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A DISSERTATION

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THE ABILITY OF FIFTH AND SIXTH GRADERS TO

LEARN SELECTED TOPICS IN PROBABILITY

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DEDICATION

This study is lovingly and respectfully dedicated to my parents, Mr. and Mrs. John H. Winston.

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CHAPTER I

INTRODUCTION AND PROBLEM

Introduction

In the past two decades a revival of interest in school mathematics has been exhibited by practically everyone in society. Mathematical scholars, educators, subject matter specialists and psychologists are but a few among these. Beginning with the secondary school program, reformers have also directed efforts toward effecting changes at the elementary school level.

Curriculum revision has been the major emphasis of most reformers of school mathematics. As a result, a wide variety of "new" topics, among which are probability and statistics, have been introduced in the elementary school program. These areas had not been included in the traditional courses of elementary school mathematics and even as late as 1959, a search of courses of study for the first six grades for any specific reference to probability and statistics would have resulted in little success.

Hardly anything was done in probability and statistics at the junior and senior high level at the time.[1]

Beginning in 1959, reference to the study and teaching of probability and statistics in the elementary school can be found. Rationales to support the inclusion of topics in these areas at the elementary school level have been given, and suggestions of topics to be included and methods for teaching probability and statistics have rapidly evolved. Some commercial textbooks have included topics in probability and statistics in their elementary series and research concerning the feasibility of teaching topics in probability and statistics in the elementary school has been conducted.

Most developments with respect to teaching probability and statistics to elementary school children have been built on subjectivity. There is also no real agreement on what can and should be taught and how it should be taught. Whether elementary school children possess the ability to understand certain topics in probability and statistics has not been given much attention. This, however, is not surprising, since the ability of precollege students to understand certain ideas related to these areas

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[1]Richard S. Pieters and John J. Kinsella, "Statistics," The Growth of Mathematical Ideas, Grades K-12, Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1959), p. 272.

has not been given sufficient attention.[2]

A paramount consideration for the inclusion of subject matter at any level of schooling is the ability of students to comprehend the ideas involved. However, the use of this consideration as the sole criterion for the inclusion of a subject area is not adequate. The areas of probability and statistics are said to be of value to the elementary school program for several reasons. One of the most important of these is the value of the areas as tools for society. If it is indeed the case that a knowledge of these areas is rapidly becoming more and more essential to the general education of all citizens as well as to the vocational preparation of an increasing number of future specialists, then there is valid reason for giving them more attention in the schools.[3] Referring to revisions in the secondary school mathematics program, the Commission on Mathematics of the College Entrance Examination Board states:

> So great is the current scientific and industrial importance of probability and statistical inference that the Commission does not believe valid objection can be offered to its inclusion in the curriculum.[4]

Because of their wide utility it is believed that everyone

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[2] _Ibid_., p. 274.

[3] _Ibid_., p. 273.

[4] Commission on Mathematics of the College Entrance Examination Board, _Program for College Preparatory Mathematics_ (New York: College Entrance Examination Board, 1959), p. 32.

should have some knowledge (no matter how little) of probability and statistics; thus, to insure this, it is felt that the teaching of these areas should commence in the elementary grades.

In addition to the importance of probability and statistics to society, other justifications for including them in the elementary school program are:

1. The introduction of probability and statistics in the elementary school program provides novel, interesting, meaningful and exciting topics as well as practical and useful application of arithmetical processes.[5]

2. Too much time is spent in the conventional program in an effort to develop speed and accuracy in use of the manipulative skills. The introduction of probability and statistics will aid in these developments and new subject matter is obtained as well.[6]

3. Many of the questions students face give the impression that every question has a single definite answer. The study of probability and statistics opens up questions which have answers of an indefinite character. As a

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[5]David A. Page, "Probability," The Growth of Mathematical Ideas, Grades K-12, Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1959), p. 230.

[6]Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, Published for Educational Services, Inc. (Boston: Houghton Mifflin Company, 1963), p. 16.

result, their study also may help alleviate many poor patterns of reasoning common in ordinary life.[7]

4. After leaving elementary school fewer and fewer students will study mathematics. A move in the direction most suitable for those who take mathematics for only a few years after elementary school must be attempted. Of particular importance is an intuitive feeling for probability and statistics.[8]

The reasons cited above do lend support for including probability and statistics in the elementary school program. Whether the reasons are valid has not been sufficiently substantiated. Even if valid, the reasons do not suggest the concepts that should be taught and approaches that should be used. A review of available information pertaining to the study and teaching of probability and statistics in the elementary school reveals suggestions for content and methodology.

Review of Related Materials in Probability for the Elementary School

Since 1959, support for the inclusion of probability and statistics in the elementary school program has been received from those concerned with revamping the

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[7]Jack D. Wilkinson and Owen Nelson, "Probability and Statistics--Trial Teaching in Sixth Grade," The Arithmetic Teacher, XII (February, 1966), 106.

[8]Goals for School Mathematics, p. 9.

mathematics curriculum in general and those interested in probability and statistics in particular. As a result, the period since 1959 has been marked by a growth of ideas for the study and teaching of probability and statistics in the elementary school. Concepts to be taught, methodology, and materials to be used have been suggested. A more recent development is the inclusion of topics in probability and statistics by some commercial textbooks for the elementary school. The review of the related materials and research concentrates on probability in the elementary school since the present study was limited to this area.

David Page[9] has written a unit on probability in which suggestions for presenting some of the proposed concepts for the elementary school are given. One purpose of the unit is "to familiarize the student with some of the fundamental ideas of probability and expose him to real and imagined experiments. . . ."[10] Page feels that it is not now feasible to teach probability as a pure mathematical subject in either elementary or high school. Rather, he believes that many notions, important in probability theory, can be grasped in a clear and intuitive way by children.

Page believes that in the elementary school the

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[9] Page, "Probability," pp. 229-271.

[10] Ibid., p. 230.

approach to probability should be basically through experiments. Through experiments with coins and dice, for example, experience in recording data and with graphs and coordinate geometry can be provided. The elementary school child can also be introduced to computing probabilities of mutually exclusive and independent events.

Page points out that his unit on probability does not mean that he favors a separate course. The subject matter of probability is primarily important, noted Page, in so far as it can be used to add interest, understanding, and pleasant "calisthenics" to the more fundamental parts of the curriculum.[11]

In 1963, a prestigious group of scholars and applied scientists outlined in the Report of the Cambridge Conference on School Mathematics a mathematics curriculum believed to be feasible by approximately the year 2000. This report clearly indicates that the study of chance situations is a vital and appropriate part of the mathematics program of the elementary school. Recognizing that probability theory can be regarded as a branch of pure mathematics, the report nevertheless recommends that it be treated as a branch of applied mathematics, aimed toward statistical inference and decision making. It also suggests that probability be taught in four phases through the curriculum. The first phase, in the elementary school, should

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[11]Ibid., p. 229.

consist of the empirical study of the statistics of repeated chance events, coupled with some arithmetic study of the workings of the law of large numbers.[12]

The subject of probability is more than just useful, states Lola J. May.[13] It is fun for the elementary school student, especially if taught through examples which have real meaning for him. She believes that it will make a welcome change from the boredom of daily computational work and at the same time teach a lot of mathematics. May suggests that a unit on probability can consist of a set of experiments written out to enable students to work independently or in small groups. Questions should be phrased in such a manner as to guide the student toward an understanding of basic probability concepts.

One of the most refreshing developments in mathematics in the last twenty-five years, states Ranucci,[14] has been the introduction of simple aspects of serious mathematics into the curriculum of the elementary school. Among the more stimulating of these is probability. Creating activities for four areas in the "new math," Ranucci included activities on probability and suggests, as did

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[12] Goals for School Mathematics.

[13] Lola J. May, "Probability--Chance for a Change," Grade Teacher, LXXXVI (January, 1969), 31-32, 34.

[14] Ernest R. Ranucci, "Activities for . . . Four Areas in the New Math," The Instructor, LXXIII (February, 1964), 63-64.

Page, that one approach to the study of probability is through use of the number line.

All of the works cited above were articles obtained from journals or yearbooks and represent the opinions of the authors. Limited in scope and objectivity though these works are, they serve the purpose of providing suggestions for content and methodology for the teaching of probability in the elementary school, and thus lend support for their inclusion at this level. More support comes from the commercial textbooks that now include probability topics and from publications prepared exclusively on probability for the elementary school.

The most extensive work published in probability for the elementary school is that by the School Mathematics Study Group (SMSG). When the Cambridge Report recommended that some basic ideas of probability be introduced very early in the school program, SMSG undertook the task of developing lessons in the area for grades kindergarten through high school.[15] Two books have been written for use at the elementary school level. One is for use in the primary grades (kindergarten-three) and is titled <u>Probability for Primary Grades</u>;[16] the other is to be used in the intermediate grades (four-six) and is titled

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[15]School Mathematics Study Group, <u>Probability for Primary Grades</u> (Palo Alto, California: Stanford University Press, 1966). Preface.

[16]<u>Ibid</u>.

Probability for the Intermediate Grades.[17]

The writers of the SMSG lessons point out that probability has many practical uses and on a practical level even quite young children are not strangers to probability. The lessons are designed to aid in using and improving the child's intuitive knowledge of what constitutes "an equal chance" or "a better chance" and to show that there is mathematical justification for many of their ideas. The concepts of certainty and uncertainty, likelihood of various events, combining events, number possibilities, ordered arrangements, and probability are presented in eleven lessons in the unit prepared for the primary grades. The unit for the intermediate grades is an extension of the ideas presented in the primary unit and in addition includes activities designed to help children learn about chance, the use of graphs and numbers to express probability, complementary events, Pascal's triangle, and conditional probability. The authors also suggest that several fringe benefits can be derived as a result of studying probability. These include:[18]

1. It should be fun for the children.
2. It should help develop systematic thinking.
3. Arithmetical skills may be practiced.

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[17]School Mathematics Study Group, Probability for the Intermediate Grades (Palo Alto, California: Stanford University Press, 1966).

[18]Probability for Primary Grades, p. 2.

4. It offers opportunities for independent investigation.

The examination of works in probability for the elementary school reveals that most of those not prepared exclusively on probability do not begin the study prior to the sixth grade. One exception to this is Sets and Numbers[19] by Patrick Suppes. Probability is begun in the fourth grade textbook of this series and the concepts of outcomes, probability of outcomes and events, and combinations of events are introduced. The fifth grade textbook extends the concepts introduced in the fourth grade and in the sixth grade there is an extension of the concepts introduced in the previous two grades; also included are the concepts of equally likely, impossible, and independent events.

Another work dealing with probability in the elementary school is the textbook Mathematics[20] by Gundlach, et al. The sixth-grade textbook is the only one of the elementary series which presents any probability concepts. The concepts of probability presented are outcome and probability of a successful outcome.

Also including topics in probability is the textbook,

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[19] Patrick Suppes, Sets and Numbers (Atlanta: The L. W. Singer Company, 1966).

[20] Bernard H. Gundlach, et al., Mathematics (Summit, New Jersey: Laidlaw Brothers, 1968).

Elementary School Mathematics,[21] written by Eicholz and O'Daffer. The over-all objective of the lessons in probability is to provide experiences with simple concepts of probability and develop an intuitive feeling for some of the basic ideas of probability such as randomness, outcome, bias, sample space, events and chance. The authors feel that this objective can best be obtained by approaching the study of probability through experiments. In the experiments, objects such as coins, rods and paper fasteners are used and resource materials include encyclopedias and almanacs.

Seeing Through Arithmetic[22] (STA) by Hartung, et al. and Modern School Mathematics: Structure and Use[23] by Duncan, et al., both include work on probability. In the former, the child is not asked to perform a single experiment, but is expected to understand the concepts through the use of hypothetical experiments. The probability concepts introduced include outcome, equally likely outcomes, probability and complementary events. The latter textbook, unlike the former, is completely problem-solving oriented. The approach utilizes neither real nor imagined

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[21]Robert E. Eicholz and Phares G. O'Daffer, Elementary School Mathematics (2nd ed.: Menlo Park, California: Addison-Wesley, 1968).

[22]Maurice L. Hartung, Henry Van Engen, and Lois Knowles, Seeing Through Arithmetic (Chicago: Scott, Foresman and Company, 1968).

[23]Ernest R. Duncan, et al., Modern School Mathematics: Structure and Use (Boston: Houghton Mifflin Company, 1967).

experiments and without any preliminaries the authors proceed directly into the presentation of the probability of an outcome.

The review of the existing publications for the study and teaching of probability in the elementary school indicates the absence of any consensus. It is evident that the basic question as to whether the study of probability should be a part of the elementary school mathematics program remains unanswered. If the answer to this question is in the affirmative, then other questions, so far unanswered, are generated. Among these questions are:

1. At what grade level shall instruction begin?
2. What concepts shall be included at a particular grade level?
3. What degree of rigor and sophistication shall be required?
4. What methods shall be used?
5. Shall probability be taught as a unit complete in itself or shall it be broken into small units and taught throughout the entire mathematics program?

Review of Research Related to the Study

A review of the research indicates that partial answers to some of the questions posed above do exist. The present study is concerned with determining if fifth and sixth grade children can learn selected topics in probability. Thus, research concerning the ability of elementary school children to learn certain topics in probability is important. Also research confirming the success of various approaches and materials used is of value. Research attesting to the following other behavioral changes as a result of studying probability is also important:

1. attitude toward mathematics in general and probability in particular.
2. ability to perform the arithmetic skills, and
3. problem solving ability.

Studies have been conducted which present a developmental description of the probabilistic concepts held by children prior to formal instruction. These studies, however, were conducted for a reason different from that of the present study, since they are concerned principally with determining the characteristic mode of response of a subject in a probabilistic situation. Although not concerned with changes in performance during experimental sessions, they do reveal age levels at which children begin to possess the ability to perform successfully in probabilistic situations. Studies of Siegel and Andrews,[24] Yost, et al.,[25]

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[24] S. Siegel and J. M. Andrews, "Magnitude of Reinforcement and Choice Behavior in Children," Journal of Experimental Psychology, LXIII (1962), 63, 337-341.

[25] Patricia A. Yost, Alberta Engvall Seigel, and Julia McMichael, "Nonverbal Probability Judgments by Young Children," Child Development, XXXIII (December, 1962), 769-780.

Messick and Salley,[26] Stevenson and Zigler,[27] and Cohen[28] represent this type. These studies sought to determine if young children are able to utilize the concept of probability. The results indicate that children, even as young as four years of age, do demonstrate some understanding of probability if the appropriate conditions are presented. These conditions include:

1. Use of appropriate amounts of reinforcements.
2. Use of the proper amounts of incentives or rewards.
3. An opportunity for non-verbal decision-making.

Studies have also been conducted in an effort to determine if elementary school children possess knowledge of probability prior to formal instruction in it. Studies of this type include those of Leake,[29] Doherty,[30] and

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[26]Samuel J. Messick and Charles M. Salley, Probability Learning in Children," The Journal of Genetic Psychology, LXX (February, 1957), 23-32.

[27]Harold W. Stevenson and Edward F. Zigler, "Probability Learning in Children," Journal of Experimental Psychology, LVI (September, 1958), 185-192.

[28]John Cohen, "Subjective Probability," Scientific American, CXCVII (November, 1957), 128-38.

[29]Lowell Leake, Jr., "The Status of Three Concepts of Probability in Children of Seventh, Eighth and Ninth Grades," The Journal of Experimental Education, XXXIV (Fall, 1965), 72-80.

[30]John Doherty, "Levels of Four Concepts of Probability Possessed by Children of Fourth, Fifth and Sixth Grade before Formal Instruction" (unpublished dissertation, University of Missouri, 1965).

Leffin.[31]

The experiment performed by Leake involved the use of seventy-two seventh, eighth and ninth graders. The three probabilistic concepts assessed were sample space, probability of a simple event, and probability of the union of two or more mutually exclusive events. Leake observed that the students had acquired knowledge of problems about probability and the ability to deal with problems of a probabilistic nature. He concluded that this knowledge was not due to formal school instruction in probability but must have come from other experiences in the lives of the subjects.

Doherty assessed the levels of the concepts of sample space, probability of a simple event, probability of the union of mutually exclusive events, and difference in mutually independent and mutually exclusive events possessed by fourth, fifth and sixth grade children. He also concluded that the children in the study were familiar with the selected concepts of the mathematics of probability and had acquired the ability to deal with probability problems. Doherty felt this acquaintance was not due to formal instruction but must have come from everyday experiences.

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[31]Walter William Leffin, "A Study of Three Concepts of Probability Possessed by Children in the Fourth, Fifth, Sixth and Seventh Grades," (unpublished dissertation, University of Wisconsin, 1968).

Leffin examined the status of three concepts of probability possessed by fourth, fifth, sixth and seventh grade children. The concepts investigated were points of a finite sample space, probability of a simple event in a finite sample space, and the quantification of probabilities. The study was carried out during the first semester of the 1967-68 academic year in the Wausau, Wisconsin Public School System. The children in the study were classified into subgroups according to sex, I.Q. (three categories) and grade (four), thus, making a total of twenty-four subgroups. The most significant outcome of the study was that the children did possess knowledge about the three concepts and could apply them in a variety of situations. Since the children possessed no formal training in notions of probability, Leffin attributed their understanding to their background, experience and intuition.

Evidence furnished by the status studies indicates that children do possess knowledge of probability prior to formal instruction in it. Since they do, perhaps there are some topics that are not too difficult to include in the elementary school program. However, there is need for the actual teaching of the assessed concepts at the levels specified to help determine if additional learning takes place and if actual teaching of the concepts has any effect on their ability to apply them. In addition, it might be possible to determine if the teaching of these concepts has any effect on other facets of the total learning program.

In the summer of 1966, Shulte[32] prepared a 199-page unit, Mathematics of Uncertainty, and a 97-page teachers' guide for the unit. During the 1966-67 school year, he undertook a research study to investigate the results of introducing a substantial unit on probability and statistics into the ninth grade general mathematics course, in an attempt to effect an improvement in that course by providing new, interesting, and meaningful topics. The probability concepts included in the unit consisted of the probability of simple, independent and mutually exclusive events; conditional probability, permutations and combinations.

The unit was taught in eighteen classes of ninth grade general mathematics in five districts of Oakland County, Michigan. Seventeen classes of ninth grade general mathematics students in the same five districts, using their regular materials, were used as a comparison group. The writer originally intended the unit to constitute about eight or nine weeks of the school year; however, teaching time for the unit was left to the discretion of the teachers, and was dependent upon internal

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[32] Albert P. Shulte, "The Effects of a Unit in Probability and Statistics on Students and Teachers of Ninth Grade General Mathematics," The Mathematics Teacher, LXII (January, 1970), 56-64.

factors in the several school districts. Thus, actual teaching time ranged from five to about fifteen weeks.

Results revealed that significant improvement was shown on the variables of probability, permutations and combinations, thus indicating that ninth grade general mathematics students can learn topics in probability. Findings also revealed that the comparison group improved significantly in the computational skills (adding, subtracting, multiplying and dividing). The experimental group, however, improved significantly only in multiplication, declined significantly in division, and there was no significant change in their ability to add and subtract.

Schulte also found that scores on the Aiken-Dreger scale (Attitude toward Mathematics) of students studying the unit on probability and statistics decreased significantly. The study revealed further the absence of any striking effect of these topics on the attitude of the students toward the usefulness of mathematics.

Even though conducted on ninth grade subjects, the study of Schulte carries several implications for teaching probability at the elementary school level and for research that can be conducted at this level. The study not only sought to answer questions about the feasibility of teaching topics in probability and statistics to ninth graders but also tried to ascertain if the introduction of these topics into the mathematics program carried with it the attainment of certain by-products.

One of the most extensive studies conducted on the teaching and learning of probability and statistics in the elementary school is that of Shepler.[33] The purpose of the study was to test the feasibility of teaching probability and statistics to sixth grade students. The unit was designed by the researcher and consisted of examining certain probability and statistical concepts and principles, breaking these concepts and principles into their learning components, then arranging the components into a hierarchy.

The unit was taught to twenty-five sixth grade students attending school in a farming village near Madison, Wisconsin, and possessing average to above average ability (mean I.Q. was 117.7, S.D. was 6.8). Shepler identified fourteen behavioral objectives for assessment as a result of studying the unit on probability and statistics. The goal of the instruction was to demonstrate "mastery of learning" of the behavioral objectives. Mastery learning of an objective had been acquired, theoretically, if 90% of the students achieved 90% of the behavioral objectives. However, for practical purposes, this level of mastery had to be altered depending upon the number of students and the number of activities designed to measure the objective.

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[33]Jack Lee Shepler, "A Study of Parts of the Development of a Unit in Probability and Statistics for the Elementary School" (unpublished dissertation, University of Wisconsin, 1969).

The concepts and principles were taught in nine lessons over a four-week period. The first lesson, designed to introduce much of the vocabulary of the unit, also utilized the models that were to be used in future lessons. In this lesson the students' concepts of certainty, uncertainty, and impossibility were to be strengthened by class discussion and written exercise. In the next lessons the children carried out experiments using the models and making graphs of the results. The a priori probability of an event in a one-dimensional space was introduced in the fourth and fifth lessons. In lesson six, counting procedures (using tree-diagrams) for two- and three-dimensional outcome spaces and the probability of an event in these sample spaces were presented. The ability of the children to determine if two events are equally likely or if one event is more likely than another was considered in lessons seven and eight. The law of averages and the meaning of the probability of an event were introduced in the final lesson.

Results revealed that instruction was successful on ten of the behavioral objectives, very close to being successful on one and unsuccessful on three. The results of the objective "to identify an instance of the law of averages" almost satisfied the criteria of mastery learning. The results of the objectives of "count the number of outcomes of an event," "specify the estimated probability of an event, given the data from an experiment," and identify an estimate of the true probability given a set of data from an experiment" did not satisfy the criteria. The over-all results, nevertheless, support the fact that the sixth grade students used in the study did possess the ability to learn most of the topics included in the study.

Wilkinson and Nelson[34] conducted trial teaching in probability and statistics using a class of twenty-two sixth graders enrolled in the Malcolm Price Laboratory School of the State College of Iowa, Cedar Falls. The experiment was conducted the last three weeks of the first semester of the 1964-65 school year with each teaching session lasting forty-five minutes. The most basic consideration in the experiment was that the ideas and experiences have real meaning for the children. Ideas were initiated at a low level of sophistication and carried out only as far as class interest was maintained. They felt that the students should develop ideas through actual experimentation and as far as possible use accepted mathematical terms and notations. They wanted the students to have experiences with the probability concepts of randomness, sample space, event, outcome, probability, odds, bias, law of large numbers, levels of confidence, sampling, and equally likely

[34]Wilkinson and Nelson, "Probability and Statistics," pp. 100-106.

and unequally likely events. Skills they wanted the students to develop included recording and interpreting data, making and testing hypotheses, observing and differentiating between relevant and irrelevant data, and communicating information to others in a clear and precise manner.[35] Many of the concepts were dealt with on an intuitive level, without definition or use of vocabulary.

The experiment was judged successful; however, since there was no indication of formal testing for the assessment of outcomes, all conclusions seem to have been subjectively drawn. The researchers felt that probability experiences in the elementary school are worthwhile since they open up questions which have answers of an indefinite character. They thought it important that elementary school children be exposed to events which have a degree of uncertainty.[36]

As a result of the experiment, Wilkinson and Nelson made the following recommendations:[37]

1. Don't let intuition lead you too far.
2. Avoid pre-prejudicial situations.
3. Keep vocabulary useful and simple.
4. Spread the teaching out (three week unit is not the best way).

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[35] Ibid., p. 101.

[36] Ibid., p. 106.

[37] Ibid., p. 105.

5. Don't overstructure experiences.

6. Use meaningful experiences.

An experiment with third and fifth graders using certain concepts of probability was conducted by Ojemann, et al.[38] The general plan of the study consisted of administering pretests to both an experimental control group and presenting a series of experiences (motion pictures) for the control group. The students were not required to assign a probability value to events; there was more concern with determining the effect on their behavior in probabilistic situations of observing patterns of outcomes of various experiments. The basic principles developed were:[39]

1. When a random selection is made from a group of equally likely available alternatives, each alternative has an equal chance of being chosen.

2. When there are more of one kind of item than another and the selection is random, the former has a greater chance of being selected.

3. Additional pertinent information usually helps in making decisions.

4. As the number of factors affecting a situation

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[38]Ralph H. Ojemann, E. James Maxey and Bill C. F. Snider, "Effects of a Program of Guided Learning Experiences in Developing Probability Concepts at Third Grade," Journal of Experimental Education, XXXIII (1965), 321-30; and Ojemann, Maxey and Snider, "Effects of Guided Learning Experiences in Developing Probability Concepts at the Fifth Grade Level," Perceptual and Motor Skills, XXI (1965), 415-27.

[39]Ibid., p. 416.

increases, combining their efforts tends to become more difficult.

Results indicated that third and fifth graders in the experimental group evidenced a significantly greater ability to relate their predictions of probable outcome to information available than did the control group. Both grade levels also showed a significantly greater ability to refrain from making a final prediction when only a limited amount of information was available and displayed the tendency to "maximize success" without the use of extraneous rewards.

Need for the Study

Reasons why probability should be included in the elementary school program, content that should comprise the area, and methods of teaching probability to elementary school children have been given by those interested in including probability in the elementary school mathematics program. Recently, some elementary textbooks have included topics in probability and studies have been conducted in an effort to ascertain the extent of the knowledge of probability possessed by elementary school children prior to formal training in probability. Research has also been conducted in an effort to determine the effect on the elementary school child of introducing a unit on probability. However, these developments in the study and teaching of probability in the elementary school show no concensus on the concepts that should be included and materials and methods that should be used. Research in the area is sparse and most of that existing furnishes little objective evidence to support the conclusions which have been drawn.

Through the years there have been several instances where curriculum revision has been attempted through opinion or judgments formed through uncontrolled experimentation. This has led to the accretion of courses in the school program. Before new subject matter is placed in the curriculum, some knowledge of limits of achievement is necessary.[40] Actual experimentation, preferably in the classroom, is necessary to test the validity of claims that concepts of probability can be learned by elementary school children.[41] Without concrete evidence of the ability of elementary school children to learn topics in probability, including them may result in unwarranted frustrations by all involved. The results of this study will furnish some objective evidence of the ability of some fifth and sixth graders to learn certain topics in probability. As a result, this study will also help to answer the question concerning appropriate methods for teaching the topics. Ultimately, it is hoped that this study will help aid in answering the question as to whether probability should be a part of the elementary school arithmetic program.

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[40] Charles M. Bridges, Jr., "Statistics for Selected Secondary-School Students," The Mathematics Teacher, LII (May, 1969), 321.

[41] Pieters and Kinsella, "Statistics," p. 274.

Statement of the Problem

Can fifth and sixth graders learn the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events through the use of experiments?

Limitations of the Study

This study was limited to students in the fifth and sixth grades at the University School of the University of Oklahoma, Norman, Oklahoma. It was further limited to the presentation of the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events through use of the method of experiments.

CHAPTER II

METHODS AND PROCEDURES

This chapter presents a general description of the study. Included is an overview of the method used to formulate the lessons in probability and the over-all design of the lessons. Also, this chapter contains a description of the evaluation instrument, its construction, and the method of obtaining reliability and validity. The subjects, teacher, and time involved are discussed, as well as the procedure used in obtaining, analyzing, and presenting the data.

Formulation and Design of the Lessons

Preparation of lessons used to teach probability to fifth and sixth grade students was begun in the early spring of 1971. The content and methodology were based upon suggestions found in the review of the related materials and research. It was impossible to use all of the suggested concepts; therefore, two other criteria were used to select the concepts. These criteria were:

1. the frequency with which a concept was suggested, and

2. inclusion of those concepts required for understanding a concept selected according to the first criterion.

As a result of these considerations, the five concepts chosen were outcome, outcome space, event, probability of a finite event, and mutually exclusive events.

The use of experimentation by children seemed to be widely suggested. Thus, all concepts selected for presentation were developed through experiments. A total of eight basic experiments, listed below, comprised the lessons:

Experiment I: Outcome, Outcome Space, Event

Experiment II: Coins

Experiment III: Spinner

Experiment IV: Cubes

Experiment V: Ordered Pairs

Experiment VI: Equally Likely Outcomes

Experiment VII: Probability of a Finite Event

Experiment VIII: Mutually Exclusive Events

Some of the basic experiments consisted of a single experiment designed to be performed by each child. Others of the basic experiments consisted of several experiments, each to be performed by each child in some cases and by groups of children in other cases. (Appendix A)

The ultimate purpose of the experiments was to help the children learn those concepts selected for study; however, each individual basic experiment had its specific objectives. The experiments were also developed for the purpose of enjoyment and to utilize arithmetic skills as much as possible. No attempt, however, was made to measure the extent to which these latter goals were achieved.

Included in each experiment were the purposes, materials needed, vocabulary to be introduced, suggested time to be spent on the experiment, discussion exercises, and activities for the children. The discussion exercises were developed as a guide for the teacher. Most of the exercises were in the form of questions, carefully worded and sequenced in order to help provoke thought and interest as well as to make the transition from the learning of one concept to another accurate and smooth.

The materials and the suggested time to be spent on the experiments were not fixed. The length of time to be spent on an experiment depended upon the ability of the children to comprehend the ideas presented.

The activities sheets were designed to be used by the children following completion of each basic experiment. They were to help acquaint the students with hypothetical experiments and also help them draw conclusions from experiments of this type. After the activity sheet was completed, there was an additional discussion period to help clear up misunderstandings.

In late July, 1971, the lessons were given to the teacher of the fifth and sixth grade arithmetic classes at the University School. After thoroughly studying the experiments, she and the researcher met and discussed the experiments and made plans for teaching them.

Evaluation Instrument

Construction

To measure whether the children had learned the selected concepts an objective test, consisting of multiple choice and true-false items, was constructed. The test supplied subscores for the five concepts and these subscores were combined to form a total probability test score. There was a total of 54 test items. The concepts measured and the number of items employed to measure them are as follows:

1. Outcome ------------------------ 6 items
2. Outcome space ------------------ 8 items
3. Event -------------------------- 11 items
4. Probability of a finite event -- 12 items
5. Mutually exclusive events ------ 17 items.

Reliability

In order to obtain reliability, the method of split-half was used; the split used in this case was odd-even. This method was used because it yields a coefficient of internal consistency and these types of coefficients are very suitable for use in computing the reliability of academic tests.[42] The test items were not numbered successively. However, this did not prevent the assigning of an odd and even number to each test item since the only reason for the numbering system used was to help prevent misunderstanding on the part of the students. Reliability was computed using 23 scores of students in the same grades as those used in the study. A scatterplot of the scores indicated that a Pearson Product-Moment Correlation Coefficient could be used appropriately, and a correlation coefficient of .80 was obtained. The reliability of a test is directly related to its length and the reliability coefficient of .80 was equivalent to that for a test one-half the length of the original test. To make the correction for this effect, the Spearman-Brown Formula was used.[43] The corrected reliability coefficient was computed to be .89. A reliability coefficient of this size indicates that the test possesses a fairly high degree of internal consistency.

Validity

The test was designed to determine if the concepts taught had been learned by the subjects in the study. One method for determining if a test, in particular an achievement

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[42] N. M. Downie and R. W. Heath, Basic Statistical Methods (2nd ed.; New York: Harper and Row, publishers, 1965), pp. 218-19.

[43] Ibid., p. 218.

test, measures what it purports to measure is to ascertain if it possesses content validity. In connection with content validity, Woods states:

> Whether one thinks in terms of sampling curriculum content and objectives in an effective way, the necessity for human judgment at some point is inescapable. . . . In respect to content validity judgment is applied more directly to the question of the relationship between the universe of behavior comprising the goals. Judgment based on a thorough consideration of the purposes of the test, with analysis into detailed components, is the most useful.[44]

After selecting samples of items representing each of the concepts, the test items were distributed to five persons with competence in the following areas:

1. elementary, secondary, undergraduate, and graduate levels of teaching;

2. designing courses of study for elementary and secondary school mathematics; and

3. probability.

They were asked to judge the test items using these four criteria:

1. Is each item representative of the concept it seeks to measure?

2. Has the concept been adequately tested?

3. Is each item clear?

4. Are the "choice items" well selected?

To help the judges make an accurate appraisal of the test

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[44]Dorothy Adkins Woods, Test Construction (Columbus, Ohio: Charles E. Merrill Books, Inc., 1960), pp. 18-19.

items using the above criteria, they were also given a copy of the eight experiments. The test items were deleted or revised until each judge felt that the items satisfied the four criteria.

The Teaching and Learning Environment

Most of the ideas presented in the lessons were new to the fifth and sixth graders. The problem of the study was to determine if the children could learn the selected concepts of probability in a learning environment approximating as closely as possible that generally used in the arithmetic classes of the children involved in the study. It was important that factors such as teacher, time and length of the arithmetic period remain as they were prior to the beginning of the study.

Subjects

Students in the fifth and sixth grades at the University School of the University of Oklahoma participated in the study. There were 21 fifth graders and 23 sixth graders. Because of frequent absences of one sixth grader, only the scores of the 22 in regular attendance were used in the computation and analysis of the data.

To obtain an approximation of the academic potential of the children, I.Q. scores were secured for all but four fifth graders and four sixth graders. The I.Q. of the fifth graders ranged from 102 to 136 with a mean of 119 and standard deviation of 8.6. The I.Q. of the sixth graders ranged from 94 to 162 with a mean of 123 and standard deviation of 15.5.

The Instructor

The teacher of the children involved in the study was the regularly employed intermediate mathematics specialist for the University School. This was her third year at the school and she was at the time of the study pursuing the Master of Arts Degree in Elementary Education at the University of Oklahoma. She received the Bachelor of Science Degree in Mathematics from Webster College, St. Louis, Missouri, in 1968. While attending Webster she was a workshop instructor in the Madison Project. Prior to accepting the position at the University School she was enrolled in a graduate program of mathematics (completing 28 hours) at the University of Oklahoma.

Time

The experiment was begun October 13, 1971. Pre- and post-testing time lasted two class periods for each of the grades--one period for the pre-test and one for the post-test.

The classes met each of the five days of the school week but were officially dismissed a few days during the study. The fifth grade class met at 8:30 a.m., the first period of the school day, and the sixth grade class met directly following it at 9:30 a.m. Both the fifth and sixth grade classes lasted a total of 50 minutes per day. The study lasted 21 and 17 teaching days for the fifth and sixth graders respectively, ending for the fifth grade on November 19, 1971 and for the sixth grade on November 15, 1971.

Procedure for Obtaining, Analyzing and Presenting the Data

Obtaining Data

Data were collected by means of pre-tests administered immediately prior to teaching the lessons in probability. Post-tests were given immediately following the teaching of the probability lessons. All but five of the teaching sessions were observed by the researcher. Those observations of importance to the study are reported herein.

Hypotheses

Four basic null hypotheses were tested; when broken into their separate components, these yielded a total of thirteen hypotheses. It was hoped that the children would show significant improvement on the variables analyzed. It was also felt that an extra year of educational training would influence performance, with the sixth graders performing significantly better on the total test than the fifth graders. Thus, all hypotheses are stated in directional form. The four basic hypotheses are:

Ho_1: There is no statistically significant gain from pre-test to post-test in the scores of fifth graders on the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events.

Ho_2: There is no statistically significant gain from pre-test to post-test in the scores of sixth graders on the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events.

Ho_3: There is no statistically significant gain from pre-test to post-test in the scores of fifth graders and the scores of sixth graders on the total probability test.

Ho_4: There is no statistically significant difference in the gain of the fifth graders from pre-test to post-test on the total probability test score and the gain of sixth graders from pre-test to post-test on the total probability test score.

Analytic Procedures

The statistical technique used to analyze basic hypotheses one, two, and three was the Wilcoxon T-Test. This test can appropriately be used with correlated data; takes into consideration relative magnitude as well as direction of differences; and allows for judgment of "greater than" between the performances of any pair of values to be made.[45] This test was used for these hypotheses since all assumptions for its use were satisfied and the desired information could be obtained through its use; however, the assumptions of the parametric counterpart were not met ($N \leq 25$ and departure for normality).

To analyze the fourth basic hypothesis the Mann-Whitney U-Test was used. This test is the most powerful alternative to the parametric analog and may be used to test whether two independent groups have been drawn from the same population.[46] It was used to test this hypothesis since assumptions of its parametric counterpart were not met ($N \leq 25$ or 30 and departure for normality). This data did satisfy the assumptions of the Mann-Whitney U-Test and the desired information could be obtained from its use.

The one-tailed test for the Wilcoxon T-Test and the Mann-Whitney U-Test was used since the direction was predicted in advance of examining the data. The criterion for rejecting the null hypothesis was set at the .01 level of significance.

Method of Presenting Data

Tables were used to present the following data:

1. Pre- and post-test scores of the fifth and

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[45] Sidney Siegel, <u>Nonparametric Statistics for the Behavioral Sciences</u> (New York: McGraw-Hill Book Company, Inc., 1956), pp. 75-83.

[46] <u>Ibid.</u>, pp. 116-126.

sixth grades for each concept measured.

2. Pre- and post-test scores of the fifth and sixth graders on the total probability test.

3. Differences in the pre- and post-test scores of the fifth and of the sixth graders on the total probability test.

CHAPTER III

PRESENTATION AND ANALYSIS OF DATA

An analysis of data on the fifth and sixth graders used in the study is the subject of this chapter. Pre-test and post-test scores were obtained on the five probabilistic concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events for the fifth and sixth graders. The scores on the individual concepts were combined to obtain a total probability test score and these data are included for both grades. A comparison of pre- and post-test scores on the total probability test of fifth and sixth graders was made. The differences in the pre-test and post-test scores on the total probability test of the fifth graders were compared with the differences in the pre-test and post-test scores on the total probability test of the sixth graders. All raw scores and computations of the statistics appear in Tables 4-16 of Appendix C.

A total of thirteen specific hypotheses were tested. Twelve dealt with the comparison of pre- and post-test scores; the thirteenth dealt with a comparison of the differences in the pre- and post-test scores on the total probability test of the fifth graders. The first twelve hypotheses were tested using the Wilcoxon T-Test, the last using the Mann-Whitney U-Test. The criterion for rejecting the null hypothesis was set at the .01 level of significance, using the one-tailed test.

Table 1 presents a summary of the data which deals with null hypothesis Ho_1 and that part of null hypothesis Ho_3 which deals with the fifth grade. Null hypothesis Ho_1 states that there is no statistically significant gain from pre-test to post-test on the scores of fifth graders on the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events. That part of null hypothesis Ho_3, for which data are presented in Table 1, states that there is no statistically significant gain from pre-test to post-test in the scores of fifth graders on the total probability test.

TABLE 1

SUMMARY OF DATA COMPUTED ON FIFTH GRADERS

	Required Value of T	Obtained Value of T
Outcome	$T \leq 24$	15
Outcome space	$T \leq 28$	37
Event	$T \leq 24$	17
Probability of a finite event	$T \leq 33$	23.5
Mutually exclusive events	$T \leq 43$	22
Total probability test score	$T \leq 43$	7

The data computed on outcome, outcome space, event, probability of a finite event, mutually exclusive events, and the total probability test, for fifth graders, require values of T less than or equal to 24, 28, 24, 33, 43, and 43, respectively, for significance at the .01 level, one-tailed test. The obtained values of T (for these categories) were 15, 37, 17, 23.5, 22, and 7, respectively. All parts of null hypothesis Ho_1, except that dealing with the concept of outcome space were rejected at the .01 level of significance, one-tailed test. That part of null hypothesis Ho_3 dealing with the fifth grade was also rejected at the .01 level, one-tailed test. Thus, the fifth graders did gain significantly from pre-test to post-test in all of the categories except that of outcome space.

Table 2 presents a summary of the data computed for null hypothesis Ho_2 and that part of null hypothesis Ho_3 which deals with the sixth grade. Null hypothesis Ho_2 states that there is no statistically significant gain from pre-test to post-test on the scores of sixth graders on the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events. The data in Table 2 also consist of that part of null hypothesis Ho_3 which states that there is no statistically significant gain from pre-test to post-test in the scores of sixth graders on the total probability test.

TABLE 2

SUMMARY OF DATA COMPUTED ON SIXTH GRADERS

	Required Value of T	Obtained Value of T
Outcome	$T \leq 28$	21
Outcome space	$T \leq 38$	26.5
Event	$T \leq 28$	19
Probability of a finite event	$T \leq 43$	33.5
Mutually exclusive events	$T \leq 38$	0
Total probability test score	$T \leq 56$	0

For significance at the .01 level, one-tailed test, values of T less than or equal to 28, 38, 28, 43, 38, and 56 are necessary for the data computed on outcome, outcome space, event, probability of a finite event, mutually exclusive events, and the total probability test, respectively. The obtained values of T (for these categories) were 21, 26.5, 19, 33.5, 0, and 0, respectively. All parts of null hypothesis Ho_2, and that part of null hypothesis Ho_3 which deals with the sixth grade were rejected at the .01 level of significance, one-tailed test. Therefore, the sixth graders did gain significantly from pre-test to post-test on all five of the concepts and on the total probability test.

Table 3 presents a summary of the data pertinent to null hypothesis Ho_4. This hypothesis states that there is no statistically significant difference between fifth and sixth graders in gain from pre- to post-test on the total probability test. The groups are independent, thus, the Mann-Whitney U-Test was used to examine the data. Since at least one of the groups of scores exceeded 20 in number, the computation of the U statistic involved the computation of the z ratio. The value of z necessary for significance at the .01 level, one-tailed test, is 2.33 or greater. The obtained value of z was .34; thus, the null hypothesis was accepted at the .01 level of significance, one-tailed test. Hence, there was no statistically significant difference between fifth and sixth graders in the gain from pre-test to post-test on the total probability test.

TABLE 3

COMPARISON OF DIFFERENCES IN GAIN FROM PRE- TO POST-TEST TOTAL PROBABILITY TEST OF FIFTH AND SIXTH GRADERS

Sum of ranks scores of fifth graders	Sum of ranks scores of sixth graders	Required Value of z	Obtained Value of z
476	470	At least 2.33	.34

Anecdotal Observations

All experimental sessions, except five, were attended by the researcher. During the sessions attended, the children were observed and questioned. It was felt that the observing and questioning would help reveal those portions of the total design of the lessons that may have helped lead to the inability of the children to understand and enjoy them. These detections might, therefore, aid in making suggestions for improvements in the structure of the lessons.

Initially, most of the students seemed quite excited about the experiments. They had been informed that they would be studying a branch of mathematics new to them, and they seemed eager to discover what it was all about. On the very first day of the lessons, as part of the introduction, and to help stimulate interest, the children drew beads from bags containing two different colors. They were asked to guess the proportion of the different colors in the bag. On the second day, the children expressed their enjoyment of the previous day's activities and asked if they could repeat the experiment. They also seemed to like using different materials with which to experiment.

Three of the experiments were to be repeated several times in an effort to arrive at the <u>a posteriori</u> probability of an outcome. Three different models (coins, spinner, cube) were used; however, it was obvious that any two of the experiments would have sufficed. This conclusion was formed because the children expressed physical exhaustion from "all of the tossing, rolling and spinning." Also, by the time the third experiment was presented, the children knew about how many times an outcome was expected to occur.

The basic experiment involving two-dimensional outcome spaces and all subsequent experiments and problems of this nature were obviously frustrating to the fifth graders and seemed only a little less so for the sixth graders. It appeared that the presentation was a bit too rigorous for them, even though they already possessed some knowledge of ordered-pairs and "grids." It was felt, however, that a little more success was gained in this area when distinctly different models (different objects as well as different number of outcomes in the outcome space of each object) were used. There also seemed to be more understanding when the children were not required to tell "how many elements were in the outcome space" but asked to list all of the elements in the outcome space or make a "grid" to show the total number of points in the outcome space.

One observation with respect to the formulation of the questions on the activity sheet was made. If a question is in written form for the children to answer, it seems best to include all pertinent information for each separate question. For example, questions such as "How many elements are there in the outcome space of the nickle? Cube?" seemed confusing to the children. This was evident from the number of questions these forms of questions elicited and, in most instances, only the question concerning the nickle was answered.

Most of the children seemed to think that outcomes of an experiment would always have the same chance of occurring. When the experiments with outcomes not having the same chance of occurring were presented, the children invariably said, "You cheated on this one" or "This experiment was rigged in favor of. . . ." The impression was that they thought all outcomes should be equally likely. This might indicate that these children did possess some preconceived notions about probability and, at least in this case, fallacious ones.

The experiments designed to be performed in groups of four or five children seemed to be very successful. They seemed to elicit the most interest, participation and understanding. This was apparent from the performance on the activity sheets and the subsequent discussion of the experiments.

The novelty of the experimentation and probability seemed to wane with many of the children after about 2½ to 3 weeks. Even though most admitted that they still were enjoying the lessons and felt that it was a pleasant change from the usual procedures, it was observed that the attention span was growing shorter and interest was waning. As a whole, however, it seemed to be an enjoyable experience for most of the children most of the time.

CHAPTER IV

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

During the past years, it has been recommended that probability be included in the elementary school mathematics program. Reasons have been given to help support the inclusion of probability at this level, concepts of probability felt suitable for this level have been suggested, and methods of teaching topics in this area have been presented. Some elementary school textbooks now include topics in probability, and research has been conducted pertaining to the study and teaching of probability in the elementary school. However, the articles are for the most part based on opinions; the perusal of some popular elementary school arithmetic textbooks indicates little evidence of concensus in content and methodology with respect to probability, and the existing research is sparse. In particular, the feasibility of teaching certain topics in probability and the methods that can be used to teach them have been given very little attention.

This study was designed to determine if the fifth and sixth grade students used in the study could learn the probability concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events. It was also designed to determine if these concepts could be learned through the use of experiments. It was hoped that this study would help determine if some elementary school children could learn selected topics in probability, and also help in beginning to furnish an answer to the question as to whether probability should be included in the elementary school program.

The subjects in the study were 21 fifth graders and 22 sixth graders enrolled in the University School of the University of Oklahoma, Norman, Oklahoma. The classes were taught during their regularly scheduled arithmetic periods. The total time for conducting the study was 21 teaching days for the fifth grade and 17 teaching days for the sixth grade.

An objective test was constructed on the five probability concepts, and a comparison of pre-test and post-test scores on the test used to determine if the concepts had been learned by the children. Observations of the classes and conversations with the children also yielded information which, while not objective, nevertheless furnishes some information concerning the teaching of probability to elementary school children.

Conclusions

Conclusions are based on the thirteen specific hypotheses tested. Because of the limited scope of the topics and methodology and the selective nature of the children involved in the study, no attempt was made to generalize the conclusions beyond the limitations imposed on the study.

The gain from pre- to post-test scores of the sixth graders was statistically significant on the five concepts of outcome, outcome space, event, probability of a finite event and mutually exclusive events. Thus, it was concluded that the sixth graders did possess the ability to learn each of the concepts involved in the study through the method of experiments. The gain from pre- to post-tests was statistically significant on the total probability test score for the sixth graders. This indicates that, in general, they possessed the ability to learn the concepts presented through the use of experiments.

The fifth graders gained significantly from pre- to post-test on the probability concepts of outcome, event, probability of a finite event, and mutually exclusive events. They showed no significant gain on the concept of outcome space. As a result, it can be concluded that the fifth graders did possess the ability to learn all of the specific concepts presented except that of outcome space.

However, since there was significant gain from pre- to post-test on the total probability test score, this indicates that they possessed, in general, the ability to learn the probability concepts presented in the study.

There was no significant difference in the gain from pre-test to post-test on the total probability test score of the fifth graders and those of the sixth graders. Thus, the conclusion can be drawn that both grades learned about the same amount of material. It must be pointed out that the time in which the learning took place did differ for the two grades.

Recommendations

For the Study and Teaching of Probability in the Elementary School

As a result of the above conclusions, it is felt that probability should be included in the elementary school mathematics program. If probability is taught in the elementary school, it is recommended that the concepts of outcome, outcome space, event, probability of a finite event, and mutually exclusive events be among topics included at the fifth and sixth grade levels. It is also recommended that actual experimentation on the part of the children should be used to present many of the topics.

In view of the anecdotal observations, the following observations are made:

1. The children seemed to lose interest and even

expressed physical exhaustion after about 2½ to 3 weeks of experiments. One of the reasons for including probability in the elementary school program is the fun children have as a result of studying it. If it ceases to be fun this reason is defeated. Thus, it is suggested that activities should be carried out only as far as interest is maintained and that perhaps a four-week period dealing exclusively with the study of probability may not be the best method.

2. The study of probability can require the use of many different materials; thus, children need quite a bit of individual attention when taught through experiments. The teacher could profit from assistance, preferably from someone with training in probability.

3. Since the concept of two-dimensional outcome space seemed difficult for most of the children to comprehend, it is suggested that it be presented with less rigor and with the use of two distinctly different models.

4. Since some preconceived notions, some of a fallacious nature, seemed to be apparent in the children, it is suggested that an early introduction to some concepts of probability may help to alleviate these biased feelings before they are too deeply rooted.

Recommendations for Further Research

The following are recommendations for additional research in the study and teaching of probability in the elementary school:

1. To obtain different ability grouping of children this study should be replicated using a larger number of classes over a broader cross section of the school district.

2. A comparative study should be conducted using two matched groups in which one group is taught probability during a four-week period and the other group is taught the same concepts in small units broken up through the semester or the year.

3. A study using an experimental group (taught probability) and a comparison group (not taught probability) should be conducted in an effort to determine what effects the teaching of probability can have on factors such as attitude toward mathematics, arithmetic skills and problem solving ability.

4. A study in which a comparison of various methods of teaching probability (for example, actual experimentation on the part of students versus probability problems formulated from hypothetical experiments) should be conducted in an effort to determine if one method is superior to another.

5. A study using additional probability concepts should be conducted. This will help extend the knowledge of concepts that can be learned by elementary school children.

6. A study using children in grades below the fifth should be conducted. This should help determine the grade level that the teaching of probability in the elementary school might best begin.

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APPENDICES

APPENDIX A

A SYNOPSIS OF THE EXPERIMENTS DEVISED FOR THE TEACHING OF PROBABILITY TO FIFTH AND SIXTH GRADERS

DEFINITIONS OF TERMS NECESSARY TO THE EXPERIMENTS

Outcome: One possible result of one trial of an experiment.[47]

Outcome space: The total collection of all possible outcomes in an experiment.[48]

Event: Any subset of an outcome space.[49]

Equally likely events: Events E and F of the outcome space, S, are said to be equally likely if the probability of E is equal to the probability of F.[50]

Probability

A priori: The probability of an event is the number of favorable cases divided by the total number of equally possible cases.[51]

A posteriori: The ratio of the number of times an event occurs in an actual series of tests to the total number of trials.[52]

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[47]William L. Hays and Robert L. Winkler, Statistics: Probability, Inference and Decision, Vol. I (New York: Holt, Rinehart and Winston, Inc., 1970), p. 40.

[48]Secondary School Mathematics Curriculum Improvement Study, Unified Modern Mathematics, Course I, Part I (New York: Board of Trustees of Teachers College, Columbia University, 1968), p. 224.

[49]Emanuel Parzen, Modern Probability Theory and Its Application (New York: John Wiley and Sons, Inc., 1960), p. 9.

[50]Ibid., p. 25.

[51]Fred N. Kerlinger, Foundations of Behavioral Research (New York: Holt, Rinehart and Winston, Inc., 1964), pp. 117-18.

[52]Ibid., p. 118.

INTRODUCTION TO THE EXPERIMENTS

The students were informed that they would study a branch of mathematics called probability. Questions were asked the students in an effort to ascertain if they were familiar with probability and if they were aware of situations which involved the use of it.

To help the children gain experience with experiments and to provoke interest in the experiments which were to follow, they were shown two bags, each containing beads. In one bag were blue and yellow beads with the same number of each color. In the other bag were black and white beads with twice as many black as white. The children were not informed of the distribution of the color of the beads. They were instructed to withdraw some of the beads from the bag and note the number of beads drawn of each color. Finally, they were asked to guess the proportion of each color of bead found in each bag.

EXPERIMENT I: OUTCOME, OUTCOME SPACE, EVENT

PURPOSES:

To provide the child with experience in conducting simple experiments.

To introduce the concepts of outcome, outcome space, and event.

To prepare for discussion of equally likely outcomes.

MATERIALS:

Penny, 26 slips of paper (same size and a different letter of the alphabet on each), 5 marbles (3 red, 2 white and a different number from 1 to 5 on each), and an activity sheet for each child.

VOCABULARY:

Outcome, outcome space, event.

SUGGESTED TIME:

Two class periods.

DIRECTIONS:

There are three experiments to be performed. Each child should participate in each experiment.

EXPERIMENT A

Twenty-six well-shuffled slips of paper, each with a different letter of the alphabet, were placed in a paper bag. One slip was drawn by a child, the alphabet letter on it noted and the slip returned to the bag. After each withdrawal, the slips were reshuffled. This procedure was repeated until each child had drawn a slip.

EXPERIMENT B

Each child tossed the penny and observed the outcome.

EXPERIMENT C

Five marbles were placed in a paper bag. Each had a different number from 1 to 5 on it. Three of the marbles were green, two were white. Each child drew a marble, noted the color and the number and returned it to the bag.

EXPERIMENT II: COINS

PURPOSES:

To extend the experiences provided in Experiment I.

To provide experiences in collecting, tabulating, and recording data.

To determine the a posteriori probability of an outcome.

To prepare for the discussion of the a priori probability of an event.

To exercise skills in addition, subtraction, and multiplication.

MATERIALS:

A penny, copy of tables on which data for tossing the penny is to be recorded, and a copy of the activities for each child in the class.

VOCABULARY:

Predict.

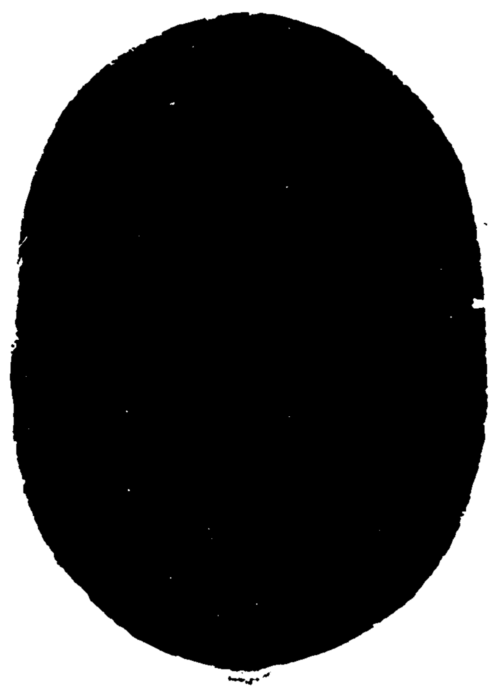
SUGGESTED TIME:

Two class periods.

THE EXPERIMENT

Each child tossed the penny a total of 50 times. After each toss the child observed the outcome and recorded it if it was heads. The totality of the outcomes of all

50 tosses of all in the class was obtained and the children determined the a posteriori chance of obtaining an outcome when a large number of trials was conducted.



50 tosses of all in the class was obtained and the children determined the a posteriori chance of obtaining an outcome when a large number of trials was conducted.

EXPERIMENT III: SPINNER

PURPOSES:

To extend the experiences provided in the previous experiments.

MATERIALS:

A spinner attached to a circular region which has been divided into four equal parts. (The parts were numbered from 1 to 4.) A table for recording and tabulating data and an activity sheet for each child.

SUGGESTED TIME:

Two class periods.

THE EXPERIMENT

Each child turned the spinner a total of 64 times. Each outcome was observed and recorded. The totality of the outcomes of all 64 spins of all in the class was obtained and the children determined the *a posterior* chance of obtaining an outcome when there was a large number of trials.

EXPERIMENT IV: CUBE

PURPOSES:

To extend the experiences provided in the previous experiments.

MATERIALS:

A cube (with the letters A, B, C, D, E, and F on the faces), a table for recording and tabulating data and an activity sheet for each child.

SUGGESTED TIME:

Two class periods.

THE EXPERIMENT

Each child rolled the cube a total of 64 times. Each outcome was observed and recorded. The totality of the outcomes of all 64 rolls of all in the class was obtained and the children determined the *a posterior* chance of obtaining an outcome when there was a large number of trials.

EXPERIMENT V: ORDERED PAIRS

PURPOSES:

To extend the experiences provided in the previous experiments.

To provide an experiment where each possible outcome is a pair.

MATERIALS:

A penny, nickel, spinner (same one in Experiment III), and an activity sheet for each child.

VOCABULARY:

Ordered pair.

SUGGESTED TIME:

One class period.

DIRECTIONS:

This basic experiment involves two experiments. Each child will perform both experiments.

EXPERIMENT A

The nickel and penny were tossed. The outcome of each was observed and recorded.

EXPERIMENT B

Each child tossed the nickel and turned the spinner. The outcome of each was observed and recorded.

EXPERIMENT VI: EQUALLY LIKELY OUTCOMES

PURPOSES:

To extend the experiences provided in the previous experiments.

To introduce the concept of equally likely outcomes.

MATERIALS:

Slips of paper of the same dimension, 7 marbles (each with a different number from 1 to 7; 5 are red and 2 are white), dictionary, spinner (the circular region is divided into four parts; parts numbered 1 and 3 are equal, parts numbered 2 and 4 are equal), an 8-inch pencil, a plain sheet of paper (8½" X 11") with 2 horizontal lines drawn 3 inches apart on it, 2 paper bags, ruler, and an activity sheet for each child.

VOCABULARY:

Equally likely outcomes.

SUGGESTED TIME:

One class period.

DIRECTIONS:

There are five experiments involved in this basic experiment. The class was divided into 5 groups. Each group performed one of the experiments and together worked on the activities. After all experiments had been performed and activities completed, a spokesman for the group described the experiment and the results to the class.

EXPERIMENT A

Each child in the group wrote his name on a slip of paper, folded it and put it in a paper bag. Without looking, the children took turns drawing names from the bag. Each name was returned before the next name was withdrawn.

EXPERIMENT B

Seven marbles were in a bag. Without looking, each child drew one from the bag. Each marble was returned before the next withdrawal.

EXPERIMENT C

Each child took turns opening a dictionary so that each page had an equal chance of being selected. The last digit of the number of the page selected was recorded.

EXPERIMENT D

Each child took turns spinning the spinner.

EXPERIMENT E

An 8-inch pencil was dropped on a piece of paper with horizontal lines drawn 3 inches apart.

EXPERIMENT VII: PROBABILITY OF A FINITE EVENT

PURPOSES:

To extend the experiences provided in the previous experiments.

To present the *a priori* probability of a given event when the possible outcomes are equally likely.

MATERIALS:

A penny, cube (with letters A, B, C, D, E, and F on its faces) and an activity sheet for each child, paper bag, 11 marbles (4 red and 7 blue) and a spinner (the circular region divided into 8 equal parts, a different symbol on each part).

VOCABULARY:

A Priori probability of an event.

SUGGESTED TIME:

Two class periods.

DIRECTIONS:

There are 5 experiments comprising this basic experiment. Experiments A, B, C and D were each performed by one of four groups of children. Experiment E was performed by each child.

EXPERIMENT A

From a bag containing 11 marbles (4 red and 7 blue), one was drawn, the number and color noted, and the marble returned to the bag. This was repeated by each member of the group.

EXPERIMENT B

One card was drawn from a pack of Old Maid cards by each member of the group. Each card was returned to the deck before the next one was drawn.

EXPERIMENT C

One card was drawn from a pack of standard playing cards by each member of the group. Each card was returned to the deck before the next one was drawn.

EXPERIMENT D

Each child tossed the penny and rolled the cube. The outcome was recorded.

EXPERIMENT VIII: MUTUALLY EXCLUSIVE EVENTS

PURPOSES:

To extend the experiences provided in the previous experiments.

To introduce the concept of mutually exclusive events.

MATERIALS:

Penny, cube (with letters A, B, C, D, E and F on its faces) and an activity sheet for each child.

VOCABULARY:

Mutually exclusive events.

SUGGESTED TIME:

One class period.

EXPERIMENT A

Each child tossed the penny one time.

EXPERIMENT B

Each child rolled the cube one time.

APPENDIX B

TEST ON PROBABILITY

DIRECTIONS

Read each question and decide which answer is correct. Circle the letter before the correct answer. If the correct answer is not given, circle the letter before the answer, "none of the above."

EXPERIMENT I

Four slips of paper are in a box. The slips are numbered from 1 to 4. You draw <u>one</u> slip.

1. Which of the following is true?

   a. $\{2\}$ is a possible outcome.

   b. $\{2,3\}$ is a possible outcome.

   c. $\{1,2,3,4\}$ is a possible outcome.

   d. $\{2,3,4\}$ is a possible outcome.

   e. all of the above.

2. A number greater than 2 but less than 4 was drawn. Which of the following is true?

   a. The outcome was $\{2\}$.

   b. The outcome was $\{3\}$.

   c. The outcome was $\{4\}$.

   d. Both a and c.

   e. None of the above.

EXPERIMENT II

Four slips of paper numbered from 1 to 4 are in a box. You draw one slip, record the number and replace it in the box. You draw another slip, record the number and replace it in the box.

1. How many possible outcomes are there?

    a. 4

    b. 8

    c. 12

    d. 16

    e. None of the above.

2. Which of the following is <u>not</u> true?

    a. $\{(1,1)\}$ is a possible outcome.

    b. $\{(1,2)\}$ is a possible outcome.

    c. $\{(1,3), (2,4)\}$ is a possible outcome.

    d. $\{(2,3)\}$ is a possible outcome.

    e. None of the above.

EXPERIMENT III

One each of the letters A, B, C, D, E and F are in a box. You draw one letter, but do not return it, then draw a second letter.

1. Which of the following is true?

   a. $\{(A,A)\}$ is a possible outcome.

   b. $\{(A,B)\}$ is a possible outcome.

   c. $\{A\}$ is a possible outcome.

   d. All of the above.

   e. None of the above.

EXPERIMENT IV

One of each of the letters A, B, C, D, E and F are in a box. You draw one letter, return it to the box then draw a second letter.

1. Which of the following is true?

   a. $\{(A,A)\}$ is a possible outcome.

   b. $\{(A,A), (B,B)\}$ is a possible outcome.

   c. $\{(A,A), (E,F)\}$ is a possible outcome.

   d. All of the above.

   e. None of the above.

EXPERIMENT V

Slips of paper are in a box. Some slips have an A written on them, some B, some C and others D. One slip is drawn and the letter on it noted.

1. How many objects are in the outcome space?

   a. 1

   b. 4

   c. Not enough information given to tell.

   d. None of the above.

2. What is the outcome space?

   a. Either $S = \{A\}$, $S = \{B\}$, $S = \{C\}$, or $S = \{D\}$.

   b. $S = \{A,B,C,D\}$.

   c. Not enough information given to tell.

   d. None of the above.

EXPERIMENT VI

There are 10 slips of paper in a box. Seven of them are numbered from 1 to 7. Each of the other 3 slips has one of the numbers from 1 to 7. One slip is drawn and the number noted.

1. How many objects are in the outcome space?

   a. 1

   b. 7

   c. 10

   d. 3

   e. None of the above.

2. What is the outcome space?

   a. $S = \{1,2,3,4,5,6,7,8,9,10\}$.

   b. $S = \{10\}$.

   c. $S = \{1,2,3,4,5,6,7\}$.

   d. $S = \{7\}$.

   e. None of the above.

EXPERIMENT VII

Three blocks, numbered 1, 2, and 3, are in a box. One block is drawn, the number recorded and the block returned to the box. Another block is drawn and the number recorded.

1. What are <u>all</u> of the possible outcomes?

   a. $S = \{(1,1), (2,2), (3,3), (4,4)\}$.

   b. $S = \{1,2,3\}$.

   c. $S = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$.

   d. All of the above.

   e. None of the above.

2. How many objects are in the outcome space?

   a. 1

   b. 3

   c. 6

   d. 9

   e. None of the above.

EXPERIMENT VIII

Three numbers, 1, 2, and 3, are in a box. One number is drawn, replaced in the box and a second number is drawn. The two numbers drawn are added.

1. What is the outcome space?

   a. {2,3,4,5,6}

   b. {2,4,6}

   c. {3,4,5}

   d. {1,2,3}

   e. None of the above.

EXPERIMENT IX

Three numbers, 1, 2, and 3, are in a box. One number is drawn, replaced in the box and a second number is drawn. The two numbers drawn are multiplied.

1. What is the outcome space?

   a. { 2,3,6 }

   b. { 1,2,3,4,6,9 }

   c. { 1,2,3 }

   d. { 1,4,9 }

   e. None of the above.

EXPERIMENT X

Fifteen slips of paper, numbered from 1 to 15, are in a box. The slips are distributed to 15 children.

1. Which of the following is true for the 3 children who receive the numbers 1, 2, and 3?
   - a. <u>All</u> had the event $\{1,2,3\}$ occur.
   - b. <u>All</u> had the event $\{1,3,5\}$ occur.
   - c. <u>All</u> had the event $\{1,2,3,4,5\}$ occur.
   - d. Both a and c.
   - e. Both a and b.

2. Which of the events below occurred for the child who received the 9?
   - a. $\{9\}$
   - b. $\{7,8,9,10\}$
   - c. $\{9,10\}$
   - d. $\{1,3,5,7,9\}$
   - e. All of the above.

3. The event $\{8,9,10\}$ occurred for the children who received which of the following numbers?

    a. 8

    b. 9

    c. 10

    d. All of the above.

    e. None of the above.

4. Which of the events below occurred for only the two children who received the numbers 2 and 4?

    a. $\{2,3,4\}$

    b. $\{1,2,3,4\}$

    c. $\{2,4\}$

    d. $\{2,4,6\}$

    e. All of the above.

5. The event $\{1,2,3\}$ occurred. Which of the following is possibly true?

    a. The event $\{1,2\}$ occurred.

    b. The event $\{1\}$ occurred.

    c. The event $\{2\}$ occurred.

    d. All of the above.

    e. None of the above.

6. The event $\{11,12,13,14,15\}$ occurred for which of the following?

   a. The one who received the number 13.

   b. All who received a 10, 11, 12, 13, 14, or 15.

   c. All who receive a 1, 2, 3, 11, 12, 13, 14, or 15.

   d. All who did not receive the 10.

   e. None of the above.

7. The numbers 6, 7, 8, 9, and 15 are received by 5 children. Which of the events did all 5 have occur?

   a. $\{6,7,8,9,15\}$

   b. $\{5,6,7,8,9,15\}$

   c. $\{6,7,8,9,10,15\}$

   d. All of the above.

   e. None of the above.

EXPERIMENT XI

Six slips of paper, numbered from 1 to 6, are in a box. A slip is drawn, the number recorded and the slip returned to the box. A second slip is drawn, the number recorded.

1. Which of the events occurred for all who drew a 1 first and either a 1, 2, or 3 second?
    a. $\{(1,2), (1,3)\}$
    b. $\{(1,1), (2,1), (3,1)\}$
    c. $\{(1,1), (1,2), (1,3)\}$
    d. All of the above.
    e. None of the above.

2. Which of the events occurred for all who drew a 5 or 6 first and a 4 second?
    a. $\{(5,4)\}$
    b. $\{(5,6)\}$
    c. $\{(4,5), (4,6)\}$
    d. $\{(5,4), (6,4)\}$
    e. None of the above.

3. Which of the following is true of the event $\{(3,4), (2,5)\}$?

    a. The event occurred for all who drew two numbers that added up to 7.

    b. The event occurred if the outcome was $\{(4,3)\}$.

    c. The event occurred if the outcome was $\{(5,2)\}$.

    d. All of the above.

    e. None of the above.

4. Which of the following is true of the event $\{(1,2), (1,3), (1,4)\}$?

    a. Only those who drew the number 1 first can have the event occur.

    b. You must draw a 2 second in order for the event to occur.

    c. For the event to occur, the second number drawn must be either a 2, 3, or 4.

    d. Both a and b.

    e. Both a and c.

EXPERIMENT XII

A cube with the letters A, B, C, D, E, and F on its faces is tossed one time. Let V = {D,E,F}, W = { A,B,C} and T = { C,D,E,F}.

DIRECTIONS:

Write T in the blank following the statement if it is true. If it is false, write F in the blank.

1. The events V and W are mutually exclusive.___
2. The events W and T are mutually exclusive.___
3. The events V and T are mutually exclusive.___
4. P(W or V) = P(W) + P(V).___
5. P(T or W) = P(T) + P(W).___
6. P(V or T) = P(V) + P(T).___
7. P(W and V) = 0.___
8. P(W and V) = 1.___
9. P(T or W) = 0.___
10. P(T or W) = 1.___
11. P(W or V) = P(W or T).___

EXPERIMENT XIII

Three slips of paper, numbered from 1 to 3, are in a box. One slip is drawn, the number recorded and returned to the box. Another slip is drawn, the number recorded.

1. Which of the events below are mutually exclusive to the event $\{(1,1), (2,2), (3,3)\}$?

   a. $\{(1,2), (2,3), (3,2)\}$

   b. $\{(3,3)\}$

   c. $\{(1,2), (1,3), (2,3), (3,2)\}$

   d. Both a and b.

   e. Both a and c.

2. Which of the events below are mutually exclusive to the event $\{(2,2)\}$?

   a. $\{(1,1), (2,2), (3,3)\}$

   b. $\{(3,3)\}$

   c. $\{(1,1)\}$

   d. Both b and c.

   e. Both a and c.

3. Which of the events below are mutually exclusive to the event $\{(2,1), (1,2)\}$?

   a. $\{(1,1), (1,2), (2,1), (2,2)\}$

   b. $\{(3,1), (3,2), (1,2), (2,1)\}$

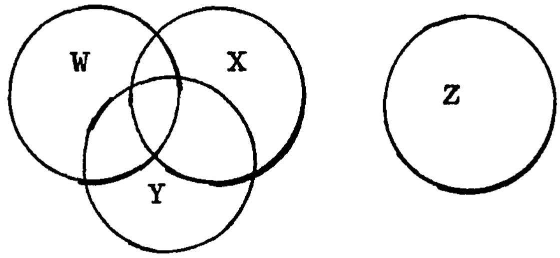
   c. $\{(2,1)\}$

   d. All of the above.

   e. None of the above.

STUDY THE DIAGRAM BELOW

Each Circle Represents a Set



1. Which of the following sets are mutually exclusive?

   a. W and X.

   b. W and Z.

   c. X and Z.

   d. Both a and b.

   e. Both b and c.

2. Which of the following sets are mutually exclusive?

   a. W and Y.

   b. X and Y.

   c. W and X.

   d. All of the above.

   e. None of the above.

3. Which of the following sets are mutually exclusive?

   a. W and Z.

   b. X and Z.

   c. Y and Z.

   d. All of the above.

   e. None of the above.

EXPERIMENT XIV

A box contains 7 balls. The balls are numbered from 1 to 7. Four balls are red, 3 are green.

1. What is the probabiliy of drawing the number 7 ball?

    a. 3/7

    b. 1/7

    c. 4/7

    d. 1

    e. None of the above.

2. What is the probability of drawing a red ball?

    a. 3/7

    b. 1/7

    c. 4/7

    d. 1

    e. None of the above.

3. What is the probability of drawing a ball numbered 2, 4, or 6?

    a. 3/7

    b. 1/7

    c. 4/7

    d. 1

    e. None of the above.

4. What is the probability of drawing a ball that is not red?

   a. 1/7

   b. 3/7

   c. 4/7

   d. 1

   e. None of the above.

5. What is the probability of drawing a blue ball?

   a. 1/7

   b. 3/7

   c. 1

   d. 0

   e. None of the above.

6. What is the probability that the event, {red, green} occurs?

   a. 2/7

   b. 1/7

   c. 1

   d. 0

   e. None of the above.

7. What is the probability that the event, {red, blue} occurs?

   a. 2/7

   b. 1/7

   c. 4/7

   d. 1

   e. None of the above.

EXPERIMENT XV

There are 5 slips of paper in a cardboard box. The slips are numbered from 1 to 5. You draw one slip, record the number and return it to the box. You draw another slip and record the number.

1. What is the probability that the two numbers drawn are the same?
    a. 1/25
    b. 2/25
    c. 5/25
    d. 3/25
    e. None of the above.

2. What is the probability that the sum of the two numbers drawn is 5?
    a. 1/25
    b. 2/25
    c. 3/25
    d. 4/25
    e. None of the above.

3. What is the probability that the product of the two numbers drawn is 30?

    a. 1/25

    b. 1

    c. 0

    d. 3/25

    e. None of the above.

4. Which sum has the lowest probability of being drawn?

    a. 2

    b. 3

    c. 10

    d. All of the above.

    e. Both a and c.

5. Which sum has the highest probability of being drawn?

    a. 5

    b. 6

    c. 10

    d. Both a and b.

    e. Both b and c.

APPENDIX C

RAW SCORES AND COMPUTATION OF STATISTICS

TABLE 4

COMPUTATION OF T FOR FIFTH GRADERS ON THE CONCEPT OF OUTCOME

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	2	2	0	-	-	-
2	2	4	-2	7		7
3	2	2	0	-	-	-
4	1	3	-2	7		7
5	1	3	-2	7		7
6	2	2	0	-	-	-
7	2	5	-3	13		13
8	3	4	-1	2		2
9	2	5	-3	13		13
10	3	5	-2	7		7
11	3	2	1	2	2	
12	1	2	-1	2		2
13	2	4	-2	7		7
14	4	1	3	13	13	
15	2	5	-3	13		13
16	0	5	-5	16		16
17	0	2	-2	7		7
18	2	4	-2	7		7
19	2	5	-3	13		13
20	3	3	0	-	-	-
21	3	3	0	-	-	-

D = Difference in pre-test and post-test scores $\Sigma R(+)=15$
R(+)=Rank associated with positive difference $\Sigma R(-)=121$
R(-)=Rank associated with negative difference $T=15$

TABLE 5

COMPUTATION OF T FOR SIXTH GRADERS ON THE CONCEPT OF OUTCOME

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	2	5	-3	13.5		13.5
2	1	4	-3	13.5		13.5
3	2	2	0	-	-	-
4	3	5	-2	7		-7
5	3	2	1	3.5	3.5	
6	1	3	-2	7		7
7	0	6	-6	17		17
8	2	5	-3	13.5		13.5
9	1	5	-4	16		16
10	3	5	-2	7		7
11	4	5	-1	3.5		3.5
12	3	3	0	-	-	-
13	2	3	-1	3.5		3.5
14	2	0	2	7	7	
15	3	3	0	-	-	-
16	3	4	-1	3.5		3.5
17	4	3	1	3.5	3.5	
18	1	4	-3	13.5		13.5
19	3	1	2	7	7	
20	4	5	-1	3.5		
21	5	5	0	-	-	-
22	2	2	0	-	-	-

$\Sigma R(+) = 21$ $\Sigma R(-) = 135.5$ $T = 21$

TABLE 6

COMPUTATION OF T FOR FIFTH GRADERS ON THE CONCEPT OF OUTCOME SPACE

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	1	3	-2	11		11
2	2	2	0	-	-	-
3	1	2	-1	5		5
4	0	4	-4	16		16
5	0	3	-3	14		14
6	0	3	-3	14		14
7	1	1	0	-	-	-
8	2	2	0	-	-	-
9	0	1	-1	5		5
10	2	3	-1	5		5
11	3	1	2	11	11	
12	1	2	-1	5		5
13	1	4	-3	14		14
14	4	3	1	5	5	
15	2	3	-1	5		5
16	0	6	-6	17		17
17	3	1	2	11	11	
18	0	1	-1	5		5
19	4	3	1	5	5	
20	1	1	0	-	-	-
21	2	1	1	5	5	

$\Sigma R(+) = 37 \quad \Sigma R(-) = 116 \quad T = 37$

TABLE 7

COMPUTATION OF T FOR SIXTH GRADERS ON THE CONCEPT OF OUTCOME SPACE

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	2	0	2	10.5	10.5	
2	5	4	1	4	4	
3	0	3	-3	15		15
4	0	1	-1	4		4
5	3	5	-2	10.5		10.5
6	3	2	1	4	4	
7	0	4	-4	17		17
8	3	5	-2	10.5		10.5
9	0	3	-3	15		15
10	3	2	1	4	4	
11	3	4	-1	4		4
12	2	7	-5	18.5		18.5
13	1	3	-2	10.5		10.5
14	1	1	0	-	-	-
15	0	3	-3	15		15
16	1	3	-2	10.5		10.5
17	2	3	-1	4		4
18	4	3	1	4	4	
19	4	4	0	-	-	-
20	6	6	0	-	-	-
21	0	5	-5	18.5		18.5
22	2	4	-2	10.5		10.5

$\Sigma R(+) = 26.5 \quad \Sigma R(-) = 163.5 \quad T = 26.5$

TABLE 8

COMPUTATION OF T FOR FIFTH GRADERS ON THE CONCEPT OF EVENT

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	3	2	1	2.5	2.5	
2	1	5	-4	14.5		14.5
3	0	4	-4	14.5		14.5
4	3	4	-1	2.5		2.5
5	4	4	0	-	-	-
6	2	4	-2	6		6
7	3	3	0	-	-	-
8	4	4	0	-	-	-
9	3	3	0	-	-	-
10	2	5	-3	10		10
11	4	2	2	6	6	
12	4	3	1	2.5	2.5	
13	2	5	-3	10		10
14	3	4	-1	2.5		2.5
15	3	6	-3	10		10
16	0	4	-4	14.5		14.5
17	4	2	2	6	6	
18	2	5	-3	10		10
19	4	4	0	-	-	-
20	0	3	-3	10		10
21	3	7	-4	14.5		14.5

$\Sigma R(+) = 17 \quad \Sigma R(-) = 119 \quad T = 17$

TABLE 9

COMPUTATION OF T FOR SIXTH GRADERS ON THE CONCEPT OF EVENT

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	4	6	-2	8.5		8.5
2	2	5	-3	11.5		11.5
3	1	1	0	-	-	-
4	5	3	2	8.5	8.5	
5	5	6	-1	3.5		3.5
6	1	5	-4	14		14
7	2	1	1	3.5	3.5	
8	3	8	-5	16.5		16.5
9	6	6	0	-	-	-
10	2	6	-4	14		14
11	7	6	1	3.5	3.5	
12	4	4	0	-	-	-
13	1	3	-2	8.5		8.5
14	1	4	-3	11.5		11.5
15	6	6	0	-	-	-
16	2	3	-1	3.5		3.5
17	2	6	-4	14		14
18	3	5	-2	8.5		8.5
19	1	1	0	-	-	-
20	3	4	-1	3.5		3.5
21	5	4	1	3.5	3.5	
22	1	6	-5	16.5		16.5

$\Sigma R(+) = 19$ $\Sigma R(-) = 134$ $T = 19$

TABLE 10

COMPUTATION OF T FOR FIFTH GRADERS ON THE CONCEPT OF PROBABILITY OF A FINITE EVENT

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	5	6	-1	2.5		2.5
2	7	2	5	14	14	
3	3	2	1	2.5	2.5	
4	3	5	-2	7		7
5	5	3	2	7	7	
6	5	5	0	-	-	-
7	1	7	-6	17		17
8	3	6	-3	10		10
9	3	9	-6	17		17
10	5	7	-2	7		7
11	2	8	-6	17		17
12	3	4	-1	2.5		2.5
13	4	6	-2	7		7
14	3	3	0	-	-	-
15	4	6	-2	7		7
16	0	5	-5	14		14
17	2	3	-1	2.5		2.5
18	1	5	-4	11.5		11.5
19	2	6	-4	11.5		11.5
20	1	6	-5	14		14
21	6	6	0	-	-	-

$\Sigma R(+) = 23.5$ $\Sigma R(-) = 147.5$ $T = 23.5$

TABLE 11

COMPUTATION OF T FOR SIXTH GRADERS ON THE CONCEPT OF PROBABILITY OF A FINITE EVENT

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	5	8	-3	16.5		16.5
2	6	7	-1	6.5		6.5
3	4	5	-1	6.5		6.5
4	8	7	1	6.5	6.5	
5	8	7	1	6.5	6.5	
6	6	4	2	14	14	
7	2	6	-4	19		19
8	6	6	0	-	-	-
9	7	9	-2	14		14
10	4	7	-3	16.5		16.5
11	9	10	-1	6.5		6.5
12	6	5	1	6.5	6.5	
13	3	5	-2	14		14
14	2	3	-1	6.5		6.5
15	7	8	-1	6.5		6.5
16	5	6	-1	6.5		6.5
17	5	6	-1	6.5		6.5
18	5	9	-4	19		19
19	5	6	-1	6.5		6.5
20	6	6	0	-	-	-
21	5	6	-1	6.5		6.5
22	6	10	-4	19		19

$\Sigma R(+) = 33.5$ $\Sigma R(-) = 176.5$ $T = 33.5$

TABLE 12

COMPUTATION OF T FOR FIFTH GRADERS ON THE CONCEPT OF MUTUALLY EXCLUSIVE EVENTS

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	2	13	-11	19		19
2	6	10	- 4	5.5		5.5
3	5	8	- 3	3.5		3.5
4	4	10	- 6	11		11
5	8	2	6	11	11	
6	1	9	- 8	15		15
7	1	12	-11	19		19
8	7	14	- 7	13		13
9	8	13	- 5	8		8
10	7	12	- 5	8		8
11	6	14	- 8	15		15
12	5	16	-11	19		19
13	5	15	-10	17		17
14	11	5	6	11	11	
15	10	10	0	-	-	-
16	0	8	- 8	15		15
17	8	9	- 1	1		1
18	6	11	- 5	8		8
19	8	12	- 4	5.5		5.5
20	3	5	- 2	2		2
21	7	10	- 3	3.5		3.5

$\Sigma R(+) = 22 \quad \Sigma R(-) = 188 \quad T = 22$

TABLE 13

COMPUTATION OF T FOR SIXTH GRADERS ON THE CONCEPT OF MUTUALLY EXCLUSIVE EVENTS

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	5	14	- 9	15.5		15.5
2	5	15	-10	17.5		17.5
3	6	6	0	-	-	-
4	8	16	- 8	13		13
5	8	12	- 4	7		7
6	7	9	- 2	2.5		2.5
7	6	14	- 8	13		13
8	4	9	- 5	9		9
9	5	17	-12	19		19
10	6	14	- 8	13		13
11	7	16	- 9	15.5		15.5
12	6	8	- 2	2.5		2.5
13	7	7	0	-	-	-
14	9	9	0	-	-	-
15	0	10	-10	17.5		17.5
16	7	13	- 6	10.5		10.5
17	7	10	- 3	4.5		4.5
18	14	15	- 1	1		1
19	5	9	- 4	7		7
20	0	6	- 6	10.5		10.5
21	7	11	- 4	7		7
22	10	13	- 3	4.5		5.5

$\Sigma R(+) = 0$ $\Sigma R(-) = 190$ $T = 0$

TABLE 14

COMPUTATION OF T FOR FIFTH GRADERS ON THE TOTAL SCORE OF THE PROBABILITY TEST

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	13	26	-13	12.5		12.5
2	18	23	- 5	2		2
3	11	18	- 7	4		4
4	11	26	-15	16		16
5	18	15	3	1	1	
6	10	23	-13	12.5		12.5
7	8	28	-20	18.5		18.5
8	19	30	-11	10		10
9	16	31	-15	16		16
10	19	32	-13	12.5		12.5
11	18	27	- 9	6		6
12	14	27	-13	12.5		12.5
13	14	34	-20	18.5		18.5
14	25	16	9	6	6	
15	21	30	- 9	6		6
16	0	28	-28	20		20
17	17	17	0	-	-	-
18	11	26	-15	16		16
19	20	30	-10	8.5		8.5
20	8	18	-10	8.5		8.5
21	21	27	- 6	3		3

$\Sigma R(+) = 7 \quad \Sigma R(-) = 203 \quad T = 7$

TABLE 15

COMPUTATION OF T FOR SIXTH GRADERS ON THE TOTAL SCORE OF THE PROBABILITY TEST

Student #	PR	PO	D	Absolute Rank of Difference	R(+)	R(-)
1	18	33	-15	17.5		17.5
2	19	35	-16	19.5		19.5
3	13	17	- 4	3		3
4	24	32	- 8	9		9
5	27	32	- 5	4.5		4.5
6	18	23	- 5	4.5		4.5
7	10	31	-21	21.5		21.5
8	18	33	-15	17.5		17.5
9	19	40	-21	21.5		21.5
10	18	34	-16	19.5		19.5
11	30	41	-11	13.5		13.5
12	21	27	- 6	6.5		6.5
13	14	20	- 6	6.5		6.5
14	15	17	- 2	1		1
15	16	30	-14	15.5		15.5
16	18	29	-11	13.5		13.5
17	20	28	- 8	9		9
18	27	36	- 9	11.5		11.5
19	18	21	- 3	2		2
20	19	27	- 8	9		9
21	22	31	- 9	11.5		11.5
22	21	35	-14	15.5		15.5

$\Sigma R(+) = 0$ $\Sigma R(-) = 253$ $T = 0$

TABLE 16

COMPUTATION OF U FOR THE DIFFERENCE IN PRE- AND POST-TEST SCORES ON THE TOTAL PROBABILITY TEST OF FIFTH AND SIXTH GRADERS

Difference Scores of Fifth Graders	Difference Scores of Sixth Graders	R_f	R_s
13	15	27.5	34
5	16	8	37.5
7	4	13	6
15	8	34	15
-3	5	2	8
13	5	27.5	8
20	21	39.5	41.5
11	15	24	34
15	21	34	41.5
13	16	27.5	37.5
9	11	18.5	24
13	6	27.5	11
20	6	39.5	11
-9	2	1	4
9	14	18.5	30.5
28	11	43	24
0	8	3	15
15	9	34	18.5
10	3	21.5	5
10	8	21.5	15
6	9	11	18.5
	14		30.5

R_f= Rank of scores of fifth graders with respect to all scores.
R_s= Rank of scores of sixth graders with respect to all scores.
$\Sigma R_f=476$ $\Sigma R_s=470$