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THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

AN INVESTIGATION OF LEARNING APPROACHES OF NONTRADITIONAL STUDENTS IN MATHEMATICS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

by

Xuan Le

Norman, Oklahoma

1997

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AN INVESTIGATION OF LEARNING APPROACHES OF NONTRADITIONAL STUDENTS IN MATHEMATICS

A DISSERTATION
APPROVED FOR THE DEPARTMENT OF
INSTRUCTIONAL LEADERSHIP AND ACADEMIC CURRICULUM

By

Anne Reynolds

Marilyn Breen

M. J. Fleener

[illegible]

Larry E. Foothaker

To my parents

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Anne Reynolds, for providing me with guidance, freedom, and support in my research project. Also, I appreciate the time and the helpful suggestions that my committee members, Dr. Marilyn Breen, Dr. Jayne Fleener, Dr. Frank McQuarrie, and Dr. Larry Toothaker contributed to this study.

My gratitude goes to the participants in my project, especially Barb and Kelli, for sharing their experiences with me and helping me to understand more about the learning of mathematics.

My mother, father, and eleven siblings have given me their love and support throughout my education. There is no way to express my gratitude for all the sacrifices my parents have made so that their children could live and obtain an education in a democratic society. Lastly, I would like to extend a special thank you to my wonderful husband, John, for his love, support, and assistance throughout this project.

ABSTRACT

This study is an attempt to understand how nontraditional college students (those who are either over twenty two years of age or are returning students) approach the learning of mathematics. Attention was given to the view nontraditional students have of mathematics and to the activities they engage in when learning the subject.

Five students volunteered to participate in this study in the fall semester of 1995. Data was obtained from this group of students through observations in their College Algebra classes and in tutoring sessions with this researcher. In addition, personal interviews with each participant were recorded throughout the semester.

Analysis of the data suggests these students are serious learners who come into a course with high expectations. They approach their studies like model students: attending class regularly, taking notes of everything on the board, asking questions of their instructors, completing all the assigned work, and seeking assistance from others when needed. Despite their hard work, however, three of the five participants received a D for their final grade in College Algebra, while one dropped the course before the end of the semester. The source of their difficulties was a mismatch between their expectations coming into College Algebra and the new learning experience they had in the course. They expected their College Algebra class to be similar to their previous mathematics classes (Elementary Algebra and Intermediate Algebra), and believed they could succeed in the course if they used the same studying approach as they had in the past. The discovery that this was not the case caught these participants off guard, and created much confusion and frustration for them.

In their struggle to deal with the difficulties in their learning, these students employed various positive coping strategies that allowed them to release some of the pressure they felt in their learning. Also, they were persistent and not easily discouraged when encountering failure in their work. Two of the participants who received D's in the course and the one who dropped out chose to repeat the course so as to gain more understanding of the material.

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Chapter 1: Introduction

Statement of the problem

Before 1990, colleges and universities gave most of their attention to students in the traditional undergraduate age group of 18 to 22 years, while according less attention to smaller group of students who did not fit this category. Today these older students, usually referred to as non-traditional students, are receiving more attention due to the rapid increase in their numbers (a nontraditional student is generally defined to be any student seeking a degree from an undergraduate institution who is over 25 years of age or who was age 22 or over while a freshmen). In fact, in the 1990's, enrollment trends indicate that the number of nontraditional students seeking degrees from higher education institutions is increasing at a higher rate than the number of traditional students. According to the National Education Center for Statistics, this increase in the number of nontraditional students on college campuses is expected to continue well into the end of this decade (Hawkins and Sides, 1994).

With the change in the college student population, universities and colleges are forced to reevaluate the many services and academic programs on their campuses in order to accommodate their nontraditional clients. To accomplish these revisions, college and university officials look to the research literature for directions and guidance. However, most of the literature on nontraditional learners is focused on general areas of interest such as demographic trends, general characteristics of nontraditional students, and predictors of attrition in the nontraditional student group; while specific academic domains such as engineering, accounting, and history have received little attention from the research community.

As far as mathematics is concerned, research about nontraditional students is very limited in both quantity and quality. Past work in mathematics education

research focused more on comparing nontraditional students with traditional students in areas such as mathematical achievement, attribution style, and confidence level than on understanding the learning needs or the assumptions that nontraditional students bring with them to the classroom. More research on nontraditional students in the area of mathematics is needed before we can know how best to help nontraditional students in their learning of the subject.

Purpose of the study

This study was an attempt to gain additional understanding of nontraditional students in their learning of mathematics. Specifically, the focus was on finding out the different conceptions that these students have about mathematics, and on understanding the effects that these views have on their engagement in learning the subject.

The guiding questions for this study included:

1. What are nontraditional students' conceptions of mathematics?
2. What are the activities that nontraditional students undertake in their learning of the subject?

Significance of the study

With the increase in the number of nontraditional students on college campuses comes the necessity for understanding this group of students better. Although information is needed on how best to accommodate nontraditional undergraduates to help them reap the greatest benefit from their education, our knowledge of this group of students is still very limited. In particular, more research is needed on nontraditional students in mathematics.

This study is significant in that it offers the research community another perspective to understand nontraditional students. It helps to supplement and extend past research findings, because it attempts to gain additional understanding

of nontraditional students through the use of case study research. This investigation helps provide some explanations for the many successes that nontraditional students have achieved in their work at the lower level of mathematics, and illuminates the difficulties that these students experienced in their learning of the subject.

Assumptions for the study

1. The participants will be honest in their responses.
2. The participants will be able to reflect on their learning activities and will be able to articulate their reflections to the researcher.
3. The participants' responses during the study will be similar to their responses outside of the study.

Limitations of the study

1. The participants included only students who volunteered for the project.
2. The focus of the study was restricted to students in College Algebra.

Organization of the remaining chapters

In chapter two, the literature concerning nontraditional students both in general and in mathematics will be discussed. Chapter three will be reserved for the discussion of the research method, while chapter four will present the data. Finally, both the analysis of the data and the conclusions of the study will be presented in chapter five.

Chapter 2: Review of the literature, theoretical, and conceptual framework.

2.1 General research literature on nontraditional students

The term "nontraditional student" originated in the mid 1940's with the end of World War II. It was around this time that Congress approved the GI bill, which made it possible for servicemen to attend colleges across the country. This resulted in the first major wave of nontraditional students on college campuses. In 1955 with the arrival of community colleges, many more adults were able to participate in education at the college level without having to travel a long distance from their homes. This factor explains the high proportion of nontraditional students at two-year colleges versus four-year universities.

Until the mid 1980's, nontraditional students constituted only a small portion of the college student population in the United States. However, the picture is different today. According to the National Education Center for Statistics, nontraditional students make up about 42% of the current college student population, and this percentage is expected to continue to rise well into the end of this decade (Hawkins and Sides, 1994). This increase comes at a crucial time for higher education, because there has been a decrease in the number of high school graduates in the 1990's. With the shrinking pool of high school graduates, colleges and universities look to nontraditional students to help them fill up the numerous spaces on their campuses. However, in order to attract these learners to their institutions, it is necessary that college and university officials take into consideration their new clients' needs in structuring the academic programs and services on their campuses.

When making these accommodations, officials in higher learning institutions rely on the adult education literature for guidance and information. For the most

part, the literature on adult learners has focused on creating a profile for this group of students. That is, the work done in the past mainly examined the demographic trends of nontraditional students, the reasons that nontraditional students give for attending school, the general characteristics that nontraditional students exhibit in their learning, etc. On the whole, past work was quantitative in nature, in that the data was collected through the use of surveys and questionnaires and then was analyzed through the use of various statistical methods.

For educators of nontraditional students, the work done on adult learners' profiles can be of help in their teaching, because it can help to inform them about the characteristics of their learners. The main finding from this body of work is that nontraditional students are not just older versions of traditional college age students (Cross, 1980; Knowles, 1979; Kidds, 1974). As a group, they are more varied than the younger students in their educational background, their personal background, their reasons for attending college, and their occupational status (Bershinsky, 1990; Kasworm, 1994). Thus, it is not an easy task to define a typical nontraditional student. In the past, there have been many attempts to define the term "nontraditional student"; however, little consensus was reached among the researchers, beyond the classification of nontraditional students as students who are twenty-five years or older.

In general, researchers find nontraditional students to be more self-directed in their learning than traditional students (Knowles, 1978). According to the literature, nontraditional students have specific reasons for attending college and are focused on accomplishing goals (Kasworm, 1994). Hence they are more motivated than traditional students to seek additional information from various sources outside of the classrooms to supplement their learning when needed. In this sense, they are more independent in their studying than their younger colleagues. Therefore, researchers of adult education tend to recommend that a more student-centered approach be employed by educators in the teaching of these older learners

(Cross, 1980).

In this teaching approach, the teacher and the students have more of a facilitator-participants relationship than a lecturer-listeners relationship. That is, the teacher serves mainly as a guide to the students rather than as the "Authority" in their learning process. Class discussion and teamwork among the students are encouraged. The climate in such a classroom is more cooperative than competitive. This kind of classroom, according to researchers in adult education, seems to be better suited to older learners because it provides a more flexible and nonthreatening environment (Cross, 1980).

Another area where the two groups of students differ is in their life experiences (Kidds, 1974; Knowles, 1970; Beder and Dardenwald, 1982). Nontraditional students come to college with more life experiences that could assist or hinder them in their learning. Their experiences could assist them by helping them to relate new information to something that they are already familiar with, thus making it easier for them to understand the material; or could hinder them by reminding them of unpleasant past learning episodes and preventing them from trying new learning experiences.

Past research has pointed out some of the areas where nontraditional students are different from younger students. Such information is useful in that it can help to better acquaint us with the students in this particular group. However, much more work is needed on understanding nontraditional students in specific academic domains, such as mathematics, before better accommodations can be introduced to serve nontraditional students in their learning of these subjects.

2.2 Research on nontraditional students in mathematics

The few studies that have focused specifically on nontraditional students in mathematics have tended to support the general finding that nontraditional students and traditional ones differ somewhat in their learning processes.

In the domain of mathematical achievement, Elliott (1987) found that nontraditional students, at least at the remedial/developmental mathematics level, can perform as well as, if not better than, the younger students in the same classes. A similar finding was reported by Schoenberger (1985) and Fredricks and Eldersveld (1981). This is encouraging, considering the time gap that nontraditional students usually have between their high school mathematics classes and the ones that they take at the college level. At the moment, the research does not offer many ideas as to why these students succeeded as well as they did at lower-level mathematics. Nor is it yet known whether nontraditional students achieve much success in learning higher-level mathematics.

Besides achievement differences, the findings from past mathematics education research also suggest some other differences between nontraditional students and traditional students. For example, in his study, Elliott tested various affective variables, in addition to prior mathematics achievement, in search of possible predicting variables that can be used to predict students' performance in mathematics. These affective variables were confidence in learning mathematics, perceived usefulness of the subject, and causal attributional style. Although he did not find affective variables to offer much predictive power for the students' mathematical achievement, Elliott did notice that they seemed to have more influence in predicting the performance of nontraditional students than they had with the younger student group. Specifically, he found that for nontraditional students, previous mathematics achievement and causal attribution style together make a good predictor of mathematics achievement. In the case of traditional students, affective variables add little or nothing to the predictive power beyond that of the students' prior mathematics achievement (Elliott, 1987).

In the area of attribution style, Elliott found that nontraditional students tend to attribute their success in mathematics either to ability or effort more often than the younger students (Elliott, 1987). Such an attributional pattern, according to

the literature on attribution style, tends to lead to a higher level of achievement than one in which success is attributed to either luck or task difficulty (Weiner, 1974).

As far as the confidence variable is concerned, data from past research suggests that nontraditional students, more often than traditional ones, expressed higher confidence in their ability to learn mathematics (Elliott, 1987; Lehmann, 1985). A similar result was reported for nontraditional students in the general literature of adult learners (Dalluge, 1982). In his working experience with nontraditional students, Elliott observes that these students are usually confident in their ability to do mathematics; however, they are more likely than the traditional students to be anxious about having their work judged by someone else. Thus, the anxiety that is exhibited by these students might arise not so much from their lack of confidence in their mathematical ability as from their fear of being evaluated by others.

This interpretation of Elliott might help to explain the puzzling finding that Lehmann discovered among the older learners in her study: namely, that they continued to have low expectations for future success despite the many successes they achieved in their learning of mathematics (Lehmann, 1985). These learners seemed to believe that the law of averages holds true in their learning of the subject, so that they can expect to have the same number of successes as failures. Thus, by not expecting to achieve success in their work, these students were able to find an outlet to relieve some of the stress of being evaluated. This finding is important in that it helps us, as mathematics educators, to be aware of the possibility that our nontraditional students (and maybe the traditional ones as well) possess such a belief about learning mathematics. Hopefully, we can help these students to overcome this conception so that they can be more at ease in their learning of mathematics.

2.3 Relevant studies concerning students' conceptions of mathematics

In addition to comparisons of nontraditional students with their younger colleagues, mathematics education research on the topic of nontraditional students also includes some work in the area of students' conceptions of mathematics. For example, Buerk (1981) investigated the conception of mathematics held by five women, all of whom were in their late twenties or thirties. Using Perry's scheme as a conceptual framework, Buerk set out to help the participants in her study develop a more relativist conception of mathematics.

Perry's scheme is a sequence of positions that describes the ways that students change in respect to their views toward knowledge, value, and responsibility. The scheme was developed by Perry and his associates at Harvard as the result of their work with college male students between 1954 and 1963 (Perry, 1970). In their study, Perry and his associates interviewed students with the focus on how students interpreted the changes in their lives during the four years they spent at Harvard. The interviews were taped and then evaluated by a panel of judges to determine the sophistication level of the students' interpretation. As the result of their work with over 100 male students, Perry and his colleagues arrived at a scheme comprised of nine different positions which describe the ways students view knowledge, value, and responsibility.

Outline of Perry's scheme

A. Truth and Authority

Position 1: Basic duality

Truth and Authority are the same thing to an individual. There is little room for gray areas in one's thinking; a thing is either right or wrong.

Position 2: Multiplicity Pre-legitimate

Acceptance of multiplicity in the learning process slowly emerges; however, its

presence in one's learning process is limited.

Position 3: Multiplicity subordinate

Uncertainty and multiplicity are accepted as parts of the learning process; however, they are considered as representing the areas where Authority has not found the answer yet.

B. Truth and context

Position 4: Multiplicity correlate or Relativism subordinate

The individual goes through a period in which he either believes that "everyone has the right to his opinion", or he believes that "this is the way that his teacher wants him to do it."

Position 5: Relativism correlate, competing, or diffuse

The individual discovers that truth depends on the context in which it is embedded. Thus, more than one truth is possible in a given situation, depending on how one looks at it.

C. Truth and self

Position 6: Commitment foreseen

One makes a decision as to how to deal with the many uncertainties in a relativist world. Commitment is foreseen at this stage as a possible solution.

Position 7: Initial commitment

One attempts to make some initial commitments in certain areas of her life.

Position 8: Orientation in implications of commitment

One makes decisions as to how to fulfill the commitments that one has made.

Position 9: Commitment and responsibility

Commitment and responsibility describe how one leads his life in a relativist world. The individual is able to defend his opinion of truth while at the same time being aware of others' opinions of truth.

As can be seen, the emphasis in Perry's scheme is on the development of autonomy in one's learning. Perry refers to the first three positions in the scheme as the dualist stage because they are rigidly structured and are heavily dependent on authority for validation in the learning process. The next two positions are considered to be the relativist stage because they signal the acceptance of multiplicity and its importance in the learning process. The remaining positions in the scheme represent the commitment stage because they emphasize independence and flexibility in the learning process.

According to Perry and his colleagues, as the students go through the different positions, it is not unusual for them to backtrack to a previous position when they feel threatened in their current position. Also, it is not uncommon for them to remain in a certain position for a long time, since they might feel the need to gather additional strength to face the challenges in the next position. And finally, the possibility of total escape from the responsibility of decision-making is also allowed for in Perry's scheme.

In addition to these alternatives to growth, Perry also points out the possibility that one can be at different positions of the scheme in the various domains of his life. For example, one can possess characteristics of position five or higher in his everyday reasoning, but he can be quite dualist when it comes to learning mathematics. This phenomenon was documented by Buerk in her work with nontraditional students in the area of mathematics.

In her study, Buerk (1981) reported that although her participants were judged to be at least at position five in Perry's scheme by a panel of judges, their views of mathematics were very dualistic in nature. During her interaction with the five participants, Buerk encouraged them to employ various approaches to solve mathematical problems, to make connections between related ideas, to use visualization as part of their problem-solving process, and to share their thinking with others in the group. Through the use of problem-solving sessions, Buerk was

able to help most of the participants in her study to become more confident in their ability to do mathematics and to see some meaning in the learning of the subject.

Meyerson (1977) also employed Perry's scheme in his work, which was done mainly with students in a teaching methods course. Like Buerk, Meyerson attempted to assist students to see mathematics in a broader view than the dualistic one that they currently possessed, by introducing doubts into their learning in order to force them to reevaluate their conception of mathematics.

For his study, Meyerson used questionnaires and interviews to gather information on the students' conceptions of mathematics. In addition, he collected data from sources such as the students' homework, observation of the ways students interact within the classroom, and the diaries that students kept for the class. Throughout the study, Meyerson used problem-solving exercises and classroom debating sessions to constantly monitor the students' conceptions of mathematics and the changes that took place. At the end of the study, some of the students did experience growth in their conception of mathematics, while others only acquired additional strengths that would enable them to leave the dualist stages of the scheme in the near future, and still others experienced little change in their conception of mathematics.

Oaks (1987) also used Perry's scheme in her investigation of college students' conceptions of mathematics. She worked with four undergraduates who were between eighteen and twenty-one years old. All of these students had previously taken Oaks' Math I class, which was a course on various topics from precalculus and analytic geometry. Through her work with these students, Oaks found that they tended to view mathematics as a subject in which the goal is to find the correct answers rather than to understand the concepts that lie behind the given problems. Thus, these students focused on memorizing the various steps that are involved in solving mathematical problems instead of on paying more attention to

the meaning of their work. As a result, they experienced difficulties in examinations when they could not do problems that were different in format from the ones that they had in their homework. Hence failures were inevitable for these students in their learning of mathematics, despite the hard work they put into memorizing different methods to solve problems.

As the number of their failures increased, the students started to feel threatened by the possibility that they might not have the necessary ability to do mathematics. Thus, in order to preserve their self-esteem and to relieve some of the stress that was caused by the presence of this possibility, students opted to attribute their failure to their lack of effort in learning the subject. Unfortunately, this lack of effort eventually led to the avoidance of mathematics altogether, so that there was little chance for the students to develop a more positive view of mathematics.

Schoenfeld also investigated the influence of mathematical conceptions upon the students' activities in learning mathematics. For example, he did several studies, at both the college and high school levels, in which the focus was on understanding the effect that students' beliefs about mathematics has on their activities in learning mathematics. According to Schoenfeld, a belief system is "a set of (not necessarily conscious) determinants of an individual's behavior that includes such things as belief about self, about the environment, about the topic, and about mathematics" (Schoenfeld, 1985).

In his work with students in problem solving sessions, Schoenfeld noticed several beliefs that were regularly expressed by the students in his study. One was the belief that informal reasoning has no place in the solving of mathematical problems (Schoenfeld, 1985). Such a belief tends to discourage students from using their own reasoning in their work even though their reasoning may be correct. For instance, Oaks reported an incident in which a student gave a correct explanation of the solution to a problem that was asked in class, but he was not confident

that his answer was correct because he thought of it himself instead of using the definitions in his mathematics book to do the task.

Another belief that was often expressed by students is the belief that "all mathematics problems have to be solved in less than ten minutes if they are to be to be solved at all" (Schoenfeld, 1985). As a result of this belief, students tend to give up doing a problem if they have not solved it after having tried for ten minutes. Thus, they are less likely to solve problems whose solutions are not straightforward, even though they have the ability and the knowledge to do such problems successfully.

A third typical belief, according to Schoenfeld, is that mathematics is an elite domain in that only geniuses are capable of doing it (Schoenfeld, 1985). Hence for ordinary people, mathematics is to be learned without questioning the meaning behind the different mathematical concepts, which were derived by the elite group of mathematicians who know what they are doing. Students with this belief usually focus on the mechanical component of mathematics instead of on its meaning in their learning of the subject.

The work discussed above provides us with evidence that many of our students have a dualistic view of mathematics, in which learning mathematics is viewed as nothing more than learning to manipulate various symbols. Moreover, they have certain beliefs about the learning of mathematics which could hinder them in the learning process if left as they are. It is not easy to help students to develop more positive beliefs about the learning of mathematics once these negative beliefs are formed, since they are deeply rooted in the students' past experience. However, the task could be accomplished under certain conditions of instruction (Buerk, 1981; Meyerson, 1978). Past studies also provide some evidence that, in order to protect their self-esteem as learners, students will attribute their failures in learning mathematics to lack of effort on their part.

Overall, past research on students' conceptions of mathematics has given us

some understanding of the effect that these conceptions can have on the learning process. However, there still remain many questions on this topic as far as nontraditional students are concerned. Additional information, going beyond the usual formats of self-report questionnaires and inventory scales, is needed with this group of students if we are to understand the needs and the difficulties that they have in their learning of mathematics.

2.4 Theoretical and conceptual framework of the study

The purpose of this research is to gain additional knowledge of nontraditional students in their learning of mathematics. The focus here is on understanding the activities that nontraditional students undertake in their learning of the subject and more importantly, on understanding the reasons behind their engagement in these activities. In this work, Skemp's learning theory and the conceptual framework in Oaks' study will serve as the guide in the interpretation of the collected data because they include affective variables (e.g. motivation, self-esteem, etc.) in the discussion of the learning process.

1. Skemp's learning theory

The assumption underlying Skemp's theory is that most human actions are goal-directed instead of stimulus-determined. That is, we control our actions so as to bring about desirable states. Therefore in explaining the actions of an individual, Skemp's theory would focus not on observable behavior but rather on the rationale that lies behind it.

In this theory, learning is defined to be a process in which one strives to bring the present state of a situation closer to that of the goal state through the use of one's schemas. A schema is a mental construction which one creates to organize and to make sense of incoming information. A schema can be viewed as a network of knowledge because its components are not just pieces of information, but are also meaningful to the learner and are connected to one another either

through associate or conceptual links. According to Skemp, associate links are formed when the learner memorizes the learned material through rote without seeing a pattern in the given information. For example, if one had to learn the sequence of numbers 15, 82, 413, 89, 586, 7093 one would have little choice but to memorize both the numbers and their order in the sequence because they follow no apparent pattern. If, however, the sequence of numbers was 15, 17, 19, 21, 23, 25 then the learner would have little difficulty remembering it since there is an obvious pattern among the numbers: namely, each term in the sequence is two less than the following term. Moreover, the learner could also produce additional terms in the sequence if needed. In such a situation, the learned material is said to be connected through conceptual links. According to Skemp, both types of links are needed in the learning process; however, in the learning of mathematics it is more desirable that most of the links in the schemas be conceptual rather than associate, so as to make less demands on the learner's memory.

Skemp believes that schemas have three roles in the learning process. Firstly, they play an important role in the construction of understanding by providing the learner with resources for interpreting incoming information. Secondly, they assist in co-operation between learners by providing a basis for communication in working towards a common goal. And thirdly, they act as catalysts for their own growth. We now discuss each of these roles in turn.

Skemp defines understanding as follows: "To understand something means to assimilate it into an appropriate schema" (Skemp 1987, p 29). That is, understanding occurs when one transforms newly acquired information into knowledge which can then be connected to the other parts of a schema. When this happens, one has more flexibility in planning one's actions so as to close the gap between the goal state and the current one. If we were to use the analogy of a city map to explain this flexibility, it would be similar to being able to find many paths connecting one location in the city to another. This increased flexibility in turn

makes it easier to understand new material. In that sense, there is an interdependent relationship between the learner's schemas and his or her understanding of the material, for the growth of one is not possible without the other. For example, suppose a student constructs a schema for addition of whole numbers by connecting together knowledge of the definition of addition, the definition of whole numbers, and the properties of place values in numbers. Now if that student further assimilates the definition of integer numbers into this schema, his or her understanding of addition has increased, since now he or she knows how to add two or more integers. Likewise, his or her schema of addition will be enlarged and strengthened due to the addition of a new component with new connections to the other components in the original schema.

Skemp argues that without the presence of appropriate schemas, one's learning of mathematics is reduced to rote memorization which can result in mechanical understanding of the material. The learner who has a mechanical understanding of the material know how to apply rules and procedures correctly in solving problems but do not know the concepts underlying the problems. His or her goal in the learning process is to learn to obtain the correct answers to problems; and when the learner reaches this goal, he or she believes he or she has understood the material. This type of understanding is undesirable in the learning of mathematics, especially beyond the introductory level, because it makes great demands on the learner's memory by requiring memorization a large number of different types of problems and the many possible ways to solve them. Also, mechanical understanding is undesirable because it is not transferable to new learning situations: a learner with this kind of understanding would not have resources for solving problems that are phrased differently from the ones memorized, even when they cover the same concepts. Also, he or she would not be able to use much previous knowledge in learning new material since there are no connections in his or her knowledge structure between the pieces of information memorized. With concep-

tual understanding, on the other hand, the components of the learner's schemas are connected to each other in ways that have meaning. Rather than relying on memorization in learning, the learner focuses instead on mastering mathematical concepts. This greatly reduces the amount of information to be remembered and gives the learner flexibility to work with a variety of problems.

In addition to helping us construct our understanding in the learning process, schemas play an important role in our work with other people. According to Skemp, individuals whose schemas are not similar would have a hard time cooperating to accomplish a goal, because not being able to interpret information in the same way would make it difficult for them to communicate. This is not to imply that everyone in a group should have the same schemas in order to complete a group task, for doing the work might require more than one kind of knowledge. Thus, it is important that there are people with different schemas within the group who can share their knowledge with each other and help each other in achieving the group goal.

Finally, schemas also play an important role in their own development. New material which is comparable to the knowledge in one of the learner's schemas will have a better chance of being understood and remembered. If, however, the material has nothing in common with the learner's previous knowledge, then it will not make sense to him or her and will be forgotten as soon as it is no longer required. When one understands something, one transforms it into a part of one's knowledge structure and thus enlarges and strengthens it. Skemp points out that this property of schemas, besides contributing to their strength, also has drawbacks in that it makes it difficult to reconstruct schemas once they have been established. Thus, if one understands something incorrectly, it is difficult (but not impossible) for one to alter one's understanding, since this requires reconstructing one's schemas. This has an important implication for teaching: namely, that students should be helped to construct appropriate schemas the first time they

encounter new material, so as to help them in their understanding of the material and to reduce their chances of forming schemas that must be reconstructed later. By providing appropriate guidance to students in their learning, and by engaging them in suitable learning activities such as making conjectures about the material, creating ways to test their conjectures, and exploring the relationships within the material, we may help them construct appropriate schemas for understanding the presented material.

Besides schemas, Skemp also addresses the importance of emotional factors in the learning process. For example, a learner who is able to understand the information presented in a course experiences pride and pleasure. However, a learner who is not able to make sense of the information reacts with anxiety and frustration. The force underlying these reactions is the feeling of self-control the learner has in the learning process. A learner who feels he or she has control over learning outcomes feels secure in his or her ability to comprehend new learning material, is more confident in his or her learning, and has more tolerance for obstacles and errors in his or her work. On the other hand, a learner who does not understand the material presented does not know how to plan his or her actions to reach the desired goals, causing anxiety in his or her work. A learner who overcomes this difficulty in a positive manner, such as by seeking assistance from others, and succeeds in achieving his or her goal, will then feel relief and satisfaction at being able to regain some control over his or her work. Otherwise, self-doubt begins to make it harder for the learner to engage in further learning experiences, as his or her fear of failure grows. The effect of repeated failures on one's learning performance can be damaging, for it can lead to self-fulfilling prophecies of failure: the learner goes into new learning experiences with the expectation that he or she will fail no matter how hard he or she works. Such a consequence is hard on the learner's self-esteem; in some cases, the urge to protect self-esteem is so great that the learner refocuses his or her learning goals in favor of protecting self-esteem,

or purposely handicaps his or her own potential to succeed so that failures can be considered the results of causes other than lack of ability (Covington, 1986; Jones and Berglas, 1978). According to Jones and Berglas, this purposeful impeding of one's own performance can take the form of neglecting to do the necessary work, reducing one's effort in the work, or neglecting one's physical well-being before an important test by not getting enough sleep or using drugs. Using one of these strategies protects one from the threat of having to expose one's true ability both to others and to one's self. In fact, one's ego could actually be boosted if one did achieve success without much effort (Riggs, 1982).

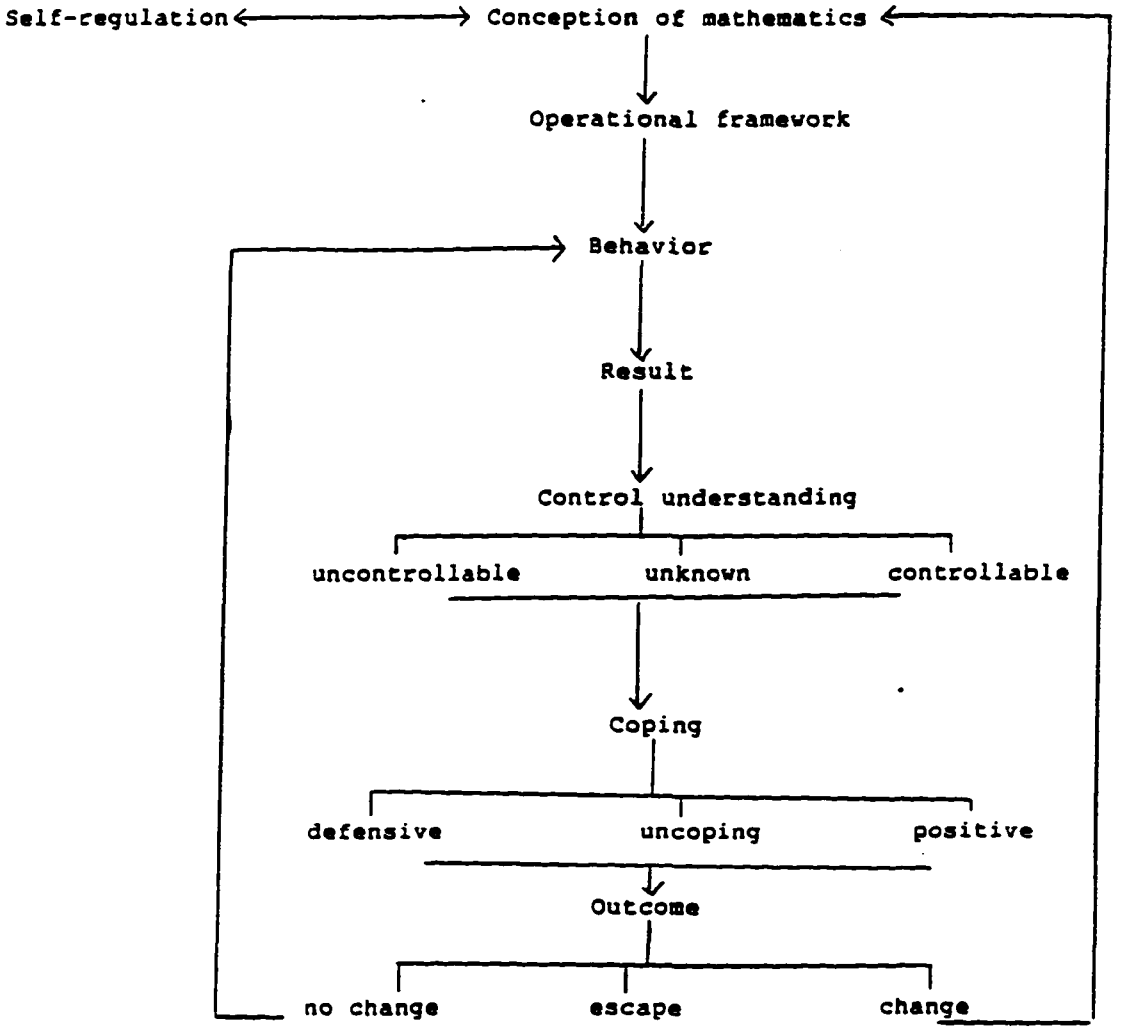
We should not underestimate the effect that self-esteem and self-control have in the learning process, for these can provide us with clues in understanding our students' actions in their studies, especially in cases where the students experience difficulties in their work. To help guard our students' self-esteem in the learning process, we should strive to provide our students with a safe learning environment in which to grow and to explore new concepts. The instruction should be structured in a way that leaves no gap between the students' previous knowledge and the new material. Although assignments should challenge the students and encourage them to apply what they have learned to new situations, Skemp cautions that the difficulty level of these new tasks should not be beyond the students' reach; otherwise, the students' motivation could decrease. Skemp also points out that it is important that we help our students learn from their mistakes. For many students, mistakes are viewed as undesirable instead of as opportunities for additional exploring and strengthening their understanding. These students should be helped to realize that making mistakes is a part of the learning process, and that the focus should not be on how many mistakes are made, but rather on understanding what caused them. As students gain more confidence in their learning, they will become more independent in seeking out and using appropriate resources to answer their questions. Skemp believes that the gratification that the

learners experience from succeeding in such tasks is enough in itself to motivate the learners to continue engaging in new learning activities.

2. Oaks' conceptual framework

In Oaks' conceptual framework (see figure 2), locus of control occupies an important part in the learning process. Locus of control, according to social learning theorists, is the perception that one has concerning one's ability to regulate the outcomes of events in one's life (Lefcourt, 1982). The emphasis here is on the perception that one has about one's ability to regulate a particular outcome and not on whether one has the required ability.

Figure 2: Oaks' Scheme



Past research related to the concept of self-regulation often made use of Weiner's attributional model. In this model, different sources that one believes to be the causal agents for academic successes and failures are discussed in relation to two dimensions of control: locus (internal versus external) and stability (stable versus unstable). In the locus dimension, ability and effort are internally controlled, while luck and task difficulty are externally controlled. On the other hand, ability and task difficulty are stable causes of success or failure, while effort and luck are unstable causes. Thus in Weiner's attribution model, ability is considered an internal-stable causal agent, effort is internal-unstable, task difficulty is external-stable and luck is external-unstable (Weiner, 1979).

According to Weiner, one's perceptions concerning the causes of one's successes or failures can influence one's future achievement (Weiner, 1979). For example, if one believes that failure on a test is caused by an internal-stable variable like ability, then he or she will be less likely to study differently for the next test. This is probably due to the belief that ability is something that one either has or does not have and there is little that one can do to make up for it. On the other hand, if one attributes failures to an internal-unstable factor like lack of effort, then one will be more likely to approach study differently in the future, because one feels that one could improve test outcome if one invested more effort into it.

In this attributional model, there is an underlying assumption that one can identify the causes of failures or successes; however, this is not necessarily true in all cases. Connell (1985) suggests that a third dimension should be added to the construct of locus of control to account for the possibility that the cause of one's failure or success is not known to a person. This dimension is called the unknown control dimension. Taken together, the work of Weiner and Connell helps to provide us with some idea of the students' attributional style; such information is useful for our understanding of the various factors that motivate students to work and of the ways they chose to cope with failure.

Although the research on motivation is extensive, our knowledge concerning the factors that motivate one to learn is still limited. Work in this domain offers neither easy nor clear solutions to the problem of motivating students in the classroom; however, it does shed some light on the complexities of the phenomenon of motivation in the learning process and on the factors that could affect it.

Several theories of motivation have been offered by researchers over the past few decades. Behaviorists such as Pavlov and Skinner believed that the principal motive for one's actions lies in one's physical needs. To these investigators, all human actions can be explained in relation to physical necessities. Other researchers hold, however, that behaviorists have only considered one aspect of human needs and thus have left unanswered the question of what motivates a person to act when all that person's physical needs are already met.

One of the theories that emerged from the investigation of this question suggests that curiosity is the main motive for a person's actions (Harlow, Harlow, and Meyer, 1950; Berlyne, 1960; Hunt, 1965). That is, a person engages in activities mainly to satisfy curiosity. Another theory suggests that the motives behind one's actions have to do with one's desire to be competent and to have control over one's environment (White, 1959). The assumption underlying these two theories is that one's motives for one's actions are not exclusively externally regulated, as the behaviorists believe, but can also be internally regulated. Hence, this way of viewing human motivation is not as restrictive as that advocated by the behaviorists. Because of this flexibility, researchers in the field after the 1950s tended to favor these two theories over behaviorism as a guide in their investigations of intrinsic and extrinsic motivation (Boggiano and Pittman, 1992).

The idea of internal regulation was again stressed in achievement motivation theory, which holds that one acts in a certain manner in order to satisfy one's needs to achieve and to avoid failure (Atkinson and Feather, 1966; Atkinson and Raynor, 1974). In addition, it calls attention to the influences that external rewards can

have upon one's willingness to participate in an achievement-related task at a particular time. For example, one would probably be more willing to do one's homework early if one knows such action would bring one money or praise. This raises an important question: namely, what happens to the students' willingness to engage in achievement-related tasks when external incentives are removed from the learning process? In other words, what effects do external rewards have upon one's intrinsic motivation?

When a person is intrinsically motivated in his or her learning, the process is viewed as an end in itself; however if a person is extrinsically motivated, the process then becomes a means to reach some end (Boggiano and Pittman, 1992). According to Deci and Ryan (1987), humans are intrinsically motivated to learn; however, a shift to external motivation can occur if there is an absence of self-competence and self-regulation in the learning process. That is, intrinsic motivation cannot prevail in situations in which one feels neither competent in one's work nor responsible for one's achievement. Researchers such as Ryan and Deci believe that the school environment does not promote the growth of intrinsic motivation in the learning process because of the control (e.g. grades, praise) that it exerts on the students.

Ryan and Deci use the term "internalization" to describe the process in which students accommodate and eventually assimilate the goals of the classroom into their own goals of learning (Ryan and Deci, 1987). Such development takes place on a continuum ranging from external regulation at one end to internal regulation at the other end. Students who are extrinsically regulated rely mainly on external sources such as rewards or authority for motivation to work. As students move beyond this level, the motivation to work shifts inwards, towards themselves. At the introjected regulation level, the main motive in the students' work is the students' self-esteem. That is, they work because they believe that their self-worth is contingent upon their academic achievement. In order to leave this level

to reach the next one, identification, students have to internalize the classroom goals to the extent that such goals are a part of them. At this stage, students work mainly to satisfy their own needs and not to please others. Here, the students' locus of control is internally regulated instead of externally controlled. Thus, it is reasonable to expect self-identified students to have an approach to failures in learning that differs from that of their colleagues who are extrinsically regulated.

According to Connell and Tero (1984), there are several ways students can choose to cope with failures in their learning. One can either choose to cope with them positively, or negatively, or not at all. Positive coping means dealing with failures by seeking help from various sources (such as teachers, tutors, or books) to understand the learning material better. Negative coping means dealing with failures by blaming poor performances on sources such as the teacher or the absence of luck. Finally, noncoping students have neither the initiative to seek help from others nor the desire to blame others for their failures; instead, they believe that their failures are due to lack of ability. Individuals with this particular outlook are regarded by researchers as displaying the characteristic of learned-helplessness.

Learned-helplessness is the state in which one believes that, because one is unable to do the work, there is nothing one can do to regulate the outcomes of learning (Seligman, 1975). The feeling of not being able to improve upon one's performance, even if one wanted to, can have serious effects upon one's life. Such effects can take the form of depression, anxiety, or loss of self-confidence, all of which pose possible threats to one's self-image (Covington, 1986). Thus, in order to relieve some of the stress that comes with this feeling of helplessness, and to protect one's self-image, one is motivated to seek variables other than ability on which to blame failure.

Past work on motivation in learning has provided us with some evidence that self-esteem plays a significant role in students' decisions concerning the failures or the difficulties that they encounter in their learning process. However we still

have little idea of the role of self-esteem in the learning process for nontraditional students. Although the general literature on adult learners suggests that nontraditional students tend to be more motivated in their learning than their younger classmates, it is not yet clear how much of that motivation relates to these students' self-esteem. Moreover, we still do not know how nontraditional students choose to cope with failures and difficulties in their studies. How do these students view failures in their learning and how do they react to such situations? Do the maturity and the work experiences that these students have gained during their years away from an academic environment help them or hinder them in dealing with difficulties that arise in their learning? Addressing such questions can further our understanding of nontraditional students in the area of mathematics, and may help us to structure programs that assist nontraditional students towards success in their learning of mathematics.

Chapter 3: Description of the study

This study is an attempt to gain some understanding of nontraditional students' learning of mathematics. The focus is on the conceptions of mathematics that nontraditional students have and on the affect that these conceptions have upon the students' learning activities. Case studies were conducted with five volunteer nontraditional students who enrolled in College Algebra at the University of Oklahoma in Norman. Data were collected during the sixteen weeks of the study from transcripts of interviews with students and the researcher's field notes of classroom observations and of tutoring sessions. While this study is similar to Oaks' (1987) in that it focuses on the relationship between college students' conceptions of mathematics and their learning activities in the subject, it differs from Oaks' study in that it focuses on nontraditional students rather than traditional ones.

This chapter consists of two parts. First definitions of various components in Oaks' conceptual framework, and a discussion of Perry's scheme which was utilized in Oaks' study as applied to mathematics will be presented. The second part contains a discussion of the research methodology of this study.

3.1 Definitions for the conceptual framework

This study used the conceptual framework employed by Oaks in her study on remedial/developmental mathematics students as the guide in the interpretation of the data. Oaks defines the components of her scheme as follows (Oaks 1987, pp 57 - 62):

Self-regulation:	the factors that motivate a student to work, ranging from extrinsic motivation to intrinsic motivation.

Conception of mathematics:	a student's view of the nature of mathematics and the activities connected to doing mathematics.
Operational framework:	the set of goals, intentions, and expectations brought to mathematical activity by the student.
Behavior:	the set of activities of a student in a mathematical setting.
Results:	feedback a student receives as a result his or her activity in the mathematics classroom.
Control understanding:	the student's view of his ability to influence his learning results.
Coping:	activities students engage in when dealing with failure.
Outcome:	the result of the interaction of all the constructs listed above.

In the area of students' conception of mathematics, the focus will be on the beliefs that students have concerning their roles, the teacher's role, and the required work for the course. As for the category of operational framework, the focus will also include the students' actual knowledge of mathematics (e.g., factoring) and the meaning behind the students' use of language in their learning of mathematics. The activities in the behavior category mainly refer to such things as the students' study habits. The main interest in the result category is on the consequences (e.g. grades, recognition) that accompany the students' activities in the learning process.

The category of control understanding includes the subcategories of controllable, unknown, or uncontrollable, which are classified according to whether the

students know the sources of their failure/success and whether they feel they can regulate those sources. Students in the controllable group believe that they know the sources of their successes/failures and can regulate those sources. Students in the unknown group do not know the sources that dictate their learning outcomes and so do not know whether regulation is possible on their part. Students in the uncontrollable group believe that they know the causes of their successes/failures, but do not believe that they have the ability to alter undesirable outcomes in their learning.

The categories of coping and of outcome also have subdivisions, which include positive coping, negative coping, and noncoping for the former category; and change, no change, and escape for the latter. The differences among the subgroups within each of these two categories are closely linked to the students' attributional styles. For example, if a person believes that ability is the cause of failures in learning mathematics, then that person might choose to deal with failures by not doing much to improve the situation, or might even purposively impede his or her own performance to divert attention from ability as the cause of failure. If a person chooses the first route, then he or she may eventually avoid taking mathematics altogether; in which case there is little possibility to develop a more positive conception of the subject. On the other hand, if the second route is chosen, the opportunity to develop a positive conception of mathematics is not completely out of reach; however, such a positive conception will not come about without a major transformation on the part of that person. In the case where change occurs as the outcome, the student not only experiences a change in the level of his or her knowledge of mathematics, but also in the approach to learning the subject.

Perry's scheme and mathematics

As mentioned above, this study will use Perry's scheme as one of the guides to the interpretation of the students' conception of mathematics. This scheme

comprises nine different positions describing how one views knowledge and ranging from basic duality to personal commitment. According to Perry, changes in one's conception of knowledge, especially in the first five positions, require both time and doubt in one's learning. That is, the process of change can take time, since before any changes can take place one must first recognize the need to re-evaluate one's view of what constitutes knowledge and truth. Such recognition usually takes the form of self-doubt, in which the focus is not so much on one's learning ability as it is on the set of criteria that one uses to judge truth in one's learning. It is only through this struggle that students gain a sense of direction and responsibility in their learning.

In this study, Oaks' list of evidence for interpreting students' conceptions of mathematics will serve as a guide in the data collection process. This list of evidence describes the characteristics that distinguish students who hold a dualistic view of mathematics from those who see mathematics in a relativist manner. Oaks focused mainly on the dualist and the relativist stages of Perry's Scheme since she believes it was less likely that students in the early levels of mathematics possess a view of the subject that reflects commitment in the learning process.

Using the characteristics of the dualist stage in Perry's scheme as the guide, Oaks created a list of evidence that depicts the concept of dualistic thinking in the area of mathematics. According to Oaks, students who have a dualistic conception of mathematics tend to assume that a solution exists for every mathematics question and that the goal of learning mathematics is to find such correct answers. In addition, they assume that there is only one right way to solve a problem and that a different answer will result if they solve the same problem using another method. In the cases where there are more than one appropriate method to do the task, these students insist that there must be one that is "the best" method to solve the problem.

Because these students believe that the goal of learning mathematics is to find

the correct answers, they are mainly interested in learning the right algorithms in their studying of the subject. Their learning focus is on getting the right solution methods from their teachers and on memorizing and practicing such methods until they can use them to solve problems quickly. Homework and textbooks have little value to these students other than for the purpose of practicing. As a result, homework means little to these students other than a way to gain mastery of the different algorithms.

For students who hold a dualistic view of mathematics, understanding refers not so much to conceptual comprehension of the material, but more to the ability to do problems correctly. Memorization, on the other hand, is often viewed by these students as the remembering of rules and formulas. Since the meanings behind the terms "understanding" and "memorizing" are limited to the mastery of algorithms, students with a dualistic view of mathematics engage in few activities in their work which involve strategy planning, exploring, hypothesizing, or checking of solutions. Mistakes in the work are usually looked upon by these students as either "stupid" or as signals that they need more practice with certain methods.

Practice in doing problems plays a central role in the learning process for students who view mathematics dualistically. These students tend to believe that mathematics is little more than a set of rules and that if they remember all the necessary rules and know how to apply them correctly, then they will have little difficulty in learning the subject. Hence, the responsibility of the learners is limited to knowing the various procedures and practicing them until mastery is achieved. The teacher's responsibility, on the other hand, is one of providing the students with different algorithms and assigning exercises for practicing purposes.

There is evidence (e.g. Meyerson 1974, Buerk 1981, Oaks 1987) that students with a dualistic view of mathematics believe that learning mathematics means learning different methods to do problems. As a result, these students have intentions and expectations which differ from those of their colleagues who view

mathematics in a relativistic manner.

According to Perry, one aspect of relativistic thinking is the ability to reflect upon one's thoughts and actions (Perry, 1970). In the area of mathematics, this includes the ability to reflect on the subject and to compare one's view of it to the views of mathematics held by others, or to one's own previous view of mathematics. Further evidence of relativistic thinking is the emergence of personal responsibility in the learning process, as one starts to realize that one's responsibilities in one's studies extend well beyond the memorizing of rules and the practicing of algorithms. Thus although students still strive to get the correct answers in their work, they are now more focused on understanding the material, and on the reasoning that underlies the process of solving problems. More importantly, they now realize that although the teacher can help them understand the material by presenting it in a certain manner, they themselves bear most of the responsibility of constructing meaning. In that sense, it is the student who has the control over his or her learning outcomes.

Another hallmark of relativist thinking in the domain of mathematics is the presence of flexibility in working with mathematics problems. Students who have a relativist view of the subject not only realize that there can be more than one way to solve a problem but also understand that the choice of an appropriate solution method depends on the context in which the problem is embedded. Thus, the emphasis is not on memorizing different algorithms and then trying to match them up with different mathematics problems, but rather on understanding the nature of the problem before generating a solution method.

Overall, students with a relativistic view of mathematics place heavy emphasis on the understanding of concepts rather than on getting correct answers in their learning process. In addition, they are more responsible in their studying and are more active in seeking answers to their questions from sources other than their teacher. Memorization of algorithms does not play a large part in these students'

mathematical learning process, for they believe such a task to be secondary to the understanding of concepts.

3.2 The research design and methodology

This section on the research methodology of this study is divided into several subsections. The first presents the justification for the use of case studies as the research method. The second discusses trustworthiness issues, and the third describes the participants and the setting of the study. The final subsection outlines the design for the data collection and analysis.

Justification of the research method

The goal of this study is to gain further understanding of nontraditional students' learning of mathematics. Case studies were used to gather information on nontraditional students' conceptions of mathematics and the activities they undertake in their learning of the subject. The case study method has been chosen because it reflects the goal of the study and because it offers flexibility in the data collection process. In addition, it provides the researcher with a way to verify the participants' interview responses.

The assumption that motivated the use of case study as a research method has to do with the belief that everyone constructs his or her own understanding in the learning process. That is, one constructs the meaning of the presented material in a way that is personally meaningful. Such a construction might or might not be similar to that of the others, and might or might not be accepted by the others as a reasonable interpretation of the material; nevertheless, its existence is justified and is significant within its creator's perspective. Thus if we are to understand how nontraditional students approach the study of mathematics, it is important that we view the investigation from these students' perspective for it could help us in our search for the "whats" and "whys" behind these students' learning behaviors.

According to Perry, students at the dualist level of Perry's scheme are less

likely to give a direct account of their conception of knowledge (Perry, 1970). This is because in order to articulate their view of knowledge, they need to be able to reflect on their thinking. That is, they need to have the ability to stand back and view the situation from different angles as part of their decision making. Such detachment from the situation is not possible for students at the dualist level of Perry's scheme due to the level's rigid structure. Since data cannot be gathered directly from these individuals, it is necessary to use a more flexible data collecting process with them. For example, data can be gathered from observation of the students' study habits and of the activities in which they choose to engage while learning the subject. Flexibility is also needed in the structuring of the interviews with the participants, because the majority of the interview questions are based on their responses.

Besides the advantage of flexibility, the case study method also offers the researcher a way to verify and clarify the participants' responses. Schoenfeld, in his work with students both at the high school and college levels, suggests that students do acquire different rhetorics from their mathematics classrooms (Schoenfeld, 1984). For example, they may have internalized their teacher's comments that understanding is important in the learning of mathematics, and may come to believe that their successes in learning are due to their understanding of the concepts. However, when asked to clarify what they mean by understanding mathematics, students may focus mainly on the memorization of rules and algorithms, as has been documented by researchers such as Schoenfeld (1984) and Oaks (1987) through their interviews and observations of students. The case study method provides the researcher with opportunities to uncover the rhetorics in the students' responses, so that the researcher's interpretation will more accurately reflect the students' beliefs about the learning of mathematics.

Trustworthiness issues

In qualitative research, the term "trustworthiness" deals with the rigorous-

ness of the study. In answering questions concerning the merits of the research, trustworthiness issues that arise are credibility, transferability, dependability, and confirmability (Lincoln and Guba, 1985).

1. Credibility

The focus of the credibility domain is on the "truth" of the study findings. The issue here is whether the investigator has provided the readers with a picture that truthfully depicts the phenomena under study. According to Lincoln and Guba, there are several activities that one can do to enhance the probability of highly credible findings: prolonged engagement, persistent observation, and triangulation. These activities were carried out in this study during a period of sixteen weeks, during which time data was collected through the use of interviews and observations (including tutoring sessions and classroom observations). The researcher's field notes were used to record the observations of the participants, while audiotapes were used to record all the interviewing sessions.

In addition to these activities, Lincoln and Guba also recommend other techniques to satisfy the credibility criterion. These include peer debriefing, negative case analysis, referential adequacy, and member checking (Lincoln and Guba, 1985). Peer debriefing refers to the disclosing of one's study to one's colleagues in order to clarify any aspect of the study that might puzzle the readers. Negative case analysis deals with the revisions of hypotheses to make them more appropriate for describing the phenomena under study. Referential adequacy is mainly concerned with the storage of the data for later analysis or comparison, and member checking refers to the informal verifying of the researcher's interpretations with the participants in the study.

In this study, the above criterion was addressed in the following manner. In the domain of peer debriefing, the researcher met with her thesis advisor biweekly to discuss her work, so that the advisor could help in checking her research methodology, revising her research questions, clarifying her interpretations of the

findings, and questioning her implicit assumptions during the study. For negative case analysis, insights that occurred during the study were recorded in the researcher's field notes for future reference and analysis. For referential adequacy, data were stored in either the researcher's field notes or audiotapes throughout the study. And lastly, the author checked with each participant periodically throughout the study on her interpretations of their responses in order to assure that the member checking category was included in the study.

2. Transferability

Transferability is concerned with the question of generalizing the findings of the research to other contexts or to other populations of interest. According to Lincoln and Guba, qualitative research findings are not transferable due to the nature of the research paradigm. Thus, the responsibility of generalizing the findings does not rest upon the researcher, but rather on the readers themselves. That is, readers have to make a decision as to how the study applies to their situations and what actions need to be taken. The researcher has the responsibility of providing a clear description of the study so that the readers can use it in their decision-making process. In this study, the author provided a detailed description of the participants so that interested readers can judge whether the findings from this work might fit into their research interest.

3. Dependability

The focus of the dependability category is on the consistency of the findings. That is, it deals with the question of whether the same finding can be achieved by different investigators using the same method. While this question makes sense for quantitative research, it does not make sense for qualitative research because the nature of qualitative research does not guarantee that investigators using the same methods will reach the same conclusions. The interpretation of the data in qualitative research depends on the researcher, the participants, and the context of the study. Hence, it is not unusual for different researchers studying

the same phenomena and using the same research method to arrive at different conclusions. According to Guba and Lincoln, dependability in qualitative research can be checked by using an outside "inquiring auditor" to evaluate the process of collecting and interpreting data and to ensure that the study is done in a fair manner (Guba and Lincoln, 1985). The author had her advisor act as the inquiring auditor in this study to ensure dependability.

4. Confirmability

Confirmability deals with the objective issues in the study. As in the category of dependability, an "inquiring auditor" was also suggested by Guba and Lincoln for dealing with the issue of objectivity in qualitative research (Lincoln and Guba, 1985). In order to keep personal biases from interfering with the analysis in this study, the author checked her interpretations with the participants periodically throughout the study, and had her thesis advisor serve as the auditor.

Participants and Setting

1. The Participants

The participants in this study were five nontraditional students in College Algebra who volunteered to participate in this research. Of the five, two were females and three were males; their ages ranged from twenty-five to forty-five. All were married with three having children under ten years, and two having children between ten and twenty years old. All except one participant had a spouse who worked full-time last semester (fall 1995), and all had been full-time workers at sometimes between their high school graduation and their study at the university.

2. The Setting

The University of Oklahoma is a research institution that has its main campus located in Norman, Oklahoma. Norman is a suburb of Oklahoma City that has approximately 80,000 residents. The Norman campus has about 800 full-time faculty and approximately 20,000 students. The university offers both undergraduate and graduate degrees in various academic disciplines. In addition, the university also

offers educational programs to adult learners through the College of Continuing Education in an effort to meet the needs of learners whose interests are different from the ones that are being offered in the regular programs at the university. There are about 5,000 nontraditional students at the Norman campus; however, the majority of them are part-time students.

Collection and Analysis of Data

In this study, data were collected in two stages: the preliminary observation stage was followed by an in-depth tutoring and data collection phase. The preliminary stage lasted approximately four weeks at the beginning of the semester. During this time, the investigator observed the students daily in their math classes and interviewed them bi-weekly outside of class. Each observation lasted for about one hour and each interview for about half an hour. The immediate analysis of the data from the preliminary stage served as the guide in constructing follow-up interview questions and sharpening the focus of observations in the second stage.

The second stage of data collection lasted approximately twelve weeks. During this time, the researcher observed the participants inside the classroom and interviewed them outside of it. Also, she worked with the three participants who had requested her to help them review for the tests in the course. For the tutoring activity, the investigator provided each of the three participants with two one-hour review sessions for each of the tests following the first in-class exam. These tutoring sessions provided the investigator with opportunities to observe and interact with the participants as they studied for their mathematics tests outside of the classroom. Moreover, these tutoring sessions provided the investigator with opportunities to verify and explore further the participants' interview responses and to clarify the meaning these students construct in their learning of mathematics. Observations from these tutoring sessions were helpful in the construction of questions that the investigator used in her subsequent interviews with all participants.

Data Collection

Interview Sessions

The interview questions for the background interview included the following:

A. What are the reasons for your returning to college? What do you plan to major in and why?

B. What were some of the difficulties you encountered in making the decision to go back to school? How did you deal with them?

C. What are your learning goals in college? If you have been in college before, have your learning goals changed since then?

D. Describe your past experience with mathematics learning.

E. Did you experience any difficulties in your previous learning of mathematics? If so, what were they? Do you know the sources of these difficulties? Were you able to resolve any of them, and if so, how?

F. What are some concerns that you have in returning to the math classroom as an older learner?

G. What do you expect to get out of this course? What is your plan for fulfilling that expectation?

H. Why did you agree to participate in this study?

Upon completion of the course, students participated in an exit interview. The interview questions included:

A. Are you satisfied with the outcome of your final exam? Explain your response.

B. If you are not satisfied with the final exam outcome, do you think you could have done something differently to get a better result?

C. What difficulties did you experience in your math class this past semester? To what do you attribute these difficulties? Were you able to resolve any of these difficulties, and if so, how?

D. What is your reaction to the grade you received in this math class? How important are grades to you? Explain your response.

E. Do you feel that you have succeeded in fulfilling the goals that you set for yourself at the beginning of the semester? To what would you attribute your result in the course?

F. Describe the experience of being back in a math classroom this semester. What were some of the challenges that confronted you as an older learner? How did you deal with these challenges?

G. What services were you aware of that are available to students on the campus? How did you learn about them? Did you use any of them? If so, how often and for what purpose?

H. Did you learn other things from your math class besides mathematical concepts this past semester? If so, explain.

I. Do you plan to continue taking math courses in the future? If you do, what are the reasons?

J. Are you satisfied with the way you studied for mathematics this past semester? Would you study differently in your next math course? Why or why not?

During the second stage of data collection, the guiding questions for the interview included the following:

A. Student's conception of mathematics.

1. Finish the statements below:

   a. Mathematics is like .....

   b. I like mathematics because .....

   c. I dislike mathematics because .....

   d. Learning mathematics involves .....

B. Student's learning activities

1. Classroom activities.

   a. Do you attend class regularly?

b. What do you think your teacher's role is in your math class? What do you expect your teacher to do in his or her teaching?

c. What do you think your role is as a student in a math class?

d. What do you do when you have a question concerning the lecture? Did you act similarly when you were younger?

e. Do you take notes in class? If so, what is your focus in note-taking?

2. Math textbook vs. lecture notes.

   a. Do you use the text book in this class? If so, for what purpose and how often do you use it?

   b. Do you use the lecture notes? If so, for what purpose and how often do you use it?

   c. Which of these two, textbook or lecture notes, do you tend to use more? Explain your response.

   d. Do you own a copy of the textbook for the course?

3. Homework.

   a. How often do you do your homework in a week? How many hours do you spend studying for math outside of the classroom?

   b. How do you usually approach doing the homework? (By imitating the examples in the lecture notes? Using examples in the textbook?)

   c. Do you use the solution manual? If so, how often and for what purpose?

   d. If you do not use the solution manual, explain the reason behind your decision.

   e. What are some of the difficulties that you experience while doing your homework? How do you deal with them? How long do you usually spend trying to resolve an obstacle in doing a homework problem, excluding the time you spent in the tutoring sessions?

   f. What is a typical amount of time that you spend doing a homework problem? Is there a limit as to how much time you are willing to spend

doing a math problem before giving up?

g. What is your focus in doing homework problems?

h. How can you tell, after doing a problem, that your answer is correct?

i. Do you use other resources in doing your homework besides the textbook, the lecture notes, and the solution manual?

4. Test preparation.

a. When does test preparation begin for you? When does it end?

b. How do you go about preparing for the test?

c. What is your focus in preparing for the test?

d. What do you use to help you in the preparation process?

e. How do you know that you have understood the concepts that are in the chapters that you will be tested on?

f. What does understanding a concept in mathematics mean to you? Compare your definition of understanding a math concept to your definition of understanding a concept in another area such as, say, sociology.

g. How do you know when you have studied enough?

5. The day after the exam was taken.

a. How did you feel on the day of the test? Explain your response.

b. Did you feel prepared for the test? Explain your response.

c. What were some of the difficulties you experienced while taking the test? Were you able to resolve any of these difficulties? If so, how?

d. Did you have enough time to take the test?

e. How did you feel after you handed in the test? Did you have any idea as to how you did on it?

f. Do you have any plans as far as preparation for the next exam is concerned?

6. The day the exam was handed back.

a. What was your reaction to your test result? Explain your response.

b. Did you feel your test score reflects your understanding of the material in the chapters that the test covered?

c. Do you understand the mistakes that you made on the test? Explain your response.

d. Did the test result agree with your prediction of your performance?

e. To what would you attribute your test result?

f. Now that you have seen the test result, do you have any plans concerning your preparation for the next exam?

g. What have you learned from this past exam?

Tutoring session observations

In the observation of the students, the focus was on the following:

1. Use of vocabulary (e.g. understand, theorem, algorithm) in their learning of mathematics.
2. Understanding of mathematical concepts (e.g. conceptual understanding vs. mechanical understanding).
3. Approach to learning mathematics (their goals, their intentions, their expectation).
4. Confidence in their learning of the subject.
5. Use of coping strategies to deal with setbacks or difficulties in their learning.
6. Study habits in their math courses.
7. Beliefs as to the causes of their difficulties and their successes.
8. Difficulties that experienced in their transition back into a math classroom.
9. Assumptions brought to their math class concerning the nature of mathematics and the learning of mathematics.
10. Expectations they had of their teacher and of themselves in the classroom.

Data Analysis

Data in this study was analyzed across the different categories of Oaks' conceptual scheme for all of the participants as a group. In addition, the analysis focused on discerning emerging patterns in the nontraditional students' activities in learning mathematics and in the reasonings behind their activities.

The following served as an outline of the data analysis in this study:

I. The general nature of mathematics.

A. What are the reactions that the students have as to why we do mathematics? Do they recognize the application of mathematics or the beauty of it?

B. Does the student believe that creativeness and relativeness can exist in the area of mathematics?

II. Classroom mathematics.

A. Conception of activities in the classroom.

1. What does the student think is the primary goal of the classroom? Does he or she learn material for immediate or for future use?
2. What does the student expect the pupil's role to be in the classroom? the teacher's role?
3. Does the student see much meaning in the rules and the procedures in his or her learning of mathematics or just accept what presented as truth to memorize?

B. Conception of problems - what goal does the student have in working a math problem? Was the student able to see meaning attached to the problem?

1. Methods of solution.

a. How does the student react to a problem where there are more than one way to solve the problem? How does the student decide which methods of solution to use?

b. Does the student devise his or her own method to solve a math problem? How does he or she feel about devising personal solution methods?

2. Answers.

   a. Does the student believe that there is an answer to every problem in mathematics?

   b. How does the student react to the case where there is more than one solution?

III. Self-regulation.

A. Reasons that the student has for doing mathematics.

1. For a reward?

   a. Was it because of personal interest?

   b. Was it to complete the requirement for his or her a major?

   c. Was it to show oneself/others that one could; to enhance self-esteem?

2. Because someone else expected of the student?

3. Because the student expects it of himself or herself?

IV. Operational framework.

A. Resources.

   1. Is the student able to make the connection between previous knowledge of mathematics and the new learned material? Does the student use previous knowledge of mathematics to help in making sense in learning of the subject?

   2. Does the student use correct vocabulary in the learning of mathematics? Is he or she aware of the meanings behind the different terms in mathematics?

B. Expectations brought to mathematical activity.

   1. What goal(s) does the student have in learning mathematics? in doing problems?

   2. Does the student have any expectation concerning the correct method and the correct answer to a problem as far as the format is concerned?

C. Intentions while doing mathematics: What are the intentions that the student

has in

1. preparing for a test?
2. taking a test?
3. attending class?
4. doing homework?

V. Behavior.

A. What does the student do while

1. learning mathematics.
2. doing problems.
3. taking tests.
4. attending class.

VI. Results of learning activities.

A. Does the student experience success in the learning of mathematics? What difficulties arise in the learning process?

B. What is the student's emotional reaction to the result (i.e grades) that he or she received?

VII. Control understanding - what does the student think it takes to do well in mathematics?

A. Effort.

B. Ability.

C. Outside help.

D. Luck.

E. Unknown.

F. Other.

VIII. Coping - how the student deals with poor grades.

A. Positive coping.

1. Studying harder.
2. Seeking help from others.

B. Non-coping.

    1. Feeling bad about himself or herself.

C. Defensive coping.

    1. Denial.

    2. Blaming others.

IX. Outcome.

A. Has the student stayed in mathematics or escaped?

B. What effect has involvement with mathematics had on the student's self-esteem?

C. What is the student's emotional reaction to mathematics?

Chapter 4: Presentation of the study's findings

This chapter contains profiles of the participants and their College Algebra class, followed by a presentation and analysis of the data. Additional analysis and synthesis of the data will be presented in the next chapter.

4.1 Introduction to the participants and the course

The participants

a. Kelli is a 35 year old junior majoring in Business Management. She is married and has a 3 year old daughter. Her husband is a college graduate currently working for an oil company.

b. Barbara is a 45 year old junior majoring in Accounting. She is married and is a mother of three sons, one of whom is in the seventh grade, one in the ninth grade, and the oldest in his first year at the university. Her husband is a college graduate and he works in the local area.

c. Stan is a 28 year old student who is a junior in Chemistry. Stan is married and has two young sons whose ages are three and five. His wife is also a student at the university and like Stan, she is working on finishing her bachelor's degree.

d. David is a 35 year old sophomore who is majoring in International Business. He is married and his wife is currently working as an elementary teacher for a school in Norman. In addition, he has two children who are in their early teens.

e. James is a 25 year old freshman who is majoring in Management Information Systems. He is married and has a daughter who is almost two years old. James's wife already has her college degree and is now working for a company in the local area. In addition to being both a husband and a father, James also is a staff sergeant in the Marines.

The course

College Algebra is a three-hour credit course that students at the University of Oklahoma can take to fulfill a portion, if not all, of the mathematics requirement for their major. Students are admitted into this class if they pass either Intermediate Algebra or the university mathematics placement exam, or make a certain score on the ACT test.

In the fall of 1995 (the semester in which the study took place), the class spent the first three weeks of school (about 9 hours total) reviewing topics from Intermediate Algebra. The rest of the semester was spent investigating functions: the definition of function, functional notations, concepts such as domain and range, operations on functions, use of function in solving application problems, and graphing of functions.

As it has since the fall of 1994, the mathematics department offered College Algebra in Fall 1995 as a uniform course: there is a coordinator who is responsible for insuring that the course is uniformly delivered as regards its course syllabus, objectives and policies, homework assignments, examinations and grading scale. Also in the semester in which the research took place, the course coordinator visited and observed each of the instructors in their class. Although all instructors were to follow a set schedule for teaching the material, they were free to decide how best to present the material in their classroom: all chose the lecturing method. That is, the instructors explained the course material while the students listened and took notes, and the instructors also provided their own notes to the students and answered any questions the students might ask.

The grading in this course consisted of 100 points for each of the three regular tests, 200 points for the final exam, and 100 points for class work. Of these categories, class work was the only one that varied among the instructors of the course. In this category, each instructor decided whether the students' regular class work grade in the class would take the form of quizzes, homework, or both,

and whether these would be given or assigned weekly or less regularly. (There was an uniform assignment at the end of each section.)

According to the course coordinator, students in College Algebra in fall 1995 were required to attend class and instructors were required to take attendance at each class meeting. However, this policy was not rigorously enforced, since attendance records were not used at all in determining the students' final course grade. Since then, there has been a change in the course syllabus giving attendance some influence on the final course grade in the case when there is a curve at the end of the semester. For example, students with less than three absences in the entire semester could add a few extra points on their total points in the course which could make a different in their final course grade.

In College Algebra, students have many opportunities to get assistance. Several different free tutoring services on campus are listed in the course syllabus, and the students are encouraged to use them when needed. Also, on the office doors of each of the instructors of the course are listed open office hours, during which times students from any sections of College Algebra can come in for help with their work. Each instructor is required to have one open office hour a week besides his two regular office hours. Although the open office hour is available to all students, it was made clear to the students in the course syllabus that each instructor is to give priority to his or her students in the cases when more than one student comes at the same time.

In fall 1995, all the tests and the final exam were constructed by the instructors who taught the course. For each test, each instructor was assigned to certain sections of the chapter by the course coordinator and asked to create a number of multiple choice questions and some open-ended test questions on the material in those sections. Then, the course coordinator collected all of the questions and selected ones to appear on a test draft. All instructors were asked to review this draft before it was finalized.

Students who had the same teacher took the test together in a designated room. All teachers handed out the test at the same time and collected it after ninety minutes. According to the course coordinator, each regular test was a one hour test; however, students were allowed up to ninety minutes to complete it.

For the multiple choice questions on the test, students were to record their answers on both the test and the Scantron sheet. Instructions on filling out the Scantron sheet were provided by the instructor administering the test. In addition, the instructor reminded the students to show their work on the portion of the test containing open-ended questions.

Students were allowed to use scratch paper and calculators on the test, however, all scratch paper was to be handed in with the test. Students were encouraged to write their work right on the test so as to have a record of their work for future test review purposes. The tests were laid out so that there was room for that purpose. Scratch papers were kept by the course coordinator for recording purposes. She reported using them on occasion to settle disagreements or questions that students had concerning their test grades.

After a test was given, the course coordinator collected the worked exam papers from all the instructors. She would send all the Scantron forms to the computing services on campus for evaluation the next morning and collect them in the afternoon. In the meantime, she graded the open-ended questions on the test for her own class to get an idea of how to set up the grading guideline for that portion of the test. Then, she wrote the guideline and sent it to every instructor of the course to use in grading the last page of their students' test.

Students who missed taking the tests were allowed to do make-up exams if their reasons for absence satisfied the criteria listed in the course syllabus. In cases where permission was granted for a make-up exam, students were required to take the test within one week of the original test date.

4.2 Findings of the study

The participants' background information

There are several similarities in the participants' personal background information. All participants are twenty-five years old or older, with at least five years experience in the work force. All are married with dependent children, and have spouses who are either college graduates who work at the moment, or are college students like themselves. According to the participants, their spouses are very supportive of them going to college, and are willing to help in any way they can. This includes helping them with the household chores, taking care of the children, and providing income for the family. Such support seemed to play a crucial role in the participants' decision to continue with their studies.

In making the decision to go back to school, all participants reported having concerns about the effect their studies might have on their families. A primary concern was whether they would have time for both their families and their studies. This was a major issue for participants who had young children, because of the problem of providing child care during the times they were in class. Some of the participants solved the problem by using private child care centers; however, this created its own problem since it added another expense to the family's budget. Others chose to handle this problem by arranging their class and their study schedule around both their children's school schedule and their spouses' schedule, so that the children would always have adult supervision when they were at home.

In addition to time, money was another concern that all but James had in making their decision to come back to school. (In James case, he continued to receive regular pay from the Marines while he is in school to prepare for officers' training.) Before enrolling in college, each participant had worked full-time outside of the home and thus had contributed significantly to their family's income. Therefore, when they went back to school, their family's income decreased sig-

nificantly; which, together with the necessity of paying schooling expenses, often put a greater responsibility on their spouses for bringing in income. To deal with the decrease in their income and to help pay for their schooling expenses, some participants took out loans, sought federal grants, or did odd jobs in the summer. Despite the reduction in their family's income and the time available to spend with their family, none of the participants seemed to regret their decision to come back to school. Instead, wanting to make these sacrifices worthwhile seemed to motivate them to do well in their studies.

For several of the participants, last semester (fall 1995) was their first time in college as full–time students; however, all have been to college before. Barb and Stan attended college immediately after high school graduation, but went mainly to please their parents and as a result, found themselves dropping out of college after their first two years to pursue other interests. After a few years away from school, however they, in addition to the ones who did not choose college immediately after high school, found themselves wanting to obtain a college education.

The main reason the participants had for attending college was to have an opportunity to improve their employment prospects. A college degree would enable some of the participants to secure a promotion in their line of work, while it would help to introduce others to new job possibilities. In addition to career advancement, personal satisfaction was also cited by some of the participants as their reason for wanting a college education, either because they enjoy learning in general or because they feel it can help to enhance their self-esteem.

Not only did the participants have specific reasons for coming back to school, they also had clear goals when it came to their studies. Two often-cited goals were to earn a high grade-point average and to gain a good understanding of the material in their classes. The participants all expressed the desire to make all A's in their class work, even though lower grades had once been acceptable to many of them. Likewise, they no longer wanted to do just enough work to pass a course,

but instead demanded of themselves to do their best in all their classes. These high self-expectations stemmed from their belief that they could do A level work and that good grades are important to their chance of gaining good employment or being accepted into graduate school once they finish with their undergraduate studies. They attributed such changes in their self-expectations to their being more mature in their learning and also to being responsible themselves for their schooling expenses.

Along with the desire to do well in their courses, the participants also expressed the wish to understand the reasons behind their work, for they now considered this understanding to be an important component of their learning process. For Stan, this desire also extended to include wanting to know how he can apply what he learns in class to other domains of his life; or at least to other classes. The participants reported demanding of themselves to know and to be comfortable with all the material presented in their courses.

To achieve the goals they set for themselves, the participants planned to attend their classes regularly and to complete the assignments in the courses in a timely manner. They believed they had to be focused and work hard in their study because they are responsible for their educational expenses and because they have to make the best use they can of a limited amount of time in which to finish their studies. Therefore, they followed their teachers' instructions closely in doing their work, and attended closely to their advisors' suggestions in choosing their courses.

The participants' mathematical experiences before College Algebra

For most of the participants, mathematics is a subject they take mainly in order to satisfy their graduation requirements. Of the five participants, Barb, David, and Stan recalled that their experience with mathematics in high school was neither positive nor negative. They reported putting in just enough work

to pass their mathematics classes, and were not bothered by the B's or C's they received for the final course grades. Also, they were more interested in getting the correct answers than they were in understanding the meanings in their work. Moreover, some even reported ignoring any difficulties they had in their study of mathematics in the hope that they would disappear by themselves. Thus as long as they obtained the correct answers, they were content and did not see much need to go beyond that point in their work.

As for the other two participants, James "sailed" through his mathematics classes with little difficulty while Kelli struggled to pass hers. According to James, he never experienced any difficulty with mathematics while growing up. In fact, he was able to do well in his mathematics class during his senior year even though he came to class only on days when there was a test. Although he was not motivated to attend class regularly, James did keep up with the work in the course independently outside of class, by asking his friends for the assignments and doing them himself. Looking back, James regrets not taking his class attendance more seriously because he now believes he needs to attend all his classes if he is to achieve the study goals he sets for himself.

Of the five participants, only Kelli reported having constant struggles with mathematics while she was in high school. She believed her difficulties with mathematics started when she was in the third grade, where she received inadequate instruction in the subject. According to Kelli, her teacher was neither comfortable with the subject nor familiar with the technique for teaching the "new math" curriculum. Thus, her teacher preferred to spend class time doing art work rather than mathematics since she was not able to explain things well to the students. As a result, Kelli felt very insecure about her ability to do well in mathematics, and would avoid taking the subject whenever she could. In fact, she was able to graduate from high school having taken only one algebra class. By contrast, all the other participants had taken at least one mathematics course beyond algebra

by the time they finished high school.

Despite the differences in their high school mathematics background, all participants took remedial mathematics classes when they started taking mathematics in college. All but James took two remedial mathematics courses before College Algebra. Some of these individuals were not required to take two remedial mathematics courses; however, they chose to do so because they wanted to have a better understanding of the basic concepts in algebra, which they felt would be necessary for their success in higher mathematics courses in college.

They felt their algebra skills were "rusty" in comparison to those of the younger students because it had been longer since they had had algebra in high school. Barb, Kelli, and David had been out of high school for more than ten years, during which time they did not need to use any mathematics beyond arithmetic. Thus, they had forgotten most, if not all, of their high school mathematics by the time they returned to college, and had to begin college taking remedial mathematics courses. These participants reported feeling better about being in a remedial class after they discovered that others in the class were at the same level as they were.

> **B.** (In Elementary Algebra at the beginning,) I was intimidated by my lack of knowledge. I did not remember anything. I had College Algebra at OCU but I have forgotten most of it. So I kind of felt a little silly, but then I realized that everyone in the class was at the same level (as me) and that made me feel better.

Unlike Barb and the other participants, James reacted to his having to take remedial mathematics with much frustration. He determined to prove to others, and to himself, that he was beyond that level of mathematics. Thus, he was quite satisfied with the A he earned in the course because he felt he had succeeded in justifying his mathematical ability to others and to himself.

> **J.** I did not like (being in the class) so much. The whole thing seemed to me

remedial ... When I was growing up, I was always in the honor courses from the fourth grade on and so I was expecting to get an A easily. And then just to get to college and find out that my background has not prepared me for college level algebra and it's kind of like a big slap in the face. So it's kind of like (I am) running into a wall ...

X. How did you deal with that frustration?

J. I just went to class. I ended up getting an A out of the course. But just the fact that (I had to take this class) gave me much more incentive to do well. I think I kind of (worked hard to) get an A in that class just to prove that I do not belong here ... .

Thus even though he was not happy about being in the course, James was able to turn his frustration into striving to meet a challenge in his study.

According to the participants, the focus of their mathematics learning in college is quite different from what it was in high school. Whereas in high school, getting the correct answers was the main priority and everything else was secondary, the focus in college has shifted more to understanding how the correct answer is derived. That is, they have become just as interested, if not more interested, in understanding the reasoning behind their work as in knowing the correct answer. They believe that such an understanding is necessary in their learning of mathematics because without it they will not be able to understand the subsequent concepts in their learning. This belief seems to stem from their view of mathematics as a subject in which one has to master the concepts in a step-by-step manner.

The participants' study habits were also altered when they found themselves back in the mathematics classroom as older learners. They became more serious in their study because they wanted to keep up with the younger students in their mathematics classes. They reported attending class regularly and taking notes of everything written on the board during the lecture. In addition, they reported

keeping up with the assigned homework and often doing extra problems in order to feel comfortable with the material. All read the textbook, and most did the margin exercises for each of the sections covered in class. (Margin exercises are practice problems that are presented in the margin of the book for the students to do after they have finished reading the section. The students were encouraged by their instructors to do these problems and to check their answers against those in the book before they did their homework.)

In doing the homework, the participants checked their answers against the answers given in the back of the book. If these matched, they continued. If not, they would go back and recheck their work for any mistake. In checking their work, the participants usually looked to see how similar problems were solved in the book, and compared the book's solutions to their own. If, after doing this, they still were not able to settle the discrepancy between their answer and that in the book, they would then seek assistance from either a solution manual, their instructors, or family or friends. The ones who used the solution manual in doing their homework reported using it only after they had tried to do the problems first themselves. If they were not able to do the problems, they then would look to see how similar problems were done in the solution manual before they redid their own problems using similar procedures. If the second attempt failed to produce the answer they wanted, they would compare their work to the work in the solution manual and try to understand why their answer did not match the one in the book. According to the participants, in the mathematics classes they took before College Algebra, they were almost always able to get their questions on one homework set answered before it was time to move on to the next.

Overall, the participants reported that their experiences in their remedial mathematics courses were good. In a few cases, they reported having some difficulties because the instructors did not explain things in detail. According to the participants, their instructors assumed the students already knew all the material

in the course, and thus did not feel the students needed much explanation of the material presented. As a result, these instructors did little in the classroom other than copy material from the textbook onto the blackboard for the students to take as notes. For some of these instructors, however, the problem extended to include responding negatively to the students' questions in the classroom. Comments such as "You ought to already know this" or "You still do not get this? Well, I do not have time to go over it with you" discouraged many of the participants from asking questions.

There were a variety of things the participants did in order to cope with the difficulties they had in their remedial mathematics courses. James reported getting together with his classmates and demanding that their teacher slow down in his lecturing and explain the material more. Often, James himself would stop his instructor during the lecture to ask for further clarification of the material. As for the other participants, they chose to respond to the problems with their instructors by taking it upon themselves to learn the material. They did this by doing extra homework problems and by doing the assigned homework problems over and over until they could do them quickly and accurately. In doing this task, they sought assistance from their family, their friends, and especially from the solution manual. Barb, Kelli, and David used the solution manual as their major resource in their Intermediate Algebra class because it was a convenient and a detailed reference.

Although all participants except James were aware of the free tutoring service offered on campus, only Kelli reported using it while taking remedial mathematics courses. Some reported not needing it, while others did not use it because their schedule conflicted with the hours the center was open. Also, many hesitated to use the service because they were informed by their friends who had used the service that the tutors were not good at explaining the material and often confused the students more than helped them. Kelli who did use the center reported quitting

it after using it a few times, because she had a different tutor each time she went, and was being confused by receiving so many different explanations.

Barb, Stan, and David did not have major problems completing their remedial mathematics courses. Kelli and David had to retake their Intermediate Algebra class after their first try with the course, because they were not able to understand the material covered, but they succeeded the second time around and were quite proud of themselves. Like the other participants, they attributed their success to their own hard work and to the type of instruction they received. Of these two variables however, the latter was emphasized more by the participants when they discussed either their success or their enjoyment in a mathematics course. In fact, for some of the participants, this variable influenced not only the attitude they had towards the course, but also towards mathematics in general.

The participants' experience in College Algebra

According to all the participants, College Algebra was the first college mathematics course they had to take for the mathematics requirement for their majors. After this course, all except Stan still have to take Business Calculus I, Business Calculus II, and Business Statistics; and Stan has to take Precalculus, Calculus I, and Calculus II to meet particular program requirement.

I. Coming into the course

A. The participants' concerns coming into the course

Although the participants did well in their remedial mathematics courses, all participants, except James, were a little apprehensive when they entered College Algebra. They worried that, having been introduced to the material while they were in high school, the younger students might have an advantage over them in learning the course material.

D. ... I am amazed at some of the stuff that they have taught them coming out of high school (now) versus with what we were taught when I was in high school. There was just so much more. We were taught just the basics, x minus y and stuff like that. If I remember correctly, we did very little graphing while I was in high school. These kids nowadays, really, they are way ahead of me. I am still struggling to get caught up ... My biggest concern now is that I don't know what is coming around the corner. I haven't really had problems with this (mathematics) class yet, but my biggest concern is whether I will know the next section. At least these kids have seen it all before; I haven't, and that is my biggest concern.

Also, some of the participants were afraid of appearing foolish in front of their younger classmates when they could not understand the presented material.

K. (In College Algebra, I had) the same fear (that I had in my previous mathematics class) that I wouldn't understand, and that with the younger crowd, (I would be) afraid to ask questions because they would look at me and say "Oh, that old woman, what is she doing here anyway?"

B. The participants' goals coming into the course

Although the participants were concerned about keeping up with the younger students, they were confident that they would do well in their College Algebra class. Moreover, they did not anticipate any major difficulties with the course material, since they had been able to do well in their previous mathematics classes. Barb, Kelli, James, and Stan started the course with the goal of making an A in it, while David strived for a B. In addition, they all expressed the desire to understand and to feel comfortable with the course material by the time they finished with the class. When asked what were their study goals in the course, some of the participants responded as follows:

B. A good grade. I expect to get an A out of this course. I expect to

understand every concept that I am supposed to. That is what I want to get out of this course.

K. The knowledge so that I can go onto the next course ... I need to know this. I don't want to fly by the seat of my pants. I want to be comfortable with this, where I know it so that I can go on to the next class ... My goal in this class is to get an A.

J. (I) pretty much (want to have) more of the basic fundamental (understanding of the material) because, you know, if she had her final exam today, I could do everything on that final exam probably. But I think I would have better understanding of each individual step by going to the course ... I am shooting for straight A's this semester.

In order to achieve these goals, the participants planned to work hard in their study. They planned to attend class, to do the assigned work, and to prepare for the exams in the course.

B. (I planned to) meet the teacher's requirements. For instance, today, she said to read the book before class. Well, in the other class, we did not do that, you know, so I will just do whatever I need to do, as much as possible, to learn the material and to pass the test.

D. ... I do what my teacher says. I plan to follow through and keep my nose to the grindstone. One thing I learned about this teacher is that if you show up for class, you pay attention in class, and you do what she says in class then you are going to learn and be alright ...

J. (I plan) just to keep up with the homework. I do homework everyday. Usually, what I have been doing is I do homework a few hours at night, whatever I can squeeze in. I will be getting up at four in the morning and doing my homework, because that way I would be the only one awake in the house and I would not get bothered by the baby and my wife would not bug me about something. So I can study pretty much uninterrupted.

Not only did James stress the need to do his homework in the course, he also stressed the need to attend class regularly. In fact, he viewed regular attendance as important for getting the grade he wanted in the course.

J. I think I need to attend every class because my long-term goal is not only to get an A in the course, but to get the top A in the course ...

Although the other participants did not share James' desire to make the top A in the course, they did, however, share with him the recognition that regular attendance was an important component of their studies.

C. The participants' beliefs coming into the course

Along with these personal goals, the participants also had various beliefs coming into the course concerning mathematics as a subject and the learning of mathematics, the structure of their College Algebra class, the instructor's responsibilities in a mathematics course, and the students' responsibilities in the learning of mathematics.

1. Beliefs about mathematics

To the participants, mathematics seemed to be a subject that mainly concerns numbers and formulas, and in which one learns to correctly manipulate abstract symbols using appropriate rules. In his explanation of his wanting to teach a leadership class instead of a mathematics course, James said:

J. ... I do not necessarily like to get in there (the classroom) and show people how to do formulas and how to do this (mathematics problem). I would rather teach them how to interact with people and how to inspire and lead people.

The above response not only gives us some hints as to how James viewed mathematics as a subject, but also some clues as to what he expected from his mathematics teacher. These expectations of the instructor will be discussed later in the section. In addition to viewing mathematics as an abstract subject which mainly involves manipulation of symbols, the participants viewed it as a subject

with a very rigid structure.

D. ... in English, you can write almost anything you want to say in a paper as long as you have your nouns, your adjectives, your subjects and your verbs all in the right place. But in mathematics, it's got to be that way (in the exact order). It is kind of like "please excuse my dear aunt Sally": you have got to do parentheses (first), (then) exponents, (then) divides, and all of that stuff and then it works. But you wouldn't get away with it if you don't go in that order.

Indeed, it was exactly because of this type of rigid structure that some participants found mathematics to be quite appealing as a subject. For example, Barb enjoyed doing her mathematics homework because she felt it was a very structured type of work in comparison to the other work she had to do.

B. ... as an adult, I find that I like working with numbers.

X. Why do you like working with numbers?

B. I like the organization of it. This may sound very funny, but my life is very busy and chaotic, and last year I discovered that I love to go to the library and sit and do my algebra because it is very therapeutic, you know, it is a very strange thing, and I realized that is why I like accounting because my life is very busy but this, this is very organized and it follows rules and it is very cut and dried ...

To some participants, however, the objectivity characteristic of mathematics is what attracted them to the subject. They liked the idea that a result in mathematics is not easily influenced by environmental factors. As James mentioned once, if something is accepted as the answer to a given problem, then it is accepted as the answer to that particular problem regardless of where the problem was solved or who solved it.

In addition to the belief that mathematical results are universally applied, the participants also believed mathematics to be a subject in which the answers

had to be exact. That is, there is little room for anything but precision in doing mathematics.

D. ... (in mathematics) there is no guess, maybe, or "I think so" answer. It is either yes or no. It is either .7 or .8, there is no in between. You might get some approximate values from the calculator but there is a correct answer on there so it is either the correct or it is not the correct answer...

Not only did this response reflect the expectation of exactness in the solution, it also seemed to suggest that when working with problems in mathematics, one can expect the solution to exist. Indeed, one of the participants' goals in doing the homework was to find the correct answers. While they did expect to find the solutions in their work, the participants did not expect each problem in mathematics to have only one answer. This flexibility to the number of solutions (or answers) a problem can have might be due to the participants' experience of working with other polynomial equations besides linear equations. Although the participants acknowledged the possibility that some problems in mathematics might have no solution, they did not expect to encounter many of these in their learning, because they had encountered few in the past.

To some of the participants, the idea of exactness in mathematics also extended to the notion that every answer in mathematics is provable.

J. Mathematics is very exact in that it is provable. There is a variety of ways to do it and you can always check your answer to make sure you got the right one. There is always a way for you to check (the answer) ...

In this response, James was focused mostly on the checking of solutions in mathematics if such solutions were to exist. To him, if a problem has an answer, then there are a number of ways to check the validity of it. Thus, he seemed to recognize that there is some flexibility in mathematics when it comes to deciding the validity of an answer. The other participants also seemed to share James' view; they reported having several ways to check their work once they were finished with

a problem. Sometimes, they checked their solution by substituting it back into the equations in the problem to see if it worked; other times, they checked their answer by going over each of the steps in their written work looking for any computational or logical errors. Most of the time, however, this checking task mainly involves verifying their answers with the answers in the solution manual or the back of their textbook.

2. Beliefs about the learning of mathematics

The participants believed that one has to learn mathematics in a step-by-step manner. That is, a person must master one concept before she can go on to the next higher one. Any questions or difficulties with a concept must be resolved before the learning of new material can be introduced; otherwise, full comprehension of the new material would be impeded.

> **S.** If you do not have your question answered, it will always be nagging at you. It will usually keep you from something down the road. Math is a step-by-step process: if you miss one step, then you usually aren't able to come up with the right answer somewhere down the road.

> **D.** ... in math, if you can't learn A, you don't learn B, and so it is no use to go onto B because you have to master A before you go to B ...

This view of mathematics learning as a step-by-step process seemed to embrace the idea that understanding of mathematical concepts develops from the simpler to the more complicated ones. Moreover, any interruption in this sequence, especially at the beginning, can negatively affect a learner's understanding of the material.

> **K.** Well, if you don't (understand a concept), if you don't get it, (then) you cannot really (go on). It (the lack of understanding on a concept) is always going to haunt you if you don't (resolve it) ... I think in math, you have got to have a handle on (the material) at the very beginning, the very, very beginning.

In describing what understanding a concept meant in the learning of mathematics, all participants focused on being able to do problems. They believed that they understood a concept in mathematics if they could recognize the problems that dealt with the concept, and know what they had to do with such problems to arrive at the answers.

J. Understanding a concept, I think, is where I don't need any notes and just by seeing a problem, I can do it ...

S. (To understand a concept in mathematics means to) be able to do any problem that they throw at you. No matter how they are asked or how they are worded.

B. Understanding a concept (in mathematics) means being able to look at it and recognize it, and understand what I am supposed to do in the problem. When I had to sit down and spend a lot of time looking up formulas, looking up to see how do you do this and going over the section again, going over and over, that wastes — well, it doesn't waste time because that is the only way I learn it. But it takes a lot of time.

Although the participants were not much concerned about the format of the problems in their definitions of understanding, they were very critical of this aspect when it came to evaluating their own understanding of the course material on the exam. This discrepancy between their conceptions of understanding and in their behavior in learning mathematics will be discussed in a later section. The above responses seemed to suggest that tasks like searching for information from the textbook, testing different conjectures, and creating different strategies in doing mathematical problems had little role, if any, in the participants' understanding of mathematical concepts. Rather, they were the means the participants used to reach understanding and therefore, were not considered as being a part of the understanding itself.

Because they believed understanding in mathematics meant being able to

recognize problems and to know how to do such problems, the participants felt they had to put in a lot of hard work practicing and memorizing the course material in order to learn the subject. Repetition was often stressed by the participants in describing their approach to study for their mathematics class, especially when it came to preparing for an exam.

> **B.** Learning mathematics involves repetition of redoing problems, redoing, redoing, and redoing. One of the things I learned is to take the simpler problems and I would make a list of those and then I would solve them. The next day, I am going to do it again, and the next day I do it again. By the time I have done that two or three times, I can whip through (those problems) like it was nothing, and I understand it and then move on to the next concept.

Indeed, the participants seemed to pay a great deal of attention to doing problems, viewing it as the main activity, if not the only one, in the learning of mathematics. That is, they seemed to believe learning mathematics involves learning to master the different types of procedures and rules to solve problems. Moreover, memorization of such procedures (or algorithms) and rules were necessary once one had learned to work the problems. Thus, understanding of the concepts was nothing more than a memory exercise in which one learns to match the appropriate procedures and rules to the given problems before carrying out all the necessary steps of the procedure to arrive at the correct answers.

3. Beliefs about the course's structure

Coming into the course, all participants expected their College Algebra class to have the same structure as their previous mathematics courses. That is, the teacher lectures and assigns homework, while the students listen to the lecture and then go home and work on the assignment. Students' understanding of the course material will be evaluated through the use of regular tests and the final exam. The participants expected to do well on these exams if they could do the problems on the assignments and on the practice exams. In fact, they expected to succeed in

the course if they studied for it as they had for their previous mathematics classes. Unfortunately, all participants except James experienced some frustration in the course as the semester progressed. We will discuss this further in a later section.

4. Beliefs about the instructor's responsibilities in the course

There were several things the participants expected of the instructor when they started the course. For one, they expected this person to be the liaison between the book and the students. That is, they wanted the instructor to help them make sense of the material in the textbook. Also, they wanted this person to have the patience to handle the students' repeated questions, and to be aware of the students' level of familiarity with the course material.

S. The teacher supposed to be a liaison between the book and your learning ability.

J. (The responsibility of the teacher is) to facilitate learning. Not only to present the material, but to present it in a way that the students can understand it. You can throw the formulas up there, but if they don't understand it, you haven't done anything. So you need to not only teach it, but you need to understand their level of comprehension and to make sure that they understand it.

B. (I am looking for) a teacher that is patient and that can explain things. Basically, that is all: one that would explain and have the patience for the repeated questions. (Also,) one that is knowledgeable about the field and one that understands this very elementary level that I am in right now, who doesn't talk above me ...

According to the participants, their instructor is very important to them in their learning of mathematics. In fact, many of them believed their success in a mathematics course depended heavily on who they have as the instructor.

S. (The teacher's role) is really important. The teacher makes the math class. If the teacher is ill-prepared, like in Math 0123, the teacher was a very nice

lady but she read from the book. I ended up failing the first two tests, and passed the last one because I took upon myself to go and talk to other people and to study myself.

K. ... When I took Elementary Algebra, I had a teacher who taught the class like a charm. I am the type of person that I have to have everything spelled out for me and that was the way how he did it. When I took Intermediate Algebra, I had somebody that I won't name, and all he did was to come in and just write exercises on the board and that was it. And I flunked it (the class) ...

The above responses suggested that the participants view their mathematics instructor as someone who knows the course material and has the responsibility to present that information to the students in a clear and detailed manner. He or she plays a crucial role in the participants' learning of mathematics because without him or her, they would have difficulties making sense of the course material.

5. Beliefs about the students' responsibilities in the course

Although the participants expected their instructor to play a crucial role in their learning of mathematics, this expectation did not extend much beyond the task of explaining the course material. They did not expect their instructor to do the learning for them, because they believed that task was their responsibility. In fact, they believed the students had several responsibilities in the learning process. For one, they believed it is the students' responsibility to buy the required material for the course and to have it with them when they go to class. Also, they believed the students should attend class regularly, bringing writing material with which to take notes, and should pay attention and actively participate while they are in class. Outside of class, the students' responsibilities include doing the assigned work, seeking assistance from others if they have questions, and preparing for the exams.

J. I think (the student's role in the learning) is to pay attention, to do the

homework, and to participate.

B. ... I think the role of the student is to attend class, to put extra (studying) time in for a test, to complete every assignment, to be there (in the class) and just to have a positive outlook, you know, and trying to understand (the course material).

According to the participants, the students should actively participate in the classroom because education is a two-way process, in which the students participate as much as the instructor does. Thus, they should respond to their instructor's questions during the lecture and ask their instructor any questions they have about the presented material.

J. (You should participate because) education is a two-way thing. It is not "The teacher gives it to you and that's it, and that is where it ends." You are paying, or your parents are paying, or somebody is paying for you to go to school to get this education. (Education is just) like any message, if I send someone a message and they don't get it, then we haven't had communication. So in order for information to be passed to the students, the students have to be actively involved in receiving it ...

B. (You should participate in class) because you shouldn't be a bump on a log, I mean, you should go (to class) prepared, maybe having read the chapter before you go in, because you are not being fair to your teacher when you are coming into (the class) like a bunch of idiots who don't care to take the time to do the work ...

II. During the semester

A. The participants' study habits

Throughout the semester, all the participants except David chose to sit in the front row seats. They believed such seating positions would help them to be more focused during the lecture. David, however, could not sit in the front because he

needed room for his learning material. Usually, he had his books, calculators and writing materials spread out around him so he could use them during the lecture. The other participants also had many of these materials with them, but did not have them on their desks until they needed them.

Of the five participants, three (Barb, Kelli, and James) attended class regularly during the semester, each having one or no absences the entire semester. Stan missed about six classes, mostly toward the end of the semester, due to illness in his family; and David missed almost the entire second half of the semester because he had to return to work. According to the participants who missed class, they learned the material from the classes they missed mostly by themselves, using the book and the study guide as their resources.

All the participants, except for Barb, usually arrived to their College Algebra class at least five minutes early. Barb usually arrived to class just as it started because she had a class right before College Algebra in a different building on campus. Upon arriving to the classroom, the participants first laid out their writing material to get themselves ready for the lecture; then some would read their textbook or the school newspaper while others looked over their homework. Once the lecture started, however, all focused their attention upon the instructor and the board.

During the lecture, the participants busied themselves with note-taking and listening to their instructor. All participants, except James, took all the given notes in the lecture. James chose to focus his attention more on listening to the lecture and only took notes on material that was new to him, or as he thought necessary.

> J. In the first part of the semester, I wrote down pretty much everything she said. However after that, she was pretty much talking about stuff that I am familiar with, so I thought it would be more important for me to understand what she is trying to show me instead of trying to write down everything;

because if I was to try to write everything, I might not be comprehending what she is talking about. I think it is better for me to write down the notes that I needed to remember. Other than that, I basically want to see how she attacks a problem and to understand her (working) process rather than just focusing all my attention on note-taking.

Unlike James, Kelli and David not only took all the notes given in class but also rewrote them once they got home. David also took some notes from his reading of the material in the textbook before he came to class.

D. I took notes in her class and then I would go home and reorganize them.

X. Do you take all the notes in the lecture?

D. Yeah. A lot of times, in her lecture it would be the third time that I take notes.

All the participants reported that the only time they used their class notes was in doing their homework. Mainly they used their notes to see how to solve problems similar to the ones they were working on in their homework. Even then, they only used their notes after they had checked their textbook for examples that were similar to the homework problems they were having difficulties with.

Besides taking notes and listening to the lecture in class, the participants were also attentive to the questions that were posed by their classmates and to the answers given by their teacher. They also asked questions, and answered some of the ones posed by their teacher when they felt they knew the answer; however, such occasions happened only a few times during the semester. One exception to this pattern was James who, unlike the other participants, tended to respond and ask questions of his teacher in class on a regular basis.

According to James, he had learned most of the material in College Algebra in his Precalculus class at the Marine preparatory school he attended in the summer before he started at the University. Thus, he was familiar with almost all the material that was covered in College Algebra and was quite comfortable in

responding to his teacher's questions; more than once, he was the only one in class who did. James also asked questions in class, not hesitating to raise his hand or ask a question whenever he was puzzled by the presented material. Sometimes his questions were about how to do a certain problem; other times they were about the reasoning behind the teacher's work on the board. More than once, he pointed out mistakes his teacher had made on the board, and sometimes even suggested a different way to explain the material so as to make it easier for the other students to understand.

Outside the classroom, the participants spent most of their studying time in mathematics by doing the homework. Of the five, Kelli spent the most hours on this task - about sixteen hours a week - while James spent the least - at five hours a week. Stan, Barb, and David reported spending six, ten, and twelve hour per week, respectively, on the course outside the classroom. On weeks when there was an exam, the participants added from one to six more hours to their study time.

In doing their homework, the participants reported they mainly did the problems in their assignment, checking their answer against the one in the back of the textbook after finishing each of the problems. They continued working in this pattern until they came to a homework problem that they did not know how to do. In such a situation, they would look through their textbook or their class notes to see if they could find similar examples to use as a reference.

> J. Usually what I do is I just go to a problem and then I try to solve it. If I get to a snag, or I am not solving it quickly or easily, then I flip through the chapter of the book and see where they explain it. So I would try to do a problem first and I would go back. I do the problem and then I check it against the back of the book. Now if I am right, then I don't worry about it. If I am wrong, then I'll go back to the chapter to see where it explains it.

If their answer did not match with the answer in the back of the textbook, all participants reported going back to their textbook to see how they were to solve

such a type of problem. In addition, they checked their work for computational errors. If they were not able to find anything wrong with their work, Kelli and Barb would immediately assume their answers were incorrect and would then compare their work with the solution manual to see where they went wrong in the problem. The other participants, however, would not immediately assume their work was incorrect at this point but would check with their instructor in the next class meeting on the validity of their answers.

J. (If my answer were different from the textbook's answer), then I will check on how to do (the problem) in the actual chapter where they show you how to do it. And if I am following all the same steps that they are, and I check my work and it matches up with what they are doing, and if I know that my figures are right, then I am just going to leave my answer because the book must be wrong.

S. (If my answer were different from the textbook's answer), then I would go back through (my work) and check all the signs, and the way that I did the problem. It usually is those little things that throw it off.

X. What would happen if you did all of that, and your answer was still different from the answer in the textbook?

S. Then I would go and talk to my teacher.

In doing their homework, the participants reported not putting a limit on the amount of time they would spend working on a problem. Usually they would stay with a problem until they were no longer making progress on it. Most of them, if not able to solve the problem the first time, would work it a few times before quitting.

J. I usually don't put a time limit on doing a problem. I would do a problem until I could solve it. On this one (referring to one of the homework problems he did not complete), I didn't have any basis to work on because I had never done that before. But if I come to some problem like this (pointing to one of

the homework problems he had done), then I know that I've got a foundation, and it is just a matter of time to do it.

D. How long would I stay with a problem? I just go until I can no longer proceed. I have been known to erase the paper that has the problems on it: ask my teacher! I use the recycled kind of paper and I just wear it out with erasing.

X. Do you ever put a limit on the time to do a problem in doing your homework?

B. No, now with my class schedule I might have an hour to do it and I would do it as best as I could. But (if) I don't understand something, I don't go further. If I have 10 problems to do in an hour or so, and I only get 5 done; well, I have to find another hour in the week to do those additional problems. So I would take as much time as I need and (do) as much as I can.

X. When you are stuck on a problem in doing your homework, you say that you just stop and go no further?

B. Well, no. I mean if I don't understand that (problem), I would set it aside and go on to (the next one). No, I don't (stop) unless every problem for the next five problems is the same. Then, I probably don't get it, then I would have to stop. But no, I would do what I can and then I will ask somebody.

Kelli reported she was frustrated at the beginning of the semester whenever she encountered a homework problem that she could not do.

K. (At the beginning of the semester) I just drove myself crazy (if I could not do a problem), I wouldn't go on until I got that one problem solved because I felt like if I don't do that one, I couldn't go on and do the next one ...

As the semester progressed, however, Kelli abandoned the idea of staying with a homework problem until she solved it before moving on to the next one. This was because she needed a lot of time to get the homework completed since she was having problems understanding the material in the lectures. In addition, she

was under pressure to turn in a complete set of homework each week because she believed she needed a good grade on the homework to pass the course. Barb was also in a similar situation as the semester progressed because she, too, was not able to do well on her exams.

The participants always showed their work when they did a homework problem. They believed showing their work was necessary because they wanted to be able to see how they had arrived at the answer, and wanted to have the work written down in detail for use in studying for the exam.

D. ... I usually don't have any difficulty (in doing the homework) because I go step-by-step in working my problems. When I started skipping steps, then I get into trouble.

X. I see; so that is why you always show all of the steps in doing a problem?

D. Yes, maybe it will take an additional 5 minutes, but it makes a whole lot of difference when you are in the 13th week of the semester and you're trying to figure out how you did a problem in the second week.

In writing up the assignment, the participants paid close attention to the format that their instructor requested of the students. All the problems were done in the order they were assigned, with each one clearly numbered and separated from the rest, and neatly written out in pencil on only one side of the paper. The work for each problem was presented in a step-by-step manner, with the answer either boxed or highlighted to differentiate it from the rest of the work. Problems the participants had difficulties with were usually marked with question marks until the questions were answered. The participants also usually made a note of these problems on either a separate piece of paper or on the front page of their homework, to remind themselves to seek assistance.

For some of the participants, it was not unusual to turn in more than ten pages of work each week. David, for example, often was teased by his teacher because he used a lot of paper in writing up his homework. He often put only one

or two problems on one page of his homework paper although there were usually sixty to seventy problems in one homework set. He did not seem to mind others teasing him about the large quantity of paper he used, because he wanted his work to be readable to both his teacher and himself. The other participants did not use as much paper as David did, but they did use enough so that their writing would be legible.

According to the participants, they always had most, if not all, of their assignment done by the time it was due. Most of them reported their focus in doing the homework was to get it done and to understand the material. James, David, and Stan reported feeling good after their completion of the homework because they felt they had understood the material. Barb and Kelli, however, felt otherwise. They were frustrated at their lack of understanding of the course material, and this frustration increased as the semester progressed. These two participants' difficulties with the homework in the course will be discussed in a later section.

In addition to doing the homework, some participants also read the textbook outside of the classroom. Also, some of them even underlined and took notes on the reading material to get a feel for the material that would be covered in the next class meeting. All of the participants were unable to understand the reading material most of the time; especially when it came to definitions and theorems. However, most still read the book because they wanted to have some ideas of where the important material was located in the chapters, and also because it was required by their teacher.

> **D.** I understand (the reading material) just about half of the time. I seldom recognize it. I read it but it was just like reading the back of a milk carton because it gives you something to read. If it interests you, then you go back and reread it. (I read it) just so I know where everything is in the chapter.
>
> **K.** I read the book, but I could not understand it. And the theorems, they don't make sense, that if a is less than b and b is greater than zero, then a is

equal b and b equals c, and I am like "What is this?". I mean if they would do it in numbers, then maybe (I might understand it).

In reading the textbook, the participants mainly focused on finding out how to do certain homework problems. That is, they read the textbook to find examples that were similar to their homework problem.

B. My focus of reading the book? It's hard to understand the material in the book. My focus is to get the homework problems done ...

J. The book is just (for getting the problems for) the homework. Although when I come to a problem that I don't quite understand, that is when I would flip through the book and read where it shows the examples to do those.

The participants worked hard to prepare themselves for the exams. Since 5/6 of the course grade depended on the exam scores, the participants were under a lot of pressure to do well on the tests. They saw test preparation as an on-going process in which attending class and doing the homework were the key components.

K. ... The way I look at it is, test preparation is going to class, doing your homework, and taking good notes so that whenever it is test time, you are not freaking out because you are not prepared, because you don't know what the material is that you covered. So test time starts at the very beginning – like right now, I am preparing for the second test ...

According to the participants, the focus of their test preparation was on doing problems. For the first two tests in the course, they mainly focused on being able to do all the problems on the practice exam in their study guide (the practice exams were the tests from the previous semester of College Algebra.) In working the practice exam, the participants used their old homework and the textbook to help them with the problems that they did not remember or did not know how to do. Some of them also made use of private tutoring for their test preparation.

K. ... What I did with this test (test one) was I tried to work (the practice exam), and (for a problem) that I could not get, I went back to the book to

find out what section it came from. I looked to see how it was worked and then I went back to the practice test and if I could not work it, then I would go back to the exercise problems (in the homework) and look in the solution manual to see how it was worked and then, I would try to rework it and then I checked it against the key that I got. I got the key for that test (from the test file) ...

S. (In studying for test one), I did the practice exam and then put it away. I made sure that I could do all the problems. I did the practice exam and that was it, just for the practice. I also memorized the formulas and any way to use them. That's all I did.

Some of the participants also prepared for the first two tests in the course by reading from the textbook before they worked the practice exam. The purpose of this task was to get a quick review of the topics that would be covered on the test and also to find out which material they should focus on in their test preparation.

B. (For test one) I started a week (before the) test and I went over each chapter. I reread it and I highlighted the notes (in) the areas that I was weak in. I took notes on this material and I made flash cards for (them and) a few formulas. Then I did the practice exam.

Unlike the other participants, however, James and David also did problems that were not on the practice exam in preparing for the first two tests. Before the first test, James did the chapter tests in the book and looked over the old homework problems, especially the hard ones, to make sure he knew how to do all of them. For the second test, however, he did neither of these tasks because he was confident that he would do well without them. This confidence was due partly to his being the only person in class who responded to most of his teacher's questions during the review session and partly to his being able to do the practice exam in about twenty minutes. As for David, he reported keeping track of the homework problems that he had difficulties with and redoing them as part of his

test preparation for both exams one and two.

According to the participants, they did not do the practice exam in their study guide for the third test, on the advice of their teacher, who said the practice exam contained material that had not been covered in class. All participants except David reported studying for their third test by doing the chapter tests in the book. James and Stan also reported going back and looking over the old homework to make sure they could do all of the problems. David did not take the third test because he was busy with his job at the time.

When asked what criteria they used to determine when they had prepared enough for the test, some participants focused on how well or how much they could do on the practice exam.

S. If I can recognize at least three quarters of the problems right off for what they are asking for, then I know I have prepared correctly.

Some participants, however, were not able to determine when they had prepared enough for the test. They just studied until either they ran out of time or until they started to have doubts about their knowledge of the material.

X. In this course, how do you know when you have studied enough for a test?

B. In this course, I did not feel that. In other courses, I do feel that. That is why I would like to start early ... if I get in a panic like I did in this test, I wouldn't have that extra time. For this test (test two), I did not feel ready for it.

K. I stop study when I get to a point where I am sick of it or when I start to second guess myself. And when I do that I know that I had enough and if I don't quit, I am gonna do worse ...

All participants except James reported starting their preparation for the final exam about a week in advance. According to the participants, their preparation mainly consisted of redoing the three old exams and working the chapter tests in the book. The number of hours spent in doing this preparation, as reported by

the participants, ranged from ten hours to over twenty hours with James at the low end of the spectrum and Kelli at the high end. This was about double the amount of hours they spent studying for the course (outside of class) during the weeks when there was an exam.

Barb, Kelli, and James had to do a large set of homework just before the final exam because their instructor assigned it to help all the students in the class in their preparation for the final. In this assignment, they were to finish the homework on the last three sections of chapter six. In addition, they were to rework all the old exams in the following manner: for each of the problems they missed on the test, they were to copy down the problem, identify the mistakes, explain why those mistakes were made, and then redo the problem. For James, this task was not difficult since he did not miss many problems on his old tests, and on the ones he missed, he knew exactly what he did wrong and how to fix it. Barb and Kelli, however, had made quite a number of mistakes on all their tests, especially the third one. On most of the problems they missed, they not only did not know how to correct their mistakes but did not even know why they had been wrong. Thus, the task was quite time-consuming and difficult for them. In fact during the week before the final, they spent most of their time in the school library doing this assignment, and were quite upset that they had to take time out of their final preparation schedule for it.

Despite their frustration with the last homework set, Kelli and Barb did finish it and handed it in. They believed they were the only people in their class who completed this assignment, because they both turned in about fifty pages of work while the other students' homework papers seemed to consist of only about ten or twelve pages. Moreover, they were surprised to find their teacher actually graded the assignment. Although they both received a perfect score on this work, neither saw much benefit in having done it. Instead, they were furious that it had taken up about half of the study time they had reserved for preparing for the final exam.

K. I am telling you, this is ridiculous because of this "book" (the last set of homework) that we had to do,

X. How many pages, by the way?

K. This ended up being 48 pages ... we had to write down the (problem) that we missed which I think is really juvenile. We had to write down the (problem) we missed. We had to write out what our original answer was. We had to show, I believe, how you got that. You had to tell how you screwed up and then you had to work it the correct way. Well, by then, it was Sunday night. This was due on Monday. This was (done) at Sunday about four o'clock in the afternoon - and I mean, a lot of these – like on this one?

X. Uh huh.

K. I wrote, "I honestly don't remember because I worked the problem on a piece of scratch paper." And on a problem with logarithms, I put "I didn't know my butt from a hole in the ground on this log stuff and I thought that I did." You know, this is so juvenile. So basically all I did was set it down, and I have not reworked this (test 3) even though it seemed like I had.

X. It looks like you had, and if I would have looked at that paper only, I would have thought that you did.

K. And I hadn't, and don't understand it either.

X. So you just filled it in to get the grade?

K. (nodded) You bet. And it is the same thing on this, there are incorrect answers on here (referring to the rest of her 48 pages of work).

X. But she wouldn't be able to find it.

K. That is what I am telling you: this is ridiculous. We spent all this time and I got 100 on it. But there are incorrect answers on here, there are answers that I got out of the students' solution manual, there are answers that just haven't got answered out, there are problems that aren't worked, but I got a 100, but I don't know.

X. So how is that going to help you in your preparation for the final exam?

K. Well, see, it's not going to help me at all. This is trash as far as I am concerned. Absolute trash. That's Thanksgiving, that's Friday, Saturday, Sunday, and all last week.

Like Kelli, Barb reacted strongly to the last set of homework because she saw little benefit in doing it other than to get the grade.

B. ... although we got our points out of that, it was meaningless and worthless to me. If I could have taken that time and studied (for the final instead) ...

X. So a lot of that was just for the homework grade, that was your purpose for doing it?

B. That was the only reason I did it. I plowed through to get the grade for the homework.

In general, the participants' approach to studying for exams stayed consistent throughout the semester. Although there were some changes between one test to the next, most were minor, and the preparation always focused on the same goal: to practice doing problems. In practicing, repetition was heavily emphasized because the participants not only wanted to know how to solve certain types of problems, but also wanted to be able to do these problems quickly. They believed they had to work fast on the test in order to get all the problems finished.

The major change that occurred in the participants' test preparation during the semester was that they began doing the problems in the chapter tests instead of the ones on the practice exam. Even this change came only after the second test was given, and it was not totally the participants' decision to make the change, for their instructors did specifically ask the students in the course not to use the third practice exam.

The participants seemed to have the expectation that the actual test would look similar, if not almost identical, to their practice exams: identical not in the sense of having exactly the same numbers in the problems, but rather in the

sense of having problems phrased in the same way. They often reported feeling confident going into the test if they were comfortable taking the practice exam. After taking the test, however, the participants expressed frustration because it was not what they had expected. They believed the test was "tricky" in that it contained problems they did not remember seeing either in the homework or in class. This frustration with the tests was the major difficulty the participants had in the course, and it weighed heavily on their minds since the test scores made up most of the course grade. More discussion of this difficulty will be presented later.

On the day of the test, all participants reported arriving to the test site at least fifteen minutes early with their pencils and calculators ready. Upon receiving the exam, Stan and James started immediately working the problems, while others read through the test before starting, in order to see what kind of problems were on the test and how much time they would need for each question. After reading through the problems, they would then start to work on the problems they felt comfortable with, leaving the others for later.

For all participants except James, the test usually took them all or almost all of the given time to complete. Usually, they worked slowly so as to avoid errors, and if time allowed, checked their work after finishing each problem by either putting the answers back into the given equation or going over all the steps in their written work.

> J. I do check my answer ... usually what I did is I checked my answer before I moved to the next one. I do all the work. I come up with an answer. I see the answer on one of the four multiple choices and I circle it. But then, I go back and I go through the problem again, following each step and saying "Ok, was this rational?" or "Can I do this?" because I caught myself, I think twice, where I made an error as far as putting in the minus or not putting it in ...
>
> S. I check my answers as I go along and then I go back and go through them

one by one to make sure that if there was a negative or a plus in the problem, I make sure that the problem was done correctly because the signs will get you every time. And then I make sure that all the (answers) on the Scantron (sheet) agreed with what I had circled already.

For Barb and Kelli, checking their work was not often possible because they usually did not have enough time on any of the tests in the course.

B. ... I work very slowly when I take a test, because I slow down for accuracy so I won't make any inadvertent little mistakes. (On test one) I did not feel like there was quite enough time. I didn't finish the test ... I looked and saw that it was like another 30 minutes to go, it kind of made me mad because I think I could have done some checking, may be not much, but I would have like to have checked a few (problems) because I made little notations and I like to go back and check what I wasn't a hundred percent sure. So I thought "Well, I will go back and check", but I didn't have enough time to do any of that.

Like Barb, all participants reported marking the problems that they felt needed additional checking or work as they went through the test, and came back to these problems when they had finished with the other ones, or just guessed at the answers if time ran out before they could come back. Before handing in the test, the participants checked to make sure the answers they had on the test matched with the ones they had on the Scantron sheet, and also checked to see if their names were on any scratch paper they turned in.

After taking an exam, participants would make plans concerning their preparation for the next test, but always at the heart of these plans lay a concern for practice in doing problems: concern as to how many practice problems they should do and how much time they should spend on this task.

B. (For test two, I) will try to be more efficient with each of the sections (in the chapter) and doing the study guide problems, and doing more problems in

the homework section. I mean I am forcing myself to take more time with each section, doing the margin exercises, rereading the material in the chapters, doing more problems from the homework, if that is possible.

X. Do you plan to meet more with the tutor?

B. I probably will. Money will be a problem. I can't afford to be paying a tutor, so I am going to do whatever I can. I am going to start going up to her office. What I am doing now too is in each section (of the chapter), I have a piece of paper and I write down any questions that I have, and I am just going to go and park myself in her office and let her explain them to me.

Like Barb, Stan reported planning to get more practice in doing problems to prepare himself for the second exam shortly after he took the first test.

S. I am going to go to the test file for the next test (test two).

X. Oh, you did not this time?

S. No, I did not go this time because we have the practice test and it was actually the spring semester's test. But then, that test wasn't a fair reflection of the test that I took. So I plan to go to the test file and get as many, as far back as I can, to see how they do it. I will go through all of them. You know how it is, you know there is like a rhythm of how test goes, and if you take a bunch of them, then you will get used to how it goes. It is like a cycle, the questions, or type of questions that they ask ... it will give you a different view point and you will be looking for how they ask the questions.

Unfortunately, all of these plans were not carried out fully due to the limited amount of study time the participants had to devote to their mathematics class and for Barb and Kelli, it was also due to their being uneasy about approaching their instructors for assistance.

Of the participants, only David and James reported reworking the problems they missed on a test immediately after getting it back from their instructor. David did this only for the first test, while James did it for all three of the tests. While

David and James (especially James) were able to figure out what their mistakes were on the test and fix them by themselves, the other participants could not do this. Instead, they reported looking at their test after they got it back only to see which problems they missed and what their scores were. Their reasons for not doing more than this included not having the time, not knowing where the mistakes were, not knowing how to fix the mistakes, and not feeling proud of their test result. However, all participants did report that they reworked all three of the in-class exams as part of their preparation for the final exam.

B. The difficulties encountered by the participants

There were several difficulties the participants experienced during the semester. Some of these were personal problems while most of them were with the course. Almost all of these problems were not resolved by the end of the semester, if anything, some of them became worse as time progressed.

1. Problems concerning the homework

Overall, the participants reported encountering several difficulties in their College Algebra class. Two of the five participants had problems with the course pace and the homework, while all except James had problems with the tests.

Since the class met three times a week, the instructor usually covered about three sections of the book a week. The exception was the first two and a half weeks of the course, during which time, the class covered all of chapter one and the first four sections of chapter two. The reason for this rapid pace was that the mathematics department expected all students to already have mastery of the material in chapter one by the time they entered the course. Thus, except for this period in the semester, a weekly assignment usually consisted of sixty to seventy problems, some of which were made up of several parts.

Class work in College Algebra such as quizzes and homework assignments was counted as only one sixth of the final course grade, and the decision of how to evaluate it was left to each individual instructor. Some instructors chose to use

only homework in their evaluation of the students' class work, some used mainly quizzes, and some employed both of these in their grading. Also, some instructors did their evaluation on a weekly basis while others did theirs less regularly throughout the semester.

All of the five participants except Stan had the same instructor for College Algebra, even though they were enrolled in different sections of the course. These four individuals had to hand in their homework and take a quiz on almost every Friday of the semester. Stan's instructor, on the other hand, did not collect homework or give quizzes on a weekly basis. Stan did not think that there was too much homework to do in the course. The others however, did think that there was a lot, but only Barb and Kelli voiced their concern about the large volume of it.

According to Barb and Kelli, it was quite difficult for them to get their homework completed during the weeks when there was an exam in the course. Usually in those weeks, their attention was more on the test preparation than it was on the assignment. Thus, most of their assignment was not completed until after the test was finished. Since the tests were on Thursday evenings and the homework was due on Friday mornings (the test does not cover this set of homework), they usually had to stay up late on Thursday night and get up early on Friday morning in order to complete the assignment.

One of the reasons why Barb and Kelli were frustrated with the homework had to do with the course's fast pace. They often felt at a loss in the course even though they attended class regularly and put considerable effort into their study.

> **K.** This class goes by a lot faster than (Intermediate Algebra). I covered more material in this class than what I had covered in Intermediate. I had more time (in Intermediate Algebra) to spend trying to figure out and if I couldn't figure it out, then the next class period we redid what we did in the night before ... And it wasn't that we were on a schedule like "No, we can't veer

off, we have to forge ahead" ...

Even though Barb and Kelli were not happy with the fast pace of the course, they still struggled to keep up with the class as best as they could. Although they did not understand much of the material, they did not ask many questions in class, for several reasons. Firstly, they were embarrassed at not being able to understand the material presented in the lecture as quickly as their classmates could. Also, they were aware that their teacher had to follow a set schedule since College Algebra was taught uniformly, and this schedule did not seem to allow their instructor much time to respond to students' questions in class. And lastly, they felt intimidated by their instructor.

Partly this was because of comments she made in class such as "Mathematics is fun," "Mathematics is easy," or "The homework in this section should take you only half an hour to complete." To Barb and Kelli, mathematics was not easy: the homework usually took them a long time to finish; often two or more hours for just one of the three sections due each week. They were also intimidated when their teacher responded to students' questions with comments such as "Oh, you should know that," or "Well, you had a homework problem like that."

> B. If I did have a question about something, I did get an answer either through my son, my husband, or Tai. With Ms. ZZZ, I think the only thing is I feel a little bit embarrassed because I feel a little ignorant ... I mean she is a good teacher, I mean I am not just saying that; because I think the rest of the class seems to understand it so much quicker. They are way up on this chapter and I am three or four sections back still trying to understand it and so it is kind of embarrassing to go to your teacher and say "I know that you are in this chapter and I am way, way, way back here ..."

Despite their discontent with the fast pace of the course and the large volume of homework, Barb and Kelli continued to hand in a complete set of homework each week; even in the final week of class when a big set of homework was assigned.

Moreover, they always wrote up their work clearly and showed their work in a step-by-step manner, even though they were not able to understand all the steps. In doing their homework, sometimes they sought assistance from their private tutor, but most of the time they used the solution manual because it was a convenient and detailed resource. They liked the fact that it showed them how to work the problems in a step-by-step manner and it did not omit too many steps in the solution procedure.

Throughout the semester (fall 1995), the solution manual played an important part in Barb and Kelli's learning of the material. In fact, they even believed that it taught them more than their instructor did about working the different problems in the course. Their dependency on the manual partly stemmed from their not being able to understand much of the class lectures, and partly due to their needing to get a high homework grade in the course. In fact, both Barb and Kelli were relying on the homework grade to help them pass the course, and were determined to turn in all their homework as completely as possible, even if this meant they would have to sacrifice their understanding of the material in the process.

> **K.** (When I had a problem in doing the homework) I just have to keep going on because I never know what is going to happen. It's like I got to have this cranked out. I have to get it done in time to turn it in and I don't feel like I have the luxury to say "Oh, I don't understand problem number 2, let's go back and reread this chapter and see." I mean, I would love to, but I couldn't ...

Although they usually received high or perfect scores on the homework, and invested a large amount of effort in doing it, neither Barb nor Kelli felt they benefited from doing the homework. Rather, they felt they had to rush from one assignment to the next one in order to turn them in for credit. In fact, as the semester progressed they saw little purpose in doing the homework other than just to get the grade. On one occasion, they tried to focus more on understanding

the homework than on just getting it done, but they were not able to keep this focus for more than a week because they were afraid it might jeopardize their chance to pass the course.

> **B.** (My purpose for doing the homework was) to hand it in as complete as I can. (At one point) I thought "Well, don't worry too much about getting it done," but if that can make a difference, then I had to do it. That is the only reason I did it because when we checked the grade and I had a 72 or 73 (percent in the class), I almost died because I had made the 2 D's on those two first tests. So I had a C in there at the time (before the third test), and so I thought, "Well, I guess I better keep trying to keep up with the homework and that might make a difference", and at that point I figured that if I made a C on the third test and a C on the final, I could make a C in the class. Well, that hope has just completely vanished (because I failed the third test).

Like David and Stan, Barb and Kelli did not focus on doing the harder homework problems because they had a limited amount of time to spend on the homework, because they believed these problems would be less likely to appear on the test, and most importantly, because they did not know how to do these problems.

The problem of keeping up with the lectures and the homework persisted for both Barb and Kelli throughout the semester. They tried various things to rectify the situation but were not able to resolve it. They sought assistance from a private tutor but were not able to do this regularly because of the expense. They also bought other mathematics books to supplement their textbook: Kelli for example, reported spending about one hundred dollars on tutoring last semester and about fifty dollars on the supplemental mathematics books.

These extra books dealt more with topics in algebra than with trigonometry. Of all the extra books they bought, Barb and Kelli reported that the ones they used most often were Cliff's Notes for Algebra I and Algebra II. They liked the fact that all the problems in the Cliff's Notes editions were solved in a detailed

manner with all steps provided. Moreover, the explanations in these notes were very concrete, in that everything was explained in terms of numbers instead of abstract symbols.

> **B.** ... Kelli and I have bought one of those Cliff's Notes to kind of help us in the first part of this course. We went through the Algebra I and then when I saw this (Cliff's Notes for Algebra II) in the book store a couple weeks ago, I picked it up and it's like "Oh yeah, this is what we are doing". And so, this has been a godsend. So what I was going to do this afternoon is to finish reading it, because again it has a very simple (way to show) how to do it ... (I like to use it) because it cuts out all the nonsense that you usually see and it just explains things very simply ... It didn't go into as much depth as the text book. It just explains very succinctly, you know: do this, and it gives you an example; do this, and it gives you another example. I think sometimes (the course textbook) explains things and I have trouble following it. It's just like I have to go back and reread it and reread it ...

The Cliff's Notes for Algebra I and Algebra II mainly teach how to mechanically manipulate the symbols in a problem so that the correct solution can be obtained. Explanation of the material is kept at a concrete level as much as possible, with much emphasis placed on working various examples. Thus although Barb and Kelli got a basic idea of how to solve problems from the Cliff's Notes, they would need more than the mechanical explanations found there to do the more complicated homework problems. Therefore, the Cliff's Notes editions only offered Barb and Kelli a temporary solution to their difficulties with the homework in the course.

2. Problems concerning the tests

In addition to the fast pace of the course and the heavy amount of homework assigned, Barb and Kelli also had problems with the exams in the course. In this they were not alone; all five participants had some difficulties in this area. James'

difficulties were the least; he only had problems with the second exam, and even then they were not as severe as the others' problems. According to James, he was overconfident coming into the second test because he was able to answer most of his teacher's questions during the review session and because he did the practice exam in only twenty minutes. After he took the test and found that he got only eighty-five percent on it, James determined to prepare more for his third exam than he had for the second one. He did indeed carry out this study plan and was quite pleased with his performance on the third exam.

The other participants had problems with all the tests in the course. In general, they found them to be "tricky" in that the problems were not phrased in the same way as the ones on their assignments or their practice exam.

> **S.** I don't like the way they asked the test questions. They seemed to want to put in trick questions. There is nothing on the homework that was on the test ... A lot of times, what you see on the test, you won't see on the homework. I have never run across anything on the test that I had hit on the homework ...

Stan was not the only participant who felt this way. In fact, his view of the test was also shared by David, Barb, and Kelli. In order to have a better understanding of what Stan believed to be "tricky" about the test, it might be helpful to hear Kelli's definition of the term:

> **K.** ... the practice exam wasn't a true example of the test. I had difficulty doing the problems (on the test) because they were asked or given differently than the way they were presented by the instructor, the book, and the practice exam. This is what I mean by "tricky".

Kelli's statement gives us some hints as to what the participants expected to see on the test: they had expectations not only about the types of problems but about their format as well. That is, they expected the questions on the test to be phrased exactly as the ones they had in the lecture, in the homework, and more

specifically, the practice exam. The fact that this was not the case was the reason why all participants except James were upset after taking a test in the course. Another reason why the participants did not think the test was fair was that they did not think their test score reflected their understanding of the material. All of them came into the test expecting to do well since they were able to do the practice exam. For the first two tests, all worked the practice exam; Barb and David reported doing their second exams three times before they took the actual test.

D. I did not think my test score reflected my understanding of the material. I probably knew the material better than my test score reflected.

X. And based on what do you have that feeling?

D. Before I walked into that test, you and I went over the practice exam and I knew every problem there was on there ... I did it three times ... The actual test was ten times harder than the practice exam and I don't understand it. The practice exam should be harder than the test because you can spend more time working through the practice exam than the test, so it is backward. That is what I am saying about the tricks and curves.

David's response emphasized the expectation that one should do well on the test if one did the practice exam more than once. Moreover, it suggested the expectation that the test should be easier than the practice exam since it has a time limit. Recall that in doing their homework, the participants tended to focus more on doing the simpler problems because they believed these problems would be more likely to appear on the test than the harder ones.

D. ... (In doing the homework, I usually) skipped some of those problems near the end.

X. But you are skipping the harder one?

D. Are the harder one always on the test? No! What is on the test? The basic ones!

B. (In studying for test two) I felt confident going into the test because I felt I had studied a little bit more specifically and had worked the practice exam three times. To me, the test generally will not use terribly complex problems like we see in our homework. And so I decided to quit worrying about those because I don't have time to get everything. So I try to focus on the chapter to really be sure I can do the simpler stuff.

David also believed the test should not be too difficult because College Algebra is still a lower level mathematics course. Thus, the tests should focus more on determining whether the students have mastered the basic concepts and can solve the simpler problems concerning the covered material.

D. ... I think it (the second test) is still a little bit harder than it needed to be for this level of mathematics. I think that there were too many tricks thrown in there. I think they just need to test the students to see what they know, to see if they have the basics so that they can work the problem and do what they're supposed to ... These are low level math courses. I am a low level person when it comes to math. I kind of expect to be taught the basics. If I want to know the in's and out's of parabolas, hyperbolas, and stuff like that, then I would take upper level courses. I would be majoring in mathematics ...

David also believed the test's focus should match the emphasis of the presented material in the lectures. That is, he wanted the test to address the different concepts according to the length of time spent on them in the lectures and to the amount of homework that was assigned on them. He did not want to spend a lot of time learning a concept to discover that only one or two questions on the test addressed it. Consistency was important to David in his learning of mathematics and he expected to find it not only in the homework but on the exam as well.

X. What about the second test was tricky?

D. They threw a lot of curves in there. That was the most difficult out of the

three tests.

X. I don't have an idea what curve you are talking about.

D. Well, they give you the basic problems, they teach you how to work the basic problems. And they throw like a negative exponent (problem) in there to see if the students can do it. That is not fair!

X. But didn't you have homework problems that had negative exponents in them?

D. A few were; so what? They weren't word problems. In the practice exam, they give you a word problem, and then you go in there and do some totally different word problem. You wouldn't even use the same formula that you did before (in the practice exam) ... I remember like the division problem, there were five of them on the homework and two on the test. I didn't remember doing something that was similar to the ones on the test. Well, now: the first problem (on the test), like the cattle question?

X. Yes.

D. There were none in the homework, but there was one of them on the test.

X. So did you expect to get -

D. No, it's just, I thought that it laid it out different I guess. I thought that if we could do the homework, then we should be able to do the ones on the test.

X. But didn't they expect you to be responsible for everything?

D. Yes, they did; and that is fine. If you want to throw a hard problem in the test, that is fine. But don't come in here and teach me one thing and expect me to know something else. Let's say I come in here and I give you the answers to 19 problems on the homework, and I give you some of the answers 3 times, and the test is in 20 minutes, which ones are you going to remember?

X. The three times.

D. See what I am saying? The emphasis was being put in the wrong place

... If you are in a math class and you see this formula that crops up 8 times in your homework, and one that is once or twice on your homework, which one are you going to write down? Why are we spending hours, hours, and hours upon a problem that we are likely never see again, but we just spent 1 or 2 hours on something that we will see over, over, and over? ... I mean every homework in the course has been absolute value, square root, function, problem solving. It's all been there and so what am I going to concern myself with: the midpoint or the circle formula, or the absolute value? See what I am saying? Why should I spend 9 hours on the midpoint or the circle formula? Do you see what I am saying? You got to key in on something. It doesn't really matter which but that is what I am saying, you got to have consistency through repetitiveness.

Again, the importance of repetitiveness was stressed in the learning of mathematics. Its importance was apparent not only in the domain of homework but also in the test preparation as well. Another expectation some participants had concerning the test was that each test would be independent from the other ones. That is, one would not need to refer back to previous knowledge in going from one test to another. Stan, for example, expressed this expectation in his response when asked why he found the third exam to be tricky. According to him, the third test was tricky because it contained problems that he did not recognize. He did, however, admit that these problems could have been on the homework and that he might have missed them in his work.

X. You missed some problems on the test, meaning what? You forgot them but you have done some problems like that, or what?

S. Yes, or it was termed differently and I didn't recognize it. And on a lot of those, they wanted you to do more steps than what you are used to on the homework. You are supposed to go to the prior homework, prior tests and work your way back to get it into the form that you need.

X. And you didn't expect to have to carry the knowledge from one test to another?

S. Right.

X. But one time you said that learning mathematics is a step-by-step process?

S. It is.

X. So that would mean you will always use some of the stuff that you have learned?

S. Right, right, but if you were not expecting it and you have a time limit on the test, then that puts it a little differently. And then when you are not used to it, since you took your practice exam, which was last year's test, and there was nothing like it on this semester's test, it's like throwing a curve. It throws you off ...

With these expectations going into a test, it was not surprising that many of the participants found the exam to be tricky or unfair. Some of them even reported experiencing panic after they saw the test questions; however, they did manage to finish the test by doing the questions they felt they could do and guessing on the others.

As they had for the homework, the participants did make some plans for dealing with their difficulties on the tests. Stan, for example, planned to get all the old course exams for the course from the campus test files to practice doing them before the second test. He believed the level of difficulty of the tests usually alternate between easy and hard from one semester to the next, and that if he took all the older exams, he would get practice on tests that were equal in difficulty to those of the current semester. This plan, however, was not carried out because Stan did not have enough time for it. Instead, he studied for his second exam mainly by solving and reworking the problems on the practice exam until he felt comfortable with them.

David, on the other hand, did not plan to alter the way he studied for the

test in the course going from first test to the second one except that he wanted to arrive to the test site at least an hour early to give himself plenty of time to locate the test room in case it changed at the last minute, as it had in the first test. According to David, his teacher changed the test location on the first exam without him knowing about it. Thus, he had to spend about half an hour searching for the new test room, and was quite upset by the time he found it.

Barb and Kelli, after taking the first test, planned to spend more time working and reworking problems from the homework and the practice exam. Barb, for instance, planned to spend about two hours each day studying College Algebra although she recognized that it would be hard for her to pack additional hours into her already busy schedule. She planned to go back to the way she learned in her previous mathematics course, since she had experienced success with that style of study.

B. ... I think the mistake I made (on the first test) was that the problems on the test are usually a lot simpler than a lot of the problems that we do for the homework. I need to go back to the way that I best learned in the other courses: I take the simpler problems that represent the concepts and know those really well. The way I do that is I would write the problem down and then I would write the solution on another piece of paper, and then I keep solving those problems until I really understand it, just quickly like that (snapping her fingers). If the problems on the test were more difficult, then I can apply the simple problems to the more complicated problems. Do you see what I mean?

X. You would just study the simpler problems and you would keep doing these over and over until you can do them quickly?

B. Yeah, repetitive. And then if it is a difficult problem (on the test) then ... in the past, I was able to figure it out (based on studying the simple problems on the homework) instead of concentrating on these really hard,

horrible problems. Like some of the word problems (in the homework) are just totally out of my league and I spent so much time worrying about those when I should have been studying some of the simpler things and understanding them better. You see what I mean?

X. Yes, but if you did a problem so many times to the point where you memorize how to do it and you could do it quickly -

B. (nodding) Then I don't have to spend the time figuring it out every time.

X. So is that understanding or it is just memorizing?

B. It is probably more memorizing, although with that, as I can look at a problem and not have to spend a lot of time trying to figure out how to do everything, then it becomes understanding the more you do it. Then it would become understanding and routine. And that is starting to happen, by the way.

X.Oh, is it?

B. Uh huh. In a few instances ... not as much as I need to get through this course.

Barb's response gives us some hints as to what the participants meant when they said they understood a concept in their learning of mathematics. This way of looking at understanding in the learning process was in fact, common to all the participants, except perhaps James. Further discussion of the differences between the way the participants defined understanding and what they actually viewed as understanding in their learning of mathematics will be provided in a later section.

Another way the participants planned to improve their test preparation was to seek assistance from others. Some planned to work with their friends, others with their private tutors, and some also planned to receive tutoring from this researcher after the first test. According to the participants, they always worked by themselves first before getting together with their friends or their tutors to have their questions answered. Moreover, they reported having a list of questions with

them ready for the tutor so they could make the best use of their tutoring time and also be sure of having all their questions answered.

After the second exam, many participants planned to shift their focus from doing the problems on the practice exam to doing the ones in the chapter tests in the textbook. This change was prompted by the belief that the practice exam was not an accurate reflection of the actual test because it usually was too easy.

> **S.** (The practice exam does not) have any problems like what was on the actual test. It kind of gives you a false sense of security. I (learned) not to trust the practice exams they gave us because they are really easy. That is not good because when you go in and you are expecting, because the practice exam was last semester's test, and you kind of judge yourself on what the (practice) test was on, and (then the actual test) was totally different. All the tests have been harder than the tests that they have taken (last semester) ...

All of the various plans the participants had to improve their test preparation focused on the importance of practicing doing problems until one could work them quickly and accurately. This emphasis on doing repetitive work had its roots in the participants' approach to studying mathematics in their previous classes. However, what might have worked in previous classes now failed to produce similar results in their College Algebra class. Further discussion of this topic appears in a later section.

3. Other problems in the course

a. Job related problems

In addition to problems with the homework and the tests, some participants also reported having personal problems. David, for example, had to discontinue his studies after the first half of the semester because he had to go back to work in order to keep his job. Recall that David came back to school because he was injured on his job and had to change his occupation. For the past two years, he

had been under doctor's care and also had undergone several operations for his back. As long as he was under the supervision of a physician, both his medical and schooling expenses were being paid by his employer. However, his injury settlement could not go to court until he had left the doctor's care due to some reasons concerning his employer and the insurance company.

Around the middle of last semester, David had to go back to work because his doctor released him from his supervision. Also, there was a change in the management at David's work place, and the new managers demanded that David choose between going back to work or quitting his position at the company. It was quite a difficult decision for David, and took him quite a while to make.

> **D.** There were a lot of things going on during that time; it involved a large sum of money and not just my salary, but the insurance pay and it involved a lot of legal aspects. Legally I still I have to do what EEE says, or else I quit. (But) that is not an option for me until we settle the law suit. On one end of the spectrum, I wasn't supposed to be working for EEE according to the doctor's orders. EEE says "OK, we are going to bring you back but we are going to give you another job description; therefore, we can get around the doctor's orders and you would be working for EEE." Well, in the process, EEE is being sold. EEE was sold to some private investors and nobody could either fire me or hire me because of the law. That is if I am fired, or let go, or suspended, they have to pay me for 17 weeks. Then, at that time I would just go and be off. So EEE says "Just keep working him until he drops" or whatever. Now we are being tended an offer from the insurance company to hire me debt-free. Once we get the money in, we will be debt-free and have a few hamburgers to eat and some extras to put in the bank, and then everybody will be happy. It just took too long to happen.

According to David, he thought his injury case would only take a few weeks in the court; however, it stretched out much longer than he had expected. In

fact, the settlement was still being negotiated at the time the semester ended and David did not know how much longer it would take. He recalled consulting with his close friends and family many times during the second half of the semester as to what his options were and what he should do.

D. I have talked to many close friends as to what I should do. Do I stay at EEE because of job security, 401K retirement and health insurance, or do I just go back to school full time and not have that 401K and job security and the retirement? And go back and get a degree in something that I already know, that I already got what most of these kids are trying to get in 2 years when they get out. Or you know, what do I do? Should I just take the money and buy my own business, I really don't see (the solution). I change every day. I change every hour. As soon as I thought that I got it made up, I change my mind.

Although David did go back to work, he still tried to attend classes whenever he could and to keep up with the homework. When he first started back to work, he was worried over whether he could keep up with the work in his College Algebra class, but he determined to keep up with the workload as long as possible, hoping to make it to the end of the semester.

D. (I worry whether) I am getting in over my head. Like I said before, I do not have a good background in mathematics, and now that the job is starting to kick in again, I might not ride it out. And I might not have time. I mean, between my job and school right now, I am putting in about 70 or 80 hours a week. And you just can't do that for 16 weeks. I am going to snap somewhere. I have a real problem, but I will just take a week (at a time) and just get out of this semester and then go on from there.

Despite his efforts, however, David was not able to keep up with any of his classes. He often worked overtime and thus, had little time for his studies.

D. It is really hard getting up like 3:30 in the morning and working till 6 at

night and then spending about 45 minutes in the evening (studying) when you get home, but I tried ...

David contacted all his instructors and explained to them about his absence from class and his current working schedule. At the time, he was working more than forty hours a week, which left him with little energy or time for his studies. In addition, he was having trouble concentrating in the classroom because he was thinking about what he would have to do on his job after getting out of class that day. Nevertheless, he continued to attend classes whenever possible because he felt it would improve his understanding of the material the next time he took the course.

Despite his heavy working schedule, David was able to make a B on his second exam in College Algebra. He was proud of this accomplishment because he was able to achieve it even though he missed quite a number of class sessions and had to learn some of the material on his own. He did not take the third exam in the course because he was not able to keep up with the homework; however, he did take the final exam at the end of the semester. According to David, he wanted to take the final exam because he was curious as to the kind of problems that would be on it and because he believed it would help him know what to expect on the final exam when he retook the course.

According to David, he took the final exam without much preparation: his preparation was done off and on over a period of four days, and mainly consisted of mentally going over the material he had learned in the course. When he took the final, he discovered to his surprise that he could recognize and do many of the problems. Afterwards, checking the test key, he was pleased to find that he had got many of the answers correct despite not having much time to prepare for the exam.

Looking back over the semester, David reported that not being able to attend class regularly was a big hindrance to his studies. He did not like to miss so

many class sessions, but he had had little choice. He looked forward to the spring semester in hopes that by then his injury settlement would be settled and he could resume his studies full time.

b. Problems related to self-esteem

Since Kelli did not have a very strong background in mathematics, she was often uncertain about her ability to do well in the subject. According to Kelli, her success in a mathematics course depends greatly on the instructor. She needs to have someone who can explain the material clearly, present the material in a detailed manner, and have patience for the students' questions.

In College Algebra, Kelli attended class regularly and did all the assigned homework. Moreover, she usually received high scores on her homework. Yet she was not able to do well on either the quizzes or the exams. She tried hard to improve her performance by buying additional Algebra books, working with private tutors, and spending more time studying, but still she was unable to keep up with the class.

As the semester progressed, Kelli's confidence that she would make an A in the course slowly turned into self-doubt; especially when exam time came near. She came close to changing her major after scoring low on the first test, but chose to continue with the course, reasoning that she had two more chances to improve her test score average before the final exam. She did indeed improve her test score on the second exam, but failed to do so on the third exam.

By the time of the third exam, Kelli was very depressed and discouraged with the course. She felt the fast pace and heavy amount of homework made it very difficult, if not impossible, for her to succeed. Her confidence that she could do well on the exams was diminished by that time, along with her willingness to spend a lot of time and effort trying to work the problems on the exam that she had difficulty with. In fact, she even believed that she would fail the third exam.

Despite this belief, Kelli still studied for her third mathematics test as best

as she could. After taking it, she recalled not experiencing much emotion when she discovered that she had, indeed, failed the test.

X. What was your reaction when you checked the test key and found out your test result?

K. Oh, I was just checking it. I mean I was really emotionless. I wasn't mad. I wasn't upset. I was just like "Well, damn!" Because just like what Barb said, I felt that I had wasted the whole day ... I felt like I had wasted my whole life. I mean my grade shows it. My grade shows just how much good all that work did. That is why I was so depressed ...

This feeling of defeat, for Kelli, continued well into the final exam. During her preparation for the final test, Kelli expressed little confidence that she would do well on it.

K. (On the final exam) I am going to flunk logarithms. I am going to flunk Chapter 5. And I am going to flunk definitely Chapter 6 no ifs, ands, or buts about it. I am going to do ok, but I am not 100 percent on Chapters 1, 2, 3. I don't even remember what was in Chapter 4.

X. I think it was about the graphing of the parabola and the circle.

K. Oh yeah, I am really weak on that too, inverses and all that kind of junk ...

On the day of the final, Kelli came to the exam suffering from a severe cold or flu. She reported guessing on most of the problems and when she had difficulty with a problem, she did not try much to struggle with it.

K. ... (On the final exam) after I got the test and started working at it, I just felt dejected ... I didn't sit there and struggle with it. I answered the ones that I knew and the ones that I didn't, I wasn't going to sit there and rack my brain trying to figure how to answer them. I just guessed ...

Kelli finished the final exam but did not check the test key after she handed the test in. By that time in the semester, she was too depressed and frustrated with the

course to care how well she did on the test. She felt that her negative experience in College Algebra had reaffirmed that she was not good in mathematics. Moreover, it made her feel like a failure even though she had worked hard throughout the semester.

> **K.** (My failure in College Algebra) is very defeating. It is very much on my self-esteem. I talked to my brother and I was telling him that I was going to flunk it, and he said "That is a hell of an attitude," and I said, "I know it is, isn't it? But that is the truth." And I said that I know I am not stupid but, this class is making me feel stupid! This class has depressed me. It is really and truly has. It has played havoc on my family. It plays havoc on me. I mean this class has literally brought me to my knees. With Barb, she's got her other classes that she is making A's and B's in, but with me, this is it! This is the money that I spent. The books, the tutor that I spent the money on - and for what? What did I get out of it?

In fact, Kelli felt she had not learned anything from the course that she had not already learned from her previous mathematics courses.

> **K.** ... I don't feel like I have learned anything in this class. It didn't teach me a dilly darn thing. I think that I am just tired. I am really tired ... It just like what I said before: you are under the impression that if you go to class and you do your homework, and you put forth the effort to trying to learn this stuff, that you are going to accomplish something. I have such a sense of defeat. You know, I was defeated is what I was. Actually I have felt this way before.

Although Kelli did finish the course, she was exhausted and frustrated with the whole experience. After sixteen weeks of struggling to maintain a positive attitude in the class and of trying to improve her performance, Kelli finally summed up her experience with the course as follows:

> **K.** You can just report that I am overwhelmed in every aspect of this class.

I am overwhelmed ...

Like Kelli, Barb experienced some difficulties in maintaining a positive outlook in her College Algebra class; especially towards the end of the semester. She also worked hard and tried to seek additional resources to help her in studying, but was very concerned about her test performance in the course throughout the whole semester.

Of the five participants, Barb was the one who always appeared enthusiastic about planning ways to improve her next test performance after she had finished taking a test. Throughout the semester, she wanted to invest more time in studying for her mathematics class. Also, she wanted to spread her work in College Algebra more evenly over the week so that she would not have to cram the last minute to get her homework done or to complete her test preparation. Unfortunately, Barb was not able to carry out many of her plans because she was busy with the three other courses she was taking at the time.

Despite the low grades she received on her work in the course, Barb tried hard to maintain a positive outlook in her study. Her outlook however, slowly darkened as the semester progressed and she found herself continuing to perform poorly on the exams. By the time of the third exam, she reported feeling very discouraged with the whole situation and unsure that she could do well on the upcoming test.

> **B.** I am so discouraged now. I know it is coming up Thursday and it's like: oh well, I mean there is some material that I will need to go over like the logarithm, just go over it and do what I can. I am not going to worry about it, but I will at the end. That is what I am saying, it is so frustrating when you get in a situation when you start feeling powerless almost to control the outcome. I have almost a defeating attitude that I am not going to do well in this class ...

Like Kelli, Barb spent a few days studying for the third exam although she did not have much confidence that she would do well on it. She was very upset

when she discovered she had failed it, because she believed she had lost all chance of making a grade higher than a D in the course. In fact, the last two weeks of class was a very trying period for both her and Kelli, as they tried to get through the class before completely wearing out from all of the struggling they had done during the semester. In studying for the final exam, Barb did not even attempt to try to understand the material that she had difficulty with; instead, she mainly wanted to focus on mastering the material that she felt comfortable with so that she could at least get it right on the test.

> **B.** (For the final exam preparation) I am going to muddle through this material but it is not sinking in. I don't have the confidence. This is what it is, I just feel very defeated that I can just study, study, study, and study and sit down and take the test and go blank. And so it's like: "Why bother, why not take those precious hours and work on something that I know I can master, which is the beginning, the first part of the book." That is what I am going to do.

Barb did her best to prepare for the final exam, and after she finished taking it, she reported feeling relieved that she had made it to the finish line. Barb was not as discouraged with the course as Kelli, for she was already thinking about retaking it even before the semester ended. She did, however, recognize that it had been very helpful for her to have Kelli in the same class because Kelli gave her some of the support and assistance she needed to finish the course.

> **B.** It would have been difficult (without Kelli) because I enjoyed (working with her). I mean, I would really feel bad being the only one being so ignorant in that class. It really helps to have a friend to work with.

Indeed, this partnership in learning was quite important to both Kelli and Barb for it carried them through some rough times in the course. Their partnership started in Intermediate Algebra class and continued well into College Algebra where its strength and durability were tested many times. According to Barb

and Kelli, they visited with each other almost daily over the phone throughout the semester and helped each other with the homework and the test preparation whenever possible. Near the end of the semester, when Barb heard that Kelli might change her college major and thus, might not be taking the remaining three mathematics courses that Barb had to take in the future, she was sad but nevertheless understood and respected her friend's decision.

X. Next semester, what would happen if Kelli is not going to be in your math class?

B. ... I told her that I hope she does because it has been really helpful to have her to study with. But if she choose not to, then I can't, I wouldn't blame her because she is just at a different point in her life right now than I am. I am hunkering down and I am going to get this finished one way or the other, of course I might be retiring and still trying to get out of College Algebra ...

Indeed, this friendship between Barb and Kelli was very helpful to both of them in their coping with the difficulties they had in the course. For, unlike the others in this study, these two participants could share the frustrations they had with the course between themselves, with each knowing exactly what the other felt or meant to say about the situation.

IV. The coping strategies used by the participants

A. Coping with the heavy amount of homework

One of the frustrations Barb and Kelli constantly encountered during the semester was the amount of homework they had to do in the course. Usually they had about sixty to seventy problems to do in a week, some of which were multi-part problems. It might not be difficult for someone who understood the material in the course to do this amount of homework; but for Barb and Kelli, who were not able to understand much of the material, it was a lot of work to do in one week.

In coping with the large volume of homework, Barb and Kelli reported refocusing their goals : from doing the homework to gain understanding of the material, to doing it just to get the points to pass the course. They were not happy with this change in their focus, but reported having little choice since they wanted to get a passing grade in the course.

> **B.** ... (The material) is making less and less sense as the semester goes on, so my goal (for doing the homework) is probably misdirected but my goal is just to finish it. And I still think that it is wrong, but my goal is to get the homework done because it might improve my grade and give me more options instead of being locked into something that does not ...

Since they were not able to understand much of the course material, Barb and Kelli had to use the solution manual often in doing the homework. Most of the time, they were able to do their homework problems by imitating the steps in the solution manual. However, there were times when they just copied down the work without having much understanding of the steps that were used in working the problems. They felt they had to have all the steps written down in order to receive full credit on their homework. Thus although they were not able to understand much of what they were doing in the course, Barb and Kelli took some comfort in the fact that they had a good chance to pass the class since they had handed in all the assignments and had received good scores on almost all of them.

B. Coping with the low test scores

There were several ways the participants coped with their low test scores during the semester, the most popular one being revising the process they used to prepare for the tests. Some participants obtained additional old exams from the campus test file in order to get more practice doing problems, while others sought assistance from private tutors or from their friends. For all the participants, however, the focus of this revision was on increasing the amount of time they spent preparing for the exam.

Another way the participants coped was persisting in their work and maintaining the hope of improving their performance on the next test. For example, Barb reacted to her score of 69.3% on the first test, as follows:

> **B.** ... It's still a D, and it still is an unacceptable score; I mean there is no two ways about it. But then if you tried to look positively on it, if it would have been an F, I would have been really depressed. I would wonder whether I would be able to get a little closer to grasping the concepts. Now with the higher of a very bad score, I am thinking if I change a few things, I have the ability to at least make a C in this course. Which I don't want to make a C. I want to make an A or B ...

Kelli also responded optimistically to her first test score of 60%, noting that she was one point above a failing grade.

> **K.** Today I was pleased that at least I did not get an F. I mean, at least, it is one point above it. So I guess that pleases me, made me a little bit more pleased.

And at the time, although she did not have much of a new plan for preparing for the second test, Kelli still determined to continue trying her hardest to improve her future test performance and more importantly, to continue focusing on this task until the class was finished.

> **K.** I am just going to try my best ... I would love it if I could just say "OK, every day I am going to set aside two hours to study," but things just come up and I can't fight that. So I am just going to try. All I am going to do is try, just try my hardest, and try to be familiar with the material. And I guess if I am not familiar with it, I am just going to make myself a nuisance to you and to Tai (her tutor). I am not giving up! I am not giving up until I take that final and they tell me that I have to take this class again, and I am not going to take this class again!

Despite this resolution however, Kelli did retake College Algebra in the spring

semester through the University Continuing Education Program. She chose to go this route instead of taking regular class because it would allow her eighteen months, rather than only sixteen weeks, to complete the course.

Barb and Kelli's low test scores did not seem to dampen their determination to finish the course; if anything, they helped to heighten it. Indeed, with each additional low test result, Barb and Kelli became more determined to stay with the course until the end of the semester. After receiving D's on the first two exams in the course, Barb responded to her third test result as follows:

> **B.** ... I just didn't think that I did it that bad. I felt like I would probably make a C on it. And then I thought "Well, if I make what I have been making, then I probably got a D." But I never imagined that I would fail it. I cannot even imagine that ... I figured that if I made a C on this test and a C on the final that I could make a C in the class. But that hope has just completely vanished. The only reason I even mentally feel like picking myself up and not giving up is because I am going to learn the material one way or the other, even if I have to stand on my head every day!

Even Kelli, who was very discouraged by the time the third test was finished, was still determined to see the course through. In fact, she appeared enthusiastic in talking about her plan to study for the final exam in the course right after she discussed her failing grade on the third exam.

> X. Do you have any plans right now concerning the preparation for the final exam or do you not even think about it yet?
>
> **K.** Well, oh yeah. We came up, we brainstormed and what we want to do is to make copies of the chapter tests for chapters 1 through 6, and to have you look at it and highlight the problems that you think are really, really truly what needs to be worked on. We will individually work the chapter tests and then get together with you to ask whatever questions each of us had, and then we will get together and explain things to each other ...

X. Oh I see, you are going to split the preparation up?

K. Yes, split it up so that we will have enough time. I don't want us to be frantically trying to get (this done before the final). I mean, if we have the game plan to where you know, it's not quite that much of a big task to deal with at once. She will take Chapter 1 and I will take Chapter 2 and then get together and maybe that won't be so overwhelming ...

Like Barb and Kelli, Stan chose to remain in the course till the end of the semester despite doing poorly on all the exams. He was always cheerful after having taken a test and confident that he would make about a B on it. His confidence, however, usually turned into puzzlement and sometimes even surprise when he found out what his test score actually was. Although his test performance was poor, Stan was not bothered by it because he took comfort in the fact that he had studied for it.

S. ... If I go and fail the test and I had prepared for it, then at least I have prepared for it. It will teach me what to look for the next time. But if I did not prepare for it and I failed it then well ...

This attitude of acceptance was repeatedly expressed by Stan throughout the semester. He did not appear distraught upon discovering that he did poorly on another test because to him, it was enough that he had tried his best on the exam.

S. I am not sure what caused my low test scores, but I don't care very much about the test scores ... It upsets me that I didn't make a C or B, but I don't cry about it. I tried hard and that's it.

Unlike Kelli and Barb, Stan did not feel pressure to get his homework done on time to hand in each week for a grade. However, he did report having his homework finished each week although his teacher only collected it a few times during the semester. As for Barb and Kelli, getting the homework turned in on time was another way they dealt with their low test performance. By the end of the semester, their purpose for doing the homework had changed from trying to

understand the material to trying to raise their grade.

Barb and Kelli also dealt with their low test results by taking comfort in the support and the assistance they received from their friendship with each other and from their families. Throughout the semester, the two continually encouraged and assisted each other; especially when it was time to study for a test. They also reported receiving a lot of support from their families. For example, Kelli reported that her husband was frustrated for her because she put a lot of time and effort into this class, and yet was not getting much of a positive result. Barb recalled her youngest son's inquiry as to how well she had done after the first exam:

> **B.** That night when I got home, my little boy, he was asleep and I went in and kissed him good night and he said "How did you do on your test?" I was so embarrassed to tell him that I didn't do good. He was the one that got the prize in math (at his school). Yes, the kids and my husband were all interested in how I did.

The participants also dealt with their frustration with the low test scores by looking forward to the next semester. That is, rather than viewing their failed efforts to improve their test performance last semester as a complete waste, they believed instead that their work would benefit them the next time they took the course. For example, David took the final exam at the end of the semester although he had not kept up with the class after the first half of the semester. Moreover, he had already dropped out of school shortly before the semester ended and thus, was not required to take the final exam. And lastly, he did not have much time to prepare for the final exam because of his work.

> **D.** I wasn't able to do that well on the final exam, but I wanted to go and see what the final was going to be like so I could have a better idea next semester.

Thus, David was able to see some value in taking the final exam although he was not able to finish the course. Besides going to the final exam, David also reported attending classes whenever he could even after dropping out of school

in the second half of the semester. Again, his reason for this was to improve his chance of success with these classes when he retook them.

Barb also believed that her efforts to improve her test scores last semester were not a waste of time. Rather, she believed they would definitely help her in the next time she took the course. For example, she was able to see some value in studying for the final exam although she was discouraged with the whole course by that time.

> **B.** For the final, I decided that I was going to do the chapter test in each of the chapters and really concentrate on knowing and understanding what I am comfortable with. If it was a concept that just looks so foreign to me then I just (didn't worry much about it). If I have only 10 hours to study, I would rather know really well parts of the material. That way if I have to repeat the course or if I go on, at least I am not lost about everything, but if I spent those 10 hours on some kind of crazy stuff like trigonometry, I am still not going to know anything about trigonometry because that was how I reacted through this whole course. So at the end of that amount of time, I am not going to know anything about that, and I am not going to have the refreshment on all of this other stuff, and so I would fail on those too, and so I will completely fail on everything. This way I have at least half a chance of maybe (passing it). I want to come away knowing something so that I won't be an idiot in my next math class.

Despite her frustration with the course, Barb determined to finish it. She saw little benefit in dropping the course since then she would still have the same difficulties the next time she took it. On the other hand, if she stayed and finished it, she might have a better chance to succeed the next time around.

> **B.** (Dropping out of the course) wasn't an option to me. There was no other option available at the university, and this makes me kind of angry. At the end of the first test, and even at the second test ... if you didn't do well

(you were encouraged) to go ahead and drop it. That is not an option! I do not understand how anybody can feel that is an option. It is like they need to have an (alternative slower-pace) course, or encourage the kids to switch to auditing it. They need to do something because all I could think about was "OK, I am not doing well in here but if I drop it, I will either be here again next semester or quit going to school, because I have to have this to graduate," and most everybody does (have to have it). There is no option here! I don't think kids understand that if they drop the course that they have to be back in here next semester or a semester down the road. I mean, I at least am a little more ahead than them because now I could sit down and take the course and probably do very well in it. So that is how I view this whole semester in the back of my mind before I made up my mind to retake it, that this was just my refresher course.

Indeed, Barb did retake the course the next semester and did well in it. She reported having little difficulty in keeping up with the work in the class and more importantly, in understanding the material.

Another coping strategy that Barb used was to refocus her goal in taking the final exam. She came into the final determined to score eighty-seven of the two hundred possible points so that she could get a D in the course.

B. ... I mean I was going to, come heck or high water, get that 87 points on the final exam. I am not going to get an F on my transcript! So every time that I did a problem, if I knew that I got the answer correct, then I would make a little mark over here on my scratch paper. I did it until I got up to 87 and then I took a sigh of relief. That was how I worked the final. I went through to where I could get enough to get an 87 and then I didn't even care, and that was the truth.

On the whole Barb and Kelli dealt positively with their low test scores, despite the discouragement and self-esteem problems they caused. They were able to focus

their attention on finishing the class with a passing grade, and put forth the effort needed to accomplish this.

III. The end of the semester

A. The outcome and the participants' reactions

Of the five participants, James was the only one who did well in the course. Not only did he get an A, but his grade was the top grade in his class. Moreover, he felt he had gained a more thorough understanding of the material from having gone through the course. He attributed his success to the hard work he invested in the course.

J. I did all the homework even though some of them were not required to hand in. Like she would say "For this homework, you are not going to have to turn it in," I would still do it.

X. Every problem?

J. Yeah, every problem because she must be assigning it for a reason. If I want to understand the material, you know, her job is to make sure that I understand the material and my job is to be actively involved in doing it ... I think it is my duty, I think if they assigned me homework, then I would feel embarrassed to come to class if I haven't done it. I don't have this belief that you can just put it off so long as you can feel that you can do the homework. I think the reason that you do good on the test is because you do the homework. They wouldn't assign any homework if there wasn't a purpose.

Indeed, James was quite actively involved not only in doing his homework, but also in participating in class as well. He was quite proud of his achievement because he felt it reflected both the level of understanding he had of the course material and the amount of effort he invested in his study.

As for the rest of the participants, all except David got a D in the course; David did not finish the class because he went back to work. Of the three who got

D's, Stan was the only one who was happy with his grade because it meant he had finally passed the course after trying three times. According to Stan, he was not bothered by the D he received in the class, because getting a high grade was not as important to him as obtaining the understanding he needed for chemistry. Thus, although he barely passed the course, Stan decided to take the next mathematics class, Precalculus, in the spring semester because he felt he had enough knowledge from College Algebra to be able to work with the formulas in his chemistry classes. According to Stan, he planned to approach his study of mathematics in the spring semester in the same way he did last semester, except perhaps for putting more time into it.

The other three participants were upset with their overall course performance because they felt they did not have the opportunity to do as well in the course as they would have liked. David was frustrated that he had to devote so much of his time to his job that he was not able to concentrate on his study.

> **D.** (The problem I had with this course) was that I was not able to be there (regularly). I feel that it was a hindrance, but I didn't have the option of not going back to work: it would have cost me a ton of money. (My employers) will be darned if they are going to pay for me to go to school and work at the same time.

For Barb and Kelli, on the other hand, the source of frustration was the course's fast pace and the heavy amount of homework assigned.

> **B.** (One of the) difficulties with the course would be the heavy amount of homework. It was frustrating ... Since I didn't know the stuff, I was not able to discover the way to take my lecture notes, study them, understand them, do my homework and turn it in. I always felt this heavy burden of accomplishing all of these. Usually we would have about 25 pages of homework a week and getting them done took up all my studying time slot for the class. The other frustration with the class was that it was geared towards the people who have

more knowledge of the material. I have never been in a situation where I was the dummy in the class. I didn't expected her to stop the lecture just for me, or one or two people, but it was very frustrating because (although) I am not afraid to ask question in class, I did feel somewhat intimidated when I had a question. It was like "Well, is this something that everybody else already learned five years ago?" I mean, you do get a certain degree of intimidation when you get this far behind in the course.

Barb and Kelli's frustration and disappointment with their overall course performance clearly showed in their reaction to their course result at the end of the semester.

B. I am devastatingly disappointed in myself. I mean failure is not a graceful thing (to accept). It is very difficult ... I didn't ace it. I failed the class and I consider a D is a failing grade ...

K. You can just report that I am overwhelmed in every aspect of this class. I am overwhelmed. I feel disgust, frustration, lack of accomplishment ... that four months was a waste of my time that I could have spent doing something else. Uh, financially broke. Totally burned out, totally pissed off, just mad at the whole situation.

Nevertheless, like David, they both decided to retake the course in the spring semester in order to gain more understanding of the course material.

D. I am thinking about retaking this class because I just don't feel like I know the subject (well enough) to go on to my next math class.

K. Well, right now my degree is still important to me. I might change my degree. I might not. I am going to try one more time, whether it will be through the correspondence course or through somewhere else. I do not want to give this department another chance for I do not see that (the situation) will get any better.

B. ... I decided that even if I could pull it off to do better on the final, I

would just go ahead and retake it. I just didn't feel comfortable moving on to my next math class.

In the spring semester, Kelli retook the course as a correspondence course while Barb and David retook it as a regular class. Barb succeeded in the class the second time around while David and Kelli did not. However, David took the course again in fall 1996 but Kelli did not.

An additional difficulty in the course for Barb and Kelli was the effect their low test scores had on their self-esteem. For Kelli, in particular, it was quite a struggle to maintain a positive attitude. Recall that she strongly believed she needed to have a teacher who could provide clear and detailed instruction in order to succeed in studying mathematics.

K. For you, this math is not a problem, you are just geared toward that, but I think people like me and Barb (who) have to work at it, I just think that we are set up for failure. We are just set up for failure in this class and this is very hard on your self-esteem, very hard ...

One way they dealt with this pressure on their self-esteem was to shift the cause of their problems in the course from them onto the course's structure. That is, they viewed their problems as caused mainly by the course's structure.

B. I made the decision that — and this is a self-preservation thing — that this (problem) is not with me, it is with the course ... what I am trying to explain is that I don't have any power or control, and when you get to that point, you just (say) "I'm in for the duration".

This coping strategy offered them only limited relief for their frustration in the course. In fact, they reported not being able to resolve much of their frustration in the course because they did not know how. According to them, they approached the course the only way they knew how: the same way they approached their previous mathematics courses. They believed that if a studying method had worked well for them in the past, then it should also work in their College Algebra class.

K. I will be the first to admit that I'm probably not doing it right, but that's how I was taught to do: to take notes you write everything that is on the board. You get everything down ... it's just a habit that I had, that I continue to do, and it works well in my other classes. Maybe what is my problem is I basically do the same, cause I mean, I have always thought that if you go to class and you do your homework, you are going to do OK. And if you work at it, then you will do OK. But that doesn't hold true in this class. I don't feel like it does.

X. So for your next math course, are you planning to be doing the same thing as you did this semester?

K. Oh yeah, I mean it's like if you don't, if that is what you know, then of course you are going to do the same thing. Unless somebody tells me a different way to study, I honestly do not know of a different way to study.

When asked how the course's structure might be altered to give students like themselves a better chance to succeed, Barb and Kelli were not able to come up with many ideas other than to split the course into two semesters to give the students more time to understand the covered material. They were willing to spend the time and the money to take the two courses because they wanted to feel comfortable with the material in College Algebra before moving on to their next mathematics class.

B. I thought that this was a natural progression (going from Intermediate Algebra to College Algebra), but it was not. I feel like I went from first grade to third grade and missed the second grade. I feel like I have missed something. I feel like if they had taken the first half of the material, or maybe the first four chapters, and stretched it into two semesters, it would have made all the difference in the world. I would have time to concentrate and work on getting this stuff ... I would have taken it (if it was offered that way.)

X. But that would cost you more money.

B. I don't care! I don't care because as it is now, I am going to as best come out with a C in this course. That doesn't make me at all happy. I am very angry about that. I should be making an A or a B. I am smart enough to do that and I know that ... And you know, then the answer to that is "Well, study more." Well, that's true, and I don't want to sound like I am whining, but you can only have a given amount of hours to work ... So I feel like that for myself that there is just too much material presented in this course and, what is making me angry is I truly want to understand this. I want to know this. I don't want to go to my next class and be an idiot because this class moves along at a pace that is a little bit more accelerated and this is what is going to happen. So my options are to retake this course and I am seriously considering that, you know, even if I make a C in it. That is how much that it has upset me and angered me ...

Like Barb, Kelli was also desperate in her search for solution to her problem in the course.

K. ... It's either the way that the class is taught or they need to rethink, regroup and try it from another approach for people that aren't, I mean there has just got to be something for this class. There's just got to be.

Chapter 5: Analysis of the study's findings and conclusion

This chapter contains an analysis and synthesis of the data of the study, followed by a discussion of the implications of the findings and suggestions for future work.

5.1 Discussion of the participants' difficulties in the course

Overall, the participants seemed to fit the description of "model students". They were self-motivated to do well in their studies and were willing to invest effort, money, and time in their work. In addition, they were not easily discouraged when they first encountered difficulties in their study; if anything, they became more determined to succeed in their work. Yet, at the end of the semester, only one made a grade higher than a D, and two students with passing grades chose to repeat the course because they felt they did not understand the material. Why did these students fail to succeed in their College Algebra class even though their study approach mirrored that of successful students? It is on this question that we will now focus our attention; to answer it, we need to reexamine the participants' study approach in the course and understand the reasons behind it.

I. Explanations of the problems with the homework in the course

Recall that all participants came into College Algebra expecting it to have a structure similar to that of their previous mathematics courses. Mainly, they expected to hear a lecture in the classroom and then to go home and do the assignment. Barb and Kelli also expected that the similarity to their previous classes would extend to the pace of presentation and the amount of homework assigned. But they were not able to keep up with the presented material, even though they invested the same amount of effort and time into their study as they had in previous mathematics classes, and hence became very frustrated with

College Algebra.

Their difficulty with the course started almost in the first week of class when they had to cover several sections of Chapter 1. This chapter was a review of material that they supposedly already had in Intermediate Algebra. The difficulty with this task was that they did not have the mastery of the material at the level that was expected of them, since their introduction to the material in Intermediate Algebra had been brief and geared toward mastering the easier problems. Also, they had been introduced to the material in the second half of the semester with the important concepts presented near the end. According to Barb, both she and Kelli had difficulties doing the homework toward the end of their Intermediate Algebra class because the material was getting more complicated. Thus, it was difficult for them to view the material in Chapter 1 as transitional work between Intermediate Algebra and College Algebra.

Indeed, the presentation of Chapter 1 in College Algebra was not a review for Barb and Kelli, as it was a quick presentation of material of which they had little knowledge, including some material they had never seen before. For example, they recalled their first encounter with the infinity symbol with embarrassment:

> **B.**(When we first saw the symbol for infinity,) Kelli and I, we were looking at each other and trying to draw the symbol and we would go like, "Now, is this what it is?" She put it on the board and I think everybody else probably had it, but Kelli and I had to go and ask a bunch of people trying to find out what she was talking about, and it was kind of embarrassing ...

Even when dealing with material she had already learned in previous mathematics courses, Barb reported having to spend more time than the younger students on remembering the use of number signs and rules.

> **B.** I was doing homework this week and I had my son helping me, and the questions that I would ask would be questions that he has had and knows so well that he does not even think about them anymore, whereas I had to sit

down for little things like changing signs, things like that. I find that I need to do a lot of memorization because it has been so long since I had it. It is not quite as natural to me. I just have to think for everything I do. I have to stop to think about signs, I have to think about double checking the formulas, and double checking everything. It is just not real ingrained in me.

As a matter of fact, the above description fits all the participants in their learning of mathematics with the exception of James, who had more mathematical training than the others. Coming into the course, the participants felt they had less familiarity with the material than the younger students, and were concerned about keeping up with them. This helps explain why memorization played such a critical role in the participants' learning of mathematics.

Since Barb and Kelli were not familiar with the material in Chapter 1, they found the pace of the presented material to be quite fast. To make matters worse, there was a lot of homework assigned on that chapter. Thus, they had already begun the struggle to keep up with the homework that would continue throughout the semester despite their efforts to catch up.

Recall that Barb and Kelli often complained they had to spend a lot of time completing the homework. On the surface, their complaint seemed to be with the amount of hours they had to spend doing the homework; but close examination of their work in previous mathematics courses and their expectations of themselves as students reveals that this was not really what bothered them. In fact, all participants reported that in the past they had done more homework problems than required if they did not feel comfortable with the material in their mathematics classes. More importantly, they did so voluntarily, and did not mind the work.

B. ... What I did last semester, the teacher said "Do as much as you want to so that you will learn it," and probably a lot of kids didn't do much of it, but I did. I sat down and I worked, and reworked the homework. I did at least every other homework problem, every odd (problem in) every single

section ... Even if I felt I understood the concept, I made myself do every other problem and even though it was kind of routine on some of them, the more you do, the quicker and easier it is to get it.

Like Barb, David reported doing extra homework problems in his Intermediate Algebra class (the first time he took it) in order to learn the material. According to David, his instructor did not explain much of the course material because she believed the course was a review class for the students.

X. What did you do in Intermediate Algebra when you weren't able to understand the material?

D. Obviously the teacher wasn't going to slow down and teach the students. So therefore, it falls upon the student's shoulders to pass the class. He has to do it on his own. There were several things I did. I did double homework.

X. What do you mean by double homework?

D. She assigned every other odd problem, but I did every one of them. I went and bought the solution manual. I did the problems over, over, and over ...

Thus, not only were the participants willing to do extra homework in order to feel more comfortable with the course material, they also believed it was their responsibility to do so as part of the learning process. This fact, together with the expectations the participants had of themselves as students, suggests that Barb and Kelli's frustration with the homework arose not so much from the amount they had to do, but rather with the lack of meaning they saw in the work.

X. You consider spending between six to nine hours a week on doing the homework in this course to be a lot?

B. Probably not. I guess it just seems to be a lot because I have thought about that before too, that is about what they say to spend. Maybe the time is so doggone frustrating, I don't know.

Even Kelli, who was very frustrated in College Algebra last semester, reported not minding doing the homework if she could see meaning in the activity.

K. ... I told Barb that when I do understand it, I love – I don't really love, I like to do factoring because it is challenging but I know that I can do it. There are certain steps that you do, and I know the steps, and I got pretty good at it. It's just kind of like working on a crossword puzzle to me. (Doing) this other stuff, I don't understand the steps to get from point A to point B. It doesn't make sense to me. I just know that you are supposed to do it like this.

Another clue to Barb and Kelli's frustration with the homework can be seen in the fact that neither of them expected to spend more time in College Algebra than in their previous mathematics course. They thought they came into the class knowing the prerequisite material, and thus would not have to spend much more time or effort than in their previous mathematics classes in order to succeed.

B. ... I thought that I could go into this class and expect to put so many hours, five or six hours a week into studying the material and come out with an A or a B ... I don't even mind spending a few extra hours, but even putting in all the hours and to come with D and C ...

K. (The transition between Intermediate Algebra and College Algebra) wasn't easy because it was two different types of classes ... There was too much material that was presented in this class ... In the previous class, you didn't have as much homework. You do have homework, but not as much. In this class you have overabundance and above it ... There was so much that you had to do ... I have a specific amount of time that I can work on problems. I don't have all day or all night. I can't stay up till 12 or 2 o'clock in the morning and then get up and take care of a 3 year old. I can't do it and I am not going to do it. I didn't have to do it last semester and I don't see why I have to do it this semester. I am not a graduate student. I am not in law school. I am not in medical school. I am taking a freshman college undergraduate course.

Thus, without having mastery of the material in the first chapter of College Algebra , and not expecting that it would be necessary to have it upon coming into the course, Barb and Kelli inevitably encountered difficulties in keeping up with both the lectures and the assignments. They fell behind in the class when they took the time to do all the assigned work in Chapter 1, believing that if they did not, they might have difficulty in understanding the later concepts. While they were attending to this task, the class continued to advance through the textbook, and it was not long before they started to fall behind in the course.

As the semester progressed, Barb and Kelli found it more difficult to understand the presented material and to keep up with the assignments. Often, they found themselves running from one assignment to the next without having much understanding of the work. As a result, they usually were exhausted by the time they took the test and found themselves having difficulty remembering the material. Since they were not able to do well on the tests, Barb and Kelli turned to their homework grade as their hope for passing the course. This in turn, put pressure on them to complete the homework despite having little understanding of the material. Thus, the cycle of their frustration continued, leaving them feeling less able to alter the result with each additional low test score.

II. Explanations of the difficulties with the exams in the course

Problems with the exams in the course were shared by all the participants. They thought their test scores did not reflect their understanding of the material and that the tests were "tricky". Of the five participants, James had the least frustration with the exams, most of his being due to not being able to get a perfect score until the final exam. He thought the tests were not too difficult, except perhaps for the second one, which happened also to be the one for which he was least prepared.

In order to understand the other participants' frustration with the exams, we need to reexamine their past learning experience in mathematics. According

to these participants, they usually prepared for exams in previous mathematics classes by redoing problems from the homework and the chapter test (especially the latter), until they could do them quickly and accurately. Also, Barb and David mainly focused on mastering the simpler problems because they believed those were more likely to appear on the tests. With this method of preparation, the participants reported having success or at least making passing grades on their tests in both Elementary and Intermediate Algebra. However, this success seemed to be narrowly defined, for it did not go beyond the mastery of basic algebra rules and procedures.

Recall that the participants reported not taking their studies seriously in high school. They took just enough mathematics to meet the graduation requirements, and in the classes they did take, they put in just enough effort to make a passing mark. Their goal in studying was to get their homework done, and in doing this they would do whatever their teacher told them even if they did not understand what they were doing. This focus, however, changed once they were in college as older learners, for they reported taking their study more seriously, and wanting to know the reasons in their work.

For these participants, Elementary Algebra and Intermediate Algebra were the first two mathematics courses they had at the university. These classes serve as a quick review, or as a quick presentation, of basic algebra concepts for students who need them before taking a college level mathematics class. Although they cover many mathematical concepts, the coverage is kept at a basic level, with emphasis on mastery of doing simple problems. Thus, the students can do the homework correctly just by imitating the procedures taught in class. Moreover, it is possible to do well on the tests by matching test questions with the appropriate procedures and then carry out all the necessary steps in order to arrive at the final answer. Thus, the ability to memorize different procedures and rules might be all the students need to do well on, or at least pass, the tests in these courses.

Since they were able to do well on the tests, the participants did not have much reason to question the level of understanding they had of the concepts presented in these courses. As far as they were concerned, they had a good understanding of the material because they could do the problems on the homework and the exams. Indeed, it seems natural that this conclusion should be reached considering the participants' weak background in mathematics and the type of instruction they received in the subject, mainly emphasizing the mastery of basic algebra skills. Thus, learning mathematics to them suggested learning to manipulate variables and numbers in a structured manner to obtain the correct answer. So mathematical concepts did not appear as ideas and relationships, but rather as the abstract symbols that were used to represent them. Moreover, the meaning and significance of mathematical concepts were not stressed much in the learning process, making it difficult for the students to see the connections in their work beyond those at the procedural level.

An example of this can be seen in a complaint that Barb and Kelli had about the section on trigonometry in their College Algebra class. They reported not even knowing what a right triangle was, let alone, what was a hypotenuse or a leg of a right triangle. Upon being shown that they had indeed, covered these ideas in their Intermediate Algebra class in the discussion of the Pythagorean theorem, Kelli replied that she knew the Pythagorean formula but mainly as an equation that has a, b, and c in it. Thus, if one was to give her numerical values for two of these three variables and ask her to find the missing one, she would know how to manipulate the variables to get the answer. But beyond this computation, Kelli was not able to appreciate the relationship expressed in the formula; to her, it was just a string of symbols that has little meaning other than computational work.

Another example demonstrating that the students had little recognition of ideas and relationships in working with formulas and equations had to do with finding roots of a polynomial function. Last semester (fall 1995) in College Alge-

bra, one of the instructors whose class the researcher observed gave a quiz that had two parts. In the first part, the students were to work the problems individually, while in the second part, they were to split into two groups to find the roots of a given polynomial function. In working the second part of the quiz, there was a lot of confusion among the students as to what the question asked of them and how to work the problem. Instead of working together as a group, some students tried to work the problem individually or in groups of two, while most only stood around and looked for someone who could help them do the problem. Whenever the instructor made a comment to a student that she or he was on the right track, everyone in the group gravitated toward that person and tried to see his work with the hope of finding out how to do the problem themselves. As the class came to an end, many students frantically copied the work of the student in their group who was able to solve the problem, or at least who they thought had solved it, so that they can turn it in for credit.

In talking with the researcher after class, this instructor reported being surprised and somewhat amazed that her students had so much difficulty with the second part of the quiz. She did not expect this to happen, because the students had had similar problems in their homework, and she had explained to them in class several times that finding the root of a function means setting the function equal to zero and solving for the variable x. In fact, when she went over the quiz in the next class period, the students seemed to recognize the steps she used in her work. Perhaps more students might have been able to do the quiz problem if the problem had been written as an equation that was to be solved for the variable x, for then they could have seen some meaning in the work: namely, treating the equation as a rule and carrying out the necessary algebraic computation.

In working with Barb and Kelli in the tutoring sessions, the researcher found they had difficulty finding x and y intercepts while graphing functions. They knew that in order to find an intercept of a function, they needed to set one of the two

variables in the function equal to zero and then solve the resulting equation for the other variable. However, they did not know why they had to do this and did not seem to be aware of this question until they were asked by the researcher to explain their work. Up until that time, they did not seem to have made the connection between the x and y intercepts and the points on the graph where the function crosses the two axes, and thus seen the significance of setting one of the variables equal to zero in finding the intercepts of a function.

Like Barb and Kelli, David and Stan had difficulty in explaining their work, beyond the mechanical aspects of the solving process. On the third test in College Algebra, Stan was puzzled as to why he got credit on a problem that he thought he had answered incorrectly. The problem was to graph a given function and its inverse together on the same x and y coordinate plane. According to Stan, his graphs crossed each other, but he believed they should not, since that was what he had read in the textbook. When asked to give additional explanation, Stan was not able to give any other than to restate the book's instructions. Thus even though he was able to find the inverse of a given function by using the procedure that was taught in class, Stan was not aware of the relationship between the graph of a function and its inverse apart from what the book told him.

The above examples suggest the participants' view that learning mathematics mainly involves learning to work with abstract symbols in a very structured manner. Mathematics symbols were viewed as the concepts themselves instead of representations of the concepts. Thus, relationships between mathematical ideas were rarely, if at all, recognized beyond their computational aspects in the learning process. It is this view of learning that shaped the participants' perception of understanding in their study of mathematics, and which in turn helped explain why they thought the tests in College Algebra were tricky.

Recall the participants' idea of understanding mathematical concepts was being able to recognize problems presented to them and knowing what to do.

S. (Understanding a concept in mathematics means) to be able to do any problem that they throw at you no matter how they were asked or worded. Understanding comes with practice. The more I use it, the more understanding I get.

J. Understanding a concept, I think, is where I don't need any notes. Just by seeing a problem, I can do it.

B. Understanding a concept means being able to look at it and recognize it, and understand what I am supposed to do in a problem. When I had to sit down and spend a lot of time looking up formulas to see how to do this and that, going over the sections again, going over and over, that wastes, well, it doesn't waste time because that is the only way I learn it, but it takes a lot of time.

On the surface, these responses seem to suggest the format of the problem was not an important issue when one understood the covered concepts; however the participants' explanation of the unfairness in their College Algebra's exams revealed that they were, indeed, quite particular about how the problems were phrased. This can be seen best in Kelli's explanation of the "trickiness" that all participants except James referred to when discussing the tests in College Algebra.

K. The practice exam wasn't a true example of the test. I had difficulty doing the problems because they were asked or given differently than the way they were presented by the instructor, the book, and the practice exam. This is what I mean by "tricky".

Thus, the main problem the participants had with the exams in College Algebra was that the questions were phrased differently than they had expected. Recall that the participants mostly focused on doing the practice exam in their preparation for the first two tests. They worked and reworked these same problems until they felt comfortable doing them; sometimes this task was done three times before the actual test. They reasoned that if they could do the practice

exam which was a test from the preceding semester, then they should be able to do well, or at least make a decent grade, on this semester's test.

Such a rationale seems appropriate and indeed, would be expected from someone who had understood the test material beyond the procedural level. That is, if one was able to understand the relationships between the concepts, then the format of the problems should not be a major issue on the test. This is because he or she would not have to rely on the format for cues as to how the problem should be solved, but on the contrary, would have the flexibility of being able to approach the problem from more than one angle. In that sense, all tests on the same concepts would appear the same to this person although the problems might be phrased differently.

A person understanding the material at this level would be described by Skemp as having conceptual understanding, in which the emphasis is on ideas and relationships rather than on rules and procedures (Skemp, 1987). With this kind of understanding, one can recognize the significance of the information being conveyed in the abstract symbols which represent a concept, and can connect it to one's previous knowledge of the material. Thus, a person can approach learning with flexibility and with a confidence that he or she has the ability to work with a variety of problems dealing with the same concept.

A student who had conceptual understanding of the material would not have much difficulty on the tests in College Algebra, even if the only test preparation was doing the practice exams. However, having conceptual understanding of the material is only a sufficient, not a necessary condition for doing well on the tests. Since the majority of the test questions require little beyond procedural understanding of the material, little is required of the students other than the ability to know which procedure to use and to do the computational work. The more complicated problems usually only number one or two on each exam, these too are similar enough to the homework problems that a student would probably be

able to do them if he could do the harder homework problems.

Since there were so few challenging questions on the tests, not answering these questions correctly would not significantly affect a student's test performance if he or she correctly answered all the others. Therefore, one may do well on the tests in College Algebra by memorizing procedures and rules, and key words that associate them to the problems, and then matching them appropriately and carrying out the necessary computation. In that case, the test would be nothing more than an evaluation of how well a student could memorize meaningless information and be able to match it correctly to the test questions. If Barb, Kelli, David, and Stan had memorized more than just the material on the practice exams and chapter tests, they probably would have scored better on the exams than they did.

Since the tests' structure placed heavier emphasis on routine work than on problem solving, it is likely that the difficulty these participants experienced in taking the exams was not due to lack of procedural understanding, but rather to the difference between their expectation of the tests and what was actually on them. Recall that their preparation for the exams consisted mainly of practicing doing the same set of problems repeatedly until they felt comfortable with them. Homework was used in test preparation mainly to provide reference to some of the problems in the set.

In the case of Barb and Kelli, the matter was more complicated in that they did not feel the homework helped them much in the comprehension of the material. Thus although they used the homework as a resource in their test preparation, they did not review all of it except for the ones that are similar to problems on the practice exam. Like David and Stan, they focused in working the problems on the practice exams, and later the chapter tests, because they believed these problems would be similar to the ones that would appear on the actual tests. This belief motivated them to work hard at these problems, and their confidence increased with each additional reworking of the problems.

In their intensive work on redoing these problems, the participants probably went from expecting to see similar problems on the test to expecting to see identical problems, except perhaps for a slight changes in the numbers used. This transformation of expectation probably occurred inadvertently for the most part, although some of it was conscious. For example, Kelli did not review the material on circles because it was not on the practice exam. Similarly, David and Barb did not pay much attention to the harder homework problems in doing their test preparation, expecting that they would not appear on the test. Stan did not even expect to have to use previous knowledge while taking a test, because he thought the tests were independent of each other.

Thus all participants except James entered the test room with a very specific expectations, which essentially reflected what had been on the practice exams. Indeed, it was this expectation that underlined the participants' frustration with the exams.

D. I did not think my test score reflect my understanding of the material. I probably knew the material better than my test score reflected.

X. And based on what do you have that feeling?

D. Before I walked into that test, you and I went over the practice exam and I knew every problem there was on there ... I did it three times ... The actual test was ten times harder than the practice exam and I don't understand it. The practice exam should be harder than the test because you can spend more time working through the practice exam than the test, so it is backward. That is what I am saying about the tricks and curves.

Not only did these participants have difficulty with the harder problems on the test, they were struggling with some of the easier ones as well, because they were not able to recognize them as problems they had done before. They had difficulty getting enough cues from the format of the questions to determine which procedure to use; partly because they did not have conceptual understanding of

the material, and partly because they did not practice enough on other problems besides what were on the practice exam. Barb suggested as such in reflecting on the third exam.

> **B.** ... If you rely on cramming at the last moment, you are not going to do as well and that is why I said it would have been nice if at the point we were on Wednesday, if that would have been the weekend. Then we would have had a few more days to review and go over it again and get it to set in concrete up here (pointing to her head) ...

Barb's reflection on her understanding in the course also suggests that having the time to practice doing problems and to rework them was an important component in her learning process.

> **B.** ... When I went back and relooked at it again, I realize I understand this stuff, but I just don't know it well enough. I am just at the beginning stages. It's just kind of like the light is starting to go on right now, but I don't know it well enough to be able to give it back on the exam ... I was surprised when I went back and studied for the final and I realized I picked up more than I thought, but it was not enough. I am just at that point where it's just starting to make sense. And now is the time for me to sit down and do all those pages, pages of homework, then I think I could get it. But at the time, it didn't help. I didn't understand it.

Indeed, having the time to work problems and to rework them as often as needed seemed to play an important role in these participants' learning of mathematics. They reported that in the past they had studied mathematics using this method and found it to be quite effective. However they could not do the same in College Algebra because there was more homework assigned and the problems were more involved than in Intermediate Algebra. Thus, once they finished with the homework, they did not have extra time to work additional problems or to rework the ones they had done already.

Since these participants were not able to see many relationships in their learning besides procedures and rules, they had to rely greatly on memorization to do well on exams. They were frustrated with the exams in College Algebra because they did not think they were being tested on what they knew, or more specifically, what they had memorized, and concluded that their test result reflected neither their understanding of the material nor their effort in studying for it.

In some ways, these beliefs are justified if we view the situation from these participants' perspective. If we interpret understanding of the material as being able to do the problems on the practice exam through memorization of rules and procedures, then we can see why the participants did not think the test evaluated them on what they knew: what they knew was exactly what was on the practice exam. They did not seem to realize that the test was not intended to evaluate them on their mastery of these particular problems, but rather on their mastery of the concepts that are behind these problems.

The justification of their belief that their grade did not match their efforts lies in the fact that memorization, in general, is a time-consuming task requiring much mental effort. For someone who can not see the relationships or the patterns in the material they are learning, the task of memorizing a lot of unrelated information can be quite tedious. For example in learning a foreign language, if one has to learn a set of words with some characteristics in common, but is not able to recognize the pattern among them, then he or she has to memorize how each and every one of them is spelled. Not only would this take a lot of time and effort, but one's familiarity with the words would fade quickly without daily practice.

The above considerations, when taken together, illuminate the participants' frustration with the exams in College Algebra. It is not clear whether these students would have reacted differently in a mathematics class where the teacher stressed the learning of relationships behind the concepts rather than the manipulation of abstract symbols. In that case, they could, inadvertently or purposely,

block out their teacher's explanations by focusing only on the mechanical aspects of the instruction — on how their instructor used the procedures and rules to work the different problems. In such a situation, the instructor would still have little success in helping the students to recognize the relationship between the concepts in their learning, and to connect these to their previous knowledge of mathematics.

Another possible explanation for the participants' belief that the test result did not reflect their knowledge of the material had to do with the tests' structure. In College Algebra, the tests consisted mostly of multiple-choice questions, together with only two or three open-ended questions. On the multiple-choice part of the test, only the answers were graded, while on the open-ended questions, both the work and the answers were graded. Since almost eighty percent of the test points came from the multiple-choice questions, it was very important that the students got correct answers on this part of the test. Hence there would be more chance for the students to score poorly on this kind of test than on a test where partial credit is given because on the latter, they could at least get credit for having some procedural knowledge even if they do not know how to do the problem.

Thus it could be possible that the participants' low test scores resulted not from their lack of the knowledge to do the problems, but rather from their making minor mistakes either through carelessness, or through being pressured to complete the test on time. For example, David and James reported discovering they had some problems wrong on the test because they missed a sign in the computation, or because they forgot to carry a sign through an entire expression in parentheses. Such mistakes are easy to make, especially when one is working under a time constraint.

There were other minor reasons why some participants were frustrated with their test results. For example, David was upset that the location of the first test was changed at the last minute without him knowing about it. He reported that

he arrived to the test site fifteen minutes early, only to discover there was no one there, and then had to spend twenty minutes searching for the new test location. According to David, when he actually was taking the test he had difficulty focusing on it because he had not yet calmed down from the panic he had just experienced. He strongly believed his test score of sixty percent would have been higher had he been informed of the change in the test location.

Part of Barb's frustration with the tests had to do with her being exhausted by the time she took them. During the semester, she had three other classes besides College Algebra, which together required her to work on many projects, homeworks, and tests; some of which were due on the same weeks as the tests in College Algebra. To make matters worse, Barb was not comfortable with the material in College Algebra and thus wanted to spend much time preparing for the test. However, finding extra time in her already packed schedule proved to be so difficult that despite her best effort, she always ended up cramming for the tests at the last minute.

Like Barb, Kelli usually arrived to the tests feeling exhausted; however, her exhaustion was due to trying to keep up with the course while taking care of her family. According to Kelli, her typical day started at five-thirty in the morning and ended at eleven-thirty at night, with most of that time spent taking care of her three-year-old daughter and doing housework. She studied two hours daily at home during the week, and six to seven hours on Sunday at the school library. However on the weeks when there was an exam, she spent a few more hours studying because she had to prepare for the exam in addition to getting the weekly homework completed. Although she did not cram for the tests as Barb did, she still had difficulty preparing for the tests since she could not understand much of the material. Although she occasionally expressed the desire to invest more time in her study, she was not able to do it because she did not want to neglect her responsibilities to her family.

The above considerations are of only limited importance in helping us to understand the participants' frustration with the test, for they alone, could not explain the failure grades that Barb, Kelli, and Stan got on some of the tests. Indeed for all the participants, their expectation of what the test would be like was the variable with the most stable and direct influence on the way they prepared for the tests, and so is of crucial importance in understanding their frustration with the test results.

5.2 Discussion of James' success in College Algebra

Of the five participants, only James finished College Algebra with the satisfaction of having understood all the presented material and of having achieved the grade he desired. His success in the course did not come without hard work, for he did indeed work hard in his study even though he was familiar with most of the presented material.

Unlike the other participants, James had a strong mathematics background coming into College Algebra. In high school, he took more mathematics courses than any of the others did and was successful in all of his classes. According to James, mathematics was his strongest subject in high school and he made good grades in it even when he did not attend his classes on a regular basis.

In one instance, James made an A in his mathematics class during his senior year of high school without coming to class more than once or twice a week. He attributed his poor attendance in that particular class to the fact that daily attendance only accounted for five percent of the final course grade and to his being lazy during his last year of high school. Although his attendance was poor in this course, James performed well on all his tests because he was keeping up with the homework independently: he would get the assignments from his friends who attended class and then would complete them at home using the textbook as his guide.

Despite his success with mathematics in high school, James had to take a remedial mathematics course when starting college because he had forgotten most of the mathematics he had learned in high school. At first, he was upset at being in a remedial mathematics class when he thought his high school mathematics background had prepared him for a more advanced course. But, his frustration soon turned into a determination to do well in the course so as to prove to others, and to himself, that he was beyond the remedial mathematics level.

According to James, he attended class daily and did all the assignments. He asked questions when he had them and did not hesitate to stop his teacher during the lectures to ask for further clarification of the presented material. He finished the course with an A and was proud of having succeeded in proving that he did not belong in a remedial mathematics class.

James' next college mathematics class was Precalculus, which he took in the Marine Preparatory School before coming to Oklahoma. According to James, the material discussed in that class ranged from College Algebra to Precalculus, and even a little bit into Calculus itself. All of this was covered in a nine-week period at the rate of one chapter in the textbook every week.

Because most of the material was new to James, he had to work hard to keep up with the class pace. He attended class daily and tried to take all the notes that were given in the lecture. After the class was over, he would immediately go find a quiet spot in which to sit down, fill in the missing parts of his notes, and highlight the important parts.

To supplement his regular lecture notes, James used three-by-five notecards to record important parts of the presented material, such as important formulas or definitions. He had learned to do this in his study skills course at the preparatory school. In fact, he used notecards for each of his classes at the preparatory school, keeping all of them in a plastic box with separate sections for each of the subjects. He carried this box with him every time he went to school, and would take out

some cards to study whenever he had free time during the day. James found that this study method worked very well for him, and he continued using it in his College Algebra class.

According to James, he had to spend many hours doing the homework for Precalculus. Often, he had to stay up into the early hours of the morning to finish it. This task took a lot of James' time because he always showed all his work in doing a problem and would do additional problems if he felt they would help him understand the material. The task became even more intense when it came to test time, for then James would do the chapter tests in the textbook in addition to his regular homework.

Whenever he could not understand the material presented in class, James would ask questions of his instructor, or if necessary would go to the instructor's office hours and continue with his inquiry until all his questions were answered. James reported that his teacher got quite used to him asking questions both inside and outside of the classroom by the time the semester ended.

After nine weeks of intensive work, James finished the course with the top grade in the class and was quite proud of his accomplishment. He attributed his success both to the hard work he put into the course and to the excellent instruction he received from his teacher. In fact, he considers his Precalculus teacher to be the best mathematics instructor that he has had. One thing about this teacher that impressed James was his ability to explain things clearly while using language the students were familiar with. Moreover, he was always willing to help the students both in and out of the classroom whenever they needed it. Thus, it was not unusual to find him in his office past ten o'clock at night helping students, especially during the week of an exam.

With the knowledge he acquired in preparatory school, James did not have to take College Algebra when he came to the University of Oklahoma. However, he did enroll in this course because his supervisor in the Marine Corps had suggested

that James take it easy during his first semester in Oklahoma while getting used to his new surroundings.

For his part, James did not mind taking College Algebra because he felt it would give him an opportunity to gain a more through understanding of the material that was covered in his Precalculus class. According to James, he had not had much time to understand the reasoning behind his work in Precalculus because the material was new to him and it took almost all his effort during the nine weeks of class just to keep up with the assignments.

Although James came into College Algebra already familiar with the course material, he still approached his study as he had in his previous mathematics courses. That is, he still attended class regularly and did all the assignments whether or not they were graded. Moreover, he regularly asked questions in class, and answered his teacher's questions on the material as well.

While the other participants prepared for exams by doing either the practice exam or the chapter tests, but not both; James would work both the practice exam and the chapter tests. Also, he would look over all his old homework problems, especially the difficult ones, to refresh his memory; and sometimes would even rework some of the old homework problems to make sure he knew how to do them.

On the tests, James reported working carefully and always checking his work after he finished working a problem. According to him, he would go through all the steps in his work and ask himself whether he could give a justification for each one. When he could not justify some of the steps in his work, James would try to modify his work or to solve the problem differently.

James made very few mistakes in his work on the tests throughout the semester. Moreover, when he did miss a question, he often knew what he had done wrong and how to fix it. Unlike the other participants in the study, James always had his mistakes on a test corrected shortly after the test was returned.

By the time the semester ended, James felt he understood more of the presented material than he did in his Precalculus class since he had had more time in College Algebra to think about and to practice using what he had learned. Moreover, he was pleased that he got the top grade in his class, feeling that his hard work had paid off.

For the most part, James' success in College Algebra probably had to do with his being more familiar with the material than the other participants coming into the course. However, this advantage alone could not have fully explained the satisfaction James felt with his success in the class. Indeed, if he had done nothing in College Algebra other than taking the exams, his understanding of the material would probably have remained the same as it was when he began the course. In that sense, hard work was responsible for James' success in his study and, more importantly, for his satisfaction with the outcome of the course.

Conclusion

This study was an attempt to gain more understanding of the way nontraditional students approach the learning of mathematics. Attention was given to the views nontraditional students have of mathematics and the possible influences of their views on their learning of the subject. The guiding questions for the study were as follows:

1. What are nontraditional students' conceptions of mathematics?
2. What are the activities that nontraditional students undertake in their learning of mathematics?

Interpretation of the collected data was done using Oaks' and Skemp's conceptual framework of the learning of mathematics. In this section, a summary of the findings will be presented, followed by accounts of the implications of the findings and limitations of the study, and finally suggestions for future research.

5.3 Summary of the study's findings

I. Findings related to the participants' conceptions of mathematics

Several findings resulted from this study: some were similar to findings of past research, others were not, and some gave new information concerning nontraditional students in their learning of mathematics.

One finding that this study had in common with Oaks' study and other previous studies had to do with the students' conceptions of mathematics. The participants in this study appear to have a rigid view of the nature and learning of mathematics: to them, mathematics is a subject dealing with numbers and formulas, in which an answer is either right or wrong and its validity can always be proved or disproved. Thus, learning mathematics means learning to work with numbers and abstract symbols in a structured manner so that correct answers can be obtained. Moreover, accomplishing this task is the same thing as understand-

ing the concepts. Thus, success in mathematics requires spending a lot of time and effort into memorizing rules and practicing these rules in solving assigned homework problems.

Of the five participants, only James appeared to move away from this rigid view of mathematics. Unlike the other participants, James changed the focus of his learning in going from his previous math class to College Algebra: from a focus on applying rules and procedures correctly to a focus on understanding the relationships behind the presented material.

According to James, he had already mastered in his Precalculus class the material which was covered in College Algebra, however, his mastery was only at the mechanical level because he had not had time in Precalculus to go beyond that. Therefore, in College Algebra he focused on knowing the strategies his teacher used in solving various problems and on understanding the reasons used to justify these approaches.

This change in James' learning focus suggested that he was now approaching his study in a relativist manner. However, reexamination of his beliefs about mathematics suggests that there are still many traces of dualistic thinking in his learning of the subject. For example, he reported disliking the idea of teaching mathematics because he did not find much excitement in teaching people how to use different formulas. This response gave the impression that mathematics has little meaning to James beyond the routine manipulation of abstract symbols in a structured manner. However, in expressing the desire to know why things work in College Algebra beyond the manipulation of abstract symbols, James seemed to show awareness that the symbols used in mathematics do have meanings behind them. Even though James' learning goals suggest a different view of mathematics than is expressed in his beliefs about mathematics, he did not seem to be aware of this contradiction. This is probably because he was in a transitional stage between the dualist and relativist perspectives in his learning process. Thus, the

significance of the relativist view of the learning of mathematics was not yet fully realized in James at the time.

As for the other participants, they appeared to have some flexibility in their study of mathematics despite their dualist view of the subject. For example, they liked the idea of having more than one way to solve a problem because it gave them some sense of personal freedom and control in their study. Another sign of flexibility in the participants' learning of mathematics is in their acceptance of informal reasoning. They referred to this type of reasoning as "common-sense thinking" and were comfortable using it in their study. In fact, they even showed signs of excitement and pride whenever they could solve a problem correctly using this type of reasoning. These examples show that the participants' conceptions of mathematics were not completely dualistic and had good potential to develop into more relativist thinking under proper instruction such as was described in Buerk's (1981) and Meyerson's (1977) works.

II. Findings related to the participants' activities in their learning of mathematics.

The findings of this project concerning the students' expectations of their instructor and of themselves in the learning of mathematics were similar to those of Oaks' study. The students expected the teacher to supply them with the necessary information and instruct them on the use of it, while they were to memorize and practice using the given information until, when presented with a problem, they could tell at a glance what to do to solve it.

These expectations motivated the participants to work hard at following their teacher's instructions closely in doing the assignments. In fact, their study behavior in College Algebra was that of the "model student": attending class, taking notes, actively participating in class, preparing for tests, and completing all the assigned homework. Despite their great efforts, however, all but James had difficulty with the exams in College Algebra. Analysis of the data revealed that the

source of their difficulty lay in the fact that they did not have enough conceptual understanding of the material to recognize the similarity between the questions on the tests and the ones on the practice exams, nor did they have enough practice doing different types of problems to compensate for their lack of conceptual understanding. Some of the participants had difficulty maintaining a positive attitude in their study since their efforts to improve the low test scores often failed to bring positive results. In addition, some of the participants' difficulties were compounded by their responsibilities toward their jobs and families. For example, David had to go back to work during the middle of the semester to avoid losing his job, and Kelli had to spend most of her time taking care of her three year-old daughter and doing household chores.

Despite their frustration with the exams in College Algebra, the participants did not give up in their studying as did some of the participants in Oaks' research. Instead, they employed a variety of coping techniques in trying to resolve their difficulties in the course. These ranged from trying to increase the amount of time spent studying for the tests to looking at their low test scores in a positive manner; however, none seemed to bring them the result they wanted. The problem lay in the participants' expectations of the tests and their meaning for the term "understanding" in the learning of mathematics.

To the participants, understanding meant the ability to do problems on the test quickly and accurately. This, together with their belief that the phrasing of the questions on the tests would be similar to that on the practice exam, explains why these participants focused on doing the practice exams (or the chapter tests) in their test preparation, and why their confidence coming into a test greatly depended on how comfortable they were in doing the practice exam.

Although this test preparation method failed to bring them positive results, the participants continued to use it for two reasons. Firstly, it had worked well for them in the past; for example, it had enabled them to do well or pass their

exams in Elementary and Intermediate Algebra. Thus, it was natural for them to expect similar results in College Algebra. Secondly, this was the only learning method they had experienced so far in their study of mathematics. That is, to them learning mathematics meant learning to follow their teacher's instructions on the use of rules and procedures. The word "practice" was heavily stressed in their study of mathematics: practice doing more problems and practice redoing the same sets of problems until automatic performance was achieved. When they failed to do well on a test, the suggestion they often received from others, including their instructors, was to get more practice in doing problems so that they could do them quickly on the exam. In the past, they had found following this advice worked well, since they had the time to practice doing other problems besides the ones that were on the practice exam, and since most of the problems in those classes were simpler than the ones in College Algebra.

These two factors, past success and lack of alternative, combined to make rote learning the basis for these participants' study of mathematics. To them, it represents security since it is something they are familiar with and have had success with in the past. When their grades are at stake, they would be unlikely to venture too far away from the security of rote learning: if anything their attachment to it would be stronger since it provides them some comfort and control in their study. In College Algebra, this security was threatened when the course's fast pace and heavy amount of homework made it difficult for some of the participants to keep up with the class work or to do well on the exams. All except James and David expressed the desire to change their studying approach because they felt their old studying method was no longer effective in this course. However, these participants did not have any idea how to change their study method. To make matters worse, they felt uneasy about experimenting with new studying approaches because they feared an untried approach might prove ineffective and jeopardize their chances of improving their course grades. Thus, they struggled to hold on to what they were

familiar with, hoping that it would at least help them to finish the course with a passing grade.

Thus, even though the participants in this project coped better with their failures in the learning of mathematics than did Oaks' participants, they too relied greatly on the memorization of rules and algorithms in their learning of the subject. In fact, their strong regard for authority and their strong desire for a good grade in the course, made it hard for them to find any other study method to use. On the other hand, these two characteristics could also have motivated the participants to experiment with other learning approaches if they had enough time to experiment and if they had appropriate instruction. For example, rote memorization would have had a less important role in their learning of mathematics if they had been in a mathematics class in which the instructor stressed conceptual understanding of the material. In such a class, the teacher does not interpret the meaning behind the material for the students, but rather guides the students in their making sense of it with the aid of suitable questioning and homework problems. The assignments are selected in a way that helps the students to connect new information with their previous knowledge and to apply what they have learned to new tasks. Thus, the homework serves as an opportunity for the students to explore using the newly acquired information in solving problems, instead of as practice in imitating the instructor's use of rules and procedures. The exams in such a course are constructed so that one would not be able to pass them by memorizing the material and matching what they have memorized with the questions on the test. In such a course, it is important that the students have enough time to explore and to solidify their understanding of the material before new concepts are introduced, so that they have suitable tools with which to learn the new information.

5.4 Implications of the findings

Several implications emerged from the findings of this study. First, the model of "practice makes perfect" did not result in positive learning results if "practice" was interpreted by the students to mean redoing the same sets of problems over and over until automaticity on the use of the presented rules and algorithms was reached. Moreover, interpretation of "practice" in this manner restricted the students' learning in that it did not promote growth in the learning process beyond that of mastering meaningless computational tasks.

Second, assigning more homework will not necessarily help the students perform better if they are not given enough time to do it, and especially if they are not given the opportunity to at least feel comfortable with the procedural aspect of the presented material. Kelli and Barb were quite frustrated in College Algebra because they did not understand the work they were doing on their assignments. For them, understanding meant being able to use rules and algorithms correctly in solving problems. Having such procedural knowledge would at least have given these participants some security and control in their study to wanting to explore the presented material beyond the algebraic manipulation aspect. Thus if a great deal of homework is assigned with the intention of helping the students practice what they have learned in class, it will do more harm than good if the students are not given time to make sense of the information. In such situations, homework ceases to serve the purpose for which it was intended and instead becomes just another meaningless activity that one has to do for a grade.

Third, responsibility for making sense of the presented materials should be shifted from the teacher to the students in the learning process. This is especially critical in the early stages of mathematics learning if the students are to learn that they, and not their teacher, have the responsibility for making sense of the material. The students should be helped to view their instructor as a guide to the material instead of as the official source of knowledge. That is, the instructor's main role

is to provide the students with the different learning experiences and guidance to help them in their interpretation of the material. This might be accomplished by helping the students to read the textbook and to partially make sense of the reading material on their own using their previous knowledge of mathematics. In addition, students should be helped to value tasks such as searching for additional information from other resources in doing their work, making conjectures about the presented material, devising ways to test their conjectures, checking their work, and making generalizations about and connections between the learned material. This might be done through giving students homework problems that require them to do more than practice the use of rules and procedures. In doing such problems, students should be encouraged to discuss their ideas with each other and share information. Through engaging in these activities, the students may come to realize that there is more to the learning of mathematics than computational work. When learning takes place in such a situation, the term "good mathematics teacher" would evoke more from the students than just an image of someone who can show others in detail how to use rules and algorithms correctly to solve routine problems.

Finally, data from this study suggests that the students' use of language in their study of mathematics (at least in low level courses) can be quite different from that of the instructor. That is, in discussing mathematics students and teachers might give different interpretations to the same words. For example, understanding of mathematical concepts has a different meaning to the students than to their instructor, as do such terms as mathematical theorem or mathematical principle. Hence, miscommunication between teacher and students can easily occur and lead to frustrations in the learning process. Thus, it is necessary for mathematics instructors to be aware of the possibility that they and their students might not interpret words alike even though both are using the same vocabulary in discussing mathematics. In addition, it is important that instructors listen closely

to their students' use of language in their study of mathematics and to explore the meaning behind it, for it might give them some clues as to the kind of difficulties their students are experiencing. Also, such an activity would help teachers find out what their students already know or do not know about a new topic so that appropriate class activities or homework could be planned in advance.

5.5 Limitations of the study

There were some limitations of this study that needed to be mentioned. One concerns the participants' inability to address the topic of their conception of mathematics in a direct manner. Throughout the study, questions aimed at getting the participants to discuss their conceptions of mathematics elicited short responses, and attempts to obtain clarification met with little success. Thus inferences about the participants' conceptions of mathematics had to be made through observing their studying behavior and noticing the contradictions that sometimes appeared between their responses and the activities they engaged in while studying mathematics.

This limitation was to be expected considering the participants' mathematical backgrounds and the type of instruction they had received in their learning of the subject. It was hard for these participants to discuss aspects of mathematics beyond the mechanical ones when they had had no experience of mathematics as anything but the mechanical manipulation of abstract symbols. Hence the use of case studies as a research method was critical to the success of this study because it provided the flexibility that this researcher needed to collect data about the participants' conception of mathematics.

As the semester progressed, the researcher also encountered problems in gathering information from two of the participants, David and Stan, concerning their studying habits in College Algebra. With David, the difficulty lay in the fact that he had to go back to work in the second half of the semester. The pressure of

working more than forty hours per week and trying to keep up with 15 hours of classes at the same time made it difficult for him to make time for this research project, and even when he could find time to meet with the researcher, he was exhausted. Thus, it was not possible to obtain detailed information from David about his work in College Algebra after the first half of the semester.

As for Stan, the difficulty appeared to be in the way he chose to cope with his low performance in the course. As the semester progressed, the researcher observed some changes in Stan's behavior. He began to appear more reluctant to meet the researcher for our regular discussions about his work in College Algebra, and twice when we did finally set up an appointment he failed to show up for it. When asked about his absence, Stan just apologized and said that he had forgotten the appointment. On one occasion, he canceled an appointment on the same day we were to meet, saying he had to take his son to the doctor for a checkup which had been scheduled some time in advance.

Also, Stan became more vague in his responses during the interview sessions, and attempts to get him to clarify them further ended in failure. Often, his response to such a request was "I don't know" along with a shrug. On two or three occasions, he even contradicted his own responses from another interview session, and did not seem to be bothered with the change in his responses when it was brought to his attention. For example, after the first exam in College Algebra, Stan reported planning to get all the old College Algebra exams from the test file office so he could practice working them before the second test. His theory was that the test difficulty level in a course tended to go in a cycle and so if one worked enough of the old exams, he would be familiar with all the difficulty levels and thus would be able to do well on the current exams. In the interview session after the second test was given, Stan reported that he did not have time to carry out this plan. However, in a later interview session after the third exam was given, Stan reported that he did indeed carry out this particular plan before the second

exam, but not before the third exam. This response again was altered in our last interview session when he was asked whether there were any differences in his test preparation for the three exams in the course. According to him, he had prepared for all three tests in the same way, except that he had focused on the practice exams for the first two exams and the chapter tests for the third exam: for none of the three tests, he said, had he used old exams from the test file office.

Stan's responses throughout the semester also show a change in his attitude towards grades. At the beginning of the semester, he reported wanting to get A's in all his classes because good grades were important to his chances of getting accepted into graduate school. However as the semester progressed and he continued to perform poorly on the tests in College Algebra, Stan became more accepting of the D's he received on his work. In fact by the time the semester ended, Stan had become convinced that getting good grades was no longer important to him. Instead, it was enough that he gave his best effort. Thus, he was content to continue on to his next math class even though he had barely passed College Algebra.

Changing his views of the importance of grades may have been a way for Stan to cope with his poor performance in the course, allowing him to protect his self-esteem and to avoid stress despite not being able to achieve the goals he had set at the beginning of the semester. This may also explain his increasing reluctance to meet with the researcher after the first exam and the vagueness of his responses. Thus, for students like Stan, we need a different research approach or a different set of interview questions to help us understand their conception of mathematics and the way they approach the learning of this subject.

5.6 Suggestions for future research

This study set out to gather more information about the way nontraditional students approach their learning of mathematics. In addition, attention was given to the types of difficulties they experienced in their learning of the subject. Several

interesting findings have resulted from the study; however, there were some areas the researcher was not able to explore due to the limit on both time and resources.

One is to explain why some students succeed in a mathematics class that they are taking for the second time after having failed it once already. Of the five participants in this study, two reported having failed their Intermediate Algebra class the first time they took it but having succeeded in passing it the second time. In College Algebra, none of the participants failed the course, but two considered that their grade of "D" meant they had failed to learn the material and so chose to repeat the class. Of the three who repeated the course (in the semester after the study was completed - two with grade of "D" the first time and one who dropped out of the course the first time), one finished the course with a grade of "B" on her second try in spring 1995, and reported liking the course the second time much more than the first time. Both the other two dropped out of College Algebra after enrolling in the course a second time in spring 1995, but one of them enrolled in the course again in the fall of 1996. It would be interesting and useful to know why some of these students were able to succeed on their second attempt after not being able to succeed the first time. Did their success result from their having more familiarity with the material through additional practice working with problems? Was it due to them having more time to memorize rules and procedures for solving problems? Was it due to a different type of instruction in the course the second time? Answers to these questions could help us structure these courses to increase our students' chances of succeeding on the first try. This would not only help them to save time and money, but more importantly would help prevent a loss of self-esteem in their learning process.

Another area that could be a possible future project is to evaluate the use of cooperative learning with nontraditional students in mathematics. On one hand, by providing them the opportunity to interact with their younger colleagues, this type of instruction might help to increase their confidence in sharing and discussing

their interpretation of the material with their classmates rather than being intimidated by them. On the other hand, this type of instruction might not work with nontraditional students, since they might experience more difficulties than the younger ones in adapting to a mathematics classroom environment where the instructor is a facilitator instead of a lecturer. Information on this question could help us in designing the kind of instruction that best meets our nontraditional students' needs in their learning of mathematics.

A third possibility for future research could be to investigate the conceptions of mathematics that college mathematics instructors, especially those who teach introductory mathematics courses, have of the subject. In the past, although work has been done on the topic of teachers' conceptions of mathematics, it has focused on high school or elementary school teachers. Moreover there is evidence from past work (Thompson, 1984; Kesler, 1985) to suggest that there is a connection between teachers' conceptions of mathematics and their teaching activities. Thus, information on the conception of mathematics held by college instructors might be useful in helping us understand the goals that these instructors have for their students and the teaching activities that they engage in.

Another future research project could be to investigate why some students pass Intermediate Algebra at a high level but fail to do well in College Algebra. For example, Barb, Kelli, and Stan finished Intermediate Algebra with a grade of B or better, but all received D's in College Algebra despite the hard work they invested in the course. Why did these students have difficulties in College Algebra when they had not only succeeded passing Intermediate Algebra, but did so at a high level? To answer this question, it may be helpful to evaluate the material covered in both Intermediate Algebra and College Algebra and the depth of the coverage to see whether there is a gap between the two courses. In addition, we could evaluate the tutoring services on campus to see whether there is a need to provide special training for tutors who work with students at the introductory

mathematics level. For example, some tutors might not be aware that many students at this level of mathematics are easily confused if they receive different explanations from the tutors on how to do the same mathematics problem. Also, evaluation of the content of College Algebra could show whether some topics could be eliminated to make additional time for the remaining ones. With fewer topics and more time to learn them, the students would have a better chance to make sense of the material. Answers to these questions could help us to find ways to help students like Barb, Kelli, and Stan to succeed in College Algebra the first time they take it.

5.7 Epilogue

I began this project with the goal of gathering information on nontraditional students in their learning of mathematics, in hopes that my findings could be used to help inform mathematics educators working with nontraditional students. Not much research had been done on this topic previously, and I thought that knowing more about how nontraditional students view and learn mathematics would help mathematics educators help these students in their learning.

Information from the participants played an important role in sharpening my research focus as the project progressed. Also, my experience as a learner and as an instructor of mathematics guided me in my interpretation of the collected data by providing me with clues to the rationales behind the participants' beliefs and behaviors. I found the nontraditional students I worked with to be more studious than the younger students to whom I had taught College Algebra in the past. Moreover, I found them to be persistent and not easily discouraged when encountering failure in their work. However, I did find that their beliefs about the nature of mathematics and the learning of mathematics were similar to those held by the students of traditional college age in Oaks' and Schoenfeld's studies. That is, the participants believed mathematics to be a collection of rules and procedures,

which must be memorized if one is to succeed in his learning of the subject. In fact, memorization may be even more ingrained in the learning habits of nontraditional students than students of traditional age, since the nontraditional students have been learning this way for a longer time.

Based on my experiences in this study and in teaching College Algebra, I now believe that some of the difficulties students have in the course can be traced to miscommunication between them and their instructors. For example, instructors and students can interpret the words "practice" and "understanding" differently, without realizing the difference. When Kelli and Barb asked their instructor for suggestions on how to prepare themselves for the next class exam, she recommended they get more practice doing problems. They worked hard at following this recommendation, but were frustrated when their test results did not improve. By "getting more practice doing problems", the instructor had meant doing a greater variety of problems; but Kelli and Barb interpreted her to mean redoing the same set of problems (the ones on the practice exam) a greater number of times. Their interpretation of the teacher's words might have been influenced by the success they had had with this method of test preparation in their Elementary Algebra and Intermediate Algebra classes.

This project has taught me to pay more attention to the students' interpretation of "understanding" in their learning of mathematics. I learned that I cannot be certain of my students' understanding of the material unless I see that they can apply their knowledge to new tasks. I believe instructors of mathematics courses at the introductory level should be careful in their evaluation of their students' understanding because at this level, the students' ability to do routine homework problems by imitating the instructor's use of mathematical rules, procedures, and vocabulary can be mistaken for conceptual understanding of the material.

In doing this project, I had an opportunity to reflect upon the role of homework and its possible effects in the learning of mathematics. I learned that the

model "the more, the better" can do more harm than good to students if the students have no understanding of the material to begin with. When the students cannot make sense of the learning material, assigning them a lot of homework to do in a short amount of time encourages them to look up the answers in solution manuals and study the material by memorizing the procedures they find there. Besides making the students dependent on rote memorization in their learning, this situation increases their frustration as they find themselves unable to do the problems on their own. Hence, the instructor should limit the number of problems in the assignments so that the students have time at least to be comfortable with the procedural aspects of the material, giving them a chance to go on and try some of the nonroutine problems.

Finally, this project taught me that fear or anxiety towards mathematics which students develop in elementary or middle school can have long-term effects on their subsequent learning of the subject. For example, Kelli reported that because her third grade teacher did not spend much class time on mathematics and did not explain mathematics well, she had felt uncomfortable with mathematics ever since. As a result, she still does not want to take any more mathematics courses than are required for her major; and in the mathematics classes she does take, she depends heavily on the instructor for direction in understanding the material and in doing the homework.

To prepare their students to develop into independent learners later, elementary and middle school mathematics teachers should not only teach arithmetic skills, but should also teach problem-solving skills and promote their students' self-confidence. Such instruction would benefit all students, including the ones who are good at using rules and procedures. But to be able to provide such instruction, the teachers themselves need better instruction in problem-solving skills as part of their preservice training, as well as more support and resources from the school system and from the public.

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