MODEL SELECTION FOR VALUE-AT-RISK:

UNIVARIATE AND MULTIVARIATE

APPROACHES

By

SANG JIN LEE

Bachelor of Science in Mathematics
Yonsei University
Seoul, Republic of Korea
1999

Master of Business Administration
Yonsei University
Seoul, Republic of Korea
2004

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Thesis Approved:

Timothy L. Krehbiel

Thesis Adviser

B. Wade Brorsen

Zhigang Zhang

A. Gordon Emslie

Dean of the Graduate College

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This thesis is meaningful to me in two aspects; first, through this thesis, I have had a chance to summarize what I learned in the Master of Science in Quantitative Financial Economics program at Oklahoma State University. Second, my long-term objective has been achieved. I finished my undergraduate degree in 1999 just after the Asian financial crisis. After that crisis, everything in the Korean financial market changed. Korea moved from a Japanese business model to an American model. The rules of the game changed. At that time I decided that someday I would go to the US to learn the new rules. This thesis is a product of beginning to learn them.

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INTRODUCTION

Value-at-Risk (VaR) is the most popular tool in risk management because it is easy to communicate and easy to comprehend. The importance of VaR is rapidly increasing because the international agreement in banking industry the, so-called, Basel Accord heavily uses VaR methodology. To manage market risk, the Basel Accord requires a financial institution to have capital in proportion to the total value of its risk-adjusted asset which is basically measured by VaR in the internal models approach. This rule is accepted by the Group of Ten (G-10) countries[1] and many other countries. So, banks in those countries evaluate their risk exposure using this VaR methodology.

Jorion (2000) intuitively defined VaR as the summary of the worst loss over a target horizon with a given level of confidence. For example, the chief financial officer of a financial company might say that the VaR of the bank is $10 million at a 95 percent confidence level over one day horizon, which means that there is a 5% probability for a loss greater than $10 million to happen under normal market conditions. So, since VaR is a single number summarizing the amount of risk the company is exposed to, it is very easy to understand and communicate.

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[1] G-10 countries are Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, United Kingdom, and the United States, plus Luxembourg and Switzerland.

However, the VaR methodology requires distributional assumptions for the relevant risk factors. Moreover, the VaR estimate depends on not only the assets class constituting portfolio, but also the model used to estimate the volatility of those assets. In this regard, it is valuable to investigate which volatility model produce superior risk measurement for a given portfolio.

In the past study, Sarma et al (2003) studied the model selection for VaR estimation in the S&P 500 and India's NSE-50. Using a two-stage model selection procedure, they compared the performance of candidate volatility models such as equally weighted moving average(EQMA) model, exponentially weighted moving average(EWMA) model, the GARCH model, and the historical simulation(HS) model. They found that the EWMA model worked best among those models. Angelidis and Alexandros (2004) analyzed the application of several volatility models to forecast daily VaR both for single assets and portfolios. They considered models such as the GARCH model, the EWMA model, the exponential GARCH model, the threshold ARCH model, the extreme value theory (EVT), and the HS model. They found that the best model depended on portfolio.

Some researchers warned of limitations of the VaR approach in risk management. Bedder (1995) examined eight VaR estimates for three hypothetical portfolios. He found that VaR is very effective measure in risk management, but since it depends on parameters, data, assumptions, and methodology, it might be dangerous in its application. Hendricks (1996) compared 12 different VaR evaluation approaches using simulated data from eight foreign exchange markets. He concluded that in almost all cases these approaches covered the risk that they were intended to cover. However, he also observed that VaR estimates from different approaches were quite different, which also implies that VaR approach is overall good tool for risk management, but needs some cautions in application.

The comparison of the univariate approach with the multivariate approach of VaR evaluation might also be a very interesting question. In evaluating the portfolio VaR, the multivariate model can have some advantages over the univariate model. According to Bauwens et al. (2004), one advantage of a multivariate model is that once we get the covariance matrix by the multivariate approach, we do not need to calculate again the covariance matrix even if the weights of each asset are changed; under the univariate model, we should evaluate the variance of portfolio again whenever the weights of each asset are changed. Another advantage is that a multivariate model may improve the evaluation performance in updating the variances and correlations by considering the individual characteristics of the portfolio's components and estimating their linear comovement. According to the Longin and Solink (1995), the markets become more closely related during periods of high volatility. In this period, considering the individual correlation among stocks might increase the model accuracy. So, it is a good research question whether multivariate models perform better than univariate models.

In the literature of the multivariate VaR approach, Manfredo and Leuthod (1999) investigated various VaR estimation techniques for the agricultural enterprise portfolio; the EQMA model, the EWMA model, the GARCH model, the implied volatility model, and the HS model as univariate model, and constant conditional correlation model as multivariate models. They found that the EWMA model and the HS model provided reasonably good estimates. Brooks and Persand (2000) found that the multivariate

GARCH(1,1) model worked best to get a VaR estimate relative to other models like the HS model, the RiskMetrics approach, and the modified RiskMetrics approach using daily closing stock prices of five Southeast Asian countries. Engle (2002) compared VaR estimates from various methods such as the BEKK[2] model, the Dynamic Conditional Correlation (DCC) model, and the orthogonal GARCH (O-GARCH) model, the multivariate EQMA model, and the multivariate EWMA model. He observed that the DCC model overall performed best to evaluate VaR under the various situations. Rombouts and Verbeek (2004) examined the usefulness of the multivariate semi-parametric GARCH models for portfolio selection under a Value-at-Risk (VaR) constraint. They also examined several alternative multivariate GARCH models for daily returns on the S&P 500 and NASDAQ indexes.

To tell a good model from a bad model, we need some criteria. One obvious property that a good model should have is predictability. That is, a good model should do a good job in predicting future risk exposure. Another criterion is whether the model uses all information available. If a prediction model does not use all information available, its prediction ability will be lowered, which means that the model is inferior. There are various statistical methods based on these ideas. We will review them in the later section.

So, this thesis will address two questions:

1. Which univariate models are appropriate to evaluate VaR of the Dow Jones Industrial Average (DJIA).
2. Considering multivariate volatility models such as the DCC model and the O-GARCH model, which incorporate conditional correlations among assets, as

[2] BEKK came from Baba, Engle, Kraft and Kroner who were contributors to the model.

well as univariate models, which models are appropriate to evaluate VaR for a hypothetical portfolio.

For the first question, we will focus on the univariate model. After that, we will turn our attention to the multivariate model for the second question. For these questions, we need some judging criteria, which will be introduced later.

In the following sections, we will review the VaR concepts, which will be followed by a review of the various methods to evaluate VaR. After that, we will move to the model discerning criteria. The empirical result of the univariate models will be presented and discussed first. Then, the result of the multivariate models will follow, and the comparison of both models will be discussed. Then, we will draw some conclusions and implications.

This thesis is different from the existing studies from two points: data and comparison. This thesis uses two sets of data: the DJIA and a hypothetical portfolio. Most past research used a portfolio consisting of two, three or, at most, five stocks. But, in this thesis we use a hypothetical portfolio consisting of 30 stocks to test the performances of the multivariate models. One other point is that it seems that little study has been done about the comparison of the multivariate VaR estimate methods with the univariate VaR estimate methods *using the same portfolio* in the VaR literature. So, the most distinctive point of this thesis is that comparison. This will allow us to determine the value of conditional correlation estimation in this VaR application.

CHAPTER II

Value-at-Risk

Jorion (2000) formally defines VaR as the description of the quantile of the projected distribution of gains and losses over the target horizon. If *c* is the selected confidence level, VaR corresponds to the 1 – *c* lower-tail levels. Mathematically, it can be formulated like this:

$$c = \int_{-VaR}^{\infty} f(x)dx \tag{1}$$

where *x* is a random variable of the profit/loss of portfolio, *f(x)* is the distribution of *x*, and *c* is the selected confidence level. If the profit/loss distribution of portfolio is assumed to follow a normal distribution with zero mean, then we can get a VaR estimate in a very easy way as follows.

2.1 Single Asset

If the profit/loss distribution of an asset is assumed to follow a normal distribution with zero mean, then

$$VaR = -z_c \times V_0 \times \sigma \tag{2}$$

where z_c is the critical value at confidence level c, V_0 is the initial value of the portfolio, and σ is the estimated standard deviation of the portfolio's return. For instance, assume that the initial value of an asset is $1, the return of the asset follows a normal distribution with zero mean and standard deviation of σ. The tomorrow's VaR estimate at 95% confidence level over one day horizon is 1.645σ in figure 1.

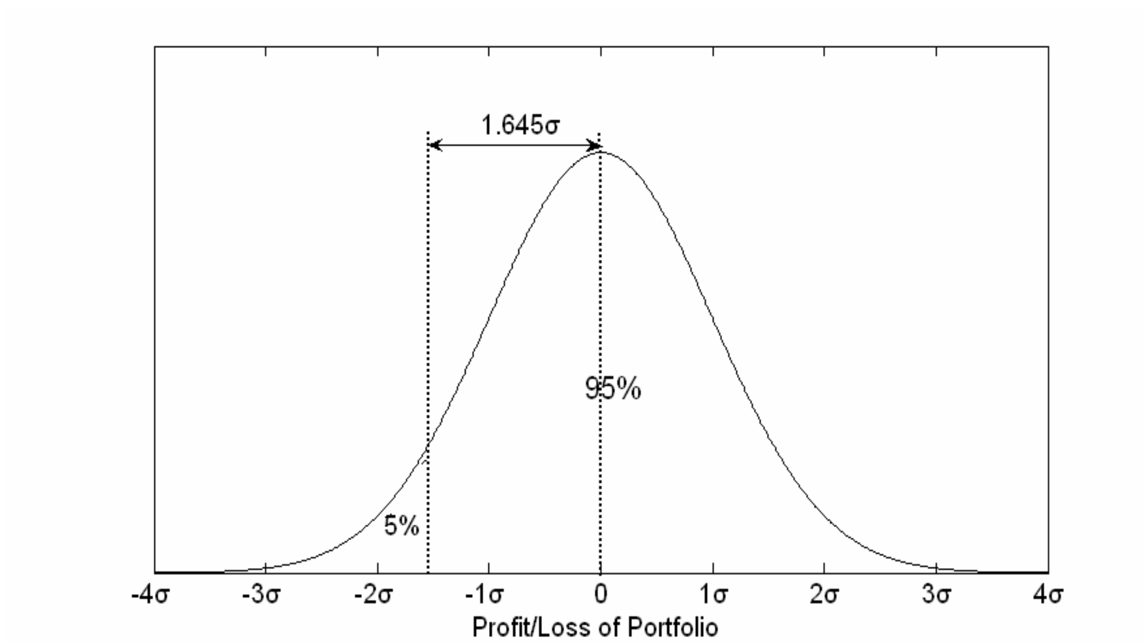

Figure 2.1: VaR Estimate and Profit/Loss Distribution

2.2 Portfolio

In the portfolio theory, the return (r_p) of a portfolio is defined as follows:

$$r_p = \sum_{i=1}^{n} w_i r_i \qquad (3)$$

where n is the number of assets in the portfolio, r_i is the return of ith asset for i = 1..n, and w_i is the weight of ith asset in the portfolio for i = 1..n. The variance (σ_p^2) of the portfolio can be calculated as follows:

$$\sigma_p^2 = \sum_{i=1}^{n} (w_i \sigma_i)^2 + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{i,j} \tag{4}$$

where σ_i is the variance of *i*th asset, $\sigma_{i,j}$ is the covariance between *i*th asset and *j*th asset. In equation (4), the first term is called diversifiable risk or non-systematic risk which can be eliminated through diversification and the second term is called undiversifiable risk or systematic risk which can not be eliminated through diversification.

To investigate the power of diversification[3], we consider a strategy where weights are equal to 1/n. Then

$$\sigma_p^2 = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{n} \sigma_i^{\ 2} + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \frac{1}{n^2} \sigma_{i,j} \tag{5}$$

We define the average variance ($\bar{\sigma}^2$) and average covariance ($\bar{C}$) of the assets as

$$\begin{aligned} \bar{\sigma}^2 &= \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2 \\ \bar{C} &= \frac{1}{n(n-1)} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \sigma_{i,j} \end{aligned} \tag{6}$$

, then portfolio variance can be expressed as

$$\sigma_p^2 = \frac{1}{n} \bar{\sigma}^2 + \frac{n-1}{n} \bar{C} . \tag{7}$$

Here, when the number of assets in the portfolio increases, the first term, non-systematic risk, will disappear, but the second term, systematic risk, will converge to $\bar{C}$. In general, we can say that the risk of a well-diversified portfolio comes from only systematic risk or covariance part in equation (7).

The variance of the portfolio can be expressed in the matrix form as

[3] You can see more detailed discussion on this in Chapter 8 of Bodie et al (2002).

$$\sigma_p^2 = \begin{bmatrix} w_1 & w_2 & ..... & w_n \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \dots & \dots & \sigma_{1,n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \sigma_{n,1} & \dots & \dots & \sigma_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ .. \\ w_n \end{bmatrix} \tag{8}$$

or

$$\sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}$$

where $\boldsymbol{w}$ is the vector of the weights of the portfolio, and $\boldsymbol{\Sigma}$ is the covariance matrix. If risk factors follow a normal distribution with zero mean, the VaR of the portfolio can be calculated as

$$VaR_p = -z \times V_0 \times \sigma_p \tag{9}$$

We note that the relation between the mean and the standard deviation depends on the length of the time horizon. Since the volatility grows with the square root of time and the mean with time for independent identically distributed processes, the mean will dominate the volatility over long horizons. Over short horizons, such as a day, volatility dominates. This provides a rationale for focusing on volatility ignoring expected returns or assuming that those are zero when we evaluate VaR measures using daily data. In this thesis, since we will use the daily returns of 30 stocks, we also assume that the expected return of the daily return of each stock is zero. The expected return of the portfolio can be assumed zero because the expected return of the portfolio is the weighted average of the expected returns of the stocks. This leads us to just focus on the standard deviations of the stocks and portfolio to evaluate VaR.

CHAPTER 3

VaR Evaluation Models

There are two approaches to evaluate the VaR in a portfolio sense. The first approach is to create a univariate return series for the portfolio using the weight of each asset, and then we can use univariate models which will be reviewed. The other approach is to estimate a multivariate variance-covariance matrix, and then we can evaluate VaR by using equation (7).

We will use five univariate models and two multivariate models to evaluate VaR. Five univariate models are the EQMA model, the EWMA model, the GARCH model with a normal distribution, the GARCH model with a t-distribution, and the HS model[4].

As multivariate models, we will use the O-GARCH model and the DCC model because these can be easily applied to a portfolio consisting of many assets. In fact, most frequently used and cited multivarite volatility models are the Vech[5] model, the BEKK model, the DCC model, and the O-GARCH model. However, as indicated in Table 3.1, the Vech model and the BEKK model are practically not available for a portfolio consisting of many assets, which leads us to use just the DCC model and the O-GARCH model.

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[4] Here, we do not consider the Monte Carlo simulation method because we will use the linear portfolio, in which case the result from the Monte Carlo simulation should be the same as the result from variance-covariance approach.

[5] Vech is the name of a mathematical operator.

Table 3.1: Number of Parameter Needed According to the Number of Assets

#. Assets	Vech Model	BEKK Model	DCC Model	O-GARCH Model
n	$\frac{n(n+1)}{2}+2\left(\frac{n(n+1)}{2}\right)^2$	$\frac{n(n+1)}{2}+2n^2$	$3n+3$	$3n$
30	432,915	2265	93	90

Note that in this table we assume that the lags of all ARCH and GARCH parameters are 1

3.1 Univariate Models

We will review five models: the EQMA model, the EQMA model, the GARCH model with a normal distribution, the GARCH model with a t-distribution, and the HS model.

3.1.1 Equally Weighed Moving Average Model

The simplest one is the equally weighted moving average model, where today's volatility is calculated by the average of the volatility over the given time window. The mathematical formula is[6]

$$\sigma_t^2 = \frac{1}{m}\sum_{i=1}^{m}\left(r_{t-i}\right)^2 \tag{10}$$

where σ is the standard deviation, n is the number of daily rate changes used to calculate standard deviation, r_t is daily return. In fact, this model gives equal weight 1/m to each volatility of the past. So, that is why this model is called equally weighted moving average model.

[6] By using *m* instead of *m*-1 in the denominator, we assume that the volatility estimate in equation (5) is the maximum likelihood estimator, not the unbiased estimator. See Hull (2003) chapter 17.

3.1.2 Exponentially Weighted Moving Average Model

It seems more reasonable to assume that today's volatility is more affected by the more recent events. To incorporate this into the model, we should give more weight to the more recent events and less weight to the latter events. One of these weight schemes is an exponential scheme. The model using this exponential weight scheme is

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2 \quad (11)$$

where σ_t is the standard deviation for day n, r_{t-1} is the daily shock for day *n*-1, and λ is the decay factor[7]. In the iterative way, we can easily show that[8]

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{m} \lambda^{t-1} (r_{t-i})^2 \quad (12)$$

, which shows that the weights for the r_t 's decline at rate λ as we move back through time. This model is called exponentially weighted moving average model. According to the technical document of the RiskMetrics (1996), it uses the EWMA model with λ = 0.94 for updating daily volatility estimates. In this thesis, we used λ = 0.94 because we used daily data.

3.1.3 GARCH Model

There is another weight scheme called generalized autoregressive conditional heteroskedasticity (GARCH) model, which is proposed by Bollerslev (1986). The GARCH (*p*,*q*) process is then given by

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[7] We note that since in this thesis the expected mean of the price is assumed to be zero, the daily shock is equal to the daily return, that is, $\varepsilon_n = r_n$ for day n. So, hereafter the daily shock means the daily return, and vice versa.

[8] Here, we assume that the volatilities before time (*t*-*m*) are so small that they can be ignored.

$$\varepsilon_t \mid \psi_{t-1} \sim \mathrm{N}(0, \sigma_t),$$
$$\sigma_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i} \tag{13}$$

where $p \geq 0, q > 0, \alpha_0 > 0, \alpha_i \geq 0, \beta_i \geq 0$. When p=q=0, ε_t is simply white noise. It is generally accepted that in most cases GARCH(1,1) model is enough to model the volatility of financial market. So, we will use GARCH(1,1) model in this thesis, and hereafter GARCH model means GARCH(1,1) model. The meaning of GARCH model is that today's volatility(σ_t) is updated by yesterday's volatility(σ_{t-1}) and yesterday's shock(ε_{t-1}). We note that in fact the EWMA model is a particular case of the GARCH(1,1) model where α_0 =0, α_1 = 1 - λ, and α_2 = λ.

Another variation of the standard GARCH model is to use the student t distribution instead of normal distribution as the conditional distribution which the daily return follows. In fact, the conditional distribution as well as the unconditional distribution of the daily return is generally considered to have fatter tails than a normal distribution. So, if we use the student t distribution instead of a normal distribution as the conditional distribution the daily return follow, we are supposed to get more realistic result.

3.1.4 Historical Simulation model

The historical simulation method uses historical data to build the distribution of the risk factor, and then evaluate VaR from that distribution. In the case of the single index, we get historical movements or series of returns ($r_t, t = 1,..,n$) of the index. Based on those movements, we can get the simulated tomorrow's value of the index V_{n+1} as:

$$V_{n+1} = (r_t + 1) * V_n \tag{14}$$

where $t = 1,..,n$ and V_n is the today's portfolio value. After that, we can construct the distribution of the *change* in the portfolio ($= V_{n+1} - V_n = (r_t + 1) * V_n - V_n = r_t, t = 1,..,n$). And, then sort $r_t (t = 1,..,n)$ observations from the biggest loss to the biggest gain. This arrangement can be considered as the distribution of the risk factor. If we want to get 95% VaR, the 5th quantile of that distribution is what we want to get.

3.2 Multivariate Model.

In multivariate models, we might consider weight schemes that are similar to weight schemes in the univariate models like a moving average and GARCH. However, we note that if we use the multivariate variance-covariance matrix proposed by RiskMetrics (2001), equally and exponential weighted schemes produce the same result in both approaches[9], that is, the volatility from the univariate return series is always equal to the volatility from the multivariate variance-covariance matrix because they use the same methods in updating the volatility and the covariance. So, we will review just two GARCH type models: the O-GARH model and the DCC model.

3.2.1 Orthogonal GARCH Model

According to Alexander (2001), the orthogonal method uses principal component analysis (PCA) approach to construct covariance matrices[10]. In the orthogonal GARCH model, the time-varying covariance matrix $\boldsymbol{H_t}$ of the original system is approximated by

$$\boldsymbol{H_t = AD_t A'} \tag{15}$$

[9] Comparison between univariate approach and multivariate approach is in appendix A.
[10] See the appendix B for details of PCA approach.

where ***A*** is the matrix of rescaled factor weights and $\boldsymbol{D_t}$ is the time-varying diagonal matrix of variances of the principal components of original multivariate series. The diagonal matrix $\boldsymbol{D_t}$ of variances of principal components is estimated using a GARCH model.

We note that basically PCA technique is a linear transformation from one space measured on real-world basis into the other space measured on so-called principal component basis. In the latter world, we can analyze real-world data in a different point based on principal components, which are mutually orthogonal; hence we don’t need to pay attention to correlation between components. So, the correlations in the real world are transformed into the variances of the principal components. Hence, analyzing dynamics of variances of principal components implicitly incorporate dynamics of correlations of real-world data.

3.2.2 Dynamic Conditional Correlation model

Engle (2002) proposed a new class of multivariate GARCH models named dynamic conditional correlation model. The DCC model evolved from the constant conditional correlation (CCC) model by Bollerslev (1990). The CCC model estimates conditional covariance matrix H_t as:

$$H_t = D_t R D_t \;\; where \;\; D_t = diag\{\sqrt{h_{i,t}}\} \tag{16}$$

where ***R*** is a constant correlation matrix, $h_{i,t}$ is the conditional covariance of univariate time series of asset *i* in the portfolio at time *t*.

The DCC model assumes the correlation matrix is time-varying, that is, R_t instead of constant R and uses the GARCH scheme to incorporate that time-varying property of the correlation into the model as follows:

$$H_t = D_t R_t D_t \;\; where \;\; D_t = diag\{\sqrt{h_{i,t}}\}$$

$$R_t = diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1} \tag{17}$$

$$Q_t = (1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n)\overline{Q} + \sum_{m=1}^{M} \alpha_m (\varepsilon_{t-m} \varepsilon'_{t-m}) + \sum_{n=1}^{N} \beta_n R_{t-1}$$

where $\boldsymbol{H_t}$ is a time-varying covariance matrix, α_m is the ARCH coefficient, β_n is the GARCH coefficient, M is the order of ARCH parameter, N is the order of GARCH parameter, $\overline{\boldsymbol{Q}}$ is the unconditional covariance of the standardized residuals, and ε_{t-m} is the standardized residual from univariate time series, and $diag\{X\}$ mean a diagonal matrix of matrix $\boldsymbol{X}$. According to Engle (2002), if $\alpha_m \geq 0$, $\beta_n \geq 0$, and $(1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n) \geq 0$, then R_t will be positive semi-definite. If any one of them is positive, then R_t will be positive definite.

Engle and Sheppard (2001) stated that the DCC model was designed to allow for two stage estimation. In the first stage, univariate GARCH models are estimated for each residual series. Then, in the second stage, residuals, transformed by their standard deviation estimated during the first stage, are used to estimate the parameters of the dynamic correlation.

CHAPTER 4

Backtesting

We can get the various VaR estimates depending on the model we used to measure volatility. So, we need some criteria to decide which is better. According to Campbell (2005), a good model should have two properties based on the result of rolling backtesting; unconditional coverage property and independence property.

4.1 Rolling Backtesting Procedure

First, we will describe rolling backtesting. The following example will best explain the procedure of rolling backtesting. Suppose we have 1500 observations of past returns of a portfolio and we use 1000 observations to estimate tomorrow's VaR estimate. Using observations from the first to the 1000th, we got tomorrow's VaR estimate of $1,000 and the tomorrow's realized observation or the 1001st observation is $1,010, then *we say that an exception*[11] *is realized or there is an exception*; if the 1001st observation is less than the VaR estimate, we say that there is no exception. Next, using observations from the second to the 1001st, we can do the same comparison whether the VaR estimate is exceeded by the 1002nd observations or next day's realized loss of the portfolio. Continuing this comparison from the 500th to the 1499th observation with 1500th

[11] Some authors use a term, "exceedance" instead of "exceptions" because the realized loss exceeds the expected loss or the VaR estimate.

observation, each comparison is regarded as one backtesting. Then we have A total of 500 backtestings. The procedure is described in figure 2. With this binomial sequence (exception or no-exception), we can do the unconditional coverage property and independence property.

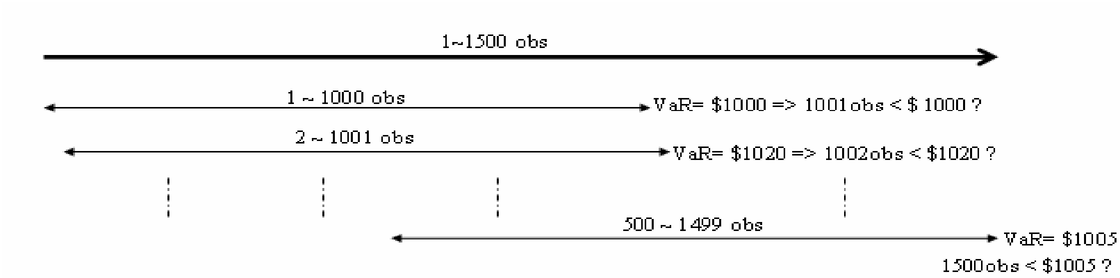

Figure 4.1: The Procedure of the Rolling Backtesting

4.2 Proportion of Failures Test

Unconditional coverage property means that the number of realized VaR exceptions of rolling backtesting with past data must be equal to the expected number of VaR exceptions indicated by the VaR model within statistical tolerance. For example, 1-day 99% VaR with 500 backtesting trials expects 5 exceptions (= (1 - 99%) × 500). If the realized exception is out of the range in which both are statistically equal[12], then we can conclude that the model is inappropriate.

Kupiec (1995) proposed the proportion of failures (PF) test as the unconditional coverage, which we will use in this thesis. The LR statistic for testing the null hypothesis that the realized ratio (*p*) of the exceptions of VaR over the past data is equal to the probability *p** of the exceptions of VaR is the following:

$$PF = -2\,\mathrm{Ln}[(1-p^*)^{n-x}(p^*)^x] + 2Ln[(1-\frac{x}{n})^{n-x}(\frac{x}{n})^x] \qquad (18)$$

[12] The null hypothesis of the statistical test is that the number of exceptions is 10.

where x is the realized number of exceptions in the sample, n is the total number of backtesting trials. Under the null hypothesis, $p = p^*$, the PF test has a chi-square distribution with 1 degree of freedom.

If the null hypothesis is rejected because the realized ratio is greater than the expected ratio, we can say that the model underestimates VaR. On the other hand, the null hypothesis is rejected because the realized ratio is less than the expected ratio, we can say that the model overestimates VaR. One possible reason of underestimation is that the distribution of the return series of financial asset usually has a fatter tail than the normal distribution and the model fails to incorporate that fat fail fully.

4.3 Runs Test

Independence property means that the exceptions of the backtesting should occur in a *random* way. If the occurrence of the exceptions is not distributed randomly across time, we can find some patterns, which a good model should incorporate with its prediction schemes. In this thesis we will use the runs test to test a randomness of the exceptions[13].

Runs can be defined as a sequence within a series in which one of the alternatives occurs on consecutive trials. Using the example of a coin toss, if a series look like this: “ H H T H H T T T H T”, then “HH”, “T”,”HH”,”TTT”,”H”, and ”T” are runs. The null hypothesis of a runs test is whether the distribution of a series of binary events in a population is random. In order to calculate the test statistics, one must determine the number (n_1,n_2) of times each of the two alternatives appears in the series and the

[13] You can find more details about the runs test in Sheskin (2003).

number(r) of runs in the series. The basic idea of a runs test is that the number of runs should be within the appropriate range for the series to be random. In the above example, the number (n_1) of heads is 5 and the number (n_2) of tails is 5, and the number(r) of runs is 6. If the number of runs is too small, say, 2, then it might be difficult to say that the series is random because the series should be "H H H H H T T T T T" or "T T T T T H H H H H". The normal distribution can be employed with a large sample size to approximate the exact distribution of the runs test as the following:

$$z = \frac{r - [\frac{2n_1 n_2}{n_1 + n_2} + 1] - .5}{\sqrt{\frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}} \qquad (19)$$

where r is the number of run, n_1 is the number representing alternative 1 which can be defined as non-exception occurrence of backtesting trials in this thesis, and n_2 is the number representing alternative 2 which can be defined as exception occurrence of backtesting trials in this thesis. In the example above (n_1=5, n_2 =5, r=2), z-score is -3.0187, which obviously results in rejecting the null hypothesis or non-randomness.

Possible reasons of the rejection of null hypothesis are clustering of exceptions and increase of the number of exceptions. As exceptions are clustered, the number of runs will decrease, and as the number of exceptions increases, n_1 will decrease and n_2 will increase in equation (19), which results in higher chance of the rejection of null hypothesis. For example, when n_1 is 3532, n_1 is 51, and r is 99, z-statistics is -1.8235(p-value=.0684). When n_1 is 3531, n_1 is 52, and r is 100, z-statistics is -2.3418(p-value=.0192). However, when n_1 is 3531, n_1 is 52, and r is 99, z-statistics is -2.9286(p-value=.0034).

CHAPTER 5

Data

We will use the DJIA index to address which univariate VaR model performs best. The DJIA data came from the Yahoo finance website. The time horizon of data is from 10/31/1986 to 12/31/2004 and the total number of sample of the DJIA closing price is 4584. We used the daily logarithmic return such as

$$R_t = 100 \times [\log(P_t) - \log(P_{t-1})] \qquad (20)$$

where P_t is the closing price on day t.

For the multivariate analysis, a hypothetical portfolio is considered to compare the performance of a univariate model with that of a multivariate model. We used 30 stocks of the DJIA components at 12/31/2004[14] and gave the same weight 1/30 to each stock to construct the portfolio. We got data from Center for Research in Security Prices (CRSP). The time horizon of data is from 10/31/1986 to 12/312004 and the number of observations is 4583[15]. We use the daily logarithmic return for each stock as follows:

$$R_{i,t} = 100 \times [\log(P_{i,t}) - \log(P_{i,t-1})] \qquad (21)$$

[14] The roster of DJIA has changed over time. So, we took a snap-shot at 12/31/2004. The same company is identified by the same PERMNO which is given by CRSP database. See data description guide for the CRSP US stock database and the CRSP US indices database. Here is the list of the companies: Alcoa Inc, AIG, American Express Inc, Boeing Co, Citigroup Inc, Caterpillar Inc, Du Pont E I De Nem, Walt Disney-Disney C, General Electric Co, General Motors, Home Depot Inc, Honeywell Intl Inc, Hewlett Packard Co, IBM, Intel Cp, Johnson And Johns Dc, JP Morgan Chase Co, Coca Cola, Mcdonalds Cp, 3M Company, Altria Group Inc, Merck Co Inc, Microsoft Cp, Pfizer Inc, Procter Gamble Co, SBC Communications, United Tech, Verizon Commun, Wal Mart Stores, Exxon Mobil Cp.

[15] The 5-24-1994 data for Altria Group was not available. So, all the data for that day were removed.

where $P_{i,t}$ is the adjusted price for factors like stock split and spin-offs, and includes dividend because we want to focus on the price movements which are not caused by corporate events such as stock split and spin-offs. Table 5.1 shows the descriptive statistics. The histograms and plots are in figure 5.1.

Here, we want to note several facts. First, both means of the DJIA daily returns and the hypothetical portfolio are so small relative to standard deviation that our assumption to ignore the mean of daily returns seems reasonable. Secondly, Ljung-Box Q test statistics show that there are autocorrelations also in both cases. ARCH LM test statistics show that there are ARCH effects in both cases. Thirdly, Jarque-Bera test statistics show that the unconditional distribution of daily returns is far from normal in both cases. We can also confirm that by the histograms in figure 5.1. Finally, we would like to pay attention to the tail property of both portfolios. The left tail and the right tail of both portfolios are fatter than those of a normal distribution as indicated in the QQ-plot of figure 5.2. Also, we can observe that the left tails of both are more deviated from the normal distribution than the right tail. In Table 5.2 and figure 5.2, though the biggest and the smallest return of the DJIA are respectively greater and less than those of the hypothetical portfolio, overall the hypothetical portfolio has a fatter tail than the DJIA.

Table 5.1: Descriptive Statistics of DJIA Daily Returns

Variables	DJIA	Hypothetical Portfolio
Observations	4583	4582
Mean	0.0381	0.0558
Maximum	9.6662	8.1318
Minimum	-25.632	-23.522
Std. Dev.	1.1283	1.1841
Skewness	-2.76534	-1.9532
Kurtosis	66.1222	41.6577
Jarque-Bera	**766,697.4** [a] **(<.0001)** [b]	**288,222.9** **(0.0000)**
Q(12)	**233.5851** **(<.0001)**	**378.9437** **(<.0001)**
ARCH-LM	**186.6556** **(<.0001)**	**276.921** **(<.0001)**

[a] Bold means that the number is statistically significant.
[b] The number in parenthesis is p-value.

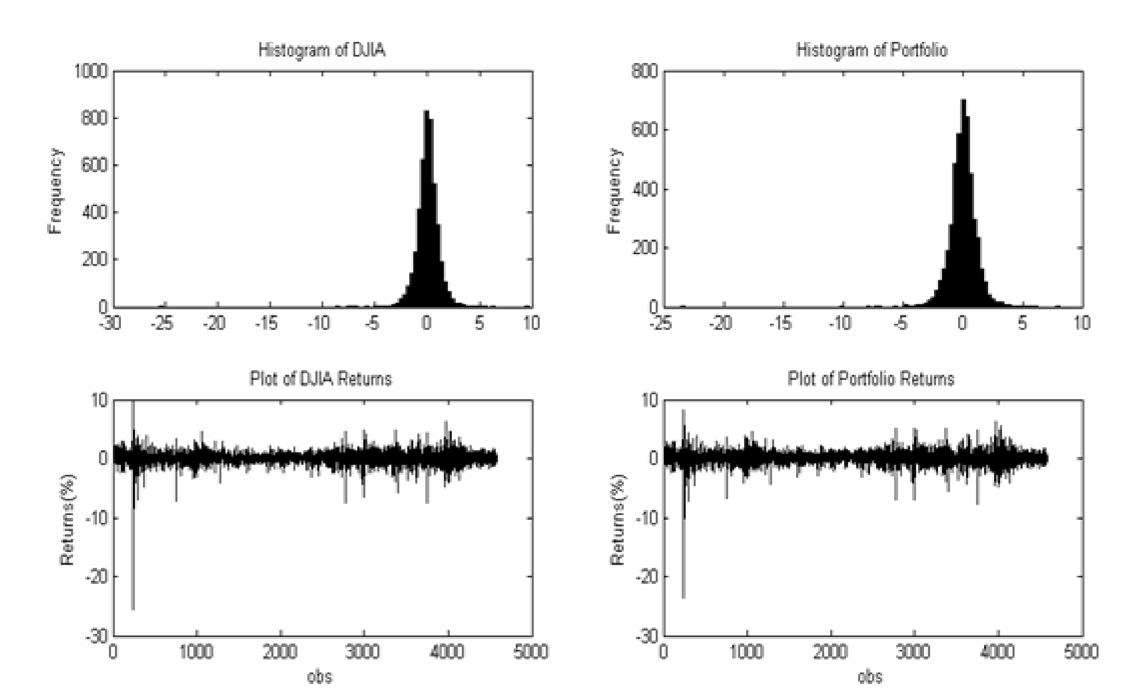

Figure 5.1: Histograms and Plots of DJIA and Portfolio

Table 5.2: Returns Sorted from the Biggest to the Smallest

Percentile	DJIA	Hypothetical Portfolio
Biggest	9.6662	8.1318
99%	2.8854	3.0996
98%	2.2521	2.4435
97%	1.9949	2.1032
95%	1.6469	1.7536
…..	……	……..
5%	-1.6275	-1.705
3%	-2.0137	-2.0808
2%	-2.3061	-2.354
1%	-2.8905	-2.9701
Smallest	-25.632	-23.522

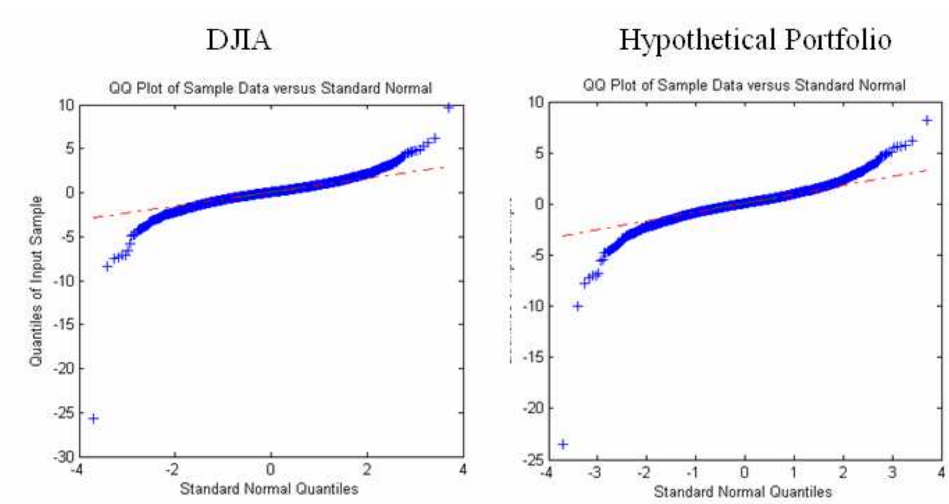

Figure 5.2: QQ-Plot of the DJIA and the Hypothetical Portfolio

CHAPTER 6

Empirical Result

We examine the univariate return series of the DJIA index and the hypothetical portfolio of 30 stocks using univariate models. Then, we examine the multivariate return series of 30 stocks using multivariate models. For each examination, we investigate the left tail of the distribution of a portfolio value which is relevant to the holder of a long position. We also investigate the right tail which is relevant to the holder of a short position.

6.1 Univariate Approach

We will consider two univariate return series; one is from the DJIA for the first research question and the other is from the hypothetical portfolio consisting of 30 stocks for the second question.

6.1.1 Dow Jones Industrial Average

We backtested the appropriateness of 99% 1-day VaR calculated from each univariate model in the section 3.1. We used 1000 observations to calculate the VaR estimate at each backtesting trial, so the total number of backtesting trials is 3583 [=

4583(the total number of observations) – 1000(observations to get one VaR backtesting trial)].

The VaR estimate for each model is in figure 6.1 and figure 6.2 for each tail when we invested $100 for each trial. In figure 6.1, the results using three models (the EWMA model, the GARCH model with a normal distribution, and the GARCH model with a t-distribution) seem very similar to each other. Overall, the VaR estimate using EQMA is the smallest. In figure 6.3 and 6.4, you can compare the VaR estimate with the realized loss. Though realized losses are smaller than VaR estimates in most cases, there are some cases that realized losses are greater than VaR estimates, which cases are thought of as exceptions.

The result of the PF test and runs test are presented in Table 6.1. In the left tail, all models except the EQMA model were not rejected with the PF test at a 99% confidence level[16]. All models except the EWMA model and the GARCH model with a normal distribution were not rejected with the runs test at a 99% confidence level. As a result, two models, the GARCH model with a t-distribution and the HS model were not rejected with both tests. Among these, the HS model show the nearest number of exceptions to the expected number of exceptions 36 which is 1% of the number of backtesting trials 3582.

In the right tail, all models except the EQMA model were not rejected with the PF test at a 99% confidence level. All models were not rejected with the runs test at a 99% confidence level. So, all models except the EQMA model were not rejected with the PF test and the runs test.

---

[16] Strictly speaking, we should say that the null hypothesis of the PF test (or the runs test) related to a model was rejected or not rejected with the PF test (or the runs test). But, in this thesis, if we have no problem in communication, for convenience, we would like to say that a model was rejected or not rejected with the PF test (or the runs test) to mean the same thing.

As a result, in both right and left tails the GARCH model with a t-distribution and the HS model were not rejected with both the PF test and the runs test. Other three models were rejected or inappropriate to evaluate VaR of the DJIA index portfolio with respect to either the PF test or the runs test.

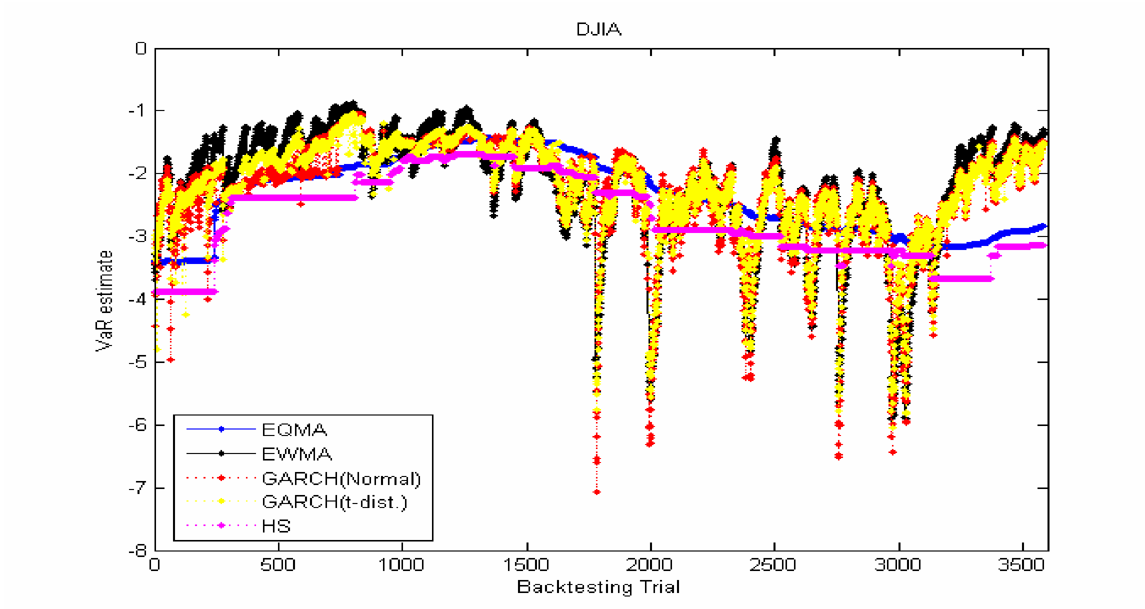

Figure 6.1: VaR Estimates for DJIA When $100 was Invested (Left Tail)

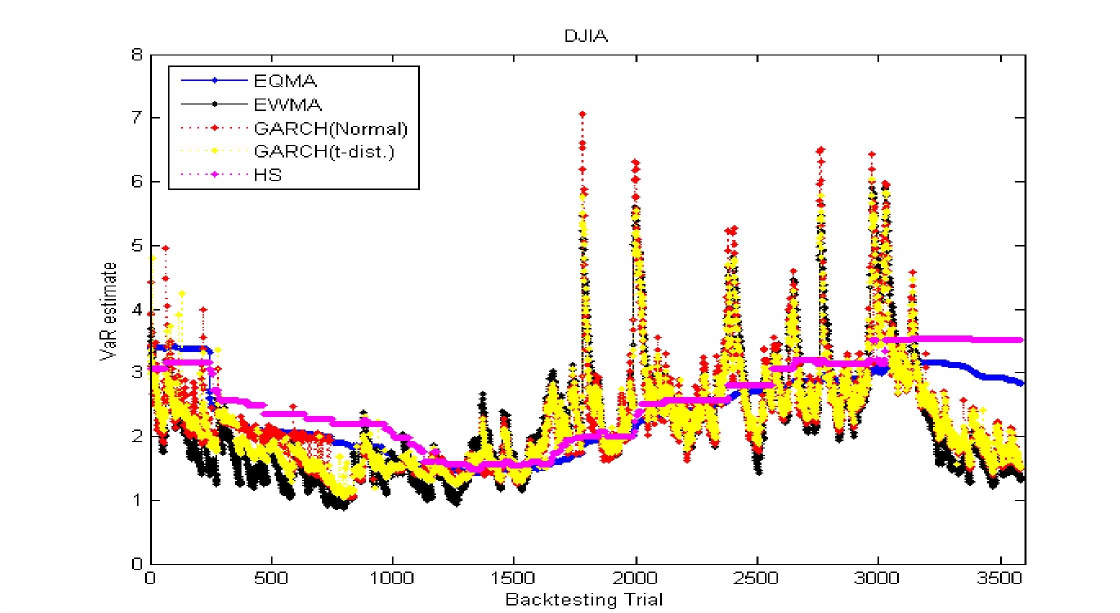

Figure 6.2: VaR Estimates for DJIA When $100 was Invested (Right Tail)

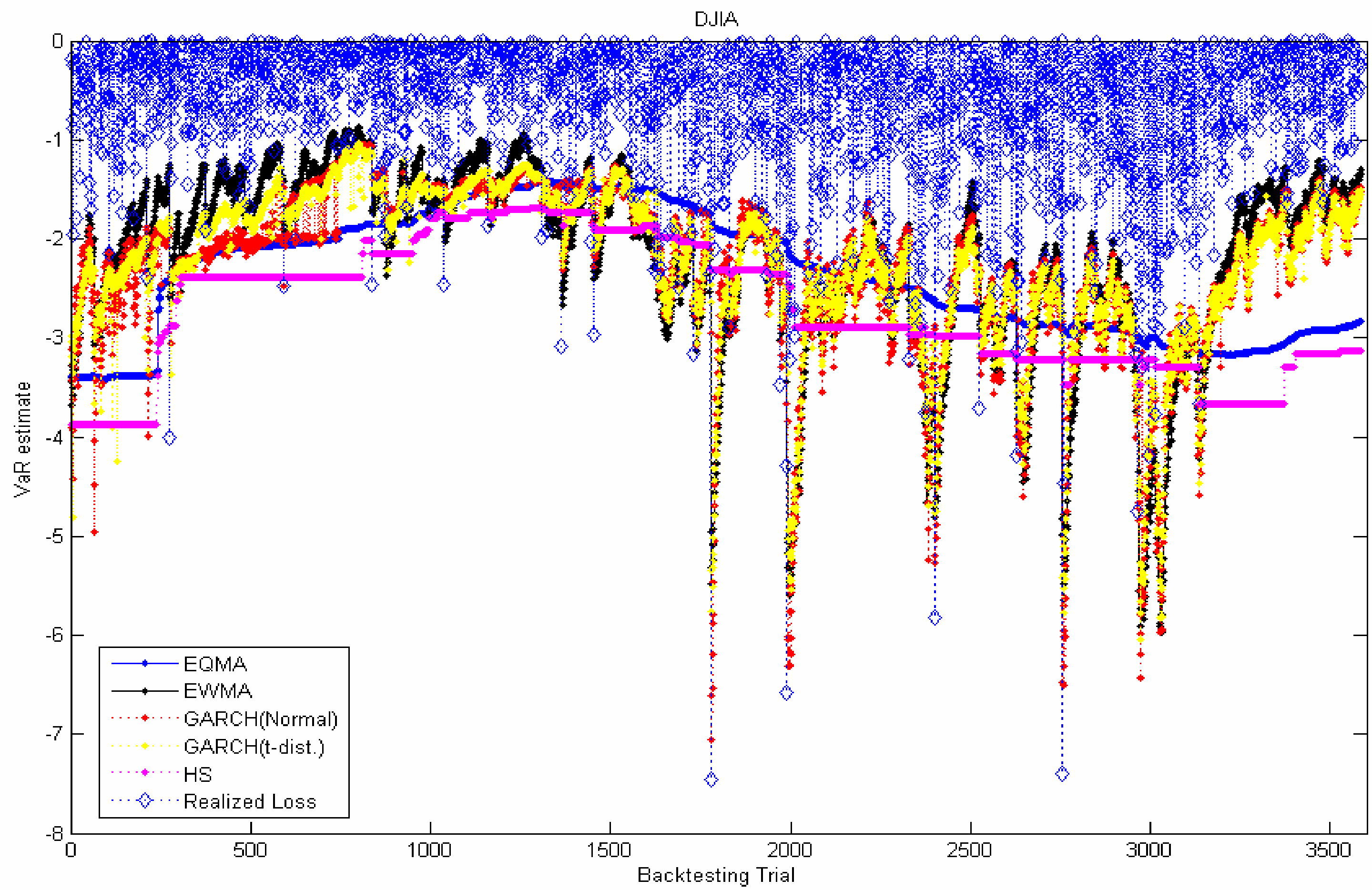

Figure 6.3: VaR estimates for DJIA with Realized Loss (Left Tail)

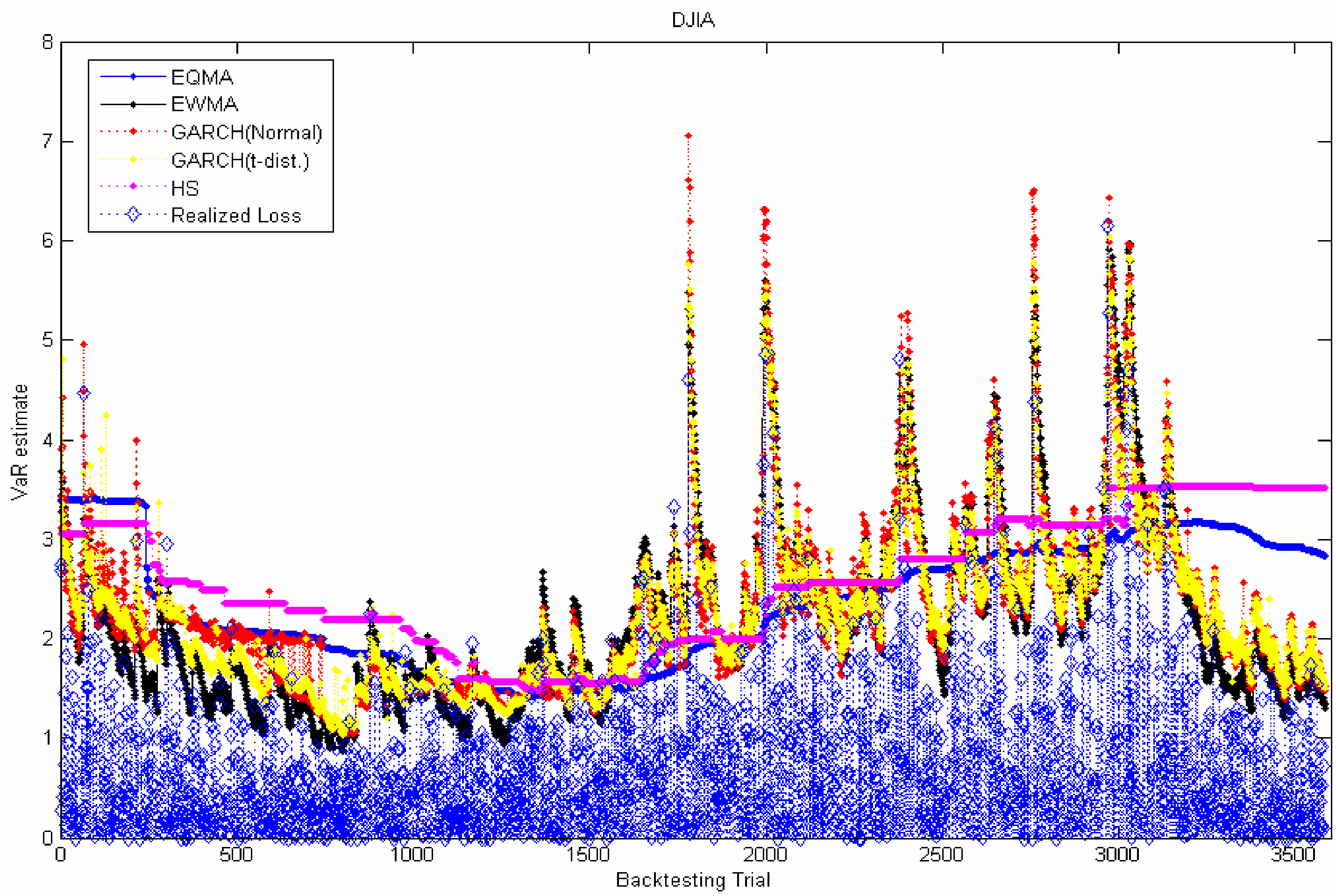

Figure 6.4: VaR estimates for DJIA with Realized Loss (Right Tail)

Table 6.1: Result of the PF test and Runs test of Univariate Models for DJIA

			PF Test		Runs Test				
Tail	Model Name	Exceptions	F-statistics	p-value	No Exceptions	Exceptions	Runs	z- statistics	p-value
Left	Equally weighted(EQMA)	59	**12.664** [a]	**0.0004**	3524	59	115	-1.3243	0.1854
	Exponentially weighted(EWMA)	52	6.4695	0.0110	**3531**	**52**	**99**	**-2.9286**	**0.0034**
	GARCH (Normal Dist.)	52	6.4695	0.0110	**3531**	**52**	**99**	**-2.9286**	**0.0034**
	GARCH (t Dist.)* [b]	47	3.2032	0.0735	3536	47	91	-2.1192	0.0341
	Historical Simulation*	38	0.13018	0.7183	3545	38	75	-1.3574	0.1747
Right	Equally weighted(EQMA)	58	**11.671**	**0.0006**	3525	58	113	-1.3813	0.1672
	Exponentially weighted(EWMA)*	44	1.7544	0.1853	3539	44	89	0.4021	0.6876
	GARCH (Normal Dist.) *	40	0.47242	0.4919	3543	40	81	0.2993	0.7647
	GARCH (t Dist.) *	41	0.72003	0.3961	3542	41	83	0.3256	0.7447
	Historical Simulation*	44	1.7544	0.18533	3539	44	85	-2.3681	0.0179

[a] Bold means we reject the null hypothesis in each test or model does not work well in each test at a 99% confidence level.
[b] Asterisk means model work well in both tests at the same time at a 99% confidence level.

We note that as indicated in the data analysis of the DJIA, the distribution of the DJIA returns has a fatter tail than a normal distribution. And, the left tail shows more deviation from the normal distribution than the right tail (i.e. negative skewness), which means that the left tail have more extreme events or stronger volatility clustering than the right tail. In the left tail, the EWMA model and the GARCH model with a normal distribution might fail to incorporate volatility clustering completely and exceptions were more likely to be clustered, so that the null hypotheses of the runs tests of those models were rejected. However, in the left tail the models such as the GARCH model with a t-distribution and the HS model, which are more like to incorporate extreme events or volatility clustering, were not rejected with the runs test as well as the PF test.

In the right tail which has less extreme events or weaker volatility clustering than the left tail, not only the GARCH model with a t-distribution and the HS model, but also the EWMA model and the GARCH model with a normal distribution were not rejected with the runs test, which means that these models can handle appropriately extreme events or volatility clustering in the right tail.

6.1.2 Hypothetical Portfolio

We will compare the univariate models with the multivariate models using the same portfolio because we want to know whether the multivariate models are more appropriate to evaluate VaR than the univariate models or conditional covariance estimation improves risk measurement. For the univariate models, we created the single portfolio returns using 30 stocks giving 1/30 weight to each stock as mentioned in the data section. So, here, we will analyze the univariate hypothetical portfolio first.

We backtested the appropriateness of 99% 1-day VaR calculated from each univariate model in the section 3.1. We used 1000 observations to evaluate VaR at each backtesting trial, so the total number of backtesting trials is 3582 [= 4582(the total number of observations) – 1000(observations to get one VaR backtesting trial)].

The VaR estimates for each model are in figure 6.5 and 6.6 for each tail when we invested $100 for each trial. In those figures, the result using three models (the EWMA model, the GARCH model with a normal distribution, and the GARCH model with a t-distribution) seem very similar to each other. Overall, the VaR estimates using the EQMA model are the smallest. In figure 6.7 and 6.8, you can compare VaR estimates with realized losses. Though realized losses are less than VaR estimates in most cases, there are some cases that realized losses are greater than VaR estimates, which cases are thought of as exceptions.

The result of the PF test and the runs test are in Table 6.2. In the left tail, all models except the EQMA model were not rejected with the PF test at a 99% confidence level. However, only EWMA model was not rejected with the runs test at a 99% confidence level. As a result, in left tail the only EWMA model was not rejected with both tests.

In the right tail, only HS model was not rejected with the PF test at a 99% confidence level. However, all models were not rejected with the runs test at a 99% confidence level. So, in the right tail only HS model was not rejected with both tests.

In sum, all models were rejected with the PF test or with the runs test in the left or right tail, which means no model is appropriate to evaluate VaR of the univariate hypothetical portfolio of 30 stocks for both tails.

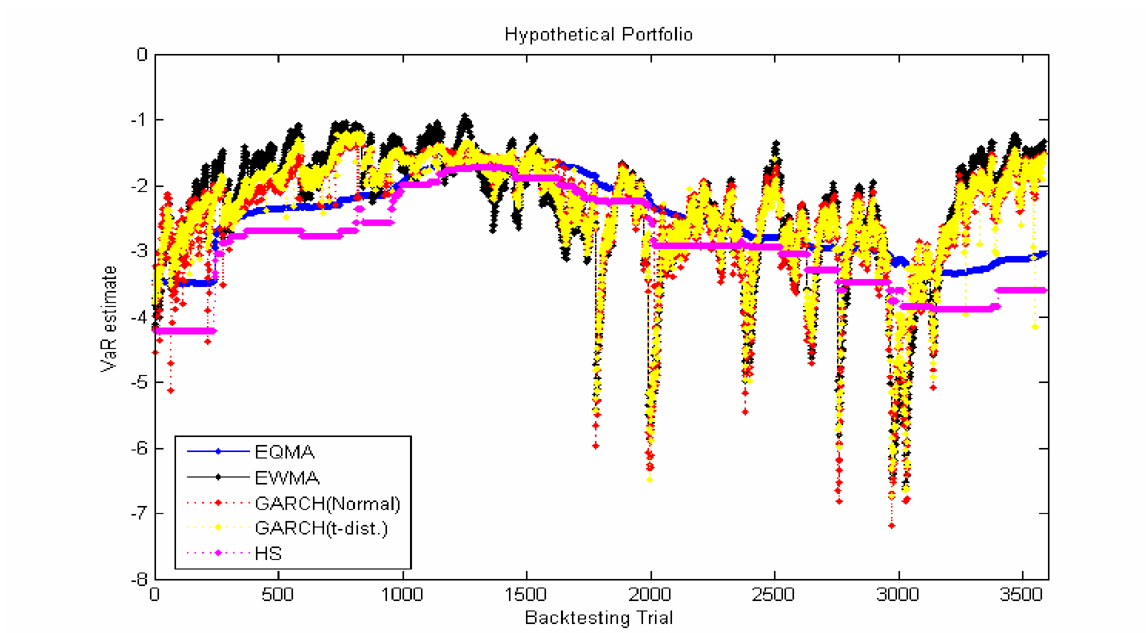

Figure 6.5: VaR Estimates for Univariate Hypothetical Portfolio when $100 was Invested (Left Tail)

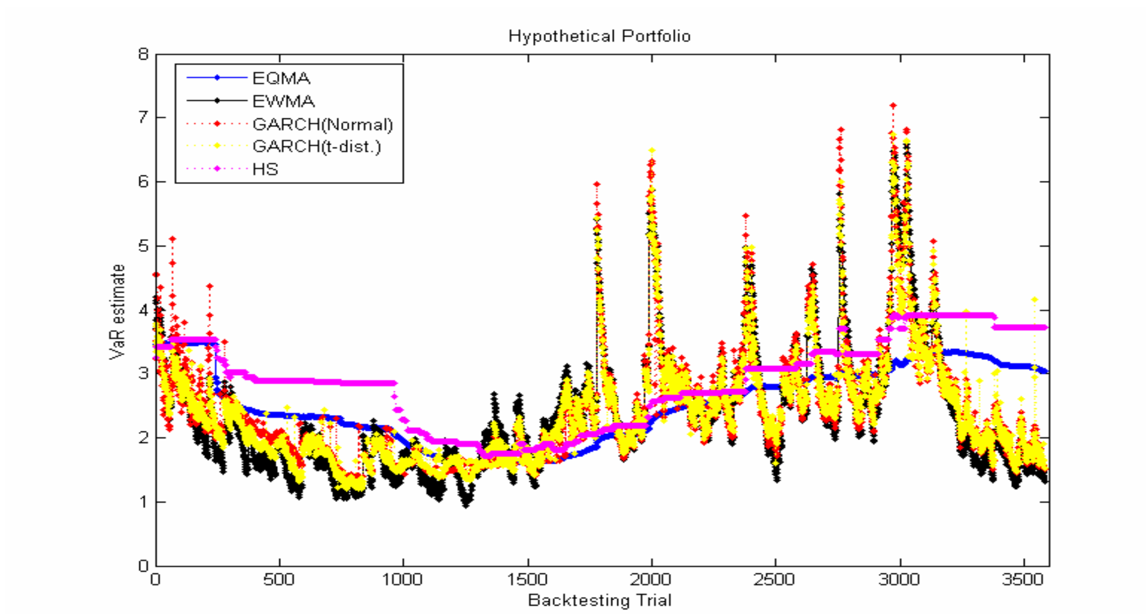

Figure 6.6: VaR Estimates Univariate Hypothetical Portfolio when $100 was Invested (Right Tail)

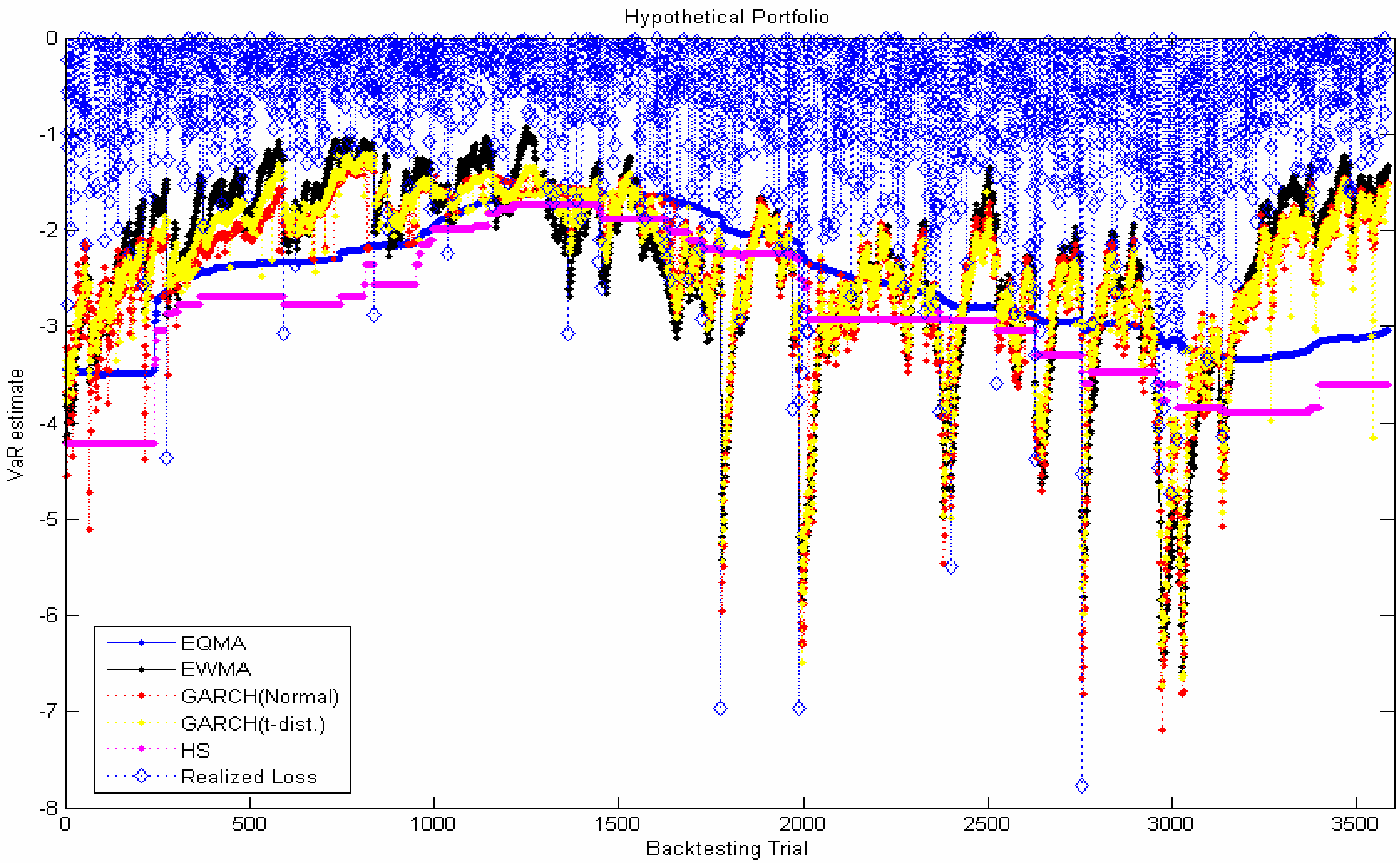

Figure 6.7: VaR Estimates for Univariate Hypothetical Portfolio with Realized Loss (Left Tail)

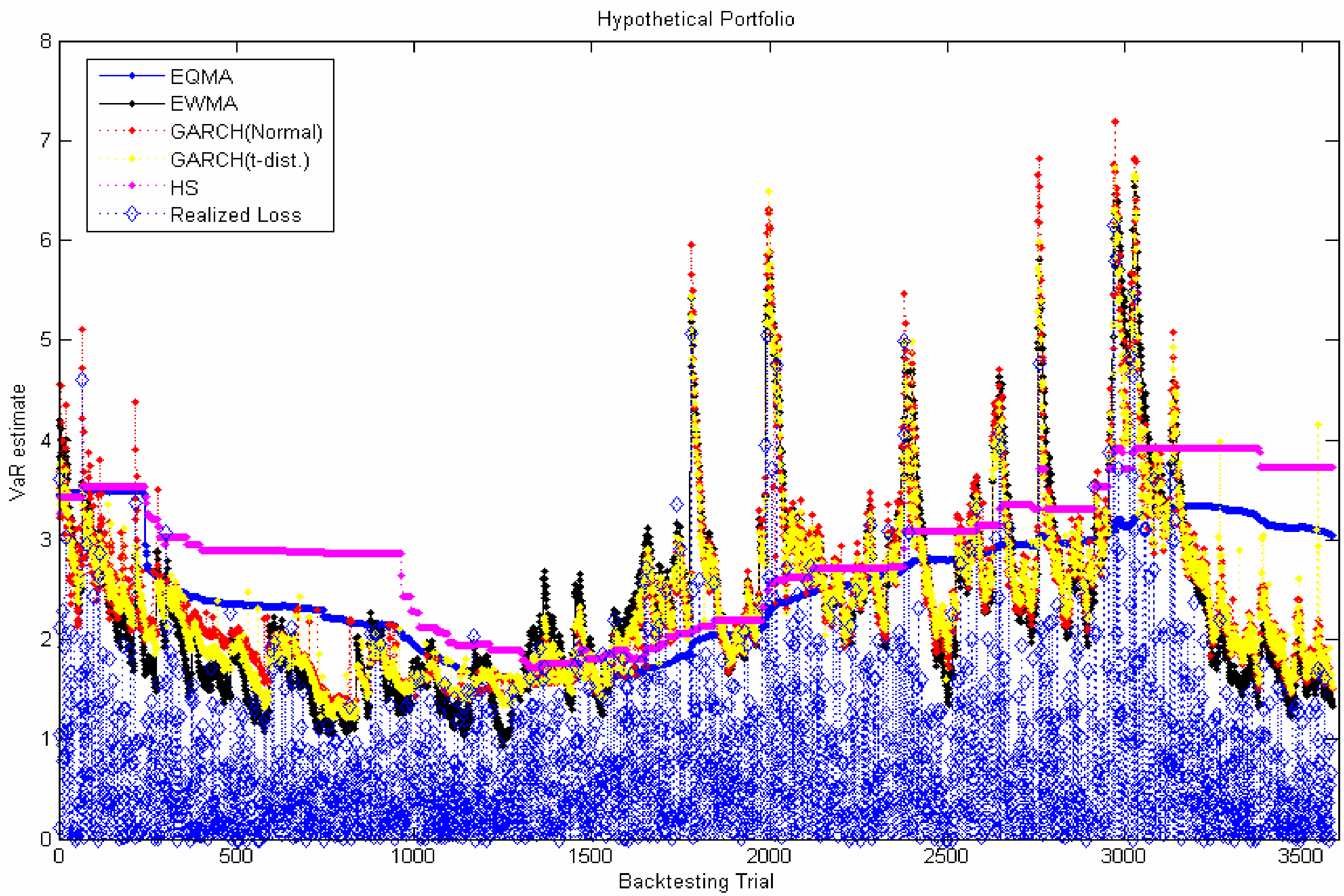

Figure 6.8: VaR Estimates for Univariate Hypothetical Portfolio with Realized Loss (Right Tail)

Table 6.2: Result of the PF test and Runs test of Univariate Models for the Hypothetical Portfolio

			PF Test		Runs Test				
Tail	Model Name	Exceptions	F-statistics	p-value	No Exceptions	Exceptions	Runs	z- statistics	p-value
Left	Equally weighted(EQMA)	60	**13.7060** [a]	**0.0002**	**3522**	**60**	**113**	**-3.3056**	**0.0009**
	Exponentially weighted(EWMA)* [b]	50	5.0484	0.0246	3532	50	97	-1.8935	0.0583
	GARCH (Normal Dist.)	49	4.3938	0.0361	**3533**	**49**	**91**	**-4.4555**	**0.0000**
	GARCH (t Dist.)	45	2.1978	0.1382	**3537**	**45**	**83**	**-4.9904**	**0.0000**
	Historical Simulation	39	0.2772	0.5986	**3543**	**39**	**73**	**-4.4117**	**0.0000**
Right	Equally weighted(EQMA)	64	**18.1530**	**0.0000**	3518	64	123	-2.0135	0.0441
	Exponentially weighted(EWMA)	60	**13.7060**	**0.0002**	3522	60	115	-2.2869	0.0222
	GARCH (Normal Dist.)	56	**9.8017**	**0.0017**	3526	56	111	-0.4084	0.6830
	GARCH (t Dist.)	58	**11.6840**	**0.0006**	3524	58	115	-0.3275	0.7433
	Historical Simulation*	43	1.3662	0.2425	3539	43	85	-1.0398	0.2985

[a] Bold means we can reject the null hypothesis in each test at a 99% confidence interval.
[b] Asterisk means model work well in both tests at the same time at a 99% confidence level.

We note that the results of the hypothetical portfolio are different from the DJIA in two aspects; one is that the EWMA model and two GARCH models are rejected with the PF test in the right tail, which may be caused by the fact that in the right tail the distribution of the hypothetical portfolio return has a fatter tails than that of the DJIA return as indicated at the data analysis section. The fatter tail makes it difficult for models to adequately estimate VaR with respect to both the PF test.

The other is that in the left tail the GARCH model with a t-distribution and the HS model were rejected with the runs test though these models were not rejected with the PF test. We note that in the DJIA case the number of runs is 91(GARCH model with t-distribution) and 75(HS model), but in the hypothetical portfolio that number dropped to 83(GARCH model with t-distribution) and 73(HS model), which means that exceptions are more clustered in the hypothetical portfolio, which resulted in the rejection of randomness null hypothesis.

6.2 Multivariate Approach

The correlation between returns of assets in a portfolio is an essential characteristic of multivariate models. So, first we will discuss the correlation estimation of multivariate models. Then, we will discuss the empirical result of multivariate approach in calculating the VaR estimate.

6.2.1 Correlation Estimation of Multivariate Models

Figure 6.9 shows average correlation using various models; the first panel is calculated by the DCC model, the second panel by O-GARCH model, and the others are calculated over sliding windows of 126(6 month), 500(2 years), and 1000(4 years). As the time window increases in the panel 3 to 5 of figure 6.9, the graph of the estimated correlation is less erratic and correlations are more centered to the overall mean. The overall level of correlation using the DCC model based on the past 1000 observations is analogous to the correlation using sliding windows of 1000 observations, but the graph using the DCC model is more erratic than that of the graph using sliding windows of 1000 observations. Overall, the correlation estimated by the DCC model seems reasonable.

Figure 6.10 and 6.11 show portfolio standard deviation and the average volatilities of 30 stocks and average correlation among 30 stocks which are calculated using the DCC model and the O-GARCH model over the entire horizon of data respectively, where for the comparison purpose we multiplied 6 to the average correlation. First, we note that the Longin and Solnik's (1995) observation that the correlation rises in periods of high volatility seems to hold over the entire period. Second, the average volatility of stocks represents a diversifiable risk or non-systematic risk of the portfolio and the average correlation represents an undiversifiable or systematic risk of the portfolio[17]. In this regard, in figure 6.10 and 6.11 we can also observe how the standard deviation of the portfolio estimated by the DCC model and the O-GARCH model incorporate the non-systematic risk and systematic risk. In figure 6.10 and figure 6.11, we also note that systematic risk (=portfolio standard deviation – average volatility) is a more dominant component in portfolio variance than variances of each stock, non-

[17] Rigorously speaking, systematic risk of a portfolio is the average covariance of stocks. However, since correlation is the most integral component of covariance, we can say that correlation represents systematic risk.

systematic risk because the portfolio consists of 30 stocks, so this is a relatively well-diversified portfolio.

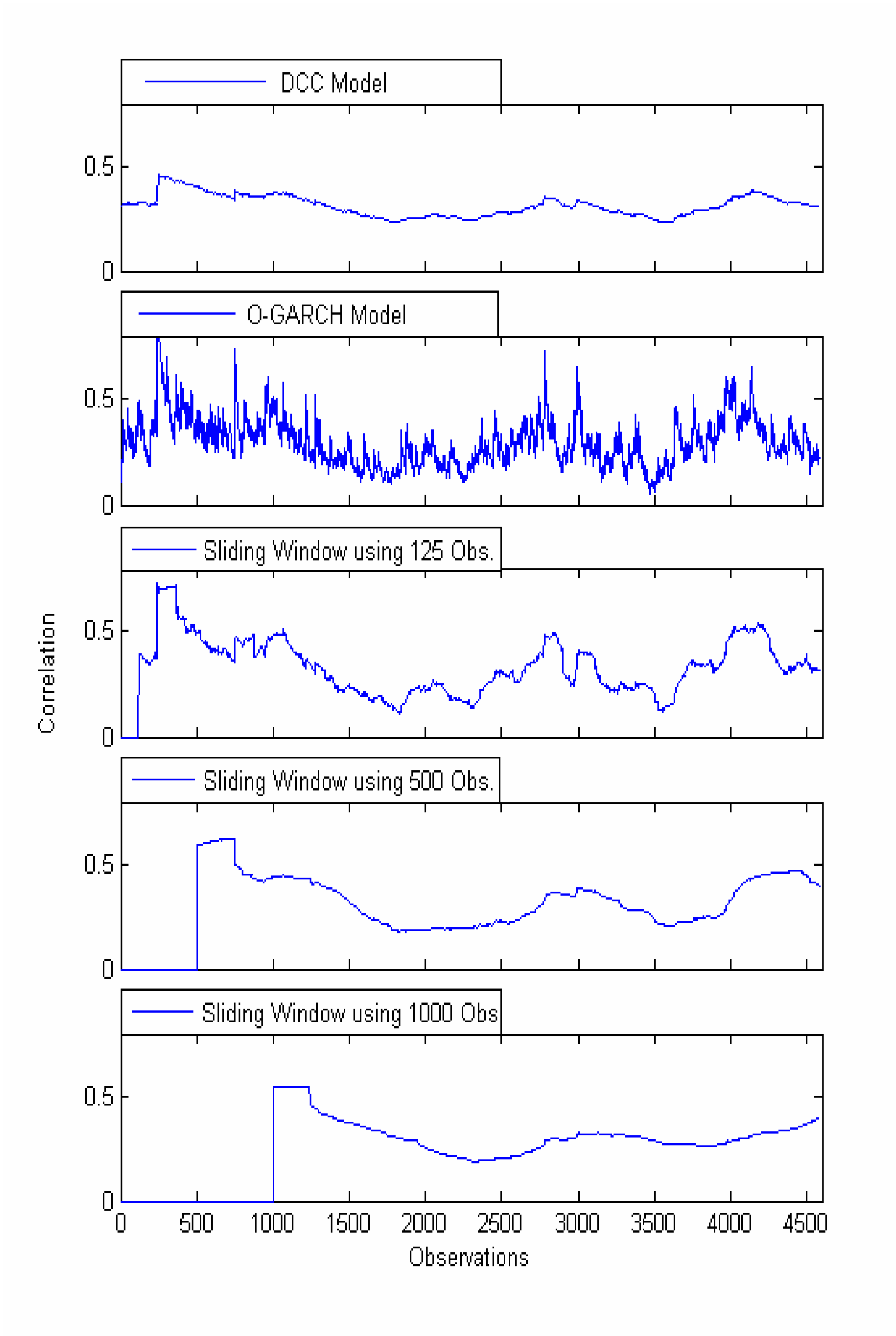

Figure 6.9: Correlations using Various Models

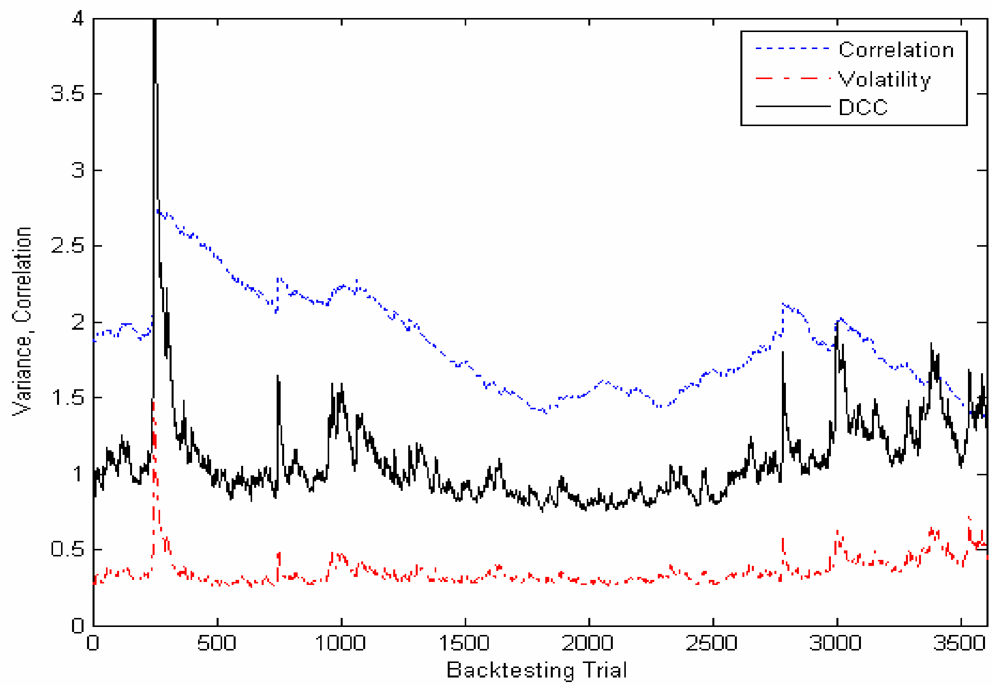

Figure 6.10: Portfolio Standard Deviation and Scaled Average Correlations and Volatility Using the DCC Model

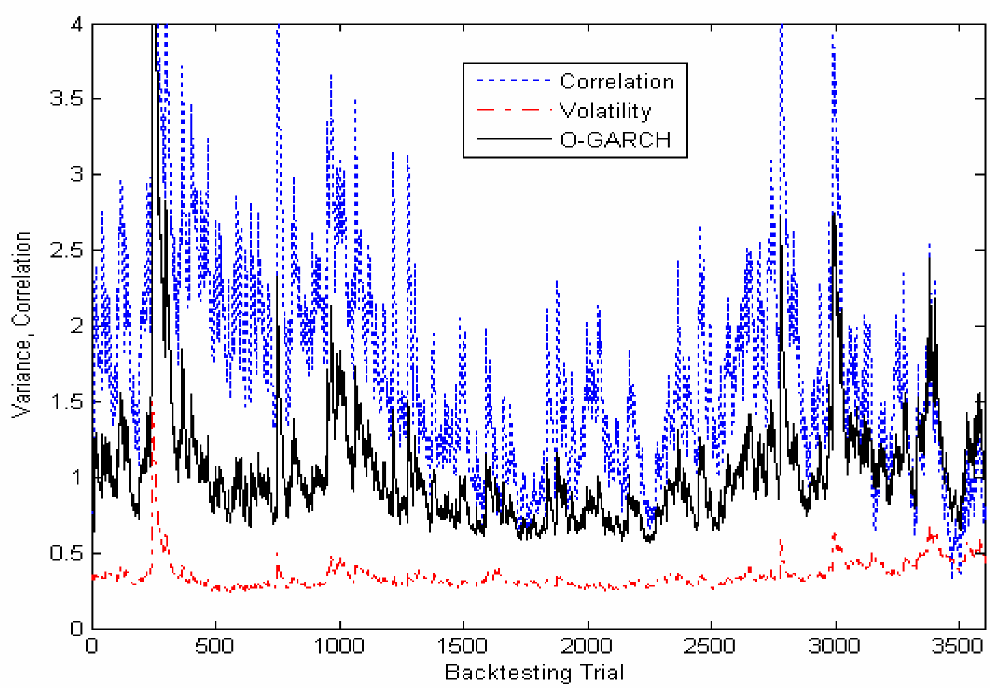

Figure 6.11: Portfolio Standard Deviation and Scaled Average Correlations and Volatility Using the O-GARCH Model

6.2.2 Multivariate Approach

In this sub-section, we will analyze the multivariate approach of evaluating the VaR estimate of the portfolio consisting of 30 stocks, the result of which is in Table 6.3. The comparison of this result with the result from the univariate hypothetical portfolio consisting of the same 30 stocks will let us address the second research question.

We backtested the appropriateness of 99% 1-day VaR calculated from multivariate model in the section 3.2. We used 1000 observations to evaluate VaR at each backtesting trial, so the total number of backtesting trials is 3582 [= 4582(the total number of observations) – 1000(observations for getting one VaR backtesting)].

The result of the PF test and the runs test are in Table 6.3. In the left tail, both the DCC model and the O-GARCH model were not rejected with the PF test at a 99% confidence level. But, both were rejected with the runs test at a 99% confidence level. As a result, both multivariate models were rejected with the PF test or with the runs test.

In the right tail, both the DCC model and the O-GARCH model were not rejected with the PF test at a 99% confidence level. Both also were not rejected with the runs test at a 99% confidence level. So, both models were not rejected with both tests time in the right tail.

In sum, two multivariate models were rejected with the PF test or with the runs test in the left or right tail, which means that both models are inappropriate to evaluate VaR of the hypothetical portfolio for both tails though both models are appropriate to evaluate VaR of the hypothetical portfolio just for right tail. One possible reason of this result is that those models failed to incorporate volatility clustering in the left tail as in the univariate hypothetical portfolio case.

Table 6.3: Result of the PF test and Runs test of Multivariate Models for the Hypothetical Portfolio

Tail	Model Name	Exceptions	PF Test		Runs Test				
			F-statistics	p-value	No Exceptions	Exceptions	Runs	z- statistics	p-value
Left	DCC	49	4.3938	0.0361	**3533**	**49**	**93**	**-3.2108**	**0.0013** [a]
	O-GARCH	48	3.7806	0.0519	**3534**	**48**	**89**	**-4.5819**	**0.0000**
Right	DCC* [b]	49	4.3938	0.0361	3533	49	95	-1.9662	0.0493
	O-GARCH*	47	3.2096	0.0732	3535	47	93	-0.8215	0.4114

[a] Bold means we can reject the null hypothesis in each test at a 99% confidence interval.
[b] Asterisk means model work well in both tests at the same time at a 99% confidence level.

6.3 Univariate vs. Multivariate

Table 6.4 represents the results of the PF test and the runs test of univariate and multivariate models for the hypothetical portfolio, which are reproduced from Table 6.2 and Table 6.3.

In the left tail, the EQMA model was rejected with both the PF test and the runs test at a 99% confidence level. The other models except the EWMA model and the EQMA model were not rejected with the PF test, but were rejected with the runs test at a 99% confidence level. Only EWMA model was not rejected with both the PF test and the runs test in the left tail.

In the right tail, the EQMA model, the EWMA model, and the GARCH models were rejected with the PF test at a 99% confidence level though these models were not rejected with the runs test at a 99% confidence level. The HS model and two multivariate models were not rejected with both tests at a 99% confidence level. Overall, in the right tail the HS model and two multivariate models were not rejected with both tests at a 99% confidence level.

If we consider both left and right tail at the same time, all models could be rejected with the PF test or with the runs test. The EWMA model could be rejected with the PF test in the right tail though that model was not rejected with both tests in the left tail. On the other hand, the HS model and two multivariate models were not rejected with both tests in the right tail, but could be rejected with the runs test in the left tail. So, we could reject all models considered in this thesis with the PF test or with the runs test in the left or right tail, which means that no univariate and multivariate models are appropriate to evaluate VaR of the hypothetical portfolio for both tails.

Table 6.4: Result of the PF test and Runs test of Univariate and Multivariate Models for the Hypothetical Portfolio

* Results are reproduced from Table 6.2 and Table 6.3

Tail		Model Name	Exceptions	PF Test		Runs Test	
				F-statistics	p-value	z- statistics	p-value
Left	Univariate model	Equally weighted(EQMA)	60	**13.7060** [a]	**0.0002**	**-3.3056**	**0.0009**
		Exponentially weighted(EWMA)* [b]	50	5.0484	0.0246	-1.8935	0.0583
		GARCH (Normal Dist.)	49	4.3938	0.0361	**-4.4555**	**0.0000**
		GARCH (t Dist.)	45	2.1978	0.1382	**-4.9904**	**0.0000**
		Historical Simulation	39	0.2772	0.5986	**-4.4117**	**0.0000**
	Multivariate model	DCC	49	4.3938	0.0361	**-3.2108**	**0.0013**
		O-GARCH	48	3.7806	0.0519	**-4.5819**	**0.0000**
Right	Univariate model	Equally weighted(EQMA)	64	**18.1530**	**0.0000**	-2.0135	0.0441
		Exponentially weighted(EWMA)	60	**13.7060**	**0.0002**	-2.2869	0.0222
		GARCH (Normal Dist.)	56	**9.8017**	**0.0017**	-0.4084	0.6830
		GARCH (t Dist.)	58	**11.6840**	**0.0006**	-0.3275	0.7433
		Historical Simulation*	43	1.3662	0.2425	-1.0398	0.2985
	Multivariate model	DCC*	49	4.3938	0.0361	-1.9662	0.0493
		O-GARCH*	47	3.2096	0.0732	-0.8215	0.4114

[a] Bold means we can reject the null hypothesis in each test at a 99% confidence interval.
[b] Asterisk means model was not rejected with both tests at the same time at a 99% confidence level.

We note that two multivariate models were not rejected in both tails if we consider only PF test, which means that the way the DCC model and the O-GARCH model incorporate the conditional correlation movements of the individual stocks as well as the conditional variance can improve at least the unconditional property of models compared with the way of parametric univariate models such as the EWMA model and the univariate GARCH models. Since the international standard, Basel Accord, in the banking industry considers only unconditional coverage property in its risk management regulatory mandates, two multivariate models could be useful in that application.

In addition, we would like to discuss the pattern of exceptions of the PF test result using the DCC model, the O-GARCH model, and the GARCH model with a normal distribution. Figure 6.12 compares binomial sequences of exceptions occurring in rolling backtesting of the VaR estimates using the GARCH model with a normal distribution and the DCC model. Figure 6.13 shows the same thing in the case of the O-GARCH model instead of the DCC model. In the stem diagrams of figure 6.12 and 6.13, ‘1’ means that exception occurs at that point. In both figures, the average correlation is calculated using the DCC model. The circle means that exception occurs at that point in that tail and that model, but exception does not occur in the same tail in the other model.

In figure 6.12, the overall patterns of exceptions resulting from the DCC model look interesting; more exceptions occur when correlation increases. However, in the case of GARCH model with a normal distribution, more exceptions occur when correlation decreases. The VaR estimate using the DCC model is above the VaR estimate using the GARCH model when correlation decreases. On the other hand, when correlation increases, the situation is reversed; the VaR estimate using the GARCH model is above the VaR estimate using the DCC model, which means that the change of the VaR estimation using the DCC model is smaller than the change of the VaR estimation using the GARCH model when correlation changes. In Table 6.5, the VaR estimate using the DCC model has the least standard deviation among three models. The reason is that as you can see in the specification of the DCC model in equation (17), the DCC model uses the GARCH specification in modeling the conditional correlation, which results in the long memory of correlation or slow response to the change of correlation.

Table 6.5: Mean and Standard Deviation of the VaR estimates

Variables	GARCH(Normal)	DCC model	O-GARCH model
Mean	2.4081	2.4361	2.4403
Standard Deviation	0.8890	0.6257	0.8723

As a result, we note that the performance improvement of DCC models relative to univariate models occurs in the period when correlation decreases, which is the opposite observation we expect; in fact, we expected that the performance improvement would occur when correlation increased.

Comparing the O-GARCH model with the univariate GARCH model, our calculation result indicates that the VaR estimates using the O-GARCH model seem more sensitive to market catastrophic event or market risk because only the seventeenth (backtesting trial 3031 in figure 6.13) out of the biggest thirty VaR estimates using the univariate GARCH model is greater than the VaR estimates using the O-GARCH model on the same day. However, in figure 6.13 the occurrence pattern of exception resulting from the O-GARCH model is not much different from the occurrence pattern of exception resulting from the GARCH model in catastrophic events (around backtesting trials 2000 through 3000)

Compared with the DCC model with respect to correlation, the O-GARCH model seems to have shorter memory than the DCC model. The early backtesting horizon of the VaR estimate using O-GARCH model in figure 6.13 seems to remember the past big correlation caused by the Black Monday stock crash, which you can check in figure 6.10. So, in that horizon the VaR estimate using the O-GARCH model is greater than the VaR estimate using the univariate GARCH model. However, after that period, the O-GARCH model looks more apt to respond to the change of correlation, even more than the univariate GARCH model.

As a result, all univariate models and multivariate models were rejected with the PF test or with the runs test in the left or right tail. However, if we consider only the PF test, which is more important than the runs test with respect to real application, the multivariate models, the DCC models and the O-GARCH model, were not rejected in both tails. We note that the performance improvement of DCC models relative to univariate models with respect to the PF test occurred when correlation decreased. The performance improvement of O-GARCH model also occurred when correlation decreased, but overall VaR estimates using O-GARCH model were greater than the VaR estimate using the univariate GARCH model when correlation increased.

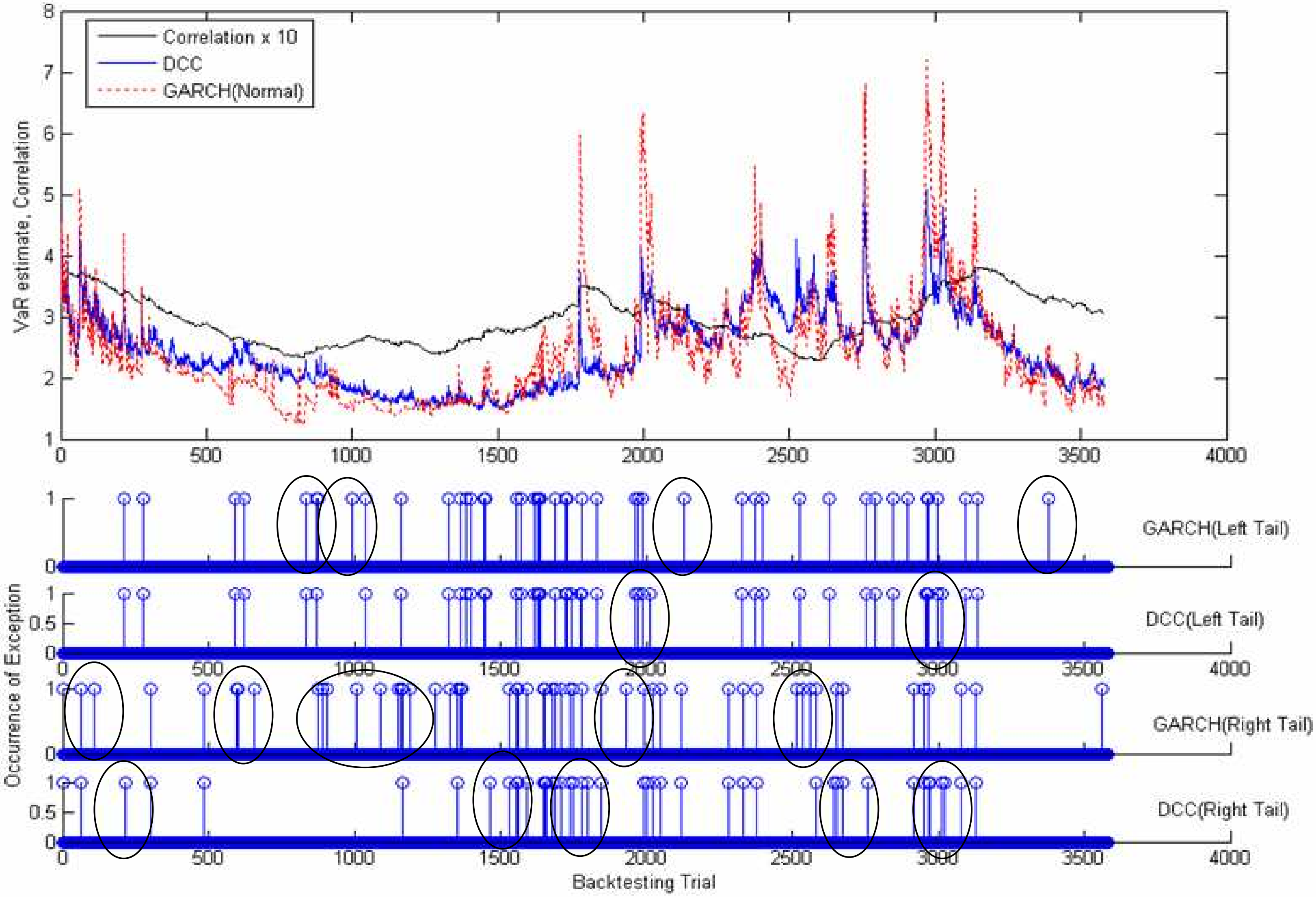

Figure 6.12: Exception Occurrence of DCC and GARCH (Normal)

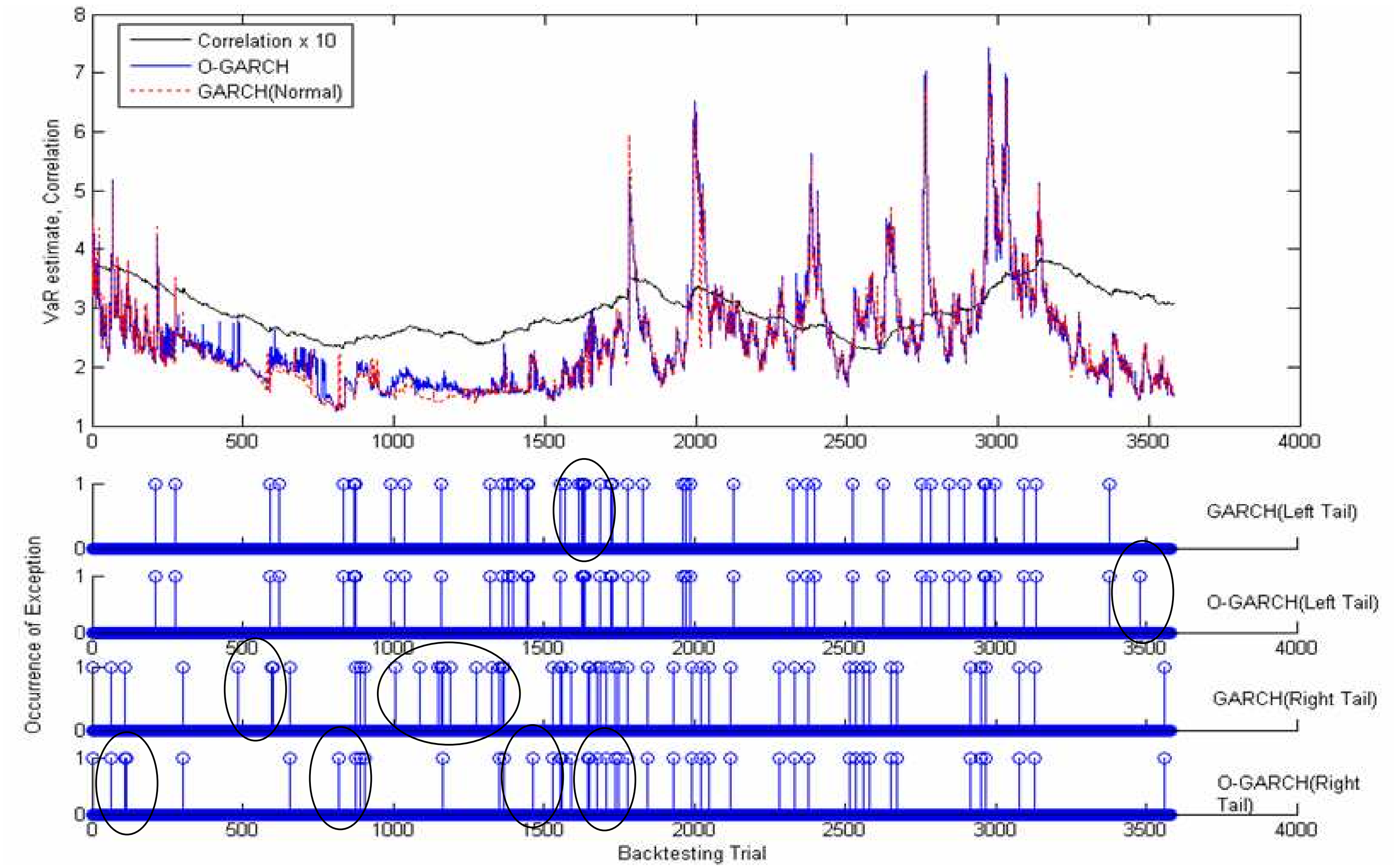

Figure 6.13: Exception Occurrence of O-GARCH and GARCH (Normal)

CHAPTER 7

Conclusion

This thesis sought to determine the best among various models in estimating VaR. Models were evaluated in terms of both accurate probabilities of extreme events and lack of correlation among exceptions. For accurate probabilities of extreme events, we used the proportion of failures (PF) test proposed by Kuiec (1995), and for lack of correlation among exceptions we used the runs test.

We compared five univariate models using the DJIA index and the hypothetical portfolio of 30 stocks; the five models are the equally weighted moving average (EQMA) model, the exponentially weighted moving average (EWMA) model, the GARCH model with a normal distribution, the GARCH model with a t-distribution, and the historical simulation (HS) model.

In DJIA index portfolio, two models (the GARCH model with t-distribution, the HS model) were not rejected in both right and left tails with the PF test and the runs test. Other models were rejected, which means that rejected models are inappropriate to evaluate VaR of the DJIA index portfolio. This result makes sense if we consider the fact that the two models are more robust to the fat-tail characteristic of financial time series than the other models. However, in the case of the hypothetical portfolio which has fatter tails than the DJIA index portfolio, all models were rejected with the PF test or with the runs test in the left or right tail, which means that no univariate models are appropriate to evaluate VaR of the hypothetical portfolio.

The difference of results between the DJIA index portfolio and the hypothetical portfolio came from the fact that in the right tail the distribution of the hypothetical portfolio returns has a fatter tail or stronger volatility clustering than that of the DJIA returns. These properties make it difficult for models to adequately estimate the VaR number with respect to both the PF test and the runs test.

Here, we note two facts regarding the VaR estimation. In the same portfolio, the VaR estimate using one model differs from the VaR estimate using another model, which implies the model dependency of the VaR estimation; in the DJIA index portfolio case, two model were not rejected with the PF test and the runs test, but in the hypothetical portfolio case, all models were rejected, which means that no model did a uniformly good job regardless of portfolio. It also turned out that the result depends on the length of periods of backtesting[18]. These observations confirm the research results by Bedder (1995) and Hendricks (1996) that VaR estimates from different parameters, data, assumptions, and methodology were quite different.

We compared the results using the univariate models, which evaluated VaR based on the univariate return series of the portfolio of 30 stocks, with the results using the multivariate models, which evaluated VaR based on the multivariate return series of the same portfolio. In each tail all univariate models and multivariate models could be rejected with the PF test or with the runs test in the left or right tail, which means that no univariate models or multivariate models are appropriate to evaluate VaR of the hypothetical.

[18] See appendix D.

However, we note that though the DCC model and the O-GARCH model were rejected with the runs test, which might be caused by strong volatility clustering in the left tail of the distribution of returns of the hypothetical portfolio, if we consider only the PF test, which is more important than the runs test with respect to real applications like the Basel Accord, the multivariate models, the DCC models and the O-GARCH model, were not rejected in both tails, which means that the way the DCC model and the O-GARCH model incorporate the conditional correlation movements of the individual stocks as well as the conditional variance can improve at least the unconditional property of models compared with the way of parametric univariate models such as the EWMA model and the univariate GARCH models.

However, improvement of DCC models relative to univariate models occurred when average correlation between assets in the portfolio decreases. Though that improvement of O-GARCH model also occurred when average correlation between assets in the portfolio decreased, overall VaR estimates using O-GARCH model were greater than VaR estimates using the univariate GARCH model when average correlation between assets in the portfolio increased.

As a concluding remark, VaR is a very convenient tool to manage a company's risk because it is easy to understand and communicate. However, the VaR estimate turns out to depend on the models used, its assumptions and the portfolio of the company, etc. So, we can say that VaR is a good starting point for risk management, not a final and perfect tool in risk management.

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APPENDIX A

Univariate Approach vs. Multivariate Approach Using the EWMA model

We compare the univariate approach with the multivariate approach using the exponential model; the equally weighted model case follows a similar procedure. For simplicity, we just consider the two-asset case. We can extend this idea for general N-asset case.

Let r_t^p be a portfolio return at time t, then $r_t^p = w_1 r_t^1 + w_2 r_t^2$

where r_t^1 is the return of asset 1, w_1 is weight of asset 1, r_t^2 is the return of asset 2, and w_2 is weight of asset 2.

Then, by the definition in the exponentially weighted scheme[19],

$$(\sigma_p)_t^2 = (1-\lambda)\sum_{i=1}^{m} \lambda^{i-1}(r_{t-i}^p)^2 \quad \text{(A.1.1)}$$

$$= (1-\lambda)\sum_{t=1}^{m} \lambda^{i-1}(w_1 r_{t-i}^1 + w_2 r_{t-i}^2)^2 \quad \text{(A.1.2)}$$

$$= (1-\lambda)\{\lambda^0 (w_1 r_1^1 + w_2 r_1^2)^2 + \lambda^1 (w_1 r_2^1 + w_2 r_2^2)^2 + ... + \lambda^{m-1}(w_1 r_{t-m}^1 + w_2 r_{t-m}^2)^2\} \quad \text{(A.1.3)}$$

$$= (1-\lambda)[\lambda^0\{(w_1 r_1^1)^2 + (w_2 r_1^2)^2 + 2w_1 w_2 r_1^1 r_1^2\} + \lambda 1\{(w_1 r_2^1)^2 + (w_2 r_2^2)^2 + 2w_1 w_2 r_2^1 r_2^2\} + ... + \lambda^{m-1}\{(w_1 r_{t-m}^1)^2 + (w_2 r_{t-m}^2)^2 + 2w_1 w_2 r_{t-m}^1 r_{t-m}^2\}] \quad \text{(A.1.4)}$$

$$= (1-\lambda)[\{\lambda^0 (w_1 r_1^1)^2 + \lambda^1 (w_1 r_2^1)^2 + ... + \lambda^{m-1}(w_1 r_{t-m}^1)^2\} + \{\lambda^0 (w_2 r_1^2)^2 + \lambda^1 (w_2 r_2^2)^2 + ... + \lambda^{m-1}(r_{t-m}^2)^2\} + \quad \text{(A.1.5)}$$

[19] According to RiskMetrics (1996), λ = 0.94 in the case of daily data.

$$\{\lambda^0 2w_1w_2r_1^1r_1^2 + \lambda^1 2w_1w_2r_2^1r_2^2 + ... + \lambda^{m-1} 2w_1w_2r_{t-m}^1r_{t-m}^2\}]$$

$$= (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}(w_1r_{t-i}^1)^2 + (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}(w_2r_{t-i}^2)^2 + (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}2w_1w_2r_{t-i}^1r_{t-i}^2 \tag{A.1.6}$$

$$= \begin{pmatrix} w_1 & w_2 \end{pmatrix} \begin{pmatrix} (1-\lambda)\sum_{t=1}^{m}\lambda^{t-1}(r_t^1)^2 & (1-\lambda)\sum_{t=1}^{m}\lambda^{t-1}r_t^1r_t^2 \\ (1-\lambda)\sum_{t=1}^{m}\lambda^{t-1}r_t^1r_t^2 & (1-\lambda)\sum_{t=1}^{m}\lambda^{t-1}(r_t^2)^2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{A.1.7}$$

At (A.1.6), the first term is the volatility form the asset 1, the second term is the volatility from asset 2, and the last term is the volatility form both or covariance. This explains that the volatility of the portfolio can be decomposed into volatilities of each asset, and covariance. Covariance matrix in (A.1.7) is exactly the same as multivariate covariance matrix by RiskMetrics (2001), which means that the volatility from the univariate model (A.1.1) is equal to the volatility from the multivariate model (A.1.7).

In sum, if we follow the methodology by RiskMetrics to get a covariance matrix, the volatility of a portfolio from the univariate portfolio returns will be equal to that from the multivariate approach.

APPENDIX B

Principal Component Analysis

Principal component analysis (PCA)), which is a kind of linear transformation, is a very popular method in dealing with multivariate variables. In the main text, we introduced the orthogonal GARCH (O-GARCH) model. That multivariate model is also based on PCA. In this appendix, we would like to explain PCA in detail to help reader understand the O-GARCH model.

Let $\boldsymbol{X}$ be the standard normalized return vector series of the original return vector series $\boldsymbol{Y}$. By the spectral theory of linear algebra, there is a orthogonal matrix $\boldsymbol{W}$ such that

$$\boldsymbol{X'XW=W\Lambda} \qquad \text{(A2-1)}$$

where $\boldsymbol{W}$ is a matrix of eigenvectors of $\boldsymbol{X'X}$, $\boldsymbol{\Lambda}$ is the associated diagonal matrix of eigenvalues.

Then, the principal components matrix $\boldsymbol{P}$ of $\boldsymbol{Y}$ can be defined as

$$\boldsymbol{P = XW} \qquad \text{(A2-2)}$$

One excellent property of the principal components matrix is that each column vector of that matrix is orthogonal with the others, which can be proved by

$$\boldsymbol{P'P = (XW)'(XW) =W'X'XW=W'W\Lambda= \Lambda} \qquad \text{(A2-3)}$$

Note that since $\boldsymbol{W}$ is orthogonal, $\boldsymbol{W'W = I}$ and $\boldsymbol{\Lambda}$ is diagonal.

From (A2-2),

$$\boldsymbol{X = PW^{-1}} \qquad \text{(A2-4)}$$

or $\boldsymbol{x_i} = w_{i1}\boldsymbol{p_1} + w_{i2}\boldsymbol{p_2} + \ldots\ldots\ldots\ldots + w_{in}\boldsymbol{p_n}$

where $\boldsymbol{x_i}$ is the ith column vector of $\boldsymbol{X}$, $\boldsymbol{p_i}$ is the ith column vector of **P**, and $\boldsymbol{w_{ij}}$ is ijth elements of $\boldsymbol{W^{-1}}$, which can be re-expressed with respect to Y as following:

$$\boldsymbol{y_i} = \mu_i + q_{i1}\boldsymbol{p_1} + q_{i2}\boldsymbol{p_2} + \ldots\ldots\ldots\ldots + q_{in}\boldsymbol{p_n} \qquad \text{(A2-5)}$$

where $\boldsymbol{y_i}$ is the ith column vector of $\boldsymbol{Y}$, $q_{ij} = \mathrm{w}_{ij} \times$ standard deviation of $\boldsymbol{y_i}$, and μ_i is mean adjustment. Taking variances of (A2-5) gives

$$\boldsymbol{H = ADA'} \qquad \text{(A2-6)}$$

where $\boldsymbol{H}$ is the covariance matrix of $\boldsymbol{Y}$, $\boldsymbol{A} = (q_{ij})$ is the matrix of denormalized factor weights, and $\boldsymbol{D} = \mathrm{diag}(\mathrm{Var}(\boldsymbol{p_1}),\ldots, \mathrm{Var}(\boldsymbol{p_n}))$. Note that $\boldsymbol{D}$ is a diagonal matrix because the principal components are uncorrelated. According to Alexander (2001), $\boldsymbol{H}$ will always be positive semi-definite.

APPENDIX C

GARCH Model Fitting Results Using Various Computer Software Packages

In this thesis, we used univariate GARCH model a lot. In fact, even the multivariate model is based on the result of a univariate GARCH model. So, the accuracy of GARCH model fitting is very critical in this thesis.

There are several software packages that offer GARCH modeling function such as SAS, EVIEWS, and Matlab. Software packages like GAUSS do not offer GARCH modeling function directly, but it is possible to create codes for it. In fact, it is also possible to create codes for GARCH model in Matlab[20]. It is very important to understand software packages used in the research because different software packages might produce different estimated GARCH models with the same data set. So, here we compared those using simulated data sets.

The model simulating data sets of returns is GARCH(1,1) in (8). The coefficients for GARCH(1,1) model generating each data set is in Table C.1. The number of observations is 1000. Data set 1 is designed to have more ARCH effect, but less GARCH effect. However, Data set 3 will have less ARCH effect, but more GARCH effect. Data set 2 is in the middle of those two data sets. Table C.2 contains the descriptive statistics of each data. Figure C.1 shows plots of each data set.

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[20] In this thesis, we usually take advantage of the Matlab GARCH codes written by Kevin K. Sheppar who is co-author of the DCC model article because this is relatively fast, and easy to modify.

Table C.1: The Coefficient Used to Simulate Data Set

Data Set	Constant	ARCH coefficient	GARCH coefficient
Data Set 1	0.07	0.35	0.60
Data Set 2	0.07	0.15	0.80
Data Set 3	0.07	.02	0.96

Table C.2: Descriptive Statistics for Simulated Data Set

Variables	Data Set 1	Data Set 2	Data Set 3
Observations	1000	1000	1000
Mean	0.0102	0.0076	0.0035
Std. Dev.	1.2526	1.2513	1.8714
Skewness	-0.3064	0.1134	0.0202
Kurtosis	20.8009	6.5543	3.1651
Jarque-Bera	13218.68 *[a] (0.0000) [b]	528.5309* (0.0000)	1.2038 (0.5478)
Q(12)	1640.4263* (0.0000)	817.8450* (0.0000)	35.3884* (0.0004)
ARCH LM	481.2426* (0.0000)	253.1163* (0.0000)	29.4383* (0.0034)

[a] Asterisks means that number is statistically significant.
[b] Numbers in parenthesis is t-statistics.

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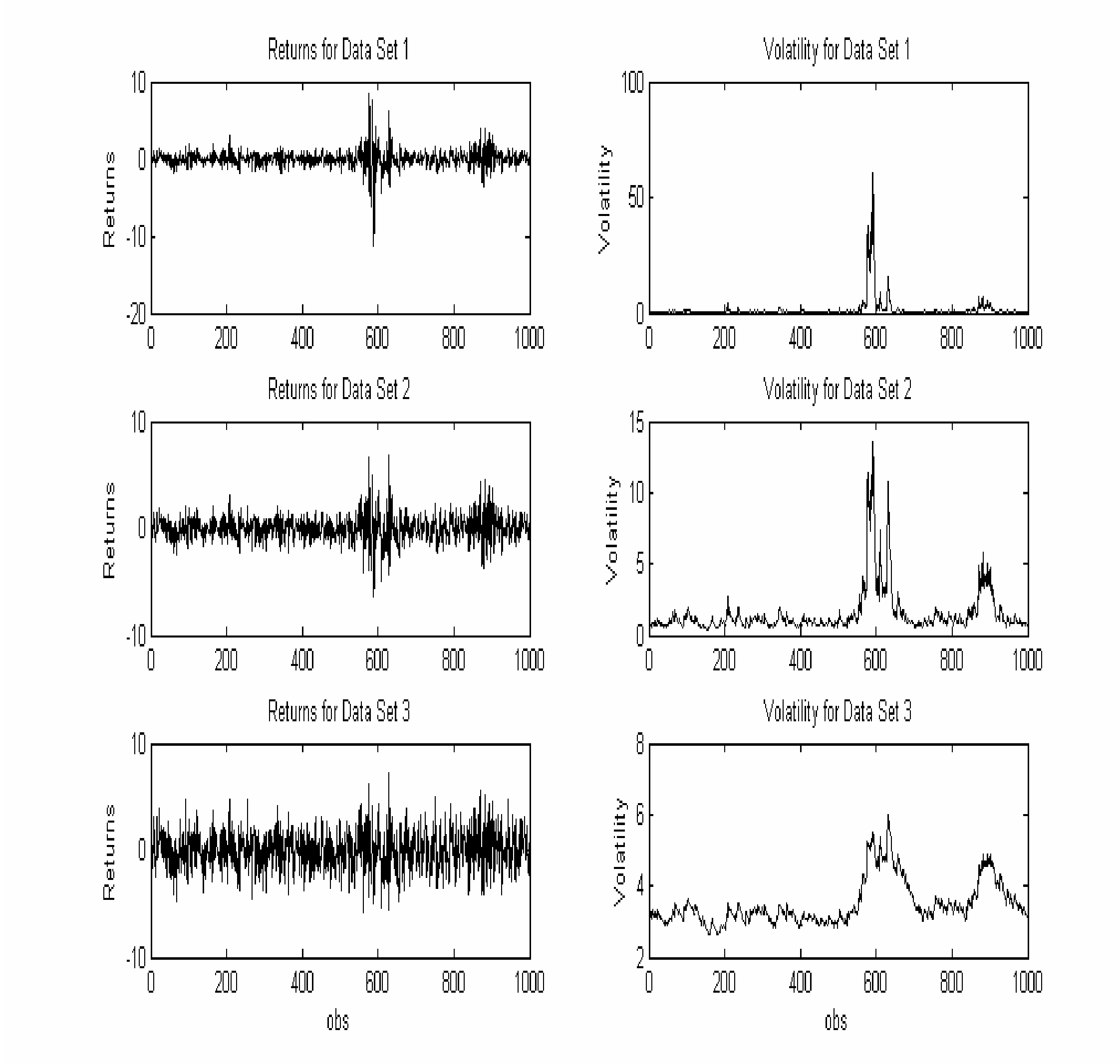

Figure C.1: Plots of Simulated Data Set

We note that data set 3 looks more likely to be normal than data set 1 in Table C.2. We can also see stronger ARCH effect in the data set 1 than data set 3. In figure C.1, data set 1 shows relatively low volatility, but high peaks. Data set 3 shows relatively high volatility, but low peaks.

Table C.3 shows the result of GARCH model fitting of each data set from various software packages[21]. With respect to coefficients and VaR figures, most software

[21] The tolerance of convergence in all cases is set to be 0.001.

package worked reasonably well except EVIEWS. EVIEWS worked badly especially with data set 3. It produced negative coefficient values[22], which are far from the true value.

Table C.3: GARCH Model Fitted Result

Data Set	Variables	Original	Matlab Embedded GARCH	Matlab Programmed GARCH	GAUSS	SAS	EVIEW
1	Constant	0.0700	**0.0704** [a]	**0.0698**	**0.0704**	**0.0705**	**0.0693**
			(4.3597) [b]	(4.4435)	(4.5171)	(4.3700)	(4.3158)
	ARCH Coefficient	0.3500	**0.3266**	**0.3255**	**0.3158**	**0.3250**	**0.3223**
			(7.3518)	(6.6872)	(6.7963)	(7.3500)	(7.2953)
	GARCH Coefficient	0.6000	**0.6124**	**0.6140**	**0.6165**	**0.6128**	**0.6177**
			(13.1747)	(12.8650)	(6.7963)	(13.2000)	(13.2245)
	Log Likelihood		-1245.2	-1245	-1244.7933	-1245.1498	-1244.105
	VaR [c]	$12,622.19	$12,731.05	$12,724.67	$12,735.22	$12,732.62	$12,749.76
2	Constant	0.0700	0.0329	0.0337	0.0340	0.0347	0.0334
			(2.2359)	(2.3913)	(2.4684)	(2.3300)	(2.3061)
	ARCH Coefficient	0.1500	**0.1069**	**0.1011**	**0.0986**	**0.1019**	**0.0994**
			(5.0925)	(4.6983)	(4.6855)	(5.0000)	(4.9445)
	GARCH Coefficient	0.8000	**0.8690**	**0.8736**	**(0.8741)**	**0.8716**	**0.8758**
			(32.0237)	(32.3864)	32.6323	(32.1600)	(32.9230)
	Log Likelihood		-1495.6	-1495.2	-1495.2026	-1495.4698	-1494.695
	VaR	$18,838.72	$18,709.01	$18,900.38	$18,834.25	$18,871.31	$18,964.03
3	Constant	0.0700	0.0479	0.0532	0.0478	0.0483	**6.730109**
			(1.1371)	(1.6927)	(1.5653)	(0.2522)	(15.5311)
	ARCH Coefficient	0.0200	0.0230	**0.0221**	**0.0226**	0.0229	-0.012882
			(2.2753)	(2.7619)	(2.8556)	(0.0231)	(-1.112199)
	GARCH Coefficient	0.9600	**0.9633**	**0.9630**	**0.9634**	0.9632	**-0.913815**
			(49.4405)	(72.7501)	(73.6365)	(<.0001)	(-8.94576)
	Log Likelihood		-2033.8	-2033.7	-2033.7361	-2033.7596	-2044.326
	VaR	$41,012.21	$40,607.76	$40,902.55	$40,433.34	$40,532.25	$43,155.78

[a] bold means that number is statistically significant.
[b] the numbers in parenthesis is t-statistics.
[c] VaR is 99% 1-day Value-at-Risk figure when $1,000,000 is invested

[22] I contacted to the technical support team of EViews company. They said current version (5.1) of EViews did not have the ability to restrict directly coefficients values to be positive. However, other approaches such as setting starting value to something more reasonable might be possible to achieve that.

APPENDIX D

The Length of Periods of Backtesting

The result of the proportion of failures (PF) test depends on the length of periods of backtesting. Using the hypothetical portfolio of 30 stocks and 1000 observations for backtesting, we tested how the result of the PF test would be changed when periods of backtesting is changed from 8/15/2000 ~ 12/31/2004 (1100 observations) to 11/3/1986 ~ 12/312004 (4582 observations), so the number of backtesting trials changes from 100 to 3582: five models are the EQMA model, the EWMA model, the GARCH model with a normal distribution, the GARCH model with a t-distribution, and the HS model. The result is in figure D.1. There is a non-rejection band that represents the region within the null hypothesis of the PF test are not rejected.

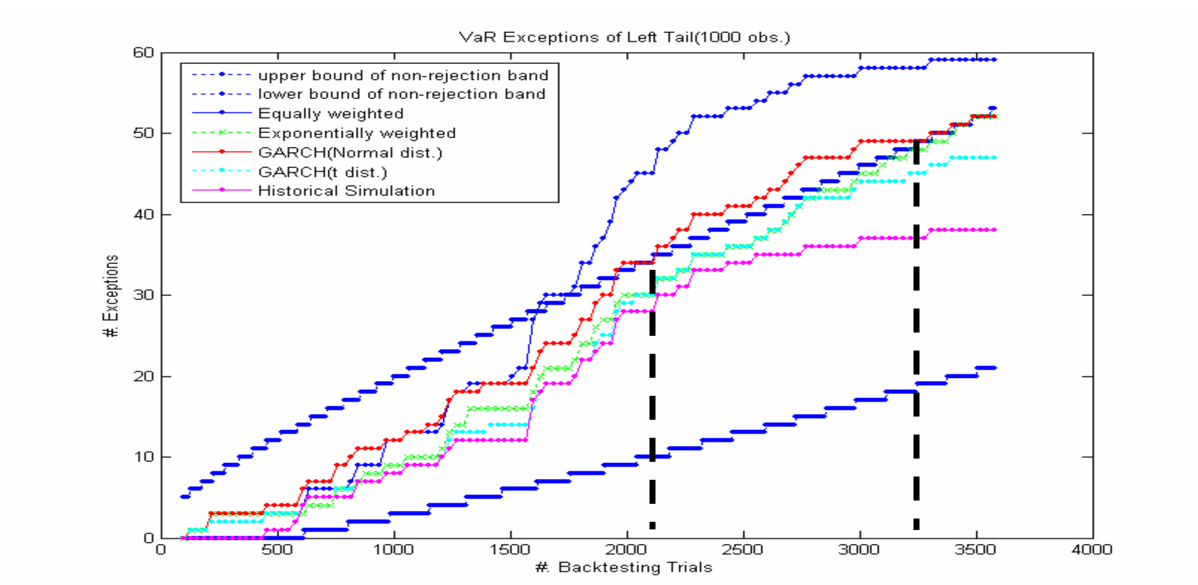

Figure D.1: The result of the PF test (Left Tail, Univariate)

In figure D.1, the result using GARCH model with a normal distribution looks very interesting. When the number of backtesting trials is less than about 2200, the PF test using that model failed to reject the null hypothesis because the number of exception from that model is in the non-rejection bands. Then, when the number of backtesting trials is around between 2200 and 3100, the test can reject the null hypothesis because the number of exceptions using that model is out of the non-rejection bands. However, the model works well after 3100. We can analyze other cases in the same way.

Apparently, we get some knowledge about the model selection from figure D.1 though the results of the PF test vary depending on the number of the backtesting trials. The GARCH model with a t-distribution show superiority over the most models in the most cases, which is consistent with our knowledge. The scheme like exponentially weighted model and GARCH model that gives more weights on the current events also does a better job than the scheme that give the same weights to the whole time horizon.

In these regards, we can compare the overall model performance among the GARCH models in figure D.2 and D.3 with respect to the PF test. In figure D.2, if we assume a normal distribution, orthogonal GARCH model works slightly better than the univariate GARCH model. DCC model works better in some ranges, but worse in other ranges. However, we can observe that GARCH model with t-distribution apparently work better job after 2000 of the backtesting trials. In figure D.3, orthogonal GARCH model works better regardless of the assumption about the distribution. However, DCC model, again, works better in some ranges, but worse in other ranges. As a result, multivariate models work reasonably well in the many cases regardless of the assumption about the distribution. But, in some cases, we can observe that is not true. So, we can not say that multivariate models are always superior over the univariate models.

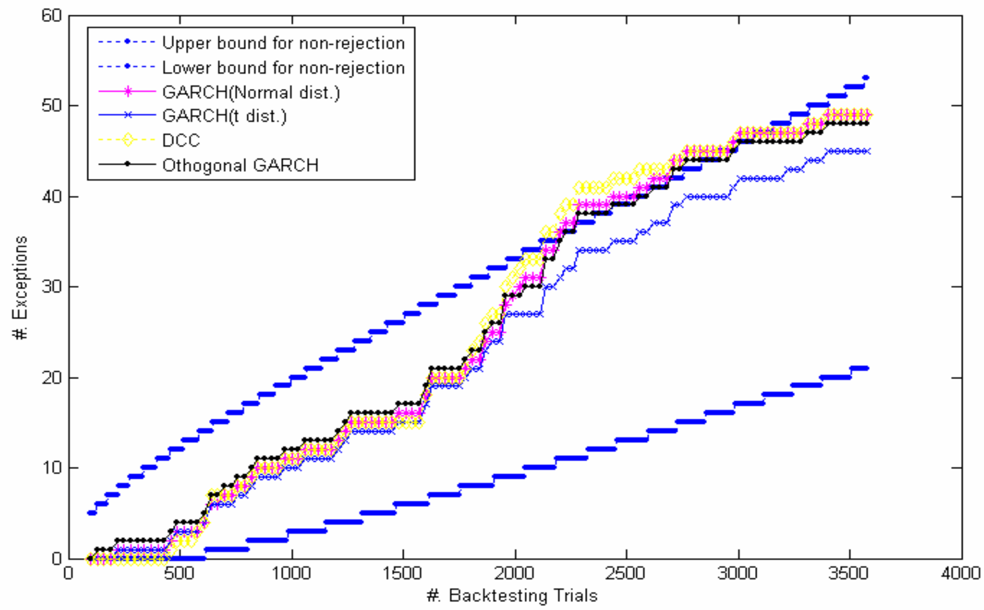

Figure D.2: The result of Backtesting Trials among GARCH models (Left Tail)

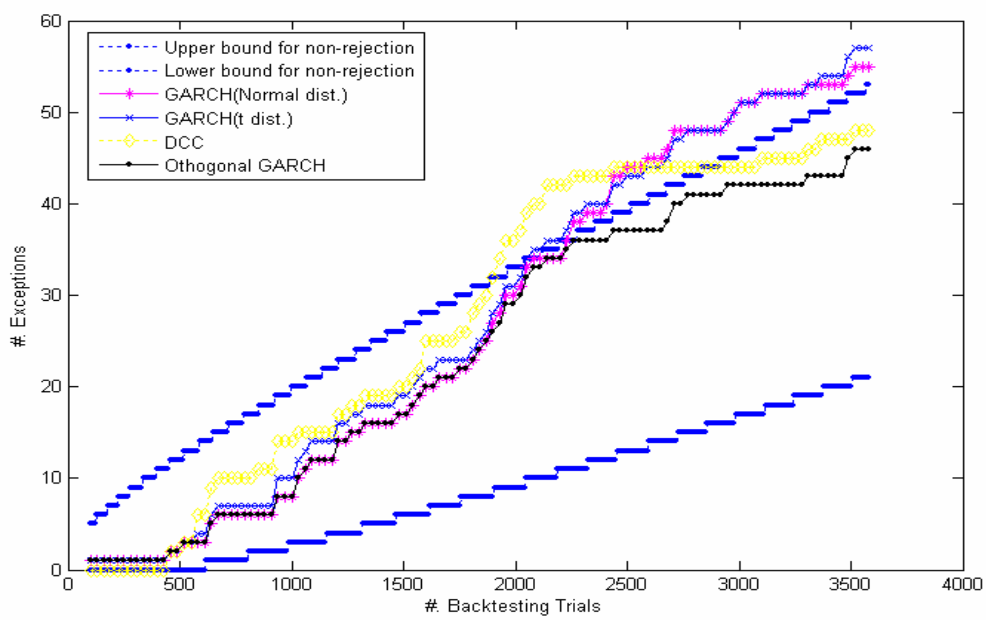

Figure D.3: The result of Backtesting Trials among GARCH models (Right Tail)

VITA

Sang Jin Lee

Candidate for the Degree of

Master of Science

Thesis: MODEL SELECTION FOR VALUE-AT-RISK: UNIVARIATE AND MULTIVARIATE APPROACHES

Major Field: Quantitative Financial Economics

Biographical:

Personal Data: Born in Seoul, Republic of Korea, on August 6, 1973, the son of Man Jong Lee and Jum Ryun Jung

Education: Received the B.S. degree from Yonsei University, Seoul, Republic of Korea in 1999 in Mathematics; Received the Master degree(Specialization : Corporate Finance, MIS) from Yonsei University, Seoul, Republic of Korea in 2004 in Business Administration; Completed the requirement for the Master of Science degree with a major in Quantative Financial Economics at Oklahoma State University in December, 2005

Experience: Graduate Associate at Oklahoma State University in 2005; Associate at Financial Supervisory Service, Seoul, Republic of Korea, from May 2000 to August 2004; Programmer at Dae Woo Information System, Seoul, Republic of Korea, from February 1999 to May 2000.

Professional Memberships: Phi Kappa Phi, Global Association of Risk Professionals.

Name: Sang Jin Lee Date of Degree: December, 2005

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: MODEL SELECTION FOR VALUE-AT-RISK: UNIVARIATE AND MULTIVARIATE APPROACHES

Pages in Study: 66 Candidate for the Degree of Master of Science

Major Field: Quantitative Financial Economics

Scope and Method of Study: Empirical Study

Findings and Conclusions:

This thesis sought to determine the best among various models in estimating VaR. Models were evaluated in terms of both accurate probabilities of extreme events and lack of correlation among exceptions.

In DJIA index portfolio, two models (the GARCH model with t-distribution, the HS model) were not rejected in both right and left tails with the PF test and the runs test. Other models were rejected, which means that rejected models are inappropriate to evaluate VaR of the DJIA index portfolio. This result makes sense if we consider the fact that these two models are more robust to the fat-tail characteristic of financial time series than the other models. However, in the case of the hypothetical portfolio which has fatter tails than the DJIA index portfolio, all other models were rejected with the PF test or with the runs test in each tail, which means that no univariate models are appropriate to evaluate VaR of the hypothetical portfolio.

We compared the results using the univariate models, which evaluated VaR based on the univariate return series of the portfolio of 30 stocks, with the results using the multivariate models, which evaluated VaR based on the multivariate return series of the same portfolio. All univariate models and multivariate models could be rejected with the PF test or with the runs test in the left or right tail, which means that no univariate models and multivariate models are appropriate to evaluate VaR of the hypothetical portfolio with respect to the PF test and the runs test.

However, we note that though the DCC model and the O-GARCH model were rejected with the runs test, which might be cause by strong volatility clustering of the distribution of returns of the hypothetical portfolio, if we consider only the PF test, which is more important than the runs test with respect to real application, the multivariate models, the DCC models and the O-GARCH model, were not reject in both tails.

As a concluding remark, VaR is a very convenient tool to manage a company's risk because it is easy to understand and communicate. However, the VaR estimate turns out to depend on the models used, its assumptions and the portfolio of the company, etc. So, we can say that VaR is a good starting point for risk management, not a final and perfect tool in risk management.

ADVISER'S APPROVAL: Timothy L. Krehbiel