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A DISSERTATION APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

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Abstract

Disruptions are large-scale stochastic events that rarely happen but have a major effect on supply networks' topology. Some examples include: air traffic being suspended due to weather or terrorism, labor unions strike, sanctions imposed or lifted, company mergers, etc. Variations are small-scale stochastic events that frequently happen but only have a trivial effect on the efficiency of flow planning in supply networks. Some examples include: fluctuations in market demands (e.g. demand is always stochastic in competitive markets) and performance of production facilities (e.g. there is not any perfect production system in reality).

A fail-safe supply network is one that mitigates the impact of variations and disruptions and provides an acceptable level of service. This is achieved by keeping connectivity in its topology against disruptions (structurally fail-safe) and coordinating the flow through the facilities against variations (operationally fail-safe). In this talk, I will show that to have a structurally fail-safe supply network, its topology should be robust against disruptions by positioning mitigation strategies and be resilient in executing these strategies. Considering "*Flexibility*" as a risk mitigation strategy, I answer the question "*What are the best flexibility levels and flexibility speeds for facilities in structurally fail-safe supply networks*?" Also, I will show that to have an operationally fail-safe supply network, its flow dynamics should be reliable against demand- and supply-side variations. In the presence of these variations, I answer the question "*What is the most profitable flow dynamics throughout a supply network that is reliable against variations?"* The method is verified using data from an engine maker. Findings include: i) there is a tradeoff between robustness and resilience in profit-based supply networks;

ii) this tradeoff is more stable in larger supply networks with higher product supply quantities; and iii) supply networks with higher reliability in their flow planning require more flexibilities to be robust. Finally, I will touch upon possible extensions of the work into non-profit relief networks for disaster management.

Chapter 1: Frame of References – Forward and After-sales Supply Networks in Stochastic Environment

High competition in markets forced companies to focus more on their core competencies and work as a member of a supply network. However supply networks induce price reduction and quality increment in the companies (improve some of their competitive advantages), their decentralized nature makes service level preservation much more challenging (worsen some of their competitive advantages). Therefore, preserving appropriate service levels is necessary for supply networks. Service level of a supply network can be improved in two ways:

- by providing after-sales services for the customers, and
- by preserving a constant flow throughout its network from upstream to downstream against all uncertainties – having fail-safe supply networks.

Therefore, "***incorporating after-sales operations***" and "***having fail-safe networks***" against uncertainties are imperative for the success of supply networks. In this dissertation, we deal with the problem of improving service levels in the supply networks through "incorporating and coordinating after-sales services – Chapters 3 and 4 –" and "having fail-safe networks – Chapters 2 and 5 –". Chapter 1 is about the literature of these two topics to figure out the existing gaps and highlight the contributions of this dissertation.

1.1. Literature of after-sales services

1.1.1. Importance of after-sales services

In highly competitive markets, products manufactured by rivals become almost homogeneous from quality and price perspectives. In such markets, to differentiate from

rivals and to leverage competitive advantages, increasing number of companies try to provide better pre- and after-sales services for their customers (Tsay and Agrawal, 2000; Cachon and Harker, 2002; Bernstein and Federgruen, 2004; Davies, 2004; Penttinen and Palmer, 2007; Johnson and Mena, 2008; Bijvank et al., 2010). This marketing strategy has been called "servitization" in the literature (Vandermerwe and Rada, 1988). Product-service system (PSS) is introduced by Baines et al. (2007) as an especial case of the servitization. The servitization motivates customers to buy and stimulates demand. In competitive markets with homogeneous products (from quality and price facets), customers tend to buy from a rival providing better service commitment. To stimulate demand, service commitment must be guaranteed. To keep a brand reputation, the actual service experienced by customers in pre- and after-sales markets can be higher than the commitment but should never be lower.

The servitization is an important marketing strategy for most of the pioneer manufacturers. For example, Rolls-Royce supplies its jet engines to airlines under service commitments to repair and maintain them for many years (Davies et al., 2006). In high tech product markets, Lenovo provides after-sales maintenance services for the customers of its PCs (Li et al., 2014). Dell Company sells its laptops under a default hardware warranty that states "1 Yr Ltd Warranty, 1 Yr Mail-In Service, and 1 Yr Technical Support". However at the additional price of $119, customers are offered an optional 3 year warranty plan (dell.com, 2010). After-sales services are critical in the automobile industry. Hyundai Company offers a 5 year/60.000 mile bumper-to-bumper and 10 year/100,000 mile power train protection warranty for all of its automobiles sold in US. In the same industry, Nissan Company is offering 10 years/unlimited mileage warranty

for its cars (Nissan warranty information Booklet, 2011). Retailers of companies like General Motor, Volkswagen and Toyota provide 4S services (sale, spare parts, service and survey) for their customers (Li et al., 2014). The after-sales service is one of strategies used by manufacturers to assure customers of products quality. Hyundai Motor Company changed customers' perception by providing an extensive warranty. This warranty signaled customers that the quality of its cars had improved to match the very best in the industry (Business Week, 2004). Khajavi et al. (2013) and Vargo and Lusch (2004) believe that in today's markets, the focus of competition shifts from quality and price to delivery of value and the customer value requires having a high probability of having a working product.

In the past, the after-sales services were considered as a necessary cost generator but today this role has been changed and they are considered as a source of competitive advantages and business opportunity (Lele, 1997; Armistead and Clark, 1991 and 1992). The after-sales service is also considered an important income resource. The yearly income of after-sales markets of electronic devises, PCs, power tools and vacuum cleaners is USA is around $6 to $8 billion (Alexander et al., 2002). The Aberdeen Research Group (2005) estimated the market for spare parts management software to be more than $100 million in 2005 and it would be much greater in 2014. According to Gallagher et al. (2005), providing after-sales services by supplying spare parts for household appliances, automobiles, copy machines, heating and air conditioning, etc. is a huge business and today's worldwide market is worth more than $200 billion. In 2009 based on the data of the United States Logistics and Material Readiness Office, the US military spent $194 billion on the spare parts supply chain (SC) and logistics, with $104,

$70 and $20 billion related to supply, repair and transportation respectively. At the end of that year the value of the spare parts inventory was $94 billion.

The after-sales business is an important part of the economy and is almost twice as profitable as the original product business is. Based on the work of Dennis and Kambil (2003), $9 billion of GM's after-sales revenue generated $2 billion profit. This is much greater than GM's profit from $150 billion revenue from its car sales. On average, after-sales services contribute 25 percent of total revenue but generate more than 40 to 50 percent of total profit. It is commonly believed that spare parts constitute one third of total sale, but create two-third of profit (Suomala et al., 2002). In the European car markets, 40 to 50 percent of the total revenue is related to the after-sales services provided by companies. Gross profit of this income is much higher than the one resulting from new cars sales (Bohmann et al., 2003).

According to Anon (1999), each year almost $7 billion is paid to maintain Boeing planes. Fiat use TNT Post to handle its spare parts distribution in Europe and South America. TNT has 2000 employees and 3 million square feet of warehouse space, handles 120,000 tons of shipments and processes 34.6 million order lines a year on Fiat's behalf (Parket, 2002). The importance of the after-sales services is much more in the capital intensive industries such as aerospace, defense and industrial equipment manufacturers. For example, in the defense industry, only 28 percent of the system's total cost is related to its development and procurement and the rest (more than 72 percent of cost) is due to its operate and maintenance (GAO report 2003). The USA Department of Defense has a budget around $70 billion (in 2007) to operate and maintenance of its systems. That is

why there is a severe competition among its supporting industries in providing better after-sales services.

The after-sales service is also considered as "one of the few constant connections that customers have with a brand" (Gallagher et al., 2005) and its critical role in continuous improvement of product design and quality should not be ignored (Armistead and Clark, 1992; Thoben et al., 2001). After-sales services build long-term relationship with the customers in the most profitable way without any marketing effort. As highlighted by Alenxander et al. (2002), Goffin and New (2001) and Goffin (1999), after-sales activities act as a lever to improve the success possibility when new products are introduced.

On the other hand, to protect consumers' rights some governmental regulations force some of companies to provide warranty for their customers. Congress of the USA passed the Magnusson Moss Act and recently European Union passed new legislation requiring two-year warranty for all products.

Based on these numbers, we conclude that even a small improvement in the after-sales services of companies can lead to a significant gain in their profitability.

The after-sales service capacity is provided in two different ways:

i) In-house service: in-house service means a company itself provides the requirements (such as spare parts availabilities and repair and service capacities) to fulfill the after-sales service requests. This in-house capacity

should be ready before the after-sales service demand realization which is called "*prior service capacity*",

ii) Outsourcing after-sales services: outsourcing after-sales services is usually called "*service spot market*". In this case, service provision is done after demand realization (Kosnik et al., 2006).

Although the spot market is usually introduced as a hedge against service demand uncertainty, its cost and service capacity are inherently uncertain. That is why most of the companies with well-known brands prefer to use prior service capacity (in-house option). This option not only is more reliable but also helps them to keep their intellectual properties. These companies build suitable prior service capacity which maximizes their expected profit.

For these reasons, providing after-sales services is an unavoidable part of the daily operations in successful companies. The number of companies providing after-sales services for their customers and servicing after-sales markets is getting more every day. These companies have both after-sales and forward supply chains (SCs) / networks (SNs). While forward SCs / SNs deal with producing and supplying the original products to target pre-markets, after-sales SCs / SNs provide the required spare parts to fulfill after-sales commitments. Flow planning in companies with both forward and after-sales SCs / SNs is much more complicated. Not only do they have to deal with two SCs / SNs, but also these chains / networks are not independent; what is happening in one SC / SN affects the performance of the other chain / network. For example, improving the after-sales service level imposes more cost to the production system of the after-sales SC / SN, but

on the other hand, it stimulates pre-market demands. Higher product sale quantity in pre-markets augments the spare parts or repair requests in the after-sales SC / SN. Considering these strong interactions between the forward and after-sales SCs / SNs, there is a huge synergy in their concurrent flow planning. In this dissertation, this synergy will be explored by concurrent flow planning in the forward and after-sales SCs / SNs.

Appropriate flow planning throughout the forward and after-sales SCs / SNs is critical to provide desirable services in pre- and after-sales markets. Although a company's pre-market service level is usually defined as the product's demand fulfillment rate to avoid lost sales, after-sales service is a function of: i) warranty length; and ii) just-in-time fulfillment of repair requests (called after-sales service level henceforth).

According to Boone et al. (2008), Aberdeen Research Group (2008), Cohen and Agrawal (2006) and Wangner et al. (2008), the lack of: i) systematic approaches for spare parts management; ii) considering SC relationships; iii) accurate models for predicting the demand for spare parts; and iv) practical models for determining appropriate inventory levels are the main challenges in the after-sales domain. We believe that considering the interactions between forward and after-sales SCs / SNs significantly improves the operations of both pre- and after-sales markets by improving demand predictions and integrated flow management. A Delphi study was done by Boone et al. (2008) in 18 industries. In this study, senior service part managers are asked about the challenges in their industries. The top challenge mentioned is "*lack of holistic perspective and system integration among SC partners*". Gaiardelli et al. (2007) highlight that SC and process-oriented literature dealing with after-sales service is very limited and overcoming obstacles of this industry, mainly related to relationships between involving entities, is

necessary. This highlights a strong need to improve integration in the after-sales operations (Zomerdijk and de Viries, 2003). The important gap that exists in the literature is the paucity of research with an integrated perspective (Bacchetti and Saccani, 2012). Even though this integration increments the complexity of the problem, it significantly improves the companies' overall performance.

1.1.2. Review of previous efforts in after-sales services

For capital goods such as computer networks and complex technical systems such as medical or defense systems: i) material contracts; ii) performance based warranties; and iii) end-of-life (EOL) warranties are the most well-known after-sales services offered by manufacturers. In these systems operational disruptions can lead to a huge loss and the longer the duration of the disruption, the greater the loss. In material contracts, customers pay the manufacturer for parts, other resources, labor, etc. (Kim et al., 2007). In the performance-based warranties, there is an agreement with respect to the availability of the system in the field (Jung and Park, 2005; Yeh, et al., 2005; Chien, 2005; Chen and Chien, 2007; Jhang, 2005; Jung and Park, 2005; de Smidt-Destombes et al., 2004, 2006, 2007, and 2009; Chakravarthy and Gomez-Corral, 2009; Kuo and Wan, 2007; Nourelfath and Dutuit, 2004; Nourelfath and Ait-Kadi, 2007; Cantoni et al., 2000; Marseguerra et al., 2005; Li and Li, 2010; Finkelstein, 2009; Monga and Zuo, 1998; Oner et al., 2010; Wang et al., 2009). For more detail refer to the review papers of Cho and Parlar (1991), Dekker et al. (1997), Pham and Wang (1996), and Wang (2002). The EOL warranties assure the after-sales service without a time limit. The company provides the required service as long as the products are in use even if the production has been discontinued

(Kim and Park, 2008). For more detail, refer to Teunter and Fortuin (1999) and Hasselbach et al. (2002).

For durable consumer goods that are considered in this dissertation: i) rebate warranties; and ii) failure free warranties are the most common after-sales policies. Rebate warranty is usually used for non-repairable goods and manufacturers commit to refund customers some portion of the sale price if the product fails during the warranty period. Goods such as automobile batteries and tires are usually sold with this type of warranty. Failure-free warranties are usually used for household appliances and electronic devices. In this warranty, manufacturers commit to repair product free of charge during the warranty period. As highlighted by Cohen and Agrawal (2006), Wagner (2002), Sanders and Manrodt (2003), Niemi et al. (2009), and Wagner et al. (2008), very little work has been done on warranty service and spare parts management for failure-free warranties. To review the literature of spare parts classifications and demand predictions for stock control refer to Bacchetti and Saccani (2012).

Research on the after-sales services covers the following streams as shown in Figure 1-1:

- ✓ ***Maintenance and replacement activities*:** These papers include activities done to prevent system failures and preserve acceptable performance (Wang et al., 2009; Wang 2012; Bensoussan and Selthi, 2007; Park et al., 2013; Jack and Murthy; 2007; Shahanaghi et al., 2013; Vahdani et al., 2013; Rao, 2011; Chien, 2005).

- ✓ ***Repair services in systems failures***: These papers include the activities that should be done in a system / product failure to recover it (Oner et al., 2010; Sahba and

Balcioglu, 2011; Diaz and Fu, 1997; Graves, 1985; Sherbrooke, 1968; Sleptchenko et al., 2002; van Ommeren et al., 2006; Rappold and Roo, 2009; Gross and Pinkus, 1979; Aggarwal and Moinzadeh, 1994; Moinzadeh and Aggarwal, 1997; Avsar and Zijm, 2000; Sherbrooke, 1968).

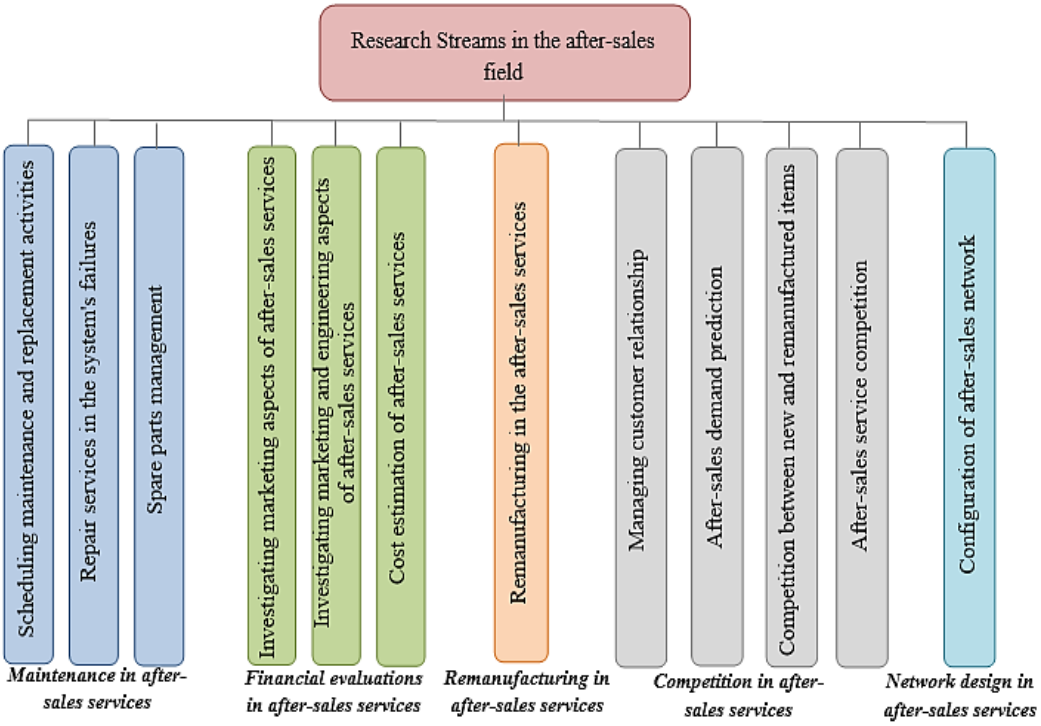


Figure 1-1: Research streams in the after-sales field.

- ✓ ***Spare parts management to fulfill after-sales commitments*:** These papers deal with inventory management (ordering time and quantity) of spare parts to fulfill after-sales demands (Thonemann et al., 2002; Chien and Chen, 2008; Kleber et al., 2011; Lieckens et al., 2013; Muchstadt and Thomas, 1980; Muckstadt, 1973; Cohen and Lee, 1990; Cohen et al., 2000). As mentioned by Boylan and Syntetos (2010), spare parts are very varied and have different costs, demand patterns, and requirements. So

classification of spare parts is critical for appropriate inventory management (Gelders and Van Looy, 1978; Huiskonen, 2001; Partovi and Anandarajan, 2002; Braglia et al., 2004; Eaves and Kingsman, 2004; Syntetos et al., 2005; Ramanathan, 2006; Zhou and Fan, 2006; Ng, 2007; Snyder, 2002; Willemain et al., 2004; Kalchschmidt et al., 2003; Kalchschmidt et al., 2006).

- ✓ ***Marketing aspect of the warranty:*** Authors of these papers by considering warranty as a marketing factor, try to select the best warranty strategy for companies along with other factors such as price, service level, etc. (Menke, 1969; Glickman and Berger, 1976; Menezes and Currim, 1992; Mesak, 1996; Mitra and Patankar, 1997; Matis et al., 2008; Zhou et al., 2009; Chu and Chintaganta, 2009; Chun and Tang, 1995; Majid et al., 2012; Su and Shen, 2012; Jack and Murthy, 2001; Huang and Yen, 2009; Chen et al., 2012; Hua et al., 2007; Hartman and Laksana, 2009; Jiang and Zhang, 2011; Li et al., 2012). These papers by considering the tradeoff of its cost and income, investigate the warranty from the marketing perspective.

- ✓ ***Marketing and engineering aspects of the warranty:*** Authors of these papers by considering that engineering factors such as product reliability and quality have an important role in the after-sales service cost, simultaneously consider the marketing and engineering aspects of the after-sales services (Murthy and Nguyen, 1987; Nguyen and Murthy, 1988; Murthy, 1990; Dockner and Gaunersdorfer, 1996; Mendez and

Narasimhan, 1996; Teng and Thompson, 1996; Mi, 1997; Monga and Zuo, 1998; Pohl and Dietrich, 1999; Zhao and Zheng, 2000; Chen and Chu, 2001; Hussain and Murthy, 2003; Shue and Chien, 2005; Balachandran and Radhakrishnan, 2005; Kamrad et al., 2005; Lin and Shue, 2005; Wu et al., 2006; Huang et al., 2007; Oner et al., 2010).

- ✓ ***Cost estimation of the after-sales services:*** These researchers only concentrate on minimizing the warranty cost by scheduling appropriate maintenance (replace and repair) activities (Murthy and Nguyen, 1987; Zuo et al., 2000; Rao, 2011; Iskandar and Murthy, 2003; Hartman and Laksana, 2009; Vahdani et al., 2011; Tsoukalas and Agrafiotis, 2013; Sahin and Zahedi, 2001a, b; Chen and Popova, 2002; Yun et al., 2002; Jack et al., 2003; Bai and Pham, 2004; Bai and Pham, 2005; Baik et al., 2004; Chukova et al., 2004; Chukova and Hayakawa, 2004a, b; Chukova and Hayakawa, 2005; Huang and Zhuo, 2004; Buczkowski et al., 2005; Iskandar et al., 2005; Rai and Singh, 2005; Chukova and Johnstone, 2006; Jiang et al., 2006; Wu and Croome, 2007; Wu and Li, 2007; Sheu and Lin, 2005; Chen and Lo, 2006; Mitra and Patankar, 2006; Chukova et al., 2007; Williams, 2007; Wu et al., 2007; Jung and Park, 2005; Yeh et al., 2005; Chen and Chien, 2007; Jhang, 2005; Wang et al., 2008).

- ✓ ***Remanufacturing process in the after-sales services:*** These researchers concentrate on the remanufacturing process of a system's failed parts (Muckstadt, 1973; Muckstadt and Thomas, 1980; Sherbrooke, 1986; Slay,

1984; van Harten and Sleptchenko, 2000; Avser and Zijm, 2002; Gross et al., 1983 and 1987; Albright, 1989).

- ✓ ***Managing customer relationships:*** These researchers illustrate the value of understanding how marketing dollars affect customer profitability and why this focus may lead to very different conclusions than those obtained from traditional approaches (Gupta and Lehmann, 2007).

- ✓ ***After-sales demand prediction:*** Demand of large portion of spare parts is lumpy and intermittent which requires new forecasting methods. On the other hand, their demands depend on some explanatory variables such as the product's failure probability and system's maintenance activities (Bartezzaghi et al., 1999; Gutierrez et al., 2008; Hua et al., 2007; Ghodrati and Kumar, 2005; Tibben-Lemke and Amato, 2001; Dolgui and Pashkevich, 2008; Chu and Chintagunta, 2009; Barabadi et al., 2014).

- ✓ ***Competition between new and remanufactured items:*** Remanufacturing failed items is very prevalent in the after-sales industries because inside warranty failed parts are almost new and usually are worth remanufacturing (Wu, 2012; Atasu et al., 2008; Debo et al., 2005; Ferrer and Swaminathan, 2006; Majumder and Groenevelt, 2001; Mitra and Webester, 2008).

- ✓ ***After-sales service competition:*** These papers are about modeling competition of rivals in markets by considering the after-sales services as

one of their marketing strategies (Cohen and Whang, 1997; Kameshwaran et al., 2009; Kurata and Nam, 2010; Kurata and Nam, 2013).

- ✓ ***Configuration of the after-sales network:*** These researchers address the problem of determining the configuration of after-sales SCs / SNs with respect to the activities should be carried out within them (Khajavi et al., 2013; Saccani et al., 2007; Armistesd and Clark, 1991; Loomba, 1996 and 1998; Goffin, 1999; Nordin 2005; Amini et al., 2005). One of the important decisions made in some of these papers is selecting appropriate strategy: i) selecting manufacturing strategy (Hayes and Wheelwrigh, 1984; Hill, 1995; Bozarth and McDermott, 1998) or ii) selecting service strategy (Schmenner, 1986; Chase and Hayes; 1991 and 1992; Fitzsimmonds and Fitzsimmonds, 1998; Silvestro et al., 1992; Johansson and Olhager, 2006).

As seen in the literature review, the focus of the previous work is mainly on downstream operations of after-sales services such as scheduling maintenance and repair activities, inventory management of spare parts, investigating financial burden and advantages of after-sales services, analyzing its competitive advantages, etc. But upstream facilities supporting these downstream operations are ignored. Ignoring upstream facilities leads to lack of holistic and process-oriented consideration in the after-sales operations. This gap is filled in this dissertation by considering all facilities involving in after-sales operations in the form of an after-sales SC / SN.

Also in the literature the after-sales operations are planned independently from pre-market operations. In this dissertation we show that there are some important interactions between operations of pre- and after-sales markets which should be reflected in planning their corresponding SCs / SNs. We fill this gap, by concurrent planning of flow in the all including facilities of pre- and after-sales operations in the form of forward and after-sales SCs / SNs.

1.1.3. Existing gaps in after-sales services literature and research questions

Based on the literature review in Section 1.1.2, it is clear that the manufacturing facilities supporting after-sales services are mainly ignored in the literature which leads to lack of a holistic integration and comprehensive planning in the facilities supporting these services. On the other hand, the interactions of forward and after-sales SCs / SNs, product-service interplays, are completely ignored in the existing papers.

In this dissertation, we fill this gap by considering the after-sales SC / SN including all the involving facilities supporting the after-sales services. Not only do we consider interactions of facilities in the after-sales SC supporting after-sales services, but also we consider the interplays of this chain / network with the forward SC / SN by concurrently flow planning throughout their chains/ networks.

Research Questions will be answered in this dissertation in the "after-sales services" context are as follows:

- **Research Question 1:** what are the important flow transitions among the facilities supporting after-sales services?

- **Research Question 2:** what are the important interactions between forward and after-sales SNs justifying the necessity of their concurrent flow planning?
- **Research Question 3:** how do these interactions affect planning flow dynamics in the forward and after-sales SNs of non-repairable goods?
- **Research Question 4:** how do these interactions affect planning flow dynamics in the forward and after-sales SNs of repairable goods?

1.2. Literature of uncertainty management in the supply network management

1.2.1. Importance of uncertainty management in the supply network management

Companies are improving their competitiveness by reducing production costs, having higher productivity, and improving products quality through concentrating on their core competencies and increasing their flexibility with respond to rapidly changing expectations of customers. All these requirements disperse traditional centralized production systems into a network of core-competency-centered companies called a SC / SN. Along with all the advantages of SCs / SNs, decentralization reduces their controllability and makes them more vulnerable to uncertainties. This highlights the importance of uncertainty management in SCs / SNs to predict, control and mitigate negative effects of uncertainties on their performance. Uncertainty management capability of a SC / SN is mainly reflected in one of its performance metrics called service level.

Recently service level has become an important competitive advantage and many companies attempt to improve their market shares by providing better service levels. For example two well-known book retailers, Amazon and Barnes and Noble, who share more than 85 percent of online sales, initiated competition by promising the same business day delivery in different parts of the country. Blockbuster, a well-known company in the

video rental industry, advertises its high fill rate and backs its promise up with a free rental guarantee. The same is happening in the fast food industry, Domino's, for example, guarantees 30-minute delivery or free delivery. Black Angus restaurants advertise free lunch if the customer's order is not served in 10 minutes. Retailers such as Lucky emphasize their short checkout times. Well Fargo Bank guarantees less than five minutes wait for its customers or gives them a $5 reward. Airline companies advertise based on their percentage of on-time arrival. Several independent internet sites provide information about the company performance such as their service level warranties, back-up chargeback agreements, etc. Moreover, specifying a delivery window is common in business-to-business settings.

Thus, service level is becoming one of the most important competition factors. Service level is the capability of a company to balance demand and supply quantities. This balancing is not easy in reality because both demand and supply processes are stochastic. By assuming perfect production systems, supply side uncertainty is usually ignored in the extensive service level literature. But in reality, there is no perfect production system. Increasing the rate of production increases the likelihood of machinery and labor failures leading to a higher rate of non-conforming items produced (Sana, 2010). Decentralized and multi-echelon production systems of SCs / SNs amplify the probability of non-conformation.

Also recently the number of natural and man-made disasters disrupting SCs' / SNs' supply processes has been increased dramatically (Baghalian et al., 2013). Disruption in SCs leads to huge lost sales in target markets and adversely affects their brand reputations. These trends demand more accurate approaches to determining and preserving

appropriate service levels in SCs / SNs. In this dissertation, by considering different uncertainties (affecting SCs / SNs both integration and coordination), we want to respond to this new need of the business environment which as will be shown later is mainly ignored in the literature.

1.2.2. Review of previous efforts in the uncertainty management in the supply network management

There are several uncertainties in SCs / SNs. They can be classified as:

i) **Operational level uncertainties (variations)**: Operational level uncertainties include uncertain customer demand with a fixed mean, uncertain supply quantities of facilities due to their imperfect production systems, expected variations in raw material prices, etc. These uncertainties are expected, occur frequently and have significant probabilities. These uncertainties only in a limit scale affect the coordination process of facilities in a network and its flow dynamics;

ii) **Strategic level uncertainties (disruptions)**: Disruptions refer to unexpected events with very low probabilities and very extensive effects changing a SC's / SN's topology such as earthquakes, floods, hurricanes, terrorist attacks, economic crises, or strikes. These events make parts of a network, some of its nodes and links, inoperative and out-of-use.

In this dissertation, we consider both of these uncertainty groups to have a fail-safe network against disruptions threatening its integration and variations threatening the coordination of its involving facilities (Figure 1-2).

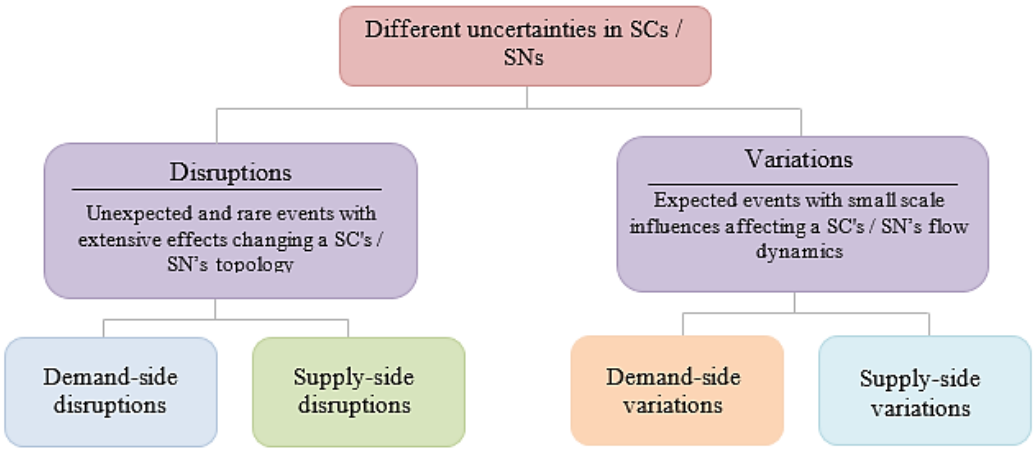


Figure 1-2: Different uncertainties in SCs / SNs.

There has been much work in the literature on operational uncertainties in SCs. Many researchers only consider demand-side variation (Sabri and Beamon, 2000; Miranda and Garrido, 2004; Shen and Daskin, 2005; Daniel and Rajendran, 2006; Romeijn et al., 2007; Ko and Evans, 2007; Shen and Qi, 2007; You and Grossmann, 2008; Schutz et al., 2009; Pan and Nagi, 2010; Park et al., 2010; Cardona-Valdes et al., 2010; Hsu and Li, 2011). In our work, in addition to demand-side variations, different supply-side variations will also be considered. Difficulties with supply in one entity disrupt production schedules in all the subsequent entities of a SC / SN which leads to delay in fulfilling customers' demands. Poor service levels lead to lost sales and long-term demand attenuation. Hence, appropriate strategies mitigating the negative effects of the supply-side variations in flow planning, especially in SCs / SNs with multiple supply echelons, are imperative. The approach presented here not only does significantly improve the service level estimation in SCs / SNs, but also improves systems reliability in preserving that service level and improving competition capabilities. We assume that production systems in SCs' / SNs'

echelons are accompanied with stochastic percentage of wastage and nonconforming outputs making their qualified supply quantities uncertain.

Supply-side uncertainty management in a SC / SN has a richer literature on disruptions rather than operational uncertainties (Santoso et al., 2005; Azaron et al., 2008; Yu et al., 2009; Li et al., 2010; Xanthopoulos et al., 2012; Baghalian et al., 2013). There are few works in the field of operational supply-side uncertainty in SCs/ SNs. Chopra et al. (2007) consider product flow planning in a SC consisting of a buyer and two suppliers. The first supplier is cheaper, but prone to unreliability and the second supplier is completely reliable, but more expensive. Demand in the markets is assumed to be deterministic. In this paper, supply-side uncertainty is considered and the necessity of decoupling operational and disruption supply risk is highlighted. Disruption is modeled by scenarios and operational supply uncertainty is considered as a random variable with a given distribution function. Schmitt and Snyder (2010) consider optimal ordering and the required amount of the reserve product of a two-echelon SN of a firm and its suppliers. One supplier is unreliable whereas the second is completely reliable and available but more expensive. They compare single-periods and multi-periods and discuss the advantages of considering multi-periods. Dada et al. (2007) consider a company with several potential suppliers both reliable and unreliable and decisions about supplier selection and order splitting are made in a way to maximize the company's expected profit. Ross et al. (2008) consider the ordering policy of a firm with a Poisson arrival demand and a single supplier with a random supply process. Supply and demand processes have time-dependent probabilities. They set a time varying ordering policy to decrease the total cost of the system. Li and Chen (2010) develop a model for inventory

management of a SC with an unreliable supplier and a retailer. They investigate the impact of supply-side uncertainty and customer differentiation on minimizing average annual cost. In existing research, SCs' / SNs' supply process is restricted to one echelon and variation in the facility performance of that echelon. Based on this literature, lack of modeling supply-side variations in SCs / SNs with multiple stochastic echelons and its effect in improving service level estimation is clear.

On the other hand, the existing work of the literature only focuses on the flow coordination in SCs / SNs and their influencing variations or considers disruptions affecting the topology of a SC / SN and mitigation of their effects. But we believe to preserve an appropriate service level, we need to architecture a fail-safe SC / SN. A fail-safe SC / SN mitigates the impacts of both disruptions and variations and provides an acceptable level of service. This is achieved by controlling its topology (structurally fail-safe) and coordinating the flow (operationally fail-safe) through the facilities. In this dissertation, we show that to have a structurally fail-safe supply network, its topology should be robust against disruptions by positioning mitigation strategies and be resilient in executing these strategies. Also we show that to have an operationally fail-safe SC / SN, its flow dynamics should be reliable against demand- and supply-side variations

Three uncertainties are mainly considered in the after-sales research field:

- ✓ ***Failure time / rate of products / systems to determine the after-sales demand*** (Barabadi et al., 2014; Glickman and Berger, 1976; Huang et al., 2007; Kim et al., 2007; Menke, 1969; Murthy, 1990; Nguyen and Murthy, 1984; Nguyen and Murthy, 1988; Oner et al., 2010; Sahba and Balcioglu, 2011; Wang, 2012; Wang

et al., 2009; Anderson, 1977; Diaz and Fu, 1997; Sleptchenko et al., 2002; Hussain and Murthy, 2003; Lieckens et al., 2013; Lin and Shue, 2005; van Ommeren et al., 2006; Rappold and Roo, 2009; Wu et al., 2009; Faridimehr and Niaki, 2012; van Jaarsveld and Dekkrer, 2011; Avsar and Zijm, 2000; Sleptchenko et al., 2003; Park et al., 2013; Matis et al., 2008; Jack and Murthy, 2007; Wu et al., 2006; Vahdani et al., 2013; Su and Shen, 2012; Hartman and Laksana, 2009; Zhao and Zheng, 2000; Rao, 2011; Chu and Chintagunta, 2009).

- ✓ ***Repair time of products / systems*** (Oner et al., 2010; Sahba and Balcioglu, 2011; Diaz and Fu, 1997; Sleptchenko et al., 2002; Lieckens et al., 2013; van Ommeren et al., 2006; Rappold and Roo, 2009; Avsar and Zijm, 2000; Sleptchenko et al., 2003; Sherbrooke, 1968; van Harten and Sleptchenko, 2000; Gross and Pinkus, 1979; Graves, 1985; Perlman et al., 2001).
- ✓ ***Repair cost*** (Zhou et al., 2009)

As noticed above, most of the work in the literature does not include holistic view and only concentrates on downstream of after-sales SCs / SNs such as repair demand and repair process and their corresponding uncertainties and ignores the upstream production facilities producing and providing the requirements (such as spare parts) for the after-sales services. In this dissertation, we consider the upstream production facilities of after-sales SCs and their corresponding uncertainties.

Three groups of operational uncertainties are considered in this dissertation: 1) demand-side variations; 2) supply-side variations and disruptions and 3) uncertainty in the performance of product's components. Demand-side variations include the

uncertainty in the prediction of product demand in pre-markets and spare parts demand in after-sales markets. We assume that product demand in pre-markets is a stochastic function of product's retail price, warranty length and service levels. The spare parts demands are stochastic and depend on the total product supply by the forward SC / SN and quality of product's components. Supply-side variations include imperfect production systems of production facilities such as suppliers and manufacturers which lead to stochastic qualified outputs and supply quantities of these facilities. Supply-side disruption refers to disruption possibility in the supply facilities of SCs / SNs.

1.2.3. Existing gaps in the uncertainty management in the supply network management and research questions

In operational supply-side uncertainty management literature, SCs' / SNs' supply process is restricted to one echelon and uncertainty in the facility performance of that echelon. However in actuality, most SCs / SNs have longer production chains / networks involving several echelons of suppliers of suppliers, suppliers, components manufacturers, assemblers, etc. To fill the gap, we consider SCs / SNs with multi-echelon supply processes servicing markets with uncertain demands. The SCs' / SNs' multi-echelon supply process includes production facilities' with uncertain production systems.

In such a complex network-based production system, uncertainties in the production facilities are accumulated by moving the material / product flow from the SC's / SN's upstream to its downstream leading to a larger and larger bias. As shown in the sample SC of Figure 1-3, due to the uncertainty in the production system of the supplier, determining the conforming output of the supplier for a given input level is not possible. Qualified output of the supplier can change in a given range. This uncertain output of the

supplier is input to the manufacturer which also has an uncertain manufacturing system. Thus the uncertainty in the manufacturer's production system is added to its uncertain input level which leads to a greater uncertainty in its qualified output. The same story repeats in the SC's downstream echelons. We call this phenomenon "*uncertainty propagation*" in SCs / SNs. In such a SC / SN not only the local effects of these uncertainties on the performance of their corresponding entities should be investigated, but also their global effects on the performance of the whole SC / SN should be governed.

At first glance the consequences of supply-side uncertainty propagation which is introduced in this dissertation and Bullwhip Effect was already introduced in 1960's look so similar to each other. But their reasons and what is amplified in these two phenomena are completely different. Details are as follows:

- ✓ ***Bullwhip effect***: two factors lead to Bullwhip Effect in a SC / SN: i) uncertainty in market demand (demand side uncertainty is only considered in this phenomenon); and ii) existence of time lag in the information transaction among a SC's / SN's echelons. This means that all the facilities in the SC / SN do not recognize demand variations simultaneously. Due to this reason, inventory volume in the SC's / SN's facilities propagates by moving from downstream to upstream. Uncertain production system of facilities or in the other word supply-side uncertainty does not have any role in this phenomenon.

- ✓ ***Supply-side uncertainty propagation***: this phenomenon happens due to the uncertainty in the performances of production facilities and their qualified output volumes in a SC / SN with multi-echelon production process. This uncertainty in

the qualified and acceptable flow volume propagates by moving of material and product from upstream to downstream. The speed of information transaction among the SC's / SN's facilities does not have any role in this phenomenon.

In this work, we contributes in the following ways to the uncertainty management in SCs / SNs literature. First the supply-side variations in SCs / SNs with a multi-echelon supply process is considered. In the literature all supply-side variation work in the context of SCs / SNs is restricted to a single echelon supply process. In SCs / SNs with a multi-echelon supply process, the phenomenon of uncertainty propagation is introduced and quantified. The importance of uncertainty propagation in the global performance of SCs / SNs is demonstrated.

In addition to supply-side variations, we also consider the possibility of disruption in supply facilities of SCs / SNs. We show that to preserve the availability of the required facilities, a SC / SN needs to have a robust network. Robustness of a SC / SN depends on the flexibilities levels of its facilities. To minimize a SC / SN injury after disruption, its facilities should be resilient. Resilience of a facility shows how fast the capacity of that facility can be ramped up – flexibility speed. We developed a comprehensive model to select the best flexibility levels and speeds for SCs' / SNs' facilities to redesign the most profitable robust and resilient network for them.

Variations and disruptions have been investigated separately in the literature. But we show that to have a fail-safe SC / SN, its structure should be robust and resilient against disruptions and the coordination of its facilities should be reliable against variations.

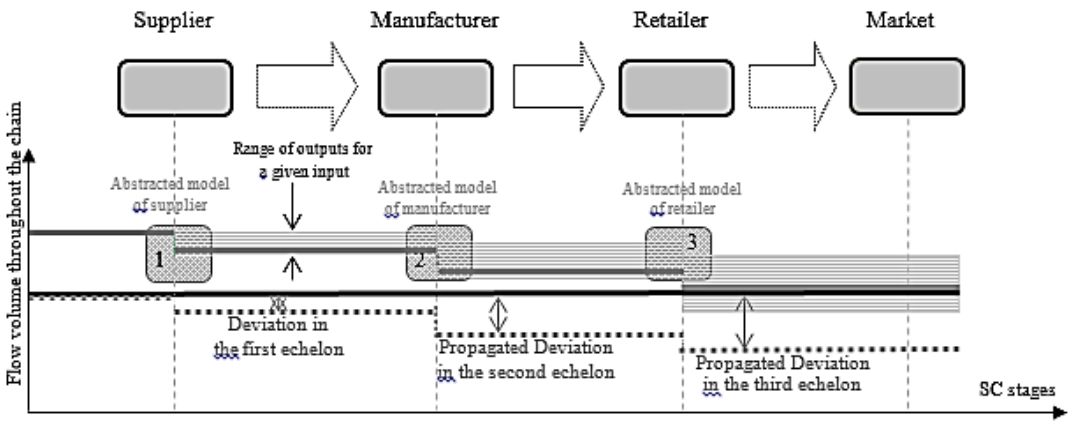


Figure 1-3: Uncertainty propagation in a sample SC.

Black line: Product flow planned by the deterministic model;
Dashed line: Product flow that is happening in reality;
Grey line: Solution of reliable model expected to be obtained in this paper.

Detailed information of mostly related papers to our problem is summarized in Table 1-1. In Columns 2-5 of the table, we explain which types of products (capital or durable consuming goods) are considered in papers and if their repair / replacement time is incorporated in modeling or not. Columns 6-7 are about the number of echelons and items considered in the after-sales operations. In Columns 8-11, we show which kinds of warranty (rebate, failure free, EOL, and Performance based) is considered for products. In Columns 12-29, we represent decisions (determining warranty parameters, repair process, spare parts inventory management, demand prediction, network topology, etc.) made in the papers. In Columns 30-31, objective functions and constraints of models are explained respectively. In Columns 32, we explain which uncertainties (failure times and numbers, repair times, demands, etc.) are considered in problems of papers.

Research questions will be answered in this dissertation in the "uncertainty management" context are as follows:

- **Research Question 5:** what are the necessities of having fail-safe SNs?
- **Research Question 6:** what are the characteristics of fail-safe SNs against disruptions – characteristics of structurally fail-safe SNs?
- **Research Question 7:** what are the characteristics of fail-safe SNs against variations – characteristics of operationally fail-safe SNs?

Table 1-1: Literature review table.

After-sale services																																	
Paper	spare						Warranty type				Decisions (outputs)																				Objective	Constraints	Uncertainty
																	Inventory		Repair capacity														
	Belongs to capital goods	Belongs to durable consuming goods	considering repairing / production time	Repairing time is ignorable / replacement	# of echelons	# of items	Rebate Warranty	Free failure warranty	EOL warranty	Performance – based logistics	Topology of after-sale network	Price of the spare / product	Warranty length	Warranty parameters	Warranty reserve funding / Warranty cost per product	Reliability allocation	Location	Volume	Location	Volume	Maintenance schedule	Availability / service level	Demand prediction	Spare part ordering time (from customer view)	Spare part manufacturing plan	Spare parts repairing plan	Buyback price of returned products	Redundancy allocation	Replacement time (based on deterioration degree and residual warranty period)	Production / repair option selection			
Barabadi et al. (2014)	*			*	1	1																	*								Min Spare part number and cost	-	Failure time
Chien & Chen (2008)		*		*	1	1	*																	*							Min per unit time cost & Max cost effectiveness	-	Life time of a product & lead time for delivering a spare

Glickman & Berger (1976)		*			1	1		*				*	*																Max Profit	-	Number of repairs under warranty
Huang et al. (2007)		*		*				*				*	*			*													Max Profit	-	Failure time
Kim & Park (2008)	*			*	1	1			*			*	*											*					Max Profit	-	-
Kim et al. (2007)	*		*		2	*n*				*				*			*												Max Utility	Service Level	Failure time
Kleber et al. (2011)		*	*		2	1																		*		*			Max Profit	Flow and price selection constraints	-
Menke (1969)		*		*	1	1	*					*			*														-	-	Failure time of product
Murthy (1990)		*		*	1	1	*					*	*			*													Max Profit	-	Failure time of product
Nguyen and Murthy (1984)		*		*	1	1		*							*														-	-	Failure time of product
Nguyen and Murthy (1988)		*		*	1	1	*	*								*													Min manufacturing + servicing costs	-	Failure time of product
Oner et al. (2010)	*		*		2	1				*						*		*											Min cost	Reliability boundary	Failure and repair time
Sahba & Balcioglu (2011)	*		*		2	*n*				*	*							*											Min cost	-	Failure and repair time
Wang (2012)	*			*	1	1												*		*										Min inventory + shut down costs	-	Plant failure
Wang et al. (2009)	*			*	1	1												*		*										Min cost	-	Uncertain deterioration of each unit
Anderson (1977)		*		*	1	1		*				*	*																Max Profit	Price is more than manufacturing cost	Failure time
Diaz and Fu (1997)	*		*		*n*	1												*											-	Service level	Failure rate & repair time

Graves (1985)	*		*		2	1												*											-	Service level	Failure rate & repair time
Sherbrooke (1968)	*		*		2	1												*											Max system availability	Service level	Failure rate & repair time
Perlman et al. (2001)	*		*		2	1																			*				*	Min sum of backorders	-	Failure rate & repair time
Sleptchenko et al. (2002)	*		*		*n*	*n*											*	*												Max Availability	-	Failure rate & repair time
Hussain & Murthy (2003)		*		*	1	1		*								*														Min manufacturing + warranty cost	-	Failure rate & reliability improvement of parts
Hussain & Murthy (2000)		*		*	1	1		*																			*			Min manufacturing + warranty cost	-	Failure rate & reliability of parts
Ming et al. (2000)		*		*	1	1		*																				*		Min warranty cost	-	Failure & deterioration rate of parts
Lieckens et al. (2013)	*		*		*n*	*n*				*	*						*	*	*	*					*	*				Max Profit	Service level	Failure rate & processing time
Lin & Shue (2005)		*		*	1	1		*				*	*																	Max Profit	-	Failure rate
Ommeren et al. (2006)		*	*		2	1					*							*	*	*										Min total expected cost	Service level	Demands and repair times
Rappold and Roo (2009)		*	*		2	1					*							*	*	*										Min total expected cost	-	Demands and repair times
Gross & Pinkus (1979)	*		*		2	1					*						*	*	*	*										Min total expected cost	Service level	Failure rates and repair times
Chen et al. (2012)		*		*	2	1		*				*[1]	*[2]																	Max Profit	-	-

[1] Whole sale price of manufacturer.
[2] Warranty length of retailer.

Wu et al., (2009)		*		*	1	1		*				*	*											*						Max Profit	-	Failure rate
Faridimehr & Niaki (2012)		*		*	1	1		*				*	*											*						Min Cost	-	Failure rate
Kurata & Nam (2010)		*		*	2	1		*					*																	Max profit	-	-
Kurata & Nam (2013)		*		*	2	1		*					*																	Max profit		Customer needs
Aggarwal & Moinzadeh (1994)		*	*		2	1												*											*	Min cost	-	Demand and service time
Moinzadeh & Aggarwal (1997)		*	*		2	1												*											*	Min cost	-	Demand and service time
Khajavi et al. (2013)	*			*	1	1					*																			Min cost	-	-
Jaarsveld and Dekkrer (2011)	*		*		1	n																*								Min cost (downtime and holding)	-	Demands for spare parts
Avsar and Zijm (2000)	*		*		2	1												*												Min stock keeping units or spare parts	Service level (Resource constraints (limited repair capacity)	Failure rate and Processing time
Acsar and Zijm (2002)	*		*		2	1												*												Min stock keeping units or spare parts	Service level (Resource constraints (limited repair capacity)	Failure rate and Processing time
Sleptchenko et al (2003)	*		*		n	n											*	*	*	*										Max Availability	Finite repair capacity	Failure rate & repair time
Harten and Sleptchenko (2000)	*		*		1	n																			*					Max Availability	-	Failure rate and service rate
Saccani et al., (2007)		*		*	2	n											*	*							*					-	-	-

Heese et al., (2005)		*		*	1	1						*																	Max Profit	-	-
Park et al., (2013)	*		*		1	1														*									Min expected cost rate per unit time	-	Failure intensity
Matis et al., (2008)	*		*		1	1						*	*															*	Max Profit	-	Failure time
Jack and Murthy (2007)	*			*	1	1		*				*								*							*		Min expected cost rate	-	Failure rate
Huang and Yen (2009)		*		*	1	1								*	*														Max Profit	-	Breakdown process of each unit
Muckstadt (1973)	*		*		2	n											*								*					Min total expected base backorders	System investment	Demand
Muckstadt and Thomas (1980)	*		*		2	n											*													Min total expected backorders	Service Level, and Budget constraint	Demand
Wu et al., (2006)		*		*	1	1		*				*	*																	Max Profit	-	Lifetime of a product(random variable with normal distribution)
Sherbroooke (1986)	*		*		2	n											*													Min expected backorders	-	Failure rate & repair time
Ray et al. (2005)		*		*	1	n		*				*		*																Max profit	-	-
Vahdani et al., (2013)	*		*		1	1		*																			*			Min manufacturer's expected warranty serving cost	-	Failure and deterioration rate
Ferrer and Swaminathan (2006)		*		*	1	1						*																*		Max profit	In any period, the number of remanufactured products is limited by the number of cores collected from previous period sales.	

Shahanaghi et al., (2013)	*			*	1	1		*											*									Min expected servicing costs	-	-
Su and Shen (2012)		*		*	1	n		*						*														*	Max expected profit	-	Failure rate
Tsoukalas and Agrafiotis (2013)		*		*	1	1		*							*														-	-	-
Hartman and Laksana (2009)		*		*	1	1								*	*														Max profit	-	Failure time
Zhao and Zheng (2000)		*		*	1	n						*																	Max expected revenue	-	Demand rate
Zhou et al., (2009)		*		*	1	1		*				*	*																Max profit	-	Uncertain repair costs & Number of failures
Rao (2011)		*		*	1	1		*																				*		Min warranty serving cost	-	Failure time
Chu & Chintagunta (2009)		*		*	1	n		*							*								*						Max profit	-	Demand (customers' preferences)
Balachandran & Radhakrishnan (2005)		*		*	1	1																								Max expected profit	the penalty is less than the buyer's external failure cost	-

1.3. Organization of the dissertation

In this dissertation we want to design / redesign a fail-safe SN. A fail-safe network is one which mitigates the impact of uncertainties and provides an acceptable level of service in markets (pre- and after-sales markets). This is achieved by controlling its topology (structurally fail-safe) and coordinating the flow (operationally fail-safe) through the facilities.

In Chapter 2, we show that to have an operationally fail-safe SC and SN, its flow dynamics should be reliable against demand- and supply-side variations – small scale expected events. In Chapters 3 and 4, we show that how the concept of operationally fail-safe SC developed in Chapter 2 can be extended to service both pre-markets – forward SC – and after-sales markets – after-sales SC. In Chapter 5, we develop a model to plan flow dynamics in operationally fail-safe SN servicing both pre- and after-sales markets.

Chapter 6 is about redesigning a structurally and operationally fail-safe SN. In this chapter, we show that to have a structurally fail-safe supply network, its topology should be robust against disruptions – large scale unexpected events – by positioning mitigation strategies and be resilient in executing these strategies. Considering "*Flexibility*" as a risk mitigation strategy, we answer the question "*What are the best flexibility levels and flexibility speeds for facilities in structurally fail-safe supply networks*?" Figure 1-4 depicts the flow of information through the chapters of this dissertation. As seen in the figure, in Chapters 2 and 6 we develop an operationally and structurally fail-safe SC / SN respectively servicing only pre-markets. Chapters 3, 4, and 5 extend the SC and SN problem to service the after-sales markets as well.

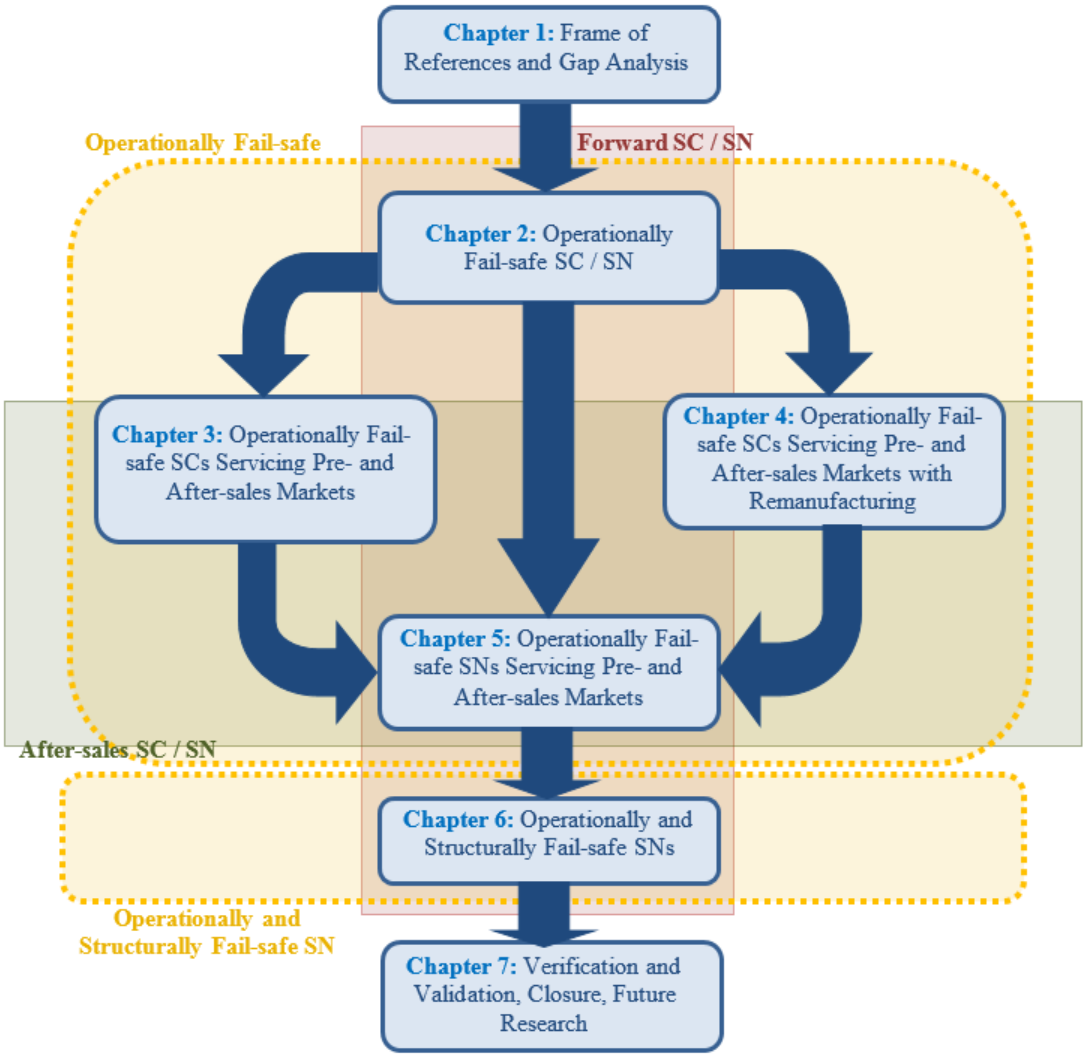


Figure 1-4: Outline of this dissertation.

In Chapter 2 – Operationally fail-safe SN – we only concentrate on forward SNs (and ignore after-sales SNs) to simplify the problem. In this chapter, at first we consider a simple forward SC with only one facility in each echelon servicing a market with a stochastic demand. We assume that the performance of production systems in the echelons of this SC is not perfect and includes stochastic rate of non-conforming output. In this chapter, we show that how we can quantify uncertainty propagation through this chain and use it to quantify qualified supply quantity in the last echelon. Then we use it

to determine the most profitable service level for the whole chain and it's supporting local reliabilities of stochastic facilities. Finally we extend this method to SNs with more than one facility in each echelon (Figure 1-5).

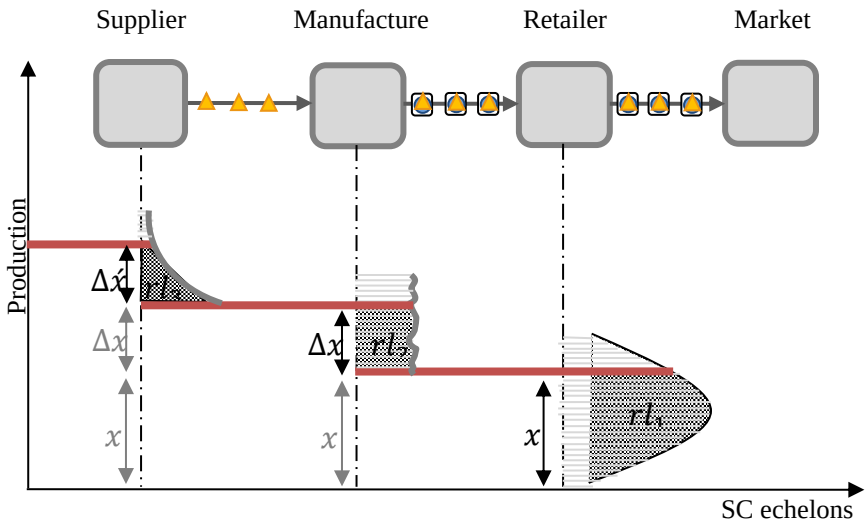


Figure 1-5: Problem will be investigated in Chapter 2.

In Chapter 3, we extend the model of Chapter 2 to include an after-sales SC as well. In this chapter, we are going to consider a company including two SCs: i) a forward SC producing and supplying products to a pre-market. These products are sold under a specific price and warranty strategies; and ii) an after-sales SC producing and supplying spare parts to fulfill after-sales commitments. Again it is assumed that the performance of production facilities and demands of the pre- and after-sales markets are stochastic. In this problem, there is one facility from each type in each echelon (Figure 1-6).

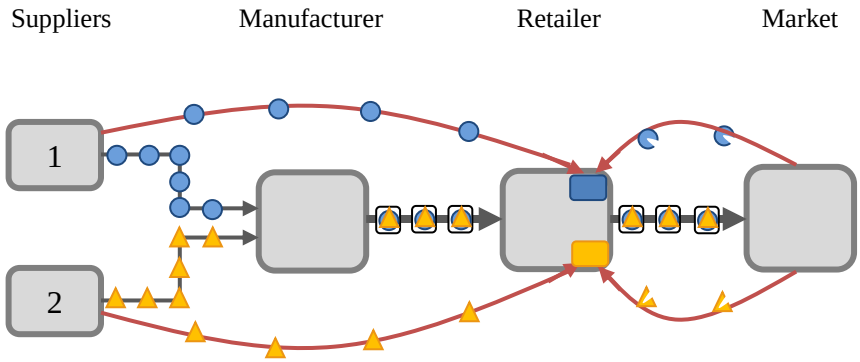


Figure 1-6: Problem will be investigated in Chapter 3.

In Chapter 4, we extend the problem of Chapter 3 to include the remanufacturing possibility of defective parts of the products returned by customers inside the warranty period. In Chapter 3, only new spare parts are used to service after-sales demands. But in this chapter remanufactured parts also can be used to service these commitments. Again it is assumed that the performance of production facilities and demands of the markets are stochastic and there is one facility from each type in each echelon (Figure 1-7).

In Chapter 5, we extend the problem of Chapter 3 and 4 to SNs with more than one facility in each echelon. In this case, the size of the problem and the number of its binary variables increase significantly in comparison with the models of the previous sections. Thus the solving methods of the previous chapters are not efficient for the model of this chapter. Therefore, a specific algorithm is proposed in this chapter to solve the model. At the end of each chapter, a test problem is used to check the model and solution method. Sensitivity analysis of the results leads to some managerial insights.

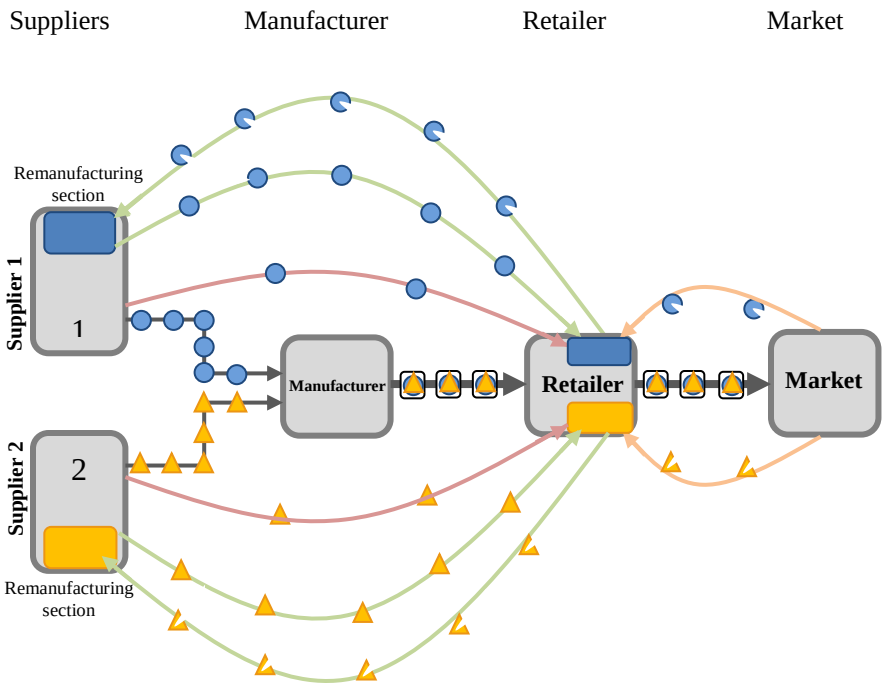


Figure 1-7: Problem will be investigated in Chapter 4.

In Chapter 6, we extend the previous problems to concurrently redesign the SN topology (integration in the SN) to be fail-safe against disruptions and plan flow dynamics throughout its network (coordination in the SN) to be fail-safe against variations. We redesign the SN topology in a way to be robust against supply side disruptions and be resilient to minimize their negative effects after occurrence. By considering demand and supply side variations and their propagated effect, we will plan a reliable flow throughout the SN's network. We will develop a comprehensive mathematical model to concurrently make these decisions in the most profitable way.

In Chapter 7, we have closing remarks and talk about verification and validation, future research, possible extensions, and other applications for the problems of this dissertation.

The research questions of this dissertation will be answered in the following chapters:

Table 1-2: Answers of the research questions.

Research Questions (RQs)	***Chapter 1***	***Chapter 2***	***Chapter 3***	***Chapter 4***	***Chapter 5***	***Chapter 6***	***Chapter 7***
RQ1: what are the important flow transitions among the facilities supporting after-sales services?	✓		✓	✓	✓		✓
RQ2: what are the important interactions between forward and after-sales SNs justifying the necessity of their concurrent flow planning?	✓		✓	✓	✓		✓
RQ3: how do these interactions affect planning flow dynamics in the forward and after-sales SNs of non-repairable goods?	✓		✓		✓		✓
RQ4: how do these interactions affect planning flow dynamics in the forward and after-sales SNs of repairable goods?	✓			✓			✓
RQ5: what are the necessities of having fail-safe SNs?	✓	✓				✓	✓
RQ6: what are the characteristics of fail-safe SNs against disruptions – characteristics of structurally fail-safe SNs?	✓					✓	✓
RQ7: what are the characteristics of fail-safe SNs against variations – characteristics of operationally fail-safe SNs?	✓	✓					✓

Chapter 2: Operationally Fail-safe Supply Chains / Networks

In this chapter, we deal with having "Operationally Fail-safe SNs". Flow planning through these SNs is fail-safe against variations. In this chapter, we want to answer the seventh research question by determining the characteristics of these SNs:

- ✓ **Research Question 7**: *what are the characteristics of fail-safe SNs against variations?*

First in Section 2.1, we determine different kinds of variations affecting the flow planning in SCs / SNs. Then in Sections 2.1.1, 2.1.2 and 2.1.3, we explain that how the variations affect the performance of facilities in the first echelon, e.g. the retailer, the second echelon, e.g. the manufacturer, and the third echelon, e.g. the supplier, of the SC. In these section, we show how uncertainties propagate through the SC and how this phenomenon adversely affect its performance. In Section 2.1.4, we develop a mathematical model to neutralize the negative effect of uncertainty propagation. In Section 2.1.5, we propose an approach to linearize and solve the model. The model is tested on an example in Section 2.1.6. The solution approach is extended from SC to SN in Section 2.2. In Section 2.2.4, we explore the design space to determine correlations exist between the price and service level in the SC / SN. Run time of the models is analyzed in Section 2.2.5.

2.1. Operations and variations in a forward supply chain

In this chapter, we consider a SC with a multi-echelon supply process including a sequence of facilities, supplier and manufacturer with imperfect production systems (supply-side variations). Components are procured from the supplier, and, after being manufactured to the final product by the manufacturer, they are supplied to a market with a stochastic demand by a retailer (demand-side variations). The stochastic demand is an

increasing function of service level and decreasing function of price. The production system of the SC's manufacturer has a stochastic percentage of defective output. After set up, each supplier's production system deteriorates after a stochastic time and shifts from in-control to out-of-control leading to a stochastic percentage of nonconforming products. To have an operationally fail-safe SC, the demand- and supply-side variations should be incorporated in its flow planning.

In this problem, service level is defined as a percentage of the market's demand which can be fulfilled immediately by the retailer's on-hand inventory and is a function of the local reliability levels of the SC's facilities. Higher reliability levels in each facility improve the SC's global performance (service level) in charge of imposing costs on the system. The goal is to determine: (i) the service level providing the highest SC profit by considering local and propagated uncertainties; (ii) the combination of reliability levels in the SC's echelons to ensure economic service level (iii) economic production planning to preserve the local reliability of facilities and the SC's service level. Products for each production planning period are produced, transported and stored in the SC's retailer before the start of that period. In the rest of this section, we elaborate our general strategy to deal with problem.

Optimizing service level is much more difficult in these SCs due to uncertainty propagation. Each facility in the SC is assigned an appropriate reliability level representing the probability that it is able to fulfill the order of its downstream facility completely. rl_1, rl_2 and rl_3 are the reliability levels of the SC's retailer, manufacturer and suppliers respectively. The retailer selects the product stock quantity to ensure, with rl_1 probability, that this stock level can fulfill the market's demand, and the manufacturer

selects its component procurement and final product manufacturing quantities to guarantee that the qualified output is equal to the retailer's requirements with rl_2 probability. rl_3, the supplier's reliability level means that its material procurement and component production quantity can fill the manufacturer's order with rl_3 probability. Thus the SC's supplier is sure with rl_3 probability that it can provide the manufacturer's complete order. The manufacturer is sure with probability rl_2 that it can provide the retailer's order and the retailer is sure with probability rl_1 that its product stock quantity will fulfill the market demand. The SC's service level is: $sl = rl_1.rl_2.rl_3$. In Operationally fail-safe SCs, not only determining the optimal sl is important, but also it is necessary to determine the optimal reliability level combination, (rl_1, rl_2, rl_3), to preserve that service level.

Based on the probability distribution function of the market's demand and chosen reliability level rl_1, the retailer selects the best *x* product order quantity from the manufacturer. SC's manufacturer receives an *x* product order from the retailer, but due to the probability of defective product production in its own manufacturing system, the manufacturer plans to manufacture extra product Δx and orders $x + \Delta x$ components from the supplier. This protects the SC against propagated uncertainty in the demand and the manufacturer's production system. The supplier receives $x + \Delta x$ order from the manufacturer. To compensate its imperfect production system, the supplier produces $\Delta \acute{x}$ more components. Δx and $\Delta \acute{x}$ are determined by the stochasticity in the production systems and reliability levels rl_2 and rl_3, and protect the SC against the propagated effects of the uncertainties (Figure 2-1). Therefore, considering both demand- and supply-

side variations in SCs leads to uncertainty propagation through their networks which should be quantified for service level estimation.

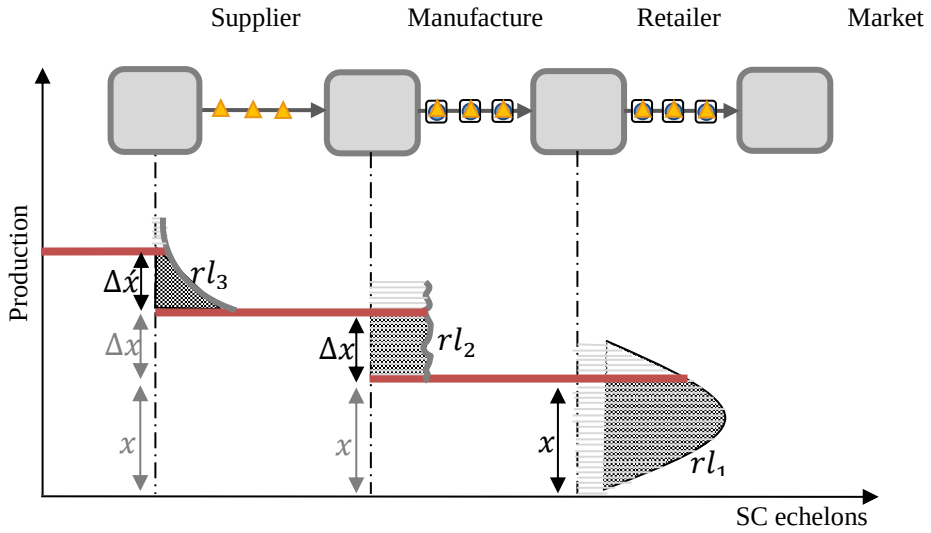


Figure 2-1: Uncertainty propagation in the SC.

In Sections 2.1.1, 2.1.2 and 2.1.3, the SC's facilities are considered separately step by step from downstream to upstream and production planning for each facility is discussed. The results of these sections are aggregated and formulated into a comprehensive mathematical model in Section 2.1.4. The notation used in formulating this problem is summarized in Table 2-1.

2.1.1. Reliable production planning in the supply chain's retailer

Demand of the SC is a stochastic function of its service level and retail price. The service level is a fraction of the market's realized demand that can be satisfied from the retailer's on-hand inventory.

The expected market demand, $D(sl, p)$ is an increasing function of the chain's service level, sl, and a decreasing function of the retail price, p. However the actual demand, $\widehat{D}(sl, p)$, is stochastic. According to Bernstein and Federgruen (2004 and 2007) demand

is formulated as $\widehat{D}(sl,p) = D(sl,p) \times \varepsilon$. ε is a general continuous random variable with $G(\varepsilon)$ cumulative distribution function independent of the SC's service level and price. Without loss of generality, $E(\varepsilon)$ is normalized to $E(\varepsilon) = 1$ which implies $E(\widehat{D}(sl,p)) = D(sl,p)$. The price of the product is fixed in the market. The retailer's order is released and fulfilled by the manufacturer before the beginning of the planning period. After realizing the period's real demand, unit holding cost, h^+, and unit shortage cost, h^-, are paid by the retailer for each end-of-period inventory or backlogged demand. Then, the total cost (summation of inventory holding and shortage costs) of the retailer, Π, is:

$$MIN \qquad \Pi = h^+ . E\left[x - \widehat{D}(sl,p)\right]^+ + h^- . E\left[\widehat{D}(sl,p) - x\right]^+ \tag{2-1}$$

$$S.T. \qquad \Pr[\widehat{D}(sl,p) \leq x] \geq rl_1 \tag{2-2}$$

Table 2-1: Notation for the SC problem.

$\widehat{D}(sl,p)$	Demand of the SC's market as a function of its service level
$D(sl,p)$	Expected demand of the SC's market
ε	Continuous random variable representing the uncertain part of the demand function
$G(\varepsilon)$	Cumulative distribution function of ε
p	Price of the product in the market
h^+	Unit holding cost in the SC's retailer
h^-	Unit shortage cost in the SC's retailer
Π	Expected total cost of the retailer
β	Maximum wastage ratio in the production system of the SC's manufacturer
$\acute{G}(.)$	Cumulative distribution function of wastage in the production system of the SC's manufacturer
μ	Rate of shifting to an out-of-control state in the SC's supplier
γ	Percentage of defect production in the out-of-control state of the supplier
N	Available production schemes in the supplier, $N = \{n_i, i = 1,2, \dots, \lvert N \rvert\}$
a_1	Unit procurement cost in the SC's supplier
a_2	Unit production cost in the SC's supplier
a_3	Set up cost in the SC's supplier
h_1	Unit inventory cost for a time unit in the SC's supplier
b_1	Unit transportation cost from the supplier to the manufacturer
b_2	Unit manufacturing cost of the SC's manufacturer
h_2	Unit inventory cost for a time unit in the SC's manufacturer
c_1	Unit transportation cost from the manufacturer to the retailer
c_2	Unit handling cost in the SC's retailer
PR_1	Production rate in the SC's supplier

PR_2	Production rate in the SC's manufacturer
SL	Set of scenarios defined for the service level of the SC, $SL = \{sl^1, sl^2, \dots, sl^{\lvert SL \rvert}\}$
RL^{sl^i}	Set of scenarios defined for service level sl^i distribution among the SC's echelons as their reliability levels, $RL^{sl^i} = \left\{RL_1^{sl^i} = (rl_{11}^{sl^i}, rl_{12}^{sl^i}, rl_{13}^{sl^i}),\ RL_2^{sl^i} = (rl_{21}^{sl^i}, rl_{22}^{sl^i}, rl_{23}^{sl^i}), \dots,\ RL_{\lvert RL^{sl^i} \rvert}^{sl^i} = (rl_{\lvert RL^{sl^i} \rvert_1}^{sl^i}, rl_{\lvert RL^{sl^i} \rvert_2}^{sl^i}, rl_{\lvert RL^{sl^i} \rvert_3}^{sl^i})\right\}$
Variables	
rl_1	Reliability level in the SC's retailer
rl_2	Reliability level in the SC's manufacturer
rl_3	Reliability level in the SC's supplier
sl	SC's Service level in the market
x	Ordering volume of the retailer from the manufacturer
Δx	Extra production in the manufacturer to compensate for the wastage in its production system
$\Delta \acute{x}$	Extra production units in the supplier to compensate the wastage of its production system
y_i	1 if production scheme n_i is selected by the supplier; otherwise 0
z_{sl^i}	1 if scenario sl^i ($\forall sl^i \in SL$) is selected as the service level of the SC, otherwise 0
$w_{RL_j^{sl^i}}$	1 if scenario $RL_j^{sl^i} = (rl_{j1}^{sl^i}, rl_{j2}^{sl^i}, rl_{j3}^{sl^i})$ ($\forall RL_j^{sl^i} \in RL^{sl^i}$) is selected to distribute service level sl^i among the SC's echelons as their reliability levels, 0 otherwise

In this model *x* represents the ordering quantity of retailer from manufacturer. The ordering volume $x = D(sl,p).G^{-1}(\frac{h^-}{h^-+h^+})$ minimizes the retailer's expected cost. To conserve the reliability level of the retailer we should have $x \geq D(sl,p).G^{-1}(rl_1)$, so the best order is:

$$x = D(sl,p).G^{-1}\left(Max\ \left\{rl_1, \frac{h^-}{h^-+h^+}\right\}\right) \tag{2-3}$$

By substituting equation (2-3) into (2-1), the cost of the retailer can be rewritten as follows:

$$\Pi = \left(h^+.E\left[G^{-1}\left(Max\ \left\{rl_1, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ + h^-.E\left[\varepsilon - G^{-1}\left(Max\ \left\{rl_1, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right).D(sl,p) \tag{2-4}$$

Thus, the SC's retailer, by ordering $x = D(sl,p).G^{-1}\left(Max\left\{rl_1, \frac{h^-}{h^-+h^+}\right\}\right)$ products from the manufacturer will be sure with rl_1 probability that its product stock will fulfill all the realized demand. In Figure 2-2 a sample probability distribution function is assumed for the market's demand. As shown in this figure, ordering quantity *x* should be selected in a way that the probability of the market's demand is equal or less than x is rl_1. The approach of this section explains how demand-side variations should be dealt in operationally fail-safe SCs.

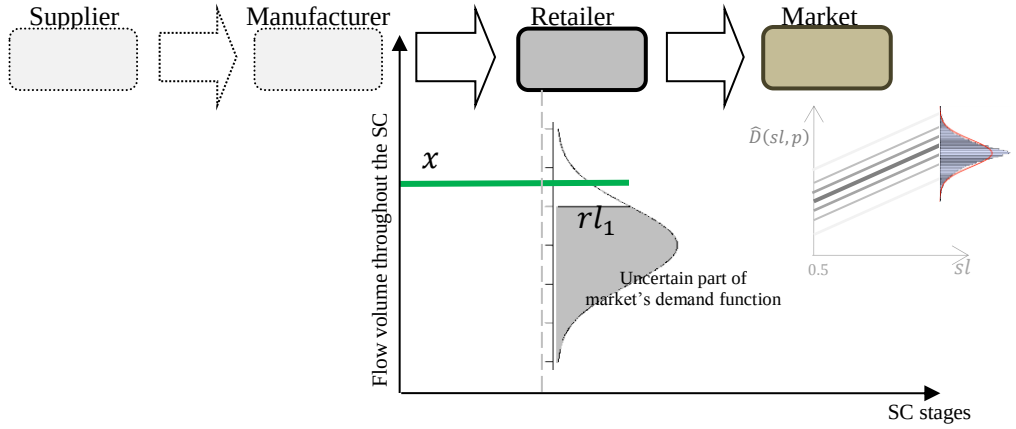


Figure 2-2: Order volume of the retailer based on its reliability level.

2.1.2. Reliable production planning in the supply chain's manufacturer

The SC's manufacturer receives an order of *x* products from the retailer. Without loss of generality, it is assumed that a single unit of component is required per product. The production system of the manufacturer always has some wastage which is determined by the general state of its machinery and varies in range [0, β%] with a cumulative distribution function *G'*(.).

A manufacturer should compensate for the wastage by manufacturing more products and consequently ordering more components from the supplier. Thus, component ordering and production volumes of the manufacturer include a surplus, Δx. If the manufacturer produces *x* units, this batch may contain $\Delta x \in [0, x.\beta\%]$ flawed units. To compensate for this wastage, the manufacturer orders $\Delta x + x$ units from the SC's supplier. Increasing Δx improves the probability of the manufacturer to fulfill all *x* product ordered by the retailer; this is its reliability level, rl_2. If the rl_2 reliability level is assigned to the manufacturer, the manufacturer should order $x.G'^{-1}(rl_2) + x$ units from the supplier, Figure 2-3. Thus $x.G'^{-1}(rl_2)$ surplus order and production quantity of the manufacturer preserves rl_2 reliability level for the manufacturer against the variation in its production system.

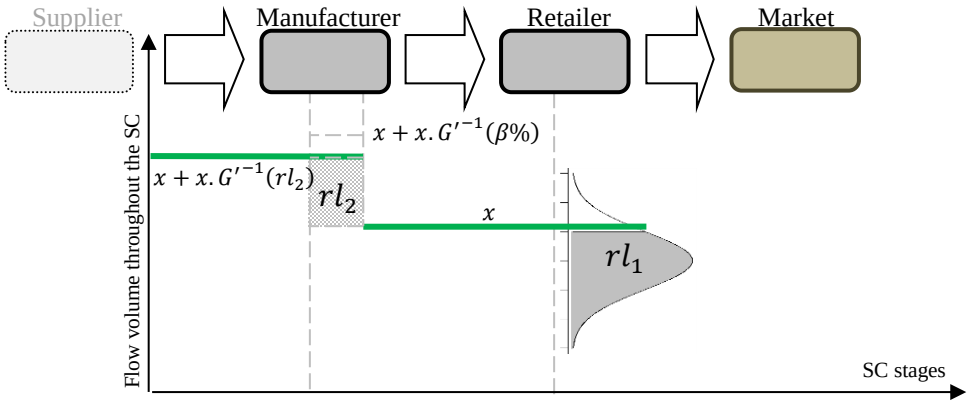


Figure 2-3: Order volume from the supplier against the retailer's and manufacturer's propagated uncertainties.

2.1.3. Reliable production planning in the supply chain's supplier

It is assumed that SC's supplier can use $|N|$ possible schemes to produce the order for the manufacturer, $N = \{1, 2, \dots, |N|\}$. The binary variable y_i is defined as the production scheme selection which is equal to 1 if production scheme $i \in \{1, 2, \dots, |N|\}$ is selected

by the supplier; otherwise 0. $y_i = 1$ means that manufacturer's order is divided into *i* equal parts and these parts are produced in *i* runs after setting up the machinery. Therefore $\sum_{i=1}^{|N|} y_i = 1$. Using the assumptions of Rosenblatt and Lee (1986) and Lee and Rosenblatt (1987) about the production process of the supplier, after setting up machinery, production runs start in the in-control state. But the machinery starts to deteriorate and become out-of-control after a stochastic while with an exponential distribution with a mean $1/\mu$. All the product units produced in the in-control state are satisfactory but γ percent of those produced in the out-of-control state are defective. Once the process shifts to the out-of-control state, it stays in this state until the batch is completed because interrupting the run is either impossible or expensive. Hence, the first production scheme, $y_1 = 1$, which produces the whole order at once, has lower set up costs but leads to greater numbers of flawed units in the output and the other schemes (producing the order in $i > 1$ runs) reduces the flawed product units at the price of higher set-up cost.

Therefore, in each run of the supplier's production system the number of flawless components to be produced is $\frac{\Delta x + x}{\sum_{i=1}^{|N|} y_i . i}$. But to compensate for flawed components, the supplier produces more components $\frac{\Delta \acute{x} + \Delta x + x}{\sum_{i=1}^{|N|} y_i . i}$. An extra volume $\Delta \acute{x}$ is added to the production system of the supplier to replace the defective component units. If it is assumed that the production rate of the supplier is PR_1, it will take $\frac{\Delta \acute{x} + \Delta x + x}{\left(\sum_{i=1}^{|N|} y_i . i\right) . PR_1}$ time units to produce this volume. $\Delta \acute{x}$ preserves the reliability level rl_3 of the supplier:

$$rl_3 = \Pr(flawless\ product\ unit\ in\ \frac{\Delta \acute{x} + \Delta x + x}{\left(\sum_{i=1}^{|N|} y_i . i\right) . PR_1}\ time\ unit \geq \frac{\Delta x + x}{\sum_{i=1}^{|N|} y_i . i})$$

$$= \Pr\left[PR_1.t + (1-\gamma).PR_1.\left(\frac{\Delta\acute{x} + \Delta x + x}{\left(\sum_{i=1}^{|N|} y_i.i\right).PR_1} - t\right) \geq \frac{\Delta x + x}{\sum_{i=1}^{|N|} y_i.i}\right]$$

$$= \Pr\left[t \geq \left(\frac{\Delta x + x}{\sum_{i=1}^{|N|} y_i.i}\right).\frac{1}{PR_1} - \left(\frac{1-\gamma}{\gamma.PR_1}\right).\left(\frac{\Delta\acute{x}}{\sum_{i=1}^{|N|} y_i.i}\right)\right]$$

$$= EXP\left[-\mu.\left(\left(\frac{\Delta x+x}{\sum_{i=1}^{|N|} y_i.i}\right).\frac{1}{PR_1} - \left(\frac{1-\gamma}{\gamma.PR_1}\right).\left(\frac{\Delta\acute{x}}{\sum_{i=1}^{|N|} y_i.i}\right)\right)\right] \quad (2\text{-}5)$$

Based on Equation (2-5), $\Delta\acute{x} = \frac{\gamma}{1-\gamma}\left[\frac{PR_1.(\sum_{i=1}^{|N|} y_i.i)}{\mu}\ln(rl_3) + (\Delta x + x)\right]$ units extra production in the supplier with $\sum_{i=1}^{|N|} y_i.i$ production scheme ensures rl_3 reliability. By producing $\Delta\acute{x}$, the supplier is able to fulfill the entire manufacturer's order with rl_3 probability.

In Figure 2-4 the probability function of qualified components in the production system of the SC's supplier is shown. The extra production $\Delta\acute{x}$ should be selected in a way that the probability of having $\Delta x + x$ qualified output equals rl_3. By producing $\Delta\acute{x}$ extra components the supplier will be able to fulfill the whole order of the manufacturer with probability rl_3, with an Δx extra product production the manufacturer will be able to fulfill the whole order of the retailer with probability rl_2 and this amount of product stock in the retailer will allow responding to the market's demand with rl_1 probability and $\Delta\acute{x} + \Delta x + x$ production volume in the supplier leads to a volume of product in the retailer that can respond to the market's demand with $rl_1.rl_2.rl_3$ probability and preserves service level $sl = rl_1.rl_2.rl_3$ for the whole SC. This attracts $\widehat{D}(rl_1.rl_2.rl_3, p)$ demand for the SC. This leads to the following equations for x, Δx and $\Delta\acute{x}$:

$$x = D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^-}{h^- + h^+}\right\}\right) \quad (2\text{-}6)$$

$$\Delta x = G'^{-1}(rl_2).D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^-}{h^- + h^+}\right\}\right) \quad (2\text{-}7)$$

$$\Delta \acute{x} = \frac{\gamma}{1-\gamma}\left[\frac{PR_1.(\sum_{i=1}^{|N|} y_i.i)}{\mu}\ln(rl_3) + \left(G'^{-1}(rl_2) + 1\right).D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^-}{h^- + h^+}\right\}\right)\right] \quad (2\text{-}8)$$

The approach of Section 2.1.2 and 2.1.3 explains how supply-side variations should be dealt in operationally fail-safe SCs.

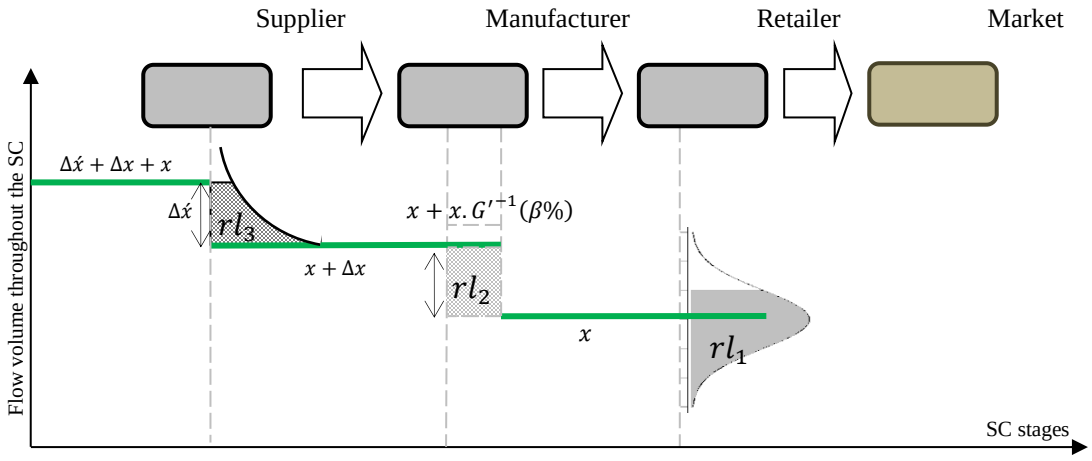


Figure 2-4: Production volume of the supplier based on the whole SC's propagated uncertainties.

2.1.4. Mathematical model for reliable flow planning in the supply chain

In Sections 2.1.1, 2.1.2 and 2.1.3, we found the relationship between the reliability levels of the SC's entities and their production levels. The appropriate selection of reliability levels is important because it determines service level and its captured demand and income and affects the SC's production levels and manufacturing cost. A mathematical model of reliable production planning determines the SC's best reliability levels by

considering the tradeoff between captureable income and manufacturing cost. The mathematical model of this problem is formulated:

$$\boldsymbol{Max}\quad Z = \left(p - h^{+}.E\left[G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right).D(rl_1.rl_2.rl_3, p) - \left[a_1.(x+\Delta x+\Delta\acute{x}) + a_2.(x+\Delta x+\Delta\acute{x}) + a_3.\left(\sum_{i=1}^{|N|} y_i\,.\,i\right) + \frac{h_1.(x+\Delta x)^2}{2.PR_1.\left(\sum_{i=1}^{|N|} y_i.i\right)^2} + b_1.(x+\Delta x) + b_2.(x+\Delta x) + \frac{h_2.(x)^2}{2.PR_2} + c_1.x + c_2.x\right] \tag{2-9}$$

Where

$$x = D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \tag{2-10}$$

$$\Delta x = G'^{-1}(rl_2).D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \tag{2-11}$$

$$\Delta\acute{x} = \frac{\gamma}{1-\gamma}\left[\frac{PR_1.(\sum_{i=1}^{|N|} y_i.i)}{\mu}\ln(rl_3) + \left(G'^{-1}(rl_2) + 1\right).D(rl_1.rl_2.rl_3, p).G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right] \tag{2-12}$$

Subject to

$$\sum_{i=1}^{|N|} y_i = 1 \tag{2-13}$$

$$0 \le rl_1,\ rl_2,\ rl_3 \le 1 \tag{2-14}$$

$$y_i \in \{0,1\} \qquad (\forall i \in \{1,2,\dots,|N|\}) \tag{2-15}$$

In this objective function, the total profit of the SC is maximized. The first term addresses the profit of the chain in the market by selling the supplied products and their

corresponding shortage and extra inventory costs. The first and second parts of the second term are the procurement and production costs in the SC's supplier. The third and fourth parts are the set up cost of the supplier's machinery and their inventory holding costs. The fifth and sixth parts are the transportation costs from the supplier to the manufacturer and the manufacturing cost in the manufacturer. The seventh, eighth and ninth parts are inventory holding costs in the manufacturer, transportation cost from the manufacturer to the retailer and handling costs in the retailer. Equations (2-10) - (2-12), as shown in the previous sections, specify the relationship between production quantities in the SC's facilities and their reliability levels. Based on constraint (2-13), only one production scheme in the chain's supplier is selected. This is a mixed integer nonlinear model with a highly nonlinear objective function. In the next section, an approach is proposed to solve this model.

Notice that there are some critical functions in this model such as $D(sl, p)$ function and cumulative distribution functions (G and G') used to quantify variation in different echelons' facilities. To implement the model of this chapter in reality these functions should be identified appropriately. Usually historical data of the same or different but similar product can be used to identify $D(sl, p)$ function. For example by having historical triples of (*demand*, *price*, *service level*) we can find the best fitting $D(sl, p)$ function by using different statistical approaches such as regression. By having nonconforming production rate of a facility in the previous production periods, statistical methods such as "goodness of fit" can be used to fit the best cumulative distribution function to quantify the uncertainty of its production system.

2.1.5. Solution for the supply chain's reliable flow planning model

In this section a solution approach is proposed for the model in the previous section. Important continuous design variables in this model are rl_1, rl_2 and rl_3 which take values on the [0, 1] interval or, it would be more rational to assume, the [0.5, 1.0] interval. Having range-restricted design variables makes discretization an efficient solution approach. After discretization, nonlinear parts of the model's objective function become linear ones. Linear models are very well-formed mathematical models and can be solved globally. To discretize the model we define $SL = \{sl^1, sl^2, \dots, sl^{|SL|}\}$, a set of scenarios for the SC's service level. For each member of SL, a set of reliability levels is defined to preserve that service level for the SC:

$$RL^{sl^i} = \left\{ RL_1^{sl^i} = (rl_{11}^{sl^i}, rl_{12}^{sl^i}, rl_{13}^{sl^i}),\ RL_2^{sl^i} = (rl_{21}^{sl^i}, rl_{22}^{sl^i}, rl_{23}^{sl^i}), \dots,\ RL_{\left|RL^{sl^i}\right|}^{sl^i} = \right.$$

$$\left. (rl_{\left|RL^{sl^i}\right|_1}^{sl^i}, rl_{\left|RL^{sl^i}\right|_2}^{sl^i}, rl_{\left|RL^{sl^i}\right|_3}^{sl^i}) \right\}.$$

z_{sl^i} $(\forall sl^i \in SL)$ and $w_{RL_j^{sl^i}}$ $(\forall RL_j^{sl^i} \in RL^{sl^i})$ are new binary design variables to select service level, sl^i, and reliability level distribution, $RL_j^{sl^i}$. By defining the above new design variables, the following terms in the mathematical mode of the problem can be revised as:

$$x = \sum_{SL} \sum_{RL^{sl^i}} z_{sl^i}.w_{RL_j^{sl^i}}.\left[D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i}, p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i}, \frac{h^-}{h^- + h^+}\right\}\right)\right] \tag{2-16}$$

$$x^2 = \sum_{SL} \sum_{RL^{sl^i}} z_{sl^i}.w_{RL_j^{sl^i}}.\left[D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i}, p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i}, \frac{h^-}{h^- + h^+}\right\}\right)\right]^2$$

(2-17)

$$\Delta x =$$

$$\sum_{SL}\sum_{RL^{sl^i}} z_{sl^i}.w_{RL_j^{sl^i}}.\left[\left(G^{'-1}\left(rl_{j2}^{sl^i}\right).D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i},\frac{h^-}{h^-+h^+}\right\}\right)\right]\right.$$

(2-18)

$$\Delta x + x = \sum_{SL}\sum_{RL^{sl^i}} z_{sl^i}.w_{RL_j^{sl^i}}.\left[\left(G^{'-1}\left(rl_{j2}^{sl^i}\right)\right.\right.$$

$$\left.\left.+1\right).D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i},\frac{h^-}{h^-+h^+}\right\}\right)\right]$$

(2-19)

$$(\Delta x + x)^2 = \sum_{SL}\sum_{RL^{sl^i}} z_{sl^i}.w_{RL_j^{sl^i}}.\left[\left(G^{'-1}\left(rl_{j2}^{sl^i}\right)\right.\right.$$

$$\left.\left.+1\right).D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i},\frac{h^-}{h^-+h^+}\right\}\right)\right]^2$$

(2-20)

$$\Delta\acute{x} + \Delta x + x = \sum_{N}\sum_{SL}\sum_{RL^{sl^i}} y_k.z_{sl^i}.w_{RL_j^{sl^i}}.\frac{\gamma}{1-\gamma}\left[\frac{PR_1.k}{\mu}\ln\left(rl_{j3}^{sl^i}\right)\right.$$

$$+\left(G^{'-1}\left(rl_{j2}^{sl^i}\right)\right.$$

$$\left.\left.+\ 1\right).D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right).G^{-1}\left(Max\left\{rl_{j1}^{sl^i},\frac{h^-}{h^-+h^+}\right\}\right)\right]$$

(2-21)

The first five of these equations are linear functions of $z_{sl^i}.w_{RL_j^{sl^i}}$ and the last term is a linear function of $y_k.z_{sl^i}.w_{RL_j^{sl^i}}$ $(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N)$. By defining $zw_{sl^i,RL_j^{sl^i}} = z_{sl^i}.w_{RL_j^{sl^i}}$ and $yzw_{k,sl^i,RL_j^{sl^i}} = y_k.z_{sl^i}.w_{RL_j^{sl^i}}$ Equations (2-16)-(2-21) become completely linear. However, the following constraints must be added:

$$\left(z_{sl^i} + w_{RL_j^{sl^i}} - 1\right) \leq zw_{sl^i,RL_j^{sl^i}} \leq \frac{z_{sl^i} + w_{RL_j^{sl^i}}}{2} \tag{2-22}$$

$$zw_{sl^i,RL_j^{sl^i}} \leq M.z_{sl^i} \tag{2-23}$$

$$zw_{sl^i,RL_j^{sl^i}} \leq M.w_{RL_j^{sl^i}} \tag{2-24}$$

$$zw_{sl^i,RL_j^{sl^i}} \in \{0,1\} \tag{2-25}$$

$$\left(y_k + z_{sl^i} + w_{RL_j^{sl^i}} - 2\right) \leq yzw_{k,sl^i,RL_j^{sl^i}} \leq \frac{y_k + z_{sl^i} + w_{RL_j^{sl^i}}}{3} \tag{2-26}$$

$$yzw_{k,sl^i,RL_j^{sl^i}} \leq M.z_{sl^i} \tag{2-27}$$

$$yzw_{k,sl^i,RL_j^{sl^i}} \leq M.w_{RL_j^{sl^i}} \tag{2-28}$$

$$yzw_{k,sl^i,RL_j^{sl^i}} \leq M.y_k \tag{2-29}$$

$$yzw_{k,sl^i,RL_j^{sl^i}} \in \{0,1\} \tag{2-30}$$

By substituting these equations into the mathematical model (2-9)-(2-15), the model becomes:

$$\textbf{Max}\quad Z = \sum_{SL}\sum_{RL^{sl^i}}[zw_{sl^i,RL_j^{sl^i}} \times D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times$$

$$\left(p - h^{+}.E\left[G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)-\varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right)]$$

$$-a_1.(x+\Delta x+\Delta\acute{x}) - a_2.(x+\Delta x+\Delta\acute{x}) - a_3.\left(\sum_{i=1}^{|N|} y_i.i\right)$$

$$-\frac{h_1(x+\Delta x)^2}{2.PR_1.(\sum_{i=1}^{|N|} y_i.i)^2} - b_1.(x+\Delta x) - b_2.(x+\Delta x) - \frac{h_2(x)^2}{2.PR_2} - c_1.x - c_2.x \quad (2\text{-}31)$$

Where

$$x = \sum_{SL}\sum_{RL^{sl^i}} zw_{sl^i,RL_j^{sl^i}}.\left[D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right] \quad (2\text{-}32)$$

$$x^2 = \sum_{SL}\sum_{RL^{sl^i}} zw_{sl^i,RL_j^{sl^i}}.\left[D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^2 \quad (2\text{-}33)$$

$$\Delta x + x = \sum_{SL}\sum_{RL^{sl^i}} zw_{sl^i,RL_j^{sl^i}}.\left[\left(G'^{-1}\left(rl_{j2}^{sl^i}\right)+1\right)\times D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right] \quad (2\text{-}34)$$

$$(\Delta x + x)^2 = \sum_{SL}\sum_{RL^{sl^i}} zw_{sl^i,RL_j^{sl^i}}.\left[\left(G'^{-1}\left(rl_{j2}^{sl^i}\right)+1\right)\times D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^2 \quad (2\text{-}35)$$

$$\Delta\acute{x} + \Delta x + x = \sum_{N}\sum_{SL}\sum_{RL^{sl^i}} yzw_{k,sl^i,RL_j^{sl^i}}.\frac{\gamma}{1-\gamma}\left[\frac{PR_1.n_k}{\mu}\ln\left(rl_{j3}^{sl^i}\right)+\left(G'^{-1}\left(rl_{j2}^{sl^i}\right)+1\right)\times D\left(rl_{j1}^{sl^i}.rl_{j2}^{sl^i}.rl_{j3}^{sl^i},p\right)\times G^{-1}\left(Max\ \left\{rl_{j1}^{sl^i},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right] \quad (2\text{-}36)$$

Subject to:

$$\sum_{i=1}^{|N|} y_i = 1 \tag{2-37}$$

$$\sum_{SL} z_{sl^i} = 1 \tag{2-38}$$

$$\sum_{RL^{sl^i}} w_{RL_j^{sl^i}} = z_{sl^i} \qquad \left(\forall RL_j^{sl^i} \in RL^{sl^i}, \forall sl^i \in SL\right) \tag{2-39}$$

$$\left(z_{sl^i} + w_{RL_j^{sl^i}} - 1\right) \leq zw_{sl^i, RL_j^{sl^i}} \leq \frac{z_{sl^i} + w_{RL_j^{sl^i}}}{2} \quad \left(\forall RL_j^{sl^i} \in RL^{sl^i}, \forall sl^i \in SL\right)$$

(2-40)

$$zw_{sl^i, RL_j^{sl^i}} \leq M . z_{sl^i} \qquad \left(\forall RL_j^{sl^i} \in RL^{sl^i}, \forall sl^i \in SL\right)$$

(2-41)

$$zw_{sl^i, RL_j^{sl^i}} \leq M . w_{RL_j^{sl^i}} \qquad \left(\forall RL_j^{sl^i} \in RL^{sl^i}, \forall sl^i \in SL\right)$$

(2-42)

$$\left(y_k + z_{sl^i} + w_{RL_j^{sl^i}} - 2\right) \leq yzw_{k, sl^i, RL_j^{sl^i}} \leq \frac{y_k + z_{sl^i} + w_{RL_j^{sl^i}}}{3}$$

$$\left(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N\right) \tag{2-43}$$

$$yzw_{k, sl^i, RL_j^{sl^i}} \leq M . z_{sl^i} \qquad \left(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N\right) \tag{2-44}$$

$$yzw_{k, sl^i, RL_j^{sl^i}} \leq M . w_{RL_j^{sl^i}} \qquad \left(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N\right) \tag{2-45}$$

$$yzw_{k, sl^i, RL_j^{sl^i}} \leq M . y_k \qquad \left(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N\right) \tag{2-46}$$

$$z_{sl^i}, w_{RL_j^{sl^i}}, y_i, zw_{sl^i, RL_j^{sl^i}}, yzw_{k, sl^i, RL_j^{sl^i}} \in \{0,1\}$$

$$\left(\forall sl^i \in SL,\ \forall RL_j^{sl^i} \in RL^{sl^i}, \forall k \in N\right) \tag{2-47}$$

The only nonlinear term in this model is $\frac{h_1(x+\Delta x)^2}{2.PR_1.(\sum_{i=1}^{|N|} y_i.i)^2}$ in the objective function. By the above substitutions its numerator is linearized and by using the same approach elaborated above for linearizing the $z_{sl^i}.w_{RL_j^{sl^i}}$ multiplication, the dominator can be linearized too. Thus, this term transforms into a linear fractional term. Several approaches have been proposed in the literature to linearize fractional linear terms. We utilized the approach proposed by Chang (2001). Based on constraint (2-38), only one service level scenario can be selected by the SC. According to constraint (2-39), only one reliability level distribution scenario for the selected service level can be selected. Thus the model becomes linear with binary design variables.

2.1.6. Example: Computational results

In this section, a sample SC is considered. The price of its product is \$12.00, holding cost of dead inventory at the end of planning period is \$0.30 and the cost of unmet demand is \$0.70. The SC's supplier procures the required material with a cost of a_1=\$2.50 and manufactures the component with a a_2=\$1.50 cost. The production rate is $PR_1 = 9000$ (components per time unit). The supplier's machinery has a setup cost $a_3 = \$100$, and starts in an in-control state. After an exponential time with $\mu = 2$ (average number of shifts in time unit), the machinery shifts to an out-of-control state with $\gamma = 20\%$ of non-conforming production. Qualified components are transported to the manufacturer with unit cost $b_1 = \$0.5$. The manufacturer produces the final product at a rate of $PR_2 = 8000$ (products per time unit), a unit manufacturing cost $b_2 = \$2.00$ and conveys them to the retailer with unit transportation cost $c_1 = \$1.00$. The manufacturer's production system has a wastage percentage uniformly distributed on [0, $\beta = 10\%$]. The retailer's unit handling costs are $c_2 = \$1.50$. Only one production scheme is possible for the supplier

and all planned material is produced at once, $|N| = 1$. Unit inventory holding costs of the supplier and manufacturer per unit time are $h_1 = \$0.40$ and $h_2 = \$0.40$ respectively. The stochastic part of the demand in the market, ε, follows a uniform distribution on the [0.7, 1.3] interval. As seen in Table 2-2, in this problem $SL = \{sl^1 = 0.82, sl^2 = 0.83, \dots, sl^{|SL|} = 0.93\}$. Reliability level sets for some of service level values are listed below:

- ✓ Reliability levels of facilities preserving $sl^1 = 0.82$ is $RL^{0.82} = \{RL_1^{0.82} = (rl_{11}^{0.82} = 0.91, rl_{12}^{0.82} = 0.91, rl_{13}^{0.82} = 0.99), RL_2^{0.82} = (rl_{21}^{0.82} = 0.91, rl_{22}^{0.82} = 0.99, rl_{23}^{0.82} = 0.91), RL_3^{0.82} = (rl_{31}^{0.82} = 0.99, rl_{32}^{0.82} = 0.91, rl_{33}^{0.82} = 0.91)\}$.
- ✓ Reliability levels of facilities preserving $sl^1 = 0.91$ is $RL^{0.91} = \{RL_1^{0.91} = (rl_{11}^{0.91} = 0.91, rl_{12}^{0.91} = 1.0, rl_{13}^{0.91} = 1.0), RL_2^{0.91} = (rl_{21}^{0.91} = 1.0, rl_{22}^{0.91} = 0.91, rl_{23}^{0.91} = 1.0), RL_3^{0.91} = (rl_{31}^{0.91} = 1.0, rl_{32}^{0.91} = 1.0, rl_{33}^{0.91} = 0.91)\}$.

The mathematical model of this problem ((2-31)-(2-47)) is formulated and solved on a Intel(R)Core(TM)4 Duo CPU, 3.6 GHz, with 12276 MB RAM using the default settings. CPLEX is used to solve the linearized mathematical model of the problem and it took less than 1 minute to solve it. The solution obtained is, $rl_1 = 1.0$, $rl_2 = 1.0$ and $rl_3 = 0.90$.

The SC's profit with respect to its service level is shown in Figure 2-6; it is equal to $rl_1.rl_2.rl_3$. When the service level is less than 0.90, incrementing the service level leads to higher profit. When the service level is 0.90, the SC has the greatest profit. Beyond 0.90, incrementing the service level leads to lower profit which means that the negative effect of service level augmentation on the system's cost is more than its positive effect

on the system's income. Thus 0.90 is the best choice for this SC (Table 2-2). However, finding the best service level is not enough. There are many rl_1, rl_2 and rl_3 combinations with $rl_1.rl_2.rl_3 = 0.90$ (see the white arrow in Figure 2-6) but the profit of the SC is different for each combination, Table 2-3. Formulating and solving the mathematical model of this problem helps us find the best combination of reliability levels in the different echelons of the SC (black dot in Figure 2-5). $rl_1 = 1.0$, $rl_2 = 1.0$ and $rl_3 = 0.90$ are the best reliability levels for service level 0.90 in this SC (Row 22 in Table 2-3).

Table 2-2: The best captureable profit in the SC with respect to its service level.

Service level	**0.82**	**0.83**	**0.84**	**0.85**	**0.86**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**
Profit	\$6056.7	\$6079.9	\$6105.6	\$6125.3	\$6146.6	\$6165.5	\$6179.3	\$6193.5	\$6199.6	\$6158.9	\$6118.6	\$6078.5

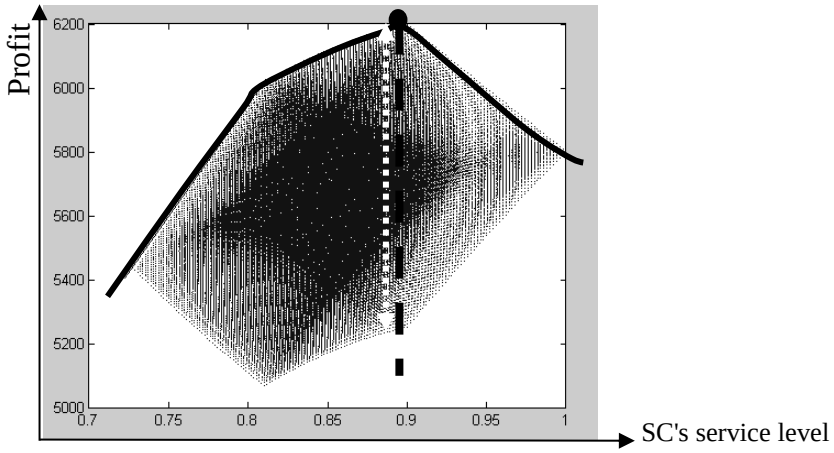


Figure 2-5: Profit of the SC with respect to its service level.

Table 2-3: Profit of the SC with respect to different reliability level combinations in service level 0.90.

Row	rl_1	rl_2	rl_3	Profit	Row	rl_1	rl_2	rl_3	Profit
1	0.970	0.940	0.985	$5637	2	0.970	0.945	0.980	$5662
3	0.970	0.955	0.970	$5712	4	0.970	0.960	0.970	$5720
5	0.970	0.975	0.950	$5812	6	0.980	0.915	1.000	$5600
7	0.980	0.920	0.995	$5626	8	0.980	0.940	0.975	$5729
9	0.985	0.910	1.000	$5619	10	0.985	0.930	0.980	$5723
11	0.990	0.905	1.000	$5638	12	0.990	0.910	0.995	$5665
13	0.990	0.920	0.990	$5701	14	0.990	0.930	0.975	$5769
15	0.995	0.900	1.000	$5657	16	0.995	0.915	0.985	$5736
17	0.995	0.925	0.975	$5789	18	0.995	0.945	0.955	$5892
19	1.000	0.900	0.995	$5702	20	1.000	0.900	1.000	$5687
21	1.000	0.910	0.985	$5755	22	1	1	0.900	$6199

2.2. Operations and variations in a forward supply network

In this section, we extend the problem to a three-echelon SC consisting of several suppliers, manufacturers and retailers which is called Supply Network (SN) henceforth (Figure 2-7). Retailers order their products before the beginning of each planning period. Manufacturers integrate the orders received from retailers and order the required components from suppliers and manufacture products and supply them to retailers.

As in Section 2.1, it is assumed that the demands of the markets are stochastic increasing functions of service levels and decreasing functions of retail price. In addition to this demand-side variation, it is assumed that the production systems of manufacturers always include a stochastic percentage of deficient output and the suppliers' production systems deteriorate after exponential times and shift from in-control to out-of-control state which leads to a percentage of nonconforming component production. The aim is to determine: (i) the most appropriate service level set for the SN to balance the costs of unmet demand and supply costs and (ii) the appropriate reliability level (and production

and ordering quantities) in the SN's facilities to provide reliable material and product flow to preserve the desired service levels and maximize the captureable profit .

2.2.1. Mathematical model for reliable flow planning in the supply network

To formulate the planning problem in the SN, we modify the approach developed for the SC with single-facility echelons. A SN is a composite of SCs with single-facility echelons. In this paper, the constituent SCs with single-facility echelons of the SN are called potential supply routes. Each route starts from a supplier in the third echelon and passes through a manufacturer in the second echelon and ends at a retailer and its market in the first echelon (Figure 2-6). For production planning in the SN the following decisions are needed:

i) Which potential routes should be selected?

ii) How many products should be supplied by each selected route?

After selection of routes and their assigned supply quantities, flow augmentation through each route due to propagated uncertainty is determined using the method of Section 2.1. The notation is in Table 2-4.

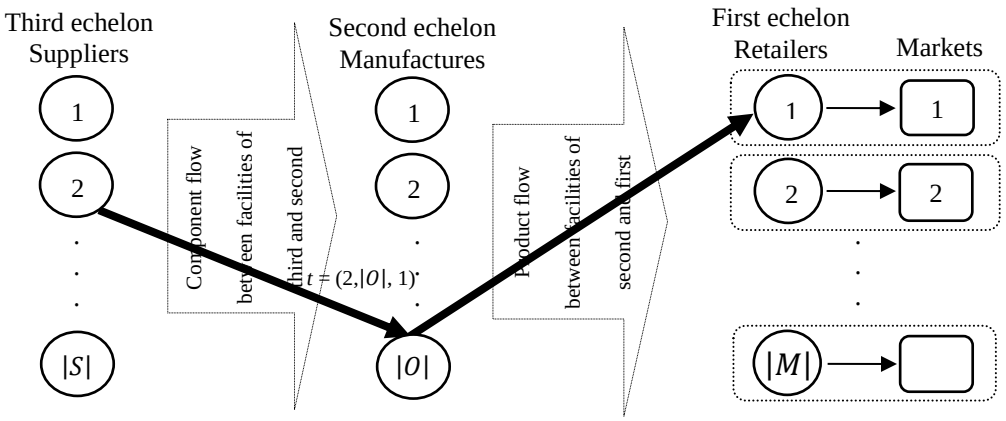


Figure 2-6: Network structure of the SN with multiple facilities in each echelon (sample potential route *t* = (2, $|O|$, 1) is shown in the SN).

Table 2-4: Notation for the SN problem.

S	Set of suppliers in the third echelon of the SN, $S = \{1, 2, \dots, |S|\}$
O	Set of manufacturers in the second echelon of the SN, $O = \{1, 2, \dots, |O|\}$
M	Set of markets and their corresponding retailers in the first echelon, $M = \{1, 2, \dots, |M|\}$
T	Set of potential routes in the SN. Each route, *t*, starts from a supplier, *s*, in the third echelon and passes a manufacturer, *o*, in the second echelon and ends to a retailer and its corresponding market, *m*, in the first echelon. So each potential route of this set is a triple of entities in different echelons: $t = (s, o, m)$ $(\forall t \in T)$, $T = \{1, 2, \dots, |T|\}$
T^s	Subset of SN's potential routes starting from supplier *s*, $T^s = \{t \in T | first\ element\ in\ triple\ t = s\}$
T^o	Subset of SN's potential routes passing through manufacturer *o*, $T^o = \{t \in T | second\ element\ in\ triple\ t = o\}$
T^m	Subset of SN's routes ending in market *m*, $T^m = \{t \in T | last\ element\ in\ triple\ t = m\}$
$\widehat{D}_m(sl_m, p_m)$	Demand in the SN's market *m* as a function of its service level and retail price, $\widehat{D}_m(sl_m, p_m) = D_m(sl_m, p_m) \times \varepsilon_m$ $(\forall m \in M)$
$D_m(sl_m, p_m)$	Expected demand in the SN's market *m* $(\forall m \in M)$
ε_m	Continuous random variable represents the uncertain part of demand at the SN's market *m* $(\forall m \in M)$
$G_m(\varepsilon_m)$	Cumulative distribution function of ε_m $(\forall m \in M)$
N_s	Available production schemes for the SN's supplier *s*, $N_s = \{n_{si}, i = 1,2, \dots, |N_s|\}$ $(\forall s \in S)$
a_{1s}	Unit procurement cost in the SN's supplier *s* $(\forall s \in S)$
a_{2s}	Unit production cost in the SN's supplier *s* $(\forall s \in S)$
a_{3s}	Set up cost in the SN's supplier *s* $(\forall s \in S)$
Ih_{1s}	Unit Inventory holding cost per time unit in the SN's supplier *s* $(\forall s \in S)$
b_{1so}	Unit transportation cost from the supplier *s* to the manufacturer *o* $(\forall s \in S, \forall o \in O)$
b_{2o}	Unit manufacturing cost in the SN's manufacturer *o* $(\forall o \in O)$
Ih_{2o}	Unit Inventory holding cost per time unit in the SN's manufacturer *o* $(\forall o \in O)$
h_m^+	Unit holding cost of extra inventory in retailer *m* at the end of planning period $(\forall m \in M)$
h_m^-	Unit shortage cost of unmet demand in retailer *m* at the end of planning period $(\forall m \in M)$

c_{1om}	Unit transportation cost from manufacturer *o* to retailer *m* ($\forall o \in O, \forall m \in M$)
c_{2m}	Unit handling cost in the SN's retailer *m* ($\forall m \in M$)
p_m	Price of product in the SN's market *m* ($\forall m \in M$)
PR_{1s}	Production rate of the SN's supplier *s* ($\forall s \in S$)
PR_{2o}	Production rate of the SN's manufacturer *o* ($\forall o \in O$)
β_o	Maximum rate of flawed product production in the SN's manufacturer *o* ($\forall o \in O$)
$\acute{G}_0(.)$	Cumulative distribution of flawed production in the SN's manufacturer *o* ($\forall o \in O$)
μ_s	Rate of shifting to the out-of-control state in the SN's supplier *s* ($\forall s \in S$)
γ_s	Percentage of nonconforming production in the out-of-control state of supplier *s* ($\forall s \in S$)
SL_m	Set of service level scenarios for market *m*, $SL_m = \left\{sl_m^1, sl_m^2, \dots, sl_m^{\lvert SL_m \rvert}\right\} (\forall m \in M)$
$RL_{sl_m^i}$	Set of scenarios defined for service level sl_m^i distribution among the SN's entities in different echelons as their reliability levels, $RL_{sl_m^i} = \left\{rl_{sl_m^i}^1 = (rl1_{sl_m^i}^1, rl2_{sl_m^i}^1, rl3_{sl_m^i}^1), \dots, rl_{sl_m^i}^{\left\lvert RL_{sl_m^i} \right\rvert} = (rl1_{sl_m^i}^{\left\lvert RL_{sl_m^i} \right\rvert}, rl2_{sl_m^i}^{\left\lvert RL_{sl_m^i} \right\rvert}, rl3_{sl_m^i}^{\left\lvert RL_{sl_m^i} \right\rvert})\right\}$. In all scenarios $rl1_{sl_m^i}^j . rl2_{sl_m^i}^j . rl3_{sl_m^i}^j = sl_m^i \ \left(\forall j = 1, 2, \dots, \left\lvert RL_{sl_m^i} \right\rvert\right)$;
PER_m^t	Set of scenarios defined for the percentage of market *m*'s demand that can be assigned to path *t* ($\forall m \in M, \forall t \in T^m$), $PER_m^t = \left\{per_{m1}^t, per_{m2}^t, \dots, per_{m\lvert PER_m^t \rvert}^t\right\}$
Variables	
rl_{1m}	Reliability level in the SN's retailer *m* ($\forall m \in M$)
rl_{2o}	Reliability level in the SN's manufacturer *o* ($\forall o \in O$)
rl_{3s}	Reliability level in the SN's supplier *s* ($\forall s \in S$)
sl_m	Service level of the SN in market *m* ($\forall m \in M$)
x_t	Quantity of product supplied by route *t* to market *m* ($\forall m \in M, \forall t \in T^m$)
$x_t + \Delta x_t$	Production quantity in the manufacturer of route *t* to fulfill its order ($\forall t \in T$)
$x_t + \Delta x_t$ $+ \Delta \acute{x}_t$	Production quantity in the supplier of route *t* to fulfill the order of its manufacture ($\forall t \in T$)
y_t	1 if route *t* is selected to supply products to market *m*; 0 otherwise ($\forall m \in M, \forall t \in T^m$)
z_{si}	1 if production scheme *i* is selected by supplier *s*; 0 otherwise ($\forall s \in S, \forall i \in N_s$)
v_m^i	1 if service level scenario sl_m^i is selected for market *m*; 0 otherwise ($\forall m \in M, \forall sl_m^i \in SL_m$)
$w_{sl_m^i,t}^j$	1 if service level distribution scenario $rl_{sl_m^i}^j = (rl1_{sl_m^i}^j, rl2_{sl_m^i}^j, rl3_{sl_m^i}^j)$ is selected by route *t* to provide service level sl_m^i to its market *m* ($\forall m \in M, \forall t \in T^m, \forall sl_m^i \in SL_m, \forall rl_{sl_m^i}^j \in RL_{sl_m^i}$); 0 otherwise
$\acute{y}_{mk}^t$	1 if scenario per_{mk}^t is selected as the market *m*'s percentage of demand assigned to route *t* ($\forall m \in M, \forall t \in T^m, \forall per_{mk}^t \in PER_m^t$); 0 otherwise

In each market there is a stochastic demand $\widehat{D}_m(sl_m, p_m) = D_m(sl_m, p_m).\varepsilon_m$ which is an increasing function of the retailer's service level, sl_m, and a decreasing function of retail price, p_m. ε_m is the stochastic part of demand and is a random variable with $G_m(.)$ cumulative density function. The mathematical model of this problem is:

$$Max \quad Z = \{\sum_{m=1}^{|M|}(p_m - h_m^+.E\left[G_m^{-1}\left(Max\left\{rl_{1m}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right) - \varepsilon_m\right]^+$$

$$- h_m^-.E\left[\varepsilon_m - G_m^{-1}\left(Max\left\{rl_{1m}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right)\right]^+) \times$$

$$D_m\left(\prod_{t=1}^{|T^m|} sl_m.y_t + (1 - y_t), p_m\right)\} - \sum_{s=1}^{|S|}(a_{1s} + a_{2s}).\sum_{t=1}^{|T^S|}(x_t + \Delta x_t + \Delta\acute{x}_t) -$$

$$\sum_{s=1}^{|S|} a_{3s}.\left(\sum_{i=1}^{|N_s|} z_{si}.i\right) - \sum_{s=1}^{|S|}\frac{Ih_{1s}.\left(\sum_{t=1}^{|T^S|} x_t + \Delta x_t\right)^2}{2.PR_{1s}.\left(\sum_{i=1}^{|N_s|} z_{si}.i\right)^2} - \sum_{s=1}^{|S|}\sum_{0=1}^{|O|}\sum_{t=1}^{|T^S \cap T^O|} b_{1so}.(x_t +$$

$$\Delta x_t) - \sum_{o=1}^{|O|}\sum_{t=1}^{|T^O|} b_{20}.(x_t + \Delta x_t) - \sum_{o=1}^{|O|}\frac{Ih_{2o}\left(\sum_{t=1}^{|T^O|} x_t\right)^2}{2.PR_{2o}}\sum_{o=1}^{|O|}\sum_{m=1}^{|M|}\sum_{t=1}^{|T^O \cap T^m|} c_{1om}.x_t -$$

$$\sum_{m=1}^{|M|} c_{2m}.\sum_{t=1}^{|T^m|} x_t$$

(2-48)

Where

$$\Delta x_t = \acute{G_0}^{-1}(rl_{2o}).x_t \qquad (\forall 0 \in O, \forall t \in T^o) \qquad (2\text{-}49)$$

$$\Delta\acute{x}_t = \frac{\gamma_s}{1-\gamma_s}\left[\frac{PR_{1s}.\left(\sum_{i=1}^{|N_s|} z_{si}.i\right)}{\mu_s}\ln(rl_{3s}) + \Delta x_t + x_t\right].y_t \quad (\forall s \in S, \forall t \in T^S) \qquad (2\text{-}50)$$

S.T.

$$x_t \leq M.y_t \qquad (\forall t \in T) \qquad (2\text{-}51)$$

$$\sum_{t=1}^{|T^m|} y_t \geq 1 \qquad (\forall m \in M) \qquad (2\text{-}52)$$

$$\sum_{t=1}^{|T^m|} x_t = D_m(sl_m, p_m).G_m^{-1}\left(Max\left\{rl_{1m}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right) \quad (\forall m \in M) \qquad (2\text{-}53)$$

$$\sum_{i=1}^{|N_s|} z_{si} = 1 \qquad (\forall s \in S) \qquad (2\text{-}54)$$

$$y_t.(rl_{3s}.rl_{2o}.rl_{1m}) \leq sl_m \leq y_t.(rl_{3s}.rl_{2o}.rl_{1m}) + (1 - y_t)$$

$$(\forall m \in M, \forall t \in T^m, t = (s, o, m)) \qquad (2\text{-}55)$$

$$0 \leq rl_{1m} \leq 1 \qquad (\forall m \in M) \qquad (2\text{-}56)$$

$$0 \leq rl_{2o} \leq 1 \qquad (\forall o \in O) \qquad (2\text{-}57)$$

$$0 \leq rl_{3s} \leq 1 \qquad (\forall s \in S) \qquad (2\text{-}58)$$

$$y_t, z_{si} \in \{0,1\} \qquad (\forall t \in T, \forall s \in S, \forall i \in N_s) \qquad (2\text{-}59)$$

$$sl_m,\ x_t,\ \Delta x_t,\ \Delta \acute{x}_t\ \geq 0 \qquad (\forall s \in S, \forall m \in M, \forall o \in O, \forall t \in T) \qquad (2\text{-}60)$$

In the objective function the total profit is maximized. The income of the SN after discarding the shortage cost of unmet demand and the inventory holding cost of dead inventory in the retailers is computed with the first term. The second term is the sum of procurement and production cost in the SN's suppliers. Production volume in supplier *s* is the sum of propagated flows in the selected routes originating from that supplier, $\sum_{t=1}^{|T^s|}(x_t + \Delta x_t + \Delta \acute{x}_t)$. The third and fourth terms address the sums of set up costs in suppliers and their inventory holding costs. The fifth and sixth terms are the sum of transportation costs of components from suppliers to manufacturers and the sum of manufacturing costs in the SN's manufacturers. The seventh, eighth and ninth terms are the sum of inventory holding costs in the manufacturers, transportation costs of products from manufacturers to retailers and retailers' handling costs. The production quantity augmentation in each potential route of the SN with respect to the reliability levels of the facilities throughout that route are given in Equations (2-49)-(2-50).

Based on constraint (2-51), a product can flow only through the selected routes of the SN. According to constraint (2-52), at least one of the potential routes ending in each market is selected. Constraint (2-53) requires that the demand of each market is fulfilled by the flow in routes ending in that market. Based on constraint (2-54), one production scheme is selected for each supplier. Constraint (2-55) ensures that the reliability levels

of the facilities in the selected routes ending in a market will preserve that market's service level. Constraints (56-60) determine the bounds and the nature (binary or continuous) of the design variables. This model is mixed integer and nonlinear with a highly nonlinear objective function and nonlinear constraints, (2-53) and (2-55).

2.2.2. Solution approach for the supply network's reliable flow planning model

The SN model is similar to the SC with single-facility echelons model in that it has very range-restricted design variables, rl_{3s}, rl_{2p} and rl_{1m}. These variables are discretized similarly. The main difference in the SN model is that more than one route can fulfill the demand of each market. To discretize this part of the model, a new discrete (binary) design variable is defined, $\acute{y}_{mk}^{t}$, representing the percentage of market *m*'s demand assigned to route *t* ending to that market. This variable $\acute{y}_{mk}^{t}$ is defined for each member of set $PER_m^t = \left\{per_{m1}^t, per_{m2}^t, \dots, per_{m|PER_m^t|}^t\right\}$ which is the set of scenarios defined for the percentage of market *m*'s demand that is assigned to route *t* ($\forall m \in M,\ \forall t \in T^m$). Using this new variable we can discretize and consequently linearize the SN model. The discretized SN model is:

$$\boldsymbol{Max}\quad \boldsymbol{Z} = \sum_{m=1}^{|M|}\sum_{i=1}^{|SL_m|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|}\sum_{t=1}^{|T^m|} w_{sl_{m,t}^i}^{j}.\Big(p_m -$$

$$h_m^+.E\left[G_m^{-1}\left(Max\ \left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right) - \varepsilon_m\right]^+ - h_m^-.E\left[\varepsilon_m -\right.$$

$$\left. G_m^{-1}\left(Max\ \left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right)\right]^+\Big).\frac{D_m\left(\prod_{t=1}^{|T^m|} sl_m.y_t + (1-y_t), p_m\right)}{\sum_{t=1}^{|T^m|} y_t} - \sum_{s=1}^{|S|}(a_{1s} +$$

$$a_{2s}).\sum_{t=1}^{|T^s|}(x_t + \Delta x_t + \Delta\acute{x}_t) - \sum_{s=1}^{|S|} a_{3s}.\left(\sum_{i=1}^{|N_s|} z_{si}\,.\,i\right) - \sum_{s=1}^{|S|}\frac{Ih_{1s}.\left(\sum_{t=1}^{|T^s|} x_t + \Delta x_t\right)^2}{2.PR_{1s}.(\sum_{i=1}^{|N_s|} z_{si}.i)^2} -$$

$$\sum_{s=1}^{|S|}\sum_{o=1}^{|O|}\sum_{t=1}^{|T^s \cap T^o|} b_{1so}.(x_t + \Delta x_t) - \sum_{o=1}^{|O|}\sum_{t=1}^{|T^o|} b_{2o}.(x_t + \Delta x_t) -$$

$$\sum_{o=1}^{|O|} \frac{Ih_{2o}\left(\sum_{t=1}^{|T^o|} x_t\right)^2}{2.PR_{2o}} - \sum_{o=1}^{|O|} \sum_{m=1}^{|M|} \sum_{t=1}^{|T^o \cap T^m|} c_{1om}.x_t - \sum_{m=1}^{|M|} c_{2m}.\sum_{t=1}^{|T^m|} x_t$$

(2-61)

Where

$$x_t = \sum_{i=1}^{|SL_{m|m\in t}|} \sum_{j=1}^{\left|RL_{sl_m^i}\right|} \sum_{k=1}^{|PER_m^t|} w_{sl_m^i,t}^{j}.\acute{y}_{mk}^{t}.per_{mk}^{t}.D_m\left(sl_m^i, p_m\right).G_m^{-1}\left(Max\left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right)$$

$$(\forall t \in T) \qquad (2\text{-}62)$$

$$\left(\sum_{t=1}^{|T^o|} x_t\right)^2 =$$

$$\sum_{t=1}^{|T^o|} \sum_{i=1}^{|SL_{m|m\in t}|} \sum_{j=1}^{\left|RL_{sl_m^i}\right|} \sum_{k=1}^{|PER_m^t|} w_{sl_m^i,t}^{j}.\acute{y}_{mk}^{t}.\left[per_{mk}^{t}.D_m(sl_m^i, p_m).G_m^{-1}\left(Max\left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right)\right]^2 +$$

$$\sum_{(\forall t, \acute{t}\in T^o, t\neq \acute{t})} \sum_{i=1}^{|SL_{m|m\in t}|} \sum_{j=1}^{\left|RL_{sl_m^i}\right|} \sum_{k=1}^{|PER_m^t|} \sum_{\acute{\imath}=1}^{|SL_{\acute{m}|\acute{m}\in \acute{t}}|} \sum_{\acute{\jmath}=1}^{\left|RL_{sl_m^i}\right|} \sum_{\acute{k}=1}^{|PER_{\acute{m}}^{\acute{t}}|} w_{sl_m^i,t}^{j}.\acute{y}_{mk}^{t} \times w_{sl_{\acute{m}}^{\acute{\imath}},\acute{t}}^{\acute{\jmath}}.\acute{y}_{\acute{m}\acute{k}}^{\acute{t}} \times$$

$$\left\{\left[per_{mk}^{t}.D_m\left(sl_m^i, p_m\right).G_m^{-1}\left(Max\left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right)\right] \times\right.$$

$$\left.\left[per_{\acute{m}\acute{k}}^{\acute{t}}.D_{\acute{m}}\left(sl_{\acute{m}}^{\acute{\imath}}, p_{\acute{m}}\right).G_{\acute{m}}^{-1}\left(Max\left\{rl1_{sl_{\acute{m}}^{\acute{\imath}}}^{\acute{\jmath}}, \frac{h_{\acute{m}}^-}{h_{\acute{m}}^- + h_{\acute{m}}^+}\right\}\right)\right]\right\} + \cdots$$

$$(\forall o \in O) \qquad (2\text{-}63)$$

$$\Delta x_t = \sum_{i=1}^{|SL_{m|m\in t}|} \sum_{j=1}^{\left|RL_{sl_m^i}\right|} \sum_{k=1}^{|PER_m^t|} w_{sl_m^i,t}^{j}.\acute{y}_{mk}^{t}.\acute{G}_{o|o\in t}^{\ -1}\left(rl2_{sl_m^i}^{j}\right).per_{mk}^{t}.D_m\left(sl_m^i, p_m\right).$$

$$G_m^{-1}\left(Max\left\{rl1_{sl_m^i}^{j}, \frac{h_m^-}{h_m^- + h_m^+}\right\}\right) \qquad (\forall t \in T) \qquad (2\text{-}64)$$

$$\left(\sum_{t=1}^{|T^s|} x_t + \Delta x_t\right)^2 =$$

$$\sum_{t=1}^{|T^s|}\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|}\sum_{k=1}^{|PER_m^t|} w^j_{sl_m^i,t}.\acute{y}^t_{mk}.\left[per^t_{mk}.D_m\left(sl_m^i,p_m\right).G_m^{-1}\left(Max\left\{rl1^j_{sl_m^i},\frac{h_m^-}{h_m^-+h_m^+}\right\}\right)\right]^2$$

$$\times\left(1+\acute{G}_{o|o\in t}{}^{-1}\left(rl2^j_{sl_m^i}\right)\right)^2$$

$$+\sum_{(\forall t,\acute{t}\in T^s,t\neq\acute{t})}\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|}\sum_{k=1}^{|PER_m^t|}\sum_{\acute{\imath}=1}^{|SL_{\acute{m}|\acute{m}\in\acute{t}}|}\sum_{\acute{\jmath}=1}^{\left|RL_{sl_{\acute{m}}^{\acute{\imath}}}\right|}\sum_{\acute{k}=1}^{|PER_{\acute{m}}^{\acute{t}}|} w^j_{sl_m^i,t}.\acute{y}^t_{mk}.w^{\acute{\jmath}}_{sl_{\acute{m}}^{\acute{\imath}},\acute{t}}.\acute{y}^{\acute{t}}_{\acute{m}\acute{k}}\times$$

$$\left\{\left[per^t_{mk}.D_m\left(s_m^i,p_m\right).G_m^{-1}\left(Max\left\{rl1^j_{sl_m^i},\frac{h_m^-}{h_m^-+h_m^+}\right\}\right).\left(1+\right.\right.\right.$$

$$\left.\left.\left.\acute{G}_{o|o\in t}{}^{-1}\left(rl2^j_{sl_m^i}\right)\right)\right]\times$$

$$\left[per^{\acute{t}}_{\acute{m}\acute{k}}.D_{\acute{m}}\left(s_{\acute{m}}^{\acute{\imath}},p_{\acute{m}}\right).G_{\acute{m}}^{-1}\left(Max\left\{rl1^{\acute{\jmath}}_{sl_{\acute{m}}^{\acute{\imath}}},\frac{h_{\acute{m}}^-}{h_{\acute{m}}^-+h_{\acute{m}}^+}\right\}\right).\left(1+\acute{G}_{\acute{o}|\acute{o}\in\acute{t}}{}^{-1}\left(rl2^{\acute{\jmath}}_{sl_{\acute{m}}^{\acute{\imath}}}\right)\right)\right]\}+\cdots$$

$$(\forall s\in S) \qquad (2\text{-}65)$$

$$\Delta\acute{x}_t=\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|}\sum_{e=1}^{|N_{s|s\in t}|} w^j_{sl_m^i,t}.z_{se}.\left(\frac{\gamma_s}{1-\gamma_s}\right).\left[\left(\frac{PR_{1s}\times e}{\mu_s}\right).\ln\left(rl3^j_{sl_m^i}\right)+x_t+\Delta x_t\right]$$

$$(\forall t\in T) \qquad (2\text{-}66)$$

Subject to:

$$\sum_{i=1}^{|SL_m|} v_m^i=1 \qquad (\forall m\in M) \qquad (2\text{-}67)$$

$$\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl_m^i,t}=v_m^i.y_t \qquad \left(\forall m\in M,\forall t\in T^m,\forall sl_m^i\in SL_m\right) \qquad (2\text{-}68)$$

$$\left(\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl_m^i,t}.rl1^j_{sl_m^i}\right)\leq rl_{3s}\leq\left(\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl_m^i,t}.rl1^j_{sl_m^i}\right)+$$

$$1.(1-y_t) \qquad (\forall s\in S,\forall t\in T^s) \qquad (2\text{-}69)$$

$$(\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl^i_{m,t}}.rl2^j_{sl^i_{m,t}}) \le rl_{2o} \le (\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl^i_{m,t}}.rl2^j_{sl^i_{m,t}}) +$$

$$1.(1-y_t) \qquad (\forall o \in O, \forall t \in T^o) \qquad (2\text{-}70)$$

$$(\sum_{i=1}^{|SL_{m|m\in t}|}\sum_{j=1}^{\left|RL_{sl_m^i}\right|} w^j_{sl^i_{m,t}}.rl3^j_{sl^i_{m,t}}) \le rl_{1m} \le (\sum_{i=1}^{|S_{m|m\in t}|}\sum_{j=1}^{\left|RL_{s_m^i}\right|} w^j_{sl^i_{m,t}}.rl3^j_{sl^i_{m,t}}) +$$

$$1.(1-y_t) \qquad (\forall m \in M, \forall t \in T^m) \qquad (2\text{-}71)$$

$$\sum_{k=1}^{|PER_m^t|} \acute{y}^t_{mk} = y_t \qquad (\forall m \in M, \forall t \in T^m) \qquad (2\text{-}72)$$

$$\sum_{t=1}^{|T^m|}\sum_{k=1}^{|PER_m^t|} \acute{y}^t_{mk}.per^t_{mk} = 1 \qquad (\forall m \in M) \qquad (2\text{-}73)$$

$$x_t \le M.y_t \qquad (\forall t \in T) \qquad (2\text{-}74)$$

$$\sum_{t=1}^{|T^m|} y_t \ge 1 \qquad (\forall m \in M) \qquad (2\text{-}75)$$

$$\sum_{e=1}^{|N_s|} z_{se} = 1 \qquad (\forall s \in S) \qquad (2\text{-}76)$$

$$y_t, z_{se}, v^i_m, w^j_{sl^i_{m,t}}, \acute{y}^t_{me}, \in \{0,1\}$$

$$(\forall t \in T, \forall s \in S, \forall e \in N_s, \forall sl^i_m \in SL_m, \forall rl^j_{sl^i_m} \in RL_{sl^i_m} \forall per^t_{me} \in PER^t_m) \qquad (2\text{-}77)$$

$$rl_{1m},\ rl_{2o},\ rl_{3s},\ x_t,\ \Delta x_t,\ \Delta \acute{x}_t\ \ge 0$$

$$(\forall s \in S, \forall m \in M, \forall 0 \in O, \forall t \in T) \qquad (2\text{-}78)$$

In this model the multiplication of binary variables is linearized by the approach described in Equations (2-22)-(2-30). In constraint (2-66) there are multiplications of continuous variables (x_t and Δx_t) and binary variables ($w^j_{sl^i_{m,t}}$ and z_{se}). To linearize nonlinear term $w^j_{sl^i_{m,t}}.z_{se}.x_t$, we define a new variable such as $wzx^j_{sl^i_{m,t,se}} = w^j_{sl^i_{m,t}}.z_{se}.x_t$ and the following constraints should be added to the model:

$$wzx^j_{sl^i_{m,t,se}} \le M.w^j_{sl^i_{m,t}} \qquad (2\text{-}79)$$

$$wzx^{j}_{sl^{i}_{m},t,se} \leq M.z_{se} \quad (2\text{-}80)$$

$$x_t + M.\left(w^{j}_{sl^{i}_{m},t} + z_{se} - 2\right) \leq wzx^{j}_{sl^{i}_{m},t,se} \leq M.\left(2 - w^{j}_{sl^{i}_{m},t} - z_{se}\right) + x_t \quad (2\text{-}81)$$

The multiplication of $w^{j}_{sl^{i}_{m},t}.z_{se}.\Delta x_t$ can be linearized in the same way. After these manipulations the above model is transformed to a mixed integer linear programming with binary variables which can be solved globally.

2.2.3. Computational results: An example from the automotive industry

Consider a SN involved in the procurement and supply process of an automotive industry in the Middle East. Variations, especially on the supply side are more prevalent in this region. IKC and SAC are two large automotive manufacturers in this region (for reasons of confidentiality, the names of these companies are omitted). SMAC is one of the well-known suppliers producing and supplying fifth gear pins to the markets. IKC and SAC are the main customers of SMAC. However, recently some external suppliers of fifth gear pins entered the market with comparable prices. SMAC procures its component, CK45 steel, from two suppliers: YIIC and FMC with similar production systems but different production costs. The fifth gear pins are supplied to two markets. SMAC has many customers in each market but its main customers in the first and second markets are IKC and SAC respectively. Recently, due to the entrance of external suppliers, the markets have become competitive. Thus, the appropriate selection of service levels is important. However, determining and implementing the best service level is not straightforward because in addition to demand variations they are faced with variations in their production systems and those of their suppliers. SMAC's network is shown in Figure 2-7. The costs are summarized in Table 2-5.

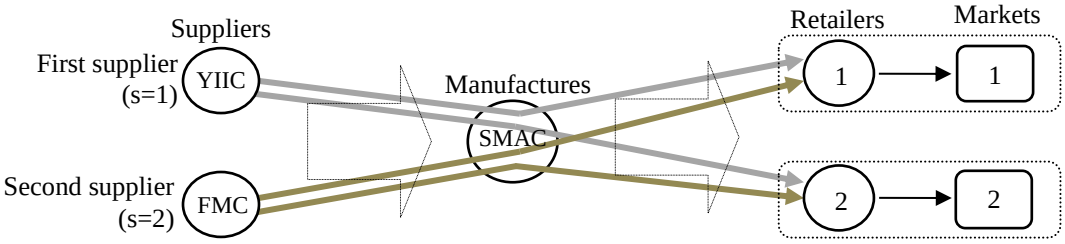


Figure 2-7: SMAC's network structure and its potential usable routes.

Table 2-5: Costs of SMAC's network.

Parameter	Amount of parameter	Parameter	Amount of parameter	Parameter	Amount of parameter	Parameter	Amount of parameter
β	10%	a_{31}	100	PR_{12}	9000	c_{111}	0.900
γ_1	0.15	a_{32}	100	PR_{21}	8000	c_{112}	0.700
γ_2	0.15	a_{21}	1.50	$p_{1\ and\ 2}$	17.00	c_{21}	1.500
μ_1	2.00	a_{22}	1.25	b_{111}	0.500	c_{22}	1.400
μ_2	2.00	a_{11}	2.50	b_{121}	0.500		
Ih_{11}	0.40	a_{12}	2.50	b_{21}	2.000		
Ih_{12}	0.30	PR_{11}	9000	Ih_{21}	0.400		

Materials are produced by the first and second suppliers with \$2.50 and \$2.50 procurement costs; \$1.50 and \$1.25 production costs; and \$100 and \$100 set-up costs respectively. Qualified components are transported to the manufacturer with \$0.50 unit transportation cost. A manufacturer produces the final product at rate of 8000 (units per time) and cost of \$2.00. Flawless products are transported to the first and second retailers with a cost of \$0.90 and \$0.70. Unit handling costs in the first and second retailers are \$1.50 and \$1.40. It is assumed that there are two production schemes for the SN's suppliers. In the first, all the material is produced at once and in the second, the required material is produced in two batches: $N_{1\ and\ 2} = \{1,\ 2\}$. Unit inventory holding costs are \$0.40, \$0.30 and \$0.40 in the first supplier, the second supplier and the manufacturer respectively. Based on historical data, the demand functions in the first and second markets, ε_1 and ε_2, follow uniform distributions on [0.8, 1.2] and [0.7, 1.3]. Demand in the first and second markets are $\widehat{D}_1(sl_1,\ p) = [100 + 6400.sl_1 - 640.(p - 17)].\varepsilon_1$ and

$\widehat{D}_2(sl_2,\ p) = [120 + 6500.sl_2 - 640.(p - 17)].\varepsilon_2$. SMAC's production system yields a percentage of defective output uniformly distributed on [0, 10%]. The suppliers' production systems deteriorate after exponential times with $\mu_{1\ and\ 2} = 2.0$ and shift from in-control to out-of-control leading to $\gamma_{1\ and\ 2} = 15\%$ of nonconforming component production. Sl_m and $Rl_{sl_m^i}$ sets in this problem are defined in the similar way as the SC with single-facility echelons problem. New part of this model is PER_m^t set which is defined as follow:

$$PER_m^t = \left\{per_{m1}^t = 0, per_{m2}^t = 0.2, \dots, per_{m|PER_m^t|}^t = 1.0\right\}$$

The best solution is 0.9 reliability levels in the SN's both suppliers, 1.0 reliability levels in the manufacturer and first retailer and 0.95 reliability level in the second retailer. This leads to 0.81 and 0.73 service levels in the first and second markets. In the first market this is produced by 1187.8 product units in route t_{111} and 4751.4 product units in route t_{211}. Service level of 0.73 in the second market is obtained by 1155.8 product units of route t_{112} and 4623.1 product units of route t_{212} (Figure 2-8).

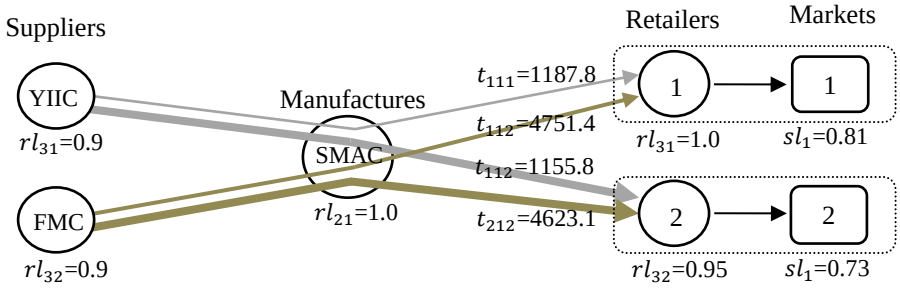


Figure 2-8: Reliable material and product planning in SMAC's network.

In Figures 2-9 and 2-10 the profit of the SMAC's network with respect to its service level in the first and second markets respectively is displayed. As seen in Figure 2-10, there is a clear tradeoff between the cost of improving service level in the first market

(production and distribution cost of more extra production in the facilities locating throughout the routes ending to this market) and the income resulted in this market through providing better service for its customers. A service level of 0.81 in the first market leads to the best combination of cost and income which results in the highest profit for the company in this market. But it seems that in the second market, the cost of improving the service level is more than in the first one. Hence, the service level 0.73 is assigned by the SN to this market.

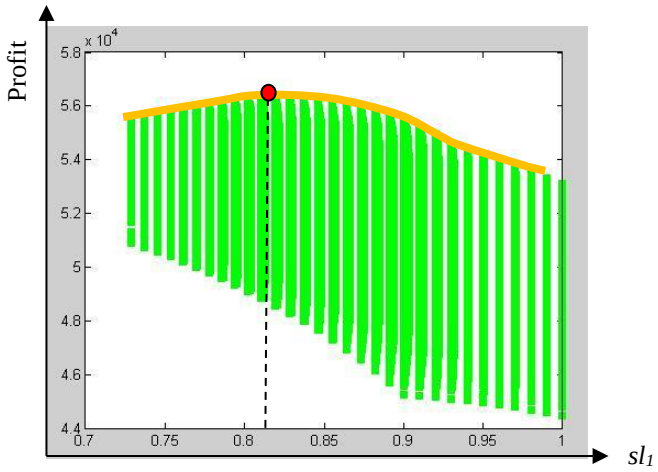


Figure 2-9: Profit of SMAC's network in the first market with respect to sl_1.

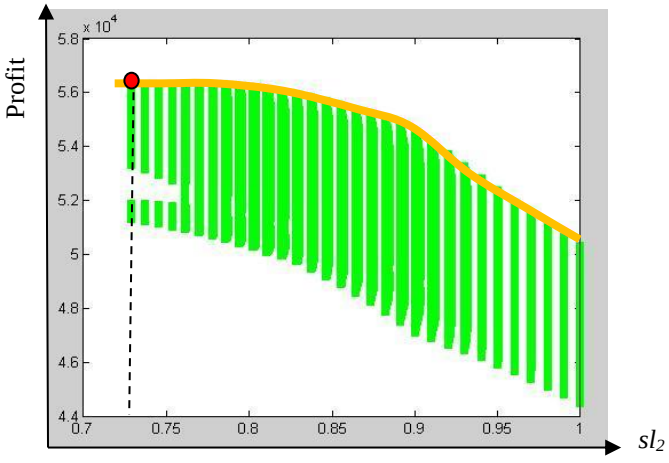


Figure 2-10: Profit of SMAC's network in the second market with respect to sl_2.

2.2.4. Optimal price determination and exploring the design space

In these models, the product price is a given exogenous factor. However retail price is an important bargaining chip for companies. Determining an appropriate retail price is difficult due to its conflicting effects on marginal profit and demand volume. In this section the outputs of the proposed models are extended by determining the product's optimal retail price by sensitivity analysis. Companies are not completely free to determine their prices. Usually there is a given interval whose bounds are determined by factors such as the prices of similar products or governmental regulations. In the SMAC problem, the appropriate price interval is [$17, $23]. This problem is solved for different values of price from this interval, Table 2-6 and Figure 2-11. In the interval [$17.0, $20.5], the positive effect of the price increment on sale profitability is greater than its negative effect on demand reduction which leads to greater profits. The profit reaches its highest value at p = $20.50 and then declines.

Table 2-6: Optimal price for the SMAC problem (all values are in dollars).

Price ($)	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23
Profit ($)	56,258	58,208	60,115	61,771	62,823	64,239	65,063	65,197	64,643	63400	61,467	58,845	55,534

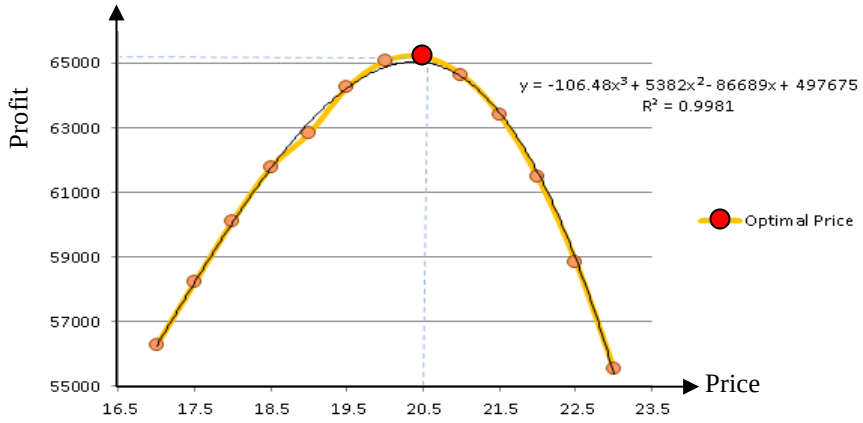


Figure 2-11: Optimal price for the SMAC problem.

To determine the relationship between the optimal retail price and service levels and to explore the design space, we ran the model for different values of the retail price and, drew the profit function of the company with respect to the markets' assigned service levels. In Figure 2-12 the company's profit with respect to the first market's service level for different retail price values is shown. Black dots represent the service levels for which the priorities of at least two price strategies change. In the price interval [\$17, \$23] the highest profit is related to the price strategies p = \$18.50, \$19.00, \$19.50, \$20.00 and \$20.50. Stars represent the first market's service levels at which the best retail price strategy changes in the SMAC problem. The results are summarized: i) for $sl_1 \leq 0.77$ the most profitable retail price is p = \$18.50; ii) for $0.77 < sl_1 \leq 0.84$ the most profitable retail price is p = \$19.0; iii) for $0.84 < sl_1 \leq 0.93$ the most profitable retail price is p = \$19.5; iv) for $0.93 < sl_1 \leq 0.98$ the most profitable retail price is p = \$20.0; and v) for $0.98 < sl_1$ the most profitable retail price is p = \$20.50.

In Figure 2-13, the profit of the company with respect to the second market's service level for different retail prices is shown. Red dots in this figure represent the second market's service level values in which the best retail price strategy changes. The results

are: i) for $sl_2 \leq 0.758$ the most profitable retail price is p = \$18.50; ii) for $0.758 < sl_2 \leq 0.792$ the most profitable retail price is p = \$19.00; iii) for $0.792 < sl_2 \leq 0.877$ the most profitable retail price is p = \$19.5; iv) for $0.877 < sl_2 \leq 0.980$ the most profitable retail price is p = \$20.0; and v) for $0.98 < sl_2$ the most profitable retail price is p = \$20.50. Combining the results in Figures 2-12 and 2-13 the service level of the first and second markets must be selected from the highlighted regions in the matrix in Figure 2-14. For each highlighted region we determine the optimal retail price strategy to maximize the company's profit. As expected, service level increases allow the company to select higher retail prices.

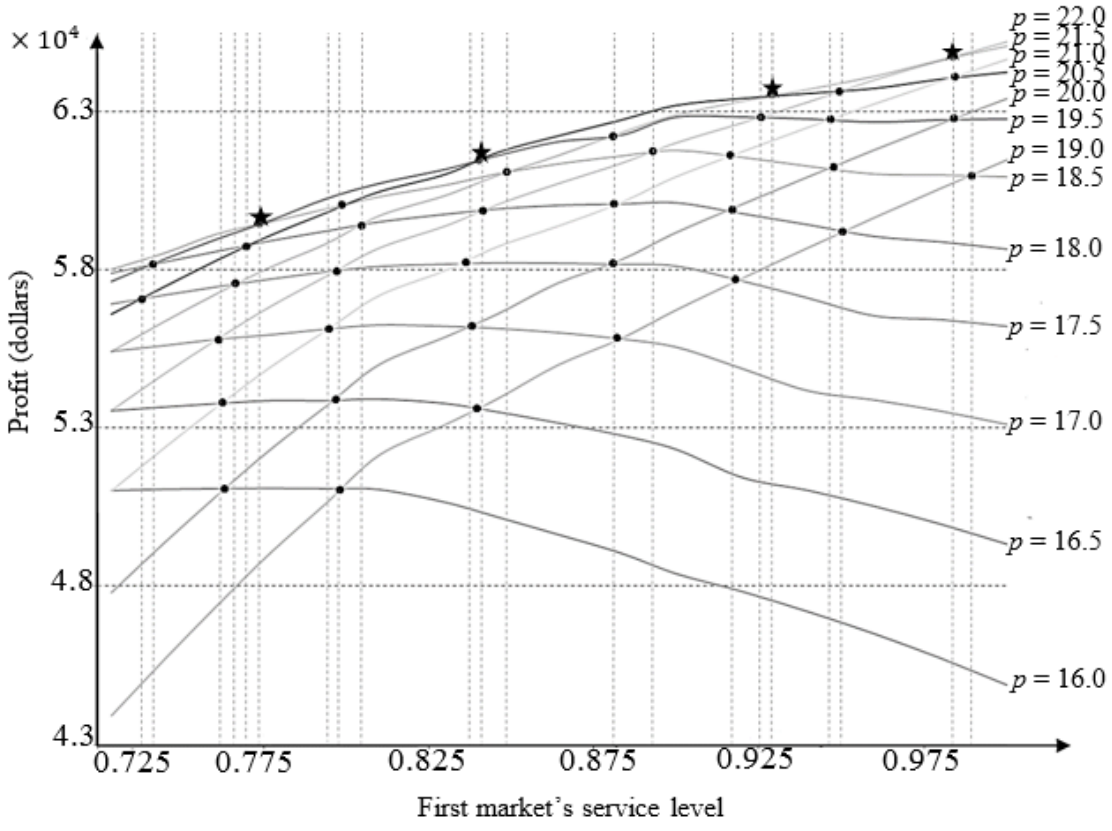


Figure 2-12: SMAC profit with respect to the first market's service level in different product prices.

2.2.5. Run time of the supply network model

The linearized model of SN problem is a mixed integer linear mathematical model with binary variables. The computational time of this kind of model mainly depends on the number of binary variables. The model of SMAC problem is solved by a computer with the following feature: Intel(R)Core(TM)4 Duo CPU, 3.6 GHz, with 12276 MB RAM using the default settings and the time of solving the model is less than a second. However it is clear that by increasing the size of the problem and consequently the number of binary variables the model's computational time increments. So to demonstrate the size of the problems which are solvable globally by this approach, we increased the size of the SMAC problem gradually by adding some new facilities to the problem. Data related to these new facilities generated randomly in consistent with the data of the facilities in the SMAC problem. Results are summarized in Table 2-7.

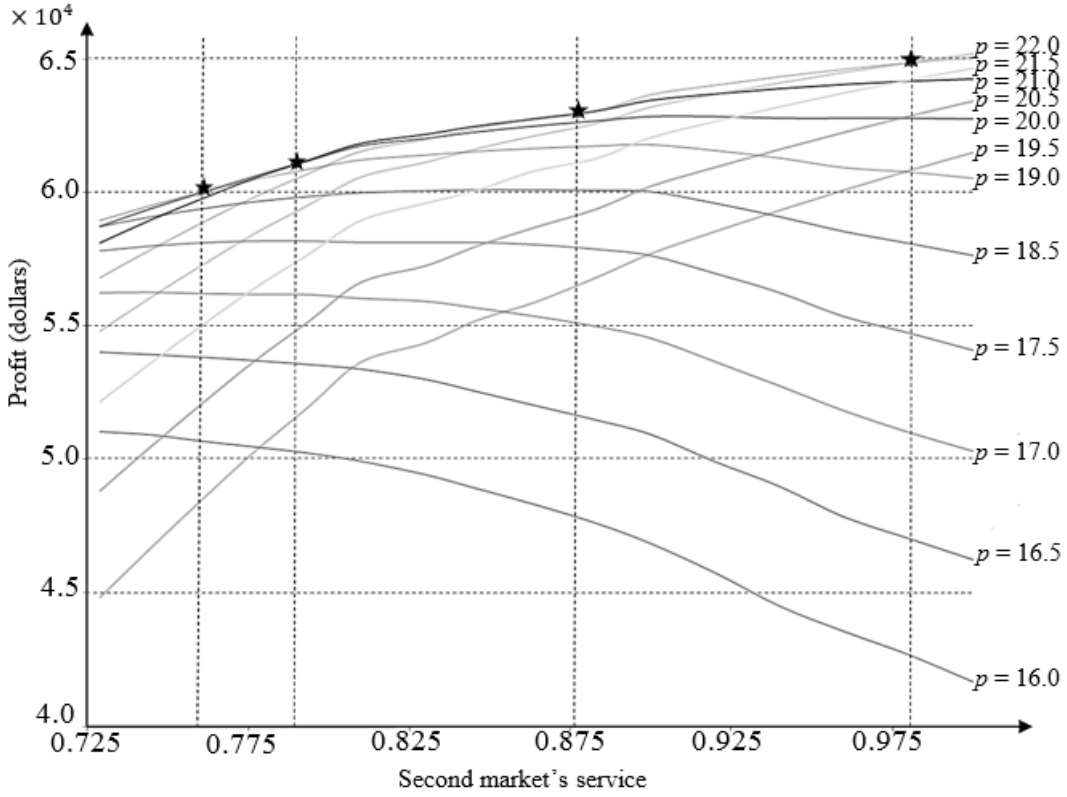


Figure 2-13: SMAC profit with respect to the second market's service level in different product prices.

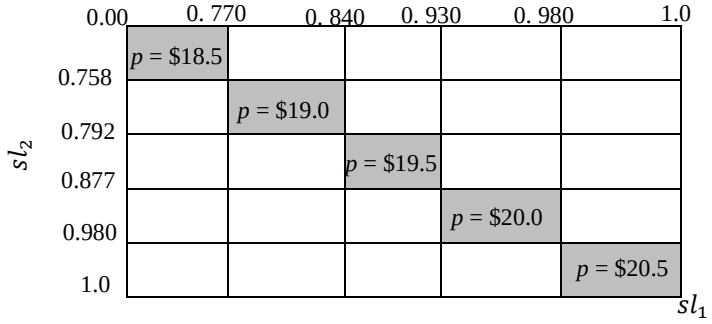


Figure 2-14: Service level matrix for the SMAC problem.

Table 2-7: Computational time for randomly generated test problems.

Problem	No. of suppliers	No. of manufacturers	No. of retailers	No. of paths	Solving time of problem
1	2	1	2	4	<1"
2	2	1	4	6	4"
3	2	1	8	13	3':34"
4	3	2	12	21	46':51"
5	3	2	16	27	5:29':48"
6	4	2	20	35	31:56':43"
7	5	3	30	54	>72

Computation time for the model of SMAC problem is less than a second. As expected by increasing the size of the problem the computational time increments. For the last problem (last row in Table 2-7), we could not gain a solution even after 72 hours. Based on these results, it seems that for bigger problems using heuristic or meta-heuristic approaches to find local optimal solution instead of global optimal solution is more rational.

2.3. Closure of Chapter 2

Controlling material / product flow in SCs / SNs is difficult because of their decentralized production systems. It is much more complicated when there are also variations in the performance of the entities inside the chain / network and variations in environmental factors. Operationally fail-safe SCs / SNs are able to handle the variations appropriately and preserve the most profitable service levels in markets to improve their competitive advantages. In this chapter we answer the following question:

- ✓ **Research Question 7**: *what are the characteristics of fail-safe SNs against variations?*

In Section 2.1, we show that there are two variation groups in SCs / SNs:

1. ***Supply-side variations***: Performance of the production system inside the facilities of SCs / SNs is not perfect. Imperfect production system leads to stochastic qualified output in these facilities.
2. ***Demand-side variations***: Demand of markets is stochastic and always has some fluctuations around its mean value.

In Sections 2.1.1, 2.1.2, and 2.1.3, we show that how these variations affect the performance of the facilities in the first echelon, e.g. the retailer, the second echelon, e.g. the manufacturer, and the third echelon, e.g. the supplier. We show that in SCs / SNs with stochastic facilities, qualified flow quantity depreciates by moving from upstream to downstream which adversely affects its service level in markets. To neutralize uncertainty propagation and flow depreciation, we suggest that orders should be amplified from downstream to upstream of the SCs / SNs. In Sections 2.1.5 and 2.1.6 respectively, we develop a mathematical model to formulate order amplification and to solve it. This model and its solution approach are extended from SC to SN in Section 2.2 by using path concepts.

In this chapter we show that in SCs / SNs with demand- and supply-side variations calculation and determination of service level is critical but not easy. In these SCs / SNs investigating the local effects of the variations on the performance of the corresponding facilities may not be enough; it is necessary to consider their cumulative effects on the SC / SN performance.

This problem is an example in the business environment, but it can easily be applied to problems in other fields in which variation has a significant role and where a high service level is critical. For example, it can be applied to humanitarian relief planning where a high service level is critical. Also this model can be applied by transportation companies dealing only with product distribution. These companies do not have production facilities but variation also exists in the performance of transportation and warehousing facilities in the distribution process. Transportation and inventory holding processes always include stochastic percentage of broken, lost, spoiled and even expired items which makes their qualified output uncertain. The approach in this chapter will not only improve service level estimation for the SCs / SNs but also offers the foundations for service level improvement.

In this chapter we only consider the problem of having an operationally fail-safe forward SCs / SNs. In the next chapter, Chapter 3, we extend it to a company including both forward and after-sales SCs / SNs.

Chapter 3: Operationally Fail-safe Supply Chains Servicing Pre- and After-sales Markets

In this chapter, we deal with having "Operationally Fail-safe SNs" in companies supplying product – service package to markets. These companies have two SNs: 1) forward SN dealing with producing and supplying original products to pre-markets; and 2) after-sales SN dealing with fulfilling the after-sales commitments of the company. Having two highly convoluted SNs complicates the process of flow planning in these companies. These complications are discussed in this chapter and a quantitative method is proposed to have an operationally fail-safe flow planning in these companies. Therefore, in this chapter we answer the first, second and third research questions for companies having two convoluted SCs – SNs with a single facility in each echelon – dealing with forward and after-sales markets:

- ✓ **Research Question 1:** *what are the important flow transitions among the facilities supporting after-sales services?*
- ✓ **Research Question 2:** *what are the important interactions between forward and after-sales SCs justifying the necessity of their concurrent flow planning?*
- ✓ **Research Question 3:** *how do these interactions affect planning flow dynamics in the forward and after-sales SCs of non-repairable goods?*

In Section 3.1, we answer the first and second research questions by explaining the operations in the forward and after-sales SCs to determine: 1) the flow transactions exist between the facilities of the SCs; and 2) the interplays exist between the operations of the two SCs. These interactions between the facilities and SCs are quantified in Section 3.2. In Sections 3.2.1-3.2.5, we formulate equations explaining performance of the forward

SC's facilities and how they are affected by the operations of the after-sales SC. In Sections 3.2.6-3.2.7, we formulate equations explaining performance of the after-sales SC's facilities and how they are affected by the operations of the forward SC. The equations derived in Section 3.2, are used in Section 3.3 to develop a mathematical model that concurrently determines an operationally fail-safe flow planning in the forward and after-sales SCs. An appropriate approach is proposed in Section 3.4 to solve the model. The model is tested on an example in Section 3.5 and results are discussed. Result analysis leads to some interesting managerial findings.

3.1. Operations and variations in a forward and after-sales supply chains

In this problem, a company producing and supplying a durable product to a target market is considered. Production and distribution processes of this product are done through the facilities of the forward SC. This product includes *r* critical components manufactured in suppliers of the first echelon. The components are transported to a manufacturer in the second echelon and, after assembly, the final product is supplied to the final customers through a retailer (Figure 3-1). The products of each sale period are produced, transported and stored in the SC's retailer before the beginning of that period.

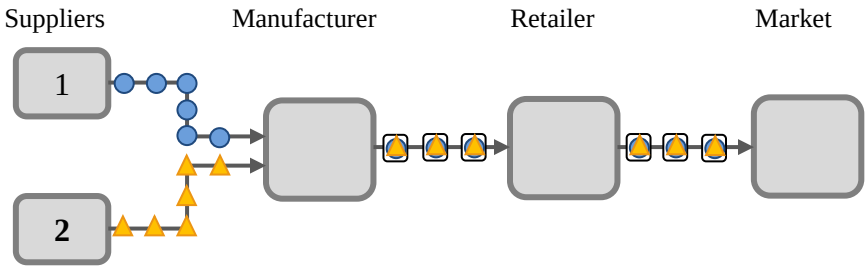


Figure 3-1: Network structure and flow dynamics through the forward SC (for a product with two critical components).

The product demand is stochastic and depends on the product's price, its availability in the pre-market (called the pre-market service level), the spare parts' availability in the after-sales (called the after-sales service level), and warranty length. Whenever a product is sold, a failure-free warranty is provided which is implementable from the time of sale. Within this warranty time any failure in the product, which is mainly caused by the failure of its key components, is repaired without charge. Without loss of generality, it is assumed that typically the first $n_i\,(i = 1, 2, \ldots, r)$ failures of these components are repaired but then failed components are substituted with new ones stored in the retailer.

Producing and supplying the required components to provide the after-sales services for the customers are done by the company's after-sales SC. The required components to fulfill the after-sales commitments of each sale period are produced by the suppliers and directly transported to the retailer and stored there before the beginning of that period (Figure 3-2) – first research questions. Accurate prediction of the required components is an important element of this problem and has a key role in preserving a recommended after-sales service level.

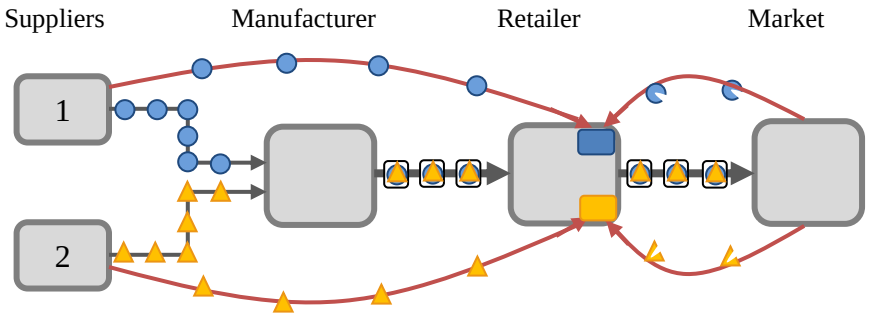


Figure 3-2: Network structure and flow dynamics through the after-sales SC (for a product with two critical components).

Two important interactions between these two SCs are (second research question): 1) the demand of the forward SC in the pre-market depends on the service level provided by the after-sales SC; and 2) the after-sales demand of the components depends on the total products supplied by the forward SC to the market and the quality of the product's components. These interactions are incorporated in the concurrent flow planning of these two SCs.

In this problem we consider several different sources of uncertainty: 1) *Demand-side variation*: there are several sources of demand-side variation in this problem. The first variation is related to the product's demand in the pre-market. The pre-market's demand is assumed to be a stochastic function of price, warranty length and service levels in the pre- and after-sales markets. The after-sales demands for spare parts are functions of the quantity of product sales in the pre-market and the quality of the product's components. Both of these are nondeterministic. We assume failure times of the product's components are stochastic and follow given density functions depending on their reliability parameters. 2) *Supply-side variation*: to make the problem more compatible with actual conditions, it is assumed the production systems of the SCs' facilities are not perfect and their output always has stochastic percentage of nonconforming production. In our problem, the performance of the suppliers and the manufacture includes stochastic percentage of nonconforming output.

In a company with these specifications, it is important to make the following decisions in order to maximize its total profit: 1) the best marketing strategy for this company (price, warranty length, and, service levels); and 2) the best reliable flow dynamics throughout the SCs preserving its service levels in the pre- and after-sales markets.

3.2. Mathematical model for the problem

This problem includes two distinct but highly interconnected parts: the forward SC and the after-sales SC. There are several interactions between the forward and after-sales SCs (second research question). For example, total product sales in the forward SC determines the potential demand for the spare parts in the after-sales market. Also the after-sales services provided by the after-sales SC such as warranty and spare parts availability have important role in the forward SC's captured demand in the pre-market. Therefore, there is considerable synergy in simultaneous flow planning of the forward and after-sales SCs.

In the rest of this section, first we deal with planning flow dynamics through the forward SC with stochastic facilities and then shift to the after-sales SC. Thereafter, by considering the interactions between these two SCs a comprehensive mathematical model is proposed which yields the most profitable marketing strategies (price, warranty, and service levels) and their preserving flow plan for the company under consideration. In this mathematical model, we see that how the operations in the forward and after-sales SCs affect each other (third research question). The solution of this model includes the synergy of concurrent coordination in comparison with hierarchical decision making processes which is much easier but leads to sub-optimal solutions for this problem.

The notations used in this section are summarized in Table 3-1.

Table 3-1: Notation for the concurrent forward and after-sales SCs problem.

Variables	
rl_1	Reliability level of the retailer
rl_2	Reliability level of the manufacturer
rl_3	Reliability level of the suppliers
sl_p	Service level in the pre-market
sl_a	Service level in the after-market
w	Warranty time
x	Product order quantity by the retailer

Δx	Extra production volume in the manufacturer
$\Delta \acute{x}_i$	Extra production volume in Supplier *i* for forward SC (*i* = 1, 2, 3, …, *r*)
x_i	Order quantity of Component *i* by retailer (*i* = 1, 2, 3, …, *r*)
$\Delta x_i''$	Extra production volume in Supplier *i* for after-sales SC (*i* = 1, 2, 3, …, *r*)
y_{rl1^i}	Binary variable equal to 1 if the reliability level $rl1^i$ is selected from set $RL1$ for the retailer; 0 otherwise ($\forall rl1^i \in RL1$)
y_{rl2^i}	Binary variable equal to 1 if the reliability level $rl2^i$ is selected from set $RL2$ for the manufacturer; 0 otherwise ($\forall rl2^i \in RL2$)
y_{rl3^i}	Binary variable equal to 1 if reliability level $rl3^i$ is selected from set $RL3$ for the suppliers; 0 otherwise ($\forall rl3^i \in RL3$)
z_{w^i}	Binary variables equal to 1 if warranty length w^i is selected from set W ($\forall w^i \in W$)
Parameters	
$\acute{\Pi}$	Profit of the company
Π	Total cost of retailer
T	Production planning period
p	Price of product in the pre-market
$\widehat{D}(p, sl_p, sl_a, w)$	Stochastic product demand function in the pre-market
$D(p, sl_p, sl_a, w)$	Expected product demand in the pre-market
ε	Random part of the pre-market demand
$G(.)$	Cumulative distribution function of ε
h^+	Unit holding cost of extra product inventory at the end of planning period in the retailer
h^-	Unit shortage cost of lost product sale at the end of planning period in the retailer
r	Number of critical components in the product
$\acute{G}(.)$	Cumulative distribution function of wastage ratio in the manufacturer
β	Maximum wastage ratio in the manufacturer of the sample problem
μ_i	Average number of failures in the time unit in the Supplier *i* (*i* = 1, 2, 3, …, *r*)
γ_i	Defective component ratio in the out-of-control state of Supplier *i* (*i* = 1, 2, 3, …, *r*)
a_{1i}	Unit procurement cost of material in the Supplier *i* (*i* = 1, 2, …, *r*)
a_{2i}	Unit production cost of Component *i* in the Supplier *i* (*i* = 1, 2, …, *r*)
h_{1i}	Unit inventory holding cost for a time unit in the Supplier *i* (*i* = 1, 2, …, *r*)
b_{1i}	Unit transportation cost of product from Supplier *i* to the manufacturer (*i* = 1, 2, …, *r*)
b_2	Unit product manufacturing cost in the manufacturer
h_2	Unit inventory holding cost for a time unit in the manufacturer
c_1	Unit transportation cost of product from the manufacturer to the retailer
c_2	Unit handling cost of product in the retailer
c_{3i}	Unit transportation cost of Component *i* from Supplier *i* to the retailer (*i* = 1, 2, …, *r*)
PR_{1i}	Production rate of the Supplier *i* (*i* = 1, 2, …, *r*)
PR_2	Production rate of the manufacturer
λ_i	Reliability parameter of the Component *i* (*i* = 1, 2, 3, …, *r*)
$f_i(.)$	Density function of failure time of the Component *i* (*i* = 1, 2, 3, …, *r*)
$F_i(.)$	Cumulative distribution function of failure time of the Component *i* (*i* = 1, 2, 3, …, *r*)

$F_i^{(m)}(.)$	Cumulative distribution function of total time to the m^{th} failure of the Component i (i = 1, 2, 3, …, r)
n_i	Number of first failures of Component i that are repairable (i = 1, 2, 3, …, r)
cn_i	Unit repair cost of Component i (i = 1, 2, 3, …, r)
cr	Average repair cost of the product unit;
Num_i	Random number of Component i substitutions for a product unit in warranty time (i = 1, 2, 3, …, r)
E_i	Average number of Component i substitutions for a product unit in warranty time (i = 1, 2, 3, …, r)
σ_i^2	Variance of number of Component i substitutions for a product unit in warranty time (i = 1, 2, 3, …, r)
D_i	Average number of Component i substitutions in warranty time in the after-sales market (i = 1, 2, 3, …, r)
k_1	Number of planning periods inside the warranty time
$\acute{T}$	Biggest time period inside the planning period in which it is logical to assume that product demand occurs at its beginning
k_2	Number of $\acute{T}$s inside the planning period
D_{ij}	Required quantity of Component i to repair product lot x/k_2 in the j^{th} period $\acute{T}$ of its selling time;
$RL1$	Sets of scenarios for the reliability level of the retailer $RL1 = \{rl1^1, rl1^2, \dots, rl1^{\lvert RL1 \rvert}\}$
$RL2$	Sets of scenarios for the reliability level of the manufacturer $RL2 = \{rl2^1, rl2^2, \dots, rl2^{\lvert RL2 \rvert}\}$
$RL3$	Sets of scenarios for the reliability level of suppliers $RL3 = \{rl3^1, rl3^2, \dots, rl3^{\lvert RL3 \rvert}\}$
W	Set of warranty length $W = \{w^1, w^2, \dots, w^{\lvert W \rvert}\}$

3.2.1. Forward supply chain formulation

In this section, only decisions related to the flow dynamics in the forward SC will be considered. As has been mentioned, there are several sources of variation in the forward SC: i) variation in the product demand in the pre-market; and ii) variation in the performance of the manufacturer's and suppliers' production systems. In the rest of this section, all the forward SC's facilities are sequentially investigated from downstream to upstream and a procedure for reliable flow planning is done in each facility against its corresponding uncertainty. In addition to investigating the local effects of these uncertainties, we also investigate their global effects on the performance of the whole forward SC.

As shown in Figure 2-3, the forward SC considered here has three echelons and the facilities in each echelon are faced with some uncertainties. The retailer of the first echelon faces with uncertain market demand with a given distribution function. The production system of the manufacture in the second echelon is always accompanied with some stochastic waste. After setting up, the production processes of the suppliers in the third echelon start their machinery in-control. But the state of the machinery deteriorates and it shifts to an out-of-control state after a stochastic while which leads to a stochastic percentage of nonconforming output. Due to the imperfect production systems of the suppliers, the exact volume of their qualified component output for given material input quantity cannot be determined. Thus, the qualified output volumes can change and are stochastic. The output components of the suppliers are the input for the manufacturer. Variation in the input volume of the manufacturer is amplified because of the stochastic wastage ratio in the manufacturer's production system and it leads to a higher variation in the qualified product output of the manufacturer. This process continues by moving material, components, and product from upstream to downstream in multi-echelon SCs with imperfect facilities. We call this phenomenon "*uncertainty propagation"* which leads to the qualified flow depreciation throughout the SCs' networks (see Figure 3-3).

In such a SC, determining an optimal service level is much more difficult due to the flow depreciation which occurs by moving the flow from upstream to downstream. In such a network with multiple stochastic facilities, a local reliability is assigned to each facility to manage the uncertainty of its own system. It is assumed that rl_1, rl_2, and rl_3 represent the local reliability in the retailer, manufacturer, and suppliers of the SC

respectively (Without loss of generality, we consider similar reliabilities for the suppliers. For different reliabilities, the same logic can be applied).

In this problem, we are exploiting the newsboy problem style for managing the inventory system of the retailer. Based on this system before the beginning of each sale period and realizing its actual demand, the products should be procured and stocked by the retailer and extra product transfer between the manufacturer and retailer is not possible during the period. So rl_1, the local reliability of the retailer, means that before the beginning of the next sale period, the retailer must select its product stock quantity to be sure with rl_1 probability that this stock level can respond to the market demand. The retailer orders the required products from the manufacturer. Furthermore, the rl_2 local reliability for the manufacturer means that the manufacturer must manufacture the appropriate product quantity to guarantee the qualified output is equal to the order of the retailer with rl_2 probability. The rl_3 local reliability in each supplier means that the material procurement and component production quantity should preserve the order of the manufacturer with rl_3 probability. In this case, the suppliers will be sure with ${rl_3}^r$ probability that they can fulfill the manufacturer's component orders. The manufacturer will be sure with rl_2 probability that it provides the complete order of the retailer and the retailer is sure with rl_1 probability that its product stock quantity can fulfill the demand of the market. Therefore, the final service level of the forward SC in the pre-market is: $sl_p = rl_1.rl_2.(rl_3)^r$. In this problem, not only determining the optimal sl_p is important, but also it is essential to govern the optimal local reliability combination, (rl_1, rl_2, rl_3), which preserves that service level.

3.2.2. Retailer in the forward supply chain

The company positions itself in the market by choosing its service levels in the pre- and after-sales markets, its warranty time, and retail price. The expected product demand in the pre-market, $D(p, sl_p, sl_a, w)$, in the sale period, *T*, is an increasing function of the chains' service levels and warranty time and a decreasing function of the product's price. Therefore the after-sales service level affects the product demand in the pre-market (second research question). Because customers are mainly willing to buy from a company providing better after-sales services. However, the actual demand is a stochastic function and has some deviation from its mean value. It is assumed that the stochastic demand function of the pre-market has a multiplicative form as $\widehat{D}(p, sl_p, sl_a, w) = D(p, sl_p, sl_a, w) \times \varepsilon$ (Bernstein and Federgruen, 2004 and 2007). Where ε is a general continuous random variable with a stationary distribution function and a cumulative distribution function, $G(\varepsilon)$, which are independent of the service levels, warranty time, and price. Without loss of generality, $E(\varepsilon) = 1$ is normalized which implies that $E[\widehat{D}(p, sl_p, sl_a, w)] = D(p, sl_p, sl_a, w)$.

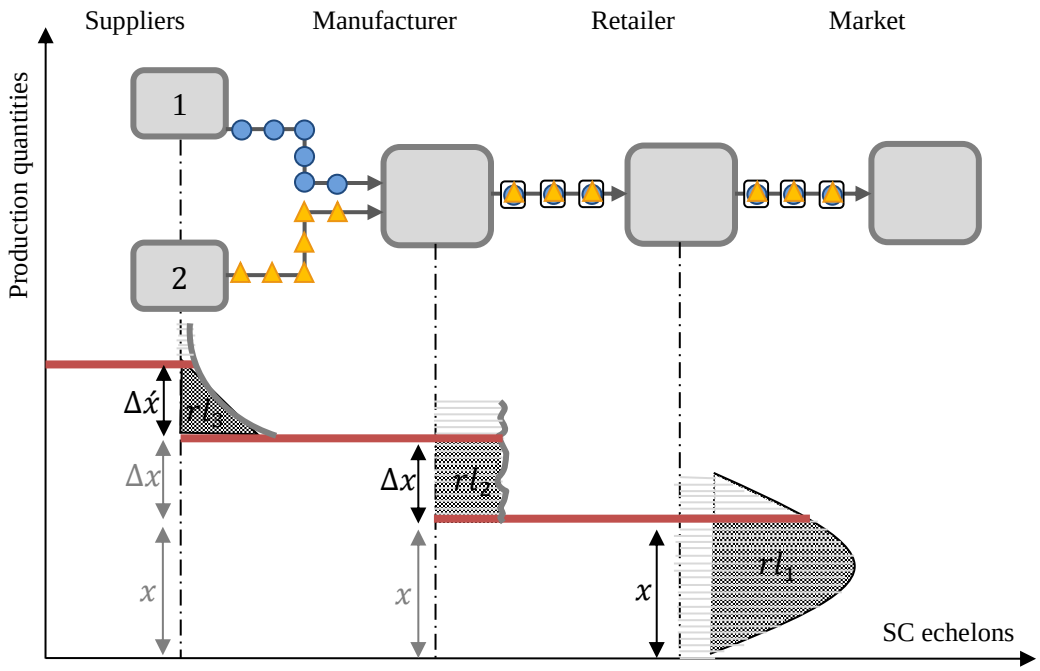


Figure 3-3: Uncertainty propagation in the forward SC.

In this section, we only focus on the operation of the forward SC. Therefore, the pre-market's service level is the focus here. The pre-market's service level is defined as the fraction of pre-market's realized product demand that can be satisfied from the on-hand product inventory available in the retailer. The retailer must order the product stock, x, from the manufacturer before the beginning of the sale period. By realizing the period's real product demand, unit holding cost, h^+, and unit shortage cost, h^-, are paid by the retailer for each end-of-period extra inventory and lost sale respectively.

The expected value of the retailer's cost, Π, is computed with Equation (3-1). Constraint (3-2) preserves the retailer's local reliability which guarantees that in a rl_1 percentage of time the retailer's product stock can fulfill the pre-market's product demand. Thus, the product order quantity of the retailer from the manufacturer is computed:

$$MIN \quad \Pi = h^{+}.E\left[x - \widehat{D}\left(p, sl_p, sl_a, w\right)\right]^{+} + h^{-}.E\left[\widehat{D}\left(p, sl_p, sl_a, w\right) - x\right]^{+} \text{(3-1)}$$

$$S.T. \quad \Pr[\widehat{D}\left(p, sl_p, sl_a, w\right) \leq x] \geq rl_1 \text{(3-2)}$$

Based on the objective function $x = D\left(p, sl_p, sl_a, w\right)$. $G^{-1}(\frac{h^{-}}{h^{-}+h^{+}})$ minimizes the expected cost of the retailer and for preserving the constraint there should be $x \geq D\left(p, sl_p, sl_a, w\right).G^{-1}(rl_1)$. Accordingly, the best product ordering amount of the retailer from the manufacturer is:

$$x = D\left(p, sl_p, sl_a, w\right).G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \text{(3-3)}$$

By substituting Equation (3-3) into (3-1), the least total cost of the retailer is calculated:

$$\Pi = \left(h^{+}.E\left[G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} + h^{-}.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right).D\left(p, sl_p, sl_a, w\right) \text{(3-4)}$$

When the retailer orders *x* product units from the manufacturer, this protects the pre-market's product demand can be fulfilled from the retailer's on-hand product inventory with rl_1 probability (see the retailer in Figure 3-3). In Section 3.2.3, it is shown how this product's flow quantity must be amplified by moving backward to the manufacturer in the forward SC.

3.2.3. Manufacturer in the forward supply chain

The forward SC's manufacturer receives an order of *x* product units from the retailer and then orders the required components from the suppliers. Without loss of generality; it is assumed that for producing one unit of product, one unit of each component is required.

However, the production system of the manufacture is always accompanied by some wastage. The ratio of wastage to qualified product depends on the general state of its machinery which varies from time to time. It is assumed that the wastage ratio of the manufacturer's output changes over the range $[0,\beta]$ with a cumulative distribution function $G'(.)$. The manufacturer tries to compensate for this wastage in its production system by manufacturing extra product and consequently orders extra components form the suppliers.

If the manufacturer produces x product units, this production lot contains less than $\Delta x = \alpha.x$ ($\alpha \in [0,\beta]$) flawed product units with a $G'(\alpha)$ probability. Therefore, the manufacturer plans to produce $\Delta x + x$ product units to be sure with $G'(\alpha)$ probability to fulfill the whole order of the retailer. Since the local reliability of the manufacturer is assumed to be rl_2 (=$G'(\alpha)$), the extra production quantity of the manufacturer is $\Delta x = \acute{G}^{-1}(rl_2).x$. Thus, the manufacturer should order $\Delta x + x$ component units from each supplier in the forward SC. As mentioned before, the manufacturer only fulfills the x product order of the retailer before the beginning of the next period and extra product acquisition during the next sale period is impossible;

Procuring and producing $\Delta x + x$ product units by the manufacturer ensures that it can fulfill the x product order of the retailer with rl_2 probability (see the manufacturer in Figure 3-3). In Section 3.2.4, it is shown how these components' flow quantities will be amplified by moving backward to the suppliers of the forward SC.

3.2.4. Suppliers in the forward supply chain

Each supplier receives an order of $\Delta x + x$ component units from the retailer. After setting up the system, the production run starts in an in-control state of Supplier *i*'s machinery (*i* = 1, 2, …, *r*). But the machinery state deteriorates and shifts to an out-of-control state after a while. The time for deterioration is stochastic and roughly has an exponential distribution with mean $1/\mu_i$ (Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987). All the component units produced in the in-control state are qualified but from the units produced in the out-of-control state, γ_i percent are defective. Once the process shifts to the out-of-control state, it stays in this state until the whole production batch is finished because interrupting the machinery is either impossible or too expensive.

Each supplier should produce $\Delta x + x$ flawless component units. To compensate for the flawed component production in its production system, the supplier should plan to produce some more components, $\Delta\acute{x}_i + \Delta x + x$. The extra quantity of units, $\Delta\acute{x}_i$, is added to the production system of Supplier *i* to replace the defective component units. If it is assumed that the production rate of Supplier *i* is PR_{1i}, it takes $\frac{\Delta\acute{x}_i+\Delta x+x}{PR_{1i}}$ time units to produce this component volume. The extra volume $\Delta\acute{x}_i$ should be determined in a way to preserve the local reliability of the supplier, rl_3:

$$rl_3 = \Pr(flawless\ component\ units\ produced\ in\ \frac{\Delta\acute{x}_i + \Delta x + x}{PR_{1i}}\ time\ unit \geq \Delta x + x)$$

$$= \Pr\left[PR_{1i}.t + (1-\gamma_i).PR_{1i}.\left(\frac{\Delta\acute{x}_i+\Delta x+x}{PR_{1i}} - t\right) \geq \Delta x + x\right] = \Pr\left[t \geq \left(\frac{\Delta x+x}{PR_{1i}}\right) - \left(\frac{1-\gamma_i}{\gamma_i.PR_{1i}}\right).(\Delta\acute{x}_i)\right]$$

$$= EXP\left[-\mu_i.\left(\left(\frac{\Delta x+x}{PR_{1i}}\right)-\left(\frac{1-\gamma_i}{\gamma_i.PR_{1i}}\right).(\Delta\acute{x}_i)\right)\right] \quad (3\text{-}5)$$

Based on the equation above, $\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_3) + (\Delta x + x)\right]$ units of extra component production in Supplier *i*, ensures rl_3 local reliability for that supplier. This means that with this amount, $\Delta\acute{x}_i$, the supplier is able to fulfill the order of the manufacturer in rl_3 percent of time and preserve local reliability rl_3 for itself (see the suppliers in Figure 3-3). Therefore, with these amounts of $\Delta\acute{x}_i$ ($i = 1, 2, \ldots, r$) the suppliers are able to fulfill the orders of the manufacturer with ${rl_3}^r$ probability. With the amount of Δx determined in Section 3.2.3, the manufacturer is able to fulfill the product order of the retailer with rl_2 probability. With x product volume, the retailer is able to respond the realized product demand of the pre-market with rl_1 probability. Thus, $\Delta\acute{x}_i + \Delta x + x$ ($i = 1, 2, \ldots, r$) production volumes of the suppliers are able to fulfill the product demand in the pre-market with $rl_1.rl_2.{rl_3}^r$ probability and preserve service level $sl_p = rl_1.rl_2.{rl_3}^r$ for the whole forward chain against uncertainty propagation in its entities.

3.2.5. After-sales supply chain formulation

In this section, flow planning decisions in the after-sales SC are considered. This flow planning is done by considering transitions exist between the after-sales SC's facilities. This means first research question is answered in this section. The after-sales SC has several variations: i) variation in the demands of spare parts in the retailer to repair or substitute failed components of returned products and ii) variation in the performance of the production systems in the suppliers. In the rest of this section, the performance of the after-sales SC's facilities is formulated sequentially from the retailer in the downstream

to the suppliers in the upstream. Here, flow planning in the after-sales SC is determined that not only locally assures appropriate reliabilities for the chain's facilities against their uncertainties but also yields an acceptable performance for the whole after-sales SC (first research question).

3.2.6. Retailer in the after-sales supply chain

Based on Section 3.2.1, if it is assumed that rl_1 and rl_3 represent the local reliabilities in the retailer and suppliers respectively, then the service levels provided by the forward and after-sales SCs are $sl_p = rl_1.rl_2.{rl_3}^r$ and $sl_a = (rl_1.rl_3)^r$ respectively. Similarly to the forward SC, in the after-sales SC, the first after-sales operation starts in the retailer. Variation in the after-sales SC's retailer is related to the demand for spare parts. The demand for spare parts in the retailer is caused by the failed components in returned products which require part substitution. Thus, the demand for spare parts in the after-sales market is a function of total product sale in the pre-market and the reliability of the product's key components. Now for a given product sale in the pre-market, x, and a given component reliability, λ_i (i =1, 2, …, r), it is necessary to find an appropriate density function for the demand of the component.

It is assumed that the performance of the components in the product is independent. The failure time of Component i has density function f_i and cumulative density function F_i including the reliability parameter λ_i (i =1, 2, …, r). Lower values of the λ_i parameter imply higher reliability and lower failure of Component i. It is assumed that typically in each product the first n_i failures of Component i are repaired in the retailer with repair cost cn_i but after that, the failed component is replaced with a new one. Note that n_i = 0

implies a non-repairable component in the product. We also assume that the breakdown probability of a failed component does not change after repair and the time required for repair or substitution of components is negligible in comparison to the warranty time, *w* (Nguyen and Murthy, 1984).

If $F_i^{(m)}$ is defined as the cumulative density function of the total time to the m^{th} failure and $Num_i(w)$ represents the random number of failures in [0, *w*], then we have (Nguyen and Murthy, 1984):

$$\Pr\{Num_i(w) = m\} = F_i^{(m)}(w, \lambda_i) - F_i^{(m+1)}(w, \lambda_i) \quad (\forall i = 1, 2, \dots, r) \tag{3-6}$$

Based Equation 3-6, it is shown that:

Lemma 1: The average number of Component *i* substitutions, $E_i(w, n_i)$, for a product unit is calculated as follows:

$$E_i(w, n_i) = \sum_{j=n_i+1}^{+\infty} F_i^{(j)}(w, \lambda_i) \qquad (\forall i = 1, 2, \dots, r) \tag{3-7}$$

Lemma 2: The variance in the of number of Component *i* substitutions, $\sigma_i^2(w, n_i)$, for a product unit is calculated as follows:

$$\sigma_i^2(w, n_i) = \sum_{j=n_i+1}^{+\infty} [2.(j - n_i) - 1].F_i^{(j)}(w, \lambda_i) - [\sum_{j=n_i+1}^{+\infty} F_i^{(j)}(w, \lambda_i)]^2$$

$$(\forall i = 1, 2, \dots, r) \tag{3-8}$$

Now the total number of required Component *i* substitutions for a lot size of x product units can be estimated to represent the demand for Component *i* in the after-sales market,

D_i. D_i is the sum of required Component *i* substitutions for *x* individual units. Since *x* is large, based on the central limit theorem, it is claimed that:

Lemma 3: The demand of Component *i* in the after-sales market, D_i, can be approximated as being normally distributed with $x.E_i(w,n_i)$ mean and $x.\sigma_i^2(w,n_i)$ variance.

$$D_i \sim Normal\left(\mu_{D_i} = x.E_i(w,n_i), \sigma_{D_i}^2 = x.\sigma_i^2(w,n_i)\right) \quad (\forall i = 1,2,\dots,r) \tag{3-9}$$

Thus, the after-sales SC faces normally distributed random demand for components. Based on Equation (3-9), the spare parts demands in the after-sales market are functions of the total product, x, supplied by the forward chain to the pre-markets. This is another interaction existing between forward and after-sales SCs (second research question). Since local reliability rl_1 is assumed for the retailer, the stock quantity of Component *i* that preserves this local reliability in the retailer is:

$$x_i = x.E_i(w,n_i) + \left(z_{rl_1}.\sqrt{x.\sigma_i^2(w,n_i)}\right) \qquad (\forall i = 1,2,\dots,r) \tag{3-10}$$

We assume that the retailer provides the same reliability for both forward and after-sales SCs. Assigning different reliabilities for the retailer would simplify the problem because, in that case, service levels of the forward and after-sales SCs are independent.

Assuming that the first n_i failures of Component *i* in each product are repaired by the retailer with repair cost cn_i, the average repair cost of the product in the retailer is:

$$cr = \sum_{i=1}^{r}\sum_{j=1}^{n_i} j.cr_i.\Pr\{Num_i(w) = j\} = \sum_{i=1}^{r}\sum_{j=1}^{n_i} j.cr_i.\left[F_i^{(j)}(w,\lambda_i) - F_i^{(j+1)}(w,\lambda_i)\right] \tag{3-11}$$

In Equation (3-10), the prediction of the demand for spare parts in the sale period, *T*, is based on *w* which is usually longer than the sale period: $w = k_1.T$.

3.2.7. Suppliers in the after-sales supply chain

In the previous section, it is shown that for local reliability rl_1 in the after-sales SC's retailer, the following stock quantity of Component *i* is required:

$$x_i = x.E_i(w, n_i) + \left(z_{rl_1}.\sqrt{x.\sigma_i^2(w, n_i)}\right) \qquad (\forall i = 1, 2, \dots, r)$$

These quantities of components are ordered directly by the retailer from the corresponding suppliers. Hence, the supplier of Component *i* not only should produce and supply $\Delta x + x$ units of Component *i* to the manufacturer to assemble and produce the final product, but also should produce and supply x_i units of Component *i* to the chain's retailer to substitute the failed Components *i* of the returned products which have already been repaired n_i times. So the total component order received by Supplier *i* is $x_i + \Delta x + x$ units. To compensate for the nonconforming output of its production system, it should plan to produce some extra components represented by $\Delta\acute{x}_i$. In Section 3.2.4, the quantity of $\Delta\acute{x}_i$ was determined by assuming that $\Delta x + x$ component units are ordered to this supplier. As explained here, in addition to this order for the forward SC another order with x_i quantity is received from the after-sales SC. In this section, we revise the quantity of $\Delta\acute{x}_i$ in order to considering the after-sales SC. By following the approach described in Section 3.2.4 and the local reliability of the suppliers, rl_s, the extra production quantity of the suppliers should be modified as follows:

$$\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_3) + x_i + \Delta x + x\right] \quad (\forall i = 1, 2, \dots, r) \qquad (3\text{-}12)$$

We assume that the shortage in fulfilling the component order is divided proportionally between the order of the manufacturer and the order of retailer. In this case, we are sure with rl_3 probability that the conforming output of Supplier *i* can fulfil the order of the retailer. With x_i stock of Component *i*, the retailer is sure with rl_1 probability that it can respond to all Component *i* substitutions needed to repair the returned products. Therefore, the after-sales SC is sure with $rl_1.rl_3$ probability that it will be able to respond to all Component *i* substitutions needed for the returned products inside the sale period. By considering all key components of the product, the after-sales SC's service level is: $sl_a = (rl_1.rl_3)^r$.

In Sections 3.2.6 and 3.2.7 we answer the first research question by showing that how orders should be amplified through the facilities of the after-sales SC to deal with uncertainty propagation – qualified flow depreciation – in its stochastic facilities.

3.3. Mathematical model for concurrent flow planning in the supply chains

The appropriate selection of local reliabilities in different echelons of the SCs and the warranty time is very critical for our problem. As described in the previous sections, the chains' service levels in the pre- and after-sales markets are functions of these reliabilities. This means service levels in the pre- and after-sales markets depend on each other. This is another interaction between forward and after-sales SCs (second research question). Higher local reliabilities improve service levels and consequently the quantity of sales of the company in the pre-market. On the other hand, higher reliabilities lead to higher production volumes in the facilities which incur more costs to the system. The same issue is true for the warranty time. Longer warranty times make the product more attractive to

the customers and also it improves the pre-market's demand quantity. On the other hand, it imposes more after-sales costs on the system. By considering all these tradeoffs and interactions between the forward and after-sales SCs, we develop a comprehensive mathematical model to determine the best service levels and warranty time for the company in its pre- and after-sales markets and their preserving best local reliabilities and flow plan in a way to maximize the company's total profit. This mathematical model incorporates the quantified interactions existing between the operations of the forward and after-sales SCs (third research question). This mathematical model is formulated as follows:

Max $\acute{I}\Pi =$

$$\left(p - h^{+}.E\left[G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+} - cr\right).D(p, rl_1.rl_2.{rl_3}^{r}, (rl_1.rl_3)^{r}, w) - \left[\sum_{i=1}^{r}(a_{1i}+a_{2i}).(x+\Delta x+x_i+\Delta\acute{x}_i) + \sum_{i=1}^{r}\frac{h_{1i}.(x+\Delta x+x_i)^2}{2.PR_{1i}} + \sum_{i=1}^{r} b_{1i}.(x+\Delta x) + b_2.(x+\Delta x) + \frac{h_2.(x)^2}{2.PR_2} + (c_1+c_2).x + \sum_{i=1}^{r} c_{3i}.x_i\right] \quad (3\text{-}13)$$

Subject to

$$x = D(p, rl_1.rl_2.{rl_3}^{r}, (rl_1.rl_3)^{r}, w).G^{-1}\left(Max\left\{rl_1, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \quad (3\text{-}14)$$

$$\Delta x = G'^{-1}(rl_2).x \quad (3\text{-}15)$$

$$x_i = x.E_i(w, n_i) + \left(z_{rl_1}.\sqrt{x.\sigma_i^2(w, n_i)}\right) \qquad (\forall i = 1, 2, \dots, r) \quad (3\text{-}16)$$

$$\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_3) + x + \Delta x + x_i\right] \qquad (\forall i = 1, 2, \dots, r) \tag{3-17}$$

$$0 \leq rl_1,\ rl_2,\ rl_3 \leq 1 \tag{3-18}$$

$$w \geq 0 \tag{3-19}$$

The first term of the objective function is used to compute the profit captured by the retailer of the company in the pre-market. In this term, the average extra inventory, average shortage and average repair costs are removed from the captured income (see Equations 3-4 and 3-11). The second term is the sum of procurement, production, inventory holding, and transportation costs throughout the forward and after-sales SCs. The first item of the second term is the sum of procurement and production costs in the suppliers. Its second and fifth items are the inventory holding costs in the suppliers and the manufacturer respectively. The third and seventh items are the product transportation costs from the suppliers to the manufacturer and the spare parts transportation costs from the suppliers to the retailer respectively. The fourth term is the manufacturing cost in the manufacturer. The sixth term is the sum of transportation costs from the manufacturer to the retailer and the handling cost in the retailer. Equations (3-14)-(3-17) represent the relationships between the local reliability of the facilities and their production volumes.

This model is a nonlinear formulation with highly nonlinear terms in the objective function and constraints. The forms of some of these terms are not fixed and depend on the density functions of the uncertainties (Equations 3-14 and 3-15). Solving this type of models is not straightforward. But our model has some special characteristics which differentiate it from other models. In the next section, we propose a solution approach to solve the model.

3.4. Solution approach

The model proposed in Section 3.3 for concurrent flow planning in the forward and after-sales SCs is not only highly nonlinear but also the mathematical forms of some of its nonlinear terms such as Equations (3-14) and (3-15) depend on the density functions considered for modeling uncertainty. This means that by changing the type of density function, the mathematical form of these terms change. This makes it more challenging to solve. On the other hand, important design variables such as rl_1, rl_2, and rl_3 take value on a very restricted interval [0,1]; it is even more reasonable to assume that this interval is [0.5, 1.0]. Also in reality, 6 months, 1 year, 18 months and 2 years warranty lengths are common. These properties of this model makes discretizing it an appropriate method for solving it.

To discretize the model, it is necessary to define some new notations. $RL3 = \{rl3^1, rl3^2, \dots, rl3^{|RL3|}\}$, $RL2 = \{rl2^1, rl2^2, \dots, rl2^{|RL2|}\}$, and $RL1 = \{rl1^1, rl1^2, \dots, rl1^{|RL1|}\}$ are defined as sets of scenarios for the local reliability of the suppliers, the manufacturer, and the retailer respectively. For scenario selections from these sets, we need to define some new binary variables. Binary variables y_{rl1^i} ($\forall rl1^i \in RL1$), y_{rl2^i} ($\forall rl2^i \in RL2$), and y_{rl3^i} ($\forall rl3^i \in RL3$) are equal to 1 if the local reliability $rl1^i$, $rl2^i$ and, $rl3^i$ are selected from the sets $RL1$, $RL2$, and $RL3$ for the retailer, manufacturer and suppliers respectively; and 0 otherwise. In the same way, we define a set of warranty length $W = \{w^1, w^2, \dots, w^{|W|}\}$ and binary design variables z_{w^i} ($\forall w^i \in W$) for warranty selection from this set. Only one local reliability and warranty length can be selected from these sets:

$$\sum_{i=1}^{|RL1|} y_{rl1^i} = 1 \tag{3-21}$$

$$\sum_{i=1}^{|RL2|} y_{rl2^i} = 1 \tag{3-22}$$

$$\sum_{i=1}^{|RL3|} y_{rl3^i} = 1 \tag{3-23}$$

$$\sum_{i=1}^{|W|} z_{w^i} = 1 \tag{3-24}$$

By defining these new sets and variables, we revise Equations (3-14)-(3-18) representing the relationships between the production volume and local reliability of the SCs' facilities:

$$x = \sum_{i=1}^{|RL1|} \sum_{j=1}^{|RL2|} \sum_{k=1}^{|RL3|} \sum_{t=1}^{|W|} y_{rl1^i}.y_{rl2^j}.y_{rl3^k}.z_{w^t}.D\left(p, rl1^i.rl2^j.(rl3^k)^r, (rl1^i.rl3^k)^r, w^t\right). G^{-1}\left(Max\left\{rl1^i, \frac{h^-}{h^-+h^+}\right\}\right) \tag{3-25}$$

$$\Delta x = \sum_{i=1}^{|RL1|} \sum_{j=1}^{|RL2|} \sum_{k=1}^{|RL3|} \sum_{t=1}^{|W|} y_{rl1^i}.y_{rl2^j}.y_{rl3^k}.z_{w^t}.G'^{-1}(rl2^j).D(p, rl1^i.rl2^j.(rl3^k)^r, (rl1^i.rl3^k)^r, w^t). G^{-1}\left(Max\left\{rl1^i, \frac{h^-}{h^-+h^+}\right\}\right) \tag{3-26}$$

$$x_i = \sum_{i=1}^{|RL1|} \sum_{j=1}^{|RL2|} \sum_{k=1}^{|RL3|} \sum_{t=1}^{|W|} y_{rl1^i}.y_{rl2^j}.y_{rl3^k}.z_{w^t}.$$

$$\left[D\left(p, rl1^i.rl2^j.(rl3^k)^r, (rl1^i.rl3^k)^r, w^t\right).G^{-1}\left(Max\left\{rl1^i, \frac{h^-}{h^-+h^+}\right\}\right).E_i(w^t, n_i) + \left(z_{rl1^i}.\sqrt{D(p, rl1^i.rl2^j.(rl3^k)^r, (rl1^i.rl3^k)^r, w^t).G^{-1}\left(Max\left\{rl1^i, \frac{h^-}{h^-+h^+}\right\}\right).\sigma_i^2(w^t, n_i)}\right)\right] \quad (\forall i = 1, 2, \dots, r) \tag{3-27}$$

$$\Delta \acute{x}_i = \sum_{i=1}^{|RL1|}\sum_{j=1}^{|RL2|}\sum_{k=1}^{|RL3|}\sum_{t=1}^{|W|} y_{rl1^i}.y_{rl2^j}.y_{rl3^k}.z_{w^t}.\left(\frac{\gamma_i}{1-\gamma_i}.\left[\frac{PR_{1i}}{\mu_i}\ln\left(rl3^k\right)\right.\right.$$
$$+\left(G'^{-1}\left(rl2^j\right)+1\right).D\left(p,rl1^i.rl2^j.(rl3^k)^r,(rl1^i.rl3^k)^r,w^t\right).G^{-1}\left(Max\left\{rl1^i,\frac{h^-}{h^-+h^+}\right\}\right)$$
$$+D\left(p,rl1^i.rl2^j.(rl3^k)^r,(rl1^i.rl3^k)^r,w^t\right).G^{-1}\left(Max\left\{rl1^i,\frac{h^-}{h^-+h^+}\right\}\right).E_i(w^t,n_i)$$
$$\left.\left.+z_{rl1^i}.\sqrt{D(p,rl1^i.rl2^j.(rl3^k)^r,(rl1^i.rl3^k)^r,w^t).G^{-1}\left(Max\left\{rl1^i,\frac{h^-}{h^-+h^+}\right\}\right).\sigma_i^2(w^t,n_i)}\right]\right)$$
$$(\forall i=1,2,\dots,r) \qquad \text{(3-28)}$$

After substituting these equations in the objective function (Equation 3-13) and linearizing the multiplication of binary variables, the mathematical model of the problem is transformed to a mixed integer linear model with binary variables which can be solved globally using software such as CPLEX, GAMS, GROOBI and LINGO. We used CPLEX to solve it.

3.5. Example, results, and discussion: Correlations among marketing factors

In this section, a company is considered which produces and supplies a durable consumer product to a target market with a stochastic and elastic demand function for the retail price $p = \$10.00$. This product includes two critical components: Component 1 and Component 2. Component 1 and 2 are manufactured by Supplier 1 and 2 with procurement and production costs of $a_{11} + a_{21} = \$3.50$ and $a_{12} + a_{22} = \$2.50$ respectively. Then, these components are transported to the manufacturer and assembled into the final product with the cost of $b_2 + b_{11} = b_2 + b_{12} = \$1.00$. After that the final products are transported and handled in the retailer with the cost of $c_1 + c_2 = \$0.5$. Based on historical sales, the average product demand in the pre-market is treated as a linear

function of the retail price, warranty time and service levels: $D(p, sl_p, sl_a, w) = 500 + 200 \times w - 250 \times (p - 10) - 500 \times (1 - sl_a) - 900 \times (1 - sl_p)$.

The products of this company are offered with a warranty. The company has four options for the warranty length: 6, 12, 18, and 24 months. Dead inventory and lost sales at the end of the sale period impose $h^- = \$0.10$ and $h^+ = \$0.15$ costs on the company respectively. Component 1 and 2 of this product have reliability parameters $\lambda_1 = 0.1$ and $\lambda_2 = 0.4$. Component 1 is not repairable. Thus, if the failure of a returned product inside the warranty time is due to Component 1, then that part is replaced with a new one by the retailer. But for Component 2, the story is different. It is more economic to repair Component 2 the first time it fails, but after the first failure, it is substituted with a new component. Similarly to the final product, the required components for repairing returned products should be produced and stored in the retailer before the beginning of the sale period. The components are produced in the first and second suppliers with the production rates of $PR_1 = 8000$ (number per time unit) and $PR_2 = 9000$ (number per time unit) respectively. The average deterioration times in the first and second suppliers are similar and equal to $1/\mu_{1\ and\ 2} = 0.5$. After deterioration, 10 and 20 percent of Component 1 and Component 2 production in the first and second suppliers is non-conforming ($\gamma_1 = 0.10$ and $\gamma_1 = 0.20$). The uncertain part of the pre-market's demand function, ε, is normally distributed with mean of 0.0 and variance 1.0. Also the flawed production rate in the manufacturer is uniformly distributed over the range [0,$\beta = 0.15$]. Components 1 and 2 produced in Suppliers 1 and 2 for after-sales market operations are transported directly to the retailer with transportation costs $c_{31} = c_{32} = \$1.00$. Solving the mathematical model of this problem leads to the following results: the local reliabilities in the retailer,

manufacturer, and suppliers are $rl_1 = 0.99$, $rl_2 = 0.86$ and $rl_3 = 0.85$ respectively. The best warranty option is 6 months. To preserve these local reliabilities $x = 632.9$ product units are ordered by the retailer from the manufacturer. For fulfilling this order of the retailer, the manufacturer plans to manufacture $\Delta x = 81.64$ extra product units to compensate for the malfunction of its system. To produce this product volume, the required components are ordered to the corresponding suppliers. In addition to this component order from the manufacturer, suppliers receive another order from the retailer, x_i ($i = 1$ and 2), to provide the required components for repairing returned products. Similarly, the first and second suppliers plan to procure and produce $\Delta x_1' = 12.13$ and $\Delta x_2' = 0.908$ extra Component 1 and Component 2 volumes to compensate for defective production in their production systems respectively. The results are summarized in Figure 3-4.

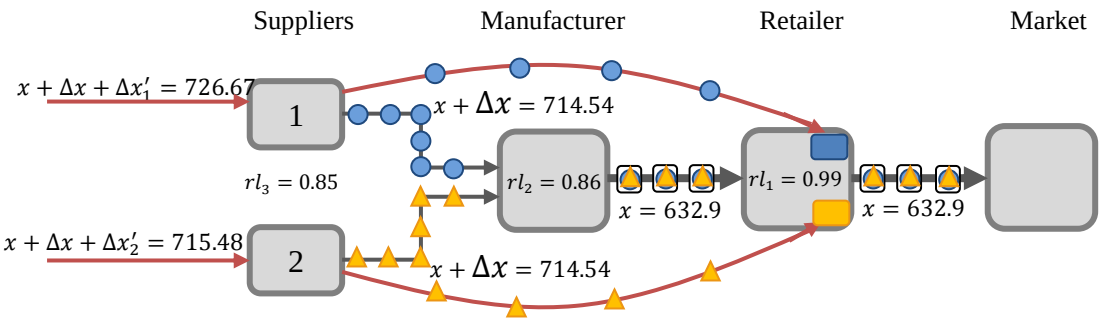


Figure 3-4: Flow dynamics in the SCs.

This flow planning leads to $\acute{I}I = \$615.60$ profit for the company which is the highest in retail price $p = \$10.00$.

In the model used in this chapter and also in the sample problem investigated in this section, the retail price of the product is assumed to be a fixed exogenous factor. However,

the retail price has always been one of the most important competition factors for rivals in the markets. Determining appropriate retail price is not straightforward because of its conflicting effects on the company's sales volume and unit marginal profit. The price increment augments the unit marginal profit of each sale but, on the other hand, reduces the attractiveness of the product for customers and leads to lower sales volume. In the rest of this section, we analyze the correlation between the retail price and the after-sales service of the company by defining several hypotheses. It is assumed that retail price of the product can be selected on the [$9.0, $13.5] interval. This interval is determined by different factors such as the retail price of similar rival or substitutable products and governmental regulations to support domestic production or customers.

Observation 1: *Price increment or reduction has non-homogenous effects on the company's profit in a given warranty option*.

Observation 2: *The trend of the profit function changes with respect to the price is homogeneous for different warranty options.*

To test these observations, the mathematical model of the problem is solved for different values of the price on the [$9.0, $13.5] interval and different options of the warranty time. The results are presented in Figure 3-5. As seen in Figure 3-5, the price increment has almost the same effect on the company's profit for different warranty length options which is consistent with Observation 2. At first, the price increment leads to higher profit in the company because the positive effect of unit marginal profit increment on the profit of the company dominates the negative effect of the reduction of sales. So gradually the company's profit starts to increase. In the 6-month warranty

length, the highest profit is achieved is $p = \$10.75$ which is equal to $\acute{I}I = \$782.20$. At this price, the difference between positive and negative effects of the price increment become zero and beyond that, its negative effect dominates the positive effect. Thus, the company's profit starts to decrease. Therefore as claimed in Observation 1, the retail price increments have non-homogeneous effects on the company's profit in a given warranty length. Based on these observations, we conclude that "price increments or reductions have non-homogeneous effects on the profit of a company in a given warranty length, but the trend of these changes are almost similar for all warranty options".

Observation 3: *The priority of the warranty options with respect to the profit changes in different price intervals.*

After solving the mathematical model of the problem for different values of the retail price and different warranty length options, a function representing the profit function of the company for each warranty length with respect to the retail price values is fitted. These functions are displayed in Figure 3-5. These profit functions have several intersections indicated by red dots in this figure. These dots represent the critical retail price values at which the priority and profitability of the warranty options changes. In this problem, these critical price values are as follows:

- *If* $p < p_1 = \$11.25$ *then the priority of the warranty options is: 6, 12, 18 and 24 months.*
- *If* $p_1 < p \leq p_2 = \$11.57$ *then the priority of the warranty options is: 12, 6, 18 and 24 months.*

- *If* $p_2 < p \leq p_3 = \$11.82$ *then the priority of the warranty options is: 12, 18, 6 and 24 months.*
- *If* $p_3 < p \leq p_4 = \$12.07$ *then the priority of the warranty options is: 18, 12, 24 and 6 months.*
- *If* $p_4 < p \leq p_5 = \$12.37$ *then the priority of the warranty options is: 18, 24, 12 and 6 months.*
- *If* $p_5 = 12.37 < p$ *then the priority of the warranty options is: 24, 18, 12 and 6 months.*

As claimed in Observation 3, the priority of the warranty options with respect to profit changes over different price intervals.

Observation 4: *Appropriate selection of the warranty length is more important in price-sensitive markets.*

To test this observation, the price sensitivity parameter of the market is doubled (is increased from 250 to 500) and all of the models are re-computed with this new parameter. The results are summarized in Figure 3-6. As seen in this figure, the optimal price in all the warranty functions shifts to the left. This means that in price sensitive markets, the highest profit of the company occurs at lower retail prices regardless of the warranty length. On the other hand, the differences among the profitability of the warranty options become more significant. This means that an inappropriate selection of the warranty length leads to a higher profit loss in this market in comparison with less price sensitive markets. Also the intervals between the critical retail price values in which the priority of the warranty options changes become smaller. In this kind of market, the

priority of the warranty options is more fragile and changes faster with retail price variations. These outcomes are consistent with Observation 4.

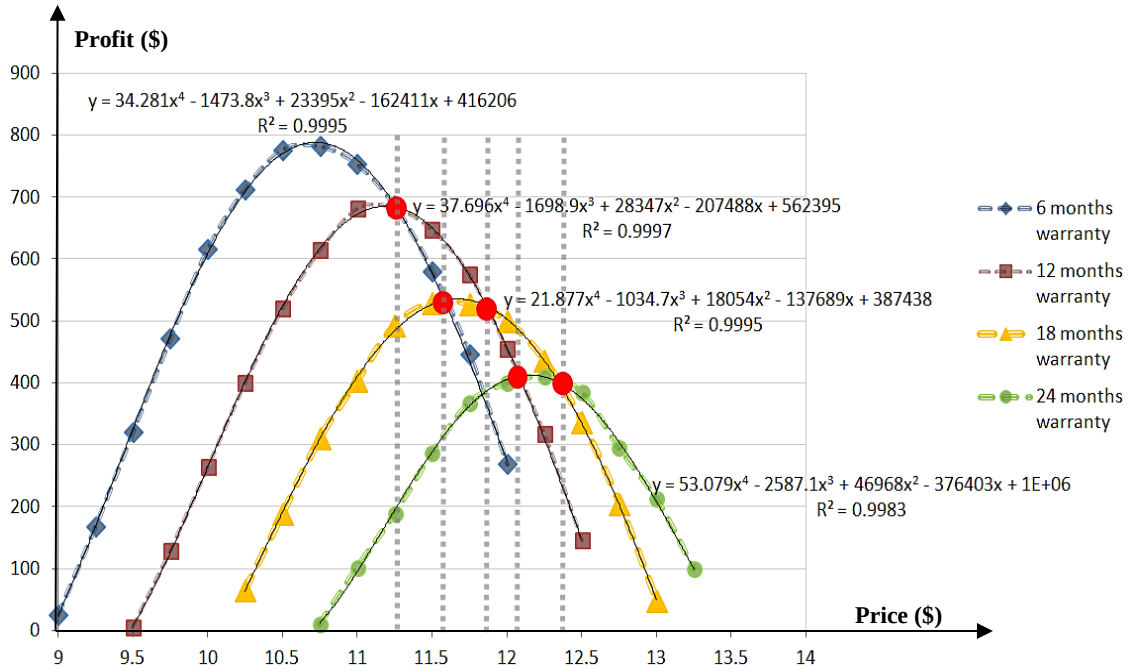


Figure 3-5: Profit of the company with respect to the price in different warranty lengths.

In the price sensitive market, the critical price values are as follows:

- *If* $p < p_1 = \$10.90$ *then the priority of the warranty options is: 6, 12, 18 and 24 months.*
- *If* $p_1 < p \leq p_2 = \$11.05$ *then the priority of the warranty options is: 12, 6, 18 and 24 months.*
- *If* $p_2 < p \leq p_3 = \$11.20$ *then the priority of the warranty options is: 12, 18, 6 and 24 months.*
- *If* $p_3 < p \leq p_4 = \$11.38$ *then the priority of the warranty options is: 18, 12, 24 and 6 months.*

- *If* $p_4 < p \leq p_4 = \$11.55$ *then the priority of the warranty options is: 18, 24, 12 and 6 months.*
- *If* $p_4 < p$ *then the priority of the warranty options is: 24, 18, 12 and 6 months.*

Therefore, we conclude that in price sensitive markets: i) an inappropriate selection of the warranty length leads to higher profit loss; and ii) the priority of the warranty options from the profit perspective is more fragile with respect to price variations.

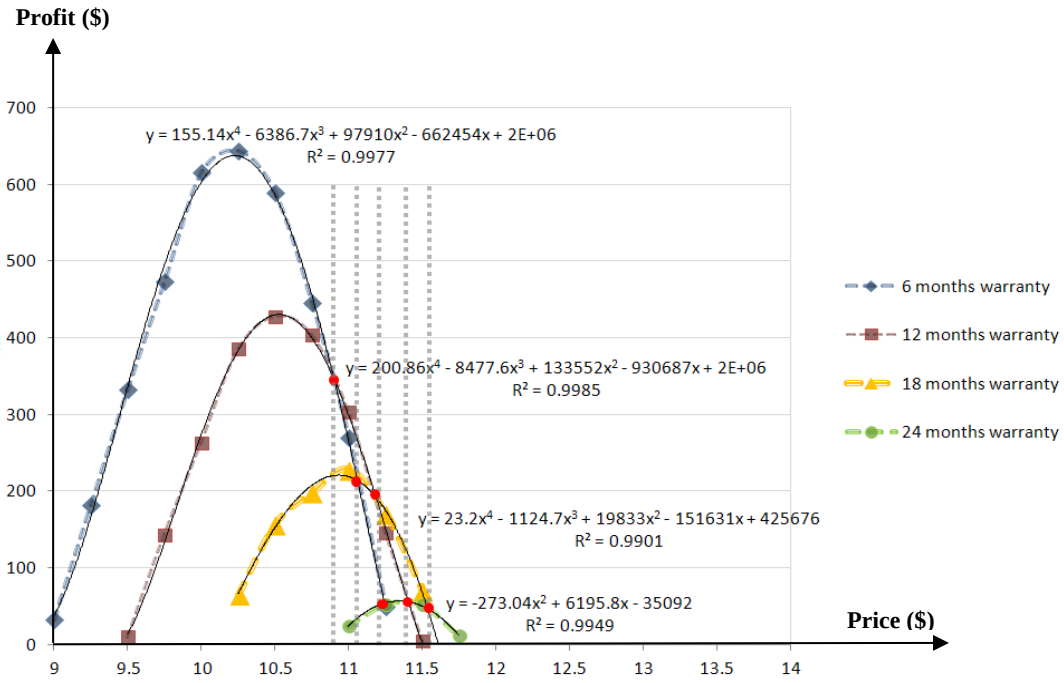


Figure 3-6: The profit of the company with respect to the price in price sensitive markets.

Observation 5: *In warranty sensitive markets, optimal retail prices are higher.*

In order to test this observation, the warranty sensitivity parameter of the market in the problem is doubled (is increased from 200 to 400) and all of the models are re-

computed with this new parameter. The results are summarized in Figure 3-7. As seen in this figure, in this case, the warranty options with higher lengths are more attractive and the highest profit is achieved with a 24 months warranty option. Furthermore, greater warranty lengths justify the optimality of higher retail prices in this market as the positive effect of the warranty increment dominates the negative effects of price augmentation on the market's demand volume. The optimal retail price is $p = \$13.30$ in this case. In warranty sensitive markets, the critical priority changing price points become farther from each other. This means that the warranty strategies are more stable in this market and the priority of the warranty options is more stationary with respect to variations of the retail price. This corroborates Observation 5.

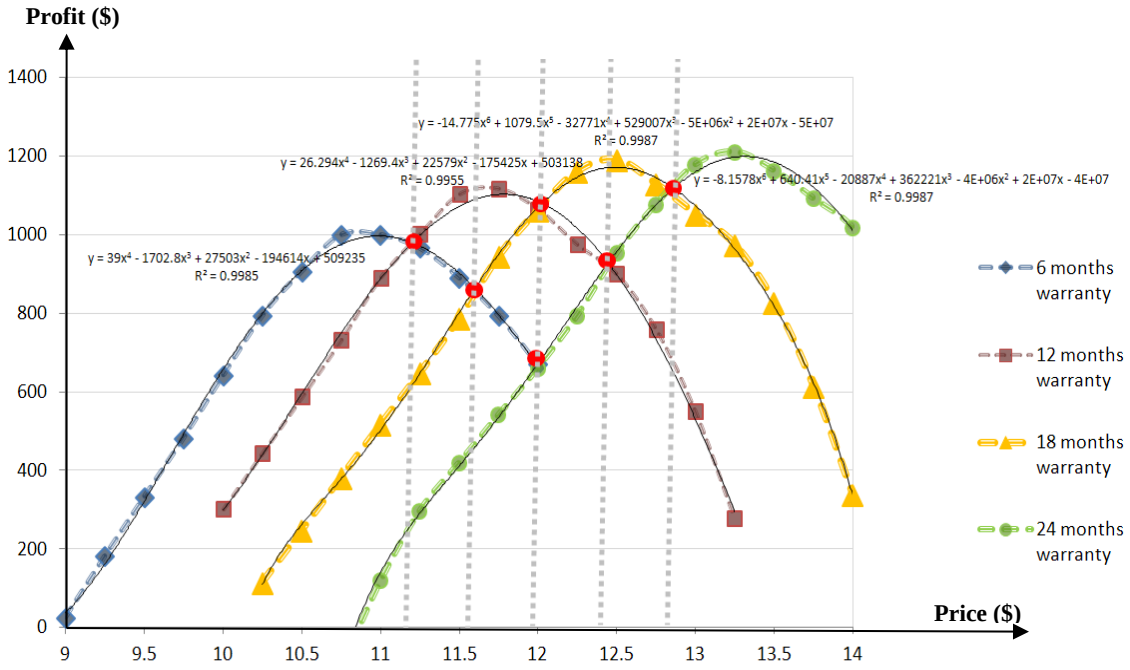


Figure 3-7: The profit of the company with respect to the price in warranty sensitive markets.

In the warranty sensitive market, critical price values are as follows:

- *If* $p < p_1 = \$11.17$ *then the priority of the warranty options is: 6, 12, 18 and 24 months.*
- *If* $p_1 < p \leq p_2 = \$11.57$ *then the priority of the warranty options is: 12, 6, 18 and 24 months.*
- *If* $p_2 < p \leq p_3 = \$11.95$ *then the priority of the warranty options is: 12, 18, 6 and 24 months.*
- *If* $p_3 < p \leq p_4 = \$12.40$ *then the priority of the warranty options is: 18, 12, 24 and 6 months.*
- *If* $p_4 < p \leq p_4 = \$12.75$ *then the priority of the warranty options is: 18, 24, 12 and 6 months.*
- *If* $p_4 < p$ *then the priority of the warranty options is: 24, 18, 12 and 6 months.*

Therefore, we conclude that in warranty sensitive markets: i) optimal retail prices are higher; and ii) the priority of the warranty options from the profit perspective is more stable with respect to price variations.

3.6. Closure of chapter 3

In this chapter, a company is considered that produces and supplies its products to the customers of a market under a failure-free warranty. Hence, producing and providing enough spare parts to repair the returned products of the customers inside the warranty time is an important responsibility of this company. While the product is produced through the forward SC, the required spare parts for repairing its failures are produced through the after-sales SC. In this chapter, we show that the operations of these two SCs

are not independent and there is a huge synergy in their concurrent flow planning. To demonstrate the necessity of this concurrent planning, we answer the following questions:

- ✓ **Research Question 1:** *what are the important flow transitions among the facilities supporting after-sales services?*
- ✓ **Research Question 2:** *what are the important interactions between forward and after-sales SCs justifying the necessity of their concurrent flow planning?*
- ✓ **Research Question 3:** *how do these interactions affect planning flow dynamics in the forward and after-sales SCs of non-repairable goods?*

In this chapter we answer these questions in the following ways:

- ✓ **Answer of Research Question 1:** In Section 3.1, we explain the operations and flow transaction through the facilities of the after-sales SCs. In the three-echeloned test problem, we show that suppliers and retailer are involved in the after-sales operations. By analyzing the failure probability of the supplied products and total product supply quantity through the forward SC, the retailer makes decision about the required spare parts quantity. Orders of the retailer are fulfilled by the suppliers.
- ✓ **Answer of Research Question 2:** In Section 3.1, we show that there are two important interactions between the forward and after-sales SCs: 1) the demand of the forward SC in the pre-market depends on the service level provided by the after-sales SC; and 2) the after-sales demand of the components depends on the total products supplied by the forward SC to the market and the quality of the product's components..

- ✓ **Answer of Research Question 3:** The interactions between the facilities of the forward SC are qualified in Sections 3.2.1-3.2.5 and these interactions in the after-sales SC are quantified in Sections 3.2.6-3.2.7. The interplays between the forward and after-sales SCs are considered in modeling product and spare parts demands in the pre- and after-sales markets. These equations are used in Section 3.3 to develop a mathematical model for concurrent flow planning in the SCs.

We show that in SCs with stochastic facilities, qualified flow depreciates by moving from upstream to downstream. To neutralize its negative effect and plan a reliable flow dynamics throughout the chains' networks, we develop an approach which amplifies the orders between the facilities from downstream to upstream. This method is incorporated in the mathematical model of Section 3.3. The outcomes of this model are as follows: 1) the best retail price, warranty length, and service levels for the company in its pre- and after-sales markets to maximize the company's total profit; and 2) the appropriate local reliabilities in the echelons of the forward and after-sales SCs and their corresponding flow planning to preserve the company's service levels. Analyzing the computational results of the model reveals some interesting insights:

- ✓ ***Effect of the retail price on the profitability of the warranty options:*** Price increments or reductions may have non-homogeneous effects on the profit of the company in a given warranty length. But the trend of these changes are almost similar for all warranty options.

- ✓ ***Priority of the warranty options in different price intervals:*** Priority of the warranty options with respect to profit changes in the critical price values.

Therefore in the price intervals between sequential critical price values, they have different priority (or profitability order).

- ✓ ***Importance and stability of the warranty options in price-sensitive markets:*** In price sensitive markets, an inappropriate selection of the warranty length leads to higher profit loss. This means that an appropriate warranty length selection is more important in price-sensitive markets. However, the priority of the warranty options from the profit perspective is more fragile with respect to price variations in these market.
- ✓ ***Optimal price and stability of the warranty options in warranty-sensitive markets:*** In warranty-sensitive markets, optimal retail prices are higher and the priority of the warranty options from the profit perspective is more stable with respect to price variations.

Although the focus of this chapter is on durable consumer products for which repairing the returned products is the main responsibility of the after-sales SCs and a failure-free warranty strategy is considered, this formulation can be modified for other product types with different warranty strategies, e.g., non-repairable products with rebate warranties. In addition, the concepts developed here can be modified to make it applicable for capital goods such as computer networks, medical and defense systems, infrastructure, and so on, for which performance-based contracts are usual. In these industries developing, installing, or constructing systems are done by the forward SCs and maintaining the system to keep them performing at an acceptable level of availability is the responsibility of after-sales SCs.

Chapter 4: Operationally Fail-safe Supply Chains Servicing Pre- and After-sales Markets of Repairable Products

In this chapter, we consider a company producing and supplying a product-service package to a target market. The service provided for customers is a failure-free warranty. Inside the warranty period, repair requests of the sold products are fulfilled free of charge. In spite of Chapter 3, we assume that the product is repairable and the failed components of the defective products returned by the customers inside the warranty period can be repaired and used in the repair process of the future returned products. The repair process of a failed component is done in the repair section of its corresponding supplier. In this problem, the company has two SCs: 1) a forward SC deals with producing and supplying the products to the pre-market; and 2) an after-sales SC deals with producing and supplying required components to fulfill the after-sales repair request of products failed inside the warranty period. Since failed components of defective products are repairable, there are two component flows in the after-sales SC: flow of repaired components and flow of new components. The new components are used when repaired ones are not available. Having two highly convoluted flow types complicates the operations throughout the after-sales SC and its flow planning problem and the interactions exist between the forward and after-sales SCs. Therefore, in this chapter we are going to answer the fourth research question for SCs:

- ✓ **Research Question 4:** *how do the interactions between the forward and after-sales SCs of repairable goods affect planning their flow dynamics?*

In Section 4.1, we define the problem in detail and talk about its assumptions, objective function, and constraints. The flow transactions between the facilities of the forward and

after-sales SCs and the interactions between the operations of the SCs are explained qualitatively in this section. These flow transactions and interactions are mathematically quantified in Section 4.2. Section 4.2.1 is dealing with flow transactions between the facilities of the forward SC. In Section 4.2.2, we model the new and repaired components' flow transactions between the facilities of the after-sales SC. An integrated mathematical model is developed in Section 4.2.3 dealing with simultaneous flow planning in the forward and after-sales SCs considering their interactions. In Section 4.3, we propose an approach to solve the integrated model. The model and its solution approach are applied for a test problem from automobile industry in Section 4.4. By analyzing the results, we investigate the correlations between the marketing strategies – price, service levels, and warranty period – of the company and find the best combinations.

4.1. Operations and variations in the supply chains of repairable products

In this problem, we consider a company producing and supplying a product to a target market through its forward SC. This product is sold to the customer under a retail price and a warranty period. This product includes several key components which are produced by suppliers in the first echelon. These components are transported to a manufacturer in the second echelon. After assembly, final products are supplied to the market through a retailer. The flow of components and final products through the forward SC are displayed in Figure 4-1 for a sample product with two key components.

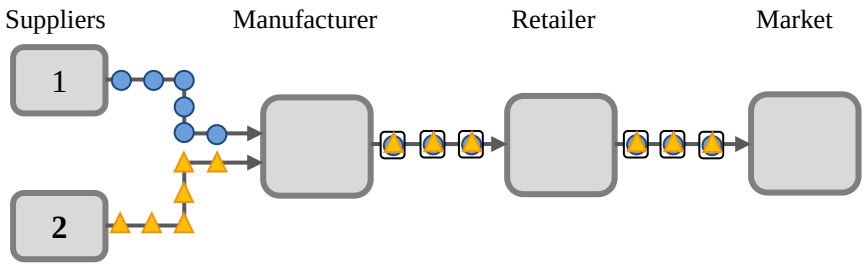


Figure 4-1: The flow of components and products through the forward SC (for a product with two key components).

The products of this company are sold under a warranty and all the defective products returned inside the warranty period must be fixed free of charge. The flow of returned defective products is represented by orange lines in Figure 4-2. Spare parts required to fix these returned products are provided through the after-sales SC. The after-sales SC has repair sections inside the suppliers to repair failed components of the returned products. As seen in Figure 4-2, defective components are sent by the retailer to the repair sections for repair. Then, the repaired components are returned and stored in the retailer for use in repairing the next defective product.

If there is not a repaired component in the retailer, new components provided and stored by the suppliers in the retailer are used for the repairs. The storage of new components in the retailer preserves an appropriate service level for the after-sales SC. The flow of the repaired and new components through the after-sales SC are displayed in Figure 4-2 by the green and pink lines respectively.

The required products and new components needed to fulfill the product demand and inside-warranty repair requests for each sales period are produced by the forward and after-sales SCs respectively and stored in the retailer before its beginning. Before the

beginning of each sales period, the retailer orders the required products and components from the manufacturer and suppliers respectively. Based on the retailer's order and the performance of its production system, the manufacturer orders the required components from the suppliers. This means that suppliers receive two orders: one order from the manufacturer and another order from the retailer. Then, based on the capabilities of their production systems, the suppliers estimate and order the required material.

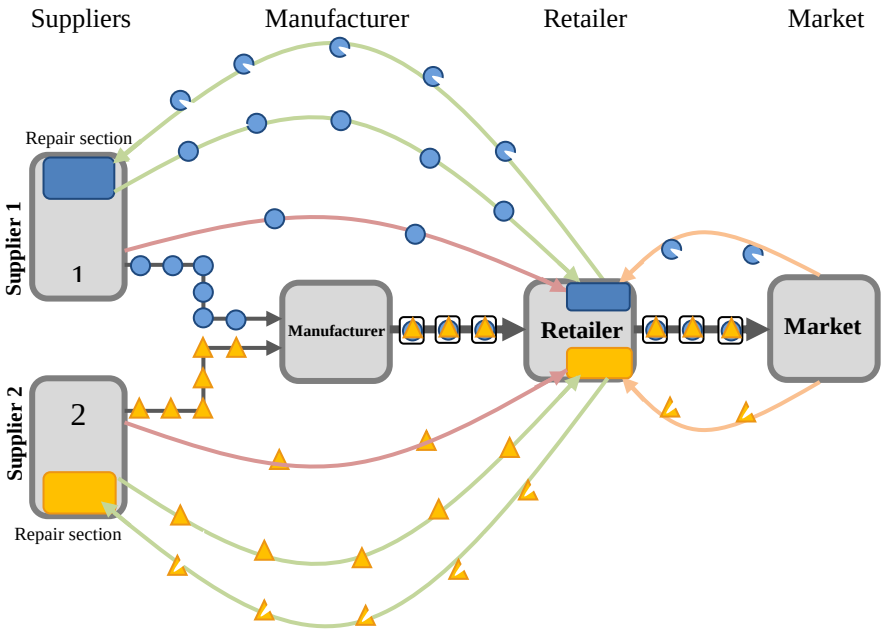


Figure 4-2: The flow of new and repaired components through the after-sales SC (for a product with two key components).

We consider two types of variation in this problem: i) demand-side variations; and ii) supply-side variations. The demand-side represents the variation in the prediction of product demand in the pre-market and the prediction of demand for spare parts in the after-sales. Supply-side variations are related to imperfect production systems in the SCs' production facilities (e.g., the suppliers and the manufacturer). Production in the production facilities is always accompanied by a stochastic percentage of non-

conforming output which depends on the state of the machinery and labor and varies from time to time. The variations in the qualified output of the facilities accumulate and become larger and larger by moving the flow from the upstream to the downstream of the chains. In this chapter we term this "uncertainty propagation". Due to uncertainty propagation, the quantity of the qualified flow depreciates by moving from the upstream to the downstream which leads to a stochastic qualified supply quantity in the last echelon (see Figure 4-3). The capability of the forward and after-sales SCs in balancing the stochastic supply and demand quantities in the pre- and after-sales markets is the pre- and after-sales service levels respectively. These service levels represent the capability of the chains to fulfill demand. The product demand in the pre-market is an increasing function of the warranty length and service levels and a decreasing function of the retail price.

In this complex production system, which includes two interactive SCs with multiple stochastic facilities and services pre- and after-sales markets with stochastic demand, we want to determine the best marketing strategies (price, warranty, and service levels) for the company and the best reliable flow dynamics through its SCs preserving the marketing strategies in the most profitable way.

4.2. Mathematical model for the problem

This problem has two critical parts with different missions: i) the forward SC servicing the pre-market and ii) the after-sales SC fulfilling the after-sales commitments. Operations in the forward and after-sales SCs are analyzed separately in Section 4.2.1 and 4.2.2 respectively. Finally with the help of the equations derived in these two sections, we develop a mathematical model in Section 4.2.3 for concurrent reliable flow planning

through the networks of these SCs. Notation used in this chapter are summarized in Table 4-1.

Table 4-1: Notation for the forward and after-sales SCs of repairable products.

Variables	
w	Warranty length of product;
s_i	Safety stock of Component *i* in the retailer ($i = 1, 2, \dots, N$);
rl_r	Local reliability of retailer;
rl_m	Local reliability of manufacturer;
rl_s	Local reliability of suppliers;
sl_a	After-sales SC's service level;
sl_p	Forward SC's service level;
x	Product order quantity of retailer from manufacturer;
Δx	Extra product assembly quantity in the manufacturer;
$\Delta \acute{x}_i$	Extra component production in Supplier *i* ($i = 1, 2, \dots, N$);
p	Price of the product supplied to the market by the company;
y_{rl1^i}	1 if reliability scenario $rl1^i$ is selected from $RL1$ set and 0 otherwise;
y_{rl2^i}	1 if reliability scenario $rl2^i$ is selected from $RL2$ set and 0 otherwise;
y_{rl3^i}	1 if reliability scenario $rl3^i$ is selected from $RL3$ set and 0 otherwise;
z_{w^t}	1 if warranty scenario w^i is selected from W set and 0 otherwise;
Parameters and Functions	
N	Number of product's components ($i = 1, 2, \dots, N$);
K	Number of sale periods inside the warranty;
$\acute{K}$	Number of time units inside the sale period;
T	Sale period;
$\acute{T}$	Time unit;
$\hat{D}(p, sl_p, sl_a, w)$	Stochastic function of product demand in the pre-market. We assume that $\hat{D}(p, sl_p, sl_a, w) = D(p, sl_p, sl_a, w) \times \varepsilon$ and $E[\hat{D}(p, sl_p, sl_a, w)] = D(p, sl_p, sl_a, w)$;
ε	Random variable representing the stochastic part of product demand function;
$G(.)$	Cumulative density function of ε variable;
h^+	Unit holding cost of extra inventory at the end of sale period in the retailer;
h^-	Unit shortage cost of lost sale at the end of sale period in the retailer;
β	Maximum defective assembly rate in the manufacturer;
$G'(.)$	Cumulative density function of defective assembly rate in the manufacturer;
μ_i	Average rate of shifting from in-control to out-of-control for the machineries of Supplier *i* in producing each production batch ($i = 1, 2, \dots, N$);
γ_i	Average rate of non-conforming production in the out-of-control state of Supplier *i*'s machineries ($i = 1, 2, \dots, N$);
PR_{1i}	Production rate of Supplier *i* (i = 1, 2, …, N);
PR_m	Production rate of manufacturer;
τ_i	Reliability index of Component *i* (i = 1, 2, …, N);
F_i	Cumulative distribution function of Component *i*'s failure time ($i = 1, 2, \dots, N$);
$F_i^{(m)}$	Cumulative distribution function of total time up to the m^{th} failure in Component *i* ($i = 1, 2, \dots, N$);

$Num_i(w)$	Random variable represents the number of Component *i*'s failures inside the warranty interval ($i = 1, 2, \dots, N$);
$E_i(w)$	Average number of failures for a unit of Component *i* inside the warranty time ($i = 1, 2, \dots, N$);
λ_i	Expected failure number of Component *i* during each sale period ($i = 1, 2, \dots, N$);
t_i	Random variable represents the repair time of Component *i* in the repair section of its corresponding supplier ($i = 1, 2, \dots, N$);
N_1^i	Steady state number of Component *i* in the in-pipeline ($i = 1, 2, \dots, N$);
N_2^i	Steady state number of Component *i* in the repair section of Supplier *i* ($i = 1, 2, \dots, N$). These parts are either waiting in the queue or being serviced;
$N_3^i(t)$	Steady state number of Component *i* in the out-pipeline ($i = 1, 2, \dots, N$);
N_4^i	Steady state inventory level at the repair section of Supplier *i* ($i = 1, 2, \dots, N$);
N_5^i	Steady state inventory level of Component *i* in the retailer ($i = 1, 2, \dots, N$);
B_0^i	Steady state backorder level in the repair section of Supplier *i* ($i = 1, 2, \dots, N$);
B_1^i	Steady state backorder level of Component *i* in the retailer ($i = 1, 2, \dots, N$);
$E[t_i^s]$	s^{th} moment of service time in the repair Section *i* ($i = 1, 2, \dots, N$);
O_i	Average shipment time between retailer and repair section of Supplier *i* ($i = 1, 2, \dots, N$);
ρ_i	Utilization of repair section of Supplier *i* ($i = 1, 2, \dots, N$);
a_0^i	Number of success parameter of negative binomial distribution used to approximate first convolution;
b_0^i	Success probability parameter of negative binomial distribution used to approximate first convolution;
a_1^i	Number of success parameter of negative binomial distribution used to approximate second convolution;
b_1^i	Success probability parameter of negative binomial distribution used to approximate second convolution;
μ_0^i	Expected number of Component *i* in the corresponding in-pipeline and repair section ($i = 1, 2, \dots, N$);
$\sigma^{2}{}_0^i$	Variance of number of Component *i* in the corresponding in-pipeline and repair section ($i = 1, 2, \dots, N$);
μ_1^i	Expected number of Component *i* backordered by the repair section or transferring to the retailer through the out-pipeline ($i = 1, 2, \dots, N$);
$\sigma^{2}{}_1^i$	Variance of number of Component *i* backordered by the repair section or transferring to the retailer through the out-pipeline ($i = 1, 2, \dots, N$);
G_{NB}^i	Negative binomial cumulative distribution function used to approximate density of $N_3^i(t) + B_0^i(t)$;
TC_r	Total inventory and shortage cost in the retailer;
cr_i^{rs}	Unit transportation cost of Component *i* from retailer to Supplier *i* ($i = 1, 2, \dots, N$);
cr_i^s	Unit service cost in the repair section of Supplier *i* ($i = 1, 2, \dots, N$);
cr_i^{sr}	Unit transportation cost of Component *i* from Supplier *i* to the retailer ($i = 1, 2, \dots, N$);
cr_i	Total cost of repairing unit of Component *i* ($i = 1, 2, \dots, N$);
cr	Average repair cost of the product unit;
Π	Total profit of whole company;
ca_{1i}	Unit procurement cost of material in Supplier *i* (*i* = 1, 2, …, *N*);
ca_{2i}	Unit production cost of Component *i* in Supplier *i* (*i* = 1, 2, …, *N*);
ch_{1i}	Unit inventory holding cost for a time unit in Supplier *i* (*i* = 1, 2, …, *N*);
cb_{1i}	Unit transportation cost of a component from Supplier *i* to the manufacturer (*i* = 1, 2, …, *N*);
cb_2	Unit product assembling cost in the manufacturer;

ch_m	Unit inventory holding cost for a time unit in the manufacturer;
cc_1	Unit transportation cost of product from manufacturer to the retailer;
cc_{3i}	Unit transportation cost of Component *i* from Supplier *i* to the retailer (*i* = 1, 2, …, *N*);
$RL1$	Set of discretized values that can be selected as the local reliability of retailer, $RL1 = \{rl1^1, rl1^2, \dots, rl1^{\lvert RL1 \rvert}\}$;
$RL2$	Set of discretized values that can be selected as the local reliability of manufacturer; $RL2 = \{rl2^1, rl2^2, \dots, rl2^{\lvert RL2 \rvert}\}$;
$RL3$	Set of discretized values that can be selected as the local reliability of suppliers; $RL3 = \{rl3^1, rl3^2, \dots, rl3^{\lvert RL3 \rvert}\}$;
W	Set of available options for warranty, $W = \{w^1, w^2, \dots, w^{\lvert W \rvert}\}$;

4.2.1. Forward supply chain formulation for repairable products

In this section, we only focus on the process of producing and supplying products through the forward SC. The forward SC, as shown in Figure 4-1, includes a retailer, a manufacturer and suppliers. Each of these facilities faces a variation. The retailer faces variation in the product demand in the pre-market. The manufacturer always has a stochastic percentage of defective assemblies in its production system. In the suppliers, the production process starts in an in-control state after setting up the machinery. But after a stochastic time, it shifts to an out-of-control state in which a given percent of output is non-conforming. Due to the imperfect performance of the facilities along the SC, the quantity of the qualified output (variation in the qualified output) decreases (accumulates and increases) by moving the flow from the upstream to the downstream. In this section, we propose an approach to neutralize the negative effects of this flow depreciation (uncertainty propagation) though the chain. Based on this approach, the order quantities are amplified by moving from the downstream to the upstream of the chain.

To model the flow deprecation, we assume rl_r, rl_m, and rl_s represent the local reliabilities of the retailer, manufacturer and suppliers respectively. The market's actual demand in each sales period is stochastic with a given density function. Before the beginning of each period, the retailer orders the required products, *x*, from the

manufacturer based on its local reliability, rl_r. This amount of product stock ensures that the entire product demand will be fulfilled by the retailer with rl_r probability. Therefore, the manufacturer receives an order of *x* products from the retailer. To compensate for the defective assemblies in its production system, the manufacturer must plan to manufacture extra products, Δx. The size of Δx depends on the local reliability of the manufacturer, rl_m.

By manufacturing $x + \Delta x$ products, the manufacturer must be sure with rl_m probability that it can fulfill the whole order of the retailer. Thus the manufacturer orders $x + \Delta x$ components from each supplier. Supplier i ($i = 1, 2, \dots, N$) receives an order of $x + \Delta x$ Component *i* units from the manufacturer. To compensate for the non-conforming output of its production system, Supplier *i* plans to produce extra components, $\Delta \acute{x}_i$. The local reliability of the supplier, rl_s, governs the amount of $\Delta \acute{x}_i$. By $\Delta \acute{x}_i$ extra production, Supplier i will be sure with rl_s probability that it can fulfill the whole order of the manufacturer. As seen above, we neutralize the negative effect of the flow depreciation by amplifying the orders transferred between the facilities form the downstream to the upstream.

In this case, the manufacturer will be sure with ${rl_s}^N$ probability that it will receive all the ordered components. With Δx extra production, the manufacturer will be sure with rl_m probability that it can fulfill the whole order of the retailer. Product stock x ensures that the retailer will be able to fulfill the whole product demand with rl_r probability. Therefore, the forward SC will be able to fulfill the pre-market's demand with $sl_p = {rl_s}^N . rl_m . rl_r$ probability which is its service level, sl_p. The forward SC's service level,

sl_p, determines the percent of the pre-market's demand that is fulfilled immediately by the retailer's on-hand inventory. The forward SC's service level depends on the reliability of its included facilities

In Figure 4-3, we represent the qualified flow depreciation throughout the forward SC. In this problem, we assume the facilities only fulfill the order of their downstream facilities and more flow transitions during the sales period is not possible. We analyze the relationship among rl_r, rl_m, and rl_s (local reliabilities) and x, Δx, and $\Delta \acute{x}_i$ (order and production quantities) in the retailer, manufacturer, and suppliers in next sections.

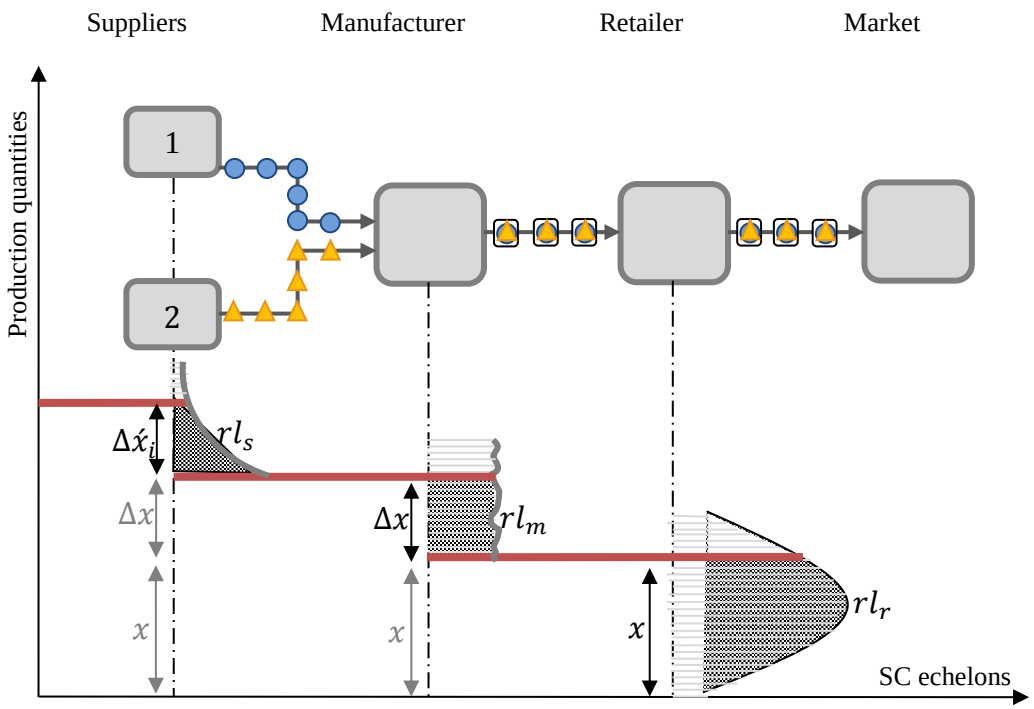


Figure 4-3: Flow depreciation in the forward SC.

Relationship between the order quantity of the retailer and its local reliability

In this section, we analyze the retailer's performance in the forward SC. In each sales period, the product demand in the pre-market is $\widehat{D}(p, sl_p, sl_a, w) = D(p, sl_p, sl_a, w) \times \varepsilon$. $D(p, sl_p, sl_a, w)$ is a deterministic decreasing function of the price (p) and an increasing function of the pre-market service level (sl_p), the after-sales service level (sl_a), and the warranty length (w). Therefore, the average product demand in the pre-market depends on the service level provided by the after-sales SC. This is one of the interactions considered between forward and after-sales SCs (fourth research question). Because customers are mainly interested to purchase from the companies providing better after-sales services. ε is a random variable with a given cumulative distribution function, $G(\varepsilon)$, which is independent of p, sl_p, sl_a, and w. Without loss of generality, we assume that $E(\varepsilon) = 1$ which implies $E\left[\widehat{D}(p, sl_p, sl_a, w)\right] = D(p, sl_p, sl_a, w)$. Before the beginning of each sales period and based on its local reliability, the retailer selects its product stock quantity represented by x. Higher x means higher reliability in the retailer to fulfill the entire demand and increases the probability of having extra inventory at the end of the period. Unit holding cost h^+ is incurred by the retailer for each extra inventory unit. Lower values for x increase the probability of lost sales at the end of the period. The unit shortage cost h^- is incurred by the retailer for each lost sales unit. To make an appropriate tradeoff between these two cost components, the retailer selects its stock quantity as follows:

$$MIN \qquad TC_r = h^+ . E\left[x - \widehat{D}(p, sl_p, sl_a, w)\right]^+ + h^- . E\left[\widehat{D}(p, sl_p, sl_a, w) - x\right]^+ \tag{4-1}$$

$$S.T. \qquad \Pr[\widehat{D}(p, sl_p, sl_a, w) \leq x] \geq rl_r \tag{4-2}$$

Objective function (4-1) is the sum of expected extra inventory cost and expected lost sales cost which should be minimized. Constraint (4-2) preserves the retailer's local reliability.

The product order quantity $x = D(p, sl_p, sl_a, w).G^{-1}(\frac{h^-}{h^-+h^+})$ minimizes the expected total cost of the retailer. To conserve the retailer's local reliability, we should have $x \geq D(p, sl_p, sl_a, w).G^{-1}(rl_r)$. Accordingly, the best product order quantity of the retailer from the manufacturer is:

$$x = D(p, sl_p, sl_a, w).G^{-1}\left(Max\left\{rl_r, \frac{h^-}{h^-+h^+}\right\}\right) \tag{4-3}$$

Substituting Equation (4-3) into (4-1) leads to the following least cost in the retailer:

$$TC_r = \left(h^+.E\left[G^{-1}\left(Max\left\{rl_r, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ + h^-.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_r, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right).D(p, sl_p, sl_a, w) \tag{4-4}$$

In the above equation, the first term, $h^+.E\left[G^{-1}\left(Max\left\{rl_r, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ + h^-.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_r, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+$, is the unit average handling cost of the product in the retailer. Equation (4-3) represents the relationship between the retailer's local reliability, rl_r, and its product order quantity, x. By ordering *x* product units from the manufacturer, the retailer is able to fulfill the realized product demand with rl_r probability (see the retailer in Figure 4-3). In the next section, we describe how the order of the retailer is amplified in the manufacturer.

Relationship between the production quantity of the manufacturer and its local reliability

The manufacturer receives an order of x product units from the retailer. But the manufacturer knows that its production system is always accompanied with a stochastic percentage of defective assembly. To compensate for the defective assemblies, the manufacturer must plan to produce some extra products, Δx, and consequently order some extra components from the suppliers. We assume the defective rate of assembly in the manufacturer is in the range $[0,\beta]$ with a given cumulative distribution function, $G'(.)$. Also without loss of generality; we assume that to produce a product unit, a unit of each component is required.

Producing x product units by the manufacturer leads to at most $\alpha.x$ ($\alpha \in [0,\beta]$) defective assemblies with $G'(\alpha)$ probability. Therefore, $\Delta x = \acute{G}^{-1}(rl_m).x$ extra production enables the manufacturer to fulfill the whole order of the retailer with rl_m probability. Assembling $\acute{G}^{-1}(rl_m).x + x$ product units preserves rl_m local reliability for the manufacture (see the manufacturer in Figure 4-3).

Equation

$$\Delta x + x = [\acute{G}^{-1}(rl_m) + 1].x \tag{4-5}$$

represents the relationship between the local reliability of the manufacturer and its production quantity. For producing $\Delta x + x$ product units, the manufacturer orders $\Delta x + x$ component units from each supplier. In the next section, we describe how the orders of the manufacturer are amplified in the suppliers.

Relationship between the production quantity of the suppliers and their local reliabilities

Each supplier receives an order of $\Delta x + x$ component units from the manufacturer. But the production system of the suppliers is not perfect. According to Rosenblatt and Lee (1986) and Lee and Rosenblatt (1987), we assume the production run of each supplier starts in an in-control state after setting up its equipment. But they deteriorate and shift to an out-of-control state after a stochastic time following exponential distribution with $1/\mu_i$ (i = 1, 2, …, N) mean. However, in-control production systems only produce conforming components, γ_i (i = 1, 2, …, N) percentage of the components produced in the out-of-control state is nonconforming. Once the production system shifts to an out-of-control state, it stays in that state until the end of the production period, because interruption of machines is prohibitively expensive.

Supplier i (i = 1, 2, …, N) receives an order of $\Delta x + x$ component units from the manufacturer. To compensate for the nonconforming components of its production system, Supplier i plans to produce $\Delta \acute{x}_i + \Delta x + x$ units of Component i. $\Delta \acute{x}_i + \Delta x + x$ production units in Supplier i should preserve with rl_s probability that this supplier will have $\Delta x + x$ sound output to fulfill the order of the manufacturer. Thus, we have

$$rl_s = \Pr\left[sound\ component\ units\ produced\ in\ \frac{\Delta \acute{x}_i + \Delta x + x}{PR_{1i}}\ time\ unit \geq \Delta x + x\right]$$

$$= \Pr\left[PR_{1i}.t + (1-\gamma_i).PR_{1i}.\left(\frac{\Delta\acute{x}_i+\Delta x+x}{PR_{1i}} - t\right) \geq \Delta x + x\right] = \Pr\left[t \geq \left(\frac{\Delta x+x}{PR_{1i}}\right) - \left(\frac{1-\gamma_i}{\gamma_i.PR_{1i}}\right).(\Delta\acute{x}_i)\right] = EXP\left[-\mu_i.\left(\left(\frac{\Delta x+x}{PR_{1i}}\right) - \left(\frac{1-\gamma_i}{\gamma_i.PR_{1i}}\right).(\Delta\acute{x}_i)\right)\right]$$

(4-6)

where PR_{1i} is the production rate in Supplier *i* (i = 1, 2, …, N). Based on Equation (4-6), to preserve rl_s local reliability, Supplier *i* should plan to produce

$$\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_s) + (\Delta x + x)\right] \quad (4\text{-}7)$$

extra components in its production system (see the suppliers in Figure 4-3).

$\Delta\acute{x}_i$ (i = 1, 2, …, N) extra production ensures that Supplier *i* will be able to fulfill the order of the manufacturer with rl_s probability. In this case, the manufacturer will be sure with ${rl_s}^N$ probability that it will receive all the component orders issued to the suppliers. With Δx extra product assembly, the manufacturer will be sure with rl_m probability that it can fulfill the whole order of the retailer. By ordering x product units, the retailer will be able to fulfill the whole product demand of the pre-market with rl_r probability. Therefore, (rl_r, rl_m, rl_s) the local reliability combination in the retailer, manufacturer and suppliers provides $sl_p = rl_r.rl_m.{rl_s}^N$ service level for the forward SC in the pre-market. The equation, $sl_p = rl_r.rl_m.{rl_s}^N$, is used to determine the relationship between the service level of the forward SC and the local reliabilities of its stochastic facilities. Equations (4-3), (4-5), and (4-7) indicate the way orders should be amplified from the downstream to the upstream of the forward SC to neutralize the negative effect of flow

depreciation throughout its network. Similar equations are developed for the after-sales SC in the next section.

4.2.2. After-sales supply chain formulation for repairable products

Since failure free warranty is provided, the company must also provide the required spare parts to repair defective products returned inside the warranty period. These parts are produced and provided through the after-sales SC. The prerequisite for production planning in the after-sales SC is estimating the after-sales demands of the spare parts. First, we describe the failure processes to estimate after-sales demand for the product and its components. Then we model the performance of the repair sections in the suppliers to compute the percentage of the after-sales demands can be fulfilled by repaired components. After that we determine how many new components should be ordered by the retailer from the suppliers to preserve a given after-sales service level. Finally we show how the orders of the retailer should be amplified in the suppliers.

Product failure

Demand of each component in the after-sales depends on: i) the total number of products supplied through the forward SC to the pre-market (this constitutes the potential demand for each component in the after-sales market); and ii) the reliability index of that component, τ_i ($i = 1, 2, \ldots, N$).

We assume the performance of the components is independent and the failure time of each Component i ($i = 1, 2, \ldots, N$) is a random variable with an F_i cumulative distribution function. F_i is a function of the component's reliability index, τ_i. Lower τ_i value implies higher reliability and vice versa. When a product with a defective Component i is returned

inside the warranty period, its defective part is removed and immediately substituted with another repaired or new component if the inventory level of Component *i* in the retailer is positive. Otherwise, the customer must wait until a repaired component is sent to the retailer from the repair section. The removed defective Component *i* is sent to the repair section of Supplier *i* for repair. Also it is assumed the probability of failure of the component does not change after repair.

We define $F_i^{(m)}$ as the cumulative distribution function of total time up to the m^{th} failure in Component *i*. $Num_i(w)$ is a random variable representing the number of failures inside the warranty interval, $[0,w]$. Based on Nguyen and Murthy (1984), we have:

$$\Pr\{Num_i(w) = m\} = F_i^{(m)}(w, \tau_i) - F_i^{(m+1)}(w, \tau_i) \qquad (\forall i = 1, 2, \dots, N) \tag{4-8}$$

According to Equation (4-8), the average number of failures, $E_i(w)$, for a unit of Component *i* inside the warranty time is:

$$E_i(w) = \sum_{j=1}^{+\infty} F_i^{(j)}(w, \tau_i) \qquad (\forall i = 1, 2, \dots, N) \tag{4-9}$$

In each sales period, at most x product units are supplied to the market through the forward SC. Therefore, the average number of Component *i* failures for the product lot size of each sales period, x, inside the warranty period, λ_i, is:

$$\lambda_i = x.E_i(w) \qquad (\forall i = 1, 2, \dots, N) \tag{4-10}$$

Assuming the total cost for repairing a unit of Component *i* (this is the sum of the unit transportation cost from the retailer to Supplier *i* (cr_i^{rs}), the unit service cost in Supplier

i (cr_i^s) and the unit transportation cost from Supplier i to the retailer (cr_i^{sr})) is cr_i ($\forall i = 1, 2, \dots, N$), the average repair cost for a unit of product, cr, is:

$$cr = \sum_{i=1}^{N} \sum_{n=1}^{\infty} n.cr_i.\Pr\{Num_i(w) = n\} = \sum_{i=1}^{N} \sum_{n=1}^{\infty} n.cr_i.\left[F_i^{(n)}(w, \tau_i) - F_i^{(n+1)}(w, \tau_i)\right] \quad (4\text{-}11)$$

In this problem, we consider a single sales period and need to determine the number of failures in the components inside that period. For this purpose, we assume the warranty period is an integer multiple of the sales period which is consistent with what happens in reality, $w = K.T$ (K is an integer number). In the same way, we consider the sales period as an integer multiple of time unit, $\acute{T}$, which means $T = \acute{K}.\acute{T}$ ($\acute{K}$ is an integer number). If we assume the pre-market rate of demand is almost constant, then in each time unit x/K products are supplied to the market. In Figure 4-4, we consider the beginning of a sales period as the origin of the time on the horizontal axis. We want to determine how many Component *i* failures will be received during this sales period. First, we do it for the first time unit of the sales period. The procedure for the other time units is similar. As shown in Figure 4-4, the warranty period for the supply lot size x/K which was sold $\acute{K}.K$ time units before is finished. But warranty period for the other lot sizes are as follows:

- For lot size x/K sold $\acute{K}.K - 1$ time units before, we will receive $\frac{x}{K}.\left[E_i(w) - E_i\left(\frac{\acute{K}.K-1}{\acute{K}.K}w\right)\right]$ failures;
- For lot size x/K sold $\acute{K}.K - 2$ time units before, we will receive $\frac{x}{K}.\left[E_i\left(\frac{\acute{K}.K-1}{\acute{K}.K}w\right) - E_i\left(\frac{\acute{K}.K-2}{\acute{K}.K}w\right)\right]$ failures;

- For lot size x/K sold $\acute{K}.K-3$ time units before, we will receive $\frac{x}{K}.\left[E_i\left(\frac{\acute{K}.K-2}{\acute{K}.K}w\right)-E_i\left(\frac{\acute{K}.K-3}{\acute{K}.K}w\right)\right]$ failures;

- …

- For lot size x/K sold 0 time units before, we will receive $\frac{x}{K}.\left[E_i\left(\frac{\acute{K}.K-(\acute{K}.K-1)}{\acute{K}.K}w\right)-E_i(0)\right]$ failures;

Therefore, $\frac{x}{K}.E_i(w)$ failures will be received in the first time unit of the sales period. There are K time units inside the sales period. Thus, the retailer will receive $x.E_i(w)$ Component i failures in each sales period ($\forall i = 1, 2, \dots, N$). This means the after-sales demand for each component depends on the total product units, x, supplied by the forward SC to the pre-market. This is the other interaction between the forward and after-sales SCs that is considered in the concurrent flow planning model in Section 4.2.3 (fourth research question).

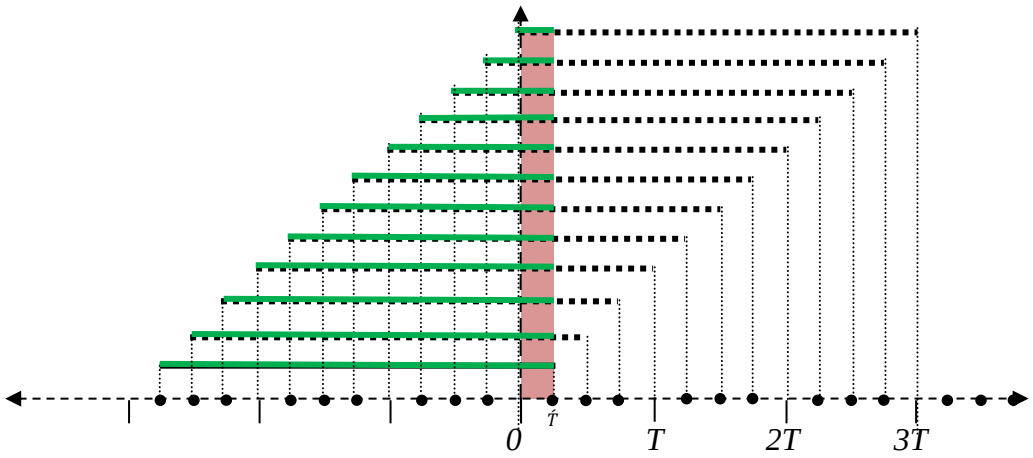


Figure 4-4: Previous supply lot sizes for which warranty commitment have not been expired by the end of [0, $\acute{T}$] time interval (In this figure it is assumed that $w = 3T$ and $T = 4\acute{T}$).

Repair process of the defective components

Defective components of the returned products are sent to the repair sections of their corresponding suppliers for repair. Repair process of each component is treated a two echelon system with one server center (the repair section) and one user (the retailer). When a defective product is returned to the retailer, first fault diagnosis is preformed to discover the source of problem. Assume that the problem is related to Component i ($i = 1, 2, \dots, N$). Then the retailer sends the defective component to the repair section of Supplier i. When the failed component enters the repair section, if there is no queue, it immediately receives the repair service. Otherwise, it waits in a queue. The repair time, t_i, is stochastic with a given distribution function.

When the repair process is completed, the repaired component is sent back to the retailer. There is storage capacity only in the retailer. Also the retailer has a safety stock, s_i $(i = 1, 2, \dots, N)$, this includes new components manufactured and stocked by the

supplier in before the beginning of each sales period. This safety stock preserves local reliability rl_r for the retailer in the after-sales services. In Figure 4-5, we represent the queuing system in the repair section of Component i ($i = 1, 2, \ldots, N$).

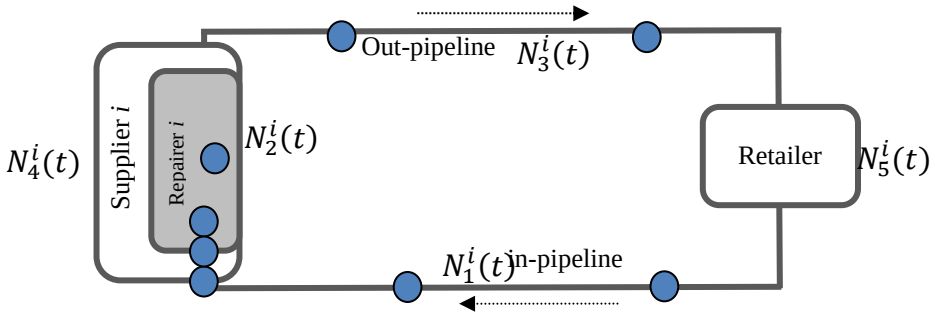


Figure 4-5: Queuing system in the repair section of Component *i*.

The inventory policy of the components in the retailer is (*S*, *S*-1). This means whenever a failed component is found in a returned product, the retailer sends it to the supplier's repair section and applies a repaired one from the repair section. $N_k^i(t)$ is a random variable that represents the number of components in state k (k = 1, 2, 3, 4, and 5) at time *t*. These states are shown in Figure 4-5. $N_1^i(t)$, $N_2^i(t)$, and $N_3^i(t)$ represent respectively the number of components in the in-pipeline transferring the defective components from the retailer to the repair section, the number of waiting components or components being serviced in the repair section, and the number of repaired components in the out-pipeline being transferred from the repair section to the retailer. $N_4^i(t)$ and $N_5^i(t)$ are the inventory levels at the repair section and retailer respectively. Demands are fulfilled when the inventory levels are positive. Otherwise, they become outstanding orders. $B_0^i(t)$ and $B_1^i(t)$ represent the backorder levels in the repair section and retailer respectively. Therefore, we have:

$$B_0^i(t) = Max\{-N_4^i(t), 0\} = Max\{N_1^i(t) + N_2^i(t), 0\} \quad (4\text{-}12)$$

$$B_1^i(t) = Max\{-N_5^i(t), 0\} = Max\{N_3^i(t) + B_0^i(t) - s_i, 0\} \quad (4\text{-}13)$$

In Equation (4-12), term $N_1^i(t) + N_2^i(t)$ represents the total number of the components in the server and in-pipeline. For each of these components, the repair section received an order from the retailer which has not been fulfilled yet. In Equation (4-13), terms $N_3^i(t)$ and $B_0^i(t)$ represent the released but not fulfilled orders of the retailer which shows Component *i*'s demand in the retailer and s_i represents the stock quantity in the retailer. Therefore, the retailer's backorder is the difference between these two terms. To show that we are only dealing with steady-state quantities of the above system, we remove the *t* argument henceforth. In this problem, the probability of having no backorders in the retailer, $\Pr(N_3^i(t) + B_0^i(t) \leq s_i)$, is important because it represents the retailer's local reliability, rl_r, in the after-sales. To compute this probability, it is critical to find the distribution function of $N_3^i(t) + B_0^i(t)$. If we assume a mutual independence between the pipelines and the repair section's server, the above problem reduces to two convolutions: i) obtaining the distribution of $B_0^i(t)$ at the repair section from the distribution of the components in the in-pipeline ($N_1^i(t)$) and the distribution of the components being repaired ($N_2^i(t)$) in the server; and ii) obtaining the distribution of inventory at the retailer ($N_5^i(t)$) from the repair section's backorder distribution ($B_0^i(t)$) which is derived from the first convolution and the distribution of the components in the out-pipeline ($N_3^i(t)$).

Diaz and Fu (1997) show that a negative binomial distribution approximates both of these convolutions with great accuracy. We use this approximation to simplify the calculations. Then the mean and variance of the outstanding orders in Repair Section *i*

must be calculated to determine the parameters a_0^i and b_0^i of the negative binomial approximating the distribution function of B_0^i.

$$E\left[B_0^i\right] = \sum_{x=0}^{\infty}(x).\frac{\Gamma(x+a_0^i)}{x!\Gamma(a_0^i)}{b_0^i}^{a_0^i}\left(1-b_0^i\right)^x \tag{4-14}$$

Where $a_0^i = {\mu_0^i}\Big/\left[\left({\sigma^2}_0^i\Big/\mu_0^i\right)-1\right]$, $b_0^i = {\mu_0^i}\Big/{\sigma^2}_0^i$, $\mu_0^i = E[N_1^i(t)] + E[N_2^i(t)]$ and ${\sigma^2}_0^i = Var[N_1^i(t)] + Var[N_2^i(t)]$.

In the same way, parameters a_1^i and b_1^i are computed to generate the negative binomial distribution function of B_1^i in the retailer. Then, the expected backorder and backorder probability in the retailer is:

$$E\left[B_1^i\right] = \sum_{x=s_i}^{\infty}(x-s_i).\frac{\Gamma(x+a_1^i)}{x!\Gamma(a_1^i)}{b_1^i}^{a_1^i}\left(1-b_1^i\right)^x \tag{4-15}$$

$$Pr(N_3^i(t) + B_0^i(t) \geq s_i) = \sum_{x=s_i}^{\infty}\frac{\Gamma(x+a_1^i)}{x!\Gamma(a_1^i)}{b_1^i}^{a_1^i}\left(1-b_1^i\right)^x \tag{4-16}$$

where $a_1^i = {\mu_1^i}\Big/\left[\left({\sigma^2}_1^i\Big/\mu_1^i\right)-1\right]$, $b_1^i = {\mu_1^i}\Big/{\sigma^2}_1^i$, $\mu_1^i = E[N_3^i(t)] + E[B_0^i(t)]$ and ${\sigma^2}_1^i = Var[N_3^i(t)] + Var[B_0^i(t)]$.

If we assume the *M*/*G*/1 queuing system for the repair process in the repair section in which $E[t_i^s]$ represents the s^{th} moment of the service time, then we have:

$$E[N_2^i] = \rho_i + \frac{(\lambda_i)^2.E[t_i^2]}{2.(1-\rho_i)} \qquad \rho_i = \lambda_i.E[t_i] \tag{4-17}$$

$$Var[N_2^i] = \frac{(\lambda_i)^3.E[t_i^3]}{3.(1-\rho_i)} + \frac{(\lambda_i)^4.(E[t_i^2])^2}{2.(1-\rho_i)^2} + \frac{(\lambda_i)^3.E[t_i^2].E[t]}{(1-\rho_i)} + \frac{(\lambda_i)^2.E[t_i^2].(3-2\rho_i)}{(1-\rho_i)} + \rho_i - (E[N_2^i])^2 \quad (4\text{-}18)$$

Considering *M*/*G*/1 queuing system for the repair process requires the assumption that the failure time of Component i follows an exponential distribution. This means the failure mode of this component is Poisson. We also model the in-pipeline and the out-pipeline as *M*/*G*/∞ queuing systems which is consistent with the assumption of independence of the numbers of components in the server and in the pipelines. Therefore, we have $E[N_1^i(t)] = Var\left[N_1^i(t)\right] = \lambda_i.O_i$ and $E[N_3^i(t)] = Var\left[N_3^i(t)\right] = \lambda_i.O_i$ (Mirasol, 1963). In these equations, O_i represents the shipment time between the retailer and the repair section of Supplier i ($i = 1, 2, \dots, N$). The main objective of the above calculations is to determine the distribution function of $N_3^i(t) + B_0^i(t)$. This negative binomial distribution is used to determine the relationship between the retailer's local reliability and its safety stocks. If we assume G_{NB}^i represents the negative binomial cumulative distribution function approximating the density of $N_3^i(t) + B_0^i(t)$, then we have:

$$\Pr\left(N_3^i(t) + B_0^i(t) \le s_i\right) = rl_r \quad (4\text{-}19)$$

$$s_i = {G_{NB}^i}^{-1}(rl_r) \quad (4\text{-}20)$$

This means to preserve the rl_r local reliability for the retailer in the after-sales SC in each sales period, the retailer should order s_i ($i = 1, 2, \dots, N$) new Component i units from Supplier *i* before the beginning of that period. In the next section, we explain how this order of the retailer will be amplified in the suppliers.

Safety stock production in the suppliers

Each supplier not only receives the order of $\Delta x + x$ component units from the manufacturer to produce new products, but also receives the order of s_i component units from the retailer to fulfill a part of the after-sales demand that cannot be fulfilled by the repaired components. Therefore, each supplier should produce $s_i + \Delta x + x$ component units for the forward and after-sales SCs. Based on this, a new order, s_i, is issued by the retailer from the suppliers, and we modify the extra production quantity of the suppliers (Equation 4-7) as follows:

$$\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_s) + (s_i + \Delta x + x)\right] \qquad (i = 1, 2, 3, \dots, N) \qquad (4\text{-}21)$$

If a supplier does not fulfill the whole $s_i + \Delta x + x$ order, the unfulfilled part of this order is divided proportionally between the forward ($\frac{\Delta x + x}{s_i + \Delta x + x}$) and after-sales SCs ($\frac{\Delta s_i}{s_i + \Delta x + x}$). Therefore, each supplier is able to fulfill the component order of the retailer with rl_s probability. The retailer by ordering s_i component units from the supplier is sure with rl_r probability that the order can fulfill the whole after-sales demand of Component i. In this case, the fulfillment rate of Component i's demand is $rl_r.rl_s$. Since the product includes N critical components, the after-sales SC's service level in fulfilling the after-sales demand of all components is $sl_a = (rl_r.rl_s)^N$.

Therefore, the service levels in the forward and after-sales SCs are completely convoluted and both are functions of the local reliabilities. This is the other interaction that should be incorporated in the concurrent flow planning of the forward and after-sales SCs (fourth research question).

4.2.3. Mathematical model

In this section, by the help of the equations derived in Sections 3.1 and 3.2 (Equations (4-3), (4-4), (4-5), (4-11), (4-20), and (4-21)), we concurrently determine the best flow dynamics through the network of forward and after-sales SCs in a way to maximize the total profit of the whole company. The model is as follows:

Max $\Pi =$

$$\left[\left(p - h^{+}.E\left[G^{-1}\left(Max\left\{rl_r, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\left\{rl_r, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+} - cr\right) \times D\left(p, (rl_s{}^{N}.rl_m.rl_r), (rl_r.rl_s)^{N}, w\right)\right] - \left\{\left[\sum_{i=1}^{N}(ca_{1i} + ca_{2i}).(x + \Delta x + s_i + \Delta\acute{x}_i)\right] + \left[\sum_{i=1}^{N}\frac{ch_{1i}.(x+\Delta x+s_i)^2}{2.PR_{1i}}\right] + \left[\sum_{i=1}^{N} cb_{1i}.(x + \Delta x)\right] + \left[\, cb_2.(x + \Delta x)\right] + \left[\frac{ch_m.(x)^2}{2.PR_m}\right] + \left[cc_1.x\right] + \left[\sum_{i=1}^{N} cc_{3i}.s_i\right]\right\} \quad (4\text{-}22)$$

Where

$$x = D\left(p, (rl_s{}^{N}.rl_m.rl_r), (rl_r.rl_s)^{N}, w\right).G^{-1}\left(Max\left\{rl_r, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \quad (4\text{-}23)$$

$$\Delta x = G'^{-1}(rl_m).D\left(p, (rl_s{}^{N}.rl_m.rl_r), (rl_r.rl_s)^{N}, w\right).G^{-1}\left(Max\left\{rl_r, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \quad (4\text{-}24)$$

$$s_i = {G_{NB}^{i}}^{-1}(rl_r) \qquad (\forall i = 1, 2, \dots, N) \quad (4\text{-}25)$$

$$\Delta\acute{x}_i = \frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\ln(rl_s) + {G_{NB}^{i}}^{-1}(rl_r) + \left(G'^{-1}(rl_m) + 1\right).D\left(p, (rl_s{}^{N}.rl_m.rl_r), (rl_r.rl_s)^{N}, w\right).G^{-1}\left(Max\left\{rl_r, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]$$

$$(\forall i = 1, 2, \dots, N) \quad (4\text{-}26)$$

Subject to:

$$0.5 \le rl_r,\ rl_m,\ rl_s \le 1 \quad (4\text{-}27)$$

$$p, w \geq 0 \tag{4-28}$$

The first term in the objective function (4-22) represents the average profit captured by the retailer through selling the products in the pre-market. This term is equal to the retailer's income, $p.D(p, sl_p, sl_a, w)$, minus the average handling cost, Equation (4-4), and average repair cost, Equation (4-11), of the products in the retailer. The second term of (4-22) represents the cost of producing and supplying the products and components in the SCs' first and second echelons. The first item in the second term is the cost of procuring material and producing the components in the suppliers. The second item in the second term is the average holding cost of the qualified components produced and stocked in the suppliers. The third item is the transportation cost of the qualified components from the suppliers to the manufacturer. The fourth item is the cost of assembling the products in the manufacturer. The fifth term is the average holding cost of the qualified products in the manufacturer. The sixth and seventh terms respectively represent the transportation cost of the products and components from the manufacturer and suppliers to the retailer.

Equations (4-23), (4-24), (4-25), and (4-26) explained before show the relationship between the local reliabilities of the echelons and their production quantities. This mathematical model determines the best local reliabilities for the SCs' facilities (and consequently the best pre- and after-sales service levels), price, and warranty length for the company to maximize the total profit. This formulation of the problem is a mathematical model with a strictly nonlinear objective function and continuous variables. In Section 4.3, an approach is proposed to solve this model.

4.3. Solution approach

The mathematical model formulated for the problem in Section 3 includes a strictly nonlinear objective function. As you may know, finding the best solution is not straightforward for nonlinear models. Analyzing the problem's model shows the most important variables which mainly appear in the nonlinear terms of the model are rl_r, rl_m and rl_s. These variables take value from a very restricted range, [0.5, 1]. Having a very restricted feasible range justifies discretizing these variables. By discretizing on the [0.5, 1] range, substituting this interval with a set of discrete values, and assuming rl_r, rl_m, and rl_s variables only take values from this set, we transform the problem's nonlinear model to a linear one which is much easier to solve globally.

The other variable in the model is warranty length, w. This variable does not have a restricted feasible range but, in reality, few warranty options are available in markets and usually offered by companies for customers such as 6, 12, 18 and 24 months. But the price variable, p, neither has a restricted feasible range, nor has few options. Therefore in this section, we assume the product price is given exogenously. By introducing price as a parameter in the model, discretizing reliability and warranty variables looks an appropriate technique to linearize and globally solve this model. In Section 4.4, we determine the best price for the company by sensitivity analysis of the results.

We discretize the feasible continuous range of rl_r by defining a set of discrete values $RL1 = \{rl1^1, rl1^2, \dots, rl1^{|RL1|}\}$. To use this set, we define new binary variables y_{rl1^r} ($\forall rl1^r \in RL1$) for selecting scenarios from this set. Variable y_{rl1^r} is equal to 1 if the reliability scenario $rl1^i$ is selected from this set and 0 otherwise. In the same way, sets $RL2 = \{rl2^1, rl2^2, \dots, rl2^{|RL2|}\}$ and $RL3 = \{rl3^1, rl3^2, \dots, rl3^{|RL3|}\}$ and their

corresponding binary variables y_{rl2^m} ($\forall rl2^m \in RL2$) and y_{rl3^s} ($\forall rl3^s \in RL3$) are defined to discretize the continuous ranges of rl_m and rl_s. Set $W = \{w^1, w^2, \dots, w^{|W|}\}$ represents the available warranty length options and binary variables z_{w^t} ($\forall w^t \in W$) are defined for warranty strategy selection from this set. By defining these new sets and variables, the important nonlinear terms of Model (4-22)-(4-28) become:

$$x = \sum_{r=1}^{|RL1|}\sum_{m=1}^{|RL2|}\sum_{s=1}^{|RL3|}\sum_{t=1}^{|W|} y_{rl1^r}.y_{rl2^m}.y_{rl3^s}.z_{w^t}.\left[D\left(p,\left(rl1^r.rl2^m.rl3^{s^N}\right),(rl1^r.rl3^s)^N,w^t\right)\right] \times \left[G^{-1}\left(Max\left\{rl1^r,\frac{h^-}{h^-+h^+}\right\}\right)\right] \quad (4\text{-}30)$$

$$\Delta x = \sum_{r=1}^{|RL1|}\sum_{m=1}^{|RL2|}\sum_{s=1}^{|RL3|}\sum_{t=1}^{|W|} y_{rl1^r}.y_{rl2^m}.y_{rl3^s}.z_{w^t}.[G'^{-1}(rl2^m) \times D\left(p,\left(rl1^r.rl2^m.rl3^{s^N}\right),(rl1^r.rl3^s)^N,w^t\right) \times G^{-1}\left(Max\left\{rl1^r,\frac{h^-}{h^-+h^+}\right\}\right)] \quad (4\text{-}31)$$

$$s_i = \sum_{r=1}^{|RL1|}\sum_{t=1}^{|W|} y_{rl1^r}.\left[{G_{NB}^{i}}^{-1}(rl_r)\right] \qquad (\forall i = 1,2,\dots,N) \quad (4\text{-}32)$$

$$\Delta\acute{x}_i = \sum_{r=1}^{|RL1|}\sum_{m=1}^{|RL2|}\sum_{s=1}^{|RL3|}\sum_{t=1}^{|W|} y_{rl1^r}.y_{rl2^m}.y_{rl3^s}.z_{w^t}.\left[\frac{\gamma_i}{1-\gamma_i}\left[\frac{PR_{1i}}{\mu_i}\mathrm{Ln}(rl3^s) + {G_{NB}^{i}}^{-1}(rl_r) + \left(G'^{-1}(rl2^m)+1\right) \times D\left(p,\left(rl1^r.rl2^m.rl3^{s^N}\right),(rl1^r.rl3^s)^N,w^t\right) \times G^{-1}\left(Max\left\{rl1^r,\frac{h^-}{h^-+h^+}\right\}\right)\right]\right] \qquad (\forall i = 1,2,\dots,N) \quad (4\text{-}33)$$

Since the product of binary variables can be linearized easily, the above items appearing in the second term of the objective function will be linear. Also the first term of the objective function can be rewritten as:

$$\left[\left(p - h^{+}.E\left[G^{-1}\left(Max\ \left\{rl_{r}, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\ \left\{rl_{r}, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+} - cr\right) \times D\right]$$

$$= \sum_{r=1}^{|RL1|}\sum_{m=1}^{|RL2|}\sum_{s=1}^{|RL3|}\sum_{t=1}^{|W|} y_{rl1^{r}}.y_{rl2^{m}}.y_{rl3^{s}}.z_{w^{t}}.\left[D\left(p, \left(rl1^{r}.rl2^{m}.rl3^{s^{N}}\right), (rl1^{r}.rl3^{s})^{N}, w^{t}\right)\right.$$

$$\times\left(p - h^{+}.E\left[G^{-1}\left(Max\ \left\{rl1^{r}, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+}\right.$$

$$\left.\left. - h^{-}.E\left[\varepsilon - G^{-1}\left(Max\ \left\{rl1^{r}, \frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+} - cr\right)\right] \qquad (4-34)$$

In this way, the first term of the objective function will be linear too. Also notice that only one reliability and one warranty option can be selected from the sets. Therefore, the following constraints are added:

$$\sum_{r=1}^{|RL1|} y_{rl1^{r}} = 1 \qquad (4\text{-}35)$$

$$\sum_{m=1}^{|RL2|} y_{rl2^{m}} = 1 \qquad (4\text{-}36)$$

$$\sum_{s=1}^{|RL3|} y_{rl3^{s}} = 1 \qquad (4\text{-}37)$$

$$\sum_{t=1}^{|W|} z_{w^{t}} = 1 \qquad (4\text{-}38)$$

By treating the price as an exogenously given factor and discretizing the feasible range of the warranty and reliability variables, the mathematical model of the problem is transformed to a mixed integer linear model with binary variables which can be solve

globally by the available software such as CPLEX, GAMS, GROOBI and LINGO. We used CPLEX to solve it. In the next section, by analyzing the sensitivity of the company's profit with respect to the price, we determine the optimal value for the product's price.

4.4. Computational results

4.4.1. An example from the automotive industry

The problem in this chapter is based on the need of a company, SMAC (due to confidentiality issues, we do not disclose the names of companies), located in the Middle East and supplying products to the regional automotive manufacturers of that area such as IKC. SMAC is a well-known Reverse Idler Gear Shaft (RIGS) supplier in the automotive industry in that region. However, recently the entrance of some new external suppliers with comparable prices and warranties has made the markets more competitive. In such competitive markets, determining the best price and warranty length and providing appropriate pre- and after-sales service levels is mandatory to keep customers. Due to low efficiency and the high rate of defective production, variations in the qualified output of the production facilities is significant. Therefore, considering supply-side variations in balancing demand and supply and estimating service levels is necessary.

The main components of RIGS are CK45 steel and barbed pins procured from the companies YIIC and AKC respectively. After shipping the conforming CK45 steel order from YIIC to SMAC, several processes are performed on the steel such as stretching it to the required diagonal, cutting stretched steel to suitable lengths, and rough grinding. Then the first puncturing, bathing, milling, second puncturing, and tapping are done on the work pieces. After plating and smoothing, the work piece is assembled with barbed pin

procured from AKC. Then the final product is cleaned and inspected. In the inspection process, defective products are removed from the batch and returned to the manufacturing process. Qualified products are sent to the retailer to supply to the market. The network structure of RIGS SC is shown in Figure 4-6. This SC includes CK45 and barbed pin suppliers in the third echelon (YIIC and AKC), one RIGS manufacturer in the second echelon (SMAC), and a retailer in the first echelon supplying the SC's product to the market.

This company provides a failure free warranty for its customers. The products returned inside the warranty period are checked by the retailer to determine whether the problem is related to the work piece made from CK45 steel or the barbed pin. If it is related to the steel work piece, the defective work piece is sent to SMAC's repair section and a repaired piece is ordered from this section. If the problem is related to the barbed pin, the defective pin is sent to AKC's repair section and a repaired pin is ordered from it (Figure 4-6).

This product includes two critical components: Component 1 (CK45) and Component 2 (barbed pin). Component 1 and 2 are manufactured with the procurement and production costs of $ca_{11} + ca_{21} = \$3.5$ and $ca_{12} + ca_{22} = \$2.5$ respectively. The sound components are shipped to the manufacturer and assembled into the final products with cost a of $cb_{11} = cb_{12} = \$0.2$ and $cb_2 = \$0.8$. After inspection, the qualified final products are shipped to the retailer with transportation cost $cc_1 = \$0.5$. Analyzing the company's historical sales data shows the pre-market's demand can be approximated as a linear function of price, warranty length, and service levels: $D(p, sl_p, sl_a, w) = 500 + 200 \times w - 250 \times (p - 10) - 500 \times (1 - sl_a) - 900 \times (1 - sl_p)$. The company has

four options for warranty length - 6, 12, 18, and 24 months. Cost components $h^- = \$0.10$ and $h^+ = \$0.15$ are considered for unit extra inventory and unit lost sales at the end of each sales period.

The CK45 work piece and the barbed pin respectively have reliability indices $\tau_1 = 0.1$ and $\tau_2 = 0.3$. The repair cost of components and the moments of their service time in the repair sections are: $cr_1 = \$1.5$, $cr_2 = \$1.0$, $E[t_1^2] = 0.0044$, $E[t_1^3] = 0.0003$, $E[t_2^2] = 0.00027$, and $E[t_2^3] = 0000046$. The transportation times of the defective components from the retailer to the repair sections of SMAC and AKC are $O_1 = 0.05\ (month)$ and $O_2 = 0.05\ (month)$ respectively.

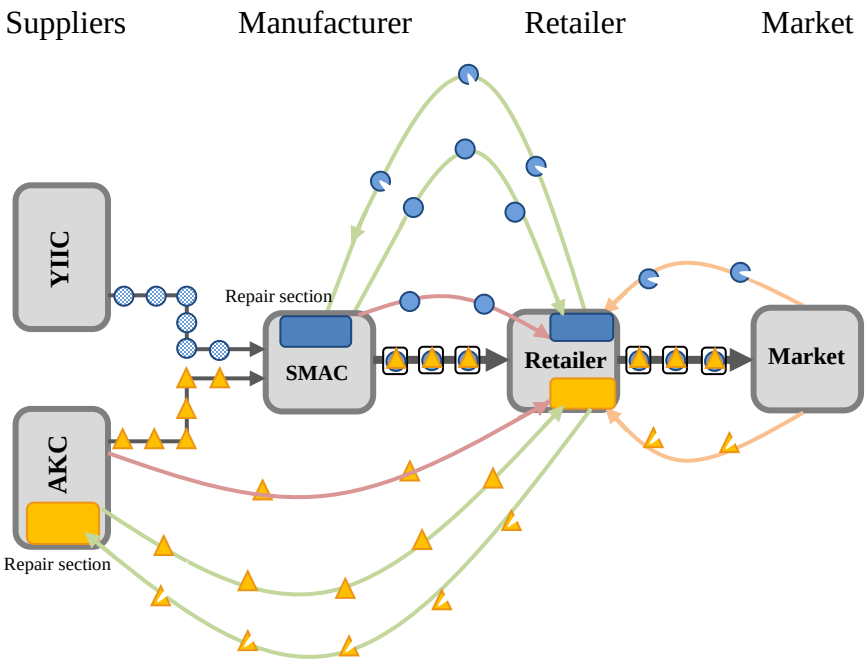


Figure 4-6: Network structure of RIGS SC.

The CK45 work pieces and the barbed pins are produced in the suppliers with $PR_1 = 8000$ (number in time unit) and $PR_2 = 9000$ (number in time unit) production rates. The average deterioration time in the production system of the suppliers is equal to

$1/_{\mu_{1\ and\ 2}} = 0.5$. In the out-of-control state, the rates of nonconforming production for CK45 and barbed pin are $\gamma_1 = 0.10$ and $\gamma_2 = 0.20$ respectively. The stochastic part of the pre-market demand, ε, follows a normal distribution with mean 0.0 and variance 1.0. SMAC's defective assembly rate has uniform density in in the range [0, $\beta = 0.15$]. The transportation cost of the repaired components from SMAC and AKC to the retailer is $cc_{31} = cc_{32} = \$1$.

In this problem, the flow of defective CK45 components is somewhat different. Instead of the supplier, they are returned to the manufacturer, SMAC. Thus, we modify Equations (4-24) and (4-26) as follows:

$$\Delta x = G'^{-1}(rl_m).\left[D\left(p,\left(rl_r.rl_m.{rl_s}^N\right),(rl_r.rl_s)^N,w\right).G^{-1}\left(Max\left\{rl_r,\frac{h^-}{h^-+h^+}\right\}\right)+ {G_{NB}^1}^{-1}(rl_r)\right] \quad (4\text{-}39)$$

$$\Delta\acute{x}_1 = \frac{\gamma_1}{1-\gamma_1}\left[\frac{PR_{11}}{\mu_1}\ln(rl_s)+\left(G'^{-1}(rl_m)+ 1\right).\left[D\left(p,\left(rl_r.rl_m.{rl_s}^N\right),(rl_r.rl_s)^N,w\right).G^{-1}\left(Max\left\{rl_r,\frac{h^-}{h^-+h^+}\right\}\right)+ {G_{NB}^1}^{-1}(rl_r)\right]\right] \quad (4\text{-}40)$$

$$\Delta\acute{x}_2 = \frac{\gamma_2}{1-\gamma_2}\left[\frac{PR_{12}}{\mu_2}\ln(rl_s)+{G_{NB}^2}^{-1}(rl_r)+\left(G'^{-1}(rl_m)+ 1\right).D\left(p,\left(rl_r.rl_m.{rl_s}^N\right),(rl_r.rl_s)^N,w\right).G^{-1}\left(Max\left\{rl_r,\frac{h^-}{h^-+h^+}\right\}\right)\right] \quad (4\text{-}41)$$

Solving the mathematical model of this example leads to the following results: local reliabilities in the SCs' echelons are $rl_r = 0.95$, $rl_m = 0.95$ and $rl_s = 0.94$ respectively.

For retail price $p = \$16.0$, the optimal warranty strategy is 6 months. Based on local reliabilities, $x = 263$ RIGS units are ordered by the retailer at the beginning of each sales period. To fulfill this order and the required CK45 work pieces as the retailer's safety stock ($s_1 = 3$), SMAC plans for $\Delta x = 41$ extra production. The required CK45 and barbed pins are ordered from YIIC and AKC respectively. In addition to SMAC's order, AKC receives another barbed pin order from the retailer, $s_2 = 6$, to preserve the retailer's local reliability in the after-sales market. In the same way, YIIC and AKC plan to procure and produce $\Delta\acute{x}_1 = 6$ and $\Delta\acute{x}_2 = 7$ extra units of CK45 and barbed pin to compensate for their non-conforming production. The results are summarized in Figure 4-7. This flow planning leads to $\Pi = \$1841.4$ profit for the company which is the highest for the retail price $p = \$16.0$.

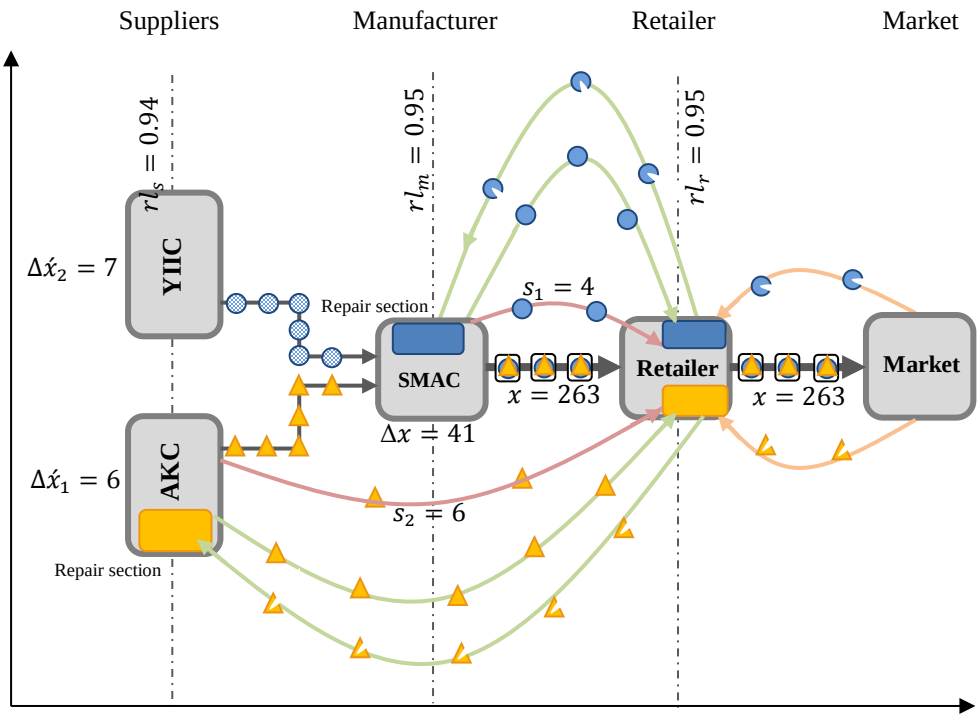


Figure 4-7: Results of solving RIGS model.

4.4.2. Sensitivity analysis

In Section 4.3, it is assumed the price of the product is given exogenously. However, price always has been one of the strongest competitive advantages for companies in the markets. In reality, the appropriate selection of the retail price is critical. Therefore in this section, we consider the price as a variable to be optimized by the company. We assume the price is selected in the range $[p_{min} = \$15, p_{max} = \$17.65\ ]$. Several factors should be considered to determine this feasible range for price, for example, the product's manufacturing cost, the prices of rival products in the market, and the governmental regulations supporting consumers' rights. In the rest of this section, first we analyze the sensitivity of the company's profit with respect to price and warranty length to determine the correlation between these two marketing strategies. For different values of price in the feasible range and warranty options, we solve the model. The results are summarized in Figure 4-8, 4-9, and 4-10. In Figure 4-8 the profit of the company with respect to the price for different warranty options is shown.

Based on Figure 4-8, for a 6 month warranty the best price that leads to the highest profit ($1934) is $16.32. However, solving the model for different combinations of price and warranty leads to better results. As seen in Figure 4-8, the best price and warranty combination is $p = \$17.12$ and $w = 18$ months which yields the highest profit $\Pi^* = \$2017$ for the company. In different warranty options, the behavior of the profit function with respect to the price is similar but shifts to right by the warranty length increment. This means changing the warranty does not change the effect of price on the company's profitability. By increasing price, first the company's profit starts to increase because the positive effect of the price increment on the marginal profit is more than its

negative effect on the demand. The difference of these effects becomes zero at the best price for that warranty option. After the best price, the negative effect of the price increment dominates its positive effect. Therefore, the profit starts to decrease. As expected, a longer warranty length leads to a higher best price.

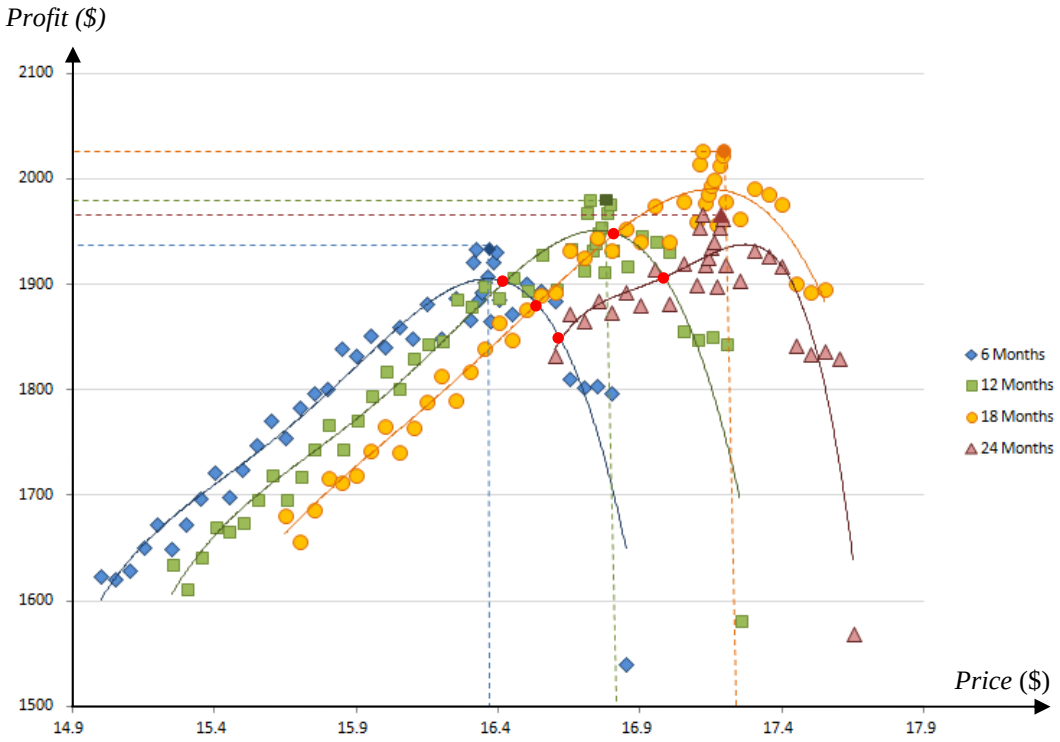


Figure 4-8: The company's profit with respect to the price in different warranty options.

The red dots in Figure 4-8 represent the intersections of the profit functions for different warranty options. These dots show the critical price values at which the priority (or, in the other words, the profitability) of the warranty options changes. Based on these price values, the priority of the warranty options in different price intervals is as follows:

- *If* $p < p_1 = \$16.41$ *then the priority of the warranty options is: 6, 12, 18 and 24 (month).*

- *If* $p_1 < p \leq p_2 = \$16.54$ *then the priority of the warranty options is: 12, 6, 18 and 24 (month).*
- *If* $p_2 < p \leq p_3 = \$16.63$ *then the priority of the warranty options is: 12, 18, 6 and 24 (month).*
- *If* $p_3 < p \leq p_4 = \$16.82$ *then the priority of the warranty options is:12, 18, 24 and 6 (month).*
- *If* $p_4 < p \leq p_5 = \$16.95$ *then the priority of the warranty options is:18, 12, 24 and 6 (month).*
- *If* $p_5 < p \leq p_6 = \$17.65$ *then the priority of the warranty options is:18, 24, 12 and 6 (month).*
- *If* $p_6 < p$ *then the priority of the warranty options is: 24, 18, 12 and 6 (month).*

In Figure 4-9, we represent the profit of the company with respect to the pre-market's service level, $sl_p = rl_r.rl_m.{rl_s}^2$, for different warranty options. As seen in this figure, the behavior of the profit function with respect to the service level is similar for all the warranty options without any significant shift to the left or right. This means all of these profit functions have almost similar optimal service levels.

Therefore, finding the best service level for one warranty option gives us a good approximation of the best service level for the other options. Therefore, it is seen that there is a very weak correlation between the warranty length and service level and they can be selected separately. Based on Figure 4-9, the highest profit corresponds to an 18 month warranty and occurs in $sl_p^* = 0.865$. However, the functions cross each other a

few times, the red dots represents the pre-market's service level values at which the priority (or, in the other words, the profitability) of the warranty options changes.

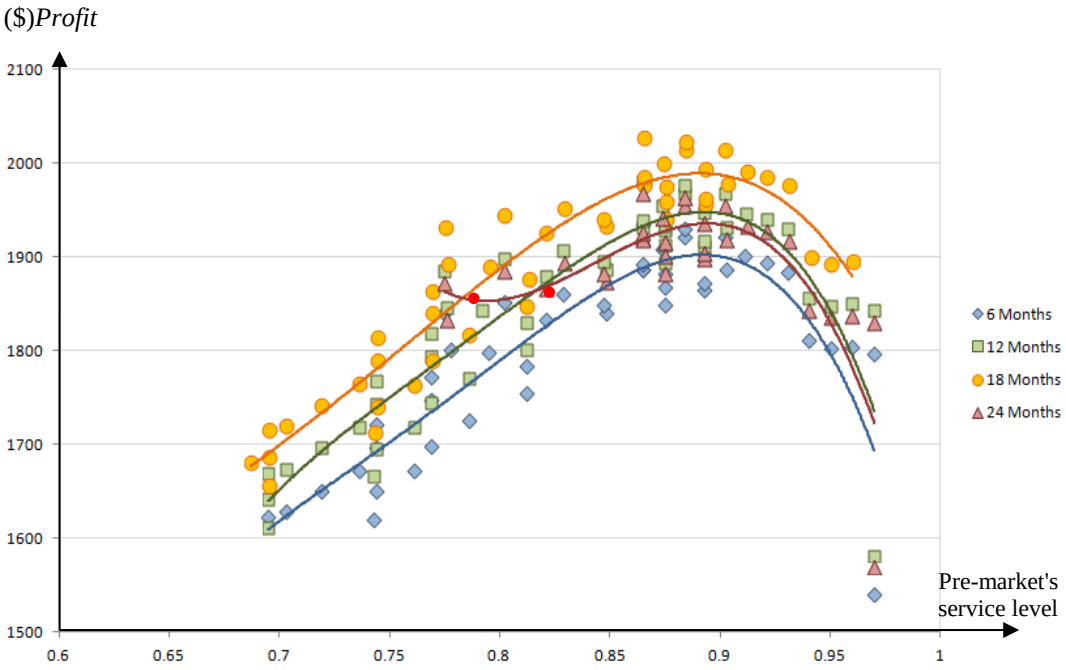


Figure 4-9: The company's profit with respect to the service level in different warranty options.

Based on these results, we have:

- *If* $sl_p < sl_p^1 = 0.785$ *then the priority of the warranty options is: 24, 18, 12, and 6 (month).*
- *If* $sl_p^1 < sl_p \leq sl_p^1 = 0.817$ *then the priority of the warranty options is: 18, 24, 12, and 6 (month).*
- *If* $sl_p^1 < sl_p$ *then the priority of the warranty options is: 18, 12, 24, and 6 (month).*

Summarizing the results in Figure 4-8 and 4-9 leads to the following best combinations of the price, service level, and priority of the warranty options (Figure 4-10):

In Figure 4-11 the positive correlation between the price and service level for different warranty options is shown. As seen in the figure, the trend of this correlation is similar for different warranty options. Increasing the warranty length only shifts the price and service level function to the right. This means that regardless of the warranty length, a given increment in the service level leads to almost the same increment in the price. However the ratio of the best price increment to the best service level increment decreases at higher prices.

	$\$16.82 < p \leq \$16.95$	$\$16.95 < p \leq \$17.65$	$\$17.65 < p$
$sl_p \leq 0.785$	-	-	24, 18, 12 and 6
$0.78 < sl_p \leq 0.817$	-	18, 24, 12 and 6	
$0.81 < sl_p$	18, 12, 24 and 6	-	-

Figure 4-10: Combinations of the best price, service level, and priority of warranty options.

As expected, for a given warranty length, increasing the product's price is always accompanied with a service level increment because the positive effect of the service level increment compensates for the negative effect of the price increment on the market's demand. Also for a given retail price, the service level improvement leads to reduction in the warranty length. For a given service level, the price increment leads to selecting a longer warranty. All of these results demonstrate that this model behaves rationally.

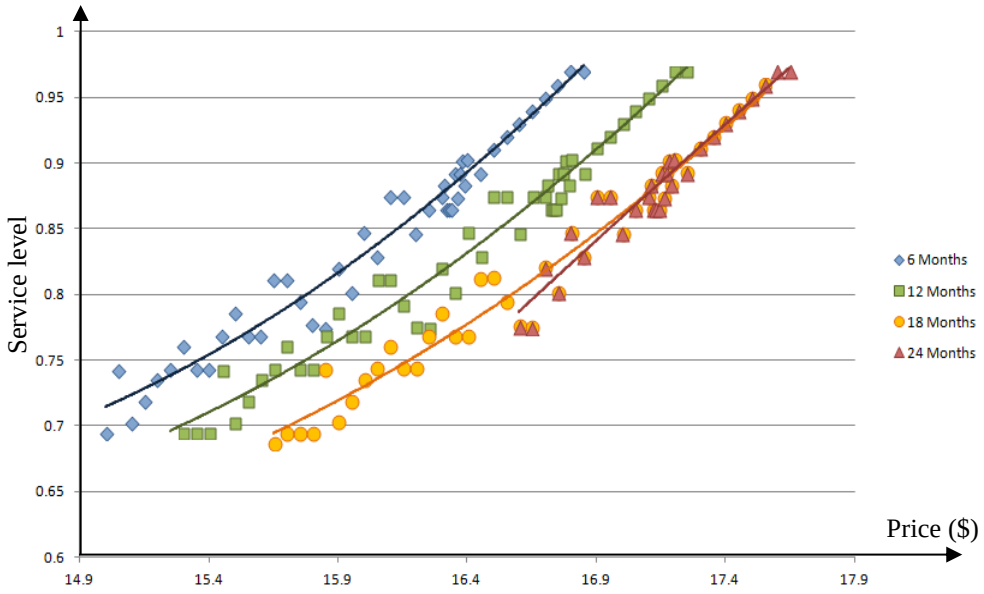


Figure 4-11: The price and service level correlation in different warranty options.

4.5. Closure of chapter 4

In this chapter, we model the flow of repaired components in the after-sales SC of repairable products. The repaired components are used to substitute the failed components of defective products returned by customers inside the warranty period. In the after-sales SCs of the repairable products, there are two types of component flow: 1) flow of repaired components; and 2) flow of new components. A new component is used when there is not any repaired component. The new components help the company to keep an appropriate after-sales service level in the after-sales markets. Having two highly convoluted flow types in the after-sales SC complicates the problem of managing operations in companies providing repairable product-service package to their customers. Therefore, in this chapter we dealt with the following question:

- ✓ **Research Question 4:** *how do the interactions between the forward and after-sales SCs of repairable goods affect planning their flow dynamics?*

We answered this question as follows:

- ✓ **In Section 4.2.2 and under "*Product failure*" title**: we determine the demand for the after-sales service in the after-sales market by considering the failure possibility in the sold products and the total product supply quantity through the forward SC.
- ✓ **In Section 4.2.2 and under "*Repair Process of Defective Components*" title**: we model the process of transferring defective components from the retailer to the repair section of the corresponding supplier, repairing defective components in the repair sections, and transporting repaired components to the retailer. These processes determine the flow of repaired components in the after-sales SC.
- ✓ **In Section 4.2.2 and under "*Safety Stock Production in the Suppliers*" title**: we determine how many new components are required to be used in the cases repaired components are not available. Existence of new components help the after-sales SC to preserve an appropriate service level in the after-sales market.
- ✓ **In Section 4.2.3**: we integrate the forward SC equations developed in Section 4.2.1 and the after-sales SC equations developed in Section 4.2.2 to formulate an integrated mathematical model for concurrent flow planning in the forward and after-sales SC.

In this problem, we consider different variations: i) supply-side variations related to the imperfect performance of the production systems in the SCs' production facilities; and ii) demand-side variations related to the stochastic demand for the product in the pre-

market and for the spare parts in the after-sales market. We show that supply-side variations propagate by moving flow from the SCs' up to downstream which yields qualified flow depreciation throughout the networks. We suggest the approach of order amplification between the SCs' facilities from down to upstream to neutralize the negative effects of the flow depreciation. This approach is used in the mathematical model to plan reliable flow throughout the SCs' networks.

The results of applying this model for an example in the automobile industry reveal the following insights:

- ✓ **The effect of warranty length on the trend of profit changes with respect to the price**: In different warranty options, the behavior of the profit function with respect to the price is almost similar but only shifts to right by the increment of warranty length. This means that changing the warranty length does not change the price effects on the company's profitability
- ✓ **The effect of warranty length on the trend of the change of profit with respect to the service level**: The behavior of the profit function with respect to the service level is similar for all the warranty options without any significant shift to the left or right. This means that all of these profit functions have almost the same optimal service level. Therefore, finding the best service level for one warranty option gives us a good approximation of the best service level for other options. This shows there is a very weak correlation between the warranty length and service level and they can be selected separately.
- ✓ **The effect of warranty length on the correlation between the price and service level:** The trend of the price and service level correlation is similar for

different warranty options. Increasing the warranty length only shifts the price and service level function to the right. This means that regardless of the warranty length, a given increment in the service level and price leads to almost the same increment in the price and service level respectively. However the ratio of the best price increment to the best service level increment decreases in higher prices.

A failure free warranty is the after-sales service considered in this chapter. However, the procedures in this chapter can be extended to cover other kinds of after-sales services such as rebate warranties, end-of-life (EOL) warranties, and performance-based logistics. Considering supply-side variations and their propagated effects significantly improves the accuracy of the service level estimation. Therefore, the methods presented in this chapter for reliable flow planning can be extended to non-profit domains in which providing a high service level is critical such as in humanitarian logistics. In the problem presented here, we only consider the supply-side variations in the performance of production facilities. There are similar variations in the connecting links between the SCs' facilities. Considering the variations in the connecting links of the chains improves the flow planning reliability.

Chapter 5: Operationally Fail-safe Supply Networks Servicing Pre- and After-sales Markets

In this chapter, we consider a product-service providing company with two supply networks (SNs): 1) a forward SN dealing with producing and supplying original products to multiple pre-markets; and 2) an after-sales SN dealing with fulfilling the after-sales commitments. A SN is a SC with more than one facility in each echelon. In this chapter, we show that how the model and solution approach developed in Chapter 3 for forward and after-sales SCs can be extended to forward and after-sales SNs. Therefore in this chapter, we answer the following questions for a company with forward and after-sales SNs:

- ✓ **Research Question 1:** *what are the important flow transitions among the facilities supporting after-sales services?*
- ✓ **Research Question 2:** *what are the important interactions between forward and after-sales SNs (SCs with more than one facility in each echelon) justifying the necessity of their concurrent flow planning?*
- ✓ **Research Question 3:** *how do these interactions affect planning flow dynamics in the forward and after-sales SNs (SCs with more than one facility in each echelon) of non-repairable goods?*

In Section 5.1, we explain the operations through the facilities of the forward and after-sales SNs. It this section, we qualitatively describe the flow transactions between the facilities of the SNs and the interactions exist between the forward and after-sales SNs. In Section 5.2, we introduce the concept of "path" in SNs and propose a "path-based" approach to model flow through the SNs. In this approach, a SN is considered as a set of

SCs. Therefore, the SCs model developed in Chapter 3 can be readily extended to model flow through the SNs.

In Section 5.2.1, we quantify the flow transactions through the paths of the forward SN. Flow modeling through the paths of the after-sales SN is explained in Section 5.2.2. In Section 5.2.3, we incorporate the equations derived in the previous sections to develop an integrated mathematical model for concurrent flow planning through the forward and after-sales SNs. A solution approach is proposed in Section 5.3 to solve the integrated model. The model and its solution approach are tested on an example from engine industry in Section 5.4.

5.1. Operations and variations in supply networks

In this problem, we consider a company producing and supplying products to objective pre-markets through a forward SN. These products are sold to the customers under a specific retail price and warranty strategy. This product includes several key components which are produced by suppliers of the first echelon. These components are transported to manufacturers in the second echelon and after assembling, the final products are supplied to the pre-markets through retailers. The products are sold with a failure-free warranty and all the defective products returned by the customers inside the warranty period should be fixed free of charge. Spare parts required to fix the returned products are provided by an after-sales SN. The after-sales SN has two echelons (first research question): i) the suppliers in the first echelon produce the required components to fix the returned products; and ii) these parts are transported to the retailers in the second echelon for substitution and repair. This is the flow transactions among the facilities of the after-sales SN to support the company's warranty commitments – first research question. The

required products and spare parts to fulfill the pre-market product demands and the warranty repair requests (called the after-sales market demands) of each sales period are produced in these forward and after-sales SNs and stored in the retailers before the beginning of that sales period.

Before the beginning of each sales period, the retailers order the required products of the pre-markets and the spare parts of the after-sales markets from the manufacturers and suppliers respectively. Based on the retailers' orders and performance of their production systems, the manufacturers order the required components from the suppliers. The suppliers receive the orders of the manufacturers and suppliers and based on the performance of their own production systems order the required materials from outside suppliers. This is the sequence of order transition among the facilities of the forward and after-sales SNs to fulfill the product and spare parts demands in the pre- and after-sales markets – first research question. We consider different variations in modeling this problem: i) variation in the pre- and after-sales market demands; ii) variation in the qualified supply quantities of the suppliers; iii) stochastic flow deterioration in the intermediate manufacturing nodes; and iv) variation in the performance of the sold products' components. The demand-side variations include uncertainty in the product demand prediction in the pre-markets and the spare parts demand prediction in the after-sales markets to repair the defective products returned by customers. The variations in the supply and intermediate manufacturing facilities are related to their imperfect production systems. These systems include a stochastic percent of nonconforming production. Thus, the qualified flow deteriorates by moving from upstream to

downstream in these networks and this deterioration increases as the uncertainty propagates.

In such complex production systems by considering all of these variations, the following questions arise:

1. What are the best service levels for the forward and after-sales SNs?
2. What are the best local reliabilities for the SNs' stochastic facilities supporting their service levels?
3. What are the best material, component, and product flow through the SNs supporting the local reliabilities of the facilities?
4. What are the best price and warranty strategies for the company?
5. What are the correlations between the best marketing strategies (service levels, price, and warranty)?

5.2. Mathematical model for concurrent flow planning in supply networks

Without loss of generality and for the purpose of modeling the problem, we consider a sample three-echelon forward SN including suppliers, manufacturers, and retailers. The modeling approach proposed here is applicable for any kind of network with any number of echelons. In Figure 5-1, a sample forward SN is shown with three suppliers ($S = \{s_1, s_2, s_3\}$), one manufacturer ($M = \{m_1\}$), and two retailers ($R = \{r_1, r_2\}$). The product of this SN includes two critical components, $N = \{n_1, n_2\}$. The first component is provided by a first group of suppliers, $S^{(n_1)} = \{s_1, s_2\}$, including the first and second suppliers. The second component is provided by the third supplier which alone is considered as a second group of suppliers, $S^{(n_2)} = \{s_3\}$. Flow streams of components starting from

the suppliers of the first echelon are assembled in the manufacturer and as final products transported to the retailers of the last echelon to supply to the markets. In the structure of the forward SN, there are several potential paths that can be used to produce and supply products to the markets.

We use the concept of "path" to model this problem. In the sample SN of Figure 5-1, each path starts from a set of suppliers in the first echelon (one supplier for each component), passes through the manufacturer in the intermediate echelon, and ends at a retailer in the last echelon. The potential paths of the sample forward SN are shown in Figure 5-1. Here each path corresponds to a triple, $t = (s, \acute{s}, r)\ \forall s \in S^{(n_1)}, \forall \acute{s} \in S^{(n_2)}$ and $\forall r \in R$. It includes the starting suppliers of the first and second components and the ending retailer. As there is a single manufacturer in this example, it is not included in the path definition. However this must be considered in a problem with several manufacturers.

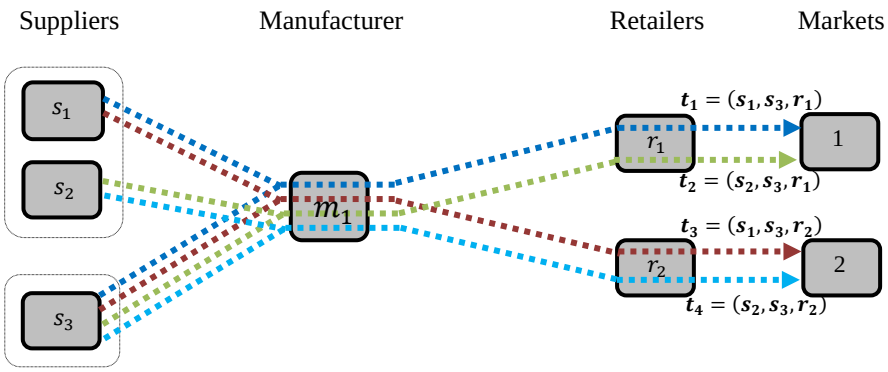


Figure 5-1: Potential paths available in the structure of a sample forward SN.

Using the concept of path in modeling this problem helps us to be able to use the developed mathematical model for any kind of network after a little manipulation. In a

different network, we only need to modify the definition of path and apply it in a same way in the mathematical model.

The set of potential paths for the sample SN of Figure 5-1 is $T = \{t_1 = (s_1, s_3, r_1), t_2 = (s_2, s_3, r_1), t_3 = (s_1, s_3, r_2), t_4 = (s_2, s_3, r_2)\}$. The most profitable subset of these paths must be selected to produce and supply the products to the pre-markets. The products of this chain are supplied to the market with a specific price, *p*, and failure free warranty, *w*. Eventually a stochastic percentage of the supplied products is returned by the customers to the retailers and their defective components should be fixed free of charge. The components required to fix these defective products must be provided by the suppliers. We assume that the required components to fix the defective items supplied by a path should be provided by the corresponding suppliers of that path.

For example, if we assume that t_1 is a selected active path in the sample forward SN in Figure 5-1 and its flow quantity is x_{t_1}, then the required first and second components to repair the returned items of these x_{t_1} products, which are represented by $\acute{x}_{t_1}^{(n_1)}$ and $\acute{x}_{t_1}^{(n_2)}$, will be supplied directly by the associated suppliers of path t_1 (s_1 and s_3) to its ending retailer, r_1 (Figure 5-2). So by determining the selected paths of the forward SN and their assigned flow quantities, the active paths of the after-sales SN and their corresponding flow quantities are determined automatically.

Table 5-1: Notation for the forward and after-sales SNs.

Sets:	
$S=\{s\}$	Set of suppliers in the supply network;
$M=\{m\}$	Set of manufacturers in the supply network;
$R=\{r\}$	Set of retailers in the supply network;
$N=\{n\}$	Set components in the product;
$S^{(n)} \subseteq S$	Subset of suppliers producing component n ($\forall n \in N$);

$T = \{t\}$	Set of potential paths in the supply network which can be used to fulfill markets. Each potential path starts from suppliers (one supplier per component) in the first echelon and after passing a manufacturer in the second echelon ends to a retailer in the third echelon to fulfill the demand of its corresponding retailer $t = (s_1 \in S^{(1)}, s_2 \in S^{(2)}, \dots, s_{\lvert N\rvert} \in S^{(\lvert N\rvert)}, m \in M, r \in R)$;
$T^{(s)} \subseteq T$	Subset of potential paths starting from Supplier *s* ($\forall s \in S$), $T^{(s)} = \{t \lvert s \in t\}$;
$T^{(m)} \subseteq T$	Subset of potential paths passing through Manufacturer *m* ($\forall m \in M$), $T^{(m)} = \{t \lvert m \in t\}$;
$T^{(r)} \subseteq T$	Subset of potential paths ending to retailer *r* ($\forall r \in R$), $T^{(r)} = \{t \lvert r \in t\}$;
$SL = \{sl = (sl_p, sl_a)\}$	Set of possible scenarios for the service level strategy of the company in the pre and after-sales markets;
$W = \{w\}$	Set of company's possible warranty strategies;
$S1 = \{s1\}$	Set of all the path selection possibilities in the network to fulfill the demand of all markets;
$S2^{(s1)} = \{s2\}$	Set of facilities' local reliabilities that can provide sl_p service level in the pre-markets and sl_a service level in the after-sales markets

***Givens*:**

p	Price of product;
$D_r(sl_p, sl_a, w, p) = \varepsilon \times \bar{D}_r(sl_p, sl_a, w, p)$	Demand of retailer *r*'s market which is considered as a product of a deterministic function, $\bar{D}_r(sl_p, sl_a, w, p)$, and a stochastic variable, ε. Without loss of generality we assume $E\left(D_r(sl_p, sl_a, w, p)\right) = \bar{D}_r(sl_p, sl_a, w, p)$;
$\bar{D}_r(sl_p, sl_a, w, p)$	Average demand of retailer *r*'s market. Retailer's average demand is an increasing function of service level and warranty length and decreasing function of price ($\forall r \in R$);
ε_r	Stochastic variable representing the uncertain part of retailer *r*'s demand ($\forall r \in R$);
$G_r(.)$	Cumulative distribution function of ε_r variable ($\forall r \in R$);
$D_{m,t\lvert m\in t}(x_t)$	Defective product quantity in manufacturer *m* in the manufacturing process of its passing path t ($m \in t$) order which is a stochastic increasing function of the path's flow, $x_{t\lvert m\in t}$;
$G'_{m,t}(.)$	Cumulative distribution function of $D_{m,t}$ ($\forall m \in M, \forall t \in T$);
$D_{s,t\lvert s\in t}(x_t)$	Nonconforming component quantity in supplier *s* in the production process of its ending path t ($s \in t$) order which is a stochastic increasing function of the path's flow, $x_{t\lvert s\in t}$, and its reliability level, rl_s;
$G''_{s,t}(.)$	Cumulative distribution function of $D_{s,t}$ ($\forall s \in S, \forall t \in T$);
z'_α	z-score of standard normal distribution for probability of α;
a_1^s	Unit procurement cost in supplier *s* ($\forall s \in S$);
a_2^s	Unit production cost in supplier *s* ($\forall s \in S$);
a^m	Unit manufacturing cost in manufacturer *m* ($\forall m \in M$);
a_{sm}^t	Unit transportation cost between supplier *s* and manufacturer *m* ($\forall s \in S, \forall m \in M$);
a_{mr}^t	Unit transportation cost between manufacturer *m* and retailer *r* ($\forall m \in M, \forall r \in R$);
a_{sr}^t	Unit transportation cost between supply *s* and retailer *r* ($\forall s \in S, \forall r \in R$);
h_r^+	Unit holding cost of extra product inventory at the end of planning period in retailer *r* ($\forall r \in R$);
h_r^-	Unit cost of product shortage at the end of planning period in retailer *r* ($\forall r \in R$);
β_m	Maximum wastage ratio in manufacturer *m* ($\forall m \in M$);
μ_s	Average number of deterioration in the time unit in supplier *s* ($\forall s \in S$);

γ_s	Defective component ratio in the out-of-control state of supplier *s* ($\forall s \in S$);
PR_s	Production rate of supplier *s* ($\forall s \in S$);
θ_n	Reliability parameter of Component *n* ($\forall n \in N$);
$f_n(.)$	Density function of failure time of Component *n* ($\forall n \in N$);
$F_n(.)$	Cumulative distribution function of failure time of Component *n* ($\forall n \in N$);
$F_n^{(m)}(.)$	Cumulative distribution function of total time to the m^{th} failure of Component *n* ($\forall n \in N$);
λ_n	Number of first failures of Component *n* that are repairable ($\forall n \in N$);
cr_n	Unit repair cost of Component *n* ($\forall n \in N$);
$Num_n(w)$	Random number of Component *n* failures in warranty time ($\forall n \in N$);
$AD_n(w)$	Average number of Component *n* substitution for a product unit in warranty time ($\forall n \in N$);
$VD_n(w)$	Variance of number of Component *n* substitution for a product unit in warranty time ($\forall n \in N$);
$\acute{D}_t^n$	After-market demand of component *n* of path *t* ($\forall n \in N;\ \forall t \in T$);
Π_r	Total cost of retailer *r* at the end of each sale period ($\forall r \in R$);
***Variables*:**	
y_t	1 if potential path *t* is used to supply products, 0 otherwise ($\forall t \in T$);
x_t	Product flow through path *t* ($\forall t \in T$);
z_{sl}	1 if service level strategy *sl* is selected by the company, 0 otherwise ($\forall sl \in SL$);
v_w	1 if warranty strategy *w* is selected by the company, 0 otherwise ($\forall w \in W$);
Δx_t	Extra production of path *t* in its corresponding manufacturer ($\forall w \in W, \forall t \in T$);
$\Delta \acute{x}_t^{(s)}$	Extra production of path *t* in supplier *s* of this path ($\forall s \in t, \forall t \in T$);
rl_s	Reliability level of supplier *s* ($\forall s \in S$);
rl_m	Reliability level of manufacturer *m* ($\forall m \in M$);
rl_r	Reliability level of retailer *r* ($\forall r \in R$);
$x_t^{\prime(n)}$	Component *n* flow through path *t* to fulfill after-sales demand ($\forall t \in T, \forall n \in N$);

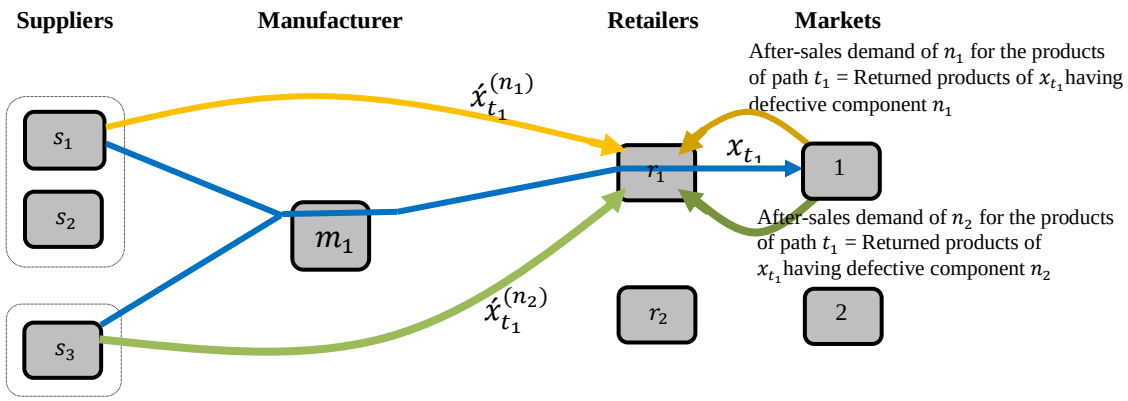


Figure 5-2: After-sales services provided by active Path t_1.

In this problem, we consider demand- and supply-side variations by assuming that demand prediction in the demand nodes is stochastic and the performance of the production systems in the supply and intermediate manufacturing facilities is imperfect. Imperfect production systems in the supply and manufacturing facilities means their qualified output quantities are stochastic. Having several uncertain echelons in a SN leads to a problem which we call uncertainty propagation. Considering and quantifying this propagation of uncertainty is critical for determining service levels in pre- and after-sales markets. The uncertainty propagation occurs through all the active paths of the networks. We display one of the paths of the forward SN as a sample in Figure 5-3. In the rest of this section, we describe the process of quantifying uncertainty propagation throughout this path of the forward SN.

5.2.1. Forward supply network formulation

The pre- and after-sales markets' service levels show the global reliabilities of the forward and after-sales networks against all the variations and their propagated effect. The service level which represents the capability of a network in balancing supply and demand quantities depends on the local reliabilities of its constituting facilities. In this problem, we introduce and use the concept of path to produce and supply products and spare parts to markets. Therefore in this section (Section 5.2.1) and Section 5.2.2, respectively we explain that how to manage the flow in the paths of the forward and after-sales SNs against variations. This means in Section 5.2.2, we answer the first research question by modeling the flow transactions among the facilities of the after-sales SN. Common variables in the equations of these two sections determine the interactions exist between the forward and after-sales SNs – second research questions. These interactions justify

the necessity of concurrent flow planning in the SNs. Finally, we use the outcomes of these sections in Section 5.2.3 to develop a comprehensive mathematical model to manage the performance of the entire system. This mathematical model avoids sequential decision making in the SNs – first forward and then after-sales SNs – which ignores the interactions (third research question).

In this section, we elaborate a way to quantify uncertainty propagation and plan a reliable flow through the paths of the forward SN. The paths of the forward network include a retailer, a manufacturer, and suppliers (one supplier for each component). However, in each path of the after-sales SN there are a retailer and suppliers (one supplier for each component, Figure 5-2). We assume that the local reliability of Retailer *r*, Manufacturer m, and Supplier s are represented by $rl_r, rl_m,$ and rl_s respectively. To quantify uncertainty propagation through each path, we start from the last echelon including a retailer, then variations of the manufacturer and suppliers are addressed later.

<u>Uncertainty management in the retailers</u>

The company positions itself in the markets by choosing its pre- and after-sales service levels, warranty length, and retail price. This means the service level provided by the after-sales SN directly affects the demand and sale quantity in the pre-markets – one of the interactions between the forward and after-sales SNs (second research question). The average product demand in Market *r*, $\bar{D}_r(sl_p, sl_a, w, p)$, in a sales period is an increasing function of the service levels, (sl_p, sl_a), and warranty length, w, and a decreasing function of price, p. However, the realized actual demand is stochastic and has a deviation from its mean. Consistently with Bernstein and Federgruen (2004 and 2007), we assume that the stochastic actual demand in the market is multiplicative as $D_r(sl_p, sl_a, w, p) =$

$\varepsilon_r \times \bar{D}_r(sl_p, sl_a, w, p)$. Where ε_r is a general continuous random variable with a cumulative distribution function, $G_r(\varepsilon_r)$, which is independent of the service levels, warranty length, and retail price. Without loss of generality, we assume $E(\varepsilon_r) = 1$ which means $E[D_r(sl_p, sl_a, w, p)] = \bar{D}_r(sl_p, sl_a, w, p)$.

Before the beginning of each sales period, Retailer r ($\forall r \in R$) orders the required products from the manufacturers. These products are provided by the active paths ending at this retailer, $\sum_{T(r)} x_t$, before the beginning of the period. Additional product transactions during the period and after real demand realization are not possible. The demand of Market r is stochastic with $G_r(.)$ cumulative distribution function (demand-side uncertainty). Extra inventory and inventory shortage at the end of each sales period impose unit cost h_r^+ and h_r^- on the retailer respectively. Thus, subject to the local reliability of Retailer r (rl_r), the product ordering quantity of the retailer, $\sum_{T(r)} x_t$, should be determined to minimize its end-of-period total cost.

Product ordering quantity of Retailer r is:

$$MIN \quad \Pi_r = h_r^+ . E\left[\sum_{T(r)} x_t - D_r(sl_p, sl_a, w, p)\right]^+ + h_r^- . E\left[D_r(sl_p, sl_a, w, p) - \sum_{T(r)} x_t\right]^+ \tag{5-1}$$

$$S.T. \quad \Pr[D_r(sl_p, sl_a, w, p) \leq \sum_{T(r)} x_t] \geq rl_r \tag{5-2}$$

The first term of the objective function (5-1) is the expected holding cost of the end-of-period extra inventory and the second term is the expected shortage cost in Retailer r. Therefore, the objective function is minimizing the total cost in the retailer. Constraint (5-2) preserves the local reliability of the retailer (Figure 5-3). Minimizing the model's

objective function without considering constraint (5-2) leads to $\sum_{T(r)} x_t = \bar{D}_r(sl_p, sl_a, w, p).G_r^{-1}(\frac{h_r^-}{h_r^- + h_r^+})$. Also to preserve the local reliability of the retailer, we have $\sum_{T(r)} x_t \geq \bar{D}_r(sl_p, sl_a, w, p).G_r^{-1}(rl_r)$.

Accordingly, the best amount of the product should be ordered by the retailer is $\sum_{T(r)} x_t = \bar{D}_r(sl_p, sl_a, w, p).G_r^{-1}\left(Max\left\{rl_r, \frac{h_r^-}{h_r^- + h_r^+}\right\}\right)$. This order is distributed among the active paths ending at this retailer and Path t's share from this order is x_t (assuming that Path *t* ends at Retailer *r*). Therefore, x_t products must be provided by the manufacturer of this path. In the next section we study the manufacturer's performance with respect to the retailers' order.

Uncertainty management in the manufacturers

Order share of each path should be produced by the manufacturer of that path. By assuming that Path *t* is passing through Manufacturer *m*, so this manufacturer should produce x_t qualified products for this path. But the production system of the manufacturer is not perfect and is always accompanied by stochastic percentage of defective items. To compensate these defective items, manufacturer should plan to produce some extra products such as Δx_t. Amount of Δx_t depends on the local reliability of Manufacturer *m*. Δx_t should be determined in a way that manufacturer will be sure with rl_m probability that it can fulfill the whole product order assigned to the path. So the probability that defective product quantity in the manufacturing process of Path *t*'s ordered products, $D_{m,t}$, would be less than Δx_t should be equal to rl_m ($\acute{G}_{m,t}$ is the cumulative distribution function assumed for $D_{m,t}$):

$$\Pr\left(D_{m,t} \le \Delta x_t\right) = rl_m \qquad \rightarrow \qquad \Delta x_t = \acute{G}^{-1}_{m,t}(rl_m) \tag{5-3}$$

For example if we assume that defective production rate in Manufacturer m is a stochastic variable, α_m, uniformly distributed in $[0, \beta_m]$ range, then appropriate Δx_t is computed as follow:

$$\Pr(\alpha_m . x_t \le \Delta x_t) = \Pr\left(\alpha_m \le \frac{\Delta x_t}{x_t}\right) = rl_m \qquad \rightarrow \qquad \Delta x_t = rl_m . \beta_m . x_t \tag{5-4}$$

So to preserve local reliability rl_m, manufacturer should plan to produce $x_t + \Delta x_t$ products for Path t. accordingly it should order $x_t + \Delta x_t$ components from the suppliers of this path. In the next section, we study the performances of Path t's suppliers respect to the component orders received from the manufacturer.

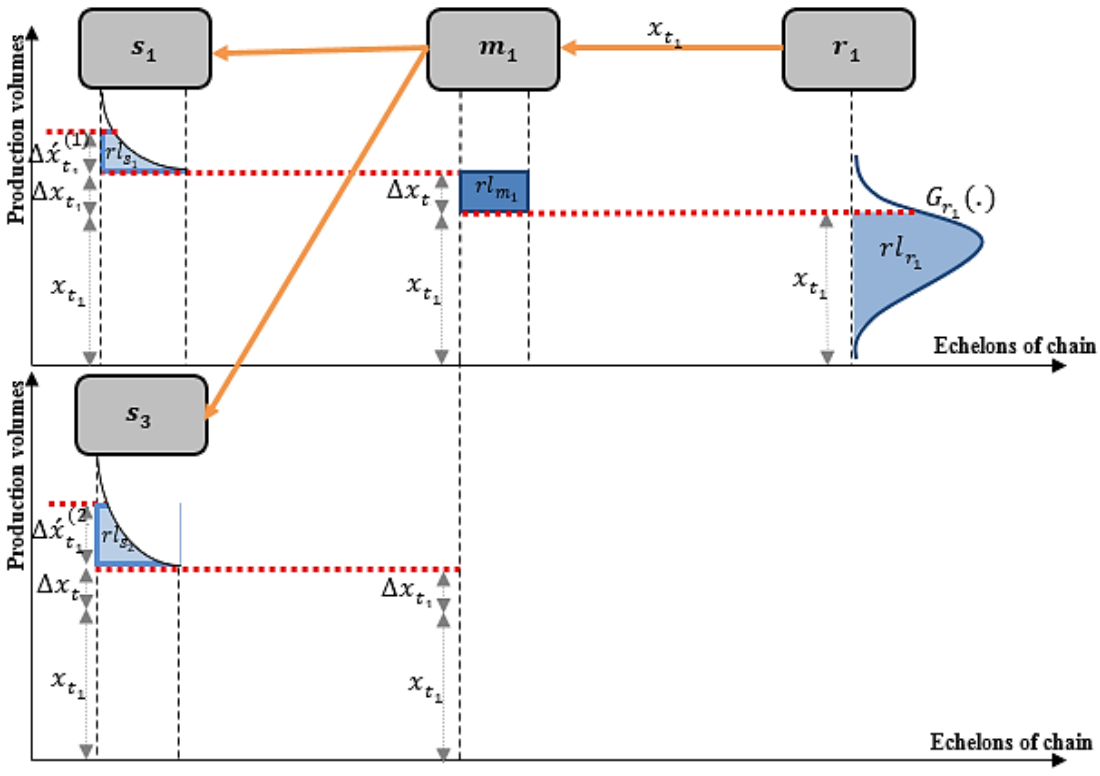


Figure 5-3: Uncertainty propagation in Path $t_1 = (s_1, s_3, r_1)$ of the forward SN.

Uncertainty management in the suppliers

We assume that Supplier *s* is a supplier of Path *t*. This supplier receives an order of $x_t + \Delta x_t$ component units from the manufacturer. But we know that its production system is not perfect and has some nonconforming output. To compensate for these nonconforming items, the supplier plans to produce extra components, $\Delta \acute{x}_t^{(s)}$. The amount of $\Delta \acute{x}_t^{(s)}$ depends on the local reliability of Supplier *s*. $\Delta \acute{x}_t^{(s)}$ insures the supplier with rl_s probability that it can fulfill the manufacturer's order. Therefore, the probability that the nonconforming component quantity in the production process of Path *t*'s order, $D_{s,t}$, is less than $\Delta \acute{x}_t^{(s)}$ and is equal to rl_s ($G''_{s,t}$ is the cumulative distribution function assumed for $D_{s,t}$):

$$\Pr\left(D_{s,t} \leq \Delta \acute{x}_t^{(s)}\right) = rl_s \qquad \rightarrow \qquad \Delta \acute{x}_t^{(s)} = {G''_{s,t}}^{-1}(rl_s) \tag{5-5}$$

Assume that in the supplier after setting up the machines to produce the required components, they start to work in an in-control state in which all the components produced are qualified. Gradually their state deteriorates and after a stochastic time, they shift to an out-of-control state in which γ_s percent of components is nonconforming. We assume the deterioration time follows exponential distribution with $1/\mu_s$ mean. After shifting to the out-of-control state, they stay in that state until the whole batch is completed because interrupting the machines is prohibitively expensive (Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987). To fulfill the component order of Path *t*, $\Delta \acute{x}_t^{(s)} + \Delta x_t + x_t$ components should be produced by this supplier. By considering PR_s as the

production rate of the supplier, it takes $\frac{\Delta \acute{x}_t^{(s)} + \Delta x_t + x_t}{PR_s}$ time units to produce this batch. Assuming rl_s as the supplier's local reliability, the probability that the quantity of non-conforming components produced during this time period is less than $\Delta \acute{x}_t^{(s)}$ should be equal to rl_s. Thus, the probability that the conforming component quantity is greater than or equal to $\Delta x_t + x_t$ should be equal to rl_s:

$$rl_s = \Pr(conforming\ component\ units\ produced\ in\ \frac{\Delta \acute{x}_t^{(s)} + \Delta x_t + x_t}{PR_s}\ time\ unit \geq \Delta x_t + x_t)$$

$$= \Pr\left[PR_s.t + (1-\gamma_s).PR_s.\left(\frac{\Delta \acute{x}_t^{(s)} + \Delta x_t + x_t}{PR_s} - t\right) \geq \Delta x_t + x_t\right]$$

$$= \Pr\left[t \geq \left(\frac{\Delta x_t + x_t}{PR_s}\right) - \left(\frac{1-\gamma_s}{\gamma_s.PR_s}\right).\left(\Delta \acute{x}_t^{(s)}\right)\right] = EXP\left[-\mu_s.\left(\left(\frac{\Delta x_t + x_t}{PR_s}\right) - \left(\frac{1-\gamma_s}{\gamma_s.PR_s}\right).\left(\Delta \acute{x}_t^{(s)}\right)\right)\right] \quad (5\text{-}6)$$

$$\rightarrow \qquad \Delta \acute{x}_t^{(s)} = \frac{\gamma_s}{1-\gamma_s}\left[\frac{PR_s}{\mu_s}\ln(rl_s) + (\Delta x_t + x_t)\right] \quad (5\text{-}7)$$

This means that with this $\Delta \acute{x}_t^{(s)}$ extra production, Supplier *s* will be sure with rl_s probability that it can fulfill the order of the manufacturer.

To sum up, with $\Delta \acute{x}_t^{(s)}$ $(\forall s \in t - \text{all the suppliers of Path } t)$ extra production, the suppliers of Path *t* in the first echelon will be sure with $\prod_{(\forall s \in t)} rl_s$ probability that they can fulfill the whole component order of this path's manufacturer. Also the manufacturer by producing Δx_t extra products will be sure with rl_m probability that it can fulfill the product order of the path's retailer. By ordering x_t products from this path, the retailer

will be sure with rl_r probability that it can fulfill a $x_t / \sum_{T^{(r)}} x_t$ portion of the corresponding pre-market's demand in the coming sales period. The global reliability provided by this path is:

$$Global\ relaibility\ of\ Path\ t = \left(\prod_{(\forall s \in t)} rl_s\right) \times rl_m \times rl_r \quad (\forall t \in T) \tag{5-8}$$

The demand of each pre-market and the order of its corresponding Retailer *r* can be fulfilled by all the potential paths ending at that retailer, $\forall t \in T^{(r)}$. To determine the active paths of this set, we define binary variables y_t ($\forall t \in T$). Variable y_t is 1 if potential Path *t* is active and used to produce and supply products and 0 otherwise.

Therefore, Retailer *r* will be sure with $\prod_{(\forall t \in T^{(r)})} \left[\left(\left(\prod_{(\forall s \in t)} rl_s\right) \times rl_m \times rl_r\right).y_t + (1 - y_t)\right]$ probability that it can fulfill the demand of its corresponding market in the next sales period. Thus, the service level (demand fulfillment rate) of the forward SN in the pre-market of Retailer *r* will be:

$$Pre - market\ service\ level\ in\ the\ market\ of\ retailer\ r$$

$$= \prod_{(\forall t \in T^{(r)})} \left[\left(\left(\prod_{(\forall s \in t)} rl_s\right) \times rl_m \times rl_r\right).y_t + (1 - y_t)\right]$$

$$(\forall r \in R) \tag{5-9}$$

5.2.2. After-sales supply network formulation

We assume that the after-sales services of the products supplied by a path to a market should be provided by that path. In this section, we answer the first research question by modeling the flow transactions among the after-sales SN's facilities. Also we show that

how these operations are affected by the decisions made in the forward SN – Second research question. These displays will be incorporated in the model of Section 5.2.3 for concurrent flow planning – third research question. The first step for planning flow dynamics in the after-sales SN is to predict the after-sales requests for the products of each path. After determining the after-sales flow of each path, this flow is amplified from downstream to upstream to deal with uncertainty propagation in that path.

After-sales demand prediction and spare parts order quantity

Assume that x_t products are supplied by Path $t \in T^{(r)}$ of the forward SN to the pre-market of Retailer *r*. The required components to repair the defective products of x_t returned by the customers inside the warranty period is the after-sales demand for Path *t*. Here, we compute the quantity of this demand for each component. This demand depends on the product quantity supplied by Path *t* in the forward SN (this is one of the interactions between the forward and after-sales SNs), the length of the warranty (this is another interaction between the forward and after-sales SNs) and the reliability of the components represented by θ_n $(\forall n \in N)$. We assume that the performance of the product's components is independent and the failure time of Component *n* is a random variable with $f_n(\theta_n)$ density and $F_n(\theta_n)$ cumulative density function. Lower θ_n means higher reliability for Component *n* and longer time between failures. We assume the first λ_n failures of Component *n* are repairable but after that it is more economical to replace it with a new one. The repair cost of Component *n* is cr_n. We assume that behavior of the components do not change after repair; the repaired and new components have similar breakdown behavior. Assuming that $F_n^{(m)}$ and $Num_n(w)$ represents the cumulative

distribution function of total time up to the m^{th} failure and the number of failures of Component *n* in [0,*w*] interval, we have (Nguyen and Murthy, 1984):

$$\Pr\{Num_n(w) = m\} = F_n^{(m)}(w, \theta_n) - F_n^{(m+1)}(w, \theta_n) \qquad (\forall n \in N) \quad (5\text{-}10)$$

Then the average number of new Component *n* required to repair a unit of product inside the warranty period, $AD_n(w, \theta_n, \lambda_n)$, is:

$$AD_n(w, \theta_n, \lambda_n) = \sum_{m=\lambda_n+1}^{+\infty} F_n^{(m)}(w, \theta_n) \qquad (\forall n \in N) \quad (5\text{-}11)$$

In the same way, the variance of the number of new Component *n* required to repair a unit of product inside the warranty period, $VD_n(w, \theta_n, \lambda_n)$, is:

$$VD_n(w, \theta_n, \lambda_n) = \sum_{m=\lambda_n+1}^{+\infty} [2.(m - \lambda_n) - 1].F_n^{(m)}(w, \theta_n) - [\sum_{m=\lambda_n+1}^{+\infty} F_n^{(m)}(w, \theta_n)]^2$$

$$(\forall n \in N) \quad (5\text{-}12)$$

By using the Central Limit theorem, the total Component *n* required in the after-sales market of Path *t*, $\acute{D}_t^n$, has a normal distribution with the following features:

$$\acute{D}_t^n \sim Normal\left(\mu_{\acute{D}_t^n} = x_t.AD_n(w, \theta_n, \lambda_n), \sigma_{\acute{D}_t^n}^2 = x_t.VD_n(w, \theta_n, \lambda_n)\right)$$

$$(\forall n \in N;\ \forall t \in T) \quad (5\text{-}13)$$

This means the spare parts demands in the after-sales markets depend on the product quantity supplied by the forward SN to the pre-markets (This is another interaction between forward and after-sales SNs). If Path *t* ends at Retailer *r* ($r \in t$) and its local reliability is rl_r, the quantity of Component *n* ordered by Retailer *r* from Path *t* is:

$$x_t^{\prime(n)} = x_t.AD_n(w,\theta_n,\lambda_n) + \left(z'_{rl_r}.\sqrt{x_t.VD_n(w,\theta_n,\lambda_n)}\right) \qquad (\forall n \in N) \quad (5\text{-}14)$$

By ordering $x_t^{\prime(n)}$ units of Component *n*, the retailer will be sure with rl_r probability that it is able to fulfill the after-sales demand of Component *n* for path *t*'s products.

Performance of the suppliers in the after-sales network

Retailer *r* not only orders x_t $(t \in T^{(r)})$ products from the manufacturer of Path *t*, but also orders $x_t^{\prime(n)}$ $(\forall n \in N)$ units of Component *n* from the path's corresponding Supplier *s* ($s \in t$ and $s \in S^{(n)}$) providing Component *n* for this path. Supplier *s* receives an order of $\Delta x_t + x_t$ component units from the manufacturer of this path (forward SN) and an order of $x_t^{\prime(n)}$ component units from the retailer of this path (after-sales SN). Thus the total order received by Supplier *s* includes $x_t^{\prime(n)} + \Delta x_t + x_t$ component units. To compensate for the nonconforming output of its production system, it plans to produce extra components $\Delta\acute{x}_t^{(s)}$. In Section 5.2.1, the quantity of $\Delta\acute{x}_t^{(s)}$ is determined by assuming that $\Delta x_t + x_t$ component order is received by this supplier. But in addition to this order of the forward SN another order with $x_t^{\prime(n)}$ quantity is received from the after-sales SN. In this section, we revise the quantity of $\Delta\acute{x}_t^{(s)}$ to consider the order of the after-sales SN:

$$rl_s$$

$$= \Pr\left(conforming\ component\ units\ produced\ in\ \frac{\Delta\acute{x}_t^{(s)} + x_t^{\prime(n)} + \Delta x_t + x_t}{PR_s}\ time\ units \geq x_t^{\prime(n)} + \Delta x_t + x_t\right)$$

$$= \Pr\left[PR_s.t + (1-\gamma_s).PR_s.\left(\frac{\Delta\acute{x}_t^{(s)} + x_t^{\prime(n)} + \Delta x_t + x_t}{PR_s} - t\right) \geq x_t^{\prime(n)} + \Delta x_t + x_t\right]$$

$$= \Pr\left[t \geq \left(\frac{x_t^{\prime(n)}+\Delta x_t+x_t}{PR_s}\right) - \left(\frac{1-\gamma_s}{\gamma_s.PR_s}\right).\left(\Delta \acute{x}_t^{(s)}\right)\right] = EXP\left[-\mu_s.\left(\left(\frac{x_t^{\prime(n)}+\Delta x_t+x_t}{PR_s}\right) - \left(\frac{1-\gamma_s}{\gamma_s.PR_s}\right).\left(\Delta \acute{x}_t^{(s)}\right)\right)\right] \quad (5\text{-}15)$$

$$\rightarrow \qquad \Delta \acute{x}_t^{(s)} = \frac{\gamma_s}{1-\gamma_s}\left[\frac{PR_s}{\mu_s}\ln(rl_s) + (x_t^{\prime(n)} + \Delta x_t + x_t)\right] \quad (5\text{-}16)$$

This means that by this $\Delta \acute{x}_t^{(s)}$ extra production, Supplier *s* is sure with rl_s probability that it can fulfill the whole orders of the forward and after-sales SNs.

Thus, with $\Delta \acute{x}_t^{(s)}$ ($\forall s \in t - s$ is supplying Component n) extra production, the supplier of Path *t* is sure with rl_s probability that it can fulfill the whole Component *n* order of this path's retailer. By ordering $x_t^{\prime(n)}$ units of Component *n* from the path's supplier, the retailer is sure with rl_r probability that it can fulfill the whole after-sales demand of Component *n* to repair the defective products of Path *t*. Therefore, the fulfill rate of Component *n* in Path *t* is:

$$fulfill\ rate\ of\ component\ n\ in\ path\ t\ = rl_s \times rl_r \qquad (t \in T^{(r)}) \quad (5\text{-}17)$$

There are *n* components in the product. The fulfill rate of all components by Path *t* will be:

$$fulfill\ rate\ of\ all\ components\ in\ path\ t\ = \prod_{(\forall n \in N,\ s \in S^{(n)} | s \in t)} (rl_s \times rl_r) = (rl_r)^{|N|}.\prod_{(\forall n \in N,\ s \in S^{(n)} | s \in t)} rl_s \qquad (t \in T^{(r)}) \quad (5\text{-}18)$$

The after-sales demand in Retailer *r* is fulfilled by all the potential active paths ending at that retailer, $\forall t \in T^{(r)}$. Therefore, the service level (demand fulfillment rate) of the after-sales SN in Retailer *r* is:

$$after-sales\ service\ level\ in\ retailer\ r$$

$$= \prod_{(\forall t \in T^{(r)})} \left[\left((rl_r)^{|N|} . \prod_{(\forall n \in N,\ s \in S^{(n)} | s \in t)} rl_s \right) . y_t + (1 - y_t) \right]$$

$$(\forall r \in R) \qquad (5\text{-}19)$$

Based on Equations (5-19) and (5-9), the service levels in the pre- and after-sales markets are convoluted and are functions of local reliabilities. This is the other interaction that is considered in the mathematical model of the next section.

5.2.3. Concurrent flow planning in the forward and after-sales supply networks

With the help of the equations formulated in Sections 5.2.1 and 5.2.2, in this section we develop a comprehensive mathematical model to simultaneously determine the best marketing strategies and their preserving flow dynamics throughout the forward and after-sales SNs. In reality, there are common options for the warranty length that are usually offered, such as 6, 12, 18, and 24 months. Therefore, in this problem we define a new set, $W = \{w\}$, including all options available for warranty length. In the same way, we define a similar set for the service levels in the pre- and after-sales markets. Set SL= $\{sl = (sl_p, sl_a)\}$ includes all possible options for the service level of the company in the pre- and after-sales markets. The options offered by the markets' rivals and government regulations are considered in determining these sets. To make decision about warranty and service levels strategy, we define two new binary variables v_w and z_{sl}. Variable v_w

is 1 if Warranty *w* is selected, 0 otherwise ($\forall w \in W$). Variable z_{sl} is 1 if Service level *sl* is selected, 0 otherwise ($\forall sl \in SL$). The model of this concurrent planning is:

$$\textbf{MAX} \quad \sum_R \sum_W \sum_{SL} v_w . z_{sl} . \overline{D}_r(sl_p, sl_a, w, p) . \left[p - h_r^+ . E\left(G_r^{-1}\left(MAX\left\{ rl_r, \frac{h_r^-}{h_r^+ + h_r^-} \right\} \right) - \varepsilon_r \right)^+ - h_r^- . E\left(\varepsilon_r - G_r^{-1}\left(MAX\left\{ rl_r, \frac{h_r^-}{h_r^+ + h_r^-} \right\} \right) \right)^+ \right] - \sum_N \sum_{S^{(n)}} \sum_{T^{(s)}} (a_1^s + a_2^s) . \left[x_t + \Delta x_t + x_t^{\prime(n)} + \Delta \acute{x}_t^{(s)} \right] - \sum_M \sum_{T^{(m)}} a^m . (x_t + \Delta x_t) - \sum_S \sum_M \sum_{T^{(s)} \cap T^{(m)}} a_{sm}^t . (x_t + \Delta x_t) - \sum_M \sum_R \sum_{T^{(m)} \cap T^{(r)}} a_{mr}^t . x_t - \sum_N \sum_{S^{(n)}} \sum_R \sum_{T^{(s)} \cap T^{(r)}} a_{sr}^t . x_t^{\prime(n)} \tag{5-20}$$

Subject To:

$$\sum_W v_w = 1 \tag{5-21}$$

$$\sum_{SL} z_{sl} = 1 \tag{5-22}$$

$$\sum_{T^{(r)}} y_t \geq 1 \qquad (\forall r \in R) \tag{5-23}$$

$$x_t \leq BM . y_t \qquad (\forall t \in T) \tag{5-24}$$

$$\sum_{T^{(r)}} x_t = \left[\sum_W \sum_{SL} v_w . z_{sl} . \overline{D}_r(sl_p, sl_a, w, p) \right] . G_r^{-1}\left(Max\left\{ rl_r, \frac{h_r^-}{h_r^- + h_r^+} \right\} \right) \qquad (\forall r \in R) \tag{5-25}$$

$$\Delta x_t = G_{m,t}^{\prime\ -1}(x_t, rl_m) . y_t \qquad (\forall t \in T, m \in t) \tag{5-26}$$

$$x_t^{\prime(n)} = x_t . AD_n(w, \theta_n, \lambda_n) + z_{rl_r}^{\prime} . \sqrt{x_t . VD_n(w, \theta_n, \lambda_n)} \qquad (\forall t \in T, \forall n \in N) \tag{5-27}$$

$$\Delta \acute{x}_t^{(s)} = G_{s,t}^{\prime\prime\ -1}\left(x_t + \Delta x_t + x_t^{\prime(n)}, rl_s \right) . y_t \qquad \left(\forall t \in T, \forall n \in N, s \in t, s \in S^{(n)} \right) \tag{5-28}$$

$$\sum_{SL} z_{sl} . sl_p = \prod_{(\forall t \in T^{(r)})} \left[\left(\left(\prod_{(\forall s \in t)} rl_s \right) \times rl_m \times rl_r \right) . y_t + (1 - y_t) \right] \qquad (\forall r \in R) \tag{5-29}$$

$$\sum_{SL} z_{sl} . sl_a = \prod_{(\forall t \in T^{(r)})} \left[\left((rl_r)^{|N|} . \prod_{(\forall n \in N,\ s \in S^{(n)} | s \in t)} rl_s \right) . y_t + (1 - y_t) \right]$$

$$(\forall r \in R) \quad (5\text{-}30)$$

$$v_w, z_{sl}, y_t \in \{0,1\} \qquad (\forall w \in W, \forall sl \in SL, \forall t \in T) \quad (5\text{-}31)$$

$$x_t, \Delta x_t, \Delta \acute{x}_t^{(s)}, x_t^{'(n)} \geq 0 \qquad (\forall t \in T, \forall s \in S, \forall n \in N) \quad (5\text{-}32)$$

In these equations, *BM* is a large constant. The first term of the objective function (5-20) represents the profit which is captured in the pre-markets. This term is equal to the income minus the shortage and holding cost of the inventory shortage and extra inventory at the end of the sales period. The second term is the sum of procurement and production costs in the suppliers. Manufacturing costs of the products in the manufacturers is computed in the third term. The fourth, fifth, and sixth terms compute the sum of transportation costs of the forward SN's components from the suppliers to the manufacturers, the forward SN's products from the manufacturers to the retailers, and the after-sales SN's components from the suppliers to the retailers. This objective function maximizes the net profit of the whole company.

Based on Constraints (5-21) and (5-22), only one warranty and service level strategy can be selected by the company. Constraint (5-23) ensures that at least one path is activated to fulfill the demand of each market. According to Constraint (5-24), product flow is only possible in the activated paths. Based on Constraint (5-25), the sum of the product flow through the paths ending at a retailer is equal to the pre-market demand of that retailer. Constraint (5-26) shows flow amplification in the manufacturer of each path. Constraint (5-27) is used to calculate the component requests of each path in the after-sales markets. Constraint (5-28) represents flow amplification in the suppliers of each path. Based on Constraint (5-29), local reliabilities assigned to the facilities must preserve

the company's selected pre-market service level. In the same way, local reliabilities assigned to the facilities should preserve the company's selected after-sales service level (Constraint 5-30).

This mathematical model determines the best warranty and service level strategies in the pre- and after-sales markets and their preserving local reliabilities and flow in the system's facilities to maximize the company's total profit. This model is a mixed integer nonlinear mathematical model. Solving this kind of models is not straightforward. Especially the form of nonlinear terms in this model depends on the cumulative distribution functions defined for the stochastic parts of the problem. This means that by changing these distribution functions, the mathematical forms of these terms also change. In Section 5.3, we propose an efficient approach to solve this model and find the solution.

5.3. Solution approach

In this section, we develop a five-step approach to solve the model proposed in the previous section (Figure 5-4). In this approach, for each $sl = (sl_p, sl_a) \in SL$ and each $w \in W$, the following steps should be done:

Step 1: Define a new set, $S1 = \{s1\}$, including all the path selection possibilities in the network to fulfill the demand of all markets. The largest size for this set is:

$$|S1| = \prod_{\forall r \in R} 2^{(|T^{(r)}|-1)} \quad (5\text{-}33)$$

Step 2: For each $s1 \in S1$, determine a set of facilities' local reliabilities that can provide sl_p service level in the pre-markets and sl_a service level in the after-sales markets, $S2^{(s1)} = \{s2\}$. Notice that:

$$s2 = \left({rl_r}^{(s2)} \ (\forall r \in R), {rl_m}^{(s2)} \ (\forall m \in M), {rl_s}^{(s2)} \ (\forall s \in S) \right) \quad (5\text{-}34)$$

Determining these feasible local reliability combinations is initiated by discretizing the continuous interval of the local reliabilities. For example, by assuming that the least possible pre-market service level is 0.75 and the facilities have the same lower bounds for their local reliabilities, the lower interval bound for the local reliabilities is 0.9. After discretizing [0.9, 1.0] interval by an acceptable step such as 0.01, these feasible local reliability combinations is determined as follows:

For $rl_r = 0.9{:}\ 0.1{:}\ 1.0 \ \ (\forall r \in R)$
  For $rl_m = 0.9{:}\ 0.1{:}\ 1.0 \ \ (\forall m \in M)$
    For $rl_s = 0.9{:}\ 0.1{:}\ 1.0 \ \ (\forall s \in S)$
      IF $sl_p \cong \prod_{\left(\forall t \in (s1 \cap T^{(r)})\right)} \left(\left(\prod_{(\forall s \in t)} rl_s \right) \times rl_{m|m \in t} \times rl_{r|r \in t} \right)$ and
      $sl_a \cong \prod_{\left(\forall t \in (s1 \cap T^{(r)})\right)} \left((rl_r)^{|N|} . \prod_{(\forall n \in N, \ s \in S^{(n)} | s \in t)} rl_s \right)$
      $(\forall r \in R)$
        Add
        $\left({rl_r}^{(s2)} = rl_r \ (\forall r \in R), {rl_m}^{(s2)} = rl_m \ (\forall m \in M), {rl_s}^{(s2)} rl_s \ (\forall s \in S) \right)$ into set *S2*
        End;
      End;
    End;
  End; $\quad (5\text{-}35)$

Having restricted feasible intervals for local reliability variables justifies the rationality of using discretizing in this step.

Step 3: For each $\forall s1 \in S1$ and $\forall s2 \in S2^{(s1)}$, solve the following linear model with continuous variables:

$$\boldsymbol{MIN} \quad Cost^{(s1,s2)}(w,sl) = \overline{D}_r(sl,w,p).\left[h_r^+.E\left(G_r^{-1}\left(MAX\left\{rl_r^{(s2)},\frac{h_r^-}{h_r^+ + h_r^-}\right\}\right) - \varepsilon_r\right)^+ + h_r^-.E\left(\varepsilon_r - G_r^{-1}\left(MAX\left\{rl_r^{(s2)},\frac{h_r^-}{h_r^+ + h_r^-}\right\}\right)\right)^+\right] + \sum_N \sum_{S^{(n)}} \sum_{T^{(s)}} (a_1^s + a_2^s).\left[x_t + \Delta x_t + x_t^{'(n)} + \Delta \acute{x}_t^{(s)}\right] + \sum_M \sum_{T^{(m)}} a^m.[x_t + \Delta x_t] - \sum_S \sum_M \sum_{T^{(s)} \cap T^{(m)}} a_{sm}^t.[x_t + \Delta x_t] + \sum_M \sum_R \sum_{T^{(m)} \cap T^{(r)}} a_{mr}^t.x_t + \sum_N \sum_{S^{(n)}} \sum_R \sum_{T^{(s)} \cap T^{(r)}} a_{sr}^t.x_t^{'(n)} \quad (5\text{-}36)$$

***Subject To*:**

$$\sum_{\forall t \in s1 | r \in t} x_t = \overline{D}_r\left(sl_p, sl_a, w, p\right).G_r^{-1}\left(MAX\left\{rl_r^{(s2)},\frac{h_r^-}{h_r^+ + h_r^-}\right\}\right) \quad (\forall r \in R) \quad (5\text{-}37)$$

$$\Delta x_t = G_{m|m \in t,t}^{'-1}\left(x_t, rl_m^{(s2)}\right) \quad (\forall t \in s1) \quad (5\text{-}38)$$

$$x_t^{'(n)} = x_t.AD_n(w,\theta_n,\lambda_n) + z'_{rl_r^{(s2)}}.\sqrt{x_t.VD_n(w,\theta_n,\lambda_n)} \quad (\forall t \in s1, \forall n \in N) \quad (5\text{-}39)$$

$$\Delta \acute{x}_t^{(s|s \in t, s \in S^{(n)})} = G_{s,t}^{''\,-1}\left(x_t + \Delta x_t + x_t^{'(n)}, rl_s^{(s2)}\right) \quad (\forall t \in s1, \forall s \in t) \quad (5\text{-}40)$$

$$x_t, \Delta x_t, \Delta \acute{x}_t^{(s)}, x_t^{'(n)} \geq 0 \quad (\forall t \in s1, \forall s \in \{\forall s \in S | s \in s1\}, \forall n \in N) \quad (5\text{-}41)$$

Step 4: Compute the minimum possible cost of each $sl = (sl_p, sl_a) \in SL$ and $w \in W$ as follows:

$$MCost(w,sl) = \underset{\forall s1 \in S1}{\text{MIN}} \; \underset{\forall s2 \in S2^{(s1)}}{\text{MIN}} \; Cost^{(s1,s2)}(w,sl) \quad (5\text{-}42)$$

The best path selection, flow assignment, and local reliability assignment corresponding to $MCost(w, sl)$ are represented by $Y^*(w, sl)$, $X^*(w, sl)$ and $RL^*(w, sl)$ respectively.

Step 5: After computing $MCost(w, sl)$ for each $\forall sl = (sl_p, sl_a) \in SL$ and $\forall w \in W$, use the following linear binary model to find the best warranty and service level strategies (w^*, sl^*):

$$MAX \quad \sum_R \sum_W \sum_{SL} v_w . z_{sl} . \overline{D}_r(sl_p, sl_a, w, p).p - \sum_W \sum_{SL} v_w . z_{sl} . MCost(w, sl) \quad (5\text{-}43)$$

Subject to:

$$\sum_W v_w = 1 \quad (5\text{-}44)$$

$$\sum_{SL} z_{sl} = 1 \quad (5\text{-}45)$$

$$v_w, z_{sl} \in \{0.1\} \quad (5\text{-}46)$$

By solving this model, the best service level, sl^*, and warranty, w^*, strategies are determined. Therefore, the best path selection, flow assignment, and local reliability assignment of the networks are $Y^*(w^*, sl^*)$, $X^*(w^*, sl^*)$, and $RL^*(w^*, sl^*)$. The flowchart for this algorithm is shown in Figure 5-4.

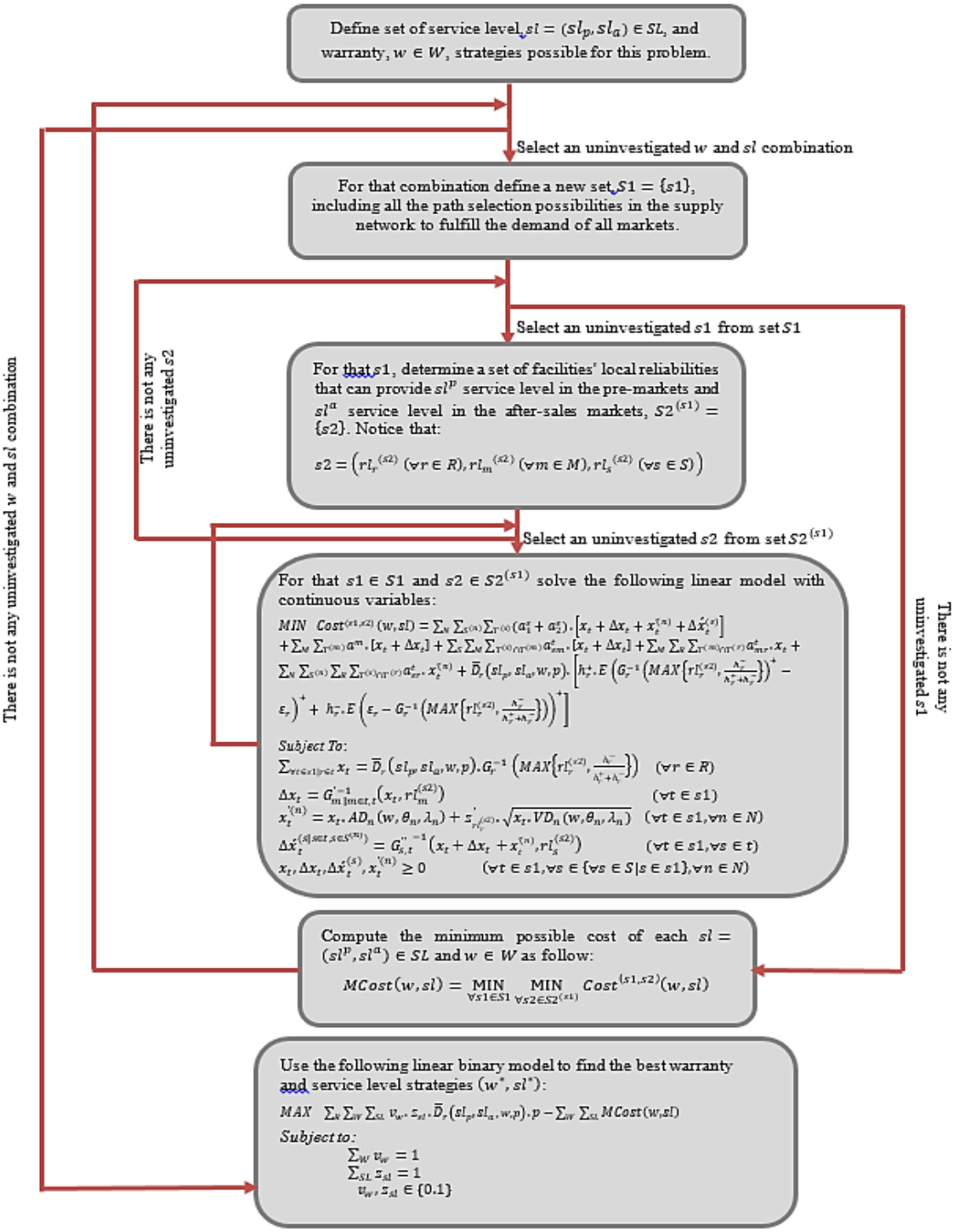


Figure 5-4: Flowchart of solution algorithm.

5.4. Numerical analysis

5.4.1. Case study problem: Engine industry

This problem is developed for a company manufacturing different kinds of engines. Tracking the quality of the products due to their long and complicated manufacturing process is not easy. However this company, to preserve its reputation, tries to satisfy its customers as much as possible by providing after-sales services. Therefore providing a suitable warranty is critical. Recently due to high rates of after-sales costs, this company decided to revise its after-sales services. By analyzing the data about the sales and return rates of the previous sales periods, the company wants to make scientific decisions about its marketing strategies such as retail price, warranty length, and service levels. In this section, we concentrate on one of the important engine groups of this company which has a greater share of production compared to the others.

This engine group has two critical components provided by external suppliers, *n1* and *n2* ($N = \{n1, n2\}$). This company has two supplier options for procuring *n1* and for providing *n2*, only one supplier exists which means $S = S^{(n1)} \cup S^{(n2)}, S^{(n1)} = \{s1, s3\}$, and $S^{(n2)} = \{s2\}$. Only two manufacturing centers of this company are capable to assemble this engine group, $M = \{m1, m2\}$, then they are supplied to two important markets by their corresponding retailers, $R = \{r1, r2\}$. The structure of the forward SN and its potential paths, $T = \{t_{1,2,1,1}, t_{1,2,1,2}, t_{3,2,2,1}, t_{3,2,2,2}\}$, are shown in Figure 5-5. $t_{s,ś,m,r}$ is the path starting from Suppliers *s* and *s'* (providing *n1* and *n2* respectively), passing through Manufacturer *m* and ending at Retailer *r*.

Analyzing the quadruples of (*price*, *service levels*, *warranty*, *average demand*) in the previous sales periods by regression shows that the following functions fit well with the historical demand data of this engine group. Assessing the differences between the actual realized demands and their average values by "Goodness-of-fit" tests shows that the stochastic deviations fit with normal density functions with 90 percent confidence limit.

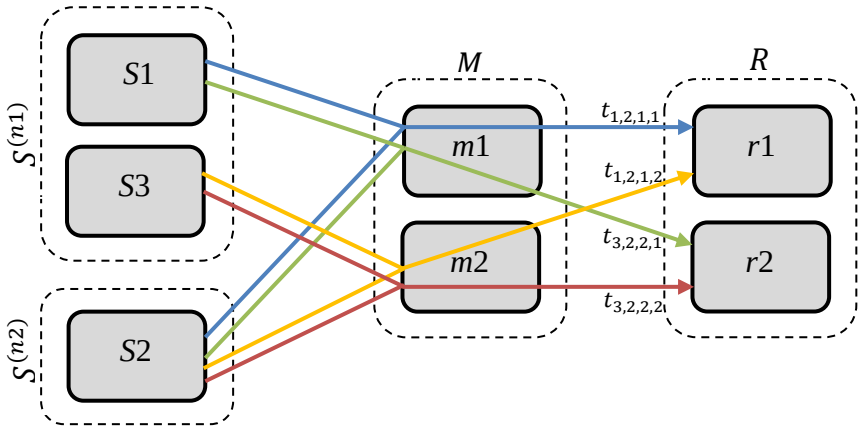


Figure 5-5: Potential supply paths in the forward SN of the engine problem.

$$D_1\big(p, sl = (sl_p, sl_a), w\big) = \big(500 + 200.w - 250.(p - 10) - 500.(1 - sl_a) - 900.(1 - sl_p)\big).\varepsilon_1 \quad (5\text{-}47)$$

$$\varepsilon_1 \sim Normal\big(\mu_{\varepsilon_1} = 0, \sigma^2_{\varepsilon_1} = 0.1\big) \quad (5\text{-}48)$$

$$D_2\big(p, sl = (sl_p, sl_a), w\big) = \big(400 + 200.w - 250.(p - 10) - 500.(1 - sl_a) - 900.(1 - sl_p)\big).\varepsilon_2 \quad (5\text{-}49)$$

$$\varepsilon_2 \sim Normal\big(\mu_{\varepsilon_2} = 0, \sigma^2_{\varepsilon_2} = 0.1\big) \quad (5\text{-}50)$$

In the company three-year data record is available for the claims have been made by the customers for this engine group. To figure out the failure rates of the engine, we used the statistical analysis approach proposed by Lawless (1998). By using his method we

calculate the mean and variance of the failure rate in different ages of the product. The mean and three sigma confidence limits of failure rates for components of this product are displayed in Figures 5-6 and 5-7.

The deterioration time in $S1$, $S2$, and $S3$ has exponential distribution with $\mu_1 = 2$, $\mu_2 = 2$, and $\mu_3 = 3$. The non-conforming production rate in the out-of-control state of their machines is $\gamma_1 = 10\%$, $\gamma_2 = 20\%$, and $\gamma_3 = 5\%$. Production rates of the first, second, and third suppliers are 8000, 8000 and 9000 component units per time unit. The cost components of this problem are summarized in Table 5-2.

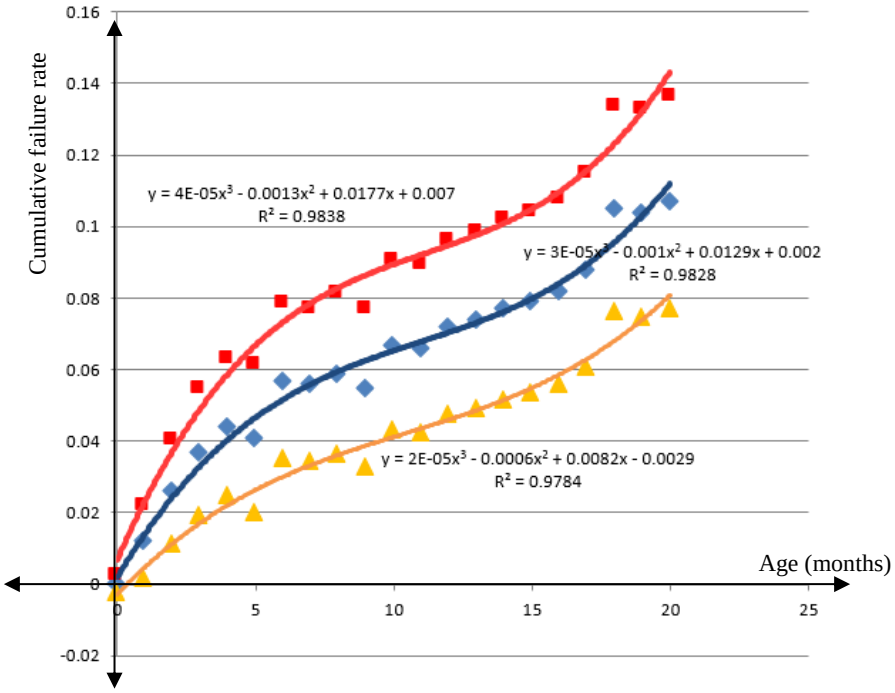


Figure 5-6: Failure rate of the first component with respect to age.

Table 5-2: Cost components of the engine problem.

Cost parameter	*Value*	*Cost parameter*	*Value*	*Cost parameter*	*Value*
$a_1^{s=1}$	\$ 0.50	$a_{s=1,m=1}^{t}$	\$ 0.05	$a_{s=1,r=1}^{t}$	\$ 0.07
$a_2^{s=1}$	\$ 0.60	$a_{s=2,m=1}^{t}$	\$ 0.08	$a_{s=1,r=2}^{t}$	\$ 0.07
$a_1^{s=2}$	\$ 0.60	$a_{s=2,m=2}^{t}$	\$ 0.08	$a_{s=2,r=1}^{t}$	\$ 0.07
$a_2^{s=2}$	\$ 0.70	$a_{s=3,m=2}^{t}$	\$ 0.06	$a_{s=2,r=2}^{t}$	\$ 0.07
$a_1^{s=3}$	\$ 0.55	$a_{m=1,r=1}^{t}$	\$ 0.05	$a_{s=3,r=1}^{t}$	\$ 0.07
$a_2^{s=3}$	\$ 0.70	$a_{m=1,r=2}^{t}$	\$ 0.04	$a_{s=3,r=2}^{t}$	\$ 0.07
$a^{m=1}$	\$ 2.00	$a_{m=2,r=1}^{t}$	\$ 0.05	$h_{r=1\ and\ 2}^{+}$	\$ 0.11
$a^{m=2}$	\$ 2.15	$a_{m=2,r=2}^{t}$	\$ 0.05	$h_{r=1\ and\ 2}^{-}$	\$ 0.05

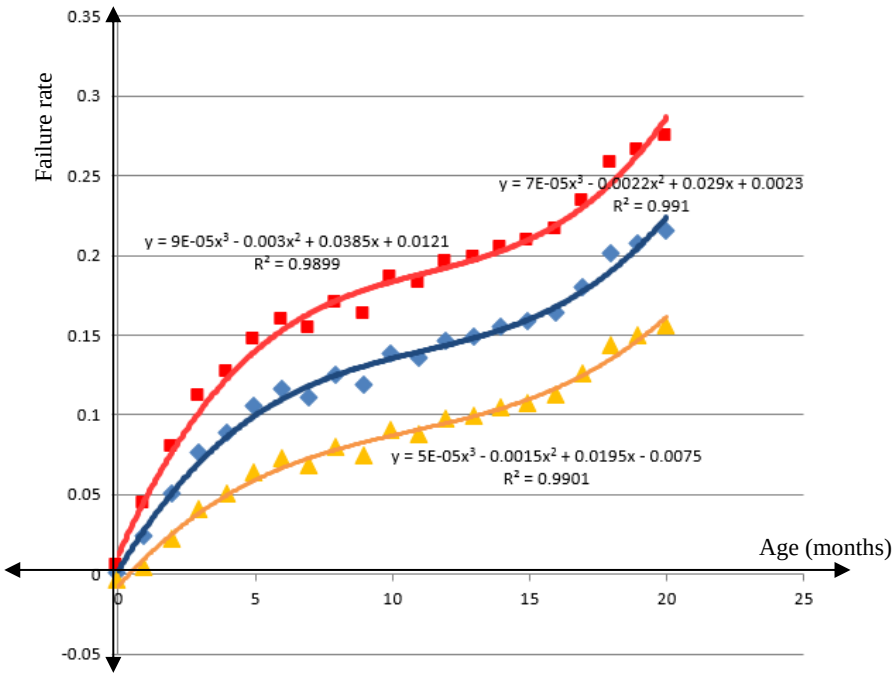


Figure 5-7: Failure rate of the second component with respect to age.

The manufacturers have imperfect production systems. Defective production rate in the first and second manufacturer has a uniform distribution with (0, $\beta_{m=1}$=0.15) and (0, $\beta_{m=2}$=0.08). First, we assume the product price in the markets is fixed at its current value, $p = \$\ 10$. In the next section by analyzing the sensitivity of the results with respect to the

product price, the best price strategy is determined. In this problem, we assume the available warranty options are $W = \{w_1 = 0.5\ (year), 1.0\ (year), 1.5\ (years), 2\ (years)\}$. The available service level options are $SL = \{sl_1 = (sl_{1p} = 0.98, sl_{1a} = 0.96), sl_2 = (sl_{2p} = 0.90, sl_{2a} = 0.95), sl_3 = (sl_{3p} = 0.85, sl_{3a} = 0.91)\}$.

Solving the mathematical model yields the following results: the most profitable service level and warranty strategies are $sl^* = (sl_p^* = 0.85, sl_a^* = 0.91)$ and $w^* = 1.0$ (*year*). The least costly reliabilities of the facilities preserving this service level strategy are $rl_{s=1} = 1.00$, $rl_{s=2} = 1.00$, $rl_{s=3} = 0.94$, $rl_{m=1} = 0.99$, $rl_{m=2} = 0.93$, $rl_{r=1} = 0.99$ and $rl_{r=2} = 0.99$. The best flow through the paths of the forward SN are $x_1 = 49.94$, $x_2 = 40.64$, $x_3 = 949.04$, and $x_4 = 772.24$ (Figure 5-8). The best flow through the paths of the after-sales SN are $\acute{x}_1^{(1)} = 6.17$, $\acute{x}_1^{(2)} = 8.31$, $\acute{x}_2^{(1)} = 5.34$, $\acute{x}_2^{(2)} = 7.18$, $\acute{x}_3^{(1)} = 63.47$, $\acute{x}_3^{(2)} = 87.70$, $\acute{x}_4^{(1)} = 53.05$, and $\acute{x}_4^{(2)} = 73.21$ (Figure 5-9). Flow amplification in these networks' facilities are $\Delta x_1 = 7.41$, $\Delta x_2 = 6.03$, $\Delta x_3 = 70.61$, $\Delta x_4 = 57.45$, $\Delta\acute{x}_1^{(1)} = 7.06$, $\Delta\acute{x}_1^{(2)} = 16.41$, $\Delta\acute{x}_2^{(1)} = 5.78$, $\Delta\acute{x}_2^{(2)} = 13.46$, $\Delta\acute{x}_3^{(3)} = 47.23$, $\Delta\acute{x}_3^{(2)} = 276.83$, $\Delta\acute{x}_4^{(3)} = 36.69$, and $\Delta\acute{x}_4^{(2)} = 225.72$.

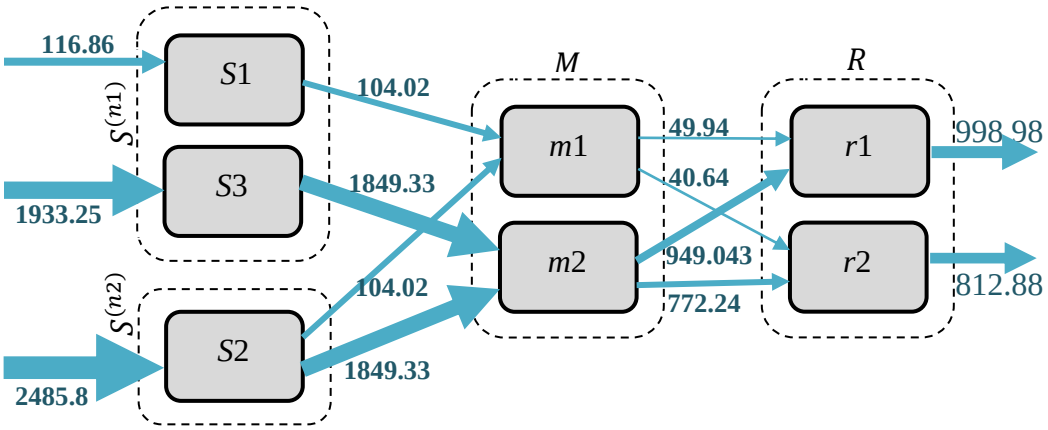


Figure 5-8: Flow through the forward SN.

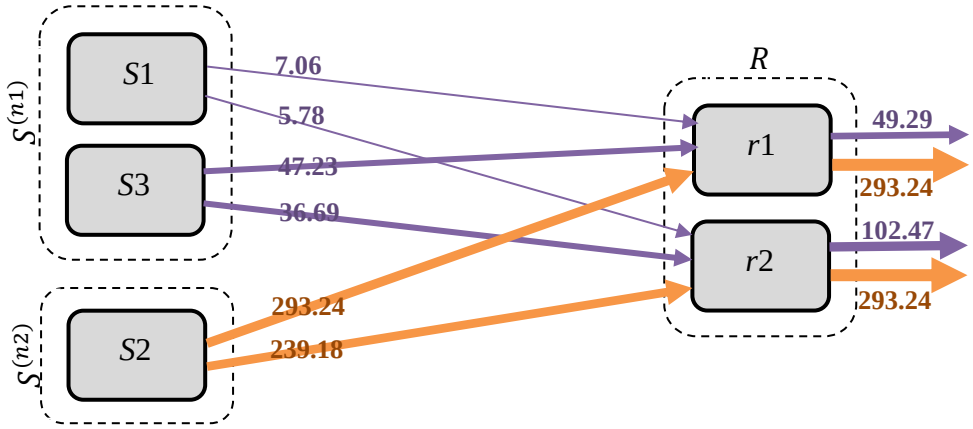


Figure 5-9: Flow through the after-sales SN.

5.4.2. Optimal price strategy determination

In the previous analysis, we assumed the product price in the markets is fixed at $p = \$\ 10$. In this section, by checking the sensitivity of the model with respect to the price, we determine the best price strategy for the company.

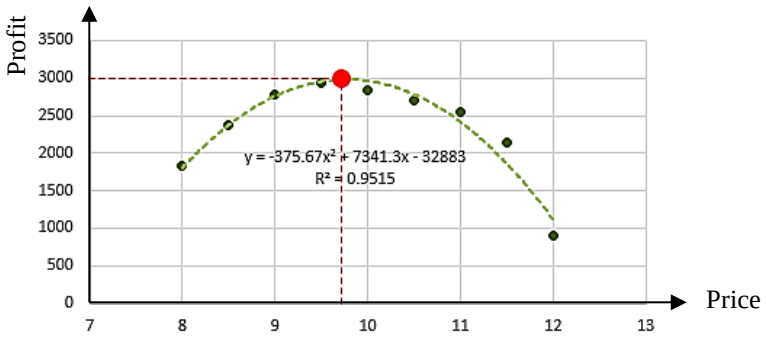


Figure 5-10: Profit with respect to price.

Based on the product's manufacturing cost and rival product prices in the markets, we assume the price should be selected from [$8, $12] range. For some sample price values from this range, we solve the mathematical model of the problem and get the results. Dark green points in Figure 5-10 represent the highest profit for these sample price values. Based on the results, a two order polynomial function fits very well with these points. To find the best price, we find the maximum point of this fitted function which gives $p^* = \$9.77$.

5.4.3. Correlation between price and warranty strategies

In this section, we analyze the correlation between the best warranty and the best price strategies in different service level options. For this purpose, for each combination of the service level and warranty options, the mathematical model is solved for some sample values in the feasible price range [$8, $12]. The resulting profit points and their fitted function are displayed for the third service level option ($sl_{3p} = 0.85, sl_{3a} = 0.91$) in Figure 5-11. The intersections of these functions show the critical price values in which the priority of the warranty options changes. Based on these results, the priority of the warranty options with respect to the price values is as follows:

- If $p \leq \$8.90$ Then the priority of warranty options is $0.5, 1.0, 1.5$ *and* 2.0.
- If $\$8.90 < p \leq \$10.10$ Then the priority of warranty options is $1.0, 0.5, 1.5$ *and* 2.0.
- If $\$10.10 < p \leq \$10.50$ Then the priority of warranty options is $1.0, 1.5, 0.5$ *and* 2.0.
- If $\$10.50 < p \leq \$10.80$ Then the priority of warranty options is $1.5, 1.0, 0.5$ *and* 2.0.
- If $\$10.80 < p \leq \$11.10$ Then the priority of warranty options is $1.5, 1.0, 2.0$ *and* 0.5.
- If $\$11.10 < p \leq \$11.45$ Then the priority of warranty options is $1.5, 2.0, 1.0$ *and* 0.5.
- If $\$11.45 < p$ Then the priority of warranty options is $2.0, 1.5, 1.0$ *and* 0.5.

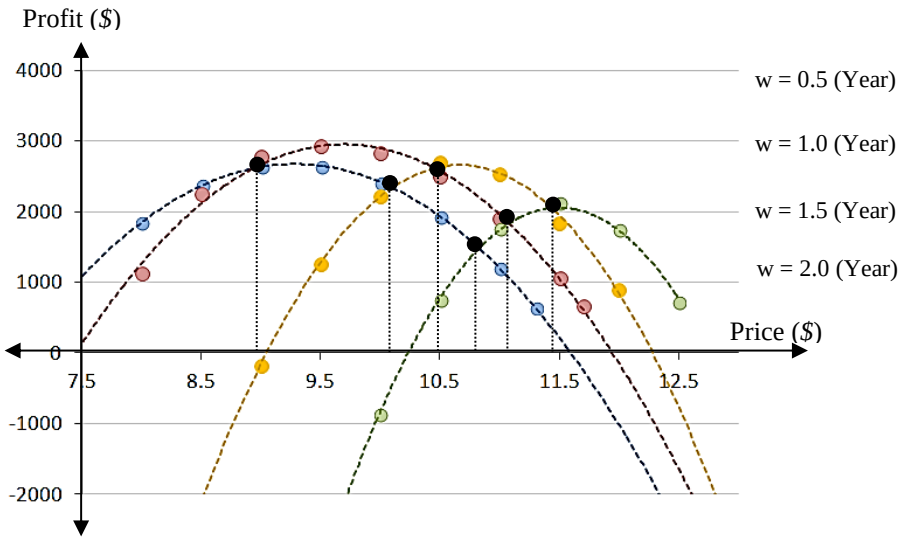


Figure 5-11: Profit variation with respect to the warranty and price in the third service level strategy.

According to Figure 5-11, *price increment imposes almost the same trend of changes on the profit function of the all warranty options*. Increasing price first improves the profitability of the company in each warranty option. But after the optimal price of that warranty option, the profit reduces by price increment. This means *changing the warranty length does not significantly affect the trend of changes in the profit function with respect to the price*. However, the profit function shifts to the right by increasing the warranty length. Therefore, *in the price intervals between two sequential critical price values, the effect of the price increment on the profit functions of different warranty lengths may be different*. For example in price interval [9.00, 10.10], while the profit function of 1.5 (year) warranty option increases by the price increment, the profit function of 0.5 (year) warranty option decreases, and the profit function of 1.0 (year) warranty increase at first and decrease after a while.

Results of solving the mathematical model for different combinations of warranty and price options at the second service level option, ($sl_{2p} = 0.90, sl_{2a} = 0.95$), and at the first service level option, ($sl_{1p} = 0.98, sl_{1a} = 0.96$), are represented respectively in Figures 5-12 and 5-13. The critical price values in the second service level option are as follows:

- If $p \leq \$10.50$ Then $w = 0.5, 1.0, 1.5\ and\ 2.0$
- If $\$10.50 < p \leq \$11.15$ Then $w = 1.0, 0.5, 1.5\ and\ 2.0$
- If $\$11.15 < p \leq \$11.60$ Then $w = 1.0, 1.5, 0.5\ and\ 2.0$
- If $\$11.60 < p \leq \$11.70$ Then $w = 1.5, 1.0, 0.5\ and\ 2.0$
- If $\$11.70 < p \leq \$12.00$ Then $w = 1.5, 1.0, 2.0\ and\ 0.5$
- If $\$12.00 < p \leq \$12.25$ Then $w = 1.5, 2.0, 1.0\ and\ 0.5$

- If $\$12.25 < p$ Then $w = 2.0, 1.5, 1.0\ and\ 0.5$

The critical price values in the first service level option are as follows:

- If $p \leq \$11.83$ Then $w = 0.5, 1.0, 1.5\ and\ 2.0$
- If $\$11.83 < p \leq \$12.20$ Then $w = 1.0, 0.5, 1.5\ and\ 2.0$
- If $\$12.20 < p \leq \$12.41$ Then $w = 1.0, 1.5, 0.5\ and\ 2.0$
- If $\$12.41 < p \leq \$12.55$ Then $w = 1.5, 1.0, 0.5\ and\ 2.0$
- If $\$12.55 < p \leq \$12.75$ Then $w = 1.5, 1.0, 2.0\ and\ 0.5$
- If $\$12.74 < p \leq \$12.85$ Then $w = 1.5, 2.0, 1.0\ and\ 0.5$
- If $\$12.85 < p$ Then $w = 2.0, 1.5, 1.0\ and\ 0.5$

Comparison of the critical price values in these three service level options reveals that by increasing the service levels the intervals between the sequential critical price values do mainly decrease. This means *the correlation between the price and warranty becomes tighter by increasing the service levels. Therefore in higher service levels, the priority of the warranty options stays stable for a smaller price interval and is more sensitive with respect to the price variations*.

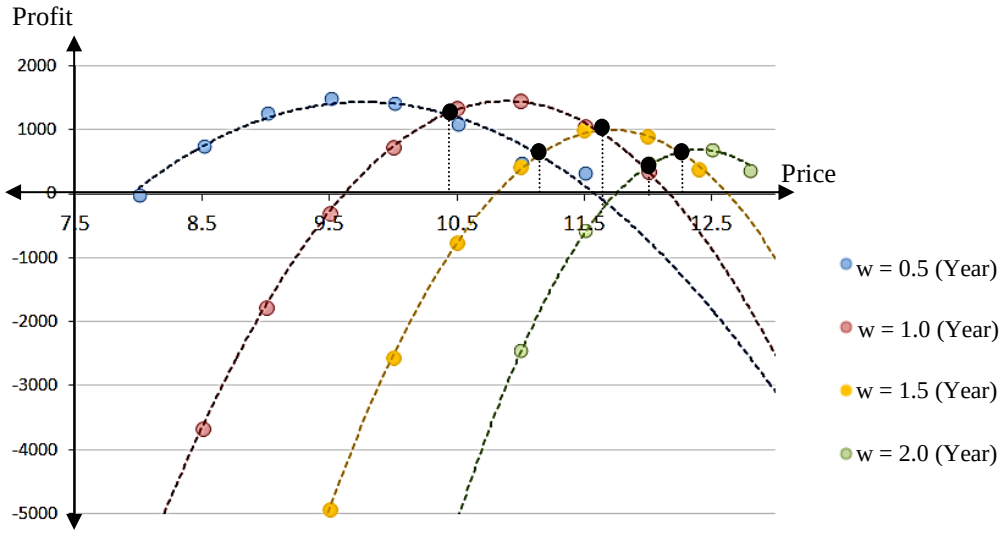


Figure 5-12: Profit variation with respect to the warranty and price in the second service level strategy.

Comparison of the profit functions in Figures 5-11, 5-12, and 5-13 shows that by increasing the service levels the overlaps among the profit functions decrease and they become more separate. The profit function of each warranty option has a connected price interval inside which the profit of that warranty is positive. By increasing the service levels, these intervals of the warranty options become more distinct. This means *the feasible range of price is divided to some more distinct intervals in each only one warranty option is profitable. Therefore, in higher service levels the positively profitable warranty options available in each price value for managers to select is much less.*

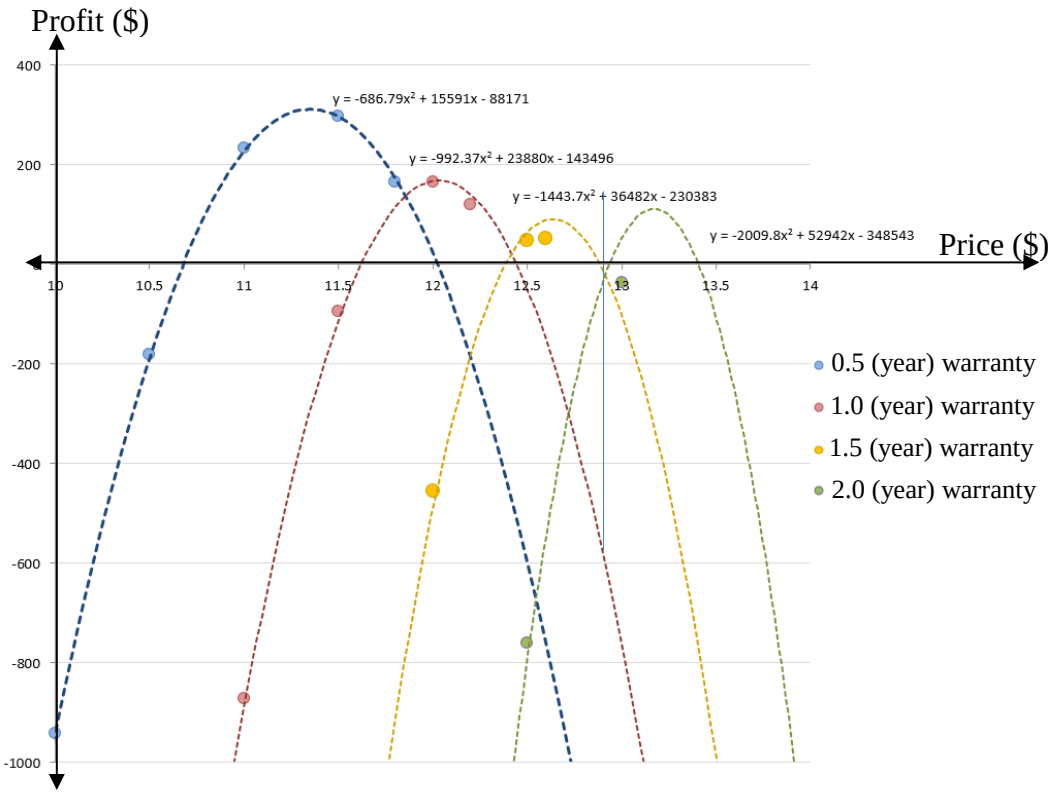


Figure 5-13: Profit variation with respect to the warranty and price in the first service level strategy.

5.4.4. Correlation between service level and warranty strategies

In this section, we analyze the relationship between the warranty length and service level in a fixed price, $p = 10$. Results of solving the mathematical model for different combinations of the warranty and service level options at $p = 10$ are represented respectively in Figure 5-14. There is no intersection among the profit functions of different service level options. This means that the priority of service level options is not changing with respect to the warranty variations. The highest profit always corresponds to the third, lowest, service level option.

Based on these results we conclude that for a given price, the priority of the service level options does not significantly change with warranty length variation and in our test

problem, always the third service level option is the best. In the other words: *the priority of the service level options is very stable and is not affected easily by warranty variations. In this problem the warranty-service level tradeoff is much more stable than the price-warranty tradeoff. However the stability of the warranty-service level tradeoff may change by increasing the service level sensitivity parameter in the demand function*.

We summarize the outcomes of these analyses in Figure 5-15. In this figure, we show the relationships between two marketing strategies in a given option of the third one. For example *in a given warranty length option, the best price strategy is increasing with respect to the service level but the trend of this increment is different for warranty options. In shorter warranty lengths, the rate of price increment is a convex increasing function of the service levels. But this function tends to become a linear increasing and then a concave increasing by the warranty length increment*.

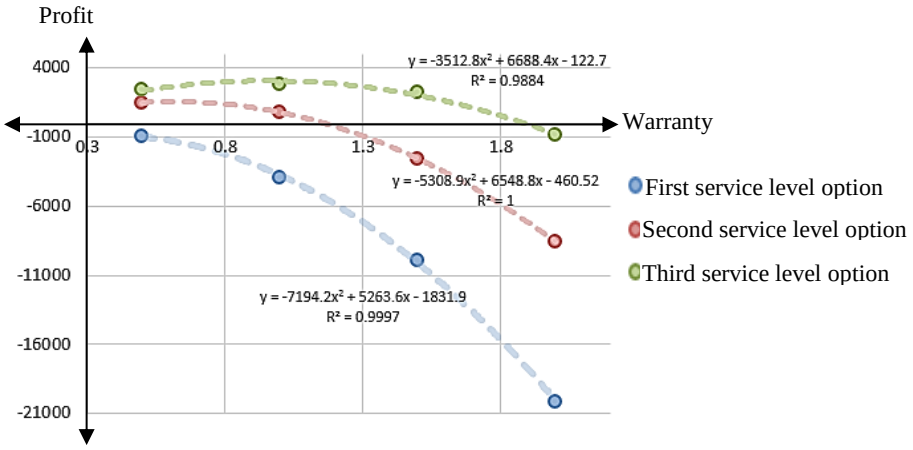


Figure 5-14: Warranty and service level correlation in $p = 10$ price strategy.

In the same way *for a given service levels option, the best price strategy is increasing with respect to the warranty length but the trend of this increment is different for service*

level options. In lower service levels, the rate of price increment is a convex increasing function of the warranty length. But this function tends to become a linearly increasing and then a concave increasing by the increment in the service levels.

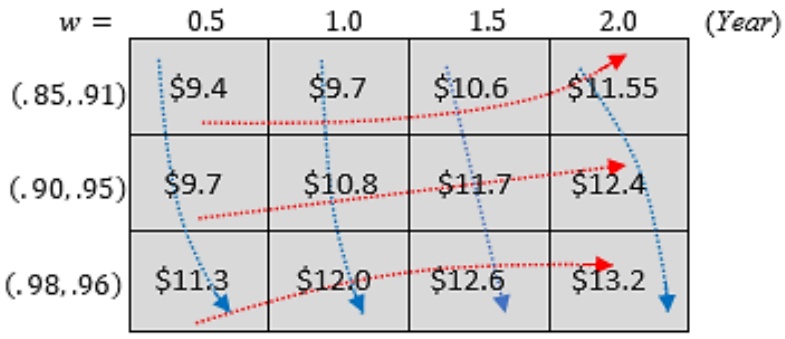


Figure 5-15: Variations of the three marketing strategies: price, service level and warranty length.

5.5. Closure of Chapter 5

In this chapter, we consider a company with forward and after-sales SNs. A SN is considered as a SC with more than one facility in each echelon. Therefore, in this chapter we extend the model and solution approach developed in Chapter 3 for a company with forward and after-sales SCs to a company with forward and after-sales SNs. Thus, again we should answer the research questions of Chapter 3, but this time for SNs:

- ✓ **Research Question 1:** *what are the important flow transitions among the facilities supporting after-sales services?*
- ✓ **Research Question 2:** *what are the important interactions between forward and after-sales SNs (SCs with more than one facility in each echelon) justifying the necessity of their concurrent flow planning?*

- ✓ **Research Question 3:** *how do these interactions affect planning flow dynamics in the forward and after-sales SNs (SCs with more than one facility in each echelon) of non-repairable goods?*

We answered these questions as follows:

- ✓ **Answer of Research Question 1:** Flow transactions among the facilities of the after-sales SN are qualitatively explained in Section 5.1. In Section 5.2, we introduce "Path" concept in SNs and propose a "Path-based" approach for flow planning through SNs. This approach helps us to readily extend the SC model developed in Chapter 3 to SNs with any network structures. In Sections 5.2.1 and 5.2.2, we quantify flow transaction through the paths of the forward and after-sales SNs.
- ✓ **Answer of Research Question 2:** Interactions between the forward and after-sales SNs' operations are explained in Section 5.1 and modeled in Sections 5.2.1 and 5.2.2.
- ✓ **Answer of Research Question 3:** The interactions between the forward and after-sales SNs are considered in Section 5.2.3 to integrate equations derived in former sections and develop an integrated mathematical model for concurrent flow planning in the forward and after-sales SNs.

The developed model and its solution approach are tested on an example problem from engine industry. The results are used to investigate the correlations among the marketing factors – price, service levels, and warranty – which leads to the following insights:

Correlation between the price and warranty

- ✓ The correlation between the price and warranty becomes tighter by increasing the service levels. In higher service levels, the priority of the warranty options stays stable for a smaller price interval and is more sensitive with respect to price variations.
- ✓ By increasing the service levels, the overlaps among the profit functions decrease and they become more separate. This means the feasible range of price is divided to some more distinct intervals in each only one warranty option is profitable. Therefore, in higher service levels the positively profitable warranty options available in each price value for managers to select is much less.

Correlation between the service levels and warranty

The priority of the service level options is very stable and is not affected easily by warranty variations. In the engine problem, the warranty-service level tradeoff is much more stable than the price-warranty tradeoff. However the stability of the warranty-service level tradeoff may change by increasing the service level sensitivity parameter in the demand function.

Correlation among the three marketing factors

In a given warranty length option, the best price strategy is increasing with respect to the service level but the trend of this increment is different for warranty options. In shorter warranty lengths, the rate of price increment is a convex increasing function of the service levels. But this function tends to become a linearly increasing and then a concave increasing by the warranty length increment.

In the same way for a given service levels option, the best price strategy is increasing with respect to the warranty length but the trend of this increment is different for service level options. In lower service levels, the rate of price increment is a convex increasing function of the warranty length. But this function tends to become a linearly increasing and then a concave increasing by the increment in the service levels.

In this problem, we assume that the spare parts required for the after-sales operations are new and directly supplied by the suppliers. However, another option is remanufacturing the defective components which are mainly new. Including the remanufacturing option in the after-sales SN is an important future research for this chapter.

Chapter 6: Operationally and Structurally Fail-safe Supply Networks

Uncertainties affecting the performance of SNs can be categorized into two main groups: 1) Disruptions; and 2) Variations. By disruptions, we mean large-scale stochastic events happen rarely but they are large enough to change the topology of SNs by inactivating a subset of their nodes, production and distribution facilities, or links, transportation possibilities between facilities. By variations, we mean small-scale stochastic events happen frequently but only affect and decrease efficiency of flow dynamics in SNs.

To have a fail-safe SN, it should be:

- ✓ **Structurally fail-safe against disruptions**: This means the topology of the SN should be designed / redesigned in a way to be safe – Robust and Resilient – against possible disruptions.
- ✓ **Operationally fail-safe against variations**: This means the flow dynamics of the SN should be planned in a way to be safe – Reliable – against possible variations.

We explain the requirements of these two characteristics, "Operationally Fail-safe" and "Structurally Fail-safe", in Section 6.1 to answer the following research question:

- ✓ **Research Question 5: what are the necessities of having fail-safe SNs?**

In Section 6.2, we explain how disruptions affect the performance and flow dynamics in SNs. First in Section 6.2.1, we explain the flow dynamics planning in a SN without any disruptions – normal condition. Then in Section 6.2.2, we explain the flow dynamics planning in the SN under disrupted conditions to answer the following question:

- ✓ **Research Question 6:** what are the characteristics of fail-safe SNs against disruptions – characteristics of structurally fail-safe SNs?

In this chapter, we show that to have a "structurally fail-safe SN", its topology should be redesigned to incorporate appropriate amount of risk mitigation strategies. These risk mitigation strategies reduce the vulnerability of the SN against disruptions which is measured by its "robustness" index. Also the SN should be agile enough in employing the risk mitigation strategies to reduce the SN's loss in the transient period from the normal to the disrupted flow plan. This SN agility is measured by a "resilience" index. By considering "flexibility" as the only risk mitigation strategy, we develop a mathematical model to find the best robustness and resilience for the "structurally fail-safe" SN and analyze the correlations between robustness, resilience, and reliability (reliability already investigated and quantified in the previous chapters).

Definitions

Flexible facility: A flexible facility is able to increase or decrease its processing capacity as needed. Flexibility Level is an indicator of how much the throughput can be increased or decreased when extra capacity is needed or when there is unused capacity. The Flexibility Speed is an indicator of how fast the facility is able to increase or decrease its capacity.

Robust supply network: A robust supply network is one that is made relatively insensitive to disruptions by triggering mitigation strategies thereby making it possible to continue delivering the level of service as before in disruptions. Robustness of a supply network is a function of its including facilities flexibility levels.

Resilient supply network: Resilience of a supply network embodies the speed (and therefore cost) with which the network employing the mitigating strategies after a disruption. Resilience of a supply network is a function of its including facilities flexibility speeds.

Reliable flow planning in a supply network: In the presence of the required facilities, an operationally reliable flow planning permits the coordination of flow among the supply network's facilities to assure an appropriate service level when coordination-disturbing uncertainties are present.

6.1. Disruptions in supply networks

SNs are undeniable parts of competitive and globalized markets. Companies improve their competition advantages through decentralization and working as a member of a SN leading to lower production cost, higher product quality, and higher responsiveness with respect to the rapidly changing needs and expectations of the customers (Chopra and Sodhi, 2004). On the other hand, because they are distributed, they are more vulnerable against uncertainties in business and working environments (Schmitt and Snyder, 2010; Peng et al., 2011; Baghalian et al., 2013). Hence, risk management is critical for successful SNs. There are many examples of risks in SNs.

According to Sarkar et al. (2002), during the labor strike in 2002, 29 ports on the west coast of the United States were shut down which led to the closure of the new United Motor manufacturing production factory. During the destructive earthquake in Japan in 2011, the Toyota Motor Company had to cease production in twelve assembly plants

which led to a production loss of 140,000 automobiles. The main cause for the loss is attributed to the disruption of its SN's manufacturing subsystems. In addition to the impairment of production facilities and factories throughout Japan, many Japanese companies had a problem with the supply of required material, fuel and power. In these types of catastrophes, supply and manufacturing disruptions are huge problems for companies. As mentioned by Norrmann and Jansson (2004), a fire in one of the major suppliers of the Ericsson Company created serious problems for this company and shut down its manufacturing plants for several days. Dole suffered revenue declines after their banana plantations were destroyed by Hurricane Mitch in 1998; Ford was forced to close five plants for several days after terrorist attacks on September 11 suspended air traffic in 2001; The 1999 earthquake in Taiwan displaced power lines to the semiconductor fabrication facilities responsible for more than 50 percent of worldwide supplies of memory chips, circuit boards, flat-panel displays and other computer components. Many hardware manufacturers including HP, Dell, Apple, IBM, Gateway and Compaq suffered. A Motorola cell phone factory in Singapore closed after an employee came down with SARS. For more details, see Joseph and Subbakrishna (2002) and Monahan et al. (2003). In another instance, Ericsson lost 400 million Euros after their supplier's semiconductor plant caught on fire in 2000; Apple was unable to fulfill many orders during a supply shortage of DRAM chips after an earthquake hit Taiwan in 1999; the 2002 longshoreman union strike at a U.S West Coast port, for example, interrupted transshipments and deliveries to many U.S.-based firms, with port operations and schedules not returning to normal until 6 months after the strike had ended. For more details, see Cavinato (2004). Hendricks and Singhal (2005) quantify negative effects of uncertainties through empirical

analysis as follows: 33 to 40% lower stock returns; 107% drop in operating income, 7% lower sales growth and 11% growth in cost. Clearly, there are numerous sources of risk in a SN and we suggest that current methods are ill-equipped to handle them.

Uncertainties in SNs are classified in different ways. Chopra and Sodhi (2004) categorize potential SN risks into nine categories: a) disruptions (e.g., natural disaster, terrorism, war, etc.), b) delays (e.g., inflexibility of the supply source), c) systems (e.g., information infrastructure breakdown), d) forecast (e.g., inaccurate forecasting, the bullwhip effect, etc.), d) intellectual property (e.g., vertical integration), e) procurement (e.g., exchange rate risk), f) receivables (e.g., number of customers), g) inventory (e.g., inventory holding costs, demand and supply uncertainties, etc.), h) capacity (e.g., cost of capacity). Waters (2007) divides SN risk sources to ***internal risks*** (can be controlled) and ***external risks*** (cannot be controlled). Internal risks appear in normal operations, such as late deliveries, excess stock, poor forecast, human error, faults in IT systems, etc. External risks come from outside a supply network, such as earthquakes, hurricanes, industrial actions, wars, terrorist attacks, price rises, problems with trading partners, shortage of row materials and crime. Moreover, Waters (2007) introduces another three-category of risk sources: a) ***environmental risk sources*** which involve any uncertainties arising from the environment interaction of the SN. These may be the result of accidents (e.g., fires), socio-political actions (e.g., fuel protests or terrorist attacks) or acts of God (e.g., extreme weather or earthquakes), b) ***organizational risk sources*** lie within the boundaries of the SN's facilities and range from labor (e.g., strikes) or production uncertainties (e.g., machine failure) to IT-system uncertainties, and c) ***network-related risk sources*** arise

from interactions among organizations within a SN. Kar (2010) believes risks of a SN can also be categorized into two groups: a) ***systematic risks*** arising from unavoidable environmental factors. Companies do not have any control over factors such as demand-side uncertainty; supply-side disruption; regulatory, legal, and bureaucratic changes; catastrophic events, and infrastructure disruption. b) ***non-systematic risks*** dealing with factors that can be controlled to a large extent by a company such as facility disruptions in manufacturing subsystems. The preceding classification schemes are not adequate for grounding a theory for designing fail-safe SNs. Therefore, we introduce and use another classification that is appropriate for the design of fail-safe SNs.

In this classification, risks are categorized into two groups based on the nature of SNs' decisions affected by them:

- ***Disruptions in a SN***: Disruptions refer to rare and unexpected events with extensive effects which mainly impact the topology of a SN. A SN's topology is determined by strategic level network design decisions. Network design decisions deal with determining the number, location and capacity of facilities in the SN's echelons. Supply-side disruptions are related to events that make some of the facilities or connecting links of a SN completely or partially inoperative. We summarize some of the most recent work that has been done in this domain: Tomlin (2006) investigates the unavailability of a supplier in a two echelon SN including a manufacturer and two suppliers. Chopra et al. (2007) analyze the appropriate selection of mitigation strategies in a two echelon SN including a buyer who can be serviced by two suppliers. One is reliable and the other is unreliable but cheaper. Qi et al. (2009) consider inventory holding

problem in a SN with a single retailer. The orders of the retailer are fulfilled with a single supplier who is prone to disruption. Peng et al. (2011) develop a model to design a SN topology that perform well under normal condition and perform relatively well when disruption strikes unreliable facilities. Baghalian et al. (2013) propose a path-based approach to design a robust SN topology under disruption possibility in the facilities and connecting links. The main strategy in these papers is reallocation the production activities among the facilities under disruptions. But the flexibility that is required in the production capacity of the facilities to handle this reallocation is ignored.

Demand-side disruptions are related to sudden and significant shifts, increases or decreases, in the average demand of markets due to the unavailability of an existing rival or the entrance of a new rival into the market. For example, Chen and Xiao (2009) develop two models to coordinate a SN after demand disruption. The SN consists of a manufacturer and several retailers. Hsu and Li (2011) study the problem of production reallocation in a SN under different fluctuating demands. Adjusting production to the demand disruptions requires flexible facilities in the SN. However this connection is mainly blinked.

Therefore, efficient handling of these disruptions necessitates significant changes in: i) the production/distribution capacities of the SN's facilities and ii) production reallocation in the network. Both need flexible facilities in SNs. There are some risk mitigation strategies that can be used by a SN to neutralize the negative effects of these disruptions. Risk mitigation strategies are pre-disruption activities that are done to provide a robust network for a SN. Robust networks preserve the availability of the

required facilities for SNs in all conditions, even under disruption. Prevalent risk mitigation strategies against disruptions include:

- *Keeping emergency stocks*: These stocks are kept for use in emergency situations and disruptions. Determining the locations and amounts of these stocks is critical (You and Grossmann, 2008; Park et al., 2010; Schmitt, 2011).

- *Multi-sourcing/having back-up facilities*: In this case, key activities of SNs are assigned to more than one facility. When one of these facilities is inoperative, the others substitute for it (Yu et al., 2009; Li et al., 2010; Schmitt and Snyder, 2010; Peng et al., 2011; Schmitt, 2011).

- *Reserving extra capacity*: Having extra capacity in some of the SN's facilities enables them to be able to compensate for the unavailability of others (Chopra et al., 2007; Romejin et al., 2007; Hsu and Li, 2011).

Incorporating each of these risk mitigation strategies requires flexibility in the production/storage capacities of a SN's facilities. Therefore, there is a close relationship between flexibility of a SN's facilities and the robustness of its whole network. The literature on the relationship between two, however, is sparse at best. In this chapter, we fill this gap in the literature. We consider a SN with supply-side disruption risk. Hence, first secondary question for the sixth primary research question is: ***What level of flexibility in the SN's facilities provides the most profitable robust network?***

Having a robust network alone does not guarantee good performance for a SN under disruptions. Robust networks only preserve the availability of required facilities.

However, the SN's performance mainly depends on its post-disruption response; it requires a plan for shifting from operating in normal conditions to deploying the disruption plan. The transient period is usually called the recovery mechanism. Hishamuddin et al. (2014) and (2013) investigate recovery in the inventory system of a serial SN including a manufacturer and a retailer. Chen and Miller-Hooks (2014) analyze recovery of an intermodal freight transport network. Losada et al. (2012) develop a model to allocate protection resources in an uncapacitated median type facility system taking into account the role of facility recovery time. Gong et al. (2014) analyze the relationships between a SN and infrastructures (e.g. transportation and communications) in its recovery process under disruptions.

In this paper, a SN with a short recovery time is called a resilient SN. Resilient SNs are elastic enough to shift quickly from normal operations to emergency operations. We believe that the topology of a fail-safe SN must be robust and resilient against disruptions. Resilience of a SN mainly depends on the flexibility speeds of its facilities, i.e., how fast these facilities can ramp up their capacities. The literature on the relationship between two is also sparse at best. This gap is fulfilled in this paper. Hence, second secondary question for the sixth primary research question is: ***What level of flexibility speed in the SN's facilities provides the most profitable resilient network?***

- ***Variations in a SN*:** Variations refer to frequent and expected events with less significant impacts. These variations mainly affect the flow dynamics in a SN. Flow dynamics in the SN refers to production quantities in the SN's facilities and transportation quantities among facilities. SNs which are able to preserve the most profitable and

serviceable flow through their networks against demand- and supply-side variations are called “operationally fail-safe SNs”.

We believe a fail-safe SN should have the following features (fifth research question):

1) The design of its topology should be “structurally fail-safe” against disruptions;
2) The planning of flow throughout its network topology should be “operationally fail-safe” against variations.

Flow planning through operationally fail-safe SNs is discussed in Chapters 2, 3, 4, and 5. Therefore, in this chapter we focus on designing / redesigning structurally fail-safe SNs. To have a structurally fail-safe SN in a highly stochastic environment which is prone to disruptions, its topology should be (sixth research question):

i) ROBUST against disruptions by incorporating appropriate amount of risk mitigation strategies in facilities (appropriate amount of flexibility level in facilities) and
ii) RESILIENT against disruptions by employing the risk mitigation strategy fast enough in its facilities (appropriate amount of flexibility speed in facilities). SNs with these features are called structurally fail-safe SNs.

6.2. Operations in supply networks

We consider a SN dealing with producing and supplying a product to target markets. This network includes two manufacturers, *M1* and *M2*, producing products for this network.

This network has four target markets which are serviced by these two manufacturers. *M1* fulfills the demands of the first and second markets through first retailer, *R1*. The third and fourth markets' demands are fulfilled by *M2* through second retailer, *R2*. The components required by these two manufacturers are provided by two suppliers, *S1* and *S2*. *S1* and *S2* supply component needs of *M1* and *M2* respectively. In Figure 6-1, the existing network structure of this network and connections of its facilities are shown:

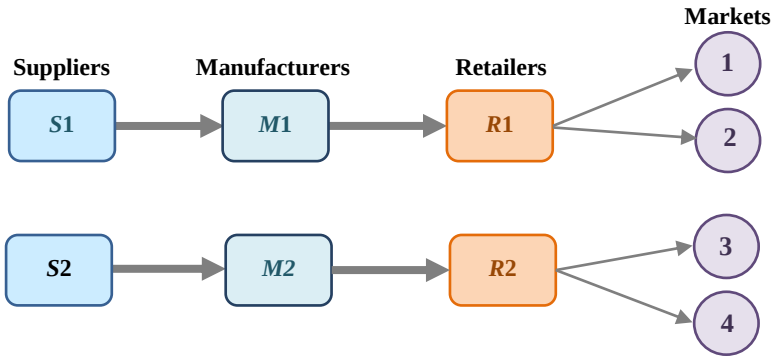


Figure 6-1: The network structure of the SN example.

Product demand in the markets is stochastic functions of the network's marketing factors, e.g., price and service level. Before the beginning of each sales period, retailers determine the quantities of product required and then issue the orders to the corresponding manufacturers. The manufacturers receive the orders from the retailers and plan to produce the ordered products. We assume the performance of the manufacturers' production systems is imperfect and they produce a stochastic percentage of defective output which brings our problem closer to reality (Rezapour et al., 2015).

As highlighted by Sana (2010), a higher rate of production increases the likelihood of machinery and labor failures in production systems leading to higher rates of non-

conforming items in a production system. To compensate for the defective output of their systems, manufacturers should plan to produce some extra products. To produce products, manufacturers order the required components from their corresponding suppliers. After setting up suppliers' machineries, they start to work in an in-control state in which all the output is almost sound. After a stochastic time, the machineries deteriorate to an out-of-control state in which γ percent of outputs is non-conforming. In the same way to compensate for non-conforming output of their systems, the suppliers plan to produce some surplus components.

In this chapter, uncertainty in the market demand is termed demand-side variation and uncertainty in the qualified product quantities available to be supplied to the markets in the SN's last echelon is termed supply-side variation. Supply-side variation is due to imperfect manufacturer and supplier production systems. In a SN with multiple imperfect production facilities, the qualified flow depreciates by moving from the network upstream to its downstream. Modeling this flow depreciation is necessary to quantify the qualified product volumes that can be supplied in the last echelon and to determine the best service level which balances the stochastic product demand and supply in the most economical way. To preserve an appropriate service level in the markets, reliable flow throughout the network is required against demand- and supply-side variations. To offer reliable flow dynamics throughout the SN in the presence of required facilities, we determine:

- ✓ The best service level for the SN maximizing its total profit,
- ✓ The best local reliabilities in the SN's stochastic facilities backing up its service level in the most economical way,

- ✓ The flow dynamics through stochastic facilities preserving their local reliabilities.

In addition to operational level variations, we also consider the possibility of disruptions in the SN's facilities. In this SN, *M1* is completely reliable but *M2* is prone to disruption. *M2* may be unavailable and unable to fulfill the orders of *R2*. There are several reasons that this can occur, e.g., the failure of its machinery or the inability of its supplier, *S2*, to procuring material therefore being unable to supply ordered components. In the unavailability of *M2*, the third and fourth markets are lost which leads to a huge loss in the SN's profitability and brand reputation. To avoid this possible loss and to improve the stability of the SN, we want to redesign a robust network for the SN. To have a robust network, we want to modify the production capabilities of its reliable facilities (*M1* and *S1*) to be able to compensate for the unavailability of its unreliable facilities (*M2* and *S2*). For this purpose, the production capacities of *M1* and *S1* should be flexible enough to be ramped up, when needed, to compensate for the unavailability of unreliable facilities and be ramped down when the unreliable facilities are available. In this problem, we want to determine the best flexibility levels in the reliable facilities, *M1* and *S1*, to redesign a robust network (first secondary research question). Redundancy in the capacity of reliable facilities is the risk mitigation strategy used to have a robust network.

The agility of the flexible facilities is ramping up their capacities after disruption, is measured by an index called resilience. Resilience of the SN in employing the redundancy mitigation strategy depends on the speed of its flexible facilities in ramping up their capacity after disruption. Therefore, the other important decisions made in this problem

are the best flexibility speeds in the reliable facilities, *M1* and *S1*, to redesign a resilient network (second secondary research question).

To formulate the model for this problem, we consider two conditions: i) the *Without Disruption Conditions* in which all the entities are available and ii) the *Disrupted Conditions* in which *M2* is unable to fulfill its assigned markets' demands and *M1* compensates for its unavailability.

6.2.1. Operations ***in the supply network under without disruption conditions***

Under the without disruption conditions, all the facilities (*M1*, *M2*, *S1* and *S2*) are available. In this case, there are two product supply paths in this SN (Figure 6-1):

I) $[S1 \rightarrow M1 \rightarrow R1]$ is the "first supply path", in which the flow of components starts from the first supplier, *S1*. These components then pass through and become finished products in the first manufacturer, *M1*, and are transported to the first retailer, *R1*, to supply to the first and second markets and fulfill their demands.

II) $[S2 \rightarrow M2 \rightarrow R2]$ is the "second supply path", in which the flow of components starts from the second supplier, *S2*. These components then pass through and become finished products in the second manufacturer, *M2*, and are transported to the second retailer, *R2*, to supply to the third and fourth markets and fulfill their demands.

In this section, we discuss reliable flow dynamics through the first path against demand and supply side variations under without disruption conditions. In Section 6.2.2, we

discuss that how this flow dynamics will be changed under disrupted condition when second supply path is inoperative.

The first path includes three kinds of facilities: the supplier (*S1*), the manufacturer (*M1*) and the retailer (*R1*). Each of these facilities faces a specific kind of variation. The retailer faces stochastic demand in the markets. The supplier and manufacturer encounter stochastic unqualified output of their production systems. For each of these facilities a desired local reliability must be chosen to deal with its corresponding uncertainty. Service level of the whole supply path is a function of these local reliabilities. We assume that $rl_{S1}^{WD}, rl_{M1}^{WD}$ and rl_{R1}^{WD} represent the local reliabilities of the first supply path's supplier, manufacturer and retailer respectively under without disruption conditions. In the rest of this section, the performance of each of these facilities against its corresponding uncertainty is investigated from downstream to upstream of the supply path.

Retailer in the first supply path, R1

The first supply path services the first and second markets. The most important marketing factors in these target markets are the price, p, and service level, sl. The service level is the fraction of the realized demand that can be fulfilled from on-hand product inventory available in the retailer. Therefore, the expected demand of each sale period in market *k* ($k = 1$ and 2), $D_k(p, sl^{WD})$, is assumed to be function of these two factors. sl^{WD} represents the service level provided by the SN under without disruption conditions. The retailer of the first supply path ($R1$) fulfills the sum of demands of first and second markets. Hence, the average demand of *R1* is $\sum_{k=1}^{2} D_k(p, sl^{WD})$. However the actual realized demand is stochastic and deviates from this mean value. This deviation is treated

as a random variable, ε, with a cumulative distribution function $G_{R1}(\varepsilon)$. The realized actual demand of *R1* is $\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon$. Without loss of generality we assume $E(\varepsilon) = 1$ which implies $E[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon] = \sum_{k=1}^{2} D_k(p, sl^{WD})$ (Bernstein and Federgruen, 2004 and 2007).

Before the beginning of each sales period, the retailer must make a decision about the quantity of its product stock for the next period which is represented by x^{WD} and issue an order to the corresponding manufacturer, *M1*. After realizing the actual demand, unit holding cost, h^+, and unit shortage cost, h^-, are paid by the retailer for each end-of-period inventory and lost sale, respectively. Therefore, the expected total cost of the retailer, Π_{R1}^{WD}, that should be minimized is Equation (6-1):

$$MIN \quad \Pi_{R1}^{N} = h^+ . E[x^N - \sum_{k=1}^{2} D_k(p, sl^N) \times \varepsilon]^+ + h^- . E[\sum_{k=1}^{2} D_k(p, sl^N) \times \varepsilon - x^N]^+ \tag{6-1}$$

$$S.T. \quad \Pr[\sum_{k=1}^{2} D_k(p, sl^N) \times \varepsilon \leq x^N] \geq rl_{R1}^{N} \tag{6-2}$$

The constraint in Equation (6-2) preserves the retailer's local reliability which guarantees that in rl_{R1}^{WD} percentage of time the retailer's product stock can fulfill the realized demand. The first term in the objective function, Equation (6-1), represents the expected end-of-period inventory holding cost in the retailer. The second term in (6-1) shows expected lost sale cost. $x^{WD} = [\sum_{k=1}^{2} D_k(p, sl^{WD})] . G_{R1}^{-1}(\frac{h^-}{h^- + h^+})$ the quantity of product ordered minimizes *R1*'s expected total cost. On the other hand, to satisfy the

constraint in Equation (6-2), we should have $x^{WD} \geq [\sum_{k=1}^{2} D_k(p, sl^{WD})]. G_{R1}^{-1}(rl_{R1}^{WD})$. Accordingly, the quantity of product to that *R1* must order is:

$$x^N = [\sum_{k=1}^{2} D_k(p, sl^N)]. G_{R1}^{-1}\left(Max\ \left\{rl_{R1}^N, \frac{h^-}{h^- + h^+}\right\}\right) \quad (6\text{-}3)$$

By substituting Equation (6-3) into (6-1), the least total cost of $R1$ will be:

$$\Pi_{R1}^N = \left(h^+. E\left[G_{R1}^{-1}\left(Max\ \left\{rl_{R1}^N, \frac{h^-}{h^- + h^+}\right\}\right) - \varepsilon\right]^+ + h^-. E\left[\varepsilon - G_{R1}^{-1}\left(Max\ \left\{rl_{R1}^N, \frac{h^-}{h^- + h^+}\right\}\right)\right]^+\right) \times [\sum_{k=1}^{2} D_k(p, sl^N)] \quad (6\text{-}4)$$

Ordering x^{WD} product units from *M1* enables *R1* to fulfill the product demand in the next sales period with rl_{R1}^{WD} probability. In the next section, it is shown how this product flow quantity must be amplified by moving backward to the manufacturer in this path. We assume that each facility only fulfills the order of its downstream facility issued before the beginning of the sales period. Extra product transaction between facilities during the sales period is not possible.

Manufacturer in the first supply path, M1

M1 receives an order including x^{WD} product units from *R1*. This order by retailer is produced in O_{M1} production runs including y^{WD} items in each production batch (Figure 6-2).

The production system in *M1* is not complete and is always accompanied with some wastage. *M1*'s wastage ratio, α_{M1}, depends on the general condition of its machinery and skills of its labors and is a random variable with the G'_{M1} cumulative distribution function.

The batch size of each production run must be determined to preserve its local reliability, rl_{M1}^{WD} (α_{M1}^{i} represents the value of random variable α_{M1} realized in production run $i = 1, 2, \ldots, O_{M1}$):

$$rl_{M1}^{N} = \Pr\left(\alpha_{M1}^{1}.y^{N} + \alpha_{M1}^{2}.y^{N} + \alpha_{M1}^{3}.y^{N} + \cdots + \alpha_{M1}^{O_{M1}}.y^{N} \leq O_{M1}.y^{N} - x^{N}\right) \quad (6\text{-}5)$$



Figure 6-2: Production systems in *M*1.

To preserve rl_{M1}^{WD} local reliability, the number of defective items in all production runs ($\alpha_{M1}^{1}.y^{WD} + \alpha_{M1}^{2}.y^{WD} + \alpha_{M1}^{3}.y^{WD} + \cdots + \alpha_{M1}^{O_{M1}}.y^{WD}$) must be less than the extra production volume ($O_{M1}.y^{WD} - x^{WD}$) with rl_{M1}^{WD} probability, Equation (6-5). Without loss of generality, we assume that for producing one unit of product, one unit of component is required. Since *M1* will produce $O_{M1}.y^{WD}$ product units, it will issue an order including $O_{M1}.y^{WD}$ component units to it supplier, *S1*. In the next section, it is shown how the quantity of flow of this component must be amplified by moving backward to the supplier.

Supplier in the first supply path, S1

In the first supply path, *S1* receives an order for $O_{M1}.y^{WD}$ unites of component from *M1*. To fulfill this order in *S1*, O_{S1} production runs must be performed with z^{WD} items in each production batch. After setting up *S1*'s machines to produce z^{WD} items in each batch, all machines start to work in an in-control state in which all the produced components are

sound. Gradually the state of the machines deteriorates and after a stochastic timeframe shifts to an out-of-control state in which γ_{S1} percent of the produced components are non-conforming. This deterioration time represented by *t* is a random variable with a G''_{S1} distribution. After shifting the production system to the out-of-control state, it stays in that state until the production of that whole batch is completed because interrupting the machines is prohibitively expensive (Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987). Cap_{S1}^{WD} represents the production capacity of *S1* in each production run including T time units. Hence, the production rate of *S1* is $Cap_{S1}^{WD}/_{T}$ and in total it will take $T.z^{WD}\Big/_{Cap_{S1}^{WD}}$ time units to produce each production batch. Before the production system deteriorates, all the output are sound but after that, γ_{S1} percent are non-conforming. Therefore, the total number of defective output in product batch *i* ($i = 1, 2, \dots, O_{S1}$) is $\left(T.z^{WD}\Big/_{Cap_{S1}^{WD}} - t_i\right).(\gamma_{S1}.Cap_{S1}^{WD})$. t_i represents the value of random variable *t* realized in production run *i* ($i = 1, 2, \dots, O_{S1}$). To preserve the local reliability of *S1*, the following constraint is needed:

$$rl_{S1}^{N} = \Pr\left(\sum_{i=1}^{O_{S1}}\left(T.z^{N}\Big/_{Cap_{S1}^{N}} - t_i\right).(\gamma_{S1}.Cap_{S1}^{N}) \le (O_{S1}.z^{N}) - (O_{M1}.y^{N})\right) =$$

$$\Pr\left((T.\gamma_{S1} - 1).O_{S1}.z^{N} + O_{M1}.y^{N} \le \gamma_{S1}.Cap_{S1}^{N}.\sum_{i=1}^{O_{S1}} t_i\right) \quad (6\text{-}6)$$

Constraint (6-6) ensures that with rl_{M1}^{WD} probability the total number of non-conforming components produced by *S1* will be less than its surplus production quantity

$O_{S1}.z^{WD} - O_{M1}.y^{WD}$. The value of the z^{WD} variable should be selected to preserve the local reliability of *S1*.

The component production batch size (z^{WD}) satisfying Constraint (6-6) ensures that *S1* will be able to fulfill the entire component order of *M1* with rl_{S1}^{WD} probability. The product production batch size (y^{WD}) satisfying Constraint (6-5) guarantees that *M1* will be able to fulfill the product order of *R1* with rl_{M1}^{WD} probability. The product stock quantity (x^{WD}) satisfying Constraint (6-3) assures that *R1* will be able to fulfill the demand of the market in the next sale period with rl_{R1}^{WD} probability. In this case, the SN will be sure with $rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD}$ probability that it can respond to the demand of the market. In this problem, this demand fulfillment rate is termed the service level:

$$sl^N = rl_{S1}^N.rl_{M1}^N.rl_{R1}^N \tag{6-7}$$

Equation (6-7) represents the relationship between local reliabilities of the stochastic facilities in the first supply path and the SN's service level in the markets serviced by this path.

Mathematical model of flow planning under without disruption condition

In this section, a mathematical model is presented for planning reliable flow dynamics in the entire first supply path of the SN by using the analysis and the relationships presented in the previous sections:

$$\textbf{Max} \quad \Psi^N = \left(P - h^+.E\left[G_{R1}^{-1}\left(Max\left\{rl_{R1}^N, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ - h^-.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{rl_{R1}^N, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^N)\right] - c_{S1}.(O_{S1}.z^N) - c_{S1,M1}.(O_{M1}.y^N) - c_{M1}.(O_{M1}.y^N) - c_{M1,R1}.(x^N) \quad (6\text{-}8)$$

Subject To:

$$O_{S1}.z^N \geq O_{M1}.y^N \quad (6\text{-}9)$$

$$O_{M1}.y^N \geq x^N \quad (6\text{-}10)$$

$$x^N = \left[\sum_{k=1}^{2} D_k(p, sl^N)\right].G_{R1}^{-1}\left(Max\left\{rl_{R1}^N, \frac{h^-}{h^-+h^+}\right\}\right) \quad (6\text{-}11)$$

$$rl_{M1}^N = \Pr\left(\alpha_{M1}^1.y^N + \alpha_{M1}^2.y^N + \alpha_{M1}^3.y^N + \cdots + \alpha_{M1}^{O_{M1}}.y^N \leq O_{M1}.y^N - x^N\right) \quad (6\text{-}12)$$

$$rl_{S1}^N = \Pr\left(\sum_{i=1}^{O_{S1}}\left(T.z^N \Big/ Cap_{S1}^N - t_i\right).(\gamma_{S1}.Cap_{S1}^N) \leq (O_{S1}.z^N) - (O_{M1}.y^N)\right) \quad (6\text{-}13)$$

$$sl^N = rl_{S1}^N.rl_{M1}^N.rl_{R1}^N \quad (6\text{-}14)$$

$$y^N \leq Cap_{M1}^N \quad (6\text{-}15)$$

$$0 \leq rl_{S1}^N, rl_{M1}^N \text{ and } rl_{R1}^N \leq 1 \quad (6\text{-}16)$$

$$x^N, y^N \text{ and } z^N \geq 0 \quad (6\text{-}17)$$

The objective function, Equation (6-8), is used to maximize the total profit under the without disruption conditions. The first term of Equation (6-8) is used to compute the capturable income after discarding the inventory holding cost for the end-of-period extra inventory and the shortage cost for end-of-period lost sales. The second term is the procurement and production cost of components in *S1*. The third cost is the transportation cost of components from *S1* to *M1*. The fourth term is the cost of manufacturing products in *M1*. The fifth term represents the transportation cost of products from *M1* to *R1*. Based

on the constraint in Equation (6-9), the product production quantity in *M1* should be less than the number of components planned to be produced by *S1*. According to the constraint in Equation (6-10), the product production quantity in *M1* should be more than the product order quantity by *R1*. The constraints in Equations (6-11), (6-12) and (6-13) represent the relationship between order and production quantities in *R1*, *M1* and *S1* and their corresponding local reliabilities respectively. The relationship between service level in the without disruption conditions and local reliabilities of stochastic facilities are shown in the constraint in Equation (6-14). Equation (6-15) and (6-16) is used to insure that that the production quantity in each run of *M1* and *S1* is less than its capacity, Cap_{M1}^{WD} and Cap_{S1}^{WD}, respectively. Equation (6-17) is used to insure that local reliabilities of facilities are selected from the [0, 1] interval.

Solution procedure for the flow planning model under without disruption conditions

The mathematical model proposed in the previous section is a stochastic nonlinear program. The objective function and some of the constraints in this model (such as Equations (6-11) and (6-14)) are highly nonlinear. This model also includes two stochastic terms (joint probability distributions) in Equations (6-12) and (6-13) and they do not have a closed form equations. Solving this mathematical model is not straightforward and needs a special solution approach. In this section, we propose a way to linearize and solve this model. Solving linear models is straightforward and fast. One of the important stochastic constraints in the model (6-8)-(6-17) is the constraint in Equation (6-12). A statistical approximation for this constraint is:

$$rl_{M1}^{N} = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}.y^{N} \le O_{M1}.y^{N} - x^{N}\right) = \frac{\sum_{k=1}^{K} rl_{M1,k}^{N}}{K} \quad (6\text{-}18)$$

$$BM.\left(rl_{M1,k}^{N} - 1\right) \le (O_{M1}.y^{N} - x^{N}) - y^{N}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i} \le BM.rl_{M1,k}^{N}$$

$$(\forall k = 1, \dots, K \; and \; \forall i = 1, \dots, O_{M1}) \; \left(\alpha_{M1}^{i} \sim G'_{M1}\right) \quad (6\text{-}19)$$

$$rl_{M1,k}^{N} \in \{0,1\} \qquad (\forall k = 1, \dots, K) \quad (6\text{-}20)$$

In Equation (6-18), the probability of an event is defined as "*left hand side of the inequality in Equation (6-18) being less than its right hand side*" is replaced by the ratio of its occurrence in a sample including *J* observations. Increasing the size of the sample, *J*, increases the accuracy of this statistical approximation. To determine the number of times in which term $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is positive, a new binary variable $rl_{M1,j}^{N}$ is defined. Variable $rl_{M1,j}^{N}$ is 1 if the term $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is positive based on the realized values of α_{M1}^{i} $(\forall i = 1, \dots, O_{M1})$ in observation *j* and 0 otherwise. Increasing the accuracy of this approximation enhances the number of these new variables. Therefore selecting the least *J* that assures an acceptable accuracy is necessary.

The constraint in Equation (6-13) is linearized in the same way. First it is simplified algebraically and rewritten as:

$$rl_{S1}^{N} = \Pr\left(\gamma_{S1}.Cap_{S1}^{N}.\left(\sum_{i=1}^{O_{S1}} t_i\right) \ge O_{S1}.(\gamma_{S1} - 1).z^{N} + O_{M1}.y^{N}\right) \quad (6\text{-}21)$$

Then it is replaced with the following constraints:

$$rl_{S1}^{N} = \Pr\left(\gamma_{S1}.Cap_{S1}^{N}.\left(\sum_{i=1}^{O_{S1}} t_i\right) \ge O_{S1}.(\gamma_{S1} - 1).z^{N} + O_{M1}.y^{N}\right) = \frac{\sum_{k=1}^{K} rl_{S1,k}^{N}}{K}$$

(6-22)

$$BM.\left(rl_{S1,k}^{N}-1\right)\leq\gamma_{S1}.Cap_{S1}^{N}.\left(\sum_{i=1}^{O_{S1}}t_i\right)-O_{S1}.(\gamma_{S1}-1).z^{N}-O_{M1}.y^{N}\leq BM.rl_{S1,k}^{N}$$

$$(\forall k=1,\dots,K\ and\ \forall i=1,\dots,O_{S1})\quad (t_i\sim G_{S1}'') \tag{6-23}$$

$$rl_{S1,k}^{N}\in\{0,1\}\qquad (\forall k=1,\dots,K) \tag{6-24}$$

To check the accuracy of this approximation and give some sense about appropriate values for the sample size, *J*, we do some numerical analysis and compute the average error of this approximation for different density functions. Results are summarized In Table 1.

Table 6-1: Average error for different density functions.

Normal Density Function				*Uniform Density Function*				*Exponential Density Function*			
J	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***
1	0.220	**60**	0.034	**1**	0.230	**60**	0.033	**1**	0.210	**60**	0.032
5	0.129	**65**	0.033	**5**	0.134	**65**	0.031	**5**	0.131	**65**	0.030
10	0.084	**70**	**0.031**	**10**	0.082	**70**	**0.029**	**10**	0.085	**70**	0.030
15	0.065	**75**	0.029	**15**	0.067	**75**	0.028	**15**	0.064	**75**	0.029
20	0.061	**80**	0.028	**20**	0.061	**80**	0.028	**20**	0.060	**80**	0.028
25	**0.051**	**85**	0.028	**25**	**0.050**	**85**	0.027	**25**	0.050	**85**	0.027
30	0.048	**90**	0.026	**30**	0.049	**90**	0.025	**30**	0.048	**90**	0.026
35	0.045	**95**	0.025	**35**	0.043	**95**	0.025	**35**	0.043	**95**	0.026
40	**0.041**	**100**	0.025	**40**	0.041	**100**	0.024	**40**	0.039	**100**	0.025
45	0.039	**120**	0.023	**45**	**0.039**	**120**	0.022	**45**	0.037	**120**	0.023
50	0.038	**140**	**0.020**	**50**	0.036	**140**	**0.019**	**50**	0.036	**140**	0.022
55	0.035	**150**	0.019	**55**	0.034	**150**	0.018	**55**	0.034	**150**	0.019

Based on these results when J belongs to $[25, 30]$ interval, the average error of this approximation is less than equal to 5 percent. To reduce the error to less than 4, 3, and 2 percent, J should be selected from $[40, 45]$, $[65, 70]$, and $[140, 150]$ intervals.

To linearize the objective function in Equation (6-8) and the constraints in Equations (6-11) and (6-14), we discretize facilities' local reliability variables,

$rl_{S1}^{WD}, rl_{M1}^{WD}$ and rl_{R1}^{WD}. These variables have a very restricted feasible range. They only take on values in the [0.5, 1] interval; this very restricted feasible range is used to justify the feasibility of their discretization. Set $RL = \{rl\}$ includes all discrete values that can be assumed by these variables. To select one of these options, we define new binary variables $\theta_{S1}^{WD,rl}$, $\theta_{M1}^{WD,rl}$ and $\theta_{R1}^{WD,rl}$. Variable $\theta_{S1}^{WD,rl}$ is 1 if reliability option $rl \in RL$ is selected for *S1* and 0 otherwise. Only one of these options can be selected for *S1*:

$$\sum_{rl=1}^{|RL|} \theta_{S1}^{rl} = 1 \tag{6-25}$$

Variable $\theta_{M1}^{WD,\,rl'}$ is 1 if reliability option $rl' \in RL$ is selected for *M1* and 0 otherwise. Only one of these options can be selected for *M1*:

$$\sum_{rl'=1}^{|RL|} \theta_{M1}^{rl'} = 1 \tag{6-26}$$

Variable $\theta_{R1}^{WD,rl^{"}}$ is 1 if reliability option $rl^{"\prime} \in RL$ is selected for *R1* and 0 otherwise. Only one of these options can be selected for *R1*:

$$\sum_{rl^{"}=1}^{|RL|} \theta_{R1}^{rl^{"}} = 1 \tag{6-27}$$

By defining these new variables, the objective function is rewritten as:

$$\textbf{\textit{Max}} \quad \Psi^N = \sum_{rl=1}^{|RL|} \sum_{rl'=1}^{|RL|} \sum_{rl^{"}=1}^{|RL|} \theta_{S1}^{rl} . \theta_{M1}^{\acute{r}l} . \theta_{R1}^{rl^{"}} . \left[\left(P - h^{+} . E \left[G_{R1}^{-1} \left(Max \left\{ rl^{"}, \frac{h^{-}}{h^{-}+h^{+}} \right\} \right) - \varepsilon \right]^{+} - h^{-} . E \left[\varepsilon - G_{R1}^{-1} \left(Max \left\{ rl^{"}, \frac{h^{-}}{h^{-}+h^{+}} \right\} \right) \right]^{+} \right) \times \left[\sum_{k=1}^{2} D_k \left(p, rl . rl' . rl^{"} \right) \right] \right] \tag{6-28}$$

After defining these new binary variables, the constraint in Equation (6-11) is linearized:

$$x^N =$$

$$\sum_{rl=1}^{|RL|}\sum_{rl'=1}^{|RL|}\sum_{rl"=1}^{|RL|}\theta_{S1}^{rl}.\theta_{M1}^{\acute{rl}}.\theta_{R1}^{rl"}.\left[\left(\sum_{k=1}^{2}D_k\left(p,rl.rl'.rl"\right)\right).G_{R1}^{-1}\left(Max\left\{rl",\frac{h^-}{h^-+h^+}\right\}\right)\right] \quad (6\text{-}29)$$

The constraint in Equation (6-14) is rewritten:

$$sl^N = \sum_{rl=1}^{|RL|}\sum_{rl'=1}^{|RL|}\sum_{rl"=1}^{|RL|}\theta_{S1}^{rl}.\theta_{M1}^{\acute{rl}}.\theta_{R1}^{rl"}.\left[rl.rl'.rl"\right] \quad (6\text{-}30)$$

After defining these new variables and using statistical approximations, the mathematical model (6-8)-(6-17) becomes mixed integer linear model which is more easily solved.

<u>Computational result: Test problem</u>

In this section, we assume that in the first supply path, $[S1 \rightarrow M1 \rightarrow R1]$, the performance of the production systems in *M1* and *S1* are imperfect. In *S1* after setting the equipment up, the machinery starts to work in an in-control state and all of the components produced are sound. But after a stochastic time following an exponential distribution with $\mu = 2$, the machinery shifts to an out-of-control state in which $\gamma_{S1} = 10\%$ of output is non-conforming. In *M1*, the product assembling process always accompanies with stochastic number of defective products. This percentage is a random variable with a uniform distribution in [0, $\beta = 0.15$] interval.

The total demand of the first and second markets to be fulfilled by *R1* is a stochastic linear function of price, $p = \$14$, and service level, sl^{WD}: $\sum_{k=1}^{2}D_k(p, sl^{WD}).\varepsilon = [1000 - 150 \times (p - 14) + 1000 \times (sl^{WD} - 0.85)].\varepsilon$. ε is a normally distributed

random variable with a mean of 1 and a variance of 1. From regression studies for historical triples ($\sum_{k=1}^{2} D_k, p, sl^{WD}$), it was shown that a linear function fits very well for this data. Biases of the real and the estimated mean demand in these triples are analyzed by a goodness-of-fit statistical test to determine the best distribution which represents these biases. The unit production cost in *S1* is \$1.40. The unit transportation cost for moving the component unit from *S1* to *M1* is \$0.50. The unit assembling cost in *M1* and the unit transportation cost from *M1* to *R1* is \$1.00 and \$0.60 respectively. The unit extra inventory and unit shortage costs in *R1* are \$0.10 and \$0.30 respectively. Demand in each period is fulfilled by $O_{S1} = 3$ and $O_{M1} = 4$ production runs.

Formulating the mathematical model for this problem and solving it leads to the following results: the best service level for the without disruption condition is 80 percent (corresponding to the highest profit in Figure 6-3). As shown in Figure 6-3, there are different combinations of local reliabilities of facilities, (rl_{S1}^{WD}, rl_{M1}^{WD}, rl_{R1}^{WD}), that lead to the same service level of $rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD} = 0.8$. For all points on line AB, the service level is 0.8 but they correspond to different local reliability combinations of facilities and their profit levels are significantly different. Therefore, in such a supply path with multiple stochastic facilities only finding the best service level is not enough. We also need to find the least costly local reliability combination to support that service level.

The mathematical model of this problem helps us to find this best local reliability combination which is $rl_{S1}^{WD} = 1$, $rl_{M1}^{WD} = 1$ and $rl_{R1}^{WD} = 0.8$. To preserve the local reliability of *R1*, its product order quantity from *M1* must be $x^{WD} = 1748$. The best production quantity in each production run of *M1* is 496.15 which means that *M1*

produces 236.6 extra units ($4.y^{WD} - x^{WD} = 236.6$). This extra production preserves its local reliability which is equal to 1. The best component production quantity in each production run of *S1* is 684.78. This production quantity leads to the extra production of 70 units in *S1* ($3.z^{WD} - 4.y^{WD} = 70$). This extra production assures 1 local reliability for *S1*.

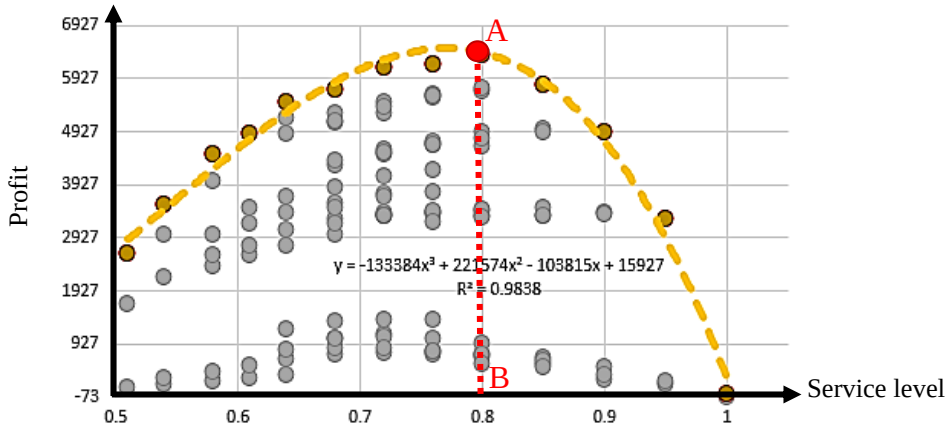


Figure 6-3: Profit of the first supply path with respect to the service level.

In the rest of this section, we analyze the relationships between local reliabilities of facilities in the supply path and its profitability. For this purpose, we solve the model for different values of local reliabilities. The results are displayed in the graphs of Figure 6-4. Based on these graphs, we conclude that:

- For a given local reliability of the retailer, the patterns which determine the supply path's profit change with respect to the local reliability of the supplier, are almost similar for all local reliabilities of the manufacturer. This means *for a given quantity of product ordered, the most profitable local reliabilities of the supplier and the manufacturer are almost independent*. Hence the best local reliabilities of

these stochastic facilities can be determined separately. This feature significantly decreases the size and computational burden of the mathematical model.

- For a given local reliability of the retailer, the effects of the local reliabilities of the manufacturer and supplier on the path's profit are almost similar. For instance, if reduction in the supplier's local reliability leads to a profit reduction for the path, a reduction in the manufacturer's local reliability also leads to a profit reduction in the path and vice versa. If reduction in the supplier's local reliability first increments the path's profit and then reduces it, a reduction in the manufacturer's local reliability imposes almost the same pattern of changes on the path's profit. Therefore *determining the best local reliability for one of these facilities provides a good estimate of the tentative local reliability of the other one*. Using this insight significantly reduces the search interval for the local reliability of the later facility.

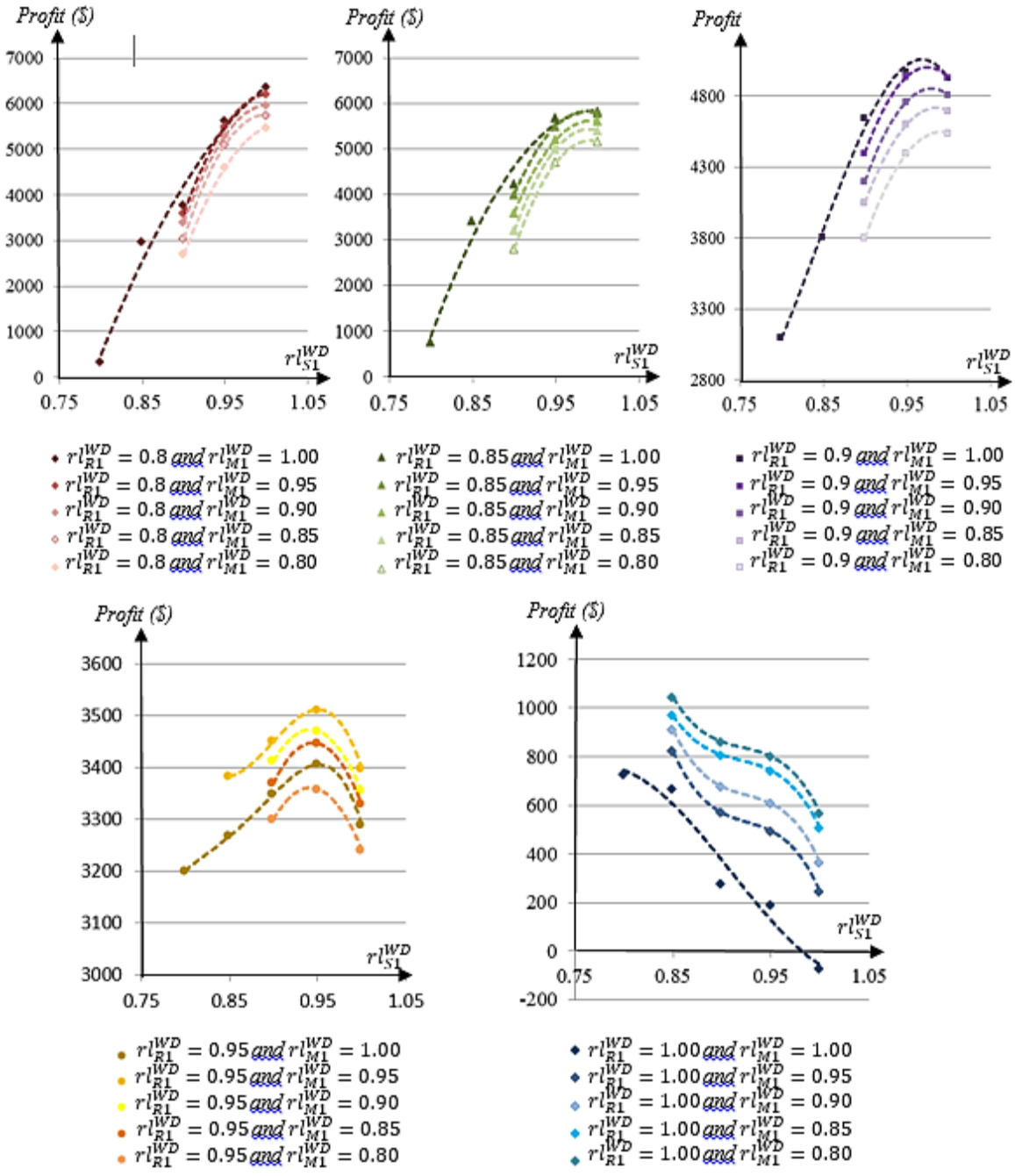


Figure 6-4. Relationships among the local reliabilities of facilities in the supply path and its profitability.

6.2.2. Operations in the supply network under disrupted conditions

When the SN is disrupted, *M2* or *S2* is unavailable. In this case, the second supply path $[S2 \rightarrow M2 \rightarrow R2]$ is inoperative and is unable to fulfill the demands of the third and fourth markets. Thus the only active supply path is $[S1 \rightarrow M1 \rightarrow R1]$ which can be used to fulfill the demands of all markets (Figure 6-5).

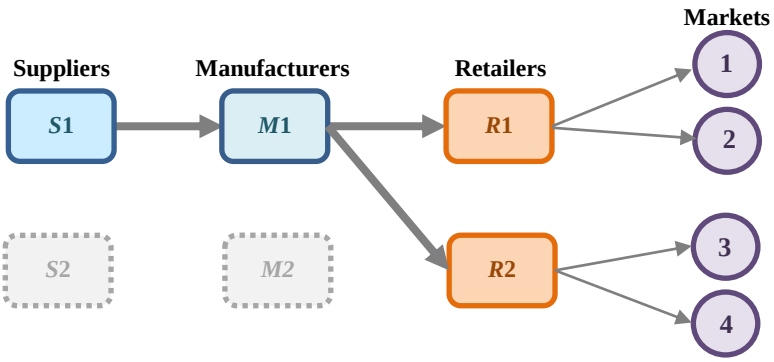


Figure 6-5: Network structure of the SN under disrupted conditions.

To answer the first secondary question and redesign a robust network for the SN, the first supply path must not only service the first and second markets but must also fulfill the demands of the third and fourth markets under disrupted conditions. For this purpose, its production facilities, *S1* and *M1*, need flexible capacities. After disruption, the capacities of these facilities should be ramped up to service both retailers and after disruption they should be ramped down to only service the first retailer. The measure of how much the capacity of a facility can be ramped up during a disruption is its flexibility level and the time pattern of this increment is its flexibility speed. The robustness and resilience of a SN is determined by the flexibility level and speed of its facilities respectively. In Figure 6-6, some of the possible flexibility speed options for capacity

ramp up in *M1* are shown. In this figure, it is assumed that one period including four production runs, $O_{M1} = 4$, is the maximum time available to ramp up capacity and flexibility level of *M1* is equal to $O_{M1}.\Delta_{M1} = 4\Delta_{M1}$.

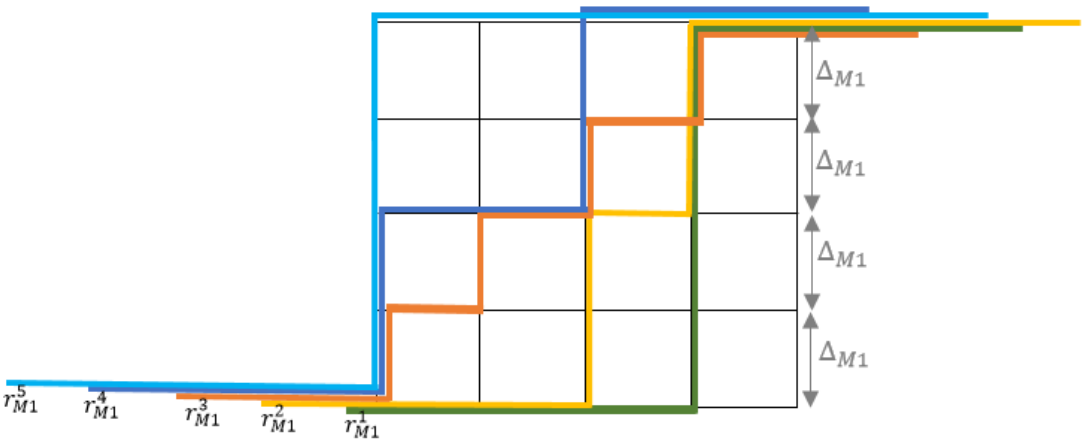


Figure 6-6: Sample flexibility speed options for capacity ramp up in *M1*.

In Figure 6-6, four different time patterns for capacity ramp up in *M1* are shown; these are available flexibility speed options for *M1*:

- ***In the first flexibility speed option shown with r^1_{M1} in Figure 6-6*:** a time equal to three production runs is given to *M1* to provide the extra capacity. In this extreme case, all of *M1*'s extra capacity, $O_{M1}.\Delta_{M1}$, is added at the beginning of the last (fourth) production run. The time pattern of capacity ramp up in the production runs of the period for this flexibility speed option is $r^1_{M1} = (r1^1_{M1} = 0, r2^1_{M1} = 0, r3^1_{M1} = 0, r4^1_{M1} = O_{M1}.\Delta_{M1})$. This means that capacity ramp up in the first ($r1^1_{M1}$), second ($r2^1_{M1}$), and third ($r3^1_{M1}$) production runs are equal to 0 and in the last run ($r4^1_{M1}$) is equal to $O_{M1}.\Delta_{M1}$;

- ***In the second flexibility speed option shown with r^{2}_{M1} in Figure 6-6*:** time equal to two production runs is given to *M1* to provide the extra capacity. This extra capacity is provided equally at the beginning of the third and fourth production runs. The time pattern of capacity ramp up for this option is $r^{2}_{M1} = (r1^{2}_{M1} = 0, r2^{2}_{M1} = 0, r3^{2}_{M1} = O_{M1}.\Delta_{M1}/2, r4^{2}_{M1} = O_{M1}.\Delta_{M1}/2)$;

- ***In the third flexibility speed option shown with r^{3}_{M1} in Figure 6-6*:** the capacity ramp up in *M1* is completely uniform. The time pattern for this option is $r^{3}_{M1} = (r1^{3}_{M1} = O_{M1}.\Delta_{M1}/4, r2^{3}_{M1} = O_{M1}.\Delta_{M1}/4, r3^{3}_{M1} = O_{M1}.\Delta_{M1}/4, r4^{3}_{M1} = O_{M1}.\Delta_{M1}/4)$;

- ***In the fourth flexibility speed option shown with r^{4}_{M1} in Figure 6-6*:** the capacity ramp up in *M1* is more drastic. Half of it, $O_{M1}.\Delta_{M1}/2$, is added at the beginning of the first production run and the rest is added in the second run. The time pattern for this option is $r^{4}_{M1} = (r1^{4}_{M1} = O_{M1}.\Delta_{M1}/2, r2^{4}_{M1} = O_{M1}.\Delta_{M1}/2, r3^{4}_{M1} = 0, r4^{4}_{M1} = 0)$;

- ***In the fifth flexibility speed option shown with r^{5}_{M1} in Figure 6-6*:** all the manufacturer's capacity increment, $O_{M1}.\Delta_{M1}$, is added at the beginning of the first production run. Hence the time pattern of this extreme option is $r^{5}_{M1} = (r1^{5}_{M1} = O_{M1}.\Delta_{M1}, r2^{5}_{M1} = 0, r3^{5}_{M1} = 0, r4^{5}_{M1} = 0)$;

Therefore, we define a new set $RO_{M1} = \{r_{M1}\}$ including all the flexibility speed options of *M1*. Providing extra production capacity is more costly in the early production

runs following a disruption. Acquiring the extra machinery and labor force to increase capacity in a short time is not easy and can be more costly. On the other hand, an early increment in capacity leads to the availability of a higher capacity in the rest of production runs and subsequently more feasible production plans will be available for selection and more uniform production quantities in the later production runs are possible. Hence we assume that the unit capacity increment cost is higher for early production runs. This assumption is consistent with the observations in the manufacturing systems. Based on the work of Koren and Shpitalni (2014), the unit capacity cost is low for the dedicated manufacturing systems but the speed of responsiveness to a required increase in capacity is also low. In a flexible manufacturing system with higher cost, the speed of responsiveness is much greater. This tradeoff between economical manufacturing and speed of responsiveness is considered in our problem.

Assuming that parameter cap_{M1}^{i} $(i = 1, 2, \dots, O_{M1})$ represents the unit extra capacity cost in *M1*'s production run *i*, we have $cap_{M1}^{1} > cap_{M1}^{2} > cap_{M1}^{3} > \cdots > cap_{M1}^{O_{M1}}$. To select the flexibility speed option and to answer the second secondary question, binary variables $w_{M1}^{r_{M1}}$ $(r_{M1} \in RO_{M1})$ are used. Variable $w_{M1}^{r_{M1}}$ is 1 if the flexibility speed option r_{M1} is selected for *M1* and 0 otherwise. In the same way, $O_{S1}.\Delta_{S1}$ represents the flexibility level in *S1* and different flexibility speed options are available for it which are included in the set $RO_{S1} = \{r_{S1}\}$. Assuming that parameter cap_{S1}^{j} $(j = 1, 2, \dots, O_{S1})$ represents the unit extra capacity cost in *S1*'s production run *j*, we have the extra capacity costs $cap_{S1}^{1} > cap_{S1}^{2} > cap_{S1}^{3} > \cdots > cap_{S1}^{O_{S1}}$. For selecting the flexibility speed option in *S1*, binary

variables $w_{S1}^{r_{S1}}$ ($r_{S1} \in RO_{S1}$) are used. Variable $w_{S1}^{r_{S1}}$ is 1 if the flexibility speed option r_{S1} is selected for *S1* and 0 otherwise.

When a disruption occurs in the second supply path, the capacity of the first supply path's *M1* and *S1* shifts from normal capacity, Cap_{M1}^{WD} and Cap_{S1}^{WD}, to the capacity suitable for the disrupted conditions, Cap_{M1}^{D} and Cap_{S1}^{D}, based on its selected flexibility speed options. The period in which Cap_{M1}^{WD} and Cap_{S1}^{WD} shifts to Cap_{M1}^{D} and Cap_{S1}^{D} respectively is defined here as the ramp-up disruption period. The production capacity of *M1* and *S1* is not fixed during this ramp-up disruption period and may change from production run to production run. In the next section, we elaborate the production plan in the first supply path's facilities in the ramp-up disruption period. After ramp-up period, capacity Cap_{M1}^{D} and Cap_{S1}^{D} is available for *M1* and *S1* in all the production runs as long as the disruption lasts. These disrupted periods after ramp-up period are called normal-disruption periods. Then, we elaborate production a plan in the first supply path's facilities for a normal-disruption period. When disruption ends, the extra capacity is not needed in the facilities of the first supply path. Therefore capacity of *M1* and *S1* reduces from Cap_{M1}^{D} and Cap_{S1}^{D} to the capacity of without disruption conditions, Cap_{M1}^{WD} and Cap_{S1}^{WD}, respectively. This period after disruption is called ramp-down period. The ramp-down period is a without disruption period and the only difference is that extra capacity is available.

In Figure 6-7 we show these periods for r_{M1}^{3} when the disruption only lasts for two periods. In this case, there is only one ramp-up, one normal disruption, and one ramp-

down period. In longer disruptions, the number of normal disruption periods is more than one.

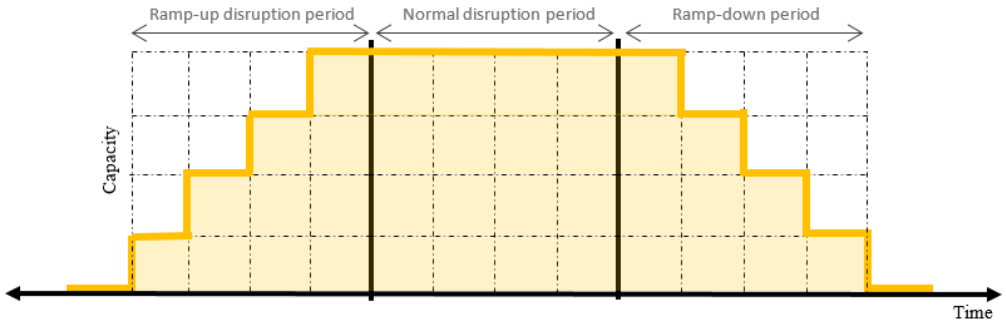


Figure 6-7: Ramp-up, normal disruption, and ramp-down periods for a disruption lasting for two periods.

The ramp-up disruption period (see Figure 6-7)

The capacities of facilities in the first supply path (*S1* and *M1*) in each production run of the ramp-up disruption period depend on the selected flexibility level options. Assume that y_i^{RUD} and z_i^{RUD} variables represent the production quantities in production run i of *M1* and *S1* respectively. During the ramp-up disruption period, each facility's production quantity in each production run must be less than its available capacity. Hence the following restrictions are required for these facilities:

$$y_i^{RUD} \leq Cap_{M1}^N + \sum_{r_{M1}=1}^{|RO_{M1}|}\left(\sum_{j=1}^{i} rj_{M1}^{r_{M1}}\right).w_{M1}^{r_{M1}} \qquad (i = 1, 2, \dots, O_{M1}) \qquad (6\text{-}31)$$

$$z_i^{RUD} \leq Cap_{S1}^N + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}} \qquad (i = 1, 2, \dots, O_{S1}) \qquad (6\text{-}32)$$

It is clear that only one of the available options for the flexibility speed of each facility can be selected. Hence:

$$\sum_{r_{M1}=1}^{|RO_{M1}|} w_{M1}^{r_{M1}} = 1 \quad (6\text{-}33)$$

$$\sum_{r_{S1}=1}^{|RO_{S1}|} w_{S1}^{r_{S1}} = 1 \quad (6\text{-}34)$$

In the disrupted periods, the total product order received by *M1*, x^D, is as follows:

$$x^D = x_1^D + x_2^D \quad (6\text{-}35)$$

$$x_1^D = [\sum_{k=1}^{2} D_k(p, sl^D)].\, G_{R1}^{-1}\left(Max\ \left\{rl_{R1}^D, \frac{h^-}{h^- + h^+}\right\}\right) \quad (6\text{-}36)$$

$$x_2^D = [\sum_{k=3}^{4} D_k(p, sl^D)].\, G_{R2}^{-1}\left(Max\ \left\{rl_{R2}^D, \frac{h^-}{h^- + h^+}\right\}\right) \quad (6\text{-}37)$$

In these equations, x_1^D and x_2^D represent the orders issued by the first and second retailer respectively. As explained before, Equation (6-36) and (6-37) determine the ordering quantities of the retailers in a way to preserve their local reliabilities under disrupted conditions, rl_{R1}^D and rl_{R2}^D.

sl^D, rl_{R1}^D, and rl_{R2}^D represent the service level, local reliability of the first retailer, and local reliability of the second retailer during the disruption respectively. To preserve the local reliabilities of *M1* and *S1* under disruptions, rl_{M1}^D and rl_{S1}^D, the following equations are required:

$$rl_{M1}^D = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^i.\, y_i^{RUD} \le \sum_{i=1}^{O_{M1}} y_i^{RUD} - x^D\right) \quad (6\text{-}38)$$

$$rl_{S1}^D = \Pr\Big((\gamma_{S1}.T - 1).\sum_{i=1}^{O_{S1}} z_i^{RUD} + \sum_{j=1}^{O_{M1}} y_j^{RUD} \le \sum_{k=1}^{O_{S1}} \gamma_{S1}.t_k.\Big(Cap_{S1}^N + \sum_{r_{S1}=1}^{|RO_{S1}|}\Big(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\Big).w_{S1}^{r_{S1}}\Big)\Big) \quad (6\text{-}39)$$

Based on Equation (6-38), the sum of defective products in all the production runs of ramp-up period is less that its extra manufacturing quantity, $\sum_{i=1}^{O_{M1}} y_i^{RUD} - x^D$, with a probability of rl_{M1}^D. Equation (6-39) is used to ensure that the number of non-conforming components in all the production runs of *S1* during the ramp-up disruption period is less than its extra production quantity with a probability of rl_{S1}^D probability. Equation (6-39) is the simplified version of the following equation which is the extended version of Equation (6-13):

$$rl_{S1}^D = \Pr\left(\sum_{k=1}^{O_{S1}}\left(\frac{T.z_k^{RUD}}{Cap_{S1}^N + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{k} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}}} - t_k\right).\gamma_{S1}.\left(Cap_{S1}^N + \right.\right.$$

$$\left.\left.\sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{k} rj_{S1}^{k}\right).w_{S1}^{r_{S1}}\right) \le \left(\sum_{i=1}^{O_{S1}} z_i^{RUD}\right) - \left(\sum_{j=1}^{O_{M1}} y_j^{RUD}\right)\right) \quad (6\text{-}40)$$

Similarly to the without disruption conditions shown in Equation 6-14, the service level provided by *R1* and *R2* to its markets in the ramp-up disruption period is $rl_{S1}^D.rl_{M1}^D.rl_{R1}^D$ and $rl_{S1}^D.rl_{M1}^D.rl_{R2}^D$ respectively. Without loss of generality, we assume that same service level is provided for all markets which means that $rl_{R1}^D = rl_{R2}^D$. Hence rl_R^D represents the local reliability in both retail facilities. Assuming similar service levels makes it easier to analyze the relationship between service level, flexibility levels and flexibility speeds in the SN. Using this assumption, the service level of all markets in the disrupted conditions is:

$$sl^D = rl_{S1}^D.rl_{M1}^D.rl_R^D \quad (6\text{-}41)$$

The total profit that is captured in the ramp-up disruption period is:

$$\Psi^{RUD} = \left\{\left(P - h^{+}.E\left[G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^-+h^+}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^D)\right] + \left(P - h^{+}.E\left[G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^-+h^+}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=3}^{4} D_k(p, sl^D)\right]\right\}$$

$$\begin{aligned}
&- c_{S1}.\left(\sum_{i=1}^{O_{S1}} z_i^{RUD}\right) - c_{S1,M1}.\left(\sum_{i=1}^{O_{M1}} y_i^{RUD}\right) - c_{M1}.\left(\sum_{i=1}^{O_{M1}} y_i^{RUD}\right) \\
&- c_{M1,R1}.x_1^D - c_{M1,R2}.x_2^D \\
&- \sum_{i=1}^{O_{M1}} cap_{M1}^{i}.\left(\sum_{r_{M1}=1}^{|RO_{M1}|} ri_{M1}^{r_{M1}}.w_{M1}^{r_{M1}}\right) - \\
&\sum_{j=1}^{O_{S1}} cap_{S1}^{j}.\left(\sum_{r_{S1}=1}^{|RO_{S1}|} rj_{S1}^{r_{S1}}.w_{S1}^{r_{S1}}\right) \\
&- \sum_{i=1}^{O_{M1}} h_{M1}^{i}.\left(Cap_{M1}^{N} + \sum_{r_{M1}=1}^{|RO_{M1}|}\left(\sum_{j=1}^{i} rj_{M1}^{r_{M1}}\right).w_{M1}^{r_{M1}} - y_i^{RUD}\right) \\
&- \sum_{i=1}^{O_{S1}} h_{S1}^{i}.\left(Cap_{S1}^{N} + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}} - z_i^{RUD}\right)
\end{aligned} \tag{6-42}$$

Most of the terms in this function have been explained before, however the last four terms are new. The first two terms of these new terms represent the cost of adding capacity in the production runs of *M1* and *S1* respectively. The last two of these terms are related to unused capacity costs in *M1* and *S1* respectively.

<u>The normal disruption period (see Figure 6-7)</u>

If the disruption continues after the ramp-up disruption period, there is at least one normal disruption period. The capacities of *M1* and *S1* in all the production runs of this period

are $Cap_{M1}^{N} + O_{M1}.\Delta_{M1}$ and $Cap_{S1}^{N} + O_{S1}.\Delta_{S1}$ respectively. The total product order received by *M1* in the normal disruption period is similar to the ramp-up period:

$$x^D = x_1^D + x_2^D \tag{6-43}$$

$$x_1^D = [\sum_{k=1}^{2} D_k(p, sl^D)].G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) \tag{6-44}$$

$$x_2^D = [\sum_{k=3}^{4} D_k(p, sl^D)].G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) \tag{6-45}$$

Variables y^{ND} and z^{ND} represent the production quantity in the production runs of normal disruption period in *M1* and *S1* respectively. The production in each run of facilities must be less than their available capacities. Hence, the following restrictions are required for the facilities:

$$y^{ND} \leq Cap_{M1}^{N} + O_{M1}.\Delta_{M1} \tag{6-46}$$

$$z^{ND} \leq Cap_{S1}^{N} + O_{S1}.\Delta_{S1} \tag{6-47}$$

As discussed before, it is assumed that rl_{S1}^D, rl_{M1}^D and rl_R^D represent local reliabilities of the first supply path's supplier, manufacturer and retailers respectively during the disruption. To preserve these local reliabilities during normal disruption periods, the following equations are required:

$$rl_{M1}^D = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^i.y^{ND} \leq O_{M1}.y^{ND} - x^D\right) \tag{6-48}$$

$$rl_{S1}^D = \Pr\left(\sum_{i=1}^{O_{S1}}\left(\frac{T.z^{ND}}{Cap_{S1}^{N} + O_{S1}.\Delta_{S1}} - t_i\right).\gamma_{S1}.(Cap_{S1}^{N} + O_{S1}.\Delta_{S1}) \leq (O_{S1}.z^{ND}) - (O_{M1}.y^{ND})\right)$$

$$= \Pr\left((T.\gamma_{S1} - 1).O_{S1}.z^{ND} + O_{M1}.y^{ND} \leq \gamma_{S1}.(Cap_{S1}^{N} + O_{S1}.\Delta_{S1}).\sum_{i=1}^{O_{S1}} t_i\right) \quad (6\text{-}49)$$

The total profit that is captured in the normal disruption period is:

$$\Psi^{ND} = \left\{\left(P - h^{+}.E\left[G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^D)\right] + \left(P - h^{+}.E\left[G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=3}^{4} D_k(p, sl^D)\right]\right\}$$

$$- c_{S1}.(O_{S1}.z^{ND}) - c_{S1,M1}.(O_{M1}.y^{ND}) - c_{M1}.(O_{M1}.y^{ND})$$

$$- c_{M1,R1}.x_1^D - c_{M1,R2}.x_2^D$$

$$- \sum_{i=1}^{O_{M1}} h_{M1}.(Cap_{M1}^{N} + O_{M1}.\Delta_{M1} - y^{ND})$$

$$- \sum_{i=1}^{O_{S1}} h_{S1}.(Cap_{S1}^{N} + O_{S1}.\Delta_{S1} - z^{ND}) \quad (6\text{-}50)$$

<u>The ramp-down disruption period (see Figure 6-7)</u>

In the ramp-down periods, the disruption is terminated and the second supply path is available again to service its corresponding markets. During these periods the production plan is similar to the without disruption periods discussed before. The only difference is that there are extra production capacities and the corresponding cost components for the production facilities. Hence the total profit of a ramp-down period is:

$$\Psi^{RD} = \Psi^{N^*} - \sum_{i=1}^{O_{M1}} h_{M1}.\left[\sum_{r_{M1}=1}^{|RO_{M1}|}\left(\sum_{j=1}^{i} rj_{M1}^{r_{M1}}\right).w_{M1}^{r_{M1}}\right] -$$

$$\sum_{i=1}^{O_{S1}} h_{S1}.\left[\sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}}\right] \quad (6\text{-}51)$$

Ψ^{WD^*} is the best solution of the without disruption period model shown in Equations (6-8)-(6-17) which represents the highest profit that is captured during each without disruption period. The second and third terms of Equation 6-51 are the unused capacity costs in *M1* and *S1* respectively.

Mathematical model for flow planning in disrupted conditions

We define different scenarios for the length of disruptions. The number of normal disruption periods is different in these scenarios. Set $SCE = \{s\}$ includes all possible scenarios. In Figure 6-8, set SCE is assumed to include four scenarios, $\{s_1, s_2, s_3, s_4\}$. Scenario s_1 is the without disruption case. The rest of scenarios are as follows:

- ✓ **In Scenario s_2:** the disruption lasts only one period. Therefore, there is no normal disruption period. In this case, the planning horizon including four periods has one ramp-up, one ramp-down and two without disruption periods.
- ✓ **In Scenario s_3:** the disruption lasts two periods. Thus there is one normal disruption period. In this case, the planning horizon includes one ramp-up, one ramp-down, one normal disruption and one without disruption period.
- ✓ **In Scenario s_4:** the disruption lasts three periods and there are two normal disrupted periods. In this case, the planning horizon includes one ramp-up, one ramp-down and two normal disruption periods.

Each of these disruption scenarios, $s \in SCE$, occurs with a probability of pr_s. It is clear that:

$$\sum_{s=1}^{|SCE|} p_s = 1 \tag{6-52}$$

Parameters num_s^{WD}, num_s^{RUD}, num_s^{ND} and num_s^{RD} respectively show the number of without disruption, ramp-up, normal disruption and ramp-down periods in scenario *s*. Flexibility level decisions (represented by Δ_{M1} and Δ_{S1} variables) and flexibility speed decisions (represented by $w_{M1}^{r_{M1}}$ and $w_{S1}^{r_{S1}}$) in the first supply path's facilities should be made in a way to maximize the expected profit in all the possible disruption scenarios. Therefore, the objective function becomes:

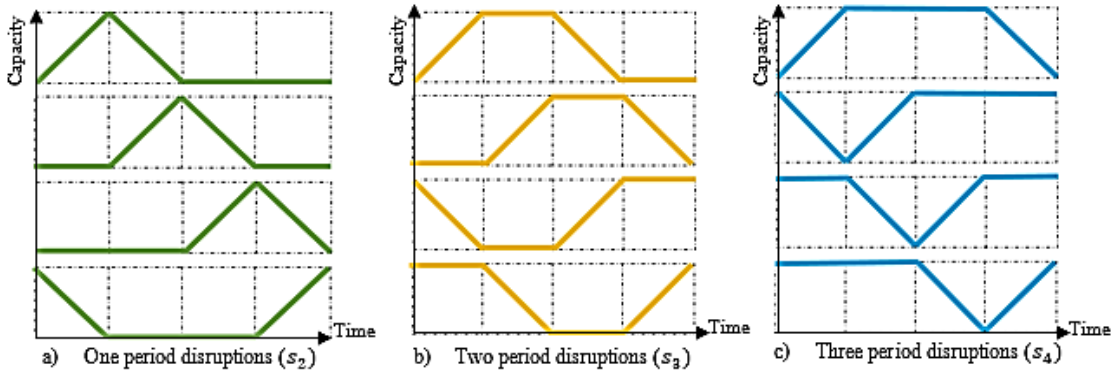


Figure 6-8: Sample scenarios for the length of disruption.

$$\textbf{\textit{Max}} \quad \Psi = \sum_{S=1}^{|SCE|} p_s.\left[num_s^N.\Psi^{N^*} + num_s^{RUD}.\Psi^{RUD} + num_s^{ND}.\Psi^{ND} + num_s^{RD}.\Psi^{RD}\right] \tag{6-53}$$

Subject to: (6-31)-(6-39), (6-41) and (6-43)-(6-49)

$$\Delta_{M1}, \Delta_{S1}, y_i^{RUD}, z_j^{RUD}, \ y^{ND}, z^{ND}, x^D, x_1^D, x_2^D, sl^D, rl_{S1}^D, rl_{M1}^D. rl_R^D \geq 0$$

$$(i = 1, 2, \dots, O_{M1} \quad and \quad j = 1, 2, \dots, O_{S1}) \qquad (6\text{-}54)$$

$$w_{M1}^{r_{M1}}, w_{S1}^{r_{S1}} \in \{0,1\} \qquad (\forall r_{M1} \in RO_{M1}, \forall r_{S1} \in RO_{S1}) \qquad (6\text{-}55)$$

The mathematical model of the disrupted conditions is a stochastic nonlinear programming similar to the model of without disruption periods. The objective function for this model and constraints shown in Equations 6-36, 6-37, and 6-41 are non-linear. The constraints in Equations 6-38, 6-39, 6-48 and 6-49 are stochastic and their forms depend on the probability distribution functions of the facilities and markets. This model is linearized using the approach described in Section 6.2.1.

Computational result: Extension of Test Problem

In this section, we extend the problem investigated in Section 6.2.1. We assume that disruption is possible in the second supply path in which the total order of the third and fourth markets, $\sum_{k=3}^{4} D_k(p, sl^D) . \varepsilon = [850 - 150 \times (p - 14) + 900 \times (sl^D - 0.85)]. \varepsilon$, is fulfilled by the first supply path. ε is a normal random variable with mean of 1 and variance of 1. Four different scenarios for the length of disruptions are possible in this problem, $SCE = \{s_1, s_2, s_3, s_4\}$. There is no disruption in Scenario s_1. Scenarios s_2, s_3 and s_4 represent disruptions with zero, one, and two normal disruption periods. The probabilities of these scenarios are assigned values of: $p_{s_1} = .83$, $p_{s_2} = .04$, $p_{s_3} = .10$ and $p_{s_4} = .03$.

The cost of adding unit capacity in each production run of *M1* is $cap_{M1}^1 = \$1$, $cap_{M1}^2 = \$0.8$, $cap_{M1}^3 = \$0.65$, and $cap_{M1}^4 = \$0.55$ respectively. The cost of adding unit capacity in the first, second, and third production run of *S1* is $cap_{S1}^1 = \$1$, $cap_{S1}^2 = \$0.7$

and $cap_{S1}^{3} = \$0.50$. The extra capacity cost in both *S1* and *M1* is $h_{S1} = h_{M1} = \$0.10$. The production and transportation cost components are similar to those in the Test Problem discussed in Section 6.2.1. The only new cost component is $c_{M1,R2} = \$0.70$. Based on the best production quantities of production runs in the Test Problem, we assume $Cap_{S1}^{N} = 800$ and $Cap_{M1}^{N} = 500$.

Five different flexibility speed options are assumed for the manufacturer as $r_{M1}^{1} = (r1_{M1}^{1} = 0, r2_{M1}^{1} = 0, r3_{M1}^{1} = 0, r4_{M1}^{1} = O_{M1}.\Delta_{M1})$, $r_{M1}^{2} = \left(r1_{M1}^{2} = 0, r2_{M1}^{2} = 0, r3_{M1}^{2} = O_{M1}.\frac{\Delta_{M1}}{2}, r4_{M1}^{2} = O_{M1}.\frac{\Delta_{M1}}{2}\right)$, $r_{M1}^{3} = \left(r1_{M1}^{3} = O_{M1}.\frac{\Delta_{M1}}{4}, r2_{M1}^{3} = O_{M1}.\frac{\Delta_{M1}}{4}, r3_{M1}^{3} = O_{M1}.\frac{\Delta_{M1}}{4}, r4_{M1}^{3} = O_{M1}.\frac{\Delta_{M1}}{4}\right)$, $r_{M1}^{4} = \left(r1_{M1}^{4} = O_{M1}.\frac{\Delta_{M1}}{2}, r2_{M1}^{4} = O_{M1}.\frac{\Delta_{M1}}{2}, r3_{M1}^{4} = 0, r4_{M1}^{4} = 0\right)$, and $r_{M1}^{5} = (r1_{M1}^{5} = O_{M1}.\Delta_{M1}, r2_{M1}^{5} = 0, r3_{M1}^{5} = 0, r4_{M1}^{5} = 0)$. Also for *S1*, five different flexibility speed options are considered as $r_{S1}^{1} = (r1_{S1}^{1} = 0, r2_{S1}^{1} = 0, r3_{S1}^{1} = O_{S1}.\Delta_{S1})$, $r_{S1}^{2} = \left(r1_{S1}^{2} = 0, r2_{S1}^{2} = O_{S1}.\frac{\Delta_{S1}}{3}, r3_{S1}^{2} = 2.O_{S1}.\frac{\Delta_{S1}}{3}\right)$, $r_{S1}^{3} = \left(r1_{S1}^{3} = O_{S1}.\frac{\Delta_{S1}}{3}, r2_{S1}^{3} = O_{S1}.\frac{\Delta_{S1}}{3}, r3_{S1}^{3} = O_{S1}.\frac{\Delta_{S1}}{3}\right)$, $r_{S1}^{4} = \left(r1_{S1}^{4} = 2.O_{S1}.\frac{\Delta_{S1}}{3}, r2_{S1}^{4} = O_{S1}.\frac{\Delta_{S1}}{3}, r3_{S1}^{4} = 0\right)$ and $r_{S1}^{5} = (r1_{S1}^{5} = O_{S1}.\Delta_{S1}, r2_{S1}^{5} = 0, r3_{S1}^{5} = 0)$.

The mathematical model of this problem is formulated and solved on a Intel(R)Core(TM)4 Duo CPU, 3.6 GHz, with 12276 MB RAM using the default settings. CPLEX is used to solve the linearized mathematical model of the problem. Solving the model of this problem leads to the following results: the best service level for the disruption condition is 80 percent and it's the best supporting local reliability combination

is $rl_{S1}^{D} = 1$, $rl_{M1}^{D} = 1$ and $rl_{R}^{D} = 0.8$. To preserve these local reliabilities, the required flexibility level in *S1* and *M1* is $O_{S1}.\Delta_{S1} = 555.2$ and $O_{M1}.\Delta_{M1} = 634.9$ respectively. The best flexibility speed to ramp up capacity in *M1* is $w_{M1}^{3} = 1$ which means uniform capacity scalability is preferred for this facility. The best flexibility speed to ramp up capacity in *S1* is $w_{M1}^{5} = 1$ which means that all the extra capacity is added at the beginning of the first production run after disruption. Ordering and production quantities in the production runs of the first supply path's facilities are represented in the ramp-up and without disruption periods in Figure 6-9 and 6-10 respectively.

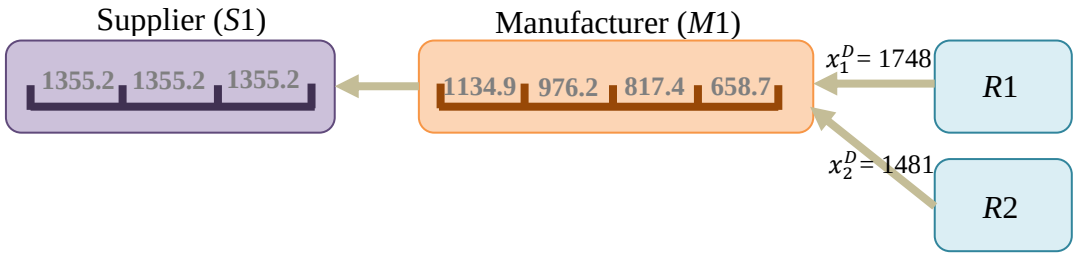


Figure 6-9: Flow dynamics in the first supply path during the ramp-up period.

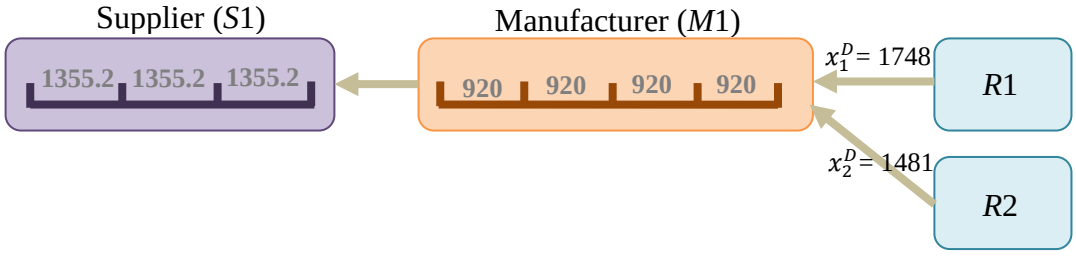


Figure 6-10: Flow dynamics in the first supply path during the without disruption period.

The average profit of the first supply path with respect to the service level under disruption is displayed in Figure 6-11. Comparing Figures 6-3 and 6-11 it can be seen

that the profit reduction on both sides of the most profitable service level point is gentler in disruption in comparison with normal condition. This gentler reduction is due to: i) the higher potential demand assigned to this path during the disruption in which the first supply path services the first, second, third and fourth markets and ii) the lower sensitivity of the third and fourth markets with respect to the service level.

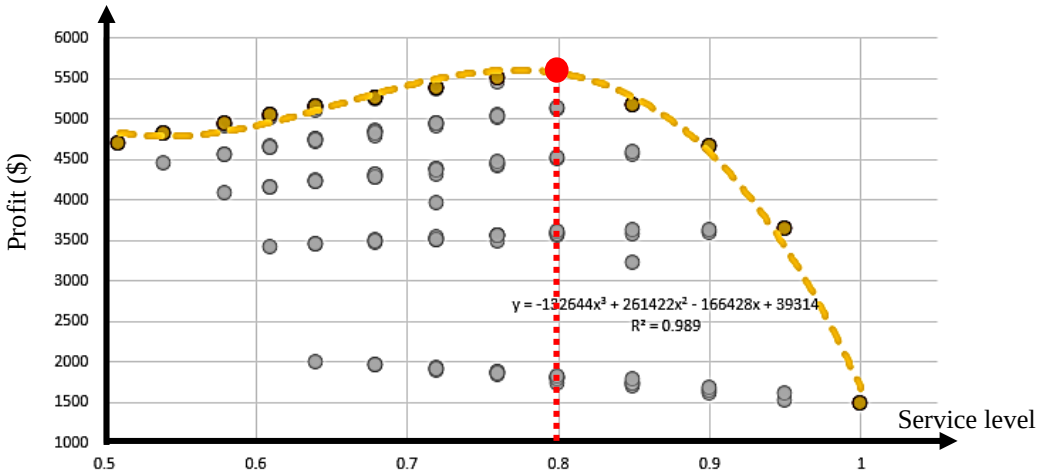


Figure 6-11. Average profit of the first supply path with respect to the service level under disrupted conditions.

In this problem, there are three important indices managing the behavior of the SN against uncertainties:

I) **Robustness** of the SN's network against disruptions: this characteristic of the SN is managed by the flexibility levels of its facilities,

II) **Resilience** of the SN's network against disruptions: this characteristic of the SN is managed by the flexibility speeds of its facilities,

III) **Reliability** of flow dynamics throughout the SN's network against demand- and supply-side variations: this characteristic of the SN is managed by the local reliabilities assigned to its stochastic facilities.

In the rest of this section, the correlations among these three indices are investigation.

Correlation between robustness and resilience of the supply network

First we start with analyzing the relationship between the flexibility level and the flexibility speed assigned to the SN's flexible facilities, *M1* and *S1*. We solve the mathematical model of the problem for different values of the service level and different local reliabilities of facilities supporting these service levels. As expected, by increasing the local reliability of the retailer, more products are ordered in the first supply path and consequently greater extra capacity, flexibility levels, is needed in its facilities if a disruption occurs. Hence, the flexibility level of the facilities start to increase. In the output of the model we follow the trend of changes in the flexibility speed of facilities to determine whether there is a correlation between the flexibility level of facilities and their flexibility speed. The results are summarized in Figure 6-12.

In Figure 6-12, the trends of changes in the resilience of *S1* and *M1* with respect to their flexibility levels are displayed for different values of the local reliabilities in the retailers. For instance, in 80 percent local reliability in the retailers, when flexibility level of *S1*, $O_{S1}.\Delta_{S1}$, is less than 70 (capacity units), its selected flexibility speed option is rl_{S1}^{5}. This means the most rapid ramp-up, high flexibility speed, is selected for this facility. But in the cases that $70 \leq O_{S1}.\Delta_{S1} < 153$, the flexibility speed of this facility reduces to rl_{S1}^{4}.

By increasing $O_{S1}.\Delta_{S1}$ to more than 153, the flexibility speed of this facility reduces more to rl_{S1}^{3}. The other bars of this figure are interpreted in the same way.

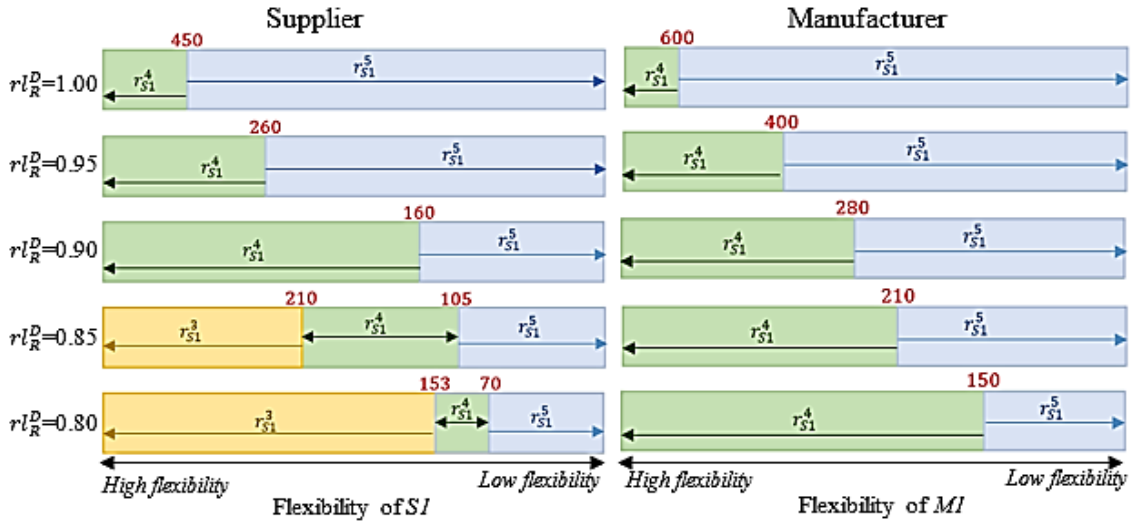


Figure 6-12: Correlation between flexibility level of facilities and their flexibility speeds (each color is corresponding to one flexibility speed option).

Based on the results summarized in Figure 6-12, we conclude:

- For a given product order quantity (local reliability of retailers), when the flexibility level in a facility's capacity is low, higher flexibility speed is generally preferred for that facility. This means that lower required extra capacities are mainly added in the early production runs after disruptions. But when the required flexibility level increases, part of it should be assigned to the later production runs to avoid the high cost of adding capacity in the early production runs. Adding more flexibility leads to greater usage of late production runs to add extra capacity, this is less flexibility speed. Hence for a given product order quantity,

there is a negative correlation between the flexibility level and the flexibility speed of facilities. Summing up for all the facilities in the SN, *higher robustness leads to lower resilience in profit-based SNs*. This tradeoff between robustness and resilience should be considered in designing/redesigning profit –based SNs.

- By increasing product order quantity (local reliability of retailers), the flexibility levels differentiating each subsequent pair of flexibility speed options in the facilities increment. Red numbers in Figure 6-12 represent these differentiating flexibility levels. For instance for 80 percent local reliability in the retailers, the flexibility level of *S1* differentiating rl_{S1}^{5} and rl_{S1}^{4} resilience options is equal to 70 (capacity units). But by increasing the retailers' local reliability to 85 percent, this differentiating flexibility level increments to 105 (capacity units). This means that higher production rates make the facility's flexibility speed more stable against the flexibility levels of its capacity. To change the flexibility speed of this facility, more flexibility level increment is required. Summing up for all the facilities in the SN, larger SNs with higher production rates are able to absorb higher levels of flexibility level in their facilities without changing their flexibility speed. Higher flexibility level in facilities means higher robustness in the SN. Therefore, *tradeoff of robustness and resilience is more stable for larger SNs with higher production rates*.

Correlation between flexibility levels and local reliabilities

For different values of local reliabilities in the stochastic facilities, *S1*, *M1*, and *R*s, we solve mathematical model (6-53)-(6-55) and find the best flexibility levels assigned to *S1*

and *M1*. In Figures 6-13 and 6-14, we respectively represent the flexibility levels of *M1* and *S1* with respect to the local reliabilities of the first supply path's stochastic facilities. Analyzing the graphs of Figure 6-13 and 6-14 leads to some new managerial insights which are summarized as follows:

- Based on Figure 6-13, increasing the local reliability in the retailers leads to higher flexibility in the production capacities of *M1* and *S1*. Higher reliability in the retailers leads to higher product ordering quantity in the first supply path and fulfilling this higher demand requires higher capacities in its flexible facilities.
- Based on Figure 6-13, increasing local reliability in *S1* leads to higher flexibility levels in *M1*. This means that regardless of the local reliability assigned to *M1*, there is a positive correlation between the local reliability of *S1* and flexibility level of *M1*. Comparison of *M1*'s flexibility level increments due to increase in the local reliability of the retailers and *S1*, it is concluded that increasing the reliability of the retailers imposes more flexibility level to *M1*.
- Based on Figure 6-14, increasing local reliability in *M1* leads to higher flexibility levels in *S1*. This means that regardless of the local reliability assigned to *S1*, there is a positive correlation between the local reliability of *M1* and flexibility level of *S1*. Comparison of *S1*'s flexibility increments due to increase in the local reliability of the retailers and *M1*, it is concluded that increasing the reliability of the retailers imposes more flexibility level to *S1*.
- Based on Figures 6-13 and 6-14, higher local reliabilities in *M1* and *S1* respectively lead to higher flexibility levels in *M1* and *S1*. But these flexibility

level increments are much less than the extra flexibilities imposed by increasing the local reliability of the retailers. All of these outcomes reveal that increasing the local reliability of the retailers leads to more significant increments in the flexibilities of the path's facilities.

Based on the abovementioned points, we conclude that in stochastic SNs there is a positive correlation between the local reliabilities of the stochastic facilities and the flexibility levels must be added to the facilities to make their networks robust.

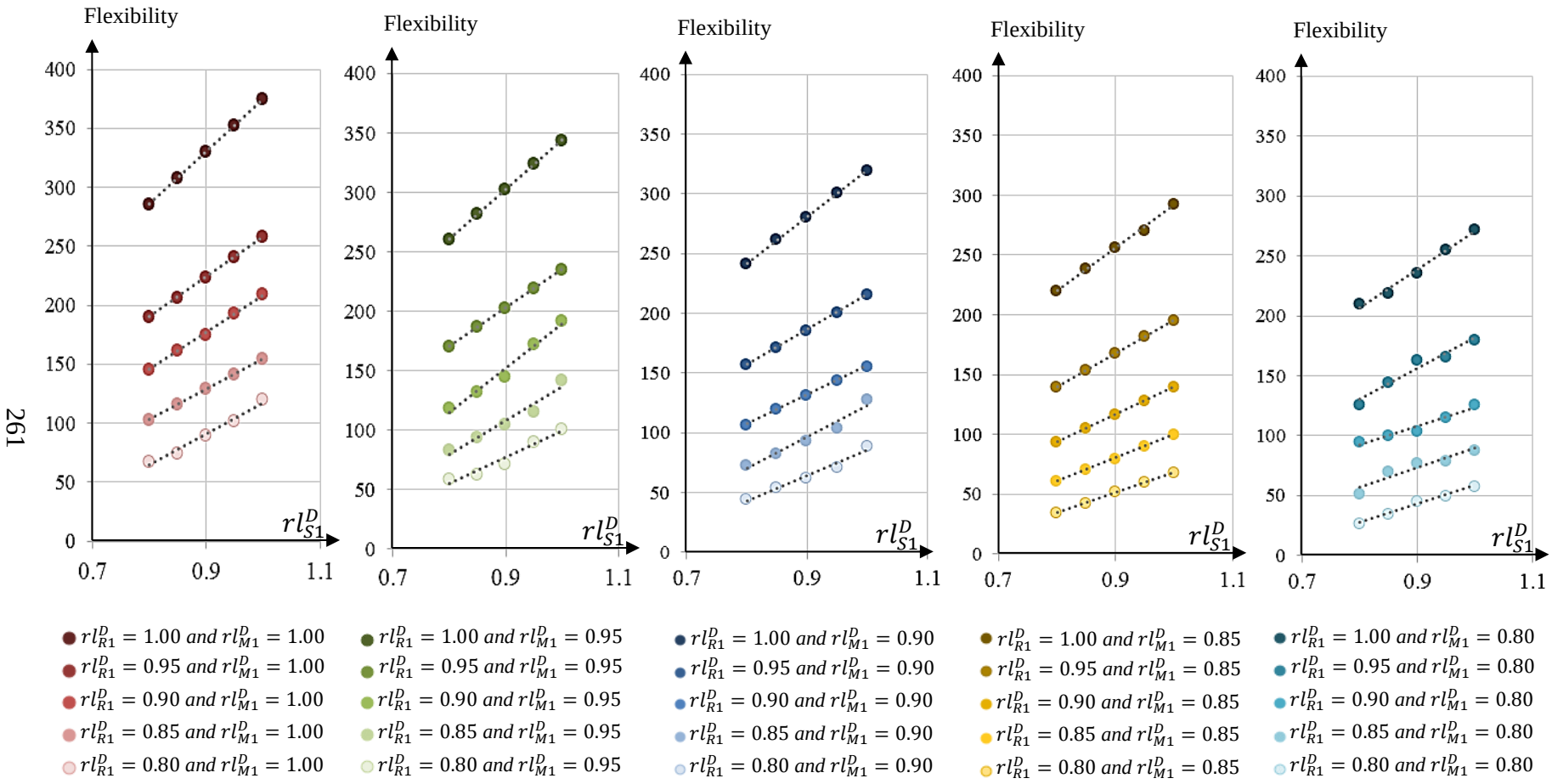


Figure 6-13: Flexibility of *M1* with respect to the local reliabilities of facilities.

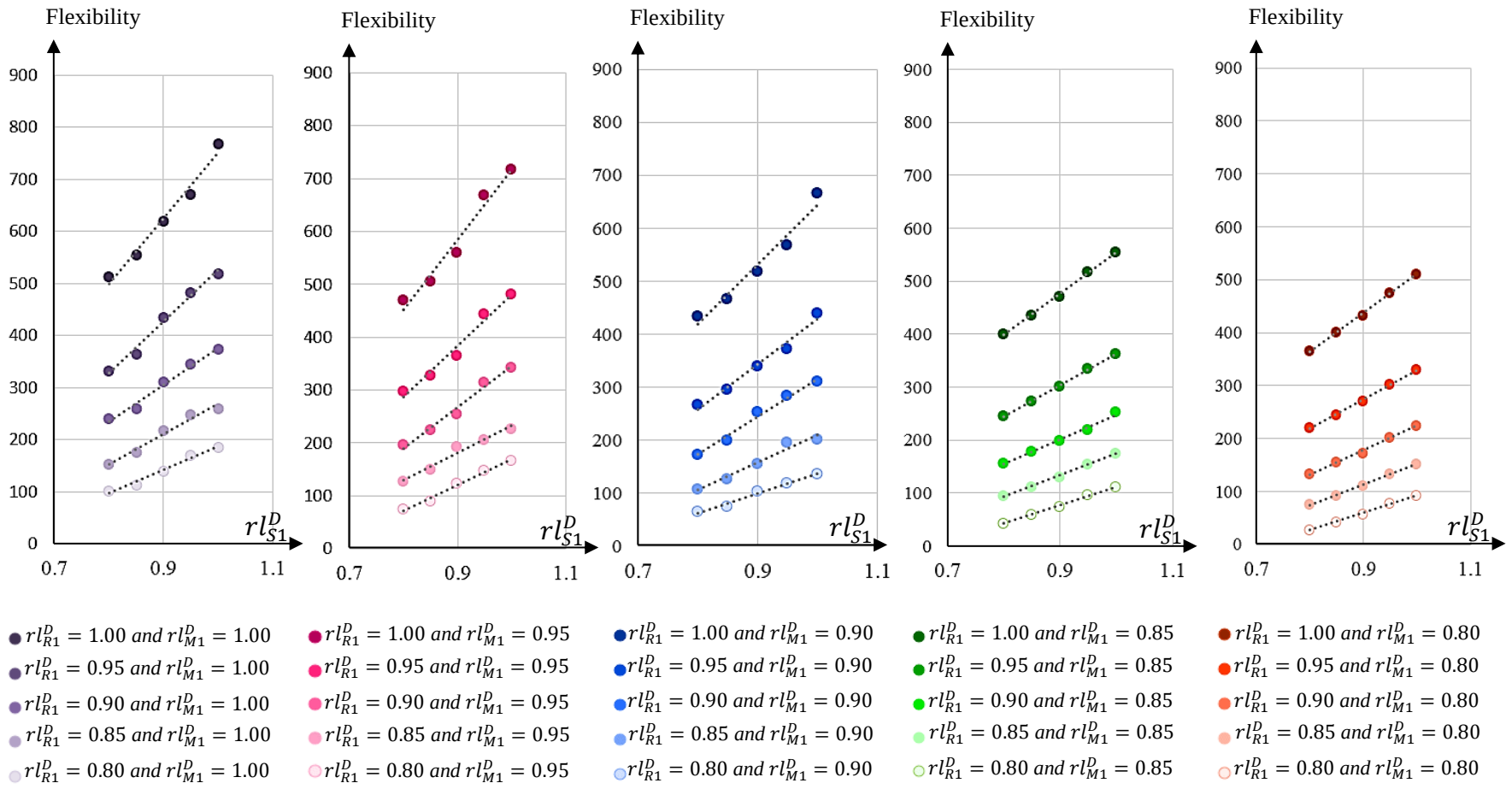


Figure 6-14: Flexibility level of *S1* with respect to the local reliabilities of facilities.

6.3. Closure of chapter 6

In this chapter, we show that being "Operationally Fail-safe" against variations is not enough for having fail-safe SNs. There is another group of uncertainties called disruptions. Disruptions are large enough to change the topology of SNs by inactivating a subset of its facilities (nodes or links). By investigating the effects of disruptions on SNs, we answer the following question in this chapter:

- ✓ **Research Question 5:** what are the necessities of having fail-safe SNs?

In Section 6.1, we answer this question and show that the topology of SNs should be designed / redesigned in a way to be able to handle disruptions appropriately. This new characteristics of SNs is called "Structurally Fail-safe". By analyzing the necessities of being "Structurally Fail-safe" in Section 6.2.2, we answer the following question:

- ✓ **Research Question 6**: what are the characteristics of fail-safe SNs against disruptions – characteristics of structurally fail-safe SNs?

In section 6.2.2, we answer this question as follows:

- ✓ The topology of a structurally fail-safe SN should be "**Robust**" against disruptions: Robustness means appropriate amount of risk mitigation strategies should be incorporated in the topology of SNs to reduce their vulnerability after disruptions.
- ✓ The topology of a structurally fail-safe SN should be "**Resilient**" against disruptions: Resilience means SNs should be agile enough in employing risk mitigation strategies to reduce their loss in the transient period from normal to disrupted flow plan.

We show that the stability of a SN's topology against disruptions not only depends on its pre-disruption robustness in incorporating an appropriate mitigation strategy, but also is determined by its post-disruption resilience in employing this strategy. Having a robust and resilient topology against disruptions is necessary but not enough to preserve an appropriate performance for a SN. A successful SN needs to have a reliable flow dynamics throughout its network against variations. We show that the robustness and resilience of the SN depend on the flexibility levels and ramp-up speeds of its facilities respectively. To quantify these relationships, two stochastic, nonlinear, and mixed integer mathematical models are developed to determine the most profitable flexibility levels (first secondary research question) and ramp-up speeds (second secondary research question) for the network's facilities and reliable flow dynamics throughout its network. Reliable flow planning preserves the highest profit for the network by selecting the best service level and supporting local reliabilities in the stochastic facilities. Computational analysis of the models leads to the following insights:

About redesigning robust and resilient network for the SN

- For a given product order quantity, there is a negative correlation between the flexibility level of each facility and its resilience. This means that longer ramp-up times are more profitable for facilities with larger flexibility levels and vice versa. Summing up on all the SN's facilities, we conclude that there is a tradeoff between a SN's robustness and its resilience.

- By increasing production quantity in a facility, the minimum required flexibility in that facility to increase its ramp-up time becomes larger. Summing up on all the SN's

facilities, larger SNs with higher production rates are able to absorb higher levels of flexibility before reducing their resilience. This means that the tradeoff of robustness and resilience is more stable for larger SNs.

- There is a positive correlation between the local reliability of each stochastic facility and its flexibility level. Also increasing the reliability of each facility positively affects the flexibility levels of the other facilities in the network. This means that in stochastic SNs there is a positive correlation between the local reliabilities of the stochastic facilities and the flexibility levels must be added to the facilities to make their networks robust. SNs with higher reliability in their flow need more flexibility to be robust.

About planning reliable flow dynamics for the SN

- For a given product order quantity (local reliability of the retailers), the effect of a stochastic facility's local reliability on the SN's profit is not significantly influenced by the reliabilities of the other facilities. This outcome highlights that independent local reliability selection for the SN's stochastic facilities leads to a good (not the best) solution. But this independent reliability selection significantly decreases the size of the model and its computational time.

In this chapter, we only consider one risk mitigation strategy, having flexible capacity, to redesign a robust network. However this work can be extended by incorporating other risk mitigation strategies such as holding emergency stocks in the SN or having back-up facilities.

Chapter 7: Closure

7.1. A summary of the dissertation

In this dissertation, we deal with architecting "Fail-safe" supply networks. A fail-safe network is one which mitigates the impact of uncertainties and provides an acceptable level of service. This is achieved by controlling its topology (structurally fail-safe) and coordinating the flow (operationally fail-safe) through the facilities. In this dissertation, we show that to have a structurally fail-safe supply network, its topology should be robust against disruptions – large scale unexpected events – by positioning mitigation strategies and be resilient in executing these strategies. Also we show that to have an operationally fail-safe supply network, its flow dynamics should be reliable against demand- and supply-side variations – small scale expected events.

In Chapter 1, we review the literature of supply chains / networks from 1) flow planning; and 2) uncertainty management perspectives. We show that considering supply chain / network relationships among after-sales operations and interactions of the forward and after-sales chains / networks are the important gaps of the pertinent literature which are fulfilled in this dissertation by concurrent flow planning of these two chains / networks.

In the uncertainty management literature, considering both disruptions and variations to respectively have structurally and operationally fail-safe supply chain / network is a critical gap. Also in the variation literature, it is mainly assumed that the performance of the facilities in the chain / network is perfect and deterministic. This means supply-side uncertainties in the output of production facilities are ignored. However, there is not any perfect production system in reality. Considering both demand- and supply-side

uncertainties not only improve service level estimation in the operational level but also improves the reliability of the flow dynamics in its coordination process.

Chapter 2 of this dissertation is dealing with planning reliable flow dynamics in a forward supply chain / network in the presence of demand- and supply-side variations (being operationally fail-safe – Research Question 7). This reliable flow preserves the most profitable service level in the chain / network in the presence of uncertainty in the performance of facilities and demand of markets (Figure 7-1). In Chapter 3, we extend the problem of Chapter 2 to include both forward and after-sales supply chains (Figure 7-1). Modeling the interaction of these two chains is the important part of this reliable flow planning problem (Research Questions 1, 2, and 3).

In Chapter 4, we consider the possibility of repairing defective parts in the after-sales operation (Research Questions 1, 2, and 4). In this case, two flow types exist in the after-sales chain: the flow of new parts and the flow of repaired parts (Figure 7-1). The problem of Chapters 3 is extended in Chapter 5 from supply chains to supply networks (Research Questions 1, 2, and 3). Due to increasing the size of the problem, a special solution approach is developed to handle the larger mathematical model of this problem (Figure 7-1). In Chapter 6, not only a reliable flow is planned through the supply network (being operationally fail-safe) but also disruption possibility in the network's facilities is considered (being structurally fail-safe – Research Questions 5 and 6).

In this section, we summarize the six problems solved in this dissertation along with their assumptions, new knowledge, and key managerial insights.

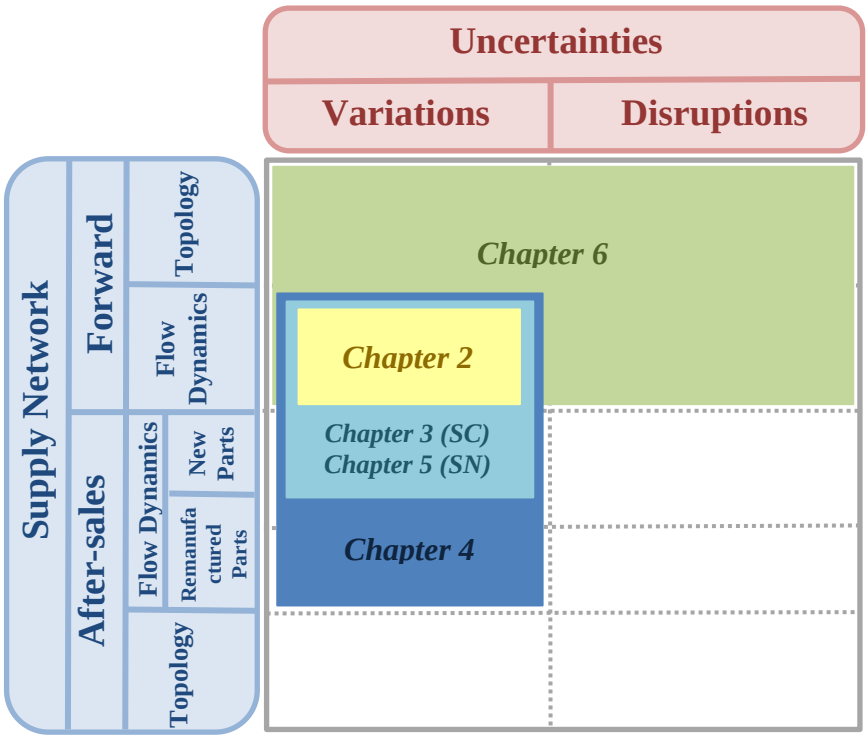


Figure 7-1: Boundary of the six problems solved in this dissertation.

7.1.1. Operationally fail-safe supply chains / networks (Chapter 2 – Research question 7)

- ***Problem description***

In this chapter, first we consider a supply chain including a supplier, a manufacturer, and a retailer servicing a market. The performance of the production systems inside the supplier and manufacturer is not perfect and is along with stochastic percentage of non-conforming components and defective products respectively. This means the qualified output of these facilities is stochastic. Also the demand of the market should be fulfilled by the retailer is stochastic and follows a given density function.

In such a supply chain with stochastic facilities and market demand, we want to determine the most profitable service level and its supporting reliable flow dynamics throughout the chain. Finally, we extend the problem to supply networks with more than one facility in each echelon as well.

- ***Outcomes of the chapter***

We show that in supply chains / networks with stochastic facilities, the uncertainties propagate and the qualified supply quantities depreciate by moving flow from upstream to downstream. We develop a method to quantify the qualified flow depreciation and service level estimation in the chain / network. This method amplifies the order quantities between the chain's / network's facilities from downstream to upstream. By the help of this method, we develop two mathematical models to find the post profitable service level and its supporting reliable flow dynamics in the supply chains / networks respectively. In this problem, we quantify the following relationships / models in supply chains / networks with stochastic facilities:

- How much the order quantities should be amplified from downstream to upstream of the supply chains / networks to neutralize the negative effect of the flow depreciation;

- Relationship between the service level of the chain / network and the local reliabilities of its stochastic facilities;

- Relationship between the local reliabilities of the facilities and their flow dynamics;

- Mathematical models selecting the most profitable service level and its supporting reliable flow dynamics in supply chains / networks.

➢ ***Managerial insights***

Using the computational results of the developed mathematical models, we conclude the following insights:

- In supply chains / networks with stochastic facilities, service level in downstream is a function of local reliabilities in the upstream facilities.

- In supply chains / networks with stochastic facilities, finding the best service level is not enough. We need to determine the least costly local reliability combination in the stochastic facilities supporting that service level as well.

7.1.2. Operationally fail-safe supply chains servicing pre- and after-sales markets (Chapter 3 – Research questions 1, 2, and 3)

➢ ***Problem description***

In this chapter, a company is considered that produces and supplies its products to the customers of a market under a failure-free warranty. Hence, producing and providing enough spare parts to repair the returned products of the customers inside the warranty time is an important responsibility of this company. While the product is produced through the forward supply chain,

the required spare parts for repairing its failures are produced through the after-sales supply chain. Here concurrent flow planning for these supply chains considering their strong interactions and convoluted sources of demand- and supply-side uncertainty has been done.

- ***Outcomes of the chapter***

In this chapter, we show that there are some important interactions between the operations of the forward and after-sales supply chains. Two important interactions considered in this chapter are: i) the service level provided by the after-sales supply chain directly affects the product demand in the pre-market of the forward supply chain; and ii) the after-sales demands are a function of total products supplied to the market through the forward supply chain. These relationships are quantified in this problem. Using these relationships, a mathematical model is developed for the problem. Using this model, we make the following decisions concurrently:

- The best retail price, warranty length, and service levels for the company in its pre- and after-sales markets to maximize the company's total profit;

- The appropriate local reliabilities in the echelons of the forward and after-sales supply chains and their corresponding reliable flow planning to preserve the company's service levels.

➢ ***Managerial insights***

Using the computational results of the developed mathematical model, we conclude the following insights:

- ***Effect of the retail price on the profitability of the warranty options:*** Price increments or reductions may have non-homogeneous effects on the profit of the company in a given warranty length. But the trend of these changes are almost similar for all warranty options.

- ***Priority of the warranty options in different price intervals:*** Priority of the warranty options with respect to profit changes in the critical price values. Therefore in the price intervals between sequential critical price values, they have different priority (or profitability order).

- ***Importance and stability of the warranty options in price-sensitive markets:*** In price sensitive markets, an inappropriate selection of the warranty length leads to higher profit loss. This means that the appropriate warranty length selection is more important in price-sensitive markets. However, the priority of the warranty options from the profit perspective is more fragile with respect to price variations in these market.

- ***Optimal price and stability of the warranty options in warranty-sensitive markets:*** In warranty-sensitive markets, optimal retail prices are higher and the priority of the warranty options from the profit perspective is more stable with respect to price variations.

7.1.3. Operationally fail-safe supply chains servicing pre- and after-sales markets for repairable products (Chapter 4 – Research questions 1, 2, and 4)

- ***Problem description***

In this problem, we consider a company producing and supplying a product to a target market through its forward supply chain including suppliers, a manufacturer, and a retailer. This product is sold to the customer under a retail price and a warranty period. All the defective products returned by the customers inside the warranty period should be fixed free of charge. Spare parts required to fix these returned products are provided through an after-sales supply chain. The after-sales supply chain has remanufacturing sections inside the suppliers to repair the failed components of the returned products. Defective components are sent by the retailer to the remanufacturing sections to get repaired. Then, the remanufactured components are sent back and stored in the retailer to be used in the repair process of the next defective products.

In the cases there is not any available remanufactured component in the retailer, new components provided and stored by the suppliers in the retailer are used to do repairs. Storage of new components in the retailer preserves an appropriate service level for the after-sales supply chain. The required products and new components to fulfill the product demand and inside-warranty repair requests of each sale period are produced by the forward and after-sales supply chains respectively and stored in the retailer before its beginning. In this problem there are two flow types in the after-sales supply chain: i) the flow of new

components from the suppliers to the retailer; and ii) the flow of defective and remanufactured components between remanufacturing sections and retailer. Again two groups of demand- and supply-side uncertainties are considered in this problem.

For this company, we want to determine the most profitable marketing strategies and supporting flow dynamics in the forward and after-sales supply chain in the presence of demand-side uncertainties (product demand in the pre-market and components demands in the after-sales markets) and supply-side uncertainties (in the performance of the manufacturing systems in the production facilities).

- ***Outcomes of the chapter***

In this chapter, we model the performance of the remanufacturing sections of the suppliers to quantify the flow of remanufactured components in the after-sales supply chain. Then we determine the relationship between the after-sales service level and the flow of new components required in the after-sales supply chain. Based on these equations, a mathematical model is developed for the problem. This model makes the following decisions concurrently:

- The best retail price, warranty length, and service levels for the company in its pre- and after-sales markets to maximize the company's total profit;

- The appropriate local reliabilities in the echelons of the forward and after-sales supply chains to preserve the company's service levels;

- Reliable flow dynamics in the forward and after-sales supply chains (including remanufacturing sections) preserving the local reliabilities of their facilities.

➢ ***Managerial insights***

Applying the mathematical model for an example in the automobile industry, we conclude the following insights:

✓ **Effect of warranty length on the trend of profit changes with respect to the price**: In different warranty options, the behavior of the profit function with respect to the price is almost similar but only shifts to right by the warranty length increment. This means changing the warranty length does not change the price effect on the company's profitability

✓ **Effect of warranty length on the trend of profit changes with respect to the service level**: The behavior of the profit function with respect to the service level is almost similar for all the warranty options without any significant shift to the left or right. This means all of these profit functions have almost the same optimal service level. Therefore, finding the best service level for one warranty option gives us a good approximation about the best service level for the other options. This reveals there is a very weak correlation between the warranty length and service level and they can be selected separately.

- ✓ **Effect of warranty length on the correlation between the price and service level:** The trend of the price and service level correlation is almost similar for different warranty options. Increasing the warranty length only shifts the price and service level function to the right. This means regardless of the warranty length, a given increment in the service level (price) leads to almost the same increment in the price (service level). However the ratio of the best price increment to the best service level increment decreases in higher prices.

7.1.4. Operationally fail-safe supply networks servicing pre- and after-sales markets (Chapter 5 – Research questions 1, 2, and 3)

- ➢ ***Problem description***

The problem of Chapters 3 is extended in this chapter from supply chains to supply networks (including more than one facility in each echelon). Due to increasing the size of the problem, a special solution approach is developed to handle the larger mathematical model of this problem.

- ➢ ***Outcomes of the chapter***

In this section, we show that a supply network can be represented as a set of paths. Each path starts from a set of suppliers in the first echelon (one supplier for each component), passes through a manufacturer in the intermediate echelon, and ends at a retailer in the last echelon. Using path concept not only generalizes our model to be applicable for any networks with any structures, but also helps us to be able to use the model of Chapter 3 which was developed for a supply chain. Each path of the supply network problem can be interpreted as a supply chain. Here, we

extend the model of Chapter 3 including one path to a set of paths called a network. This model determines the most profitable marketing strategies for the company and the least costly flow dynamics throughout its networks preserving the marketing strategies.

The model is a mixed integer nonlinear mathematical model. Solving this kind of models is not straightforward. Especially the form of nonlinear terms in this model depends on the cumulative distribution functions defined for the stochastic parts of the problem. This means that by changing these distribution functions, the mathematical forms of these terms also change. In this chapter, we propose an efficient approach to solve this model and find the best solution. Finally the model is tested on a test problem defined in engine industry.

- ***Managerial insights***

Applying the mathematical model for the test problem in the automobile industry, we conclude the following correlations among the marketing strategies:

Correlation between the price and warranty:

- ✓ The correlation between the price and warranty becomes tighter by increasing the service levels. In higher service levels, the priority of the warranty options stays stable for a smaller price interval and is more sensitive with respect to price variations.
- ✓ By increasing the service levels, the overlaps among the profit functions decrease and they become more separate. This means the feasible range of price is divided to some more distinct intervals in each only one warranty

option is profitable. Therefore in higher service levels, the positively profitable warranty options available in each price value for managers to select is less.

Correlation between the service levels and warranty:

The priority of the service level options is very stable and is not affected easily by warranty variations. In the engine problem, the warranty-service level tradeoff is much more stable than the price-warranty tradeoff. However the stability of the warranty-service level tradeoff may change by increasing the service level sensitivity parameter in the demand function.

Correlation among the three marketing factors:

In a given warranty length option, the best price strategy is increasing with respect to the service level but the trend of this increment is different for warranty options. In shorter warranty lengths, the rate of price increment is a convex increasing function of the service levels. But this function tends to become a linearly increasing and then a concave increasing by the warranty length increment.

In the same way for a given service levels option, the best price strategy is increasing with respect to the warranty length but the trend of this increment is different for service level options. In lower service levels, the rate of price increment is a convex increasing function of the warranty length. But this function tends to become a linearly increasing and then a concave increasing by the increment in the service levels.

7.1.5. Operationally and structurally fail-safe supply networks (Chapter 6– Research questions 5 and 6)

- ***Problem description***

A fail-safe network is one which mitigates the impact of uncertainties and provides an acceptable level of service. This is achieved by controlling its topology (structurally fail-safe) and coordinating the flow (operationally fail-safe) through the facilities. In this chapter we show that to have a structurally fail-safe supply network, its topology should be robust against disruptions – large scale unexpected events – by positioning mitigation strategies and be resilient in executing these strategies. Considering "*Flexibility*" as a risk mitigation strategy, we answer the question "*What are the best flexibility levels and flexibility speeds for facilities in structurally fail-safe supply networks*?" Also we show that to have an operationally fail-safe supply network, its flow dynamics should be reliable against demand- and supply-side variations – small scale expected events. In the presence of these variations, we answer the question "*What is the most profitable flow dynamics throughout the supply network which is reliable against variations?"*

- ***Outcomes of the chapter***

In addition to operational level variations, in this chapter we also consider the possibility of disruptions in the SN's facilities. In this SN, one of manufacturers is completely reliable but the other is prone to disruption. Prone to disruption manufacturer may be unavailable and unable to fulfill the orders of its

corresponding retailer. There are several reasons that this can occur, e.g., the failure of its machinery or the inability of its supplier to procuring material therefore being unable to supply ordered components. In the unavailability of this manufacturer, the markets of its corresponding retailer are lost which leads to a huge loss in the SN's profitability and brand reputation.

To avoid this possible loss and to improve the stability of the SN, we want to redesign a robust network for the SN. To have a robust network, we want to modify the production capabilities of its reliable facilities to be able to compensate for the unavailability of its unreliable facilities. For this purpose, the production capacities of reliable facilities should be flexible enough to be ramped up, when needed, to compensate for the unavailability of unreliable facilities and be ramped down when the unreliable facilities are available. In this problem, *we want to determine the best flexibility levels in the reliable facilities to redesign a robust network.* Redundancy in the capacity of reliable facilities is the risk mitigation strategy used to have a robust network.

The agility of the flexible facilities is ramping up their capacities after disruption, is measured by an index called resilience. Resilience of the SN in employing the redundancy mitigation strategy depends on the speed of its flexible facilities in ramping up their capacity after disruption. Therefore, *the other important decisions made in this problem are the best flexibility speeds in the reliable facilities to redesign a resilient network.*

In this chapter, we show that the stability of a SN's topology against disruptions not only depends on its pre-disruption robustness by incorporating an appropriate mitigation strategy, but also is determined by its post-disruption resilience in employing this strategy. Having a robust and resilient network is necessary but not enough to preserve an appropriate performance for the SN. A successful SN needs to have reliable flow dynamics throughout its network against operational variations. We show that the robustness and resilience of the SN depend on the flexibility levels and ramp-up speeds of its facilities respectively. To quantify these relationships, two stochastic, nonlinear, and mixed integer mathematical models are developed to determine the most profitable flexibility levels and ramp-up speeds for the network's facilities and reliable flow dynamics throughout its network. Reliable flow planning preserves the highest profit for the network by selecting the best service level and supporting local reliabilities in the stochastic facilities.

- ***Managerial insights***

Computational analysis of the models leads to the following insights:

<u>About redesigning robust and resilient network for the SN</u>

- For a given product order quantity, there is a negative correlation between the flexibility level of each facility and its resilience. This means that longer ramp-up times are more profitable for facilities with larger flexibility levels and vice versa. Summing up on all the SN's facilities, we conclude that there is a tradeoff between a SN's robustness and its resilience.

- By increasing production quantity in a facility, the minimum required flexibility in that facility to increase its ramp-up time becomes larger. Summing up on all the SN's facilities, larger SNs with higher production rates are able to absorb higher levels of flexibility before reducing their resilience. This means that the tradeoff of robustness and resilience is more stable for larger SNs.

- There is a positive correlation between the local reliability of each stochastic facility and its flexibility level. Also increasing the reliability of each facility positively affects the flexibility levels of the other facilities in the network. This means that in stochastic SNs there is a positive correlation between the local reliabilities of the stochastic facilities and the flexibility levels must be added to the facilities to make their networks robust. SNs with higher reliability in their flow need more flexibility to be robust.

About planning reliable flow dynamics for the SN

- For a given product order quantity (local reliability of the retailers), the effect of a stochastic facility's local reliability on the SN's profit is not significantly influenced by the reliabilities of the other facilities. This outcome highlights that independent local reliability selection for the SN's stochastic facilities leads to a good (not the best) solution. But this independent reliability selection significantly decreases the size of the model and its computational time.

7.2. Verification and validation in this dissertation

Here, we generally describe the method that will later be utilized to validate the methods / models of this dissertation, namely the Validation Square (Figure 7-2). The Validation Square is a method to prove the usefulness of a design method considering whether the method provides design solutions 'correctly' (effectiveness), and whether it provides 'correct' design solutions (efficiency).

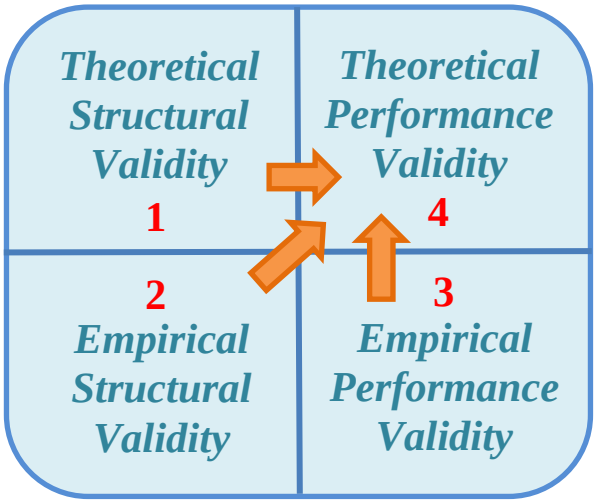


Figure 7-2: Verification and validation square.

This square has two "structural" and "performance" horizontal splits. The structural split, including first and second quadrants, checks the logical structure of the design method by qualitative testing. The performance split, including third and fourth quadrants, checks the ability of the design method to produce useful results by quantitative testing. Also the square has two "theoretical" and "empirical" vertical splits. The theoretical split, including first and fourth quadrants, deals with validity of the design method for a generalized problem. The empirical split, including second and third quadrants, deals with validity of the design method for specially chosen examples.

Therefore, the detailed description of these four quadrants are as follows:

- ***Theoretical Structural Validity:*** This quadrant checks the internal consistency of the design methods, i.e., the logical soundness of its constructs both individually and integrated.
- ***Empirical Structural Validity:*** This quadrant checks the appropriateness of the chosen example problem(s) intended to test design method.
- ***Empirical Performance Validity:*** This quadrant checks the ability of the design method to produce useful results for the chosen example problems.
- ***Theoretical Performance Validity:*** This quadrant checks the ability to produce useful results beyond the chosen example problem(s). This requires a "leap of faith" which is eased by the process of the previous quadrants to build confidence in the general usefulness of the design method.

In the rest of this section, we want to show that how these four quadrants of the validation square have been covered in this dissertation (Figure 7-3).

Theoretical Structural Validity

- ***In Chapter 1***: We justify the necessity of investigating the problems and the advantages of solving these problems to the practical world.
- ***In Chapter 1***: We do literature review to discover the existing gaps and show that how solving these problems can fill parts of the existing gaps (both from supply network design and uncertainty management perspectives).
- ***In Chapter 1***: We define the general structure of modeling the problem: included sub-problems, their outputs and inputs, assumptions, constraints, and flow transactions.

Empirical Structural Validity

- ***Chapter 2***: Two test problems are solved in this section to show the process of quantifying "uncertainty propagation" in supply chains and networks respectively and their consistency with the problems of this section is discussed.
- ***Chapter 3***: A test problem is solved in this section to show the process of modeling the interactions of the forward and after-sales supply chains and its consistency with the problem of this section is discussed.
- ***Chapter 4***: A test problem from automobile industry is solved in this section to show the process of modeling the remanufacturing sections and its consistency with the problem of this section is discussed.

Empirical Performance Validity

- ***Chapter 5:*** A comprehensive test problem from automobile industry is solved in this section to show the process of modeling reliable flow dynamics through the structure of forward and after-sales supply networks.
- ***Chapter 6****:* A comprehensive test problem is solved in this section to show the process of integrating being structurally and operationally fail-safe in supply networks.

Empirical Performance Validity

- ***Chapter 7***: In this chapter, we discuss about the other possible applications for the models developed in this dissertation.

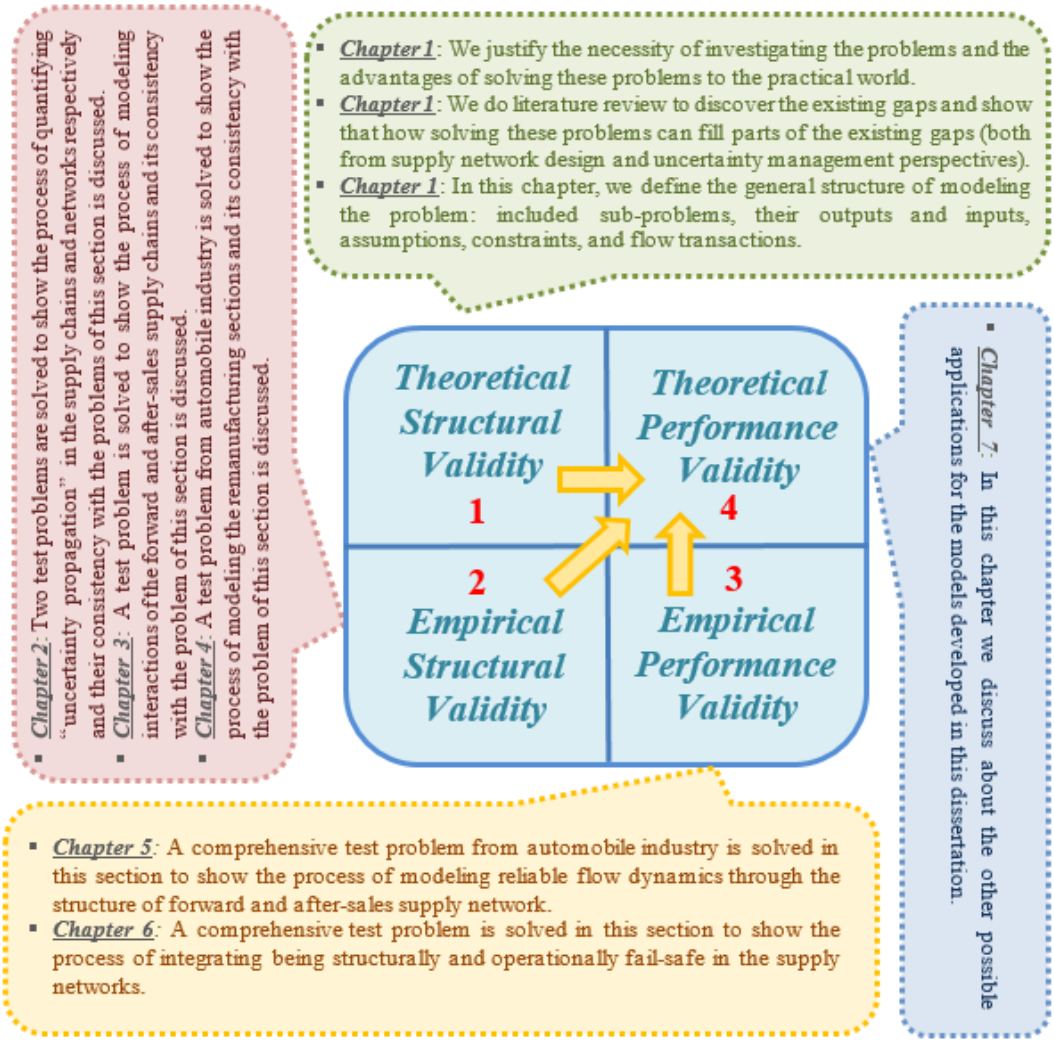


Figure 7-3: The process of verifying and validating the content of the dissertation.

7.3. Critical evaluation and recommendations

While the research in this dissertation covers a relatively broad spectrum within risk management in networks, there are some shortcomings that can be covered in future research.

1) **Use redundancy as another risk mitigation strategy**: In this dissertation, I show that to have a fail-safe network, it should be (Figure 7-4):

- ✓ Structurally fail-safe: which means the integration of its topology should be *robust* against disruptions – large scale unexpected events – by positioning appropriate amount of risk mitigation strategies and be *resilient* in executing these strategies;
- ✓ Operationally fail-safe: which means the coordination of its facilities should be fail-safe against variations – small scale expected events – to preserve a reliable flow dynamics throughout its topology.

Two kinds of risk mitigation strategies can be incorporated to make networks' topology robust and resilient against disruptions (Figure 7-4):

- ✓ ***Redundancy***: redundancy as a risk mitigation strategy means keeping extra resources (e.g., stock or capacity) in systems that can be used in disrupted conditions;
- ✓ ***Flexibility***: which means having flexible facilities which are able to ramp-up and ramp-down their processing capabilities when it is needed – in disrupted and normal conditions respectively.

In this dissertation, I only focus on the second risk mitigation strategy – Flexibility. However, "**Redundancy**" may be more appropriate for SCs / SNs of non-perishable and cheap products. Therefore, considering redundancy as another risk mitigation strategy to architecture robust / resilient SCs / SNs is one of the important extensions for the problem of this dissertation.

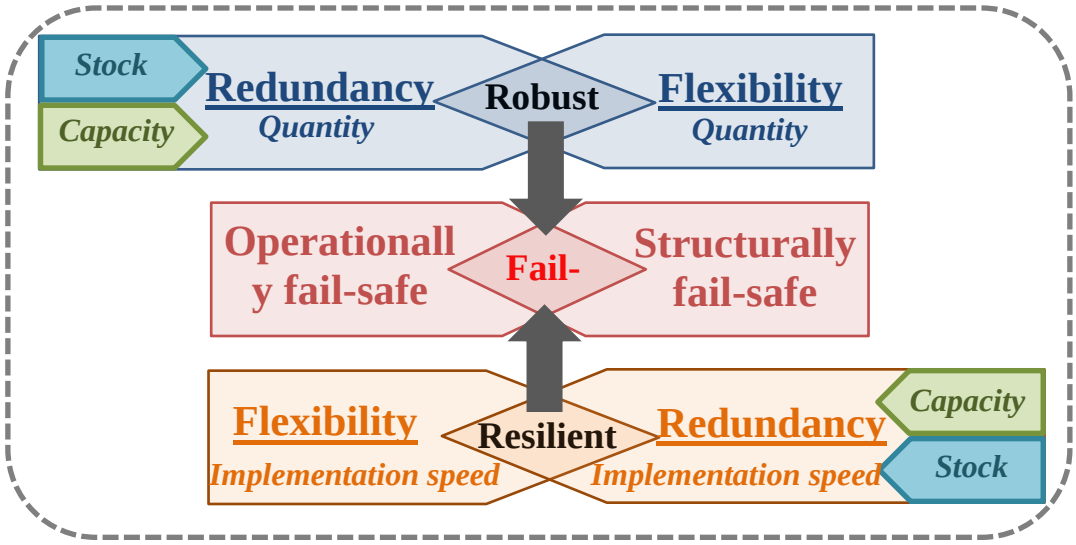


Figure 7-4: Architecting fail-safe systems.

To consider redundancy as a risk mitigation strategy, the following steps should be taken (see Figure 7-5): a) we should determine how much redundancy, i.e., inventory, should be added to facilities of the SN. Adding redundancy imposes some cost to the system. Higher redundancy means higher cost and higher robustness. Networks with higher robustness are able to preserve their performance in bigger disruptions. In this step, we should develop a model to find the most appropriate robustness for the SN and redundancy for its facilities by considering the tradeoff between their imposing costs and improving servitization; b) we should determine where and in what facilities, redundancy should be added. Redundancy may impose different costs to different facilities. In this step, we should develop a model to find the most economic places to locate redundancy in the SN by considering the tradeoff between their imposing costs and improving servitization; c) we integrate the models developed in the previous steps to concurrently find the most appropriate robustness and resilience for the SN by considering redundancy

as a risk mitigation strategy. This model simultaneously determines the best place and quantity to impose redundancy to the SN. This model helps us to analyze the correlation between robustness and resilience.

It is possible to integrate the redundancy model with the flexibility model developed in this dissertation (see Figure 7-5). This model includes two risk mitigation strategies and provides opportunity for decision makers to select the most appropriate risk mitigation strategy for the SN's facilities. This model should determine:

- What facilities in the SN need risk mitigation strategies?
- What is the best risk mitigation strategy for each facility?
- What is the best quantity of each risk mitigation strategy that should be assigned to each facility?

2) **Architecting fail-safe SNs with several conflicting goals**: Being fail-safe is important for a broad spectrum of network-oriented systems. Network-oriented systems can be classified as follows:

- ✓ Profit-based network-oriented systems: in profit-based networks, the most important goal is maximizing profit or minimizing cost. Some examples of these networks that are running on profit or cost are multi-stage manufacturing systems, transportation networks, SCs / SNs, energy networks, etc.
- ✓ Nonprofit network-oriented systems: in non-profit networks, there are other goals along with the economic goal. Some examples of network-oriented

systems that have non-profit goals are urban traffic, civil infrastructures, humanitarian / contingency logistics, etc.

In this dissertation, I only concentrate on SCs / SNs for which maximizing profit is the only goal. Customizing the models of this dissertation for non-profit network-oriented systems such as civil infrastructure, urban traffic, contingency logistics, etc. is very interesting future research (see Figure 7-5). I believe that the behavior of the developed models with respect to the correlations among "reliability", "robustness", and "resilience" would be completely different in non-profit networks concentrating more on improving service level rather than profit.

3) **Architecting fail-safe SNs under competition:** In this dissertation, I only investigate the impact of being fail-safe on SCs' / SNs' performance and profitability. The major harm to a SN after a disruption comes not from the direct damage to facilities but in the market share lost to competitors. It is because SN disruptions could prevent a firm from capitalizing on strong market demand due to unavailability of products and consequently the market share is lost to the competitors. Investigating the marketing benefits of being fail-safe for SNs is another interesting future research. Being fail-safe not only works to the advantage of SNs but also customers benefit from it. For example, having fail-safe SNs in markets results in price reduction and minimizes price fluctuations that customer can enjoy. In my opinion, the following questions should be answered in this area:

- ✓ What is the impact of being fail-safe on stabilizing the SCs' / SNs' market shares?
- ✓ What is the contribution of each risk mitigation strategy on stabilizing the SCs' / SNs' market shares?
- ✓ What is the impact of being fail-safe on stabilizing the products' retail price in markets?
- ✓ What is the contribution of each risk mitigation strategy on stabilizing the products' retail price in markets?

4) **Architecting decentralized fail-safe SNs**: In all the models developed in this dissertation, I assume that the investigated SCs / SNs are centralized. In the centralized SCs / SNs, all decisions are made by a single leadership team. In the practical world, some SCs / SNs are decentralized. In decentralized systems, there are more than one decision makers with conflicting interests. Using game theory to model bargaining among facilities in decentralized SCs / SNs is another interesting future research. In my opinion, the following questions should be answered in this area:

- ✓ What risk mitigation strategies can be used by each facility in the SC / SN?
- ✓ What is the impact of the risk mitigation strategy selected by each facility on the performance of the other facilities in the SC / SN?
- ✓ What is the most equilibrating risk mitigation strategy for each facility in the SC / SN that prevents other facilities from taking more actions?

- ✓ What is the difference between centralized and decentralized risk management in SCs / SNs?

I believe that incorporating the abovementioned points in the mathematical models developed in this dissertation may lead to interesting future research with very useful managerial insights in the area of risk management in SNs.

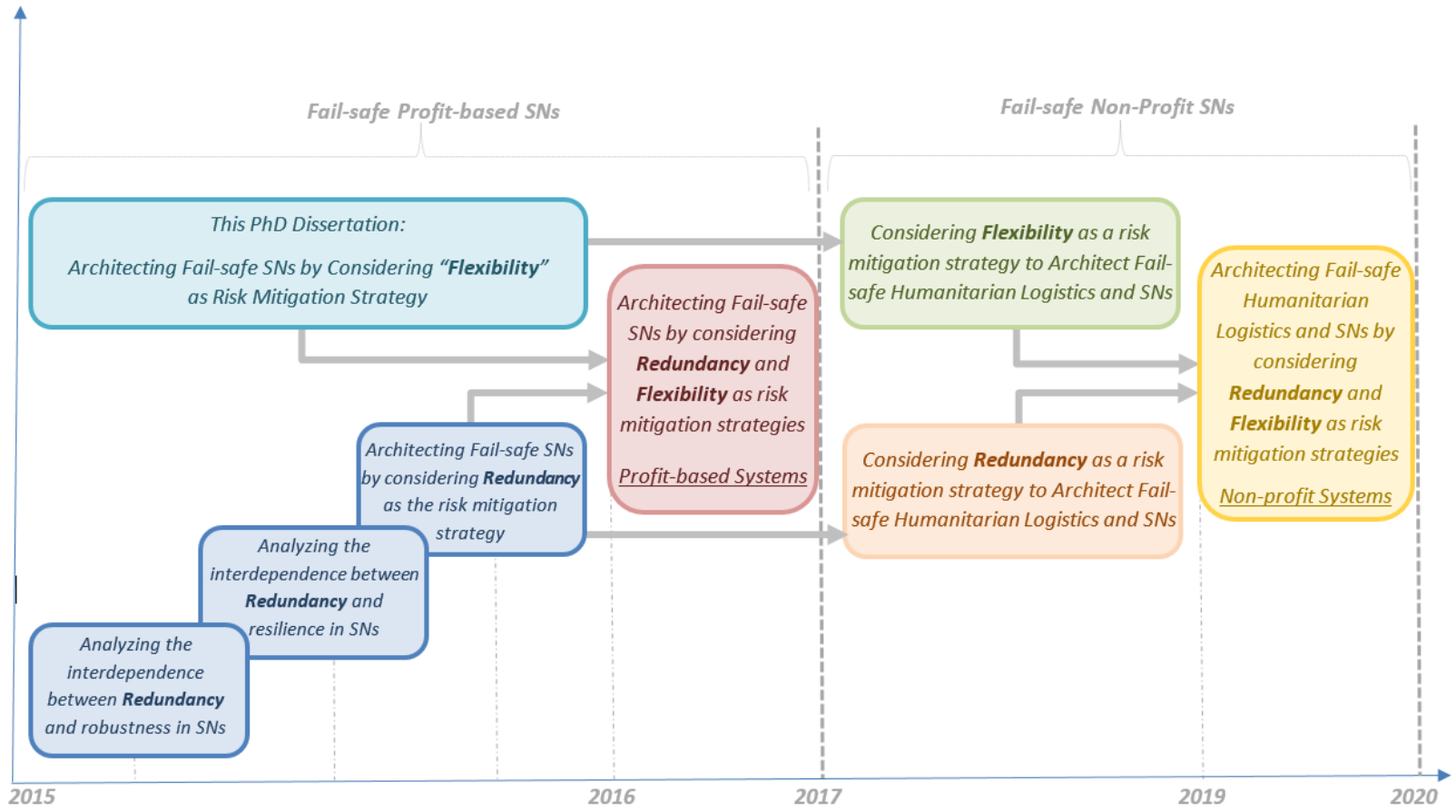


Figure 7-5: Roadmap for future research.

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Appendix

In Section 5.3, a five-step approach is developed to solve the mathematical model proposed in Chapter 5. The MATLAB codes developed for Steps 2 and 3 of this approach are as follows:

MATLAB code for Step 2

```
clc
clear all

y1=1;
y2=1;
y3=1;
y4=1;
n=1;
for rls1=0.93:0.01:1
    for rls2=0.93:0.01:1
        for rls3=0.93:0.01:1

           for rlm1=0.93:0.01:1
                for rlm2=0.93:0.01:1

                    for rlr1=0.93:0.01:0.99
                        for rlr2=0.93:0.01:0.99

                            pservicelevelpath1=(rls1*rls2)*rlm1*rlr1;
                            pservicelevelpath3=(rls1*rls2)*rlm1*rlr2;
                            pservicelevelpath2=(rls3*rls2)*rlm2*rlr1;
                            pservicelevelpath4=(rls3*rls2)*rlm2*rlr2;

aservicelevelpath1=(rls1*rlr1)*(rls2*rlr1);

aservicelevelpath3=(rls1*rlr2)*(rls2*rlr2);

aservicelevelpath2=(rls2*rlr1)*(rls3*rlr1);

aservicelevelpath4=(rls2*rlr2)*(rls3*rlr2);


slp1=y1*y2*(pservicelevelpath1*pservicelevelpath2)+(1-
y1)*y2*(pservicelevelpath2)+(1-y2)*y1*(pservicelevelpath1);

sla1=y1*y2*(aservicelevelpath1*aservicelevelpath2)+(1-
y1)*y2*(aservicelevelpath2)+(1-y2)*y1*(aservicelevelpath1);

slp2=y3*y4*(pservicelevelpath3*pservicelevelpath4)+(1-
y4)*y3*(pservicelevelpath3)+(1-y3)*y4*(pservicelevelpath4);

sla2=y3*y4*(aservicelevelpath3*aservicelevelpath4)+(1-
y4)*y3*(aservicelevelpath3)+(1-y3)*y4*(aservicelevelpath4);
```

```
                                if slp1>=0.975 & slp1<=0.985 & slp2>=0.975
& slp2<=0.985 & sla1>=0.955 & sla1<=0.965 & sla2>=0.955 & sla2<=0.965

                                    rrls1(n)=rls1;
                                    rrls2(n)=rls2;
                                    rrls3(n)=rls3;
                                    rrlm1(n)=rlm1;
                                    rrlm2(n)=rlm2;
                                    rrlr1(n)=rlr1;
                                    rrlr2(n)=rlr2;

                                    n=n+1;
                                end;

                            end;
                        end;

                    end;
                end;

        end;
    end;
end;

for i=(n-1):-1:1
   for j=(i-1):-1:1
       if rrls1(i)>=rrls1(j) & rrls2(i)>=rrls2(j) & rrls3(i)>=rrls3(j)
& rrlm1(i)>=rrlm1(j) & rrlm2(i)>=rrlm2(j) & rrlr1(i)>=rrlr1(j) &
rrlr2(i)>=rrlr2(j)
          rrls1(i)=5;
       end;
   end;
end;

for k=1:1:(n-1)
    if rrls1(k)~=5

       'reliability level supplier 1'
        rrls1(k)
       'reliability level supplier 2'
        rrls2(k)
       'reliability level supplier 3'
        rrls3(k)
       'reliability level manufacturer 1'
        rrlm1(k)
       'reliability level manufacturer 2'
        rrlm2(k)
       'reliability level retailer 1'
        rrlr1(k)
       'reliability level retailer 2'
        rrlr2(k)

    end;
end;
```

MATLAB code for Step 3

```
clc
clear all

mua=0.05;
vara=0.05;
mub=0.0704;
varb=0.085;
cr=0.201;

cpps1=1.10;
cpps2=1.30;
cpps3=1.25;

cm1=2.00;
cm2=2.15;

ct11=0.05;
ct21=0.08;
ct22=0.08;
ct32=0.06;

hh=0.11;
 h=0.05;

ctt11=0.05;
ctt12=0.04;
ctt21=0.05;
ctt22=0.05;

cttt11=0.07;
cttt21=0.07;
cttt22=0.07;
cttt32=0.07;

beta1=0.15;
beta2=0.08;

PR1=8000;
landa1=0.1;
mmu1=2;

PR2=8000;
landa2=0.2;
mmu2=2;

PR3=9000;
landa3=0.05;
mmu3=3;

w=1.0;
p=10.0;
```

```
rls1=1;
rls2=1;
rls3=0.94;
rlm1=0.99;
rlm2=0.93;
rlr1=0.99;
rlr2=0.99;

o1=1;
o2=1;
o3=1;
o4=1;

pservicelevelpath1=(rls1*rls2)*rlm1*rlr1;
pservicelevelpath3=(rls1*rls2)*rlm1*rlr2;
pservicelevelpath2=(rls3*rls2)*rlm2*rlr1;
pservicelevelpath4=(rls3*rls2)*rlm2*rlr2;

aservicelevelpath1=(rls1*rlr1)*(rls2*rlr1);
aservicelevelpath3=(rls1*rlr2)*(rls2*rlr2);
aservicelevelpath2=(rls2*rlr1)*(rls3*rlr1);
aservicelevelpath4=(rls2*rlr2)*(rls3*rlr2);

slp1=o1*o2*(pservicelevelpath1*pservicelevelpath2)+(1-
o1)*o2*(pservicelevelpath2)+(1-o2)*o1*(pservicelevelpath1);
sla1=o1*o2*(aservicelevelpath1*aservicelevelpath2)+(1-
o1)*o2*(aservicelevelpath2)+(1-o2)*o1*(aservicelevelpath1);

slp2=o3*o4*(pservicelevelpath3*pservicelevelpath4)+(1-
o4)*o3*(pservicelevelpath3)+(1-o3)*o4*(pservicelevelpath4);
sla2=o3*o4*(aservicelevelpath3*aservicelevelpath4)+(1-
o4)*o3*(aservicelevelpath3)+(1-o3)*o4*(aservicelevelpath4);

dm1=500+200*w-250*(p-10)-500*(1-slp1)-900*(1-sla1);
dm2=400+200*w-250*(p-10)-500*(1-slp2)-900*(1-sla2);

n=1;

for per1=0.05:0.05:0.95
    for per2=0.05:0.05:0.95

        pper1(n)=per1;
        pper2(n)=per2;

        x1(n)=dm1*norminv(rlr1,0,0.8)*per1;
        x3(n)=dm1*norminv(rlr1,0,0.8)*(1-per1);

        x2(n)=dm2*norminv(rlr2,0,0.8)*per2;
        x4(n)=dm2*norminv(rlr2,0,0.8)*(1-per2);

        xx1(n)=x1(n)*beta1*rlm1;
        xx2(n)=x2(n)*beta1*rlm1;

        xx3(n)=x3(n)*beta2*rlm2;
```

```
        xx4(n)=x4(n)*beta2*rlm2;

        y1(n)=mua*x1(n)+norminv(rlr1,0,1)*(x1(n)*vara)^0.5;
        z1(n)=mub*x1(n)+norminv(rlr1,0,1)*(x1(n)*varb)^0.5;

        y3(n)=mua*x3(n)+norminv(rlr1,0,1)*(x3(n)*vara)^0.5;
        z3(n)=mub*x3(n)+norminv(rlr1,0,1)*(x3(n)*varb)^0.5;

        y2(n)=mua*x2(n)+norminv(rlr2,0,1)*(x2(n)*vara)^0.5;
        z2(n)=mub*x2(n)+norminv(rlr2,0,1)*(x2(n)*varb)^0.5;

        y4(n)=mua*x4(n)+norminv(rlr2,0,1)*(x4(n)*vara)^0.5;
        z4(n)=mub*x4(n)+norminv(rlr2,0,1)*(x4(n)*varb)^0.5;


        yy1(n)=((landa1/(1-
landa1))*(PR1*log(rls1)*(1/mmu1)+y1(n)+x1(n)+xx1(n)))*o1;
        yy2(n)=((landa1/(1-
landa1))*(PR1*log(rls1)*(1/mmu1)+y2(n)+x2(n)+xx2(n)))*o2;

        yy3(n)=((landa3/(1-
landa3))*(PR3*log(rls3)*(1/mmu3)+y3(n)+x3(n)+xx3(n)))*o3;
        yy4(n)=((landa3/(1-
landa3))*(PR3*log(rls3)*(1/mmu3)+y4(n)+x4(n)+xx4(n)))*o4;

        zz1(n)=((landa2/(1-
landa2))*(PR2*log(rls2)*(1/mmu2)+z1(n)+x1(n)+xx1(n)))*o1;
        zz2(n)=((landa2/(1-
landa2))*(PR2*log(rls2)*(1/mmu2)+z2(n)+x2(n)+xx2(n)))*o2;

        zz3(n)=((landa2/(1-
landa2))*(PR2*log(rls2)*(1/mmu2)+z3(n)+x3(n)+xx3(n)))*o3;
        zz4(n)=((landa2/(1-
landa2))*(PR2*log(rls2)*(1/mmu2)+z4(n)+x4(n)+xx4(n)))*o4;

        if yy1(n)<0
           yy1(n)=0;
        end;

        if yy2(n)<0
           yy2(n)=0;
        end;

        if yy3(n)<0
           yy3(n)=0;
        end;

        if yy4(n)<0
           yy4(n)=0;
        end;

        if zz1(n)<0
           zz1(n)=0;
        end;
```

```
        if zz2(n)<0
           zz2(n)=0;
        end;

        if zz3(n)<0
           zz3(n)=0;
        end;

        if zz4(n)<0
           zz4(n)=0;
        end;


cost(n)=cpps1*((x1(n)+xx1(n)+y1(n)+yy1(n))+(x2(n)+xx2(n)+y2(n)+yy2(n))
)+cpps2*((x1(n)+xx1(n)+z1(n)+zz1(n))+(x2(n)+xx2(n)+z2(n)+zz2(n))+(x3(n
)+xx3(n)+z3(n)+zz3(n))+(x4(n)+xx4(n)+z4(n)+zz4(n)))+cpps3*((x3(n)+xx3(
n)+y3(n)+yy3(n))+(x4(n)+xx4(n)+y4(n)+yy4(n)))+cm1*(x1(n)+xx1(n)+x2(n)+
xx2(n))+cm2*(x3(n)+xx3(n)+x4(n)+xx4(n))+ct11*(x1(n)+xx1(n)+x2(n)+xx2(n
))+ct21*(x1(n)+xx1(n)+x2(n)+xx2(n))+ct22*(x3(n)+xx3(n)+x4(n)+xx4(n))+c
t32*(x3(n)+xx3(n)+x4(n)+xx4(n))+ctt11*x1(n)+ctt21*x3(n)+ctt12*x2(n)+ct
t22*x4(n)+cttt11*(y1(n)+y2(n))+cttt21*(z1(n)+z2(n))+cttt22*(z3(n)+z4(n
))+cttt32*(y3(n)+y4(n));
        z(n)=cost(n);

        n=n+1;
    end;
end;


zz=min(z);

for i=1:(n-1)
    if z(i)==zz
        'cost'
         z(i)
        pper1(i)
        pper2(i)

        x1(i)
        x3(i)

        x2(i)
        x4(i)

        xx1(i)
        xx2(i)

        xx3(i)
        xx4(i)

        y1(i)
        z1(i)
```

```
            y3(i)
            z3(i)

            y2(i)
            z2(i)

            y4(i)
            z4(i)

            yy1(i)
            yy2(i)

            yy3(i)
            yy4(i)

            zz1(i)
            zz2(i)

            zz3(i)
            zz4(i)
        end;
    end;
```