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A RADIATIVE HEAT TRANSFER ANALYSIS OF

DIATHERMANOUS SHEETS AND COATINGS

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ABSTRACT

In the work of previous investigators, the analysis of dielectric coatings and sheets of dielectrics has been through the solution of the transport equation as applied to flux rather than intensity. In this way the directional dependency of intensity is neglected.

In this investigation the directional dependency of intensity is accounted for through the solution of the transport equation at descrete angular positions. The model employed is infinite in extent but of finite thickness, therefore allowing the assumption of axial symmetry. Initially all interfaces are assumed to be perfectly amooth specular reflectors.

Fresnel's law is used to determine the directional emittance of the conductor into various dielectrics, and these are found plotted in the thesis. Fresnel's law is also used to determine the reflectance of the dielectric-air interface.

Emittance of various coating-conductor combinations with varying thicknesses are calculated, and the curves are presented. These are numerically integrated to obtain the hemispherical emittance and a comparison is given to those values found by using the attenuated flux approach.

The analysis is then carried on to nonisothermal coatings with both linear and parabolic temperature profiles imposed on the coating. The dimensionless flux ratio is plotted for specific cases.

The same procedure is followed on a sheet of dielectric material, and in addition the directional transmittance is obtained.

The analysis of the coatings is then modified using the combined

diffuse and specular reflection at the dielectric-conductor interface. These values are plotted for hemispherical values only with two values of surface roughness.

For the isothermal coating, scattering is considered with the scattering medium being spherical air bubbles. These results are also plotted for hemispherical values.

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CHAPTER I

INTRODUCTION

The current activities of the aerospace industries in the area of ceramic coatings as a heat shield have increased interest in the effect of ceramic coatings on thermal radiation. Perhaps even more interest has been generated through the need of either highly reflective or highly absorbing coatings on satellites. Still another area of interest is in the study of frequency selective coatings. They might also find application as paints or windows in houses or automobiles functioning in different locations as either good solar absorbers for cold climates or good solar reflectors for the more arid regions.

For years physicists have studied radiation in transparent media in relation to their interest in optics. While most of their interest has been in thin film optics, they have also done work considering coatings of thickness much greater than a wavelength.

The study of the effect of protective coatings on optical instruments and the effect of corrosive, oxide or liquid films on the optical properties of materials has been a major factor in encouraging the work in thin film optics. The physicist has been involved in the optics of sheets of a dielectric in relation to the glassmaking industry for years, but their interest in optically thick coatings has been centered around problems such as the desire to know the effect that a backup or holding plate for a sample of a dielectric might have on the measurements of the dielectric's refractive index.

Radiative heat transfer owes much of its theory to the work of physicists working in the area of optics and electromagnetic wave theory, but this indebtedness has often times become a handicap. For example, most, if not all, of the equations in optics involve the use of the refractive index of the medium in which the photons of light are traveling. Since this is normally air with a refractive index of unity, the formulas have become nearly standardized with unity substituted for the refractive index. When engineers work in media other than air, they have a tendency to continue to use these simplified formulas. Nearly all of the heat transfer texts have directional emittance curves for both dielectrics and conductors, but these curves are for emittance into air. When the dielectric or conductor emits into a medium other than air, the emittance curves change form since the refractive index of the emitter to the surrounding medium determines the shape of the emittance curve. These curves for emittance into dielectrics are not readily available.

It has long been known that a dielectric coating on a conductor tends to change the form of the emittance curve from that of the conductor to more nearly that of the dielectric. In the past this has often been explained by stating that emittance is a surface property, or just as often it has been used to verify the assumption that emittance is a surface property. However, the more rigorous investigators, realizing that any emission of energy would necessarily be associated with some elemental volume of mass, have stated that the emittance is different because of the combined emission of the conductor and the dielectric. While this statement is quite reasonable, there has been to the author's knowledge no theoretical justification, as for example curves showing the change in directional emittance for different coatings and coating thicknesses on various conductors. What little work has been done in the radiative heat transfer analysis of coatings has largely used the approach of Hamaker (9). In this approach the transport equation is applied to the flux rather than the intensity and therefore the directional dependency of the intensity is neglected.

It is the purpose of this work to contribute to the state of the art of radiative heat transfer analysis of diathermanous materials by supplying some analytical tools which will provide a more accurate approach to the problem.

CHAPTER II

LITERATURE SURVEY

Investigators in the area of radiant heat transfer have one definite advantage over engineers working in many other areas. This advantage is in the development of a literature survey. The advent of the space age has greatly renewed interest in radiant heat transfer. However, the amount of work done in this area still does not even compare to the volumes of material available in such areas as stress analysis, fluid flow or even the other modes of heat transfer. Lest one should be led astray, it should be pointed out that even though there has been less work done in the radiation area than some other areas, the amount of literature available is extensive. Vast amounts of literature related to this paper exist in all areas of radiative heat transfer and even in areas one might think are unrelated. It is not the purpose of this work to make a complete literature survey in the radiant heat transfer area; it is only hoped that those articles which are pertinent to the particular problems involved in this work have been uncovered.

Several investigators have made studies relating the effect thickness has on the emittance of a dielectric sheet. McMahon (17) developed analytically the apparent emissivity, transmissivity and reflectivity of partially transparent bodies by considering an infinite number of internal reflections. His approach gives these properties for unidirectional radiation only as a function of thickness and wavelength. Funk (7) has developed general formulas for the absorption and emission of arbitrary absorbers and specifically for slabs. Through the consideration of each term of the transport equation separately and assuming an infinite number of internal reflections, his equations are formulated considering both internal and external reflections and a gray medium. Gardon (8), whose approach is most similar to one used in this paper, has developed equations for both the directional and hemispherical emittance. His approach utilized the volume emissive power of the dielectric attenuated by the well-known exponential relation. He uses the boundary condition considering an infinite number of internal reflections and also considers the effect of the refractive index on the emission. Laikin (13) has studied the reflection and transmission of thin dielectric films. His studies are for films less than one wavelength in thickness for normal incidence alone. The reflectance and transmittance for various thicknesses of a dielectric sheet has been evaluated by Boehringer (3); however, his analysis is for hemispherical properties considering scattering.

In all of the papers mentioned, consideration was given only to the isothermal sheet. However, in this work an analytical approach will be given which will consider both an isothermal and a nonisothermal dielectric considering the surfaces as specular reflectors, and the assumption of an infinite number of internal reflections is avoided.

Until recently the major interest has been in the dielectric itself rather than in a dielectric coating on a conductor. Hamaker (9) studied radiation and combined radiation and conduction in light-scattering material. His approach is to apply the transport equation to the flux rather than the intensity considering the surfaces as diffuse reflectors. Richmond (18) has used a similar approach considering the interfaces to reflect specularly by Fresnel's law. Both of these approaches consider only isotropic scattering. The work of Cess and Sotak (4) solves the transport equation between two parallel plates separated by an emitting and absorbing medium. One plate is an isothermal specular reflector and a diffuse emitter, while the other plate is an isothermal black body.

The investigators who have applied the transport equation to the flux have ignored the directional dependency of the intensity. In this paper the transport equation for intensities is solved for both the isothermal and nonisothermal coatings. The analysis is for both an absorbing and emitting medium and for an absorbing, emitting and scattering medium. The method of solution follows the work of Love (16) and includes his analysis of anisotropic scattering. The air-dielectric interface is assumed to be a specular reflector and dielectric-conductor interface is first assumed to be a specular reflector and then a combination of both a diffuse and specular reflector as outlined by Birkebak (1).

This paper also includes directional transmittance and absorbance of coatings and dielectric sheets. Also the temperature profile is determined by considering unidirectional radiation incident on the surface and combined conduction and radiation within the medium. Viskanta (23) has studied combined conduction and radiation in an isotropic scattering medium. His dimensionless form of the governing equation has been used in this work, but the radiation term is quite different.

CHAPTER III

DEFINITIONS AND CONCEPTS

In examining the literature in the radiant heat transfer area, one finds a wide variation in the definitions of the various properties. This ambiguity in the terminology can cause one a great deal of difficulty when gathering a literature survey in the area.

While it would be most beneficial to have a standardized nomenclature in this area, it is not the intent of this work to attempt such a standardization. However, in reading the literature, one does soon realize the value of clearly defining all the terminology to be used in any particular paper. With this in mind, it is the intent of this chapter to clearly define all of the physical quantities involved. The definitions given here will follow explicitly those given by Love (15).

The word monochromatic, meaning one color, will be used in this work to denote that the property is being observed at a particular frequency given by the Greek symbol υ. The choice of constant frequency over constant wavelength is made because the frequency does not change when going from one medium to another. Remembering that the frequency and wavelength of radiation are interrelated and that all of the laws of experimental optics except that of diffraction are incorporated in the study of radiant heat transfer, one realizes that the larger the wavelength of the radiation to be considered, the larger in size must be the bounding space.

1) I_υ Monochromatic intensity of radiation. The radiating electromagnetic energy leaving a differential element of area of an imaginary plane within the time interval t, t + dt, having a direction of propagation contained in the differential solid angle $d\omega$ about a normal direction to the plane and characterized by a frequency within the frequency interval υ, $\upsilon + d\upsilon$. Mathematically it may be expressed as

$$I_\upsilon = \frac{e_\upsilon}{d\omega\, d\upsilon dtA}$$

where e represents a quentity of monochromatic radiant energy.

2) I Intensity of radiation. The radiating electromagnetic energy leaving a differential element of area of an imaginary plane within the time interval t, t + dt, having a direction of propagation contained in the differential solid angle $d\omega$ about a normal direction to the plane. Mathematically expressed, it is

$$I = \frac{e}{d\omega\, dtdA} = \int_0^\infty I_\upsilon d\upsilon.$$

Here one must show care in the size of the accounting period. According to Planck (14), the time interval is even more important than the dimensions of the system since intensity is the energy transmitted by the ray per unit time. This implies the assumption that the unit of time chosen is large compared with the period of vibration corresponding to the color of the ray.

To avoid this difficulty, the period of time used in defining the intensity in differential form is large compared with the frequencies contained.

3) q_ν Monochromatic radiation flux. The radiating electromagnetic energy leaving a differential element of surface area of an imaginary plane in all directions contained in the hemispherical solid angle bounded by the plane, with the time interval t, t + dt, and characterized by a frequency in the interval ν, $\nu + d\nu$.

$$q_\nu = \int_0^{2\pi} I_\nu \cos\theta d\omega$$

where θ is the angle between the direction of the intensity and the normal to the imaginary plane. Noting that in spherical coordinates $d\omega = \sin\theta d\theta d\phi$, where ϕ is an azimuthal angle,

$$q_\nu = \int_0^{2\pi} \int_0^{\pi} I_\nu \cos\theta \sin\theta d\theta d\phi.$$

This is often written letting $\mu = \cos\theta$.

$$q_\nu = \int_0^{2\pi} \int_0^{1} I_\nu \mu d\mu d\phi$$

4) Q_ν Net monochromatic radiation flux. The difference formed by subtracting the monochromatic radiation flux leaving the arbitrarily designated negative side of a differential element of area of an imaginary surface from the monochromatic radiation flux leaving the positive side.

$$Q_\nu = q_\nu^+ - q_\nu^- = \int_0^{2\pi}\int_{-1}^{+1} I_\nu \mu d\mu d\phi$$

5) q Radiation flux. The radiating electromagnetic energy leaving a differential element of an imaginary surface within the time interval t, t + dt in all directions contained within the hemispherical solid angle bounded by the surface.

$$q = \int_0^\infty q_\nu d\nu$$

6) Q Net radiation flux. The difference formed by subtracting the radiation flux leaving the arbitrarily designated negative side of a differential element of area of an imaginary surface from the radiation flux leaving the positive side.

$$Q = \overline{q^+} - q^- = \int_0^{2\pi}\int_{-1}^{+1} I\mu d\mu d\phi$$

Having presented the definitions of the basic radiation properties, attention will now be given to a few basic concepts, some of which are developed in most texts in the area and others which are not so readily found in the literature.

It can easily be shown that for an isolated, evacuated enclosure, which is in thermodynamic equilibrium with its bounding surfaces, the radiation field must be isotropic and, even more restrictive, must correspond to the intensity of radiation emitted by a black surface at the temperature of the enclosure. This relationship was first developed by Kirchhoff as a consequence of the second law of thermodynamics. Next, consider an enclosure in thermodynamic equilibrium containing a different medium. Using the same reasoning, namely that there cannot be a directed flow of energy within the enclosure, one determines that the radiation field is again isotropic. A directed flow is not permissable in the state of thermodynamic equilibrium, since every ray must have just the same intensity as the one traveling in exactly the opposite direction.

Since the radiation is isotropic in any homogeneous enclosure in thermodynamic equilibrium, it follows that if two different media are within the enclosure, the radiation within each will be isotropic. This can be shown by fictitiously placing a perfect insulator between the two media. The insulator in no way disturbs the thermodynamic equilibrium; therefore, the intensity also must be isotropic in each medium when the insulator is removed.

Having determined that the intensity within an enclosure in thermodynamic equilibrium is isotropic, it now becomes desirable to know the relationship between any two media within the enclosure. To determine this relationship, consideration is given to a perfectly smooth interface between two media in thermodynamic equilibrium. (See Figure 1) The intensity within the first medium will be denoted by I_{υ_1} and the intensity within the second medium by I_{υ_2}. Similarly the intensity I_{υ_1} is contained within the solid angle $d\omega_1$ and I_{υ_2} is contained within the solid angle $d\omega_2$. The interface will have a directional reflectivity and emissivity given by Fresnel's equations, but for simplicity consideration will be given to a monodirectional oblique ray, and therefore, the coefficient of reflection in medium 1 will be given by ρ_1 and in medium 2 will be given by ρ_2.

The fraction transmitted through the interface from medium 2 into medium 1 will be given by τ_2, where Kirchhoff's relationship $\tau_2 = 1 - \rho_2$.

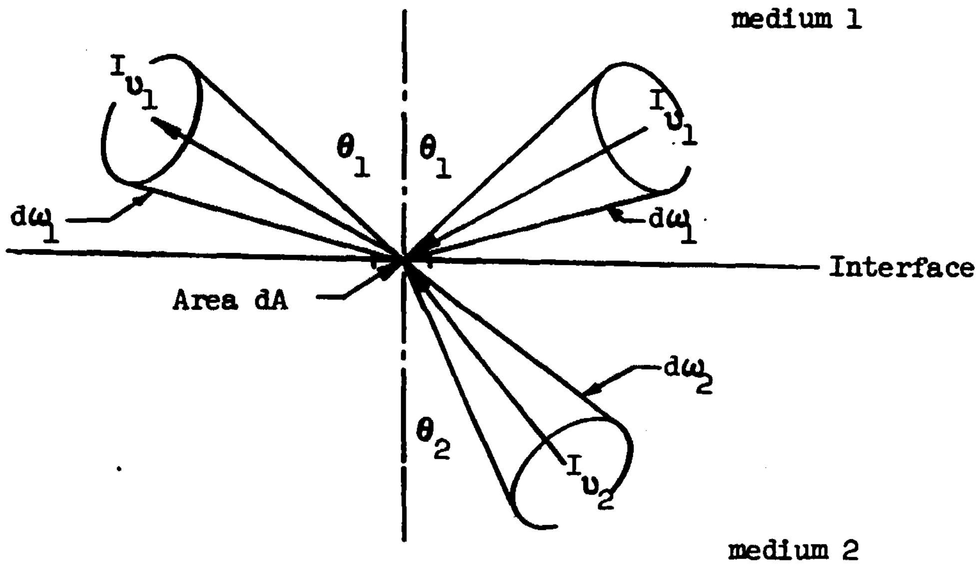


Figure 1

By the previous definitions, the energy leaving the element of area in the direction θ_1 into medium 1 is given by

$$dAdtd\upsilon\cos\theta_1 d\omega_1 I_{\upsilon_1} .$$

This energy is composed of that energy incident on the interface from medium 1 at the same angle θ_1, and reflected into θ_1, which is given by

$$\rho_1 dAdtd\upsilon\cos\theta_1 d\omega_1 I_{\upsilon_1} ,$$

and that energy transmitted from medium 2 at the angle θ_2 and refracted into medium 1 at the angle θ_1, which is given by

$$(1 - \rho_2)dAdtd\upsilon\cos\theta_2 d\omega_2 I_{\upsilon_2}.$$

Therefore,

$$dAdtd\upsilon\cos\theta_1 d\omega_1 I_{\upsilon_1} = \rho_1 dAdtd\upsilon\cos\theta_1 d\omega_1 I_{\upsilon_1} + (1 - \rho_2)dAdtd\upsilon\cos\theta_2 d\omega_2 I_{\upsilon_2},$$

or

$$I_{\upsilon_1}\cos\theta_1 d\omega_1(1 - \rho_1) = I_{\upsilon_2}\cos\theta_2 d\omega_2(1 - \rho_2),$$

which gives

$$\frac{I_{\upsilon_1}}{I_{\upsilon_2}} = \left[\frac{1 - \rho_2}{1 - \rho_1}\right] \frac{\cos\theta_2 d\omega_2}{\cos\theta_1 d\omega_1}.$$

By definition,

$$d\omega_2 = \sin\theta_2 d\theta_2 d\phi \text{ and } d\omega_1 = \sin\theta_1 d\theta_1 d\phi.$$

By now recalling Snell's law of refraction for a perfectly smooth surface, namely

$$n_1\sin\theta_1 = n_2\sin\theta_2,$$

where n_1 and n_2 are the respective refractive indices of medium 1 and medium 2, a further simplification can be performed. Squaring both sides of the equation yields

$$n_1^2\sin^2\theta_1 = n_2^2\sin^2\theta_2,$$

and upon differentiating both sides, the equation becomes

$$n_1^2 2\sin\theta_1 \cos\theta_1 d\theta_1 = n_2^2 2\sin\theta_2 \cos\theta_2 d\theta_2,$$

or

$$\frac{\sin\theta_2 \cos\theta_2 d\theta_2}{\sin\theta_1 \cos\theta_1 d\theta_1} = \frac{\cos\theta_2 d\omega_2}{\cos\theta_1 d\omega_1} = \frac{n_1^2}{n_2^2} .$$

Substituting this into the relationship for intensities yields

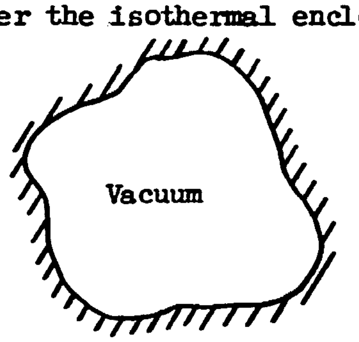
$$\frac{I_{\upsilon_1}}{I_{\upsilon_2}} = \frac{1 - \rho_2}{1 - \rho_1} \left[\frac{n_1}{n_2} \right]^2 .$$

However, the coefficients of reflection on both sides of the interface are equal as a result of Helmholtz's reciprocity law. This means that the intensities of two different media in thermodynamic equilibrium are related directly to each other by inverse ratio of their refractive indices squared. Namely,

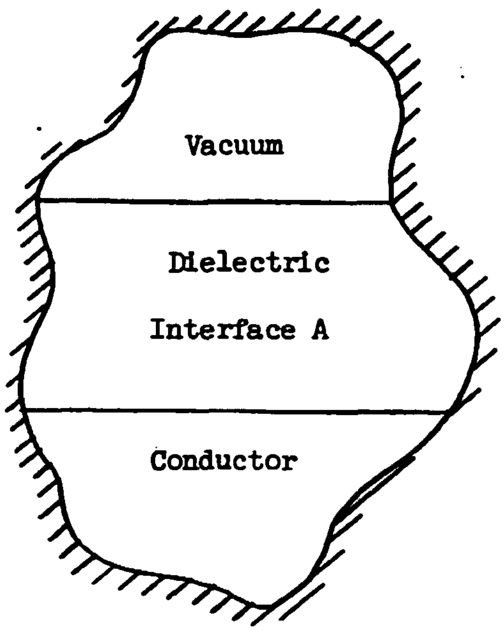
$$I_{\upsilon_1} = \left[\frac{n_1}{n_2} \right]^2 I_{\upsilon_2} .$$

This useful concept allows one to look at a more specific concept which will be needed to correctly analyze the problem of dielectric coatings.

Consider the isothermal enclosure shown with a vacuum within. It is readily shown that the intensity within the enclosure is that of a black surface emitting at the same temperature, $I_{bb}(T)$.



Now extend the enclosure to include a metal coated with a dielectric as shown. The intensity in the evacuated portion is still $I_{bb}(T)$, but the intensity in the dielectric is $n^2 I_{bb}(T)$. At the dielectric-conductor interface A, if it is assumed that the interface reflects specularly, the intensity can be given by



$$n^2 I_{bb}(T) = n^2 I_{bb}(T)\rho_A + \text{emitted energy}$$

or

$$n^2(1 - \rho_A) I_{bb}(T) = \text{emitted energy}$$

Therefore, the emitted energy must be $n^2 \epsilon_A I_{bb}(T)$, where ϵ_A equals $(1 - \rho_A)$ by definition.

This concept, so far as the author has been able to determine, has not been brought out clearly anywhere in the literature. It should be noted from the above discussion, that it is possible to have intensities greater than black body intensity since the black body concept is merely a body which absorbs all incident rays.

It also follows from the previous argument that for an enclosure in thermodynamic equilibrium containing more than one medium, since

$$I_{\nu_1} = \left[\frac{n_1}{n_2}\right]^2 I_{\nu_2},$$

$$\frac{I_{\nu_1}}{n_1^2} = \frac{I_{\nu_2}}{n_2^2} \text{ or } \frac{I_\nu}{n^2}$$

is constant throughout the enclosure. Therefore, if part of the enclosure is evacuated, $\frac{I_\nu}{n^2}$ would be equal to

$$\frac{I_{bb_\nu}(T)}{(1)^2} = I_{bb_\nu}(T)$$

or the monochromatic black body intensity. Hence,

$$\frac{I_\nu}{n^2} = I_{bb_\nu}(T)$$

for all media in thermodynamic equilibrium. Now, by writing the emission coefficient as

$$j_\nu = \kappa_\nu I_{bb_\nu}(T),$$

the absorption coefficient as k_ν, and following Kourganoff (12) the intensity I_ν in the medium is given by

$$I_\nu = \frac{j_\nu}{k_\nu} .$$

It follows, therefore, that

$$\kappa_\nu = n^2 k_\nu \text{ or } j_\nu = n^2 k_\nu I_{bb_\nu}(T).$$

This is a point very often neglected by authors in this area as pointed out by Gardon (8). Most authors have used the simplified version where n is assumed to be one. This is accurate enough when dealing with gases as a medium, but there is a tendency to continue to use the simplified form of the relation when it is used in materials with refractive indices sufficiently greater than one. As will be seen later in this work, often the refractive index drops out of the equation of transport and can therefore be neglected in the first place without loss of accuracy. However, the end does not justify the means, and consideration should always be given the refractive index in the equations, even when it is near unity.

All of the concepts mentioned in this chapter are for a homogeneous medium in thermodynamic equilibrium, but often times the actual case is nonisothermal and/or the medium is hetrogeneous enough to encounter scattering. Planck points out that, "small impurities, as for instance, particles of dust, increase the influence of scattering without, however, appreciably affecting its general character. Hence, so-called 'turbid' media, i.e., such as contain foreign particles, may be quite properly regarded as optically homogeneous, provided only that the linear dimensions of the foreign particles, as well as the distances of neighboring particles are sufficiently small compared with the wavelength of the rays considered."

In order to apply Kirchhoff's law to a medium not in thermodynamic equilibrium, one assumes that the medium is in local thermodynamic equilibrium. This says that there is a region small enough at any point and at some temperature T such that Kirchhoff's law holds. Thus, in a nonisothermal enclosure, the temperature may vary from point to point, but each point may be characterized by a definite temperature T, so that an element of matter at each point is behaving as if in local thermodynamic equilibrium at temperature T as stated in Viskanta (23).

CHAPTER IV

THERMAL RADIATION IN AN ABSORBING AND EMITTING MEDIUM

The most simplified form of the equation of transport is termed Beer's law. This law states that the change of radiant intensity across an absorbing medium is directly proportional to the product of the intensity and the distance traveled. In this form the equation of transport considers only absorption. To account for emission the transport equation becomes a modification of Beer's law.

For a homogeneous absorbing and emitting medium the equation of transport becomes

$$\frac{dI_\nu(s)}{ds} = - \rho k_\nu I_\nu(s) + \rho \kappa_\nu I_{bb_\nu}(T),$$

which is the equation for conservation of radiant energy along a ray in the arbitrary direction s, where emission and absorption are accounted for, but scattering is neglected.

k_ν	is the monochromatic absorption coefficient.
ρ	is the mass density of the medium.
K_ν	is the monochromatic mass absorption coefficient.
κ_ν	is the monochromatic emission coefficient.
$I_\nu(s)$	is the monochromatic intensity of radiation.
$I_{bb_\nu}(T)$	is the monochromatic intensity of a black body.

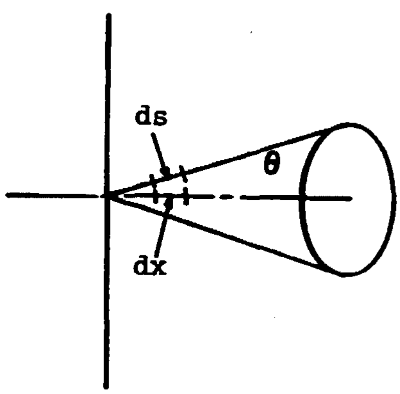
n is the refractive index of the medium.

By applying Kirchhoff's law, the equation of transport becomes

$$\frac{dI_\upsilon(s)}{ds} = - \rho k_\upsilon \left[I_\upsilon(s) - n^2 I_{bb_\upsilon}(T) \right].$$

Here it is assumed that even though the medium may not be in thermodynamic equilibrium, it is in local thermodynamic equilibrium.

The problems considered in this paper will be for infinite parallel geometry. Therefore, the intensity is axially symmetric. In the axially symmetric case, the intensity is a function of s and θ only.



$$\cos\theta = \mu$$

$$ds\,\mu = dx$$

$$ds = \frac{dx}{\mu}$$

The equation of transport can be written in the form

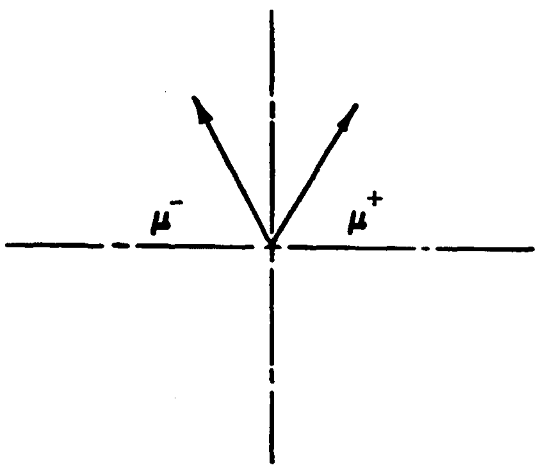
$$\frac{dI_\upsilon(x,\mu)}{dx} = - \rho k_\upsilon \left[I_\upsilon(x,\mu) - n^2 I_{bb_\upsilon}(T) \right],$$

where one now replaces $\rho k_\upsilon dx$ by its equivalent $d\tau$ or $\tau = \int_{x_1}^{x_2} \rho k_\upsilon dx$, which is the optical depth.

The final form of the equation of transport becomes

$$\mu \frac{dI_\upsilon(\tau,\mu)}{d\tau} = - I_\upsilon(\tau,\mu) + n^2 I_{bb_\upsilon}(T).$$

Following the suggestion of Love (15) and writing the equations in terms of the intensities directed along the positive and negative directions separately, one avoids the anticipated discontinuities for $\mu = 0$ at the boundries. These equations then become



$$\mu \frac{I_\upsilon^+(\tau,\mu)}{d\tau} = - I_\upsilon^+(\tau,\mu) + n^2 I_{bb_\upsilon}(T)$$

$$- \mu \frac{I_\upsilon^-(\tau,\mu)}{d\tau} = - I_\upsilon^-(\tau,\mu) + n^2 I_{bb_\upsilon}(T),$$

for which the isothermal solution is

$$I_\upsilon^+(\tau,\mu) = c_1 e^{-\tau/\mu} + n^2 I_{bb_\upsilon}(T)$$

$$I_\upsilon^-(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu} + n^2 I_{bb_\upsilon}(T),$$

and since $T = f(\tau)$, the nonisothermal solution is

$$I_\upsilon^+(\tau,\mu) = c_1' e^{-\tau/\mu} + \frac{n^2}{\mu} \int_o^\tau e^{(t-\tau)/\mu} I_{bb_\upsilon}(t)dt$$

$$I_\upsilon^-(\tau,\mu) = c_2' e^{(\tau-\tau_o)/\mu} + \frac{n^2}{\mu} \int_\tau^{\tau_o} e^{(t-\tau)/\mu} I_{bb_\upsilon}(t)dt.$$

This solution is restricted to the axially symmetric case.

EMISSION OF ISOTHERMAL DIELECTRIC COATING ON A CONDUCTOR

1. Emission of an isothermal coating on a conductor.

The problem consists of an isothermal dielectric coating infinite in extent of optical thickness τ_o on the surface of a conductor. The directional emission of the conductor will be considered, and both the dielectric-air and dielectric-conductor interface will be assumed to be perfectly smooth specular reflectors.

Assuming the interfaces are perfectly smooth surfaces implies Fresnel's laws of reflection hold. For the dielectric-air interface, all radiation striking the surface at an angle equal to or greater than the critical angle will be totally reflected. The concept of the critical angle, of course, only applies when speaking of rays striking the surface from within the optically denser medium.

The problem now arises as to how to apply Fresnel's reflection laws. Fresnel's laws are for rays for which one knows the polarization of the radiant energy upon intercepting the surface. Thermal radiation, on the other hand, is perfectly random vibration with no predictable order. On the average the effect of the vibrations is completely symmetrical about the axis of propagation. From this one can see that natural or unpolarized radiation may be assumed to consist of equal amounts of two components that are plane polarized in mutually perpendicular azimuths. Hence the proportion of natural radiation that is reflected at the interface between dielectric media is given by the following adaptation of Fresnel's laws:

$$\frac{I(\theta')}{I_o(\theta)} = \frac{1}{2}\left[\frac{\sin^2(\theta - \theta')}{\sin^2(\theta + \theta')} + \frac{\tan^2(\theta - \theta')}{\tan^2(\theta + \theta')}\right] = \frac{1}{2}\left[R_s + R_p\right]$$

$I(\theta')$ = intensity of reflected radiation

$I_o(\theta)$ = intensity of incident radiation

θ = angle of incidence

θ' = angle of refraction

R_s = reflectance of the plane polarized component with vibrations parallel to the plane of incidence

R_p = reflectance of the plane polarized component with vibrations perpendicular to the plane of incidence

The angle of refraction θ' is found by Snells' law of refraction.

$n_1 \sin \theta = n_2 \sin \theta'$

n_1 = refractive index of medium 1

n_2 = refractive index of medium 2

To simplify notation, henceforth the ratio $\frac{I(\theta')}{I_o(\theta)}$ will be denoted by $\rho_2(\theta,\theta')$ or simply ρ_2.

This relationship gives the directional specular reflectance of an interface for which the refractive index is real. Of course, all materials have a complex portion to their refractive index to some extent since the complex portion is a measure of its absorptive ability. This would be especially true in the proposed problem, since absorption is being considered. However, on most dielectric materials, even though absorption is being considered, the complex portion of the refractive index is so small that it may be neglected with very little error produced.

Fresnel's laws not only give the reflectivity at the dielectric-air interface, but they will also give directional emission of the conductor into the dielectric. Several investigators have used Fresnel's laws to study the emission of a conductor, but to this author's knowledge none have studied the variation of the emission curve as the medium into which the conductor is emitting is changed. The same relationships hold, although they are more complicated, since now the refractive index of the conductor is complex as is the angle of refraction. Therefore, Snells' law becomes

$$n_2 \sin\theta = \bar{n} \sin\bar{\theta}'$$

where $\bar{n}$ is the complex refractive index of the conductor and $\bar{\theta}'$ is the complex angle of refraction. n_2 is the refractive index of the dielectric.

$$\sin\theta = \frac{\bar{n}}{n_2} \sin\bar{\theta}' \text{ and } \sin\bar{\theta}' = \frac{n_2 \sin\theta}{\bar{n}}$$

$$\theta' = \alpha + i\beta \qquad \sin\theta' = \frac{n_2\, \bar{n}^* \sin\theta}{\bar{n}\,\bar{n}^*} \qquad \bar{n}^* = \text{conjugate } \bar{n}$$

$$\bar{n} = n_1 - ik_1 \qquad \sin\bar{\theta}' = \frac{n_2(n_1 + ik_1)\sin\theta}{n_1^2 + k_1^2}\,.$$

The reflection function for the perpendicular component then becomes

$$R_s = \left[\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}\right]^2 = \left[\frac{\sin(\theta - \alpha - i\beta)}{\sin(\theta + \alpha + i\beta)}\right]^2$$

$$R_s = \left[\frac{\sin(\theta - \alpha)\cosh\beta - i\cos(\theta - \alpha)\sinh\beta}{\sin(\theta + \alpha)\cosh\beta + i\cos(\theta + \alpha)\sinh\beta}\right]^2.$$

Recalling

$$\cosh^2\beta = 1 + \sinh^2\beta$$

$$R_s = \frac{\sin^2(\theta - \alpha)\cosh^2\beta + \cos^2(\theta - \alpha)\sinh^2\beta}{\sin^2(\theta + \alpha)\cosh^2\beta + \cos^2(\theta + \alpha)\sinh^2\beta}$$

$$R_s = \frac{\sin^2(\theta - \alpha) + \sinh^2\beta}{\sin^2(\theta + \alpha) + \sinh^2\beta}.$$

Similarly, the parallel component can be written as

$$R_p = \left[\frac{\tan(\theta - \theta')}{\tan(\theta + \theta')}\right]^2 = \left[\frac{\sin(\theta - \theta')\cos(\theta + \theta')}{\sin(\theta + \theta')\cos(\theta - \theta')}\right]^2$$

$$R_p = \left[\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}\right]^2 \left[\frac{\cos(\theta + \theta')}{\cos(\theta - \theta')}\right]^2 = R_s \left[\frac{\cos(\theta + \theta')}{\cos(\theta - \theta')}\right]^2$$

$$R_p = R_s \left[\frac{\cos^2(\theta + \alpha)\cosh^2\beta + \sin^2(\theta + \alpha)\sinh^2\beta}{\cos^2(\theta - \alpha)\cosh^2\beta + \sin^2(\theta - \alpha)\sinh^2\beta}\right].$$

Recalling

$$\cosh^2\beta = 1 + \sinh^2\beta,$$

$$R_p = R_s \frac{\cos^2(\theta + \alpha) + \sinh^2\beta}{\cos^2(\theta - \alpha) + \sinh^2\beta},$$

and

$$\rho_2(\theta',\theta) = \frac{1}{2}(R_s + R_p)$$

$$\epsilon_2(\theta) = 1 - \frac{1}{2}(R_s + R_p).$$

It is common practice by many investigators to use an approximation for Fresnel's law for conductors given by

$$\epsilon(\theta) = 1 - \frac{1}{2}(R_s + R_p),$$

where

$$R_s = \frac{(n^2 + k^2) - 2n \cos \theta + \cos^2\theta}{(n^2 + k^2) + 2n \cos \theta + \cos^2\theta}$$

and

$$R_p = \frac{(n^2 + k^2)\cos^2\theta - 2n \cos \theta + 1}{(n^2 + k^2)\cos^2\theta + 2n \cos \theta + 1} .$$

This approximation is developed considering Snell's law

$$(n_1 - ik_1)\sin \bar{\theta}' = n_2 \sin \theta$$

$$\sin \bar{\theta}' = \frac{n_2}{n_1 - ik_1} \sin \theta$$

$$\frac{n_1}{n_2} = n \qquad \frac{k_1}{n_2} = k .$$

Therefore, since

$$\sin \bar{\theta}' = \frac{\sin \theta}{n - ik} ,$$

if one multiplies by the complex conjugate of $\bar{n}$,

$$\sin \bar{\theta}' = \frac{(n + ik)\sin \theta}{n^2 + k^2} = \sin(\alpha + i\beta)$$

is obtained. Now if one considers

$$n^2 + k^2 \gg 1,$$

then

$$\sin \bar{\theta}' = \bar{\theta}' ,$$

or

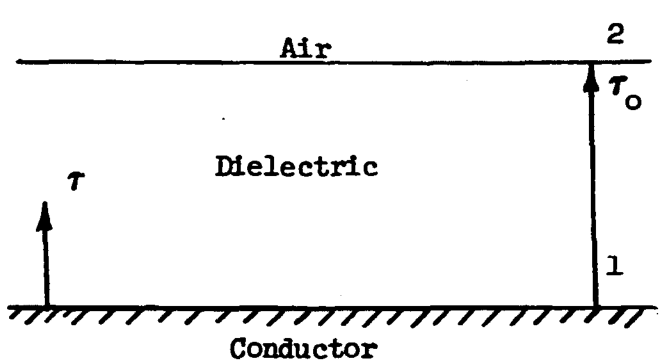
$$\alpha = \frac{n \sin \theta}{n^2 + k^2} \qquad \text{and} \qquad \beta = \frac{k \sin \theta}{n^2 + k^2} .$$

Similarly,

$$\sin \bar{\theta}' = \bar{\theta}' \qquad \text{and} \qquad \cos \bar{\theta}' = 1.$$

It is obvious that this equation loses accuracy as the refractive index of the dielectric increases since n and k are relative refractive and absorption indexes. This possible lack of accuracy urges one to use the exact Fresnel equations to determine the angular emission into dielectrics. These are shown in Figures 2 through 6.

It is noted that as the conductor emits into a more optically dense medium, the shape of the emission curve changes form to more nearly that of a dielectric. One can understand this more easily if he remembers that the complex portion of the refractive index of a conductor gives its emission curve a different shape than that of the dielectric. This complex portion of the refractive index denotes the absorption of the conductor, but in the emission studies the quantity which determined the shape of the directional emission curves was the relative refractive index. This means that as a conductor emits into a more optically dense medium, the relative absorption is becoming less until the emission approaches that of a dielectric (no absorption).



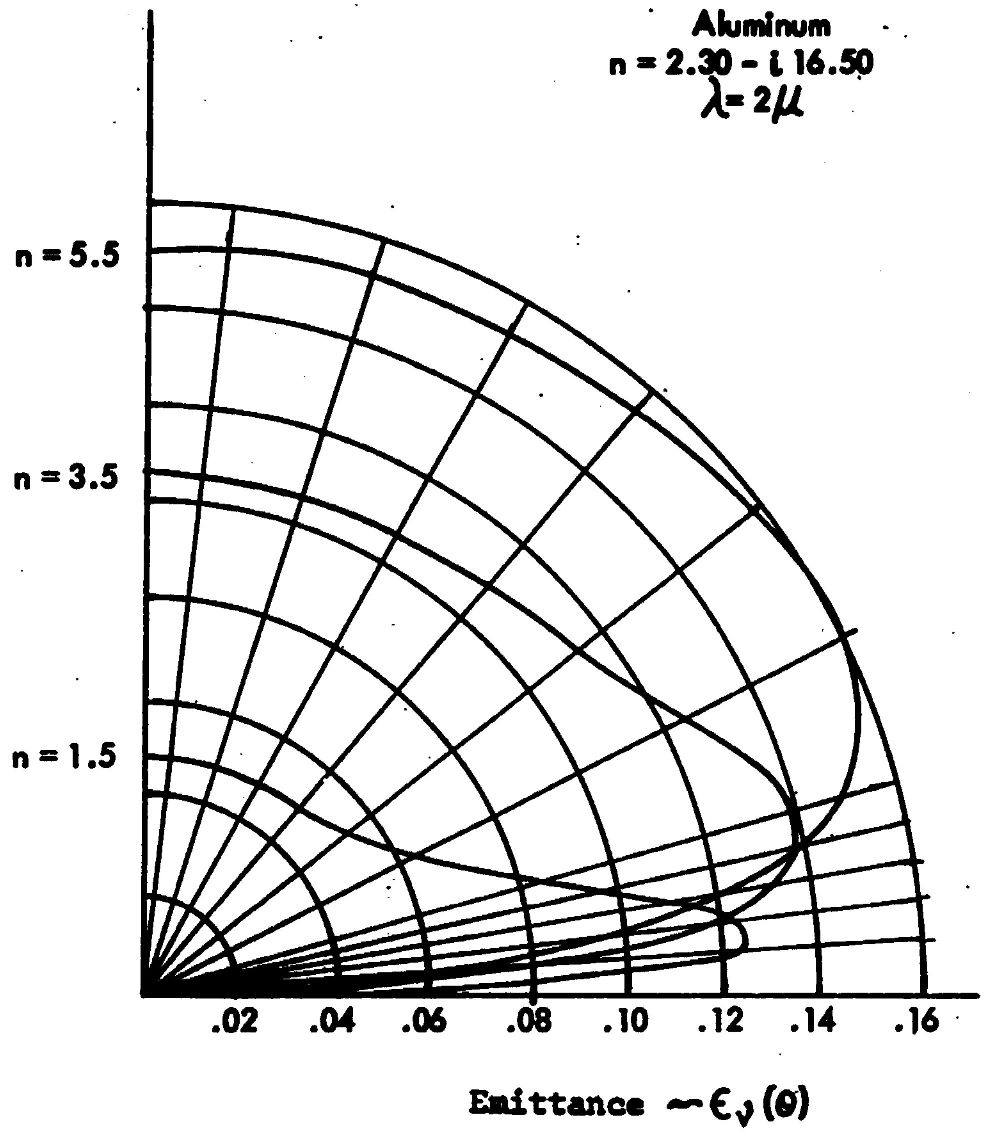


Figure 2

Monochromatic Directional Emittance of
Aluminum into Different Dielectrics

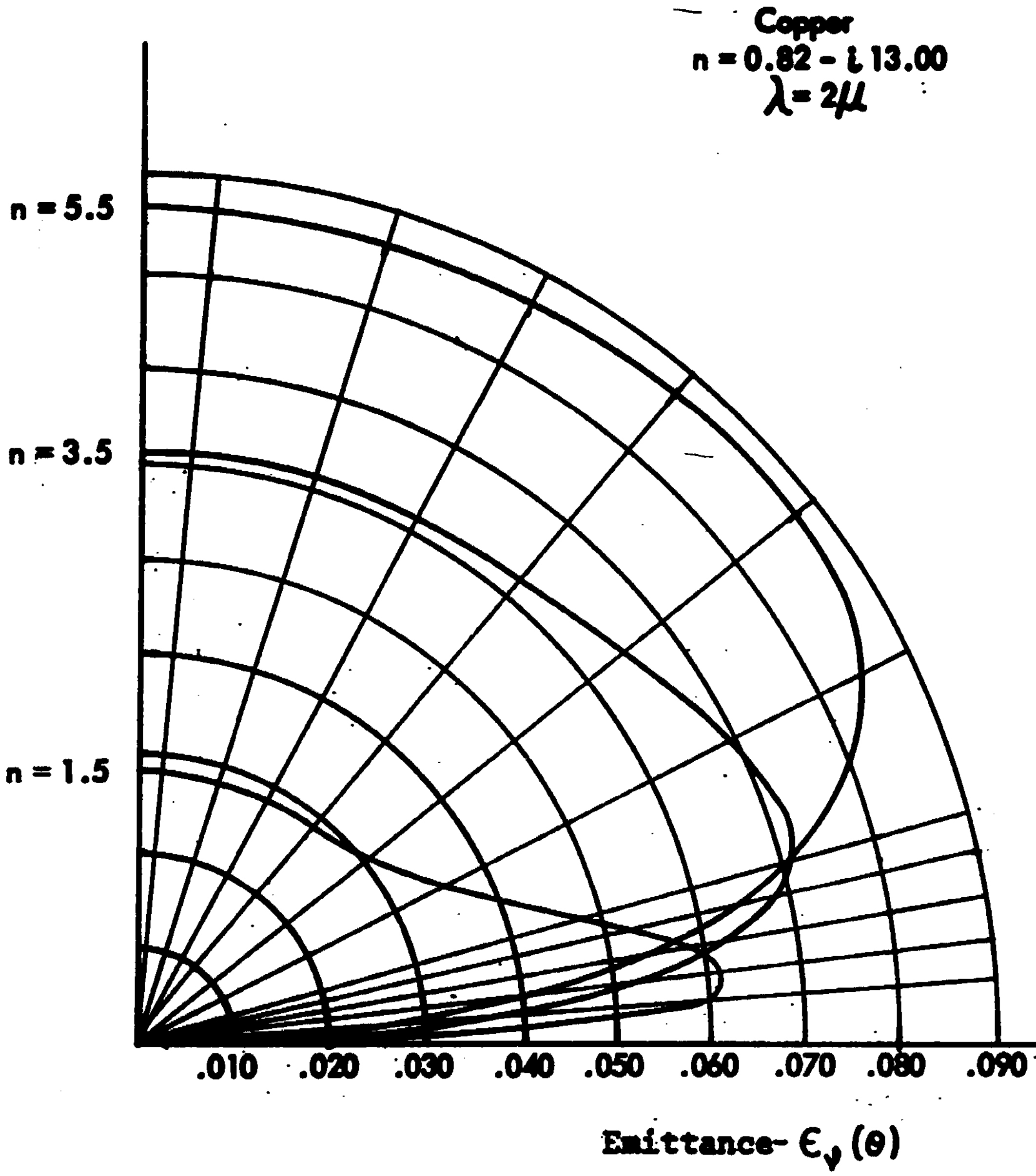


Figure 3

Monochromatic Directional Emittance of Copper into Different Dielectrics

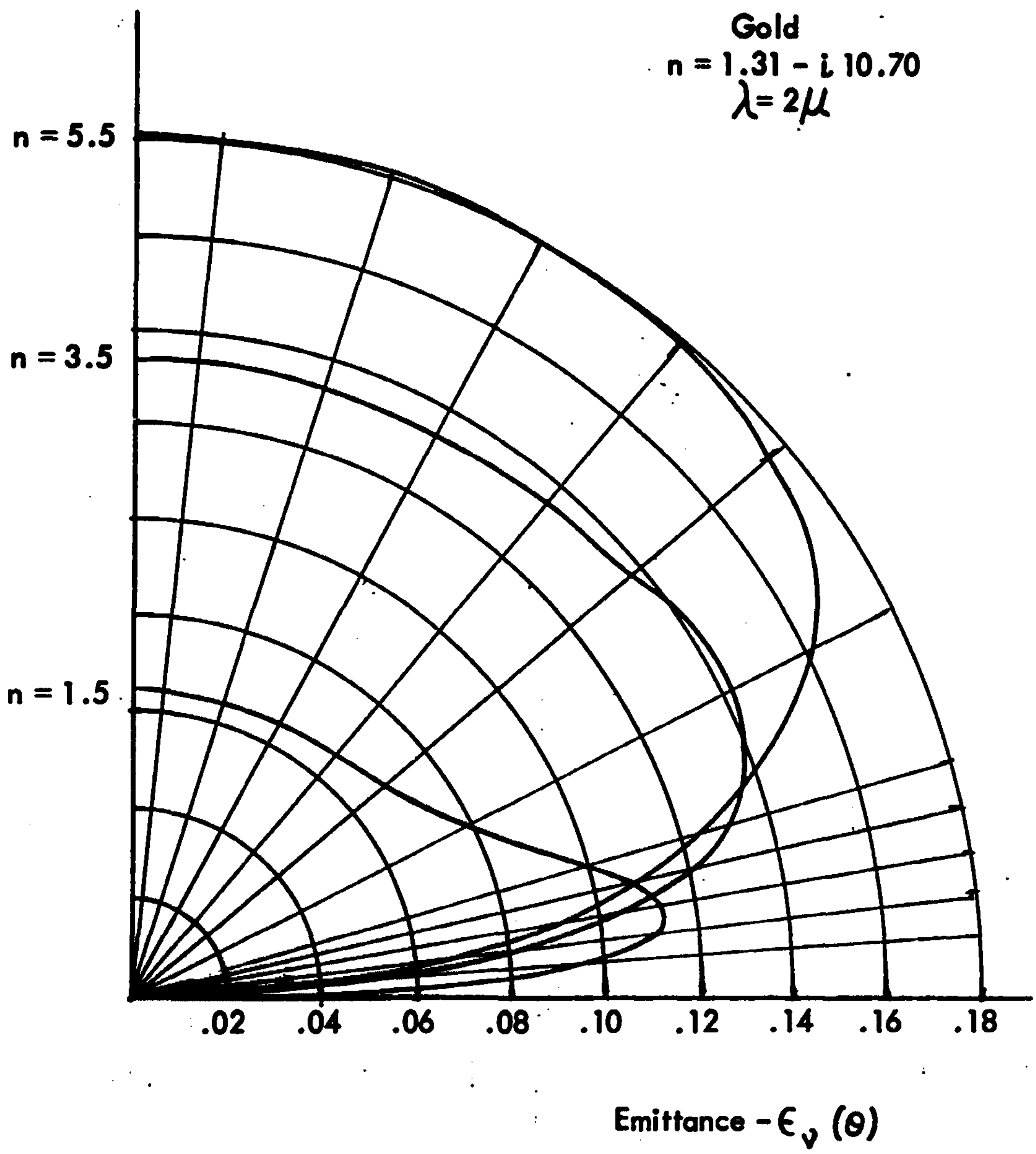


Figure 4

Monochromatic Directional Emittance of
Gold into Different Dielectrics

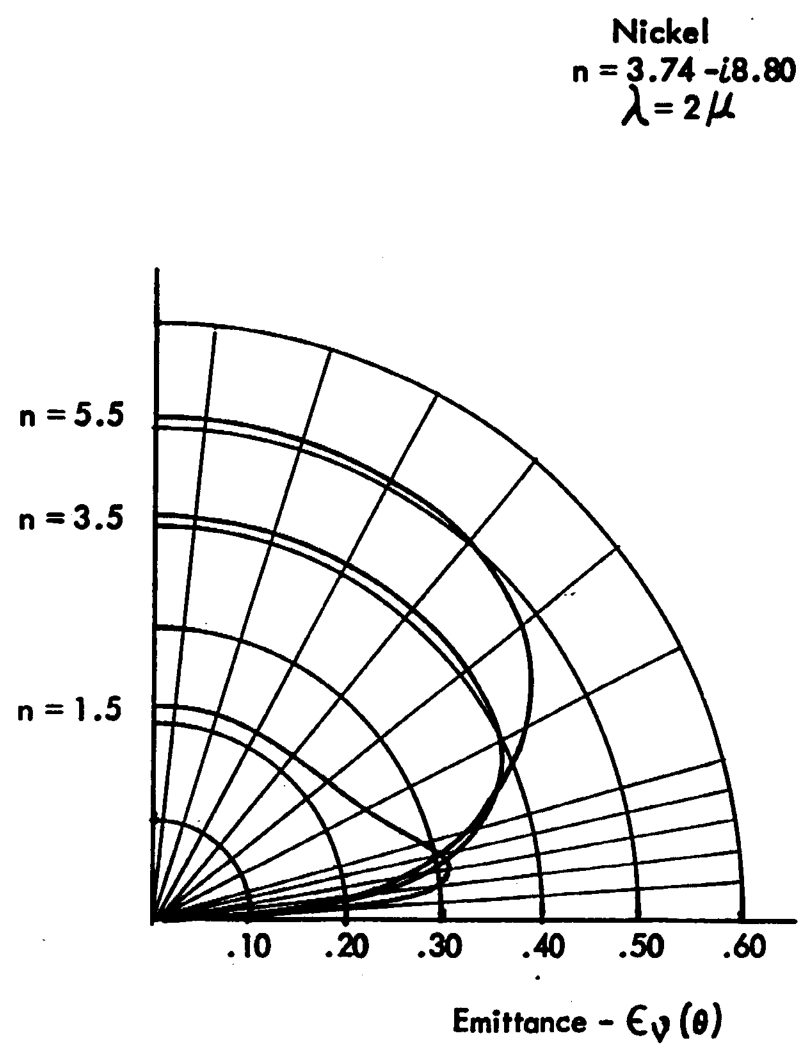


Figure 5

Monochromatic Emittance of Nickel
Into Different Dielectrics

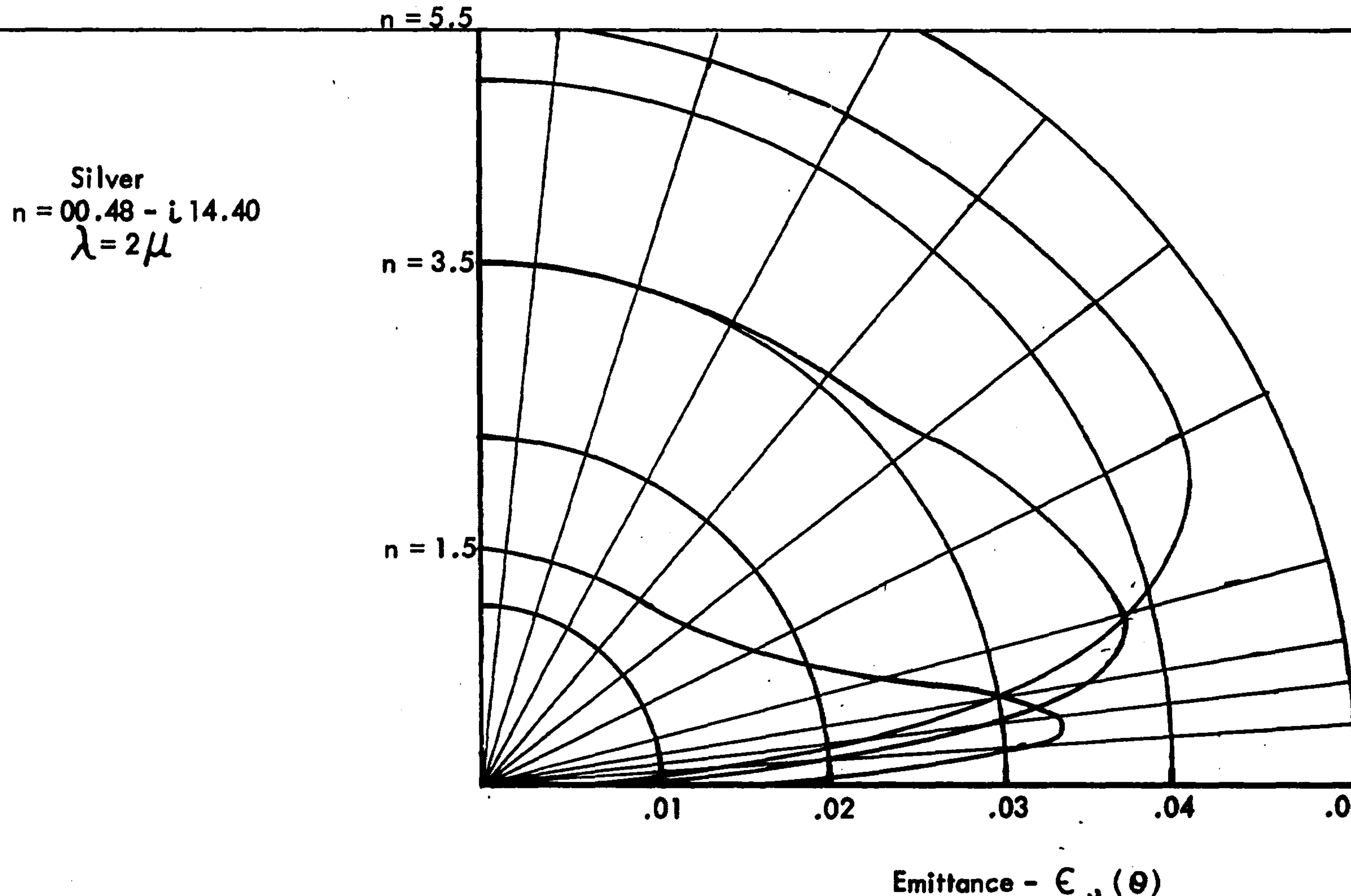


Figure 6

Monochromatic Emittance of Silver
Into Different Dielectrics

Having developed the equations for emission and reflection within the dielectric, one can use these relationships in the analysis of the problem. The boundary conditions involved for the isothermal dielectric coating on a conductor as seen in Figure 7 are as follows. At the dielectric-conductor interface, $\tau = 0$, the intensity in the positive direction is equal to that emitted from the conductor in addition to the intensity in the negative direction which is reflected into the same path. This relationship is given by

$$I_\nu^+(o,\mu) = n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T) + \rho_{1_\nu}(\mu)I_\nu^-(o,\mu),$$

or upon substitution for the values of intensity

$$c_1 + n^2 I_{bb_\nu}(T) = n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T) + \rho_{1_\nu}(\mu)\left[c_2 e^{-(\tau_o/\mu)} + n^2 I_{bb_\nu}(T)\right],$$

which when simplified is

$$c_1 = c_2\rho_{1_\nu}(\mu)e^{-(\tau_o/\mu)},$$

for all μ. Here $\epsilon_1(\mu)$ is the directional emission of the conductor into the dielectric and

$$\rho_{1_\nu}(\mu) = 1 - \epsilon_{1_\nu}(\mu).$$

The boundary condition at the dielectric-air interface ($\tau = \tau_o$) is

$$\rho_{2_\nu}(\theta',\theta)I_\nu^+(\tau_o,\mu) = I_\nu^-(\tau_o,\mu),$$

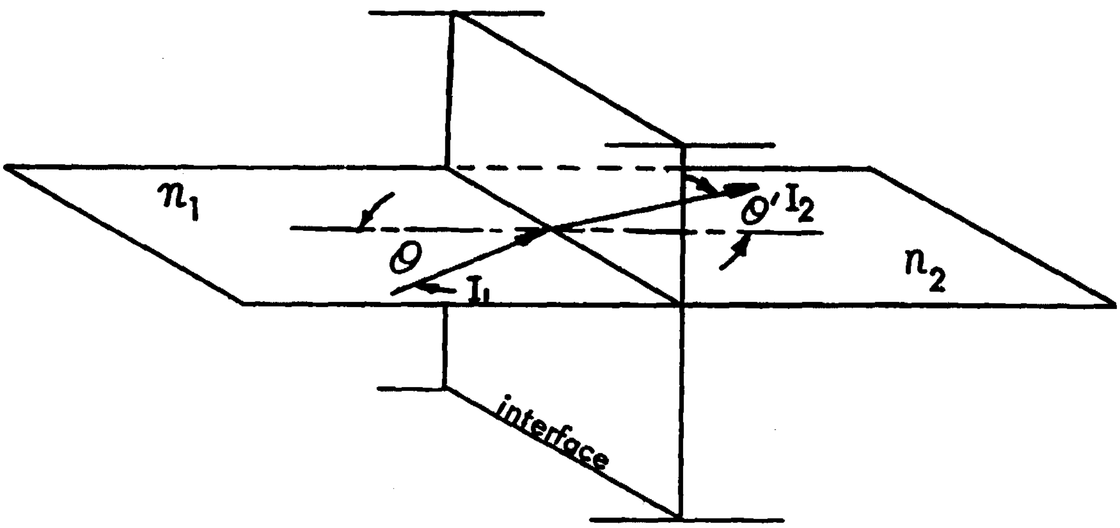


Figure 7A

COORDINATE SCHEME FOR DIELECTRIC INTERFACE

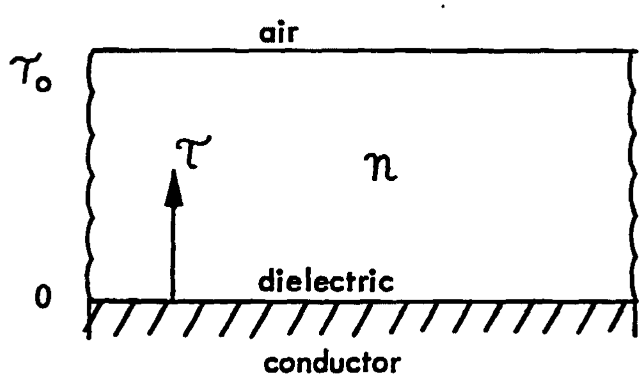


Figure 7

SHEET OF DIELECTRIC MATERIAL WITH PARALLEL
SPECULARLY REFLECTING SURFACES

which upon substitution yields

$$\rho_{2_\upsilon}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + n^2 I_{bb_\upsilon}(T)\right] = c_2 + n^2 I_{bb_\upsilon}(T),$$

or

$$c_2 = \rho_{2_\upsilon}(\theta',\theta) c_1 e^{-(\tau_o/\mu)} + n^2 I_{bb_\upsilon}(T)\left[\rho_{2_\upsilon}(\theta',\theta) - 1\right].$$

c_2 is found by combining the two boundary conditions

$$c_2 = \frac{n^2 I_{bb_\upsilon}(T)\left[\rho_{2_\upsilon}(\theta',\theta) - 1\right]}{1 - \rho_{2_\upsilon}(\theta',\theta)\rho_{1_\upsilon}(\mu) e^{-(2\tau_o/\mu)}}$$

for $\mu > \mu_c$. For $\tau = \tau_o$ and $\mu \leq \mu_c$, there is total reflection. Therefore,

$$I_\upsilon^+(\tau_o,\mu) = I_\upsilon^-(\tau_o,\mu),$$

or upon substitution

$$c_1 e^{-(\tau_o/\mu)} = c_2.$$

Since

$$c_1 = c_2 \rho_1(\mu) e^{-(\tau_o/\mu)}$$

from the boundary conditions at $\tau = 0$ holds for all μ. The two boundary conditions may be combined to give

$$c_1 = c_2 = 0.$$

The emittance for $\mu > \mu_c$ is determined by finding the net intensity,

$$I_{net_\upsilon}(\tau_o,\mu) = I_\upsilon^+(\tau_o,\mu) - I_\upsilon^-(\tau_o,\mu) = c_1 e^{-(\tau_o/\mu)} - c_2,$$

outside the dielectric. If n_d is the refractive index for the dielectric and n_a is the refractive index for air, then,

$$n_a^2 I_{net_\upsilon}(\tau_o,\mu)_{inside} = n_d^2 I_{net_\upsilon}(\tau_o,\mu')_{outside} ,$$

or

$$I_{net_\upsilon}(\tau_o,\mu')_{outside} = \frac{I_{net_\upsilon}(\tau_o,\mu)_{inside}}{n^2} .$$

Here

$$n = \frac{n_d}{n_a} .$$

Therefore, the effective emittance is

$$\epsilon_{eff_\upsilon}(\mu') = \frac{I_{net_\upsilon}(\tau_o,\mu)}{n^2 I_{bb_\upsilon}(T_2)} = c_1' e^{-(\tau_o/\mu)} - c_2',$$

where

$$c_1' = \frac{c_1}{n^2 I_{bb_\upsilon}(T_2)} \text{ and } c_2' = \frac{c_2}{n^2 I_{bb_\upsilon}(T_2)} .$$

One should recall that $\epsilon_{1_\upsilon}(\mu)$ is plotted $\epsilon_{1_\upsilon}(\mu')$ where μ' is given by

$$\mu' = \left[1 - n^2(1 - \mu^2)\right]^{\frac{1}{2}} .$$

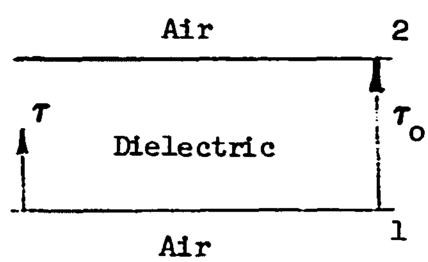
$$(1 - \mu^2) = \frac{1 - \mu'^2}{n^2}$$

$$\mu^2 = \frac{1 - \mu'^2}{n^2}$$

In this way the refraction of the radiation as it leaves the dielectric and enters the air is taken into account. The curves for directional emission of an isothermal dielectric coating on a conductor are given in Figures 8 through 22.

2. Emission of an Isothermal Sheet of Dielectric.

The same conditions hold as in the coating problem, but as is seen in the following figure, the directional emission of the conductor is not needed since there is no conductor present. The purpose will be to determine the dependency of directional emissivity on optical thickness of a sheet of dielectric. Gardon has done this successfully, but his approach is quite different and it is in the author's opinion, much more clumbersome.



The governing equations are again

$$I_\upsilon^+(\tau,\mu) = c_1 e^{-(\tau/\mu)} + n^2 I_{bb_\upsilon}(T_2)$$

and

$$I_\upsilon^-(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu} + n^2 I_{bb_\upsilon}(T_2).$$

The boundary conditions for $\mu \leq \mu_c$ are as follows at $\tau = \tau_o$

$$I_\upsilon^+(\tau_o,\mu) = I_\upsilon^-(\tau_o,\mu)$$

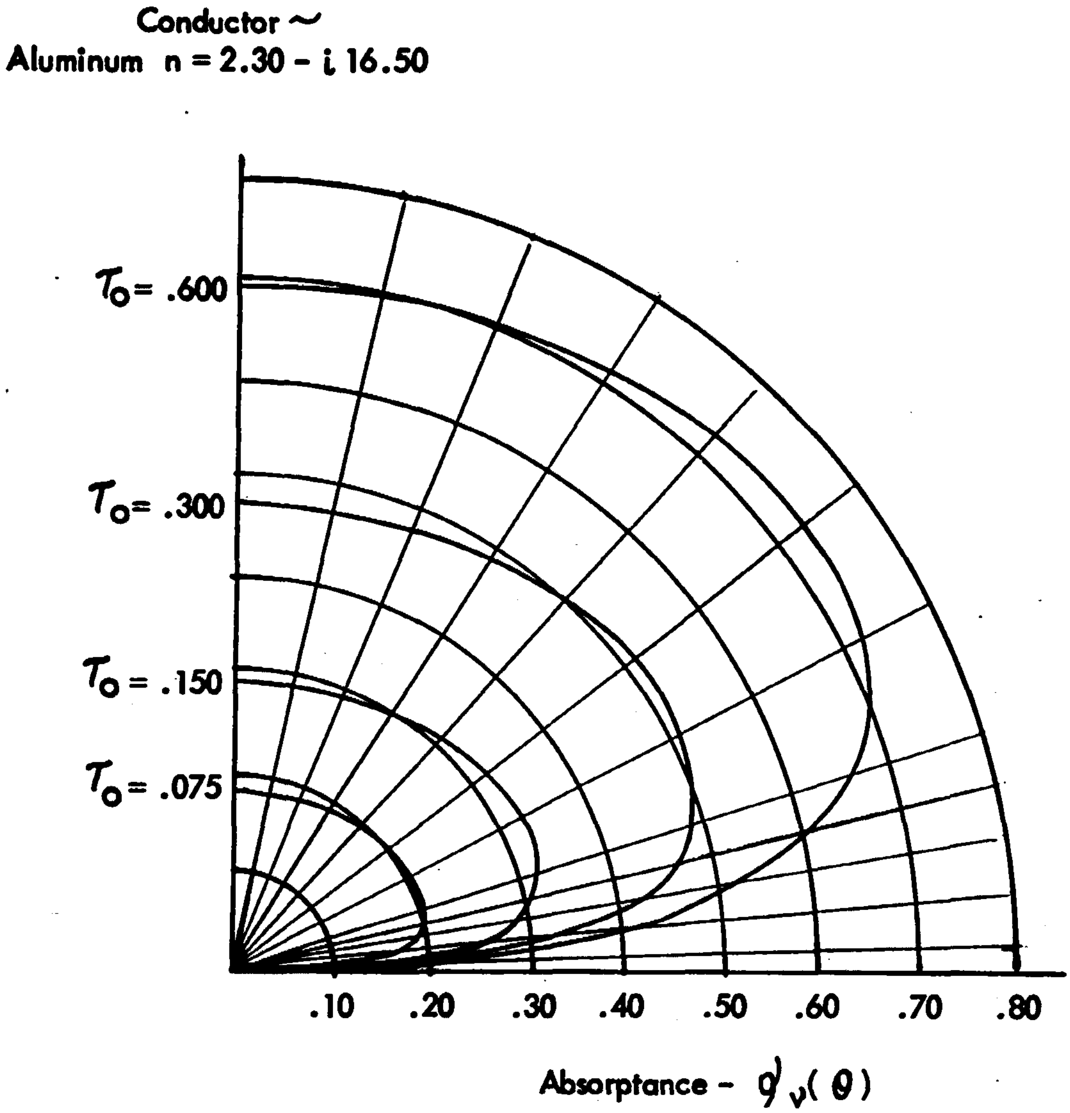


Figure 8

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 1.5 on Aluminum

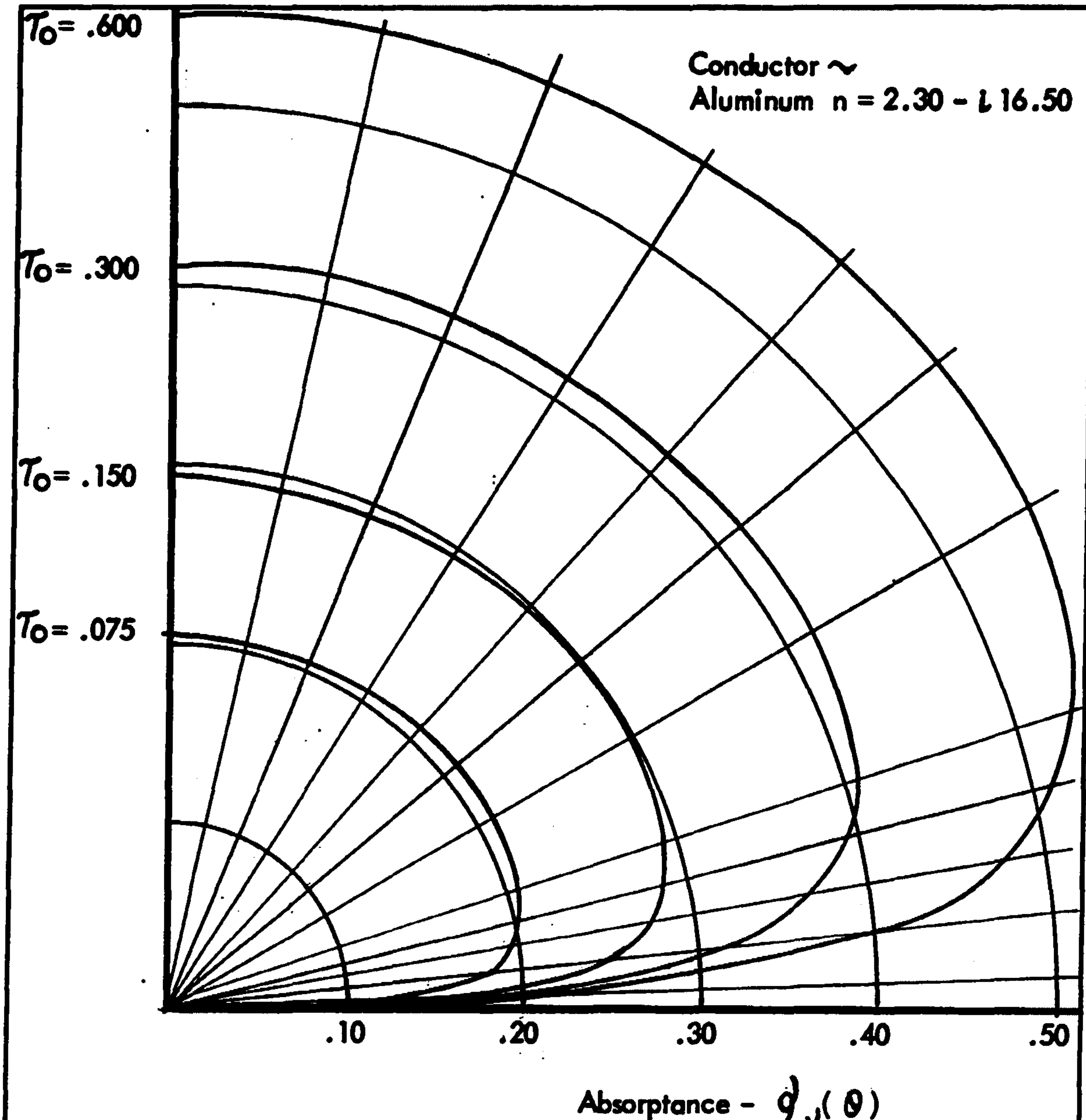


Figure 9

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 3.5 on Aluminum

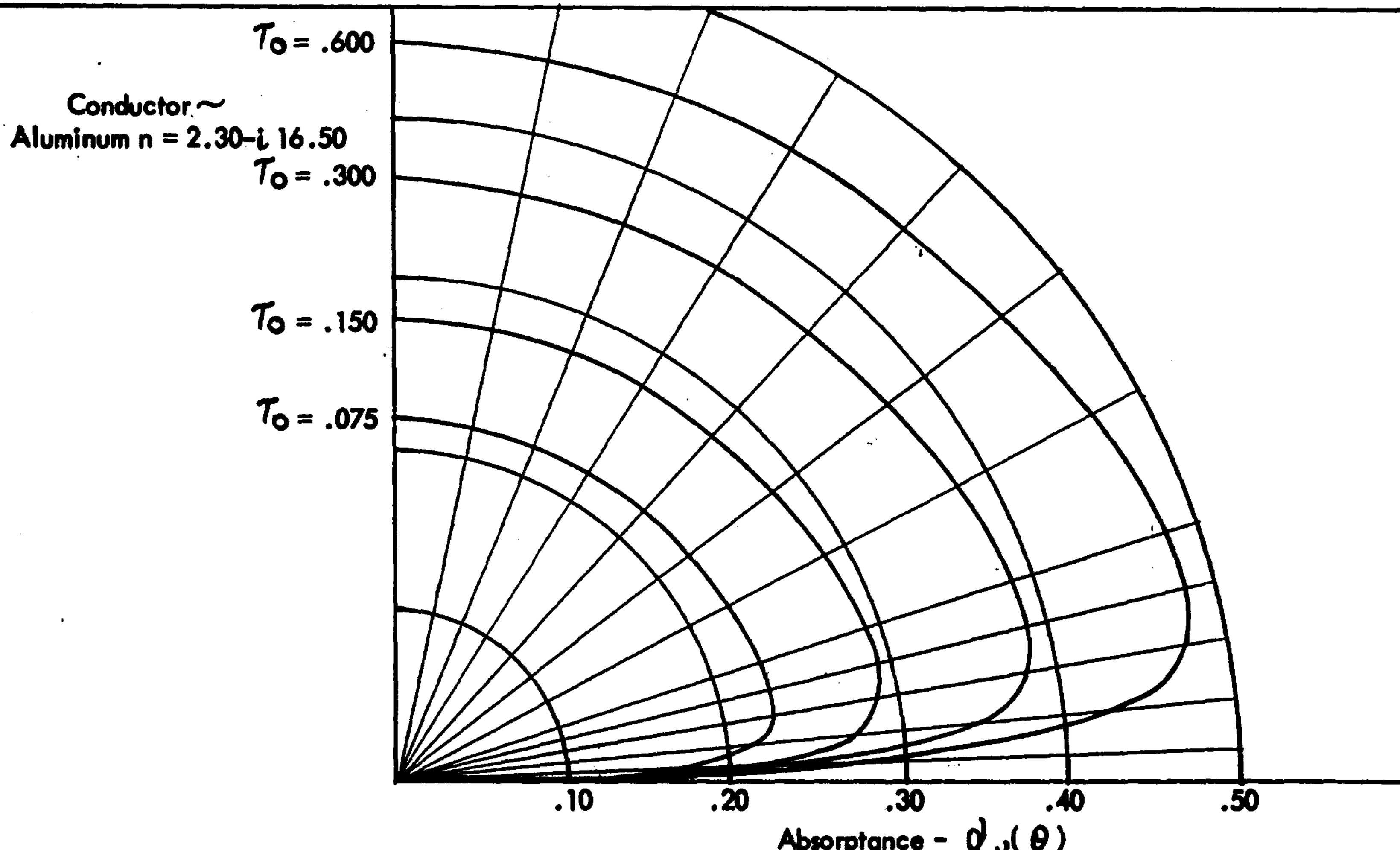


Figure 10

Monochromatic Directional Absorptance for a
Dielectric Coating with a Refractive Index of 5.5 on Aluminum

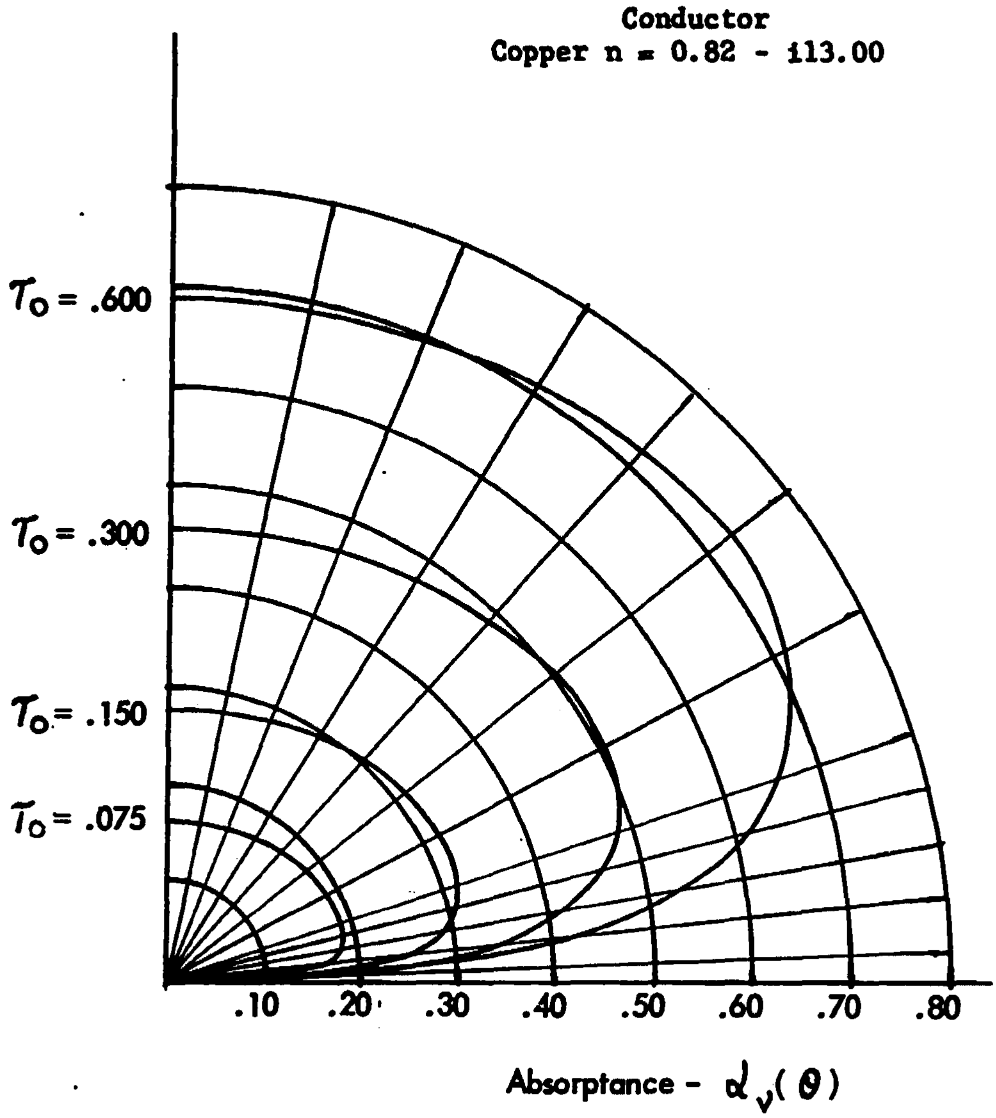


Figure 11

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 1.5 on Copper

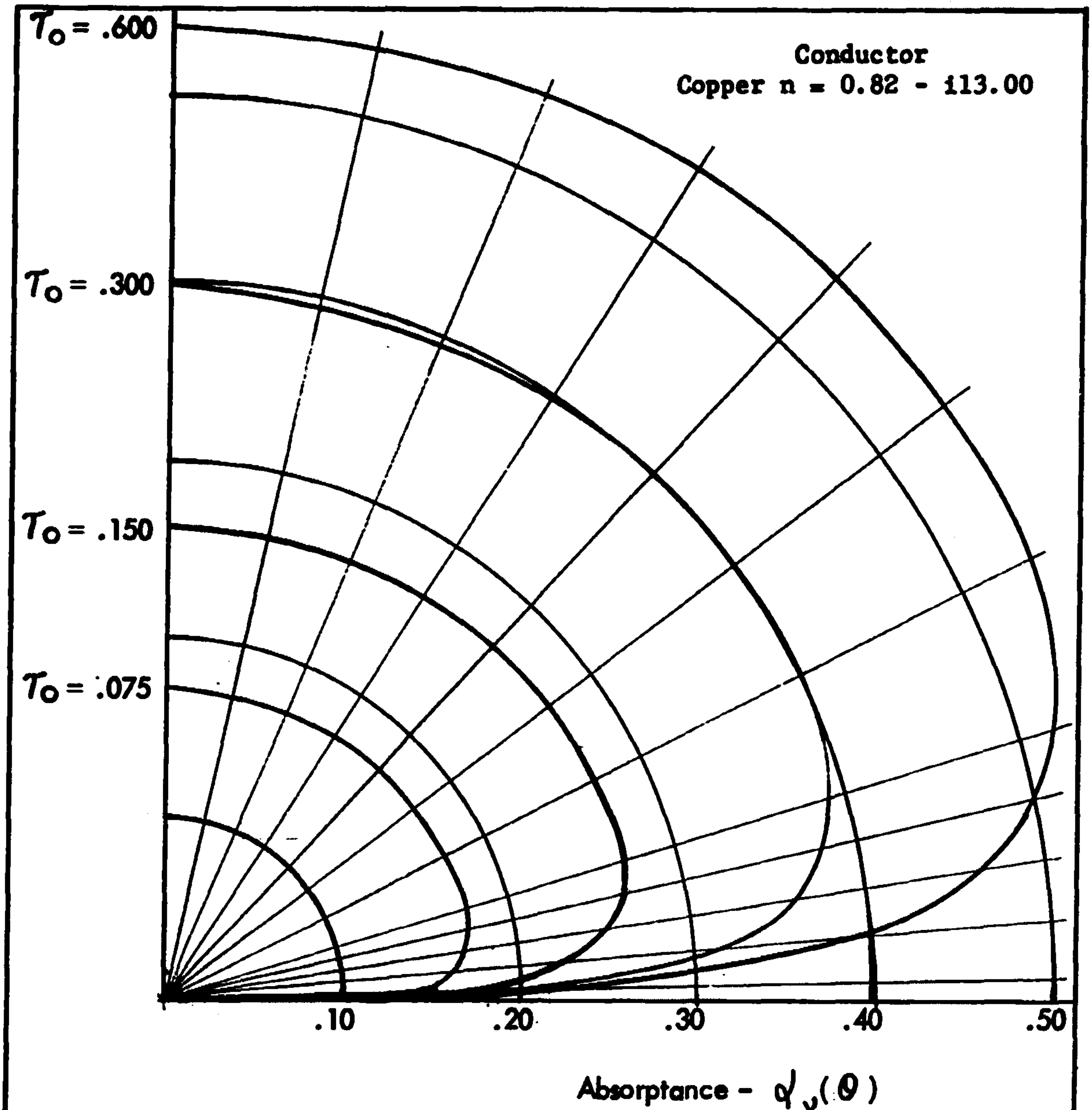


Figure 12

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 3.5 on Copper

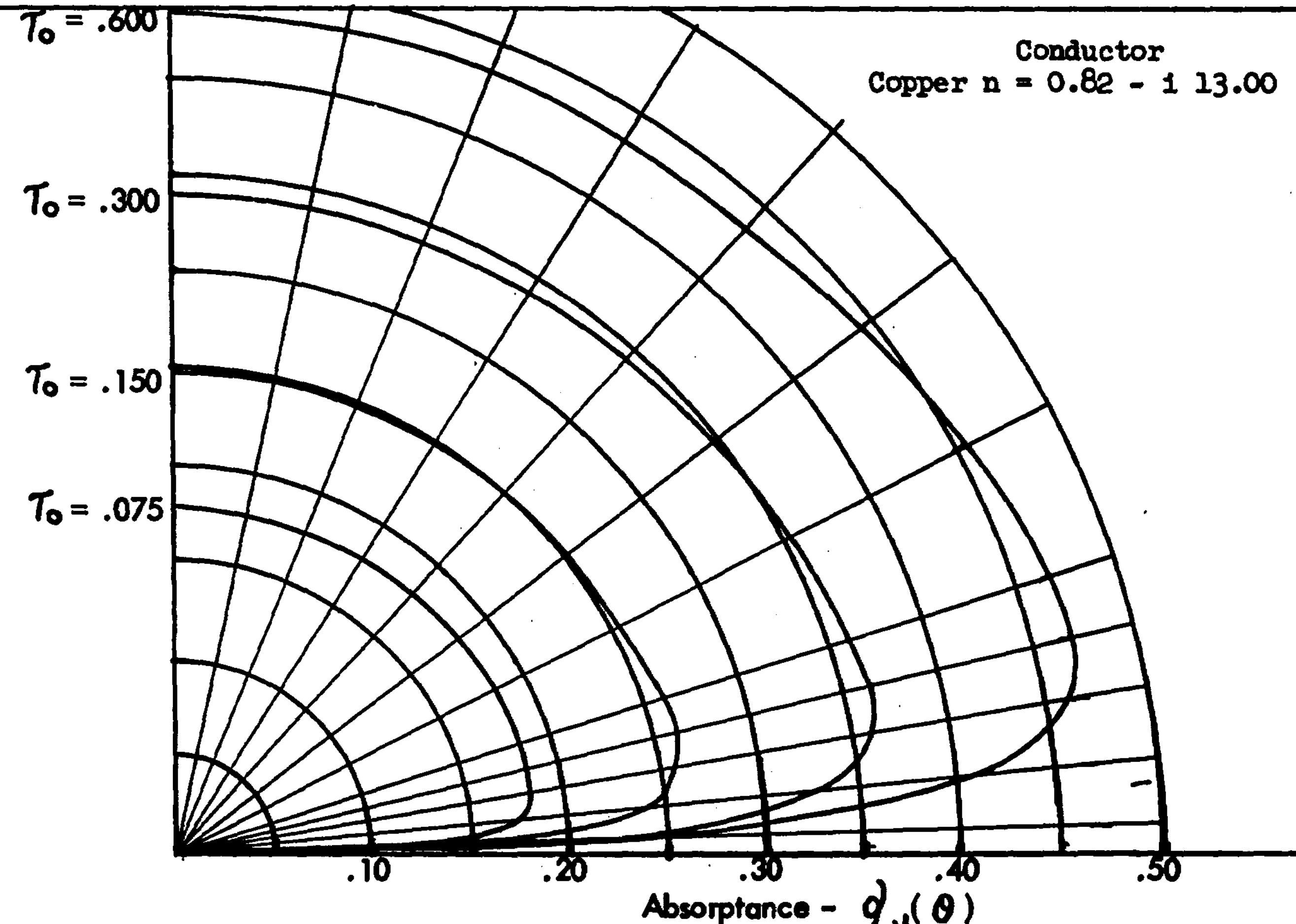


Figure 13

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 5.5 on Copper

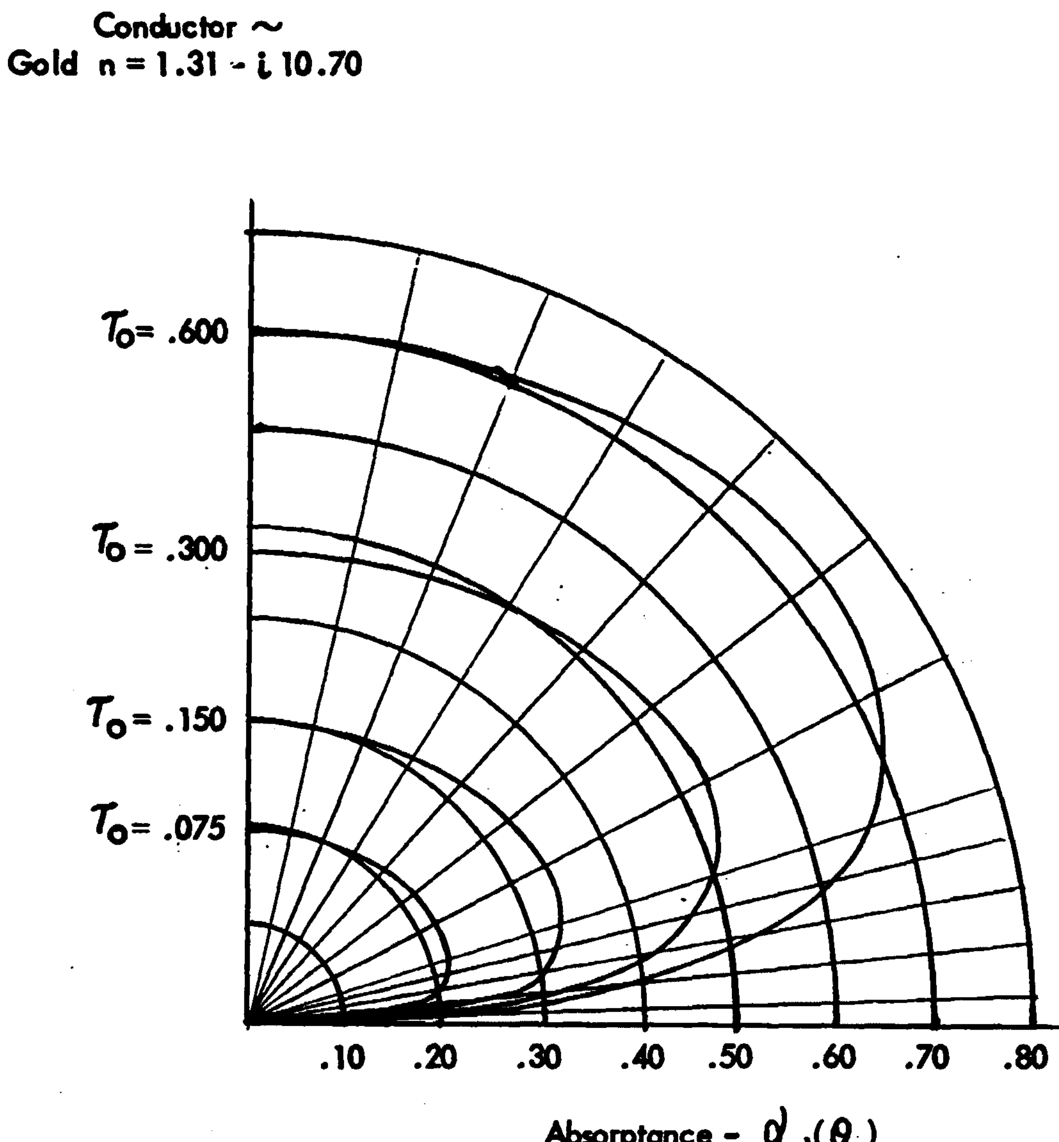


Figure 14

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 1.5 on Gold

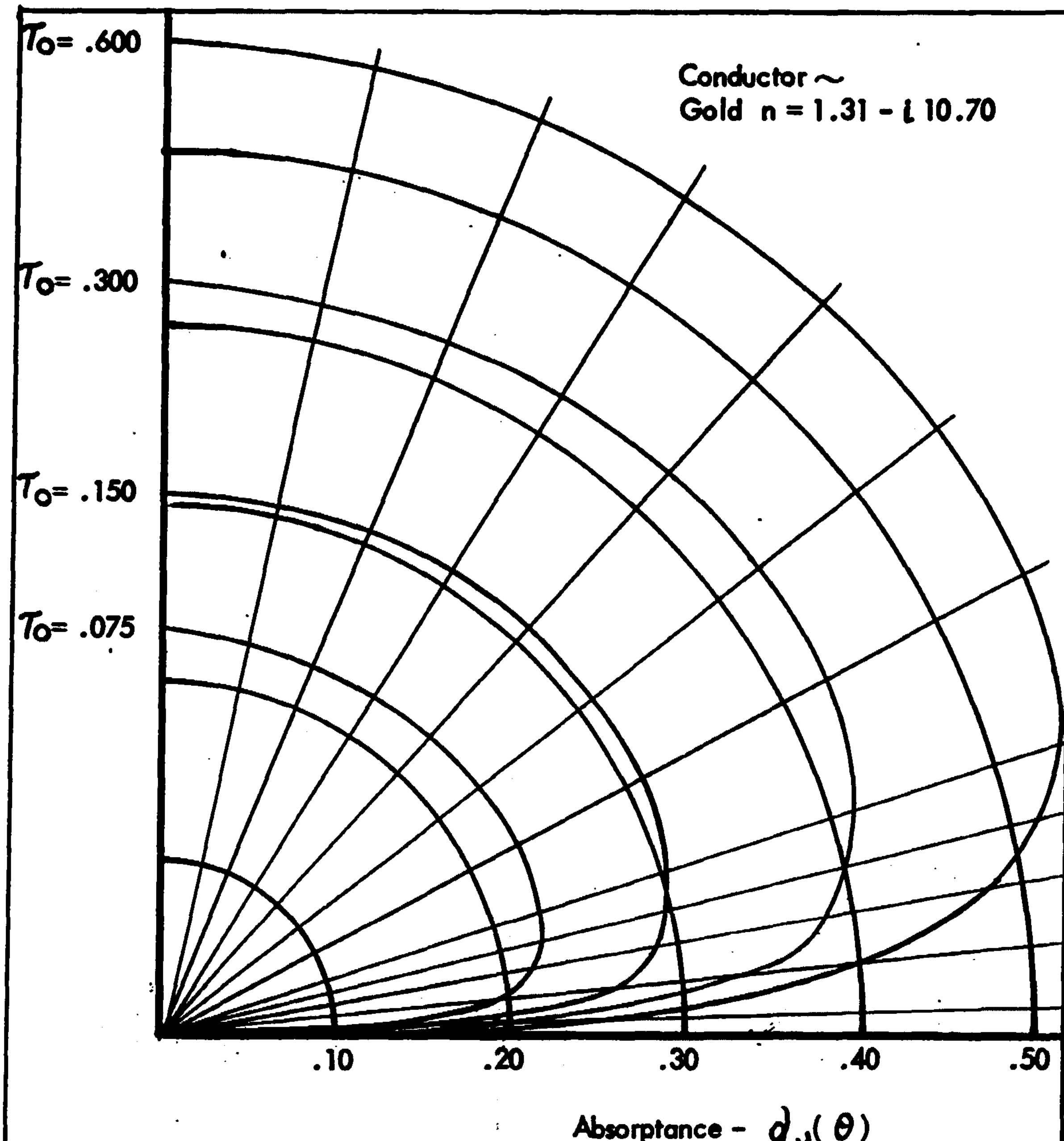


Figure 15

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 3.5 on Gold

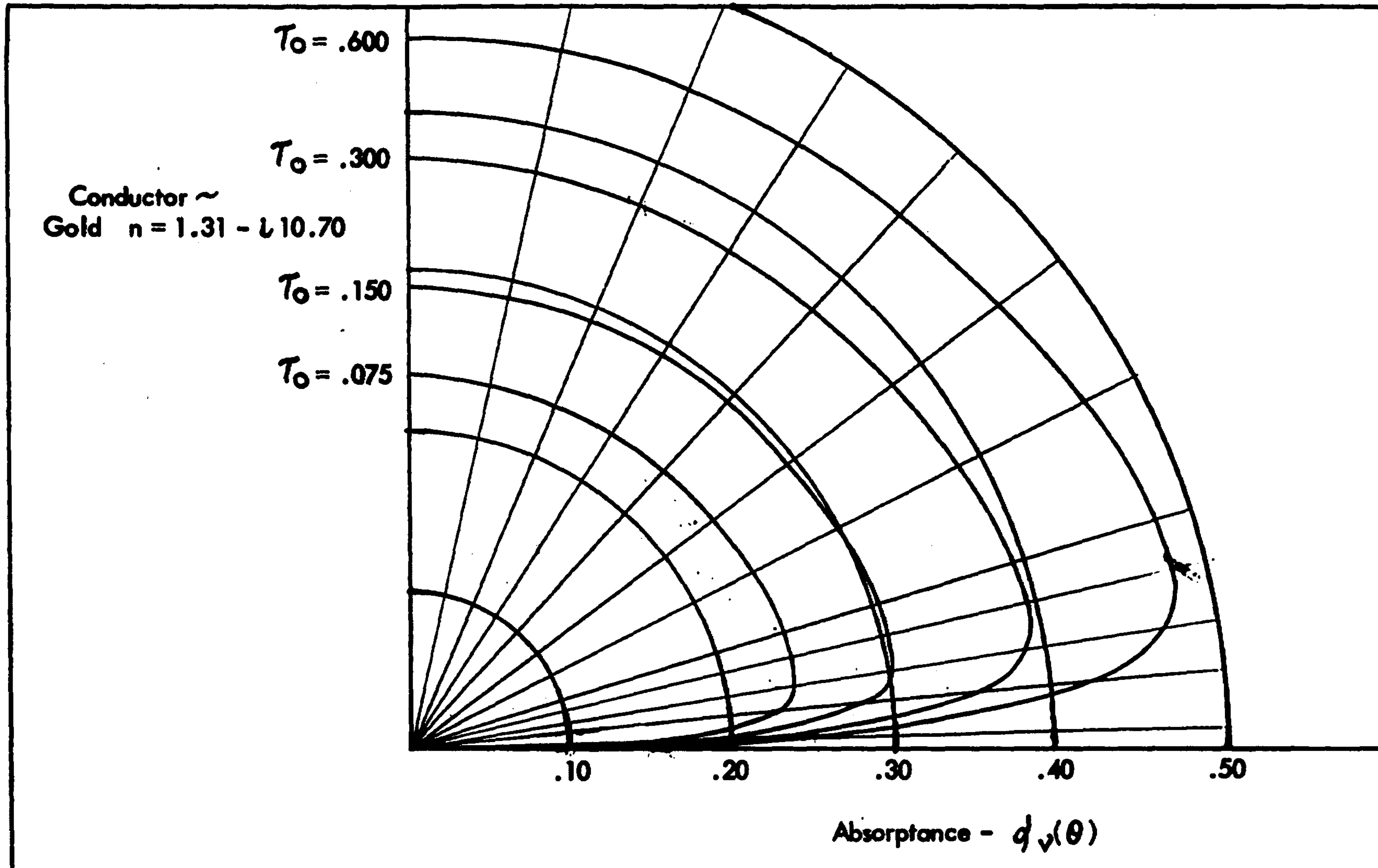


Figure 16
Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 5.5 on Gold

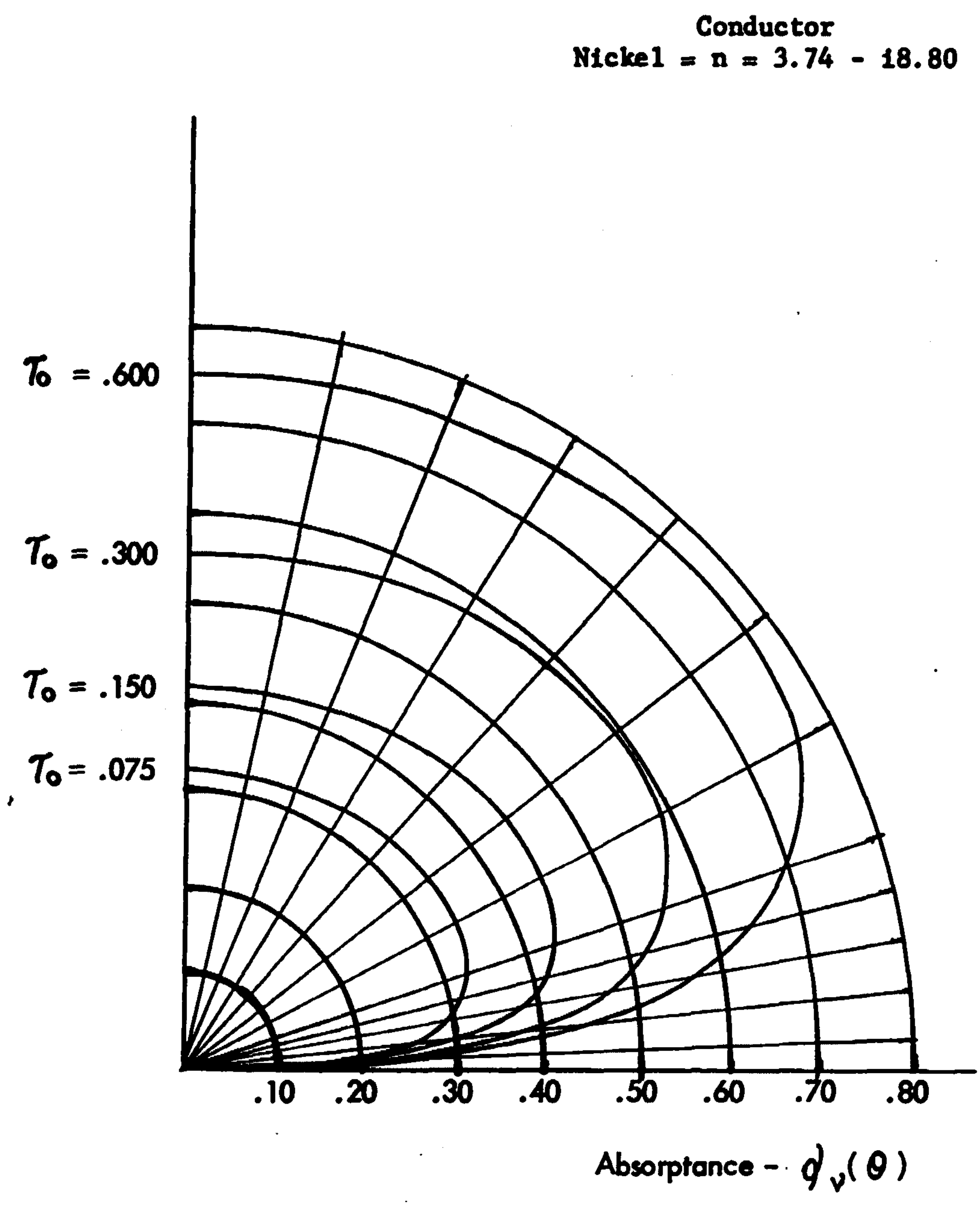


Figure 17

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 1.5 on Nickel

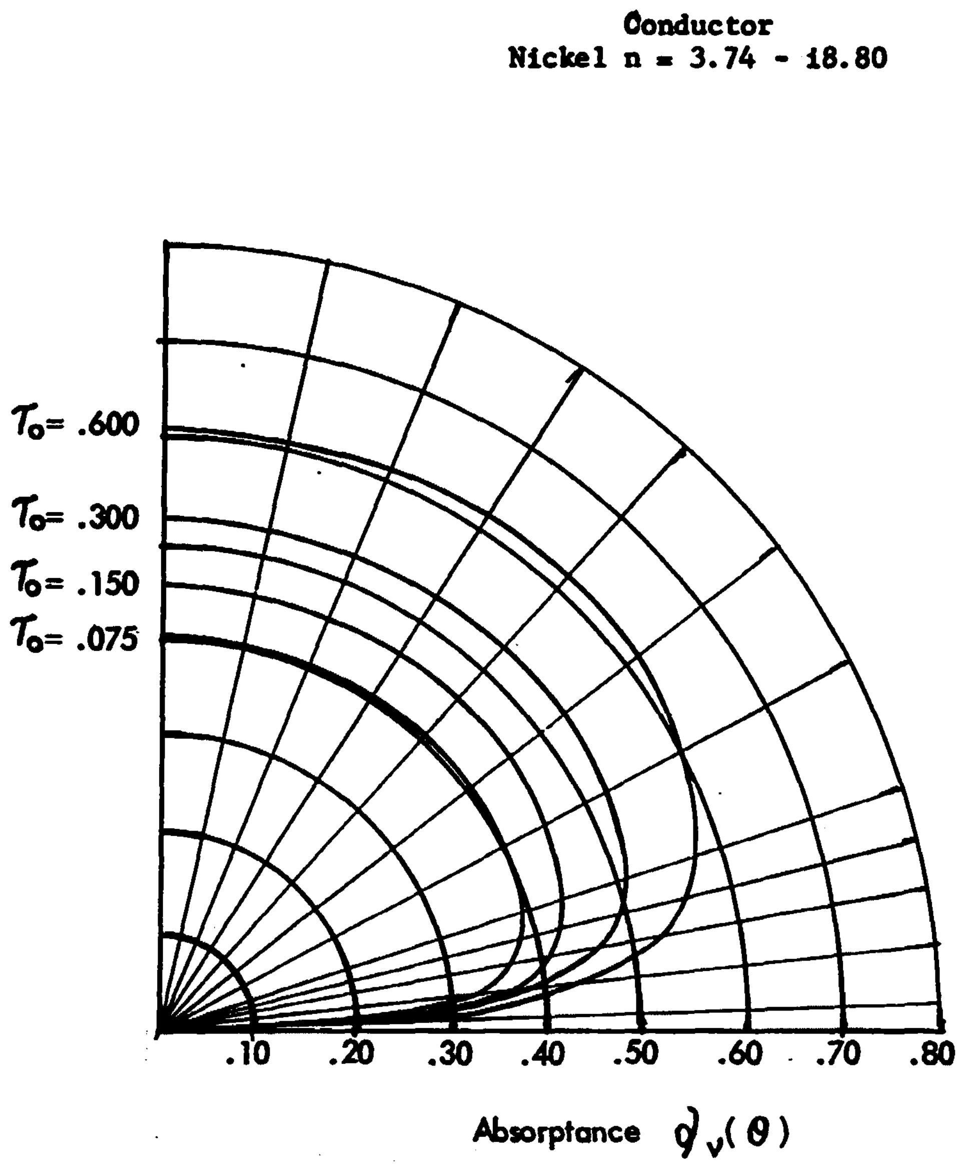


Figure 18

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 3.5 on Nickel

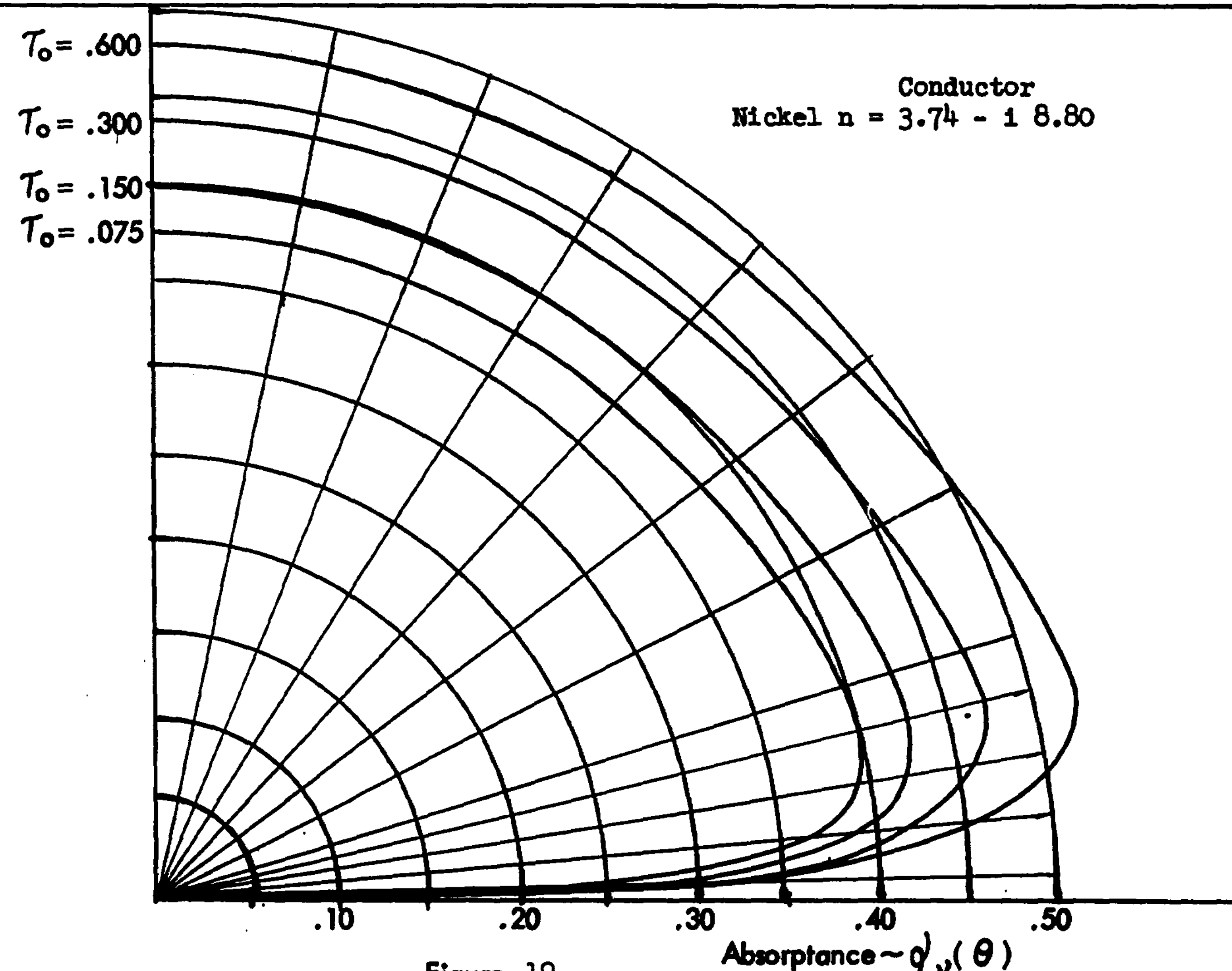


Figure 19
Monochromatic Directional Absorptance for a Dielectric
Coating with a Refractive Index of 5.5 on Nickel

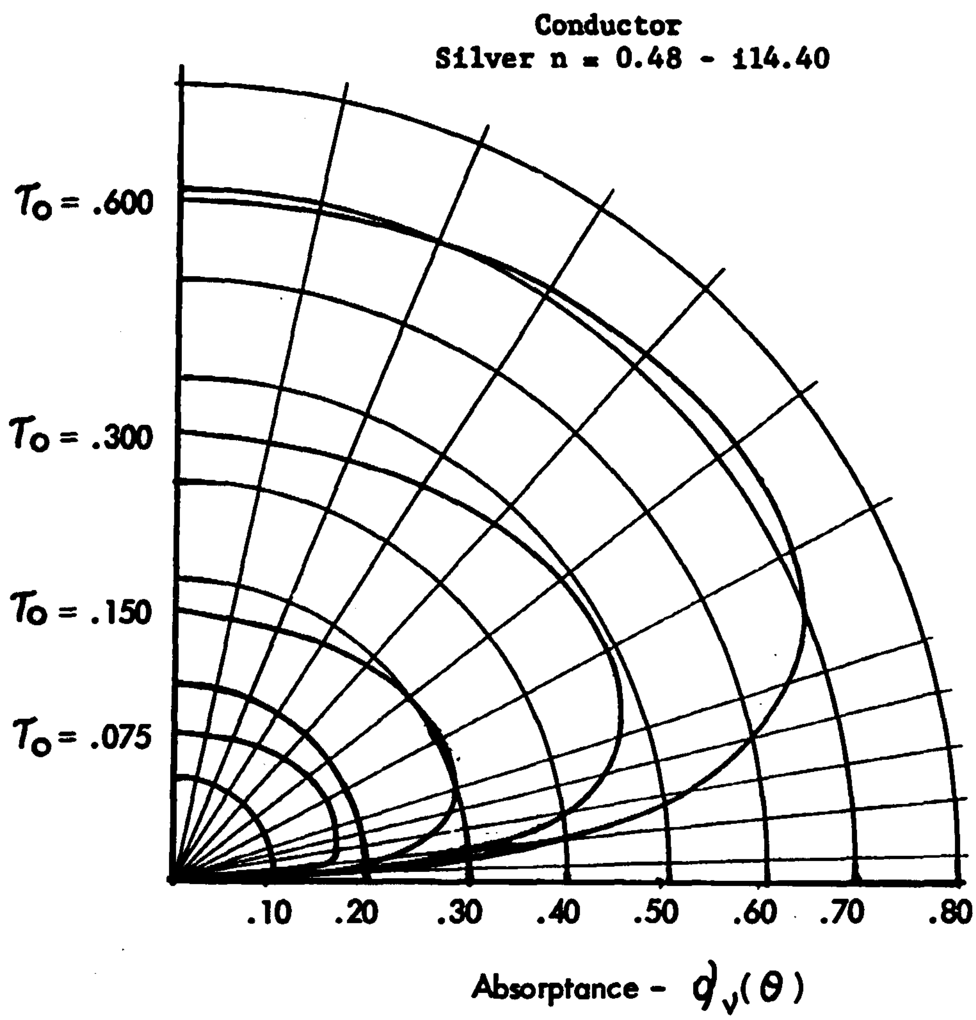


Figure 20

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 1.5 on Silver

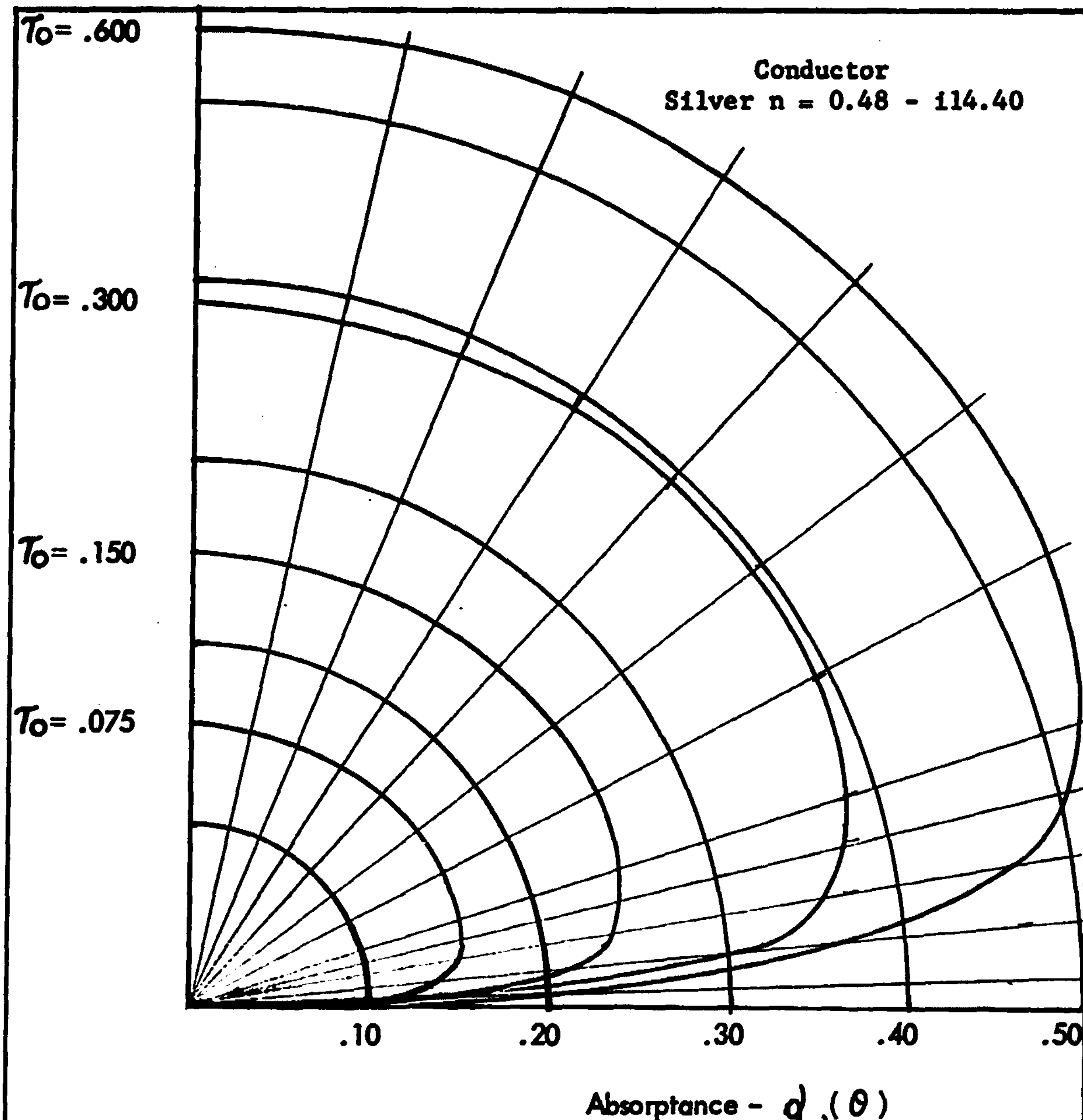


Figure 21

Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 3.5 on Silver

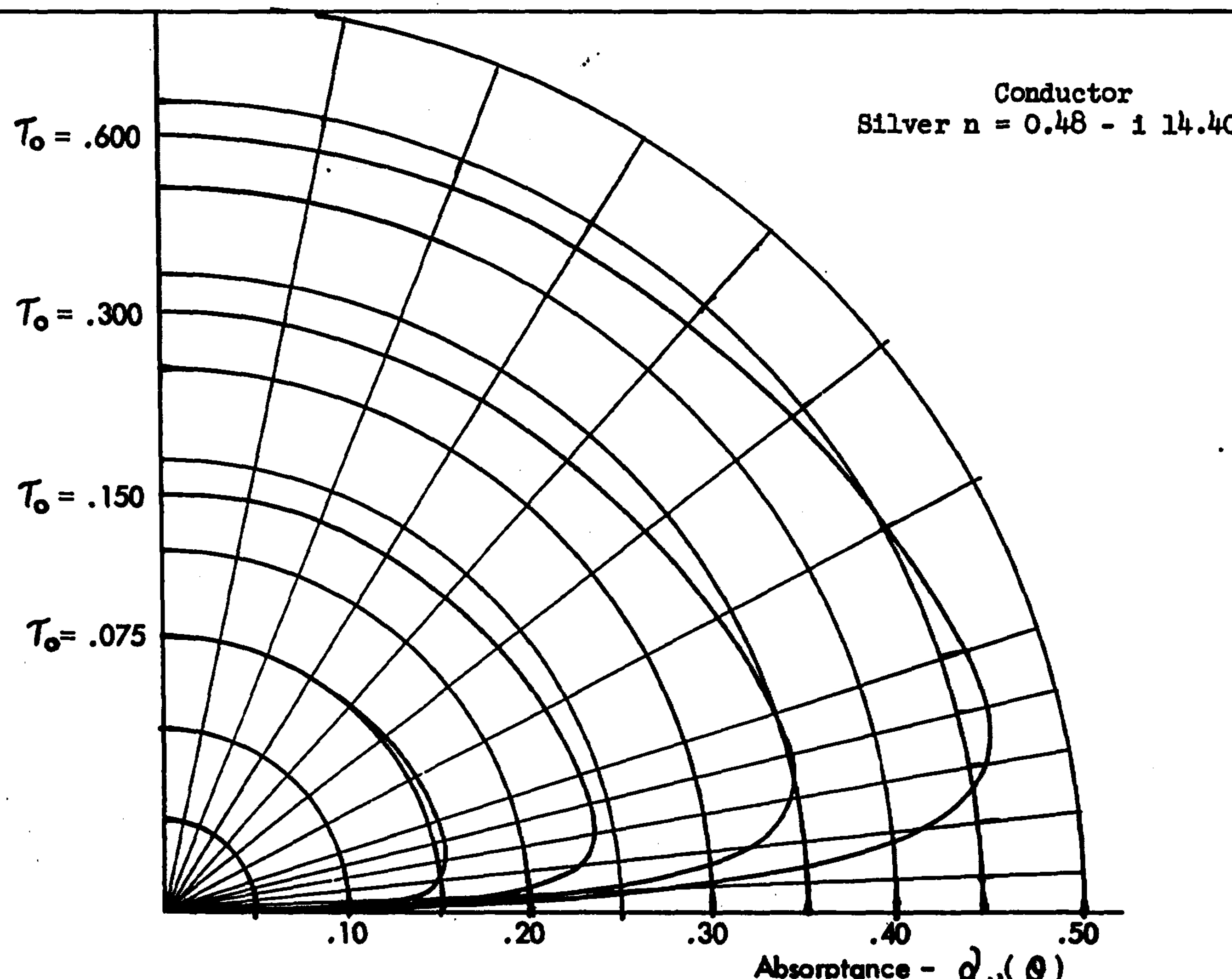


Figure 22
Monochromatic Directional Absorptance for a Dielectric Coating with a Refractive Index of 5.5 on Silver

or upon substitution

$$c_1 e^{-(\tau_o/\mu)} = c_2,$$

and at $\tau = 0$

$$I_\upsilon^+(o,\mu) = I_\upsilon^-(o,\mu)$$

or upon substitution

$$c_1 = c_2 e^{-(\tau_o/\mu)}.$$

Therefore, after combining the two boundary conditions, one finds

$$c_1 = c_2 = 0.$$

The boundary conditions for $\mu > \mu_c$, $\tau = \tau_o$, and $\tau = 0$ are

$$\rho_{2_\upsilon}(\theta',\theta) I_\upsilon^+(\tau_o,\mu) = I_\upsilon^-(\tau_o,\mu)$$

and

$$\rho_{2_\upsilon}(\theta',\theta) I_\upsilon^-(o,\mu) = I_\upsilon^+(o,\mu)$$

respectively. Therefore, upon substitution for the intensities

$$c_2 = \rho_{2_\upsilon}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + n^2 I_{bb_\upsilon}(T_2)\right] - n^2 I_{bb_\upsilon}(T_2),$$

and

$$c_1 = \rho_{2_\upsilon}(\theta',\theta)\left[c_2 e^{-(\tau_o/\mu)} + n^2 I_{bb_\upsilon}(T_2)\right] - n^2 I_{bb_\upsilon}(T_2).$$

The combination of the two above relations yields

$$c_1 = c_2 = \frac{n^2 I_{bb_\upsilon}(T_2)\left[\rho_{2_\upsilon}(\theta',\theta) - 1\right]}{1 - \rho_{2_\upsilon}(\theta',\theta) e^{-(\tau_o/\mu)}}.$$

The effective emittance

$$\epsilon_{eff_\nu}(\mu) = \frac{I_{net_\nu}(\tau_o,\mu')}{n^2 I_{bb_\nu}(T_2)} = \frac{I^+_\nu(\tau_o,\mu) - I^-_\nu(\tau_o,\mu)}{n^2 I_{bb_\nu}(T_2)} ,$$

can be simplified to

$$\epsilon_{eff_\nu}(\mu') = c_1'(e^{-(\tau_o/\mu)} - 1).$$

Here again

$$c_1' = \frac{c_1}{n^2 I_{bb_\nu}(T_2)} .$$

These results are shown in Figures 23 through 25.

It is observed from the graphs that the emissivity of a sheet of dielectric material increases with optical depth. The same is true with the isothermal coating on a conductor. For the isothermal sheet of dielectric which has an emittance given by

$$\epsilon_{eff_\nu}(\mu) = \frac{\left[\rho_{2_\nu}(\theta',\theta) - 1\right] \left[e^{-(\tau_o/\mu)} - 1\right]}{1 - \rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}}$$

as the optical depth τ_o goes to the limit

$$\operatorname*{Limit}_{\tau_o \to \infty} \epsilon_{eff_\nu}(\mu) = 1 - \rho_{2_\nu}(\theta',\theta),$$

which is Fresnel's emission of an opaque dielectric which is infinite in extent.

In the case of the dielectric coating on a conductor, the emission is given by

$$\epsilon_{eff_\nu} = c_1 e^{-(\tau_o/\mu)} - c_2,$$

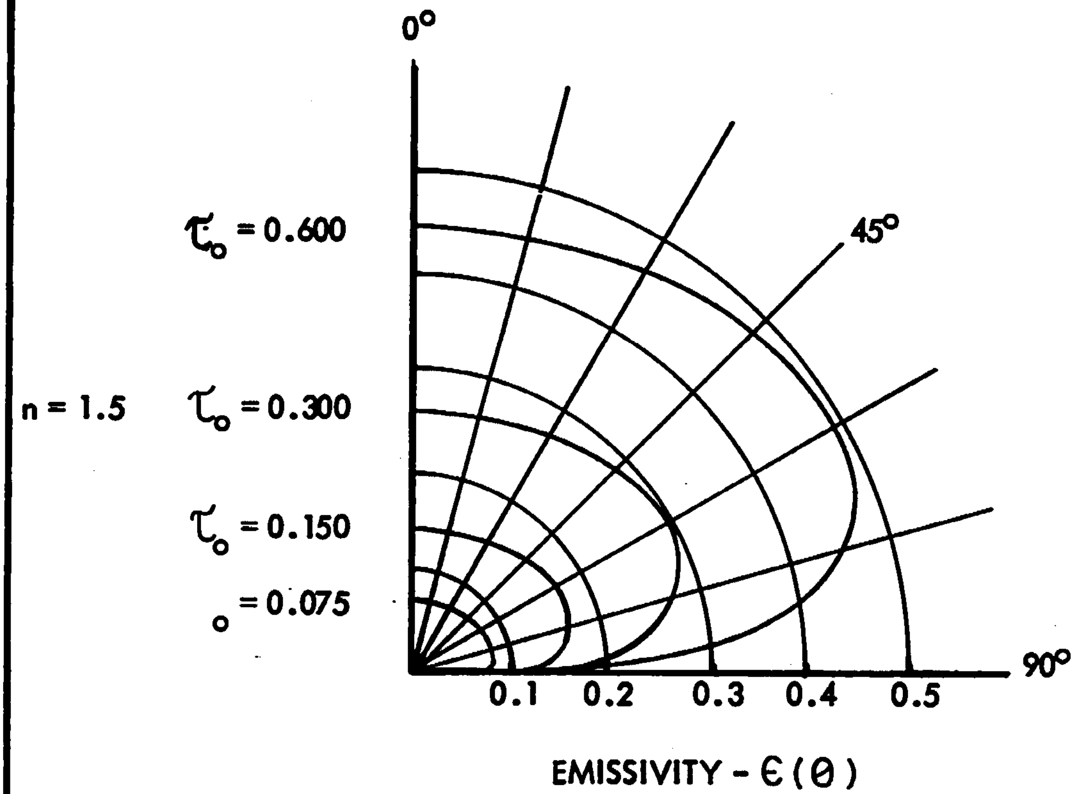


Figure 23

Monochromatic Directional Emissivity
For Various Optical Depths of an Isothermal
Dielectric with a Refractive Index of 1.50

Dielectric Sheet
n = 3.50

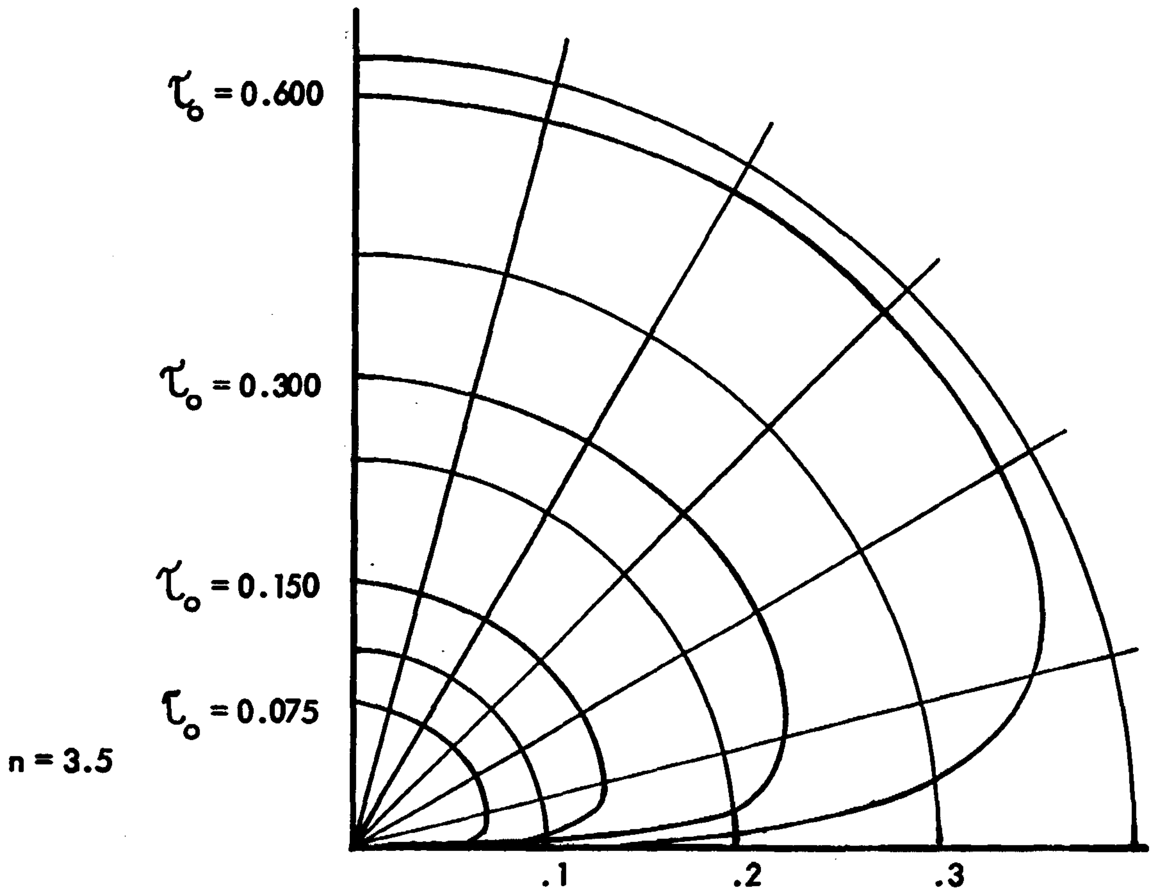


Figure 24

Monochromatic Directional Emissivity
For Various Optical Depths of an Isothermal
Dielectric with a Refractive Index of 3.5

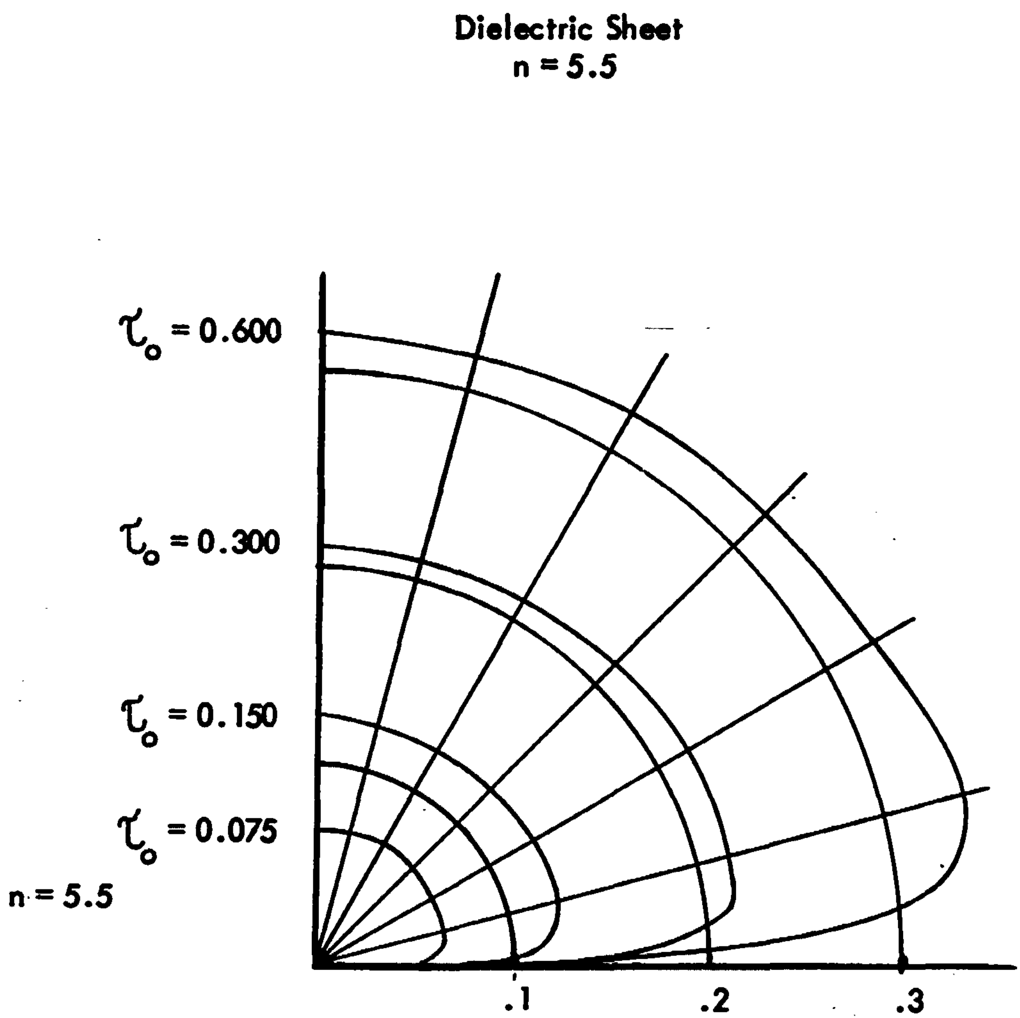


Figure 25

Monochromatic Directional Emissivity
For Various Optical Depths of an Isothermal
Dielectric with a Refractive
Index of 5.5

where

$$c_1 = c_2 \left[1 - \epsilon(\mu)\right] e^{-(\tau_o/\mu)}$$

and

$$c_2 = \frac{n^2 \left[\rho_{2_\upsilon}(\theta',\theta) - 1\right]}{1 - \rho_{2_\upsilon}(\theta',\theta)\rho_{1_\upsilon}(\mu)e^{-(2\tau_o/\mu)}} .$$

Here again as $\tau_o \longrightarrow \infty$

$$\lim_{\tau_o \longrightarrow \infty} \epsilon_{\text{eff}_\upsilon}(\mu) = 1 - \rho_{2_\upsilon}(\theta',\theta)$$

the same as before.

3. Flux Comparison.

Several investigators have used the assumption that the flux and not the intensity is attenuated through the optical depth τ_o. In this way they avoid the directional dependency of the intensity. It is now desirable to compare the results obtained by the two methods. Therefore, the flux approach will be applied to an absorbing and emitting isothermal medium.

The transport equation written in terms of fluxes will take the form

$$\frac{dQ(\tau)}{d\tau} = - Q(\tau) + n^2 \pi I_{bb_\upsilon}(T).$$

Solving the equation when in the form of positive and negative directed fluxes yields

$$q_\upsilon^+(\tau) = c_1 e^{-\tau} + n^2 \pi I_{bb_\upsilon}(T)$$

$$q_\upsilon^-(\tau) = c_2 e^{(\tau-\tau_o)} + n^2\pi I_{bb_\upsilon}(T).$$

The boundary conditions are as follows at $\tau = \tau_o$,

$$q^- = \rho_D q^+$$

where ρ_D is equal to the hemispherical reflectivity of the dielectric-air interface, which upon substitution gives

$$c_2 + n^2\pi I_{bb_\upsilon}(T) = \rho_D \left[c_1 e^{-(\tau_o)} + n^2\pi I_{bb_\upsilon}(T)\right] .$$

Rearranging the terms gives

$$c_2 + n^2(1 - \rho_D)\pi I_{bb_\upsilon}(T) = c_1\rho_D e^{-(\tau_o)} .$$

The boundary condition at $\tau = 0$ is

$$q^+(o) = n^2\pi\epsilon_H I_{bb_\upsilon}(T) + \rho_H q^-(o),$$

where ϵ_H equals the hemispherical emissivity of the conductor into the dielectric.

$$\rho_H = 1 - \epsilon_H$$

The boundary condition can now be written as

$$c_1 + n^2\pi I_{bb_\upsilon}(T) = n^2\pi\epsilon_H I_{bb_\upsilon}(T) + \rho_H \left[c_2 e^{-(\tau_o)} + n^2\pi I_{bb_\upsilon}(T)\right]$$

or

$$c_1 = n^2 \pi I_{bb_\upsilon}(T) \left[\epsilon_H + \rho_H - 1\right] + \rho_H c_2 e^{-(\tau_o)},$$

where

$$c_1 = \rho_H c_2 e^{-(\tau_o)}$$

and

$$c_2 = \frac{n^2(1 - \rho_D)\pi I_{bb_\upsilon}(T)}{\rho_D \rho_H e^{-(2\tau_o)} - 1}.$$

Now by denoting

$$c_2' = \frac{c_2}{\pi I_{bb_\upsilon}(T)}$$

and

$$c_1' = \rho_H c_2' e^{-(\tau_o)}.$$

The net flux can be written as

$$Q_{\upsilon_{net}}(\tau_o) = q_\upsilon^+(\tau_o) - q_\upsilon^-(\tau_o) = c_1 e^{-(\tau_o)} - c_2 = c_2(\rho_H e^{-(2\tau_o)} - 1)$$

and the flux ratio as

$$\frac{Q_{\upsilon_{net}}(\tau_o)}{\pi I_{bb_\upsilon}(T)} = c_2'(\rho_H e^{-(2\tau_o)} - 1).$$

The flux ratio is the hemispherical emittance of the coating combination while ρ_H is the hemispherical reflection of the conductor-dielectric interface given by

$$\rho_H = 1 - \epsilon_H = 1 - 2\int_o^1 \mu\epsilon(\mu)d\mu,$$

where $\epsilon(\mu)$ is given by Fresnel's relation for emission of metals into dielectrics as given previously.

The hemispherical reflectance of the dielectric-air interface is ρ_D given by

$$\rho_D = 1 - \epsilon_D = 2\int_{\mu_c}^{1} \mu\rho_2(\theta',\theta)d\mu.$$

Again $\rho_2(\theta',\theta)$ is the Fresnel relation developed previously.

Walsh has integrated from 0 to 1 to obtain ρ_D', which is found to be

$$\rho_D' = \frac{1}{2} + \frac{(n-1)(3n-1)}{6(n+1)^2} + \left[\frac{n^2(n^2-1)^2}{(n^2+1)^3}\right] \ln\frac{(n-1)}{(n+1)} -$$

$$- 2n^3 \frac{(n^2+2n-1)}{(n^2+1)(n^4-1)} + \frac{8n^4(n^4+1)}{(n^2+1)(n^4-1)^2} \ln n.$$

Here it is noted that

$$n^2(1 - \rho_D) = (1 - \rho_D').$$

The results for hemispherical emittance of the conductor-coating combination obtained by this method are compared with those obtained by considering the directional intensities in Figures 26 through 30.

As the coating thickness increases toward infinity, the results of both methods agree as they both approach the integral of Fresnel's equation. On the other hand, as the coating thickness approaches zero, the attenuated flux equation approaches

$$\epsilon_{eff_H} = \frac{(1-\rho_H)(1-\rho_D)}{1-\rho_H\rho_D}$$

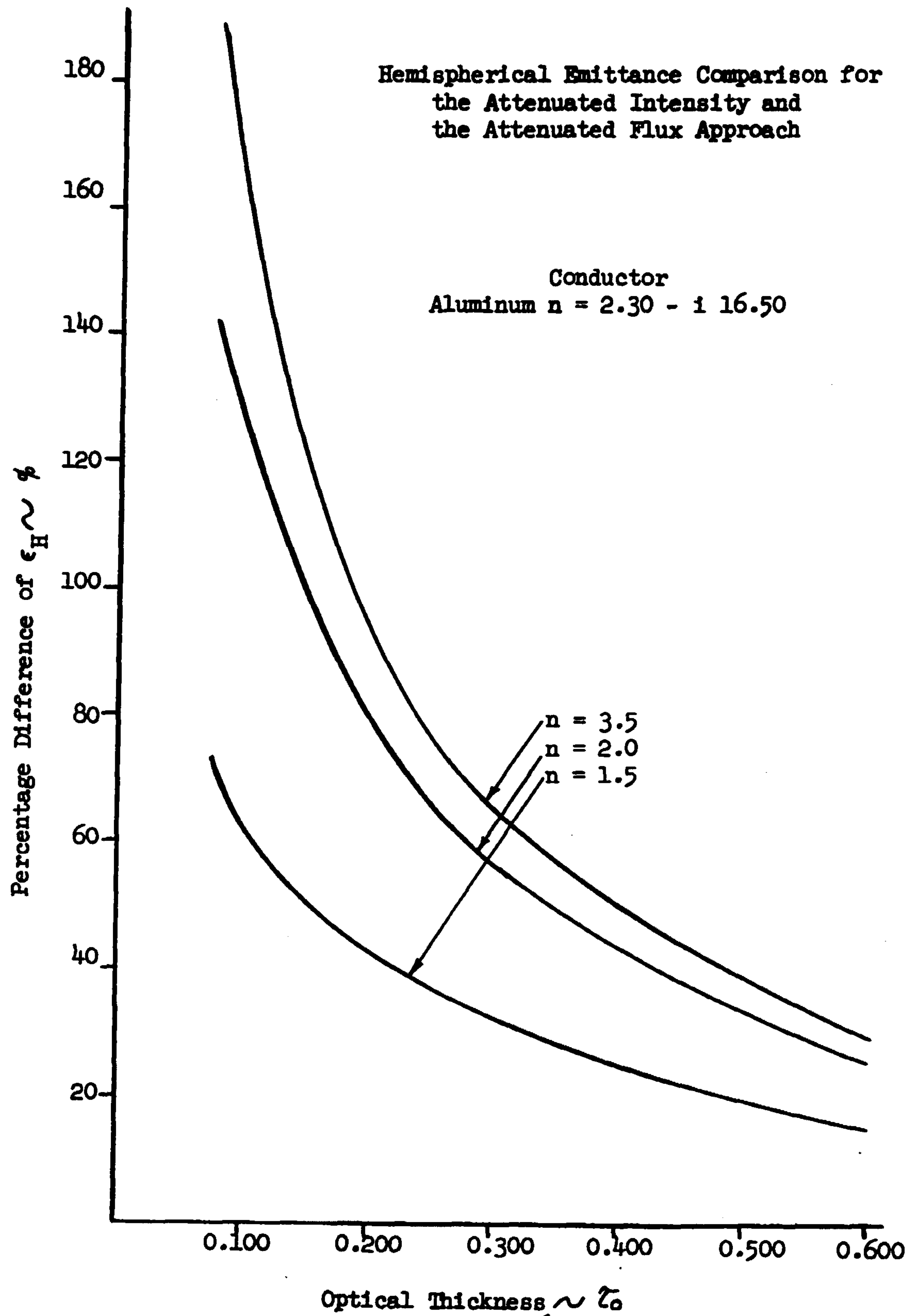

Hemispherical Emittance Comparison for
the Attenuated Intensity and
the Attenuated Flux Approach
Conductor
Aluminum n = 2.30 - i 16.50
n = 3.5
n = 2.0
n = 1.5
180
160
140
120
100
80
60
40
20
Percentage Difference of $\epsilon_H \sim$ %
0.100
0.200
0.300
0.400
0.500
0.600
Optical Thickness $\sim \tau_o$

Figure 26

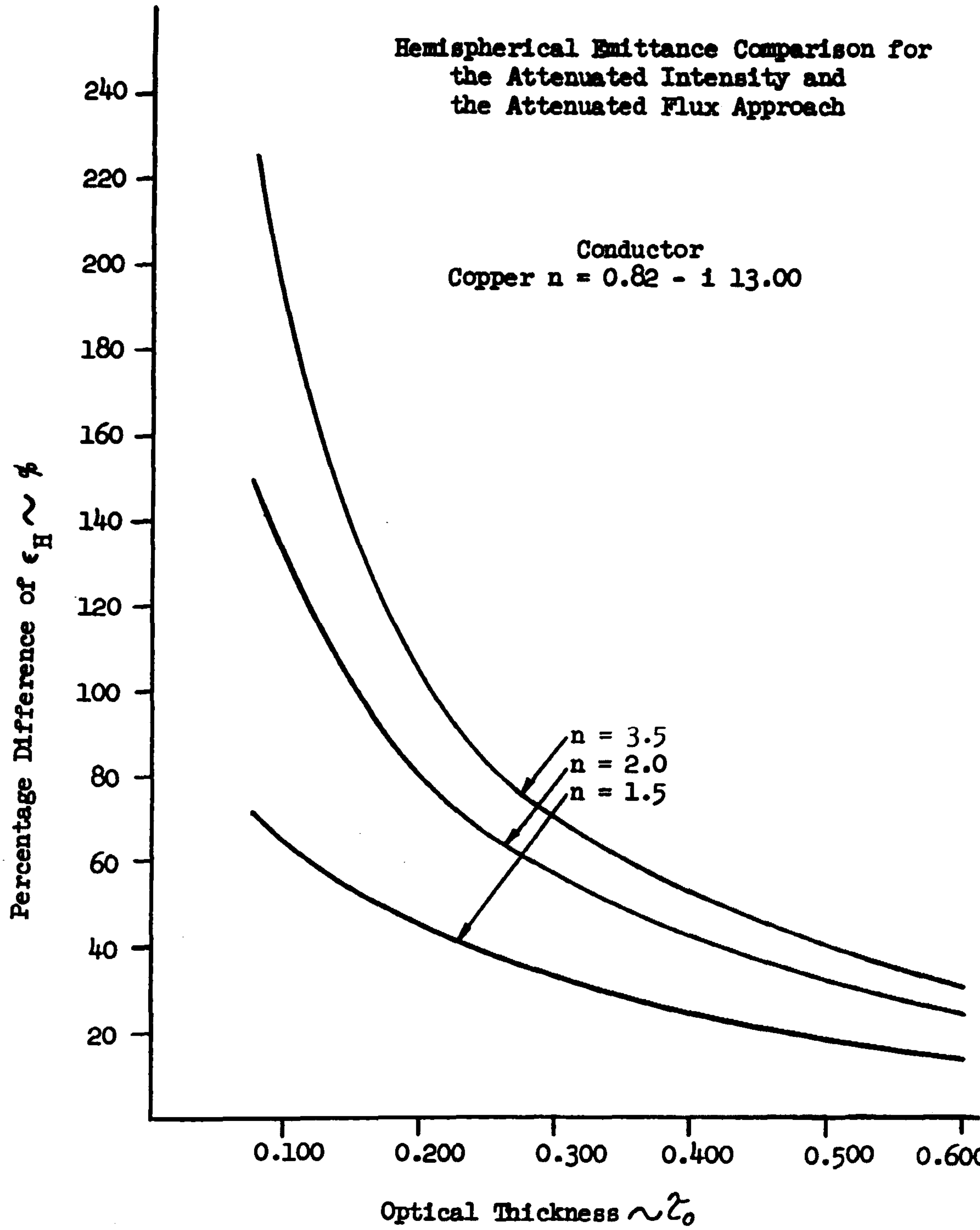

Hemispherical Emittance Comparison for
the Attenuated Intensity and
the Attenuated Flux Approach
Conductor
Copper n = 0.82 - i 13.00
n = 3.5
n = 2.0
n = 1.5
Percentage Difference of ϵ_H ~ %
240
220
200
180
160
140
120
100
80
60
40
20
0.100
0.200
0.300
0.400
0.500
0.600
Optical Thickness ~ τ_o

Figure 27

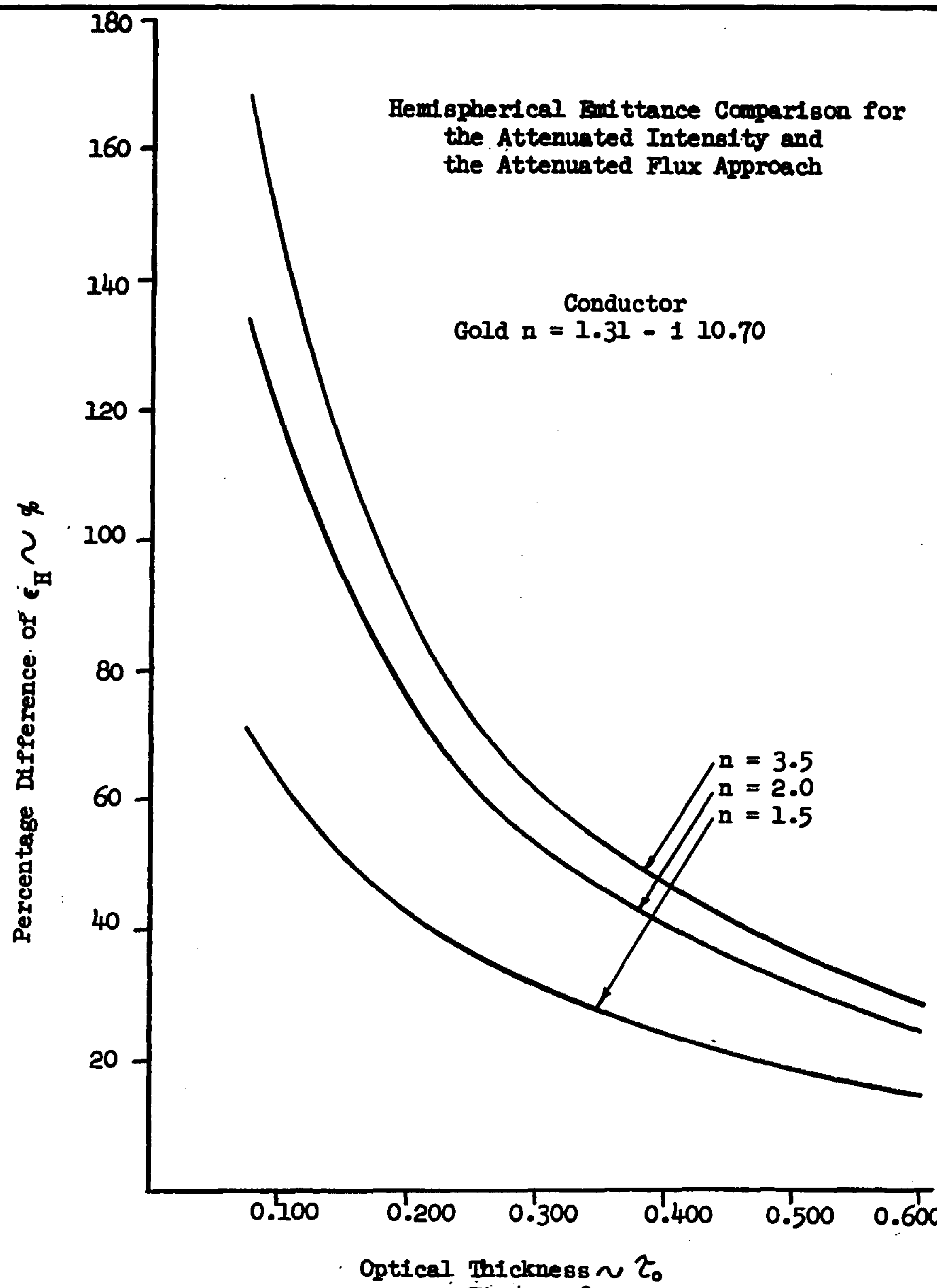
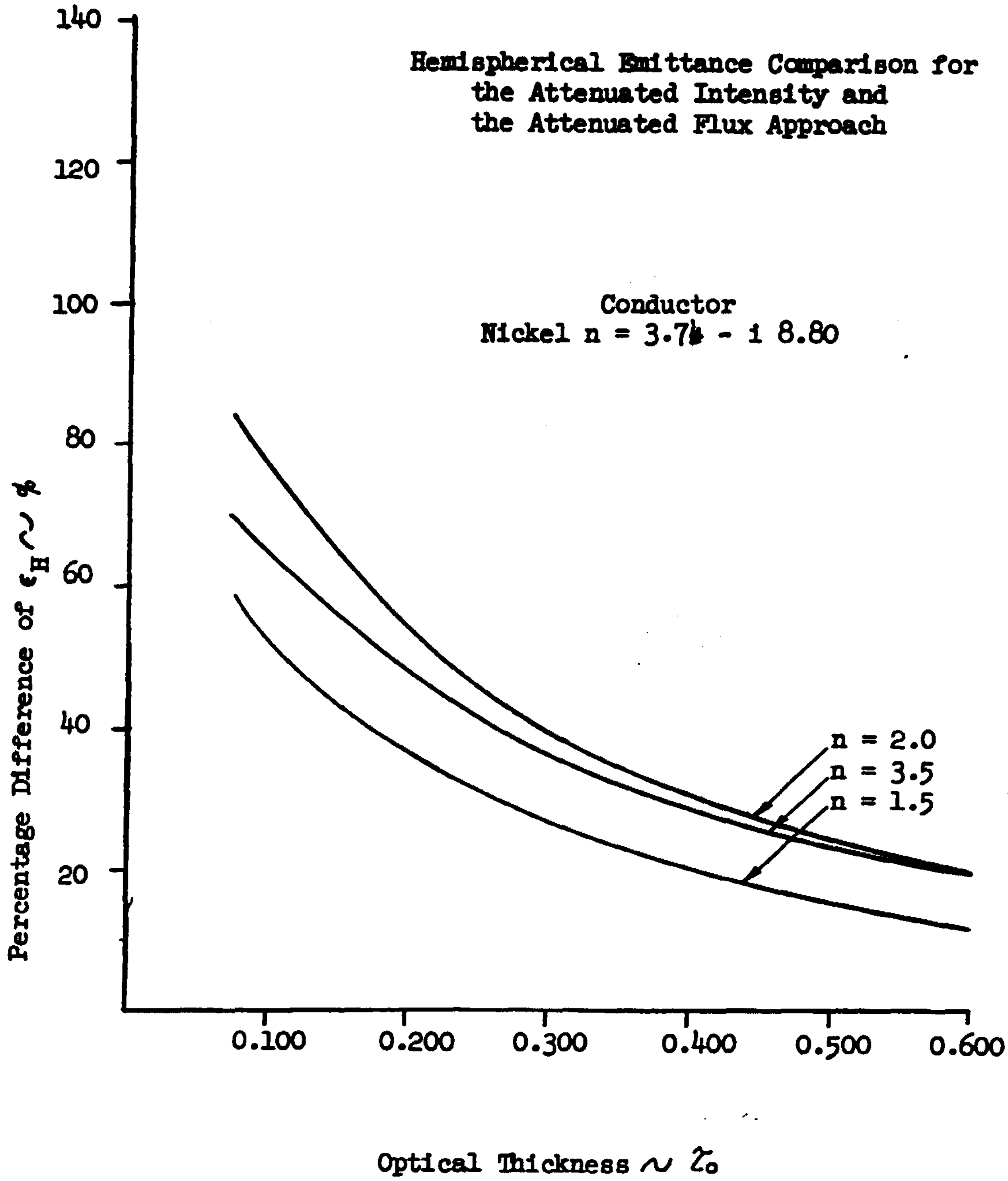

Hemispherical Emittance Comparison for
the Attenuated Intensity and
the Attenuated Flux Approach
Conductor
Gold n = 1.31 - i 10.70
n = 3.5
n = 2.0
n = 1.5
180
160
140
120
100
80
60
40
20
Percentage Difference of ϵ_H ~ %
0.100
0.200
0.300
0.400
0.500
0.600
Optical Thickness ~ τ_o

Figure 28



Hemispherical Emittance Comparison for
the Attenuated Intensity and
the Attenuated Flux Approach
Conductor
Nickel n = 3.74 - i 8.80
n = 2.0
n = 3.5
n = 1.5
140
120
100
80
60
40
20
Percentage Difference of ϵ_H ~ %
0.100
0.200
0.300
0.400
0.500
0.600
Optical Thickness ~ τ_o

Figure 29

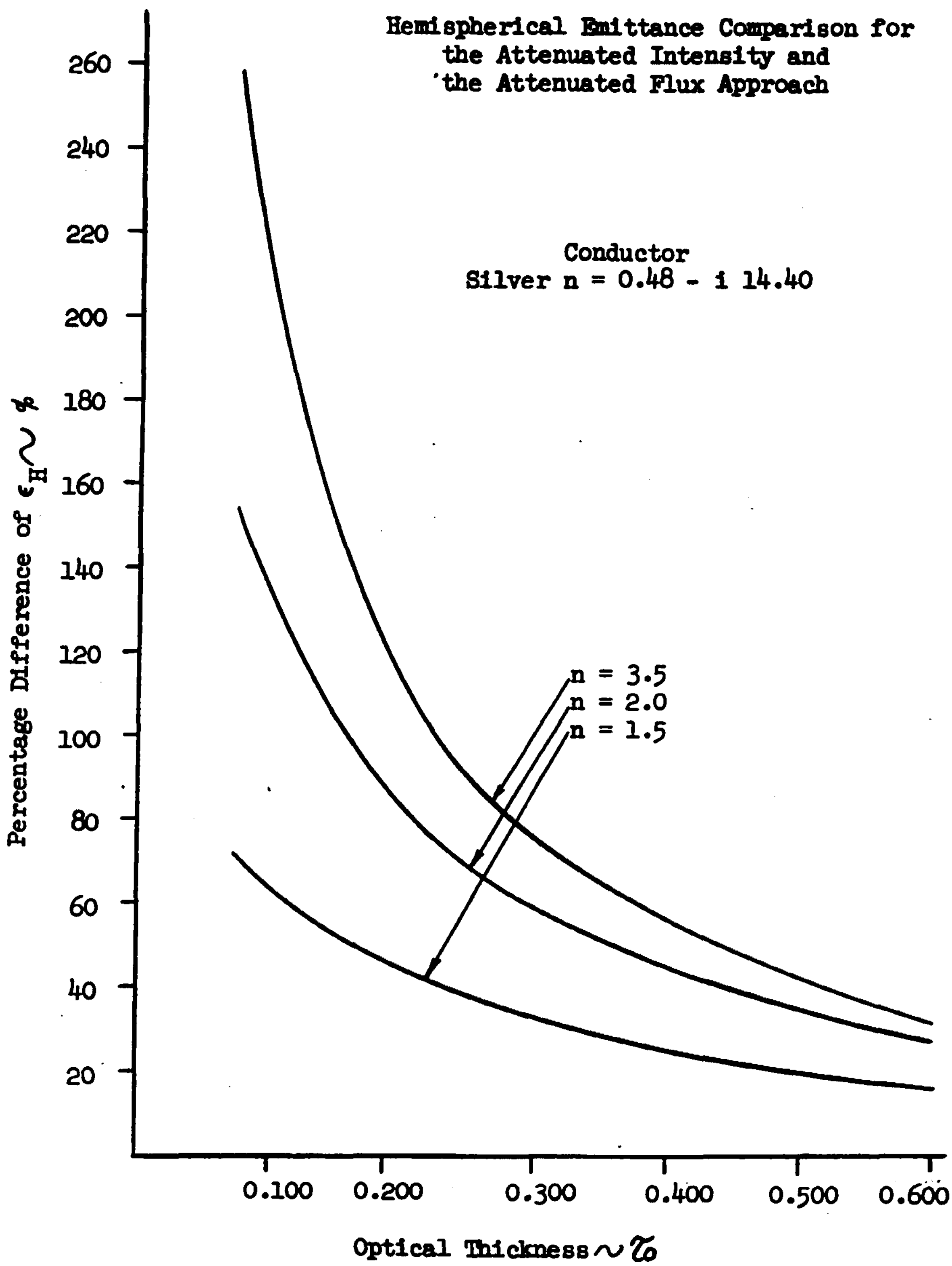

Hemispherical Emittance Comparison for
the Attenuated Intensity and
the Attenuated Flux Approach
Conductor
Silver n = 0.48 - i 14.40
Percentage Difference of $\epsilon_H \sim$ %
260
240
220
200
180
160
140
120
100
80
60
40
20
n = 3.5
n = 2.0
n = 1.5
0.100
0.200
0.300
0.400
0.500
0.600
Optical Thickness $\sim \tau_o$

Figure 30

and the attenuated intensity method approaches

$$\epsilon_{eff_H} = 2\int_o^1 \frac{\left[1 - \rho_2(\theta',\theta)\right]\left[1 - \rho_{1_\upsilon}(\mu)\right]}{1 - \rho_2(\theta',\theta)\rho_1(\mu)}\,\mu d\mu.$$

This explains the increasing difference in the two methods as $\tau_o \longrightarrow 0$. Actually, in both cases if $\tau = 0$, $\rho_2(\theta',\theta) = 0$, since no coating is present and the results would then be the emittance of the conductor.

It seems obvious that for the flux equation of transport to agree with the concepts developed in the previous chapter, the $\pi I_{bb_\upsilon}(T)$ term should be multiplied by n^2. However, while $n^2 I_{\upsilon_1} = I_{\upsilon_2}$ at an interface in the thermodynamic equilibrium, it is also true that $Q_{\upsilon_{net_1}} = Q_{\upsilon_{net_2}}$ at the same interface. Here it is desirable to show that the n^2 factor is not required at the interface. At an interface in thermodynamic equilibrium

$$I_{\upsilon_1} = n^2 I_{\upsilon_2}$$

and by substitution

$$I_{\upsilon_1} = n^2 \frac{j_\upsilon}{k_\upsilon}\ .$$

The flux at the interface in medium 1 is given by

$$Q_{\upsilon_1} = 2\pi\int_o^1 I_{\upsilon_1}\mu d\mu = Q_{\upsilon_1} = \pi I_{bb_\upsilon}(T_1),$$

and in medium 2 by

$$n^2 Q_{\upsilon_2} = 2\pi\int_{\mu_c}^1 n^2 I_{\upsilon_2}\mu d\mu,$$

or

$$Q_{\nu_2} = 2\pi \frac{j_\nu}{k_\nu} \int_{\mu_c}^{1} \mu d\mu = 2\pi \frac{j_\nu}{k_\nu} \left[\frac{1}{2} - \frac{\mu_c^2}{2}\right].$$

The two fluxes must be equal or

$$Q_{\nu_2} = Q_{\nu_1} = 2\pi \frac{j_\nu}{k_\nu} (\tfrac{1}{2})(1 - \mu_c^2).$$

Substituting the relationships

$$\mu_c = (1 - \sin^2 \theta_c)^{\frac{1}{2}}$$

and

$$\sin \theta_c = \frac{1}{n^2} ,$$

yields

$$\pi \frac{j_\nu}{k_\nu} = Q_{\nu_2} = Q_{\nu_1} = n^2 \pi I_{bb_\nu}(T_1)$$

or

$$\frac{j_\nu}{k_\nu} = n^2 I_{bb_\nu}(T_1).$$

Therefore,

$$\hbar_\nu = k_\nu n^2,$$

for the case when fluxes are considered. This point also has never been made clear in the literature to the author's knowledge.

4. Absorption and Transmission of a Dielectric Slab.

Absorption is dependent only upon the surface characteristics and the internal characteristics of the material. It is dependent on temperature only in that at extreme temperatures the surface characteristics may change. When considering absorption one must remember the difference between the absorption and the absorption coefficient. Materials can be poor absorbers and have a large absorption coefficient. This is exampled in conductors which reflect most of the incident energy but that energy which does enter the conductor is absorbed. Other materials such as certain dielectrics have low reflectivities, and therefore a large amount of the incident energy enters the dielectric. If the dielectric is optically thin, the absorption is small and part of the energy is transmitted on through the dielectric, but if the dielectric is optically thick, there is very little transmission and the absorption is quite large. Since the absorption is independent of temperature, for simplification the temperature will be assumed to be absolute zero. The governing equations are

$$I_\upsilon^+(\tau,\mu) = c_1 e^{-(\tau/\mu)}$$

$$I_\upsilon^-(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu}.$$

The boundary condition at $\tau = 0$ is the same as before, but at $\tau = \tau_o$ the boundary condition changes. The boundary conditions are

$$I_\upsilon^+(o,\mu) = \rho_{2_\upsilon}(\theta',\theta) I_\upsilon^-(o,\mu)$$

and

$$I_\upsilon^-(\tau_o,\mu) = \rho_{2_\upsilon}(\theta',\theta) I_\upsilon^+(\tau_o,\mu) + \left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] I_{i_\upsilon}(\theta') n^2.$$

Again, $\rho_{2_\upsilon}(\theta',\theta)$ is Fresnel's reflection coefficient for the dielectric. $I_{i_\upsilon}(\theta')$ is the directional incident radiation intensity. Substituting into the boundary conditions yields

$$c_1 = c_2 e^{-(\tau_o/\mu)} \rho_{2_\upsilon}(\theta',\theta)$$

and

$$c_2 = \rho_{2_\upsilon}(\theta',\theta) c_1 e^{-(\tau_o/\mu)} + \left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] n^2 I_{i_\upsilon}(\theta'),$$

which upon combining yields

$$c_1 = \frac{n^2 \left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] \rho_{2_\upsilon}(\theta',\theta) e^{-(\tau_o/\mu)} I_{i_\upsilon}(\theta')}{1 - \rho^2_{2_\upsilon}(\theta',\theta) e^{-(2\tau_o/\mu)}}.$$

The net intensity at $\tau = \tau_o$ is

$$I_{net_\upsilon}(\tau_o,\mu) = I^-_\upsilon(\tau_o,\mu) - I^+_\upsilon(\tau_o,\mu) = c_2 - c_1 e^{-(\tau_o/\mu)}$$

or

$$I_{net_\upsilon}(o,\mu) = c_2 e^{-(\tau_o/\mu)} - c_1 = c_1 \left[\frac{1}{\rho_{2_\upsilon}(\theta',\theta)} - 1\right].$$

The transmission is denoted as $\tau_\upsilon(\mu)$ and given by

$$\tau_\upsilon(\mu) = \frac{I_{net_\upsilon}(o,\mu)}{n^2 I_i(\theta')}$$

or

$$\tau_\upsilon(\mu) = \frac{\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right]\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] e^{-(\tau_o/\mu)}}{1 - \rho_{2_\upsilon}(\theta',\theta)^2 e^{-(2\tau_o/\mu)}}.$$

The apparent reflectivity will be used to denote that energy reflected from the surface in addition to that energy which enters the dielectric and is internally reflected until it finally comes back out the interface at $\tau = \tau_o$. The apparent reflectivity is denoted as

$$\rho_\upsilon^*(\mu) = \rho_{2_\upsilon}(\theta',\theta) + \frac{I_\upsilon^+(\tau_o,\mu)}{n^2 I_{1_\upsilon}(\theta')} \left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] .$$

The absorptivity is given by an adaptation of Kirchhoff's law.

$$\alpha_\upsilon(\mu) = 1 - \rho_\upsilon^*(\mu) - \tau_\upsilon(\mu),$$

where $\alpha_\upsilon(\mu)$ is the monochromatic directional absorption. Graphs of $\tau_\upsilon(\mu)$ are given in Figures 31 through 33, and $\alpha_\upsilon(\mu)$ is plotted as $\epsilon_\upsilon(\mu)$ in Figures 23 through 25. (See pages 55 through 57.)

5. <u>Absorption of a Dielectric Coating on a Conductor</u>.

The governing equations remain unchanged. Namely,

$$I_\upsilon^+(\tau,\mu) = c_1 e^{-(\tau/\mu)}$$

and

$$I_\upsilon^-(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu} .$$

However, the boundary condition at $\tau = 0$ now becomes

$$I_\upsilon^+(o,\mu) = \epsilon_\upsilon(\mu) I_{bb_\upsilon}(T) + \rho_\upsilon(\mu) I_\upsilon^-(o,\mu),$$

which can be written as

$$c_1 = \rho_\upsilon(\mu) c_2 e^{-(\tau_o/\mu)} .$$

Here $\rho_\upsilon(\mu)$ is Fresnel's reflection coefficient for the metal. The boundary condition at $\tau = \tau_o$ remains unchanged, namely

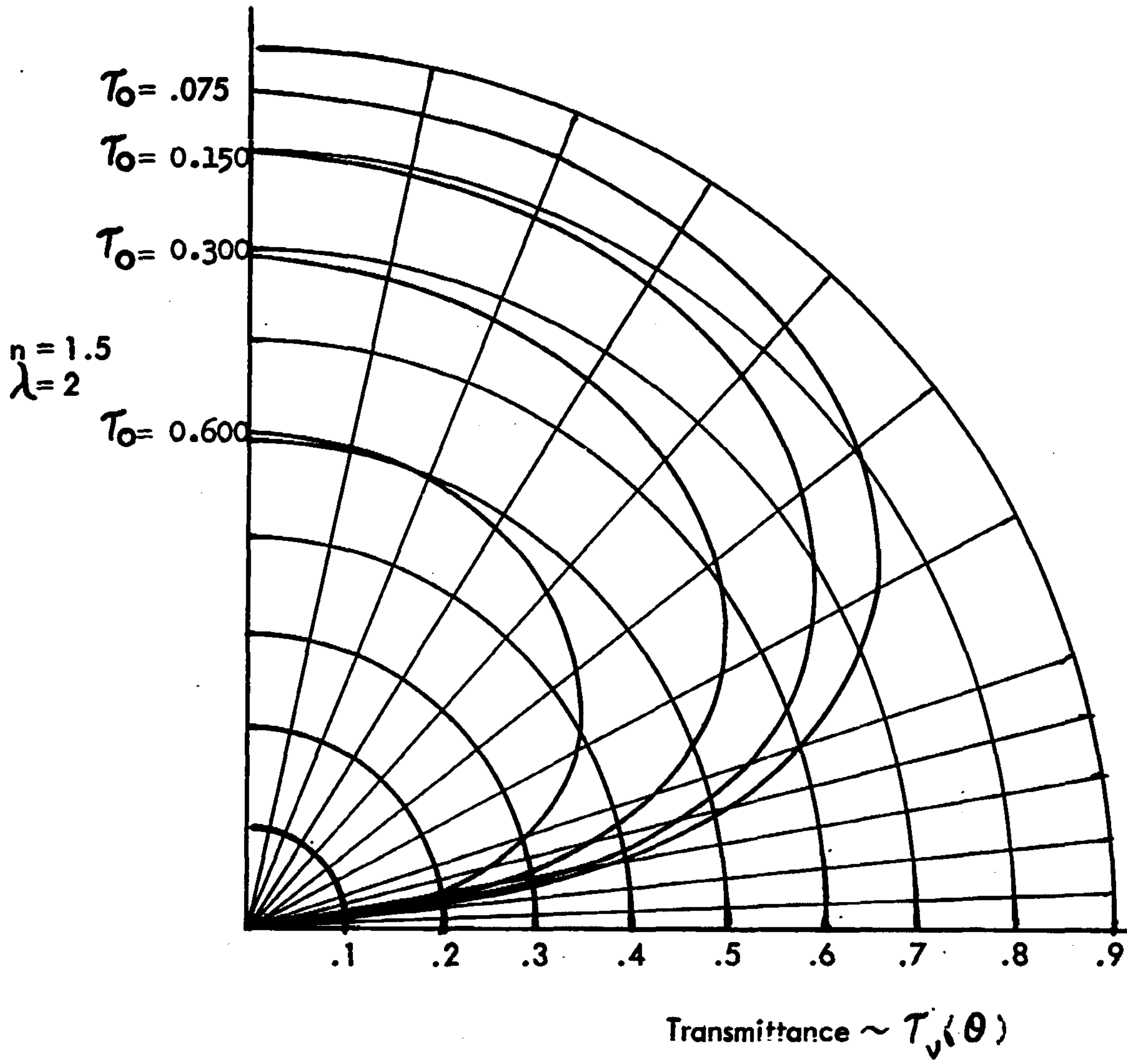


Figure 31

Monochromatic Directional Transmittance of a Dielectric of Varying Optical Thickness

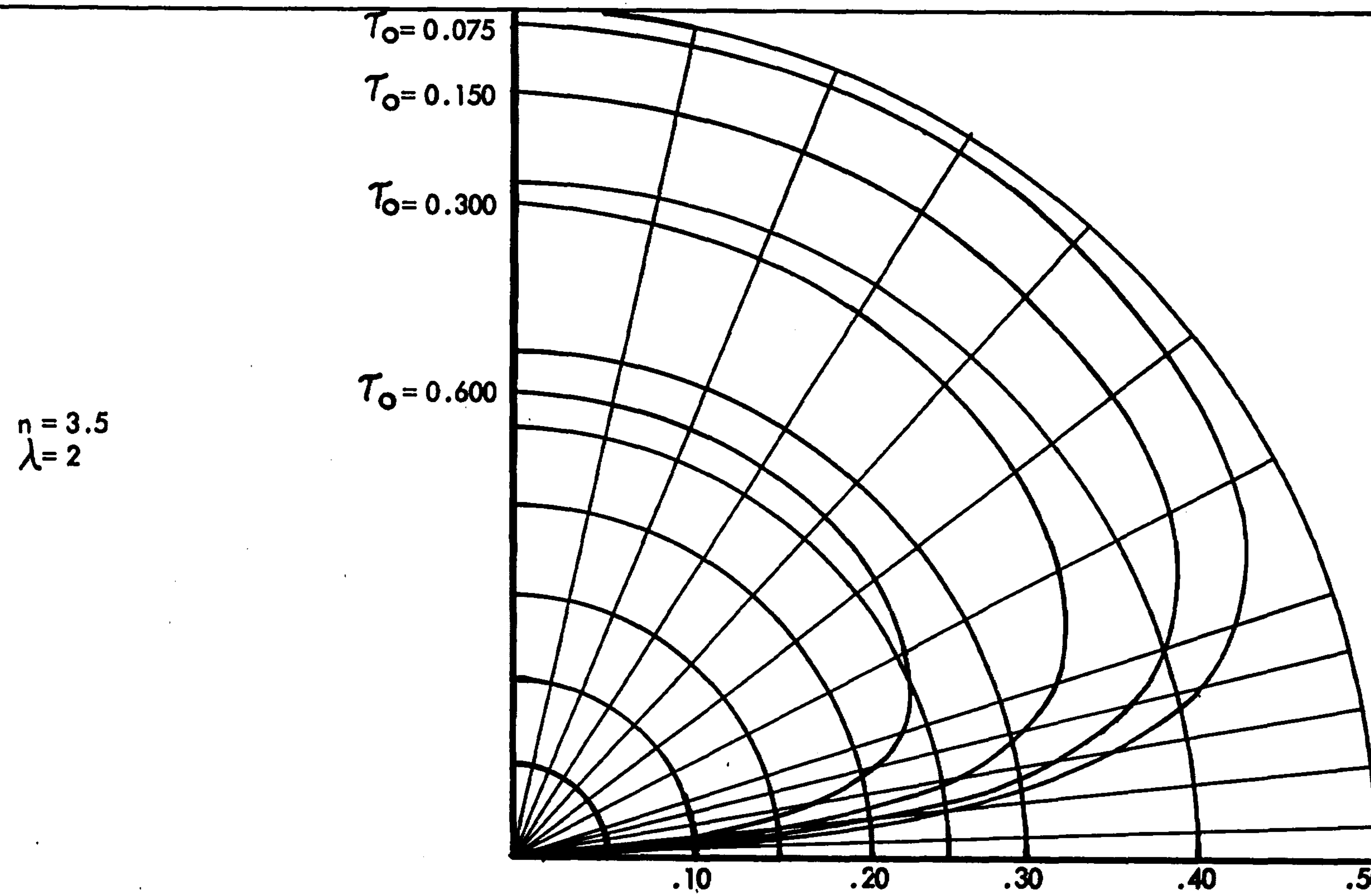


Figure 32

Monochromatic Directional Transmittance of a Dielectric of Varying Optical Thickness

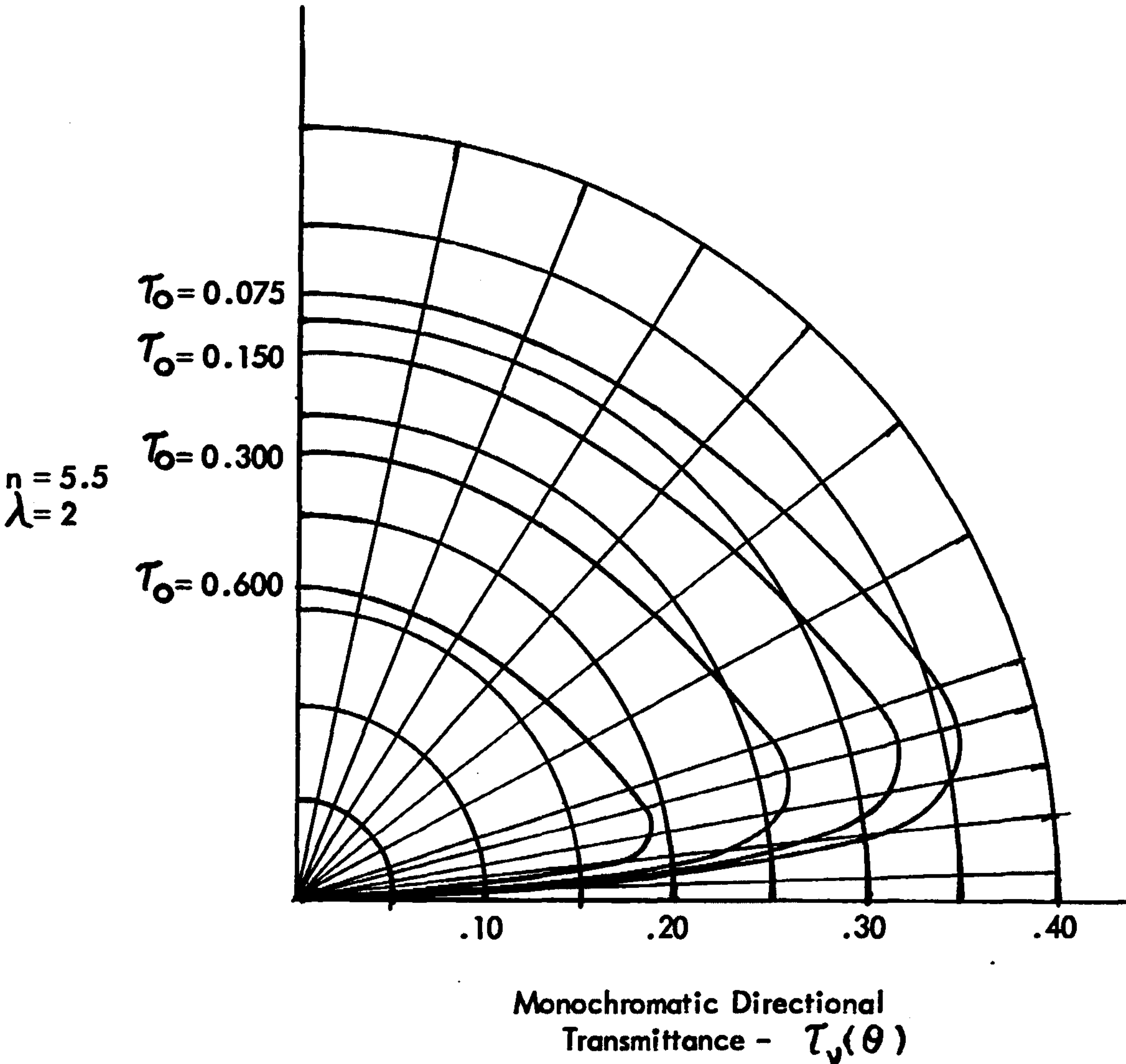


Figure 33

Monochromatic Directional Transmittance of a Dielectric of Varying Optical Thickness

$$c_2 = \rho_{2_\upsilon}(\theta',\theta)c_1 e^{-(\tau_o/\mu)} + n^2 I_{i_\upsilon}(\theta')\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] ,$$

which upon substitution yields

$$c_2 = \frac{n^2\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] I_{i_\upsilon}(\theta')}{1 - \rho_{2_\upsilon}(\theta',\theta)\rho_\upsilon(\mu)e^{-(2\tau_o/\mu)}} .$$

The net intensity is given by

$$I_{\upsilon_{net}}(\tau_o,\mu) = I_\upsilon^-(\tau_o,\mu) - I_\upsilon^+(\tau_o,\mu) = c_2\left[1 - e^{-(2\tau_o/\mu)}\rho_{2_\upsilon}(\theta',\theta)\right] .$$

As before,

$$\rho_\upsilon^*(\mu) = \rho_{2_\upsilon}(\theta',\theta) + \frac{I_\upsilon^+(\tau_o,\mu)}{n^2 I_{i_\upsilon}(\theta')}\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] ,$$

since the surface seems opaque $\tau_\upsilon(\mu) = 0$. Applying Kirchhoff's law again

$$\alpha_\upsilon(\mu) = 1 - \rho_\upsilon^*(\mu).$$

The absorptivity $\alpha_\upsilon(\mu)$ for the dielectric coated conductors is plotted as $\epsilon_\upsilon(\mu)$ in Figures 8 through 22. (See pages 38 through 52.)

CHAPTER V

NONISOTHERMAL DIELECTRIC

Having determined the emission and absorption of an isothermal absorbing and emitting dielectric, the next step is to impose a temperature profile on the dielectric. The problem now becomes what temperature profile to impose. Throughout this paper the profiles used will be linear and parabolic. They are given by

$$T = \frac{T_2 - T_1}{\tau_o} \tau + T_1$$

and

$$T = \frac{T_1 - T_2}{\tau_o^2} \tau^2 + \frac{2(T_2 - T_1)}{\tau_o} \tau + T_1,$$

where

T_2 = temperature of the dielectric surface and

T_1 = temperature of the conductor surface.

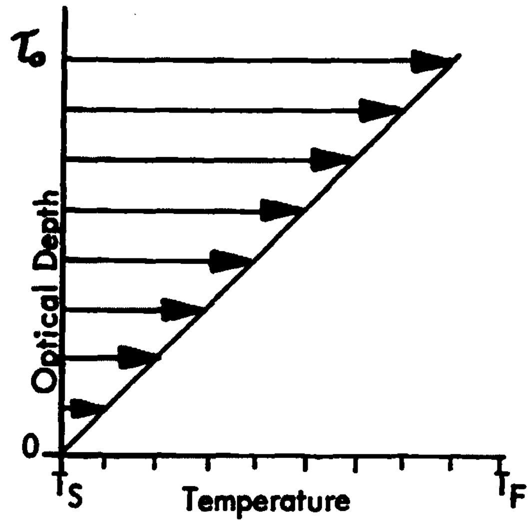
These are plotted in Figure 34.

Since the medium is nonisothermal, the simplifying assumption of local thermodynamic equilibrium as discussed in Chapter III will be required so the equation of transport may be used. Also, the monochromatic refractive index and absorption coefficient are assumed to be independent of temperature. The equation of transport is the same as before, but now the emission term is dependent upon optical thickness.

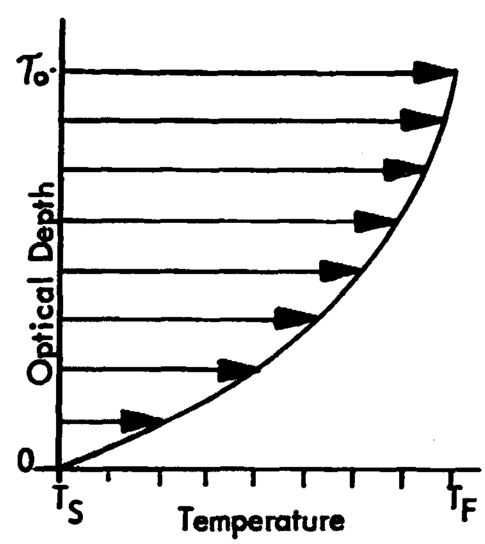
Figure 34

Temperature Profiles

$$T = \frac{T_F - T_S}{\tau_o}\tau + T_S$$



$$T = \frac{T_S - T_F}{\tau_o^{\,2}}\tau^2 + \frac{2(T_F - T_S)}{\tau}\tau_o + T_S$$



The solution to the differential equation by the use of the integrating factor now becomes

$$I_{\upsilon}^{+}(\tau,\mu) = c_1 e^{-\tau/\mu} + \frac{n^2}{\mu}\int_0^{\tau} e^{(t-\tau)/\mu}\, I_{bb_{\upsilon}}(t)dt$$

and

$$I_{\upsilon}^{-}(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu} + \frac{n^2}{\mu}\int_{\tau}^{\tau_o} e^{-(t-\tau)/\mu}\, I_{bb_{\upsilon}}(t)dt.$$

Both the isothermal and nonisothermal solutions are given in detail in Appendix I.

1. Emission of a Dielectric Coating on a Conductor

The coating will be of optical thickness, τ_o, which is much greater than one wavelength of the radiation considered. The boundary conditions are the same as before. At $\tau = 0$,

$$I_{\upsilon}^{+}(o,\mu) = n^2\epsilon_{1_{\upsilon}}(\mu)I_{bb_{\upsilon}}(T_1) + \rho_{1_{\upsilon}}(\mu)I_{\upsilon}^{-}(o,\mu),$$

or

$$c_1 = \rho_{1_{\upsilon}}(\mu)\left[c_2 e^{-(\tau_o/\mu)} + \frac{n^2}{\mu}\int_0^{\tau_o} e^{-(t/\mu)}\, I_{bb_{\upsilon}}(t)dt\right] + n^2\epsilon_{1_{\upsilon}}(\mu)I_{bb_{\upsilon}}(T_1).$$

For simplicity, henceforth the integral

$$\frac{n^2}{\mu}\int_0^{\tau_o} e^{-(t/\mu)}\, I_{bb_{\upsilon}}(t)dt$$

will be denoted by L. Therefore, c_1 may be written as

$$c_1 = \left[c_2 e^{-(\tau_o/\mu)} + L\right] \rho_{1_{\upsilon}}(\mu) + n^2\epsilon_{1_{\upsilon}}(\mu)I_{bb_{\upsilon}}(T_1).$$

At $\tau = \tau_o$ and $\mu \quad \mu_c$,

$$I_\nu^+(\tau_o,\mu) = I_\nu^-(\tau_o,\mu),$$

or

$$c_1 e^{-(\tau_o/\mu)} + \frac{n^2}{\mu}\int_o^{\tau_o} e^{(t-\tau_o)/\mu}\, I_{bb_\nu}(t)dt = c_2.$$

The integral

$$\frac{n^2}{\mu}\int_o^{\tau_o} e^{(t-\tau_c)/\mu}\, I_{bb_\nu}(t)dt$$

will be denoted by I2. Therefore c_2 may be written as

$$c_2 = c_1 e^{-(\tau_o/\mu)} + I2.$$

Therefore, for $\mu \leq \mu_c$

$$c_1 = \left[c_1 e^{-(2\tau_o/\mu)} + I2e^{-(\tau_c/\mu)} + L\right]\rho_{1_\nu}(\mu) + n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T_1)$$

$$c_1 = \frac{I2e^{-(\tau_o/\mu)}\rho_{1_\nu}(\mu) + n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T_1)}{1 - e^{-(2\tau_o/\mu)}\rho_{1_\nu}(\mu)}$$

$$c_2 = c_1 e^{-(\tau_o/\mu)} + I2.$$

For $\mu > \mu_c$ and $\tau = \tau_o$,

$$I_\nu^-(\tau_o,\mu) = \rho_{2_\nu}(\theta',\theta)I_\nu^+(\tau_o,\mu)$$

$$c_2 = \rho_{2_\nu}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + I2\right],$$

and from before for all μ at $\tau = 0$,

$$c_1 = \left[c_2 e^{-(\tau_o/\mu)} + L\right]\rho_{1_\nu}(\mu) + n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T_1)$$

$$c_2 = \rho_{2_\nu}(\theta',\theta)\left\{\left[(c_2 e^{-(\tau_o/\mu)} + L)\rho_{1_\nu}(\mu) + n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T_1)\right] e^{-(\tau_o/\mu} + I2\right\}$$

$$c_2 = \frac{\rho_{2_\nu}(\theta',\theta)\left\{\left[L\rho_{1_\nu}(\mu) + n^2\epsilon_{1_\nu}(\mu)I_{bb_\nu}(T_1)\right] e^{-(\tau_o/\mu)} + I2\right\}}{1 - \rho_{2_\nu}(\theta',\theta)\rho_{1_\nu}(\mu)e^{-(\tau_o/\mu)}}$$

$$I_{net_\nu}(\tau_o,\mu) = I_\nu^+(\tau_o,\mu) - I_\nu^-(\tau_o,\mu)$$

$$I_{net_\nu}(\tau_o,\mu) = c_1 e^{-(\tau_o/\mu)} + I2 - c_2.$$

For the net flux at τ_o

$$Q_{\nu_{net}}(\tau_o) = \int_o^{2\pi}\int_o^1 I'_{net_\nu}(\tau_o,\mu')\mu' d\mu' d\phi$$

$$Q_{\nu_{net}}(\tau_o) = 2\pi\int_o^1 I'_{net_\nu}(\tau_o,\mu')\mu' d\mu',$$

where $I'_{net_\nu}(\tau_o,\mu')$ is the net intensity leaving the dielectric, or

$$I'_{net_\nu}(\tau_o,\mu') = \frac{I_{net_\nu}(\tau_o,\mu)}{n^2}.$$

The net flux may also be found using $I_{net_\upsilon}(\tau_o,\mu)$ by noticing that above the critical angle the net flux is zero. If the cosine of the critical angle is denoted by μ_c, the net flux can be written as

$$Q_{\upsilon_{net}}(\tau_o) = 2\pi \int_{\mu_c}^{1} I_{net_\upsilon}(\tau_o,\mu)\mu d\mu.$$

Throughout this work the flux integrals will be evaluated numerically. The method of integration used will be the Gauss four point quadrature which gives an exact solution if the integrand can be expressed as a polynomial of degree seven or less. This quadrature still will give good results if the integrand cannot be expressed by a polynomial of this low an order. A more complete discussion of quadrature is given in Appendix II. This same quadrature will be used to evaluate the integrals L and L2.

In order to have a comparison with the isothermal case, it is desirable to define a directional emission term. This term will be called the apparent emission, $\epsilon_\upsilon^*(\mu)$, and will be defined as the ratio of the directional intensity leaving the coating to the blackbody intensity at the hotter surface. Therefore,

$$\epsilon_\upsilon^*(\mu) = \frac{I'_{net_\upsilon}(\tau_c,\mu)}{I_{bb_\upsilon}(T_{Hot})} .$$

2. Emission of a Dielectric Sheet

The same transport equation governs as in the coating, but once again the boundary conditions change. At $\tau = 0$,

$$I_\upsilon^+(o,\mu) = \rho_{2_\upsilon}(\theta',\theta) I_\upsilon^-(o,\mu),$$

and at $\tau = \tau_o$

$$I_\upsilon^-(\tau_o,\mu) = \rho_{2_\upsilon}(\theta',\theta) I_\upsilon^+(\tau_o,\mu).$$

The boundary equation at $\tau = 0$ yields

$$c_1 = c_2 e^{-(\tau_o/\mu)} + \frac{n^2}{\mu}\int_o^{\tau_o} e^{(t-\tau_o)/\mu}\, I_{bb_\upsilon}(t)dt,$$

and at $\tau = \tau_o$,

$$c_2 = c_1 e^{-(\tau_o/\mu)} + \frac{n^2}{\mu}\int_o^{\tau_o} e^{-(t/\mu)}\, I_{bb_\upsilon}(t)dt.$$

Again calling

$$\frac{n^2}{\mu}\int_o^{\tau_o} e^{-(t/\mu)}\, I_{bb_\upsilon}(t)dt = L,$$

and

$$\frac{n^2}{\mu}\int_o^{\tau_o} e^{(t-\tau_o)/\mu}\, I_{bb_\upsilon}(t)dt = L2,$$

the boundary conditions can be rewritten as

$$c_1 = \rho_{2_\upsilon}(\theta',\theta)\left[c_2 e^{-(\tau_o/\mu)} + L\right]$$

$$c_2 = \rho_{2_\upsilon}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + L2\right].$$

Substitution yields

$$c_1 = \rho_{2_\upsilon}(\theta',\theta)\left\{\rho_{2_\upsilon}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + L2\right] e^{-(\tau_o/\mu)} + L\right\},$$

which can be written as

$$c_1 = \frac{\rho_{2_\upsilon}(\theta',\theta)\left[\rho_{2_\upsilon}(\theta',\theta) I2 e^{-(\tau_o/\mu)} + L\right]}{1 - \rho_{2_\upsilon}(\theta',\theta)^2 e^{-(2\tau_o/\mu)}} .$$

The net intensity is given by

$$I_{net_\upsilon}(\tau_o,\mu) = I_\upsilon^+(\tau_o,\mu) - I_\upsilon^-(\tau_o,\mu)$$

$$I_{net_\upsilon}(\tau_o,\mu) = c_1 \left\{ e^{-(\tau_o/\mu)} \left[1 - \rho_{2_\upsilon}(\theta',\theta)\right]\right\} + I2\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] .$$

The apparent emissivity as mentioned before will be given by

$$\epsilon_\upsilon^*(\mu) = \frac{I_{net_\upsilon}(\tau_o,\mu)}{I_{bb_\upsilon}(T_{Hot})} .$$

When considering the dielectric sheet it is convenient to also determine the emission from the interface at $\tau = 0$. This in effect merely reverses the temperature profile. The net intensity at $\tau = 0$ is given by

$$I_{net_\upsilon}(o,\mu) = I_\upsilon^-(o,\mu) - I_\upsilon^+(o,\mu)$$

$$I_{net_\upsilon}(o,\mu) = c_2 e^{-(\tau_o/\mu)} + L - c_1 .$$

Upon substitution

$$I_{net_\upsilon}(o,\mu) = \rho_{2_\upsilon}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + I2\right] e^{-(\tau_o/\mu)} + L - c_1$$

$$I_{net_\upsilon}(o,\mu) = c_1\left[\rho_{2_\upsilon}(\theta',\theta)e^{-(2\tau_o/\mu)} - 1\right] + L + L2\rho_{2_\upsilon}(\theta',\theta)e^{-(\tau_o/\mu)},$$

which can also be written as

$$I_{net_\upsilon}(o,\mu) = c_2 e^{-(\tau_o/\mu)}\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right] + L\left[1 - \rho_{2_\upsilon}(\theta',\theta)\right].$$

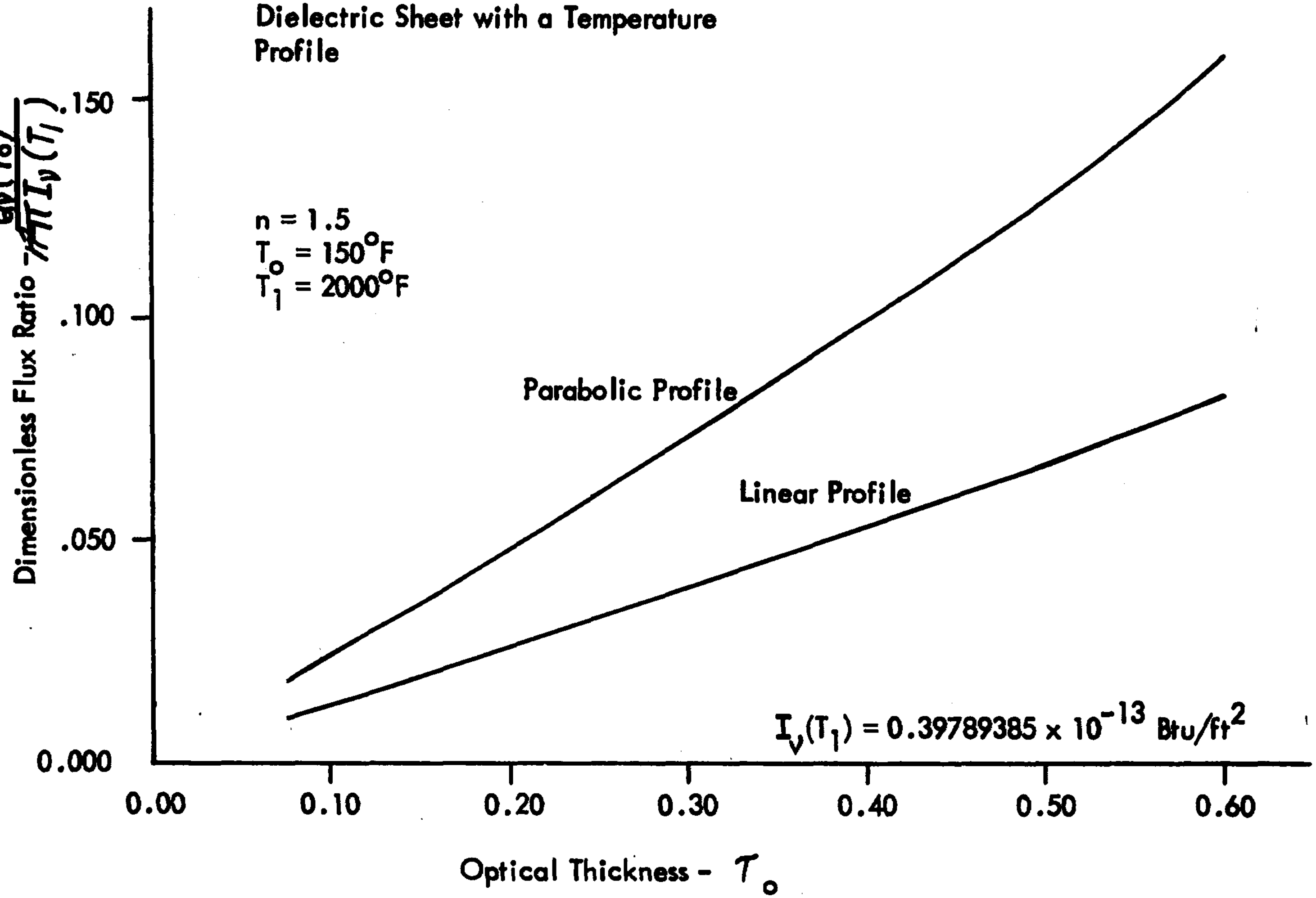
The apparent directional emissivity then becomes

$$\epsilon^*_\upsilon(\mu) = \frac{I_{net_\upsilon}(o,\mu)}{I_{bb_\upsilon}(T_{Hot})}.$$

The results of this chapter for various temperature profiles are plotted in Figures 35 through 42.

Figure 35

Dimensionless Flux Ratio for a Dielectric Sheet with a Temperature Profile



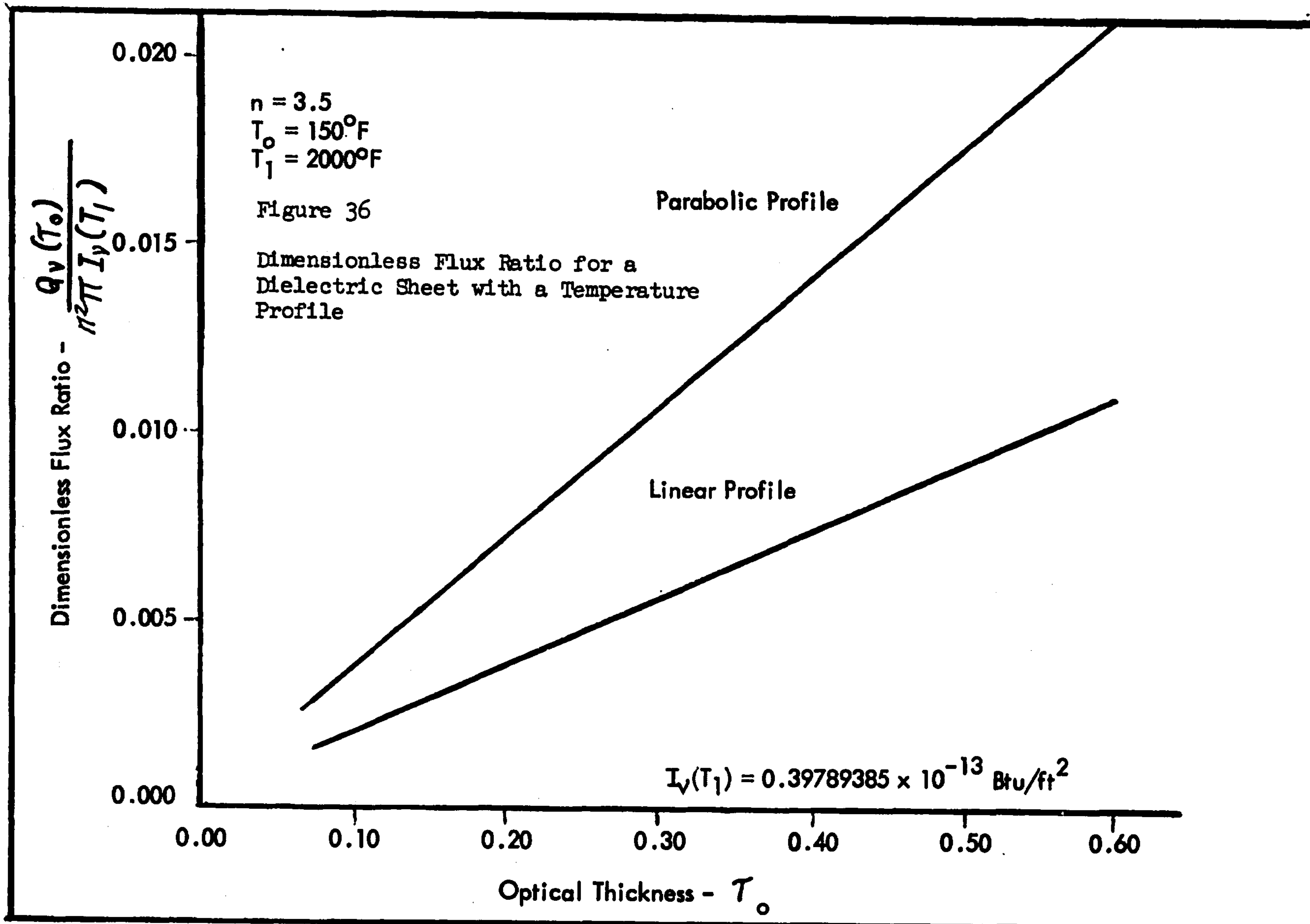


Figure 36

Dimensionless Flux Ratio for a Dielectric Sheet with a Temperature Profile

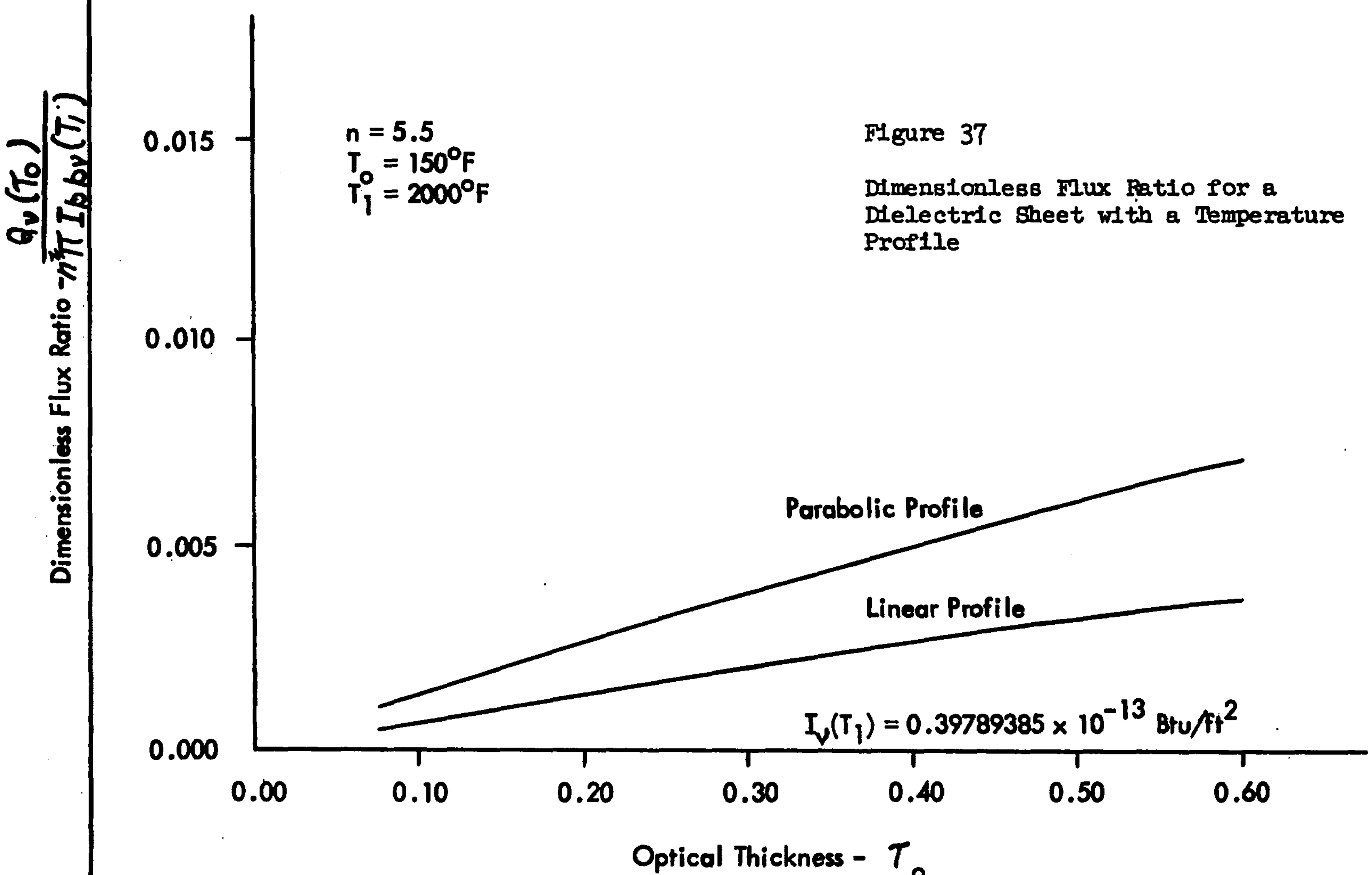
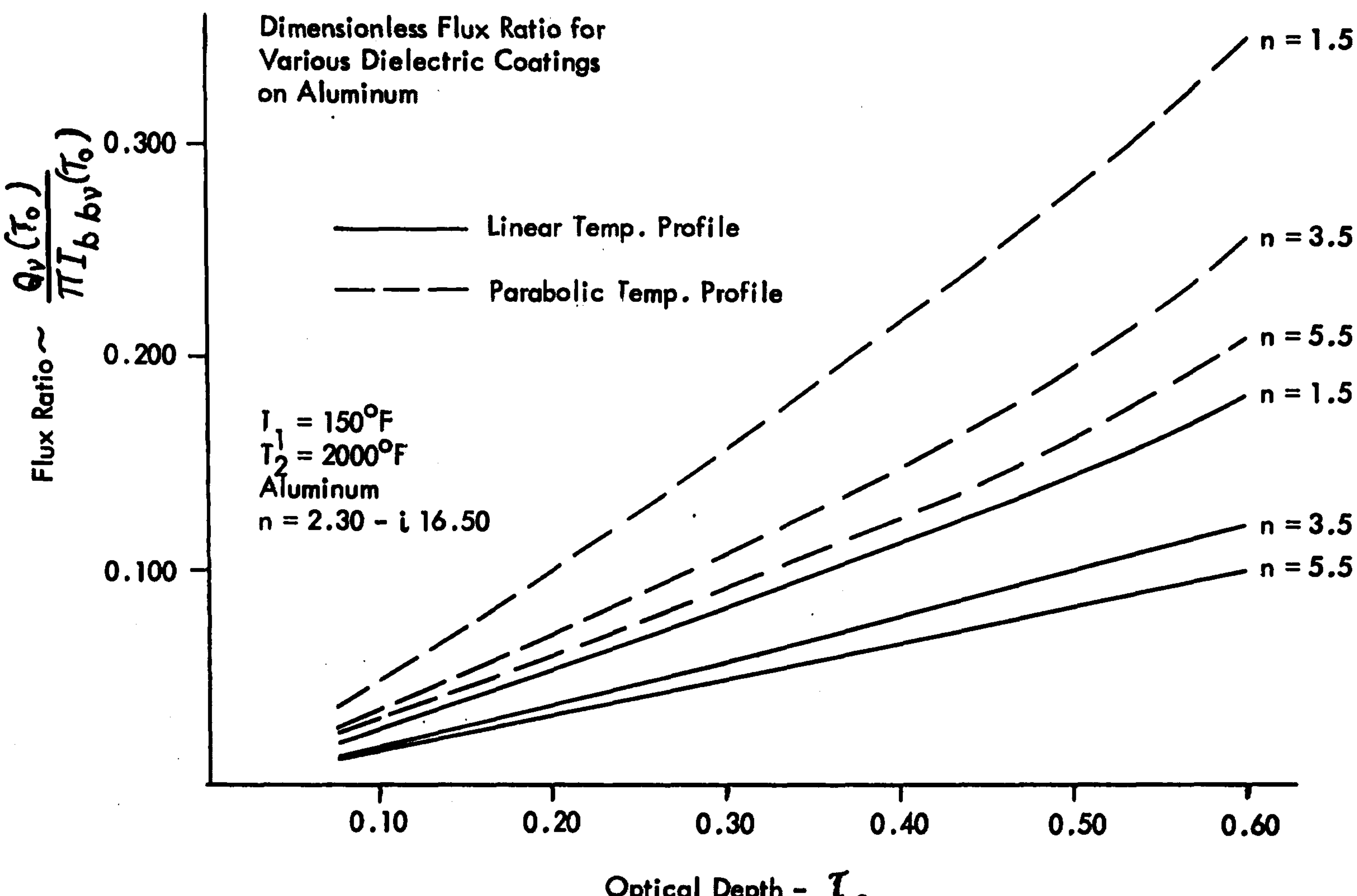


Figure 37

Dimensionless Flux Ratio for a Dielectric Sheet with a Temperature Profile

Figure 38

Dimensionless Flux Ratio for Various Dielectric Coatings on Aluminum



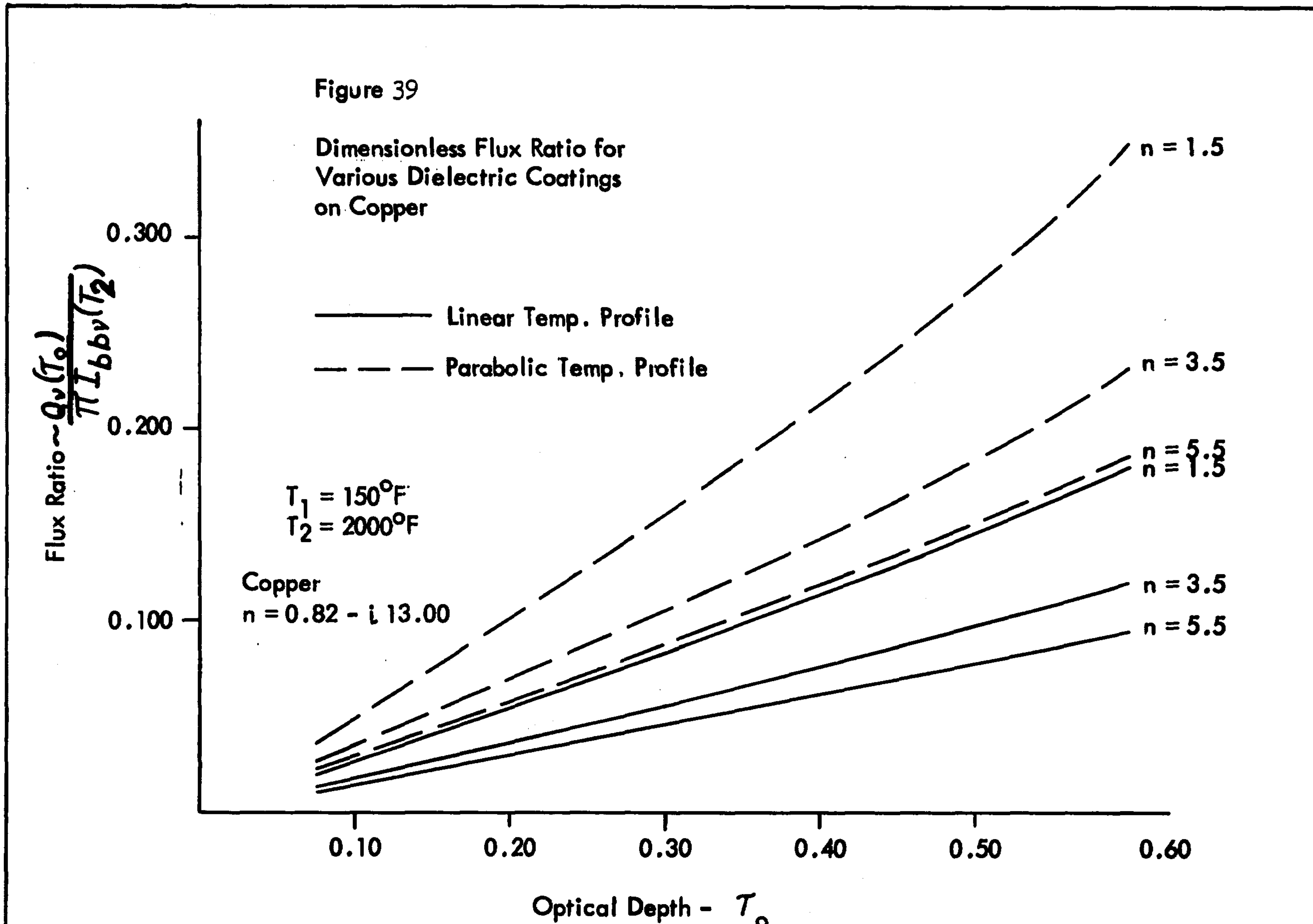


Figure 39

Dimensionless Flux Ratio for Various Dielectric Coatings on Copper

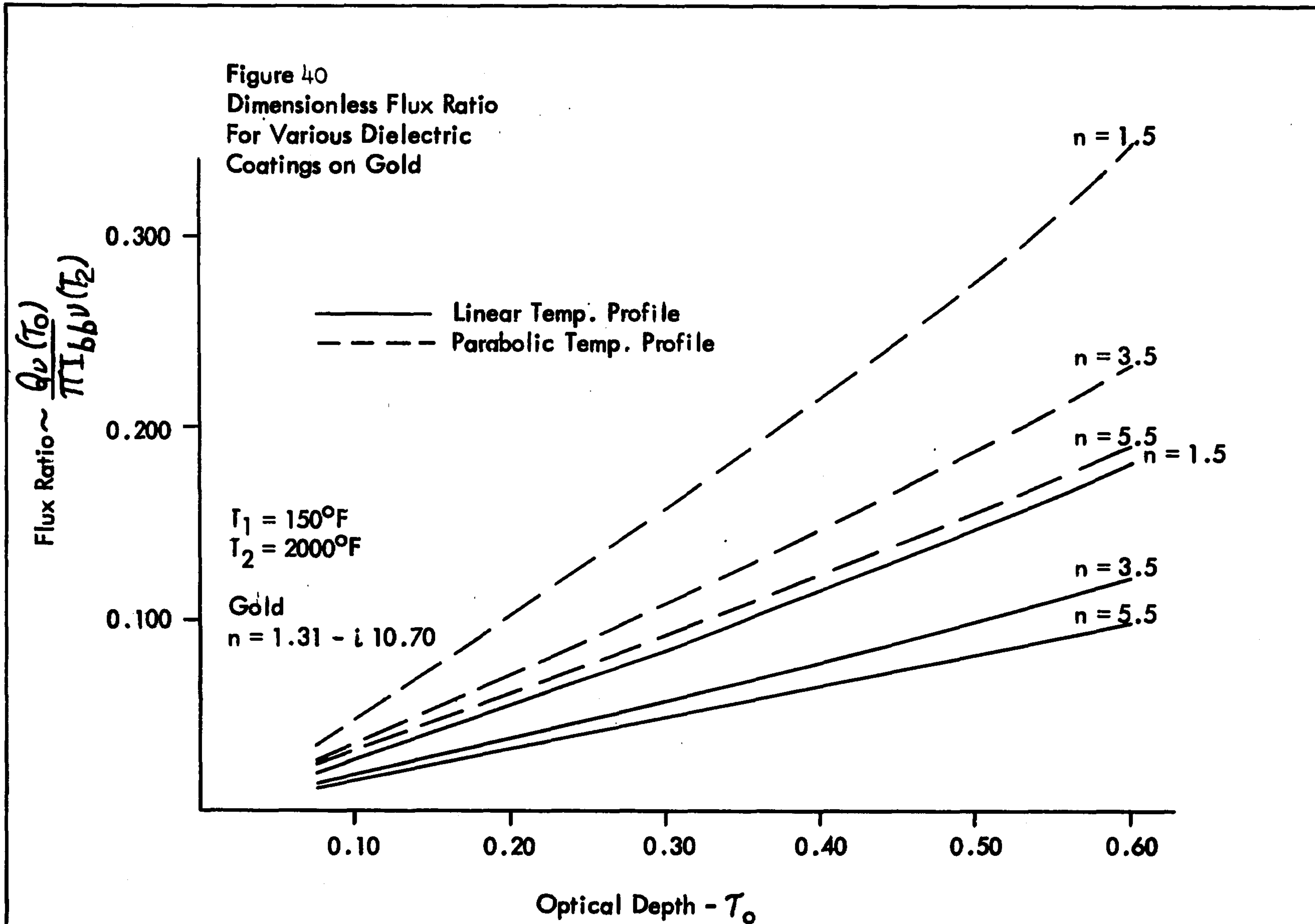


Figure 40
Dimensionless Flux Ratio
For Various Dielectric
Coatings on Gold

Figure 41

Dimensionless Flux Ratio for Various Dielectric Coatings on Nickel

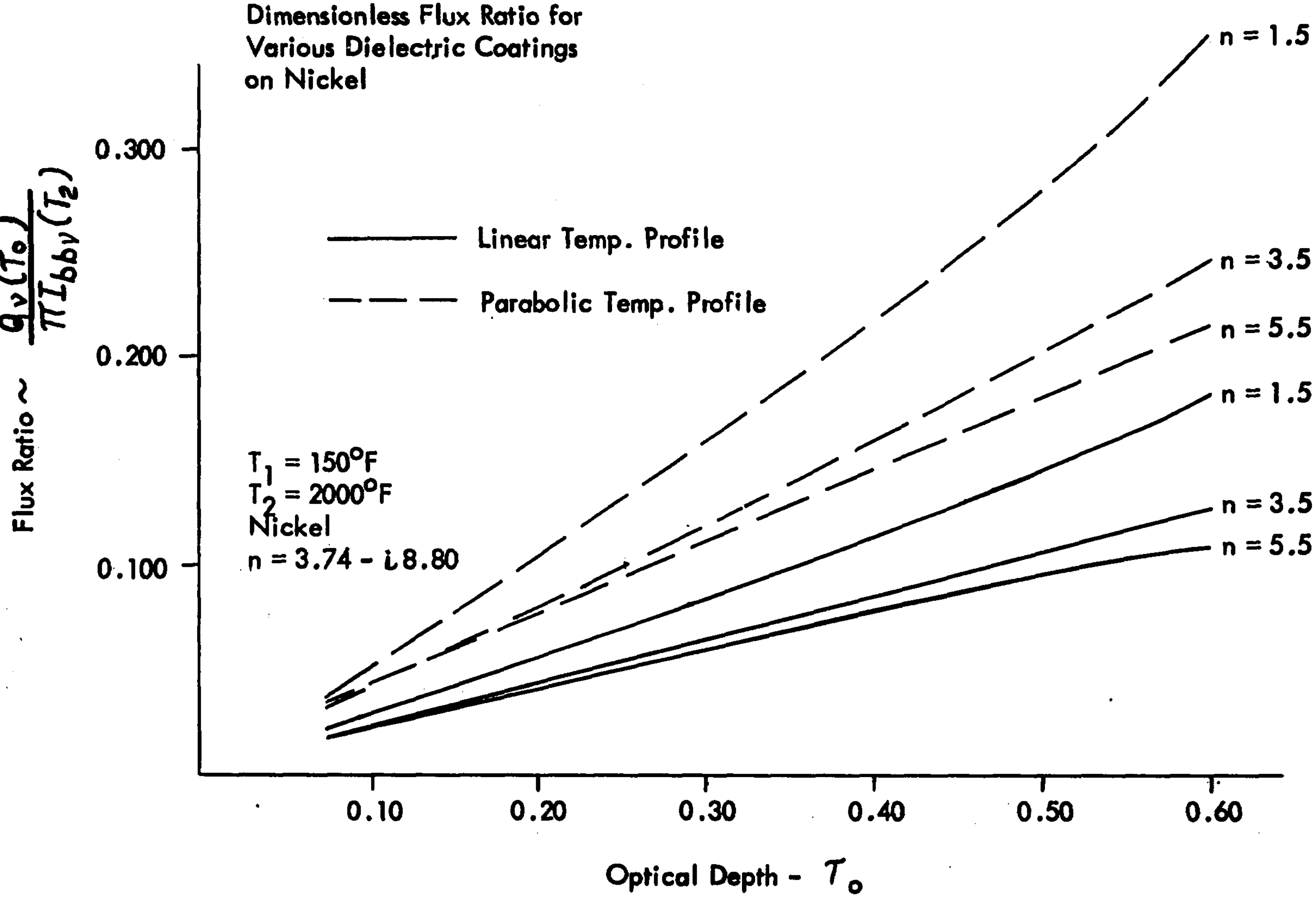
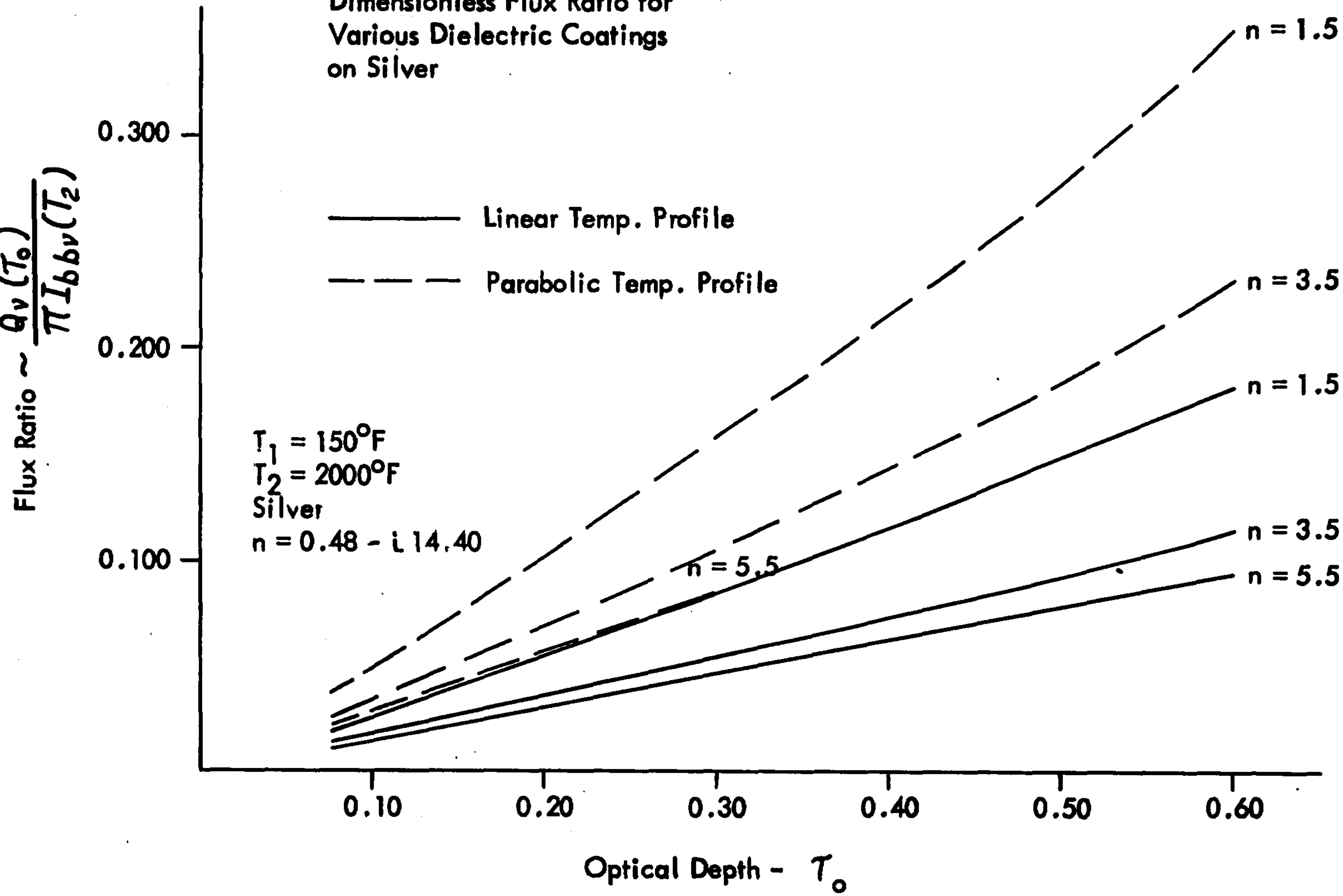


Figure 42

Dimensionless Flux Ratio for Various Dielectric Coatings on Silver



CHAPTER VI

COMBINED DIFFUSE AND SPECULAR REFLECTION

In all of the problems previously considered, all interfaces have been assumed to reflect specularly. This implies the interfaces are perfectly smooth, which is a physical impossibility. For the dielectric to air interface, this is a reasonable assumption and should give fairly accurate results, but it seems likely that the conductor-dielectric interface would not be quite so smooth. In fact there would probably be some intensional roughness in the conductor surface so that the dielectric would bond well to it.

Most surfaces normally encountered have some degree of roughness. These real surfaces reflect neither specularly nor diffusely, but are described by a reflection function which thus far has only been obtained experimentally. Authors such as Birkebak and Torrance have measured the directional reflection of various materials and reported their findings. These experimentally determined curves indicate that true surfaces can be approximated by the combination of both diffuse and specular reflection. Birkebak has done this and some data for the diffuse and specular coefficients are found in his work.

1. Isothermal Coating

Using Birkebak's notation, the boundary condition at $\tau = 0$ becomes

$$I_\nu^+(o,\mu) = 2\int_o^{+1} \rho_D I_\nu^-(o,\mu)\mu d\mu + \rho_s I_\nu^-(o,\mu) + n^2 \epsilon_\nu(\mu) I_{bb_\nu}(T)$$

ρ_D = diffuse reflectivity

ρ_s = specular reflectivity

The solution to the transport equation for the isothermal case is

$$I_\nu^+(\tau,\mu) = c_1 e^{-(\tau/\mu)} + n^2 I_{bb_\nu}(T)$$

$$I_\nu^-(\tau,\mu) = c_2 e^{(t-\tau_o)/\mu} + n^2 I_{bb_\nu}(T).$$

For $\mu \leq \mu_c$,

$$I_\nu^+(\tau_o,\mu) = I_\nu^-(\tau_o,\mu) \text{ Total Reflection.}$$

At $\tau = \tau_o$

$$I_\nu^-(\tau_o,\mu) = \rho_{2_\nu}(\theta',\theta) I_\nu^+(\tau_c,\mu)$$

$$c_2 + n^2 I_{bb_\nu}(T) = \rho_{2_\nu}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + n^2 I_{bb_\nu}(T)\right]$$

$$c_2 - \rho_{2_\nu}(\theta',\theta) c_1 e^{-(\tau_o/\mu)} = n^2 I_{bb_\nu}(T)\left[\rho_{2_\nu}(\theta',\theta) - 1\right].$$

From the boundary condition at $\tau = 0$,

$$c_1 + n^2 I_{bb_\nu}(T) = \rho_s\left[c_2 e^{-(\tau_o/\mu)} + n^2 I_{bb_\nu}(T)\right] + n^2 \epsilon_\nu(\mu) I_{bb_\nu}(T) +$$

$$+ 2\int_0^1 \rho_D \left[c_2 e^{-(\tau_o/\mu)} + n^2 I_{bb_\upsilon}(T)\right] \mu d\mu.$$

Kirchhoff's law for an opaque body is $\epsilon = 1 - \rho$. Therefore, by definition

$$\epsilon_\upsilon(\mu) = \left[1 - \rho_s - \rho_D\right],$$

where obviously ρ_s and ρ_D are spectral and directional quantities. Therefore the boundary conditions become

$$c_1 + n^2 I_{bb_\upsilon}(T) \left[1 - \rho_s - \rho_D - \epsilon_\upsilon(\mu)\right] = \rho_s c_2 e^{-(\tau_o/\mu)} + 2\int_0^1 \rho_D c_2 e^{-(\tau_o/\mu)} \mu d\mu$$

which can be reduced to

$$c_1 = \rho_s c_2 e^{-(\tau_o/\mu)} + 2\int_0^1 \rho_D c_2 e^{-(\tau_o/\mu)} \mu d\mu.$$

Multiplying both sides by

$$\rho_{2_\upsilon}(\theta',\theta) e^{-(\tau_o/\mu)}$$

yields

$$c_2 - n^2 I_{bb_\upsilon}(T) \left[\rho_{2_\upsilon}(\theta',\theta) - 1\right] = \rho_s \rho_{2_\upsilon}(\theta',\theta) c_2 e^{-(2\tau_o/\mu)} +$$

$$+ 2\rho_{2_\upsilon}(\theta',\theta) e^{-(\tau_o/\mu)} \int_0^1 \rho_D c_2 e^{-(\tau_o/\mu)} \mu d\mu,$$

or

$$c_2 \left[1 - \rho_s \rho_{2_\upsilon}(\theta',\theta) e^{-(2\tau_o/\mu)}\right] - 2\rho_{2_\upsilon}(\theta',\theta) e^{-(\tau_o/\mu)} \int_0^1 \rho_D c_2 e^{-(\tau_o/\mu)} \mu d\mu =$$

$$= n^2 I_{bb_\upsilon}(T) \left[\rho_{2_\upsilon}(\theta',\theta) - 1\right].$$

It is obvious that this integral equation would be quite difficult if not impossible to solve exactly. However, since in most engineering problems the net flux is of major concern, an approximate solution should be sufficient.

Before even an approximate solution can be attempted, one must know more about the specular and diffuse reflection coefficients ρ_s and ρ_D. As mentioned before, both ρ_s and ρ_D must be spectral directional quantities. The diffuse reflection coefficient ρ_D implies that the energy reflected in all angles is equal and this at first thought seems to contradict the statement that ρ_D is directional. Here the adjective directional means that the magnitude of ρ_D is dependent upon the direction of the incident radiation. In other words the incident energy is reflected equally in all directions, but the amount of energy which is reflected is dependent upon the incident angle.

The specular reflection coefficient ρ_s can be determined as a function of Fresnel's reflection coefficient. This has been done by H. Davies and is presented by Torrance.

$$\left[\frac{\rho_s(\theta)}{\rho_{s,\rho}(\theta)}\right]_{coh} = \exp - \left[(4\pi \cos\theta \frac{\sigma_o}{\lambda})^2 \right]$$

$\rho_{s,\rho}(\theta)$ is the specular reflection given by Fresnel

θ is the angle of incidence

λ is the wavelength of the radiation

σ_o is the surface roughness in microns, defined as the root mean square of the surface profile from the mean level

coh is the subscript denoting coherent radiation

The use of this relationship not only allows one to consider the directional dependency of the specular reflection coefficient of a rough surface, but it also considers the reflection in mediums other than air.

While there is a nice relationship between the specular component and Fresnel's law, there is no such relationship to determine the directional dependency of the diffuse component. However, if one assumes the same amount of total energy is reflected independent of the angle of incidence, the diffuse component can be found. This is not exactly true. However, from Birkebak's data for the surface roughnesses used, there is not too great of an error induced in this assumption. Therefore, in this paper $\rho_D = \rho_T - \rho_S$, where ρ_T is assumed constant and taken from the literature.

Again using Gauss four point quadrature, the integral equation can be written at four descrete values of μ to obtain

$$c_2(1)\left[1 - \rho_s(1)\rho_{2_\upsilon}(1)e^{-\tau_o/\mu(1)}\right] -$$

$$- 2\rho_{2_\upsilon}(1)e^{-\tau_o/\mu(1)}\left[\sum_{i=1}^{4} a_{(i)}\rho_D(i)c_2(i)e^{-\tau_o/\mu(i)}\ \mu(i)\right] =$$

$$= n^2 I_{bb_\upsilon}(T)\left[\rho_{2_\upsilon}(1) - 1\right]$$

$$c_2(2)\left[1 - \rho_s(2)\rho_{2_\upsilon}(2)e^{-\tau_o/\mu(2)}\right] -$$

$$- 2\rho_{2_\upsilon}(2)e^{-\tau_o/\mu(2)}\left[\sum_{i=1}^{4} a_{(i)}\rho_D(i)c_2(i)e^{-\tau_o/\mu(i)}\ \mu(i)\right] =$$

$$= n^2 I_{bb_\upsilon}(T)\left[\rho_{2_\upsilon}(2) - 1\right]$$

$$c_2(3) \text{ etc.}$$

where $a_{(i)}$ and $\mu_{(i)}$ are the weight factors and the points of the quadrature. Also, $\rho_{2_\upsilon}(i)$ corresponds to $\rho_{2_\upsilon}(\theta',\theta)$ at $\cos\theta = \mu(i)$.

These equations can be solved simultaneous to obtain $c_2(i)$ which with the following will yield $c_1(i)$.

$$c_1(i) = \frac{1}{\rho_{2_\upsilon}(i)e^{-\tau_o/\mu(i)}} \left\{ n^2 I_{bb_\upsilon}(T) \left[1 - \rho_{2_\upsilon}(i)\right] + c_2 \right\}$$

The net intensity at $\tau = \tau_o$ leaving is

$$I_{net_\upsilon}(\tau_o,\mu) = \frac{I_\upsilon^+(\tau_o,\mu) - I_\upsilon^-(\tau_o,\mu)}{n^2}$$

or

$$\underset{\text{leaving}}{I_{net_\upsilon}(\tau_o,\mu)} = \frac{c_1 e^{-(\tau_o/\mu)} - c_2}{n^2}$$

as before.

The directional emissivity is given by

$$\epsilon_\upsilon(\mu) = \frac{I_{net_\upsilon}(\tau_o,\mu)}{n^2 I_{bb_\upsilon}(T)}$$

and the hemispherical emissivity is obtained by integration

$$\epsilon_{H_\upsilon} = 2\int_o^1 \epsilon_\upsilon(\mu)\mu d\mu.$$

Values of hemispherical emissivity with these boundary conditions are found in Figures 43 and 44.

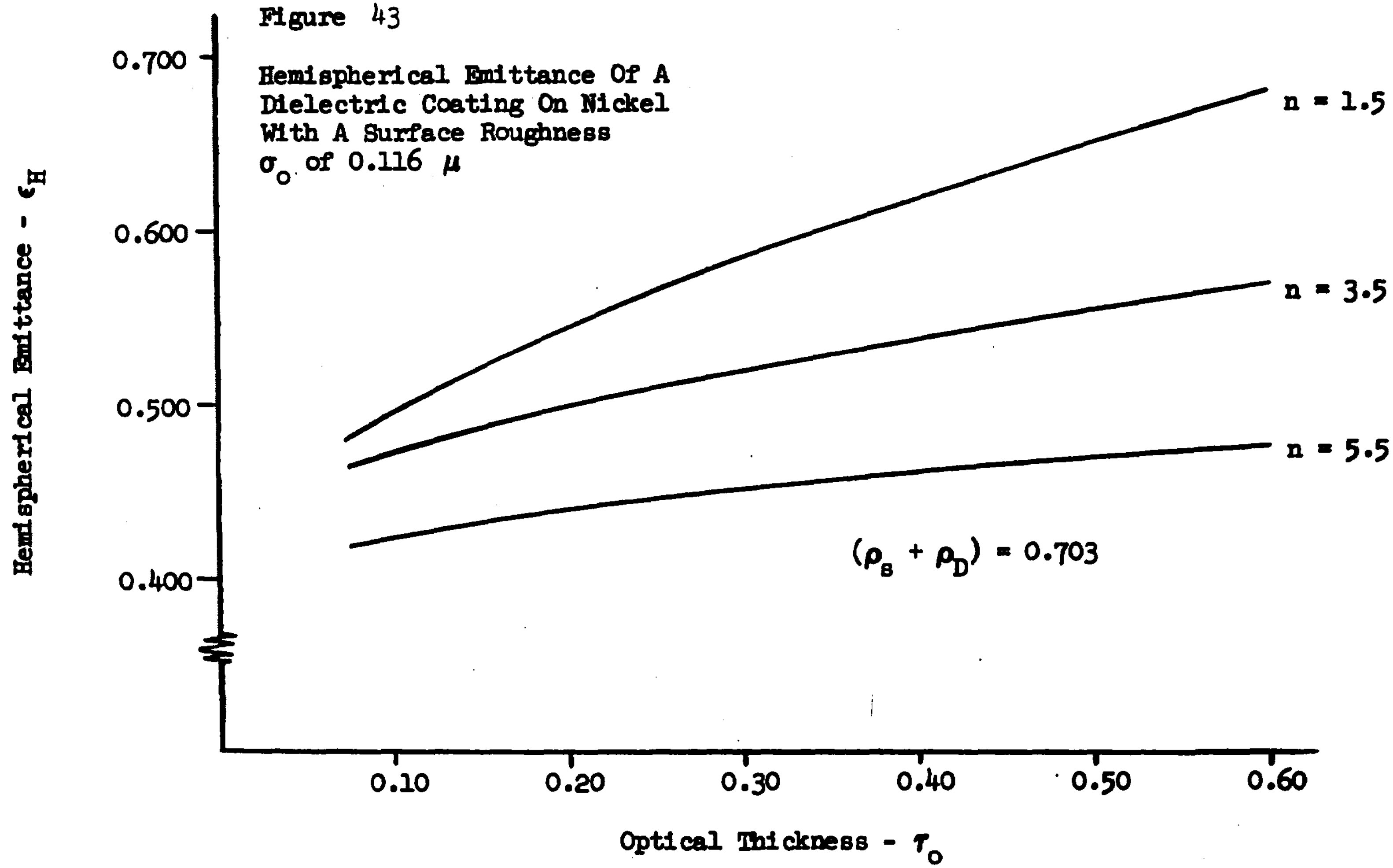
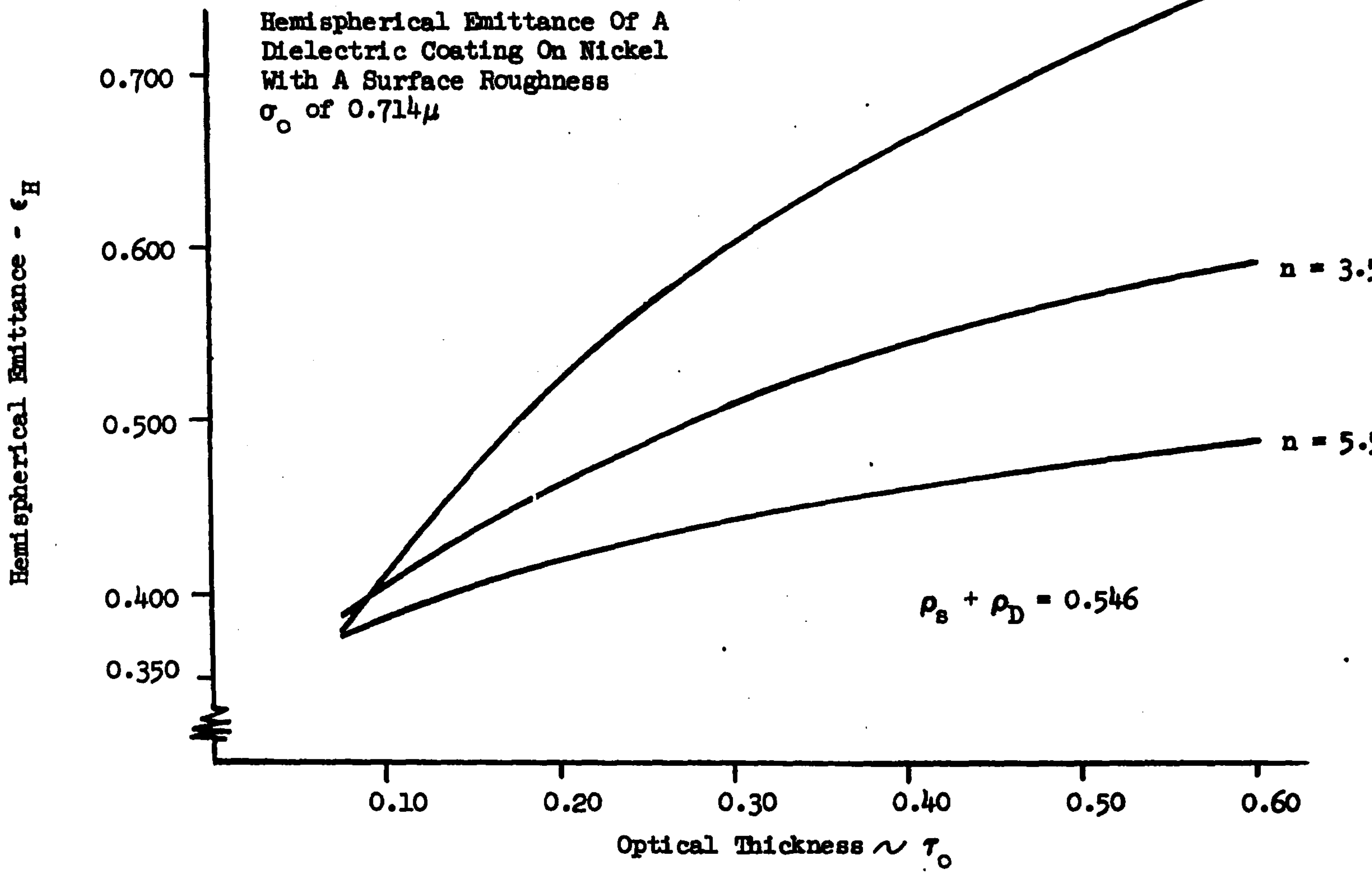


Figure 43

Hemispherical Emittance Of A Dielectric Coating On Nickel With A Surface Roughness σ_o of 0.116 μ

Figure 46

Hemispherical Emittance Of A Dielectric Coating On Nickel With A Surface Roughness σ_o of 0.714μ



The data used from Birkebak is

σ_m	σ_o	ρ_T
0.14	0.116	0.703
0.86	0.714	0.546

This data is taken at 100^{o}F and is for total rather than spectral properties as would be required for this work. The only comparison that could be made is that at small optical depths the combined diffuse and specular solution should be greater than the solution considering only specular reflection. This is observed to be true in both cases. A correction is made in the computer program to account for the change in ρ_T with the change in coatings.

2. Nonisothermal Coating

The solutions to the governing equations for the nonisothermal coatings are

$$I_\upsilon^+(\tau,\mu) = c_1 e^{-(\tau/\mu)} + \frac{n^2}{\mu}\int_0^\tau e^{(t-\tau)/\mu}\, I_{bb_\upsilon}(t)dt$$

$$I_\upsilon^-(\tau,\mu) = c_2 e^{(\tau-\tau_o)/\mu} + \frac{n^2}{\mu}\int_\tau^{\tau_o} e^{-(t-\tau)/\mu}\, I_{bb_\upsilon}(t)dt.$$

The boundary conditions remain unchanged.

$$I_\upsilon^-(\tau_o,\mu) = \rho_{2_\upsilon}(\theta',\theta) I_\upsilon^+(\tau_o,\mu)$$

$$c_2 = \rho_{2_\nu}(\theta',\theta)\left[c_1 e^{-(\tau_o/\mu)} + I2\right],$$

where

$$I2 = \frac{n^2}{\mu}\int_o^{\tau_o} e^{(t-\tau_o)/\mu}\, I_{bb_\nu}(t)dt$$

as before.

$$I_\nu^+(o,\mu) = 2\int_o^1 \rho_D I_\nu^-(o,\mu)\mu d\mu + \rho_s I_\nu^-(o,\mu) + n^2\epsilon_\nu(\mu)I_{bb_\nu}(T_1)$$

or

$$c_1 = 2\int_o^1 \rho_D\left[c_2 e^{-(\tau_o/\mu)} + L\right]\mu d\mu + \rho_s\left[c_2 e^{-(\tau_o/\mu)} + L\right] + n^2\epsilon_\nu(\mu)I_{bb_\nu}(T_1).$$

Multiplying by $\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}$ and combining the two boundary conditions yields

$$c_1\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)} = 2\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}\int_o^1 \rho_D\left[c_2 e^{-(\tau_o/\mu)} + L\right]\mu d\mu +$$

$$+ n^2\epsilon_\nu(\mu)I_{bb_\nu}(T_1)\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)} + \rho_s\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}\left[c_2 e^{-(\tau_o/\mu)} + L\right]$$

or

$$c_2\left[1 - \rho_s\rho_{2_\nu}(\theta',\theta)e^{-(2\tau_o/\mu)}\right] - 2\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}\int_o^1\rho_D\left[c_2 e^{-(\tau_o/\mu)} + L\right]\mu d\mu =$$

$$= n^2\epsilon_\nu(\mu)I_{bb_\nu}(T_1)\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)} + \rho_{2_\nu}(\theta',\theta)I2 + \rho_s\rho_{2_\nu}(\theta',\theta)e^{-(\tau_o/\mu)}L.$$

The reflectances ρ_s and ρ_D are the same as in the previous section.

However, $\epsilon_\upsilon(\mu)$ is determined by

$$\epsilon_\upsilon(\mu) = 1 - (\rho_s + \rho_D).$$

The equation can be written as before for the four descrete points given by the quadrature, but this time L and L2 are also dependent on μ. The general form of the four equations is

$$c_2(i)\left[1 - \rho_s(i)\rho_{2_\upsilon}(i)e^{-2\tau_o/\mu(i)}\right] -$$

$$- 2\rho_{2_\upsilon}(i)e^{-\tau_o/\mu(i)} \sum_{j=1}^{4} \rho_D(j)\left[c_2(j)e^{-\tau_o/\mu(j)} + L(j)\right]\mu(j)a(j) =$$

$$= n^2\epsilon_\upsilon\left[\mu(i)\right] I_{bb_\upsilon}(T_1)\rho_{2_\upsilon}(i)e^{-\tau_o/\mu(i)} + \rho_{2_\upsilon}(i)L2(i) + \rho_{2_\upsilon}(i)e^{-\tau_o/\mu(i)} L(i)\rho_s(i).$$

The four equations can be solved simultaneously to obtain $c_2(i)$ which with the following yields $c_1(i)$

$$c_1(i) = \left[\frac{c_2(i)}{\rho_{2_\upsilon}(i)} - L2(i)\right] e^{\tau_o/\mu(i)} .$$

The net intensity at $\tau = \tau_o$ is

$$I_{net_\upsilon}(\tau_o,\mu) = I_\upsilon^+(\tau_c,\mu) - I_\upsilon^-(\tau_o,\mu)$$

or

$$I_{net_\upsilon}(\tau_o,\mu) = c_1 e^{-(\tau_o/\mu)} + L2 - c_2.$$

The net flux leaving the surface is given by

$$Q_\upsilon(\tau_o) = \int_0^1 I_{net_\upsilon}(\tau_o,\mu)\mu d\mu = \sum_{i=1}^{4} a_{(i)}\left[c_{1_{(i)}} e^{-\tau_o/\mu(i)} + L2(i) - c_2(i)\right]\mu(i)$$

and the apparent hemispherical emissivity is given by

$$\epsilon^*_{H_\nu} = \frac{Q_\nu(\tau_o)}{I_{bb_\nu}(T_{Hot})} \ .$$

The temperature profiles used are the same as before and are shown in Figure 34. (See page 77.) The apparent hemispherical emissivity is plotted in Figures 45 and 46.

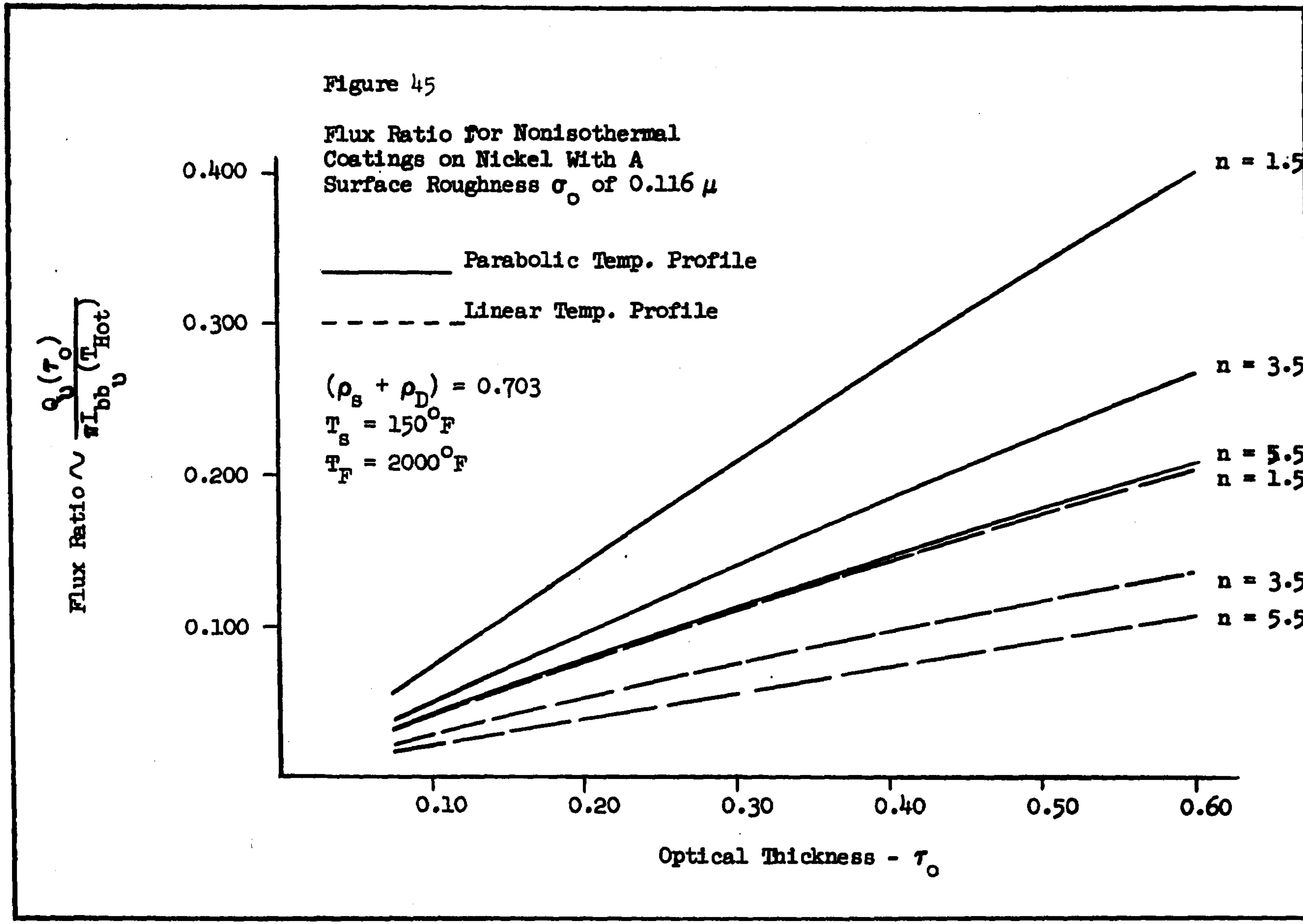
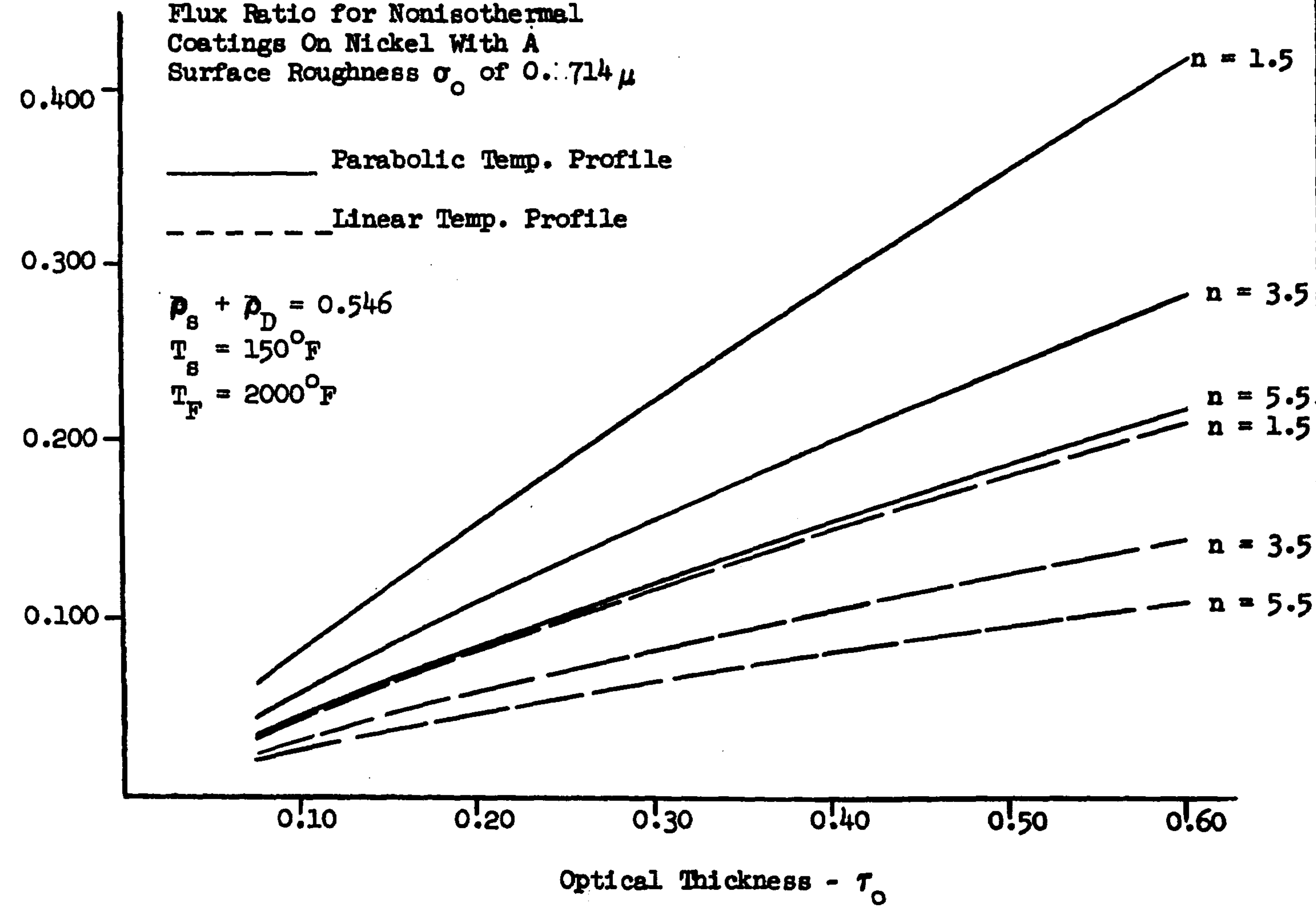


Figure 45

Flux Ratio for Nonisothermal Coatings on Nickel With A Surface Roughness σ_o of 0.116 μ

Figure 46

Flux Ratio for Nonisothermal Coatings On Nickel With A Surface Roughness σ_o of 0.714 μ



CHAPTER VII

SCATTERING IN AN ISOTHERMAL DIELECTRIC

In general, radiation studies in scattering media have considered only two types of media. The scattering medium most commonly studied considers a uniform dispersion of particles suspended in a non-absorbing substance with a refractive index of unity. The same equation of transport will hold in the second most commonly considered medium, which is an absorbing and emitting medium which scatters electromagnetic energy through local inhomonogenities in the medium. In this second medium, there is no way to differentiate the scattering particles from the suspending medium.

The problem this work is concerned with involves an absorbing and emitting dielectric which has scattering particles suspended within it. These particles are assumed to be spherical air bubbles. Neither of the two most common cases correspond to this problem. However, it will be advantageous to revert to the equation of transport used in the other cases to aid in developing an equation of transport for this particular problem.

Love (16) has written the equation of transport in the following manner:

For a medium consisting of a uniform dispersion of particles suspended in a non-absorbing substance with a refractive index of unity, the equation of transport along a distance s is

$$\frac{dI_\nu(s,\theta,\varphi)}{ds} = -\rho\beta I_\nu(s,\theta,\varphi) +$$

$$+\frac{\rho\sigma}{4\pi}\int_0^{\pi}\int_0^{2\pi} s_\nu(\theta,\varphi,\theta',\varphi')I_\nu(s,\theta',\varphi')\sin\theta' d\varphi' d\theta' + \rho\kappa I_{bb_\nu}(s).$$

The left side of the equation represents the rate of decrease of intensity along the pencil of ray; the first term on the right represents the extinction of the ray; the second term on the right represents the energy scattered into the ray in the direction (θ,φ) from the radiation in all other directions, and the last term represents the emission of the particles.

ρ is the mass of scattering matter per unit volume of mixture

κ is the monochromatic emission coefficient

σ is the monochromatic scattering coefficient

β is the monochromatic extinction coefficient
The decrease in intensity is the extinction of the ray and is the result of scattering and absorption. $\beta = (\sigma + \kappa)$

$s(\theta,\varphi,\theta',\varphi')$ is the probability function that a photon traveling in the direction (θ',φ') will be scattered into the direction (θ,φ).

In a similar sense σ is the probability that a photon will be scattered and κ is the probability that a photon will be trapped. For a more detailed discussion of scattering in general and a more thorough development of this equation, refer to Love (16).

For a uniform medium consisting of a uniform distribution of scattering particles in an absorbing medium, the equation must be modified to include the absorption and emission of the suspending medium. As mentioned earlier in this work the particles will be assumed to be air bubbles, and absorption and emission of the bubbles themselves is so small that they will be neglected. Figure 47 is taken from the work of Chromey (5) and gives the monochromatic scattering coefficient σ as a function of the particles size parameter, where $\alpha = \frac{2\pi r}{\lambda}$ and r is the radius of the particle (air bubble). The exact way in which the absorption of the suspending medium enters into the transport equation is not known. However, for a first assumption one would assume the relationship to be linear. That is

$$\rho\beta = (\rho_s\beta_s + c\rho_m k_m).$$

Here the subscript (s) denotes the scattering particle and (m) denotes the suspending medium.

To the author's knowledge there is no information available at the present time to help determine the proportionality constant c. In this work c will be assumed equal to unity. The equation of transport now becomes

$$\frac{dI_\upsilon(s,\theta,\varphi)}{ds} = -(\rho_s\sigma_s + \rho_m k_m)I_\upsilon(s,\theta,\varphi) +$$

$$+ \frac{\rho_s\sigma_s}{4\pi}\int_0^\pi\int_0^{2\pi} s_\upsilon(\theta,\varphi,\theta',\varphi')I_\upsilon(s,\theta',\varphi')\sin\theta' d\varphi' d\theta' + \rho_m k_m n^2 I_{bb_\upsilon}(s).$$

This follows from the relationships

$$\rho_s\beta_s = \rho_s(\sigma_s + \cancelto{0}{k_s}) = \rho_s\sigma_s$$

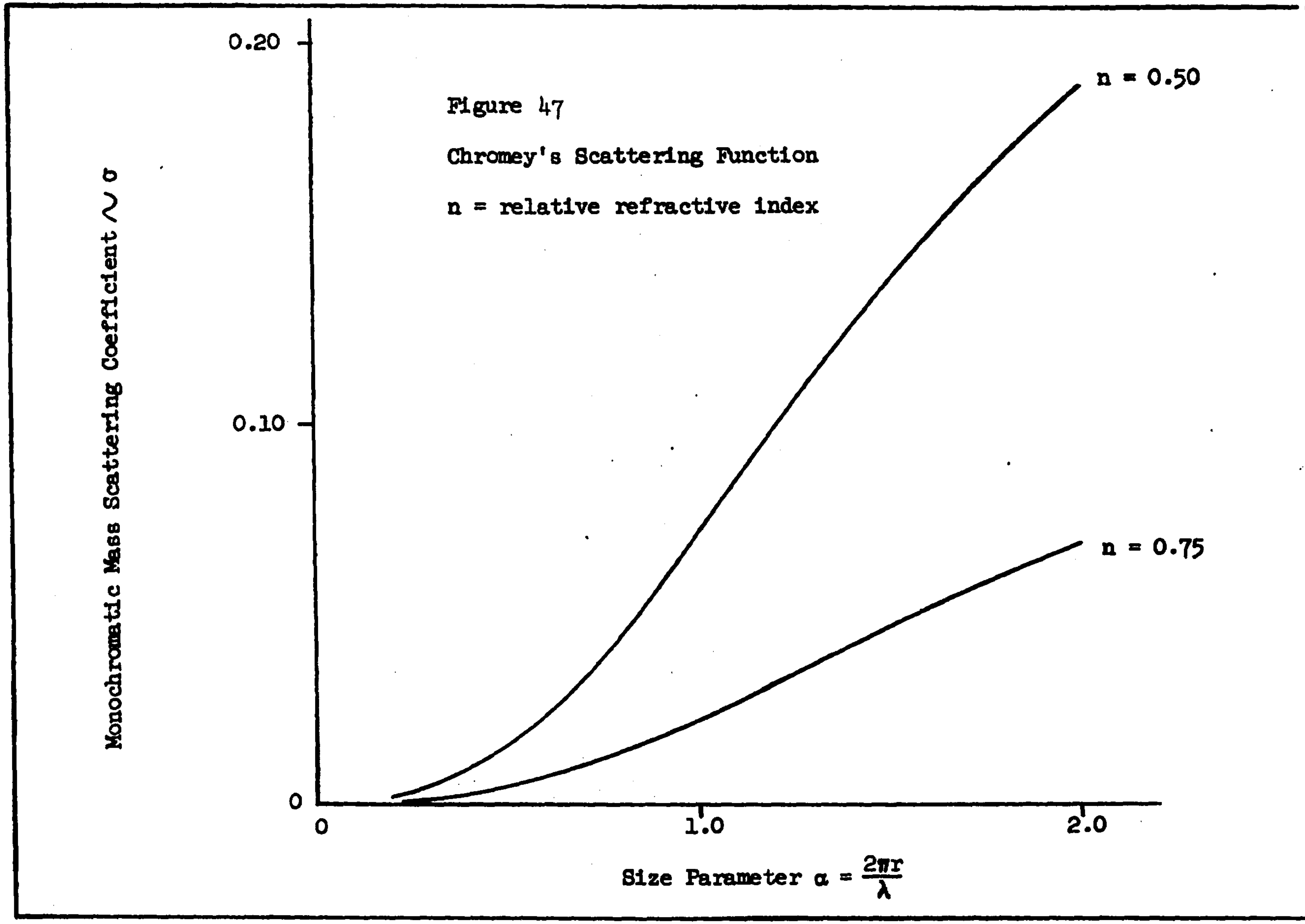


Figure 47

Chromey's Scattering Function

n = relative refractive index

$$\rho_m \beta_m = \rho_m(\cancelto{0}{\sigma_m} + k_m) = \rho_m k_m$$

$$(\rho_s \sigma_s + \rho_m k_m) = \rho\beta,$$

since the problem under consideration is the plane parallel case with axially symmetric boundary conditions with the use of the axially symmetric scattering function and the coordinate system shown in Figure 48. With the definition $\tau = \int_0^x \rho\beta dx$, the equation of transport can be transformed to

$$\mu \frac{dI_\nu(\tau,\mu)}{d\tau} - I_\nu(\tau,\mu) + \frac{\rho_s \sigma_s}{4\pi\rho\beta} \int_{-1}^{+1} I_\nu(\tau,\mu') \int_0^{2\pi} s(\theta) d\varphi' d\mu' + \frac{\rho_m k_m n^2 I_{bb_\nu}(\tau)}{\rho\beta}$$

where θ is the angle between the incident and scattered ray, and $s(\theta)$ is the axially symmetric scattering function of the medium.

Letting

$$s(\mu,\mu') = \frac{1}{2\pi} \int_0^{2\pi} s(\theta) d\varphi',$$

where

$$\cos \theta = \mu\mu' + (1 - \mu^2)^{\frac{1}{2}}(1 - \mu'^2)^{\frac{1}{2}} \cos(\varphi - \varphi')$$

the equation of transport can be written as

$$\mu \frac{dI_\nu(\tau,\mu)}{d\tau} = - I_\nu(\tau,\mu) + \frac{\rho_s \sigma_s}{2\rho\beta} \int_{-1}^{+1} I(\tau,\mu') s(\mu,\mu') d\mu' + \frac{\rho_m k_m n^2 I_{bb_\nu}(\tau)}{\rho\beta}.$$

The exact solution to this integro-differential equation in most cases would be quite difficult. However, Love (16) suggests an approximate

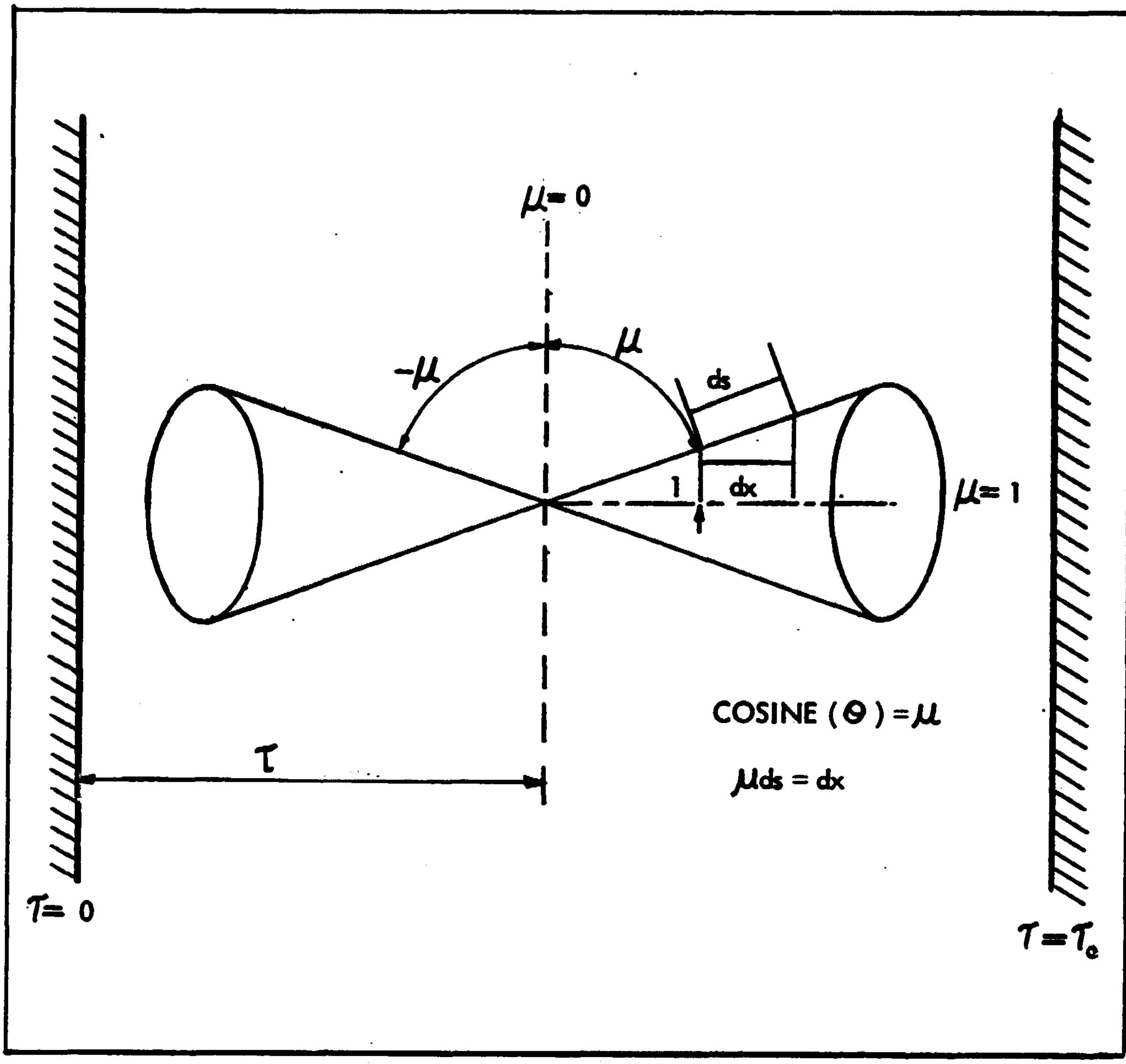


Figure 48

COORDINATE SCHEME FOR AXIALLY SYMMETRIC CASE solution using the Gaussian quadrature which should be quite suitable for engineering problems. His method of solution also has an advantage in that experimental data of the scattering function at descrete points can be readily applied.

The integro-differential equation can be written as follows.

$$\mu_i \frac{dI_\upsilon(\tau,\mu_i)}{d\tau} = - I_\upsilon(\tau,\mu_i) + \frac{\rho_s\sigma_s}{2\rho\beta} \sum_{j=1}^{n} a_j s(\mu_i,\mu_j) I_\upsilon(\tau,\mu_j) + \frac{\rho_m k_m n^2}{\rho\beta} I_{bb_\upsilon}(\tau)$$

where μ_i and μ_j are the descrete ordinates of the quadrature and a_j is the weight function.

The integro-differential equation is now represented by n first order simultaneous differential equations. As in the non-scattering medium, discontinuities are anticipated at $\mu = 0$ at the boundaries so that the equations will be written in the positive direction for the interval $0 < \mu \leq 1$ and in the negative direction for the integral $- 1 \leq \mu < 0$ as follows.

$$+ \mu_i \frac{dI(\tau,+\mu_i)}{d\tau} = - I(\tau,+\mu_i) + \frac{\rho_s\sigma_s}{2\rho\beta} \sum_{j=1}^{n} a_j s(+\mu_i,\mu_j) I_\upsilon(\tau,\mu_j) +$$

$$+ \frac{\rho_s\sigma_s}{2\rho\beta} \sum_{j=1}^{n} a_j s(+\mu_i,-\mu_j) I(\tau,-\mu_j) + \frac{\rho_m k_m n^2}{\rho\beta} I_{bb_\upsilon}(\tau),$$

$$- \mu_i \frac{dI(\tau,-\mu_i)}{d\tau} = - I(\tau,-\mu_i) + \frac{\rho_s\sigma_s}{2\rho\beta} \sum_{j=1}^{n} a_j s(-\mu_i,\mu_j) I_\upsilon(\tau,\mu_j) +$$

$$+ \frac{\rho_s\sigma_s}{2\rho\beta} \sum_{j=1}^{n} a_j s(-\mu_i,-\mu_j) I(\tau,-\mu_j) + \frac{\rho_m k_m n^2}{\rho\beta} I_{bb_\upsilon}(\tau).$$

Here it should be noted that a_j, μ_j and μ_i are not the same as before, but now they refer to the double Gaussian quadrature (Appendix II).

Love suggests a solution of the following form for the homogeneous solutions.

$$I_{\upsilon}(\tau,+\mu_i) = x_i e^{\gamma\tau}$$

$$I_{\upsilon}(\tau,-\mu_i) = x_{(i+n)} e^{\gamma\tau}$$

He goes on to write these simultaneous equations in matrix form and shows that solution takes the form

$$I_{\upsilon}(\tau,+\mu_i) = \sum_{\alpha=1}^{2n} c_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau}$$

$$I_{\upsilon}(\tau,-\mu_i) = \sum_{\alpha=1}^{2n} c_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau}$$

where the 2n values of γ are the eigenvalues of the matrix and the 2n values of x_i, $x_{(i+n)}$ are the eigenvectors. The values of c_α are determined by the boundary conditions. The eigenvectors and eigenvalues used in this work will be obtained from a program developed by Han M. Hsia which will appear in his dissertation at The University of Oklahoma before June 1, 1965.

To obtain the particular solution associated with a constant temperature medium, the solution is assumed to be an arbitrary constant c. Substituting into either of the two governing differential equations, one obtains

$$0 = -\,c + \frac{\rho_s \sigma_s}{2\rho\beta}\, c \sum_{j=1}^{n} \left[a_j s(+\mu_i,+\mu_j) + a_j s(+\mu_i,-\mu_j)\right] + \frac{\rho_m k_m}{\rho\beta}\, n^2 I_{bb_\upsilon}(T).$$

For conservative scattering

$$\frac{1}{4\pi}\int_{-1}^{+1}\int_{0}^{2\pi} s(\theta)d\varphi' d\mu' = 1,$$

or

$$\sum_{j=1}^{n} a_j \left[s(+\mu_i,+\mu_j) + s(+\mu_i,-\mu_j)\right] = 2,$$

or

$$c\left(1 - \frac{\rho_s\sigma_s}{\rho\beta}\right) = \frac{\rho_m k_m}{\rho\beta} n^2 I_{bb_\upsilon}(T)$$

$$c = \frac{\rho_m k_m}{\rho\beta - \rho_s\sigma_s} n^2 I_{bb_\upsilon}(T)$$

$$c = n^2 I_{bb_\upsilon}(T).$$

The complete solution may be written as

$$I_\upsilon(\tau,+\mu_i) = \sum_{\alpha=1}^{2n} c_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau} + n^2 I_{bb_\upsilon}(T)$$

$$I_\upsilon(\tau,-\mu_i) = \sum_{\alpha=1}^{2n} c_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau} + n^2 I_{bb_\upsilon}(T).$$

The boundary conditions must now be investigated to determine the constants c_α. They are, at $\tau = 0$,

$$I_\upsilon(0,+\mu_i) = n^2\epsilon_\upsilon(\mu) I_{bb_\upsilon}(T) + \rho_\upsilon(\mu) I_\upsilon(0,-\mu_i)$$

$$\sum_{\alpha=1}^{2n} c_\alpha x_{i,\alpha} + n^2 I_{bb_\upsilon}(T) = n^2\epsilon_\upsilon(\mu) I_{bb_\upsilon}(T) + \rho_\upsilon(\mu)\left[\sum_{\alpha=1}^{2n} c'_\alpha x_{(i+n),\alpha} + n^2 I_{bb_\upsilon}(T)\right]$$

$$\sum_{\alpha=1}^{2n} c'_\alpha x_{i,\alpha} = \rho_\upsilon(\mu) \sum_{\alpha=1}^{2n} c'_\alpha x_{(i+n),\alpha}.$$

At $\tau = \tau_o$ and $\mu > \mu_c$,

$$I_\upsilon(\tau_o,-\mu_i) = \rho_2(\theta',\theta) I_\upsilon(\tau_o,+\mu_i)$$

$$\sum_{\alpha=1}^{2n} c_\alpha x_{(i+n),\alpha} e^{\gamma_\alpha \tau_o} + n^2 I_{bb_\upsilon}(T) = \rho_{2_\upsilon}(\theta',\theta) \left[\sum_{\alpha=1}^{2n} c_\alpha x_{i,\alpha} e^{\gamma_\alpha \tau_o} + n^2 I_{bb_\upsilon}(T)\right].$$

The two sets of equations become

$$\sum_{\alpha=1}^{2n} c_\alpha e^{\gamma_\alpha \tau_o} \left[x_{(i+n),\alpha} - \rho_{2_\upsilon}(\theta',\theta) x_{i,\alpha}\right] = n^2 I_{bb_\upsilon}(T) \left[\rho_{2_\upsilon}(\theta',\theta) - 1\right],$$

and

$$\sum_{\alpha=1}^{2n} c'_\alpha \left[x_{i,\alpha} - \rho_\upsilon(\mu) x_{(i+n),\alpha}\right] = 0,$$

since

$$c'_\alpha = \frac{c_\alpha}{n^2 I_{bb_\upsilon}(T)}.$$

The equations may be solved simultaneously to obtain the c_α's.

The directional emissivity can be given by

$$\epsilon_\upsilon(\mu) = \frac{I_{net_\upsilon}(\tau_o,\mu)}{n^2 I_{bb_\upsilon}(T)} = \frac{I_\upsilon(\tau_o,\mu) - I_\upsilon(\tau_o,-\mu)}{n^2 I_{bb_\upsilon}(T)}$$

$$\epsilon_\upsilon(\mu) = \sum_{\alpha=1}^{2n} c'_\alpha \left[x_{i,\alpha} - x_{(i+n),\alpha}\right] e^{\gamma_\alpha \tau_o}.$$

The hemispherical emissivity is given by

$$\epsilon_{H_\upsilon} = 2\int_0^1 \mu\epsilon_\upsilon(\mu)d\mu = 2\sum_{i=1}^{n} a_j\epsilon_\upsilon(\mu_i)\mu_i .$$

Figure 47 (see page 110) is a plot of the scattering coefficient versus the particle size parameter taken from Chromey (5). The hemispherical emissivity over refractive index squared for different particle sizes, optical depths, and dielectric suspensions are given in Figure 49.

The values of n in Figures 49 and 47 do not agree. This is due to the lack of available data. The computations were made based upon tabulated values of distribution coefficient and these did not agree with the values of n chosen by Chromey for his scattering coefficient. The values of σ required were found by interpolation from Figure 47. The refractive index n on Figure 49 is for the dielectric medium, but the relative refractive index for scattering medium to dielectric is 0.93 for all cases.

Additional graphs for hemispherical properties of the various problems follow this chapter.

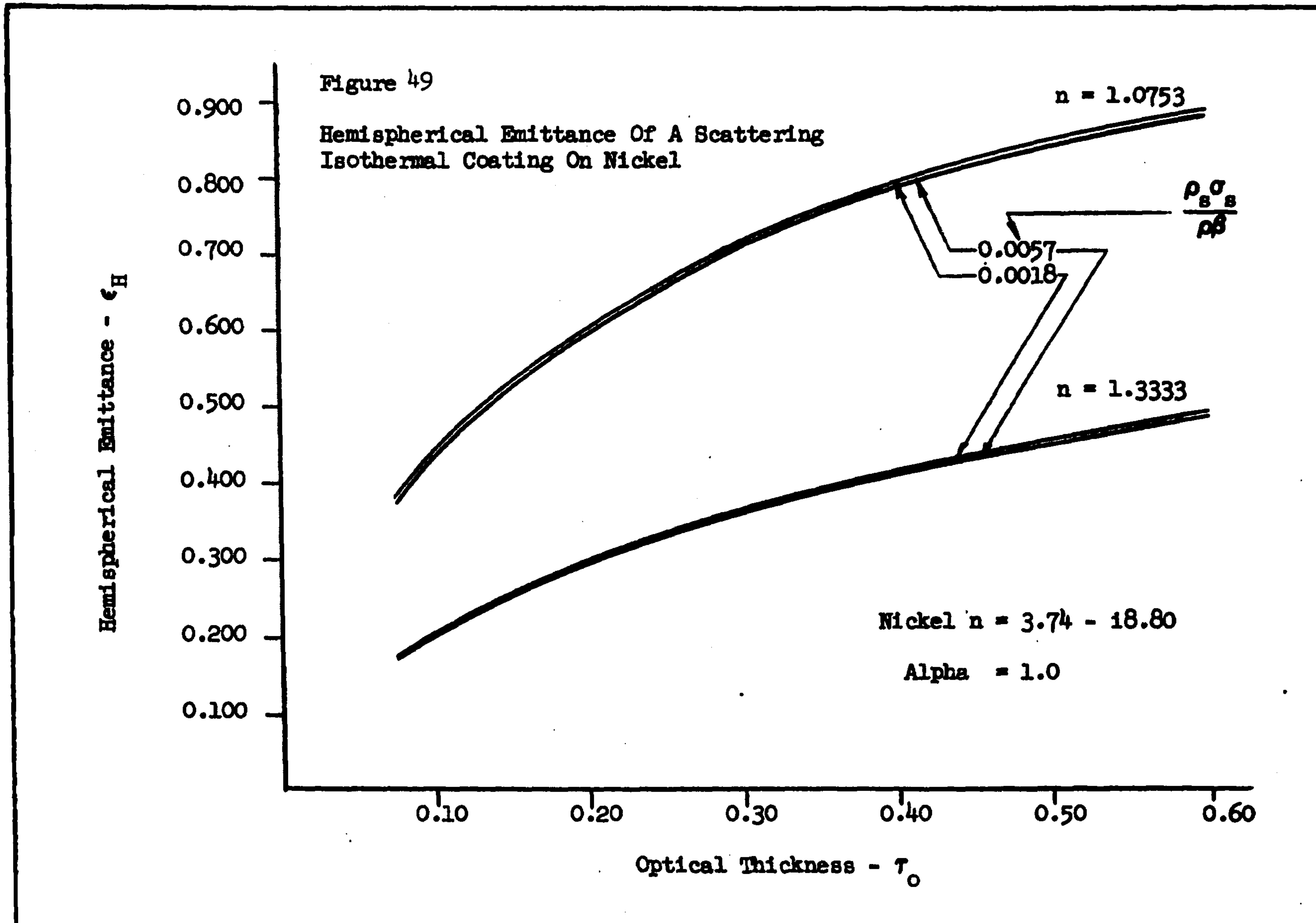


Figure 49

Hemispherical Emittance Of A Scattering Isothermal Coating On Nickel

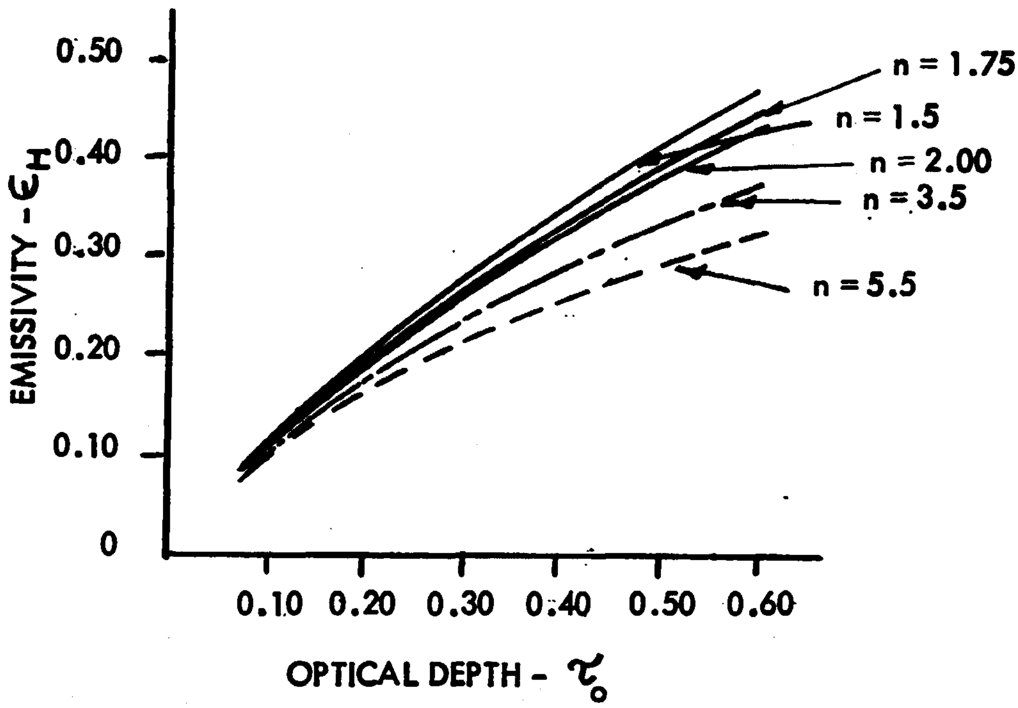


Figure 50

Monochromatic Hemispherical Emissivity of an Isothermal Sheet of Dielectric

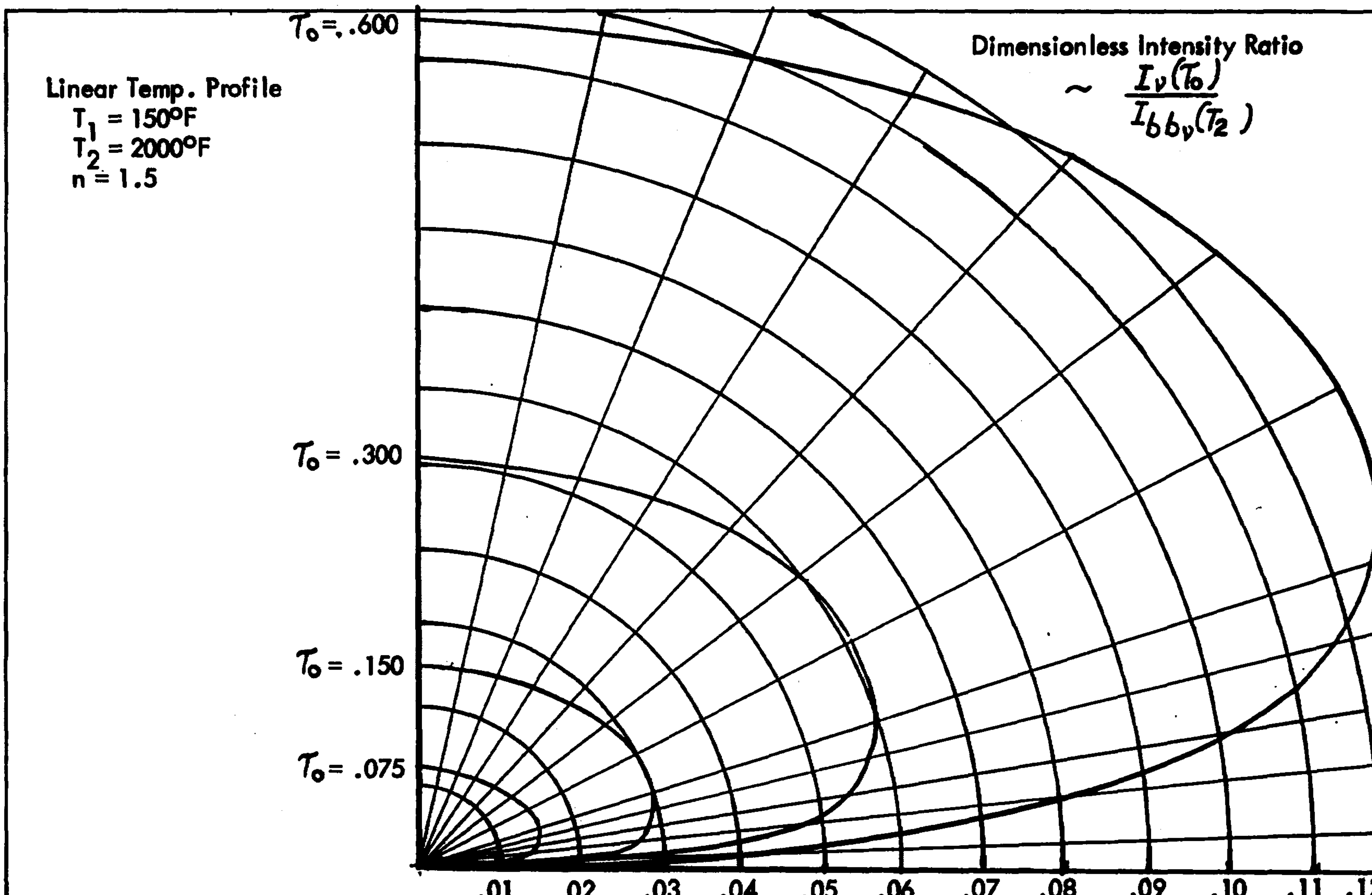


Figure 51
Dimensionless Intensity Ratio for a Linear Temp. Profile in a Dielectric Sheet

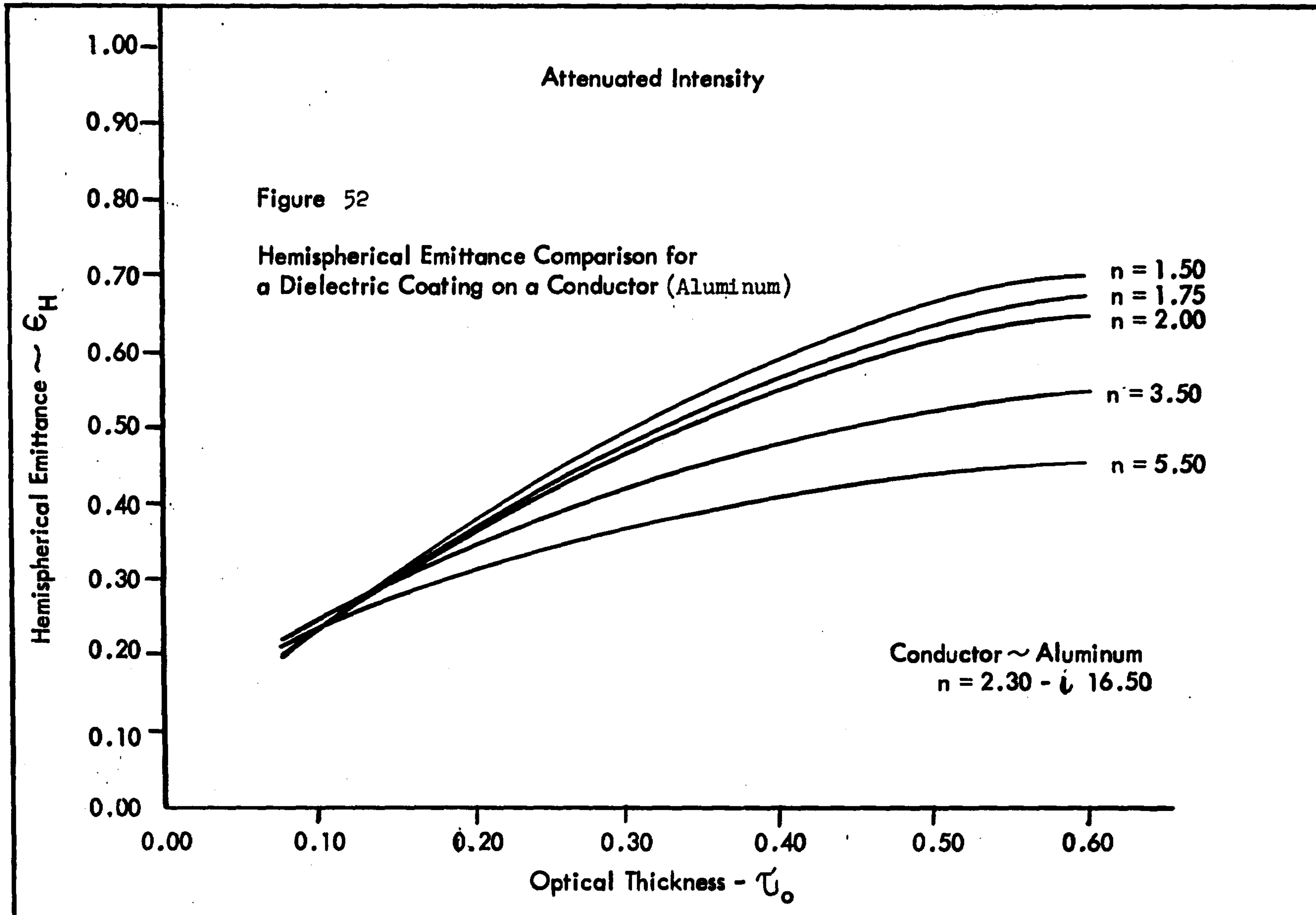


Figure 52

Hemispherical Emittance Comparison for a Dielectric Coating on a Conductor (Aluminum)

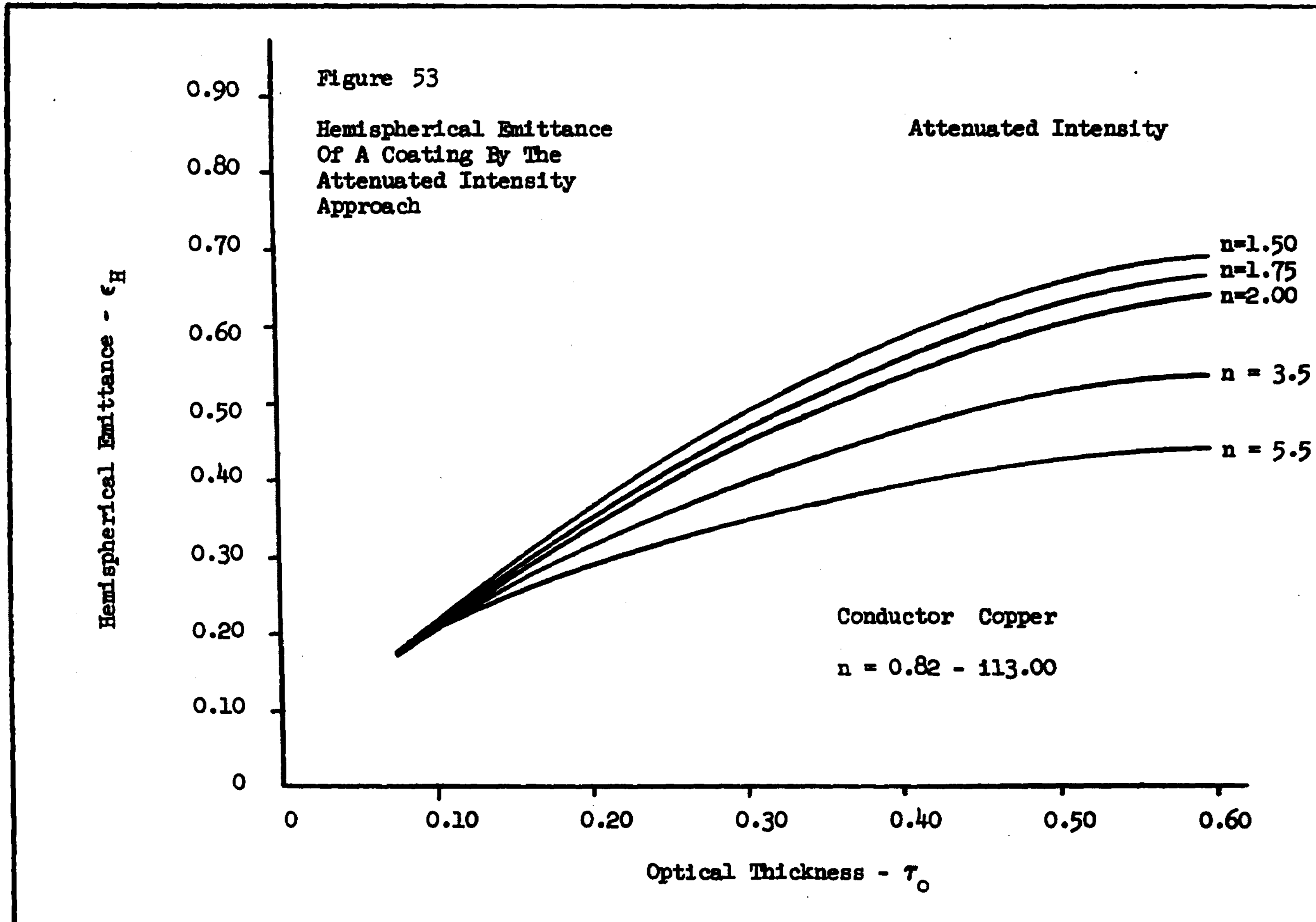


Figure 53

Hemispherical Emittance Of A Coating By The Attenuated Intensity Approach

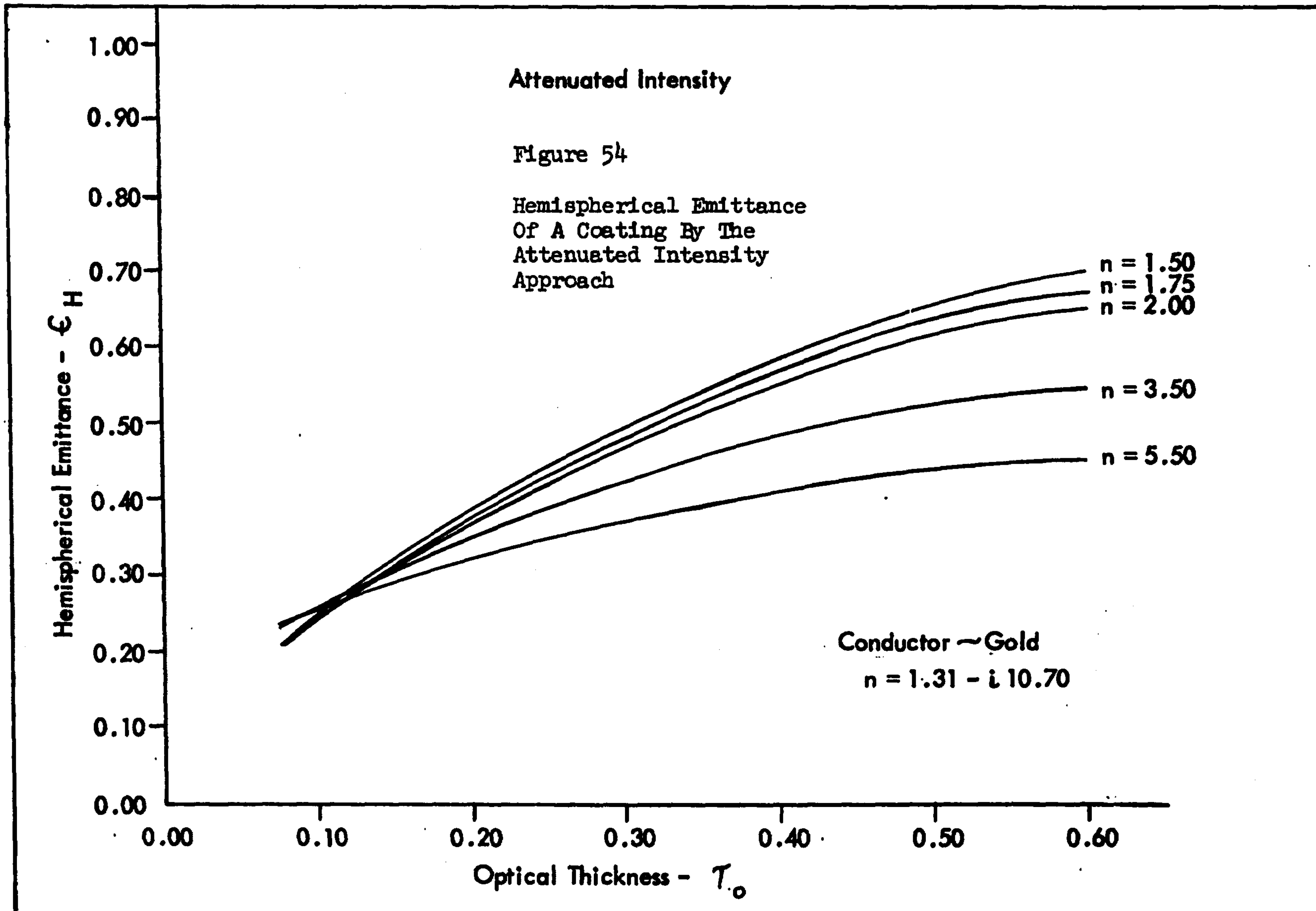


Figure 54

Hemispherical Emittance Of A Coating By The Attenuated Intensity Approach

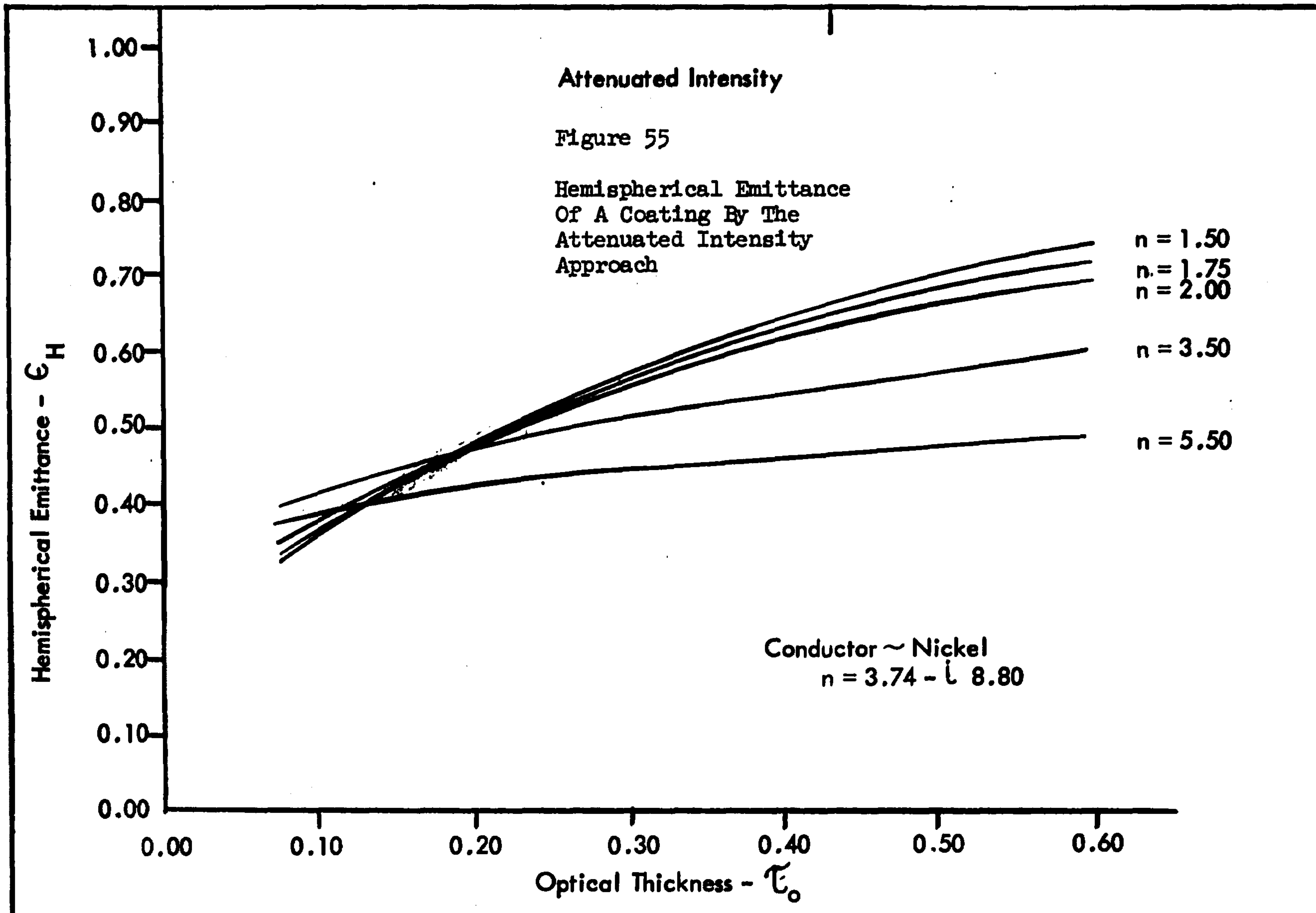
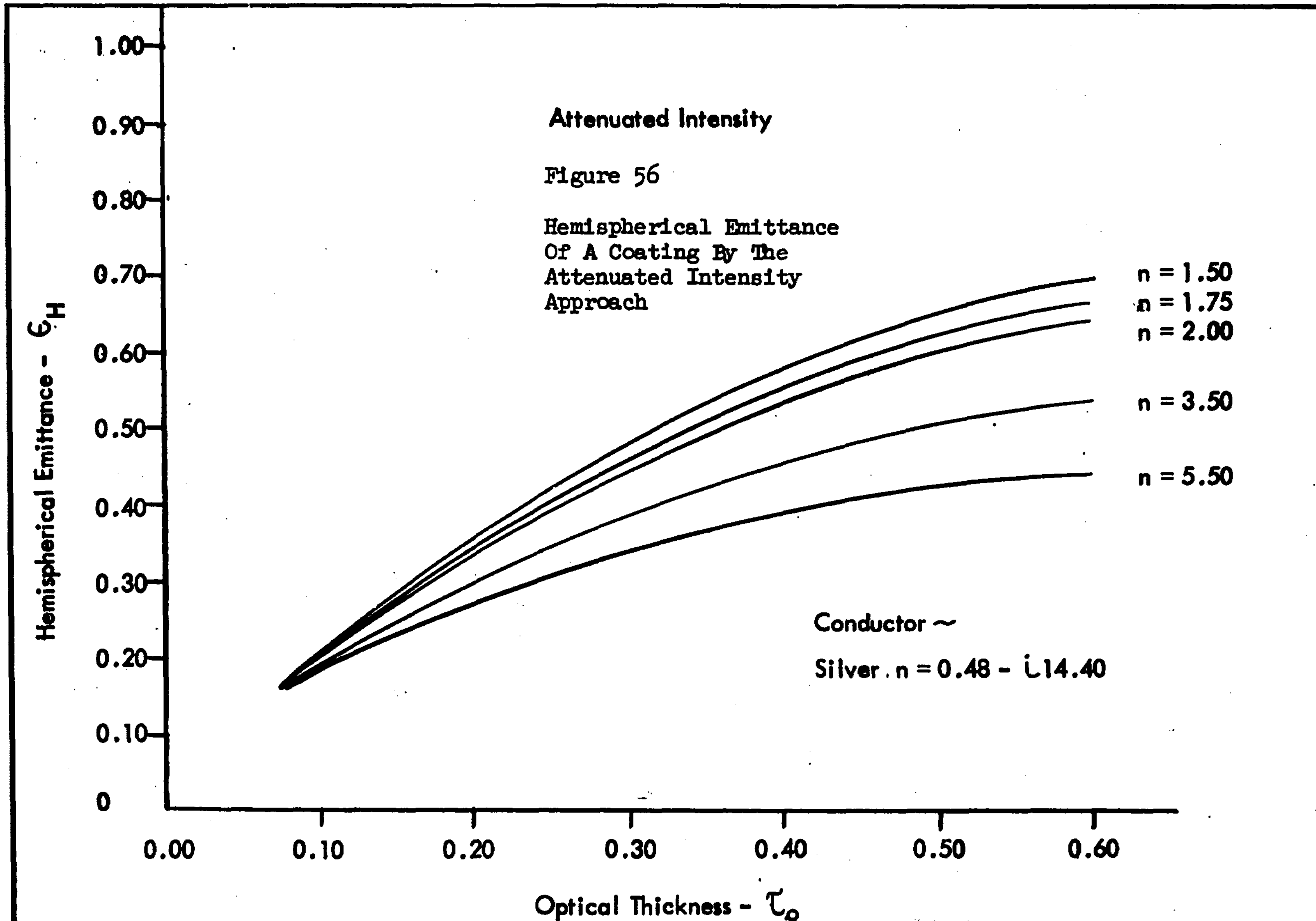


Figure 55

Hemispherical Emittance
Of A Coating By The
Attenuated Intensity
Approach


Attenuated Intensity
Figure 56
Hemispherical Emittance
Of A Coating By The
Attenuated Intensity
Approach
n = 1.50
n = 1.75
n = 2.00
n = 3.50
n = 5.50
Conductor ~
Silver . n = 0.48 - i 14.40
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
0.00
0.10
0.20
0.30
0.40
0.50
0.60
Hemispherical Emittance - ϵ_H
Optical Thickness - τ_o

CHAPTER VII

DISCUSSION OF RESULTS

To the author's knowledge the directional emittance of conductors into various dielectrics has not been used before in the literature. The analysis of the sheet of dielectric material, while a somewhat different approach than used by Gardon (8), does check with his results. The variation between the hemispherical emittance of isothermal air-dielectric coatings on a conductor found by the author and that found by using the solution of Richmond (18) is shown in Figures 26 through 30. (See pages 62 through 66.) As the optical depth approaches infinity, both solutions approach the emittance of the coating, but as the optical thickness approaches zero, the author's solution approaches the integral of a function made up of products and divisions of the directional properties of both interfaces, while Richmond's solution approaches the same functional relationship of the integrated or hemispherical properties of these interfaces. Therefore, the large discrepancy between the two solutions at small optical depths is due to the fact that the integral of a product of two functions is not equal to the product of the integrals of the functions.

The directional intensity ratio of nonisothermal coatings were not plotted. However, one plot of the directional intensity ratio for a sheet of dielectric with a linear temperature profile was made in order to indicate how the curves would appear. The nonisothermal results for coatings varied very little from conductor to conductor because of the shape of the temperature profile. That is, the conductor contributed very little. The profile chosen was based on the assumption of a hot coating surface and therefore, a cold conductor. This caused the contribution of the conductor to be small.

Both the scattering results and the combined diffuse-specular interface results were limited for lack of data. Their main contribution is in affording a method of solution which can be used when more data becomes available. The present combined diffuse-specular data is for a very limited number of materials and for total rather than spectral properties. The values used for ρ_T and σ_o are taken from Birkebak's work and are for total properties. Therefore, little can be said quantitatively for the results. As the surface roughness increases, ρ_T decreases as does ρ_s, but the results seem to indicate that the hemispherical emissivity of the combination increases as the total reflectivity of the conductor increases rather than with the surface roughness. The problem soon becomes a mathematical game where any values of ρ_T and σ_o that seem reasonable may be used but with no experimental validity.

The results of the combined diffuse-specular interface are somewhat higher than the results found considering all interfaces specular. This agrees with the work of Cess and Sotak (4). However, in their results they found the difference to increase with optical thickness, while for this paper, the difference decreased with optical thickness. This can be explained in that at increasing optical thicknesses the conductor plays a decreasing role in the solution.

The data available for calculating the scattering functions is quite limited for relative refractive indicies less than unity. What data does exist on scattering coefficients and distribution coefficients does not always agree. Added to this problem, the nonsymmetric matrix solution for the scattering function will not converge for many values of particle size. The large variation in the results with refractive index is due to the plot being of hemispherical emittance over the refractive index squared versus the optical thickness. This tends to spread the curves out for easier reading. The same is true in Figures 35 through 37. (See pages 85 through 87.)

Further work is planned for the study of combined radiation and conduction in the same problems. Inversion of the temperature profile in the coating problem to allow the conductor to play a larger role; a more complete analysis of the combined diffuse-specular boundary problem as more data becomes available; and the scattering coefficients and distribution coefficients for relative refractive indicies less than unity will be evaluated in order to obtain more results when considering scattering.

TABLE 1

MONOCHROMATIC DIRECTIONAL EMITTANCE OF ALUMINUM

INTO VARIOUS DIELECTRICS

Angle θ		Refractive Index of the Dielectric		
Radians	Degrees	1.5000	3.5000	5.5000
0.10	5.73	0.0480	0.1053	0.1519
0.30	17.19	0.0482	0.1053	0.1520
0.50	28.65	0.0485	0.1059	0.1525
0.70	40.11	0.0497	0.1079	0.1541
0.90	51.57	0.0530	0.1129	0.1572
1.10	63.03	0.0616	0.1233	0.1600
1.30	74.49	0.0843	0.1371	0.1478
1.35	77.36	0.0947	0.1372	0.1368
1.40	80.22	0.1076	0.1318	0.1198
1.45	83.08	0.1208	0.1158	0.0951
1.50	85.95	0.1207	0.0826	0.0614
1.55	88.81	0.0578	0.0280	0.0194

TABLE 2

MONOCHROMATIC DIRECTIONAL EMITTANCE OF COPPER

INTO VARIOUS DIELECTRICS

Angle θ		Refractive Index of the Dielectric		
Radians	Degrees	1.5000	3.5000	5.5000
0.10	5.73	0.0282	0.0612	0.0863
0.30	17.19	0.0282	0.0612	0.0864
0.50	28.65	0.0284	0.0615	0.0866
0.70	40.11	0.0291	0.0625	0.0870
0.90	51.57	0.0311	0.0651	0.0874
1.10	63.03	0.0360	0.0694	0.0849
1.30	74.49	0.0482	0.0708	0.0701
1.35	77.36	0.0532	0.0678	0.0623
1.40	80.22	0.0584	0.0615	0.0521
1.45	83.08	0.0616	0.0504	0.0394
1.50	85.95	0.0547	0.0332	0.0243
1.55	88.81	0.0220	0.0105	0.0074

TABLE 3

MONOCHROMATIC DIRECTIONAL EMITTANCE OF GOLD

INTO VARIOUS DIELECTRICS

Angle θ		Refractive Index of the Dielectric		
Radians	Degrees	1.5000	3.5000	5.5000
0.10	5.73	0.0642	0.1332	0.1792
0.30	17.19	0.0643	0.1333	0.1791
0.50	28.65	0.0647	0.1338	0.1790
0.70	40.11	0.0662	0.1353	0.1782
0.90	51.57	0.0701	0.1384	0.1743
1.10	63.03	0.0798	0.1416	0.1613
1.30	74.49	0.1011	0.1318	0.1237
1.35	77.36	0.1082	0.1223	0.1079
1.40	80.22	0.1139	0.1073	0.0887
1.45	83.08	0.1129	0.0852	0.0663
1.50	85.95	0.0928	0.0550	0.0406
1.55	88.81	0.0353	0.0173	0.0124

TABLE 4

MONOCHROMATIC DIRECTIONAL EMITTANCE OF NICKEL

INTO VARIOUS DIELECTRICS

Angle θ		Refractive Index of the Dielectric		
Radians	Degrees	1.5000	3.5000	5.5000
0.10	5.73	0.2139	0.4032	0.5053
0.30	17.19	0.2140	0.4032	0.5050
0.50	28.65	0.2150	0.4034	0.5029
0.70	40.11	0.2185	0.4038	0.4954
0.90	51.57	0.2279	0.4031	0.4759
1.10	63.03	0.2493	0.3948	0.4302
1.30	74.49	0.2897	0.3487	0.3288
1.35	77.36	0.3000	0.3212	0.2838
1.40	80.22	0.3045	0.2819	0.2406
1.45	83.08	0.2920	0.2264	0.1832
1.50	85.95	0.2376	0.1584	0.1153
1.55	88.81	0.0960	0.0496	0.0363

TABLE 5

MONOCHROMATIC DIRECTIONAL EMITTANCE OF SILVER

INTO VARIOUS DIELECTRICS

Angle θ		Refractive Index of the Dielectrics		
Radians	Degrees	1.5000	3.5000	5.5000
0.10	5.73	0.0136	0.0301	0.0434
0.30	17.19	0.0136	0.0301	0.0435
0.50	28.65	0.0137	0.0303	0.0436
0.70	40.11	0.0141	0.0309	0.0440
0.90	51.57	0.0151	0.0324	0.0448
1.10	63.03	0.0176	0.0352	0.0447
1.30	74.49	0.0241	0.0377	0.0386
1.35	77.36	0.0270	0.0369	0.0348
1.40	80.22	0.0303	0.0342	0.0294
1.45	83.08	0.0331	0.0286	0.0225
1.50	85.95	0.0307	0.0191	0.0139
1.55	88.81	0.0128	0.0061	0.0042

TABLE 6

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 1.5 ON ALUMINUM

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1111	0.1378	0.1598	0.1739
84	0.1737	0.2493	0.3328	0.4003
80	0.1940	0.2943	0.4195	0.5339
76	0.2025	0.3158	0.4669	0.6156
72	0.2060	0.3263	0.4935	0.6663
62	0.2054	0.3313	0.5160	0.7216
52	0.1998	0.3243	0.5121	0.7308
42	0.1934	0.3144	0.5005	0.7240
32	0.1877	0.3052	0.4881	0.7131
22	0.1833	0.2979	0.4780	0.7029
12	0.1805	0.2932	0.4712	0.6958

TABLE 7

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 3.5 ON ALUMINUM

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1281	0.1552	0.1828	0.2043
84	0.1839	0.2454	0.3227	0.3960
80	0.1993	0.2737	0.3734	0.4753
76	0.2055	0.2856	0.3961	0.5129
72	0.2084	0.2914	0.4074	0.5321
62	0.2106	0.2960	0.4171	0.5497
52	0.2107	0.2964	0.4186	0.5534
42	0.2102	0.2957	0.4180	0.5536
32	0.2096	0.2947	0.4169	0.5528
22	0.2090	0.2939	0.4158	0.5519
12	0.2086	0.2933	0.4150	0.5512

TABLE 8

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 5.5 ON ALUMINUM

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1610	0.1922	0.2278	0.2585
84	0.2142	0.2734	0.3515	0.4303
80	0.2240	0.2896	0.3787	0.4718
76	0.2257	0.2933	0.3835	0.4792
72	0.2250	0.2911	0.3816	0.4763
62	0.2216	0.2856	0.3721	0.4619
52	0.2191	0.2814	0.3652	0.4515
42	0.2176	0.2791	0.3614	0.4459
32	0.2160	0.2778	0.3595	0.4433
22	0.2165	0.2773	0.3587	0.4423
12	0.2163	0.2770	0.3583	0.4419

TABLE 9

MONOCHROMATIC DIRECTIONAL ABSORPTANCE OF A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 1.5 ON COPPER

Angle	Optical Thickness τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1068	0.1357	0.1591	0.1737
84	0.1632	0.2427	0.3297	0.3993
80	0.1809	0.2850	0.4144	0.5320
76	0.1881	0.3050	0.4605	0.6131
72	0.1908	0.3146	0.4862	0.6633
62	0.1892	0.3182	0.5074	0.7177
52	0.1833	0.3107	0.5028	0.7264
42	0.1767	0.3005	0.4908	0.7192
32	0.1709	0.2911	0.4781	0.7079
22	0.1665	0.2837	0.4677	0.6975
12	0.1637	0.2789	0.4607	0.6902

TABLE 10

MONOCHROMATIC DIRECTIONAL ABSORPTANCE OF A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 3.5 ON COPPER

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1160	0.1483	0.1799	0.2033
84	0.1599	0.2287	0.3135	0.3924
80	0.1714	0.2530	0.3611	0.4701
76	0.1760	0.2632	0.3823	0.5668
72	0.1780	0.2680	0.3927	0.5256
62	0.1795	0.2717	0.4016	0.5427
52	0.1793	0.2718	0.4028	0.5461
42	0.1787	0.2709	0.4020	0.5462
32	0.1780	0.2699	0.4008	0.5453
22	0.1774	0.2690	0.3997	0.5444
12	0.1770	0.2683	0.3989	0.5436

TABLE 11

MONOCHROMATIC DIRECTIONAL ABSORPTANCE OF A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 5.5 ON COPPER

Angle	Optical Thickness τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1394	0.1787	0.2212	0.2562
84	0.1777	0.2467	0.3360	0.4238
80	0.1843	0.2598	0.3608	0.4640
76	0.1854	0.2620	0.3651	0.4712
72	0.1849	0.2611	0.3633	0.4684
62	0.1826	0.2565	0.3547	0.4544
52	0.1808	0.2530	0.3483	0.4443
42	0.1797	0.2510	0.3448	0.4389
32	0.1791	0.2500	0.3430	0.4363
22	0.1787	0.2494	0.3422	0.4352
12	0.1786	0.2491	0.3418	0.4348

TABLE 12

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 1.5 ON GOLD

Angle	Optical Thickness τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1144	0.1393	0.1604	0.1740
84	0.1818	0.2544	0.3353	0.4011
80	0.2044	0.3016	0.4235	0.5353
76	0.2140	0.3244	0.4720	0.6176
72	0.2182	0.3357	0.4993	0.6687
62	0.2183	0.3417	0.5229	0.7248
52	0.2130	0.3352	0.5196	0.7344
42	0.2068	0.3256	0.5084	0.7280
32	0.2012	0.3166	0.4963	0.7172
22	0.1969	0.3095	0.4863	0.7073
12	0.1941	0.3048	0.4796	0.7003

TABLE 13

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 3.5 ON GOLD

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1349	0.1592	0.1847	0.2049
84	0.1982	0.2556	0.3283	0.3982
80	0.2162	0.2863	0.3810	0.4785
76	0.2236	0.2995	0.4047	0.5167
72	0.2271	0.3058	0.4165	0.5362
62	0.2299	0.3111	0.4268	0.5542
52	0.2301	0.3117	0.4284	0.5580
42	0.2297	0.3111	0.4279	0.5582
32	0.2291	0.3101	0.4269	0.5575
22	0.2286	0.3094	0.4259	0.5566
12	0.2283	0.3088	0.4252	0.5559

TABLE 14

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 5.5 ON GOLD

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1689	0.1974	0.2304	0.2594
84	0.2285	0.2839	0.3578	0.4329
80	0.2396	0.3014	0.3860	0.4750
76	0.2415	0.3044	0.3909	0.4825
72	0.2408	0.3032	0.3890	0.4796
62	0.2369	0.2971	0.3791	0.4649
52	0.2341	0.2927	0.3720	0.4544
42	0.2325	0.2902	0.3681	0.4488
32	0.2317	0.2889	0.3662	0.4462
22	0.2313	0.2883	0.3654	0.4451
12	0.2311	0.2881	0.3650	0.4447

TABLE 15

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 1.5 ON NICKEL

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1373	0.1513	0.1652	0.1754
84	0.2481	0.2978	0.3571	0.4085
80	0.2931	0.3654	0.4592	0.5487
76	0.3153	0.4009	0.5178	0.6360
72	0.3271	0.4205	0.5522	0.6910
62	0.3364	0.4373	0.5863	0.7537
52	0.3349	0.4358	0.5887	0.7677
42	0.3304	0.4290	0.5811	0.7644
32	0.3259	0.4218	0.5715	0.7562
22	0.3223	0.4160	0.5633	0.7480
12	0.3200	0.4121	0.5578	0.7423

TABLE 16

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 3.5 ON NICKEL

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1788	0.1878	0.1992	0.2102
84	0.3102	0.3383	0.3773	0.4189
80	0.3569	0.3946	0.4487	0.5087
76	0.3777	0.4202	0.4821	0.5522
72	0.3881	0.4331	0.4991	0.5747
62	0.3976	0.4448	0.5148	0.5958
52	0.3998	0.4474	0.5183	0.6008
42	0.4002	0.4478	0.5188	0.6017
32	0.4001	0.4475	0.5184	0.6015
22	0.3999	0.4471	0.5178	0.6010
12	0.3996	0.4467	0.5174	0.6006

TABLE 17

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 5.5 ON NICKEL

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.2340	0.2434	0.2563	0.2699
84	0.3665	0.3900	0.4241	0.4627
80	0.3962	0.4238	0.4644	0.5111
76	0.4015	0.4298	0.4717	0.5199
72	0.3995	0.4275	0.4689	0.5165
62	0.3894	0.4159	0.4550	0.4997
52	0.3821	0.4076	0.4450	0.4878
42	0.3783	0.4032	0.4398	0.4815
32	0.3766	0.4012	0.4374	0.4787
22	0.3759	0.4005	0.4364	0.4776
12	0.3757	0.4002	0.4361	0.4772

TABLE 18

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 1.5 ON SILVER

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1033	0.1341	0.1586	0.1735
84	0.1553	0.2378	0.3274	0.3985
80	0.1711	0.2782	0.4107	0.5307
76	0.1774	0.2971	0.4559	0.6113
72	0.1795	0.3059	0.4809	0.6611
62	0.1774	0.3087	0.5010	0.7149
52	0.1712	0.3007	0.4960	0.7231
42	0.1645	0.2904	0.4836	0.7156
32	0.1587	0.2807	0.4707	0.7041
22	0.1542	0.2732	0.4601	0.6935
12	0.1513	0.2684	0.4531	0.6861

TABLE 19

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 3.5 ON SILVER

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1062	0.1430	0.1776	0.2026
84	0.1420	0.2164	0.3068	0.3898
80	0.1509	0.2380	0.3523	0.4664
76	0.1544	0.2469	0.3724	0.5025
72	0.1560	0.2511	0.3822	0.5209
62	0.1569	0.2542	0.3905	0.5377
52	0.1565	0.2541	0.3915	0.5410
42	0.1558	0.2531	0.3906	0.5410
32	0.1551	0.2520	0.3894	0.5400
22	0.1545	0.2510	0.3882	0.5390
12	0.1540	0.2503	0.3873	0.5382

TABLE 20

MONOCHROMATIC DIRECTIONAL ABSORPTANCE FOR A DIELECTRIC COATING WITH A REFRACTIVE INDEX OF 5.5 ON SILVER

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1230	0.1688	0.2166	0.2546
84	0.1519	0.2283	0.3255	0.4195
80	0.1567	0.2394	0.3487	0.4589
76	0.1575	0.2413	0.3527	0.4659
72	0.1571	0.2404	0.3510	0.4631
62	0.1553	0.2365	0.3429	0.4494
52	0.1539	0.2334	0.3369	0.4395
42	0.1530	0.2316	0.3335	0.4342
32	0.1525	0.2306	0.3318	0.4316
22	0.1522	0.2301	0.3310	0.4306
12	0.1520	0.2298	0.3306	0.4302

TABLE 21

ABSORPTANCE AND EMITTANCE FOR A DIELECTRIC SHEET WITH n = 1.5

Angle	Optical Depth - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.0668	0.0999	0.1327	0.1581
84	0.0851	0.1477	0.2331	0.3252
80	0.0893	0.1619	0.2717	0.4072
76	0.0905	0.1672	0.2895	0.4515
72	0.0903	0.1689	0.2977	0.4758
62	0.0877	0.1662	0.2997	0.4951
52	0.0839	0.1599	0.2914	0.4896
42	0.0801	0.1531	0.2808	0.4769
32	0.0768	0.1472	0.2711	0.4638
22	0.0743	0.1426	0.2635	0.4530
12	0.0727	0.1397	0.2585	0.4459

TABLE 22

ABSORPTANCE AND EMITTANCE FOR A DIELECTRIC SHEET WITH n = 3.5

Angle	Optical Depth - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.0594	0.0957	0.1375	0.1754
84	0.0691	0.1237	0.2039	0.3002
80	0.0711	0.1304	0.2230	0.3435
76	0.0718	0.1329	0.2308	0.3626
72	0.0721	0.1340	0.2344	0.3720
62	0.0721	0.1345	0.2369	0.3797
52	0.0717	0.1340	0.2366	0.3804
42	0.0712	0.1332	0.2355	0.3795
32	0.0708	0.1325	0.2343	0.3781
22	0.0704	0.1318	0.2333	0.3768
12	0.0701	0.1313	0.2326	0.3759

TABLE 23

ABSORPTANCE AND EMITTANCE FOR A DIELECTRIC SHEET WITH n = 5.5

Angle	Optical Depth - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.0619	0.1041	0.1578	0.2117
84	0.0684	0.1241	0.2086	0.3145
80	0.0694	0.1272	0.2179	0.3361
76	0.0695	0.1278	0.2194	0.3398
72	0.0694	0.1275	0.2187	0.3382
62	0.0690	0.1262	0.2153	0.3306
52	0.0686	0.1252	0.2127	0.3250
42	0.0683	0.1245	0.2111	0.3218
32	0.0681	0.1240	0.2101	0.3201
22	0.0679	0.1237	0.2096	0.3193
12	0.0678	0.1235	0.2094	0.3189

TABLE 24

HEMISPHERICAL EMITTANCE OF AN ISOTHERMAL SHEET OF DIELECTRIC

Refractive Index	Optical Thickness - τ_o			
	0.0750	0.1500	0.3000	0.6000
1.50	0.0813	0.1543	0.2797	0.4680
1.75	0.0779	0.1477	0.2675	0.4469
2.00	0.0759	0.1437	0.2595	0.4317
3.50	0.0711	0.1327	0.2336	0.3747
5.50	0.0684	0.1249	0.2121	0.3241

TABLE 25

HEMISPHERICAL EMITTANCE OF AN ISOTHERMAL DIELECTRIC COATING
ON A CONDUCTOR SURFACE BY ATTENUATED INTENSITY APPROACH

Conductor	Coating Refractive Index n	Optical Thickness - τ_o			
		0.0750	0.1500	0.3000	0.6000
Aluminum n = 2.30 - i16.50	1.50	0.1935	0.3120	0.4901	0.6991
	1.75	0.1926	0.3043	0.4721	0.6695
	2.00	0.1939	0.3004	0.4595	0.6458
	3.50	0.2086	0.2927	0.4121	0.5437
	5.50	0.2186	0.2807	0.3643	0.4504
Copper n = 0.82 - 13.00	1.50	0.1774	0.2987	0.4810	0.6947
	1.75	0.1741	0.2891	0.4617	0.6644
	2.00	0.1732	0.2835	0.4480	0.6402
	3.50	0.1776	0.2684	0.3967	0.5366
	5.50	0.1804	0.2524	0.3475	0.4432
Gold n = 1.31 - i10.70	1.50	0.2066	0.3227	0.4975	0.7028
	1.75	0.2072	0.3162	0.4803	0.6736
	2.00	0.2098	0.3133	0.4684	0.6501
	3.50	0.2278	0.3077	0.4218	0.5482
	5.50	0.2336	0.2920	0.3711	0.4534

TABLE 25 -- Continued

Conductor	Coating Refractive Index n	Optical Thickness - τ_o			
Nickel $n = 3.74 - i8.80$	1.50	0.3258	0.4211	0.5655	0.7362
	1.75	0.3389	0.4249	0.5554	0.7109
	2.00	0.3520	0.4302	0.5488	0.6900
	3.50	0.3946	0.4409	0.5099	0.5901
	5.50	0.3814	0.4068	0.4441	0.4867
Silver $n = 0.48 - i14.40$	1.50	0.1655	0.2889	0.4743	0.6914
	1.75	0.1605	0.2780	0.4541	0.6607
	2.00	0.1581	0.2712	0.4396	0.6360
	3.50	0.1551	0.2510	0.3856	0.5315
	5.50	0.1536	0.2329	0.3360	0.4384

TABLE 26

HEMISPHERICAL EMITTANCE BY THE ATTENUATED FLUX APPROACH

Conductor	Coating Refractive Index n	Optical Thickness - τ_o			
		0.0750	0.1500	0.3000	0.6000
Aluminum $n = 2.30 - i16.50$	1.50	0.3350	0.4763	0.6466	0.8026
	1.75	0.4082	0.5436	0.6906	0.8119
	2.00	0.4696	0.5920	0.7144	0.8080
	3.50	0.6027	0.6644	0.6787	0.7019
	5.50	0.5385	0.5506	0.5595	0.5627
Copper $n = 0.82 - i13.00$	1.50	0.3051	0.4561	0.6361	0.7985
	1.75	0.3730	0.5222	0.6805	0.8082
	2.00	0.4316	0.5708	0.7052	0.8048
	3.50	0.5770	0.6330	0.6750	0.7007
	5.50	0.5385	0.5506	0.5596	0.5627
Gold $n = 1.31 - i10.70$	1.50	0.3539	0.4889	0.6536	0.8053
	1.75	0.4291	0.5565	0.6967	0.8140
	2.00	0.4908	0.6040	0.7196	0.8096
	3.50	0.6113	0.6493	0.6811	0.7019
	5.50	0.5385	0.5506	0.5596	0.5627

TABLE 26 -- Continued

Conductor	Coating Refractive Index n	Optical Thickness - τ_o			
Nickel $n = 3.74 - i8.80$	1.50	0.5164	0.6037	0.7175	0.8309
	1.75	0.5957	0.6661	0.7531	0.8348
	2.00	0.6480	0.7024	0.7676	0.8268
	3.50	0.6725	0.6836	0.6958	0.7068
	5.50	0.5566	0.5596	0.5627	0.5657
Silver $n = 0.48 - i14.40$	1.50	0.2835	0.4417	0.6284	0.7956
	1.75	0.3470	0.5065	0.6731	0.8057
	2.00	0.4028	0.5552	0.6984	0.8028
	3.50	0.5549	0.6235	0.6725	0.6995
	5.50	0.5233	0.5415	0.5566	0.5627

TABLE 27

TRANSMISSIVITY FOR n = 1.5

Angle	Optical Thickness - τ_0			
θ	0.0750	0.1500	0.3000	0.6000
88	0.0658	0.0487	0.0314	0.0170
84	0.2373	0.2021	0.1522	0.0936
80	0.3871	0.3418	0.2702	0.1745
76	0.5081	0.4551	0.3675	0.2432
72	0.6018	0.5430	0.4433	0.2977
62	0.7445	0.6776	0.5616	0.3865
52	0.8085	0.7399	0.6198	0.4353
42	0.8363	0.7689	0.6499	0.4646
32	0.8483	0.7828	0.6666	0.4835
22	0.8534	0.7896	0.6761	0.4958
12	0.8555	0.7929	0.6813	0.5030

TABLE 28

TRANSMISSIVITY FOR n = 3.5

Angle	Optical Thickness - τ			
θ	0.0750	0.1500	0.3000	0.6000
88	0.0931	0.0744	0.0523	0.0312
84	0.2594	0.2288	0.1823	0.1227
80	0.3553	0.3198	0.2624	0.1826
76	0.4089	0.3708	0.3076	0.2168
72	0.4390	0.3994	0.3331	0.2363
62	0.4695	0.4285	0.3593	0.2566
52	0.4781	0.4370	0.3671	0.2631
42	0.4810	0.4400	0.3703	0.2661
32	0.4822	0.4414	0.3719	0.2678
22	0.4827	0.4421	0.3728	0.2690
12	0.4829	0.4425	0.3734	0.2697

TABLE 29

TRANSMISSIVITY FOR n = 5.5

Angle	Optical Thickness - τ_o			
θ	0.0750	0.1500	0.3000	0.6000
88	0.1328	0.1107	0.0819	0.0513
84	0.2971	0.2651	0.2150	0.1480
80	0.3486	0.3142	0.2584	0.1808
76	0.3585	0.3236	0.2669	0.1872
72	0.3549	0.3201	0.2637	0.1849
62	0.3364	0.3026	0.2483	0.1733
52	0.3237	0.2907	0.2378	0.1655
42	0.3174	0.2847	0.2326	0.1617
32	0.3147	0.2822	0.2305	0.1602
22	0.3137	0.2813	0.2299	0.1598
12	0.3135	0.2812	0.2298	0.1598

TABLE 30

$\frac{1}{n^2}$ · DIMENSIONLESS FLUX RATIO FOR A DIELECTRIC SHEET WITH A TEMPERATURE PROFILE

$T_o = 150^oF$ $\qquad$ $T_1 = 2000^oF$

		Optical Thickness - τ_o			
Refractive Index	Temperature Profile	0.075000	0.150000	0.300000	0.600000
1.5	Parabolic	0.177500	0.035870	0.073770	0.159170
	Linear	0.009617	0.019300	0.039144	0.082074
3.5	Parabolic	0.002798	0.005466	0.010608	0.021041
	Linear	0.001517	0.002946	0.005647	0.010920
5.5	Parabolic	0.001083	0.002057	0.003817	0.007131
	Linear	0.000587	0.001109	0.002032	0.003704

Surface Intensity = $I_\nu(T_1) = 0.39789285 \times 10^{-13}$

TABLE 31

FLUX RATIO FOR VARIOUS NONISOTHERMAL COATINGS WITH SPECULAR INTERFACES ON ALUMINUM

$T_s = 150^{\circ}F$ $\qquad$ $T_F = 2000^{\circ}F$

Refractive Index	Optical Thickness	Linear Profile	Parabolic Profile
1.5	0.0750	0.0201	0.0370
	0.1500	0.0408	0.0757
	0.3000	0.0842	0.1588
	0.6000	0.1807	0.3505
3.5	0.0750	0.0142	0.0264
	0.1500	0.0286	0.0532
	0.3000	0.0580	0.1090
	0.6000	0.1211	0.2333
5.5	0.0750	0.0129	0.0239
	0.1500	0.0252	0.0468
	0.3000	0.0491	0.0922
	0.6000	0.0981	0.1889

TABLE 32

FLUX RATIO FOR VARIOUS NONISOTHERMAL COATINGS

WITH SPECULAR INTERFACES ON COPPER

$T_s = 150^{\circ}F$ $T_F = 2000^{\circ}F$

Refractive Index	Optical Thickness	Linear Profile	Parabolic Profile
1.5	0.0750	0.0200	0.0369
	0.1500	0.0407	0.0756
	0.3000	0.0841	0.1585
	0.6000	0.1806	0.3503
3.5	0.0750	0.0139	0.0256
	0.1500	0.0280	0.0519
	0.3000	0.0570	0.1072
	0.6000	0.1200	0.2313
5.5	0.0750	0.0119	0.0219
	0.1500	0.0235	0.0435
	0.3000	0.0466	0.0876
	0.6000	0.0955	0.1839

TABLE 33

FLUX RATIO FOR VARIOUS NONISOTHERMAL COATINGS

WITH SPECULAR INTERFACES ON GOLD

$T_S = 150^{o}F$ $T_F = 2000^{o}F$

Refractive Index	Optical Thickness	Linear Profile	Parabolic Profile
	0.0750	0.0201	0.0371
	0.1500	0.0408	0.0759
1.5	0.3000	0.0843	0.1589
	0.6000	0.1808	0.3506
	0.0750	0.0145	0.0268
	0.1500	0.0291	0.0539
3.5	0.3000	0.0586	0.1101
	0.6000	0.1217	0.2346
	0.0750	0.0134	0.0246
	0.1500	0.0259	0.0481
5.5	0.3000	0.0501	0.0940
	0.6000	0.0991	0.1909

TABLE 34

FLUX RATIO FOR VARIOUS NONISOTHERMAL COATINGS

WITH SPECULAR INTERFACES ON NICKEL

$T_s = 150^\circ F$ $T_F = 2000^\circ F$

Refractive Index	Optical Thickness	Linear Profile	Parabolic Profile
1.5	0.0750	0.0204	0.0377
	0.1500	0.0414	0.0769
	0.3000	0.0851	0.1604
	0.6000	0.1816	0.3522
3.5	0.0750	0.0167	0.0309
	0.1500	0.0328	0.0609
	0.3000	0.0641	0.1205
	0.6000	0.1279	0.2464
5.5	0.0750	0.0175	0.0322
	0.1500	0.0330	0.0612
	0.3000	0.0606	0.1139
	0.6000	0.1110	0.2138

TABLE 35

FLUX RATIO FOR VARIOUS NONISOTHERMAL COATINGS

WITH SPECULAR INTERFACES ON SILVER

$T_s = 150^{\circ}F$ $T_F = 2000^{\circ}F$

Refractive Index	Optical Thickness	Linear Profile	Parabolic Profile
1.5	0.0750	0.0200	0.0369
	0.1500	0.0406	0.0755
	0.3000	0.0841	0.1584
	0.6000	0.1805	0.3501
3.5	0.0750	0.0136	0.0251
	0.1500	0.0275	0.0510
	0.3000	0.0563	0.1059
	0.6000	0.1193	0.2300
5.5	0.0750	0.0111	0.0205
	0.1500	0.0223	0.0413
	0.3000	0.0450	0.0845
	0.6000	0.0938	0.1806

TABLE 36

HEMISPHERICAL EMITTANCE OF A DIELECTRIC COATING ON NICKEL

WITH A SURFACE ROUGHNESS σ_o OF 0.116 μ

$(\rho_s + \rho_D) = 0.703$

Refractive Index	Optical Thickness	ϵ_H
1.5	0.0750	0.4826
	0.1500	0.5202
	0.3000	0.5852
	0.6000	0.6835
3.5	0.0750	0.4637
	0.1500	0.4848
	0.3000	0.5201
	0.6000	0.5704
5.5	0.0750	0.4189
	0.1500	0.4306
	0.3000	0.4500
	0.6000	0.4773

TABLE 37

HEMISPHERICAL EMITTANCE OF A DIELECTRIC COATING ON NICKEL

WITH A SURFACE ROUGHNESS σ_o OF 0.714 μ

$(\rho_s + \rho_D) = 0.546$

Refractive Index	Optical Thickness	ϵ_H
1.5	0.0750	0.3795
	0.1500	0.4672
	0.3000	0.6002
	0.6000	0.7569
3.5	0.0750	0.3877
	0.1500	0.4361
	0.3000	0.5080
	0.6000	0.5915
5.5	0.0750	0.3757
	0.1500	0.4028
	0.3000	0.4425
	0.6000	0.4877

TABLE 38

PERCENTAGE DIFFERENCE BETWEEN THE ATTENUATED INTENSITY AND THE ATTENUATED FLUX APPROACHES

Conductor	Coating Refractive Index	Optical Thickness τ_o			
		0.075	0.150	0.300	0.600
Aluminum	1.50	73.12	52.66	31.94	14.80
n = 2.30 - i16.50	2.00	142.18	97.07	55.47	25.14
	3.50	188.93	120.16	64.69	29.10
Copper	1.50	71.98	52.70	32.25	14.94
n = 0.82 - i13.00	2.00	149.19	101.34	57.41	25.71
	3.50	224.89	135.95	70.15	30.58
Gold	1.50	71.30	57.50	31.38	14.58
n = 1.31 - i10.70	2.00	133.90	92.79	53.66	24.53
	3.50	168.35	111.02	81.47	28.04
Nickel	1.50	58.50	43.36	26.88	12.86
n = 3.74 - i8.80	2.00	84.09	63.27	39.87	19.83
	3.50	69.65	55.05	36.46	19.71
Silver	1.50	71.30	52.89	32.49	15.07
n = 0.48 - i14.40	2.00	154.77	104.72	58.87	26.23
	3.50	257.77	148.41	74.40	31.61

TABLE 39

RATIO FOR NONISOTHERMAL COATINGS ON NICKEL

WITH A SURFACE ROUGHNESS σ_o OF 0.116 μ

$(\rho_s + \rho_D) = 0.703$

$T_s = 150^\circ F$ $T_F = 2000^\circ F$

Refractive Index	Optical Depth	Linear Temperature Profile	Parabolic Temperature Profile
1.5	0.075	0.0301	0.0555
	0.150	0.0578	0.1074
	0.300	0.1090	0.2053
	0.600	0.2072	0.4017
3.5	0.075	0.0209	0.0386
	0.150	0.0401	0.0743
	0.300	0.0749	0.1407
	0.600	0.1400	0.2698
5.5	0.075	0.0161	0.0296
	0.150	0.0307	0.0570
	0.300	0.0576	0.1081
	0.600	0.1080	0.2079

TABLE 40

FLUX RATIO FOR NONISOTHERMAL COATINGS ON NICKEL

WITH A SURFACE ROUGHNESS OF $\sigma_o = 0.714\ \mu$

$(\rho_s + \rho_D) = 0.546$

$T_S = 150^oF$ $T_F = 2000^oF$

Refractive Index	Optical Depth	Linear Temperature Profile	Parabolic Temperature Profile
1.5	0.075	0.0340	0.0627
	0.150	0.0644	0.1197
	0.300	0.1185	0.2233
	0.600	0.2172	0.4213
3.5	0.075	0.0244	0.0450
	0.150	0.0458	0.0850
	0.300	0.0830	0.1560
	0.600	0.1486	0.2863
5.5	0.075	0.0188	0.0346
	0.150	0.0352	0.0653
	0.300	0.0638	0.1197
	0.600	0.1144	0.2203

TABLE 41

HEMISPHERICAL EMITTANCE

OF A SCATTERING ISOTHERMAL COATING ON NICKEL

$n = 3.74 - i8.80$

Refractive Index	Optical Depth	ϵ_H	Alpha	$\frac{\rho_s \sigma_s}{\rho \beta}$
	0.0750	0.37712		
	0.1500	0.52497		
1.07527	0.3000	0.71058	1.0	0.0057
	0.6000	0.88355		
	0.0750	0.38005		
	0.1500	0.52888		
1.07527	0.3000	0.71557	1.0	0.0018
	0.6000	0.88915		
	0.0750	0.17936		
	0.1500	0.25692		
1.3333	0.3000	0.36659	1.0	0.0018
	0.6000	0.49470		
	0.0750	0.17582		
	0.1500	0.25125		
1.3333	0.3000	0.35910	1.0	0.0057
	0.6000	0.48782		

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APPENDIX I

SOLUTION TO THE TRANSPORT EQUATION

The equation of transport is given by

$$\mu \frac{dI_{\upsilon}(\tau,\mu)}{d\tau} = - I_{\upsilon}(\tau,\mu) + n^2 I_{bb_{\upsilon}}(T)$$

where T could be a function of τ. If μ is fixed, the equation is an ordinary differential equation in τ, which may be written as

$$\frac{dI_{\upsilon}}{d\tau} = - \frac{I_{\upsilon}}{\mu} + \frac{n^2}{\mu} I_{bb_{\upsilon}}(T).$$

From ordinary differential equations, the equation

$$\frac{dy}{dx} + Py = 0$$

may be written as

$$\frac{dy}{y} + Pdx = 0.$$

The solution becomes

$$\ln y + \int pdx = \ln c \qquad ye^{\int pdx} = c,$$

or

$$y = ce^{-\int p dx}.$$

The integrating factor is $e^{\int p dx}$ or in the equation of transport

$$e^{\int p dx} = e^{\int (1/\mu) d\tau)} = e^{\tau/\mu}.$$

Therefore, multiplying by $e^{\tau/\mu}$ yields

$$e^{\tau/\mu} \frac{dI_\upsilon}{d\tau} = - e^{\tau/\mu} \frac{I_\upsilon}{\mu} + \frac{n^2 e^{\tau/\mu} I_{bb_\upsilon}(T)}{\mu}$$

but

$$\frac{d}{d\tau}(e^{\tau/\mu} I_\upsilon) = e^{\tau/\mu} \frac{dI_\upsilon}{d\tau} + \frac{I_\upsilon e^{\tau/\mu}}{\mu} .$$

Therefore, the equation may be written as

$$\frac{d}{d\tau}(e^{\tau/\mu} I_\upsilon) = \frac{n^2 e^{\tau/\mu} I_{bb_\upsilon}(T)}{\mu}$$

or

$$d(e^{\tau/\mu} I_\upsilon) = \frac{n^2 e^{\tau/\mu} I_{bb_\upsilon}(T) d\tau}{\mu} .$$

If I_{bb_υ} is independent of τ, solving

$$e^{\tau/\mu} I_\upsilon = n^2 e^{\tau/\mu} I_{bb_\upsilon}(T) + c$$

$$I_\upsilon = n^2 I_{bb_\upsilon}(T) + ce^{-\tau/\mu}.$$

If $T = f(\tau)$, the solution becomes

$$I_\nu = ce^{-\tau/\mu} + \frac{e^{-\tau/\mu}}{\mu} \int e^{\tau/\mu} n^2 I_{bb_\nu}(t)dt$$

$$I_\nu = ce^{-\tau/\mu} + \frac{n^2}{\mu} \int e^{(t-\tau)/\mu} I_{bb_\nu}(t)dt.$$

APPENDIX II

NUMERICAL INTEGRATION

By definition, numerical integration when applied to a function of a single variable is a mechanical quadrature process and when computing a double integral of a function of two variables is a mechanical cubature process. In most forms of numerical integration, the intervals are equally spaced, but Gauss decided there must be some better way to subdivide the interval in order to get a more accurate value for the integral. His efforts resulted in what is known as Gauss' quadrature formula, which is the most accurate of the quadrature formulas Gauss based his quadrature on the zeros of the Legendre polynomials which forces the error term to zero if the integrand is a polynomial of order 2n - 1 or less. Here n is the number of intervals. In Gauss' quadrature the subdivisions are not equal, but they are symmetric about the midpoint of the interval.

If $I = \int_a^b f(x)dx$ is the integral to be computed, Gauss' formula says

$$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \phi(u)du = \sum_{j=1}^{n} w_j \phi(u_j).$$

Therefore, making the substitution

$$x = (b - a)u + \frac{a + b}{2} \quad \text{and} \quad dx = (b - a)du,$$

I becomes

$$I = \int_a^b f(x)dx = (b - a) \int_{-\frac{1}{2}}^{\frac{1}{2}} \phi(u)du = (b - a) \sum_{j=1}^{n} w_j \phi(u_j)$$

where

$$\phi(u) = f\left[(b - a)u + \frac{a + b}{2}\right].$$

For the work in this paper the four point quadrature was chosen and assumed to be sufficiently accurate.

For $n = 4$,

$w_1 = 0.3260725774$ $\qquad$ $u_1 = -\ 0.1699905218$

$w_2 = 0.3260725774$ $\qquad$ $u_2 = +\ 0.1699905218$

$w_3 = 0.1739274226$ $\qquad$ $u_3 = -\ 0.4305681558$

$w_4 = 0.1739274226$ $\qquad$ $u_4 = +\ 0.4305681558$

Values for n up to ten are given in Scarborough (15).

In this work the interest is in evaluating the flux integral and the integral across the optical depth. For the flux the integral is of the form

$$\int_{\mu_c}^{1} f(\mu)d\mu.$$

Rewriting the quadrature to fit the integral, the coefficients become

$$\int_{\mu_c}^{1} f(\mu)d\mu = (1 - \mu_c) \sum_{j=1}^{n} w_j f(\mu_j).$$

For $n = 4$,

$w_1 = 0.1739274226$ $\qquad$ $\mu_1 = 0.0694318442 + 0.9305681558\mu_c$

$w_2 = 0.3260725774$ $\qquad$ $\mu_2 = 0.3300094782 + 0.6699905218\mu_c$

$w_3 = 0.3260725774$ $\qquad$ $\mu_3 = 0.6699905218 + 0.3300094782\mu_c$

$w_4 = 0.1739274226$ $\qquad\qquad$ $\mu_4 = 0.9305681558 + 0.0694318442\mu_c$

It is observed that as μ_c goes to zero, the quadrature becomes the so-called double Gaussian quadrature and is widely applicable where flux caluclations are made with no critical angle consideration.

For integrating across the optical depth, the integral has the form

$$I = \int_0^{\tau_0} g(t)dt$$

for which the quadrature becomes

$$\int_0^{\tau_0} g(t)dt = \tau_0 \sum_{j=1}^{n} w_j g(t_j).$$

Where for n = 4,

$w_1 = 0.1739274226$ $\qquad\qquad$ $t_1 = 0.0694318442\tau_0$

$w_2 = 0.3260725774$ $\qquad\qquad$ $t_2 = 0.3300094782\tau_0$

$w_3 = 0.3260725774$ $\qquad\qquad$ $t_3 = 0.6699905218\tau_0$

$w_4 = 0.1739274226$ $\qquad\qquad$ $t_4 = 0.9305681558\tau_0$

In order to determine the total radiant flux, it is necessary to integrate overall wavelengths. It would be desirable to have a quadrature formula suitable for this integration which would enable one to determine the intensities at the frequencies of the quadrature. This would be particularly useful in the isothermal case in which the refractive index is a function of frequency alone.

To perform this integration, A. Reiz (see Love 16) has developed a quadrature based on the zeros of the Laguerre polynomials. His quadrature again reduces the error term to zero if the integrand can be expressed as a polynomial of order 2n - 1 or less. The number of intervals will be restricted to five here as any number larger than five would cause extensive calculations.

For n = 5,

$$\int_0^\infty e^{-x}f(x)dx = \sum_{j=1}^{n} a_j f(x_j).$$

$x_1 = 0.2635603$ $\qquad$ $a_1 = 0.5217556$

$x_2 = 1.4134030$ $\qquad$ $a_2 = 0.3986668$

$x_3 = 3.5964258$ $\qquad$ $a_3 = 0.0759424$

$x_4 = 7.0858102$ $\qquad$ $a_4 = 0.0036118$

$x_5 = 12.6408007$ $\qquad$ $a_5 = 0.0000234$