DEHUMIDIFYING COIL MODELS FOR ENERGY SIMULATION AND DESIGN

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Stillwater, Oklahoma

2012

Submitted to the Faculty of the

Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
May, 2012

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SIMULATION AND DESIGN

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1. INTRODUCTION

This thesis presents a summary and analysis of the development of dehumidifying fin and tube coil models. The goal is to compare and assess the models that are presented in the available literature and evaluate the implementation and performance of some select models. By bringing all of these works into one central location and comparing one author's work to another and experimental data, differences in the models can be evaluated for increased accuracy of calculated heat transfer rates, ease of implementation, and range of applicability. This will facilitate the need to clearly identify assumptions and methods that each author uses and determine if the assumptions are acceptable.

A. Overview

Heat transfer between air and a fluid separated by an impermeable surface has a well-defined calculation method when the heat transfer is sensible only. The sensible heat transfer is driven by a temperature potential between the air and fluid. Several methods have been developed to calculate the performance of a coil such as the Log Mean Temperature Difference (LMTD) and the Effectiveness-NTU (Eff-NTU). Some difficulty is introduced when calculating the performance of a coil that dehumidifies the air. The dehumidification process introduces complexities in the heat and mass transfer calculations.

To solve for the combined heat and mass transfer problem, assumptions are made in order to derive a workable set of equations. The assumptions used have developed over time depending on computing resources available at the time. Early works mainly used the LMED and Eff-NTU methods based on the enthalpy potential assumption. Later works have relied on computational power of more modern computers and not rely on simplifying assumptions such as the enthalpy potential. Also, as the computational power of computers increased, authors subdivided the heat exchangers into smaller sections for a more accurate solution.

This review will bring continuity to the body of dehumidifying coil literature by:

- clearly identifying the underlying assumption associated with each author's work
- assessing the performance of some select models to experimental data
- evaluating the differences between models

2. REVIEW OF THE LITERATURE

Twenty four papers are included in this review with the earliest being Goodman (1938). Goodman presented the enthalpy potential assumption and the LMED method. Threlkeld (1962) presented a LMED method that is more detailed than Goodman's by accounting for factors such as the tube conduction, condensate film conduction, and fin efficiency. Threlkeld also used the term 'fictitious enthalpy' which aids in the understanding of his derivations. Several authors reference Threlkeld's work such as Elmahdy & Mitalas (1977), Braun, et al (1989), Domanski (1991), Mirth & Ramadhyani (1993), Theerakulpisut & Priprem (1998), Vardhan & Dhar (1998), and Morisot, et al. (2002).

A. Methods and Assumptions

A complete listing of the papers included in the review is shown in Table 1. The table shows the methods that are used to develop the models as well as some of the common assumptions that are made. With one exception, the methods are independent of the assumptions. Since the enthalpy potential assumption is used to derive the LMED method, the use of the LMED method implies the use of the enthalpy potential assumption.

Table 1 - Summary of methods and assumptions used by various authors

Author	Date	Method				Assumptions		
		LMED	LMTD	Eff-NTU	Fundamental Equation	Enthalpy Potential	Le = 1	Fictitious Enthalpy
Goodman	1938	✓				✓	✓	✓
Ware	1960		✓(1)			✓	✓	✓
Threlkeld	1962	✓				✓	✓	✓
McQuiston	1975		✓(2)					
Elmahdy	1977	✓				✓	✓	✓
Braun	1989			✓		✓	✓	✓
Domanski	1991		✓(1)				✓	
Hill	1991				✓			✓
Mirth	1993				✓			
Theerakulpisut	1998	✓				✓		✓
Vardhan	1998				✓	✓	✓	✓
McQuiston	2000	✓(3)				✓	✓	
AHRI	2001	✓				✓	✓	
Wang	2003			✓		✓	✓	
Oliet	2007			✓			✓	✓
Wang	2007			✓			✓	
Xia	2009		✓			✓		✓

(1) – Application of method not explicitly defined
(2) – Used LMTD and LMwD
(3) – Used LMED (air to surface) and LMTD (surface to cold fluid)

Additional assumptions are typically made in the calculation of the thermal resistance from the free stream air to the cold fluid. There are a number of possible combinations.

Figure 1 illustrates each thermal resistance in the network as a solid line between two points representing different temperature measurement locations in the network. A vertical line between two points indicates that the temperatures of the two points are the same indicating that the thermal resistance between those two points is assumed to be negligible. Tube and film thermal resistances are typically neglected as illustrated in Figure 1 (a), (b), (d). Table 2 shows the resistances that each author includes in their work.

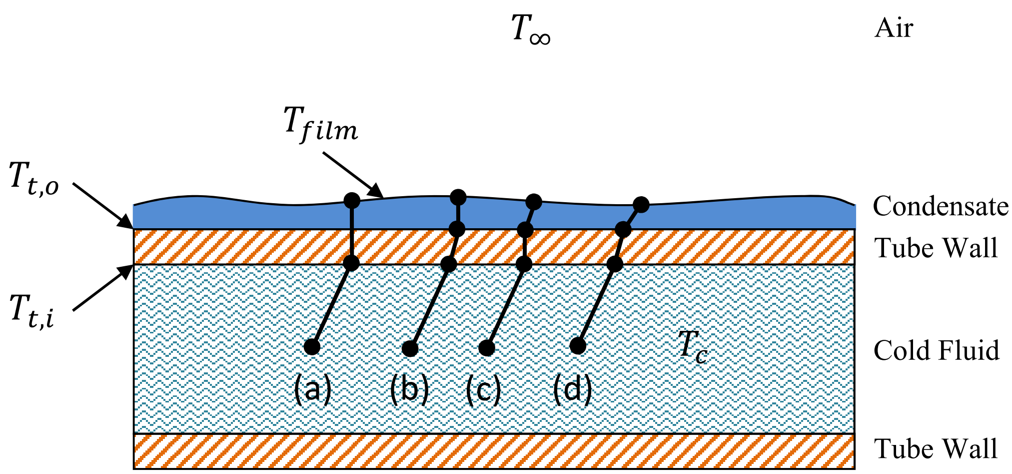

Figure 1 - Temperature gradients with different resistances included.

Table 2 - Various heat transfer resistances used by various authors

Author	Date	Resistances included					
		Air side convection	Condensate film	Tube wall	Cold fluid convection	Contact	Fouling
Goodman	1938	X			X		
Ware	1960	X			X		
Threlkeld	1962	X	X		X		X
McQuiston	1975	x		X	x		
Elmahdy	1977	X		X	X		X
Braun	1989	X	Lumped term*				
Domanski	1991	X	X	X	X	x	
Hill	1991	X	Lumped term*				
Mirth	1993	X		X	X		
Theerakulpisut	1998	X	X	X	X		
Vardhan	1998	X		X	X		
McQuiston	2000	X			X		
AHRI	2001	X		X	X		X
Wang	2003	X		X	X		
Oliet	2007	X	X	X	X		X
Wang	2007	Lumped Term*	Lumped term*				
Xia	2009	X			X		

* - combined UA term with no detailed description

B. Nomenclature

A representative section of a dehumidifying coil is show in Figure 2. This figure will aid in clearly identifying where temperatures or other properties are located.

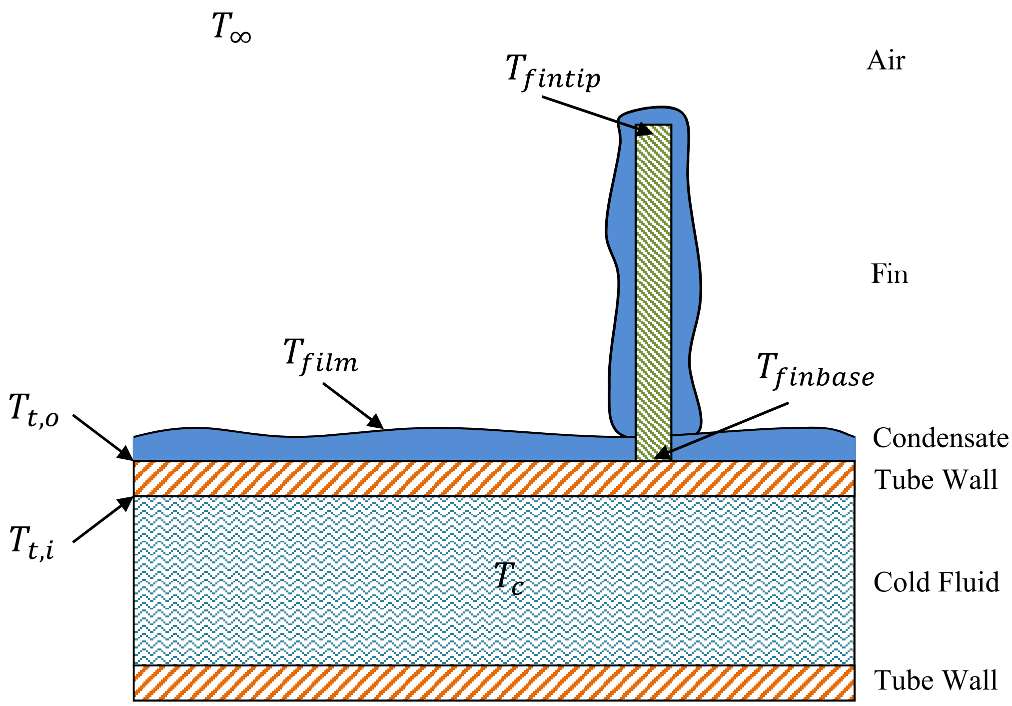

Figure 2 - Cut away view of a representative dehumidifying coil

One of the common points of confusion is the use of a 'surface' temperature. There are several surfaces shown in Figure 2 such as the film surface, fin surface, and tube surface. When an author assumes no thermal resistance in the tube and/or condensate layer, the term 'surface' gets even vaguer.

A complete table of nomenclature used in this thesis is included at the end.

C. Fundamental Equation and Basic Assumptions

The governing equation for dehumidifying air is given by equation (2-1) ASHRAE (2009). This equation is the starting point for all the dehumidifying coil models discussed in this paper.

$$\frac{\dot{Q}}{A} = h(T_\infty - T_{film}) + h_m(w_\infty - w_{film})i_{fg,water} \tag{2-1}$$

Some authors Domanski (1991), Hill & Jeter (1991), Mirth & Ramadhyani (1993), Wang, et al., (2007), and Oliet, et al. (2007) have developed methods that use the fundamental equation [equation (2-1)] as a basis for their starting point and did not try to simplify this equation to make it more manageable. This leads to a computationally intensive equation set but fewer assumptions are used, implying a more accurate solution. All of the other authors use a simplified form of the fundamental equation called the enthalpy potential which will be discussed later.

Wang & Hihara (2003) developed a new methodology to calculate the heat and mass transfer of a dehumidifying coil. Their method, the equivalent dry bulb temperature method (EDT), finds an equivalent sensible only heat transfer process and uses that process to determine the total capacity. In Figure 3 the dashed green line represents the actually dehumidifying process and the red line is the equivalent sensible only process. Wang defines three states of the coil, completely dry, partially wet, and completely wet, and defines how to determine which state the coil is in by the entering air conditions. This is convenient because it results in a non-iterative process to determine if the coil is wet or dry. One of Wang's reasons for developing this methodology is to simplify the calculations. Although it is hard to determine if this method is simpler, it does present a different way to analyze the coil state. A more detailed analysis of Wang's method is found in Appendix D.

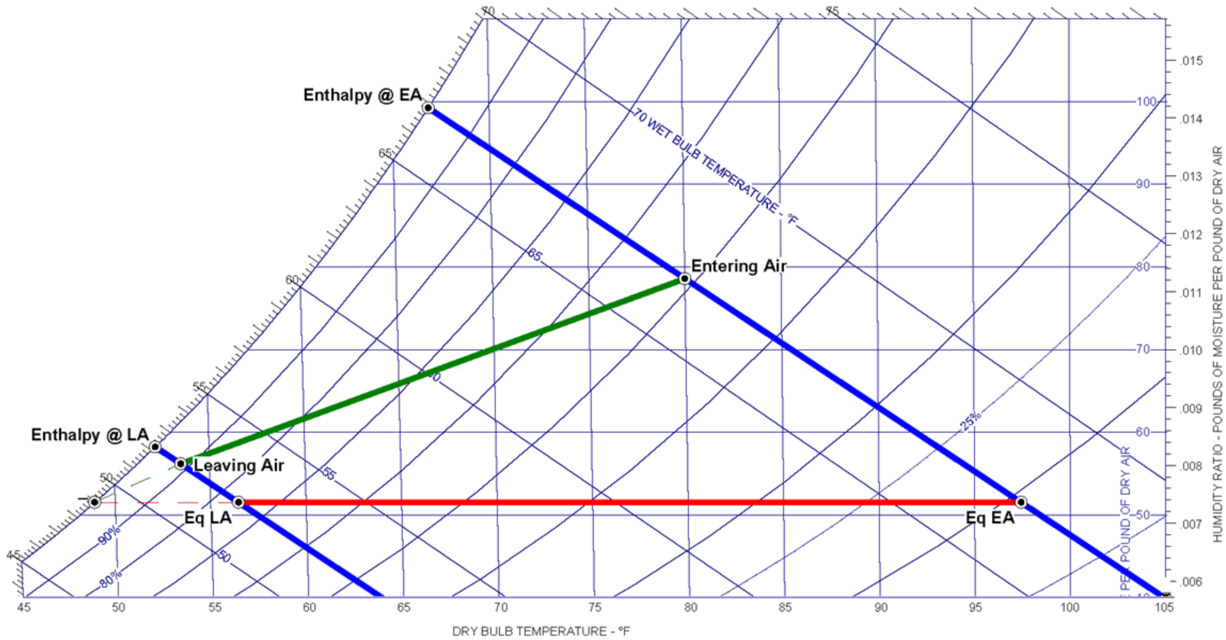

Figure 3 - Representation of the EDT method

i. Lewis Number

Evaluating the mass transfer coefficient in equation (2-1) is difficult. By introducing the Lewis number, the ratio of the heat transfer coefficient to the mass transfer coefficient can be established.

$$Le^{2/3} = \frac{h}{c_{p,\infty} h_m} \tag{2-2}$$

ASHRAE (2009) states the value of the Lewis number for air is about 0.845 for an air and water vapor mixture. Thus leading to $0.845^{2}/_{3} = 0.894 = \frac{h}{c_{p,\infty} h_m}$. For most authors, the Lewis number is assumed to be unity, mainly for convenience and simplification.

Few authors discuss the significance of assuming the Lewis number of unity. Xia, et al. (2009) does give some analysis in justifying his Modified LMED method. Xia states that there is significant deviation (range of -18% to 15% of total capacity) for Lewis number values that range from 0.6 to 1.4, although he is not clear what air side operating conditions produced the desired

Lewis numbers. The deviation is compared to a numerical. The effects of the Lewis number will be revisited in Chapter 5.

ii. Enthalpy Potential Assumption

In order to simplify equation (2-1) a Lewis number of unity is typically assumed. If a Lewis number of unity is not assumed, the derivation becomes more complicated. The enthalpy of the free stream air and the saturated air at the film surface is then introduced as shown in equation (2-3).

$$\frac{\dot{Q}}{A} = h_m\left[\left(i_\infty - i_{film}\right) - \left(w_\infty - w_{film}\right)i_{f,water} - w_\infty c_{p,vapor}\left(T_\infty - T_{film}\right)\right] \tag{2-3}$$

By neglecting the enthalpy change in the liquid water and superheated vapor, equation (2-3) can be simplified into equation (2-4). According to McQuiston, et al. (2000) the neglected enthalpy change typically accounts for about 0.5% of the total heat transfer and is therefore relatively insignificant. Equation (2-4) is commonly referred to as the enthalpy potential equation.

$$\frac{\dot{Q}}{A} = h_m\left(i_\infty - i_{film}\right) \tag{2-4}$$

Since the enthalpy potential equation accounts for heat and mass transfer in one equation, it is also referred to as a 'single potential' model. Most of the earliest papers presented here use the enthalpy potential as a basis for their heat exchanger model including Goodman (1938), Ware & Hacha (1960), Threlkeld (1962), Elmahdy & Mitalas (1977), Braun, et al (1989), Theerakulpisut & Priprem (1998), Vardhan & Dhar (1998), McQuiston, et al. (2000), Wang & Hihara (2003), and Xia, et al. (2009). A more detailed derivation of the enthalpy potential is found in Chapter 3.

D. Log Mean Enthalpy Difference

When Goodman presented his enthalpy potential assumption, heat exchanger calculations were still being performed by hand, so having a simple methodology for determining heat exchanger performance was necessary. By using the enthalpy potential assumption, the Log Mean Enthalpy Difference (LMED) methodology is derived. This methodology is analogous to the Log Mean Temperature Difference (LMTD) Cengel & Ghajar (2010).

$$\dot{Q} = h_m A \Delta i_{mean} \tag{2-5}$$

$$\Delta i_{mean} = \frac{(\Delta i_2 - \Delta i_1)}{ln\left[\frac{\Delta i_2}{\Delta i_1}\right]} \tag{2-6}$$

Where,

$\Delta i_1 = i_{\infty,1} - i_{film,1}$
$\Delta i_2 = i_{\infty,2} - i_{film,2}$

The form of the LMED method as shown in equations (2-5) and (2-6) calculates the heat transfer rate from the condensate film to the air. The temperature of the film must be determined by other means. McQuiston (1975) suggests using the LMED and the LMTD methods as shown in Figure 4 to calculate the heat and mass transfer from the free stream air to the cold fluid. This method requires equating the heat transfer rates determined from the LMED and LMTD methods. An iterative solution scheme is typically used to determine the film temperature that satisfies both equations.

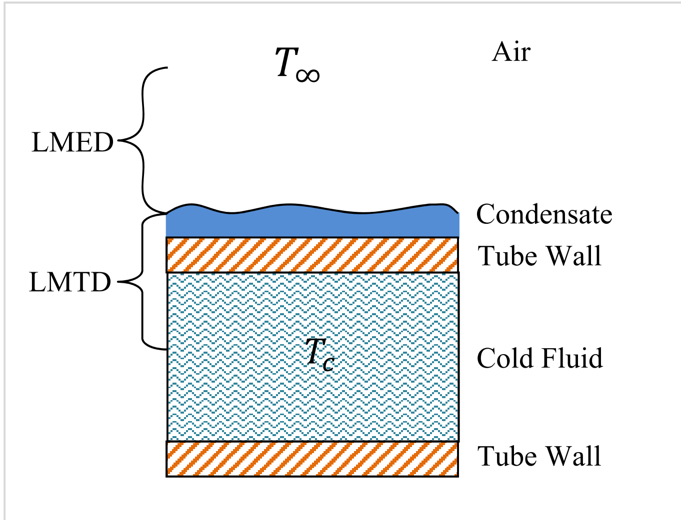

Figure 4 - Application of LMED and LMTD methods to a heat exchanger

Recently, Xia, et al. (2009) presented a LMED method with the Lewis number as an input. Xia's work resulted in a Lewis number adjustment factor that is applied to the air side convection coefficient. While Xia's work started with the enthalpy potential equation (which assumed a Lewis number of unity) and then reintroduced the Lewis number, it would have been easier to understand if Xia started with the fundamental equation (2-1) and derived the enthalpy potential assumption without the Lewis number set to unity. The end result is a LMED method that accounts for varying values of the Lewis number as shown in equations (2-7) and (2-8).

$$\dot{Q} = \frac{hA}{c_{p,\infty}} Le_{adjustment}\left(i_\infty - i_{film}\right) \tag{2-7}$$

$$Le_{adjustment} = \left[1 - \frac{Le^{2/3} - 1}{\frac{\dot{Q}_{sensible}}{\dot{Q}_{latent}} + Le^{2/3}}\right] \tag{2-8}$$

The concern with this method is determining the value of the Lewis number. The method used in Xia's work is described in Pirompugd, et al. (2007) which is a curve fit of the Lewis number for a specific coil geometry. As a result, the correlation is not applicable beyond the original data set.

i. Fictitious Air Enthalpy

An alternative to the combined LMED and LMTD approach shown in Figure 4 applies the LMED from the free stream air to the cold fluid. In order to do this, all heat transfer potentials must be defined in terms of air enthalpy so they can be added together in series. This leads to the introduction of a fictitious air enthalpy. The fictitious air enthalpy is the saturated air enthalpy evaluated at a temperature other than the air temperature. This allows a temperature node to be expressed in terms of an air enthalpy, thus meeting the requirement to express the entire process from refrigerant to air in terms of an enthalpy difference. The fictitious air enthalpy takes the form of a linear approximation, as shown in equation (2-9), due to the complexities that a higher order polynomial representation of the saturation line introduces when solving the differential equations. The fictitious air enthalpy is defined as the tangent to the saturation line at a particular temperature.

$$i' = a + bT \tag{2-9}$$

Ideally there should be an enthalpy approximation for each temperature node shown in Figure 1, Threlkeld (1962). Linear approximations for the cold fluid temperature, the fin base temperature, and the mean fin temperature are shown in Figure 5. The dashed green line is tangent to the saturated air curve at 45°F, the gold line is tangent at 50°F, and the blue line is tangent at 51°F. These temperatures are a fair representation of a typical HVAC cooling coil.

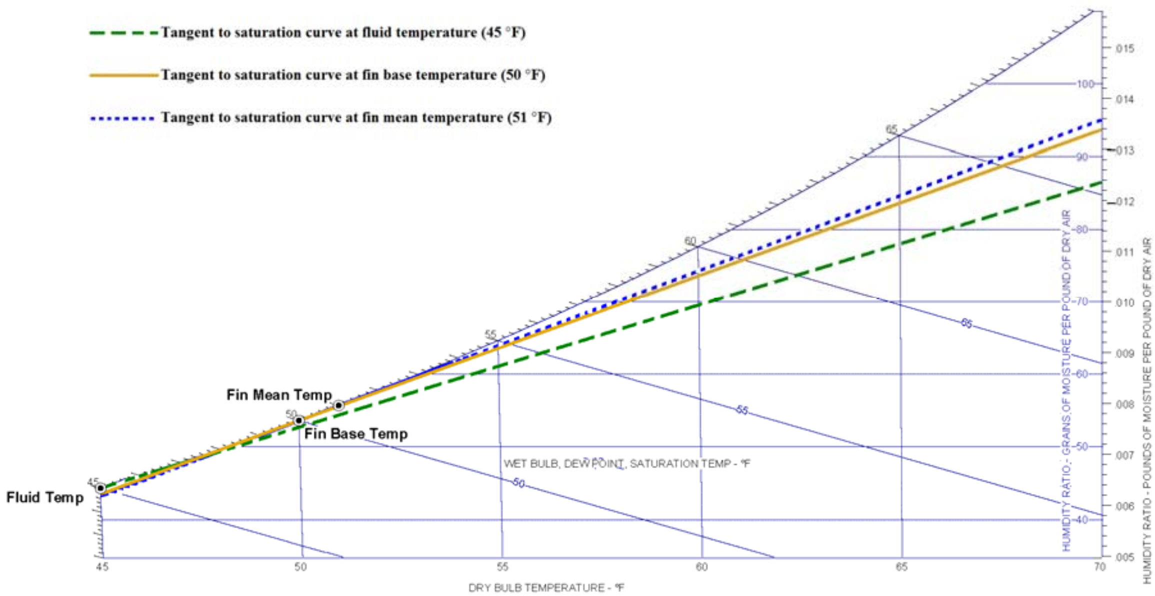

Figure 5 - Linear approximation of air enthalpy

The following equations are presented to give an example of the application of the fictitious air enthalpy. Equation (2-10) describes the heat transfer between the cold fluid and the inner tube surface in terms of the temperature potential.

$$\dot{Q} = Ah_i\left(T_{t,i} - T_c\right) \qquad (2\text{-}10)$$

By applying the fictitious enthalpy of equation (2-9) to equation (2-10), equation (2-11) is formed, which describes the heat transfer in terms of fictitious air enthalpy.

$$\dot{Q} = Ah_i\left[\left(\frac{i'_{t,i} - a_{t,i}}{b_{t,i}}\right) - \left(\frac{i'_c - a_c}{b_c}\right)\right] \qquad (2\text{-}11)$$

All papers either assume the 'a' coefficients cancel out or can just be neglected except for Hill & Jeter (1991). Hill's method uses a fictitious humidity ratio instead of a fictitious air enthalpy but the fictitious is used in the same manner as fictitious enthalpy. Authors will also make some simplifications to the 'b' coefficient such as Goodman (1938), Ware & Hacha (1960), and Elmahdy & Mitalas (1977) use a single 'b' coefficients will be approximated by an average value. Threlkeld (1962) uses two different 'b' coefficients instead of combining them into one. The percent change in the green dashed line compared to the blue dotted line is about 10%. This equates to a 2% change in total resistance so it is a simplification that is used as suggested by Ware & Hacha (1960).

Assuming that $a_{t,i} \approx a_c$ and $b_{t,i} \approx b_c$ equation (2-11) reduces to equation (2-12).

$$\dot{Q} = \frac{Ah_i}{b_c}\left(i'_{t,i} - i'_c\right) \tag{2-12}$$

Ware & Hacha (1960) predates the fictitious enthalpy presented by Threlkeld but he does present similar work regarding the 'b' term of equation (2-9). Ware presents a theoretical analysis that justifies the combining of several different 'b' terms into one. He shows that the assumptions do not affect the overall thermal resistance by more than 1.5%. The end result of Ware's work is the LMTD method for a wet coil, which will be discussed in the next section.

One of the issues with the fictitious enthalpy is that if it needs to be solved at an internal temperature node (not the air or cold fluid), the solution now becomes iterative. An initial guess of the internal temperature is used to evaluate the desired fictitious enthalpy then the heat transfer rate can be determined. Once the heat transfer rate is determined, the internal temperatures are then calculated via the resistance network and verified with any initial guesses.

Braun, et al (1989) used a fictitious enthalpy but in a different form. Braun uses differential equations to build his method so he essentially uses the differential of equation (2-9). Because of this, only the 'b' term of equation (2-9) is seen is Braun's work. Braun implies, though does not explicitly state, that the final form of his method evaluates the 'b' term at a single point and uses that slope for all temperatures in the coil.

Elmahdy & Mitalas (1977) is the only model that requires the 'a' coefficient to solve the set of equations. This is due to the way they chose to form the equation set. Elmahdy writes an equation to determine an intermediate surface temperature that is a function of the entering air enthalpy and water temperature. He converts the fictitious enthalpy to a temperature in the equation. Threlkeld, on the other hand, defines an equation in terms of the entering air enthalpy and fictitious enthalpies and allows the user to determine how to convert the fictitious enthalpy into a temperature. The user could use the fictitious enthalpy equation or use their own psychrometric routines.

Since this fictitious air enthalpy is a linear approximation of the saturation line at a particular point, it can only be applied over a small range of temperatures without introducing significant error. Threlkeld (1962) suggests that this approximation only be applied to any temperature in the thermal network that deviates by less than 10°F from the temperature at which the fictitious enthalpy is defined. This refers to the temperature change of the cold fluid from the inlet to outlet and any other temperature node where the fictitious air enthalpy is applied such as the tube surface or condensate temperatures. While Threlkeld (1962) did not give any justification for the 10°F limit, at 60°F, the max percent error for a range of ±5°F is -0.7% and the max percent error for a range of 0-10°F is -3.3% as shown in Figure 6.

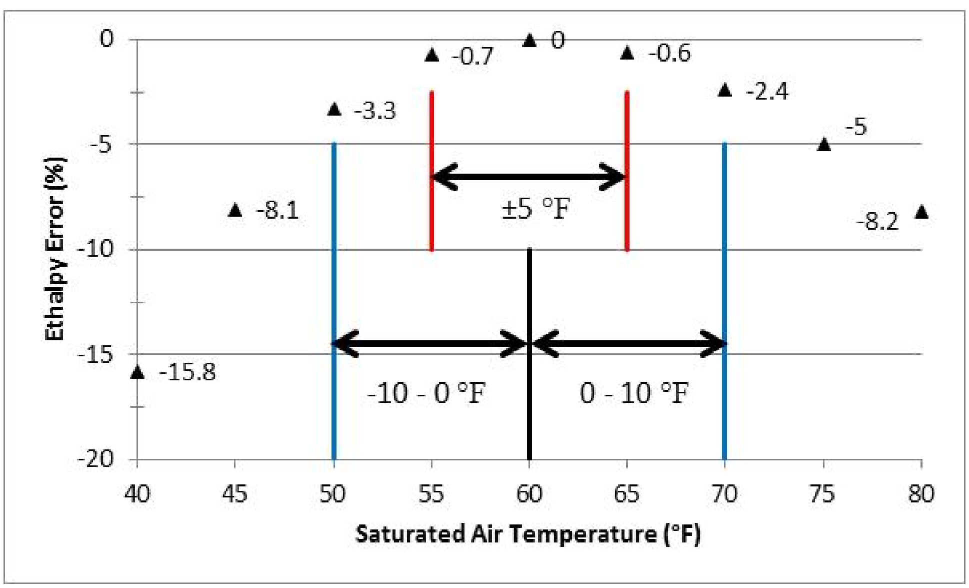

Figure 6 - Percent error between actual air enthalpy value and linear approximation

E. LMTD

While the fictitious enthalpy is usually used to convert a temperature to an enthalpy, it can also be used to convert an enthalpy to a temperature. Some authors Ware & Hacha (1960) Vardhan & Dhar (1998) have taken the enthalpy potential equation and then applied the fictitious enthalpy approximation to change the enthalpy potential into a temperature potential. This requires that the enthalpy for air is constant for a given wet bulb temperature. By substituting the fictitious enthalpy into equation (2-4), the following equation is produced.

$$\dot{Q} = hA\left(b_{\infty}T_{\infty,wb} + a_{\infty} - b_{film}T_{film} + a_{film}\right) \tag{2-13}$$

For simplicity, the authors use an average value of b_{∞} and b_{film} defined by equation (2-14).

$$b_{ave} = \frac{i_{\infty} - i_{film}}{T_{\infty,wb} - T_{film}} \tag{2-14}$$

$$\dot{Q} = hAb_{ave}\left(T_{\infty,wb} - T_{film}\right) \tag{2-15}$$

Technically, this allows a dehumidifying heat transfer process to be determined using the LMTD method, but the free stream air temperature is its wet bulb temperature instead of its dry bulb temperature.

An earlier work by McQuiston (1975) presented a method where the sensible and latent heat transfer rates are solved independently. The sensible heat transfer is solved by the LMTD method, using the dry bulb temperature, and the latent heat transfer is solved by a log mean humidity ratio difference (LMwD). The UA term as presented in the paper for the sensible heat transfer includes the air side convection coefficient, the tube conduction, and the cold fluid convection coefficient. This means that the temperature potential is from the air side to the cold fluid. While a temperature potential from the free stream air to the condensate boundary applies to a sensible load, a temperature potential from the condensate to the cold fluid applies to a sensible and latent load. The equation for the sensible heat transfer in this paper appears to be incorrect.

F. Eff-NTU

Both the LMTD and the LMED methods need the input and output conditions of the two fluids. When determining the performance of a given coil, this causes the solution to be iterative. For a dry coil, the Eff-NTU method is not iterative as only the inlet fluid conditions are needed. Taking the Eff-NTU methodology and applying to a wet coil is, unfortunately, still an iterative process. When Braun, et al (1989) and Oliet, et al. (2007) developed an Eff-NTU method for a wet coil, they both used the fictitious enthalpy. This fictitious enthalpy must be evaluated at various intermediate temperatures, such as at the fin, tube surface, or leaving cold fluid.

Hill & Jeter (1991) used equation (2-1) to derive what they called a 'generalized effectiveness-NTU heat flux equation for combined heat and mass transfer'. In order to simplify the equation set, the humidity ratio of the saturated air is assumed to be a linear function of temperature. Hill shows that his equation reduces to the Eff-NTU for a dry coil with Cmin always calculated from the air side. This is a limiting assumption of his method. Hill also assumes that the cold fluid temperature can be represented by a single temperature. When the cold fluid is undergoing a phase change, the temperature is constant, but if the fluid is not undergoing a phase change, then some equivalent cold fluid temperature must be defined.

G. Additional Points of Interest

i. Definition of a Wet Coil

All of the coil models mentioned previously apply to a wet or partially wet coil, the definition of which may vary. For a coil to have condensate, part of the coil that is exposed to the air must have a temperature that is less than the dew point of the air. The simplest way to determine if the coil is wet is to use the tube outer surface temperature Goodman (1938), Ware & Hacha (1960), Threlkeld (1962), Elmahdy & Mitalas (1977), Braun, et al (1989), Domanski (1991), Hill & Jeter (1991). Since this temperature is the minimum temperature at the fin base, it is possible to have a partially wet fin.

Some authors, Threlkeld (1962) and Oliet, et al. (2007), use the mean fin temperature, which is determined from the fin efficiency, to determine if the coil is wet or dry. Other authors, Jiang, et al. (2006) and Vardhan & Dhar (1998), use the equivalent surface temperature, which is determined from the surface effectiveness.

In order to get more detailed dry/wet analysis, additional information about the fin is needed since the fin efficiency changes when the fin is dry or wet. Mirth & Ramadhyani (1993) and Wang & Hihara (2003) used methods that give more details about the fin so their definition of a wet coil changes. These authors define three regions of a coil; wet, dry, and partially wet and define a coil as dry when the surface temperature at the fin base is above the dew point of the air. A coil is defined as wet when the fin tip is below the dew point of the free stream air. Any other condition results in a partially wet coil.

ii. Application of the Methods

There are several different ways to apply the methods mentioned above. The simplest way to apply these methods is the 'slab' application. This application assumes that some properties such as convection coefficients and fin efficiencies are constant throughout the entire coil. In addition, wet/dry coil definitions must be applied to the entire coil. These assumptions are not always valid but they simplify the evaluation of the heat exchanger performance.

There are several circumstances where evaluating a coil as a 'slab' may not be appropriate. If the properties of the fluids cannot be assumed constant, the coil is partially wet, or the use of the fictitious enthalpy assumption is not in a valid range; the coil needs to be divided into smaller sections and each section individually evaluated. One way to divide the coil into sections is by rows. For example, a five row coil could be split into two sections; one that is three rows and the other that is two rows Threlkeld (1962).

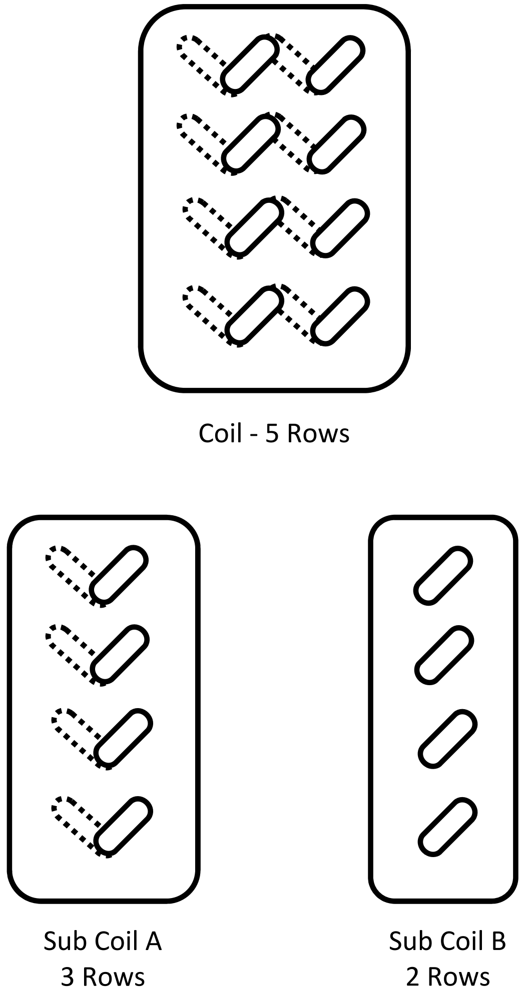

Figure 7 - Schematic of coil split into smaller sections

For a counter flow coil, the solution of the coil now becomes more iterative as the fluid outlet conditions of subcoil A is now the inlet fluid conditions of subcoil B. Also, the outlet air conditions of coil B are now the inlet air conditions of coil A.

Not all authors suggest dividing the coil at a physical boundary such as between rows. Dividing a coil into a non-condensing (dry) section and a condensing (wet) section, Elmahdy & Mitalas

(1977) suggests determining the percent area that is dry and wet even if the division is not at a physical boundary. Elmahdy presents an equation that determines the water temperature at the boundary between a dry and wet coil. With this key temperature an iterative solution is developed to determine the percent areas for each section of the coil.

The slab approach works well for a refrigerant coil that is undergoing a phase change as long as the refrigerant stays as a two phase fluid for its entire time in the coil. With the fluid undergoing a phase change, the temperature of the fluid is nearly constant. While there are some applications where this does occur, the refrigerant commonly exists as either a liquid and two phase fluid or gas and two phase fluid in the common applications of HVAC refrigerant coils. Theerakulpisut & Priprem (1998) present a slab approach for an direct expansion evaporator coil using three sections, a dry two-phase section, a wet two phase section, and a superheated section, where two-phase and superheated refer to the refrigerant and dry/wet refer to air side condensate.

The next reasonable step to a more accurate solution is to evaluate the performance of each individual tube of the coil which is an application Domanski (1991) presented. This application is commonly referred to as the 'tube by tube' approach. This application has several benefits over the 'slab' approach. First is the ability to account for different circuiting options of a coil. The slab approach assumes that every tube has the same water flow rate and convection coefficient. If there is a coil that has a circuitry where the water flow rate varies from tube to tube, this approach can account for it. Another benefit is the 'tube by tube' approach can account for a non-uniform airflow between tubes, however it cannot account for non-uniform airflow along the length of the tube. In contrast, the 'slab' approach assumes uniform airflow across the entire coil face area.

Even in a tube by tube approach, the heat exchanger may not be divided into small enough sections to account for all property variations such as refrigerant changing from a liquid to a two phase fluid and then to a super-heated vapor. Dividing the tube into smaller sections leads to the formation of a 'segment by segment' approach. Vardhan & Dhar (1998) and Jiang, et al. (2006) present details on segment by segment approaches. Only with a segment by segment approach can air distribution variations across the length of the tube be accounted for. Another benefit of the 'segment by segment' approach is that the application of the linearized fictitious enthalpy is more accurate since the temperature difference across a segment is rather small. Increasing the number of segments will lead to a more accurate but will cost much more computational time. The type of coil can also affect the required number of segments. A coil with two-phase refrigerant only (fairly constant heat transfer properties) would need fewer segments than a coil with both two-phase and single phase (variable heat transfer properties).

Any of the calculation methods described in previous sections (LMED, LMTD, Eff-NTU) can be used with any of the applications mention in this section ('slab', 'tube by tube', and 'segment by segment'). The preferred method and application depends on the desired level of tolerance and available computational resources of the user.

The one final application is a 'finite difference' application. This application is similar to the 'segment by segment' application except the segments are so small, the temperatures across the segment are essentially constant. This allows the fundamental equation [equation (2-1)] to be used in its original form as described by Mirth & Ramadhyani (1993). Mirth also assumes that the coil can be modeled as a pure counter flow heat exchanger (see Figure 41) as a representative heat exchanger), if the coil has more than three rows. This treats the coil like a slab application where varying airflows and cold fluid flow rates cannot be accounted for but divides the coil into many sections to account for varying fluid properties.

iii. Energy Modeling

For an energy simulation software purposes, an ideal model needs to be quick, have minimal inputs, and be accurate. To use the previously mentioned methods, the thermal resistance must be calculated which required many inputs, specifically, many details about the coil geometry. There have been models developed that can take a few operating points and build a simple coil model. Some of these include Morisot, et al. (2002), Lemort, et al.(2008), and Brandemuehl (1993). These models determine thermal resistances from catalog data and ratio the performance based on parameters like air and refrigerant rates. These models must make some assumptions and can vary from assuming 'current technology' or ignoring the performance of the cold fluid.

iv. Sensible Load

When using the LMED or Eff-NTU method to calculate the performance of a dehumidifying coil, the total heat and mass transfer is calculated but the split between latent and sensible is not known. To determine the split, the following equation is given by AHRI (2001).

One of the previously mentioned methods must first be used to determine the outlet air enthalpy. Then an equivalent film surface enthalpy is determined by the following equation.

$$i_{film,eqv} = i_{\infty,i} - \frac{i_{\infty,i} - i_{\infty,o}}{1 - e^{-NTU}} \tag{2-16}$$

Where,

$$NTU = \left(\frac{A_o h}{\dot{m}_\infty c_{p,\infty}}\right)$$

Then equation (2-17) is used to determine the leaving air dry bulb temperature, where $T_{film,eqv}$ is the saturated temperature with an enthalpy of $i_{film,eqv}$.

$$T_{\infty,db,o} = T_{film,eqv} + \left(T_{\infty,db,i} - T_{film,eqv}\right)e^{-NTU} \tag{2-17}$$

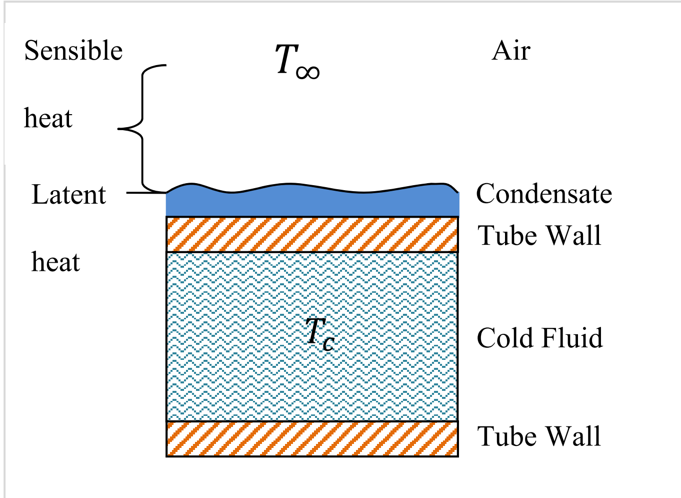

Figure 8 - Sensible and latent heat transfer locations.

H. Summary

As seen from the sections above, there are many different ways authors build a dehumidifying coil models. Each model is unique as they apply different assumptions, use different heat transfer potentials, and even use equivalent temperatures and processes to model a heat exchanger. A comparison between models is needed to see if the added details of each model show an improvement to experimental data.

3. MATHEMATICAL BASIS OF THE MODELS

Determining the performance of a heat exchanger is one of the central problems in unitary equipment design. There are basic techniques that can be found in almost any heat and mass transfer book, such as the Log Mean Temperature Difference (LMTD) and the Effectiveness-NTU Method (Eff-NTU). These methods have certain assumptions and a specific range of application. When mass transfer gets introduced into the heat exchanger problem, as in the HVAC industry with a dehumidifying coil, these two methods must undergo some modifications to apply to this situation. These methods have been developed over many years, but it is often unclear as what the basic equations and assumptions are for a particular form of the methods. In addition, some of the 'advancements' in these heat exchanger models may be just undoing early assumptions which were made to facilitate performing the calculations by hand. This chapter will lay out the fundamental equations used to derive these state-of-the-art heat exchanger models so that newer models can be compared.

Our basic heat exchanger problem involves two fluids of different temperature separated by a tube or channel wall. The simplest example of this would be a coaxial heat exchanger. A cold fluid is flowing through a tube; that cold fluid and tube is placed inside of another tube that has a hot fluid inside of it. Figure 9 shows a schematic of a coaxial heat exchanger in a parallel flow configuration.

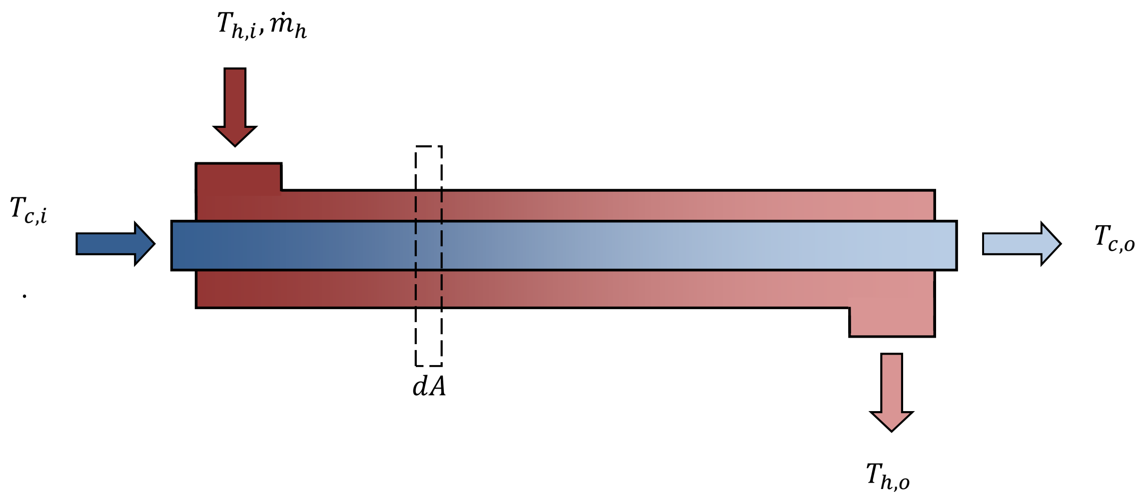

Figure 9 - Coaxial heat exchange with parallel flow

Cengel & Ghajar (2010) present the derivations that develop the basic LMTD and Eff-NTU methods. The derivations are shown in Appendix B and Appendix C to provide the context necessary when working with more the complex combined heat and mass transfer scenarios.

A. Enthalpy Potential

For a cooling coil, there is a chance of water condensing out of the air stream and onto the coil. At times this is a desired effect to maintain comfort conditions in an occupied space.

A 'dry' cooling coil cools the air but not below the dew point of the air. Therefore, no moisture will condense out of the air stream.

The water side heat transfer rate can be written as:

$$\dot{Q} = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) \tag{3-1}$$

The air side heat transfer rate can be written as:

$$\dot{Q} = \dot{m}_\infty c_{p,\infty}(T_{\infty,o} - T_{\infty,i}) \tag{3-2}$$

A 'wet' cooling coil cools the air below the dew point of the air and moisture does condense out the air stream. Equation (3-2) does not represent the total heat transfer rate. However, if an equation could be defined in terms of an enthalpy potential (instead of a temperature potential) this would aid in determining the performance of a wet cooling coil. Defining equation (3-3) will hold true if Δi_{mean} is properly defined and appropriate assumptions are made.

$$\dot{Q} = UA\Delta i_{mean} \tag{3-3}$$

The following set of equations which uses the methodology presented by McQuiston (1976) will lead to the definition of Δi_{mean} and define $\dot{Q}$ in terms of an enthalpy difference.

Figure 10 shows a cut away of a fluid to air heat exchanger with condensate. All temperatures have been labeled for identification purposes.

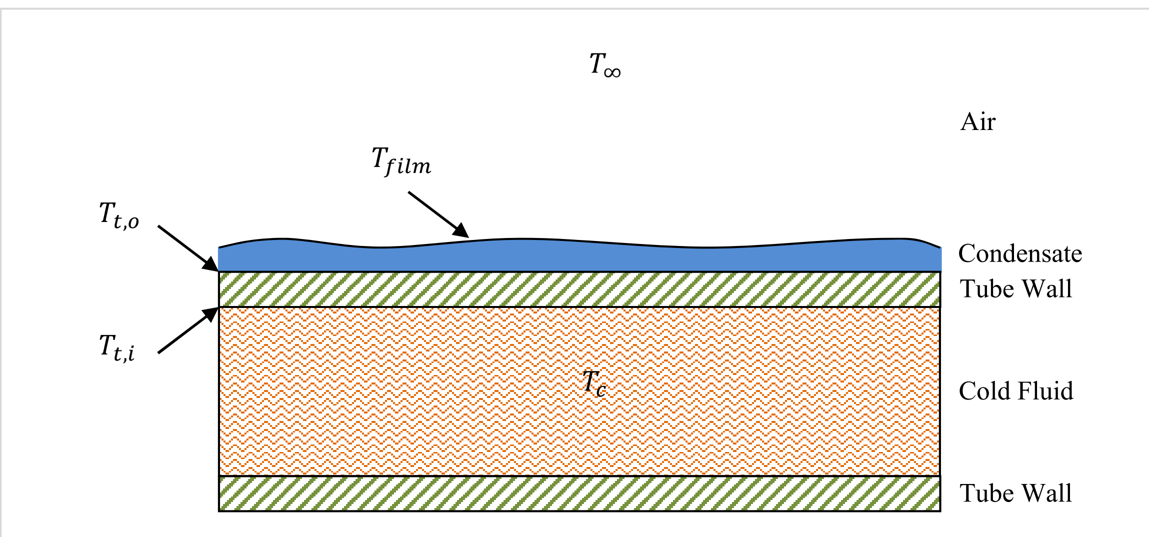

Figure 10 - Cross section of a heat exchanger with air side condensate

The heat transfer rate between the condensate and air can be written as.

$$\frac{\dot{Q}}{A} = h\left(T_\infty - T_{film}\right) + h_m\left(w_\infty - w_{film}\right)i_{fg,water} \tag{3-4}$$

Equation (3-4) is based on the conditions at the condensate-air boundary as seen in Figure 10. It accounts for both the convection heat transfer and the phase change mass transfer.

To aid in the simplification of equation (3-4), the Lewis number, which is defined as the thermal diffusivity divided by the mass diffusivity, is used in the following equation.

$$Le^{2/3} = \frac{h}{c_{p,\infty} h_m} \tag{3-5}$$

The Lewis number has an approximate value of 0.845 for air ASHRAE (2009). This value will change based on temperatures but is fairly insensitive to temperature variations for air-water mixtures. Even though the Lewis number is not unity, it is often approximated as unity as discussed in the previous chapter. This assumption simplifies the relationship between the heat transfer coefficient and the mass transfer coefficient.

Assuming $Le = 1$ and substituting equation (3-5) into equation (3-4), gives the following equation.

$$\frac{\dot{Q}}{A} = h_m\left[c_{p,\infty}\left(T_\infty - T_{film}\right) + \left(w_\infty - w_{film}\right)i_{fg,water}\right] \tag{3-6}$$

The enthalpy at the film is defined as the enthalpy of the dry air plus the enthalpy of the moisture in the air at the wall. It is assumed that the air is saturated at the film temperature. Enthalpies and humidity ratios are therefore evaluated at the film temperature. The total enthalpy of the moisture in the air is then defined as the enthalpy of the water vapor at the wall temperature plus the latent heat of evaporation also at the wall temperature.

$$i_{film} = c_{p,\infty}\left(T_{film} - T_{ref}\right) + w_{film}\left[\left(i_{f,water}\right) + i_{fg,water}\right] \tag{3-7}$$

The variable T_{ref} refers to an arbitrary base point where the enthalpy equals zero. As the derivation proceeds, the reference variables will cancel out of the equations.

The enthalpy of the free stream is also evaluated as the enthalpy of the dry air plus the enthalpy of the moisture in the air. The enthalpies for water should be computed at the dew point temperature of the free stream air. In order to obtain a comparable reference baseline, enthalpies are instead evaluated at the film temperature. While using the film temperature to evaluate the enthalpies introduces some error, the errors tend to compensate for each other and the equation is still a good approximation McQuiston, et al. (2000). Once again it is assumed that all of the moisture in the air is due to evaporation at the film temperature. A third term in the moisture energy calculation is included to account for the change in temperature of the water vapor from its dew point (the film temperature) to the free stream temperature.

$$\begin{aligned} i_{\infty} = {} & c_{p,air}\left(T_{\infty} - T_{ref}\right) \\ & + W_{\infty}\left[\left(i_{f,water}\right) + i_{fg,water} \right. \\ & \left. + c_{p,water}\left(t_{\infty} - t_{film}\right)\right] \end{aligned} \tag{3-8}$$

Subtracting the enthalpy of the air at the film interface from the total potential enthalpy of the free stream gives the total enthalpy transfer from the free stream to the film which produces equation (3-9).

$$\begin{aligned} i_{\infty} - i_{film} = {} & c_{p,\infty}\left(T_{\infty} - T_{film}\right) + \left(w_{\infty} - w_{film}\right) i_{fg,water} \\ & + \left(w_{\infty} - w_{film}\right) i_{f,water} \\ & + w_{\infty} c_{p,water}\left(T_{\infty} - T_{film}\right) \end{aligned} \tag{3-9}$$

Solving for $c_{p,air}(T_\infty - T_{film})$ in equation (3-9) and then substituting into equation (3-6) gives the following equation.

$$\frac{\dot{Q}}{A} = h_m\left[(i_\infty - i_{film}) - (w_\infty - w_{film})i_{f,water} - w_\infty c_{p,water}(T_\infty - T_{film})\right] \tag{3-10}$$

The last two terms of equation (3-10) account for a very small percentage of total heat transfer (approximately 0.5%) so they can be neglected to simplify the equation McQuiston, et al. (2000). With this last assumption, the heat transfer can be approximated by difference in the enthalpy of the free stream and the enthalpy of saturated air at the film temperature. This difference in enthalpy is called the 'enthalpy potential'.

$$\frac{\dot{Q}}{A} \approx h_m(i_\infty - i_{film}) = \frac{h}{c_{p,\infty}}(i_\infty - i_{film}) \tag{3-11}$$

The enthalpy of the free stream air and saturated air is not constant along the heat exchanger. This leads to rewriting equation (3-11) in the form of equation (3-12) where Δi_{mean} is some mean enthalpy difference that produces the correct heat transfer rate for the heat exchanger.

$$\dot{Q} = h_m A \Delta i_{mean} \tag{3-12}$$

B. Log Mean Enthalpy Difference

Equation (3-12) is analogous to equation (8-4) so it is reasonable to assume that Δi_{mean} can be shown to have a form similar to equation (8-26) where temperature variables are replaced with enthalpy variables.

$$\Delta T_{mean} = \frac{(\Delta T_2 - \Delta T_1)}{ln\left[\frac{\Delta T_2}{\Delta T_1}\right]} \tag{8-26}$$

In order to follow a development that is similar to the LMTD method, we must have two equations describing the heat transfer rate with one in terms of free stream enthalpies and another in terms of saturated air enthalpies.

Defining the heat transfer rate in terms of free stream enthalpies yields the following equations:

$$\dot{Q}_h = C_h\left(i_{\infty,i} - i_{\infty,o}\right) \tag{3-13}$$

$$C_h = \dot{m}_h \tag{3-14}$$

Defining the heat transfer rate in terms of saturated air enthalpies is more difficult because the mass flow is a fictitious mass flow. The heat transfer rate using saturated air enthalpies can be written as shown in equation (3-15). The variable $\dot{m}_x$ is an artificial mass flow that allows equation (3-15) to be true.

$$\dot{Q}_c = C_c\left(i_{film,o} - i_{film,i}\right) \tag{3-15}$$

$$C_c = \dot{m}_x \tag{3-16}$$

The fictitious flow rate, $\dot{m}_x$, will be defined later in the derivation. Using equations (3-13) and (3-15) and the same methodology for defining ΔT_{mean} shown in Appendix B, Δi_{mean} can be defined as shown in equation (3-17).

$$\Delta i_{mean} = \frac{(\Delta i_2 - \Delta i_1)}{ln\left[\frac{\Delta i_2}{\Delta i_1}\right]} \tag{3-17}$$

For a parallel flow heat exchanger:

$$\Delta i_1 = i_{\infty,i} - i_{film,i} \tag{3-18}$$

$$\Delta i_2 = i_{\infty,o} - i_{film,o} \tag{3-19}$$

For a counter flow heat exchanger:

$$\Delta i_1 = i_{\infty,i} - i_{film,o} \tag{3-20}$$

$$\Delta i_2 = i_{\infty,o} - i_{film,i} \tag{3-21}$$

Using equations (3-12) and (3-17) through (3-21) define the *Log Mean Enthalpy Difference (LMED)* method. This method is only valid when the hot fluid is air and water is condensing out of the air stream. The air side restriction is due to fact that the Lewis number is assumed to be unity. Since the LMED method, as presented in this section, accounts for the heat transfer between the free stream air and the film surface, another method, such as the LMTD, is needed to determine the heat transfer between the condensate film and the cold fluid. Then a condensate film temperature would need to be determined that calculates the same heat transfer rate for the LMED and LMTD methods.

C. Eff-NTU Wet

Equations (3-13) through (3-16) can be used to derive the Eff-NTU method. The effectiveness as defined in equation (8-54) will be the same when using equations (3-13) through (3-16). The difference will be the definition of C_c and C_h. C_h is defined by equations (3-14). Now $\dot{m}_x$ needs to be defined so C_c can be calculated.

Since the assumption is made that all of the heat is transferred from the air (hot fluid) to the cold fluid and none is lost to the surroundings; the heat transfer rate for the cold fluid can be written and then equated to equation (3-15).

$$\dot{Q}_c = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) \tag{3-22}$$

$$\dot{m}_x\left(i_{film,o} - i_{film,i}\right) = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) \tag{3-23}$$

Rearranging equation (3-23) to solve for $\dot{m}_x$ leads to the following equation.

$$\dot{m}_x = \frac{\dot{m}_c c_{p,c}}{\frac{(i_{film,o} - i_{film,i})}{(T_{c,o} - T_{c,i})}} \tag{3-24}$$

One of the benefits with the Eff-NTU method is that it can be solved explicitly without iteration. Only the inlet conditions are needed to solve for the heat transfer rate. With the definition of the fictitious mass flow rate, we have introduced a term that will require either iteration or simplification in order to solve for the heat transfer rate. The term $\frac{(i_{film,o}-i_{film,i})}{(T_{c,o}-T_{c,i})}$ needs to be determined from the outlet conditions.

By assuming the thermal resistance between the outer film surface and the inner tube fluid is negligible, i_{film} can be evaluated at the corresponding fluid temperature, T_c, based on coil location. This is an assumption that Braun, et al (1989) made in their derivation of this same method. While their derivation uses a different procedure, the results are the same as presented here.

With equations (3-16) and (3-24), C_c is defined so the Eff-NTU methodology can be used. One of the major benefits of the Eff-NTU method is that no outlet conditions are needed for the capacity calculation. With the wet coil case, C_c is now a function of the outlet fluid temperature and therefore the heat transfer rate must be solved for iteratively.

D. Threlkeld's Fictitious Enthalpy

The LMED method accounts for thermal resistances between the condensate film and the free stream air as compared to the previously described LMTD method that accounts for the thermal resistances between the cold fluild and the free stream air. Threlkeld (1962) describes a method that combines the LMED method and all thermal resistances of a heat exchanger into one set of equations. To do this, he assumed that the enthalpy of saturated air can be approximated by a linear function that is tangent to the saturation line at temperature, T.

$$i_{saturated\ air} = a + bT \tag{3-25}$$

Figure 11 shows the saturated air enthalpy curve compared to the linear approximation based at 60°F. The approximation always results in a lower value than the actual value. Threlkeld suggested that his approximation only be applied to a narrow temperature range of approximately 10°F. For this example, the linear approximation inside the 60 ±5°F range has an error of less than 1%. Once outside of this range, the error grows rapidly as shown in Figure 12

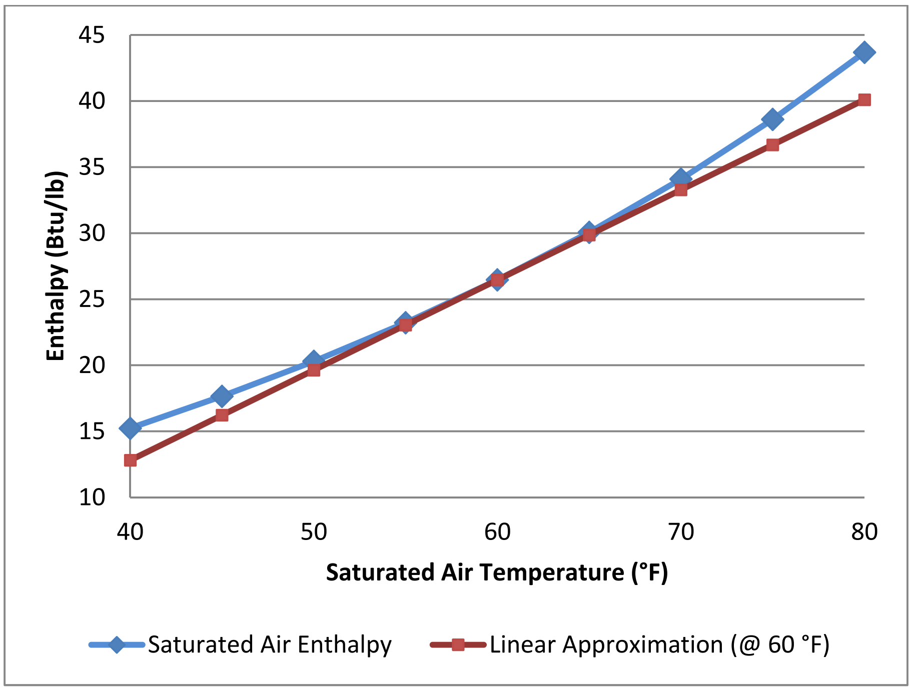

Figure 11 - Saturated air enthalpy with linear approximation

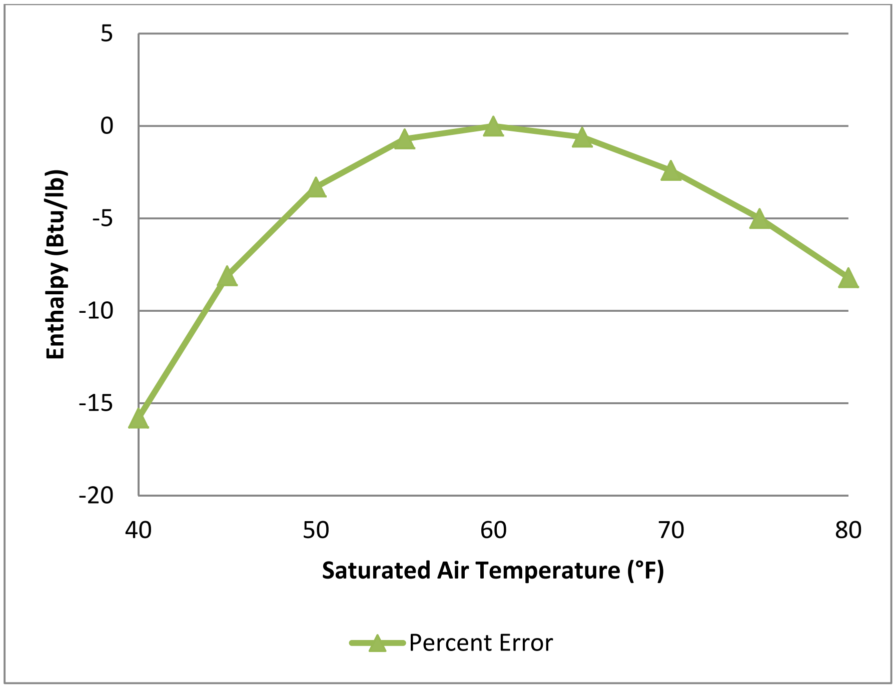

Figure 12 - Saturated air enthalpy linear approximation percent error

The temperature of the cold fluid must be related to an air enthalpy to apply the LMED method. This leads to the definition of a 'fictitious' enthalpy.

$$i'_{fictitious} = a + bT \tag{3-26}$$

It is called a fictitious enthalpy because the terms a and b are evaluated at the temperature T which is not an air temperature. The temperature may be a tube surface temperature, a fin temperature, or some average temperature.

The heat transfer is given by the following equation.

$$\dot{Q} = UA\Delta i_m \tag{3-27}$$

The UA term includes interior and exterior convection coefficients, and it also includes correction factors when using the airside enthalpies as the driving force for the heat transfer. In Threlkeld's application of the method, correction factors are the b term in equation (3-26) evaluated at the appropriate temperature.

Equation (3-27) is of the same format as equation (3-12) upon which the LMED method is based. In this case the definition of Δi_m and UA will be different.

Threlkeld made the assumption that the thermal resistance of the tube wall is negligible. By that assumption, $T_{t,i} = T_{t,o}$ as defined in Figure 10. Since $T_{t,i} = T_{t,o}$ the tube temperature will be defined as T_t.

The local heat transfer between the cold fluid and the tube is described in the following equation.

$$\delta\dot{Q} = h_i dA_{t,i}(T_t - T_c) \tag{3-28}$$

Two fictitious air enthalpies are defined. One is evaluated at the tube temperature (T_t) and the other is evaluated at the cold fluid temperature (T_c).

$$i_t' = a_t + b_t T_t \tag{3-29}$$

$$i_c' = a_c + b_c T_c \tag{3-30}$$

Since $T_t - T_c$ is usually rather small, the a and b terms in equations (3-29) and (3-30) are approximately equal to each other ($b_c \approx b_t$ and $a_c \approx a_t$) and b_c will be used. Combining this assumption and equations (3-28) through (3-30) leads to the following equation, that describes the local heat transfer between the cold fluid and the tube in terms of air enthalpy.

$$\delta\dot{Q} = \left(\frac{h_i dA_{t,i}}{b_c}\right)(i_t' - i_c') \tag{3-31}$$

The enthalpy potential equation gives the heat transfer from the free stream air to the tube condensate film as follows:

$$\delta\dot{Q}_{tube} = \left(\frac{h dA_{t,o}}{c_{p,\infty}}\right)\left(i_\infty - i_{film}'\right) \tag{3-32}$$

The condensate film on the tube has the same heat transfer rate and is given by the following equation.

$$\delta\dot{Q}_{tube} = dA_{t,o}\left(\frac{k_w}{y_w}\right)\left(T_{film} - T_t\right) \tag{3-33}$$

Replacing the temperature terms in equation (3-33) with fictitious enthalpies to describe the heat transfer as an enthalpy potential leads to:

$$\delta\dot{Q}_{tube} = dA_{t,o}\left(\frac{k_w}{y_w}\right)\left(\frac{i'_{film} - a_{film}}{b_{film}} - \frac{i'_t - a_t}{b_t}\right) \tag{3-34}$$

Again the assumption is made that the a and b terms in equation (3-34) are approximately equation to each other ($b_{film} \approx b_t$ and $a_{film} \approx a_t$) and b_{film} will be used.

$$\delta\dot{Q}_{tube} = dA_{t,o}\left(\frac{k_w}{y_w b_{film}}\right)\left(i'_{film} - i'_t\right) \tag{3-35}$$

The following equation is produced by combining equations (3-32) and (3-35) into one equation to describe the heat transfer from the free stream air to the tube surface.

$$\delta\dot{Q}_{tube} = dA_{t,o}\left(\frac{1}{\frac{c_{p,\infty}}{h} + \frac{y_w b_{film}}{k_w}}\right)(i_\infty - i'_t) \tag{3-36}$$

The same method can be applied to the heat transfer from the free stream air to the fin surface.

$$\delta\dot{Q}_{fin} = dA_F\left(\frac{1}{\frac{c_{p,\infty}}{h} + \frac{y_w b_{F,film,mean}}{k_w}}\right)\left(i_\infty - i'_{F,film,mean}\right) \tag{3-37}$$

Again the assumption is made that the a and b terms in equations (3-36) and (3-37) are approximately equation to each other ($b_{t,film} \approx b_{F,film,mean}$ and $a_{film} \approx a_{F,film,mean}$) and $b_{F,film,mean}$ will be used.

The total heat transfer is the sum of the heat transfer through the tube and the fin.

$$\delta\dot{Q} = \delta\dot{Q}_{tube} + \delta\dot{Q}_{fin} \tag{3-38}$$

The following equation can be written by substituting equations (3-36) and (3-37) into equation (3-38).

$$\delta\dot{Q} = h'_o dA_{t,o}(i_\infty - i'_t) + h'_o dA_F\left(i_\infty - i'_{F,film,mean}\right) \tag{3-39}$$

Where,

$$h'_o = \frac{1}{\dfrac{c_{p,\infty}}{h_{c,o}} + \dfrac{y_w b_{F,film,mean}}{k_w}} \tag{3-40}$$

h'_o is a heat transfer coefficient that accounts for the convection coefficient and conduction through the water film. The variable is defined slightly differently from how Threlkeld defines it. In this form it is easier to see that the b term is a result of the fictitious enthalpy assumption.

The introduction of fin efficiency allows the heat transfer through the tube and through the fin to both be calculated in terms of the free stream enthalpy and the tube surface temperature which then becomes a fictitious enthalpy. The fin efficiency is defined as,

$$\phi_{fin,wet} = \frac{i_\infty - i'_{F,film,mean}}{i_\infty - i'_t} \tag{3-41}$$

There are various methods for calculating the fin efficiency, which is a function of fin geometry and operating conditions. The methods are not discussed in this paper. Substituting equation (3-41) into equation (3-39) results in the following equation.

$$\delta \dot{Q} = h_o' \left(dA_{t,o} + \phi_{fin,wet} dA_F\right)(i_\infty - i_t') \tag{3-42}$$

In order to simplify the final form of the heat transfer rate equation, a new variable called surface efficiency is defined as:

$$\phi_{surface,wet} = \frac{dA_{t,o} + \phi_{fin,wet} dA_F}{dA_{t,o} + dA_F} = \frac{A_{t,o} + \phi_{fin,wet} A_F}{A_{t,o} + A_F} \tag{3-43}$$

Applying the surface efficiency to equation (3-42) leads to the following.

$$\delta \dot{Q} = h_o' \phi_{surface,wet} dA_o (i_\infty - i_t') \tag{3-44}$$

Combining equations (3-44) and (3-31) in the form of equation (3-45) leads to the definition of $d[UA]$ as shown in equation (3-46).

$$\delta \dot{Q} = d[UA](i_\infty - i_c') \tag{3-45}$$

$$d[UA] = \frac{1}{\dfrac{b_c}{dA_{t,i} h_i} + \dfrac{1}{h_o' \phi_{surface,wet} dA_o}} \tag{3-46}$$

$d[UA]$ is written with the brackets to show that the U and A are not separable in this definition. For the U and A terms to be separated, the A term must be defined. A can be defined as any area on the heat exchanger, as example, A_o or $A_{t,i}$. The following equation shows how the U and A terms can be separated once the area is defined as A_o. This results in all areas being described as a fraction of A_o.

$$d[UA] = U_{A_o} \cdot dA_o = \frac{1}{\frac{dA_o b_c}{dA_{t,i} h_i} + \frac{1}{h'_o \phi_{surface,wet}}} \cdot dA_o \tag{3-47}$$

The heat transfer is now defined as an enthalpy potential between the free stream air and the cold fluid and includes all thermal resistances of the heat exchanger.

Equation (3-48) is of the form that leads to the LMED equations as described in previous sections.

$$\dot{Q} = UA\Delta i_m \tag{3-48}$$

Where,

$$UA = \frac{1}{\frac{b_c}{A_{t,i} h_i} + \frac{1}{h'_o \phi_{surface,wet} A_o}} \tag{3-49}$$

From previous derivations, Δi_m from equation (3-48) is defined for a counter flow heat exchanger as,

$$\Delta i_m = \frac{\left(i_{\infty,1} - i'_{t,2}\right) - \left(i_{\infty,2} - i'_{t,1}\right)}{ln\left(\frac{i_{\infty,1} - i'_{t,2}}{i_{\infty,2} - i'_{t,1}}\right)} \tag{3-50}$$

An initial guess of the mean fin film temperature ($t_{F,film,mean}$) must be used to solve for the heat transfer. This temperature varies throughout the coil surface so an average of the inlet and outlet temperature must be used. If the temperature difference between the inlet and outlet is greater than about 10°F then the coil should be separated into smaller heat exchangers and the performance calculated individually. The following equation can then be used to see if the initial guess is an appropriate value.

$$i'_{F,film,mean} = i_{\infty,1} - \frac{c_{p,\infty} h_{o,w} \phi_{fin,wet}}{b_{F,film,mean} h_{c,o}} \left(1 - \frac{b_c UA}{h_i A_{t,i}}\right) \left(i_{\infty,1} - i'_{t,1}\right) \qquad (3\text{-}51)$$

An average cold fluid temperature should be used to calculate the value of b_c. An initial guess of the average cold fluid temperature will have to be made and then verified it is valid. The temperature difference should not be greater than about 10°F due to the limitation of the linear approximation of the saturated air enthalpy.

Threlkeld noted that the term ${}^{y_w}/_{k_w}$ in equation (3-40) is usually small so that a precise value of the thickness of the water film is not necessary.

E. Modified LMED

In order to develop a more accurate LMED methodology, Xia, et al. (2009) developed a modified LMED method. This modified method does not assume that $Le^{2/3} = 1$. By not making this assumption, L. Xia is really changing the enthalpy potential derivation and not modifying the LMED method. Equation (3-6) from the Enthalpy Potential derivation is where his modified method starts to differ with the standard LMED derivation.

Xia & Jacobi (2005) assumes that the ratio of sensible heat transfer to latent heat transfer is constant throughout the entire coil. He states that the local sensible heat transfer ratio does vary throughout the coil but that this approximation deviated from their experimental results by only a 3%.

$$\frac{\delta \dot{Q}_{sensible}}{\delta \dot{Q}_{latent}} \approx \frac{\dot{Q}_{sensible}}{\dot{Q}_{latent}} = constant \qquad (3\text{-}52)$$

Following the same methodology as presented in the Enthalpy Potential section and including the assumption stated in equation (3-52), the following equation for the heat transfer is defined.

$$\delta\dot{Q} = \frac{hdA_o}{c_{p,\infty}} Le_{adjustment}\left(i_\infty - i_{film}\right) \tag{3-53}$$

Where,

$$Le_{adjustment} = \left[1 - \frac{Le^{2/3} - 1}{\frac{Q_{sensible}}{Q_{latent}} + Le^{2/3}}\right] \tag{3-54}$$

If $Le^{2/3} = 1$ in equation (3-53) then equations (3-11) and (3-53) are the same. The term $Le^{2/3} = \frac{h_{c,o}}{c_{p,\infty} h_m} = 1$ will have to be substituted into equation (3-11) to get the two equations identical.

The $Le_{adjustment}$ term only gets applied to the air side convection coefficient to account for the heat and mass transfer of the condensing air stream. Threlkeld's LMED derivation can be applied and results in the following equations.

$$Q = UA\Delta i_m \tag{3-55}$$

Where,

$$UA = \frac{1}{\frac{b_c}{A_{t,i} h_i} + \frac{c_{p,\infty}}{\phi_{surface,wet} A_o h_{c,o} Le_{adjustment}}} \tag{3-56}$$

Xia ignored the conduction resistances through the condensate film and tube walls.

The difficulty with this method is how to define the value of Le. Xia uses an equation given by Pirompugd, et al. (2007).

$$Le^{2/3} = A \cdot B \cdot C \cdot D \tag{3-57}$$

$$A = 2.282 N^{0.2393} \tag{3-58}$$

$$B = \left(\frac{S_f}{D_o}\right)^{(0.0239N+0.4332)} \quad (3\text{-}59)$$

$$C = \left(\frac{A_a}{A_{a,t}}\right)^{(0.0321N+0.0747)} \quad (3\text{-}60)$$

$$D = Re_{D_o}^{\left(-0.01833N-0.1094\left(\frac{S_f}{D_o}\right)-0.0026\left(\frac{p_l}{D_o}\right)-0.03012\left(\frac{p_t}{D_o}\right)+0.0418\right)} \quad (3\text{-}61)$$

Where,

$N = number\ of\ tube\ rows\ of\ an\ air\ cooling\ coil$

$S_f = fin\ spacing$

$D_o = outside\ tube\ diameter$

$A_a = total\ air\ side\ area$

$A_{a,t} = outside\ tube\ area$

$p_l = longitudinal\ tube\ pitch$

$p_t = transverse\ tube\ pitch$

This equation is dependent on coil geometry and air properties and is derived from test data. So the application of this equation is limited to the test data set and the tested coil geometry.

4. DESCRIPTION OF THE MODELS

A. Introduction

In order to compare the relative accuracy and robustness of the models discussed in the previous chapters, four representative models were developed. These models are based on the work on Threlkeld (1962), Elmahdy & Mitalas (1977), Wang & Hihara (2003), and (Hill & Jeter, 1991) as shown in

Table 3. The models were selected to assess the significance of key parameters, illustrate the accuracy of the models, and illustrate the level of difficulty to implement the models.

Table 3 - Abridged version of Table 1 showing authors used for programming

Author	Date	Method				Assumptions		
		LMED	LMTD	Eff-NTU	Fundamental Equation	Enthalpy Potential	Le = 1	Fictitious Enthalpy
Threlkeld	1962	✓				✓	✓	✓
Elmahdy	1977	✓				✓	✓	✓
Hill	1991				✓			✓
Wang	2003			✓		✓	✓	

The objective of the study is to compare each model to existing experimental data. This will verify the implementation and provide a baseline for assessing model accuracy.

A second objective is to evaluate parameters such as Lewis number and the use of a fictitious enthalpy to see if the current assumptions are acceptable. The Lewis number is usually assumed to be one, and several different fictitious enthalpy values are usually approximated by a single value.

A third objective is to assess the potential limits of the models and range of applications. A model that works well in a coil design software might not be acceptable in an annual energy simulation program.

B. Model Inputs and Outputs

The basic input parameters for all models shown in Figure 13. These parameters are used to determine other model parameters. Equations for determining coil parameters such as fin surface area, total air side surface area, tube inner surface area can be found in AHRI (2001). These are in turn used in part to calculate the heat transfer resistances of the coil. The values for all of these inputs are listed in Elmahdy's experimental setup.

The outputs for the models are simply the leaving air conditions and the leaving water temperatures.

Parameters like the fin efficiency, air convection coefficient, water convection coefficient are not listed as inputs as they are calculated with values based on coil geometry and air and water conditions.

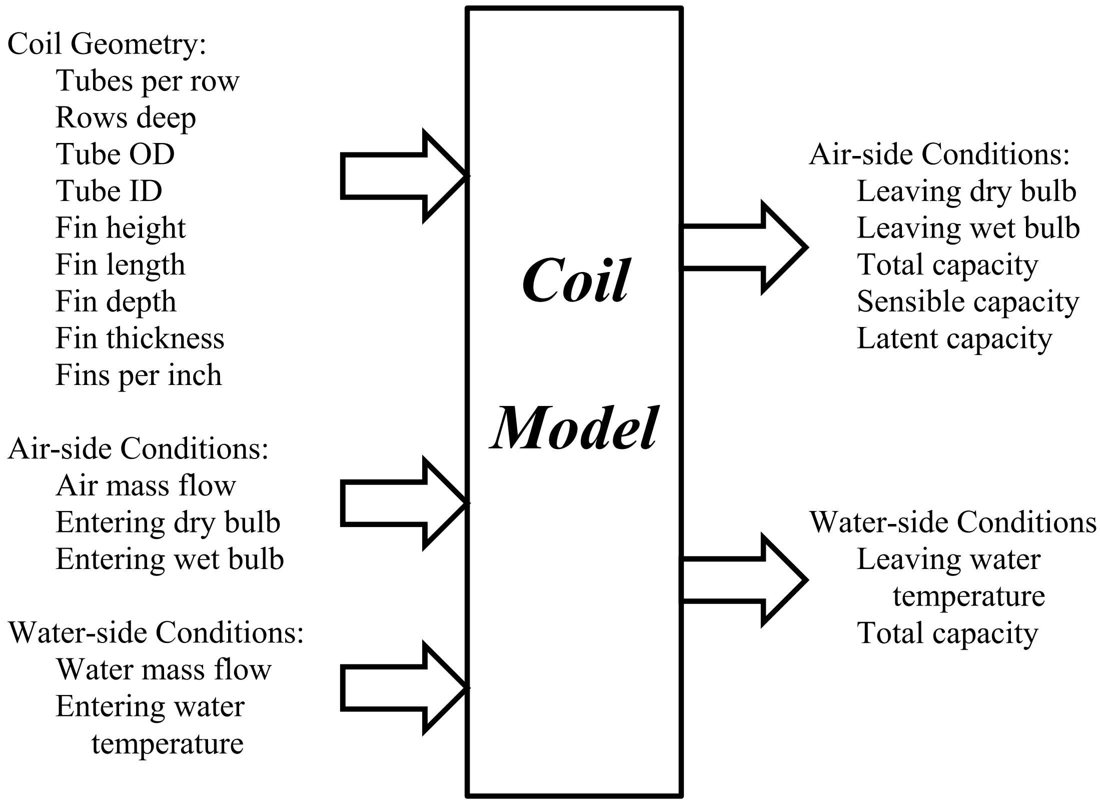

Figure 13 - Schematic showing basic inputs and outputs for coil models

The following table will correlate several authors' nomenclature with the nomenclature given in Figure 2 for reference. It also shows the implications of which resistances are used in the calculations.

Table 4 - Nomenclature of select variables

Author	Temperature Location Nomenclature				
	Free Stream Air	Condensate Film	Outer Tube Surface	Inner Tube Surface	Cold Fluid
Kastl	T_∞	T_{film}	$T_{t,o}$	$T_{t,i}$	T_c
Threlkeld	t	$t_{w,m}$	t_P		t_R
Elmahdy	t_a	t_s			t_w
Wang	T_a	T_{avf}	T_{bo}	T_{bi}	T_{fin}
Hill	T_a	T_i	*	*	T_r

**Not explicitly defined*

i. Application of Methods

All of the models were coded for a finite difference application as described by Mirth & Ramadhyani (1993). Since the coils used were four and eight rows deep and the circuiting is such where every tube sees the same conditions for a given row, the overall coil can be described as a counter flow heat exchanger. Figure 14 illustrates the application of the finite difference method with the counter flow assumption to the cooling coil. Since the heat exchanger is assumed to be counter flow, the details of the conditions inside the heat exchanger are lost. For example, the condition of the air between the second and third rows cannot be determined. Only the outlet conditions of the heat exchanger are calculated.

This application cannot account for any variation in the coil circuiting. This means that each tube must have the same water flow rate and water entering temperature as all other tubes in the row. Also, the air must be evenly distributed across the face of the coil so that each tube and fin has the same air velocity. All of these requirements were met for Elmahdy's experimental data.

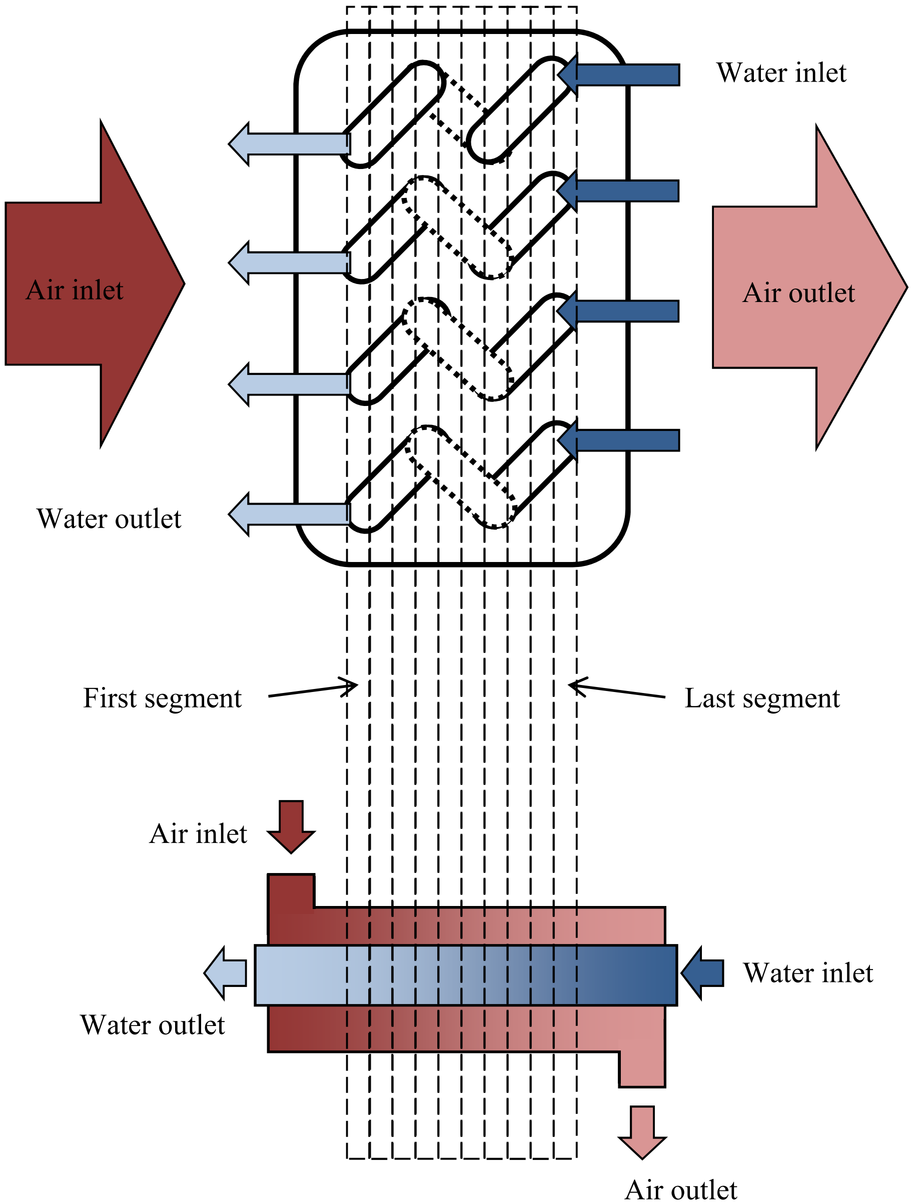

Figure 14 - Representation of finite difference application

The segment numbering is relative to the air flow direction so the first segment is at the air inlet and the water outlet as shown in Figure 14. Starting at the first segment, the water temperatures at the inlet and outlet of the segment are initially known. In order to apply the finite difference approach to a counter flow heat exchanger, the leaving water temperature of the coil must first be guessed as shown in Figure 15. Next, the first segment's entering water temperature must be determined so that the first segment's leaving water temperature to matches the initial guess. A schematic of the coil segment's known and unknown temperatures are shown in Figure 15. After segment number one's entering and leaving temperatures are known, they are passed to the next segment and the process for the segment repeats. This continues on until the end of the heat exchanger. Once at the end, the entering water temperature is then checked against the known entering water temperature. If it does not match, a new guess of the leaving water temperature is used and the process repeated. A flow diagram of this entire process is presented in Figure 16.

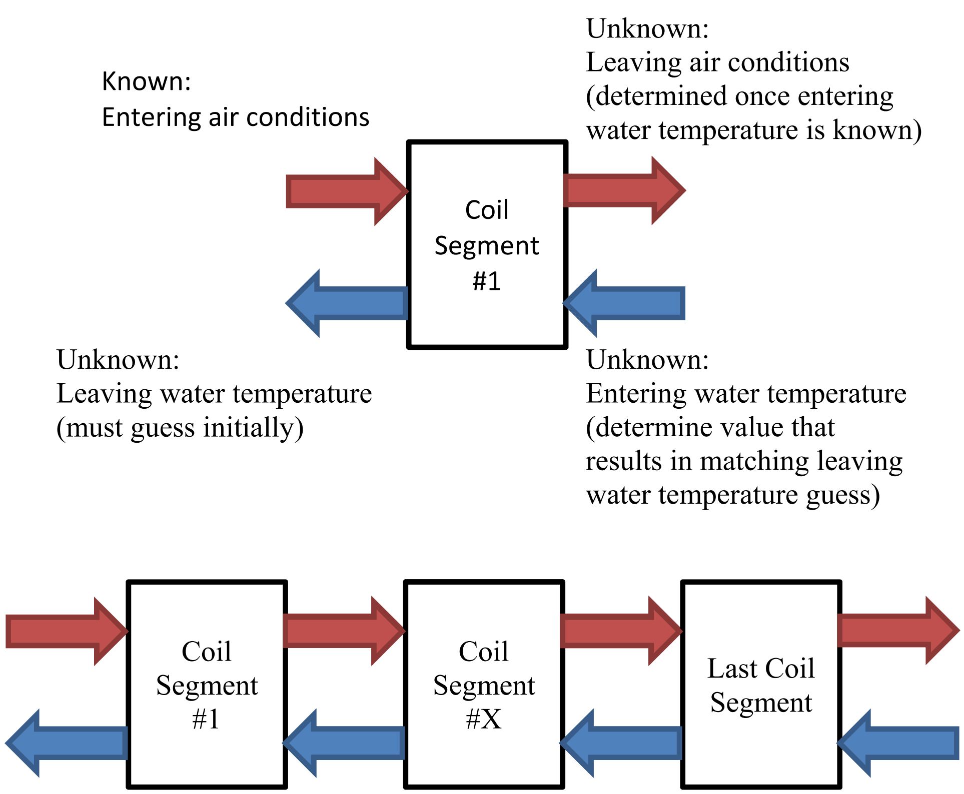

Figure 15 - Detail of segment #1 and #2 for solving multi-segment counter flow coil

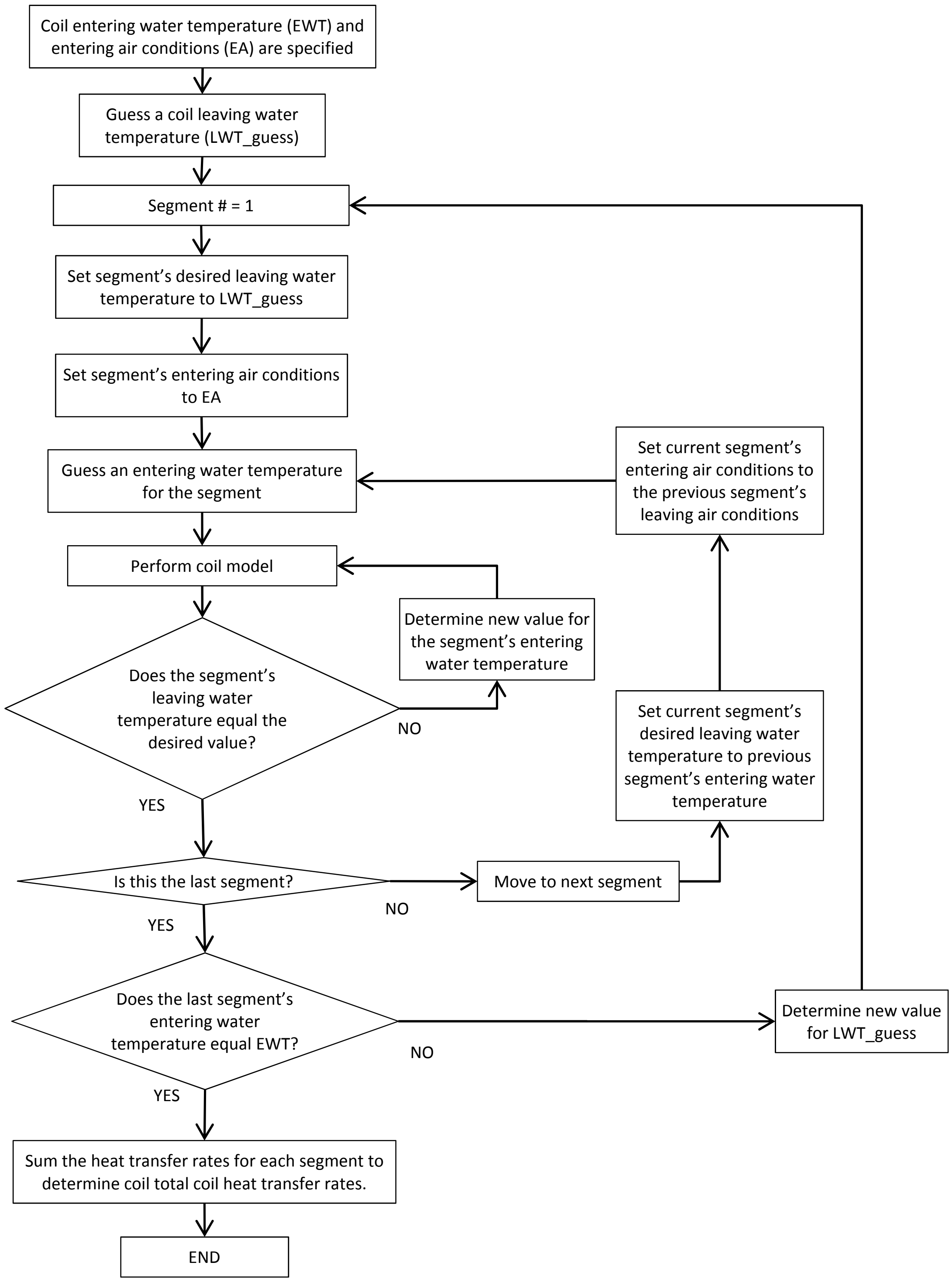

Figure 16 - Flow diagram for finite difference application

To facilitate inter-model comparisons, certain parameters were standardized and may differ from the one presented in the source papers for the model. The modified parameters include the air side and water side convection coefficients, Lewis number, and fin efficiency. These parameters are discussed in the following sections.

All models use the same air and water convection coefficient equations as described in Elmahdy & Mitalas (1977).

ii. Air Convection Coefficients

The air convection coefficient on a dry surface is given by the following equation.

$$h = \frac{J \cdot G \cdot c_{p,air}}{Pr^{2/3}} \tag{4-1}$$

Where,

$$J = 0.101\ Re^{-0.369} \tag{4-2}$$

G = maximum air mass flux

The Reynolds and Prandtl numbers are evaluated at the air side conditions. The coefficients used in (4-2) only apply to the coils used in the experiment. Elmahdy determined the coefficients after the experiments where performed.

The convection coefficient on a wet surface is given by the following equation.

$$h_{wet} = h_{dry} \cdot C \tag{4-3}$$

Where,

$$C = 1.425 - 0.51\left(\frac{Re}{1000}\right) + 0.263\left(\frac{Re}{1000}\right)^2 \tag{4-4}$$

Again, the coefficients used in (4-4) only apply to the coils used in the experiment. Elmahdy determined the coefficients after the experiments where performed.

iii. Water Side Convection Coefficients

The water convection coefficient is given by the following equation.

$$Nu = 0.023 * Re^{0.8} Pr^{0.4} = \frac{h_i\, D}{k} \tag{4-5}$$

The Reynolds and Prandtl number are evaluated at the water side conditions.

iv. Fin Efficiency

The fin efficiency equation is provided by McQuiston (1975).

$$\phi_{fin} = \frac{\tanh(Ml)}{Ml} \tag{4-6}$$

Where

$$M = \left[\frac{2h}{kt}\left(1 + \frac{C\, \cdot\, i_{fg}}{c_p}\right)\right]^{1/2} \tag{4-7}$$

$$C = \frac{w_\infty - w_{film}}{T_\infty - T_{film}} \tag{4-8}$$

When the coil is dry, $C = 0$

v. Lewis Number

In the Threlkeld, Elmahdy, and Wang models, the Lewis number is assumed to be unity since these three models used the enthalpy potential as the basis for their models. In order to account for a Lewis number not equal to unity, the Lewis number adjustment factor as described by Xia, et al. (2009) is applied to these three works. Xia's method adjusts the air side convection coefficient to account for a Lewis number other than unity when using the LMED method. Since this adjustment factor is a function of the sensible to latent heat transfer ratio it must first be assumed and then checked once the heat transfer rates are calculated.

Hill's model is not built on the enthalpy potential assumption and he does not assume a Lewis number of unity. So no additional work is needed to account for a varying Lewis number.

C. Model 1 – Threlkeld

Model 1 is based from the works of Threlkeld (1962). This is the first version of a model from the literature that implements Threlkeld's 'fictitious enthalpy'. It is a LMED method that converts all intermediate temperatures into an enthalpy via the fictitious enthalpy. The enthalpy potential in this method is from the free stream air to the cold fluid.

Figure 17 and Figure 18 show the flow of logic for this Threlkeld method. This method begins by evaluating the coil or segment as if it were wet. To evaluate a wet coil, there are several values that must be first assumed with an initial guess and then checked for convergence. These values include the Lewis number adjustment factor, the total heat transfer rate, and the mean film temperature. The Lewis number adjustment factor must first be assumed because it is a function of the sensible and latent heat transfer rates which are unknown initially. Next, the total heat transfer rate must be assumed because the LMED method needs the input and output conditions of the air and cold fluid. Since only the inlet air and water conditions are known, the outlet conditions must be assumed. Finally, the initial mean film temperature must be assumed in order to calculate the UA of the coil. The mean film temperature is used by Threlkeld to determine a fictitious enthalpy that built into the UA term. After the UA term is evaluated, it is used to determine the mean film temperature. If the initial guess and the calculated value are not within the desired tolerance, a new guess is used until convergence is achieved. After the mean film temperature is converged the total heat transfer rate is calculated with the UA and LMED terms. Convergence is then checked between the calculated value and the initial guess. Again, if convergence is not achieved then a new initial value is determined until convergence is achieved.

The final convergence check is the Lewis number adjustment factor. At this point the total heat transfer rate is known and so are the sensible and latent heat transfer rates. The Lewis number adjustment factor is then calculated with these heat transfer rates and compared with the initial value. Again, if convergence is not achieved then a new initial value is determined until convergence is achieved. To determine the sensible heat transfer rate, the method provided in AHRI (2001) is used.

Once these three loops are converged, the mean film temperature is compared to the entering air dew point. If the mean film temperature is less than the entering air dew point, the coil is wet and the calculated heat transfer rates are valid. If the mean film temperature is higher than the entering air dew point, the coil is reevaluated as a dry coil.

The flow diagram is much simpler for a dry coil compared to a wet coil. Since the coil is dry, there is no Lewis number to account for and there are no fictitious enthalpies. This means two of the iteration loops in the wet coil are not present in the dry coil model.

To calculate the dry coil heat transfer rate, an initial guess of the heat transfer rate is determined and used to calculate the leaving air and water conditions for the LMTD method. Then the UA and LMTD are used to calculate the heat transfer rate. If the calculated value and the initial guess are within tolerance then solution is final.

This method is not presented in a manner that facilitates programming. One of the major drawbacks of this method is the determination of the mean film temperature. Having several converging loops inside of each other is computationally intensive, especially if tolerances are tight.

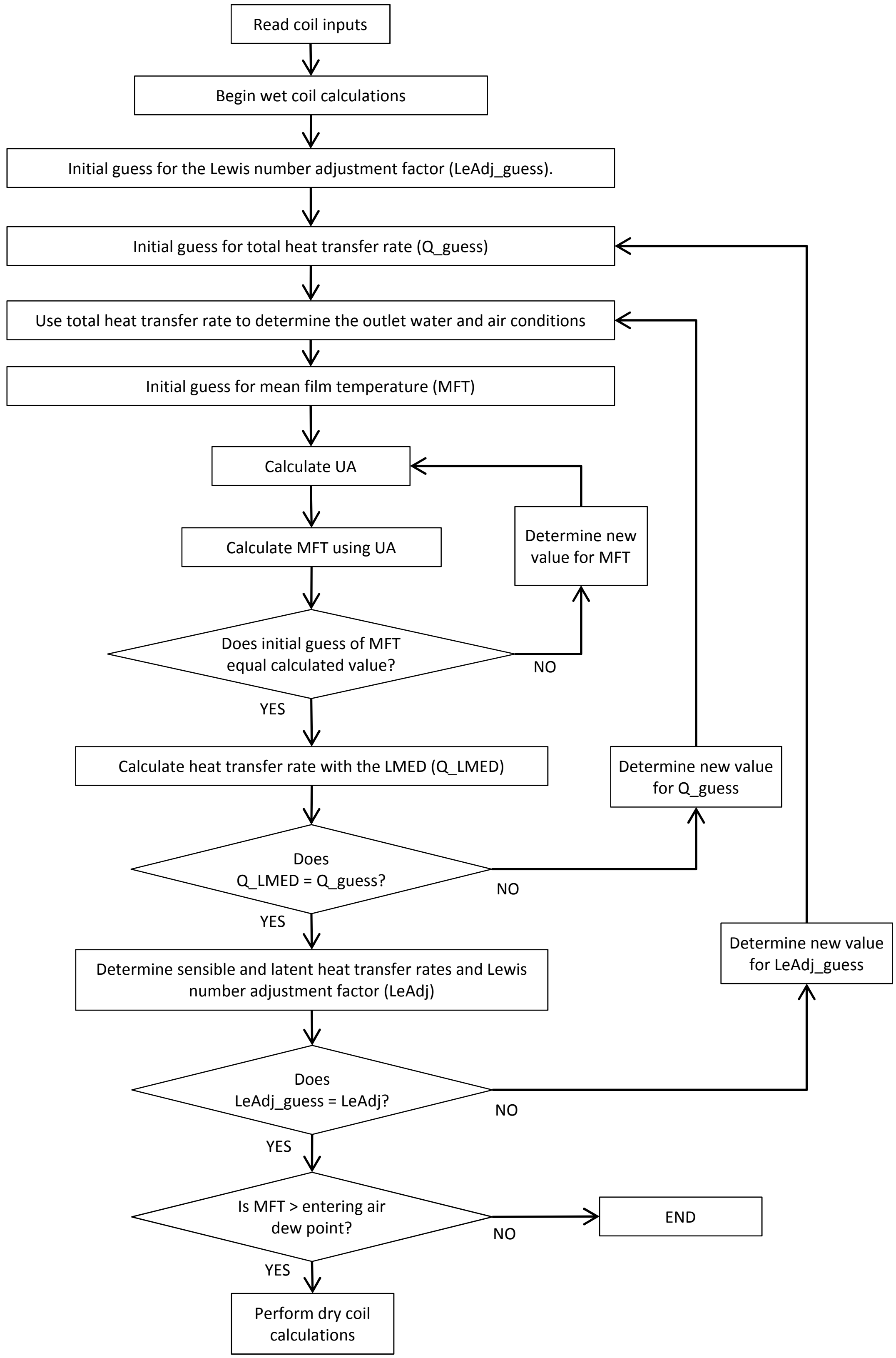

Figure 17 - Flow diagram for Threlkeld's method (wet coil)

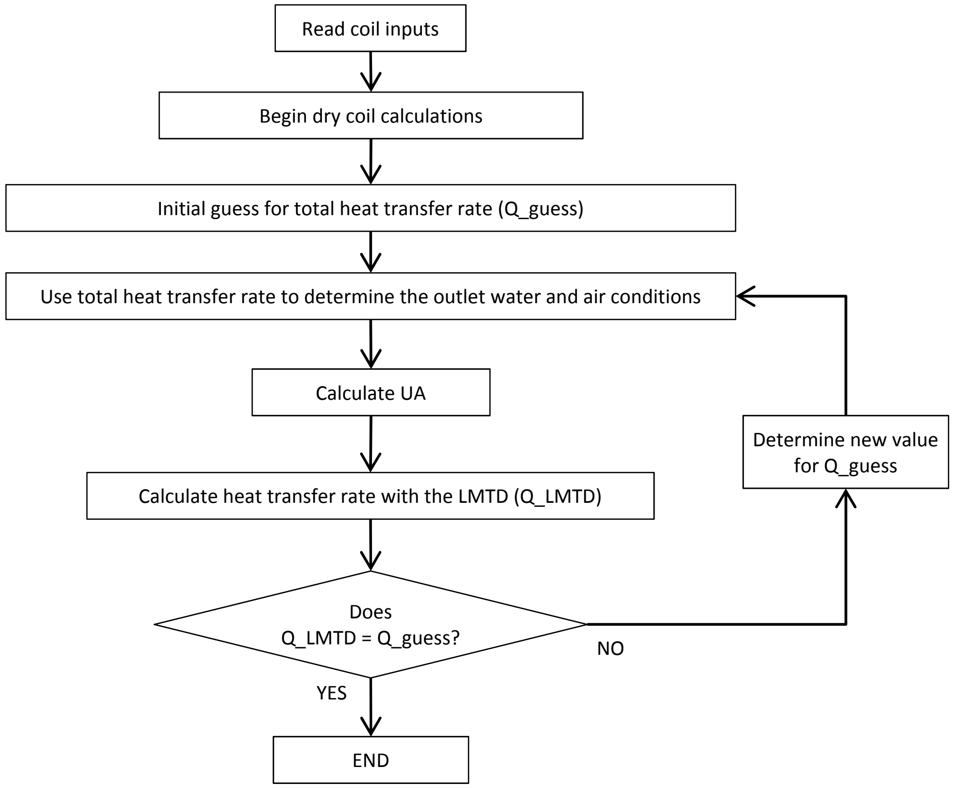

Figure 18 - Flow diagram for Threlkeld's method (dry coil)

D. Model 2 – Elmahdy

Model 2 is based on the work of Elmahdy & Mitalas (1977). This method was presented for ease of programming. The equations are may be hard to following while reading because there are many intermediate variables that are defined that have no significant meaning but they significantly aid in programming.

The flow diagrams for Elmahdy's method are very similar to the diagrams for Threlkeld's method. The one difference is the evaluation of the mean film temperature. Elmahdy does not have to determine this temperature prior to evaluating the wet coil UA. Elmahdy assumes that all of the fictitious enthalpies essentially have the same 'b' and 'a' terms from equation (2-9), which are evaluated at the water temperature. While Elmahdy does not explicitly describe which water temperature to use (entering, leaving, average), the average water temperature was used to evaluate the ‘b’ term for the fictitious enthalpy. This resulted in less computational time compared to the Threlkeld method as a result of not having an additional convergence loop.

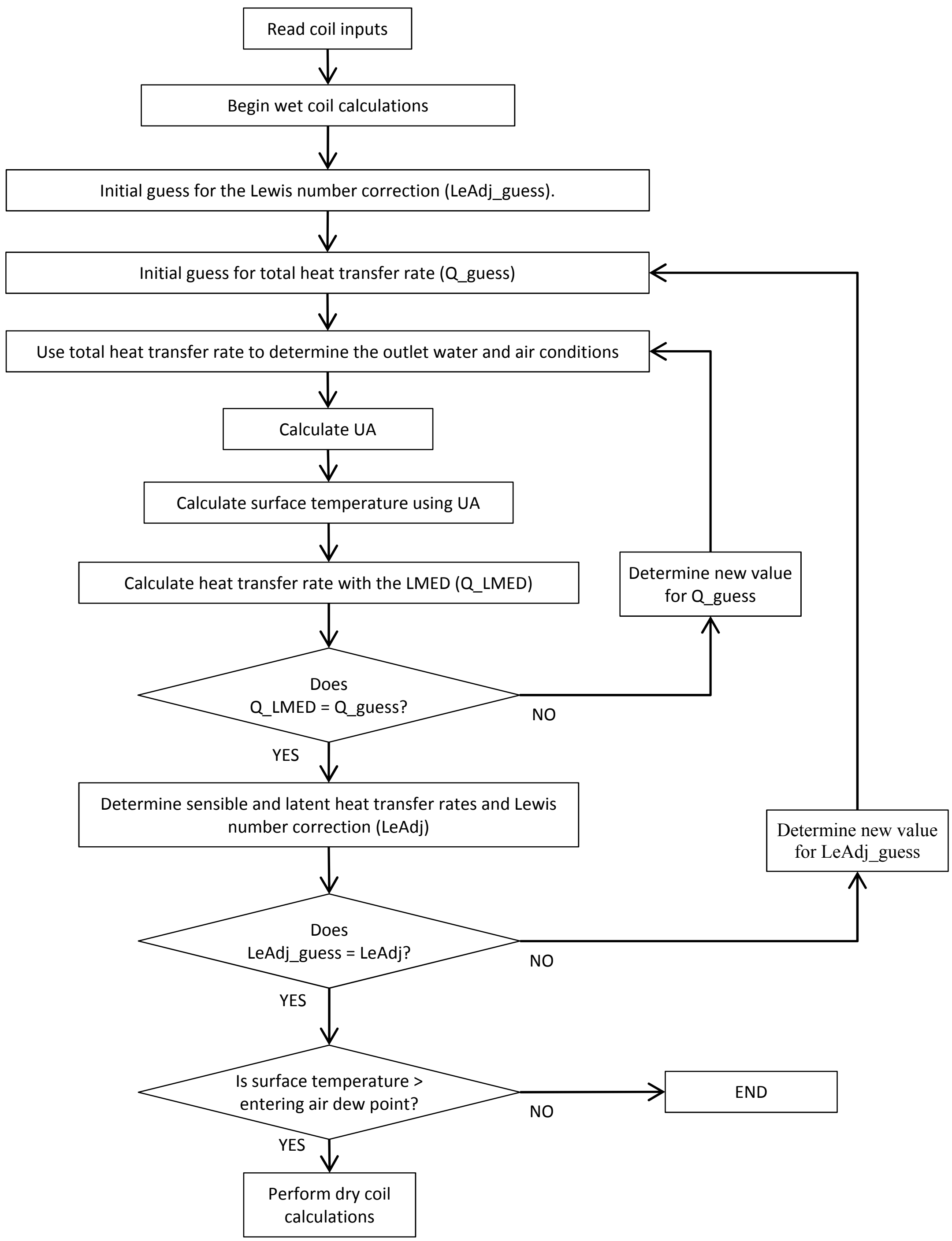

Figure 19 - Flow diagram for Elmahdy method (wet coil)

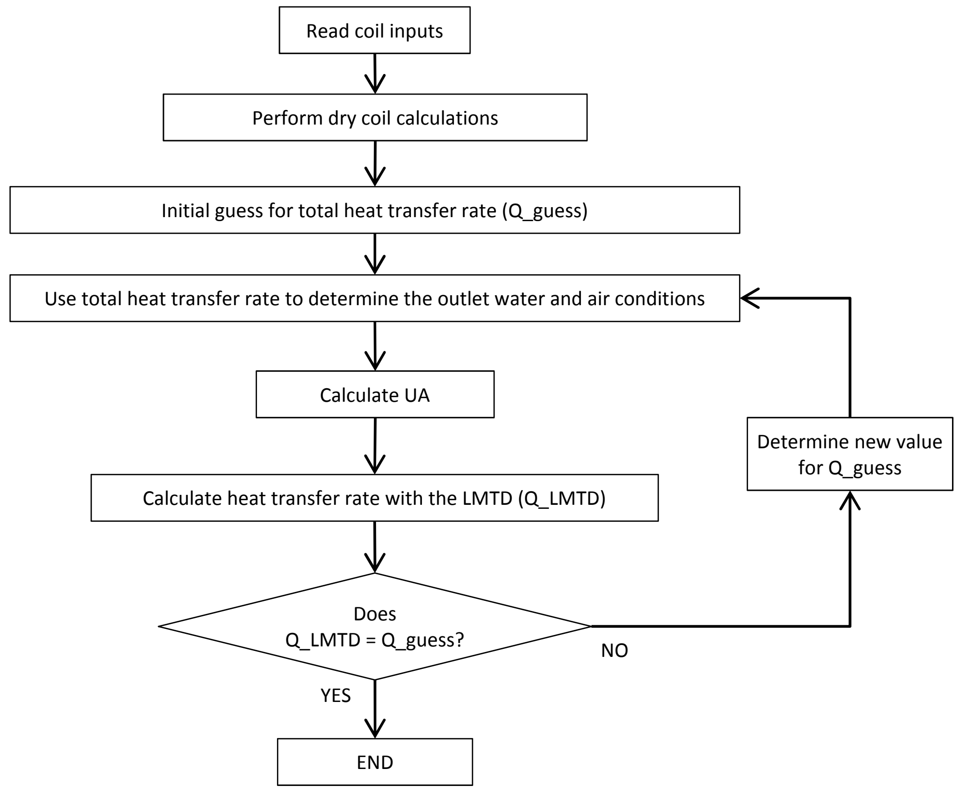

Figure 20 - Flow diagram for Elmahdy method (dry coil)

E. Model 3 – Wang

Model 3 is based on the work of Wang & Hihara (2003). This method is unique in that is converts a wet cooling coil to an equivalent capacity dry coil. The ease of this method is that there are no fictitious enthalpies to determine. Wang's method determines equivalent temperatures that are then used in an Eff-NTU application. The temperature potential for the heat transfer is between the equivalent entering air dry bulb and the water temperature.

The flow diagram for Wang's method is shown in Figure 21. This method has a more direct path than either of the previous two methods. The only major convergence loop is the Lewis number adjustment factor. The Lewis number adjustment factor is introduced into this method since the basis for Wang's method is the enthalpy potential. First an initial guess of the Lewis number adjustment factor must be determined. Then TD_1, TD_{23}, and TD_4 are determined which are key dry bulb temperatures that determine the state of the coil. Next the entering air dry bulb is compared to TD_4 and TD_1. If the entering air dry bulb is less than TD_4 then the coil is completely wet. The UA and heat transfer rates for a wet coil can be determined. If the entering air dry bulb is greater than TD_1 then the coil is completely dry. The UA and heat transfer rates for a dry coil can be determined. If neither is true then the coil is partially wet and the UA and heat transfer rates can be determined with the given equations in the method. Since an Eff-NTU method is used, outlet states do not have to be known to solve for the heat transfer rates. This makes this method less iterative and quicker to solve. After the total heat transfer rate is determined, the sensible heat transfer rate needs to be determined. This method provides an equation set to determine the sensible load which differs from the one provided in AHRI (2001). Then the Lewis number adjustment factor can be compared to the initial value and checked for convergence.

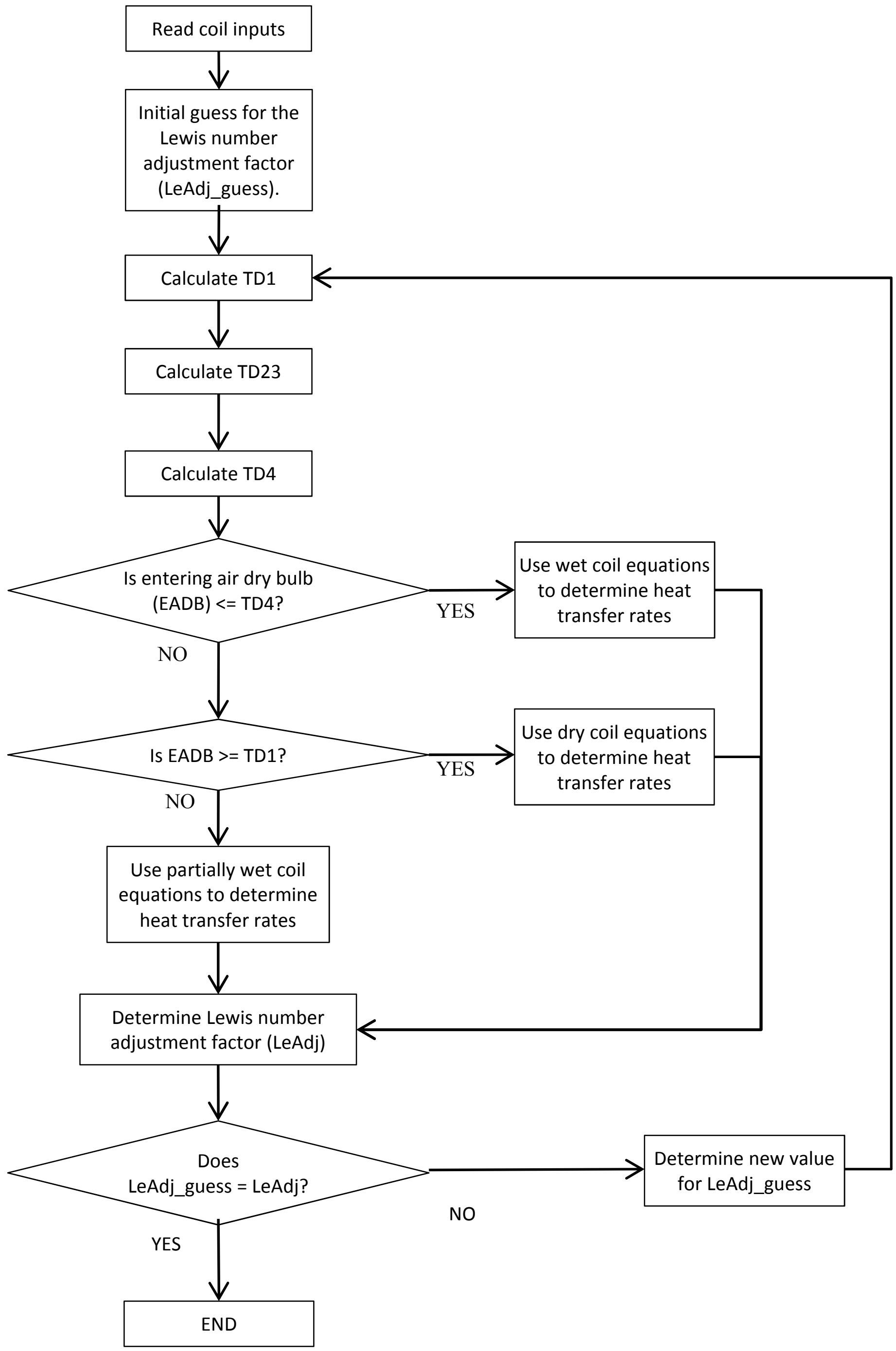

Figure 21 - Flow diagram for Wang's method (dry and wet coil)

F. Model 4 – Hill

Model 4 is based on the work of Hill & Jeter (1991). This method does not use the enthalpy potential and does not assume the Lewis number is unity. The method also does not use a fictitious enthalpy, but does linearize the humidity ratio based on temperature, which is a similar concept to fictitious enthalpy. The derived heat transfer equation takes a form that is similar to the Eff-NTU equation and requires only the inlet states of the fluids to be known. This allows for less computation time compared to the Threlkeld and Elmahdy's methods.

The flow diagram for Hill's method is shown in Figure 22. This method first assumes the coil is wet like Threlkeld's and Elmahdy's methods. The first step is to assume an initial value for the interface temperature, which is the key temperature to determine if the coil is truly wet. Next the interface temperature is used to calculate the UA for a wet coil. The equations for the UA have the Lewis number as an input so the Lewis number adjustment factor is not needed in the method. Next, the UA is used to calculate the interface temperature. If the calculated value is not within tolerance of the initial guess then a new initial value is determined until convergence is met. After the interface temperature is determined, it is compared to the entering air dew point. If the interface temperature is less than the entering air dew point, the coil is wet and the heat transfer rates are calculated with the wet coil equations. If the interface temperature is greater than the entering air dew point, the coil is dry. The heat transfer rates are calculated with the dry coil equations. For a wet coil, Hill provides an equation set to determine the sensible heat transfer rate which differs from the one provided in AHRI (2001).

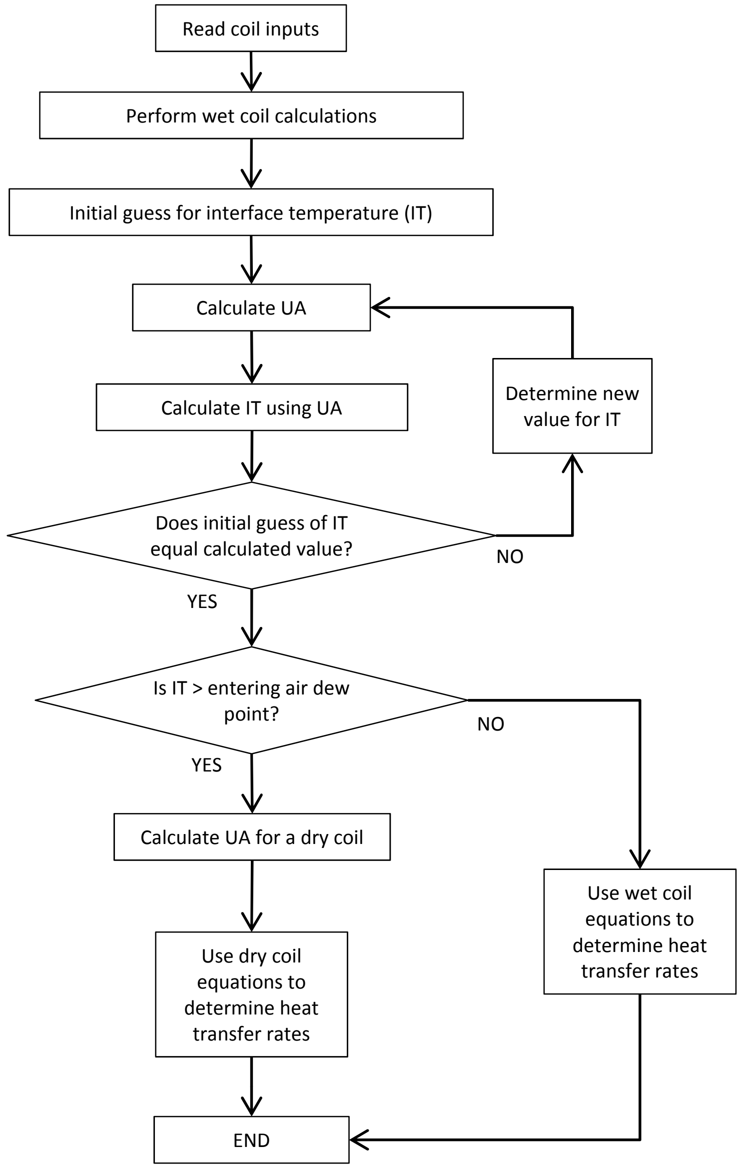

Figure 22 - Flow diagram for Hill's method (dry and wet coils)

5. EVALUTION OF MODEL PERFORMANCE

A. Introduction

All four models are compared to known experimental data provided by Elmahdy & Mitalas (1977) and Elmahdy (1975). The Elmahdy data consist of a four row and an eight row chilled water coil. Each coil is operated at twelve different conditions that result in a combination of dry and wet coils.

After the models have been compared to the experimental data, other operating conditions will be examined with Intra-model comparisons to discuss any trends that appear in order to understand more about the models. These will include the sensible/latent capacity split, the prediction of the wet/dry interface and the impact of the Lewis model on predicted results. The Lewis number will be varied through all four modes and the results will be compared to see if the Lewis number has the same sensitivity to all models and if it does change the heat transfer rates.

B. Experimental Data

Elmahdy data consists of twelve data points using a four row coil and twelve data points using an eight row coil for total of twenty four data points. These coils were operated at a wide range of operating conditions as summarized in Table 5. Water flow rates where adjusted to keep the change in the water temperature from inlet to outlet to no more than about 10°F. The air flow rates and entering temperatures varied to achieve both wet and dry coil surfaces. The total heat transfer rate was calculated from measuring the air flow rate, air dry bulb temperature and air dew point temperature. The uncertainty in the measured value is about ±1%, while the heat balance between the air side and water sides total heat transfer rates is an average ±2%.

Table 5 - Elmahdy's experimental data used for model comparison

Test Number	Coil Rows	Velocity (fpm)	Entering Air		Entering Water Temp (°F)	Leaving Water Temp (°F)	Heat Transfer Rates (MBtu/hr)		
			DB (°F)	WB (°F)			Total	Sensible	Latent
1	4	312	96.1	66.9	47.1	49.9	52.9	52.9	0.0
2	4	177	81.2	60.9	47.2	49.4	23.2	23.2	0.0
3	4	482	77.6	60.2	47.2	49.6	46.0	46.0	0.0
4	4	605	71.5	57.6	47.2	49.4	43.0	43.0	0.0
5	4	367	95.9	77.9	47.3	55.0	78.7	48.1	30.6
6	4	544	85.9	73.6	47.2	54.6	77.1	47.5	29.6
7	4	667	76.6	69	47.2	53.8	68.1	38.0	30.1
8	4	818	88.2	75	47.2	56.2	93.6	64.3	29.3
9	4	857	96.9	79.7	47.2	58.1	114.3	84.2	30.1
10	4	736	74	67.5	47.2	53.6	62.9	35.1	27.8
11	4	603	86.8	74.4	47.3	55.2	82.4	51.3	31.1
12	4	584	75.4	68.9	47.2	53.5	63.0	33.3	29.7
13	8	310	77.5	60.6	47.0	49.2	39.5	39.5	0.0
14	8	475	77.2	60.3	46.9	50.1	56.5	56.5	0.0
15	8	618	82.4	61.7	46.7	52.0	82.9	82.9	0.0
16	8	381	93.6	66.1	46.7	51.5	74.1	74.1	0.0
17	8	250	97.7	75.9	47.3	51.9	84.3	50.3	34.0
18	8	286	93.1	77.4	46.8	53.4	102.5	50.8	51.7
19	8	470	78.7	69.5	46.8	53.2	100.3	52.1	48.2
20	8	285	86.6	71	47.2	51.4	75.0	44.0	31.0
21	8	445	99	73.4	47.2	53.5	115.3	83.0	32.3
22	8	590	84.4	68	47.6	53.5	106.4	75.0	31.4
23	8	675	70.1	62.3	47.2	51.7	79.8	50.0	29.8
24	8	819	84.7	68.2	47.3	54.4	127.6	96.3	31.3

C. Model Performance: Total, Sensible, and Latent Capacities

In this section, predicted total, sensible and latent capacities are compared to experimental data presented by Elmahdy & Mitalas (1977).

As shown in Figure 23, all four models calculate the total capacity is within -5% and +7% of the experimental data with an average percent difference of .±1.9%.

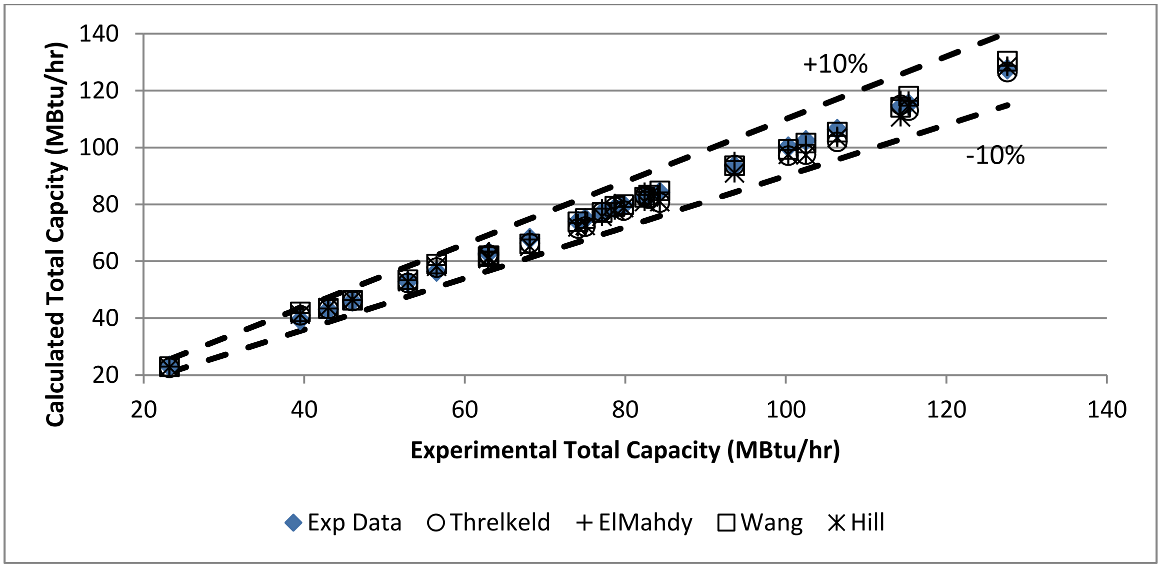

Figure 23 - Total capacity results from the four models

This is a good indicator that the methods have been programmed correctly but does not tell the entire story. There may be some trends or weakness of a model that are not apparent in Figure 23.

Figure 24 shows the percent difference of total capacity for each data point and each method. This figure is plotted to see if there is one model that consistently shows better or worse performance than another.

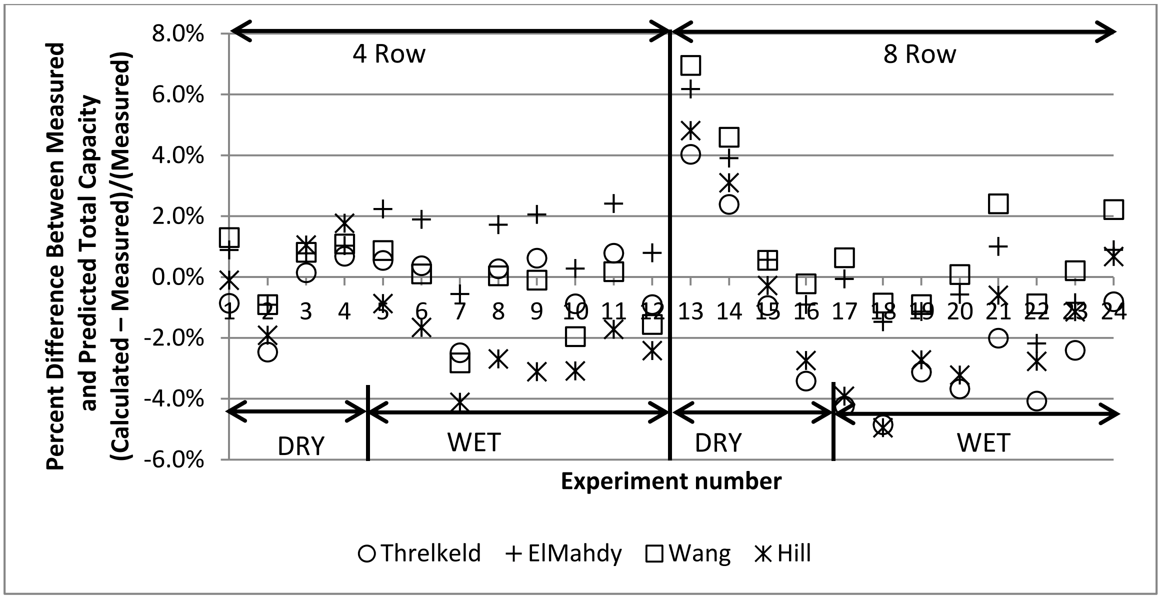

Figure 24 - Percent difference of total capacity from experimental data

As stated previously, Elmahdy's average uncertainty of the experimental data is ±2%. A good portion of the data falls within this range. Experiment 13 and 14 are a couple of outliers for all four models. For these two data points, all four models predict that the coil is very slightly wet while Elmahdy's experimental data states the coil is dry. The higher percent difference for these two points is due to the latent capacity the models predict but is not present in the experimental data.

One interesting point is to notice that Threlkeld's results are consistently lower than Elmahdy's. Both of these methods are similar in the fact they use an LMED with some fictitious enthalpies so it is expected that the results should be very similar. The first item to investigate is the fictitious enthalpy assumption that is used between the two models. Threlkeld uses two fictitious enthalpies in his model, one evaluated at the water temperature and the other evaluated at a mean film temperature. Elmahdy on the other hand only uses one, which is evaluated at the water temperature. To see if this is the cause of the difference, Threlkeld's model was restructured to use one fictitious enthalpy evaluated at the mean water temperature, similar to Elmahdy's. With this change, the total heat transfer rate increased 0.9% on average while the average difference between Threlkeld's and Elmahdy's methods is about 2.1%. This does not make up the complete difference between the two models. After further investigation, it is believe to be the way the two models are structured and how the intermediate values are calculated. Unlike Elmahdy's method, Threlkeld's method is not presented with computer programming in mind, so it is up to the reader to structure the equation set. The main difference between the two methods is Threlkeld's method has more iterative loops. The extra iterative loops could lead to a slow creep in the solution. Since the calculations divided the coil into 50 segments, a small percent difference could add up over the entire coil.

The results from Wang's method are consistently within the ±2% experimental uncertainty. Like Threlkeld's and Elmahdy's methods, Wang's method makes use of the enthalpy potential assumption. Since Wang's method moves the wet coil process line to an equivalent dry process with the same enthalpy potential, there is no need for a fictitious enthalpy.

To check the validly of the assumption that the four row coil can be modeled as a counter flow heat exchanger, the calculated heat transfer rates for the four row coil are compared to the calculated heat transfer rates of the eight row coil. If the assumption is not valid, we would expect to see a lower percent different on the eight row calculated values compared to the four row calculated values. Figure 25 and Figure 26 show the percent difference values for the four row and eight row coils respectively as is summarized in Table 6.

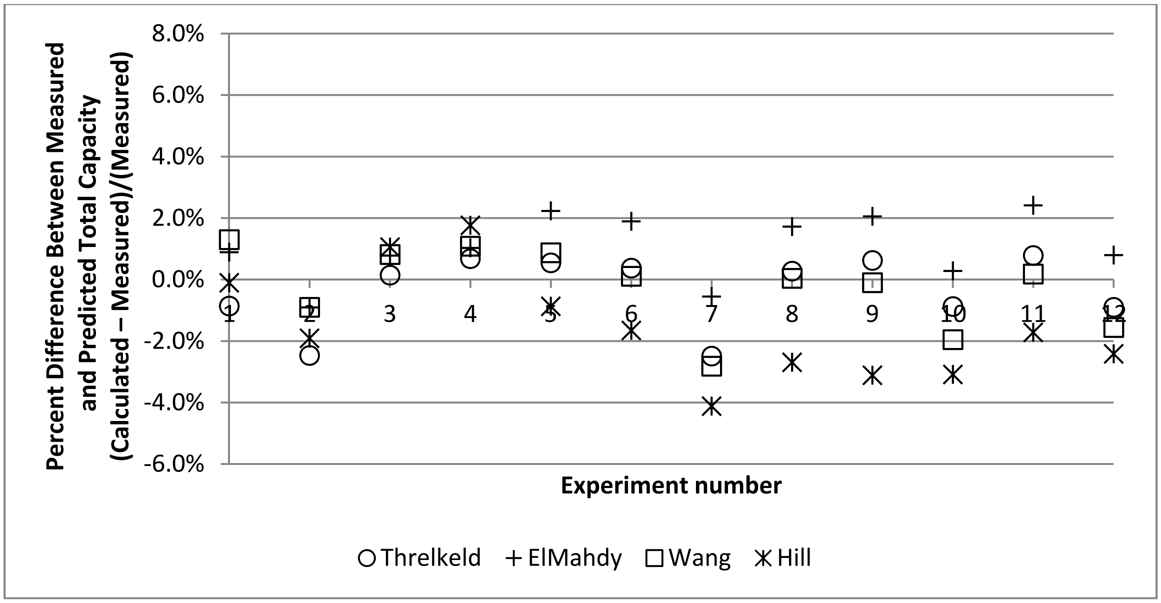

Figure 25 - Percent difference of total capacity from experimental data, four row coil data only

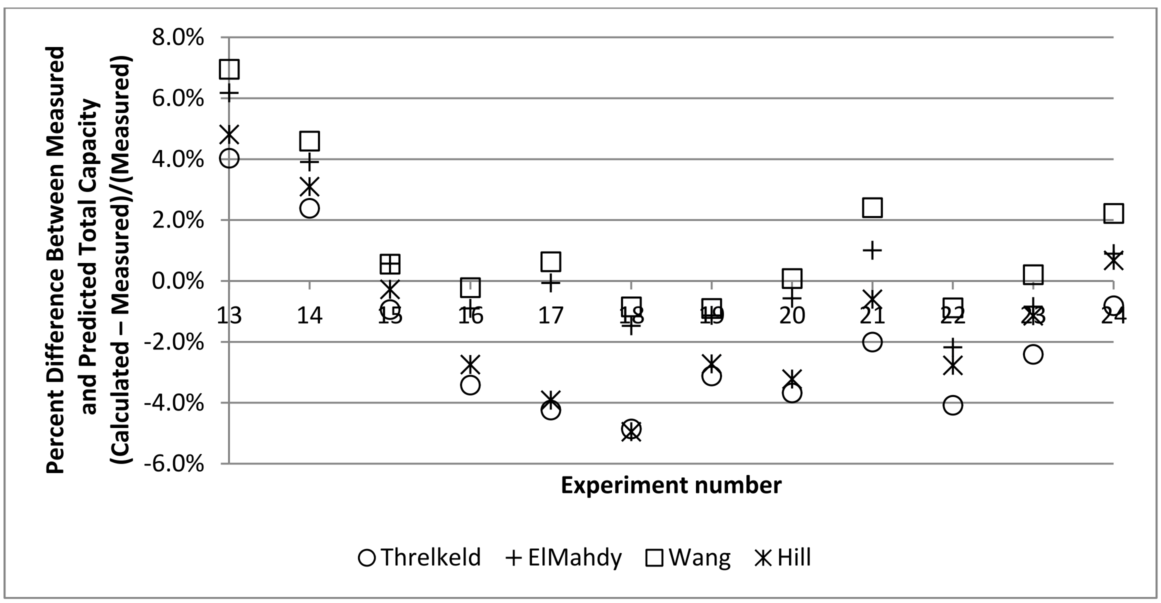

Figure 26 - Percent difference of total capacity from experimental data, eight row coil data only

Only Elmahdy's method and Wang's method show a noticeable reduction in error between the four row and eight row coils. Elmahdy's method goes from an average 1.1% percent difference to 0.4% difference and Wang's method goes from an average -1.9% to 0.5% difference. These two methods may show that the counter flow assumption may be in question, but the other two methods do not support that claim. Hill's method has almost no change in percent difference between the four row and eight row data. Threlkeld's method has a greater negative percent difference when calculating the eight row coil verses the four.

Table 6 - Average percent difference for each model

	4 row average % difference	8 row average % difference
Threlkeld	-0.3%	-1.9%
Elmahdy	1.1%	0.4%
Wang	-1.9%	0.5%
Hill	-1.8%	-1.9%

Since all four models do not have consistent improvement of the error when comparing a four row coil to an eight row coil, it can be said that the counter flow assumption is a valid assumption.

For all four models, the sensible capacity is within -5.0% and +14.4% of the experimental data as shown in Figure 27.

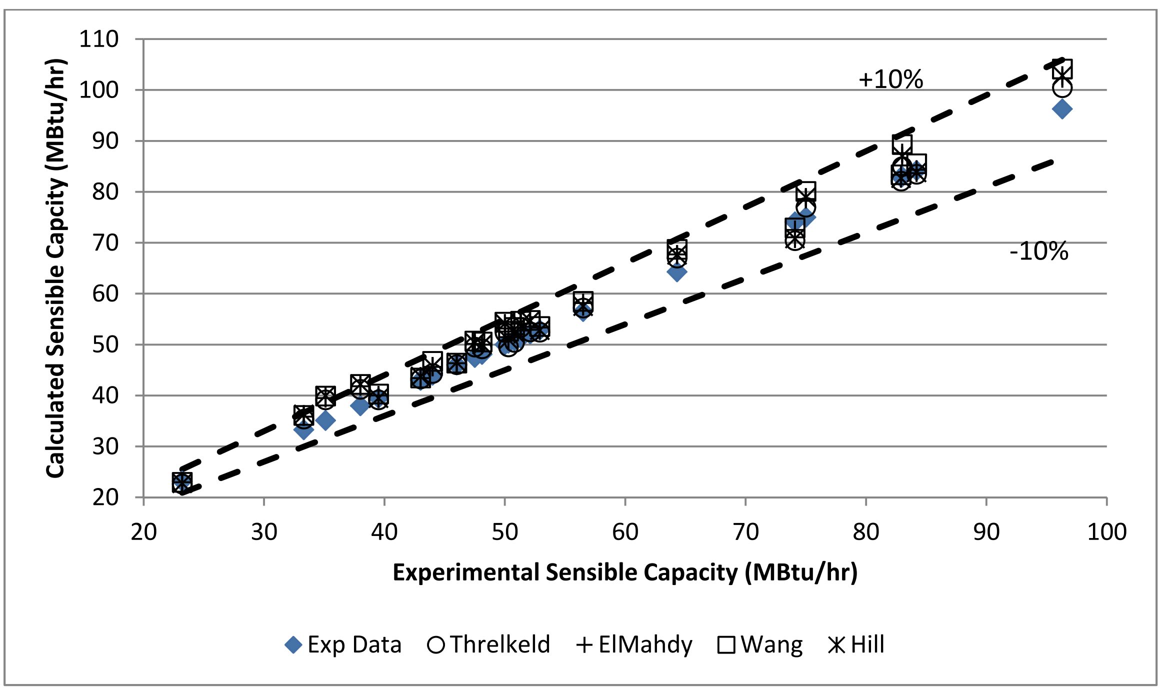

Figure 27 - Sensible capacity results from the four models

All four methods use the total heat transfer rate or a value derived from it as an input to the sensible heat transfer rate calculation. This means any error in the total heat transfer rate is propagated to the sensible heat transfer rate. Threlkeld's and Elmahdy's methods were programed using the method described in AHRI (2001) for determining the sensible heat transfer rate. This method determines an equivalent air side surface enthalpy that will produce the determined heat transfer rate. Then an equivalent surface temperature is determined by the saturated air temperature that produces the equivalent surface enthalpy. The equivalent surface temperature and the entering air dry bulb temperature are used to determine the leaving air dry bulb temperature. Wang's method uses a linear approximation that he derives with his equivalent temperatures. Hill's method determines the leaving air dry bulb with an Eff-NTU like equation he derived. With the leaving air enthalpy and temperature, the leaving air state is known.

Another factor that is affecting the sensible heat transfer rate is at what entering water temperature the latent heat transfer begins. For some of the coils that are specified as a dry coil, the models will predict that there is some latent heat transfer. This will cause the models to use the wet coil equations and therefore assign part of the total heat transfer rate as latent and reduce the sensible. This will compound the percent difference in the sensible heat transfer rate. For these reasons, it is expected to see a higher sensible heat transfer rate percent difference when compared to the total heat transfer rate percent difference.

Again there is no noticeable increase in accuracy of the models when calculating performance of a four row coil versus an eight row coil as seen in Figure 28 and Figure 29.

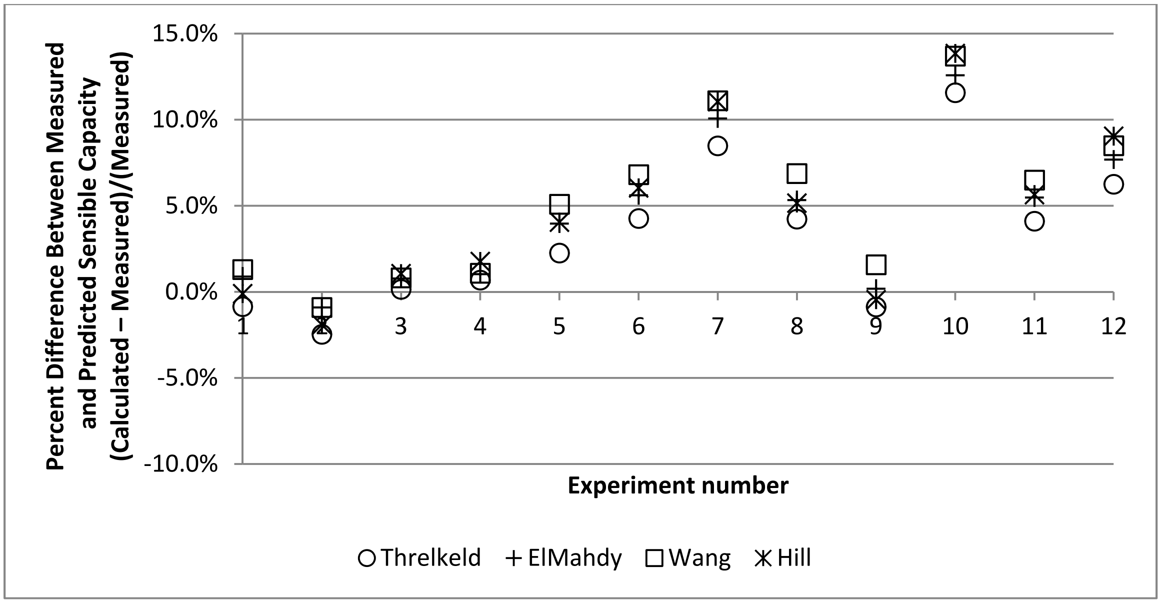

Figure 28 - Percent difference of sensible capacity from experimental data, four row coil data only

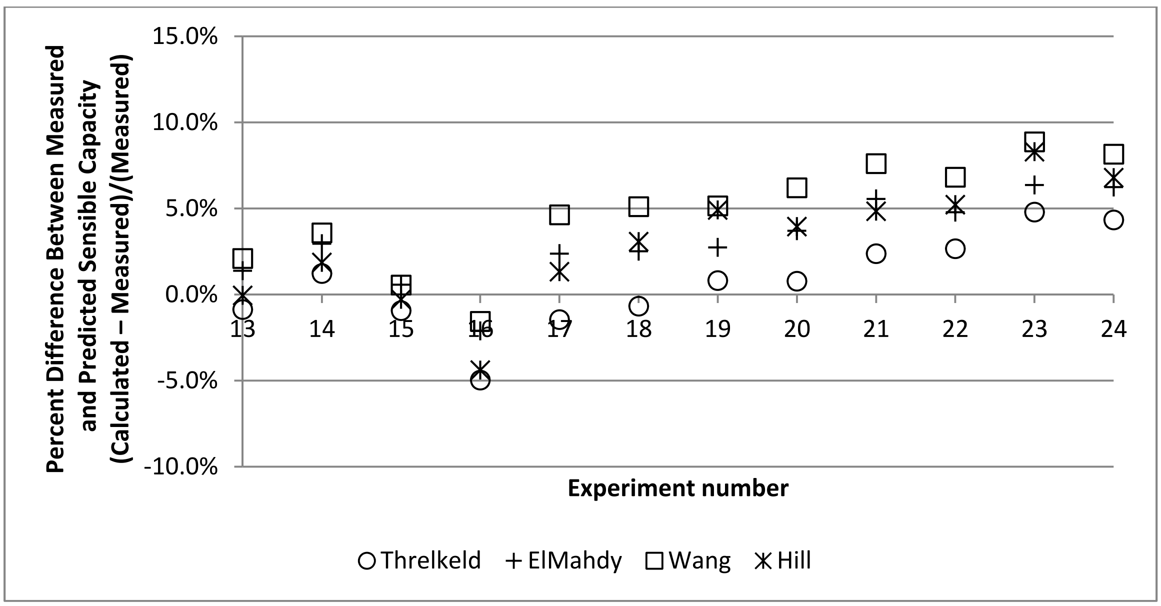

Figure 29 - Percent difference of sensible capacity from experimental data, eight row coil data only

It is worth noting that the deviation of the eight row, wet coil runs have a lower deviation compared to the four row wet coil runs. With an eight row coil, there is more surface for heat transfer which increases the latent heat transfer rate for a cooling coil. It is a common practice among HVAC manufactures to increase the number of coil rows in order to increase the latent capacity of the coil. With most of the coil being wet (at least 80% of the segments as predicted by the models), the error introduced by determining the incorrect surface temperature for the onset of latent heat removal is minimized.

For all four models the latent capacity is within -25.8% and +7.3% of the experimental data as shown in Figure 30.

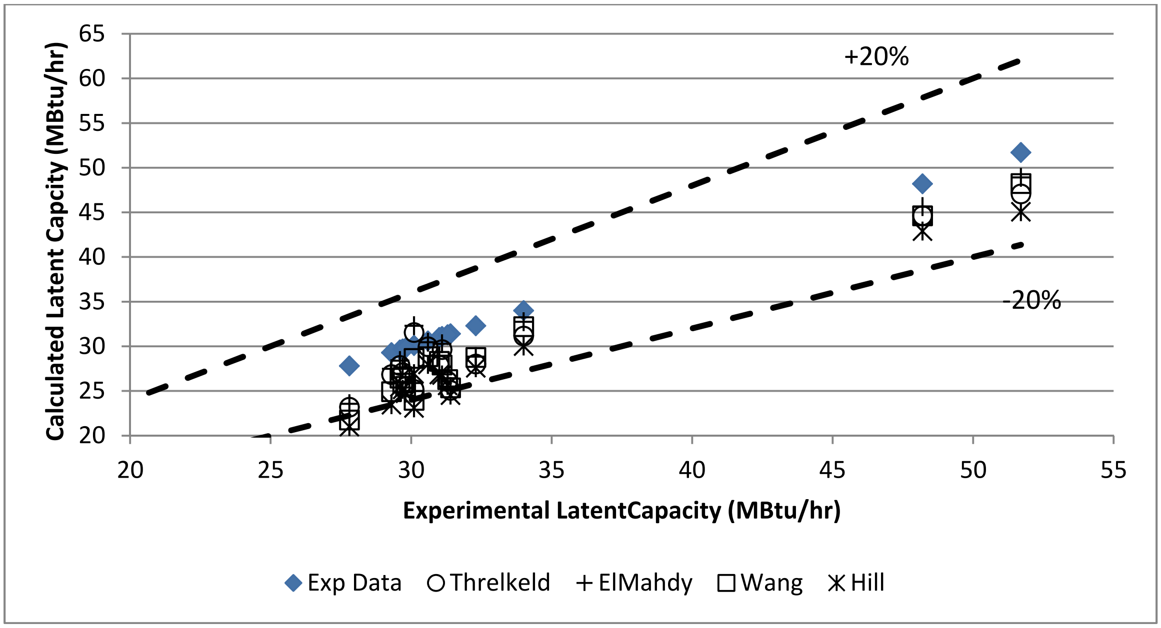

Figure 30 - Latent capacity results from the four models

The higher percent difference of the latent heat transfer rate is a factor of several items. The latent heat transfer rate is usually calculated as the difference between the total and sensible heat transfer rates. So any error in those two heat transfer rates is present in the latent heat transfer rate. The latent heat transfer is usually a smaller value than the sensible so any change in the latent heat transfer rate is usually a higher percentage. For example, if the experimental values for an experiment are as follows: total heat transfer rate of 10,000 Btu/hr, sensible heat transfer rate of 8000 Btu/hr, and latent heat transfer rate of 2000 Btu/hr. If a model predicted heat transfer rates of 10,000 Btu/hr total, 8400 Btu/hr sensible, and 1600 Btu/hr latent; the percent differences would be 0% total, +5% sensible, and -20% latent.

Another factor that can contribute to this percent difference is that each author has different criteria for when latent heat transfer begins. Wang's and Threlkeld's methods use the mean fin temperatures while Elmahdy's method uses the fin base temperature. Hill's method uses an equivalent surface temperature to determine when the latent heat transfer occurs.

Figure 31 shows the latent capacity at different entering water temperatures for all four models. Each model has a different entering water temperature at which the onset of latent heat transfer occurs.

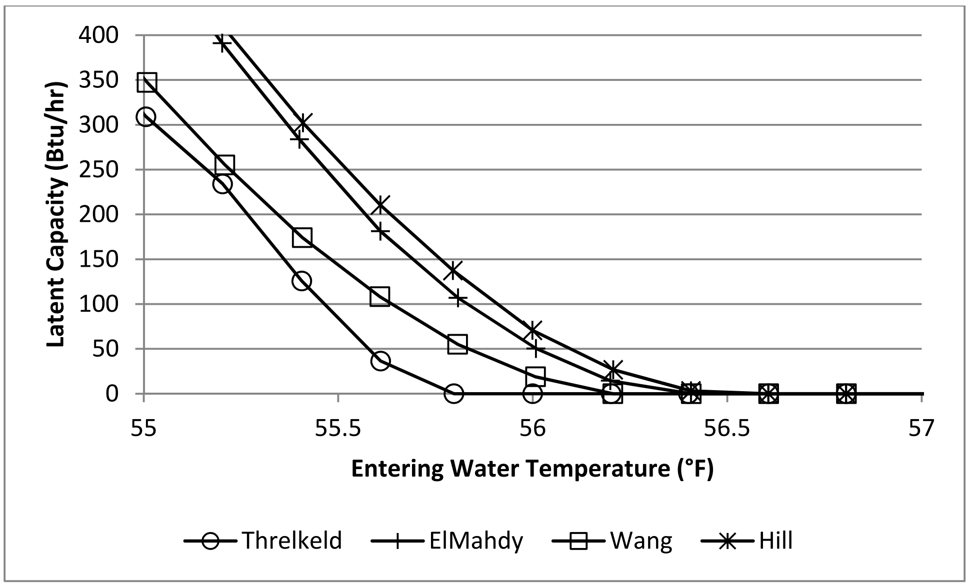

Figure 31 - Latent heat transfer at varying entering water temperatures

Figure 31 was generated by taking the four row coil and holding the air side conditions (temperatures and flow rate) and water flow rate constant while adjusting the entering water temperature. Even though the models use different methods to determine the onset of latent heat transfer, they are relatively close to each other, within ±0.3°F.

D. Wet/Dry Discontinuity

One of the issues that may arise when using these coil models in a simulation environment is the existence of a dry/wet discontinuity. This occurs when the model switches from a dry coil or segment to a wet coil or segment. Figure 32 shows the discontinuity in the total heat transfer rate when using Threlkeld's model in a slab (single segment) application. A single segment application will have the greatest discontinuity since the coil is not subdivided into smaller pieces.

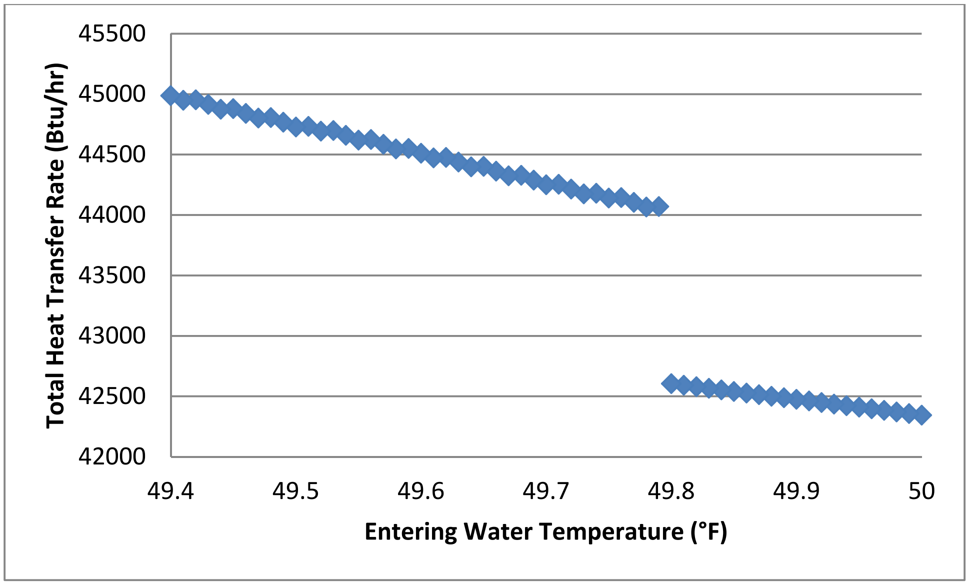

Figure 32 - Dry/wet discontinuity in Threlkeld's method as applied as a slab

This discontinuity occurs when the model's key surface temperature is at the dew point of the entering air temperature. When the entering water temperature is slightly above this point the coil is dry and when it is slightly below the coil is wet. By applying Threlkeld's method as a slab coil (single segment), it assumes the entire coil is completely wet or completely dry. Of course the coil really has a transition where part of the coil surface area is wet and part of it is dry. As a result, the model is not unconditionally stable. For example, if the simulation needed to find an entering water temperature where the total heat transfer rate of the coil is 43,500 Btu/hr, a solution would not be found.

One way to minimize the effect of the discontinuity is specify very loose convergence tolerances. In this case if the target heat transfer rate is 43,500 Btu/hr, the tolerances for convergence would have to be about ±750 Btu/hr or about 1.8%. A typical value for convergence is usually less than 1% so having a convergence tolerance of 1.8% is usually not acceptable. Both the size of the discontinuity and the minimum tolerance for convergence are dependent on model inputs such as air side convection coefficients and thermal resistances.

A better way to minimize the effect of the discontinuity is to break the coil up into enough segments so that the magnitude of the discontinuity is smaller than the convergence tolerance. Figure 33 shows the discontinuity with changing segment sizes. The lines labeled 'discontinuity' show when the model is switching between a wet and dry segment. As the number of segments increases, the magnitude of the discontinuity decreases and the location of the discontinuity changes.

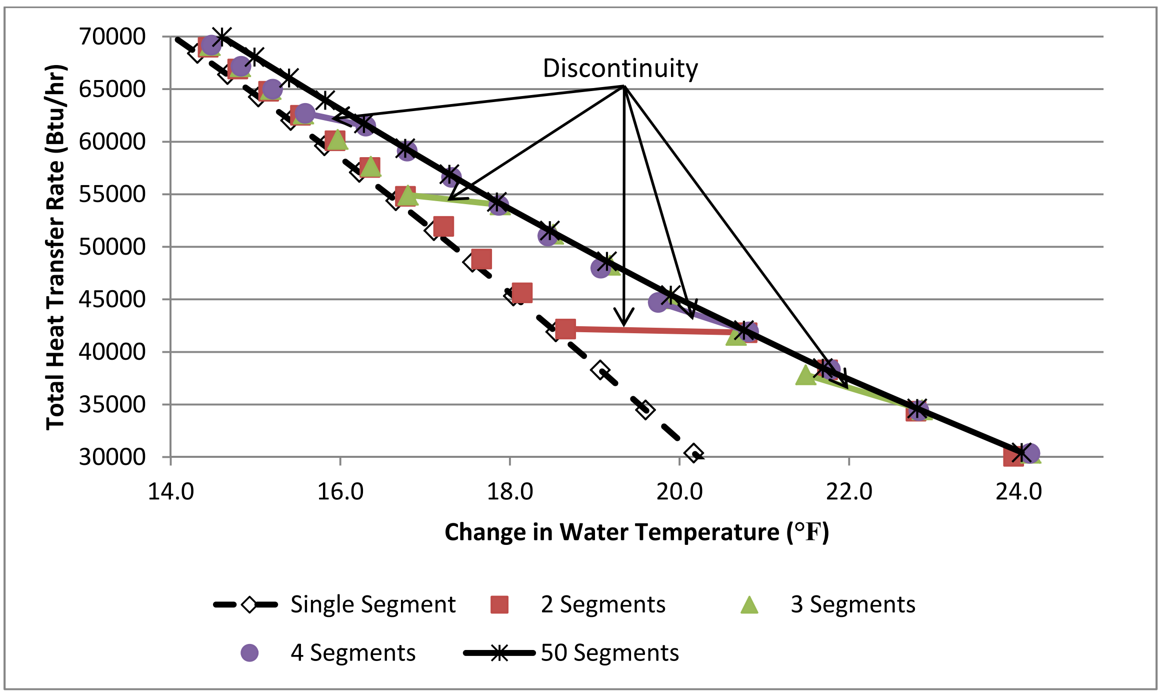

Figure 33 - Dry/wet discontinuity shown for different number of coil segments

The best way to remove the discontinuity is to use a model that has a smooth transition from wet to dry. This would require air side convection coefficients, fin efficiencies and thermal resistances that smoothly transition from dry to wet correlations.

E. Fictitious Enthalpy Assumption

When Threlkeld introduced the fictitious enthalpy, he somewhat arbitrarily stated that it can only be applied to a temperature that does not vary more than 10°F. Figure 34 shows the error of the fictitious enthalpy assumption when taken out of range. A coil was operated at constant conditions except for the water flow rate. The water flow rate was varied to achieve a range of leaving water temperatures in 0.5gpm increments. The entering air and water temperatures are set to result in a completely wet coil for all water flow rates. The entering air is set at 80°F dry bulb and 75°F wet bulb and the entering water temperature is set to 45°F. This set up was performed 3 times with 1, 2, and 50 segments. The percent difference in total capacity is shown in Figure 34 as compared to the 50 segment run.

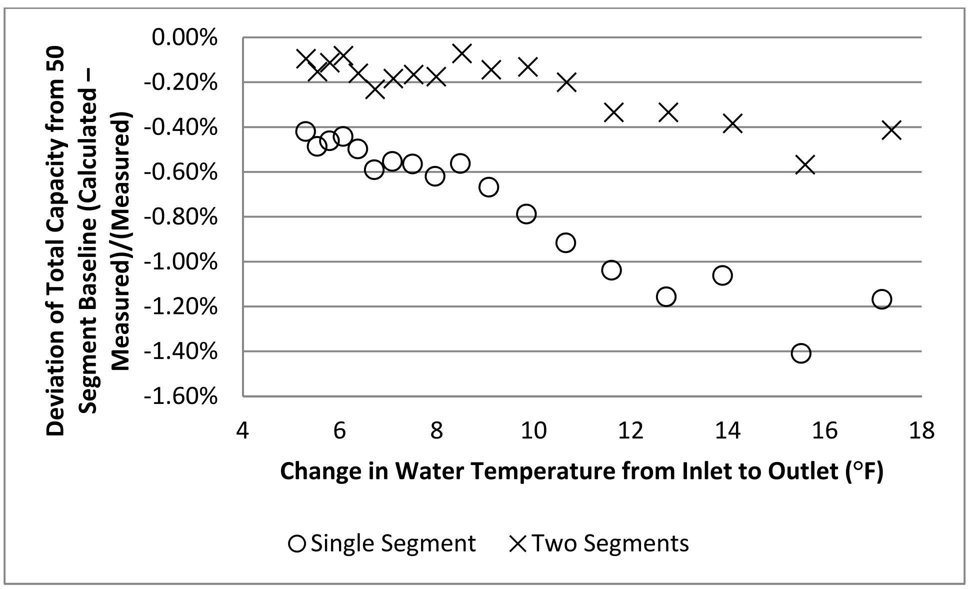

Figure 34 - Threlkeld's model with various segment numbers with a 50 segment baseline

The single segment model under predicts the total heat transfer rate as compared to the 50 segment model. The total percent difference is less than 1% until a water temperature difference of 10°F is present. The percent difference is less than 1.5% at a water temperature difference of 17°F which is nearly double the initial limit Threlkeld suggests. By dividing the coil into two segments, each segment will only see a part of the total change in the water temperature, therefore is within the range of application of the fictitious enthalpy.

F. Effects of the Lewis Number

The Lewis number is usually assumed to be unity for convenience but ASHRAE (2009) cites the value of 0.845 for air. Since the Lewis number is assumed to be unity to derive the enthalpy potential, Threlkeld's, Elmahdy's, and Wang's methods use the adjustment factor that is presented by Xia, et al. (2009) to account for a Lewis number other than unity. To show the effects of the Lewis number on the total and latent heat transfer rates, three different Lewis numbers are used. The test conditions from the experimental data provided by Elmahdy are ran with a Lewis number of 1, 0.845, and 1.155 with the Lewis number equal to unity being the base case. The results for each model are shown below for each model.

The result of changing the Lewis number from unity to 0.845 is an increase of latent heat transfer of about 2% on average and an increase of total heat transfer rate of about 0.9% for Threlkeld's, Elmahdy's models.

For Wang's model, the total heat transfer rate only increased about 0.5% and the latent heat transfer rate increased about 0.6%. Wang's model is less sensitive to the Lewis number correction in part of the partially wet condition in the model. The Lewis number correction is applied to this region of the model even if the model determines the region is mostly dry. If this occurs, the Lewis number is applied to a dry coil segment when it should not be. Through the iterations, the effect of the Lewis number correction is reduced.

The computation time saved by assuming a Lewis number of unity will be preferred over the extra precision in the heat transfer rate values. These extra iterations took about 3 times as long compared to assuming a Lewis number of unity.

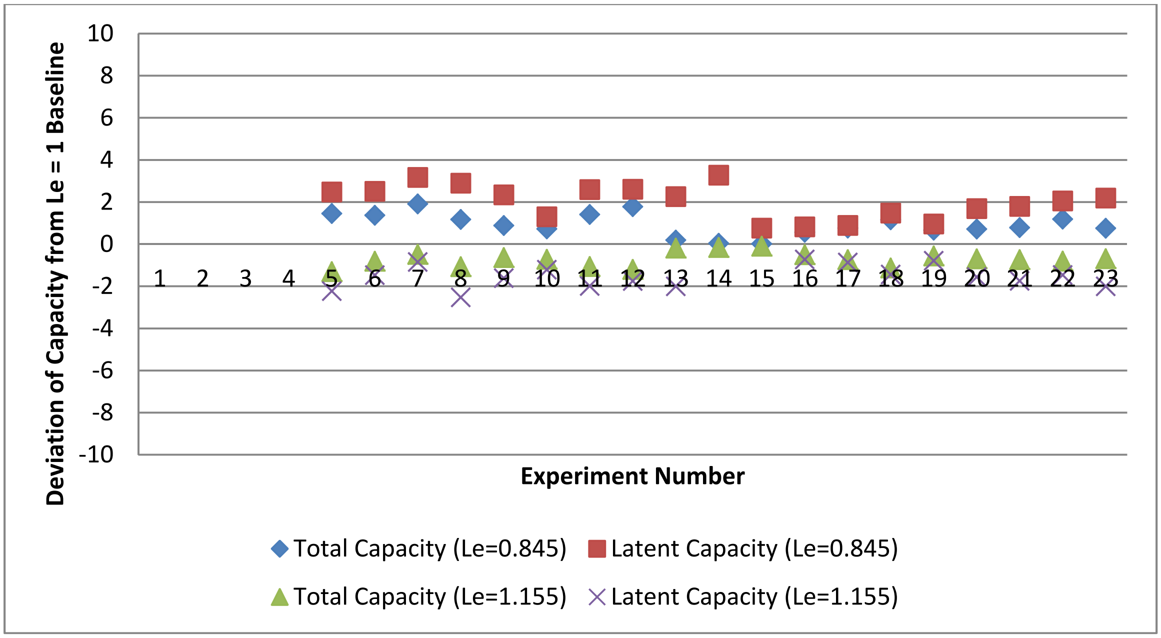

Figure 35 - Effects of Lewis number on total and latent heat transfer on Threlkeld's model

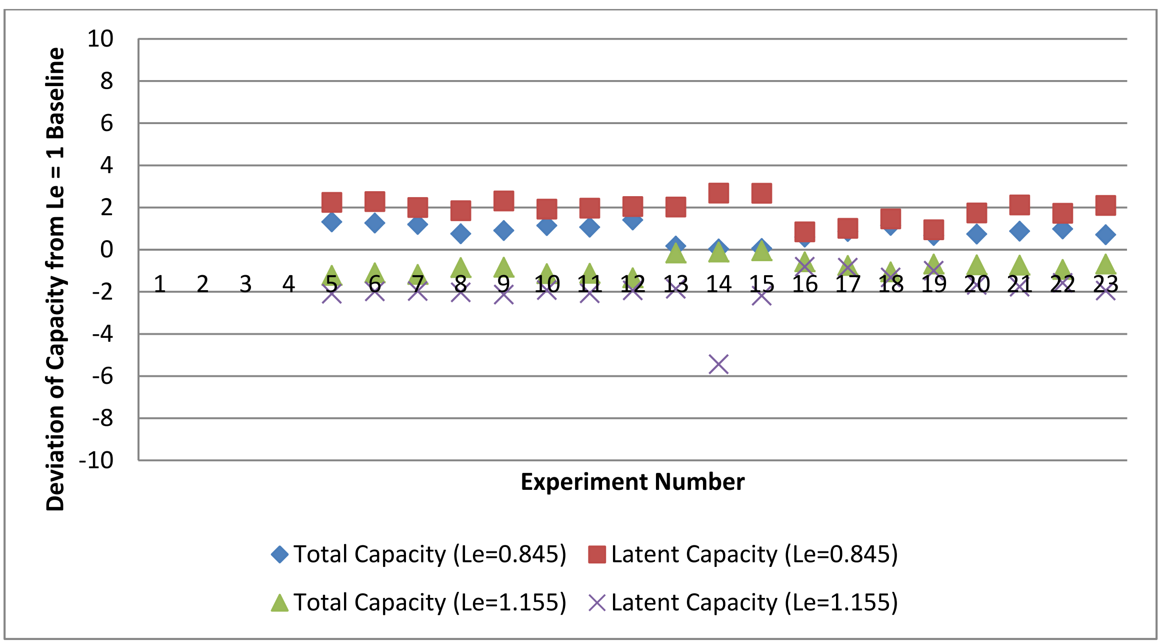

Figure 36 - Effects of Lewis number on total and latent heat transfer on Elmahdy's model

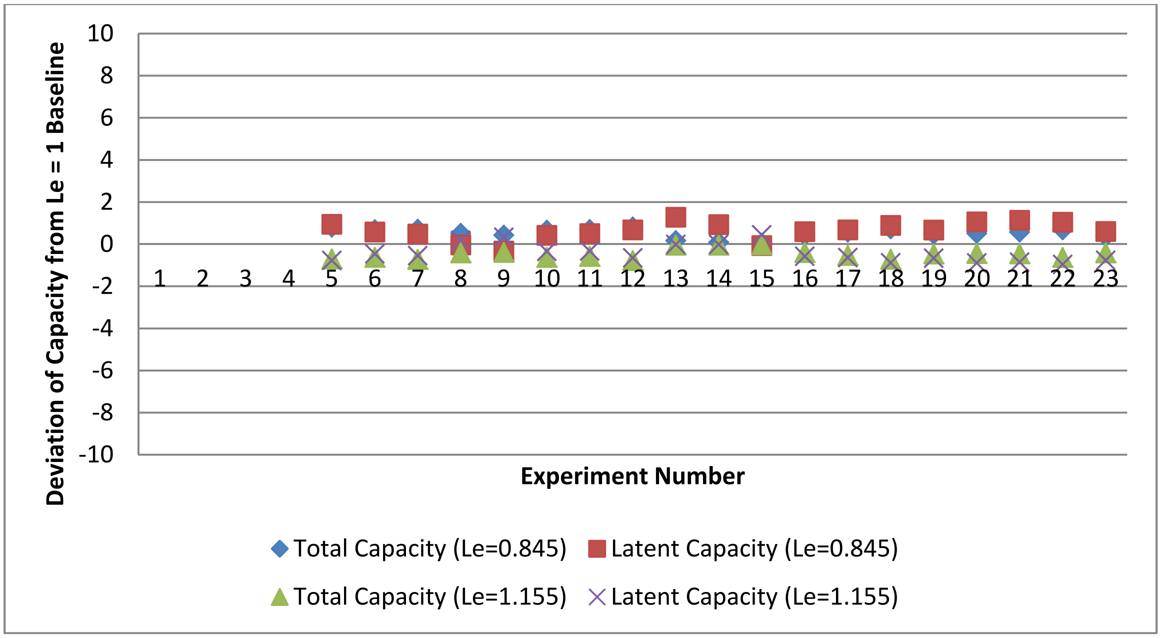

Figure 37 - Effects of Lewis number on total and latent heat transfer on Wang's model

Hill's model has the Lewis number as an input since the enthalpy potential assumption was not used. This leads to no extra Lewis number iterations as seen in the three other models. Figure 38 shows that the latent capacity is about 3% higher when using a Lewis number of 0.845 as compared to unity. This is a significant increase compared to other three models but is still relatively small. The effect of the Lewis number is higher in the model due to the fact that the Lewis number is directly used to determine the sensible/latent heat transfer rate split of a coil segment. The other models are built with the assumption of a Lewis number of unity then the model is modified with an additional adjustment factor. This model requires no extra computation time to evaluate a coil with a Lewis number not equal to unity.

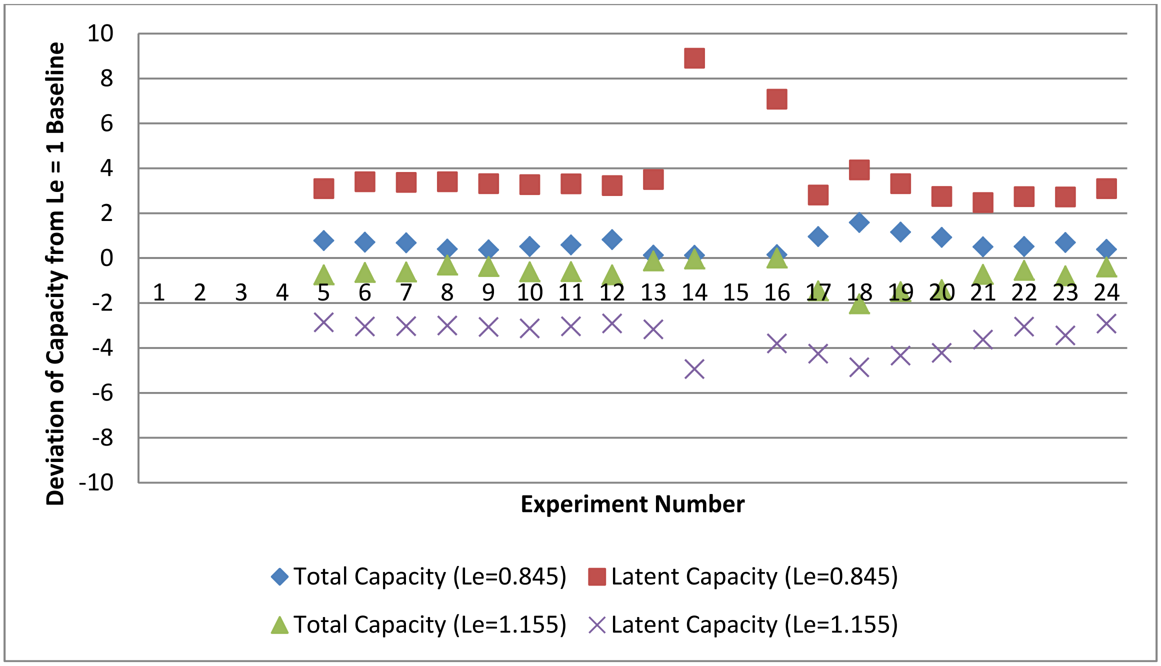

Figure 38 - Effects of Lewis number on total and latent heat transfer on Hill's model

G. Summary

All four methods predicted the total heat transfer rate within ±10% of the experimental results with an average percent difference of ±2%. For all of the different assumptions (fictitious enthalpy, Lewis number, enthalpy potential) and methods (LMED, equivalent sensible process, Eff-NTU, application of fictitious enthalpy) used, all of the results have a similar average percent difference. Other consideration must be made when selecting a model to use, such as computation time, ease of implementation, and available inputs.

The effort required to implement a variable Lewis number is large, and the resulting change in results is relatively small, about 1%. Implementing Xia's Lewis number adjustment factor to Threlkeld's, Elmahdy's, and Wang's methods increased the computation time by four fold. Of course this all depends on how efficient the solver is and how good the initial guesses are. The fact that the Lewis number correction factor depends on the ratio of the sensible to latent heat transfer rates makes solving variable Lewis number in this way time consuming. The computational cost to directly solve for a variable Lewis number is less since in this case the

Lewis number is built into the equation set. The computation time is only increased by about 25%.

The biggest gain in model accuracy seems to be from using the finite difference approach. Any assumptions that are used only apply over a small area so the introduction of error, if any, is very small. As long as the assumptions are re-evaluated at each new segment, the assumptions can be applied over a larger area.

6. SUMMARY AND CONCLUSIONS

A major contribution of this work has been to carefully synthesize and characterize the body of literature. This thesis reviewed twenty four different works on the subject of dehumidifying coils. These papers were characterized by which basic assumptions and methods that were used and a detailed explanation of each assumption and method are provided. With an understanding of the assumption and methods, this provides some insight to how each body of work is different and its possible limitations.

Understanding the fundamentals of the dehumidifying coil models is important because there are assumptions that are sometimes hidden in an author's work which may be important to the reader. Of the four models that were programmed, only two mentioned the Lewis number and its value. Threlkeld (1962) stated it is assumed to be unity, Hill & Jeter (1991) stated he used a value of 0.854, and both Elmahdy & Mitalas (1977) and Wang & Hihara (2003) made no mention of what valued is used. Elmahdy and Wang both used the enthalpy potential which has the assumption of a Lewis number equal to unity built into it. Having all of these works summarized in one location will aid in the future development of dehumidifying coils.

Another contribution is to assess the importance of various models parameters and recommend models for various applications. There are many dehumidifying coil models that have been developed since Threlkeld first presented his. The more recent models have a tendency to rely on computational power of personal computers to iteratively solve a system of equations. While these models may produce a more accurate result, there are situations that call for a compact and quick coil models. Comparison of the four selected models shows that even though they use various assumptions and methods, all have about the same average percent error of about ±2%. Within a reasonable range the effect of the Lewis number on latent heat transfer rate was less than 3%. The effect of the fictitious enthalpy was found to be is less than 1.5%

Another contribution of this work has been to identify the limits of the models and the application of the models. With certain combinations of models and applications, potential problems could arise. Threlkeld's model applied to a single segment application can produce a discontinuity at the dry wet interface, which could make a simulation program unstable. A different application of the model should be selected, such as 'segment by segment' or 'finite difference'.

The next step in this work is to compare model performance against direct expansion evaporator coils where the fluid in the coil tube is undergoing a phase change. The work would investigate if the constant temperature of the two phase fluid increases the accuracy of the models. It would also investigate if the coil models can accurately predict when the fluid changes from a two phase fluid to a single phase. By accurately predicting when the phase change occurs, the coil model would then be able to accurately predict the superheat state of the refrigerant. The superheat state is an important parameter in designing packaged rooftop HVAC equipment.

Another future work would be to determine if a more detailed fin efficiency equation is beneficial. A more detailed fin efficiency equation could be used to help bridge the dry wet discontinuity but the computation time may be a limiting factor.

Another step in this line of work would be to investigate if the models accurately calculate when the onset of latent heat transfer occurs. An experiment would have to be performed where a coil maintained constant air side conditions and the cold fluid temperature would be varied. The cold fluid temperature would have to be such that no latent heat transfer would occur and then incremented lower until the entire coil surface area is forming condensation. The cold fluid would need to be incremented in small amounts, such as 0.1°F. This will allow for a detailed determination of the onset of latent heat transfer.

7. REFERENCES

AHRI. (2001). *Standard For Forced-Circulation Air-Cooling and Air-Heating Coils, 410-2001.*

ASHRAE. (2009). *ASHRAE handbook of fundamentals.* Atlanta: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc.

Brandemuehl, M. J. *HVAC Toolkit: Algorithms and Subroutines for Secondary HVAC Systems Energy Calculations.* ASHRAE.

Braun, J. E., Klein, S. A., & Mitchell, J. W. (1989). Effectiveness Models for Cooling Towers and Cooling Coils. *ASHRAE Transactions , 95* (2), 3270-3280.

Cengel, Y. A., & Ghajar, A. J. (2010). *Heat and Mass Transfer, 4th Edition.* New York: McGraw-Hill.

Domanski, P. A. (1991). Simulation of an Evaporator with Nonuniform One-dimensional Air Distribution. *ASHRAE Transactions* , 793-802.

Elmahdy, A. H. (1975). Analytical and Experimental Multi-Row Finned-Tube Heat Exchanger Performance During Cooling and Dehumidifying Processes. Ph.D. Thesis, Carleton University, Ottawa, Canada.

Elmahdy, A. H., & Mitalas, G. P. (1977). A simple model for cooling and dehumidifying coils for use in calculating energy requirements for buildings. *ASHRAE Transactions , 83* (2), 103-117.

Goodman, W. (1938). Performance of Coils for Dehumidifying Air. *Heating, Piping, and Air Conditioning , 10,11*, 697-707.

Hill, J. M., & Jeter, S. M. (1991). A linear subgrid cooling and dehumidification coil model with emphasis on mass transfer. *ASHRAE Transactions , 97* (2), 118-128.

Jiang, H., Aute, V., & Radermacher, R. (2006). CoilDesigner: a general-purpose simulation and design tool for air-to-refrigerant heat exchangers. *International Journal of Refrigeration , 29*, 601-610.

Lemort, V., Lebrun, J., Cuevas, C., et al. (2008). Development of simple cooling coil models for simulation of HVAC systems. *ASHRAE Transactions , 114* (1), 319-328.

McQuiston, F. C. (1975). Fin Efficiency with Combined Heat and Mass Transfer. *ASHRAE Transactions , 81* (1), 350-355.

McQuiston, F. C. (1976). Heat, mass and momentum transfer in a parallel plate dehumidifying exchanger. *ASHRAE Transactions , 82* (2), 87-105.

McQuiston, F. C., Parker, J. D., & Spitler, J. D. (2000). *Heating, ventilating, and air conditioning analysis and design, 5th edition.* New York: John Wiley & Sons, Inc.

Mirth, D. R., & Ramadhyani, S. (1993). Comparison of methods of modeling the air-side heat and mass transfer in chilled-water cooling coils. *ASHRAE Transactions , 99* (2), 285-299.

Morisot, O., Marchio, D., & Stabat, P. (2002). Simplified model for the operation of chilled water cooling coils under nonnominal conditions. *HVAC&R Research , 8* (2), 135-158.

Oliet, C., Perez-Segarra, C. D., Danov, S., et al. (2007). Numerical simulation of dehumidifying fin-and-tube heat exchangers; Semi-analytical modelling and experimental comparison. *International Journal of Refrigeration , 30*, 1266-1277.

Pirompugd, W., Wang, C., & Wongwises, S. (2007). Finite circular fin method for heat and mass transfer characteristics for plain fin-and-tube heat exchangers under fully and partially wet surface conditions. *International Journal of Heat and Mass Transfer , 50*, 552-565.

Theerakulpisut, S., & Priprem, S. (1998). Modeling Cooling Coils. *International Communications in Heat and Mass Transfer , 25* (1), 127-137.

Threlkeld, J. L. (1962). *Thermal Environmental Engineering, 1st Ed.* Englewood Cliffs, NJ: Prentice-Hall.

Vardhan, A., & Dhar, P. L. (1998). A New Procedure For Performance Prediction of Air Conditioning Coils. *International Journal of Refrigeration , 21* (1), 77-83.

Wang, G., Liu, M., & Claridge, D. E. (2007). Decoupled modeling of chilled-water cooling coils. *ASHRAE Transactions , 113* (1), 484-493.

Wang, J., & Hihara, E. (2003). Prediction of air coil performance under partially wet and totally wet cooling conditions using equivalent dry-bulb temperature method. *International Journal of Refrigeration , 26*, 293-301.

Ware, C. D., & Hacha, T. H. (1960). Heat Transfer From Humid Air to Fin and Tube Extended Surface Cooling Coils. *ASME Paper No. 60-HT-17* .

Xia, L., Chan, M. Y., Deng, S. M., et al. (2009). A modified logarithmic mean enthalpy difference (LMED) method for evaluating the total heat transfer rate of a wet cooling coil under both unit and non-unit Lewis factors. *International Journal of Thermal Sciences , 48*, 2159-2164.

Xia, Y., & Jacobi, A. M. (2005). Air-side data interpretation and performance analysis for heat exchangers with simultaneous heat and mass transfer: Wet and frosted surfaces. *International Journal of Heat and Mass Transfer , 48*, 5089-5102.

8. APPPENDICES

A. Appendix A – Variable Nomenclature

Variables

T	Temperature	$°F$
i	Enthalpy	$\frac{Btu}{lbm}$
i'	Fictitious air enthalpy	$\frac{Btu}{lbm}$
h	Air side heat transfer coefficient	$\frac{Btu}{hr\ ft^2\ °F}$
h_m	Air side mass transfer coefficient	$\frac{lb}{hr\ ft^2}$
h_i	Cold fluid heat transfer coefficient	$\frac{lb}{hr\ ft^2}$
Le	Lewis Number	
Pr	Prandtl Number	
Nu	Nusselt Number	
Re	Reynolds Number	
k	Conduction coefficient	$\frac{Btu}{hr\ ft\ °F}$
c_p	Specific heat	$\frac{Btu}{lbm\ °F}$
$\dot{Q}$	Heat transfer rate	$\frac{Btu}{hr}$
A	Surface area	ft^2
w	Humidity ratio	
i_{fg}	Enthalpy of evaporation	$\frac{Btu}{lbm}$
i_f	Saturated liquid enthalpy	$\frac{Btu}{lbm}$
a	Constant for linear approximation of fictitious enthalpy	$\frac{Btu}{lbm}$
b	Slope for linear approximation of fictitious enthalpy	$\frac{Btu}{lbm\ °F}$
$\dot{m}$	Mass flow rate	$\frac{lbm}{hr}$
ϕ_{fin}	Fin efficiency	
$\phi_{surface}$	Fin effectiveness	
ε	Effectiveness	

Subscripts

c	Cold fluid
h	Hot fluid
t	Tube
i	Inner surface
o	Outer surface
$film$	Outer condensate film surface
$finbase$	Fin base
$fintip$	Fin tip
fin	Fin
F	Fin
∞	Free stream air
$water$	Liquid water
$vapor$	Water vapor
db	Dry bulb
wb	Wet bulb
$dewpoint$	Dew point
1	Inlet
2	Outlet
dry	Dry coil
wet	Wet coil
pw	Partiall wet coil
s	Saturated air
eqv	Equivalent
eq	Equivalent

B. Appendix B – LMTD Derivation

When determining the heat transfer rate between two constant temperatures, the heat transfer rate can be calculated by the following equation.

$$\dot{Q} = UA(T_h - T_c) \tag{8-1}$$

Equation (8-1) as also be written as

$$\dot{Q} = UA\Delta T \tag{8-2}$$

The value of U includes several different heat transfer coefficients such as conduction coefficients of materials between the two temperatures and convection coefficients of any moving fluids.

As we expand the use of equation (8-1) to heat exchangers as seen in Figure 9, it is realized that the temperatures are not constant; therefore, equation (8-1) cannot be used in that particular form. The differential form of equation (8-1) does hold true for this heat exchanger, which is given below.

$$\delta\dot{Q} = U(T_h - T_c)dA \tag{8-3}$$

How T_h and T_c change with respect to dA must be determined. If an appropriate mean temperature difference could be calculated for the two fluids in the heat exchanger, equation (8-2) could be used to determine the heat transfer rate of a heat exchanger.

$$\dot{Q} = UA\Delta T_{mean} \tag{8-4}$$

The heat transfer rate of the hot and cold fluids can be calculated knowing the entering and leaving conditions.

For this example there are some assumptions made for the heat exchanger.

- Each fluid is a liquid during its time in the heat exchanger
- The mass flow of each fluid is constant
- No heat is lost to the surroundings. All heat transferred is from the hot fluid to the cold fluid.
- The specific heat of each fluid is constant.

The hot fluid heat transfer rate is defined as:

$$\dot{Q}_h = C_h\left(T_{h,1} - T_{h,2}\right) \tag{8-5}$$

where:

$$C_h = \dot{m}_h c_{p,h} \tag{8-6}$$

The cold fluid heat transfer rate is defined as:

$$\dot{Q}_c = C_c\left(T_{c,2} - T_{c,1}\right) \tag{8-7}$$

where:

$$C_c = \dot{m}_c c_{p,c} \tag{8-8}$$

For a given heat exchanger, it is common to assume that there is no heat loss to the surroundings. Any heat loss from the hot fluid must be a heat gain for the cold fluid. Since we are defining a hot and cold side, the direction of heat transfer is known and the directional sign convention is not needed. Therefore, all of the $\dot{Q}$ terms will be positive and equation (8-9) can be written.

$$\dot{Q} = \dot{Q}_h = \dot{Q}_c \tag{8-9}$$

Using a parallel flow heat exchanger, the equation for ΔT_{mean} will be derived.

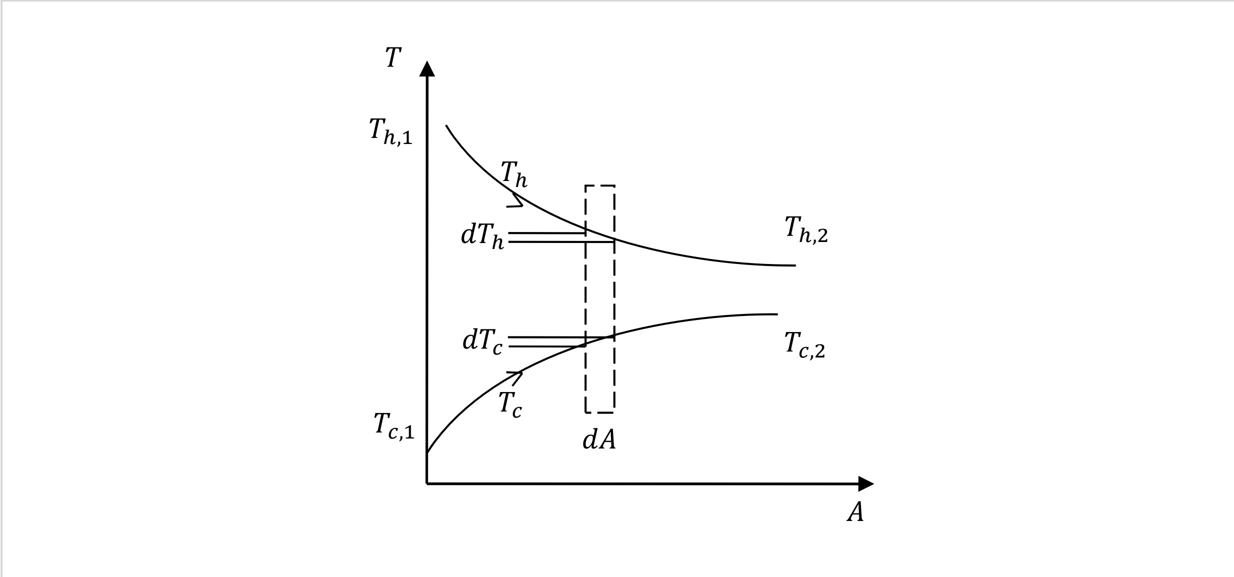

Figure 39 - Representative temperature distribution for a parallel flow heat exchanger

Figure 39 shows the temperature distribution in a parallel flow heat exchanger. This is useful to see the temperature change of a fluid for a differential area of the heat exchanger.

Taking the differential of equation (8-5) and then solving for dT_h leads to the following equations.

$$\delta\dot{Q} = -C_h dT_h \tag{8-10}$$

$$\frac{\delta\dot{Q}}{-C_h} = dT_h \tag{8-11}$$

There is a negative sign introduced into equation (8-10) because the hot fluid temperature is decreasing as dA goes from 0 to A, dT_h is negative, and we desire all $\dot{Q}$ terms to be positive.

Taking the differential of equation (8-7) and then solving for dT_c leads to the following equations.

$$\delta\dot{Q} = C_c dT_c \tag{8-12}$$

$$\frac{\delta\dot{Q}}{C_c} = dT_c \tag{8-13}$$

There is no negative sign in equation (8-13) as there is in equation (8-11) because the cold fluid is increasing as dA goes from 0 to A, dT_c is positive.

As mentioned earlier, equation (8-1) does not hold true to most heat exchangers because the temperatures are not constant.

We now have equations that define $\delta\dot{Q}$in terms of dT_h, dT_c, and dA, equations (8-11), (8-13), and (8-3).

Taking the difference of equation (8-11) and (8-13) leads to the following.

$$dT_h - dT_c = d(T_h - T_c) = \delta\dot{Q}\left(\frac{1}{-C_h}\right) - \left(\frac{1}{C_c}\right) \tag{8-14}$$

Substituting equation (8-3) into (8-14) gives.

$$d(T_h - T_c) = U(T_h - T_c)dA\left[\left(\frac{1}{-C_h}\right) - \left(\frac{1}{C_c}\right)\right] \tag{8-15}$$

Simplifying equation (8-15) and moving all temperature references to the left side of the equals sign gives

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -UdA\left[\left(\frac{1}{C_h}\right) + \left(\frac{1}{C_c}\right)\right] \tag{8-16}$$

Integrating equation (8-16) over the entire surface area of the heat exchanger leads to.

$$ln\left[\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}}\right] = -UA\left[\left(\frac{1}{C_h}\right) + \left(\frac{1}{C_c}\right)\right] \tag{8-17}$$

If we rearrange equations (8-5) and (8-7) to solve to C_h and C_c, the following equations are produced.

$$C_h = \frac{\dot{Q}}{\left(T_{h,1} - T_{h,2}\right)} \tag{8-18}$$

$$C_c = \frac{\dot{Q}}{\left(T_{c,2} - T_{c,1}\right)} \tag{8-19}$$

Substituting equations (8-18) and (8-19) into equation (8-17) gives.

$$ln\left[\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}}\right] = -UA\left[\left(\frac{T_{h,1} - T_{h,2}}{\dot{Q}}\right) + \left(\frac{T_{c,2} - T_{c,1}}{\dot{Q}}\right)\right] \tag{8-20}$$

Rearranging equation (8-20)

$$ln\left[\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}}\right] = \frac{-UA}{\dot{Q}}\left(T_{h,1} - T_{h,2} + T_{c,2} - T_{c,1}\right) \tag{8-21}$$

Define the following variables.

$$\Delta T_1 = T_{h,1} - T_{c,1} \tag{8-22}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} \tag{8-23}$$

Substituting equations (8-22) and (8-23) into (8-21) and removing the negative sign leads to.

$$ln\left[\frac{\Delta T_2}{\Delta T_1}\right] = \frac{UA}{\dot{Q}}(\Delta T_2 - \Delta T_1) \tag{8-24}$$

Rearrange equation (8-24).

$$\dot{Q} = UA\frac{(\Delta T_2 - \Delta T_1)}{ln\left[\frac{\Delta T_2}{\Delta T_1}\right]} \tag{8-25}$$

Comparing equation (8-2) to equation (8-25), ΔT_{mean} can now be defined for a parallel flow heat exchanger.

$$\Delta T_{mean} = \frac{(\Delta T_2 - \Delta T_1)}{ln\left[\frac{\Delta T_2}{\Delta T_1}\right]} \tag{8-26}$$

Due to the form of equation (8-26), this methodology for calculating the performance of a heat exchanger is called the *Log Mean Temperature Difference (LMTD)*.

The same methodology can be applied to a counter flow heat exchanger. Taking the same heat exchanger in Figure 9 and reversing the flow of the cold fluid produces counter flow exchanger, see Figure 40.

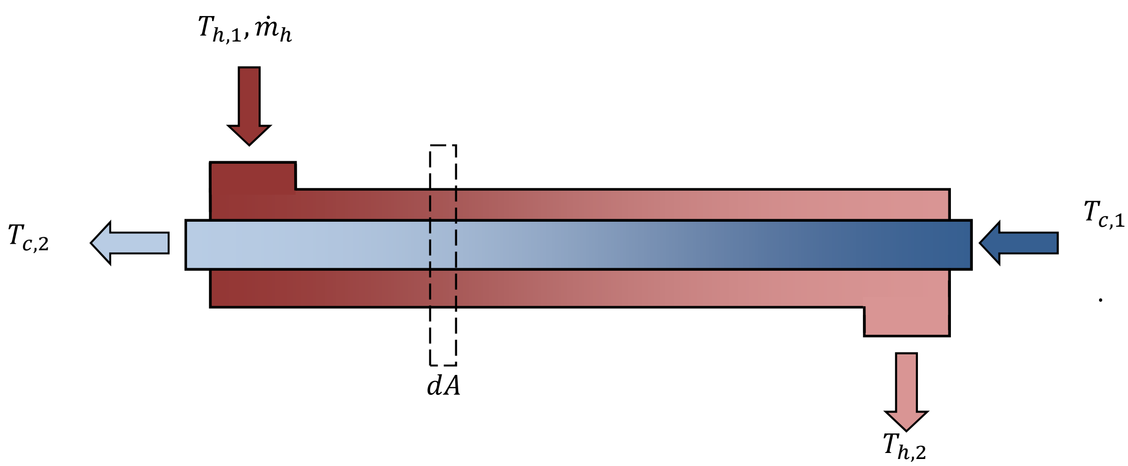

Figure 40 - Coaxial heat exchange with counter flow

Figure 41 shows the temperature distribution of a counter flow heat exchanger. The hot fluid inlet is on the same side as the cold fluid outlet. This flow arrangement produces a more constant temperature differential between the hot and cold fluids. For a given set of inlet and outlet temperatures, the counter flow arrangement will produce a higher log mean temperature difference.

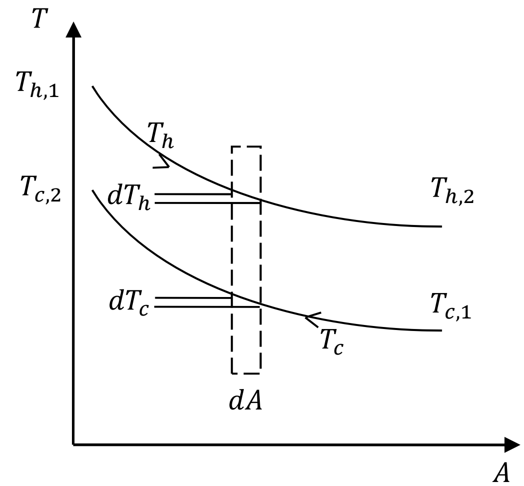

Figure 41 - Representative temperature distribution for a counter flow heat exchanger

The counter flow derivation starts with the same equations as the parallel flow derivation. Only when the configuration of the heat exchanger is used to form the equation do some of the equations differ. Equations (8-1) through (8-9) are valid for both derivations. Since the hot fluid is flowing in the same direction compared to our area reference in Figure 41, equations (8-10) and (8-11) are the same.

$$\delta\dot{Q} = -C_h dT_h \tag{8-10}$$

$$\frac{\delta\dot{Q}}{-C_h} = dT_h \tag{8-11}$$

Taking the differential of equation (8-7) and then solving for dT_c leads to the following equations.

$$\delta\dot{Q} = -C_c dT_c \tag{8-27}$$

$$\frac{\delta\dot{Q}}{-C_c} = dT_c \tag{8-28}$$

There is a negative sign introduced into equation (8-27) because the cold fluid temperature is decreasing as dA goes from 0 to A, dT_c is negative, and we desire all $\dot{Q}$ terms to be positive.

Taking the difference of equation (8-11) and (8-28) leads to the following.

$$dT_h - dT_c = d(T_h - T_c) = \delta\dot{Q}\left(\frac{1}{-C_h}\right) - \left(\frac{1}{-C_c}\right) \tag{8-29}$$

Substituting equation (8-3) into (8-29) and reorganizing the equation produces the following.

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = UdA\left[\left(\frac{1}{-C_h}\right) + \left(\frac{1}{C_c}\right)\right] \tag{8-30}$$

Integrating equation (8-30) over the entire surface area of the heat exchanger and substituting in equations (8-18) and (8-19) leads to.

$$ln\left[\frac{T_{h,2} - T_{c,1}}{T_{h,1} - T_{c,2}}\right] = UA\left[-\left(\frac{T_{h,1} - T_{h,2}}{\dot{Q}}\right) + \left(\frac{T_{c,2} - T_{c,1}}{\dot{Q}}\right)\right] \tag{8-31}$$

Define the following variables.

$$\Delta T_1 = T_{h,1} - T_{c,2} \tag{8-32}$$

$$\Delta T_2 = T_{h,2} - T_{c,1} \tag{8-33}$$

Substituting equations (8-32) and (8-33) into (8-31) produces equation (8-24) again.

$$ln\left[\frac{\Delta T_2}{\Delta T_1}\right] = \frac{UA}{\dot{Q}}(\Delta T_2 - \Delta T_1) \tag{8-24}$$

From equation (8-24) the definition of ΔT_{mean} is produced. The equation for ΔT_{mean} is the same for a counter and parallel flow heat exchanger. ΔT_1 and ΔT_2 can be defined as the temperature difference between the two fluids at the two sides of the heat exchanger. Depending on the configuration of the heat exchanger, this may be an outlet of one to the inlet or outlet of the other fluid.

The trouble with the LMTD method is that the entering and leaving temperatures of each fluid must be known. This leads to an iterative solution when trying to determine the performance of a heat exchanger. Typically, determining the leaving fluid conditions is why a mathematical model is built. At first glance, it looks like the LMTD method does not depend on the mass flow of either fluid since an $\dot{m}$ term does not appear in equation (8-25). The mass flow of the fluid is accounted for in the equation in the UA term.

C. Appendix C – Eff-NTU Derivation

Another mathematical heat exchanger model can be derived that does not depend on leaving fluid temperatures to calculate the heat transfer rate. This approach will compare the actual heat transfer rate to the theoretical maximum. Defining the maximum heat transfer rate will require only inlet conditions which will eliminate the need to iterate to find a solution. Again, this derivation is included in most heat and mass transfer books Cengel & Ghajar (2010) but will be included here to provide the necessary context for new model development.

The maximum heat transfer rate between the hot and cold fluids would be if the hot fluid cooled down to the cold fluid inlet temperature or if the cold fluid heated up to the hot fluid inlet temperature. With that being said we can write the following equation.

$$\dot{Q}_{max} = C\left(T_{h,1} - T_{c,1}\right) \tag{8-34}$$

Where C is either C_h or C_c.

Once one of the fluids reaches the maximum temperature difference, there will be no more heat transfer between the two fluids. The first fluid to reach this condition would be the one with the smaller C value, which we will define as C_{min}. Equation (8-34) can now be written as the following.

$$\dot{Q}_{max} = C_{min}\left(T_{h,1} - T_{c,1}\right) \tag{8-35}$$

where

$$C_{min} = min(C_h \, , C_c) \tag{8-36}$$

Since the actual heat transfer rate cannot exceed the maximum, it is fair to say it must be a portion of the max. We can define a heat transfer rate effectiveness, ε.

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} \tag{8-37}$$

If we substitute equations (8-5), (8-7), and (8-35) into equation (8-37) we can define different forms of the heat transfer rate effectiveness.

$$\varepsilon = \frac{C_h\left(T_{h,1} - T_{h,2}\right)}{C_{min}\left(T_{h,1} - T_{c,1}\right)} \tag{8-38}$$

$$\varepsilon = \frac{C_c\left(T_{c,2} - T_{c,1}\right)}{C_{min}\left(T_{h,1} - T_{c,1}\right)} \tag{8-39}$$

To aid the derivation, we define a new variable C_r, capacitance ratio.

$$C_r = \frac{C_{min}}{C_{max}} \tag{8-40}$$

Where:

$$C_{max} = max(C_h\, , C_c) \tag{8-41}$$

From this point on we will assume $C_{min} = C_h$. The derivation will hold true no matter which value C_{min} is. With this assumption, equation (8-38) can be written in terms of temperature only.

$$\varepsilon = \frac{\left(T_{h,1} - T_{h,2}\right)}{\left(T_{h,1} - T_{c,1}\right)} \tag{8-42}$$

Substituting equations (8-18) and (8-19) into (8-40) leads to C_r being expressed in terms of temperatures only.

$$C_r = \frac{\left[\dfrac{\dot{Q}}{\left(T_{h,1} - T_{h,2}\right)}\right]}{\left[\dfrac{\dot{Q}}{\left(T_{c,1} - T_{c,1}\right)}\right]} = \frac{\left(T_{c,2} - T_{c,1}\right)}{\left(T_{h,1} - T_{h,2}\right)} \tag{8-43}$$

This derivation starts at equation (8-17) as where everything before this step is identical between this derivation and the LMTD method. This derivation will be different in the fact we will remove all references of the outlet temperatures. This will give a different mathematical model that will not require iteration to solve.

Taking equation (8-17) and converting C_h and C_c to C_{min}, C_{max}, and C_r gives the following equation.

$$ln\left[\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}}\right] = \frac{-UA}{C_{min}}(1 + C_r) \tag{8-44}$$

Define the variable NTU, number of transfer units.

$$NTU = \frac{UA}{C_{min}} \tag{8-45}$$

Now combining equation (8-45) into (8-44) and moving the ln to the right side of the equation gives;

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = e^{[-NTU(1+C_r)]} \tag{8-46}$$

Now we can focus on the $\frac{T_{h,2}-T_{c,2}}{T_{h,1}-T_{c,1}}$ term to put the inlet temperatures in terms of other variables.

If we add a $T_{h,1}$ and a $-T_{h,1}$ term of the numerator of the left hand side of equation (8-46), we can get the following equation.

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = \frac{T_{h,2} - T_{h,1}}{T_{h,1} - T_{c,1}} + \frac{T_{h,1} - T_{c,2}}{T_{h,1} - T_{c,1}} \tag{8-47}$$

Equation (8-47) can be simplified as follows:

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = -\varepsilon + \frac{T_{h,1} - T_{c,2}}{T_{h,1} - T_{c,1}} \tag{8-48}$$

Reorder equation (8-43) to solve for $T_{c,2}$ gives:

$$T_{c,2} = C_r\left(T_{h,1} - T_{h,2}\right) + T_{c,1} \tag{8-49}$$

Substituting (8-49) into (8-48) gives:

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = -\varepsilon + \frac{T_{h,1} - \left[C_r\left(T_{h,1} - T_{h,2}\right) + T_{c,1}\right]}{T_{h,1} - T_{c,1}} \tag{8-50}$$

Reorganizing equation (8-50) gives:

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = -\varepsilon + \frac{T_{h,1} - T_{c,1}}{T_{h,1} - T_{c,1}} - \frac{C_r\left(T_{h,1} - T_{h,2}\right)}{T_{h,1} - T_{c,1}} \tag{8-51}$$

Simplifying and substituting equation (8-42) into equation (8-51)

$$\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}} = -\varepsilon + 1 - C_r\varepsilon \tag{8-52}$$

Now we can combine equations (8-46) and (8-52) to get rid of the $\frac{T_{h,2} - T_{c,2}}{T_{h,1} - T_{c,1}}$ term. The following equation is produced.

$$-\varepsilon + 1 - C_r\varepsilon = e^{[-NTU(1+C_r)]} \tag{8-53}$$

Reordering to move ε to the left side produces the following equation.

$$\varepsilon = \frac{1 - e^{[-NTU(1+C_r)]}}{1 + C_r} \tag{8-54}$$

The effectiveness of a parallel flow heat exchanger, as defined in equation (8-54), is a function of UA, C_{min}, C_{max} so it can be calculated without knowing any outlet conditions. Due to the form of equation (8-54), this methodology for calculating the performance of a heat exchanger is called the *Effectiveness-NTU (Eff-NTU)* method.

A similar methodology can be applied for a counter flow heat exchanger. The form of equation (8-54) will be slightly different but it will be a function of the same terms.

D. Appendix D – Equivalent dry-bulb temperature method

Wang & Hihara (2003) introduced a method that takes a wet coil psychrometric process and transforms it to an equivalent dry process. This method was developed to simplify the calculations and distinguish three different cooling modes. A graphical representation of this method is shown in Figure 42.

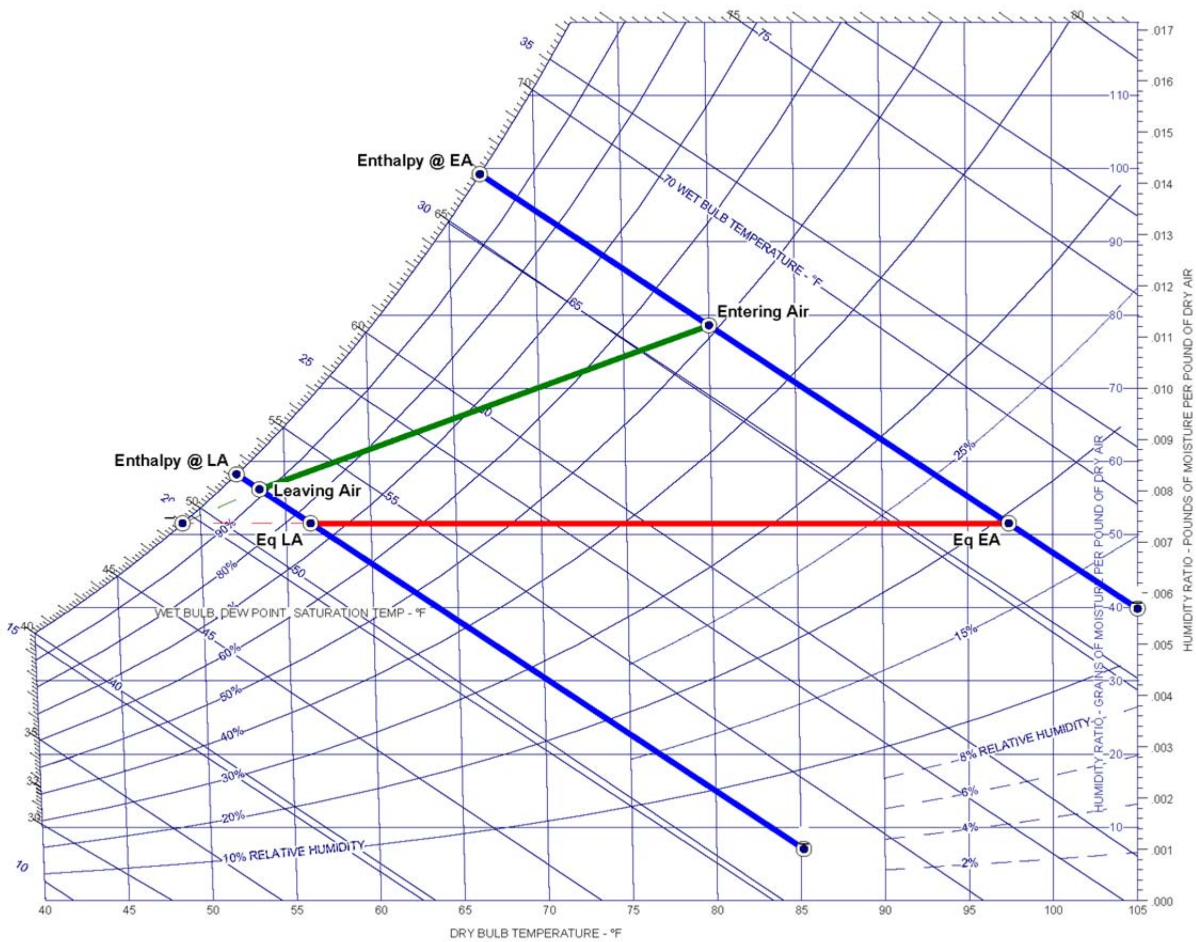

Figure 42 - Representation of wet cooling coil process and equivalent dry process

Line Entering Air – Leaving Air is the actual dehumidifying process of the cooling coil and line Eq EA – Eq LA is the equivalent dry process. The equivalent dry process is at a dew point less than a critical dew point temperature that defines the transition between a dry coil and a wet coil.

The equation for the actual dehumidifying process can be described by the enthalpy potential equation.

$$\dot{Q} = \frac{hA}{c_{p,\infty}}\left(i_{\infty} - i_{film}\right) \tag{8-55}$$

The enthalpy difference in equation (8-55) can be described by using the specific heat of air and a temperature difference. An equivalent entering air temperature must be determined to produce the same capacity.

$$i_{\infty} - i_{film} = c_{p,\infty}\left(T_{\infty,eq} - T_{film}\right) \tag{8-56}$$

Substituting equation (8-56) into equation (8-55) leads to the following.

$$\dot{Q} = hA\left(T_{\infty,eq} - T_{film}\right) \tag{8-57}$$

There are four different cooling modes that are defined by this methodology.

1. Dry coil
2. Partially wet coil without net vapor condensate
3. Partially wet coil with net vapor condensate
4. Completely wet coil

The next step is to find the key entering air dry bulb temperatures that define the transition between different cooling modes.

A dry coil is occurs when the dew point temperature of the free stream air is less than or equal to the outer tube surface temperature.

$$T_{t,o} \geq T_{\infty,dewpoint} \tag{8-58}$$

A partially wet coil occurs when the dew point temperature of the free stream air is greater than the outer tube surface temperature and less than the fin tip temperature.

$$T_{fintip} \geq T_{\infty,dewpoint} > T_{t,o} \tag{8-59}$$

Since an average fin temperature will be used to determine the performance of the coil, there will only be a calculated latent load when the average fin temperature is below the free stream air dew point temperature.

$$T_{fintip} \geq T_{\infty,dewpoint} > T_{fin,mean} > T_{t,o} \tag{8-60}$$

When the free stream air dew point temperature is less than or equal to the average fin temperature, there is no calculated latent load.

$$T_{fintip} > T_{fin,mean} \geq T_{\infty,dewpoint} > T_{t,o} \tag{8-61}$$

A completely wet coil occurs when the dew point temperature of the free stream air is greater than the fin tip temperature.

$$T_{\infty,dewpoint} > T_{fintip} \tag{8-62}$$

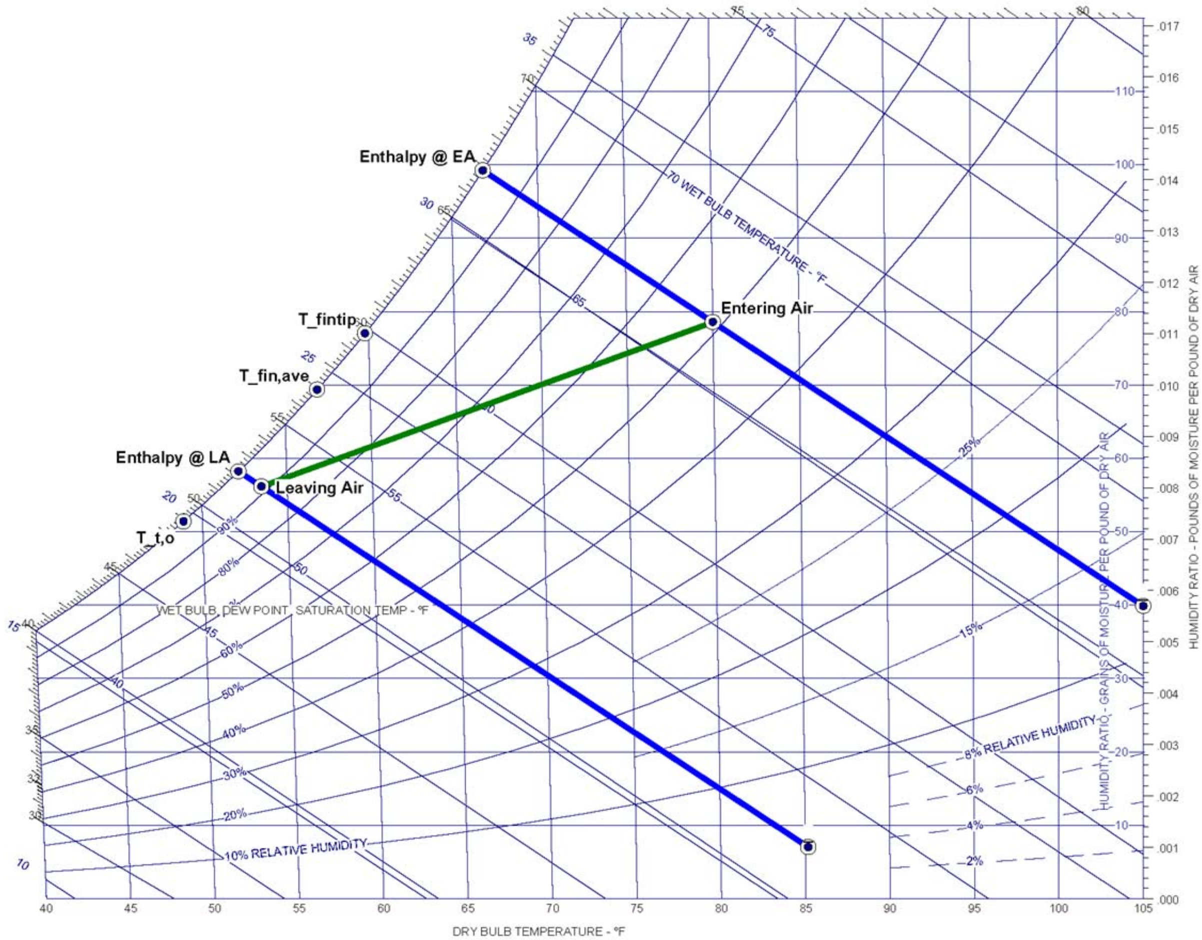

Figure 43 - Critical temperatures for different cooling modes

For a given coil and entering air enthalpy, we can find the key entering air dry bulb temperatures that define the transition between different cooling modes.

Define $TD1$ as a psychrometric point that has the enthalpy of the entering air and a dew point of the tube outer surface temperature when the coil is dry. $TD1$ is the lowest entering air dry bulb temperature that will produce a dry coil for a given coil, entering air enthalpy, and air flow.

$TD1$ can be determined by ratio of thermal resistances to ratio of temperature potentials.

$$\frac{TD_1 - T'_{t,o}}{TD_1 - T_c} = \frac{\frac{1}{\phi_{surface,dry} h_{dry} A_t}}{\frac{1}{\phi_{surface,dry} h_{dry} A_o} + \frac{Ln(D_o \,/\, D_i\,)}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-63}$$

Where $T'_{t,o}$ is the dew point of TD_1.

Since the air side convection coefficient changes when the coil is wet, there is a slightly different value for the tube outer surface temperature and that defines the transition between a dry and partially wet coil.

Define T^e_{ad}as a psychrometric point that has the enthalpy of the entering air and a dew point of the tube outer surface temperature when the coil is wet. T^e_{ad} is the dry bulb temperature for the equivalent dry psychrometric process as described by Figure 42.

T^e_{ad} can be determined by ratio of thermal resistances to ratio of temperature potentials.

$$\frac{T^e_{ad} - T_{t,o}}{T^e_{ad} - T_c} = \frac{\frac{1}{\phi_{surface,wet} h_{wet} A_t}}{\frac{1}{\phi_{surface,wet} h_{wet} A_o} + \frac{Ln(D_o \,/\, D_i\,)}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-64}$$

Where $T_{t,o}$ is the dew point of T^e_{ad}

It appears that Wang assumes the conduction through the condensate on the film is negligible and therefore excluded from the thermal resistance network. Wang does make the assumption that the air side sensible heat transfer coefficients are the same for all cooling modes (dry, partially wet, and wet).

To find the maximum equivalent dry bulb temperature to produce a completely wet coil, the temperature at the fin tip must be determined. Wang states the following relationships can be used to determine the fin tip temperature.

$$\frac{i_\infty - i'_{fintip}}{i_\infty - i'_c} = \frac{1}{\cosh(MH)} \tag{8-65}$$

Where,

$$M = \left(\frac{2h_{wet}C_{wet}}{k_{fin}\delta_{fin}}\right)^2 \tag{8-66}$$

$$C_{wet} = \frac{b_{fin,mean}}{c_{p,\infty}} \tag{8-67}$$

$$H = equivalent\ fin\ height$$

Wang uses a wet fin efficiency that is not constant for a various wet coil conditions. The methodology that Wang uses varies the wet fin efficiency with the mean fin temperature.

Once $i'_{s,fintip}$ is determined, its saturated air temperature is the temperature of the fin tip. Then $TD4$ can be defined as a psychrometric point that has the enthalpy of the entering air and a dew point of the fin tip temperature. $TD4$ is the maximum dry bulb temperature that will result in a completely wet coil with the given coil, entering air enthalpy, and air flow.

T^e_{avf} can be determined by ratio of thermal resistances to ratio of temperature potentials for the equivalent dry process.

$$\frac{T^e_{ad} - T^e_{avf}}{T^e_{ad} - T_c} = \frac{\frac{1}{h_{wet}A_t}}{\frac{1}{\phi_{surface,wet}h_{wet}A_t} + \frac{Ln(D_o\ /\ D_i\ )}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-68}$$

After T^e_{avf} has been determined, its value along with the dew point of $T_{t,o}$ can be used to determine the enthalpy at T^e_{avf} on the equivalent dry process line. The saturated air temperature with the afore mentioned enthalpy is the mean fin temperature, $T_{fin,mean}$.

$TD23$ is defined as a psychrometric point that has the enthalpy of the entering air and a dew point of the average fin temperature.

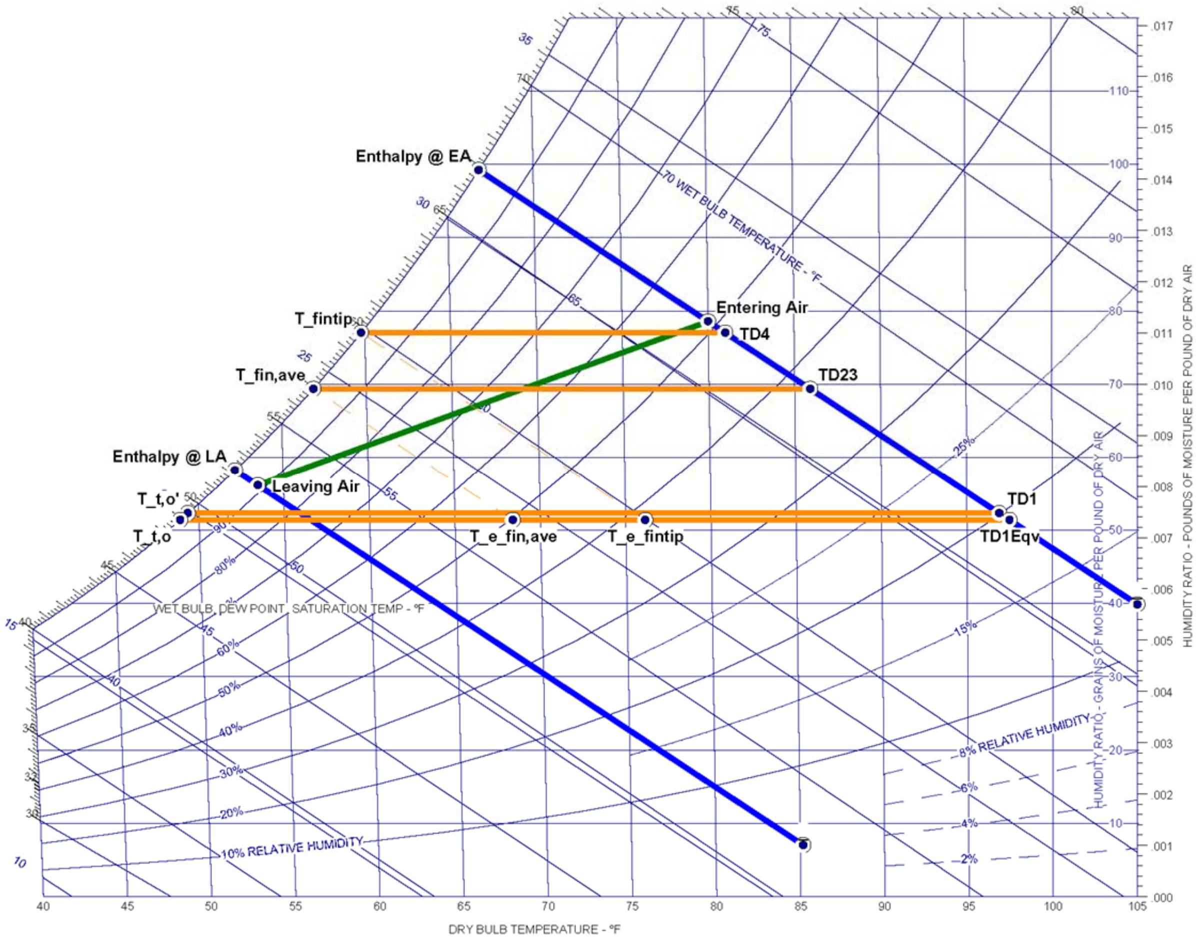

Figure 44 - Relevant temperatures for EDT method

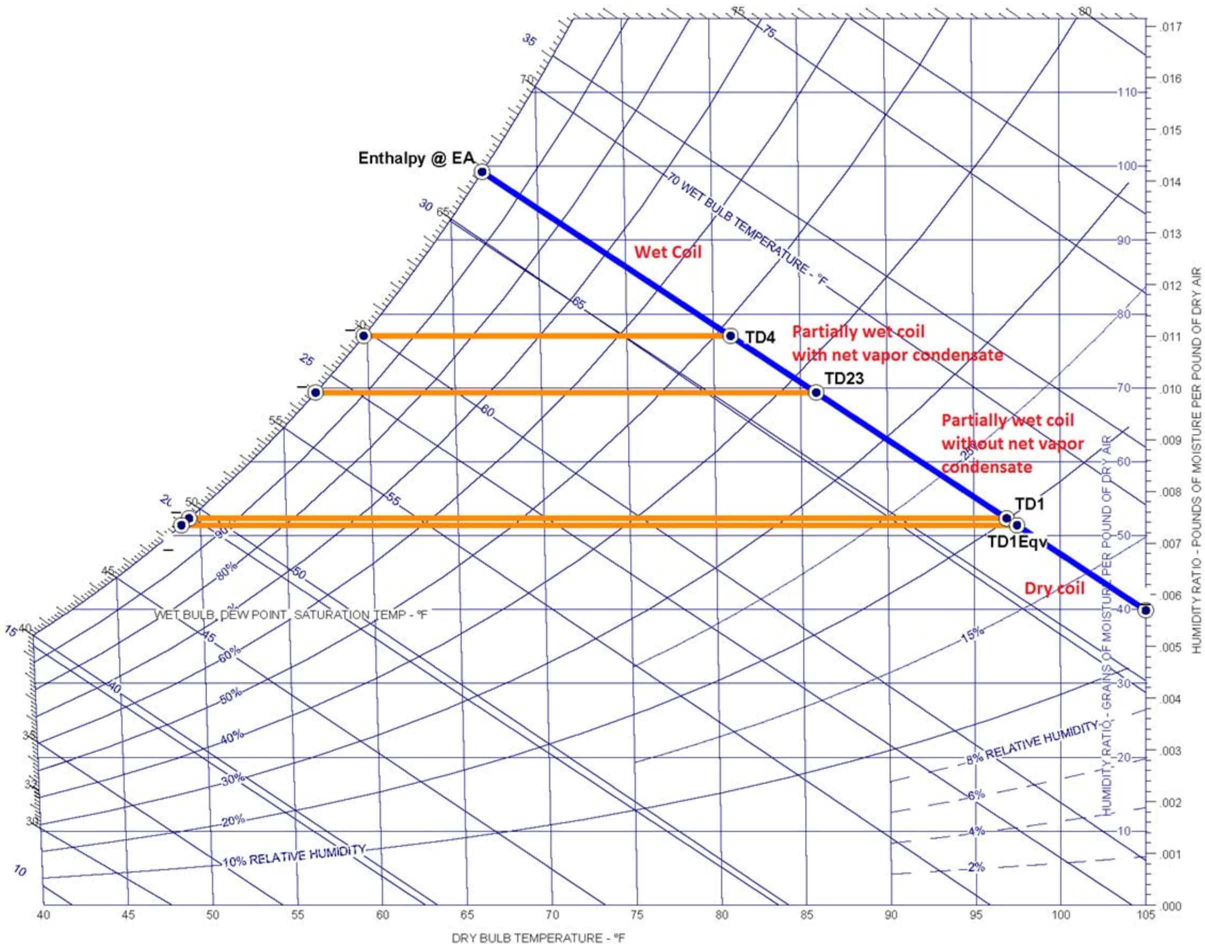

Figure 45 - Four cooling modes of the EDT method

Wang presents the following capacity equations in a way that is used in a segment by segment model which has some assumptions made that will be discussed later. In this section of the paper, these capacity equations will be presented in a way where the equations can be used on the entire coil.

ε represents the effectiveness calculated by the Eff-NTU method.

If $T_{\infty} \geq TD_1$ (dry coil) then

$$UA = \frac{1}{\frac{1}{\phi_{surface,dry} h_{dry} A_t} + \frac{Ln(D_o \,/\, D_i\,)}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-69}$$

$$Q = C_{min}\varepsilon(T_\infty - T_c) \tag{8-70}$$

If $T_\infty \leq TD_4$ (completely wet coil) then

$$UA = \frac{1}{\frac{1}{\phi_{surface,wet} h_{wet} A_t} + \frac{Ln(D_o \,/\, D_i\,)}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-71}$$

$$Q = C_{min}\varepsilon(T_\infty^e - T_c) \tag{8-72}$$

If $TD_4 < T_\infty < TD_1$ (partially wet coil) then

$$UA = \frac{1}{\frac{1}{\phi_{surface,pw} h_{pwet} A_t} + \frac{Ln(D_o \,/\, D_i\,)}{2\pi k_t L_t} + \frac{1}{h_i A_{t,i}}} \tag{8-73}$$

$$Q = C_{min}\varepsilon\left(\Delta T_{pwet}^e\right) \tag{8-74}$$

$$\Delta T_{pw}^e = (TD_1 - T_c) + \frac{T_\infty^e - TD_1}{TD_1 - TD_4}(TD_1 - T_\infty) \tag{8-75}$$

To calculate the partially wet surface efficiency [equation (3-43)], a slight change to the fin efficiency equation is needed. The M term in equation (8-65) is now defined as.

$$M = \left(\frac{2h_{wet}C_{pwet}}{k_{fin}\delta_{fin}}\right)^2 \tag{8-76}$$

Where,

$$C_{pwet} = 1 + \frac{C_{wet} - 1}{TD_1 - TD_4}(TD_1 - T_\infty) \tag{8-77}$$

Now the total heat transfer is known but not the sensible heat ratio. Wang states that for the dry coil and partially wet coil without net vapor condensate, the psychrometric process is a constant humidity ratio process. Therefore the leaving air enthalpy and entering air humidity ratio can be used to get the leaving air dry bulb.

For the wet coil and partially wet coil with net vapor condensate, a linear approximation is used to get the leaving air dry bulb temperature.

$$\frac{T_{\infty,1} - T_{\infty,2}}{T_{ad1}^{e} - T_{ad2}^{e}} = \frac{T_{\infty,1} - T_{avf}}{TD_{23} - T_{avf}} \tag{8-78}$$

VITA

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Master of Science

Thesis: DEHUMIDIFYING COIL MODELS FOR ENERGY SIMULATION AND DESIGN

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Scope and Method of Study:
This thesis presents a summary and analysis of the development of dehumidifying fin and tube coil models. The goal is to compare and assess the models that are presented in the available literature and evaluate the implementation and performance of some select models. By bringing all of these works into one central location and comparing one author's work to another and experimental data, differences in the models can be evaluated for increased accuracy of calculated heat transfer rates, ease of implementation, and range of applicability. This will facilitate the need to clearly identify assumptions and methods that each author uses and determine if the assumptions are acceptable.

Findings and Conclusions:
A major contribution of this work has been to carefully synthesize and characterize the body of literature. This thesis reviewed twenty four different works on the subject of dehumidifying coils. These papers were characterized by which basic assumptions and methods that were used and a detailed explanation of each assumption and method are provided. Comparison of four selected models showed that even though they used various assumptions and methods, all have about the same average percent error of about ±2%. Within a reasonable range the effect of the Lewis number on latent heat transfer rate was less than 3%. The effect of the fictitious enthalpy was found to be is less than 1.5%

ADVISER'S APPROVAL: Dr. Daniel E. Fisher