

Scalable Multipartite Entanglement Using Squeezed Light

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Chapter 1

Introduction

Quantum light sources such as squeezed states and entangled states that allow correlations beyond the classical limit are attractive on several fronts. For example, the reduced noise properties of squeezed states allow high precision measurements[1], to achieve good resolution in quantum imaging[2, 3] and enhance the sensitivity of measurements in optical devices[4, 5]. On the other hand the strong nonlocality of entangled states makes it a key ingredient in quantum communications.[6].

Nonclassical correlations and entanglement existing between multiple spatial or temporal modes are promising for multichannel and long distance quantum communications. Different techniques for the generation of multipartite entanglement have been proposed and experimentally implemented both in the discrete variable(DV) and continuous variable(CV) regime. Squeezed light is a good source of CV entanglement. A standard technique for the generation of CV multipartite entanglement is based on mixing squeezed states on linear beam splitters[7, 8] or using cascaded amplifier networks[9]. However, these techniques limit the number of modes that can be entangled, as the correlations can degrade due to the mode mismatch and certain generation methods can introduce additional optical losses . To overcome this limitation, one or two quantum states composed of multiple modes can be used, and this has been experimentally achieved with temporal modes from a parametric oscillator[10] . Entangled multimodes have also been generated in different spatial regions of a single beam[11]. Further advancing from this, a scalable technique for the generation of genuine CVME has been demonstrated by time bin multiplexing of squeezed light sources in temporal domain[12].

A prior work from our group proposed a theoretical scheme for the generation of genuine scalable CVME based on spatial modes[13]. This proposal consists of an $SU(1,1)$ -like interferometer with two four wave mixing (FWM) processes serving as the source and mixing element for the multiple spatial modes contained in two different beams. The setup contains only two active elements and offers the advantage of providing a more straightforward and scalable technique for the generation and verification of genuine multipartite entanglement in the spatial domain. In this report, we present our preliminary experimental results on the implementation of this technique[13].

Chapter 2

States of Electromagnetic Field

We are interested in the quantum properties of light. In case of an optical field the measurable quantity is the quadratures associated with the electric field. So, in this chapter, we first talk about the uncertainties that the quadratures of an electric field will satisfy. Later we will have a detailed discussion on coherent states and squeezed states of light.

2.1 Uncertainties in field quadratures

Three Hermitian operators $\hat{A}$, $\hat{B}$ and $\hat{C}$, satisfying the following commutation relation,

$$[\hat{A}, \hat{B}] = i\hat{C} \quad (2.1)$$

gives a lower bound for the product of variances of the operators as

$$\langle \Delta \hat{A}^2 \rangle \langle \Delta \hat{B}^2 \rangle \geq \frac{1}{4} \langle \Delta \hat{C}^2 \rangle. \quad (2.2)$$

This is called the uncertainty relation and the states for which the equality sign holds are called minimum uncertainty states. The variance of the operators in Eqn.2.2 gives a measure of the noise associated with the measurement of observable $\hat{A}$ of that quantum state.

The electric field of light can be expressed in terms of operators $\hat{X}$ and $\hat{Y}$, called as the amplitude and phase quadrature respectively. The magnitude of these quadratures are the experimentally measurable quantities of the field. For a field with frequency ω , we can write the electric field operator in terms of quadratures as [14]

$$\hat{E}(z, t) = \epsilon(\hat{X} \cos \omega t + \hat{Y} \sin \omega t). \quad (2.3)$$

The quadratures can be conveniently expressed in terms of non-hermitian annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$. The field quadratures in terms of these operators are given by $\hat{X} = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$ and

$\hat{Y} = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$. Since photons are bosons, they obey the bosonic commutation relations $[\hat{a}, \hat{a}^\dagger] = 1$. From the commutation relations, it can be deduced that the quadratures $\hat{X}$ and $\hat{Y}$ satisfy $[\hat{X}, \hat{Y}] = \frac{i}{2}$. And using Eqn.2.2, we can show that the quadratures obey the uncertainty relation,

$$\langle \Delta \hat{X}^2 \rangle \langle \Delta \hat{Y}^2 \rangle \geq \frac{1}{16}. \quad (2.4)$$

A vacuum state minimizes the uncertainty product in Eqn.2.2 with equal noise in both quadratures.i.e.,

$$\langle \Delta \hat{X}^2 \rangle = \frac{1}{4} = \langle \Delta \hat{Y}^2 \rangle. \quad (2.5)$$

2.2 Coherent States

A field produced by a laser is a coherent state, meaning it has a well defined amplitude and phase. An appropriate basis state to represent such a field is a coherent state, denoted by $|\alpha\rangle$. This state minimize the uncertainty relation for the two field quadratures given in Eqn.2.1 with equal uncertainties in each of the quadratures. That is, for a coherent state, the quadrature variances are $\langle \Delta \hat{X}^2 \rangle = \langle \Delta \hat{Y}^2 \rangle = \frac{1}{4}$. This shows that coherent states have quadrature fluctuations equal to that of a vacuum state given in Equation (2.5). The equality in quadrature variances will lead to an isotropic phase space diagram for a coherent state as shown in Fig.2.2(a). It is the closest possible quantum mechanical state to a classical field. The fluctuations of this state are completely random and they cancels out when we take an intensity-intensity difference measurement. The noise measurement of this state with equal fluctuations in both quadratures is called the standard quantum limit(SNL).

Coherent states can be generated by displacing a vacuum state using unitary displacement operator, $\hat{D}(\alpha)$

$$|\alpha\rangle = \hat{D}(\alpha) |0\rangle \quad (2.6)$$

$$\hat{D}(\alpha) = e^{(\alpha \hat{a}^\dagger - \alpha^* \hat{a})} \quad (2.7)$$

where the complex amplitude, $\alpha = |\alpha|e^{i\phi}$. The real part of α will give us the expectation value of $\hat{X}$ quadrature (or equivalently the amplitude of the electric field), whereas the imaginary part of α gives the measure of $\hat{Y}$ quadrature.

2.3 Single mode Squeezed State

It is possible to reduce the noise in one quadrature below that of a coherent state at the expense of increased noise in the other quadrature so as to satisfy the uncertainty relation in Fig.2.1. A squeezed state is such a state. As shown in Fig.2.1 if the noise in phase quadrature is reduced with an increased

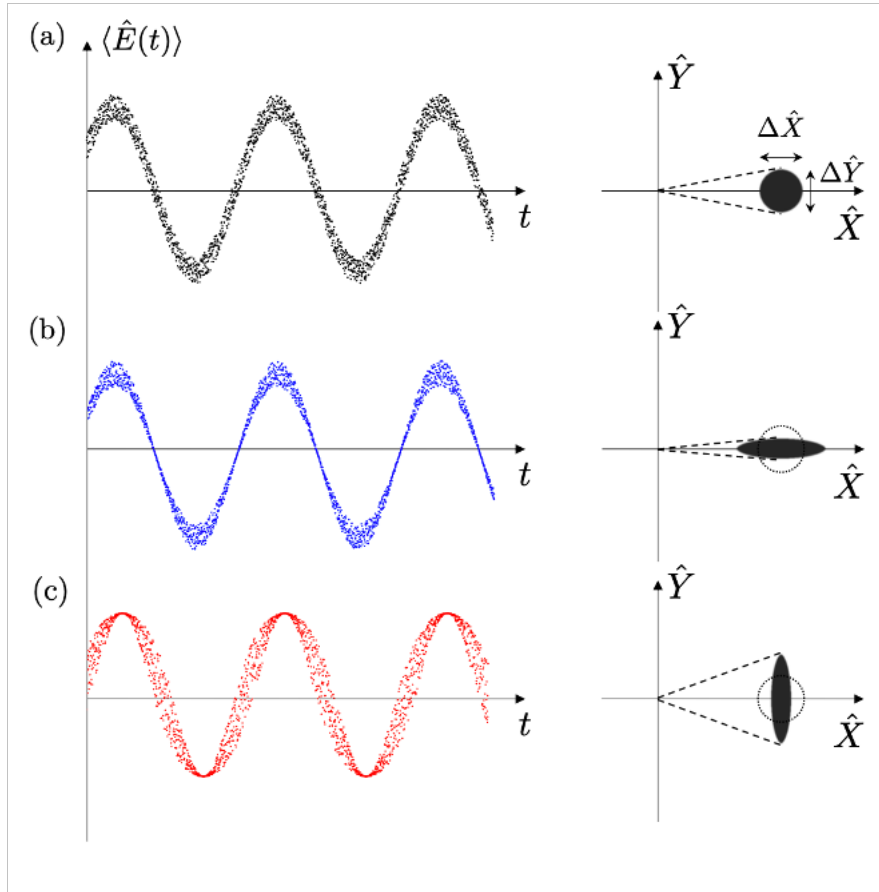


Figure 2.1: Electric field versus time graph and phase space diagram showing uncertainty in phase and amplitude quadratures for a) Coherent state b) Phase squeezed state c) Amplitude squeezed state [adapted from [15]]

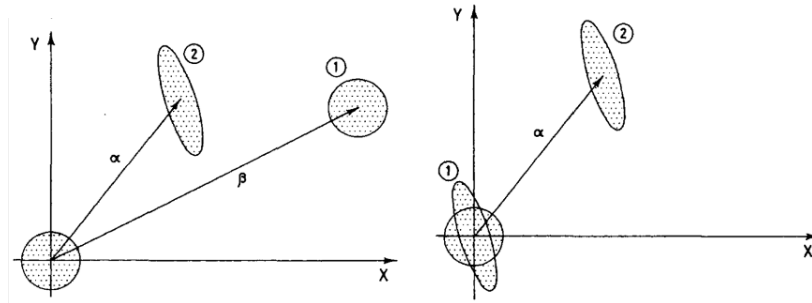


Figure 2.2: Phase space diagram for a squeezed coherent state. a) Displace the vacuum state to a coherent state with amplitude β and then squeeze the coherent state. b) Squeeze the vacuum state and then displace the squeezed vacuum state to a coherent state with amplitude α [adapted from [14]]

noise in amplitude quadrature it is called a phase squeezed state . On the other hand if the fluctuations in amplitude is decreased with increased fuctuations in phase, it is called an amplitude squeezed light. The unitary squeezing operator for a single mode , $\hat{S}(\epsilon)$ is defined as

$$\hat{S}(\epsilon) = \exp\left(\frac{1}{2}\epsilon^* \hat{a}^2 - \frac{1}{2}\epsilon \hat{a}^{\dagger 2}\right) \quad (2.8)$$

where r is the degree of squeezing, $\epsilon = r e^{i\theta}$ is the squeezing parameter and θ is the squeezing phase. The amount of squeezing depends on the value of r . If $\hat{X}$ is the squeezed and $\hat{Y}$ is the antisqueezed quadrature, then

$$\langle \Delta \hat{X}^2 \rangle < \frac{1}{4} \quad (2.9)$$

$$\langle \Delta \hat{Y}^2 \rangle > \frac{1}{4}. \quad (2.10)$$

Apart from this, if they satisfy the equality sign in ??, it is a minimum uncertainty state and is called an ideal squeezed state. To visualize the effect of squeezing, we can plot the uncertainties(or variances) of the quadratures in phase space. For a coherent state, the equal uncertainties on both quadratures will lead to an uncertainty circle centered on the complex amplitude α . For a squeezed state, the compression and elongation along two axes lead to an uncertainty ellipse. This is shown in Figure ??.

A squeezed state can be generated by applying the squeezing operator either to a vacuum state (squeezed vacuum state) or to a coherent state(squeezed coherent state). For generating a squeezed coherent state, we can either displace the vacuum and squeeze the displaced state or squeeze the vacuum and displace the squeezed state [14]. These operations can be written as

$$|\beta, \epsilon\rangle = \hat{S}(\epsilon) \hat{D}(\beta) |0\rangle \quad (2.11)$$

$$|\alpha, \epsilon\rangle = \hat{D}(\alpha) \hat{S}(\epsilon) |0\rangle \quad (2.12)$$

where α and β are the complex amplitudes of the coherent state.

2.4 Two mode squeezed state

Instead of squeezing a single mode of the electromagnetic field, two or more modes of the field can be squeezed simultaneously. The simplest example is the two mode squeezed state. Analogous to a single-mode squeezing operator, we can define a two mode squeezing operator, as

$$\hat{S}_{\hat{a}\hat{b}}(\epsilon) = \exp\left(\epsilon^* \hat{a} \hat{b} - \epsilon \hat{a}^{\dagger} \hat{b}^{\dagger}\right) \quad (2.13)$$

with the operators a and b are the annihilation operators of two different modes. They satisfy the commutators $[\hat{a}, \hat{a}^\dagger] = 1$, $[\hat{b}, \hat{b}^\dagger] = 1$ and $[\hat{a}, \hat{b}^\dagger] = 0$. In contrast to single mode squeezing, here squeezing does not exist in single modes but in the correlated states created by the multimodes (two in this case) [16]. So if we measure the noise in the individual quadratures of the two modes, it will go above the vacuum fluctuations,

$$\langle \Delta \hat{X}_a^2 \rangle > \frac{1}{4}, \quad \langle \Delta \hat{X}_b^2 \rangle > \frac{1}{4} \quad \text{and} \quad \langle \Delta \hat{Y}_a^2 \rangle > \frac{1}{4}, \quad \langle \Delta \hat{Y}_b^2 \rangle > \frac{1}{4}$$

and each mode by itself is not squeezed. Here $\hat{X}_a, \hat{Y}_a$ and $\hat{X}_b, \hat{Y}_b$ corresponds to the quadratures of the two modes a and b respectively.

And also, the two mode squeezing operator $\hat{S}(\epsilon_{a,b})$, given in Equation (2.13) cannot be written as the product of two single mode squeezing operators $\hat{S}(\epsilon_a)$ and $\hat{S}(\epsilon_b)$ of the individual modes. However we can rewrite $\hat{S}(\epsilon_{a,b})$ as a factorized product of two new single mode squeezing operators as

$$\exp(\epsilon \hat{a}^\dagger \hat{b}^\dagger - \epsilon^* \hat{a} \hat{b}) = \exp\left(\frac{\epsilon}{2} \hat{d}^{\dagger 2} - \frac{\epsilon^*}{2} \hat{d}^2\right) \exp\left(\frac{-\epsilon}{2} \hat{c}^{\dagger 2} + \frac{\epsilon^*}{2} \hat{c}^2\right). \quad (2.14)$$

Here $\hat{c}$ and $\hat{d}$ are commuting operators that are linear combinations of $\hat{a}$ and $\hat{b}$, $\hat{c} = \frac{\hat{a}-\hat{b}}{\sqrt{2}}$ and $\hat{d} = \frac{\hat{a}+\hat{b}}{\sqrt{2}}$ with $[\hat{c}, \hat{c}^\dagger] = 1, [\hat{d}, \hat{d}^\dagger] = 1, [\hat{c}, \hat{d}^\dagger] = 0$. They also satisfy the properties $\langle a^2 \rangle = \langle b^2 \rangle = 0, \langle a \rangle = \langle b \rangle = 0$. From Equation (2.14) it is evident that in two mode squeezed state, squeezing exists between different combinations of quadratures. Therefore in order to observe squeezing we need to define the combined quadratures

$$\hat{X} = \frac{\hat{X}_a - \hat{X}_b}{\sqrt{2}}, \quad \hat{Y} = \frac{\hat{Y}_a + \hat{Y}_b}{\sqrt{2}}. \quad (2.15)$$

Just as we generate a single mode squeezed state by displacing vacuum, we can generate a two mode squeezed state (TMSS) as follows

$$|\epsilon, \alpha, \beta\rangle = D(\alpha) D(\beta) S(\epsilon_{ab}) |0, 0\rangle. \quad (2.16)$$

by displacing each mode of a two mode squeezed vacuum (TMSV) state.

Chapter 3

Quantum Entanglement

The properties of a quantum system can depend on the method or interaction through which it is generated. The consequences of such interactions lead to an interesting quantum mechanical phenomenon called entanglement. Sometimes, when two or more physical systems interact with each other certain nonclassical correlations between the systems that can persist even if they are separated in space (spatial separation) or time (temporal separation). In other words entanglement describes an ensemble of systems (two or more) that are not independent with respect to one or more of its properties, such as position, momentum, or spin. The simplest case of entanglement is a bipartite system where two systems are entangled with each other. If more than two systems are entangled, we call it multipartite entanglement (ME). Now, the next fundamental question to ask is which states are entangled and which ones are not. There are certain criteria which when violated or satisfied, can prove the existence or nonexistence of ME. These criteria mainly determine whether different combinations of subsystems in a composite system are separable or not. Hence they are also called inseparability criteria. The violation of inseparability criteria is sufficient to prove the existence of entanglement in a composite system. Several of these criteria can be experimentally verified in laboratories.

In this chapter, we will look at the definition of entanglement and inseparability of quantum systems, for which we will first introduce the idea of pure and mixed states. Later we will briefly discuss some inseparability inequalities and the existence of nonclassical correlations in two or higher order squeezed modes. Finally, we will discuss the methodology we are adopting for producing multipartite entanglement using squeezed states of light.

3.1 Entangled states

The first postulate of quantum mechanics states that the state of a physical system is specified, at any time t , by a state vector $|\psi(t)\rangle$ in a Hilbert space $\mathcal{H}$ (infinite-dimensional space); $|\psi(t)\rangle$ contains all the needed information of the system. Any superposition (linear combination) of state vectors is also a state vector.

However, if two or more systems are entangled, it is impossible to describe them by independent state vectors. This possibility of superposition of state vectors suggests that, unlike in classical mechanics, a quantum system can have a non-zero probability of being in two different states simultaneously..

3.1.1 Pure and mixed states

Inseparability criteria's are defined separately for two different types of two quantum systems, called pure and mixed states. A state is called a pure quantum state when we can get complete information about the system from its wave function, $|\psi(t)\rangle$. However, a pure state is unphysical because of its loss of information due to interaction with its surroundings, and in real situations, we unavoidably deal with mixed states. They are a statistical mixture of pure states and can be represented in terms of the hermitian linear statistical operator $\hat{\rho}$, defined by the statistically weighted outer product of wave functions where

$$\hat{\rho} = \sum_{n=-\infty}^{+\infty} p_n |\psi_n\rangle \langle \psi_n|. \quad (3.1)$$

Here, $p_n \geq 0$ is the probability for the system to be in the state of $|\psi_n\rangle$ and $\sum_n p_n = 1$. Note that in Equation (3.1), $\hat{\rho}$ is not the superposition of the state vectors since there is no interference between them. Rather it is a mixture of states that is inevitable in real situations because a system cannot be completely isolated from the environment. Hence, it is important to understand the inseparability in the case of mixed states.

3.1.2 Inseparability

The spectrum of the field quadrature variables is continuous. Therefore the squeezed state is a good resource for continuous variable(CV) quantum entanglement. Squeezing gives rise to a correlation between the field quadratures. Now we need to understand how we can confirm the existence of CV entanglement between different states of the field. Experimentally, we can demonstrate this by looking at the violations of certain inseparability criteria briefly discussed below.

Pure states

Consider the simplest case of bipartite entanglement, with two subsystems, A and B. If orthonormal bases for each subsystem exist and the total state vector of the system can be written as a Schmidt decomposition of these orthonormal bases as,

$$|\psi\rangle = \sum_n c_n |\psi_n(A)\rangle |\psi_n(B)\rangle \quad (3.2)$$

with atleast one non-zero Schmidt coefficient, c_n , then the composite system is said to be separable. In Equation (3.2), the summation is over the smaller of the dimensionalities of A and B. The Schmidt coefficients c_n are real and non-negative and they satisfy $\sum_n c_n^2 = 1$. This can be extended to any number of subsystems.

Mixed States

We can generalize the factorizability of the total state vector to the mixed states through the inseparability of the total density operator. A mixed quantum state is said to be separable if and only if its total density operator can be expressed as a mixture or convex sum of product states

$$\hat{\rho} = \sum_n p_n \hat{\rho}_{n1} \otimes \hat{\rho}_{n2} \otimes \hat{\rho}_{n3} \cdots \otimes \hat{\rho}_{nm} \quad (3.3)$$

with any $p_n \geq 0$. Here m corresponds to the number of subsystems. If a mixed quantum state does not satisfy Equation (3.3), it means they are inseparable, and the state is said to be entangled.

Inseparability Criteria

Equations 3.2 and 3.3 give a theoretical definition of inseparability. However, detection of entanglement requires a different approach. A practical approach for the inseparability of continuous variable states was first derived by Duan[17]. These inequalities involving the total variances was derived directly from Heisenberg's uncertainty relation. For a two-party, two-mode case, Duan showed that the necessary separability condition for a CV state is

$$\langle [\Delta(\hat{x}_1 - \hat{x}_2)]^2 \rangle \langle [\Delta(\hat{p}_1 + \hat{p}_2)]^2 \rangle \geq 2 |\langle [\hat{x}, \hat{p}] \rangle| \quad (3.4)$$

with $[\hat{x}_i, \hat{p}_j] = \frac{i\delta_{ij}}{2}$. The $\hat{x}_i$ and $\hat{p}_i$ in the above equation are analogous to the $\hat{X}_i$ and $\hat{Y}_i$ quadratures of the field, which are experimentally accessible quantities. This condition is necessary and sufficient to prove entanglement for a bipartite system.

We cannot measure the quadratures using a simple Later this type of inequalities was extended to the multiparty multimode continuous variable state by Loock and Furusawa [18]. These inequalities, commonly called the Loock and Furusawa inequalities, are sufficient for verifying GME. For N parties and N modes, violation of (N-1) inequalities shown below

$$\langle [\Delta(\hat{x}_1 - \hat{x}_2)]^2 \rangle \langle [\Delta(\hat{p}_1 + \hat{p}_2 + g_3 \hat{p}_3 + \cdots + g_N \hat{p}_N)]^2 \rangle < 1 \quad (3.5)$$

$\vdots$

$$\langle [\Delta(\hat{x}_{N-1} - \hat{x}_N)]^2 \rangle \langle [\Delta(g_1 \hat{p}_1 + g_2 \hat{p}_2 + g_3 \hat{p}_3 + \cdots + g_{N-2} \hat{p}_{N-2} + \hat{p}_{N-1} + \hat{p}_N)]^2 \rangle < 1 \quad (3.6)$$

that are expressed in terms of variances of particular combinations of all involved quadratures are sufficient to verify full inseparability. In the above equations, $\hat{x}_i'$'s and $\hat{p}_i'$'s are the two quadratures of different modes, and the g_i' 's are coefficients in the quadrature linear combinations which can be chosen conveniently..

3.2 Multimode squeezing as a source of CV entanglement

In the previous chapter, we discussed two mode squeezing. We cannot express a two mode squeezed state as a factorized product of states of the individual modes. Such a state is an entangled state. In order to understand how two or higher order squeezing serves as a source of entanglement, we need to discuss in detail how nonclassical correlations are shared by the combination of quadratures. Let us look at the simplest example of a two mode squeezed vacuum(TMSV). Operating the two mode squeezing operator given by Equation (2.13) on TMSV will give us[19],

$$\hat{S}(\epsilon_{a,b}) |0, 0\rangle = \frac{1}{\cosh r} \exp \left(e^{i\phi} \tanh r \hat{a}^\dagger \hat{b}^\dagger \right) |0, 0\rangle. \quad (3.7)$$

By expanding the exponential, we will get an explicit result for TMSV state as

$$|\epsilon\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \exp(i n \phi) (\tanh r)^n |n, n\rangle. \quad (3.8)$$

This state of a two mode squeezed vacuum cannot be further simplified using schmidt decomposition. Experimentally, we need to measure this particular combination to see squeezing below SNL. The correlated states created as a result of squeezing makes it a good source for entanglement. Squeezing does not always imply that the states are entangled. Whether a squeezed state is entangled or not depends on the degree of squeezing and other experimental factors. However, if entangled, the entanglement in the state of two modes depends on the degree of squeezing r . In the limit of infinite squeezing, i.e., $r \rightarrow \infty$, this state will become a perfectly correlated or maximally entangled state which is practically impossible. For any number of modes or quadratures involved, a particular combination of the individual quadratures exhibits squeezing. And the Looock and Furusawa inequalities are sufficient to prove the existence of entanglement between them.

In the previous chapter, we discussed two mode squeezing and the presence of quantum correlations between the two individual modes that make up TMSS. Hence it is a good source of CV entanglement. For the

Chapter 4

Experimental Implementation

Introduction

The electric response of a nonlinear medium can be written in terms of electric dipole moment per unit volume (Polarization, $\tilde{P}(t)$) as

$$\tilde{P}(t) = \epsilon_0 [\chi^{(1)} \tilde{E}(t) + \chi^{(2)} \tilde{E}^2(t) + \chi^{(3)} \tilde{E}^3(t) + \dots]. \quad (4.1)$$

Here χ is the susceptibility of the medium. The first term on RHS is the linear response whereas the second, third and other higher order terms indicate the nonlinear response of the medium. Depending on the nonlinear coefficients of a medium, different orders of nonlinear effects contribute to the behaviour of a system.

A crystal is said to be centrosymmetric or to display inversion symmetry if it possess a center of inversion and an inversion around that point will be symmetric. The second-order nonlinear interactions occur only in mediums that do not display inversion symmetry. We are using a typical light atom interaction process called four wave mixing (FWM) for the generation of squeezed light. Due to the inversion symmetry of atoms, $\chi^{(2)}$ term vanishes and hence FWM is a third order nonlinear process which arises due to the contribution of the third term in the RHS of Eqn.4.1.

In this chapter, we discuss the different experimental techniques adopted for our work. It is possible to encode arbitrary patterns into the light beam creating different independent spatial modes (Spatially separated regions) in a single beam. In this chapter, first we will discuss the FWM process and how we can generate twin beams with multi-spatial modes. Later we will have a discussion on our approach for the generation of scalable multipartite entanglement.

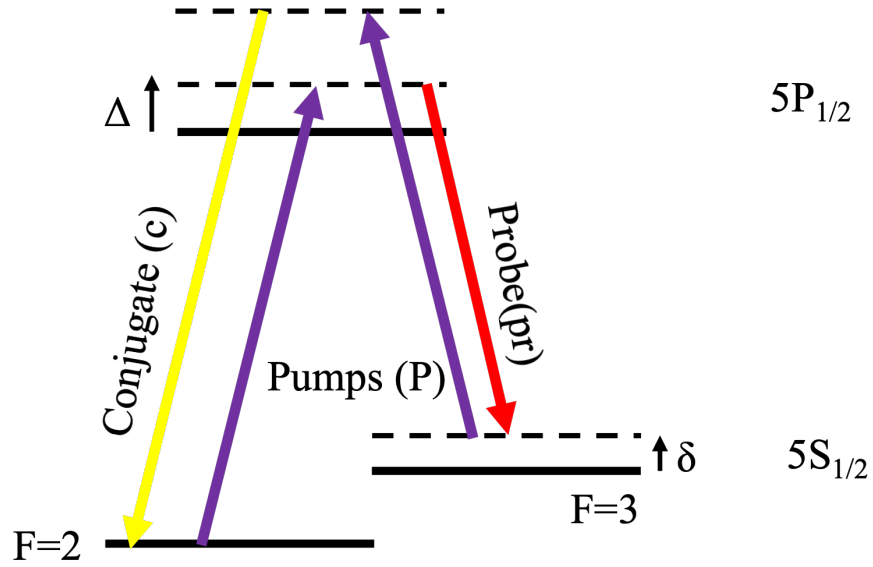


Figure 4.1: Double lambda configuration for FWM in ^{85}Rb . Two pump photons, P (purple) are destroyed with the simultaneous generation of a probe, pr (red) and conjugate, c (yellow).

4.1 Four Wave Mixing

In FWM process an intense light beam called pump with frequency ω_P interacts with an atomic medium, resulting in the destruction of two pump photons and the simultaneous generation of a probe and conjugate photon, also called the twin beams. The frequencies of annihilated and created photons satisfy

$$2\omega_P = \omega_{pr} + \omega_c \quad (4.2)$$

so that energy is conserved. Here ω_P , ω_{pr} and ω_c are the frequencies of pump, probe and conjugate respectively. Fig.4.1 shows the double lambda configuration for the FWM process in the D1 line of ^{85}Rb . The difference in frequency, between the pump and probe fields is equal to the two photon detuning, δ plus the difference between ground levels of the hyperfine splitting i.e., $\omega_P - \omega_{pr} = \omega_{HF} + \delta$ where ω_{HF} is the energy difference between the hyperfine levels F=2 and F=3 of $5S_{1/2}$. Efficient FWM happens when the frequency of the field is in resonance with atomic transition. However, absorption which is a linear process may suppress the nonlinear FWM on resonance. To avoid this we slightly detune the frequency of the pump by Δ as shown in Fig. 4.1.

From Fig.4.1 it is evident that an input probe is not necessary for the occurrence of FWM. It is possible to seed the process with no beams, either probe or conjugate, or both beams together. If the process is not seeded with either beams then we will have two mode squeezed vacuum discussed in the previous chapter. However, if we seed the FWM process with an input probe we will get a squeezed displaced vacuum state

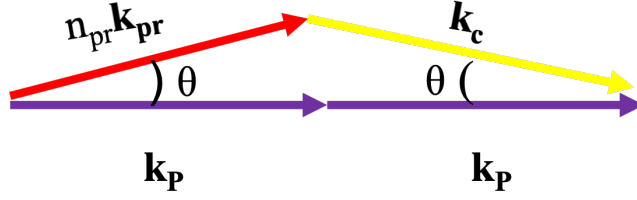


Figure 4.2: Phase matching condition in FWM process between pump(purple), probe (red) and conjugate(yellow) by tuning the angle θ between the beams.

which contains a higher number of mean photons. Hence this two mode squeezed state is called bright two mode squeezed states(bTMSS).

4.1.1 Phase matching and spatial correlations

The FWM process is a parametric process. In a parametric process, the state of the system before and after the process is the same or in other words the momentum and energy is conserved. There is no loss of photons due to its interaction with the atomic medium. The momentum conservation is more generally written in terms of wave vectors, k . For copropagating beams, the phase matching condition (or equally momentum conservation) is

$$\Delta k = 2k_P - k_{\text{Probe}} - k_{\text{Conjugate}}. \quad (4.3)$$

Here k_P , k_{Probe} and $k_{\text{Conjugate}}$ is the momentum vector of the pump, probe and conjugate respectively. At $\Delta k = 0$ the FWM occurs effectively. However, in general cases $\Delta k \neq 0$ and this phase mismatch reduces the efficiency of FWM. The phase mismatch is due to the change in the effective refractive index of the medium as seen by a beam when it propagates through a medium. This is the result of the dispersive behaviour of the medium. Because of this we need to define a new phase matching condition. As can be seen from Fig.4.1, the frequency of pump, probe and conjugate photons are different. Hence we need to consider the frequency dependence of the index of refraction for all three beams. The susceptibility, χ of the medium given in Eqn.4.1 does not play a role far off resonance. Therefore we can safely neglect the dispersive behaviour of pump by assuming that the effective index of pump is close to 1 since it optically pump the atoms towards the off resonant transitions[20]. The conjugate is also far off resonance. So by

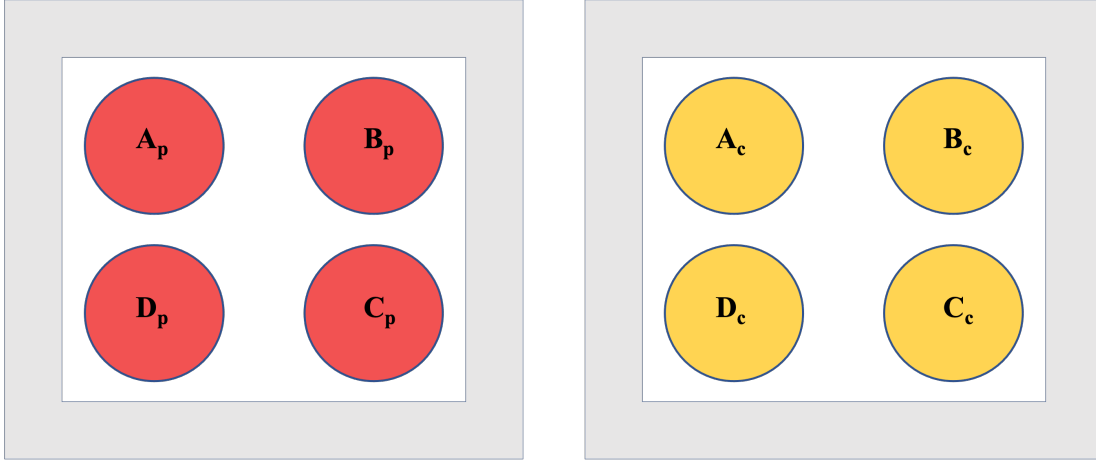


Figure 4.3: Near field correlations for multi-spatial modes: Left half image shows probe (red) with multiple independent spatial modes. After FWM conjugate(yellow) with the same spatial pattern will be generated. This is shown in the right half of the image. Each mode in probe will be correlated with the corresponding mode in conjugate. A_p, B_p, C_p and D_p will be correlated with A_c, B_c, C_c and D_c respectively at the center of the cell(near field). Here A_p, B_p, C_p, D_p and A_c, B_c, C_c, D_c are used to label the different spatial modes of probe and conjugate

neglecting any change in behaviour for pump and conjugate , the new phase matching condition becomes,

$$\Delta k = 2k_p - n_p k_{\text{Probe}} - k_{\text{Conjugate}} = 0. \quad (4.4)$$

To accomodate the effective change in index of probe and to satisfy the new phase matching condition we can tune the angle θ between pump and probe when two pump photons are copropagating as shown in the figure[20]. Hence at the center of a rubidium vapor cell, the pump and probe beam intersects at an angle θ to undergo efficient FWM as shown in Fig.4.3.

4.2 Multi-Spatial Modes in Twin beams

The input seed can be encoded with arbitrary patterns to generate different spatially independent modes within a single probe beam. When this spatially modulated beam undergoes FWM, conjugate with same spatial pattern but opposite momentum is generated at the same spatial locations. However after the FWM, the twin beams will symmetrically propagate away from each other due to phase matching. This phase matching condition given in Eqn.4.4 also governs the spatial distribution of the probe and conjugate. As a result the twin beams will have position dependent quantum correlations. Hence the spatial correlations between the twin beams depend on the analysis plane,i.e., whether we are measuring the correlations in the near field or the far field. The center of the cell where the probe is amplified and conjugate is

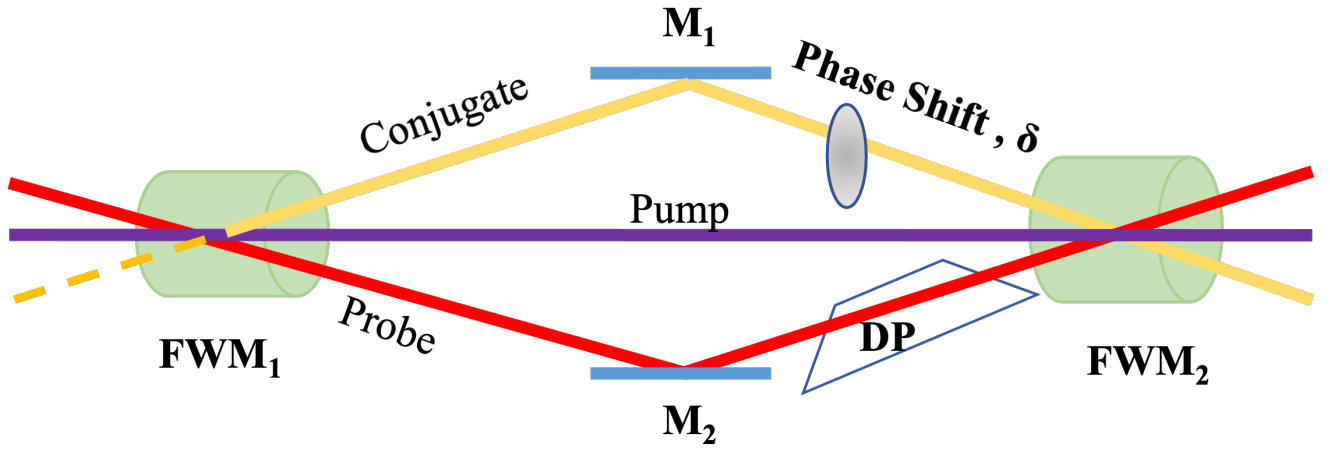


Figure 4.4: The SU(1,1) like interferometer with a dove prism (DP) in one of the arms for rotating the probe beam . In addition to the generation of newly entangled pairs after the second FWM ,the modes entangled after the first FWM will preserve its correlation. M_1 and M_2 are mirrors.

generated simultaneously is called the near field. Since they are generated at the same spatial location the correlated spatial regions will be the ones with same orientation. As shown in Fig.4.2 the corresponding modes of probe and conjugate will be correlated. Throughout our experiment, we are measuring near field correlation i.e., in the imaging plane .

4.3 Generation of scalable multipartite entanglement

In a traditional quantum interferometer, two beam splitters (BS) with a phase shift introduced in between serve as the splitting and the mixing element for the input light beam. Unlike this, in an SU(1,1) interferometer two BS's are replaced by parametric amplifiers (PA). These parametric amplifiers are a result of three or four wave mixing (FWM in our case) and they can split and mix two input fields coherently for interference. The second parametric process ensures that the correlations are preserved. In our experiment we are using an SU(1,1) like interferometer for the generation of scalable multipartite entanglement. After the FWM certain pairs of probe and conjugate spatial modes will be entangled as shown in Fig.4.2 [21]. To entangle uncorrelated pairs of spatial modes in twin beams that are not already entangled after the first FWM process ,we are doing a second FWM process. For example $[A_p, B_c]$, $[B_p, C_c]$, $[C_p, D_c]$ and $[D_p, A_p]$ are some pairs that can be entangled through the second FWM if we rotate the probe by 90 degrees clockwise.

As shown in Fig.?? the probe beam is rotated after the first FWM before injecting it to the second FWM. This means that the input spatial modes of probe to the second FWM have different spatial locations compared to the input seed to the first FWM. From Fig.4.2 only 4 pairs of probe and conjugate modes

can be correlated after first FWM. However using two FWM, we can create 8 pairs of correlated probe and conjugate. The implementation of these will be discussed in more detail in the following chapter.

Chapter 5

Experimental setup and preliminary results

To generate the beams, we use a CW Ti: Sapphire laser. A portion of the beam is separated using a beam sampler and passed through an acoustic optical modulator(AOM) which is tuned in order to generate the probe beam at the desired frequency. The probe beam(red) and the intense pump beam(purple) are then passed through single mode optical fibers to obtain a clean spatial mode before the FWM process.

The probe coming out of the fiber is projected onto the DLP to create different spatial modes out of a single beam. The DLP is a digitally controlled micromirror device that is also a spatial light modulator(SLM). The desirable image is fed into the DLP screen as a bitmap, with the required pattern having a binary value of 1 and the rest of the background with a binary value of 0. The DLP has two tilt angles depending on the binary value of the pixels. A binary value of 0 and 1 will result in a tilt angle of -12° and $+12^\circ$, respectively. So we can get to only reflect the pattern in one particular direction, thereby getting rid of the rest of the background. This beam containing a specific pattern is then used as the input seed beam for the four wave mixing process. For this experiment, the probe beam is modulated to contain two spatially independent circles. Since we are doing analysis in the near field, the beam at the DLP is imaged to the center of the first cell using a 4f imaging system using L_1 and L_2 as shown in Fig.5.

The orthogonally polarized intense pump beam and weak probe beam intersect at an angle[20] inside the center of the cell. The cell is 12mm long, and the temperature of the cell is kept high to increase the atomic density to provide enough atoms to participate in the process. As a result of FWM the probe beam will have a gain with the simultaneous generation of conjugate(yellow). After the FWM process, both of these beams will move away from each other by momentum conservation, and they are spatially separated and imaged to the second cell using another 4f imaging system (L_3 and L_4).

Using a dove prism in one of the arms we are rotating the probe image before injecting it to the second FWM. As a result of rotating one of the beams, the mutually uncorrelated mode pairs of probe and conjugate will overlap for the second FWM process.

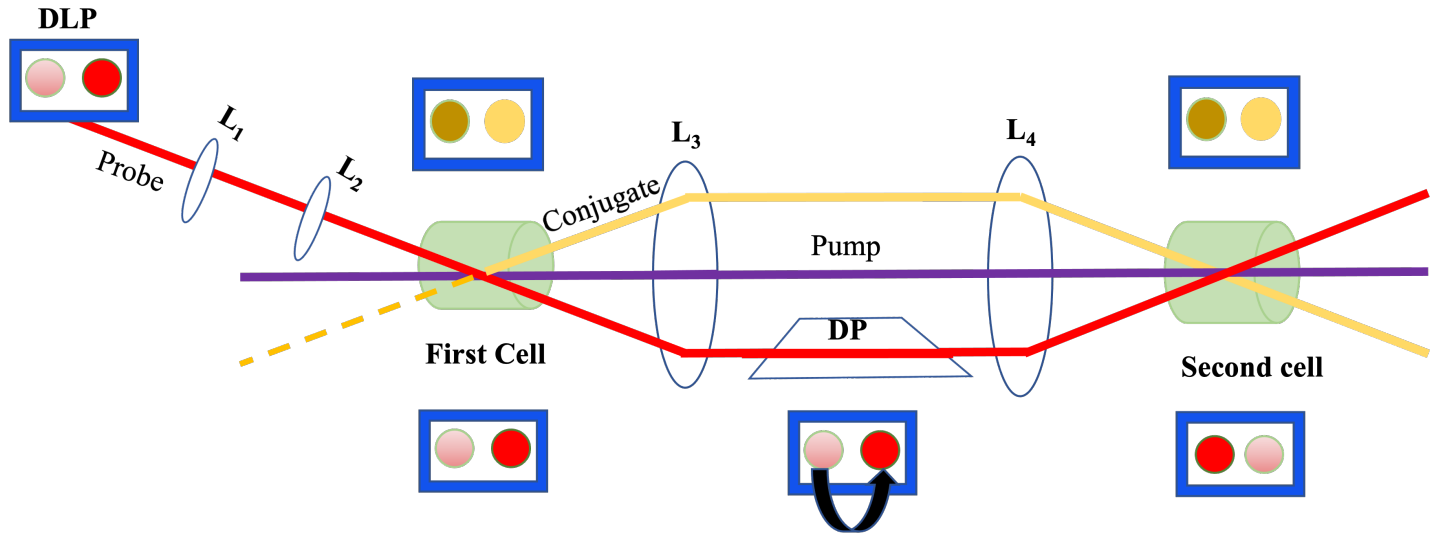


Figure 5.1: Two four wave mixing setup. Red is the probe, purple is the pump, and yellow is the conjugate beam. The probe and pump intersect at the center of the first cell by simultaneously generating the conjugate. All three beams are imaged from the center of the first cell to the center of the second cell using a 4f imaging system. Using a dove prism, the probe image is rotated after the first FWM, thereby swapping the position of individual circles in one beam, whereas the conjugate remains the same. After the second FWM, all possible pairs of the different modes of probe and conjugate will be mutually correlated. This is shown below the center of the second cell.

Preliminary Experimental Results

If the different modes of probe and conjugate are not separately correlated (i.e., cross talk exists between the modes), clipping the beam to select correlated subregions will decrease the amount of squeezing. In order to verify the multi-spatial mode nature of the twin beams, we blocked one pair of correlated twin beams at a time and measured the noise of the other pair. To measure the noise, the beams are separated after the first FWM, imaged to the clipping plane, and then sent into a detector which is connected to a spectrum analyzer. These results are shown in Figure ?? and Figure ?. In Figure ??, we can see the correlations between the corresponding circles of probe and conjugate. A squeezing of 5db below the shot noise is achieved without any clipping. The level of squeezing is maintained for a single pair of circles indicating that the circles can be seen as different spatial modes. Multiple parameters such as pump power, the temperature of the cell, and detuning were optimized to obtain efficient FWM. For the data plotted in Figure ?? and Figure ??, all parameters affecting the FWM were kept constant to make the squeezing comparable. The data was taken for a pump power of 350 mW and a cell temperature of 107°C. This verifies that there is no overlap indicating that the spatial regions are independent and can be treated as separate spatial modes. In contrast, if we measure the noise of anti-correlated beams, the noises of the individual modes add up. This is verified from Figure ?. This figure shows the noise spectrum of the left circle of the probe and the right circle of the conjugate, which is not correlated after the first

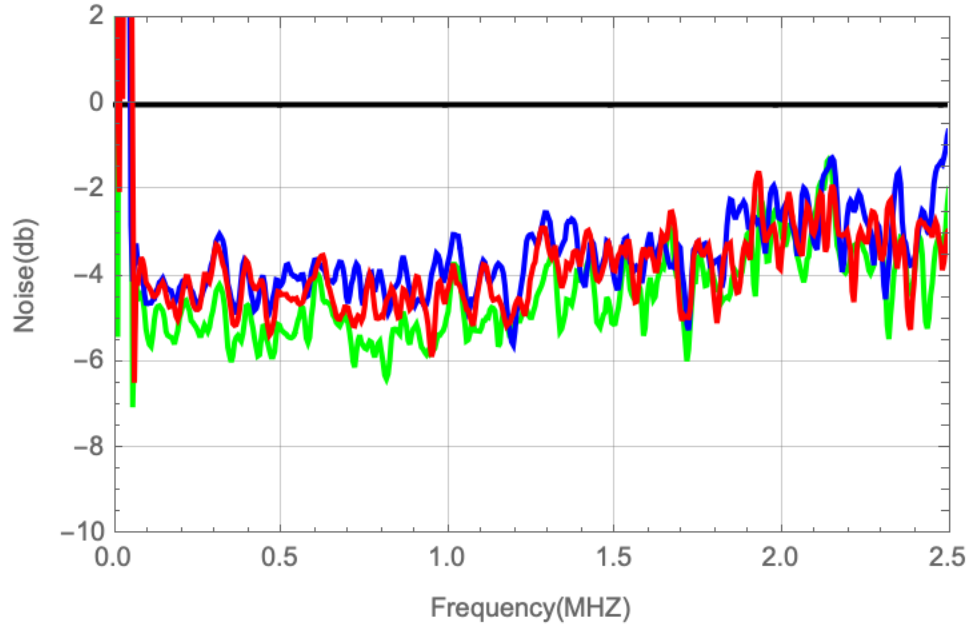


Figure 5.2: Black line is the standard noise limit. Green trace shows the noise of probe and conjugate after the first FWM. Red trace is the noise with only the right circle of probe and conjugate. Blue is the noise spectrum of the left circles of twin beams.

FWM.

5.1 Conclusion

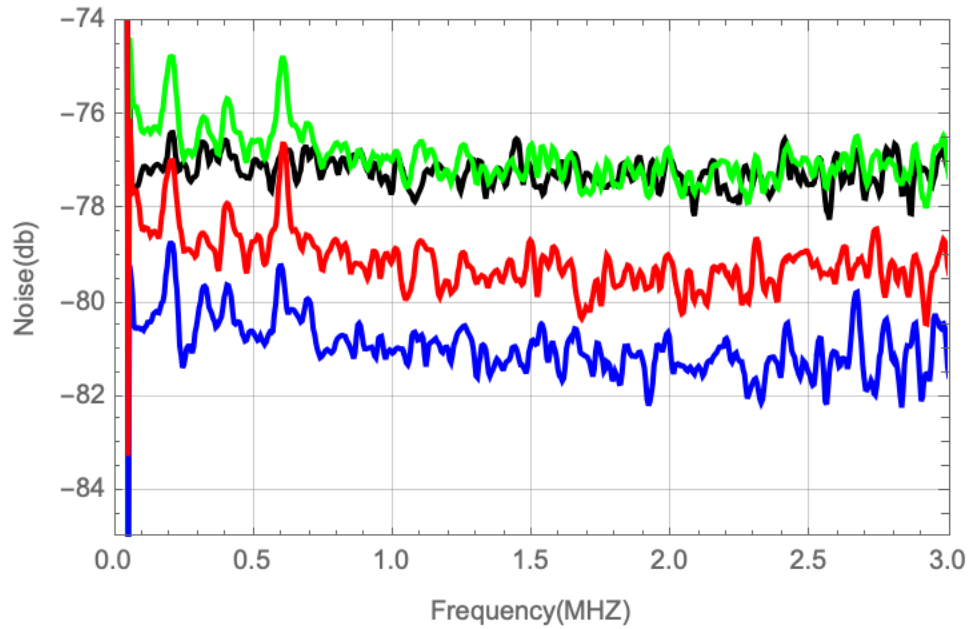


Figure 5.3: Red and blue traces corresponds to the noise spectrum of the left circle of probe and conjugate respectively. Black dotted line trace shows the added noise of the red and blue traces. Black solid line is the noise corresponding to the left circle of probe and the right circle of conjugate. As expected, the noises add up verifying that they are independent and not correlated.

Chapter 6

Future work

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