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CREATING SPACES FOR LEARNING IN THE MATHEMATICS CLASSROOM: A PHENOMENOLOGICAL STUDY OF PRE-SERVICE TEACHERS

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CREATING SPACES FOR LEARNING IN THE MATHEMATICS CLASSROOM:

A PHENOMENOLOGICAL STUDY OF PRE-SERVICE TEACHERS

A Dissertation APPROVED FOR THE
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DEDICATION

To my parents, Max and Jutta Tankersley, who worked extremely hard to offer me as many opportunities as possible so that I could lead a happy and successful life. Their passion for education and loving ways will be forever appreciated and this dissertation is dedicated to them.

To my husband, Eric Richardson, whose supportive nature and faith in my ability to succeed has kept me going throughout this challenging process. Without him, none of this would have been possible and he is truly my best friend.

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ABSTRACT

This is a phenomenological study of the types of relationships which emerged with a group of elementary education majors. The phenomenon studied was "being in the world with mathematics" and the atmosphere in which the phenomenon was studied was that of a problem-centered learning environment containing pre-service teachers. Students in the environment were accustomed to conversation, exploration, and collaboration. The instructor of the course promoted a dynamic environment which enabled students to experience mathematics education from a variety of perspectives.

Themes were selected as part of an emergence of student relationships during the course of the semester. Each theme was reflective of a relationship which seemed to have the most prevalence among student experiences. Themes emerged from data sources such as conversations, interviews, field notes, assignments, and classroom observations made by the researcher.

The Pirie-Kieren model for the dynamical growth of mathematical understanding and Brent Davis' mathematical mentalities were used as inquiry guides for this study. The work of Pirie and Kieren offered a way to engage students in their thinking about their experiences with mathematics, while Davis' mentalities helped frame questions to explore students' phenomenological experiences of being with mathematics.

The findings indicate that meaningful experiences are important to students' ways of being in the world with mathematics. These experiences are discussed through emerging relationships such as folding back (Pirie & Kieren, 1994),

language, relevance, and open-endedness. Other findings indicate ways to combine both theory and practice so that classroom teachers and mathematics educators may find value in the phenomenon of being in the world with mathematics.

Implications for mathematics education may best be discussed in terms of curriculum. These findings call for a different perspective to be taken concerning the mathematics curriculum of schools and universities. This perspective involves viewing mathematics as a space for Richness, Recursion, Relations, and Rigor (Doll, 1993) to occur. It is hoped that this study will enrich the mathematical conversations of a variety of individuals ranging from seasoned practitioners and theorists to first year teachers.

Chapter I

INTRODUCTION

In one part of the book *Feed* by M. T. Anderson (2002), the main character Titus and his love interest Violet, who come from very different backgrounds, are trying to fit into one another's social groups. The two find themselves in an argument about his group of friends and Titus says to Violet, "It sometimes feels like you're watching us, instead of being us."

Are we ever really watching something? Is the individual ever really separated from what or who she/he comes into contact with? Isn't this what we try to do, however, in education? Many models of schooling lend themselves to the banking metaphor of education, where the teachers are the depositors and the students the depositees of information. Here, teachers are on the "outside" of learning – watching, evaluating, judging. Creating this inside/outside perspective is prevalent among the discourses of mathematics. *Watching* mathematics has essentially been the common practice over *being with* mathematics.

This *watching* of mathematics occurs in the traditional classroom in a variety of ways. The teacher lists examples on the board while the students watch. The teacher demonstrates one way to work a problem, and then offers worksheets with a large quantity of the same type of problem for students to imitate. Success on this drill and practice is seen as a way to "know" if students know mathematics. Knowing, itself, assumes the "outside" perspective – a demonstration evaluated by objective measures that leave little room for ambiguity. More important questions, however, to ask of teachers are: How do your students experience mathematics? How do they live in the world with mathematics? How are they being WITH mathematics?

Being with mathematics entails a very different understanding of students' interactions with mathematics and their world, with implications for instruction and assessment. It is not just another side of a dualism but a denial that we can ever achieve anything but a personal connection with that which we come to know. Making connections, forming relationships, and seeing patterns are large parts of a type of classroom that goes beyond dualisms and embraces the connectedness of the inner life. *Being with* mathematics can involve deep exploration into a problem or idea which may include conversation, experimentation, manipulation of objects, and/or all of these.

Using the interpretive frameworks of curriculum theorists like Davis (1996), Kieren (1994, 1995), and Pirie (1994,1995, 2000), this phenomenological study is designed to problematize mathematical perceptions and experiences with the goal of changing how we think and talk about mathematics instruction. Problematizing perceptions and experiences involves the inclusion of a variety of perspectives which may allow educators to view mathematical discourse as an integrated and relational discipline. This is an important area to look at because many pre-service teachers have strong, negative views toward mathematics but will be teaching it in their future classrooms. I will take the stance that mathematics classrooms pre-K through college should create spaces for learning which allow students to problematize their mathematical perceptions and experiences as a vehicle for understanding being with mathematics.

My ideas arise from studies which problematize what it means to perceive and/or experience mathematics. Perceptions as relevant to this study, however, are not to be interpreted from the "outside" as manifestations of behavior, but as complex ways of interacting with and of interpreting our own experiences being-with mathematics. Likewise, experiences of and with mathematics are not inside feelings or interpretations alone but dynamic relationships that include participation in a discourse domain of mathematics (Applebee, 1996). Authors such as Thomas Kieren (1994, 1995) and Susan Pirie (1994, 1995, 2002), for example, will be included in my literature review. Each of these theorists have focused their attention on non-dichotomous thinking and how this type of thinking allows for the embodiment of mathematics, as being with mathematics is interpreted as more complex than either the inside or outside perspectives might suggest. Also, research done on how students develop early number concepts, by Grayson Wheatley and Anne Reynolds (1996, 1997, 1999), among others, provides an insight into opening up spaces for constructing number as something beyond the inner/outer dichotomy. Their work, especially from a practitioner's standpoint, has not only been useful in the classroom, but has been vital to the practitioner's personal perceptions and experiences of being in the world with mathematics.

PURPOSE OF THE STUDY

Implications of the study are how to open up spaces and/or create new spaces and new ways of thinking about mathematics instruction in the mathematics classroom. The purpose of this study is to explore how looking at students' experiences with mathematics may provide insights into their mathematics learning and ultimately into how mathematics education could be taught. Tearing down walls and/or opening up spaces for mathematics learning to occur may include a variety of components that look like constructivist teaching approaches: students and/or teachers having a conversation about a particular issue, students expressing an idea in a mathematics journal, working a problem on the chalk board, manipulating a set of objects to gain a new perspective on an idea, or taking some sort of a field trip that relates to a particular notion being discussed in the class. An important component of opening up these learning spaces, explored through the phenomenological experiences of individuals' mathematical perceptions, however, goes beyond a cognitivist approach to understanding learning. Phenomenology, in terms of qualitative research, describes the subjective experience of an individual or individuals from the point of view of the researcher (Schwandt, 2001). The idea is to discover students' perceptions and experiences *of* mathematics as they have lived them. The particular approach of this study, incorporating a phenomenological methodology within an existential hermeneutical frame, provides the potential to understand how students' experiences with mathematics, how their being in the world with mathematics, may impact their understandings of and beliefs and feelings about mathematics, as well as their performances in mathematics classes. Thus, the purpose of my study is to explore the mathematical experiences and beliefs of education majors from the perspective of their being with mathematics. Guiding questions in my study are: What are students' perceptions about their experiences with mathematics? How can students' perceptions about and experiences with mathematics inform their understandings of mathematics?

I explored these perceptions and experiences of being with mathematics through interviews, written responses, assignments given by the instructor, and other observations made by me (the researcher) throughout the semester of students in an elementary education teacher preparation class. I also discuss the Pirie-Kieren model for the dynamical growth of mathematical understanding and Davis' mathematical mentalities as inquiry guides for my study. The work of Pirie and Kieren offers a way to engage students in their thinking about their experiences with mathematics, their being "with" mathematics while Davis' mentalities helps me frame my questions to explore their phenomenological experiences of being with mathematics and hermeneutical understandings of what being with mathematics may be like.

RATIONALE

In our modern society comes a certain status with success in mathematics classes. This success is held by the minority rather than the majority of students and is apparent in the number of remedial mathematics classes offered at universities across the country. The University of Oklahoma alone offers 3 different remedial classes with approximately 15 sections containing 25-35 students per class despite an increase in high school graduation and admissions requirements that all students have three high school credits in mathematics. Students receive no credit hours for passing these "zero-level" classes, which meet for an entire semester and cover skills determined necessary for success in a college algebra and basic mathematics. Students are required, despite their high school transcripts, to take a placement exam in order to enroll in any mathematics classes, where performance on the mathematics placement exam, not prior mathematics courses, determines students' placements in college-level classes. Students in the zero-level classes are placed in those classes as a result of their placement exam scores.

What is occurring prior to college that is leading students to the path of remedial mathematics classes besides the other issue of over-reliance on one measure for placement? From a mainstream perspective, answers to this question may be discussed in a variety of ways. Some of these ways include students lack self discipline, students lack of appropriate study habits, the curriculum provides insufficient preparation in high school, high school instruction is impeded by poor teaching/grade inflation leading students to negative feelings about mathematics, and/or students have a false sense of accomplishment in their mathematics classes. Despite "feel good" instruction at the pre-collegiate level, as those who object to "humanistic" approaches associated with constructivist teaching methods argue, students still come to college with little confidence and desire to learn mathematics at the college level. Despite the interplay among students' previous mathematical experiences, classes, instruction, and expectations, students' lack of performance or desire to learn mathematics typically is blamed on the individual either as an expression of personal choice or of innate intelligence (failing to possess the "math gene"). Few challenge the meaningfulness of mathematics for our children as related to the meaningfulness of mathematics for society or the role mathematics has played in establishing what we value and who we are.

As a former middle school mathematics teacher, I experienced at the beginning of each school year anti-math sentiments expressed by nearly everyone around me. During open house, parents would unashamedly bring their child into my classroom, proclaiming that no one in their family possessed the "math gene" and that I should be aware of this fact. I often found myself wondering if that same parent paraded their child into the language arts classroom professing the absence of the "reading gene." Other teachers would call out my name during faculty meetings if there was some type of a calculation that needed to be done because they were seemingly very proud of the fact they "couldn't do math." At the college level, I was shocked to find that my own students (pre-service elementary teachers) felt as long as they could balance their checkbook, they knew enough mathematics, or, more depressing, that that was all the mathematics any average person needed.

Mathematics, for most people, is not a way of being in the world but is a performance or set of discrete facts that have little relevance to everyday life beyond the ability to "do" simple calculations or routine mathematical tasks. How does being-in-the-world with mathematics change our ideas about the relevance of mathematics or even about what mathematics is? One way to address this question is by tracing the history of mathematics. This can be done through Davis' (1996) mathematical mentalities.

Mathematical Mentalities in the History of Mathematics

Davis' notion of the five mentalities traces a historical route into the beginnings of mathematics, the present focus of mathematics, and future becomings of mathematics in regards to the role it plays in society. Understanding these mentalities gives us insight into why so many in our modern society experience mathematics as meaningless and disjoint from our ways of being in the world.

The first mentality, Davis (1996) calls Oral Pre-civilization, deals primarily with mathematics as an ontological premise. Humans in oral traditions experience mathematics through culture, activity, and general life experiences. However in this mentality, there is no epistemological component, so mathematics is not formally written and used as a separate component of study or way of knowing. The same goes for the second mentality, called Pre-Formalism, in which writing begins to surface, number becomes a symbol rather than a verbal description and logical deduction comes into play (Davis, 1996). In both of these mentalities, however, mathematics is considered as a number or as a place-holder for "things." Mathematics does not come to represent a way of understanding the world, nor does mathematics take on its predictive potential to map the world or, as we see with Galileo, a way of "reading the mind of God." Mathematics in oral and pre-formal traditions is an ontological statement of fact. Being with mathematics or being in the world in mathematical ways has little bearing on the lives of individuals in pre-formal, pre-modern cultures.

Epistemologically speaking, the third mentality, Formalism, separates mathematical ideas apart from experience, thus diminishing the ontological component (Davis, 1996). During this time, around the 1600s, the invention of mechanical time specifically led to the mathematization of reality, which contributed to ideas about the quantifiable and intimately connected natures of space and time (Fleener, 2002). Time itself allowed us to quantify the universe and this led to time becoming isolated. Mathematics, in particular, was used as a way of abstractly describing natural phenomena, which lead to the destruction of culture and various alternatives. Being with mathematics in a formalist realm entails formalizing relationships in mathematical ways to understand, control, and predict. Knowing the laws of gravity, for example, allow us, in a formalist sense, to predict motions and forces, map our universe, and predict consequences. Lost is the essence of relationship or the "things" themselves as the mathematical objects take on their own meanings.

Continuing with the epistemological focus is the fourth mentality, Hyper-Formalism, in which power is associated with knowing mathematics. Beyond a map for understanding the universe and predicting relationships, hyper-formalism confuses the map for the territory, elevating the mathematics as a way of understanding these relationships as the underlying relationships themselves. All other ways of knowing become subservient to mathematical knowing and, ultimately, if something cannot be expressed in mathematical terms, it is not worth expressing at all. In the formalist and hyper-formalist mentalities, being with the world in mathematical ways is in actuality a separation of experience from knowing and the abstraction of mathematics is elevated to a higher status of knowing than bodily experience or feelings.

The hyper-formalist mentality is very much a part of our modern society today and mathematics continues to be thought of as a separate entity of study in which we must know rather than experience. Our being in the world in mathematical ways is subverted by the hyper-formalist perspective which emphasizes the formal properties of mathematics separated from experience. Of utmost importance, from the hyper-formalist view, is "knowing" mathematics with little relevance to "being with" mathematics. Applications and practicalities of mathematics are only secondary as hyper-formalism emphasizes the mathematical relationships among mathematical structures.

Inherent in the hyper-formalist mentality are issues of power and privilege associated with the knowing of mathematics. During this time, the goal is to ignore mathematical experience because it is seen as too open to interpretation and a hindrance on one's deductive abilities (Davis, 1996). Current focus on students' "real world" experiences with mathematics (NCTM, 2000) is confused within a discourse that emphasizes and values hyper-formalist perspectives.

As hyper-formalism emphasizes mathematical form as an epistemological frame, other ways of knowing, especially those that cannot be "formalized" as mathematical representation, are considered of less value and, at the extreme, irrelevant. Women in particular have suffered under this privileged imposition of scientific/mathematical knowing. We see this in women's health, for example, as described by Code (1996)

> [T]he masculine superiority of it practitioners constructed the new science of medicine as an authority over women's lives…knowledge itself, increasingly, became a commodity of privilege. With the rise of the middle classes and the establishment of (middle-class) women's private domestic realm, the prestige of science extended rapidly into the domains of housework and child rearing….women began to draw "prodigiously on the advice of male experts in an attempt to lay the basis for a *science* of childraising and a *science* of housework. (p. 207)

In mathematics, many women and minorities who do not find the abstraction of hyper-formalist ways of being as meaningful are not typically as successful as their male counterparts. Instead of challenging what mathematics has become and our interpreted ways of being with mathematics, educators, politicians, and researchers have "blamed" the "victims," so to speak. A variety of blame is put on girls ranging from hormonal attributes to genetic pre-disposition as explanations for the fewer numbers of women who do not do well in mathematics classes or who do not choose careers in STEM (science, technology, engineering, mathematics) fields. Various feminist scholars argue that, far from being the "neutral" discipline it is held to be, mathematics is biased culturally, socially, and by gender (Davis, 1996). As Davis (1996) described:

> Valerie Walkerdine articulates a feminist critique of the preeminent place assigned to mathematics within the modern curriculum-a practice which she regards at the foundation of the "bourgeois and patriarchal rule of science, it is indeed inscribed with domination." Social ecologist Murray Bookchin comments on the devastating cultural consequences of our privileging of an ostensibly neutral "conventional reason" modeled after mathematical thought. (p. 88)

<u>Post-Formalism</u>, Davis' fifth mentality emphasizes the recursive nature of mathematics in which our mathematical experiences are very much a part of our mathematical knowledge as an unfolding and continuous process. The components of the mentality include nature, dynamics, chaos, recursion, enactivism, change, relationships, creativity, and community. Consistent with the organicism of Whitehead (1929) and Dewey, mathematics from a post-formalist perspective is a more general science, and number, quantity, and space are all interconnected. Mathematics, from a post-formalist way of being in the world, then becomes dynamic, emergent, and recursive (Doll, 1993). The teaching of mathematics in schools might, from a post-formalist perspective, emphasize students being in the world in mathematical ways, experiencing mathematics as a meaningful way of understanding themselves in relationship to their world (Fleener, 2002). The focus of this study and the guiding questions of the inquiry will be consistent with a post-formalist mentality of mathematics. School mandates and social expectations of mathematics instruction, however, are typically driven by a hyper-formalist mentality.

The National Council for Teachers of Mathematics (NCTM) has entered into the conversation about providing more meaningful experiences for students in mathematics without adequately challenging the underlying assumptions of goals or standards that drive the curriculum. Beginning in 1920, NCTM has undergone several phases of revamping to meet the needs of our ever changing society. The most recent set of changes occurred in 2000 which involved a uniform set of Principles and Standards that applies to all grade levels, pre-K through 12. As taken from NCTM (2000), below are the Principles:

1. Equity. Excellence in mathematics education requires equity—high expectations and strong support for all students.
2. Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
3. Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
4. Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
5. Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
6. Technology. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

As stated by NCTM (2000), “Each Principle is discussed separately, but the power of these Principles as guides and tools for decision making derives from their interaction in the thinking of educators. The Principles will come fully alive as they are used together to develop high-quality school mathematics programs” (p. 12).

The NCTM Standards (2000) are divided into two sections:

Content Standards
1. Number and Operations
2. Algebra
3. Geometry
4. Measurement
5. Data Analysis & Probability

Process Standards
6. Problem Solving
7. Reasoning & Proof
8. Communication
9. Connections
10. Representation

NCTM stresses the importance of not just the content, but the process that goes with the content. In other words, it is not enough to just repeat geometric terms and their definitions, it is equally as important to problem solve, for example, with perhaps those terms as part of the process. While the vision of the NCTM Standards may have been born of a hyper-formalist need, they can serve as a useful frame for our post-formalist conversations about mathematics teaching and learning as important for students' being with mathematics. In fact, I would claim that it is only from a post-formalist perspective that the NCTM framework makes sense. With these three ideas in mind, the following chapter will explore what it means to be in the world with mathematics from a post-formalist perspective.

CHAPTER II

RELATED LITERATURE

When initially confronted with my interests in mathematics education, I harbored a variety of assumptions. These assumptions ranged anywhere from the importance of memorization to the thought that all students need to learn the same "math." When initially asked in a graduate level course, "What is mathematics?" I quickly jotted down a few sentences. I was bored with the question because I thought everyone would have the same response as me. Interestingly enough, some of my colleagues who were a little further along in their programs challenged my assumptions of mathematics consisting of only content. Their perceptions of mathematics included a more relational and integrated explanation. Many used words such as: manipulating, finding, discovering, and exploring. For me, mathematics was this separate thing that consisted of the content of subject-area studies in geometry, algebra and calculus. There was little room in my previous understandings of mathematics learning for discovery or meaning – mathematics was performance and doing. I was fortunate, however, as I was one who mostly made A's in mathematics. I gave little thought to expanding my ideas about mathematics. Some of my colleagues who expressed such dynamic responses had a lot of negative mathematical experiences. It occurred to me that I had simply been successful at playing the traditional mathematics game, that I had not really explored my perceptions and experiences of mathematics.

In this chapter, I will review related literature in mathematics anxiety. I will also review literature on how students come to know mathematics, which includes students' experiences and understandings of mathematics.

NOTIONS OF PERCEPTION AND EXPERIENCE

How have mathematics education researchers explored the idea of perceptions and experiences of mathematics? There are insights to be gained both in terms of how one studies "intangibles" such as perceptions and feelings about mathematics and how hyper-formalist perspectives create an inside/outside dualism with respect to the meaningful study of these phenomena. One such example is that of mathematics anxiety.

Mathematics Anxiety Research

According to Newstead (1998) mathematics anxiety was first defined in the early 1970s by Richardson and Suin who felt "it was feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 54). However, other literature (Kaiser & Rogers, 1995) suggests this term was coined by Sheila Tobias in 1978 to describe "a psychological fear or anxiety associated with engaging in mathematical activity" (p. 6). Studies and literature that focus on mathematics anxiety tend to give credit to Richardson and Suin's work if gender is *not* included in their research while Tobias' receives the distinction of the term if gender *is* included. Some literature however, does address both Richardson and Suin *and* Tobias. Regardless of who deserves the credit, some researchers do address the "pre-mathematics anxiety" work of attitudes toward mathematics, done in 1969 by Neale, as the beginning of this look into mathematics anxiety (Ma, 2003).

This beginning has since led to a variety of quantitative studies with various measurement techniques and some qualitative studies indicating some themes. The very definition has been expanded upon and emerging components of mathematics anxiety have been and are currently being studied.

The two terms of "math anxiety" and "math avoidance" in the work of many researchers (Kelly & Tomhave, 1985; Hembree, 1990; Sloan, Daane, & Giesen; 2002) tend to refer to math avoidance as something people *do* and math anxiety as more of a condition that people are afflicted *with* causing them to avoid math. Most of the research found treats these two terms in this cause and effect manner. The anxiety label tends to indicate a "medical" affliction of some type which encourages people to think there is some sort of a "treatment" out there for the "sufferer". "The bulk of the studies on math anxiety have been conducted on students in the United States and have emphasized its debilitating effect on mathematics performance" (Ho et al., 2000, p. 364).

Math avoidance can also take on a different, less cause and effect characteristic among feminist discussions because it is a less psychopathologizing term. In Rogers and Kaiser's 1995 work, for example, they use the terms in the following way:

> The early work of Tobias (1978)…espouses the goal of getting girls to choose mathematics and to persist in it, by focusing on an attitude of the individual student, namely mathematics anxiety, as the source of her mathematics avoidance, by assuming that this attitude is learned and therefore changeable, and by acting to help students unlearn it. (p. 157)

In terms of quantitative research, an important component to discuss is the MARS which is the Math Anxiety Rating Scale. First developed in 1972 by Richardson and Suinn, this Likert-type scale has the participant decide on the degree of anxiety aroused, using the dimensions of "not at all," "a little," "a fair amount," "much," or "very much"

(Vinson, 2001). Consisting of 98 items and considered a self rating scale, the questions are scenario based and the individual rates their anxiety reactions to these scenarios (Kelly & Tomhave, 1985). This scale is used in varying levels as a single entity or in other ways like a pre and post test along with interviews based on the results.

With 'math anxiety as a construct' being criticized as unidimensional (Newstead, 1998), two researchers in gender and mathematics, Fennema and Sherman, developed a Mathematics Attitude Scale (MAS) which consisted of nine scales measuring constructs related to the cognitive performance of females (Forgasz & Leder, 1999). One of these scales is a mathematics anxiety scale that tends to ask questions more relational and contextual to that typically associate with females' ways of knowing (Fennema & Leder, 1990).

Newstead (1998) looked at math anxiety among children in traditional classrooms and those in alternative classrooms. Traditional approaches mean students are taught standard, pencil-and-paper methods of computation, by teacher demonstration followed by individual practice (Newstead, 1998). Alternative approaches indicate that pupils use and discuss their own strategies for solving word sums, which are used as the principal learning vehicle. Solving non-routine problems and discussing strategies in small groups are of primary importance (Newstead, 1998). These two approaches are believed to have strong implications on whether a student possesses or may develop mathematics anxiety. It is believed the traditional approach encourages students to separate one's self from the mathematics by focusing on the answers and procedures which can lead to mathematics anxiety. Data (e.g., Newstead, 1998) indicates mathematics anxiety can develop at a very early age for students and that high levels of it can impair performance.

Alternative approaches stray away from rote memorization, timed tests and repetitive procedures. These approaches encourage social interaction and communication about mathematics and mathematical thinking (Newstead, 1998). Such a support system may help to reduce anxiety and encourage social norms which enable pupils to express their ideas without risk of embarrassment or humiliation (Newstead, 1998). Preservice teachers who are exposed to alternative teaching approaches (e.g., Vinson, 2001) indicate a dramatic decrease in mathematics anxiety

A consistent theme seen in most of the mathematics anxiety research is that the majority rather than the minority of students are struggling with modern mathematics. With that being said, although anxiety research may be valuable at understanding how students do not interact in meaningful ways with mathematics, the research doesn't help us understand how students can be in the world in mathematical ways. Nor do the studies have clear implications for instruction with regard to encouraging students to be in the world mathematically.

One approach is to take an enactivist perspective on how we might study students' being in the world mathematically and how we might create a post-formalist environment that supports their growth in being with mathematics.

Studies Exploring Students Coming-to-Know Mathematics

The work of Wheatley, Reynolds, Pirie, and Kieren, among others, lie in not what a student knows, meaning the outside perspective of knowing, but rather on the "coming to know" or the dynamic "knowing" that portrays the growth of understandings (Davis, 1994). Coming to know may involve perceptions, understandings, communications, experiences, and perhaps seeing patterns, forming relationships, and making connections (Wheatley and Reynolds, 1999).

Perceptions about mathematical ability is placed on students as individual learners which often times causes them to feel inferior to those who possess mathematical knowing, placing "other" in a position of privilege. This "us and them" dualism is instilled at such an early age and still remains steadfast in them today. Looking at mathematics mindlessly, mechanically, and abstractly, without personal meaning, rather than mindfully (Langer, 1986) as connected to self and relational, is what most of our students have been used to.

"Coming to Know Number" (Wheatley and Reynolds, 1999) exposes students and teachers to problem-centered learning and discusses the thought processes of how students think about mathematics. Just being able to see how a student adds using his or her *own* algorithm has a profound impact. Through this approach, students are encouraged to discuss an open-ended problem posed to them with other persons in their group, work on various ideas, and present their findings in a very non-traditional way. Many times the same answer to a problem is presented in a variety of ways and students find themselves spending most of their time discussing their thought processes rather than the final answer. They suddenly realize they are no less privileged than a mathematician in the way they approach a problem. The experiences they bring to the question posed to them are very important and group members and the instructor are encouraged to nurture these experiences. This type of enactivism (Davis, 1996) allows for students to revisit problems, revisit ideas, and revise projects they are working on (I will expound more on enactivism in the next section).

Particular studies done by Wheatley (1990, 1999) and Wheatley and Reynolds (1994, 1996, 1997, 1999) focused on problem centered learning environments in which number sense and spatial sense were developed. Within number sense and spatial sense may lie reflection, symbolization, and meaning making. Number sense may include number relationships and forming ten as an abstract unit. One example is a student who figures out that adding can be done in tens rather than adding from the ones column and then carrying to the tens column. When asked how a student added 19 + 11, the student may reply, "I changed the 19 to a 20 and the 11 to a 10 and added them to get 30."

Spatial sense may best be defined as "an intuitive feel for one's surroundings and the objects in them" (Yackel & Wheatley, 1990, p. 52). One activity offered by Wheatley (1999) is Quickdraw in which the student is asked to draw what s/he remembered seeing after being shown a geometric image for a few seconds. This helps in developing the student's spatial sense and also helps students realize the variety of ways mathematics can be approached (Reynolds & Wheatley, 1999).

Wheatley and Reynolds have had a direct impact on classrooms because their work essentially connects "theory to practice" as viewed by many educators. Although this type of theory versus practice thinking may still be prevalent among many classroom teachers, the work of Wheatley and Reynolds has perturbed how teachers and students think about mathematics.

Problematizing Inside/Outside Dualisms in Hyper-Formalist Settings

Looking at the work of Wheatley and Reynolds, just as with the NCTM Standards, it is not obvious that a different way of thinking about mathematics is required. The conundrum of much of educational research in mathematics education is insights that mathematics needs to be more relational and meaningful to students are met with approaches to teaching and learning that emphasize mathematics in a hyper-formalist sense. Just as with the research on mathematics anxiety, as differences in meaning and usefulness of mathematics are explored, inherent dualisms are created. Women versus men. Science versus humanities. Drill-and-kill versus constructivism. Knowing versus meaning.

How do we get beyond our hyper-formalist perspectives? How might we consider mathematics, mathematics teaching and learning, and mathematics classrooms from a post-formalist perspective? How do we engage in research practices of post-formalist mathematics without creating the dualisms that seem inherent to the hyper-formalist perspective?

Enactivism, also noted by Davis' (1994) work among others (Maturana and Varela, 1980, 1992; Varela, Thompson and Rosch, 1991; Sumara, 1994) can best be described as a position where cognition is seen in terms of action in, as living in a world of significance with others (Kieren et al., 1995). Enactivism implies that interpreting mathematical thinking, for example, involves the problem, the solution, the learner, the teacher, the situation, the structure, and the interaction all in relationship with one another, thus moving beyond dualisms.

Enactivist mathematics education research, as with other types of enactivist research, can be thought of as situational in nature. In other words, the researcher demonstrates that notions of perceptions and experiences are formed based off of an understanding between the varieties of relationships in any given situation. These relationships may or may not open up spaces for new ways of thinking but the fundamental belief in enactivist research is the attempt to do so. The following section elaborates on this attempt to open up spaces by describing enactivist approaches from the perspectives of mathematics education researchers.

Exploring Students Being-With Mathematics

Recall that Brent Davis' (1996) post-formalist mentality includes nature, dynamics, chaos, recursion, enactivism, change, relationships, creativity, and community. It also has a recursive nature in which our mathematical experiences are very much a part of our mathematical knowledge. Being-with mathematics involves all of these components in some way or another. In this next discussion, I will present a model that enables us to explore the phenomenon of students' being-with mathematics.

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding

What is meant by perceiving and experiencing mathematics? What is involved in perception and experience? Perception is defined as the process, act or faculty of becoming aware of directly through any of the senses, especially to see or hear. The process of taking notice of; observe; detect (Morris, 1975). Experience is defined as the apprehension of an object, thought, or emotion through the senses or mind (Morris, 1975). It can also be defined as active participation in events or activities, leading to the accumulation of knowledge or skill (Morris, 1975). Whether one agrees or disagrees with the two dictionary definitions I have listed, their initial meaning is a way to begin the conversation.

The work of Kieren and Pirie (1994, 1995) uses a more general term that encompasses both perception and experience: understanding. They developed the Pirie-Kieren model for the dynamical growth of mathematical understanding (Pirie & Kieren, 1994). Based on the notion that understanding is central to the learner, other factors such as the classroom, peers, teachers and the interactions of all of these can be seen as integrated into this understanding.

Consisting of circles which include eight parts, the model is designed to assist in the coming to know of students' thinking (See figure 1). "Under its use understanding is viewed not as a state but as a dynamical, non-linear, non-monotonic process involving eight unfolding and enfolded modes of mathematical activity" (Kieren, 1995). The most inner circle, Primitive Knowing, is the beginning of the process of coming to understand or the starting place for the growth of any particular mathematical understanding (Pirie & Kieren, 1994). An example used by Pirie and Kieren (1994) is with the concept of fractions and how a student might bring to the lesson their knowledge of fractional words, and types of fractional reasoning.

Image Making occurs when the learner makes distinctions in her/his previous experiences and uses it in different or new ways (Piere & Kieren, 1994). For example, as stated by Piere and Kieren (1994), students may use previous part-part-whole knowing to combine fractional quantities into other such quantities.

Image Having involves the idea of mentally constructing ideas without having to act on these ideas in a manner which requires the manipulation of an object or problem. Subtracting or adding fractions mentally would be an example of Image Having.

Property Noticing is the culmination of Imagings which allow the learner to notice properties about the phenomenon being studied. As stated by Pirie and Kieren (1994):

> [U]sing her image of addition as finding subparts which fit and her idea of equivalent fractions to generate a means of performing addition. (p. 170)

Primitive Knowing, Image Making, Image Having, and Property Noticing are considered less formal and more local modes of mathematics activity. While, Formalizing, Observing, and Structuring, are more formal, sophisticated, and abstract modes (Kieren, Calvert, Reid, & Simmt, 1995).

Formalizing includes abstracting some of the properties noticed by using or stating a method about the idea being discussed. As an example, a student may "see that addition is something that can be done using only the number concepts and symbols related to fractions" (Pirie & Kieren, 1994, p. 171).

Observing involves the idea of reflection and taking various experiences within the activity and creating patterns or seeing relationships. Seeing fractional patterns is a good example of this because it aides in the Formalizing component. Observing and Formalizing are difficult to differentiate because each work so closely with one another.

Structuring takes the reflecting and theorizes about it. Because of the patterns found, a person may form a theory about their findings.

The final, Inventizing, indicates a possibility for a person to develop new diverging mathematical ideas (Kieren, Reid, & Pirie, 1995).

It is important to note that the layered circles in the model indicate perceiving and experiencing mathematics are not a linear process and students may dance between the various circles, create a unique pathway among them, and/or have disjointed jumps from an outside circle back to an inner one. As stated by Martin, Towers, & Pirie (2000):

> Growth occurs through a continual movement back and forth through the levels of knowing, as the individual reflects on and reconstructs his or her current and previous knowledge. When faced with a problem at any level that is not immediately solvable, an individual will need to return to an inner layer of understanding. This shift to working at an inner layer of understanding actions is termed folding back and enables the learner to make use of current outer layer knowing to inform inner understanding acts, which in turn enable further outer layer understanding. (p. 226)

"A person folding back, say from formalizing to image making or image having, brings with her or him both the methods and results from the more formal level which they now use for different less formal intents" (Kieren, et. al. 1995, p. 21).

The phenomenon of folding back within and between Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Observing, Structuring, and Inventizing provides a way for the enactivist researcher to problematize mathematical understandings. It could be argued that within enactivism's phenomenological roots lies a vast complexity of being-with mathematics. It could also be argued that looking for classroom settings which encourage these complexities provides even more richness to the conversation.

Settings that Encourage Being-With Mathematics: Post-Formalist Classrooms

"Coemergence; Four Enactive Portraits of Mathematical Activity" was a study that included two undergraduate students named Stacey and Kerry which utilized the Pirie-Kieren model. True to form, this was a dynamic study in the sense that an entire research team was used to work with these students. Consisting of four articles, Kieren headed the project along with three other colleagues and the emerging themes were centered on enactivism and coemergence. In terms of mathematical cognition, it is much easier for many to think of givens and uniformity, such as prescribed formulas, than think of the coming together of a variety of ideas. The study emphasized enactivism as not an either/or, but rather a coemergence of ideas.

Coemergence refers to a combining of perceptions, understandings, and experiences. The most notable example lies with two students in the study, Stacey and Kerry, who both had very different views about mathematics. Kerry would define it as rule based with logical patterns that everyone should see in order to find the most efficient route for solving. Stacey, on the other hand, was more of an explorer who felt mathematics had discoveries embedded in it just waiting to surface. The two worked on a series of problems together and each was a huge help to the other. From reading the dialogue between the two students, it is apparent why the NCTM (2000) standards are designed to meet the needs of both content and process. During their investigations, Kerry tended to bring prior experiences with content such as formulas to the table, while Stacey tended to play off the content by thinking of a new route to approach the problem. When being interviewed about their mathematical philosophies, each complimented one another's approaches and noted how helpful they were.

In one of the final articles in the study, Kieren looked at Kerry and Stacey using the Pirie-Kieren model for the dynamical growth of mathematical understanding as an interpretive framework (see figure 2). The figure itself indicated ebb and flow between various notions that seemed to take a unique path and indicated folding back among the participants. By looking at the model and reading the conversations between Stacey and

Kerry, the connections between the words they were speaking and the lines on the figure definitely took on a dynamic pattern all its own. For example, Kieren, et. al. (1995), listed some of the conversation in his study:

> Finally, Kerry began to speak, "I guess this is uh, --uh,-:" "Trial and error", interrupted Stacey.
>
> "No. We'll go- Let's assign variables to these then. Do you want-" "Go for it!" encouraged Stacey.
>
> "We'll start with A, B, and C"…"And I guess we'll get a system of equations out of that. And we'll try and- And we'll subtract one equation from another and try to deduce one variable and plug it back in. OK, so that's the sum of A and B."
>
> "Yup." Stacey helped as Kerry started writing equations. (p. 23)

In relation to figure 2, this small portion of the conversation can be connected with 1, 5 and 6. Not all of the dialogue was included in the study, but the part above gives examples of how the model traces the students' experiences.

Participating in a different study by Kieren, et al. (1995), Kerry and Stacey were posed with a problem involving the Fibonacci sequence. The first setting involved specific tasks concerning the problem and the second setting, 15 months later, involved the same problem but only one general prompt about the problem. The model for dynamical growth was also used in this study and the first session demonstrated "disjoint jumps rather than as a more continuous pathway" (Kieren, et al., 1995, p. 4). The second session produced a more similar model to the one in the previous study where more of a dynamic pathway was developed. As taken from the article (Kieren, et. al., 1995), Kerry and Stacey saw a difference when asked about the experience:

Kerry: Yeah. Cause we- When we walked away from it [setting one] neither one of us felt we'd really climbed a mountain or conquered anything.

Stacey: No.

Kerry: But now, I'm quite happy with it now. The Fibonacci sequence is allowed back in my life.

Stacey: Yeah [laughter]. (p. 8)

From reading the conversations between Stacey and Kerry, the notion of communication is in dynamic process with the Pirie-Kieren model. Sfard (2000) defines communication as "an activity in which one is trying to make his or her interlocutor act or feel in a certain way" (p. 300). She (2000) clarifies her stance by saying:

> First, the action intended by the speaker need not be overt or public. For my purpose, paying attention, thinking, and attempting to remember are also forms of action. Second, I wish to emphasize that a speaker's intention is usually quite general; that is, there is more than one (re)action that senders should consider to be matches for their intentions. (p. 301)

In this communicative process, Sfard (2000) adds:

> Finally, the student who was asked to solve a mathematical problem may propose many different solutions, all of them discursively appropriate even if mathematically faulty. (p. 301)

Mathematical perception and experience are part and parcel to mathematical understanding and mathematical communication. All of these lie on a continuum unique to the individual.

Davis' enactivism and Kieren and Pirie's techniques to explore students' experiences with mathematics both support the vision of the post-formalist classroom and provide new ways of thinking about mathematics education research.

CHAPTER III

METHODOLOGY

Research Perspective

Experience and perception are at the heart of the type of research I am interested in at this point in my work. There are varying definitions of what it means to experience something depending on the interpretative framework being used. Based on my understanding of post-formalist mathematics, enactivism, and approaches to exploring connectedness of experiences and knowing, I have chosen a phenomenological approach.

Phenomenology is an approach which is very much grounded in bodily experience. There are different types of phenomenology such as transcendental phenomenology as introduced by Edmund Husserl (1859-1938), hermeneutical phenomenology as explored by Martin Heidegger (1889-1976), and existential phenomenology developed by Maurice Merleau-Ponty (1908-1961) (Schwandt, 2001). My research can best be described as falling under the existential phenomenology frame.

Existentialism is a category of ontological thought, or thought about the conditions of being. Phenomenology is an epistemological category, designating a primary concern with the logic and meaning of events rather than that of structures. Thus, Existential Phenomenology, while difficult to define, may be thought of as "oriented more toward describing the experience of everyday life as it is internalized in the subjective consciousness of individuals" (Schwandt, 2001, p. 192). This combining of ontology and epistemology fits well with the post-formalist mentality (Davis, 1996), and the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding (Pierie & Kieren, 1994). One reason for this is because a key existential question in the

Pirie-Kieren model asks, "What in the nature of human beings allows them to understand mathematics?" (Kieren, 1995, p. 20) Another reason existential phenomenology seems to fit the kind of research I will be doing is expressed by Davis (1996) as he describes the five mentalities. The journey between each mentality leads us through ontology to epistemology and finally to a combination of both in post-formalist thought. Post-formalism essentially serves as a vision for mathematics curriculum reflecting a more connected, integrated way of being in the world with mathematics. Components of Pirie and Kieren's work provide a useful approach to engage and explore interactive and recursive characteristics of one's participation with mathematics as a meaningful activity. Using the post-formalist framework and Pirie and Kieren's work for exploring students' phenomenological experiences with mathematics, I explore the following questions: What are students' perceptions about their experiences with mathematics? How can students' perceptions about and experiences with mathematics inform their understandings of mathematics?

Context

During the process of choosing the type of environment I wanted to be in for my research, one of my committee members stressed that it needed to be in a place where I had strong connections. For me, these connections meant having an existing relationship with the students I was observing. Therefore, I chose to work with students taking the third mathematics methods course. Students in this course had me for their previous course. All of the students in this course were in their final semester of coursework and field experiences prior to their student teaching internship. Field experience at the

University of Oklahoma means the students spend a minimum of ten hours per week in a school observing and/or teaching lessons for five weeks.

The instructor's theoretical background lied in constructivism and she and I had many of our classes together and knew each other well. She agreed to allow me to observe in her class for an entire semester and told her students I would be there to make observations and possibly ask them for an interview. All but one of the students knew me because they had me for a previous class. Students were told that my presence did not affect their grade and that I was there to see how they investigated mathematical ideas. They were also told that not participating in interviews did not affect their grade and that the confidentiality of the individuals' identity was maintained through the use of pseudonyms in my dissertation (see Appendix G).

Student Participants

Twenty-five participants were involved in the study in some way. These ways involved participating in interviews and/or being involved in the weekly occurrences of the classroom situation. Eighteen students in particular were interviewed. The rest were unable to participate in the interview process due to their time constraints. Not all eighteen students were included in the findings because thirteen in particular seemed to be more reflective and willing to articulate their experiences of being with mathematics.

Twenty-four of the participants were female and one was male and twenty-four of them were traditional, while one was non-traditional. The non-traditional student had received a previous bachelor's degree in the engineering field and was also from another country besides the United States.

Data Collection

During the semester, I observed in the class by taking field notes, collected student artifacts from assignments, and engaged in personal interactions. I conducted various interviews with groups and/or pairs of students about their mathematical beliefs. Each interview was taped using an audio tape recorder and took around 30 minutes. Upon the completion of transcription, tapes were destroyed. Transcripts were kept in a locked drawer in my office and destroyed after the study was completed.

Students were chosen based on my observations and interactions. During some class periods, the instructor for the class had students do problem solving tasks while I conducted observations. During other class periods, students presented completed assignments assigned by the instructor in which I conducted observations and gathered artifacts. Assignments for the class are listed in the syllabus in appendix B. I interviewed students about the various activities they experienced in the class, focused on any activities which seemed to reflect, from my standpoint, their being with mathematics. I only interviewed students in pairs or groups because I felt listening to conversations between students was vital to this study.

In addition to the conversational type of interview I just described, I also asked a series of questions (see appendix A), which demonstrated a semi-formal type of interview. For example, I asked the students, "What is Mathematics?" The reason for the more formal questioning was so I was able to engage in a broader view about their mathematical perceptions and experiences.

Data Analysis

How specifically did I analyze the data? One way to address this question is by using a qualitative technique called coding. "Coding is a procedure that disaggregates the data, breaks it down into manageable segments, and identifies or names those segments" (Schwandt, 2001, p. 26). The codes I assigned were based on emerging themes in my data. Either during or after my observations of a problem solving activity, student presentation, or any other classroom interaction, I coded each occurrence. The codes I developed will be discussed in the next chapter.

Over all, the data collection and data analysis of my work included conversations, reflections, the unfolding of students' ideas, and the context of their being with mathematics, all from the perspectives of problem solving and/or any other class activities or assignments.

In the following chapters, I will focus on both pairs of students and groups of students, depending on the types of observations that emerged from my data.

CHAPTER IV

AN ELEMENTARY MATHEMATICS METHODS CLASS

"..In every event of learning or self-transformation, the learner is always and already entangled in a complex web of relationality." – Brent Davis, Dennis Sumara, & Rebecca Luce-Kapler

As discussed in chapter two, perception and experience can be thought of in a variety of ways. Davis, Sumar, and Luce-Kaplar (2000) note:

> Perception is not about channeling information into the brain; rather, perception is more a matter of expectation and past experience…perception has to do with relationships - or more precisely, with expected relationships. (p. 4)

Therefore, as I explore the mathematical perceptions of and experiences among students in this chapter, I will be operating from the central theme of relationship, which I believe best represents perception and experience. To gain a better understanding of why I have arrived at relationship as my central theme, my mathematical subjectivity statement in appendix C may shed some light as to why relationship is such an important idea in my being with mathematics.

The connections among relationship, perception and experience is inherent in post-formalist approaches to mathematics education (Davis, 1997). Pirie and Kieren (1994) operate under the notion that mathematics learning is a recursive process as students constantly revisit ideas and re-form new ones. This study will draw on Pirie and Kieren's ideas of folding back and non-linear approaches to mathematical understandings from a post-formalist perspective which focuses on relationship. The central purpose of my data analyses is to establish the types of relationships students have with mathematics in the context of an elementary mathematics methods class for pre-service elementary education majors. Through observations and interviews, the following research questions will be explored:

1. What are students' perceptions about their experiences with mathematics?
2. How can students' perceptions about and experiences with mathematics inform their understandings of mathematics?

The following section will discuss the processes by which the themes occurred.

Themes of Experience and Relationship

As stated in an earlier section, it only makes sense to discuss the NCTM Principles and Standards (2000) from a post-formalist perspective. These principles and standards are at the heart of the methods courses taken by the students in this study. Remember they include: number and operations, algebra, geometry and measurement, data analysis and probability, problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000). The particular problem-centered environment I chose to observe involved much more than just *one* of these NCTM components which we see so often in public schooling. The elementary education majors in this study, in my view, were encouraged to be with their mathematics not only through problem solving, for example, but through historical presentations, newsletter assignments, and reflections based off their own field experiences incorporating and intertwining the full range of principles and standards. For me, it made sense to look at the classroom I observed in as a whole rather than studying one particular component of the mathematics curriculum or of the professional standards for becoming a teacher of mathematics.

Students in this class were all majoring in elementary education and were in their final semester of field experiences prior to the student-teaching internship. Students spent the first eight weeks attending class, three weeks observing and teaching in a school, then four weeks back in class. The methods class can be characterized as being very collaborative. Because of the collaborations and opportunities for conversations about mathematics and reflections on mathematics teaching and learning, the class was conducive to interviews and in-class observations. Appendix B includes a complete syllabus of the class which lists in detail each assignment and expectations of the course which reflect an emphasis on collaboration, communication, and reflection.

Data were gathered over a 12 week time period and came from interviews and observations. The interviews were designed to be conversational between myself and the interviewees and among the interviewees themselves. I probed students about various group activities and had conversations with them about their general mathematical ideas which included both their understandings of and experiences with mathematics and the way mathematics, in their opinion, should be taught. In addition to interview data, I also collected field notes based on my observations of various activities during the class and artifacts pertaining to the activities. The artifacts included such items as the newsletter assignment (see appendix E), their class notes, field notes and their historical presentations (see appendix F).

I interviewed students in groups of two or three after class in the same room where class met because I wanted to make sure students could point to various objects and/or prompts in the room to help them describe their experiences. We (the three or four of us) sat at a hexagonal shaped table as we discussed their experiences. All interviews were audio-taped for later review (see appendix G for copies of student informed consent). Guiding questions explored how groups came to know their mathematics and probed their individual experiences with various tasks as related to previous mathematics experiences (see Appendix A).

Upon completion of the interviews, I transcribed all conversations, and looked for emerging themes. With "relationship" as an overarching idea in my data analysis, I chose to use the *types* of relationships I found as my sub-themes which were directly chosen from students' responses and my own observations. Students' responses were generated from questions like "How did your group approach this activity?", "What do you want for your future students of mathematics?", and "What were your previous mathematical experiences?" Based on the types of questions I have just noted and other observations I made about class activities and assignments, student responses and actions generated the following sub-themes: **folding back**, **language**, **relevance**, **and open-endedness**.

In the next sections I will discuss the relationships I saw building throughout the semester, followed by explanations of each relationship which may include excerpts of taped transcriptions, observations from my field notes, and student artifacts such as journals and assignments. I have separated the themes for the sake of discussion and listed examples of student experiences that I feel are the most representative of each theme. However, the themes which emerged from my perspective are deeply intertwined with one another.

Folding Back

As a matter of practice in the classroom in which I observed, students were offered multiple opportunities for and freedom to problem solve, explore, connect, create, and communicate in ways that were meaningful to them. None of the groups was caught up in getting the precise answer to a problem or the precise way to explore an idea but rather identifying and pursuing processes that worked for them. Relating these creative spaces with one another and with the class was an essential aspect of the unfolding and dynamics of classroom conversations and practices. As described by Pirie and Kieren (1994) with individual students and small student dyads, the class itself was continually unfolding, folding back, and recursing through process, discourse and reflection.

The emergence of experiences which involved **folding back** (Pierie and Kieren, 1994) was especially apparent as student groups explored problems posed during class by the instructor. One such example of a "folding back experience" was when students were asked to find the area of the classroom. The context of the activity is detailed in Appendix D.

During their description of the area activity, many students folded back (Pirie & Kieren, 1994) to their previous experiences with area by discussing a similar activity in another class or mentioned the formula of length times width. Ashley and Dima were two traditional female students who frequently worked together during classroom activities. As noted in the following conversation, they were good examples of the folding back relationship:

> Ashley: We had done something similar in another class like filling the room up with ping pong balls. We were supposed to see how many we could fill the room up with, so we had to get the size of the room. So, it was kind of similar to that but um, we started out just taking sections of the room. Like this section of the

door over, we like sectioned the room out and we took all the measurements we had and added them all together to get the total square feet.

Kerri: What kind of previous knowledge did you bring into the lesson?

Dima: We knew the formula was length times width for area, so that's our knowledge. I think from experience I had seen a video and the kids get up and use a ruler to count the squares and I guess that's a prior knowledge. But instead of getting a ruler we saw tiles and said hey let's count the tiles instead of getting a ruler because it can't fit one wall to the other side. So we decided to count the tiles then measure the tiles and then do that.

Ashley: I guess most of mine was from the other math class we had to take when we had to fill the room with ping pong balls. But like we had to measure the perimeter of the room so I knew that we to take out like for the door. Like that square had to be taken out and that triangle had to be taken out. So I guess that would be more of the prior knowledge I had because I had done a similar activity. But we had just counted the perimeter so we didn't do the whole area of the room, at first it started out with just the perimeter.

Kerri: So when did you formalize what was going on? In other words, when did things start to come together?

Dima: What we did was we assigned each person to count the length and to count the width and then one person count the area of one square. And after that we got together and told each other our results. And then we drew a diagram and wrote down the 24 tiles and 27.5 tiles and I guess in the beginning not everyone understood what we were doing. I guess some people did but other people just kind of did what we asked them to do. And when we got together and discussed it that is when everyone actually understood what we were trying to get at. So we did that and we couldn't figure out that corner near the door. So we did that and we were like oh no there's something missing and we decided to go back and count again and subtract the tiles. So when you were saying how we organized it, we first found the formula. First we converted, let me see (looking at notes) first we found how many tiles there are then we found the area of the tiles, then we divided it by two. So 635 tiles times 144 inches squared and that's the area of one square and I forgot how we found out people. Oh that's how many tiles there are! 660 tiles minus 25 tiles then subtract that (pointing to the area outside of the door) and then we divided by 2 to get 317 people.

Kerri: Why did you divided by 2?

Dima: Because for the pencil example, the pencil was twice the size so you divide by two.

Ashley: One person per two tiles, right?

Dima: Yeah

Ashley: Because we had 655.64 square feet so we said it would be one person for the small pencil so it would be 655 and then for the 2 tiles it would be 327.

--Transcription from taped interview

Ashley and Dima folded back (Pirie & Kieren, 1994) in a variety of ways which was one type of relationship during their explorations of the classroom. Folding back (Pirie & Kieren, 1994) to a previous activity informed their experiences by giving them some type of a framework to begin with; a starting point for their conversations. Ashley and Dima were the kind of students who stayed after class to discuss ideas with the instructor and frequently problematized a variety of issues concerning mathematics education. They were both very concerned about their own experiences in the mathematics classroom and how those experiences will affect their future students.

John, the only male in the class, and the members of his group also had their own experiences of folding back:

> John: For us, I knew the tiles were 1 foot by 1 foot.
>
> Kerri: How did you know that?
>
> John: Common knowledge I guess. I think I've done something like this before like in high school or something. We always just used them as a basis. We just divided the room up into different sections. I think we had six sections. Like there are different shapes in the room, there are triangles and squares. So we divided the room up into individual triangles and squares and added them all up and that is how we found the entire area of the whole room.
>
> Kerri: Did your group talk about the ceiling?
>
> John: I kind of led the group. I didn't think the tiles on the ceiling were a good idea because you couldn't get up there and measure them and know the exact measurement. And then also because it would've been a lot harder to decide if the edges or the ones on the sides of the room were actually the right size.
>
> --Transcription from taped interview

While Ashley and Dima's experiences tended to unfold with disjointed jumps (Pirie & Kieren, 1994) which cycled back through to ideas they agreed on to reach a solution, John's description of his previous experiences was done so in a more linear, abstracted, non-relational fashion. John was a student who spoke fondly of his experiences with mathematics and often bragged about his successes in the traditional classroom setting. Each group, though, folded back (Pirie & Kieren, 1994) in ways that led their experiences.

Other students immediately got out of their seats and moved around the room while folding back (Pirie & Kieren, 1994) in ways related to their physical surroundings. One pair in particular, Diane and Lacy, stood out as physically engaging in classroom investigations. While less vocal, these two traditional students usually tended to keep to themselves and rarely ventured into the happenings of other groups. Some of their ideas are noted in the following conversation:

> Diane: The first thing that came to my mind is get up and explore. Okay the first thing is to find the area of the room by counting the tiles on the floor.
>
> Lacy: I had done it before so I automatically stood up. I started counting the tiles but then I remembered about the ceiling. Our group broke the room up to where the walls are indented in and we just counted the tiles.
>
> Diane: Yeah like you all did a certain part and me and Karen did a certain part. And then we brought them all together. We included the tiles against the wall.
>
> Diane: I thought it was interesting how we all counted different but we still came around the same answer. Cause like one group counted those, but we didn't we counted them as halves and broke them down into quarters. But we all still came up with the same answer. Like we took away the piece that was sticking out and they didn't but we still came up with the same answer.

Lacy: In Ellen's class we found how many ping pong balls it would take to fill up a room but I don't remember how I did that. I just remember thinking about ping pong balls.

--Transcription from taped interview

The beginning of Ashley, Dima and John's experiences, as described by them in the interviews, started with folding back via group conversations. Diane and Lacy's approach, however, involved bodily movement for the beginnings of their folding back, as was described in their interview. It is important to note, though, that each group approached the area question from part or all of the following perspectives: How objects and ideas related to one another, how those objects and ideas related to their previous experiences, how they related to physical features of the classroom, and the possible connections and patterns between all of those components. This was apparent with Deidre, Susan, and Fiona. Deidre and Susan often looked to Fiona for guidance as noted in the following dialogue:

Kerri: Where did you start with the area activity?

Deidre & Susan: The floor (laughing).

Kerri: So you started with the floor?

Deidre: *WE* started with the floor. Fiona went right to the ceiling.

Fiona: I never got out of the chair. I just sat here and looked at the relationship between the ceiling and the floor.

Kerri: Fiona, were you working with Deidre & Susan?

Deidre: Yes, that's why we went from the floor and then we looked at her and were like oh that's way easier!

Kerri: So why did you think Fiona's way was easier?

Susan: Visually

Deidre: Yeah, there's so many things in this room, you can easily see how many tiles there are on the ceiling. But on the floor, you know, you've got shelves…

Susan: Cabinets and desks

Deidre: Yeah, whatever this little wall is, I don't even know what you call it.

Susan: a divider?

Kerri: What was some of the previous knowledge you brought into this lesson?

Fiona: I was looking at relationships and you can see that 4 tiles makes almost the same size to one of the ceiling tiles. So we just averaged and calculated the value of each tile.

You multiply whatever you have in the tile by the amount on the floor.

--Transcription from taped interview

As they continued to describe the process of solving the problem, they addressed the next part of the question: How many people could fit in the room? Person-size itself was a problem for estimating both size of a person based on the dimensions of a pencil the person might hold and then how much space each person would occupy. As they described their approaches to the problem, they discussed their group conversational dynamics.

Kerri: What kinds of conversations did you all have between one another?

Susan: We argued for a while.

Deidre: We argued over how many people to put in each tile on the floor. Like some wanted one and some were like no, we need two tiles for each person.

Kerri: Tell me more about that argument.

Deidre: It was more of a comfort thing I think because a lot of the people in the group did not want to be smashed.

Susan: Yeah, cause I get claustrophobic. To be smashed up against a million other people would not have been comfortable and someone would've gotten punched in the face or something (laughing).

Deidre: Me and Sophia didn't care and they wanted the 2 tiles so we were like that's fine.

Susan: We figured out the answer for both of them..

Deidre: Yeah, we did end up doing both of them, too.

Fiona: Yeah in the room we were counting how to add or take off out of the column.

Kerri: How did you deal with the column and angle?

Fiona: Just kind of draw a sketch of how it looks. So we were discussing how we should take the whole rectangle and look at the complete numbers like in the complete drawing. And then look at the numbers in an incomplete one so how many are missing? So we just tried to cut out the triangle out of the ceiling and then we calculated everything.

Deidre: I just did that row, then that row, then there's two triangles so they make one square. And then with this pillar there's a fourth out of each one so that's a half gone.

Fiona: I was showing them the whole thing and taking pieces away and they finally figured it out.

Kerri: What was it like to have to explain it to the class?

Susan: It wasn't that bad actually because we were able to do it on the board. Everyone could see our method of doing it as we talked about it and did it on the board.

Deidre: Other groups had done their problem on the floor and we were like take 4 tiles on the floor and that will be one on the ceiling. So they could see the…

Fiona: Relationship.

Deidre: Yeah, the relationship.

--Transcription from taped interview

Fiona's experiences and perceptions particularly echoed the theme of folding back. Through the rhythm of classroom dynamics, she was placed in a leadership position in many activities and conversations. This was not surprising because of her first degree which was in engineering. Her previous experiences with mathematics as an engineering major and as an international student provided a unique background for how Fiona approached and expressed her mathematics. Her approach to experiencing mathematics relationally is noted in one of our conversations:

Kerri: Now you didn't go to school in the United States, you went to school in?

Fiona: In [South American country]. Like they were very strict and there was no time for going through things in many ways. So they would just tell it to you straight, one way. And I never liked it and I was bad at testing and all that kind of stuff because I couldn't write what I was thinking. The teacher was always wanting me to show what I was thinking but I could only tell her. So that was always frustrating and things changed when I got to high school. I had a teacher who encouraged me to explain things in many ways. So I got through high school and went through college doing engineering and that was also a big problem. I need to find a relationship between the math I am doing and the reality. When they would give us a bunch of formulas and stuff, I would find the answer but I did it my way. And they would say that is not the way I am teaching. So everything I did was wrong all the time. So even when I got my test grade back and it was bad I would have to take my test to the dean and have them grade it again so I could get a fair grade because they don't say what way you should use, they just say solve it. That's what I did. It was crazy!

--Transcription from taped interview

Fiona's perceptions about her experiences with mathematics reflected her need to fold back (Pirie & Kieren, 1994) from a variety of perspectives. This creation of multiple spaces for her thinking can inform her understandings by allowing for discourse about her mathematical perspectives and the mathematical perspectives of others.

Fiona, Deidre, and Susan (and sometimes the class as a whole) tended to relate and connect in ways led by Fiona's experiences. Deidre and Susan frequently expressed being able to experience problems in ways they hadn't thought of due to Fiona's guidance. As one student, Sophia, who sat in a different group noted, "She [Fiona] has a higher order of mathematical thinking." Having a mentor such as Fiona, informed their understandings of mathematics, allowing their own folding back through prompts in their thinking by Fiona.

In addition to the area problem, another example of the informing of mathematical understandings through Fiona's prompts was when working a problem of the week (see appendix H). Students had a variety of ways to solve the problem which were discussed in groups, worked on the board and presented to the class. One student said "I started guessing and checking and then working backwards" while another said "I worked it algebraically," both writing their solutions on the board. The majority of the class, though, expressed using the guess and check method. As noted in my field notes, Dima in particular expressed to others in her group that she was uncomfortable with her "guess and check" method because it seemed inefficient to her. Upon hearing Fiona's explanation, which again demonstrated Fiona's unique background in comparison to others in the class, Dima's experience with the problem of the day helped her see another way of working the problem even though she expressed some criticism of herself upon discussing the problem. As taken from my field notes during the problem of the week activity, Dima said, "Oh I see, you add 1.25 eight times and you get $10. I always second guess myself."

Although Dima seemed somewhat discouraged that *she* didn't find the most efficient method, it could be argued that Fiona's folding back experiences greatly added to the conversations and experiences of students like Dima.

A variety of different folding back experiences were discussed in each area investigation and problems of the day. Some used a more linear, abstracted, non-relational approach, while others had some disjointed jumps (Pirie & Kieren, 1994) in their conversations which eventually cycled back through to a justifiable solution. From my own observations on folding back experiences made during the students' explorations, I noticed a variety of approaches which included students arguing (as noted with Fiona, Deidre, and Susan), various calculations being made, students physically engaging in an investigation (as noted with Diane and Lacy), and most importantly, conversations about ideas and findings among the groups.

During the course of the semester, another strong theme emerged: **language**. As students argued about the patterns they found, connections they made, and their ways of folding back, they did so through language. The next section discusses student experiences with language.

Language

Throughout the semester, language itself allowed students to see mathematics differently and was noticeable in a variety of student experiences. I first came to know this theme during a class discussion about the multiplication of fractions. One student inquired as to why numbers became larger when multiplying whole numbers but smaller when multiplying fractions. Everyone in the class knew the procedure behind multiplying two fractions together which is to multiply the denominators and the numerators of each fraction. However, they had never experienced thinking about why this procedure was taught to them in their mathematics classes. One student went up to the board and explained that it was the language that had been a problem for her. She said, "Think of it as one-half *of a* fourth rather than one-half *times* one-fourth and you should see it better." Another student asked her to explain because she had always struggled with the concept herself. The student explaining quickly drew a circle on the board, divided it into fourths, and then divided it into eighths to illustrate her language and her answer. It was as if a light bulb turned on for much of the class and many tried working another problem in the same way and found that it made sense to them because they finally understood why they were taught that procedure.

From examples such as the fraction discussion, students came to understand their experiences with mathematics in their previous schooling as limited because it was presented in a way that was rote and routine. This particular discussion on fractions was not rote and routine because of the impact of the language. The language and the drawing interacted in a way that allowed the student(s) to see differently (the analysis and discussion chapter will discussing “seeing differently” in greater detail).

Another example of the language sub-theme was noticeable during the presentations of the newsletter assignment (see appendix B for requirements and appendix E for examples). The newsletter assignment was appropriate for students in this study (remember students were in their third methods course). Because of the dynamic interplay between their mathematical experiences *as students* and the assignment given to them to offer those experiences *as teachers*. This is not to say, however, that language emerged as a dualism between student versus teacher but rather as a dynamic relationship. In other words, language was reflective of the elementary education majors taking on both the student and teacher roles. Although it could be argued the newsletters themselves contained the sub-theme of language, it is even more important for this discussion to include the presentations of the newsletters and the relationships that emerged from those presentations.

The process standards which involve problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000) were a basis in each groups’ presentation. Presentations focused particularly on mathematics and how they felt it should be taught and learned in their classroom. In many presentations, their ideas expressed the longing to allow for exploration, pattern finding, connection making, and relationship forming through mathematical discourse. The usage of terms such as the ones I have just listed indicated a change in thinking about mathematics as being a more integrated and relational discipline. A student changing the way s/he thinks *and* speaks about mathematics is perhaps a step in a new direction.

Other language chosen by the groups to describe their classrooms included some of the following terms: reflection, communication and listening skills, reading, student-centered, problem-solving in a real world situation, students creating classroom rules, students constructing their own formulas, meaningful learning environments, activating prior knowledge, discussion, allowing children to voice their ideas, integration, goals, and number sense. Mathematics, in their future classrooms will not be about memorization and dualistic thinking, according to the newsletters.

Each of the elementary education majors expressed to the "parents" their constructivist philosophies. This included various projects they planned on doing for the upcoming year. Through the language of explaining, students in this study were able to articulate the way they saw the mathematics classroom and this language was also reflective of their own experiences. For example, one student said "we don't teach in the traditional way because we will work as a community of learners." When asked by the instructor of the class how they will work as a community of learners, math journaling was included as a part of their responses to her. Math journaling and working in groups were a common experience among all of the students in the class. Groups also discussed rules, classroom management, websites, classroom theme activities, expectations, how children learn, supplies needed, and assessment.

In connection with many of Katie and Deidre's previous experiences (as noted in interviews and field notes), they laid out their newsletter in such a way that nothing was hierarchical. The basics of the classroom were outlined on page one in a nonlinear fashion and the more specific components were on page two outlined in the same way. The newsletter invited parents to be a part of the classroom in a way that presented mathematics in a different type of language; the language of experiencing mathematics, the language of *being with* mathematics. My thoughts about Katie and Deidre's newsletter emerged during our interview about the area activity as noted in the following dialogue:

> Kerri: Since entering this math education program, how have your mathematical thought processes changed?
>
> Deidre: Yes, in that the way I've always *felt* I wanted to do it has been confirmed as a good way to do it. Like it just um, that people can do things how they want and its okay if it is in a manner that will get them the correct answer each time. So, like, when I add numbers, big numbers, I take a part of the whole and make an even number like a solid number and then add the rest and people never understood that, like "why would you do that, that seems so much more complicated than just adding down the lines" but I think this program has shown me that it doesn't matter how you do it just so long as it comes out right and it will makes sense to you and you'll get the right answer consistently.
>
> Katie: I'm one of those people who adds down once, that's how I do it, and trying to take my thought process and trying to group them into here's a whole number now let's add up the rest is more difficult to me. But I understand not everyone

thinks in that same way and while I need that, its almost like I need that structure in order to do it then there are people who can just look at it and break it down real quick and put it back together and have the answer in half the time it would take me to write everything out. So, knowing that, but at the same time, I am, I feel like we do a lot of theory but I don't know how to teach somebody a different way. It's like going down the lines is the only thing I understand and know because that is just what I do.

Kerri: So, with that in mind, how do you envision teaching mathematics?

Deidre: I'm struggling with that.

Kerri: Your newsletter reflected a very relational curriculum. You didn't even have a linear order in it. You had it in categories and you had a lot of different components to your classroom, but you didn't say "we are doing this first" everything related quite well.

Deidre: I think we think as a whole relational. I don't think we think in pieces, I think we think of the whole picture and you break it down as you go. I mean I don't know if you think that way necessarily.

Katie: Yeah, that's like, with the newsletter I do see all of the parts floating together. Like for me having the activities next to each other and things like that was kind of a way, an order to it. But as far as trying to come up with things like that kids can do where they can be creative in finding their stuff, like coming up with using George Tran's book, that was all hard, it was like "how about these math books" and so I tried to figure out a way to have the kids make it into an algebraic equation.

Deidre: We're a good fit! (laughing) We fit very well! (laughing)

--Transcription from taped interview

The reflective nature of Katie and Deidre's comments indicated they are still struggling with how their own experiences will surface in their future classrooms. However, one way Katie and Deidre seemed to be experiencing mathematics was through language because they always sat together and conversation was a large part of their ways of making sense of classroom experiences.

The sub-theme of language also emerged in a more personal sense not related to specific assignments in the class. As noted in the following conversation:

Kerri: Now tell me about your previous math experiences in your life.

Susan: I moved around a whole bunch when I was in grade school like every six months sometimes. When I got to one math class they were doing something totally different than what they had been doing at another school. So I pretty much taught myself the concepts we were learning but I did it differently. So I was always getting in trouble because I wasn't doing it the teacher's way because I had no background of what they had been doing. So up until middle school all the way through high school I was teaching myself how to do it. But it didn't help me because when I got to high school I didn't understand it any different way than how I could do it. Like when I took calculus the teacher explained it one way and I can't do that. I have to have at least three or four ways of doing something or else I feel like I just don't know it. Like I can do the work but I just don't know it, it is like plug and chug.

Kerri: What about college?

Susan: Ellen Johnson was my god send. When we did the problems of the week with partners and one person did it one way and I did the other, but they were both right, and she didn't care! Thank you!

Kerri: Fiona, what about you?

Fiona: I think my math was from my childhood because my dad was always playing with us. He was always asking how to do mental math all of the time. He was always getting us to think. When I was in school long ago, it was horrible.

Kerri: Tell me about yours.

Deidre: I would say it was all textbook like do the problems and whatever. Like I always got good grades and like I always got an A or B in math. Always. I think it was because I would memorize the formula but I can't even tell you half the stuff we did but I did have teachers who would let us partner up in middle and high school. So sometimes I could understand a formula better if someone else was showing me than what the teacher was saying.

--Transcription from taped interview

Relationship was an important part of each student's experiences. More specifically, though, were the interactions that took place within those relationships. Those interactions (Fiona and her father, Susan and Ellen, Deidre and her peers) were something that stood out as experiences for them and language was a component of those interactions.

It could be argued that language was found in all components of the classroom in this study. My focus, though, was particularly on the relationships between language and students' experiences. Throughout the semester, the relationships of folding back (Pirie &

Kieren, 1994) and language were deeply interconnected and built on one another in a recursive process. In the next section, another sub-theme part of this recursive process is explored, listing the most prevalent examples.

Relevance

When asked about how their thought processes have changed was how the students (elementary education majors) were able to express the relevance of mathematics in their own lives and the lives of their future students. Over and over, students expressed how they were never allowed to experience mathematics in multiple ways until taking methods courses. For example:

> Kerri: What is mathematics?
>
> Susan: For me it is the logical and numerical relationship to the outside world. I don't think the world would function without math.
>
> --Transcription from taped interview

The relevance of mathematics in Susan's world indicates that she is thinking about mathematics in new ways. During a class discussion, she commented on how she felt mathematics was only about numbers when first asked the question in another class. She expressed that mathematics was an important component to her everyday experiences and her ideas on teaching mathematics were similarly reflected in the newsletter assignment. Other student notions of mathematics were noted in the following interviews:

> Kerri: What is mathematics?
>
> Alicia: It is everything almost. Our everyday life is dependent upon mathematics. The people that say you are never going to use this in the real world…that is just wrong. There is always going to be something that has to do with mathematics.

Karen: I would say that math is not only around us everywhere but connected. I have had to accept that is so connected with everything around us. And I also agree with what she says.

--Transcription from taped interview

Kerri: What is mathematics to you personally?

John: I think it is a way of thinking, kind of. It's everywhere and either you're really good at it but most people aren't. If you're really good, you can do a lot of things with it.

Angie: I don't think you're good at it or you're really not. I think everyone has some degree, they just haven't found it. Not everyone has figured it out yet. I don't know if you can separate it out as a specific thing. It's everything. I don't know how to say it (laughing). It is connected to everything. Even if it's not like complex like calculus. You do math everyday like figuring out how much money you need to take out of the bank or whatever.

--Transcription from taped interview

Kerri: What do you want for your future students of mathematics?

Ashley: I think what I want is for them to realize that there is more than one way to work a problem and not just be told this is how you do it and don't ask why. I think that it is important that they question and explore concepts. I also think it is important that they relate it to their real life. Like make connections.

Dima: Yeah like making connections to real life because I don't think I had those connections when I was young.

Kerri: So with that in mind, what is mathematics?

Dima: Anything you look at is math.

Ashley: There's not like one answer to that question.

Dima: Math is everywhere you go, math, science, social studies, you'll see it.

Ashley: Before I didn't think there was math outside of the classroom but being in these classes you see it everywhere. Like you can build lesson plans around just about anything.

--Transcription from taped interview

Students' notions of mathematics in relation to relevance were not only revealed in interviews, they also emerged as a central theme in the historical presentations (see appendix E). Through the interview process, as one may see in the above transcriptions, relevance emerged from students explanations about their own theories on what mathematics is to them personally. Their historical presentations, in some ways, bridged students' theories into practice by demonstrating one way of enriching experiences through mathematical history. Topics were chosen by students on the basis of what relevance those topics had to them and were then shared with their peers.

Examples of presentations included the following: the abacus, tessellations, pi, the metric system, time, Roman arches, the tower of Hanoi, money, pyramids, latitude and longitude, calculators, geometry, fractals, and the coordinate system. Each presentation demonstrated the relevance theme throughout. For example, Ashley and Dima presented the history of latitude and longitude. Before, as noted in their presentation, they had both taken latitude and longitude for granted and were just told in school what it was but not why we have it or how it came about in our world. The rest of the class during the presentation was surprised to find out about the struggles especially with longitude and the rich history behind it. This particular presentation especially generated a lot of conversation about the topic. In my field notes, I noted a student saying, "I had no idea it had been such a controversial issue in our world." In the written portion of the presentation the following is noted and also included in appendix E:

> Why does it [latitude and longitude] matter in our lives?
>
> - If you were to ask to find where a particular place was in the world, how would you do it?
> - If a pilot or ship's captain wants to specify position on a map, how would they do that and how would they travel? They will need to know and understand the "coordinates" they use.
> - The measurement is important to both cartography and navigation; the discovery of how to measure it accurately was among the important discoveries of the 1600s and 1700s.
>
> What would it be like for the field of mathematics and our world in general if this discovery had not been made?
>
> - No navigation
> - No map
> - Effects time zones, weather, climates
> - No understanding of clocks
> - Where to locate a country, state, city, or county on a map
> - How to travel from one place to another
> - How to calculate the time it takes to go to one place or another
> - How to measure the distance from the North Star to the equator
> - To know your position on the earth
>
> -Taken from student artifact of historical presentation

It was a requirement of the instructor for each group to include in their presentation why their particular topic was relevant to our world. The interactive nature of each presentation encouraged discussion and a variety of interactions between students. The presentations seemed to draw the semester to a close because students (elementary education majors) reflected a sense of knowing how to offer the discipline of mathematics to their future students in ways that may be non-threatening, less rigid, and more experiential than their own previous experiences in school. This creating of spaces for their future students to experience relevance within mathematics stood out as a key feature.

Another example of student experiences with relevance was in relation to their three week field experiences. They were each placed in a school in the surrounding community at grade levels varying from first through eighth. During that time, education majors were involved with elementary students for various activities depending on the supervising teacher they were assigned. Some noted teaching almost immediately while others were "eased" into the climate of the classroom in which they were placed. Upon returning from the field experiences, the instructor of the methods class invited discussion about where they had been, what they had experienced, and any other thoughts pertaining to their assigned schools.

One student discussed the lack of ownership her teacher seemed to have with the curriculum in her classroom. She explained it as if the teacher was just going through the motions and felt the students weren't really experiencing anything, just being fed information. She said this was particularly noticeable during math time because it stood out as being very separate from the rest of the curriculum. There were few connections being made between ideas and math did not appear, in her view, to be relevant to the students' lives in any way except in terms of achieving a passing grade.

Another education major, however, found herself in a fifth grade classroom where the mathematics curriculum was quite challenging for her as an adult. The curriculum was a new one that had recently been implemented by the school district which attempted to present mathematics more relationally. She expressed how difficult one particular concept concerning integers had been for her but finally understood what the teacher was presenting. She noted that her biggest surprise was how quickly some of the fifth graders grasped onto the idea presented and almost "ran with it," as she stated during the methods class discussion. The education major went on to say how she saw the importance of thinking about mathematics in new ways than she had been taught because it opened up so many other avenues for her as both a student and teacher.

Relevance, as described through interviews concerning notions of mathematics, experiences with the history of mathematics, and field experiences in elementary classrooms, was a theme which emerged in a variety of ways. Students in this study experienced mathematical relevance as more than just a "real world" problem given to them by an instructor. In the next section the final theme is presented which discusses one of the most important spaces created for students in this study to experience folding back, language, and relevance: open-endedness.

Open-endedness

"Creativity, like its consequences of evolution and cognition, is a process of diversification, of expanding the realm of the possible…..so understood, creativity is not a trait that is specific to some and not to others. Rather, all living forms are inherently creative beings" (Davis, et. al. 2000, pp. 116-117). Operating under Davis', et. al. (2000) idea that creativity is "a process of diversification," the problem-centered classroom in this study was presented by the instructor in such a way that diversification was made possible. The open-endedness of various assignments and activities throughout the semester allowed students to create spaces which involved a variety of approaches for building upon their relationships and experiences with mathematics. Why is creating space so important in mathematics education? To some, open-endedness and mathematics may seem like an oxymoron, but experiencing mathematics through ways that allow for freedom and open-endedness are vital to thinking and speaking about mathematics in new ways.

Open-endedness in this study, while varied and open to a wide variety of interpretations, emerged *with* the sub-themes of folding back, language and relevance. It could be argued that every mathematical experience or perception noted in the study had some creative or diversifying element contained in it. Therefore, it made the most sense to discuss open-endedness in terms of its relationships with other themes. The sub-themes of folding back, language, and relevance built on one another throughout each assignment, conversation, and experience in the creative spaces afforded by the instructor and students. Choosing open-endedness as my final theme to discuss was purposefully done to demonstrate the recursing of students' experiences through my perspective.

The structure of many mathematics classrooms and mathematics methods classes involves an emphasis on common routines. These routines may include beginning the class with going over homework, a question and answer session, examples listed on the board of how to do a problem, and teacher led discussions. Particularly relevant to this study, many methods classes are centered on the "how to" of education (how to teach the multiplication of fractions, etc.). The methods class studied here, though, tended to be centered more on the "why" of education. Why do we multiply the numerators and denominators of fractions? With the underlying notion of the "why" being at the heart, open-endedness was a prevalent sub-theme through each exploration, discussion, and assignment. It was a theme that enabled students to **fold back** (Pirie & Kieren, 1994) as noted with Ashley and Dima's disjointed jumps, John's non-relational approach, Diane and Lacy's focus on physical surroundings, and Fiona's dynamic strategies.

Open-endedness was also the theme that enabled the use of **language** by allowing for the "why" questions to surface without any threat of being judged for not understanding a particular idea. Thinking of two fractions being multiplied as a *half of a fourth* rather than *one-half times one-fourth* and not being told by the instructor that it must be said and thought of in one way was just one example.

Being given only a few criteria for the newsletter assignment, for example, students were able to use language that had meaning for their mathematical experiences. This language of explaining to others in the class about how they felt their experiences could best be shared was a challenge for the education majors. One student in particular commented after class one day that she felt the assignment had been very difficult but in a good way. I asked her why it had been difficult and she said because there were so many possibilities and she really needed to think about what was the most important to her.

The simple yet complex question, "what is mathematics?" may have initially had characteristics of exact and non-varying answers, but students came to realize this to be an open-ended question with a variety of responses having **relevance** to their world. When asked the "what is mathematics" question during the interviews, many of the responses had personal overtones reflective of student experiences.

Relationships, including folding back, language, and relevance are at the heart of the mathematical experiences among this particular group of education majors. The four sub-themes identified were deeply intertwined with one another and difficult to separate.

Looking at the third methods course experienced by the elementary education majors in this study as a whole was one way to explain the phenomenon of being in the world with mathematics.

Chapter V

ANALYSIS AND DISCUSSION

"Newton's clockwork universe and the mathematization of our world are not rich enough to handle the complexity and evolving nature of the world around us and of which we are a part." – M. Jayne Fleener

In addressing my research questions:

1. What are students' perceptions about their experiences with mathematics?
2. How can students' perceptions about and experiences with mathematics inform their understandings of mathematics?

I realized how various mathematics educators' ideas connected with my own beliefs about the phenomenon of being in the world with mathematics (Matney, 2004). What I found is that being in the world with mathematics is not just a complex notion; it is, more specifically, a phenomenon which may be studied and reified in ways that add to the mathematical conversation. At the same time, I also realized how deeply entrenched my own beliefs were in my findings and interpretations of my findings.

My findings indicate that meaningful experiences are important to students' ways of being in the world with mathematics. Their perceptions about their experiences, as indicated by the data, included their descriptions of how they approached a particular problem posed to them in class (e.g., the area problem), their emergent conversations (e.g., the multiplication of fractions), and ways in which they described how they hope to share their personal experiences with their future students (e.g., newsletters, interviews, historical presentations), just to name a few. As stated by Fiona during one of the interviews, "I need to make a connection between the mathematics I am doing and the reality," which demonstrated her way of perceiving her experiences with mathematics.

My findings also specifically generated types of relationships found in students' mathematical experiences. These sub-themes, as I call them, generated ways to inform their understandings through folding back (Pirie & Kieren, 1994), language, relevance, and open-endedness.

With folding back (Pirie & Kieren, 1994), I found students constantly revisiting ideas and reforming new ones. Students folded back to previous lessons, conversations, and ideas during the course of the semester. With language, students regularly involved themselves in rich conversations and spoke about mathematics in new ways as indicated by the numerous formal and informal discussions. Simply speaking about mathematics was something I found to be a regular occurrence in their experiences. Relevance was a recurring sub-theme throughout the semester because this type of a relationship seemed prevalent among instances of folding back and language in particular. During conversations, personal thoughts about occurrences in and outside of the classroom helped students make sense of their ideas. Folding back, language, and relevance all seemed to occur within the open-endedness of the classroom.

I found open-endedness as becoming the most prevalent type of relationship in this study. The reason for this is because without it, I do not feel the other sub-themes would not have emerged as often as they did or maybe not at all. In fact, I believe this would have been a very different study without it.

The next section will offer reasons why open-endedness was so important through a discussion of the classroom context. Following sections will continue with a visionary look at my findings by offering thoughts about curriculum and ways in which students can be encouraged to have more meaningful experiences with mathematics. By addressing limitations of a dynamic curriculum, I will then go on to problematize "method" (Doll, 2005) and offer ideas on teacher education. Final parts of the chapter include revisiting ideas on creating spaces for mathematics learning, thinking methods anew, and final thoughts on the study as a whole.

Classroom Context

One of the main reasons for the dynamic relationships found in this study may have been because the instructor constantly kept an open invitation to the mathematical conversation. Students also engaged in keeping this conversation open by working together, reflecting on experiences, and approaching ideas from a variety of perspectives. This type of open-endedness was vital to the classroom climate because it reflected opportunities for students to interact with mathematics in meaningful ways. The instructor's constructivist theory base surfaced through practice by guiding education majors in the development of ways to experience mathematics and to share those experiences with their future students.

Even more telling was the open-endedness of the conversations themselves as noted with the fraction discussion and with the historical presentation on latitude (chapter 4). The instructor could have easily encouraged the class to go in a different direction, but instead, she "went with it" and allowed students to construct their own ideas on fractions and latitude.

The setting was that of a problem-centered classroom and was ideal to find types of relationships with students, the instructor, and the mathematics. This finding leads to another important discussion in terms of being in the world with mathematics. What type of an environment needs to be in place for students to experience mathematics in new ways reflective of a post-formalist vision? Post-formalism emphasizes the recursive nature of mathematics in which our mathematical experiences are very much a part of our mathematical knowledge as an unfolding and continuous process. The components of the mentality include nature, dynamics, chaos, recursion, enactivism, change, relationships, creativity, and community. The following discussion will offer a vision not only for a way of thinking and speaking about mathematics, but also settings that encourage *being with* mathematics.

Dynamic Curriculum

Mathematics has long been viewed as a complicated system but should be viewed as a complex system (Davis et al., 2000). In other words, instead of viewing mathematics as a system distinct from physical experience, it should be viewed as being in dynamic process with the body and mind (Davis et al., 2000). One facet of a complex system, in terms of curriculum, involves Richness, Recursion, Relations, and Rigor (Doll, 1993). Richness is in the variety of investigations in which a student is allowed to construct or explore an idea that is meaningful for them. These multiple interpretations (Doll, 1993) allow the student a way to explore patterns and experience relationships.

During investigations, Recursion describes the revisiting of a specific idea a student has constructed. This “looping”, as Doll states, allows thoughts to reify and extend one’s ability to grasp reasoning. Relations are essentially a “result” of this recursive way of being among the student, thus the two work closely together. “One perspective of relations in curriculum reveals the connections among emergent patterns of the curriculum process” (Fleener, 2002, p. 171). For example, during the fraction discussion (chapter 4), one student noticed a pattern in the language used to express the multiplication of fractions. She expressed the pattern through illustrations on the chalk board using several different examples.

Rigor, in terms of mathematics as a complex system, represents a post-modern notion of constantly revisiting and discovering new ways of thinking. According to Fleener (2002), "it also means examining assumptions, challenging traditions, and encouraging creative solutions" (p.173). As Doll (1993) states, these four post-modern frameworks of curriculum are an alternative to the modern framework of the three R's; "'Readin', 'Ritin', and 'Rithmetic'" (p. 174).

Far too often mathematics is being presented as an exact science rather than a way to make connections, explore, and see relationships as Doll's four R's describe. Without this time to investigate ideas, students are being stripped of their inherent ability to apply meaning to their discoveries. Fiona in particular was a good example of how a student can become disenchanted with mathematics because of negative experiences and demands for conformity. Luckily, though, she was finally able to be in an environment which nurtured her ways of being in the world with mathematics.

As I reflect on my research questions, I am reminded not only of Doll's post-modernism; I am also reminded of Fleener's curriculum dynamics. She has entered into the conversation about new ways to envision mathematics by offering ideas about relationship, systems, and meaning. She holds the belief that for students to be able to experience mathematics and then make sense of them, dynamic relationships must occur within curricular structures (Fleener, 2002).

Fleener (2002) describes three underlying logics of postmodernism: the logic of relationship, the logic of systems, and the logic of meaning. Her vision of "recreating the heart of schooling" includes seeing curriculum not as a *thing* but as a *relationship* intertwined with infinite *meanings* all part of a complex *system*. She promotes a change in the way we think and speak about mathematics by moving beyond the factory metaphor of schooling so engrained in our thinking and moving toward a world of process and relationship. Fleener (2003) notes:

> Exploring relationships rather than examining things has implications for every facet of life. The logic of relationship has its own demands for value, purpose, and method. Examining and shifting our ideas about the purpose of schooling with an underlying logic of relationship has the potential to infuse the curriculum with meaning and purpose, rekindling the spirit and heart of learning. (p. 80)

Her thoughts on the idea of *Weltbild* or "seeing as" as noted by Wittgenstein, describe not seeing our world as specific things, but *seeing* them *as* "exhibited through our actions." For example, it is a common notion among many people that mathematics primarily involves numbers. When I have asked my own methods students "what is mathematics?" at the beginning of the semester, I hear a lot of the same answers. However, toward the end of the semester I ask the same question again and get more varied responses. In my view, students begin to *see* mathematics *as* more related to their ways of being. They also experience from this exercise how their peers *see as* which may or may not be different from their own ideas.

Fleener not only advocates thinking about mathematics differently, she also envisions *speaking* about mathematics differently for the purpose of meaning. She supports this notion by bringing forth Wittgensteinian ideas on language. Described as language games, Fleener (2003) notes:

> Wittgenstein's language-games approach neither tries to capture the meaning of words nor provides the rules to follow in its method. Rather, this approach unravels the difficulties in philosophy by exploring how meanings and confusions occur…The meaning of a word and learning how to use it occur within the language game rather than prior to it. There are, from this perspective, no underlying meanings. (p. 131)

She commonly speaks about curriculum and even mathematics in a metaphoric sense. Having studied with her, we [my peers and I] paired up during class and developed metaphors for schooling (this is just one example) and were asked to illustrate and describe them on a large piece of butcher paper. I found this task to be very challenging because I had always thought there was one set meaning for everything in education and was afraid I would be wrong in my metaphor. Quickly, I realized how interested she was in the varied relationships each group discussed. It wasn't about being right or wrong; it was about *seeing* schooling *as* something different than we had before. The metaphoric exercises that took place in her curriculum class were a way for us as students to problematize our own experiences. Some of us would present a very mechanistic approach to our ideas on education and she would not tell us we were wrong, but would simply challenge us to explore a new or different path in our thinking, which would perhaps add to our conversations.

Doll and Fleener's visions of curriculum take on a very organic approach with a variety of influences from Dewey, Whitehead, and Wittgenstein, just to name a few. Doll's four R's and Fleener's logics support one another well and are both valuable aspects of the curriculum conversation. Their work in curriculum theory, from my perspective, envisions the possibilities for students to experience mathematics in new ways reflective of a post-formalist classroom.

The next issue is, however, what sort of teacher education is needed to create such a classroom? With so many “how to” methods classes in universities across the country, it is challenging to find one which takes a new approach to mathematics learning. The following section problematizes the very notion of methods from the perspective of a dynamic curriculum.

Preparing Mathematics Teachers

What does it mean to teach a methods course? How do you teach people how to teach mathematics when their perceptions are so engrained? Does even the name “method” imply linearity to teaching that reinforces those engrained perceptions and beliefs? How do you get beyond method? In order to address these types of questions it is important to first trace the beginnings of the idea of method to give a clearer picture as to why methods classes have arrived at their current state.

Beginning with a Renaissance scholar named Peter Ramus, his philosophies on the way students should learn and teachers should teach has had a major impact on methods as we know them today (Doll, 2005). His philosophies came out of a frustration with a lack of order and logic in the field of philosophy. Interestingly enough, though, Ramus’ ideas which were essentially designed to be “teacher-proof” have had a major impact on educational history.

Known as branching, Ramus’ approach can be demonstrated through mathematics, for example. Mathematics becomes arithmetic and geometry, the arithmetic branch becomes numbers and fractions and the geometry branch becomes the study of plain and solid (Doll, 2005). As you further delineate the specific content of mathematical study, a “tree” is created. This curriculum “tree” has become our traditional curriculum sequence chart known to all 20th century teachers and used by textbook publishers to break-apart and organize the curriculum. Ramus’ ideas centered on this linear, step by step order to education, changing how we have come to understand the organization of knowledge. The Puritans were intrigued with this type of approach because “it provided for them a frame of both morality and simplicity – virtues they embraced (Doll, 2005, p. 27).”

As noted by Doll (2005):

> As Walter Ong, the noted Jesuit scholar and Ramus historian, has pointed out, over the centuries Ramism became diffused throughout many countries, amalgamated with various other instructional procedures, and enfolded into and mixed up with scientific method and a strong enthusiasm about method in general. (p. 22)

Another Ramist who wanted to follow in this “categorizing and presenting of knowledge” approach was John Amos Comenius. Comenius essentially used Ramism to infuse Christianity with education, enabling him to spread the word in a step by step manner. He felt the education of boys and girls should be kept very simple and the complexities of the Bible taught when they were much older (Doll, 2005). Comenius’ work had an impact on the way religions, especially many denominations of Christianity, are taught today.

Doll (2005) enters into the conversation concerning methods by taking on a new way of thinking and speaking about it. Consistent with Fleener’s logic of meaning, Doll uses the term "agency" alongside method in his ideas. Agency entails interactive characteristics, not just the teacher "passing on" her "knowledge" to the student. This recursiveness between and among content, learner, and agency (method) involves a certain spirit (Doll, 2005). As noted by Doll (2005):

> I do think there is a "spirit" to the subject matter we teach in methods courses and that finding and utilizing this spirit brings knowledge alive, liberating it from the banal. (p. 32)

Doll speaks about methods in a similar fashion as Fleener speaks about students. Both methods and students are not *things* to them; they are spirits, relationships, and more importantly a complexity of both. Doll (1993) also comments on the role of the teacher and possible curriculum implications in a "spirited" classroom:

> One curriculum implication is that if cooperative, purposeful behavior (which leads to higher levels of organization) suddenly appears at critical threshold points, then teachers need to work toward finding these junctions in their own group interactions. And if autocatalysm and iteration take over at some point, so that a given class generates its own order and methods of development, then finding these junctions might well be one of the most important tasks a teacher has. (p. 105)

Finding junctions, as stated by Doll, connects with my thinking and what it means to teach a methods course. For me, it means to view the entire system as a way for my students to learn but also as a way for me to learn. Every semester, I gain so much from the system as a whole; my interactions with students, their interactions with one another, my experiences, their shared experiences, and so on. I feel my students are often surprised at how much they have to offer during our classroom conversations.

How do you teach people how to teach mathematics when their perceptions are so engrained? Again, I view this question from the perspective of the system as a whole. One way my colleagues and I have approached this issue in the teaching of our methods classes is by setting an initial tone or a "spirit" in the classroom. One of our favorite questions is to ask students, "What is mathematics?" and have them reflect on the question through writing and discussing. This may seem rather simple to an outsider, but it is a space which is created for the students to really problematize their past experiences with mathematics.

How do you get beyond method? I am not sure getting *beyond* method is necessarily the goal. However, I do think re-contextualizing it in a way that is less hierarchical and on a more level playing field is perhaps a step in a new direction. The power so deeply entrenched within method is what is problematic for me. For example, telling students the best way to teach the subtraction of three digit numbers is best done by borrowing is ridiculous in my thinking. The reason for this is that many of the same pre-service teachers who would teach this in their future classrooms never really made sense of this procedure themselves. It is as if an arbitrary procedure gets carried on generation after generation without space ever being created for students and their teachers to enter into the conversation. Opening up these spaces invites pre-service teachers to investigate why the procedure was such a problem for them.

In the preparation of mathematics teachers, regardless of the grade level they aspire to teach, it is not enough to encourage them to *see* their experiences; they must *see* their experiences *as* a complex set of relationships and interactions (Fleener, 2002). The next section discusses the *seeing as* in terms of creating spaces for learning in the mathematics classroom.

Creating Spaces

In both creating spaces with methods and providing opportunities for students, how do we get people to interact with mathematics in ways that encourage them to be with mathematics and be in the mathematical conversation with others? My initial hoped for implications for this study were how to open up spaces and/or create new spaces and new ways of thinking about instruction in the mathematics (methods) classroom. The amount of data collected and relationships which formed were much more than I had anticipated. The daunting task of making sense of all of the occurrences throughout the semester was sometimes tumultuous and frustrating. However, it was this type of perturbation which resulted in the opening up of new spaces for my own thinking about mathematics education.

Creating space for students to experience the unfolding of mathematics was demonstrated through emerging themes with a variety of conversations about those themes. Again, the themes included folding back (Pirie & Kieren, 1994), language, relevance, and open-endedness. The implications are tangible and have significance because students in this study appeared to have come to know mathematics from a variety of ways not known to them in their past experiences. One way we can get students to be with mathematics rather than be a passive observer is by establishing a climate of open-endedness. This climate should involve a sense of discovery, exploration, recursion, and even chaos. “Finding the junctions” or creating space is where the teacher must assume her/his role in maintaining the post-formalist classroom culture.

Because of the deeply engrained modernist beliefs toward education in our country, though, creating space in the mathematics classroom is constrained. The Ramus map is so engrained in our thinking about the curriculum and in our approaches to instruction, that our very ideas about the structure of the discipline and “good teaching” have become almost intractable. The next and final section problematizes those limitations and constraints and summarizes ways we can deal with these curricular challenges.

Thinking Methods Anew

As a former teacher in the public school system, I distinctly remember the harsh realities of my day to day routine. For several years in particular I taught in a middle school in which I had 80 seventh graders throughout the course of one day. Not only was I responsible for following the mandated curriculum (“covering” the curriculum, as it were), I was also expected to fill out attendance sheets, gather missed assignments for absent students, perform recess duty three to four times a week, be on committees, keep up with eligibility for student athletes, attend a host of meetings, fill in for other teachers when no substitute could be found, and maintain a healthy relationship with parents, colleagues, and students. My days were completely filled with something at every moment, giving me little time to think about the two most important aspects; the students themselves and the curriculum in which they were a part.

I am often faced with a variety of questions from my own students in the methods classes I teach about the realities of public schooling. I emphasize that nearly everything is situational, meaning it is impossible for me to tell them *the* way in dealing with any given issue. What I do offer in these types of conversations, though, is an invitation for students to offer their experiences. Without this offering there would be few spaces for reflection to occur. The students in my classes are also enrolled in their second field experience and are able to shed light onto issues that may arise. Since they are spending time in the schools, they have an awareness of the current system and the types of experiences children are having in the mathematics classroom. While I do offer my own experiences, I offer them carefully so I do not sound as if I have *the* answer to their problem.

Another limitation in opening up spaces for students to problematize their ways of being with mathematics is simply time. Many universities across the country offer only one methods course. In the higher education environment I was educated at both as an undergraduate and graduate, I had numerous spaces created for my thinking largely due to a variety of courses offered to me. Students may find that they need more time to make sense of new ways of thinking and speaking about mathematics. There is not one simple solution to this problem but maybe one step in a new direction would be to offer pre-service teachers various resources to continue on with well into their internships. For example, one colleague of mine created perhaps one of the best undergraduate mathematics education websites I've seen (www.youngzones.org/Elaine/). To this day, I suggest the site as a resource for both students and current classroom teachers. I often get feedback from these students and teachers that the site was and is very helpful in getting them to continue thinking about their own constructivist philosophies.

A third limitation is dealing with the current curriculum and the way in which it is expected to be followed by the current system. One way I have encouraged students to deal with this issue is by listing each NCTM principle and standard (2000) on their lessons. They grow tired of having to do this but realize with issues of accountability and so forth, they must do so to satisfy whoever is supervising them at the time. Students often come back and tell me that during their internship experiences they were able to "get away" with implementing something their supervising teacher had never seen before simply because they had all of the skills listed.

Few spaces for reflection, lack of time to problematize preconceived notions about mathematics, and dealing with current curriculum structures are limitations I perceive as being the most prevalent in preventing a post-modern method to emerge. With that being said, one guiding quest still remains and that is we can't have openness if we can't see differently. As stated by Fleener (2002) at the beginning of this chapter, "Newton's clockwork universe and the mathematization of our world are not rich enough to handle the complexity and evolving nature of the world around us and of which we are a part" (p. 98).

Final Thoughts

The value of phenomenology in this study has helped me to see how deeply entrenched the researcher is with the types of conversations she or he is generating. It has also helped me to see that the phenomenon of being in the world with mathematics is open to numerous interpretations.

As I reflect on the completion of this study, I am reminded of my own experiences throughout the process and the ways in which I chose to express my *being* *with* mathematics. Asking my guiding questions (what are students' perceptions about their experiences with mathematics, and how can students' perceptions about and experiences with mathematics inform their understandings of mathematics) were an important start and continuation in my thinking. For me, it first made sense to address these questions by tracing the history of mathematics through Davis' ideas and then to explore the work of others such as Pirie and Kieren. By utilizing their ideas as inquiry guides, I was then able to bring theory into practice by experiencing the unfolding of the curriculum in a pre-service elementary mathematics methods class. The works of Doll and Fleener finally helped me to make sense of the results and I found I was able to discuss both implications and limitations from the perspective of mathematics education as a complex system.

For me, my most important finding was that meaningful experiences are important to students' ways of being in the world with mathematics. I was able to discuss these experiences more specifically through emerging relationships such as folding back (Pirie & Kieren, 1994), language, relevance, and open-endedness. Emerging relationships will be at the heart of my continued explorations in my own mathematics education research and my hope is to constantly find new ways of thinking and speaking about mathematics.

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Figure 1: The Pirie-Kieren model for the dynamical growth of mathematical understanding

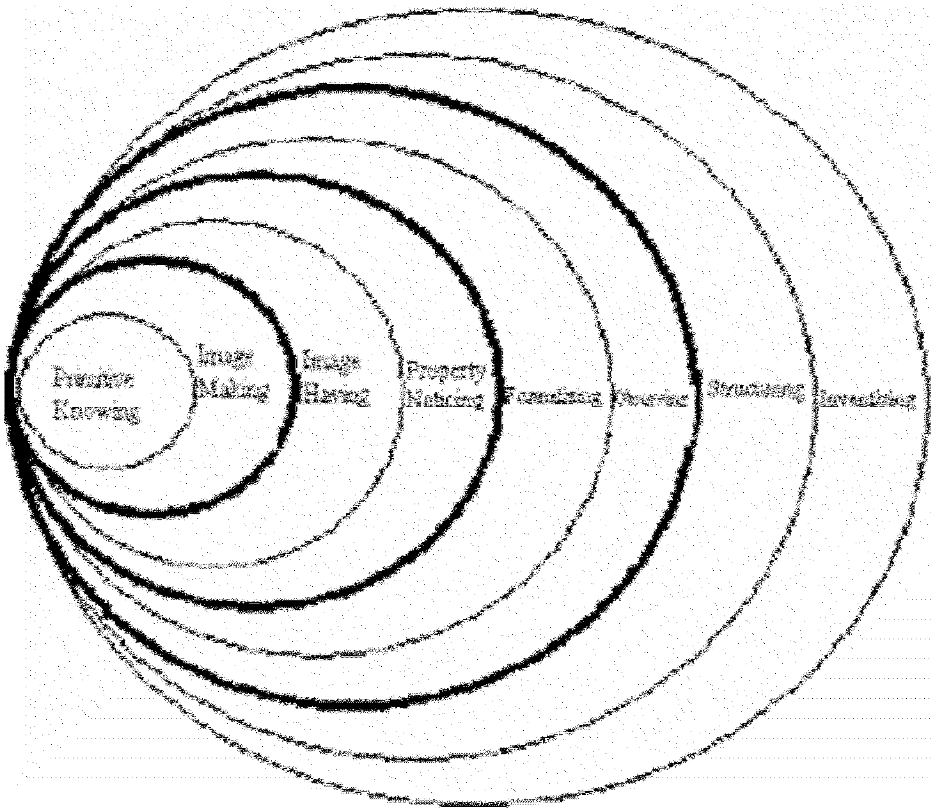
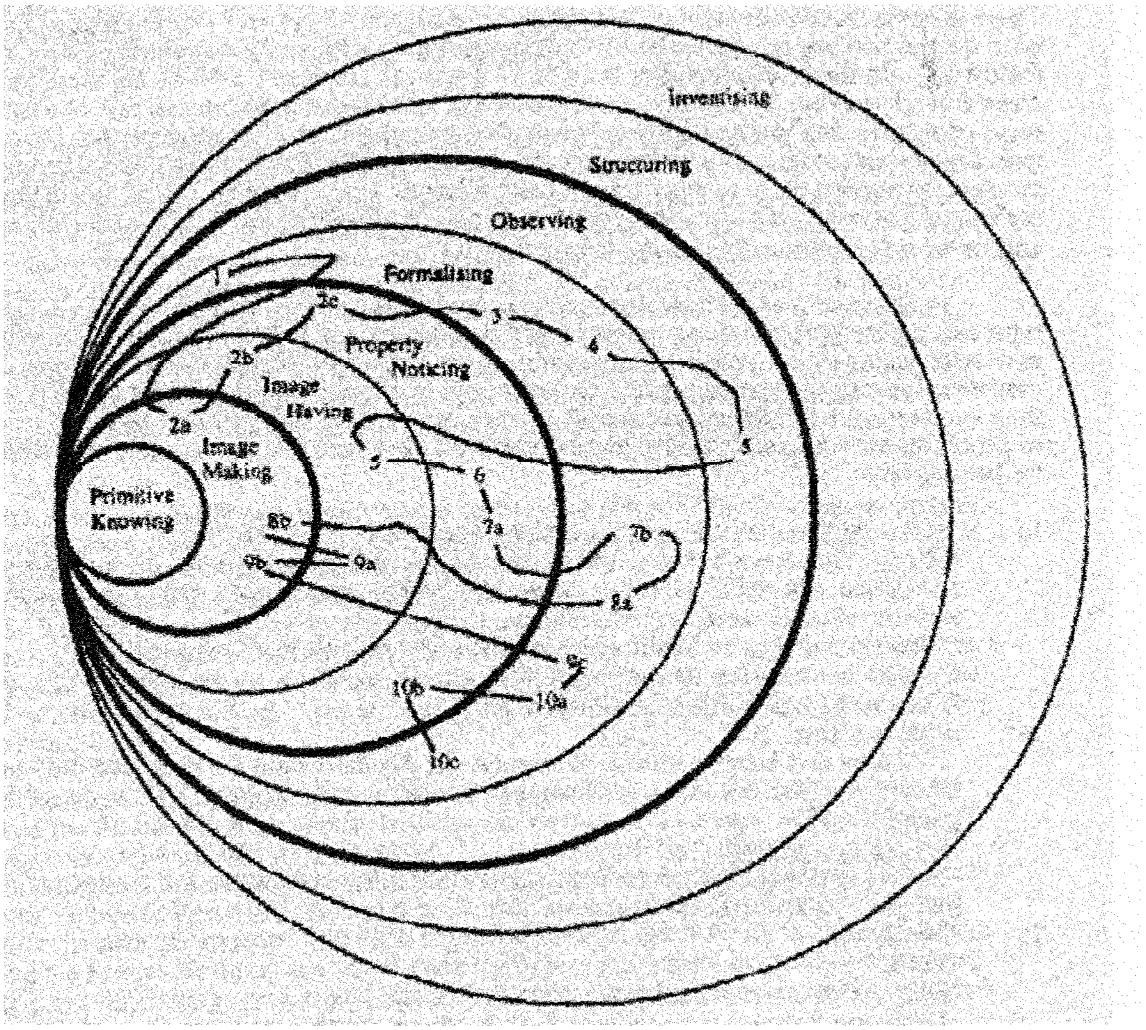


Figure 2: The Pirie-Kieren model for the dynamical growth of mathematical understanding as indicated by Stacey & Kerry's changing understanding of the problem.



Appendix A

Types of questions I asked for the interview component:

1. Describe your experiences with mathematics prior to this class. (Possible follow up questions: How about your earlier college classes? How about a remedial course such as MATH 0123?)
2. Since entering the education program, have your mathematical thought processes changed? If so, why? If not, why?
3. Describe how you envision teaching mathematics.
4. In your view, what is mathematics?
5. From your perspective, what is the purpose of teaching mathematics?
6. What is the role of mathematics in society to you personally?
7. For you, what does it mean to be good at mathematics?
8. What do you want for your future students of mathematics?
9. Based on your opinion, how should mathematics be taught?

Appendix B

College of Education
Instructional Leadership and Academic Curriculum

Syllabus for [redacted]

Elementary Mathematics Curriculum

[redacted]

Course Instructor: Office Hours: Monday 1:00 – 3:30
Thursday 9:00 – 3:00 (or by appointment)

Phone: (messages only)

E-mail:
(E-mail is the preferred method of contact)

"It is not enough to have a good mind; the main thing is to use it well."
-Rene Descartes

"Treat a man as he is and he will remain as he is. Treat a man as he can be,
and he will become as he can and should be."
-Goethe

The free, exploring mind of the individual human is the most valuable thing in the world,
the freedom of the mind to take any direction, undirected...
-Steinbeck, *East of Eden*, 1952

BLOCK PORTFOLIOS ARE DUE IN THE ILAC OFFICE BY 5:00 PM ON [redacted]

Course Description:
This course is designed to offer the student means for planning, implementing, and evaluating mathematics curriculum and instruction in the elementary school. Specific strategies and materials for teaching mathematical content will be explored. In addition, topics of special concern to the mathematics educator will receive attention. Infused throughout the course will be materials and discussions relevant and sensitive to ethnic and racial diversity, special populations, and communications in mathematics. Other areas of focus will be the appropriate use of technology in the teaching and learning of mathematics and the inclusion of historical and interdisciplinary topics in teaching mathematics.

Goals for the Elementary Curriculum Block:

This course is a part of your Elementary Curriculum Block experience which has been organized around several common goals. These common goals include a shared focus on:

1. understanding the culture of schooling;

2. connecting theory and practice;

3. curriculum integration; and

4. professional development

Course Goals and Objectives:
The goals of the Elementary Curriculum Block experience are intimately connected with the specific course objectives. Specifically, we will work toward an integration of your previous experiences, your methods and block experiences, and life-long professional development opportunities throughout your career. Personal, block, and class goals will be accomplished through the integration of your university classroom and field placement experiences.

1. Students have an understanding of 4-8 mathematics curriculum as presented in the Priority Academic Student Skills prepared by the Oklahoma State Department of Education.

2. Students will be able to discuss and apply implementation strategies for the NCTM Standards in mathematics education for grades 4-8.

3. Students will prepare and present mathematics lessons that are consistent with the NCTM Standards and developmental theory.

4. Students will use ideas/research findings from journals, books, internet and videotapes to prepare and present lessons to intermediate school children.

5. Students will prepare lessons and materials that make appropriate use of various technologies including calculators and the internet.

6. Students will understand and utilize a variety of formal and informal assessment strategies.

7. Students will be able to discuss and apply topics from the history of mathematics in their mathematics instruction.

Required Materials:

1. Tsuruda, Gary. (1994). *Putting it together: Middle school math in transition.* Portsmouth, NH: Heinemann Press. ISBN 0-435-08355-4.
2. Course packet from King Kopy.
3. Principles and Standards for School Mathematics. (2000). National Council of Teachers of Mathematics, Inc. ISBN: 0-87353-480-8. http://www.nctm.org
4. Colored pencils, markers or crayons.
5. Packet of Construction paper or cardstock and a standard 12" ruler or tape measure (one used for sewing is fine).
6. Pocket folder with brads and notebook paper for in-class writing
7. Oil Pastels.
8. Empty food boxes – any size, but you need to have at least three of each size you collect (bring to class as early as possible).
9. Supplies for teaching in-class lessons, unless these items are included in materials found in the classroom (PHSC 355).

Suggested Materials:

1. Burns, M. (1992). *About teaching mathematics: A K-8 resource.* Math Solutions Publication. ISBN 0-941355-05-5.
2. Nagel, Greta. (1998). *The Tao of teaching: The ageless wisdom of Taoism and the art of teaching.* Penguin Group. ISBN 0-452-28095-8 (Plume).

Course Requirements:
All assignments need to be word processed in Times New Roman – 12 point font

1. Class Participation and Assignments – 260 pts

Students are expected to attend and be prepared for **every** class. Poor attendance, lack of punctuality, or inadequate preparation may result in the lowering of your grade. **If you miss more than one class (including field experiences), you will be asked to research a topic in mathematics from a research journal and write a five page paper to avoid receiving a failing grade (Journal and topic will be determined by instructor and student if this becomes necessary).** In addition to attendance, the class participation grade will be determined by **readiness for discussion, in-class written assignments and activities as well as preparation of homework assignments.**

One of my main objectives in this class is to support your development into critical thinkers, challenging what you believe to be true and providing nurturing challenges to other class members as you progress from learner/teacher into teacher/learner.

The breakdown of the assignments for this semester is as follows:

Daily attendance (130 pts. total)
Participation (130 pts. total)
Curriculum library textbook comparison (100 pts.)
In-class historical lesson presentation, activity and write-up (100 pts.)
Two lessons taught in class to peer groups (100 points each for 200 pts. total)
Tsuruda Write-up (100 pts.)
Mathematics Internet Sites – to be sent to everyone through blackboard (30 pts.)
Mathematics Lessons (2) and reflections taught during Field Experience (100 pts.)
Mathematics newsletter to parents (200 pts.)
Field Experience Journal – Must include at least 8 entries – a minimum of two sent every week by Friday evening while you are in the schools (80 pts.)
Current article of interest regarding teaching Mathematics, Resilience Education or NCLB - This can be a newspaper clipping, journal article, etc. (100 pts.)

<u>Note on grading</u>: All assignments are considered late if they are not turned in at the beginning of class on the date they are due. Turning in lessons or homework when they are due does not guarantee full credit and late assignments (turned in after class on the date they are due) will result in a 50 % reduction of possible points awarded. All assignments must be complete. If you cannot be in class the day an assignment is due then you are expected to have that assignment to me through email or in my office by noon on Monday of the week it is due. Anything submitted through blackboard or through email to me needs to have your name, section #, and the assignment and due date in the subject line.

<u>Note on Lesson Write-ups</u>: 100 pts each – 200 pts total (Prepare lessons for 4th grade or older)

Each student will prepare and present two assigned lessons to cooperative groups in class. You will choose these lessons from the lesson section of your packet and modify them to the requirements for lesson plans outlined in the syllabus. Make certain that no lessons are duplicated in your group (each group member will need to teach a different lesson both times you are to teach in your group – no duplication of lessons taught). Short student write-ups will be turned in for lessons presented. These lessons are to be prepared and taught individually and not with your field experience partner. These lessons must be taught in their entirety during the class time and not just discussed within your group. If you are absent when you are expected to teach a lesson, only partial credit will be given for the assignment write-up. Expectations for lesson write-ups will be discussed in class but are to minimally include the items listed below. Finally, time will be provided in class for you to submit a short reflection on what you did especially well and what you would do differently if you were to teach the lesson again. You will occasionally be required to write a peer reflection on one of the lessons you were taught in class.

The format for your lesson plans is:

- Name of lesson and your name
- Grade Level
- Objective/s
- NCTM Standards (These need to be written out and not just referenced)
- PASS Objectives (These need to be written out and not just referenced)
- Materials List
- Preparation (include any data collection sheets, worksheets used, etc. as an addendum to your lesson and include on the last page)
- Procedures (do not include your questions in this section)
- Sample problems or examples you will model
- Questions for discussion
- Expected outcomes or possible solutions for the lesson (include this section even if the answers will vary)
- Connections
- Assessment
- Extensions

Include all of these sections in your lesson plans in the order above as your goal is for your lesson plans to have the ability to be replicated for anyone using the lesson.

2. **Curriculum Library Textbook Comparison – 100 pts**
During the week of February 14th - 16th you will sign up to visit the curriculum library in Cross Center (Buchanan entrance- 1st floor) and review pre-selected mathematics textbooks for comparison. Your assignment will be to review a minimum of two different textbooks, making comparisons of similarities and differences but also what you like about them and how you could use them as a resource for a problem centered mathematics learning environment. You should try to sign up with your dyad partner because this information will need to be included within your newsletter (validation for not

using the textbook but yet having it as a resource). **The library assignment can be accomplished using a rubric or chart depicting pros and cons with a short write-up validating and supporting how you would utilize the text in your classroom.**

3. **Mathematics Lessons and Reflections (taught during Level 3 Field Experience) – 100 pts**

During your Field Experience you (and your partner) will teach two mathematics lessons as described in the *Goals for the Elementary Curriculum Block* handout. These lessons along with your reflections are due during your first mathematics class meeting following the Field Experience. Each dyad should turn in one copy of the lesson with individual reflections attached.

4. **Field Experience Journal – 80 pts**

This is a compilation of your field experience. This can be reflections on lessons taught, specific experiences and interactions with students, interactions with teachers or parents, your reflective process of the time you are in the school, etc. This is the time for you to reflect on what you are experiencing and your interpretations of your experiences. Type your journal and submit at least two entries to me through email every Friday evening while you are in the schools. Your journal needs to be about what you experience in the school and should be thorough and capture your thoughts and feelings as an educator.

5. **Current article of interest regarding teaching Mathematics, Resilience Education or NCLB – 100 pts – (Subscribe to ascd@smartbrief.com for great resources)**

This is where you show me what your interests are and how the topic of the article will effect you as an educator. The article should be from a newspaper, teaching journal, online journal, etc. You will submit a copy of the article (with complete citations) along with a two - three page paper telling me how this affects you as an educator.

6. **Tsuruda Write-Up – 100 pts**

Topics and questions will be discussed regarding the Tsuruda book. Your write-up will be determined by the questions generated during the discussion. Three page write-up. You will need to address the questions generated through discussion but also apply it to your own mathematical experiences.

7. **Internet Site Assignments – 30 pts**

During the semester you will be required to find three web sites (make sure you do not duplicate a specific site or activity found in your packet or one that someone else sends via blackboard) to turn in through blackboard. One site needs to target teaching geometry, one site that targets teaching algebra, and one of your choosing. Send a short description of the site along with the address. You are to send these to the class through blackboard.

8. Mathematics Newsletter – 200 pts (make copies of this for everyone in class)(4^{th} grade or older)

For this assignment you will be asked to create a newsletter to parents describing your mathematics curriculum as well as a brief description of your philosophy on teaching and learning. This will be a very informative newsletter telling parents what will be happening in your classroom this year as well as validating and supporting your stance with theory from any readings you have in your packet or have done in other classes (be sure to include complete references). With your dyad partner you will present this to the class (they will be the parents) complete with an activity they will participate in and a question/answer session (I realize that many of the questions and answers will be similar in nature but I would like to give everyone the opportunity to have the chance to have this experience). Much of the information that you can use will be available in your packet.

Included in this newsletter will be:

- Description of your mathematics curriculum
- Activities you will be using in the classroom with your students
- Activities that you want to encourage parents to do with their child
- Validation behind a problem centered learning environment
- Support (theory) for cooperative groups, not ability grouping
- Supporting research validating how children learn
- Classroom management techniques
- Classroom rules and how they were derived
- Cross curriculum integration (at least two content areas; can include music and art)
- Goals for the class including PASS objectives and NCTM standards being met
- What parents can expect to see with their child
- Assessments used – include a modified report card that more adequately depicts a problem centered learning environment; i.e., number sense, problem solving and strategies, etc.
- Call for supplies or special needs for your class
- Welcome to parents coming to the classroom and best times to contact you

These are minimum requirements for the newsletter. Remember that school is just beginning and you need to inform your parents of your classroom practices and beliefs and how those affect their children. You will need to take into consideration that not all of your parents will know what you know about education and how children best learn. You want to win these parents over while keeping in mind that some parents will be more involved than other parents with their children's learning, so you will have to address all of them.

10. Historical Lesson Presentation – 100 pts

This assignment will include a write-up of the information including all references to be presented to the class, including an activity pertaining to the topic you have chosen that you will do with the class. You can do this assignment with your dyad partner but you may not use any of the historical mathematicians or discoveries in your packet. For this assignment you will be asked to address the following along with any other pertinent information you find:

a. When did this discovery happen?
b. What was going on in the world that made this discovery such a major event or even necessary?
c. Why does it matter to our lives?
d. What would it be like for the field of mathematics and our world in general if this discovery had not been made?

**** It is the responsibility of the student to contact the instructor and make any necessary and appropriate arrangements if any class or assignment is missed.**

Grading:

Grading is difficult in any class. A variety of assessment strategies have been included to provide a more valid understanding of your performance, professionalism, and potential. Collaboration and collegiality are essential components of a teaching professional and therefore are considered as part of your experiences and assessment in this class. Other aspects of professionalism include punctuality, readiness, participation, thoroughness, informed and grounded practice, and respect for teaching colleagues.

In accordance with the national governing boards of both the National Council of Teachers of Mathematics and the Interstate New Teacher Assessment and Support Consortium (INTASC) for mathematics, there are three broad areas of expectation that will be assessed in this class:

1. Knowledge of mathematics (including understanding of mathematical ideas and concepts, mathematical processes including problem-solving, reasoning,

communication and connections, and mathematical perspectives including historical, societal and cultural influences),

2. Knowledge of teaching mathematics (including pedagogical understandings and dispositions through which teachers of mathematics utilize effective practices), and

3. Professionalism of teaching (including your attitude, participation, cooperation, collaboration, conduct and dress code in the public school experience).

There is no way to separate your understanding of methods of teaching from knowledge of content. Therefore, your understanding of key mathematical concepts and processes as well as your knowledge of pedagogy will be assessed in the class along with your professionalism.

The following intervals will be used in grading:

1143 - 1270	A
1016 - 1142	B
889 - 1015	C
762 - 888	D
761 and below	F

Statement of Accommodation:
Any student in this course who has a disability that may prevent him or her from fully demonstrating his or her abilities should contact me personally as soon as possible so we can discuss accommodations necessary to ensure full participation and facilitate your educational opportunities.

University Absence Policy:

It is the policy of the University to excuse absences of students that result from religious observances and to provide without penalty for the rescheduling of examinations and additional required class work that may fall on religious holidays. Please advise me as early as possible so alternate arrangements can be made.

Inclement Weather:

If the university is closed, class will be cancelled. In case you are unsure, check your e-mail – I will e-mail everyone to confirm class cancellations by 8:00 the morning of the scheduled class time. If you do not have access to a computer at home make arrangements with someone in class to call you in the event class is cancelled.

Tentative Schedule (Additional Readings may be assigned throughout the Semester)

Week	Week Of…	Topics for Class and Assignments Due
1	January 18 & 19	*Go over syllabus. *Read the following articles from your packet for discussion next week: 1) Constructing Knowledge in the Classroom, 2) Writing, Reading and Talking Mathematics: One Interdisciplinary Approach, and 3) Teacher-Clinicians Encourage Children to Think as Mathematicians. **Always have your construction paper, markers, ruler, and oil pastels with you in every class!**
2	January 24 & 25	*Discussion of articles *In class writing assignment *Direction/listening activity *Cooperative learning, higher-order thinking skills, mathematics as communication *Internet Sites Due – send to everyone through Blackboard before class
3	January 31 & February 1	*Start Reading Tsuruda's Book - Putting it all Together. *Teach first of two lessons to peer groups ***Sign up for mathematics topics to be addressed in NEWSLETTER so that none are duplicated** ***Sign up for mathematics topics to be addressed in HISTORICAL PRESENTATIONS so that none are duplicated**
4	February 7 & 8	*Teach second of two lessons to peer groups *Tsuruda topics decided upon in class
5	February 14, 15, & 16	*Visit curriculum library * Write –up of this assignment due on February 21 & 22
6	February 21 & 22	*Tsuruda Write-up due * Write –up of curriculum assignment due
7	February 28 & March 1	*Newsletters and activities presented in class ***All Newsletters are due to me whether you present this week or next week**
8	March 7 & 8	*Newsletters and activities presented in class
9	March 14 & 15	Spring Break – no class
10	March 21 & 22	*Field Experience –minimum of two journal entries due on Friday

	& 22	
11	***March 28 & 29***	**Field Experience-minimum of two journal entries due by Friday*
12	***April 4 & 5***	**Field Experience-minimum of two journal entries due by Friday (you should have submitted a total of eight by this time)*
13	***April 11 & 12***	** Mathematics lessons (taught during FE) and reflections due* **Current topic of interest paper and copy of article due regarding teaching Mathematics, Resilience Education or NCLB - This can be a newspaper clipping, journal article, etc (2 - 3 page paper describing how this affects you as an educator)*
14	***April 18 & 19***	**Historical presentations due to present in class* ****All Historical Presentations are due to me whether you present this week or next week***
15	***April 25 & 26***	**Historical presentations due to present in class*
16	***May 2 & 3***	*Closure to class.*

Appendix C - Subjectivity Statement

What led me to my beliefs that mathematics involves connections, patterns, and relationships? I certainly didn't hold this belief as a K-12 student nor as a public school teacher. At a recent presentation a mathematics education person challenged me on this notion stating that mathematics was the study of numbers and space. At the time I emphasized that for me it is what happens within that space and with those numbers that really constitutes mathematics. Having thought more about it, I can see that the person was treating mathematics as an object and ignoring it's interconnectedness with nature and our being in the world. Our being in the world is what led me to my belief system and taking a course on chaos and complexity theory and courses on feminist theory is where I like to think that my mathematical journey began.

Prior to the class, I held the common objectifying notions that many hold in our modern society about mathematics. What changed for me? The Mandelbrot set in particular had an impact. This simple function with complex behaviors made me realize mathematics was not an exact science. There was both unpredictability and predictability, relationships, connections, and patterns. The idea that fractals were found in nature had the largest impact of all on me. Thinking of a leaf as mathematical opened an entirely new door for my views. Instead of viewing mathematics and my world being very separate experiences, I started to gain an understanding of the relationships between the two.

Edward Lorenz's work which found that it was impossible to predict long term weather patterns also had an impact me. His story helped me realize that mathematics, as I knew it at the time, wasn't the yard stick for everything in our world. I came to see that relationships must be a large part of mathematics for it even to be constituted as mathematics. If left alone with no conversation, mathematics is essentially useless; in fact it is not longer mathematics, in my view.

Another influence was from that of my mentor Jayne Fleener and her work as a curriculum theorist. In her book *Curriculum Dynamics*, she discussed her change in thinking concerning students and schooling in general. Her journey from thinking about "things" to "relationships" had a major impact on how she viewed the world. Fleener (2002) states:

> A shift in thinking about "things" to thinking about "relationships" and recognizing the impact of interconnectedness and context are more than a change in perspective, however. These shifts in thinking represent a fundamentally different way of seeing and approaching the world and all that is in it. Exploring relationships rather than examining things has implications for every facet of life. The logic of relationship has its own demands for value, purpose, and method. Examining and shifting our ideas about the purpose of schooling with an underlying logic of relationship has the potential to infuse the curriculum with meaning and purpose, rekindling the spirit and heart of learning. (p. 80)

I admit that it took the majority of my Doctor of Philosophy program to really absorb a lot of her ideas and that of other curriculum theorists because the perturbation to my thinking was many times overwhelming. So many of my previous ideas about the world were entrenched within me and not only did taking classes from Dr. Fleener help me, attending conferences and reading suggested theorists who inspired her, helped as well.

Susan Laird, another one of my mentors, opened me up to the world of feminist theory. It was in these classes that I started making connections between occurrences in my classroom teaching experiences and the theory I was being presented. Through the works she assigned, I was able to see many correlations between ecological feminism and chaos and complexity theory. I don't know that I would have ever been able to make my connections within mathematics education without feminist theory.

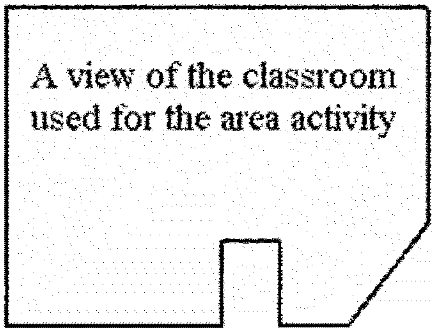
My experiences with public school teaching, chaos and complexity theory, and feminist theory led me to my beliefs that mathematics is more than just numbers; it is a way of being in the world.

Appendix D – The Area Problem and Problem of the Day 40

The Area Problem

Taped to the chalk board in the classroom were two pencils; one large and one small. The large one was similar to the type one would find as a novelty item from a keepsake store, while the small one was the size of a regular pencil. The instructor first asked students what they could tell her about the person who would have the bigger pencil. Responses included the person having big hands, being less coordinated, having a silly sense of humor, having smaller hands, being a bean stalk, being twice as big as a regular sized person, being fatter. The instructor then went on to ask, "How many people can we fit in the room who use the smaller pencil?" and she went on to ask the same question but with the bigger pencil. She wrote both questions on the board followed by asking them to find the area of the classroom.

Involving bodily movement, open-endedness, dialogue, and problem solving, the area activity was just one component to the relationship building occurring in the classroom. Bodily movement included students getting up out of their chairs to explore the classroom; open-endedness meant students were not told by the instructor how to approach the task; dialogue included a variety of conversations among groups and with the instructor about the problem-centered task; and problem solving involved elements of the process and content standards pertaining to measurement and problem solving (NCTM, 2000). Bodily movement, open-endedness, dialogue, and problem solving all involve a certain sense of being with mathematics – a way of engaging in both the social dynamics of mathematical conversation and a way of embodying (Davis, 1997) mathematics as a meaning-making activity.



Problem of the Day 40

Hank did not plan his finances well for his day in the city. First he spent half the money he had with him on breakfast in the train station. Then he spent half of what remained on the subway token. Then he checked his finances after drinking a $.75 glass of juice. He had only $.50 left. How much money did Hank have with him when he started the day?

Welcome to [redacted] Fifth Grade!	
Curriculum In this unit, we will be working on algebra, mainly equations. We will be using lots of group and individual problem solving, manipulatives, and other activities that will support cooperative learning. We use problem-centered learning in our classroom because we have found it to be the most effective way for students to learn, understand, and apply mathematical concepts.	**Welcome for Parents** We encourage any parent to come to our classroom. We want you to feel welcome and know that you are important to the success of your child's education. Contact us anytime through e-mail or weekdays after five/on weekends by phone. goodteachers@fakeaddress.com (405) 123-4567
Classroom Rules Our approach to rules is the same as our approach to learning: the students should have a voice. After a class meeting, the students decided that the golden rule would be the key to our classroom rules.	**Classroom Management** Classroom meetings and positive discipline are the management tools we use in our classroom.
NCTM Standards grades 3-5 algebra expectations • describe, extend, and make generalizations about geometric and numeric patterns; • represent and analyze patterns and functions, using words, tables, and graphs.	**PASS Standards** **5:1:1**~Describe rules that produce patterns found in tables, graphs, and models, and use variables (e.g., boxes, letters, pawns, number cubes, or other symbols) to solve problems or to describe general rules in algebraic expression or equation form. **5:1:2**~ Use algebraic problem-solving techniques (e.g., use a balance to model an equation and show how subtracting a number from one side requires subtracting the same amount from the other side) to solve problems.

How Children Learn	**What to Expect with Your Child**
Children learn math best through activities that encourage them to: • explore • think about what they are exploring; • solve problems using information they have gathered themselves; • explain how they reached their solutions. http://www.edu.gov.on.ca/eng/document/brochure/earlymath/howchildren.html	• Increased mathematical confidence • Better problem-solving skills • Deeper understanding of mathematical concepts • Ability to work well with other children/work towards a common goal
Classroom Activities	**Home Activities**
• Balancing "equations" on a scale • History of Algebra group project • Graphing temperatures over time to work on graph skills • Problems of the day • Group problem solving will take up the majority of the unit • Greg Tang's "Grapes of Math" problem creation	http://www.gamequarium.com/prealgebra.html http://www.kidsmath.com/Algebra.html http://www.funbrain.com/algebra/index.html
Assessments	**Supplies/Classroom Needs**
• Participation in class discussions/group and individual work • Journal entries • Completed group/individual class work • Quizzes • One end-of-unit test	• Weights for the balancing of equations • Markers for each child • Parent volunteers are always welcome. Just contact us through e-mail or phone to let us know you're coming so we can arrange an activity for you to assist.

March 2005: Place Value Unit

[redacted]

Description of Mathematics Curriculum

The number one goal for your children in math is for them to learn as much as they possibly can in the best environment for each child. In this classroom the mathematics curriculum will be primarily problem centered. Each student will be encouraged in mathematics at their own level and be pushed to make new goals and succeed in achieving those goals.

Support for cooperative Groups and not ability grouping

Cooperative groups are a great way to help the students reach their full capacity. It is so important that students fell comfortable in the groups that they are in and not just feel that they are the best/worst at mathematics. Doing cooperative groups can help the more advanced children actually help teach the ones that may not understand as much.

Validation behind problem centered Learning Environment

In Problem Based Learning (PBL) environments, students act as professionals and confront problems as they occur - with fuzzy edges, insufficient information, and a need to determine the best solution possible by a given date. This is the manner in which engineers, doctors, and, yes, even teachers, approach problem solving, unlike many classrooms where teachers are the "sage on the stage" and guide students to neat solutions to contrived problems.

Supporting research validating how children learn and what the teacher can do to help them

- Be committed to the idea that all children can become proficient in math.
- Develop and deepen your understanding of math, of student thinking, and of techniques that promote math proficiency.
- Use an instructional program and materials that, based on the best available scientific evidence, support the development of math proficiency.
- Teach mathematics for a sufficient amount of time (e.g., an hour a day).

Classroom Rules & Management

- Be respectful of other people and their property.
- Follow directions and work hard.
- Use inside voices in the school building.
- Pick up after yourself.

The students developed these classroom rules on the fist day of school. They began by brainstorming ideas for rules, which were based off of previous experiences and new ideas. Then, the students worked to eliminate statements covered elsewhere and combine rules in order to shorten their list. After working with these rules for a couple of weeks and making changes as needed, a committee of students created a "Rules Poster" to hang in the classroom.

Classroom management is an important part of having a productive day. With the use of these rules I (the teacher) must enforce the rules. Management is a daily process but needs to be consistent. If the student's name is spoken aloud that is the 1st warning, then the second time the student's name is called they know that they only have one more chance and on the third time they must retreat to the hallway where I will talk with them within 10 minutes.

Activities & Integration

Activities that we will use in the classroom

Place Value Bingo (the activity that will be done tonight)

- In order to prepare for this game, the teacher must create a set of index cards. For the first set of cards, write "ones" and "0". The next card will have "ones" and "1" on it, continue this process through number nine. Then create a set of cards for the "tens" 0-9. Make a set of cards up to and including the millions place.
- Each student will create a seven-digit number with each digit being a different number. The students will write their number in large print across a piece of paper.
- The teacher will shuffle the place value cards and will draw from the stack.
- The students who have this digit in their number may cover it with bingo chips. For example, if the teacher draws "9 thousands" all students who have a nine in the thousands place would cover that number.
- This repeats until one person covers their entire number. If they can call back each digit correctly and each was originally called, that student wins the game.
- At the completion of one round, the students may trade their number cards and play the game again.

Cheerios and Apple Jacks place value

We will be making our own place value necklaces using cereal. We will measure put 9 Cheerios on then the tenth will be an Apple Jack and so on. Using those we will count by tens to see how long their necklaces are.

Curriculum Integration

It is very important to integrate other subjects into the math curriculum. This means that in this classroom all the subjects (math, literature, social studies, science, music, and art) will be taught together, rather than in divided time slots. An activity we carry out throughout the school year allows students to develop their mathematical thinking and practice their writing skills.

- Students are asked to keep a learning log of their mathematics activities. In this log they will write about new concepts, problem solving strategies, and thought processes. Students use this writing activity to help understand their mathematical thinking.

This is an example of a short-term activity that involves science reasoning and mathematical reasoning. We are always completing activities such as this one.

- In the activity "Fat Finders" students will use food labels to determine the amount of fat in a meal they have chosen. This will be compared to the recommended 30% limit of calories from fat. From a mathematical standpoint, students use whole number operations, equalities/inequalities, ratios, decimals, percent, graphs, and estimation. From a scientific standpoint, students use observation, collecting and recording data, comparing and contrasting, and applying and learn about life science/health nutrition.

Upcoming Events

Goals for Place Value Unit

- To enhance number sense, problem solving strategies, and cooperative work (personal goal).
- Model the concept of place value through 4 digits. (PASS Standard 2, 1-a)
- Read, model and write whole numbers up to 4 digits. (PASS Standard 2, 1-b)
- Compare and order whole numbers up to 4 digits. (PASS Standard 2, 2-a)
- Understand the place-value structure of the base-ten numbers system and be able to represent and compare whole numbers and decimals. (NCTM)

Look For…

Your student should be developing an understanding for place value and what each digit within a number represents. Your child should be able to tell you things like how many tens are in 68, how many ones are in 103, how many thousands are in 5,981, and so on. The following are activities that you can do at home with your child to help enhance their understanding for place value:

- Go to this website: http://www.math.com/homeworkhelp/EverydayMath.html and click on the place value section. There are many interactive things that can be done on place value to help your child understand.

- When you are in the car driving, you and your child can play a version of the license plate game only we are going to be learning place value. You (parent) find a number on the car and ask your child what the place value is.

Assessment

I use a variety of assessment tools in order to determine what students know. I will choose various cooperative activities to grade, which is done by taking anecdotal records of a student's participation, effort, and application of known math concepts. I will also be grading some individual work by checking how the student solved a problem, the steps taken, and the ability to compute accurately. Finally, I will choose certain days to grade the students' math learning logs. On such days I will give them a specific prompt that will allow me to determine their level of understanding for a particular math concept.

My report cards look a bit different than what you may be used to. Here is an example:

Number Sense (level of understanding/application)……………………………0 through 8
Problem Solving (strategies used)…………………………………………1, 2, 3, 4, or 5
Cooperative Work (how student works with others)……………………………0, 3, 5, or 7
Explanation (ability to explain thought process)……………………………1, 2, 3, 4, or 5
Application (how students use math concepts)……………………………..1, 2, 3, 4, or 5
Total Possible Points……………………………………………………………………20

How to Help

We are always looking for supplies to use in our math activities. Please save the following items:

- Empty paper towel and toilet paper rolls
- Empty food boxes
- Index cards (any size)
- Beads and buttons
- Popsicle sticks
- Yarn

We would LOVE to have your help in the classroom anytime. The only requirement is that you call at least one day before you plan on visiting so that I may have an activity prepared for you to do. You may work with the children or help prepare materials for a lesson/activity.

Please feel free to contact me: [redacted]

I am available to accept phone calls from

- 7:00 – 7:45 am
- 10:30 – 11:45 am
- 2:45 – 4:00 pm

I look forward to hearing from you all!!

MONTHLY NEWSLETTER

[redacted]

Dear Parent,

Hello and welcome to our classroom! We would like to extend a big thank you for taking the time out of your schedule to read this and be an active part of your child's educational experience. It is our belief that children need the help of parents and educators working together to help them achieve the most from their education. We will be sending a newsletter home with your child every month to outline what we will be learning in math, as well as some ways you can work with your child at home to extend the curriculum. We are glad that you are willing to put your best foot forward to help us this year and look forward to working with you.

In this newsletter, we will outline the math curriculum that we use, as well as, our beliefs in education and some tools for you to use at home with your child. We hope that you find these newsletters helpful and productive for your child.

MATH CURRICULUM: SYMMETRY UNIT

The Math curriculum is outlined by the benchmarks identified by our state. We use many resources for each lesson. The ultimate goal of the Math curriculum is to develop a strong foundation in Mathematics that students will use to build on during their educational years. We want to make Math an enjoyable subject that does not frustrate the students. We also want our students to understand that Math is and will be an important aspect of their daily lives.

We designed this unit to provide opportunities for practical application and problem solving of geometric concepts, as well as communicating and developing mathematical reasoning. The van Hiele Theory of Geometric Thought, determined five levels of learning through which students must progress to succeed in geometry.

Level 0 (Basic Level): Visualization

Students recognize figures as total entities (triangles, squares), but do not recognize properties of these figures (right angles in a square).

Level 1: Analysis

Students analyze component parts of the figures (opposite angles of parallelograms are congruent), but interrelationships between figures and properties cannot be explained.

Level 2: Informal Deduction

Students can establish interrelationships of properties within figures (in a quadrilateral, opposite sides being parallel necessitates opposite angles being congruent) and among figures (a square is a rectangle because it has all the properties of a rectangle). Informal proofs can be followed but students do not see how the logical order could be altered nor do they see how to construct a proof starting from different or unfamiliar premises.

Level 3: Deduction
At this level the significance of deduction as a way of establishing geometric theory within an axiom system is understood. The interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof is seen. The possibility of developing a proof in more than one way is seen.

Level 4: Rigor
Students at this level can compare different axiom systems (non-Euclidean geometry can be studied). Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

This unit gives the students opportunities to improve their understanding of the first three levels: visualization, analysis, and informal deduction.

GOALS AND OBJECTIVES

Goal

Students will gain understanding and experience in real-world application of mathematical reasoning, problem solving, and principles of geometry through participation in activities that will provide instruction in the first three levels of van Hiele geometric thinking.

Objectives:
The students will:
1. Learn the word and the concept of symmetry.
2. Explore lines of symmetry.
3. Explore transformations: rotation (turn), translation (slide), reflection (flip).
4. Use problem-solving strategies to solve real-world problems.

PASS SKILLS:
Standard 4: Geometry and Measurement - The student will use geometric properties and relationships to recognize and describe shapes and use customary and metric measurements to solve problems.
1. Spatial Reasoning and Coordinate Locations
a. Describe and compare two- and three-dimensional shapes (e.g., count the edges and faces of a cube, combine or divide basic shapes to form new shapes, identify and draw congruent shapes).

NCTM STANDARDS:

- **identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;**
- **investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;**
- **explore congruence and similarity;**
- **make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.**
- **predict and describe the results of sliding, flipping, and turning two-dimensional shapes;**
- **describe a motion or a series of motions that will show that two shapes are congruent;**
- **identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.**

UNIT ACTIVITIES

Activity 1: Beautiful Butterflies (Art lesson)
Symmetry is defined as: having two parts (cut by an imaginary line) that are exactly the same.

- Child will be given a piece of paper and butterfly pattern. Have the child fold the piece of paper in half and place the butterfly pattern on the fold. The middle of the butterfly should be on the fold with wings extended outward.
- The child will trace the around the pattern.
- After the pattern is traced, the child will carefully cut the butterfly out on the line.
- The child will open the butterfly and note symmetry.
- Paint is applied to the inside of the butterfly on one side.
- The butterfly is closed and child presses on butterfly gently.
- Open the butterfly carefully to see the beautiful butterfly that has been constructed. It is symmetrical!

Activity 2: Symmetrical Sandwich

- After the child has finished his or her beautiful butterfly, a table will be set up for this edible experience.
- The child will go to the table and get a graham cracker. He or she will snap it in half so the cracker is symmetrical.
- The child will spread white icing on both pieces of the graham cracker.
- With various Skittles, the child will decorate his or her edible insect... making sure it is symmetrical. YUM! YUM!

Activity 3: On The Trail With Symmetry (Social Studies lesson)

Cowboy Math: Anticipatory Set - We will ask, "What is a cowboys best friend?" Take suggestions from class. Finally, tell class that a bandanna is a cowboy's best friend. Then talk to class about "Cowboy Math", a cowboy who likes to be perfect, so insists that when he wears a bandanna it must be symmetrical. The class will be helping out Cowboy Math today.

- First they will do the "Square Meal, Square Deal", folding their bandanna's in half making a fine napkin or a crumb catcher. Does this new design have a line of symmetry?
- A cowboy's food, or "grub" was piled onto a tin plate. Fold your bandanna in half again. Now you can put those hot plates right on your lap when you eat. Is this design symmetrical?
- You're full and need a hot drink for the cold, chilly night. Fold the bandanna in half and put it around you cup of hot chocolate. It will keep you hand from being burned. Is this design symmetrical?
- Cowboys spent their day outdoors. Bandannas helped them look cool and keep cool in hot weather. Make a triangle with your bandanna. You could dampen this triangle with cold water and put it under your cowboy hat to keep cool. You could also put it around your neck to mop your face when you sweat. Is this design symmetrical?
- A bandanna comes in handy when you need a bag. Or when you need a tent for you pet snake. Fold the bandanna to make a triangle. Is this design symmetrical?
- Fold it again. Tie both ends in a knot. (You can open the "bag" so that one side has three layers of cloth and the other only one.) Is this design symmetrical?
- Say you've found some chunks of gold. Or maybe you're carrying a pocket watch your great grand - daddy gave you. Wrap your keepsakes in your bandanna by folding each corner into the middle. Place your precious loot in the first layer of folds. Is this design symmetrical?

WHAT PARENTS CAN DO TO HELP

Some activities that you as parents can do with your children in order to help them to understand, explore and extend the concept of symmetry can include:

1. Parents can ask their children to find different symmetrical objects around the house and to explain why they think they are symmetrical.
2. After children work with the bandanna at school parents can ask their children to find other ways to fold the bandanna symmetrically. They can share their discoveries with the entire family. Then ask them also to find ways to fold the bandanna that aren't symmetrical.

We are going to work using collaborative learning which is a method of teaching and learning in which students team together to explore a significant question or create a meaningful project. Cooperative learning, which will be one of the important focus and approaches to be use during the activities in this unit, is a specific kind of collaborative learning. In cooperative learning, students work together in small groups on a structured activity. They are individually accountable for their work, and the work of the group as a whole is also assessed. Cooperative groups work face-to-face and learn to work as a team. It is very important to note that in small groups, students can share strengths and also develop their weaker skills. They develop their interpersonal skills. They learn to deal with conflict. When cooperative groups are guided by clear objectives, students engage in numerous activities that improve their understanding of subjects explored.

For years people have been doing research on how children learn. Many studies have shown that students learn best when they participate in hands on activities like using manipulatives, blocks, beads, to just name a few. The NCTM Curriculum and Evaluation Standards encourage teachers to allow students to work with patterns, sort, and count, classifying objects, using number games, and exploring geometric figures. According to R.N Caine and G. Caine in their Understanding the Brain Based Approach to Learning and Teaching article, exposing children to problem-centered learning, students are allowed to establish a foundation in mathematics that they can build on year after year.

CLASSROOM BEHAVIOR CONTRACT

During the first days of school, we involved the students in creating our classroom rules. The rule-making process begins when we posted four questions to the students:

- How do you want us to treat you?
- How do you want to treat on another?
- How do you think we want to be treated?
- How should we treat one another when there's a conflict?

Students' shared their thoughts about those questions in small groups, and then with the entire class. Responses were posted on a large sheet of chart paper. As an idea is repeated, a checkmark or star is placed beside it. After we finished, we have all the students sign the 'poster' as a commitment to follow the class rules. Then we took it to the local copy center and have it reduced to notebook size. We made enough copies for everyone. Students keep their copies in their notebooks. Some of the rules are:

1. Follow Directions
2. Complete Work On Time
3. Respect Fellow Classmates
4. Raise Your Hand And Wait To Be Called On
5. Stay On Task
6. Respect Other People's Property

7. Always Do Your Best

We also set a list of consequences for breaking a classroom rule because we consider they are at least as important as the rule itself. Every teacher must create consequences with which they are comfortable (or follow set school procedures). Our teacher's list of consequences for breaking classroom rules follows:
-First time: Name on board. Warning.
-Second time: Student fills out a form that asks them to identify the rule they've broken and what they plan to do to correct the situation. (Teacher keeps the form on file.)
-Third time: Isolation from class/team.
-Fourth time: Call home to parents.
-Fifth time: Office referral.

ASSESSMENT:

To assess the students on this unit,we will have them construct a rug using symmetrical figures. Each student will draw a symmetrical object and then as a class, they would put all the symmetrical objects drawn together. This would then create a rug that we would hang on the wall for display. This assessment uses problem solving.

To assess students in a problem solving curriculum, there are changes that differ from the traditional classroom. We will be giving test and there will be homework, but the test will be much like quizzes and the homework will be more like outside of class observations and group activity assignments.

Much of our assessment will weigh heavily on portfolios, observations, journals, group assignments, and participation. It is not simply enough for your child to have the right answer or the correct amount of point, but rather to have a sound understanding of the subject matter.

MATERIALS:

- Butterfly shapes cutouts, paint, and brushes.
- Graham crackers, icing and Skittles.
- Bandanna (one per two students), paper, pencil

Other materials needed for the class:

- paper
- notebooks for journals
- binders for portfolios
- pencils
- rulers
- compasses
- Other items will be asked for throughout the semeter

We hope that you are as anxious and excited for this semester as we are looking forward to meeting you and working with you throughout the semester. Our door is always open and we would love to visit with you. There is not a problem that is neither too small nor too big. We will always be there to work with you and your child. If you do come to our classroom, please give us a days notice so that we can make sure the children are in the room. We do many hands on activities and sometimes this takes us outside of the classroom. We would love to have you and will put you to work. The kids like it best when you are working with them side by side. We want you to enjoy your visit so if for any reason you would like to just observe let us know.

Remember, if you need ANYTHING from us, please call us and let us know how we can help. Thank you again for your time and your concern.

[redacted]

Historical Presentation

[redacted]

Longitude and Latitude

The Colledege will the whole world measure,
Which most impossible conclude,
And Navigators make a pleasure
By finding out the longitude,
Every Tarpalling shall then with ease
Sayle any ships to th 'Antipodes

1. When did the discovery happen?
o This happen
o In 1421 The Year China discovered the World
2. What was going on in the world that made this discovery such a major event or even
necessary?
o In 1421, when the fleets of hong baou and zhou Man had sailed south-west from the entrance to the Caribbean towards the coast of South American, they had left the fleet of Admiral Zhou Wen taking a course to the north-west following the northern branch of the equatorial current.
o China discovered the World by using the lunar eclipses technique
o The Chinese navigator and astronomers use the lunar eclipse to determine longitude
o By determining the differences in the time when the event took place, as observed from the separate location they could then calculate the difference in longitude.
o Use clocks to calculate the longitude, clepsydras (water clock)
o In the Mediterranean it was difficult to go very far astray, and in western and northwest Europe navigation coastal.
o The only reference points when traveling were on the high seas were the stars and Sun.
o The Portuguese pioneered the method of navigating by latitude.
o Ships used the instruments astrolabes, cross staffs to measure the altitudes of stars of the Sun.

Latitude:
o has a great effect on climate and weather
o Dates back Ancient Greece
o Latitude is basically an inherent property of the earth. It is just a measure of the angle between the horizon and the North Star. It measures 0 degrees at the equator and 90° at the North Pole. So, wherever you are on the earth, all you have to do is measure the angle

to the North Star, convert to miles (or km), and you know how far you are from the equator or the North Pole.o This has been known since antiquity; the ancient Greeks knew the earth was round, about how big it was, and about the North Star, which is pretty easy.

Longitude:

o The first effective solution for mapmaking was achieved by Giovanni Domenico Cassinni starting in 1681

o Using Galileo's method based on the satellites of Jupiter.

o It all started with clocks.

o Eratosthenes calculated the Earth's circumference and he was the first to attempt to produce a map of the World based on a system of lines of latitude and longitude.

o Hipparchus was the first to specify the positions of places on the Earth using latitude and longitude as coordinates. His work on spherical trigonometry led him to this system. He suggested measuring latitude, the distance north and south of the equator, by determining the ratio of the longest to the shortest day at the place

o Measuring longitude was problem until the l8 century, when the British Parliament issued a huge monetary prize, that the problem was solved by **John Harrison.** He won the longitude prize

o In the middle of 1 7th century in England: English attack on the longitude problem: Important people met at the Academic Royale and discussed about the problems of longitude and latitude.

o Longitute is an entirely different matter. In order to know your position on the earth you need two coordinates, one for the X axis and one for the *Y* axis. The trouble is that measuring in the other axis is dependent on time, because that earth rotates. Also, there is no fixed, natural zero point; that is, a place to start measuring.

o The problem of measuring longitude then, was dependent on good clocks. It wasn't until the 18th Century, when the British Parliament issued a huge monetary prize, that the problem was solved by a man named John Harrison.

3. Why does **it matter to our lives?**

o If you were to ask to find where a particular place was in the world, how would you do it?

o If a pilot or ship's captain wants to specify position on a map, how would they do that and how would they travel? They will need to know and understand the "coordinates" they use.

o The measurement is important to both cartography and navigation; the discovery of how to measure it accurately was among the important discoveries of the 1600's and 1700s.

4. What would it be like for the like for the field of mathematics and our world in general if this discovery **had not been made?**

o No navigation

o No map

o Effects time zones, weather, climates

o No understanding of clocks
o Where to locate at country, state, city, or county on a map
o How to travel from one place to another
o How to calculate the time it takes to go to one place to another
o How to measure the distance from the North Star to the equator
o To know your position on the earth.
Interesting facts to read and work on:
• This is what allows ship and airline pilots to pinpoint their locations as they travel. a If you enjoy hiking or skiing, it is also the magical numbers that will allow rescuers to find you with a GPS (global positioning system) device if you were to get lost or caught in an avalanche.
• Full story about longitude: By Dava Sobel

OKLAHOMA
Ada 34°47'N 96°41'W
Altus AFB **340** 39' N **990** 16' W
Ardmore **340** 18'N **970** 1'W
Bartlesville 36° *45'* N 96° 0' W
Chickasha *35*° 3' N **970** *55'* W
Enid, Vance AFB 36° 21' N 97° *55'* W
Lawton AP **340** 34' N 98° *25'* W
McAlester **340** *50'* N 95° 55' W
MuskogeeAP *35°40'N* 95°22'W
Norman *35°15'N* 97°29'W
Oklahoma City AP (S) **350** 24' N 97° 36' W
Ponca City 36°44'N 97°6'W
Seminole *35*° 14' N 96° 40' W
Stiliwater (S) 36° 10' N **970** *5'* W
Tulsa AP 36° 12' N 95° *54'* W
Woodward 36° 36' N 99° 31' W

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6. http://www.nationalgeographic.com/xpeditions/lessons/0l /gk2/longlat.html
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11. http://www.ncgia.ucsb.edu/education/curricu1a/giscc/units/u0 1 4/uO 1 4.html
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• The history of the discovery of longitude
• John Harrison: the invention chronometer and H:5 longitude time-keeper

[redacted]
Historical presentation

Geometry History

Geometry in ancient times was recognized as part of everyone's education.

Egyptians
c. 2000 - 500 B.C.
Ancient Egyptians demonstrated a practical knowledge of geometry through surveying and construction projects. The Nile River overflowed its banks every year, and the river banks would have to be re-surveyed. In the Rhind Papyrus, pi is approximated.
The best-known sources of ancient Egyptian mathematics in written format are the Rhind Papyrus and the Moscow Papyrus. The sources provide undeniable proof that the later Egyptians had intermediate knowledge of the following mathematical problems, applications to surveying, salary distribution, calculation of area of simple geometric figures' surfaces and volumes, simple solutions for first and second degree equations. They found pi to equal C/D or 4($/9) whereas a equals 2. The method for ancient peoples arriving at this numerical equation was fairly easy. They simply counted how many times a string that fit the circumference of the circle fitted into the diameter, thus the rough approximation of 3. The biblical value of pi can be found in the Old Testament (I Kings vii.23 and 2 Chronicles iv.2) in the following verse "Also, he made a molten sea often cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about." The molten sea, as we are told is round, and measures thirty cubits round about (in circumference) and ten cubits from brim to brim (in diameter). Thus the biblical value for pi is 30/10 3.

Babylonians
c. 2000 - 500 B.C.
Ancient clay tablets reveal that the Babylonians knew the Pythagorean relationships. One clay tablet reads
4 is the length and 5 the diagonal. What is the breadth? Its size is not known. 4 times 4 is 16. 5 times 5 is 25. You take l6from 25 and there remain 9. What times what shall I take in order to get 9? 3 times 3 is 9. 3 is the breadth.

Greeks
c. 750-250 B.C.
Ancient Greeks practiced centuries of experimental geometry like Egypt and Babylonia had, and they absorbed the experimental geometry of both of those cultures. Then they created the first formal mathematics of any kind by organizing geometry with rules of logic. Euclid's (400BC) important geometry book The Elements formed the basis for most of the geometry studied in schools ever since.

The Fifth Postulate Controversy
c. 400 B.C.- 1800A.D.
There are two main types of mathematical (including geometric) rules: postulates (also

called axioms), and theorems. Postulates are basic assumptions - rules that seem to be obvious and are therefore accepted without proof. Theorems are rules that must be proved.
Euclid gave five postulates. The fifth postulate reads: Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line. Euclid was not satisfied with accepting the fifth postulate (also known as the parallel postulate) without proof. Many mathematicians throughout the next centuries unsuccessflilly attempted to prove Euclid's Fifth.

The Search for pi
??? B.C. - present
It seems to have been known from most ancient of times that the ratio of the circumference and diameter of a circle is a constant, but what is that constant? A search for a better answer to that question has intrigued mathematicians throughout history.

Coordinate Geometry
c. 1600 A.D.
Descartes made one of the greatest advances in geometry by connecting algebra and geometry. A myth is that he was watching a fly on the ceiling when he conceived of locating points on a plane with a pair of numbers. Maybe this has something to do with the fact that he stayed in bed everyday until 11:00 A.M. Fermat also discovered coordinate geometry, but it's Descartes' version that we use today.

Non-Euclidean Geometries
c. early 1800's
Since mathematicians couldn't prove the 5th postulate, they devised new geometries with "strange" notions of parallelism. (A geometry with no parallel lines?!?) Bolyai and Lobachevsky are credited with devising the first non-Euclidean geometries.

Differential Geometry
c. late 1800's-1900's
Differential geometry combines geometry with the techniques of calculus to provide a method for studying geometry on curved surfaces. Gauss and Riemann (his student) laid the foundation of this field. Einstein credits Gauss with formulating the mathematical fundamentals of the theory of relativity.

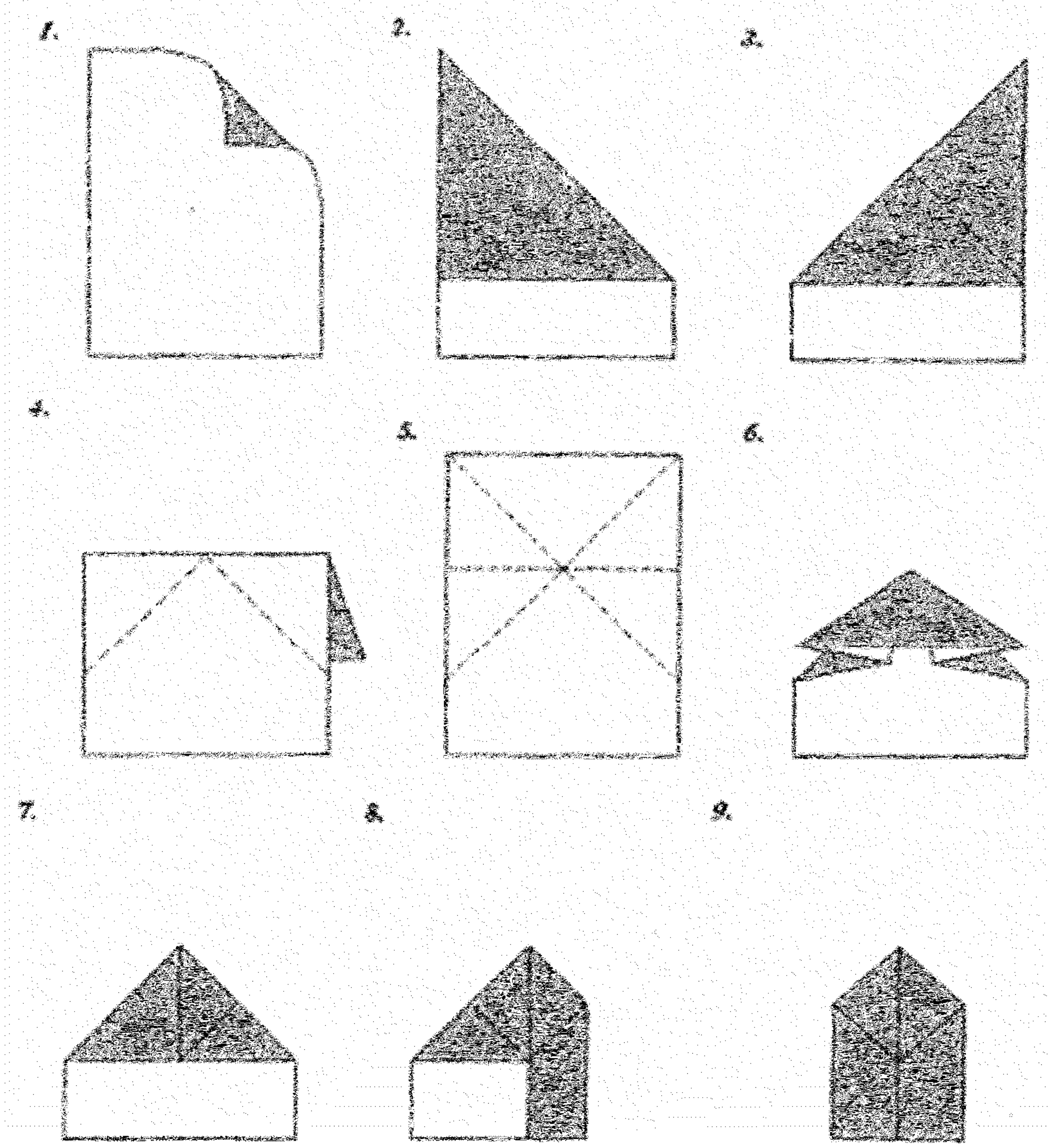
Fractal Geometry
c. late 1800's-1900's
Fractals are geometric figures that model many natural structures like ferns or clouds. The invention of computers has greatly aided the study of fractals since many calculations are required. Mandlebrot is one of the researchers of fractal geometry
To build a tall building, one requires a strong foundation; and, to get the American students at a height that the reformists hope them to achieve, students need to start constructing the foundations of understanding at the earliest. This means that we have to start at the base with elementary students if we want to meet the raised expectations for middle and high school mathematics.

It is during their elementary years that young children begin to lay down those habits of reasoning upon which later achievement in mathematics will crucially depend. The power to reason mathematically is a natural human capacity. Young children enter school already curious about number and size, and with ideas about how to join, remove, and split quantities. Mathematics instruction in the elementary years can should be designed to cultivate this curiosity.

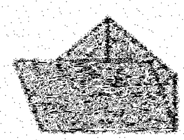
Hence, it is our duty as educators to make sure that we present young minds with every opportunity to make exciting discoveries and grow intellectually into students who have the understanding to appreciate the value of mathematics and enjoy the challenges that come with it. To make an effect, we need to introduce reforms at the beginning because anything later than the earliest might be too late.

For our activity we decided to do a frog using an origami technique (see attached pages.)

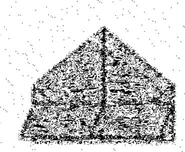
Make an Origami Jumping Frog



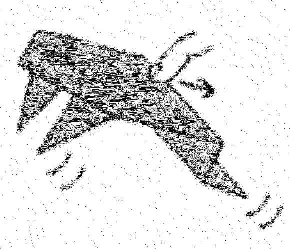
10.



11.



12.



The drawing shows how to make the frog hop: run your finger down his back, compressing his back legs like a spring.

Appendix G – Consent Forms

INFORMED CONSENT FORM FOR RESEARCH BEING CONDUCTED UNDER THE AUSPICES OF THE UNIVERSITY OF OKLAHOMA-NORMAN CAMPUS

INTRODUCTION: This study is entitled Creating Spaces for Learning in the Mathematics Classroom: A Phenomenological Study of Pre-Service Teachers. The person(s) directing this project is (are) Kerri Richardson, Graduate Research Assistant, and Neil Houser, Instructional Leadership and Academic Curriculum. This document defines the terms and conditions for consenting to participate in this study.

DESCRIPTION OF THE STUDY:
[This study explores the phenomenon of being in the world with mathematics.]

Procedures:

1. Participant observation will occur during the semester.
2. Voluntary interviews will take place during a time that is convenient for both the interviewer and interviewee.
3. The interview will take approximately thirty minutes and the instructor will ask approximately 7 questions, giving the interviewee plenty of time to verbally answer. The responses will be taped using an audio tape recorder. After transcription, tapes will be destroyed. Pseudonyms will be used.
4. Responses will be used in a dissertation of the researcher using fictitious names to differentiate between responses.

RISKS AND BENEFITS: Individual participants may gain insight into their own learning and aid in finding what is meaningful to them from participating in the study by discussing their ideas and introspection concerning particular contexts or concerns. Students will also gain important insights into the teaching of mathematics to their future students. This should be a significant benefit to society in general. No risks are anticipated.

CONDITIONS OF PARTICIPATION: Participation is voluntary. Refusal to participate will not involve penalty or loss of benefits to which the subject is otherwise entitled. Furthermore, the participant may discontinue participation at any time without penalty or loss of benefits to which the participant is otherwise entitled.

CONFIDENTIALITY: Findings will be presented in aggregate form with no identifying information to ensure confidentiality.

CONTACTS FOR QUESTIONS ABOUT THE STUDY: Participants may contact Kerri Richardson, 325-2599, krichardson@ou.edu or Neil Houser, 325-1498, nhouser@ou.edu with questions about the study.

For inquires about rights as a research participant, contact the University of Oklahoma-Norman Campus Institutional Review Board (OU-NC IRB) at 405/325-8110 or irb@ou.edu.

PARTICIPANT ASSURANCE: I have read and understand the terms and conditions of this study and I hereby agree to participate in the above-described research study. I understand my participation is voluntary and that I may withdraw at any time without penalty.

Signature of Participant	Date
Printed Name of Participant	Researcher Signature

TAPE RECORDED INTERVIEW CONSENT SCRIPT

Date

Dear _________________:

I am a graduate student under the direction of Professor Neil Houser in the Instructional Leadership & Academic Curriculum Department at The University of Oklahoma-Norman Campus. I invite you to participate in an interview as part of a research study being conducted under the auspices of the University of Oklahoma-Norman Campus *[sponsored by The College of Education ILAC Department]*, entitled Creating Spaces for Learning in the Mathematics Classroom: A Phenomenological Case Study of Pre-Service Teachers. The purpose of this study is to explore mathematical experiences and beliefs of education majors.

Your participation will involve sitting at a table with me and verbally answering a series of about 7 questions and the interview will be audio-tape recorded. It should only take about 30 minutes. Your involvement in the study is voluntary, and you may choose not to participate or to stop at any time. The results of the research study may be published, but your name will not be used. In fact, the published results will be presented in summary form only. All information you provide will remain strictly confidential and released only with explicit written permission.

The findings from this project will provide information on mathematics teaching and learning with no cost to you other than the time it takes for the interview.

If you have any questions about this research project, please feel free to call me *[or Dr. Houser]* at (405) 325-2599 or send an e-mail to krichardson@ou.edu. Questions about your rights as a research participant or concerns about the project should be directed to the Institutional Review Board at the University of Oklahoma-Norman Campus at (405) 325-8110 or irb@ou.edu.

I would like to audio-tape this interview. Do I have your permission to audio tape the interview?

Thanks for your help!

Sincerely,

Researcher's Name ***Kerri Richardson***
Researcher's Title ***Graduate Research Assistant***

The University of Oklahoma

OFFICE OF HUMAN RESEARCH PARTICIPANT PROTECTION

January 14, 2005

Ms. Kerri Richardson
820 Van Vleet Oval, Rm 100
Norman, OK, 73069

RE: Exempt from IRB Review
IRB Number: FY2005-209
Title: Creating Spaces for Learning in the Mathematics Classroom: A Phenomenological Study of Pre-Service Teachers

Dear Ms. Richardson:

The Institutional Review Board considers that this research is exempt in accordance with the Code of Federal Regulations, Title 45, Part 46, Sub-part 101 (b), Category:

1. Research conducted in educational settings involving normal educational practices, such as research on instructional strategies, techniques, curricula, or classroom management methods.
2. Research using cognitive, diagnostic, aptitude, and educational achievement tests, or surveys, interviews, or observations of public behavior, unless human subjects are identifiable, and disclosure of responses would put them at risk of liability, or damage to their reputations or financial standing.

as revised November 13, 2001. Further review of this study by the IRB is not required unless the protocol changes with regards to the use of human subjects. In that case, the study must be resubmitted immediately to the Board. Please inform the IRB when this research is completed.

If you have any questions related to this research or the IRB, you may telephone the IRB staff at 405.325.8110 or visit our web site out irb@ou.edu.

Cordially,

Lynn Devenport, Ph.D.
Vice Chair
Institutional Review Board – Norman Campus (FWA #00003191)

FY2005-209

cc: Dr. Neil Houser, Instructional Leadership & Academic Curriculum

660 Parrington Oval, Suite 316, Norman, Oklahoma 73019-3085 PHONE: (405) 325-8110 FAX: (405) 325-2373

The University of Oklahoma

OFFICE FOR HUMAN RESEARCH PARTICIPANT PROTECTION

February 21, 2005

Ms. Kerri Richardson
820 Van Vleet Oval, Rm 100
Norman, OK, 73069

SUBJECT: **"Creating Spaces for Learning in Mathematics Classroom: A Phenomenological Study of Pre-Service Teachers"**

Dear Ms. Richardson:

The Institutional Review Board has reviewed and approved the requested revision to the subject protocol:

- Add scale: Pirie-Kieren Model for Dynamic Understanding

Please note that this approval is for the protocol and informed consent form initially approved by the Board on January 12, 2005, and the revision(s) included in your request dated February 21, 2005. If you wish to make other changes, you will need to submit a request for revision to this office for review.

If you have any questions, please contact me at 325-8110.

Cordially,

Gregory [illegible], Ph.D.
Vice Chair
Institutional Review Board - Norman Campus (FWA #00003191)

FY2005-299

cc: Dr. Neil Houser, Instructional Leadership & Academic Curriculum

[illegible]