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Brian Stade
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SEA CLUTTER MODELING FOR THE EVALUATION OF MARITIME
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BY THE COMMITTEE CONSISTING OF

Dr. Justin Metcalf, Chair

Dr. Nathan Goodman

Dr. Yan Zhang

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“The wonderful arrangement and harmony of the cosmos would only originate in the plan of an almighty omniscient being. This is and remains my greatest comprehension.”

-Sir Issac Newton

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Abstract

Evaluating the performance of a radar system before the construction of prototypes or deployment is a critical step in the design process that guides the design choices, enabling optimal performance in the desired deployment environment. For maritime deployments, radar systems struggle with backscattered radiation from the ever-changing surface of the ocean, which obscures targets of interest, preventing positive detections from occurring. This thesis seeks to predict the performance of a maritime deployed radar system based on a dynamic simulation with real-world inputs to assess the operational capacity of a radar in the presence of sea clutter. Through a study of electromagnetic scattering characteristics and ocean wave behavior, the simulation presented in this thesis will serve as a launching point for gauging the performance of an X-band radar system. To aid in detection capabilities, clutter mitigation and cancellation techniques will be utilized to unmask small targets in the scene that cohabit the cluttered environment, and Constant False Alarm Rate (CFAR) detection techniques will be implemented to output a binary decision about the presence of a target in the scene. By simulating the expected returns from the sea surface with the expected returns from a small target near the ocean surface in a geometrically accurate simulation, the radar system's ability to perform in the harsh environment will be evaluated, and system design choices or constraints can be identified before a substantial investment is made into prototyping.

Chapter 1

Introduction

1.1 Overview

Radio Detection and Ranging (RADAR) uses electromagnetic waves to detect the range of a target of interest. Classically, the radar system transmits and listens for echoes to determine the distance of a target of interest by measuring the time difference between the transmission and reception of the wave. This was often done using magnetron-based systems that are capable of producing high-power electromagnetic waves. More recently, advances in radar technology have allowed the generation of precise waveforms through solid-state technology. This precise control over the transmitted waveform allows for phase-coherent Doppler processing of the reflections, which adds another dimension of data relevant to the user in determining the location of targets of interest. With Doppler processing, radar systems can see scatterers in both range and velocity, allowing the end user to separate undesired return signals from the environment. This data is commonly viewed in the form of a range-Doppler map where signal energy is plotted as a function of range and velocity. The undesired return signals from the environment are often called "clutter" and can occur in any deployment environment. Clutter is characteristically difficult to deal with due to the nature of its manifestation.

The term "noise" is often used to characterize the corruption of the data due to temperature within the radar system and processing sets in the receive chain, and it can easily be mitigated by increasing the power of the transmitted waveform. However, clutter is inherently different in that an increase in transmitted power results in an increase in reflected power from clutter sources. Ground-based radar systems deal with mostly stationary clutter sources; trees, buildings, terrain, etc. Doppler processing of coherent data allows return signals from moving targets to significantly stand out from stationary clutter sources; however, the clutter returns from a maritime environment necessitate the utilization of other processing techniques in order to acquire positive detections on moving targets. Because the ocean surface is constantly moving, randomized in its movement patterns, and highly reflective, Doppler processing does not always allow simple identification of moving targets within the scene. Often, the clutter from the sea surface obscures reflections from smaller targets as a result of the strong sidelobe interference from Doppler processing.

Sea clutter has long been a complex problem to model in a simulated environment, thus few models exist that prove to be dynamic in capability and robust in outputting valid results across a spectrum of user-defined inputs to gauge potential system performance. Due to the computational complexity required to model sea clutter data and the limited capabilities in such simulations, radar system design trade-offs are difficult to evaluate until a prototype has been built for testing. The clear drawbacks of this approach to the design process are that significant resources must be allocated before any conclusive design decisions can be validated. Therefore, performance prediction modeling is a vital resource for any system designer to gauge system effectiveness in a maritime environment. Typical simulations of this type consist of modeling amplitude statistics by generating

temporally and spatially correlated random variables with some random phase shift applied to simulate movement of the sea surface. The key parameters radar sea clutter simulations must endeavor to replicate are the mean backscatter or reflectivity, the amplitude statistics, and the Doppler characteristics of the ocean surface. Several factors of simulation geometry influence these characteristics, such as grazing angle, polarization, center frequency, spatial correlations, spatial resolution, wind and swell characteristics, and sea spike characteristics [1]. Modeling simulations for maritime environments are scarce due to the complexity and sometimes narrow application, but a general use simulation could be beneficial for any user-defined parameters of a potential maritime deployed radar system. Work in this field of study has been ongoing for some time with [1]–[4], just to name a few, dedicating enormous amounts of time and effort to deriving, implementing and compiling the ocean characteristics as they relate to radar. Others, [2], [5]–[7], have dedicated their work to simply characterizing mean backscatter of the ocean surface, paving the way for sea clutter simulations to take place. Currently, [3], [8], [9] show that simulations are feasible and guide understanding in the field that allow end users to analyze radar performance in a given scenario. Although there are many advances, there is still much to be done within the field related to Doppler characteristics, detection, and electromagnetic modeling, and more [10].

This thesis seeks to analyze the effect of sea clutter on a monostatic pulsed Doppler radar system through accurate simulation of reflectivity, amplitude statistics, and Doppler characteristics and investigates mitigating sea clutter to accurately detect small targets in a maritime environment. The key to analyzing sea clutter is creating a robust simulation in which the characteristics of real-world components can be fully tested before a final system is produced. Thus, an emphasis will be placed on the accurate modeling of sea clutter given real radar

parameters to predict the performance of an X-band radar within a maritime environment. Targets of interest, similar to birds, will be simulated as point targets, and multiple clutter mitigation and cancellation techniques will be used to separate sea clutter from the target to achieve an accurate detection.

1.2 Contributions

This thesis contains the following contributions:

- The characteristics and models of sea clutter scattering were studied to produce an adaptable simulation for a variety of system configurations.
- Real-world antenna designs were imported from an electromagnetics simulator to reproduce the realistic effects of sea clutter as it relates to sidelobe and back-radiation of a maritime deployed radar system.

1.3 Thesis Outline

This document is organized into the following chapters:

Chapter 2 gives an overview of radar operation including waveforms, bandwidth, range equation, Doppler effect, and signal noise comparisons in order to build the foundation for the simulation geometry and data collection methods. In addition, ocean wave characteristics are given an introduction with some context on how they are formed, quantified, and their scattering properties.

Chapter 3 introduces the parameters needed to simulate a maritime environment starting with the physical parameters: clutter bin geometry, sea surface modeling, and antenna gain. Next, three parameters necessary for simulating sea clutter are discussed, which include: amplitude statistics, Doppler characteristics, and mean backscatter or reflectivity. With a focus on X-band, these parameters

and their equations will be discussed to give insight into how the sea clutter simulation was constructed.

Chapter 4 discusses the generation of sea clutter returns in relation to the models given in the previous chapter including the amplitude statistics and Doppler characteristics. Subsequent data processing techniques are explained in this chapter. After Doppler processing, the clutter will be evident; thus, techniques to mitigate and cancel the clutter will be derived and implemented.

In Chapter 5, a brief overview of detection theory is given as well as an implementation of Cell Averaging Constant False Alarm Rate (CA-CFAR). Following this, the detection results are given for each of the clutter cancellation techniques.

Chapter 6 concludes the thesis with information about what was accomplished and what lies ahead for this area of research.

Appendix B gives a more in-depth discussion of how to calculate the parameters of the Evolving Doppler model in [3]. In addition, tables of constants are supplied here to guide the reader on how to calculate the Doppler spectrum of a maritime environment.

Chapter 2

Background

2.1 Radar Background

2.1.1 Transmitted Waveform

Radar systems rely on the transmission and reception of electromagnetic energy to detect targets within a scene. Time-varying electromagnetic waves are radiated from an antenna element, which acts as a spatial filter for what the radar system is able to detect from the scene. The emitted signal is modified by the environment and/or targets of interest in the scene and returned to the radar, which samples the returned signal as a complex voltage. The signal transmitted, both real valued and band-limited, can be represented by the equation,

$$\tilde{x}(t) = A(t)\cos(2\pi F_o t + \theta(t) + \theta_o) \quad (2.1)$$

where F_o is the center frequency of the transmitted signal, $A(t)$ and $\theta(t)$ are the amplitude and phase functions which are modulated in time, t , and θ_o is the initial phase of the transmitted signal. Conventionally, the signal is processed at a lower intermediate frequency, where the signal is mixed down from F_o , the higher carrier frequency. This conversion removes the need for a high-speed analog-to-

digital converter (ADC), as Nyquist's Theorem requires the sampling rate to be twice the highest frequency content of the signal being sampled. The signal, at complex baseband, is represented in terms of the complex envelope and can be expressed by,

$$x(t) = A(t)e^{j\theta(t)} \quad (2.2)$$

which shows the same modulated amplitude and frequency in time, but removes the dependence on the center frequency and ignores the random phase offset, θ_o , assuming that the waveform generator can arbitrarily generate the waveform with a known starting phase. Modern radar systems transmit the signal at the maximum value of A for the given pulse duration to ensure the amplifier is operating in the saturation region which maximizes the energy of the signal for the duration of the pulse.

For waveforms that transmit at a singular frequency, the bandwidth of the transmitted pulse is inversely proportional to the time duration of the pulse and is given by,

$$\beta \approx \frac{1}{T_p} \quad (2.3)$$

where T_p is the time duration of the non-frequency modulated pulse. The bandwidth of linear frequency modulated waveforms is defined as the span of frequencies traversed during the total time duration of the pulse. Lastly, for phase-coded waveforms, the bandwidth is similar to Equation 2.3 in that the bandwidth is inversely proportional to the time duration of each phase chip. This is expressed as,

$$\beta \approx \frac{1}{T_c} \quad (2.4)$$

where T_c is the time duration of the phase chip. Since the amplitude of the transmitted waveform is kept at its maximum values, phase modulation schemes

are the primary factor in waveform design.

2.1.2 Range Resolution and Bins

The radar's ability to distinguish two targets at two different ranges depends on the range resolution of the transmitted waveform. The range resolution is inversely proportional to the bandwidth of the transmitted signal and is given by the expression,

$$\Delta R = \frac{c}{2\beta} \quad (2.5)$$

where c is the speed of light propagation in the given medium, and β is the bandwidth of the transmitted signal, $x(t)$. The range bins can then be defined as the radial volume of space extending from the phase center of the antenna, spaced by increments in range resolution ΔR . Within each range bin, all contributions of every scattering object within the range bin are coherently summed into one echo observed by the radar. Radar signals generally have much higher bandwidths than other applications in order to achieve finer range resolutions, which can prove challenging as the Analog-to-Digital Converter sampling rate must be at least twice the highest frequency content of the complex envelope to avoid aliasing.

2.1.3 Radar Equation

The power received from any given point scatterer in the scene can be given by the radar range equation modeled as,

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 L_s L_a R^4} \quad (2.6)$$

where, P_t is the power transmitted in watts, G_t and G_r is the gain on transmit and receive, respectively, λ is the wavelength in meters, σ is the radar cross section

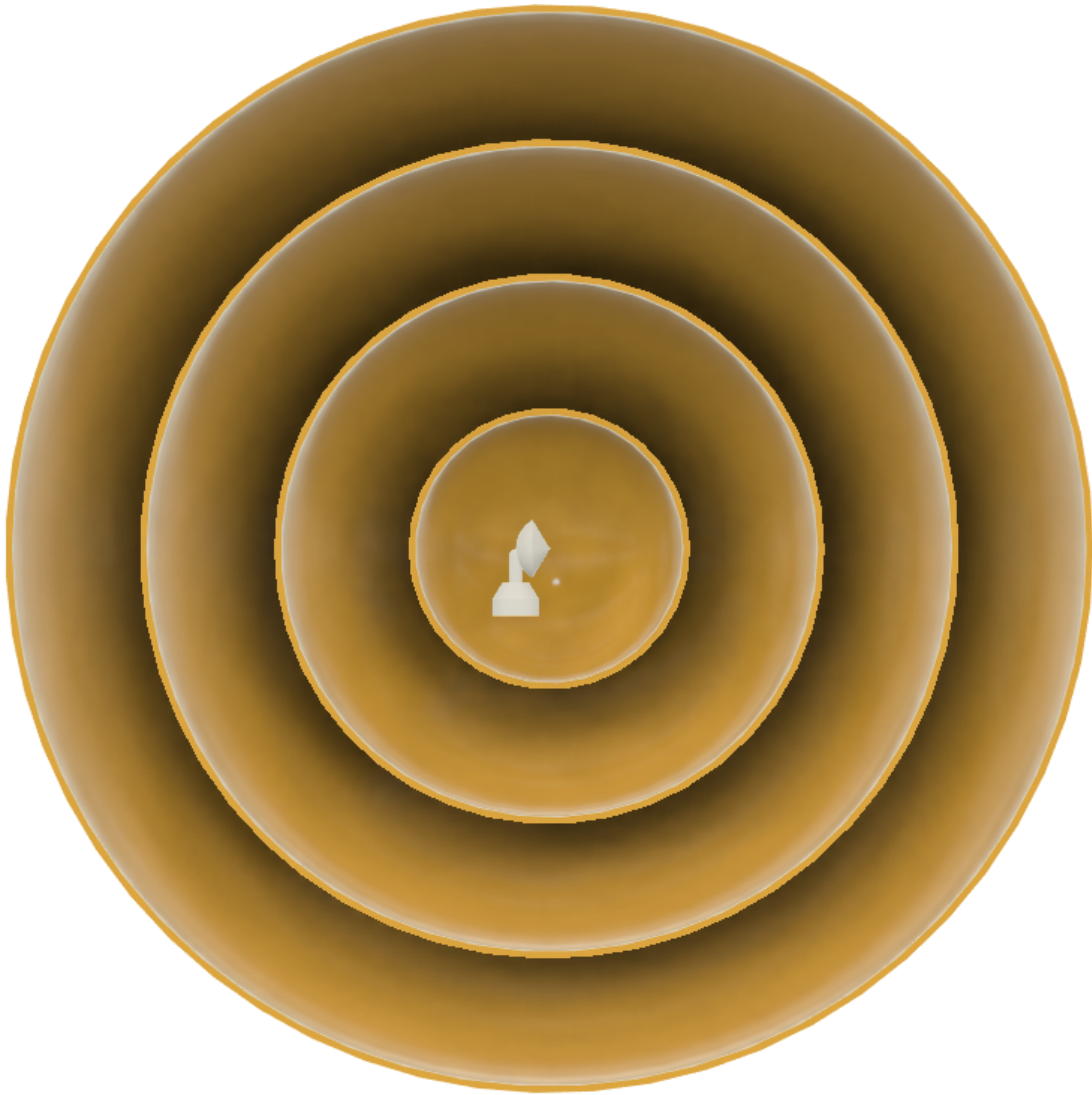


Figure 2.1: Range resolution depends on the bandwidth of the transmitted waveform, while range bins represent discrete distance intervals measured from the antenna's phase center.

(RCS) value of the given target, L_s is the losses due to the system, L_a is the atmospheric losses and lastly, R is the radial range of the scatterer relative to the radar. The radar range equation has few features that merit observing.

Gain is defined as the ratio of the maximum power density to the omnidirectional density[11]. For monostatic radar, the gain on transmit and the gain on receive are equivalent, so G_t and G_r can be replaced with G^2 . However, the gain when transmitting or receiving at any particular point, $P(R, \theta, \phi)$, in the scene has an associated gain, $G(\theta, \phi)$, which is dependent on the elevation and azimuth angles to the point P . Therefore, when referencing the radar range equation, the isotropic backscattered power density at some point P depends on the particular gain associated with that direction. Since antennas act as a spatial filter for the radar, they are generally purpose-built for the application in which they will be used. Highly directive antennas allow pinpoint focus on an area of interest, blocking out returns from any scatterer in the scene that does not lie in the mainlobe or sidelobes. Wide-beam antennas transmit and receive on a broader scale, capturing returns from a greater collection of scatterers in the scene. Antenna radiation patterns of different kinds have their application-specific purpose; however, when dealing with small target detection in maritime radar applications, narrow main beams with low sidelobe levels (SLL) are often preferred to spatially reject clutter returns from the ocean surface.

The RCS value represents the power that is reradiated back to the antenna from the scatterer. RCS is more appropriately described as a random variable where the value of the radar cross section of a scatterer is dependent on a myriad of factors such as aspect angle, center frequency, polarization, surface texture, etc. Often when measuring the returns from a particular target, the incident wave on the target might have been transmitted in one polarization, but when the electro-

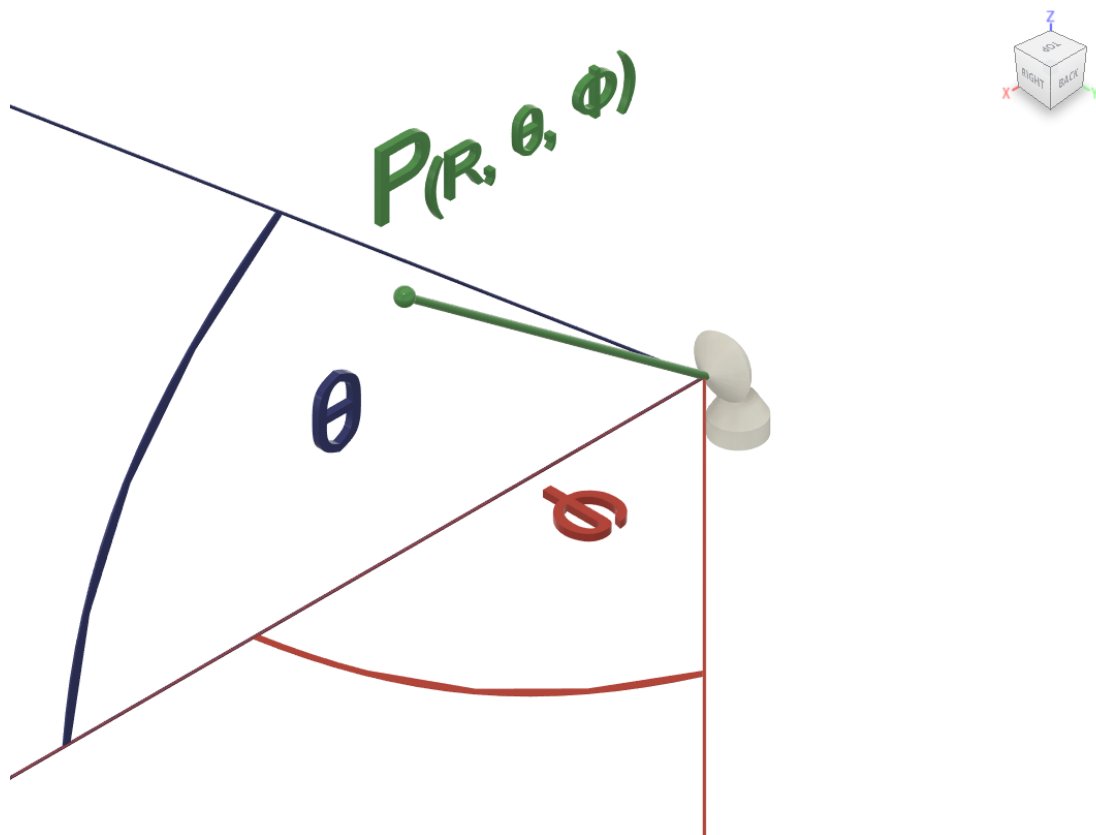


Figure 2.2: Any point in the scene can be referenced from the phase center of the antenna using spherical coordinates, defined by range (R), elevation angle (θ), and azimuth angle (ϕ).

magnetic wave is reflected off of the target, it is common to observe a portion of the energy reradiated as the opposite polarization, termed cross-polarization. In the real world, it is common to see this isotropic backscattered radiation from a target in both vertical and horizontal polarizations, which decreases the potential reradiated energy back to the radar if the system only supports a singular polarization; however, for Equation 2.6 it is assumed the target is a point target with one RCS value and only one polarization considered, thus it does not account for the cross-polarization effect. For land and sea surfaces, the RCS value is often approximated by defining an average or median value for the RCS denoted as the surface reflectivity, σ^o . The unit-less measure of reflectivity is given by the equation,

$$\sigma^o = \frac{\sigma}{A_s} \left[\frac{m^2}{m^2} \right] \quad (2.7)$$

where σ is the RCS value of the surface patch and A_s is the area of the surface patch observed by the radar. Reflectivity of a surface is highly dependent on the texture of the surface, the grazing angle, the center frequency of the radar, and the polarization of the radar. The texture of the surface is influenced by the geometry of the terrain, weather conditions, and surface type, which is why defining an average or median value of reflectivity is beneficial, as few models will suffice to accurately capture the information about scatterers on the reflective surface. The grazing angle, or depression angle, is also important to the reflectivity of the surface and is defined as the negative elevation angle to that surface. The terms “grazing” angle and “depression” angle are used interchangeably and have the same meaning. The reflectivity of a given surface tends to increase as the grazing angle increases, and the radar begins to look perpendicular to the surface. Because of the complex interaction between these inputs, it is better to utilize a model that accurately describes the reflectivity of a surface rather than solely relying

on a probability density function (pdf) to describe the surface. For maritime deployments, backscattered radiation from the sea surface is called sea clutter, and its reflectivity heavily depends on the surface conditions of the ocean.

Lastly, the radial range to the target, R , is the distance in meters from the radar to the scatterer. Losses in the power received are most common in radar applications through the quadratic relationship range has to the power received. Since $P_r \propto 1/R^4$, the received signal strength drops dramatically as the radar looks farther in range. Thus, small RCS targets become difficult to detect at long ranges due to the low amount of backscattered power received from the already small power reflected from the target itself.

2.1.4 Received Waveform

When $x(t)$ is transmitted from the radiating element, it propagates outward in all directions with a gain equal to $G(\theta, \phi)$ of the antenna radiation pattern. The electromagnetic wave travels at the speed of light, reflecting off of scatterers within the scene. These reflections are then received by the radar as the return waveform is incident on the radiating element. The waveform received is considered to be a copy of the transmitted waveform that is both delayed in time and shifted in frequency.

$$y(t) = \hat{A}(t - \tau_o)e^{[j(2\pi F_o(t - \tau_o) + \theta(t - \tau_o) + \phi(t))]} + N(t) \quad (2.8)$$

where $\hat{A}$ is the amplitude of the received signal scaled by factors within the environment: propagation losses, target RCS, clutter reflectivity, etc. can cause the amplitude of the received signal to be scaled differently. The time delay, τ_o , represents the two-way round trip time delay of the waveform from a scatterer whose range corresponds to $R_o = c\tau_o/2$. Lastly, $N(t)$ is simply the noise added to the received waveform, which will be expanded later in Section 2.1.6.

2.1.5 Doppler Frequencies and the Coherent Pulse Train

In pulsed Doppler radar systems, it is common to process multiple pulses together to gauge the target velocity through the use of the Fourier transform (FT). One pulse is transmitted at a known starting phase, followed by a time duration where the radar is listening for an echo, then the next pulse is transmitted as the cycle repeats. The Coherent Pulse Train (CPT) denotes the linkage of these pulses with a known starting phase by some time interval denoted as the Pulse Repetition Interval (PRI). Specifically, the PRI denotes the time duration that the radar has to transmit and receive one waveform iteration in Equation 2.1 before sending the next iteration. The PRI can also be described in frequency given by the Pulse Repetition Frequency (PRF), where $PRF = 1/PRI$, which denotes the number of pulses per second the radar transmits. Figure 2.3 shows a CPT composed of four pulses plotted over time.

The Doppler effect on $x(t)$ is due to the time difference of the arrival of two different points on the waveform $x(t)$. As a target is in motion, the location of the target when the beginning of the waveform begins to reflect is different than the location of the target when the end of the waveform is reflected back at the radar, assuming the target has a nonzero radial velocity relative to the radar. This difference in position due to the target's velocity causes the waveform to have a different expectation of arrival time from the two different points on the waveform, and either a compression or expansion is applied to the waveform. Thus, the difference in arrival times between the two different points on the waveform is called the Doppler effect. Then, the difference between the original center frequency of $x(t)$ and the new center frequency of the reflected waveform is called the Doppler

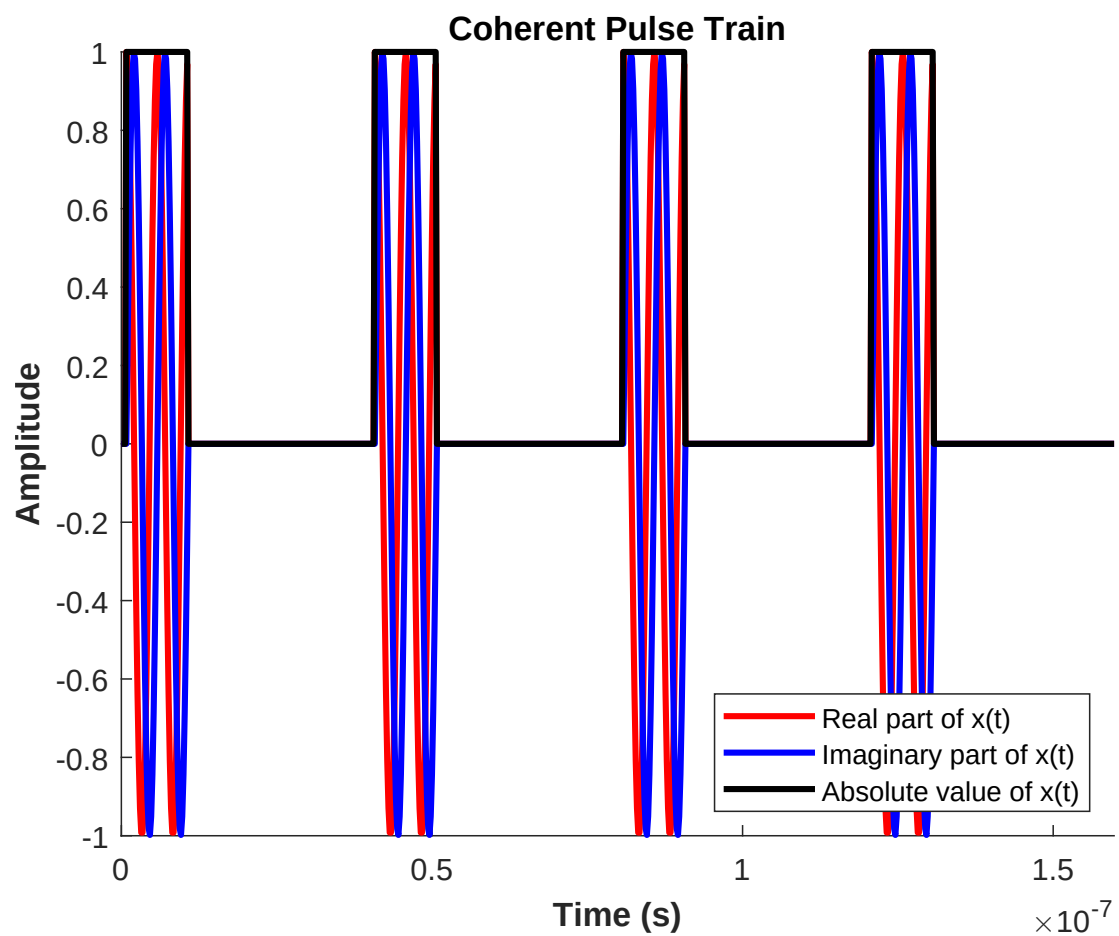


Figure 2.3: A Coherent Pulse Train (CPT) of four pulses linked together spaced in time by the Pulse Repetition Interval (PRI).

shift. The Doppler shift caused by a target in motion is given by,

$$F_D = \frac{2v}{c - v} F_o \quad (2.9)$$

where v is the radial velocity of the target relative to the radar and F_o is the original center frequency of the waveform in Equation 2.1. Equation 2.9 accounts for all potential radial velocities of the target, but it is common when $v \ll c$ to express the Doppler shift as,

$$F_D = \frac{2v}{\lambda} \quad (2.10)$$

where both equations are based on the center frequency of the radar through λ . Conventionally, a positive Doppler shift correlates to a target moving towards the radar, and a negative Doppler shift correlates to a target moving away from the radar.

Modern pulsed Doppler radar systems measure the Doppler shift of successive pulses to confidently determine the velocity of a target. The ability to resolve distinct velocities of a target, or Doppler shift frequencies of that target, is directly proportional to the PRF of the radar. The Doppler resolution can then be given by,

$$\Delta F_D = \frac{PRF}{m} = \frac{1}{mPRI} \quad (2.11)$$

where m is the number of pulses processed in the coherent processing interval (CPI). This relation reveals that the longer the radar observes the scene, the better it can resolve two distinct velocities.

2.1.6 Thermal Noise and the Signal-to-Noise Ratio

Thermal noise, denoted $N(t)$ in Equation 2.8, is a result of electron motion in a material with a temperature above 0° K. In a radar system, noise is additively in-

roduced throughout every component of the receive chain, thus noise compounds the more components the receive signal must travel through. The noise power at the output of the receiver can be approximated by the equation,

$$P_n = k_B \beta F T_s [W] \quad (2.12)$$

where $k_B = 1.38 \cdot 10^{-23} J/K$ is the Boltzmann constant, β is the receiver bandwidth in Hertz, F is the noise figure and T_s is the system temperature measured in Kelvin. The noise figure is a unit-less scalar value that is the ratio of the input signal-to-noise ratio (SNR) to the output SNR of the collective components in the receive chain. The SNR at the receiver output is then a ratio of power received to the noise power, and before any modification or processing is given by the equation,

$$\text{SNR} = \frac{P_r}{P_n} = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k_B \beta F T_s} \quad (2.13)$$

Figure 2.4 shows a simulated range profile with an embedded target to show the SNR.

2.1.7 Noise Versus Clutter

In real maritime radar deployments, noise is not the dominant interferer, clutter is. Noise differs from clutter in that noise is considered to be Additive White Gaussian Noise (AWGN), and is an effect of receiver temperature and characteristics. Clutter is a result of the deployment environment and behaves differently from noise, as it is considered to be multiplicative, not additive, thus the strategies for overcoming noise will not transfer to clutter.

Given the relation in Equation 2.13, overcoming the noise power is done simply by causing an increase in the power received. For an uncontrolled environment,

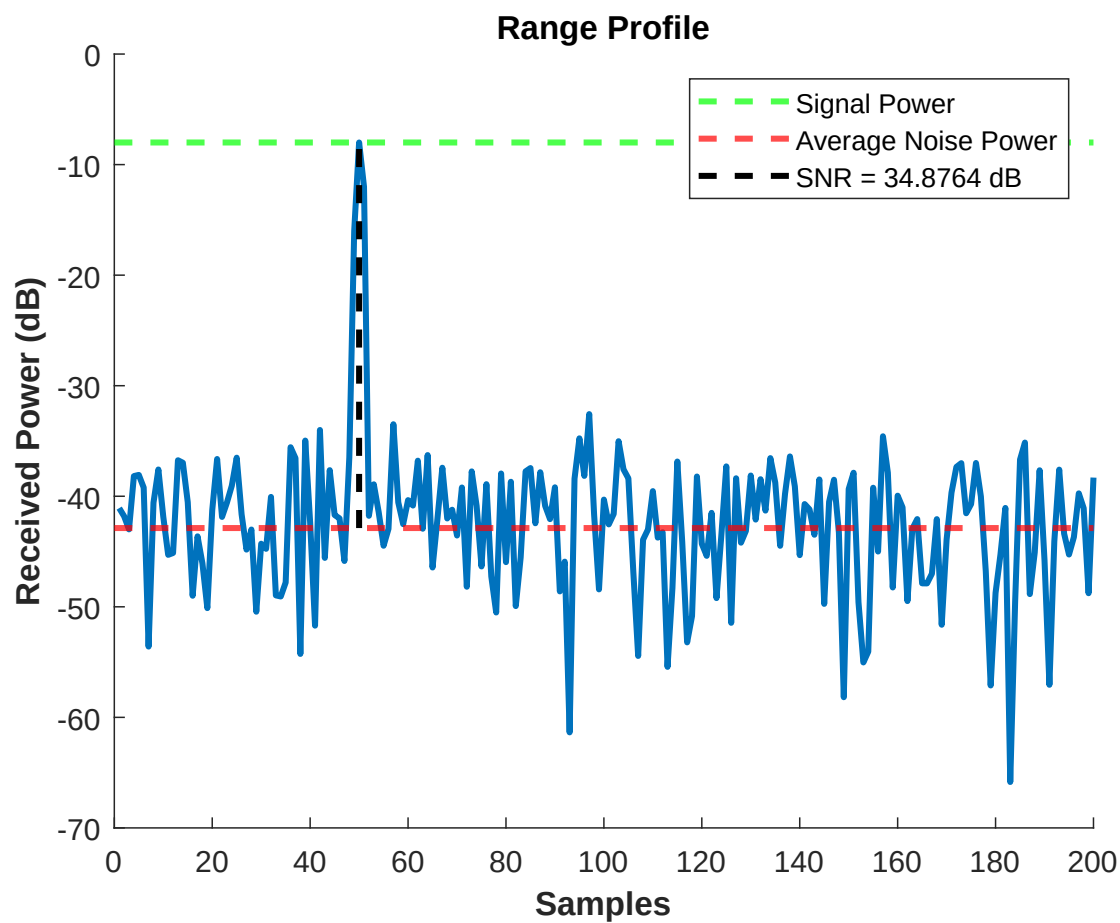


Figure 2.4: Example range profile illustrating the signal-to-noise ratio (SNR).

Equation 2.6 shows that the simplest way to increase the power of the received signal from the desired target is to increase the transmit power, the gain of the antenna, or the wavelength of the radar. All solutions yield an increased SNR, allowing a user to separate targets of interest from the mean value of the AWGN, or the noise floor.

Clutter presents a much more challenging task to overcome. Richards defines the equation below for distributed scatterers contained within an area that is pulse-limited [11],

$$P_r(t_o) = \frac{P_t G^2 \lambda^2 \sigma^o \Delta R \phi_n}{(4\pi)^3 R_o^3 L_s L_a(R_o) \cos \delta} \quad (2.14)$$

This equation, though similar to Equation 2.6 for the individual point target where RCS is denoted σ , associates an area with reflectivity as σ^o with the RCS. Equation 2.14 is derived by integrating Equation 2.6 over the radial range difference denoted ΔR , and the azimuth resolution ϕ_n , of the area of interest. The input, t_o , exists as a reference to the time delay to the center of the clutter cell with an associated range, $R_o = ct_o/2$. The grazing angle, δ , from the phase center of the radiating element to the center of the reflective surface is also relevant to the power received as $P_r \propto 1/\cos \delta$. Equation 2.14 shows that power varies as R^{-3} because the area of the resolution cell only increases in the cross-range dimension as R increases due to a fixed ΔR as a result of the range resolution of the transmitted waveform. Power transmitted, gain, wavelength of the radar, system and atmospheric losses, (P_t , G , λ , L_s , and L_a) all follow the same form as Equation 2.6 and utilize the same units. The pulse-limited case is used here to ensure proper simulation of the sea surface with respect to the radar as scattering surfaces will be accounted for in each range bin. [11] also contains a beam-limited version of Equation 2.14, but it is not considered for this work as simulation of the sea surface at low grazing angles causes the beam-limited case to provide sparse data about scattering surfaces as

the range increases. Simple techniques to overcome noise will cause the clutter returns to be amplified also, therefore, a different approach is needed to extract the targets of interest from a maritime scene.

The signal-to-interference-plus-noise ratio (SINR) is an important metric in gauging the performance of clutter cancellation and subsequently the radar's ability to detect a target in the scene. SINR is similar to Equation 2.13 and is given by,

$$\text{SINR} = \frac{P_r}{P_n + P_c} \quad (2.15)$$

where the addition of the term P_c in this case accounts for the power received by the radar due to clutter. Interference in SINR can often be a broad term, representing any unwanted signal such as clutter returns, interference from other devices, or jamming. In this work, the interferer will be cluttered returns from the sea surface, therefore, the power received from the ocean will follow the relation of the pulse-limited case for area scatterers given in Equation 2.14. Sea clutter simulations and, later, clutter cancellation techniques have found their niche in performance prediction modeling to provide insight into the behavior of the maritime radar system before the system is fully built and deployed, and SINR is an important metric for weighing the effectiveness of clutter removal techniques.

2.2 Ocean Wave Characteristics

Several factors influence the characteristics of waves on the sea surface: wind speed, wind duration, and fetch, which is the distance that the blowing wind covers. In order for waves to form, the wind needs both strength and time to influence the surface of the water with any reasonable impact. Because waves do not form or grow instantly due to increased wind speeds, it is common to characterize the sea surface with the average wind speed, not transient wind gusts.

As the wind blows on the sea surface, energy is transferred from the wind to the waves. This can occur either as a direct push on the waves or friction drag from the air to the water, both causing an increase in energy of the waves [12].

The height of the wave, H , is the elevation difference between the peak and the trough of a wave. Similarly, ocean waves have an associated period T . The average speed of a singular water particle riding on a wave is $c_p = \pi H/T$, and the wave speed as it relates to a given wavelength, L , is $c_w = L/T$, which allows for the relation between the water particle speed and wave speed given by the equation,

$$\frac{c_p}{c_w} = \frac{\pi H}{L}. \quad (2.16)$$

However, in a maritime radar deployment, we are often concerned with the celerity of the wave, the speed at which the wave's peak and trough travel through the water, rather than individual particles riding on the surface. In this case, a similar relation can be built on Equation 2.16 to identify the celerity of waves as it relates to wavelength, L , and water depth, h . For deep water, where $h > L/2$, the celerity is given by the equation,

$$c_d = \sqrt{\frac{gL}{2\pi}} \quad (2.17)$$

where $g = 9.81ms^{-1}$ is the acceleration due to gravity. For shallow water, where $h < L/20$, the celerity is given by the equation,

$$c_s = \sqrt{gh} \quad (2.18)$$

where it can be seen that the celerity of the waves in shallow water is independent of the wavelength or period [12].

As waves continue to gain energy, their height will not scale with the period, but the period will continue to grow as the height becomes more restricted due

to adverse forces on the waves such as gravity and cross currents. For radar applications, the scattering characteristics of the ocean surface become more homogeneous as the wave period increases with respect to the range resolution of the radar, causing the surface to appear as one continuous surface. However, when considering a “choppy” ocean, wave height and a short period cause challenges for a radar system as electromagnetic scattering becomes more erratic. When considering a larger swath of ocean, many times ocean wave periods are intermingled and overlay on each other. Since waves can travel great distances, one particular section of the ocean surface is often considered to be the superposition of multiple different wave fronts, each with varying height and period from multiple sources that could be hundreds of miles apart. Simply put, the ocean surface manifests itself randomly due to a myriad of influencing factors but can be described as the summation of individual sinusoidal components as explained by a Fourier series.

Ocean waves can break in two different ways: through plunging breaks and through spilling breaks. Plunging breakers are commonly seen on beaches, and they generally degrade, losing form soon after breaking. They are characteristically shaped with a rounded back and a concave front. Spilling waves are of greater importance for the deployment of a maritime radar. These waves are concave on both sides, reaching a peak where the top of the wave breaks over just slightly. After breaking, the wave retains its form and continues onward [12]. Spilling breakers are more commonly called “whitecaps”, and can occur in the same wave multiple times as the wave reforms, steepens, and breaks again. During the break of a whitecap, water is launched from the breakage often at a higher velocity than the wave itself. For maritime radar applications, these water particles can prove to be the source of the Doppler spikes seen in range-Doppler maps [3].

Waves can be classified in many different ways, and one common way is by

period. Table 2.1 shows how waves are classified by period and frequency. The smallest waves, capillary waves, are short high-frequency waves that are the first to develop in a calm sea. The classification of waves goes all the way up to trans-tidal waves, which are caused by storm systems, eruptions of volcanoes, shifts in tectonic plates, Coriolis force, or even the Sun and Moon. Waves can also be classified by wavelength, L , and significant wave height, $H_{1/3}$, which is defined as the average height of the highest third of waves. Proper classification of waves is difficult, yet necessary, to account for the scattering characteristics of the waves.

Table 2.1: A classification of ocean surface waves by period and frequency [12]

Wave Name	Period Band (sec)	Frequency Band (cps)
Capillary	$0 \rightarrow 1 \times 10^{-1}$	$\infty \rightarrow 1 \times 10^1$
Ultragravity	$1 \times 10^{-1} \rightarrow 1 \times 10^0$	$1 \times 10^1 \rightarrow 1 \times 10^0$
Gravity	$1 \times 10^0 \rightarrow 3 \times 10^1$	$1 \times 10^0 \rightarrow 3.33 \times 10^{-2}$
Infragravity	$3 \times 10^1 \rightarrow 3 \times 10^{2*}$	$3.33 \times 10^{-2} \rightarrow 3.33 \times 10^{-3}$
Long Period	$3 \times 10^2 \rightarrow 8.64 \times 10^{4**}$	$3.33 \times 10^{-3} \rightarrow 1.16 \times 10^{-5}$
Trans-tidal	$8.64 \times 10^4 \rightarrow \infty$	$1.16 \times 10^{-5} \rightarrow 0$

* 5 minutes

** 24 hours

2.2.1 Wave Types

As described above in Section 2.2, different wave types have different periods, and the wind causes waves to develop with both strength and time. Long-period waves are generally of little interest in gauging the performance of a maritime radar system, and thus capillary waves and gravity waves will be examined further. For any oscillation of an ocean wave to exist, three components must be present: an equilibrium, a disturbing force, and a restoring force to restore equilibrium [12].

Having all three components present allows for oscillation of a wave, and each type of wave has differing components, which drives their creation and propagation.

Capillary waves form as a precursor to other types of waves assuming that the surface of the water is calm and previously unaffected. These waves are caused by light winds blowing on the surface of the water which causes short, high-frequency waves to form. Capillary waves are often thought of as ripples in the water, and they travel a short distance at a very slow speed. The surface tension of the water acts as the restoring force of the capillary waves, pulling the ripples back to a calm equilibrium. These waves have a very short peak-to-peak wavelength, less than 1.73 cm, with a significant wave height of just a few millimeters and die off quickly as the wind disappears [3], [12]. They can also be embedded onto other, longer-period wave structures, such as gravity waves.

All types of gravity waves are generated by the continuous action of the wind on the sea surface. However, gravity waves are classified into two subsets: *sea* and *swell*. *Sea* gravity waves are higher-frequency gravity waves that have just formed from capillary waves. They are jagged and rough with very steep faces. By the time a *swell* gravity wave develops, the wind behavior is less effective in altering the wave, unlike capillary waves. If the wind stopped blowing, the *swell* gravity waves would continue with the energy they had gained, and the inertia of the gravity waves would prevent the waves from dissipating due to a gust of wind in the opposite direction [12]. The restoring force of all gravity waves is gravity. These waves have a widely varied significant wave height, as the terminology of ultragravity, gravity, and infragravity waves covers a wide range of possible wave characteristics, and as previously mentioned, wave height begins to taper out as wave period continues to increase. For maritime radar applications, these are the greatest extent of waves that matter as longer period waves have less effect on sea

clutter returns.

2.2.2 Sea Scales

Sea scales are a way to generalize sea surface conditions through the description of wind and wave conditions. Two popular sea scales are the Douglas Sea Scale and Beaufort Wind Force Scale. Traditionally, the Douglas Sea Scale has been used to refer to sea states as they relate to deployed maritime radar systems. Table 2.2 shows the Douglas sea states along with the wind and wave conditions of the associated sea state [3]. Attempting to classify the sea surface in this way allows for discretizations of wind and wave conditions; however, the sea surface in fact is hardly ever placed in one singular category at any given time. Therefore, Table 2.2 represents the wind and wave conditions for a *fully developed* sea. This terminology means that there are no intermediate or transitional states present, and the swath of ocean considered can be placed into one of these discrete sea states.

Sea state	Description	Sig. wave height (m)	Wind speed (m/s)	Fetch (NM)	Duration (h)
0	Glassy	no waves	0	-	-
1	Calm	0–0.1	0–3	50	5
2	Smooth	0.1–0.5	3–6	120	20
3	Slight	0.5–1.25	6–7.5	150	23
4	Moderate	1.25–2.5	7.5–10	200	25
5	Rough	2.5–4.0	10–12.5	200	25
6	Very rough	4.0–6.0	12.5–17	300	27
7	High	6.0–9.0	17–25	500	30
8	Very high	9.0–14.0	>25	-	-
9	Phenomenal	14.0+	≥ 25	-	-

Table 2.2: Douglas sea scale with associated wind speeds of a fully developed sea. Adapted from [3]

2.2.3 Scattering Characteristics

The physics of ocean waves are relevant to understanding the scattering characteristics observed by a maritime radar system. Whitrow [13], in his work, concluded the scattering sources of ocean waves as they relate to transmit polarizations. Both vertical and horizontal polarizations exhibit scattering due to the Bragg phenomenon and “whitecaps” or spilling breakers; however, for horizontally polarized radar systems, Whitrow notes the contribution of a “sea spike” component that is not as dominant in vertical polarizations. This component of sea spikes is caused by specular scattering of wave crests that do not or will not break. Whitrow also notes that whitecap scattering in the vertical polarization is much greater in magnitude than the horizontal polarization [13].

Bragg scattering is a phenomenon in which a received signal is amplified by coherent superposition through constructive interference of the received signal if the scattering surface exhibits a periodic structure similar to that of the transmitted waveform. For Bragg scattering to exist, the periodic distance of the scattering subsurface must follow the equation,

$$d = \frac{\lambda}{2 \cos(\theta_{\text{dep}}^\circ)} \quad (2.19)$$

where d is the wavelength of the periodic scattering subsurfaces, and $\theta_{\text{dep}}^\circ$ is the depression angle to the scattering surface in degrees. In the case of radar sea clutter, the distance of interest, d , is the wavelength of an ocean wave, previously referred to as L . Equation 2.19 describes the key dependence on both the center frequency of the radar and the incident angle of the transmitted waveform to produce a coherent superposition of return signals. Figure 2.5 shows the Bragg scattering distance as a function of both parameters, and Figure 2.6 shows the

Bragg scattering distance as a function of the depression angle for an X-band radar operating at 9.41 GHz. These plots show that for small depression angles, capillary waves with a wavelength generally less than 1.73 cm pose a significant source for Bragg scattering. The contribution due to Bragg scattering generally appears in a range-Doppler map as the zero-mean or near-zero-mean Doppler component of sea clutter, and due to capillary waves, it often smears across Doppler in a range-Doppler map instead of representing a singular Doppler frequency.

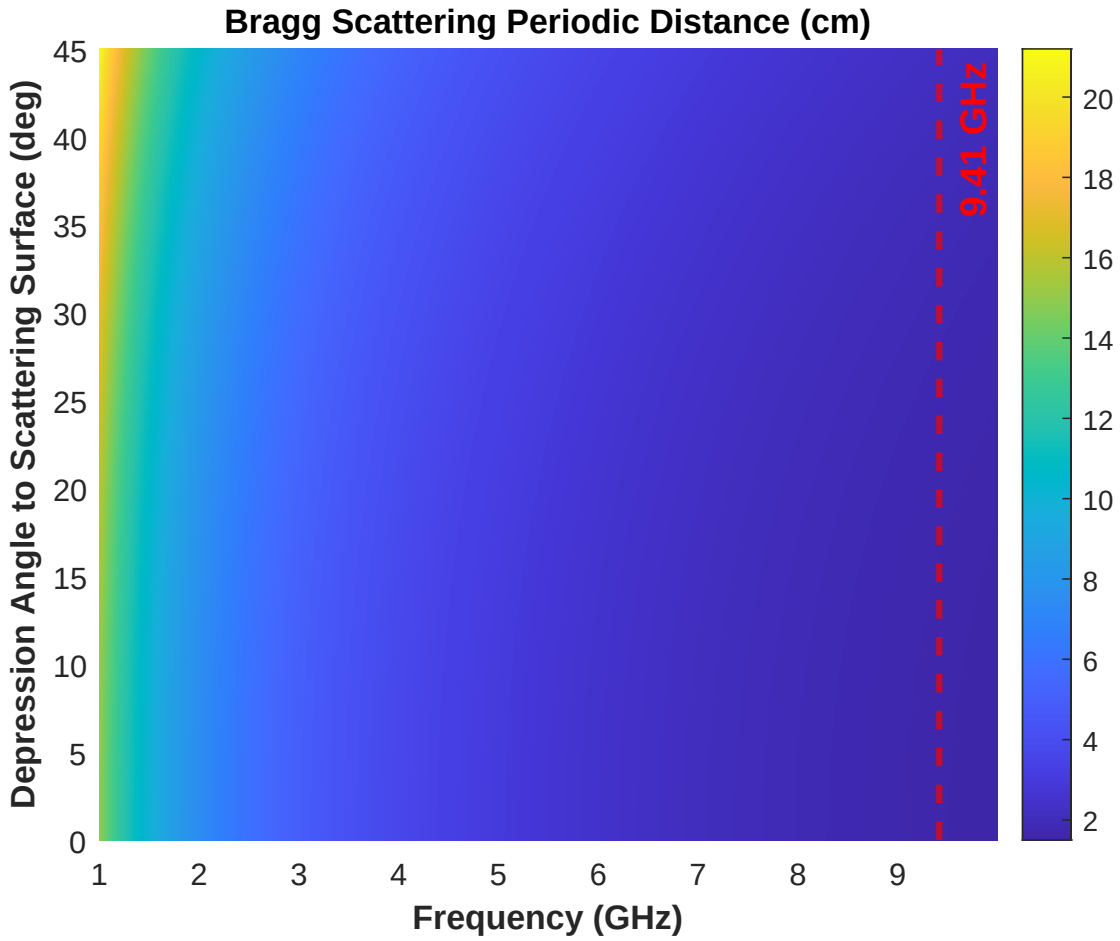


Figure 2.5: Bragg scattering distance as a function of frequency and grazing angle ranging from 1 – 10 GHz and $0^\circ - 45^\circ$

Spilling breakers, often termed whitecaps, are responsible for the *fast* scattering seen on a range-Doppler map. Because highly reflective water particles are

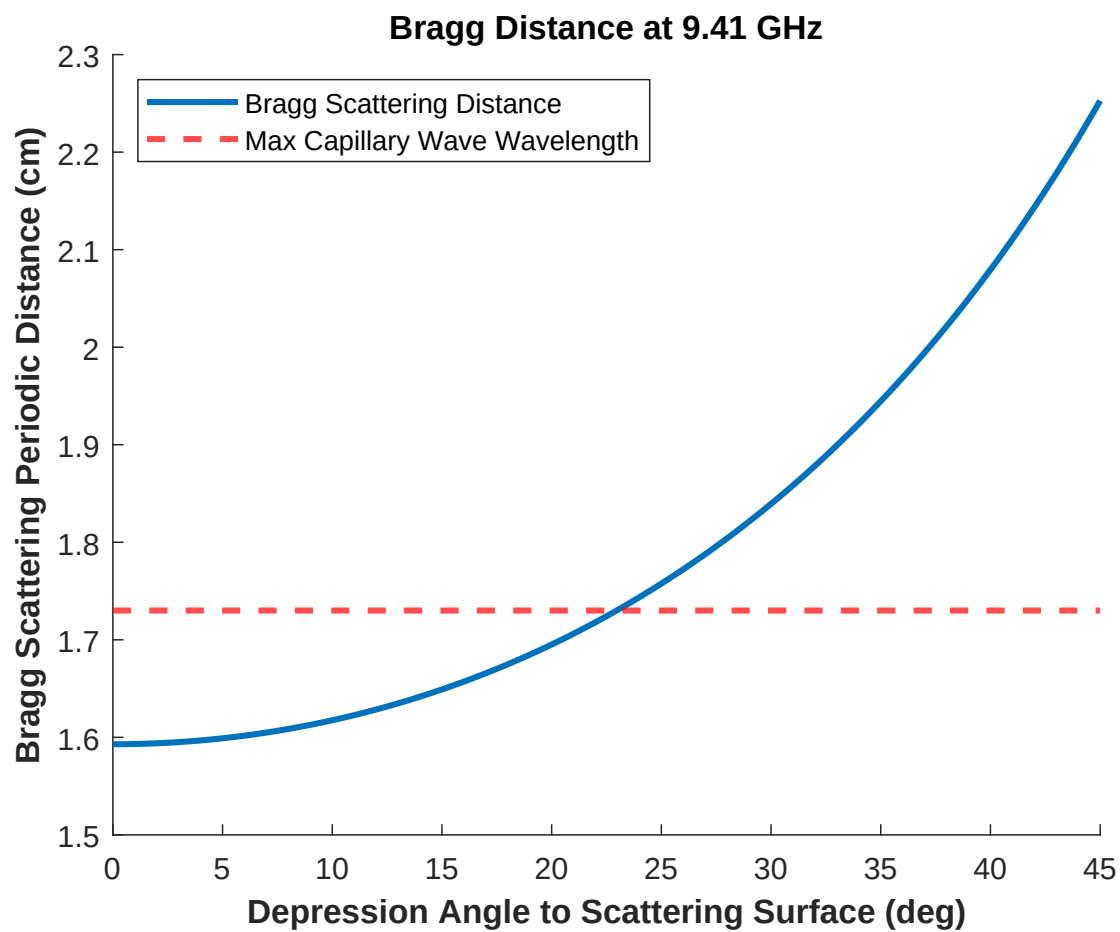


Figure 2.6: Bragg scattering distance for an X-band system operating at 9.41 GHz.

launched into the air generally at faster speeds than the wave itself, reflections occur at higher Doppler frequencies causing fast moving outliers in the range-Doppler map. Whitecaps, when they occur, decorrelate quickly with time as do unbroken peaks which are responsible for specular scattering, but for the duration they are present, they can overcome the return signals from targets of interest due to their reflectivity and generally higher mean Doppler component. The temporal decorrelation model helps to explain the Doppler characteristics and correlation time of these scattering types. Table 2.3 provides the coefficients needed to calculate the mean temporal correlation given by the equation,

$$T_{temp} = (a_{temp} + b_{temp}U + c_{temp}H_{1/3}) \times 10^{-3} \quad (2.20)$$

where U and $H_{1/3}$ are the wind speed and wave height, respectively. T_{temp} , the mean temporal correlation as measured at the $1/e$ point, is then used in the calculation of the autocorrelation model which can be represented by a Gaussian function given by,

$$\rho_n = \exp \left(- \left(\frac{|n|}{T_{temp}PRF} \right)^2 \right) \quad (2.21)$$

and the negative exponential function given by,

$$\rho_n = \exp \left(- \frac{|n|}{T_{temp}PRF} \right) \quad (2.22)$$

where $n > 0$ is the time delay. This model validates the trend that the Doppler spread increases, as it is proportional to the wave height divided by the wave period. The model does have its limitations with the coefficients used in Table 2.3, and can only consider $15^\circ - 45^\circ$ and a zero mean Doppler spectrum. However, there are other models that more fully model the Doppler spectrum, which will be covered in Section 3.3.

Pol.	a_{temp}	b_{temp}	c_{temp}
HH	6.34	−0.20	−0.23
HV	5.58	−0.10	−0.21
VV	5.83	−0.10	−0.21

Table 2.3: Decorrelation coefficients for the temporal decorrelation model.

Chapter 3

Sea Clutter Simulation

3.1 Simulation

Sea clutter is often simulated using statistical methods that provide a practical balance between realism and computational efficiency. Although ray-tracing techniques (as they relate to the tracing of isotropic backscattered radiation within a scene) are becoming more prevalent, their implementation remains limited by significant computational demands. Previous work has explored ray tracing in simulated environments [14]–[16], though only [14] addresses effects such as multipath propagation and self-occlusion in a meaningful way. As a result, statistical approaches remain the standard for most sea clutter simulations. These are based on models that characterize the amplitude distribution, Doppler behavior, and surface reflectivity of the clutter. When combined, these components allow for the generation of realistic clutter profiles with relatively low computational overhead. An accurate representation of the geometry of the sea surface is also important, as the simulation must assign physically consistent properties to each clutter patch based on its spatial context. By assigning a geometry to the simulation, defining the amplitude statistics of the received power, associating expected Doppler characteristics, and deriving proper reflectivity of the clutter bins, the simulation can

account for the backscattered returns to the radar system of a maritime environment.

3.1.1 Clutter Bins

The scene of any clutter simulation has to be divided into smaller sections for processing. These subsections of the scene are parsed in range by a scalar factor of the waveform-dependent range resolution of the radar and some predefined azimuth section, $\Delta\phi$, and are called clutter patches or clutter bins. By placing numerous scatterers within a single range bin, a homogeneous return signal can be synthesized, emulating the response expected from a maritime radar system. A high density of clutter patches per range bin yields more realistic returns that approximate a continuous, multivariate surface. However, while increasing the number of scatterers enhances realism, it also raises computational cost. Moreover, if too many scatterers are included, the central limit theorem causes the amplitude statistics to converge toward a purely Gaussian distribution, potentially masking non-Gaussian features of interest. Figure 3.1 shows how the clutter bins are segmented in a simulated environment.

Once the scene is divided into clutter bins, the geometry relating the clutter patches to the radar is calculated. The average reflectivity values, σ^o discussed in further detail in Section 3.4, depend on the depression angle to the clutter bin relative to the phase center of the antenna and the area of each clutter bin. The equation relating the depression angle from the center of the i th range bin to the radar is given by,

$$\theta_{\text{dep}}^o(i) = \sin^{-1} \left(\frac{H}{\sqrt{H^2 + R(i)^2}} \right) \quad (3.1)$$

where H represents the z-coordinate of the phase center of the antenna, and R represents the horizontal distance to the center of the i th range bin. Furthermore,

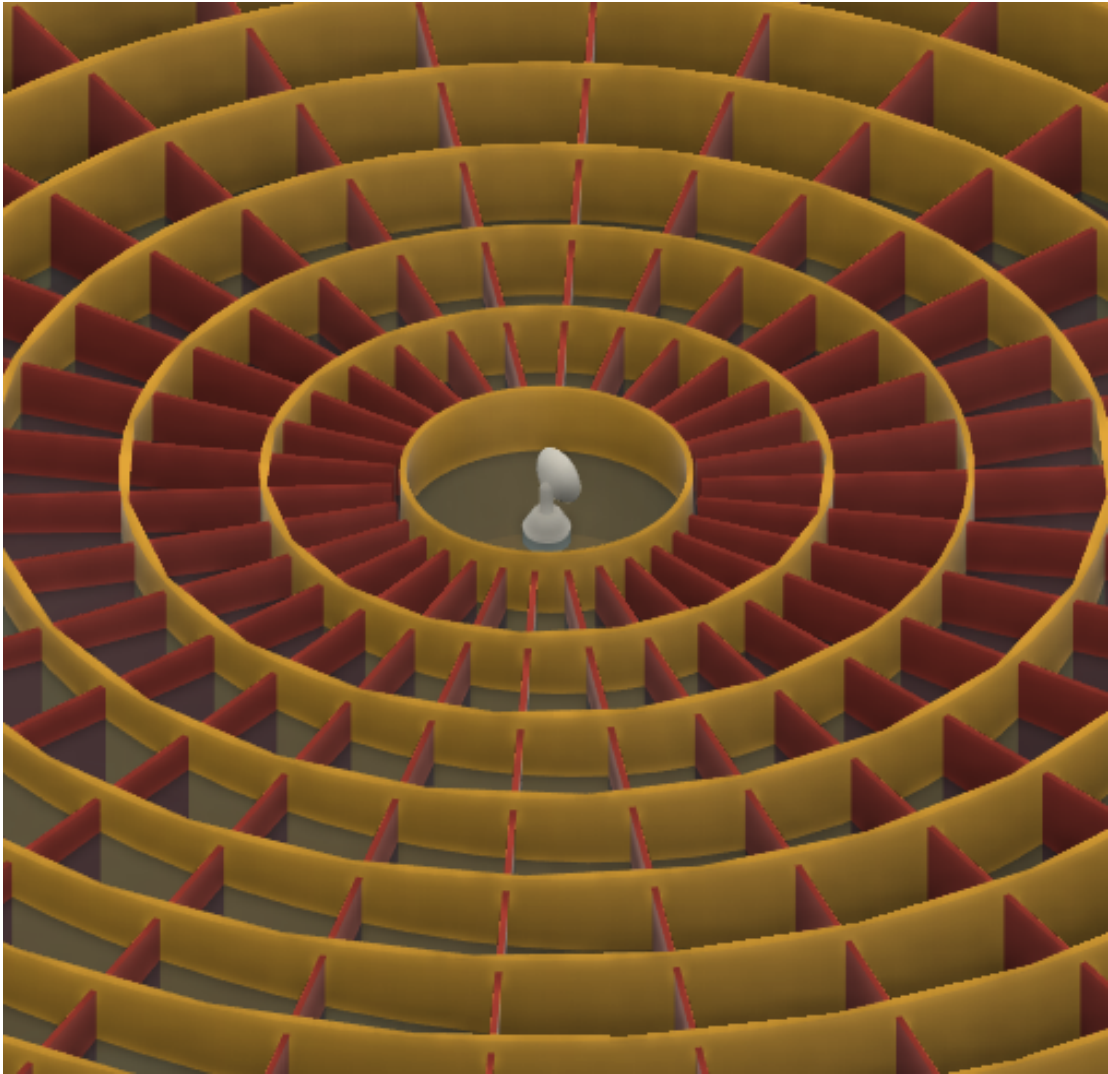


Figure 3.1: Clutter bins are defined in increments of the range resolution divided up by the desired azimuth resolution to create clutter cells. These clutter cells house the scattering characteristics needed to calculate the sea clutter returns for performance prediction modeling.

the two-way propagation time delay to the clutter bin, τ_c , must also be calculated to account for the time delay portion of Equation 2.8. Once completed, the geometric relationship of each clutter bin to the radar system is fully described, and the clutter simulation can proceed to account for the characteristics of amplitude, Doppler and reflectivity of the sea surface.

3.1.2 Sea Surface Modeling

Modeling the sea surface for a ray tracing focused simulation can be done by treating the sea surface as a closed-form bounded surface. When defining the texture of the surface, a specific point can be described by the exponential distribution [12]. The considered swath of ocean is often estimated to be stationary Gaussian due to the central limit theorem when extrapolating to nearby points within the ocean surface [12]. However, as the sea state increases because of stronger disturbing forces, the surface becomes more turbulent and complex as the periodicity of the waves grows, which undermines the stationary Gaussian assumption, limiting its ability to represent the actual ocean geometry. In this case, the ocean surface cannot be considered strictly Gaussian because waves are limited in height by breaking and wave motion must satisfy the nonlinear Bernoulli equation and the free-surface boundary conditions [12].

Self-occlusion occurs when part of the sea surface becomes obscured from view due to interference from another part of the sea surface. This can occur from the perspective of the radar when the waves peak in height before breakage. Self-occlusion is particularly an issue for maritime radar systems at low grazing angles, as the peaks of the waves directly interfere with the line of sight from the phase center of the radiating element to the troughs of the waves past a given peak. Ray tracing based models can account for the self-occlusion through a basic line-of-

sight algorithm that relates to Equation 3.1 by,

$$\mathcal{L}(s) = \begin{cases} 1, & \text{if } \theta_{\text{dep}}^{\circ}(s) < \theta_{\text{dep}}^{\circ}(s-1) \\ 0, & \text{otherwise} \end{cases} \quad \text{for } s = 2, \dots, n \quad (3.2)$$

where n is the number of potential scatterers represented in the simulation. $\theta_{\text{dep}}^{\circ}(s)$ is then the depression angle to a particular scatterer, s , along one section, $\Delta\phi$, in azimuth. By definition, $\mathcal{L}(1) = 1$ since the first clutter cell is always visible by the radar.

Multipath reflections occur when transmitted electromagnetic waves reflect isotropically off of multiple points from the ocean surface before returning to the antenna, and when multipath reflections occur, the time delay of the received signal can increase depending on how many surfaces the wave reflects off of. The received signals, when comparing both direct and multipath, can exhibit different phases, amplitudes, and polarizations, which can ultimately obscure the received signal from the target of interest through constructive or destructive interference. The degree to which electromagnetic waves scatter in this way is called the multipath factor and is quantified in three separate regions: the interference region, the transition region, and the diffraction region [17]. Each region occurs as the distance to the sea surface increases, with the diffraction region being the furthest out, towards the horizon.

3.1.3 Antenna Gain

Radar simulations often use abstract system parameters to evaluate performance in a given environment, with antenna gain patterns being a key factor. When high accuracy is not critical, many simulations approximate the antenna gain pattern using a sinc function. As shown in Figure 3.2, this approach provides an

acceptable approximation for environments where the gain can be assigned based on angle. However, the sinc approximation is less suitable for maritime radar systems, where the structure and directional intensity of sidelobes significantly influence near-in clutter characteristics.

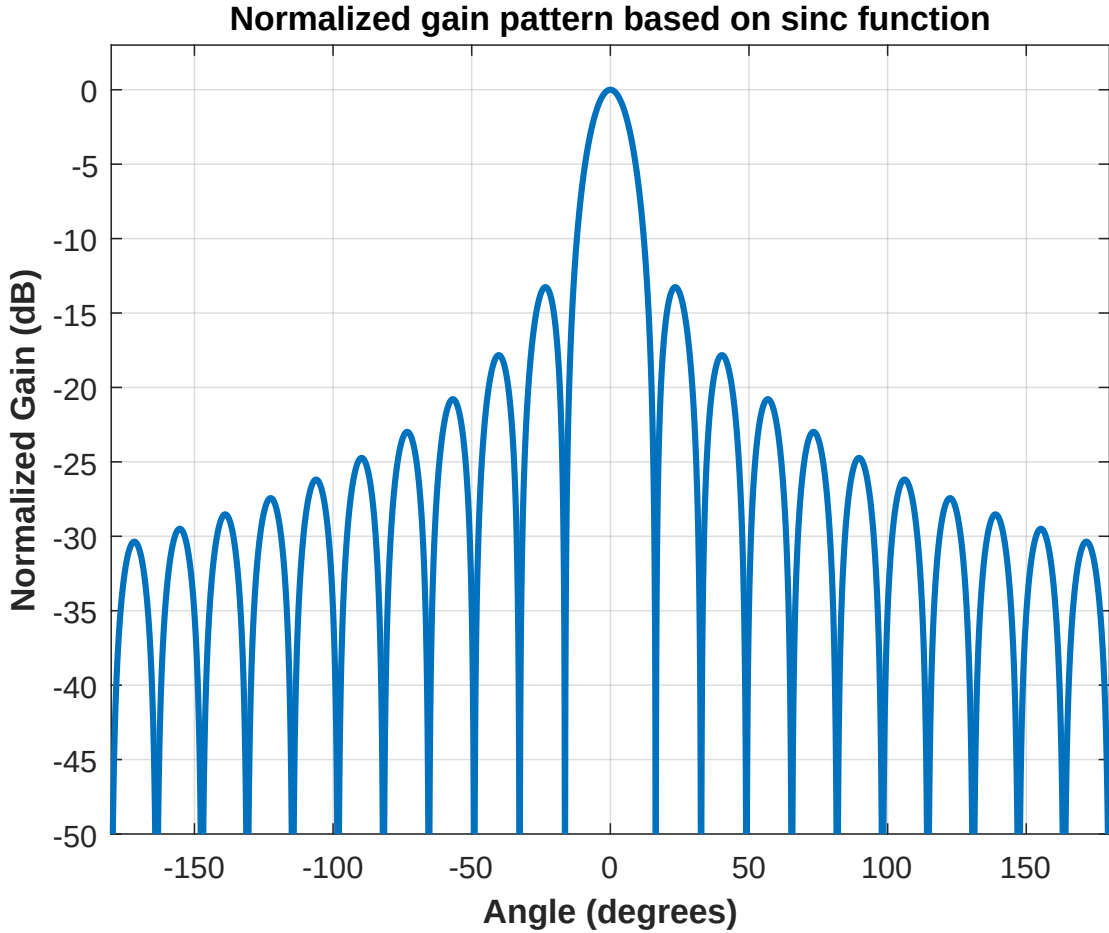


Figure 3.2: Sinc antenna gain pattern generally used in simulations.

Electromagnetic simulators, such as ANSYS HFSS, allow for complex simulation of antennas by numerically solving Maxwell's equations over finely meshed three-dimensional geometries. This enables the design and optimization of antennas under realistic conditions, including material properties and operating frequencies. One key output from HFSS is the far-field radiation pattern, which

describes the antenna gain as a function of direction in both azimuth and elevation. These radiation patterns can be exported from HFSS in standard formats and imported into MATLAB for further processing. Within MATLAB, the patterns can be visualized, rotated, interpolated, and integrated into larger radar system simulations. This allows for a more accurate representation of the antenna's behavior in the operational environment, especially in scenarios where sidelobe levels, null locations, or polarization characteristics are critical, such as in sea clutter modeling. By combining high-fidelity radiation data with simulation environments, designers can better predict and mitigate real-world performance limitations. Figure 3.3 is an antenna radiation pattern designed in HFSS [18], imported into MATLAB, which allows realistic gain values to be associated with the clutter bins of the simulation.

Once a radiation pattern is imported into MATLAB from HFSS, a few operations must take place: a spherical rotation, a translation of the antenna coordinate system to the location of the phase center of the antenna with respect to the simulated scattering surface, and an interpolation of gain values dependent on simulation geometry in order to assign a gain value to a clutter bin based on the pulse-limited representation of surface scatterers in Equation 2.14. To effectively perform a single line ray trace to the desired clutter bin, a spherical rotation must also occur to allow the main beam to coincide with the $+x$ axis. Then, the translation of the antenna radiation pattern to the desired height of the phase center of the antenna aperture can take place. This translation can be stepped in time to simulate platform motion if desired.

The portion of the radiation pattern used in the simulation is dependent on the simulation geometry itself. Importing the entire radiation pattern does not mean that it is used all the time, since reflectivity models and simulation geom-

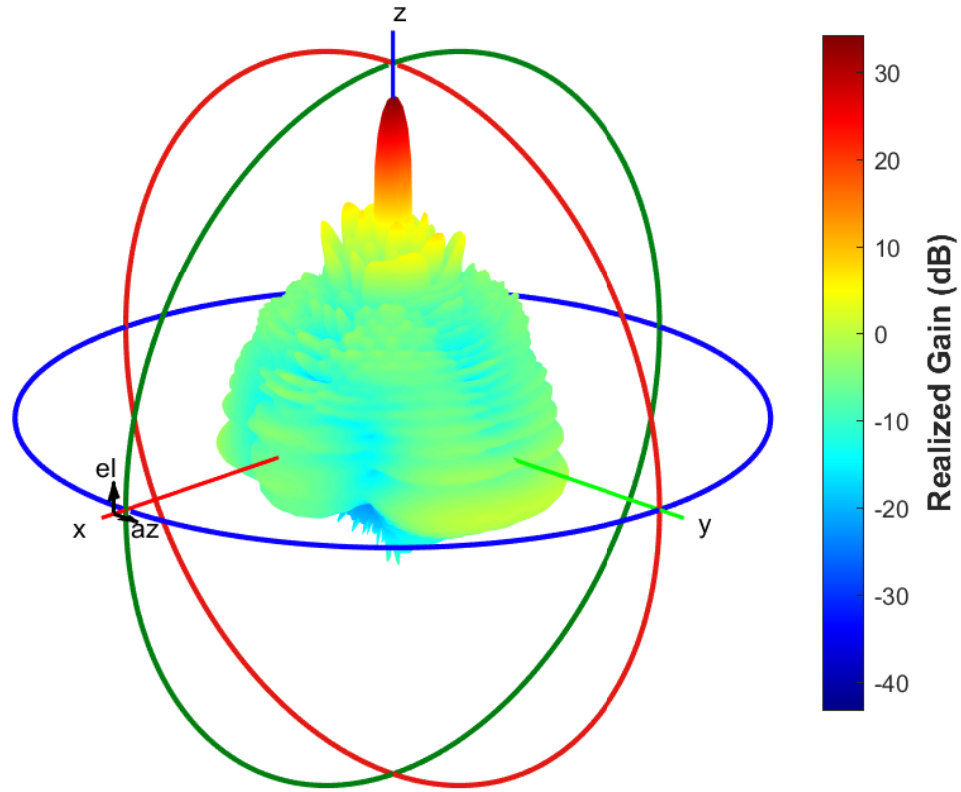


Figure 3.3: This figure displays a real world button-hook parabolic dish antenna radiation pattern designed by [18] in HFSS. The antenna used here is preferred due to its narrow mainbeam with a gain of 34.295 dB and a beamwidth of 4.24° in azimuth and 3.12° in elevation. The sidelobe structure of the antenna is preferred since the strongest sidelobes are -26.97 dB down from the mainbeam and are placed strategically such that they will not contribute greatly to the near-in sea clutter returns at low look angles due to the minimum range constraints of the radar. This radiation pattern is the raw input from HFSS plotted in MATLAB before transformations to enable compatibility with simulation in MATLAB.

etry bind the plausible angles that can be utilized. For example, for low-altitude and low-grazing-angle simulations, only a few degrees of elevation are pertinent. When using a monostatic radar, this is due to the waveform-dependent minimum range and PRF-limited maximum range, as both relate to the altitude of the radar system, which restricts the usable elevation angles of the radiation pattern. Establishing a simulation geometry in this manner forces the use of two frames of reference for the simulation. First, elevation with respect to the antenna and elevation with respect to the observation geometry. Elevation with respect to the antenna is a set axis where, for example, 0° in elevation will always intersect the mainlobe of the beam when considering all azimuth angles. This frame of reference is useful for associating realized gain values independently of the simulation geometry. Secondly, and independent of the former, the elevation with respect to the simulation is the elevation angles of the antenna radiation pattern pertinent to the calculation of radar returns despite rotations or changes in the look direction of the antenna. An antenna can be turned to point upward, but the angles calculated with respect to the simulation geometry remain the same, thus changing the look angle of the radar forces an interaction between these two frames of reference causing the simulation to simply factor in different gain values of the rotated radiation pattern. This interaction is critical in visualizing how sidelobes interact with the ocean surface and can cause spikes or minimal clutter returns with certain look angles. Figure 3.4 shows an example of how portions of the radiation pattern of the antenna designed by [18] are most likely to be factored into the simulation for a low-grazing angle maritime radar deployment. The figure shows the copolarized realized gain elevation pattern of the main beam where the main beam is pointed at both 0° and 25° for *a)* and *b)*, respectively, yet the depression angles for the gain calculated with respect to the ocean surface

(enclosed in the acute angles of the red lines) remain the same for both instances. Figure 3.4 also shows how sidelobes or nulls can factor into sea clutter returns for a maritime deployed system. By using an antenna pattern generated from an electromagnetic simulator, like HFSS, the performance of a maritime radar system can be predicted with reasonable accuracy before large scale, costly changes are implemented.

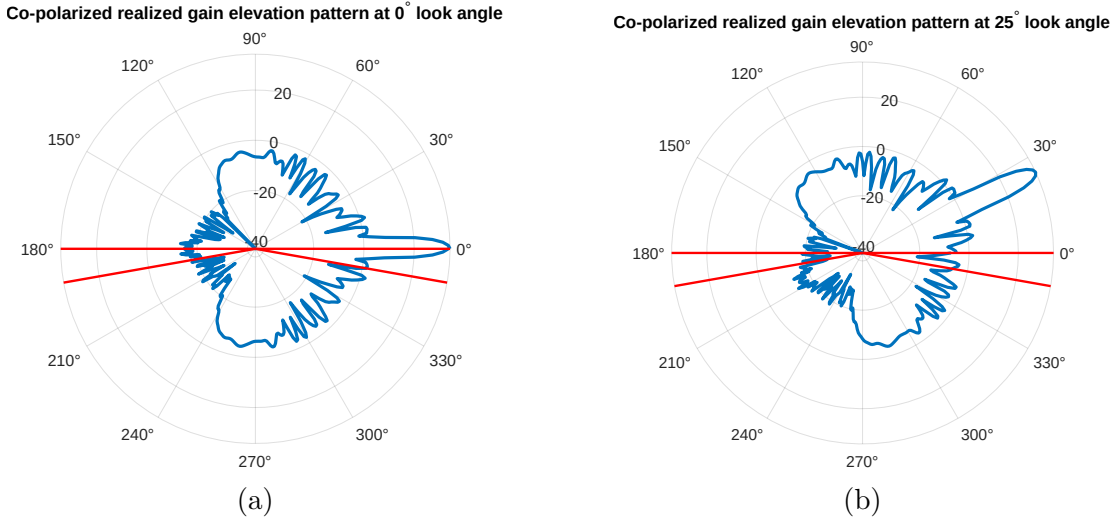


Figure 3.4: Co-polarized realized gain elevation pattern for the peak of the main-beam of the buttonhook antenna designed in ANSYS HFSS by [18]. Subfigure (a) shows the antenna pointed at a 0° look angle in elevation while subfigure (b) shows a 25° angle. The gain values enclosed in the acute angles of the red lines show the approximate values used in calculation for a low altitude maritime radar deployment of a few meters off the ocean surface.

3.2 Clutter Return Signal Amplitude Distributions

The amplitude statistics of sea clutter are commonly modeled using a small set of probability density functions (pdf(s)). Among these, the K-distribution, the log-normal, and Weibull distributions are the most widely used for simulation purposes. [1]–[3], [8] indicate that the K-distribution provides the best agreement with measured data, particularly in high sea states, due to its ability to capture

the heavy-tailed, spiky nature of sea clutter. These statistical models are used to generate radar cross section (RCS) values for individual scatterers within clutter patches, directly influencing the simulated amplitude of the received radar signal. Overall, non-Gaussian pdfs are preferred over Gaussian models because they more accurately represent the occasional large returns, commonly referred to as sea spikes, that are characteristic of real maritime clutter.

3.2.1 K-Distribution

The K-distribution is a Gaussian random process with a power that is Gamma distributed, and the amplitude pdf is given by the equation,

$$\begin{aligned} P(r) &= \frac{2b}{\Gamma(v)} \left(\frac{br}{2} \right)^v K_{v-1}(br) \\ &= \frac{b^{v+1} r^v}{2^{v-1} \Gamma(v)} K_{v-1}(br) \end{aligned} \quad (3.3)$$

and the cumulative distribution function (CDF) is given by,

$$F(r) = 1 - \frac{2}{\Gamma(v)} \left(\frac{br}{2} \right)^v K_v(br) \quad (3.4)$$

where $K(\cdot)$ is the modified Bessel function of the second kind of order $v - 1$ and the order v for pdf and CDF, respectively [1]–[3], [19]. The shape and scale parameters, v and b , characterize the envelope of the K distribution. When v is small, the K distribution becomes “heavy tailed”; however, as $v \rightarrow \infty$, the distribution becomes Gaussian. The K-distribution has been described in [2] in detail, which explains the distribution as a type of modulated Gaussian whose process is modulated by a generalized Chi distribution, equivalent to a power-modulated Rayleigh pdf of the Gaussian signal by use of a Gamma distributed random vari-

able. The Gamma distributed random variable then represents the mean value, while the Rayleigh distribution characterizes the spikes in amplitude. The Gamma distribution features scale and shape parameters which, when changed, will yield different pdf shapes of the K-distribution to fit sea clutter returns [1]. Figure 3.5 shows the K-distribution with a varying shape parameter. MATLAB has built in functions *gamrnd* and *raylrnd* to assist with the generation of K distributed samples; however, [19] outlines the way to generate Spherically Invariant Random Variables (SIRVs) if the convenience of built-in functions is not available.

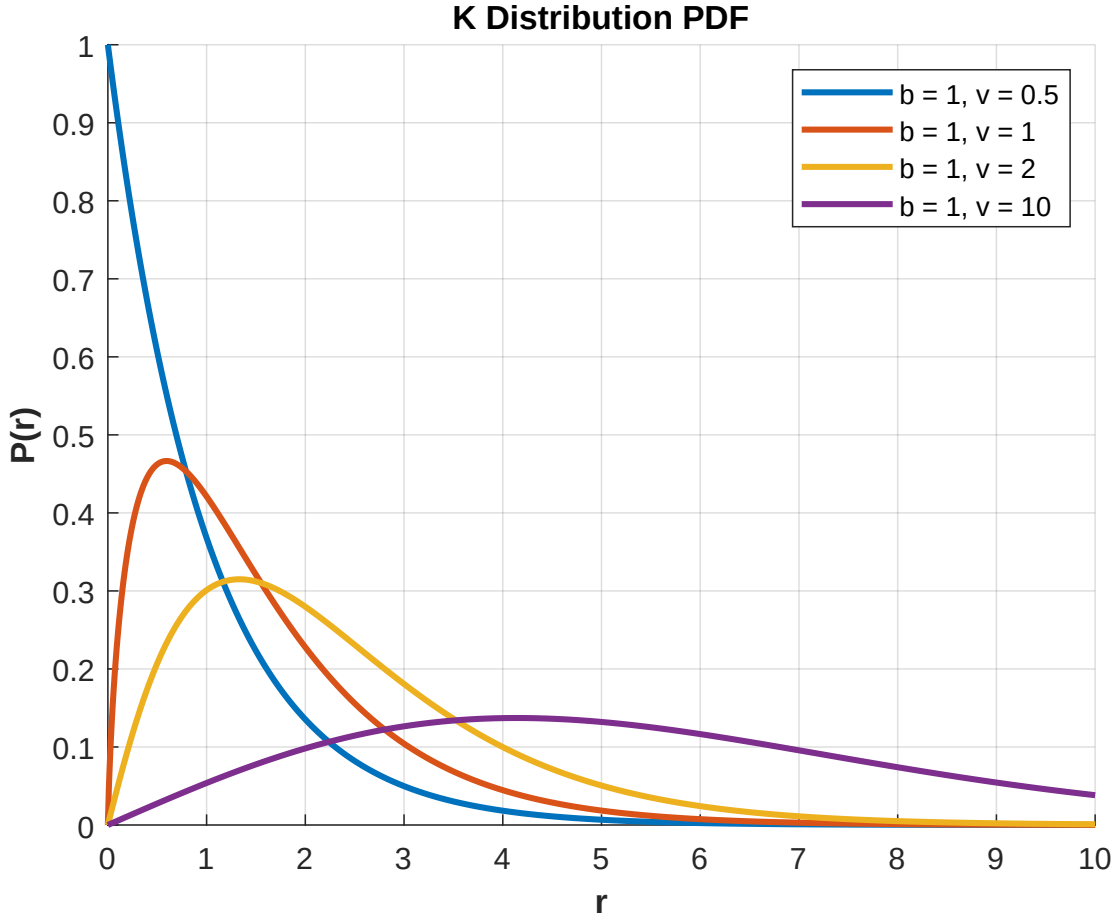


Figure 3.5: K Distribution PDFs with varying shape parameter, v .

3.2.2 Log-Normal Distribution

The log-normal distribution associates a Gaussian distribution, with its own mean, μ and variance, σ^2 , to the logarithm yielding the amplitude pdf given by,

$$P(z) = \frac{1}{z\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln(z) - \mu)^2}{2\sigma^2}\right); z \geq 0 \quad (3.5)$$

and the CDF is given by,

$$F(z) = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{\ln(z) - \mu}{\sigma\sqrt{2}}\right) \right) \quad (3.6)$$

with $\operatorname{erf}(\cdot)$ as the error function. Then the n th moments of the log-normal distribution are given by the equation,

$$\mathbb{E}[z^n] = \exp\left(n\mu + \frac{n^2\sigma^2}{2}\right) \quad (3.7)$$

Figure 3.6 shows the log-normal distribution with a mean of $\mu = 0$ and differing variance σ^2 .

3.2.3 Weibull Distribution

The Weibull distribution is also a commonly used distribution for modeling land and sea clutter which can be shown by the equation,

$$P(x) = \frac{v_w x^{v_w-1}}{b_w^{v_w}} \exp\left(-\left(\frac{x}{b_w}\right)^{v_w}\right); v_w, b_w > 0 \quad (3.8)$$

and the CDF is given by,

$$F(x) = 1 - \exp\left(-\left(\frac{x}{b_w}\right)^{v_w}\right) \quad (3.9)$$

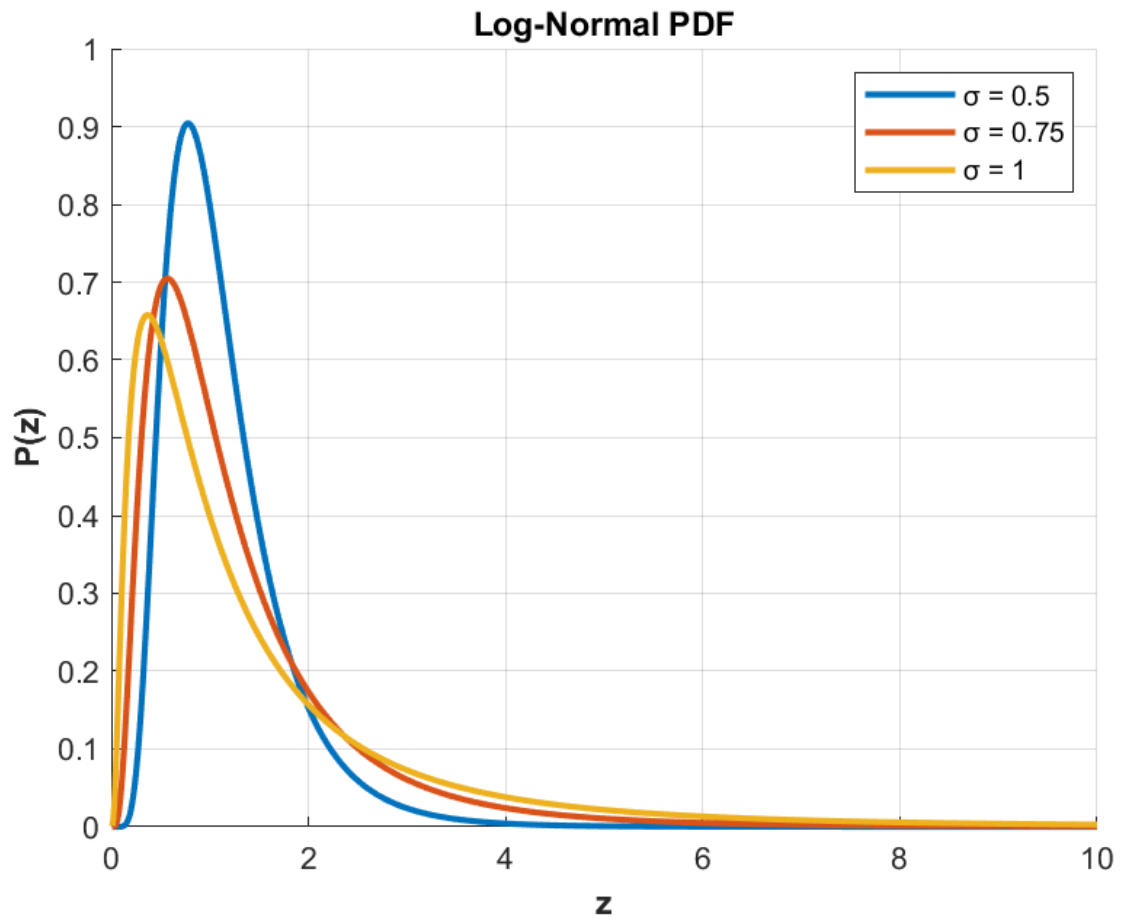


Figure 3.6: Log-Normal Distribution PDFs with different variance σ^2 and constant mean $\mu = 0$.

Then, the n th order moments of the Weibull distribution are given by the equation,

$$\mathbb{E}[x^n] = b_w^n \Gamma\left(1 + \frac{n}{v_w}\right) \quad (3.10)$$

The Weibull distribution features shape and scale parameters similar to the K distribution, and the Weibull distribution can be equivalent to the K distribution for some of the parameter values. Additionally, for shape parameters $v = 1$ or 2 , the Weibull distribution can be used to acquire the exponential and Rayleigh distributions, respectively. Figure 3.7 shows the Weibull distribution with varying shape parameters.

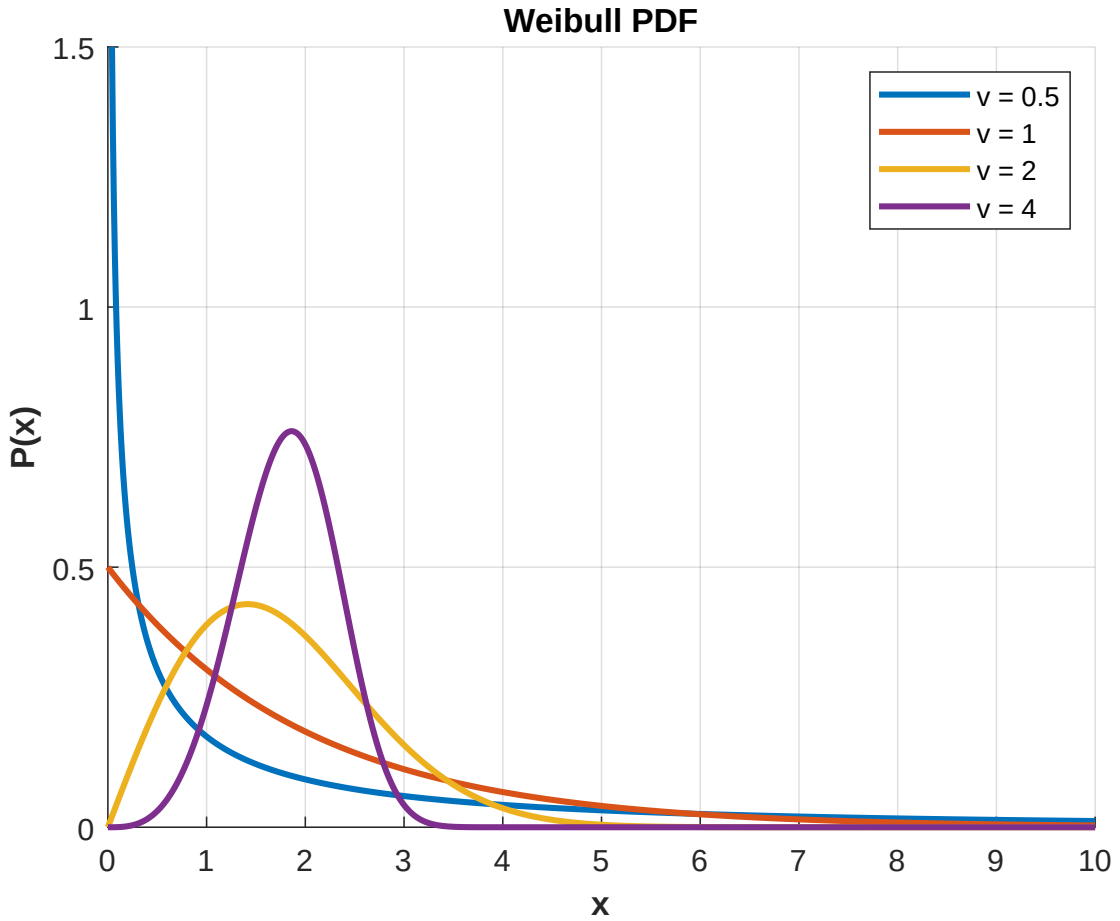


Figure 3.7: Weibull Distribution PDFs with different shape parameter v .

3.2.4 Shape Parameters

The estimation of shape parameters is valuable to both the K distribution and the Weibull distribution. Since both pdfs take on vastly different shapes as their respective shape parameter changes, it is critical to estimate the proposed shape parameter to match the characteristics of the clutter. [1], [8], [20], [21] give insight to the estimates shape parameter of the K-distribution as it represents clutter-plus-noise as,

$$v = \frac{2(\mathbb{E}[r] - P_n)^2}{\mathbb{E}[r^2] - 2\mathbb{E}[r]^2} \quad (3.11)$$

This estimate of the shape parameter comes from matching the first two intensity moments of the data samples to samples of a K-distribution in [1]. Where expressing the moments of clutter intensity in terms of clutter-plus-noise comes from the derived relation between the received complex signal and the clutter signal plus noise,

$$\mathbf{S} = \mathbf{S}_c + \mathbf{N} \quad (3.12)$$

where $\mathbf{S}$ is the received signal, $\mathbf{S}_c$ is the clutter contribution of the received signal, and $\mathbf{N}$ is considered the additive white Gaussian noise (AWGN) that corrupts the signal.

Similarly, the estimation of the shape parameter for the Weibull distribution can be done by using the parameter U in [1]. The first moment of the Weibull distribution is, $\mathbb{E}[x] = b_w \Gamma(1 + 1/v_w)$, therefore the shape parameter, v_w is estimated by,

$$U = \mathbb{E}[\log x] - \log x \quad (3.13)$$

Then, U relates to the shape parameter by,

$$U(v_w) = -\frac{\gamma}{v_w} - \log \left(1 + \frac{1}{v_w} \right) \quad (3.14)$$

where γ represents Euler's constant. Equation 3.14 is then inverted to find the shape parameter, v_w .

3.3 Doppler Characteristics

As the radar transmits and receives pulses, the motion of the ocean causes the return waveform to have an induced phase shift that can be seen across pulses. The Doppler spectrum coherent processing interval (CPI), or a burst of pulses, can be estimated from the ocean surface. Visualizing this relation is done using the power spectral density (PSD) of the Doppler spectrum. The PSD is calculated by taking the Fourier transform of the autocorrelation function between two or more pulses, and thus the PSD shows how power is distributed across a range of possible Doppler frequencies. For radar applications, the frequencies are dictated by the PRF and range from $-\text{PRF}/2$ to $\text{PRF}/2$, centered on 0. Defining a power spectral density of the sea clutter will allow fine tuning of the ocean's Doppler characteristics.

Often, the Doppler spectrum of sea clutter can be generalized by a power spectral density of a Gaussian distribution with an associated mean and standard deviation. The mean is generally between $1/4$ and $1/6$ of the wind velocity, and the standard deviation is usually between 0 and 1 m/s, which for an X-band system is about 65 Hz [3]. Additionally, the temporal decorrelation model in Section 2.2.3 can be used to model both exponential or Gaussian representations of the sea clutter. These can be used to quickly generate a Doppler spectrum for a simulated dataset; however, it is often useful to intertwine deeper characteristics linked to the sea state, wind speed and direction, grazing angle, and local clutter intensity.

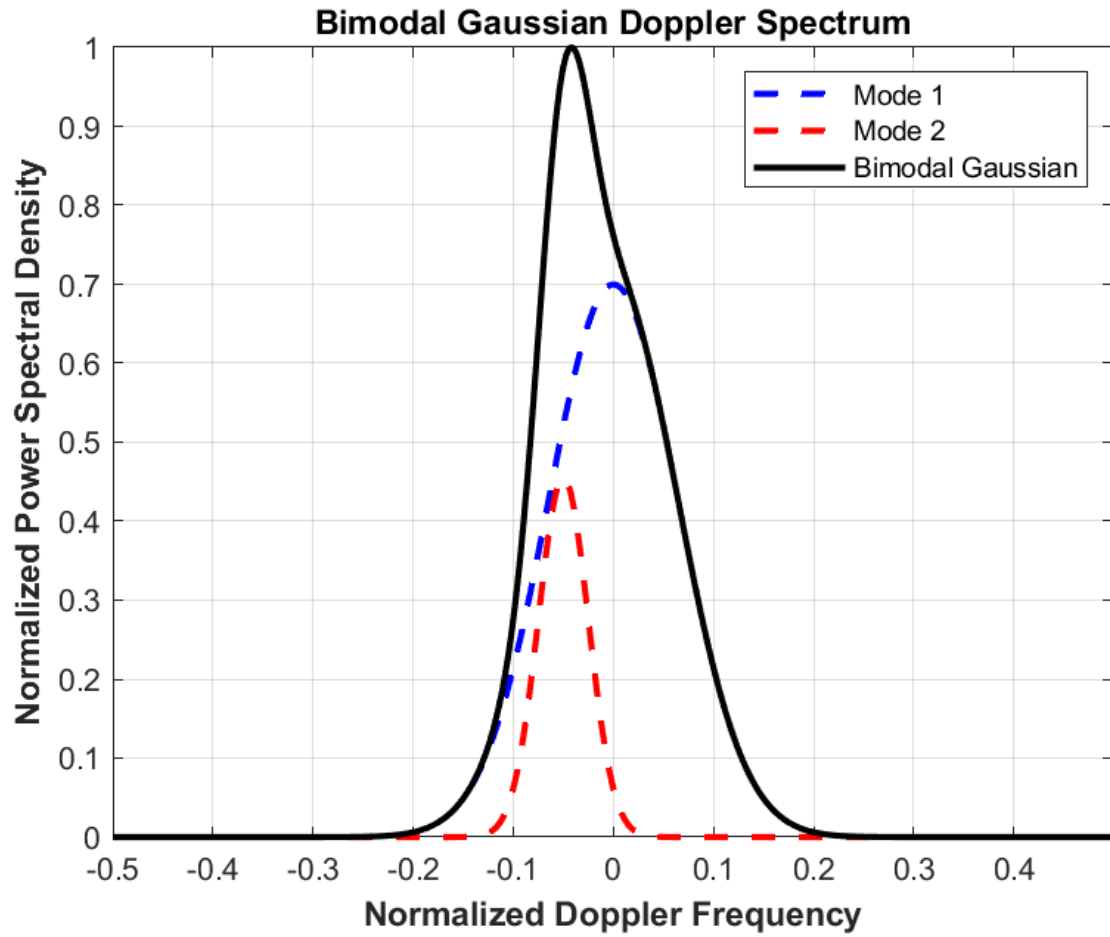


Figure 3.8: Bimodal Gaussian distribution represented as a power spectral density. Although the modes are slightly exaggerated for visualization purposes, Mode 1 and 2 correspond to the Bragg and fast scattering components respectively.

3.3.1 Evolving Doppler Spectrum Model

Rosenburg, in [3], developed a model to characterize the Doppler spectrum of the sea surface based on the conditions mentioned above. Rosenburg's model, called the Evolving Doppler Model, partitions the Doppler characteristics of the ocean surface into two parts, a Bragg scattering component and a fast scattering component. The Evolving Doppler Model is then a proposed combination of these two components into a bimodal Gaussian distribution that fully captures the power spectral density of the Doppler spectrum of the sea clutter. Figure 3.8 shows an example of a bimodal Gaussian distribution where *Mode 1* and *Mode 2* resemble the Bragg and fast scattering components, respectively. In the traditional sense, summing pdfs leads to the overall distribution being Gaussian through the central limit theorem. However, the bimodal Gaussian distribution is different from the sum of two Gaussian distributions and is generally called a *mixture* of two Gaussian distributions to properly articulate the two distinct contributions to the overall shape. The bimodal Gaussian power spectral density of the Evolving Doppler Model is given by the equation,

$$G_{bi}(f) = \frac{\alpha\tau}{\sqrt{2\pi} w_1} \exp\left(-\frac{(f - m_{f_1}(\tau))^2}{2w_1^2}\right) + \frac{(1 - \alpha)\tau}{\sqrt{2\pi} w_2} \exp\left(-\frac{(f - m_{f_2}(\tau))^2}{2w_2^2}\right) \quad (3.15)$$

where α is the weighting factor between the two distributions, τ is the local clutter intensity, w_1 and w_2 are the standard deviations of the distributions, f are the possible Doppler frequencies of the power spectral density, and m_{f_1} and m_{f_2} are the mean Doppler frequencies of the distributions. Both mean frequencies are functions of the local clutter intensity and are related to the normalized mean

speckle power, $\tilde{\tau} = \tau/\mathbb{E}[\tau]$, by the equations,

$$m_{f_1}(\tau) = \begin{cases} A_c + B_c \tilde{\tau} + f_{\text{bi}}, & \tilde{\tau} \leq \tau_{\text{bi}} \\ A_c + B_c \tau_{\text{bi}} + f_{\text{bi}}, & \tilde{\tau} > \tau_{\text{bi}} \end{cases} \quad (3.16)$$

$$m_{f_2}(\tau) = A_c + B_c \tilde{\tau} + f_{\text{bi}}. \quad (3.17)$$

where f_{bi} is a zero mean Gaussian random variable with an associated standard deviation of σ_{bi} . This additional Gaussian factored into the mean of the two Gaussian distributions of the model helps to illustrate the noisy relation between the mean Doppler and local clutter intensity. All constant parameters of the model (A_c , B_c , τ_{bi} , α , and σ_{bi}) are derived from a series of two equations whose dependence is ultimately determined by the wind speed, significant wave height, grazing angle, and swell angle relative to the radar. The chosen generic constant parameter is calculated by the equation,

$$X(\theta, \phi \mid U, H_{1/3}) = \left(\frac{\phi}{\phi_0} \right)^{\gamma_{\text{gen}}} \left(a_{\text{gen}} + b_{\text{gen}} \cos \theta + c_{\text{gen}} \cos(2\theta) + d_{\text{gen}} \cos(\theta - \theta_{\text{sw}}) + e_{\text{gen}} \cos(2(\theta - \theta_{\text{sw}})) \right) \quad (3.18)$$

where ϕ is the grazing angle to the clutter patch, ϕ_0 is the reference grazing angle set at 10° , θ and θ_{sw} are the azimuth and swell angles, with the former relative to the latter to account for the radial velocity relative to the radar. The constants a_{gen} , b_{gen} , c_{gen} , d_{gen} , e_{gen} , and γ_{gen} of Equation 3.18 are each calculated using the equation,

$$Y = f_{\text{gen}} + g_{\text{gen}} \log_{10}(U) + h_{\text{gen}} H_{1/3} \quad (3.19)$$

which derives its dependence on the wind speed U , and significant wave height $H_{1/3}$, of the simulated sea state found in Table 2.2. Finally, the constants of Equa-

tion 3.19, f_{gen} , g_{gen} , and h_{gen} , can be found in Appendix B, where each constant varies depending on the constant of Equation 3.18 is being calculated. The constants of Equation 3.19 change with polarization with supported configurations being HH, HV, and VV. The Evolving Doppler Model was created from data collected by the Ingara X-band radar system; thus, the model was designed and validated for medium grazing angles and Douglas Sea States 2 - 6; however, the model still produces acceptable power spectral densities for differing geometries.

3.4 Reflectivity Models

Fred E. Nathanson published in his book the most comprehensive set of empirical sea clutter reflectivity data in Tables 7.2 through 7.8 of his book [4]. These tables outline the normalized mean sea backscatter coefficient based on varying sea states, polarizations, frequencies, and grazing angles. To this day, most reflectivity models seek to match this set of empirical data because it is the most exhaustive set of data on the topic of sea surface reflectivity, and it sets the gold standard for reflectivity models unless a model is specifically configured to accommodate a specialized system design.

As mentioned in Section 2.1.3, reflectivity, defined as the unitless measure of RCS per square meter, is dependent on surface conditions, grazing angle, center frequency, and polarization. Several models for mean backscatter are listed in Table 3.1, each with their own limitations in center frequency, grazing angle, wind direction, and Douglas Sea State. These models are generally derived from matching real data to fit equations or analytically applying electromagnetic scattering theory to the sea surface. Since this work primarily focuses on low to medium grazing angle X-band applications, the three most applicable models chosen are the Georgia Institute of Technology (GIT) model, the Sittrop model, and the

Naval Research Lab (NRL) model. These models are most applicable due to their accommodation of X-band frequencies and the wide range of simulation geometries the models account for. However, each model differs slightly from each other, offering benefits for a certain subset of simulation environments.

Table 3.1: Summary of mean backscatter models, with the published regions of validity. Adapted from Rosenberg [3].

Model	Freq. (GHz)	Grazing (°)	Wind direction (°)	Douglas Sea state
RRE [1] (1974)	9–10	0.1–10	Averaged	1–6
Sittrop [4] (1977)	8–18	0.1–10	Up/cross-wind	–
GIT [5] (1978)	1–100	0.1–10	0–360	0–5
Hybrid [14] (1990)	0.5–35 ^a	0.1–30	0–360	3–6
TSC [8] (1990)	0.5–35	0–90	0–360	0–5
Masuko [11] (1986)	10, 34	0–70	0–360	2–6
IRSG [12] (2008)	8–12	20–45	0–360	2–6
NRL [9] (2012)	0.5–35	0.1–60	Averaged	0–8
DSTG cont. [13] (2017)	8–12	0.1–45	0–360	2–6

^a This was later extended to 70 GHz in the NAAWS model **NAAWS**.

3.4.1 Georgia Institute of Technology (GIT) Model

Nathanson describes in Chapter 7 of his book [4] the model for reflectivity, proposed by Horst et al. in [6], termed the Georgia Institution of Technology (GIT) model. This model explicitly accounts for average wave height, H_{avg} , and wind speed, U , instead of a discrete Douglas Sea State. The model assumes a fully developed sea with consistent wind and swell directions that are correlated with the average wave height. The significant wave height relates to the average wave height through the equation,

$$H_{1/3} = 1.6 \times H_{avg} \quad (3.20)$$

which can then be related to the wind speed through,

$$H_{1/3} = 0.00724 \times U^{0.25} \quad (3.21)$$

or inversely,

$$U = 7.18 \times H_{1/3}^{0.4} \quad (3.22)$$

as [5] describes. Then, an approximation is then made in [3] in chapter four about the average wave height and wind speed as it relates to sea state expressed as,

$$H_{avg} = 0.00452U^{2.5} \quad (3.23)$$

then, the sea state is related by,

$$U = 3.16S_{ss}^{0.8} \quad (3.24)$$

where S_{ss} is the chosen Douglas Sea State.

The GIT model then requires four constants to be calculated dependent on wind speed, grazing angle, average wave height, wind speed, and wind direction, which the aforementioned equations will aid in calculating. The first constant which accounts for multipath reflections and interference is given by,

$$a_{GIT} = \frac{\sigma_{GIT}^4}{1 + \sigma_{GIT}^4} \quad (3.25)$$

where the wave heights are assumed to be Gaussian, the multipath texture parameter is calculated using,

$$\sigma_{GIT} = \frac{(14.4\lambda + 5.5)\theta_{dep}^\circ H_{avg}}{\lambda + 0.015} \quad (3.26)$$

Then the sea direction factor is calculated by,

$$b_{\text{GIT}} = \begin{cases} \exp \left(0.2 \cos \theta (1 - 2.8 \theta_{\text{dep}}^{\circ}) (\lambda + 0.015)^{-0.4} \right), & 1 \leq F_o < 10 \\ \exp \left(0.25 \cos \theta (1 - 2.8 \theta_{\text{dep}}^{\circ}) (\lambda + 0.015)^{-0.33} \right), & 10 \leq F_o < 100 \end{cases} \quad (3.27)$$

which is under the assumption of an aligned wind and swell direction in azimuth, θ . Lastly, the wind speed factor is given by,

$$c_{\text{GIT}} = \begin{cases} \left(\frac{1.94U}{1+U/15.4} \right)^{1.1(\lambda+0.015)^{-0.4}}, & 1 \leq F_o < 10 \\ \left(\frac{1.94U}{1+U/15.4} \right)^{1.93(\lambda+0.015)^{-0.4}}, & 10 \leq F_o < 100 \end{cases} \quad (3.28)$$

Finally, the reflectivity values for the supported polarizations (HH and VV) can be calculated using the piecewise equations given by Rosenburg in [3] Chapter four, which avoids discontinuities at 10 GHz for HH and both 3 and 10 GHz for VV. The mean reflectivity for both polarizations is given by,

$$\sigma_{\text{HH,dB}}^0 = \begin{cases} 10 \log_{10} \left(\lambda \theta_{\text{dep}}^{\circ 0.4} a_{\text{GIT}} b_{\text{GIT}} c_{\text{GIT}} \right) - 54.089, & 1 \leq F_o < 10 \\ 10 \log_{10} \left(\theta_{\text{dep}}^{\circ 0.547} a_{\text{GIT}} b_{\text{GIT}} c_{\text{GIT}} \right) - 54.381, & 10 \leq F_o < 100 \end{cases} \quad (3.29)$$

$$\sigma_{VV,dB}^0 = \begin{cases} \sigma_{HH,dB}^0 - 1.73 \ln(H_{avg} + 0.015) + 3.76 \ln \lambda \\ \quad + 2.46 \ln(\theta_{dep}^\circ + 0.0001) + 22.2, & 1 \leq F_o < 3 \\ \\ \sigma_{HH,dB}^0 - 1.05 \ln(H_{avg} + 0.015) + 1.09 \ln \lambda \\ \quad + 1.27 \ln(\theta_{dep}^\circ + 0.0001) + 9.7, & 3 \leq F_o < 10 \\ \\ \sigma_{HH,dB}^0 - 1.38 \ln(H_{avg}) + 3.43 \ln \lambda \\ \quad + 1.31 \ln \theta_{dep}^\circ + 18.55, & 10 \leq F_o < 100 \end{cases} \quad (3.30)$$

3.4.2 Sittrop Model

Sittrop's model offers tailored mean clutter reflectivity values specifically for X-band and Ku-band, and in his model, Sittrop assumes the sea clutter returns are predominately caused by capillary waves formed by the wind. Because of this, Sittrop's model is considered to be more reliable when specific wind speeds are used instead of sea states, since capillary waves, therefore Bragg scattering, cease to exist without the presence of wind [2]. Sea clutter returns using this model are expected to be lower than in practice, since gravity waves persist as a source of sea clutter after the wind has died down. The Sittrop model accounts for both vertical and horizontal polarizations and up-wind and cross-wind configurations. The equation for reflectivity is given by,

$$\sigma_{db}^o = a_{sit} + b_{sit} \log_{10} \left(\frac{\phi}{\phi_o} \right) + \log_{10} \left(\frac{U}{U_o} \right) \left(c_{sit} \log_{10} \left(\frac{\phi}{\phi_o} \right) + d_{sit} \right) \quad (3.31)$$

where ϕ_o is the reference grazing angle established at 0.5° and U_o is the reference wind speed set at 5 m/s. The coefficients a_{sit} , b_{sit} , c_{sit} , and d_{sit} , for X-band and

Ku-band frequencies are given in Table 3.2.

Table 3.2: Sittrop’s mean backscatter X- and Ku-band model coefficients [7]. Adapted from Rosenburg [3].

Pol.	Freq. band	Wind direction	a_{Sit}	b_{Sit}	c_{Sit}	d_{Sit}
HH	X	Up-wind	−50	12.6	−13.2	34
HH	X	Cross-wind	−53	6.5	0	34
VV	X	Up-wind	−49	17	−12.4	30
VV	X	Cross-wind	−58	19	−33	50
HH	Ku	Up-wind	−46	13.6	−10	26
HH	Ku	Cross-wind	−48	13	−6.6	21
VV	Ku	Up-wind	−46	17	−9	22
VV	Ku	Cross-wind	−47	13	−8.4	22

3.4.3 Naval Research Laboratory (NRL) Model

The Naval Research Lab developed a model for the sea surface reflectivity to quantify the scattering characteristics of the ocean surface for different Douglas Sea States. The model was derived through perceived issues in the GIT model’s method of addressing reflectivity. Vilhelm Gregers-Hansen cites: lack of dependence on sea state as the main input, generalizations with no basis in the publication, and an overly optimistic representation of reflectivity at low sea states as the reasons why the GIT model is insufficient in performance prediction of a radar system [5]. Consequently, Vilhelm Gregers-Hansen proposed the NRL model with a strict focus on Nathanson’s empirical sea surface reflectivity data.

The NRL model constants listed in Table 3.3 are used as inputs to the equation

given by,

$$\begin{aligned}
\sigma_{dB}^o = & c_1 + c_2 \log_{10}(\sin \theta_{\text{dep}}) \\
& + \frac{(27.5 + c_3 \theta_{\text{dep}}^o) \log_{10} \left(\frac{F_o}{10^9} \right)}{1 + 0.95 \theta_{\text{dep}}^o} \\
& + c_4 (1 + S)^{\frac{1}{2+0.085 \theta_{\text{dep}}^o + 0.033 S}} \\
& + c_5 \left(\theta_{\text{dep}}^o \right)^2
\end{aligned} \tag{3.32}$$

where θ_{dep} is the depression angle in radians, θ_{dep}^o is the depression angle in degrees, F_o is the center frequency of the radar in Hertz, and S is the Douglas Sea State to be modeled. The five constants, $c_1, c_2, \dots, c_5$, were tuned to minimize the absolute deviation from Nathanson's data across both vertical and horizontal polarizations. Each term in the model equation captures a distinct physical or empirical aspect of the reflectivity behavior. The first provides a baseline bias, while the second introduces a logarithmic dependence on the depression angle. The third term captures the mutual dependence between the center frequency and the depression angle. The fourth incorporates sea state effects along with a correction for grazing angle. Finally, the fifth term models the rapid rise in reflectivity as the radar increasingly points toward the ocean surface. Figures 3.9 and 3.10 show how the reflectivity at 9.41 GHz changes as a function of grazing angle for both vertical and horizontal polarizations, respectively.

Constant	H Polarization	V Polarization
c_1	-73.00	-50.79
c_2	20.78	25.93
c_3	7.351	0.7093
c_4	25.65	21.58
c_5	0.00540	0.00211

Table 3.3: NRL model constants for horizontal and vertical polarizations [5]

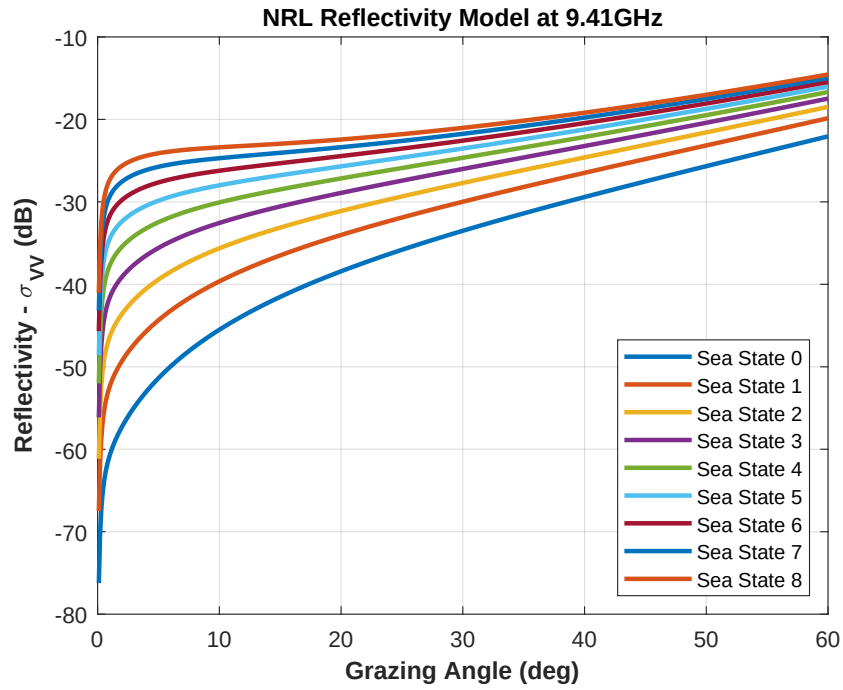


Figure 3.9: Naval Research Laboratory Reflectivity Model for VV Polarization at X band.

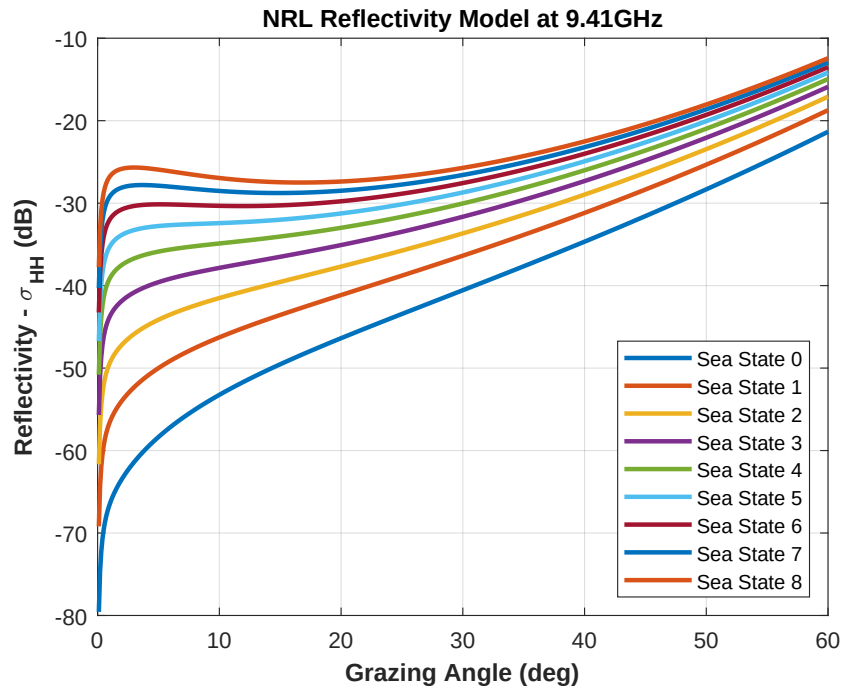


Figure 3.10: Naval Research Laboratory Reflectivity Model for HH Polarization at X band.

Chapter 4

Doppler Processing and Clutter Mitigation

4.1 Data Processing

After defining the system parameters of the radar system to be simulated, the inputs and environment can be built up to the point of data generation and processing. The characteristics of sea clutter are calculated before hand as described in Chapter 3, and predictive performance analysis can be performed by generating and analyzing the received data in the simulated maritime environment. When generating the returned signal data, the returns from every scatterer within the scene (ocean surface and target of interest) will be summed together in the proportion of the power received from each individual component. With each scatterer contributing a scaled amplitude in proportion to the radar cross section (RCS) of the scatterer and a Doppler shift dependent on the motion, the total return signal represents the superposition of each of these components from every scatterer in the scene. This data is sampled at the ADC, and with each pulse, the analog-to-digital converter (ADC) samples L_{ADC} a number of samples which correspond to the range of the radar. Thus, the ideal data format to be generated by the simulation is the fast-time slow-time data of size $L_{\text{ADC}} \times [N_p \cdot \text{PRI}]$ where N_p is the pulses per coherent processing interval (CPI).

For the data simulated, the ocean characteristics were set to mimic Douglas Sea State 2, and for models that required wind speed or wave height, a randomly selected value that falls within the bounds defined in Table 2.2 was chosen assuming that for the duration of data collection, the wind speed or average wave height did not change drastically. Each CPI contains 128 pulses that have been processed together, which allows for a collection time of 71.1 ms. The radar system parameters and the simulated target parameters are described in Tables 4.1 and 4.2, respectively.

Table 4.1: Radar system parameters

Parameter	Value
Carrier Frequency	9.41 GHz
Bandwidth	50 MHz
Noise Figure	12 dB
System Temperature	290 K
Antenna Mainbeam Gain	34.3 dBi
Polarization	Horizontal
Elevation Look Angle	0 degrees
Power Transmitted	150 W
Pulse Repetition Frequency (PRF)	1800 Hz
Max Range	5000 m
Height from Mean Wave Height	3.2 m

Table 4.2: Simulated target parameters

Parameter	Value
Type	Point Target
Range	1000 m
Velocity	4 m/s
Radar Cross Section	-15 dBm

4.1.1 Data Generation

Fast-time slow-time data is generated by taking the waveform in Equation 2.1 and applying the time delay and Doppler shift to achieve the received waveform of

Equation 2.8. Figure 4.1 shows how the data is laid out for subsequent processing. Each block in the data cube represents a real and imaginary component of the complex voltage measured at the receiver. Although both fast-time and slow-time dimensions are quantified in time, the distinction is between *what* is happening on that time scale. Fast-time denotes the time it takes for the signal in Equation 2.1 to travel the round trip distance between the radar and scatterers in the scene at the speed of light and is analogous to range. Slow-time denotes the time corresponding with the duration to transmit the number of pulses in the given CPI, thus is correlated with the pulse repetition interval (PRI) time of the radar system.

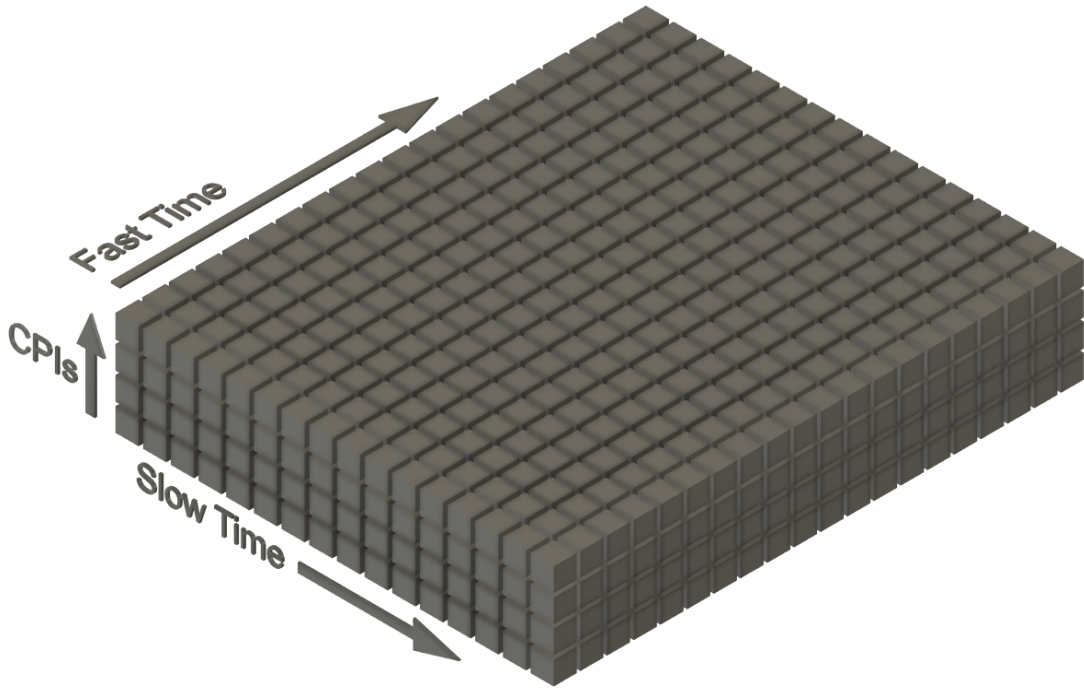


Figure 4.1: Radar data cube structure comprised of 4 CPIs each with 16 pulses containing 20 ADC fast time samples for each pulse.

To achieve the data format in Figure 4.1, the transmitted signal $x(t)$ must be modified by the environment and the returned signal sampled by the receiver.

The modified amplitude, $\hat{A}(t - \tau_o)$, of Equation 2.8 is found by applying Equation 2.6 and is given by,

$$\hat{A} = \sqrt{\frac{P_r}{P_t}} e^{j\phi} = \sqrt{\frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}} e^{j\phi} \quad (4.1)$$

where $\hat{A}$ depends on the specified range, R or a subsequent time delay not specified here, τ_o , and ϕ is a uniformly distributed random phase offset between 0 and 2π .

Assuming the fast-time receive samples is of length L_{ADC} for all PRIs of the CPI, applying the time delay and Doppler shift can be done in the frequency domain through these steps:

1. Perform an L_{ADC} point Fourier transform of the transmitted waveform, $x(t)$.
2. Center the Fourier transformed transmit signal, $X(F)$, on the frequency axis using a function like MATLAB's **fftshift()**.
3. Configure a range-frequency axis, F , that corresponds to $X(F)$.
4. Multiply $X(F)$ with $\exp(-j2\pi F(\tau - t_{\min}))$ where $t_{\min}$ is the minimum time delay for a monostatic radar to receive the signal.
5. Return the modified signal to the time domain through the inverse fast Fourier transform (FFT) shift and the inverse FFT.
6. Apply the attenuation and Doppler shift by multiplying the modified time domain signal by: $\hat{A} \exp(j2\pi F_D t)$ where t is the time axis associated with the fast-time receive vector.

This process yields a fast-time receive vector corresponding to the attenuated, delayed, Doppler-shifted waveform for a particular scatterer within the scene. When simulating sea clutter, this process is repeated for each scatterer placed within the scene, each with its own $\hat{A}$ and F_D that obey the chosen model parameters set out in Sections 3.2, 3.4, and 3.3, respectively. The data per CPI is then in the format

$\mathbf{x} \in \mathbb{C}^{L_{\text{ADC}} \times N_p}$ which can be further processed. Once the data is generated in this manner, the matched filter can be applied to maximize the signal-to-noise ratio (SNR), Doppler processing can extract the Doppler frequencies of the scatterers within the scene, and clutter mitigation/cancellation can be performed to extract targets in the presence of sea clutter.

4.1.2 The Matched Filter

The matched filter is the optimal linear filter for detecting a known transmitted waveform in the presence of additive noise. It is defined by the equation:

$$h(t) = x^*(T_p - t) \quad (4.2)$$

where T_p is the duration of the waveform $x(t)$, and the condition $T_p \geq t$ ensures that the filter remains causal. Equation 4.2 represents the time-reversed and conjugated form of the transmitted signal. The matched filtering operation is performed along the fast-time axis of the radar data cube. Given the received signal matrix $\mathbf{x} \in \mathbb{C}^{L_{\text{ADC}} \times N_p}$, where L_{ADC} is the number of fast-time samples and N_p is the number of pulses in the CPI, the matched filter output for each pulse is given by,

$$\mathbf{y}(:, n) = \mathbf{x}(:, n) * h(t), \quad \text{for } n = 1, 2, \dots, N_p \quad (4.3)$$

The matched filter maximizes the SNR at the point of detection and is typically implemented by convolving the received signal with $h(t)$. When the matched filter is applied along the fast-time axis of the received signal $y(t)$, corresponding to each pulse in the radar data cube (e.g., Figure 4.1), the result is a set of range-resolved returns. These form the range-slow-time matrix, where each range bin contains the progression of the signal across successive pulses, enabling Doppler processing

for motion detection.

4.2 Spectral Shaping and Doppler Processing

Section 3.3 described the Doppler behavior of ocean clutter in relation to wind and wave conditions. To connect the power spectral density (PSD) of the ocean surface returns to the observed Doppler frequencies, the first step is to synthesize spectrally shaped noise according to the modeled PSD. Once the synthetic data reflects the appropriate Doppler characteristics, conventional Doppler processing techniques can then be applied to the resulting data cube to generate a range-Doppler map for each CPI. The structure of the Doppler PSD and how it is applied across range bins is discussed further in this section.

Doppler processing involves performing spectral analysis on the returns from a single range cell over consecutive pulses. This technique is used to mitigate clutter and improve the detectability of moving targets in clutter-dominated scenarios since Doppler processing separates returns from moving and stationary targets in a distinct manner. Doppler processing assumes coherent radar operation, where each waveform has a known starting phase. This allows the target velocity to be observed through the phase progression of the returns across successive pulses. Targets that fall outside of the clutter will be seen with ease; however, in maritime applications, the Doppler characteristics of the sea can cause the clutter to more easily mask targets of interest.

4.2.1 Spectrum Shaping

The PSD from Equation 3.15 cannot directly model the Doppler component of sea clutter, as it represents only a single realization of the Doppler spectrum under given conditions. Instead, the PSD should be used to shape uncorrelated

Gaussian noise. Once the target PSD is determined, the white Gaussian noise can be shaped accordingly to match the desired Doppler characteristics. To do this, a matrix of Doppler steering vectors is constructed given by,

$$\mathbf{V}_{F_D} = \exp(j2\pi n f_d) \quad (4.4)$$

where n is a normalized pulse index vector ranging from 0 to $N_p - 1$ with N_p being the number of pulses in the CPI. Then, f_d is the normalized Doppler spectrum ranging from 0 to $N_{fft} - 1$ with N_{fft} points, where N_{fft} is the number of basis functions computed in the Fourier transform. Therefore, $\mathbf{V}_{F_D} \in \mathbb{C}^{N_p \times N_{fft}}$. After the Doppler steering vector is formed and the power spectrum of the clutter as it behaves across f_d is defined, the clutter covariance matrix can be calculated. If the power spectrum of the clutter is given by $\mathbf{P}_c \in \mathbb{R}^{1 \times N_{fft}}$, then the associated diagonal matrix $\mathbf{P}_s \in \mathbb{R}^{N_{fft} \times N_{fft}}$, formed from $\mathbf{P}_c$, can be used to compute the clutter covariance matrix as,

$$\mathbf{R}_c = \mathbf{V}_{F_D} \mathbf{P}_s \mathbf{V}_{F_D}^H \quad (4.5)$$

where $\mathbf{V}_{F_D}^H$ is the Hermitian transpose of the Doppler steering vector. Then, the complex white noise can be shaped to the desired spectrum by,

$$\text{PSD}_c = \sqrt{\mathbf{R}_c} \times \mathbf{N} \quad (4.6)$$

where the complex white Gaussian noise is $(1/\sqrt{2})(\mathcal{N}(0, 1) + j\mathcal{N}(0, 1))$. Figure 4.2 shows the result of uncorrelated Gaussian noise shaping to the example bimodal Gaussian spectrum in Figure 3.8.

Equation 4.6 only represents one realization from Equation 3.15 which depends

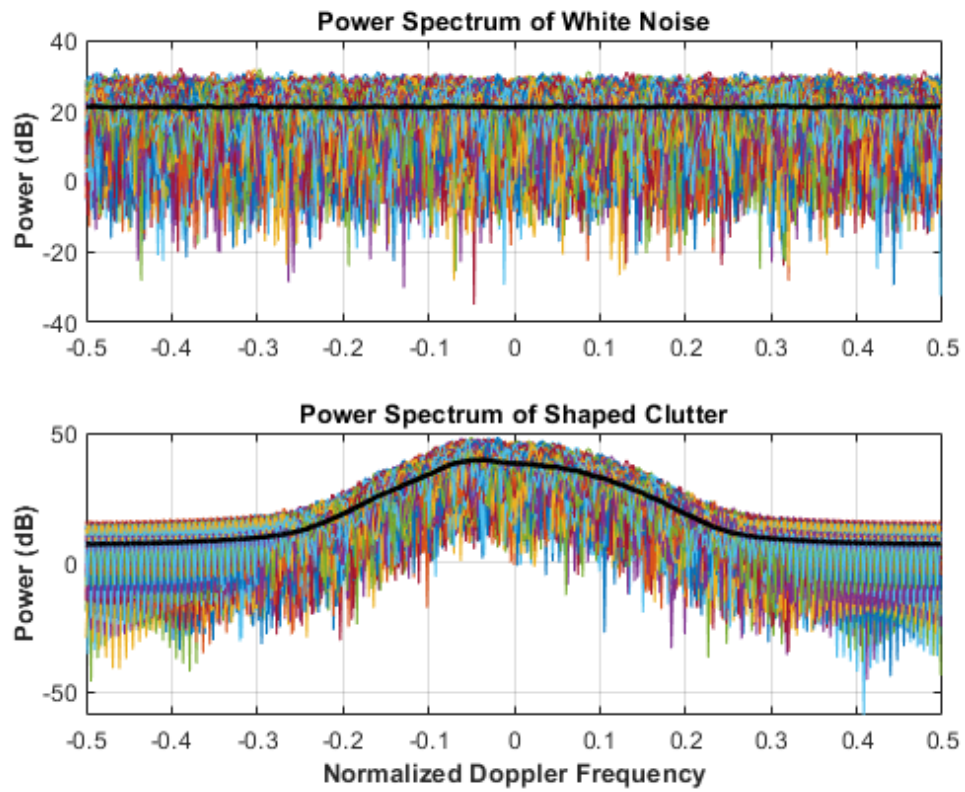


Figure 4.2: Uncorrelated white Gaussian noise (top) that has been shaped by the power spectral density (PSD) into the power spectrum of the clutter (bottom) for one realization of a bimodal Gaussian spectrum.

on the local clutter intensity. In order to properly model the Doppler characteristics of a particular CPI of L_{ADC} range samples, the PSD of the clutter must be recomputed per range bin to acquire a proper estimate of Doppler characteristics as it relates to the local clutter intensity of that range bin. Figure 4.3 depicts the Fourier transform of the spectrally shaped clutter according to the Evolving Doppler Model, which exemplifies how the parameters of the Evolving Doppler Model change with the local clutter intensity and grazing angle.

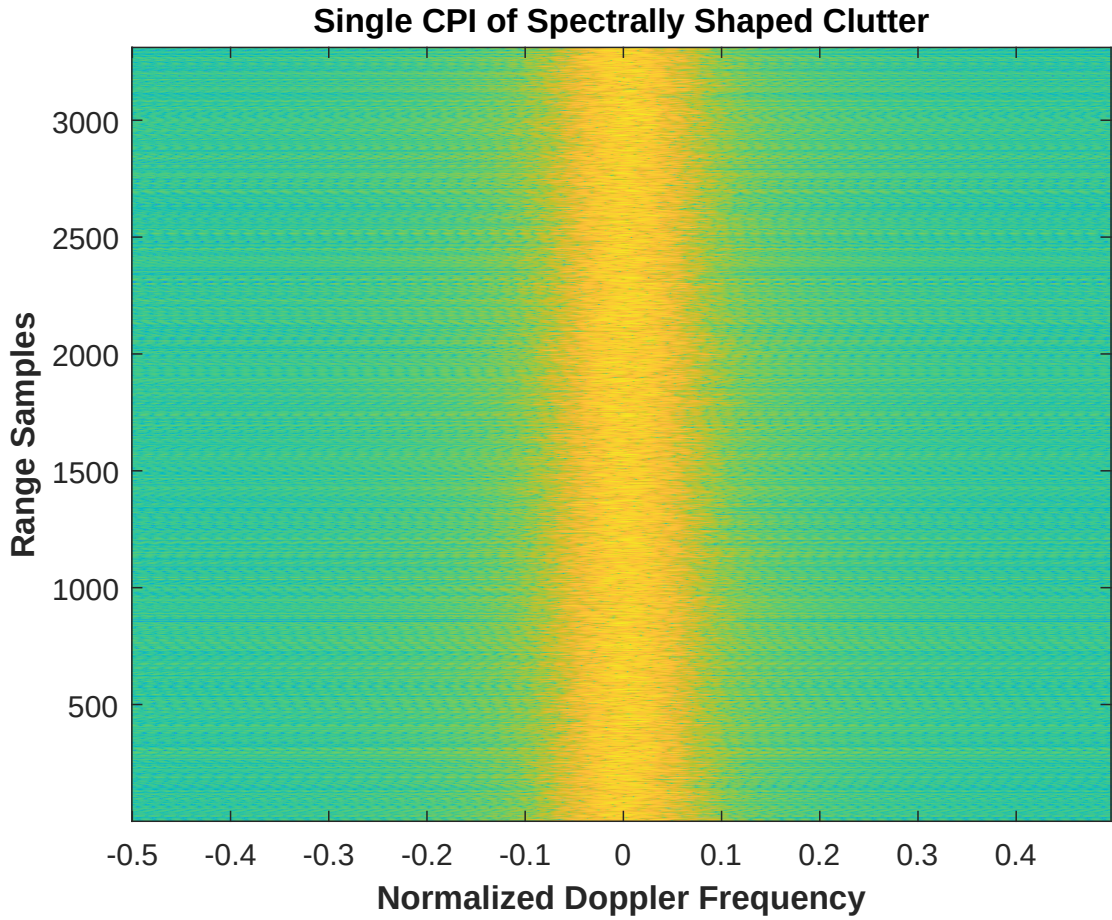


Figure 4.3: Spectrally shaped clutter with varying parameters for the bimodal Gaussian of the Evolving Doppler model as it relates to local clutter intensity per range bin and grazing angle.

4.2.2 Doppler Processing and the Range-Doppler Map

Once the data cube in Figure 4.1 has been generated, Doppler processing can be performed to achieve a range-Doppler map (RDM). Doppler processing is the spectral analysis of returns received by the radar for a singular range bin over successive pulses and is performed across the slow-time dimension of the data cube so as to analyze the Doppler frequency content of the received signal. The spectral analysis is done through the discrete Fourier transform (DFT) along the slow-time dimension to convert slow-time to Doppler bins. It is common to oversample the DFT by increasing the number of basis functions used in the Fourier transform, as the number of frequency bins can exceed the number of pulses in slow-time. Doppler resolution remains unchanged by this oversampling, as the Doppler resolution depends on the observation time, as seen in Equation 2.11. At least two pulses are required for Doppler processing, but with additional pulses for each CPI processed, the Doppler resolution increases. The inherent trade-off in this relationship is that high Doppler resolution can be achieved through long observation times; however, target returns become obscured over time in the range-Doppler map because of their changing velocity and range, causing target returns to smear over long CPIs.

Figure 4.5 shows the resultant range-Doppler map of the sea clutter generated after Doppler processing. The sea clutter uses the K-distribution to form the amplitude statistics, the Evolving Doppler model, and the NRL's model for reflectivity. This data was simulated under the conditions of Douglas Sea State 2 while the radar system parameters and target parameters were described in Tables 4.1 and 4.2. The reflectivity of the sea surface completely swamps out the target signature since the sidelobe interference on near-in sea clutter is so impactful as seen in Figure 4.6. The Signal-to-Interference-plus-Noise Ratio (SINR) will be

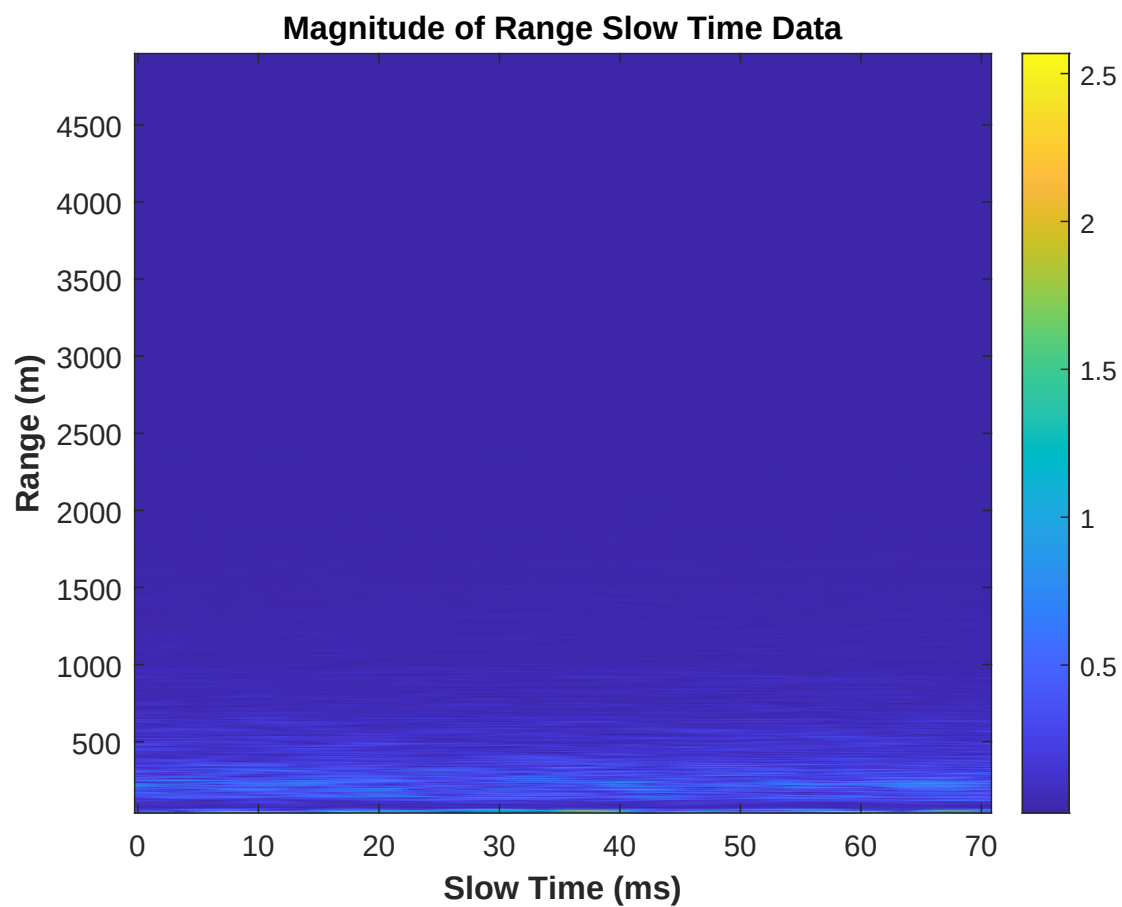


Figure 4.4: Range slow-time data before Doppler processing across successive pulses.

discussed more in Section 4.6, but

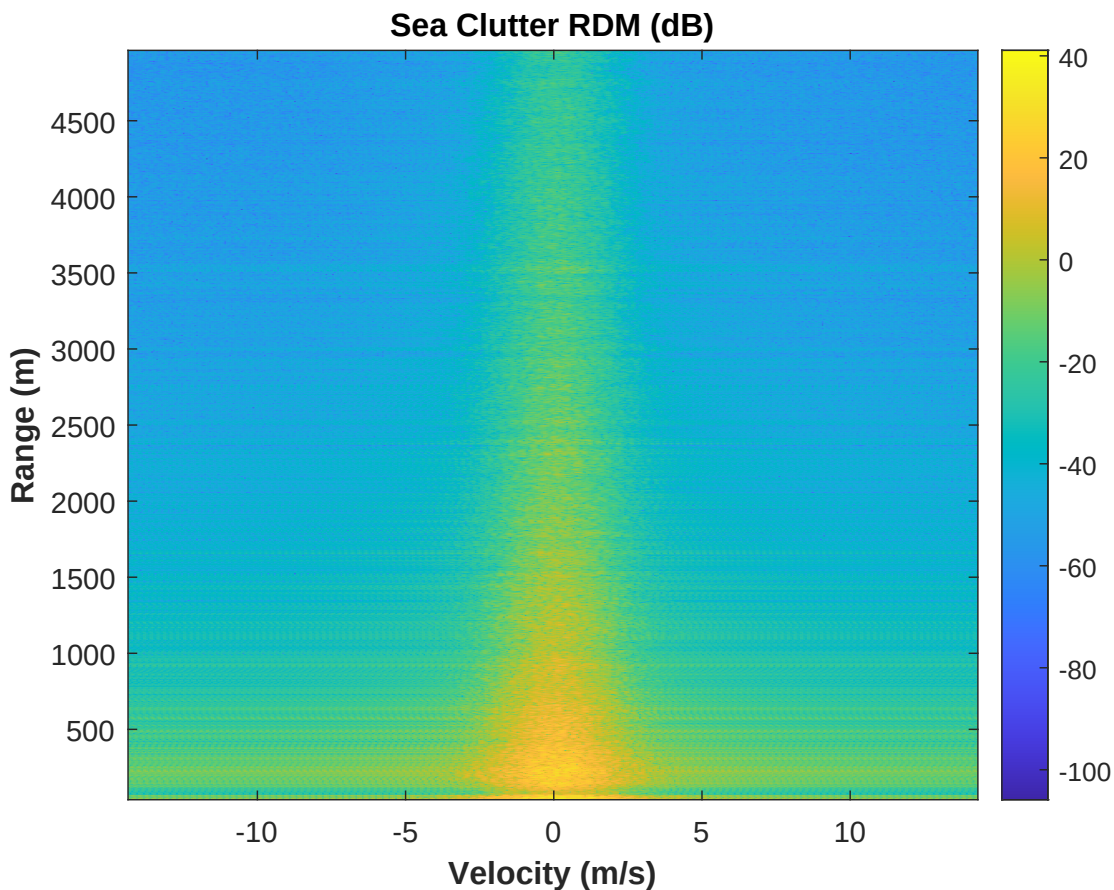


Figure 4.5: Range-Doppler map of sea clutter with the given parameters described in Section 4.2.2. With 128 pulses, the number of the basis functions in the DFT were over sampled by a factor of four to achieve 512 frequency bins.

4.3 Clutter Mitigation

Clutter mitigation is usually the first step to detect potential targets within a range-Doppler map. The term mitigation implies utilizing techniques that work around the dominant interfering aspects of the existing clutter without removing it. Steps to mitigate clutter are generally taken first as computational time to mitigate clutter out performs clutter cancellation techniques where matrix inversion

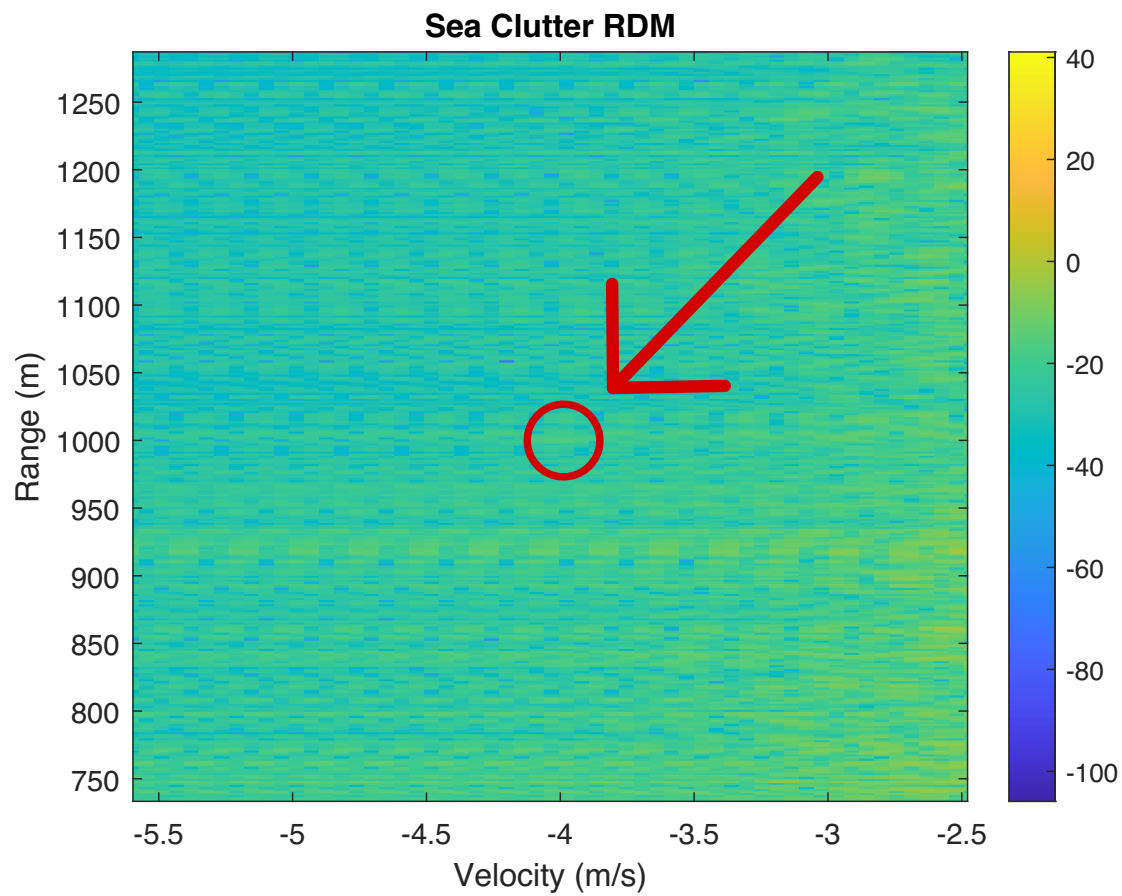


Figure 4.6: Zoomed in version of Figure 4.5 to show proposed target location. Filtering will be applied later to fully uncover the target range and velocity.

may be required. The most popular mitigation technique that will be observed here in more detail is windowing.

4.3.1 Windowing

Applying a window function to the data is done before Doppler processing and serves to suppress sidelobe leakage of the sinc-like response of the Doppler processed returns. Because of the time-frequency duality, rectangular windows in time with sharp transitions result in a sinc function in frequency with significantly large sidelobes; however, smooth transitions in time result in lower sidelobes at the expense of widening the energy at the mainlobe. Windowing serves to smooth the edges of the rectangular shape of the returns, causing sidelobes to be drastically reduced. Data can be windowed in both range and slow-time depending on the desired result.

Different windowing functions can be used, with some popular ones being: Hann, Chebyshev, Hamming, and Blackman. Figure 4.7 shows how the window applied in slow time affects the magnitude of the range slow time data. It can be seen that the magnitudes of the data at the beginning and end of the CPI begin to taper in response to the window. This causes the resulting range-Doppler map shown in Figure 4.8 to have decreased sidelobes compared to Figure 4.5 at the expense of a wider mainlobe response. SINR is improved over the unfiltered range-Doppler map as the windowed range-Doppler map shows an SINR of -21.24 dB. Windowing can aid in the detection of targets that reside strictly in the sidelobes; however, as the target is positioned closer to the main clutter ridge, detectability fails as the mainlobe of the clutter widens, overcoming the target signature. In that case, clutter mitigation is no longer sufficient and clutter cancellation techniques must be considered to attempt to resolve the target.

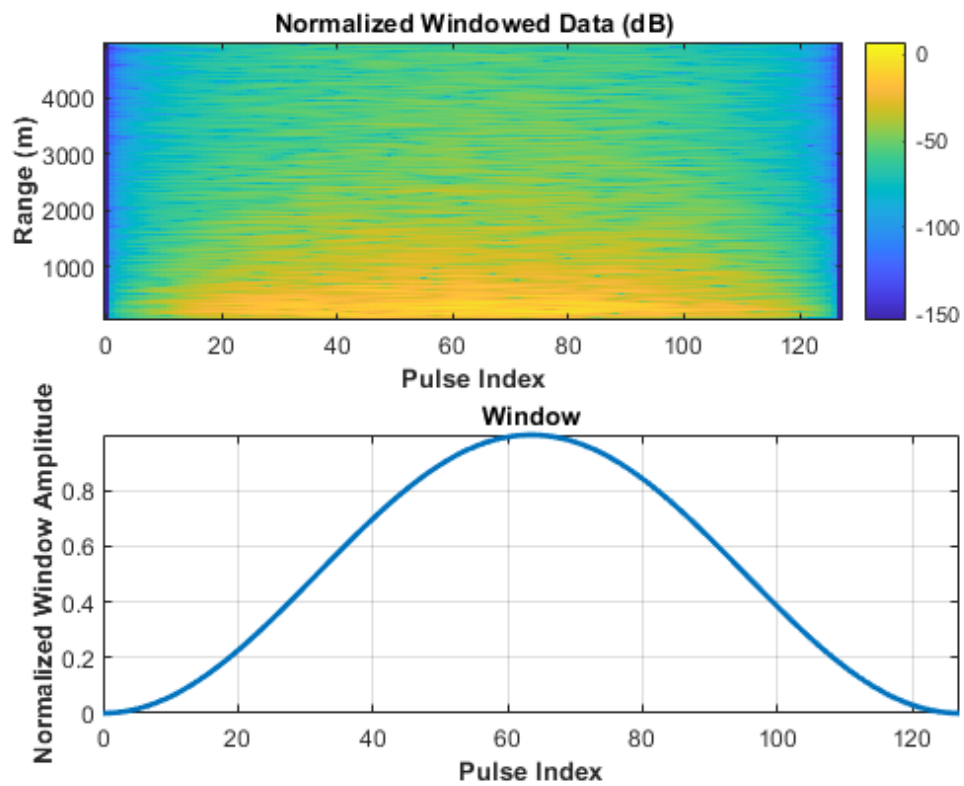


Figure 4.7: A hanning window fit to a 128 pulse CPI is applied across the slow-time dimension of the data. By tapering the data in slow-time, the sidelobe in Doppler are attenuated in the resulting range-Doppler map.

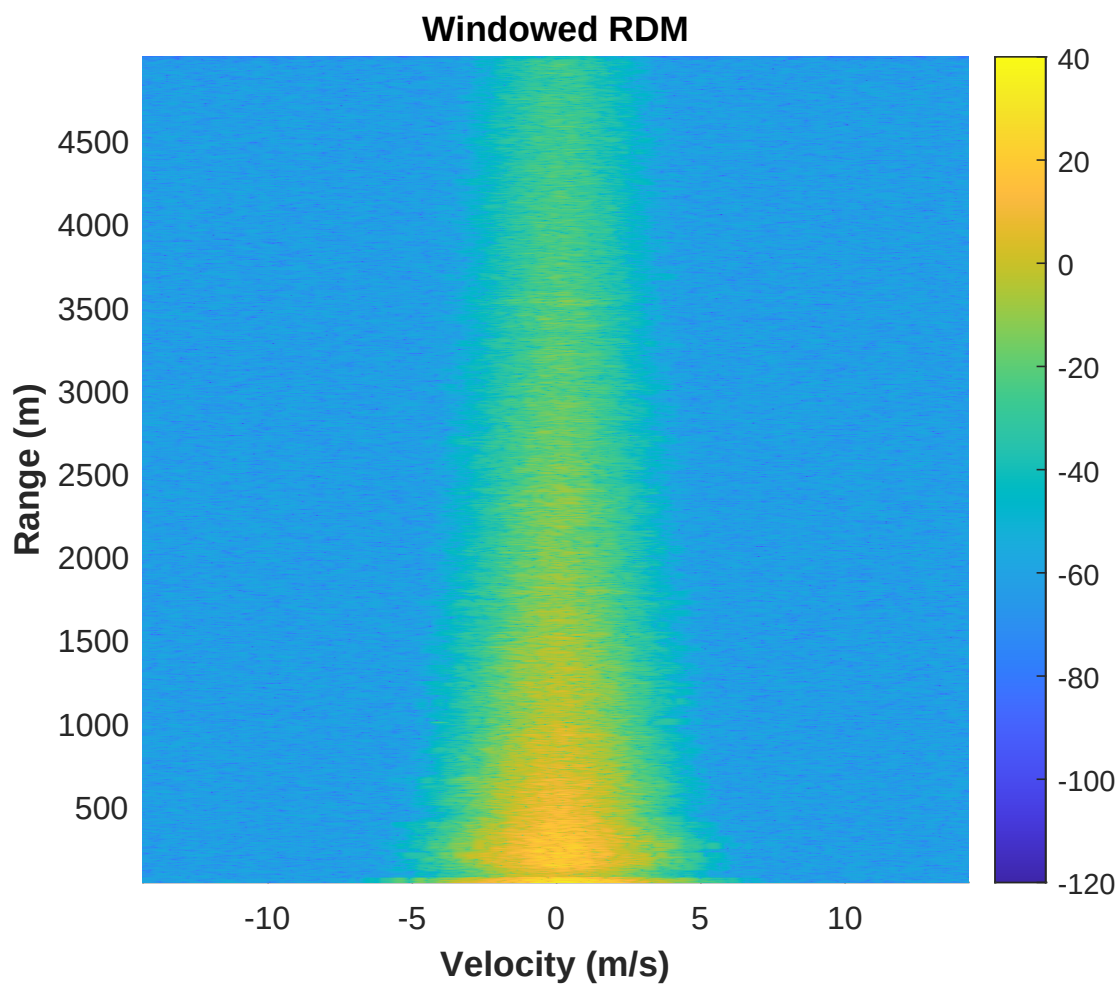


Figure 4.8: The range-Doppler map of the windowed sea clutter data exhibits vastly decreased sidelobes in comparison to Figure 4.5 at the expense of a broader mainlobe.

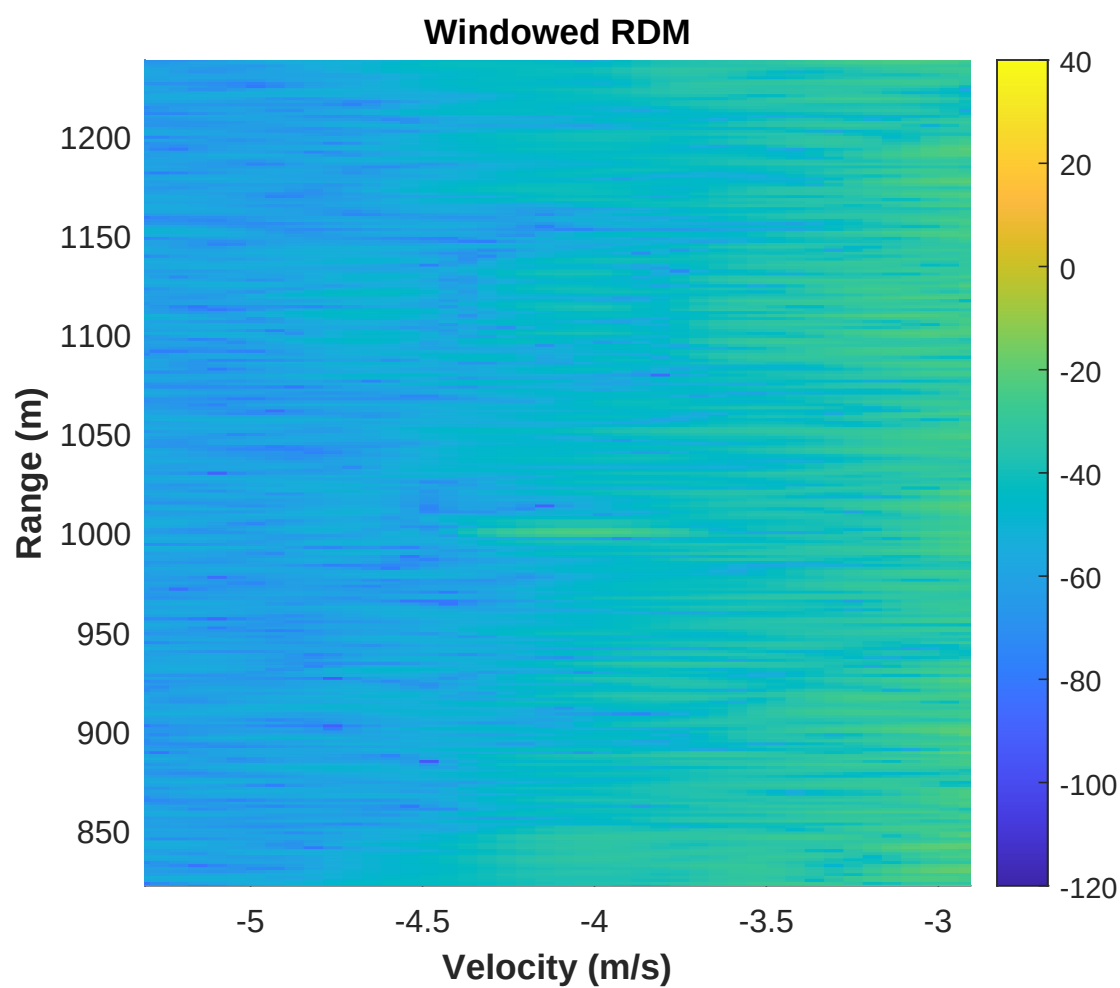


Figure 4.9: Zooming into the target location shows the target signature begins to be visible after windowing.

4.4 Clutter Cancellation

Clutter cancellation is a step deeper than mitigation, where filters are designed specifically to negate the effects the clutter has on the range-Doppler map. Often, statistical relationships are derived using the covariance matrix of the signal data, which shows how each pulse of the dataset varies from one another. The covariance is then generally used to create filter weights that best fit the clutter characteristics. Eigenvalues and vectors can also be used in the generation of clutter cancellation filters to notch the frequencies associated with the most dominant clutter signals. Three techniques were used in an attempt to cancel sea clutter for a maritime radar system to attain detections of low RCS targets in the midst of the clutter, these are adaptive filtering and clutter notching.

4.4.1 Adaptive Filtering

Adaptive filtering is a technique taken from Space-Time Adaptive Processing (STAP) [22], [23] where adaptive filter weights for each Doppler bin are calculated based on clutter interference statistics. These weights optimize the ratio of signal power to interference power and take the form,

$$\mathbf{w}(f_k) = \hat{\beta} \mathbf{R}^{-1} \mathbf{V}_{F_D} \quad (4.7)$$

where $\hat{\beta}$ is a scalar, $\mathbf{R}$ is the RDM covariance matrix between pulses, and $\mathbf{V}_{F_D}$ is the Doppler steering vector as described in 4.4. The covariance matrix is commonly computed by,

$$\mathbf{R} = \frac{1}{P} \sum_{m=1}^P \mathbf{x}_m \mathbf{x}_m^H \quad (4.8)$$

where $\mathbf{R}$ is normalized by a factor of P , and the data, $\mathbf{x}$, contributes to the calculation of the covariance yielding $\mathbf{R} \in \mathbb{C}^{N_p \times N_p}$. The scalar $\hat{\beta}$ is important in

deriving the appropriate filter for an advantageous SINR. If following the square-law output, $\hat{\beta}$ can be calculated by the expression,

$$\hat{\beta} = \frac{1}{\sqrt{\mathbf{V}_{F_D}^H \mathbf{R}^{-1} \mathbf{V}_{F_D}}} \quad (4.9)$$

which yields the expected value,

$$\eta = |\mathbf{w}^H \mathbf{x}|^2 = \frac{|\mathbf{V}_{F_D}^H \mathbf{R}^{-1} \mathbf{x}|^2}{\mathbf{V}_{F_D}^H \mathbf{R}^{-1} \mathbf{V}_{F_D}} \quad (4.10)$$

which is called the adaptive matched filter, which also exemplifies constant false alarm rate properties. Accounting for the scalar, the final filter weights across Doppler bins are given by the expression,

$$\mathbf{w}(f_k) = \frac{\mathbf{R}^{-1} \mathbf{V}_{F_D}}{\sqrt{\mathbf{V}_{F_D}^H \mathbf{R}^{-1} \mathbf{V}_{F_D}}} \quad (4.11)$$

The manifold of weighted steering vectors can be applied across the data for the RDM by multiplying the data with the conjugate of the filter. If $\mathbf{x} \in \mathbb{C}^{L_{\text{ADC}} \times N_p}$ and $\mathbf{w} \in \mathbb{C}^{N_p \times N_{fft}}$, the output yields the RDM $\in \mathbb{C}^{L_{\text{ADC}} \times N_{fft}}$.

The SINR of the adaptively filtered range-Doppler map is greatly improved as it is increased to 10.95 dB. However, this filtering method requires the inversion of the signal covariance matrix, thus the covariance in Doppler space, $\mathbf{R} \in \mathbb{R}^{N_p \times N_p}$, has a complexity given by the equation,

$$O(n^3) \quad (4.12)$$

where n is the dimensionality of the square matrix to be inverted, in this case $n = N_p$. However, if the computational complexities can be sustained, adaptive filtering proves to be an efficient implementation of clutter cancellation as the

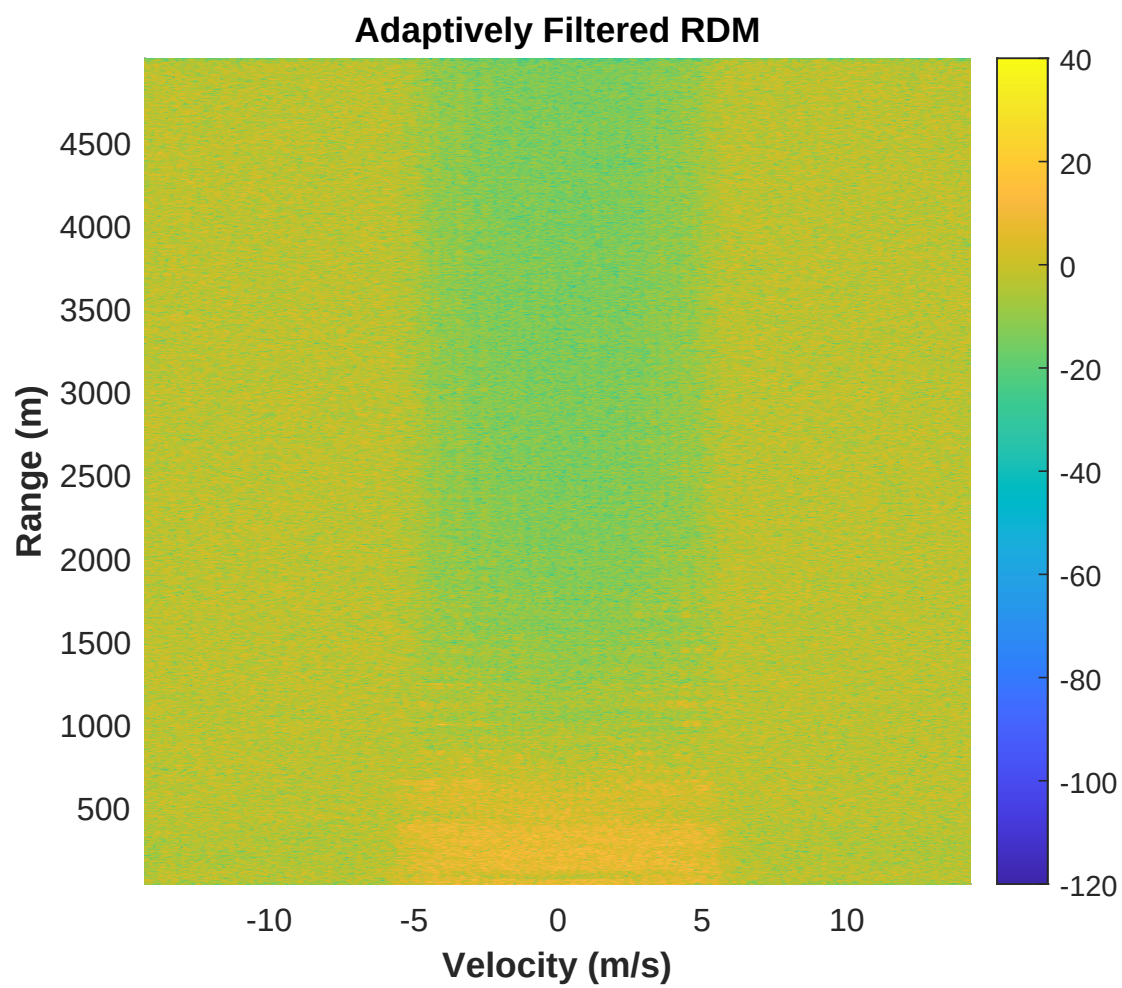


Figure 4.10: RDM achieved from adaptive filtering in the Doppler dimension.

filter weights converge on the adaptive matched filter giving optimal performance.

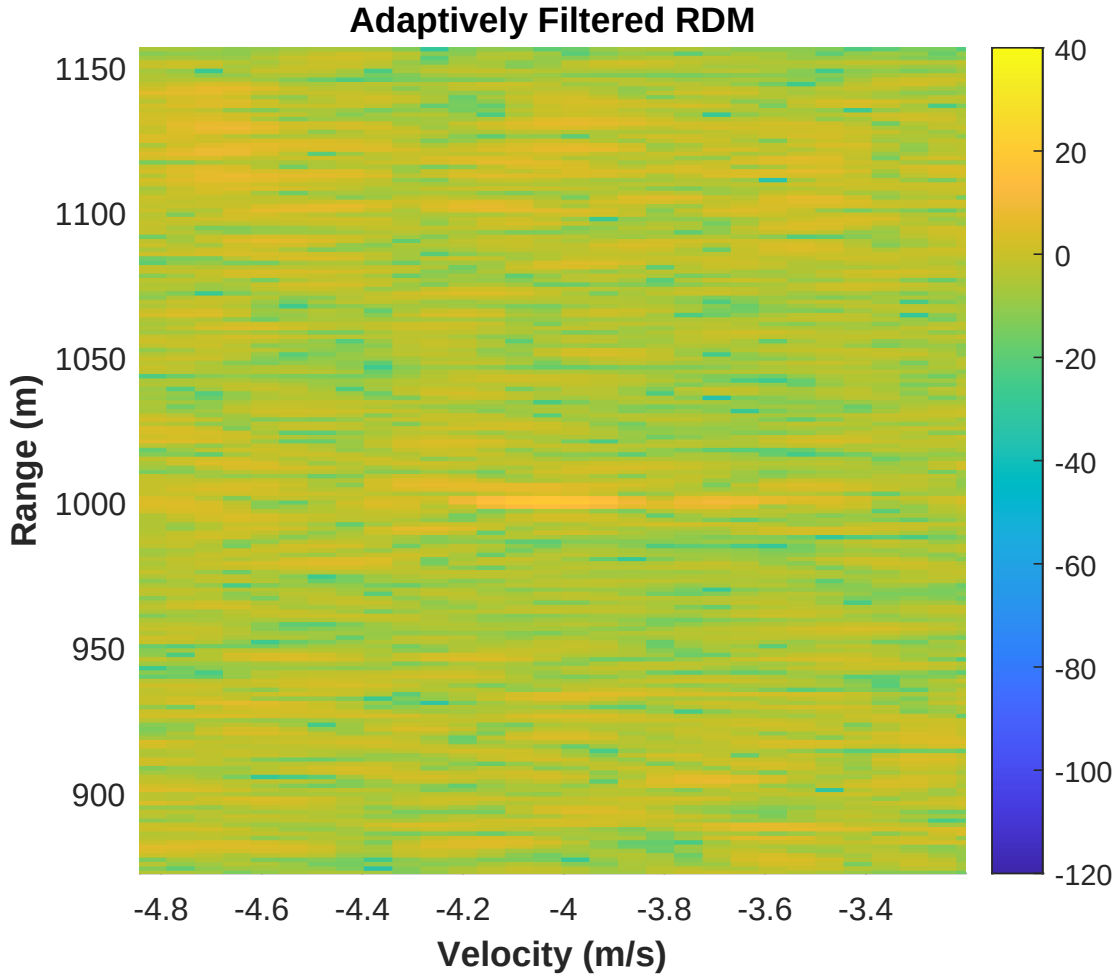


Figure 4.11: Zoomed in image of the RDM in Figure 4.11 showing the target revealed after performing adaptive filtering.

4.4.2 Clutter Notching

Clutter notching is a common term that refers to the nulling out of certain Doppler frequencies that the radar system is expected to ignore before passing off the data to the detector. In most use cases, ground clutter can easily be nulled out at 0 Hz Doppler to allow any target that falls within the minimum detectable velocity to be isolated from the clutter signature. For sea clutter, the unpredictability makes

choosing a specific Doppler frequency to null difficult, so often times, nulling a particular Doppler is dependent on what is expected to be seen in the scene, and thus is more suited for post-processing. The nulling technique presented here is done by using singular value decomposition (SVD).

To create a null in Doppler using SVD vectors, the steering vector must first be formed in a manner similar to spectrum shaping. The Doppler steering matrix takes the form of Equation 4.4 with the exception that f_d represents the Doppler frequencies associated with the null bins. Then, performing a singular value decomposition of the steering matrix,

$$\mathbf{A} = \mathbf{U}\mathbf{E}\mathbf{V}^H \quad (4.13)$$

where $\mathbf{E} \in \mathbb{R}^{N_p \times N_{fft}}$ are the singular values of $\mathbf{A}$, while $\mathbf{U} \in \mathbb{C}^{N_p \times N_p}$ and $\mathbf{V} \in \mathbb{C}^{N_{fft} \times N_{fft}}$ are the left and right singular vectors. Then, to define the clutter subspace $\mathbf{U}_c$, the dominant left singular vectors are selected by thresholding their singular values given by,

$$\mathbf{U}_c = [(10 \log_{10}(\sigma_i^2) > \max(10 \log_{10}(\sigma_i))) - d] \in \mathbb{C}^{N_p \times d} \quad (4.14)$$

where σ_i are the diagonal elements of $\mathbf{E}$, and d is the desired dynamic range threshold (in dB) of the selected singular vectors. Lastly, an orthogonal projection matrix used to cancel the clutter subspace can be constructed by,

$$\mathbf{P} = \mathbf{I} - \mathbf{U}_c \mathbf{U}_c^H \in \mathbb{C}^{N_p \times N_p} \quad (4.15)$$

where $\mathbf{I}$ is the identity matrix. The projection matrix is then applied to the data in the slow-time dimension which removes the components of the signal data most

aligned with the clutter subspace. Depending on the choices of f_d and d , the notch can be of different widths and depths; however, Figure 4.12 shows the RDM of the sea clutter with the notching applied through the decomposition of singular values. If desired, the notch can be shifted in frequency by compensating for the normalized Doppler component of the steering vector, f_d . For the widened notch in Figure 4.13, which is obtained by expanding the frequencies, f_d , represented in $\mathbf{A}$, the SINR increases to 20.44 dB. While this is a substantial increase, the effort required to gauge the width and depth of the clutter notch to make this example a best-case scenario must not be understated. Clutter notching is best applied when the exact width and depth are known or if post-processing of the data allows time to decide on an optimal configuration. Other techniques that adaptively isolate the target from the clutter might be preferential based on the application.

4.5 Spectral Refinement

Spectral refinement can be performed to decrease interfering sidelobes to a target that may be obscured through constructive or destructive interference from a dominant clutter source. Various algorithms exist to refine the Doppler spectrum of a range-Doppler map, but Reiterative Superresolution (RISR) will be considered here.

4.5.1 Reiterative Superresolution (RISR)

Reiterative Superresolution (RISR) is a superresolution technique that can be repeated multiple times to effectively reduce sidelobes to resolve target energy signatures. RISR was developed in [24] as a Direction of Arrival (DOA) algorithm for STAP techniques; however, it can be applied on the range-Doppler level

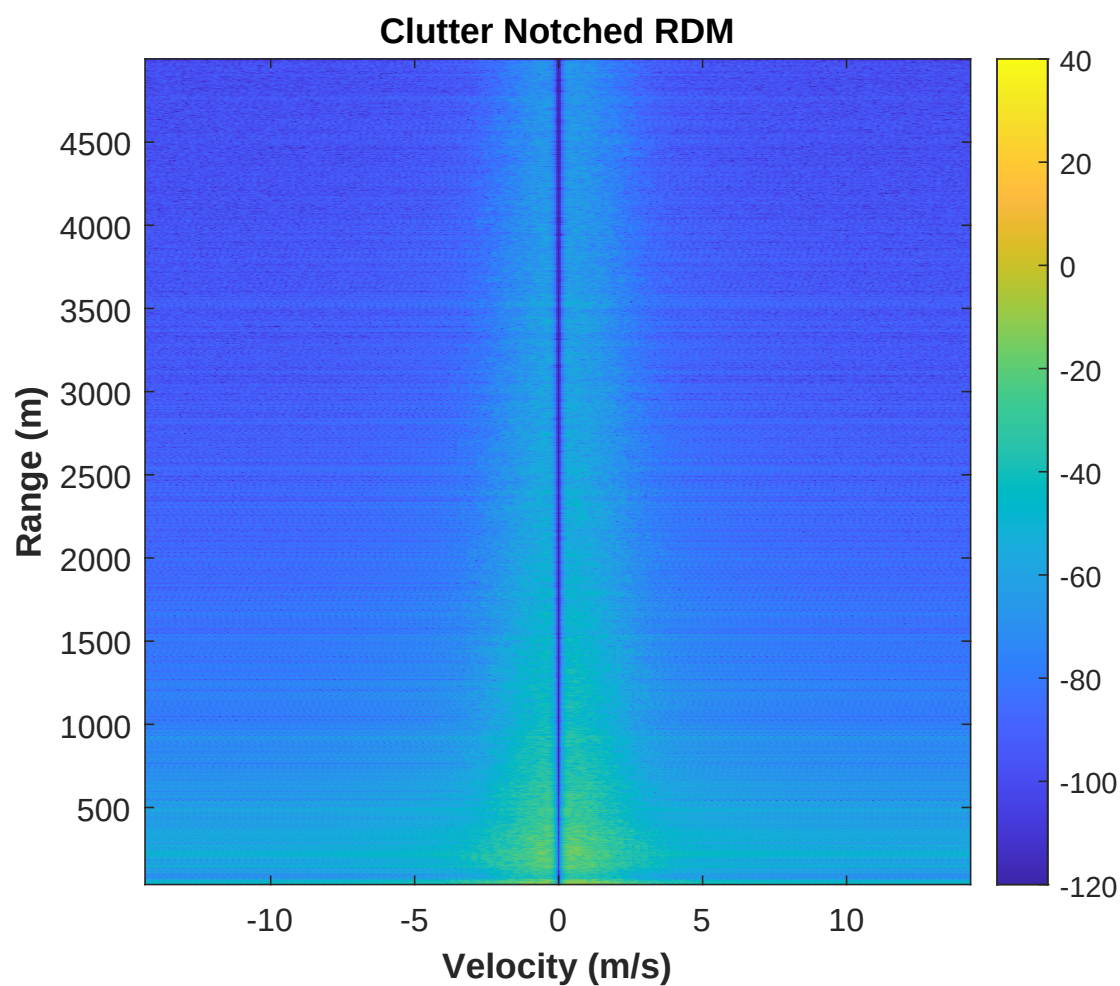


Figure 4.12: SVD notching performed on the sea clutter RDM with the 0 Hz and surrounding clutter bins notched.

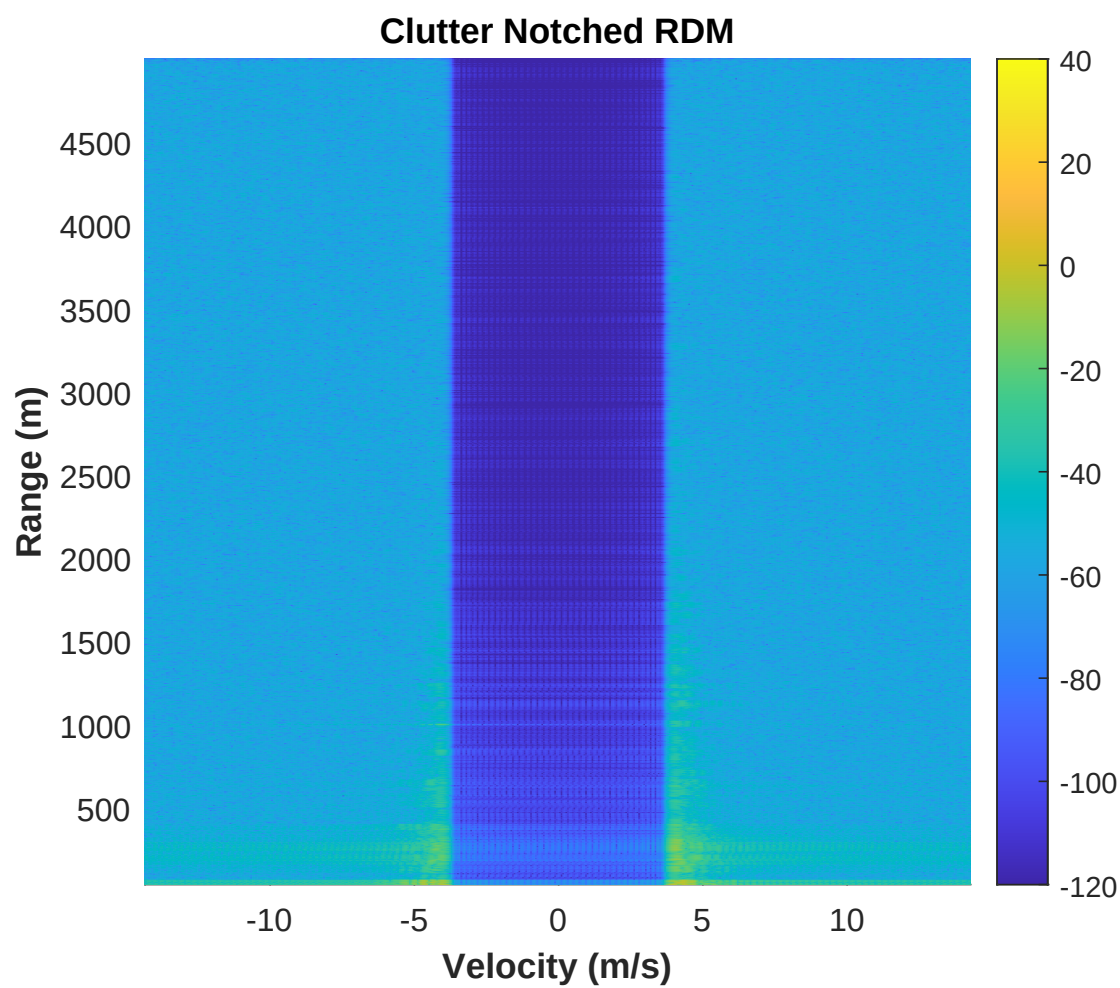


Figure 4.13: SVD notching with a wider notch to accommodate the width of the Doppler spectrum from the sea clutter.

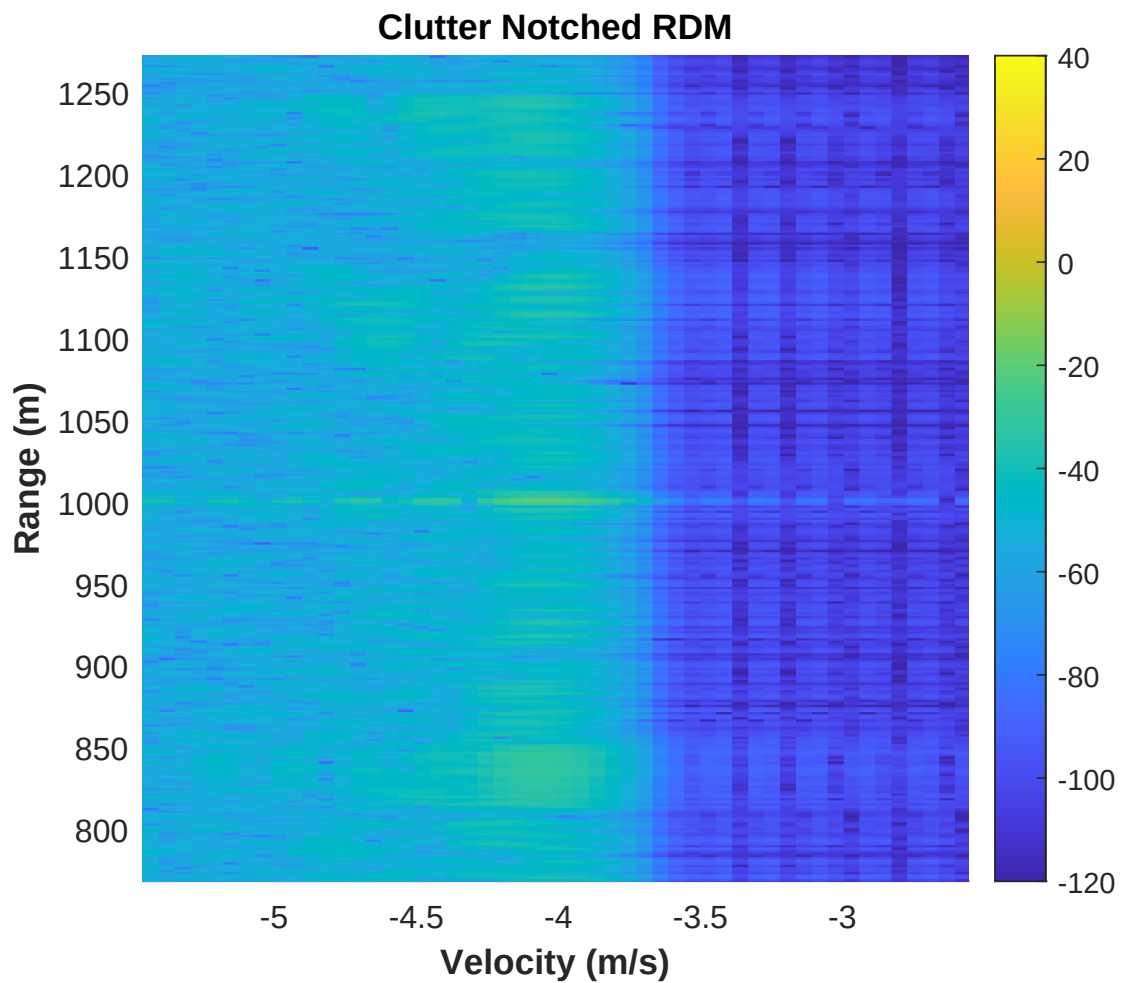


Figure 4.14: After widening the notch by expanding the frequency values in the calculation of the Doppler steering matrix, sidelobes from the sea clutter become reduced to a level where the target is visible to the eye.

to super-resolve targets in either dimension. The technique is derived from the Minimum Mean Square Error (MMSE) formulation that estimates a profile (either range or Doppler in this case) to be reconstructed and then formulates a filter specifically fit to that bin which minimizes the error through MMSE. After applying the filter, the estimate is updated for further iterations as the estimate is continually improved. The filters produced, through various iterations, place nulls at locations found to contain high levels of energy as a result of interference and sidelobes; therefore, in effect, smaller targets can be resolved in the presence of high-energy sidelobes. The RISR aggregate filter produced for each bin is given by [24],

$$\mathbf{W}(l) = (\mathbf{S}\mathbf{P}\mathbf{S}^H + \mathbf{R}_n)^{-1}\mathbf{s}(l)\mathbf{p}(l) \quad (4.16)$$

where $\mathbf{S}$ is the steering vector manifold associated with the profile being reconstructed, $\mathbf{P}$ is the signal covariance matrix for $\mathbf{P} = \text{diag}(\mathbf{p})$, $\mathbf{p}$ is then the current profile estimate, $\mathbf{R}_n$ is the noise covariance matrix (assumed to be $\mathbf{R}_n = \sigma^2\mathbf{I}$ for white noise), and lastly, $\mathbf{s}$ is the steering vector for the l th bin. Each iteration produces a new filter bank $\mathbf{W}$, followed by an updated estimation of the signal covariance $\mathbf{P}$ for subsequent iterations.

Similarly to the adaptive filtering technique described in Section 4.4.1, inversion of a square matrix is a required step in the process which introduces the same computational complexity requirements of Equation 4.12 for each iteration of RISR. RISR variants are in development by [25] that rely on reducing the dimensionality of the matrix before inversion to meet computational demands. Reduced-Dimension Reiterative Superresolution would allow for more real-time applications of this filtering method.

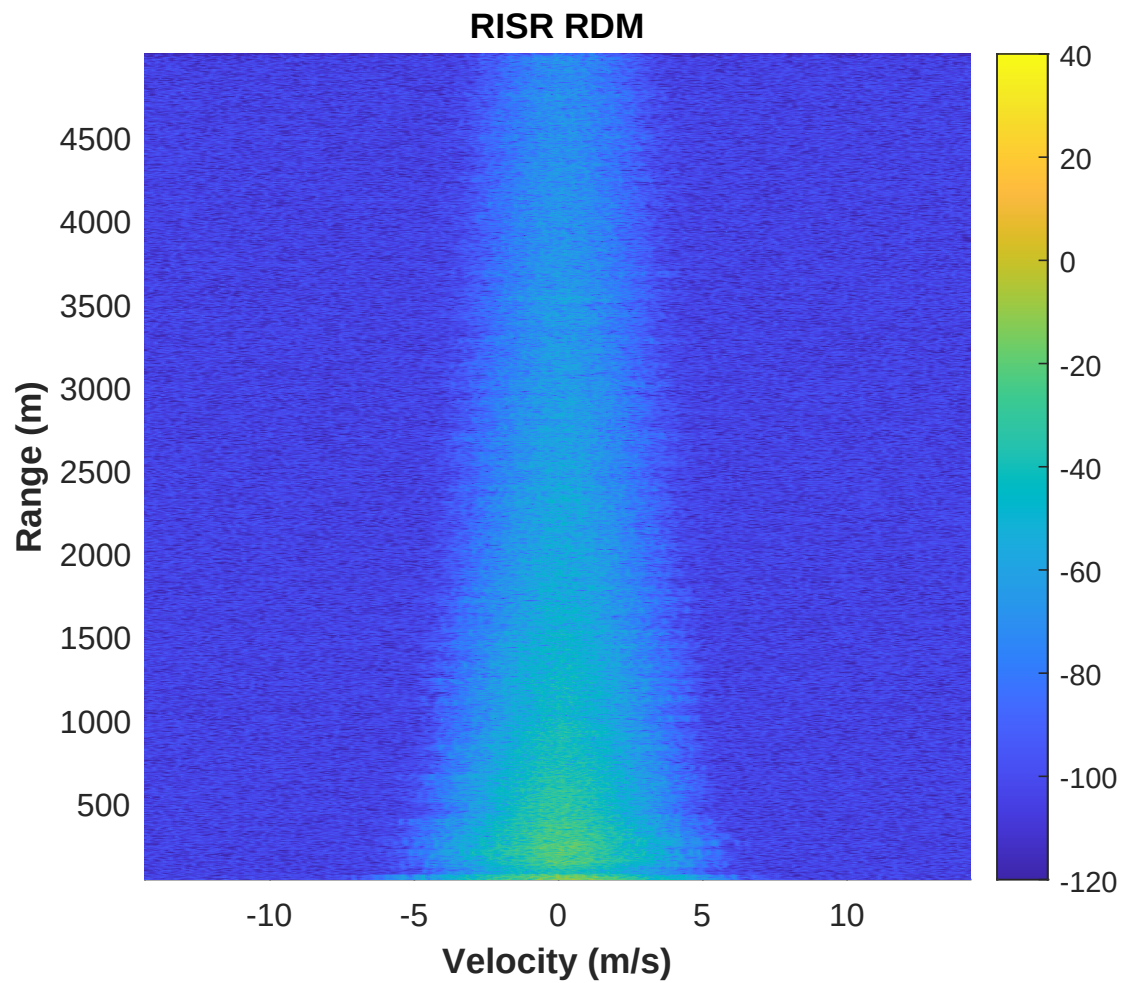


Figure 4.15: RISR algorithm with 10 iterations performed on sea clutter data to superresolve the range Doppler map in the Doppler dimension.

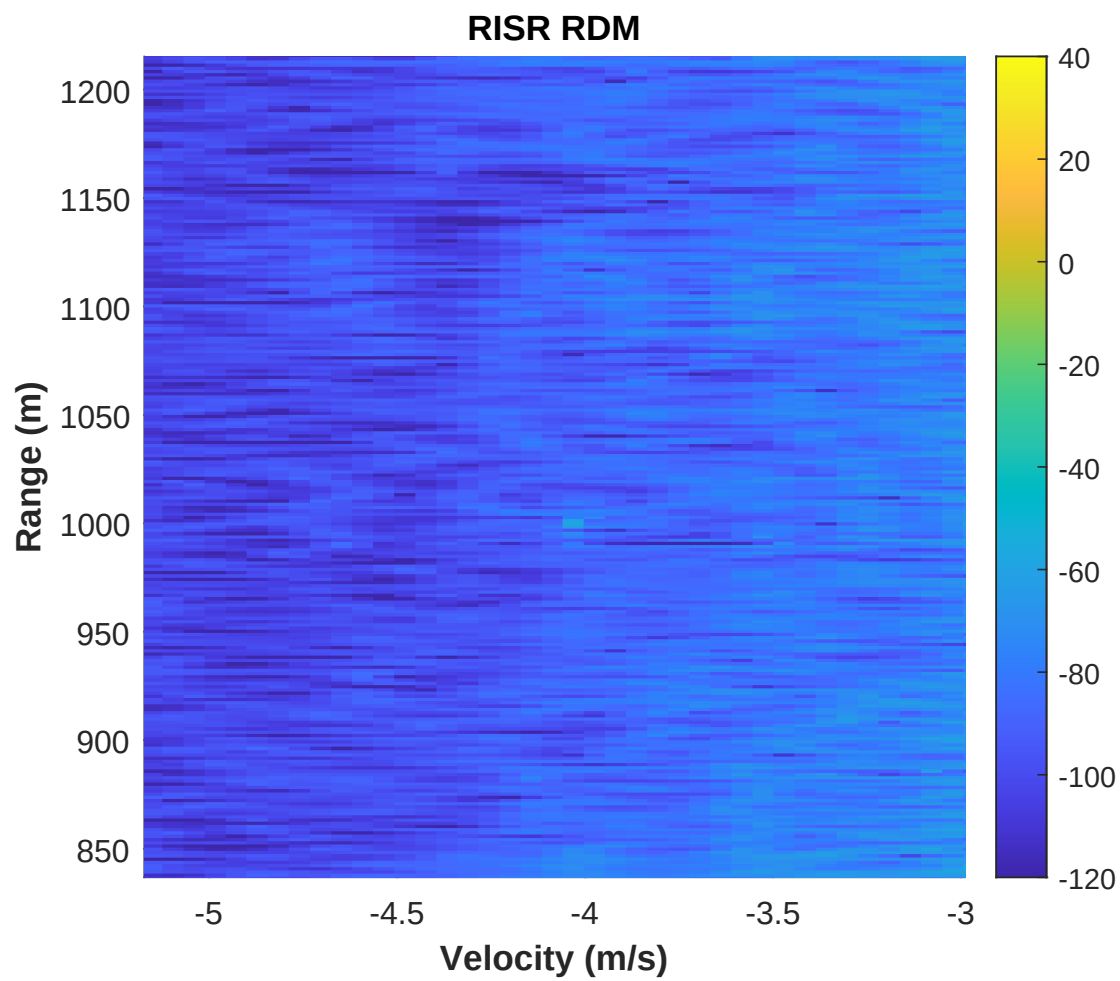


Figure 4.16: After the RISR algorithm superresolved the RDM in Doppler with 10 iterations, the target is revealed.

4.6 Signal-to-Interference-Plus-Noise Ratio (SINR)

The Signal-to-Interference-Plus-Noise Ratio (SINR) is used to quantify the proposed benefit of each clutter processing technique. Equation 2.15 outlines the ratio as the proportion of the signal power received from the target, P_r , to the combined power contributions from clutter and noise, P_c and P_n . To calculate these throughout each phase of the simulation, the contributions of power received from the target are separated from the clutter and noise power to determine the starting SINR. For the fast-time slow-time data cube, this is simple as the target returns are kept separate from the clutter and noise power; therefore, by summing the magnitude squared of each contribution independently, P_r , P_c , and P_n can be determined. However, once Doppler processing occurs, the calculation becomes difficult to determine with exact precision. In the case of Figure 4.5, the clutter-dominated scene will bias the SINR calculation. While SINR will increase due to Doppler processing, constructive and deconstructive interference of the clutter sidelobes will offset the estimate of the power received from the target. In this case, the best option is to simply choose the range and Doppler bin where the target appears and declare that bin and its surrounding bins as the target power. Then the clutter and noise power is attributed as the rest of the data excluding the target window. That is, after Doppler processing, the SINR calculations will be optimistic since the target, clutter, and noise power are no longer separated, allowing constructive interference from strong clutter sidelobes to bias the target returns. Table 4.3 shows the calculated SINR and the improvement over the raw data after processing.

Table 4.3: Signal-to-Interference-Plus-Noise results with improvement over raw data.

Method	SINR	Improvement
Raw Data	-76.63 dB	n/a
Doppler Processing	-18.94 dB	57.69 dB
Windowing	-21.24 dB	55.39 dB
Adaptive Filtering	10.95 dB	87.58 dB
Clutter Notching	20.44 dB	97.07 dB
RISR	-15.49	61.14 dB

Chapter 5

Target Detection

5.1 Detection

The ideal output of any detector is a Boolean true or false decision about whether a target within a range and Doppler bin is present or not. The received signal, corrupted with noise and contributions from clutter that are not relevant to the targets of interest in the scene, must be analyzed to determine whether the target exists and at which range and velocity it appears. Detectors generally use statistical models to determine the presence of a target among other interfering sources of energy, but a common way detectors sift out unwanted returns is through threshold detection. Thresholding is a method to sift out the effect of unwanted returns by setting a limit where a Boolean true is assigned if the returns from that particular bin exceed the set threshold. This can work assuming the interference has an average level that is below the targets of interest the deployed radar system would expect to see. However, in a maritime deployed system, sea spikes would make thresholding very difficult to implement successfully for detection decisions [26]. Aside from sea spikes, which show their own difficulty, determining the threshold itself would prove a challenge, as sea surface conditions are constantly evolving with the influence of winds and currents. For maritime radar, the more effective

detector is often a Constant False Alarm Rate (CFAR) detector, where the detector assigns a probability of false alarms that is acceptable. The CFAR detector is preferred because thresholding alone assumes a known interference level, yet the interference is often unknown, which is why CFAR is sometimes referred to as “adaptive” or “automatic” detection [11]. In reality, the false alarm rate for a maritime radar system must be constantly adjusted as ocean conditions change, further proving a challenge to detect targets in a maritime environment [3].

Detection decisions, often statistical in nature, are described by two hypotheses; the given sample is a measurement of the interference only, or the sample under test is a measurement of both returns from the target of interest and interference. The detector must examine each sample and come to one of the two conclusions. General analysis of this model is given by the following probability density functions with the aforementioned hypotheses given by,

$$p_y(y|H_0) = \text{PDF of } y \text{ given that a target } \textit{was not} \text{ present.}$$

$$p_y(y|H_1) = \text{PDF of } y \text{ given that a target } \textit{was} \text{ present.}$$

where y is the sample tested by the detector. Together, these create a pdf that is dependent on the number of samples in the dataset and the fulfillment of one of the two hypotheses. From these pdfs, several outcomes can be true:

1. Probability of detection, P_D : The probability that a target is detected when there is a target present. In other words, H_1 is chosen and is a correct assessment of the presence of the target.
2. Probability of false alarm, P_{FA} : The probability that a target is detected when there is *not* a target present. In this case, H_1 is chosen when H_0 was the correct decision.

3. Probability of miss, P_M : The probability that a target is not detected when, in fact, a target is present. This case occurs when H_0 is chosen when H_1 was the correct hypothesis.

Figure 5.1 shows each outcome as related to K-distributed sea clutter as the primary interference. As the threshold is adjusted, so are the probabilities of detections and misses. Given the relation,

$$P_D = 1 - P_M \quad (5.1)$$

lowering the threshold would surely increase the probability that the target is detected; however, the probability that the radar has a false alarm is greatly increased. Hypothesis H_0 depends on the clutter statistics described in Section 3.2, and as the distribution becomes more heavy-tailed with more violent ocean conditions and sea spikes, the ability to detect the same target deteriorates. In contrast to this, the more the two pdfs are isolated from each other, the easier it will be for the detector to make an accurate detection of the target. There are two primary ways of increasing the probability of detection for a given threshold: increasing the mean of $p_y(y|H_1)$ through an increased P_r of the target effectively increasing SINR, or reducing the variance of the two pdfs through a reduction of noise power, which will widen the gap between the two pdfs as the variance of both pdfs will be reduced.

The Neyman-Pearson detection rule is used to determine the threshold that maximizes the probability of detection for a given, acceptable probability of false alarm. This approach is used in the Likelihood Ratio Test (LRT) where,

$$\frac{p_y(y|H_1)}{p_y(y|H_0)} \underset{H_0}{\overset{H_1}{>}} -T$$

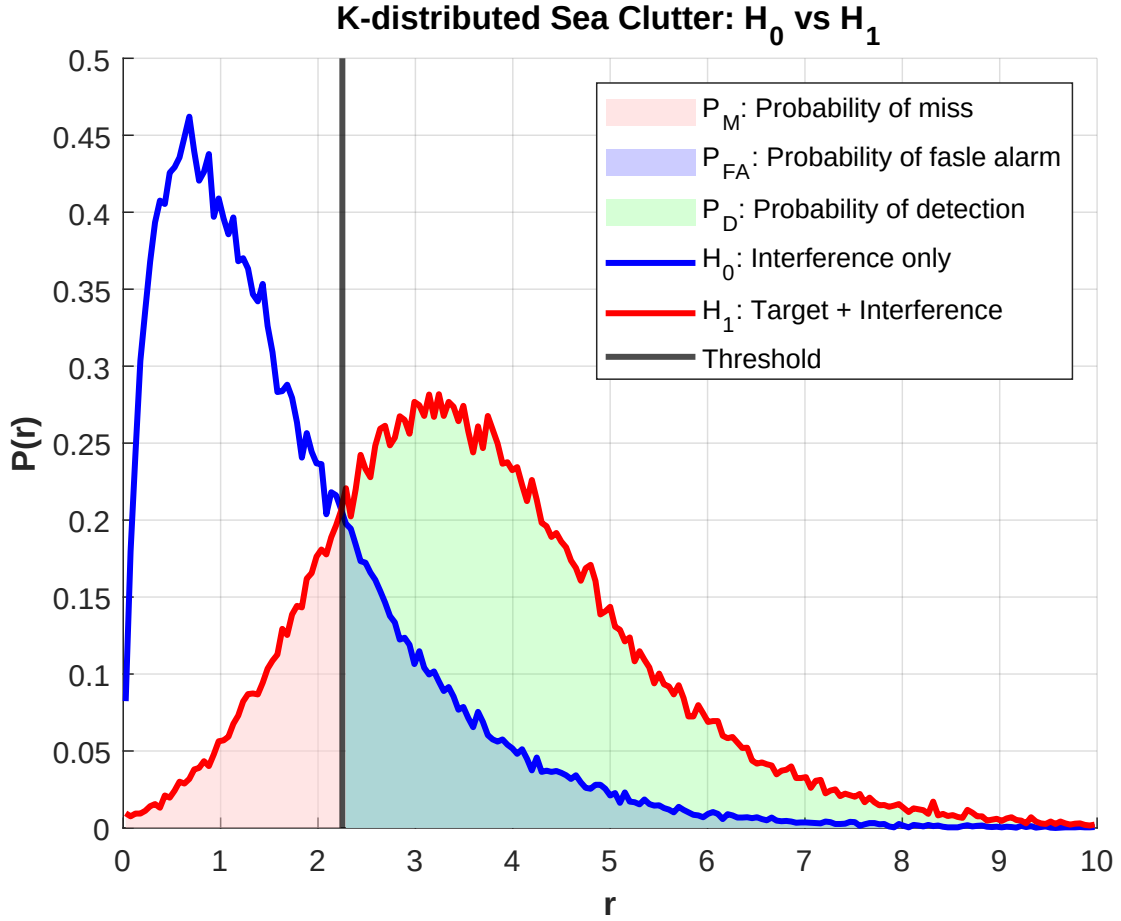


Figure 5.1: Detection hypotheses using K-distributed interference and a Gaussian-modeled target signal in the presence of noise and clutter. The outcome subspace is divided between the two probability distributions, H_0 (interference only) and H_1 (target + interference), based on the detection threshold applied to the test statistic r , which approximates signal amplitude but is plotted in arbitrary units.

gives the relationship of each hypothesis based on the threshold, T . The log-likelihood ratio test provides the same form as the previous equation but with the natural logarithm taken on both sides. From this, the desired threshold that follows this rule for Gaussian noise can be calculated based on a set probability of false alarm by,

$$T = \sqrt{2N\beta^2} \operatorname{erf}^{-1}(1 - 2P_{FA}) \quad (5.2)$$

conversely the probability of false alarm based on the threshold is,

$$P_{FA} = \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{T}{\sqrt{2N\beta^2}} \right) \right) \quad (5.3)$$

where N is the number of samples and β is the noise variance of the samples before matched filtering.

Detection in this manner tends to fall apart if the noise is not independent and identically distributed, which is the case for sea clutter. Setting a hard threshold while the sea surface could cause a multiplicity of false alarms as the sea states and the wind increase or decrease over time. A more adaptive approach must be taken to ensure that local interference levels are not factored into an absolute detection decision. Different techniques must be adapted to handle the non-Gaussian correlated nature of the clutter to achieve proper detections. CFAR is the most common approach, as the detector is given a set probability of false alarm and adapts a threshold based on local clutter intensity.

5.1.1 Cell Averaging Constant False Alarm Rate (CA-CFAR)

Constant False Alarm Rate (CFAR) detectors are frequently used in standard radar system configurations, since interference statistics are generally not an a priori input to the system. Cell Averaging Constant False Alarm Rate (CA-CFAR)

detectors analyze mean local interference levels to estimate the likelihood that a target is present while maintaining a specified false alarm rate. The CA-CFAR structure, illustrated in Figure 5.2, is composed of a sliding reference window that calculates the mean interference level of the Cell-Under-Test (CUT) (shown in red) of the surrounding windows cells (shown in blue). Additional guard cells are placed around the CUT (shown in yellow) are not factored into the mean level of the CUT to avoid contamination of the mean levels from the sidelobes of the target. The CA-CFAR detector decides if a target is present based on how target returns bias the reference window as they relate to the local mean interference level as the CUT moves across the location of a target. Depending on the probability of false alarm, a detection decision is made about the target based on the adaptive threshold.

For CA-CFAR, the threshold and probability of false alarm relation for a single raw sample with a non-fluctuating target in Gaussian interference is given by,

$$P_{FA} = \exp\left(\frac{-T}{\sigma_w^2}\right) \Rightarrow T = -\sigma_w^2 \ln(P_{FA}) \quad (5.4)$$

where σ_w^2 is the variance of the noise [11]. This relation inherently falls apart in sea clutter scenarios, as the assumptions made when applying CA-CFAR are [27]:

1. The interference present in the window cells and CUT is IID.
2. When the CUT is placed on a target, the window cells do not contain returns from other targets present in the scene that might bias the averaging.

Because sea clutter is characteristically not IID, CA-CFAR can only perform so well in complex sea clutter environments where the returns are correlated and non-Gaussian. A better representation of the probability of false alarm is given by [28] where the probability of false alarm for a single sample, y , is influenced by

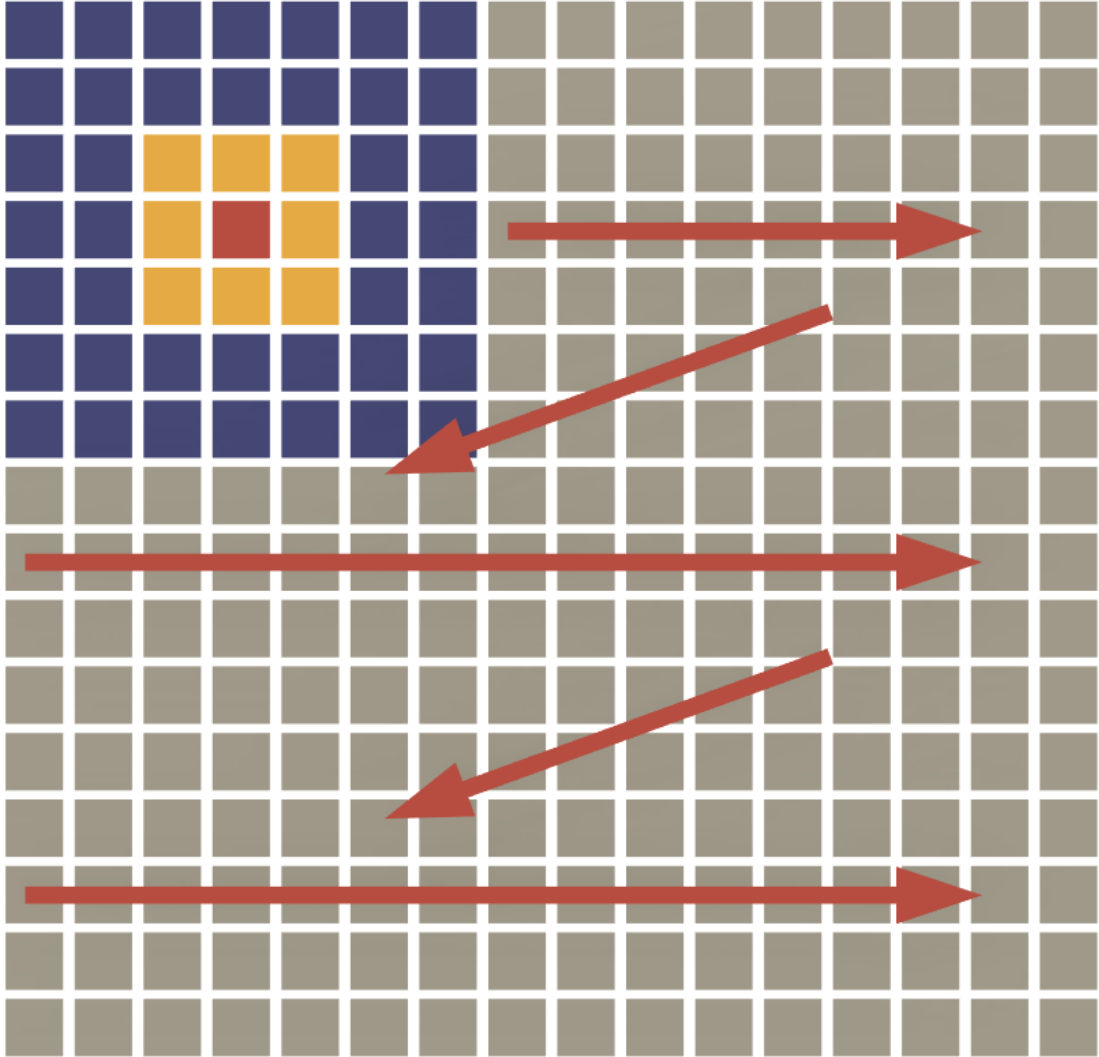


Figure 5.2: CA-CFAR requires a window to be moved across the dataset to generate an noise/interference estimate for each cell under test. The cell under test and guard cells are red and yellow respectively, while the blue cells are the window cells. The structuring element does not have to be square as shown, and the number of guard and window cells is assigned according to the application.

the K-distributed amplitude returns and is given by,

$$P_{FA}(y) = \frac{2(vy)^{v/2}}{\Gamma(v)} K_v(2\sqrt{vy}) \quad (5.5)$$

where instead of a set dependence on the exponential tails of a Gaussian, the true probability of false alarm shows dependence on the heavy-tailed K-distribution.

CA-CFAR often has difficulty handling transitions from noise to clutter, as heightened returns from clutter bias the mean interference level when targets are close to the clutter ridge. Some variants of CFAR that can help mitigate the aforementioned issues are [3], [27]:

1. **Greatest-of CA-CFAR (GOCA-CFAR):** Selects the larger of the two mean estimates from a double-sided CA-CFAR window. GOCA-CFAR is optimized to cancel the effects of clutter ridge false alarms.
2. **Smallest-of CA-CFAR (SOCA-CFAR):** Opposite to GOCA-CFAR, this uses the smaller of the two mean level estimates.
3. **Trimmed mean (TM) or censored (CS) CFAR:** (TM) CFAR ranks the samples of the reference window in magnitude and discards a predetermined number of values from the top and bottom of the ranking before calculating the mean level. (CS) CFAR does the same, but it does not remove values from the lowest ranking magnitudes in the reference window.
4. **Ordered statistics (OS) CFAR:** OS-CFAR ranks the samples in the reference window by magnitude with the local clutter power estimated by the i th largest sample.

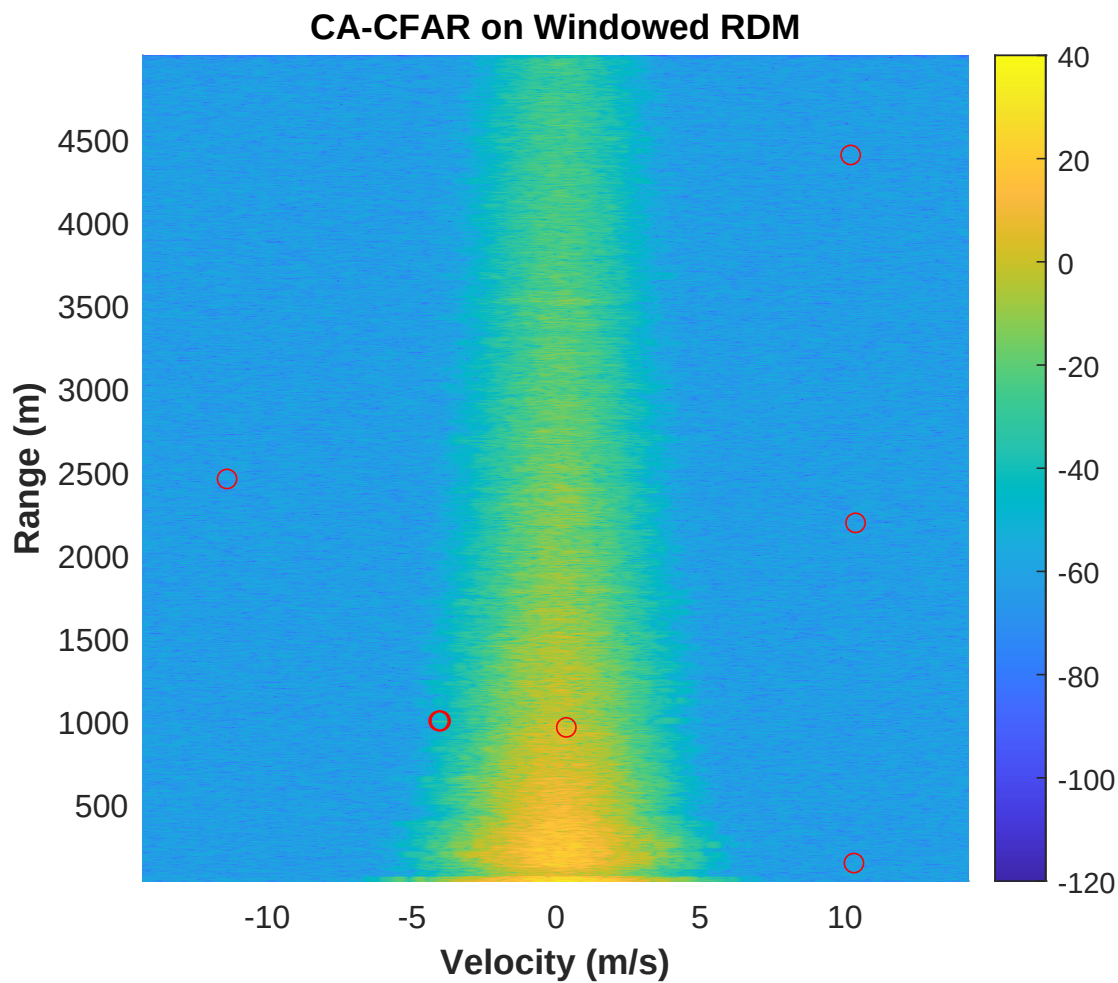


Figure 5.3: CA-CFAR detection applied to the windowed dataset. The target is successfully detected, yet a few false alarms in the range-Doppler map exist with a realized false alarm rate of $2.95e^{-6}$. These false alarms are reduced due to the reduction in sidelobe energy from the clutter ridge.

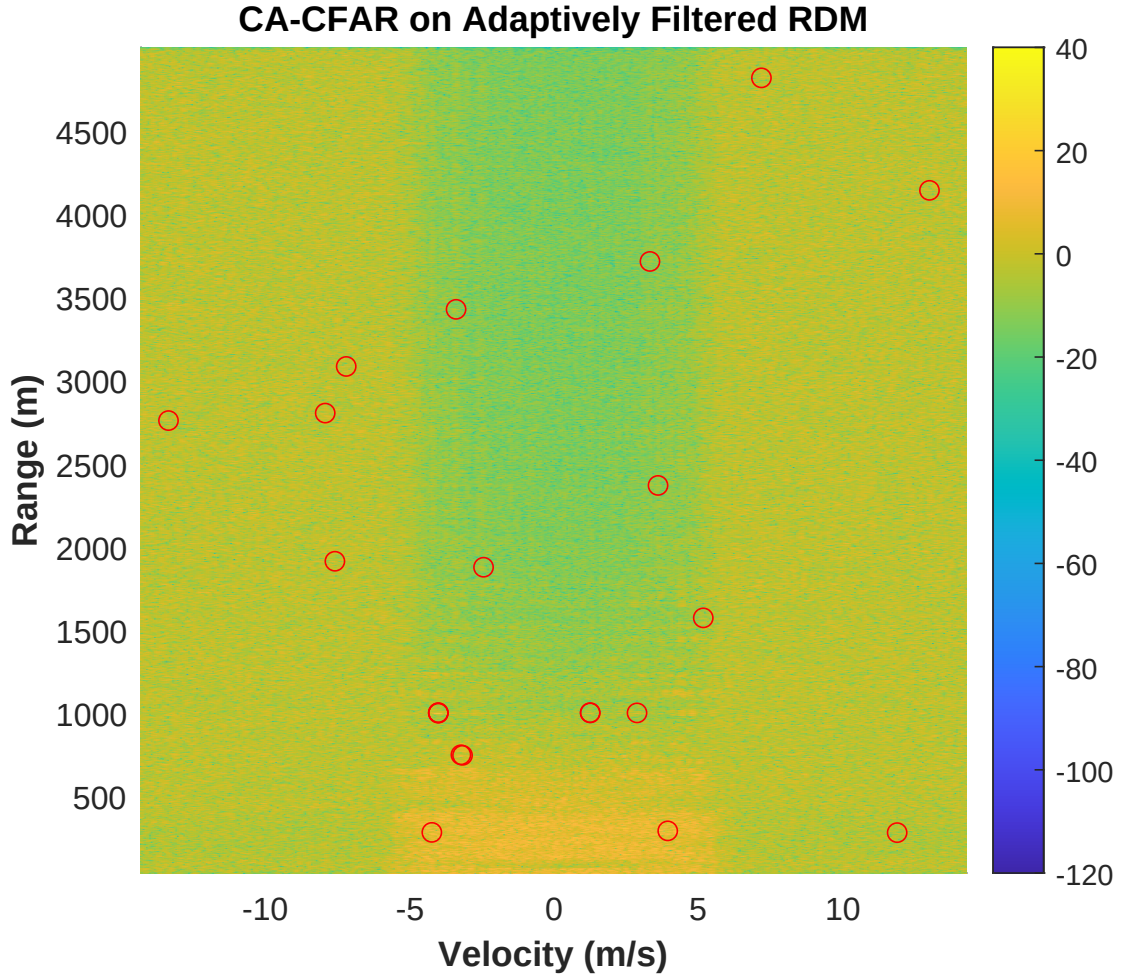


Figure 5.4: CA-CFAR detection applied to the adaptively filtered dataset yields a positive detection with a realized false alarm rate of $11.21e^{-6}$. Although this is close to the set P_{FA} of the detector, the biased near-in clutter returns cause the filter to overcompensate in range, allowing for more false alarms.

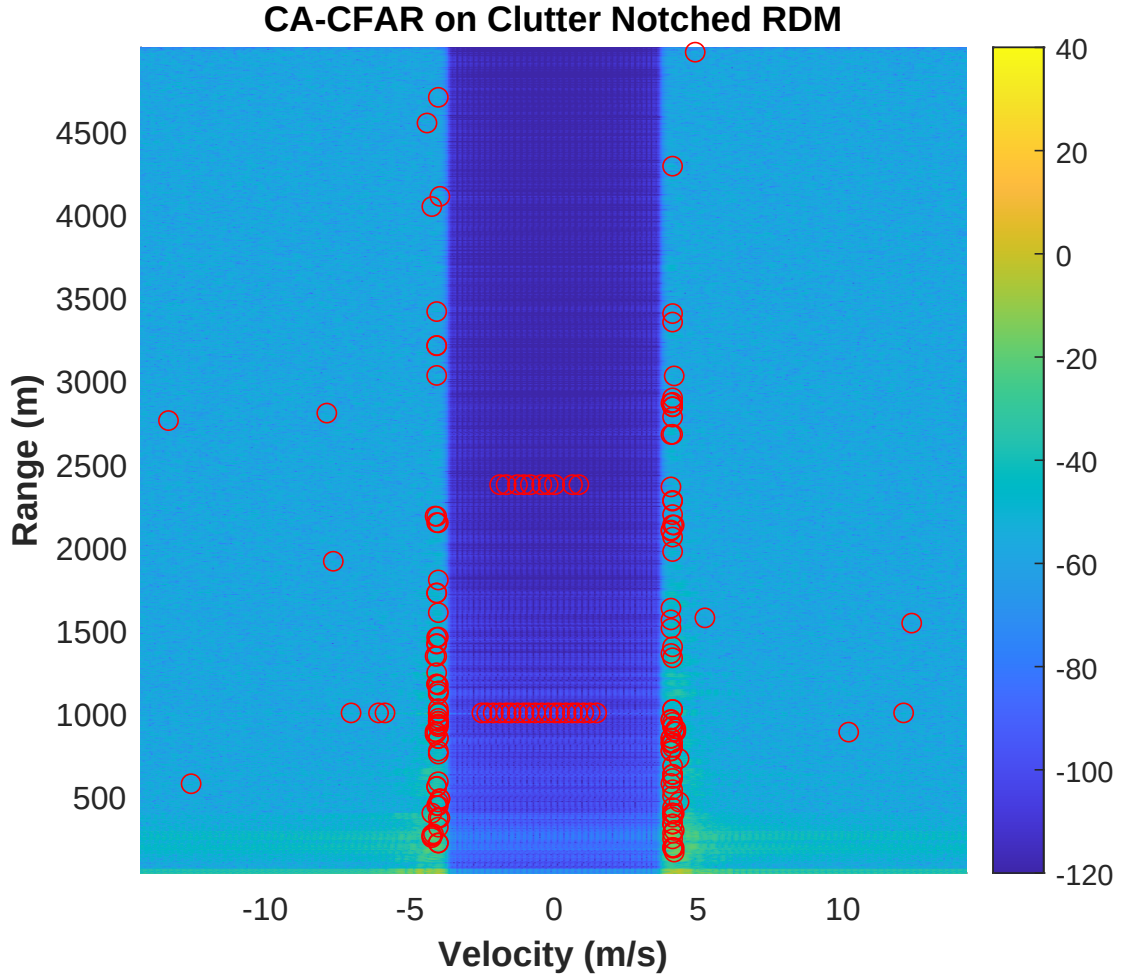


Figure 5.5: CA=CFAR dection applied to the SVD notched range-Doppler map. Although a positive detection is present, the realized false alarm rate rose to $10.15e^{-5}$. Additionally, the CA-CFAR failed to perform well at the boundary of the notch causing false alarms to appear in abundance at the edges of the notch.

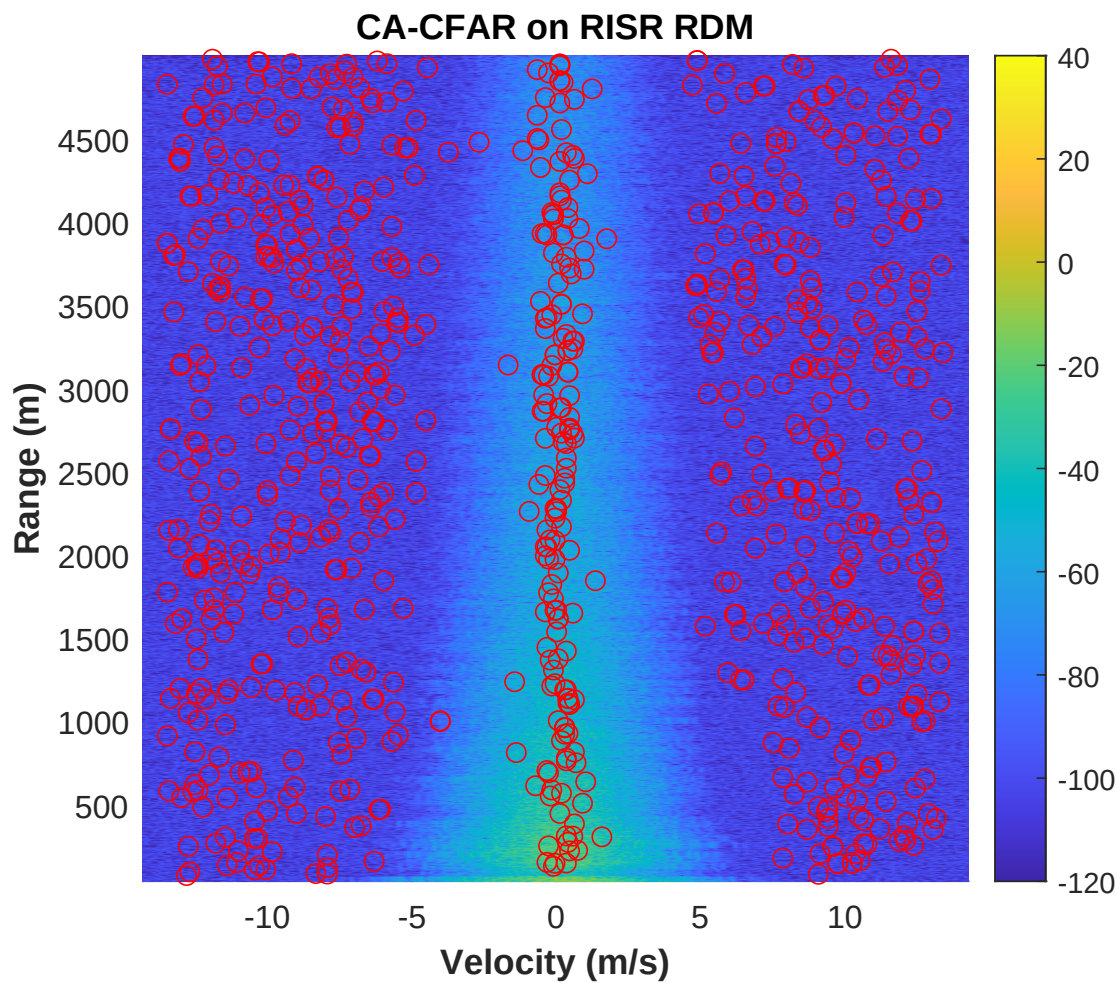


Figure 5.6: Although a positive detection on the target was made, the false alarm rate skyrocketed to $50.44e^{-5}$ showing that spectral refinement alone is not sufficient for a data set that is then passed into a CA-CFAR detector.

5.1.2 Order Statistics Constant False Alarm Rate (OS-CFAR)

Order Statistics Constant False Alarm Rate (OS-CFAR) is a more robust CFAR technique in complex clutter environments. Because the assumption with CA-CFAR is that the noise is independent and identically distributed, CA-CFAR detectors throw off more false alarms in the presence of sea clutter given the heavy-tailed nature of the K distribution. To mitigate this, OS-CFAR suppresses target masking by addressing the reference window in a different way. For OS-CFAR, there are no guard cells as in Figure 5.2, but every sample in the window (apart from the CUT) is ranked. Once ranked, the OS-CFAR detector selects the k th sample as the CFAR statistic [27]. To reduce losses, the OS-CFAR detector performs best when $k = 3N/4$, with N being the number of samples in the reference window [27]. This relation allows the OS-CFAR detector to reject $N/4$ sources that interfere with the CUT. This is particularly beneficial for sea clutter, as the OS-CFAR detector will remove the samples associated with clutter ridges as the CUT passes over the target. OS-CFAR handles clutter boundaries better than CA-CFAR for this reason, and it is worth considering when dealing with sea clutter returns if the computational time increase of a sorting algorithm can be tolerated.

5.1.3 Detection Results and Comparison

As previously mentioned in Tables 4.1 and 4.2, a point target of -15 dBsm was placed at 1 km with a velocity of 4 ms^{-1} approaching the radar system. The target is assumed to be in the main beam for the duration of the CPI with 34.295 dB of gain.

The CA-CFAR detector was set with a reference window size of 31×21 in

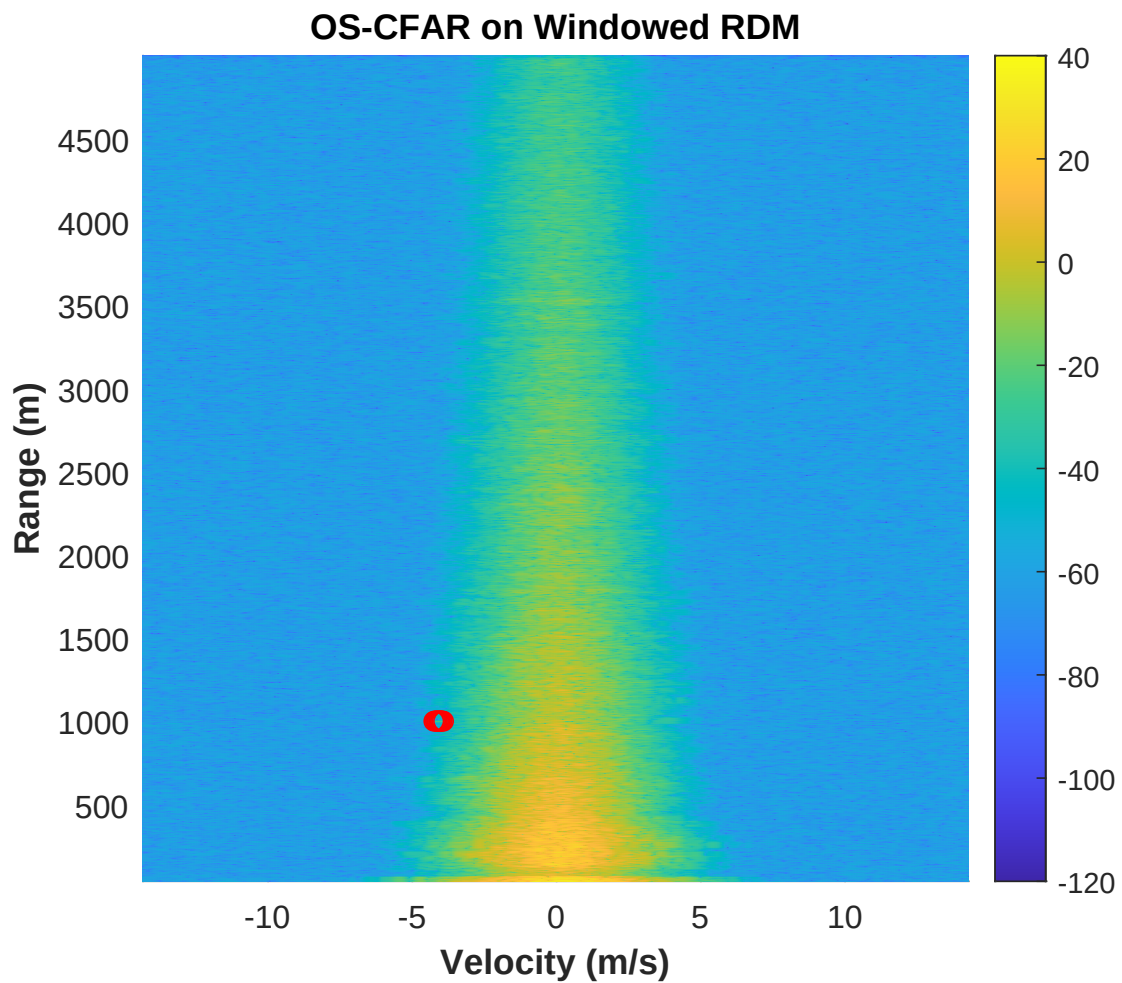


Figure 5.7: Using OS-CFAR, the windowed range-Doppler map shows no false alarms as calculated by Equation 5.6. If the computational cost can be sustained, OS-CFAR proves to be an effective detector.

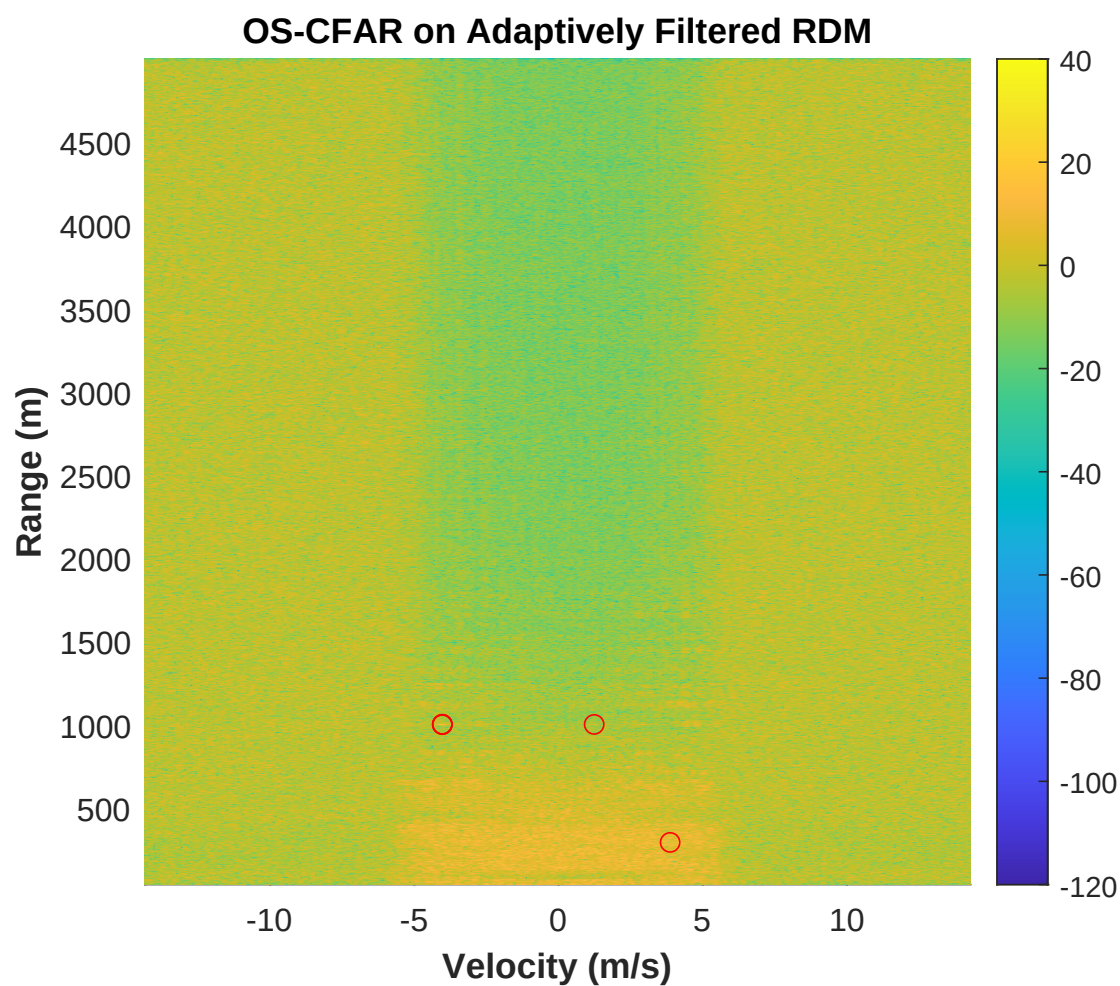


Figure 5.8: OS-CFAR as applied to the adaptively filtered dataset drops the realized false alarm rate down to $1.18e^{-6}$.

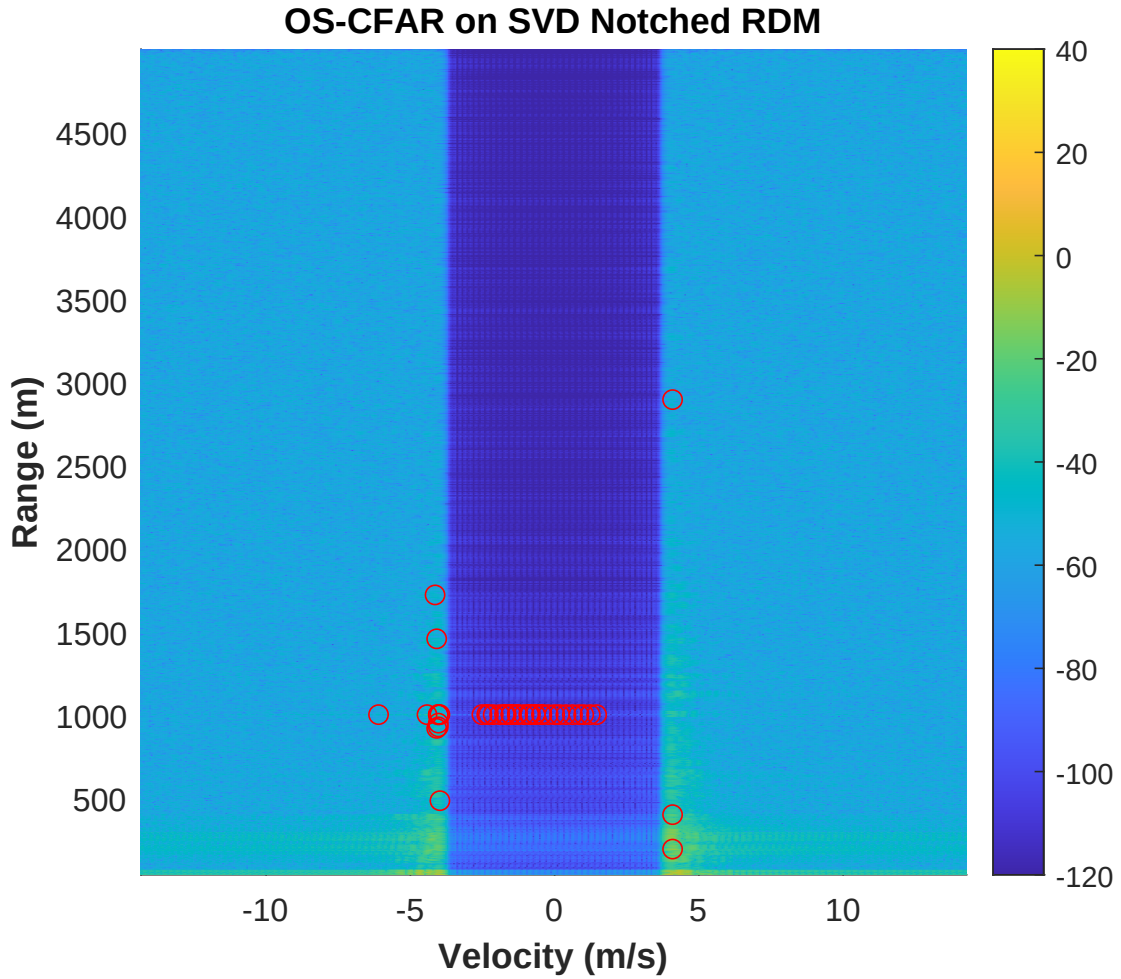


Figure 5.9: OS-CFAR on the clutter notched data shows how this detector performs well at clutter boundaries. The realized false alarm rate dropped to $2.24e^{-5}$, and nearly all false alarms caused by the boundary are eliminated. Most false alarms in this detector output are caused by the sidelobes of the target itself and are present in the notch where the interference level was drastically reduced.

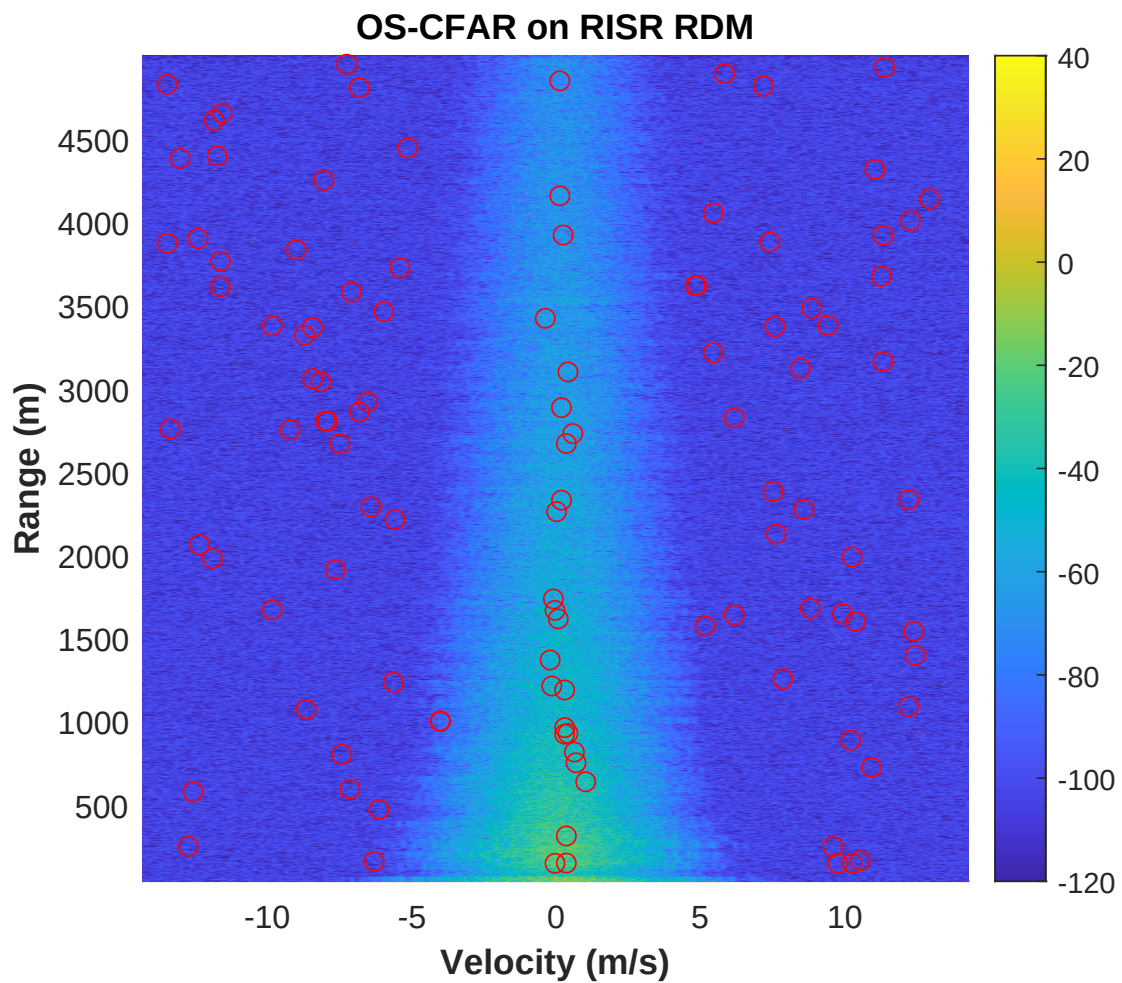


Figure 5.10: Using OS-CFAR on the RISR data, the realized false alarm rate dropped down to $6.31e^{-5}$ showing that the detector outperforms CA-CFAR in a sea clutter environment.

Doppler and range, with a 5×3 window of guard cells protecting the CUT. All processing techniques underwent the same CA-CFAR with the same set probability of false alarm, $10e^{-6}$, to gauge the effectiveness of each processing technique. Although the goal set for the detector was $P_{FA} = 10e^{-6}$, CA-CFAR generally produced worse results due to clutter statistics, as expected. Table 5.1 shows the realized false alarm rate of both detectors with CA-CFAR held at a constant window size and P_{FA} to gauge detector performance as a result of the processing techniques utilized on the range-Doppler map. The realized false alarm rate was calculated by finding the ratio of false detections to the number of samples in the data set. This is given by the equation,

$$\text{Realized False Alarm Rate} = \frac{N_{FA}}{N} \quad (5.6)$$

where N_{FA} is the total number of false alarms present in the detector output and N is the total number of samples, in this case $N = L_{ADC} \times N_{fft}$. Positive detections directly adjacent to the target cell were allowed to converge on the central sample, causing a singular detection to be produced at the target location when the target was detected.

Consequently, the OS-CFAR detector outperformed the CA-CFAR detector in every scenario, highlighting the need for various detection techniques in complex environments. Since OS-CFAR takes a longer amount of time than CA-CFAR due to the utilization of a sorting algorithm, an argument can be made for merging windowing with CA-CFAR since both are fast techniques that require relatively low computational resources. This could be implemented in a basic detect function, where once a target is detected, more complex tracking algorithms can take over to maintain focus on the detected target. However, depending on the use case, OS-CFAR might be the better detector to use. The clutter notched and

RISR data sets saw massive reductions in false alarms with the OS-CFAR detector, highlighting the effectiveness at the clutter boundaries.

Overall, the decision of which detector to use is more of a question of what the end goals are for the system. For a fast-performing system, windowing and CA-CFAR might be the answer; however, accuracy in a maritime environment might require more computationally intensive operations like adaptive filtering and OS-CFAR. Table 5.1 serves as a performance gauge for this generated data set, but the results are likely to differ depending on the radar and simulation scene parameters and the CA-CFAR parameters such as the chosen window size and the input P_{FA} .

Table 5.1: This table shows the realized false alarm rate in the presence of sea clutter for both detectors across all clutter mitigation techniques. Both window and guard cell dimensions were in (Doppler $\times$ Range). The window size was 31×21 , and for CA-CFAR, the guard cells were 5×3 . The size of the window was constant across all clutter mitigation techniques and detectors.

Technique	CA-CFAR	OS-CFAR
Windowing	$2.95e^{-6}$	0
Adaptive Filtering	$11.21e^{-6}$	$1.18e^{-6}$
Clutter Notching	$10.15e^{-5}$	$2.24e^{-5}$
RISR	$50.44e^{-5}$	$6.31e^{-5}$

Chapter 6

Conclusion

6.1 Conclusion

A sea clutter simulation was created with the goal of modeling real-world deployment scenarios in order to gauge system performance. The behavior of ocean waves and how it affects electromagnetic scattering was studied in detail to create an accurate, realistic simulation that models both the amplitude return statistics and the Doppler spectrum of the sea surface. With a primary focus on X-band, the radar system was simulated with a realistic antenna radiation pattern, designed in HFSS by [18], to more accurately model near-in clutter as it relates to the sidelobes and geometry relative to the ocean surface. The clutter return amplitude was modeled using the K-distribution, the most popular of three pdfs used to describe the sea surface returns, and the Evolving Doppler model was used to characterize the Doppler spectrum of the sea surface dependent on wind and wave height conditions. The Naval Research Lab's reflectivity model was then used to find the mean backscatter based primarily on the center frequency and the grazing angle. Each of these components allowed for an accurate and realistic simulation of a maritime environment for the radar system, and further processing could take place. Once the data was simulated based on the ocean conditions,

Doppler processing was performed to attempt to uncover a low RCS point target close to the clutter ridge. In the worst-case scenario simulated, where the main beam intersects the sea surface, the low RCS target was concealed, provoking the use of clutter mitigation and cancellation techniques to uncover the target. Windowing, adaptive filtering, clutter notching, and RISR were performed to suppress the sidelobes of the clutter in the range-Doppler map. After clutter suppression, the improved SINR allowed CA-CFAR and OS-CFAR detection techniques to be used to test detection performance in the harsh sea clutter environment. The production of a high-resolution simulation for sea clutter that accounts for densely placed scatterers that follow the K-distribution, each with a referenced gain from a realistic radiation pattern that is spatially consistent with the radar geometry relative to the scene, was demonstrated here, and with varying simulation geometry and radar parameters, a flexible system configuration can be reached that enables performance prediction in a maritime environment. Analyzing the performance of a radar system in this manner will cut the costs of building a maritime radar system on the front end by informing the user of the proposed radar configuration performance before costly prototypes are built.

6.2 Future Work

Although sea clutter has been studied in detail, opportunities remain to further understand the ocean surface and the models derived to simulate it. The obvious next step would be to fully implement a ray tracing simulation that accounts for self-occlusion and multipath scattering. Ray tracing simulations that can account for isotopically backscattered radiation instead of just a single transmit and receive ray are on the rise with state-of-the-art graphics processing units, and this technology could be utilized to produce high resolution simulations with varied

geometry. However, simulations in this manner and detail would beg the question of model integrity since most models discussed in this work factor in multipath propagation which leads to another course of future work, data collection. Data sets in radar signal processing are already sparse as-is, and many times a dataset is collected, it leaves much to be desired in the way of fully configuring the environment. Software-defined radios (SDRs) could pave the way for future modeling of the sea surface by collecting mass amounts of data from the sea surface under different configurations and system geometry. With most reflectivity models still referencing Nathanson's tables of reflectivity, an open avenue of exploration could be model refinement through the collection of new data in an inexpensive way by implementing reconfigurable SDRs. These steps would provide a more complete and flexible framework for evaluating the performance of radar systems in maritime environments.

Furthermore, implementing more CFAR techniques in this work would allow more detection schemes to be tested against the background of the sea clutter. Although CA-CFAR and OS-CFAR were tested here, the mass generation of data within this simulation would allow for a receiver operating characteristic (ROC) curve to be built out to quantify detection performance as it relates to SINR. Because sea clutter simulation takes a substantial computational time, data generation for this purpose could take a significant amount of time, but a framework is presented here to perform it. Adding more CFAR techniques would allow the evaluation of their performance in the midst of sea clutter that is progressively stepped in sea states to determine if a particular detector and data processing combination works better at higher or lower sea states.

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Appendix A

List of Acronyms and Abbreviations

ADC	Analog-to-digital converter
AWGN	Additive White Gaussian Noise
CA-CFAR	Cell Averaging Constant False Alarm Rate
CDF	Cumulative Distribution Function
CFAR	Constant False Alarm Rate
CPI	Coherent Processing Interval
CPT	Coherent Pulse Train
CS-CFAR	Censored Cell Averaging Constant False Alarm Rate
CUT	Cell Under Test
DFT	Discrete Fourier Transform
DOA	Direction of Arrival
FFT	Fast Fourier Transform
FT	Fourier Transform
GIT	Georgia Institute of Technology
GOCA-CFAR	Greatest of Cell Averaging Constant False Alarm Rate
LRT	Likelihood Ratio Test
MMSE	Minimum Mean Square Error
NRL	Naval Research Laboratory
OS-CFAR	Ordered Statistics Cell Averaging Constant False Alarm Rate

pdf	Probability Density Function
PRF	Pulse Repetition Frequency
PRI	Pulse Repetition Interval
PSD	Power Spectral Density
RADAR	Radio Detection And Ranging
RCS	Radar Cross Section
RISR	Reiterative Superresolution
ROC	Receiver Operating Characteristics
SDR	Software Defined Radio
SINR	Signal-to-Interference-plus-Noise Ratio
SLL	Sidelobe Level
SNR	Signal-to-noise ratio
SOCA-CFAR	Smallest of Cell Averaging Constant False Alarm Rate
STAP	Space-Time Adaptive Processing
SVD	Singular Value Decomposition
TM-CFAR	Trimmed Mean Cell Averaging Constant False Alarm Rate

Appendix B

Evolving Doppler Model

As mentioned in Section 3.3, the Evolving Doppler model developed by Rosenberg in [3] provides a comprehensive way to model the power spectrum of the sea surface radar returns given the wind speed and significant wave height inputs. The model accounts for Bragg and fast scattering components and is modeled by the bimodal Gaussian distribution given by,

$$G_{bi}(f) = \frac{\alpha\tau}{\sqrt{2\pi} w_1} \exp\left(-\frac{(f - m_{f_1}(\tau))^2}{2w_1^2}\right) + \frac{(1-\alpha)\tau}{\sqrt{2\pi} w_2} \exp\left(-\frac{(f - m_{f_2}(\tau))^2}{2w_2^2}\right) \quad (\text{B.1})$$

The input, $f \in \mathbb{R}^{N_{fft} \times 1}$, is a vector corresponding to the number of Doppler frequencies associated with the range-Doppler map which ranges from $-\text{PRF}/2$ to $\text{PRF}/2$ with N_{fft} points. Each mode of the Gaussian has its own mean and variance denoted m_f and w^2 respectively. The variance can be modeled as a random variable that can be correlated over time as it is linked to texture intensity. The mean values are calculated with the equations,

$$m_{f_1}(\tau) = \begin{cases} A_c + B_c \tilde{\tau} + f_{bi}, & \tilde{\tau} \leq \tau_{bi} \\ A_c + B_c \tau_{bi} + f_{bi}, & \tilde{\tau} > \tau_{bi} \end{cases} \quad (\text{B.2})$$

$$m_{f_2}(\tau) = A_c + B_c \tilde{\tau} + f_{bi}. \quad (\text{B.3})$$

The input to the mean frequency is dependent on the local clutter intensity and is related to the normalized mean clutter intensity by $\tilde{\tau} = \tau/\mathbb{E}[\tau]$. The added parameter f_{bi} is a zero-mean Gaussian random variable with standard deviation σ_{bi} .

Given these equations, several primary constants must be calculated: A_c , B_c , τ_{bi} , α , and σ_{bi} ; however, to calculate these, we first need a subset of secondary constants, with each primary constant having their own subset of secondary constants. These secondary constants are a_{gen} , b_{gen} , c_{gen} , d_{gen} , e_{gen} , and γ_{gen} . To calculate the given secondary constant, the following equation must be satisfied, with each secondary constant having their own tertiary constants f_{gen} , g_{gen} , and

h_{gen} . The equation is given by,

$$Y = f_{\text{gen}} + g_{\text{gen}} \log_{10}(U) + h_{\text{gen}} H_{1/3} \quad (\text{B.4})$$

Now, with the secondary constants calculated (a_{gen} , b_{gen} , c_{gen} , d_{gen} , e_{gen} , and γ_{gen}) with their respective constants (f_{gen} , g_{gen} , and h_{gen}), the desired primary constant can be calculated by the equation,

$$X(\theta, \phi | U, H_{1/3}) = \left(\frac{\phi}{\phi_0} \right)^{\gamma_{\text{gen}}} \left(a_{\text{gen}} + b_{\text{gen}} \cos \theta + c_{\text{gen}} \cos(2\theta) + d_{\text{gen}} \cos(\theta - \theta_{\text{sw}}) + e_{\text{gen}} \cos(2(\theta - \theta_{\text{sw}})) \right) \quad (\text{B.5})$$

where the equation has a dependence on the wind speed and the significant wave height, U and $H_{1/3}$, through Equation B.4, and the inputs θ and ϕ are the azimuth and grazing angles to the clutter cell. The reference grazing angle, ϕ_0 , is fixed at 10° , while θ_{sw} is the swell angle relative to the wind. Once A_c , B_c , τ_{bi} , α , and σ_{bi} have been calculated using Equation B.5, all parameters for the bimodal Gaussian spectrum have been defined to calculate the power spectral density in Equation B.1. The tables of tertiary constants to calculate the secondary constants of the primary constants are given below.

		A_c			B_c		
Pol.		f_{gen}	g_{gen}	h_{gen}	f_{gen}	g_{gen}	h_{gen}
HH	γ_{gen}	-2.38	2.77	0.044	-1.88	1.44	0.059
	a_{gen}	127.9	-125.1	-0.46	-1.98	5.62	-0.018
	b_{gen}	-262.3	225.7	3.43	148.9	-103.6	-5.03
	c_{gen}	126.1	-123.2	-1.68	-85.09	81.25	2.92
	d_{gen}	6.81	-15.26	3.69	-28.09	38.42	4.21
	e_{gen}	6.71	-4.54	-0.38	4.50	-4.30	-0.26
HV	γ_{gen}	5.85	-3.73	-0.54	2.65	-2.95	-0.094
	a_{gen}	-18.1	1.97	-0.075	-1.03	5.20	-0.69
	b_{gen}	-40.31	24.39	-2.56	147.3	-101.3	-4.32
	c_{gen}	18.41	-16.56	-0.64	-47.35	41.75	-0.92
	d_{gen}	-37.14	35.17	-0.93	6.71	11.31	-1.04
	e_{gen}	11.2	-10.07	-0.094	-21.95	21.32	-0.76
VV	γ_{gen}	-3.01	3.86	-0.23	-2.79	3.22	-0.10
	a_{gen}	0.16	-0.028	0.28	7.45	1.24	-0.12
	b_{gen}	-8.63	-13.5	0.85	16.3	6.29	-0.86
	c_{gen}	-4.33	5.26	-2.15	-9.02	-1.23	2.23
	d_{gen}	-0.16	-2.15	-0.85	-2.90	-5.56	-3.22
	e_{gen}	-2.08	3.93	-1.43	0.13	-1.15	1.16

Table B.1: Tertiary coefficients for A_c and B_c by polarization.

Pol.		τ_{bi}			α		
		f_{gen}	g_{gen}	h_{gen}	f_{gen}	g_{gen}	h_{gen}
HH	γ_{gen}	5.27	−4.48	−0.11	7.41	−5.96	−0.28
	a_{gen}	−0.55	2.15	0.027	−0.21	0.65	0.0025
	b_{gen}	−0.24	0.40	0.035	0.15	−0.25	−0.0098
	c_{gen}	0.42	−0.45	0.045	0.25	−0.37	−0.024
	d_{gen}	−0.21	0.45	0.00	0.069	−0.12	−0.0028
	e_{gen}	−0.024	0.0016	0.024	−0.034	0.13	−0.066
HV	γ_{gen}	0.41	0.25	−0.084	−1.96	2.03	−0.0048
	a_{gen}	0.59	0.56	0.030	0.62	0.11	−0.018
	b_{gen}	0.096	−0.22	0.020	0.019	0.15	0.021
	c_{gen}	0.026	−0.11	0.045	0.48	−0.54	0.026
	d_{gen}	0.017	−0.031	0.012	0.76	−0.72	−0.026
	e_{gen}	−0.31	0.35	−0.019	0.095	−0.11	0.0021
VV	γ_{gen}	0.031	0.22	−0.065	0.097	−0.080	−0.0039
	a_{gen}	1.00	0.111	0.01	0.89	0.18	0.018
	b_{gen}	0.10	0.11	0.0095	−0.064	0.03	0.0023
	c_{gen}	0.23	0.31	0.014	0.043	0.016	0.059
	d_{gen}	0.25	−0.40	0.03	0.059	−0.089	−0.0061
	e_{gen}	0.13	−0.16	−0.0087	−0.038	−0.0058	0.061

Table B.2: Tertiary coefficients for τ_{bi} and α by polarization.

		σ_{bi}		
Pol.		f_{gen}	g_{gen}	h_{gen}
HH	γ_{gen}	-1.051	0.25	0.12
	a_{gen}	27.37	-5.67	1.43
	b_{gen}	3.89	-3.84	0.67
	c_{gen}	1.84	-3.96	-0.19
	d_{gen}	-9.40	11.21	0.80
	e_{gen}	3.48	-3.75	-0.29
HV	γ_{gen}	0.16	-0.56	0.062
	a_{gen}	24.72	-4.52	1.28
	b_{gen}	3.79	-4.98	0.85
	c_{gen}	-2.30	0.22	-0.24
	d_{gen}	-4.72	3.73	1.43
	e_{gen}	11.54	-11.49	-0.32
VV	γ_{gen}	-0.21	0.22	0.0037
	a_{gen}	10.05	1.40	0.26
	b_{gen}	1.16	0.77	0.63
	c_{gen}	-1.99	3.38	-0.20
	d_{gen}	-2.00	3.19	0.41
	e_{gen}	2.21	-4.03	0.65

Table B.3: Tertiary coefficients for σ_{bi} by polarizations.