UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

HEAT INTEGRATION ACROSS PLANTS IN THE TOTAL SITE

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

By

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Norman, Oklahoma

2001

UMI Number: 3001316

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HEAT INTEGRATION ACROSS PLANTS IN THE TOTAL SITE

A Dissertation APPROVED FOR THE
SCHOOL OF CHEMICAL ENGINEERING AND MATERIALS SCIENCE

BY

To

My parents

With love for looking after me from Heaven

"If you are headed in the right direction, each step, no matter how small,

is getting you closer to your goal."

ANONYMOUS

ACNOWLEDMENTS

I gratefully acknowledge the University of Oklahoma for the funds that were made available for this project. I appreciate the companionship and support of all my friends, especially those that belong or were part of the Process Design and Plant Operations group.

I would like to thank all of the Chemical Engineering Faculty and Staff at the School of Chemical Engineering and Materials Science for sharing their knowledge and providing me with helpful advice. I grateful acknowledge Dr. Roger Harrison, Dr. Dimitrios Papavassiliou, Dr Pakize Pulat, and Dr. Theodore Trafalis for their participation on my dissertation committee. I also would like to thank Dr. Theodore Trafalis for providing me with the necessary tools to successful conduct my research. It is with immense gratitude and appreciation that I acknowledge Dr. Miguel Bagajewicz for his generosity from the first day I arrived to Norman. His advice, guidance, and friendship have been the reasons for my success. I will be forever grateful.

I would like to thank my brothers, my sister-in-law, and my nephew and niece for being always present in my thoughts giving me the strength to persevere in the seeking of my goals. Finally, I would like to thank my girlfriend Paola for her companionship and love. You are and always will be the best thing to ever happen to me.

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ABSTRACT

The quest for energy-saving opportunities has driven academia to develop methods for energy-efficient design of individual plants. Several practical applications of these now-available tools have been proved useful to the industry. On the other hand, the scarce knowledge about the potential energy savings that can be obtained by integration of many plants in a complex (i.e. the "total site") usually has been attributed to difficulties in implementing these savings. Hence, a gap exists between the absence of information about integration in the "total site" and the actual practical instances in which this integration is implemented.

As a step forward in closing this gap, the purpose of this thesis is to discuss heat integration across plants in the "total site". This integration can be accomplished either directly using process streams or indirectly using intermediate fluids. By applying pinch analysis to a system of two plants, it is first shown that the heat transfer that effectively leads to energy savings occurs at temperature levels between pinch temperatures. In some cases, however, heat transfer in other regions is required to attain maximum energy savings. Therefore, a systematic procedure is presented to identify energy-saving targets, and it is followed by a strategy to determine the minimum number of intermediate-fluid circuits needed to achieve maximum energy savings. Next, an MILP problem is proposed to determine the optimum location of these intermediate-fluid circuits. Subsequently, the targets identified are employed in the synthesis of multipurpose heat exchanger networks that are capable of operating each plant stand-alone as well as both plants integrated. An

economic analysis shows that the use of a single intermediate-fluid circuit sometimes can be economically more beneficial than direct integration.

Then, the models previously developed for the two-plant case are extended to many plants. These models lead to the identification of the maximum energy saving targets, establish the minimum number of connections required between the two-plant combinations, and determine the location of independent intermediate-fluid circuits. Alternative solutions exist that allow flexibility for the subsequent design of a multipurpose heat exchanger network. For cases in which the "total site" is partially shut down, the optimal location of multi-operation circuits that allow flexibility of operation is presented. The use of steam as an intermediate fluid is briefly discussed within the heat integration framework. Finally, the new concept of a "heat belt," which is a single pipe circuit used to extract heat from and release it to the plants, is introduced.

CHAPTER 1

Background

1.1 Overview

Since the onset of heat integration as a tool for process synthesis, energy-saving methods have been developed for the design of energy-efficient individual plants. Heat integration across plants (i.e., involving streams from different plants in a complex) has always been considered impractical for various reasons. One of the arguments used is the fact that plants are physically separated from each other and, because of this separation, pumping and piping costs are high. However, an even more powerful argument against integration is the fact that different plants have different startup and shutdown schedules. For example, if integration is done across two plants and one of the plants is put out of service, the other plant may have to resort to an alternative heat exchanger network to reach its target temperatures. Plants may also operate at different production rates that depart from design conditions and require additional heat exchangers to reach desired operating temperatures. All these discouraging aspects of the problem led practitioners and researchers to leave opportunities for heat integration between plants unexplored.

Notwithstanding the aforementioned objections, in several practical instances, these savings opportunities are actually implemented either directly using process streams (Siirola, 1998; Zecchini, 1997) or indirectly through the use of the steam system in what has been called the steam belt (Robertson, 1998). The first attempt to study the recovery of energy through integration between processes was made by Morton and Linnhoff (1984), who considered the overlap of grand composite curves to show the maximum possible heat recovery using steam. Later, Ahmad and Hui (1991) extended this concept to both direct and indirect heat integration, also utilizing the overlapping of grand composite curves. In addition, they proposed a systematic approach to generate different heat recovery schemes for inter-process integration.

The concept of "total site" was introduced by Dhole and Linnhoff (1992) to describe a set of processes serviced by and linked through a central utility system. Using site source and site sink profiles (based on the combination of modified grand composite curves of the individual processes), they set targets for the generation and use of steam between processes. However, the elimination of process-to-process heat exchange zones, also called "pockets," from the grand composite curves of the individual processes reduces the opportunities for energy recovery in certain cases.

One of the questions in total-site integration is whether a process fluid should be used to perform the heat transfer or whether an intermediate fluid should be used. In addition, the question of how to preserve energy efficiency when nonsimultaneous shutdowns take place needs to be addressed. Then the objective is to have a multipurpose design in which both heat integration and independent operation are achievable. In their approach to the problem, Rodera and Bagajewicz (1999a) establish energy targets to calculate the maximum savings for direct and indirect heat integration for the particular case of two plants. The nonsimultaneous operation of both plants is directly related to the ability of the heat exchanger network in both plants to operate in both modes, i.e., integrated and nonintegrated. Therefore, Rodera and Bagajewicz (1999b) propose a design procedure for the synthesis of multipurpose heat exchanger networks based on their previously obtained targets (Rodera and Bagajewicz, 1999a). Extension of the energy saving targets to a "total site" consisting of a system of *n* plants is also addressed by these authors (Bagajewicz and Rodera, 2000a, 2001).

1.2 Direct vs. Indirect Heat Integration

In principle, direct transfer of heat from one plant to another may involve many process streams, which results in many heat exchangers. The incentives to use intermediate fluids are:

- ***Multiple pumps and compressors***: Integration among plants may require the transfer of heat from a number of streams in one plant to a number of streams in the other. Thus, the cost of integration can be high because of the use of multiple pump and compressor units.
- ***Pumping and compression costs***: A fluid with a larger heat capacity than the process streams will result in a smaller flowrate of liquids to be pumped across plants. When process streams are gases, then the installation of compressors to cover large distances can be more expensive than the equivalent pumping of an intermediate fluid.
- ***Safety***: Process streams being pumped large distances may pose a hazard should any spill occur.
- ***Control***: Piping process streams long distances also introduces long delays, which would eventually make process control more difficult. The use of intermediate fluids simplifies the problem.

There are, however, some disadvantages worth mentioning:

- ***Savings***: The use of an intermediate fluid reduces the interval of effective heat transfer (i.e., between extreme pinches) by a multiple of the minimum temperature difference ($\Delta T_{\min}$). Therefore, compared with the direct integration case, smaller savings can be obtained.
- ***Heat Exchangers***: The number of heat exchangers involved in a setup that uses intermediate fluids can also be higher than using direct heat exchange. In this case, in the absence of other incentives, the trade-off is between the new number of heat exchangers and the pumping costs.

In many cases, steam can be used as an intermediate fluid. This offers many possibilities, as the steam system is already in place. Recent work regarding the use of the utility system for the indirect integration of different processes (Hui and Ahmad, 1994) focuses on the generation and use of steam to reduce utility costs. Later, Rodera and Bagajewicz (1999a) introduced targets based on fixed steam pressures that are usually available in the plants.

1.3 Targeting for Energy Savings

1.3.1 Heat Integration across Two Plants

As the starting point in discussing heat integration across plants, the particular case of a system of two plants was considered by Rodera and Bagajewicz (1999a). Their objective was to take advantage of the simplicity of the system and to gain insights into the problem that could then be applied to the general system of *n* plants. By applying pinch analysis to the system of two plants, the authors show that the heat transfer that effectively leads to energy savings occurs at temperature levels between the pinch points of both plants (from right to left with the plants sorted by increasing pinch temperature as shown in Figure 1.1).

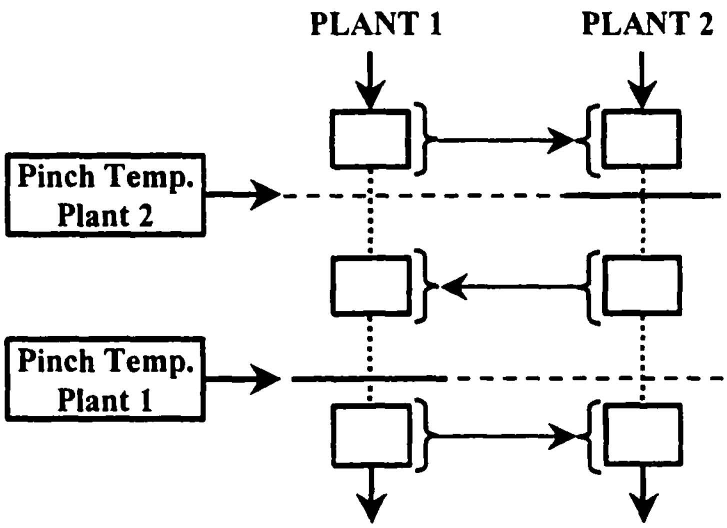

Figure 1.1. Heat integration across two plants

However, in some cases, heat transfer in other regions is required to attain maximum savings. Figure 1.1 shows the possibility of heat transfer above and below both pinch temperatures (from left to right) that debottlenecks the heat cascades of plant 1 and plant 2, respectively. These assisting heat transfers make possible maximum effective heat transfer between pinches.

In their work, Rodera and Bagajewicz (1999a) present a systematic procedure to identify energy saving targets for both the direct and indirect forms of heat integration.

This is followed by the formulation of an MILP problem to determine the optimum location of the intermediate fluid circuits for the indirect integration case. Finally, they propos a strategy to determine the minimum number of intermediate fluid circuits needed to achieve maximum savings. The use of steam as intermediate fluid was briefly discussed by the authors and comparisons with the use of circuits were made.

1.3.2 Generalization for More than Two Plants

The concepts explored by Rodera and Bagajewicz (1999a) can be extended to the case in which many plants are considered for integration. The increase in complexity is evident, because in principle, all possible combinations of two plants have to be evaluated. As a starting point, consider the example of heat integration across three plants. First, the plants are ordered by increasing pinch temperatures, and the highest and lowest pinches are identified. For the unassisted cases, the zone delimited by these pinches is the region in which all the integration can take place. The three possible ways of heat transfer are shown in Figure 1.2.

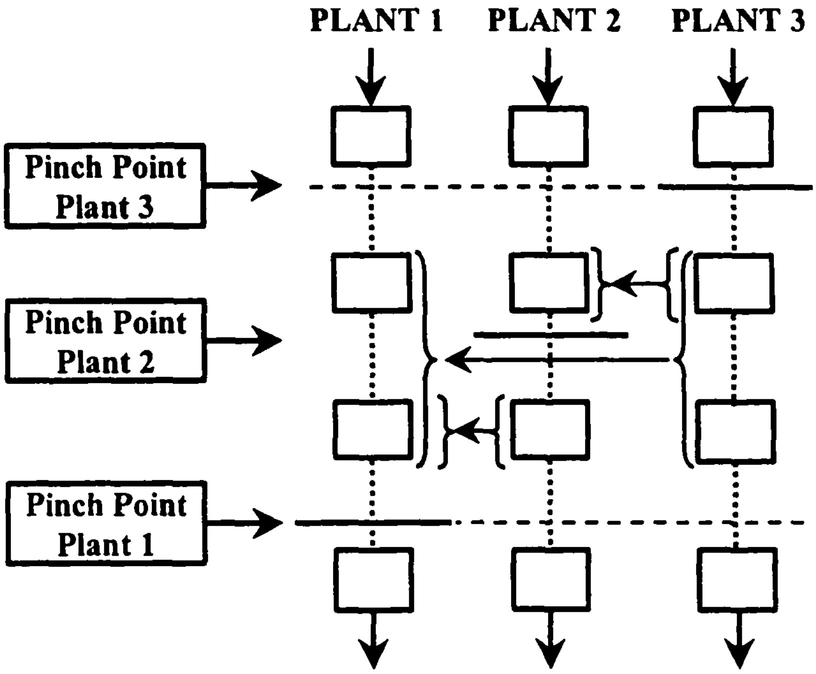

Figure 1.2. Heat integration across three plants

In turn, assisted cases will require transfer of heat in the zones above or below the pinches of the plants involved in the assistance. To predict maximum possible savings, a reformulation of the mathematical programming model by Rodera and Bagajewicz (1999a) is required to account for each one of the combinations of two plants and the respective directions in which the heat will be transferred. The results of this problem are useful targets for models that will determine the heat exchanger network needed to accomplish the predicted savings.

In the case of indirect integration, new shifts of scales are required. As a first approach solution, three circuits can be established that account for the three possible combinations of two plants in the unassisted cases. A downward shift is performed in the temperature scale of the second and third plants to make transfer heat feasible from the first plant. In turn, a second downward shift is needed in the third plant in order to transfer heat from the second plant. Other alternatives may consist of the use of a single circuit that branches prior to entering plant 2 and plant 3, picks up the required heat in each of these plants, and then performs a similar split when the heat is to be released. Therefore, the concepts and tools developed by Rodera and Bagajewicz (1999a) for the two-plant system are a stepping-stone for the generalized integration of a set of *n* plants.

1.4 Design of Multipurpose Heat Exchanger Networks

Rodera and Bagajewicz (1999a) discuss the opportunities for direct and indirect integration and propose methods to determine how much savings can be accomplished using intermediate fluids. A methodology is presented to determine the minimum number of intermediate fluid circuits needed to achieve maximum savings. While all these studies determine the target savings, there is still a need to synthesize a heat exchanger network that can accomplish minimum energy consumption while the plants are integrated as well as when they are functioning separately. This must take place at a minimum investment cost. The problem is also constrained by the fact that the same heat exchanger network should operate satisfactorily for a stand-alone plant as well as when it is integrated.

Ahmad and Hui (1991) propose the overlapping of grand composite curves for targeting and discusse the use of mathematical programming to address the integration. They introduced a modification of the objective function used in the transshipment model

(Papoulias and Grossmann, 1983) by considering weighting factors for those matches that are established between plants. However, they only mention the limitations of the model in predicting cyclic matches and do not further analyze the complications that arise in the construction of the heat exchanger networks. In addition, they do not guarantee the flexibility of stand-alone operation of each plant.

In a recent work, Rodera and Bagajewicz (1999b) employed the targets that they previously identified (Rodera and Bagajewicz, 1999a) in the synthesis of multipurpose heat exchanger networks that are capable of operating each plant in a stand-alone mode as well as integrating the plants. The authors present several mathematical programming models to design these multipurpose heat exchanger networks and consider both forms of integration (i.e., direct integration using process streams and indirect integration using intermediate fluids). The proposed models feature the minimum number of units and account for unassisted and assisted forms of integration. Although better heat exchanger models can be used, the simplicity of models featuring maximum energy recovery and the minimum number of units allowed the authors to discuss the complexity of the problem in a more straightforward fashion. Moreover, in an economical analysis, Rodera and Bagajewicz (1999c) show that the use of intermediate fluids sometimes can be economically more beneficial than direct integration.

1.5 Targeting for Energy Savings in the Total Site

Total site integration is the name coined when referring to the complex problem of heat integration across plants that can be accomplished either directly, using process streams, or indirectly, using intermediate fluids such as steam or dowtherms. Early studies by Dhole and Linnhoff (1992) and Hui and Ahmad (1994) on total site heat integration help to determine levels of generation of steam to integrate indirectly different processes. Because the generation and use of steam has to be performed at a fixed temperature level, opportunities for integration are sometimes lost. Rodera and Bagajewicz (1999a) developed targeting procedures for direct and indirect integration in the special case of two plants and demonstrated the drawbacks of using steam as an intermediate fluid. The heat transfer that effectively leads to savings is demonstrated to occur at temperature levels between the pinch points of both plants by applying pinch analysis (Rodera and Bagajewicz, 1999a). In some other cases, however, heat transfer in the external regions is also required to attain maximum savings (assisted heat integration). The use of cascade diagrams for each plant allows for the detection of unassisted and assisted cases. Distinction between these two cases is not accomplished by procedures that make use of combined grand composite curves (it was overlooked by Dhole and Linnhoff, 1992) or by methods developed to determine heat transfer between zones (Ahmad and Hui, 1991; Amidpour and Polley, 1997). In addition, Rodera and Bagajewicz (1999b,c) presented a methodology to design multipurpose heat exchanger networks that can realize these savings and function in the two scenarios, i.e., integrated and not integrated.

Extensions to *n* plants of LP and MILP models previously developed for the two-plant case (Rodera and Bagajewicz, 1999a) are presented by Bagajewicz and Rodera (2000a, 2001). First, they generalize the LP model to consider all possible heat transfer across pairs of plants that leads to savings. This formulation identifies energy-saving targets for direct and indirect integration by determining the amounts of heat to be transferred within established temperature intervals. Then, Bagajewicz and Rodera (2001) introduce an MILP model that makes use of these targets to establish the minimum number of connections between the two-plant combinations. For indirect integration, the original MILP model that locates single intermediate fluid circuits between two plants is generalized by Bagajewicz and Rodera (2000a, 2001) to locate independent circuits between any pair of plants. Its computational burden can be diminished by a reformulation that decomposes the model into the heat that enters and exits the circuits leading to a reduction of the number of heat intervals (Bagajewicz and Rodera, 2001). Finally, Bagajewicz and Rodera (2000a, 2001) show that the optimal location of these circuits in order to allow flexibility of operation can be easily added to these formulations.

1.6 The "Heat Belt" alternative

As previously discussed, indirect heat integration across plants via intermediate fluids like steam or dowtherms is, in many cases, a preferable alternative to direct integration using process streams. Bagajewicz and Rodera (2000a, 2001) discuss the use of independent circuits to transfer heat between the two-plant combinations of a "total site" composed of *n* plants. Moreover, by adding to their formulation cases in which any of the plants are shut down, the authors consider the optimal location of multi-operation circuits that allow flexibility of operation.

A reduction of the piping and pumping costs can be expected if a single pipe collects and delivers heat to and from the plants. In cases in which independent circuits transfer heat from the same plant to many other plants, a pair of pipes has to be used for each transfer. Additionally, more heat exchangers may be necessary. The relative location of the plants to each other also plays an important role. Simplicity in many aspects can then be obtained by using a single belt system that takes advantage of the existing location of the plants. Rodera and Bagajewicz (1999c) present a case study on integration across two plants that supports this idea. In their study, piping and pumping costs are such that the use of one circuit instead of two is more economical, even though one circuit does not achieve all the possible energy savings.

In a new approach to the problem, Bagajewicz and Rodera (2000b) introduce the concept of a heat belt as an alternative to the use of independent or multi-operation circuits. This heat belt consists of a single pipe circuit used to extract heat from and release it to the plants. The concept is derived by restricting multi-operation circuits to the use of a single pipe arrangement (Bagajewicz and Rodera, 2000a, 2001). The analysis is restricted to three plants and unassisted heat integration cases.

1.7 Objectives of Research

The objectives of this research are:

(a) to investigate the theoretical and practical aspects of the heat integration across plants in the "total site", and

(b) to develop mathematical programming model formulations to solve the problem.

1.8 Structure of Dissertation

- Chapter 2 presents a systematic mathematical programming procedure to evaluate targets for the energy savings that can be attained by direct or indirect heat integration across plants. As the starting point of this study, only the particular case of a system of two plants is considered.
- In Chapter 3, the already identified targets for the system of two plants are employed in the synthesis of multipurpose heat exchanger networks capable of operating each plant stand-alone as well as both plants integrated. The models presented feature the minimum number of units for both direct and indirect integration.
- Chapter 4 extends the results originally developed for the particular case of two plants to the general system of *n* plants. Generalized mathematical programming models are presented to evaluate the energy savings targets for the "total site."
- In Chapter 5, the concept of a "heat belt" is introduced to take advantage of the use of a single-pipe intermediate fluid circuit to heat integrate the whole system of *n* plants. A mathematical programming model to locate the heat-belt circuit for the particular case of three plants is presented.
- Finally, Chapter 6 summarizes the accomplished results and discusses planning for the implementation of energy savings in the "total site."

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CHAPTER 2

Targeting Procedures for Energy Savings by Heat Integration across Plants

2.1 Introduction

In this chapter, pinch analysis is used to establish target maximum energy savings for either direct or indirect integration for the case of two plants. First, temperature intervals where heat transfer should take place are identified, together with the identification of which plant should be the source. Then, an LP problem is set up to determine these targets. The design of the intermediate fluid circuits is considered next. The possibility of using a single intermediate fluid circuit is then evaluated. Finally, an MILP model is introduced to determine the location of the minimum number of circuits needed to achieve the target savings. To illustrate these concepts, examples using heat integration problems from the literature are solved. In addition, an example consisting of the integration between a crude unit and an FCC plant is solved.

2.2 Maximum Transferable Heat

Consider two plants and suppose that minimum utility targets are obtained independently for each of the plants using LP transportation or transshipment models (Cerdá et. al, 1983; Papoulias and Grossmann, 1983). When all the streams from both plants are included in the same set (combined plant), the total minimum heating is usually lower.

2.2.1 Location of the Combined-plant Pinch Temperature

Without loss of generality, assume that plant 2 has a pinch temperature that is higher than the pinch temperature of plant 1. Therefore, the pinch temperature of the combined plant can fall in any of these three regions: (a) above the pinch temperature of plant 2, (b) between pinch temperatures, or (c) below the pinch temperature of plant 1 (Figure 2.1).

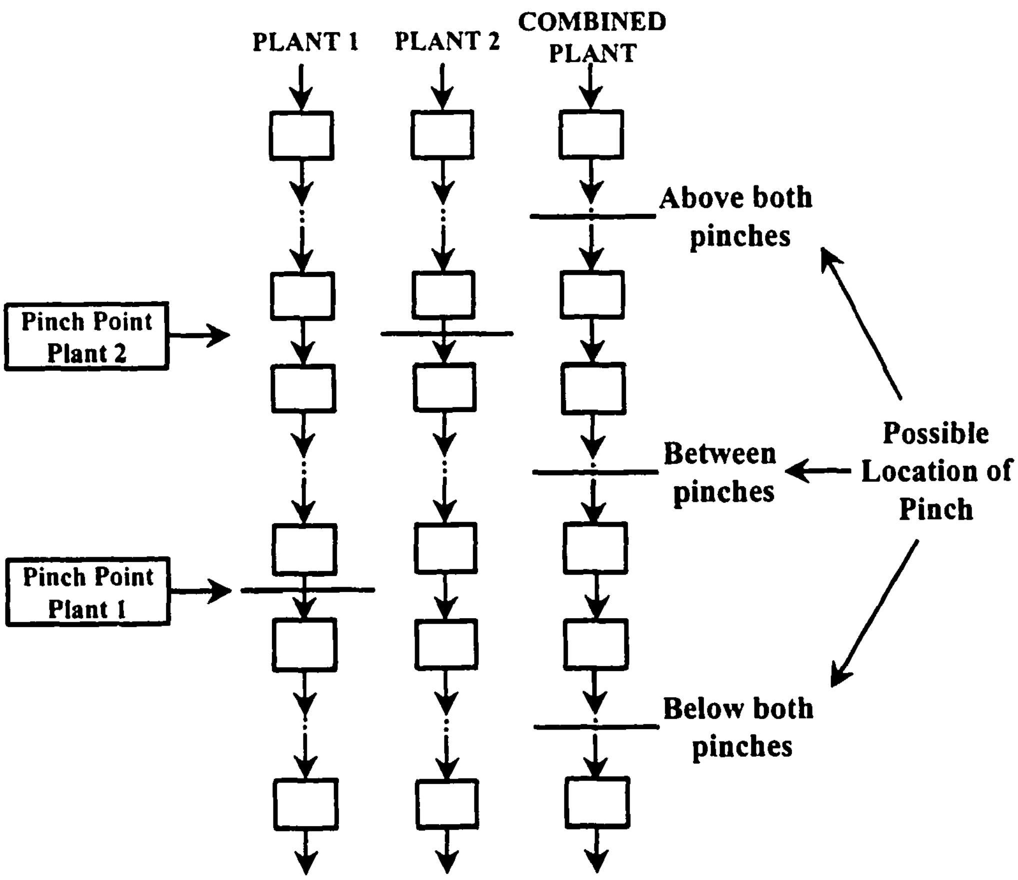


Figure 2.1. Possible location of the combined-plant pinch temperature

Consider first the case in which all intervals above the respective pinch temperatures of the individual plants are sinks of heat. If the corresponding intervals of the plants are added, the combined plant will also have sinks of heat in these intervals. Therefore, the system considered has a combined-plant pinch temperature located no higher than the original pinch temperature of plant 2. A similar analysis can be made for the region below the pinch temperature of plant 1. When the above conditions are not met, the combined-plant pinch temperature can be located in any place. The implications of this will be further investigated.

The first intuitive conclusion one can make is that heat should be transferred at temperatures between the pinch temperatures of the original plants. Indeed, this is the only region where plant 2 is a heat source while plant 1 is a heat sink. This intuitive conclusion is in principle correct, but sometimes heat transfer in the opposite direction (from plant 1 to plant 2) above or below the region between plant pinch temperatures is required to assist the realization of maximum savings.

2.2.2 Transfer of Heat outside the Region between Pinch Temperatures

Either transferring heat above the higher pinch temperature or below the lower pinch temperature does not decrease the utility usage. Only an equivalent amount of corresponding utility is shifted from one plant to the other. Figure 2.2 illustrates the effect of an amount of heat Q_A transferred from plant 2 to plant 1 in the upper zone (without loss of generality, intervals are lumped to allow clarity of illustration).

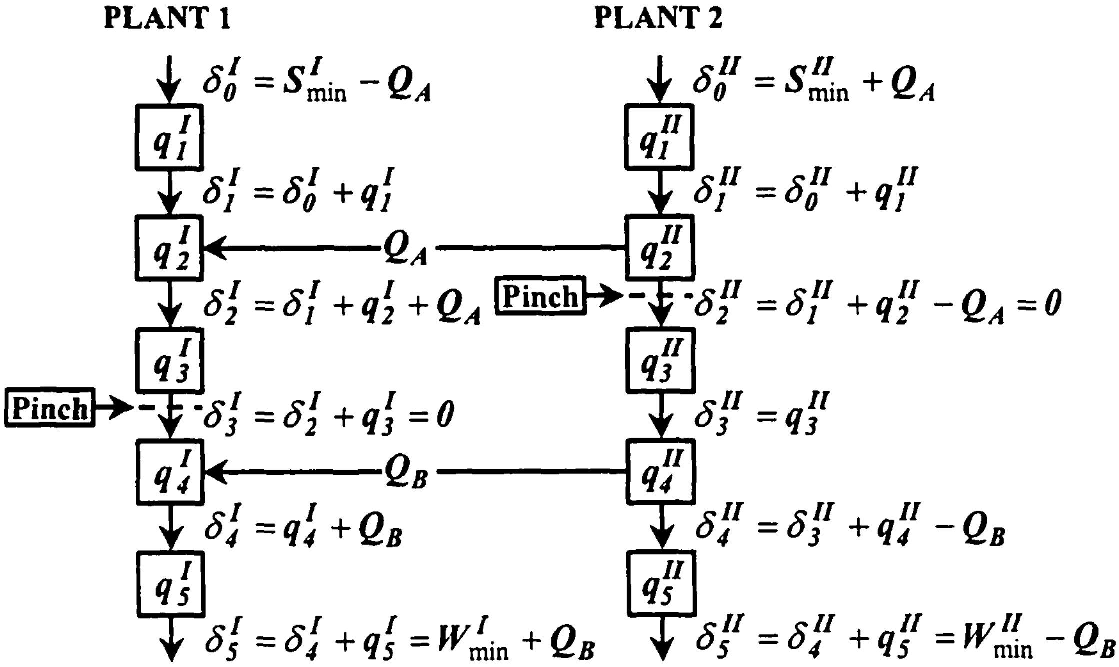


Figure 2.2. Effect of transferring heat outside the region between pinch temperatures

An increase of the amount of heating utility in the plant that releases the heat is followed by a reduction of the same amount of heating utility in the plant that receives the heat. The same effect on the cooling utility needs is observed if a certain amount of heat Q_B is transferred from plant 2 to plant 1 in the lower zone (Figure 2.2). In addition, as the temperature level at which the heat transfer in the upper zone takes place is lowered, a maximum amount that can be transferred exists. For example, if the transfer is made in the first interval, the amount that can be transferred is constrained by the original utility usage of plant 1, $S^{I}_{\min}$. If the heat is transferred in an interval below the first one, in interval i for example, the upper limit will be smaller because some of the heating utility used by plant 1 is used to satisfy the heat demand of the first i-1 intervals. Therefore, to compute this upper limit, one should subtract from $S^{I}_{\min}$ all the intervals that are heat sinks (negative values) above the interval of transfer. Similar upper limits for the transfer of heat are found if the lower zone is considered.

In conclusion, no savings can be obtained by transferring heat in the regions above the higher pinch temperature or below the lower pinch temperature. However, transfer from plant 1 to plant 2 in these regions is needed in some cases to facilitate the transfer of heat in the region between both pinch temperatures. This is explored next.

2.2.3 Transfer of Heat between Pinch Temperatures

As illustrated in Figure 2.3, a certain amount of heat Q_E is transferred from plant 2 to plant 1 between pinch temperature. This transfer has the effect of reducing the heating utility in plant 1 and cooling utility in plant 2. In addition, transferring heat from plant 2 to plant 1 has the effect of reducing the lowest level of the heating utility demand on plant 1, which is usually the cheapest. Finally, it has no effect on the heating utility demand of plant 2 or the cooling utility of plant 1.

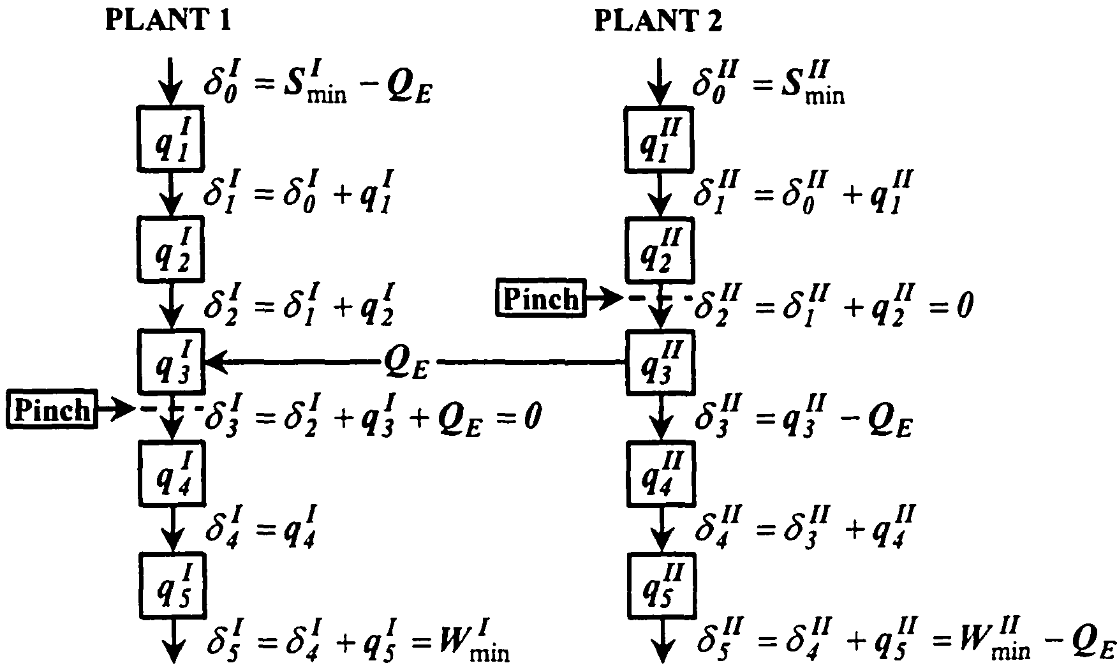


Figure 2.3. Effect of transferring heat in the region between pinch temperatures

2.2.4 Assisted and Unassisted Heat Transfer

We will now investigate the upper limits for the heat that plant 1 can accept and the upper limits for the heat that plant 2 can deliver in the region between pinches. Consider the case in which there are only sink intervals above the pinch temperature of plant 2 and only source intervals below the pinch temperature of plant 1. The maximum heat that plant 1 can receive is the actual sum of the demands it has in the intervals between pinch temperatures. Similarly, the maximum amount that plant 2 can transfer is the resulting available heat that it has between pinch temperatures. *Since any heat that is* *transferred to plant 1 at any temperature interval can be cascaded down to lower temperatures, the real limitation on how much can be transferred is given by the ability of plant 2 to fulfill the demand at each interval.* Because all the intervals above the pinch temperature of plant 2 are sinks, the whole demand of heat in plant 1 is only satisfied by utility or by plant 2 from the intervals between pinch temperatures. Likewise, since all intervals below the pinch temperature of plant 1 are sources of heat, plant 2 does not need to use heat from the intervals between pinch temperatures to satisfy any demand below the pinch temperature of plant 1. Therefore, the amount of heat that can be transferred to plant 1 is not limited by such demand. This motivates the following definition:

Definition: *Unassisted Heat Transfer across Plants* takes place when only heat transfer between pinch temperatures is needed to achieve maximum savings.

When plant 1 has only sink intervals above the pinch temperature of plant 2 and only source intervals below the pinch temperature of plant 1, it is special case of unassisted heat transfer across plants. Unassisted cases can also take place even though some intervals in plant 1 are sources of heat above the pinch temperature of plant 2 or some intervals in plant 2 are sinks of heat below the pinch temperature of plant 1. If a case is unassisted, the combined-plant pinch temperature lies between pinch temperatures. Indeed, the addition of all intervals and the fact that there is heat transfer across the location of the pinch temperature of plant 2 indicates that the combined plant pinch temperature does not lie above this temperature. The same can be said for the region below the pinch temperature of plant 1.

Assume now that some of the intervals in plant 1 above the location of the pinch temperature of plant 2 are sources of heat. Furthermore, assume such heat sources are enough to produce a surplus that in the absence of integration among plants is effectively transferred in plant 1 through the location of the pinch temperature of plant 2. In other words, the surplus of heat above the pinch temperature of plant 2 needs to be used to satisfy the heat demand of plant 1 between pinch temperatures. In turn, this may limit the amount that can be transferred from plant 2, and therefore limit the maximum savings that can be obtained. To prevent such limitation, one can transfer the surplus heat from plant 1 to plant 2, reducing the heating utility of plant 2, and allowing maximum heat transfer between pinch temperatures. The heat transfer outside the region between pinch temperatures does not realize any savings, only shifts utility load from one plant to the other. In fact, if the surplus is larger than the heating utility of plant 2, the amount Q_A is limited by $S^{II}_{\min}$, and the surplus may become an effective limitation to realize all the potential for savings. Similarly, if the heat demand of plant 2 in the corresponding intervals is not sufficiently large, the total surplus that can be transferred is limited. An exact symmetric case happens below the pinch of plant 1. Some of the surplus from plant 1 below its pinch temperature can eventually be used to satisfy this demand and therefore to free the heat from plant 2 which will be completely available to realize savings through transfer to plant 1 between pinch temperatures.

These two cases motivate the following definition:

Definition: *Assisted Heat Transfer across Plants* takes place when heat transfer between pinch temperatures needs to be assisted by heat transfer outside this region to attain maximum savings.

The existence of assisted cases has been overlooked by Ahmad and Hui (1991) who only showed that sometimes, more than one steam level is required for maximum indirect recovery between processes. However, they do not further explore the significance of the assisted transfer in order to realize maximum savings. Dhole and Linnhoff (1992) constructed site-source and site-sink profiles based on the combination of modified grand composite curves of the individual processes. In these modified curves, process-to-process heat integration zones or "pockets" are eliminated. Consequently, in the presence of an assisted heat integration case, opportunities for realizing maximum savings are lost and only limited savings between pinch temperatures can be pursued.

A model to predict the exact amount of heat that needs to be transferred in each region will be presented later. First, some illustrative examples are shown.

2.2.5 Unassisted and Assisted Heat Integration Examples

2.2.5.1 Example 2.1

Table 2.1 shows the interval balances, the heat cascade to determine the utilities, and the actual value of these utilities for each of the plants as well as for the combined plant. Sink intervals are located above the pinch temperature and source intervals are located below the pinch temperature in either plant 1 or plant 2. Therefore, this is an unassisted case and only transfer between pinch temperatures is needed in order to obtain maximum savings. These savings are obtained by subtracting the combined-plant utility from the sum of the individual utilities. Pinch temperature locations are shown with filled lines. As expected, the combined-plant pinch temperature is in between the original plant pinch temperatures.

Table 2.1. Example 2.1

Temp. scale	Plant 1			Plant 2			Combined Plant		
	q_i^I	γ_i^I	S_{min}^I	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
140									
120	-12	-12	**30**	-19	-19	**20**	-31	-31	**38**
100	-1	-13		-1	-20		-2	-33	
80	-15	-28	W_{min}^I	10	-10	W_{min}^{II}	-5	-38	W_{min}^{CP}
60	-2	-30		5	-5		3	-35	
40	2	-28	**2**	1	-4	**16**	3	-32	**6**
Maximum potential savings between pinches =12						Maximum possible savings =12			

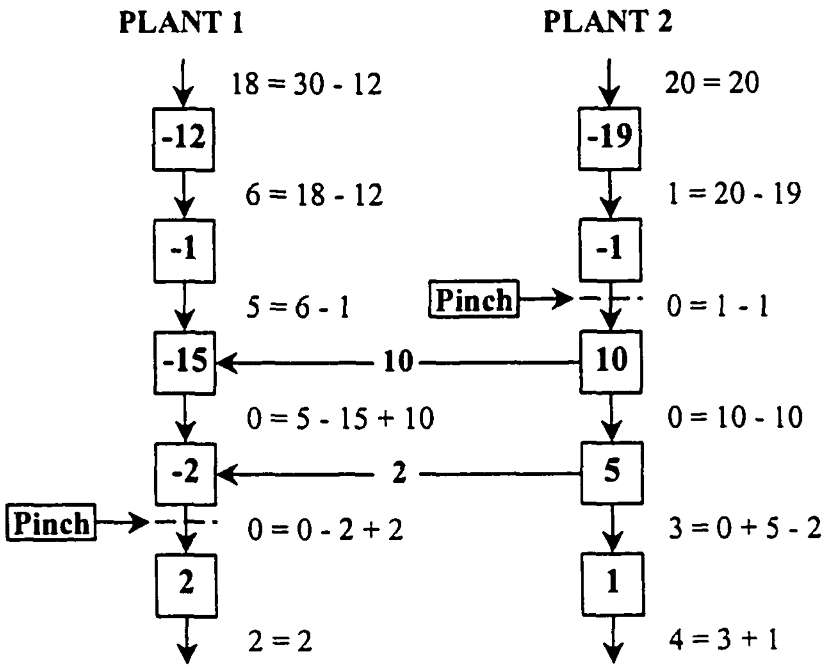


Figure 2.4. Cascade diagram solution for Example 2.1

Figure 2.4 shows the cascade diagram solution after the integration is conducted. In order to compare the results obtained using the cascade diagram with methods that make use of grand composite curves, the approach of Ahmad and Hui (1991) is employed. Figure 2.5 shows the countercurrent profiles for the grand composite curves of the two plants. The grand composite curve of plant 1 has been inverted to be able to establish the maximum amount of direct heat transfer. The extent of the maximum possible savings is reached whenever the profiles coincide in a point as shown. Unassisted cases are therefore readily tractable with the reported method.

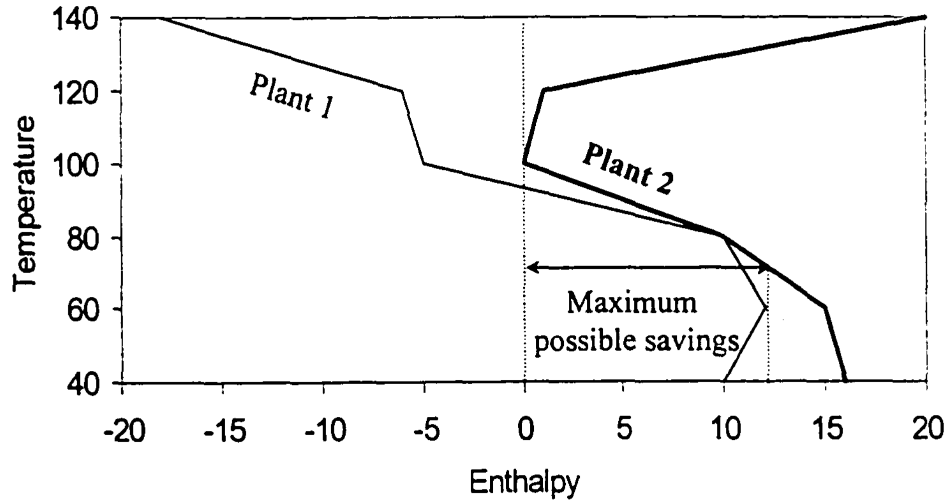


Figure 2.5. Countercurrent composite curve profiles for Example 2.1

2.2.5.2 Example 2.2

Table 2.2 presents the data corresponding to Example 2.2. A source interval is located in the region above the pinch temperature of plant 2 (higher pinch) in plant 1. This source interval prevents plant 1 from receiving all the potential heat available to be transferred between pinch temperatures. However, a transfer of the necessary amount of heat from plant 1 to plant 2 above the higher pinch temperature allows maximum potential savings to be realized. Assisted cases below the two pinch temperatures are similar in nature and therefore examples are omitted. The combined-plant pinch temperature lies between pinch temperatures. This is a result of the fact that all the limitation for transfer between pinch temperatures can be completely removed.

Table 2.2. Example 2

Temp. scale	Plant 1			Plant 2			Combined Plant		
	q_i^I	γ_i^I	S_{min}^I	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
140									
120	-7	-7	**20**	-10	-10	**20**	-17	-17	**23**
100	-5	-2	W_{min}^I	-10	-20	W_{min}^{II}	-5	-22	W_{min}^{CP}
80	-15	-17		14	-6		-1	-23	
60	-3	-20		10	4		7	-16	
40	3	-17	**3**	5	9	**29**	8	-8	**15**
Maximum potential savings between pinches =17						Maximum possible savings =17			

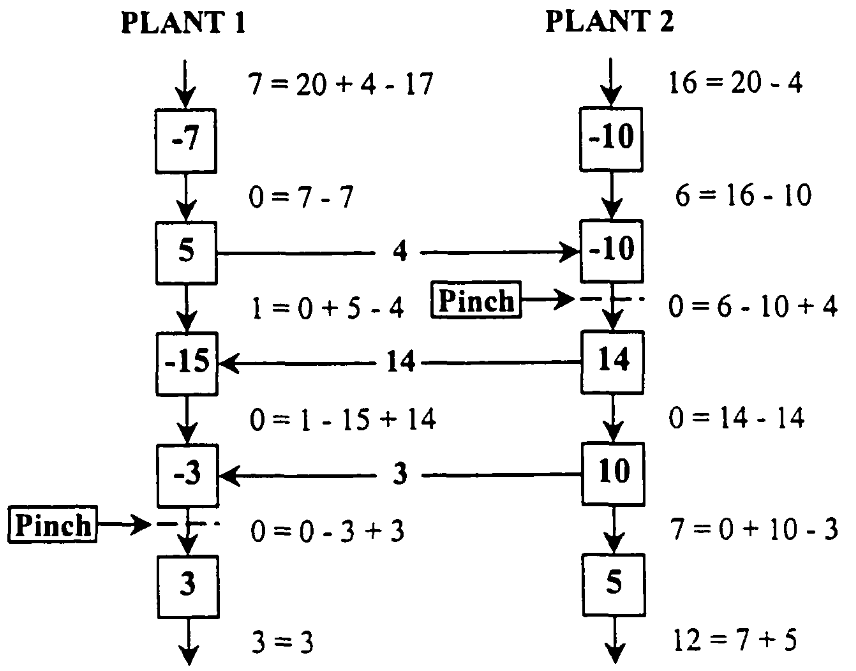
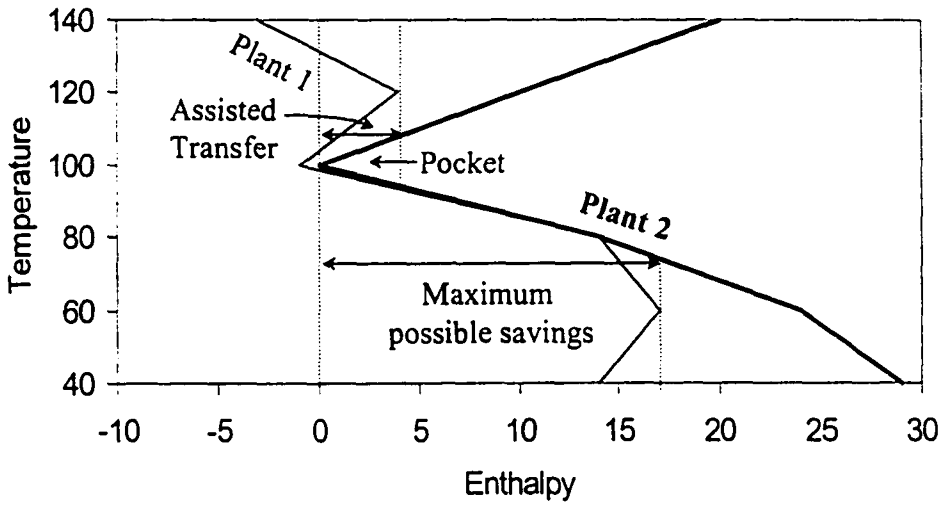
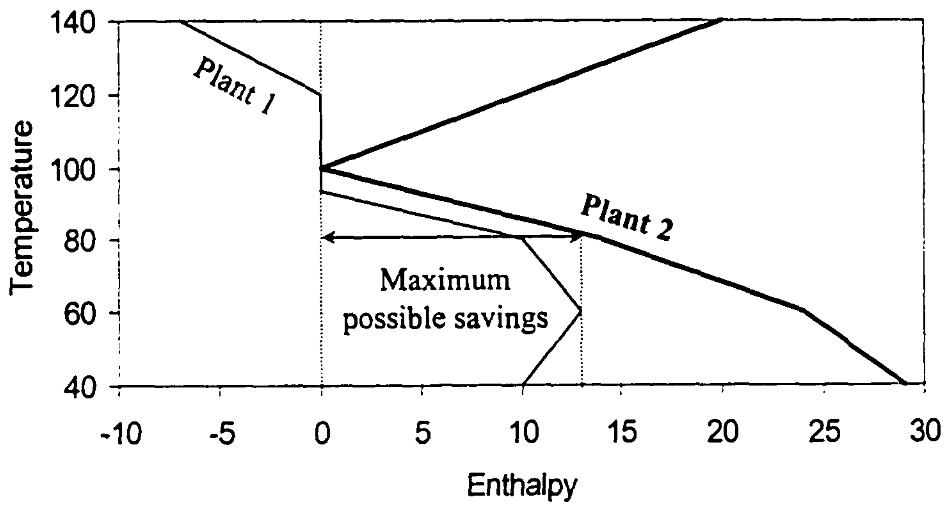


Figure 2.6. Cascade diagram solution for Example 2.2

Figure 2.6 shows the cascade diagram solution after the integration is conducted. When a comparison with the method that uses grand composite curves is performed, the diagram of Figure 2.7a is obtained.



(a)



(b)

Figure 2.7. Countercurrent composite curve profiles for Example 2.2

The pocket present in the composite curve of plant 1 has not been removed since it makes the assisted transfer to plant 2 possible and allows full transfer of heat between pinch temperatures. These situations have been overlooked by Ahmad and Hui (1991). Moreover, in the procedure introduced by Dhole and Linnhoff (1992) to indirectly integrate the total site through the utility system, pockets are eliminated prior to the construction of the site source and site sink profiles. Therefore, whenever the pockets are eliminated, the possibility of realizing maximum savings has been lost. This is illustrated in Figure 2.7b.

2.2.5.3 Example 2.3

Table 2.3 presents the data for Example 2.3. A source interval in plant 1 is found in the region above the pinch temperatures of plant 2. Thus, this is an assisted heat integration case. However, a limit imposed by plant 2 arises in the heat that plant 1 can transfer above the higher pinch temperature. Therefore, the limitation to obtain maximum potential savings cannot be totally removed.

Table 2.3. Example 2.3

Temp. scale	Plant 1			Plant 2			Combined Plant		
	q_i^I	γ_i^I	S_{min}^I	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
140									
120	-18	-18	20	-19	-19	20	-37	-37	37
100	5	-13		-1	-20		4	-33	
80	-5	-18	W_{min}^I	10	-10	W_{min}^{II}	5	-28	W_{min}^{CP}
60	-2	-20		5	-5		3	-25	
40	2	-18	2	1	-4	16	3	-22	15
Maximum potential savings between pinches =7						Maximum possible savings =3			

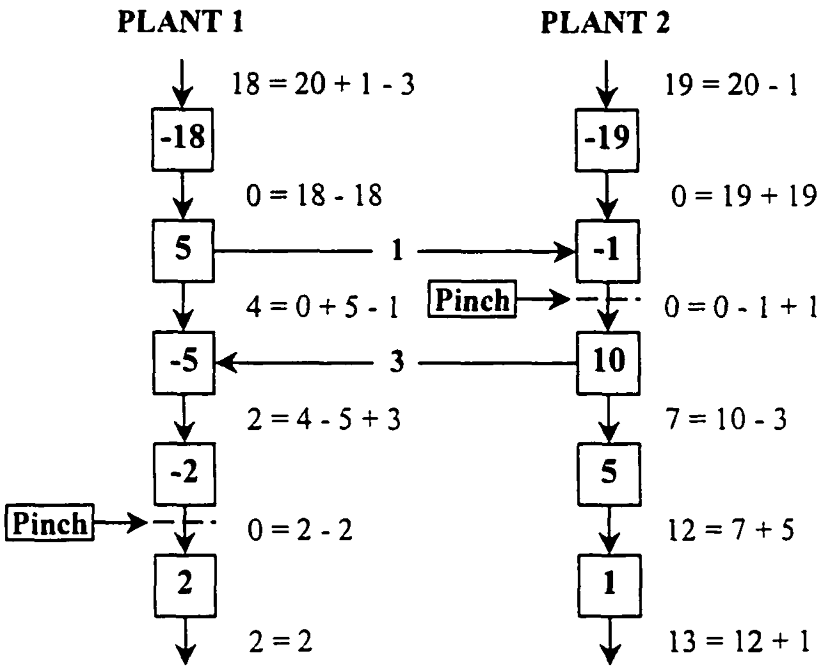


Figure 2.8. Cascade diagram solution for Example 2.3

Figure 2.8 shows the cascade diagram solution after the integration is conducted. The composite curves for this example are shown in Figure 2.9a. In this case, the presence of a pocket in plant 1 allows the partial removal of the limitations in the transfer between pinch temperatures. The elimination of this pocket prevents the realization of the maximum possible savings as it is shown in Figure 2.9b.

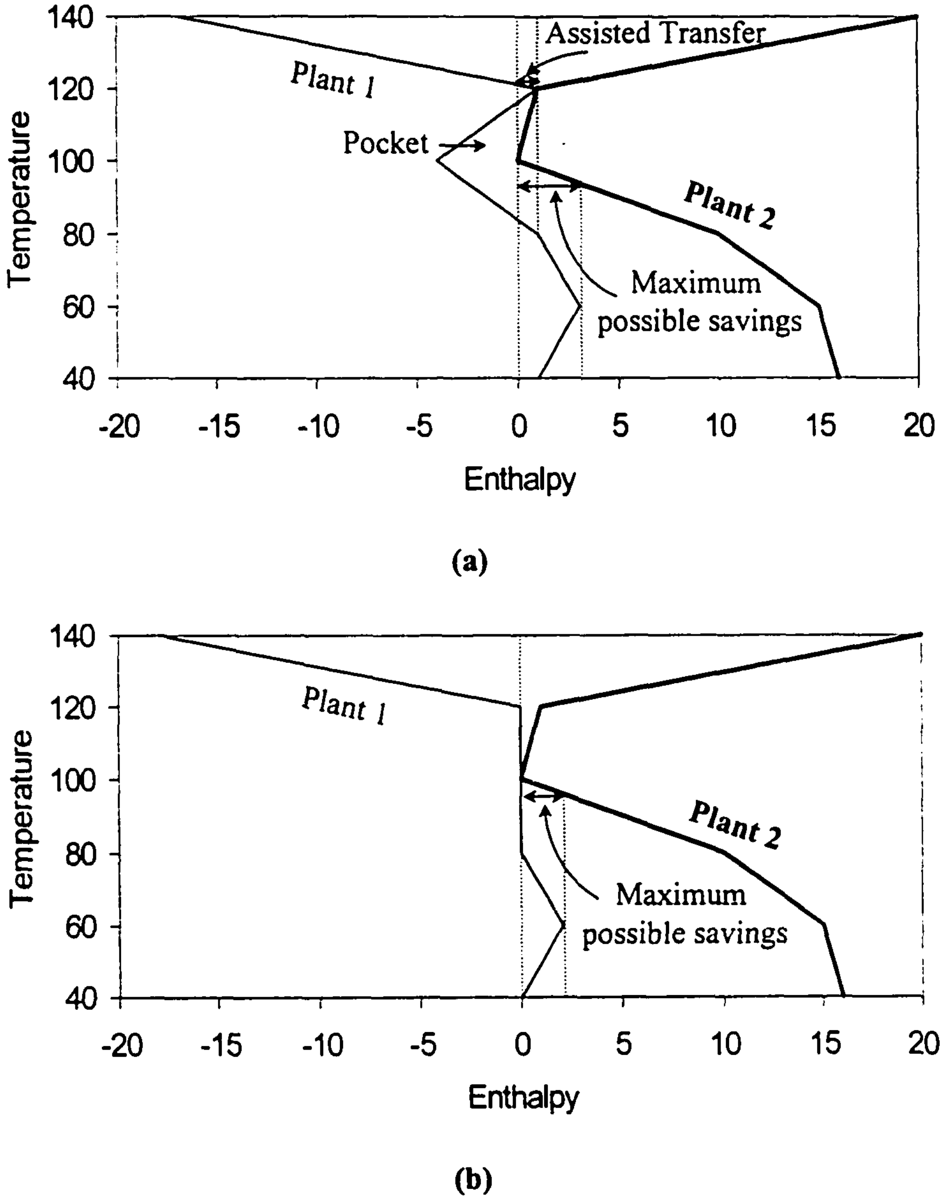


Figure 2.9. Countercurrent composite curve profiles for Example 2.3

2.3 Targeting Model for Heat Integration

In this section, a model that allows the automatic determination of unassisted and assisted cases is presented. This model predicts the amount of heat that needs to be transferred in each interval to achieve maximum savings. Application to either direct or indirect integration is possible. In order to facilitate the computations, the temperature intervals are constructed using inlet and outlet temperatures of all streams from both plants (i.e., $m^{I} = m^{II} = m$).

2.3.1 Maximum Energy Savings

Maximum savings that can be obtained by integration are computed by subtracting the combined-plant minimum heating utility from the summation of the individual plants minimum heating utilities. To obtain the amount of heat that has to be transferred in each interval, a model is constructed where heat can be transferred independently within each interval (Figure 2.10). A single direction of heat transfer is allowed: from plant 2 to plant 1 between pinch temperatures, and from plant 1 to plant 2 outside this region.

The task is now to determine what amount is transferred at each interval to achieve maximum savings. To do that, an LP model is proposed. Let $\hat{\delta}_0^{I}$ and $\hat{\delta}_0^{II}$ be the original minimum heating utility of plant 1 and plant 2 respectively when no integration across plants is assumed. These values are $S_{\min}^{I}$ and $S_{\min}^{II}$, the results obtained by solving the LP transportation or transshipment models for each of the plants separately. In the same way, let $\hat{\delta}_m^{I}$ and $\hat{\delta}_m^{II}$ be the original cooling utilities ($W_{\min}^{I}$ and $W_{\min}^{II}$ values). Also, let δ_i^{I} and δ_i^{II} be the new heat transferred between intervals after integration across plants is implemented. Finally, let q_i^{E} be the heat transferred between pinch temperatures in interval i from plant 2 to plant 1, and let q_i^A and q_i^B be the heat transferred in the inverse direction above the higher pinch temperature and below the lower pinch temperature, respectively.

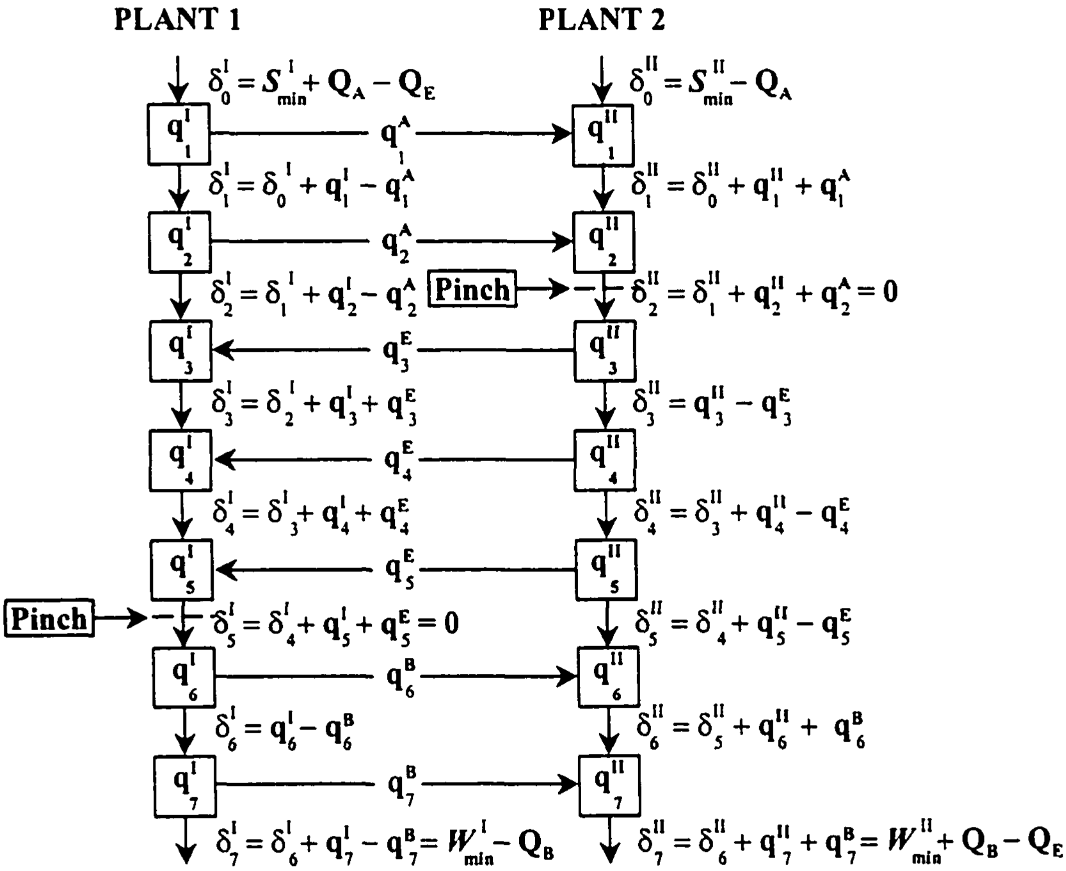


Figure 2.10. Splitting the heat transfer among intervals

The model that predicts the maximum possible energy savings that effectively occur between pinch temperatures Q_E, and the eventual minimum amount of heat Q_A and Q_B to be transferred in the regions outside the one between pinch temperatures is:

$$
\begin{aligned}
\mathbf{P2.1} = & \, Min\,(\delta_0^{I} + \delta_m^{II}) \\
& st \\
& \delta_0^{I} = \hat{\delta}_0^{I} + Q_A - Q_E \\
& \delta_0^{II} = \hat{\delta}_0^{II} - Q_A \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{A} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{A} \end{aligned}\right\} \quad \forall i = 1,\ldots, p^{II} \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} + q_i^{E} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{E} \end{aligned}\right\} \quad \forall i = (p^{II} + 1),\ldots, p^{I} \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{B} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{B} \end{aligned}\right\} \quad \forall i = (p^{I} + 1),\ldots, m \\
& \delta_m^{I} = \hat{\delta}_m^{I} - Q_B \\
& \delta_m^{II} = \hat{\delta}_m^{II} + Q_B - Q_E \\
& \delta_i^{I}, \delta_i^{II}, q_i^{A}, q_i^{E}, q_i^{B} \geq 0
\end{aligned}
\tag{2.1}
$$

In this formulation p^{I} and p^{II} are the respective pinch temperature levels, and it is assumed that $p^{I} < p^{II}$. The problem considers the conditions of minimum utility usage for both plants as the starting point. The objective function used needs some explanation. Minimizing the heating utility needed in plant 1 serves two purposes:

(a) to reduce the utility in the amount transferred from plant 2 between pinches, and

(b) to make sure that the amount of heat that is needed to be transferred from plant 1 to plant 2 is strictly the minimum necessary.

When a higher amount of heat than the minimum needed is transferred above pinch temperatures in the assisted case, the excess consists of a simple shift of utility from plant 1 to plant 2. Such shifting requires equipment and therefore represents additional investment without a benefit and should be avoided. The same result is obtained if one solves the problem by minimizing the amount of cooling utility of plant 2. In this case, the transfer to plant 2 is maximized while the transfer from plant 1 to plant 2 below pinch temperatures is kept at its minimum necessary to assist in the savings. Finally, for each unit of heat transferred between plants, both values are reduced by the same amount simultaneously. This implies that independent reductions of these utilities are not possible. Hence, adding them to form the objective function of problem **P2.1** is possible. A simple balance around plant 1 proves that the summation of the solutions (heat transferred amounts q_i^E) will represent the total possible amount of heat to be transferred between pinches Q_E.

Remark: *The LP problem presented has degenerate solutions. Indeed, when transferring heat surplus from an interval in plant 2 to any interval in plant 1 between pinch temperatures, the heat can be transferred from plant 2 to plant 1 first and then transferred down, or transferred down in plant 2 first, and then transferred to plant 1 at a lower interval. The same situations occur in an inverse manner when the transfer takes place in any of the regions outside the region between pinch temperatures. Therefore, many different paths are available. This degeneracy is actually a flexibility that can be exploited later when a design is attempted.*

The results from the above models can now be used as target values for models that will determine the heat exchanger network needed to accomplish such savings. In particular, the knowledge of what are the intervals at which the heat transfer from one plant to the other should take place (in addition to the direction of such transfer) is a useful input for these models. These models, which are presented in Chapter 3, will address the design of systems featuring minimum number of exchangers to accomplish a dual operation (with and without integration).

2.3.2 Direct Integration Examples

2.3.2.1 Example 2.4

In this example, Test Case #2 from Linnhoff & Hindmarsh (1983) is plant 1 and problem 4sp1 is plant 2. The data for the separate plants are shown in Table 2.5 and Table 2.6, respectively.

Table 2.5. Data for plant 1 in Example 2.4

Test Case #2				
Streams	**F(kW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(kW)**
H1(Hot)	2.0	150	60	180.0
C2(Cold)	2.5	20	125	262.5
H3(Hot)	8.0	90	60	240.0
C4(Cold)	3.0	25	100	225.0
S(Steam)	-	270	270	107.5
CW(Water)	0.9	38	82	40.0
ΔT_{min} =20°C				

Table 2.6. Data for plant 1 in Example 2.4

Problem 4spl				
Streams	**F(kW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(kW)**
C1(Cold)	7.62	60	160	762
H2(Hot)	8.79	160	93	589
C3(Cold)	6.08	116	260	876
H4(Hot)	10.55	249	138	1171
S(Steam)	-	270	270	128
CW(Water)	5.68	38	82	250
ΔT_{min} =10°C				

Note that ΔT_{min} for plant 1 is 20°C, while ΔT_{min} for plant 2 is 10°C. Pinch temperatures and minimum utility consumption for each of the plants are shown in Table 2.7.

Table 2.7. Individual plant pinch analysis for Example 2.4

Problem	**Pinch Temp.(°C)**	**Heating Utility (kW)**	**Cooling Utility (kW)**
Test Case #2	90	107.5	40.0
4spl	249	128.0	250.0

Table 2.8 shows the results of the pinch analysis for the direct integration. The interval between pinch temperatures goes from 90°C to 249°C. After considering all streams in a single set, the resulting combined-plant pinch temperature is 249°C (upper limit of the interval between pinch temperatures). This is the consequence of the large availability of heat to transfer that plant 2 has in all the intervals between pinch temperatures. This amount of heat is sufficient to supply the entire demand of plant 1.

Table 2.8. Pinch analysis for direct integration in Example 2.4

T (°C)	Test Case #2			4sp1			Combined Plant		
	q_i^I (kW)	γ_i^I (kW)	S_{min}^I (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S_{min}^{II} (kW)	q_i^C (kW)	γ_i^C (kW)	S_{min}^{CP} (kW)
270	0	0	**107.5**	-127.8	-127.8	**127.8**	-127.8	-127.8	**127.8**
249	0	0		353.1	225.3		353.1	225.3	
170	0	0		-31.5	193.8		-31.5	193.8	
160	0	0		56.4	250.2		56.4	250.2	
150	10.0	10.0		28.2	278.4		38.2	288.4	
145	-3.5	6.5		39.5	317.9		36.0	324.4	
138	-6.0	0.5		-58.9	259.0		-64.9	259.5	
126	-3.0	-2.5		7.0	266.0		4.0	263.5	
120	-94.5	-97.0		31.6	297.6		-62.9	200.6	
93	-10.5	-107.5		-22.9	274.7		-33.4	167.2	
90	90.0	-17.5	W_{min}^I	-152.4	122.3	W_{min}^{II}	-62.4	104.8	W_{min}^{CP}
70	45.0	27.5		0	122.3		45.0	149.8	
60	-82.5	-55.0	(kW)	0	122.3	(kW)	-82.5	67.3	(kW)
45	-12.5	-67.5	**40.0**	0	122.3	**250.0**	-12.5	54.8	**182.5**

Therefore, the maximum possible heat savings for the direct integration are 107.5 kW that is the original minimum utility of plant 1. This is also the result obtained by solving problem **P2.1**.

2.3.2.2 Example 2.5

In this case, an example taken from Trivedi (1988) is plant 1 and example 1 from Ciric and Floudas (1991) is plant 2. The data for the separate plants are shown in Table 2.9 and Table 2.10, respectively.

Table 2.9. Data for plant 1 in Example 2.5

Trivedi				
Streams	**F (kW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(kW)**
H1(Hot)	7.032	160	110	351.6
H2(Hot)	8.44	249	138	936.8
H3(Hot)	11.816	227	106	1429.7
H4(Hot)	7.0	271	146	875.0
C1(Cold)	9.144	96	160	585.2
C2(Cold)	7.296	115	217	744.2
C3(Cold)	18	140	250	1980.0
S(Steam)	-	300	300	404.8
CW(Water)	34.43	70	90	688.6
ΔT_{min} =20°C				

Table 2.10. Data for plant 2 in Example 2.5

Ciric & Floudas				
Streams	**F(kW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(kW)**
H5(Hot)	10	300	200	1000
H6(Hot)	120	200	100	12000
C4(Cold)	15	70	270	3000
C5(Cold)	25	70	190	3000
C6(Cold)	50	70	180	5500
S(Steam)	-	300	300	600
CW(Water)	105	70	90	2100
ΔT_{min} =20°C				

Pinch temperatures and minimum utility consumption for each of the plants are shown in Table 2.11.

Table 2.11. Individual plant pinch analysis for Example 2.5

Problem	**Pinch Temp.(°C)**	**Heating Utility (kW)**	**Cooling Utility (kW)**
Trivedi	160	404.8	688.6
Ciric & Floudas	200	600	2100

Table 2.12 shows the results of the pinch analysis for the direct integration. The interval between pinch temperatures goes from 160°C to 200°C. After considering all streams in a single set, the resulting combined-plant pinch temperature is 200°C (upper limit of the interval between pinches). The maximum possible savings for direct integration are 104.4 kW. Solving problem **P2.1** gives the targeting values of the heat to be transferred in each of the zones. A minimum of 52.9 kW (Q_A) has to be transferred in the zone above both pinch temperatures in order to attain maximum possible savings.

Table 2.12. Pinch analysis for direct integration in Example 2.5

T (°C)	Trivedi			Ciric & Floudas			Combined Plant		
	q_i^I (kW)	γ_i^I (kW)	S^I_{min} (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S^{II}_{min} (kW)	q_i^C (kW)	γ_i^C (kW)	S^{CP}_{min} (kW)
300	0	0	**404.8**	100.0	100.0	**600.0**	100.0	100.0	**900.4**
290	0	0		-95.0	5.0		-95.0	5.0	
271	7.0	7		-5.0	0.0		2.0	7.0	
270	-231.0	-224		-105.0	-105.0		-336.0	-329.0	
249	-30.7	-254.7		-60.0	-165.0		-90.7	-419.7	
237	-98.6	-353.3		-50.0	-215.0		-148.6	-568.3	
227	33.3	-320.0		-85.0	-300.0		-51.7	-620.0	
210	19.6	-300.4		-300.0	-600.0		-280.4	-900.4	
200	39.2	-261.2		600.0	0.0		639.2	-261.2	
180	-143.7	-404.8		600.0	600.0		456.3	195.2	
160	249.9	-155.0		420.0	1020.0		669.9	865.0	
146	86.8	-68.2		240.0	1260.0		326.8	1191.8	
138	7.2	-61.0		90.0	1350.0		97.2	1289.0	
135	184.4	123.4		570.0	1920.0		754.4	2043.4	
116	113.1	236.5	W^I_{min} (kW)	180.0	2100.0	W^{II}_{min} (kW)	293.1	2336.5	W^{CP}_{min} (kW)
110	47.3	283.8		120.0	2220.0		167.3	2503.8	
106	0	283.8		180.0	2400.0		180.0	2683.8	
100	0	283.8	**688.6**	-900.0	1500.0	**2100.0**	-900.0	1783.8	**2684.1**

2.3.2.3 Example 2.6

This example consists of a crude unit processing 150,000 bbl/day and a FCC unit processing 40,000 bbl/day. The crude unit is plant 1 while a FCC unit is plant 2. The data for the separate plants are shown in Table 2.13 and Table 2.14, respectively.

Table 2.13. Data for plant 1 in Example 2.6

Crude Unit				
Streams	**F(MW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(MW)**
C1(Cold)	0.6230	30.0	127.3	60.64
C2(Cold)	0.6945	127.3	239.3	77.78
C3(Cold)	0.7855	239.3	352.9	89.24
H1(Hot)	0.0655	127.3	37.8	5.86
H2(Hot)	0.3053	143.5	26.7	35.67
H3(Hot)	0.1439	261.4	37.8	32.18
H4(Hot)	0.0334	326.7	37.8	9.64
H5(Hot)	0.3400	347.3	268.3	26.85
H6(Hot)	0.2744	163.3	79.6	22.98
H7(Hot)	0.1771	194.5	142.6	9.20
H8(Hot)	0.2617	261.4	206.3	14.42
H9(Hot)	0.1221	336.3	239.8	11.78
F(Fuel)	124.2856	427.2	426.7	69.05
CW(Water)	0.8968	15.6	26.7	9.96
ΔT_{min} =5.6°C				

The ΔT_{min} in this case is 5.6°C (equivalent to 10°F) for both plants, and the downward shift of plant 2 during intermediate fluid integration is 5.6°C.

Table 2.14. Data for plant 2 in Example 2.6

FCC Unit				
Streams	**F(MW/°C)**	**T_s(°C)**	**T_t(°C)**	**Q(MW)**
C4(Cold)	0.0831	471.1	532.2	5.07
H10(Hot)	0.0083	348.2	21.1	2.73
H11(Hot)	0.0078	243.9	21.1	1.73
H12(Hot)	0.0773	147.2	48.9	7.59
H13(Hot)	0.0252	348.2	115.5	5.86
H14(Hot)	0.0362	313.2	232.2	2.93
H15(Hot)	0.1503	190.1	107.2	12.46
F(Fuel)	9.1262	538.3	537.8	5.07
CW(Water)	2.9990	15.6	26.7	33.32
ΔT_{min} =5.6°C				

Pinch temperatures and minimum utility consumption are shown in Table 2.15.

Table 2.15. Individual plant pinch analysis for Example 2.6

Plant	**Pinch Temp.(°C)**	**Heating Utility (MW)**	**Cooling Utility (MW)**
Crude Unit	143.5	69.0	10.0
FCC Unit	471.1	5.1	33.3

Considerable energy recovery is possible due to the big temperature difference between pinches (471.1°C to 143.5°C). The FCC unit needs a great amount of cooling utility below its pinch temperature due to the high temperatures of the streams emanating from the reactor. On the other hand, the crude unit needs a great amount of heating utility in order to heat up its streams during the fractionation process. Table 2.16 shows the results of the pinch analysis for the direct integration. The intervals below the lower pinch temperature have been merged since heat recovery is not performed here. The resulting combined-plant pinch temperature is 163.3°C, and is the result of a compensation of the heat in the first interval of plant 1 by heat provided by plant 2. After this interval, the heat that plant 2 has available in the rest of the intervals between pinches is not sufficient to supply the demand of the corresponding intervals of plant 1. Therefore, the maximum possible heat savings are 15.1 MW obtained by solving problem **P2.1**. Note that 13.8 MW are transferred above the combined-plant pinch temperature, and 1.3 MW below the combined-plant pinch temperature.

Table 2.16. Pinch analysis for direct integration in Example 2.6

T (°C)	Crude Unit			FCC Unit			Combined Plant		
	q_i^I (MW)	γ_i^I (MW)	S_{min}^I (MW)	q_i^{II} (MW)	γ_i^{II} (MW)	S_{min}^{II} (MW)	q_i^C (MW)	γ_i^C (MW)	S_{min}^{CP} (MW)
532.2	0	0	**69.1**	-5.1	-5.1	**5.1**	-5.1	-5.1	**59.0**
471.1	-8.1	-8.1		0.0	-5.1		-8.1	-13.2	
348.2	-0.7	-8.8		0.0	-5.0		-0.7	-13.9	
347.3	-4.9	-13.7		0.4	-4.7		-4.5	-18.4	
336.3	-3.1	-16.8		0.3	-4.4		-2.8	-21.2	
326.7	-3.9	-20.7		0.5	-3.9		-3.5	-24.6	
313.2	-13.0	-33.7		3.1	-0.8		-9.9	-34.5	
268.3	-4.3	-38.1		0.5	-0.3		-3.9	-38.4	
261.4	-3.7	-41.8		1.2	0.9		-2.6	-40.9	
244.9	-0.1	-41.9		0.1	0.9		-0.1	-41.0	
243.9	-0.5	-42.5		0.3	1.2		-0.2	-41.2	
239.8	-1.9	-44.4		0.6	1.8		-1.3	-42.6	
232.2	-6.6	-51.0		1.1	2.9		-5.5	-48.1	
206.3	-6.1	-57.1		0.5	3.4		-5.6	-53.8	
194.5	-1.5	-58.6		0.2	3.6		-1.3	-55.1	
190.1	-9.1	-67.8		5.1	8.7		-4.0	-59.0	
163.3	-1.1	-68.8	W_{min}^I	3.1	11.8	W_{min}^{II}	2.0	-57.0	W_{min}^{CP}
147.2	-0.2	-69.1	(kW)	1.0	12.8	(kW)	0.7	-56.3	(kW)
143.5	0.2	-68.8		0.3	13.0		0.5	-55.8	
26.7	9.8	-59.1	**10.0**	15.2	28.2	**33.3**	24.9	-30.9	**28.2**

2.4 Indirect Heat Integration

The focus is now on the case of indirect heat integration by the use of intermediate fluid circuits. The design parameters for these circuits, namely flowrate and inlet and outlet temperatures, have to be calculated.

2.4.1 Shift of Scales

When an intermediate fluid is used, new streams appear in each plant. Consider the region between pinch temperatures first. In plant 1, the intermediate fluid acts as a hot stream, whereas in plant 2, it acts as a cold stream. The temperature of the intermediate fluid leaving plant 1 (registered in its hot scale) should be equal to the starting temperature of the same fluid in plant 2, requiring the coincidence between the respective hot and cold scales. Thus, a shift consisting of moving the hot scale of plant 2 (and with it, the cold scale too) downward ΔT_{min} degrees in the region below its pinch is performed. Consider now the possibility of assisted heat integration cases. In the region above the higher pinch temperature and below the lower pinch temperature, the fluid circulates in the inverse direction than between pinch temperatures. Therefore, a match between the cold scale of plant 1 and the hot scale of plant 2 is required in these two regions. To accomplish this, the hot scale of plant 2 (and with it, the cold scale) is shifted upward ΔT_{min} degrees in the zone above its pinch temperature. Similarly, in the region below the lower pinch temperature, a shift of the hot scale of plant 1 (and with it, the cold scale) downward is needed. However, the hot scale of plant 2 was already shifted by ΔT_{min}. Therefore, a shift of 2 ΔT_{min} degrees downward of the hot scale of plant 1 (and with it its cold scale) has to be performed. Finally, as in the direct integration case, the temperature intervals are constructed using inlet and outlet temperatures of all streams of both plants.

As a result of these temperature shifts, smaller savings than in the direct integration case may be achieved. If the use of intermediate fluids is not mandatory (due to safety or other considerations), then this reduction in savings potential may or may not be compensated by the reduction in piping, pumping, and/or compression costs.

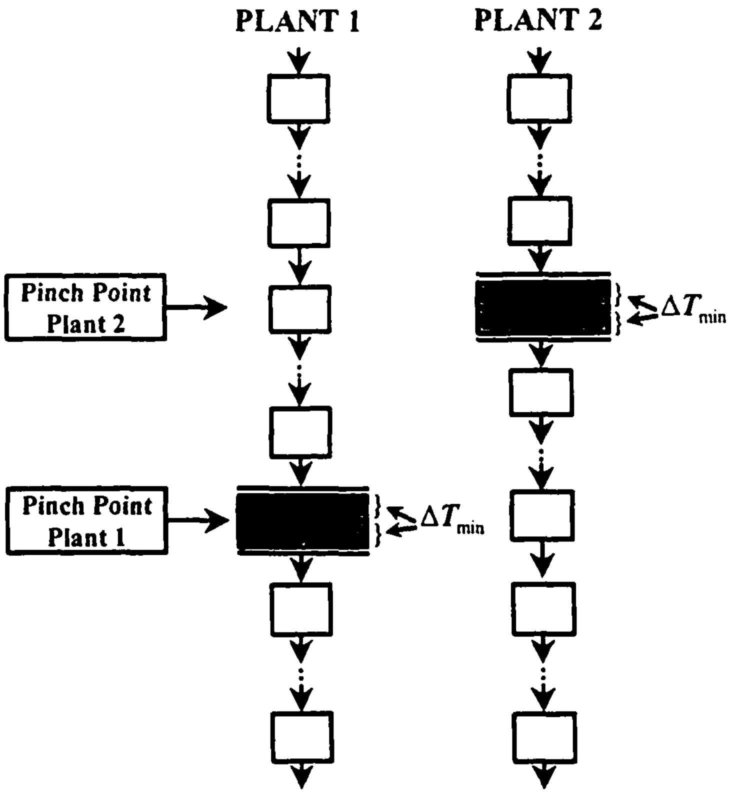


Figure 2.11. Shift of scales to allow the use of intermediate fluids

Summarizing, the scale shifts required are:

(a) a shift of both hot and cold scales downward by ΔT_{min} degrees in plant 2 below its pinch,

(b) a shift of both hot and cold scales upward by $\Delta T_{\min}$ degrees in plant 2 above its pinch, and

(c) a shift of both hot and cold scales downward by $2\Delta T_{min}$ degrees in plant 1 below its pinch.

These shifts create gaps in the scales as depicted in Figure 2.11.

2.4.2 Maximum Energy Savings

The maximum savings that can be obtained by integration can be computed by subtracting the combined-plant minimum heating utility using the shifted scales from the summation of the individual plants minimum heating utilities. Next, to establish the regions in which heat transfer should be made to accomplish the overall target, problem **P2.1** is solved.

Remark: *The solution to problem **P2.1** can be implemented in practice. Indeed, a circuit can be established for each interval that has a nonzero heat transfer* q_i^E, q_i^A *or* q_i^B. *However, one is interested in performing the transfer with the smallest amount of circuits possible. The issue is investigated in the next sections.*

2.4.3 Feasibility of a Single Fluid Circuit

Even though the target value for Q_E can always be transferred between pinch temperatures and the eventual target amounts Q_A and Q_B can always be transferred outside this region, the question is whether these transfers can be achieved with a single circuit in each region.

Figure 2.12a shows the unassisted case with m_E intervals within the region between pinch temperatures $T_{p''}$ and $T_{p'}$. The assisted cases are represented in Figure 2.12b.

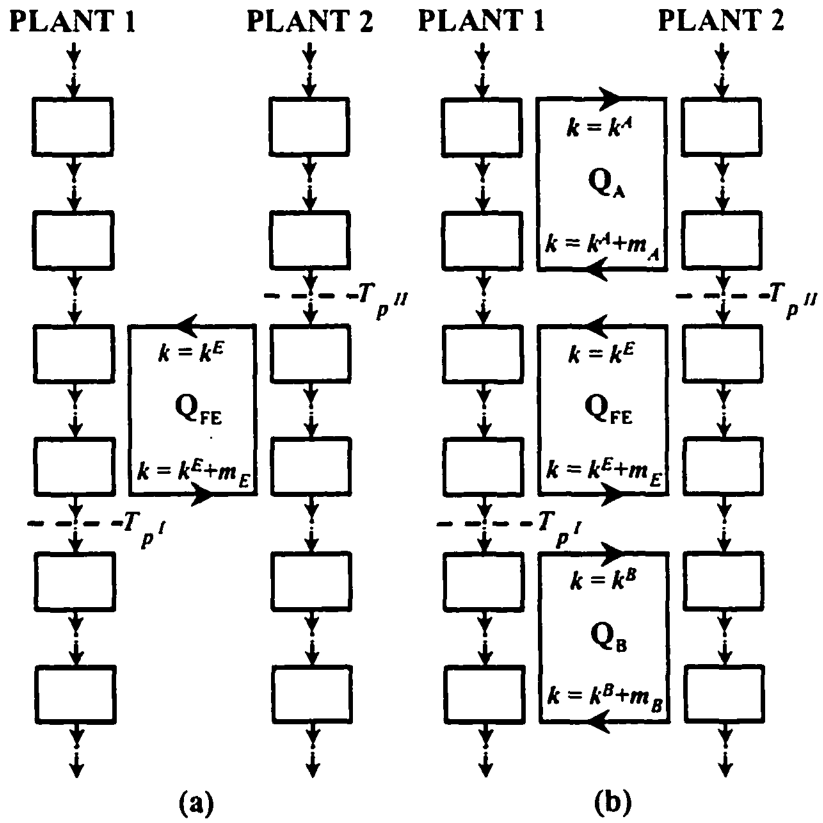


Figure 2.12. Circuits of intermediate fluids in unassisted and assisted heat integration cases (gaps are omitted for simplicity)

In the region between pinch temperatures, the heat released in each interval from plant 2 to the intermediate fluid does not have to be the same as the heat released by the fluid to plant 1 in the same interval. Likewise, in the region above the higher pinch temperature and the region below the lower pinch temperature, the heat released from plant 1 in the same interval does not have to be the same as the heat received by plant 2.

A generalization for any of the regions comes next. The use of separate variables for the heat transferred to and from the intermediate fluid are defined as follows: q_i^{FC} is the heat received by the fluid in interval i, and q_i^{FH} is the heat released by the fluid in interval i. The total heat transferred Q_F is already given by the targeting procedure, that is:

$$Q_F = \sum_{i=k^F}^{k^F+m_K} q_i^{FC} = \sum_{i=k^F}^{k^F+m_K} q_i^{FH} \tag{2.2}$$

where k^F is the first interval in which the transfer between plants takes place and m_K represents the number of intervals covered by a circuit in any of the regions.

2.4.4 Temperature Constraints

Let us first explore the second law constraints regarding q_i^{FC} and q_i^{FH}. A circuit covering m_E intervals between pinch temperatures receives heat from plant 2 and delivers it to plant 1. In the assisted cases, a circuit covering either m_A or m_B intervals performs the inverse task carrying heat from plant 1 to plant 2. Therefore, this can be generalized for a set of m_K intervals. The plant that is providing the heat is considered the "heat-source plant", while the one receiving that heat is considered the "heat-sink plant". The following constraints prevent the temperature of the fluid to go higher than the interval temperature T_{k-l} in the heat-source plant.

$$F(T_{k-1} - T_0^{FC}) \geq \sum_{i=k}^{k^F+m_K} q_i^{FC} \quad \forall k = (k^F + m_K),\ldots,(k^F + 1) \tag{2.3}$$

$$F(T_{k^F-1}^{FC} - T_0^{FC}) = Q_F \tag{2.4}$$

In the heat-sink plant, similar constraints are introduced to prevent the temperature of the fluid going lower than the interval temperature T_k.

$$F(T_0^{FH} - T_k) \geq \sum_{i=k^F}^{k} q_i^{FH} \quad \forall k = k^F,\ldots,(k^F + m_K - 1) \tag{2.5}$$

$$F(T_0^{FH} - T_{k^F+m_K}^{FH}) = Q_F \tag{2.6}$$

In order to assure a closed circuit, it should be noticed that the following has to be verified.

$$T_0^{FC} = T_{k^F+m_K}^{FH} \tag{2.7}$$

$$T_0^{FH} = T_{k^F-1}^{FC} \tag{2.8}$$

Let us now examine what values the initial temperatures of the intermediate fluid can take. First, note that to guarantee feasibility of heat transfer for some k, T_0^{FC} and

T_0^{FH} are equal to T_{k-1} and T_k, respectively (i.e., they are confined to be end interval temperatures). Now consider the case where the heat-source plant does not have any heat demand in the first set of $(k^+ - k^F + 1)$ intervals, that is $q_i^{FH} = 0,\ \forall i = k^F,\ldots,k^+$. Then by increasing the flowrate of the intermediate fluid and without limitations in the transfer of heat from the heat-source plant, a new solution with an upper temperature smaller than the one considered initially is possible. If this is the case, the heat-source plant will only be transferring heat to the intermediate fluid at temperatures lower than T_{k^+}. To find such a solution, heat can be cascaded down from the first $(k^+ - k^F + 1)$ intervals in the heat-source plant. The values of q_i^{FC} can be transformed to a new set $\hat{q}_i^{FC}$ as follows:

$$\hat{q}_i^{FC} = 0 \quad \forall i = k^F,\ldots,k^+ \tag{2.9}$$

$$\hat{q}_{k^+ + 1}^{FC} = q_{k^+ + 1}^{FC} + \sum_{i=k^F}^{k^+} q_i^{FC} \tag{2.10}$$

$$\hat{q}_i^{FC} = q_i^{FC} \quad \forall i = k^+ + 2,\ldots,(k^F + m_K) \tag{2.11}$$

where $\hat{q}_i^{FC}$ is another degenerate solution. This solution allows the circuit to be established between the intervals $k^+ + 1$ and $(k^F + m_K)$. A similar argument can be made for the case where the last intervals in plant 2 do not transfer heat to the intermediate fluid.

Thus, one can assume without loss of generality that the initial temperatures of the intermediate fluid in the heat-source plant and heat-sink plant are, respectively:

$$T_0^{FC} = T_{k^F+m_K}^{FH} = T_{k^F+m_K} \qquad (2.12)$$

$$T_0^{FH} = T_{k^F-1}^{FC} = T_{k^F-1} \qquad (2.13)$$

With these equalities, the set of equations (2.3-2.6) become:

$$F(T_{k^F-1} - T_k) \geq \sum_{i=k^F}^{k} q_i^{FH} \quad \forall k = k^F,\dots,(k^F + m_K - 1) \qquad (2.14)$$

$$F(T_k - T_{k^F+m_K}) \geq \sum_{i=k}^{k^F+m_K} q_i^{FC} \quad \forall k = (k^F + 1),\dots,(k^F + m_K) \qquad (2.15)$$

$$F(T_{k^F-1} - T_{k^F+m_K}) = Q_F \qquad (2.16)$$

Equations (2.14-2.16) constitute a feasibility test for a single circuit that transfers the heat Q_F across plants. The flowrate F can be calculated using (2.16). This value can then be replaced in equations (2.14) and (2.15). If any of these equations are not satisfied, then a circuit between T_{k^F-1} and $T_{k^F+m_K}$ cannot transfer the maximum amount but perhaps some smaller value.

2.4.5 Candidate Heat Transfer sets

The flexibility at hand for defining the general variables q_i^{FH} and q_i^{FC} is explored by an adjusted heat-cascaded diagram (Figure 2.13).

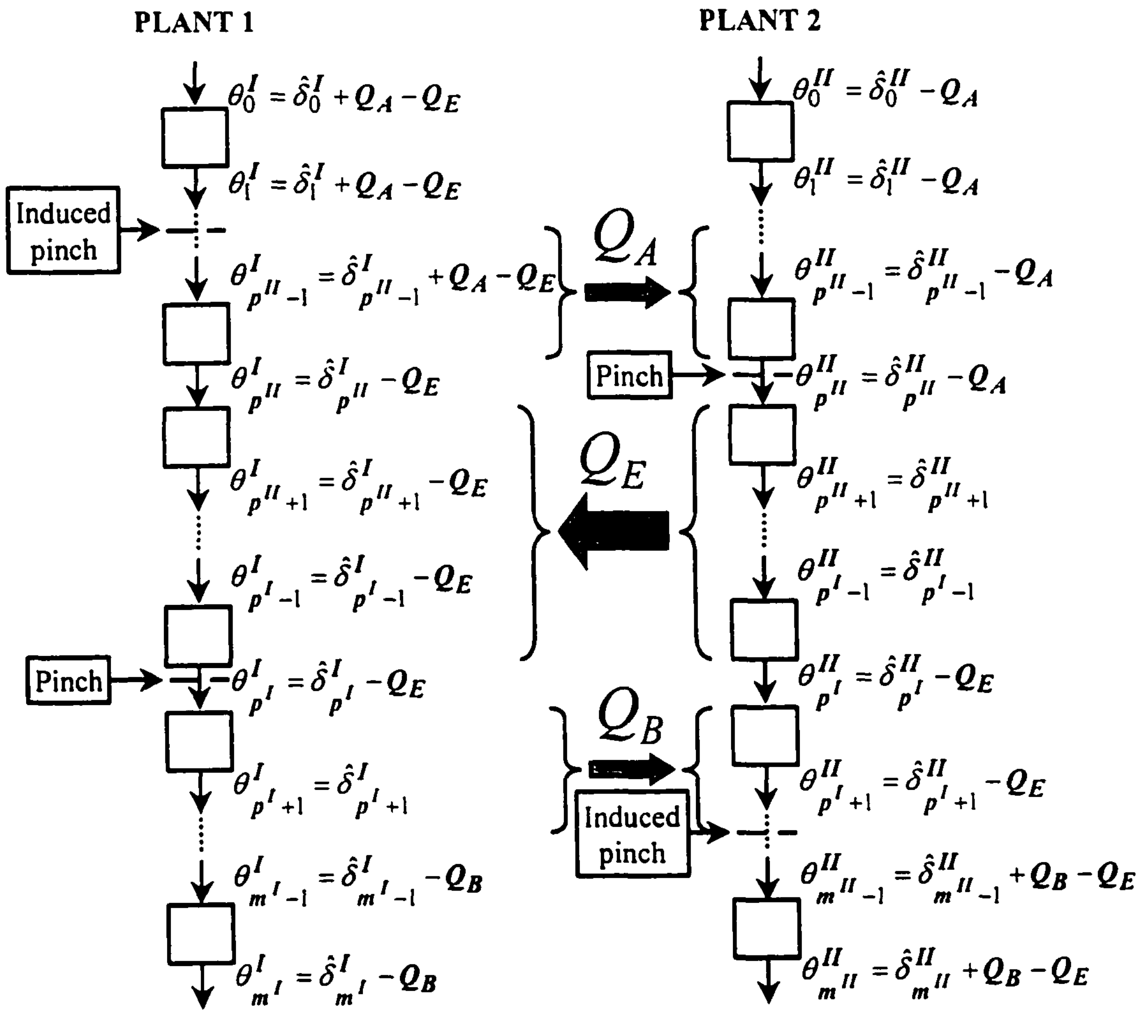


Figure 2.13. Adjusted heat-cascaded diagram

(gaps are omitted for simplicity)

The target values Q_E, Q_A and Q_B are added and subtracted in the three defined zones of the cascade. This accounts for the supplies or demand each of the zones will experience when the respective single-circuit candidates are considered. The values obtained for the different intervals are not realistic heat transfer amounts since some of them are negative, but rather a calculation aid. Moreover, because of these operations, the adjusted heat-cascaded diagram of the heat-source plant may exhibit an induced pinch temperature in each region where a circuit can be installed. In this instance, the transfer will only occur in the sub zone delimited by the real and induced pinch temperatures.

In the unassisted heat integration case, the solutions of problem **P2.1** satisfy the following relations:

$$\delta_k^{I} = \hat{\delta}_{p^{II}}^{I} - Q_E + \sum_{i=p^{II}+1}^{k} (q_i^{I} + q_i^{E}) \geq 0 \quad \forall k = (p^{II}+1), \ldots, p^{I} \tag{2.17}$$

$$\delta_k^{II} = \sum_{i=p^{II}+1}^{k} (q_i^{II} - q_i^{E}) \geq 0 \quad \forall k = (p^{II}+1), \ldots, p^{I} \tag{2.18}$$

The same set of equations can be written in terms of q_i^{EH} and q_i^{EC}:

$$\delta_k^{I} = \hat{\delta}_{p^{II}}^{I} - Q_E + \sum_{i=p^{II}+1}^{k} (q_i^{I} + q_i^{EH}) \geq 0 \quad \forall k = (p^{II}+1), \ldots, p^{I} \tag{2.19}$$

$$\delta_k^{II} = \sum_{i=p^{II}+1}^{k} (q_i^{II} - q_i^{EC}) \geq 0 \quad \forall k = (p^{II}+1), \ldots, p^{I} \tag{2.20}$$

Similarly, for the assisted heat integration cases, the relations for the upper and lower zones are:

$$\delta_k^I = \hat{\delta}_0^I + Q_A - Q_E + \sum_{i=1}^{k}(q_i^I - q_i^{AC}) \geq 0 \quad \forall k = 1,\ldots,p^{II} \tag{2.21}$$

$$\delta_k^{II} = \hat{\delta}_0^{II} - Q_A + \sum_{i=1}^{k}(q_i^{II} + q_i^{AH}) \geq 0 \quad \forall k = 1,\ldots,p^{II} \tag{2.22}$$

$$\delta_k^I = \sum_{i=p^I+1}^{k}(q_i^I - q_i^{BC}) \geq 0 \quad \forall k = (p^I + 1),\ldots,m \tag{2.23}$$

$$\delta_k^{II} = \hat{\delta}_{p^I}^I - Q_E + \sum_{i=p^{II}+1}^{k}(q_i^{II} + q_i^{BH}) \geq 0 \quad \forall k = (p^I + 1),\ldots,m \tag{2.24}$$

Thus, any non-negative set of generalized values q_i^{FH} and q_i^{FC} that satisfies (2.19) and (2.20), (2.21) and (2.22), or (2.23) and (2.24), and the balance (2.2) is an acceptable candidate for a single circuit. If in addition, (2.14-2.16) are satisfied, the candidate set will be a feasible single-circuit solution in any of the zones. A few generalized candidate sets will be presented next:

One candidate set is given by the solution of problem **P2.1**, i.e.:

$$q_i^{FH} = q_i^{FC} = q_i^F \tag{2.25}$$

Other sets can be found by making use of degeneracy. In particular, one can choose a set that prioritizes the heat transfer to the intermediate fluid over the heat transfer to the interval below in the source plant. This solution is called the higher- circuit solution because the circuit starts and ends at the higher possible intervals. A lower-circuit solution will be presented later. The maximization of the heat delivered to the intermediate fluid is the purpose of constructing a higher-circuit solution.

Therefore, to establish the maximum amount of heat that each interval can provide to the intermediate fluid, the deficit of heat in the intervals below it need to be taken into account. The heat availability ω_k^H at each interval is then defined as follows: Let $\lambda_k^H = Min\{q_k^H, 0\}$ and $\sigma_k^H = Max\{q_k^H, 0\}$ $\forall k = k^F + 1, \ldots, k^F + m_K$. In addition, let $\lambda_{k^F}^H = 0, \sigma_{k^F}^H = q_{k^F}^H + \hat{\delta}_{k^F-1}^H$. Thus, the availability ω_k^H is given by:

$$z_{k^F+m_K}^H = \begin{cases} \lambda_{k^F+m_K}^H + \sigma_{k^F+m_K}^H & q_{k^F+m_K}^H + \lambda_{k^F+m_K}^H < 0 \\ 0 & q_{k^F+m_K}^H + \lambda_{k^F+m_K}^H \geq 0 \end{cases} \tag{2.26}$$

$$z_k^H = \begin{cases} z_{k+1}^H + \lambda_k^H + \sigma_k^H & q_k^H + z_{k+1}^H + \lambda_k^H < 0 \\ 0 & q_k^H + z_{k+1}^H + \lambda_k^H \geq 0 \end{cases} \quad \forall k = k^F + 1, \ldots, k^F + m_K - 1 \tag{2.27}$$

$$z_{k^F}^H = \begin{cases} z_{k^F+1}^H + \lambda_{k^F}^H + \sigma_{k^F}^H & q_{k^F}^H + z_{k^F+1}^H + \lambda_{k^F}^H < 0 \\ 0 & q_{k^F}^H + z_{k^F+1}^H + \lambda_{k^F}^H \geq 0 \end{cases} \tag{2.28}$$

$$\omega_k^H = Max\{q_k^H + z_{k+1}^H + \lambda_k^H, 0\} \qquad \forall k = k^F, \ldots, k^F + m_K \tag{2.29}$$

In these equations, z_k^H is an auxiliary variable that helps determine the amount of cumulative demand (from the bottom) at every interval. To illustrate this, consider first the situation depicted in Table 2.17 for which $\hat{\delta}_{k^F-1}^H = 0$, that is, either a higher circuit between pinch temperatures or a circuit above pinch temperatures with an induced pinch temperature.

Table 2.17. Determination of heat availability

Interval	q_i^H	λ_k^H	σ_k^H	$q_k^H + z_{k+1}^H + \lambda_k^H$	z_k^H	ω_k^H
k^F	12	0	12	8	0	8
k^F+1	-1	-1	0	-5	-4	0
k^F+2	-1	-1	0	-4	-3	0
k^F+3	15	0	15	-2	-2	0
k^F+4	-15	-15	0	-32	-17	0
k^F+5	-2	-2	0	-4	-2	0
k^F+6	20	0	20	20	0	20

Under such conditions, all the resulting heat flows cascaded down are positive. On the source side, the higher-circuit solution is given by:

$$q_{k^F}^{FC} = Min\{\omega_{k^F}^H, Q_F\} \tag{2.30}$$

$$q_k^{FC} = Min\{\omega_k^H, Q_F - \sum_{i=k^F}^{k-1} q_i^{FC}\} \qquad \forall k = k^F + 1, \ldots, k^F + m_K \tag{2.31}$$

Equation (2.30) states that at the first interval, all the heat available will be used provided it is lower than the overall maximum. Equation (2.31) states that at every interval, the maximum that can be transferred is the surplus. In turn, for the heat-sink plant, the higher-circuit solution is:

$$q_{k^F}^{FH} = Q_F \tag{2.32}$$

$$q_k^{FH} = 0 \qquad \forall k = k^F + 1, \ldots, k^F + m_K \tag{2.33}$$

This means that if the heat-sink plant can transfer all the maximum possible heat Q_F in the first interval of transfer k^F, then the higher-circuit solution will consist of this interval only.

At the other extreme, we have a solution that maximizes the transfer of heat to the interval below in the heat-source plant, minimizing the transfer to the intermediate fluid. This solution is called thc lower circuit solution because it starts and ends at the lowest intervals possible. In such case, the solutions for the heat-sink plant and the heat-source plant are somewhat related. Indeed, when transferring heat at lower intervals in the heat-source plant, one must make sure that the heat-sink plant does not need the heat at such intervals. The adjusted cascaded heat values already account for this. Then, the lower-circuit solution for the heat-sink plant is given by:

$$q_{k^F}^{FH} = Max\{-\theta_{k^F}^{C}, 0\} \tag{2.34}$$

$$q_k^{FH} = Max\left\{-\theta_k^C - \sum_{i=k^F}^{k-1} q_i^{FH}, 0\right\} \quad \forall k = k^F + 1, \ldots, k^F + m_K \tag{2.35}$$

Now let k^+ be the first interval with nonzero heat transferred from the intermediate fluid to the heat-sink plant, i.e. k^+ is such that $q_k^{FH} = 0$, $\forall\ k = k^F, \ldots, (k^+ - 1); q_{k^+}^{FH} \neq 0$. Next, all the surplus heat in the heat-source plant can be transferred down until interval k^+ is reached. From then on, only the minimum amount of heat should be transferred to the intermediate fluid. This minimum should be at least equal to q_i^{FH} to guarantee that temperature constraints have a chance of being satisfied. Thus, the lower-circuit solution for the heat-source plant is:

$$q_k^{FC} = 0 \qquad \forall k = k^F, \ldots, (k^+ - 1) \tag{2.36}$$

$$q_k^{FC} = q_k^{FH} \qquad \forall k = k^+, \ldots, k^F + m_K \tag{2.37}$$

By construction, no one-circuit solution can:

(a) start at a lower interval, and

(b) transfer less heat from the intermediate fluid to the sink plant at any interval defined by the lower-circuit solution.

Because of the calculation of the cumulative heat demands, a limit for the starting point of the unique circuit is established. In view of the above, a second test for the feasibility of a single circuit transferring the maximum savings Q_F in any of the regions consists of constructing the higher-circuit solution or the lower-circuit solution and checking if the following equations are satisfied:

$$Q_F \frac{(T_{k^F-1} - T_k)}{(T_{k^F-1} - T_{k^F+m_K})} \geq \sum_{i=k^F}^{k} q_i^{FH} \qquad \forall k = k^F,\ldots,(k^F + m_K - 1) \tag{2.38}$$

$$Q_F \frac{(T_k - T_{k^F+m_K})}{(T_{k^F-1} - T_{k^F+m_K})} \geq \sum_{i=k}^{k^F+m_K} q_i^{EC} \qquad \forall k = (k^F + m_K),\ldots,(k^F + 1) \tag{2.39}$$

These equations have been obtained by substituting (2.16) in (2.14) and (2.15).

Remark: *The lower solution obtained by the above procedure is not always feasible. Some other lower solutions might exist, not necessarily covering the last intervals of the region between pinches, but covering a region that ends somewhere above it.*

2.4.6 Indirect Integration Examples

2.4.6.1 Example 2.4 (continued)

After the hot temperature scale in plant 2 is shifted down 10°C, the interval between pinch temperatures is from 239°C to 90°C. Table 2.18 shows the pinch analysis for the indirect integration. The combined-plant pinch temperature is at the upper bound (239°C). Fewer intervals than in the case of direct integration are found since some of the extreme temperatures now coincide due to the shift. The solution of problem **P2.1** is Q_F=107.5 kW, which is equal to the maximum possible savings that can be obtained either with direct or indirect integration. Therefore, in this case the shift does not have any effect in reducing the amount that can be transferred between pinch temperatures. Table 2.18 shows that there is no demand in the upper intervals of plant 1, and the shift does not appreciably decrease the large availability of heat in plant 2 in the region between pinches.

Table 2.18. Pinch analysis for indirect integration in Example 2.4

	Test Case #2			4sp1			Combined Plant		
T (°C)	q_i^I (kW)	γ_i^I (kW)	S^I_{min} (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S^{II}_{min} (kW)	q_i^C (kW)	γ_i^C (kW)	S^{CP}_{min} (kW)
260	0	0	**107.5**	-127.8	-127.8	**127.8**	-127.8	-127.8	**127.8**
239	0	0		353.1	225.3		353.1	225.3	
160	0	0		-31.5	193.8		-31.5	193.8	
150	10.0	10.0		28.2	222.0		38.2	232.0	
145	-8.5	1.5		95.8	317.8		87.3	319.3	
128	-4.0	-2.5		-39.3	278.5		-43.3	276.0	
120	-14.0	-16.5		-19.6	258.9		-33.6	242.4	
116	-91.0	-107.5		30.4	289.3		-60.6	181.8	
90	31.5	-76.0	W^I_{min}	8.2	297.5	W^{II}_{min}	39.7	221.5	W^{CP}_{min}
83	103.5	27.5		-175.3	122.2		-71.8	149.7	
60	-82.5	-55.0	(kW)	0	122.2	(kW)	-82.5	67.2	(kW)
45	-12.5	-67.5	**40.0**	0	122.2	**250.0**	-12.5	54.7	**182.5**

An implementation of this indirect integration follows. The test of feasibility for a single circuit is applied first to a circuit covering all the intervals between pinches. This solution is feasible and Table 2.19 shows the values obtained for the parameters (ending temperatures and rate-heat capacity product).

Table 2.19. Some of the indirect solutions to Example 4

Solution	N° of Intervals	T_{up}(°C)	T_{down}(°C)	F(kW/°C)
All Intervals	7	239	90	0.721
Lower Circuit	5	150	90	1.792
Higher Circuit	2	239	150	1.208

The higher- circuit solution is presented in Table 2.20.

Table 2.20. Higher-circuit solution to Example 4

Q_E=107.5 kW	Test Case #2		4sp1		
Interval	q_k^I (kW)	q_k^{EH} (kW)	q_k^{II} (kW)	ω_k^{II} (kW)	q_k^{EC} (kW)
p^{II}+1=2	0	107.5	353.1	321.6	107.5
p^{II}+2=3	0	0	-31.5	0	0
p^{II}+3=4	10	0	28.2	28.2	0
p^{II}+4=5	-8.5	0	95.8	36.9	0
p^{II}+5=6	-4.0	0	-39.3	0	0
p^{II}+6=7	-14.0	0	-19.6	0	0
p^{II}+7=8	-91.0	0	30.4	30.4	0

The position of the resulting circuit is shown in Figure 2.14. Note that equations (2.38) and (2.39) are satisfied. Moreover, the intermediate solutions between the circuit spanning all intervals and the highest possible circuit are feasible.

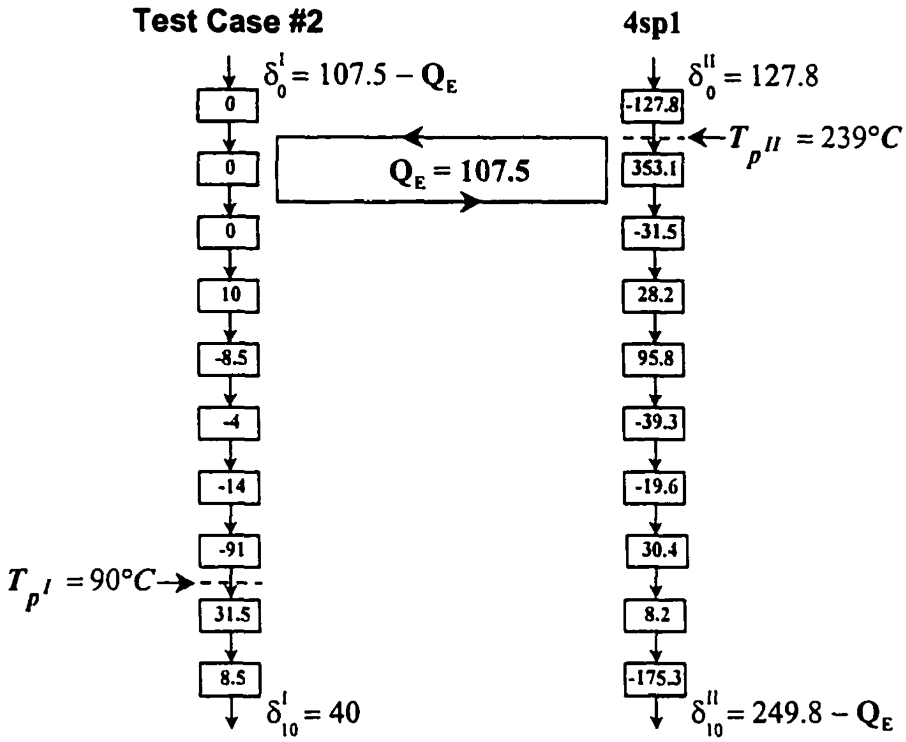


Figure 2.14. Higher-circuit solution for Example 4

Finally, the lower-circuit solution is presented in Table 2.21 and its position is shown in Figure 2.15.

Table 2.21. Lower-circuit solution to Example 4

Q_E=107.5 kW	Test Case #2			4sp1	
Interval	q_k^I (kW)	θ_k^I (kW)	q_k^{EH} (kW)	q_k^{II} (kW)	q_k^{EC} (kW)
p^{II}+1=2	0	0	0	353.1	0
p^{II}+2=3	0	0	0	-31.5	0
p^{II}+3=4	10	10	0	28.2	0
p^{II}+4=5	-8.5	1.5	0	95.8	0
p^{II}+5=6	-4.0	-2.5	2.5	-39.3	2.5
p^{II}+6=7	-14.0	-16.5	14.0	-19.6	14.0
p^{II}+7=8	-91.0	-107.5	91.0	30.4	91.0

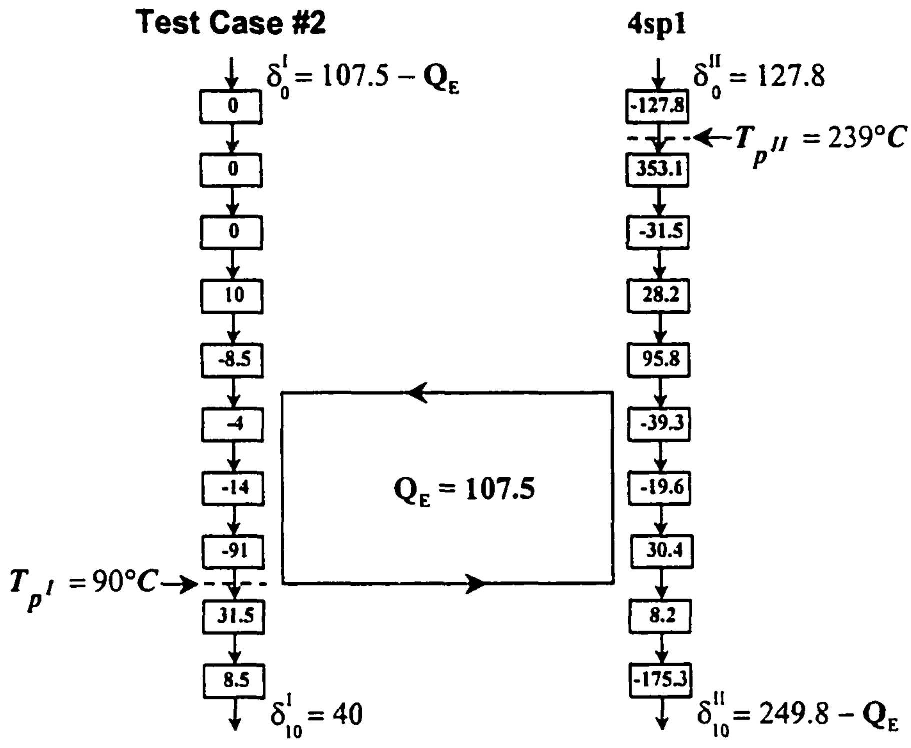


Figure 2.15. Lower-circuit solution for Example 4

The intermediate circuits between the circuit spanning all intervals and the lowest possible circuit are proven feasible. Other solutions can be found each time that a certain amount of heat could be cascaded, and the single-circuit solutions that result are feasible. However, no solution will be able to start below the limit established by the lower-circuit solution. In this sense, the problem has a large finite number of possible solutions that require further analysis taking into account the resulting heat exchanger network and the economic aspects.

2.4.6.2 Example 2.5 (continued)

In this case, the hot temperature scale in plant 2 below its pinch is shifted down 20°C, while above the pinch, the same scale is shifted up 20°C. A gap of 40°C is then created in plant 2, and no integration is possible in this zone. The interval between pinch temperatures is from 160°C to 180°C (hot scale of plant 1). Table 2.22 shows the pinch analysis for the indirect integration.

Table 2.22. Pinch analysis for indirect integration in Example 2.5

T (°C)	Trivedi			Ciric & Floudas			Combined Plant		
	q_i^I (kW)	γ_i^I (kW)	S^I_{min} (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S^{II}_{min} (kW)	q_i^C (kW)	γ_i^C (kW)	S^{CP}_{min} (kW)
300	0	0	**404.8**	100.0	100.0	**600.0**	100.0	100.0	**939.6**
310	0	0		-195.0	-95.0		-195.0	-95.0	
271	7.0	7		-5.0	-100.0		2.0	-93.0	
270	-231.0	-224		-105.0	-205.0		-336.0	-429.0	
249	-30.7	-254.7		-60.0	-265.0		-90.7	-519.7	
237	-69.0	-323.7		-35.0	-300.0		-104.0	-623.7	
230	-29.6	-353.3		-90.0	-390.0		-119.6	-743.3	
227	13.7	-339.6		-210.0	-600.0		-196.3	-939.6	
220	78.4	-261.2		GAP			78.4	-261.2	
180	-143.7	-404.8		600.0	0.0		456.3	-404.8	
160	249.9	-155.0		420.0	420.0		669.9	265.0	
146	86.8	-68.2		240.0	660.0		326.8	591.8	
138	7.2	-61.0		90.0	750.0		97.2	689.0	
135	184.4	123.4		570.0	1320.0		754.4	1443.4	
116	113.1	236.5	W^I_{min}	180.0	1500.0	W^{II}_{min}	293.1	1736.5	W^{CP}_{min}
110	47.3	283.8	(kW)	120.0	1620.0	(kW)	167.3	1903.8	(kW)
106	0	283.8		780.0	2400.0		780.0	2683.8	
80	0	283.8	**688.6**	-900.0	1500.0	**2100.0**	-900.0	1783.8	**2723.3**

Again, the combined-plant pinch temperature is at the upper bound of this interval (180°C). The maximum possible savings from indirect integration, 65.2 kW, are 38 %

lower than the savings from direct integration. Solving problem **P2.1** gives the targeting values of the heat to be transferred in each of the zones. A minimum of 13.7 kW (Q_A) is to be transferred in the zone above both pinch temperatures to attain the maximum possible savings. This represents 26% of the amount to be transferred in the direct case.

An implementation of this indirect integration is found in Table 2.23 that shows the adjusted cascade heats. Only intervals above the pinch temperature of plant 1 are shown since these intervals include the two zones of interest (upper and between pinch temperatures). Since an induced pinch temperature appears in plant 1, a single interval is left for the transfer of the amount Q_A. In this case, a coincidence in the higher and lower circuits is found. A transfer of 13.7 kW is required in the upper zone. Once this amount is transferred, a circuit is established in the interval between pinch temperatures. This circuit will transfer 65.3 kW.

Table 2.23. Adjusted cascaded heats for Example 2.5

Interval	Trivedi		Ciric & Floudas	
	q_k^I (kW)	θ_k^I (kW)	q_k^{II} (kW)	θ_k^{II} (kW)
1	0	353.3	100.0	686.3
2	0	353.3	-195.0	491.3
3	7.0	360.3	-5.0	486.3
4	-231.0	129.3	-105.0	381.3
5	-30.7	98.6	-60.0	321.3
6	-69.0	29.6	-35.0	286.3
7	-29.6	0	-90.0	196.3
8	13.7	13.7	-210.0	-13.7
9	78.4	-	GAP	-
10	-143.7	-65.3	600.0	600.0

Then, the solution for the indirect integration is shown in Figure 2.16.

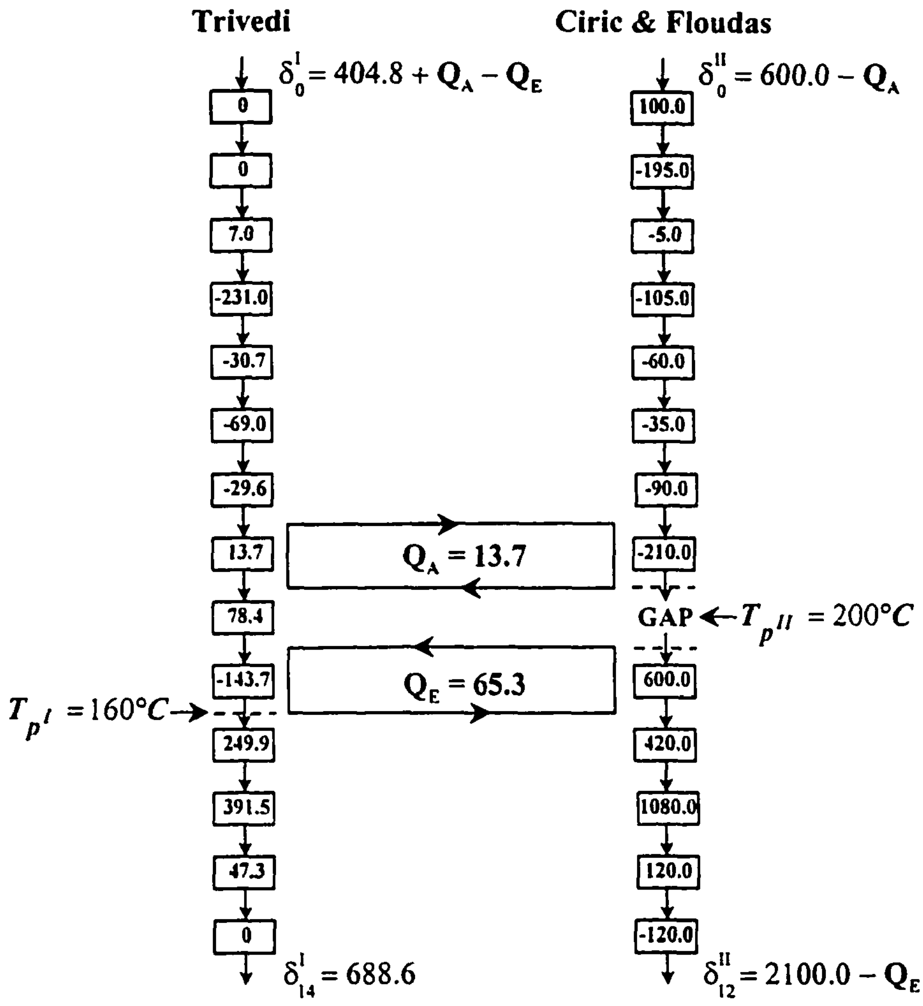
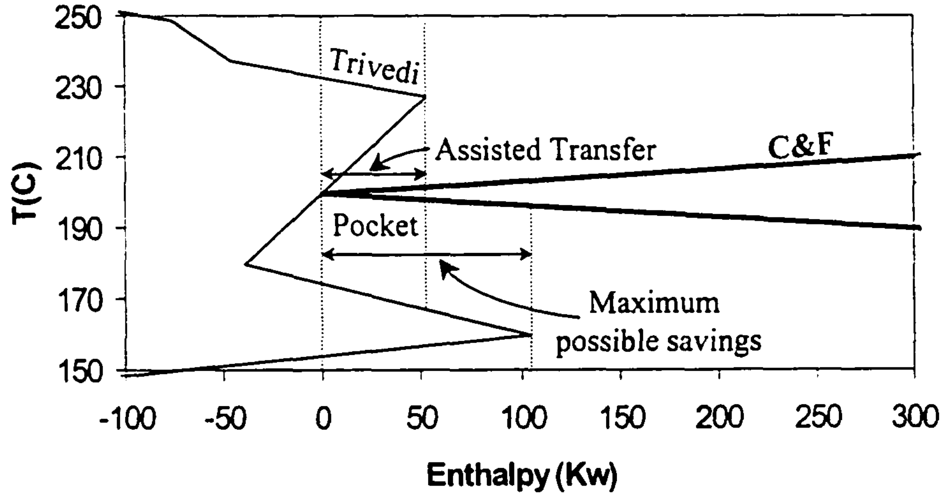
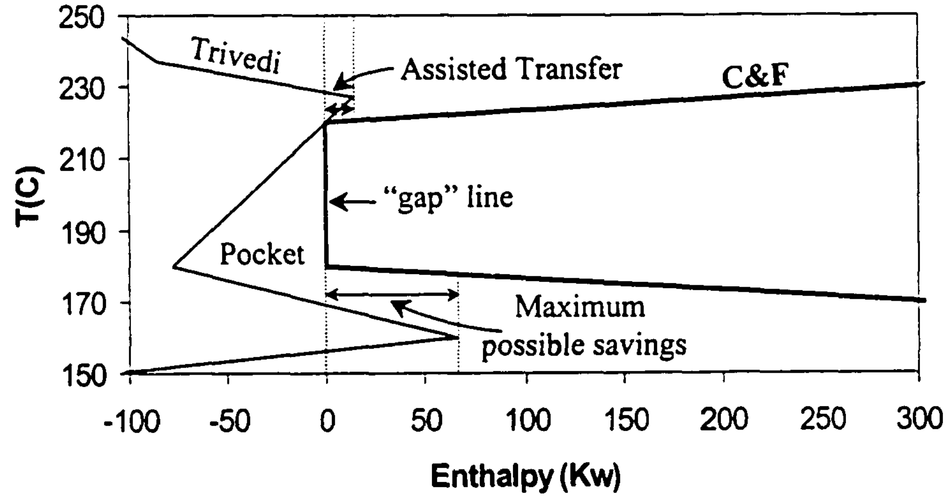


Figure 2.16. Assisted-circuit solution for Example 2.5

Use of Composite Curves: For comparison, the method that uses countercurrent profiles for the grand composite curves is also applied to the direct integration of the plants (Figure 2.17a). A pocket in plant 1 allows the transfer of the required amount of heat above both pinches that makes possible the realization of maximum savings. Figure 2.17b shows how the same method can be used for the indirect transfer of heat across plants. The vertical line in the profile of plant 2 represents the gap where no integration is possible. Still an amount of heat can be transferred from plant 1 to plant 2 inside the pocket to allow the realization of the maximum savings for the indirect integration.



(a)



(b)

Figure 2.17. Countercurrent composite curve profiles for Example 2.5

2.4.6.3 Example 2.6 (continued)

Table 2.24 shows the results of the pinch analysis for the indirect integration. After the hot temperature scale in plant 2 is shifted down 18°C, the interval between pinch temperatures is from 465.6°C to 143.5°C. The maximum possible savings obtained by solving problem **P2.1** are now 13.9 MW. This amount is 1.2 MW smaller than the maximum possible heat found for the direct integration.

Table 2.24. Pinch analysis for indirect integration in Example 2.6

T (°C)	Crude Unit			FCC Unit			Combined Plant		
	q_i^I (MW)	γ_i^I (MW)	S_{min}^I (MW)	q_i^{II} (MW)	γ_i^{II} (MW)	S_{min}^{II} (MW)	q_i^C (MW)	γ_i^C (MW)	S_{min}^{CP} (MW)
526.7	0	0	**69.1**	-5.1	-5.1	**5.1**	-5.1	-5.1	**60.1**
465.6	-8.8	-8.8		0	-5.1		-8.8	-13.9	
347.3	-2.1	-10.9		0	-5.1		-2.1	-15.9	
342.7	-2.8	-13.7		0.2	-4.9		-2.6	-18.6	
336.3	-3.1	-16.8		0.3	-4.5		-2.8	-21.3	
326.7	-5.5	-22.3		0.6	-3.9		-4.9	-26.2	
307.7	-11.4	-33.7		2.7	-1.2		-8.7	-34.9	
268.3	-4.3	-38.1		0.5	-0.7		-3.9	-38.8	
261.4	-3.7	-41.8		1.2	0.5		-2.6	-41.3	
244.9	-0.7	-42.5		0.4	0.8		-0.3	-41.6	
239.8	-0.4	-42.8		0.1	0.9		-0.3	-41.9	
238.3	-3.0	-45.8		0.9	1.8		-2.1	-44.0	
226.7	-5.2	-51.0		0.8	2.7		-4.4	-48.3	
206.3	-6.1	-57.1		0.5	3.2		-5.6	-54.0	
194.5	-3.4	-60.5		0.4	3.6		-3.0	-56.9	
184.6	-7.2	-67.8	W_{min}^I	4.1	7.6	W_{min}^{II}	-3.2	-60.1	W_{min}^{CP}
163.3	-1.3	-69.1	(kW)	3.8	11.4	(kW)	2.5	-57.6	(kW)
143.5	0.2	-68.8		0.2	11.6		0.4	-57.2	
142.6	9.8	-59.1	**10.0**	16.6	28.2	**33.3**	26.4	-30.8	**29.3**

Figure 2.18 shows the result of the higher circuit solution. Without loss of generality, some of the intervals have been lumped to clarify the illustration in the upper part of the zone between pinch temperatures and below plant 1 pinch temperature. This candidate solution covers all the intervals and transfers in plant 2 all the heat in each interval except in the last one, where a less amount of heat is transferred. In this last interval, the value of the maximum possible heat Q_E is reached.

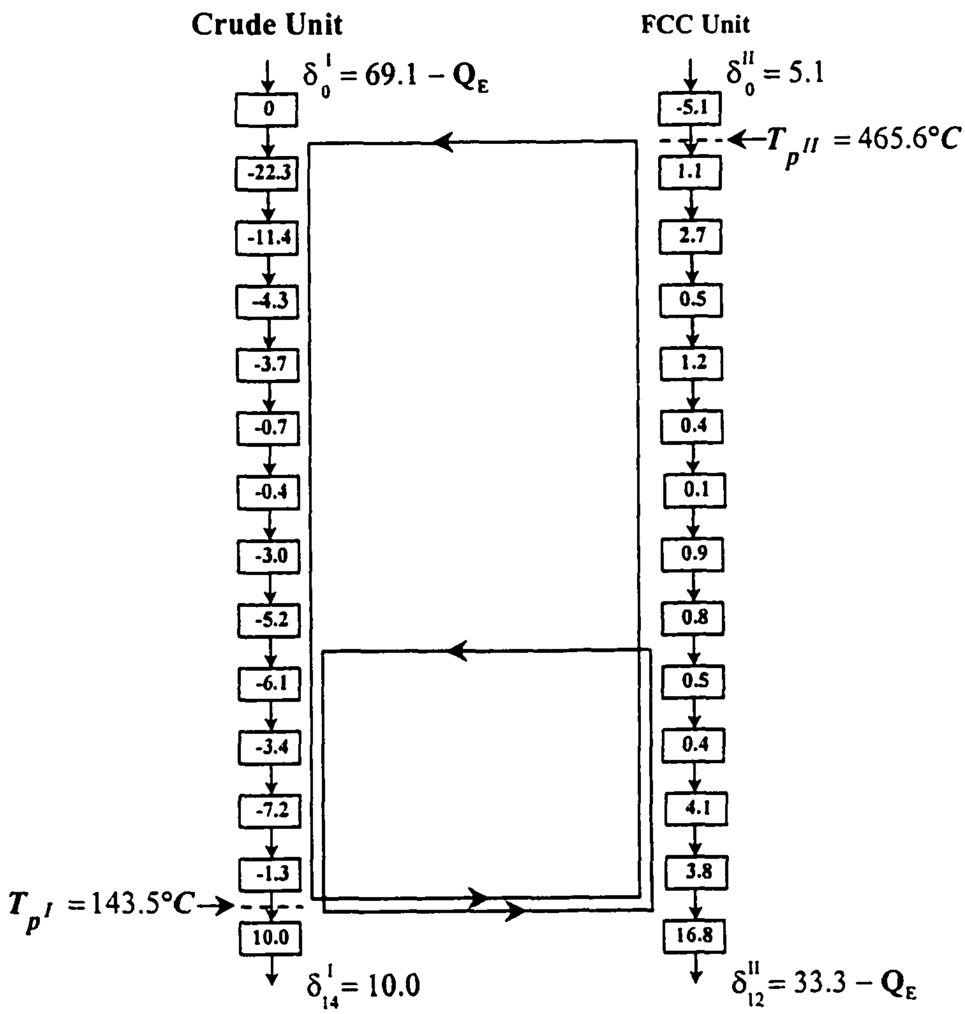


Figure 2.18. Higher and lower candidate single-circuit solutions for Example 2.6

The lower circuit solution is also shown in Figure 2.18. Only four intervals are required to transfer the maximum possible heat. In this case, the test with equations (2.38) and (2.39) fails for both candidate solutions. Then, a single circuit will transfer a smaller amount of heat than the maximum Q_E. This is determined next.

2.5 Maximum Amount Transferred by a Single Circuit

In the case where a single circuit cannot realize the entire target savings, one may still consider establishing a single circuit and realize only a portion of these total savings. Consider the unassisted case first. The maximum amount of heat transferred by a single circuit, for which its location (starting and ending intervals) is known, can be obtained by solving the following problem:

$$
\begin{aligned}
&\mathbf{P2.2} = Min\ (\delta_0^I + \delta_m^{II}) \\
&\quad s.t \\
&\quad \delta_0^I = \hat{\delta}_0^I - Q_E \\
&\quad \delta_0^{II} = \hat{\delta}_0^{II} \\
&\quad \delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = 1,\ldots,(k^E - 1); j = \mathrm{I}, \mathrm{II} \\
&\quad \left.\begin{aligned} \delta_i^I &= \delta_{i-1}^I + q_i^I + q_i^{EH} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{EC} \end{aligned}\right\} && \forall i = k^E,\ldots,(k^E + m_E) \\
&\quad \delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = (k^E + m_E + 1),\ldots,m; j = \mathrm{I}, \mathrm{II} \\
&\quad \delta_m^I = \hat{\delta}_m^I \\
&\quad \delta_m^{II} = \hat{\delta}_m^{II} - Q_E && (2.40) \\
&\quad F_E \sum_{i=k^E}^{k} \Delta T_i \geq \sum_{i=k^E}^{k} q_i^{EH} && \forall k = k^E,\ldots,(k^E + m_E - 1) \\
&\quad F_E \sum_{i=k^E}^{k^E+m_E} \Delta T_i = \sum_{i=k^E}^{k^E+m_E} q_i^{EH} \\
&\quad F_E \sum_{i=k}^{k^E+m_E} \Delta T_i \geq \sum_{i=k}^{k^E+m_E} q_i^{EC} && \forall k = (k^E + 1),\ldots,(k^E + m_E) \\
&\quad F_E \sum_{i=k^E}^{k^E+m_E} \Delta T_i = \sum_{i=k^E}^{k^E+m_E} q_i^{EC} \\
&\quad \delta_i^I, \delta_i^{II}, q_i^{EH}, q_i^{EC} \geq 0
\end{aligned}
$$

where $\Delta T_i = T_{i-1} - T_i$

The equalities that correspond to the heat balances in each interval have been split into two sets of equalities. The first one considers only those intervals in which all the heat cascades down. The second set consists of the intervals in which the transfer is taking place. This problem is linear and offers no major difficulties.

For assisted cases, the sets of equations for the balances in each interval in problem **P2.2** are replaced by the following new sets:

$$
\begin{aligned}
&\delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = 1,\ldots,(k^A - 1); j = I, II \\
&\left.\begin{aligned} \delta_i^I &= \delta_{i-1}^I + q_i^I + q_i^{AC} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{AH} \end{aligned}\right\} && \forall i = k^A,\ldots,(k^A + m_A) \\
&\delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = (k^A + m_A + 1),\ldots,(k^E - 1); j = I, II
\end{aligned}
\tag{2.41}
$$

$$
\begin{aligned}
&\delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = (k^E + m_E + 1),\ldots,(k^B - 1); j = I, II \\
&\left.\begin{aligned} \delta_i^I &= \delta_{i-1}^I + q_i^I + q_i^{BC} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{BH} \end{aligned}\right\} && \forall i = k^B,\ldots,(k^B + m_B) \\
&\delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = (k^B + m_B + 1),\ldots,m; j = I, II
\end{aligned}
\tag{2.42}
$$

In addition to these equations, the following two new sets of constraints have to be added in order to account for feasibility of a single circuit in each of the aforementioned regions:

$$
\begin{aligned}
&F_A \sum_{i=k^A}^{k} \Delta T_i \geq \sum_{i=k^A}^{k} q_i^{AC} && \forall k = k^A,\ldots,(k^A + m_A - 1) \\
&F_A \sum_{i=k^A}^{k^A+m_A} \Delta T_i = \sum_{i=k^A}^{k^A+m_A} q_i^{AC} && \\
&F_A \sum_{i=k}^{k^A+m_A} \Delta T_i \geq \sum_{i=k}^{k^A+m_A} q_i^{AH} && \forall k = (k^A + 1),\ldots,(k^A + m_A) \\
&F_A \sum_{i=k^A}^{k^A+m_A} \Delta T_i = \sum_{i=k^A}^{k^A+m_A} q_i^{AH} &&
\end{aligned}
\tag{2.43}
$$

$$
\begin{aligned}
&F_B \sum_{i=k^B}^{k} \Delta T_i \geq \sum_{i=k^B}^{k} q_i^{BC} \qquad \forall k = k^B, \ldots, (k^B + m_B - 1) \\
&F_B \sum_{i=k^B}^{k^B+m_B} \Delta T_i = \sum_{i=k^B}^{k^B+m_B} q_i^{BC} \\
&F_B \sum_{i=k}^{k^B+m_B} \Delta T_i \geq \sum_{i=k}^{k^B+m_B} q_i^{BH} \qquad \forall k = (k^B + 1), \ldots, (k^B + m_B) \\
&F_B \sum_{i=k^B}^{k^B+m_B} \Delta T_i = \sum_{i=k^B}^{k^B+m_B} q_i^{BH}
\end{aligned}
\tag{2.44}
$$

2.6 Optimum Location of a Single Circuit

Problem **P2.2** is expressed for known fixed starting and ending intervals. An MILP formulation to determine the optimum points of insertion of such a circuit in any of the regions is presented next. The case of a circuit between pinch temperatures is presented.

Consider the following general binary variables:

$$
Y_i^{FH} = \begin{cases} 1 & \text{Interval } (i+1) \text{ is the starting interval} \\ 0 & \text{Otherwise} \end{cases}
$$

$$
Y_i^{FC} = \begin{cases} 1 & \text{Interval } i \text{ is the ending interval} \\ 0 & \text{Otherwise} \end{cases}
$$

To guarantee that only an interval is a starting/ending one, the following inequalities are introduced:

$$\sum_{i=p''}^{p'-1} Y_i^{FH} = 1 \tag{2.45}$$

$$\sum_{i=p''+1}^{p'} Y_i^{FC} = 1 \tag{2.46}$$

In addition, the following variables are defined:

$$Z_i^F = \begin{cases} 1 & \text{Interval } i \text{ is in the circuit} \\ 0 & \text{Otherwise} \end{cases}$$

These variables are related to Y_i^{FH} and Y_i^{FC} by the following equalities:

$$Z_{p''+1}^F = Y_{p''}^{FH} \tag{2.47}$$

$$Z_i^F = Z_{i-1}^F + Y_{i-1}^{FH} - Y_{i-1}^{FC} \qquad \forall i = (p''+2),\ldots,p' \tag{2.48}$$

These equalities are needed to restrict the values of the heat transferred to and from the intermediate fluid to be zero for intervals that are not in the circuit. For example, consider a circuit starting in the first interval below the pinch temperature of plant 2. Then:

$$Y_{p''}^{FC} = 1 \quad \text{and} \quad Z_{p''+1}^F = Y_{p''}^{FH} = 1$$

Otherwise, $Y^{FH}_{p''} = 0$ and the first interval of transfer will be located at a low level. That is $Z^{F}_{p''+1} = 0$. Let's consider now that the third interval is the starting one. Then:

$$Y^{FC}_{p''+2} = 1 \quad \text{and} \quad Z^{F}_{p''+3} = Z^{F}_{p''+2} + Y^{FH}_{p''+2} - Y^{FC}_{p''+2} = 0 + 1 + 0 = 1$$

In any case, $Y^{FC}_{p''}$ must be zero in the interval in which $Y^{FH}_{p''}$ is one in order for the circuit to span at least an interval. Finally, consider that the circuit ends in the fifth interval. Then:

$$Y^{FC}_{p''+5} = 1 \quad \text{and} \quad Z^{F}_{p''+6} = Z^{F}_{p''+5} + Y^{FH}_{p''+5} - Y^{FC}_{p''+5} = 1 + 0 - 1 = 0$$

Therefore, there will not be a transfer in the sixth interval.

An optimization problem based on the above binary variables to solve the unassisted case is then proposed.

$$
\begin{aligned}
\mathbf{P2.3} = Min\ & (\delta_0^I + \delta_m^{II}) \\
s.t\ & \\
& \delta_0^I = \hat{\delta}_0^I - Q_E \\
& \delta_0^{II} = \hat{\delta}_0^{II} \\
& \delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = 1,\dots,p^{II}; j = I,II \\
& \left.\begin{aligned} \delta_i^I &= \delta_{i-1}^I + q_i^I + q_i^{EH} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{EC} \end{aligned}\right\} && \forall i = (p^{II}+1),\dots,p^I \\
& \delta_i^j = \delta_{i-1}^j + q_i^j && \forall i = (p^I+1),\dots,m; j = I,II \\
& \delta_m^I = \hat{\delta}_m^I \\
& \delta_m^{II} = \hat{\delta}_m^{II} - Q_E \\
& F_E \sum_{i=p^{II}+1}^{k} Z_i^E \Delta T_i \geq \sum_{i=p^{II}+1}^{k} q_i^{EH} && \forall k = (p^{II}+1),\dots,(p^I-1) \\
& F_E \sum_{i=p^{II}+1}^{p^I} Z_i^E \Delta T_i = \sum_{i=p^{II}+1}^{p^I} q_i^{EH} && (2.49) \\
& F_E \sum_{i=k}^{p^I} Z_i^E \Delta T_i \geq \sum_{i=k}^{p^I} q_i^{EC} && \forall k = (p^{II}+2),\dots,p^I \\
& F_E \sum_{i=p^{II}+1}^{p^I} Z_i^E \Delta T_i = \sum_{i=p^{II}+1}^{p^I} q_i^{EC} \\
& q_i^{EH} - UZ_i^E \leq 0 && \forall i = (p^{II}+1),\dots,p^I \\
& q_i^{EC} - UZ_i^E \leq 0 && \forall i = (p^{II}+1),\dots,p^I \\
& Z_{p^{II}+1}^E = Y_{p^{II}}^{EH} \\
& Z_i^E = Z_{i-1}^E + Y_{i-1}^{EH} - Y_{i-1}^{EC} && \forall i = (p^{II}+2),\dots,p^I \\
& \sum_{i=p^{II}}^{p^I-1} Y_i^{EH} = 1 \\
& \sum_{i=p^{II}+1}^{p^I} Y_i^{EC} = 1 \\
& \delta_i^I, \delta_i^{II}, q_i^{EH}, q_i^{EC}, Z_i^E \geq 0 \\
& Y_i^{EH}, Y_i^{EC} \in \{0,1\}
\end{aligned}
$$

In these equations, U is an upper bound of the total heat that can be transferred. This is a mixed integer nonlinear problem having a single nonlinearity consisting of the product of a continuous variable times a binary variable. The following constraints are introduced to eliminate this nonlinearity:

$$\left.\begin{array}{l} B_i = F^E Z_i^E \\ F^E \geq 0 \\ Z_i^E = (0,1) \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} B_i - Z_i^E\,\Omega \leq 0 \\ B_i \geq 0 \\ (F^E - B_i) - (1 - Z_i^E)\,\Omega \leq 0 \\ (F^E - B_i) \geq 0 \\ Z_i^E = (0,1) \end{array}\right. \tag{2.50}$$

where Ω is a sufficiently large number.

Assisted cases can also be solved by introducing similar constraints in the appropriate temperature intervals.

2.6.1 Example of Location of a Single Circuit

2.6.1.1 Example 2.6 (continued)

The linear formulation of problem **P2.3** by introducing the simplification (2.50) is implemented in GAMS (Brooke et. al., 1996). The MILP model obtained is solved with the CPLEX solver. The two optimum solutions found are shown in Table 2.25 and Figure 2.19.

Table 2.25. Single-circuit solutions for Example 2.6

Solution	N° of Intervals	T_{up}(°C)	T_{down}(°C)	F(MW/°C)
Optimum #1	4	226.7	163.3	0.199
Optimum #2	3	206.3	163.3	0.293

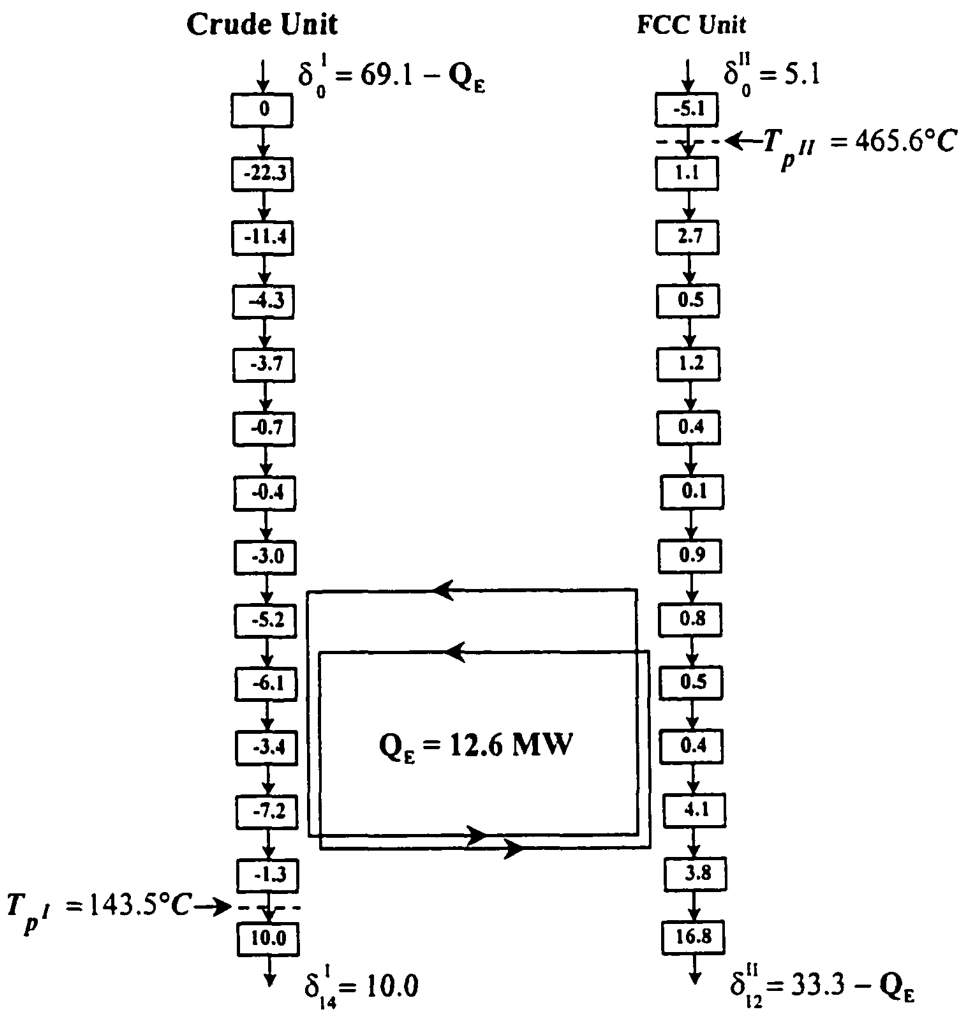


Figure 2.19. The two alternative single-circuit solutions for Example 2.6

Either of the circuits is capable of transferring 12.6 MW, which represents 91% of the total possible savings predicted by problem **P2.1**. Since a single circuit is not capable of transferring the maximum possible heat, a new formulation is presented that achieves this target with the minimum number of circuits.

2.7 Optimum Location of Many Circuits

When a single circuit is not capable of realizing the maximum target savings, a step-by-step increase of the number of circuits seems a logical procedure to reach the minimum required. Then at each step, the optimal location of an increasing number of circuits is to be found by maximizing the overall heat transfer. The following modification of problem **P2.3** is proposed in order to find the location of a number n of circuits:

$$
\mathbf{P2.4} = Min\ (\delta_0^I + \delta_m^{II})
$$

$$
\begin{aligned}
& s.t \\
& \delta_0^I = \hat{\delta}_0^I - Q_E^{(l)} \\
& \delta_0^{II} = \hat{\delta}_0^{II} \\
& \delta_i^J = \delta_{i-1}^J + q_i^J \qquad \forall i = 1,\dots, p^{II};\ \mathrm{j} = \mathrm{I,II} \\
& \left.\begin{aligned}
& \delta_i^I = \delta_{i-1}^I + q_i^I + \sum_{l=1}^{n} q_{i,l}^{EH} \\
& \delta_i^{II} = \delta_{i-1}^{II} + q_i^{II} - \sum_{l=1}^{n} q_{i,l}^{EC}
\end{aligned}\right\} \quad \forall i = (p^{II}+1),\dots,p^I \\
& \delta_i^J = \delta_{i-1}^J + q_i^J \qquad \forall i = (p^I+1),\dots,m;\ \mathrm{j} = \mathrm{I,II} \\
& \delta_m^I = \hat{\delta}_m^I \\
& \delta_m^{II} = \hat{\delta}_m^{II} - Q_E^{(l)} \\
& \left.\begin{aligned}
& F_E^{(l)} \sum_{i=p^{II}+1}^{k} Z_{i,l}^E \Delta T_i \geq \sum_{i=p^{II}+1}^{k} q_{i,l}^{EH} \quad \forall k = (p^{II}+1),\dots,(p^I-1) \\
& F_E^{(l)} \sum_{i=p^{II}+1}^{p^I} Z_{i,l}^E \Delta T_i = \sum_{i=p^{II}+1}^{p^I} q_{i,l}^{EH} \\
& F_E^{(l)} \sum_{i=k}^{p^I} Z_{i,l}^E \Delta T_i \geq \sum_{i=k}^{p^I} q_{i,l}^{EC} \qquad \forall k = (p^{II}+2),\dots,p^I \\
& F_E^{(l)} \sum_{i=p^{II}+1}^{p^I} Z_{i,l}^E \Delta T_i = \sum_{i=p^{II}+1}^{p^I} q_{i,l}^{EC} \\
& q_{i,l}^{EH} - UZ_{i,l}^E \leq 0 \qquad\qquad \forall i = (p^{II}+1),\dots,p^I \\
& q_{i,l}^{EC} - UZ_{i,l}^E \leq 0 \qquad\qquad \forall i = (p^{II}+1),\dots,p^I \\
& Z_{p^{II}+1,l}^E = Y_{p^{II},l}^{EH} \\
& Z_{i,l}^E = Z_{i-1,l}^E + Y_{i-1,l}^{EH} - Y_{i-1,l}^{EC} \quad \forall i = (p^{II}+2),\dots,p^I \\
& \sum_{i=p^{II}}^{p^I-1} Y_{i,l}^{EH} = 1 \\
& \sum_{i=p^{II}+1}^{p^I} Y_{i,l}^{FC} = 1
\end{aligned}\right\} \forall l = 1,\dots,n \qquad (2.51) \\
& \delta_i^I, \delta_i^{II}, q_{i,l}^{EH}, q_{i,l}^{EC}, Z_{i,l}^E \geq 0 \\
& Y_{i,l}^{EH}, Y_{i,l}^{EC} \in \{0,1\}
\end{aligned}
$$

The strategy to find the optimum number of circuits consists of a trial procedure. At each step, the value obtained by solving problem **P2.4** is compared with the maximum heat possible to be transferred. If the difference is not zero, then the value of l is increased to approach the target. The minimum number of circuits resulting from this procedure will be less than or equal to the number of intervals between pinches.

2.7.1 Example of Location of Many Circuits

2.7.1.1 Example 2.6 (continued)

If the formulation presented above is now applied, a minimum of two circuits is obtained. This set of two circuits will transfer all the heat predicted by problem **P2.1**. The location of both circuits for one of the possible solutions (first alternative) to problem **P2.4** is shown in Figure 2.20.

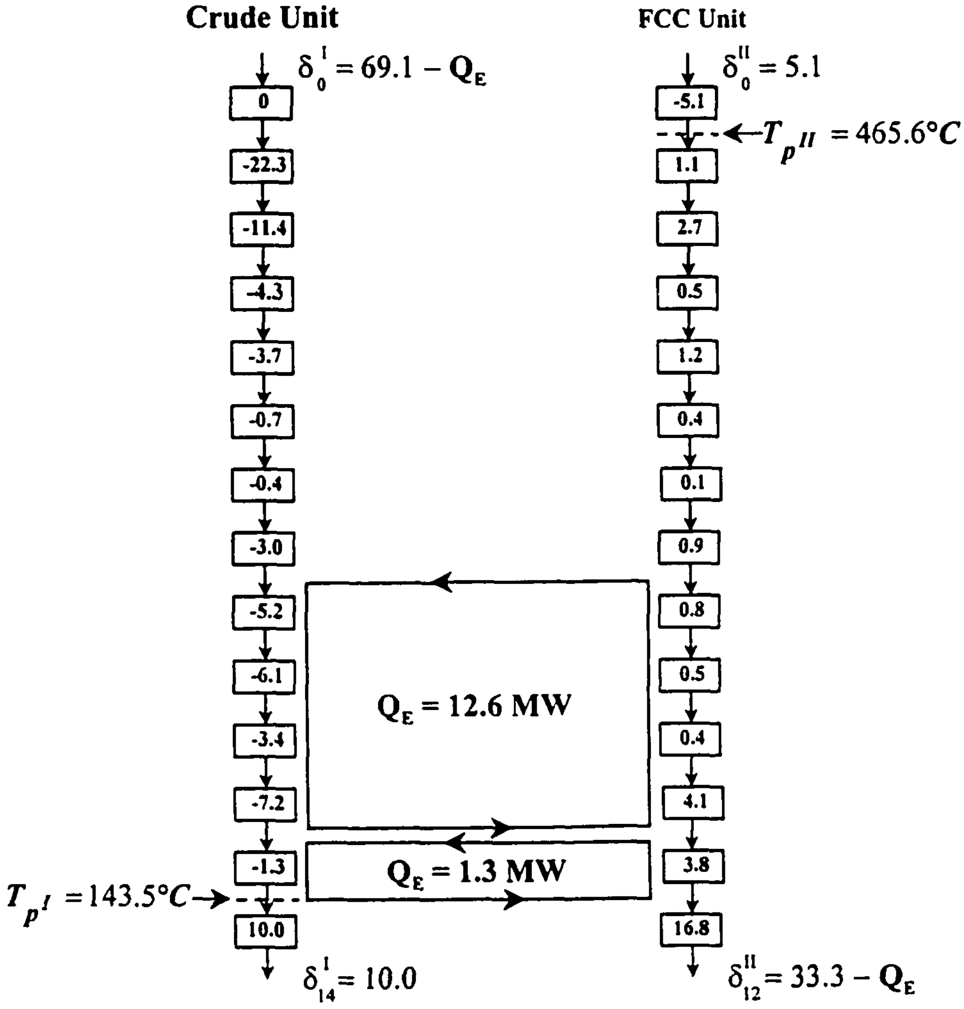


Figure 2.20. Two-circuits solution for Example 2.6 (first alternative)

As illustrated, this solution is the combination of one of the possible solutions obtained for a single circuit (covering four intervals) and a circuit covering the last interval. Another possible solution (Second alternative) is shown in Figure 2.21.

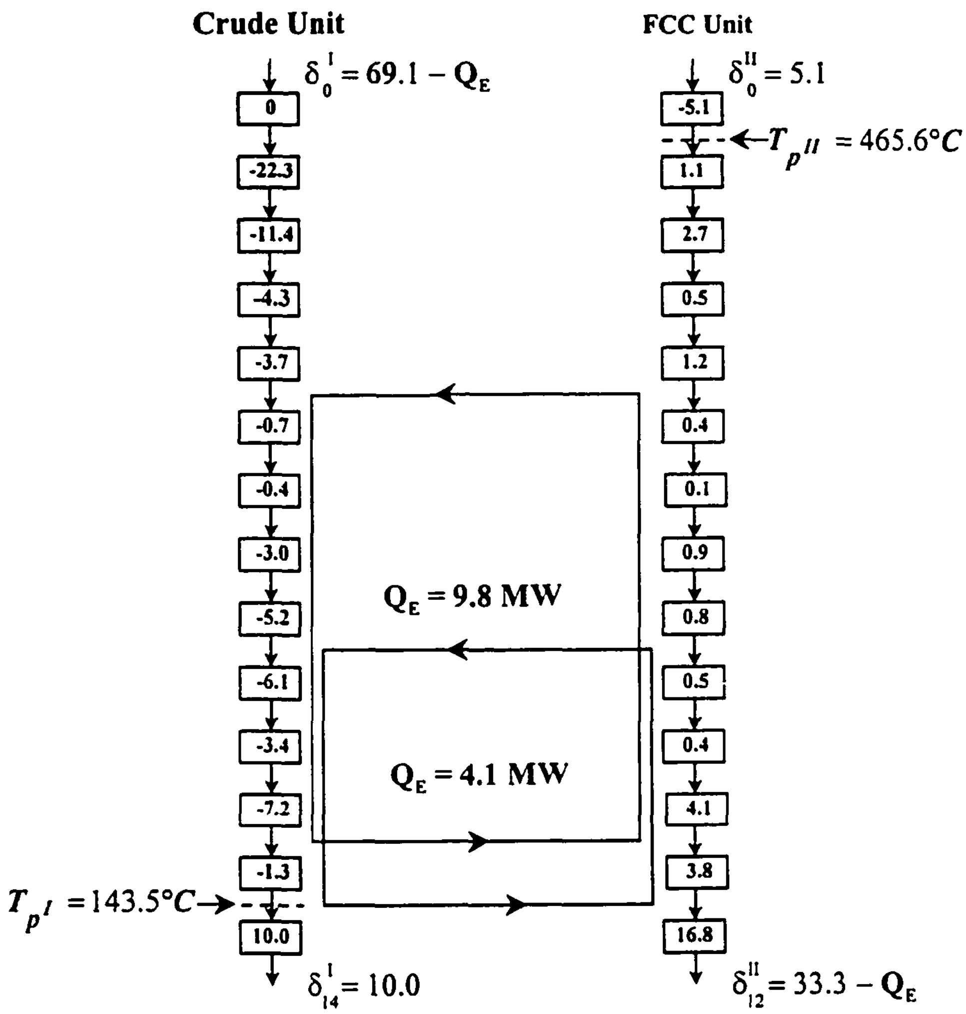


Figure 2.21. Two-circuits solution for Example 2.6 (second alternative)

In this alternative, the two circuits overlap. Due to degeneracy, there is a large number of possible solutions. The two alternatives considered here were obtained using GAMS with CPLEX.

2.8 Indirect Integration Using Steam

The use of steam for indirect integration imposes extra restrictions in the maximum amount of heat that can be transferred. Consider that steam at fixed pressure levels is generated by the plant having an excess of heat (higher pinch temperature). For simplicity, assume first that a single steam-temperature level is specified between pinches, and that is the only indirect fluid used (Figure 2.22).

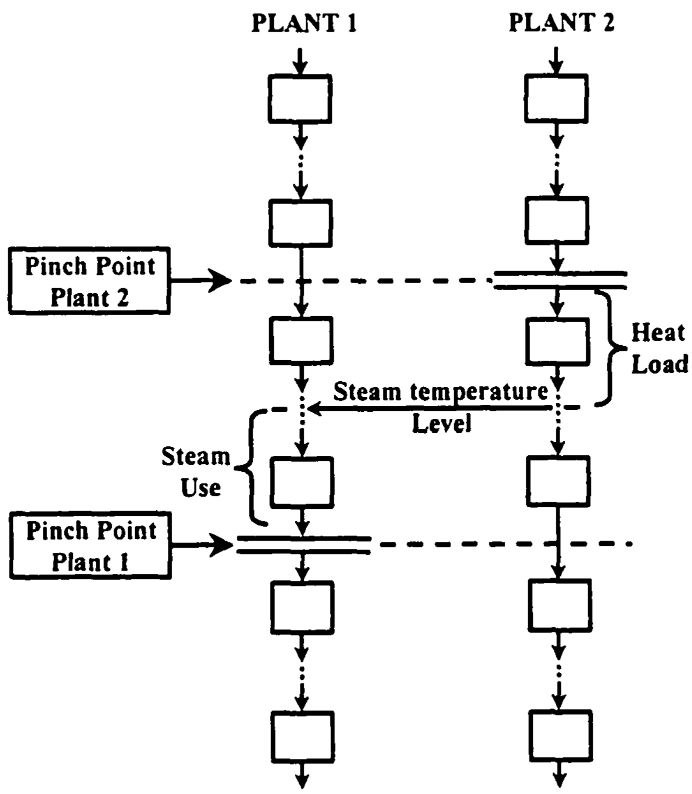


Figure 2.22. Indirect integration using a single steam-temperature level

By computing the cooling utility required for the source plant from its pinch temperature down to the level of steam generation, the heat load of this steam can be established. This amount is the maximum heat this steam will be able to transfer to the sink plant. Consider now the sink plant and the zone between pinch temperatures. This plant will be able to use the steam coming from the source plant to reduce its heating utility demands only if this steam temperature is above its pinch temperature. In addition, the maximum load that the sink plant can accept is the deficit it presents between the steam level and its pinch temperature. Thus, the use of latent heat of a single steam stream may reduce the opportunities of integration.

An alternative is the use of the utility system to balance the steam supply and demand of source and sink plants respectively (Hui and Ahmad, 1994). In any case, the difficulties arise when more than steam level is considered. Hui and Ahmad (1994) consider the utility as a "market" selling and buying utilities at fixed prices from the processes. Treating every single plant individually, they applied a procedure for multiple utilities optimization (Parker, 1989).

2.9 Conclusions

There is a large incentive to perform heat integration across plants. Models that account for maximum energy savings by direct and indirect heat integration, including in this last case the location of the fluid circuits for the case of two plants, were presented. Consequently, a strategy to capture these savings was developed. While all of these studies determine the target savings, there is still a need to determine a heat exchanger network that can accomplish minimum energy consumption while the plants are integrated, as well as when they are functioning separately. This must take place at a minimum investment cost. Among many other options, dual-use heat exchanger networks featuring minimum number of units accomplish such goal. The design of such networks is attempted in Chapter 3

2.10 Nomenclature

F = product of heat capacity and flow
i = temperature interval
k = auxiliary temperature intervals
k^A = first transfer interval in the zone above both pinch temperatures
k^B = first transfer interval in the zone below both pinch temperatures
k^E = first transfer interval in the zone between pinch temperatures
k^F = generalized first transfer interval for indirect heat integration
k^+ = first interval with nonzero heat transferred from the intermediate fluid
m = total number of intervals
m^I = total number of intervals plant 1
m^{II} = total number of intervals plant 2
p^I = last interval above the pinch of plant 1
p^{II} = last interval above the pinch of plant 2
Q_A = total heat transferred in the zone above both pinch temperatures
Q_B = total heat transferred in the zone below both pinch temperatures
Q_E = total heat transferred in the zone of effective transfer of heat (between pinch temperatures)
Q_F = total heat transferred in the generalized zone
q = heat surplus or heat demand / heat transferred
q^I = heat surplus or heat demand in plant 1
q^{II} = heat surplus or heat demand in plant 2
q^C = heat demand in the heat sink plant
q^{CP} = heat surplus or heat demand in the combined plant
q^H = heat surplus in the heat-source plant
S_{min} = minimum heating utility
T_0 = initial temperature of the intermediate fluid
T = temperature
T_s = supply temperature
T_t = target temperature
T_{up} = upper temperature of a fluid circuit
T_{down} = lower temperature of a fluid circuit
U = upper bound of the total heat that can be transfer
W_{min} = minimum cooling utility
Y^{FH} = binary variable starting interval of a fluid circuit

Y^{FC} = binary variable starting interval of a fluid circuit
Z = binary variable denoting interval that belongs to a fluid circuit
z^H = auxiliary variable to determine the amount of cumulative demand

Greek Letters

ΔT_{min} = minimum temperature approach
δ_0 = minimum surplus to the first interval
$\hat{\delta}_0$ = original minimum surplus to the first interval
δ = minimum cascaded heat
$\hat{\delta}$ = original minimum cascaded heat
δ^I = minimum cascaded heat in plant 1
δ^{II} = minimum cascaded heat in plant 2
$\hat{\delta}^C$ = original minimum cascaded heat in the heat-sink plant
$\hat{\delta}^H$ = original minimum cascaded heat in the heat-source plant
γ = variable used in the cascade of heat
θ = cumulative heat demands
θ^I = adjust cascaded heat in plant 1
θ^{II} = adjust cascaded heat in plant 2
ω^H = heat availability in the heat-source plant
λ^H = heat demand in the heat-source plant

Superscripts

A = zone above both pinches
AC = cold fluid stream in the zone above both pinch temperatures
AH = hot fluid stream in the zone above both pinch temperatures
B = zone below both pinch temperatures
BC = cold fluid stream in the zone below both pinch temperatures
BH = hot fluid stream in the zone below both pinch temperatures
E = zone of effective transfer of heat (between pinch temperatures)
EC = cold fluid stream in the zone of effective transfer of heat
EH = hot fluid stream in the zone of effective transfer of heat
FC = cold fluid stream in the general case
FH = hot fluid stream in the general case
j = chemical plant

Subscripts

A = zone above both pinch temperatures
B = zone below both pinch temperatures
E = zone of effective transfer of heat (between pinch temperatures)
K = generalized zone
i = temperature interval

$j =$ chemical plant
$k =$ auxiliary temperature intervals
$p^{I} =$ last interval above the pinch temperature of plant 1
$p^{II} =$ last interval above the pinch temperature of plant 2
$r =$ hot stream
$s =$ cold stream

2.11 References

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CHAPTER 3

Multipurpose Heat Exchanger Networks for Heat Integration across Plants

3.1 Introduction

In the previous chapter, opportunities for energy savings between two plants were identified making use of a systematic mathematical programming procedure. In this chapter, the targets identified are employed in the synthesis of multipurpose heat exchanger networks that are capable of operating each plant stand-alone as well as both plants integrated. Several mathematical programming models are presented for the design of these multipurpose heat exchanger networks. The proposed models feature the minimum number of units for both direct and indirect heat integration and consider unassisted and assisted forms of integration. Although better heat exchanger models can be used, the simplicity of models featuring maximum energy recovery and the minimum number of units allows the discussion of the complexity of the problem in a more straightforward fashion. To illustrate these concepts, three examples are considered for which energy target were calculated in the previous chapter. The first two examples show the integration of small problems taken from literature. The third example is an integration of a crude unit and a FCC plant. Finally, a simplified economical analysis on this last example is introduced to compare direct integration using the crude stream and indirect integration using an intermediate fluid.

3.2 Target Savings

In this section, we briefly review the conclusions of the targeting procedures for energy savings by heat integration across two plants presented in the previous chapter. The following conclusions have been established:

(a) Energy savings across plants are effectively accomplished by transferring heat between plant pinch temperatures.

(b) An LP model to determine the maximum savings that can be achieved by transferring heat from plant 2 to plant 2 between pinch temperatures is presented. Flexibility for the design of multipurpose heat exchanger networks is a consequence of the degenerate solutions of this problem.

(c) The existence of assisted heat transfer, omitted by other researchers, is pointed out. This assisted heat transfer consists of transferring heat above or below the zone between pinch temperatures, which is needed to enable maximum heat transfer between pinch temperatures.

(d) For indirect integration, a procedure is presented to evaluate the feasibility of a single circuit in order to achieve maximum energy savings.

(e) An MILP model is presented and solved to determine the location of a single circuit. Then the model is extended to multiple circuits to include the case in which a single circuit is not capable of realizing maximum energy savings.

Examples 2.4, 2.5 and 2.6 are selected from the previous chapter and the savings obtained in the different integration scenarios are reported in Table 3.1.

Table 3.1. Target savings

Scenario	Example 2.4	Example 2.5	Example 2.6
Direct Integration	107.5 kW	104.5 kW	15.1 MW
Indirect Integration	107.5 kW	65.3 kW	13.9 MW
One Intermediate Fluid Circuit	107.5 kW	51.6 kW	12.6 MW
Two Intermediate Fluid Circuit	-	65.3 kW	13.9 MW

In example 2.4, maximum energy savings can be accomplished using either direct or indirect heat integration. In the case of indirect integration, the use of a single intermediate fluid circuit between pinch temperatures guarantees the transfer of the total heat target amount. Therefore, this is an example of an unassisted heat transfer case. There are different solutions due to degeneracy, and the result is different possible ways of implementing the intermediate-fluid circuit. Higher and lower circuit solutions can be obtained by algorithmic methods.

Example 2.5 shows the instance of an assisted heat integration case. Direct integration requires the transference of assisting heat in the region above both pinch temperatures to attain maximum savings by transferring between pinch temperatures. Because of a gap resulting from the shifts in the temperature scales, the heat amount that plant 1 can receive is reduced. Therefore, indirect integration accomplishes fewer savings when compared to direct integration. A single circuit in the zone between pinch temperatures is not capable of transferring all the maximum possible heat for indirect integration. This limitation is removed by adding another circuit in the zone above pinches.

In the third example, direct integration can attain larger savings because for indirect integration the region between pinch temperatures reduces to make possible the use of the intermediate-fluid circuit. The use of a single circuit can transfer up to 91% of the total indirect integration possible savings. Two degenerate solutions were identified. They provide flexibility in the selection of the intermediate fluid flowrate. Total savings can be achieved by the implementation of a system of two intermediate fluid circuits. The degeneracy of the two circuits problem produces many possible solutions giving a larger flexibility to the design.

3.3 Multipurpose Heat Exchanger Networks

The design of multipurpose heat exchanger networks is now considered. An extension of the "minimum matching approach" (Papoulias and Grossmann, 1983) is developed to obtain the minimum number of matches for the system and translate this result to a network with the minimum number of heat exchangers. The transshipment model that finds the minimum number of matches among a set of streams has the advantage of being simple and computationally tractable. This model has limitations on the estimation of global cost optimality because of the use of the same minimum temperature approach for both the energy recovery and the design of the resulting heat exchanger network. This is pointed out by Gundersen and Grossmann (1990), who also mention the need of differentiating between the various heat exchanger network structures that can be obtained after the transshipment model is solved. However, for the purposes of showing the interaction between two plants when a multipurpose heat exchanger network is designed, this simple model is a good starting point. The complexity of more elaborate models would put a shadow on the intricacies of the multipurpose design problem being attempted.

3.3.1 Sets and Constraints

In order to distinguish regions in which indirect/direct heat transfer across plants takes place the subscript r is used. The values $r = e_u$ and $r = e_l$ correspond to the regions immediately above and below the combined-plant pinch temperature. These regions span between pinch temperatures, and are the ones in which heat that leads to savings is effectively transferred. In turn, the values $r = a$ and $r = b$ correspond to the regions above and below both pinch temperatures in which assisted heat transfer takes place. Moreover, values $r = u_1 / u_2$ corresponding to regions above the pinch temperature and $r = l_1 / l_2$ corresponding to regions below the pinch temperature of a single plant are used. These regions consist of the addition of the corresponding heat transfer regions across plants. All these regions are depicted in Figure 3.1.

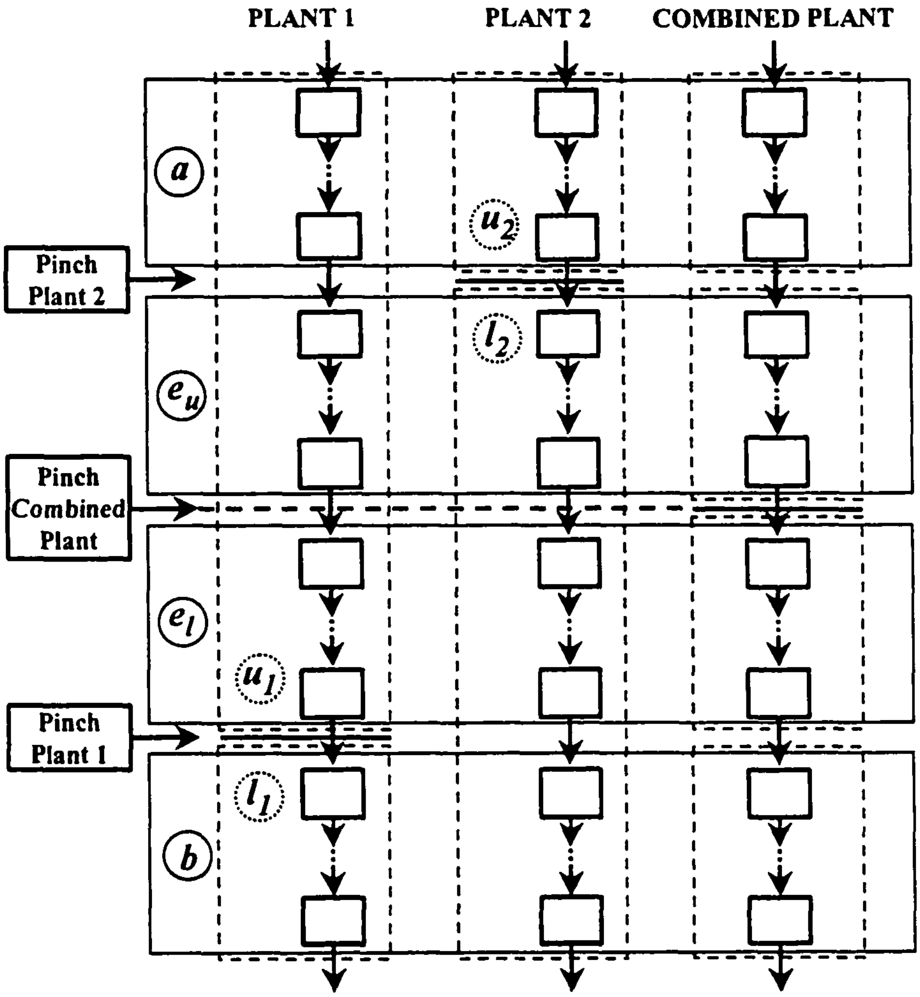


Figure 3.1. Heat transfer regions

For convenience the following general sets are defined:

Stand-alone plants

- N_r : set of temperature intervals corresponding to region r.
- S_r : set of hot streams including heating utilities present in region r.
- D_r : set of cold streams including cooling utilities present in region r.

- S_t: set of hot streams including heating utilities present in temperature interval $\bar{t} \le t; \bar{t}, t \in N_r$.
- D_t: set of cold streams including cooling utilities present in temperature interval $t \in N_r$.

Integration across plants

- $\hat{S}_r$: set of hot streams used for integration present in region r.
- $\hat{D}_r$: set of cold streams used for integration present in region r.
- $\hat{S}_t$: set of hot streams used for integration present in temperature interval $\bar{t} \le t; \bar{t}, t \in N_r$.
- $\hat{D}_t$: set of cold streams used for integration present in temperature interval $t \in N_r$.

Consider the constraints used by Papoulias and Grossmann (1983) in their transshipment model for minimum number of stream matches. The left-hand side of these constraints is rewritten using a compact notation as follows:

Equations

$$\mathbf{B}_{\mathrm{T}}(S_r, \hat{S}_r, D_r, \hat{D}_r, N_r) = \left[\left. \begin{matrix} \left. \begin{matrix} \delta_{it}^{\mathrm{T}} - \delta_{i(t-1)}^{\mathrm{T}} \\ + \sum\limits_{j \in D_t \subseteq D_r} V_{ijt}^{\mathrm{T}} + \sum\limits_{k \in \hat{D}_t \subseteq \hat{D}_r} V_{ikt}^{\mathrm{T}} - W_{it}^{(H)\mathrm{T}} \end{matrix} \right\} \forall i \in S_t \subseteq S_r \\ \sum\limits_{i \in S_t \subseteq S_r} V_{ijt}^{\mathrm{T}} + \sum\limits_{k \in \hat{S}_t \subseteq \hat{S}_r} V_{kjt}^{\mathrm{T}} - W_{jt}^{(C)\mathrm{T}} \quad \forall j \in D_t \subseteq D_r \end{matrix} \right\} \forall t \in N_r \right] \quad (3.1)$$

Inequalities

$$\mathbf{G}_{\mathrm{T}}(S_r, \hat{S}_r, D_r, \hat{D}_r, N_r) = \begin{bmatrix} \sum_{t \in N_r} V_{ijt}^{\mathrm{T}} - U_{ijr}^{\mathrm{T}} Y_{ijr}^{\mathrm{T}} & \forall i \in S_r, \forall j \in D_r \\ \sum_{t \in \hat{N}_r} V_{kjt}^{\mathrm{T}} - U_{kjr}^{\mathrm{T}} Y_{kjr}^{\mathrm{T}} & \forall k \in \hat{S}_r, \forall j \in D_r \\ \sum_{t \in \hat{N}_r} V_{ikt}^{\mathrm{T}} - U_{ikr}^{\mathrm{T}} Y_{ikr}^{\mathrm{T}} & \forall i \in S_r, \forall k \in \hat{D}_r \\ \left.\begin{matrix} -\delta_{it}^{\mathrm{T}} & \forall i \in S_t \subseteq S_r \\ -V_{ijt}^{\mathrm{T}} & \forall i \in S_t \subseteq S_r, \forall j \in D_t \subseteq D_r \\ -V_{kjt}^{\mathrm{T}} & \forall k \in \hat{S}_t \subseteq \hat{S}_r, \forall j \in D_t \subseteq D_r \\ -V_{ikt}^{\mathrm{T}} & \forall i \in S_t \subseteq S_r, \forall k \in \hat{D}_t \subseteq \hat{D}_r \end{matrix}\right\} & \forall t \in N_r \end{bmatrix} \tag{3.2}$$

Binary Variables

$$\mathbf{I}_{\mathrm{T}}(S_r, \hat{S}_r, D_r, \hat{D}_r) = \begin{bmatrix} Y_{ijr}^{\mathrm{T}} & \forall i \in S_r, \forall j \in D_r \\ Y_{kjr}^{\mathrm{T}} & \forall k \in \hat{S}_r, \forall j \in D_r \\ Y_{ikr}^{\mathrm{T}} & \forall i \in S_r, \forall k \in \hat{D}_r \end{bmatrix} \tag{3.3}$$

where the script **T** denotes the type of integration considered. These types are:

T=I, No integration across plants

T=II, Indirect integration across plants using an intermediate fluid

T=III, Direct integration across plants.

Notice that extra terms are added to the balances in equations (3.1) to consider not only heat transfer within the individual plants but also heat that is used for integration across plants. Moreover, separated constraints are written in equations (3.2) and (3.3) to account for the different stream matches. In addition, Finally, the specific sets that are included in the general sets previously defined are as follows:

Single Plant

- H_r^p. Hot/heating utility streams present in region r of plant p.
- C_r^p. Cold/cooling utility streams present in region r of plant p.

In these sets, the values $p = 1,2$ correspond to plant 1 and plant 2, respectively.

Intermediate Fluid

To account for the intermediate fluid streams that are used in the circuits established by targeting in each of the regions, the sets considered are:

- H_a^F. Hot intermediate fluid streams above the pinch temperature of plant 2, which correspond to circuits used in assisted cases. These are hot streams in plant 2
- C_a^F. Cold intermediate fluid streams above the pinch temperature of plant 2, which correspond to circuits used in assisted cases. These are cold streams in plant 1.
- $H_{e_u}^F$. Hot intermediate fluid streams between pinch temperatures above the combined-plant pinch temperature. These are hot streams in plant 1
- $C_{e_u}^F$. Cold intermediate fluid streams between pinch temperatures above the combined-plant pinch temperature. These are cold streams in plant 2
- $H_{e_l}^F$. Hot intermediate fluid streams between pinch temperatures below the combined-plant pinch temperature. These are hot streams in plant 1

- $C^F_{e_l}$. Cold intermediate fluid streams between pinch temperatures below the combined-plant pinch temperature. These are cold streams in plant 2
- H^F_b. Hot intermediate fluid streams below the pinch temperature of plant 1, which correspond to circuits used in assisted cases. These are hot streams in plant 1
- C^F_b. Cold intermediate fluid streams below the pinch temperature of plant 1, which correspond to circuits used in assisted cases. These are cold streams in plant 2.

3.3.2 Mathematical Model for Single Plant Integration

Based on the general constraints defined above, the transshipment model minimizing the number of matches (Papoulias and Grossmann, 1983) can be expressed in the following compact form.

$$\mathbf{P3.1} = Min \sum_{i \in H^p_r, j \in C^p_r; r = u_p, l_p; p=1,2} Y^{\mathrm{I}}_{ijr} \tag{3.4}$$

$$s.t.$$

$$\left.\begin{aligned} &\mathbf{B}_{\mathrm{I}}(H^p_r, \varnothing, C^p_r, \varnothing, N^p_r) = 0 \\ &\mathbf{G}_{\mathrm{I}}(H^p_r, \varnothing, C^p_r, \varnothing, N^p_r) \le 0 \\ &\mathbf{I}_{\mathrm{I}}(H^p_r, \varnothing, C^p_r, \varnothing) = \{0,1\} \end{aligned}\right\} \begin{aligned} &r = u_p, l_p \\ &p = 1,2 \end{aligned} \tag{3.5}$$

Note that since no integration is considered the supplementary sets $\hat{S}_r$, $\hat{D}_r$ and $\hat{N}_r$ are empty. Moreover, problem **P3.1** is by construction separable, that is, four separate problems can be considered with each of them accounting for a single plant region.

3.3.3 Variety of stream matches

With the purpose of gaining a better understanding of the different possible matching schemes in the condition of integration and stand-alone operation for the system of two plants, a description of different structures with stream matches follows. Consider the case of hot stream i and cold stream j in plant 1 in the region above the combined-plant pinch temperature. Let k correspond to some hot stream used for heat integration between plants, either intermediate fluid or a hot stream from plant 2. Table 3.2 shows the different matching possibilities in non-integrated and integrated cases.

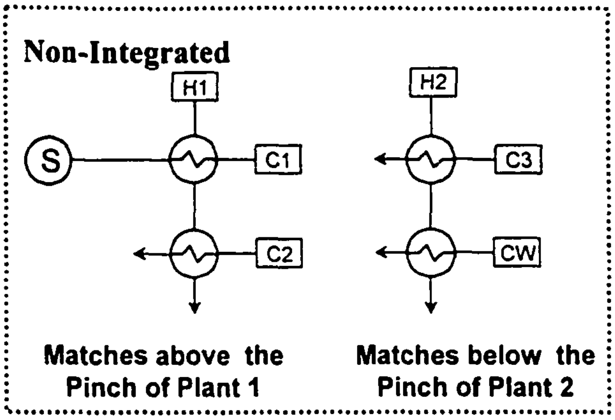
Table 3.2: Matching possibilities

CASE NUMBER	*MATCHES IN PLANT 1*	
	NON-INTEGRATED	*INTEGRATED*
1	(i,j)	(i,j)
2	(i,j)	(k,j)
3	(i,j)	(i,j) and (k,j)
4	(i,j)	-
5	-	(i,j)
6	-	(k,j)
7	-	(i,j) and (k,j)

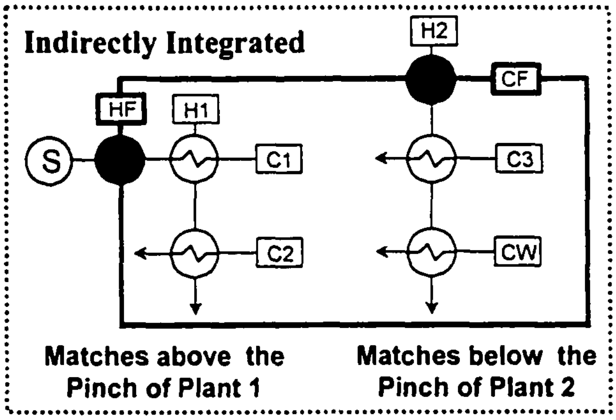
Case 1 indicates that the match *(i,j)* is present in both situations. The second case accounts for the match *(i,j)* not present in the integrated case, with stream j exchanging heat with the intermediate fluid or hot streams from plant 2 instead. Case 3 indicates that a new match is added to stream j. The next three cases account for a match not being present in one of the networks. Finally, the last case indicates that both matches are present in the integrated plant, but the match (i,j) is not present in the non-integrated plant. Similar situations arise if plant 2 is considered.

3.3.4 Additive Heat Exchanger Networks

Assume for example that the targeting procedure has revealed opportunities for indirect integration using an intermediate fluid that collects heat from plant 2 and delivers it to plant 1. Both the heating utility of plant 1 and cooling water of plant 2 demands are reduced. Consider now the network of Figure 3.2a. It corresponds to a subset of matches of a non-integrated plant. If one wants to preserve the existing structure, new heat exchangers should be added to obtain the network for the integrated operation. Thus, for the case of Figure 3.2a, the addition of two new exchangers is proposed (Figure 3.2b).



(a)



(b)

Figure 3.2. Additive heat exchanger networks

This motivates the following definition:

Definition: An additive multipurpose heat exchanger network is a network in which the heat exchangers matching hot and cold streams belonging to the plant under consideration and used for stand-alone integration are also present in conditions of integration, and they exchange heat between the same streams. Heat exchangers used for any type of integration (direct or not) are new heat exchangers.

Notice that in additive networks, it is possible to find cases in which some of the heat exchangers have zero heat loads in one of the modes of operation. Networks in which the same heat exchanger can be used for different purposes will be discussed later.

3.3.5 Mathematical Models for Indirect Heat Integration

The shift of scales used in the targeting procedure is not needed, because each plant can be solved independently. Moreover, the intermediate fluid streams may create interval partitions when they are included in the corresponding intervals by using their initial temperatures. These temperatures were already determined by the targeting procedures.

Heat-sink Plant: the following model is presented to obtain the minimum number of matches required during the integrated state by making use of the concept of additive heat exchanger networks:

$$\mathbf{P3.2}_{\mathrm{I}} = Min \left\{ \sum_{i \in H^{1}_{a \oplus e_u} \cup H^{F}_{e_u}, j \in C^{1}_{a \oplus e_u} \cup C^{F}_{a}} Y^{\mathrm{II}}_{ijr} + \sum_{i \in H^{1}_{e_l} \cup H^{F}_{e_l}, j \in C^{1}_{e_l}} Y^{\mathrm{II}}_{ijr} + \sum_{i \in H^{1}_{b}, j \in C^{1}_{b} \cup C^{F}_{b}} Y^{\mathrm{II}}_{ijr} \right\} \tag{3.6}$$

$s.t.$

$$\left. \begin{array}{l} \mathbf{B}_{\mathrm{I}}(H^{1}_{r}, \emptyset, C^{1}_{r}, \emptyset, N_r) = 0 \\ \mathbf{G}_{\mathrm{I}}(H^{1}_{r}, \emptyset, C^{1}_{r}, \emptyset, N_r) \le 0 \\ \mathbf{I}_{\mathrm{I}}(H^{1}_{r}, \emptyset, C^{1}_{r}, \emptyset) = \{0,1\} \end{array} \right\} r = a \oplus e_u, e_l, b \tag{3.7}$$

$$\begin{array}{l} \mathbf{B}_{\mathrm{II}}(H^{1}_{a \oplus e_u} \cup H^{F}_{e_u}, \emptyset, C^{1}_{a \oplus e_u} \cup C^{F}_{a}, \emptyset, N_{a \oplus e_u}) = 0 \\ \mathbf{G}_{\mathrm{II}}(H^{1}_{a \oplus e_u} \cup H^{F}_{e_u}, \emptyset, C^{1}_{a \oplus e_u} \cup C^{F}_{a}, \emptyset, N_{a \oplus e_u}) \le 0 \\ \mathbf{I}_{\mathrm{II}}(H^{1}_{a \oplus e_u} \cup H^{F}_{e_u}, \emptyset, C^{1}_{a \oplus e_u} \cup C^{F}_{a}, \emptyset) = \{0,1\} \\ \mathbf{B}_{\mathrm{II}}(H^{1}_{e_l} \cup H^{F}_{e_l}, \emptyset, C^{1}_{e_l}, \emptyset, N_{e_l}) = 0 \\ \mathbf{G}_{\mathrm{II}}(H^{1}_{e_l} \cup H^{F}_{e_l}, \emptyset, C^{1}_{e_l}, \emptyset, N_{e_l}) \le 0 \\ \mathbf{I}_{\mathrm{II}}(H^{1}_{e_l} \cup H^{F}_{e_l}, \emptyset, C^{1}_{e_l}, \emptyset) = \{0,1\} \\ \mathbf{B}_{\mathrm{II}}(H^{1}_{b}, \emptyset, C^{1}_{b} \cup C^{F}_{b}, \emptyset, N_b) = 0 \\ \mathbf{G}_{\mathrm{II}}(H^{1}_{b}, \emptyset, C^{1}_{b} \cup C^{F}_{b}, \emptyset, N_b) \le 0 \\ \mathbf{I}_{\mathrm{II}}(H^{1}_{b}, \emptyset, C^{1}_{b} \cup C^{F}_{b}, \emptyset) = \{0,1\} \end{array} \tag{3.8}$$

$$\mathbf{I}_{\mathrm{II}}(H^{1}_{r}, \emptyset, C^{1}_{r}, \emptyset) \ge \mathbf{I}_{\mathrm{I}}(H^{1}_{r}, \emptyset, C^{1}_{r}, \emptyset) \quad r = a \oplus e_u, e_l, b \tag{3.9}$$

The set of constraints (3.7) correspond to the heat-sink plant (plant 1) in conditions of no integration, while the set (3.8) corresponds to the plant operating in the integrated state including the intermediate-fluid streams present in the corresponding heat transfer regions. Notice that the supplementary sets $\hat{S}_r$, $\hat{D}_r$ and $\hat{N}_r$ are empty because the intermediate-fluid streams are added to the respective hot and cold stream sets in the corresponding heat transfer across plants regions. Finally, constraint (3.9) requires that matches existing during stand-alone operation also be present during integration. This last constraint conveys the basic concept of additive heat exchanger network. Because one set of constraints is included in the other, the objective function counts the matches for the integrated case only.

Heat-source Plant: The model is similar to the case of the heat-sink plant and is presented without further explanation.

$$\mathbf{P3.2}_2 = Min \left\{ \sum_{i \in H_a^2 \cup H_a^F, j \in C_a^2} Y_{ijr}^{\mathrm{II}} + \sum_{i \in H_{e_u}^2, j \in C_{e_u}^2 \cup C_{e_u}^F} Y_{ijr}^{\mathrm{II}} + \sum_{i \in H_{e_l \oplus b}^2 \cup H_b^F, j \in C_{e_l \oplus b}^2 \cup C_{e_l}^F} Y_{ijr}^{\mathrm{II}} \right\} \tag{3.10}$$

$s.t.$

$$\left.\begin{aligned} &\mathbf{B}_{\mathrm{I}}(H_r^2, \varnothing, C_r^2, \varnothing, N_r) = 0 \\ &\mathbf{G}_{\mathrm{I}}(H_r^2, \varnothing, C_r^2, \varnothing, N_r) \le 0 \\ &\mathbf{I}_{\mathrm{I}}(H_r^2, \varnothing, C_r^2, \varnothing) = \{0,1\} \end{aligned}\right\} r = a, e_u, e_l \oplus b \tag{3.11}$$

$$\begin{aligned} &\mathbf{B}_{\mathrm{II}}(H_a^2 \cup H_a^F, \varnothing, C_a^2, \varnothing, N_a) = 0 \\ &\mathbf{G}_{\mathrm{II}}(H_a^2 \cup H_a^F, \varnothing, C_a^2, \varnothing, N_a) \le 0 \\ &\mathbf{I}_{\mathrm{II}}(H_a^2 \cup H_a^F, \varnothing, C_a^2, \varnothing) = \{0,1\} \\ &\mathbf{B}_{\mathrm{II}}(H_{e_u}^2, \varnothing, C_{e_u}^2 \cup C_{e_u}^F, \varnothing, N_{e_u}) = 0 \\ &\mathbf{G}_{\mathrm{II}}(H_{e_u}^2, \varnothing, C_{e_u}^2 \cup C_{e_u}^F, \varnothing, N_{e_u}) \le 0 \\ &\mathbf{I}_{\mathrm{II}}(H_{e_u}^2, \varnothing, C_{e_u}^2 \cup C_{e_u}^F, \varnothing) = \{0,1\} \\ &\mathbf{B}_{\mathrm{II}}(H_{e_l \oplus b}^2 \cup H_b^F, \varnothing, C_{e_l \oplus b}^2 \cup C_{e_l}^F, \varnothing, N_{e_l \oplus b}) = 0 \\ &\mathbf{G}_{\mathrm{II}}(H_{e_l \oplus b}^2 \cup H_b^F, \varnothing, C_{e_l \oplus b}^2 \cup C_{e_l}^F, \varnothing, N_{e_l \oplus b}) \le 0 \\ &\mathbf{I}_{\mathrm{II}}(H_{e_l \oplus b}^2 \cup H_b^F, \varnothing, C_{e_l \oplus b}^2 \cup C_{e_l}^F, \varnothing) = \{0,1\} \end{aligned} \tag{3.12}$$

$$\mathbf{I}_{\mathrm{II}}(H_r^2, \varnothing, C_r^2, \varnothing) \ge \mathbf{I}_{\mathrm{I}}(H_r^2, \varnothing, C_r^2, \varnothing) \quad r = a, e_u, e_l \oplus b \tag{3.13}$$

3.3.6 Idle Heat Exchangers

The use of inequality (3.9) in problem $\mathbf{P3.2}_1$ and inequality (3.13) in problem $\mathbf{P3.2}_2$ makes it possible for some of the heat exchangers to become idle in the integrated mode of operation. Whenever a match between two streams is included in the stand-alone integration, the constraints force it to also be present in the integrated mode. Then, instances of case 1 are readily accounted for, but the models cannot represent instances of case 4 because a match is not allowed to be absent in the integrated mode when it is present in the stand-alone mode. Thus, the only possibility for case 4 is when the exchanger not required is idle during the integrated mode of operation. Therefore, case 4 is contained in case 1. The same can be said of case 3, which includes case 2 and leads to the presence of an idle exchanger whenever a case 2 instance appears.

Given the additive nature of models $\mathbf{P3.2_1}$ and $\mathbf{P3.2_2}$, the situation represented by cases 5 and 7 results in the existence of unnecessary exchangers in the stand-alone mode. These exchangers will then be idle during this mode of operation. Finally, case 6 represents the purely additive case.

The possibility of reducing the number of heat exchangers by identifying exchangers that become idle in one of the modes of operations and their possible assignment to some other matches are explored later.

3.3.7 Heat Loops

Consider now the situation depicted by Figure 3.3a where part of the network of plant 1 above the combined-plant pinch temperature that reveals opportunities for integration is presented.

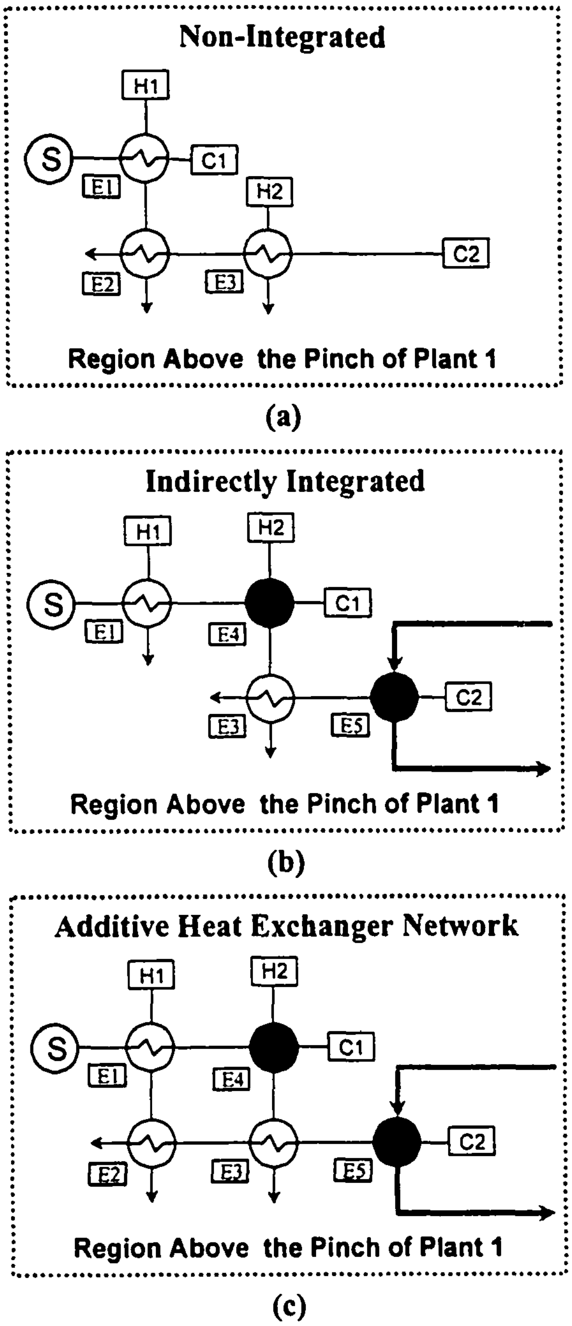


Figure 3.3. Heat Loops

The corresponding solution for the indirect integration with plant 2 requires the addition of two exchangers (Figure 3.3b). The first exchanger (E4) is between hot stream

H2 and cold stream C1 and represents a new match that uses hot and cold streams belonging to plant 1 (case 5). The second new exchanger (E5) is between the intermediate fluid stream and cold stream C2. Exchangers E3 and E5 correspond to case 3. The heat transfer by these exchangers is sufficient to fulfill the heating demand of cold stream C2. Therefore, exchanger E2, which is required when plant 1 works in stand-alone mode, is no longer required during integration (case 4).

The application of the concept of additive heat exchanger networks by the solution of model **P3.2**$_1$ for plant 1 is shown in Figure 3.3c. The result is the direct addition of exchangers E4 and E5 to the original network of plant 1. A heat loop involving exchangers E1, E2, E3 and E4 has been established. The existence of this loop has the advantage that the heat can be accommodated through these exchangers such that the overall cost is minimal. That is, exchanger E2 can still be used during integration to fulfill part of the heat demand of stream C2 if the load on exchanger E3 is reduced and the load on exchanger E4 is increased. In the case of exchanger E4, its use would be the consequence of a decrease in the load on exchanger E2 and an increase in the load on exchanger E3. On the other hand, one could choose to leave exchangers E2 or E4 idle so that cleaning can be performed on them when they are not used. However, models **P3.2**$_1$ and **P3.2**$_2$ are not favoring one solution or the other. This motivates a modification of the objective function discussed in the next section.

3.3.8 Loop Elimination

In order to guarantee the identification of the exchangers that in one mode of operation are not required (that is they can become idle without influencing operation) changes in the objective function and constraints of models **P3.2$_{\text{I}}$** and **P3.2$_{\text{2}}$** are necessary. For cases presenting a loop as the one shown in Figure 3.3c, this identification is equivalent to the elimination of the loop in each of the integration conditions.

Consider first the case in which an exchanger used during integration can become idle when the plants are working independently. The objective function used in model **P3.2$_{\text{I}}$** counts only the matches used during integration. The following modification of this function prevents matches only existing for integration purposes from transferring heat in the non-integrated case.

$$Min \left\{ \sum_{i \in H^{\text{I}}_{a \oplus e_u} \cup H^F_{e_u}, j \in C^{\text{I}}_{a \oplus e_u} \cup C^F_a} Y^{\text{II}}_{ijr} + \sum_{i \in H^{\text{I}}_{e_l} \cup H^F_{e_l}, j \in C^{\text{I}}_{e_l}} Y^{\text{II}}_{ijr} + \sum_{i \in H^{\text{I}}_b, j \in C^{\text{I}}_b \cup C^F_b} Y^{\text{II}}_{ijr} + \varepsilon \left[\sum_{i \in H^{\text{I}}_r, j \in C^{\text{I}}_r; r = a \oplus e_u, e_l, b} Y^{\text{I}}_{ijr} \right] \right\} \quad (3.14)$$

This new objective function not only counts the matches for the integrated case, but also the ones required for independent integration. Since a weighting factor ε is introduced (0.01 for example) and the matches used in the independent case are included in the integrated case, this results in an extra penalty for these matches. Therefore, this objective function makes possible the identification of idle exchangers (only used during conditions of integration).

The modification in the objective function previously presented does not guarantee the elimination of heat loops during conditions of stand-alone operation. Thus, an additional modification is needed. A new binary variable Z_{ijr}^{T} is defined which plays the same role as Y_{ijr}^{T}. The following additional constraints are also introduced:

Additional Inequalities

$$\mathbf{A}_{\mathrm{T}}(S_r, D_r, N_r) = \left[\left. \begin{matrix} \sum_{t \in N_r} V_{ijt}^{\mathrm{T}} - U_{ijr}^{\mathrm{T}} Z_{ijr}^{\mathrm{T}} \\ L_{ijr}^{\mathrm{T}} Z_{ijr}^{\mathrm{T}} - \sum_{t \in N_r} V_{ijt}^{\mathrm{T}} \end{matrix} \right\} \forall i \in S_r, \forall j \in D_r \right] \tag{3.15}$$

Additional Binary Variables

$$\mathbf{Z}_{\mathrm{T}}(S_r, D_r) = \left\lfloor Z_{ijr}^{\mathrm{T}} \quad \forall i \in S_r, \forall j \in D_r \right\rfloor \tag{3.16}$$

Therefore, the models for indirect integration that identify idle heat exchangers in either of the modes of operation are as follows:

Heat-sink Plant:

$$\mathbf{P3.3}_{\mathrm{I}} = Min \left\{ \begin{matrix} \sum_{i \in H_{a \oplus e_u}^{\mathrm{I}} \cup H_{e_u}^{F}, j \in C_{a \oplus e_u}^{\mathrm{I}} \cup C_a^F} Y_{ijr}^{\mathrm{II}} + \sum_{i \in H_{e_l}^{\mathrm{I}} \cup H_{e_l}^{F}, j \in C_{e_l}^{\mathrm{I}}} Y_{ijr}^{\mathrm{II}} + \sum_{i \in H_b^{\mathrm{I}}, j \in C_b^{\mathrm{I}} \cup C_b^F} Y_{ijr}^{\mathrm{II}} \\ + \varepsilon \left[\sum_{i \in H_r^{\mathrm{I}}, j \in C_r^{\mathrm{I}}; r = a \oplus e_u, e_l, b} \left(Y_{ijr}^{\mathrm{I}} + Z_{ijr}^{\mathrm{II}} \right) \right] \end{matrix} \right\} \tag{3.17}$$

s.t.

Constraints (3.7) through (3.9)

$$\left. \begin{matrix} \mathbf{A}_{\mathrm{II}}(H_r^{\mathrm{I}}, C_r^{\mathrm{I}}, N_r) \leq 0 \\ \mathbf{Z}_{\mathrm{II}}(H_r^{\mathrm{I}}, C_r^{\mathrm{I}}) = 0,1 \end{matrix} \right\} r = a \oplus e_u, e_l, b \tag{3.18}$$

Heat-source Plant:

$$\mathbf{P3.3}_2 = Min \left\{ \begin{array}{c} \sum_{i \in H_a^2 \cup H_a^F, j \in C_a^2} Y_{ijr}^{II} + \sum_{i \in H_{e_u}^2, j \in C_{e_u}^2 \cup C_{e_u}^F} Y_{ijr}^{II} + \sum_{i \in H_{e_l \oplus b}^2 \cup H_b^F, j \in C_{e_l \oplus b}^2 \cup C_{e_l}^F} Y_{ijr}^{II} \\ + \varepsilon \left[\sum_{i \in H_r^2, j \in C_r^2; r = a, e_u, e_l \oplus b} \left(Y_{ijr}^{I} + Z_{ijr}^{II} \right) \right] \end{array} \right\} \tag{3.19}$$

$s.t.$

Constraints (3.11) through (3.13)

$$\left. \begin{array}{l} \mathbf{A}_{II}(H_r^2, C_r^2, N_r) \leq 0 \\ \mathbf{Z}_{II}(H_r^2, C_r^2) = 0,1 \end{array} \right\} r = a, e_u, e_l \oplus b \tag{3.20}$$

The use of the new binary variable serves the purpose of avoiding matches not required during integration that are forced to exist due to the inclusion of constraints (3.9) or (3.13). Model $\mathbf{P3.3_p}$ is adopted for this paper.

3.3.9 Mathematical Model for Direct Heat Integration

The model for direct integration that minimizes the number of matches and identifies exchangers that become idle during one of the modes of operation is presented.

$$\mathbf{P3.4} = Min \left\{ \begin{array}{c} \sum\limits_{i\in H_r^1, j\in C_r^1; r=a\oplus e_u, e_l, b} Y_{ijr}^{\mathrm{III}} + \sum\limits_{i\in H_r^2, j\in C_r^2; r=a,e_u,e_l\oplus b} Y_{ijr}^{\mathrm{III}} + \sum\limits_{i\in H_r^2, j\in C_r^1; r=e_u, e_l} Y_{ijr}^{\mathrm{III}} + \sum\limits_{i\in H_r^1, j\in C_r^2; r=a,b} Y_{ijr}^{\mathrm{III}} \\ + \varepsilon\left[\sum\limits_{i\in H_r^1, j\in C_r^1; r=a\oplus e_u, e_l, b} \left(Y_{ijr}^{\mathrm{I}} + Z_{ijr}^{\mathrm{III}}\right) + \sum\limits_{i\in H_r^2, j\in C_r^2; r=a,e_u,e_l\oplus b} \left(Y_{ijr}^{\mathrm{I}} + Z_{ijr}^{\mathrm{III}}\right) \right] \end{array} \right\} \quad (3.21)$$

s.t.

Constraints (3.7) and (3.11)

$$\left.\begin{array}{l} \mathbf{B}_{\mathrm{III}}(H_{a\oplus e_u}^1, H_{e_u}^2, C_{a\oplus e_u}^1, C_a^2, N_{a\oplus e_u}) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_{a\oplus e_u}^1, H_{e_u}^2, C_{a\oplus e_u}^1, C_a^2, N_{a\oplus e_u}) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_{a\oplus e_u}^1, H_{e_u}^2, C_{a\oplus e_u}^1, C_a^2) = \{0,1\} \\ \mathbf{B}_{\mathrm{III}}(H_{e_l}^1, H_{e_l}^2, C_{e_l}^1, \varnothing, N_{e_l}) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_{e_l}^1, H_{e_l}^2, C_{e_l}^1, \varnothing, N_{e_l}) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_{e_l}^1, H_{e_l}^2, C_{e_l}^1, \varnothing) = \{0,1\} \\ \mathbf{B}_{\mathrm{III}}(H_b^1, \varnothing, C_b^1, C_b^2, N_b) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_b^1, \varnothing, C_b^1, C_b^2, N_b^1) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_b^1, \varnothing, C_b^1, C_b^2) = \{0,1\} \end{array}\right. \quad (3.22)$$

$$\left.\begin{array}{l} \mathbf{B}_{\mathrm{III}}(H_a^2, H_a^1, C_a^2, \varnothing, N_a) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_a^2, H_a^1, C_a^2, \varnothing, N_a) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_a^2, H_a^1, C_a^2, \varnothing) = \{0,1\} \\ \mathbf{B}_{\mathrm{III}}(H_{e_u}^2, \varnothing, C_{e_u}^2, C_{e_u}^1, N_{e_u}) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_{e_u}^2, \varnothing, C_{e_u}^2, C_{e_u}^1, N_{e_u}) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_{e_u}^2, \varnothing, C_{e_u}^2, C_{e_u}^1) = \{0,1\} \\ \mathbf{B}_{\mathrm{III}}(H_{e_l\oplus b}^2, H_b^1, C_{e_l\oplus b}^2, C_{e_l}^1, N_{e_l\oplus b}) = 0 \\ \mathbf{G}_{\mathrm{III}}(H_{e_l\oplus b}^2, H_b^1, C_{e_l\oplus b}^2, C_{e_l}^1, N_b) \le 0 \\ \mathbf{I}_{\mathrm{III}}(H_{e_l\oplus b}^2, H_b^1, C_{e_l\oplus b}^2, C_{e_l}^1) = \{0,1\} \end{array}\right. \quad (3.23)$$

$$\left.\begin{array}{l} \mathbf{A}_{\mathrm{III}}(H_r^1, C_r^1, N_r) \le 0 \\ \mathbf{Z}_{\mathrm{III}}(H_r^1, C_r^1) = 0,1 \end{array}\right\} r = a \oplus e_u, e_l, b \quad (3.24)$$

$$\left.\begin{array}{l} \mathbf{A}_{\mathrm{III}}(H_r^2, C_r^2, N_r) \le 0 \\ \mathbf{Z}_{\mathrm{III}}(H_r^2, C_r^2) = 0,1 \end{array}\right\} r = a, e_u, e_l \oplus b \quad (3.25)$$

$$\mathbf{I}_{\mathrm{III}}(H_r^1, \varnothing, C_r^1, \varnothing) \ge \mathbf{I}_{\mathrm{I}}(H_r^1, \varnothing, C_r^1, \varnothing) \qquad r = a \oplus e_u, e_l, b \quad (3.26)$$

$$\mathbf{I}_{\mathrm{III}}(H_r^2, \varnothing, C_r^2, \varnothing) \ge \mathbf{I}_{\mathrm{I}}(H_r^2, \varnothing, C_r^2, \varnothing) \qquad r = a, e_u, e_l \oplus b \quad (3.27)$$

Model **P3.4** first considers the set of constraints (3.7) and (3.11) corresponding to the regions of each plant in conditions of no integration. Then, constraints (3.22) and (3.23) account for the same regions when integration is performed. To avoid matches not required during integration, constraints (3.24) and (3.25) are included. Finally, constraints (3.26) and (3.27) require that matches between streams belonging to one of the plants present when they are working independently to also be present in conditions of integration across plants.

3.3.10 Intersecting Heat Exchanger Networks

An alternative solution to the problem shown in Figure 3.3 is to prevent exchangers E2 and E4 from exchanging heat in the mode of operation in which they are not required to do so. The existence of exchangers that become idle in any of the working conditions constitutes a degree of flexibility in the operation of a heat exchanger network. Switch of modes can be in some circumstances a way of putting the units that become idle out of operation for cleaning purposes. Another use of this property is the utilization of a single exchanger unit that works with different streams in the non-integrated and integrated modes. This motivates the following definition:

Definition: An intersecting multipurpose heat exchanger network is a network for which the heat exchangers matching hot and cold streams belonging to the plant under consideration are present in both stand-alone and across plants integration, either to exchange heat between the same streams in both cases or by changing them with the change in the mode of operation. Heat exchangers used for any type of integration (direct or not) are not necessary new heat exchangers.

3.4 Results

In this section, all the concepts previously presented are applied to the design of multipurpose heat exchanger networks for the three examples reviewed in the target savings section. These examples were selected from the previous chapter in which hot and cold stream data is available.

3.4.1 Example 3.1

This example consists of Test Case #2 (L&H) from Linnhoff & Hindmarsh (1983) (plant 1) and problem 4sp1 (plant 2). Applying pinch analysis resulted in a combined-plant pinch located at the same temperature as the pinch temperature of plant 2 for both direct and indirect integration (Table 2.4 and Table 2.18 in Chapter 2). Therefore, three of the four heat transfer regions depicted in Figure 3.1 are considered for the design (i.e., no partition of the region between pinches is required). As a result, the integration takes place in a single effective heat transfer region e_l below the combined-plant pinch temperature and above the pinch temperature of plant 1.

Solution to model **P3.3**$_1$ results in a network containing eight heat exchangers units for plant 1, which includes the additional heat exchanger unit using the hot intermediate-fluid stream that is only required during indirect integration. Similarly, solution to model **P3.3**$_2$ results in a network containing six heat exchangers units for plant

2, which includes the heat exchanger unit using the cold intermediate-fluid stream. Figure 3.4 shows the resulting design for heat exchanger network of the entire system for the case in which the minimum possible flowrate times heat capacity product for the intermediate fluid is used (0.72 kW/°C). This corresponds to a targeting circuit solution covering all intervals between pinch temperatures.

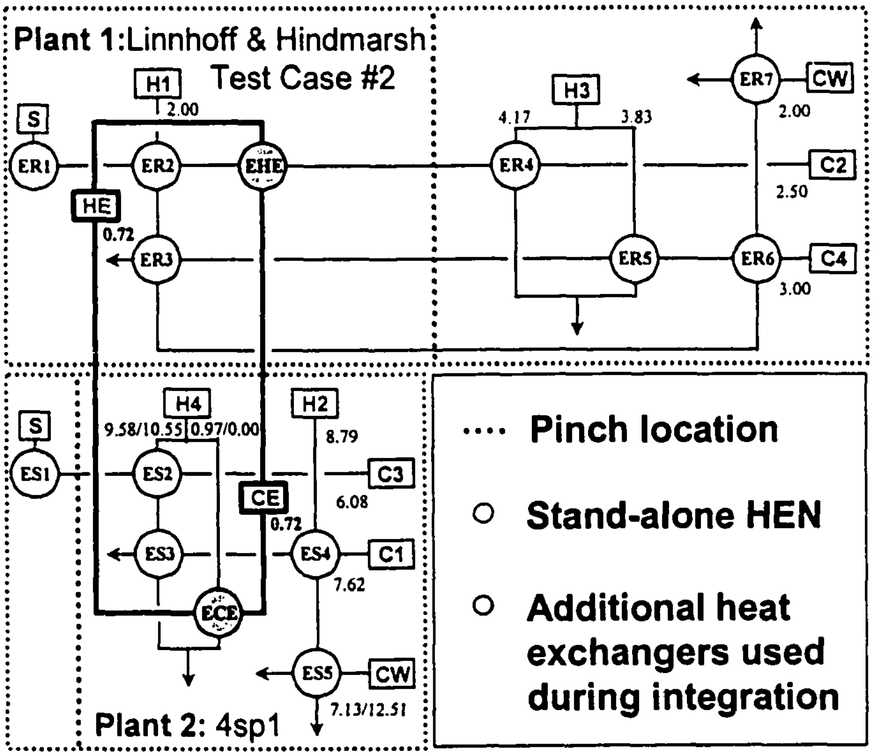


Figure 3.4. Indirect integration HEN for Example 3.1

Heat exchanger unit specifications for the network of Figure 3.4 are presented in Table 3.3. Notice that the steam heater unit used during plant 1 stand-alone operation becomes idle during integration because the intermediate fluid is capable of supplying the total amount of heating demand. This situation is predicted by the solution to model **P3.3**$_1$ that results in a network requiring seven heat exchangers units for plant 1 when is working under integration conditions. The concept of intersecting heat exchanger networks can be applied here by using a single heat exchanger unit that works with different streams depending on the mode of operation.

Table 3.3. Indirect integration HEN specifications for Example 3.1

Heat Exchanger	Heat Load (kW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	0.0/107.5	NA/270	NA/125	NA/270	NA/82
ER2	30.0	150	125/82	135	113/70
ER3	90.0	135	100	90	70
ER4	125.0	90	70	60	20
ER5	115.0	90	70	60	31.7
ER6	40.0	90	31.7	80	25
ER7	20.0	80	40	60	20
ES1	127.7	270	260	270	239
ES2	747.8	249	239	170.9/178.1	116
ES3	315.7/423.2	170.9/178.1	160	138	118.6/104.5
ES4	446.3/338.8	160	118.6/104.5	109.2/121.5	60
ES5	142.6/250.1	109.2/121.5	40	93	20
EHE	107.5/0.0	239/NA	113/NA	90/NA	70/NA
ECE	107.5/0.0	249/NA	239/NA	138/NA	90/NA

Integration / Stand alone operation NA: non-applicable

The heat exchanger network design obtained after solving the direct integration model **P3.4** is shown in Figure 3.5. It consists of thirteen heat exchanger units with only an additional unit used during integration. In this case, instead of using an intermediate-fluid circuit, a match between stream C2 of plant 1 and stream H4 of plant 2 is required during integration conditions.

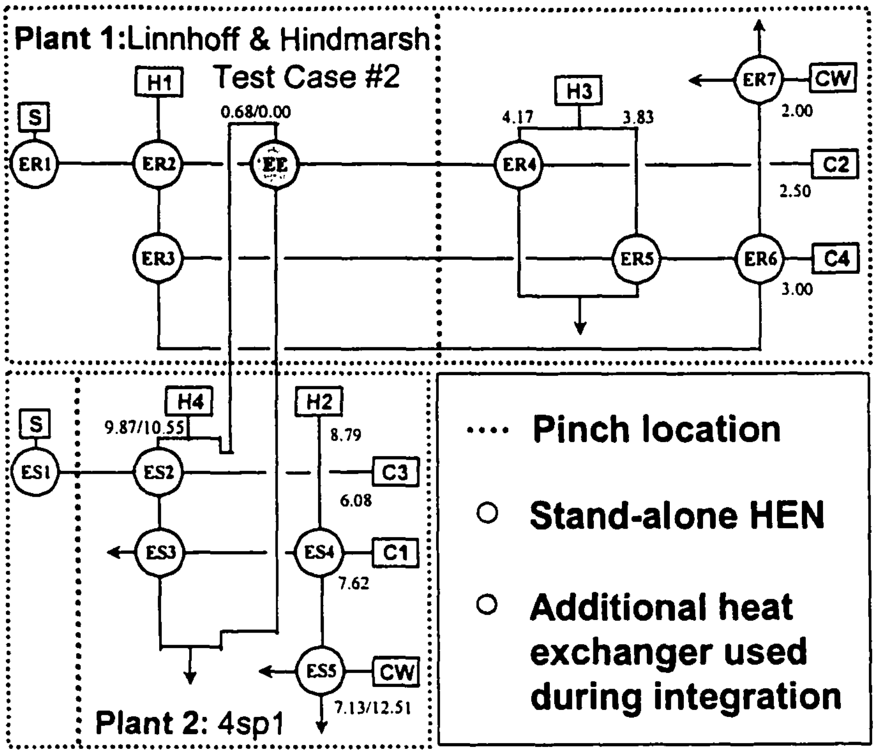


Figure 3.5. Direct integration HEN for Example 3.1

A branch of hot stream H4 is chosen to extend across plants to deliver the effective heat from plant 2 to plant 1. This results in a minimum flowrate times heat capacity product of 0.68 kW/°C that is lower than the minimum possible value for the case of indirect integration using an intermediate fluid. The explanation of a higher value for the minimum flowrate times heat capacity when the intermediate fluid is used stems from the additional minimum temperature difference required to extract the heat from hot stream H4. The disadvantage of having to pump at a higher flowrate for indirect integration (when the heat capacities of the process stream and intermediate fluid are equal) and an additional heat exchanger unit are the price paid for using a close circuit that may have process control and security advantages. However, a trade-off analysis of the pumping cost incurred in delivering the effective heat need to take into account that when the heat capacity of the intermediate fluid is higher that the process stream, it can be expected the pumping cost to compensate the cost of the additional heat exchanger unit. Further analysis is not performed because the problems used in this example are not related to real situations.

Table 3.4 shows the heat exchanger unit specifications for the direct integration network of Figure 3.5. As in indirect integration, the steam heater unit used during plant 1 stand-alone operation becomes idle during integration. The solution to model **P3.4** predicts this result by requiring a network containing only twelve heat exchangers units for the entire network when is working under integration conditions

Table 3.4. Direct integration HEN specifications for Example 3.1

Heat Exchanger	Heat Load (kW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	0.0/107.5	NA/270	NA/125	NA/270	NA/82
ER2	30.0	150	125/82	135	113/70
ER3	90.0	135	100	90	70
ER4	125.0	90	70	60	20
ER5	115.0	90	70	60	31.7
ER6	40.0	90	31.7	80	25
ER7	20.0	80	40	60	20
ES1	127.7	270	260	270	239
ES2	747.8	249	239	173.2/178.1	116
ES3	315.7/423.2	173.2/178.1	160	141.2	118.6/104.5
ES4	446.3/338.8	160	118.6/104.5	109.2/121.5	60
ES5	142.6/250.1	109.2 / 121.5	40	93	20
EE	107.5/0.0	249/NA	113/NA	90/NA	70/NA

Integration / Stand alone operation NA: non-applicable

3.4.2 Example 2

In this example, a problem from Trivedi (1988) is plant 1 and example 1 from Ciric and Floudas (1991) is plant 2. Again, the combined-plant pinch is located at the same temperature as the pinch temperature of plant 2 for both direct and indirect integration (Table 2.12 and Table 2.22 in Chapter 2). The effective transfer takes place in a single effective heat transfer region e_l below the combined-plant pinch temperature and above the pinch temperature of plant 1. The presence of a gap in the combined temperature scale for the heat cascade of plant 2 during indirect integration only represents a reduction of this region when an intermediate-fluid circuit is used. Assisting heat transfer takes place in the region $\boldsymbol{a}$ above both pinch temperatures.

When model **P3.3$_1$** is solved a network containing eighteen heat exchangers units for plant 1, which includes three additional heat exchanger units used only during indirect integration. These units are: a unit matching hot stream H3 with cold stream C2, a unit transferring effective heat to plant 1 by the use of a hot intermediate-fluid stream (effective intermediate-fluid circuit), and a unit extracting assisting heat from plant 1 by the use of a cold intermediate-fluid stream (assisting intermediate-fluid circuit). Similarly, solving model **P3.3$_2$** for plant 2 results in a network containing nine heat exchangers units, which includes two additional heat exchanger units. A cold intermediate-fluid stream is used to extract the effective heat from plant 2, and a hot intermediate-fluid stream is used to transfer assisting heat to plant 2, respectively in these exchangers (effective and assisting intermediate-fluid circuits). The resulting heat exchanger network for the entire system in which the previously obtained targeting values of flowrate and temperatures for the effective and assisting intermediate-fluid circuits are used (Chapter 2) is shown in Figure 3.6.

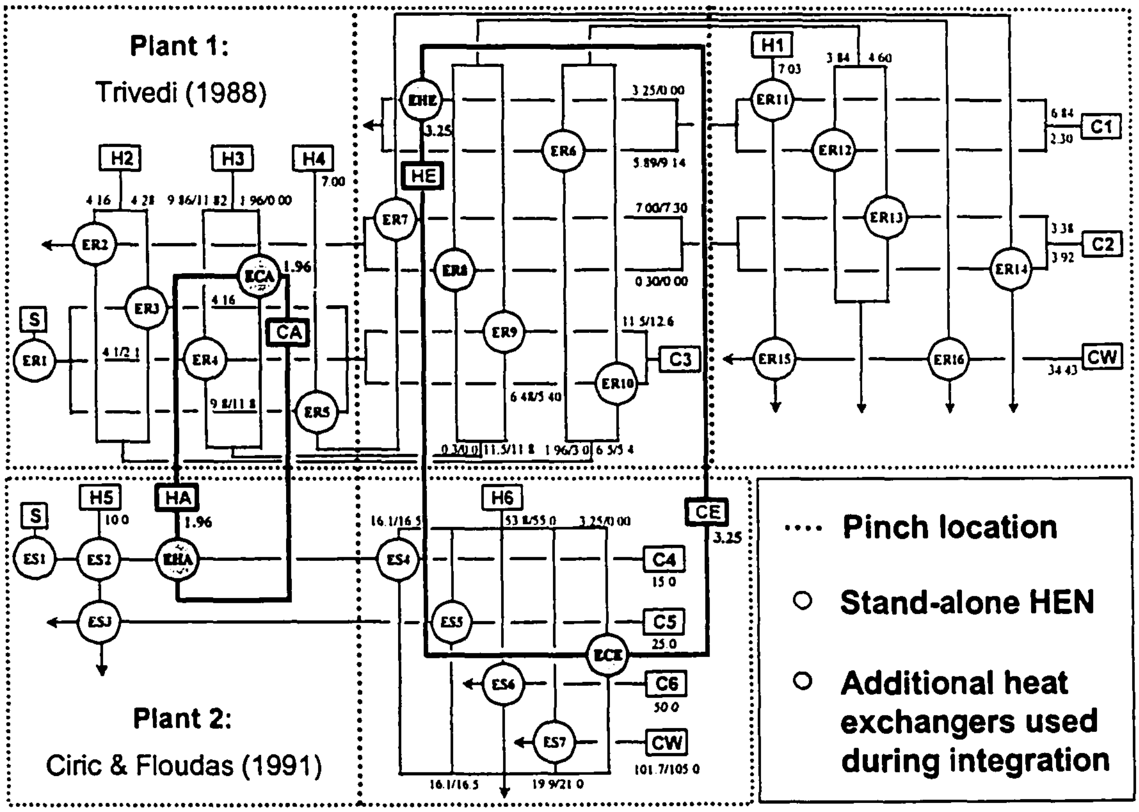


Figure 3.6. Indirect integration HEN for Example 3.2

Table 3.5 shows the heat exchanger unit specifications for the network of Figure 3.6. A pure additive case is present, and the additional heat exchangers used during indirect integration become idle during stand-alone operation.

Table 3.5. Indirect integration HEN specifications for Example 3.2

Heat Exchanger	Heat Load (kW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	353.3/404.8	300	250	300	230.4/227.5
ER2	124.0	249	217	220	200
ER3	120.7	249	229	220	200
ER4	69.0/35.2	227	217	220/224	200
ER5	357.0/339.2	271	236.5/228.2	220/222.5	200
ER6	117.6/182.9	220	160	160	140
ER7	420.0/437.8	220/222.5	200	160	140
ER8	17.8/0.0	220/NA	200/NA	160/NA	140/NA
ER9	691.2/756.5	220/224	200	160	140
ER10	388.8/323.5	220	200	160	140
ER11	301.1	160	140	117.2	96
ER12	101.3	160	140	138	96
ER13	84.4	160	140	138	115
ER14	98.0	160	140	146	115
ER15	50.5	117.2	90	110	70
ER16	638.1	160	88.5	106	70
ES1	586.3/600.0	300	270	300	230.9/230
ES2	750.0	300	230.9/230	225	180.9/180
ES3	250.0	225	190	200	180
ES4	1650.0	200	180	97.8/100	70
ES5	2750.0	200	180	97.8/100	70
ES6	550.0	200	180	97.8/100	70
ES7	2034.7/2100.0	200	180	97.8/100	70
ECA	13.7/0.0	227/NA	207/NA	220/NA	200/NA
EHA	13.7/0.0	207/NA	180.9/NA	200/NA	180/NA
EHE	65.3/0.0	239/NA	113/NA	90/NA	70/NA
ECE	65.3/0.0	249/NA	239/NA	138/NA	90/NA

Integration / Stand alone operation NA: non-applicable

Figure 3.7 shows the heat exchanger network design obtained after solving the direct integration model **P3.4** for this assisted heat integration example. The design consists of twenty-five heat exchanger units with three additional units used during integration. Two of these additional heat exchangers unit transfer the total amount of effective heat and match hot stream H6 of plant 2 with cold streams C2 and C3 of plant 1. The third additional heat exchanger unit transfers assisting heat and matches hot stream H4 of plant 1 with cold stream C4 of plant 2.

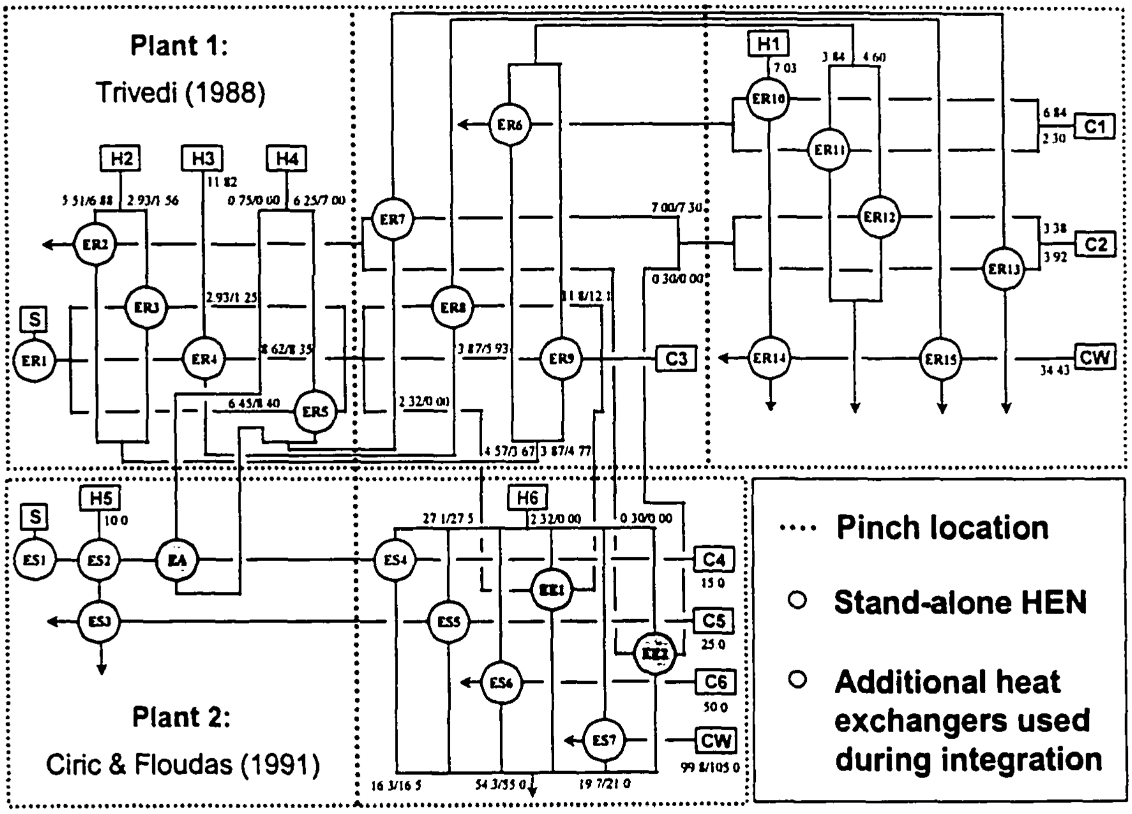


Figure 3.7: Direct integration HEN for Example 3.2

A comparison of the direct integration heat exchanger network of Figure 3.7 with the indirect integration heat exchanger network of Figure 3.6 follows. Two intermediate-fluid circuits are required in indirect integration, while direct integration requires three process streams to extend across plants. The summation of flowrates, however, is smaller for the direct integration case and the savings as determine by targeting are 60% higher when this form of integration is used. Similar to Example 3.1, the problems used in this example are not related to real situations, and further analysis is not performed. Heat exchanger unit specifications for the direct integration network of Figure 3.7 are shown in Table 3.6.

Table 3.6. Direct integration HEN specifications for Example 3.2

Heat Exchanger	Heat Load (kW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	353.3/404.8	300	250	300	230.4/227.5
ER2	270.0	249	217	200/209.8	180
ER3	143.6/61.2	249	229	200/209.8	180
ER4	319.0/308.8	227	217	200/200.9	180
ER5	444.1/485.2	271	248.9/237.8	200/201.7	180
ER6	182.9	200/209.8	160	160	140
ER7	280.0/291.8	200/201.7	180	160	140
ER8	472.6/482.8	200/200.9	180	160	140
ER9	154.7/237.2	200/209.8	180	160	140
ER10	301.1	160	140	117.2	96
ER11	101.3	160	140	138	96
ER12	84.4	160	140	138	115
ER13	98.0	160	140	146	115
ER14	50.5	117.2	90	110	70
ER15	638.1	160	88.5	106	70
ES1	547.1/600.0	300	270	300	233.5/230
ES2	750.0	300	233.5/230	225	183.5/180
ES3	250.0	225	190	200	180
ES4	1650.0	200	180	98.6/100	70
ES5	2750.0	200	180	98.6/100	70
ES6	550.0	200	180	98.6/100	70
ES7	1995.5/2100.0	200	180	98.6/100	70
EA	52.9/0.0	271/NA	233.5/NA	200/NA	180/NA
EE1	92.6/0.0	200/NA	180/NA	160/NA	140/NA
EE2	11.8/0.0	200/NA	180/NA	160/NA	140/NA

Integration / Stand alone operation NA: non-applicable

3.4.3 Example 3.3

The last example consists of a crude unit processing 150,000 bbl/day and a FCC plant processing 40,000 bbl/day. The crude unit is plant 1 while a FCC unit is plant 2. Analysis of the combined-plant pinch reveals that it is located at a temperature between the pinch temperatures of the plants for both direct and indirect integration (Table 2.16 and Table 2.24 in Chapter 2). All the regions depicted in Figure 3.1 have to be considered for the design of a heat exchanger network. In order for the combined-plant pinch to exist during integration, the maximum amounts of savings obtained by the targeting procedures must be used (Table 3.1). Therefore, in the case of indirect integration the targeting solution containing two intermediate-fluid circuits is considered first.

The solution to model $\mathbf{P3.3_1}$ gives a network containing twenty-five heat exchangers units for plant 1, which includes three additional heat exchanger units used only during indirect integration. One of these additional units is used to transfer heat from hot stream H9 to cold stream C3. The other units are transferring effective heat to cold stream C2 of plant 1 by the use of two hot intermediate-fluid streams. The corresponding two effective intermediate-fluid circuits are located one above the combined-plant pinch temperature and the other below this temperature. In turn, solving model $\mathbf{P3.3_2}$ for plant 2 results in a network containing thirteen heat exchangers units, which includes six additional heat exchanger units. Two cold intermediate-fluid streams, one using five units and the other using one unit, extract the effective heat from several hot streams of plant 2. They correspond to the two effective intermediate-fluid circuits expanding across plants at both sides of the combined-plant pinch. Figure 3.8 shows the resulting heat exchanger network for the entire system. The flowrate times heat capacity for the circuits are the minimum possible in order to minimize pumping costs.

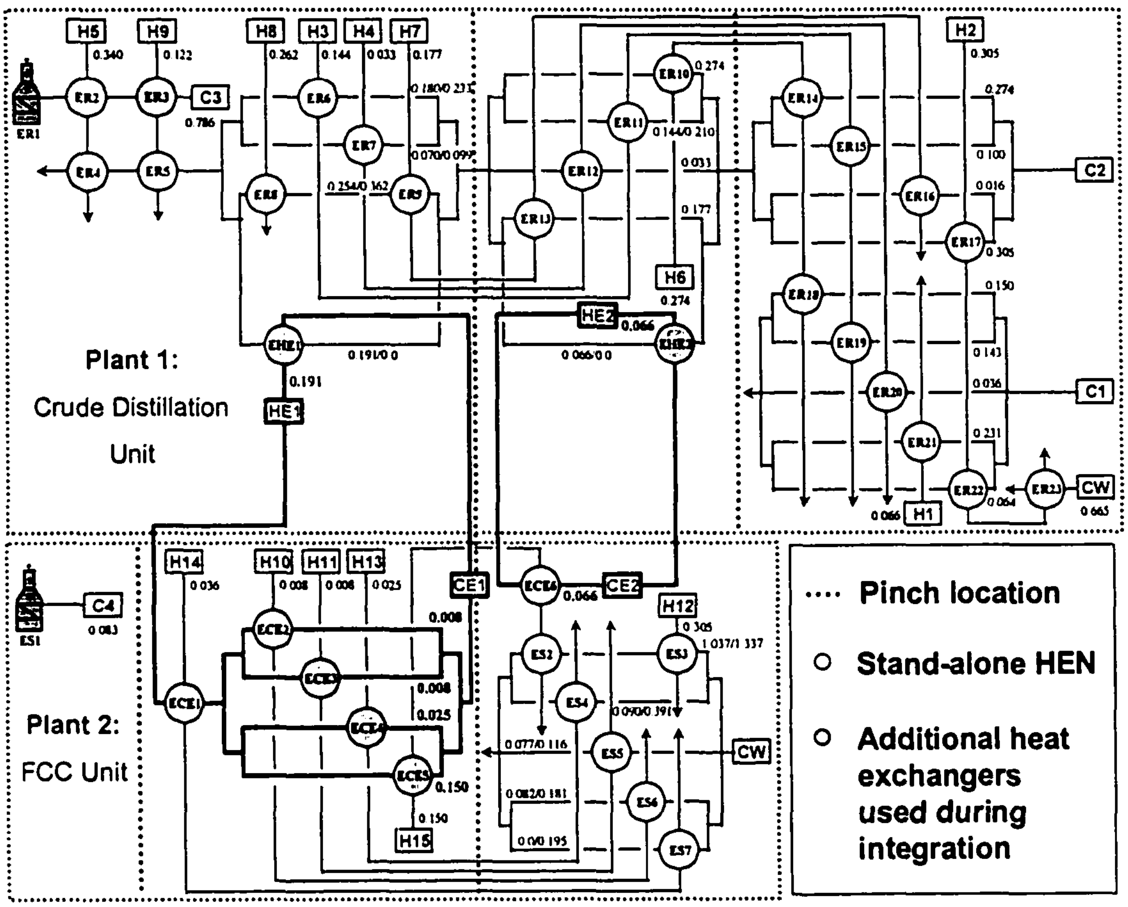


Figure 3.8. Two intermediate-fluid circuits HEN for Example 3.3

Heat exchanger unit specifications for the network of Figure 3.8 are shown in Table 3.7. The presence of idle heat exchangers during conditions of stand-alone operation is obvious from the additive nature of the model employed. However, notice that the unit that is used to transfer heat from hot stream H5 to cold stream C2 during stand-alone operation of plant 1 becomes idle during indirect integration. A heat loop is present comprising exchangers ER2 to ER5, and the heat loads are accommodated in the loop to leave unit ER3 during stand-alone operation and ER4 idle during integration. The existence of the loop, as it was previously discussed, can be a way of reducing the overall cost of the network by still using units ER3 and ER4 to fulfill part of the demand of heat of the corresponding cold streams in the modes of operation in which they become idle. Breaking the loop, by applying the concept of intersecting multipurpose heat exchanger networks, results in one less heat exchanger unit. Another heat exchanger that becomes idle during integration conditions is unit ES7 that represents a cooler use to bring stream H14 to its target temperature. The existence of idle exchangers present during integration conditions is identified by the solution to models **P3.3$_1$** and **P3.3$_2$** that predict a network with twenty-four and twelve heat exchanger units, respectively.

Table 3.7. Two intermediate-fluid circuits HEN specifications for Example 3.3

Heat Exchanger	Heat Load (MW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	55.03/69.04	427.2	352.9	427.2	282.9/265
ER2	26.86/20.19	347.3	282.9/265	268.3/287.9	248.7/239.3
ER3	7.34/0.0	336.3/NA	248.7/NA	276.1/NA	239.3/NA
ER4	0.0/6.67	NA/287.9	NA/239.3	NA/268.3	NA/229.7
ER5	4.42/11.78	276.1/336.3	239.3/229.7	239.8	232.9/212.7
ER6	14.12/12.82	261.4	236.3/212.7	163.3/172.3	157.7
ER7	5.46	326.7	236.3/212.7	163.3	157.7
ER8	14.42	261.4	236.3/212.7	206.3	157.7
ER9	5.53	194.5	179.5/173	163.3	157.7
ER10	5.43	163.3	157.7	143.5	137.9
ER11	2.85/4.15	163.3/172.3	157.7	143.5	137.9
ER12	0.66	163.3	157.7	143.5	137.9
ER13	3.51	163.3	157.7	143.5	137.9
ER14	2.91	143.5	137.9	132.9	127.3
ER15	1.06	143.5	137.9	136.1	127.3
ER16	0.16	143.5	137.9	142.6	127.3
ER17	3.24	143.5	137.9	132.9	127.3
ER18	14.63	132.9	127.3	79.6	30
ER19	14.15	136.1	129.3	37.8	30
ER20	3.53	143.5	129.3	37.8	30
ER21	5.86	127.3	121.7	37.8	30
ER22	22.45	132.9	127.3	59.4	30
ER23	9.98	59.4	30	26.7	15
ES1	5.080	538.3	532.2	538.3	471.1
ES2	7.97/12.46	168.9/243.9	30	107.2	22.3/20.7
ES3	7.59	147.2	22.3/20.7	48.9	15
ES4	1.35/5.86	168.9/348.2	30	115.5	15
ES5	1.15/1.74	168.9/190.1	30	21.1	15
ES6	1.23/2.72	168.9/348.2	30	21.1	15
ES7	0.0/2.93	NA/313.2	NA/30	NA/232.2	NA/15
EHE1	12.71/0.0	229.6/NA	224/NA	163.3/NA	157.7/NA
EHE2	1.30/0.0	163.3/NA	157.7/NA	143.5/NA	137.9/NA
ECE1	2.93/0.0	313.2/NA	229.6/NA	232.2/NA	214.3/NA
ECE2	1.50/0.0	348.2/NA	342.6/NA	168.9/NA	163.3/NA
ECE3	0.60/0.0	243.9/NA	238.3/NA	168.9/NA	163.3/NA
ECE4	4.52/0.0	348.2/NA	342.6/NA	168.9/NA	163.3/NA
ECE5	3.19/0.0	190.1/NA	184.5/NA	168.9/NA	163.3/NA
ECE6	1.30/0.0	168.9/NA	163.3/NA	160.2/NA	143.5/NA

Integration / Stand alone operation NA: non-applicable

After solving the direct integration model **P3.4** the heat exchanger network design obtained shown in Figure 3.9 is obtained. The design consists of thirty-six heat exchanger units with seven additional units used only during integration. One of these additional heat exchanger units (as it was the case for indirect integration) transfers heat from hot stream H9 to cold streams C3 in plant 1. The other six additional heat exchanger units transfer heat from plant 2 to plant 1. Five units are used above the combined-plant pinch temperature to transfer heat from several hot streams of plant 2 to cold stream C2 of plant 1. Below the combined-plant pinch temperature, transfer of heat from hot stream H15 of plant 2 to cold stream C2 of plant 1 takes place in the sixth unit.

Exactly the same minimum flowrate times heat capacity values used for the two intermediate-fluid circuits are possible to be obtained for the two splits of cold stream C2 that extend across plants. Therefore, if equal heat capacities for the intermediate fluid and cold stream C2 are assumed, it can be expected a lower total cost for the multipurpose heat exchanger network that performs direct integration. Two additional units are required and the resulting savings are 8% lower during indirect integration.

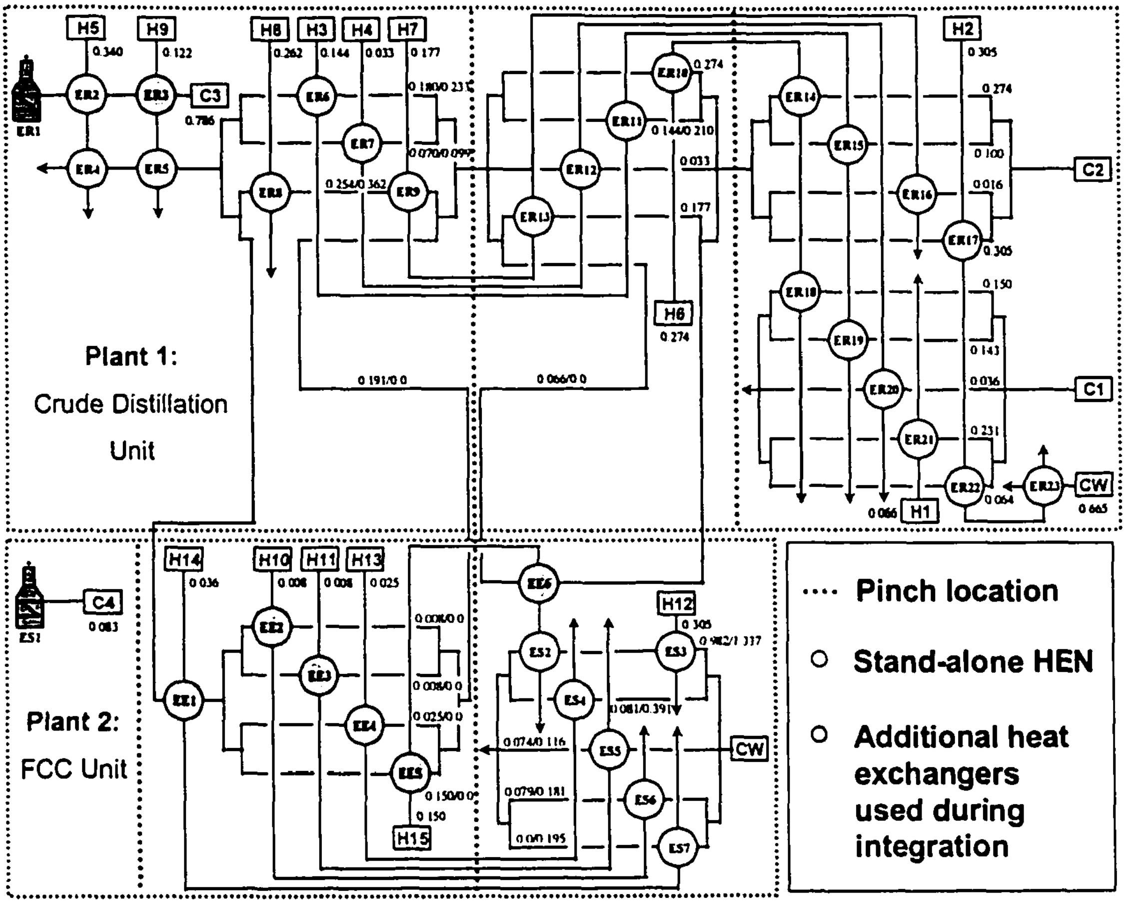


Figure 3.9. Direct integration HEN for Example 3.3

Table 3.8 shows the heat exchanger unit specifications for the direct integration network of Figure 3.9. The loop previously described for indirect integration also exists and the previously presented analysis is valid for direct integration. Moreover, in addition to heat exchanger ER4, heat exchanger ES7 becomes idle during direct integration, as it was the case during indirect integration. The solution to model **P3.4** predicts, as expected, a network with thirty-four heat exchanger units.

Table 3.8. Direct integration HEN specifications for Example 3.3

Heat Exchanger	Heat Load (MW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	53.96/69.04	427.2	352.9	427.2	284.2/265
ER2	26.86/20.19	347.3	284.2/265	268.3/287.9	250/239.3
ER3	8.41/0.0	336.3/NA	250/NA	267.4/NA	239.3/NA
ER4	0.0/6.67	NA/287.9	NA/239.3	NA/268.3	NA/229.7
ER5	3.37/11.78	267.4/336.3	239.3/229.7	239.8	234.2/212.7
ER6	14.12/12.82	261.4	236.3/212.7	163.3/172.3	157.7
ER7	5.46	326.7	236.3/212.7	163.3	157.7
ER8	14.42	261.4	236.3/212.7	206.3	157.7
ER9	5.53	194.5	179.5/173	163.3	157.7
ER10	5.43	163.3	157.7	143.5	137.9
ER11	2.85/4.15	163.3/172.3	157.7	143.5	137.9
ER12	0.66	163.3	157.7	143.5	137.9
ER13	3.51	163.3	157.7	143.5	137.9
ER14	2.91	143.5	137.9	132.9	127.3
ER15	1.06	143.5	137.9	136.1	127.3
ER16	0.16	143.5	137.9	142.6	127.3
ER17	3.24	143.5	137.9	132.9	127.3
ER18	14.63	132.9	127.3	79.6	30
ER19	14.15	136.1	129.3	37.8	30
ER20	3.53	143.5	129.3	37.8	30
ER21	5.86	127.3	121.7	37.8	30
ER22	22.45	132.9	127.3	59.4	30
ER23	9.98	59.4	30	26.7	15
ES1	5.080	538.3	532.2	538.3	471.1
ES2	7.13/12.46	163.3/243.9	30	107.2	22.7/20.7
ES3	7.59	147.2	22.7/20.7	48.9	15
ES4	1.21/5.86	163.3/348.2	30	115.5	15
ES5	1.11/1.74	163.3/190.1	30	21.1	15
ES6	1.18/2.72	163.3/348.2	30	21.1	15
ES7	0.0/2.93	NA/313.2	NA/30	NA/232.2	NA/15
EE1	2.93/0.0	313.2/NA	229.6/NA	232.2/NA	214.3/NA
EE2	1.53/0.0	348.2/NA	342.6/NA	163.3/NA	157.7/NA
EE3	0.63/0.0	243.9/NA	238.3/NA	163.3/NA	157.7/NA
EE4	4.66/0.0	348.2/NA	342.6/NA	163.3/NA	157.7/NA
EE5	4.03/0.0	190.1/NA	184.5/NA	163.3/NA	157.7/NA
EE6	1.30/0.0	163.3/NA	157.7/NA	154.7/NA	137.9/NA

Integration / Stand alone operation NA: non-applicable

The location of the combined-plant pinch temperature between pinch temperatures in both types of integration has the disadvantage of decomposing the multipurpose heat exchanger network design of plant 1 above its pinch temperature. Two heat exchanger units have to be used for the same predicted match if the match is present at both sides of the combined-plant pinch temperature. This is the case of the matches between hot streams H3, H4, and H7 and cold stream C2 in plant 1. Thus, three less heat exchanger units can be used if a certain amount of heat is allowed to pass through the combined-plant pinch temperature during integration conditions. Lower maximum amount of savings are obtained, but the decrease in the number of units drastically reduces the capital costs.

The starting point for indirect integration is to consider the targeting solution that makes use of a single intermediate fluid circuit (Table 3.1). The resulting heat exchanger network design without considering combined-plant pinch temperature partition is shown in Figure 3.10. It consists of twenty-one units, with two additional units only used during integration one transferring heat from hot stream H9 to cold stream C3 and the other transferring effective heat from the intermediate fluid to cold stream C2. In order to minimize pumping costs, the flowrate times heat capacity for the circuit is the minimum possible.

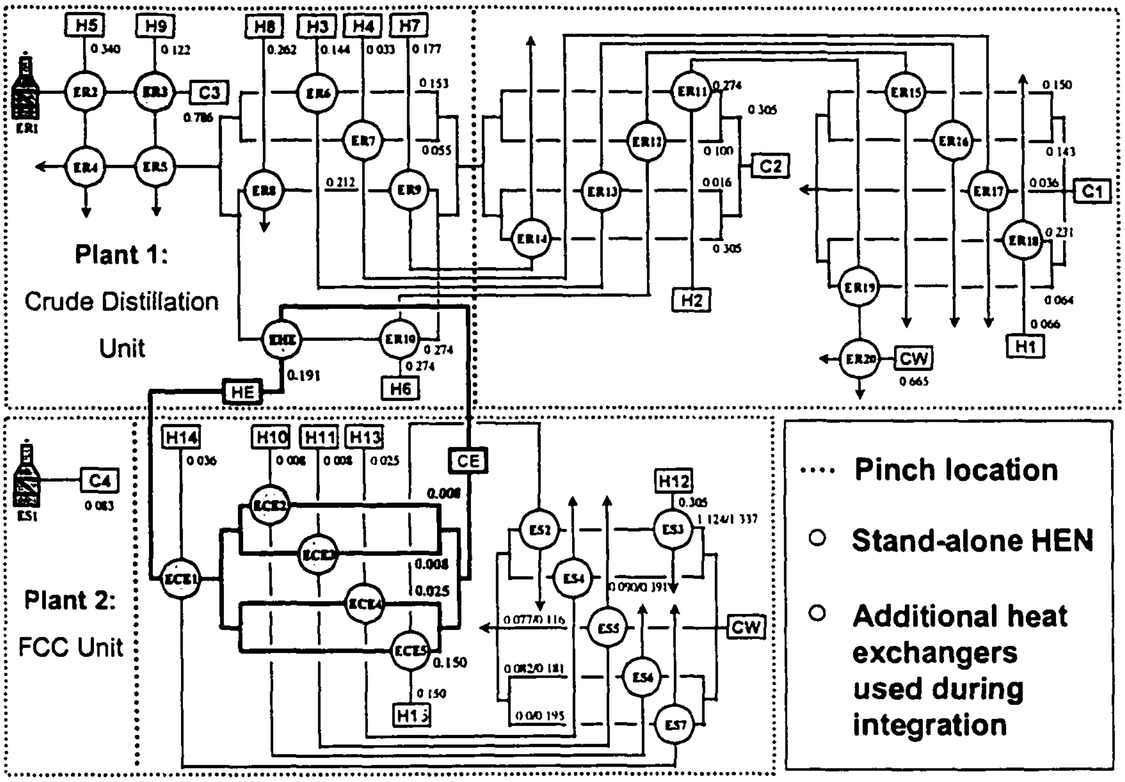


Figure 3.10. Single intermediate-fluid circuit HEN for Example 3.3

Table 3.9 shows the heat exchanger unit specifications for the network of Figure 3.10. The same analysis performed for the case of two intermediate-fluid circuits (Table 3.8) is valid in this case. Additionally, notice that in the heat exchanger network of plant 1 only the heat exchanger units participating in the loop rearrange their heat loads to switch from indirect integration to stand-alone operation mode.

Table 3.9. Single intermediate-fluid circuit HEN specifications for Example 3.3

Heat Exchanger	Heat Load (MW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	56.33/69.04	427.2	352.9	427.2	281.2/265
ER2	26.86/20.19	347.3	281.2/265	268.3/287.9	247/239.3
ER3	6.07/0.0	336.3/NA	247/NA	286.5/NA	239.3/NA
ER4	0.0/6.67	NA/287.9	NA/239.3	NA/268.3	NA/229.7
ER5	5.72/11.78	286.5/336.3	239.3/229.7	239.8	231.1/212.7
ER6	16.97	261.4	248.6	143.5	137.9
ER7	6.12	326.7	248.6	143.5	137.9
ER8	14.42	261.4	248.6	206.3	180.6
ER9	9.03	194.5	180.6	143.5	137.9
ER10	5.43	163.3	157.7	143.5	137.9
ER11	2.91	143.5	137.9	132.9	127.3
ER12	1.06	143.5	137.9	136.1	127.3
ER13	0.16	143.5	137.9	142.6	127.3
ER14	3.24	143.5	137.9	132.9	127.3
ER15	14.63	132.9	127.3	79.6	30
ER16	14.15	136.1	129.3	37.8	30
ER17	3.53	143.5	129.3	37.8	30
ER18	5.86	127.3	121.7	37.8	30
ER19	22.45	132.9	127.3	59.4	30
ER20	9.98	59.4	30	26.7	15
ES1	5.080	538.3	532.2	538.3	471.1
ES2	9.27/12.46	168.9/243.9	30	107.2	21.8/20.7
ES3	7.59	147.2	21.8/20.7	48.9	15
ES4	1.35/5.86	168.9/348.2	30	115.5	15
ES5	1.15/1.74	168.9/190.1	30	21.1	15
ES6	1.23/2.72	168.9/348.2	30	21.1	15
ES7	0.0/2.93	NA/313.2	NA/30	NA/232.2	NA/15
EHE1	12.71/0.0	229.6/NA	224/NA	163.3/NA	157.7/NA
ECE1	2.93/0.0	313.2/NA	229.6/NA	232.2/NA	214.3/NA
ECE2	1.50/0.0	348.2/NA	342.6/NA	168.9/NA	163.3/NA
ECE3	0.60/0.0	243.9/NA	238.3/NA	168.9/NA	163.3/NA
ECE4	4.52/0.0	348.2/NA	342.6/NA	168.9/NA	163.3/NA
ECE5	3.19/0.0	190.1/NA	184.5/NA	168.9/NA	163.3/NA

Integration / Stand alone operation NA: non-applicable

For the case of direct integration, a similar reduction of three heat exchanger units as in indirect integration can be achieved if partial energy savings are considered. By allowing a certain amount of heat to pass through the combined-plant pinch temperature matches located at both sides can be joined. The split of stream C2 that collects the greater amount of heat from plant 2 (similar to the single intermediate-fluid circuit solution) is therefore considered. Targeting savings are 13.8 MW, a 10% higher than the targeting savings for a single intermediate-fluid solution (Table 3.1). The resulting network consisting of thirty-three heat exchanger units is shown in Figure 3.11, and the corresponding heat exchanger unit specifications in Table 3.10.

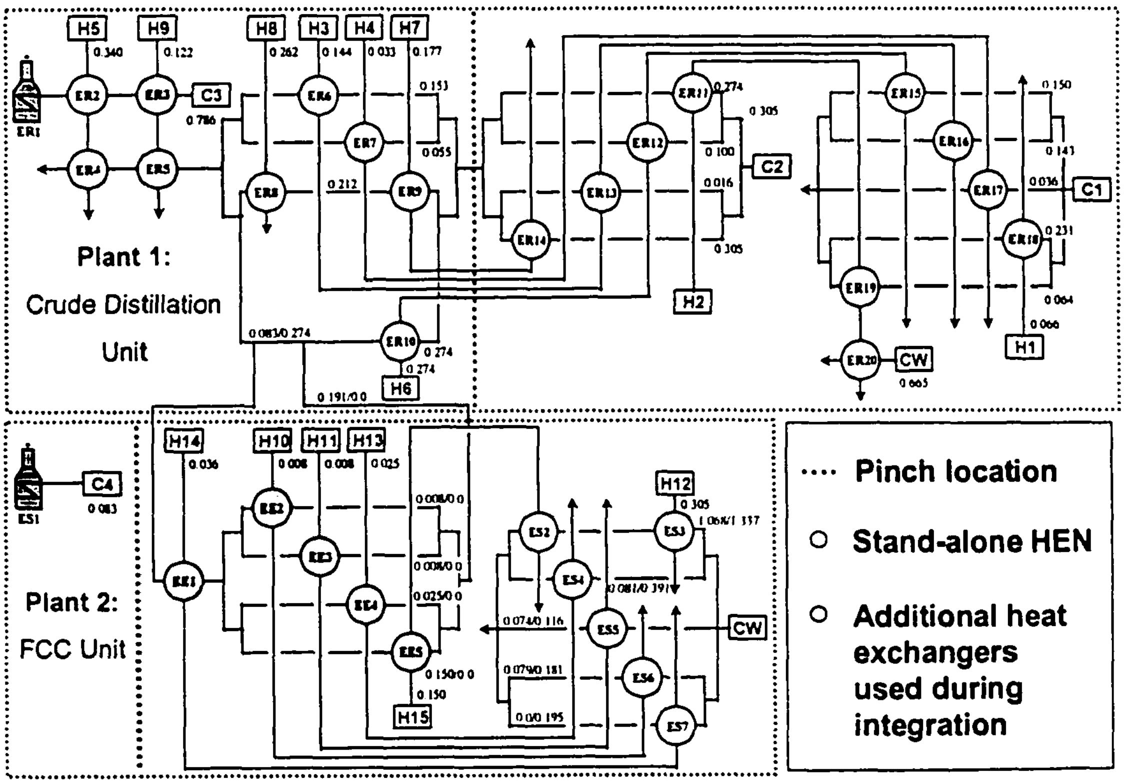


Figure 3.11. Partial direct integration HEN for Example 3.3

Table 3.10. Partial direct integration HEN specifications for Example 3.3

Heat Exchanger	Heat Load (MW)	Hot side Temperatures (°C)		Cold side Temperatures (°C)	
ER1	55.26/69.04	427.2	352.9	427.2	282.6/265
ER2	26.86/20.19	347.3	282.6/265	268.3/287.9	248.4/239.3
ER3	7.15/0.0	336.3/NA	248.4/NA	277.7/NA	239.3/NA
ER4	0.0/6.67	NA/287.9	NA/239.3	NA/268.3	NA/229.7
ER5	4.62/11.78	277.7/336.3	239.3/229.7	239.8	232.6/212.7
ER6	16.97	261.4	248.6	143.5	137.9
ER7	6.12	326.7	248.6	143.5	137.9
ER8	14.42	261.4	248.6	206.3	180.6
ER9	9.03	194.5	180.6	143.5	137.9
ER10	5.43	163.3	157.7	143.5	137.9
ER11	2.91	143.5	137.9	132.9	127.3
ER12	1.06	143.5	137.9	136.1	127.3
ER13	0.16	143.5	137.9	142.6	127.3
ER14	3.24	143.5	137.9	132.9	127.3
ER15	14.63	132.9	127.3	79.6	30
ER16	14.15	136.1	129.3	37.8	30
ER17	3.53	143.5	129.3	37.8	30
ER18	5.86	127.3	121.7	37.8	30
ER19	22.45	132.9	127.3	59.4	30
ER20	9.98	59.4	30	26.7	15
ES1	5.080	538.3	532.2	538.3	471.1
ES2	8.43/12.46	163.3/243.9	30	107.2	22.1/20.7
ES3	7.59	147.2	22.1/20.7	48.9	15
ES4	1.21/5.86	163.3/348.2	30	115.5	15
ES5	1.11/1.74	163.3/190.1	30	21.1	15
ES6	1.18/2.72	163.3/348.2	30	21.1	15
ES7	0.0/2.93	NA/313.2	NA/30	NA/232.2	NA/15
EE1	2.93/0.0	313.2/NA	229.6/NA	232.2/NA	214.3/NA
EE2	1.53/0.0	348.2/NA	342.6/NA	163.3/NA	157.7/NA
EE3	0.63/0.0	243.9/NA	238.3/NA	163.3/NA	157.7/NA
EE4	4.66/0.0	348.2/NA	342.6/NA	163.3/NA	157.7/NA
EE5	4.03/0.0	190.1/NA	184.5/NA	163.3/NA	157.7/NA

Integration / Stand alone operation NA: non-applicable

3.5 Economical Analysis

As follows from the previous section, the flowrate time heat capacity of the split of stream C2 that extends across to plant 2 is equal to the minimum value for the intermediate fluid circuit. Therefore, a priori comparison of the two networks favors direct integration over indirect integration if only economical reasons are accounted. Besides the 10% higher energy savings, direct integration requires one less heat exchanger unit. Pumping costs only can be higher if the intermediate fluid has a greater heat capacity than the crude stream. Usual intermediate fluids like dowtherms® perform with similar heat capacity than the crude for the range of temperatures considered in Example 3.3. The real advantages of having a circuit using these intermediate fluids are from the point of view of control and security. Moreover, the clean performance of intermediate fluids can save cost of maintenance of the heat exchangers used in the integration. The simplified analysis that follows gives an idea of the magnitude of the differences in total cost of the single-circuit indirect integration and partial direct integration heat exchanger networks studied in Example 3.3.

3.5.1 Installed Cost

The installed cost of the individual heat exchanger units shown in Table 3.11 is computed using the following simplified formula (Douglas 1988):

$$InstalledCost\ (\$) = \frac{M\ \&\ S}{280}\, 21.6(A^{0.65})(2.29 + F_c) \tag{3.28}$$

where M & S is the Marshall and Swift cost index value for chemical and petrochemical plants (a value of 1,130 is considered), and F_c is a correction factor. In computing the installed cost for the heat exchangers in which the heat load changes from integration to stand-alone mode of operation the largest heat exchange areas are considered. In the case of the furnaces the following formula is used (Douglas 1988):

$$InstalledCost\ (\$) = \frac{M\ \&\ S}{280} 1945(Q^{0.85})(1.27 + F_c) \tag{3.29}$$

The pipe installation cost included in Table 3.11 is calculated for a circuit or a split of stream C2 of a length of 1000 m. Dowtherm® A (average heat capacity 2.06 kJ/kg.°C) is considered as the intermediate fluid, and the cost of the fluid mass used in the circuit also is included in Table 3.11. The crude is assumed to have a heat capacity of 2.6 kJ/kg.°C. The estimated optimum economic pipe diameter is twelve inches for both the circuit and split pipes. In estimating the cost of piping, the following formula is used (Peters and Timmerhaus 1991):

$$Installed\ Cost\ of\ Piping\ (\$) = \frac{M\ \&\ S}{904}[(1 + F)XD_i^n K_F]L \tag{3.30}$$

where D_i is the diameter, L the length of the pipe, and F, X and K_F are cost correction factors. In accounting insulation costs, the obtained value is doubled using a rule of thumb. Finally, the total cost of installation for indirect and direct integration is shown in Table 3.11.

Table 3.11. Installation costs for Example 3.3

Cost ($)	Single-circuit Indirect integration	Partial Direct integration	Difference %
Total HEN Installation	21,700,000	20,450,000	5.8
Total Pipe Installation	150,000	150,000	0.0
Price Intermediate Fluid	405,000	-	100.0
Total Installation	22,255,000	20,600,000	7.4

3.5.2 Operating Cost

In order to compute the total operating cost, furnace and pumping costs are calculated. The following formula is used for the estimation of the pumping costs (Peters and Timmerhaus 1991):

$$Pumping\ Cost\ (\$) = \frac{M\ \&\ S}{904}\left(\frac{3.7x10^{-5}\cdot q_f^{2.84}\rho^{0.84}\mu_c^{0.16}K(1+J)H_y}{D_i^{4.84}E}\right) \tag{3.31}$$

where q_f is the volumetric flowrate, μ_c the viscosity, and K, J, H_y, and E are cost correction factors. Notice that the piping cost is 75% higher for the case of the intermediate fluid, but a comparison with the magnitude of the heating utility costs reduces the difference to a 1.9%.

Table 3.12. Operating cost for Example 3.3

Cost ($)	Single-circuit Indirect integration	Partial Direct integration	Difference %
Furnace	4,130,000	4,058,000	1.7
Pumping	18,200	10,400	75.0
Total Operating	4,148,200	4,068,400	1.9

3.5.3 Total Annual Cost

Considering an amortization of 10% for the installation cost of Table 3.11, a total investment cost can be estimated. Table 3.13 shows the total annual cost after adding the operating and investment costs. A 3.8% difference favors direct integration and this is the price to pay for the use of an intermediate fluid circuit during indirect integration.

Table 3.13. Total annual cost comparison for Example 3.3

Cost ($)	Single-circuit Indirect integration	Partial Direct integration	Difference %
Total Operating	4,148,200	4,068,400	1.9
HEN Amortization	2,225,500	2,060,000	7.4
Total Annual Cost	6,373,700	6,128,400	3.8

3.6 Conclusions

The analysis of energy integration opportunities across plants has been somewhat dismissed in the past because of practical considerations. Nevertheless, some of these opportunities have been implemented without any theoretical counterpart. Chapter 2 discussed the targeting procedures for energy savings, while this chapter presented a methodology to design multipurpose heat exchanger networks featuring minimum number of heat exchangers units. Indirect integration using an intermediate fluid and direct integration using process streams were compare and their advantages and disadvantages discussed. A final practical example considering the integration of a crude distillation unit and an FCC unit gives a comparative idea of the magnitude of the economics of the two types of integration.

3.7 Nomenclature

A = maximum heat exchanger area (m^2)
D_i = standard pipe diameter (inch)
E = efficiency of motor and pump expressed as a fraction
F = ratio of total costs for fitting and installation to purchase cost for new pipe
F_c = correction factor for heat exchanger installed cost
H_y = hours of operation per year
i = hot process stream
J = frictional loss due to fitting and bend, expressed as equivalent fractional loss in a straight pipe
j = cold process stream
K = cost of electrical energy ($/kWh)
K_f = annual fixed charges including maintenance, expressed as a fraction of initial cost of completely installed pipe
k = auxiliary hot/cold process stream
$L^{\mathbf{T}}_{ijr}$ = lower bound in the heat transfer between hot stream i and cold stream j within region r for type of integration **T** (kW)
p = chemical plant
Q = absorbed duty for a process furnace (MW)
q_f = fluid volumetric flowrate (m^3/s)
r = heat transfer region
t = temperature interval
$U^{\mathbf{T}}_{ijr}$ = upper bound in the heat transfer between hot stream i and cold stream j within region r for type of integration **T** (kW)
$V^{\mathbf{T}}_{ijt}$ = heat transfer from hot stream i to cold stream j in interval t for type of integration **T** (kW)
X = purchase cost of new pipe per foot of one-inch pipe length ($/m)

$Y_{ijr}^{\mathbf{T}}$ = heat transfer match between hot stream i and cold stream j within region r for type of integration **T**

$Z_{ijr}^{\mathbf{T}}$ = heat transfer match between hot stream i and cold stream j within region r for type of integration **T**

$\delta_{it}^{\mathbf{T}}$ = cascaded heat of hot stream i from interval t for type of integration **T** (kW)

ρ = fluid density (kg/m^3)

μ_c = fluid viscosity (Pa.s)

3.8 References

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CHAPTER 4

Targeting Procedures for Energy Savings by Heat Integration in the Total Site

4.1 Introduction

In this chapter, generalized mathematical models are presented that extend the results originally developed for two plants (Chapter 2) to the case of multiple plants. First, an LP model that considers all possible heat transfer among plants leading to savings is presented. This formulation identifies energy-saving targets for direct and indirect integration by determining the amounts of heat to be transferred within established temperature intervals. Then, an MILP model that makes use of these targets is introduced to establish the minimum number of connections between the two-plant combinations. For indirect integration, another MILP model is proposed that locates single intermediate-fluid circuits. The computational burden for this last model can be diminished by a reformulation that decomposes the model into the heat that enters and exits the circuits leading to a reduction of the number of heat intervals. Finally, it will be shown that the optimal location of these circuits that allows flexibility of operation easily can be added to these formulations. Examples showing the different features of this approach are presented. The direct and indirect targets for the heat integration of an entire oil refinery comprised of seven process units are calculated, and the practical issues related with its implementation are discussed.

4.2 Maximum Transferable Heat

Background material on the analysis of the maximum possible transferable heat for the case of two plants was presented in Chapter 2. The extension to a site consisting of a set of n plants is introduced next.

4.2.1 Heat Transfer Region Leading to Savings

Consider a set of n plants sorted from left to right in order of increasing pinch temperatures for which minimum utility targets have been calculated independently. Figure 4.1 shows this for the case of three plants.

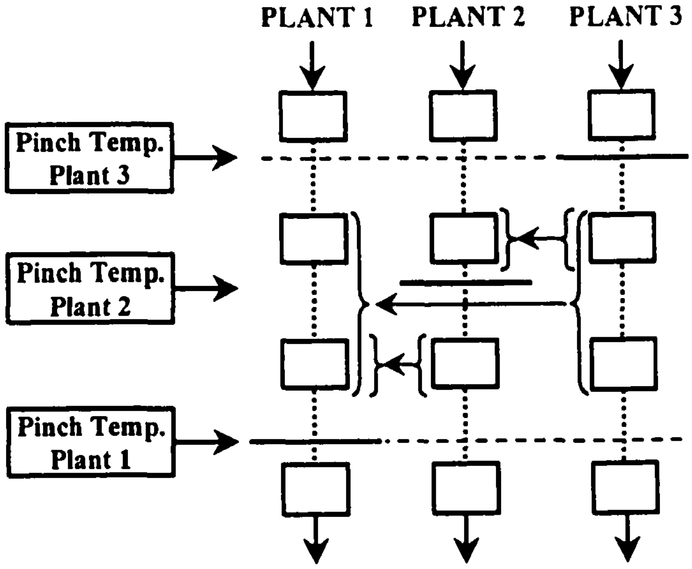


Figure 4.1. Effective Heat transfer

When any two plants of the set are taken into account, the region between pinch temperatures is the region in which effective transfer leading to utility savings takes place. Indeed, these regions are the only ones in which the plant with the higher pinch temperature is a net heat source and the plant with the lower pinch temperature is a net heat sink. This type of heat transfer is called *effective heat transfer.* The existing relationship between the plants during integration requires the following definitions:

Effective-supplier plant: plant that releases effective heat to the plant in which savings are obtained by a reduction of its heating utility demands.

Effective-receiver plant: plant that receives effective heat from the plant in which savings are obtained by a reduction of its cooling utility demands.

4.2.2 Unassisted and Assisted Heat Transfer

When maximum energy savings are attained solely by transferring heat in the region between pinch temperatures, an unassisted heat transfer case is present. Figure 4.2 shows an instance of unassisted heat transfer among a set of three plants. In this diagram, plant 2 and plant 3 are effective-supplier plants for plant 1, while plant 3 is the effective-supplier plant for plant 2. Moreover, plant 1 is the effective-receiver plant for plant 2, while plant 1 and plant 2 are effective-receiver plants for plant 3.

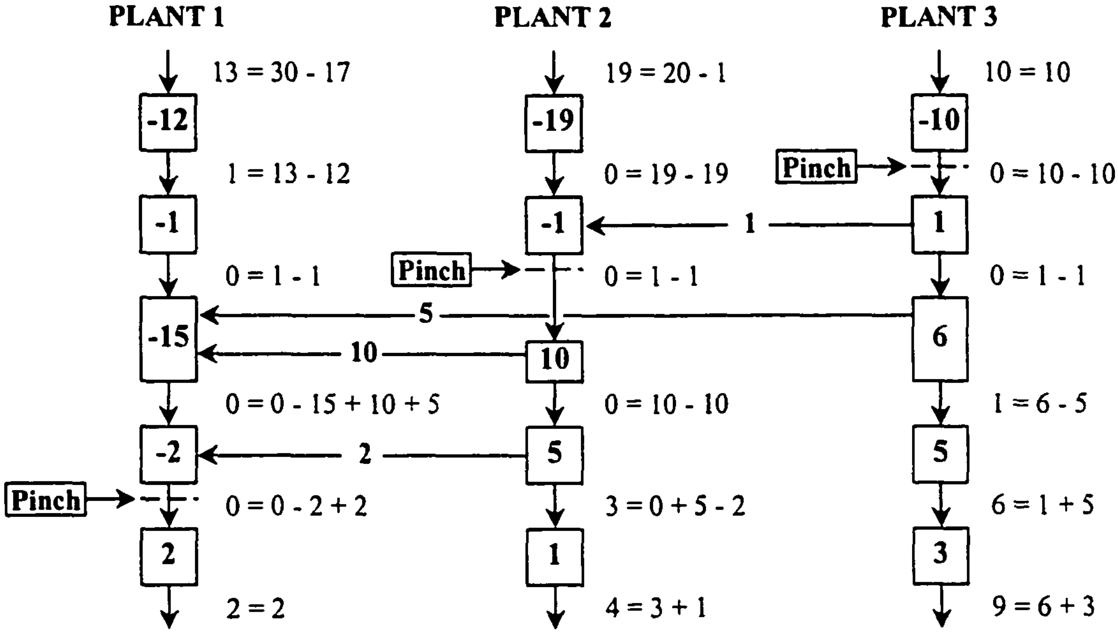


Figure 4.2. Unassisted heat transfer

In Chapter 2, it was shown that to attain maximum energy savings in certain cases, effective heat transfer across two plants (i.e., taking place between pinch temperatures) must be accompanied by heat transfer in the reverse direction and outside the region between pinch temperatures. The interaction among plants becomes more complex for the case of more than two plants. In this case, assisting heat does not have to come from the same plant that is receiving the heat, as it can be supplied by other plants. As it is not known a priori which plant will be providing assisting heat, we introduce the following definitions.

Assisted Plant: plant that releases assisting heat above its pinch temperature, or receives assisting heat below its pinch temperature.

Assisting Plant: plant that receives assisting heat above its pinch temperature, or releases assisting heat below its pinch temperature.

Figure 4.3 shows an instance of assisted heat transfer among a set of three plants in the opposite direction to the heat received in order to attain effective savings. In this case, plant 2 is simultaneously an assisting plant and an effective-supplier plant for plant 1. Conversely, plant 1 is simultaneously an assisted plant and an effective-receiver plant for plant 2.

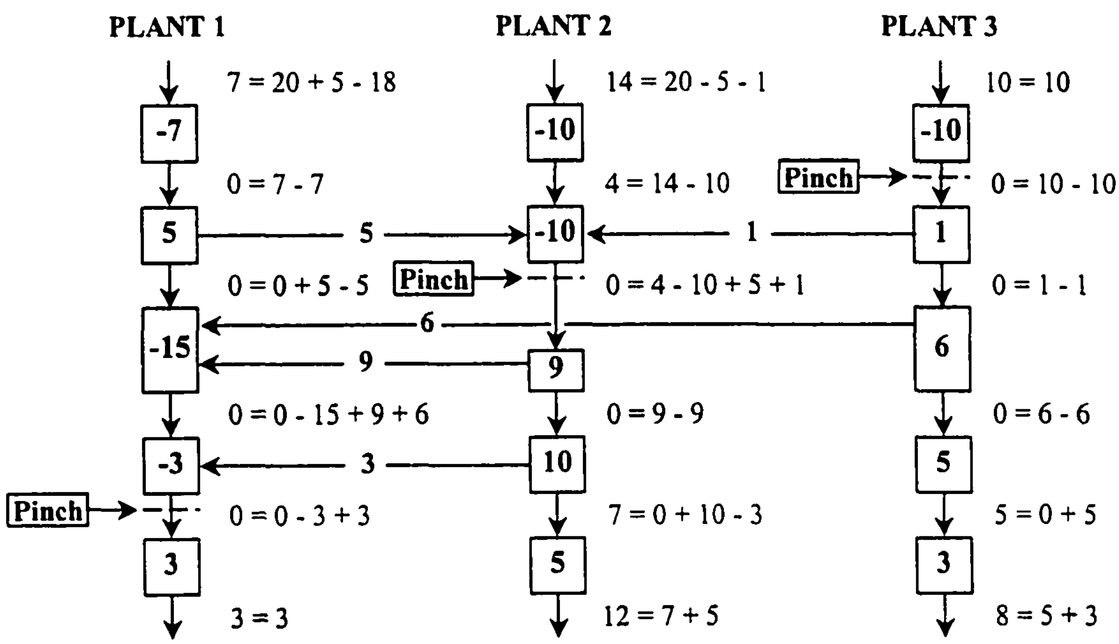


Figure 4.3. Assisted heat transfer opposite to effective heat transfer

A case in which the assisted heat transfer from plant 3 to plant 2 is parallel to the effective heat transfer between these plants is shown in Figure 4.4. In this example, plant 2 and plant 3 are effective-supplier plants for plant 1. For plant 2, plant 3 is simultaneously an effective-supplier plant and an assisting plant. Conversely, for plant 3, plant 2 is simultaneously an effective-receiver plant and an assisted plant.

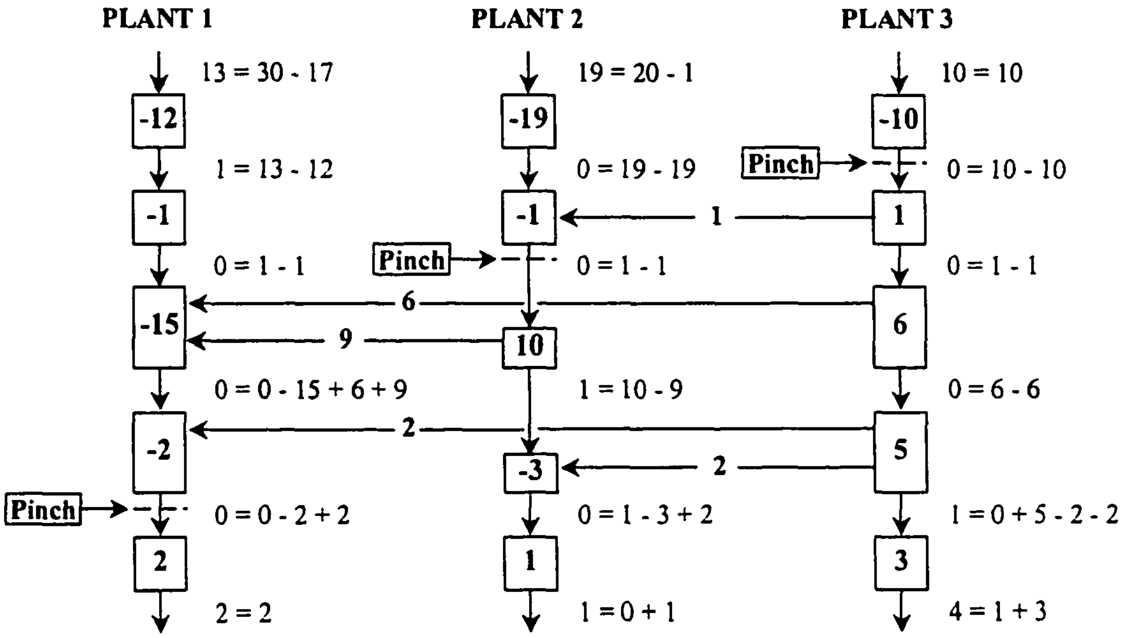


Figure 4.4. Assisted heat transfer parallel to effective heat transfer

In summary, we have shown that in principle there are different types of assisted heat transfer with properties more general than the ones previously described in Chapter 2 for the particular case of two plants.

4.3 Targeting Models for Heat Integration

Background material regarding the analysis of the maximum possible transferable heat for the case of two plants was presented in Chapter 2. The extension to a site consisting of a set of *n* plants is the purpose of this section. The maximum energy savings model for the case of two plants is the starting point for developing the general model for n plants.

4.3.1 Maximum Energy Savings Model for the Two-plants Case

A transshipment model was introduced in Chapter 2 to establish the amount of heat that can be transferred within each interval for the particular case of two plants. Figure 4.5 illustrates the notation for total heat amounts transferred in each region.

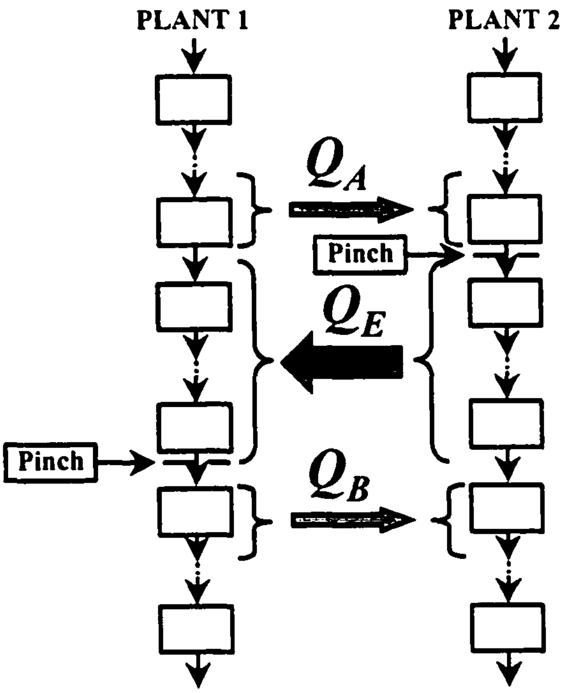


Figure 4.5. Directions of heat transfer for the two-plants case

When considering energy minimization, the obvious objective function is the sum of all the heating utilities used (i.e., $\delta_0^{I} + \delta_0^{II}$). This objective function, however, does not minimize the flow of assisted heat. Therefore, the objective function used for the two-plant model (i.e., $\delta_0^I + \delta_m^{II}$) consists of minimizing the sum of the heating utility of plant 1 (the effective-receiver plant), and the cooling utility of plant 2 (the effective-supplier plant). While minimizing the heating utility of the receiver plant is clearly necessary, minimizing the cooling utility of the supplier plant is unnecessary in the absence of assisting heat. Adding the cooling utility to the objective function accomplishes the goal of reducing the assisting heat below both pinch temperatures to the minimum strictly necessary. At the same time, the minimization of the heating utility of plant 1 reduces the assisting heat to plant 2 above both pinch temperatures to the strictly necessary minimum.

The flow of heat between intervals of n plants can be generalized without much difficulty from the equations of the model for two plants. However, the objective function needs to be revisited. The starting point is the use of an alternative objective function. Notice first that:

$$\delta_o^I = \hat{\delta}_o^I + Q_A - Q_E \tag{4.1}$$

$$\delta_m^{II} = \hat{\delta}_m^{II} + Q_B - Q_E \tag{4.2}$$

Therefore, the following objective functions are equivalent

$$Min\,(\delta_0^I + \delta_m^{II}) \quad \Longleftrightarrow \quad Max\,\{2 \cdot Q_E - (Q_A + Q_B)\} \tag{4.3}$$

In the alternative objective function, the total effective heat is counted twice, because savings are attained both on heating and on cooling utilities. The simultaneous maximization of the heat that effectively leads to savings and minimization of the assisting heat amounts are clearly achieved. The purpose of the assisting heat is to debottleneck the heat cascade of the corresponding assisted plant. Therefore, in an assisted heat integration case, the increase of effective heat in one unit is achieved by transferring exactly one unit of assisting heat. Sometimes, however, the plants need to be simultaneously debottlenecked by the transfer of assisting heat within both the regions above and the below pinch temperatures, as Figure 4.6 illustrates.

In this example, the optimal case (Figure 4.6a) features $Q_E = 19$, $Q_A = 4$ and $Q_B = 3$. The sub-optimal case (Figure 4.6b) features $Q_E = 18$, $Q_A = 3$ and $Q_B = 2$. The value of the objective function is clearly the same for both cases. Therefore, even though the objective function used in Chapter 2 fails to distinguish optimal from sub-optimal solutions, the illustrative examples used in that chapter are correct. The special case illustrated in Figure 4.6 was not considered in the analysis. Indeed, only when a reduction of the effective heat transfer can be accompanied by the same amount of reduction in assisting heat above both pinches and below both pinches does this objective function fail to identify the optimum. While the case presented is very special, it prompts revisiting the study of the proper objective function.

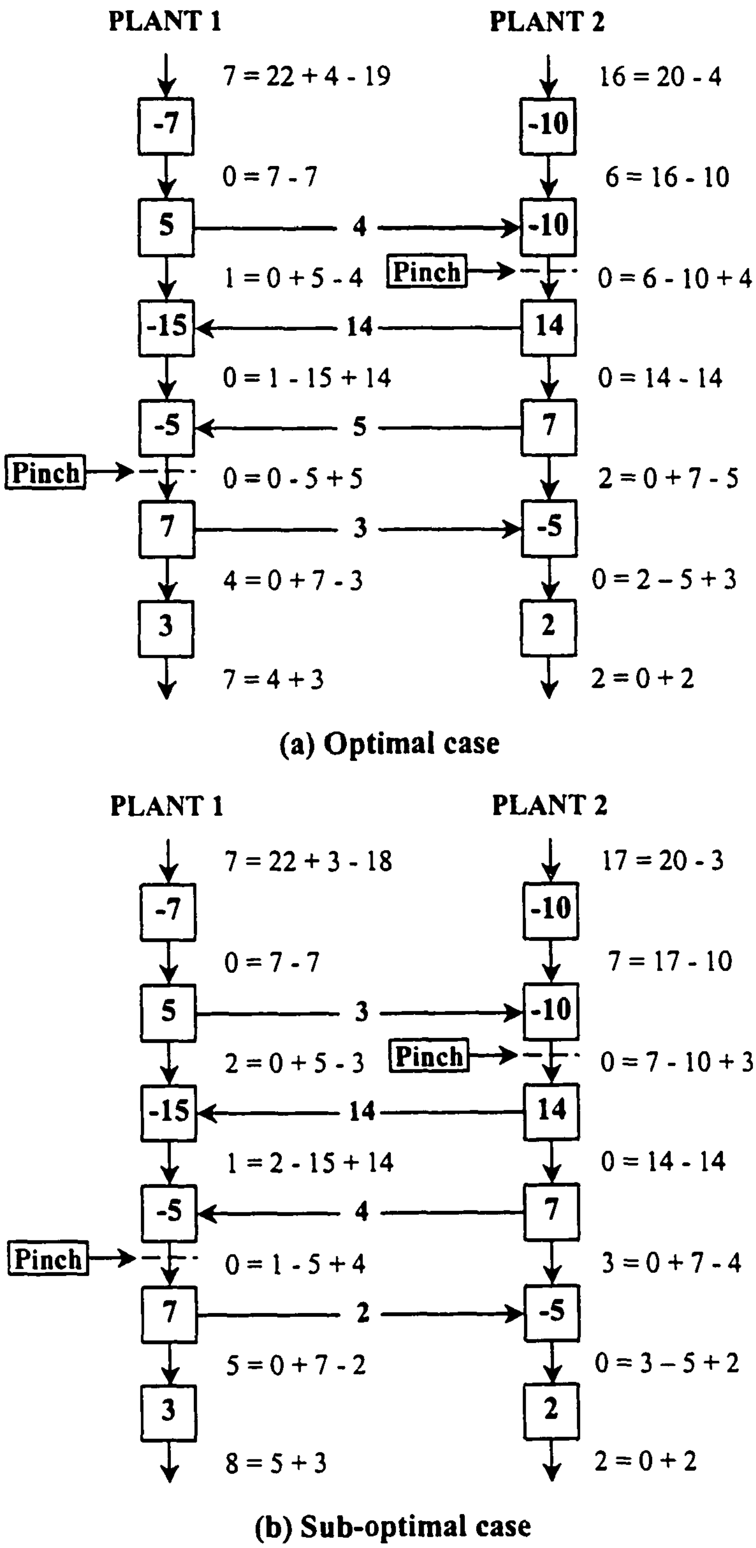


Figure 4.6. Cascade example of simultaneous assisted heat

To remedy the aforementioned shortcomings, we return to the sum of heating utilities objective function that in general for n plants is $\sum_{\forall j \in P} \delta_0^j$. A small mathematical manipulation shows that this is equal to Q_E for the case of two plants. Thus the problem that needs to be solved is:

$$
\begin{aligned}
\mathbf{P4.1} = & \, Min\,(\delta_0^{I} + \delta_0^{II}) \\
& s.t \\
& \delta_0^{I} = \hat{\delta}_0^{I} + Q_A - Q_E \\
& \delta_0^{II} = \hat{\delta}_0^{II} - Q_A \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{A} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{A} \end{aligned}\right\} \quad \forall i = 1,\ldots,p^{II} \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} + q_i^{E} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{E} \end{aligned}\right\} \quad \forall i = (p^{II} + 1),\ldots,p^{I} \\
& \left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{B} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{B} \end{aligned}\right\} \quad \forall i = (p^{I} + 1),\ldots,m \\
& \delta_m^{I} = \hat{\delta}_m^{I} - Q_B \\
& \delta_m^{II} = \hat{\delta}_m^{II} + Q_B - Q_E \\
& \delta_i^{I}, \delta_i^{II}, q_i^{A}, q_i^{E}, q_i^{B} \geq 0
\end{aligned}
\tag{4.4}
$$

The only problem with this objective function is that it is invariant to the value of the assisting heat above the threshold established by the value that allows the de-bottlenecking of the cascade. For example, take the solution shown in Figure 4.6a and increase the assisting heat. There is no effect on the value of the total heating utility $(\delta_0^{I} + \delta_0^{II})$. However, since Q_A = 4 is the threshold, a reduction of Q_A below this value affects the total energy consumption. To fix the value of assisting heat to the minimum, a new problem needs to be solved. Let Q_E^* be the optimal value of effective heat transfer between the two plants as determined using problem **P4.1**. Then the following problem minimizes the assisting heat above and below both pinch temperatures:

$$\begin{aligned} \mathbf{P4.2} = & \ Min\,(Q_A + Q_B) \\ & s.t. \\ & Q_E = Q_E^* \\ & \textit{All constraints of problem } \mathbf{P4.1} \end{aligned} \tag{4.5}$$

A penalty function version for **P4.2** is

$$\begin{aligned} \mathbf{P4.3} = & \ Min\,\{(Q_A + Q_B) + \mu\, f(Q_E)\} \\ & s.t \\ & \textit{All constraints of problem } \mathbf{P4.1} \end{aligned} \tag{4.6}$$

Because $Q_E^* \geq Q_E$, then a linear penalty function for **P4.2** is possible; that is, $f(Q_E) = (Q_E^* - Q_E)$, and because Q_E^* is a constant, it can be dropped. Therefore, the problem can be rewritten as follows:

$$\begin{aligned} \mathbf{P4.3'} = & \ Max\,\{\, Q_E - \varepsilon(Q_A + Q_B)\} \\ & s.t \\ & \textit{All constraints of problem } \mathbf{P4.1} \end{aligned} \tag{4.7}$$

where $\varepsilon = 1/\mu$. Thus, if the proper value of μ is used, the solution of problem **P4.3** is the same as the solution obtained solving **P4.1** and **P4.2** in sequence. The issue is then to determine the proper value of μ. The answer is $\mu > 2$. Indeed, as it can be seen from the example of Figure 4.6, when $\mu = 2$, all cases, except the ones that require double assistance (above and below both pinch temperatures simultaneously) will render the correct optimal solution. In the special case illustrated in Figure 4.6, the problem is degenerate, as was illustrated above. Thus, for $\mu > 2$, the effective heat (that always produces two units of total savings per unit of assisting heat) will have a larger weight than the assisting heat. This very same analysis can be made for the case of multiple plants, as the worst case scenario is that a certain amount of effective heat can be transferred only if a debottlenecking takes place in the effective supplier and effective receiver plants. The only difference is that the assisted heat now can be provided by any plant.

4.3.2 Maximum Energy Savings Model for the Total-Site

Let us introduce the following sets of plants:

- P : Set of n plants considered for direct or indirect integration.
- R_j^A : Set of assisting plants k receiving heat from plant j in the region above both the pinch temperature of plant j and the pinch temperature of plant k.
- S_j^A : Set of assisted plants k supplying heat to plant j in the region above both the pinch temperature of plant k and the pinch temperature of plant j.

- ➢ R_j^E : Set of effective-receiver plants k receiving heat from plant j in the region between the pinch temperature of plant j and the pinch temperature of plant k.
- ➢ S_j^E : Set of effective-supplier plants k supplying heat to plant j in the region between the pinch temperature of plant k and the pinch temperature of plant j.
- ➢ R_j^B : Set of assisted plants k receiving heat from plant j in the region below both the pinch temperature of plant k and the pinch temperature of plant j.
- ➢ S_j^B : Set of assisting plants k supplying heat to plant j in the region below both the pinch temperature of plant j and the pinch temperature of plant k.
- ➢ R_{ij}^A : Set of assisting plants $k \in R_j^A$ present in interval i.
- ➢ S_{ij}^A : Set of assisted plants $k \in S_j^A$ present in interval i.
- ➢ R_{ij}^E : Set of effective-receiver plants $k \in R_j^E$ present in interval i.
- ➢ S_{ij}^E : Set of effective-supplier plants $k \in S_j^E$ present in interval i.
- ➢ R_{ij}^B : Set of assisted plants $k \in R_j^B$ present in interval i.
- ➢ S_{ij}^B : Set of assisting plants $k \in S_j^B$ present in interval i.

Based on these sets, the model that considers independent transfer of heat within each interval is presented next. This model accounts for transference from effective-supplier plants to effective-receiver plants, from assisted to assisting plants above both pinch temperatures, and from assisting to assisted plants below both pinch temperatures. The model follows:

$$\mathbf{P4.4} = Max\,\{Q_E - \varepsilon(Q_A + Q_B)\}$$

s.t.

$$Q_E = \sum_{j\in P}\sum_{k\in S_j^E} Q_{kj}^E$$

$$Q_A = \sum_{j\in P}\sum_{k\in R_j^A} Q_{jk}^A$$

$$Q_B = \sum_{j\in P}\sum_{k\in R_j^B} Q_{jk}^B$$

$$\left.\begin{aligned}
&Q_{kj}^E = \sum_{i=p^k+1,\dots,p^j} q_{ikj}^E && k\in S_j^E\\
&Q_{jk}^A = \sum_{i=1,\dots,p^k} q_{ijk}^A && k\in R_j^A\\
&Q_{jk}^B = \sum_{i=p^j+1,\dots,m} q_{ijk}^B && k\in R_j^B\\
&\delta_o^j = \hat{\delta}_o^j - \sum_{k\in S_j^E} Q_{kj}^E + \sum_{k\in R_j^A} Q_{jk}^A - \sum_{k\in S_j^A} Q_{kj}^A\\
&\delta_i^j = \delta_{i-1}^j + q_i^j + \sum_{k\in S_{ij}^E} q_{ikj}^E - \sum_{k\in R_{ij}^A} q_{ijk}^A + \sum_{k\in S_{ij}^A} q_{ikj}^A && \forall i = 1,\dots,p^j\\
&\delta_i^j = \delta_{i-1}^j + q_i^j - \sum_{k\in R_{ij}^E} q_{ijk}^E + \sum_{k\in S_{ij}^B} q_{ikj}^B - \sum_{k\in R_{ij}^B} q_{ijk}^B && \forall i = (p^j+1),\dots,m\\
&\delta_m^j = \hat{\delta}_m^j - \sum_{k\in R_j^E} Q_{jk}^E + \sum_{k\in S_j^B} Q_{kj}^B - \sum_{k\in R_j^B} Q_{jk}^B
\end{aligned}\right\}\forall j\in P \tag{4.8}$$

$$\delta_i^j, q_{ijk}^A, q_{ikj}^E, q_{ijk}^B \geq 0$$

where the value of ε is smaller than 0.5 following the analysis of the previous section. The overall effective heat transfer amount Q_E and the eventual assisted heat amounts Q_A and Q_B are each a summation of the corresponding heat amounts transferred between all the pair combinations. These overall amounts of heat transferred between plants Q_{kj}^E, Q_{jk}^A, and Q_{jk}^B, are related to the heat amounts transferred in each interval q_{ikj}^E, q_{ijk}^A, and

q_{ijk}^{B} as found by simple addition. Finally, the model contains the well-known cascade heat balance equations.

In its original form for the case of two plants (Chapter 2), indirect integration and particularly the issue of having intermediate circuits transferring heat in different directions were resolved by shifting the temperature scales of the plants. These temperature scale shifts cannot be applied to more than two plants, as the multiple shifts conflict with each other. Therefore, in these models indirect integration (and particularly the issue of having intermediate circuits transferring heat in different directions) is resolved by considering that the variables representing heat transfer between plants correspond to upward and downward diagonal transfer. This diagonal transfer is established between intervals of the same length that are located a fixed number of intervals apart. The procedure used for obtaining this generalized structure of intervals is given in Appendix A.

4.3.3 Minimum Number of Connections

The result obtained by solving model **P4.4** represents the maximum possible savings for the whole system without accounting for the number of required connections between the plant pairs. In addition to unnecessary connections generally obtained, the amount of heat transfer via these connections is a concern. The following model introduces three different sets of binary variables (X_{kj}^{E}, X_{jk}^{A}, X_{jk}^{B}) to account for connections from effective-supplier plants to effective-receiver plants between pinch temperatures, from assisted to assisting plants above their pinch temperatures, and from assisting to assisted plants below their pinch temperatures.

$$
\begin{aligned}
&\mathbf{P4.5} = Min \sum_{j\in P}\left(\sum_{k\in S_j^E} X_{kj}^E + \sum_{k\in R_j^A} X_{jk}^A + \sum_{k\in R_j^B} X_{jk}^B \right) \\
&s.t. \\
&Q_E = Q_E^* \\
&Q_A + Q_B = Q_A^* + Q_B^* \\
&All\ constraints\ of\ problem\ \mathbf{P4.1} \\
&\left.\begin{array}{ll}
L_{kj}^E X_{kj}^E \le Q_{kj}^E \le U_{kj}^E X_{kj}^E & k\in S_j^E \\
L_{jk}^A X_{jk}^A \le Q_{jk}^A \le U_{jk}^A X_{jk}^A & k\in R_j^A \\
L_{jk}^B X_{jk}^B \le Q_{jk}^B \le U_{jk}^B X_{jk}^B & k\in R_j^B
\end{array}\right\} j\in P \\
&X_{kj}^E, X_{jk}^A, X_{jk}^B \in \{0,1\}.
\end{aligned}
\tag{4.9}
$$

The target amounts of effective and assisting heat are used to fix the total heat transferred via the connections. Notice that although assisting heat is represented with two independent variables for amounts of heat transferred above and below the pinch temperatures, the total amount of assisting heat determine by the summation of the separate targets is maintained. This allows the separate targets to be rearranged during the determination of the heat connections. Finally, additional constraints provide upper and lower bounds for the connections.

The purpose of the model is to find the minimum number of connections in order to attain maximum effective savings. However, the relaxation of the value of total effective heat transfer may lead to a reduction in the number of connections. Moreover, values of assisting heat may be obtained that also reduce the number of connections. It is clear that a trade-off between energy savings and the number of connections exists.

4.3.4 Maximum Energy Savings Examples

4.3.4.1 Example 4.1

This example, introduced by Ahmad and Hui (1991), is used here to pinpoint the differences between their procedure and our proposed targeting approach. Table 4.1 shows the results of independently applying pinch analysis to each of the "areas of integrity" that can be considered individual plants.

Table 4.1. Individual plant pinch analysis for Example 4.1

Problem	Pinch Temperature (°C)	Minimum Heating Utility (kW)	Minimum Cooling Utility (kW)
Area A	70	43	12.75
Area B	500	1.5	19.35
Area C	210	25	36.5

Figure 4.7a shows the solution obtained by solving problem **P4.4.** Both types of assisted heat integration (i.e., in the opposite direction and in the same direction as the effective heat) take place between area C and area B. These assisting heats allow effective savings, not only between these areas, but also between area C and area A, while the system reaches maximum savings. Figure 4.7b illustrates the required flows and duties between the same areas. This solution can be found using the procedure suggested by Ahmad and Hui (1991). However, to do so the appropriate choices of heats to be

disallowed must be made (i.e., no automatic solution is possible), which requires some a priori knowledge. Moreover, their procedure overlooks the insights gained by considering effective heat transfer and assisting heat transfer plant regions. When model **P4.5** is solved, four connections are obtained, and the interval heat transfers have the same values as seen in the solution of problem **P4.4**. Two connections transfer effective heat from area B to area A and from area C to area A, and two connections transfer assisting heat below pinch temperatures from area B to area C and vice versa.

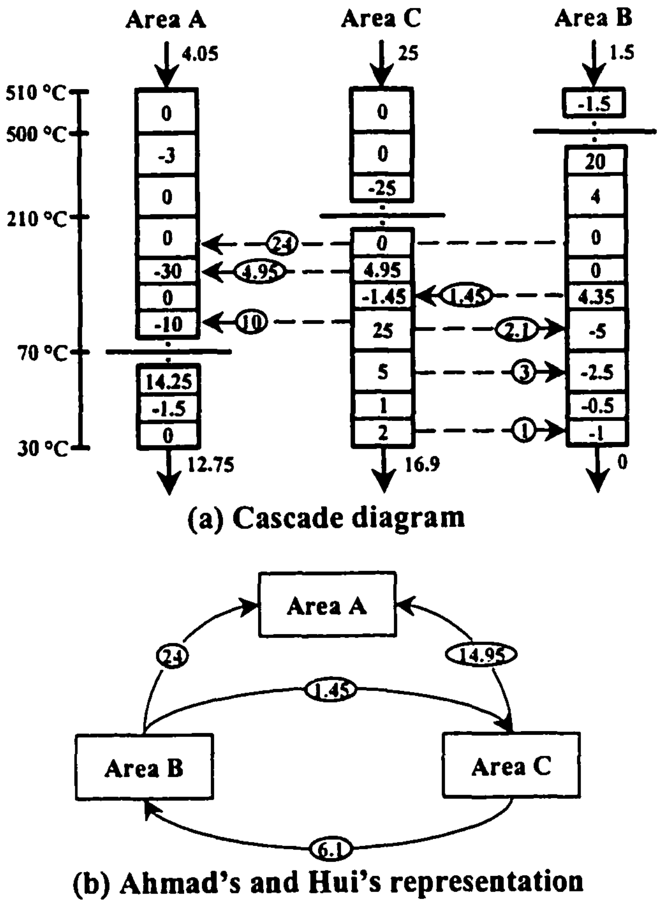
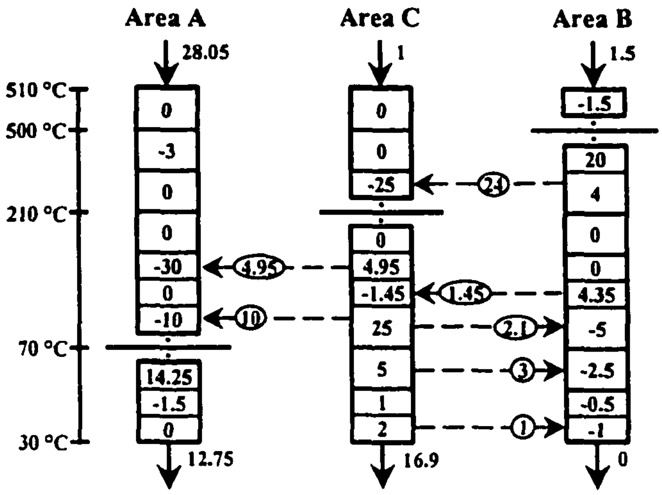
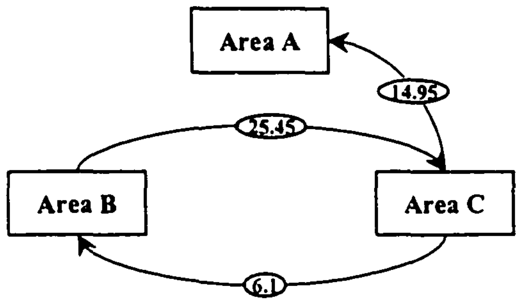


Figure 4.7. Comparison between targeting approaches for Example 4.1.



(a) Cascade diagram

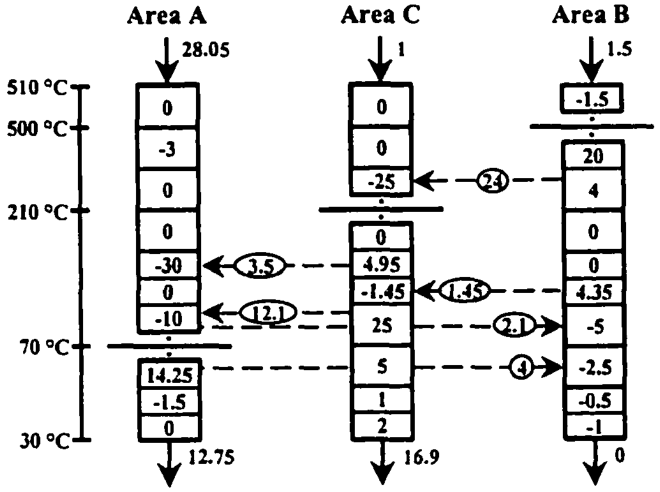


(b) Ahmad's and Hui's representation

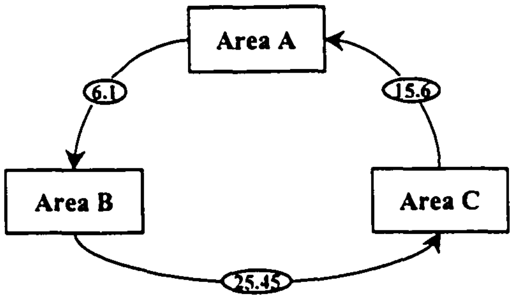
Figure 4.8. Minimum number of heat flows for Example 4.1.

The reported solution to the procedure that finds the required heat flows between the regions (Ahmad and Hui, 1991), however, results in the scheme showed in Figure 4.8b. Note that this solution requires one less flow between the areas (no flow from area B to area A exists). Nevertheless, Figure 4.8a demonstrates that this solution can be found by restricting the flow from area B to area A in problem **P4.4**. Notice that if effective and assisting connections are separately considered, the result is an alternative solution to the one presented in Figure 4.7a. As this flow of 24 units can be separated from the assisted flow of 1.45 units, because they are transferred in different regions, the number of connections is again four (i.e., two effective and two assisting below pinch temperatures).

Another alternative solution is shown in Figure 4.9.



(a) Cascade diagram



(b) Ahmad's and Hui's representation

Figure 4.9. Minimum number of heat flows for Example 4.1 (alternative solution)

We conclude that targets can be obtained automatically by solving problem **P4.4**. Moreover, a distinction between the effective and assisting heat flows between areas is made using this model. On the other hand, the procedure presented by Ahmad and Hui (1991) requires an iterative procedure and some decision-making. Total heat flows between the areas are obtained without differentiating between effective and assisting heat transfer. The solution to problem **P4.5** will automatically determine the number of connections that distinguish between effective and assisting connections.

4.3.4.2 Example 4.2

This example was constructed using a combination of examples 2.4 and 2.5 from Chapter 2. It is used to show the integration across a set of four plants. The results of applying individual pinch analysis to each of the plants are shown in Table 4.2.

Table 4.2. Individual plant pinch analysis for Example 4.2

Problem	Pinch Temperature (°C)	Minimum Heating Utility (kW)	Minimum Cooling Utility (kW)
Test Case #2	90	107.5	40.0
Trivedi (1988)	160	404.8	688.6
Ciric & Floudas (1991)	200	600.0	2100.0
4sp1	249	128.0	250.0

The results of applying direct heat integration to this example by solving problem **P4.4** are shown in Figure 4.10. This is an instance of an assisted heat integration case, because heat is sent from plant 2 to plant 3 to debottleneck the heat cascade of plant 2.

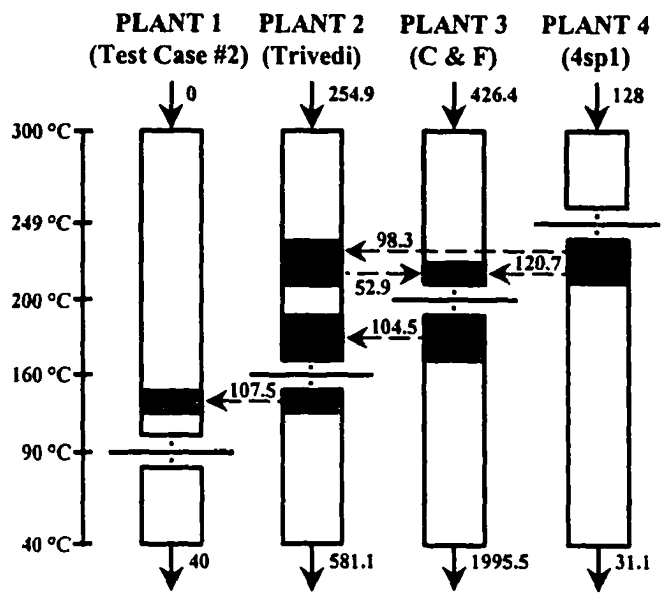


Figure 4.10. Direct integration solution for Example 4.2

Table 4.3 shows the amount of savings achieved in each of the plants as well as the maximum savings for the system.

Table 4.3. Direct integration for Example 4.2

Problem	Savings (kW)	
	Heating	Cooling
Test Case #2	107.5	0.0
Trivedi (1988)	149.9	107.5
Ciric & Floudas (1991)	173.6	104.5
4spl	0.0	219.0
Total Savings	431.0	431.0

Heat can be transferred between each pair of plants within each interval, or it can be cascaded first in one of them and then transferred within another interval. Hence, the model has alternative solutions. One of the alternatives for the direct heat integration case is shown in Figure 4.11. This solution was obtained by first disallowing heat transfer between plant 1 and plant 2 and then by solving problem **P4.4**.

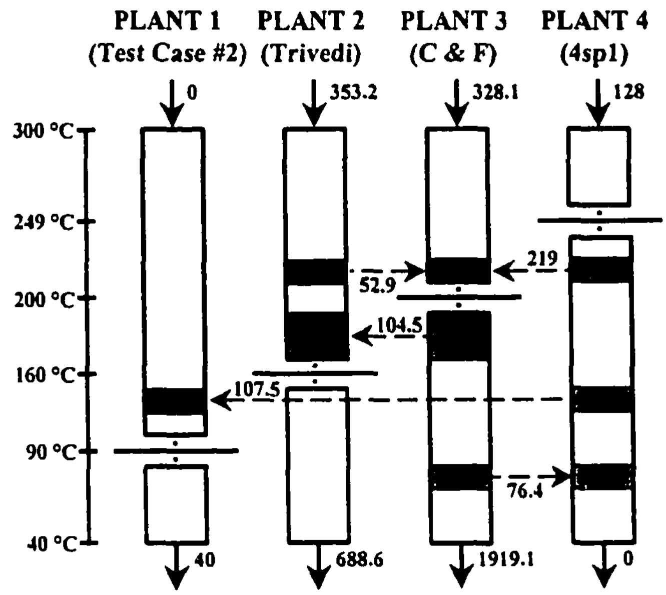


Figure 4.11. Alternative direct integration solution for Example 4.2

Table 4.4 shows the amount of savings for this case. The total amounts are equal to those in the alternative presented in Table 4.3. However, the individual savings reflect the different characteristics of this alternative.

Table 4.4. Direct integration for Example 4.1

(alternative solution)

Problem	Savings (kW)	
	Heating	Cooling
Test Case #2	107.5	0.0
Trivedi (1988)	51.6	0.0
Ciric & Floudas (1991)	271.9	180.9
4spl	0.0	250.1
Total Savings	431.0	431.0

Indirect integration applied to this example reflects a lower amount of savings than in the direct integration case. The region leading to effective savings is reduced, because diagonal transference between equal intervals is required in order to use an intermediate fluid.

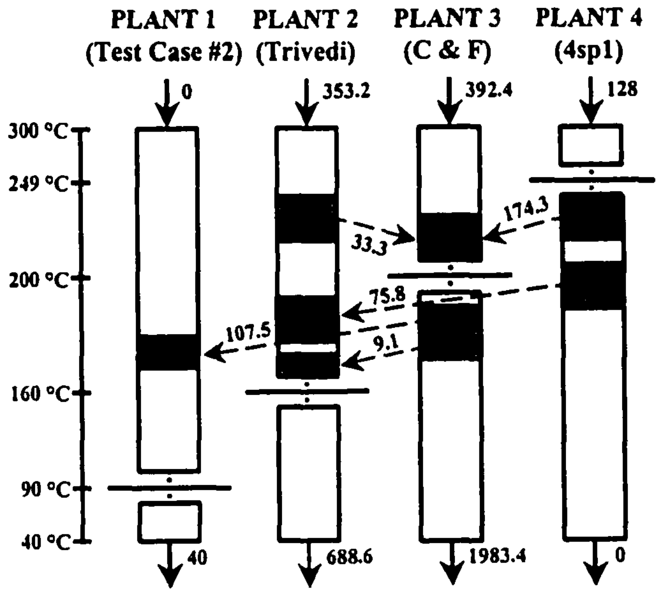


Figure 4.12. Indirect integration solution for Example 4.2

The solution for the assisted indirect heat integration after solving problem **P4.4** is shown in Figure 4.12. Notice that although plant 3 continues to assist plant 2, the pattern of effective heat transfer changes with respect to direct heat integration. Table 4.5 shows the amount of savings achieved in each of the plants as well as the maximum savings achieved when the system is indirect heat integrated.

Table 4.5. Direct integration for Example 4.2

Problem	Savings (kW)	
	Heating	Cooling
Test Case #2	107.5	0.0
Trivedi (1988)	51.6	0.0
Ciric & Floudas (1991)	207.5	116.6
4sp1	0.0	250.1
Total Savings	366.7	366.7

The indirect integration solution presented does not consider the minimization of the number of effective and assisting connections between plants. A solution featuring the minimum number of connections is obtained after solving problem **P4.5** and is shown in Figure 4.13. This alternative solution contains only four connections: three effective connections and one assisting connection from plant 2 to plant 3.

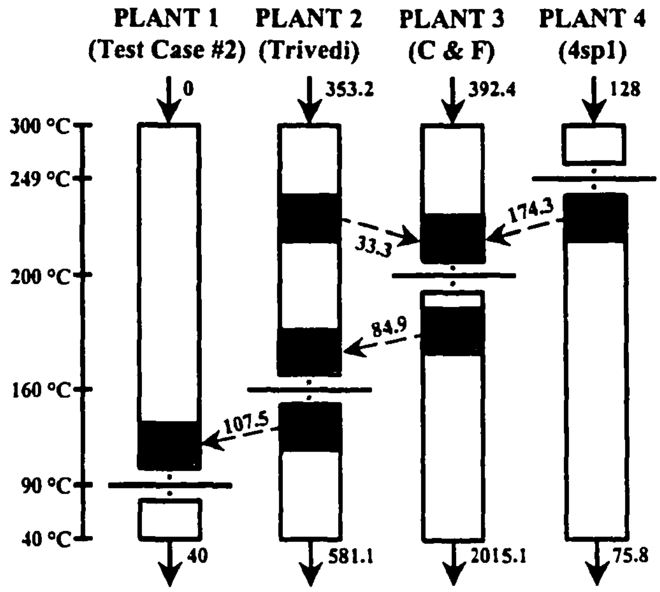


Figure 4.13. Indirect integration solution for Example 4.2

(minimum number of connections)

The amount of savings is equal to those in the alternative previously presented. However, the individual savings reflect the unique characteristics of this alternative (Table 4.6).

Table 4.6. Indirect integration for Example 4.2

(minimum number of connections)

Problem	Savings (kW)	
	Heating	Cooling
Test Case #2	107.5	0.0
Trivedi (1988)	51.6	107.5
Ciric & Floudas (1991)	207.6	84.9
4spl	0.0	174.3
Total Savings	366.7	366.7

4.4 Targeting Model for Circuit Location

The location of the intermediate fluid circuits used for indirect integration is found by solving an extension of the MILP model that was presented in Chapter 2 for the special case of two plants. This model locates single independent circuits by considering all possible plant-pair combinations. In this extension, temperature constraints are added to model **P4.4** to guarantee that any single circuit between two plants follows the second law restrictions. Moreover, the upper and lower temperatures for these circuits are represented by binary variables Y_{ikj}^{EU} and Y_{ikj}^{EL}, respectively. These variables allow heat transfer where the circuits span by setting variables Z_{ikj}^{EU} to one; these variables are related to the interval heat transfer amounts by big M constraints. All these constraints are direct extensions of the ones developed for the case of two plants. The reader is referred to Chapter 2 for a detail explanation. The model is presented next:

P4.6 = $Max\ Q_E$

$s.t.$

$$Q_E = \sum_{j\in P}\sum_{k\in S_j^E} Q_{kj}^{EH}$$

$$
\left.\begin{array}{l}
Q_{kj}^{EH} = \sum\limits_{i=p^k+1,\dots,p^j} q_{ikj}^{EH} \qquad k\in S_j^E \\
\delta_o^j = \hat{\delta}_o^j - \sum\limits_{k\in S_j^E} Q_{kj}^{EH} \\
\delta_i^j = \delta_{i-1}^j + q_i^j + \sum\limits_{k\in S_{ij}^E} q_{ikj}^{EH} \qquad \forall i = 1,\dots,p^j \\
\delta_i^j = \delta_{i-1}^j + q_i^j - \sum\limits_{k\in R_{ij}^E} q_{ijk}^{EC} \qquad \forall i = (p^j+1),\dots,m \\
\delta_m^j = \hat{\delta}_m^j - \sum\limits_{k\in R_j^E} Q_{jk}^{EC} \\
\left.\begin{array}{ll}
F_{kj}^E \sum\limits_{i=p^k+1}^{r} Z_{ikj}^E \Delta T_i \ge \sum\limits_{i=p^k+1}^{r} q_{ikj}^{EH} & \forall r = (p^k+1),\dots,(p^j-1) \\
F_{kj}^E \sum\limits_{i=p^k+1}^{p^j} Z_{ikj}^E \Delta T_i = \sum\limits_{i=p^k+1}^{p^j} q_{ikj}^{EH} & \\
q_{ikj}^{EH} - U_{ikj}^{EH} Z_{ikj}^E \le 0 & \forall i = (p^k+1),\dots,p^j
\end{array}\right\} k\in S_j^E \\
\left.\begin{array}{ll}
F_{jk}^E \sum\limits_{i=r}^{p^k} Z_{ijk}^E \Delta T_i \ge \sum\limits_{i=r}^{p^k} q_{ijk}^{EC} & \forall r = (p^j+2),\dots,p^k \\
F_{jk}^E \sum\limits_{i=p^j+1}^{p^k} Z_{ijk}^E \Delta T_i = \sum\limits_{i=p^j+1}^{p^k} q_{ijk}^{EC} & \\
q_{ijk}^{EC} - U_{ijk}^{EC} Z_{ijk}^E \le 0 & \forall i = (p^j+1),\dots,p^k
\end{array}\right\} k\in R_j^E \\
\left.\begin{array}{ll}
Z_{(p^k+1)kj}^E = Y_{(p^k)kj}^{EU} & \\
Z_{ikj}^E = Z_{(i-1)kj}^E + Y_{(i-1)kj}^{EU} - Y_{(i-1)kj}^{EL} & \forall i = (p^k+2),\dots,p^j \\
\sum\limits_{i=p^k}^{p^j-1} Y_{ikj}^{EU} = 1 & \\
\sum\limits_{i=p^k+1}^{p^j} Y_{ikj}^{EL} = 1 &
\end{array}\right\} k\in S_j^E
\end{array}\right\} \forall j \in P \qquad (4.10)
$$

$$\delta_i^j, q_{ikj}^{EH}, q_{ijk}^{EC}, Z_{ikj}^E \ge 0$$

$$Y_{ikj}^{EU}, Y_{ikj}^{EL} \in \{0,1\}$$

For simplicity, model **P4.6** and its decomposition consider unassisted heat integration only. Consideration of assisted cases requires the addition of the term $-\varepsilon(Q_A + Q_B)$ in the objective function and of similar constraints for circuits involving assisted heat. This model establishes single independent circuits between any two plants. Absence of integration is represented by a circuit that although has no heat transfer involved, it may extend across two plants. This is an MINLP model, and it becomes linear by replacing the product of continuous variables times binary variables with a set of linear constraints (Chapter 2). The strategy applied to generate equal intervals for the case of targeting also has to be applied to establish the intervals used by model **P4.6**. However, this leads to the use of too many binary variables, thus making the MILP problem difficult to converge. A heat supplied and heat demand decomposition is therefore proposed to alleviate the computational burden.

For each plant, we propose to write a set of equations that establish the single circuits, making the equations independent of the interval partitioning. The equations are:

$$
\left.\begin{array}{l}
\left.\begin{array}{ll}
F_{kj}^{E} \sum_{i=p^k+1}^{r} Z_{ikj}^{EH} \Delta T_i \geq \sum_{i=p^k+1}^{r} q_{ikj}^{EH} & \forall r=(p^k+1),\dots,(p^j-1) \\
F_{kj}^{E} \sum_{i=p^k+1}^{p^j} Z_{ikj}^{EH} \Delta T_i = \sum_{i=p^k+1}^{p^j} q_{ikj}^{EH} & \\
q_{ikj}^{EH} - U_{ikj}^{EH} Z_{ikj}^{EH} \leq 0 & \forall i=(p^k+1),\dots,p^j \\
Z_{(p^j+1)kj}^{EH} = Y_{(p^j)kj}^{EHU} & \\
Z_{ikj}^{EH} = Z_{(i-1)kj}^{EH} + Y_{(i-1)kj}^{EHU} - Y_{(i-1)kj}^{EHL} & \forall i=(p^k+2),\dots,p^j \\
\sum_{i=p^k}^{p^j-1} Y_{ikj}^{EHU} = 1 & \\
\sum_{i=p^k+1}^{p^j} Y_{ikj}^{EHL} = 1 &
\end{array}\right\} k \in S_j^E \\
\left.\begin{array}{ll}
F_{jk}^{E} \sum_{i=r}^{p^k} Z_{ijk}^{EC} \Delta T_i \geq \sum_{i=r}^{p^k} q_{ijk}^{EC} & \forall r=(p^j+2),\dots,p^k \\
F_{jk}^{E} \sum_{i=p^j+1}^{p^k} Z_{ijk}^{EC} \Delta T_i = \sum_{i=p^j+1}^{p^k} q_{ijk}^{EC} & \\
q_{ijk}^{EC} - U_{ijk}^{EC} Z_{ijk}^{EC} \leq 0 & \forall i=(p^j+1),\dots,p^k \\
Z_{(p^j+1)kj}^{EC} = Y_{(p^j)kj}^{ECU} & \\
Z_{ikj}^{EC} = Z_{(i-1)kj}^{EC} + Y_{(i-1)kj}^{ECU} - Y_{(i-1)kj}^{ECL} & \forall i=(p^j+2),\dots,p^k \\
\sum_{i=p^j}^{p^k-1} Y_{ijk}^{ECU} = 1 & \\
\sum_{i=p^j+1}^{p^k} Y_{ijk}^{ECL} = 1 &
\end{array}\right\} k \in R_j^E
\end{array}\right\} \forall j \in P \qquad (4.11)
$$

Different binary variables and different variables that allow the heat transfer for the supplier and receiver sides are used. These two sets of equations are linked only by the circuit flowrates that do not depend on the intervals. Thus, the original intervals in each plant, constructed by using the starting temperatures of the individual-plant streams, are partitioned further by considering the temperatures of the plants that will eventually deliver heat to or receive heat from the plant. These additional temperatures are shifted by adding or subtracting the minimum temperature difference from the original temperatures depending on the direction of the heat transfer. The procedure is illustrated in Appendix B. The following equation is added to guarantee that the heat transfer from the circuit to the receiver is equal to the heat transfer from the supplier to the circuit.

$$\sum_{i=p^k+1}^{p^j} q_{ikj}^{EH} = \sum_{i=p^k+1}^{p^j} q_{ikj}^{EC} \quad k \in S_j^E; \forall j \in P \tag{4.12}$$

Finally, to connect both sets of constraints properly, extra relations are required to establish that the temperatures at the upper and lower parts of the circuits are the same in both the supplier and receiver plants. These constraints are:

$$\left.\begin{aligned} Y_{ikj}^{EHU} &= Y_{rkj}^{ECU} \\ Y_{ikj}^{EHL} &= Y_{rkj}^{ECL} \end{aligned}\right\} \forall (i,r) \in \{(i,r) / T_i = T_r\} k \in S_j^E; \forall j \in P \tag{4.13}$$

If numerical problems are still a concern, further reduction of the number of intervals is possible. One can partition further (using the method of Appendix B) only the set of intervals belonging to the regions containing the connections shown in the solution of problem **P4.5** and define binary variables for only these intervals. This procedure is illustrated in Appendix C. In using this approach, one has to be careful: only when the solution states that all the targeted effective heat is transferred using the intervals proposed, can it be assure that the optimal solution has been captured. When the solution states that less heat than the target is transferred, it might be possible that one circuit, positioned in other intervals, can transfer more. Thus, sub-optimal solutions are possible.

It is important to note that the solutions to problem **P4.6** can be implemented when all the plants performing the integration are operating. As the problem is known to have alternative solutions, one could try to use those solutions that maximize the savings in circumstances for which different subsets of plants are not in operation. A strategy to achieve this goal is presented next.

4.4.1 Independent Circuits Location Example

4.4.1.1 Example 4.2 (continued)

Figure 4.14 shows the results obtained after solving problem **P4.6**. Three circuits that transfer effective heat are required to achieve maximum savings, one between plants 1 and 2, the second between plants 2 and 3, and the third between plants 3 and 4. An additional circuit that transfers assisting heat from plant 2 to plant 3 is also required to debottleneck the heat cascade of plant 2. Notice that the number of effective and assisting connections is minimal, and it is obtained directly when solving problem **P4.6** for the independent circuits location. The solution of problem **P4.5**, however, can be used to reduce the number of intervals to be generated by the procedure presented in Appendix C. The amount of savings in each of the plants as well as the entire maximum savings are equivalent to the targeting values already obtained for this example when problem **P4.5** was solved in the previous section (Table 4.6).

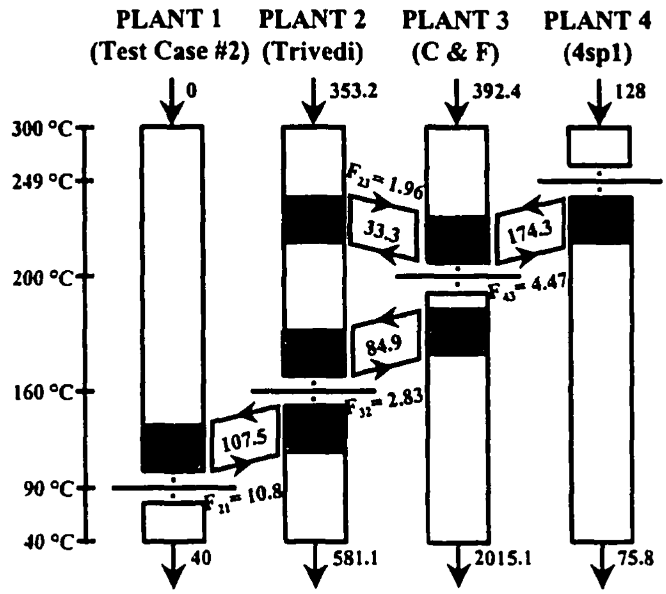


Figure 4.14. Indirect integration solution using independent circuits for Example 4.2

4.5 Multiple-Operation Circuits

Consider the cases in which one or more plants go out of service. The goal is to maximize the overall expected heat savings on all of the possible operational modes, with each of these scenarios having a given probability p_w. These scenarios are the instances in which proper subsets of all the plants (i.e. containing at least two plants) are considered to be in operation. By using the same set of binary and continuous variables to represent the span of the circuits in all scenarios, it is guaranteed that a single arrangement of circuits is selected. Restrictions to the use of a single pipe to transfer heat from a plant to all of its receivers can be easily added by asking the corresponding binary variables to take the same value at the same temperature interval. The model is shown next:

$$
\mathbf{P4.7} = Max \sum_{w\in(1,w_{max})} p_w \cdot Q_{E,w}
$$

s.t.

$$
Q_{E,w} = \sum_{j\in P_w}\sum_{k\in S^E_{j,w}} Q^{EH}_{kj,w}
$$

$$
\left.\begin{array}{l}
Q^{EH}_{kj,w} = \sum_{i=p^k+1,\dots,p^j} q^{EH}_{ikj,w} \qquad k\in S^E_{j,w} \\
\delta^j_{o,w} = \hat{\delta}^j_o - \sum_{k\in S^E_{j,w}} Q^{EH}_{kj,w} \\
\delta^j_{i,w} = \delta^j_{i-1,w} + q^j_i + \sum_{k\in S^E_{ij,w}} q^{EH}_{ikj,w} \qquad \forall i=1,\dots,p^j \\
\delta^j_{i,w} = \delta^j_{i-1,w} + q^j_i - \sum_{k\in R^E_{ij,w}} q^{EC}_{ijk,w} \qquad \forall i=(p^j+1),\dots,m \\
\delta^j_{m,w} = \hat{\delta}^j_{m,w} - \sum_{k\in R^E_{j,w}} Q^{EC}_{jk,w} \\
\left.\begin{array}{ll}
F^E_{kj,w}\sum_{i=p^k+1}^{r} Z^E_{ikj}\Delta T_i \ge \sum_{i=p^k+1}^{r} q^{EH}_{ikj,w} & \forall r=(p^k+1),\dots,(p^j-1) \\
F^E_{kj,w}\sum_{i=p^k+1}^{p^j} Z^E_{ikj}\Delta T_i = \sum_{i=p^k+1}^{p^j} q^{EH}_{ikj,w} & \\
q^{EH}_{ikj,w} - U^{EH}_{ikj,w} Z^E_{ikj} \le 0 & \forall i=(p^k+1),\dots,p^j
\end{array}\right\} k\in S^E_{j,w} \\
\left.\begin{array}{ll}
F^E_{jk,w}\sum_{i=r}^{p^k} Z^E_{ijk}\Delta T_i \ge \sum_{i=r}^{p^k} q^{EC}_{ijk,w} & \forall r=(p^j+2),\dots,p^k \\
F^E_{jk,w}\sum_{i=p^j+1}^{p^k} Z^E_{ijk}\Delta T_i = \sum_{i=p^j+1}^{p^k} q^{EC}_{ijk,w} & \\
q^{EC}_{ijk,w} - U^{EC}_{ijk,w} Z^E_{ijk} \le 0 & \forall i=(p^j+1),\dots,p^k
\end{array}\right\} k\in R^E_{j,w} \\
\left.\begin{array}{ll}
Z^E_{(p^k+1)kj} = Y^{EU}_{(p^k)kj} & \\
Z^E_{ikj} = Z^E_{(i-1)kj} + Y^{EU}_{(i-1)kj} - Y^{EL}_{(i-1)kj} & \forall i=(p^k+2),\dots,p^j \\
\sum_{i=p^k}^{p^j-1} Y^{EU}_{ikj} = 1 & \\
\sum_{i=p^k+1}^{p^j} Y^{EL}_{ikj} = 1 &
\end{array}\right\} k\in S^E_{j,w}
\end{array}\right\}
\begin{array}{l} \forall j\in P_w \\ w\in(1,w_{max}) \end{array}
\qquad (4.14)
$$

$$
\delta^j_{i,w}, q^{EH}_{ikj,w}, q^{EC}_{ijk,w}, Z^E_{ikj} \ge 0
$$

$$
Y^{EU}_{ikj}, Y^{EL}_{ikj} \in \{0,1\}
$$

The maximum possible number of scenarios w_{max} is given by $\sum_{s=2}^{n} \frac{n!}{s!(n-s)!}$, which represents all possible combinations of s plants in operation.

4.5.1 Multi-operation Circuit Example

4.5.1.1 Example 4.3

This example considers three of the plants of Example 4.2 (Test case #2, Ciric & Floudas (1991) and 4sp1 are selected) to illustrate indirect heat integration using intermediate fluid circuits and the convenience of establishing multi-operation circuits. The solution to this example only requires effective heat transfer (i.e. is an unassisted heat integration problem).

Figure 4.15a, shows the result of solving model **P4.6**. Two circuits are required to achieve maximum savings, one between plants 1 and 2, and the second between plants 2 and 3. The alternative solution is to have the arrangement shown in Figure 4.15b. However, maximum savings are not possible in this case since limitations in the amount that plant 3 can transfer below its pinch are encountered. Notice that these arrangements only work when the three plants are operating and any of the two alternatives may be preferred depending upon the shutdown schedules of the plants.

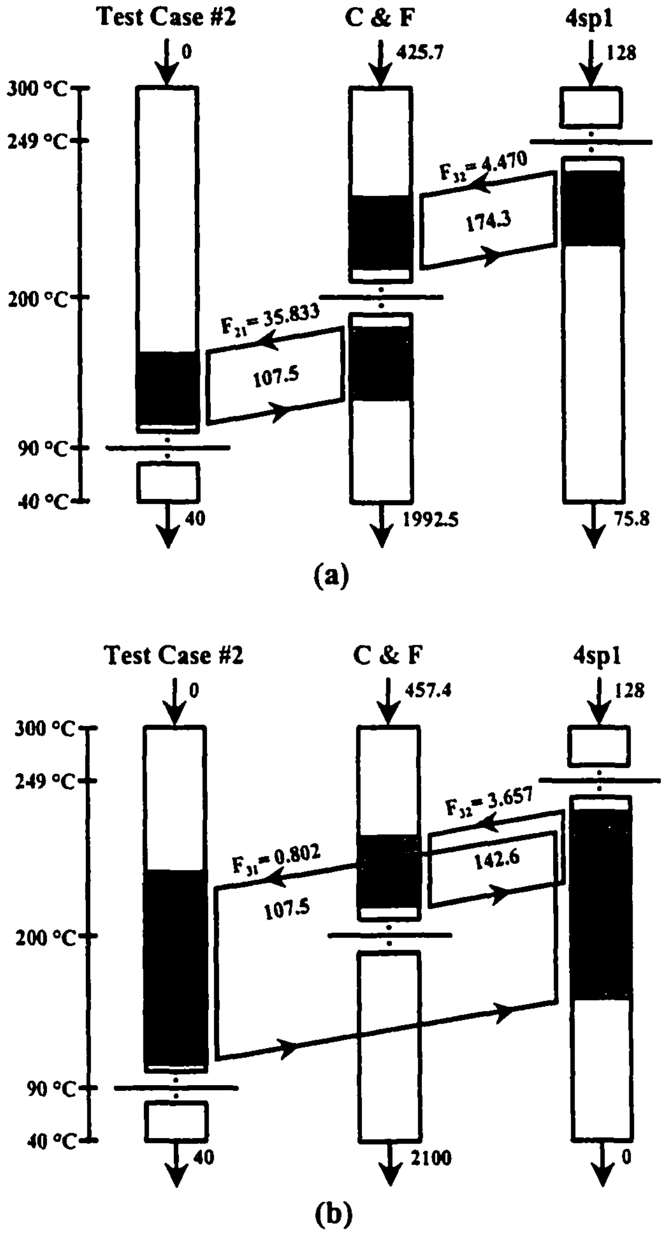
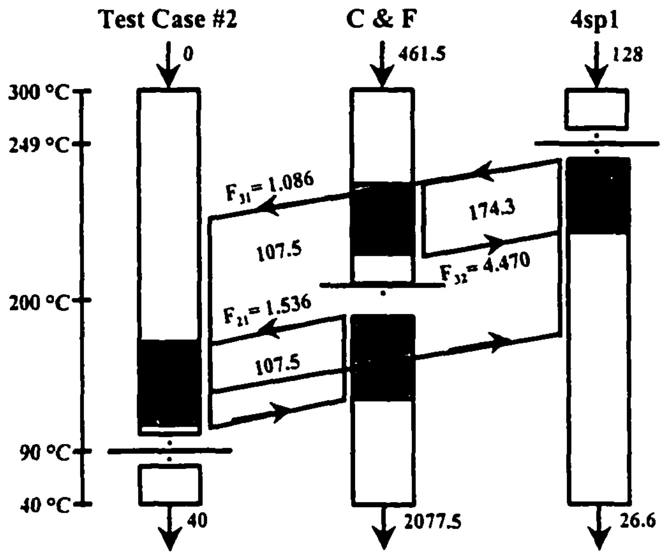
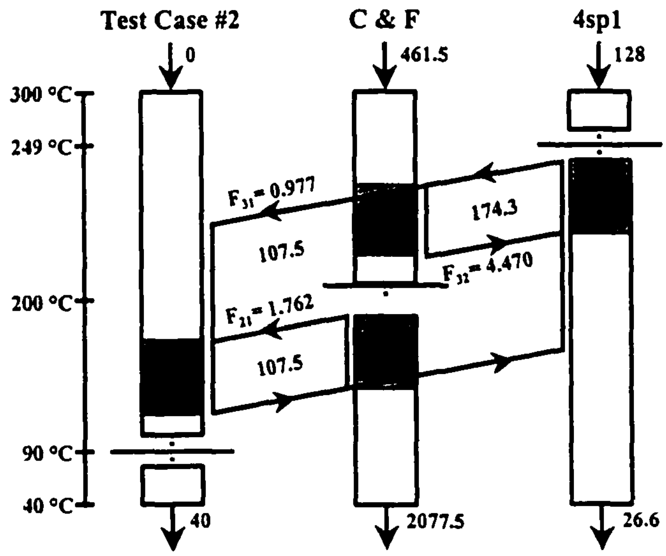


Figure 4.15. Independent circuits alternatives for Example 4.3

In order to attain the maximum possible savings in any operating condition and considering that the given probabilities of occurrence of each scenario are equal, model **P4.7** is solved. Figure 4.16a shows the set of circuits required for this task. Further improvement of this arrangement can be obtained by adding constraints in the binary variables. The resulting solution is shown in Figure 10b, in which only two pipes are used to integrate both plants 2 and 3 with plant 1.



(a)



(b)

Figure 4.16. Multi-operation circuit alternatives for Example 4.3

4.6 Entire Oil Refinery Example

To test the developed tools in a large and realistic problem the heat integration between seven units representing the instance of an entire oil refinery is considered. The data for these units can be found in Fraser and Gillespie (1992) who applied pinch technology to energy integrate the whole system. The results reported by these authors are based on current plant heating utility usage (none of the existing plants is completely energy integrated). Since their savings have a retrofit component, comparisons with their approach will not be made. Table 4.7 shows the results of applying pinch analysis to the individual plants.

Table 4.7. Individual plant pinch analysis for the entire oil refinery example

Unit	Pinch Temperature (°C)	Minimum Heating Utility (MW)	Minimum Cooling Utility (MW)
Platformer (reformer)	79.4	18.00	8.37
Visbreaker (thermal cracking)	145	6.83	3.20
Kerosene hydrotreater	176.6	0.73	4.19
Naphtha hydrotreater	177.2	4.17	7.73
Crude and vacuum distilation	272	54.94	23.94
Fluid catalytic cracking	-	0.00	20.45
Diesel hydrotreater	-	0.00	2.76
Entire Refinery	NA	84.67	70.64

When problem **P4.4** is solved, effective-direct integration savings of 19.47 MW are obtained. This represents a 23% savings of the total heating utility of the site. An amount of 3.30 MW of assisted heat is required to be transferred above pinch temperatures, and an amount of 2.34 MW below pinch temperatures. These targeting values are then use in formulating problem **P4.5** that is solved to obtain the minimum number of connections and their respective heat fluxes. Figure 4.17 shows the solution containing seventeen connections, eleven of which are the effective connections leading to savings. The rest are assisting connections in the opposite direction to the effective connections.

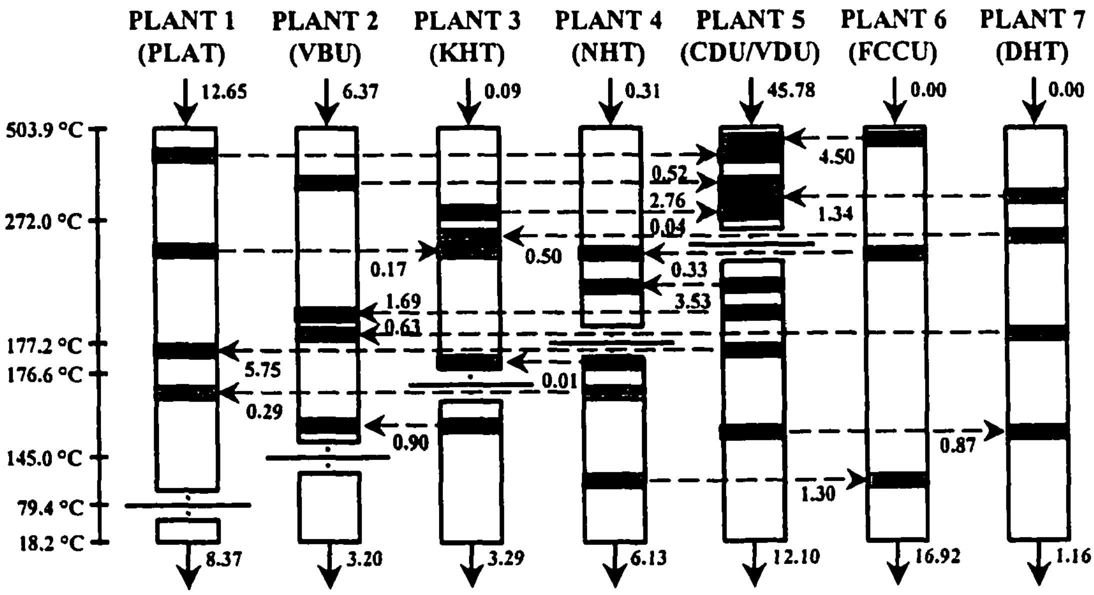


Figure 4.17. Direct integration solution for the entire oil refinery example

Table 4.8 shows the amount of savings achieved in each of the units as well as the entire oil refinery are. The excessive amount of inter-unit connections required to attain maximum savings makes questionable the practical use of this solution.

Table 4.8. Direct integration for the entire oil refinery example

Unit	Code	Savings (MW)	
		Heating	Cooling
Platformer (reformer)	**PLAT**	5.35	0.00
Visbreaker (thermal cracking)	**VBU**	0.46	0.00
Kerosene hydrotreater	**KHT**	0.64	0.90
Naphtha hydrotreater	**NHT**	3.86	1.60
Crude and vacuum distilation	**CDU/VDU**	9.16	1.60
Fluid catalytic cracking	**FCCU**	0.00	3.53
Diesel hydrotreater	**DHT**	0.00	1.60
Total Savings	-	19.47	19.47

After the interval partition procedure given in Appendix A is applied and indirect diagonal transfer is considered, problem **P4.4** is solved. Effective-indirect integration savings of 18.03 MW are obtained which represents 21% savings of the total heating utility of the site. An amount of 3.3 MW of assisted heat is required to be transferred above pinch temperatures, and an amount of 2.3 MW below pinch temperatures. The minimum number of connections and their respective heat fluxes are then obtained by solving problem **P4.5** with the use of these targeting values. Figure 4.18 shows the solution containing twenty connections, fourteen of which are the effective connections leading to savings. The rest are assisting connections in the opposite direction to the effective connections.

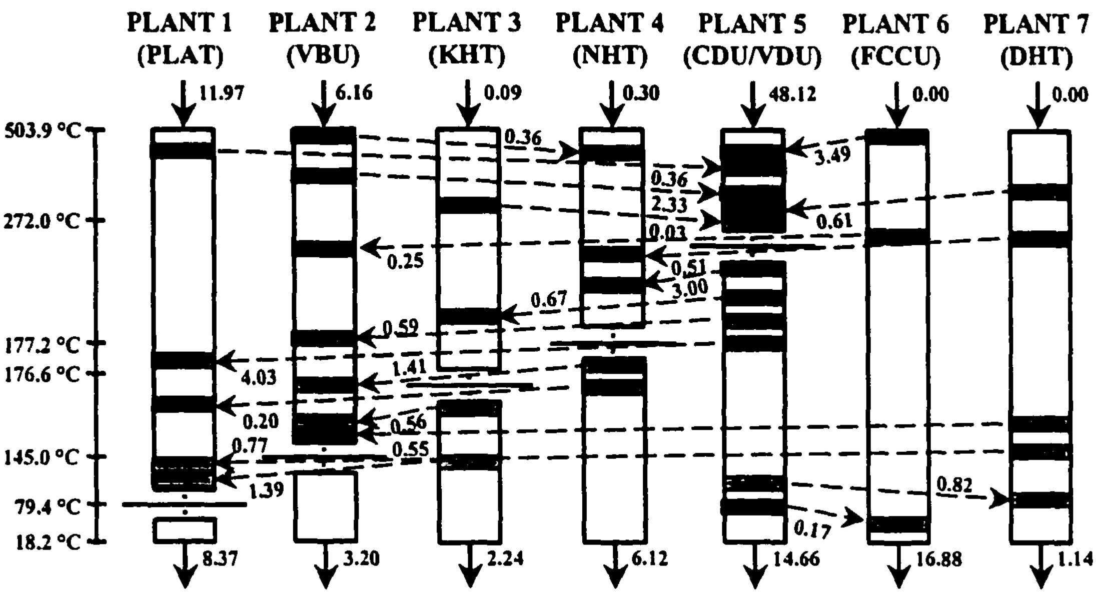


Figure 4.18. Indirect integration solution for the entire oil refinery example

Table 4.9 shows the amount of savings achieved in each of the units as well as the entire oil refinery.

Table 4.9. Indirect integration for the entire oil refinery example

Unit	Code	Savings (MW)	
		Heating	Cooling
Platformer (reformer)	PLAT	6.03	0.00
Visbreaker (thermal cracking)	VBU	0.67	0.00
Kerosene hydrotreater	KHT	0.64	1.95
Naphtha hydrotreater	NHT	3.87	1.61
Crude and vacuum distilation	CDU/VDU	6.82	9.28
Fluid catalytic cracking	FCCU	0.00	3.57
Diesel hydrotreater	DHT	0.00	1.62
Total Savings	-	18.03	18.03

Again, the excessive amount of inter-unit connections required to attain maximum savings rises the question of the practicality of implementing these solution that will required too many circuits.

4.7 Conclusions

The targeting methods for heat integration between two plants presented in Chapter 2 were extended to consider a total site composed by a set of n plants. Important new aspects are revealed. The pattern corresponding to assisted heat transfer between two plants changes for many plants. In particular, assisting heat can be transferred in both opposite and parallel directions to the effective heat transfer. For indirect integration, transfer between equal intervals that are a fixed number of intervals apart is used to account for the presence of the fluid circuits.

Finally, the resulting problem exhibits alternative solutions, and flexibility is gained by optimizing the different operational scenarios. Future work will concentrate in screening these alternatives with additional criteria, as well as exploring the concept of a heat belt.

4.8 Nomenclature

i = temperature interval
j = chemical plant
k = auxiliary chemical plant
m = total number of intervals
n = total number of chemical plants
p^j = last interval above the pinch temperature of plant j
Q_A = total heat transferred in the zone above both pinch temperatures
Q_B = total heat transferred in the zone below both pinch temperatures

Q_E = total heat transferred in the zone of effective transfer of heat (between pinch temperatures)
q = heat surplus or heat demand / heat transferred
q^j = heat surplus or heat demand in plant j
δ_0 = minimum surplus to the first interval
$\hat{\delta}_0$ = original minimum surplus to the first interval
δ = minimum cascaded heat
$\hat{\delta}$ = original minimum cascaded heat

Superscripts
A = zone above both pinch temperatures
B = zone below both pinch temperatures
E = zone of effective transfer of heat (between pinch temperatures)
j = chemical plant

Subscripts
A = zone above both pinch temperatures
B = zone below both pinch temperatures
E = zone of effective transfer of heat (between pinch temperatures)
i = temperature interval
j = chemical plant
k = auxiliary temperature intervals

4.9 References

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CHAPTER 5

On the Use of Heat Belts for Heat Integration across Many Plants in the Total Site

5.1 Introduction

In the previous chapter, generalized mathematical models extending the results originally developed for two plants (Chapter 2) to the case of multiple plants were proposed. Results of the application of an MILP model that locates the circuits that perform the indirect integration were presented. Moreover, the optimal location of intermediate-fluid circuits that allows flexibility of operation was added to the MILP model to consider cases in which any of the plants was shut down.

A reduction of the piping and pumping costs can be expected if a single pipe collects heat from and delivers it to the plants. In cases in which independent circuits transfer heat from the same plant to many other plants, a pair of pipes has to be used for each transfer. Additionally, more heat exchangers may be necessary. The relative location of each of the plants to the others also plays an important role. In many aspects, simplicity then can be obtained by using a single belt system that takes advantage of the existing location of the plants. Chapter 3 presents a case study on integration between two plants that supports this idea. In this study, piping and pumping costs are such that the use of one circuit instead of two is more economical, even though one circuit does not achieve all the possible energy savings.

In this paper, the new concept of the "heat belt" is introduced as an alternative to the use of independent or multi-operation circuits. The concept is derived by restricting multi-operation circuits to the use of a single pipe arrangement, as presented in Chapter 4. The MINLP model that locates the "heat belt" for the special case of three plants is then introduced. Linearization of this model is possible as only bilinearities consisting of the product of a binary variable and a continuous variable are present. Examples showing the different features of this approach are presented.

5.2 The Concept of Heat Belt

In Chapter 4, the use of a "heat belt" was suggested to help gather heat from some plants and deliver it to others. It was speculated that this could save some piping/pumping costs and possibly resolve the issue of flexibility, as multiple alternative solutions could be included in the belt. This would allow for maximum efficiency in multiple scenarios of plants shutting down and/or changing throughput.

To attain the maximum possible savings in any operating condition, the targeting model for circuit location was solved in the previous chapter by considering all of the possible operational modes. These modes are the instances in which two or more plants are considered to be in operation. If the restriction of the use of a single pipe arrangement is imposed, sometimes alternative circuits can be obtained that use less piping. Figure 5.1 from Chapter 4 shows an example of the circuit required for this task established across three plants. This circuit works as two independent circuits, one between plant 2 and plant 1 and the other between plant 3 and plant 1 when integration of the total site is present. The external circuit is available for the case in which plant 2 is shut down, and it allows the transfer of heat from plant 3 to plant 1.

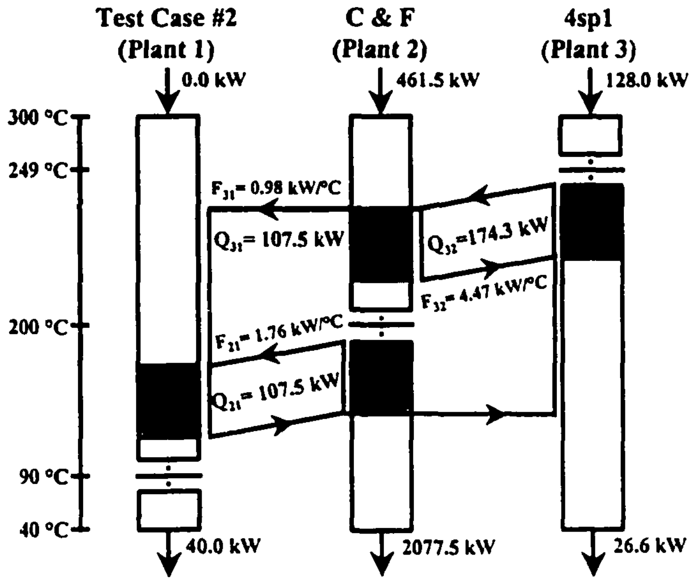

Figure 5.1. Multi-operation circuit

One of the characteristics of the scheme in Figure 5.1 is that certain parts of the two circuits are common. Thus, it is practical to join these circuit parts in a single pipe by restricting the extended MILP model. The new idea is that the heat transfer circuits can be thought of as a heat-belt circuit from which the different plants take and/or discharge fluid to extract and/or release heat. In this new approach, mixing plays an important role. As is immediately apparent, the position of the plants sometimes does not suggest a "heat belt." To illustrate this, consider Figure 5.2 in which the plants of Figure 5.1 have been rearranged. The use of independent circuits no longer suggests the location of the "heat belt."

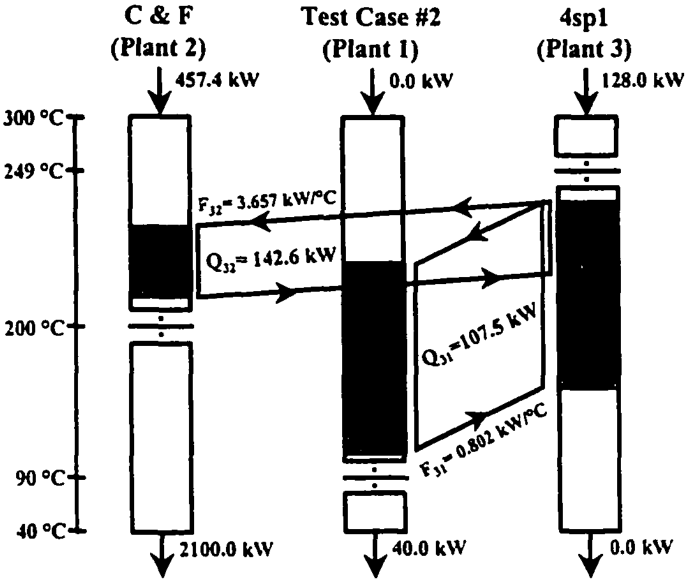

Figure 5.2. Independent circuits for rearranged plants

In the analysis that follows, *one* of the possible heat belts that can be established across a set of *n* plants is discussed. This version of a "heat belt" is formally introduced in Figure 5.3, in which *n* plants are aligned. The "heat belt" is a fixed circuit that has two main lines, the top line and the bottom line. Branches split and mix with the main lines as the belt passes by the plants. For each of the plants, two branches are present: 1) the hot line F_j^H, from which the plant receives heat from the "heat belt," and 2) the cold line F_j^C, from which the plant delivers heat to the "heat belt." Extreme plants have only one branch, because plant 1 can only receive heat from the "heat belt" (heat leading to savings is transported only from the right to the left), and plant *n* can only deliver heat to the "heat belt."

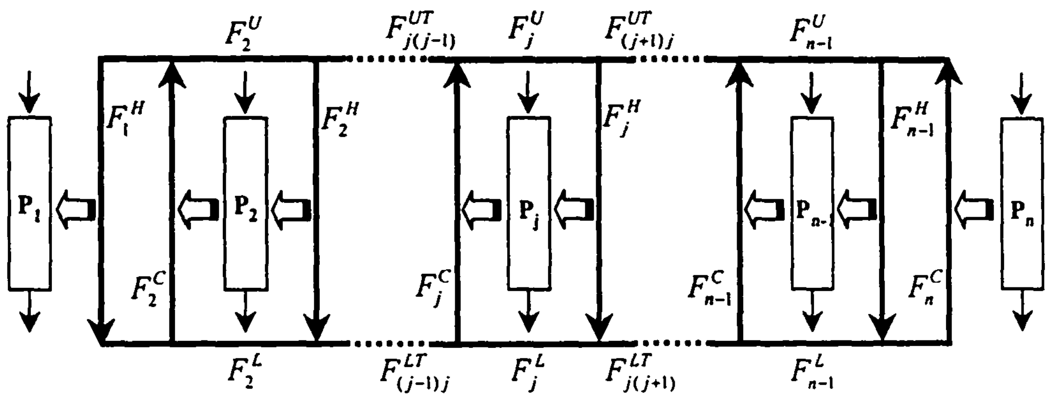

Figure 5.3. General "heat belt"

The top line starts on the extreme right with the amount of heat collected by the cold branch from plant n. As it passes through each of the plants, hot fluid branches are split to deliver heat. Each split is followed by the mixing of the remaining hot fluid with cold branches that have collected heat from the plants. Similarly, the bottom line starts on the extreme left, following the hot branch that has delivered heat to plant 1. Cold branches are split to collect heat from each of the plants. Splits are followed by mixers in which cold branches are mixed with hot branches that have delivered heat.

The "heat belt" defined in Figure 5.3 relates only to plants that are aligned horizontally. However, other arrangements are possible, and therefore, other definitions for the "heat belt" might follow. Analyses of these other alternatives are not the purpose of this chapter.

Some features of the "heat belt" are immediately apparent:

(a) because of the "heat belt" definition, it is not possible to transfer assisted heat in the direction opposite to the effective heat (i.e. from left to right).

(b) The plant with the highest pinch temperature can never receive effective heat. However, in principle it may receive assisted heat from other plants in the same direction as the effective heat. Thus, if the plant is located on the extreme left (plant 1), only the intervals above its pinch temperature can receive assisted heat in the same direction as the effective heat. If one can establish a priori that assisted heat is not needed, then the plant with the highest pinch temperature located on the extreme left cannot participate in the belt.

(c) Similarly, the plant with the lowest pinch temperature can never deliver effective heat. In principle, it may give assisted heat to other plants in the same direction as the effective heat. If it is located on the extreme right (plant *n*), only the intervals above its pinch temperature can receive heat, and in the absence of assisted heat in the same direction as the effective heat, the plant with the lowest pinch temperature located on the extreme right cannot participate in the belt.

For the special case of three plants and no assisted heat in either direction, three geographical location cases can be considered as candidates for a heat belt. These cases of relative location are described by the following situations:

(a) The pinch temperatures are increasing from left to right

(b) The plant with the intermediate pinch temperature is located either on the extreme left or on the extreme right, while the plant having the highest pinch temperature is never located on the extreme left and the plant with the lowest pinch temperature is never located on the extreme right.

A mathematical programming model for the case of three plants is presented next.

5.3 MILP for the Three Plants Case

Figure 5.3 presents the general definition for the different flowrate variables. In order to establish the upper limit in the temperature intervals for the hot and cold branches, binary variables Y_{ij}^{UH} and Y_{ij}^{UC} are defined. Similarly, binary variables Y_{ij}^{LH} and Y_{ij}^{LC} are defined to establish the lower limit in the temperature intervals for their respective branches. In turn, these binary variables determine whether or not there is heat transfer with the "heat belt" by setting the continuous variables Z_{ij}^{H} and Z_{ij}^{C} to either one or zero. Finally, the variable X_j^H represents the enthalpy of the hot branches before delivering heat, and X_j^C represents the enthalpy of the cold branches before receiving heat. The following MINLP model establishes the optimal heat-belt circuit for the special case of a given set of three plants.

$$\textbf{P5.1} = Min\ \left(\delta_o^{I} + \delta_o^{II}\right) \tag{5.1}$$

s.t.

$$\delta_o^{I} = \hat{\delta}_o^{I} - \sum_{i=1}^{m} q_{i,1}^{H}$$

$$\delta_o^{II} = \hat{\delta}_o^{II} - \sum_{i=1}^{m} q_{i,2}^{H}$$

$$\left.\begin{aligned} \delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} + q_{i,1}^{H} \\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_{i,2}^{H} - q_{i,2}^{C} \\ \delta_i^{III} &= \delta_{i-1}^{III} + q_i^{III} - q_{i,3}^{C} \end{aligned}\right\} \forall i = 1,\ldots,m \tag{5.2}$$

$$\delta_m^{II} = \hat{\delta}_m^{II} - \sum_{i=1}^{m} q_{i,2}^{C}$$

$$\delta_m^{III} = \hat{\delta}_m^{III} - \sum_{i=1}^{m} q_{,3}^{C}$$

$$\begin{aligned} F_1^{H} &= F_2^{U} + F_2^{C} \\ F_2^{U} + F_2^{H} &= F_3^{C} \\ F_3^{C} &= F_2^{H} + F_2^{L} \\ F_2^{C} + F_2^{L} &= F_1^{H} \end{aligned} \tag{5.3}$$

$$\left.\begin{aligned} & q_{ij}^{H} - U_{ij}^{H} Z_{ij}^{H} \le 0 && \forall i = 1,\ldots,m \\ & Z_{ij}^{H} = Y_{(i-1)j}^{UH} && i = 1 \\ & Z_{ij}^{H} = Z_{(i-1)j}^{H} + Y_{(i-1)j}^{UH} - Y_{(i-1)j}^{LH} && \forall i = 2,\ldots,m \\ & \sum_{i=0}^{m-1} Y_{ij}^{UH} = 1 && \\ & \sum_{i=1}^{m} Y_{ij}^{LH} = 1 && \end{aligned}\right\} j = 1,2 \tag{5.4}$$

$$\left.\begin{aligned} & q_{ij}^{C} - U_{ij}^{C} Z_{ij}^{C} \le 0 && \forall i = 1,\ldots,m \\ & Z_{ij}^{C} = Y_{(i-1)j}^{UC} && i = 1 \\ & Z_{ij}^{C} = Z_{(i-1)j}^{C} + Y_{(i-1)j}^{UC} - Y_{(i-1)j}^{LC} && \forall i = 2,\ldots,m \\ & \sum_{i=0}^{m-1} Y_{ij}^{UC} = 1 && \\ & \sum_{i=1}^{m} Y_{ij}^{LC} = 1 && \end{aligned}\right\} j = 2,3 \tag{5.5}$$

$$X_1^H = \sum_{i=0}^{m-1}\left(Y_{i,2}^{UC} F_2^C T_i + Y_{i,3}^{UC} F_3^C T_i\right) - X_2^H$$

$$X_2^H = \sum_{i=0}^{m-1} Y_{i,3}^{UC} F_2^H T_i \tag{5.6}$$

$$X_2^C = \sum_{i=1}^{m} Y_{i,1}^{LH} F_2^C T_i$$

$$X_3^C = \sum_{i=1}^{m}\left(Y_{i,1}^{LH} F_1^C T_i + Y_{i,2}^{LH} F_2^C T_i\right) - X_2^C$$

$$X_j^H - \sum_{i=1}^{m} Y_{i,j}^{LH} F_j^H T_i = \sum_{i=1}^{m} q_{i,j}^H$$

$$\sum_{i=0}^{m-1} Y_{i,j}^{UC} F_j^C T_i - X_j^C = \sum_{i=1}^{m} q_{i,j}^C \tag{5.7}$$

$$F_1^H \sum_{i=1}^{r} Z_{i,1}^H \Delta T_i + \sum_{i=0}^{r-1} Y_{i,1}^{UH}\left(X_1^H - F_1^H T_i\right) \geq \sum_{i=1}^{r} q_{i,1}^H \quad \forall r = 1,\ldots,(m-1)$$

$$F_1^H \sum_{i=1}^{m} Z_{i,1}^H \Delta T_i + \sum_{i=0}^{m-1} Y_{i,1}^{UH}\left(X_1^H - F_1^H T_i\right) = \sum_{i=1}^{m} q_{i,1}^H \tag{5.8}$$

$$X_1^H - F_1^H T_i \geq 0 \quad \forall i = 0,\ldots,m-1$$

$$F_2^H \sum_{i=1}^{r} Z_{i,2}^H \Delta T_i \geq \sum_{i=1}^{r} q_{i,2}^H \quad \forall r = 1,\ldots,(m-1)$$

$$F_2^H \sum_{i=1}^{m} Z_{i,2}^H \Delta T = \sum_{i=1}^{m} q_{i,2}^H \tag{5.9}$$

$$F_3^C \sum_{i=r}^{m} Z_{i,3}^C \Delta T_i + \sum_{i=r}^{m} Y_{i,3}^{LC}\left(F_3^C T_i - X_3^C\right) \geq \sum_{i=r}^{m} q_{i,3}^C \quad \forall r = 2,\ldots,m$$

$$F_3^C \sum_{i=1}^{m} Z_{i,3}^C \Delta T_i + \sum_{i=2}^{m} Y_{i,3}^{LC}\left(F_3^C T_i - X_3^C\right) = \sum_{i=1}^{m} q_{i,3}^C \tag{5.10}$$

$$F_3^C T_i - X_3^C \geq 0 \quad \forall i = 2,\ldots,m$$

$$F_2^C \sum_{i=r}^{m} Z_{i,2}^C \Delta T \geq \sum_{i=r}^{m} q_{i,2}^C \quad \forall r = 2,\ldots,m$$

$$F_2^C \sum_{i=1}^{m} Z_{i,2}^C \Delta T = \sum_{i=1}^{m} q_{i,2}^C \tag{5.11}$$

$$\delta_i^I, \delta_i^{II}, \delta_i^{III}, q_{ij}^H, q_{ij}^C, F_j^H, F_j^C, F_2^U, F_2^L, X_j^H, X_j^C, Z_{ij}^H, Z_{ij}^C \geq 0$$

$$Y_{ij}^{UH}, Y_{ij}^{LH}, Y_{ij}^{UC}, Y_{ij}^{UC} \in \{0,1\} \tag{5.12}$$

The objective function (5.1) minimizes the summation of the heating utilities of the first two plants, as no heating utility savings are possible in plant 3 (no assisted heat in either direction is considered). Reduction of these utilities is attained by the heat transfer from the "heat belt" to the respective plants. In turn, the respective cooling utilities are reduced by the transfer of heat from the plants to the "heat belt" (with the exception of plant 1). Equations (5.2) represent the balances established to allow all the above-mentioned heat transfer. Flowrate balances are established by equations (5.3) in all the nodes in which the branches of the "heat belt" are split or mixed. Equations (5.4) and (5.5) establish the intervals in which heat transfer from the plants to the "heat belt" and from the "heat belt" to the plants is allowed. In Chapter 2, a detailed explanation is provided when considering equivalent equations that are used to establish intervals in which a circuit is located between two plants. Expressions for all the enthalpies as a function of the respective flowrates and fixed interval temperatures are given by equations (5.6). Equations (5.7) guarantee that the heat transfer to and from the "heat belt" equals the available enthalpy difference.

The nature of the thermodynamic feasibility constraints as found in equations (5.8) through (5.11) is explained next. Similar equations are used in Chapter 2 for the feasible location of circuits between two plants. Due to the possibility of mixing of streams in the "heat belt," an extra positive term is needed in the restrictions imposed in equations (5.8) and (5.10). In the case of equations (5.8), the enthalpy of the hot branch just before delivering heat to plant 1 (X_1^H) is obtained by mixing. As Figure 5.4a shows, this enthalpy is located somewhere within the fixed temperature interval in plant 1 immediate above the temperature level at which the heat cascade starts receiving heat. Therefore, the difference between the temperature that the hot branch has (obtained by mixing) and the temperature at which the heat cascade of plant 1 starts receiving heat is not available for heat transfer. The extra term added to the feasibility constraint in equation (5.8) makes possible this temperature difference. This term must always be positive, because the temperature of the hot branch must be greater than or equal to the upper temperature of the starting interval. Similar analyses can be done for the case of equations (5.10), in which the enthalpy of the cold branch before receiving heat from plant 3 (X_3^C) is obtained by mixing. Figure 5.4b shows the location within the fixed temperature interval in plant 3 immediate below the temperature at which the cold branch starts receiving heat.

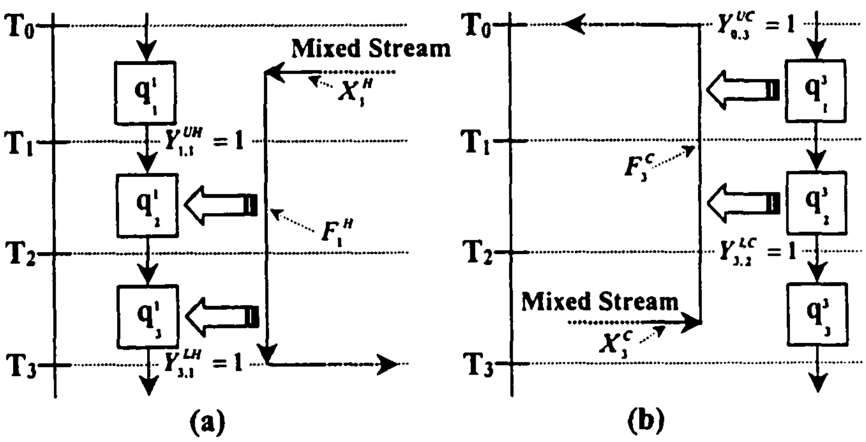

Figure 5.4. Enthalpies of the "heat belt"

As only unassisted heat integration is considered, the region in which the "heat belt" is established lies between the extreme pinch temperatures. Therefore, the zone above the highest pinch temperature and the zone below the lowest pinch temperature do not need to be considered. For the transfer of heat to and from the intermediate fluid to be possible, a shift of scales similar to the one implemented in Chapter 2 is necessary.

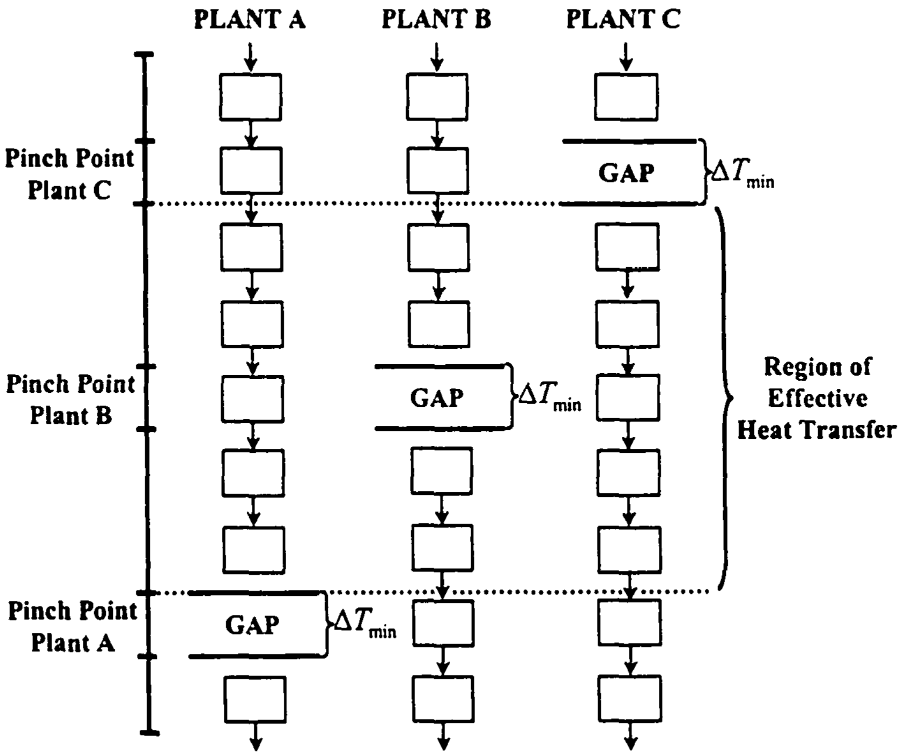

Figure 5.5. Shift of scales below the plant pinch temperatures

Consider the three-plant arrangement sorted by increasing pinch temperatures shown in Figure 5.5. The cold branches below the plant pinch temperatures zone are heated up and represent cold streams that receive heat from the hot streams located in this zone. In turn, the hot branches above plant pinch temperatures that are cooled down represent hot streams delivering heat to cold streams located in this zone. The hot scales of the zone of plants that deliver heat (i.e. below their pinch temperatures) and the cold scales of the zone of plants that receive heat (i.e. above their pinch temperatures) must coincide in order to establish the "heat belt." Therefore, the hot and cold scales below the pinch temperatures of the plants are shifted downward $\Delta T_{\min}$ degrees. The process of moving these scales down generates gaps as shown in Figure 5.5. The shift does not depend on the location of the plants relative to each other. The sequence can be altered, but the shift of the scales remains the same. That is, the scales of all the plants below their pinch temperatures are shifted downward.

Model **P5.1** is MINLP because of the presence of bilinear terms that include the product of a continuous variable times a binary variable. These terms are replaced by their equivalent set of linear inequalities, and an MILP model is obtained. Equations (5.6-5.10) render replacements similar to those employed in Chapter 2.

5.4 Heat Belt Solutions

As previously mentioned, for the particular case of three horizontally aligned plants, three locations of one plant relative to the others are possible. Consider the arrangement with the plants sorted with increasing pinch temperature location from left to right (1st case). The "heat belt" reduces to the one shown in Figure 5.6. Filled lines represent the temperature ranges for hot and cold branches in which heat transfer takes place. Therefore, the dotted parts of the circuit preserve their initial temperatures.

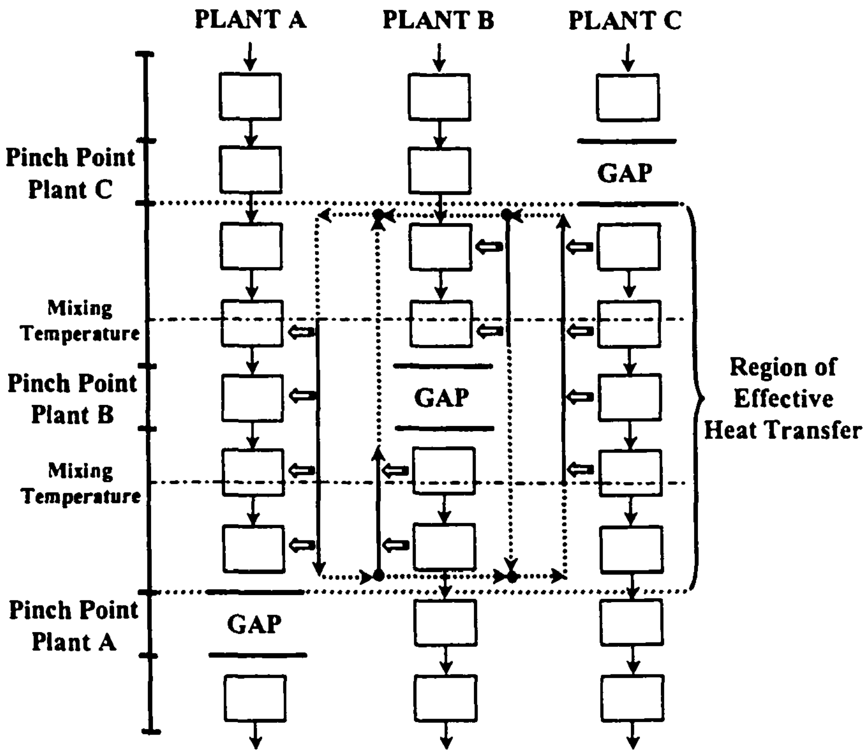

Figure 5.6. "Heat belt" for the 1st case

Now consider the arrangement in which the plant with the intermediate pinch temperature is located first on the left, followed by the plant with the lowest pinch temperature and the plant with the highest pinch temperature (2nd case). The "heat belt" for this case is presented in Figure 5.7. Notice that no cold branch is possible on the left side of the plant with the lowest pinch temperature.

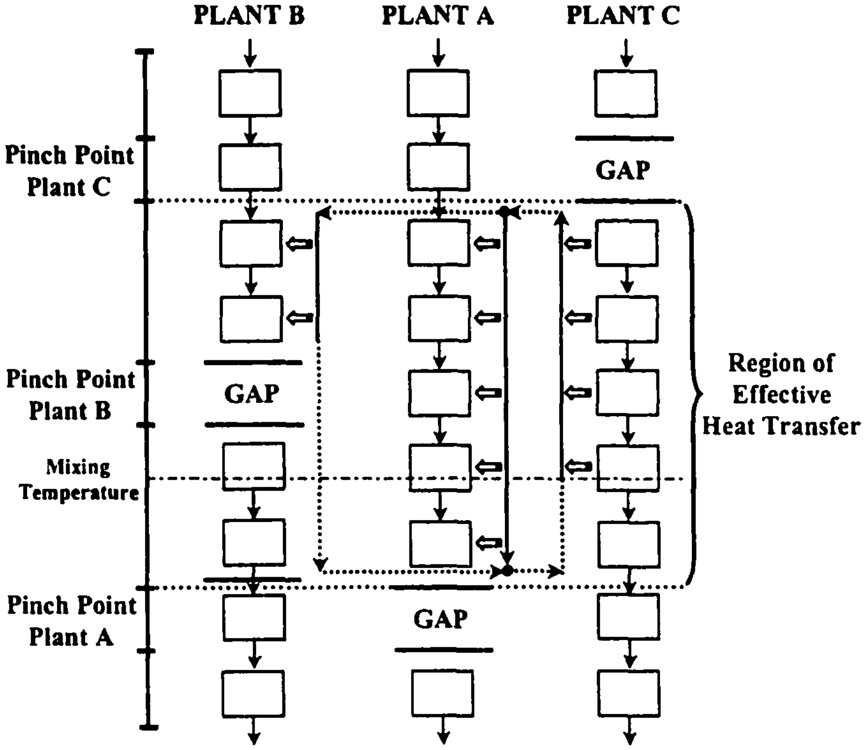

Figure 5.7. "Heat belt" for the 2nd case

In the last case (Figure 5.8), the sequence of Figure 5.6 is altered by locating the plant with the highest pinch temperature in the middle and the plant with the intermediate pinch temperature last in the sequence (3rd case). Because of its position, no hot branch is possible on the right side of the plant with the highest pinch temperature.

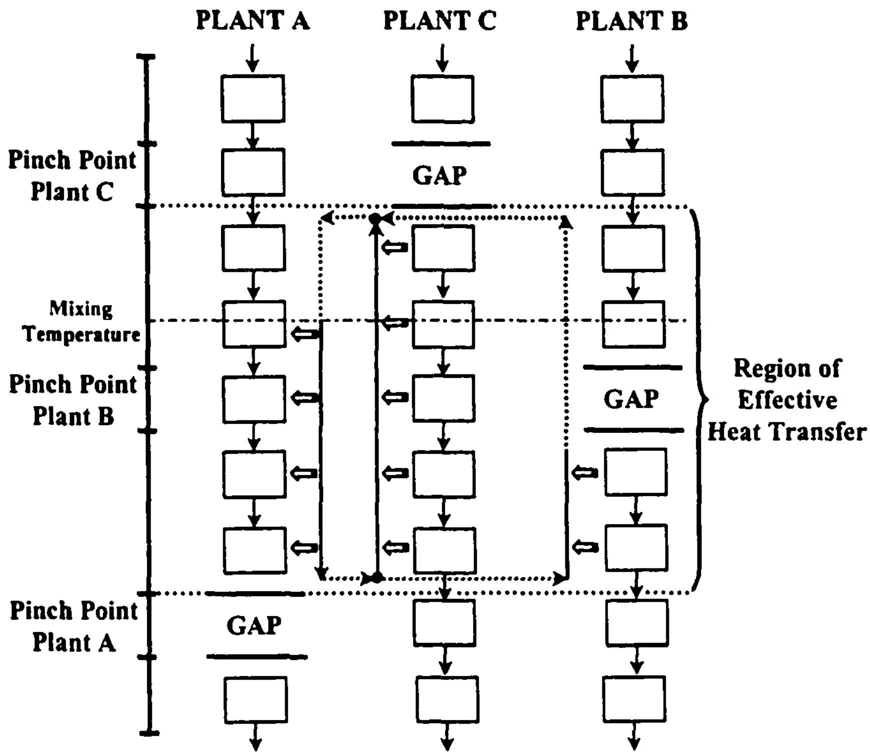

Figure 5.8. "Heat belt" for the 3rd case

5.5 Analysis using composite curves

In the analysis that follows, the heat cascade diagram previously presented to introduce the "heat belt" is compared with the traditional diagram that makes use of the composite curves of the processes. Adopting the hot streams scale of temperatures, consider the composite curves for three plants shown in Figure 5.9a. Notice the separation between the hot and cold composites that represent the gap required to make possible the indirect heat transfer between the plant and the "heat belt." These curves correspond to the 1st case arrangement (increasing pinch temperatures order). The filled lines represent the independent circuits that are obtained when only transference between pinches is allowed and the branches of the "heat belt" that bypass plant 2 are eliminated. Ahmad and Hui (1991) showed a similar diagram for the two plants case. They explained that when the heat transfer is not restricted to a constant temperature (e.g. steam as in Dhole and Linnhoff, 1992), but has a sloping profile (i.e., the intermediate fluid of the 'heat belt"), transference from the cold composite of one plant to the hot composite of the other could extend until the pinch of the latter is reached. The argument is clearly demonstrated if the composite of the heat receiving process is inverted (Ahmad and Hui, 1991).

When the general "heat belt" is considered a diagram like the one shown in Figure 5.9b is obtained. Now the heat transfer from the cold composite of plant 3 to the "heat belt" extends beyond the pinch point of plant 2. The splitter located before the heat-belt hot branch starts delivering heat to plant 2 (splitter 1), generates a filled line representing this heat transfer from the "heat belt" to plant 2 and a branch bypassing plant 2. This branch is mixed with the heat-belt cold branch receiving heat from plant 2 (mixer 1). The mixed stream (filled line) represents the heat-belt hot branch delivering heat to plant 1. This branch is then split in the heat-belt cold branch receiving heat from plant 2 and the stream bypassing plant 2 (splitter 2). The latter stream finally mixes with the heat-belt hot stream delivering heat to plant 2 to generate the heat-belt cold stream receiving heat form plant 3.

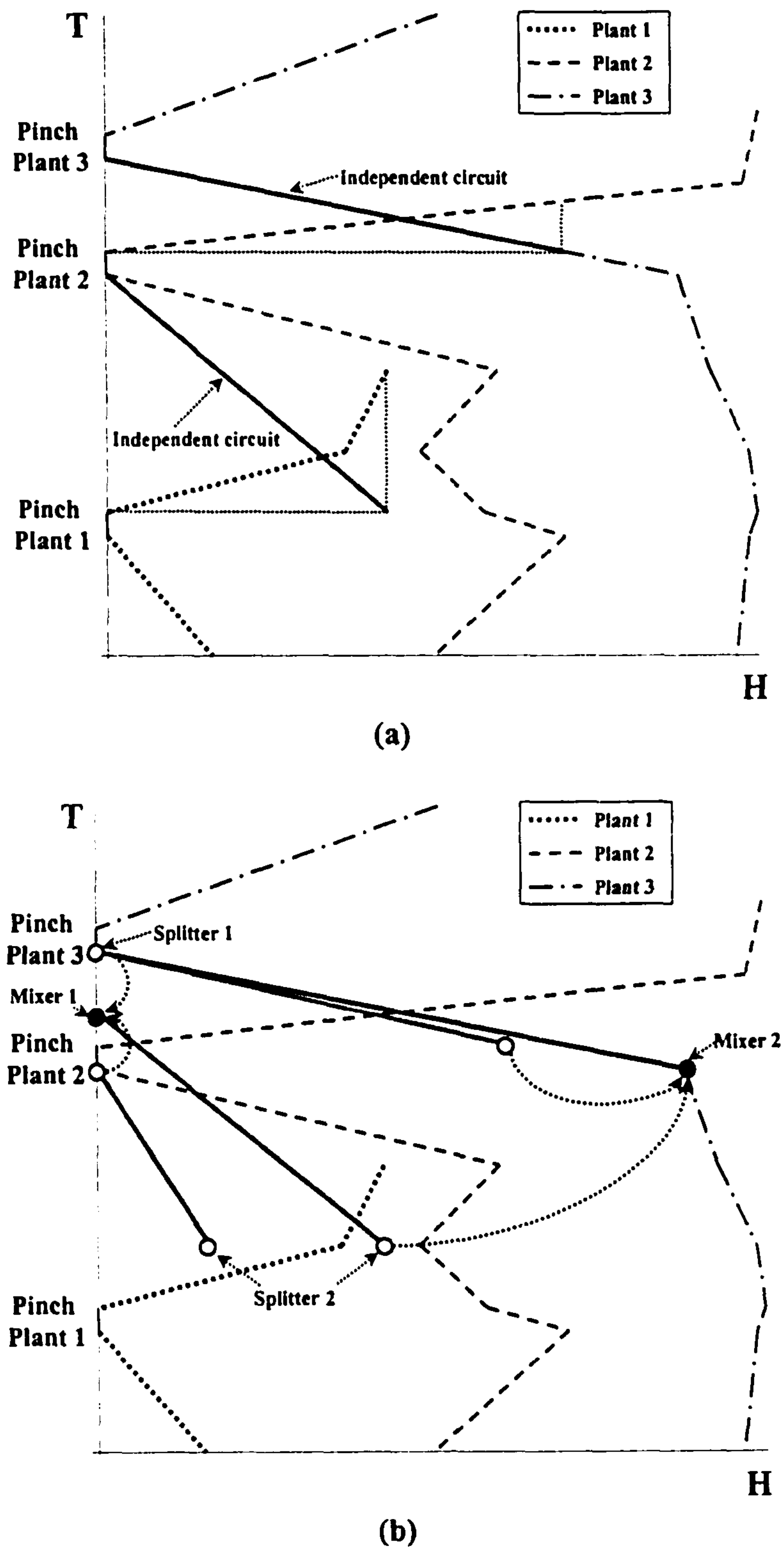

Figure 5.9. Composite curves for the 1st case

5.6 Heat Belt Examples

The concepts developed in the previous sections are now illustrated by considering examples of "total sites" composed by three plants.

5.6.1 Example 5.1

In this example Test Case #2 from Linnhoff & Hindmarsh (1983), example 1 from Ciric and Floudas (1991), and problem 4sp1 are considered for indirect heat integration using a "heat belt". The example was used in Chapter 4 to show indirect integration across a set of three plants. The results of applying individual pinch analysis to each of the plants are shown in Table 5.1.

Table 5.1. Individual plant pinch analysis for Example 5.1

Problem	Pinch Temperature (°C)	Minimum Heating Utility (kW)	Minimum Cooling Utility (kW)
Test Case #2	90	107.5	40.0
Ciric & Floudas (1991)	200	600.0	2100.0
4sp1	249	128.0	250.0

First, the plants are arranged in order of increasing pinch temperature (1st case) and model **P5.1** is solved. Figure 5.10a shows the solution that transfers the maximum possible heat. As the flowrates bypassing the plant with the intermediate pinch temperature are zero, this solution is equivalent to the one obtained in Chapter 4 using two independent circuits.

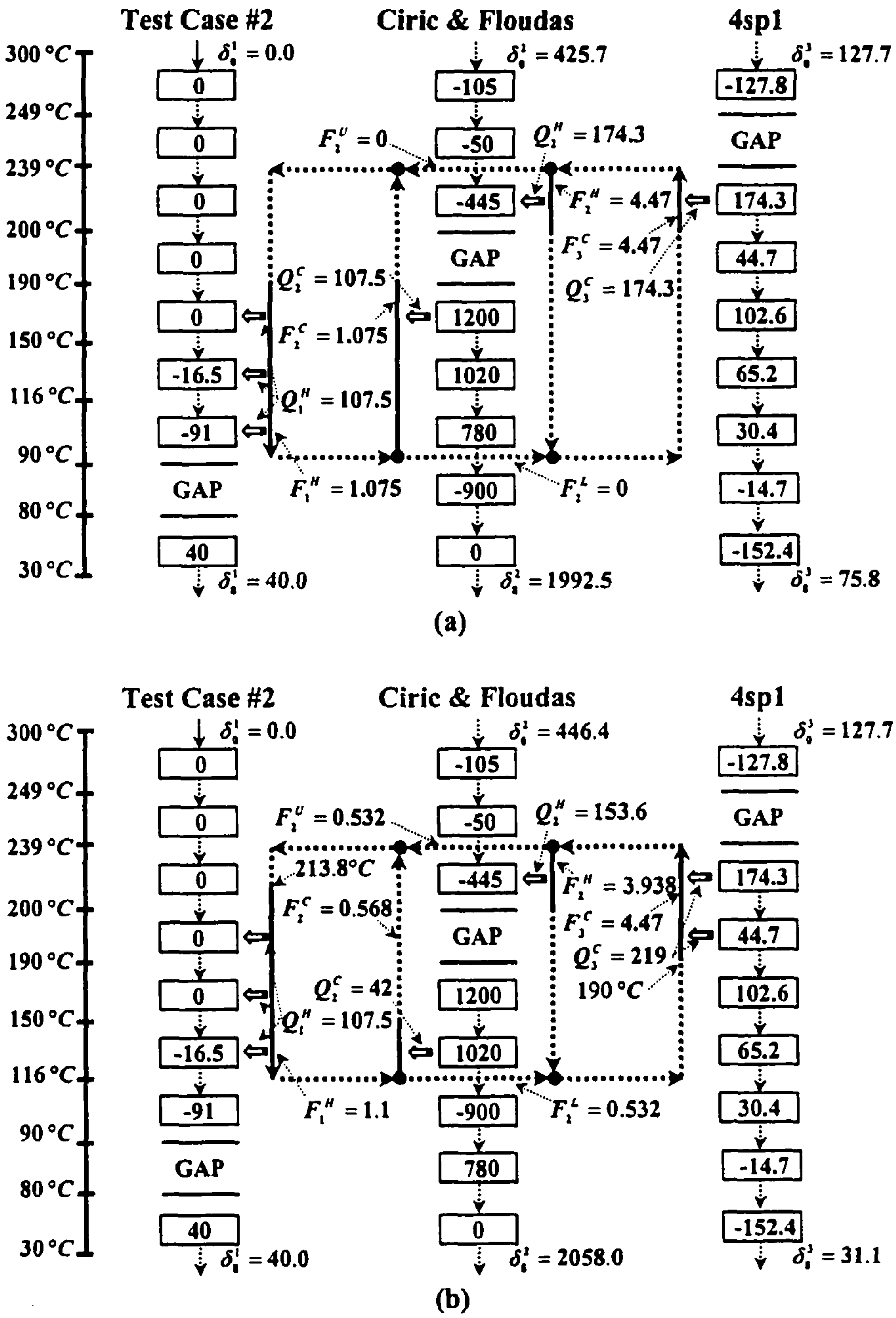

Figure 5.10. Solutions for the 1st case

If some flow is forced to bypass the plant in the middle (e.g. $F_2^U = 0.5$), a new solution is obtained. As Figure 5.10b shows, there is a decrease in the total savings. This is due to the mixing required to establish the "heat belt." Notice the structural difference between this "heat belt" and the multi-operation circuit presented in Chapter 4. The belt in Figure 5.10b is also capable of obtaining savings if plant 2 is shut down. The presence of the bypass streams not used when the system is fully integrated makes this possible.

The case in which the plant with the intermediate pinch temperature is located first in the sequence is now considered (2nd case). The solution of model **P5.1** is presented in Figure 5.11.

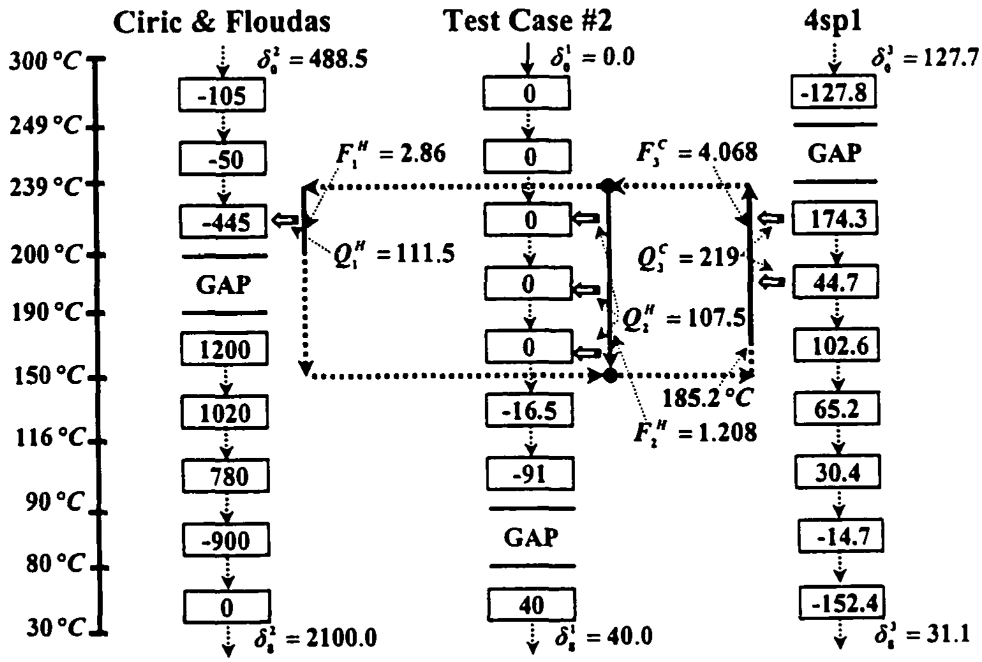

Figure 5.11. Solution for the 2nd case

Maximum total savings with this array are lower than in the previous one (increasing pinch temperatures order), because the plant delivering the heat to the "heat belt" (4sp1) is limited in the amount of heat it can transfer (cooling utility amount). Consequently, no cooling utility is required for this plant when the system is fully integrated.

Finally, the case in which the plant with the intermediate pinch temperature is located last in the sequence is considered. The solution to model **P5.1** establishes a single circuit between the plant with the highest pinch temperature (4sp1) and the plant with the lowest pinch temperature (Test Case #2). The reason for this single circuit is that no heating utility savings can be attained in the plant with the intermediate pinch temperature (Ciric and Floudas).

5.6.2 Example 5.2

This example was introduced by Ahmad and Hui (1991) to show the application of their procedure to find required heat flows between what they called "areas of integrity." Table 5.2 shows the results of independently applying pinch analysis to each of the areas.

Table 5.2. Individual plant pinch analysis for Example 5.2

Problem	Pinch Temperature (°C)	Minimum Heating Utility (kW)	Minimum Cooling Utility (kW)
Area A	70	43	12.75
Area B	500	1.5	19.35
Area C	210	25	36.5

The presence of an assisted heat integration case is detected by the solution of the targeting model for energy savings applied to indirect integration (i.e., applying the methods presented in Chapter 2). This model considers independent transfer of heat within each interval and requires simultaneous parallel and opposite heat transfer between the areas. Because model **P5.1** only considers effective heat, lower savings are expected. Moreover, limitations in the amount that can be transferred arise when a single pipe is used to collect or deliver heat to the plants.

When model **P5.1** is solved for the case in which the plants are arranged in order of increasing pinch temperature (1^{st} case), the solution shown in Figure 5.12 is obtained. Similar to the previous example, the flowrates bypassing the plant with the intermediate pinch temperature are zero, and this solution is equivalent to the use of two independent circuits. The presence of the bypass streams is useful for the case in which area C is shut down.

In the 2^{nd} case the plant with the intermediate pinch temperature is located first in the sequence. Then the solution to model **P5.1** establishes a single circuit between the plant with the highest pinch temperature (area B) and the plant with the intermediate pinch temperature (area C). Energy savings are limited in this case to the amount that area B can transfer.

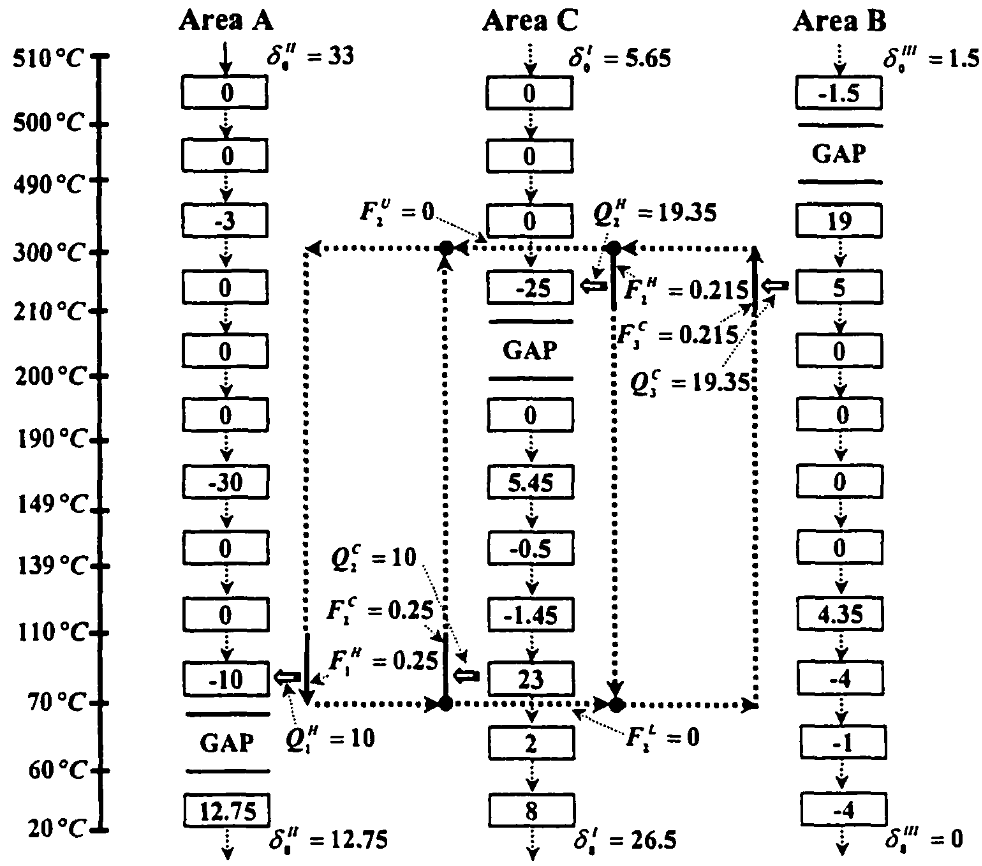

Figure 5.12. Solution for the 1st case

The final array locates the plant with the intermediate pinch temperature last in the sequence (3rd case). For this case, the solution to model **P5.1** is shown in Figure 5.13. Notice that mixing and the structural restrictions, imposed by the location of the "heat belt," reduce the energy savings when this solution is compared with the solution shown in Figure 5.12.

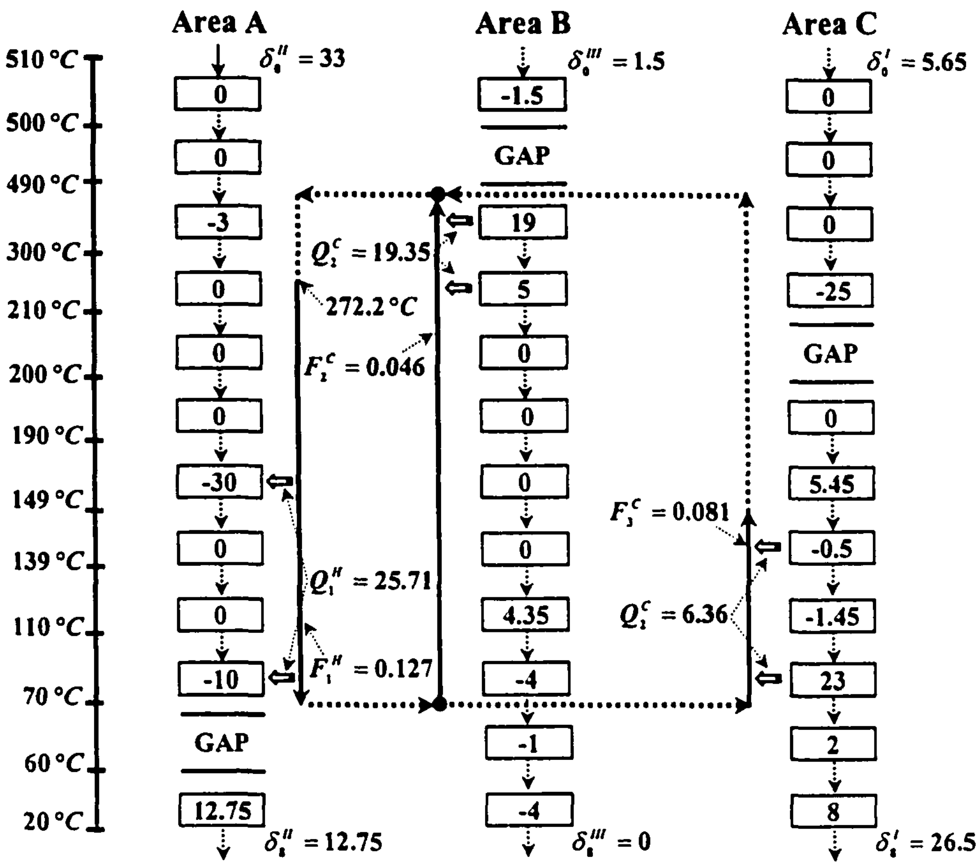

Figure 5.13. Solution for the 3rd case

5.7 Alternative Heat Belt Representations

Consider the location of the plants for Example 5.1 shown in Figure 5.14. Because the plant with the intermediate pinch temperature is located last in the sequence (3rd case), this plant cannot receive efficient heat. However, consider the inclusion of a hot branch for this plant and the possibility of this branch to transfer heat from left to right. The "heat belt" can now deliver heat that is collected from the with the intermediate pinch temperature if the direction of the lines going from this plant to the plant with the highest pinch temperature is reversed. The same savings are obtained as the result shown in Figure 5.11. It is not possible to obtain this solution by using model **P5.1,** as it represents the instance of another type of "heat belt." However, model **P5.1** first can be solved using the arrangement of Figure 5.11, and then the solution can be used to establish the circuit of Figure 5.14.

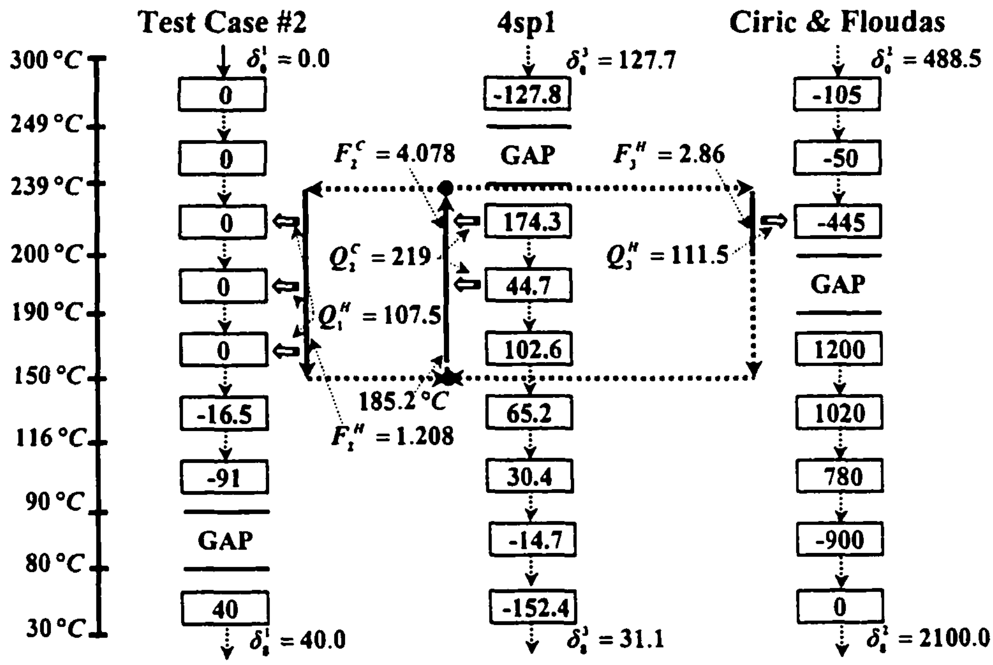

Figure 5.13. Alternative "heat belt" for the 3rd case array

5.8 Conclusions

The new concept of the "heat belt" that indirectly integrates a system of *n* plants by using an intermediate fluid was introduced. Some characteristics of this circuit were discussed. An MINLP for the special case of three plants was formulated. Replacement of binary-continuous bilinear terms by a linear set of equations is possible. The resulting MILP problem is therefore solved. Examples that show the utility of the "heat belt" for different relative locations of the three-plant system were presented.

5.9 Nomenclature

F_{jk} = product of heat capacity and flowrate for the intermediate fluid going from plant j to plant k (kW/°C)

F_j^C = product of heat capacity and flowrate for the heat-belt cold branch receiving heat from plant j (kW/°C)

F_j^H = product of heat capacity and flowrate for the heat-belt hot branch delivering heat to plant j (kW/°C)

F_j^L = product of heat capacity and flowrate for the lower heat-belt main line bypassing plant it (kW/°C)

F_j^{LT} = product of heat capacity and flowrate for the lower heat-belt main line going from plant j to plant k (kW/°C)

F_j^U = product of heat capacity and flowrate for the upper heat-belt main line bypassing plant j (kW/°C)

F_{jk}^{UT} = product of heat capacity and flowrate for the upper heat-belt main line going from plant j to plant k (kW/°C)

i = temperature interval

j, k = chemical plant

m = total number of temperature intervals

n = total number of chemical plants

P_j = chemical plant

Q_{jk} = total heat transferred from plant j to plant k via the intermediate fluid (kW)

Q_j^C = total heat transferred from plant j to the heat-belt cold branch (kW)

Q_j^H = total heat transferred from the heat-belt hot branch to plant j (kW)

$q_{i,j}^C$ = heat transferred from plant j to the heat-belt cold branch in interval i (kW)

$q_{i,j}^H$ = heat transferred from the heat-belt hot branch to plant j in interval i (kW)

T_i = lower interval temperature (°C)

U_{ij}^C = upper bound in the amount of heat transferred from plant j to the heat-belt cold branch in interval i (kW)

U_{ij}^{H} = upper bound in the amount of heat transferred from the heat-belt hot branch to plant j in interval i (kW)

X_{j}^{C} = enthalpy of heat-belt cold branch before receiving heat from plant j (kW)

X_{j}^{H} = enthalpy of heat-belt hot branch before delivering heat to plant j (kW)

Y_{ij}^{LC} = binary variable that establishes the lower limit temperature interval i for the heat-belt cold branch delivering heat to plant j

Y_{ij}^{LH} = binary variable that establishes the lower limit temperature interval i for the heat-belt hot branch delivering heat to plant j

Y_{ij}^{UC} = binary variable that establishes the upper limit temperature interval i for the heat-belt cold branch delivering heat to plant j

Y_{ij}^{UH} = binary variable that establishes the upper limit temperature interval i for the heat-belt hot branch delivering heat to plant j

Z_{ij}^{C} = continuous variable that determines whether or not there is heat transfer in temperature interval i for the heat-belt hot branch delivering heat to plant j

Z_{ij}^{H} = continuous variable that determines whether or not there is heat transfer in temperature interval i for the heat-belt hot branch delivering heat to plant j

ΔT_i = interval temperature difference (°C)

δ_0^j = minimum surplus to the first interval of plant j (kW)

$\hat{\delta}_0$ = original minimum surplus to the first interval of plant j (kW)

δ_i^j = heat cascaded from interval i in plant j (kW)

5.10 References

Ahmad S. and C. W. Hui, "Heat Recovery between Areas of Integrity," *Comput. Chem. Engng.*, **12**, 809 (1991).

Ciric A. R. and C. A. Floudas, "Heat Exchanger Network Synthesis without Decomposition," *Comput. Chem. Engng.*, **15**, 385 (1991).

Linhoff B. and E. Hindmarsh, "The Pinch Design Method for Heat Exchanger Networks," *Chem. Engng Sci.*, **38**, 745 (1983).

CHAPTER 6

Summary

6.1 Introduction

Some of the issues related to integration across plants have been discuss throughout this thesis. The focus was primarily on heat integration and the two ways in which it can be accomplish, i.e., directly by using process streams or indirectly by using and intermediate-fluid circuit. Chapter 2 concentrated in the particular case of two plants in order to gain insights of the problem and develop basic mathematical programming models. Energy-saving targets were obtained as a solution to these models, including the location of the intermediate-fluid circuits for the case of indirect integration. Based on these targets, Chapter 3 focused on the design of multipurpose heat exchanger networks capable of operating each plant stand-alone as well as both plants integrated.

Then the models developed for the two-plants case were extended to a "total site" composed of *n* plants in Chapter 4. The resulting targets determine maximum energy savings, minimum number of connections, and location of independent circuits between plant pairs. In addition, multi-operation circuits for flexibility of operation were included, and they motivated the introduction of the "heat belt" concept. This new concept was the topic of Chapter 5.

Planning for the implementation of energy savings in the "total site" to make capital investment profitable is a natural extension of the results presented in this thesis. Some insights into the models that can be applied to solve this problem are presented next.

6.2 Planning for Energy Savings in the "Total Site"

In the case of integration in the "Total Site," the aim of planning is to select the optimal policy for the implementation of energy savings by heat integration across a set of plants given forecasts of the value of the energy savings and the costs of additional investment and operation of the heat exchanger network. The analysis is performed over a long-range planning horizon that consists of a finite number of time periods during which utility prices, capital and operating cost, as well as the available investment budget can vary. The problem is then to determine the optimal net present value over this time horizon by considering the time and the size of heat-exchange capacity expansions to achieve maximum energy savings.

6.2.1 Net Present Value

The net present value (NPV) gives a measure of the return after a project has generated sufficient income to repay, among other things, the original investment and any interest charges that the invested money would otherwise have brought into the company. In the case of planning for integration in the "total site", the net present value takes into account the value of the energy savings attained because of the heat integration across plants, the expenses incurred to purchase the additional equipment, and the increase on the operating costs. The present value P of a future sum of money F is calculated by the formula:

$$P_t = F \cdot d_t \tag{6.1}$$

where d_t is the discount factor given by:

$$d_t = \frac{1}{(1+r)^t} \tag{6.2}$$

For the set of plants considered for integration, we also have to account for a finite total number of time periods T during which the value of the energy savings and the additional investment and additional operating costs can vary. Therefore, the objective function to be maximized is the net present value NPV of the project over the specified horizon consisting of T time periods. For the simplified mathematical model presented in this chapter, the objective function is the maximization of the net present value of the project that in this case is defined as:

$$NPV = \sum_{t=1}^{T} d_t \sum_{j \in P} \left(c_{jt} S_t^j - IC_{jt} \right) \tag{6.3}$$

where d_t is the discount factor at period t, c_{jt} is the heating utility cost, IC_{jt} is the investment cost and S_t^j the savings in plant j at the time period t. Total-site energy savings are defined by the difference between the original heating utility of a plant when no integration among plants is assumed and the heating utility when integration is performed, i.e.:

$$S_t^j = \hat{\delta}_o^j - \delta_{o,t}^j \quad \forall j \in P \tag{6.4}$$

For a heat exchanger network, a relationship can be established between the installed cost and the area required to transfer a certain heat load. Therefore, the investment cost IC_{jt}, can be expressed as:

$$IC_{jt} = \alpha_{jt} A_{jt} + \beta_{jt} Z_{jt} \tag{6.5}$$

where α_{jt} and β_{jt} are the variable and fixed investment costs; A_{jt} represents the heat transfer area for plant j at time period t and Z_{jt} is an integer variable that accounts for the number of expansions realized for plant j at period t.

As the amount of capital available for investment may be limited, the following constraint is used to restrict the capital investment during period t.

$$\sum_{j \in P} d_F^t \left(\alpha_{jt} \cdot A_{jt} + \beta_{jt} \cdot Z_{jt} \right) \leq INV(t) \tag{6.6}$$

6.2.2 Heat Transfer Constraints

In this section, the constraints of the targeting model for direct heat integration of n plants introduced in Chapter 4 are extended to consider diagonal heat transfer. This extended heat transfer allows the calculation of not only a target for the utility savings, but also a target for the network total area. Diagonal heat transfer between two plants is illustrated in Figure 6.1.

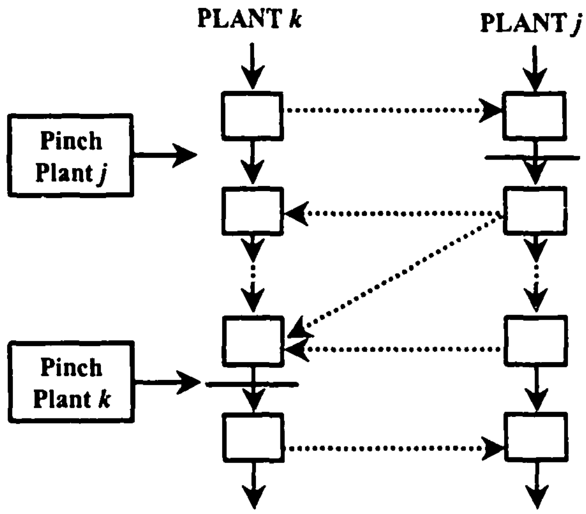

Figure 6.1. Horizontal and diagonal heat transfer

Next, the heat balances for each heat transfer zone are presented. The nomenclature for sets and variables was introduced in Chapter 4. An additional subscript accounting for different time periods is added to all variables.

To calculate the heating utility for every plant j, we have to take into account the effective heat supplied from every plant $k \in S_j^E$, the heat received from assisted plants $k \in S_j^A$ and the heat supplied to assisting plants $k \in R_j^A$. All these heat balances that include diagonal transfer are expressed as:

$$
\left.
\begin{aligned}
\delta^{j}_{o,t} &= \hat{\delta}^{j}_{o} + \sum_{k \in R^{A}_{j}} \sum_{i=1}^{\min(p^{j},p^{k})} \sum_{s=i}^{p^{k}} q^{A}_{ijskt} - \sum_{k \in S^{A}_{j}} \sum_{i=1}^{p^{j}} \sum_{s=1}^{\min(i,p^{k})} q^{A}_{skijt} - \sum_{k \in S^{E}_{j}} \sum_{i=p^{k}+1}^{p^{j}} \sum_{s=i}^{p^{j}} q^{E}_{iksjt} \\
\delta^{j}_{i,t} &= \delta^{j}_{i-1,t} + q^{j}_{i} + \sum_{k \in S^{A}_{ij}} \sum_{s=1}^{i} q^{A}_{skijt} - \sum_{k \in R^{A}_{ij}} \sum_{s=i}^{p^{k}} q^{A}_{ijskt} \quad \forall i = 1,\dots,p^{n} \\
\delta^{j}_{i,t} &= \delta^{j}_{i-1,t} + q^{j}_{i} + \sum_{k \in S^{E}_{ij}} \sum_{s=p^{k}+1}^{i} q^{E}_{skijt} + \sum_{k \in S^{A}_{ij}} \sum_{s=1}^{\min(i,p^{k})} q^{A}_{skijt} \\
&\qquad - \sum_{k \in R^{A}_{ij}} \sum_{s=i}^{p^{k}} q^{A}_{ijskt} \quad \forall i = p^{n}+1,\dots,p^{j} \\
\delta^{j}_{i,t} &= \delta^{j}_{i-1,t} + q^{j}_{i} + \sum_{k \in S^{B}_{ij}} \sum_{s=p^{k}+1}^{i} q^{B}_{skijt} - \sum_{k \in R^{B}_{ij}} \sum_{s=\max(i,p^{k}+1)}^{m} q^{B}_{ijskt} \\
&\qquad - \sum_{k \in R^{E}_{ij}} \sum_{s=i}^{p^{k}} q^{E}_{ijskt} \quad \forall i = p^{j}+1,\dots,p^{l} \\
\delta^{j}_{i,t} &= \delta^{j}_{i-1,t} + q^{j}_{i} + \sum_{k \in S^{B}_{ij}} \sum_{s=p^{k}+1}^{i} q^{B}_{skijt} - \sum_{k \in R^{B}_{ij}} \sum_{s=i}^{m} q^{B}_{ijskt} \quad \forall i = p^{l}+1,\dots,n
\end{aligned}
\right\}
\begin{array}{c} \forall j \in P \\ \forall t \end{array}
\tag{6.7}
$$

The relationships between the amount of heat transferred at one time period and the following are given next. The variable h_{ijskt} represents the expansion in the heat transferred at period t+1 when compared to period t.

$$
\left.
\begin{array}{l}
\left.\begin{aligned} q^{A}_{ijsk(t+1)} &= q^{A}_{ijskt} + h^{A}_{ijsk(t+1)} \\ q^{A}_{ijsk1} &= h^{A}_{ijsk1} \end{aligned}\right\} \quad i = 1,\dots,\min(p^{j},p^{k}); s = i,\dots,p^{k} \\
\left.\begin{aligned} q^{E}_{ijsk(t+1)} &= q^{E}_{ijskt} + h^{E}_{ijsk(t+1)} \\ q^{E}_{ijsk1} &= h^{E}_{ijsk1} \end{aligned}\right\} \quad i = p^{j}+1,\dots,\max(p^{j},p^{k}); s = i,\dots,p^{k} \\
\left.\begin{aligned} q^{B}_{ijsk(t+1)} &= q^{B}_{ijskt} + h^{B}_{ijsk(t+1)} \\ q^{B}_{ijsk1} &= h^{B}_{ijsk1} \end{aligned}\right\} \quad i = p^{j}+1,\dots,n; s = \max(i,p^{k}),\dots,n
\end{array}
\right\}
\begin{array}{c} \forall j,k \in P \\ \forall t \end{array}
\tag{6.8}
$$

Now binary variables Y^A_{jkt}, Y^E_{jkt}, and Y^B_{jkt} are defined to indicate the occurrence of the expansions of heat transferred from plant j to plant k at time period t within the respective regions, the constraints that apply are:

$$\left.\begin{aligned} Y^A_{jkt}\cdot h^{AL}_{jkt} &\le \sum_{i=1}^{\min(p^j,p^k)}\ \sum_{s=i}^{\min(p^j,p^k)} h^A_{ijskt} \le Y^A_{jkt}\cdot h^{AL}_{jkt} \\ Y^E_{jkt}\cdot h^{EL}_{jkt} &\le \sum_{i=p^j+1}^{p^k}\ \sum_{s=i}^{p^k} h^E_{ijskt} \le Y^E_{jkt}\cdot h^{EL}_{jkt} \\ Y^B_{jkt}\cdot h^{BL}_{jkt} &\le \sum_{i=\max(p^j+1,p^k+1)}^{m}\ \sum_{s=i}^{m} h^B_{ijskt} \le Y^B_{jkt}\cdot h^{BL}_{jkt} \end{aligned}\right\} \begin{aligned} &j,k\in P \\ &j\ne k \\ &t=1\ldots T \end{aligned} \qquad (6.9)$$

In these equations, lower and upper bounds for the expansions are considered. A zero-value of the binary variables forces the expansion at period t to zero. If the binary variable is equal to one, the expansion is performed.

We work in the region where it is possible to linearize the IC dependence on A. Therefore, Z_{jt} are integer variables given by:

$$Z_{jt} = \sum_{k\in R^A_j} Y^A_{jkt} + \sum_{k\in R^E_j} Y^E_{jkt} + \sum_{k\in R^B_j} Y^B_{jkt} \qquad (6.10)$$

6.2.3 Total Area Targeting

In the problem formulation, it is assumed a constant overall heat transfer coefficient, U. Therefore, for a transfer between interval i and interval s, the following equations apply:

$$\left.\begin{aligned}
A_{jkt}^{A} &= \sum_{i=1}^{\min(p^j,p^k)} \sum_{s=i}^{\min(p^j,p^k)} \frac{h_{ijskt}^{A}}{U\left(T_i - T_s + \Delta T_{\min}\right)} \\
A_{jkt}^{E} &= \sum_{i=p^j+1}^{p^k} \sum_{s=i}^{p^k} \frac{h_{ijskt}^{E}}{U\left(T_i - T_s + \Delta T_{\min}\right)} \\
A_{jkt}^{B} &= \sum_{\max(p^j+1,p^k+1)}^{m} \sum_{s=i}^{m} \frac{h_{ijskt}^{B}}{U\left(T_i - T_s + \Delta T_{\min}\right)}
\end{aligned}\right\} \begin{array}{c} j,k \in P \\ j \neq k \\ t = 1,\dots,T \end{array} \qquad (6.11)$$

Finally, additional equations are introduced to avoid area of transfer too small and to account for the total network area:

$$\left.\begin{aligned}
A_{jkt}^{A} &\geq A_{MIN}^{A} \cdot Y_{jkt}^{A} \\
A_{jkt}^{E} &\geq A_{MIN}^{E} \cdot Y_{jkt}^{E} \\
A_{jkt}^{B} &\geq A_{MIN}^{B} \cdot Y_{jkt}^{B}
\end{aligned}\right\} \begin{array}{c} j,k \in P \\ j \neq k \\ t = 1,\dots,T \end{array} \qquad (6.12)$$

$$A_{jt} = \sum_{k \in P} \left(A_{jkt}^{A} + A_{jkt}^{E} + A_{jkt}^{B} \right) \quad j \in P; j \neq k; t = 1,\dots,T \qquad (6.13)$$

APPENDIX A

Generation Procedure for the Set of Intervals Used in Indirect Heat Integration

In this appendix, we present a procedure to obtain the generalized set of intervals used for indirect heat integration when solving models **P4.4** to **P4.7**. In Chapter 2 the transfer of heat between plants was performed between intervals at the same temperature level. Such a thing can be accomplished by a special shifting of scales. This shifting cannot be performed when many plants are considered. Therefore, in Chapter 4 the transfer between plants is modeled as an upward and downward diagonal transfer between equal size intervals that are a fixed number of intervals apart. We now present a procedure to obtain equal intervals.

Consider the temperature intervals within the region between pinches for the case of two plants. A simple shift of the scales of plant 2 downward by ΔT_{min} degrees guarantees that hot streams of plant 2 are at the same temperature than cold streams of plant 1, and the use of an intermediate fluid is possible. For assisted cases, heat is transfer in the opposite direction to the effective heat transfer and in the regions above and below both pinches. This requires two additional scale shifts and two gaps are generated (Chapter 2). Figure A1a shows the final arrangement that allows horizontal heat transfer. To avoid conflicts for the case of more than two plants, this procedure is replaced in the region between pinches by diagonal transfer from an interval in plant 2 to an interval in plant 1 located ΔT_{min} degrees below. In the other regions, diagonal transfer results from an interval in plant 1 to an interval in plant 2 located ΔT_{min} degrees below. Figure A1b shows the diagonal transfer in all the regions. Note that corresponding intervals of equal size have to exist in both plants to make possible the heat transfer.

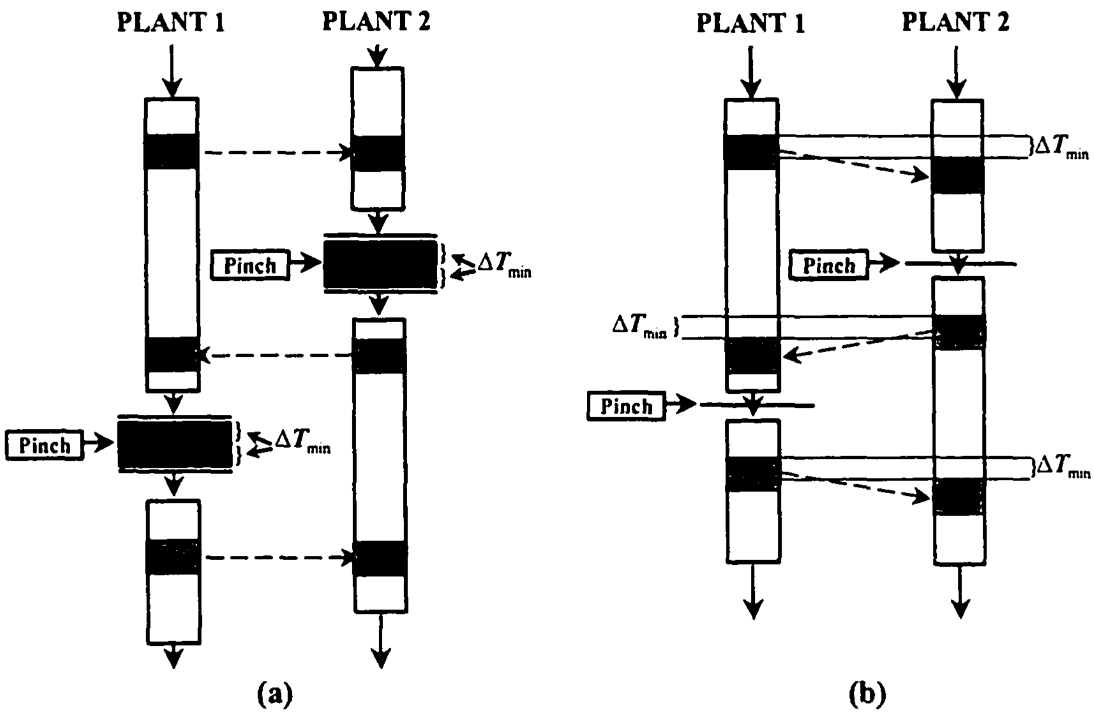

Figure A1. Horizontal and diagonal heat transfer

Starting from the uppermost temperature, new interval boundaries are generated by subtracting from each of the existing temperatures increasing number of fixed temperature differences (ΔT_{min}). All the sequences are terminated at the closest temperature above the minimum existing temperature. The same thing is done starting from the lowermost temperature. In this last case, all the sequences are terminated at the closest temperature below the maximum existing temperature. The procedure is repeated for each original interval temperature boundary. Finally, all temperatures are sorted and the procedure ends. The reader can simply verify that the procedure guarantees equal size temperature intervals located ΔT_{min} degrees one from the other in both upward and downward directions.

APPENDIX B

Generation Procedure for the Set of Intervals Used in Indirect Heat Integration (decouple circuit equations)

In this appendix, we present a procedure to obtain the generalized set of intervals used for indirect heat integration when the equations for the circuits are decupled, as shown in equations (4.11). Consider for simplicity that only effective heat transfer is present. A similar procedure is conducted when assisting heat integration is required. The original intervals in each plant constructed by using the starting temperatures of only streams belonging to each plant are shown in Figure B1a. The procedure consists of the further partition of the individual set of intervals in order to consider all the temperatures at which a circuit can start or end. From the point of view of a receiver plant, the circuit represents a hot stream delivering heat to its intervals. In addition to its original interval partition temperatures, a circuit can start or end at a supplier interval-partition temperature shifted ΔT_{min} degrees downward. This shift considers the fact that the circuit represents a cold stream for the supplier. Therefore, the temperature originally in the cold scale of the supplier will be now in the hot scale of the receiver. An equivalent analysis can be done from the point of view of the supplier where a circuit can start or end at a original interval partition or at a receiver interval partition temperature shifted ΔT_{min} degrees upward. Figure B1b shows the final interval partitions with the connecting dot lines representing possible starting/ending temperatures for the circuits.

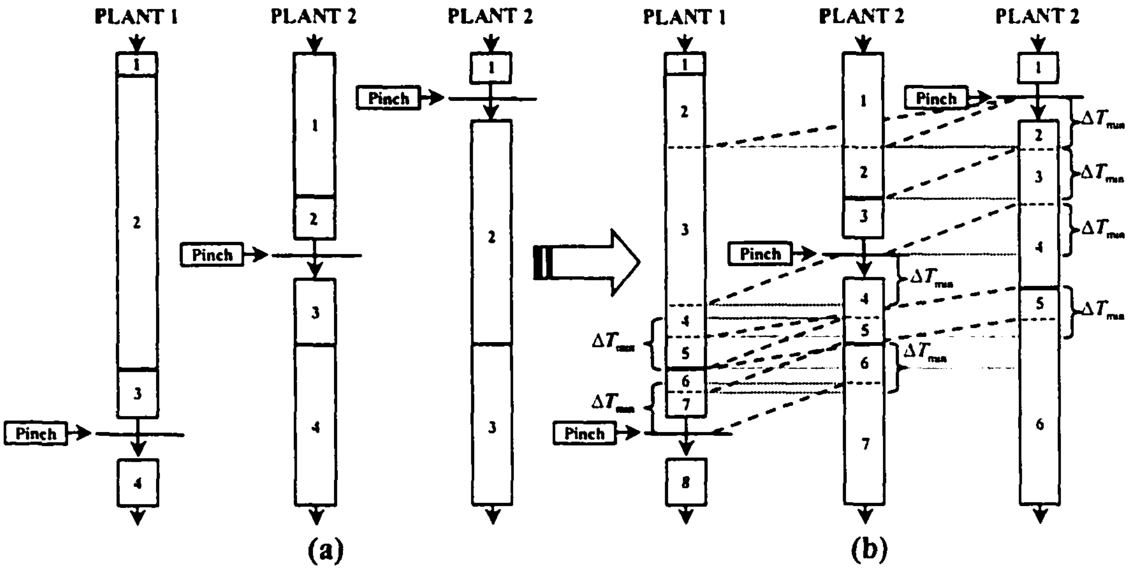

Figure B1. Generation of the set of intervals (decouple circuit equations)

This partitioning guarantees that all circuit temperatures will be included, but as Figure B1b shows, there is no direct relation between the interval number and the temperatures when a circuit is establish. Therefore, equation (4.13) is added to relate temperatures in different plants. A simplification of the partition procedure presented above, that produces a large number of intervals but has simpler implementation, is to directly consider the intervals used for direct integration and further partition them by consider temperatures shifted ΔT_{min} degrees upward and ΔT_{min} degrees downward. The resulting number of intervals for the simplified case will be at most three times the number of intervals used when direct integration is considered.

APPENDIX C

Generation Procedure for the Set of Intervals Used in Indirect Heat Integration (minimum number of connections)

In this appendix, we present a procedure to obtain a set of intervals for indirect heat integration using the solution of model **P4.5**. Figure C1a shows an example of three plants where effective heat integration from plant 2 to plant 1 and from plant 3 to plant 2 is present. Also assisting heat from plant 1 to plant 2 is required. With the use of this information, the procedure explained in Appendix B can is applied only to these regions. That is the original temperature intervals of each plant are further partitioned using temperatures of the receivers of suppliers only where heat transfer is predicted by model **P4.5**. The final interval partitions are shown in Figure C1b.

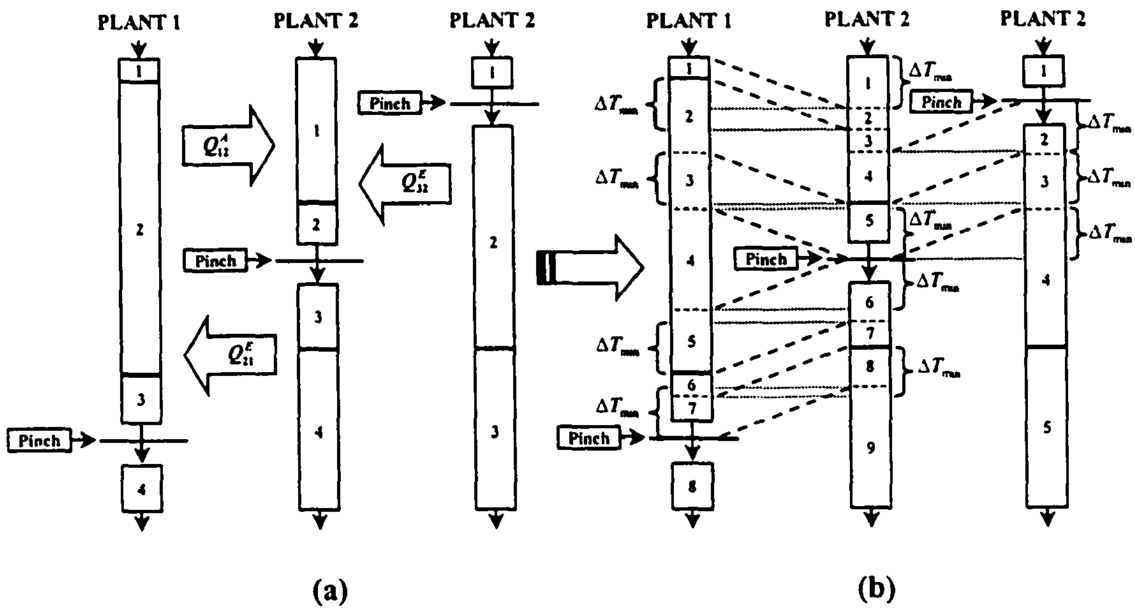

Figure C1. Generation of the set of intervals (minimum number of connections)