrast behind the tornado in all of the drawings of Fig. 18 suggests that the tornado's parent cloud system was of limited horizontal extent. The apparent diameter of the Wichita radar mesocyclone signature during its mature stage (Fig. 16) were probably of comparable magnitude to measured 16 km at 2300 GMT, corresponding in time with Fig. 18.

Hoecker's (1960a) for the Dallas tornado. In making such a comparison, however, we must note that Hoecker's tracers consisted principally of large, solid debris objects photographed at close range, while those available in the Salina

film were exclusively aggregates of small debris photographed at a distance. The Salina funnel, though, was considerably broader; its vertically-averaged mean diameter was 176 m compared with 76 m for the Dallas tornado (Hoecker, 1960b). The highest horizontal windspeed (64 ms^{-1}) was found at a larger radial distance outward from the center: $r = 309 \text{ m}$ in Fig. 16 versus $r = 39 \text{ m}$ for Dallas. Furthermore, this horizontal velocity maximum occurred at a higher elevation above the ground: 96 m for Salina versus 69 m for Dallas (Hoecker, 1960a).

Size and Shape Characteristics

A scaled time-sequence analysis of certain size and height features characteristic of the Salina tornado and its parent cloudbase structure is shown in Fig. 18. The photograph sequence (Weigel, 1974) is especially appropriate for photogrammetric study because the entire life cycle of the tornado was captured on film. More importantly, the sequence may indicate that family-outbreak tornadoes evolve through organized life cycle stages (Fujita, 1960a, b; Garrett and Rockney, 1962) similar to those in waterspouts (Golden, 1973). Four distinct stages were recognized for the Salina tornado. These are the (a) organizing stage, (b) mature stage, (c) shrinking stage, and (d) decay stage. Similar life cycle stages have also been identified for the Union City, Oklahoma tornado (Moller *et al.*, 1974; Brown, 1976).

Brilliant subcloud contrast behind the tornado in all of the drawings of Fig. 18 suggests that the tornado's parent cloud system was of limited horizontal extent. The apparent diameter of the Wichita radar mesocyclone signature measured 16 km at 2300 GMT, corresponding in time with Fig. 18.

The wall cloud parent to the Salina tornado appeared to evolve concomitantly with the funnel, becoming most pronounced as the funnel established windspeed was also measured during the shrinking stage.)

full contact with the ground. Moreover, the funnel descended from the north-west portion of the wall cloud and remained pendant from the wall cloud's north side. Maximum horizontal and vertical dimensions of the wall cloud were 2.2 km across and 0.6 km below surrounding cloudbase, at 2307 and 2305 GMT, respectively (see Fig. 18). Mean cloudbase height of the parent storm, before and after the wall cloud was present (2304-2308 GMT), was computed also from Fig. 18 to be 345 m AGL. This was consistent with a cumulonimbus base height of 343 m computed analytically using nearby Salina surface observations one hour before the tornado (Haurwitz, 1941; Moses et al., 1968). No part of the cloudbase descended significantly below this mean height, not even the wall cloud. At 2305 GMT, when the wall cloud was most pronounced, the surrounding cloudbase height had more than doubled its original mean height; however, the wall cloud base height remained nearly constant throughout the wall cloud's life history.

Diameters of the lower funnel in Fig. 18 appeared to expand from 2304 to 2307 GMT, then shrink slowly thereafter until the tornado's demise at approximately 2310. The progressive tapering of the condensation funnel and 3-minute decay period are characteristics of this tornado which are also typical of Florida Keys' waterspouts in their decay stage (Golden, 1974b).

During general expansion of the lower funnel, several diameter fluctuations were documented. These included a shrinkage at 2305 GMT, followed by a 10 second period of constant 90 m diameter, after which the diameter rapidly expanded until 2307 GMT. Concurrently, the upper funnel diameter was increasing more slowly with time. Moreover, the maximum funnel diameter (318 m) did not occur during the mature stage at 2307 GMT, but rather during the shrinking stage approximately a half-minute later. (Recall from Fig. 15 that the largest windspeed was also measured during the shrinking stage.)

Several changes in height of the apparent funnel intersection with cloudbase were also evident during the photographic sequence (Fig. 18). Heights at which the funnel appeared to vanish into cloudbase varied from 414 to 752 m. This rather large apparent variation has three possible explanations. First, small bits of cloud material could have intermittently obscured the extreme upper funnel portions. Second, since the photographs from which Fig. 18 was drawn were black-and-white, poor contrasts within the cloud (immediately above cloudbase) could have made the upper funnel outline and its precise cloudbase intersection indistinct. Finally, the wall cloud base appeared at times to be sloping or concave, resembling an inverted bowl (see Fig. 12).

Salina Tornado Decay Stage

Vortex stretching: orientation and length. By 2309 GMT, the Salina tornado was into its final decay stage. Photographer A (D. Patterson—see Appendix A) was viewing the tornado at that time (Fig. 19), as was photographer D (K. Houchin). Since then photography was simultaneous from known independent vantage points, the photographic site geometry was reproduced to scale on a horizontal plane (Fig. 20). Together, Figs. 19 and 20 provide a simple geometrical solution of the spatial orientation and total length of the tornado's projection along the ground.

Figure 19 (image plane) corresponds to the plane A'A'' in Fig. 20, as seen by Patterson at point A. In Fig. 20, the computed length of segment $T_G T_A$ is 783 m; the corrected horizontal distance, after a 6.5% angular distortion error (see Appendix B) is removed, is 732 m (Fig. 19). Upper and lower ends of the funnel, points T_A and T_G , as well as principal point P_A , are represented in Fig. 20 exactly as they are shown with respect to one another in Fig. 19. Similar points T_D , T_G , and P_D were identified in plane D'D'', as seen by Houchin at point D (see Fig. 20).

At any given instant, the upper and lower ends of the funnel were occupying unique points in space. Since Patterson's and Houchin's photographs were simultaneous, intersections of their line-of-sight azimuths through the apparent funnel intersections with the ground and with cloudbase, respectively, located points T_G and T_U uniquely in space. Golden (1973, 1974b) has

From Fig. 20, it is evident that the measured horizontal length of the tornado in Fig. 19, $\overline{T_G T_A}$, is an absolute minimum, or lower bound approximation for the true projected length along the ground, $\overline{T_G T_U}$. In fact, spatial tilt of a funnel out of the photographer's image plane would always increase the total length of the funnel relative to its apparent length, and, hence, its projection along the horizontal ground. This is true for funnel tilt either toward or away from the photographer.

There are four possible alternatives for the orientation of segment $T_G T_U$ (see Fig. 20): (a) north of plane $A'A''$; (b) on either plane $A'A''$ or $D'D''$; (c) between planes $A'A''$ and $D'D''$; (d) south of plane $D'D''$. For this case, condition (c) was satisfied. Angle $T_A T_G T_U$, between image plane $A'A''$ (Fig. 19) and the plane containing the funnel, was $12^\circ 46'$.

Finally, the length of segment $T_G T_U$ was computed directly from Fig. 20 using the Law of Sines, with $\Delta T_G T_A T_U$: $\overline{T_G T_U} = 825$ m. When the 6.5% angular distortion error was removed, the actual length of $\overline{T_G T_U}$ was 771 m. This was only 39 m longer than the original estimate computed directly from Fig. 19

alone. Table 1 illustrates how the funnel axis tip at the numbered frames

To summarize: if a photographed funnel tilts away from the image plane, it is longer than it appears in that plane. In this case, since the funnel's orientation with respect to the image plane deviated by only a small angle, its apparent horizontal length was not too severely underestimated. However, in

Hence, if looping about V_c were occurring, the following

cases where the plane containing the funnel intersects the image plane acutely, serious underestimates of funnel dimensions would be obtained. In any event, stereophotography is necessary to resolve the three-dimensional orientation of a tornado.

Apparent looping of the vortex center. Golden (1973, 1974b) has explored a peculiar phenomenon which, on rare occasions, has been observed within the spray vortex of decaying waterspouts. As the lower spray vortex weakened, the central clear eye began to slowly follow a looping path over the sea surface. This resulted in a cycloidal trailing sea-surface wake.

145 The Salina tornado movie exhibited the possibility of such apparent vortex translatory motions for a short time during the tornado's life cycle. To determine if looping motions did occur in the Salina tornado, linear translation was eliminated from the observed motion of both the debris vortex and the funnel axis aloft.

Figure 21 shows successively-tracked, apparent funnel axis positions relative to the ground as the Salina funnel was beginning to stretch early in its decay stage. Funnel movement was from right-to-left in Fig. 21. Numbered vertical marks at treetop level locate the apparent geometrical center of the visible debris cloud for frames corresponding to the funnel axes drawn above. It was, therefore, possible to compare the motion of the funnel axis with that of the associated debris cloud centroid.

Table 1 illustrates how the funnel axis tip at the numbered frames (Fig. 21) fluctuated around the fixed, unique geometrical center of the debris cloud vortex, V_c .

Only a sinusoidal variation of x' with time (increasing frame numbers) about $x' = 0$ would indicate a looping funnel axis with respect to the debris cloud center. Hence, if looping about V_c were occurring, the following

TANGENTIAL TABLE 1 Estimation

APPARENT LOOPING EFFECT IN THE
DECAYING SALINA TORNADO

Frame Number	Radial Distance x' of Axis Tip From V_c (m) ⁵	Comments
13	-34.4	Decreasing Negative
91	-23.9	
94	-20.0	
110	+ 2.1	Increasing Positive
145	-16.8	Increasing Negative
154	- 8.1	Decreasing Negative
171	-11.2	Increasing Negative
178	-37.9	
197	-42.1	

progression of change in x' only would be possible: decreasing negative, increasing positive, decreasing positive, increasing negative, decreasing negative again, and so forth. Notice that x' in Table 1 shows harmonic variation with time, although the x' values are biased toward the negative (trailing) side of the debris cloud. Nevertheless, the progression of change of x' in Table 1 is different from that above. Therefore, it was concluded that looping behavior was probably absent during the demise of the Salina tornado.

⁵ $x' > 0$ in the direction the vortex is translating. V_c is at $x' = 0$.

Figure 22(a) shows an eddy which had entered the right-rear quadrant of the debris cloud. As the eddy approached the apparent, fixed vortex center

Tangential Velocity Estimation

When a tracer is located near the funnel's apparent maximum radius in the movie image plane, little tangential motion will be perceived relative to the tornado itself.

By making certain simplifying assumptions concerning vortex symmetry and tilt, however, it is possible to compute rotational speeds for a tracer regardless of its location in orbit, provided that it is visible. A complete mathematical derivation to accomplish this may be found in Appendix C. For convenience, this derivation has also been programmed in FORTRAN IV.

During the decay stage of the Salina tornado (Fig. 17), dust and debris concentrations of high optical density were embedded within the debris cloud. These appeared in the form of small-scale vortices, or eddies, which rotated cyclonically within the debris cloud at a relatively constant radius. As many as six of these eddies were observed simultaneously. These eddies are possibly the visual manifestation of a type of Kelvin-Helmholtz flow instability, as noted by Barcilon and Drazin (1971) for a series of dust devils.

High contrast allowed eddies traversing the far side of the debris cloud to be differentiated from those on the near side relative to photographer A (see Appendix A). Several of these tracked eddies apparently rotated completely around the near and far sides of the debris cloud, crossing its apparent geometrical center on the image plane. For these eddies, tangential velocities were computed in the usual manner (directly), using the tracking and scaling procedures already outlined. Others remained near the outer edges of the debris cloud. Their velocities were computed using Program ARCLIN in Appendix C.

Figure 22(a) shows an eddy which had entered the right-rear quadrant of the debris cloud. As the eddy approached the apparent, fixed vortex center

(V_c), its tangential velocity steadily increased from 21 to 43 ms^{-1} (see Fig. 24(b)). Direct calculation using another eddy (not shown) on the far (right) side as it passed the apparent center of the debris cloud yielded a consistent 42 ms^{-1} tangential velocity estimate.

Summary of Results

Additional eddies within the debris vortex are shown in Fig. 22(b). Significant photogrammetric results pertaining to the Salina tornado and its parent storm are summarized below in Table 2. Program ARCLEN to be 29 ms^{-1} (see Fig. 24(b)); this compares with 26 ms^{-1} obtained for the same eddy as it crossed the z-axis between frames 163 and 182. Also in relatively good agreement with direct computations is eddy C, between frames 121 and 130. Figure 24(a) indicated $v = 48 \text{ ms}^{-1}$ for eddy C there. Two other eddies on the near side which crossed the z-axis showed tangential speeds of 53 and 64 ms^{-1} , respectively. Only eddy B, which was apparently approaching its turning radius from the right-front quadrant, showed a poor (i.e., conservative) tangential velocity estimate. Its tangential speed was only approximately one-fourth that indicated by eddy A, which was also on the far (right) side (see Fig. 24(b)).

Prior to formation of the primary Salina tornado, a multiple-vortex tornado was observed by photographer B (Appendix A). At least two secondary funnels rotated cyclonically about an apparent common center (Fig. 23(a)). Program ARCLEN was used to compute the rotational speed of each of these secondary funnels with respect to this fixed center. Figure 24 shows that the tangential speed for both vortices about the center was approximately 10 ms^{-1} . Their actual tangential speed values were in excellent agreement.

Finally, another secondary funnel was tracked while rotating around the apparent left-forward quadrant of this multivortex tornado (Fig. 23(b)). Similar to the debris eddy in Fig. 22(a), this secondary funnel's rotational speed appeared to increase from 14 ms^{-1} between frames 44-79 to an apparent

maximum of 23 ms^{-1} between frames 111-123 as it approached the z-axis (see Fig. 24(b)). In agreement with this, a tangential speed of $v = 21 \text{ ms}^{-1}$ was obtained by direct calculation for this funnel between frames 79-111.

Summary of Results

Significant photogrammetric results pertaining to the Salina tornado and its parent storm are summarized below in Table 2.

TABLE 2
SUMMARY OF PHOTOGRAMMETRIC RESULTS FOR THE SALINA TORNADO SYSTEM

Feature	Quantity	Magnitude ⁶	Apparent Height (AGL)	Radius
Cloud Tags (wall cloud)	v_n (min-max)	6-49 ms^{-1}		
	v_n (typical)	30-45 ms^{-1}		
	v_n (max)	49 ms^{-1}	349 m	204 m
	w (max)	45 ms^{-1}	711 m	366 m
	a_w	9.5 ms^{-2}	337 m	
Funnel Wall Perturbations	v_n (min-max)	9-38 ms^{-1}		
	w (max)	34 ms^{-1}	212 m	37 m
Dust and Debris Aggregates	v_n (min-max)	12-69 ms^{-1}		
	v_n (typical)	20-45 ms^{-1}		
	v_n (max)	69 ms^{-1}	67 m	114 m
	w (max)	31 ms^{-1}	129 m	102 m
Tornado Axis (T_c)	v_{tr} (min)	8 ms^{-1}	(organizing stage)	
	v_{tr} (max)	20 ms^{-1}		
	v_{tr} (mean)	14 ms^{-1}	(organizing through shrinking stages)	
Tornado	Apparent Diameter (max)	318 m	454 m	
Tornado	Apparent Diameter (max)	153 m	86 m	
Wall Cloud	Apparent Diameter (mean)	1975 m		
Wall Cloud			362 m (mean = 1/2 parent cloud base)	
Parent Cb			345 m (before and after wall cloud)	
Parent Cb			728 m (during wall cloud)	
Radar "Hook Echo" Width		16,000 m		
Tornado Path Length		5,500 m		
Tornado Duration		300 s		
Tornado Cyclone Speed		5.36 ms^{-1}		

⁶ All windspeeds are relative to the ground.

Kailua-Kona Tornadic-Waterspout

A motion picture film of an unusual tornadic-waterspout was taken as it was forming over Kailua Bay and moving ashore (see Appendix A). It had a sea-surface spray vortex diameter of 99 m and moved northward toward shore at an apparent 13 ms^{-1} . Photogrammetric analyses were performed on this vortex primarily as it made landfall and immediately thereafter. Vortex landfall was close enough to the observer (1 km) such that individual debris elements were visible and their motions readily trackable, unlike the debris aggregates in the Salina tornado. In addition, the density of visible, trackable debris increased severalfold when the spray vortex first moved over land.

Other portions of the film that could have been used for detailed photogrammetric analysis were limited for study. Two reasons for this were: (1) lack of surveyable landmarks for scaling when the vortex was over water, and (2) lack of vortex path information over water for range determination. During and after vortex landfall, however, visible landmarks provided reference for its position. Finally, the necessary vertical and horizontal angular displacements of visible landmarks in the film were obtained during a subsequent field survey.

Figure 25(a) shows the first debris trajectories, of mostly large, flat objects, which rapidly became airborne shortly after the vortex made landfall. The debris elements appeared to travel inland in the right-rear vortex quadrant, while rising upward in smooth curving trajectories. Maximum net and vertical velocities of 56 and 25 ms^{-1} , respectively, are indicated at the point labelled in trajectory A (Fig. 25(a)). Longer trajectories usually were associated with smaller debris particles; the net velocities generally ranged from 25 - 55 ms^{-1} . Larger debris objects did not attain the great heights or velocities that their smaller counterparts did. The greatest apparent height measured for a debris

particle in Fig. 25(a) was 62 m above sea level (MSL) in trajectory B. Apparent radii for debris in Fig. 25(a) were incalculable because the center of the vortex was obscured by sea spray and the visible vortex edges were indistinct shortly after landfall. This was also the case for Fig. 25(b)-(d).

Figure 25(b)-(d) shows solid debris particle trajectories with net velocities as indicated; the drawings appear in chronological order as the vortex moved inland (b) and out of view (d) of the eyewitness camera (see Fig. A-2, Appendix A). Although trajectories correspond only to the most visible and trackable of several hundred air tracers, a sufficient number were tracked to represent adequately the apparent velocity distribution.

Isotachs, such as in Hoecker (1960a), however, were not drawn in Fig. 25 because they would not be physically meaningful. Contouring these velocity fields would have served little purpose because the data points indicate only image-plane components of the velocity rather than the true air velocity itself. As a result, such windspeed distributions would be underestimated everywhere, except at the azimuth of the apparent vortex center. As previously stated, however, exact vortex center azimuths were indeterminable.

Most of the debris particles in Fig. 25 appear to be located in the rear or right-rear quadrant of the vortex circulation. Golden (1973, 1974a) found that the maximum radial inflow occurred within the right-rear quadrant relative to the moving vortex for cyclonic waterspouts. Furthermore, Golden and Purcell (1975) have recently substantiated this same asymmetric inflow character for a tornado. Therefore, it appears that most of the left-to-right moving trajectories in Fig. 25 were associated with a long, trailing band of high-speed, accelerating inflow, which spiraled into the vortex center near the surface. Elliptical trajectories at the left in Fig. 25(c) indicate the presence of cyclonic airflow as particles accelerated into the primary inflow from

the vortex rear. Finally, debris particles in Fig. 25(d) were apparently falling or sinking out of the circulation, slowly accelerating to terminal velocity outside the helical-upward flow in the main vortex.

CONCLUSIONS AND SUGGESTIONS

Summary of Results

Table 3, below, summarizes important windspeed, movement, and size characteristics of the Kailua-Kona tornadic-waterspout.

The primary goal of this study was to demonstrate how powerful photogrammetry can be as a tool for improving our understanding of atmospheric vortex flow structure.

TABLE 3 SUMMARY OF PHOTOGRAMMETRIC RESULTS FOR THE KAILUA-KONA TORNADIC-WATERSPOUT

Quantity	Magnitude	Apparent Height (MSL)
v_n (min-max)	1-56 ms^{-1}	
v_n (typical)	25-55 ms^{-1}	
v_n (max)	56 ms^{-1}	38 m
w (max)	25 ms^{-1}	38 m
v_{tr} (over water)	13 ms^{-1}	
r (over water)	49.5 m	sea level
t (longest debris tracking sequence)	4.6 s	

shows that the largest net velocities (which appeared to be predominantly horizontal) were located around the lower levels of the Salina vortex (below 100 m). Similar results for other cases were obtained by Hoecker (1960a), Golden and Purcell (1975), and Fujita (1975). A notable exception was the recent results of Blechman (1975), whose tangential velocities were found to be strongest at cloudbase. The highest net velocities associated with the Salina tornado were found at relatively large radii (i.e., $r = 100-300$ m), on the right-front quadrant of its circulation. In addition, strong radial asymmetry was present in

the debris distribution below $z = 150$ m throughout the tornado's mature stage.

On the other hand, vertical velocities were slightly weaker near the

CHAPTER V

ground than at cloudbase when the Salina funnel was of maximum overall size and fully extended. The

CONCLUDING REMARKS AND SUGGESTIONS

an apparent 45°- sloping helical structure in the left-rear portion of the

FOR FURTHER RESEARCH

wall cloud. However, maximum upward velocities were about comparable at middle

The primary goal of this study was to demonstrate how powerful photogrammetry can be as a tool for improving our understanding of atmospheric vortex flow structure. Reliable photogrammetric windspeed estimates from natural tornadoes are greatly improving the geometric and dynamic similitude between laboratory and atmospheric vortices (Hsu and Fattahi, 1975). For one thing, vortex modellers are becoming increasingly aware of what reasonable tornado windspeeds should be. More importantly, recently-documented flow asymmetries (Golden and Purcell, 1975) will provide an exciting, though formidable, challenge for vortex modellers in the future.

The Salina tornado provided additional insight into several complex questions which have been also raised in other recent tornado investigations (Blechman, 1975; Fujita, 1975; Golden and Purcell, 1975). For example, where are the maximum velocities usually found in a tornado? Examination of Table 2 shows that the largest net velocities (which appeared to be predominantly horizontal) were located around the lower levels of the Salina vortex (below 100 m). Similar results for other cases were obtained by Hoecker (1960a), Golden and Purcell (1975), and Fujita (1975). A notable exception was the recent results of Blechman (1975), whose tangential velocities were found to be strongest at cloudbase. The highest net velocities associated with the Salina tornado were found at relatively large radii (i.e., $r = 100-300$ m), on the right-front quadrant of its circulation. In addition, strong radial asymmetry was present in

the debris distribution below $z = 150$ m throughout the tornado's mature stage. On the other hand, vertical velocities were slightly weaker near the ground than at cloudbase when the Salina funnel was of maximum overall size and fully extended. The greatest upward velocities measured were associated with an apparent 45° -sloping helical trajectory in the left-rear portion of the wall cloud. However, maximum upward velocities were about comparable at middle and lower levels near the vortex. This was evidenced by little change between the vertical velocity maxima for funnel perturbations and for debris aggregates (Table 2). On the whole, these results indicate that the flow tended to become more helical with increasing height above the ground, while near the ground radial components apparently were large.

Since photographic tracers were confined to the spray vortex region of the Kailua-Kona tornadic-waterspout, the distribution of vertical and net velocities representative of the funnel as a whole is not known. Nevertheless, the largest vertical and net velocities in the tornadic-waterspout's debris tracers were comparable to, but slightly less than, those of the Salina tornado's aggregates. This is in some agreement with observations that the largest waterspouts are probably as strong as the weakest tornadoes (Davies-Jones and Kessler, 1974).

Recent improved techniques in photogrammetric research should be applied to future tornado cases on an even broader scale. The value of photogrammetry as applied to the tornado problem is already receiving special attention. Abbey (1975) has recently emphasized that techniques for measuring tornado/waterspout vortex characteristics have progressed to the extent that reliable windspeeds can for the first time be obtained. Along with the small-scale remote sensing capabilities of Doppler radar and Doppler lidar, advancements in photogrammetry have helped bring this measurement confidence about. Most

importantly, as a direct result of several recent photogrammetric and engineering investigations (Mehta et al., 1971; Golden and Davies-Jones, 1975; Fujita, 1975; Agee et al., 1975; Blechman, 1975; Golden and Purcell, 1975), meteorologists have generally accepted that the maximum windspeeds likely in tornadoes are probably considerably less than was once believed (up to 225 ms^{-1}).

A safe estimate on tornadic windspeed maxima near the ground may be put at 110 ms^{-1} ; reliable windspeed estimates in excess of 98 ms^{-1} (Fujita, 1975) have not been measured to this date.

In the future, high-resolution terrestrial stereophotography taken by trained storm chase teams in the field would allow considerably improved photogrammetric windspeed estimations. Moreover, future tornado photogrammetry studies should utilize state-of-the-art digitized tracking and scaling devices which are both much faster and more accurate than manual analysis techniques (Morgan, 1974).

Finally, comparing photogrammetric wind velocities around developing wall clouds to corresponding dual-Doppler velocities aloft presents one of the most exciting possibilities for the future (Brown, 1976; Zrnic et al., 1976).

It is only by combining such techniques that progress toward a better understanding of tornadoes can ever be attained.

Church, C. R., 1975: A photogrammetric study of the Parker, Indiana tornado. Proceedings of the SESAME Opening Meeting, D. K. Lilly, Ed., Boulder, Colorado, Environmental Research Laboratories, NOAA, 422-427.

Crone, D. R., 1968: Elementary Photogrammetry. New York, Frederick Ungar Publishing Co., 197 pp.

Darkow, G. L. and M. G. Fowler, 1971: Tornado proximity sounding wind analysis. Preprints, Seventh Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 148-151.

Davies-Jones, R. P. and E. Kessler, 1974: Tornadoes. Weather and Climate Modification, Chapter 16, W. N. Hess, Ed., New York, John Wiley and Sons, Inc., 842 pp.

Dergarabadian, P. and F. Fendell, 1970: Estimation of maximum wind speeds in tornadoes. Tellus, 22, 511-515.

_____, and F. Fendell, 1971: **BIBLIOGRAPHY** for rapid estimation of maximum tangential wind speed in tornadoes. Mon. Wea. Rev., 99, 143-145.

- Abbey, R. F., 1975: Establishment of maximum regional tornado criteria for nuclear power plants. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 368-375. 1-3.
- Agee, E., C. Church, C. Morris, and J. Snow, 1975: Some synoptic aspects and dynamic features of vortices associated with the tornado outbreak of 3 April 1974. Mon. Wea. Rev., 103, 318-333.
- Fawbush, E. J., R. C. Miller, and L. G. Starvick, 1951: An empirical method Air Weather Service, 1969: Use of the skew T, log p diagram in analysis and forecasting. AWS Manual 105-124, Dept. of the Air Force, Headquarters, AWS (MAC), Scott Air Force Base, Illinois, 138 pp.
- _____, and R. C. Miller, 1954: The types of air masses in which North Barclon, A. I., and P. G. Drazin, 1971: Dust devil formation. Tech. Report No. 35 to the Office of Naval Research, Geophysical Fluid Dynamics Fujita, Institute, Florida State University, Tallahassee, 16 pp. June 20, 1957. U.S. Weather Bureau Research Paper No. 42, Washington, D.C., U.S. Gov-
- Beebe, R. G., 1955: Types of air masses in which tornadoes occur. Bull. Amer. Meteor. Soc., 36, 349-350.
- _____, 1960: Mother cloud of the Fargo tornadoes of 20 June 1957. Cumulus _____, 1958: Tornado proximity soundings. Bull. Amer. Meteor. Soc., 39, 195-201.
- _____, 1973: Tornadoes around the world. Weatherwise, 26, 56-83.
- Beeth, D. R., and S. H. Hobbs, 1975: Analysis of tornado generated missiles. Topical Report B & R-001, Brown and Root, Inc., Houston, Texas, 99 pp. Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc.,
- Blechman, J. B., 1975: The Wisconsin tornado event of April 21, 1974: observations and theory of secondary vortices. Report 75-3, Dept. Meteor., University of Wisconsin, Madison, 130 pp. 1970: Palm Sunday tornadoes of April 11, 1965. Mon. Wea. Rev., 98, 29-69.
- Brown, R. A., Ed., 1976: The Union City, Oklahoma tornado of 24 May 1973. NOAA Tech Memo ERLTM-NSSL, National Severe Storm Laboratory, Norman, Oklahoma [in preparation]. Rev., 90, 231-240.
- Church, C. R., 1975: A photogrammetric study of the Parker, Indiana tornado. Proceedings of the SESAME Opening Meeting, D. K. Lilly, Ed., Boulder, Colorado, Environmental Research Laboratories, NOAA, 422-427.
- Crone, D. R., 1968: Elementary Photogrammetry. New York, Frederick Ungar Publishing Co., 197 pp. 139-147.
- Darkow, G. L. and M. G. Fowler, 1971: Tornado proximity sounding wind analysis. Preprints, Seventh Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 148-151. Tallahassee, 371 pp.
- Davies-Jones, R. P. and E. Kessler, 1974: Tornadoes. Weather and Climate Modification, Chapter 16, W. N. Hess, Ed., New York, John Wiley and Sons, Inc., 842 pp.

- Dergarabedian, P. and F. Fendell, 1970: Estimation of maximum wind speeds in tornadoes. Tellus, 22, 511-515.
- _____, and F. Fendell, 1971: A method for rapid estimation of maximum tangential wind speed in tornadoes. Mon. Wea. Rev., 99, 143-145.
- _____, and F. Fendell, 1975: Tornado dynamics. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado, Wind Eng. Research Council, sect. II, paper 1, 1-3.
- Dinwiddie, F. C., 1959: Waterspout-tornado structure and behavior at Nags Head, N.C., August 12, 1952. Mon. Wea. Rev., 87, 239-250.
- Fawbush, E. J., R. C. Miller, and L. G. Starrett, 1951: An empirical method of forecasting tornado development. Bull. Amer. Meteor. Soc., 32, 1-9.
- _____, and R. C. Miller, 1954: The types of airmasses in which North American tornadoes form. Bull. Amer. Meteor. Soc., 35, 154-165.
- Fujita, T., 1960a: A detailed analysis of the Fargo tornadoes of June 20, 1957. U.S. Weather Bureau Research Paper No. 42, Washington, D.C., U.S. Government Printing Office, 67 pp.
- _____, 1960b: Mother cloud of the Fargo tornadoes of 20 June 1957. Cumulus Dynamics, C. E. Anderson, Ed., New York, Pergamon Press Inc., 175-177.
- _____, 1973: Tornadoes around the world. Weatherwise, 26, 56-83.
- _____, 1975: New evidence from April 3-4, 1974 tornadoes. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 248-255.
- _____, D. L. Bradbury, and C. F. Van Thullenar, 1970: Palm Sunday tornadoes of April 11, 1965. Mon. Wea. Rev., 98, 29-69.
- Garrett, R. A. and V. D. Rockney, 1962: Tornadoes in northeastern Kansas, May 19, 1960. Mon. Wea. Rev., 90, 231-240.
- Glaser, A. H., 1960: An observational deduction of the structure of a tornado vortex. Cumulus Dynamics, C. E. Anderson, Ed., New York, Pergamon Press Inc., 157-166.
- Golden, J. H., 1969: The Dinner Key "tornadic waterspout" of June 7, 1968. Mariners Weather Log, 13, 139-147.
- _____, 1973: The life-cycle of the Florida Keys waterspout as the result of five interacting scales of motion. Ph.D dissertation, Florida State University, Tallahassee, 371 pp.
- _____, 1974a: The life cycle of Florida Keys' waterspouts. I. J. Appl. Meteor., 13, 676-692.

- Kimball, J., 1974b: Life cycle of Florida Keys' waterspouts. NOAA Tech. Memo ERLTM-NSSL-70, National Severe Storms Laboratory, Norman, Oklahoma, 147 pp. [COM-74-11477/8GA, Available from National Technical Information Service, Springfield, Va., 22151.]
- Lee, J. J., and D. Purcell, 1974: Photogrammetric velocity and morphological analysis of the Union City, Oklahoma tornado, May 24, 1973. EOS Transactions, Amer. Geophys. Un., 56, 1130.
- Neckey, J., and D. Purcell, 1975: Photogrammetric velocities for the Great Bend, Kansas tornado: accelerations and asymmetries. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 336-343.
- Maddox, J., and R. P. Davies-Jones, 1975: Photogrammetric windspeed analysis and damage interpretation of the Union City, Oklahoma tornado, May 24, 1973. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado, Wind Eng. Research Council, sect. II, paper 2, 1-4.
- Mehta, K. C., J. R. McDonald, J. E. Minor, and A. J. Sanger, 1971: Response to the Union City tornado. Mon. Wea. Rev., 99, 1-10.
- Goldman, J. L., 1965: The Illinois Tornadoes of 17 and 22 April 1963. Satellite and Mesometeorology Research Project Paper No. 39, University of Chicago, 80 pp.
- Miller, J., and T. Fujita, 1963: Preliminary radar and photogrammetric study of the Illinois tornadoes of April 17 and 22, 1963. Satellite and Mesometeorology Research Project Paper No. 24, University of Chicago, 7 pp.
- Groat, B. F., 1899: The force of a tornado. Mon. Wea. Rev., 27, 305.
- Hallert, B., 1960: Photogrammetry. New York, McGraw-Hill Book Company, Inc., 340 pp.
- Moller, J., J. R. McDonald, J. E. Minor, and R. Zipser, 1974: Field observation of the Union City tornado in Oklahoma. Weatherwise, 27, 1-10.
- Haurwitz, B., 1941: Dynamic Meteorology. New York, McGraw-Hill Book Company, Inc., 365 pp.
- Morgan, B. J., 1974: Tornado photographic analysis. Final Report, Grant No. NA-19-01-001-001, National Aeronautics and Space Administration, Silver Spring, Md., 77 pp.
- Hoecker, W. H., 1960a: Wind speed and air flow patterns in the Dallas tornado of April 2, 1957. Mon. Wea. Rev., 88, 167-180.
- Moses, H., W. C. Ashby, and M. A. Bognar, 1965: Dewpoint temperature inversions and the Dallas tornado. Mon. Wea. Rev., 93, 1-10.
- _____, 1960b: The dimensional and rotational characteristics of the tornadoes and their parent cloud system. The Tornadoes at Dallas, Tex., April 2, 1957, U.S. Weather Bureau Research Paper No. 41, Washington, D.C., U.S. Government Printing Office, 53-113.
- _____, 1961: Three-dimensional pressure pattern of the Dallas tornado and some resultant implications. Mon. Wea. Rev., 89, 533-542.
- Holle, R. L., 1975: Ground-based single-camera photogrammetry for tilted horizon. Unpublished manuscript, National Hurricane and Experimental Meteorology Laboratory, NOAA, Coral Gables, Florida, 5 pp.
- Price, S., 1975: Tornado funnel formation from a tornado cyclone. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 358-363.

- Kimball, H. H., 1901: Wind force in tornadoes. Mon. Wea. Rev., 29, 419. Bull. Amer. Meteor. Soc., 35, 191-197.
- Kuo, H. L., 1971: Axisymmetric flows in the boundary layer of a maintained vortex. J. Atmos. Sci., 28, 20-41. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado.
- Lee, J. T. and J. G. Galway, 1956: Preliminary report on the relationship between the jet at the 200-mb level and tornado occurrence. Bull. Amer. Meteor. Soc., 37, 327-332. Weather, 18, 8-11.
- Mackey, W. A., 1953: The Easter tornadoes at Bermuda. Weatherwise, 6, 74-75. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc.
- Maddox, R. A., 1973: A study of tornado proximity data and an observationally derived model of tornado genesis. Atmos. Sci. Paper No. 212, Dept. Atmos. Sci., Colorado State University, Fort Collins, 101 pp. U.S. National Conf. on Severe Local Storms, April 2, 1957, U.S. Weather Bureau, Washington, D.C.
- Marshall, J. D., 1932: Report of a tornado at Washington, Kansas, July 4, 1932. Bull. Amer. Meteor. Soc., 13, 148.
- Mehta, K. C., J. R. McDonald, J. E. Minor, and A. J. Sanger, 1971: Response of structural systems to the Lubbock storm. Texas Tech Univ. Storm Research Report 03, Dept. Civ. Eng., Texas Tech University, Lubbock, 428 pp. U.S. National Conf. on Severe Local Storms, April 2, 1957, U.S. Weather Bureau, Washington, D.C.
- Miller, R. C., 1972: Notes on analysis and severe-storm forecasting procedures of the Air Force Global Weather Central. Tech. Report 200 (Rev), Air Weather Service (MAC), United States Air Force, 190 pp.
- Moffitt, F. H., 1967: Photogrammetry. Scranton, Pa., International Textbook Co., 540 pp.
- Moller, A., C. Doswell, J. McGinley, S. Tegtmeier, and R. Zipser, 1974: Field observation of the Union City tornado in Oklahoma. Weatherwise, 27, 68-77.
- Morgan, B. J., 1974: Tornado photographic analysis. Final Report, Grant No. 04-4-022-8, Dept. Civ. Eng., University of Notre Dame, 47 pp. U.S. Dept. of Commerce, NOAA, 1973: Storm Data, Asheville, N.C., National Weather Service
- Moses, H., W. C. Ashby, and M. A. Bogner, 1968: Dewpoint temperature inversions. J. Appl. Meteor., 7, 206-216. U.S. National Conf. on Severe Local Storms, April 2, 1957, U.S. Weather Bureau, Washington, D.C.
- Ostby, F. P. and A. D. Pearson, 1974: The tornado season of 1973. Weatherwise, 27, 4-9. U.S. National Conf. on Severe Local Storms, April 2, 1957, U.S. Weather Bureau, Washington, D.C.
- Pautz, M. E., 1969: Severe local storm occurrences, 1955-1967. ESSA Tech Memo WBTM FCST 12, Weather Bureau Office of Meteorological Operations, Silver Spring, Md., 77 pp. [PB-187 761, Available from National Technical Information Service, Springfield, Va., 22151] U.S. National Conf. on Severe Local Storms, April 2, 1957, U.S. Weather Bureau, Washington, D.C.
- Price, S. and R. I. Sasaki, 1963: Some tornadoes, waterspouts, and other funnel clouds of Hawaii. Mon. Wea. Rev., 91, 175-190. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado
- Ramage, C. S., 1962: The subtropical cyclone. J. Geophys. Res., 67, 1401-1411.

- Reber, C. M., 1954: The South Platte Valley tornadoes of June 7, 1953. Bull. Amer. Meteor. Soc., 35, 191-197.
- Sadeh, W. Z., 1975: Drag of tornado-generated missiles. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado, Wind Eng. Research Council, sect. IV, paper 31, 1-3.
- Saunders, P. M., 1963: Simple sky photogrammetry. Weather, 18, 8-11.
- Schroeder, T. A., 1975: A recent evaluation of severe weather occurrences in Hawaii. Preprints, Ninth Conf. on Severe Local Storms, Boston, Amer. Meteor. Soc., 404-408.
- Segner, E. P., 1960: Estimates of minimum wind forces causing structural damage. The Tornadoes at Dallas, Tex., April 2, 1957, U.S. Weather Bureau Research Paper No. 41, Washington, D.C., U.S. Government Printing Office, 169-175.
- Sharp, H. O., 1951: Practical Photogrammetry. New York, The Macmillan Company, 229 pp.
- Showalter, A. K., 1953: A stability index for thunderstorm forecasting. Bull. Amer. Meteor. Soc., 34, 250-252.
- Simpson, R. H., 1952: Evaluation of the kona storm, a subtropical cyclone. J. Meteor., 9, 24-35.
- Sinclair, P. C., 1973: The lower structure of dust devils. J. Atmos. Sci., 30, 1599-1619.
- Thompson, M. M., Ed., 1966: Manual of Photogrammetry, vol. I and II. Falls Church, Va., American Society of Photogrammetry, 1199 pp.
- U.S. Dept. of Commerce, NOAA, 1973: Storm Data. Asheville, N.C., National Climatic Center, 15(9), 11-12.
- Van Tassel, E. L., 1955: The North Platte Valley tornado outbreak of June 27, 1955. Mon. Wea. Rev., 83, 255-264.
- Varney, B. M., 1926: An early morning tornado at Bermuda. Mon. Wea. Rev., 54, 428.
- Ward, N. B., 1972: The exploration of certain features of tornado dynamics using a laboratory model. J. Atmos. Sci., 29, 1194-1204.
- Weigel, E. P., 1974: Success story in Kansas. NOAA, 4(2), 34-41. [Available from Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402.]
- Wen, Y., 1975: Tornadic wind-load, wind effect and risk analysis. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado, Wind Eng. Research Council, sect. III, paper 28, 1-3.

- Williams, J. C. C., 1969: Simple Photogrammetry. London, Academic Press Ltd., 211 pp.
- Wills, T. G., 1969: Characteristics of the tornado environment as deduced from proximity soundings. Atmos. Sci. Paper No. 140, Dept. Atmos. Sci., Colorado State University, Fort Collins, 55 pp.
- Wolf, P. R., 1974: Elements of Photogrammetry. New York, McGraw-Hill Book Co., Inc., 562 pp.
- Worthley, L. E., 1967: Weather phenomena in Hawaii, Part I. Synoptic climatology of Hawaii. Final Report HIG-67-9, Contract Cwb 11373, Hawaii Institute of Geophysics, University of Hawaii, 53 pp.
- Yeh, G. C. K., 1975: Differential pressures on building walls during tornadoes. Preprints, Second U.S. National Conf. on Wind Eng. Research, Fort Collins, Colorado, Wind Eng. Research Council, sect. III, paper 5, 1-3.
- Zrnic, D. S., R. J. Doviak, D. W. Burgess, and D. Sirmans, 1976: Tornado velocities from Doppler spectra. National Severe Storms Laboratory, Norman, Oklahoma. Paper to be presented at the Seventeenth Conf. on Radar Meteorology, Seattle, Washington, October 1976 [in preparation].

				estimation for secondary funnels
			$F = 15mm/f1.5, V$	
C	Montell, F.	24 F	MU; $F = 150mm$	Size and shape characteristics
D	Houchins, E.	12 F	MU	Vertex stretching
E	Schuckman, S.	12 S	MU	
F	Sandberg, R.	52	Canon Zoom 50S; $F = 9.5-47.5mm/f1.8$	
G	Wegman, F.	12 F	Honeywell Pentax; $F = 135mm$	
H	Wendell, J.	24 F	MU	
I	Young, R. L.	52	Hinolta Autopak-B D6; $F = 50mm$	Primary data source for Kailua-Kona study
	Carlson, S. K.	12 F	MU	

SB = Super-8 Movie; F = Prints; S = 35mm slides

MU = Camera Make Unknown; F = Focal Length;

V = Variable Focal/Zoom Setting Used.

APPENDIX A

Photographers and Locations Relative to the Vortex Paths

TABLE 4

LIST OF PHOTOGRAPHERS

Photographer	Name	No. of Photos	Camera Information	Comments
A	Patterson, D.	S8	MU;V	Primary data source for Salina Study
B	Zickafoose, B.	S8	Bell and Howell Autoload; F = 15mm/f1.9;V	Tangential velocity estimation for secondary funnels
C	Mendell, F.	24 P	MU;F = 150mm	Size and shape characteristics
D	Houchin, K.	12 P	MU	Vortex stretching
E	Schuckman, S.	31 S	MU;V	
F	Sundberg, M.	S8	Cannon Zoom 518; F = 9.5-47.5mm/f1.8	
G	Shogren, B.	18 P	Honeywell Pentax; F = 135mm	
H	Engdahl, S.	20 P	MU	
I	Young, R. T.	S8	Minolta Autopak-8 D6; F = 50mm	Primary data source for Kailua-Kona study
	Carlson, N. K.	32 P	MU	

S8 = Super-8 Movie; P = Prints; S = 35mm slides

MU = Camera Make Unknown; F = Focal Length;

V = Variable Focal/Zoom Setting Used.

angle β increases, distance D also increases across the image focal (film) plane. The most accurate distances are always measured near the azimuth of the principal point (geometrical center of the image).

APPENDIX B

Correction for Azimuthal Angular Distortion in the Image-Focal and Effective-Image Planes

Angular distortion error is introduced when distances across the image plane are measured between objects located at a greater than 10° angular displacement from the principal axis. To better understand how angular distortion error occurs, one should understand the Tangent Law (Eq. 3.3). Figure B-1(a) illustrates how Eq. (3.3) originates.

If the landmark (ξ_2) farthest away from principal point PP is no greater than 10° as seen from the camera site, then the following approximation is valid with negligible error:

$$\tan \beta \approx \tan \beta_2 - \tan \beta_1$$

where

$$\tan \beta_1 = \frac{\xi_1}{f'}$$

and

$$\tan \beta_2 = \frac{\xi_2}{f'}$$

The precise form of Eq. (3.3) is then

$$\Delta X = R(\tan \beta_2 - \tan \beta_1) \quad (B.1)$$

However, Eq. (3.3) is a good approximation for Eq. (B.1) in most scaling instances. The Tangent Law is, therefore, based upon distance across the drawing screen from the azimuth of the principal point.

Now consider Fig. B-1(b), which is a more detailed representation of the lower part of Fig. B-1(a). Here, angle β is trisected: $\beta_1 = \beta_2 = \beta_3$. As

angle β increases, distance D also increases across the image focal (film) plane. The most accurate distances are always measured near the azimuth of the principal point (geometrical center of the image). To see how the linear distance D between any two points on the film varies as a function of the angle β away from the principal axis, we differentiate Eq. (3.3) with respect to β and obtain:

$$\frac{dD}{d\beta} = f \sec^2 \beta \quad (\text{B.2})$$

TABLE 5
ANGULAR DISTORTION ERROR

β (deg)	$\frac{dD}{d\beta}$ (length/deg)	Error (%)
0	1.000f	0.0
5	1.008f	0.8
10	1.031f	3.0
15	1.072f	6.7
20	1.132f	11.7
25	1.217f	17.8
30	1.333f	25.0
35	1.490f	33.0
40	1.704f	41.3
45	2.000f	50.0
50	2.420f	58.7
55	3.040f	67.1
60	4.000f	75.0

This function is tabulated (Table 5) for β -angles between 0° to 60° . Table 5 shows that the farther away from the principal axis one gets, the greater will be the error in assuming that a given distance across the film is equal to the same distance near the center.

Tangential Velocity Determination and Correction

Corrections for apparent planar foreshortening of the tangential velocity vector require the transformation of certain easily-measured tracer displacements in the image plane into arc length trajectories on a horizontal plane. Of course, such a corrected velocity estimate is still only an approximation of the true air velocity because of certain simplifying assumptions and sometimes complex debris motions. Nevertheless, it should make a considerable improvement over unrectified speeds for tracers approaching the apparent maximum (turning) radius in the image plane. Furthermore, the derivation is applicable for motion on either the near or far side of the vortex from the observer.

Figure C-1 shows a plan-view schematic containing the observer (at point O) and the circular vortex tracer orbit around the tornado center T_c . The derivation requires this plane to be horizontal, and the tracer element's motion to be circular. In addition, the vortex is assumed to be vertical since spatial tilt considerations overly complicate the derivation's geometry.

A special "critical" angle, β_c , distinguishes whether the element being followed lies on the near or far side of the vortex with respect to point O. In angular measure (see Fig. C-1), β_c is 90° as the element moves along

$$\beta_c = \sin^{-1} \left(\frac{r}{R} \right) \quad (C.1)$$

Of course, if the sense of rotation is known and the vortex debris cloud is diaphanous, then it becomes immediately obvious which side the element is on.

Therefore, point O is the apparent turning radius to an observer at point O. This observation is an important difference between the near side and far side derivations.

APPENDIX C

Tangential Velocity Determination and Correction

Finally, subscript conventions are as follows: 1 refers to the element's position at the earlier time, 2 at the later time; N indicates that the velocity vector require the transformation of certain easily-measured tracer displacement lies on the near side, and F on the far side, of the vortex from the placements in the image plane into arc length trajectories on a horizontal plane. Of course, such a corrected velocity estimate is still only an approximation of the true air velocity because of certain simplifying assumptions and sometimes complex debris motions. Nevertheless, it should make a considerable improvement over unrectified speeds for tracers approaching the apparent maximum (turning) radius in the image plane. Furthermore, the derivation is applicable for motion on either the near or far side of the vortex from the observer.

Figure C-1 shows a plan-view schematic containing the observer (at point O) and the circular vortex tracer orbit around the tornado center T_c . The derivation requires this plane to be horizontal, and the tracer element's motion to be circular. In addition, the vortex is assumed to be vertical since spatial tilt considerations overly complicate the derivation's geometry.

A special "critical" angle, β_c , distinguishes whether the element being followed lies on the near or far side of the vortex with respect to point O. In angular measure (see Fig. C-1), from 180° to 90° as the element moves along

the circular arc in the right-front view from P_N to C. However, the same angle, γ_F , varies from 90° to 0° as the element continues past point C to point

$$\beta_c = \sin^{-1} \left(\frac{r}{R} \right) \quad (C.1)$$

Of course, if the sense of rotation is known and the vortex debris cloud is diaphanous, then it becomes immediately obvious which side the element is on. Therefore, point C is the apparent debris turning radius to an observer at point O. This observation is an important difference between the near side and far side derivations.

Finally, subscript conventions are as follows: 1 refers to the element's position at the earlier time, 2 at the later time; N indicates that the element lies on the near side, and F on the far side, of the vortex from the observer.

Near Side Element

Important quantities known either from the field survey or measured directly from the image plane are:

R = observer-to-tornado range

r = orbiting radius of the tracer element

X_{1N}, X_{2N} = apparent radial distances from the tornado center of the tracer element at earlier and later times, respectively, in its orbit.

In Fig. C-1, we obtain using $\Delta X_{1N}^T O$:

$$\beta_{1N} = \tan^{-1} \left(\frac{X_{1N}}{R} \right) \quad (C.2)$$

Similarly, from $\Delta X_{2N}^T O$,

$$\beta_{2N} = \tan^{-1} \left(\frac{X_{2N}}{R} \right) \quad (C.3)$$

Notice that angle γ_N varies from 180° to 90° as the element moves along the circular arc in the right-front quadrant from P_N to C. However, the same angle, γ_F , varies from 90° to 0° as the element continues past point C to point

P_F . This is the only mathematical difference between this and the far side derivation.

Applying the Law of Sines to $\Delta AT_c O$ yields γ_{1N} (in degrees):

$$\gamma_{1N} = 180 - \sin^{-1} \left[\frac{R}{r} \sin \beta_{1N} \right]. \quad (C.4)$$

Similarly, from $\Delta BT_c O$,

$$\gamma_{2N} = 180 - \sin^{-1} \left[\frac{R}{r} \sin \beta_{2N} \right]. \quad (C.5)$$

Now,

$$\alpha_N = \alpha_{2N} - \alpha_{1N}, \quad (C.6)$$

from which it can be shown that

$$\alpha_N = \beta_{1N} - \beta_{2N} + \gamma_{1N} - \gamma_{2N}. \quad (C.7)$$

Since all right-member terms in Eq. (C.7) are known from Eqs. (C.2-C.5),

angle α_N can be computed in degrees. One should, however, immediately convert this angle into radians.

The circular arc length S traversed between points A and B (see Fig. C-1) is then the product of angle α_N in radians and orbit radius r in meters:

$$S = r\alpha_N \quad (C.8)$$

Finally, the corrected tangential velocity magnitude estimate (ms^{-1}) is

obtained by multiplying the arc length S computed from Eq. (C.8) by the constant framing rate of the camera (18 fps) and dividing by the total number of frames elapsed as the element traveled from A to B:

$$\left(\begin{array}{c} \text{Corrected} \\ \text{Tangential} \\ \text{Velocity} \end{array} \right) = \frac{(\text{Arc Length})(\text{Framing Rate})}{(\text{Total Frames Elapsed})}. \quad (C.9)$$

Far Side Element

The derivation for the far side correction is much the same as that just outlined for the near side. However, special attention must be paid to the subscript conventions—especially, to replacing N with F (see Fig. C-1).

For a tracer element moving from B' to A', angles β_{1F} and β_{2F} are computed using Eqs. (C.3) and (C.2), respectively. The only major difference is in the computation of angle γ , where on the far side,

$$\gamma_{1F} = \sin^{-1} \left[\frac{R}{r} \sin \beta_{1F} \right] \quad (C.10)$$

and

$$\gamma_{2F} = \sin^{-1} \left[\frac{R}{r} \sin \beta_{2F} \right] \quad (C.11)$$

Finally, the remainder of the derivation, is the same from Eq. (C.6) to Eq. (C.9).

Alternate Correction for a
Program ARCLLEN

Each of the preceding derivations has been programmed in standard FORTRAN IV language. Listings of these programs, referred to by the name ARCLLEN, appear in Fig. C-2. Both programs readily follow the derivations and key variable names used in them. Those not defined hitherto are:

NFR = frame number at beginning of the velocity vector

LNFR = frame number at end of the velocity vector

FRINT = frame number interval containing the velocity vector

FRATE = constant framing rate of camera

VT = corrected tangential velocity estimate.

The following conventions were adopted concerning the input of data and its resultant output in program ARCLLEN:

(1) If two or more elements on the near side are tracked over the same frame sequence, the rightmost element is listed first in the output of program ARCLEN, followed by the remaining sequences as they are positioned across the image plane from right to left. Conversely, on the far side, the leftmost sequence is listed first, followed by the remaining sequences from left to right.

(2) Frame numbers shown in ARCLEN's output (Fig. 24) are the first frames of successive frame intervals over which an individual element is tracked. Normally, the following frame number value marks the end of the preceding vector interval, or the start of the interval corresponding to the next velocity vector in the sequence. The ending frame of the last vector in a sequence is not listed in program ARCLEN, signifying that the particular element was no longer tracked.

(3) The interpretation of negative arc length values in Fig. 24 may be seen in Fig. C-3. A minus sign in front of the computed arc length value for an element on the near (far) side of the vortex from the observer indicates that the element lies to the left (right) of the principal axis; that is, lies in the mathematical third (first) quadrant. A positive arc length value denotes an element on the near (far) side which is to the right (left) of the principal axis; this corresponds to the fourth (second) quadrant, where the origin of the coordinate system is the vortex center.

Alternate Correction for a Distant Vortex

The general derivations discussed previously apply for any case in which $1 \leq \frac{R}{r} < \infty$. However, these derivations become even simpler when the tornado is so far away from the observer (i.e., $\frac{R}{r} \gg 1$) that lines-of-sight to any points on the vortex can be assumed parallel (see Fig. C-4). An alternative derivation for the latter event will now be given.

In the usual way, a scale ratio is determined using the range R to the tornado center, T_c (see Chapter III). For a particular tangential velocity vector in the image plane, the apparent radial distance from the "tail" of this vector is then measured. This distance is called X_1 .

⁷ The word "effective," as used in reference to the image plane, means that we are free to choose any range whatever at which to place the image plane from a fixed observer (Morgan, 1974).

If we were to move the effective⁷ image plane toward the observer so that it passes through the tail of this velocity vector (point T), then segment $\overline{C'T}$ also has length X_1 . Hence, $\angle C'T_c T = \alpha = \sin^{-1}\left(\frac{X_1}{r}\right)$, which will be needed later in the computation. Furthermore, the length of segment $\overline{C'T_c} = r \cos \alpha = (X_1) \cot \alpha$. Using the scale ratio just determined, we compute the apparent length in the new effective image plane of the tangential velocity vector $\overrightarrow{TH}$. Since $\Delta C'T_c T$ and $\Delta THH'$ are similar, then $\angle H'TH = \angle C'T_c T = \alpha$. Hence, the magnitude of the corrected tangential velocity vector $\overrightarrow{TH'}$ (see Fig. C-4) is

$$\overrightarrow{TH'} = \overrightarrow{TH} \sec \alpha \quad (C.12)$$

where $\overrightarrow{TH}$ and α are both known.

Figure 2. Computed values of the apparent scale features at 1200 GMT, 23 September, and 1200 GMT, 24 September 1973. Positions of

⁷The word "effective," as used in reference to the image plane, means that we are free to choose any range whatever at which to place the image plane from a fixed observer (Morgan, 1974).

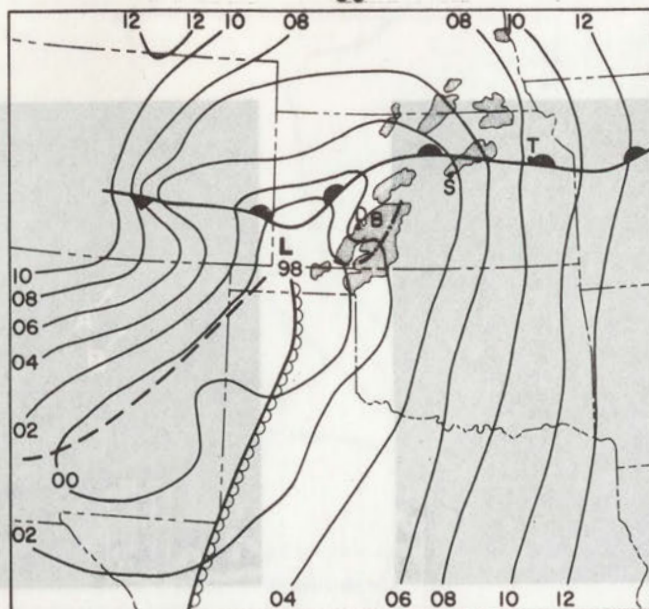


Figure 1. Hand-analyzed regional surface analysis for 0000 GMT, 26 September 1973. Wichita radar echoes are shaded. Dots marked T, S, and D locate Topeka, Salina, and Dodge City, respectively. The letter B denotes the presence of a "bubble" high.

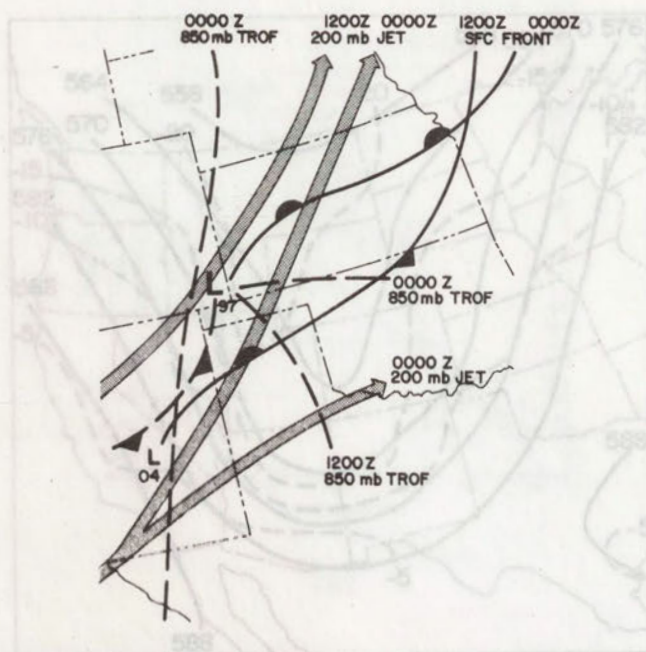
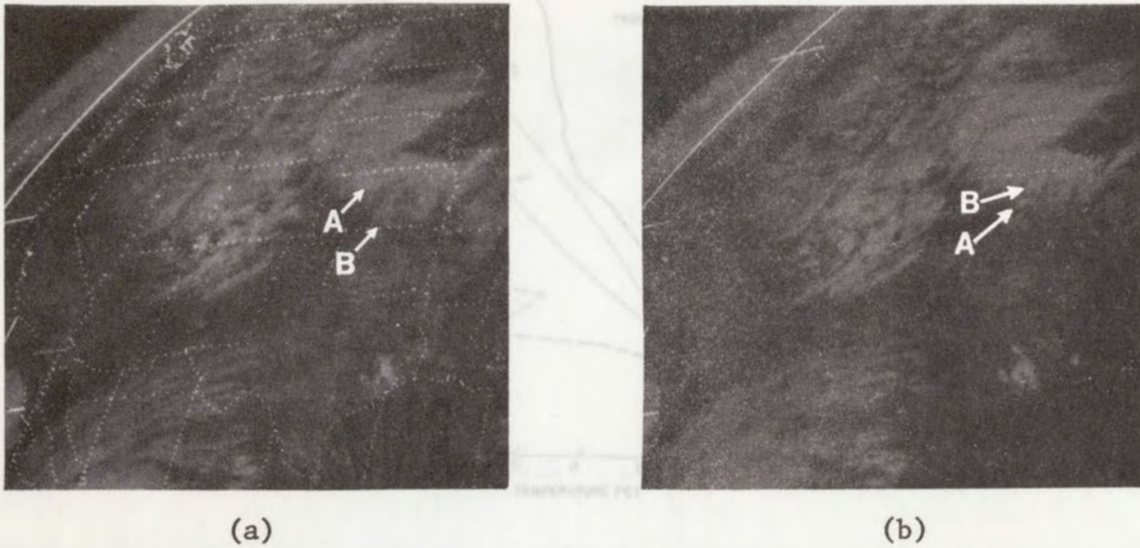


Figure 2. Composite analysis of NMC's synoptic-scale features at 1200 GMT, 25 September, and 0000 GMT, 26 September 1973. Positions of surface lows, fronts, 850 mb troughlines, and 200 mb jets are indicated.



(a)

(b)

Figure 3. ATS-3 satellite photographs for (a) 2003 GMT and (b) 2057 GMT, 25 September 1973. Arrows marked A and B are explained in the text.

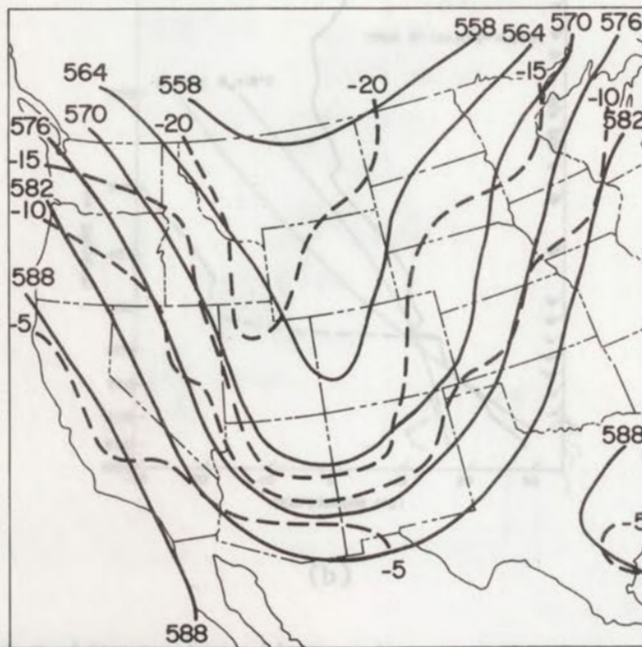
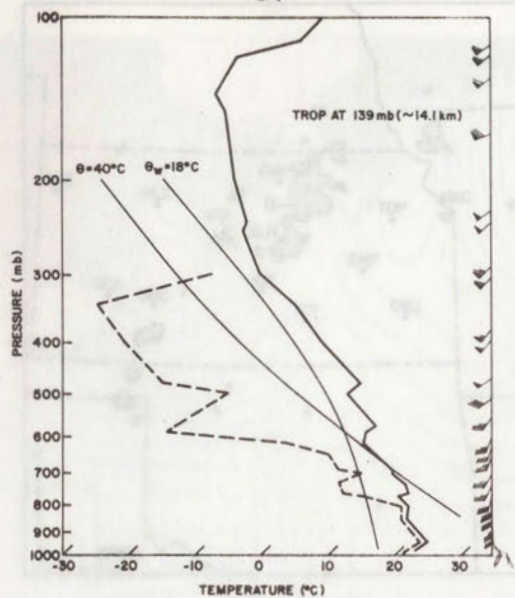
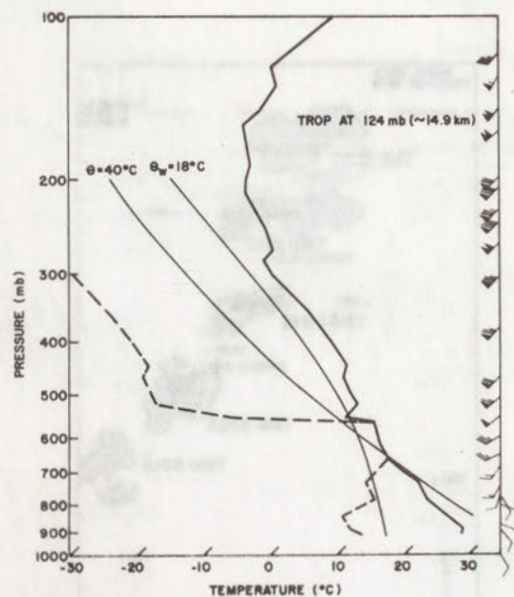


Figure 4. 500 mb analysis for 0000 GMT, 26 September 1973. Solid lines represent height contours in dekameters and dashed lines are isotherms in $^{\circ}\text{C}$.



(a)



(b)

Figure 5. Skew-T log p temperature/dew point soundings for (a) Topeka, and (b) Dodge City, Kansas, 0000 GMT, 26 September 1973. Thick solid lines are temperature, dashed lines are dew point temperature. Dry (θ) and saturation adiabats (θ_w) are thin solid lines. Vertical profiles of horizontal wind are shown at the right.

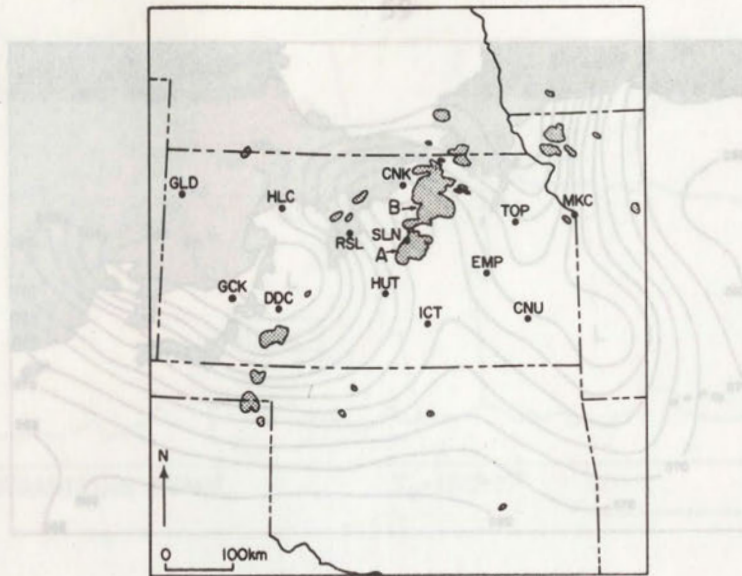


Figure 6. Regional map of surface synoptic stations in Kansas showing Wichita (ICT) radar echoes (shaded), 2230 GMT, 25 September 1973. Cells A and B were producing the Salina and Bennington series tornadoes, respectively.

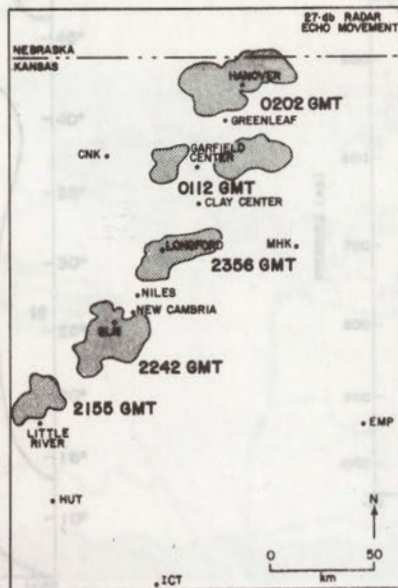


Figure 7. Sequential plot of the 27-db echoes (shaded) within the Salina tornado's parent cell at times indicated. The 2155 GMT echo is 10 minutes before the first tornado; however, the cell was tornadic for all subsequent times shown. Note the double-core after 0112 GMT.

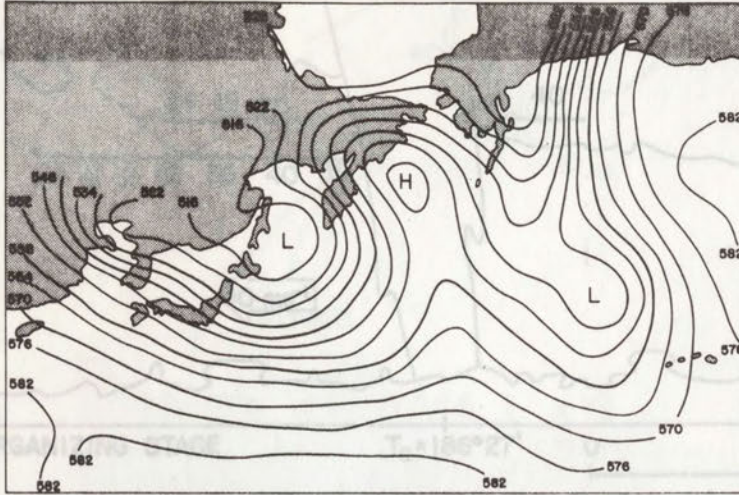


Figure 9. 500 mb analysis for 0000 GMT, 29 January 1971, over the North Pacific. Contours are in dekameters. The Hawaiian Islands are at the lower right.

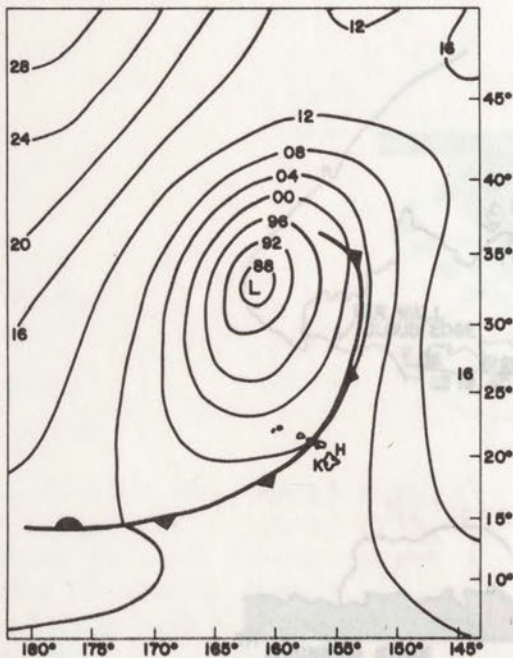


Figure 8. Surface analysis for 1200 GMT, 28 January 1971. Dots H and K denote locations of Hilo and Kailua-Kona, Hawaii, respectively.

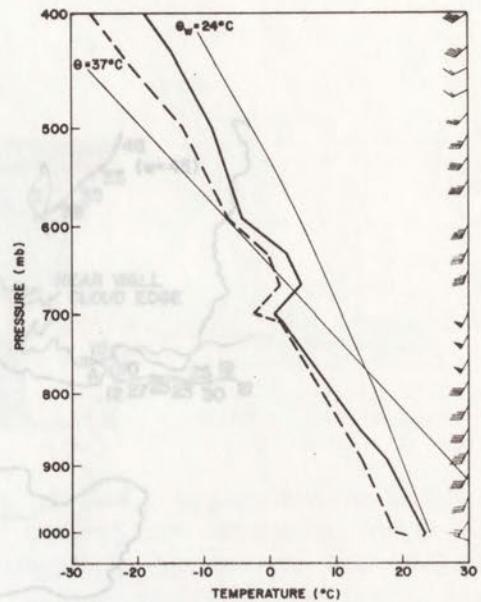


Figure 10. Temperature/dew point sounding for Hilo, Hawaii, 0000 GMT, 29 January 1971. Thick solid line is temperature, dashed line is dew point temperature. Dry (θ) and pseudoadiabats (θ_w) are thin solid lines. The vertical profile of horizontal wind is on the right.

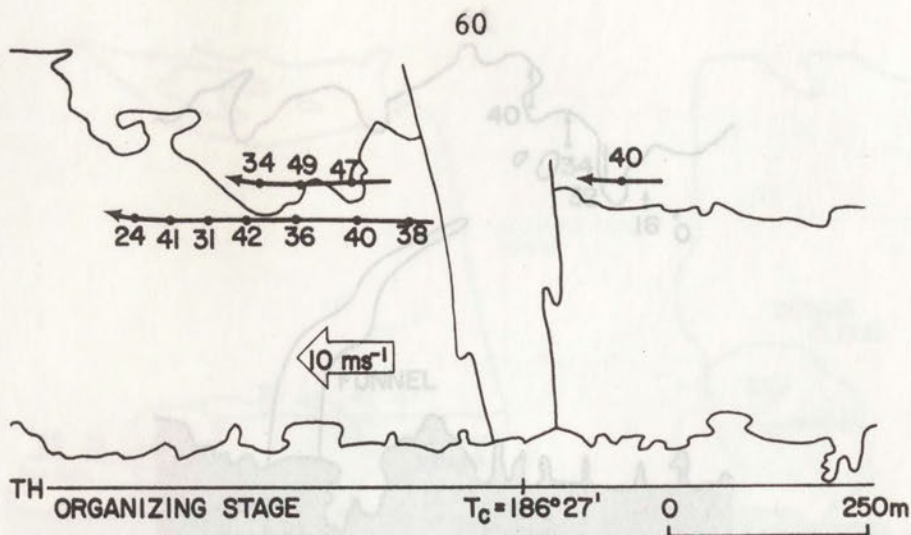


Figure 11. Net velocity trajectories for peripheral cloud tags surrounding the organizing Salina tornado. Net speeds are in ms⁻¹ at the points indicated. Funnel translation is represented by the wide arrow. The azimuth of the tornado's position is marked T_c along the true horizon line (TH).

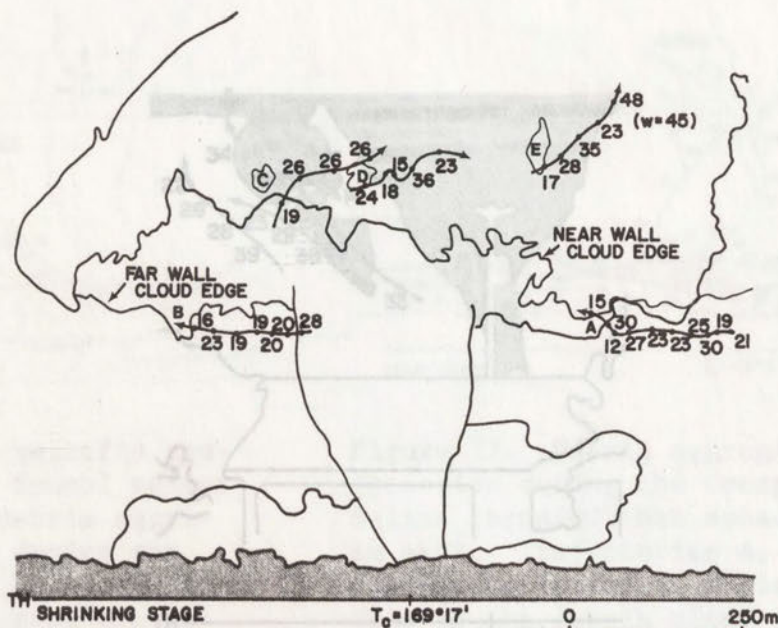


Figure 12. Net cloud-tracer velocity trajectories within the shrinking Salina tornado's wall cloud. Speeds indicated are relative to the wall cloud's apparent axis of rotation in ms⁻¹. A vertical component for a point on trajectory E appears in parentheses. Near-and-far-side lower edges of the wall cloud from the observer are labelled.

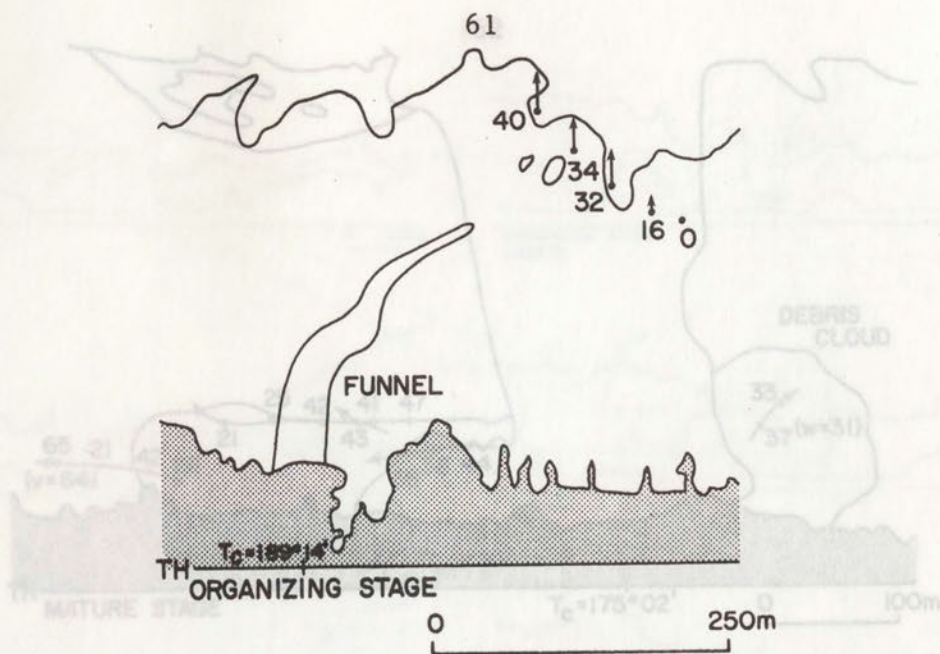


Figure 13. Apparent vertical velocity components of a cloudbase tag above a short-lived funnel prior to formation of the Salina tornado. Speeds corresponding to vectors are in ms^{-1} . Notice that the tag is accelerating upward into cloudbase.

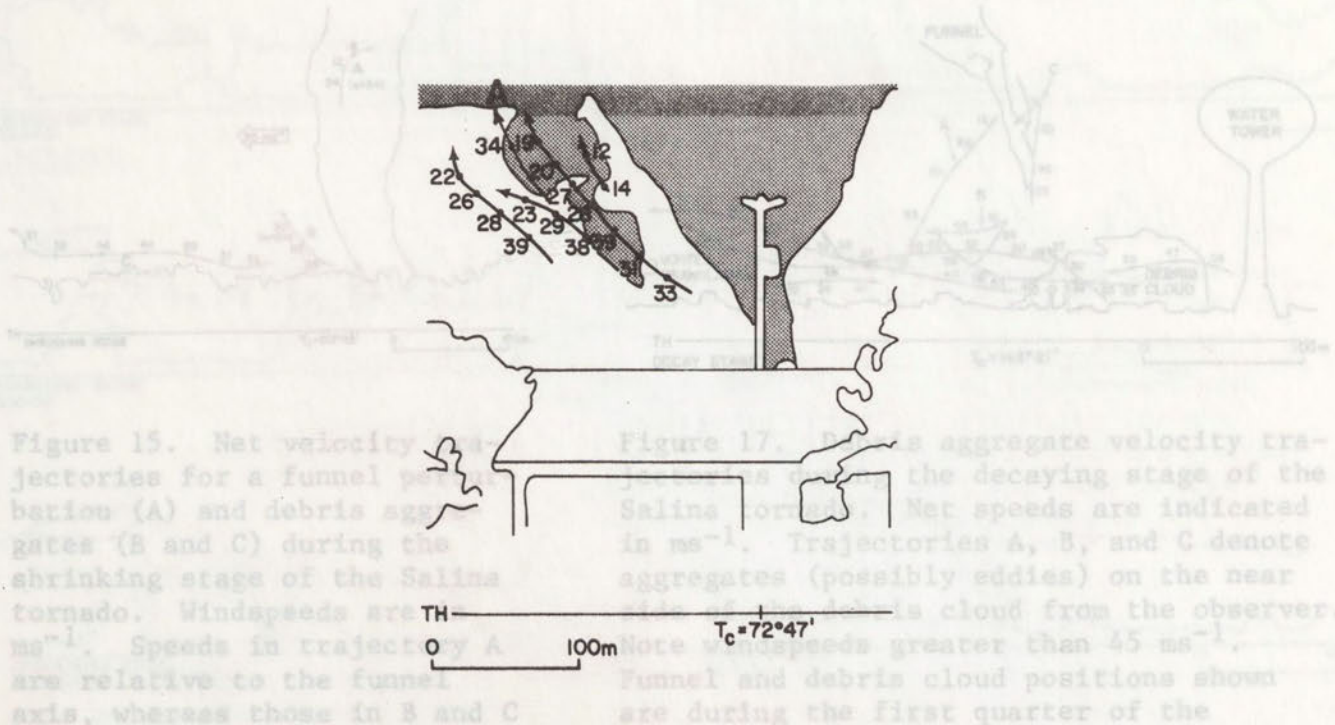


Figure 14. Net velocity trajectories for cloud tags associated with the vortex following the primary Salina tornado. The funnel and tags are shaded. Speeds are in ms^{-1} . Tag A is observed to accelerate upward.

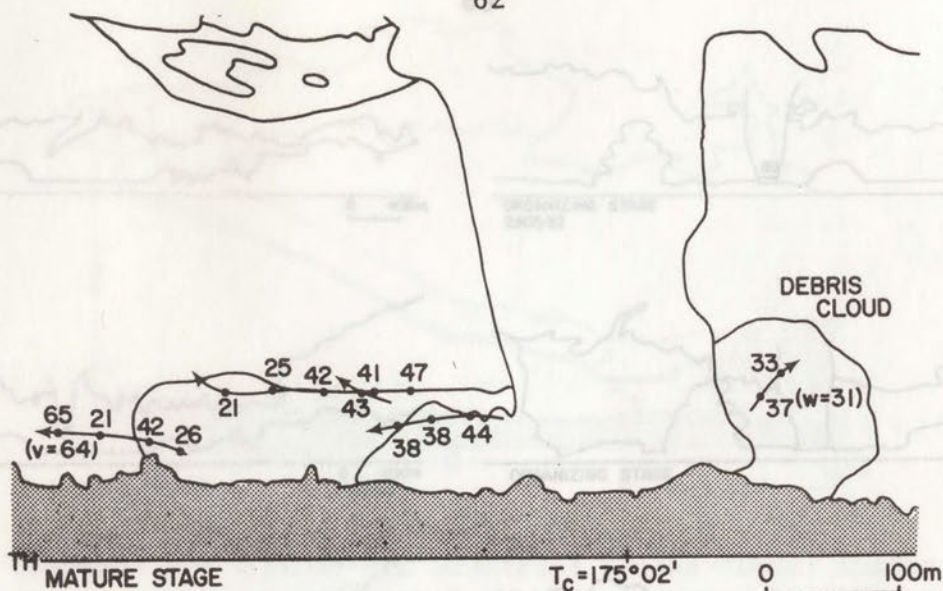


Figure 16. Net velocity trajectories for debris cloud aggregates during the mature stage of the Salina tornado. Speeds are in ms^{-1} . Maximum apparent vertical (w) and horizontal (v) components are given in parentheses.

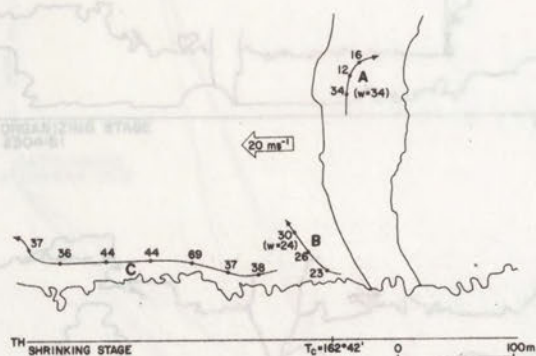


Figure 15. Net velocity trajectories for a funnel perturbation (A) and debris aggregates (B and C) during the shrinking stage of the Salina tornado. Windspeeds are in ms^{-1} . Speeds in trajectory A are relative to the funnel axis, whereas those in B and C are with respect to the ground. Funnel translation is indicated by the wide arrow. Vertical velocity components in trajectories A and B appear in parentheses.

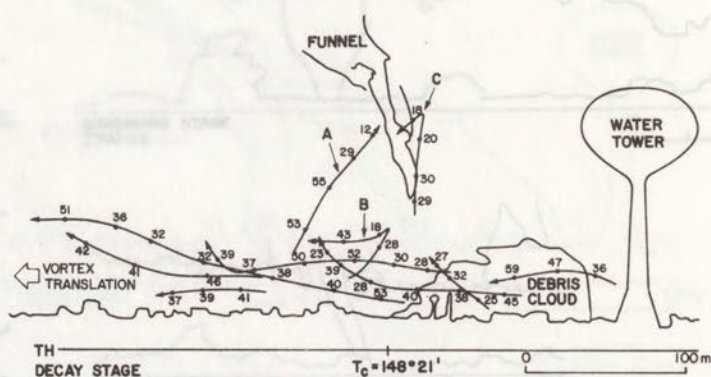


Figure 17. Debris aggregate velocity trajectories during the decaying stage of the Salina tornado. Net speeds are indicated in ms^{-1} . Trajectories A, B, and C denote aggregates (possibly eddies) on the near side of the debris cloud from the observer. Note windspeeds greater than 45 ms^{-1} . Funnel and debris cloud positions shown are during the first quarter of the tracking sequence.

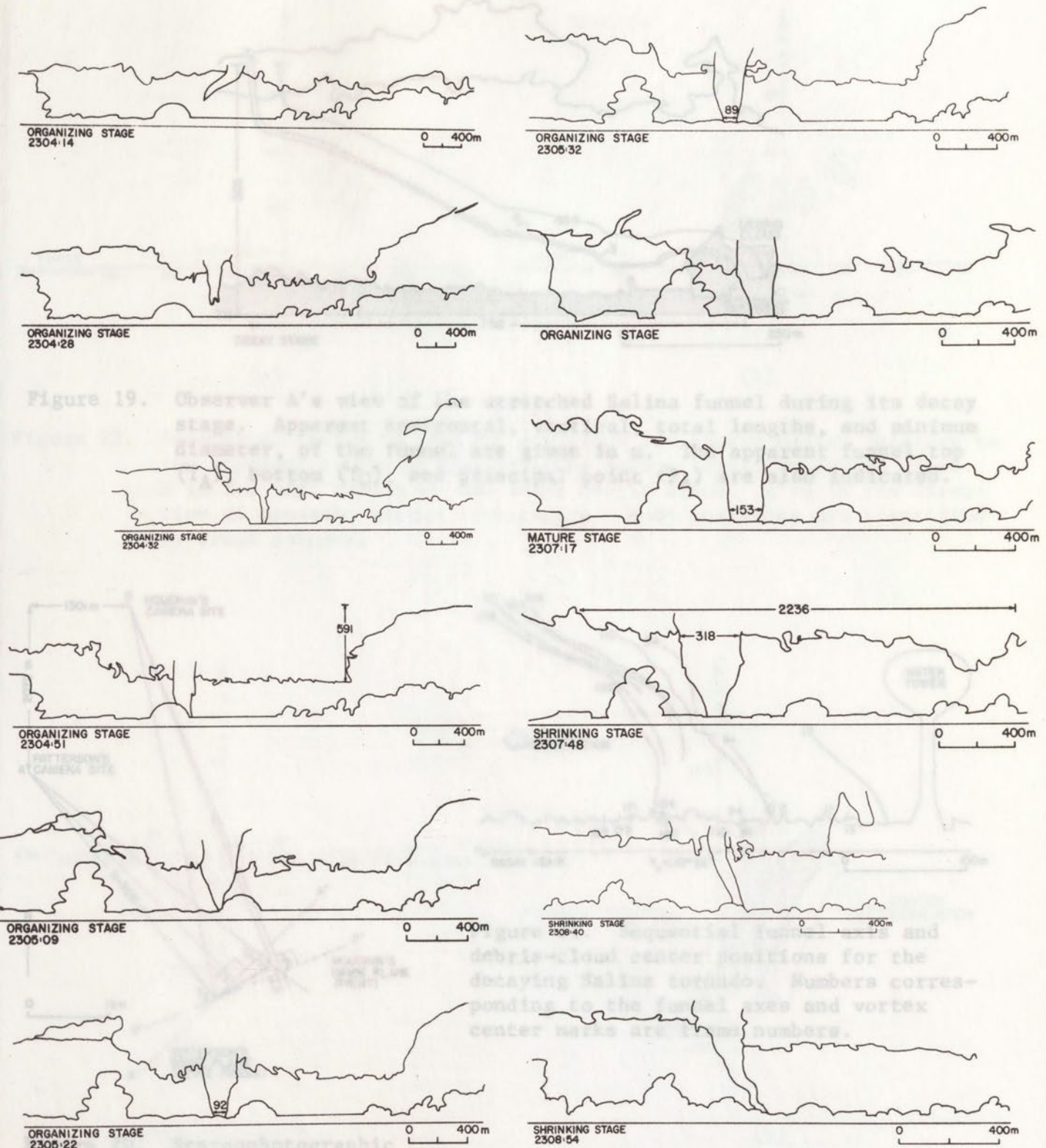


Figure 18. Size and shape characteristics of the Salina tornado and its wall cloud during evolution through sequential life cycle stages. Dimensions shown are in m. Times are given to the nearest s (GMT), but are accurate only to within ± 1 minute. Drawings were made from still photographs taken by photographer C (see Appendix A).

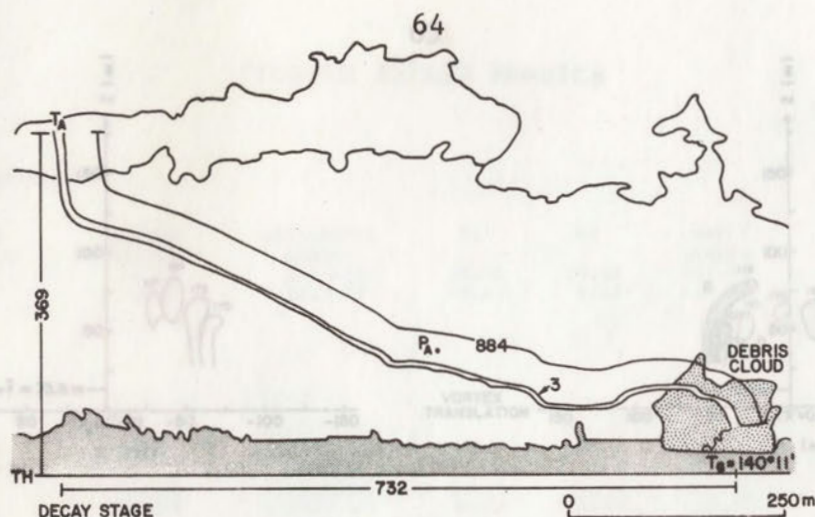


Figure 19. Observer A's view of the stretched Salina funnel during its decay stage. Apparent horizontal, vertical, total lengths, and minimum diameter, of the funnel are given in m. The apparent funnel top (T_A), bottom (T_G), and principal point (P_A) are also indicated.

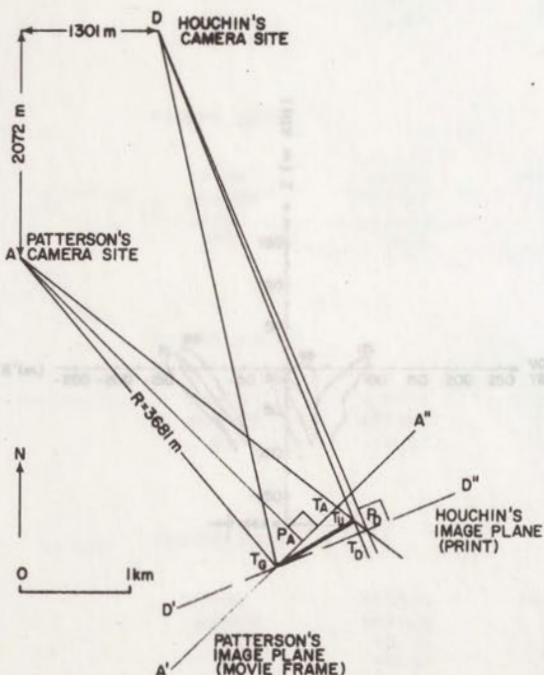


Figure 20. Stereophotographic site geometry for two independent observers viewing the stretched Salina funnel simultaneously. The drawing is to scale.

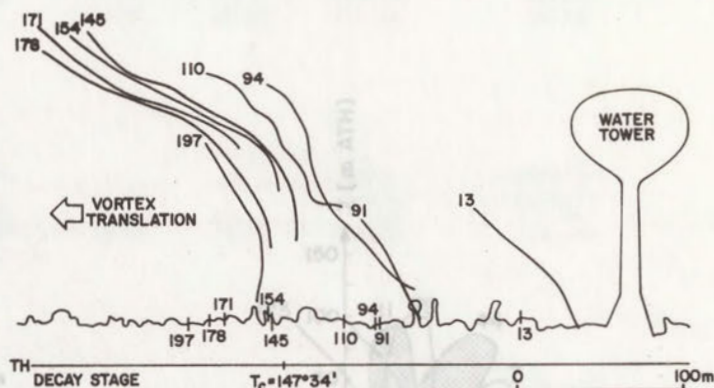
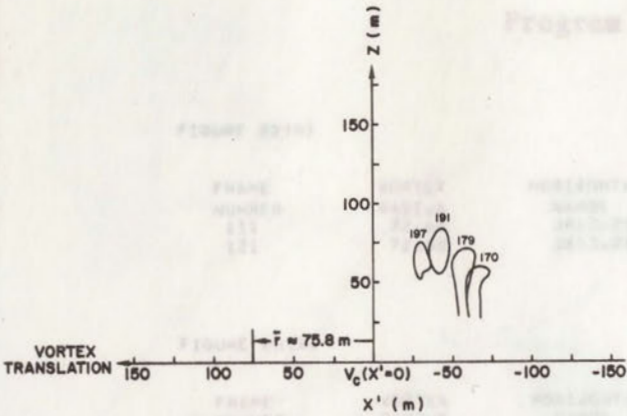
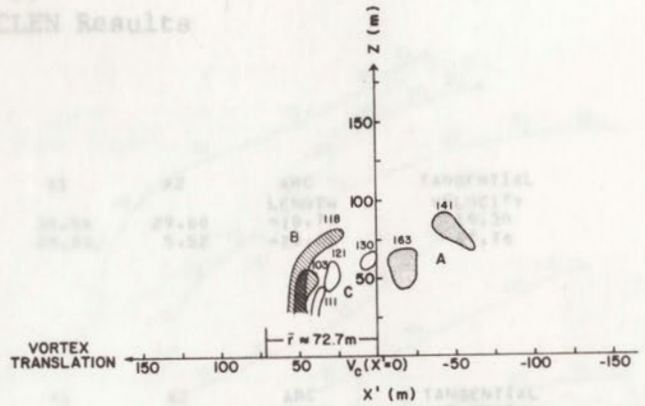


Figure 21. Sequential funnel axis and debris-cloud center positions for the decaying Salina tornado. Numbers corresponding to the funnel axes and vortex center marks are frame numbers.

Program ANCLN Results

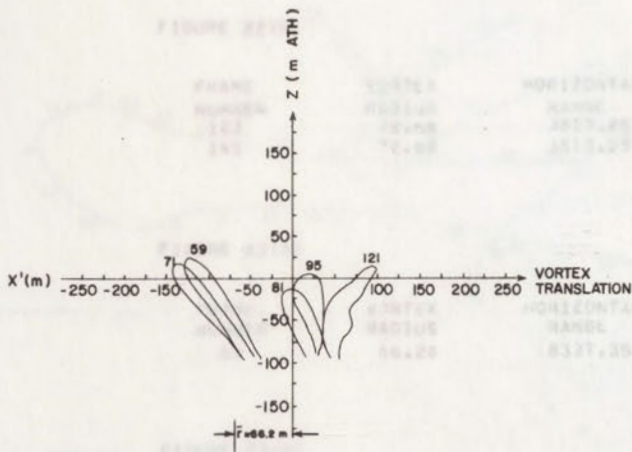


(a)

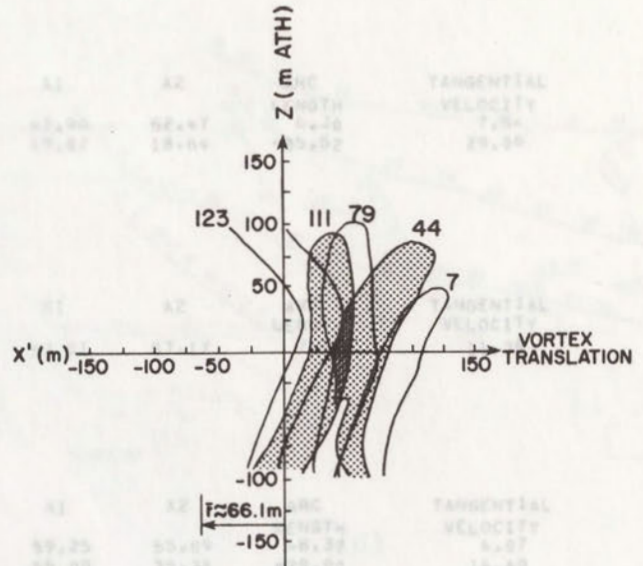


(b)

Figure 22. Sequential radius-vs.-height positions for debris eddies relative to the apparent center of the decaying Salina tornado's debris cloud. $\bar{r}$ is the mean radius of the lower debris cloud. $x' > 0$ in the direction of apparent vortex translation. Eddy positions are identified by frame numbers.



(a)



(b)

Figure 23. Sequential radius-vs.-height positions for secondary vortices relative to the center of a multifunnel vortex which preceded the primary Salina tornado. $\bar{r}$ is the mean apparent radius of the lower vortex. $x' > 0$ in the direction toward which the vortex is moving. Funnel positions are identified by frame numbers.

Program ARCLEN Results

FIGURE 22(A)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
111	72.68	3813.25	38.66	29.00	-10.75	19.36
121	72.68	3813.25	29.00	5.52	-23.85	47.70

FIGURE 23(A)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
81	66.20	8337.35	29.10	35.34	7.10	9.12
95	66.20	8337.35	35.34	46.78	14.57	10.09

ALL LENGTHS ABOVE ARE IN METERS
THE TANGENTIAL VELOCITY IS IN M/SEC

(a)

FIGURE 22(A)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
170	75.80	3807.23	63.17	56.85	-10.51	21.01
179	75.80	3807.23	56.85	43.51	-18.19	27.29
191	75.80	3807.23	43.51	31.23	-14.40	43.20

FIGURE 22(H)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
103	72.68	3813.25	47.98	52.47	6.30	7.54
141	72.68	3813.25	49.02	18.64	-35.52	29.06

FIGURE 23(A)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
59	66.20	8337.35	53.01	57.17	7.56	11.34

FIGURE 23(H)

FRAME NUMBER	VORTEX RADIUS	HORIZONTAL RANGE	X1	X2	ARC LENGTH	TANGENTIAL VELOCITY
7	66.10	8337.35	59.25	55.09	-8.37	4.07
44	66.10	8337.35	55.09	35.34	-28.00	14.40
111	66.10	8337.35	2.49	17.67	15.51	23.27

ALL LENGTHS ABOVE ARE IN METERS
THE TANGENTIAL VELOCITY IS IN M/SEC

(b)

Figure 24. Computed tangential velocity estimates for tracers on (a) near and (b) far sides of vortices with respect to two Salina area observers.

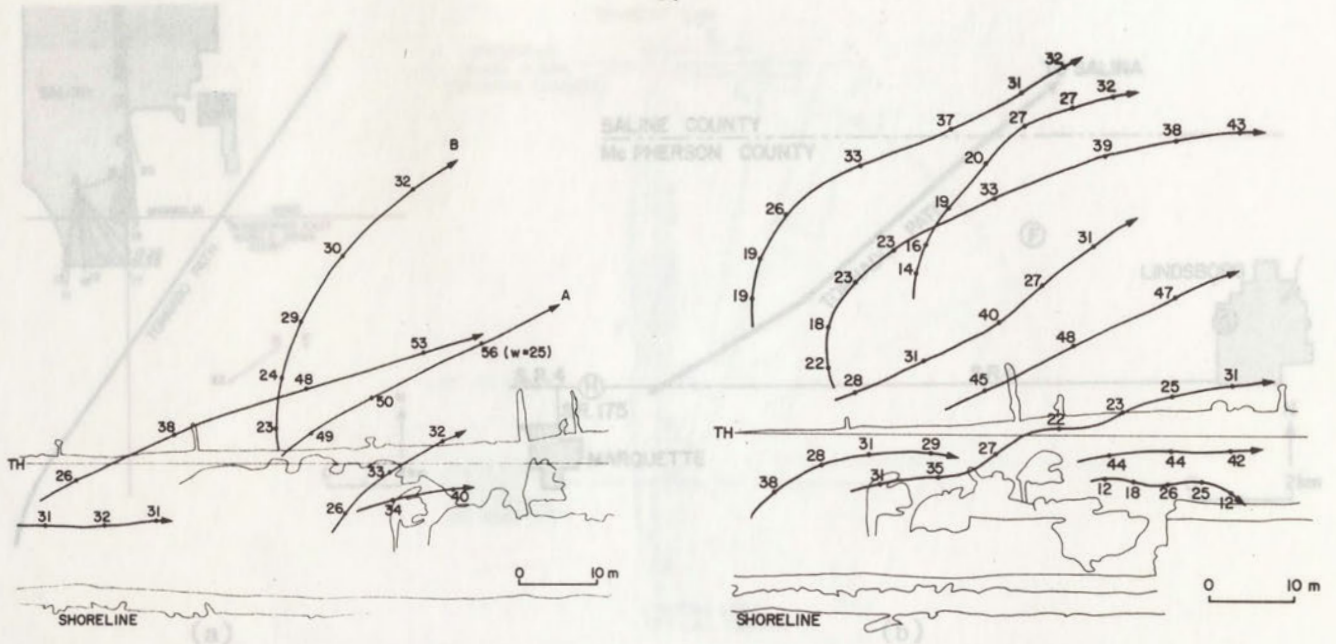


Figure A-1. Maps showing locations of contributing photographers in (a) Salina and (b) McPherson County, Kansas areas. Photographers lines-of-site in (a) correspond to figure numbers in the text. Letters in (a) and circled letters in (b) correspond to the photographers listed alphabetically in Table 4.

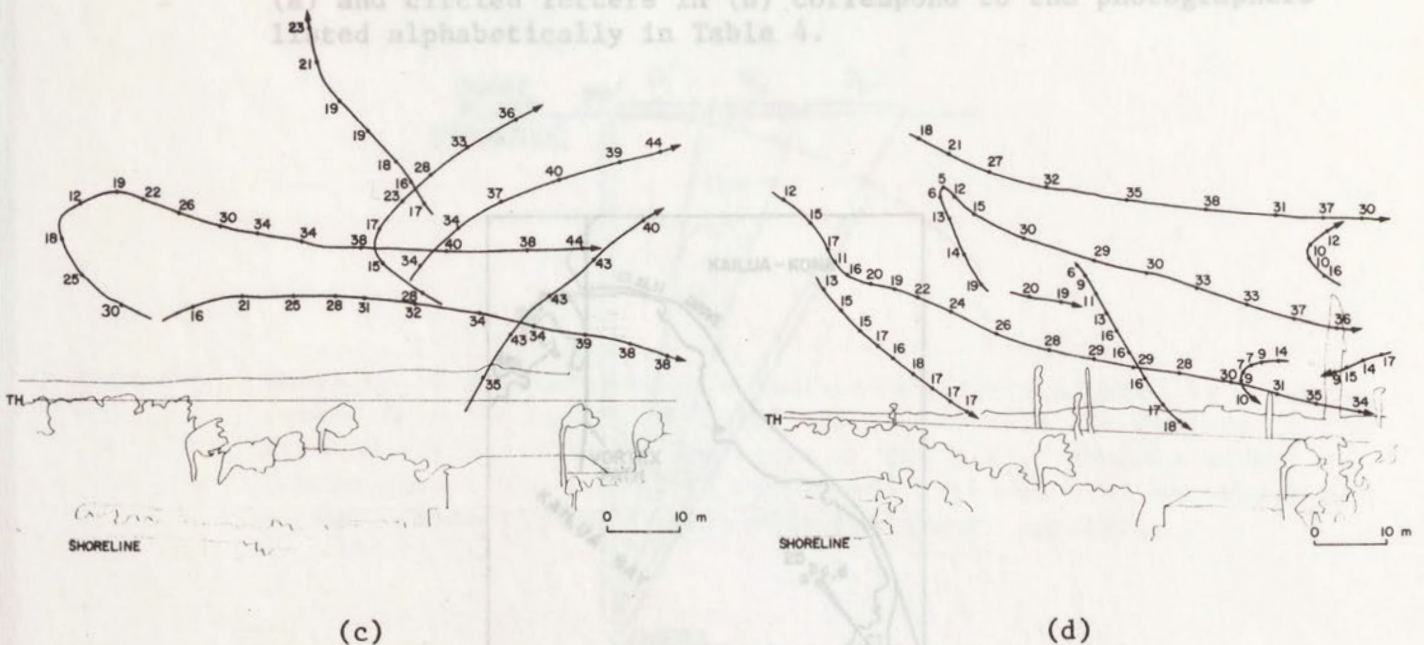


Figure 25. Trajectories of airborne solid debris with net windspeeds plotted in ms^{-1} as the Kailua-Kona vortex moves ashore and travels inland. The observer's true horizon is labelled TH. See text for explanation of trajectories A and B in (a).

Figure A-2. Map of Kona coastline showing path of tornadic waterspout. Photographic landmarks are shaded. Location of the photographer and lines-of-sight to the vortex center in Figure 25 are indicated.

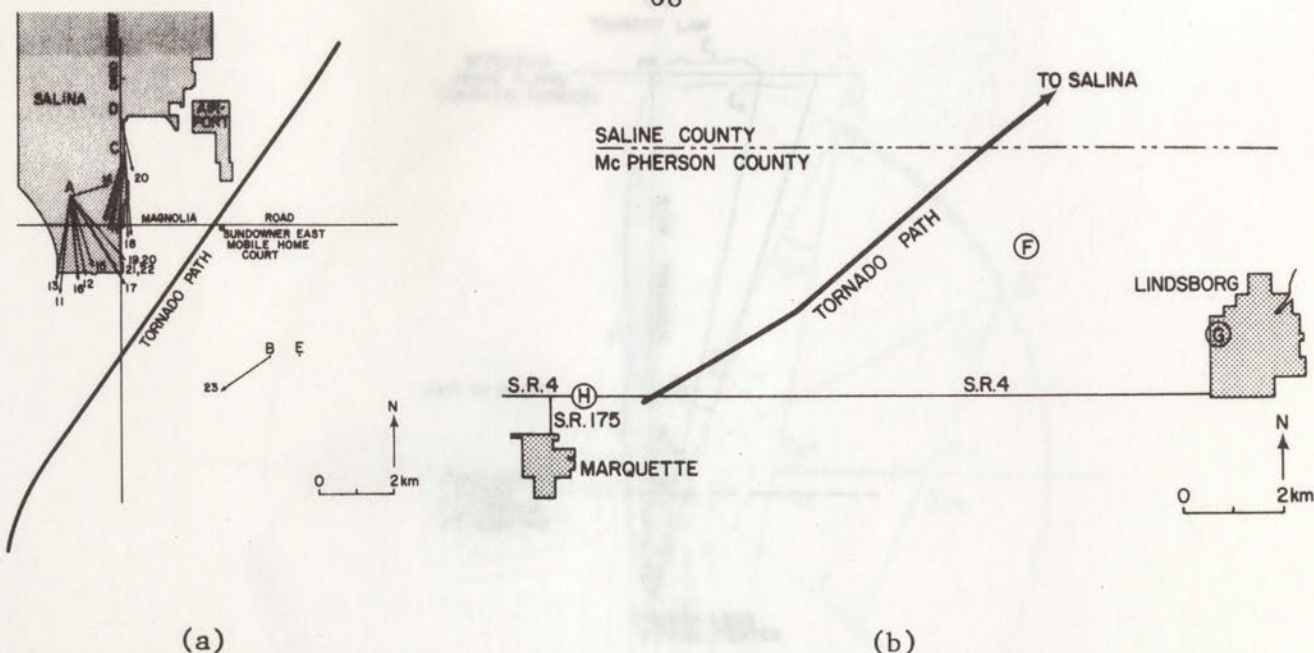


Figure A-1. Maps showing locations of contributing photographers in (a) Salina and (b) Mc Pherson County, Kansas areas. Photographers lines-of-site in (a) correspond to figure numbers in the text. Letters in (a) and circled letters in (b) correspond to the photographers listed alphabetically in Table 4.

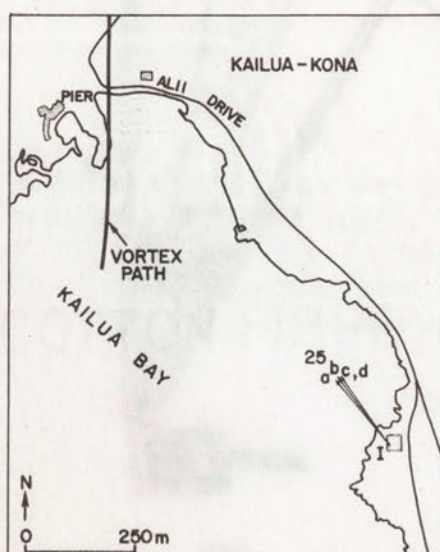


Figure A-2. Map of Kona coastline showing path of tornadic-waterspout. Photographic landmarks are shaded. Location of the photographer and lines-of-sight to the vortex center in Figure 25 are indicated.

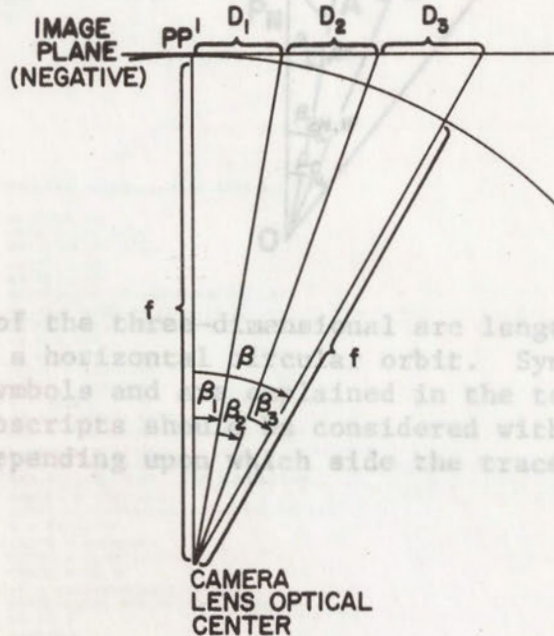
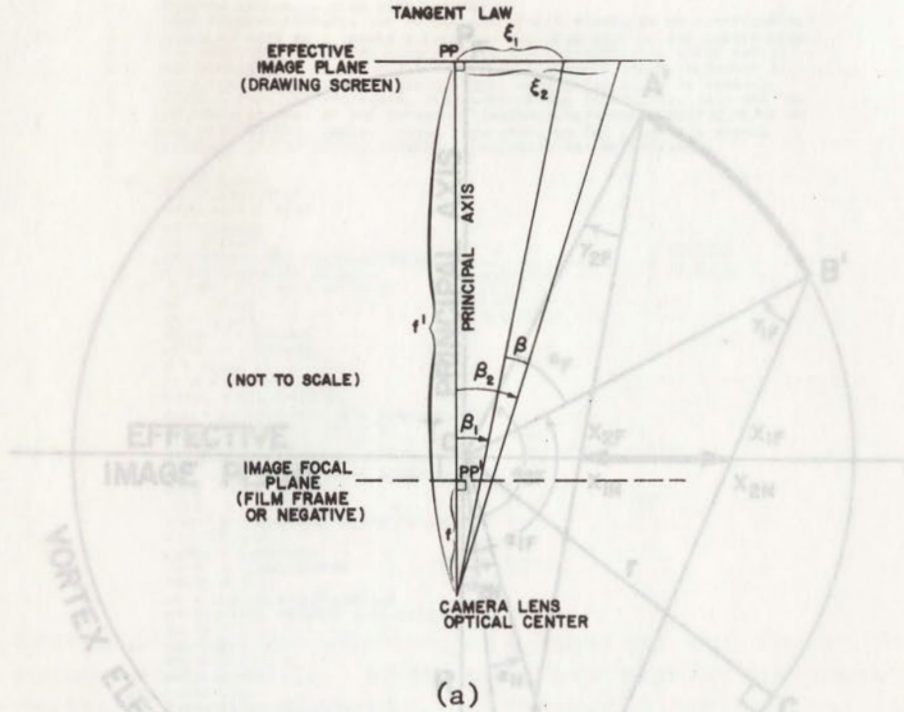


Figure C-1. Geometry of the three-dimensional arc length travelled by a vortex tracer in a horizontal orbit. Symbols are defined in the List of Symbols and defined in the text. Symbols with double subscripts should be considered with only one subscript at a time, depending upon which side the tracer appears.

Figure B-1. Illustrations of (a) the Tangent Law and (b) azimuthal angular distortion of linear distance across the photographic image. Symbols are defined in the List of Symbols.

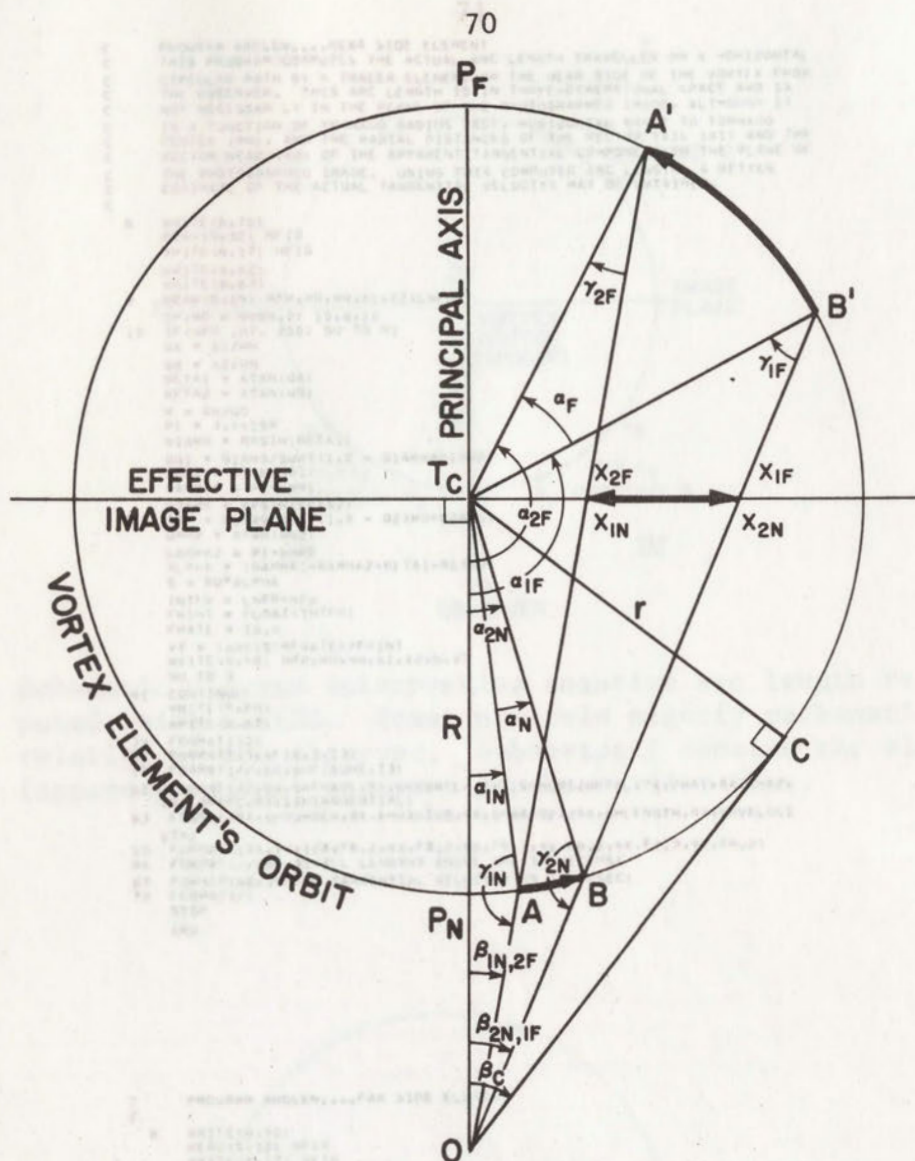


Figure C-1. Geometry of the three-dimensional arc length travelled by a vortex tracer in a horizontal circular orbit. Symbols are defined in the List of Symbols and are explained in the text. Symbols with double subscripts should be considered with only one subscript at a time, depending upon which side the tracer appears.

C PROGRAM ARCLEN...NEAR SIDE ELEMENT
 C THIS PROGRAM COMPUTES THE ACTUAL ARC LENGTH TRAVELLED ON A HORIZONTAL
 C CIRCULAR PATH BY A TRACER ELEMENT ON THE NEAR SIDE OF THE VORTEX FROM
 C THE OBSERVER. THIS ARC LENGTH IS IN THREE-DIMENSIONAL SPACE AND IS
 C NOT NECESSARILY IN THE PLANE OF THE PHOTOGRAPHED IMAGE. ALTHOUGH IT
 C IS A FUNCTION OF TORNAUO RADIUS (RD), HORIZONTAL RANGE TO TORNAUO
 C CENTER (RN), AND THE RADIAL DISTANCES OF THE VECTOR TAIL (X1) AND THE
 C VECTOR HEAD (X2) OF THE APPARENT TANGENTIAL COMPONENT IN THE PLANE OF
 C THE PHOTOGRAPHED IMAGE. USING THIS COMPUTED ARC LENGTH, A BETTER
 C ESTIMATE OF THE ACTUAL TANGENTIAL VELOCITY MAY BE OBTAINED.

```

8  WRITE(6,70)
   READ(5,32) NFIG
   WRITE(6,17) NFIG
   WRITE(6,62)
   WRITE(6,63)
3  READ(5,29) NFN,RD,MN,X1,X2,LNFM
   IF(RD = 9999.0) 10,8,10
10  IF(NFN.GT. 200) GO TO 91
   QA = X1/MN
   QB = A2/RN
   BETA1 = ATAN(QA)
   BETA2 = ATAN(QB)
   H = RN/RD
   PI = 3.14159
   GIANG = R*SIN(BETA1)
   QG1 = GIANG/SQRT(1.0 - GIANG*GIANG)
   GMM1 = ATAN(QG1)
   GAMMA1 = PI-GMM1
   G2ANG = R*SIN(BETA2)
   QG2 = G2ANG/SQRT(1.0 - G2ANG*G2ANG)
   GMM2 = ATAN(QG2)
   GAMMA2 = PI-GMM2
   ALPHA = (GAMMA1-GAMMA2+BETA1-BETA2)
   S = RD*ALPHA
   INTFW = LNFM-NFM
   FRINT = FLOAT(INTFW)
   FRATE = 18.0
   VT = (ABS(S)*FRATE)/FRINT
   WRITE(6,50) NFN,RD,MN,X1,X2,S,VT
   GO TO 3
91  CONTINUE
   WRITE(6,86)
   WRITE(6,87)
12  FORMAT(13)
29  FORMAT(13,4F10.3,13)
17  FORMAT(//,6X,6HFIGURE,13)
62  FORMAT(//,9X,5HFNAME,9X,6HVORTEX,7X,10HORIZONTAL,7X,2HA1,8X,2HX2,
18X,3HARC,9X,10HTANGENTIAL)
63  FORMAT(9X,6NUMBER,8X,6NRADIUS,4X,5HRANGE,29X,6HLENGTH,8X,6HVELOC
ITY)
50  FORMAT(10X,13,10X,F6.2,8X,F8.2,6X,F6.2,4X,F6.2,4X,F7.2,4X,F6.2)
86  FORMAT(//,6X,31HALL LENGTHS ABOVE ARE IN METERS)
87  FORMAT(8X,35HTHE TANGENTIAL VELOCITY IS IN M/SEC)
70  FORMAT(//)
   STOP
   END

```

Figure C-3. Schematic diagram illustrating negative arc length values computed using the program. Roman numerals signify mathematical quadrants relative to the observer. Subscript 1 denotes the element's (apparent) position.

```

C PROGRAM ARCLEN...FAH SIDE ELEMENT
C
8  WRITE(6,70)
   READ(5,32) NFIG
   WRITE(6,17) NFIG
   WRITE(6,62)
   WRITE(6,63)
3  READ(5,29) NFN,RD,MN,X2,X1,LNFM
   IF(RD = 9999.0) 10,8,10
10  IF(NFN.GT. 200) GO TO 91
   QA = X1/RN
   QB = A2/RN
   BETA1 = ATAN(QB)
   BETA2 = ATAN(QA)
   H = RN/RD
   GIANG = R*SIN(BETA1)
   QG1 = GIANG/SQRT(1.0-GIANG*GIANG)
   GMM1 = ATAN(QG1)
   GAMMA1 = ATAN(QG1)
   G2ANG = R*SIN(BETA2)
   QG2 = G2ANG/SQRT(1.0-G2ANG*G2ANG)
   GAMMA2 = ATAN(QG2)
   ALPHA = (GAMMA1-GAMMA2+BETA1-BETA2)
   S = RD*ALPHA
   INTFW = LNFM-NFM
   FRINT = FLOAT(INTFW)
   FRATE = 18.0
   VT = (ABS(S)*FRATE)/FRINT
   WRITE(6,50) NFN,RD,MN,X1,X2,S,VT
   GO TO 3
91  CONTINUE
   WRITE(6,86)
   WRITE(6,87)
12  FORMAT(13)
29  FORMAT(13,4F10.3,13)
17  FORMAT(//,6X,6HFIGURE,13)
62  FORMAT(//,9X,5HFNAME,9X,6HVORTEX,7X,10HORIZONTAL,7X,2HA1,8X,2HA2,
18X,3HARC,9X,10HTANGENTIAL)
63  FORMAT(9X,6NUMBER,8X,6NRADIUS,4X,5HRANGE,29X,6HLENGTH,8X,6HVELOC
ITY)
50  FORMAT(10X,13,10X,F6.2,8X,F8.2,6X,F6.2,4X,F6.2,4X,F7.2,4X,F6.2)
86  FORMAT(//,6X,31HALL LENGTHS ABOVE ARE IN METERS)
87  FORMAT(8X,35HTHE TANGENTIAL VELOCITY IS IN M/SEC)
70  FORMAT(//)
   STOP
   END

```

Figure C-4. Geometry for a distant vortex. The observer is positioned parallel to the principal axis of the vortex. The line of sight through the tracer at point 1 is shown. Symbols are explained in the text.

Figure C-2. Program ARCLEN.

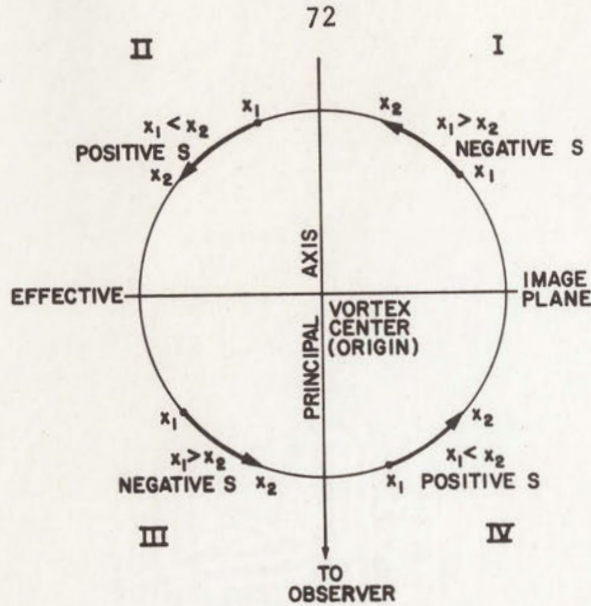


Figure C-3. Schematic diagram interpreting negative arc length values computed using ARCLLEN. Roman numerals signify mathematical quadrants relative to the observer. Subscript 1 denotes the element's (apparent radial) position at the earlier time.

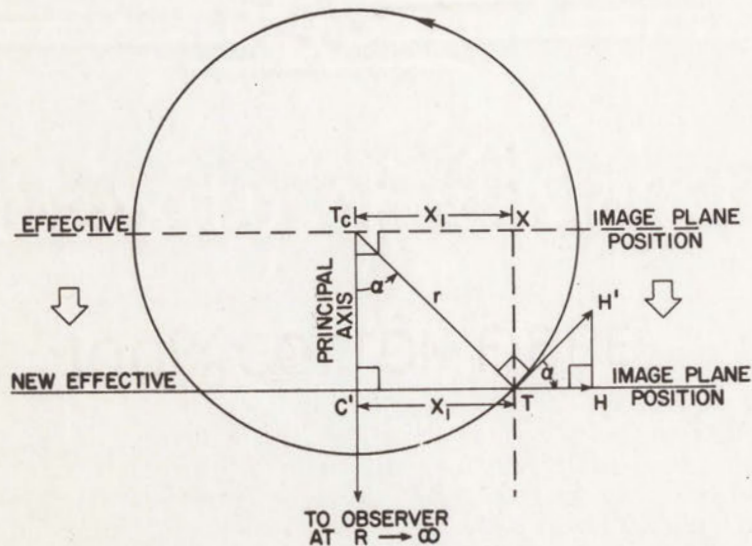


Figure C-4. Geometry of the tangential velocity correction for a distant vortex observer. The dashed line drawn parallel to the principal axis represents the observer's line-of-sight through the tracer at point T. Symbols are explained in the text.

This volume is the property of the University of Oklahoma, but the literary rights of the author are a separate property and must be respected. Passages must not be copied or closely paraphrased without the previous written consent of the author. If the reader obtains any assistance from this volume, he must give proper credit in his own work.

I grant the University of Oklahoma Libraries permission to make a copy of my thesis upon the request of individuals or libraries. This permission is granted with the understanding that a copy will be provided for research purposes only, and that requestors will be informed of these restrictions.

NAME [REDACTED]

DATE 5/5/76

A library which borrows this thesis for use by its patrons is expected to secure the signature of each user.

This thesis by Randal Allan Zipser has been used by the following persons, whose signatures attest their acceptance of the above restrictions.

NAME AND ADDRESS

DATE