# MODELING INVENTORY INFORMATION VISIBILITY IN SUPPLY CHAIN NETWORKS


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# CHAPTER 1

# INTRODUCTION

A supply chain network (SCN) is a network of firms that work together to supply the end products to the customer with a focus on both customer satisfaction and profitability of all firms. The nodes or firms involved in a SCN may be raw material suppliers, production facilities where the raw material is converted into finished products, warehouses that store the finished products, distribution centers that deliver the finished products to the retailers and retailers who satisfy the end customer demand. An example SCN is illustrated in Figure 1.1.

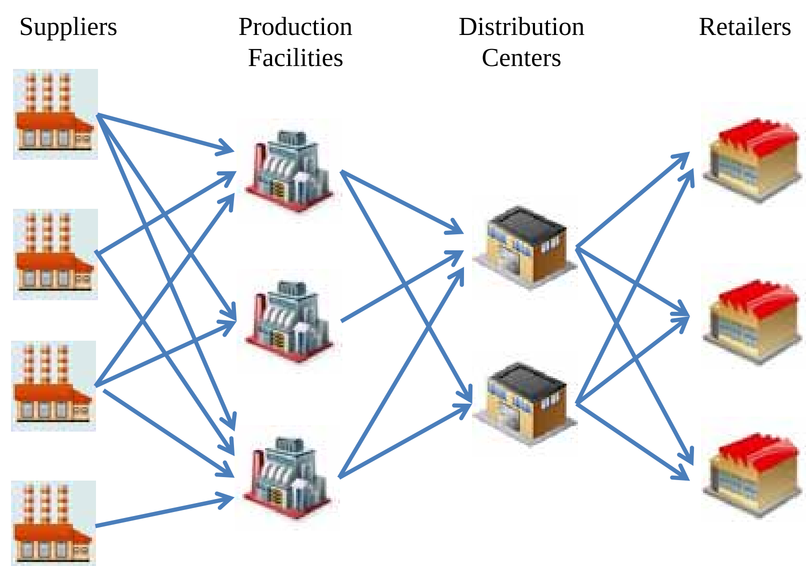



Figure 1.1: Generic Supply Chain Network [54]

The globalization and rapid growth of e-business has meant that the customers have more choices of "suppliers" for any product that they need. As a result, stock-out situation at a store may result in a customer choosing a competitor's product. A study by Corsten and Gruen [15] focused on a common problem in SCNs, namely, the shelf stock-out rate, and its effect on the behavior of the end customer. According to their article, several trade associations and joint trade-industry bodies have sponsored and released major reports on the stock-out rate for fast-moving consumer goods. The average stock-out rate at the store for all 40 studies was found to be 8.3%. Corsten and Gruen [15] also looked at consumer reactions to shelf stock-outs using 29 studies in 20 different countries. The results show that about 31% of the consumers switch the store, 15% delay their purchase, 19% substitute the product with the same brand, 26% switch the brand, while 9% of the consumers do not purchase the item and contribute to lost sales. As a result, a key focus of the firms in a SCN is to reduce the shelf stock-out rate and hence, improve end customer satisfaction.

Supply chain visibility among the SCN partners is a key area of research and is seen as a panacea to the stock-out problem. Supply chain visibility refers to the real time transparency in the supply chain facilitated through built-in information systems that allow the participants in the SCN to keep track of semi-finished/finished goods that may be in store or in transit. The global supply chain benchmark report published by Aberdeen group in 2006 [18] found that 79% of the large enterprises that were surveyed reported that lack of supply chain visibility is a critical concern. 51% of the enterprises that were surveyed identified supply chain visibility among their top three concerns. In addition, more than 77% of the enterprises were prepared to spend heavily to attain supply chain visibility. With the advances in information technology, sharing demand, capacity, and inventory information among the partners in the SCN has become quicker and cheaper [12]. There have been studies (e.g., Graves [24], Zipkin [63], Gavirneni et al. [21]) that have focused on the effects of information sharing. While most of these studies showed that information sharing has benefits, a few (e.g., Graves [24]) argued that there are no added benefits of sharing information with the upstream firms for the situations they modeled. More details are presented in Section 2.1. In this research, our focus is on inventory information sharing or inventory visibility. Inventory visibility allows companies to be informed about their partner's in-stock inventory and in-transit inventory in order to make their supply chain as effective as possible.

In a typical SCN, demand and sales information flows from the downstream firms (e.g., retailers) to the upstream firms (e.g., suppliers), while there is flow of material in the opposite direction. There can also be other information exchanges that can occur between the various firms in the SCN depending on the amount of collaboration between the partners in the SCN. The order status information can flow from the upstream firms to the downstream partners, while there can be two-way communication between firms when it comes to sharing inventory information and production plan information [41].

Several critical decisions need to be taken while designing and operating a SCN. These decisions include facility locations, production capacities, the safety stock to be held at each location, and the base-stock level at each location. Performance evaluation tools aid decision makers in making these decisions during the design and operation phases of a SCN. In addition, these models provide insight into the working of the SCN and would help in gaining a better understanding of the dynamics of the system (SCN). This research focused on analytical performance evaluation models of supply chain networks and has developed models that can explicitly capture inventory visibility in order to study the effects of sharing inventory information on the SCN performance.

Performance evaluation tools are generally used to measure key performance measures of a system (e.g., average response time, fill rate, expected inventory, and expected backorders in the case of a SCN) for any given set of parameter values (e.g., inter-arrival and service time parameters, base-stock levels, etc.) through the development and solution of analytical and simulation models. Performance evaluation tools aid system designers and operations managers in making some key decisions, while keeping in mind the goals of the company [55]. The analytical performance evaluation tools are typically based on modeling techniques such as Markov chains, stochastic Petri nets, and queueing networks [57]. Simulation models can also be used for performance evaluation, but need more detailed information for modeling and more time for model development and model execution phases. Analytical models yield results more quickly and "are appropriate for rapid and rough cut analysis" [55]. In fact, analytical and simulation models have been used in tandem to analyze and design complex systems. For example, analytical models can be used to reduce a large set of design alternatives, and the remaining few alternatives can be studied in detail using simulation models [55]. The development of performance evaluation and optimization models for supply chain networks is an active area of research (see e.g., Ettl et al. [19], Raghavan and Viswanadham [49], Dong and Chen [17], and Srivathsan [53]). However, very few performance modeling studies have explicitly addressed the issue of modeling inventory visibility in a SCN. The focus of this research will be on developing such performance evaluation models of SCNs.

## 1.1 Information Sharing in Supply Chains

Information sharing is believed to be a key component in the success of a SCN. Research on studying the effects of information sharing have dealt with demand information (e.g., [24], [21]), inventory information (e.g., [63], [20]), and a combination of demand and inventory information (e.g., [13], [12], [45]). Li et al. [41] presented a review of ten different models used to study the value of information sharing in supply chains. Lee and Whang [39] described the inventory, sales, demand forecast, order status, and production schedule information that can be shared and also discussed how and why such information is shared along with industry examples. While most of these studies revealed that there are benefits of information sharing, a few studies including Graves [24] showed that sharing demand information with upstream firms did not add value to their SCN model setup. The studies that consider inventory information sharing mostly used simulation models. Some of the research studies that focused on developing queueing models with inventory visibility include Zipkin [63] and Armony and Plambeck [4].

There is also considerable research that focuses on the effect of information distortion. The bullwhip effect, the phenomenon in which there is an increase in the mean and variability of the demand process as we proceed from the retailer to the supplier stage in a SCN, occurs due to distorted demand information. For example, Lee et al. [38] is a frequently cited study on the bullwhip effect in a SCN.

## 1.2 Performance Evaluation and Performance Optimization Models

The common analytical performance modeling tools that are used in performance evaluation include queueing models, Markov chain models, and stochastic Petri nets. Analytical models based on queueing theory have been developed for performance evaluation of production networks since the early 1950s. Prior to the 1990s, the literature on stochastic models of production networks primarily considered capacity, congestion and reliability issues, and did not explicitly model the presence of planned inventory. Similarly, the early literature on inventory theory did not consider capacity and congestion issues explicitly. Inventory theory models have been developed since the 1910s. The earliest work in this field includes the EOQ model [27] and its various extensions. Some of the recent text books that focus on inventory theory are Bramel and Simchi-Levi [10], Zipkin [65], Nahmias [46], and Axsater [5].

Since the 1990s, there has been a growing interest among researchers in developing analytical models of production networks which consider capacity, congestion and planned inventory issues in a unified manner. Recent work on modeling production-inventory networks includes models of single-stage systems developed by Buzacott and Shanthikumar [11] and models of multi-stage production-inventory networks developed by Lee and Zipkin [40], Sivaramakrishnan [51], Sivaramakrishnan and Kamath [52] and Zipkin [64]. A few research efforts also developed performance optimization models for SCNs. These include the studies of Cohen and Lee [14] and Lee and Billington [37]. More recently, queueing models have been used in tandem with optimization models to take into account the stochastic components in the SCN. For example, Ettl et al. [19] combined optimization under service level constraints along with a queueing model to support strategic level decision making in large SCNs.

### 1.3 Motivation for the Proposed Research

In the development of queueing-based performance evaluation models of production-inventory and supply chain networks, the common assumption about routing of orders from downstream stages to upstream stages is that the routing probabilities are fixed and not dependent on state information such as inventory levels. The routing based on fixed probabilities is also called Bernoulli routing. This routing policy allows the placement of an order at a production facility even when it is facing a stock-out situation. Such a routing policy is appropriate for the development of models for cases with no inventory visibility. In such a situation, the order may be placed based on historical information (say, for example, the percentage of orders placed by a downstream firm at an immediate upstream firm). In cases where the downstream firm has visibility of the immediate upstream firm's inventory, a routing policy in which orders are placed based on item availability is certainly a possibility and perhaps desirable. Similarly, when the retail stores share their inventory information with their immediate upstream "suppliers", then the latter can choose to satisfy an order from the retail store that has the least net inventory level. These situations pose very interesting modeling scenarios that have not been studied much in the queueing literature. In addition, there is a need to study the value in sharing inventory information among the various firms in the SCN, and also to quantify the value of such information sharing. When there is value in sharing inventory information among firms in the SCN, then there is also a need to find the sensitivity of the benefits to different SCN parameters such as base-stock levels, production capacities, and variability in service and arrival processes.

### 1.3.1 Problem Statement

With the advancements in the field of information technology, information sharing among firms within a SCN has become easier. Information sharing to provide inventory visibility has gained the attention of practitioners and academic researchers. There is a need to develop analytical models that can explicitly model inventory visibility in production-inventory and supply chain networks. Such models will not only give us better insight into the value of inventory visibility, but also enable us to develop performance optimization tools to maximize the value of inventory visibility by choosing the right combination of system parameter values.

## 1.4 Outline of the Dissertation

The rest of the document is structured as follows. Chapter 2 presents a review of the literature on information sharing and performance modeling of production-inventory and supply chain networks. Chapter 3 presents the research goals, objectives, and contributions. Chapter 4 presents the research approach. Chapter 5 presents the Markov chain models that were developed for a SCN with one retail store and two production facilities at different levels of inventory information sharing. Chapter 6 presents queueing-based analytical models that were developed for a SCN with one retail store and two production facilities under Poisson arrivals and exponential processing times, while Chapter 7 presents the analytical models for the case of general inter-arrival and processing time distributions. Chapter 8 presents the analytical models that were developed for a SCN configuration with two retail stores and two production facilities, while Chapter 9 extends the analytical models to include transit times in a SCN with inventory information sharing. Chapter 10 presents the research contributions and highlights the scope for future research.

# CHAPTER 2

# LITERATURE REVIEW

This chapter presents a detailed review of the literature on the study of value of information sharing in supply chains and the analytical performance modeling of production networks with planned inventory. Section 2.1 presents a review of the published studies that focused on the value of information sharing in SCNs. Section 2.2 summarizes the literature on modeling production-inventory and supply chain networks. Section 2.3 provides a summary of the literature review and identifies some gaps in the literature, which form the basis for the research conducted.

## 2.1 Literature on Value of Information Sharing

Information sharing is increasingly seen as a contributing factor to the success of a SCN. Research efforts have focused on finding if there is any value in information sharing, in assessing the actual value of information sharing, and in identifying factors influencing the value of information sharing. Studies have considered various types of information that can be shared, namely supplier status, inventory levels, demand forecasts, price, schedule and capacity information [41].

Graves [24] studied a single-item inventory system with a non-stationary demand process that behaved like a random walk. Exponential-weighted moving average was used to obtain the mean square forecast of the demand and deterministic lead-time was assumed. An adaptive base-stock policy where the base-stock is adjusted based on changes in demand forecast was proposed. The safety stock required for non-stationary demand was found to be much more than that for stationary demand.

The relationship between lead time and safety stock was found to be convex in the case of non-stationary demand indicating that more safety stock is required with increasing lead time. This single-item model was then extended to a single-item, multi-stage system and the upstream demand was found to be non-stationary with the same form as the downstream demand process. So the adaptive base-stock policy was applied to the upstream stage and the results showed that the bullwhip effect cannot be mitigated by sharing more information to the upstream stage.

Zipkin [63] studied the performance of a multi-item production-inventory system with a production facility under the first-come-first-served (FCFS) policy and longest-queue (LQ) policy. The production facility had a finite capacity and ample raw material supply. The processor at the production facility was assumed to be perfectly flexible with no setup costs. All products were assumed to be symmetric (i.e., same demand rates, processing times, etc.). The demands for the products were assumed to be Poisson and the processing times followed a general distribution. Each product was assumed to follow a base-stock policy with identical base-stock levels. Each demand for a product at a store resulted in the consumption of the product from the store and a resultant replenishment order being placed at the production facility. Backorders were allowed in the system. In the case of the FCFS policy, these replenishment orders waited at the processor's queue and were processed on a first-come-first-served basis. In the case of the LQ policy, the processor at the production facility used the inventory information and serviced the product that had the smallest net inventory level. Preemption was allowed in the case of the LQ policy and ties were resolved randomly. The sum of the standard deviations of number of outstanding orders of type $i$ ($\sigma_i$) over all products ($\sigma = \sum_{i=1}^{M} \sigma_i$) was used to study the performance of the system as it captured "the gross behavior of performance over a fairly wide range of systems, and no other measure of comparable simplicity did so" [63]. A closed form expression for $\sigma$ was developed in the case of FIFO policy. In the case of LQ policy, an approximation for $\sigma$ was developed. Numerical experiments suggested that the LQ policy performed better than the FIFO policy by about 20% in some cases. The difference in performance was the greatest when the number of product types was large, and at small SCVs. The difference was small at low utilizations, increased till 90% utilization and vanished in heavy traffic.

Cachon and Fisher [12] studied the impact of sharing demand and inventory data in a supply chain with one supplier, $N$ identical retailers and a stationary stochastic demand. Costs were associated with holding inventory and backorders. The traditional information policy that did not use shared information was compared against a full information policy that exploited the shared information. For both the models, it was assumed that the supplier's orders are always received at the retailer after a constant lead time. A stock-out at a supplier caused a resulting replenishment delay for the retailer. In the traditional information policy, the retailers and suppliers were assumed to follow ($R_r$, $nQ_r$) and ($R_s$, $nQ_s$) policies, respectively. The supplier was assumed to allocate inventory to each retailer based on a batch priority allocation. As per this allocation, if retailer $i$ ordered $b$ batches, then the first batch was given priority $b$, the second batch was given priority $b$-1, and so on. All the batches were assumed to be placed in a shipment queue based on decreasing priority order with ties broken randomly. In the full information model, the supplier could improve its order quantity decisions as well as allocation decisions based on the demand and inventory information from the retailer. The decision of allocation of batches could be improved as batches could be allocated based on the inventory position at the retailer in the period the batch is shipped as against the period it was ordered. Based on their comparison, it was found that supply chain costs were 2.2% lower on average with full information policy than with the traditional policy.

Gavirneni et al. [21] studied the periodic review inventory control problem in a two-echelon supply chain with a retailer and a supplier. The sequence of events during each period started with the supplier deciding on its production quantity for the period followed by the retailer realizing its customer demand. On satisfying the demand, the retailer placed an order with the supplier if its inventory level fell below the re-order point $s$. The order was assumed to be satisfied at the beginning of the next period. An order not satisfied by the supplier was assumed to be satisfied from some other supplier with no lead time. The $(s, S)$ inventory policy was assumed to be optimal for the retailer who incurs a fixed plus linear ordering cost, linear holding and backorder costs, while the supplier incurred a linear holding and backorder cost. They studied three different models which differ in the amount of information shared. In the first case, there was no information sharing and the supplier followed a naive approach, assuming that the retailer demand followed an i.i.d. process. In the second model, the supplier knew the number of periods that had elapsed since the last order from the retailer as well as that the retailer was using the $(s, S)$ inventory policy. In the third model, the supplier knew the number of units that had been sold by the retailer since the last order. In all three models, the optimal order up-to level was computed via simulation-based optimization using infinitesimal perturbation analysis. It was shown that the third model had the least cost with the first model having the highest cost, showing that full information sharing is beneficial.

Gavirneni [20] studied a periodic review inventory control problem in a supply chain with one capacitated supplier who supplies a single product to multiple identical retailers. The customer demands were assumed to be i.i.d. During each period, the sequence of events started with the retailer reviewing its inventory and placing an order with the supplier. The supplier responded by using the available retailer inventory information to satisfy as many demands as possible. The retailer received the supplier shipments and satisfied the customer demand. Costs were associated with holding inventory and penalties were imposed for unsatisfied demand. The author studied this system under three different levels of cooperation. In the first level (no cooperation), the only information available to the supplier was the demand from the retailers. If the total demand was less than the capacity of the supplier, then all retailers' demands were satisfied. If there were orders from the retailers in excess of the supplier capacity, the supplier was assumed to use the lexicographic allocation scheme. This scheme "ranked the consumers in the order of their importance (independent of their order quantities) and the demands were satisfied in that order" [20]. In the second level of cooperation, the supplier received the current retailer inventory level in addition to the demand. The supplier could allocate its capacity based on this information in such a way that the retailer with a larger inventory level received smaller shipments and the one with smaller inventory level received larger shipments. The third level of cooperation extended the second level model to include the possibility of transfer of inventory from one retailer to another. The research used a simulation model to search for an optimal target inventory level and optimal cost. The computational results showed that the third level of coordination resulted in the least cost and the benefits of cooperation in this supply chain decreased with increase in supplier capacity, increase in number of retailers, decrease in penalty cost, and decrease in demand variance.

Armony and Plambeck [4] studied the effect of duplicate orders on a manufacturer's estimation of demand rate and customer's sensitivity to delay, and decisions on capacity investment. They considered a SCN where a manufacturer sold its products through two distributors. The customer demand arrival process at each distributor was assumed to be a Poisson process and each demand was assumed to be for a single product. Each distributor followed one-for-one replenishment policy. When one of the distributors faced a stock-out situation, then an arriving customer had the option to immediately obtain the finished goods from the other distributor, provided the other distributor had inventory. When both the distributors were out of stock, then the customer could place an order with both distributors. When the product was delivered by one of the distributors, then the customer canceled the duplicate order placed at the other distributor. The customers were assumed to be impatient, leading to cancellations of outstanding orders after a waiting time that was exponentially distributed. They assumed that the manufacturer had knowledge of the base-stock policy used by the distributors, which would help in the inference about the inventory level and the number of outstanding orders at the distributors. They obtained the maximum likelihood estimators for the demand rate, the reneging rate, and the probability that a customer would place a duplicate order when made to wait.

Li et al. [41] presented a review of about 10 different information sharing models. These included the works of Zipkin [63], Gavirneni et al. [21], Gavirneni [20], Graves [24], Moinzadeh [45], Chen [13], Kulp [36], Cachon and Fisher [12], Schouten et al. [50], and Bourland et al. [9]. From the literature review presented here, it can be seen that only Armony and Plambeck [4] and Zipkin [63] considered the development of queueing-based performance evaluation models.

## 2.2 Modeling Production-Inventory and Supply Chain Networks

### 2.2.1 Modeling Production-Inventory Networks

Performance evaluation models of production-inventory networks consider capacity, variability, and inventory issues in a unified manner. Production-inventory network models are typically used to analyze the performance of make-to-stock systems, in which planned inventory is maintained for finished products and semi-finished products at the intermediate stages.

Gavish and Graves ([22], [23]) studied a single-stage, single-product make-to-stock production facility with Poisson demand. They focused on finding a control policy that minimizes the expected cost per unit time using an M/D/1 queueing system [22] as well as an M/G/1 queueing system [23]. The optimal decision policy for both the models was found to be a two-critical-number policy characterized by the parameters

($Q^*$, $Q^{**}$), "such that the server is turned off when the queue length is first reduced to $Q^*$, and is turned on when the queue length first reaches $Q^{**}$" [22]. Graves and Keilson [23] studied a one-product, one-machine problem under Poisson demand and exponentially distributed order size. They focused on minimizing the system cost that included setup cost, inventory holding cost, and backorder cost using a spatially constrained Markov process model.

The work of Svoronos and Zipkin [56] does not explicitly model capacity issues, but forms the basis for models developed by Zipkin [40] and others. Svoronos and Zipkin [56] modeled a multi-echelon inventory system with Poisson demand arrivals at the lowest hierarchy of the system called the leaf. Each demand reached the central depot through instantaneous replenishment orders to preceding stages. The central depot was assumed to have infinite raw material supply from an outside source. Each location was considered to be a node and each arc connecting a node with its predecessor was treated as a transit system (representing transportation or production activities). Parts are assumed to be processed sequentially and the behavior of each transit system was assumed to be independent of the demands and orders in the inventory system. The transit systems for all the arcs on the path from the outside source node to the leaf node were assumed to be mutually independent. Svoronos and Zipkin [56] applied the results of the single-location problem recursively, starting from the highest-level echelon to analyze the complete network.

Buzacott and Shanthikumar [11] presented results for general single-stage systems with unit demand and backlogging using the production authorization (PA) card concept, where a tag attached to an item is converted into a PA card when demand consumes the item. Using the number of outstanding orders in the stage at time $t$, $K(t)$, the total number of tags in the system, $S$, the inventory level at time $t$, $I(t)$, and the number of customer orders backordered at time $t$, $B(t)$, can be obtained using equations (2.1) and (2.2). Since the PA card represents an outstanding order at a stage, the number of PA cards available at time $t$, $C(t)$, can be obtained using equation (2.3). It can be seen from equations (2.1) through (2.3) that by studying the process $K(t)$, we can derive $I(t)$, $B(t)$ and $C(t)$.

$$I(t) = \max[0, S - K(t)] \tag{2.1}$$

$$B(t) = \max[0, K(t) - S] \tag{2.2}$$

$$C(t) = \min[S, K(t)] \tag{2.3}$$

The distribution of the number of outstanding orders in the system ($K$) is the same as the number of customers in a GI/G/1 queue because of the assumption of infinite raw material supply. The expected inventory level and the expected number of backorders can be obtained using equations (2.4) and (2.5).

$$E[I] = \sum_{n=0}^{S} n\, P(I = n) \tag{2.4}$$

$$E[B] = E[I] + E[K] - S \tag{2.5}$$

Buzacott and Shanthikumar [11] also modeled single-stage systems with lost sales, interrupted demand, bulk demand, machine failures and yield losses.

The analytical models for multi-stage tandem production-inventory networks developed by Lee and Zipkin [40], Sivaramakrishnan [51], Liu et al [43], and Srivathsan [53] have many common assumptions, such as one-for-one replenishment with backordering permitted at each stage, no limit on WIP queues at a stage and ample raw material supply at the first stage [30].

The approaches developed by Sivaramakrishnan [51], Liu et al. [43], and Srivathsan [53] are based on the parametric decomposition approach ([59], [61]) and share a common solution structure as explained in Kamath and Srivathsan [30].

Sivaramakrishnan and Kamath [52] modeled an M-stage tandem make-to-stock system by decomposing it into individual stages where each stage is made up of the manufacturing resource at that stage and a delay node to "capture the upstream delay experienced by an order when there was no part in the output store of the previous stage" [51]. Sivaramakrishnan [51] modeled the manufacturing unit as a GI/G/1 queue and the delay node as an M/G/$\infty$ queue.

Sivaramakrishnan [51] extended the tandem model to include (i) multiple servers at a stage, (ii) batch service, (iii) limited supply of raw material, (iv) multiple part types, and (v) service interruptions due to machine failure. Sivaramakrishnan [51] also extended his approach to tandem networks with feedback and feed-forward networks.

Liu et al. [43] modeled a multi-stage tandem manufacturing/supply network with general inter-arrival and service processes. The tandem network was modeled as a system of inventory queues and then the overall inventory in the network was optimized with service-level constraints. A decomposition scheme was proposed, in which a semi-finished product from a stage is moved into the input buffer of the downstream stage, whenever the job queue (material queue at the input buffer of node $i$ + backorder queue at node $i$) is increased by one. If the inventory store at stage $i$-1 is empty, then the request for the semi-finished part is backordered. As a result, the job queue or outstanding orders queue at stage $i$ consists of the material queue comprising of the semi-finished products from the upstream stage and a backorder queue. In this approach, the backorder distribution is used to account for any delay due to stock-out at stage $i$-1.

Lee and Zipkin [40] modeled a tandem production-inventory network with Poisson demand arrivals, and exponential service times at all stages. The multi-echelon system developed by Svoronos and Zipkin [56] was used to analyze the tandem system by assuming that stage $i$ behaved like an M/M/1 queue [40] and the sojourn times was exponentially distributed with mean $1/(\mu_i$ - $\lambda)$, where $\lambda$ is the demand rate, and $\mu_i$ is the processing rate at stage $i$. The sojourn times at all the stages were considered to be independent. Such an effective lead time decomposition approach is similar to Whitt's parametric decomposition approach [59]. The approach developed by Svoronos and Zipkin [56] that employed phase type approximations (for more details, refer to Theorem 2.2.8 in Neuts [47]) was then used to solve the above approximate system. Zipkin [64] extended this work to model tandem production-inventory networks with feedback.

For a detailed comparison of the models developed by Lee and Zipkin [40], Sivaramakrishnan [51], Liu et al. [43] and Srivathsan [53], the reader is referred to Kamath and Srivathsan [30], wherein the similarities and differences between the approaches are clearly documented along with the results of an extensive numerical study to gain better insight into the performance prediction capability of the different approximations.

Nguyen [48] analyzed the problem of setting the base-stock levels in a production system that produced both make-to-order and make-to-stock products with lost sales. Nguyen [48] derived the product-form steady-state distribution for the above network under the assumptions that each station operated under a FIFO service discipline, all processing times and inter-demand times were exponential, and all products had the same mean processing time. Nguyen [48] proposed approximations for the base-stock levels based on heavy traffic analysis of queueing networks.

Karaesmen et al. [31] assumed that the inter-arrival times and the processing times are geometrically distributed and modeled the system with advance order information for contract suppliers. They analyzed the base-stock policy ($S$, $L$) with a focus on optimization and performance evaluation of the Geo/Geo/1 make-to-stock queue, where $L$ is the release lead-time, which was used to "regulate the timing of material release into the manufacturing stage" [31].

Benjaafar et al. [8] examined the effect of product variety on the inventory cost in a finite capacity make-to-stock production-inventory network where the products were manufactured in bulk and shared a common manufacturing facility. The production facility was assumed to comprise of a batching stage and a processing stage. The production stage was viewed as a GI/G/1 queue whose arrival process was the superposition of the order arrivals for the individual products. The key findings of the study were that the total cost increased linearly with the number of products, and that the rate of increase depended on the system parameters such as demand and processing time variability and capacity levels.

Benjaafar et al. [7] studied the effect of inventory pooling in production-inventory networks with $n$ locations having identical costs. The production facility was assumed to have finite capacity and supply lead times were assumed to be endogenous. They studied the sensitivity of the cost benefits from inventory pooling to system parameters such as service levels, demand and service time variability, and structure of the production-inventory network. The different structures of the network that were studied include (i) single production facility with Poisson arrivals and exponential service times and two sets of priorities - FIFO and the longest queue first policies, (ii) single production facility with service level constraints, (iii) single production facility with non-Markovian demand and general service times, (iv) systems with multiple production facilities, and (v) systems with multiple production stages. For the system with multiple production facilities, they considered three different scenarios - (a) inventory pooling without capacity pooling, (b) inventory pooling with capacity pooling, and (c) capacity pooling without inventory pooling. In the first scenario, they considered a single inventory location supplied by $n$ production facilities, with a demand from stream $i$ resulting in a replenishment order at production facility $i$. They assumed a Markovian system and each facility was modeled as an independent M/M/1 queue. In the second scenario, both the inventory and capacity were assumed to be pooled. Under Markovian assumptions, the production system was modeled as an M/M/n queue. In the third scenario, the production facilities are consolidated, while the inventory locations were distinct. The Markovian system with $n$ identical facilities was modeled as an M/M/n multi-class FIFO production system. The results of the study showed that the benefit of inventory pooling decreased with utilization, while the benefit of capacity pooling increased with utilization.

### 2.2.2 Modeling Supply Chain Networks and its Constituents

Performance evaluation models of SCNs consider the supply, transportation and distribution operations in a SCN in addition to the capacity, congestion, and inventory issues. These models are typically used to analyze the flow of information and goods between the various stores in a SCN.

Cohen and Lee [14] developed an analytical model for integrated production-distribution systems by decomposing the network into sub-models such as the material control sub-model, production sub-model, stockpile inventory sub-model, and the distribution sub-model. The sub-models were optimized based on certain control parameters which served as links between the sub-models. These control parameters included lot sizes, reorder points and safety stocks.

Lee and Billington [37] and Ettl et al. [19] focused on developing models to capture the interdependence of base-stock levels at different stores in a SCN. Lee and Billington [37] study addressed the needs of the manufacturing managers at Hewlett-Packard in managing the material flows in their decentralized supply chains. The study focused on computing the mean and variance of the replenishment lead time for every stock-keeping unit (SKU) at a site. Lee and Billington [37] then used this information to compute the required base-stock level to attain a target service level for each SKU at that site. On the other hand, Ettl et al. [19] used a combination of analytical and queueing models to support strategic decision making for large SCNs with non-stationary demand. They considered a one-for-one replenishment policy, while ignoring order size at each store and other operational aspects of inventory management in a SCN. They developed an optimization model with service level constraints and used a queueing model to obtain the order queue distribution at each stage.

Raghavan and Viswanadham [49] used fork-join approximations to compute the mean and variance of departure processes at nodes in a SCN. They presented simple approximations for the case of deterministic arrivals and normally distributed service times. Dong [16] and Dong and Chen [17] used the PA card concept of Buzacott and Shanthikumar [11] to model an integrated logistics system with ($s$, $S$) inventory policy, where each orders had a fixed lot size. They made use of the expressions provided by Buzacott and Shanthikumar [11] for a system with batch arrivals to find the distribution of number of orders at the manufacturing stage and then used this to find the stock-out probability and fill rate.

Srivathsan [53] extended the performance evaluation models for production-inventory networks developed by Sivaramakrishnan [51] to model a generic supply chain network where each manufacturer was assumed to have an input store for raw material storage and an output store for finished product storage. An example network with three suppliers, two manufacturers and three retailers was used to illustrate the performance prediction capability of the approximations. While Sivaramakrishnan [51] used the delay block to link successive nodes in the network, Srivathsan [53] used the backorder distribution to establish the link. With one-for-one replenishment, an end customer order at the retailer triggered an instantaneous replenishment order at each of the upstream stores. Each manufacturer was modeled as a single server queue, and the lead time delays at the raw material suppliers and the transportation delays in the network were approximated by an M/G/$\infty$ model.

Ayodhiramanujan [6] developed integrated analytical models that addressed capacity, congestion, and inventory issues simultaneously in warehouse systems. The research effort focused on developing queueing network models for a shared-server system and an order-picking system. The former is an inventory store with a server performing both the storage and retrieval operations. In the order-picking system, the configuration of the unit-load that is stored (pallets) is different from that which is retrieved (cases). Ayodhiramanujan [6] also extended these models to include multi-sever cases. An integrated model was also developed to demonstrate the applicability of these two key building blocks in developing end-to-end models of warehouse systems.

Jain and Raghavan [29] considered a production-inventory system with a manufacturing plant and a warehouse. The warehouse inventory was modeled as the input control mechanism for the manufacturing plant, which was modeled as a single-stage discrete-time queueing system with an infinite waiting line. The warehouse was assumed to follow a base-stock policy with one-for-one replenishment and a production authorization card was assumed to be attached to each finished good. The customer order arrival process was assumed to be Poisson and the manufacturer was assumed to process orders from the warehouse at fixed discrete time slots. The queueing models were embedded into two optimization models, the first of which focused on minimizing the long run expected total cost per unit time (comprising of holding cost at warehouse and backorder cost). The second optimization model focused on minimizing the long run expected total cost per unit time (comprising of the holding cost alone) subject to service level constraints based on the probability of backorders.

Arda and Hennet [3] considered an enterprise network where the end-product manufacturer had several potential suppliers for components. The supply system with random arrivals of customer orders and random supplier delivery times was modeled as a queueing system. The manufacturer was assumed to follow a base-stock inventory policy. Demand at the manufacturer was assumed to follow a Poisson process and was of unit size. The manufacturer was assumed to follow a ($S$-1, $S$)

inventory policy and placed an order with supplier $i$ with a probability $\alpha_i$ as per a Bernoulli splitting process. An optimization model with an objective of minimizing the average cost of the manufacturer (expressed as the sum of mean inventory holding cost and mean backordering cost) was used to obtain the Bernoulli probabilities and the optimal base-stock value. Each supplier was assumed to follow an exponential service time and FIFO discipline. They modeled each supplier as an M/M/1 queue as the order arrival process at the supplier was Poisson because of the Bernoulli splitting process. They showed that the problem at hand was hard as the convexity of the objective function was not guaranteed. They presented optimal solutions for the make-to-order system by solving the Lagrangian of the relaxed problem. In the case of the make-to-stock system, due to the complexity of the objective function, the problem was decomposed into two parts. In the first part, they considered the Bernoulli parameters as the decision variables, while considering the base-stock level as zero (make-to-order system). The values of the Bernoulli variables from the first part were then used as an input in the second part. The decision variable in this case was the base-stock level, which was computed using a discrete version of the newsvendor problem.

## 2.3 Summary of the Literature Review

This chapter presented a detailed review of literature on modeling information sharing in a supply chain network as well as performance evaluation models of production-inventory and supply chain networks. Based on the literature review, we note the following.

Queueing models of production-inventory networks have focused on considering capacity, variability, and inventory issues in a unified manner and included the works of Graves and Kielson [25], Lee and Zipkin [40], Buzacott and Shanthikumar [11], Sivaramakrishnan [51], Liu et al. [43] and a few others. The ability of queueing models to explicitly consider inventory issue has been considerably strengthened by the works of Sivaramakrishnan [51] on modeling production-inventory networks, Srivathsan [53] on modeling supply chain networks, and Ayodhiramanujan [6] on modeling warehouse operations. Other research efforts that focused on performance evaluation of SCNs include Dong [16], Dong and Chen [17], and Jain and Raghavan [29]. The research efforts that focused on performance optimization of SCNs included Ettl et al. [19], Lee and Billington [37], and Cohen and Lee [14].

The review of the literature on modeling inventory information sharing in a SCN showed that there are only a few efforts on developing performance evaluation models of SCNs in this context. Armony and Plambeck [4] and Zipkin [63] focused on performance evaluation using queuing models. The former focused on modeling the effect of duplicate orders on SCN performance, while the latter modeled a production-inventory network with multiple products produced by a single production facility.

We concluded that there is a need for more research in explicitly modeling upstream inventory information sharing within SCN performance evaluation models. The availability of a richer set of performance evaluation models will enable us to better predict the value of sharing upstream inventory information in a SCN and understand the complex dynamics resulting from decisions related to production capacities, maximum inventory levels, and order placement policies.

# CHAPTER 3

# RESEARCH STATEMENT

The overall goals of this research were (i) to develop analytical performance evaluation models that consider inventory information sharing between SCN firms, and (ii) to study the value of inventory information sharing and identify SCN conditions under which the benefits of inventory information sharing are significant.

## 3.1 Research Objectives

In most analytical performance evaluation models of SCNs, it is assumed that a downstream firm places an order at one of its immediate upstream "suppliers" using a Bernoulli routing policy. As per this policy, orders from a retail store would be routed to say the production facilities based on fixed probabilities (based on preference or historical information) and this would allow an order to be placed at a production facility facing a stock-out situation even when there is inventory at the other facility. This routing is appropriate for SCNs with no inventory information sharing. On the other hand, if a downstream firm has information about the net inventory level at all of its immediate upstream firms, orders from the downstream firm could possibly be placed in such a way that this situation can be averted, thereby reducing the backorders and decreasing the stock-out rate. The models developed in this research consider inventory information sharing in one direction - upstream stores sharing information with the immediate downstream stores.

An analytical model that addresses inventory information sharing in a SCN can be used to quantify the value of information sharing and provide insight into the sensitivity of the value of inventory information sharing to different SCN parameters (inter-arrival and service time parameters, base-stock levels, etc.). The model could then be used to identify the ranges of the various SCN parameters where the benefit of inventory information sharing is significant.

The specific objectives of this research are as follows.

**Objective 1:** To perform a thorough investigation of the literature related to (i) the value of information sharing in a SCN, and (ii) the analytical modeling of production-inventory and supply chain networks.

**Objective 2:** To develop analytical models that can capture the effect of inventory information sharing on the performance of a two-echelon SCN comprising of one retail store that can order items from one of two upstream production facilities, each with its own inventory store and to study the benefits of inventory information sharing in this context.

As discussed in the beginning of this section, this objective focused on modeling the effect of inventory information sharing on the performance of a supply chain. We first considered a SCN with one retail store and two production facilities each with its own inventory store. Henceforth, we will refer to this two-echelon configuration as "1R/2P." For this case, we considered three levels of inventory information sharing and developed analytical models for these three levels. The performance measures from these analytical models were compared with the corresponding measures for the SCN with no visibility (henceforth referred to as NoVis) to study the value of inventory information sharing. The three levels of inventory information sharing and the assumed routing policy for each level are presented next.

**Low level of Inventory Information Sharing (LoVis)**

In a SCN with minimum level of inventory visibility, the retail store is assumed to have information about the presence of inventory or backorders at the production facilities. In such cases, an order routing policy can be adopted where the placement of orders at a production facility with backorders can be avoided when the other facility has inventory (see Table 3.1).

Table 3.1: Order Routing Policy for SCN with Low Level of Inventory Information Sharing

| Net Inventory Level at Production Facility 1 | Net Inventory Level at Production Facility 2 | Order Routing Policy |
|---|---|---|
| $\geq 1$ | $\geq 1$ | Order routed with equal probability |
| $\geq 1$ | $\leq 0$ | Order routed to production facility 1 |
| $\leq 0$ | $\geq 1$ | Order routed to production facility 2 |
| $\leq 0$ | $\leq 0$ | Order routed with equal probability |

**Medium level of Inventory Information Sharing (MedVis)**

In a SCN with medium level of inventory visibility, the retail store is assumed to also have information about the number of backorders at the individual production facilities. When both production facilities are backordered, the order is routed to the facility with the shortest backorder queue. The order routing policy in such a scenario can be modified as shown in Table 3.2.

Table 3.2: Order Routing Policy for SCN with Medium Level of Inventory Information Sharing

<table>
<tr><th>Net Inventory Level at Production Facility 1 ($i$)</th><th>Net Inventory Level at Production Facility 2 ($j$)</th><th>Condition</th><th>Order Routing Policy</th></tr>
<tr><td>$\geq 1$</td><td>$\geq 1$</td><td></td><td>Order routed with equal probability</td></tr>
<tr><td>$\geq 1$</td><td>$\leq 0$</td><td></td><td>Order routed to production facility 1</td></tr>
<tr><td>$\leq 0$</td><td>$\geq 1$</td><td></td><td>Order routed to production facility 2</td></tr>
<tr><td rowspan="3">$\leq 0$</td><td rowspan="3">$\leq 0$</td><td>$|i| < |j|$</td><td>Order routed to production facility 1</td></tr>
<tr><td>$|i| > |j|$</td><td>Order routed to production facility 2</td></tr>
<tr><td>$i = j$</td><td>Order routed with equal probability</td></tr>
</table>

**High level of Inventory Information Sharing (HiVis)**

In a SCN with a high level of inventory visibility, the retail store is assumed to have information about the number of items in stock as well as the number of backorders at the individual production facilities. In such cases, the order routing policy can be modified as shown in Table 3.3.

Table 3.3: Order Routing Policy for SCN with High Level of Inventory Information Sharing

| Net Inventory Level at Production Facility 1 ($i$) | Net Inventory Level at Production Facility 2 ($j$) | Condition | Order Routing Policy |
|---|---|---|---|
| $\geq 1$ | $\geq 1$ | $i > j$ | Order routed to production facility 1 |
| | | $i < j$ | Order routed to production facility 2 |
| | | $i = j$ | Order routed with equal probability |
| $\geq 1$ | $\leq 0$ | | Order routed to production facility 1 |
| $\leq 0$ | $\geq 1$ | | Order routed to production facility 2 |
| $\leq 0$ | $\leq 0$ | $\lvert i \rvert < \lvert j \rvert$ | Order routed to production facility 1 |
| | | $\lvert i \rvert > \lvert j \rvert$ | Order routed to production facility 2 |
| | | $i = j$ | Order routed with equal probability |

**Objective 3:** To develop analytical models that can capture the effect of inventory information sharing on the performance of a two-echelon SCN comprising of two retail stores that can order items from one of two production facilities, each with its own inventory store and to study the benefits of inventory information sharing in this context.

We considered a SCN with two retail stores that place orders with one of two production facilities. The production facilities share their inventory information with the retail stores. Henceforth, we will refer to this two-echelon configuration as "2R/2P." For this case, we considered the three levels of information sharing defined in Objective 2 and developed analytical models for these levels.

**Objective 4:** To extend the models developed in Objectives 3 and 4 to include in-transit inventory.

## 3.2 Research Scope

The scope of this research effort was limited by the following assumptions.

1. There is no limit on the size of the WIP and backorder queues.

2. Each production facility has a single-stage with a single server. Each demand is for one unit of the product.

3. As the SCN configurations modeled are considered to be of the building block type, only two-echelon SCN structures will be modeled.

## 3.3 Research Contributions

The contributions of this research effort are listed below.

1. Development of analytical queueing models that can explicitly model inventory information sharing from upstream stores to downstream stores in supply chain networks.

2. Understanding the benefits of sharing inventory information and developing insights into SCN configurations for which these benefits are significant.

3. Evaluating the significance of each incremental piece of information that becomes available in the SCN.

4. An offshoot of this research is the potential to develop good approximations for the well-known shortest queue problem.

# CHAPTER 4

# RESEARCH APPROACH

This chapter explains the overall research methodology and the various modeling approaches that were used to achieve the research objectives outlined in the previous chapter. Section 4.1 presents the research methodology, and then describes the Markov chain approach followed by the parametric decomposition approach. Section 4.2 presents a list of performance measures that were used along with their definitions and their significance in a SCN context. Section 4.3 explains the validation procedure used for the analytical models.

## 4.1 Research Methodology

This research effort involved the development of analytical performance evaluation models. A standard methodology for such developmental research was employed. First, we modeled the SCN configurations with Poisson arrivals at the retail store(s) and exponential processing times at the production facilities as these conditions make the models more analytically tractable. For this case, we initially used the Markov chain approach. If the approach did not yield a closed-form solution, we then focused on the development of approximate queueing models based on the characteristics of the model under consideration. We then relaxed the exponential assumption and considered SCN configurations with general inter-arrival and processing time distributions using the two-moment framework. Whitt's ([59], [61]) parametric decomposition (PD) method was used to solve the resulting queueing models. Each analytical model developed was validated by comparing its results to equivalent simulation estimates. The objective for such a comparison was to determine the regions of the model parameter space where the accuracy of the analytical results was good or deemed acceptable. A design of experiments approach was used to ensure adequate coverage of the parameter space from the perspective of model usage in practice and to control the number of numerical experiments that had to be conducted. The numerical experimentation had the added benefit of providing insights into the system behavior under different parameter settings. The results of the numerical investigation were used to develop additional corrections or enhancements to improve the accuracy of the analytical models. Section 4.1.1 summarizes the Markov chain approach, and Section 4.1.2 briefly describes the PD approach.

### 4.1.1 Markov Chain Approach

Continuous Time Markov Chain (CTMC) models have been widely used to develop performance evaluation models of discrete event systems. The advantage of CTMC models is that they can yield exact solutions under exponential or Markovian assumptions. Also, because the CTMC models are based on detailed state information, they are preferred when state-based decisions have to be modeled. For example, the modeling of visibility-based routing policies in SCNs requires detailed state level information and could be easily modeled using the CTMC approach.

Both transient and steady-state analysis can be performed using a CTMC model [44]. In the case of performance evaluation, steady-state analysis is done to compute the long-run performance measures. This involves the solution of the rate balance equations, which could be an issue in the case of an infinite state space. A standard method in such cases is to express all state probabilities in terms of the probability of one particular state and to use the total probability equation to find this probability. However, this is possible if the expression involving the sum of the probabilities can be simplified to yield a closed-form expression. If simplification is not possible, then we need to find ways (e.g., limit the number of backorders) to truncate the state space in order to yield a finite-state CTMC which can then be solved numerically.

Another strategy to solve the balance equations is to use the structure of the CTMC to identify similarities between the state transitions in that CTMC and other CTMC models for which closed-form solutions are known. For instance, while solving the CTMC model of the SCN under the Bernoulli routing policy, we were able to identify transitions patterns in the CTMC model that resembled the state transitions in an M/M/1 queuing model (see Section 5.6). Such similarities may allow us to guess a solution, which can be verified by substituting the guessed solution into the balance equations.

If there is symmetry in a multi-tuple system, we might be able to combine states (e.g., $(i, j)$ combined with $(j, i)$) to reduce the size of the state space. The reduced CTMC can sometimes simplify the solution of the original CTMC. This reduced CTMC model can be solved using the rate balance equations or by identifying patterns in the chain. Once the CTMC is solved and the limiting probabilities are obtained, we can use a reverse mapping to obtain the limiting probabilities of the original CTMC model.

Other methods that can be used to solve the CTMC models include the difference-equation technique and the method of generating functions. For more details on these methods, the reader is referred to Medhi [44].

### 4.1.2 Parametric Decomposition Approach

In the 1980s, Whitt [59] defined a new modeling ideology highlighted by the parametric decomposition (PD) approach. According to Whitt, "a natural alternative to an exact analysis of an approximate model is an approximate analysis of an exact model" [59]. The PD approach is a very comprehensive method for analyzing a queueing network and uses only the first two moments of both the inter-arrival and service times. This approach formed the basis for a software package developed by Whitt, called the Queueing Network Analyzer (QNA) [59].

The PD approach for open queueing networks consists of two main steps: 1) analyzing the nodes and the interaction among the nodes to obtain the mean and the squared coefficient of variation ($SCV = variance\ /\ mean^2$) of the inter-arrival times at each node, and 2) obtaining the node and system performance measures based on GI/G/1 or GI/G/m approximations ([59], [61]).

**Analyzing nodes:** In a network, nodes interact with each other because of customer movement and these interactions can be approximately captured by the flow parameters, namely, the rates and variability parameters of the arrival processes at the nodes. The total arrival rate at each node is obtained using the traffic rate equations, which represent the conservation of flow. The utilizations of each of the nodes are calculated to check for stability of the system. The system is said to be stable if all utilizations are strictly less than one. This part of the analysis is similar to the approach introduced by Jackson [28] in solving open networks and involves no approximations.

The approximations come into the picture while calculating the variability parameters related to the flow, namely, the SCVs of the inter-arrival times. The SCVs are calculated using the traffic variability equations, which involve approximations for the basic network operations, which are a) flow through a node, b) merging of flow, and c) splitting of flow. These approximations can be found in Whitt ([59], [61]).

**Calculating node and system performance measures:** The nodes are treated as stochastically independent. The performance measures at each node can be calculated from the results available for the GI/G/1 ([59], [34]) and GI/G/m queues ([60]). The expected waiting time at each node is calculated from the results provided and the expected queue length is obtained using Little's law [42]. Whitt [59] also explains how several other node and network measures can be calculated.

In our research, there is a need to incorporate state-level details while modeling the SCN under visibility-based routing policies. The two-moment framework is well-suited for queueing network type models and it could be challenging to accommodate state-level details in the analytical model developed. A strategy that is sometimes used involves the use of phase-type distributions to represent general distributions. The feature or sub-system that needs to be modeled in detail (e.g., the order routing or splitting process at the retailer) is studied in isolation. The phase-type approach enables CTMC type modeling because the phases are exponential stages. To analyze a feature or sub-model in detail, the states in the Markov chain embedded at an instant of the feature need to be identified. The solution of this Markov chain results in the stationary probability vector, which can be used in obtaining the two-moment approximations for the feature or sub-system. Such an approach can be employed when closed-form expressions exist for the Markov chain with Poisson arrivals and exponential processing times. A good example is the development of two-moment approximations for a fork-join configuration [35]. In our research, we did not consider the phase-type approach because an exact solution to the CTMC model under Poisson arrivals and exponential distributions was not possible (discussed in Chapter 5).

## 4.2 Performance Measures

The performance measures that were computed at each stage of the SCN are fill rate, expected number of backorders, expected inventory level, the expected time to fulfill a backorder and expected time spent by an order.

- Fill rate is the probability that an order will be satisfied immediately and this depends on the availability of inventory at the stage. The definition of fill rate suggests that as the fill rate increases, the stock-out rate decreases and the relationship between them is given by stock-out rate = 1 - fill rate. In our analytical approach, we approximate the fill rate by the ready rate, which is the probability that there is inventory in the system. It should be noted that for Poisson arrivals, fill rate and ready rate will be the same because of the PASTA (Poisson Arrivals See Time Averages) principle [62].

- The expected number of backorders at any stage is the average number of unsatisfied orders at the stage. The significance of this measure is that a high value could indicate a potential for loss of customer goodwill and sales.

- The expected inventory level at any stage is the average number of items in the store at that stage. The expected inventory level is a paradoxical measure as its high value would increase the inventory holding cost, while its low value could be an indicator of lower fill rates and higher backorder levels.

- The expected time to fulfill a backorder at a stage is the average time that an order has to wait before being satisfied at a store given that the store is facing a stock-out situation. It has to be noted that the expected time to fulfill an order can be considerably lower than the expected time to fulfill a backorder. This is because, majority of the orders could be satisfied instantaneously, while orders that are backordered could take significantly longer time to complete. Thus, this is a measure of the experience of a customer who faces a stock-out situation.

- The expected time spent by an order is defined as the sum of the average time that an order spends at that stage and the average time spent (as a product) in the output store at that stage. This measure is important as it is not a desirable situation to have an item sitting in stock for long periods as this would increase the holding cost.

### 4.3 Numerical Validation Procedure

Each SCN configuration that was modeled in our research effort was also simulated using a model developed in Arena 11.0 software [32]. The warm-up period was determined using Welch's procedure [58] (See Appendix A). The number of independent replications was set to 10.

The parameters of the various SCN configurations, namely, base-stock levels; variability of inter-arrival and processing times; utilization; and probabilities (if any) - were varied systematically using a design of experiments approach to cover a wide range of scenarios. For each scenario, the analytical results were compared with steady-state simulation estimates to evaluate the accuracy of the analytical results. As mentioned in Whitt [60], the two standard ways to measure the accuracy are absolute difference and relative percentage error. As Whitt [60] contends, neither procedure is appropriate for a wide range of values. When the performance measure values are themselves small (e.g., less than 0.5), the absolute difference seems to be appropriate. Whitt [60] is of the opinion that the quality of the approximations is satisfactory "if either the absolute difference is below a critical threshold or the relative percentage error is below another critical threshold" [60]. Recently, another approach, namely, normalized error has been used to evaluate the accuracy of queueing approximations ([35], [54]).

Hence, we designed the following approach to evaluate the accuracy of the analytical results. For the fill rate (bounded by 1), we used the absolute difference, $|simulation - analytical|$, expressed as a percentage. For the expected inventory level and the expected number of backorders, we used the normalized percentage error given by 100 $(|simulation - analytical|)/base\text{-}stock\ level$. For the expected time to fulfill a backorder and the expected time spent by an order, we used the relative percentage error defined as 100 $(|simulation - analytical|)/simulation$. This scheme allowed us to perform an overall analysis of the quality of the analytical results.

# CHAPTER 5

# CTMC MODELS OF THE 1R/2P SCN CONFIGURATION

This chapter presents the CTMC models of the 1R/2P SCN configuration under three levels of inventory information sharing as defined in Section 3.1. Section 5.1 presents the description of the SCN structure used in the study. The state definition used in the CTMC models is presented in Section 5.2. Sections 5.3, 5.4 and 5.5 present the CTMC models for the SCN with high (HiVis), medium (MedVis) and low (LoVis) levels of information sharing, respectively. Section 5.6 presents the CTMC model for the SCN with no information sharing (NoVis). Section 5.7 presents the results of the study on the benefits of inventory information sharing on the SCN performance. Finally, some concluding remarks about the CTMC models are presented in Section 5.8. For the sake of simplicity, we will refer to inventory information sharing as simply information sharing.

## 5.1 1R/2P SCN Structure

We consider a two-echelon SCN with one retail store and two production facilities each with its own output store to stock finished products. Each store in the SCN is assumed to operate under a base-stock control policy with one-for-one replenishment. Both the production facilities are assumed to have the same base-stock level ($S$). The base-stock level at the retail store is $R$. Each customer order is assumed to be for a single unit of the finished product. The demand inter-arrival times at the retail store as well as the processing times at the production facilities follow general distributions, but only the exponential case is considered in this chapter. The arrival of a demand consumes a finished product at the retail store if available and causes an order for replenishment to be placed at one of the two production facilities (see Figure 5.1). If available, a finished product from the output store of the production facility is instantaneously sent to the retail store and the output store sends an order for replenishment to its processing stage. As noted in Chapter 1, extensions to include transit delays will be presented in Chapter 9. We assume that the processing stage has a single server. An order can join the WIP queue or be processed as soon as it is received. There are no limits on the number of backorders at either production facility. It has to be noted that when the base-stock levels at the two production facilities are the same, the HiVis routing policy closely resembles the shortest queue problem studied in the literature (e.g. [33], [26], and [2]).

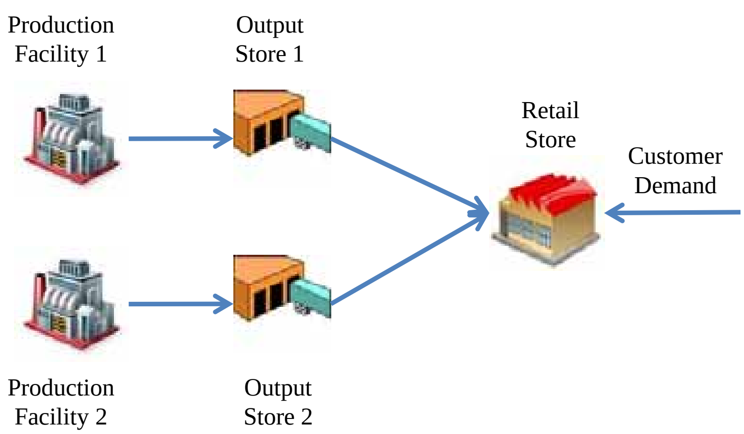



Figure 5.1: 1R/2P SCN Configuration

## 5.2 CTMC Model for Poisson Arrivals and Exponential Processing Times

This section presents the state-space definition for the CTMC model used to study the SCN with or without information sharing. This section also provides the details on obtaining the performance measures for the SCNs with information sharing (LoVis, MedVis and HiVis). The following assumptions were made in developing the CTMC models.

1. The demand/order arrival process is Poisson with rate $\lambda$.

2. The processing times at both the production facilities follow an exponential distribution with rate $\mu$ (the two production facilities are identical).

3. The transportation time from the production facility to the retail store is not modeled (relaxed later).

Ignoring the transportation delay between the production facilities and the retail store means that there is no need to consider the number of orders at the retail store while defining the state of the SCN (see explanation below). As a result, the state of the SCN at time $t$ is defined by $X(t) = \{i,\, j\}$, where $i$ and $j$ are non-negative integers representing the number of orders at production facilities 1 and 2, respectively.

The details about the net inventory levels at the various stores in the SCN can be obtained from the state definition as follows. When the number of orders at a production facility is less than the base-stock level, the output store at the production facility will have inventory and the inventory level will be given by ($S$ - $x$), where $x = i$ for production facility 1 and $x = j$ for production facility 2. Similarly, when the number of orders at a production facility exceeds the base-stock level, the output store is backordered. The number of backorders at the production facility will be given by ($x$ - $S$). The net inventory level at a production facility will be given by ($S$ - $x$). This value will be positive in the presence of inventory and negative in the presence of backorders.

As the transportation delay is ignored, when the number of orders at each production facility does not exceed the base-stock level of its output store (i.e. $0 \leq i, j \leq S$), the inventory level at the retail store will be equal to its base-stock level ($R$). When at least one of the production facilities is backordered, the inventory level at the retail store will be equal to its base-stock level less the sum of the backorders at the production facilities and the number of orders (backorders + replenishment orders) at the retail store is given by $(i - S)^+ + (j - S)^+$. The retail store will be backordered only when the sum of the backorders at the two production facilities exceeds the base-stock level at the retail store (i.e., $(i - S)^+ + (j - S)^+ > R$). The net inventory level at the retail store will be $[R - (i - S)^+ - (j - S)^+]$.

To better understand the above discussion, let us consider a case with $R = S = 3$. If the number of orders at production facility 1 is 1, and the number of orders at production facility 2 is 1, then the current state of the SCN is (1, 1). The net inventory level at the two production facilities will be 2 (i.e., 3-1), indicating that there is inventory at both the production facilities. As a result, the number of orders at the retail store is zero and the net inventory level is 3 (i.e., $[3-(1-3)^+-(1-3)^+]$).

If the number of orders at production facility 1 is 2, and the number of orders at production facility 2 is 4, then the current state of the SCN is (2, 4). In this case the net inventory level at production facility 1 is 1 (i.e. 3-2), indicating that there is one unit of inventory at production facility 1. The net inventory level at production facility 2 is -1 (i.e., 3-4), indicating that there is one backorder at production facility 2. As a result, number of orders (replenishment in this case) at the retail store is 1 (because of the backorder at production facility 2) and the net inventory level is 2 (i.e., $[3 - (2 - 3)^+ - (4 - 3)^+]$).

If the number of orders at both the production facilities is 5, the current state of the SCN is (5, 5). In this case the net inventory level at both the production facilities is -2 (i.e. 3-5), indicating that there are two backorders at each production facility. The number of orders at the retail store is 4 (sum of the backorders at the two production facilities) and the net inventory level is -1 (i.e., $[3-(5-3)^+-(5-3)^+]$), indicating one backorder at the retail store.

The following notations are used in this chapter.

| | |
|---|---|
| | ***Model Parameters*** |
| $\lambda$ | Order arrival rate at retail store |
| $R, S$ | Base-stock levels at retail store and output stores of the production facilities, respectively |

| | |
|---|---|
| | ***Model Variables (Upper case letters are random variables)*** |
| $N^r$ | Number of orders at the retail store |
| $N^p$ | Number of orders at a production facility |
| $\rho$ | Utilization of a production facility |
| $I^r$, $I^p$ | Inventory levels at the retail store and output store of a production facility |
| $B^r$, $B^p$ | Number of backorders at the retail store and output store of a production facility |
| $f^r$, $f^p$ | fill rates at the retail store and output store of a production facility |
| $W^{br}$, $W^{bp}$ | time to fulfill a backorder at the retail store and the output store of a production facility |
| $T^r$, $T^p$ | time spent by an order at the retail store and a production facility including its output store |

The CTMC models for the different levels of information sharing have some common properties that are listed below.

1. The state space of the CTMC is symmetrical as the two production facilities are identical (same base-stock level and processing rate). This means that the transition rates into (out of) the states $(i, j)$ and $(j, i)$ are the same except when $i = j$. This observation allowed us to combine the states $(i, j)$ and $(j, i)$ and

reduce the state space of the CTMC model.

2. The patterns present in the reduced CTMC models for the SCNs with information sharing (Figures 5.3, 5.5, and 5.7) could not be exploited to yield a closed form solution. As a result, the CTMC models were solved by limiting the maximum number of backorders at the retail store to $M$.

3. Once we numerically solved for $\pi_{i,j}$ by limiting the maximum number of backorders at the retail store, we computed the key performance measures for a production facility as follows.

   The fill rate at a production facility is simply the probability that an order from the retail store is satisfied immediately as shown in equation (5.1).

$$f^p = \sum_{i=0}^{S-1}\sum_{j=0}^{S-1}\pi_{i,j} + \sum_{i=S}^{S+R+M}\sum_{j=0}^{S-1}\pi_{i,j} + \sum_{i=0}^{S-1}\sum_{j=S}^{S+R+M}\pi_{i,j} \tag{5.1}$$

   The rest of the performance measures for a production facility were obtained using expressions (5.2) and (5.5).

$$E[I^p] = \sum_{i=0}^{S}\sum_{j=0}^{S+R+M}(S-i)\pi_{i,\,j} \tag{5.2}$$

   The expected backorder at a store can be calculated using the fundamental expression relating the expected inventory level, expected number of orders and the base-stock level as shown in equation (5.3).

$$\begin{aligned} E[B^p] &= E[I^p] + E[N^p] - S \\ &= \sum_{i=0}^{S}\sum_{j=0}^{S+R+M}(S-i)\pi_{i,\,j} + \sum_{i=0}^{S+R+M}\sum_{j=0}^{S+R+M} i\pi_{i,\,j} - S \end{aligned} \tag{5.3}$$

Now, using Little's law we have

$$E[W^{bp}] = \frac{E[B^p]}{(1 - f^p)\lambda/2} \tag{5.4}$$

$$E[T^p] = \frac{\sum_{i=0}^{S+R+M}\sum_{j=0}^{S+R+M} i\pi_{i,\,j}}{\lambda/2} + \frac{E[I^p]}{\lambda/2} \tag{5.5}$$

4. The arrival process at the retail store is Poisson and as per the PASTA principle [62], the ready rate can be used to calculate the fill rate. Note that an arriving customer order at the retail store would see the number of orders at the two production facilities. The fill rate at the retail store was obtained by calculating the probability that the total number of orders at the two production facilities is less than the sum of their base-stock levels and that of the retail store, i.e. $i + j < 2S + R$. The other performance measures at the retail store can be obtained using expressions (5.7) through (5.11).

$$f^r = \sum_{i,\,j \in A_1} \pi_{i,\,j} \tag{5.6}$$

where $A_1 = \{i, j : 0 \leq i + j \leq 2S + R - 1\}$

$$E[I^r] = \sum_{i=0}^{S-1}\sum_{j=0}^{S-1} R\pi_{i,\,j} + \sum_{i,\,j \in A_2} \left[R - (i\ -\ S)^+ - (j\ - S)^+\right]\pi_{i,j} \tag{5.7}$$

where $A_2 = \{i, j : 1 \leq (i\ -\ S)^+ + (j\ - S)^+ \leq R\}$

$$E[N^r] = \sum_{i,\,j \in A_3} \left[(i\ -\ S)^+ + (j\ - S)^+\right]\pi_{i,j} \tag{5.8}$$

where $A_3 = \{1 \leq (i\ -\ S)^+ + (j\ - S)^+ < \infty\}$

$$E[B^r] = E[I^r] + E[N^r] - R \tag{5.9}$$

$$E[W^{br}] = \frac{E[B^r]}{\lambda(1 - f^r)} \tag{5.10}$$

$$E[T^r] = \frac{E[I^r]}{\lambda} + \frac{E[N^r]}{\lambda} \tag{5.11}$$

Next, we present the CTMC models for all three levels of information sharing followed by the no information sharing case. Sections 5.3, 5.4 and 5.5 present the CTMC models of the SCN with high (HiVis), medium (MedVis) and low (LoVis) levels of information sharing, respectively. Section 5.6 presents the CTMC model of the SCN with no information sharing (NoVis).

## 5.3 CTMC Model of 1R/2P SCN with HiVis

In the HiVis case, it is assumed that each production facility shares complete information about the number of items in stock as well as the number of backorders. In the presence of such detailed information, a routing policy presented in Table 3.3 can be adopted.

Based on the state definition and the routing policy, we obtain the symmetric CTMC model shown in Figure 5.2. The symmetry in Figure 5.2 is exploited to obtain the reduced CTMC presented in Figure 5.3. The rate balance equations based on the different state transitions that are possible in the reduced CTMC model are presented in Table 5.1.

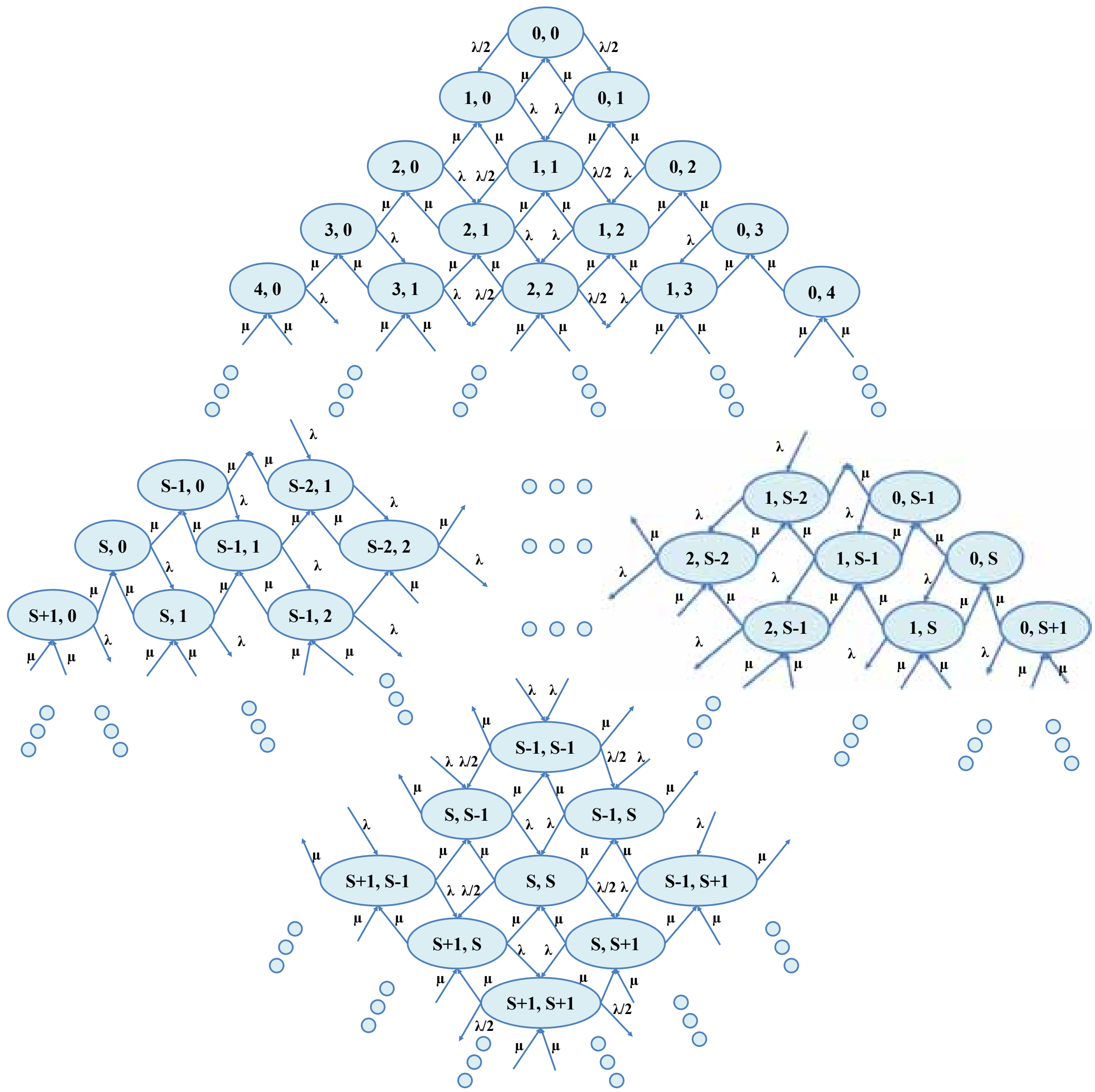



Figure 5.2: CTMC Model of the 1R/2P SCN with HiVis

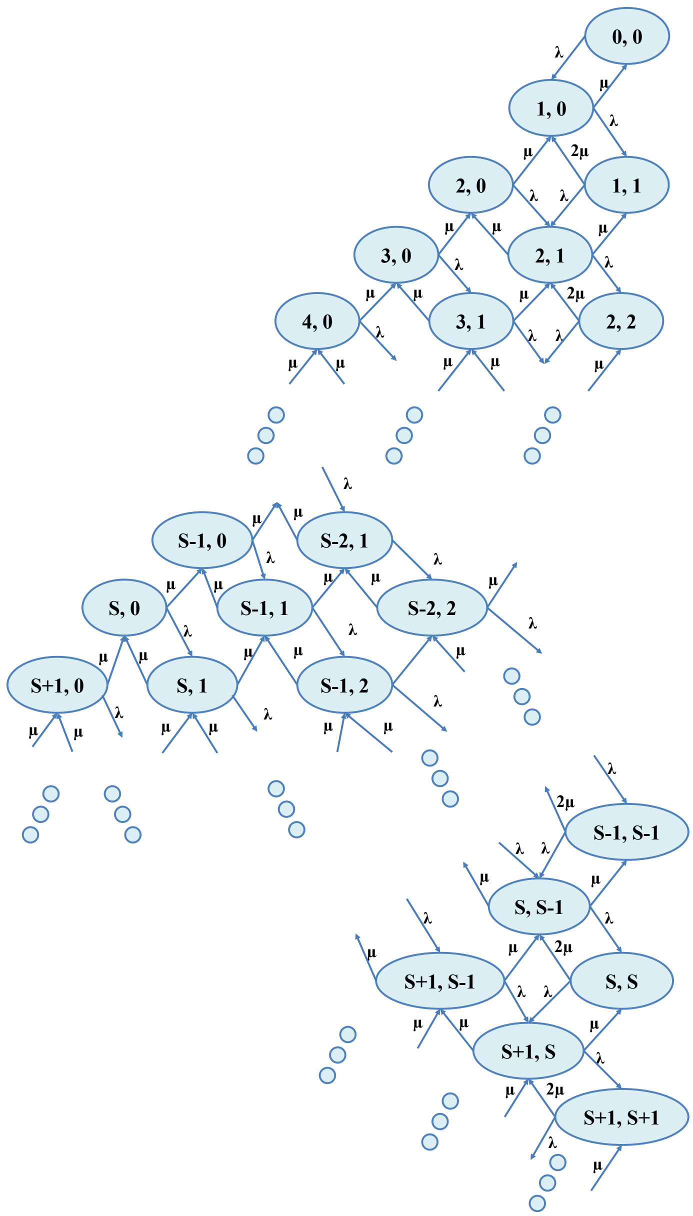
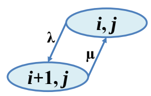
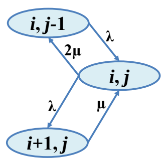
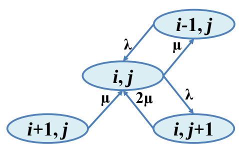
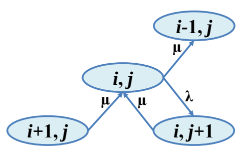
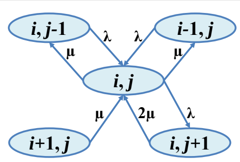
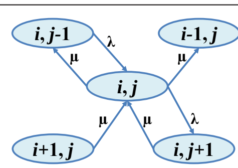



Figure 5.3: Reduced CTMC Model of the 1R/2P SCN with HiVis

Table 5.1: Rate Balance Equations for the Reduced CTMC Model of the 1R/2P SCN with HiVis

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $i = j = 0$ |  | $\pi_{i,j} = \frac{\mu\pi_{i+1,j}}{\lambda}$ |
| $i \geq 1, j \geq 1, i = j$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu\pi_{i+1,j}}{(\lambda+2\mu)}$ |
| $i = 1,\ \ j = 0$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i-1,j}+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}]}{(\lambda+\mu)}$ |
| $i \geq 2,\ \ j = 0$ |  | $\pi_{i,j} = \frac{\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+\mu)}$ |
| $i \geq 2,\ \ j \geq 1,\ \ i = j+1$ |  | $\pi_{i,j} = \frac{\lambda[\pi_{i,j-1}+\pi_{i-1,j}]+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}}{(\lambda+2\mu)}$ |
| $i \geq 3,\ \ j \geq 1,\ \ i = j+2$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+2\mu)}$ |

The reduced CTMC model revealed some state transition patterns, but we could not exploit these patterns to arrive at a closed-form solution. The different approaches that were explored to obtain a closed-form solution included the solution to the system of rate balance equations, difference-equation techniques and the method of generating functions. Hence, we fixed the maximum number of backorders at the retail store to 1,000 and numerically solved the CTMC. By fixing the maximum number of backorders, we make sure that the state space is finite. Further, a careful examination of the CTMC model shows that all the states communicate and that the CTMC model is irreducible. Hence, the CTMC is positive recurrent and has a steady state solution.

The CTMC model was validated by comparing its numerical solution to the estimates obtained from an Arena simulation model of the 1R/2P SCN with HiVis, as shown in Tables B.1 and B.2. The numerical experiments show that the analytical results match the simulation results, thus confirming the validity of the CTMC model.

## 5.4 CTMC Model of the 1R/2P SCN with MedVis

In this section, we present the CTMC model of the 1R/2P SCN with medium level of inventory information sharing. It is assumed that the information about the number of backorders at the individual production facilities is available at the retail store. In the presence of the backorder information, the routing policy presented in Table 3.2 can be adopted.

The symmetric CTMC model for this case is shown in Figure 5.4 and the reduced CTMC model is presented in Figure 5.5. In Figures 5.4 and 5.5, the transitions shown in blue are common to the HiVis CTMC model and the MedVis CTMC model, and those shown in red are the additional transitions for the MedVis CTMC model. The transition rates marked in red when the transitions are blue indicate that the transitions are common to both the HiVis and MedVis models, but the rates have changed in the case of the MedVis model. The rate balance equations based on the different state transitions that are possible in the reduced CTMC model are presented in Table 5.2.

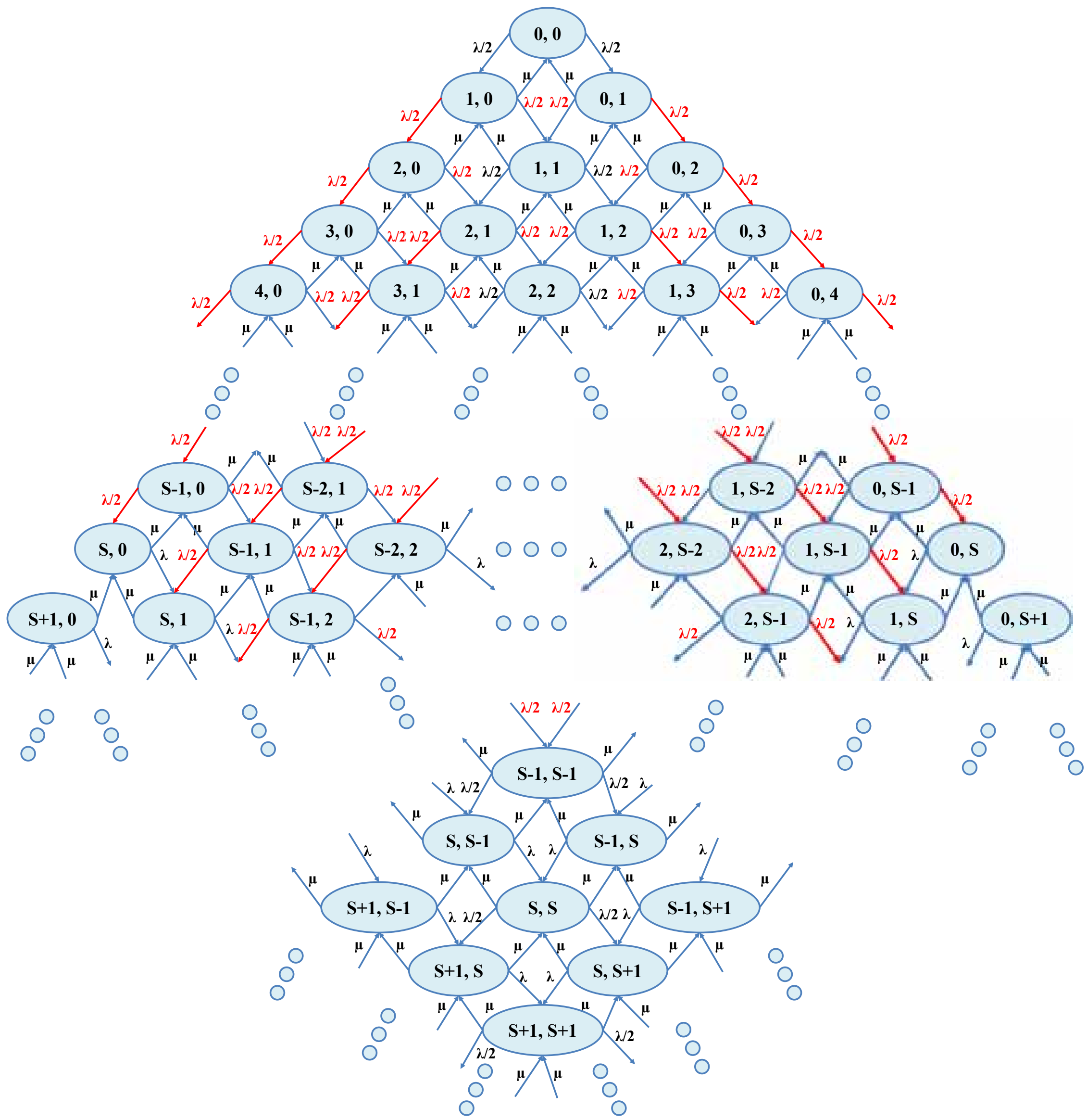



Figure 5.4: CTMC Model of the 1R/2P SCN with MedVis

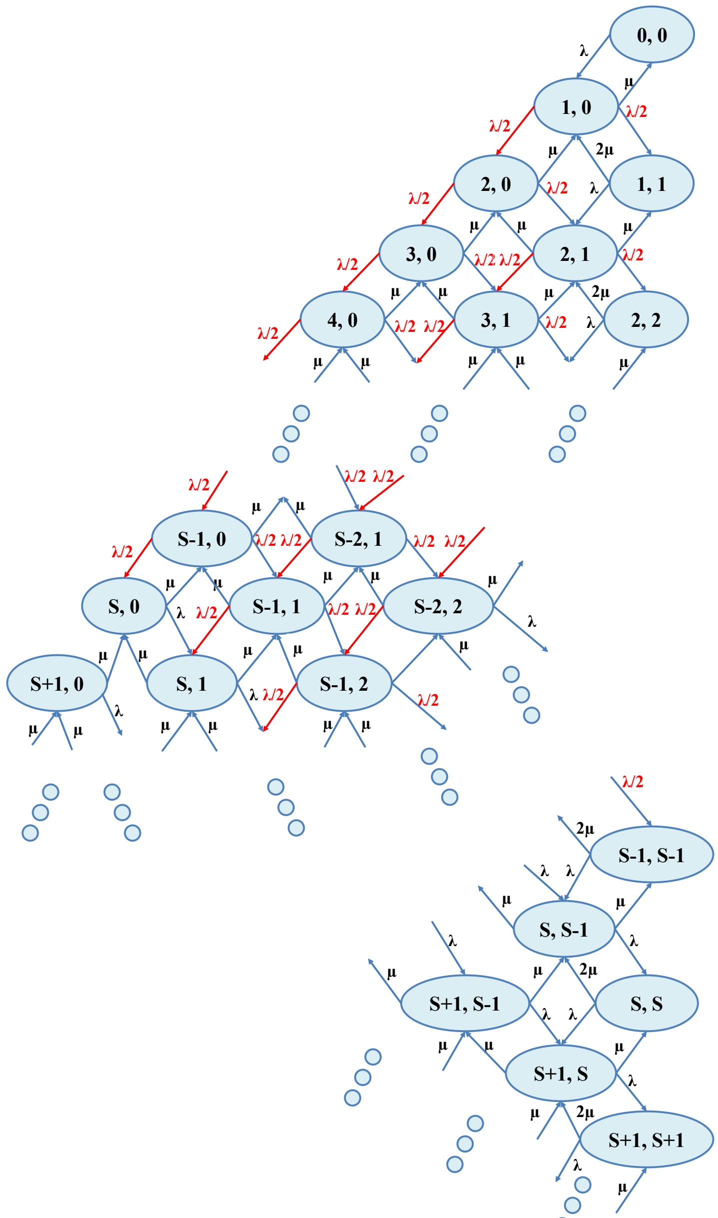
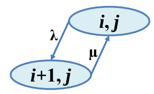
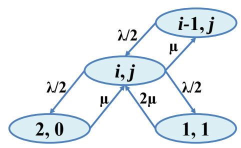
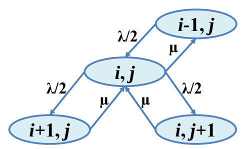
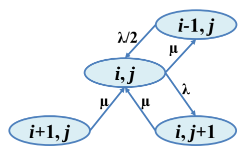
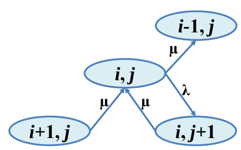
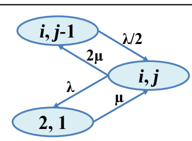
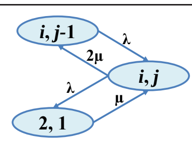
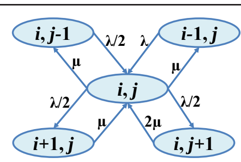
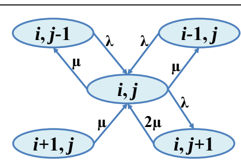


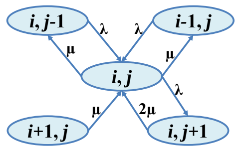
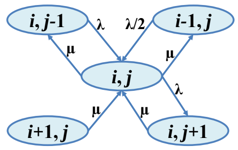
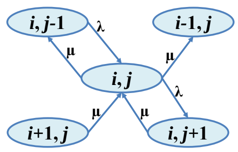

Figure 5.5: Reduced CTMC Model of the 1R/2P SCN with MedVis

Table 5.2: Rate Balance Equations of the Reduced CTMC Model of the 1R/2P SCN with MedVis

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $i = j = 0$ |   | $\pi_{i,j} = \frac{\mu\pi_{i+1,j}}{\lambda}$ |
| $i = 1, \ j = 0$ |   | $\pi_{i,j} = \frac{\lambda\pi_{i-1,j}+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}}{(\lambda+\mu)}$ |
| $2 \le i \le S-1, \ j = 0$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+\mu)}$ |
| $i = S, \ j = 0$ |   | $\pi_{S,0} = \frac{\frac{\lambda}{2}\pi_{S-1,0}+\mu[\pi_{S+1,0}+\pi_{S,1}]}{(\lambda+\mu)}$ |
| $S+1 \le i < \infty, \ j = 0$ |   | $\pi_{i,j} = \frac{\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+\mu)}$ |
| $1 \le i \le S-1, \ i = j$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i,j-1}+\mu\pi_{i+1,j}}{(\lambda+2\mu)}$ |
| $S \le i < \infty, \ i = j$ |   | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu\pi_{i+1,j}}{(\lambda+2\mu)}$ |
| $2 \le i \le S-1, \ j = i-1$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i,j-1}+\lambda\pi_{i-1,j}+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}]}{(\lambda+2\mu)}$ |
| $S \le i < \infty, j = i-1$ |   | $\pi_{i,j} = \frac{\lambda[\pi_{i,j-1}+\pi_{i-1,j}]+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}]}{(\lambda+2\mu)}$ |

Continued on next page

**Table5.2 – continued from previous page**

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $1 \leq j \leq S-3,\ i \geq j+2,$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}[\pi_{i,j-1}+\pi_{i-1,j}]+\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+2\mu)}$ |
| $i = S,\ 1 \leq j \leq S-2$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\frac{\lambda}{2}\pi_{i-1,j}+\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+2\mu)}$ |
| $S+1 \leq i < \infty,\ 1 \leq j \leq S-1$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |

The state transition patterns identified in the reduced CTMC model could not be exploited to obtain a closed-form solution. As before, we explored the following approaches to obtain a closed-form solution including the solution to the system of rate balance equations, difference-equation techniques and the method of generating functions. Hence, we fixed the maximum number of backorders at the retail store to 1,000 and numerically solved the CTMC. By fixing the maximum number of backorders, we make sure that the state space is finite. Further, a careful examination of the CTMC model shows that all the states communicate. Hence, the CTMC is irreducible and positive recurrent. This ensures that steady state solution exists for the reduced CTMC model.

The CTMC model was validated by comparing its numerical solution to estimates from an Arena simulation model of the 1R/2P SCN with MedVis, as shown in Tables B.3 and B.4. The numerical experiments show that the analytical results match the simulation results, thus confirming the validity of the CTMC model.

## 5.5 CTMC Model of the 1R/2P SCN with LoVis

In this case, the retail store is assumed to have the minimum amount of inventory information (presence or absence of inventory) from the production facilities. The order routing policy presented in Table 3.1 can be adopted in this case.

The symmetric CTMC model for this case is presented in Figure 5.6, while the reduced CTMC model is presented in Figure 5.7. In Figures 5.6 and 5.7, the transitions and rates shown in red indicate that these are common for the MedVis and LoVis models. Also, the transitions shown in blue are common to the CTMC models corresponding to all three levels of information sharing. The additional transitions and rates that are specific to the CTMC model of SCN with LoVis are shown in purple in Figures 5.6 and 5.7. The rates shown in purple when the transitions are blue indicate that the transitions are common for all three levels of information sharing, but the rate has changed for the LoVis case. The rate balance equations based on the different state transitions that are possible in the reduced CTMC model are presented in Table 5.3.

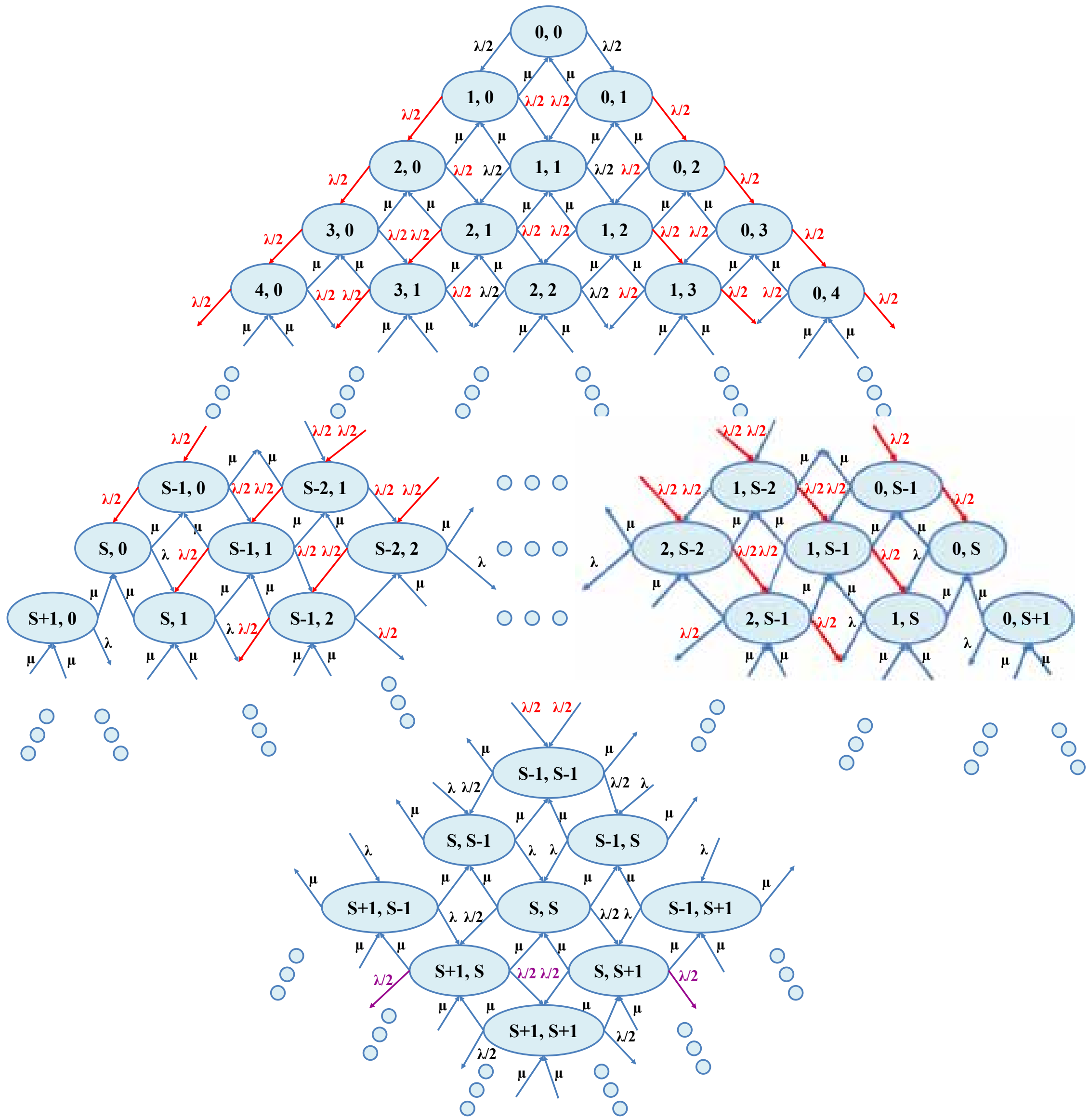


Figure 5.6: CTMC Model of the 1R/2P SCN with LoVis

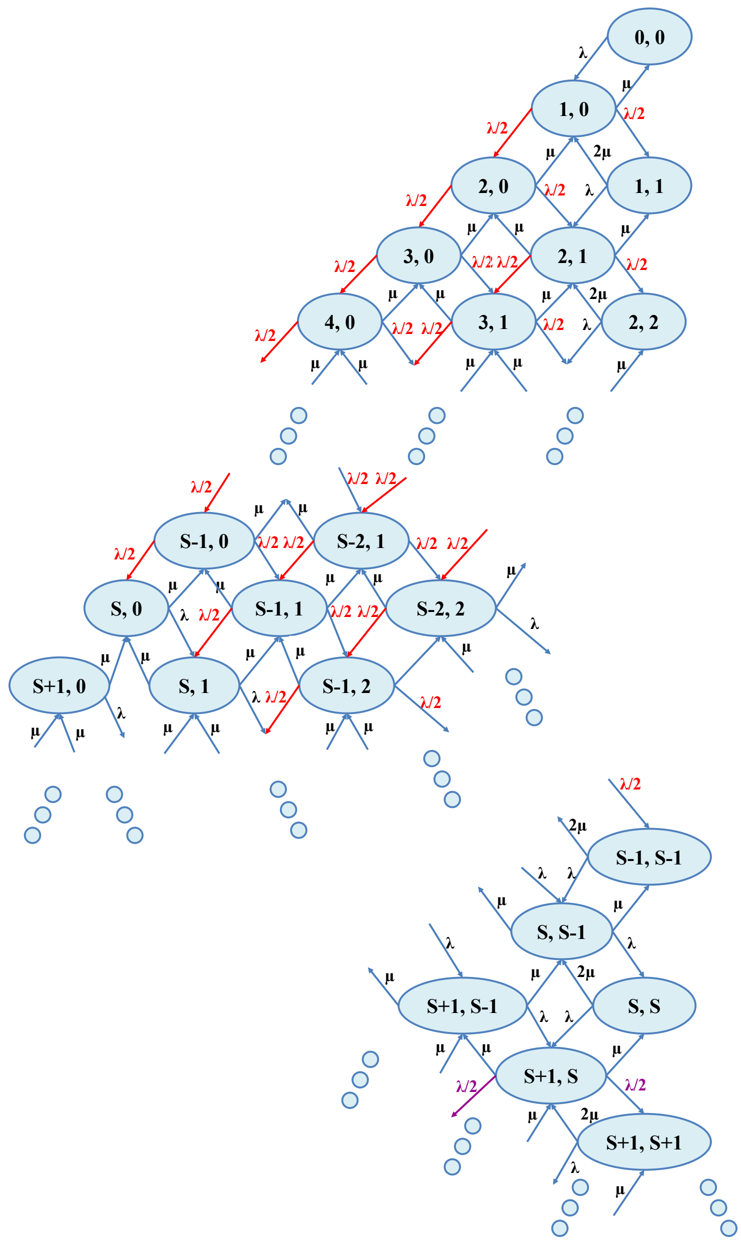
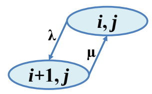
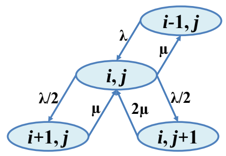
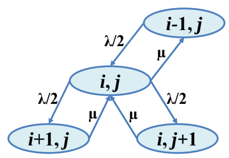
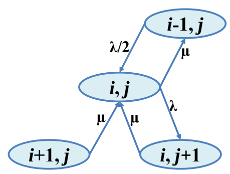
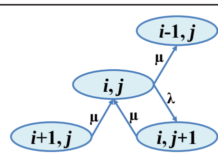
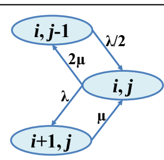
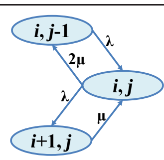
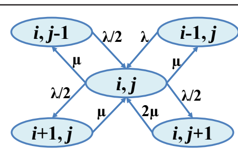


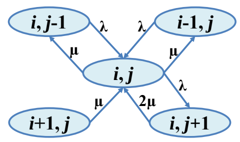
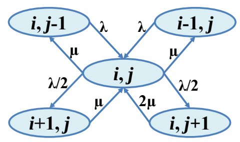
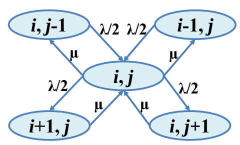
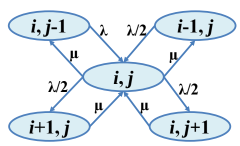
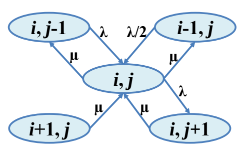
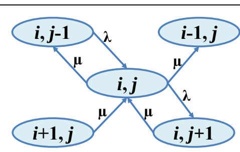

Figure 5.7: Reduced CTMC of the 1R/2P SCN with LoVis

Table 5.3: Rate Balance Equations of the Reduced CTMC Model of the 1R/2P SCN with LoVis

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $i = j = 0$ |  | $\pi_{i,j} = \frac{\mu\pi_{i+1,j}}{\lambda}$ |
| $i = 1,\ \ j = 0$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i-1,j}+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}]}{(\lambda+\mu)}$ |
| $2 \leq i \leq S-1,\ j = 0$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+\mu)}$ |
| $i = S,\ \ j = 0$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+\mu)}$ |
| $S+1 \leq i < \infty,\ \ j = 0$ |  | $\pi_{i,j} = \frac{\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+\mu)}$ |
| $1 \leq i < \infty,\ \ i \neq S,\ \ i = j$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i,j-1}+\mu\pi_{i+1,j}]}{(\lambda+2\mu)}$ |
| $i = j = S$ |  | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu\pi_{i+1,j}]}{(\lambda+2\mu)}$ |
| $2 \leq i < \infty,\ \ i \neq S, i \neq S+1, j = i-1$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i,j-1}+\lambda\pi_{i-1,j}+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}}{(\lambda+2\mu)}$ |

Continued on next page

**Table5.3 – continued from previous page**

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $i = S,\ \ j = S-1$ |   | $\pi_{i,j} = \frac{\lambda[\pi_{i,j-1}+\pi_{i-1,j}]+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}}{(\lambda+2\mu)}$ |
| $i = S+1,\ \ j = S$ |   | $\pi_{i,j} = \frac{\lambda[\pi_{i,j-1}+\pi_{i-1,j}]+\mu\pi_{i+1,j}+2\mu\pi_{i,j+1}}{(\lambda+2\mu)}$ |
| $i \leq S-1,\ \ 1 \leq j \leq S-3,\ \ i \geq j+2,\ \ and\ \ S+1 \leq j < \infty,\ \ i \geq j+2$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}[\pi_{i,j-1}+\pi_{i-1,j}]+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |
| $j = S,\ \ i \geq j+2$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\lambda\pi_{i,j-1}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |
| $i = S, 1 \leq j \leq S-2$ |   | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\lambda\pi_{i,j-1}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |
| $S+1 \leq i < \infty,\ \ 1 \leq j \leq S-1,\ \ i \geq j+2$ |   | $\pi_{i,j} = \frac{\lambda\pi_{i,j-1}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |

As before, the state transition patterns identified in the reduced CTMC model could not be exploited to obtain a closed-form solution. We fixed the maximum number of backorders at the retail store to 1,000 and numerically solved the CTMC. All states in the CTMC model communicate and the CTMC model is positive recurrent, thus confirming the existence of steady state.

The CTMC model was validated by comparing its numerical solution to estimates from an Arena simulation model of the 1R/2P SCN with MedVis, as shown in Tables B.5 and B.6. The numerical experiments show that the analytical results match the simulation results, thus confirming the validity of the CTMC model.

## 5.6 CTMC Model of the 1R/2P SCN with NoVis

In this case, we assumed that the retail store places an order with production facility 1 with a fixed probability ($p$). Because of symmetry, $p$ is 0.5. The corresponding CTMC model is shown in Figure 5.8. The rate balance equations based on the different state transitions that are possible are presented in Table 5.4.

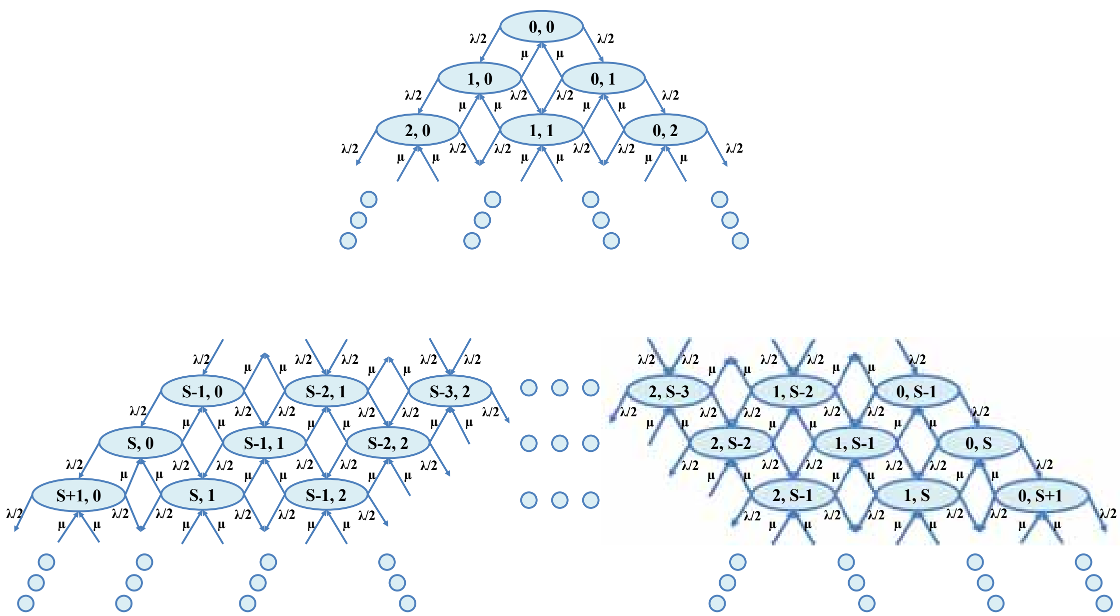
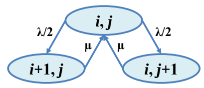
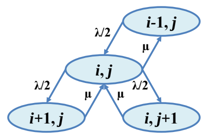
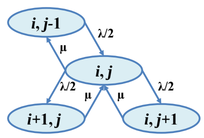
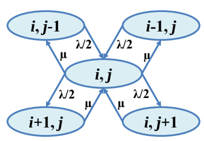



Figure 5.8: CTMC Model of the 1R/2P SCN with NoVis (p = 0.5)

Table 5.4: Rate Balance Equations of the CTMC Model of the 1R/2P SCN with NoVis

| State | State Transitions | Rate Balance Equation |
|---|---|---|
| $i = j = 0$ |  | $\pi_{i,j} = \frac{\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{\lambda}$ |
| $i \geq 1, \;\; j = 0$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i-1,j}+\mu[\pi_{i+1,j}+\pi_{i,j+1}]}{(\lambda+\mu)}$ |
| $i = 0, \;\; j \geq 1$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}\pi_{i,j-1}+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+\mu)}$ |
| $i \geq 1, \;\; j \geq 1$ |  | $\pi_{i,j} = \frac{\frac{\lambda}{2}[\pi_{i,j-1}+\pi_{i-1,j}]+\mu[\pi_{i,j+1}+\pi_{i+1,j}]}{(\lambda+2\mu)}$ |

To solve the CTMC model for the NoVis case, we used the following approach. The demand arrival process at the retail store is split into two streams according to fixed probabilities (as per the Bernoulli routing policy), each stream representing the demand arrival process at the two production facilities. Thus, the order arrival process at each production facility is a Poisson process with rate $\lambda/2$. Since the server at each production facility has an exponential processing-time distribution with mean $\mu$, each production facility can be modeled as an independent M/M/1 queue. Let $\pi_{i,\,j}$ be the steady-state joint probability that the number of orders at production facilities 1 and 2 are $i$ and $j$, respectively. The joint probability, $\pi_{i,\,j}$, is a product of the marginal probabilities (steady state solution for the M/M/1 queue) and is given by the expression in (5.12).

$$\begin{aligned}\pi_{i,j} &= \pi_i \cdot \pi_j \\ &= (1-\rho)\rho^i(1-\rho)\rho^j = (1-\rho)^2\rho^{i+j}\end{aligned} \tag{5.12}$$

Note that $\rho$ is the utilization of a production facility and is given by $\frac{\lambda}{2\mu}$

The above solution satisfies the rate balance equations shown in Table 5.4, thereby confirming the validity of the solution. Hence, the steady-state distribution for the CTMC model of a symmetric 1R/2P SCN with no information sharing has a product-form. Using this distribution, the expressions for the performance measures at a production facility as well as the retail store can be derived. Since the production facilities are symmetric, we present the expressions for only one production facility. The fill rate, the expected number of backorders, the expected inventory level, the expected time to fulfill a backorder, and the expected time spent by an order at a production facility were obtained using expressions (5.13) through (5.17).

$$\begin{aligned}f^p &= \sum_{i=0}^{S-1}\sum_{j=0}^{\infty}\pi_{i,j} \\ &= \sum_{i=0}^{S-1}\sum_{j=0}^{\infty}(1-\rho)^2\rho^{i+j} \\ &= (1-\rho)\left[(1-\rho)\sum_{j=0}^{\infty}\rho^j\right]\sum_{i=0}^{S-1}\rho^i \\ &= 1-\rho^S\end{aligned} \tag{5.13}$$

$$
\begin{aligned}
E[I^p] &= \sum_{i=0}^{S}\sum_{j=0}^{\infty}(S-i)\pi_{i,j} \\
&= (1-\rho)^2\left[S\sum_{k=0}^{\infty}\rho^k + (S-1)\sum_{k=1}^{\infty}\rho^k + (S-2)\sum_{k=2}^{\infty}\rho^k + \cdots + \sum_{k=S-1}^{\infty}\rho^k\right] \\
&= (1-\rho)^2\left[\frac{S}{1-\rho} + \frac{(S-1)\,\rho}{1-\rho} + \frac{(S-2)\,\rho^2}{1-\rho} + \cdots + \frac{\rho^{S-1}}{1-\rho}\right] \\
&= \frac{S-\rho-\rho S+\rho^{S+1}}{1-\rho}
\end{aligned}
\tag{5.14}
$$

$$
\begin{aligned}
E[B^p] &= E[I^p] + E[N^p] - S \\
&= \frac{S-\rho-\rho S+\rho^{S+1}}{1-\rho} + \frac{\lambda}{2\mu-\lambda} - S \\
&= \frac{\rho^{S+1}}{1-\rho}
\end{aligned}
\tag{5.15}
$$

$$
\begin{aligned}
E[W^{bp}] &= \frac{E[B^p]}{(\lambda/2)\,(1-f^p)} \\
&= \frac{\rho^{S+1}}{(\lambda/2)\,(1-\rho)\,\rho^S} \\
&= \frac{\rho}{(\lambda/2)\,(1-\rho)}
\end{aligned}
\tag{5.16}
$$

$$
\begin{aligned}
E[T^p] &= E[Time\ in\ facility] + E[Time\ in\ inventory\ store] \\
&= \frac{1}{\mu-(\lambda/2)} + \frac{E[I^p]}{\lambda/2} \\
&= \frac{1}{\mu-(\lambda/2)} + \frac{2\left(S-\rho-\rho S+\rho^{S+1}\right)}{\lambda\,(1-\rho)}
\end{aligned}
\tag{5.17}
$$

The reasoning presented in Section 5.2 to obtain the number of orders at the retail store was used to obtain the performance measures at the retail store. The fill rate, the expected inventory level, the expected number of backorders, the expected time to fulfill a backorder, and the expected time spent by an order at the retail store can be obtained by using expressions (5.18) through (5.23).

$$\begin{aligned}
f^r &= \sum_{i,\, j \in A_1} \pi_{i,\, j} \\
&= (1-\rho)^2 \left[1 + 2\rho + \cdots + (S+1)\,\rho^S + S\rho^{S+1} + (S-1)\,\rho^{S+2} + \cdots + \rho^{2S}\right] \\
&\quad + (1-\rho)^2 \left[\sum_{k=1}^{S} 2\rho^{S+k} + \sum_{l=1}^{S} 2\rho^{S+l+1} + \cdots + \sum_{m=1}^{S} 2\rho^{S+R+m-2}\right] \\
&\quad + (1-\rho)^2 \left[2\rho^{2S+1} + 3\rho^{2S+2} + \cdots + R\rho^{2S+R-1}\right] \\
&= (1-\rho)^2 \left[\sum_{x_1=0}^{S} (1+x_1)\,\rho^{x_1} + \sum_{x_2=1}^{S} (S+1-x_2)\,\rho^{S+x_2}\right] \\
&\quad + (1-\rho)^2 \left\{\sum_{y=1}^{R-1} \left[(1+y)\,\rho^{2S+y} + \sum_{z=1}^{S} 2\rho^{S+y+z-1}\right]\right\} \\
&= R\rho^{2S+R+1} - R\rho^{2S+R} - 2\rho^{R+S} + \rho^{2S+R} + 1
\end{aligned} \tag{5.18}$$

$$\begin{aligned}
E[I^r] &= \sum_{i=0}^{S-1} \sum_{j=0}^{S-1} R\pi_{i,\, j} + \sum_{i,\, j \in A_2} \left[R - (i - S)^+ - (j - S)^+\right] \pi_{i,j} \\
&= (1-\rho)^2 R \left[\sum_{x_1=0}^{S} (1+x_1)\,\rho^{x_1} + \sum_{x_2=1}^{S} (S+1-x_2)\,\rho^{S+x_2}\right] \\
&\quad + (1-\rho)^2 \left\{\sum_{y=1}^{R-1} (R-y) \left[(1+y)\,\rho^{2S+y} + \sum_{z=1}^{S} 2\rho^{S+y+z-1}\right]\right\} \\
&= \frac{R - 2\rho^{S+1} - R\rho + 2\rho^{R+S+1} + R\rho^{2S+R+1} - R\rho^{2S+R+2}}{1-\rho}
\end{aligned} \tag{5.19}$$

$$\begin{aligned}
E[N^r] &= \sum_{i,\, j \in A_3} \left[(i - S)^+ + (j - S)^+\right] \pi_{i,j} \\
&= \frac{2\rho^{S+1}}{1-\rho}
\end{aligned} \tag{5.20}$$

$$\begin{aligned} E[B^r] &= E[I^r] + E[N^r] - R \\ &= \rho^{S+R+1}\left(\frac{R\rho^S - R\rho^{S+1} + 2}{1-\rho}\right) \end{aligned} \tag{5.21}$$

$$\begin{aligned} E[W^{br}] &= \frac{E[B^r]}{[\lambda(1-f^r)]} \\ &= \frac{\rho^{S+R+1}\left(R\rho^S - R\rho^{S+1} + 2\right)}{\lambda\left(1-\rho\right)\left(1-f^r\right)} \end{aligned} \tag{5.22}$$

$$\begin{aligned} E[T^r] &= E[Time\ in\ facility] + E[Time\ in\ inventory\ store] \\ &= \frac{E[N^r]}{\lambda} + \frac{E[I^r]}{\lambda} \end{aligned} \tag{5.23}$$

As we have an exact solution for the NoVis CTMC model, we only present the numerical results for various cases in Tables B.7 and B.8.

## 5.7 Value of Information Sharing

To develop insights into the value of information sharing, we compared the values of the performance measures for the 1R/2P SCN for all four levels of inventory information sharing (NoVis, LoVis, MedVis, and HiVis). Figures 5.9 through 5.13 present the fill rate, expected number of backorders, expected inventory level, expected time to fulfill a backorder, and expected time spent by an order at the retail store, while Figures 5.14 through 5.18 present the corresponding values at a production facility. The detailed numerical results are presented in Appendix B as well as in Tables C.1 through C.5 in Appendix C.

For a given SCN configuration, the values of fill rate under information sharing (LoVis, MedVis and HiVis) need to be greater than the corresponding value with no information sharing (NoVis) to signify the value of information sharing. Similarly, the values for the expected number of backorders, expected time to fulfill a backorder and expected time spent by an order under information sharing need to be less than the corresponding values with NoVis.

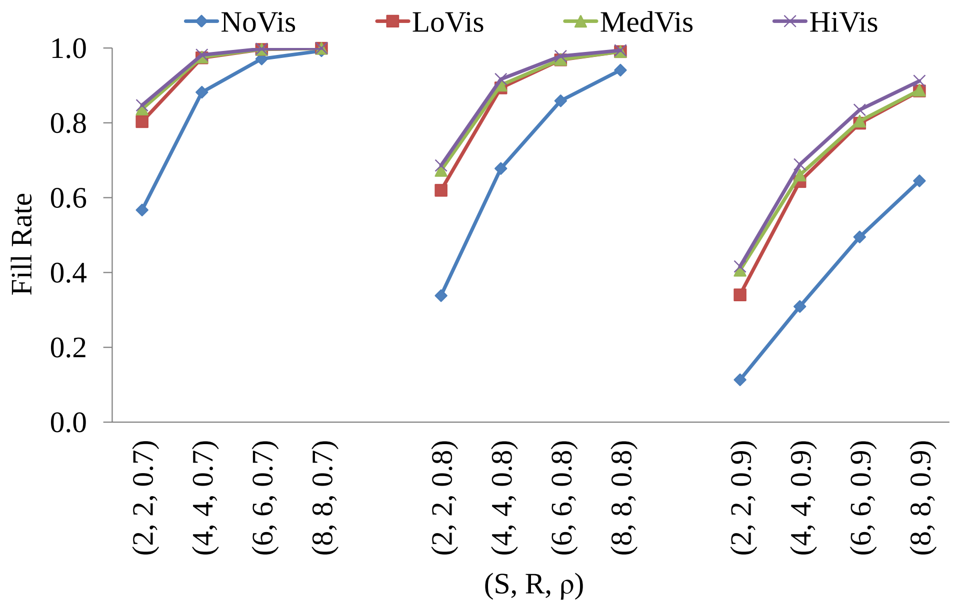


Figure 5.9: Fill Rate at the Retail Store

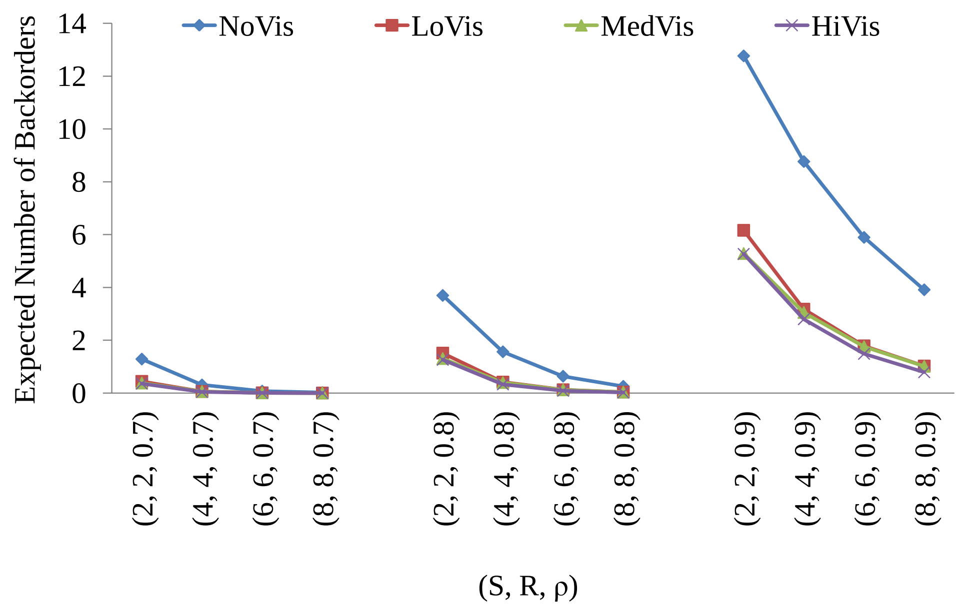


Figure 5.10: Expected Number of Backorders at the Retail Store

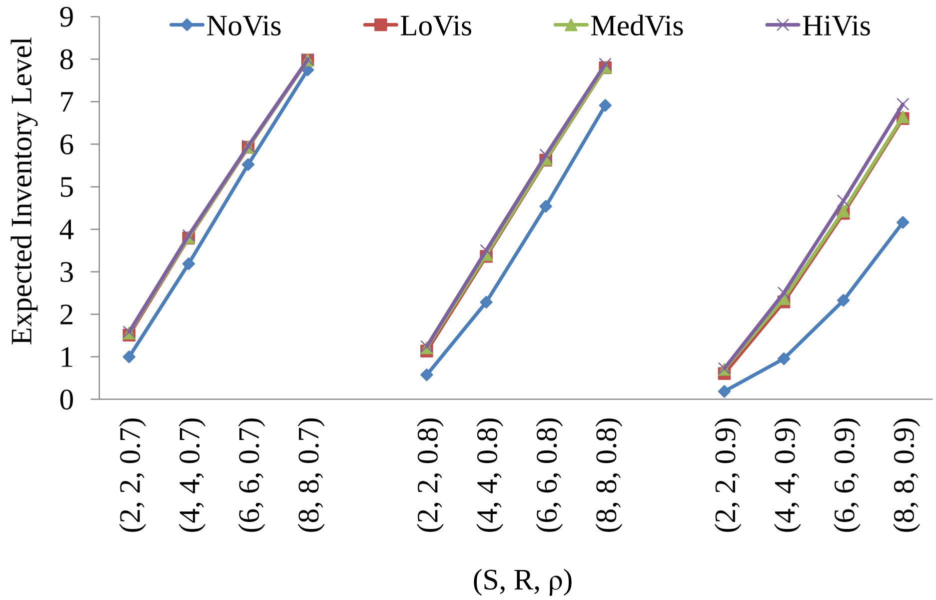



Figure 5.11: Expected Inventory Level at the Retail Store

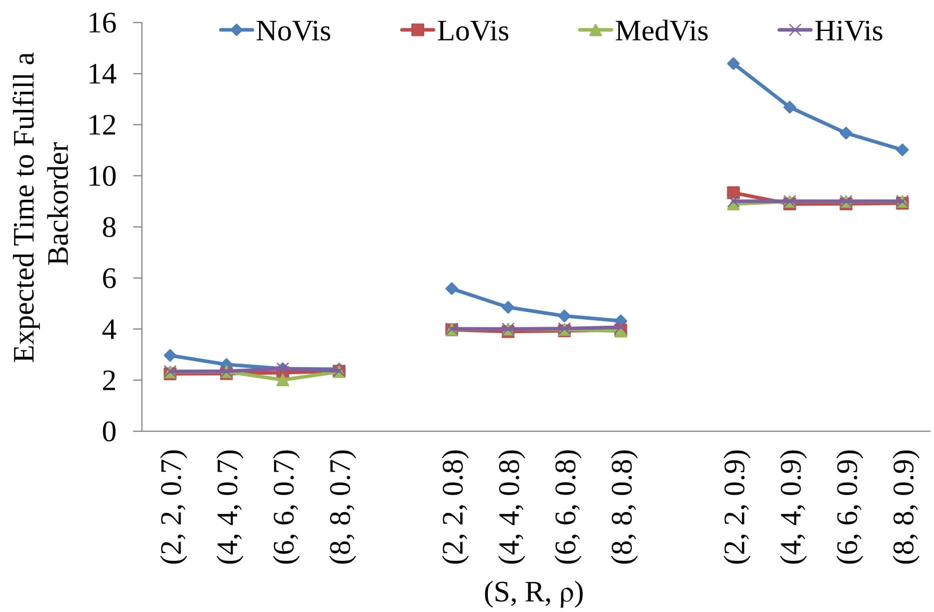



Figure 5.12: Expected Time to Fulfill a Backorder at the Retail Store

Figures 5.9 through 5.18 show that the overall performance of the SCN with inventory information sharing is better than that of the SCN without information sharing for parameter settings and is nearly equal to that of the SCN with no information sharing for a combination of low utilization levels and high base-stock levels, which is to be expected.

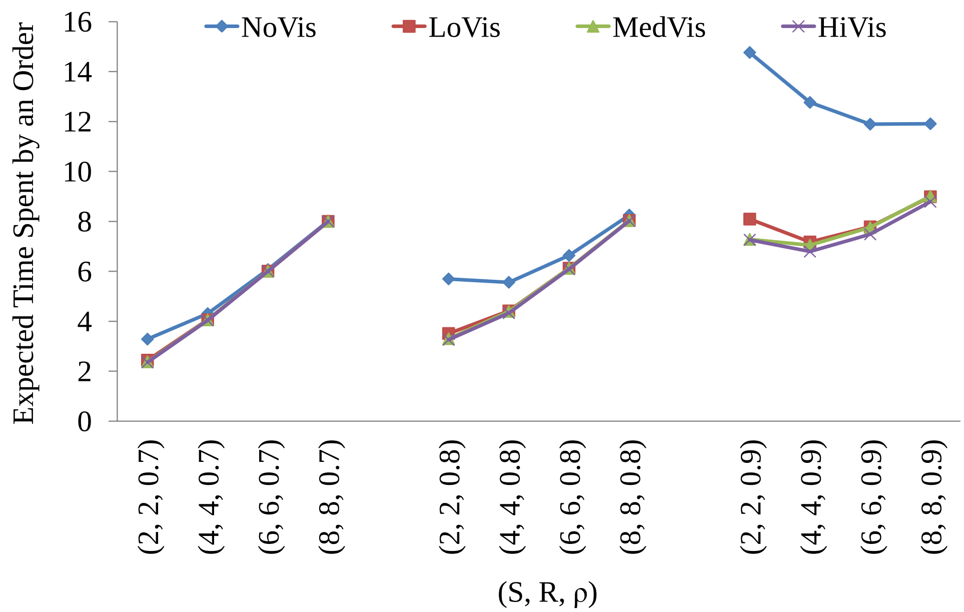


Figure 5.13: Expected Time Spent by an Order at the Retail Store

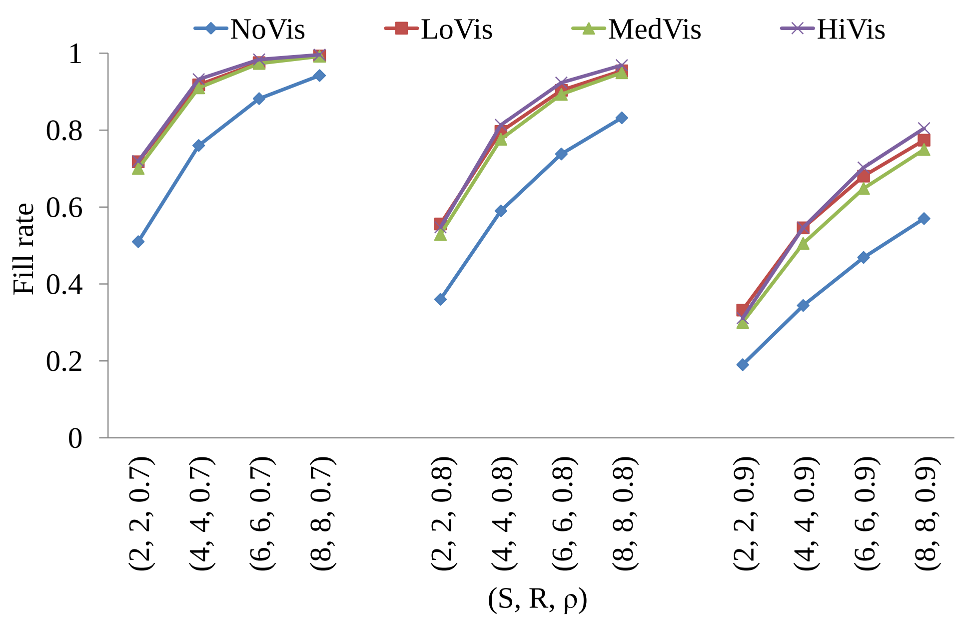


Figure 5.14: Fill Rate at a Production Facility

It can be seen from Figures 5.9 and 5.14 that when the utilization level is kept constant and the base-stock level is increased, the difference in fill rates between the SCNs with information sharing and without information sharing decreases, showing that there is significant value to inventory information sharing when the base-stock level is low and the value diminishes as the base-stock level is increased. This can be

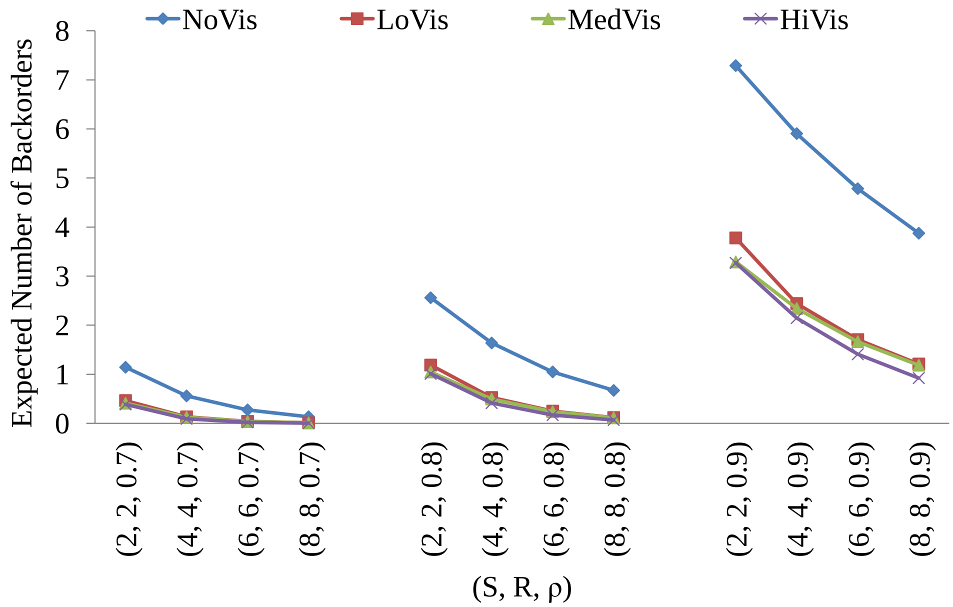



Figure 5.15: Expected Number of Backorders at a Production Facility

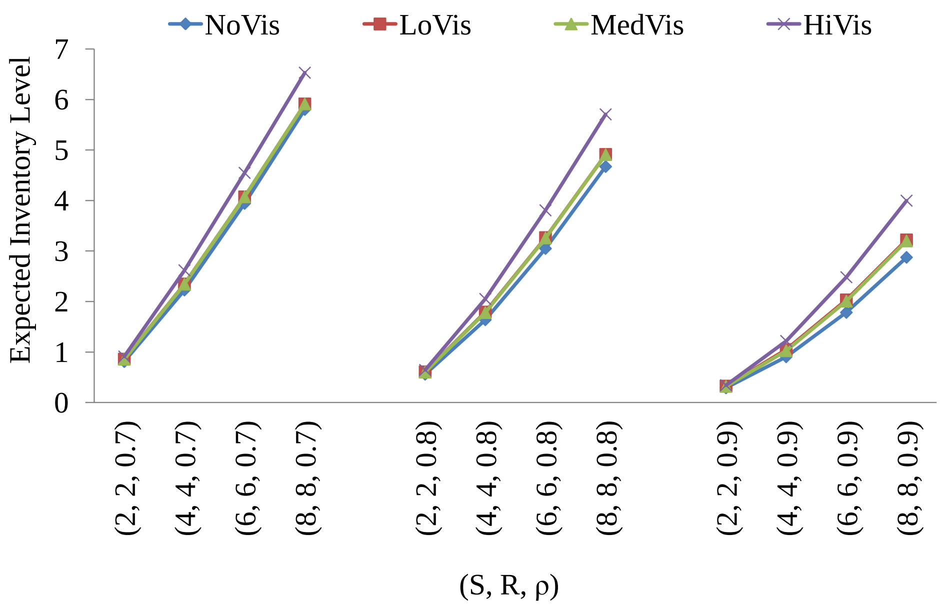



Figure 5.16: Expected Inventory Level at a Production Facility

attributed to the fact that as the base-stock level is increased, more finished goods inventory is present in the system and orders have a greater chance of being satisfied directly from the output store. In other words, as the base-stock level is increased, the SCN with information sharing starts to behave like the SCN with NoVis - the value of information sharing diminishes rapidly. Further, it can be noted that the difference

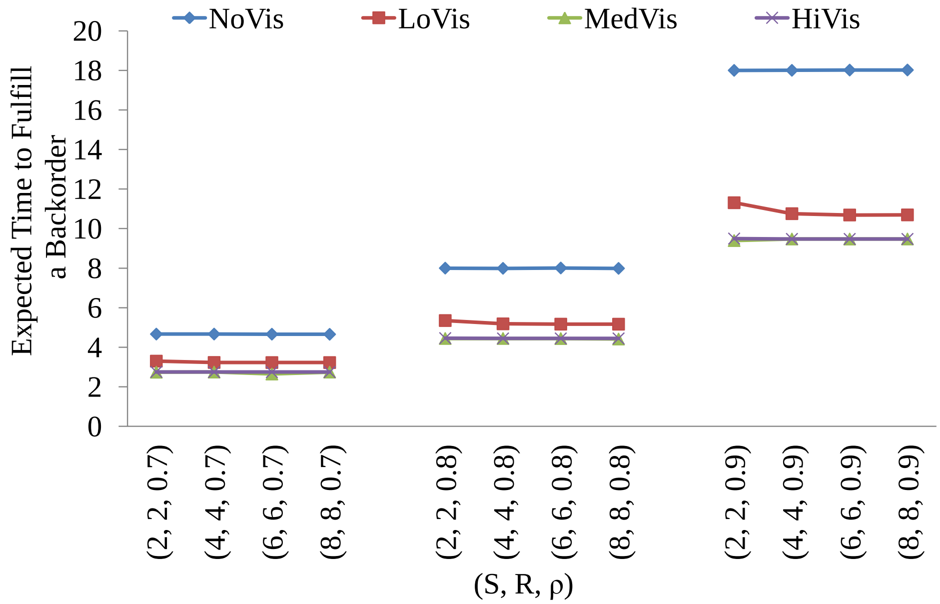



Figure 5.17: Expected Time to Fulfill a Backorder at a Production Facility

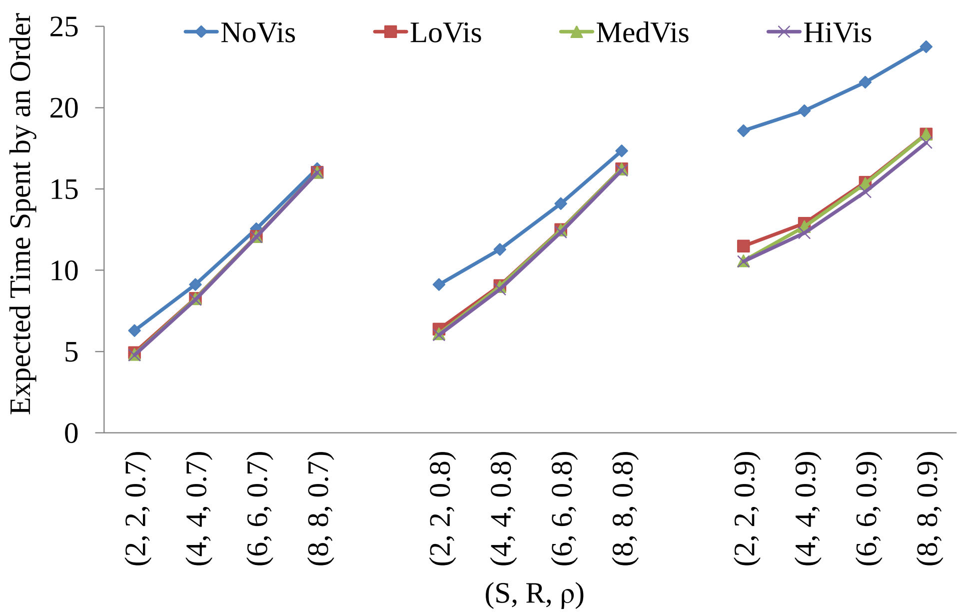



Figure 5.18: Expected Time Spent by an Order at a Production Facility

in fill rate decreases rapidly when the utilization level is low (70% in our case) and the base-stock level is increased from 2 to 8. However, at higher utilization levels, the difference in fill rates reduces with increase in base-stock level at a slower rate. The same trends can be seen for the expected number of backorders at both stores (Figures 5.10 and 5.15), the expected inventory level at the retail store (Figure 5.11), the expected time to fulfill a backorder at the retail store (Figure 5.12), and the expected time spent by an order at both stores (Figures 5.13 and 5.18).

Figure 5.17 shows that the expected time to fulfill a backorder at a production facility is almost insensitive to the base-stock level. Furthermore, the value for this measure for the SCN with MedVis or HiVis is nearly half that of the corresponding value for the SCN with NoVis. Also, the performance measure for the SCN with MedVis and the SCN with HiVis are nearly the same. This can be attributed to the fact that the order routing policies for these two levels of information sharing are identical when both the production facilities are backordered.

Figures 5.9 through 5.18 also show that a combination of a low base-stock level ($S = 2$) and high utilization ($\rho = 90\%$) yields the highest benefits. In this case, the fill rates for the four routing policies are 0.113 for SCN with NoVis, 0.340 for SCN with LoVis, 0.406 for SCN with MedVis and 0.420 for SCN with HiVis, indicating that an additional 31% of the orders in the HiVis case are satisfied instantaneously compared to the NoVis case. On the other hand, the benefits are the least for a combination of high base-stock level ($S = 8$) and low utilization ($\rho = 70\%$). In this case the fill rates for all the routing policies are above 90% (0.993 for SCN with NoVis and almost 1 for the SCNs with LoVis, MedVis, and HiVis). It can be seen that as the base-stock level is increased to 8, the fill rates and the expected inventory levels at the stores tend towards the corresponding values for the SCN with NoVis (particularly at lower utilization levels). In addition, the expected inventory level for the SCN with information sharing is greater than that with no information sharing. Although this results in increased inventory holding costs, the impact of holding less inventory is more serious as it would increase the stock-out rate in general, leading to a greater loss in customer good will and an increase in lost sales [15]. So there is a need to consider all the performance measures before deciding on the target inventory level and the use of an optimization model would be desirable for this purpose.

Figures 5.9 through 5.18 show that there is significant value to information sharing even at the lowest level (LoVis). Further, as more information becomes available at the retail store as in the case of MedVis and HiVis, the marginal benefit derived from each incremental information is significantly less. This can be attributed to the fact that even for the LoVis case, the orders are always routed to the production facility that has inventory when the other facility is backordered.

## 5.8 Conclusions

In this chapter, we have presented the CTMC models developed for the 1R/2P SCN configuration under three levels of inventory visibility and for the case with no visibility. Closed form solutions were obtained for the SCN with no inventory visibility. However, with inventory visibility, the state transition patterns in the CTMC models could not be exploited to solve the balance equations. As a result, the CTMC models were solved by fixing the maximum number of backorders at the retail store. The numerical experiments indicated that there is value to sharing inventory information among the SCN partners and that significant value can be achieved even at the lowest level of information sharing. Because, only a numerical solution was possible for CTMC models with information sharing, there is still a need to explore approximate analytical models even for the exponential case which is the subject of the next chapter.

# CHAPTER 6

# QUEUEING MODELS OF THE 1R/2P SCN CONFIGURATION WITH POISSON ARRIVALS AND EXPONENTIAL PROCESSING TIMES

In this chapter, we present the queueing models that were developed for the 1R/2P SCN configuration with information sharing (HiVis, MedVis and LoVis). As seen in Chapter 5, the CTMC models could only be solved numerically by limiting the maximum number of backorders. Queueing models, although approximate, have two distinct advantages over the CTMC models. First, they yield closed-form expressions under Markovian assumptions. More importantly, they could be extended to model general arrival processes and general processing times. Section 6.1 first presents the queueing model that was developed for the SCN with HiVis, and then presents its modification for lower levels of information sharing (MedVis and LoVis). Section 6.2 presents the queueing model developed for an asymmetric 1R/2P SCN configuration where the production facilities are not identical and the conclusions are presented in Section 6.3.

## 6.1 Queueing Model of the 1R/2P SCN Configuration with HiVis

The routing policy used under HiVis is equivalent to choosing the shortest queue when the two production facilities have identical base-stock levels. No closed form solution exists for the shortest queue problem [2]. This led us to explore the development of an approximate queueing model based on the following observations. With inventory visibility, the retail store orders cannot be routed to a production facility with an empty output store, when the other production facility has items in stock. In addition, with HiVis, the orders are always routed to the production facility with the least number of orders. Note that the production facility with the smaller number of orders will have more items in stock when there is inventory at both stores and the smaller number of backorders when both stores are backordered. Thus, the routing policy results in balancing the number of orders at the two production facilities. This is analogous to the behavior of a multi-server queue (M/M/2 queue in the case of two production facilities) where an arriving customer (order in our case) waits in a common queue before joining the first available server. Additionally, a server in a multi-server queue can never be idle when there are customers waiting to be processed. However, some of the states in the CTMC model for HiVis case, namely, $(i, 0)$ and $(0, j)$ when $i, j \geq 2$ are not modeled in an M/M/2 system. But the order routing in the HiVis case should make the steady state probabilities of the process being in these states negligible. With this knowledge, we used the M/M/2 queue as an approximation for the SCN with HiVis. The advantage of using this approximation is that closed form solutions can be obtained for the performance measures at all stores and the model can be easily extended to consider SCN with more than two production facilities. The argument for the use of the M/M/2 model as an approximation was strengthened by the work of Adan et al. [1] which compared the symmetric shortest queue problem with the shortest queue problem with threshold jockeying and the shortest queue problem with threshold blocking. Their study showed that the former provided a tight lower bound on the expected waiting time, while the latter provided a tight upper bound for the same measure. Adan et al. [1] also stated that a shortest queue problem with threshold jockeying, where the threshold is set to one, behaves like a multi-server queue.

Figure 6.1 shows the exact representation of the 1R/2P SCN configuration, while Figure 6.2 shows the aggregate representation based on the M/M/2 approximation. In essence, the two production facilities are modeled as an M/M/2 queueing system and the arrival process is the same as that at the retail store. Figure 6.2 shows that in an M/M/2 approximation, the inventory at the two production facilities would be pooled and this leads to some loss of information about inventory and orders at individual production facilities. For instance, when one order (customer) is present in the M/M/2 model, it would represent the states $(1, 0)$ and $(0, 1)$ in the original CTMC. The additional information about whether the order is at production facility 1 or 2 is lost.

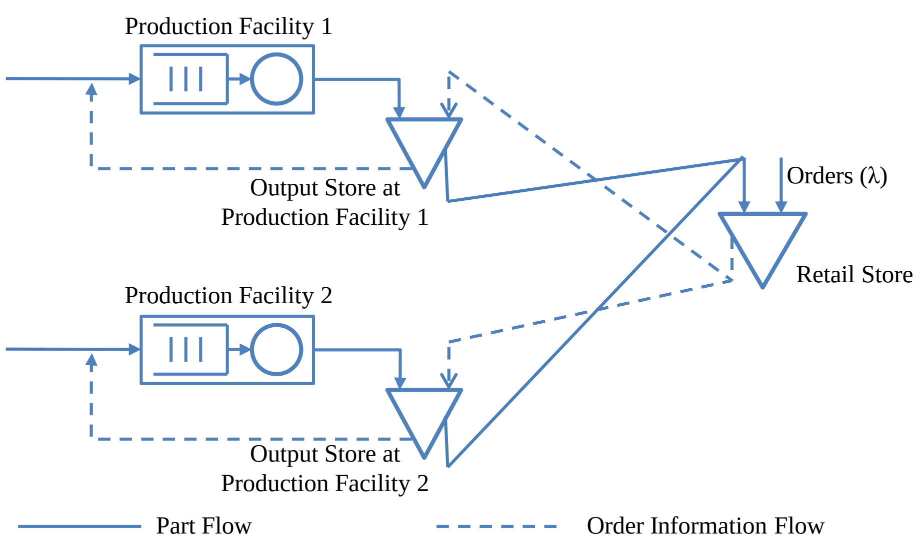



Figure 6.1: Queueing Representation of the 1R/2P SCN Configuration

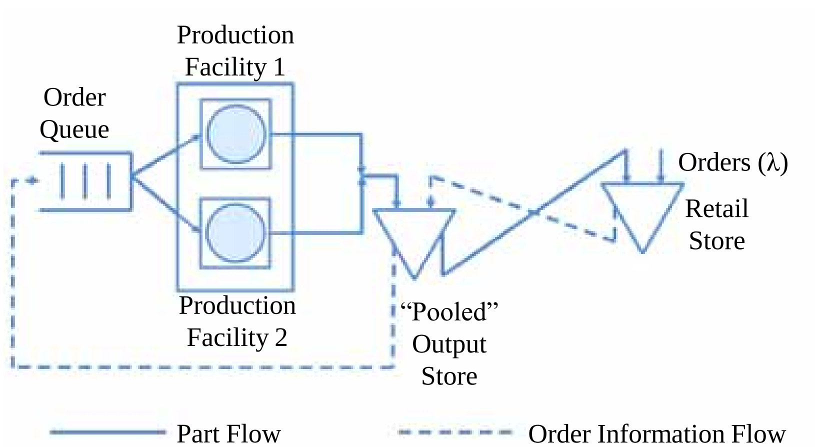



Figure 6.2: Queueing Representation of the M/M/2 Approximation

When an order is placed at a retail store, an instantaneous replenishment order is sent to the pooled output store. Since transit time is not modeled, if a finished part is available at the output store of the pooled facility, the part is instantaneously available at the retail store. As a result, an order (replenishment or backorder) at a retail store can be reflected only as a backorder at the pooled facility. This fact can be used in the calculation of the distribution of number of orders at the retail store, $N^r$, as shown in expression (6.1) where $N^p$ is the number of orders at the pooled facility. Note that the distribution of $N^r$ is the same as the backorder distribution at the pooled facility. The distribution of $N^p$ can be obtained using the expression for the number in an M/M/2 system as shown in expression (6.2) where $\rho = \frac{\lambda}{2\mu}$. The notations that will be used in the rest of the chapter are similar to those presented in Section 5.2.

$$P(N^r = i) = \begin{cases} \sum_{n=0}^{2S} P\left(N^p = n\right) & ; \quad i = 0 \\ P\left(N^p = 2S + i\right) & ; \quad i \geq 1 \end{cases} \tag{6.1}$$

$$P(N^p = i) = \begin{cases} \frac{1-\rho}{1+\rho} & ; \quad i = 0 \\ 2\rho^i \frac{1-\rho}{1+\rho} & ; \quad i \geq 1 \end{cases} \tag{6.2}$$

The performance measures, namely, the fill rate, the expected number of backorders, the expected inventory level, expected time to fulfill a backorder, and expected time spent by a product at the retail store can be obtained by using expressions (6.3) through (6.7).

$$\begin{aligned} f^r &= \sum_{i=0}^{R-1} P(N^r = i) \\ &= \frac{1 + \rho - 2\rho^{2S+R}}{1 + \rho} \end{aligned} \tag{6.3}$$

$$
\begin{aligned}
E[I^r] &= \sum_{i=0}^{R-1} iP(N^r = i) \\
&= \left\{ R\left[\frac{1+\rho-2\rho^{2S+1}}{1+\rho}\right] + 2\rho^{2S+1}\left[\frac{R-R\rho+\rho^R-1}{(1-\rho^2)}\right]\right\}
\end{aligned} \tag{6.4}
$$

$$
\begin{aligned}
E[B^r] &= \sum_{i=R}^{\infty} P(N^r = i) \\
&= \frac{2\rho^{2S+R+1}}{(1-\rho^2)}
\end{aligned} \tag{6.5}
$$

$$
\begin{aligned}
E[W^{br}] &= \frac{E[B^r]}{(1-f^r)\lambda} \\
&= \frac{\rho}{(1-\rho)\lambda}
\end{aligned} \tag{6.6}
$$

$$
\begin{aligned}
E[T^r] &= \frac{E[N^r]}{\lambda} + \frac{E[I^r]}{\lambda} \\
&= \frac{E[B^r]+R}{\lambda} \\
&= \frac{R\,(1-\rho^2)+2\rho^{2S+R+1}}{\lambda\,(1-\rho^2)}
\end{aligned} \tag{6.7}
$$

With the loss of information about the exact number of orders at the individual production facilities, the fill rate at a production facility was calculated as the fill rate at the pooled facility. The performance measures for a production facility can be obtained from expressions (6.8) through (6.12).

$$
\begin{aligned}
f^p &= \sum_{i=0}^{2S-1} P(N^p = i) \\
&= \frac{1+\rho-2\rho^{2S}}{1+\rho}
\end{aligned} \tag{6.8}
$$

$$
\begin{aligned}
E[I^p] &= 0.5 \sum_{i=0}^{2S} (2S - i) P(N^p = i) \\
&= \frac{S\ (1-\rho)^2 + \rho(2S + \rho^{2S} - 2S\rho - 1)}{1-\rho^2}
\end{aligned}
\tag{6.9}
$$

$$
\begin{aligned}
E[B^p] &= 0.5 \sum_{i=2S}^{\infty} P(N^p = i) \\
&= \frac{\rho^{2S+1}}{1-\rho^2}
\end{aligned}
\tag{6.10}
$$

$$
\begin{aligned}
E[W^{bp}] &= \frac{E[B^p]}{(\lambda/2)(1-f^p)} \\
&= \frac{\rho}{(1-\rho)\lambda}
\end{aligned}
\tag{6.11}
$$

$$
\begin{aligned}
E[T^p] &= \frac{E[N^p]}{\lambda/2} + \frac{E[I^p]}{\lambda/2} \\
&= \frac{E[B^p] + S}{\lambda/2} \\
&= \frac{S(1-\rho^2) + \rho^{2S+1}}{(\lambda/2)(1-\rho^2)}
\end{aligned}
\tag{6.12}
$$

### 6.1.1 Validation of the M/M/2 Approximation of the 1R/2P SCN Configuration with HiVis

The M/M/2 approximation was validated by comparing the analytical results with the simulation estimates for the 1R/2P SCN under HiVis. The SCN parameter values used in the numerical experimentation are presented in Table 6.1. The design consists of 12 experiments.

The results of the M/M/2 approximation and the simulation results for all three levels of visibility are presented in Appendix C and shown in Figures 6.3 through 6.12.

Table 6.1: Experiments for the 1R/2P SCN Configuration under Poisson Arrivals and Exponential Processing Times

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rate $\lambda$ | 1 | 1 |
| Base-stock levels $(S,\ R)$ | 4 | (2, 2), (4, 4), (6, 6), (8,8) |
| Utilization level $\rho$ | 3 | 70%, 80%, 90% |

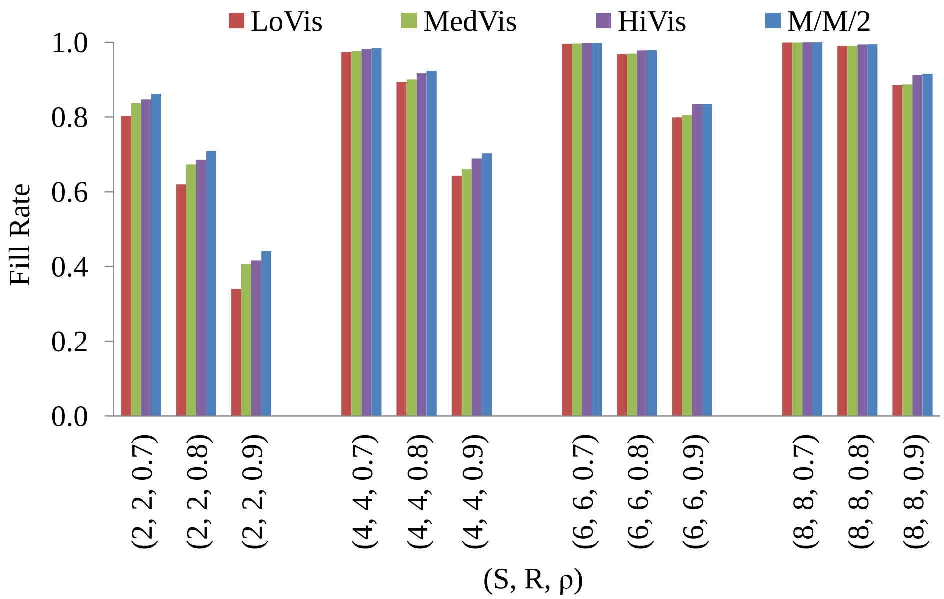



Figure 6.3: Fill Rate at the Retail Store

Numerical results indicate that the analytical values are very close to the simulation estimates for the HiVis case. The prediction error is within 15% for majority of the cases. This shows that the M/M/2 model serves as a good approximation for the HiVis case. It can also be seen from the tables in Appendix C and the plots shown in Figures 6.3 through 6.12 that the analytical model presents reasonable bounds for the MedVis and LoVis cases in most of the experiments (although this has not been shown theoretically). For the retail store, this approximation yields upper bounds for the fill rate and the expected inventory level, and lower bounds for the expected number of backorders and the expected time to fulfill a backorder. These observations also hold at a production facility except in the case of fill rate.

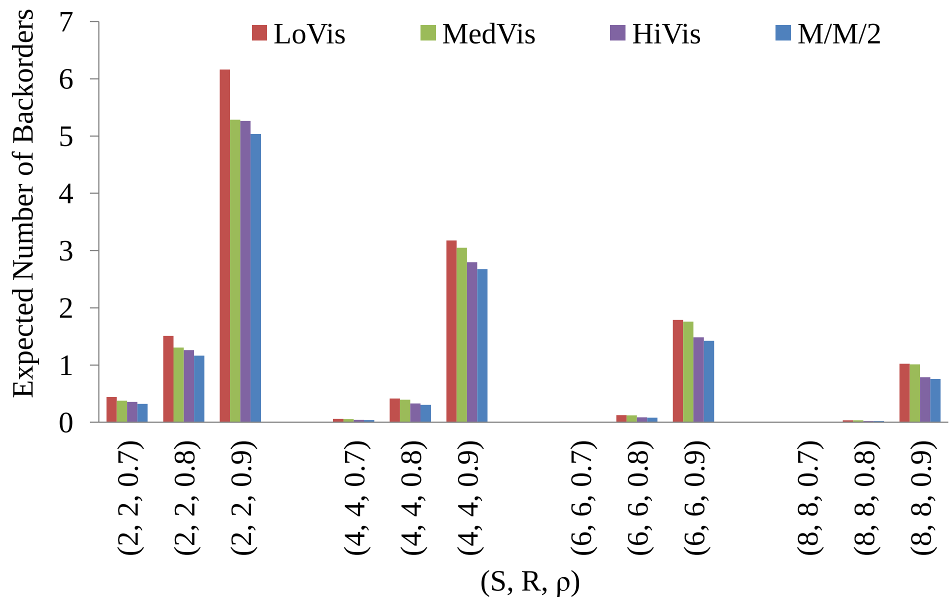



Figure 6.4: Expected Number of Backorders at the Retail Store

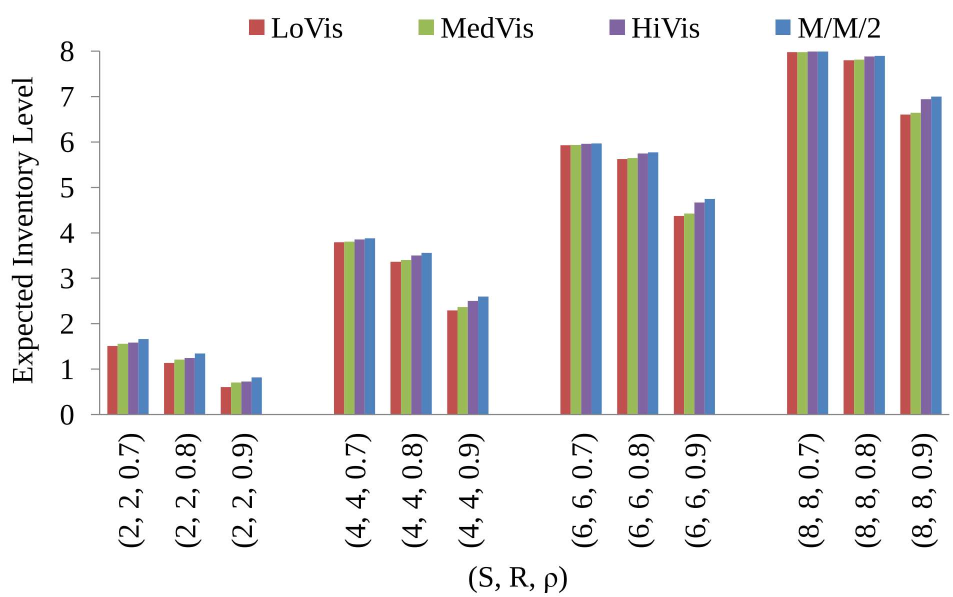



Figure 6.5: Expected Inventory Level at the Retail Store

In the following section, we present the modifications to the M/M/2 model to obtain better approximations for lower levels of visibility.

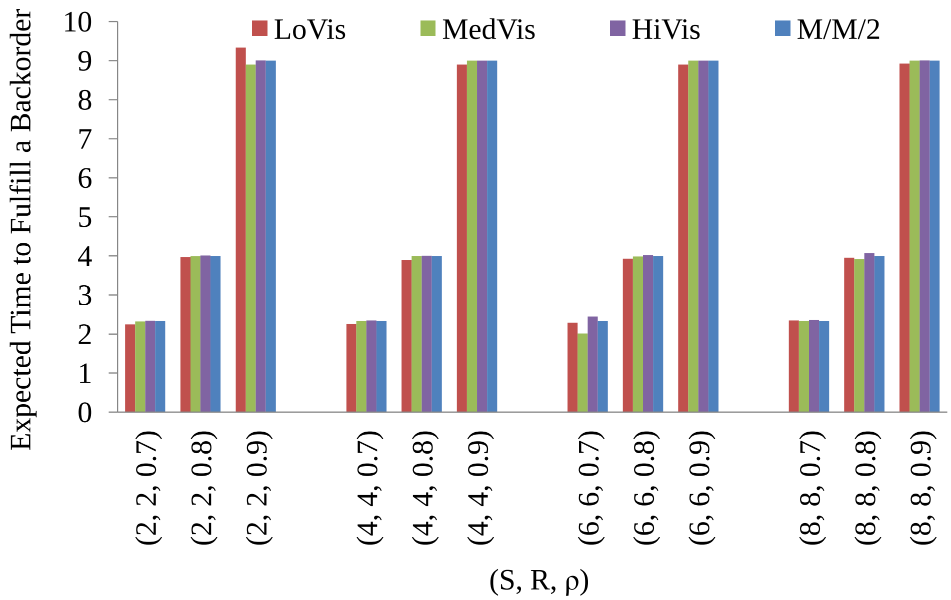



Figure 6.6: Expected Time to Fulfill a Backorder at the Retail Store

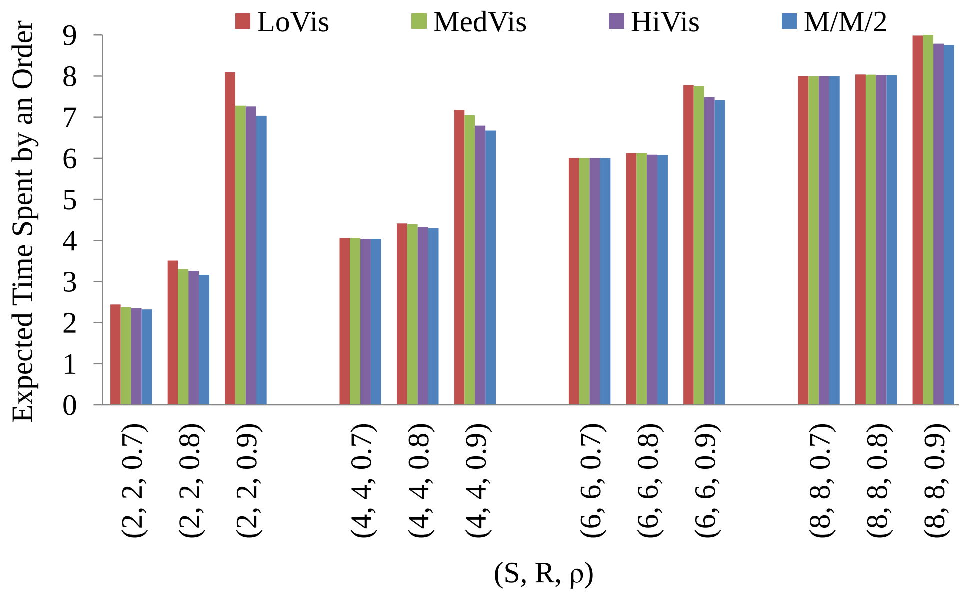



Figure 6.7: Expected Time Spent by an Order at the Retail Store

### 6.1.2 Queueing Model of SCN with Lower Levels of Information Sharing

The M/M/2 based approximation works well for the HiVis case, while providing useful bounds for the other two cases with lower levels of visibility. Here, we develop a modified M/M/2 based approximation to better predict the performance measures

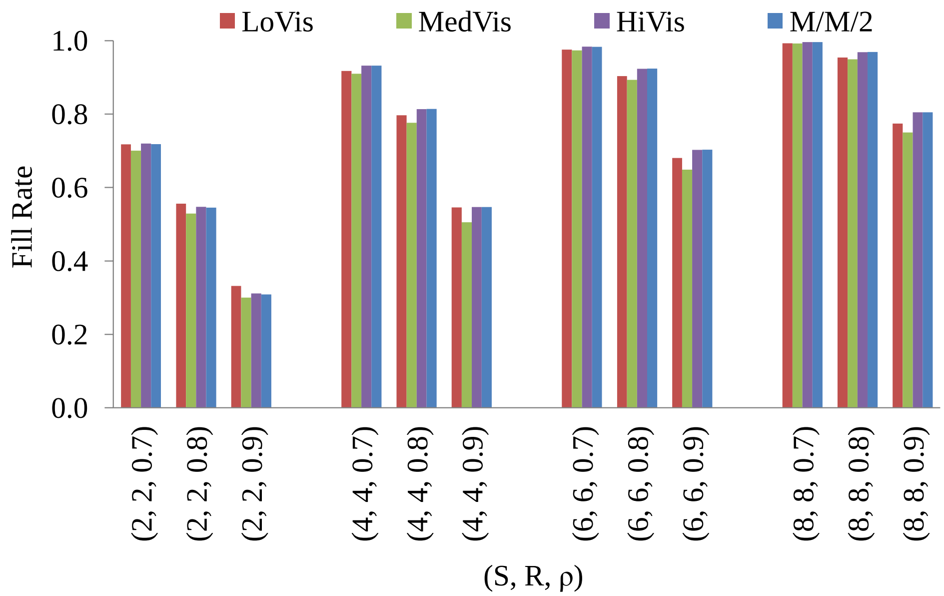



Figure 6.8: Fill Rate at a Production Facility

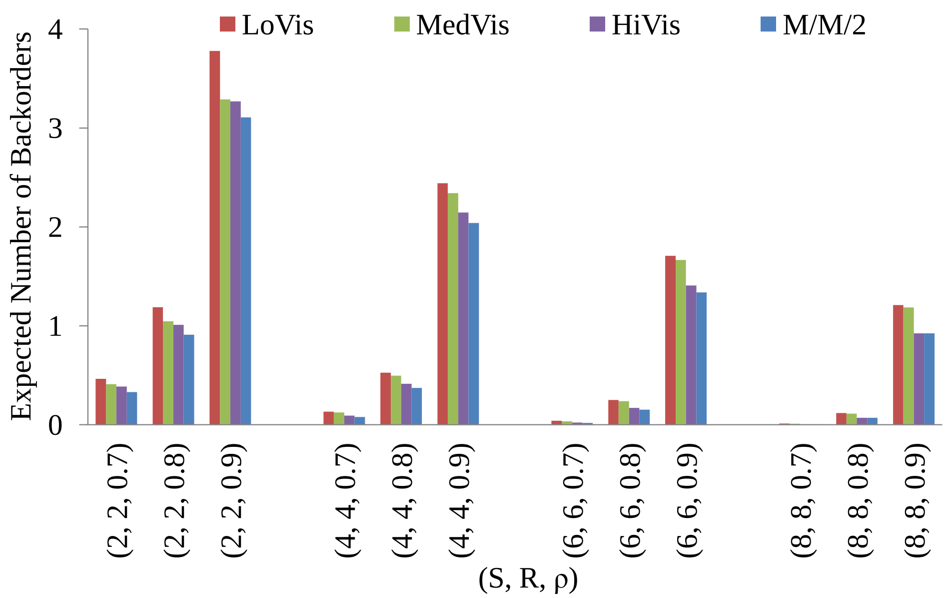



Figure 6.9: Expected Number of Backorders at a Production Facility

at the two lower levels of visibility (LoVis and MedVis). This modification involves the development of a correction factor for the steady state probabilities. The modification is based on a combination of analytical reasoning and empirical observations as explained below.

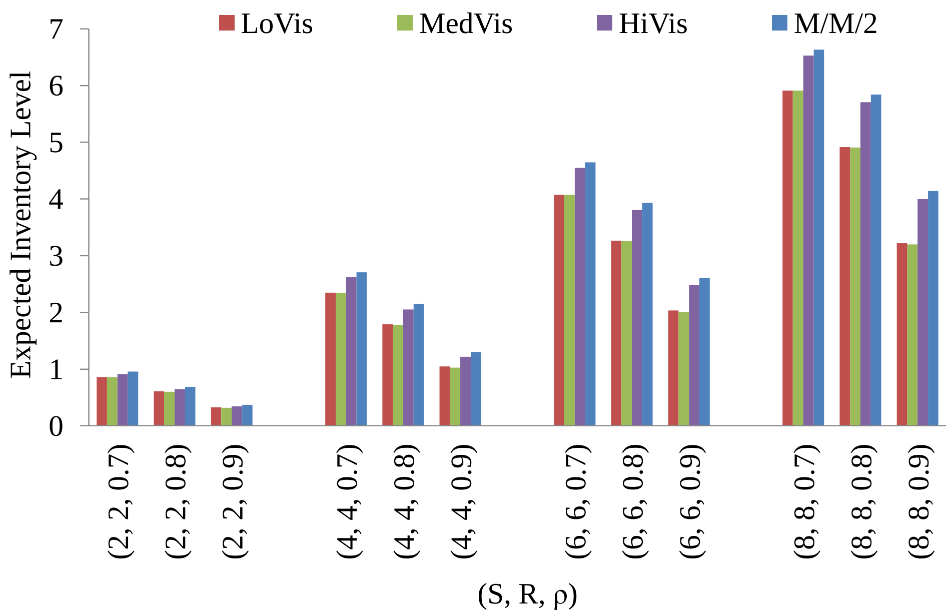



Figure 6.10: Expected Inventory Level at a Production Facility

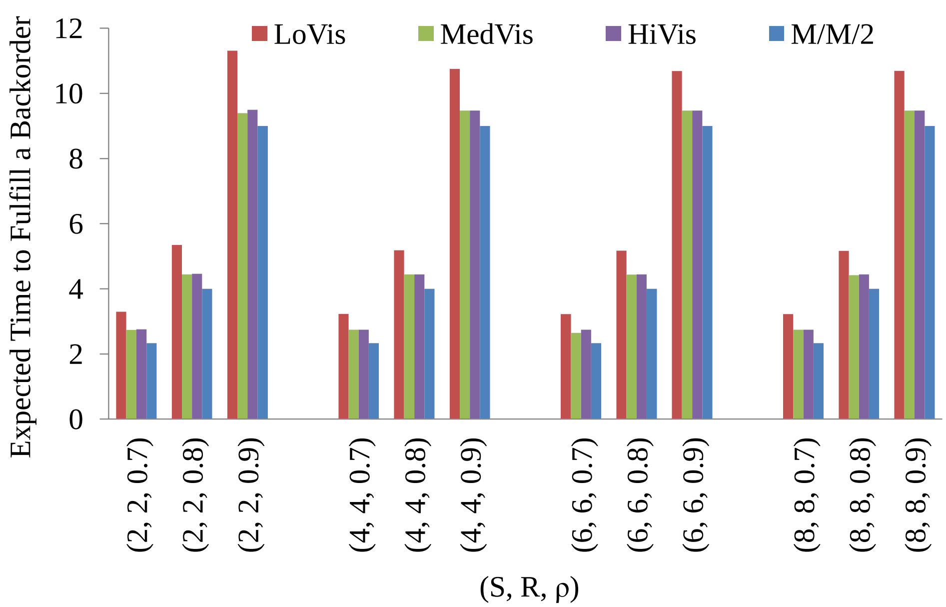



Figure 6.11: Expected Time to Fulfill a Backorder at a Production Facility

1. As the base-stock level is increased, the routing policies for LoVis and MedVis behave more like the (Bernoulli) routing policy for the SCN with no information sharing. This is because of the fact that there is always enough inventory in-stock to satisfy the customer orders and the stock-out probability becomes

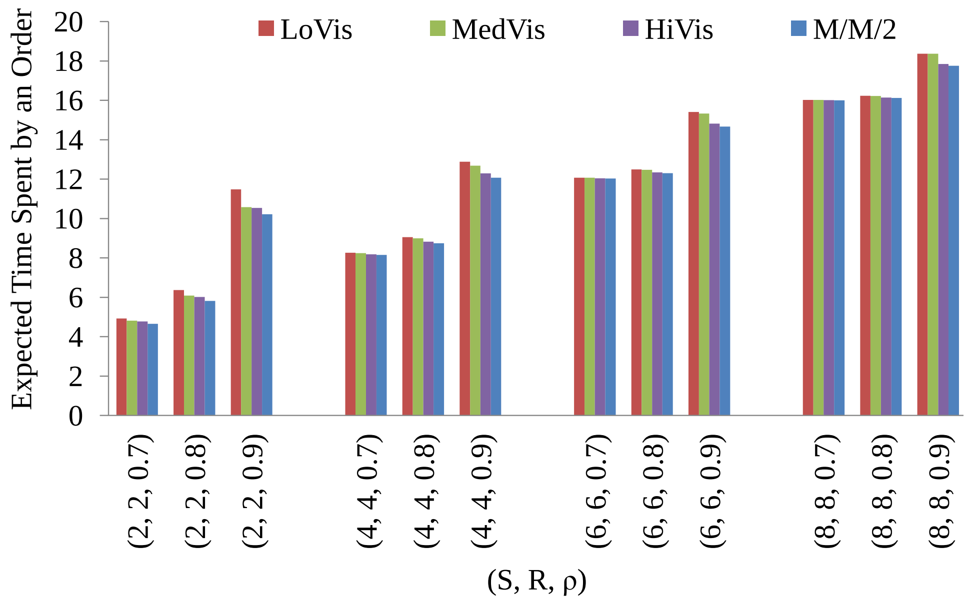



Figure 6.12: Expected Time Spent by an Order at a Production Facility

negligible.

2. In the M/M/2 model, $\pi_{0,0}$ (or $\pi_0$) depends only on the utilization and is independent of the base-stock level $S$. However, simulation and CTMC results indicated that the $\pi_{0,\,0}$ value is dependent on both the base-stock level, $S$, and the utilization, $\rho$, for the LoVis and MedVis cases. The reason for this dependence is the probabilistic routing of orders in the presence of inventory at both production facilities. Note that the structure of the Markov chain differs for each value of $S$ in these cases. In the HiVis case, $\pi_{0,\,0}$ is dependent only on $\rho$ because of the fact that irrespective of the base-stock level, the orders are always routed to the store with the maximum inventory when there is inventory at both production facilities and to the store with minimum number of backorders in the presence of backorders at both production facilities. The equivalent probability value in the M/M/2 queue is only dependent on the utilization ($P(N^p = 0) = (1-\rho)/(1+\rho)$).

3. As the base-stock level is increased, the value of $\pi_{0,\,0}$ in the LoVis and MedVis

cases approaches $(1-\rho)^2$ (corresponding value for the Bernoulli routing from equation (5.12)). Further, the values of $\pi_{0,0}$ are always less than the $P(N^p = 0)$ value of the M/M/2 model and greater than $(1-\rho)^2$ as seen in Table 6.2.

Table 6.2: Dependence of $\pi_{0,0}$ on the Base-stock Level and Production Facility Utilization

| $\rho$ | Model | S = 2 | S = 4 | S = 6 | S = 8 | S = 10 | S = 12 | S = $\infty$ |
|---|---|---|---|---|---|---|---|---|
| 70% | M/M/2 | ← 0.1765 → | | | | | | |
| | HiVis | ← 0.1565 → | | | | | | |
| | MedVis | 0.1385 | 0.1127 | 0.1011 | 0.0955 | 0.0928 | 0.0914 | 0.0900 |
| | LoVis | 0.1352 | 0.1125 | 0.1010 | 0.0955 | 0.0928 | 0.0914 | 0.0900 |
| 80% | M/M/2 | ← 0.1111 → | | | | | | |
| | HiVis | ← 0.0942 → | | | | | | |
| | MedVis | 0.0823 | 0.0633 | 0.0536 | 0.0483 | 0.0452 | 0.0434 | 0.0400 |
| | LoVis | 0.0779 | 0.0629 | 0.0535 | 0.0483 | 0.0452 | 0.0434 | 0.0400 |
| 90% | M/M/2 | ← 0.0526 → | | | | | | |
| | HiVis | ← 0.0422 → | | | | | | |
| | MedVis | 0.0366 | 0.0266 | 0.0210 | 0.0178 | 0.0158 | 0.0144 | 0.0100 |
| | LoVis | 0.0320 | 0.0260 | 0.0210 | 0.0178 | 0.0158 | 0.0144 | 0.0100 |

The above observations formed the basis for the development of a correction factor for the probability $P(N^p = 0)$. Table 6.2 clearly shows the dependency of $\pi_{0,0}$ for LoVis and MedVis on the base-stock level and the utilization as well as the insensitivity of $\pi_{0,0}$ for HiVis and M/M/2 to the base-stock level. Further, it can be seen that $\pi_{0,0}$ becomes nearly identical for the LoVis and MedVis cases as the base-stock level increases.

Since the value of $\pi_{0,0}$ is always greater than $(1-\rho)^2$, a correction factor of the form $(1-\rho)^2\,(1+\omega(\rho, S))$ was used. It is clear that the $\omega(\rho, S)$ term should vanish as the base-stock level increases and approaches infinity. Examples of such a term are an exponential decay function and a multiple of the reciprocal of the base-stock level. To obtain a better idea, we solved the CTMC for LoVis to obtain the $\pi_{0,0}$ for base-stock levels of 2 to 20 in steps of 2 at the three utilization levels of 70%, 80% and 90% and plotted the values as shown in Figure 6.13.

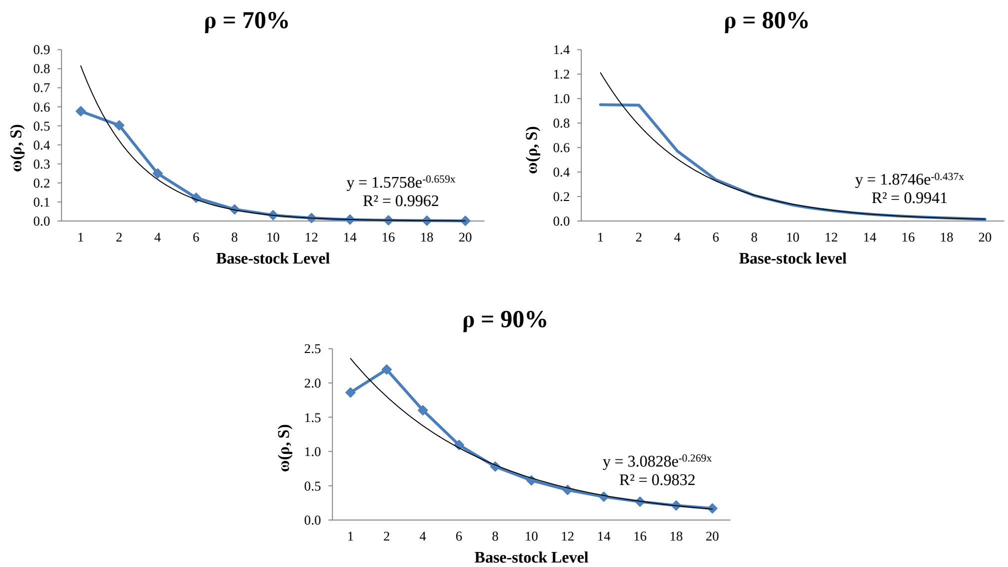



Figure 6.13: $\omega(\rho, S)$ Function

The three plots in Figure 6.13 clearly show that the $\omega(\rho, S)$ term can be approximated by an exponential decay function that depends on the utilization of the production facility and its base-stock level. With this information, we found the exponential decay function empirically and the function is presented in equation (6.13) and the scaled value of $P(N^p = 0)$ is presented in equation (6.14).

$$\omega(\rho, S) = \frac{exp^{-(1-\rho^2)S}}{2\rho(1-\rho^2)} \tag{6.13}$$

$$P(N^p = 0) = (1-\rho)\left[\frac{2\rho\,(1-\rho^2) + exp^{-(1-\rho^2)S}}{2\rho\,(1+\rho)}\right] \tag{6.14}$$

Since the $P(N^p = 0)$ value has been scaled down, there is a need to renormalize the remaining probabilities in the M/M/2 distribution so that the sum of the probabilities is one. The easiest way to achieve this is to scale all the remaining probabilities by a normalizing factor $\zeta(\rho, S)$. However, further examination of the CTMC models for

LoVis and MedVis cases (Figures 5.4 and 5.6) revealed that the probability of having one order in the system is equal to the utilization times $\pi_{0,0}$ (i.e. $\pi_{1,0} = \pi_{0,1} = \rho\pi_{0,0}$). As a result, the probability $\pi_1 = (\pi_{1,0} + \pi_{0,1})$ is fixed to the value $2\rho P(N^p = 0)$. The terms in the total probability equation are presented in equation (6.15) and the resulting normalizing factor $\zeta(\rho, S)$ is presented in equation (6.16). The resulting distribution for the random variable representing the number of orders at the pooled production facility, $N^p$, is presented in equation (6.17).

$$P(N^p = 0)\left[1 + 2\rho + 2\rho^2\zeta(\rho, S) + 2\rho^3\zeta(\rho, S) + \cdots\right] = 1 \tag{6.15}$$

$$\zeta(\rho, S) = \frac{(1+\rho)}{\rho\left[2\rho\left(1-\rho^2\right) + exp^{-(1-\rho^2)S}\right]} - \frac{(1+2\rho)\,(1-\rho)}{2\rho^2} \tag{6.16}$$

$$P(N^p = i) = \begin{cases} (2\rho)P(N^p = 0) & ; \quad i = 1 \\ 2\rho^i\zeta(\rho, S)P(N^p = 0) & ; \quad i \geq 2 \end{cases} \tag{6.17}$$

The performance measures at a production facility were obtained using equations (6.18) through (6.22). The distribution of the number of orders at the retail store was obtained using equation (6.23) and equations (6.24) through (6.28) were used to obtain the performance measures at the retail store.

$$\begin{aligned} f^p &= \sum_{i=0}^{2S-1} P(N^p = i) \\ &= \left[(1+2\rho)\,(1-\rho) + 2\rho^2\zeta(\rho, S)\left(1-\rho^{2S-2}\right)\right] P(N^p = 0) \end{aligned} \tag{6.18}$$

$$E[I^p] = 0.5\sum_{i=0}^{2S-1}(2S-i)P(N^p = i) \tag{6.19}$$

$$E[B^p] = 0.5\sum_{i=2S}^{\infty}(2S-i)P(N^p=i)$$
$$= \frac{\rho^{2S+1}\zeta(\rho,S)}{(1-\rho)^2}P(N^p=0) \tag{6.20}$$

$$E[W^{bp}] = \frac{E[B^p]}{(1-f^p)\,\lambda/2}$$
$$= \frac{\rho}{(1-\rho)\,\lambda} \tag{6.21}$$

$$E[T^p] = \frac{S+E[B^p]}{\lambda/2}$$
$$= \frac{\left(S+\frac{\rho^{2S+1}\zeta(\rho,S)}{(1-\rho)^2}P(N^p=0)\right)}{\lambda/2} \tag{6.22}$$

$$P(N^r=i) = \begin{cases} \left[1+2\rho+\frac{2\rho^2\zeta(\rho,S)\left(1-\rho^{2S-1}\right)}{(1-\rho)}\right]P(N^p=0); & i=0 \\ 2\rho^{2S+i}\zeta(\rho,S)P(N^p=0); & i\geq 1 \end{cases} \tag{6.23}$$

$$f^r = \sum_{i=0}^{R-1}P(N^r=i) = \left[P(N^r=0)+\frac{2\rho^{2S+1}\zeta(\rho,S)\left(1-\rho^{R-1}\right)}{(1-\rho)}P(N^p=0)\right] \tag{6.24}$$

$$E[I^r] = \sum_{i=0}^{R-1}(R-i)P(N^r=i)$$
$$= RP(N^r=0)+2\rho^{2S+1}\zeta(\rho,S)P(N^p=0)\left[\frac{R-R\rho+\rho^R-1}{(1-\rho)^2}\right] \tag{6.25}$$

$$E[B^r] = E[I^r] + E[N^r] - R = \frac{2\rho^{R+1}\zeta(\rho, S)}{(1-\rho)^2} P(N^p = 0) \tag{6.26}$$

$$E[W^{br}] = \frac{E[B^r]}{(1-f^r)\,\lambda} = \frac{\rho}{(1-\rho)\,\lambda} \tag{6.27}$$

$$E[T^r] = \frac{R + E[B^r]}{\lambda} = \frac{\left[R + \frac{2\rho^{R+1}\zeta(\rho,S)}{(1-\rho)^2} P(N^p = 0)\right]}{\lambda} \tag{6.28}$$

The analytical results for the modified M/M/2-based approximation along with the results for LoVis and MedVis cases are presented in Appendix C. Figures 6.14 through 6.23 present the plots for the performance measures at the retail store and a production facility. The performance measures for the M/M/2 model are included in the tables to show that the modification yields better approximations.

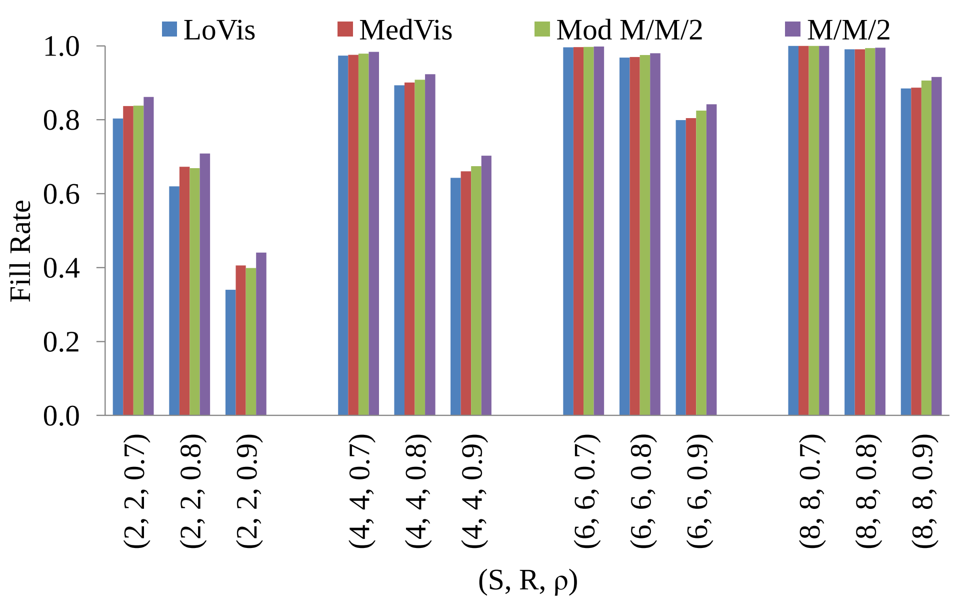



Figure 6.14: Fill Rate at the Retail Store

It can be seen from Figures 6.14 through 6.23 that the M/M/2 model is a good approximation for the LoVis and MedVis cases. But the modified M/M/2 model performs better in modeling the LoVis and MedVis cases for all performance measures

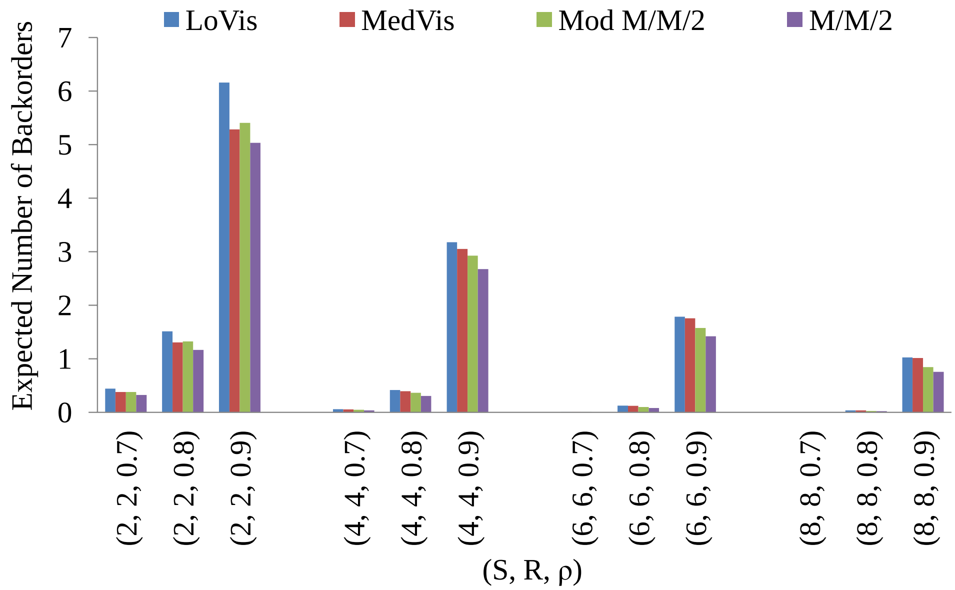



Figure 6.15: Expected Number of Backorders at the Retail Store

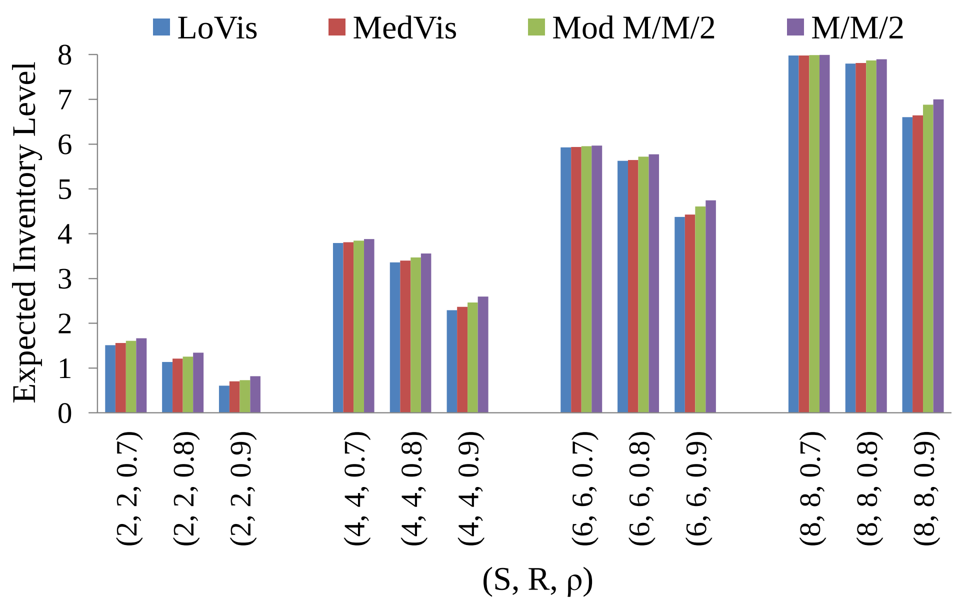



Figure 6.16: Expected Inventory Level at the Retail Store

(except expected time to fulfill a backorder) at the retail store and the two production facilities. In the case of the expected time to fulfill a backorder, the values for the modified M/M/2 model and the original M/M/2 model remain the same. This can be attributed to the fact that the time to fulfill a backorder is a conditional measure, and

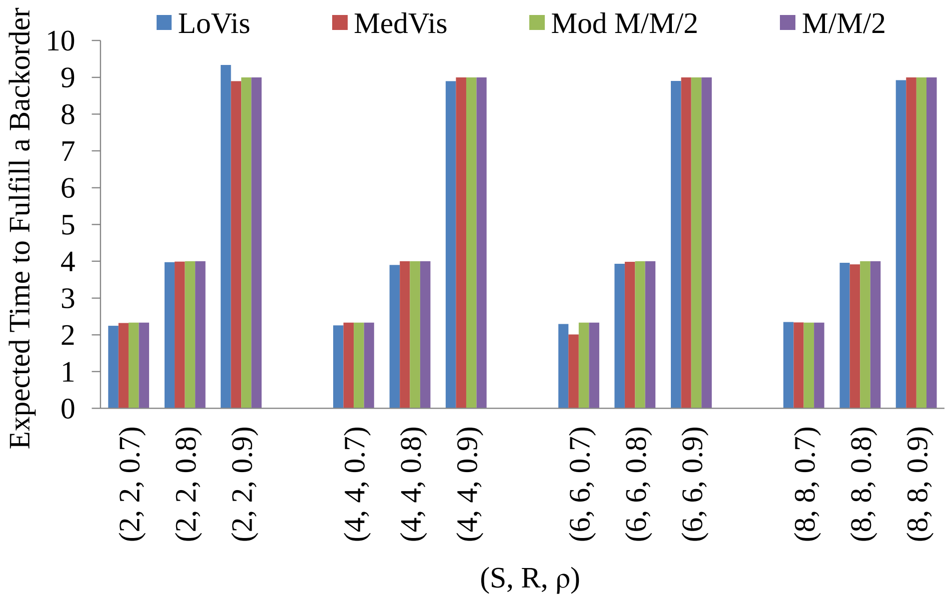



Figure 6.17: Expected Time to Fulfill a Backorder at the Retail Store

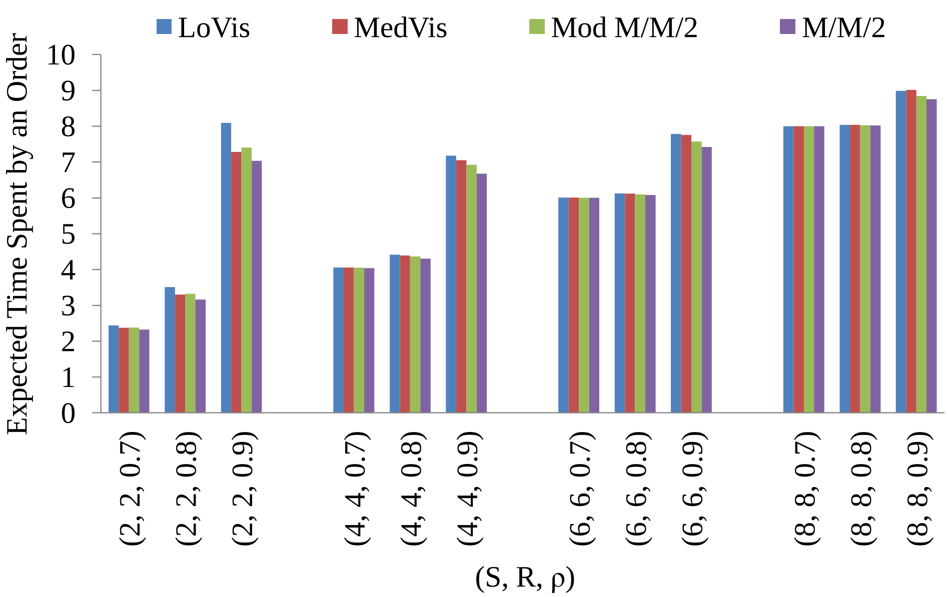



Figure 6.18: Expected Time Spent by an Order at the Retail Store

there is a cancellation effect that occurs due to the conditioning. Equations (6.21) and (6.27) present the expressions for the expected time to fulfill a backorder at a production facility and the retail store, respectively. We note that there is still an opportunity to explore additional correction factors to model the individual levels of

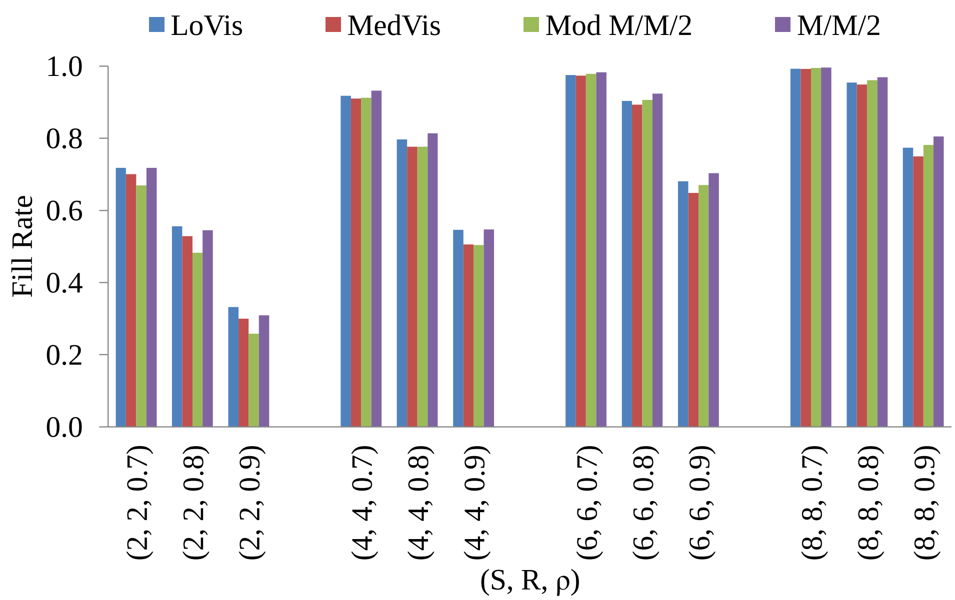



Figure 6.19: Fill Rate at a Production Facility

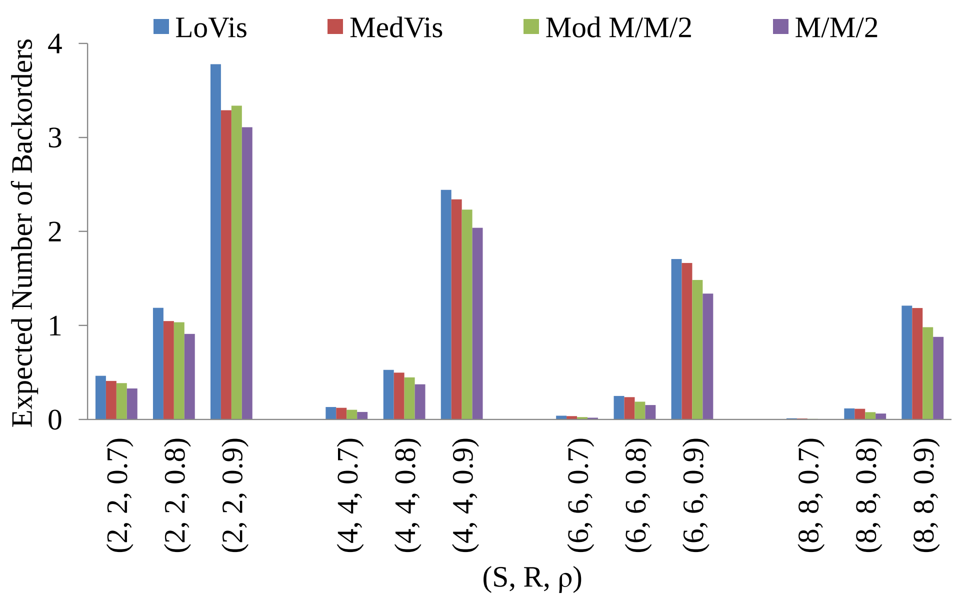



Figure 6.20: Expected Number of Backorders at a Production Facility

information sharing using our knowledge about the routing policies. For example, it is known that the difference between these two routing policies, MedVis and LoVis, is observed when both the production facilities are backordered. In such instances, the orders are probabilistically routed in a LoVis case and based on the shortest queue

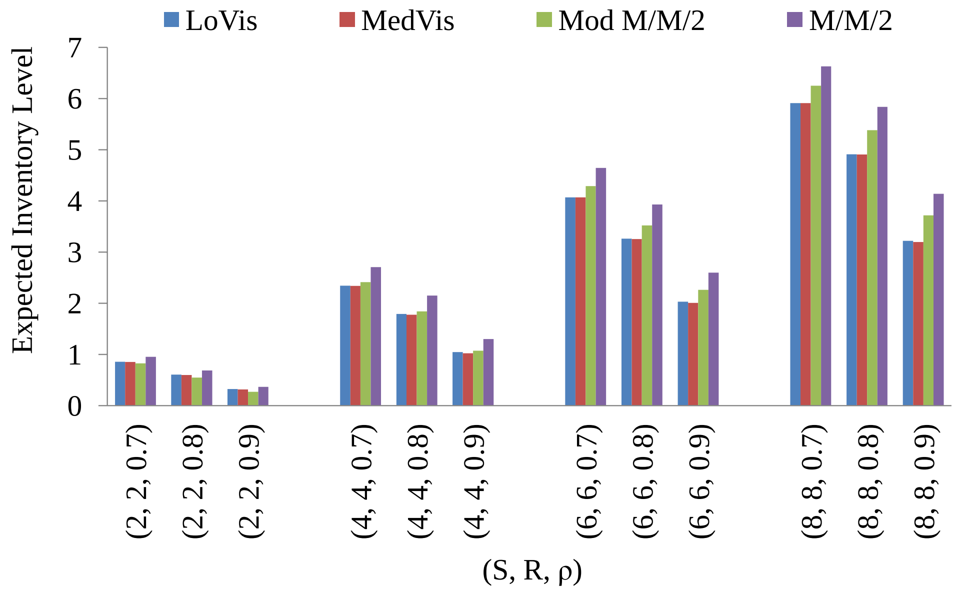



Figure 6.21: Expected Inventory Level at a Production Facility

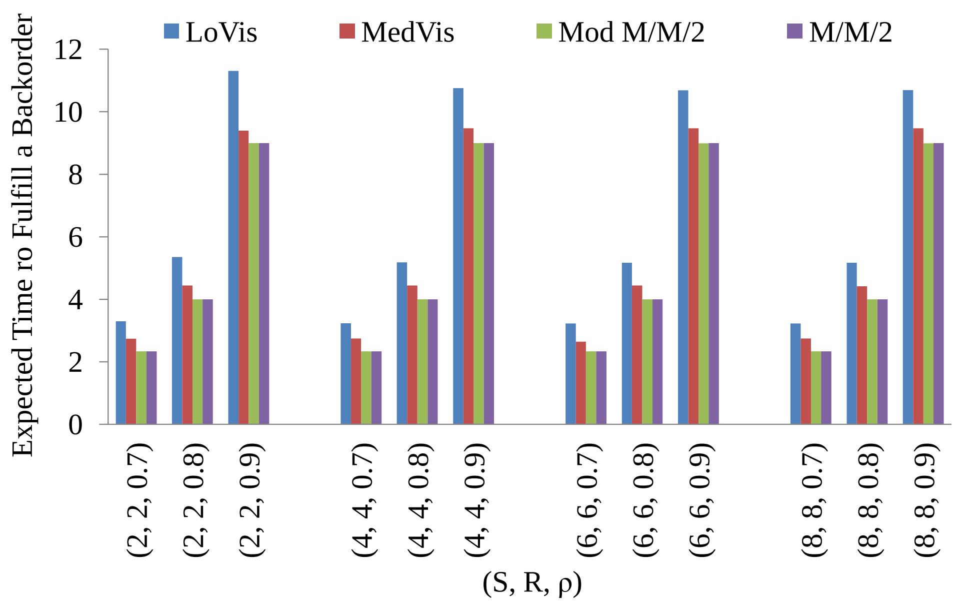



Figure 6.22: Expected Time to Fulfill a Backorder at a Production Facility

in the MedVis case. This provides us an avenue to improve the approximations for the MedVis and LoVis cases and could be explored in the future.

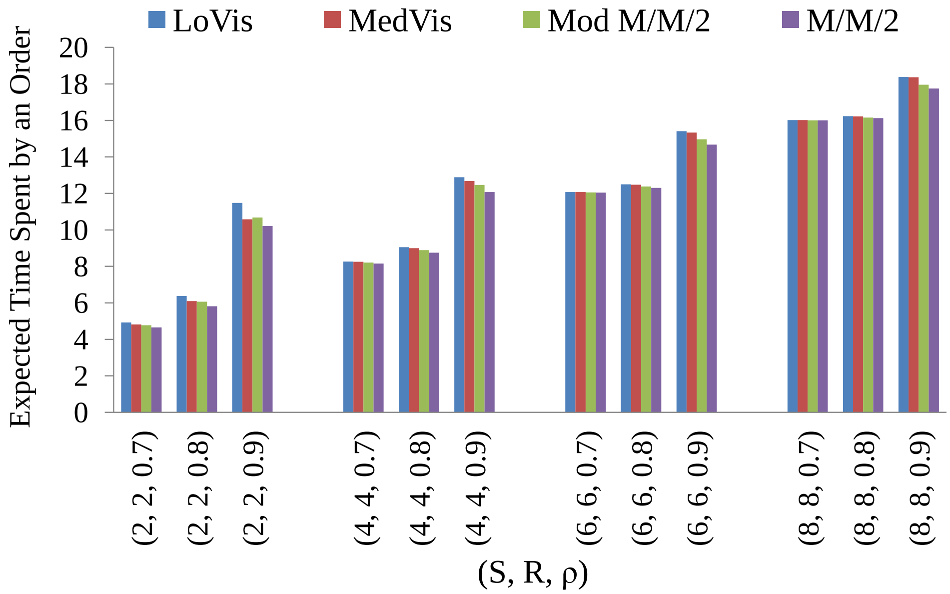



Figure 6.23: Expected Time Spent by an Order at a Production Facility

## 6.2 An Asymmetric 1R/2P SCN Configuration

In this section, we present an analytical model for a 1R/2P SCN with two non-identical production facilities. The processing time at production facility $i$ was assumed to be exponentially distributed with rate $\mu_i$. The base-stock level was assumed to be the same at the two production facilities. Since the processing time distributions at the two production facilities are not identical, the multi-server model based on the M/M/2 system is no longer appropriate. To still operate within the framework of a multi-server queueing system, we explored the idea of replacing two non-identical servers with two identical servers with an aggregate processing time distribution that represents a probabilistic mixture of the two original exponential processing time distributions. Using the properties of the exponential distribution, the probability that an order is placed at an individual production facility was first calculated. This probability was then used to obtain an aggregate processing distribution as a mixture of two exponential distributions, which in our case would be a 2-phase hyper-exponential distribution. The M/G/2 queueing system was used to model the SCN under study.

The details of the aggregation procedure are presented below. The probability of choosing the exponential distribution with rate $\mu_i$ as the processing time distribution is equal to the probability that an arriving customer joins production facility 1. This probability, $p_1$, can be obtained by conditioning on the number of busy servers (0, 1 and 2 in our case) as shown in equation (6.29).

$$
\begin{aligned}
p_1 = & P(Order\ goes\ to\ facility\ 1|both\ facilities\ are\ idle).P(both\ facilities\ are\ idle)+ \\
& P(Order\ goes\ to\ facility\ 1|facility\ 2\ is\ busy).P(facility\ 2\ is\ busy)+ \\
& P(Order\ goes\ to\ facility\ 1|both\ facilities\ are\ busy).P(both\ facilities\ are\ busy)
\end{aligned} \tag{6.29}
$$

When both the servers are idle an order is placed at production facility 1 with a probability of 0.5. Similarly, when production facility 2 is busy, an order will be placed at production facility 1 with a probability of 1. An order cannot be placed at production facility 1 when production facility 1 is busy and production facility 2 is idle. When the servers at both production facilities are busy, an order will be placed at production facility 1 when the server at production facility 1 finishes processing ahead of the server at production facility 2. Since the processing time distributions at both facilities are exponential, the probability that the server at production facility 1 finishes first is given by $\mu_1/(\mu_1+\mu_2)$. Then there is a need to calculate the probabilities of both servers being busy, both being idle and one of them being busy. We used the probabilities from the M/M/2 queueing system to obtain these probabilities as shown in equations (6.30), (6.31) and (6.32). The use of M/M/2 probabilities is consistent with the approach of Whitt [60] in which the measures from the M/M/c models are used in the calculation of the distribution of number of orders in a GI/G/c queuing system.

$$P(both\ facilities\ are\ idle) = \frac{(1-\rho)}{(1+\rho)} \tag{6.30}$$

where $\rho = \lambda/(\mu_1 + \mu_2)$.

$$\begin{aligned} P(facility\ 2\ is\ busy) &= 0.5P(one\ facility\ is\ busy) \\ &= 0.5(2\rho)\frac{(1-\rho)}{(1+\rho)} \\ &= \rho\frac{(1-\rho)}{(1+\rho)} \end{aligned} \tag{6.31}$$

$$\begin{aligned} P(both\ facilities\ are\ busy) &= 1 - [P(both\ facilities\ are\ idle) + P(one\ facility\ is\ busy)] \\ &= \frac{2\rho^2}{(1+\rho)} \end{aligned} \tag{6.32}$$

Using these probabilities, the probability with which the exponential distribution with rate $\mu_1$ is chosen, $p_1$, can be approximately obtained as shown in equation (6.33)

$$p_1 = 0.5\frac{(1-\rho)}{(1+\rho)} + 1\rho\frac{(1-\rho)}{(1+\rho)} + \frac{\mu_1}{(\mu_1+\mu_2)}\frac{2\rho^2}{(1+\rho)} \tag{6.33}$$

Thus the processing time distribution would be the exponential distribution with rate $\mu_1$ with probability $p_1$ and the exponential distribution with rate $\mu_2$ with probability $p_2(=1-p_1)$. The aggregate processing time follows a 2-phase hyper-exponential distribution and hence, the first two moments of the aggregate processing time distribution were obtained using equations (6.34) and (6.35), where $X$ is a random variable representing the aggregate processing time. The variance and SCV of the distribution can be obtained using equations (6.36) and (6.37).

$$E[X] = \sum_{i=1}^{2} \frac{p_i}{\mu_i} \tag{6.34}$$

$$E[X^2] = \sum_{i=1}^{2} \frac{2p_i}{\mu_i^2} \tag{6.35}$$

$$Var(X) = E[X^2] - (E[X])^2 \tag{6.36}$$

$$cp^2 = \frac{Var(X)}{(E[X])^2} \tag{6.37}$$

Using the parameters of the inter-arrival distribution and the aggregate processing time distribution, the expressions for the GI/G/2 queueing system provided by Whitt [60] were used to obtain the distribution of number of orders at the production facilities, $N^p$. The distribution of the number of orders at the retail store, $N^r$, can be obtained using the equation (6.38).

$$P(N^r = n) = \begin{cases} \sum_{i=0}^{2S} P(N^p = i) & ; \quad n = 0 \\ P(N^p = 2S + n) & ; \quad n \geq 1 \end{cases} \tag{6.38}$$

The performance measures at the retail store were obtained using the distribution of $N^r$ as shown in equations (6.39) through (6.43).

$$f^r = \sum_{i=0}^{R-1} P(N^r = i) \tag{6.39}$$

$$E[I^r] = \sum_{i=0}^{R} (R - i) P(N^r = i) \tag{6.40}$$

$$E[B^r] = E[I^r] + E[N^r] - R \tag{6.41}$$

where $E[N^r] = \sum_{i=0}^{\infty} iP(N^r = i)$

$$E[W^{br}] = \frac{E[B^r]}{(1 - f^r)\lambda} \tag{6.42}$$

$$E[T^r] = \frac{R + E[B^r]}{\lambda} \tag{6.43}$$

The asymmetry in the 1R/2P SCN configuration will result in different utilization levels at the individual production facilities. The utilization levels at production facility $j$ can be obtained using equation (6.44). The production facility with the lower utilization (faster facility) is the one that receives a larger proportion of orders. It is also clear that the production facility with the higher utilization (slower facility) will have a longer order queue. With the knowledge about the utilizations at the individual production facilities and their effect on the order queue lengths at the production facilities, the marginal distribution for the number of orders at each production facility was obtained. The conditional probability that there are $m$ orders at production facility $j$ given that there are $l$ orders in total at the production facilities follows a binomial distribution as shown in equation (6.46). The parameters of the binomial distribution include the probability that an order at the pooled facility belongs to production facility 1 ($\alpha_1$) or production facility 2 ($\alpha_2$) and $l$, the total number of orders at the production facilities. The probability that an order at the pooled facility belongs to production facility $j$, $\alpha_j$, was obtained by using utilizations as "weights" as shown in equation (6.45). The unconditional distribution of the number of orders at production facility $j$, $N_j^p$, was obtained by multiplying the conditional probability with the distribution of the number of orders at the production facilities, $N^p$ and summing over all values $l$ greater than or equal to $m$ as shown in equation (6.47).

$$\rho_j^p = (\lambda\, p_j)\; /\; \mu_j\;;\; j = 1, 2 \tag{6.44}$$

$$\alpha_j = \frac{\rho_j}{\sum_{i=1}^{2} \rho_i} \tag{6.45}$$

$$P\left(N_j^p = m | N^p = l\right) = \binom{l}{m} \alpha_j^m (1-\alpha_j)^{(l-m)} \quad ;\ m = 0, 1, \cdots, l \tag{6.46}$$

$$P\left(N_j^p = m\right) = \sum_{l=m}^{\infty} P\left(N_j^p = m | N^p = l\right) \cdot P\left(N^p = l\right) \tag{6.47}$$

The fill rate at a production facility was obtained by calculating the fill rate at the pooled production facility (equation (6.48)) since the orders from the retail store are always placed at the production facility that has inventory. Additionally, it is known at the time of placing an order at the production facility whether it would be satisfied instantaneously or not.

$$f_j^p = \sum_{i=0}^{2S-1} P(N^p = i) \tag{6.48}$$

The performance measures at the production facility $j$ can be obtained using this distribution as shown in equations (6.49) through (6.52).

$$E[I_j^p] = \sum_{i=0}^{S} (S-i) P(N_j^p = i) \tag{6.49}$$

$$E[B_j^p] = E[I_j^p + E[N_j^p] - S \tag{6.50}$$

where $E[N_j^p] = \sum_{i=0}^{\infty} i P(N_j^p = i)$

$$E[W_j^{bp}] = \frac{E[B_j^p]}{(1-f_j^p)\lambda\, p_j} \tag{6.51}$$

$$E[T_j^p] = \frac{\left[E[B_j^p] + S\right]}{\lambda\, p_j} \tag{6.52}$$

### 6.2.1 Model Validation

In this section, the M/G/2 approximation for the asymmetric 1R/2P SCN configuration is validated by comparing the performance measures for the analytical model with simulation estimates for the HiVis case. The SCN parameter values used in the numerical experimentation are presented in Table 6.3. The design consists of 27 experiments. Without loss of generality, we assume that the server at production facility 1 is faster than the server at the production facility 2.

Table 6.3: Experiments for the Asymmetric 1R/2P SCN Configuration

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rate $\lambda$ | 1 | 1 |
| Base-stock levels $(S,\ R)$ | 3 | (2, 1), (4, 2), (6, 3) |
| Mean processing Times $(\tau_1, \tau_2)$ | 9 | (1.3, 1.5), (1.2, 1.6), (1.1, 1.7),<br>(1.5, 1.7), (1.4, 1.8), (1.3, 1.9),<br>(1.7, 1.9), (1.6, 2.0), (1.5, 2.1) |

The numerical results for the performance measures at the retail store and the production facilities are presented in Section C.3 (see Appendix C). The summary of results is presented in Table 6.4. Figures 6.24 through 6.34 plot the various performance measures at the retail store and production facilities when the level of asymmetry $\tau_2 - \tau_1$ is varied at a base-stock setting of (2, 1) with $\tau_1 + \tau_2$ fixed at 3.6 (signifying an average utilization of 90%).

Figures 6.24 through 6.28 (refer to Table C.11 in Appendix C for numerical results) show that the analytical model clearly captures the sensitivity of the performance measures at the retail store to the heterogeneous processing time distributions. At the production facility level, the model accurately predicts the utilization, fill rate, the expected number of backorders, the expected inventory level and the expected time spent by an order. In fact, all the results for the fill rate at all stores, and utilizations of the production facilities fall within a 3% error range. As the value of $\tau_1 + \tau_2$ is kept constant and the difference between the two processing time means is increased, the

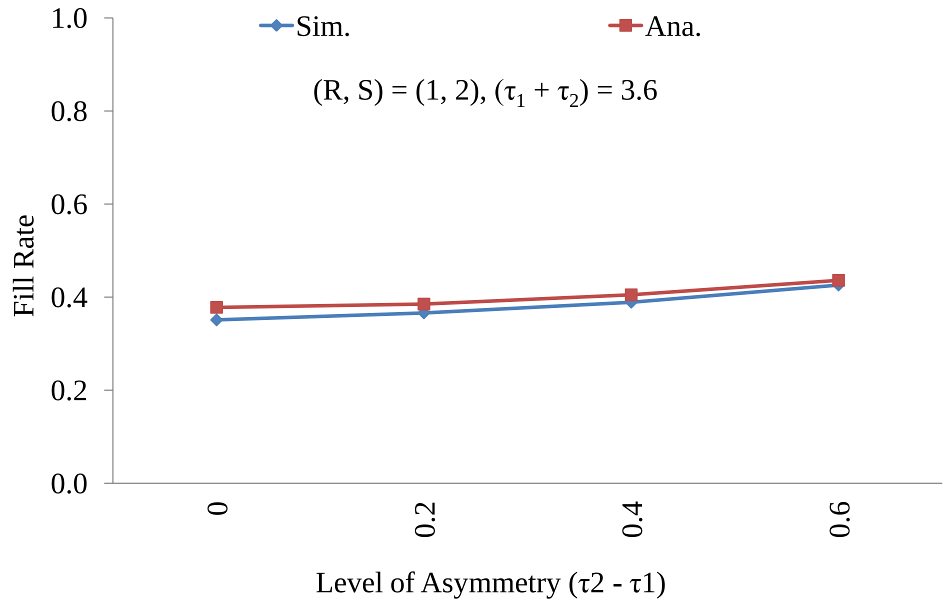



Figure 6.24: Fill Rate at the Retail Store

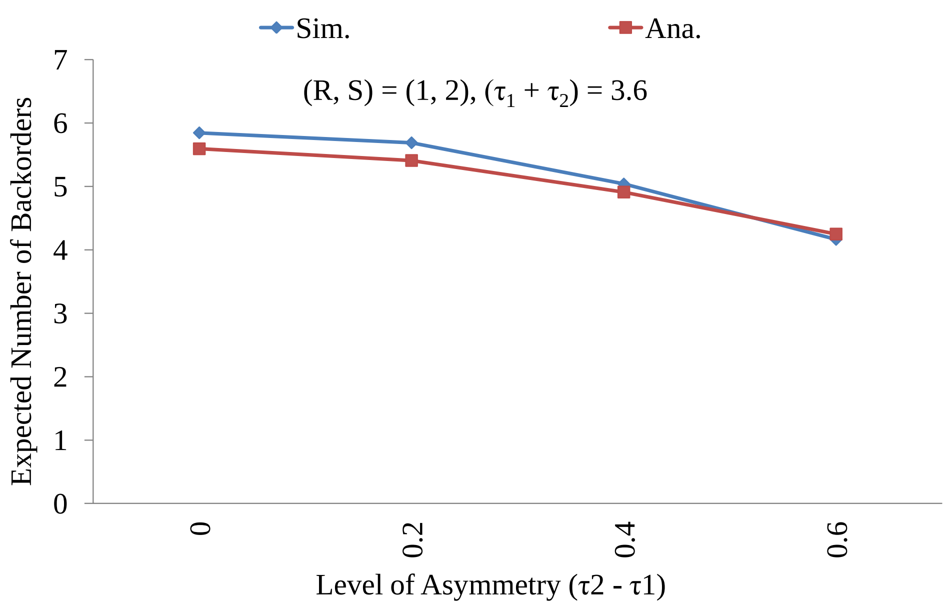



Figure 6.25: Expected Number of Backorders at the Retail Store

utilizations of the two production facilities differ significantly as seen from Table C.14 and the analytical model accurately captures the asymmetric utilization levels at the production facilities. Figures 6.29 through 6.34 present an illustration of how the analytical model can capture the trends in the various performance measures at the

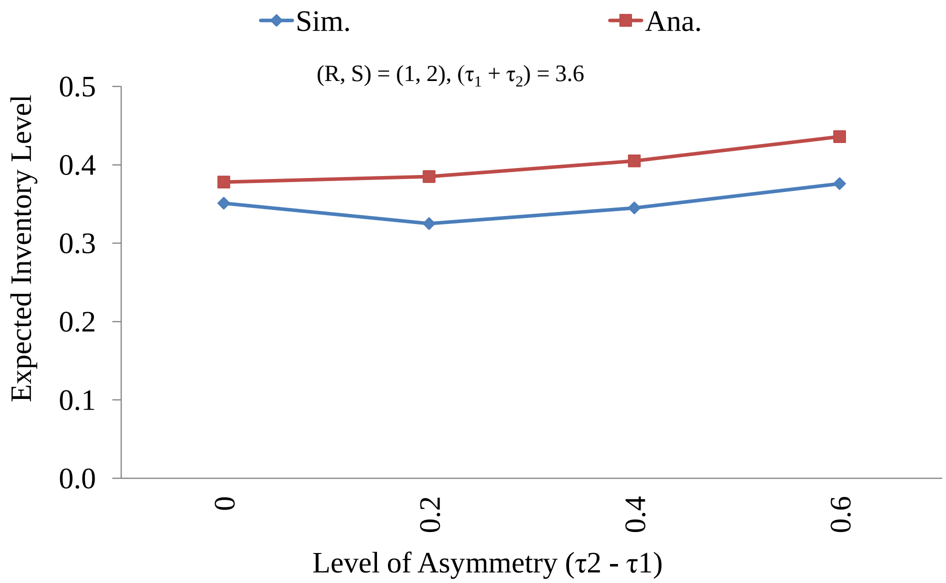



Figure 6.26: Expected Inventory Level at the Retail Store

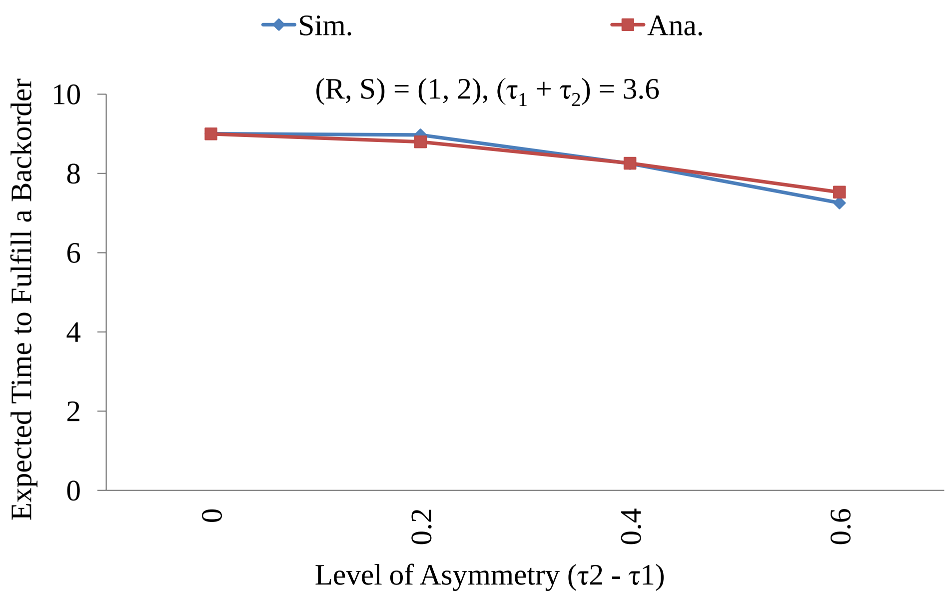



Figure 6.27: Expected Time to Fulfill a Backorder at the Retail Store

two production facilities. It can also be seen from Table 6.4 that 84% of the results for the expected time to fulfill a backorder are within an acceptable error range of 20%. This wider range of error is because of the fact that this performance measure depends on the value of two other performance measures, namely, the expected number of

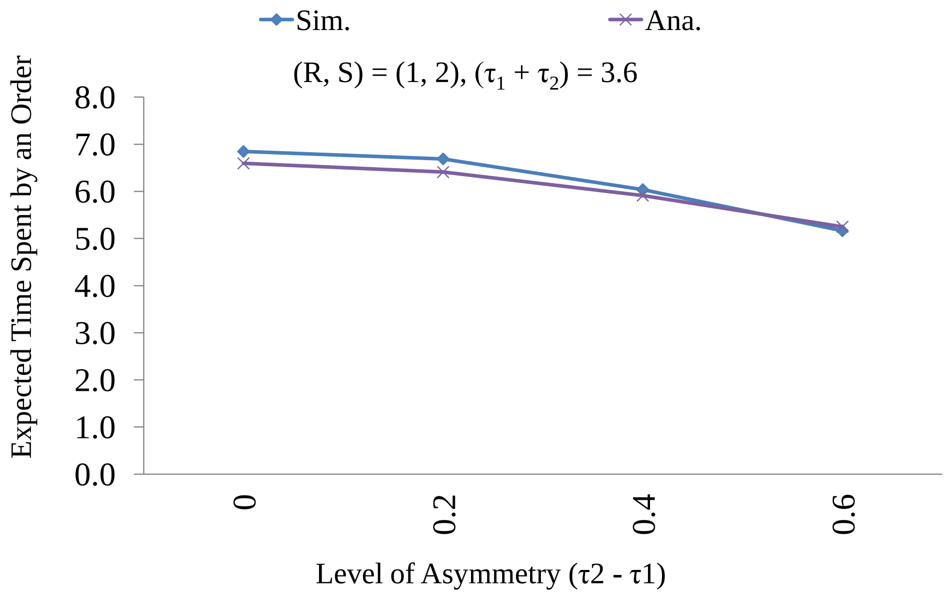



Figure 6.28: Expected Time Spent by an Order at the Retail Store

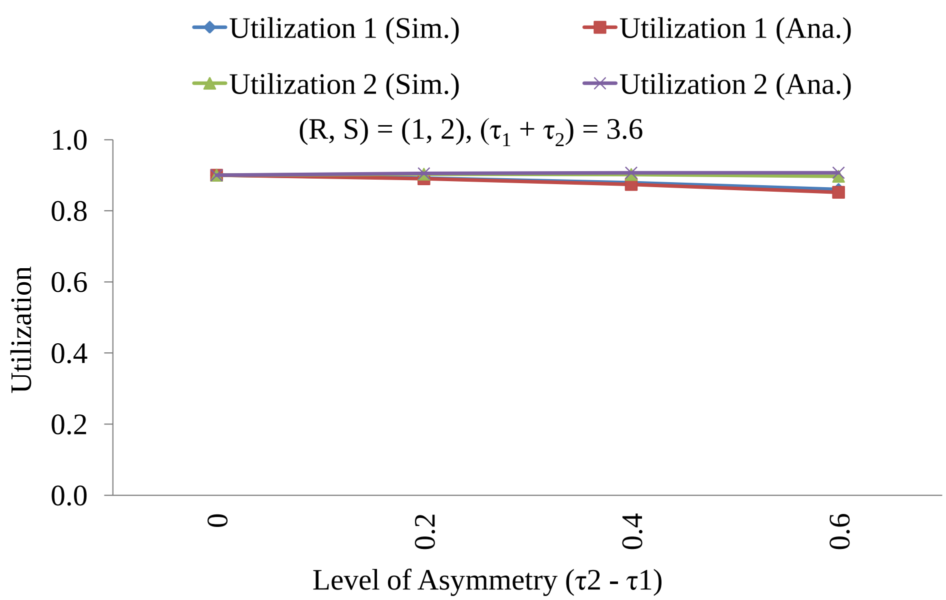



Figure 6.29: Utilization at the Production Facilities

backorders and fill rate. It has to be noted that any error in estimating these two performance measures could affect the prediction of the expected time to fulfill a backorder. In particular, even a small error in estimating the fill rate value would affect the expected time to fulfill a backorder significantly as the fill rate terms are

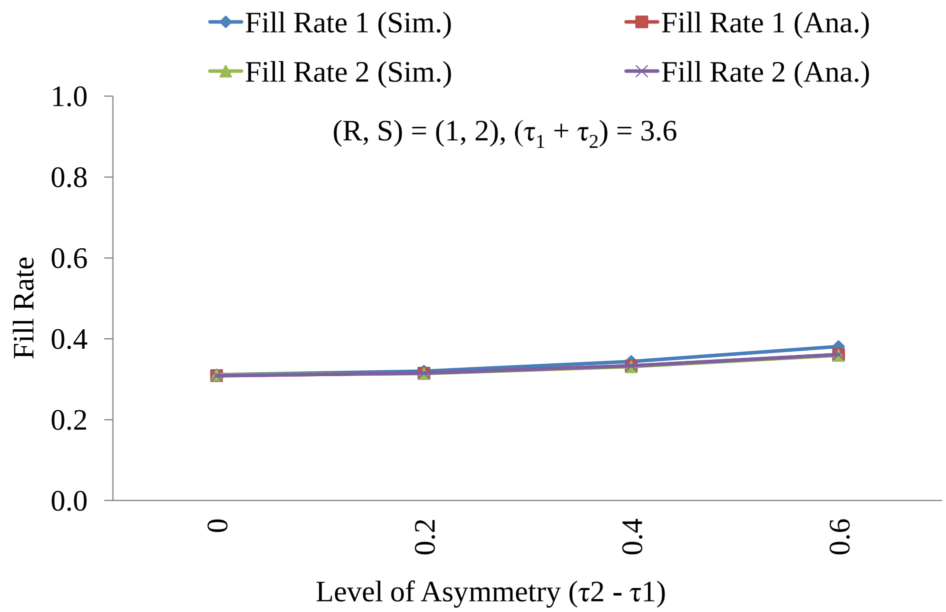



Figure 6.30: Fill Rate at the Production Facilities

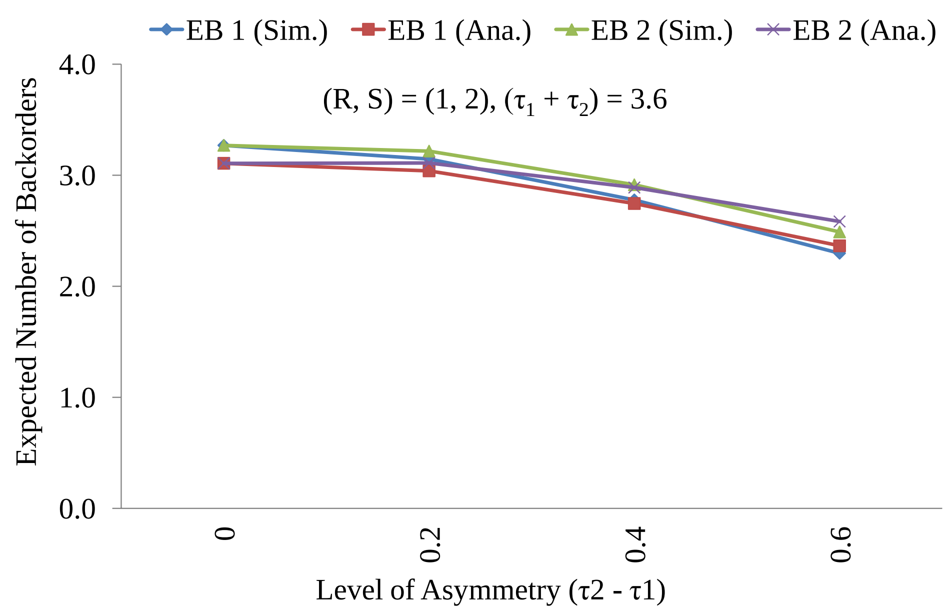



Figure 6.31: Expected Number of Backorders at the Production Facilities

always less than 1 and occur in the denominator of the expression for calculation of the performance measure under consideration.

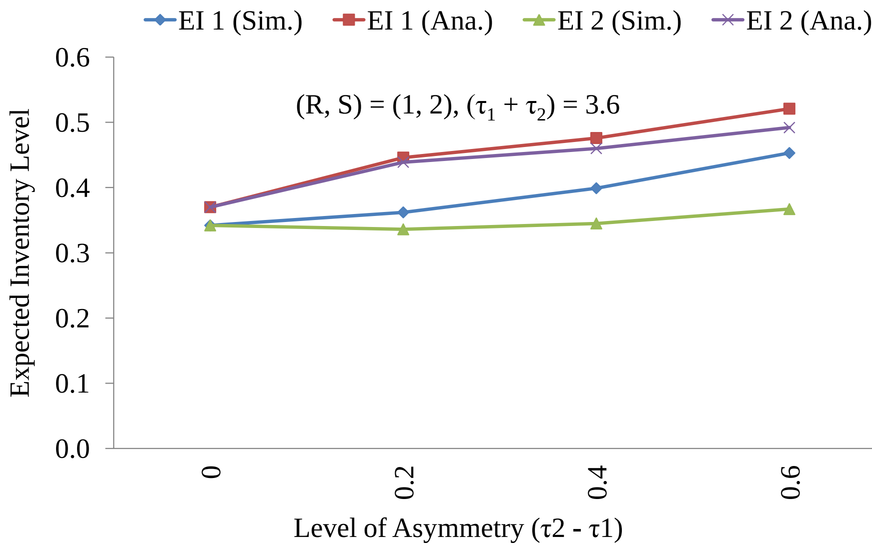



Figure 6.32: Expected Inventory Level at the Production Facilities

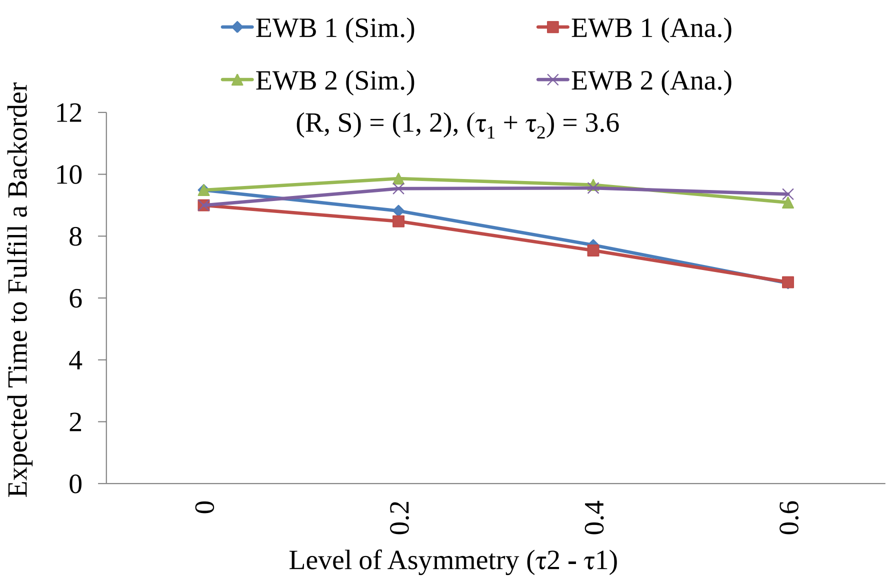



Figure 6.33: Expected Time to Fulfill a Backorder at the Production Facilities

## 6.3 Conclusions

In this chapter, we have developed approximate analytical models for the 1R/2P SCN configuration with inventory visibility under Poisson arrivals and exponential processing times. The approximate analytical model worked well for the HiVis case and also provided useful bounds for lower levels of visibility. Using a combination of analytical

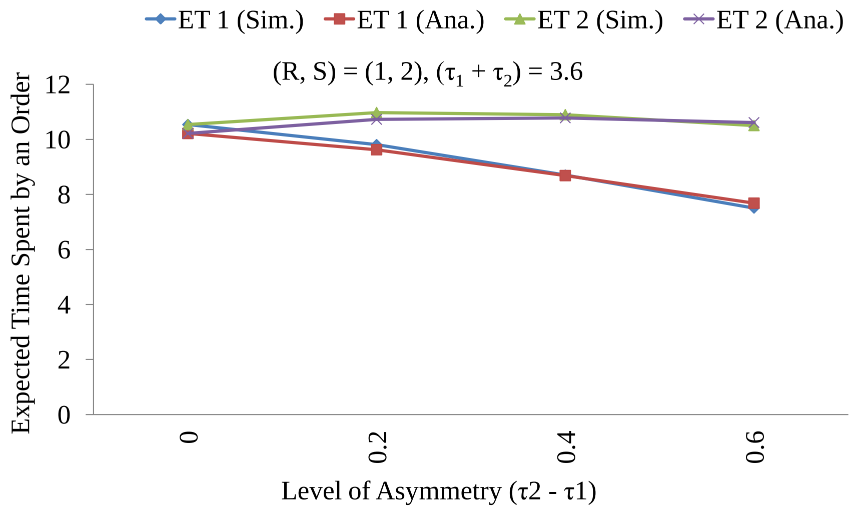



Figure 6.34: Expected Time Spent by an Order at the Production Facilities

Table 6.4: Summary of Results

| Error Range | Percentage of Results within Error Range (%) | | | | |
|---|---|---|---|---|---|
| | Fill Rate | Expected Number of Backorders | Expected Inventory Level | Expected Time to Fulfill a Backorder | Expected Time Spent by an Order |
| $< 5$ % | 100.0 | 87.7 | 76.5 | 56.8 | 100.0 |
| $< 10$% | 100.0 | 96.3 | 100.0 | 72.8 | 100.0 |
| $< 15$% | 100.0 | 98.8 | 100.0 | 79.0 | 100.0 |
| $< 20$% | 100.0 | 98.8 | 100.0 | 84.0 | 100.0 |
| $< 25$% | 100.0 | 98.8 | 100.0 | 87.7 | 100.0 |

reasoning and empirical observations, we derived a correction factor for the steady state probabilities of the M/M/2 system. This modified M/M/2 approximation was found to better model the SCN with lower levels of inventory information sharing. We also developed extensions to model non-identical production facilities and the resulting M/G/2 model was found to capture the sensitivity of the performance measures at the retail store to the heterogeneous processing time distributions as well as to the different utilizations at the production facilities. The following chapter will focus on the development of analytical models for the 1R/2P SCN configuration with HiVis under general inter-arrival and processing time distributions.

# CHAPTER 7

# QUEUEING MODELS OF THE 1R/2P SCN CONFIGURATION WITH GENERAL INTER-ARRIVAL AND PROCESSING TIME DISTRIBUTIONS

In this chapter, we present analytical models of the 1R/2P SCN configuration under general inter-arrival and processing times. As before, we considered the SCN with three levels of information sharing (defined in Section 3.1) along with the SCN with no information sharing. We considered only the symmetric configuration for the general case; the two production facilities have identical processing time distributions and the same base-stock level. Since the M/M/2 model has served as a good approximation for the three levels of information sharing in the case of Poisson arrivals and exponential processing times, the GI/G/2 model [60] was used to approximate the SCN with HiVis under general inter-arrival and processing time distributions. The model and its validation are presented in Section 7.1. The analytical model developed by Srivathsan [53] was adapted for the SCN with NoVis and is presented in Section 7.2. Section 7.3 presents the effect of the inter-arrival and processing time variability on the value of information sharing, while concluding remarks on the analytical models developed in this chapter are presented in Section 7.4.

## 7.1 Queueing Model of the General 1R/2P SCN Configuration with HiVis

The approximations presented by Whitt [60] for the GI/G/m queueing system were used to model the 1R/2P SCN configuration with HiVis. Let $P(N^p = n)$ represent the distribution of the number of orders at the pooled production facility. The distribution of the number of orders can be used to obtain the performance measures at the production facility as shown in equations (7.1) through (7.5). With symmetric production facilities, the arrival rate at the individual production facilities would be $\lambda/2$.

$$f^p = \sum_{i=0}^{2S-1} P(N^p = i) \tag{7.1}$$

$$E[I^p] = 0.5 \sum_{i=0}^{2S} (2S - i) P(N^p = i) \tag{7.2}$$

$$E[B^p] = 0.5 \sum_{i=2S}^{\infty} (i - 2S) P(N^p = i) \tag{7.3}$$

$$E[W^{bp}] = \frac{E[B^p]}{(1 - f^p)(\lambda/2)} \tag{7.4}$$

$$E[T^p] = \frac{S + E[B^p]}{\lambda/2} \tag{7.5}$$

As seen in Section 6.1, the information about the number of orders at the individual production facilities is not directly available as the inventory is "pooled" in the queueing model. We use the distribution of the number of orders at the production facilities to obtain the distribution of the number of orders at the retail store as shown in equation (7.6). The fact that we have instantaneous replenishment from the production facilities means that when the number of orders at the production facilities is less than $2S$, the number of orders at the retail store is zero. When the number of orders, $i$ at the production facilities exceeds $2S$, the number of orders at the retail store is $2S - i$. The performance measures at the retail store can be computed as shown in equations (7.7) through (7.11).

$$P(N^r = n) = \begin{cases} \sum_{i=0}^{2S} P(N_j^p = i) & ; \quad n = 0 \\ P(N_j^p = 2S + n) & ; \quad n \geq 1 \end{cases} \tag{7.6}$$

$$f^r = \sum_{i=0}^{R-1} P(N^r = i) \tag{7.7}$$

$$E[I^r] = \sum_{i=0}^{R} (R - i) P(N^r = i) \tag{7.8}$$

$$E[B^r] = \sum_{i=R}^{\infty} (i - R) P(N^r = i) \tag{7.9}$$

$$E[W^{br}] = \frac{E[B^r]}{(1 - f^r)\lambda} \tag{7.10}$$

$$E[T^r] = \frac{E[B^r] + R}{\lambda} \tag{7.11}$$

### 7.1.1 Validation of the GI/G/2 Approximation

In this section, the GI/G/2 approximation is validated by comparing the performance measures obtained from the analytical model with simulation estimates for the HiVis case. The SCN parameter values used in the numerical experimentation are presented in Table 7.1. The design consists of 108 experiments.

The results for the numerical experiments are presented in Appendix D. The GI/G/2-based model is validated by using the prediction error measure defined in Section 4.3. A summary of the results is presented in Table 7.2.

Tables D.1 through D.5 in Appendix D show that the GI/G/2-based model is a very good approximation for the HiVis case. Table 7.2 shows that more than 90% of the results for fill rate, expected number of backorders, expected inventory level, and the expected time spent by an order are within the acceptable 15% error range. In

Table 7.1: 1R/2P SCN Experiments for General Inter-arrival and Processing Time Distributions

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rate $\lambda$ | 1 | 1 |
| Base-stock levels $(S,\ R)$ | 4 | (2, 2), (4, 4), (6, 6), (8, 8) |
| Utilization level $\rho$ | 3 | 70%, 80%, 90% |
| Inter-arrival distribution at the retail store | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |
| Processing time distribution at the production facilities | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |

Table 7.2: Summary of Results

| Error Range | Percentage of Results within Error Range (%) | | | | |
|---|---|---|---|---|---|
| | Fill Rate | Expected Number of Backorders | Expected Inventory Level | Expected Time to Fulfill a Backorder | Expected Time Spent by an Order |
| $< 5\%$ | 89.8 | 72.7 | 86.6 | 37.5 | 94.0 |
| $< 10\%$ | 99.1 | 86.1 | 98.6 | 53.7 | 100.0 |
| $< 15\%$ | 100.0 | 92.6 | 100.0 | 65.7 | 100.0 |
| $< 20\%$ | 100.0 | 94.9 | 100.0 | 72.2 | 100.0 |
| $< 25\%$ | 100.0 | 97.7 | 100.0 | 75.5 | 100.0 |

the case of expected time to fulfill a backorder, it can be seen that 75% of the results fall within the 25% error range. It can also be seen from Tables D.2 and D.4 that most of the higher errors occur when the arrival process at the retail store follows an Erlang distribution (SCV = 0.25). This can be attributed to the fact that the calculations for this measure depend on the fill rate. It has to be noted that the fill rate has been approximated by the ready rate and this approximation is exact only when the arrival process is Poisson.

## 7.2 Queueing Model of a General 1R/2P SCN Configuration with NoVis

The analytical performance modeling approach for supply chain networks developed based on the parametric decomposition method by Srivathsan [53], and Srivathsan and Kamath [54] was adapted to obtain the performance measures for the SCN with NoVis. Each production facility was modeled as a GI/G/1 queue. The backorder distribution at the production facilities was used to obtain the distribution of number of orders at the retail store.

### 7.2.1 Queueing Model

The parametric decomposition method was used to analyze the SCN under consideration. The order information flow from the retail store to the production facilities was first analyzed to obtain the mean and SCV of the arrival process at the two output stores corresponding to the two production facilities. As the orders are routed from the retail store to the individual production facility with a probability of 0.5, the arrival rate at an individual production facility is $\lambda/2$. The SCV of the arrival process at a production facility, $cm_j^2$, is obtained by using the splitting approximations [59] as shown in (7.12).

$$cm_j^2 = 0.5 \; cr^2 + 1 - 0.5 = 0.5(cr^2 + 1) \tag{7.12}$$

where $cr^2$ is the SCV of the inter-arrival process at the retail store.

Each production facility was modeled as a GI/G/1 queue. The distribution of the number in system for a GI/G/1 was used to obtain the distribution of the number of orders at production facility $j$, $N_j^p$, as shown in equation (7.13). In the following equations, the utilization, $\rho_j$, at each production facility is equal to $\frac{\lambda}{2\mu}$.

$$P(N_j^p = n) = \begin{cases} 1 - \rho_j & ; \quad n = 0 \\ \rho_j \cdot (1 - \sigma_j)\, \sigma_j^{n-1} & ; \quad n \geq 1 \end{cases} \tag{7.13}$$

$$\sigma_j = \frac{\left(E[N_j^p] - \rho_j\right)}{E[N_j^p]} \tag{7.14}$$

$$E[N_j^p] = g_j \left[\frac{\rho_j^2}{(1-\rho_j)}\right] \left[\frac{\left(cm_j^2 + cp_j^2\right)}{2}\right] \tag{7.15}$$

$$g_j = \begin{cases} exp\left(-(2 \cdot (1-\rho_j)/3\rho_j) \cdot ((1-cm_j^2)^2/(cm_j^2+cp_j^2))\right) & ; \quad cm_j^2 < 1 \\ exp\left(-(1-\rho_j) \cdot (cm_j^2 - 1)/(cm_j^2 + 4cp_j^2)\right) & ; \quad cm_j^2 \geq 1 \end{cases} \tag{7.16}$$

The fill rate at production facility $j$ can be obtained using equation (7.17). The remaining performance measures at the production facilities can be obtained using equations (6.49) through (6.52) (see Section 6.2).

$$f_j^p = \sum_{i=0}^{S-1} P(N_j^p = i) \tag{7.17}$$

As in Chapters 5 and 6, there are no orders (replenishment or backorders) at the retail store as long as there are no backorders at the individual production facility that receives the order. From this argument, we can see that an order at a retail store can be either backordered at production facility 1 or production facility 2. Assuming independence among the random variables $B_1^p$ and $B_2^p$, representing the number of orders backordered at production facilities 1 and 2, we can obtain the distribution of the number of orders at the retail store as given in equation (7.18).

$$P(N^r = n) = \sum_{x=0}^{n} P(B_1^p = x).P(B_2^p = n - x) \tag{7.18}$$

The performance measures at the retail store can be obtained using equations (7.7) through (7.11).

## 7.3 Effect of Inter-arrival Time and Processing Time SCVs on the Value of Information Sharing

In this section, we study the effect of the inter-arrival and processing time SCVs on the value of information sharing. As an illustration of this, we use a base-stock level setting of (4, 4). Figures 7.1 through 7.10 present the line plots for the performance measures at the retail store and a production facility. Each plot consists of 5 lines representing the performance measures for the SCN with NoVis, LoVis, MedVis and HiVis cases, and the GI/G/2 based analytical model. We use simulation estimates for the performance measures for the SCN with LoVis, MedVis and HiVis cases.

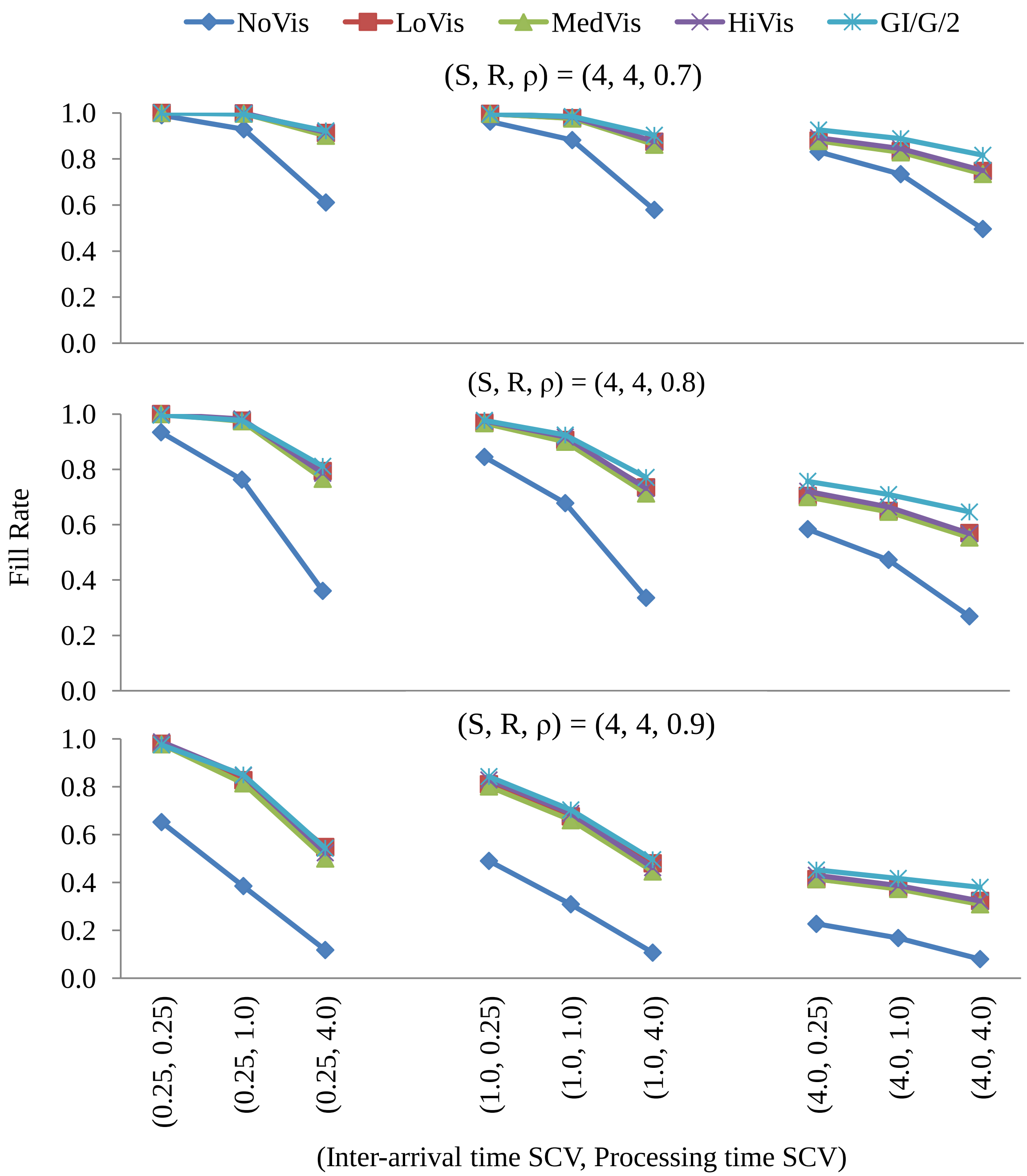



Figure 7.1: Fill Rate at the Retail Store

Figures 7.1 through 7.10 clearly show that as the inter-arrival time SCV is held constant and the processing time SCV is increased from 0.25 to 4.0, the difference between the performance measures for the SCN with NoVis and the SCN with inventory visibility (LoVis, MedVis and HiVis cases) increases. This shows that there

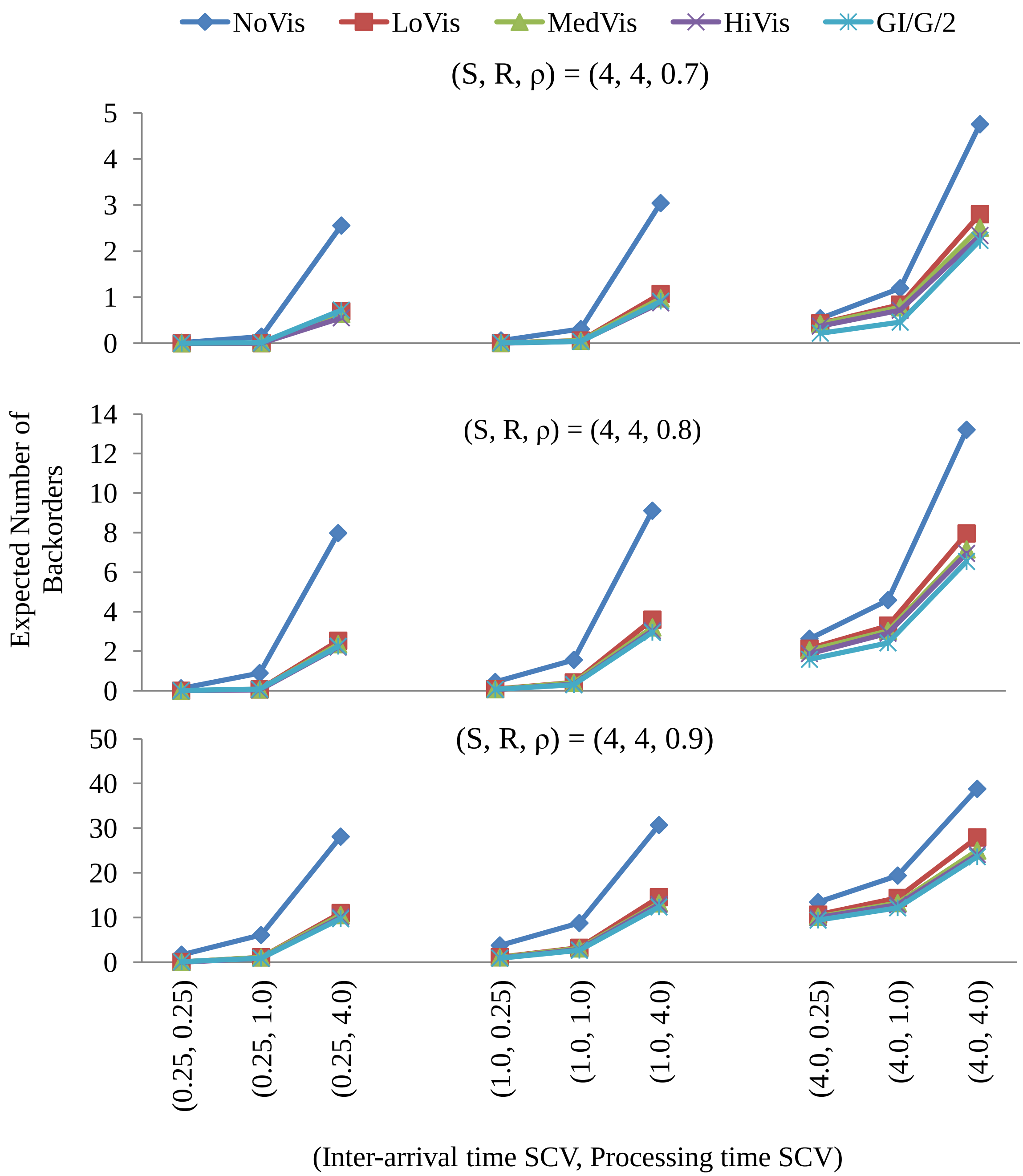



Figure 7.2: Expected Number of Backorders at the Retail Store

is significant value to information sharing at higher processing time SCVs. Similarly, when the processing time SCV is fixed and the inter-arrival time SCV is increased from 0.25 to 4.0, the difference between the performance measures for the SCN with NoVis and SCN with information sharing increases. But, it can be seen that this

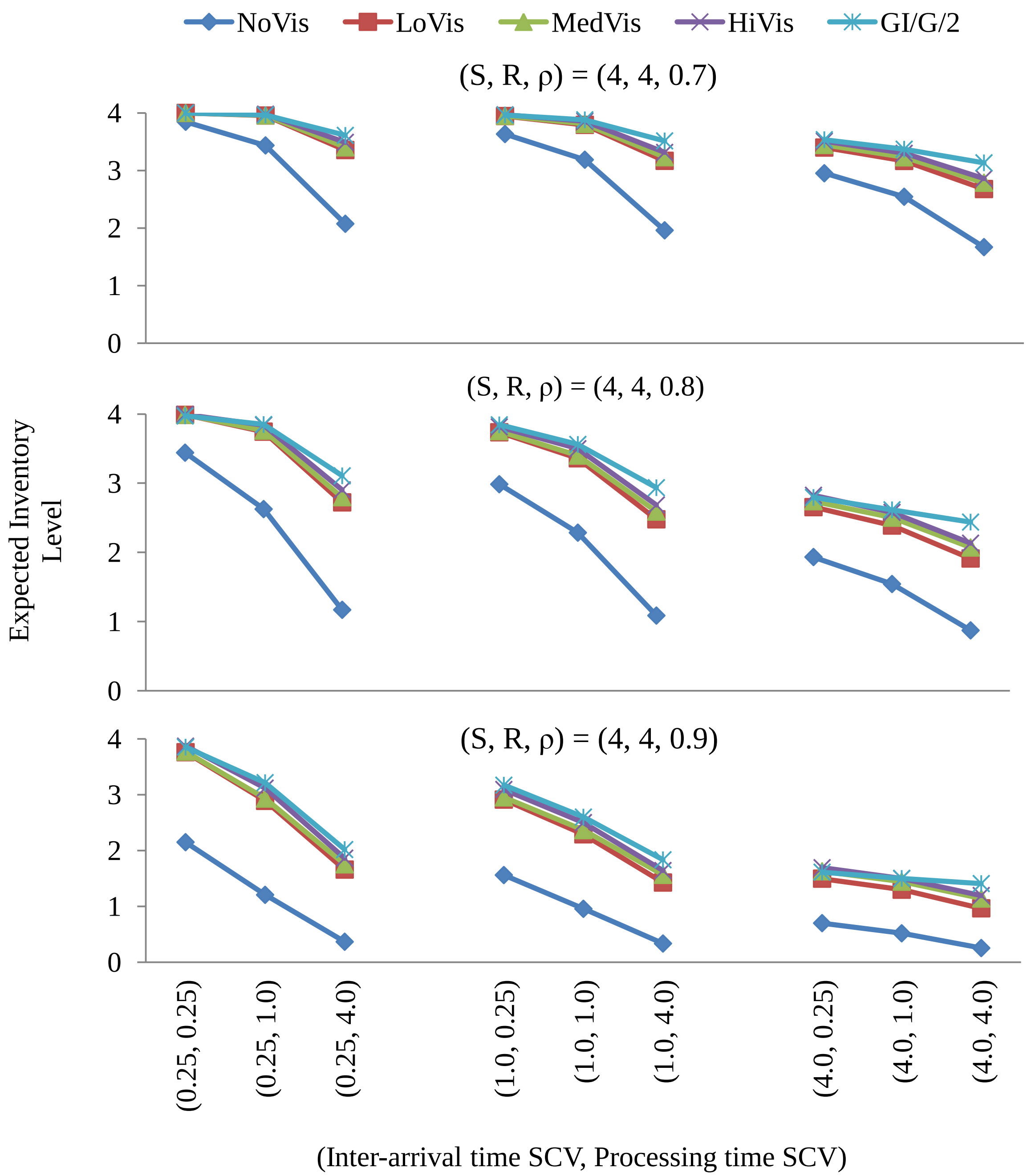



Figure 7.3: Expected Inventory Level at the Retail Store

increase is comparatively less than the increase observed with increasing processing time SCV, showing that the value for information sharing is more sensitive to processing time SCV and is somewhat insensitive to the inter-arrival time SCV. As in the case of Poisson arrivals and exponential processing times, the value of information

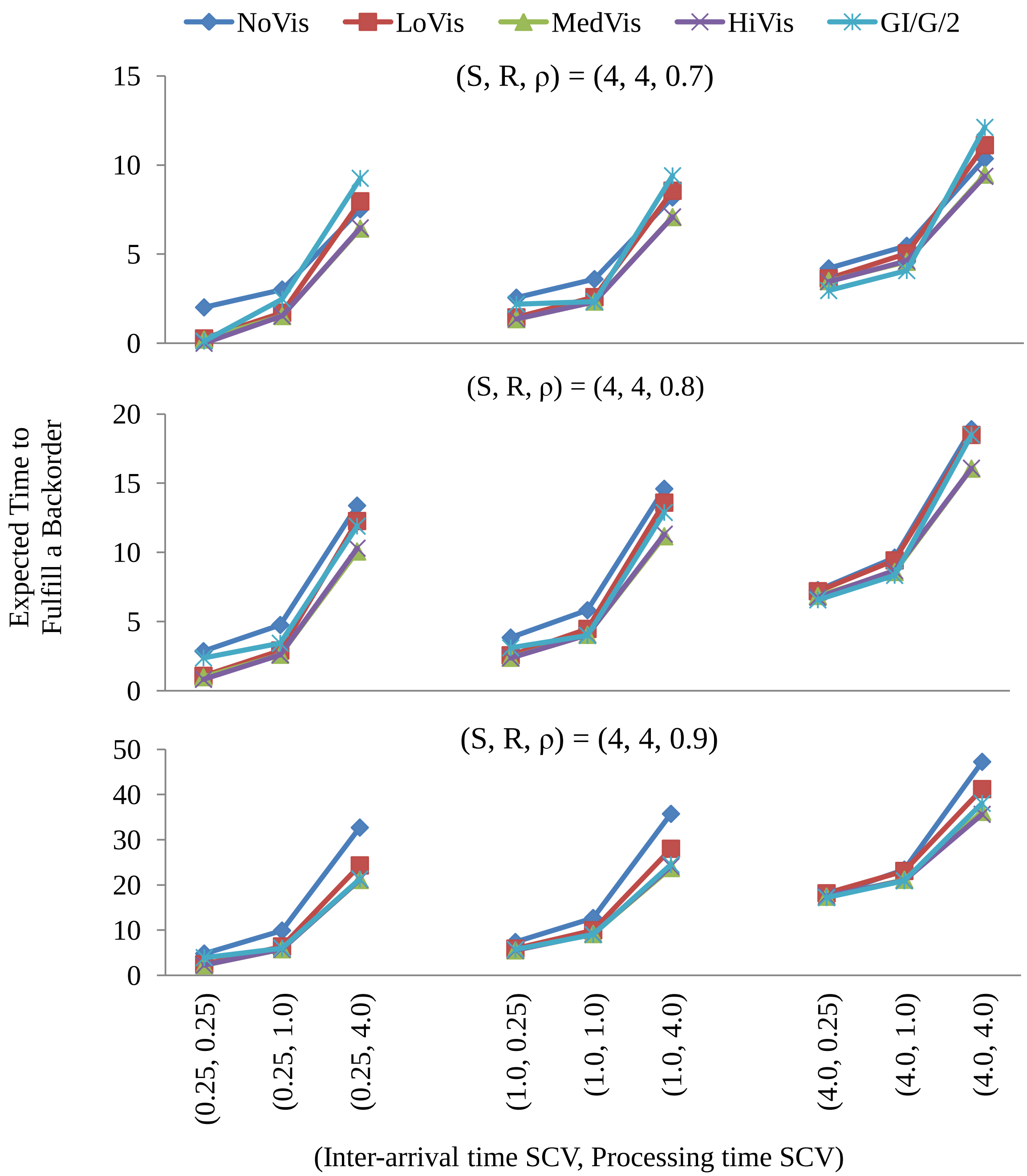



Figure 7.4: Expected Time to Fulfill a Backorder at the Retail Store

sharing increases with increase in the utilization levels (see Figures 7.1 through 7.10) and a decrease in the base-stock level (see Tables D.1 through D.5 in Appendix D).

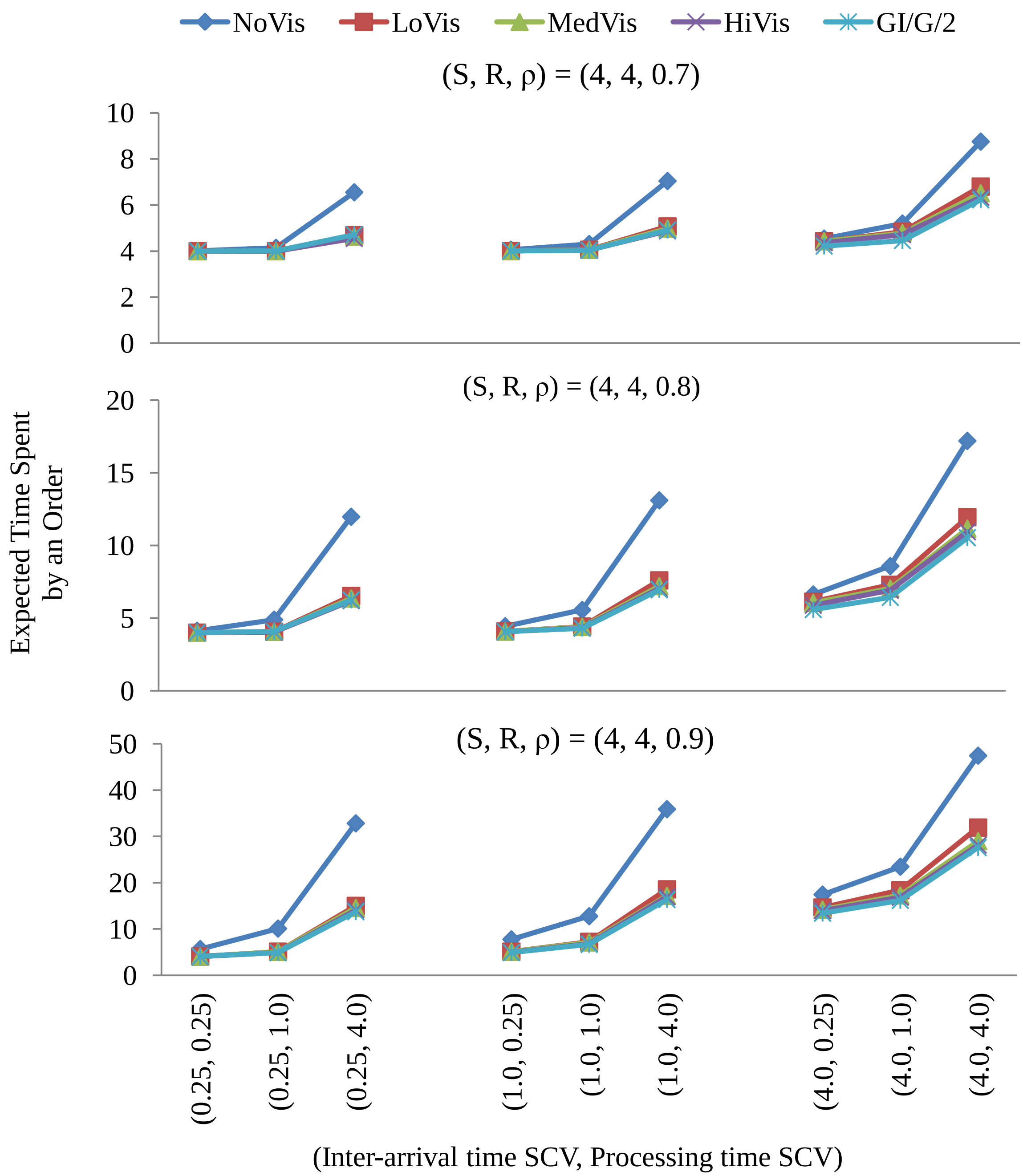



Figure 7.5: Expected Time Spent by an Order at the Retail Store

### 7.4 Conclusions

In this chapter, we have developed analytical models of the 1R/2P SCN with HiVis under general inter-arrival and processing time distributions using the approximations developed by Whitt [60]. We noted that the analytical model serves as a good

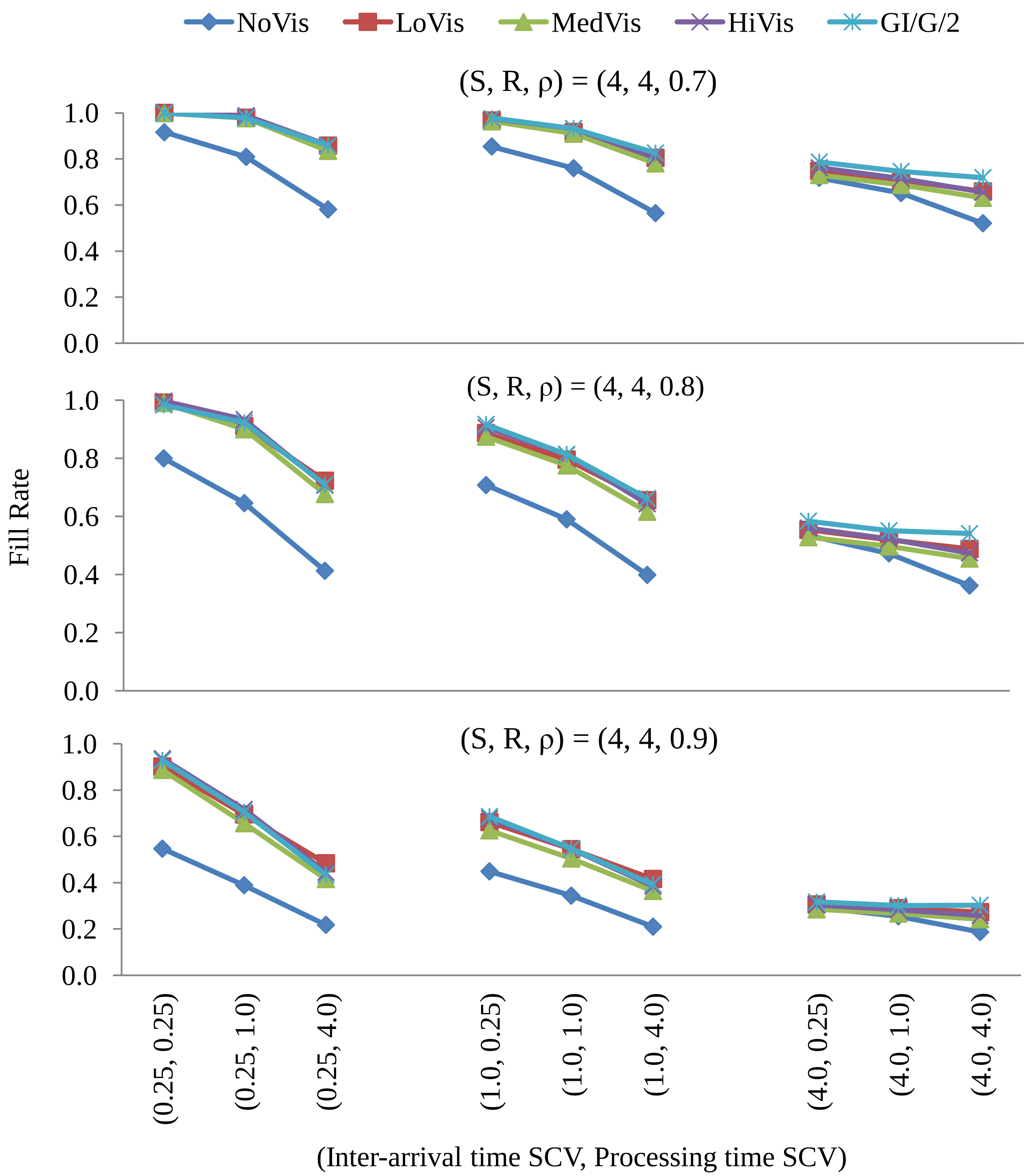



Figure 7.6: Fill Rate at a Production Facility

approximation for the SCN with HiVis with more than 90% of the results for fill rate, expected number of backorders, expected inventory level, and the expected time spent by an order falling within the acceptable 15% error range. We also studied the effect of inter-arrival time SCV and processing time SCV on the value of information shar-

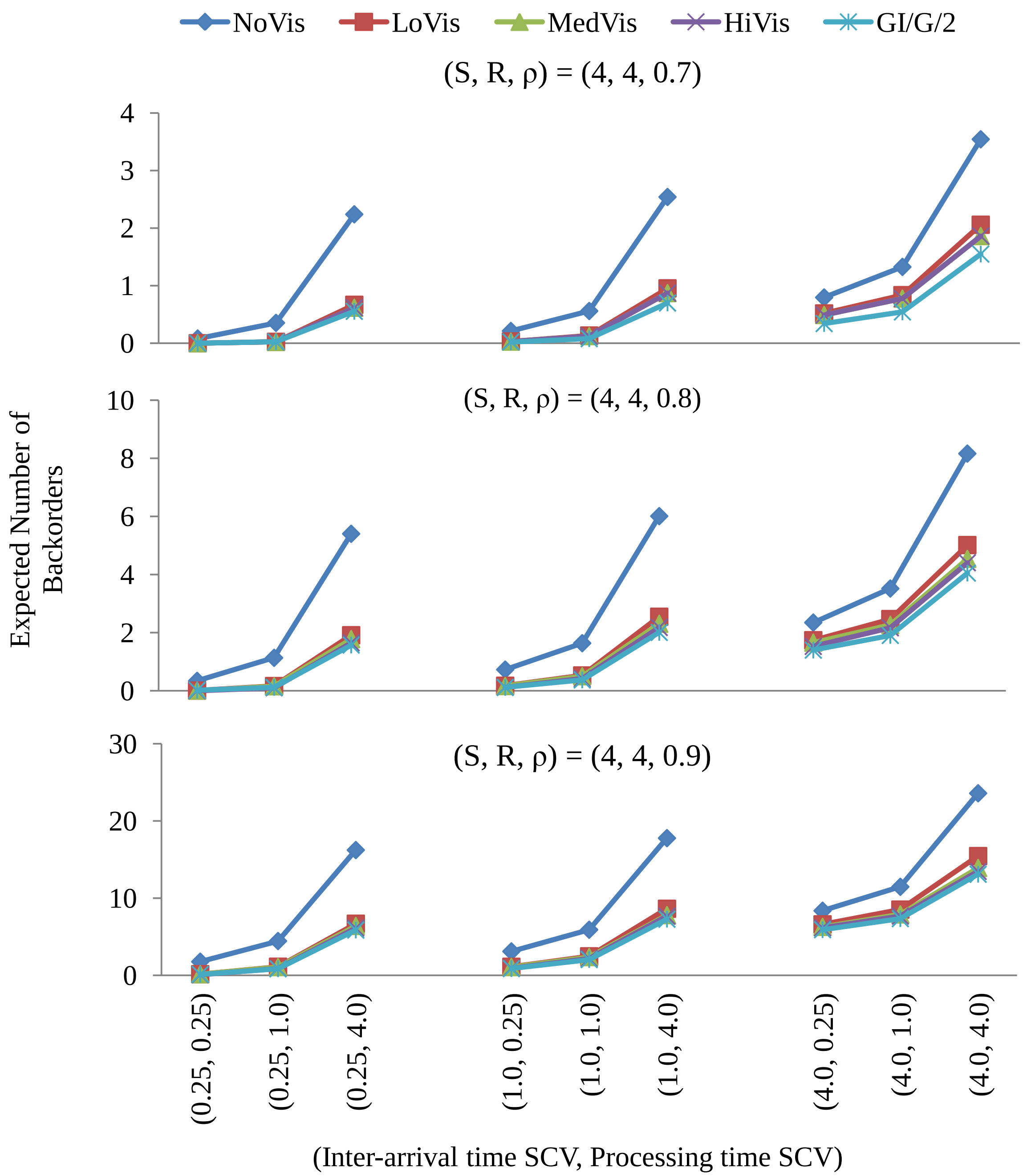



Figure 7.7: Expected Number of Backorders at a Production Facility

ing. Having explored the 1R/2P SCN configuration extensively, our focus in the next chapter will be on developing analytical models for the 2R/2P SCN configuration.

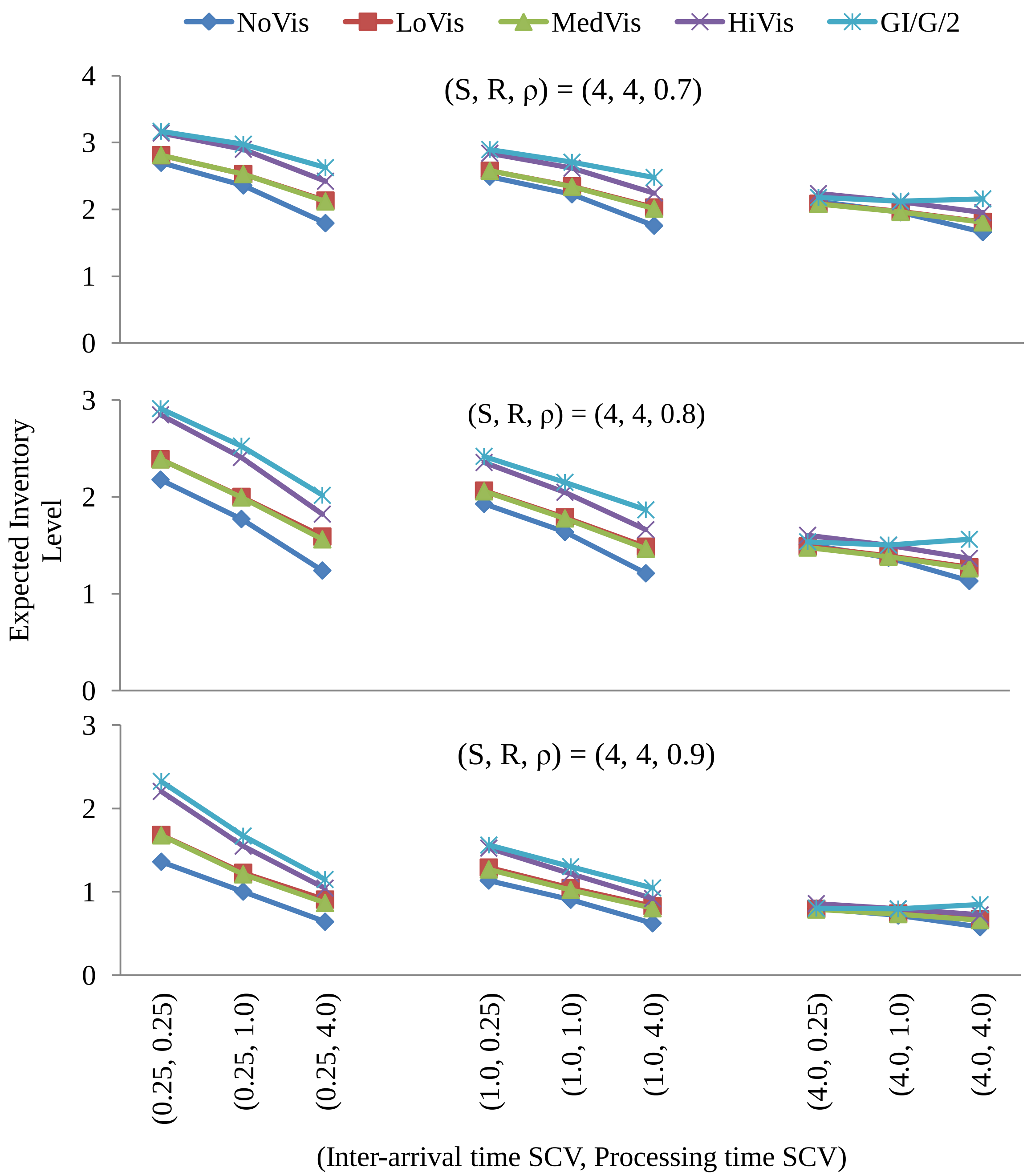



Figure 7.8: Expected Inventory Level at a Production Facility

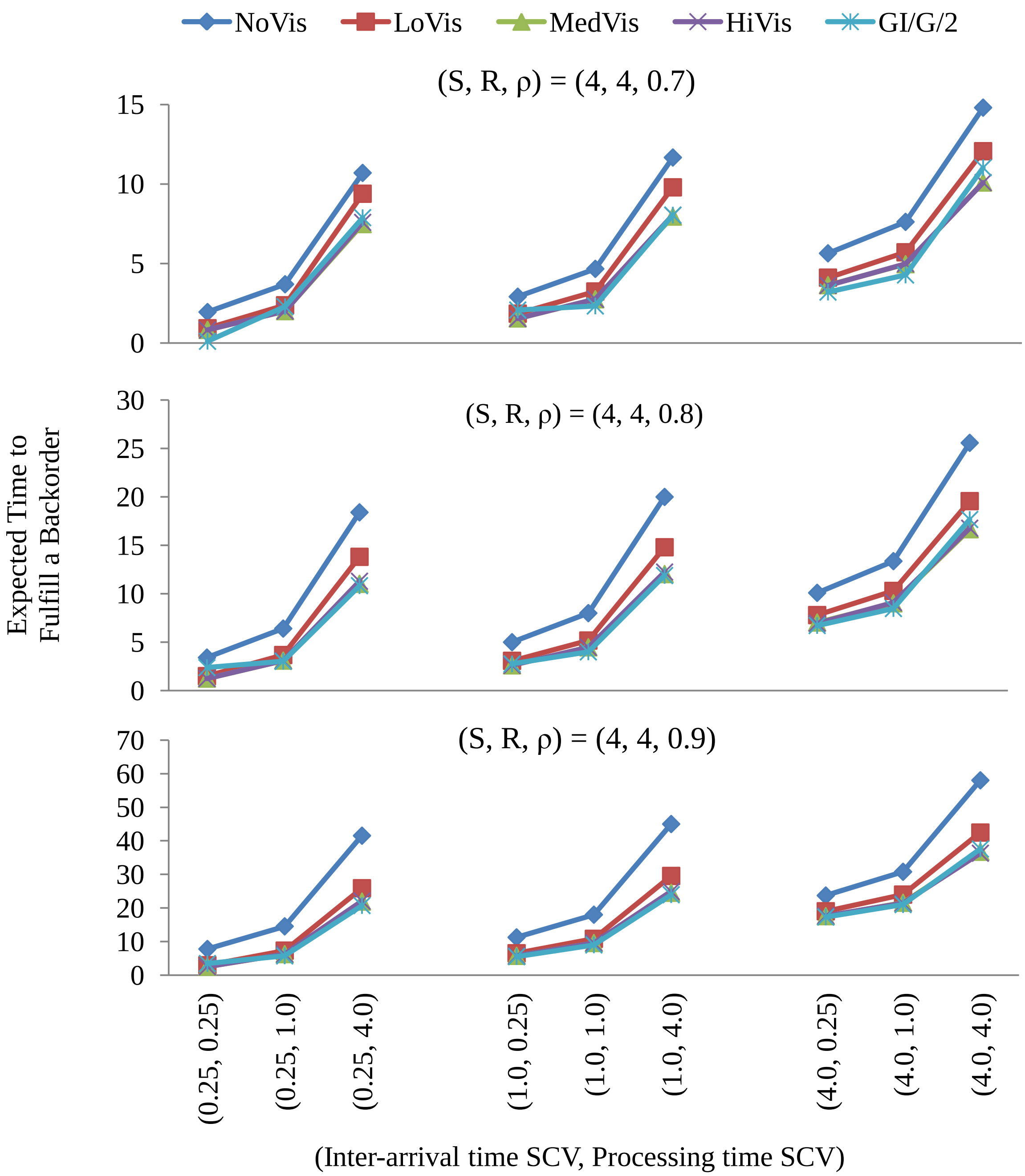



Figure 7.9: Expected Time to Fulfill a Backorder at a Production Facility

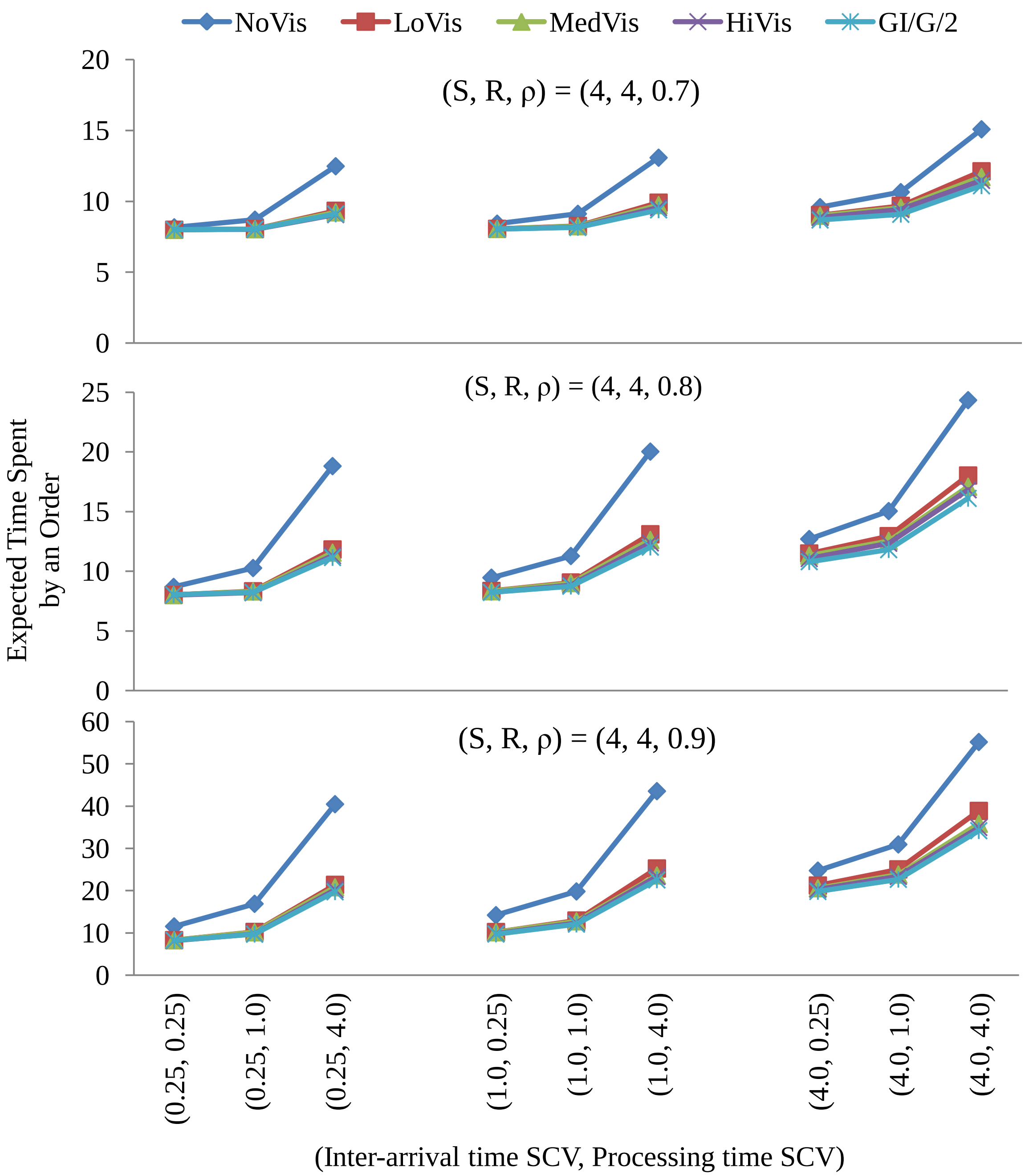



Figure 7.10: Expected Time Spent by an Order at a Production Facility

# CHAPTER 8

# ANALYTICAL MODELS OF THE 2R/2P SCN CONFIGURATION

This chapter presents extensions of the analytical models developed earlier (Chapters 6 and 7) to model a 2R/2P SCN configuration. First, the analytical models were developed for the SCN with Poisson arrivals and exponential processing times. The analytical model was then extended to accommodate general inter-arrival and processing time distributions. This chapter is organized as follows. Section 8.1 describes the 2R/2P SCN structure used in the study. Section 8.2 presents the queueing model that was developed for the SCN with HiVis, and Section 8.3 presents its modification for lower levels of information sharing (MedVis and LoVis) under Poisson arrivals and exponential processing times. We then present the analytical model for general inter-arrival and processing time distributions in Section 8.4. In Section 8.5, we present our conclusions for this chapter.

## 8.1 2R/2P SCN Structure

We consider a two-echelon SCN with two retail stores and two production facilities each with its own output store to stock finished products. Each store in the SCN is assumed to operate under a base-stock control policy with one-for-one replenishment. Both the production facilities are assumed to have the same base-stock level ($S$). The base-stock level at retail store $i$ ($i = 1, 2$) is $R_i$. Each customer order is assumed to be for a single unit of the finished product. The demand inter-arrival times at the retail stores and the processing times at the production facilities follow general distributions. The arrival of a demand consumes a finished product at the retail store, if available and causes an order for replenishment to be placed at one of the two production facilities (see Figure 8.1). If available, a finished product from the output store of the production facility is immediately shipped to the retail store and the output store sends an order for replenishment to its processing stage. In this chapter, we assume that the processing stage has a single server. An order can join the WIP queue or be processed as soon as it is received. There are no limits on the number of backorders at either production facility.

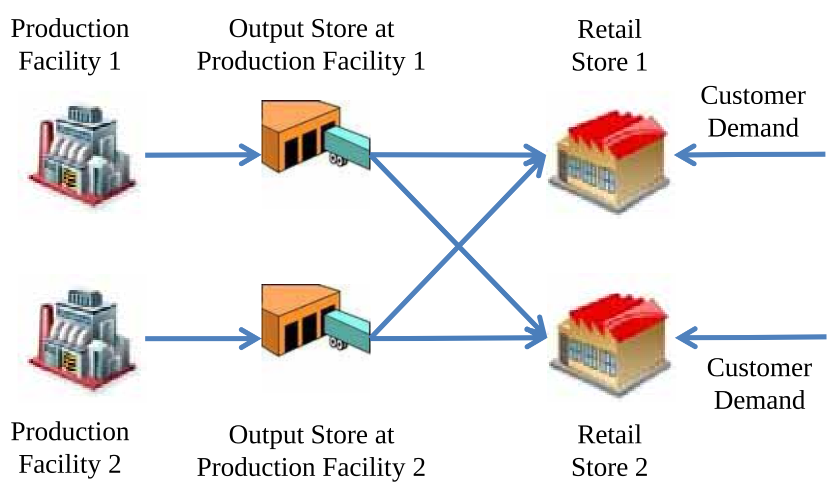



Figure 8.1: Supply Chain Instance

## 8.2 Analytical Model for Poisson Arrivals and Exponential Processing Times

In this section, we present a queueing based analytical model of the 2R/2P SCN configuration with HiVis under Poisson arrivals and exponential processing times. First, we explored the development of a CTMC model, in which, the state is given by $X(t) = \{i, j, k, l\}$, where $i$, $j$, $k$ and $l$ are non-negative integers representing the number of orders (replenishment plus backorders) at retail store 1, retail store 2, production facility 1 and production facility 2, respectively. Since there is no limit on the number of backorders, the state space of the CTMC is infinite. Furthermore, the order placement policies resulted in numerous conditions for defining the state transitions and the balance equations. Only an approximate solution was possible by numerically solving the CTMC with arbitrary limits on backorders. As a result, we focused on developing an approximate queueing-based model of the SCN with HiVis.

### 8.2.1 Queueing Model of the 2R/2P SCN Configuration with HiVis

The routing policy used under HiVis is equivalent to choosing the shortest queue when the two production facilities have identical base-stock levels. Due to the absence of closed form solutions for the shortest queue problem (see Section 6.1 for more details), we used the multi-server queue to model the production facilities as a pooled facility.

Figure 8.2 shows the exact representation of our SCN, while Figure 8.3 shows the aggregate representation based on the M/M/2 approximation. In essence, the two production facilities were modeled as an M/M/2 queueing system, where the net arrival process is the superposition of the two arrival streams from the retail stores (i.e., $\lambda = \lambda_1^r + \lambda_2^r$) and the combined arrival process is also Poisson. Figure 8.3 shows that in an M/M/2 approximation, the inventory at the two production facilities would be "pooled" and this leads to some loss of information about inventory and orders at the individual production facilities.

When an order is placed at a retail store, an instantaneous replenishment order is placed at the pooled output store. Since transit time is not modeled, if a finished part is available at the output store of the pooled facility, the part is instantaneously available at the retail store. As a result, an order (replenishment or backorder) at a retail store can be reflected only as a backorder at the pooled facility. It has to be noted that an order at the pooled facility is an aggregate order; the identity of the retail store that placed the order is lost while merging the two arrival streams from the retail stores. Hence, a "disaggregation procedure" was used to find the distribution of the number of orders pertaining to retail store $i$, $N_i^r$, at the pooled facility by

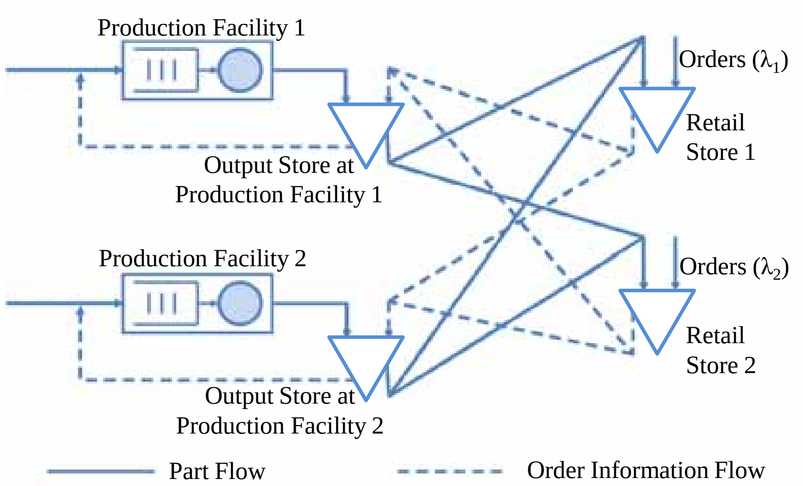



Figure 8.2: Exact Representation of the 2R/2P SCN Structure

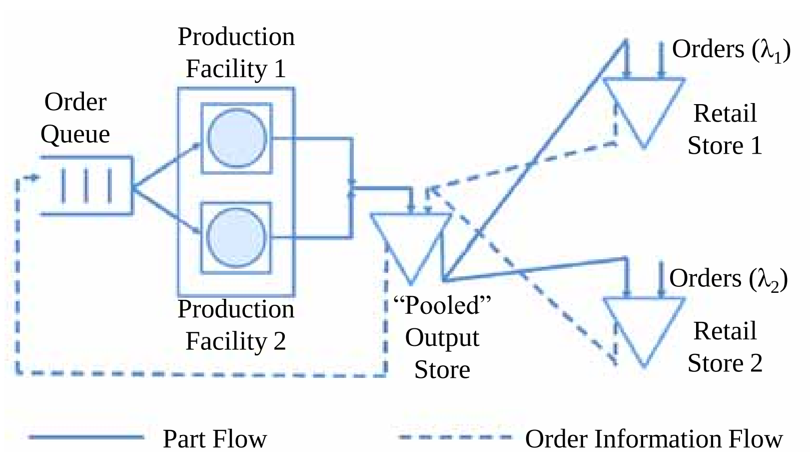



Figure 8.3: Aggregate Representation of the 2R/2P SCN Structure

conditioning on the number of backorders, $B^p$, at the pooled facility. This conditional distribution is Binomial as shown in Equations (8.1) and (8.2). The parameters of the binomial distributions are the proportions of orders from retail store $i$, $\alpha_i$, that are placed at the pooled production facility along with $l$, the total number of backorders at the pooled facility. To derive the unconditional probability that a backorder at the pooled facility belongs to retail store $i$, the conditional probabilities obtained in Equations (8.1) and (8.2) are multiplied by the backorder probability at the pooled facility and then summed over all values of $l$ (backorders) greater than or equal to $m$ as shown in Equations (8.5) and (8.6). The backorder distribution at the pooled facility can then be obtained from the distribution of the number of orders at pooled production facility $N^p$ as shown in Equation (8.4). The distribution of $N^p$ is shown later in Equation (8.12). Note that the expressions in Equations (8.5) and (8.6) are applicable only for a two-production facility SCN.

$$P\left(N_1^r = m | B^p = l\right) = \binom{l}{m} \alpha_1^m \alpha_2^{(l-m)} \quad ; \; m = 0, 1, \cdots, l \tag{8.1}$$

$$P\left(N_2^r = m | B^p = l\right) = P\left(N_1^r = l - m | B^p = l\right) \tag{8.2}$$

$$\alpha_1 = \lambda_1^r / \sum_{k=1}^{2} \lambda_k^r, \qquad \alpha_2 = 1 - \alpha_1 \tag{8.3}$$

$$P\left(B^p = l\right) = \begin{cases} \sum_{i=0}^{2S} P\left(N^p = i\right) & ; \quad l = 0 \\ P\left(N^p = 2S + l\right) & ; \quad l \geq 1 \end{cases} \tag{8.4}$$

$$P\left(N_1^r=m\right)=\sum_{l=m}^{\infty}P\left(N_1^r=m|B^p=l\right)\cdot P\left(B^p=l\right)$$
$$=\begin{cases}\left[1+2\rho\left(\frac{1-\rho^{2S}}{1-\rho}\right)\right]+\left[\frac{2\alpha_2\rho^{2S+1}}{1-\alpha_2\rho}\right]\left(\frac{1-\rho}{1+\rho}\right) & ;\ m=0\\ \left[\frac{2\alpha_1^m\rho^{2S+m}}{[1-\alpha_2\rho]^{m+1}}\right]\left(\frac{1-\rho}{1+\rho}\right) & ;\ m\geq 1\end{cases}\tag{8.5}$$

$$P\left(N_2^r=m\right)=\sum_{l=m}^{\infty}P\left(N_2^r=m|B^p=l\right)\cdot P\left(B^p=l\right)$$
$$=\begin{cases}\left[1+2\rho\left(\frac{1-\rho^{2S}}{1-\rho}\right)\right]+\left[\frac{2\alpha_1\rho^{2S+1}}{1-\alpha_1\rho}\right]\left(\frac{1-\rho}{1+\rho}\right) & ;\ m=0\\ \left[\frac{2\alpha_2^m\rho^{2S+m}}{[1-\alpha_1\rho]^{m+1}}\right]\left(\frac{1-\rho}{1+\rho}\right) & ;\ m\geq 1\end{cases}\tag{8.6}$$

As before, the retail store performance measures that are of interest include the fill rate $f_i^r$, expected number of backorders $E[B_i^r]$, expected inventory level $E[I_i^r]$, expected time to fulfill a backorder $E[W_i^{br}]$, and the expected time spent by an order $E[T_i^r]$. In our calculations, we used the ready rate (probability that an item is available) to approximate the fill rate at all stages. As noted earlier, this approximation would be exact only when the arrival process is Poisson (PASTA principle [62]). The performance measures at retail store $i$, $i=1,2$ are presented in Equations (8.7) through (8.11).

$$f_i^r=\sum_{j=0}^{R_i-1}P\left(N_i^r=j\right)\tag{8.7}$$

$$E[I_i^r]=\sum_{j=0}^{R_i-1}\left(R_i-j\right)P\left(N_i^r=j\right)\tag{8.8}$$

$$E[B_i^r]=E[N_i^r]+E[I_i^r]-R\tag{8.9}$$

where, $E[N_i^r] = \sum_{i=0}^{\infty} iP\left(N_i^r = j\right)$

$$E[W_i^{br}] = \frac{E[B_i^r]}{(1 - f_i^r)\lambda_i^r} \tag{8.10}$$

$$E[T_i^r] = \frac{R_i + E[B_i^r]}{\lambda_i^r} \tag{8.11}$$

The pooled facility is modeled as an M/M/2 queueing system and the distribution of the number of orders at the pooled facility, $N^p$, is given by Equation (8.12). The symmetry among the production facilities can be used in the calculation of the fill rate $f^p$, expected number of backorders $E[B^p]$, the expected inventory level $E[I^p]$, the expected time to fulfill a backorder $E[W^{bp}]$, and the expected time spent by an order $E[T^p]$ at the individual production facilities. In the case of expected number of backorders and expected inventory level, the performance measures for the pooled store is calculated first and then it is multiplied by 0.5 to obtain the performance measures at the individual stores. The performance measures at a production facility can be obtained from Equations (8.13) through (8.17).

$$P(N^p = i) = \begin{cases} \frac{(2\rho)^i}{i!} P(N^p = 0) & ; \quad i = 0, 1 \\ 2\rho^i P(N^p = 0) & ; \quad i \geq 2 \end{cases} \tag{8.12}$$

$$\rho = \frac{\lambda}{2\mu}, \qquad P(N^p = 0) = \frac{1 - \rho}{1 + \rho}$$

$$\begin{aligned} f^p &= \sum_{i=0}^{2S-1} P(N^p = i) \\ &= \frac{1 + \rho - 2\rho^{2S}}{1 + \rho} \end{aligned} \tag{8.13}$$

$$
\begin{aligned}
E[I^p] &= 0.5 \sum_{i=0}^{2S-1} (2S-i) P(N^p = i) \\
&= \frac{S\ (1-\rho)^2 + \rho(2S + \rho^{2S} - 2S\rho - 1)}{1-\rho^2}
\end{aligned} \tag{8.14}
$$

$$
\begin{aligned}
E[B^p] &= 0.5 \sum_{i=2S+1}^{\infty} (i-2S) P(N^p = i) \\
&= \frac{\rho^{2S+1}}{1-\rho^2}
\end{aligned} \tag{8.15}
$$

$$
\begin{aligned}
E[W^{bp}] &= \frac{E[B^p]}{\lambda/2(1-f^p)} \\
&= \frac{\rho}{2\lambda\,(1-\rho)}
\end{aligned} \tag{8.16}
$$

$$
E[T^p] = \frac{S + E[B^p]}{\lambda/2} \tag{8.17}
$$

### 8.2.2 Validation of the M/M/2 Approximation

In this section, the M/M/2 approximation was validated by comparing the analytical results with the simulation estimates for the 2R/2P SCN under HiVis case. The SCN parameter values used in the numerical experimentation are presented in Table 8.1. The design consists of 8 experiments.

Table 8.1: Experiments for the 2R/2P SCN Configuration under Poisson Arrivals and Exponential Processing Times

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rates ($\lambda_1^r$, $\lambda_2^r$) | 1 | (2, 1) |
| Base-stock levels ($S$, $R_1$, $R_2$) | 4 | (2, 2, 4), (2, 4, 2), (4, 4, 6), (4, 6, 4) |
| Utilization level $\rho$ | 2 | 80%, 90% |

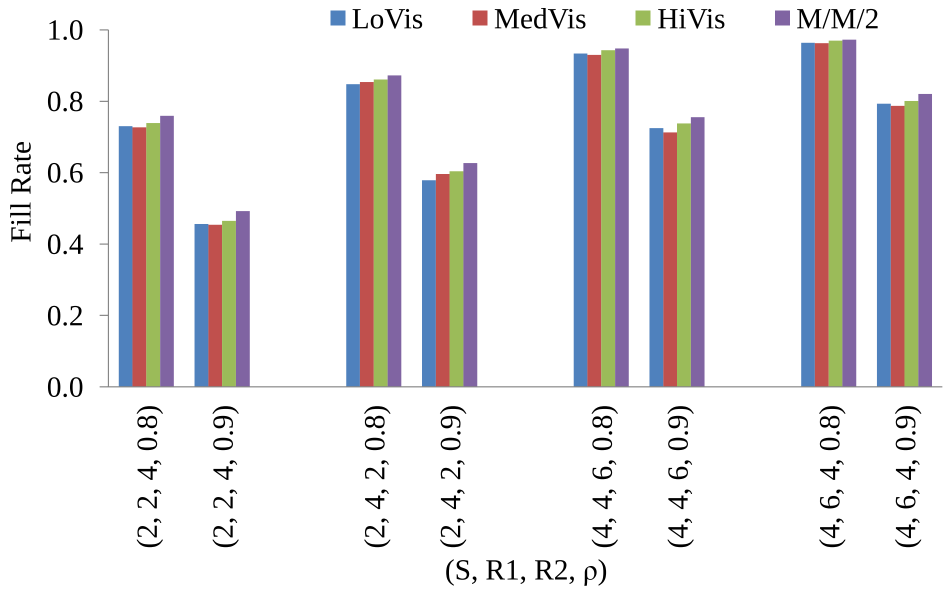



Figure 8.4: Fill Rate at Retail Store 1

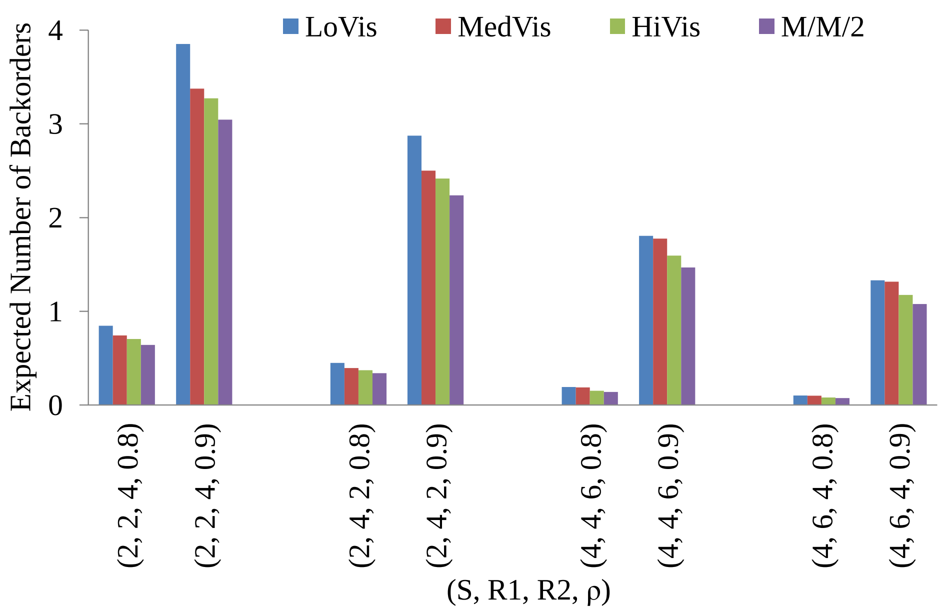



Figure 8.5: Expected Number of Backorders at Retail Store 1

The results of the numerical experiments are tabulated in Appendix E (see Section E.1). The plots presented in Figures 8.4 through 8.13 show that the analytical values are very close to the simulation estimates for the HiVis case. The maximum prediction error is within 15% for majority of the cases. This shows that the an-

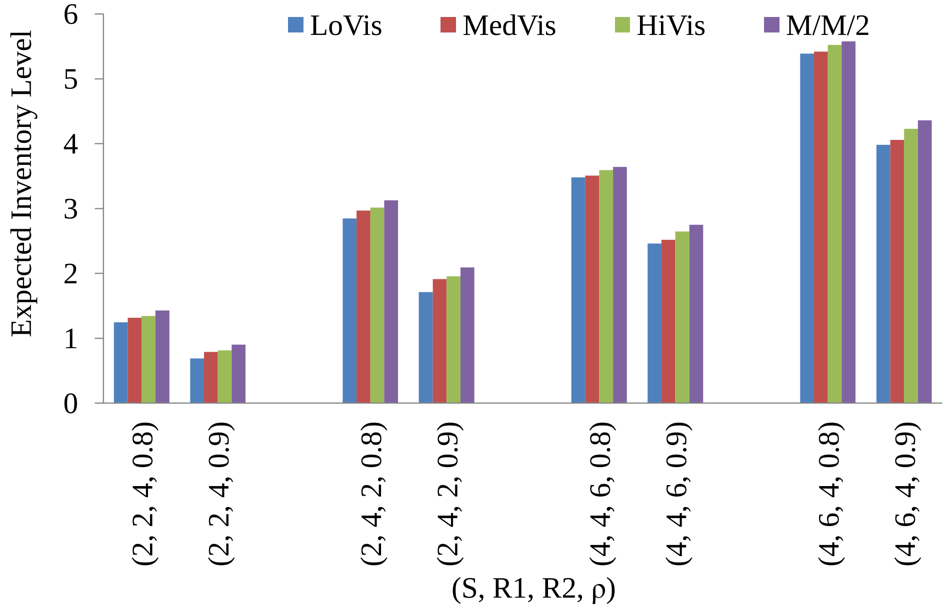



Figure 8.6: Expected Inventory Level at Retail Store 1

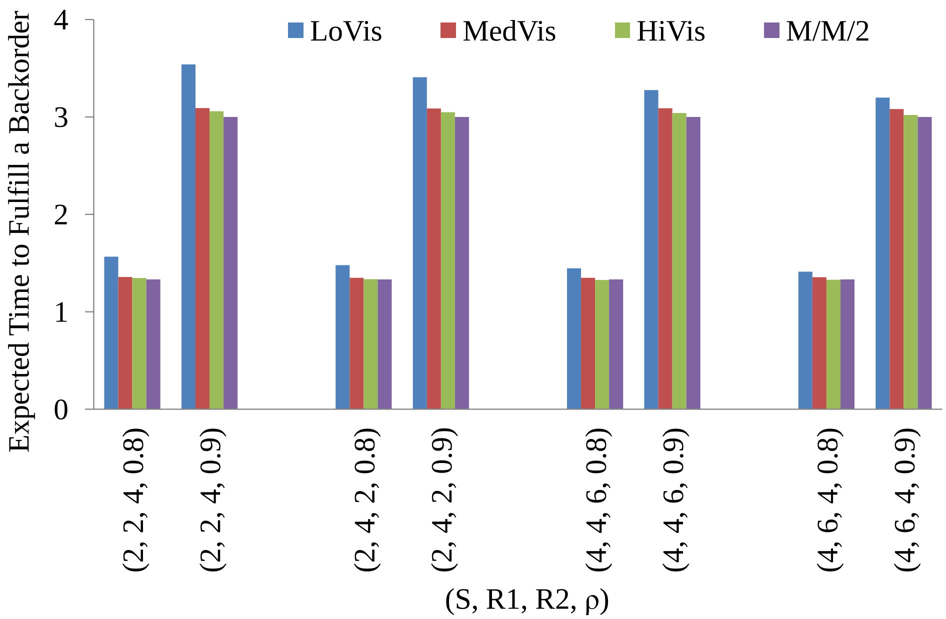



Figure 8.7: Expected Time to Fulfill a Backorder at Retail Store 1

alytical model serves as a good approximation for the HiVis case. It can also be seen from the tables that the analytical model seems to present reasonable bounds for the MedVis and LoVis cases in most of the experiments (in a manner similar to the 1R/2P SCN). The approximation yields upper bounds for fill rate and expected

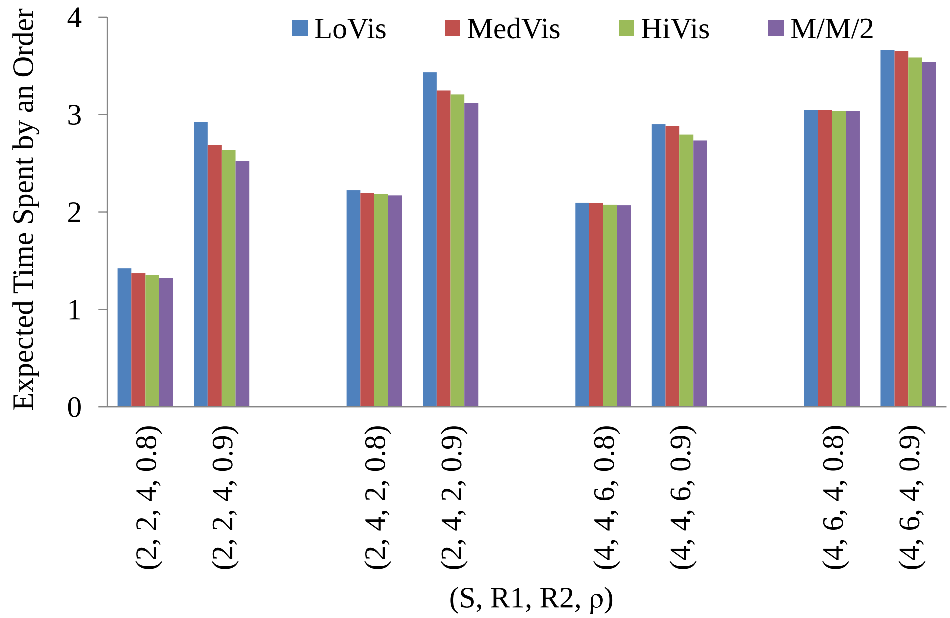


Figure 8.8: Expected Time Spent by an Order at Retail Store 1

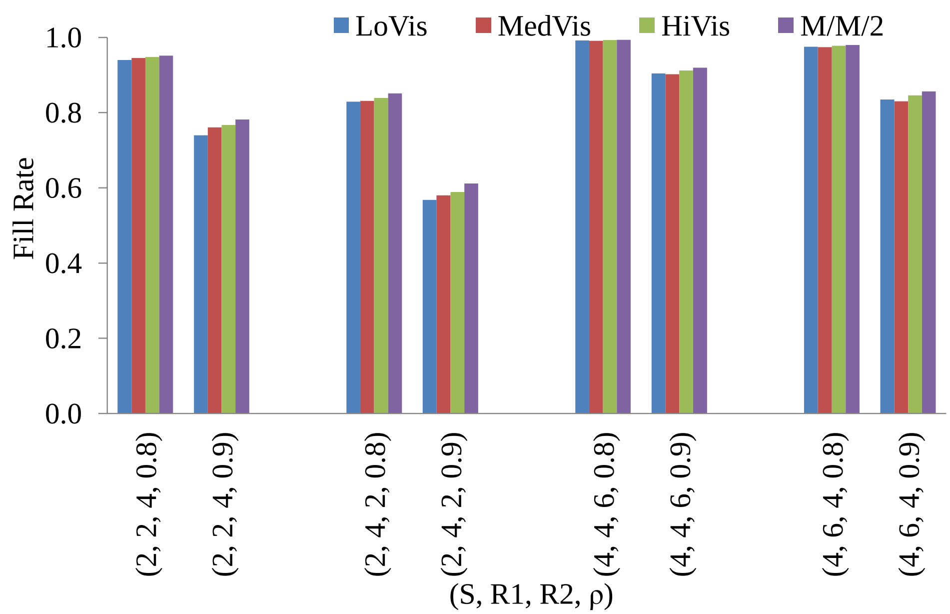


Figure 8.9: Fill Rate at Retail Store 2

inventory level, and lower bounds for expected number of backorders and expected time to fulfill a backorder at the retail stores. These observations also hold at the production facilities except in the case of fill rate.

It can be seen from Figures 8.4 and 8.9 that even though the probabilities are

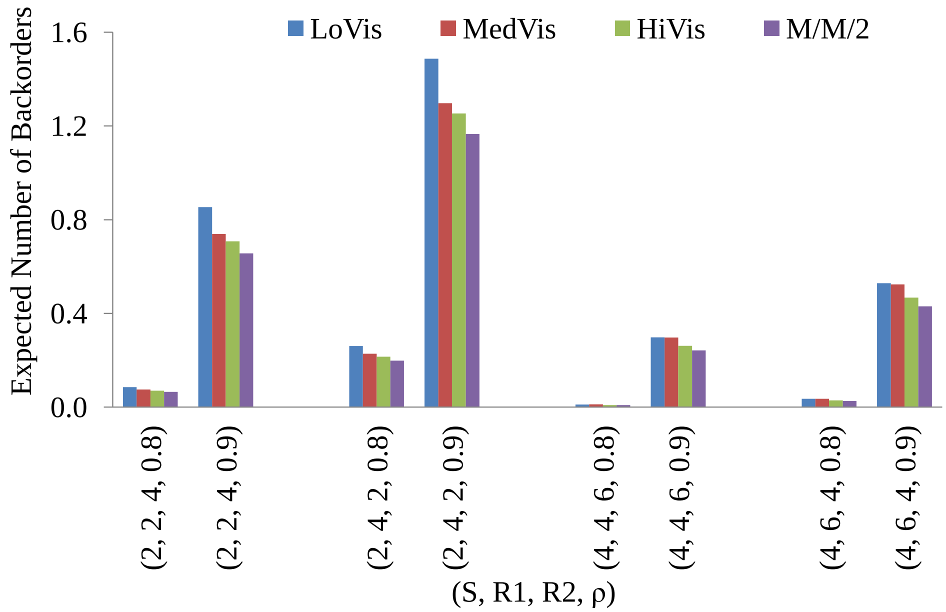



Figure 8.10: Expected Number of Backorders at Retail Store 2

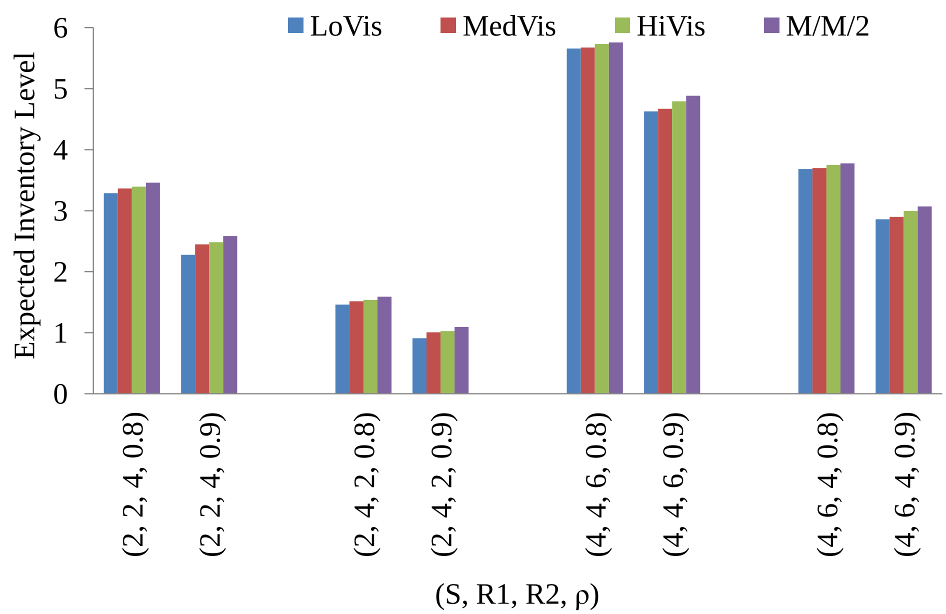



Figure 8.11: Expected Inventory Level at Retail Store 2

distributed differently based on the routing policies, the fill rates for the three cases (LoVis, MedVis and HiVis) are almost the same. Further, the fill rates for the LoVis and MedVis cases are very close due to the same order routing policy in these cases when both stores have inventory. It can also be seen from Figures 8.5 and 8.10 that

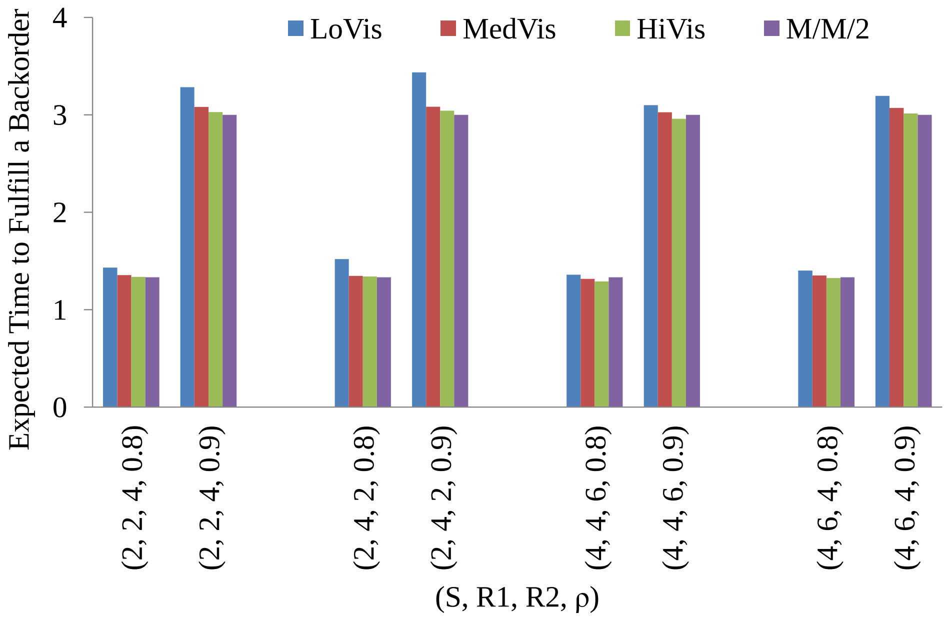


Figure 8.12: Expected Time to Fulfill a Backorder at Retail Store 2

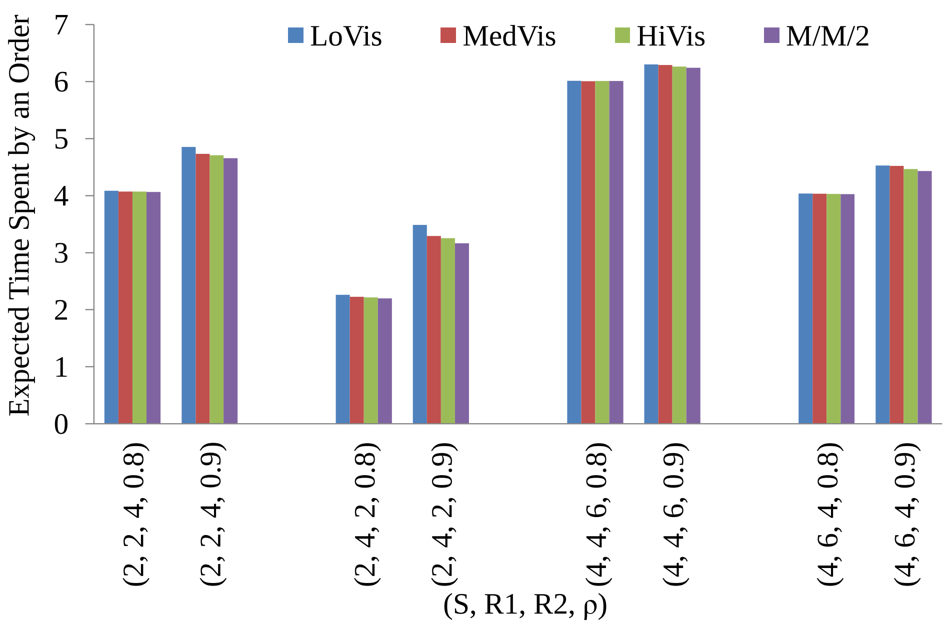


Figure 8.13: Expected Time Spent by an Order at Retail Store 2

the values of the expected time to fulfill a backorder are very close for the MedVis and HiVis due to the identical routing policies in these cases when both stores are backordered.

The M/M/2-based model thus serves as a good approximation for the SCN with

HiVis case and yields reasonable bounds for the SCN with LoVis and MedVis cases (as in the case of 1R/2P SCN configuration). We then used the modified M/M/2 model developed in Section 6.1.2 to obtain better approximations for the SCN with LoVis and MedVis cases.

## 8.3 Queueing Model of the 2R/2P SCN Configuration with Lower Levels of Information Sharing (MedVis and LoVis)

In this section, we present analytical models that can better model the 2R/2P SCN with lower levels of information sharing under Poisson arrivals and exponential processing time. The two production facilities were modeled using the modified M/M/2 queueing system developed in Section 6.1.2. The demand arrival rate at the pooled production facility is obtained as the sum of the demand arrival rates at the individual retail stores and follows a Poisson distribution as shown in Section 8.2.1. The distribution of the number of orders at the pooled production facility is obtained using equation (6.17). The performance measures for a production facility can be obtained using equations (8.18) through (8.22). The distribution of the number of orders at the retail store can be obtained using equation (8.23). The performance measures at retail store $i$ can be obtained using Equations (8.7) through (8.11).

$$\begin{aligned} f^p &= \sum_{i=0}^{2S-1} P(N^p = i) \\ &= \left[(1+2\rho)(1-\rho) + 2\rho^2\zeta(\rho, S)\left(1-\rho^{2S-2}\right)\right] P(N^p = 0) \end{aligned} \tag{8.18}$$

$$E[I^p] = 0.5 \sum_{i=0}^{2S-1} (2S-i) P(N^p = i) \tag{8.19}$$

$$
\begin{aligned}
E[B^p] &= 0.5 \sum_{i=0}^{2S-1} P(N^p = i) \\
&= \frac{\rho^{2S+1}\zeta(\rho, S)}{(1-\rho)^2} P(N^p = 0)
\end{aligned}
\tag{8.20}
$$

$$
\begin{aligned}
E[W^{bp}] &= \frac{E[B^p]}{(1 - f^p)\,\lambda/2} \\
&= \frac{\rho}{(1-\rho)\,\lambda}
\end{aligned}
\tag{8.21}
$$

$$
\begin{aligned}
E[T^p] &= \frac{S + E[B^p]}{\lambda/2} \\
&= \frac{\left(S + \frac{\rho^{2S+1}\zeta(\rho,S)}{(1-\rho)^2} P(N^p = 0)\right)}{\lambda/2}
\end{aligned}
\tag{8.22}
$$

$$
P\left(N_i^r = m\right) = \sum_{l=m}^{\infty} P\left(N_i^r = m | B^p = l\right) \cdot P\left(B^p = l\right) \tag{8.23}
$$

where $P\left(N_i^r = m | B^p = l\right)$ and $P\left(B^p = l\right)$ can be obtained using Equations (8.1) through (8.4).

The analytical results for the modified M/M/2-based approximation along with the results for the LoVis and MedVis cases are presented in Appendix E (see Section E.2). Figures 8.14 through 8.23 present the plots for the performance measures at retail stores 1 and 2. The performance measures for the M/M/2 model are included in the tables to show that the modification yields better approximations.

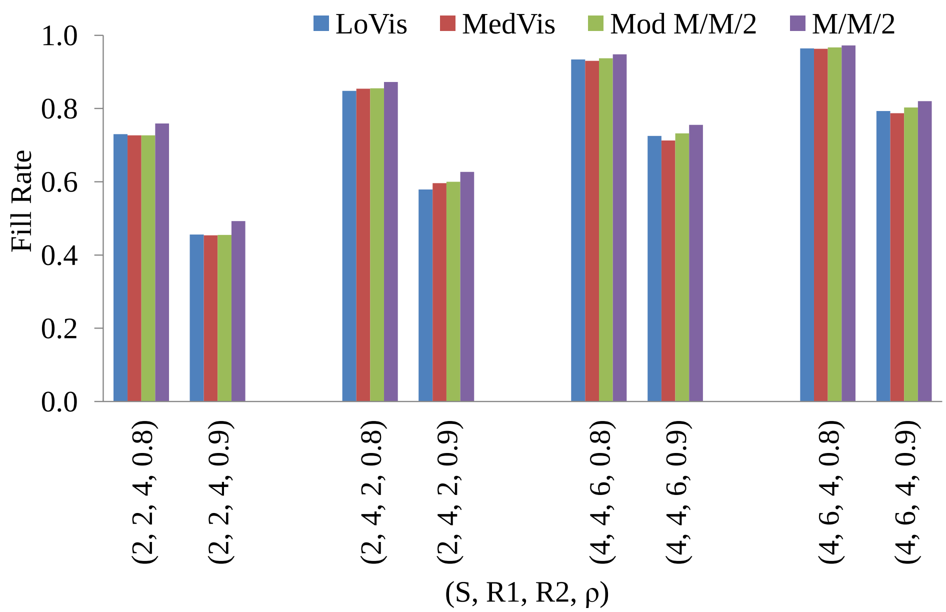



Figure 8.14: Fill Rate at Retail Store 1

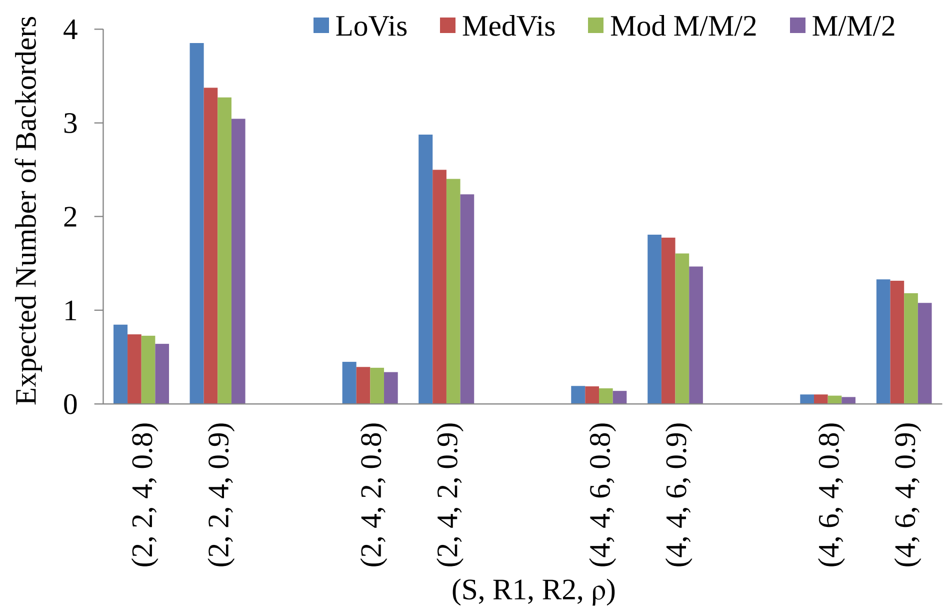



Figure 8.15: Expected Number of Backorders at Retail Store 1

Figures 8.14 through 8.23 show that the M/M/2 model is a good approximation for the LoVis and MedVis cases. But the modified M/M/2 model performs even better in modeling the LoVis and MedVis cases for all performance measures (except expected time to fulfill a backorder) at the retail store and two production facilities. In the case

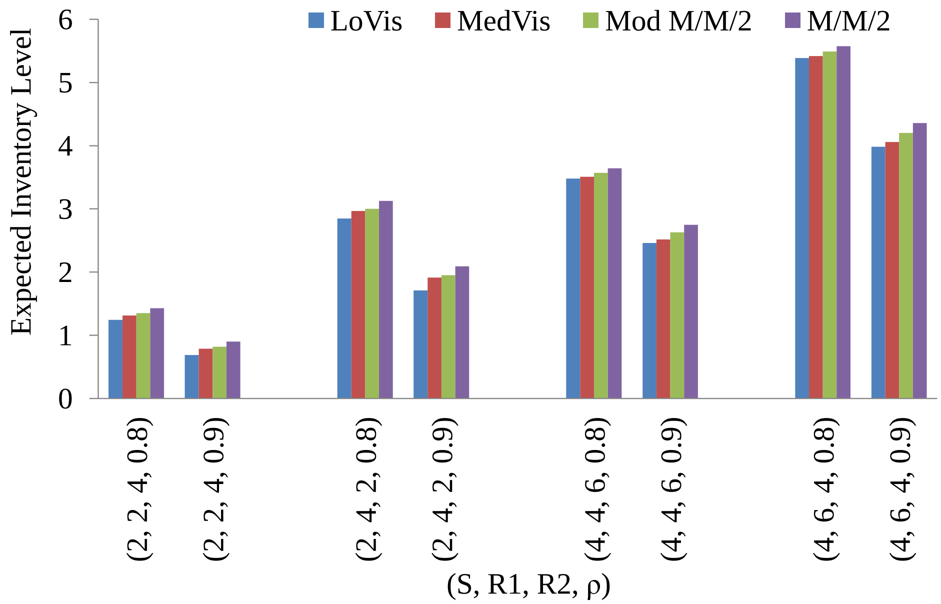



Figure 8.16: Expected Inventory Level at Retail Store 1

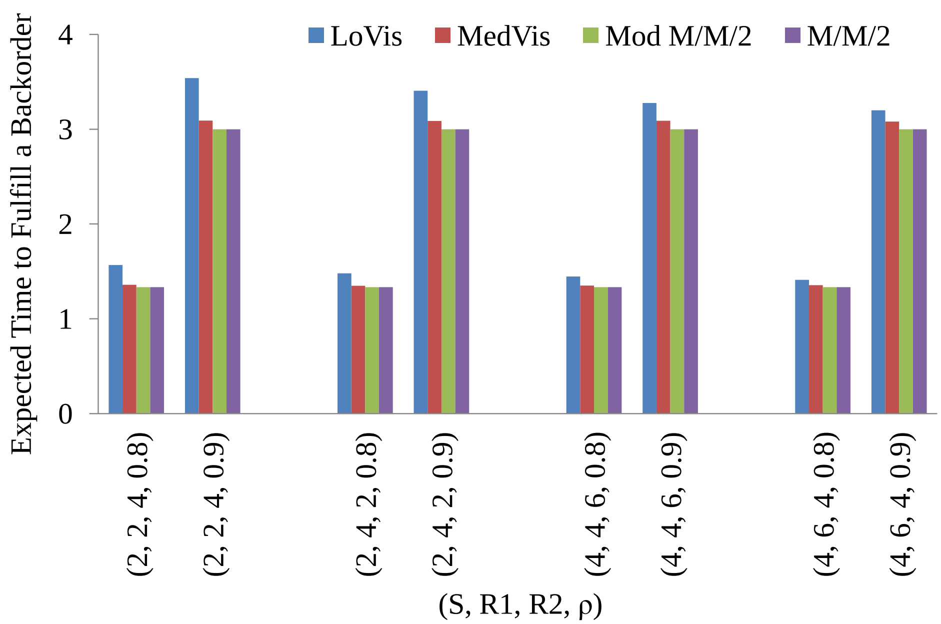



Figure 8.17: Expected Time to Fulfill a Backorder at Retail Store 1

of expected time to fulfill a backorder, the performance measures obtained from the modified M/M/2 model and the M/M/2 model remain the same (see Section 6.1.2 for details). Since the modified M/M/2 model performs better than the M/M/2-based model for the SCN with LoVis and MedVis cases, we note that there is an opportunity

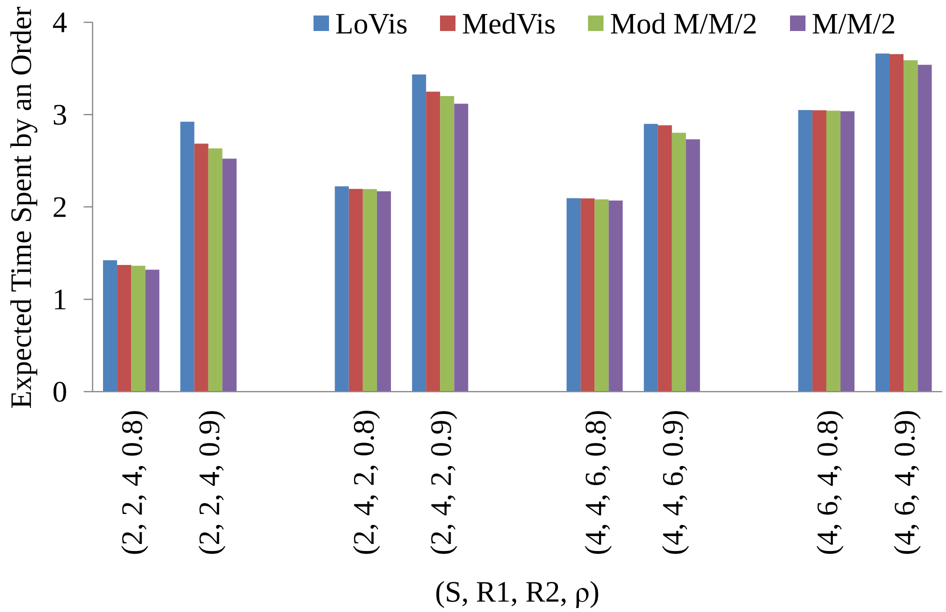



Figure 8.18: Expected Time Spent by an Order at Retail Store 1

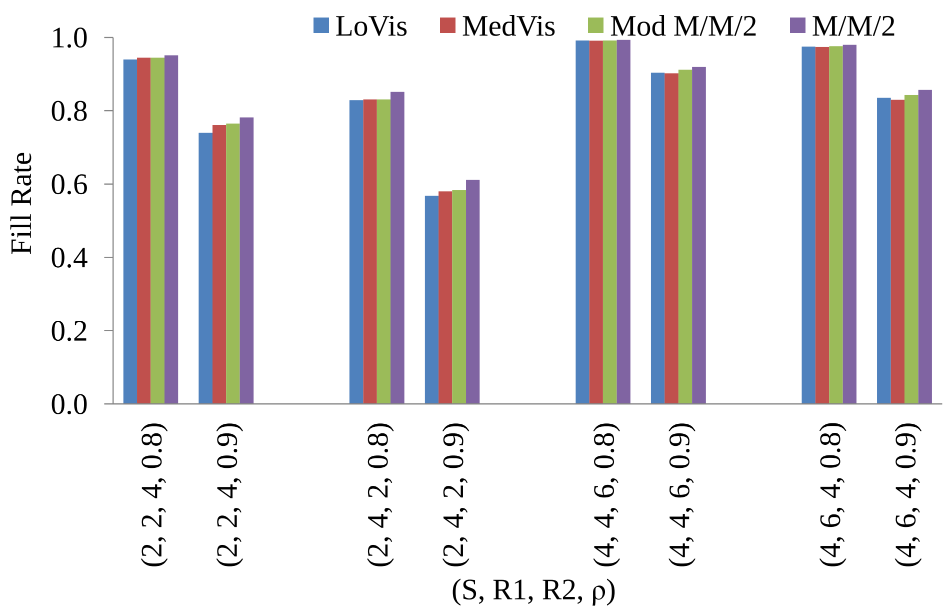



Figure 8.19: Fill Rate at Retail Store 2

to explore additional correction factors to model the individual levels of information sharing based on the specific routing policies.

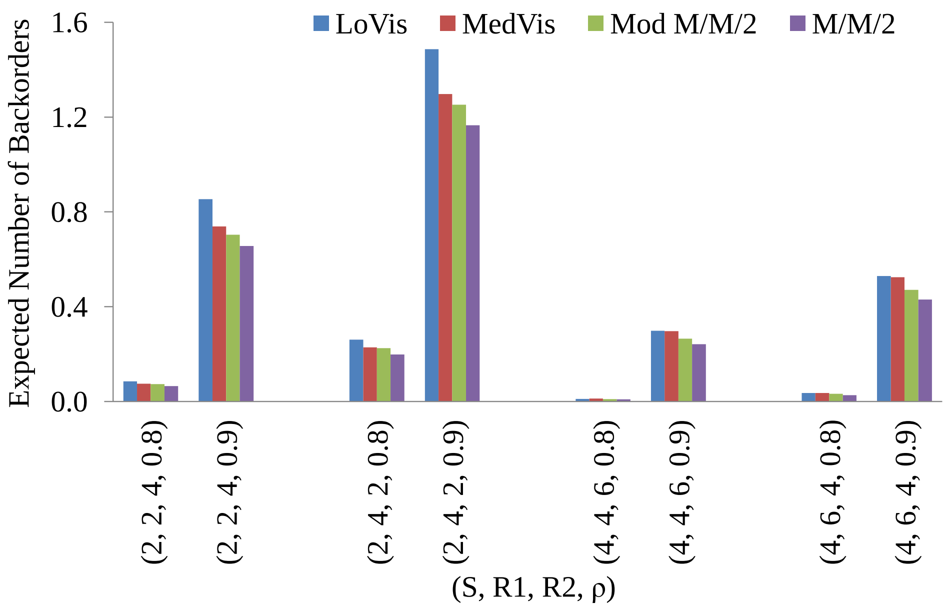



Figure 8.20: Expected Number of Backorders at Retail Store 2

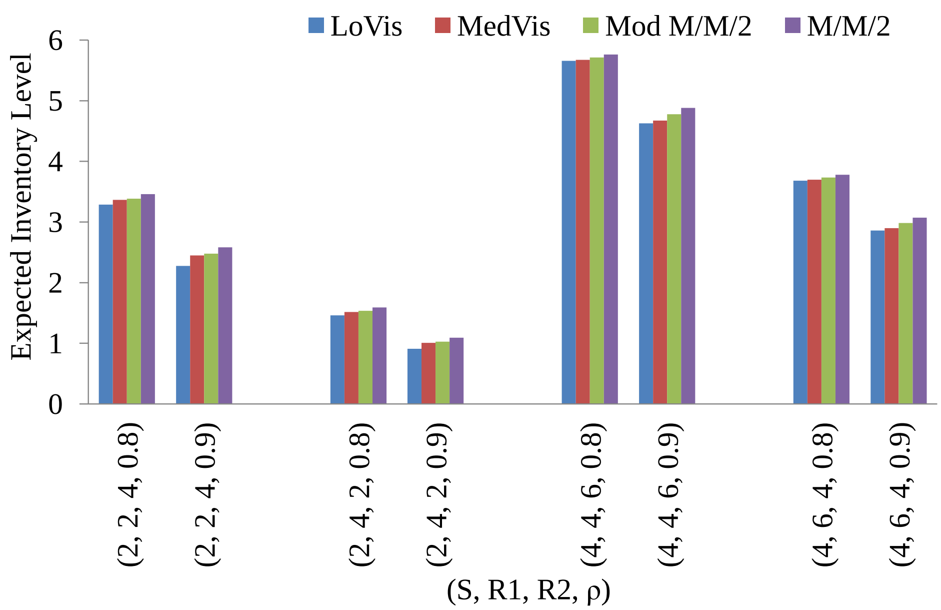



Figure 8.21: Expected Inventory Level at Retail Store 2

## 8.4 Analytical Model for General Inter-Arrival and Processing Time Distributions

In this section, we present the analytical model for the 2R/2P SCN configuration with HiVis under general inter-arrival and processing time distributions. The two

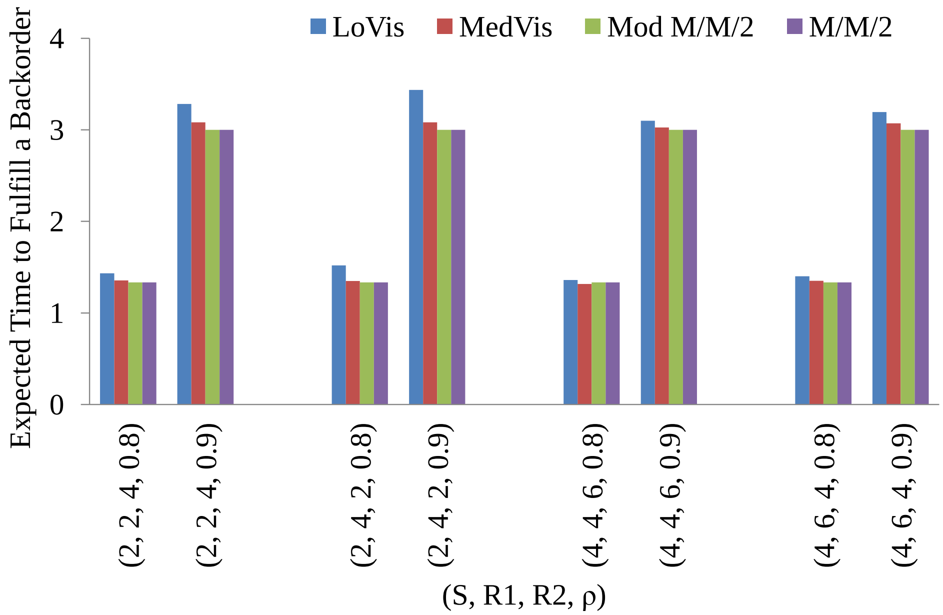



Figure 8.22: Expected Time to Fulfill a Backorder at Retail Store 2

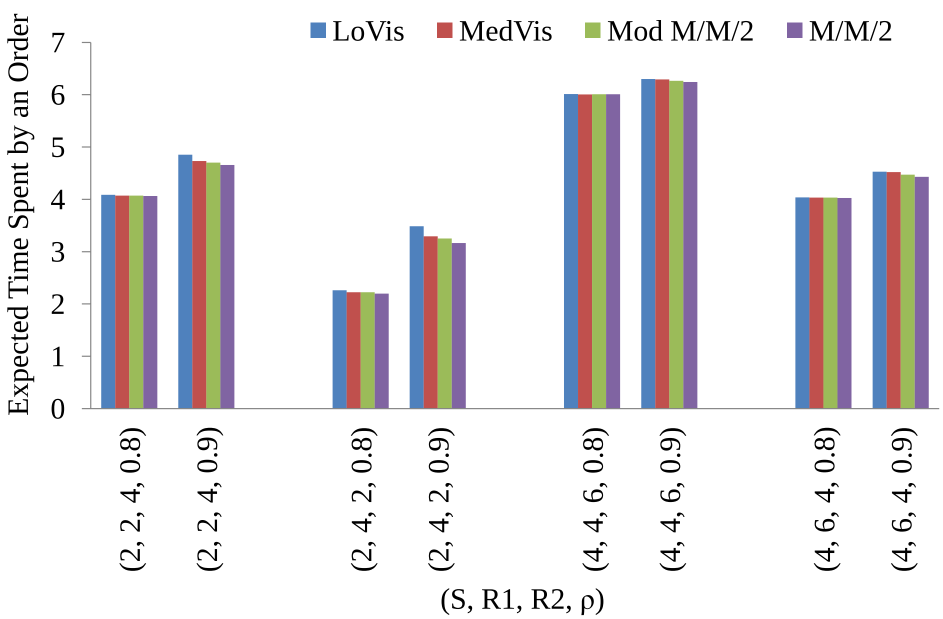



Figure 8.23: Expected Time Spent by an Order at Retail Store 2

production facilities were modeled as a GI/G/2 queueing system and the expressions provided by Whitt [60] were used to obtain the distribution of the number of orders at the pooled production facility ($N^p$). The arrival process at the GI/G/2 queue is the superposition of the arrival streams from the two retail stores. The traffic rate equations and traffic variability equations ([59]) shown in Equations 8.24 and 8.25 were used to obtain the mean and SCV ($cap^2$) of the arrival process at the pooled production facility. The distribution of the number of orders at retail store $i$ can be obtained using Equation (8.26). The performance measures at retail store $i$ can be obtained using Equations (8.7) through (8.11). The performance measures at the production facilities can be obtained using Equations (8.27) through (8.31).

$$\lambda = \lambda_1^r + \lambda_2^r \tag{8.24}$$

$$cap^2 = \alpha_1 \cdot ca_1^2 + \alpha_2 \cdot ca_2^2 \tag{8.25}$$

$$P\left(N_i^r = m\right) = \sum_{l=m}^{\infty} P\left(N_i^r = m | B^p = l\right) \cdot P\left(B^p = l\right) \tag{8.26}$$

where $P\left(N_i^r = m | B^p = l\right)$ and $P\left(B^p = l\right)$ can be obtained using Equations (8.1) through (8.4).

$$f^p = \sum_{i=0}^{2S-1} P(N^p = i) \tag{8.27}$$

$$E[I^p] = 0.5 \sum_{i=0}^{2S-1} (2S - i) P(N^p = i) \tag{8.28}$$

$$E[B^p] = E[N^p] + E[I^p] - S \tag{8.29}$$

$$E[W^{bp}] = \frac{E[B^p]}{(\lambda/2)(1 - f^p)} \tag{8.30}$$

$$E[T^p] = \frac{S + E[B^p]}{\lambda/2} \tag{8.31}$$

### 8.4.1 Validation of the GI/G/2 Approximation

The GI/G/2 approximation was validated by comparing the performance measures obtained from the analytical model with simulation estimates for the 2R/2P SCN configuration. The SCN parameter values used in the numerical experimentation are presented in Table 8.2. The design consists of 216 experiments.

Table 8.2: Experiments for 2R/2P SCN configuration under General Inter-arrival and Processing Time Distributions

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rates ($\lambda_1^r$, $\lambda_2^r$) | 2 | (1, 1), (2, 1) |
| Base-stock levels ($S$, $R_1$, $R_2$) | 4 | (2, 2, 4), (2, 4, 2), (4, 4, 6), (4, 6, 4) |
| Utilization level $\rho$ | 3 | 70%, 80%, 90% |
| Inter-arrival distribution at the retail stores | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |
| Processing time distribution at the production facilities | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |

The GI/G/2-based model is validated by using the prediction error measure defined in Section 4.3. The summary of the results is presented in Table 8.3. It can be seen from Table 8.3 that the GI/G/2-based model serves as a very good approximation for the HiVis case. The numerical results for the validation of the GI/G/2-based model are presented in Appendix E (see Section E.3). Figures 8.24 through 8.33 present the performance measures for retail stores 1 and 2 for a base-stock level of (2, 2, 4) and a utilization level of 90%.

Figures 8.24 through 8.33 confirm that the performance measures obtained from the GI/G/2-based model are close to the simulation estimates for the SCN with HiVis case under a base-stock setting of (2, 2, 4) and utilization level of 90%. The plots presented in Figures 8.24 and 8.29 indicate that the fill rates for the three cases (LoVis, MedVis and HiVis) are very close (as in the case of Poisson arrivals and exponential processing times). Similarly, Figures 8.27 through 8.32 show that the expected time to fulfill a backorder are very close for MedVis and HiVis cases. These

Table 8.3: Summary of Results

| Error Range | Percentage of Results within Error Range (%) | | | | |
|---|---|---|---|---|---|
| | Fill Rate | Expected Number of Backorders | Expected Inventory Level | Expected Time to Fulfill a Backorder | Expected Time spent by an Order |
| < 5 % | 93.5 | 85.7 | 86.1 | 37.4 | 98.8 |
| < 10 % | 100.0 | 95.2 | 100.0 | 55.4 | 100.0 |
| < 15 % | 100.0 | 98.3 | 100.0 | 67.0 | 100.0 |
| < 20 % | 100.0 | 99.2 | 100.0 | 71.0 | 100.0 |
| < 25% | 100.0 | 100.0 | 100.0 | 75.2 | 100.0 |

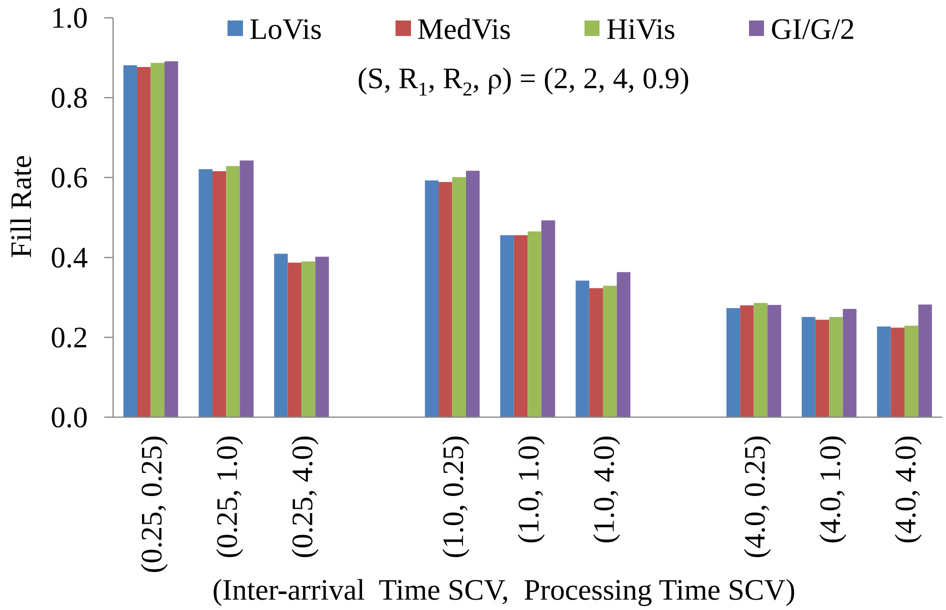



Figure 8.24: Fill Rate at Retail Store 1

results hold for other base-stock levels and utilization levels that were studied in our numerical experiments (for more details, refer to the results provided in Section E.3 of Appendix E).

### 8.4.2 Insights from the Analytical Model

The analytical model can provide valuable insights into the performance of the SCN as a function of production capacity, variability in the inter-arrival and processing times and inventory related decisions. As expected, the fill rate and the expected inventory level decrease with increase in the variability of the inter-arrival and pro-

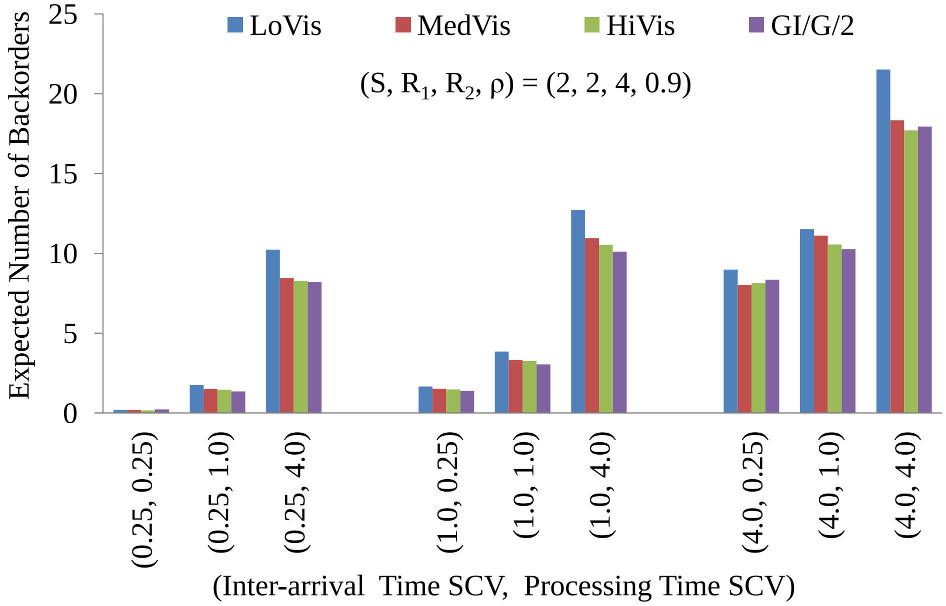



Figure 8.25: Expected Number of Backorders at Retail Store 1

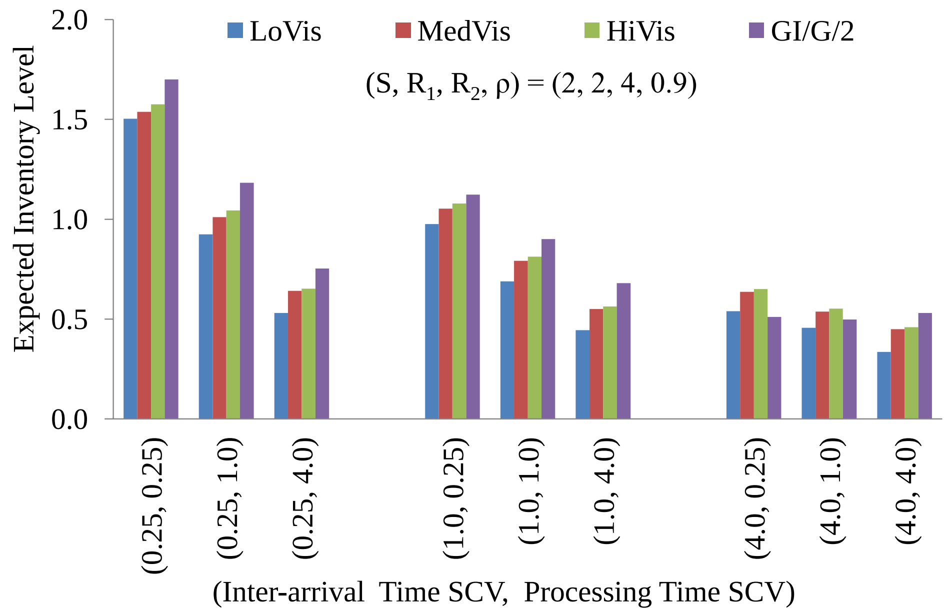



Figure 8.26: Expected Inventory Level at Retail Store 1

cessing times, while the expected number of backorders and expected time to fulfill a backorder increase with increase in the variability of the two processes. A more careful examination of the results presented in Appendix E (see Section E.3) reveals some interesting behavior. For instance, when either the inter-arrival time SCV or

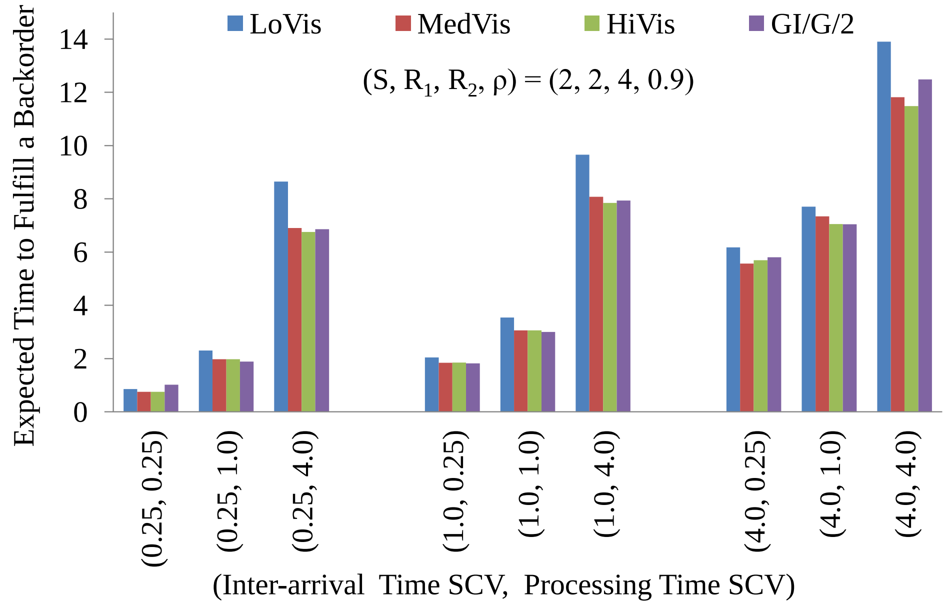



Figure 8.27: Expected Time to Fulfill a Backorder at Retail Store 1

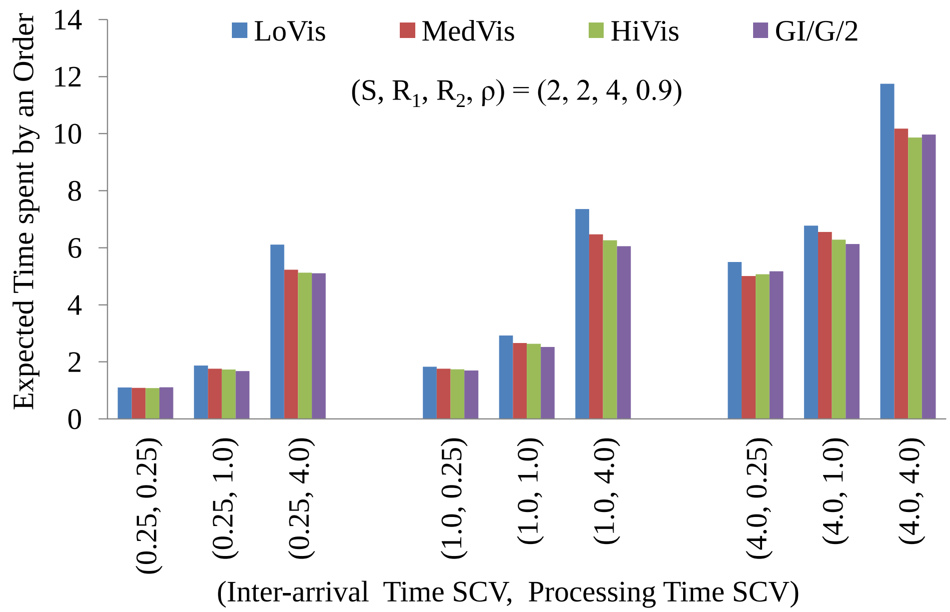



Figure 8.28: Expected Time Spent by an Order at Retail Store 1

the processing time SCV is high (4 in our case), the fill rates at retail stores 1 and 2 are nearly insensitive to changes in the other SCV. In Figures 8.34 and 8.35 (results plotted for a base-stock setting of (2, 4, 2) and a utilization of 90%), this is shown by the bottom most line being nearly flat. On the other hand, when the inter-arrival

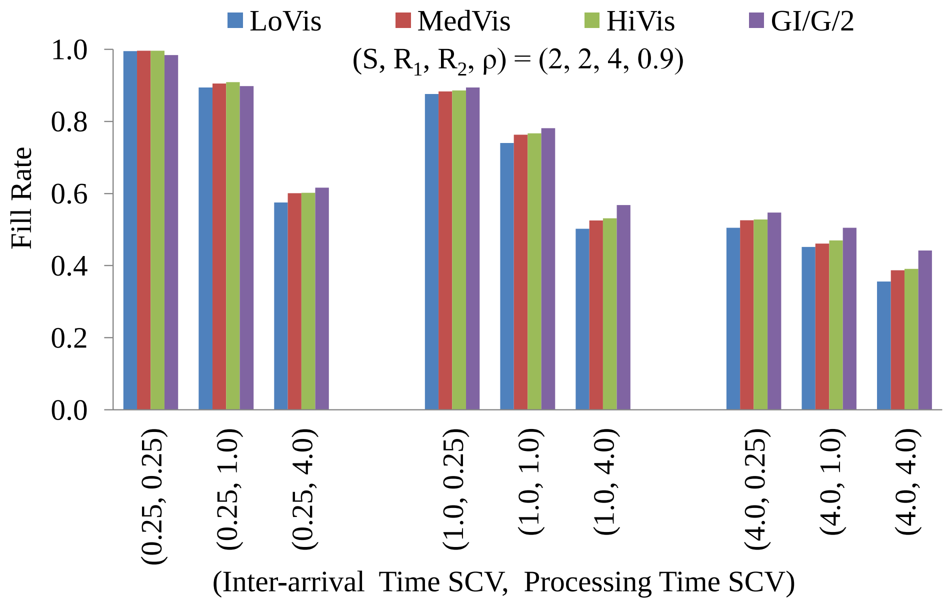



Figure 8.29: Fill Rate at Retail Store 2

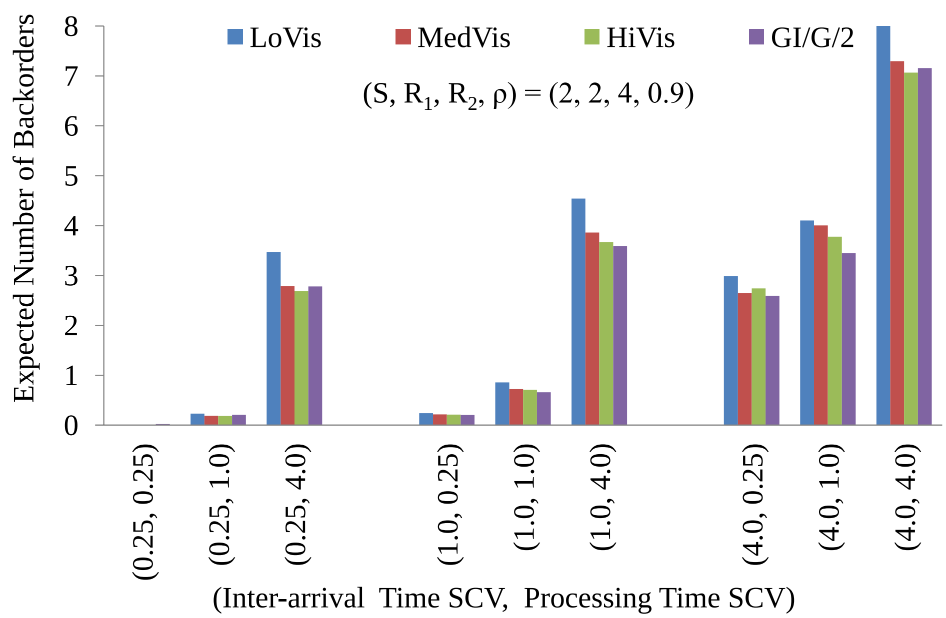



Figure 8.30: Expected Number of Backorders at Retail Store 2

time SCV or the processing time SCV is fixed at a lower value (0.25 or 1 in our case), any change in the other SCV creates a more significant change in the fill rates at retail stores 1 and 2.

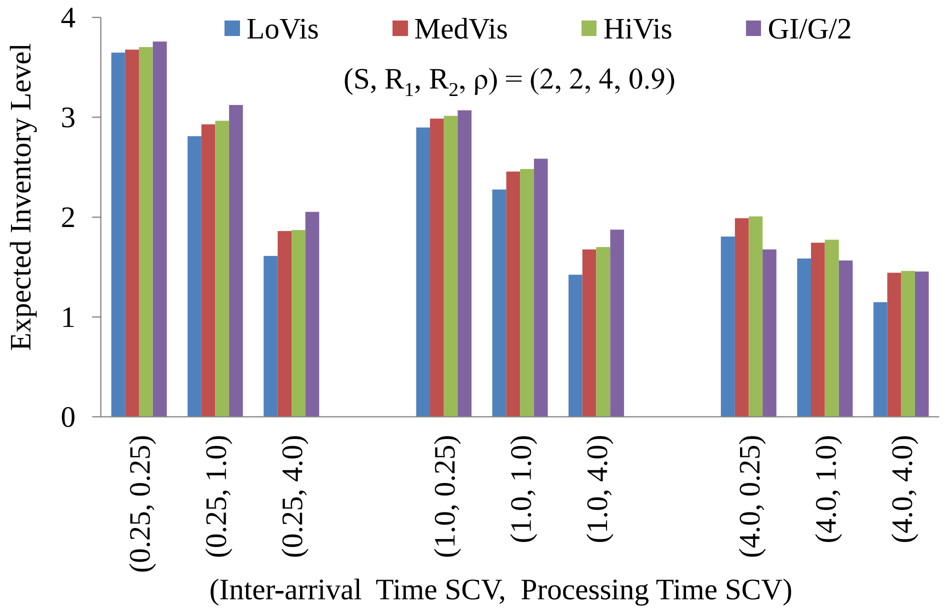



Figure 8.31: Expected Inventory Level at Retail Store 2

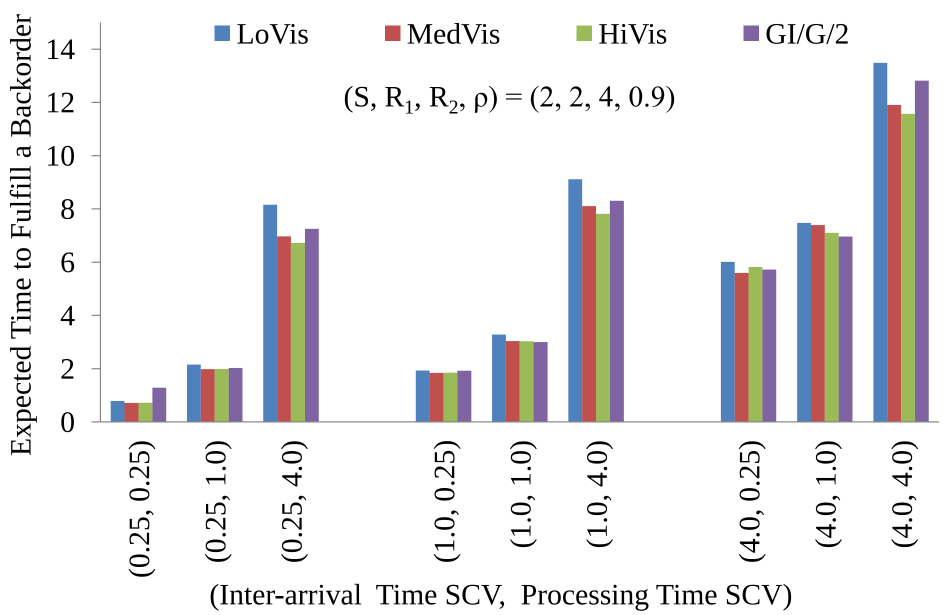



Figure 8.32: Expected Time to Fulfill a Backorder at Retail Store 2

## 8.5 Conclusions

We have developed analytical queueing models of the 2R/2P SCN configuration with high inventory visibility (HiVis). The analytical model was found to be accurate and also provides useful bounds for lower levels of inventory visibility. The modifications

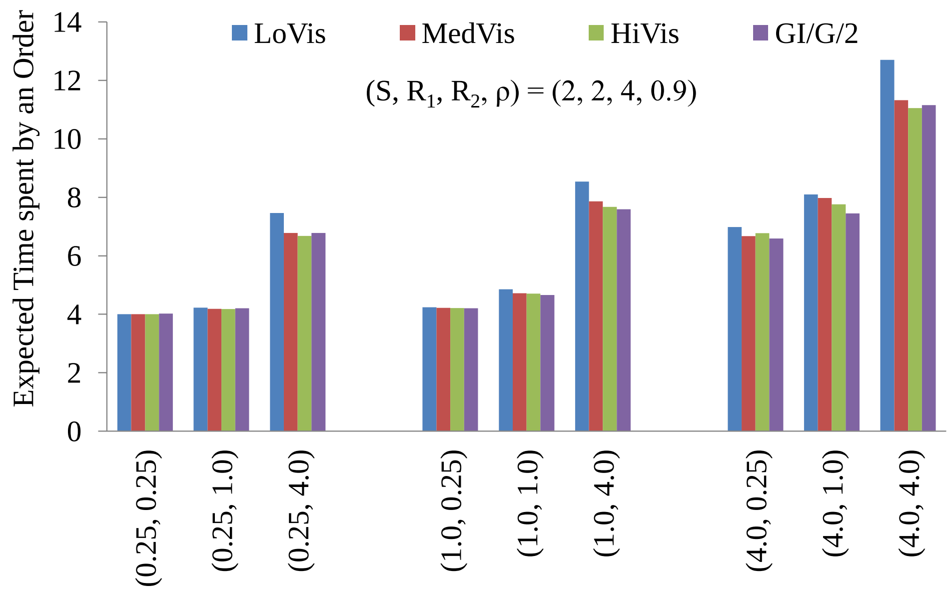



Figure 8.33: Expected Time Spent by an Order at Retail Store 2

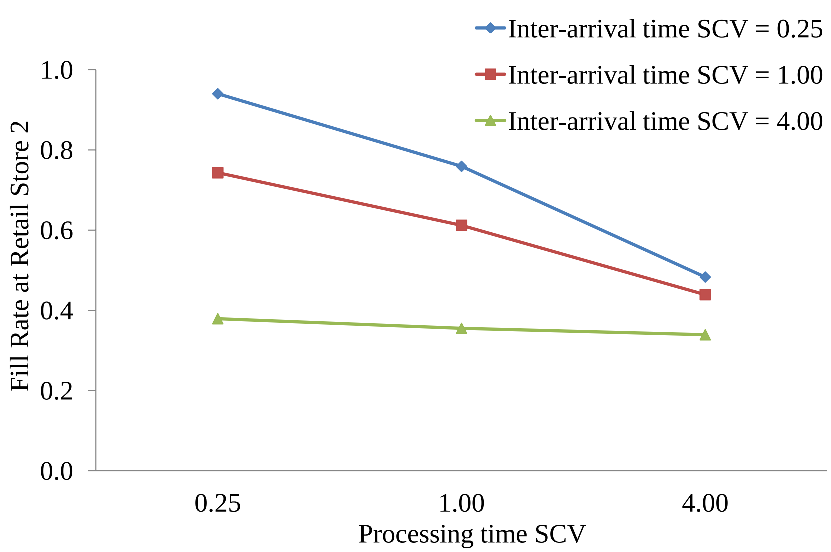



Figure 8.34: Effect of Processing Time SCV on Fill Rate

of the M/M/2 model developed in Chapter 6 were extended to yield better models for the 2R/2P SCN configuration under lower levels of information sharing. We also developed analytical models for the SCN under general inter-arrival and processing time distributions.

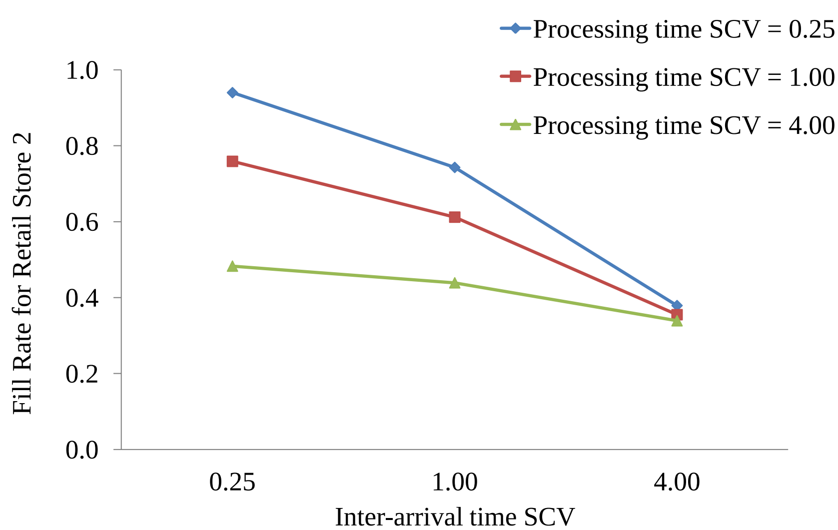



Figure 8.35: Effect of Inter-arrival Time SCV on Fill Rate

# CHAPTER 9

# MODELING TRANSIT TIMES IN SCNs WITH INVENTORY INFORMATION SHARING

In this chapter, we extend the previously developed analytical models to include transit times in SCNs operating under high level of information sharing (HiVis). The transit system behavior would be independent of the demands and orders in the SCN and each transit system will function independently of another. The two "building-block" SCN configurations (1R/2P and 2R/2P) presented in Chapters 6 and 8 were considered. The transit delays in the SCN from the production facilities to the retail store were modeled using an infinite server queue ([53], [54]). In the absence of results for the GI/G/$\infty$ queue, we approximate the delay by an M/G/$\infty$ queue. The number in system in an M/G/$\infty$ queue is known to follow a Poisson distribution. The statement by Whitt [60] that the number in system in a GI/G/$\infty$ queue can be reasonably approximated by a Poisson distribution further supports our approximation. Extensions to the analytical model developed for the 1R/2P SCN and 2R/2P SCN configurations are presented in Sections 9.1 and 9.2, respectively. Finally Section 9.3 presents the conclusions for this chapter.

## 9.1 Analytical Model of 1R/2P SCN Configuration with Transit Delay

In this section, we present the analytical model that considers transit delays in the 1R/2P SCN with HiVis under general inter-arrival and processing time distributions. In this model, the production facilities were modeled as a pooled store using the GI/G/2 system and the transit delay was modeled as an M/G/$\infty$ queueing system.

The mechanics involved in the calculation of the performance measures at the production facility are similar to those presented in Section 7.1. The distribution of the number of orders at a production facility was obtained using the approach contained in [60] and the performance measures at a production facility were obtained using equations (7.1) through (7.5). Since transit times were not considered in Section 7.1, the distribution of the number of orders at the retail store was given by the backorder distribution at the pooled store. When transit operations are considered, the order from a retail store can be either backordered or be in transit from the production facility to the retail store. Let $N^r$ be the random variable representing the number of orders at the retail store. The probability distribution of $N^r$ was obtained in equation (9.1) by assuming that the random variables corresponding to the orders in transit, $Tr^p$, and those backordered at the production facility, $B^p$, are independent.

$$P(N^r = n) = \sum_{l=0}^{n} P(B^p = l)P(Tr^p = n - l) \quad n = 0, 1, 2, \cdots \tag{9.1}$$

The probability distribution of $B^p$ is given in equation (9.2).

$$P(B^p = i) = \begin{cases} \sum_{n=0}^{2S} P\left(N^p = n\right) & ; \quad i = 0 \\ P\left(N^p = 2S + i\right) & ; \quad i \geq 1 \end{cases} \tag{9.2}$$

As the transit delay is modeled as an M/G/$\infty$ queueing system, the random variable $Tr^p$ is Poisson distributed as shown in equation (9.3). Note that the total arrival rate at the transit node is equal to the external arrival rate ($\lambda$) and $\theta^{pr}$ is the mean transit time.

$$P(Tr^p = x) = \frac{e^{-\delta}\delta^x}{x!} \quad x = 0, 1, 2, \cdots \tag{9.3}$$

where,

$$\delta = \lambda\theta^{pr} \tag{9.4}$$

The performance measures at the retail store were obtained using equations (7.7) through (7.11). The accuracy of the analytical model was evaluated by comparing the analytical results to simulation estimates. The 1R/2P SCN parameter values used in the numerical experimentation are presented in Table 9.1. The design consists of 72 experiments.

Table 9.1: Experiments for 1R/2P SCN Configuration with Transit Delay

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rates $\lambda^r$ | 1 | 1 |
| Base-stock levels $(S,\ R)$ | 4 | (2, 4), (4, 2), (3, 6), (6, 3) |
| Utilization level $\rho$ | 1 | 90% |
| Transit time distribution | 2 | Unif(1, 5), Unif(4, 8) |
| Inter-arrival distribution at the retail stores | 3 | (Erlang, Exponential, 2-phase hyper-exponential) |
| Processing time distribution at the production facilities | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |

Table 9.2: Summary of Results

| Error Range | Percentage of Results within Error Range (%) | | | | |
|---|---|---|---|---|---|
| | Fill Rate | Expected Number of Backorders | Expected Inventory Level | Expected Time to Fulfill a Backorder | Expected Time Spent by an Order |
| < 5 % | 86.8 | 38.9 | 75.7 | 56.9 | 86.8 |
| < 10% | 99.3 | 63.9 | 100.0 | 87.5 | 100.0 |
| < 15% | 100.0 | 74.3 | 100.0 | 95.1 | 100.0 |
| < 20% | 100.0 | 79.9 | 100.0 | 95.8 | 100.0 |
| < 25% | 100.0 | 85.4 | 100.0 | 95.8 | 100.0 |

The results for the numerical experiments are presented in Section F.1 of Appendix F. The summary of results is presented in Table 9.2. It can be seen that more than 90% of the results for all performance measures except expected number of backorders are within 15% error range. Thus, the analytical model is able to capture the effect of the transit delay effectively. Larger errors were observed when the inter-arrival and processing time distributions follow an Erlang distribution, an observation made even in cases with no transit delays. Comparison with the results for the SCN with no transit delay (see Chapters 5 and 6) and the results contained in tables presented in Section F.1 (see Appendix F) shows that the effect of transit delay is present only at the retail store. This result is intuitive as the production facility will not be affected by the transit delay.

### 9.1.1 Effect of Transit Delay on the Performance of the Retail Stores in the 1R/2P Configuration

In order to study the impact of transit delay on the various performance measures at the retail store, we plotted the performance measures obtained from the simulation and analytical models at three different levels of transit delay (no transit, transit delay with distribution Unif(1, 5) and transit delay with distribution Unif(4, 8)). Figures 9.1 through 9.5 present the performance measures at the retail store.

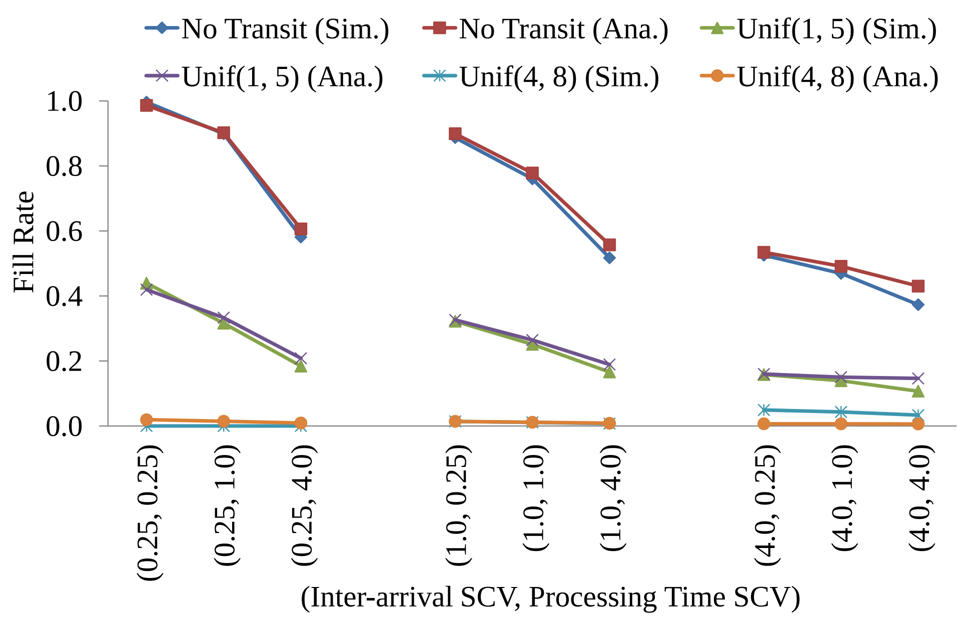


Figure 9.1: Fill Rate at the Retail Store

Figures 9.1 and 9.5 shows that as the transit delay is increased from a mean of 0 (no transit) to 6 (Unif(4, 8)), there is a significant decrease in the values of the fill rate and expected inventory level. However, with increase in transit delay, there is a

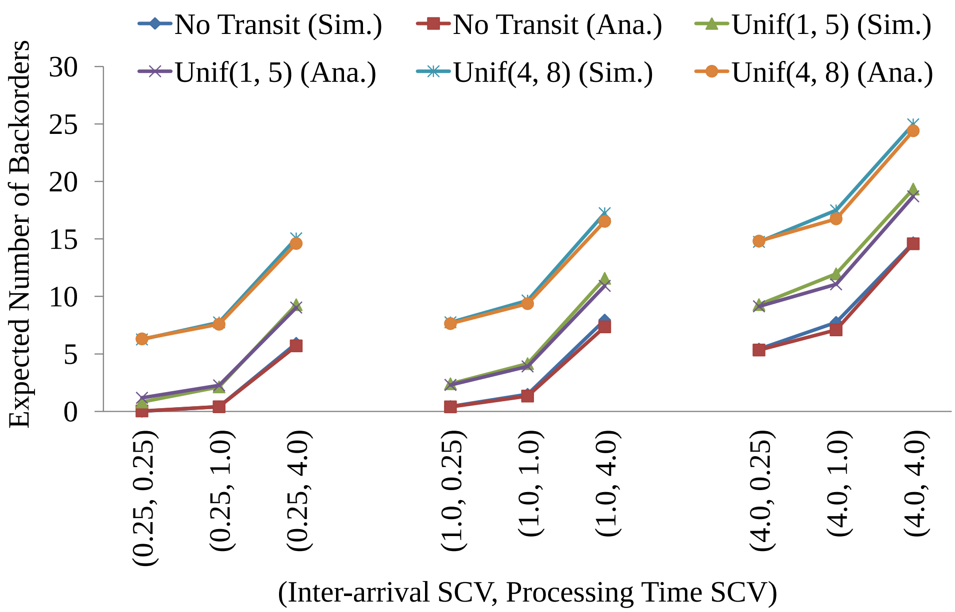



Figure 9.2: Expected Number of Backorders at the Retail Store

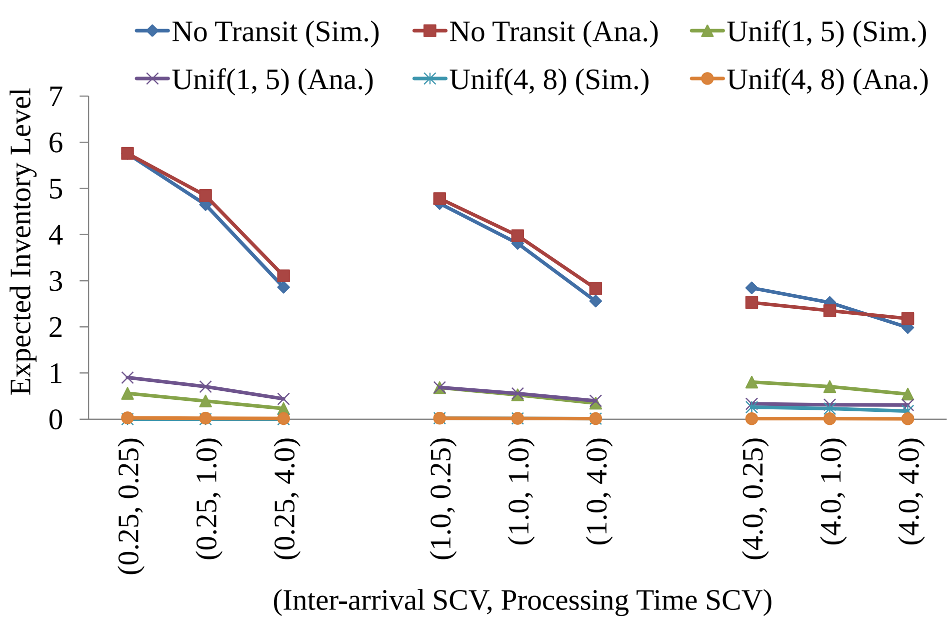



Figure 9.3: Expected Inventory Level at the Retail Store

significant increase in the values of expected number of backorders and the expected time spent by an order as shown in Figures 9.2 and 9.5, respectively. The plots also indicate that the analytical model is able to capture the trends in the various performance measures at the retail store.

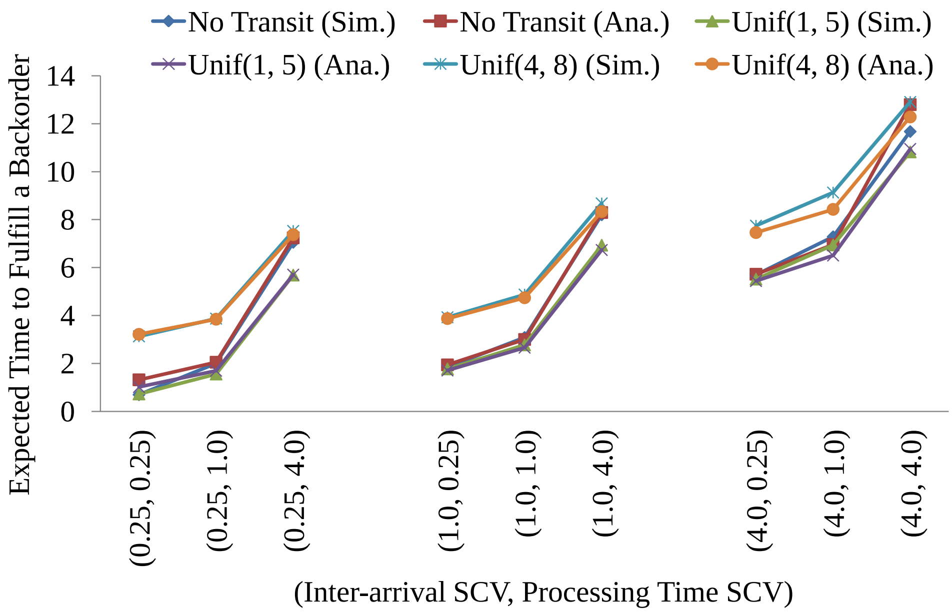



Figure 9.4: Expected Time to Fulfill a Backorder at the Retail Store

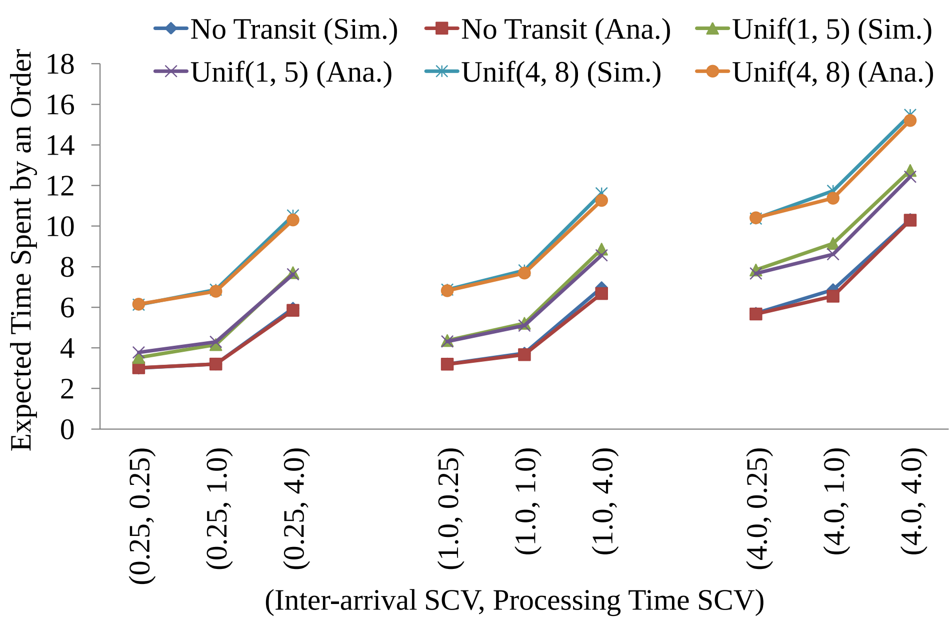



Figure 9.5: Expected Time Spent by an Order at the Retail Store

## 9.2 Analytical Model of the 2R/2P SCN Configuration with Transit Delays

In this section, we present the analytical model that considers transit delays in a 2R/2P SCN with high level of information sharing. The transit delays in the SCN from the production facilities to the retail stores are modeled using an infinite server queue as before ([53], [54]). The production facilities are modeled using the GI/G/2 queueing system.

The mechanics involved in the calculation of the performance measures at the production facility are similar to those presented in Section 8.4. The distribution of the number of orders at a production facility, $N^p$ was obtained using the approach presented in [60] and the performance measures at a production facility were obtained using equations (8.27) through (8.30). Since transit times were not considered in Section 7.1, the distribution of the number of orders pertaining to retail store $i$, $B_i^r$, backordered at the pooled facility yielded the distribution of the number of orders at retail store $i$. When transit is considered, it is clear that the order from a retail store can be either backordered or be in transit from the production facility to the retail store. Let $N_i^r$ represent the random variable representing the number of orders at retail store $i$. The probability distribution of $N_i^r$ was obtained in equation (9.5) by assuming that the random variables corresponding to the orders in transit, $Tr_i^p$, and those backordered at the production facility, $Np_i^r$, are independent.

$$P(N_i^r = n) = \sum_{l=0}^{n} P(Np_i^r = l)P(Tr_i^p = n - l) \quad n = 0, 1, 2, \cdots \tag{9.5}$$

The probability distribution of $Np_i^r$ was obtained using the "disaggregation procedure" discussed in Section 8.2.1. The orders at the pooled facility are aggregate orders since the facility does not have the information regarding the identity of the retail store that placed the order. As a result, a "disaggregation procedure" was used to find the distribution of the number of orders pertaining to retail store $i$, $N_i^r$, that are backordered at the pooled facility. This is obtained by conditioning on the number of backorders, $B^p$, at the pooled facility. This conditional distribution is Binomial as shown in Equations (8.1) and (8.2). The parameters of the binomial distributions are the proportions of orders from retail store $i$, $\alpha_i$, that are placed at the pooled production facility along with $l$, the total number of backorders at the pooled facility as shown in Equations (8.3) and (8.4). To derive the unconditional probability that a backorder at the pooled facility belongs to retail store $i$, $Np_i^r$, the conditional probabilities obtained in Equations (8.1) and (8.2) are multiplied by the backorder probability at the pooled facility and then summed over all values of $l$ (backorders) greater than or equal to $m$ as shown in Equations (9.6) and (9.7).

$$P\left(Np_1^r = m\right) = \sum_{l=m}^{\infty} P\left(N_1^r = m | B^p = l\right) \cdot P\left(B^p = l\right) \tag{9.6}$$

$$P\left(Np_2^r = m\right) = \sum_{l=m}^{\infty} P\left(N_2^r = m | B^p = l\right) \cdot P\left(B^p = l\right) \tag{9.7}$$

As the transit delay is modeled as an M/G/$\infty$ queueing system, the random variable $Tr_i^p$ is Poisson distributed as shown in equation (9.8). Note that the total arrival rate at the transit node is equal to the external arrival rate at the individual retail stores ($\lambda_i^r$) and $\theta_i^{pr}$ is the mean transit time for the production facilities to retail store $i$.

$$P(Tr_i^p = x) = \frac{e^{-\delta_i}\delta_i^x}{x!} \quad x = 0, 1, 2, \cdots \tag{9.8}$$

where,

$$\delta_i = \lambda_i^r \theta_i^{pr} \tag{9.9}$$

The performance measures at retail store $i$, $i$ = 1, 2 are presented in Equations (8.7) through (8.10). The accuracy of the analytical model was evaluated by comparing the analytical results to simulation estimates. The SCN parameter values used in the numerical experimentation are presented in Table 9.3. The design consists of 36 experiments.

The results of the analytical model for retail stores 1 and 2, and a production facil-

Table 9.3: Experiments for the 2R/2P SCN Configuration with Transit Delay

| Parameters | Levels | Parameter Values |
|---|---|---|
| Demand arrival rates ($\lambda_1^r$, $\lambda_2^r$) | 1 | (2, 1) |
| Base-stock levels ($S$, $R_1$, $R_2$) | 2 | (3, 6, 3), (3, 3, 6) |
| Utilization level $\rho$ | 1 | 90% |
| Transit time distribution | 2 | Unif(1, 5), Unif(4, 8) |
| Inter-arrival distribution at the retail stores | 3 | (Erlang, Exponential, 2-phase hyper-exponential) |
| Processing time distribution at the production facilities | 3 | (Erlang , Exponential, 2-phase hyper-exponential) |

Table 9.4: Summary of Results

| Error Range | Percentage of Results within Error Range (%) | | | | |
|---|---|---|---|---|---|
| | Fill Rate | Expected Number of Backorders | Expected Inventory Level | Expected Time to Fulfill a Backorder | Expected Time Spent by an Order |
| $< 5$ % | 94.4 | 45.4 | 79.6 | 62.0 | 89.8 |
| $< 10$% | 100.0 | 70.4 | 98.1 | 84.3 | 96.3 |
| $< 15$% | 100.0 | 88.0 | 100.0 | 91.7 | 100.0 |
| $< 20$% | 100.0 | 97.2 | 100.0 | 95.4 | 100.0 |
| $< 25$% | 100.0 | 100.0 | 100.0 | 95.4 | 100.0 |

ity are presented in Section F.2 of Appendix F. The summary of results is presented in Table 9.4. Table 9.4 shows that more than 90% of all performance measures except expected number of backorders falls within a 15% error range. However, most of the higher errors in expected inventory level occur when its value is very small (say less than 1). The analytical model clearly follows the trend in the different performance measures as indicated by the simulation results.

### 9.2.1 Effect of Transit Delays on the Performance of the Retail Stores in the 2R/2P Configuration

In this section, we compare the performance measures obtained from the simulation and analytical models for the two retail stores at three levels of transit delays (no transit, transit delay with distribution Unif(1, 5) and transit delay with distribution Unif(4, 8)). Figures 9.6 through 9.15 present the performance measures at the retail stores.

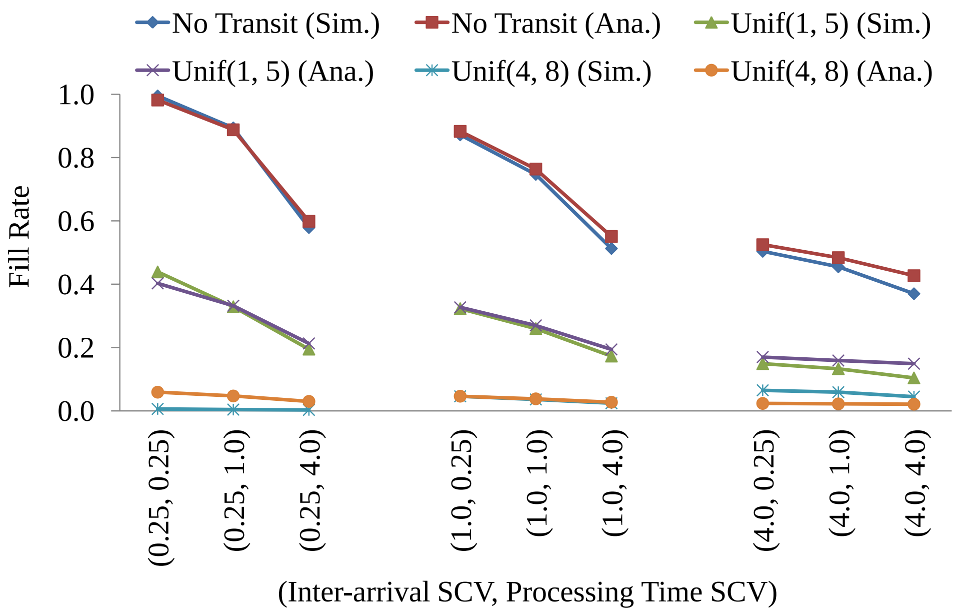



Figure 9.6: Fill Rate at Retail Store 1

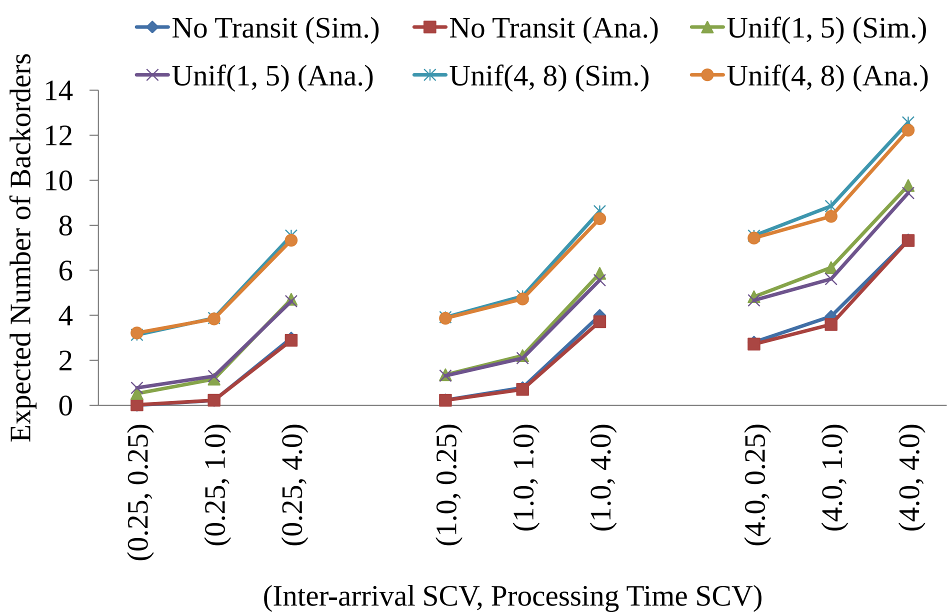



Figure 9.7: Expected Number of Backorders at Retail Store 1

As in the case of 1R/2P SCN configuration, it can be seen that the values of fill rate and expected inventory level for the two retail stores fall steeply as the mean transit delay is increased from 0 (no transit) to 6 (Unif(4, 8)). The values of expected

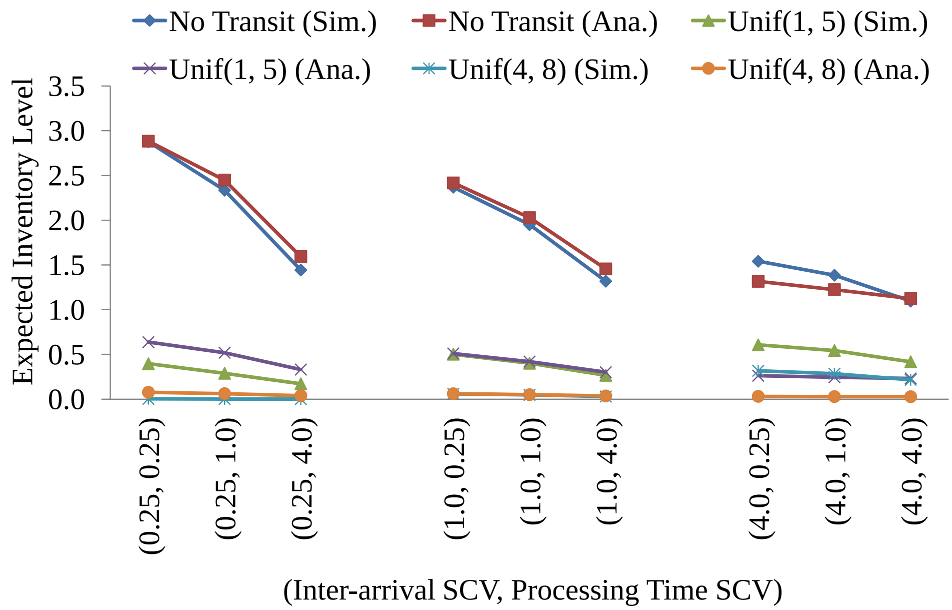



Figure 9.8: Expected Inventory Level at Retail Store 1

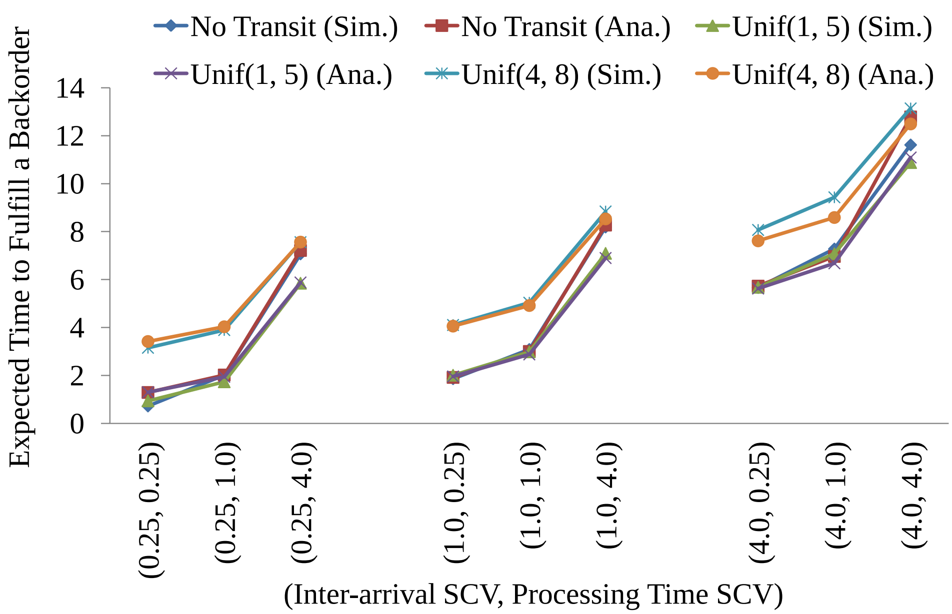



Figure 9.9: Expected Time to Fulfill a Backorder at Retail Store 1

number of backorders and the expected time spent by an order increase with increase in the mean transit time. The plots presented in Figures 9.6 through 9.15 indicate that the analytical model can effectively capture the trends in the various performance measures for the two retail stores.

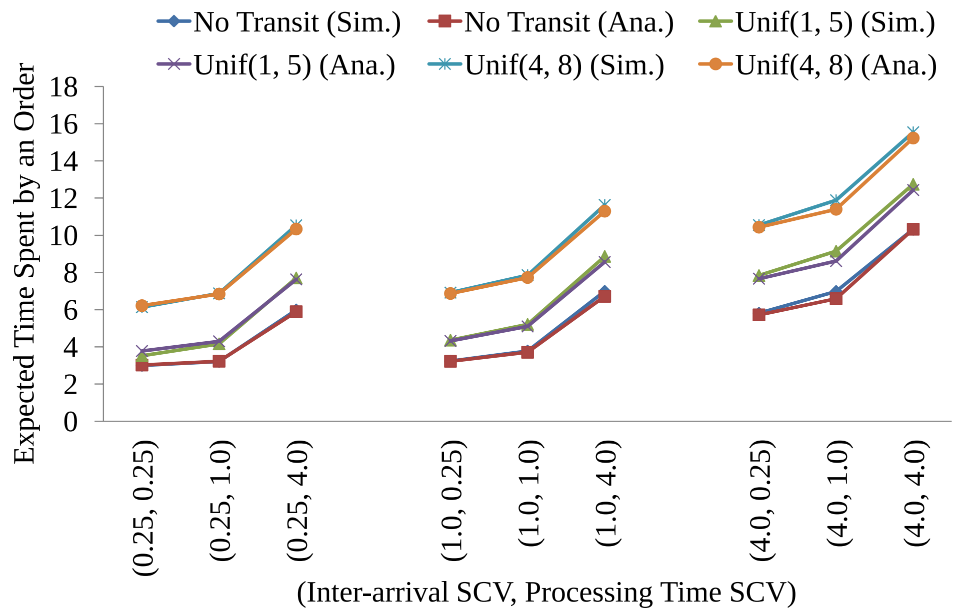



Figure 9.10: Expected Time spent by an Order at Retail Store 1

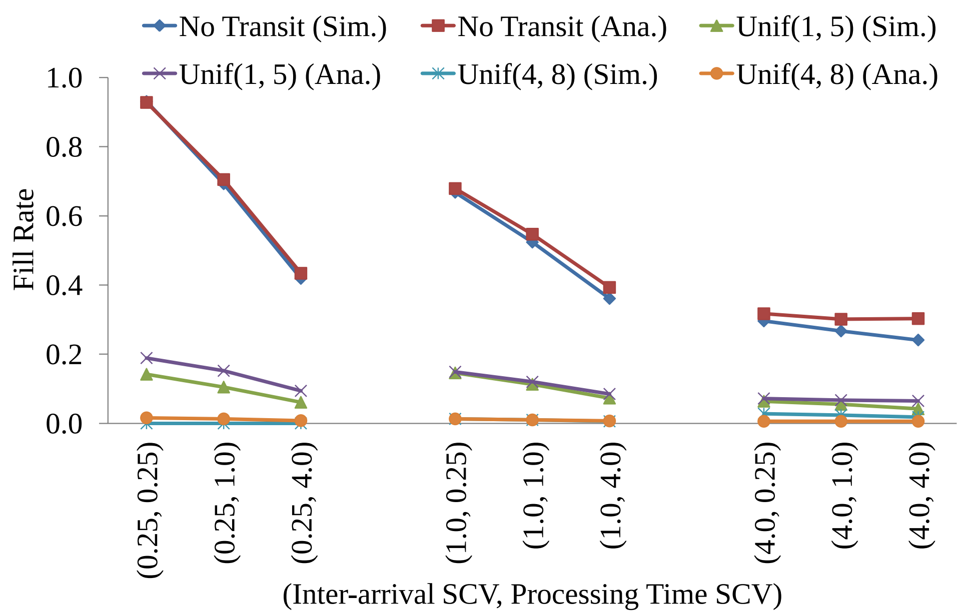



Figure 9.11: Fill Rate at Retail Store 2

## 9.3 Conclusions

In this chapter, we have extended the analytical queueing models of SCNs with inventory information sharing to model transit times. The transit delay was modeled as an infinite server queue and it was found that the analytical model effectively captured

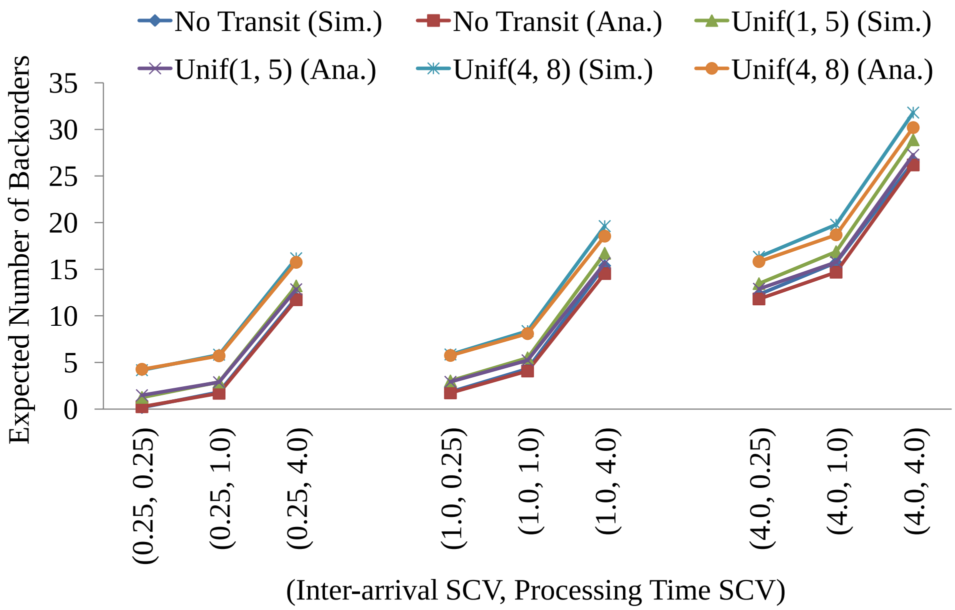



Figure 9.12: Expected Number of Backorders at Retail Store 2

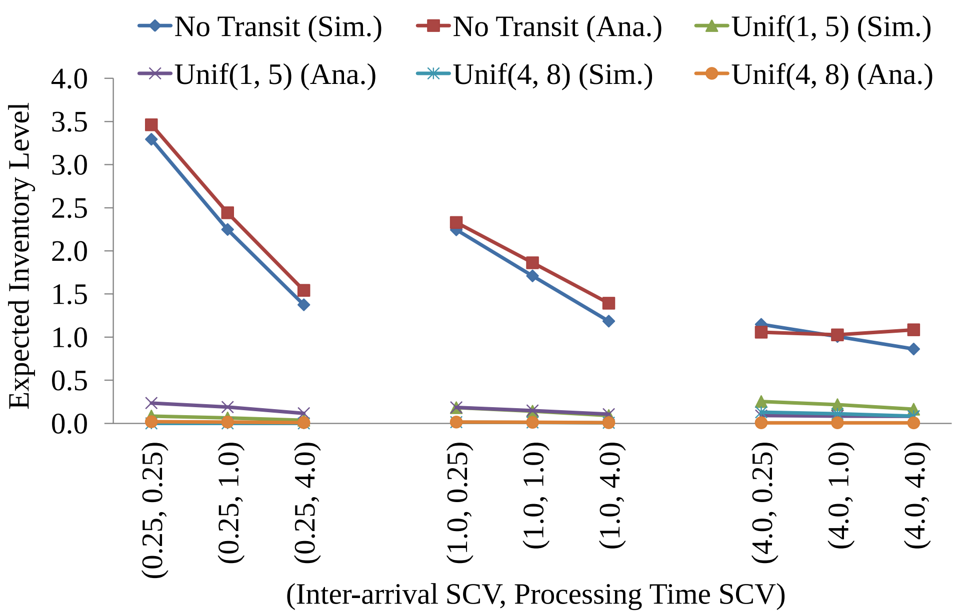



Figure 9.13: Expected Inventory Level at Retail Store 2

the effect of transit delays on the SCN performance measures.

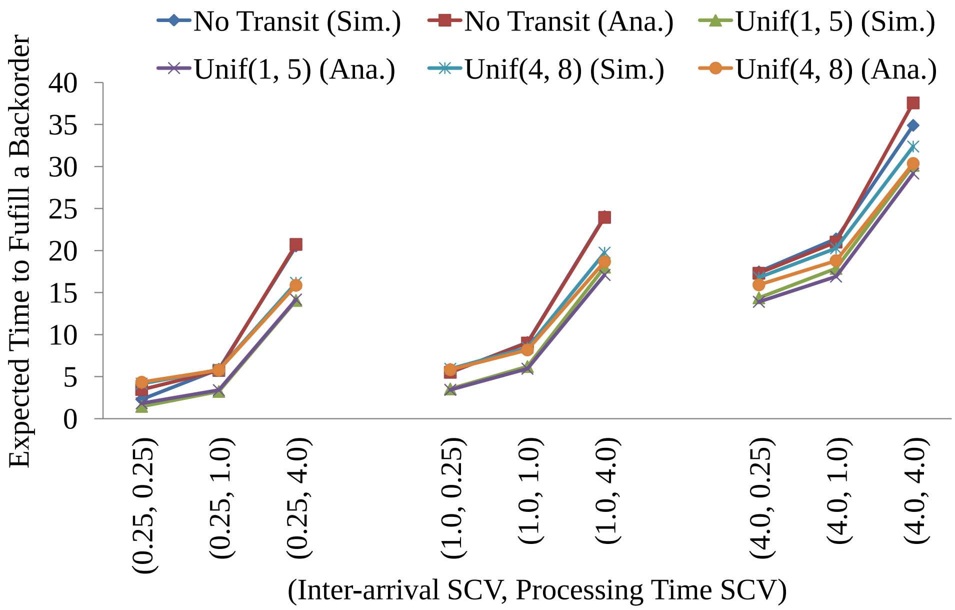



Figure 9.14: Expected Time to Fulfill a Backorder at Retail Store 2

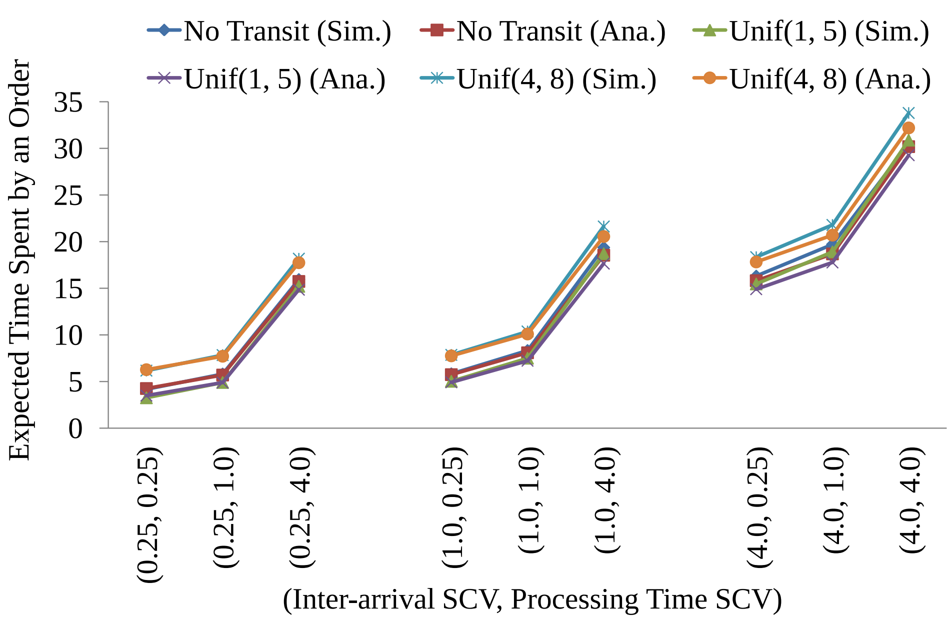



Figure 9.15: Expected Time spent by an Order at Retail Store 2

# CHAPTER 10

# CONCLUSIONS

This concluding chapter provides a summary of the research carried out in this dissertation effort, research contributions made and some directions for future research. Section 10.1 provides a summary of the research that was completed. Section 10.2 summarizes the contributions made by the successful completion of the research, and Section 10.3 provides suggestions for future research.

## 10.1 Summary of Research

The main objective of this research was to develop analytical performance evaluation models that explicitly consider inventory information sharing between SCN partners, and to study the value of inventory information sharing in order to identify SCN conditions under which the benefits of inventory information sharing are significant. In this research, we considered two "building-block" type configurations of a two-echelon SCN and three different levels of inventory information sharing or visibility - high (HiVis), medium (MedVis) and low (LoVis) (see Section 3.1 for more details). We first developed CTMC models for the two-echelon SCN with one retail store and two identical production facilities under the three levels of inventory visibility and the case with no information sharing (NoVis). We were able to obtain closed form solutions for the SCN with NoVis, while the CTMC models for the SCN under the three levels of information sharing were numerically solved by fixing the maximum number of backorders at the retail store. Numerical results indicated that there is value in sharing inventory information among the SCN partners. With no closed form analytical solutions for the CTMC models, we explored the development of approximate queueing models for the SCN under information sharing for Poisson arrivals and exponential processing times in Chapter 6. First, we developed approximate queueing models for the SCN under HiVis case by using the characteristics of the order routing policy. An M/M/2 based approximation was found to work well for the SCN under HiVis and produced reasonable bounds for the LoVis and MedVis cases. We also developed a modified M/M/2 model using analytical reasoning and empirical results to better model the SCN under lower levels of inventory visibility. This model was able to capture the trends in the performance measures at the various stores under lower levels of information sharing. Finally, in Chapter 6, we also extended the analytical models to model asymmetric production facilities. The key idea was to replace the two heterogeneous servers by two homogeneous servers by computing an aggregate processing time distribution. The resulting M/G/2 based approximation was found to capture the sensitivity of the performance measures at the retail store and the production facilities to the asymmetric utilizations at the individual production facilities and the non-identical processing time distributions. In Chapter 7, the analytical models were extended to model general inter-arrival and processing time distributions. The GI/G/2 based model was found to be a very good approximation for the HiVis case with more than 90% of the results for all performance measures except expected time to fulfill a backorder within the acceptable 15% error range. In Chapter 8, the analytical models were developed for a two-echelon SCN with two retail stores and two production facilities. The models were found to be accurate for the SCN under HiVis and captured the trends in the performance measures with asymmetry at the retail store level (retail stores with different base-stock levels and different inter-arrival rates). In Chapter 9, the analytical models developed for the 2 two-echelon configurations were extended to include transit delays from the production facilities to the retail stores. The models were found to be reasonably accurate and the analytical model adequately captured the effect of transit delays on the various performance measures at the retail stores.

## 10.2 Research Contributions

In most analytical performance evaluation models of SCNs, it is assumed that a downstream firm places an order at one of its immediate upstream "suppliers" using a Bernoulli routing policy. As per this policy, orders from a retail store would be routed to the production facilities based on fixed probabilities (based on preference or historical information) and this would allow an order to be placed at a production facility facing a stock-out situation even when there is inventory at the other facility. However, alternate order routing strategies could be adopted to avoid such circumstances in the presence of upstream inventory information at the retail store. As seen in Chapter 2, there was a need to develop performance evaluation models of SCNs under inventory visibility. The major contributions of this dissertation effort can be summarized as follows.

1. We have developed analytical queueing models that can explicitly model inventory information sharing from upstream stores to downstream stores in "building-block" type two-echelon SCN configurations.

2. We have studied the value of inventory information sharing and identified the ranges of the SCN parameters at which the value of information sharing is significant. For example, the value of information sharing is maximum for a combination of lower base-stock levels and high utilization levels at the production facilities and diminishes as the base-stock level is increased or the utilization is decreased.

3. Our study on the effect of inter-arrival time and processing time SCVs on the value of information sharing indicated that the value is more sensitive to pro-

cessing time SCV than to the inter-arrival time SCV.

4. We have extensively studied two SCN configurations - one retail store and two production facilities and two retail stores and two production facilities and believe that these can serve as building-blocks for SCN structures with more echelons.

5. We have also identified a relationship between the model for SCN under HiVis and the shortest queue problem. The models presented in Chapters 6 and 7 could be further explored to develop approximations for the shortest queue problem.

## 10.3 Future Research

The scope for future research has been broadly classified into four groups that are presented below.

**Developing analytical models for SCN under LoVis and MedVis**

In Chapter 6, we have developed a correction factor for the steady-state probabilities of the M/M/2 system to better model SCNs under LoVis and MedVis. However, there is an opportunity to explore additional correction factors to model the individual levels of information sharing using our knowledge about the routing policies. For example, it is known that the there is a difference between these two routing policies when both the production facilities are backordered. In such instances, the orders are probabilistically routed in a LoVis case and based on the shortest queue in the MedVis case.

**Modeling Asymmetry in the SCN**

We have successfully modeled asymmetric production facilities (see Chapter 6) and asymmetric retail stores (see Chapter 8). We have identified the following areas for future research with respect to asymmetry in the SCN.

1. There is a need to model asymmetry in the base-stock levels at the production facilities (i.e., $S_1 \neq S_2$). Assuming that $S_1 > S_2$, it can be seen that in the SCN with HiVis, no orders will be placed at production facility 2 until the number of orders in the system exceeds $S_1 - S_2$. This idea could be used in the calculation of probability of placing an order at production facility 1 ($p_1$). This probability could then be used in an M/G/2 queueing model to capture the performance measures at the various stores.

2. A combination of the asymmetric base-stock levels and heterogeneous processing time distributions at the production facilities is also of interest and may be more appropriate in a real world scenario. There could be added value in determining whether the effect of the two individual sources of asymmetry is additive or not through simulation studies before developing analytical models.

3. The modeling of the above two scenarios and the asymmetric processing time distribution cases under general inter-arrival and processing time distributions would be a challenging task because the nice properties of the exponential distributions can no longer be exploited to obtain the distribution of the number of busy servers (see Section 6.2).

**Modeling more Complex SCN Structures**

We have shown that the analytical models developed for the SCN with one retail store and two production facilities can be extended to model SCNs with an additional retail store. However, there is a need to explore the development of analytical models for more complex SCN configurations. The use of the multi-server system makes it easy to extend the model to consider SCNs with more than two production facilities. Similarly, a scenario where more than two retail stores placing an order at a production facility can be accommodated by using the multinomial distribution in the calculation of the distribution of the number of orders pertaining to retail store $i$, $N_i^r$, at the production facilities (see Section 8.2.1). In case of modeling additional echelons in the SCN, an approach similar to that used by Srivathsan [53], and Srivathsan and Kamath [54] where the backorder distribution was used to link successive stage in the SCN can be explored. Note that we are already using the backorder distribution to link the production facilities and the retail store(s) in Chapters 5 through 9.

**Exploring the Relationship to the Shortest Queue Problem**

In Chapter 6, we identified that there is a relationship between the SCN under HiVis and the shortest queue problem. It was also seen that the multi-server model is a good approximation for the HiVis case. There is a need to further explore the relationship between these two models.

# BIBLIOGRAPHY


[1] I.J.B.F. Adan, G.J. van Houtum, and J. van der Wal. Upper and Lower Bounds for the Waiting Time in the Symmetric Shortest Queue System. *Annals of Operations Research*, 48(2):197–217, 1994.

[2] I.J.B.F. Adan, J. Wessels, W.H.M. Zijm, and B.V. Nederlandse Philips Bedrijven. Analysis of the Symmetric Shortest Queue Problem. *Stochastic Models*, 6:691–713, 1990.

[3] Y. Arda and J.C. Hennet. Inventory Control in a Multi-Supplier System. *International Journal of Production Economics*, 104(2):249–259, 2006.

[4] M. Armony and E.L. Plambeck. The Impact of Duplicate Orders on Demand Estimation and Capacity investment. *Management Science*, 51(10):1505–1518, 2005.

[5] S. Axsater. *Inventory Control*. Kluwer Academic Publishers,, Boston, MA, USA, 2000.

[6] K. Ayodhiramanujan. *Integrated Analytical Performance Evaluation Models of Warehouses*. Phd dissertation, Oklahoma State University, Stillwater, OK, USA, 2009.

[7] S. Benjaafar, W.L. Cooper, and J-S. Kim. On the Benefits of Pooling in Production-Inventory Systems. *Management Science*, 51(4):548–565, 2005.

[8] S. Benjaafar, J-S. Kim, and N. Vishwanadham. On the Effect of Product Variety in Production-Inventory Systems. *Annals of Operations Research*, 126:71–101, 2004.

[9] K. Bourland, S. Powell, and D. Pyke. Exploiting Timely Demand Information to Reduce Inventories. *European Journal of Operational Research*, 92(2):239–253, 1996.

[10] J. Bramel and D. Simchi-Levi. *The Logic of Logistics: Theory, Algorithms, and Application for Logistics Management*. Springer Series in Operations Research, New York, NY, USA, 1997.

[11] J.A. Buzacott and J.G. Shanthikumar. *Stochastic Models of Manufacturing Systems*. Prentice Hall, Inc., Englewood Cliffs, NJ, USA, 1993.

[12] G.P. Cachon and M. Fisher. Supply Chain Inventory Management and the Value of Shared Information. *Management Science*, 46(8):1032–1048, 2000.

[13] F. Chen. Echelon Reorder Points, Installation Reorder Points, and the Value of Centralized Demand Information. *Management Science*, 44(12):S221–S234, 1998.

[14] M.A. Cohen and H.L. Lee. Strategic Analysis of Integrated Production-Distribution Systems: Models and Methods. *Operations Research*, 36(2):216–228, 1988.

[15] D. Corsten and T. Gruen. Desperately Seeking Shelf Availability: An Examination of the Extent, the Causes, and the Efforts to Address Retail Out-of-Stocks. *International Journal of Retail & Distribution Management*, 31(12):605–617, 2003.

[16] M. Dong. Inventory Planning of Supply Chains by Linking Production Authorization Strategy to Queueing Models. *Production Planning & Control*, 14(6):533–541, 2003.

[17] M. Dong and F.F. Chen. Performance Modeling and Analysis of Integrated Logistic Chains: An Analytic Framework. *European Journal of Operational Research*, 162(1):83–98, 2005.

[18] B. Enslow. Global Supply Chain Benchmark Report: Industry Priorities for Visibility, B2B Collaboration, Trade Compliance, and Risk Management. Technical report, Aberdeen Group, 2006.

[19] M. Ettl, G.E. Feigin, G.Y. Lin, and D.D. Yao. A Supply Network Model with Base-Stock Control and Service Requirements. *Operations Research*, 48(2):216–232, 2000.

[20] S. Gavirneni. Benefits of Cooperation in a Production Distribution Environment. *European Journal of Operational Research*, 130(3):612–622, 2001.

[21] S. Gavirneni, R. Kapuscinski, and S. Tayur. Value of Information in Capacitated Supply Chains. *Management Science*, 45(1):16–24, 1999.

[22] B. Gavish and S.C. Graves. A One-Product Production/Inventory Problem under Continuous Review Policy. *Operations Research*, 28(5):1228–1236, 1980.

[23] B. Gavish and S.C. Graves. Production/Inventory Systems with a Stochastic Production Rate under a Continuous Review Policy. *Computers & Operations Research*, 8(3):169–183, 1981.

[24] S.C. Graves. A Single-Item Inventory Model for a Nonstationary Demand Process. *Manufacturing & Service Operations Management*, 1(1):50–61, 1999.

[25] S.C. Graves and J. Keilson. The Compensation Method Applied to a One-Product Production/Inventory Problem. *Mathematics of Operations Research*, 6(2):246–262, 1981.

[26] S. Halfin. The Shortest Queue Problem. *Journal of Applied Probability*, 22(4):865–878, 1985.

[27] F.W. Harris. How Many Parts to Make at Once. *Operations Research*, 38(6):947–950 (reprint from Factory, The Magazine of Management, 10:2, 1913, 135–136, 152), 1990.

[28] J.R. Jackson. Networks of Waiting Lines. *Operations Research*, 5(4):518–521, 1957.

[29] S. Jain and N.R.S. Raghavan. Analysis of Base-Stock Controlled Production-Inventory System using Discrete-Time Queueing Models. In *IEEE International Conference on Automation Science and Engineering*, pages 37–42, Edmonton, Canada, Aug 2005.

[30] M. Kamath and S. Srivathsan. A Comparative Evaluation of Analytical Approximations for Production-Inventory Networks. *Computers & Industrial Engineering*, 62:644–652, 2012.

[31] F. Karaesmen, J.A. Buzacott, and Y. Dallery. Integrating advanced order information in make-to-stock production systems. *IIE Transactions*, 34:649–662, 2002.

[32] W.D. Kelton, R.P. Sadowski, and D.A. Sadowski. *Simulation with Arena*. The McGraw-Hill Companies, Inc., New York, NY, USA, 2 edition, 2002.

[33] J.F.C. Kingman. Two Similar Queues in Parallel. *Annals of Mathematical Statistics*, 32:1314–1323, 1961.

[34] W. Kraemer and M. Langenbach-Belz. Approximate Formulae for the Delay in the Queueing System GI/G/1. In *Proceedings of the 8th Int. Teletraffic Congress*, pages 235–(1/8), 1976.

[35] A. Krishnamurthy. *Analytical Performance Models for Material Control Strategies in Manufacturing Systems*. Phd dissertation, University of Wisconsin, Madison, WI, USA, 2002.

[36] S.C. Kulp. Asymmetric Information in Vendor Managed Inventory Systems. Working paper, 2000.

[37] H.L. Lee and C. Billington. Material Management in Decentralized Supply Chains. *Operations Research*, 41(5):835–847, 1993.

[38] H.L. Lee, V. Padmanabhan, and S. Whang. Information Distortion in a Supply Chain: The Bullwhip Effect. *Management Science*, 43(4):546–558, 1997.

[39] H.L. Lee and S. Whang. Information Sharing in a Supply Chain. *International Journal of Manufacturing Technology and Management*, 1(1):79–93, 2000.

[40] Y-J. Lee and P.H. Zipkin. Tandem Queues with Planned Inventories. *Operations Research*, 40(5):936–947, 1992.

[41] G. Li, H. Yan, S. Wang, and Y. Xia. Comparative Analysis on Value of Information Sharing in Supply Chains. *Supply Chain Management*, 10(1):34–46, 2005.

[42] J.D.C. Little. A Proof for the Queuing Formula: $L = \lambda W$. *Operations Research*, 9(3):383–387, 1961.

[43] L. Liu, X. Liu, and D.D. Yao. Analysis and Optimization of a Multistage Inventory-Queue System. *Management Science*, 50(3):365–380, 2004.

[44] J. Mehdi. *Stochastic Models in Queueing Theory*. Academic Press, Orlando, FL, USA, 2 edition, 2003.

[45] K. Moinzadeh. A Multi-Echelon System with Information Exchange. *Management Science*, 48(3):414–426, 2002.

[46] S. Nahmias. *Production and Operations Analysis*. McGraw-Hill, Boston, MA, USA, 4 edition, 2001.

[47] M.F. Neuts. *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach*, volume 2 of *Johns Hopkins Series in the Mathematical Sciences*. The Johns Hopkins University Press, 1981.

[48] V. Nguyen. On Base-Stock Policies for Make-to-Order/Make-to-Stock Production. Technical Report 3812-95-MSA, M. I. T., Cambridge, MA, USA, Apr 1995.

[49] N.R.S. Raghavan and N. Viswanadham. Generalized Queueing Network Analysis of Integrated Supply Chains. *International Journal of Production Research*, 39(2):205–224, 2001.

[50] F.A.D. Schouten, M.J.G. Eijs, and R.M.J. Heuts. The Value of Supplier Information to Improve Management of a Retailer's Inventory,. *Decision Sciences*, 25(1):1–14, 1994.

[51] S. Sivaramakrishnan. *Analytical Models for Performance Evaluation of Production-Inventory Systems*. Phd dissertation, Oklahoma State University, Stillwater, OK, USA, 1998.

[52] S. Sivaramakrishnan and M. Kamath. Analytical Models for Multi-Stage Make-to-Stock Systems. In *Proceedings of the 6th Industrial Engineering Research Conference*, pages 795–800, Norcross, GA, USA, 1996.

[53] S. Srivathsan. *Analytical Performance Modeling of Supply Chain Networks*. Master's thesis, Oklahoma State University, Stillwater, OK, USA, Dec 2004.

[54] S. Srivathsan and M. Kamath. An Analytical Performance Modeling Framework for Supply Chain Networks. *IEEE Transactions on Automation Science and Engineering*, 9(2):265–275, 2012.

[55] R. Suri, J.L. Sanders, and M. Kamath. Performance Evaluation of Production Networks. In A.H.G. Rinnooy Kan S.C Graves and P.H. Zipkin, editors, *Logistics of Production and Inventory*, volume 4 of *Handbooks in Operations Research and Management Science*, pages 199–286. Elsevier, 1993.

[56] A. Svoronos and P.H. Zipkin. Evaluation of One-for-One Replenishment Policies for Multi-Echelon Inventory Systems. *Management Science*, 37(1):68–83, 1991.

[57] N. Viswanadham and Y. Narahari. *Performance Modeling of Automated Manufacturing Systems*. Prentice-Hall Inc., Englewood Cliffs, NJ, USA, 1992.

[58] P.D. Welch. The Statistical Analysis of Simulation Results. In S. S. Lavenberg, editor, *Computer Performance Modeling Handbook*. Academic Press, New York, NY, USA, 1983.

[59] W. Whitt. The Queueing Network Analyzer. *The Bell System Technical Journal*, 62(9):2779–2815, 1983.

[60] W. Whitt. Approximations for the GI/G/m Queue. *Production and Operations Management*, 2(2):114–161, 1993.

[61] W. Whitt. Towards Better Multi-Class Parametric-Decomposition Approximations for Open Queueing Networks. *Annals of Operations Research*, 48:221–248, 1994.

[62] R.W. Wolff. Poisson Arrivals See Time Averages. *Operations Research*, 30(2):223–231, 1982.

[63] P.H. Zipkin. Performance Analysis of a Multi-Item Production-Inventory System under Alternative Policies. *Management Science*, 41(4):690–703, 1995.

[64] P.H. Zipkin. Processing Networks with Planned Inventories: Tandem Queues with Feedback. *European Journal of Operational Research*, 80(2):344–349, 1995.

[65] P.H. Zipkin. *Foundations of Inventory Management*. McGraw-Hill Inc., New York, NY, US, 2000.

# APPENDIX A

## DETERMINATION OF THE PARAMETERS FOR SIMULATION EXPERIMENTS

In any simulation experiment, the warm-up period, run length and number of replications are parameters that should be chosen so that the simulation estimates are statistically sound. When the warm-up period is not considered, the simulation results may be affected by some initialization bias. At the same time, if the simulation is not run for a sufficiently long time then any infrequent event may be missed and this may affect the accuracy of the resulting steady-state performance measures. Thus, it is all the more crucial and imperative that the simulation estimates must represent steady-state performance measures, especially when the accuracy of the analytical results is evaluated using these simulation estimates.

In this study, we have simulated several SCN configurations to evaluate the accuracy of the various analytical models developed. It would be time consuming to find the above mentioned parameters for each and every SCN configuration. The key SCN parameters which influence these simulation parameters are the demand inter-arrival time and the service time means and SCVs. Hence, we used a system with high variability and high utilization to determine the warm-up period and run length for all simulation experiments. We fixed the number of independent replications to 10 for all simulation experiments.

The simulation runs to determine these two parameters were performed on the 2R/2P SCN configuration with HiVis. The inter-arrival times as well as the service times followed a hyper-exponential distribution. The arrival rate at each retailer was set at one arrival per unit time and the processing time at each production facility was set as 0.90 time units, so as to obtain 90% utilization. The base-stock level at each stage was set at two and the distribution of transit delay was fixed at Unif(4, 8).

### A.1 Determination of the Warm-up Period using Welch's Method

The simulation was run for 10 independent replications, each with a run length of 30,000 time units. This resulted in approximately 30,000 orders at each retail store. The time spent by a customer order at retail store 1 was considered for the analysis. Welch's method [58] was used to determine the warm-up period. Let $Y_{ij}$ represent the time in the system for the $i^{th}$ customer order from the jth replication ($i$ = 1, 2, $\cdots$, 30,000) and ($j$ = 1, 2, $\cdots$, 10). The average time in the system for the $i^{th}$ customer order was calculated as shown in equation (A.1).

$$\overline{Y}_i = \sum_{j=1}^{10} Y_{ij}/10 \tag{A.1}$$

The averaged processes $\overline{Y}_1, \overline{Y}_2, \cdots, \overline{Y}_{30{,}000}$ have the same mean curves as the original process, but the plot has only $1/10^{th}$ of the variance of the original process. To smoothen out the high frequency oscillations in $\overline{Y}_1, \overline{Y}_2, \cdots, \overline{Y}_{30{,}000}$, the moving average $\overline{Y}_i(w)$ was calculated as shown in equation (A.2), where $w$ is a positive integer.

$$\overline{Y}_i(w) = \begin{cases} \sum_{s=-(i-1)}^{(i-1)} \frac{\overline{Y}_{i+s}}{2i-1} & ; \quad i = 1, \cdots, w \\ \sum_{s=-w}^{w} \frac{\overline{Y}_{i+s}}{2w+1} & ; \quad i = w+1, \cdots, (30,000 - w) \end{cases} \tag{A.2}$$

In our study, the value of $w$ was fixed at 1,000. The process $\overline{Y}_i(1000)$ was plotted for $i$ = 1, 2, $\cdots$, (29,000) and the value of $i$ beyond which $\overline{Y}_i(1000)$ appeared to converge ($i$ = 10,000 in our case) was identified as the warm-up period for our experiments. The plot for $\overline{Y}_i(1000)$ is presented in Figure A.1.

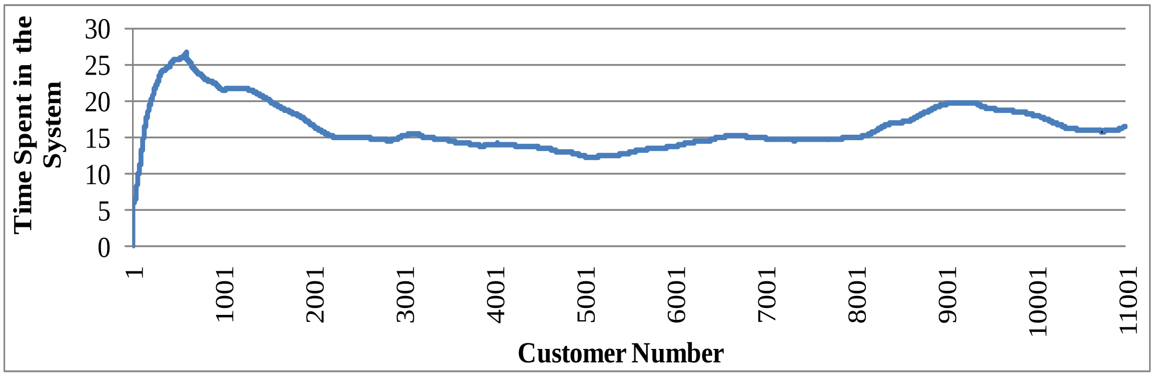



Figure A.1: Plot of Time in System

## A.2 Determination of the Run Length

Different run lengths were explored by setting the warm-up period to 10,000 time units. For each run length, a 95% confidence interval was computed for the average time in system. The goal was to find a run length that yielded a tight confidence interval (e.g., ± 2% of point estimate). A run length of 500,000 time units was chosen as the run length for all simulation experiments.

# APPENDIX B

# CTMC MODEL RESULTS FOR THE 1R/2P SCN CONFIGURATION

This appendix presents the numerical solution of the CTMC models developed in Chapter 5 for the SCNs with HiVis, MedVis, LoVis and NoVis. Sections B.1, B.2, and B.3 present the results for the CTMC model of the 1R/2P SCN configuration with HiVis, MedVis and LoVis, respectively. Section B.4 presents the results for the SCN with no visibility.

## B.1 Validation of the CTMC Model of 1R/2P SCN Configuration with HiVis

This section presents the CTMC model solution and the simulation model estimates for SCN with HiVis. The results for a production facility and the retail store are presented in Tables B.1 and B.2, respectively.

Table B.1: Validation of CTMC Model of SCN with HiVis (Production Facility)

| $S, R, \rho$ | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.720 | 0.720 | 0.386 | 0.387 | 0.911 | 0.911 | 2.754 | 2.758 | 4.773 | 4.774 |
| 2, 2, 0.8 | 0.547 | 0.547 | 1.009 | 1.010 | 0.645 | 0.645 | 4.458 | 4.462 | 6.019 | 6.022 |
| 2, 2, 0.9 | 0.311 | 0.311 | 3.270 | 3.269 | 0.342 | 0.342 | 9.494 | 9.498 | 10.539 | 10.540 |
| 4, 4, 0.7 | 0.932 | 0.932 | 0.093 | 0.093 | 2.618 | 2.618 | 2.746 | 2.748 | 8.187 | 8.188 |
| 4, 4, 0.8 | 0.814 | 0.814 | 0.414 | 0.415 | 2.050 | 2.050 | 4.445 | 4.456 | 8.829 | 8.832 |
| 4, 4, 0.9 | 0.547 | 0.547 | 2.146 | 2.146 | 1.219 | 1.220 | 9.475 | 9.476 | 12.293 | 12.295 |
| 6, 6, 0.7 | 0.984 | 0.984 | 0.022 | 0.022 | 4.547 | 4.547 | 2.745 | 2.728 | 12.046 | 12.046 |
| 6, 6, 0.8 | 0.924 | 0.924 | 0.170 | 0.171 | 3.806 | 3.805 | 4.445 | 4.464 | 12.341 | 12.343 |
| 6, 6, 0.9 | 0.703 | 0.703 | 1.408 | 1.409 | 2.481 | 2.483 | 9.474 | 9.489 | 14.817 | 14.821 |
| 8, 8, 0.7 | 0.996 | 0.996 | 0.005 | 0.005 | 6.530 | 6.529 | 2.745 | 2.653 | 16.012 | 16.012 |
| 8, 8, 0.8 | 0.969 | 0.969 | 0.070 | 0.070 | 5.706 | 5.705 | 4.445 | 4.458 | 16.141 | 16.145 |
| 8, 8, 0.9 | 0.805 | 0.805 | 0.924 | 0.926 | 3.997 | 3.999 | 9.474 | 9.496 | 17.849 | 17.857 |

Table B.2: Validation of CTMC Model of SCN with HiVis (Retail Store)

| $S, R, \rho$ | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.847 | 0.848 | 0.358 | 0.359 | 1.586 | 1.586 | 2.342 | 2.364 | 2.357 | 2.359 |
| 2, 2, 0.8 | 0.686 | 0.687 | 1.261 | 1.264 | 1.243 | 1.243 | 4.010 | 4.043 | 3.259 | 3.264 |
| 2, 2, 0.9 | 0.416 | 0.418 | 5.264 | 5.263 | 0.726 | 0.726 | 9.006 | 9.047 | 7.262 | 7.263 |
| 4, 4, 0.7 | 0.982 | 0.982 | 0.042 | 0.042 | 3.856 | 3.856 | 2.346 | 2.328 | 4.042 | 4.043 |
| 4, 4, 0.8 | 0.917 | 0.917 | 0.330 | 0.333 | 3.502 | 3.502 | 4.005 | 4.028 | 4.330 | 4.333 |
| 4, 4, 0.9 | 0.689 | 0.690 | 2.796 | 2.798 | 2.504 | 2.506 | 9.001 | 9.024 | 6.796 | 6.798 |
| 6, 6, 0.7 | 0.998 | 0.998 | 0.005 | 0.005 | 5.961 | 5.960 | 2.448 | 2.224 | 6.006 | 6.005 |
| 6, 6, 0.8 | 0.979 | 0.978 | 0.087 | 0.088 | 5.748 | 5.746 | 4.019 | 4.013 | 6.087 | 6.089 |
| 6, 6, 0.9 | 0.835 | 0.835 | 1.486 | 1.490 | 4.670 | 4.672 | 9.002 | 9.029 | 7.486 | 7.491 |
| 8, 8, 0.7 | 1.000 | 1.000 | 0.001 | 0.001 | 7.991 | 7.990 | 3.887 | 2.363 | 8.001 | 8.002 |
| 8, 8, 0.8 | 0.994 | 0.994 | 0.023 | 0.023 | 7.885 | 7.882 | 4.072 | 3.984 | 8.024 | 8.024 |
| 8, 8, 0.9 | 0.912 | 0.912 | 0.790 | 0.795 | 6.942 | 6.944 | 9.004 | 9.050 | 8.790 | 8.795 |

## B.2 Validation of the CTMC Model of 1R/2P SCN Configuration with MedVis

This section presents the CTMC model solution and the simulation model estimates for SCN with MedVis. The results for a production facility and the retail store are presented in Tables B.3 and B.4, respectively.

Table B.3: Validation of CTMC Model of SCN with MedVis (Production Facility)

| $S, R, \rho$ | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.700 | 0.700 | 0.410 | 0.413 | 0.856 | 0.856 | 2.738 | 2.757 | 4.820 | 4.827 |
| 2, 2, 0.8 | 0.529 | 0.528 | 1.047 | 1.052 | 0.602 | 0.602 | 4.443 | 4.463 | 6.094 | 6.105 |
| 2, 2, 0.9 | 0.300 | 0.299 | 3.290 | 3.327 | 0.319 | 0.318 | 9.394 | 9.490 | 10.579 | 10.656 |
| 4, 4, 0.7 | 0.910 | 0.910 | 0.124 | 0.124 | 2.343 | 2.343 | 2.745 | 2.754 | 8.247 | 8.249 |
| 4, 4, 0.8 | 0.776 | 0.776 | 0.497 | 0.499 | 1.780 | 1.780 | 4.443 | 4.463 | 8.994 | 9.004 |
| 4, 4, 0.9 | 0.506 | 0.506 | 2.342 | 2.340 | 1.025 | 1.025 | 9.473 | 9.476 | 12.683 | 12.683 |
| 6, 6, 0.7 | 0.973 | 0.973 | 0.035 | 0.037 | 4.072 | 4.071 | 2.646 | 2.738 | 12.071 | 12.076 |
| 6, 6, 0.8 | 0.893 | 0.893 | 0.238 | 0.239 | 3.257 | 3.258 | 4.439 | 4.464 | 12.475 | 12.481 |
| 6, 6, 0.9 | 0.648 | 0.649 | 1.665 | 1.668 | 2.008 | 2.009 | 9.472 | 9.499 | 15.331 | 15.339 |
| 8, 8, 0.7 | 0.992 | 0.992 | 0.011 | 0.011 | 5.912 | 5.913 | 2.744 | 2.699 | 16.022 | 16.022 |
| 8, 8, 0.8 | 0.949 | 0.949 | 0.112 | 0.114 | 4.908 | 4.909 | 4.416 | 4.481 | 16.225 | 16.230 |
| 8, 8, 0.9 | 0.750 | 0.750 | 1.185 | 1.188 | 3.199 | 3.199 | 9.474 | 9.504 | 18.371 | 18.383 |

Table B.4: Validation of CTMC Model of SCN with MedVis (Retail Store)

| $S, R, \rho$ | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.837 | 0.838 | 0.379 | 0.384 | 1.558 | 1.557 | 2.323 | 2.364 | 2.377 | 2.384 |
| 2, 2, 0.8 | 0.673 | 0.675 | 1.306 | 1.315 | 1.212 | 1.212 | 3.991 | 4.043 | 3.303 | 3.316 |
| 2, 2, 0.9 | 0.406 | 0.408 | 5.284 | 5.357 | 0.705 | 0.703 | 8.898 | 9.045 | 7.281 | 7.358 |
| 4, 4, 0.7 | 0.976 | 0.976 | 0.056 | 0.056 | 3.809 | 3.809 | 2.333 | 2.341 | 4.056 | 4.057 |
| 4, 4, 0.8 | 0.901 | 0.901 | 0.396 | 0.399 | 3.402 | 3.402 | 4.002 | 4.028 | 4.395 | 4.399 |
| 4, 4, 0.9 | 0.661 | 0.662 | 3.050 | 3.051 | 2.367 | 2.370 | 8.999 | 9.022 | 7.050 | 7.051 |
| 6, 6, 0.7 | 0.997 | 0.996 | 0.007 | 0.008 | 5.936 | 5.934 | 2.011 | 2.252 | 6.007 | 6.008 |
| 6, 6, 0.8 | 0.970 | 0.969 | 0.121 | 0.123 | 5.646 | 5.644 | 3.987 | 4.014 | 6.121 | 6.123 |
| 6, 6, 0.9 | 0.805 | 0.805 | 1.757 | 1.764 | 4.427 | 4.429 | 8.999 | 9.036 | 7.757 | 7.765 |
| 8, 8, 0.7 | 1.000 | 1.000 | 0.001 | 0.001 | 7.979 | 7.979 | 2.335 | 2.338 | 8.001 | 8.002 |
| 8, 8, 0.8 | 0.991 | 0.991 | 0.036 | 0.038 | 7.811 | 7.809 | 3.918 | 3.994 | 8.036 | 8.039 |
| 8, 8, 0.9 | 0.887 | 0.888 | 1.013 | 1.020 | 6.642 | 6.645 | 8.999 | 9.048 | 9.013 | 9.020 |

## B.3 Validation of the CTMC Model of 1R/2P SCN Configuration with LoVis

This section presents the CTMC model solution and the simulation model estimates for SCN with LoVis. The results for a production facility and the retail store are presented in Tables B.5 and B.6, respectively.

Table B.5: Validation of CTMC Model of SCN with LoVis (Production Facility)

| $S, R, \rho$ | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.718 | 0.718 | 0.465 | 0.468 | 0.859 | 0.859 | 3.298 | 3.293 | 4.931 | 4.937 |
| 2, 2, 0.8 | 0.556 | 0.556 | 1.188 | 1.188 | 0.608 | 0.608 | 5.349 | 5.359 | 6.375 | 6.379 |
| 2, 2, 0.9 | 0.332 | 0.332 | 3.779 | 3.775 | 0.325 | 0.326 | 11.308 | 11.298 | 11.484 | 11.550 |
| 4, 4, 0.7 | 0.918 | 0.918 | 0.133 | 0.133 | 2.347 | 2.348 | 3.230 | 3.242 | 8.266 | 8.269 |
| 4, 4, 0.8 | 0.797 | 0.797 | 0.527 | 0.532 | 1.791 | 1.792 | 5.184 | 5.228 | 9.054 | 9.066 |
| 4, 4, 0.9 | 0.546 | 0.547 | 2.442 | 2.443 | 1.049 | 1.050 | 10.754 | 10.774 | 12.884 | 12.883 |
| 6, 6, 0.7 | 0.976 | 0.975 | 0.040 | 0.040 | 4.073 | 4.074 | 3.225 | 3.236 | 12.079 | 12.089 |
| 6, 6, 0.8 | 0.904 | 0.904 | 0.249 | 0.251 | 3.265 | 3.267 | 5.168 | 5.198 | 12.499 | 12.507 |
| 6, 6, 0.9 | 0.681 | 0.681 | 1.707 | 1.716 | 2.033 | 2.036 | 10.687 | 10.753 | 15.408 | 15.438 |
| 8, 8, 0.7 | 0.993 | 0.993 | 0.012 | 0.012 | 5.913 | 5.913 | 3.225 | 3.1999 | 16.023 | 16.029 |
| 8, 8, 0.8 | 0.954 | 0.954 | 0.118 | 0.120 | 4.912 | 4.913 | 5.166 | 5.231 | 16.237 | 16.245 |
| 8, 8, 0.9 | 0.774 | 0.774 | 1.210 | 1.216 | 3.220 | 3.223 | 10.692 | 10.734 | 18.375 | 18.434 |

Table B.6: Validation of CTMC Model of SCN with LoVis (Retail Store)

| $S, R, \rho$ | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim | CTMC | Sim |
| 2, 2, 0.7 | 0.843 | 0.842 | 0.442 | 0.443 | 1.511 | 1.511 | 2.811 | 2.817 | 2.442 | 2.443 |
| 2, 2, 0.8 | 0.683 | 0.682 | 1.511 | 1.514 | 1.136 | 1.136 | 4.768 | 4.777 | 3.511 | 3.514 |
| 2, 2, 0.9 | 0.418 | 0.417 | 6.161 | 6.151 | 0.606 | 0.606 | 10.475 | 10.572 | 8.093 | 8.151 |
| 4, 4, 0.7 | 0.977 | 0.977 | 0.059 | 0.059 | 3.793 | 3.793 | 2.591 | 2.589 | 4.059 | 4.060 |
| 4, 4, 0.8 | 0.906 | 0.906 | 0.416 | 0.418 | 3.362 | 3.362 | 4.425 | 4.453 | 4.416 | 4.419 |
| 4, 4, 0.9 | 0.677 | 0.676 | 3.176 | 3.174 | 2.292 | 2.295 | 9.833 | 9.853 | 7.176 | 7.175 |
| 6, 6, 0.7 | 0.997 | 0.997 | 0.008 | 0.008 | 5.929 | 5.929 | 2.506 | 2.368 | 6.008 | 6.009 |
| 6, 6, 0.8 | 0.971 | 0.971 | 0.124 | 0.126 | 5.626 | 5.625 | 4.300 | 4.320 | 6.125 | 6.126 |
| 6, 6, 0.9 | 0.814 | 0.812 | 1.788 | 1.791 | 4.374 | 4.377 | 9.585 | 9.637 | 7.783 | 7.792 |
| 8, 8, 0.7 | 1.000 | 1.000 | 0.001 | 0.001 | 7.978 | 7.978 | 2.497 | 2.430 | 8.001 | 8.002 |
| 8, 8, 0.8 | 0.991 | 0.991 | 0.038 | 0.038 | 7.801 | 7.799 | 4.222 | 4.192 | 8.038 | 8.039 |
| 8, 8, 0.9 | 0.892 | 0.890 | 1.024 | 1.031 | 6.604 | 6.605 | 9.197 | 9.626 | 8.988 | 9.031 |

## B.4 CTMC Model Results for 1R/2P SCN Configuration with NoVis

This section presents the CTMC model results for the SCN with NoVis. Tables B.7 and B.8 present the performance measures for the different stores.

Table B.7: Exact Results for the SCN with NoVis (Production Facility)

| $S, R, \rho$ | $f^p$ | $E[B^p]$ | $E[I^p]$ | $E[W^{bp}]$ | $E[T^p]$ |
|---|---|---|---|---|---|
| | CTMC | CTMC | CTMC | CTMC | CTMC |
| 2, 2, 0.7 | 0.510 | 1.143 | 0.810 | 4.667 | 6.286 |
| 2, 2, 0.8 | 0.360 | 2.560 | 0.560 | 8.000 | 9.120 |
| 2, 2, 0.9 | 0.190 | 7.290 | 0.290 | 18.000 | 18.580 |
| 4, 4, 0.7 | 0.760 | 0.560 | 2.227 | 4.667 | 9.120 |
| 4, 4, 0.8 | 0.590 | 1.638 | 1.638 | 8.000 | 11.276 |
| 4, 4, 0.9 | 0.344 | 5.905 | 0.905 | 18.000 | 19.810 |
| 6, 6, 0.7 | 0.882 | 0.275 | 3.941 | 4.667 | 12.549 |
| 6, 6, 0.8 | 0.738 | 1.049 | 3.049 | 8.000 | 14.100 |
| 6, 6, 0.9 | 0.469 | 4.783 | 1.783 | 18.000 | 21.566 |
| 8, 8, 0.7 | 0.942 | 0.135 | 5.801 | 4.667 | 16.249 |
| 8, 8, 0.8 | 0.832 | 0.671 | 4.671 | 8.000 | 17.342 |
| 8, 8, 0.9 | 0.570 | 3.874 | 2.874 | 18.000 | 23.748 |

Table B.8: Exact Results for the SCN with NoVis (Retail Store)

| $S, R, \rho$ | $f^r$ | $E[B^r]$ | $E[I^r]$ | $E[W^{br}]$ | $E[T^r]$ |
|---|---|---|---|---|---|
| | CTMC | CTMC | CTMC | CTMC | CTMC |
| 2, 2, 0.7 | 0.567 | 1.285 | 0.999 | 2.968 | 3.285 |
| 2, 2, 0.8 | 0.338 | 3.696 | 0.576 | 5.583 | 5.696 |
| 2, 2, 0.9 | 0.113 | 12.766 | 0.186 | 14.390 | 14.766 |
| 4, 4, 0.7 | 0.882 | 0.308 | 3.187 | 2.610 | 4.308 |
| 4, 4, 0.8 | 0.678 | 1.562 | 2.285 | 4.851 | 5.562 |
| 4, 4, 0.9 | 0.309 | 8.765 | 0.955 | 12.680 | 12.765 |
| 6, 6, 0.7 | 0.971 | 0.071 | 5.522 | 2.448 | 6.071 |
| 6, 6, 0.8 | 0.859 | 0.636 | 4.539 | 4.511 | 6.636 |
| 6, 6, 0.9 | 0.495 | 5.894 | 2.328 | 11.670 | 11.894 |
| 8, 8, 0.7 | 0.993 | 0.017 | 7.748 | 2.429 | 8.017 |
| 8, 8, 0.8 | 0.941 | 0.255 | 6.913 | 4.322 | 8.255 |
| 8, 8, 0.9 | 0.645 | 3.910 | 4.161 | 11.010 | 11.910 |

# APPENDIX C

# QUEUEING RESULTS FOR THE 1R/2P SCN CONFIGURATION WITH POISSON ARRIVALS AND EXPONENTIAL PROCESSING TIMES

This appendix presents the results for the approximate models developed in Chapter 6 for the 1R/2P SCN configuration under all three levels of inventory visibility. Sections C.1 and C.2 present the results for the validation of the M/M/2 models (primarily for the HiVis case) and the modified M/M/2 model (for lower levels of visibility), respectively. Section C.3 presents the results for the validation of the M/G/2 based approximation for the asymmetric 1R/2P SCN configuration.

## C.1 Validation of the M/M/2 Approximation

This section presents analytical and simulation results for the validation of the M/M/2 based approximate model for the HiVis case. The values of the performance measures at the retail store and a production facility are presented in Tables C.1 through C.5.

Table C.1: Fill Rate and Expected Number of Backorders at the Retail Store

| $S, R, \rho$ | $f^r$ | | | | | $E[B^r]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NoVis | LoVis | MedVis | HiVis | Queueing | NoVis | LoVis | MedVis | HiVis | Queueing |
| 2, 2, 0.7 | 0.567 | 0.803 | 0.837 | 0.847 | 0.862 | 1.285 | 0.442 | 0.379 | 0.358 | 0.323 |
| 2, 2, 0.8 | 0.338 | 0.620 | 0.673 | 0.686 | 0.709 | 3.696 | 1.511 | 1.306 | 1.261 | 1.165 |
| 2, 2, 0.9 | 0.113 | 0.340 | 0.406 | 0.416 | 0.441 | 12.766 | 6.161 | 5.284 | 5.265 | 5.035 |
| 4, 4, 0.7 | 0.882 | 0.974 | 0.976 | 0.982 | 0.984 | 0.308 | 0.059 | 0.056 | 0.042 | 0.038 |
| 4, 4, 0.8 | 0.678 | 0.894 | 0.901 | 0.917 | 0.924 | 1.562 | 0.416 | 0.396 | 0.330 | 0.305 |
| 4, 4, 0.9 | 0.309 | 0.643 | 0.661 | 0.689 | 0.703 | 8.765 | 3.177 | 3.050 | 2.796 | 2.676 |
| 6, 6, 0.7 | 0.971 | 0.996 | 0.997 | 0.998 | 0.998 | 0.071 | 0.008 | 0.007 | 0.005 | 0.005 |
| 6, 6, 0.8 | 0.859 | 0.968 | 0.970 | 0.979 | 0.980 | 0.636 | 0.124 | 0.121 | 0.087 | 0.080 |
| 6, 6, 0.9 | 0.495 | 0.799 | 0.805 | 0.835 | 0.842 | 5.894 | 1.788 | 1.757 | 1.486 | 1.422 |
| 8, 8, 0.7 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 0.017 | 0.001 | 0.001 | 0.001 | 0.001 |
| 8, 8, 0.8 | 0.941 | 0.991 | 0.991 | 0.994 | 0.995 | 0.255 | 0.038 | 0.036 | 0.023 | 0.021 |
| 8, 8, 0.9 | 0.645 | 0.885 | 0.887 | 0.912 | 0.916 | 3.910 | 1.024 | 1.013 | 0.790 | 0.756 |

Table C.2: Expected Inventory Level and Expected Time to Fulfill a Backorder at the Retail Store

| $S, R, \rho$ | $E[I^r]$ | | | | | $E[W^{br}]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NoVis | LoVis | MedVis | HiVis | Queueing | NoVis | LoVis | MedVis | HiVis | Queueing |
| 2, 2, 0.7 | 0.999 | 1.511 | 1.558 | 1.586 | 1.664 | 2.968 | 2.248 | 2.323 | 2.342 | 2.333 |
| 2, 2, 0.8 | 0.576 | 1.136 | 1.212 | 1.243 | 1.345 | 5.583 | 3.972 | 3.991 | 4.010 | 4.000 |
| 2, 2, 0.9 | 0.186 | 0.606 | 0.705 | 0.726 | 0.819 | 14.392 | 9.337 | 8.898 | 9.006 | 9.000 |
| 4, 4, 0.7 | 3.187 | 3.793 | 3.809 | 3.856 | 3.880 | 2.610 | 2.256 | 2.333 | 2.346 | 2.333 |
| 4, 4, 0.8 | 2.285 | 3.362 | 3.402 | 3.502 | 3.560 | 4.851 | 3.900 | 4.002 | 4.005 | 4.000 |
| 4, 4, 0.9 | 0.955 | 2.292 | 2.367 | 2.504 | 2.598 | 12.685 | 8.899 | 8.999 | 9.001 | 9.000 |
| 6, 6, 0.7 | 5.522 | 5.929 | 5.936 | 5.961 | 5.967 | 2.448 | 2.292 | 2.011 | 2.448 | 2.333 |
| 6, 6, 0.8 | 4.539 | 5.626 | 5.646 | 5.748 | 5.775 | 4.511 | 3.932 | 3.987 | 4.019 | 4.000 |
| 6, 6, 0.9 | 2.328 | 4.374 | 4.427 | 4.670 | 4.746 | 11.671 | 8.900 | 8.999 | 9.002 | 9.000 |
| 8, 8, 0.7 | 7.748 | 7.978 | 7.979 | 7.991 | 7.991 | 2.429 | 2.347 | 2.335 | 3.887 | 2.333 |
| 8, 8, 0.8 | 6.913 | 7.801 | 7.811 | 7.885 | 7.896 | 4.322 | 3.956 | 3.918 | 4.072 | 4.000 |
| 8, 8, 0.9 | 4.161 | 6.604 | 6.642 | 6.942 | 7.000 | 11.014 | 8.925 | 8.999 | 9.004 | 9.000 |

Table C.3: Fill Rate and Expected Number of Backorders at a Production Facility

| $S, R, \rho$ | $f^p$ | | | | | $E[B^p]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NoVis | LoVis | MedVis | HiVis | Queueing | NoVis | LoVis | MedVis | HiVis | Queueing |
| 2, 2, 0.7 | 0.510 | 0.718 | 0.700 | 0.720 | 0.718 | 1.143 | 0.465 | 0.410 | 0.386 | 0.330 |
| 2, 2, 0.8 | 0.360 | 0.556 | 0.529 | 0.547 | 0.545 | 2.560 | 1.188 | 1.047 | 1.009 | 0.910 |
| 2, 2, 0.9 | 0.190 | 0.332 | 0.300 | 0.311 | 0.309 | 7.290 | 3.779 | 3.290 | 3.270 | 3.108 |
| 4, 4, 0.7 | 0.760 | 0.918 | 0.910 | 0.932 | 0.932 | 0.560 | 0.133 | 0.124 | 0.093 | 0.079 |
| 4, 4, 0.8 | 0.590 | 0.797 | 0.776 | 0.814 | 0.814 | 1.638 | 0.527 | 0.497 | 0.414 | 0.373 |
| 4, 4, 0.9 | 0.344 | 0.546 | 0.506 | 0.547 | 0.547 | 5.905 | 2.442 | 2.342 | 2.146 | 2.039 |
| 6, 6, 0.7 | 0.882 | 0.976 | 0.973 | 0.984 | 0.983 | 0.275 | 0.040 | 0.035 | 0.022 | 0.019 |
| 6, 6, 0.8 | 0.738 | 0.904 | 0.893 | 0.924 | 0.924 | 1.049 | 0.249 | 0.238 | 0.170 | 0.153 |
| 6, 6, 0.9 | 0.469 | 0.681 | 0.648 | 0.703 | 0.702 | 4.783 | 1.707 | 1.665 | 1.408 | 1.338 |
| 8, 8, 0.7 | 0.942 | 0.993 | 0.992 | 0.996 | 0.996 | 0.135 | 0.012 | 0.011 | 0.005 | 0.005 |
| 8, 8, 0.8 | 0.832 | 0.954 | 0.949 | 0.969 | 0.969 | 0.671 | 0.118 | 0.112 | 0.070 | 0.063 |
| 8, 8, 0.9 | 0.570 | 0.774 | 0.750 | 0.805 | 0.805 | 3.874 | 1.210 | 1.185 | 0.924 | 0.878 |

Table C.4: Expected Inventory Level and Expected Time to Fulfill a Backorder at a Production Facility

| $S, R, \rho$ | $E[I^p]$ | | | | | $E[W^{bp}]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NoVis | LoVis | MedVis | HiVis | Queueing | NoVis | LoVis | MedVis | HiVis | Queueing |
| 2, 2, 0.7 | 0.810 | 0.859 | 0.856 | 0.911 | 0.957 | 4.665 | 3.298 | 2.738 | 2.754 | 2.333 |
| 2, 2, 0.8 | 0.560 | 0.608 | 0.602 | 0.645 | 0.688 | 8.000 | 5.349 | 4.443 | 4.458 | 4.000 |
| 2, 2, 0.9 | 0.290 | 0.325 | 0.319 | 0.342 | 0.370 | 18.000 | 11.308 | 9.394 | 9.494 | 9.000 |
| 4, 4, 0.7 | 2.227 | 2.347 | 2.343 | 2.618 | 2.707 | 4.667 | 3.230 | 2.745 | 2.746 | 2.333 |
| 4, 4, 0.8 | 1.638 | 1.791 | 1.780 | 2.050 | 2.151 | 7.990 | 5.184 | 4.443 | 4.445 | 4.000 |
| 4, 4, 0.9 | 0.905 | 1.049 | 1.025 | 1.219 | 1.302 | 18.003 | 10.754 | 9.473 | 9.475 | 9.000 |
| 6, 6, 0.7 | 3.941 | 4.073 | 4.072 | 4.547 | 4.646 | 4.661 | 3.225 | 2.646 | 2.745 | 2.333 |
| 6, 6, 0.8 | 3.049 | 3.265 | 3.257 | 3.806 | 3.931 | 8.008 | 5.168 | 4.439 | 4.445 | 4.000 |
| 6, 6, 0.9 | 1.783 | 2.033 | 2.008 | 2.481 | 2.601 | 18.015 | 10.687 | 9.472 | 9.474 | 9.000 |
| 8, 8, 0.7 | 5.801 | 5.913 | 5.912 | 6.530 | 6.632 | 4.655 | 3.225 | 2.744 | 2.745 | 2.333 |
| 8, 8, 0.8 | 4.671 | 4.912 | 4.908 | 5.706 | 5.840 | 7.988 | 5.166 | 4.416 | 4.445 | 4.000 |
| 8, 8, 0.9 | 2.874 | 3.220 | 3.199 | 3.997 | 4.141 | 18.019 | 10.692 | 9.474 | 9.474 | 9.000 |

Table C.5: Expected Time Spent by an Order at the Retail Store and a Production Facility

| $S, R, \rho$ | $E[T^r]$ | | | | | $E[T^p]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NoVis | LoVis | MedVis | HiVis | Queueing | NoVis | LoVis | MedVis | HiVis | Queueing |
| 2, 2, 0.7 | 3.285 | 2.442 | 2.377 | 2.357 | 2.323 | 6.287 | 4.931 | 4.820 | 4.773 | 4.659 |
| 2, 2, 0.8 | 5.696 | 3.511 | 3.303 | 3.259 | 3.165 | 9.120 | 6.375 | 6.094 | 6.019 | 5.820 |
| 2, 2, 0.9 | 14.766 | 8.093 | 7.281 | 7.262 | 7.035 | 18.580 | 11.484 | 10.579 | 10.539 | 10.216 |
| 4, 4, 0.7 | 4.308 | 4.059 | 4.056 | 4.042 | 4.038 | 9.120 | 8.266 | 8.247 | 8.187 | 8.158 |
| 4, 4, 0.8 | 5.562 | 4.416 | 4.395 | 4.330 | 4.305 | 11.277 | 9.054 | 8.994 | 8.829 | 8.746 |
| 4, 4, 0.9 | 12.765 | 7.176 | 7.050 | 6.796 | 6.676 | 19.810 | 12.884 | 12.683 | 12.293 | 12.078 |
| 6, 6, 0.7 | 6.071 | 6.008 | 6.007 | 6.006 | 6.004 | 12.549 | 12.079 | 12.071 | 12.046 | 12.038 |
| 6, 6, 0.8 | 6.636 | 6.125 | 6.121 | 6.087 | 6.080 | 14.097 | 12.499 | 12.475 | 12.341 | 12.305 |
| 6, 6, 0.9 | 11.894 | 7.783 | 7.757 | 7.486 | 7.422 | 21.566 | 15.408 | 15.331 | 14.817 | 14.676 |
| 8, 8, 0.7 | 8.017 | 8.001 | 8.001 | 8.001 | 8.001 | 16.249 | 16.023 | 16.022 | 16.012 | 16.009 |
| 8, 8, 0.8 | 8.255 | 8.038 | 8.036 | 8.024 | 8.021 | 17.342 | 16.237 | 16.225 | 16.141 | 16.125 |
| 8, 8, 0.9 | 11.910 | 8.988 | 9.013 | 8.790 | 8.756 | 23.748 | 18.375 | 18.371 | 17.849 | 17.755 |

## C.2 Validation of the Modified M/M/2 Model

This section presents the analytical and simulation results for the validation of the modified M/M/2 model for the LoVis and MedVis cases. The values of the performance measures at the retail store and a production facility are presented in Tables C.6 through C.10.

Table C.6: Fill Rate and Expected Number of Backorders at the Retail Store

| $S, R, \rho$ | $f^r$ | | | | $E[B^r]$ | | | |
|---|---|---|---|---|---|---|---|---|
| | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| 2, 2, 0.7 | 0.803 | 0.837 | 0.838 | 0.862 | 0.442 | 0.379 | 0.378 | 0.323 |
| 2, 2, 0.8 | 0.620 | 0.673 | 0.669 | 0.709 | 1.511 | 1.306 | 1.324 | 1.165 |
| 2, 2, 0.9 | 0.340 | 0.406 | 0.399 | 0.441 | 6.161 | 5.284 | 5.406 | 5.035 |
| 4, 4, 0.7 | 0.974 | 0.976 | 0.979 | 0.984 | 0.059 | 0.056 | 0.049 | 0.038 |
| 4, 4, 0.8 | 0.894 | 0.901 | 0.908 | 0.924 | 0.416 | 0.396 | 0.367 | 0.305 |
| 4, 4, 0.9 | 0.643 | 0.661 | 0.674 | 0.703 | 3.177 | 3.050 | 2.927 | 2.676 |
| 6, 6, 0.7 | 0.996 | 0.997 | 0.997 | 0.998 | 0.008 | 0.007 | 0.006 | 0.005 |
| 6, 6, 0.8 | 0.968 | 0.970 | 0.975 | 0.980 | 0.124 | 0.121 | 0.099 | 0.080 |
| 6, 6, 0.9 | 0.799 | 0.805 | 0.825 | 0.842 | 1.788 | 1.757 | 1.575 | 1.422 |
| 8, 8, 0.7 | 1.000 | 1.000 | 1.000 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 |
| 8, 8, 0.8 | 0.991 | 0.991 | 0.994 | 0.995 | 0.038 | 0.036 | 0.026 | 0.021 |
| 8, 8, 0.9 | 0.885 | 0.887 | 0.906 | 0.916 | 1.024 | 1.013 | 0.843 | 0.756 |

Table C.7: Expected Inventory Level and Expected Time to Fulfill a Backorder at the Retail Store

| $S, R, \rho$ | $E[I^r]$ | | | | $E[W^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|
| | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| 2, 2, 0.7 | 1.511 | 1.558 | 1.607 | 1.664 | 2.248 | 2.323 | 2.333 | 2.333 |
| 2, 2, 0.8 | 1.136 | 1.212 | 1.255 | 1.345 | 3.972 | 3.991 | 4.000 | 4.000 |
| 2, 2, 0.9 | 0.606 | 0.705 | 0.731 | 0.819 | 9.337 | 8.898 | 8.995 | 9.000 |
| 4, 4, 0.7 | 3.793 | 3.809 | 3.845 | 3.880 | 2.256 | 2.333 | 2.333 | 2.333 |
| 4, 4, 0.8 | 3.362 | 3.402 | 3.472 | 3.560 | 3.900 | 4.002 | 4.000 | 4.000 |
| 4, 4, 0.9 | 2.292 | 2.367 | 2.464 | 2.598 | 8.899 | 8.999 | 8.993 | 9.000 |
| 6, 6, 0.7 | 5.929 | 5.936 | 5.955 | 5.967 | 2.292 | 2.011 | 2.333 | 2.333 |
| 6, 6, 0.8 | 5.626 | 5.646 | 5.723 | 5.775 | 3.932 | 3.987 | 4.000 | 4.000 |
| 6, 6, 0.9 | 4.374 | 4.427 | 4.609 | 4.746 | 8.900 | 8.999 | 8.985 | 9.000 |
| 8, 8, 0.7 | 7.978 | 7.979 | 7.988 | 7.991 | 2.347 | 2.335 | 2.333 | 2.333 |
| 8, 8, 0.8 | 7.801 | 7.811 | 7.870 | 7.896 | 3.956 | 3.918 | 4.000 | 4.000 |
| 8, 8, 0.9 | 6.604 | 6.642 | 6.881 | 7.000 | 8.925 | 8.999 | 8.974 | 9.000 |

Table C.8: Fill Rate and Expected Number of Backorders at a Production Facility

| $S, R, \rho$ | $f^p$ | | | | $E[B^p]$ | | | |
|---|---|---|---|---|---|---|---|---|
| | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| 2, 2, 70% | 0.718 | 0.700 | 0.669 | 0.718 | 0.465 | 0.410 | 0.386 | 0.330 |
| 2, 2, 80% | 0.556 | 0.529 | 0.483 | 0.545 | 1.188 | 1.047 | 1.034 | 0.910 |
| 2, 2, 90% | 0.332 | 0.300 | 0.258 | 0.309 | 3.779 | 3.290 | 3.337 | 3.108 |
| 4, 4, 70% | 0.918 | 0.910 | 0.912 | 0.932 | 0.133 | 0.124 | 0.102 | 0.079 |
| 4, 4, 80% | 0.797 | 0.776 | 0.776 | 0.814 | 0.527 | 0.497 | 0.447 | 0.373 |
| 4, 4, 80% | 0.546 | 0.506 | 0.504 | 0.547 | 2.442 | 2.342 | 2.232 | 2.039 |
| 6, 6, 70% | 0.976 | 0.973 | 0.978 | 0.983 | 0.040 | 0.035 | 0.025 | 0.019 |
| 6, 6, 80% | 0.904 | 0.893 | 0.906 | 0.924 | 0.249 | 0.238 | 0.188 | 0.153 |
| 6, 6, 90% | 0.681 | 0.648 | 0.670 | 0.703 | 1.707 | 1.665 | 1.483 | 1.338 |
| 8, 8, 70% | 0.993 | 0.992 | 0.995 | 0.996 | 0.012 | 0.011 | 0.006 | 0.005 |
| 8, 8, 80% | 0.954 | 0.949 | 0.961 | 0.969 | 0.118 | 0.112 | 0.078 | 0.063 |
| 8, 8, 70% | 0.774 | 0.750 | 0.782 | 0.805 | 1.210 | 1.185 | 0.981 | 0.878 |

Table C.9: Expected Inventory Level and Expected Time to Fulfill a Backorder at a Production Facility

| $S, R, \rho$ | $E[I^p]$ | | | | $E[W^{bp}]$ | | | |
|---|---|---|---|---|---|---|---|---|
| | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| 2, 2, 70% | 0.859 | 0.856 | 0.829 | 0.957 | 3.298 | 2.738 | 2.333 | 2.333 |
| 2, 2, 80% | 0.608 | 0.602 | 0.551 | 0.688 | 5.349 | 4.443 | 4.000 | 4.000 |
| 2, 2, 90% | 0.325 | 0.319 | 0.274 | 0.370 | 11.308 | 9.394 | 8.996 | 9.000 |
| 4, 4, 70% | 2.347 | 2.343 | 2.414 | 2.707 | 3.230 | 2.745 | 2.333 | 2.333 |
| 4, 4, 80% | 1.791 | 1.780 | 1.843 | 2.151 | 5.184 | 4.443 | 4.000 | 4.000 |
| 4, 4, 80% | 1.049 | 1.025 | 1.076 | 1.302 | 10.754 | 9.473 | 8.994 | 9.000 |
| 6, 6, 70% | 4.073 | 4.072 | 4.290 | 4.646 | 3.225 | 2.646 | 2.333 | 2.333 |
| 6, 6, 80% | 3.265 | 3.257 | 3.524 | 3.931 | 5.168 | 4.439 | 4.000 | 4.000 |
| 6, 6, 90% | 2.033 | 2.008 | 2.265 | 2.601 | 10.687 | 9.472 | 8.992 | 9.000 |
| 8, 8, 70% | 5.913 | 5.912 | 6.254 | 6.632 | 3.225 | 2.744 | 2.333 | 2.333 |
| 8, 8, 80% | 4.912 | 4.908 | 5.385 | 5.840 | 5.166 | 4.416 | 4.000 | 4.000 |
| 8, 8, 70% | 3.220 | 3.199 | 3.720 | 4.141 | 10.692 | 9.474 | 8.988 | 9.000 |

Table C.10: Expected Time Spent by an Order at the Retail Store and a Production Facility

| $S, R, \rho$ | $E[T^r]$ | | | | $E[T^p]$ | | | |
|---|---|---|---|---|---|---|---|---|
| | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| 2, 2, 0.7 | 2.442 | 2.377 | 2.378 | 3.249 | 4.931 | 4.820 | 4.772 | 4.659 |
| 2, 2, 0.8 | 3.511 | 3.303 | 3.324 | 2.345 | 6.375 | 6.094 | 6.069 | 5.820 |
| 2, 2, 0.9 | 8.093 | 7.281 | 7.406 | 1.819 | 11.484 | 10.579 | 10.675 | 10.216 |
| 4, 4, 0.7 | 4.059 | 4.056 | 4.049 | 4.880 | 8.266 | 8.247 | 8.204 | 8.158 |
| 4, 4, 0.8 | 4.416 | 4.395 | 4.367 | 4.560 | 9.054 | 8.994 | 8.895 | 8.746 |
| 4, 4, 0.9 | 7.176 | 7.050 | 6.927 | 3.598 | 12.884 | 12.683 | 12.463 | 12.078 |
| 6, 6, 0.7 | 6.008 | 6.007 | 6.006 | 6.967 | 12.079 | 12.071 | 12.051 | 12.038 |
| 6, 6, 0.8 | 6.125 | 6.121 | 6.099 | 6.775 | 12.499 | 12.475 | 12.376 | 12.305 |
| 6, 6, 0.9 | 7.783 | 7.757 | 7.575 | 5.746 | 15.408 | 15.331 | 14.965 | 14.676 |
| 8, 8, 0.7 | 8.001 | 8.001 | 8.001 | 8.991 | 16.023 | 16.022 | 16.012 | 16.009 |
| 8, 8, 0.8 | 8.038 | 8.036 | 8.026 | 8.896 | 16.237 | 16.225 | 16.156 | 16.125 |
| 8, 8, 0.9 | 8.988 | 9.013 | 8.843 | 8.000 | 18.375 | 18.371 | 17.961 | 17.755 |

## C.3 Validation of the M/G/2 based model for the 1R/2P SCN with Heterogeneous Production Facilities

This section presents the analytical and simulation results for the validation of the M/G/2 based approximation for the heterogeneous 1R/2P SCN configuration. The values of the performance measures at the retail store and a production facility are presented in Tables C.11 through C.14, respectively.

Table C.11: Performance Measures at the Retail Store

| | | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $(S,R)$ | $(\tau_1,\tau_2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| (2, 1) | (1.3, 1.5) | 0.791 | 0.804 | 0.503 | 0.455 | 0.749 | 0.804 | 2.408 | 2.325 | 1.504 | 1.455 |
| | (1.2 1.6) | 0.803 | 0.810 | 0.454 | 0.439 | 0.759 | 0.810 | 2.304 | 2.304 | 1.453 | 1.439 |
| | (1.1, 1.7) | 0.822 | 0.817 | 0.383 | 0.416 | 0.776 | 0.817 | 2.148 | 2.277 | 1.383 | 1.416 |
| | (1.4, 1.4) | 0.782 | 0.802 | 0.51 | 0.461 | 0.782 | 0.802 | 2.335 | 2.333 | 1.510 | 1.461 |
| | (1.5, 1.7) | 0.620 | 0.640 | 1.544 | 1.427 | 0.570 | 0.640 | 4.066 | 3.963 | 2.543 | 2.427 |
| | (1.4, 1.8) | 0.637 | 0.651 | 1.399 | 1.348 | 0.585 | 0.651 | 3.857 | 3.861 | 2.398 | 2.348 |
| | (1.3, 1.9) | 0.665 | 0.668 | 1.190 | 1.235 | 0.609 | 0.668 | 3.548 | 3.716 | 2.190 | 2.234 |
| | (1.6, 1.6) | 0.607 | 0.636 | 1.009 | 1.456 | 0.607 | 0.636 | 4.003 | 4.000 | 2.573 | 2.456 |
| | (1.7, 1.9) | 0.366 | 0.385 | 5.689 | 5.409 | 0.325 | 0.385 | 8.975 | 8.799 | 6.689 | 6.409 |
| | (1.6, 2.0) | 0.389 | 0.405 | 5.039 | 4.911 | 0.345 | 0.405 | 8.249 | 8.258 | 6.038 | 5.911 |
| | (1.5, 2.1) | 0.426 | 0.436 | 4.164 | 4.249 | 0.376 | 0.436 | 7.255 | 7.529 | 5.164 | 5.249 |
| | (1.8, 1.8) | 0.351 | 0.378 | 5.846 | 5.594 | 0.351 | 0.378 | 9.003 | 9.000 | 6.846 | 6.594 |
| (4, 2) | (1.3, 1.5) | 0.965 | 0.967 | 0.082 | 0.076 | 1.903 | 1.920 | 2.334 | 2.325 | 2.082 | 2.076 |
| | (1.2 1.6) | 0.969 | 0.969 | 0.068 | 0.072 | 1.913 | 1.924 | 2.228 | 2.304 | 2.068 | 2.072 |
| | (1.1, 1.7) | 0.976 | 0.970 | 0.051 | 0.067 | 1.928 | 1.928 | 2.064 | 2.277 | 2.051 | 2.067 |
| | (1.4, 1.4) | 0.963 | 0.967 | 0.086 | 0.078 | 1.911 | 1.919 | 2.340 | 2.333 | 2.086 | 2.078 |
| | (1.5, 1.7) | 0.875 | 0.883 | 0.496 | 0.463 | 1.695 | 1.737 | 3.976 | 3.963 | 2.495 | 2.463 |
| | (1.4, 1.8) | 0.888 | 0.890 | 0.423 | 0.426 | 1.721 | 1.751 | 3.764 | 3.861 | 2.423 | 2.426 |
| | (1.3, 1.9) | 0.906 | 0.899 | 0.325 | 0.375 | 1.759 | 1.771 | 3.450 | 3.716 | 2.325 | 2.375 |
| | (1.6, 1.6) | 0.871 | 0.881 | 0.515 | 0.477 | 1.710 | 1.732 | 4.003 | 4.000 | 2.516 | 2.477 |
| | (1.7, 1.9) | 0.627 | 0.641 | 3.325 | 3.157 | 1.178 | 1.242 | 8.896 | 8.799 | 5.324 | 5.157 |
| | (1.6, 2.0) | 0.656 | 0.664 | 2.812 | 2.773 | 1.235 | 1.287 | 8.166 | 8.258 | 4.811 | 4.773 |
| | (1.5, 2.1) | 0.700 | 0.698 | 2.149 | 2.277 | 1.321 | 1.355 | 7.165 | 7.529 | 4.149 | 4.277 |
| | (1.8, 1.8) | 0.617 | 0.633 | 3.452 | 3.303 | 1.190 | 1.225 | 9.001 | 9.000 | 5.452 | 5.303 |
| (6, 3) | (1.3, 1.5) | 0.994 | 0.995 | 0.013 | 0.013 | 2.971 | 2.976 | 2.264 | 2.325 | 3.013 | 3.013 |
| | (1.2 1.6) | 0.995 | 0.995 | 0.010 | 0.012 | 2.976 | 2.977 | 2.149 | 2.304 | 3.010 | 3.012 |
| | (1.1, 1.7) | 0.997 | 0.995 | 0.007 | 0.011 | 2.982 | 2.978 | 1.957 | 2.277 | 3.007 | 3.011 |
| | (1.4, 1.4) | 0.994 | 0.994 | 0.014 | 0.013 | 2.973 | 2.975 | 2.371 | 2.333 | 3.015 | 3.013 |
| | (1.5, 1.7) | 0.960 | 0.962 | 0.160 | 0.150 | 2.836 | 2.855 | 3.941 | 3.963 | 3.159 | 3.150 |
| | (1.4, 1.8) | 0.966 | 0.965 | 0.129 | 0.135 | 2.857 | 2.866 | 3.728 | 3.861 | 3.128 | 3.135 |
| | (1.3, 1.9) | 0.974 | 0.969 | 0.089 | 0.114 | 2.887 | 2.881 | 3.412 | 3.716 | 3.089 | 3.114 |
| | (1.6, 1.6) | 0.958 | 0.961 | 0.169 | 0.156 | 2.839 | 2.851 | 4.010 | 4.000 | 3.169 | 3.156 |
| | (1.7, 1.9) | 0.781 | 0.791 | 1.948 | 1.843 | 2.245 | 2.297 | 8.868 | 8.799 | 4.948 | 4.843 |
| | (1.6, 2.0) | 0.807 | 0.810 | 1.574 | 1.566 | 2.326 | 2.359 | 8.141 | 8.258 | 4.574 | 4.566 |
| | (1.5, 2.1) | 0.844 | 0.838 | 1.114 | 1.221 | 2.442 | 2.446 | 7.121 | 7.529 | 4.113 | 4.221 |
| | (1.8, 1.8) | 0.774 | 0.783 | 2.038 | 1.951 | 2.242 | 2.275 | 9.002 | 9.000 | 5.039 | 4.951 |

Table C.12: Fill Rate and Expected Number of Backorders at the Production Facilities

| | | $f_1^p$ | | $f_2^p$ | | $E[B_1^p]$ | | $E[B_2^p]$ | |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R)$ | $(\tau_1, \tau_2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| (2, 1) | (1.3, 1.5) | 0.727 | 0.720 | 0.719 | 0.720 | 0.364 | 0.370 | 0.391 | 0.417 |
| | (1.2 1.6) | 0.744 | 0.727 | 0.727 | 0.727 | 0.321 | 0.335 | 0.373 | 0.429 |
| | (1.1, 1.7) | 0.767 | 0.737 | 0.743 | 0.737 | 0.268 | 0.295 | 0.339 | 0.438 |
| | (1.4, 1.4) | 0.720 | 0.718 | 0.720 | 0.718 | 0.386 | 0.330 | 0.386 | 0.330 |
| | (1.5, 1.7) | 0.556 | 0.549 | 0.547 | 0.549 | 0.964 | 0.941 | 1.009 | 1.001 |
| | (1.4, 1.8) | 0.576 | 0.561 | 0.559 | 0.561 | 0.864 | 0.865 | 0.949 | 0.986 |
| | (1.3, 1.9) | 0.606 | 0.578 | 0.580 | 0.578 | 0.731 | 0.768 | 0.850 | 0.952 |
| | (1.6, 1.6) | 0.547 | 0.545 | 0.547 | 0.545 | 1.009 | 0.910 | 1.009 | 0.910 |
| | (1.7, 1.9) | 0.320 | 0.315 | 0.314 | 0.315 | 3.147 | 3.040 | 3.218 | 3.111 |
| | (1.6, 2.0) | 0.344 | 0.333 | 0.331 | 0.333 | 2.779 | 2.746 | 2.915 | 2.890 |
| | (1.5, 2.1) | 0.381 | 0.361 | 0.359 | 0.361 | 2.298 | 2.365 | 2.490 | 2.584 |
| | (1.8, 1.8) | 0.311 | 0.309 | 0.311 | 0.309 | 3.270 | 3.108 | 3.270 | 3.108 |
| (4, 2) | (1.3, 1.5) | 0.935 | 0.933 | 0.933 | 0.933 | 0.086 | 0.104 | 0.093 | 0.124 |
| | (1.2 1.6) | 0.943 | 0.935 | 0.939 | 0.935 | 0.072 | 0.090 | 0.083 | 0.130 |
| | (1.1, 1.7) | 0.953 | 0.939 | 0.948 | 0.939 | 0.054 | 0.075 | 0.069 | 0.136 |
| | (1.4, 1.4) | 0.932 | 0.932 | 0.932 | 0.932 | 0.093 | 0.079 | 0.093 | 0.079 |
| | (1.5, 1.7) | 0.820 | 0.817 | 0.816 | 0.817 | 0.391 | 0.411 | 0.409 | 0.448 |
| | (1.4, 1.8) | 0.836 | 0.825 | 0.829 | 0.825 | 0.334 | 0.365 | 0.368 | 0.439 |
| | (1.3, 1.9) | 0.859 | 0.837 | 0.849 | 0.837 | 0.261 | 0.310 | 0.305 | 0.420 |
| | (1.6, 1.6) | 0.814 | 0.814 | 0.814 | 0.814 | 0.414 | 0.373 | 0.414 | 0.373 |
| | (1.7, 1.9) | 0.557 | 0.555 | 0.553 | 0.555 | 2.050 | 2.012 | 2.096 | 2.072 |
| | (1.6, 2.0) | 0.588 | 0.578 | 0.579 | 0.578 | 1.745 | 1.767 | 1.832 | 1.886 |
| | (1.5, 2.1) | 0.635 | 0.612 | 0.621 | 0.612 | 1.357 | 1.456 | 1.472 | 1.634 |
| | (1.8, 1.8) | 0.547 | 0.547 | 0.547 | 0.547 | 2.146 | 2.039 | 2.146 | 2.039 |
| (6, 3) | (1.3, 1.5) | 0.985 | 0.984 | 0.984 | 0.984 | 0.020 | 0.029 | 0.022 | 0.037 |
| | (1.2 1.6) | 0.987 | 0.985 | 0.986 | 0.985 | 0.016 | 0.042 | 0.024 | 0.039 |
| | (1.1, 1.7) | 0.991 | 0.986 | 0.989 | 0.986 | 0.011 | 0.019 | 0.014 | 0.042 |
| | (1.4, 1.4) | 0.984 | 0.984 | 0.984 | 0.984 | 0.022 | 0.019 | 0.022 | 0.019 |
| | (1.5, 1.7) | 0.927 | 0.925 | 0.925 | 0.925 | 0.158 | 0.179 | 0.166 | 0.200 |
| | (1.4, 1.8) | 0.936 | 0.930 | 0.934 | 0.930 | 0.129 | 0.154 | 0.142 | 0.195 |
| | (1.3, 1.9) | 0.950 | 0.937 | 0.946 | 0.937 | 0.093 | 0.125 | 0.109 | 0.185 |
| | (1.6, 1.6) | 0.924 | 0.924 | 0.924 | 0.924 | 0.170 | 0.153 | 0.170 | 0.153 |
| | (1.7, 1.9) | 0.712 | 0.711 | 0.709 | 0.711 | 1.337 | 1.331 | 1.367 | 1.380 |
| | (1.6, 2.0) | 0.742 | 0.733 | 0.736 | 0.733 | 1.097 | 1.137 | 1.151 | 1.231 |
| | (1.5, 2.1) | 0.785 | 0.764 | 0.776 | 0.764 | 0.802 | 0.897 | 0.870 | 1.033 |
| | (1.8, 1.8) | 0.703 | 0.703 | 0.703 | 0.703 | 1.408 | 1.338 | 1.408 | 1.338 |

Table C.13: Expected Inventory Level and Expected Time to Fulfill a Backorder at the Production Facilities

| | | $E[I_1^p]$ | | $E[I_2^p]$ | | $E[W_1^{bp}]$ | | $E[W_2^{bp}]$ | |
|---|---|---|---|---|---|---|---|---|---|
| $(S,R)$ | $(\tau_1,\tau_2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| (2, 1) | (1.3, 1.5) | 0.943 | 1.049 | 0.890 | 1.011 | 2.539 | 2.542 | 2.931 | 3.105 |
| | (1.2 1.6) | 0.989 | 1.082 | 0.881 | 1.005 | 2.278 | 2.274 | 3.043 | 3.415 |
| | (1.1, 1.7) | 1.047 | 1.125 | 0.883 | 1.002 | 2.001 | 2.011 | 3.099 | 3.766 |
| | (1.4, 1.4) | 0.911 | 0.957 | 0.911 | 0.957 | 2.754 | 2.333 | 2.754 | 2.333 |
| | (1.5, 1.7) | 0.672 | 0.782 | 0.630 | 0.761 | 4.136 | 3.997 | 4.691 | 4.643 |
| | (1.4, 1.8) | 0.715 | 0.812 | 0.629 | 0.767 | 3.708 | 3.623 | 4.779 | 4.912 |
| | (1.3, 1.9) | 0.772 | 0.854 | 0.639 | 0.781 | 3.239 | 3.236 | 4.744 | 5.166 |
| | (1.6, 1.6) | 0.645 | 0.688 | 0.645 | 0.688 | 4.458 | 4.000 | 4.458 | 4.000 |
| | (1.7, 1.9) | 0.362 | 0.446 | 0.336 | 0.439 | 8.821 | 8.481 | 9.862 | 9.540 |
| | (1.6, 2.0) | 0.399 | 0.476 | 0.345 | 0.460 | 7.712 | 7.538 | 9.659 | 9.559 |
| | (1.5, 2.1) | 0.453 | 0.521 | 0.367 | 0.492 | 6.481 | 6.511 | 9.089 | 9.360 |
| | (1.8, 1.8) | 0.342 | 0.371 | 0.342 | 0.371 | 9.494 | 9.000 | 9.494 | 9.000 |
| (4, 2) | (1.3, 1.5) | 2.665 | 2.782 | 2.592 | 2.718 | 2.532 | 2.985 | 2.925 | 3.858 |
| | (1.2 1.6) | 2.739 | 2.837 | 2.592 | 2.705 | 2.277 | 2.589 | 3.029 | 4.379 |
| | (1.1, 1.7) | 2.833 | 2.906 | 2.613 | 2.700 | 2.002 | 2.204 | 3.080 | 5.004 |
| | (1.4, 1.4) | 2.618 | 2.707 | 2.618 | 2.707 | 2.746 | 2.333 | 2.746 | 2.333 |
| | (1.5, 1.7) | 2.098 | 2.251 | 2.030 | 2.208 | 4.127 | 4.289 | 4.675 | 5.106 |
| | (1.4, 1.8) | 2.185 | 2.312 | 2.047 | 2.220 | 3.702 | 3.844 | 4.763 | 5.495 |
| | (1.3, 1.9) | 2.303 | 2.396 | 2.093 | 2.248 | 3.234 | 3.387 | 4.715 | 5.907 |
| | (1.6, 1.6) | 2.050 | 2.151 | 2.050 | 2.151 | 4.445 | 4.000 | 4.445 | 4.000 |
| | (1.7, 1.9) | 1.265 | 1.418 | 1.215 | 1.400 | 8.822 | 8.634 | 9.860 | 9.772 |
| | (1.6, 2.0) | 1.365 | 1.496 | 1.262 | 1.456 | 7.719 | 7.661 | 9.644 | 9.855 |
| | (1.5, 2.1) | 1.512 | 1.613 | 1.348 | 1.542 | 6.486 | 6.604 | 9.069 | 9.746 |
| | (1.8, 1.8) | 1.219 | 1.302 | 1.219 | 1.302 | 9.475 | 9.000 | 9.475 | 9.000 |
| (6, 3) | (1.3, 1.5) | 4.599 | 4.707 | 4.521 | 4.631 | 2.523 | 3.505 | 2.922 | 4.795 |
| | (1.2 1.6) | 4.683 | 4.771 | 4.527 | 4.615 | 2.280 | 2.948 | 3.017 | 5.617 |
| | (1.1, 1.7) | 4.790 | 4.850 | 4.558 | 4.606 | 1.989 | 2.416 | 3.059 | 6.648 |
| | (1.4, 1.4) | 4.547 | 4.646 | 4.547 | 4.646 | 2.745 | 2.333 | 2.745 | 2.333 |
| | (1.5, 1.7) | 3.866 | 4.020 | 3.786 | 3.960 | 4.127 | 4.601 | 4.677 | 5.615 |
| | (1.4, 1.8) | 3.980 | 4.101 | 3.821 | 3.977 | 3.700 | 4.079 | 4.758 | 6.146 |
| | (1.3, 1.9) | 4.135 | 4.211 | 3.897 | 4.013 | 3.237 | 3.545 | 4.731 | 6.753 |
| | (1.6, 1.6) | 3.806 | 3.930 | 3.806 | 3.930 | 4.445 | 4.000 | 4.445 | 4.000 |
| | (1.7, 1.9) | 2.551 | 2.738 | 2.485 | 2.708 | 8.836 | 8.789 | 9.881 | 10.009 |
| | (1.6, 2.0) | 2.717 | 2.866 | 2.582 | 2.801 | 7.739 | 7.786 | 9.665 | 10.160 |
| | (1.5, 2.1) | 2.957 | 3.053 | 2.746 | 2.941 | 6.498 | 6.698 | 9.086 | 10.149 |
| | (1.8, 1.8) | 2.481 | 2.601 | 2.481 | 2.601 | 9.474 | 9.000 | 9.474 | 9.000 |

Table C.14: Expected Time Spent by an Order at the Production Facilities and Utilizations

| | | $E[T_1^p]$ | | $E[T_2^p]$ | | $\rho_1^p$ | | $\rho_2^p$ | |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R)$ | $(\tau_1, \tau_2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| (2, 1) | (1.3, 1.5) | 4.499 | 4.555 | 5.037 | 5.040 | 0.683 | 0.677 | 0.712 | 0.719 |
| | (1.2 1.6) | 4.215 | 4.326 | 5.279 | 5.279 | 0.661 | 0.648 | 0.719 | 0.736 |
| | (1.1, 1.7) | 3.944 | 4.117 | 5.502 | 5.508 | 0.633 | 0.613 | 0.723 | 0.753 |
| | (1.4, 1.4) | 4.773 | 4.659 | 4.773 | 4.659 | 0.700 | 0.700 | 0.700 | 0.700 |
| | (1.5, 1.7) | 5.645 | 5.634 | 6.336 | 6.279 | 0.788 | 0.783 | 0.808 | 0.813 |
| | (1.4, 1.8) | 5.212 | 5.273 | 6.546 | 6.539 | 0.770 | 0.761 | 0.811 | 0.822 |
| | (1.3, 1.9) | 4.764 | 4.917 | 6.677 | 6.755 | 0.746 | 0.732 | 0.811 | 0.830 |
| | (1.6, 1.6) | 6.019 | 5.820 | 6.019 | 5.820 | 0.800 | 0.800 | 0.800 | 0.800 |
| | (1.7, 1.9) | 9.808 | 9.625 | 10.977 | 10.728 | 0.892 | 0.890 | 0.903 | 0.905 |
| | (1.6, 2.0) | 8.703 | 8.685 | 10.899 | 10.783 | 0.879 | 0.874 | 0.902 | 0.907 |
| | (1.5, 2.1) | 7.506 | 7.683 | 10.505 | 10.614 | 0.860 | 0.852 | 0.897 | 0.907 |
| | (1.8, 1.8) | 10.539 | 10.216 | 10.539 | 10.216 | 0.900 | 0.900 | 0.900 | 0.900 |
| (4, 2) | (1.3, 1.5) | 7.777 | 7.886 | 8.623 | 8.599 | 0.683 | 0.677 | 0.712 | 0.719 |
| | (1.2 1.6) | 7.394 | 7.577 | 9.086 | 8.975 | 0.661 | 0.648 | 0.719 | 0.736 |
| | (1.1, 1.7) | 7.050 | 7.313 | 9.572 | 9.342 | 0.633 | 0.613 | 0.723 | 0.753 |
| | (1.4, 1.4) | 8.187 | 8.158 | 8.187 | 8.158 | 0.700 | 0.700 | 0.700 | 0.700 |
| | (1.5, 1.7) | 8.362 | 8.448 | 9.284 | 9.306 | 0.788 | 0.783 | 0.808 | 0.813 |
| | (1.4, 1.8) | 7.889 | 8.033 | 9.694 | 9.721 | 0.770 | 0.761 | 0.811 | 0.822 |
| | (1.3, 1.9) | 7.434 | 7.655 | 10.084 | 10.113 | 0.746 | 0.732 | 0.811 | 0.830 |
| | (1.6, 1.6) | 8.829 | 8.746 | 8.829 | 8.746 | 0.800 | 0.800 | 0.800 | 0.800 |
| | (1.7, 1.9) | 11.530 | 11.482 | 12.825 | 12.745 | 0.892 | 0.890 | 0.903 | 0.905 |
| | (1.6, 2.0) | 10.462 | 10.553 | 12.932 | 12.980 | 0.879 | 0.874 | 0.902 | 0.907 |
| | (1.5, 2.1) | 9.355 | 9.605 | 12.800 | 13.045 | 0.860 | 0.852 | 0.897 | 0.907 |
| | (1.8, 1.8) | 12.293 | 12.078 | 12.293 | 12.078 | 0.900 | 0.900 | 0.900 | 0.900 |
| (6, 3) | (1.3, 1.5) | 11.458 | 11.586 | 12.688 | 12.587 | 0.683 | 0.677 | 0.712 | 0.719 |
| | (1.2 1.6) | 10.925 | 11.160 | 13.392 | 13.124 | 0.661 | 0.648 | 0.719 | 0.736 |
| | (1.1, 1.7) | 10.453 | 10.801 | 14.149 | 13.648 | 0.633 | 0.613 | 0.723 | 0.753 |
| | (1.4, 1.4) | 12.046 | 12.038 | 12.046 | 12.038 | 0.700 | 0.700 | 0.700 | 0.700 |
| | (1.5, 1.7) | 11.728 | 11.835 | 12.982 | 12.974 | 0.788 | 0.783 | 0.808 | 0.813 |
| | (1.4, 1.8) | 11.155 | 11.326 | 13.631 | 13.568 | 0.770 | 0.761 | 0.811 | 0.822 |
| | (1.3, 1.9) | 10.630 | 10.879 | 14.310 | 14.153 | 0.746 | 0.732 | 0.811 | 0.830 |
| | (1.6, 1.6) | 12.341 | 12.305 | 12.341 | 12.305 | 0.800 | 0.800 | 0.800 | 0.800 |
| | (1.7, 1.9) | 13.981 | 14.002 | 15.498 | 15.491 | 0.892 | 0.890 | 0.903 | 0.905 |
| | (1.6, 2.0) | 12.924 | 13.059 | 15.858 | 15.945 | 0.879 | 0.874 | 0.902 | 0.907 |
| | (1.5, 2.1) | 11.879 | 12.140 | 16.071 | 16.284 | 0.860 | 0.852 | 0.897 | 0.907 |
| | (1.8, 1.8) | 14.817 | 14.676 | 14.817 | 14.676 | 0.900 | 0.900 | 0.900 | 0.900 |

## APPENDIX D

## QUEUEING RESULTS FOR THE SCN CONFIGURATION 1R/2P: GENERAL INTER-ARRIVAL AND PROCESSING TIME DISTRIBUTIONS

This appendix presents the results for the GI/G/2-based queueing approximations developed in Chapter 7 for the SCN configuration 1R/2P. The performance measures at the retail store are presented in Tables D.1 and D.2, while those at a production facility are presented in Tables D.3 and D.4. Table D.5 presents the expected time spent by a part at the retail store and a production facility.

Table D.1: Fill Rate & Expected Number of Backorders at the Retail Store

| | | $f^r$ | | | | | $E[B^r]$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | NoVis | LoVis | MedVis | HiVis | GI/G/2 | NoVis | LoVis | MedVis | HiVis | GI/G/2 |
| (2, 2, 70%) | (0.25, 0.25) | 0.831 | 0.997 | 0.997 | 0.998 | 1.000 | 0.193 | 0.002 | 0.002 | 0.001 | 0.000 |
| | (0.25, 1.00) | 0.643 | 0.949 | 0.946 | 0.954 | 0.949 | 0.821 | 0.097 | 0.085 | 0.072 | 0.102 |
| | (0.25, 4.00) | 0.332 | 0.798 | 0.777 | 0.785 | 0.804 | 4.894 | 1.699 | 1.439 | 1.406 | 1.415 |
| | (1.00, 0.25) | 0.719 | 0.919 | 0.916 | 0.923 | 0.944 | 0.498 | 0.128 | 0.115 | 0.105 | 0.103 |
| | (1.00, 1.00) | 0.567 | 0.842 | 0.837 | 0.848 | 0.862 | 1.285 | 0.444 | 0.385 | 0.360 | 0.323 |
| | (1.00, 4.00) | 0.313 | 0.724 | 0.706 | 0.713 | 0.760 | 5.525 | 2.436 | 2.025 | 1.995 | 1.776 |
| | (4.00, 0.25) | 0.506 | 0.619 | 0.615 | 0.626 | 0.663 | 1.805 | 1.460 | 1.328 | 1.294 | 1.168 |
| | (4.00, 1.00) | 0.417 | 0.582 | 0.575 | 0.585 | 0.636 | 2.957 | 2.248 | 1.974 | 1.933 | 1.647 |
| | (4.00, 4.00) | 0.268 | 0.553 | 0.540 | 0.547 | 0.649 | 7.604 | 5.092 | 4.209 | 4.115 | 3.694 |

Continued on next page

TableD.1 – continued from previous page

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^r$ NoVis | $f^r$ LoVis | $f^r$ MedVis | $f^r$ HiVis | $f^r$ GI/G/2 | $E[B^r]$ NoVis | $E[B^r]$ LoVis | $E[B^r]$ MedVis | $E[B^r]$ HiVis | $E[B^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 80%) | (0.25, 0.25) | 0.626 | 0.977 | 0.975 | 0.980 | 0.968 | 0.809 | 0.028 | 0.025 | 0.020 | 0.075 |
| | (0.25, 1.00) | 0.405 | 0.841 | 0.831 | 0.846 | 0.848 | 2.598 | 0.509 | 0.447 | 0.407 | 0.417 |
| | (0.25, 4.00) | 0.166 | 0.641 | 0.605 | 0.614 | 0.636 | 11.570 | 4.596 | 3.980 | 3.968 | 3.762 |
| | (1.00, 0.25) | 0.488 | 0.795 | 0.789 | 0.801 | 0.830 | 1.696 | 0.562 | 0.508 | 0.479 | 0.428 |
| | (1.00, 1.00) | 0.338 | 0.682 | 0.674 | 0.687 | 0.709 | 3.696 | 1.521 | 1.322 | 1.270 | 1.165 |
| | (1.00, 4.00) | 0.156 | 0.555 | 0.533 | 0.541 | 0.585 | 12.815 | 6.231 | 5.168 | 5.120 | 4.731 |
| | (4.00, 0.25) | 0.278 | 0.420 | 0.420 | 0.430 | 0.470 | 5.215 | 4.400 | 3.961 | 3.865 | 3.687 |
| | (4.00, 1.00) | 0.218 | 0.398 | 0.394 | 0.402 | 0.453 | 7.677 | 6.027 | 5.238 | 5.217 | 4.756 |
| | (4.00, 4.00) | 0.128 | 0.382 | 0.370 | 0.376 | 0.476 | 17.194 | 11.981 | 9.844 | 9.807 | 9.051 |
| (2, 2, 90%) | (0.25, 0.25) | 0.286 | 0.835 | 0.823 | 0.840 | 0.855 | 4.042 | 0.428 | 0.400 | 0.362 | 0.416 |
| | (0.25, 1.00) | 0.144 | 0.578 | 0.559 | 0.576 | 0.584 | 9.733 | 2.899 | 2.560 | 2.458 | 2.334 |
| | (0.25, 4.00) | 0.463 | 0.395 | 0.353 | 0.363 | 0.370 | 32.875 | 15.461 | 13.505 | 13.420 | 12.890 |
| | (1.00, 0.25) | 0.192 | 0.534 | 0.523 | 0.535 | 0.553 | 6.912 | 2.901 | 2.643 | 2.573 | 2.421 |
| | (1.00, 1.00) | 0.113 | 0.417 | 0.407 | 0.417 | 0.441 | 12.766 | 6.213 | 5.418 | 5.322 | 5.035 |
| | (1.00, 4.00) | 0.043 | 0.322 | 0.299 | 0.305 | 0.334 | 35.439 | 19.767 | 16.492 | 16.589 | 15.780 |
| | (4.00, 0.25) | 0.083 | 0.208 | 0.213 | 0.220 | 0.246 | 17.811 | 15.254 | 13.548 | 13.618 | 13.212 |
| | (4.00, 1.00) | 0.063 | 0.201 | 0.201 | 0.205 | 0.241 | 24.025 | 19.303 | 16.710 | 16.722 | 16.109 |
| | (4.00, 4.00) | 0.034 | 0.195 | 0.190 | 0.195 | 0.261 | 43.299 | 34.781 | 28.549 | 28.355 | 27.602 |
| (4, 4, 70%) | (0.25, 0.25) | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 0.011 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.929 | 0.998 | 0.997 | 0.999 | 0.995 | 0.143 | 0.004 | 0.004 | 0.002 | 0.012 |
| | (0.25, 4.00) | 0.611 | 0.913 | 0.901 | 0.915 | 0.922 | 2.554 | 0.695 | 0.637 | 0.553 | 0.719 |
| | (1.00, 0.25) | 0.962 | 0.996 | 0.996 | 0.997 | 0.995 | 0.059 | 0.006 | 0.006 | 0.004 | 0.010 |
| | (1.00, 1.00) | 0.882 | 0.977 | 0.976 | 0.982 | 0.984 | 0.308 | 0.059 | 0.056 | 0.042 | 0.038 |
| | (1.00, 4.00) | 0.579 | 0.875 | 0.862 | 0.876 | 0.903 | 3.042 | 1.069 | 0.974 | 0.878 | 0.913 |
| | (4.00, 0.25) | 0.831 | 0.880 | 0.878 | 0.892 | 0.926 | 0.542 | 0.435 | 0.425 | 0.372 | 0.218 |
| | (4.00, 1.00) | 0.734 | 0.834 | 0.829 | 0.845 | 0.888 | 1.195 | 0.838 | 0.780 | 0.717 | 0.457 |
| | (4.00, 4.00) | 0.496 | 0.748 | 0.735 | 0.750 | 0.816 | 4.753 | 2.802 | 2.511 | 2.338 | 2.232 |
| (4, 4, 80%) | (0.25, 0.25) | 0.934 | 1.000 | 1.000 | 1.000 | 0.996 | 0.123 | 0.000 | 0.000 | 0.000 | 0.009 |
| | (0.25, 1.00) | 0.763 | 0.976 | 0.974 | 0.982 | 0.977 | 0.894 | 0.071 | 0.069 | 0.046 | 0.081 |
| | (0.25, 4.00) | 0.361 | 0.794 | 0.766 | 0.786 | 0.811 | 7.976 | 2.530 | 2.354 | 2.207 | 2.254 |
| | (1.00, 0.25) | 0.845 | 0.968 | 0.967 | 0.975 | 0.977 | 0.443 | 0.082 | 0.079 | 0.060 | 0.072 |
| | (1.00, 1.00) | 0.678 | 0.906 | 0.900 | 0.917 | 0.924 | 1.562 | 0.422 | 0.403 | 0.336 | 0.305 |
| | (1.00, 4.00) | 0.336 | 0.735 | 0.713 | 0.731 | 0.771 | 9.107 | 3.597 | 3.206 | 3.043 | 2.954 |

Continued on next page

TableD.1 – continued from previous page

| (R, S, ρ) | $(cr^2, cp^2)$ | $f^r$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[B^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (4, 4, 80%) | (4.00, 0.25) | 0.584 | 0.704 | 0.700 | 0.721 | 0.757 | 2.630 | 2.128 | 2.053 | 1.880 | 1.593 |
| | (4.00, 1.00) | 0.473 | 0.650 | 0.647 | 0.664 | 0.709 | 4.583 | 3.293 | 3.032 | 2.913 | 2.425 |
| | (4.00, 4.00) | 0.269 | 0.570 | 0.554 | 0.568 | 0.646 | 13.192 | 7.956 | 7.154 | 6.948 | 6.540 |
| (4, 4, 90%) | (0.25, 0.25) | 0.652 | 0.979 | 0.977 | 0.987 | 0.975 | 1.674 | 0.053 | 0.051 | 0.029 | 0.096 |
| | (0.25, 1.00) | 0.385 | 0.827 | 0.814 | 0.846 | 0.849 | 6.088 | 1.106 | 1.076 | 0.887 | 0.903 |
| | (0.25, 4.00) | 0.118 | 0.548 | 0.500 | 0.524 | 0.543 | 28.097 | 11.002 | 10.549 | 10.100 | 9.744 |
| | (1.00, 0.25) | 0.490 | 0.810 | 0.801 | 0.828 | 0.842 | 3.756 | 1.118 | 1.099 | 0.949 | 0.901 |
| | (1.00, 1.00) | 0.309 | 0.676 | 0.660 | 0.688 | 0.703 | 8.765 | 3.227 | 3.103 | 2.848 | 2.676 |
| | (1.00, 4.00) | 0.107 | 0.480 | 0.446 | 0.462 | 0.495 | 30.676 | 14.560 | 13.165 | 12.959 | 12.371 |
| | (4.00, 0.25) | 0.227 | 0.414 | 0.414 | 0.430 | 0.452 | 13.431 | 10.631 | 10.186 | 9.979 | 9.423 |
| | (4.00, 1.00) | 0.168 | 0.377 | 0.373 | 0.387 | 0.417 | 19.378 | 14.375 | 13.236 | 12.845 | 12.180 |
| | (4.00, 4.00) | 0.080 | 0.323 | 0.309 | 0.323 | 0.380 | 38.786 | 27.918 | 24.986 | 24.109 | 23.594 |
| (6, 6, 70%) | (0.25, 0.25) | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.987 | 1.000 | 1.000 | 1.000 | 0.999 | 0.024 | 0.000 | 0.000 | 0.000 | 0.002 |
| | (0.25, 4.00) | 0.789 | 0.961 | 0.956 | 0.966 | 0.963 | 1.296 | 0.294 | 0.281 | 0.220 | 0.412 |
| | (1.00, 0.25) | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 0.007 | 0.000 | 0.000 | 0.000 | 0.001 |
| | (1.00, 1.00) | 0.971 | 0.997 | 0.996 | 0.998 | 0.998 | 0.071 | 0.008 | 0.008 | 0.005 | 0.004 |
| | (1.00, 4.00) | 0.759 | 0.938 | 0.933 | 0.944 | 0.953 | 1.632 | 0.513 | 0.466 | 0.391 | 0.528 |
| | (4.00, 0.25) | 0.948 | 0.964 | 0.963 | 0.969 | 0.987 | 0.157 | 0.127 | 0.126 | 0.107 | 0.036 |
| | (4.00, 1.00) | 0.889 | 0.934 | 0.931 | 0.942 | 0.970 | 0.467 | 0.321 | 0.313 | 0.268 | 0.120 |
| | (4.00, 4.00) | 0.671 | 0.847 | 0.839 | 0.856 | 0.896 | 2.911 | 1.672 | 1.524 | 1.344 | 1.434 |
| (6, 6, 80%) | (0.25, 0.25) | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 0.019 | 0.000 | 0.000 | 0.000 | 0.001 |
| | (0.25, 1.00) | 0.916 | 0.996 | 0.996 | 0.998 | 0.995 | 0.296 | 0.010 | 0.010 | 0.005 | 0.019 |
| | (0.25, 4.00) | 0.536 | 0.875 | 0.860 | 0.880 | 0.894 | 5.399 | 1.491 | 1.400 | 1.237 | 1.436 |
| | (1.00, 0.25) | 0.958 | 0.995 | 0.995 | 0.997 | 0.996 | 0.111 | 0.012 | 0.012 | 0.007 | 0.014 |
| | (1.00, 1.00) | 0.859 | 0.971 | 0.969 | 0.978 | 0.980 | 0.636 | 0.128 | 0.125 | 0.089 | 0.080 |
| | (1.00, 4.00) | 0.503 | 0.832 | 0.816 | 0.838 | 0.866 | 6.367 | 2.207 | 2.071 | 1.820 | 1.940 |
| | (4.00, 0.25) | 0.781 | 0.847 | 0.847 | 0.864 | 0.896 | 1.284 | 1.088 | 1.046 | 0.914 | 0.678 |
| | (4.00, 1.00) | 0.670 | 0.797 | 0.794 | 0.812 | 0.852 | 2.670 | 1.829 | 1.784 | 1.629 | 1.226 |
| | (4.00, 4.00) | 0.409 | 0.687 | 0.672 | 0.693 | 0.756 | 10.000 | 5.715 | 5.311 | 4.949 | 4.825 |

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**TableD.1 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^r$ NoVis | $f^r$ LoVis | $f^r$ MedVis | $f^r$ HiVis | $f^r$ GI/G/2 | $E[B^r]$ NoVis | $E[B^r]$ LoVis | $E[B^r]$ MedVis | $E[B^r]$ HiVis | $E[B^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (6, 6, 90%) | (0.25, 0.25) | 0.850 | 0.997 | 0.997 | 0.999 | 0.994 | 0.665 | 0.007 | 0.006 | 0.002 | 0.025 |
| | (0.25, 1.00) | 0.592 | 0.925 | 0.921 | 0.944 | 0.942 | 3.709 | 0.464 | 0.458 | 0.321 | 0.369 |
| | (0.25, 4.00) | 0.203 | 0.646 | 0.609 | 0.642 | 0.665 | 23.852 | 8.329 | 8.191 | 7.621 | 7.447 |
| | (1.00, 0.25) | 0.708 | 0.920 | 0.916 | 0.936 | 0.941 | 1.975 | 0.463 | 0.458 | 0.348 | 0.351 |
| | (1.00, 1.00) | 0.495 | 0.812 | 0.803 | 0.833 | 0.842 | 5.894 | 1.833 | 1.805 | 1.528 | 1.422 |
| | (1.00, 4.00) | 0.185 | 0.584 | 0.549 | 0.578 | 0.614 | 26.395 | 11.331 | 11.164 | 10.141 | 9.775 |
| | (4.00, 0.25) | 0.380 | 0.566 | 0.565 | 0.584 | 0.609 | 9.980 | 7.833 | 7.500 | 7.322 | 6.708 |
| | (4.00, 1.00) | 0.287 | 0.511 | 0.504 | 0.530 | 0.559 | 15.469 | 10.968 | 10.633 | 9.872 | 9.198 |
| | (4.00, 4.00) | 0.137 | 0.418 | 0.404 | 0.425 | 0.478 | 34.593 | 23.667 | 21.308 | 20.527 | 20.227 |
| (8, 8, 70%) | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 4.00) | 0.890 | 0.982 | 0.981 | 0.986 | 0.979 | 0.649 | 0.129 | 0.122 | 0.087 | 0.252 |
| | (1.00, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (1.00, 1.00) | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 0.017 | 0.001 | 0.001 | 0.000 | 0.001 |
| | (1.00, 4.00) | 0.867 | 0.970 | 0.966 | 0.975 | 0.974 | 0.864 | 0.242 | 0.236 | 0.174 | 0.325 |
| | (4.00, 0.25) | 0.985 | 0.988 | 0.988 | 0.991 | 0.998 | 0.045 | 0.042 | 0.039 | 0.031 | 0.006 |
| | (4.00, 1.00) | 0.956 | 0.973 | 0.973 | 0.978 | 0.992 | 0.180 | 0.131 | 0.125 | 0.100 | 0.031 |
| | (4.00, 4.00) | 0.792 | 0.907 | 0.901 | 0.917 | 0.937 | 1.761 | 0.984 | 0.947 | 0.774 | 0.966 |
| (8, 8, 80%) | (0.25, 0.25) | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.971 | 0.999 | 0.999 | 1.000 | 0.999 | 0.097 | 0.001 | 0.001 | 0.000 | 0.004 |
| | (0.25, 4.00) | 0.674 | 0.923 | 0.916 | 0.933 | 0.936 | 3.611 | 0.901 | 0.837 | 0.696 | 0.959 |
| | (1.00, 0.25) | 0.989 | 0.999 | 0.999 | 1.000 | 0.999 | 0.028 | 0.002 | 0.002 | 0.001 | 0.003 |
| | (1.00, 1.00) | 0.941 | 0.991 | 0.990 | 0.994 | 0.995 | 0.255 | 0.039 | 0.039 | 0.023 | 0.021 |
| | (1.00, 4.00) | 0.638 | 0.893 | 0.881 | 0.903 | 0.916 | 4.402 | 1.379 | 1.375 | 1.088 | 1.326 |
| | (4.00, 0.25) | 0.890 | 0.922 | 0.919 | 0.934 | 0.956 | 0.617 | 0.540 | 0.548 | 0.443 | 0.288 |
| | (4.00, 1.00) | 0.801 | 0.881 | 0.881 | 0.894 | 0.925 | 1.528 | 1.076 | 1.017 | 0.911 | 0.620 |
| | (4.00, 4.00) | 0.534 | 0.773 | 0.761 | 0.781 | 0.828 | 7.518 | 4.043 | 3.798 | 3.532 | 3.629 |

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**TableD.1 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^r$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[B^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (8, 8, 90%) | (0.25, 0.25) | 0.938 | 1.000 | 1.000 | 1.000 | 0.998 | 0.260 | 0.001 | 0.001 | 0.000 | 0.007 |
| | (0.25, 1.00) | 0.741 | 0.968 | 0.966 | 0.980 | 0.976 | 2.224 | 0.196 | 0.194 | 0.116 | 0.157 |
| | (0.25, 4.00) | 0.293 | 0.718 | 0.695 | 0.730 | 0.751 | 20.139 | 6.751 | 6.342 | 5.758 | 5.752 |
| | (1.00, 0.25) | 0.841 | 0.966 | 0.965 | 0.977 | 0.977 | 1.020 | 0.191 | 0.190 | 0.127 | 0.142 |
| | (1.00, 1.00) | 0.645 | 0.890 | 0.885 | 0.910 | 0.916 | 3.910 | 1.062 | 1.050 | 0.821 | 0.756 |
| | (1.00, 4.00) | 0.268 | 0.663 | 0.639 | 0.669 | 0.703 | 22.598 | 9.116 | 8.647 | 7.937 | 7.783 |
| | (4.00, 0.25) | 0.517 | 0.675 | 0.672 | 0.695 | 0.722 | 7.337 | 5.872 | 5.728 | 5.378 | 4.775 |
| | (4.00, 1.00) | 0.404 | 0.623 | 0.614 | 0.639 | 0.667 | 12.252 | 8.172 | 8.030 | 7.591 | 6.946 |
| | (4.00, 4.00) | 0.199 | 0.500 | 0.488 | 0.510 | 0.560 | 30.740 | 19.889 | 18.040 | 17.487 | 17.389 |

Table D.2: Expected Inventory Level & Expected Time to Fulfill a Backorder at the Retail Store

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{br}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 70%) | (0.25, 0.25) | 1.519 | 1.966 | 1.969 | 1.979 | 1.997 | 2.130 | 0.711 | 0.593 | 0.590 | 0.114 |
| | (0.25, 1.00) | 1.140 | 1.759 | 1.779 | 1.812 | 1.862 | 3.265 | 1.923 | 1.578 | 1.577 | 1.991 |
| | (0.25, 4.00) | 0.585 | 1.323 | 1.380 | 1.408 | 1.571 | 8.217 | 8.404 | 6.448 | 6.538 | 7.227 |
| | (1.00, 0.25) | 1.287 | 1.723 | 1.752 | 1.774 | 1.845 | 2.748 | 1.569 | 1.368 | 1.366 | 1.825 |
| | (1.00, 1.00) | 0.999 | 1.510 | 1.557 | 1.585 | 1.664 | 3.919 | 2.817 | 2.365 | 2.365 | 2.333 |
| | (1.00, 4.00) | 0.553 | 1.175 | 1.248 | 1.269 | 1.476 | 8.927 | 8.821 | 6.900 | 6.960 | 7.402 |
| | (4.00, 0.25) | 0.888 | 1.214 | 1.292 | 1.311 | 1.254 | 4.593 | 3.834 | 3.455 | 3.459 | 3.465 |
| | (4.00, 1.00) | 0.732 | 1.097 | 1.177 | 1.195 | 1.212 | 5.990 | 5.373 | 4.651 | 4.658 | 4.530 |
| | (4.00, 4.00) | 0.476 | 0.932 | 1.024 | 1.040 | 1.255 | 11.247 | 11.392 | 9.157 | 9.091 | 10.512 |
| (2, 2, 80%) | (0.25, 0.25) | 1.092 | 1.850 | 1.861 | 1.889 | 1.923 | 3.122 | 1.182 | 1.005 | 1.005 | 2.375 |
| | (0.25, 1.00) | 0.692 | 1.436 | 1.481 | 1.528 | 1.630 | 5.273 | 3.196 | 2.650 | 2.650 | 2.748 |
| | (0.25, 4.00) | 0.288 | 0.957 | 1.036 | 1.061 | 1.227 | 14.658 | 12.787 | 10.084 | 10.265 | 10.338 |
| | (1.00, 0.25) | 0.839 | 1.412 | 1.468 | 1.498 | 1.580 | 4.245 | 2.746 | 2.412 | 2.411 | 2.519 |
| | (1.00, 1.00) | 0.576 | 1.134 | 1.210 | 1.241 | 1.345 | 6.468 | 4.788 | 4.054 | 4.054 | 4.000 |
| | (1.00, 4.00) | 0.270 | 0.813 | 0.912 | 0.933 | 1.125 | 15.943 | 14.009 | 11.066 | 11.149 | 11.412 |
| | (4.00, 0.25) | 0.473 | 0.815 | 0.917 | 0.934 | 0.881 | 8.075 | 7.594 | 6.839 | 6.788 | 6.951 |
| | (4.00, 1.00) | 0.374 | 0.722 | 0.821 | 0.836 | 0.859 | 10.636 | 10.009 | 8.649 | 8.727 | 8.701 |
| | (4.00, 4.00) | 0.226 | 0.590 | 0.699 | 0.712 | 0.916 | 20.462 | 19.372 | 15.630 | 15.724 | 17.276 |

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**TableD.2 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{br}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 90%) | (0.25, 0.25) | 0.471 | 1.414 | 1.450 | 1.501 | 1.632 | 6.508 | 2.593 | 2.257 | 2.257 | 2.879 |
| | (0.25, 1.00) | 0.237 | 0.849 | 0.927 | 0.970 | 1.087 | 12.108 | 6.875 | 5.797 | 5.800 | 5.603 |
| | (0.25, 4.00) | 0.079 | 0.495 | 0.581 | 0.605 | 0.704 | 35.000 | 25.564 | 20.851 | 21.054 | 20.455 |
| | (1.00, 0.25) | 0.315 | 0.856 | 0.938 | 0.966 | 1.018 | 9.342 | 6.220 | 5.537 | 5.537 | 5.414 |
| | (1.00, 1.00) | 0.186 | 0.605 | 0.701 | 0.725 | 0.819 | 15.087 | 10.660 | 9.131 | 9.132 | 9.000 |
| | (1.00, 4.00) | 0.074 | 0.394 | 0.496 | 0.509 | 0.635 | 37.537 | 29.173 | 23.514 | 23.864 | 23.684 |
| | (4.00, 0.25) | 0.138 | 0.384 | 0.483 | 0.495 | 0.458 | 20.064 | 19.277 | 17.212 | 17.462 | 17.521 |
| | (4.00, 1.00) | 0.105 | 0.333 | 0.426 | 0.434 | 0.454 | 26.221 | 24.178 | 20.926 | 21.045 | 21.220 |
| | (4.00, 4.00) | 0.060 | 0.257 | 0.357 | 0.366 | 0.499 | 45.272 | 43.252 | 35.265 | 35.222 | 37.338 |
| (4, 4, 70%) | (0.25, 0.25) | 3.846 | 4.000 | 4.000 | 4.000 | 4.000 | 2.014 | 0.268 | 0.203 | 0.048 | 0.114 |
| | (0.25, 1.00) | 3.435 | 3.954 | 3.957 | 3.978 | 3.964 | 3.015 | 1.717 | 1.530 | 1.530 | 2.487 |
| | (0.25, 4.00) | 2.075 | 3.355 | 3.402 | 3.489 | 3.610 | 7.521 | 7.965 | 6.435 | 6.476 | 9.257 |
| | (1.00, 0.25) | 3.633 | 3.945 | 3.949 | 3.967 | 3.964 | 2.557 | 1.453 | 1.354 | 1.367 | 2.193 |
| | (1.00, 1.00) | 3.187 | 3.792 | 3.808 | 3.855 | 3.880 | 3.598 | 2.589 | 2.341 | 2.327 | 2.333 |
| | (1.00, 4.00) | 1.961 | 3.168 | 3.231 | 3.314 | 3.514 | 8.182 | 8.559 | 7.083 | 7.092 | 9.400 |
| | (4.00, 0.25) | 2.952 | 3.398 | 3.444 | 3.508 | 3.535 | 4.202 | 3.640 | 3.489 | 3.454 | 2.955 |
| | (4.00, 1.00) | 2.544 | 3.166 | 3.229 | 3.299 | 3.370 | 5.468 | 5.039 | 4.574 | 4.620 | 4.076 |
| | (4.00, 4.00) | 1.669 | 2.678 | 2.788 | 2.861 | 3.133 | 10.360 | 11.116 | 9.465 | 9.368 | 12.116 |
| (4, 4, 80%) | (0.25, 0.25) | 3.441 | 3.988 | 3.989 | 3.996 | 3.972 | 2.861 | 1.078 | 0.981 | 0.824 | 2.375 |
| | (0.25, 1.00) | 2.626 | 3.745 | 3.760 | 3.839 | 3.848 | 4.752 | 2.914 | 2.606 | 2.595 | 3.434 |
| | (0.25, 4.00) | 1.170 | 2.723 | 2.798 | 2.904 | 3.107 | 13.380 | 12.267 | 10.066 | 10.303 | 11.895 |
| | (1.00, 0.25) | 2.985 | 3.736 | 3.752 | 3.814 | 3.844 | 3.843 | 2.564 | 2.386 | 2.378 | 3.103 |
| | (1.00, 1.00) | 2.285 | 3.360 | 3.400 | 3.499 | 3.560 | 5.819 | 4.471 | 4.044 | 4.041 | 4.000 |
| | (1.00, 4.00) | 1.087 | 2.478 | 2.590 | 2.686 | 2.935 | 14.594 | 13.595 | 11.167 | 11.314 | 12.886 |
| | (4.00, 0.25) | 1.933 | 2.654 | 2.740 | 2.826 | 2.794 | 7.272 | 7.196 | 6.852 | 6.753 | 6.568 |
| | (4.00, 1.00) | 1.543 | 2.388 | 2.502 | 2.579 | 2.616 | 9.622 | 9.422 | 8.581 | 8.682 | 8.334 |
| | (4.00, 4.00) | 0.873 | 1.912 | 2.071 | 2.133 | 2.438 | 18.901 | 18.489 | 16.054 | 16.085 | 18.492 |
| (4, 4, 90%) | (0.25, 0.25) | 2.148 | 3.759 | 3.769 | 3.868 | 3.850 | 5.768 | 2.455 | 2.245 | 2.240 | 3.905 |
| | (0.25, 1.00) | 1.205 | 2.888 | 2.929 | 3.116 | 3.214 | 10.791 | 6.387 | 5.771 | 5.769 | 5.999 |
| | (0.25, 4.00) | 0.365 | 1.657 | 1.751 | 1.862 | 2.016 | 32.533 | 24.312 | 21.095 | 21.197 | 21.310 |
| | (1.00, 0.25) | 1.559 | 2.911 | 2.951 | 3.092 | 3.169 | 8.287 | 5.895 | 5.524 | 5.516 | 5.709 |
| | (1.00, 1.00) | 0.955 | 2.289 | 2.364 | 2.499 | 2.598 | 13.532 | 9.961 | 9.125 | 9.128 | 9.000 |
| | (1.00, 4.00) | 0.333 | 1.425 | 1.563 | 1.640 | 1.833 | 35.022 | 27.991 | 23.737 | 24.087 | 24.492 |

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**TableD.2 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{br}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (4, 4, 90%) | (4.00, 0.25) | 0.698 | 1.495 | 1.626 | 1.691 | 1.611 | 18.184 | 18.141 | 17.390 | 17.523 | 17.179 |
| | (4.00, 1.00) | 0.516 | 1.297 | 1.441 | 1.498 | 1.499 | 24.036 | 23.082 | 21.133 | 20.973 | 20.891 |
| | (4.00, 4.00) | 0.252 | 0.964 | 1.139 | 1.193 | 1.407 | 42.735 | 41.191 | 36.166 | 35.619 | 38.043 |
| (6, 6, 70%) | (0.25, 0.25) | 5.961 | 6.000 | 6.000 | 6.000 | 6.000 | 1.985 | 0.000 | 0.000 | 0.000 | 0.118 |
| | (0.25, 1.00) | 5.726 | 5.993 | 5.993 | 5.998 | 5.989 | 2.920 | 1.408 | 1.332 | 1.207 | 2.516 |
| | (0.25, 4.00) | 4.118 | 5.508 | 5.538 | 5.644 | 5.694 | 7.132 | 7.555 | 6.371 | 6.404 | 11.033 |
| | (1.00, 0.25) | 5.857 | 5.991 | 5.992 | 5.996 | 5.991 | 2.494 | 1.452 | 1.417 | 1.398 | 2.204 |
| | (1.00, 1.00) | 5.522 | 5.928 | 5.934 | 5.960 | 5.996 | 3.462 | 2.368 | 2.218 | 2.172 | 2.333 |
| | (1.00, 4.00) | 3.930 | 5.314 | 5.374 | 5.478 | 5.615 | 7.755 | 8.292 | 6.995 | 6.988 | 11.182 |
| | (4.00, 0.25) | 5.291 | 5.652 | 5.674 | 5.731 | 5.818 | 4.025 | 3.493 | 3.396 | 3.464 | 2.812 |
| | (4.00, 1.00) | 4.803 | 5.410 | 5.445 | 5.533 | 5.663 | 5.206 | 4.830 | 4.564 | 4.624 | 3.995 |
| | (4.00, 4.00) | 3.413 | 4.715 | 4.818 | 4.940 | 5.202 | 9.820 | 10.961 | 9.490 | 9.343 | 13.782 |
| (6, 6, 80%) | (0.25, 0.25) | 5.748 | 5.999 | 5.999 | 6.000 | 5.992 | 2.767 | 0.238 | 0.206 | 0.000 | 2.375 |
| | (0.25, 1.00) | 4.978 | 5.907 | 5.911 | 5.957 | 5.938 | 4.512 | 2.714 | 2.537 | 2.526 | 3.688 |
| | (0.25, 4.00) | 2.608 | 4.750 | 4.824 | 4.995 | 5.183 | 12.579 | 11.915 | 10.015 | 10.302 | 13.505 |
| | (1.00, 0.25) | 5.368 | 5.904 | 5.909 | 5.946 | 5.943 | 3.673 | 2.476 | 2.357 | 2.347 | 3.314 |
| | (1.00, 1.00) | 4.539 | 5.622 | 5.642 | 5.744 | 5.775 | 5.498 | 4.347 | 4.032 | 4.019 | 4.000 |
| | (1.00, 4.00) | 2.433 | 4.454 | 4.552 | 4.727 | 4.986 | 13.729 | 13.109 | 11.258 | 11.240 | 14.433 |
| | (4.00, 0.25) | 4.012 | 4.833 | 4.900 | 5.025 | 5.085 | 6.846 | 7.121 | 6.818 | 6.746 | 6.528 |
| | (4.00, 1.00) | 3.340 | 4.465 | 4.553 | 4.687 | 4.801 | 9.040 | 9.032 | 8.644 | 8.673 | 8.300 |
| | (4.00, 4.00) | 1.963 | 3.618 | 3.777 | 3.921 | 4.285 | 17.838 | 18.252 | 16.187 | 16.125 | 19.789 |
| (6, 6, 90%) | (0.25, 0.25) | 4.434 | 5.918 | 5.922 | 5.972 | 5.929 | 5.416 | 2.324 | 2.162 | 2.112 | 3.985 |
| | (0.25, 1.00) | 2.845 | 5.150 | 5.176 | 5.422 | 5.466 | 10.046 | 6.220 | 5.767 | 5.774 | 6.367 |
| | (0.25, 4.00) | 0.920 | 3.163 | 3.255 | 3.486 | 3.703 | 30.712 | 23.541 | 20.924 | 21.237 | 22.203 |
| | (1.00, 0.25) | 3.507 | 5.179 | 5.205 | 5.393 | 5.450 | 7.729 | 5.753 | 5.478 | 5.477 | 5.986 |
| | (1.00, 1.00) | 2.328 | 4.366 | 4.418 | 4.660 | 4.746 | 12.598 | 9.749 | 9.141 | 9.127 | 9.000 |
| | (1.00, 4.00) | 0.837 | 2.820 | 2.934 | 3.135 | 3.404 | 33.132 | 27.247 | 24.739 | 24.043 | 25.333 |
| | (4.00, 0.25) | 1.749 | 3.086 | 3.210 | 3.333 | 3.285 | 16.972 | 18.046 | 17.241 | 17.591 | 17.157 |
| | (4.00, 1.00) | 1.305 | 2.700 | 2.841 | 2.995 | 3.019 | 22.533 | 22.429 | 21.431 | 20.981 | 20.871 |
| | (4.00, 4.00) | 0.621 | 1.994 | 2.221 | 2.350 | 2.632 | 40.723 | 40.613 | 35.718 | 35.679 | 38.767 |

Continued on next page

**TableD.2 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{br}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (8, 8, 70%) | (0.25, 0.25) | 7.990 | 8.000 | 8.000 | 8.000 | 8.000 | 1.977 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 7.878 | 7.999 | 7.999 | 8.000 | 7.997 | 2.880 | 0.313 | 0.290 | 0.000 | 2.518 |
| | (0.25, 4.00) | 6.393 | 7.654 | 7.678 | 7.775 | 7.761 | 6.889 | 7.260 | 6.281 | 6.336 | 12.140 |
| | (1.00, 0.25) | 7.948 | 7.999 | 7.999 | 8.000 | 7.998 | 2.471 | 0.825 | 0.859 | 0.455 | 2.204 |
| | (1.00, 1.00) | 7.748 | 7.978 | 7.979 | 7.990 | 7.991 | 3.397 | 2.280 | 2.138 | 2.002 | 2.333 |
| | (1.00, 4.00) | 6.166 | 7.493 | 7.519 | 7.639 | 7.698 | 7.484 | 7.979 | 7.003 | 6.914 | 12.334 |
| | (4.00, 0.25) | 7.573 | 7.815 | 7.823 | 7.867 | 7.939 | 3.933 | 3.589 | 3.342 | 3.535 | 2.758 |
| | (4.00, 1.00) | 7.136 | 7.623 | 7.644 | 7.720 | 7.843 | 5.056 | 4.891 | 4.558 | 4.636 | 3.978 |
| | (4.00, 4.00) | 5.493 | 6.873 | 6.947 | 7.108 | 7.313 | 9.457 | 10.595 | 9.600 | 9.283 | 15.277 |
| (8, 8, 80%) | (0.25, 0.25) | 7.895 | 8.000 | 8.000 | 8.000 | 7.998 | 2.730 | 0.000 | 0.000 | 0.000 | 2.375 |
| | (0.25, 1.00) | 7.332 | 7.968 | 7.970 | 7.990 | 7.974 | 4.383 | 2.441 | 2.373 | 2.047 | 3.750 |
| | (0.25, 4.00) | 4.459 | 6.891 | 6.955 | 7.172 | 7.300 | 12.033 | 11.653 | 9.949 | 10.321 | 14.942 |
| | (1.00, 0.25) | 7.648 | 7.969 | 7.971 | 7.986 | 7.979 | 3.590 | 2.382 | 2.253 | 2.156 | 3.364 |
| | (1.00, 1.00) | 6.913 | 7.797 | 7.807 | 7.881 | 7.896 | 5.314 | 4.197 | 4.006 | 3.955 | 4.000 |
| | (1.00, 4.00) | 4.192 | 6.586 | 6.648 | 6.890 | 7.106 | 13.131 | 12.839 | 11.521 | 11.162 | 15.880 |
| | (4.00, 0.25) | 6.338 | 7.102 | 7.125 | 7.276 | 7.386 | 6.588 | 6.952 | 6.780 | 6.714 | 6.523 |
| | (4.00, 1.00) | 5.498 | 6.678 | 6.761 | 6.913 | 7.080 | 8.666 | 9.015 | 8.526 | 8.641 | 8.298 |
| | (4.00, 4.00) | 3.440 | 5.584 | 5.731 | 5.921 | 6.300 | 17.069 | 17.848 | 15.928 | 16.108 | 21.112 |
| (8, 8, 90%) | (0.25, 0.25) | 6.848 | 7.975 | 7.976 | 7.995 | 7.967 | 5.219 | 2.115 | 2.052 | 1.729 | 3.989 |
| | (0.25, 1.00) | 4.921 | 7.414 | 7.429 | 7.660 | 7.663 | 9.570 | 6.141 | 5.743 | 5.793 | 6.670 |
| | (0.25, 4.00) | 1.764 | 4.891 | 5.006 | 5.358 | 5.616 | 29.311 | 23.907 | 20.748 | 21.285 | 23.116 |
| | (1.00, 0.25) | 5.799 | 7.446 | 7.460 | 7.637 | 7.663 | 7.391 | 5.675 | 5.457 | 5.462 | 6.223 |
| | (1.00, 1.00) | 4.161 | 6.588 | 6.629 | 6.927 | 7.000 | 11.980 | 9.626 | 9.136 | 9.123 | 9.000 |
| | (1.00, 4.00) | 1.607 | 4.460 | 4.596 | 4.891 | 5.219 | 31.659 | 27.034 | 23.939 | 24.016 | 26.196 |
| | (4.00, 0.25) | 3.234 | 4.950 | 5.043 | 5.253 | 5.262 | 16.127 | 18.070 | 17.441 | 17.646 | 17.155 |
| | (4.00, 1.00) | 2.469 | 4.463 | 4.568 | 4.787 | 4.845 | 21.436 | 21.669 | 20.786 | 21.020 | 20.870 |
| | (4.00, 4.00) | 1.190 | 3.318 | 3.570 | 3.762 | 4.104 | 39.082 | 39.786 | 35.230 | 35.680 | 39.508 |

Table D.3: Fill Rate & Expected Number of Backorders at a Production Facility

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^p$ NoVis | $f^p$ LoVis | $f^p$ MedVis | $f^p$ HiVis | $f^p$ GI/G/2 | $E[B^p]$ NoVis | $E[B^p]$ LoVis | $E[B^p]$ MedVis | $E[B^p]$ HiVis | $E[B^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 70%) | (0.25, 0.25) | 0.654 | 0.963 | 0.961 | 0.973 | 0.976 | 0.337 | 0.018 | 0.016 | 0.011 | 0.001 |
| | (0.25, 1.00) | 0.546 | 0.860 | 0.849 | 0.871 | 0.845 | 0.840 | 0.169 | 0.153 | 0.130 | 0.120 |
| | (0.25, 4.00) | 0.410 | 0.737 | 0.708 | 0.722 | 0.722 | 3.155 | 1.193 | 1.030 | 0.999 | 0.922 |
| | (1.00, 0.25) | 0.585 | 0.788 | 0.771 | 0.790 | 0.810 | 0.606 | 0.202 | 0.181 | 0.165 | 0.129 |
| | (1.00, 1.00) | 0.510 | 0.717 | 0.700 | 0.719 | 0.718 | 1.143 | 0.468 | 0.414 | 0.387 | 0.330 |
| | (1.00, 4.00) | 0.402 | 0.648 | 0.621 | 0.632 | 0.662 | 3.486 | 1.638 | 1.389 | 1.362 | 1.150 |
| | (4.00, 0.25) | 0.483 | 0.469 | 0.441 | 0.456 | 0.521 | 1.458 | 1.125 | 1.018 | 0.991 | 0.957 |
| | (4.00, 1.00) | 0.446 | 0.464 | 0.440 | 0.453 | 0.520 | 2.113 | 1.576 | 1.399 | 1.369 | 1.217 |
| | (4.00, 4.00) | 0.383 | 0.476 | 0.450 | 0.460 | 0.557 | 4.564 | 3.075 | 2.593 | 2.538 | 2.220 |
| (2, 2, 80%) | (0.25, 0.25) | 0.496 | 0.882 | 0.870 | 0.896 | 0.934 | 0.859 | 0.089 | 0.082 | 0.065 | 0.076 |
| | (0.25, 1.00) | 0.390 | 0.715 | 0.689 | 0.717 | 0.680 | 1.953 | 0.536 | 0.483 | 0.440 | 0.394 |
| | (0.25, 4.00) | 0.278 | 0.581 | 0.534 | 0.547 | 0.540 | 6.641 | 2.834 | 2.471 | 2.454 | 2.267 |
| | (1.00, 0.25) | 0.429 | 0.636 | 0.605 | 0.627 | 0.626 | 1.429 | 0.574 | 0.520 | 0.491 | 0.424 |
| | (1.00, 1.00) | 0.360 | 0.555 | 0.528 | 0.546 | 0.545 | 2.560 | 1.193 | 1.056 | 1.015 | 0.910 |
| | (1.00, 4.00) | 0.273 | 0.489 | 0.454 | 0.465 | 0.489 | 7.273 | 3.706 | 3.129 | 3.095 | 2.803 |
| | (4.00, 0.25) | 0.332 | 0.308 | 0.284 | 0.295 | 0.359 | 3.371 | 2.796 | 2.523 | 2.465 | 2.403 |
| | (4.00, 1.00) | 0.304 | 0.312 | 0.290 | 0.299 | 0.363 | 4.652 | 3.636 | 3.208 | 3.192 | 2.949 |
| | (4.00, 4.00) | 0.258 | 0.326 | 0.299 | 0.307 | 0.400 | 9.491 | 6.735 | 5.573 | 5.548 | 5.068 |
| (2, 2, 90%) | (0.25, 0.25) | 0.284 | 0.664 | 0.621 | 0.656 | 0.632 | 2.786 | 0.508 | 0.475 | 0.431 | 0.392 |
| | (0.25, 1.00) | 0.209 | 0.465 | 0.421 | 0.444 | 0.407 | 5.748 | 2.023 | 1.817 | 1.744 | 1.623 |
| | (0.25, 4.00) | 0.141 | 0.356 | 0.300 | 0.313 | 0.297 | 17.830 | 8.488 | 7.462 | 7.408 | 7.093 |
| | (1.00, 0.25) | 0.236 | 0.400 | 0.358 | 0.375 | 0.360 | 4.298 | 2.026 | 1.853 | 1.803 | 1.702 |
| | (1.00, 1.00) | 0.190 | 0.331 | 0.298 | 0.311 | 0.309 | 7.290 | 3.800 | 3.358 | 3.300 | 3.108 |
| | (1.00, 4.00) | 0.138 | 0.284 | 0.246 | 0.254 | 0.268 | 19.388 | 10.597 | 8.999 | 9.039 | 8.572 |
| | (4.00, 0.25) | 0.170 | 0.153 | 0.137 | 0.144 | 0.184 | 9.840 | 8.364 | 7.533 | 7.562 | 7.377 |
| | (4.00, 1.00) | 0.155 | 0.158 | 0.143 | 0.147 | 0.190 | 13.007 | 10.514 | 9.142 | 9.145 | 8.828 |
| | (4.00, 4.00) | 0.130 | 0.169 | 0.150 | 0.155 | 0.216 | 25.242 | 17.980 | 15.095 | 14.995 | 14.551 |

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**TableD.3 – continued from previous page**

| (R, S, ρ) | $(cr^2, cp^2)$ | $f^p$ NoVis | $f^p$ LoVis | $f^p$ MedVis | $f^p$ HiVis | $f^p$ GI/G/2 | $E[B^p]$ NoVis | $E[B^p]$ LoVis | $E[B^p]$ MedVis | $E[B^p]$ HiVis | $E[B^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (4, 4, 70%) | (0.25, 0.25) | 0.916 | 1.000 | 1.000 | 1.000 | 1.000 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.809 | 0.979 | 0.977 | 0.988 | 0.979 | 0.354 | 0.025 | 0.023 | 0.012 | 0.024 |
| | (0.25, 4.00) | 0.581 | 0.858 | 0.835 | 0.860 | 0.859 | 2.239 | 0.668 | 0.617 | 0.532 | 0.554 |
| | (1.00, 0.25) | 0.854 | 0.967 | 0.964 | 0.976 | 0.978 | 0.213 | 0.031 | 0.028 | 0.018 | 0.023 |
| | (1.00, 1.00) | 0.760 | 0.917 | 0.910 | 0.932 | 0.932 | 0.560 | 0.134 | 0.124 | 0.094 | 0.079 |
| | (1.00, 4.00) | 0.565 | 0.805 | 0.781 | 0.806 | 0.826 | 2.540 | 0.953 | 0.871 | 0.781 | 0.699 |
| | (4.00, 0.25) | 0.718 | 0.748 | 0.730 | 0.762 | 0.787 | 0.795 | 0.518 | 0.491 | 0.432 | 0.342 |
| | (4.00, 1.00) | 0.652 | 0.707 | 0.687 | 0.716 | 0.746 | 1.326 | 0.835 | 0.775 | 0.709 | 0.544 |
| | (4.00, 4.00) | 0.521 | 0.659 | 0.631 | 0.655 | 0.719 | 3.542 | 2.058 | 1.862 | 1.740 | 1.549 |
| (4, 4, 80%) | (0.25, 0.25) | 0.7998 | 0.992 | 0.9907 | 0.997 | 0.984 | 0.341 | 0.006 | 0.0058 | 0.0019 | 0.019 |
| | (0.25, 1.00) | 0.646 | 0.911 | 0.900 | 0.933 | 0.923 | 1.134 | 0.163 | 0.155 | 0.103 | 0.116 |
| | (0.25, 4.00) | 0.413 | 0.723 | 0.677 | 0.707 | 0.710 | 5.403 | 1.912 | 1.778 | 1.652 | 1.574 |
| | (1.00, 0.25) | 0.708 | 0.887 | 0.875 | 0.906 | 0.917 | 0.729 | 0.173 | 0.163 | 0.123 | 0.114 |
| | (1.00, 1.00) | 0.590 | 0.796 | 0.776 | 0.813 | 0.814 | 1.638 | 0.527 | 0.502 | 0.418 | 0.373 |
| | (1.00, 4.00) | 0.399 | 0.656 | 0.616 | 0.644 | 0.662 | 6.011 | 2.548 | 2.308 | 2.180 | 2.009 |
| | (4.00, 0.25) | 0.535 | 0.554 | 0.529 | 0.560 | 0.584 | 2.348 | 1.740 | 1.657 | 1.527 | 1.399 |
| | (4.00, 1.00) | 0.473 | 0.520 | 0.497 | 0.523 | 0.551 | 3.520 | 2.467 | 2.265 | 2.168 | 1.904 |
| | (4.00, 4.00) | 0.362 | 0.488 | 0.455 | 0.474 | 0.541 | 8.164 | 5.016 | 4.540 | 4.408 | 4.051 |
| (4, 4, 90%) | (0.25, 0.25) | 0.547 | 0.902 | 0.887 | 0.936 | 0.929 | 1.763 | 0.145 | 0.141 | 0.081 | 0.123 |
| | (0.25, 1.00) | 0.389 | 0.695 | 0.657 | 0.717 | 0.705 | 4.441 | 1.112 | 1.073 | 0.886 | 0.845 |
| | (0.25, 4.00) | 0.218 | 0.484 | 0.416 | 0.445 | 0.434 | 16.230 | 6.683 | 6.399 | 6.119 | 5.864 |
| | (1.00, 0.25) | 0.449 | 0.662 | 0.626 | 0.677 | 0.686 | 3.099 | 1.102 | 1.074 | 0.929 | 0.866 |
| | (1.00, 1.00) | 0.344 | 0.545 | 0.505 | 0.546 | 0.547 | 5.905 | 2.461 | 2.370 | 2.175 | 2.039 |
| | (1.00, 4.00) | 0.210 | 0.416 | 0.365 | 0.385 | 0.393 | 17.773 | 8.620 | 7.800 | 7.659 | 7.269 |
| | (4.00, 0.25) | 0.294 | 0.306 | 0.285 | 0.304 | 0.317 | 8.369 | 6.602 | 6.280 | 6.145 | 5.906 |
| | (4.00, 1.00) | 0.255 | 0.290 | 0.267 | 0.283 | 0.301 | 11.468 | 8.515 | 7.897 | 7.675 | 7.340 |
| | (4.00, 4.00) | 0.187 | 0.273 | 0.243 | 0.258 | 0.303 | 23.588 | 15.450 | 13.921 | 13.459 | 13.094 |
| (6, 6, 70%) | (0.25, 0.25) | 0.979 | 1.000 | 1.000 | 1.000 | 1.000 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.919 | 0.997 | 0.997 | 0.999 | 0.995 | 0.149 | 0.004 | 0.004 | 0.001 | 0.006 |
| | (0.25, 4.00) | 0.703 | 0.916 | 0.902 | 0.924 | 0.922 | 1.589 | 0.393 | 0.372 | 0.288 | 0.359 |
| | (1.00, 0.25) | 0.949 | 0.995 | 0.995 | 0.997 | 0.995 | 0.075 | 0.004 | 0.004 | 0.002 | 0.005 |
| | (1.00, 1.00) | 0.882 | 0.975 | 0.973 | 0.984 | 0.984 | 0.275 | 0.040 | 0.037 | 0.023 | 0.019 |
| | (1.00, 4.00) | 0.683 | 0.880 | 0.865 | 0.888 | 0.903 | 1.851 | 0.600 | 0.546 | 0.456 | 0.456 |

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**TableD.3 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^p$ NoVis | $f^p$ LoVis | $f^p$ MedVis | $f^p$ HiVis | $f^p$ GI/G/2 | $E[B^p]$ NoVis | $E[B^p]$ LoVis | $E[B^p]$ MedVis | $E[B^p]$ HiVis | $E[B^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (6, 6, 70%) | (4.00, 0.25) | 0.846 | 0.883 | 0.874 | 0.896 | 0.926 | 0.433 | 0.238 | 0.226 | 0.188 | 0.109 |
| | (4.00, 1.00) | 0.782 | 0.838 | 0.826 | 0.853 | 0.888 | 0.832 | 0.457 | 0.434 | 0.367 | 0.229 |
| | (4.00, 4.00) | 0.629 | 0.761 | 0.738 | 0.767 | 0.816 | 2.749 | 1.471 | 1.352 | 1.203 | 1.116 |
| (6, 6, 80%) | (0.25, 0.25) | 0.921 | 0.999 | 0.999 | 1.000 | 0.996 | 0.135 | 0.000 | 0.000 | 0.000 | 0.005 |
| | (0.25, 1.00) | 0.794 | 0.972 | 0.968 | 0.985 | 0.977 | 0.659 | 0.052 | 0.050 | 0.024 | 0.040 |
| | (0.25, 4.00) | 0.522 | 0.802 | 0.768 | 0.803 | 0.811 | 4.396 | 1.361 | 1.288 | 1.121 | 1.127 |
| | (1.00, 0.25) | 0.851 | 0.964 | 0.960 | 0.976 | 0.977 | 0.372 | 0.054 | 0.051 | 0.031 | 0.036 |
| | (1.00, 1.00) | 0.738 | 0.903 | 0.892 | 0.923 | 0.924 | 1.049 | 0.254 | 0.241 | 0.173 | 0.153 |
| | (1.00, 4.00) | 0.503 | 0.749 | 0.714 | 0.750 | 0.771 | 2.167 | 1.856 | 1.760 | 1.547 | 1.477 |
| | (4.00, 0.25) | 0.676 | 0.711 | 0.693 | 0.727 | 0.757 | 1.636 | 1.125 | 1.073 | 0.945 | 0.796 |
| | (4.00, 1.00) | 0.602 | 0.664 | 0.643 | 0.676 | 0.709 | 2.663 | 1.689 | 1.615 | 1.472 | 1.213 |
| | (4.00, 4.00) | 0.451 | 0.591 | 0.558 | 0.587 | 0.646 | 7.023 | 4.062 | 3.769 | 3.514 | 3.270 |
| (6, 6, 90%) | (0.25, 0.25) | 0.713 | 0.971 | 0.966 | 0.988 | 0.975 | 1.116 | 0.044 | 0.042 | 0.015 | 0.048 |
| | (0.25, 1.00) | 0.528 | 0.819 | 0.795 | 0.857 | 0.849 | 3.432 | 0.659 | 0.641 | 0.449 | 0.451 |
| | (0.25, 4.00) | 0.289 | 0.564 | 0.501 | 0.543 | 0.543 | 14.773 | 5.599 | 5.468 | 5.067 | 4.872 |
| | (1.00, 0.25) | 0.603 | 0.803 | 0.782 | 0.833 | 0.842 | 2.234 | 0.643 | 0.627 | 0.478 | 0.451 |
| | (1.00, 1.00) | 0.469 | 0.679 | 0.647 | 0.701 | 0.703 | 4.783 | 1.724 | 1.694 | 1.434 | 1.338 |
| | (1.00, 4.00) | 0.276 | 0.503 | 0.445 | 0.481 | 0.495 | 16.293 | 7.216 | 7.116 | 6.502 | 6.185 |
| | (4.00, 0.25) | 0.400 | 0.431 | 0.409 | 0.436 | 0.452 | 7.118 | 5.386 | 5.145 | 4.995 | 4.711 |
| | (4.00, 1.00) | 0.343 | 0.396 | 0.370 | 0.399 | 0.417 | 10.111 | 7.154 | 6.897 | 6.439 | 6.090 |
| | (4.00, 4.00) | 0.240 | 0.348 | 0.315 | 0.339 | 0.380 | 22.043 | 13.826 | 12.541 | 12.089 | 11.797 |
| (8, 8, 70%) | (0.25, 0.25) | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.996 | 1.000 | 1.000 | 1.000 | 0.999 | 0.063 | 0.001 | 0.001 | 0.000 | 0.002 |
| | (0.25, 4.00) | 0.789 | 0.950 | 0.941 | 0.959 | 0.953 | 1.128 | 0.237 | 0.222 | 0.156 | 0.246 |
| | (1.00, 0.25) | 0.982 | 0.999 | 0.999 | 1.000 | 1.000 | 0.026 | 0.001 | 0.001 | 0.000 | 0.001 |
| | (1.00, 1.00) | 0.942 | 0.993 | 0.992 | 0.996 | 0.996 | 0.135 | 0.012 | 0.011 | 0.005 | 0.005 |
| | (1.00, 4.00) | 0.769 | 0.925 | 0.913 | 0.934 | 0.941 | 1.349 | 0.373 | 0.358 | 0.267 | 0.314 |
| | (4.00, 0.25) | 0.916 | 0.945 | 0.940 | 0.955 | 0.977 | 0.236 | 0.113 | 0.108 | 0.082 | 0.033 |
| | (4.00, 1.00) | 0.863 | 0.911 | 0.903 | 0.924 | 0.953 | 0.522 | 0.252 | 0.240 | 0.190 | 0.094 |
| | (4.00, 4.00) | 0.712 | 0.829 | 0.810 | 0.839 | 0.875 | 2.134 | 1.057 | 0.999 | 0.834 | 0.826 |

Continued on next page

**TableD.3 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $f^p$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[B^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (8, 8, 80%) | (0.25, 0.25) | 0.968 | 1.000 | 1.000 | 1.000 | 0.999 | 0.054 | 0.000 | 0.000 | 0.000 | 0.001 |
| | (0.25, 1.00) | 0.881 | 0.991 | 0.990 | 0.996 | 0.992 | 0.383 | 0.016 | 0.016 | 0.006 | 0.015 |
| | (0.25, 4.00) | 0.611 | 0.856 | 0.831 | 0.866 | 0.872 | 3.576 | 1.008 | 0.941 | 0.762 | 0.830 |
| | (1.00, 0.25) | 0.924 | 0.989 | 0.988 | 0.994 | 0.993 | 0.190 | 0.017 | 0.016 | 0.007 | 0.012 |
| | (1.00, 1.00) | 0.832 | 0.954 | 0.948 | 0.968 | 0.969 | 0.671 | 0.122 | 0.116 | 0.072 | 0.063 |
| | (1.00, 4.00) | 0.589 | 0.813 | 0.784 | 0.822 | 0.840 | 4.105 | 1.390 | 1.364 | 1.099 | 1.110 |
| | (4.00, 0.25) | 0.774 | 0.814 | 0.798 | 0.831 | 0.862 | 1.140 | 0.722 | 0.712 | 0.584 | 0.451 |
| | (4.00, 1.00) | 0.699 | 0.762 | 0.748 | 0.780 | 0.815 | 2.015 | 1.202 | 1.128 | 1.000 | 0.770 |
| | (4.00, 4.00) | 0.528 | 0.672 | 0.642 | 0.673 | 0.725 | 6.042 | 3.238 | 3.033 | 2.806 | 2.664 |
| (8, 8, 90%) | (0.25, 0.25) | 0.819 | 0.991 | 0.990 | 0.998 | 0.990 | 0.706 | 0.013 | 0.012 | 0.003 | 0.020 |
| | (0.25, 1.00) | 0.635 | 0.892 | 0.878 | 0.927 | 0.921 | 2.652 | 0.393 | 0.383 | 0.228 | 0.247 |
| | (0.25, 4.00) | 0.352 | 0.625 | 0.571 | 0.622 | 0.629 | 13.446 | 4.894 | 4.668 | 4.200 | 4.068 |
| | (1.00, 0.25) | 0.714 | 0.885 | 0.873 | 0.914 | 0.919 | 1.610 | 0.373 | 0.365 | 0.245 | 0.239 |
| | (1.00, 1.00) | 0.570 | 0.774 | 0.750 | 0.805 | 0.805 | 3.874 | 1.229 | 1.211 | 0.947 | 0.878 |
| | (1.00, 4.00) | 0.336 | 0.569 | 0.517 | 0.560 | 0.578 | 14.936 | 6.193 | 6.026 | 5.522 | 5.282 |
| | (4.00, 0.25) | 0.489 | 0.530 | 0.508 | 0.542 | 0.562 | 6.053 | 4.486 | 4.342 | 4.063 | 3.756 |
| | (4.00, 1.00) | 0.421 | 0.491 | 0.462 | 0.496 | 0.516 | 8.915 | 5.887 | 5.732 | 5.403 | 5.050 |
| | (4.00, 4.00) | 0.290 | 0.414 | 0.380 | 0.408 | 0.447 | 20.598 | 12.133 | 11.236 | 10.863 | 10.643 |

Table D.4: Expected Inventory Level & Expected Time to Fulfill a Backorder at a Production Facility

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 70%) | (0.25, 0.25) | 0.954 | 1.054 | 1.053 | 1.154 | 1.169 | 3.950 | 0.962 | 0.834 | 0.828 | 0.114 |
| | (0.25, 1.00) | 0.846 | 0.928 | 0.925 | 1.019 | 1.073 | 5.690 | 2.414 | 2.019 | 2.023 | 1.544 |
| | (0.25, 4.00) | 0.710 | 0.830 | 0.825 | 0.889 | 0.994 | 12.698 | 9.069 | 7.055 | 7.186 | 6.623 |
| | (1.00, 0.25) | 0.885 | 0.940 | 0.935 | 0.988 | 1.003 | 4.917 | 1.912 | 1.583 | 1.575 | 1.358 |
| | (1.00, 1.00) | 0.810 | 0.858 | 0.855 | 0.910 | 0.957 | 6.667 | 3.312 | 2.758 | 2.759 | 2.333 |
| | (1.00, 4.00) | 0.702 | 0.790 | 0.786 | 0.827 | 0.929 | 13.667 | 9.311 | 7.322 | 7.406 | 6.809 |
| | (4.00, 0.25) | 0.783 | 0.780 | 0.775 | 0.798 | 0.795 | 7.643 | 4.237 | 3.647 | 3.642 | 3.992 |
| | (4.00, 1.00) | 0.746 | 0.752 | 0.749 | 0.774 | 0.800 | 9.620 | 5.873 | 4.995 | 5.005 | 5.069 |
| | (4.00, 4.00) | 0.683 | 0.727 | 0.726 | 0.750 | 0.830 | 16.802 | 11.722 | 9.437 | 9.398 | 10.029 |

Continued on next page

**TableD.4 – continued from previous page**

| (R, S, ρ) | $(cr^2, cp^2)$ | $E[I^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 80%) | (0.25, 0.25) | 0.696 | 0.817 | 0.814 | 0.911 | 0.969 | 5.408 | 1.509 | 1.256 | 1.253 | 2.303 |
| | (0.25, 1.00) | 0.590 | 0.672 | 0.666 | 0.744 | 0.802 | 8.407 | 3.756 | 3.104 | 3.107 | 2.461 |
| | (0.25, 4.00) | 0.478 | 0.586 | 0.576 | 0.625 | 0.711 | 20.406 | 13.534 | 10.593 | 10.837 | 9.868 |
| | (1.00, 0.25) | 0.629 | 0.688 | 0.678 | 0.723 | 0.729 | 7.000 | 3.153 | 2.635 | 2.628 | 2.267 |
| | (1.00, 1.00) | 0.560 | 0.607 | 0.602 | 0.644 | 0.688 | 10.000 | 5.367 | 4.473 | 4.474 | 4.000 |
| | (1.00, 4.00) | 0.473 | 0.550 | 0.545 | 0.577 | 0.660 | 22.000 | 14.520 | 11.467 | 11.588 | 10.965 |
| | (4.00, 0.25) | 0.532 | 0.530 | 0.525 | 0.542 | 0.541 | 12.096 | 8.090 | 7.054 | 7.002 | 7.494 |
| | (4.00, 1.00) | 0.504 | 0.511 | 0.508 | 0.525 | 0.548 | 15.369 | 10.573 | 9.040 | 9.109 | 9.251 |
| | (4.00, 4.00) | 0.458 | 0.492 | 0.490 | 0.506 | 0.579 | 27.582 | 19.972 | 15.907 | 16.023 | 16.895 |
| (2, 2, 90%) | (0.25, 0.25) | 0.384 | 0.499 | 0.486 | 0.556 | 0.595 | 9.782 | 3.022 | 2.508 | 2.506 | 2.133 |
| | (0.25, 1.00) | 0.309 | 0.371 | 0.361 | 0.407 | 0.448 | 16.532 | 7.553 | 6.273 | 6.271 | 5.473 |
| | (0.25, 4.00) | 0.241 | 0.313 | 0.300 | 0.332 | 0.377 | 43.531 | 26.330 | 21.325 | 21.552 | 20.182 |
| | (1.00, 0.25) | 0.336 | 0.385 | 0.371 | 0.399 | 0.396 | 13.250 | 6.755 | 5.775 | 5.767 | 5.317 |
| | (1.00, 1.00) | 0.290 | 0.325 | 0.317 | 0.341 | 0.371 | 20.000 | 11.356 | 9.572 | 9.577 | 9.000 |
| | (1.00, 4.00) | 0.238 | 0.289 | 0.282 | 0.299 | 0.350 | 47.000 | 29.602 | 23.877 | 24.214 | 23.424 |
| | (4.00, 0.25) | 0.270 | 0.271 | 0.266 | 0.276 | 0.275 | 25.712 | 19.752 | 17.469 | 17.672 | 18.088 |
| | (4.00, 1.00) | 0.255 | 0.262 | 0.258 | 0.267 | 0.282 | 32.781 | 24.996 | 21.357 | 21.461 | 21.784 |
| | (4.00, 4.00) | 0.230 | 0.252 | 0.250 | 0.259 | 0.304 | 60.028 | 43.266 | 35.538 | 35.521 | 37.109 |
| (4, 4, 70%) | (0.25, 0.25) | 2.700 | 2.812 | 2.812 | 3.143 | 3.168 | 3.950 | 0.933 | 0.830 | 0.819 | 0.114 |
| | (0.25, 1.00) | 2.359 | 2.530 | 2.530 | 2.901 | 2.977 | 5.699 | 2.377 | 2.010 | 2.013 | 2.290 |
| | (0.25, 4.00) | 1.795 | 2.132 | 2.120 | 2.422 | 2.626 | 12.698 | 9.389 | 7.481 | 7.589 | 7.885 |
| | (1.00, 0.25) | 2.492 | 2.579 | 2.579 | 2.841 | 2.897 | 4.917 | 1.850 | 1.549 | 1.543 | 2.076 |
| | (1.00, 1.00) | 2.227 | 2.346 | 2.342 | 2.617 | 2.707 | 6.667 | 3.243 | 2.754 | 2.758 | 2.333 |
| | (1.00, 4.00) | 1.757 | 2.029 | 2.014 | 2.245 | 2.479 | 13.667 | 9.791 | 7.952 | 8.037 | 8.048 |
| | (4.00, 0.25) | 2.120 | 2.085 | 2.081 | 2.238 | 2.180 | 7.643 | 4.113 | 3.640 | 3.622 | 3.204 |
| | (4.00, 1.00) | 1.958 | 1.970 | 1.961 | 2.114 | 2.126 | 9.620 | 5.708 | 4.960 | 4.988 | 4.280 |
| | (4.00, 4.00) | 1.661 | 1.813 | 1.807 | 1.952 | 2.159 | 16.802 | 12.062 | 10.095 | 10.092 | 11.025 |
| (4, 4, 80%) | (0.25, 0.25) | 2.178 | 2.3896 | 2.3876 | 2.848 | 2.912 | 5.407 | 1.497 | 1.243 | 1.251 | 2.375 |
| | (0.25, 1.00) | 1.772 | 2.001 | 1.995 | 2.407 | 2.525 | 8.407 | 3.672 | 3.099 | 3.095 | 3.024 |
| | (0.25, 4.00) | 1.240 | 1.591 | 1.562 | 1.823 | 2.017 | 20.406 | 13.815 | 11.024 | 11.292 | 10.837 |
| | (1.00, 0.25) | 1.929 | 2.065 | 2.058 | 2.356 | 2.419 | 7.000 | 3.077 | 2.608 | 2.611 | 2.756 |
| | (1.00, 1.00) | 1.638 | 1.790 | 1.779 | 2.048 | 2.151 | 10.000 | 5.171 | 4.475 | 4.473 | 4.000 |
| | (1.00, 4.00) | 1.211 | 1.486 | 1.466 | 1.662 | 1.867 | 22.000 | 14.803 | 12.033 | 12.244 | 11.885 |

Continued on next page

**TableD.4 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (4, 4, 80%) | (4.00, 0.25) | 1.510 | 1.489 | 1.478 | 1.604 | 1.537 | 12.096 | 7.799 | 7.032 | 6.950 | 6.721 |
| | (4.00, 1.00) | 1.372 | 1.391 | 1.383 | 1.501 | 1.504 | 15.369 | 10.295 | 9.013 | 9.091 | 8.475 |
| | (4.00, 4.00) | 1.132 | 1.273 | 1.263 | 1.366 | 1.562 | 27.582 | 19.567 | 16.667 | 16.772 | 17.670 |
| (4, 4, 90%) | (0.25, 0.25) | 1.361 | 1.683 | 1.678 | 2.205 | 2.326 | 9.782 | 2.950 | 2.502 | 2.508 | 3.459 |
| | (0.25, 1.00) | 1.002 | 1.229 | 1.210 | 1.548 | 1.670 | 16.532 | 7.296 | 6.256 | 6.256 | 5.735 |
| | (0.25, 4.00) | 0.641 | 0.907 | 0.867 | 1.044 | 1.148 | 43.531 | 25.886 | 21.909 | 22.056 | 20.735 |
| | (1.00, 0.25) | 1.136 | 1.291 | 1.269 | 1.525 | 1.561 | 13.250 | 6.519 | 5.746 | 5.745 | 5.513 |
| | (1.00, 1.00) | 0.905 | 1.047 | 1.023 | 1.217 | 1.302 | 20.000 | 10.819 | 9.564 | 9.577 | 9.000 |
| | (1.00, 4.00) | 0.623 | 0.827 | 0.803 | 0.919 | 1.047 | 47.000 | 29.536 | 24.528 | 24.899 | 23.949 |
| | (4.00, 0.25) | 0.799 | 0.796 | 0.788 | 0.858 | 0.804 | 25.712 | 19.021 | 17.569 | 17.675 | 17.302 |
| | (4.00, 1.00) | 0.716 | 0.740 | 0.732 | 0.796 | 0.795 | 32.781 | 23.988 | 21.562 | 21.419 | 21.007 |
| | (4.00, 4.00) | 0.576 | 0.672 | 0.660 | 0.723 | 0.846 | 60.028 | 42.464 | 36.745 | 36.311 | 37.571 |
| (6, 6, 70%) | (0.25, 0.25) | 4.638 | 4.725 | 4.725 | 5.143 | 5.168 | 3.950 | 0.549 | 0.437 | 0.100 | 0.114 |
| | (0.25, 1.00) | 4.154 | 4.323 | 4.323 | 4.890 | 4.959 | 5.699 | 2.321 | 2.019 | 1.978 | 2.487 |
| | (0.25, 4.00) | 3.145 | 3.628 | 3.607 | 4.178 | 4.431 | 12.698 | 9.402 | 7.553 | 7.615 | 9.257 |
| | (1.00, 0.25) | 4.354 | 4.419 | 4.419 | 4.825 | 4.879 | 4.917 | 1.815 | 1.541 | 1.557 | 2.193 |
| | (1.00, 1.00) | 3.941 | 4.072 | 4.069 | 4.545 | 4.646 | 6.667 | 3.235 | 2.739 | 2.749 | 2.333 |
| | (1.00, 4.00) | 3.068 | 3.469 | 3.462 | 3.920 | 4.236 | 13.667 | 9.975 | 8.114 | 8.140 | 9.400 |
| | (4.00, 0.25) | 3.758 | 3.696 | 3.692 | 3.995 | 3.947 | 7.643 | 4.073 | 3.577 | 3.616 | 2.955 |
| | (4.00, 1.00) | 3.465 | 3.463 | 3.453 | 3.772 | 3.811 | 9.620 | 5.639 | 4.979 | 4.998 | 4.076 |
| | (4.00, 4.00) | 2.868 | 3.110 | 3.099 | 3.415 | 3.726 | 16.802 | 12.295 | 10.335 | 10.315 | 12.116 |
| (6, 6, 80%) | (0.25, 0.25) | 3.972 | 4.159 | 4.158 | 4.846 | 4.897 | 5.407 | 1.458 | 1.196 | 1.032 | 2.375 |
| | (0.25, 1.00) | 3.296 | 3.594 | 3.591 | 4.328 | 4.449 | 8.407 | 3.675 | 3.083 | 3.074 | 3.434 |
| | (0.25, 4.00) | 2.233 | 2.784 | 2.756 | 3.292 | 3.571 | 20.406 | 13.763 | 11.096 | 11.359 | 11.895 |
| | (1.00, 0.25) | 3.572 | 3.726 | 3.723 | 4.263 | 4.341 | 7.000 | 3.070 | 2.587 | 2.580 | 3.103 |
| | (1.00, 1.00) | 3.049 | 3.264 | 3.255 | 3.802 | 3.930 | 10.000 | 5.224 | 4.476 | 4.490 | 4.000 |
| | (1.00, 4.00) | 2.167 | 2.624 | 2.596 | 3.029 | 3.334 | 21.935 | 14.758 | 12.307 | 12.365 | 12.886 |
| | (4.00, 0.25) | 2.797 | 2.755 | 2.749 | 3.021 | 2.934 | 12.096 | 7.786 | 7.001 | 6.930 | 6.568 |
| | (4.00, 1.00) | 2.516 | 2.544 | 2.529 | 2.805 | 2.812 | 15.369 | 10.055 | 9.045 | 9.093 | 8.334 |
| | (4.00, 4.00) | 1.990 | 2.242 | 2.220 | 2.472 | 2.781 | 27.582 | 19.879 | 17.069 | 17.044 | 18.492 |

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**TableD.4 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (6, 6, 90%) | (0.25, 0.25) | 2.714 | 3.157 | 3.157 | 4.140 | 4.251 | 9.782 | 3.005 | 2.492 | 2.479 | 3.905 |
| | (0.25, 1.00) | 1.993 | 2.373 | 2.362 | 3.112 | 3.277 | 16.532 | 7.271 | 6.256 | 6.263 | 5.999 |
| | (0.25, 4.00) | 1.184 | 1.645 | 1.595 | 1.992 | 2.156 | 43.531 | 25.708 | 21.889 | 22.169 | 21.310 |
| | (1.00, 0.25) | 2.271 | 2.512 | 2.499 | 3.074 | 3.145 | 13.250 | 6.515 | 5.738 | 5.728 | 5.709 |
| | (1.00, 1.00) | 1.783 | 2.031 | 2.004 | 2.476 | 2.601 | 20.000 | 10.749 | 9.597 | 9.598 | 9.000 |
| | (1.00, 4.00) | 1.143 | 1.516 | 1.460 | 1.762 | 1.963 | 47.000 | 29.011 | 25.645 | 25.044 | 24.492 |
| | (4.00, 0.25) | 1.547 | 1.552 | 1.541 | 1.709 | 1.609 | 25.712 | 18.926 | 17.433 | 17.721 | 17.179 |
| | (4.00, 1.00) | 1.360 | 1.403 | 1.383 | 1.561 | 1.544 | 32.781 | 23.674 | 21.874 | 21.411 | 20.891 |
| | (4.00, 4.00) | 1.030 | 1.210 | 1.194 | 1.354 | 1.550 | 60.028 | 42.392 | 36.593 | 36.594 | 38.043 |
| (8, 8, 70%) | (0.25, 0.25) | 6.622 | 6.700 | 6.700 | 7.143 | 7.168 | 3.950 | 0.000 | 0.000 | 0.000 | 0.114 |
| | (0.25, 1.00) | 6.068 | 6.209 | 6.208 | 6.889 | 6.955 | 5.699 | 2.337 | 1.973 | 1.676 | 2.514 |
| | (0.25, 4.00) | 4.684 | 5.238 | 5.223 | 6.046 | 6.318 | 12.698 | 9.451 | 7.494 | 7.609 | 10.508 |
| | (1.00, 0.25) | 6.306 | 6.343 | 6.342 | 6.823 | 6.875 | 4.917 | 1.830 | 1.617 | 1.612 | 2.203 |
| | (1.00, 1.00) | 5.801 | 5.912 | 5.912 | 6.528 | 6.632 | 6.667 | 3.156 | 2.693 | 2.664 | 2.333 |
| | (1.00, 4.00) | 4.566 | 5.053 | 5.030 | 5.731 | 6.093 | 13.667 | 9.948 | 8.183 | 8.107 | 10.650 |
| | (4.00, 0.25) | 5.561 | 5.462 | 5.456 | 5.888 | 5.871 | 7.643 | 4.122 | 3.594 | 3.620 | 2.845 |
| | (4.00, 1.00) | 5.155 | 5.130 | 5.125 | 5.595 | 5.677 | 9.620 | 5.649 | 4.975 | 5.006 | 4.010 |
| | (4.00, 4.00) | 4.253 | 4.555 | 4.534 | 5.046 | 5.436 | 16.802 | 12.373 | 10.504 | 10.377 | 13.235 |
| (8, 8, 80%) | (0.25, 0.25) | 5.891 | 6.032 | 6.031 | 6.846 | 6.894 | 5.407 | 1.340 | 1.082 | 0.199 | 2.375 |
| | (0.25, 1.00) | 5.020 | 5.326 | 5.320 | 6.310 | 6.423 | 8.407 | 3.605 | 3.057 | 3.031 | 3.637 |
| | (0.25, 4.00) | 3.414 | 4.107 | 4.091 | 4.933 | 5.273 | 20.406 | 13.960 | 11.096 | 11.406 | 12.977 |
| | (1.00, 0.25) | 5.390 | 5.525 | 5.526 | 6.240 | 6.317 | 7.000 | 3.059 | 2.564 | 2.556 | 3.272 |
| | (1.00, 1.00) | 4.671 | 4.910 | 4.905 | 5.701 | 5.840 | 10.000 | 5.255 | 4.489 | 4.494 | 4.000 |
| | (1.00, 4.00) | 3.305 | 3.906 | 3.870 | 4.581 | 4.968 | 22.000 | 14.869 | 12.604 | 12.366 | 13.920 |
| | (4.00, 0.25) | 4.301 | 4.242 | 4.221 | 4.660 | 4.588 | 12.096 | 7.756 | 7.052 | 6.925 | 6.533 |
| | (4.00, 1.00) | 3.868 | 3.894 | 3.893 | 4.332 | 4.370 | 15.369 | 10.122 | 8.971 | 9.097 | 8.305 |
| | (4.00, 4.00) | 3.009 | 3.374 | 3.349 | 3.764 | 4.175 | 27.582 | 19.764 | 16.949 | 17.149 | 19.351 |

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**TableD.4 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[I^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (8, 8, 90%) | (0.25, 0.25) | 4.304 | 4.780 | 4.773 | 6.128 | 6.223 | 9.782 | 2.953 | 2.465 | 2.404 | 3.977 |
| | (0.25, 1.00) | 3.212 | 3.724 | 3.714 | 4.891 | 5.072 | 16.532 | 7.305 | 6.255 | 6.260 | 6.250 |
| | (0.25, 4.00) | 1.857 | 2.490 | 2.457 | 3.124 | 3.352 | 43.531 | 26.103 | 21.767 | 22.227 | 21.902 |
| | (1.00, 0.25) | 3.648 | 3.954 | 3.945 | 4.841 | 4.934 | 13.250 | 6.501 | 5.726 | 5.694 | 5.898 |
| | (1.00, 1.00) | 2.874 | 3.213 | 3.192 | 3.989 | 4.141 | 20.000 | 10.742 | 9.614 | 9.606 | 9.000 |
| | (1.00, 4.00) | 1.786 | 2.318 | 2.264 | 2.783 | 3.060 | 47.000 | 28.681 | 24.938 | 25.067 | 25.050 |
| | (4.00, 0.25) | 2.483 | 2.487 | 2.469 | 2.777 | 2.655 | 25.712 | 19.104 | 17.660 | 17.765 | 17.159 |
| | (4.00, 1.00) | 2.163 | 2.256 | 2.216 | 2.524 | 2.505 | 32.781 | 23.136 | 21.317 | 21.421 | 20.873 |
| | (4.00, 4.00) | 1.586 | 1.878 | 1.852 | 2.128 | 2.395 | 60.028 | 41.415 | 36.281 | 36.710 | 38.524 |

Table D.5: Expected Time Spent by an Order at the Retail Store and a Production Facility

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[T^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[T^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 70%) | (0.25, 0.25) | 2.193 | 2.001 | 2.001 | 2.000 | 2.000 | 4.674 | 4.034 | 4.032 | 4.021 | 4.003 |
| | (0.25, 1.00) | 2.821 | 2.097 | 2.085 | 2.071 | 2.102 | 5.681 | 4.338 | 4.306 | 4.258 | 4.240 |
| | (0.25, 4.00) | 6.894 | 3.698 | 3.439 | 3.405 | 3.415 | 10.309 | 6.388 | 6.060 | 5.999 | 5.844 |
| | (1.00, 0.25) | 2.498 | 2.128 | 2.116 | 2.105 | 2.103 | 5.211 | 4.406 | 4.363 | 4.332 | 4.258 |
| | (1.00, 1.00) | 3.285 | 2.444 | 2.385 | 2.361 | 2.323 | 6.287 | 4.938 | 4.829 | 4.776 | 4.659 |
| | (1.00, 4.00) | 7.525 | 4.437 | 4.026 | 3.996 | 3.776 | 10.972 | 7.283 | 6.777 | 6.725 | 6.300 |
| | (4.00, 0.25) | 3.805 | 3.461 | 3.330 | 3.293 | 3.168 | 6.916 | 6.251 | 6.040 | 5.979 | 5.914 |
| | (4.00, 1.00) | 4.957 | 4.248 | 3.976 | 3.935 | 3.647 | 8.225 | 7.150 | 6.797 | 6.742 | 6.435 |
| | (4.00, 4.00) | 9.604 | 7.092 | 6.214 | 6.118 | 5.694 | 13.128 | 10.143 | 9.200 | 9.077 | 8.440 |
| (2, 2, 80%) | (0.25, 0.25) | 2.809 | 2.027 | 2.024 | 2.020 | 2.075 | 5.718 | 4.177 | 4.163 | 4.130 | 4.152 |
| | (0.25, 1.00) | 4.598 | 2.508 | 2.447 | 2.406 | 2.417 | 7.906 | 5.070 | 4.965 | 4.877 | 4.787 |
| | (0.25, 4.00) | 13.570 | 6.594 | 5.979 | 5.967 | 5.762 | 17.282 | 9.670 | 8.930 | 8.910 | 8.535 |
| | (1.00, 0.25) | 3.696 | 2.562 | 2.508 | 2.479 | 2.428 | 6.857 | 5.148 | 5.040 | 4.981 | 4.848 |
| | (1.00, 1.00) | 5.696 | 3.522 | 3.323 | 3.271 | 3.165 | 9.120 | 6.388 | 6.112 | 6.029 | 5.820 |
| | (1.00, 4.00) | 14.816 | 8.232 | 7.169 | 7.121 | 6.731 | 18.545 | 11.418 | 10.266 | 10.201 | 9.605 |
| | (4.00, 0.25) | 7.215 | 6.404 | 5.965 | 5.870 | 5.687 | 10.742 | 9.597 | 9.052 | 8.940 | 8.807 |

Continued on next page

**TableD.5 – continued from previous page**

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[T^r]$ NoVis | $E[T^r]$ LoVis | $E[T^r]$ MedVis | $E[T^r]$ HiVis | $E[T^r]$ GI/G/2 | $E[T^p]$ NoVis | $E[T^p]$ LoVis | $E[T^p]$ MedVis | $E[T^p]$ HiVis | $E[T^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (2, 2, 80%) | (4.00, 1.00) | 9.677 | 8.031 | 7.243 | 7.223 | 6.756 | 13.303 | 11.282 | 10.425 | 10.391 | 9.897 |
| | (4.00, 4.00) | 19.207 | 13.976 | 11.846 | 11.809 | 11.051 | 22.981 | 17.469 | 15.155 | 15.104 | 14.135 |
| (2, 2, 90%) | (0.25, 0.25) | 6.042 | 2.426 | 2.399 | 2.361 | 2.416 | 9.572 | 5.015 | 4.948 | 4.861 | 4.784 |
| | (0.25, 1.00) | 11.733 | 4.897 | 4.558 | 4.457 | 4.334 | 15.496 | 8.044 | 7.632 | 7.484 | 7.247 |
| | (0.25, 4.00) | 35.740 | 17.457 | 15.502 | 15.416 | 14.890 | 39.661 | 20.961 | 18.925 | 18.821 | 18.186 |
| | (1.00, 0.25) | 8.912 | 4.901 | 4.643 | 4.573 | 4.421 | 12.597 | 8.053 | 7.707 | 7.607 | 7.403 |
| | (1.00, 1.00) | 14.766 | 8.214 | 7.419 | 7.324 | 7.035 | 18.580 | 11.600 | 10.721 | 10.604 | 10.216 |
| | (1.00, 4.00) | 38.850 | 21.771 | 18.495 | 18.592 | 17.780 | 42.777 | 25.194 | 21.995 | 22.073 | 21.145 |
| | (4.00, 0.25) | 19.819 | 17.262 | 15.550 | 15.629 | 15.212 | 23.681 | 20.734 | 19.069 | 19.139 | 18.754 |
| | (4.00, 1.00) | 26.118 | 21.324 | 18.726 | 18.734 | 18.109 | 30.013 | 25.046 | 22.305 | 22.304 | 21.655 |
| | (4.00, 4.00) | 50.544 | 36.805 | 30.566 | 30.370 | 29.602 | 54.485 | 39.971 | 34.214 | 34.017 | 33.103 |
| (4, 4, 70%) | (0.25, 0.25) | 4.011 | 3.999 | 3.999 | 3.999 | 4.000 | 8.164 | 7.997 | 7.998 | 7.998 | 8.000 |
| | (0.25, 1.00) | 4.143 | 4.003 | 4.003 | 4.001 | 4.012 | 8.708 | 8.048 | 8.043 | 8.019 | 8.049 |
| | (0.25, 4.00) | 6.554 | 4.694 | 4.635 | 4.552 | 4.719 | 12.478 | 9.336 | 9.226 | 9.065 | 9.109 |
| | (1.00, 0.25) | 4.059 | 4.006 | 4.007 | 4.005 | 4.010 | 8.426 | 8.061 | 8.059 | 8.039 | 8.046 |
| | (1.00, 1.00) | 4.308 | 4.060 | 4.057 | 4.043 | 4.038 | 9.120 | 8.269 | 8.251 | 8.189 | 8.158 |
| | (1.00, 4.00) | 7.042 | 5.070 | 4.975 | 4.879 | 4.913 | 13.080 | 9.907 | 9.739 | 9.562 | 9.399 |
| | (4.00, 0.25) | 4.542 | 4.435 | 4.427 | 4.370 | 4.218 | 9.590 | 9.035 | 8.987 | 8.859 | 8.684 |
| | (4.00, 1.00) | 5.195 | 4.843 | 4.782 | 4.720 | 4.457 | 10.651 | 9.682 | 9.556 | 9.424 | 9.087 |
| | (4.00, 4.00) | 8.753 | 6.802 | 6.513 | 6.341 | 6.232 | 15.084 | 12.119 | 11.719 | 11.481 | 11.099 |
| (4, 4, 80%) | (0.25, 0.25) | 4.123 | 4.000 | 4.000 | 3.999 | 4.009 | 8.682 | 8.013 | 8.01 | 8.002 | 8.037 |
| | (0.25, 1.00) | 4.894 | 4.070 | 4.068 | 4.045 | 4.081 | 10.269 | 8.322 | 8.310 | 8.203 | 8.233 |
| | (0.25, 4.00) | 11.976 | 6.528 | 6.352 | 6.206 | 6.254 | 18.806 | 11.826 | 11.558 | 11.306 | 11.147 |
| | (1.00, 0.25) | 4.443 | 4.082 | 4.080 | 4.060 | 4.072 | 9.458 | 8.349 | 8.328 | 8.245 | 8.228 |
| | (1.00, 1.00) | 5.562 | 4.423 | 4.403 | 4.337 | 4.305 | 11.277 | 9.055 | 9.009 | 8.836 | 8.746 |
| | (1.00, 4.00) | 13.108 | 7.598 | 7.207 | 7.044 | 6.954 | 20.021 | 13.099 | 12.621 | 12.373 | 12.018 |
| | (4.00, 0.25) | 6.630 | 6.133 | 6.053 | 5.887 | 5.593 | 12.697 | 11.487 | 11.315 | 11.066 | 10.799 |
| | (4.00, 1.00) | 8.583 | 7.296 | 7.037 | 6.918 | 6.425 | 15.039 | 12.940 | 12.543 | 12.345 | 11.809 |
| | (4.00, 4.00) | 17.202 | 11.955 | 11.163 | 10.950 | 10.540 | 24.328 | 18.020 | 17.081 | 16.824 | 16.103 |

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**TableD.5 – continued from previous page**

| (R, S, ρ) | ($cr^2, cp^2$) | $E[T^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[T^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (4, 4, 90%) | (0.25, 0.25) | 5.674 | 4.052 | 4.050 | 4.028 | 4.096 | 11.526 | 8.287 | 8.282 | 8.160 | 8.246 |
| | (0.25, 1.00) | 10.088 | 5.104 | 5.075 | 4.886 | 4.903 | 16.883 | 10.223 | 10.142 | 9.766 | 9.689 |
| | (0.25, 4.00) | 32.824 | 14.998 | 14.546 | 14.097 | 13.744 | 40.459 | 21.370 | 20.800 | 20.245 | 19.728 |
| | (1.00, 0.25) | 7.756 | 5.118 | 5.099 | 4.950 | 4.901 | 14.197 | 10.204 | 10.147 | 9.857 | 9.732 |
| | (1.00, 1.00) | 12.765 | 7.228 | 7.104 | 6.849 | 6.676 | 19.810 | 12.928 | 12.741 | 12.355 | 12.078 |
| | (1.00, 4.00) | 35.880 | 18.563 | 17.167 | 16.962 | 16.371 | 43.547 | 25.247 | 23.587 | 23.312 | 22.537 |
| | (4.00, 0.25) | 17.436 | 14.635 | 14.191 | 13.989 | 13.423 | 24.738 | 21.212 | 20.570 | 20.305 | 19.811 |
| | (4.00, 1.00) | 23.451 | 18.384 | 17.252 | 16.857 | 16.180 | 30.936 | 25.039 | 23.812 | 23.363 | 22.681 |
| | (4.00, 4.00) | 4.429 | 31.907 | 28.991 | 28.123 | 27.594 | 55.177 | 38.870 | 35.836 | 34.946 | 34.187 |
| (6, 6, 70%) | (0.25, 0.25) | 6.001 | 5.999 | 5.999 | 5.999 | 6.000 | 12.040 | 11.998 | 11.997 | 11.997 | 12.000 |
| | (0.25, 1.00) | 6.024 | 5.998 | 5.999 | 5.998 | 6.002 | 12.298 | 12.006 | 12.010 | 11.995 | 12.012 |
| | (0.25, 4.00) | 7.296 | 6.292 | 6.280 | 6.219 | 6.412 | 15.179 | 12.787 | 12.746 | 12.577 | 12.719 |
| | (1.00, 0.25) | 6.007 | 6.001 | 6.001 | 6.001 | 6.001 | 12.150 | 12.010 | 12.014 | 12.007 | 12.010 |
| | (1.00, 1.00) | 6.071 | 6.009 | 6.008 | 6.006 | 6.004 | 12.549 | 12.081 | 12.075 | 12.048 | 12.038 |
| | (1.00, 4.00) | 7.632 | 6.514 | 6.467 | 6.392 | 6.528 | 15.702 | 13.200 | 13.094 | 12.912 | 12.913 |
| | (4.00, 0.25) | 6.157 | 6.135 | 6.130 | 6.105 | 6.036 | 12.866 | 12.493 | 12.458 | 12.371 | 12.218 |
| | (4.00, 1.00) | 6.467 | 6.323 | 6.313 | 6.272 | 6.120 | 13.663 | 12.920 | 12.876 | 12.742 | 12.457 |
| | (4.00, 4.00) | 8.911 | 7.671 | 7.527 | 7.347 | 7.434 | 17.498 | 14.938 | 14.712 | 14.408 | 14.232 |
| (6, 6, 80%) | (0.25, 0.25) | 6.018 | 5.999 | 5.999 | 5.998 | 6.001 | 12.271 | 11.999 | 11.998 | 11.998 | 12.009 |
| | (0.25, 1.00) | 6.296 | 6.009 | 6.009 | 6.004 | 6.019 | 13.318 | 12.104 | 12.096 | 12.043 | 12.081 |
| | (0.25, 4.00) | 11.400 | 7.488 | 7.399 | 7.235 | 7.436 | 20.792 | 14.707 | 14.569 | 14.246 | 14.254 |
| | (1.00, 0.25) | 6.111 | 6.014 | 6.013 | 6.008 | 6.014 | 12.744 | 12.112 | 12.110 | 12.061 | 12.072 |
| | (1.00, 1.00) | 6.636 | 6.129 | 6.126 | 6.090 | 6.080 | 14.097 | 12.505 | 12.484 | 12.344 | 12.305 |
| | (1.00, 4.00) | 12.368 | 8.208 | 8.072 | 7.821 | 7.940 | 21.935 | 15.706 | 15.514 | 15.110 | 14.954 |
| | (4.00, 0.25) | 7.284 | 7.091 | 7.051 | 6.921 | 6.678 | 15.272 | 14.258 | 14.157 | 13.904 | 13.593 |
| | (4.00, 1.00) | 8.666 | 7.831 | 7.783 | 7.636 | 7.226 | 17.326 | 15.378 | 15.229 | 14.954 | 14.425 |
| | (4.00, 4.00) | 16.010 | 11.718 | 11.315 | 10.950 | 10.825 | 26.046 | 20.133 | 19.554 | 19.039 | 18.540 |
| (6, 6, 90%) | (0.25, 0.25) | 6.665 | 6.005 | 6.005 | 6.001 | 6.025 | 14.232 | 12.086 | 12.080 | 12.029 | 12.096 |
| | (0.25, 1.00) | 9.709 | 6.463 | 6.456 | 6.319 | 6.369 | 18.864 | 13.309 | 13.275 | 12.892 | 12.903 |
| | (0.25, 4.00) | 30.466 | 14.326 | 14.187 | 13.618 | 13.447 | 41.545 | 23.216 | 22.924 | 22.142 | 21.744 |
| | (1.00, 0.25) | 7.975 | 6.464 | 6.459 | 6.349 | 6.351 | 16.468 | 13.290 | 13.255 | 12.956 | 12.901 |
| | (1.00, 1.00) | 11.894 | 7.834 | 7.807 | 7.529 | 7.422 | 21.566 | 15.449 | 15.390 | 14.876 | 14.676 |
| | (1.00, 4.00) | 33.423 | 17.334 | 17.167 | 16.144 | 15.775 | 44.586 | 26.404 | 26.240 | 24.999 | 24.371 |

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TableD.5 – continued from previous page

| $(R,\ S,\ \rho)$ | $(cr^2, cp^2)$ | $E[T^r]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 | $E[T^p]$ NoVis | LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (6, 6, 90%) | (4.00, 0.25) | 15.984 | 13.841 | 13.512 | 13.332 | 12.708 | 26.235 | 22.784 | 22.308 | 22.007 | 21.423 |
| | (4.00, 1.00) | 21.528 | 16.966 | 16.631 | 15.883 | 15.198 | 32.223 | 26.305 | 25.789 | 24.893 | 24.180 |
| | (4.00, 4.00) | 44.707 | 29.666 | 27.304 | 26.540 | 26.227 | 56.085 | 39.648 | 37.053 | 36.209 | 35.594 |
| (8, 8, 70%) | (0.25, 0.25) | 8.000 | 7.998 | 7.998 | 7.998 | 8.000 | 16.010 | 16.005 | 16.005 | 15.995 | 16.000 |
| | (0.25, 1.00) | 8.004 | 7.998 | 7.998 | 7.998 | 8.000 | 16.126 | 16.000 | 15.997 | 15.991 | 16.003 |
| | (0.25, 4.00) | 8.649 | 8.128 | 8.120 | 8.086 | 8.252 | 18.256 | 16.466 | 16.443 | 16.312 | 16.492 |
| | (1.00, 0.25) | 8.001 | 8.001 | 8.001 | 8.001 | 8.000 | 16.053 | 16.006 | 16.005 | 16.005 | 16.002 |
| | (1.00, 1.00) | 8.017 | 8.002 | 8.003 | 8.002 | 8.001 | 16.269 | 16.028 | 16.022 | 16.015 | 16.009 |
| | (1.00, 4.00) | 8.864 | 8.243 | 8.237 | 8.176 | 8.325 | 18.698 | 16.741 | 16.702 | 16.534 | 16.627 |
| | (4.00, 0.25) | 8.045 | 8.041 | 8.041 | 8.029 | 8.006 | 16.472 | 16.226 | 16.221 | 16.157 | 16.066 |
| | (4.00, 1.00) | 8.180 | 8.131 | 8.121 | 8.105 | 8.031 | 17.044 | 16.510 | 16.472 | 16.390 | 16.188 |
| | (4.00, 4.00) | 9.761 | 8.988 | 8.947 | 8.777 | 8.966 | 20.267 | 18.129 | 17.990 | 17.671 | 17.652 |
| (8, 8, 80%) | (0.25, 0.25) | 8.003 | 7.998 | 7.998 | 7.998 | 8.000 | 16.108 | 15.998 | 15.998 | 15.997 | 16.002 |
| | (0.25, 1.00) | 8.097 | 7.999 | 7.999 | 7.999 | 8.004 | 16.765 | 16.026 | 16.032 | 16.005 | 16.030 |
| | (0.25, 4.00) | 11.611 | 8.898 | 8.834 | 8.694 | 8.959 | 23.153 | 18.014 | 17.875 | 17.529 | 17.659 |
| | (1.00, 0.25) | 8.028 | 8.003 | 8.003 | 8.002 | 8.003 | 16.379 | 16.036 | 16.036 | 16.015 | 16.024 |
| | (1.00, 1.00) | 8.255 | 8.040 | 8.040 | 8.025 | 8.021 | 17.342 | 16.249 | 16.234 | 16.141 | 16.125 |
| | (1.00, 4.00) | 12.402 | 9.381 | 9.377 | 9.089 | 9.326 | 24.211 | 18.769 | 18.731 | 18.218 | 18.221 |
| | (4.00, 0.25) | 8.617 | 8.546 | 8.547 | 8.452 | 8.288 | 18.280 | 17.459 | 17.425 | 17.186 | 16.902 |
| | (4.00, 1.00) | 9.528 | 9.077 | 9.029 | 8.918 | 8.620 | 20.030 | 18.418 | 18.277 | 18.012 | 17.540 |
| | (4.00, 4.00) | 15.523 | 12.049 | 11.809 | 11.534 | 11.629 | 28.083 | 22.488 | 22.089 | 21.623 | 21.328 |
| (8, 8, 90%) | (0.25, 0.25) | 8.260 | 7.999 | 7.999 | 7.998 | 8.007 | 17.412 | 16.022 | 16.019 | 16.003 | 16.039 |
| | (0.25, 1.00) | 10.224 | 8.194 | 8.193 | 8.115 | 8.157 | 21.303 | 16.782 | 16.764 | 16.448 | 16.495 |
| | (0.25, 4.00) | 28.657 | 14.747 | 14.338 | 13.755 | 13.752 | 42.893 | 25.785 | 25.329 | 24.408 | 24.136 |
| | (1.00, 0.25) | 9.020 | 8.192 | 8.191 | 8.129 | 8.142 | 19.221 | 16.750 | 16.734 | 16.491 | 16.478 |
| | (1.00, 1.00) | 11.910 | 9.063 | 9.052 | 8.822 | 8.756 | 23.748 | 18.460 | 18.428 | 17.903 | 17.755 |
| | (1.00, 4.00) | 31.478 | 17.118 | 16.649 | 15.940 | 15.783 | 45.872 | 28.361 | 28.033 | 27.039 | 26.564 |
| | (4.00, 0.25) | 15.340 | 13.876 | 13.732 | 13.388 | 12.775 | 28.107 | 24.981 | 24.693 | 24.145 | 23.513 |
| | (4.00, 1.00) | 20.299 | 16.190 | 16.037 | 15.602 | 14.946 | 33.830 | 27.809 | 27.473 | 26.822 | 26.100 |
| | (4.00, 4.00) | 42.387 | 27.901 | 26.055 | 25.500 | 25.389 | 57.197 | 40.281 | 38.516 | 37.758 | 37.285 |

# APPENDIX E

# QUEUEING RESULTS FOR THE 2R/2P SCN CONFIGURATION

This appendix presents the results for the approximate models developed in Chapter 8 for the 2R/2P SCN configuration. Sections E.1 and E.2 present the results for validating the M/M/2 model (primarily for the HiVis case) and the modified M/M/2 model (for lower levels of visibility), respectively. Section E.3 presents the results for the validation of the GI/G/2 based approximation.

## E.1 Results for the Validation of the M/M/2 based Queueing Model for the 2R/2P SCN Configuration

In this section, we present the numerical results for the validation of the M/M/2 based approximate model developed for the 2R/2P SCN configuration with HiVis. Tables E.1, E.3 and E.6 present the fill rates and expected inventory levels at retail stores 1 and 2, and a production facility, respectively. Tables E.2, E.4 and E.7 present the expected number of backorders and the expected time to fulfill a backorder at retail stores 1 and 2, and a production facility, respectively. Table E.5 presents the expected time spent by an order at retail stores 1 and 2, while Table E.8 presents the corresponding values at a production facility. It can be seen from Equations (8.13) through (8.16) that the performance measures at the production facilities depend on only the base-stock level at the facility and the utilization of the processing stage at that facility. As a result, we present the results for the base-stock settings of (2, 2, 4) and (4, 4, 6) in Tables E.6, E.7 and E.8.

Table E.1: Fill Rate & Expected Inventory Level at Retail Store 1

| $(S,\ R_1,\ R_2)$ | $\rho$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 0.730 | 0.727 | 0.739 | 0.759 | 1.244 | 1.314 | 1.344 | 1.428 |
| | 0.9 | 0.456 | 0.454 | 0.465 | 0.493 | 0.689 | 0.789 | 0.813 | 0.901 |
| $(2,4,2)$ | 0.8 | 0.848 | 0.854 | 0.861 | 0.873 | 2.848 | 2.967 | 3.013 | 3.126 |
| | 0.9 | 0.579 | 0.596 | 0.604 | 0.627 | 1.711 | 1.913 | 1.955 | 2.093 |
| $(4,4,6)$ | 0.8 | 0.934 | 0.930 | 0.943 | 0.948 | 3.479 | 3.508 | 3.593 | 3.642 |
| | 0.9 | 0.725 | 0.713 | 0.738 | 0.755 | 2.459 | 2.518 | 2.647 | 2.749 |
| $(4,6,4)$ | 0.8 | 0.964 | 0.963 | 0.970 | 0.972 | 5.388 | 5.420 | 5.521 | 5.577 |
| | 0.9 | 0.793 | 0.787 | 0.801 | 0.820 | 3.983 | 4.058 | 4.228 | 4.359 |

Table E.2: Expected Number of Backorders & Expected Time to Fulfill a Backorder at Retail Store 1

| $(S,\ R_1,\ R_2)$ | $\rho$ | $E[B_1^r]$ | | | | $E[W_1^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 0.846 | 0.743 | 0.704 | 0.642 | 1.567 | 1.358 | 1.347 | 1.333 |
| | 0.9 | 3.853 | 3.375 | 3.273 | 3.044 | 3.539 | 3.091 | 3.058 | 3.000 |
| $(2,4,2)$ | 0.8 | 0.450 | 0.395 | 0.372 | 0.340 | 1.479 | 1.349 | 1.334 | 1.333 |
| | 0.9 | 2.875 | 2.500 | 2.416 | 2.237 | 3.407 | 3.087 | 3.048 | 3.000 |
| $(4,4,6)$ | 0.8 | 0.192 | 0.188 | 0.152 | 0.139 | 1.446 | 1.350 | 1.327 | 1.333 |
| | 0.9 | 1.806 | 1.775 | 1.595 | 1.468 | 3.276 | 3.089 | 3.041 | 3.000 |
| $(4,6,4)$ | 0.8 | 0.101 | 0.100 | 0.080 | 0.074 | 1.411 | 1.355 | 1.329 | 1.333 |
| | 0.9 | 1.330 | 1.315 | 1.175 | 1.078 | 3.199 | 3.081 | 3.021 | 3.000 |

Table E.3: Fill Rate & Expected Inventory Level at Retail Store 2

| $(S,\ R_1,\ R_2)$ | $\rho$ | $f_2^r$ | | | | $E[I_2^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 0.940 | 0.945 | 0.948 | 0.952 | 3.286 | 3.364 | 3.393 | 3.458 |
| | 0.9 | 0.740 | 0.761 | 0.767 | 0.782 | 2.277 | 2.448 | 2.483 | 2.584 |
| $(2,4,2)$ | 0.8 | 0.829 | 0.831 | 0.839 | 0.851 | 1.461 | 1.517 | 1.538 | 1.591 |
| | 0.9 | 0.568 | 0.580 | 0.589 | 0.612 | 0.910 | 1.006 | 1.027 | 1.094 |
| $(4,4,6)$ | 0.8 | 0.992 | 0.991 | 0.993 | 0.994 | 5.657 | 5.673 | 5.732 | 5.760 |
| | 0.9 | 0.904 | 0.902 | 0.912 | 0.919 | 4.628 | 4.671 | 4.793 | 4.883 |
| $(4,6,4)$ | 0.8 | 0.975 | 0.974 | 0.978 | 0.980 | 3.681 | 3.697 | 3.751 | 3.778 |
| | 0.9 | 0.835 | 0.830 | 0.846 | 0.857 | 2.859 | 2.899 | 2.996 | 3.071 |

Table E.4: Expected Number of Backorders & Expected Time to Fulfill a Backorder at Retail Store 2

| $(S,\ R_1,\ R_2)$ | $\rho$ | $E[B_2^r]$ | | | | $E[W_2^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 0.085 | 0.075 | 0.070 | 0.065 | 1.433 | 1.355 | 1.337 | 1.333 |
| | 0.9 | 0.854 | 0.739 | 0.708 | 0.656 | 3.283 | 3.082 | 3.029 | 3.000 |
| $(2,4,2)$ | 0.8 | 0.261 | 0.228 | 0.215 | 0.198 | 1.520 | 1.348 | 1.340 | 1.333 |
| | 0.9 | 1.487 | 1.297 | 1.253 | 1.165 | 3.437 | 3.083 | 3.043 | 3.000 |
| $(4,4,6)$ | 0.8 | 0.011 | 0.012 | 0.009 | 0.009 | 1.360 | 1.316 | 1.290 | 1.333 |
| | 0.9 | 0.298 | 0.297 | 0.262 | 0.242 | 3.100 | 3.026 | 2.959 | 3.000 |
| $(4,6,4)$ | 0.8 | 0.036 | 0.036 | 0.029 | 0.027 | 1.401 | 1.351 | 1.325 | 1.333 |
| | 0.9 | 0.529 | 0.524 | 0.467 | 0.430 | 3.194 | 3.072 | 3.015 | 3.000 |

Table E.5: Expected Time Spent by an Order at Retail Stores 1 and 2

| $(S,\ R_1,\ R_2)$ | $\rho$ | $E[T_1^r]$ | | | | $E[T_2^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 1.422 | 1.371 | 1.351 | 1.321 | 4.086 | 4.071 | 4.071 | 4.065 |
| | 0.9 | 2.924 | 2.686 | 2.635 | 2.522 | 4.854 | 4.734 | 4.709 | 4.656 |
| $(2,4,2)$ | 0.8 | 2.224 | 2.196 | 2.185 | 2.170 | 2.261 | 2.225 | 2.216 | 2.198 |
| | 0.9 | 3.435 | 3.248 | 3.206 | 3.118 | 3.487 | 3.294 | 3.253 | 3.165 |
| $(4,4,6)$ | 0.8 | 2.095 | 2.093 | 2.075 | 2.070 | 6.012 | 6.005 | 6.011 | 6.009 |
| | 0.9 | 2.901 | 2.885 | 2.796 | 2.734 | 6.299 | 6.291 | 6.263 | 6.242 |
| $(4,6,4)$ | 0.8 | 3.049 | 3.048 | 3.039 | 3.037 | 4.036 | 4.032 | 4.030 | 4.027 |
| | 0.9 | 3.662 | 3.655 | 3.586 | 3.539 | 4.530 | 4.520 | 4.467 | 4.430 |

Table E.6: Fill Rate & Expected Inventory Level at a Production Facility

| $(S,\ R_1,\ R_2)$ | $\rho$ | $f^p$ | | | | $E[I^p]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2,2,4)$ | 0.8 | 0.554 | 0.525 | 0.545 | 0.545 | 0.604 | 0.597 | 0.641 | 0.688 |
| | 0.9 | 0.328 | 0.293 | 0.307 | 0.309 | 0.320 | 0.311 | 0.336 | 0.370 |
| $(4,4,6)$ | 0.8 | 0.795 | 0.773 | 0.812 | 0.814 | 1.781 | 1.770 | 2.041 | 2.151 |
| | 0.9 | 0.541 | 0.498 | 0.541 | 0.547 | 1.034 | 1.006 | 1.202 | 1.302 |

Table E.7: Expected Number of Backorders & Expected Time to Fulfill a Backorder at a Production Facility

| $(S,\ R_1,\ R_2)$ | $\rho$ | $E[B^p]$ | | | | $E[W^{bp}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Simulation | | | Analytical | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | | LoVis | MedVis | HiVis | |
| $(2, 2, 4)$ | 0.8 | 1.204 | 1.070 | 1.018 | 1.009 | 1.799 | 1.498 | 1.492 | 1.333 |
| | 0.9 | 3.856 | 3.439 | 3.343 | 3.270 | 3.819 | 3.244 | 3.214 | 3.000 |
| $(4, 4, 6)$ | 0.8 | 0.534 | 0.510 | 0.419 | 0.373 | 1.735 | 1.497 | 1.488 | 1.333 |
| | 0.9 | 2.504 | 2.441 | 2.209 | 2.039 | 3.636 | 3.233 | 3.205 | 3.000 |

Table E.8: Expected Time Spent by an Order at a Production Facility

| $(S,\ R_1,\ R_2)$ | $\rho$ | $E[T^p]$ | | | |
|---|---|---|---|---|---|
| | | Simulation | | | Analytical |
| | | LoVis | MedVis | HiVis | |
| $(2, 2, 4)$ | 0.8 | 2.135 | 2.044 | 2.012 | 1.940 |
| | 0.9 | 3.900 | 3.626 | 3.560 | 3.405 |
| $(4, 4, 6)$ | 0.8 | 3.024 | 3.004 | 2.946 | 2.915 |
| | 0.9 | 4.333 | 4.287 | 4.137 | 4.026 |

## E.2 Results for the Validation of the Modified M/M/2 Model for the 2R/2P SCN Configuration

In this section, we present the numerical results for the validation of the modified M/M/2 based approximate model developed for the 2R/2P SCN configuration with lower levels of visibility. Tables E.9, E.11 and E.14 present the fill rates and expected inventory levels at retail stores 1 and 2, and a production facility, respectively. Tables E.10, E.12 and E.15 present the expected number of backorders and the expected time to fulfill a backorder at retail stores 1 and 2, and a production facility, respectively. Table E.13 presents the expected time spent by an order at retail stores 1 and 2, while Table E.16 presents the corresponding values at a production facility. It can be seen from Equations (8.13) through (8.16) that the performance measures at the production facilities depend on only the base-stock level at the facility and the utilization of the processing store at that facility. As a result, we present the results for the base-stock settings of (2, 2, 4) and (4, 4, 6) in Tables E.14, E.15 and E.16

Table E.9: Fill Rate & Expected Inventory Level at Retail Store 1

| $(S, R_1, R_2)$ | $\rho$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 0.730 | 0.727 | 0.727 | 0.759 | 1.244 | 1.314 | 1.351 | 1.428 |
| | 0.9 | 0.456 | 0.454 | 0.455 | 0.493 | 0.689 | 0.789 | 0.819 | 0.901 |
| (2, 4, 2) | 0.8 | 0.848 | 0.854 | 0.855 | 0.873 | 2.848 | 2.967 | 3.001 | 3.126 |
| | 0.9 | 0.579 | 0.596 | 0.600 | 0.627 | 1.711 | 1.913 | 1.951 | 2.093 |
| (4, 4, 6) | 0.8 | 0.934 | 0.930 | 0.937 | 0.948 | 3.479 | 3.508 | 3.571 | 3.642 |
| | 0.9 | 0.725 | 0.713 | 0.732 | 0.755 | 2.459 | 2.518 | 2.630 | 2.749 |
| (4, 6, 4) | 0.8 | 0.964 | 0.963 | 0.967 | 0.972 | 5.388 | 5.420 | 5.492 | 5.577 |
| | 0.9 | 0.793 | 0.787 | 0.803 | 0.820 | 3.983 | 4.058 | 4.203 | 4.359 |

Table E.10: Expected Number of Backorders & Expected Time to Fulfill a Backorder at Retail Store 1

| $(S, R_1, R_2)$ | $\rho$ | $E[B_1^r]$ | | | | $E[W_1^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 0.846 | 0.743 | 0.729 | 0.642 | 1.567 | 1.358 | 1.333 | 1.333 |
| | 0.9 | 3.853 | 3.375 | 3.271 | 3.044 | 3.539 | 3.091 | 3.000 | 3.000 |
| (2, 4, 2) | 0.8 | 0.450 | 0.395 | 0.386 | 0.340 | 1.479 | 1.349 | 1.333 | 1.333 |
| | 0.9 | 2.875 | 2.500 | 2.403 | 2.237 | 3.407 | 3.087 | 3.000 | 3.000 |
| (4, 4, 6) | 0.8 | 0.192 | 0.188 | 0.167 | 0.139 | 1.446 | 1.350 | 1.333 | 1.333 |
| | 0.9 | 1.806 | 1.775 | 1.607 | 1.468 | 3.276 | 3.089 | 3.000 | 3.000 |
| (4, 6, 4) | 0.8 | 0.101 | 0.100 | 0.088 | 0.074 | 1.411 | 1.355 | 1.333 | 1.333 |
| | 0.9 | 1.330 | 1.315 | 1.181 | 1.078 | 3.199 | 3.081 | 3.000 | 3.000 |

Table E.11: Fill Rate & Expected Inventory Level at Retail Store 2

| $(S, R_1, R_2)$ | $\rho$ | $f_2^r$ | | | | $E[I_2^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 0.940 | 0.945 | 0.945 | 0.952 | 3.286 | 3.364 | 3.384 | 3.458 |
| | 0.9 | 0.740 | 0.761 | 0.765 | 0.782 | 2.277 | 2.448 | 2.478 | 2.584 |
| (2, 4, 2) | 0.8 | 0.829 | 0.831 | 0.831 | 0.851 | 1.461 | 1.517 | 1.536 | 1.591 |
| | 0.9 | 0.568 | 0.580 | 0.583 | 0.612 | 0.910 | 1.006 | 1.026 | 1.094 |
| (4, 4, 6) | 0.8 | 0.992 | 0.991 | 0.992 | 0.994 | 5.657 | 5.673 | 5.712 | 5.760 |
| | 0.9 | 0.904 | 0.902 | 0.912 | 0.919 | 4.628 | 4.671 | 4.776 | 4.883 |
| (4, 6, 4) | 0.8 | 0.975 | 0.974 | 0.976 | 0.980 | 3.681 | 3.697 | 3.734 | 3.778 |
| | 0.9 | 0.835 | 0.830 | 0.843 | 0.857 | 2.859 | 2.899 | 2.982 | 3.071 |

Table E.12: Expected Number of Backorders & Expected Time to Fulfill a Backorder at Retail Store 2

| $(S, R_1, R_2)$ | $\rho$ | $E[B_2^r]$ | | | | $E[W_2^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 0.085 | 0.075 | 0.073 | 0.065 | 1.433 | 1.355 | 1.333 | 1.333 |
| | 0.9 | 0.854 | 0.739 | 0.704 | 0.656 | 3.283 | 3.082 | 3.000 | 3.000 |
| (2, 4, 2) | 0.8 | 0.261 | 0.228 | 0.225 | 0.198 | 1.520 | 1.348 | 1.333 | 1.333 |
| | 0.9 | 1.487 | 1.297 | 1.252 | 1.165 | 3.437 | 3.083 | 3.000 | 3.000 |
| (4, 4, 6) | 0.8 | 0.011 | 0.012 | 0.010 | 0.009 | 1.360 | 1.316 | 1.333 | 1.333 |
| | 0.9 | 0.298 | 0.297 | 0.265 | 0.242 | 3.100 | 3.026 | 3.000 | 3.000 |
| (4, 6, 4) | 0.8 | 0.036 | 0.036 | 0.032 | 0.027 | 1.401 | 1.351 | 1.333 | 1.333 |
| | 0.9 | 0.529 | 0.524 | 0.471 | 0.430 | 3.194 | 3.072 | 3.000 | 3.000 |

Table E.13: Expected Time Spent by an Order at Retail Stores 1 and 2

| $(S, R_1, R_2)$ | $\rho$ | $E[T_1^r]$ | | | | $E[T_2^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 1.422 | 1.371 | 1.364 | 1.321 | 4.086 | 4.071 | 4.073 | 4.065 |
| | 0.9 | 2.924 | 2.686 | 2.635 | 2.522 | 4.854 | 4.734 | 4.704 | 4.656 |
| (2, 4, 2) | 0.8 | 2.224 | 2.196 | 2.193 | 2.170 | 2.261 | 2.225 | 2.225 | 2.198 |
| | 0.9 | 3.435 | 3.248 | 3.201 | 3.118 | 3.487 | 3.294 | 3.252 | 3.165 |
| (4, 4, 6) | 0.8 | 2.095 | 2.093 | 2.083 | 2.070 | 6.012 | 6.005 | 6.010 | 6.009 |
| | 0.9 | 2.901 | 2.885 | 2.804 | 2.734 | 6.299 | 6.291 | 6.265 | 6.242 |
| (4, 6, 4) | 0.8 | 3.049 | 3.048 | 3.044 | 3.037 | 4.036 | 4.032 | 4.032 | 4.027 |
| | 0.9 | 3.662 | 3.655 | 3.590 | 3.539 | 4.530 | 4.520 | 4.471 | 4.430 |

Table E.14: Fill Rate & Expected Inventory Level at a Production Facility

| $(S, R_1, R_2)$ | $\rho$ | $f^p$ | | | | $E[I^p]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 0.554 | 0.525 | 0.483 | 0.545 | 0.604 | 0.597 | 0.551 | 0.688 |
| | 0.9 | 0.328 | 0.293 | 0.258 | 0.309 | 0.320 | 0.311 | 0.274 | 0.370 |
| (4, 4, 6) | 0.8 | 0.795 | 0.773 | 0.780 | 0.814 | 1.781 | 1.770 | 1.840 | 2.151 |
| | 0.9 | 0.541 | 0.498 | 0.500 | 0.547 | 1.034 | 1.006 | 1.080 | 1.302 |

Table E.15: Expected Number of Backorders & Expected Time to Fulfill a Backorder at a Production Facility

| $(S, R_1, R_2)$ | $\rho$ | $E[B^p]$ | | | | $E[W^{bp}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 1.204 | 1.070 | 1.034 | 1.009 | 1.799 | 1.498 | 1.333 | 1.333 |
| | 0.9 | 3.856 | 3.439 | 3.339 | 3.270 | 3.819 | 3.244 | 3.000 | 3.000 |
| (4, 4, 6) | 0.8 | 0.534 | 0.510 | 0.450 | 0.373 | 1.735 | 1.497 | 1.333 | 1.333 |
| | 0.9 | 2.504 | 2.441 | 2.230 | 2.039 | 3.636 | 3.233 | 3.000 | 3.000 |

Table E.16: Expected Time Spent by an Order at a Production Facility

| $(S, R_1, R_2)$ | $\rho$ | $E[T^p]$ | | | |
|---|---|---|---|---|---|
| | | LoVis | MedVis | Mod M/M/2 | M/M/2 |
| (2, 2, 4) | 0.8 | 2.135 | 2.044 | 2.023 | 1.940 |
| | 0.9 | 3.900 | 3.626 | 3.559 | 3.405 |
| (4, 4, 6) | 0.8 | 3.024 | 3.004 | 3.000 | 2.915 |
| | 0.9 | 4.333 | 4.287 | 4.150 | 4.026 |

## E.3 Results for the Validation of the GI/G/2 based Model for the 2R/2P SCN Configuration

The performance measures at the two retail stores are presented in Tables E.17 through E.26. Since the performance measures at the production facilities are dependent only on the utilization level at the processing stage in the facility and the SCVs of the inter-arrival and processing time distributions, we present the results for production facility (1 or 2) only at a base-stock setting of (2, 2, 4) (Tables E.27 and E.32).

Table E.17: Fill Rate and Expected Inventory Level at Retail Store 1
$(\lambda_1^r, \ \lambda_2^r) = (1, 1)$

| | | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | $(cp^2, ca_i^2)$ | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.999 | 1.000 | 1.000 | 1.000 | 1.979 | 1.981 | 1.986 | 1.999 |
| | $(0.25, 1.00)$ | 0.979 | 0.978 | 0.981 | 0.972 | 1.841 | 1.856 | 1.875 | 1.910 |
| | $(0.25, 4.00)$ | 0.841 | 0.830 | 0.838 | 0.852 | 1.434 | 1.492 | 1.517 | 1.650 |
| | $(1.00, 0.25)$ | 0.959 | 0.958 | 0.962 | 0.970 | 1.829 | 1.847 | 1.860 | 1.900 |
| | $(1.00, 1.00)$ | 0.904 | 0.903 | 0.909 | 0.918 | 1.664 | 1.700 | 1.719 | 1.766 |
| | $(1.00, 4.00)$ | 0.778 | 0.769 | 0.774 | 0.817 | 1.325 | 1.399 | 1.414 | 1.571 |
| | $(4.00, 0.25)$ | 0.761 | 0.756 | 0.765 | 0.769 | 1.506 | 1.550 | 1.567 | 1.425 |
| | $(4.00, 1.00)$ | 0.709 | 0.703 | 0.711 | 0.733 | 1.384 | 1.439 | 1.454 | 1.365 |
| | $(4.00, 4.00)$ | 0.642 | 0.632 | 0.638 | 0.712 | 1.166 | 1.252 | 1.267 | 1.356 |

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**TableE.17 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ LoVis | $f_1^r$ MedVis | $f_1^r$ HiVis | $f_1^r$ GI/G/2 | $E[I_1^r]$ LoVis | $E[I_1^r]$ MedVis | $E[I_1^r]$ HiVis | $E[I_1^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 0.993 | 0.993 | 0.994 | 0.981 | 1.907 | 1.915 | 1.929 | 1.946 |
| | $(0.25, 1.00)$ | 0.908 | 0.908 | 0.915 | 0.908 | 1.585 | 1.624 | 1.654 | 1.740 |
| | $(0.25, 4.00)$ | 0.687 | 0.672 | 0.680 | 0.702 | 1.070 | 1.162 | 1.184 | 1.330 |
| | $(1.00, 0.25)$ | 0.875 | 0.874 | 0.881 | 0.899 | 1.593 | 1.632 | 1.652 | 1.709 |
| | $(1.00, 1.00)$ | 0.772 | 0.772 | 0.781 | 0.798 | 1.336 | 1.403 | 1.426 | 1.494 |
| | $(1.00, 4.00)$ | 0.613 | 0.604 | 0.608 | 0.655 | 0.960 | 1.069 | 1.081 | 1.236 |
| | $(4.00, 0.25)$ | 0.566 | 0.561 | 0.569 | 0.575 | 1.137 | 1.204 | 1.220 | 1.042 |
| | $(4.00, 1.00)$ | 0.517 | 0.514 | 0.523 | 0.545 | 1.009 | 1.094 | 1.109 | 0.997 |
| | $(4.00, 4.00)$ | 0.468 | 0.454 | 0.463 | 0.537 | 0.813 | 0.913 | 0.931 | 1.011 |
| 0.9 | $(0.25, 0.25)$ | 0.913 | 0.913 | 0.919 | 0.913 | 1.579 | 1.608 | 1.638 | 1.746 |
| | $(0.25, 1.00)$ | 0.673 | 0.675 | 0.685 | 0.689 | 1.022 | 1.102 | 1.132 | 1.261 |
| | $(0.25, 4.00)$ | 0.431 | 0.412 | 0.419 | 0.431 | 0.574 | 0.682 | 0.699 | 0.799 |
| | $(1.00, 0.25)$ | 0.640 | 0.641 | 0.650 | 0.668 | 1.070 | 1.141 | 1.163 | 1.208 |
| | $(1.00, 1.00)$ | 0.503 | 0.508 | 0.516 | 0.538 | 0.783 | 0.883 | 0.902 | 0.973 |
| | $(1.00, 4.00)$ | 0.361 | 0.352 | 0.359 | 0.390 | 0.488 | 0.606 | 0.621 | 0.722 |
| | $(4.00, 0.25)$ | 0.308 | 0.314 | 0.317 | 0.316 | 0.623 | 0.717 | 0.724 | 0.563 |
| | $(4.00, 1.00)$ | 0.279 | 0.281 | 0.284 | 0.300 | 0.530 | 0.627 | 0.632 | 0.542 |
| | $(4.00, 4.00)$ | 0.251 | 0.244 | 0.249 | 0.302 | 0.395 | 0.502 | 0.512 | 0.562 |
| | | | | $(S, R_1, R_2) = (2, 4, 2)$ | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 3.979 | 3.981 | 3.986 | 3.999 |
| | $(0.25, 1.00)$ | 0.997 | 0.998 | 0.998 | 0.993 | 3.823 | 3.841 | 3.862 | 3.889 |
| | $(0.25, 4.00)$ | 0.907 | 0.907 | 0.912 | 0.916 | 3.141 | 3.242 | 3.279 | 3.455 |
| | $(1.00, 0.25)$ | 0.993 | 0.993 | 0.994 | 0.993 | 3.803 | 3.824 | 3.839 | 3.879 |
| | $(1.00, 1.00)$ | 0.970 | 0.972 | 0.974 | 0.976 | 3.570 | 3.619 | 3.643 | 3.698 |
| | $(1.00, 4.00)$ | 0.861 | 0.862 | 0.865 | 0.895 | 2.942 | 3.076 | 3.097 | 3.328 |
| | $(4.00, 0.25)$ | 0.880 | 0.881 | 0.885 | 0.907 | 3.275 | 3.335 | 3.362 | 3.184 |
| | $(4.00, 1.00)$ | 0.830 | 0.833 | 0.837 | 0.871 | 3.051 | 3.139 | 3.163 | 3.049 |
| | $(4.00, 4.00)$ | 0.734 | 0.739 | 0.743 | 0.807 | 2.593 | 2.764 | 2.787 | 2.929 |
| 0.8 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 0.994 | 3.903 | 3.911 | 3.927 | 3.931 |
| | $(0.25, 1.00)$ | 0.975 | 0.977 | 0.978 | 0.969 | 3.480 | 3.532 | 3.569 | 3.657 |
| | $(0.25, 4.00)$ | 0.775 | 0.778 | 0.783 | 0.802 | 2.450 | 2.628 | 2.662 | 2.893 |
| | $(1.00, 0.25)$ | 0.961 | 0.962 | 0.965 | 0.969 | 3.476 | 3.526 | 3.552 | 3.623 |
| | $(1.00, 1.00)$ | 0.892 | 0.898 | 0.902 | 0.910 | 3.044 | 3.149 | 3.181 | 3.270 |

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**TableE.17 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.8 | $(1.00, 4.00)$ | 0.708 | 0.718 | 0.720 | 0.762 | 2.229 | 2.446 | 2.462 | 2.712 |
| | $(4.00, 0.25)$ | 0.709 | 0.712 | 0.717 | 0.742 | 2.584 | 2.691 | 2.716 | 2.450 |
| | $(4.00, 1.00)$ | 0.647 | 0.657 | 0.664 | 0.697 | 2.320 | 2.475 | 2.502 | 2.321 |
| | $(4.00, 4.00)$ | 0.557 | 0.560 | 0.568 | 0.640 | 1.871 | 2.080 | 2.112 | 2.243 |
| 0.9 | $(0.25, 0.25)$ | 0.981 | 0.982 | 0.983 | 0.969 | 3.491 | 3.529 | 3.565 | 3.665 |
| | $(0.25, 1.00)$ | 0.820 | 0.831 | 0.837 | 0.834 | 2.505 | 2.644 | 2.689 | 2.869 |
| | $(0.25, 4.00)$ | 0.509 | 0.514 | 0.521 | 0.537 | 1.390 | 1.622 | 1.652 | 1.824 |
| | $(1.00, 0.25)$ | 0.798 | 0.806 | 0.811 | 0.825 | 2.569 | 2.683 | 2.717 | 2.793 |
| | $(1.00, 1.00)$ | 0.649 | 0.670 | 0.675 | 0.691 | 1.956 | 2.149 | 2.179 | 2.285 |
| | $(1.00, 4.00)$ | 0.435 | 0.450 | 0.457 | 0.491 | 1.193 | 1.453 | 1.482 | 1.656 |
| | $(4.00, 0.25)$ | 0.418 | 0.436 | 0.438 | 0.446 | 1.492 | 1.672 | 1.682 | 1.391 |
| | $(4.00, 1.00)$ | 0.371 | 0.390 | 0.390 | 0.413 | 1.281 | 1.481 | 1.487 | 1.313 |
| | $(4.00, 4.00)$ | 0.308 | 0.319 | 0.324 | 0.378 | 0.945 | 1.184 | 1.202 | 1.280 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 4.000 | 4.000 | 4.000 | 4.000 |
| | $(0.25, 1.00)$ | 1.000 | 1.000 | 1.000 | 0.998 | 3.974 | 3.976 | 3.987 | 3.978 |
| | $(0.25, 4.00)$ | 0.948 | 0.944 | 0.953 | 0.950 | 3.520 | 3.554 | 3.623 | 3.701 |
| | $(1.00, 0.25)$ | 0.999 | 0.999 | 0.999 | 0.998 | 3.970 | 3.973 | 3.982 | 3.979 |
| | $(1.00, 1.00)$ | 0.992 | 0.992 | 0.994 | 0.994 | 3.877 | 3.886 | 3.913 | 3.928 |
| | $(1.00, 4.00)$ | 0.919 | 0.913 | 0.922 | 0.937 | 3.377 | 3.430 | 3.490 | 3.626 |
| | $(4.00, 0.25)$ | 0.945 | 0.942 | 0.950 | 0.970 | 3.663 | 3.679 | 3.723 | 3.702 |
| | $(4.00, 1.00)$ | 0.908 | 0.906 | 0.916 | 0.945 | 3.483 | 3.519 | 3.570 | 3.567 |
| | $(4.00, 4.00)$ | 0.819 | 0.812 | 0.824 | 0.870 | 3.028 | 3.119 | 3.174 | 3.297 |
| 0.8 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 0.999 | 3.993 | 3.994 | 3.998 | 3.983 |
| | $(0.25, 1.00)$ | 0.993 | 0.993 | 0.995 | 0.990 | 3.842 | 3.851 | 3.898 | 3.902 |
| | $(0.25, 4.00)$ | 0.852 | 0.839 | 0.854 | 0.867 | 2.956 | 3.021 | 3.111 | 3.277 |
| | $(1.00, 0.25)$ | 0.989 | 0.988 | 0.991 | 0.990 | 3.842 | 3.851 | 3.888 | 3.901 |
| | $(1.00, 1.00)$ | 0.953 | 0.952 | 0.960 | 0.963 | 3.570 | 3.596 | 3.662 | 3.701 |
| | $(1.00, 4.00)$ | 0.797 | 0.787 | 0.802 | 0.835 | 2.740 | 2.842 | 2.922 | 3.125 |
| | $(4.00, 0.25)$ | 0.814 | 0.808 | 0.825 | 0.852 | 3.091 | 3.133 | 3.203 | 3.085 |
| | $(4.00, 1.00)$ | 0.760 | 0.759 | 0.771 | 0.806 | 2.843 | 2.925 | 2.983 | 2.902 |
| | $(4.00, 4.00)$ | 0.656 | 0.642 | 0.658 | 0.720 | 2.316 | 2.444 | 2.515 | 2.644 |

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**TableE.17 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ LoVis | $f_1^r$ MedVis | $f_1^r$ HiVis | $f_1^r$ GI/G/2 | $E[I_1^r]$ LoVis | $E[I_1^r]$ MedVis | $E[I_1^r]$ HiVis | $E[I_1^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (0.25, 0.25) | 0.995 | 0.995 | 0.997 | 0.988 | 3.856 | 3.862 | 3.919 | 3.900 |
| | (0.25, 1.00) | 0.905 | 0.901 | 0.917 | 0.913 | 3.173 | 3.203 | 3.331 | 3.423 |
| | (0.25, 4.00) | 0.613 | 0.585 | 0.603 | 0.624 | 1.890 | 1.982 | 2.070 | 2.239 |
| | (1.00, 0.25) | 0.891 | 0.887 | 0.902 | 0.911 | 3.211 | 3.235 | 3.338 | 3.395 |
| | (1.00, 1.00) | 0.773 | 0.767 | 0.785 | 0.797 | 2.636 | 2.696 | 2.800 | 2.875 |
| | (1.00, 4.00) | 0.547 | 0.529 | 0.538 | 0.575 | 1.682 | 1.821 | 1.874 | 2.048 |
| | (4.00, 0.25) | 0.532 | 0.526 | 0.542 | 0.557 | 1.968 | 2.045 | 2.112 | 1.897 |
| | (4.00, 1.00) | 0.476 | 0.476 | 0.493 | 0.512 | 1.714 | 1.837 | 1.909 | 1.752 |
| | (4.00, 4.00) | 0.395 | 0.387 | 0.398 | 0.446 | 1.288 | 1.465 | 1.512 | 1.579 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 0.999 | 5.973 | 5.975 | 5.987 | 5.976 |
| | (0.25, 4.00) | 0.970 | 0.969 | 0.974 | 0.967 | 5.431 | 5.472 | 5.554 | 5.628 |
| | (1.00, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.970 | 5.972 | 5.982 | 5.977 |
| | (1.00, 1.00) | 0.998 | 0.998 | 0.998 | 0.998 | 5.869 | 5.879 | 5.908 | 5.923 |
| | (1.00, 4.00) | 0.949 | 0.947 | 0.953 | 0.959 | 5.244 | 5.308 | 5.382 | 5.534 |
| | (4.00, 0.25) | 0.973 | 0.972 | 0.975 | 0.989 | 5.613 | 5.628 | 5.678 | 5.673 |
| | (4.00, 1.00) | 0.947 | 0.947 | 0.953 | 0.975 | 5.384 | 5.425 | 5.487 | 5.506 |
| | (4.00, 4.00) | 0.866 | 0.864 | 0.872 | 0.908 | 4.749 | 4.872 | 4.940 | 5.096 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.993 | 5.994 | 5.998 | 5.982 |
| | (0.25, 1.00) | 0.998 | 0.998 | 0.999 | 0.996 | 5.834 | 5.844 | 5.893 | 5.891 |
| | (0.25, 4.00) | 0.895 | 0.890 | 0.900 | 0.906 | 4.676 | 4.759 | 4.874 | 5.072 |
| | (1.00, 0.25) | 0.997 | 0.997 | 0.997 | 0.996 | 5.832 | 5.841 | 5.881 | 5.892 |
| | (1.00, 1.00) | 0.979 | 0.979 | 0.982 | 0.984 | 5.514 | 5.543 | 5.617 | 5.660 |
| | (1.00, 4.00) | 0.849 | 0.846 | 0.857 | 0.881 | 4.369 | 4.506 | 4.609 | 4.867 |
| | (4.00, 0.25) | 0.876 | 0.874 | 0.885 | 0.913 | 4.862 | 4.909 | 4.997 | 4.884 |
| | (4.00, 1.00) | 0.828 | 0.831 | 0.837 | 0.874 | 4.515 | 4.620 | 4.691 | 4.619 |
| | (4.00, 4.00) | 0.714 | 0.709 | 0.722 | 0.779 | 3.720 | 3.902 | 3.997 | 4.173 |

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**TableE.17 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.9 | (0.25, 0.25) | 0.999 | 0.999 | 0.999 | 0.995 | 5.851 | 5.857 | 5.916 | 5.887 |
| | (0.25, 1.00) | 0.950 | 0.949 | 0.957 | 0.952 | 5.029 | 5.063 | 5.214 | 5.310 |
| | (0.25, 4.00) | 0.672 | 0.657 | 0.672 | 0.692 | 3.106 | 3.235 | 3.355 | 3.591 |
| | (1.00, 0.25) | 0.941 | 0.939 | 0.947 | 0.951 | 5.065 | 5.092 | 5.214 | 5.280 |
| | (1.00, 1.00) | 0.845 | 0.844 | 0.855 | 0.864 | 4.274 | 4.349 | 4.479 | 4.573 |
| | (1.00, 4.00) | 0.606 | 0.599 | 0.606 | 0.642 | 2.782 | 2.985 | 3.052 | 3.301 |
| | (4.00, 0.25) | 0.611 | 0.609 | 0.622 | 0.644 | 3.221 | 3.320 | 3.410 | 3.144 |
| | (4.00, 1.00) | 0.549 | 0.555 | 0.571 | 0.594 | 2.828 | 3.004 | 3.105 | 2.901 |
| | (4.00, 4.00) | 0.445 | 0.446 | 0.456 | 0.505 | 2.131 | 2.401 | 2.467 | 2.560 |

Table E.18: Fill Rate and Expected Inventory Level at Retail Store 1
$(\lambda_1^r, \quad \lambda_2^r) = (2, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| | $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.999 | 0.999 | 0.999 | 1.000 | 1.972 | 1.975 | 1.982 | 1.998 |
| | (0.25, 1.00) | 0.967 | 0.966 | 0.971 | 0.963 | 1.805 | 1.822 | 1.846 | 1.891 |
| | (0.25, 4.00) | 0.819 | 0.806 | 0.813 | 0.830 | 1.378 | 1.441 | 1.467 | 1.614 |
| | (1.00, 0.25) | 0.944 | 0.941 | 0.946 | 0.960 | 1.787 | 1.807 | 1.824 | 1.878 |
| | (1.00, 1.00) | 0.879 | 0.876 | 0.884 | 0.895 | 1.601 | 1.641 | 1.664 | 1.722 |
| | (1.00, 4.00) | 0.751 | 0.740 | 0.749 | 0.791 | 1.255 | 1.334 | 1.356 | 1.527 |
| | (4.00, 0.25) | 0.717 | 0.710 | 0.720 | 0.722 | 1.419 | 1.468 | 1.486 | 1.346 |
| | (4.00, 1.00) | 0.665 | 0.660 | 0.668 | 0.689 | 1.289 | 1.353 | 1.371 | 1.293 |
| | (4.00, 4.00) | 0.610 | 0.598 | 0.604 | 0.682 | 1.082 | 1.171 | 1.184 | 1.307 |

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**TableE.18 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ | | | | $E[I_1^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.8 | $(0.25, 0.25)$ | 0.987 | 0.988 | 0.990 | 0.976 | 1.882 | 1.895 | 1.913 | 1.937 |
| | $(0.25, 1.00)$ | 0.879 | 0.877 | 0.886 | 0.885 | 1.516 | 1.563 | 1.597 | 1.696 |
| | $(0.25, 4.00)$ | 0.663 | 0.641 | 0.649 | 0.673 | 1.012 | 1.104 | 1.129 | 1.286 |
| | $(1.00, 0.25)$ | 0.843 | 0.843 | 0.852 | 0.873 | 1.518 | 1.572 | 1.595 | 1.658 |
| | $(1.00, 1.00)$ | 0.733 | 0.731 | 0.741 | 0.760 | 1.250 | 1.323 | 1.349 | 1.430 |
| | $(1.00, 4.00)$ | 0.584 | 0.571 | 0.576 | 0.624 | 0.889 | 1.000 | 1.013 | 1.186 |
| | $(4.00, 0.25)$ | 0.517 | 0.517 | 0.528 | 0.526 | 1.031 | 1.116 | 1.136 | 0.967 |
| | $(4.00, 1.00)$ | 0.474 | 0.472 | 0.481 | 0.502 | 0.908 | 0.998 | 1.017 | 0.932 |
| | $(4.00, 4.00)$ | 0.436 | 0.423 | 0.432 | 0.509 | 0.728 | 0.836 | 0.853 | 0.967 |
| 0.9 | $(0.25, 0.25)$ | 0.880 | 0.876 | 0.886 | 0.891 | 1.502 | 1.537 | 1.574 | 1.700 |
| | $(0.25, 1.00)$ | 0.627 | 0.622 | 0.634 | 0.643 | 0.937 | 1.021 | 1.054 | 1.183 |
| | $(0.25, 4.00)$ | 0.412 | 0.385 | 0.392 | 0.402 | 0.533 | 0.637 | 0.654 | 0.753 |
| | $(1.00, 0.25)$ | 0.591 | 0.588 | 0.599 | 0.617 | 0.973 | 1.051 | 1.076 | 1.123 |
| | $(1.00, 1.00)$ | 0.463 | 0.462 | 0.470 | 0.493 | 0.701 | 0.802 | 0.822 | 0.901 |
| | $(1.00, 4.00)$ | 0.342 | 0.329 | 0.331 | 0.363 | 0.442 | 0.559 | 0.566 | 0.680 |
| | $(4.00, 0.25)$ | 0.273 | 0.275 | 0.282 | 0.281 | 0.538 | 0.628 | 0.639 | 0.511 |
| | $(4.00, 1.00)$ | 0.250 | 0.250 | 0.257 | 0.271 | 0.456 | 0.549 | 0.564 | 0.498 |
| | $(4.00, 4.00)$ | 0.227 | 0.223 | 0.229 | 0.282 | 0.336 | 0.446 | 0.459 | 0.531 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 3.972 | 3.974 | 3.981 | 3.998 |
| | $(0.25, 1.00)$ | 0.994 | 0.994 | 0.995 | 0.988 | 3.771 | 3.792 | 3.820 | 3.858 |
| | $(0.25, 4.00)$ | 0.879 | 0.878 | 0.882 | 0.892 | 3.009 | 3.130 | 3.167 | 3.372 |
| | $(1.00, 0.25)$ | 0.987 | 0.986 | 0.988 | 0.988 | 3.742 | 3.766 | 3.786 | 3.844 |
| | $(1.00, 1.00)$ | 0.952 | 0.954 | 0.957 | 0.961 | 3.460 | 3.519 | 3.549 | 3.619 |
| | $(1.00, 4.00)$ | 0.825 | 0.827 | 0.833 | 0.866 | 2.780 | 2.936 | 2.972 | 3.227 |
| | $(4.00, 0.25)$ | 0.838 | 0.837 | 0.843 | 0.861 | 3.108 | 3.178 | 3.208 | 3.007 |
| | $(4.00, 1.00)$ | 0.781 | 0.787 | 0.791 | 0.820 | 2.857 | 2.970 | 2.997 | 2.874 |
| | $(4.00, 4.00)$ | 0.692 | 0.695 | 0.700 | 0.769 | 2.405 | 2.591 | 2.612 | 2.805 |

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**TableE.18 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ LoVis | MedVis | HiVis | GI/G/2 | $E[I_1^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (0.25, 0.25) | 0.999 | 0.999 | 0.999 | 0.991 | 3.873 | 3.887 | 3.907 | 3.910 |
| | (0.25, 1.00) | 0.955 | 0.957 | 0.961 | 0.953 | 3.352 | 3.420 | 3.466 | 3.577 |
| | (0.25, 4.00) | 0.736 | 0.733 | 0.739 | 0.763 | 2.297 | 2.484 | 2.524 | 2.772 |
| | (1.00, 0.25) | 0.938 | 0.941 | 0.944 | 0.951 | 3.343 | 3.417 | 3.449 | 3.532 |
| | (1.00, 1.00) | 0.850 | 0.857 | 0.862 | 0.873 | 2.860 | 2.984 | 3.022 | 3.129 |
| | (1.00, 4.00) | 0.664 | 0.670 | 0.672 | 0.719 | 2.047 | 2.279 | 2.299 | 2.581 |
| | (4.00, 0.25) | 0.647 | 0.657 | 0.665 | 0.675 | 2.354 | 2.499 | 2.534 | 2.247 |
| | (4.00, 1.00) | 0.589 | 0.601 | 0.608 | 0.633 | 2.094 | 2.268 | 2.300 | 2.136 |
| | (4.00, 4.00) | 0.509 | 0.515 | 0.522 | 0.596 | 1.667 | 1.903 | 1.934 | 2.118 |
| 0.9 | (0.25, 0.25) | 0.962 | 0.963 | 0.967 | 0.955 | 3.356 | 3.404 | 3.452 | 3.587 |
| | (0.25, 1.00) | 0.760 | 0.770 | 0.778 | 0.782 | 2.288 | 2.442 | 2.495 | 2.686 |
| | (0.25, 4.00) | 0.472 | 0.468 | 0.474 | 0.489 | 1.266 | 1.496 | 1.526 | 1.690 |
| | (1.00, 0.25) | 0.737 | 0.745 | 0.751 | 0.768 | 2.340 | 2.473 | 2.512 | 2.594 |
| | (1.00, 1.00) | 0.586 | 0.604 | 0.609 | 0.627 | 1.739 | 1.942 | 1.975 | 2.093 |
| | (1.00, 4.00) | 0.399 | 0.409 | 0.409 | 0.445 | 1.063 | 1.328 | 1.337 | 1.530 |
| | (4.00, 0.25) | 0.364 | 0.379 | 0.384 | 0.385 | 1.288 | 1.461 | 1.482 | 1.230 |
| | (4.00, 1.00) | 0.324 | 0.340 | 0.347 | 0.359 | 1.096 | 1.294 | 1.324 | 1.173 |
| | (4.00, 4.00) | 0.270 | 0.284 | 0.291 | 0.342 | 0.795 | 1.047 | 1.071 | 1.185 |
| | | | | | $(S, R_1, R_2) = (4, 4, 6)$ | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 4.000 | 4.000 | 4.000 | 4.000 |
| | (0.25, 1.00) | 0.999 | 0.999 | 1.000 | 0.997 | 3.967 | 3.968 | 3.983 | 3.972 |
| | (0.25, 4.00) | 0.933 | 0.927 | 0.938 | 0.938 | 3.447 | 3.495 | 3.570 | 3.660 |
| | (1.00, 0.25) | 0.998 | 0.998 | 0.999 | 0.997 | 3.961 | 3.963 | 3.976 | 3.973 |
| | (1.00, 1.00) | 0.987 | 0.986 | 0.989 | 0.991 | 3.845 | 3.855 | 3.890 | 3.908 |
| | (1.00, 4.00) | 0.901 | 0.892 | 0.904 | 0.923 | 3.292 | 3.350 | 3.421 | 3.576 |
| | (4.00, 0.25) | 0.926 | 0.922 | 0.932 | 0.953 | 3.589 | 3.609 | 3.658 | 3.632 |
| | (4.00, 1.00) | 0.885 | 0.881 | 0.893 | 0.922 | 3.388 | 3.431 | 3.489 | 3.481 |
| | (4.00, 4.00) | 0.792 | 0.786 | 0.795 | 0.846 | 2.902 | 3.014 | 3.064 | 3.221 |

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**TableE.18 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ LoVis | $f_1^r$ MedVis | $f_1^r$ HiVis | $f_1^r$ GI/G/2 | $E[I_1^r]$ LoVis | $E[I_1^r]$ MedVis | $E[I_1^r]$ HiVis | $E[I_1^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 0.998 | 3.991 | 3.992 | 3.997 | 3.979 |
| | $(0.25, 1.00)$ | 0.987 | 0.987 | 0.991 | 0.985 | 3.804 | 3.817 | 3.875 | 3.881 |
| | $(0.25, 4.00)$ | 0.824 | 0.807 | 0.825 | 0.843 | 2.846 | 2.923 | 3.024 | 3.204 |
| | $(1.00, 0.25)$ | 0.981 | 0.982 | 0.986 | 0.986 | 3.801 | 3.819 | 3.864 | 3.880 |
| | $(1.00, 1.00)$ | 0.935 | 0.932 | 0.943 | 0.948 | 3.485 | 3.520 | 3.598 | 3.644 |
| | $(1.00, 4.00)$ | 0.771 | 0.755 | 0.770 | 0.808 | 2.631 | 2.737 | 2.821 | 3.043 |
| | $(4.00, 0.25)$ | 0.777 | 0.778 | 0.795 | 0.813 | 2.953 | 3.029 | 3.106 | 2.960 |
| | $(4.00, 1.00)$ | 0.723 | 0.719 | 0.734 | 0.765 | 2.697 | 2.782 | 2.854 | 2.777 |
| | $(4.00, 4.00)$ | 0.623 | 0.613 | 0.624 | 0.687 | 2.168 | 2.326 | 2.380 | 2.552 |
| 0.9 | $(0.25, 0.25)$ | 0.990 | 0.989 | 0.994 | 0.983 | 3.817 | 3.824 | 3.898 | 3.880 |
| | $(0.25, 1.00)$ | 0.873 | 0.866 | 0.887 | 0.887 | 3.050 | 3.086 | 3.236 | 3.333 |
| | $(0.25, 4.00)$ | 0.577 | 0.544 | 0.565 | 0.586 | 1.762 | 1.872 | 1.969 | 2.134 |
| | $(1.00, 0.25)$ | 0.858 | 0.852 | 0.872 | 0.883 | 3.084 | 3.115 | 3.233 | 3.298 |
| | $(1.00, 1.00)$ | 0.730 | 0.721 | 0.743 | 0.755 | 2.484 | 2.551 | 2.667 | 2.749 |
| | $(1.00, 4.00)$ | 0.514 | 0.490 | 0.502 | 0.537 | 1.562 | 1.705 | 1.764 | 1.946 |
| | $(4.00, 0.25)$ | 0.491 | 0.486 | 0.504 | 0.508 | 1.809 | 1.901 | 1.976 | 1.761 |
| | $(4.00, 1.00)$ | 0.440 | 0.436 | 0.453 | 0.467 | 1.567 | 1.690 | 1.755 | 1.631 |
| | $(4.00, 4.00)$ | 0.369 | 0.355 | 0.367 | 0.414 | 1.170 | 1.334 | 1.386 | 1.496 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| | $(0.25, 1.00)$ | 1.000 | 1.000 | 1.000 | 0.999 | 5.965 | 5.968 | 5.982 | 5.969 |
| | $(0.25, 4.00)$ | 0.956 | 0.953 | 0.961 | 0.957 | 5.321 | 5.379 | 5.472 | 5.565 |
| | $(1.00, 0.25)$ | 1.000 | 1.000 | 1.000 | 0.999 | 5.959 | 5.961 | 5.975 | 5.970 |
| | $(1.00, 1.00)$ | 0.995 | 0.995 | 0.996 | 0.997 | 5.830 | 5.842 | 5.880 | 5.899 |
| | $(1.00, 4.00)$ | 0.930 | 0.927 | 0.935 | 0.945 | 5.105 | 5.186 | 5.275 | 5.456 |
| | $(4.00, 0.25)$ | 0.957 | 0.956 | 0.962 | 0.979 | 5.504 | 5.531 | 5.590 | 5.580 |
| | $(4.00, 1.00)$ | 0.926 | 0.925 | 0.932 | 0.958 | 5.244 | 5.297 | 5.369 | 5.382 |
| | $(4.00, 4.00)$ | 0.837 | 0.835 | 0.841 | 0.883 | 4.558 | 4.708 | 4.769 | 4.970 |

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**TableE.18 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_1^r$ LoVis | MedVis | HiVis | GI/G/2 | $E[I_1^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 0.999 | 5.991 | 5.992 | 5.997 | 5.977 |
| | $(0.25, 1.00)$ | 0.996 | 0.995 | 0.997 | 0.993 | 5.789 | 5.802 | 5.865 | 5.864 |
| | $(0.25, 4.00)$ | 0.865 | 0.856 | 0.869 | 0.882 | 4.491 | 4.593 | 4.724 | 4.949 |
| | $(1.00, 0.25)$ | 0.993 | 0.993 | 0.995 | 0.994 | 5.786 | 5.800 | 5.851 | 5.864 |
| | $(1.00, 1.00)$ | 0.965 | 0.964 | 0.970 | 0.973 | 5.397 | 5.435 | 5.527 | 5.579 |
| | $(1.00, 4.00)$ | 0.816 | 0.809 | 0.821 | 0.852 | 4.175 | 4.327 | 4.437 | 4.726 |
| | $(4.00, 0.25)$ | 0.840 | 0.841 | 0.854 | 0.876 | 4.657 | 4.744 | 4.844 | 4.684 |
| | $(4.00, 1.00)$ | 0.786 | 0.787 | 0.798 | 0.831 | 4.279 | 4.394 | 4.487 | 4.408 |
| | $(4.00, 4.00)$ | 0.671 | 0.672 | 0.680 | 0.740 | 3.467 | 3.705 | 3.774 | 4.006 |
| 0.9 | $(0.25, 0.25)$ | 0.997 | 0.997 | 0.998 | 0.991 | 5.805 | 5.812 | 5.891 | 5.859 |
| | $(0.25, 1.00)$ | 0.921 | 0.919 | 0.932 | 0.929 | 4.838 | 4.882 | 5.064 | 5.172 |
| | $(0.25, 4.00)$ | 0.624 | 0.605 | 0.623 | 0.645 | 2.868 | 3.029 | 3.164 | 3.395 |
| | $(1.00, 0.25)$ | 0.911 | 0.909 | 0.921 | 0.927 | 4.873 | 4.907 | 5.052 | 5.132 |
| | $(1.00, 1.00)$ | 0.798 | 0.795 | 0.810 | 0.820 | 4.020 | 4.107 | 4.257 | 4.359 |
| | $(1.00, 4.00)$ | 0.561 | 0.549 | 0.559 | 0.595 | 2.558 | 2.772 | 2.851 | 3.108 |
| | $(4.00, 0.25)$ | 0.559 | 0.559 | 0.574 | 0.583 | 2.949 | 3.073 | 3.177 | 2.892 |
| | $(4.00, 1.00)$ | 0.502 | 0.503 | 0.519 | 0.536 | 2.573 | 2.749 | 2.842 | 2.669 |
| | $(4.00, 4.00)$ | 0.407 | 0.403 | 0.414 | 0.462 | 1.919 | 2.175 | 2.248 | 2.396 |

Table E.19: Expected Number of Backorders and Expected Time to Fulfill a Backorder at Retail Store 1
$(\lambda_1^r, \quad \lambda_2^r) = (1, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.354 | 0.298 | 0.292 | 0.057 |
| | $(0.25, 1.00)$ | 0.020 | 0.017 | 0.015 | 0.030 | 0.938 | 0.801 | 0.798 | 1.065 |
| | $(0.25, 4.00)$ | 0.642 | 0.547 | 0.519 | 0.572 | 4.026 | 3.216 | 3.197 | 3.862 |
| | $(1.00, 0.25)$ | 0.031 | 0.028 | 0.025 | 0.029 | 0.763 | 0.675 | 0.671 | 0.969 |
| | $(1.00, 1.00)$ | 0.131 | 0.114 | 0.107 | 0.096 | 1.358 | 1.176 | 1.177 | 1.167 |

Continued on next page

**TableE.19 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.7 | (1.00, 4.00) | 0.958 | 0.812 | 0.791 | 0.721 | 4.321 | 3.512 | 3.501 | 3.946 |
| | (4.00, 0.25) | 0.448 | 0.421 | 0.401 | 0.382 | 1.874 | 1.722 | 1.708 | 1.655 |
| | (4.00, 1.00) | 0.754 | 0.680 | 0.663 | 0.583 | 2.588 | 2.288 | 2.289 | 2.186 |
| | (4.00, 4.00) | 2.019 | 1.704 | 1.670 | 1.576 | 5.639 | 4.630 | 4.624 | 5.468 |
| 0.8 | (0.25, 0.25) | 0.004 | 0.003 | 0.003 | 0.022 | 0.590 | 0.508 | 0.507 | 1.188 |
| | (0.25, 1.00) | 0.141 | 0.123 | 0.114 | 0.133 | 1.543 | 1.344 | 1.340 | 1.450 |
| | (0.25, 4.00) | 1.952 | 1.658 | 1.619 | 1.601 | 6.238 | 5.057 | 5.060 | 5.375 |
| | (1.00, 0.25) | 0.167 | 0.151 | 0.143 | 0.133 | 1.336 | 1.200 | 1.199 | 1.322 |
| | (1.00, 1.00) | 0.529 | 0.457 | 0.442 | 0.405 | 2.317 | 2.011 | 2.017 | 2.000 |
| | (1.00, 4.00) | 2.651 | 2.183 | 2.186 | 2.038 | 6.847 | 5.517 | 5.580 | 5.905 |
| | (4.00, 0.25) | 1.577 | 1.498 | 1.470 | 1.446 | 3.638 | 3.412 | 3.414 | 3.398 |
| | (4.00, 1.00) | 2.335 | 2.074 | 2.052 | 1.946 | 4.834 | 4.265 | 4.303 | 4.273 |
| | (4.00, 4.00) | 4.993 | 4.364 | 4.267 | 4.079 | 9.404 | 7.989 | 7.941 | 8.813 |
| 0.9 | (0.25, 0.25) | 0.112 | 0.100 | 0.092 | 0.138 | 1.282 | 1.140 | 1.139 | 1.591 |
| | (0.25, 1.00) | 1.088 | 0.948 | 0.919 | 0.885 | 3.329 | 2.910 | 2.917 | 2.850 |
| | (0.25, 4.00) | 7.134 | 6.213 | 5.931 | 5.892 | 12.530 | 10.558 | 10.207 | 10.355 |
| | (1.00, 0.25) | 1.094 | 0.994 | 0.970 | 0.910 | 3.041 | 2.766 | 2.771 | 2.743 |
| | (1.00, 1.00) | 2.578 | 2.236 | 2.208 | 2.080 | 5.186 | 4.545 | 4.562 | 4.500 |
| | (1.00, 4.00) | 9.174 | 7.804 | 7.609 | 7.295 | 14.358 | 12.058 | 11.876 | 11.964 |
| | (4.00, 0.25) | 6.451 | 5.808 | 5.923 | 5.940 | 9.325 | 8.463 | 8.682 | 8.681 |
| | (4.00, 1.00) | 8.498 | 7.597 | 7.685 | 7.370 | 11.791 | 10.579 | 10.719 | 10.532 |
| | (4.00, 4.00) | 15.784 | 13.992 | 13.230 | 13.113 | 21.107 | 18.499 | 17.607 | 18.779 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.124 | 0.046 | 0.057 |
| | (0.25, 1.00) | 0.002 | 0.002 | 0.002 | 0.009 | 0.869 | 0.791 | 0.777 | 1.205 |
| | (0.25, 4.00) | 0.349 | 0.297 | 0.280 | 0.377 | 3.764 | 3.176 | 3.165 | 4.465 |
| | (1.00, 0.25) | 0.005 | 0.004 | 0.004 | 0.008 | 0.710 | 0.666 | 0.657 | 1.074 |
| | (1.00, 1.00) | 0.038 | 0.033 | 0.031 | 0.028 | 1.263 | 1.168 | 1.169 | 1.167 |
| | (1.00, 4.00) | 0.575 | 0.489 | 0.474 | 0.478 | 4.123 | 3.541 | 3.508 | 4.542 |
| | (4.00, 0.25) | 0.217 | 0.206 | 0.196 | 0.141 | 1.803 | 1.728 | 1.709 | 1.523 |
| | (4.00, 1.00) | 0.421 | 0.381 | 0.371 | 0.267 | 2.483 | 2.278 | 2.275 | 2.070 |
| | (4.00, 4.00) | 1.446 | 1.216 | 1.190 | 1.148 | 5.443 | 4.663 | 4.639 | 5.963 |

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**TableE.19 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | $E[B_1^r]$ MedVis | $E[B_1^r]$ HiVis | $E[B_1^r]$ GI/G/2 | $E[W_1^{br}]$ LoVis | $E[W_1^{br}]$ MedVis | $E[W_1^{br}]$ HiVis | $E[W_1^{br}]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.007 | 0.563 | 0.486 | 0.489 | 1.188 |
| | $(0.25, 1.00)$ | 0.036 | 0.031 | 0.029 | 0.050 | 1.449 | 1.326 | 1.317 | 1.636 |
| | $(0.25, 4.00)$ | 1.333 | 1.123 | 1.098 | 1.160 | 5.920 | 5.056 | 5.060 | 5.855 |
| | $(1.00, 0.25)$ | 0.050 | 0.045 | 0.042 | 0.046 | 1.266 | 1.194 | 1.188 | 1.480 |
| | $(1.00, 1.00)$ | 0.237 | 0.203 | 0.197 | 0.180 | 2.194 | 2.001 | 2.007 | 2.000 |
| | $(1.00, 4.00)$ | 1.920 | 1.559 | 1.568 | 1.514 | 6.585 | 5.534 | 5.601 | 6.363 |
| | $(4.00, 0.25)$ | 1.024 | 0.985 | 0.967 | 0.854 | 3.520 | 3.420 | 3.420 | 3.303 |
| | $(4.00, 1.00)$ | 1.646 | 1.456 | 1.445 | 1.269 | 4.667 | 4.241 | 4.296 | 4.183 |
| | $(4.00, 4.00)$ | 4.050 | 3.530 | 3.448 | 3.311 | 9.148 | 8.020 | 7.972 | 9.201 |
| 0.9 | $(0.25, 0.25)$ | 0.024 | 0.020 | 0.019 | 0.058 | 1.227 | 1.124 | 1.126 | 1.878 |
| | $(0.25, 1.00)$ | 0.571 | 0.490 | 0.476 | 0.492 | 3.173 | 2.900 | 2.911 | 2.964 |
| | $(0.25, 4.00)$ | 5.950 | 5.153 | 4.885 | 4.916 | 12.105 | 10.593 | 10.197 | 10.628 |
| | $(1.00, 0.25)$ | 0.593 | 0.536 | 0.523 | 0.495 | 2.930 | 2.762 | 2.768 | 2.828 |
| | $(1.00, 1.00)$ | 1.751 | 1.501 | 1.485 | 1.930 | 4.987 | 4.544 | 4.562 | 4.500 |
| | $(1.00, 4.00)$ | 7.880 | 6.651 | 6.470 | 6.228 | 13.948 | 12.102 | 11.907 | 12.224 |
| | $(4.00, 0.25)$ | 5.320 | 4.763 | 4.881 | 4.768 | 9.138 | 8.452 | 8.696 | 8.601 |
| | $(4.00, 1.00)$ | 7.249 | 6.451 | 6.540 | 6.141 | 11.533 | 10.580 | 10.711 | 10.456 |
| | $(4.00, 4.00)$ | 14.334 | 12.674 | 11.919 | 11.832 | 20.720 | 18.588 | 17.606 | 19.009 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.057 |
| | $(0.25, 1.00)$ | 0.000 | 0.000 | 0.000 | 0.002 | 0.837 | 0.787 | 0.722 | 1.252 |
| | $(0.25, 4.00)$ | 0.194 | 0.183 | 0.150 | 0.250 | 3.754 | 3.247 | 3.186 | 5.115 |
| | $(1.00, 0.25)$ | 0.001 | 0.001 | 0.000 | 0.002 | 0.690 | 0.659 | 0.640 | 1.100 |
| | $(1.00, 1.00)$ | 0.010 | 0.010 | 0.007 | 0.007 | 1.246 | 1.167 | 1.154 | 1.167 |
| | $(1.00, 4.00)$ | 0.337 | 0.310 | 0.271 | 0.325 | 4.143 | 3.556 | 3.483 | 5.188 |
| | $(4.00, 0.25)$ | 0.097 | 0.101 | 0.086 | 0.044 | 1.776 | 1.734 | 1.716 | 1.444 |
| | $(4.00, 1.00)$ | 0.226 | 0.214 | 0.191 | 0.111 | 2.464 | 2.277 | 2.276 | 2.016 |
| | $(4.00, 4.00)$ | 0.983 | 0.883 | 0.826 | 0.846 | 5.428 | 4.695 | 4.682 | 6.523 |
| 0.8 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.002 | 0.462 | 0.414 | 0.165 | 1.188 |
| | $(0.25, 1.00)$ | 0.011 | 0.010 | 0.007 | 0.018 | 1.444 | 1.313 | 1.299 | 1.780 |
| | $(0.25, 4.00)$ | 0.873 | 0.825 | 0.745 | 0.850 | 5.875 | 5.115 | 5.085 | 6.396 |
| | $(1.00, 0.25)$ | 0.014 | 0.014 | 0.010 | 0.015 | 1.238 | 1.180 | 1.158 | 1.604 |
| | $(1.00, 1.00)$ | 0.101 | 0.096 | 0.081 | 0.074 | 2.149 | 1.996 | 2.000 | 2.000 |

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**TableE.19 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (1.00, 4.00) | 1.331 | 1.207 | 1.124 | 1.135 | 6.559 | 5.667 | 5.662 | 6.879 |
| | (4.00, 0.25) | 0.670 | 0.656 | 0.604 | 0.484 | 3.606 | 3.426 | 3.440 | 3.271 |
| | (4.00, 1.00) | 1.103 | 1.024 | 1.002 | 0.806 | 4.606 | 4.253 | 4.369 | 4.156 |
| | (4.00, 4.00) | 3.134 | 2.897 | 2.731 | 2.695 | 9.092 | 8.089 | 7.989 | 9.629 |
| 0.9 | (0.25, 0.25) | 0.006 | 0.006 | 0.003 | 0.023 | 1.193 | 1.092 | 1.071 | 1.978 |
| | (0.25, 1.00) | 0.297 | 0.287 | 0.242 | 0.268 | 3.109 | 2.896 | 2.906 | 3.092 |
| | (0.25, 4.00) | 4.609 | 4.409 | 4.141 | 4.103 | 11.898 | 10.620 | 10.431 | 10.924 |
| | (1.00, 0.25) | 0.314 | 0.311 | 0.269 | 0.261 | 2.874 | 2.759 | 2.755 | 2.943 |
| | (1.00, 1.00) | 1.099 | 1.058 | 0.981 | 0.914 | 4.838 | 4.540 | 4.565 | 4.500 |
| | (1.00, 4.00) | 6.102 | 5.602 | 5.553 | 5.317 | 13.476 | 11.897 | 12.021 | 12.503 |
| | (4.00, 0.25) | 4.210 | 4.119 | 4.000 | 3.802 | 8.996 | 8.687 | 8.710 | 8.581 |
| | (4.00, 1.00) | 6.031 | 5.531 | 5.216 | 5.093 | 11.510 | 10.545 | 10.303 | 10.438 |
| | (4.00, 4.00) | 11.915 | 10.871 | 10.758 | 10.673 | 19.709 | 17.711 | 17.891 | 19.249 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.057 |
| | (0.25, 1.00) | 0.000 | 0.000 | 0.000 | 0.001 | 0.720 | 0.651 | 0.487 | 1.257 |
| | (0.25, 4.00) | 0.106 | 0.101 | 0.082 | 0.182 | 3.565 | 3.241 | 3.180 | 5.583 |
| | (1.00, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.642 | 0.646 | 0.610 | 1.101 |
| | (1.00, 1.00) | 0.003 | 0.003 | 0.002 | 0.002 | 1.175 | 1.138 | 1.118 | 1.167 |
| | (1.00, 4.00) | 0.203 | 0.188 | 0.163 | 0.233 | 3.994 | 3.545 | 3.449 | 5.663 |
| | (4.00, 0.25) | 0.047 | 0.050 | 0.042 | 0.015 | 1.751 | 1.744 | 1.704 | 1.413 |
| | (4.00, 1.00) | 0.126 | 0.120 | 0.107 | 0.049 | 2.396 | 2.272 | 2.274 | 2.000 |
| | (4.00, 4.00) | 0.705 | 0.635 | 0.592 | 0.646 | 5.254 | 4.684 | 4.647 | 7.022 |
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.003 | 0.002 | 0.002 | 0.008 | 1.376 | 1.287 | 1.235 | 1.836 |
| | (0.25, 4.00) | 0.593 | 0.563 | 0.507 | 0.645 | 5.657 | 5.110 | 5.090 | 6.877 |
| | (1.00, 0.25) | 0.004 | 0.004 | 0.003 | 0.006 | 1.173 | 1.157 | 1.127 | 1.649 |
| | (1.00, 1.00) | 0.044 | 0.043 | 0.036 | 0.033 | 2.098 | 1.991 | 2.003 | 2.000 |
| | (1.00, 4.00) | 0.961 | 0.871 | 0.811 | 0.876 | 6.359 | 5.653 | 5.656 | 7.350 |
| | (4.00, 0.25) | 0.441 | 0.431 | 0.398 | 0.284 | 3.562 | 3.424 | 3.449 | 3.064 |
| | (4.00, 1.00) | 0.776 | 0.720 | 0.710 | 0.524 | 4.521 | 4.260 | 4.362 | 4.151 |
| | (4.00, 4.00) | 2.538 | 2.356 | 2.214 | 2.225 | 8.874 | 8.098 | 7.971 | 10.044 |

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**TableE.19 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | $(0.25, 0.25)$ | 0.001 | 0.001 | 0.001 | 0.010 | 1.089 | 0.981 | 0.934 | 1.991 |
| | $(0.25, 1.00)$ | 0.153 | 0.148 | 0.125 | 0.155 | 3.043 | 2.886 | 2.908 | 3.194 |
| | $(0.25, 4.00)$ | 3.826 | 3.662 | 3.426 | 3.455 | 11.661 | 10.654 | 10.428 | 11.211 |
| | $(1.00, 0.25)$ | 0.168 | 0.167 | 0.145 | 0.146 | 2.834 | 2.753 | 2.755 | 3.001 |
| | $(1.00, 1.00)$ | 0.738 | 0.711 | 0.661 | 0.612 | 4.750 | 4.534 | 4.555 | 4.500 |
| | $(1.00, 4.00)$ | 5.201 | 4.766 | 4.732 | 4.570 | 13.211 | 11.898 | 12.008 | 12.776 |
| | $(4.00, 0.25)$ | 3.463 | 3.394 | 3.298 | 3.050 | 8.903 | 8.680 | 8.700 | 8.578 |
| | $(4.00, 1.00)$ | 5.145 | 4.697 | 4.412 | 4.241 | 11.386 | 10.550 | 10.280 | 10.436 |
| | $(4.00, 4.00)$ | 10.758 | 9.807 | 9.714 | 9.654 | 19.371 | 17.687 | 17.878 | 19.487 |

Table E.20: Expected Number of Backorders and Expected Time to Fulfill a Backorder at Retail Store 1
$(\lambda_1^r, \quad \lambda_2^r) = (2, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $(S, R_1, R_2) = (2, 2, 4)$ | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.001 | 0.000 | 0.000 | 0.000 | 0.237 | 0.201 | 0.202 | 0.039 |
| | $(0.25, 1.00)$ | 0.041 | 0.036 | 0.031 | 0.051 | 0.629 | 0.531 | 0.530 | 0.693 |
| | $(0.25, 4.00)$ | 0.992 | 0.829 | 0.799 | 0.848 | 2.736 | 2.132 | 2.138 | 2.499 |
| | $(1.00, 0.25)$ | 0.058 | 0.054 | 0.049 | 0.051 | 0.515 | 0.460 | 0.459 | 0.633 |
| | $(1.00, 1.00)$ | 0.222 | 0.194 | 0.183 | 0.164 | 0.917 | 0.786 | 0.788 | 0.780 |
| | $(1.00, 4.00)$ | 1.455 | 1.205 | 1.160 | 1.067 | 2.926 | 2.322 | 2.314 | 2.557 |
| | $(4.00, 0.25)$ | 0.700 | 0.668 | 0.639 | 0.628 | 1.237 | 1.151 | 1.142 | 1.127 |
| | $(4.00, 1.00)$ | 1.150 | 1.028 | 1.010 | 0.923 | 1.716 | 1.511 | 1.524 | 1.482 |
| | $(4.00, 4.00)$ | 2.909 | 2.439 | 2.421 | 2.279 | 3.735 | 3.038 | 3.059 | 3.584 |
| 0.8 | $(0.25, 0.25)$ | 0.010 | 0.008 | 0.007 | 0.038 | 0.390 | 0.333 | 0.333 | 0.790 |
| | $(0.25, 1.00)$ | 0.251 | 0.217 | 0.200 | 0.217 | 1.039 | 0.880 | 0.882 | 0.943 |
| | $(0.25, 4.00)$ | 2.826 | 2.401 | 2.350 | 2.295 | 4.192 | 3.340 | 3.351 | 3.509 |
| | $(1.00, 0.25)$ | 0.282 | 0.249 | 0.235 | 0.220 | 0.896 | 0.794 | 0.794 | 0.862 |

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**TableE.20 – continued from previous page**

| ρ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | $E[B_1^r]$ MedVis | $E[B_1^r]$ HiVis | $E[B_1^r]$ GI/G/2 | $E[W_1^{br}]$ LoVis | $E[W_1^{br}]$ MedVis | $E[W_1^{br}]$ HiVis | $E[W_1^{br}]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(1.00, 1.00)$ | 0.833 | 0.722 | 0.696 | 0.637 | 1.560 | 1.343 | 1.345 | 1.329 |
| | $(1.00, 4.00)$ | 3.854 | 3.192 | 3.154 | 2.906 | 4.635 | 3.721 | 3.721 | 3.862 |
| | $(4.00, 0.25)$ | 2.371 | 2.151 | 2.092 | 2.156 | 2.453 | 2.227 | 2.218 | 2.277 |
| | $(4.00, 1.00)$ | 3.373 | 3.011 | 2.933 | 2.845 | 3.211 | 2.852 | 2.831 | 2.858 |
| | $(4.00, 4.00)$ | 7.086 | 6.026 | 5.913 | 5.694 | 6.286 | 5.230 | 5.209 | 5.802 |
| 0.9 | $(0.25, 0.25)$ | 0.207 | 0.188 | 0.172 | 0.223 | 0.860 | 0.757 | 0.756 | 1.020 |
| | $(0.25, 1.00)$ | 1.671 | 1.459 | 1.414 | 1.347 | 2.242 | 1.927 | 1.933 | 1.885 |
| | $(0.25, 4.00)$ | 10.005 | 8.482 | 8.324 | 8.211 | 8.504 | 6.890 | 6.841 | 6.862 |
| | $(1.00, 0.25)$ | 1.664 | 1.514 | 1.481 | 1.392 | 2.035 | 1.840 | 1.846 | 1.817 |
| | $(1.00, 1.00)$ | 3.723 | 3.257 | 3.220 | 3.044 | 3.466 | 3.026 | 3.041 | 3.000 |
| | $(1.00, 4.00)$ | 12.588 | 10.498 | 10.577 | 10.110 | 9.563 | 7.825 | 7.907 | 7.936 |
| | $(4.00, 0.25)$ | 8.941 | 8.318 | 8.208 | 8.347 | 6.148 | 5.745 | 5.714 | 5.807 |
| | $(4.00, 1.00)$ | 11.646 | 10.428 | 10.289 | 10.268 | 7.768 | 6.953 | 6.936 | 7.041 |
| | $(4.00, 4.00)$ | 21.720 | 18.432 | 17.888 | 17.933 | 14.043 | 11.859 | 11.616 | 12.483 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.179 | 0.149 | 0.039 |
| | $(0.25, 1.00)$ | 0.007 | 0.007 | 0.006 | 0.019 | 0.577 | 0.524 | 0.521 | 0.791 |
| | $(0.25, 4.00)$ | 0.623 | 0.518 | 0.500 | 0.606 | 2.570 | 2.114 | 2.117 | 2.816 |
| | $(1.00, 0.25)$ | 0.013 | 0.012 | 0.011 | 0.017 | 0.486 | 0.457 | 0.455 | 0.710 |
| | $(1.00, 1.00)$ | 0.081 | 0.072 | 0.068 | 0.061 | 0.855 | 0.779 | 0.780 | 0.780 |
| | $(1.00, 4.00)$ | 0.979 | 0.807 | 0.776 | 0.767 | 2.798 | 2.333 | 2.324 | 2.869 |
| | $(4.00, 0.25)$ | 0.389 | 0.378 | 0.360 | 0.289 | 1.199 | 1.163 | 1.151 | 1.039 |
| | $(4.00, 1.00)$ | 0.718 | 0.645 | 0.636 | 0.504 | 1.642 | 1.512 | 1.523 | 1.401 |
| | $(4.00, 4.00)$ | 2.232 | 1.859 | 1.849 | 1.777 | 3.632 | 3.055 | 3.077 | 3.837 |
| 0.8 | $(0.25, 0.25)$ | 0.001 | 0.001 | 0.001 | 0.014 | 0.365 | 0.327 | 0.325 | 0.790 |
| | $(0.25, 1.00)$ | 0.087 | 0.074 | 0.069 | 0.098 | 0.970 | 0.873 | 0.874 | 1.055 |
| | $(0.25, 4.00)$ | 2.110 | 1.782 | 1.745 | 1.780 | 4.003 | 3.334 | 3.344 | 3.755 |
| | $(1.00, 0.25)$ | 0.106 | 0.094 | 0.089 | 0.093 | 0.850 | 0.793 | 0.792 | 0.958 |
| | $(1.00, 1.00)$ | 0.442 | 0.382 | 0.369 | 0.337 | 1.477 | 1.339 | 1.341 | 1.329 |

Continued on next page

TableE.20 – continued from previous page

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (1.00, 4.00) | 3.012 | 2.472 | 2.440 | 2.300 | 4.478 | 3.741 | 3.727 | 4.096 |
| | (4.00, 0.25) | 1.694 | 1.535 | 1.490 | 1.436 | 2.401 | 2.237 | 2.226 | 2.206 |
| | (4.00, 1.00) | 2.558 | 2.280 | 2.216 | 2.049 | 3.112 | 2.856 | 2.829 | 2.790 |
| | (4.00, 4.00) | 6.025 | 5.093 | 4.994 | 4.845 | 6.143 | 5.256 | 5.230 | 5.998 |
| 0.9 | (0.25, 0.25) | 0.062 | 0.055 | 0.051 | 0.110 | 0.822 | 0.755 | 0.754 | 1.218 |
| | (0.25, 1.00) | 1.021 | 0.881 | 0.855 | 0.850 | 2.131 | 1.917 | 1.925 | 1.946 |
| | (0.25, 4.00) | 8.737 | 7.342 | 7.196 | 7.148 | 8.270 | 6.903 | 6.844 | 7.000 |
| | (1.00, 0.25) | 1.031 | 0.936 | 0.917 | 0.863 | 1.958 | 1.837 | 1.844 | 1.863 |
| | (1.00, 1.00) | 2.760 | 2.397 | 2.373 | 2.237 | 3.334 | 3.024 | 3.038 | 3.000 |
| | (1.00, 4.00) | 11.209 | 9.267 | 9.348 | 8.960 | 9.327 | 7.845 | 7.916 | 8.070 |
| | (4.00, 0.25) | 7.691 | 7.152 | 7.051 | 7.066 | 6.044 | 5.766 | 5.728 | 5.745 |
| | (4.00, 1.00) | 10.287 | 9.174 | 9.049 | 8.943 | 7.610 | 6.956 | 6.941 | 6.981 |
| | (4.00, 4.00) | 20.179 | 17.032 | 16.500 | 16.587 | 13.810 | 11.896 | 11.638 | 12.599 |
| | $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.006 | 0.028 | 0.000 | 0.039 |
| | (0.25, 1.00) | 0.001 | 0.001 | 0.001 | 0.005 | 0.570 | 0.524 | 0.512 | 0.834 |
| | (0.25, 4.00) | 0.344 | 0.317 | 0.265 | 0.403 | 2.564 | 2.159 | 2.127 | 3.266 |
| | (1.00, 0.25) | 0.002 | 0.002 | 0.001 | 0.004 | 0.483 | 0.458 | 0.468 | 0.734 |
| | (1.00, 1.00) | 0.022 | 0.021 | 0.016 | 0.015 | 0.835 | 0.769 | 0.769 | 0.780 |
| | (1.00, 4.00) | 0.547 | 0.507 | 0.445 | 0.513 | 2.761 | 2.354 | 2.310 | 3.314 |
| | (4.00, 0.25) | 0.178 | 0.179 | 0.156 | 0.091 | 1.197 | 1.155 | 1.146 | 0.976 |
| | (4.00, 1.00) | 0.380 | 0.365 | 0.329 | 0.211 | 1.645 | 1.539 | 1.541 | 1.354 |
| | (4.00, 4.00) | 1.512 | 1.318 | 1.294 | 1.296 | 3.630 | 3.086 | 3.159 | 4.208 |

Continued on next page

**TableE.20 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.003 | 0.297 | 0.301 | 0.253 | 0.790 |
| | (0.25, 1.00) | 0.025 | 0.023 | 0.016 | 0.035 | 0.955 | 0.862 | 0.862 | 1.169 |
| | (0.25, 4.00) | 1.396 | 1.295 | 1.180 | 1.290 | 3.967 | 3.359 | 3.362 | 4.115 |
| | (1.00, 0.25) | 0.031 | 0.029 | 0.022 | 0.030 | 0.845 | 0.790 | 0.783 | 1.054 |
| | (1.00, 1.00) | 0.190 | 0.181 | 0.152 | 0.138 | 1.455 | 1.339 | 1.343 | 1.329 |
| | (1.00, 4.00) | 2.018 | 1.835 | 1.723 | 1.707 | 4.403 | 3.743 | 3.754 | 4.437 |
| | (4.00, 0.25) | 1.055 | 0.994 | 0.912 | 0.814 | 2.365 | 2.235 | 2.231 | 2.175 |
| | (4.00, 1.00) | 1.683 | 1.617 | 1.529 | 1.301 | 3.042 | 2.875 | 2.879 | 2.762 |
| | (4.00, 4.00) | 4.594 | 4.016 | 4.000 | 3.926 | 6.094 | 5.200 | 5.316 | 6.278 |
| 0.9 | (0.25, 0.25) | 0.017 | 0.016 | 0.009 | 0.044 | 0.805 | 0.749 | 0.740 | 1.313 |
| | (0.25, 1.00) | 0.533 | 0.514 | 0.434 | 0.459 | 2.088 | 1.914 | 1.927 | 2.033 |
| | (0.25, 4.00) | 6.797 | 6.242 | 6.009 | 5.952 | 8.026 | 6.846 | 6.899 | 7.194 |
| | (1.00, 0.25) | 0.549 | 0.541 | 0.471 | 0.452 | 1.931 | 1.833 | 1.837 | 1.928 |
| | (1.00, 1.00) | 1.757 | 1.691 | 1.567 | 1.468 | 3.257 | 3.029 | 3.047 | 3.000 |
| | (1.00, 4.00) | 8.863 | 8.069 | 7.986 | 7.637 | 9.130 | 7.915 | 8.021 | 8.251 |
| | (4.00, 0.25) | 6.027 | 5.931 | 5.699 | 5.635 | 5.914 | 5.778 | 5.757 | 5.723 |
| | (4.00, 1.00) | 8.246 | 7.846 | 7.505 | 7.418 | 7.366 | 6.957 | 6.861 | 6.960 |
| | (4.00, 4.00) | 17.147 | 15.174 | 15.180 | 14.954 | 13.586 | 11.753 | 11.988 | 12.758 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.039 |
| | (0.25, 1.00) | 0.000 | 0.000 | 0.000 | 0.002 | 0.509 | 0.484 | 0.467 | 0.839 |
| | (0.25, 4.00) | 0.213 | 0.201 | 0.167 | 0.308 | 2.436 | 2.151 | 2.121 | 3.545 |
| | (1.00, 0.25) | 0.000 | 0.000 | 0.000 | 0.001 | 0.493 | 0.478 | 0.493 | 0.735 |
| | (1.00, 1.00) | 0.008 | 0.008 | 0.006 | 0.005 | 0.799 | 0.754 | 0.752 | 0.780 |
| | (1.00, 4.00) | 0.376 | 0.343 | 0.299 | 0.390 | 2.693 | 2.342 | 2.290 | 3.595 |
| | (4.00, 0.25) | 0.101 | 0.101 | 0.088 | 0.039 | 1.172 | 1.158 | 1.145 | 0.954 |
| | (4.00, 1.00) | 0.237 | 0.231 | 0.208 | 0.112 | 1.596 | 1.540 | 1.540 | 1.342 |
| | (4.00, 4.00) | 1.148 | 1.011 | 1.000 | 1.046 | 3.523 | 3.074 | 3.150 | 4.474 |

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**TableE.20 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $E[B_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_1^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.001 | 0.195 | 0.139 | 0.044 | 0.790 |
| | (0.25, 1.00) | 0.008 | 0.008 | 0.005 | 0.017 | 0.916 | 0.854 | 0.852 | 1.208 |
| | (0.25, 4.00) | 1.038 | 0.966 | 0.880 | 1.036 | 3.841 | 3.353 | 3.353 | 4.372 |
| | (1.00, 0.25) | 0.011 | 0.011 | 0.008 | 0.014 | 0.820 | 0.787 | 0.776 | 1.086 |
| | (1.00, 1.00) | 0.100 | 0.096 | 0.081 | 0.073 | 1.416 | 1.334 | 1.335 | 1.329 |
| | (1.00, 4.00) | 1.587 | 1.425 | 1.339 | 1.390 | 4.311 | 3.727 | 3.742 | 4.686 |
| | (4.00, 0.25) | 0.747 | 0.708 | 0.650 | 0.537 | 2.326 | 2.232 | 2.229 | 2.169 |
| | (4.00, 1.00) | 1.269 | 1.229 | 1.162 | 0.932 | 2.972 | 2.881 | 2.880 | 2.758 |
| | (4.00, 4.00) | 3.933 | 3.395 | 3.394 | 3.380 | 5.987 | 5.190 | 5.304 | 0.490 |
| 0.9 | (0.25, 0.25) | 0.005 | 0.005 | 0.003 | 0.023 | 0.780 | 0.738 | 0.721 | 1.325 |
| | (0.25, 1.00) | 0.321 | 0.310 | 0.262 | 0.298 | 2.042 | 1.909 | 1.926 | 2.090 |
| | (0.25, 4.00) | 5.903 | 5.399 | 5.203 | 5.214 | 7.856 | 6.834 | 6.888 | 7.340 |
| | (1.00, 0.25) | 0.338 | 0.334 | 0.291 | 0.287 | 1.896 | 1.828 | 1.833 | 1.971 |
| | (1.00, 1.00) | 1.293 | 1.246 | 1.157 | 1.078 | 3.198 | 3.030 | 3.047 | 3.000 |
| | (1.00, 4.00) | 7.859 | 7.136 | 7.073 | 6.799 | 8.960 | 7.908 | 8.017 | 8.388 |
| | (4.00, 0.25) | 5.168 | 5.102 | 4.900 | 4.766 | 5.859 | 5.791 | 5.762 | 5.720 |
| | (4.00, 1.00) | 7.252 | 6.905 | 6.591 | 6.457 | 7.275 | 6.953 | 6.851 | 6.958 |
| | (4.00, 4.00) | 15.896 | 14.014 | 14.042 | 13.855 | 13.423 | 11.741 | 11.992 | 12.877 |

Table E.21: Fill Rate and Expected Inventory Level at Retail Store 2
$(\lambda_1^r,\ \lambda_2^r) = (1,\ 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $f_2^r$ LoVis | MedVis | HiVis | GI/G/2 | $E[I_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 3.979 | 3.981 | 3.986 | 3.999 |
| | (0.25, 1.00) | 0.997 | 0.998 | 0.998 | 0.993 | 3.823 | 3.841 | 3.862 | 3.889 |
| | (0.25, 4.00) | 0.907 | 0.907 | 0.911 | 0.916 | 3.141 | 3.242 | 3.279 | 3.455 |
| | (1.00, 0.25) | 0.993 | 0.993 | 0.994 | 0.993 | 3.802 | 3.823 | 3.839 | 3.879 |
| | (1.00, 1.00) | 0.970 | 0.972 | 0.974 | 0.976 | 3.571 | 3.619 | 3.643 | 3.698 |

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**TableE.21 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | MedVis | HiVis | GI/G/2 | $E[I_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.7 | (1.00, 4.00) | 0.860 | 0.862 | 0.865 | 0.895 | 2.941 | 3.075 | 3.098 | 3.328 |
| | (4.00, 0.25) | 0.880 | 0.880 | 0.886 | 0.907 | 3.275 | 3.335 | 3.362 | 3.184 |
| | (4.00, 1.00) | 0.830 | 0.833 | 0.837 | 0.871 | 3.052 | 3.139 | 3.163 | 3.049 |
| | (4.00, 4.00) | 0.734 | 0.739 | 0.743 | 0.807 | 2.593 | 2.761 | 2.788 | 2.929 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 0.994 | 3.903 | 3.912 | 3.927 | 3.931 |
| | (0.25, 1.00) | 0.975 | 0.977 | 0.978 | 0.969 | 3.480 | 3.533 | 3.569 | 3.657 |
| | (0.25, 4.00) | 0.775 | 0.778 | 0.783 | 0.802 | 2.450 | 2.628 | 2.662 | 2.893 |
| | (1.00, 0.25) | 0.961 | 0.962 | 0.964 | 0.969 | 3.476 | 3.525 | 3.551 | 3.623 |
| | (1.00, 1.00) | 0.892 | 0.898 | 0.902 | 0.910 | 3.045 | 3.149 | 3.181 | 3.270 |
| | (1.00, 4.00) | 0.708 | 0.718 | 0.720 | 0.762 | 2.229 | 2.445 | 2.461 | 2.712 |
| | (4.00, 0.25) | 0.709 | 0.712 | 0.718 | 0.742 | 2.584 | 2.691 | 2.718 | 2.450 |
| | (4.00, 1.00) | 0.647 | 0.659 | 0.664 | 0.697 | 2.319 | 2.477 | 2.503 | 2.321 |
| | (4.00, 4.00) | 0.556 | 0.560 | 0.567 | 0.640 | 1.868 | 2.081 | 2.112 | 2.243 |
| 0.9 | (0.25, 0.25) | 0.981 | 0.982 | 0.983 | 0.969 | 3.491 | 3.529 | 3.565 | 3.665 |
| | (0.25, 1.00) | 0.820 | 0.832 | 0.837 | 0.834 | 2.505 | 2.644 | 2.689 | 2.869 |
| | (0.25, 4.00) | 0.509 | 0.514 | 0.521 | 0.537 | 1.391 | 1.622 | 1.652 | 1.824 |
| | (1.00, 0.25) | 0.797 | 0.805 | 0.810 | 0.825 | 2.568 | 2.682 | 2.716 | 2.793 |
| | (1.00, 1.00) | 0.649 | 0.669 | 0.674 | 0.691 | 1.957 | 2.148 | 2.179 | 2.285 |
| | (1.00, 4.00) | 0.435 | 0.449 | 0.457 | 0.491 | 1.194 | 1.452 | 1.482 | 1.656 |
| | (4.00, 0.25) | 0.418 | 0.436 | 0.438 | 0.446 | 1.494 | 1.673 | 1.682 | 1.391 |
| | (4.00, 1.00) | 0.371 | 0.388 | 0.390 | 0.413 | 1.281 | 1.480 | 1.486 | 1.313 |
| | (4.00, 4.00) | 0.308 | 0.318 | 0.323 | 0.378 | 0.945 | 1.184 | 1.203 | 1.280 |
| | $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 1.979 | 1.981 | 1.986 | 1.999 |
| | (0.25, 1.00) | 0.979 | 0.978 | 0.981 | 0.972 | 1.841 | 1.856 | 1.875 | 1.910 |
| | (0.25, 4.00) | 0.840 | 0.830 | 0.838 | 0.852 | 1.434 | 1.493 | 1.517 | 1.650 |
| | (1.00, 0.25) | 0.959 | 0.958 | 0.962 | 0.970 | 1.829 | 1.847 | 1.860 | 1.900 |
| | (1.00, 1.00) | 0.904 | 0.903 | 0.910 | 0.918 | 1.664 | 1.700 | 1.719 | 1.766 |
| | (1.00, 4.00) | 0.778 | 0.769 | 0.774 | 0.817 | 1.325 | 1.398 | 1.414 | 1.571 |
| | (4.00, 0.25) | 0.760 | 0.756 | 0.765 | 0.769 | 1.507 | 1.549 | 1.567 | 1.425 |
| | (4.00, 1.00) | 0.709 | 0.702 | 0.711 | 0.733 | 1.384 | 1.439 | 1.454 | 1.365 |
| | (4.00, 4.00) | 0.641 | 0.631 | 0.638 | 0.712 | 1.166 | 1.251 | 1.268 | 1.356 |

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TableE.21 – continued from previous page

| ρ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | $f_2^r$ MedVis | $f_2^r$ HiVis | $f_2^r$ GI/G/2 | $E[I_2^r]$ LoVis | $E[I_2^r]$ MedVis | $E[I_2^r]$ HiVis | $E[I_2^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (0.25, 0.25) | 0.993 | 0.993 | 0.994 | 0.981 | 1.907 | 1.915 | 1.929 | 1.946 |
| | (0.25, 1.00) | 0.908 | 0.908 | 0.915 | 0.908 | 1.585 | 1.625 | 1.654 | 1.740 |
| | (0.25, 4.00) | 0.687 | 0.672 | 0.680 | 0.702 | 1.069 | 1.163 | 1.184 | 1.330 |
| | (1.00, 0.25) | 0.875 | 0.874 | 0.881 | 0.899 | 1.593 | 1.632 | 1.652 | 1.709 |
| | (1.00, 1.00) | 0.772 | 0.773 | 0.781 | 0.798 | 1.337 | 1.403 | 1.426 | 1.494 |
| | (1.00, 4.00) | 0.613 | 0.604 | 0.608 | 0.655 | 0.960 | 1.069 | 1.080 | 1.236 |
| | (4.00, 0.25) | 0.566 | 0.561 | 0.570 | 0.575 | 1.137 | 1.205 | 1.221 | 1.042 |
| | (4.00, 1.00) | 0.517 | 0.515 | 0.524 | 0.545 | 1.008 | 1.094 | 1.110 | 0.997 |
| | (4.00, 4.00) | 0.467 | 0.453 | 0.462 | 0.537 | 0.812 | 0.914 | 0.932 | 1.011 |
| 0.9 | (0.25, 0.25) | 0.913 | 0.913 | 0.920 | 0.913 | 1.579 | 1.608 | 1.638 | 1.746 |
| | (0.25, 1.00) | 0.673 | 0.674 | 0.685 | 0.689 | 1.022 | 1.102 | 1.132 | 1.261 |
| | (0.25, 4.00) | 0.431 | 0.412 | 0.420 | 0.431 | 0.574 | 0.682 | 0.699 | 0.799 |
| | (1.00, 0.25) | 0.639 | 0.640 | 0.650 | 0.668 | 1.070 | 1.141 | 1.163 | 1.208 |
| | (1.00, 1.00) | 0.503 | 0.508 | 0.516 | 0.538 | 0.783 | 0.883 | 0.902 | 0.973 |
| | (1.00, 4.00) | 0.361 | 0.352 | 0.359 | 0.390 | 0.488 | 0.606 | 0.621 | 0.722 |
| | (4.00, 0.25) | 0.308 | 0.313 | 0.317 | 0.316 | 0.624 | 0.717 | 0.724 | 0.563 |
| | (4.00, 1.00) | 0.279 | 0.280 | 0.283 | 0.300 | 0.530 | 0.627 | 0.631 | 0.542 |
| | (4.00, 4.00) | 0.252 | 0.244 | 0.249 | 0.302 | 0.395 | 0.503 | 0.513 | 0.562 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 0.999 | 5.973 | 5.975 | 5.987 | 5.976 |
| | (0.25, 4.00) | 0.970 | 0.969 | 0.974 | 0.967 | 5.430 | 5.471 | 5.555 | 5.628 |
| | (1.00, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.969 | 5.972 | 5.982 | 5.977 |
| | (1.00, 1.00) | 0.997 | 0.997 | 0.998 | 0.998 | 5.869 | 5.878 | 5.908 | 5.923 |
| | (1.00, 4.00) | 0.949 | 0.947 | 0.953 | 0.959 | 5.243 | 5.308 | 5.383 | 5.534 |
| | (4.00, 0.25) | 0.973 | 0.971 | 0.975 | 0.989 | 5.613 | 5.628 | 5.679 | 5.673 |
| | (4.00, 1.00) | 0.948 | 0.948 | 0.953 | 0.975 | 5.384 | 5.426 | 5.488 | 5.506 |
| | (4.00, 4.00) | 0.866 | 0.865 | 0.872 | 0.908 | 4.749 | 4.872 | 4.939 | 5.096 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.993 | 5.994 | 5.998 | 5.982 |
| | (0.25, 1.00) | 0.998 | 0.998 | 0.999 | 0.996 | 5.834 | 5.843 | 5.893 | 5.891 |
| | (0.25, 4.00) | 0.895 | 0.890 | 0.900 | 0.906 | 4.675 | 4.759 | 4.874 | 5.072 |
| | (1.00, 0.25) | 0.997 | 0.997 | 0.997 | 0.996 | 5.831 | 5.840 | 5.881 | 5.892 |
| | (1.00, 1.00) | 0.979 | 0.979 | 0.982 | 0.984 | 5.515 | 5.543 | 5.618 | 5.660 |

Continued on next page

**TableE.21 – continued from previous page**

| ρ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | $f_2^r$ MedVis | $f_2^r$ HiVis | $f_2^r$ GI/G/2 | $E[I_2^r]$ LoVis | $E[I_2^r]$ MedVis | $E[I_2^r]$ HiVis | $E[I_2^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | (1.00, 4.00) | 0.848 | 0.846 | 0.856 | 0.881 | 4.367 | 4.507 | 4.609 | 4.867 |
| | (4.00, 0.25) | 0.876 | 0.874 | 0.884 | 0.913 | 4.860 | 4.909 | 4.996 | 4.884 |
| | (4.00, 1.00) | 0.828 | 0.830 | 0.838 | 0.874 | 4.513 | 4.619 | 4.693 | 4.619 |
| | (4.00, 4.00) | 0.714 | 0.709 | 0.722 | 0.779 | 3.718 | 3.901 | 3.996 | 4.173 |
| 0.9 | (0.25, 0.25) | 0.999 | 0.999 | 0.999 | 0.995 | 5.851 | 5.857 | 5.916 | 5.887 |
| | (0.25, 1.00) | 0.950 | 0.949 | 0.957 | 0.952 | 5.030 | 5.064 | 5.215 | 5.310 |
| | (0.25, 4.00) | 0.672 | 0.657 | 0.672 | 0.692 | 3.107 | 3.234 | 3.355 | 3.591 |
| | (1.00, 0.25) | 0.940 | 0.939 | 0.947 | 0.951 | 5.064 | 5.091 | 5.213 | 5.280 |
| | (1.00, 1.00) | 0.844 | 0.843 | 0.855 | 0.864 | 4.273 | 4.348 | 4.479 | 4.573 |
| | (1.00, 4.00) | 0.606 | 0.599 | 0.606 | 0.642 | 2.780 | 2.987 | 3.052 | 3.301 |
| | (4.00, 0.25) | 0.611 | 0.609 | 0.622 | 0.644 | 3.223 | 3.318 | 3.410 | 3.144 |
| | (4.00, 1.00) | 0.549 | 0.554 | 0.570 | 0.594 | 2.826 | 3.005 | 3.105 | 2.901 |
| | (4.00, 4.00) | 0.445 | 0.445 | 0.455 | 0.505 | 2.130 | 2.401 | 2.462 | 2.560 |
| | $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 4.000 | 4.000 | 4.000 | 4.000 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 0.998 | 3.974 | 3.976 | 3.987 | 3.978 |
| | (0.25, 4.00) | 0.948 | 0.943 | 0.953 | 0.950 | 3.519 | 3.554 | 3.623 | 3.701 |
| | (1.00, 0.25) | 0.999 | 0.999 | 0.999 | 0.998 | 3.970 | 3.972 | 3.982 | 3.979 |
| | (1.00, 1.00) | 0.992 | 0.991 | 0.993 | 0.994 | 3.877 | 3.886 | 3.913 | 3.928 |
| | (1.00, 4.00) | 0.919 | 0.913 | 0.923 | 0.937 | 3.377 | 3.430 | 3.491 | 3.626 |
| | (4.00, 0.25) | 0.945 | 0.942 | 0.950 | 0.970 | 3.663 | 3.679 | 3.723 | 3.702 |
| | (4.00, 1.00) | 0.909 | 0.907 | 0.917 | 0.945 | 3.483 | 3.519 | 3.571 | 3.567 |
| | (4.00, 4.00) | 0.819 | 0.813 | 0.823 | 0.870 | 3.027 | 3.119 | 3.172 | 3.297 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 0.999 | 3.993 | 3.994 | 3.998 | 3.983 |
| | (0.25, 1.00) | 0.993 | 0.993 | 0.995 | 0.990 | 3.842 | 3.851 | 3.898 | 3.902 |
| | (0.25, 4.00) | 0.852 | 0.839 | 0.854 | 0.867 | 2.955 | 3.021 | 3.112 | 3.277 |
| | (1.00, 0.25) | 0.988 | 0.988 | 0.991 | 0.990 | 3.841 | 3.850 | 3.888 | 3.901 |
| | (1.00, 1.00) | 0.953 | 0.952 | 0.960 | 0.963 | 3.571 | 3.596 | 3.663 | 3.701 |
| | (1.00, 4.00) | 0.796 | 0.787 | 0.801 | 0.835 | 2.740 | 2.843 | 2.923 | 3.125 |
| | (4.00, 0.25) | 0.814 | 0.809 | 0.824 | 0.852 | 3.090 | 3.133 | 3.202 | 3.085 |
| | (4.00, 1.00) | 0.760 | 0.758 | 0.771 | 0.806 | 2.842 | 2.923 | 2.984 | 2.902 |
| | (4.00, 4.00) | 0.655 | 0.642 | 0.658 | 0.720 | 2.315 | 2.444 | 2.514 | 2.644 |

Continued on next page

**TableE.21 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | $f_2^r$ MedVis | $f_2^r$ HiVis | $f_2^r$ GI/G/2 | $E[I_2^r]$ LoVis | $E[I_2^r]$ MedVis | $E[I_2^r]$ HiVis | $E[I_2^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (0.25, 0.25) | 0.995 | 0.995 | 0.997 | 0.988 | 3.856 | 3.862 | 3.919 | 3.900 |
| | (0.25, 1.00) | 0.905 | 0.901 | 0.917 | 0.913 | 3.173 | 3.203 | 3.332 | 3.423 |
| | (0.25, 4.00) | 0.613 | 0.586 | 0.603 | 0.624 | 1.890 | 1.981 | 2.070 | 2.239 |
| | (1.00, 0.25) | 0.891 | 0.887 | 0.902 | 0.911 | 3.210 | 3.235 | 3.338 | 3.395 |
| | (1.00, 1.00) | 0.773 | 0.767 | 0.785 | 0.797 | 2.635 | 2.695 | 2.800 | 2.875 |
| | (1.00, 4.00) | 0.547 | 0.529 | 0.538 | 0.575 | 1.681 | 1.822 | 1.873 | 2.048 |
| | (4.00, 0.25) | 0.532 | 0.525 | 0.542 | 0.557 | 1.970 | 2.044 | 2.112 | 1.897 |
| | (4.00, 1.00) | 0.476 | 0.475 | 0.493 | 0.512 | 1.713 | 1.839 | 1.909 | 1.752 |
| | (4.00, 4.00) | 0.395 | 0.386 | 0.397 | 0.446 | 1.288 | 1.466 | 1.509 | 1.579 |

Table E.22: Fill Rate and Expected Inventory Level at Retail Store 2
$(\lambda_1^r, \quad \lambda_2^r) = (2,\ 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $f_2^r$ LoVis | $f_2^r$ MedVis | $f_2^r$ HiVis | $f_2^r$ GI/G/2 | $E[I_2^r]$ LoVis | $E[I_2^r]$ MedVis | $E[I_2^r]$ HiVis | $E[I_2^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 3.985 | 3.987 | 3.990 | 3.999 |
| | (0.25, 1.00) | 0.999 | 1.000 | 1.000 | 0.997 | 3.880 | 3.892 | 3.906 | 3.922 |
| | (0.25, 4.00) | 0.944 | 0.948 | 0.949 | 0.944 | 3.327 | 3.415 | 3.438 | 3.566 |
| | (1.00, 0.25) | 0.998 | 0.998 | 0.998 | 0.997 | 3.866 | 3.878 | 3.889 | 3.916 |
| | (1.00, 1.00) | 0.987 | 0.988 | 0.988 | 0.990 | 3.700 | 3.733 | 3.750 | 3.787 |
| | (1.00, 4.00) | 0.905 | 0.910 | 0.913 | 0.929 | 3.155 | 3.275 | 3.299 | 3.464 |
| | (4.00, 0.25) | 0.928 | 0.926 | 0.929 | 0.954 | 3.465 | 3.500 | 3.516 | 3.405 |
| | (4.00, 1.00) | 0.888 | 0.891 | 0.893 | 0.929 | 3.277 | 3.350 | 3.365 | 3.282 |
| | (4.00, 4.00) | 0.796 | 0.803 | 0.804 | 0.860 | 2.858 | 3.009 | 3.022 | 3.108 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 0.998 | 3.934 | 3.943 | 3.953 | 3.951 |
| | (0.25, 1.00) | 0.992 | 0.993 | 0.993 | 0.985 | 3.637 | 3.676 | 3.701 | 3.756 |
| | (0.25, 4.00) | 0.839 | 0.847 | 0.851 | 0.858 | 2.702 | 2.858 | 2.886 | 3.086 |
| | (1.00, 0.25) | 0.983 | 0.984 | 0.985 | 0.985 | 3.632 | 3.674 | 3.692 | 3.734 |
| | (1.00, 1.00) | 0.941 | 0.946 | 0.948 | 0.952 | 3.293 | 3.374 | 3.397 | 3.460 |
| | (1.00, 4.00) | 0.774 | 0.789 | 0.790 | 0.825 | 2.490 | 2.697 | 2.708 | 2.924 |

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**TableE.22 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ | | | | $E[I_2^r]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.8 | $(4.00, 0.25)$ | 0.787 | 0.792 | 0.797 | 0.834 | 2.867 | 2.965 | 2.988 | 2.768 |
| | $(4.00, 1.00)$ | 0.731 | 0.740 | 0.745 | 0.791 | 2.632 | 2.766 | 2.788 | 2.623 |
| | $(4.00, 4.00)$ | 0.624 | 0.637 | 0.642 | 0.712 | 2.151 | 2.372 | 2.396 | 2.462 |
| 0.9 | $(0.25, 0.25)$ | 0.994 | 0.995 | 0.996 | 0.984 | 3.646 | 3.675 | 3.701 | 3.759 |
| | $(0.25, 1.00)$ | 0.899 | 0.909 | 0.912 | 0.898 | 2.836 | 2.949 | 2.984 | 3.124 |
| | $(0.25, 4.00)$ | 0.580 | 0.598 | 0.604 | 0.616 | 1.621 | 1.849 | 1.877 | 2.052 |
| | $(1.00, 0.25)$ | 0.875 | 0.882 | 0.885 | 0.894 | 2.891 | 2.983 | 3.008 | 3.069 |
| | $(1.00, 1.00)$ | 0.747 | 0.767 | 0.770 | 0.781 | 2.307 | 2.477 | 2.501 | 2.584 |
| | $(1.00, 4.00)$ | 0.502 | 0.533 | 0.531 | 0.568 | 1.421 | 1.702 | 1.705 | 1.875 |
| | $(4.00, 0.25)$ | 0.505 | 0.517 | 0.521 | 0.547 | 1.809 | 1.959 | 1.976 | 1.676 |
| | $(4.00, 1.00)$ | 0.450 | 0.471 | 0.477 | 0.505 | 1.580 | 1.778 | 1.804 | 1.565 |
| | $(4.00, 4.00)$ | 0.357 | 0.383 | 0.389 | 0.442 | 1.145 | 1.433 | 1.458 | 1.454 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.986 | 1.987 | 1.990 | 1.999 |
| | $(0.25, 1.00)$ | 0.990 | 0.991 | 0.992 | 0.982 | 1.886 | 1.897 | 1.910 | 1.933 |
| | $(0.25, 4.00)$ | 0.875 | 0.873 | 0.877 | 0.882 | 1.519 | 1.576 | 1.593 | 1.703 |
| | $(1.00, 0.25)$ | 0.975 | 0.975 | 0.977 | 0.981 | 1.877 | 1.889 | 1.898 | 1.926 |
| | $(1.00, 1.00)$ | 0.936 | 0.936 | 0.940 | 0.946 | 1.747 | 1.773 | 1.788 | 1.822 |
| | $(1.00, 4.00)$ | 0.816 | 0.814 | 0.820 | 0.854 | 1.423 | 1.497 | 1.513 | 1.635 |
| | $(4.00, 0.25)$ | 0.815 | 0.809 | 0.815 | 0.834 | 1.608 | 1.637 | 1.649 | 1.539 |
| | $(4.00, 1.00)$ | 0.764 | 0.762 | 0.768 | 0.798 | 1.498 | 1.548 | 1.559 | 1.475 |
| | $(4.00, 4.00)$ | 0.690 | 0.685 | 0.688 | 0.770 | 1.287 | 1.372 | 1.380 | 1.431 |
| 0.8 | $(0.25, 0.25)$ | 0.997 | 0.998 | 0.998 | 0.988 | 1.935 | 1.943 | 1.953 | 1.959 |
| | $(0.25, 1.00)$ | 0.947 | 0.949 | 0.953 | 0.938 | 1.683 | 1.715 | 1.737 | 1.801 |
| | $(0.25, 4.00)$ | 0.733 | 0.729 | 0.735 | 0.751 | 1.168 | 1.258 | 1.276 | 1.414 |
| | $(1.00, 0.25)$ | 0.916 | 0.919 | 0.923 | 0.933 | 1.692 | 1.726 | 1.741 | 1.780 |
| | $(1.00, 1.00)$ | 0.829 | 0.834 | 0.840 | 0.852 | 1.466 | 1.524 | 1.542 | 1.593 |
| | $(1.00, 4.00)$ | 0.657 | 0.659 | 0.662 | 0.708 | 1.065 | 1.177 | 1.186 | 1.323 |
| | $(4.00, 0.25)$ | 0.631 | 0.630 | 0.637 | 0.656 | 1.268 | 1.334 | 1.348 | 1.174 |
| | $(4.00, 1.00)$ | 0.581 | 0.580 | 0.588 | 0.619 | 1.148 | 1.228 | 1.242 | 1.116 |
| | $(4.00, 4.00)$ | 0.514 | 0.509 | 0.516 | 0.588 | 0.930 | 1.044 | 1.058 | 1.093 |

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**TableE.22 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | MedVis | HiVis | GI/G/2 | $E[I_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (0.25, 0.25) | 0.952 | 0.955 | 0.959 | 0.940 | 1.682 | 1.706 | 1.729 | 1.805 |
| | (0.25, 1.00) | 0.752 | 0.761 | 0.768 | 0.759 | 1.166 | 1.238 | 1.263 | 1.382 |
| | (0.25, 4.00) | 0.471 | 0.466 | 0.473 | 0.483 | 0.652 | 0.763 | 0.778 | 0.881 |
| | (1.00, 0.25) | 0.715 | 0.721 | 0.727 | 0.743 | 1.220 | 1.282 | 1.300 | 1.340 |
| | (1.00, 1.00) | 0.575 | 0.587 | 0.593 | 0.612 | 0.924 | 1.021 | 1.037 | 1.094 |
| | (1.00, 4.00) | 0.397 | 0.404 | 0.405 | 0.439 | 0.566 | 0.699 | 0.703 | 0.800 |
| | (4.00, 0.25) | 0.364 | 0.365 | 0.370 | 0.379 | 0.753 | 0.837 | 0.847 | 0.660 |
| | (4.00, 1.00) | 0.328 | 0.332 | 0.339 | 0.355 | 0.650 | 0.751 | 0.766 | 0.626 |
| | (4.00, 4.00) | 0.280 | 0.284 | 0.289 | 0.339 | 0.471 | 0.605 | 0.618 | 0.619 |
| | $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 1.000 | 5.982 | 5.984 | 5.991 | 5.984 |
| | (0.25, 4.00) | 0.987 | 0.987 | 0.989 | 0.980 | 5.581 | 5.616 | 5.674 | 5.709 |
| | (1.00, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.979 | 5.980 | 5.987 | 5.985 |
| | (1.00, 1.00) | 0.999 | 0.999 | 0.999 | 1.000 | 5.911 | 5.917 | 5.937 | 5.947 |
| | (1.00, 4.00) | 0.973 | 0.972 | 0.976 | 0.974 | 5.439 | 5.486 | 5.543 | 5.635 |
| | (4.00, 0.25) | 0.987 | 0.986 | 0.988 | 0.996 | 5.723 | 5.731 | 5.766 | 5.774 |
| | (4.00, 1.00) | 0.972 | 0.972 | 0.974 | 0.990 | 5.556 | 5.582 | 5.624 | 5.648 |
| | (4.00, 4.00) | 0.909 | 0.911 | 0.914 | 0.939 | 5.025 | 5.138 | 5.174 | 5.270 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 5.995 | 5.996 | 5.999 | 5.988 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 0.998 | 5.889 | 5.896 | 5.929 | 5.925 |
| | (0.25, 4.00) | 0.938 | 0.938 | 0.943 | 0.939 | 4.948 | 5.020 | 5.109 | 5.290 |
| | (1.00, 0.25) | 0.999 | 0.999 | 0.999 | 0.999 | 5.886 | 5.896 | 5.922 | 5.926 |
| | (1.00, 1.00) | 0.991 | 0.992 | 0.993 | 0.994 | 5.660 | 5.681 | 5.733 | 5.762 |
| | (1.00, 4.00) | 0.902 | 0.902 | 0.908 | 0.920 | 4.704 | 4.815 | 4.892 | 5.085 |
| | (4.00, 0.25) | 0.924 | 0.924 | 0.931 | 0.956 | 5.134 | 5.188 | 5.253 | 5.169 |
| | (4.00, 1.00) | 0.888 | 0.886 | 0.893 | 0.928 | 4.852 | 4.920 | 4.982 | 4.936 |
| | (4.00, 4.00) | 0.779 | 0.784 | 0.787 | 0.838 | 4.101 | 4.300 | 4.340 | 4.455 |
| 0.9 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 0.998 | 5.899 | 5.903 | 5.944 | 5.921 |
| | (0.25, 1.00) | 0.980 | 0.980 | 0.983 | 0.976 | 5.296 | 5.320 | 5.431 | 5.490 |
| | (0.25, 4.00) | 0.742 | 0.741 | 0.751 | 0.764 | 3.439 | 3.578 | 3.683 | 3.914 |
| | (1.00, 0.25) | 0.972 | 0.972 | 0.975 | 0.976 | 5.319 | 5.338 | 5.425 | 5.471 |
| | (1.00, 1.00) | 0.907 | 0.907 | 0.914 | 0.919 | 4.655 | 4.711 | 4.812 | 4.883 |

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**TableE.22 – continued from previous page**

| $\rho$ | $(\rho, cp^2, ca_i^2)$ | $f_2^r$ LoVis | $f_2^r$ MedVis | $f_2^r$ HiVis | $f_2^r$ GI/G/2 | $E[I_2^r]$ LoVis | $E[I_2^r]$ MedVis | $E[I_2^r]$ HiVis | $E[I_2^r]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (1.00, 4.00) | 0.677 | 0.681 | 0.687 | 0.718 | 3.144 | 3.339 | 3.401 | 3.627 |
| | (4.00, 0.25) | 0.698 | 0.696 | 0.706 | 0.738 | 3.666 | 3.753 | 3.833 | 3.561 |
| | (4.00, 1.00) | 0.638 | 0.640 | 0.651 | 0.686 | 3.284 | 3.433 | 3.510 | 3.293 |
| | (4.00, 4.00) | 0.512 | 0.518 | 0.527 | 0.578 | 2.501 | 2.762 | 2.824 | 2.852 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 1.000 | 4.000 | 4.000 | 4.000 | 4.000 |
| | (0.25, 1.00) | 1.000 | 1.000 | 1.000 | 0.999 | 3.982 | 3.984 | 3.991 | 3.984 |
| | (0.25, 4.00) | 0.970 | 0.968 | 0.973 | 0.965 | 3.626 | 3.656 | 3.708 | 3.756 |
| | (1.00, 0.25) | 1.000 | 1.000 | 1.000 | 0.999 | 3.979 | 3.981 | 3.987 | 3.985 |
| | (1.00, 1.00) | 0.996 | 0.996 | 0.997 | 0.997 | 3.913 | 3.920 | 3.939 | 3.949 |
| | (1.00, 4.00) | 0.946 | 0.943 | 0.950 | 0.956 | 3.508 | 3.553 | 3.602 | 3.695 |
| | (4.00, 0.25) | 0.966 | 0.964 | 0.969 | 0.985 | 3.745 | 3.757 | 3.788 | 3.784 |
| | (4.00, 1.00) | 0.941 | 0.939 | 0.945 | 0.970 | 3.607 | 3.632 | 3.670 | 3.676 |
| | (4.00, 4.00) | 0.863 | 0.859 | 0.865 | 0.903 | 3.213 | 3.298 | 3.329 | 3.400 |
| 0.8 | (0.25, 0.25) | 1.000 | 1.000 | 1.000 | 0.999 | 3.996 | 3.996 | 3.999 | 3.988 |
| | (0.25, 1.00) | 0.998 | 0.998 | 0.999 | 0.995 | 3.891 | 3.898 | 3.930 | 3.929 |
| | (0.25, 4.00) | 0.894 | 0.889 | 0.899 | 0.902 | 3.125 | 3.183 | 3.258 | 3.397 |
| | (1.00, 0.25) | 0.995 | 0.995 | 0.996 | 0.995 | 3.891 | 3.899 | 3.924 | 3.930 |
| | (1.00, 1.00) | 0.975 | 0.974 | 0.978 | 0.980 | 3.685 | 3.704 | 3.752 | 3.780 |
| | (1.00, 4.00) | 0.847 | 0.843 | 0.853 | 0.877 | 2.947 | 3.037 | 3.100 | 3.263 |
| | (4.00, 0.25) | 0.866 | 0.865 | 0.876 | 0.906 | 3.278 | 3.322 | 3.376 | 3.278 |
| | (4.00, 1.00) | 0.819 | 0.816 | 0.827 | 0.867 | 3.061 | 3.122 | 3.172 | 3.106 |
| | (4.00, 4.00) | 0.709 | 0.709 | 0.715 | 0.775 | 2.546 | 2.693 | 2.727 | 2.808 |
| 0.9 | (0.25, 0.25) | 0.998 | 0.999 | 0.999 | 0.994 | 3.900 | 3.904 | 3.944 | 3.926 |
| | (0.25, 1.00) | 0.948 | 0.947 | 0.956 | 0.946 | 3.362 | 3.384 | 3.484 | 3.551 |
| | (0.25, 4.00) | 0.667 | 0.656 | 0.670 | 0.687 | 2.067 | 2.166 | 2.247 | 2.421 |
| | (1.00, 0.25) | 0.933 | 0.932 | 0.941 | 0.945 | 3.392 | 3.410 | 3.489 | 3.531 |
| | (1.00, 1.00) | 0.838 | 0.836 | 0.848 | 0.857 | 2.880 | 2.928 | 3.012 | 3.071 |
| | (1.00, 4.00) | 0.601 | 0.595 | 0.604 | 0.638 | 1.881 | 2.018 | 2.067 | 2.228 |
| | (4.00, 0.25) | 0.605 | 0.600 | 0.613 | 0.638 | 2.233 | 2.306 | 2.366 | 2.131 |
| | (4.00, 1.00) | 0.547 | 0.545 | 0.559 | 0.589 | 1.980 | 2.093 | 2.149 | 1.967 |
| | (4.00, 4.00) | 0.445 | 0.441 | 0.451 | 0.502 | 1.496 | 1.673 | 1.717 | 1.733 |

Table E.23: Expected Number of Backorders and Expected Time to Fulfill a Backorder at Retail Store 2
$(\lambda_1^r, \quad \lambda_2^r) = (1,\, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.341 | 0.226 | 0.196 | 0.057 |
| | (0.25, 1.00) | 0.002 | 0.002 | 0.002 | 0.009 | 0.865 | 0.791 | 0.777 | 1.205 |
| | (0.25, 4.00) | 0.350 | 0.297 | 0.282 | 0.377 | 3.766 | 3.185 | 3.178 | 4.465 |
| | (1.00, 0.25) | 0.005 | 0.005 | 0.004 | 0.008 | 0.719 | 0.668 | 0.662 | 1.074 |
| | (1.00, 1.00) | 0.038 | 0.033 | 0.031 | 0.028 | 1.274 | 1.181 | 1.183 | 1.167 |
| | (1.00, 4.00) | 0.578 | 0.488 | 0.476 | 0.478 | 4.118 | 3.536 | 3.519 | 4.542 |
| | (4.00, 0.25) | 0.217 | 0.208 | 0.197 | 0.141 | 1.803 | 1.734 | 1.716 | 1.523 |
| | (4.00, 1.00) | 0.422 | 0.381 | 0.372 | 0.267 | 2.483 | 2.279 | 2.277 | 2.070 |
| | (4.00, 4.00) | 1.453 | 1.219 | 1.189 | 1.148 | 5.461 | 4.656 | 4.636 | 5.963 |
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.007 | 0.538 | 0.491 | 0.454 | 1.188 |
| | (0.25, 1.00) | 0.036 | 0.031 | 0.029 | 0.050 | 1.446 | 1.324 | 1.317 | 1.636 |
| | (0.25, 4.00) | 1.330 | 1.124 | 1.101 | 1.160 | 5.913 | 5.059 | 5.076 | 5.855 |
| | (1.00, 0.25) | 0.050 | 0.045 | 0.042 | 0.046 | 1.267 | 1.188 | 1.187 | 1.480 |
| | (1.00, 1.00) | 0.237 | 0.204 | 0.197 | 0.180 | 2.194 | 2.005 | 2.013 | 2.000 |
| | (1.00, 4.00) | 1.919 | 1.563 | 1.566 | 1.514 | 6.576 | 5.532 | 5.585 | 6.363 |
| | (4.00, 0.25) | 1.034 | 0.987 | 0.962 | 0.854 | 3.550 | 3.429 | 3.411 | 3.303 |
| | (4.00, 1.00) | 1.648 | 1.440 | 1.444 | 1.269 | 4.671 | 4.225 | 4.305 | 4.183 |
| | (4.00, 4.00) | 4.064 | 3.522 | 3.447 | 3.311 | 9.150 | 8.003 | 7.964 | 9.201 |
| 0.9 | (0.25, 0.25) | 0.023 | 0.020 | 0.018 | 0.058 | 1.214 | 1.116 | 1.113 | 1.878 |
| | (0.25, 1.00) | 0.572 | 0.489 | 0.476 | 0.492 | 3.179 | 2.903 | 2.915 | 2.964 |
| | (0.25, 4.00) | 5.955 | 5.154 | 4.888 | 4.916 | 12.119 | 10.600 | 10.207 | 10.628 |
| | (1.00, 0.25) | 0.594 | 0.538 | 0.525 | 0.495 | 2.925 | 2.763 | 2.766 | 2.828 |
| | (1.00, 1.00) | 1.754 | 1.506 | 1.488 | 1.930 | 4.990 | 4.542 | 4.561 | 4.500 |
| | (1.00, 4.00) | 7.898 | 6.663 | 6.485 | 6.228 | 13.976 | 12.088 | 11.922 | 12.224 |
| | (4.00, 0.25) | 5.334 | 4.768 | 4.895 | 4.768 | 9.156 | 8.460 | 8.703 | 8.601 |
| | (4.00, 1.00) | 7.253 | 6.482 | 6.557 | 6.141 | 11.529 | 10.574 | 10.728 | 10.456 |
| | (4.00, 4.00) | 14.294 | 12.653 | 11.901 | 11.832 | 20.664 | 18.554 | 17.590 | 19.009 |

Continued on next page

**TableE.23 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.357 | 0.304 | 0.300 | 0.057 |
| | $(0.25, 1.00)$ | 0.020 | 0.017 | 0.015 | 0.030 | 0.937 | 0.802 | 0.797 | 1.065 |
| | $(0.25, 4.00)$ | 0.642 | 0.548 | 0.520 | 0.572 | 4.026 | 3.217 | 3.208 | 3.862 |
| | $(1.00, 0.25)$ | 0.031 | 0.028 | 0.026 | 0.029 | 0.765 | 0.679 | 0.676 | 0.969 |
| | $(1.00, 1.00)$ | 0.131 | 0.114 | 0.107 | 0.096 | 1.359 | 1.180 | 1.180 | 1.167 |
| | $(1.00, 4.00)$ | 0.961 | 0.811 | 0.792 | 0.721 | 4.320 | 3.504 | 3.506 | 3.946 |
| | $(4.00, 0.25)$ | 0.449 | 0.423 | 0.402 | 0.382 | 1.874 | 1.727 | 1.711 | 1.655 |
| | $(4.00, 1.00)$ | 0.755 | 0.681 | 0.663 | 0.583 | 2.590 | 2.288 | 2.292 | 2.186 |
| | $(4.00, 4.00)$ | 2.026 | 1.709 | 1.668 | 1.576 | 5.651 | 4.624 | 4.620 | 5.468 |
| 0.8 | $(0.25, 0.25)$ | 0.004 | 0.003 | 0.003 | 0.022 | 0.586 | 0.509 | 0.507 | 1.188 |
| | $(0.25, 1.00)$ | 0.142 | 0.123 | 0.113 | 0.133 | 1.544 | 1.339 | 1.339 | 1.450 |
| | $(0.25, 4.00)$ | 1.949 | 1.659 | 1.623 | 1.601 | 6.234 | 5.059 | 5.072 | 5.375 |
| | $(1.00, 0.25)$ | 0.167 | 0.152 | 0.143 | 0.133 | 1.334 | 1.201 | 1.201 | 1.322 |
| | $(1.00, 1.00)$ | 0.529 | 0.458 | 0.442 | 0.405 | 2.317 | 2.014 | 2.018 | 2.000 |
| | $(1.00, 4.00)$ | 2.651 | 2.187 | 2.185 | 2.038 | 6.842 | 5.514 | 5.570 | 5.905 |
| | $(4.00, 0.25)$ | 1.587 | 1.501 | 1.465 | 1.446 | 3.656 | 3.418 | 3.409 | 3.398 |
| | $(4.00, 1.00)$ | 2.337 | 2.057 | 2.051 | 1.946 | 4.840 | 4.248 | 4.310 | 4.273 |
| | $(4.00, 4.00)$ | 5.008 | 4.355 | 4.266 | 4.079 | 9.406 | 7.967 | 7.941 | 8.813 |
| 0.9 | $(0.25, 0.25)$ | 0.111 | 0.099 | 0.092 | 0.138 | 1.279 | 1.135 | 1.137 | 1.591 |
| | $(0.25, 1.00)$ | 1.089 | 0.948 | 0.920 | 0.885 | 3.333 | 2.908 | 2.920 | 2.850 |
| | $(0.25, 4.00)$ | 7.138 | 6.214 | 5.935 | 5.892 | 12.537 | 10.562 | 10.219 | 10.355 |
| | $(1.00, 0.25)$ | 1.096 | 0.997 | 0.971 | 0.910 | 3.039 | 2.769 | 2.770 | 2.743 |
| | $(1.00, 1.00)$ | 2.580 | 2.241 | 2.211 | 2.080 | 5.186 | 4.547 | 4.563 | 4.500 |
| | $(1.00, 4.00)$ | 9.192 | 7.817 | 7.624 | 7.295 | 14.380 | 12.053 | 11.888 | 11.964 |
| | $(4.00, 0.25)$ | 6.463 | 5.813 | 5.937 | 5.940 | 9.339 | 8.469 | 8.689 | 8.681 |
| | $(4.00, 1.00)$ | 8.503 | 7.629 | 7.702 | 7.370 | 11.790 | 10.582 | 10.731 | 10.532 |
| | $(4.00, 4.00)$ | 15.745 | 13.972 | 13.210 | 13.113 | 21.063 | 18.478 | 17.598 | 18.779 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.057 |
| | $(0.25, 1.00)$ | 0.000 | 0.000 | 0.000 | 0.001 | 0.688 | 0.572 | 0.506 | 1.257 |
| | $(0.25, 4.00)$ | 0.106 | 0.101 | 0.082 | 0.182 | 3.587 | 3.238 | 3.172 | 5.583 |
| | $(1.00, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.601 | 0.533 | 0.550 | 1.101 |

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**TableE.23 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.7 | (1.00, 1.00) | 0.003 | 0.003 | 0.002 | 0.002 | 1.207 | 1.149 | 1.115 | 1.167 |
| | (1.00, 4.00) | 0.204 | 0.188 | 0.161 | 0.233 | 3.988 | 3.542 | 3.451 | 5.663 |
| | (4.00, 0.25) | 0.047 | 0.050 | 0.042 | 0.015 | 1.758 | 1.757 | 1.720 | 1.413 |
| | (4.00, 1.00) | 0.126 | 0.117 | 0.106 | 0.049 | 2.406 | 2.239 | 2.268 | 2.000 |
| | (4.00, 4.00) | 0.706 | 0.630 | 0.597 | 0.646 | 5.279 | 4.666 | 4.665 | 7.022 |
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.25, 1.00) | 0.003 | 0.002 | 0.002 | 0.008 | 1.313 | 1.252 | 1.242 | 1.836 |
| | (0.25, 4.00) | 0.594 | 0.566 | 0.508 | 0.645 | 5.678 | 5.122 | 5.097 | 6.877 |
| | (1.00, 0.25) | 0.004 | 0.004 | 0.003 | 0.006 | 1.143 | 1.123 | 1.102 | 1.649 |
| | (1.00, 1.00) | 0.045 | 0.043 | 0.036 | 0.033 | 2.103 | 2.012 | 2.007 | 2.000 |
| | (1.00, 4.00) | 0.967 | 0.873 | 0.815 | 0.876 | 6.366 | 5.655 | 5.661 | 7.350 |
| | (4.00, 0.25) | 0.445 | 0.431 | 0.398 | 0.284 | 3.588 | 3.424 | 3.433 | 3.064 |
| | (4.00, 1.00) | 0.780 | 0.722 | 0.708 | 0.524 | 4.523 | 4.251 | 4.371 | 4.151 |
| | (4.00, 4.00) | 2.523 | 2.359 | 2.221 | 2.225 | 8.802 | 8.092 | 7.996 | 10.044 |
| 0.9 | (0.25, 0.25) | 0.001 | 0.001 | 0.001 | 0.010 | 1.036 | 0.948 | 0.914 | 1.991 |
| | (0.25, 1.00) | 0.154 | 0.149 | 0.126 | 0.155 | 3.063 | 2.892 | 2.909 | 3.194 |
| | (0.25, 4.00) | 3.828 | 3.657 | 3.429 | 3.455 | 11.669 | 10.641 | 10.439 | 11.211 |
| | (1.00, 0.25) | 0.169 | 0.168 | 0.145 | 0.146 | 2.828 | 2.756 | 2.754 | 3.001 |
| | (1.00, 1.00) | 0.744 | 0.714 | 0.661 | 0.612 | 4.764 | 4.537 | 4.551 | 4.500 |
| | (1.00, 4.00) | 5.221 | 4.786 | 4.745 | 4.570 | 13.226 | 11.932 | 12.036 | 12.776 |
| | (4.00, 0.25) | 3.468 | 3.425 | 3.299 | 3.050 | 8.920 | 8.733 | 8.704 | 8.578 |
| | (4.00, 1.00) | 5.138 | 4.726 | 4.433 | 4.241 | 11.363 | 10.579 | 10.310 | 10.436 |
| | (4.00, 4.00) | 10.797 | 9.822 | 9.734 | 9.654 | 19.443 | 17.683 | 17.860 | 19.487 |
| | | | | $(S, R_1, R_2) = (4, 6, 4)$ | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.057 |
| | (0.25, 1.00) | 0.000 | 0.000 | 0.000 | 0.002 | 0.819 | 0.729 | 0.717 | 1.252 |
| | (0.25, 4.00) | 0.195 | 0.183 | 0.150 | 0.250 | 3.768 | 3.245 | 3.183 | 5.115 |
| | (1.00, 0.25) | 0.001 | 0.001 | 0.000 | 0.002 | 0.670 | 0.640 | 0.626 | 1.100 |
| | (1.00, 1.00) | 0.011 | 0.010 | 0.008 | 0.007 | 1.282 | 1.185 | 1.164 | 1.167 |
| | (1.00, 4.00) | 0.337 | 0.309 | 0.269 | 0.325 | 4.146 | 3.547 | 3.473 | 5.188 |
| | (4.00, 0.25) | 0.098 | 0.101 | 0.086 | 0.044 | 1.782 | 1.743 | 1.724 | 1.444 |

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**TableE.23 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.7 | (4.00, 1.00) | 0.226 | 0.211 | 0.189 | 0.111 | 2.468 | 2.255 | 2.270 | 2.016 |
| | (4.00, 4.00) | 0.985 | 0.878 | 0.831 | 0.846 | 5.444 | 4.685 | 4.694 | 6.523 |
| 0.8 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.002 | 0.485 | 0.321 | 0.282 | 1.188 |
| | (0.25, 1.00) | 0.011 | 0.010 | 0.007 | 0.018 | 1.439 | 1.310 | 1.290 | 1.780 |
| | (0.25, 4.00) | 0.874 | 0.828 | 0.745 | 0.850 | 5.885 | 5.129 | 5.091 | 6.396 |
| | (1.00, 0.25) | 0.014 | 0.014 | 0.010 | 0.015 | 1.211 | 1.160 | 1.144 | 1.604 |
| | (1.00, 1.00) | 0.101 | 0.097 | 0.081 | 0.074 | 2.158 | 2.015 | 2.011 | 2.000 |
| | (1.00, 4.00) | 1.339 | 1.209 | 1.128 | 1.135 | 6.565 | 5.665 | 5.672 | 6.879 |
| | (4.00, 0.25) | 0.676 | 0.656 | 0.604 | 0.484 | 3.622 | 3.427 | 3.439 | 3.271 |
| | (4.00, 1.00) | 1.109 | 1.027 | 0.999 | 0.806 | 4.612 | 4.246 | 4.370 | 4.156 |
| | (4.00, 4.00) | 3.120 | 2.901 | 2.738 | 2.695 | 9.031 | 8.086 | 8.008 | 9.629 |
| 0.9 | (0.25, 0.25) | 0.006 | 0.006 | 0.003 | 0.023 | 1.167 | 1.069 | 1.043 | 1.978 |
| | (0.25, 1.00) | 0.298 | 0.288 | 0.243 | 0.268 | 3.120 | 2.902 | 2.913 | 3.092 |
| | (0.25, 4.00) | 4.612 | 4.404 | 4.144 | 4.103 | 11.905 | 10.615 | 10.441 | 10.924 |
| | (1.00, 0.25) | 0.316 | 0.312 | 0.270 | 0.261 | 2.880 | 2.764 | 2.758 | 2.943 |
| | (1.00, 1.00) | 1.107 | 1.062 | 0.982 | 0.914 | 4.857 | 4.543 | 4.563 | 4.500 |
| | (1.00, 4.00) | 6.122 | 5.622 | 5.566 | 5.317 | 13.488 | 11.924 | 12.035 | 12.503 |
| | (4.00, 0.25) | 4.214 | 4.151 | 4.001 | 3.802 | 9.006 | 8.726 | 8.714 | 8.581 |
| | (4.00, 1.00) | 6.025 | 5.560 | 5.237 | 5.093 | 11.491 | 10.583 | 10.332 | 10.438 |
| | (4.00, 4.00) | 11.955 | 10.886 | 10.781 | 10.673 | 19.777 | 17.718 | 17.873 | 19.249 |

Table E.24: Expected Number of Backorders and Expected Time to Fulfill a Backorder at Retail Store 2
$(\lambda_1^r, \quad \lambda_2^r) = (2,\, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.008 | 0.038 | 0.000 | 0.039 |
| | (0.25, 1.00) | 0.000 | 0.000 | 0.000 | 0.003 | 0.568 | 0.536 | 0.526 | 0.817 |
| | (0.25, 4.00) | 0.136 | 0.111 | 0.109 | 0.183 | 2.438 | 2.124 | 2.136 | 3.248 |
| | (1.00, 0.25) | 0.001 | 0.001 | 0.001 | 0.002 | 0.474 | 0.455 | 0.455 | 0.724 |

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**TableE.24 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | $E[B_2^r]$ MedVis | $E[B_2^r]$ HiVis | $E[B_2^r]$ GI/G/2 | $E[W_2^{br}]$ LoVis | $E[W_2^{br}]$ MedVis | $E[W_2^{br}]$ HiVis | $E[W_2^{br}]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.7 | $(1.00, 1.00)$ | 0.011 | 0.010 | 0.009 | 0.008 | 0.841 | 0.778 | 0.779 | 0.780 |
| | $(1.00, 4.00)$ | 0.256 | 0.212 | 0.203 | 0.234 | 2.681 | 2.359 | 2.336 | 3.300 |
| | $(4.00, 0.25)$ | 0.086 | 0.086 | 0.082 | 0.046 | 1.188 | 1.173 | 1.149 | 0.998 |
| | $(4.00, 1.00)$ | 0.182 | 0.166 | 0.163 | 0.098 | 1.616 | 1.529 | 1.528 | 1.367 |
| | $(4.00, 4.00)$ | 0.727 | 0.606 | 0.608 | 0.594 | 3.572 | 3.078 | 3.112 | 4.245 |
| 0.8 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.002 | 0.315 | 0.286 | 0.258 | 0.790 |
| | $(0.25, 1.00)$ | 0.008 | 0.006 | 0.006 | 0.017 | 0.962 | 0.872 | 0.869 | 1.129 |
| | $(0.25, 4.00)$ | 0.615 | 0.512 | 0.504 | 0.590 | 3.826 | 3.342 | 3.371 | 4.149 |
| | $(1.00, 0.25)$ | 0.014 | 0.013 | 0.012 | 0.015 | 0.826 | 0.786 | 0.783 | 1.018 |
| | $(1.00, 1.00)$ | 0.086 | 0.073 | 0.070 | 0.064 | 1.443 | 1.339 | 1.343 | 1.329 |
| | $(1.00, 4.00)$ | 0.978 | 0.795 | 0.784 | 0.783 | 4.317 | 3.772 | 3.735 | 4.481 |
| | $(4.00, 0.25)$ | 0.507 | 0.462 | 0.455 | 0.362 | 2.385 | 2.229 | 2.243 | 2.181 |
| | $(4.00, 1.00)$ | 0.821 | 0.743 | 0.723 | 0.580 | 3.057 | 2.865 | 2.840 | 2.768 |
| | $(4.00, 4.00)$ | 2.277 | 1.922 | 1.885 | 1.825 | 6.059 | 5.296 | 5.268 | 6.347 |
| 0.9 | $(0.25, 0.25)$ | 0.005 | 0.004 | 0.003 | 0.021 | 0.834 | 0.760 | 0.751 | 1.283 |
| | $(0.25, 1.00)$ | 0.210 | 0.175 | 0.171 | 0.206 | 2.081 | 1.924 | 1.940 | 2.024 |
| | $(0.25, 4.00)$ | 3.362 | 2.779 | 2.721 | 2.781 | 8.001 | 6.914 | 6.868 | 7.249 |
| | $(1.00, 0.25)$ | 0.239 | 0.215 | 0.211 | 0.204 | 1.910 | 1.830 | 1.837 | 1.921 |
| | $(1.00, 1.00)$ | 0.821 | 0.706 | 0.701 | 0.656 | 3.239 | 3.024 | 3.044 | 3.000 |
| | $(1.00, 4.00)$ | 4.499 | 3.676 | 3.721 | 3.590 | 9.020 | 7.864 | 7.924 | 8.307 |
| | $(4.00, 0.25)$ | 2.956 | 2.797 | 2.752 | 2.594 | 5.977 | 5.783 | 5.740 | 5.725 |
| | $(4.00, 1.00)$ | 4.134 | 3.689 | 3.644 | 3.450 | 7.521 | 6.970 | 6.971 | 6.963 |
| | $(4.00, 4.00)$ | 8.748 | 7.346 | 7.132 | 7.153 | 13.586 | 11.925 | 11.683 | 12.817 |
| | $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.264 | 0.225 | 0.210 | 0.039 |
| | $(0.25, 1.00)$ | 0.006 | 0.005 | 0.004 | 0.013 | 0.620 | 0.538 | 0.537 | 0.732 |
| | $(0.25, 4.00)$ | 0.328 | 0.272 | 0.264 | 0.320 | 2.627 | 2.137 | 2.147 | 2.713 |
| | $(1.00, 0.25)$ | 0.012 | 0.012 | 0.010 | 0.013 | 0.503 | 0.458 | 0.457 | 0.663 |
| | $(1.00, 1.00)$ | 0.057 | 0.050 | 0.047 | 0.042 | 0.892 | 0.784 | 0.785 | 0.780 |
| | $(1.00, 4.00)$ | 0.524 | 0.434 | 0.417 | 0.405 | 2.842 | 2.335 | 2.320 | 2.770 |

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**TableE.24 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ | | | | $E[W_2^{br}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.7 | $(4.00, 0.25)$ | 0.229 | 0.223 | 0.214 | 0.179 | 1.242 | 1.169 | 1.155 | 1.082 |
| | $(4.00, 1.00)$ | 0.403 | 0.364 | 0.357 | 0.290 | 1.709 | 1.533 | 1.539 | 1.437 |
| | $(4.00, 4.00)$ | 1.156 | 0.968 | 0.967 | 0.917 | 3.733 | 3.074 | 3.101 | 3.777 |
| 0.8 | $(0.25, 0.25)$ | 0.001 | 0.001 | 0.001 | 0.010 | 0.391 | 0.341 | 0.338 | 0.790 |
| | $(0.25, 1.00)$ | 0.054 | 0.045 | 0.042 | 0.062 | 1.012 | 0.889 | 0.889 | 0.995 |
| | $(0.25, 4.00)$ | 1.081 | 0.912 | 0.894 | 0.920 | 4.051 | 3.360 | 3.371 | 3.695 |
| | $(1.00, 0.25)$ | 0.073 | 0.065 | 0.061 | 0.060 | 0.875 | 0.793 | 0.792 | 0.904 |
| | $(1.00, 1.00)$ | 0.259 | 0.223 | 0.215 | 0.197 | 1.516 | 1.339 | 1.342 | 1.329 |
| | $(1.00, 4.00)$ | 1.552 | 1.275 | 1.262 | 1.183 | 4.523 | 3.739 | 3.729 | 4.044 |
| | $(4.00, 0.25)$ | 0.909 | 0.831 | 0.814 | 0.769 | 2.463 | 2.245 | 2.249 | 2.234 |
| | $(4.00, 1.00)$ | 1.338 | 1.206 | 1.176 | 1.073 | 3.192 | 2.874 | 2.854 | 2.815 |
| | $(4.00, 4.00)$ | 3.056 | 2.593 | 2.547 | 2.456 | 6.288 | 5.285 | 5.264 | 5.967 |
| 0.9 | $(0.25, 0.25)$ | 0.041 | 0.035 | 0.032 | 0.066 | 0.857 | 0.768 | 0.766 | 1.112 |
| | $(0.25, 1.00)$ | 0.540 | 0.463 | 0.450 | 0.464 | 2.175 | 1.934 | 1.942 | 1.924 |
| | $(0.25, 4.00)$ | 4.393 | 3.693 | 3.623 | 3.609 | 8.302 | 6.917 | 6.865 | 6.982 |
| | $(1.00, 0.25)$ | 0.567 | 0.514 | 0.503 | 0.474 | 1.990 | 1.839 | 1.845 | 1.847 |
| | $(1.00, 1.00)$ | 1.438 | 1.250 | 1.237 | 1.165 | 3.379 | 3.022 | 3.038 | 3.000 |
| | $(1.00, 4.00)$ | 5.644 | 4.674 | 4.719 | 4.514 | 9.359 | 7.840 | 7.919 | 8.052 |
| | $(4.00, 0.25)$ | 3.900 | 3.676 | 3.623 | 3.578 | 6.138 | 5.787 | 5.752 | 5.764 |
| | $(4.00, 1.00)$ | 5.205 | 4.663 | 4.606 | 4.511 | 7.743 | 6.981 | 6.976 | 6.999 |
| | $(4.00, 4.00)$ | 10.074 | 8.518 | 8.292 | 8.320 | 13.973 | 11.907 | 11.683 | 12.590 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.039 |
| | $(0.25, 1.00)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.208 | 0.101 | 0.840 |
| | $(0.25, 4.00)$ | 0.030 | 0.029 | 0.024 | 0.081 | 2.318 | 2.163 | 2.187 | 3.961 |
| | $(1.00, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.367 | 0.427 | 0.428 | 0.736 |
| | $(1.00, 1.00)$ | 0.001 | 0.001 | 0.000 | 0.000 | 0.718 | 0.719 | 0.711 | 0.780 |
| | $(1.00, 4.00)$ | 0.068 | 0.066 | 0.055 | 0.104 | 2.493 | 2.350 | 2.258 | 4.026 |
| | $(4.00, 0.25)$ | 0.015 | 0.016 | 0.014 | 0.003 | 1.171 | 1.171 | 1.145 | 0.938 |
| | $(4.00, 1.00)$ | 0.044 | 0.043 | 0.039 | 0.013 | 1.595 | 1.535 | 1.521 | 1.335 |
| | $(4.00, 4.00)$ | 0.306 | 0.271 | 0.268 | 0.380 | 3.385 | 3.045 | 3.111 | 5.039 |

Continued on next page

**TableE.24 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.790 |
| | $(0.25, 1.00)$ | 0.000 | 0.000 | 0.000 | 0.200 | 0.846 | 0.776 | 0.786 | 1.234 |
| | $(0.25, 4.00)$ | 0.224 | 0.211 | 0.194 | 0.302 | 3.615 | 3.379 | 3.396 | 4.913 |
| | $(1.00, 0.25)$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.787 | 0.766 | 0.767 | 1.107 |
| | $(1.00, 1.00)$ | 0.012 | 0.011 | 0.009 | 0.009 | 1.381 | 1.325 | 1.327 | 1.329 |
| | $(1.00, 4.00)$ | 0.399 | 0.365 | 0.343 | 0.417 | 4.054 | 3.725 | 3.729 | 5.239 |
| | $(4.00, 0.25)$ | 0.174 | 0.169 | 0.154 | 0.096 | 2.299 | 2.228 | 2.226 | 2.166 |
| | $(4.00, 1.00)$ | 0.328 | 0.327 | 0.307 | 0.198 | 2.931 | 2.865 | 2.866 | 2.756 |
| | $(4.00, 4.00)$ | 1.278 | 1.116 | 1.118 | 1.142 | 5.771 | 5.177 | 5.267 | 7.036 |
| 0.9 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.003 | 0.725 | 0.675 | 0.667 | 1.329 |
| | $(0.25, 1.00)$ | 0.041 | 0.038 | 0.033 | 0.053 | 2.026 | 1.919 | 1.929 | 2.188 |
| | $(0.25, 4.00)$ | 1.962 | 1.772 | 1.716 | 1.823 | 7.611 | 6.848 | 6.896 | 7.728 |
| | $(1.00, 0.25)$ | 0.052 | 0.051 | 0.045 | 0.049 | 1.857 | 1.811 | 1.822 | 2.045 |
| | $(1.00, 1.00)$ | 0.294 | 0.282 | 0.263 | 0.242 | 3.132 | 3.020 | 3.037 | 3.000 |
| | $(1.00, 4.00)$ | 2.800 | 2.530 | 2.514 | 2.473 | 8.665 | 7.916 | 8.024 | 8.762 |
| | $(4.00, 0.25)$ | 1.746 | 1.763 | 1.699 | 1.498 | 5.795 | 5.794 | 5.778 | 5.718 |
| | $(4.00, 1.00)$ | 2.590 | 2.502 | 2.380 | 2.187 | 7.138 | 6.947 | 6.826 | 6.957 |
| | $(4.00, 4.00)$ | 6.471 | 5.651 | 5.667 | 5.581 | 13.225 | 11.696 | 11.988 | 13.215 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.039 |
| | $(0.25, 1.00)$ | 0.000 | 0.000 | 0.000 | 0.001 | 0.512 | 0.532 | 0.476 | 0.838 |
| | $(0.25, 4.00)$ | 0.073 | 0.069 | 0.058 | 0.127 | 2.435 | 2.166 | 2.167 | 3.635 |
| | $(1.00, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.472 | 0.479 | 0.446 | 0.735 |
| | $(1.00, 1.00)$ | 0.003 | 0.003 | 0.002 | 0.002 | 0.818 | 0.758 | 0.751 | 0.790 |
| | $(1.00, 4.00)$ | 0.145 | 0.133 | 0.115 | 0.163 | 2.667 | 2.354 | 2.289 | 3.689 |
| | $(4.00, 0.25)$ | 0.040 | 0.042 | 0.036 | 0.014 | 1.178 | 1.166 | 1.145 | 0.957 |
| | $(4.00, 1.00)$ | 0.095 | 0.094 | 0.085 | 0.040 | 1.592 | 1.531 | 1.534 | 1.343 |
| | $(4.00, 4.00)$ | 0.483 | 0.431 | 0.423 | 0.446 | 3.518 | 3.066 | 3.125 | 4.613 |
| 0.8 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.082 | 0.065 | 0.009 | 0.790 |
| | $(0.25, 1.00)$ | 0.002 | 0.002 | 0.001 | 0.006 | 0.954 | 0.863 | 0.851 | 1.204 |
| | $(0.25, 4.00)$ | 0.402 | 0.373 | 0.343 | 0.440 | 3.804 | 3.371 | 3.389 | 4.505 |
| | $(1.00, 0.25)$ | 0.004 | 0.004 | 0.003 | 0.005 | 0.814 | 0.780 | 0.777 | 1.082 |
| | $(1.00, 1.00)$ | 0.036 | 0.035 | 0.029 | 0.026 | 1.416 | 1.339 | 1.338 | 1.329 |

Continued on next page

**TableE.24 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B_2^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W_2^{br}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(1.00, 4.00)$ | 0.659 | 0.587 | 0.551 | 0.595 | 4.294 | 3.729 | 3.741 | 4.825 |
| | $(4.00, 0.25)$ | 0.314 | 0.303 | 0.277 | 0.205 | 2.347 | 2.236 | 2.222 | 2.169 |
| | $(4.00, 1.00)$ | 0.542 | 0.529 | 0.497 | 0.367 | 2.993 | 2.869 | 2.865 | 2.758 |
| | $(4.00, 4.00)$ | 1.742 | 1.509 | 1.506 | 1.496 | 5.982 | 5.181 | 5.281 | 6.639 |
| 0.9 | $(0.25, 0.25)$ | 0.001 | 0.001 | 0.001 | 0.008 | 0.781 | 0.737 | 0.728 | 1.323 |
| | $(0.25, 1.00)$ | 0.107 | 0.102 | 0.086 | 0.114 | 2.055 | 1.923 | 1.939 | 2.106 |
| | $(0.25, 4.00)$ | 2.589 | 2.360 | 2.280 | 2.330 | 7.777 | 6.856 | 6.908 | 7.449 |
| | $(1.00, 0.25)$ | 0.126 | 0.124 | 0.108 | 0.109 | 1.886 | 1.814 | 1.822 | 1.982 |
| | $(1.00, 1.00)$ | 0.519 | 0.499 | 0.463 | 0.430 | 3.189 | 3.025 | 3.044 | 3.000 |
| | $(1.00, 4.00)$ | 3.537 | 3.209 | 3.179 | 3.074 | 8.864 | 7.918 | 8.013 | 8.496 |
| | $(4.00, 0.25)$ | 2.314 | 2.315 | 2.232 | 2.068 | 5.863 | 5.777 | 5.770 | 5.719 |
| | $(4.00, 1.00)$ | 3.286 | 3.162 | 3.018 | 2.861 | 7.257 | 6.953 | 6.846 | 6.958 |
| | $(4.00, 4.00)$ | 7.466 | 6.562 | 6.561 | 6.462 | 13.434 | 11.714 | 11.970 | 12.979 |

Table E.25: Expected Time Spent by an Order at Retail Stores 1 and 2 $(\lambda_1^r, \quad \lambda_2^r) = (1,\ 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| | | $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 2.000 | 1.999 | 1.999 | 2.001 | 4.000 | 3.999 | 3.999 | 4.000 |
| | $(0.25, 1.00)$ | 2.019 | 2.016 | 2.014 | 2.030 | 4.002 | 4.000 | 4.000 | 4.009 |
| | $(0.25, 4.00)$ | 2.641 | 2.545 | 2.517 | 2.572 | 4.350 | 4.297 | 4.281 | 4.377 |
| | $(1.00, 0.25)$ | 2.032 | 2.029 | 2.026 | 2.029 | 4.005 | 4.002 | 4.002 | 4.008 |
| | $(1.00, 1.00)$ | 2.132 | 2.115 | 2.107 | 2.096 | 4.038 | 4.031 | 4.030 | 4.028 |
| | $(1.00, 4.00)$ | 2.959 | 2.813 | 2.792 | 2.721 | 4.577 | 4.486 | 4.475 | 4.478 |
| | $(4.00, 0.25)$ | 2.447 | 2.421 | 2.405 | 2.382 | 4.217 | 4.204 | 4.196 | 4.141 |
| | $(4.00, 1.00)$ | 2.756 | 2.681 | 2.660 | 2.583 | 4.424 | 4.384 | 4.371 | 4.267 |
| | $(4.00, 4.00)$ | 4.020 | 3.704 | 3.676 | 3.576 | 5.453 | 5.212 | 5.198 | 5.148 |

Continued on next page

**TableE.25 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(0.25, 0.25)$ | 2.003 | 2.003 | 2.001 | 2.022 | 3.999 | 3.999 | 3.998 | 4.007 |
| | $(0.25, 1.00)$ | 2.140 | 2.122 | 2.113 | 2.133 | 4.035 | 4.030 | 4.028 | 4.050 |
| | $(0.25, 4.00)$ | 3.951 | 3.656 | 3.617 | 3.601 | 5.330 | 5.123 | 5.100 | 5.160 |
| | $(1.00, 0.25)$ | 2.168 | 2.152 | 2.144 | 2.133 | 4.049 | 4.043 | 4.041 | 4.046 |
| | $(1.00, 1.00)$ | 2.531 | 2.459 | 2.442 | 2.405 | 4.237 | 4.202 | 4.195 | 4.180 |
| | $(1.00, 4.00)$ | 4.654 | 4.185 | 4.187 | 4.038 | 5.919 | 5.561 | 5.563 | 5.514 |
| | $(4.00, 0.25)$ | 3.582 | 3.497 | 3.472 | 3.446 | 5.033 | 4.989 | 4.965 | 4.854 |
| | $(4.00, 1.00)$ | 4.335 | 4.072 | 4.052 | 3.946 | 5.648 | 5.449 | 5.449 | 5.269 |
| | $(4.00, 4.00)$ | 7.002 | 6.362 | 6.263 | 6.079 | 8.068 | 7.523 | 7.453 | 7.311 |
| 0.9 | $(0.25, 0.25)$ | 2.111 | 2.098 | 2.090 | 2.138 | 4.023 | 4.019 | 4.017 | 4.058 |
| | $(0.25, 1.00)$ | 3.087 | 2.947 | 2.918 | 2.885 | 4.571 | 4.488 | 4.475 | 4.492 |
| | $(0.25, 4.00)$ | 9.131 | 8.209 | 7.927 | 7.892 | 9.954 | 9.152 | 8.886 | 8.916 |
| | $(1.00, 0.25)$ | 3.096 | 2.996 | 2.971 | 2.910 | 4.594 | 4.536 | 4.523 | 4.495 |
| | $(1.00, 1.00)$ | 4.580 | 4.238 | 4.209 | 4.080 | 5.754 | 5.503 | 5.485 | 5.393 |
| | $(1.00, 4.00)$ | 11.180 | 9.810 | 9.611 | 9.295 | 11.897 | 10.658 | 10.482 | 10.228 |
| | $(4.00, 0.25)$ | 8.451 | 7.812 | 7.934 | 7.940 | 9.336 | 8.782 | 8.887 | 8.768 |
| | $(4.00, 1.00)$ | 10.502 | 9.609 | 9.678 | 9.370 | 11.256 | 10.470 | 10.551 | 10.141 |
| | $(4.00, 4.00)$ | 17.806 | 15.982 | 15.226 | 15.113 | 18.305 | 16.649 | 15.913 | 15.832 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 3.999 | 3.998 | 3.998 | 4.000 | 2.000 | 1.999 | 2.000 | 2.001 |
| | $(0.25, 1.00)$ | 4.001 | 4.000 | 4.000 | 4.009 | 2.020 | 2.016 | 2.014 | 2.030 |
| | $(0.25, 4.00)$ | 4.348 | 4.294 | 4.278 | 4.377 | 2.643 | 2.547 | 2.520 | 2.572 |
| | $(1.00, 0.25)$ | 4.007 | 4.007 | 4.005 | 4.008 | 2.031 | 2.027 | 2.025 | 2.029 |
| | $(1.00, 1.00)$ | 4.039 | 4.035 | 4.032 | 4.028 | 2.131 | 2.113 | 2.106 | 2.096 |
| | $(1.00, 4.00)$ | 4.577 | 4.491 | 4.476 | 4.478 | 2.961 | 2.810 | 2.792 | 2.721 |
| | $(4.00, 0.25)$ | 4.215 | 4.207 | 4.202 | 4.141 | 2.449 | 2.420 | 2.402 | 2.382 |
| | $(4.00, 1.00)$ | 4.425 | 4.381 | 4.366 | 4.267 | 2.756 | 2.682 | 2.663 | 2.583 |
| | $(4.00, 4.00)$ | 5.447 | 5.216 | 5.199 | 5.148 | 4.026 | 3.703 | 3.675 | 3.576 |
| 0.8 | $(0.25, 0.25)$ | 3.999 | 3.999 | 3.998 | 4.007 | 2.004 | 2.003 | 2.002 | 2.022 |
| | $(0.25, 1.00)$ | 4.035 | 4.029 | 4.027 | 4.050 | 2.141 | 2.122 | 2.113 | 2.133 |
| | $(0.25, 4.00)$ | 5.332 | 5.121 | 5.095 | 5.160 | 3.949 | 3.657 | 3.621 | 3.601 |
| | $(1.00, 0.25)$ | 4.051 | 4.047 | 4.044 | 4.046 | 2.167 | 2.151 | 2.142 | 2.133 |
| | $(1.00, 1.00)$ | 4.240 | 4.206 | 4.198 | 4.180 | 2.528 | 2.457 | 2.440 | 2.405 |

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**TableE.25 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.8 | $(1.00, 4.00)$ | 5.923 | 5.562 | 5.569 | 5.514 | 4.650 | 4.185 | 4.183 | 4.038 |
| | $(4.00, 0.25)$ | 5.030 | 4.983 | 4.969 | 4.854 | 3.587 | 3.502 | 3.467 | 3.446 |
| | $(4.00, 1.00)$ | 5.647 | 5.453 | 5.444 | 5.269 | 4.337 | 4.063 | 4.054 | 3.946 |
| | $(4.00, 4.00)$ | 8.062 | 7.529 | 7.443 | 7.311 | 7.011 | 6.355 | 6.272 | 6.079 |
| 0.9 | $(0.25, 0.25)$ | 4.022 | 4.018 | 4.016 | 4.058 | 2.111 | 2.099 | 2.091 | 2.138 |
| | $(0.25, 1.00)$ | 4.570 | 4.488 | 4.473 | 4.492 | 3.089 | 2.947 | 2.919 | 2.885 |
| | $(0.25, 4.00)$ | 9.947 | 9.149 | 8.880 | 8.916 | 9.137 | 8.212 | 7.932 | 7.892 |
| | $(1.00, 0.25)$ | 4.595 | 4.539 | 4.525 | 4.495 | 3.096 | 2.995 | 2.971 | 2.910 |
| | $(1.00, 1.00)$ | 5.754 | 5.504 | 5.487 | 5.393 | 4.580 | 4.239 | 4.209 | 4.080 |
| | $(1.00, 4.00)$ | 11.886 | 10.658 | 10.473 | 10.228 | 11.191 | 9.811 | 9.621 | 9.295 |
| | $(4.00, 0.25)$ | 9.320 | 8.768 | 8.894 | 8.768 | 8.465 | 7.826 | 7.929 | 7.940 |
| | $(4.00, 1.00)$ | 11.254 | 10.464 | 10.532 | 10.141 | 10.505 | 9.618 | 9.697 | 9.370 |
| | $(4.00, 4.00)$ | 18.357 | 16.663 | 15.916 | 15.832 | 17.755 | 15.968 | 15.222 | 15.113 |
| | $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 3.998 | 3.998 | 3.998 | 4.000 | 5.998 | 5.998 | 5.998 | 6.000 |
| | $(0.25, 1.00)$ | 3.999 | 3.999 | 3.998 | 4.002 | 5.999 | 5.999 | 5.998 | 6.001 |
| | $(0.25, 4.00)$ | 4.192 | 4.182 | 4.149 | 4.255 | 6.105 | 6.100 | 6.080 | 6.182 |
| | $(1.00, 0.25)$ | 4.004 | 4.003 | 4.001 | 4.002 | 5.997 | 5.997 | 5.998 | 6.000 |
| | $(1.00, 1.00)$ | 4.013 | 4.012 | 4.009 | 4.007 | 6.000 | 5.999 | 6.000 | 6.002 |
| | $(1.00, 4.00)$ | 4.339 | 4.313 | 4.272 | 4.325 | 6.201 | 6.185 | 6.159 | 6.233 |
| | $(4.00, 0.25)$ | 4.098 | 4.101 | 4.090 | 4.044 | 6.051 | 6.053 | 6.049 | 6.015 |
| | $(4.00, 1.00)$ | 4.227 | 4.218 | 4.189 | 4.111 | 6.129 | 6.120 | 6.105 | 6.049 |
| | $(4.00, 4.00)$ | 4.986 | 4.887 | 4.831 | 4.846 | 6.704 | 6.626 | 6.595 | 6.646 |
| 0.8 | $(0.25, 0.25)$ | 3.998 | 3.998 | 3.998 | 4.002 | 5.999 | 5.998 | 5.998 | 6.000 |
| | $(0.25, 1.00)$ | 4.008 | 4.008 | 4.005 | 4.018 | 6.001 | 6.001 | 6.000 | 6.008 |
| | $(0.25, 4.00)$ | 4.871 | 4.823 | 4.743 | 4.850 | 6.592 | 6.564 | 6.505 | 6.645 |
| | $(1.00, 0.25)$ | 4.017 | 4.016 | 4.011 | 4.015 | 6.000 | 6.001 | 6.000 | 6.006 |
| | $(1.00, 1.00)$ | 4.103 | 4.099 | 4.082 | 4.074 | 6.042 | 6.040 | 6.034 | 6.033 |
| | $(1.00, 4.00)$ | 5.335 | 5.210 | 5.125 | 5.135 | 6.963 | 6.870 | 6.812 | 6.876 |
| | $(4.00, 0.25)$ | 4.671 | 4.662 | 4.605 | 4.484 | 6.447 | 6.425 | 6.393 | 6.284 |
| | $(4.00, 1.00)$ | 5.112 | 5.025 | 5.000 | 4.806 | 6.771 | 6.727 | 6.715 | 6.524 |
| | $(4.00, 4.00)$ | 7.131 | 6.894 | 6.733 | 6.695 | 8.515 | 8.352 | 8.222 | 8.225 |

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**TableE.25 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (0.25, 0.25) | 4.004 | 4.004 | 4.002 | 4.023 | 5.999 | 5.999 | 5.999 | 6.010 |
| | (0.25, 1.00) | 4.295 | 4.285 | 4.241 | 4.268 | 6.153 | 6.147 | 6.124 | 6.155 |
| | (0.25, 4.00) | 8.605 | 8.404 | 8.137 | 8.103 | 9.825 | 9.654 | 9.426 | 9.455 |
| | (1.00, 0.25) | 4.317 | 4.313 | 4.270 | 4.261 | 6.166 | 6.165 | 6.143 | 6.146 |
| | (1.00, 1.00) | 5.102 | 5.062 | 4.983 | 4.914 | 6.741 | 6.711 | 6.658 | 6.612 |
| | (1.00, 4.00) | 10.108 | 9.607 | 9.555 | 9.317 | 11.215 | 10.780 | 10.741 | 10.570 |
| | (4.00, 0.25) | 8.218 | 8.121 | 7.999 | 7.802 | 9.473 | 9.412 | 9.299 | 9.050 |
| | (4.00, 1.00) | 10.029 | 9.538 | 9.225 | 9.093 | 11.128 | 10.727 | 10.443 | 10.141 |
| | (4.00, 4.00) | 15.919 | 14.868 | 14.788 | 14.673 | 16.799 | 15.816 | 15.740 | 15.654 |
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 5.997 | 5.997 | 5.997 | 6.000 | 3.999 | 3.999 | 3.999 | 4.000 |
| | (0.25, 1.00) | 5.998 | 5.997 | 5.997 | 6.001 | 4.000 | 4.000 | 3.999 | 4.002 |
| | (0.25, 4.00) | 6.102 | 6.099 | 6.079 | 6.182 | 4.194 | 4.183 | 4.149 | 4.255 |
| | (1.00, 0.25) | 6.004 | 6.004 | 6.002 | 6.000 | 3.999 | 3.998 | 3.999 | 4.002 |
| | (1.00, 1.00) | 6.007 | 6.006 | 6.004 | 6.002 | 4.009 | 4.008 | 4.006 | 4.007 |
| | (1.00, 4.00) | 6.207 | 6.193 | 6.164 | 6.233 | 4.335 | 4.307 | 4.267 | 4.325 |
| | (4.00, 0.25) | 6.047 | 6.050 | 6.048 | 6.015 | 4.101 | 4.103 | 4.091 | 4.044 |
| | (4.00, 1.00) | 6.128 | 6.126 | 6.104 | 6.049 | 4.228 | 4.212 | 4.189 | 4.111 |
| | (4.00, 4.00) | 6.709 | 6.641 | 6.599 | 6.646 | 4.983 | 4.874 | 4.829 | 4.846 |
| 0.8 | (0.25, 0.25) | 5.997 | 5.997 | 5.997 | 6.000 | 3.999 | 3.999 | 3.999 | 4.002 |
| | (0.25, 1.00) | 6.000 | 6.000 | 5.999 | 6.008 | 4.010 | 4.009 | 4.006 | 4.018 |
| | (0.25, 4.00) | 6.590 | 6.560 | 6.504 | 6.645 | 4.872 | 4.827 | 4.744 | 4.850 |
| | (1.00, 0.25) | 6.008 | 6.007 | 6.005 | 6.006 | 4.011 | 4.012 | 4.008 | 4.015 |
| | (1.00, 1.00) | 6.049 | 6.046 | 6.038 | 6.033 | 4.099 | 4.094 | 4.079 | 4.074 |
| | (1.00, 4.00) | 6.965 | 6.875 | 6.812 | 6.876 | 5.336 | 5.207 | 5.127 | 5.135 |
| | (4.00, 0.25) | 6.442 | 6.439 | 6.400 | 6.284 | 4.677 | 4.651 | 4.601 | 4.484 |
| | (4.00, 1.00) | 6.788 | 6.722 | 6.707 | 6.524 | 5.102 | 5.030 | 5.004 | 4.806 |
| | (4.00, 4.00) | 8.535 | 8.351 | 8.217 | 8.225 | 7.113 | 6.896 | 6.739 | 6.695 |
| 0.9 | (0.25, 0.25) | 5.998 | 5.998 | 5.998 | 6.010 | 4.005 | 4.004 | 4.002 | 4.023 |
| | (0.25, 1.00) | 6.150 | 6.145 | 6.123 | 6.155 | 4.297 | 4.287 | 4.242 | 4.268 |
| | (0.25, 4.00) | 9.821 | 9.657 | 9.421 | 9.455 | 8.609 | 8.402 | 8.141 | 8.103 |
| | (1.00, 0.25) | 6.173 | 6.171 | 6.147 | 6.146 | 4.313 | 4.310 | 4.269 | 4.261 |

Continued on next page

**TableE.25 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | $(1.00, 1.00)$ | 6.742 | 6.716 | 6.662 | 6.612 | 5.104 | 5.060 | 4.980 | 4.914 |
| | $(1.00, 4.00)$ | 11.208 | 10.772 | 10.734 | 10.570 | 10.116 | 9.616 | 9.562 | 9.317 |
| | $(4.00, 0.25)$ | 9.473 | 9.396 | 9.297 | 9.050 | 8.219 | 8.139 | 8.001 | 7.802 |
| | $(4.00, 1.00)$ | 11.142 | 10.705 | 10.422 | 10.141 | 10.016 | 9.560 | 9.246 | 9.093 |
| | $(4.00, 4.00)$ | 16.762 | 15.803 | 15.745 | 15.654 | 15.956 | 14.881 | 14.786 | 14.673 |

Table E.26: Expected Time Spent by an Order at Retail Stores 1 and 2 $(\lambda_1^r, \quad \lambda_2^r) = (2,\ 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 1.000 | 1.000 | 1.000 | 1.000 | 4.000 | 3.999 | 3.999 | 4.000 |
| | $(0.25, 1.00)$ | 1.020 | 1.018 | 1.016 | 1.026 | 4.000 | 3.999 | 3.999 | 4.003 |
| | $(0.25, 4.00)$ | 1.496 | 1.415 | 1.399 | 1.424 | 4.135 | 4.110 | 4.107 | 4.183 |
| | $(1.00, 0.25)$ | 1.029 | 1.027 | 1.026 | 1.025 | 3.998 | 3.999 | 3.998 | 4.002 |
| | $(1.00, 1.00)$ | 1.112 | 1.098 | 1.092 | 1.082 | 4.009 | 4.008 | 4.006 | 4.008 |
| | $(1.00, 4.00)$ | 1.728 | 1.603 | 1.582 | 1.533 | 4.253 | 4.211 | 4.200 | 4.234 |
| | $(4.00, 0.25)$ | 1.349 | 1.335 | 1.322 | 1.314 | 4.089 | 4.085 | 4.076 | 4.046 |
| | $(4.00, 1.00)$ | 1.576 | 1.515 | 1.506 | 1.462 | 4.180 | 4.172 | 4.169 | 4.098 |
| | $(4.00, 4.00)$ | 2.456 | 2.222 | 2.211 | 2.139 | 4.730 | 4.608 | 4.610 | 4.594 |
| 0.8 | $(0.25, 0.25)$ | 1.005 | 1.004 | 1.003 | 1.019 | 4.000 | 3.998 | 3.998 | 4.002 |
| | $(0.25, 1.00)$ | 1.125 | 1.108 | 1.099 | 1.109 | 4.007 | 4.005 | 4.005 | 4.017 |
| | $(0.25, 4.00)$ | 2.413 | 2.200 | 2.174 | 2.148 | 4.614 | 4.511 | 4.502 | 4.590 |
| | $(1.00, 0.25)$ | 1.142 | 1.125 | 1.118 | 1.110 | 4.012 | 4.011 | 4.009 | 4.015 |
| | $(1.00, 1.00)$ | 1.417 | 1.362 | 1.349 | 1.319 | 4.082 | 4.072 | 4.068 | 4.064 |
| | $(1.00, 4.00)$ | 2.929 | 2.597 | 2.579 | 2.453 | 4.974 | 4.793 | 4.781 | 4.783 |
| | $(4.00, 0.25)$ | 2.187 | 2.076 | 2.050 | 2.078 | 4.509 | 4.467 | 4.459 | 4.362 |
| | $(4.00, 1.00)$ | 2.689 | 2.506 | 2.471 | 2.423 | 4.821 | 4.748 | 4.724 | 4.580 |
| | $(4.00, 4.00)$ | 4.548 | 4.016 | 3.962 | 3.847 | 6.281 | 5.920 | 5.892 | 5.825 |

Continued on next page

| ρ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | $(0.25, 0.25)$ | 1.104 | 1.093 | 1.086 | 1.111 | 4.003 | 4.002 | 4.002 | 4.021 |
| | $(0.25, 1.00)$ | 1.836 | 1.729 | 1.707 | 1.674 | 4.208 | 4.173 | 4.169 | 4.206 |
| | $(0.25, 4.00)$ | 6.002 | 5.240 | 5.161 | 5.105 | 7.361 | 6.777 | 6.719 | 6.781 |
| | $(1.00, 0.25)$ | 1.833 | 1.758 | 1.741 | 1.696 | 4.236 | 4.214 | 4.208 | 4.204 |
| | $(1.00, 1.00)$ | 2.863 | 2.630 | 2.611 | 2.522 | 4.818 | 4.704 | 4.697 | 4.656 |
| | $(1.00, 4.00)$ | 7.297 | 6.252 | 6.292 | 6.055 | 8.493 | 7.673 | 7.715 | 7.590 |
| | $(4.00, 0.25)$ | 5.472 | 5.165 | 5.107 | 5.173 | 6.967 | 6.792 | 6.754 | 6.594 |
| | $(4.00, 1.00)$ | 6.828 | 6.217 | 6.153 | 6.134 | 8.139 | 7.689 | 7.656 | 7.450 |
| | $(4.00, 4.00)$ | 11.859 | 10.215 | 9.951 | 9.966 | 12.734 | 11.359 | 11.145 | 11.155 |
| $(S, R_1, R_2) = (2, 4, 2)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 2.000 | 2.000 | 1.999 | 2.000 | 2.000 | 1.999 | 2.000 | 2.000 |
| | $(0.25, 1.00)$ | 2.003 | 2.003 | 2.003 | 2.009 | 2.006 | 2.004 | 2.004 | 2.013 |
| | $(0.25, 4.00)$ | 2.311 | 2.259 | 2.249 | 2.303 | 2.327 | 2.272 | 2.263 | 2.320 |
| | $(1.00, 0.25)$ | 2.007 | 2.007 | 2.007 | 2.009 | 2.011 | 2.011 | 2.009 | 2.013 |
| | $(1.00, 1.00)$ | 2.042 | 2.037 | 2.036 | 2.030 | 2.056 | 2.050 | 2.046 | 2.042 |
| | $(1.00, 4.00)$ | 2.490 | 2.405 | 2.390 | 2.384 | 2.522 | 2.433 | 2.416 | 2.405 |
| | $(4.00, 0.25)$ | 2.194 | 2.190 | 2.184 | 2.145 | 2.231 | 2.222 | 2.211 | 2.179 |
| | $(4.00, 1.00)$ | 2.360 | 2.324 | 2.320 | 2.252 | 2.402 | 2.367 | 2.360 | 2.290 |
| | $(4.00, 4.00)$ | 3.119 | 2.934 | 2.925 | 2.888 | 3.158 | 2.969 | 2.967 | 2.917 |
| 0.8 | $(0.25, 0.25)$ | 2.000 | 2.000 | 2.000 | 2.007 | 2.001 | 2.000 | 2.000 | 2.010 |
| | $(0.25, 1.00)$ | 2.043 | 2.037 | 2.034 | 2.090 | 2.053 | 2.045 | 2.042 | 2.062 |
| | $(0.25, 4.00)$ | 3.055 | 2.890 | 2.872 | 2.890 | 3.080 | 2.911 | 2.893 | 2.920 |
| | $(1.00, 0.25)$ | 2.054 | 2.048 | 2.045 | 2.047 | 2.072 | 2.064 | 2.059 | 2.000 |
| | $(1.00, 1.00)$ | 2.222 | 2.193 | 2.186 | 2.168 | 2.257 | 2.222 | 2.214 | 2.197 |
| | $(1.00, 4.00)$ | 3.508 | 3.237 | 3.222 | 3.150 | 3.549 | 3.274 | 3.260 | 3.183 |
| | $(4.00, 0.25)$ | 2.848 | 2.768 | 2.750 | 2.718 | 2.910 | 2.834 | 2.818 | 2.769 |
| | $(4.00, 1.00)$ | 3.282 | 3.141 | 3.114 | 3.025 | 3.338 | 3.208 | 3.177 | 3.073 |
| | $(4.00, 4.00)$ | 5.019 | 4.550 | 4.503 | 4.423 | 5.060 | 4.592 | 4.552 | 4.456 |
| 0.9 | $(0.25, 0.25)$ | 2.031 | 2.027 | 2.025 | 2.055 | 2.040 | 2.034 | 2.031 | 2.066 |
| | $(0.25, 1.00)$ | 2.511 | 2.440 | 2.427 | 2.424 | 2.539 | 2.462 | 2.449 | 2.464 |
| | $(0.25, 4.00)$ | 6.368 | 5.669 | 5.597 | 5.574 | 6.391 | 5.691 | 5.621 | 5.609 |
| | $(1.00, 0.25)$ | 2.516 | 2.469 | 2.460 | 2.431 | 2.566 | 2.514 | 2.501 | 2.474 |
| | $(1.00, 1.00)$ | 3.382 | 3.201 | 3.188 | 3.118 | 3.436 | 3.249 | 3.234 | 3.165 |

Continued on next page

**TableE.26 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | (1.00, 4.00) | 7.608 | 6.637 | 6.678 | 6.480 | 7.639 | 6.671 | 6.715 | 6.514 |
| | (4.00, 0.25) | 5.847 | 5.582 | 5.528 | 5.533 | 5.909 | 5.672 | 5.625 | 5.578 |
| | (4.00, 1.00) | 7.148 | 6.591 | 6.534 | 6.472 | 7.209 | 6.662 | 6.616 | 6.511 |
| | (4.00, 4.00) | 12.088 | 10.515 | 10.258 | 10.294 | 12.061 | 10.531 | 10.303 | 10.320 |
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 1.999 | 1.999 | 2.000 | 2.000 | 5.998 | 5.998 | 5.998 | 6.000 |
| | (0.25, 1.00) | 2.000 | 2.000 | 2.000 | 2.002 | 5.999 | 5.998 | 5.998 | 6.000 |
| | (0.25, 4.00) | 2.171 | 2.158 | 2.133 | 2.201 | 6.028 | 6.027 | 6.022 | 6.081 |
| | (1.00, 0.25) | 2.003 | 2.003 | 2.002 | 2.002 | 5.998 | 5.998 | 5.996 | 6.000 |
| | (1.00, 1.00) | 2.013 | 2.012 | 2.009 | 2.007 | 5.998 | 5.998 | 5.996 | 6.000 |
| | (1.00, 4.00) | 2.275 | 2.255 | 2.223 | 2.256 | 6.065 | 6.063 | 6.051 | 6.104 |
| | (4.00, 0.25) | 2.090 | 2.091 | 2.080 | 2.046 | 6.007 | 6.014 | 6.016 | 6.003 |
| | (4.00, 1.00) | 2.190 | 2.186 | 2.165 | 2.105 | 6.052 | 6.038 | 6.039 | 6.013 |
| | (4.00, 4.00) | 2.756 | 2.663 | 2.647 | 2.648 | 6.311 | 6.280 | 6.269 | 6.308 |
| 0.8 | (0.25, 0.25) | 1.999 | 2.000 | 1.999 | 2.002 | 5.999 | 5.998 | 5.998 | 6.000 |
| | (0.25, 1.00) | 2.012 | 2.011 | 2.007 | 2.018 | 5.998 | 5.998 | 5.998 | 6.020 |
| | (0.25, 4.00) | 2.697 | 2.647 | 2.590 | 2.645 | 6.222 | 6.209 | 6.192 | 6.302 |
| | (1.00, 0.25) | 2.017 | 2.015 | 2.012 | 2.015 | 5.999 | 5.998 | 5.997 | 6.001 |
| | (1.00, 1.00) | 2.097 | 2.092 | 2.077 | 2.069 | 6.009 | 6.009 | 6.006 | 6.009 |
| | (1.00, 4.00) | 3.011 | 2.919 | 2.863 | 2.854 | 6.396 | 6.362 | 6.338 | 6.417 |
| | (4.00, 0.25) | 2.525 | 2.499 | 2.460 | 2.407 | 6.178 | 6.169 | 6.151 | 6.096 |
| | (4.00, 1.00) | 2.844 | 2.812 | 2.768 | 2.650 | 6.329 | 6.330 | 6.297 | 6.198 |
| | (4.00, 4.00) | 4.297 | 4.015 | 4.001 | 3.963 | 7.278 | 7.121 | 7.128 | 7.142 |
| 0.9 | (0.25, 0.25) | 2.008 | 2.008 | 2.004 | 2.022 | 5.999 | 5.998 | 5.998 | 6.003 |
| | (0.25, 1.00) | 2.266 | 2.257 | 2.216 | 2.229 | 6.039 | 6.037 | 6.031 | 6.053 |
| | (0.25, 4.00) | 5.397 | 5.120 | 5.003 | 4.976 | 7.959 | 7.770 | 7.713 | 7.823 |
| | (1.00, 0.25) | 2.276 | 2.272 | 2.237 | 2.226 | 6.049 | 6.048 | 6.041 | 6.049 |
| | (1.00, 1.00) | 2.881 | 2.847 | 2.785 | 2.734 | 6.291 | 6.279 | 6.259 | 6.242 |
| | (1.00, 4.00) | 6.436 | 6.038 | 5.996 | 5.819 | 8.797 | 8.526 | 8.508 | 8.473 |
| | (4.00, 0.25) | 5.015 | 4.969 | 4.856 | 4.818 | 7.761 | 7.758 | 7.697 | 7.498 |
| | (4.00, 1.00) | 6.124 | 5.929 | 5.756 | 5.709 | 8.590 | 8.502 | 8.382 | 8.187 |
| | (4.00, 4.00) | 10.586 | 9.589 | 9.597 | 9.477 | 12.458 | 11.639 | 11.683 | 11.581 |

Continued on next page

**TableE.26 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T_1^r]$ LoVis | MedVis | HiVis | GI/G/2 | $E[T_2^r]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (4, 6, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 2.999 | 2.999 | 2.999 | 3.000 | 3.999 | 3.999 | 3.999 | 4.000 |
| | $(0.25, 1.00)$ | 2.999 | 2.999 | 3.000 | 3.000 | 3.999 | 3.999 | 3.999 | 4.001 |
| | $(0.25, 4.00)$ | 3.106 | 3.100 | 3.083 | 3.154 | 4.072 | 4.068 | 4.057 | 4.127 |
| | $(1.00, 0.25)$ | 3.002 | 3.002 | 3.002 | 3.001 | 3.999 | 3.999 | 3.998 | 4.000 |
| | $(1.00, 1.00)$ | 3.006 | 3.006 | 3.005 | 3.003 | 4.002 | 4.001 | 4.000 | 4.002 |
| | $(1.00, 4.00)$ | 3.190 | 3.174 | 3.151 | 3.197 | 4.143 | 4.132 | 4.112 | 4.163 |
| | $(4.00, 0.25)$ | 3.053 | 3.054 | 3.047 | 3.020 | 4.040 | 4.040 | 4.037 | 4.014 |
| | $(4.00, 1.00)$ | 3.115 | 3.120 | 3.105 | 3.056 | 4.094 | 4.090 | 4.085 | 4.040 |
| | $(4.00, 4.00)$ | 3.574 | 3.511 | 3.499 | 3.523 | 4.485 | 4.438 | 4.424 | 4.446 |
| 0.8 | $(0.25, 0.25)$ | 2.999 | 2.999 | 2.999 | 3.001 | 3.999 | 3.999 | 3.998 | 4.000 |
| | $(0.25, 1.00)$ | 3.003 | 3.003 | 3.002 | 3.009 | 4.001 | 4.001 | 4.000 | 4.006 |
| | $(0.25, 4.00)$ | 3.518 | 3.482 | 3.439 | 3.518 | 4.401 | 4.372 | 4.342 | 4.440 |
| | $(1.00, 0.25)$ | 3.008 | 3.007 | 3.006 | 3.007 | 4.003 | 4.002 | 4.001 | 4.005 |
| | $(1.00, 1.00)$ | 3.052 | 3.050 | 3.042 | 3.036 | 4.034 | 4.033 | 4.027 | 4.026 |
| | $(1.00, 4.00)$ | 3.796 | 3.715 | 3.672 | 3.695 | 4.657 | 4.585 | 4.548 | 4.595 |
| | $(4.00, 0.25)$ | 3.372 | 3.357 | 3.330 | 3.269 | 4.311 | 4.303 | 4.275 | 4.205 |
| | $(4.00, 1.00)$ | 3.637 | 3.619 | 3.586 | 3.466 | 4.539 | 4.531 | 4.490 | 4.367 |
| | $(4.00, 4.00)$ | 4.970 | 4.706 | 4.699 | 4.690 | 5.742 | 5.513 | 5.513 | 5.496 |
| 0.9 | $(0.25, 0.25)$ | 3.002 | 3.002 | 3.001 | 3.012 | 4.001 | 4.000 | 3.999 | 4.008 |
| | $(0.25, 1.00)$ | 3.160 | 3.154 | 3.131 | 3.149 | 4.106 | 4.101 | 4.085 | 4.114 |
| | $(0.25, 4.00)$ | 5.950 | 5.698 | 5.601 | 5.607 | 6.587 | 6.358 | 6.278 | 6.330 |
| | $(1.00, 0.25)$ | 3.171 | 3.169 | 3.148 | 3.144 | 4.124 | 4.122 | 4.105 | 4.109 |
| | $(1.00, 1.00)$ | 3.649 | 3.626 | 3.580 | 3.539 | 4.517 | 4.497 | 4.460 | 4.430 |
| | $(1.00, 4.00)$ | 6.934 | 6.572 | 6.540 | 6.400 | 7.534 | 7.205 | 7.174 | 7.074 |
| | $(4.00, 0.25)$ | 5.585 | 5.556 | 5.458 | 5.383 | 6.326 | 6.312 | 6.230 | 6.068 |
| | $(4.00, 1.00)$ | 6.627 | 6.459 | 6.300 | 6.228 | 7.286 | 7.161 | 7.020 | 6.861 |
| | $(4.00, 4.00)$ | 10.961 | 10.010 | 10.029 | 9.927 | 11.454 | 10.551 | 10.575 | 10.462 |

Table E.27: Fill Rate and Expected Inventory Level at a Production Facility
$(\lambda_1^r, \quad \lambda_2^r) = (1, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $f^p$ | | | | $E[I^p]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.957 | 0.955 | 0.967 | 0.976 | 1.044 | 1.042 | 1.133 | 1.169 |
| | (0.25, 1.00) | 0.852 | 0.841 | 0.862 | 0.845 | 0.919 | 0.915 | 1.000 | 1.073 |
| | (0.25, 4.00) | 0.730 | 0.700 | 0.716 | 0.722 | 0.822 | 0.814 | 0.878 | 0.994 |
| | (1.00, 0.25) | 0.788 | 0.770 | 0.790 | 0.810 | 0.940 | 0.934 | 0.988 | 1.003 |
| | (1.00, 1.00) | 0.717 | 0.700 | 0.719 | 0.718 | 0.859 | 0.855 | 0.910 | 0.957 |
| | (1.00, 4.00) | 0.648 | 0.619 | 0.631 | 0.662 | 0.790 | 0.783 | 0.826 | 0.929 |
| | (4.00, 0.25) | 0.548 | 0.516 | 0.534 | 0.521 | 0.826 | 0.817 | 0.852 | 0.795 |
| | (4.00, 1.00) | 0.525 | 0.497 | 0.511 | 0.520 | 0.785 | 0.778 | 0.812 | 0.800 |
| | (4.00, 4.00) | 0.518 | 0.486 | 0.497 | 0.557 | 0.747 | 0.741 | 0.774 | 0.830 |
| 0.8 | (0.25, 0.25) | 0.871 | 0.860 | 0.883 | 0.934 | 0.809 | 0.805 | 0.888 | 0.969 |
| | (0.25, 1.00) | 0.704 | 0.680 | 0.705 | 0.680 | 0.664 | 0.658 | 0.727 | 0.802 |
| | (0.25, 4.00) | 0.572 | 0.530 | 0.543 | 0.540 | 0.578 | 0.571 | 0.617 | 0.711 |
| | (1.00, 0.25) | 0.636 | 0.605 | 0.626 | 0.626 | 0.687 | 0.677 | 0.723 | 0.729 |
| | (1.00, 1.00) | 0.556 | 0.528 | 0.546 | 0.545 | 0.608 | 0.601 | 0.644 | 0.688 |
| | (1.00, 4.00) | 0.491 | 0.454 | 0.463 | 0.489 | 0.552 | 0.545 | 0.575 | 0.660 |
| | (4.00, 0.25) | 0.378 | 0.344 | 0.357 | 0.359 | 0.570 | 0.559 | 0.585 | 0.541 |
| | (4.00, 1.00) | 0.362 | 0.334 | 0.346 | 0.363 | 0.536 | 0.530 | 0.555 | 0.548 |
| | (4.00, 4.00) | 0.367 | 0.329 | 0.341 | 0.400 | 0.513 | 0.502 | 0.528 | 0.579 |
| 0.9 | (0.25, 0.25) | 0.649 | 0.609 | 0.639 | 0.632 | 0.493 | 0.480 | 0.537 | 0.595 |
| | (0.25, 1.00) | 0.454 | 0.413 | 0.433 | 0.407 | 0.365 | 0.355 | 0.395 | 0.448 |
| | (0.25, 4.00) | 0.349 | 0.298 | 0.308 | 0.297 | 0.308 | 0.299 | 0.326 | 0.377 |
| | (1.00, 0.25) | 0.399 | 0.357 | 0.374 | 0.360 | 0.385 | 0.371 | 0.399 | 0.396 |
| | (1.00, 1.00) | 0.331 | 0.298 | 0.310 | 0.309 | 0.325 | 0.317 | 0.341 | 0.371 |
| | (1.00, 4.00) | 0.284 | 0.246 | 0.254 | 0.268 | 0.289 | 0.281 | 0.300 | 0.350 |
| | (4.00, 0.25) | 0.194 | 0.171 | 0.178 | 0.184 | 0.293 | 0.288 | 0.299 | 0.275 |
| | (4.00, 1.00) | 0.189 | 0.167 | 0.171 | 0.190 | 0.275 | 0.270 | 0.280 | 0.282 |
| | (4.00, 4.00) | 0.196 | 0.167 | 0.173 | 0.216 | 0.264 | 0.255 | 0.268 | 0.304 |

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**TableE.27 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $f^p$ LoVis | $f^p$ MedVis | $f^p$ HiVis | $f^p$ GI/G/2 | $E[I^p]$ LoVis | $E[I^p]$ MedVis | $E[I^p]$ HiVis | $E[I^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.999 | 0.999 | 1.000 | 1.000 | 2.795 | 2.794 | 3.119 | 3.168 |
| | $(0.25, 1.00)$ | 0.978 | 0.976 | 0.987 | 0.979 | 2.517 | 2.515 | 2.874 | 2.977 |
| | $(0.25, 4.00)$ | 0.856 | 0.833 | 0.859 | 0.859 | 2.125 | 2.105 | 2.402 | 2.626 |
| | $(1.00, 0.25)$ | 0.967 | 0.963 | 0.976 | 0.978 | 2.579 | 2.577 | 2.840 | 2.897 |
| | $(1.00, 1.00)$ | 0.917 | 0.910 | 0.932 | 0.932 | 2.346 | 2.342 | 2.616 | 2.707 |
| | $(1.00, 4.00)$ | 0.805 | 0.781 | 0.806 | 0.826 | 2.027 | 2.016 | 2.242 | 2.479 |
| | $(4.00, 0.25)$ | 0.788 | 0.767 | 0.799 | 0.787 | 2.198 | 2.187 | 2.378 | 2.180 |
| | $(4.00, 1.00)$ | 0.741 | 0.719 | 0.751 | 0.746 | 2.052 | 2.041 | 2.227 | 2.126 |
| | $(4.00, 4.00)$ | 0.681 | 0.652 | 0.676 | 0.719 | 1.862 | 1.851 | 2.019 | 2.159 |
| 0.8 | $(0.25, 0.25)$ | 0.991 | 0.990 | 0.997 | 0.984 | 2.375 | 2.375 | 2.817 | 2.912 |
| | $(0.25, 1.00)$ | 0.909 | 0.898 | 0.930 | 0.923 | 1.990 | 1.983 | 2.375 | 2.525 |
| | $(0.25, 4.00)$ | 0.721 | 0.677 | 0.708 | 0.710 | 1.583 | 1.555 | 1.808 | 2.017 |
| | $(1.00, 0.25)$ | 0.888 | 0.874 | 0.906 | 0.917 | 2.064 | 2.057 | 2.354 | 2.419 |
| | $(1.00, 1.00)$ | 0.796 | 0.775 | 0.813 | 0.814 | 1.790 | 1.777 | 2.046 | 2.151 |
| | $(1.00, 4.00)$ | 0.653 | 0.613 | 0.641 | 0.662 | 1.480 | 1.457 | 1.652 | 1.867 |
| | $(4.00, 0.25)$ | 0.599 | 0.568 | 0.602 | 0.584 | 1.589 | 1.571 | 1.730 | 1.537 |
| | $(4.00, 1.00)$ | 0.558 | 0.530 | 0.559 | 0.551 | 1.465 | 1.452 | 1.601 | 1.504 |
| | $(4.00, 4.00)$ | 0.514 | 0.473 | 0.498 | 0.541 | 1.320 | 1.299 | 1.434 | 1.562 |
| 0.9 | $(0.25, 0.25)$ | 0.899 | 0.885 | 0.932 | 0.929 | 1.675 | 1.667 | 2.169 | 2.327 |
| | $(0.25, 1.00)$ | 0.691 | 0.654 | 0.710 | 0.705 | 1.223 | 1.202 | 1.519 | 1.670 |
| | $(0.25, 4.00)$ | 0.482 | 0.416 | 0.442 | 0.434 | 0.904 | 0.864 | 1.027 | 1.148 |
| | $(1.00, 0.25)$ | 0.662 | 0.625 | 0.676 | 0.686 | 1.289 | 1.268 | 1.523 | 1.561 |
| | $(1.00, 1.00)$ | 0.545 | 0.505 | 0.545 | 0.547 | 1.046 | 1.021 | 1.214 | 1.302 |
| | $(1.00, 4.00)$ | 0.419 | 0.366 | 0.384 | 0.393 | 0.834 | 0.804 | 0.918 | 1.047 |
| | $(4.00, 0.25)$ | 0.342 | 0.310 | 0.334 | 0.332 | 0.864 | 0.839 | 0.938 | 0.804 |
| | $(4.00, 1.00)$ | 0.316 | 0.287 | 0.309 | 0.301 | 0.782 | 0.770 | 0.862 | 0.795 |
| | $(4.00, 4.00)$ | 0.296 | 0.260 | 0.274 | 0.303 | 0.708 | 0.692 | 0.766 | 0.846 |

Table E.28: Fill Rate and Expected Inventory Level at a Production Facility $(\lambda_1^r, \ \lambda_2^r) = (2, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $f^p$ | | | | $E[I^p]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| | | $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.956 | 0.954 | 0.966 | 0.975 | 1.040 | 1.039 | 1.129 | 1.168 |
| | $(0.25, 1.00)$ | 0.852 | 0.841 | 0.862 | 0.844 | 0.920 | 0.916 | 1.001 | 1.072 |
| | $(0.25, 4.00)$ | 0.730 | 0.702 | 0.717 | 0.721 | 0.822 | 0.817 | 0.880 | 0.992 |
| | $(1.00, 0.25)$ | 0.788 | 0.768 | 0.787 | 0.809 | 0.939 | 0.930 | 0.983 | 1.002 |
| | $(1.00, 1.00)$ | 0.718 | 0.699 | 0.718 | 0.717 | 0.859 | 0.854 | 0.909 | 0.956 |
| | $(1.00, 4.00)$ | 0.646 | 0.620 | 0.634 | 0.661 | 0.788 | 0.784 | 0.829 | 0.928 |
| | $(4.00, 0.25)$ | 0.540 | 0.506 | 0.522 | 0.520 | 0.821 | 0.809 | 0.841 | 0.794 |
| | $(4.00, 1.00)$ | 0.518 | 0.492 | 0.506 | 0.519 | 0.781 | 0.775 | 0.809 | 0.799 |
| | $(4.00, 4.00)$ | 0.515 | 0.485 | 0.495 | 0.557 | 0.746 | 0.743 | 0.772 | 0.828 |
| 0.8 | $(0.25, 0.25)$ | 0.872 | 0.863 | 0.887 | 0.935 | 0.809 | 0.811 | 0.895 | 0.970 |
| | $(0.25, 1.00)$ | 0.705 | 0.682 | 0.707 | 0.681 | 0.666 | 0.661 | 0.730 | 0.803 |
| | $(0.25, 4.00)$ | 0.573 | 0.530 | 0.544 | 0.542 | 0.580 | 0.572 | 0.619 | 0.713 |
| | $(1.00, 0.25)$ | 0.636 | 0.610 | 0.630 | 0.627 | 0.688 | 0.684 | 0.729 | 0.730 |
| | $(1.00, 1.00)$ | 0.556 | 0.529 | 0.548 | 0.546 | 0.608 | 0.603 | 0.646 | 0.689 |
| | $(1.00, 4.00)$ | 0.489 | 0.454 | 0.464 | 0.490 | 0.550 | 0.546 | 0.576 | 0.661 |
| | $(4.00, 0.25)$ | 0.370 | 0.340 | 0.354 | 0.359 | 0.565 | 0.561 | 0.586 | 0.542 |
| | $(4.00, 1.00)$ | 0.358 | 0.331 | 0.342 | 0.363 | 0.535 | 0.530 | 0.555 | 0.550 |
| | $(4.00, 4.00)$ | 0.363 | 0.329 | 0.340 | 0.401 | 0.511 | 0.504 | 0.530 | 0.580 |
| 0.9 | $(0.25, 0.25)$ | 0.649 | 0.610 | 0.640 | 0.632 | 0.493 | 0.481 | 0.539 | 0.595 |
| | $(0.25, 1.00)$ | 0.455 | 0.414 | 0.434 | 0.407 | 0.367 | 0.357 | 0.397 | 0.448 |
| | $(0.25, 4.00)$ | 0.350 | 0.299 | 0.309 | 0.297 | 0.310 | 0.300 | 0.327 | 0.377 |
| | $(1.00, 0.25)$ | 0.399 | 0.358 | 0.374 | 0.360 | 0.384 | 0.371 | 0.399 | 0.396 |
| | $(1.00, 1.00)$ | 0.332 | 0.299 | 0.311 | 0.309 | 0.326 | 0.318 | 0.342 | 0.371 |
| | $(1.00, 4.00)$ | 0.284 | 0.247 | 0.253 | 0.268 | 0.289 | 0.283 | 0.298 | 0.350 |
| | $(4.00, 0.25)$ | 0.190 | 0.167 | 0.174 | 0.184 | 0.292 | 0.284 | 0.296 | 0.275 |
| | $(4.00, 1.00)$ | 0.187 | 0.165 | 0.173 | 0.190 | 0.276 | 0.269 | 0.284 | 0.282 |
| | $(4.00, 4.00)$ | 0.191 | 0.167 | 0.173 | 0.216 | 0.260 | 0.258 | 0.272 | 0.304 |

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**TableE.28 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $f^p$ LoVis | $f^p$ MedVis | $f^p$ HiVis | $f^p$ GI/G/2 | $E[I^p]$ LoVis | $E[I^p]$ MedVis | $E[I^p]$ HiVis | $E[I^p]$ GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.999 | 0.999 | 1.000 | 1.000 | 2.788 | 2.789 | 3.115 | 3.167 |
| | $(0.25, 1.00)$ | 0.978 | 0.976 | 0.987 | 0.979 | 2.518 | 2.516 | 2.875 | 2.976 |
| | $(0.25, 4.00)$ | 0.856 | 0.836 | 0.860 | 0.859 | 2.126 | 2.116 | 2.408 | 2.624 |
| | $(1.00, 0.25)$ | 0.967 | 0.962 | 0.975 | 0.978 | 2.579 | 2.570 | 2.833 | 2.895 |
| | $(1.00, 1.00)$ | 0.918 | 0.909 | 0.932 | 0.932 | 2.348 | 2.341 | 2.614 | 2.704 |
| | $(1.00, 4.00)$ | 0.807 | 0.783 | 0.807 | 0.826 | 2.033 | 2.018 | 2.250 | 2.476 |
| | $(4.00, 0.25)$ | 0.786 | 0.764 | 0.794 | 0.786 | 2.186 | 2.169 | 2.353 | 2.177 |
| | $(4.00, 1.00)$ | 0.740 | 0.719 | 0.749 | 0.745 | 2.045 | 2.033 | 2.215 | 2.123 |
| | $(4.00, 4.00)$ | 0.680 | 0.656 | 0.676 | 0.718 | 1.860 | 1.861 | 2.016 | 2.157 |
| 0.8 | $(0.25, 0.25)$ | 0.991 | 0.991 | 0.997 | 0.984 | 2.377 | 2.386 | 2.827 | 2.913 |
| | $(0.25, 1.00)$ | 0.910 | 0.899 | 0.931 | 0.924 | 1.992 | 1.989 | 2.383 | 2.528 |
| | $(0.25, 4.00)$ | 0.721 | 0.678 | 0.708 | 0.711 | 1.584 | 1.555 | 1.814 | 2.021 |
| | $(1.00, 0.25)$ | 0.888 | 0.878 | 0.909 | 0.918 | 2.066 | 2.072 | 2.368 | 2.422 |
| | $(1.00, 1.00)$ | 0.796 | 0.777 | 0.814 | 0.814 | 1.792 | 1.782 | 2.051 | 2.154 |
| | $(1.00, 4.00)$ | 0.656 | 0.617 | 0.643 | 0.663 | 1.489 | 1.470 | 1.660 | 1.870 |
| | $(4.00, 0.25)$ | 0.596 | 0.572 | 0.606 | 0.585 | 1.579 | 1.580 | 1.737 | 1.540 |
| | $(4.00, 1.00)$ | 0.558 | 0.529 | 0.558 | 0.552 | 1.463 | 1.451 | 1.596 | 1.508 |
| | $(4.00, 4.00)$ | 0.513 | 0.478 | 0.498 | 0.542 | 1.319 | 1.313 | 1.435 | 1.566 |
| 0.9 | $(0.25, 0.25)$ | 0.899 | 0.885 | 0.933 | 0.929 | 1.676 | 1.670 | 2.173 | 2.326 |
| | $(0.25, 1.00)$ | 0.693 | 0.655 | 0.712 | 0.705 | 1.224 | 1.205 | 1.524 | 1.670 |
| | $(0.25, 4.00)$ | 0.478 | 0.415 | 0.443 | 0.434 | 0.897 | 0.862 | 1.029 | 1.148 |
| | $(1.00, 0.25)$ | 0.662 | 0.627 | 0.677 | 0.686 | 1.292 | 1.271 | 1.524 | 1.561 |
| | $(1.00, 1.00)$ | 0.546 | 0.506 | 0.546 | 0.547 | 1.049 | 1.025 | 1.217 | 1.302 |
| | $(1.00, 4.00)$ | 0.418 | 0.366 | 0.384 | 0.393 | 0.831 | 0.805 | 0.916 | 1.047 |
| | $(4.00, 0.25)$ | 0.341 | 0.311 | 0.335 | 0.317 | 0.859 | 0.840 | 0.938 | 0.804 |
| | $(4.00, 1.00)$ | 0.317 | 0.287 | 0.307 | 0.301 | 0.783 | 0.769 | 0.854 | 0.795 |
| | $(4.00, 4.00)$ | 0.295 | 0.258 | 0.273 | 0.303 | 0.709 | 0.687 | 0.762 | 0.846 |

Table E.29: Expected Number of Backorders and Expected Time to Fulfill a Backorder at a Production Facility
$(\lambda_1^r, \quad \lambda_2^r) = (1, 1)$

| | | $E[B^p]$ | | | | $E[W^{bp}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | $(cp^2, ca_i^2)$ | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.021 | 0.019 | 0.014 | 0.001 | 0.488 | 0.421 | 0.417 | 0.057 |
| | $(0.25, 1.00)$ | 0.179 | 0.161 | 0.140 | 0.120 | 1.210 | 1.015 | 1.015 | 0.772 |
| | $(0.25, 4.00)$ | 1.210 | 1.055 | 1.002 | 0.922 | 4.483 | 3.516 | 3.525 | 3.312 |
| | $(1.00, 0.25)$ | 0.202 | 0.181 | 0.165 | 0.129 | 0.955 | 0.790 | 0.786 | 0.679 |
| | $(1.00, 1.00)$ | 0.468 | 0.414 | 0.388 | 0.330 | 1.655 | 1.379 | 1.380 | 1.167 |
| | $(1.00, 4.00)$ | 1.627 | 1.414 | 1.379 | 1.150 | 4.622 | 3.708 | 3.739 | 3.404 |
| | $(4.00, 0.25)$ | 0.942 | 0.872 | 0.834 | 0.957 | 2.086 | 1.803 | 1.791 | 1.996 |
| | $(4.00, 1.00)$ | 1.370 | 1.242 | 1.209 | 1.217 | 2.886 | 2.468 | 2.470 | 2.535 |
| | $(4.00, 4.00)$ | 2.845 | 2.454 | 2.402 | 2.220 | 5.905 | 4.770 | 4.787 | 5.015 |
| 0.8 | $(0.25, 0.25)$ | 0.098 | 0.089 | 0.073 | 0.076 | 0.757 | 0.631 | 0.628 | 1.151 |
| | $(0.25, 1.00)$ | 0.557 | 0.499 | 0.460 | 0.394 | 1.882 | 1.559 | 1.559 | 1.230 |
| | $(0.25, 4.00)$ | 2.894 | 2.496 | 2.437 | 2.267 | 6.766 | 5.304 | 5.323 | 4.934 |
| | $(1.00, 0.25)$ | 0.572 | 0.520 | 0.491 | 0.424 | 1.573 | 1.316 | 1.312 | 1.134 |
| | $(1.00, 1.00)$ | 1.196 | 1.054 | 1.016 | 0.910 | 2.691 | 2.232 | 2.236 | 2.000 |
| | $(1.00, 4.00)$ | 3.697 | 3.118 | 3.104 | 2.803 | 7.264 | 5.718 | 5.777 | 5.482 |
| | $(4.00, 0.25)$ | 2.446 | 2.295 | 2.247 | 2.403 | 3.933 | 3.496 | 3.496 | 3.747 |
| | $(4.00, 1.00)$ | 3.325 | 2.972 | 2.942 | 2.949 | 5.214 | 4.463 | 4.497 | 4.625 |
| | $(4.00, 4.00)$ | 6.182 | 5.447 | 5.335 | 5.068 | 9.777 | 8.118 | 8.093 | 8.448 |
| 0.9 | $(0.25, 0.25)$ | 0.533 | 0.491 | 0.453 | 0.392 | 1.516 | 1.256 | 1.254 | 1.066 |
| | $(0.25, 1.00)$ | 2.067 | 1.846 | 1.787 | 1.623 | 3.783 | 3.145 | 3.152 | 2.737 |
| | $(0.25, 4.00)$ | 8.540 | 7.531 | 7.235 | 7.093 | 13.108 | 10.722 | 10.458 | 10.091 |
| | $(1.00, 0.25)$ | 2.028 | 1.854 | 1.807 | 1.702 | 3.379 | 2.885 | 2.885 | 2.658 |
| | $(1.00, 1.00)$ | 3.804 | 3.355 | 3.307 | 3.107 | 5.688 | 4.778 | 4.791 | 4.500 |
| | $(1.00, 4.00)$ | 10.688 | 9.204 | 8.994 | 8.572 | 14.934 | 12.204 | 12.050 | 11.712 |
| | $(4.00, 0.25)$ | 7.856 | 7.093 | 7.206 | 7.377 | 9.745 | 8.570 | 8.765 | 9.044 |
| | $(4.00, 1.00)$ | 10.009 | 8.986 | 9.061 | 8.828 | 12.347 | 10.783 | 10.922 | 10.892 |
| | $(4.00, 4.00)$ | 17.405 | 15.480 | 14.708 | 14.551 | 21.670 | 18.580 | 17.769 | 18.554 |

Continued on next page

**TableE.29 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B^p]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| $(S, R_1, R_2) = (4, 4, 6)$ | | | | | | | | | |
| 0.7 | $(0.25, 0.25)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.480 | 0.413 | 0.410 | 0.057 |
| | $(0.25, 1.00)$ | 0.027 | 0.025 | 0.013 | 0.024 | 1.193 | 1.017 | 1.011 | 1.145 |
| | $(0.25, 4.00)$ | 0.680 | 0.629 | 0.527 | 0.554 | 4.726 | 3.769 | 3.746 | 3.943 |
| | $(1.00, 0.25)$ | 0.030 | 0.028 | 0.018 | 0.023 | 0.917 | 0.768 | 0.759 | 1.038 |
| | $(1.00, 1.00)$ | 0.133 | 0.124 | 0.094 | 0.079 | 1.614 | 1.376 | 1.378 | 1.167 |
| | $(1.00, 4.00)$ | 0.966 | 0.879 | 0.780 | 0.599 | 4.953 | 4.012 | 4.016 | 4.024 |
| | $(4.00, 0.25)$ | 0.436 | 0.422 | 0.364 | 0.342 | 2.059 | 1.812 | 1.811 | 1.602 |
| | $(4.00, 1.00)$ | 0.742 | 0.693 | 0.620 | 0.544 | 2.864 | 2.471 | 2.483 | 2.140 |
| | $(4.00, 4.00)$ | 1.961 | 1.761 | 1.655 | 1.549 | 6.153 | 5.070 | 5.114 | 5.512 |
| 0.8 | $(0.25, 0.25)$ | 0.007 | 0.006 | 0.002 | 0.019 | 0.743 | 0.621 | 0.617 | 1.188 |
| | $(0.25, 1.00)$ | 0.168 | 0.159 | 0.109 | 0.116 | 1.842 | 1.558 | 1.552 | 1.512 |
| | $(0.25, 4.00)$ | 1.914 | 1.806 | 1.635 | 1.574 | 6.862 | 5.587 | 5.596 | 5.418 |
| | $(1.00, 0.25)$ | 0.172 | 0.163 | 0.122 | 0.114 | 1.532 | 1.302 | 1.298 | 1.378 |
| | $(1.00, 1.00)$ | 0.532 | 0.500 | 0.418 | 0.373 | 2.609 | 2.221 | 2.230 | 2.000 |
| | $(1.00, 4.00)$ | 2.599 | 2.366 | 2.203 | 2.009 | 7.482 | 6.104 | 6.137 | 5.942 |
| | $(4.00, 0.25)$ | 1.585 | 1.523 | 1.402 | 1.399 | 3.954 | 3.527 | 3.523 | 3.310 |
| | $(4.00, 1.00)$ | 2.268 | 2.102 | 2.017 | 1.904 | 5.135 | 4.478 | 4.577 | 4.238 |
| | $(4.00, 4.00)$ | 4.793 | 4.454 | 4.220 | 4.051 | 9.847 | 8.433 | 8.404 | 8.835 |
| 0.9 | $(0.25, 0.25)$ | 0.150 | 0.144 | 0.085 | 0.123 | 1.481 | 1.249 | 1.251 | 1.729 |
| | $(0.25, 1.00)$ | 1.126 | 1.085 | 0.911 | 0.845 | 3.647 | 3.132 | 3.143 | 2.868 |
| | $(0.25, 4.00)$ | 6.791 | 6.425 | 6.072 | 5.864 | 13.097 | 10.992 | 10.873 | 10.367 |
| | $(1.00, 0.25)$ | 1.101 | 1.076 | 0.932 | 0.866 | 3.251 | 2.873 | 2.878 | 2.756 |
| | $(1.00, 1.00)$ | 2.468 | 2.365 | 2.182 | 2.039 | 5.422 | 4.773 | 4.788 | 4.500 |
| | $(1.00, 4.00)$ | 8.469 | 7.790 | 7.686 | 7.269 | 14.579 | 12.285 | 12.477 | 11.975 |
| | $(4.00, 0.25)$ | 6.283 | 6.090 | 5.889 | 5.906 | 9.555 | 8.816 | 8.833 | 8.651 |
| | $(4.00, 1.00)$ | 8.265 | 7.708 | 7.318 | 7.340 | 12.070 | 10.808 | 10.600 | 10.503 |
| | $(4.00, 4.00)$ | 14.603 | 13.413 | 13.261 | 13.094 | 20.759 | 18.106 | 18.287 | 18.785 |

Table E.30: Expected Number of Backorders and Expected Time to Fulfill a Backorder at a Production Facility
$(\lambda_1^r, \quad \lambda_2^r) = (2, 1)$

| | | $E[B^p]$ | | | | $E[W^{bp}]$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | $(cp^2, ca_i^2)$ | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| $(S, R_1, R_2) = (2, 2, 4)$ | | | | | | | | | |
| 0.7 | (0.25, 0.25) | 0.021 | 0.020 | 0.014 | 0.001 | 0.327 | 0.282 | 0.279 | 0.039 |
| | (0.25, 1.00) | 0.178 | 0.161 | 0.140 | 0.121 | 0.804 | 0.676 | 0.676 | 0.516 |
| | (0.25, 4.00) | 1.211 | 1.042 | 1.001 | 0.926 | 2.990 | 2.334 | 2.355 | 2.211 |
| | (1.00, 0.25) | 0.203 | 0.185 | 0.169 | 0.130 | 0.637 | 0.532 | 0.529 | 0.453 |
| | (1.00, 1.00) | 0.467 | 0.415 | 0.389 | 0.331 | 1.102 | 0.920 | 0.921 | 0.780 |
| | (1.00, 4.00) | 1.652 | 1.404 | 1.355 | 1.155 | 3.114 | 2.462 | 2.467 | 2.274 |
| | (4.00, 0.25) | 0.952 | 0.893 | 0.859 | 0.961 | 1.381 | 1.206 | 1.199 | 1.335 |
| | (4.00, 1.00) | 1.384 | 1.246 | 1.219 | 1.223 | 1.915 | 1.635 | 1.646 | 1.694 |
| | (4.00, 4.00) | 2.852 | 2.433 | 2.412 | 2.229 | 3.926 | 3.153 | 3.184 | 3.351 |
| 0.8 | (0.25, 0.25) | 0.097 | 0.085 | 0.070 | 0.076 | 0.505 | 0.415 | 0.414 | 0.774 |
| | (0.25, 1.00) | 0.552 | 0.492 | 0.454 | 0.391 | 1.247 | 1.031 | 1.033 | 0.818 |
| | (0.25, 4.00) | 2.858 | 2.477 | 2.419 | 2.256 | 4.465 | 3.515 | 3.534 | 3.281 |
| | (1.00, 0.25) | 0.570 | 0.508 | 0.480 | 0.421 | 1.044 | 0.868 | 0.865 | 0.753 |
| | (1.00, 1.00) | 1.187 | 1.048 | 1.009 | 0.905 | 1.783 | 1.485 | 1.487 | 1.329 |
| | (1.00, 4.00) | 3.726 | 3.145 | 3.108 | 2.789 | 4.864 | 3.845 | 3.862 | 3.645 |
| | (4.00, 0.25) | 2.485 | 2.266 | 2.211 | 2.392 | 2.630 | 2.291 | 2.284 | 2.489 |
| | (4.00, 1.00) | 3.315 | 2.995 | 2.925 | 2.935 | 3.444 | 2.984 | 2.968 | 3.073 |
| | (4.00, 4.00) | 6.254 | 5.371 | 5.276 | 5.045 | 6.551 | 5.345 | 5.335 | 5.615 |
| 0.9 | (0.25, 0.25) | 0.534 | 0.490 | 0.450 | 0.392 | 1.015 | 0.837 | 0.834 | 0.711 |
| | (0.25, 1.00) | 2.043 | 1.833 | 1.773 | 1.623 | 2.500 | 2.086 | 2.089 | 1.824 |
| | (0.25, 4.00) | 8.577 | 7.388 | 7.257 | 7.093 | 8.792 | 7.029 | 7.005 | 6.727 |
| | (1.00, 0.25) | 2.019 | 1.848 | 1.804 | 1.702 | 2.240 | 1.920 | 1.923 | 1.772 |
| | (1.00, 1.00) | 3.769 | 3.342 | 3.299 | 3.108 | 3.761 | 3.179 | 3.191 | 3.000 |
| | (1.00, 4.00) | 10.539 | 8.957 | 9.014 | 8.572 | 9.811 | 7.938 | 8.040 | 7.808 |
| | (4.00, 0.25) | 7.741 | 7.265 | 7.173 | 7.377 | 6.377 | 5.815 | 5.789 | 6.029 |
| | (4.00, 1.00) | 9.878 | 8.895 | 8.782 | 8.828 | 8.104 | 7.103 | 7.086 | 7.261 |
| | (4.00, 4.00) | 17.379 | 14.948 | 14.551 | 14.551 | 14.310 | 11.959 | 11.739 | 12.370 |

Continued on next page

**TableE.30 – continued from previous page**

| $\rho$ | $(cp^2, ca_i^2)$ | $E[B^p]$ LoVis | MedVis | HiVis | GI/G/2 | $E[W^{bp}]$ LoVis | MedVis | HiVis | GI/G/2 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $(S, R_1, R_2) = (4, 4, 6)$ | | | | | |
| 0.7 | (0.25, 0.25) | 0.000 | 0.000 | 0.000 | 0.000 | 0.322 | 0.281 | 0.283 | 0.039 |
| | (0.25, 1.00) | 0.026 | 0.024 | 0.013 | 0.024 | 0.789 | 0.673 | 0.676 | 0.764 |
| | (0.25, 4.00) | 0.673 | 0.618 | 0.523 | 0.557 | 3.117 | 2.504 | 2.494 | 2.631 |
| | (1.00, 0.25) | 0.031 | 0.029 | 0.019 | 0.023 | 0.621 | 0.523 | 0.517 | 0.693 |
| | (1.00, 1.00) | 0.132 | 0.125 | 0.094 | 0.080 | 1.072 | 0.917 | 0.918 | 0.780 |
| | (1.00, 4.00) | 0.942 | 0.868 | 0.769 | 0.703 | 3.254 | 2.662 | 2.660 | 2.687 |
| | (4.00, 0.25) | 0.441 | 0.428 | 0.373 | 0.344 | 1.378 | 1.207 | 1.210 | 1.072 |
| | (4.00, 1.00) | 0.739 | 0.697 | 0.627 | 0.547 | 1.898 | 1.656 | 1.662 | 1.431 |
| | (4.00, 4.00) | 1.942 | 1.718 | 1.661 | 1.557 | 4.054 | 3.332 | 3.414 | 3.682 |
| 0.8 | (0.25, 0.25) | 0.007 | 0.006 | 0.002 | 0.019 | 0.496 | 0.413 | 0.410 | 0.790 |
| | (0.25, 1.00) | 0.167 | 0.155 | 0.106 | 0.115 | 1.227 | 1.029 | 1.028 | 1.006 |
| | (0.25, 4.00) | 1.923 | 1.783 | 1.619 | 1.565 | 4.591 | 3.686 | 3.696 | 3.604 |
| | (1.00, 0.25) | 0.171 | 0.158 | 0.118 | 0.113 | 1.020 | 0.860 | 0.860 | 0.917 |
| | (1.00, 1.00) | 0.530 | 0.496 | 0.415 | 0.370 | 1.737 | 1.482 | 1.488 | 1.329 |
| | (1.00, 4.00) | 2.543 | 2.324 | 2.176 | 1.998 | 4.935 | 4.042 | 4.066 | 3.952 |
| | (4.00, 0.25) | 1.575 | 1.473 | 1.354 | 1.390 | 2.600 | 2.298 | 2.292 | 2.232 |
| | (4.00, 1.00) | 2.226 | 2.121 | 2.000 | 1.893 | 3.357 | 3.007 | 3.017 | 2.815 |
| | (4.00, 4.00) | 4.791 | 4.253 | 4.200 | 4.031 | 6.566 | 5.437 | 5.594 | 5.873 |
| 0.9 | (0.25, 0.25) | 0.150 | 0.145 | 0.084 | 0.123 | 0.986 | 0.837 | 0.835 | 1.153 |
| | (0.25, 1.00) | 1.113 | 1.073 | 0.900 | 0.845 | 2.412 | 2.075 | 2.084 | 1.912 |
| | (0.25, 4.00) | 6.765 | 6.282 | 6.037 | 5.864 | 8.644 | 7.152 | 7.219 | 6.912 |
| | (1.00, 0.25) | 1.103 | 1.070 | 0.929 | 0.866 | 2.177 | 1.911 | 1.917 | 1.838 |
| | (1.00, 1.00) | 2.451 | 2.355 | 2.175 | 2.039 | 3.597 | 3.175 | 3.192 | 3.000 |
| | (1.00, 4.00) | 8.532 | 7.778 | 7.668 | 7.290 | 9.779 | 8.183 | 8.299 | 7.983 |
| | (4.00, 0.25) | 6.163 | 6.020 | 5.794 | 5.906 | 6.238 | 5.825 | 5.812 | 5.767 |
| | (4.00, 1.00) | 8.045 | 7.613 | 7.310 | 7.340 | 7.849 | 7.120 | 7.039 | 7.002 |
| | (4.00, 4.00) | 15.004 | 13.367 | 13.319 | 13.094 | 14.200 | 12.014 | 12.213 | 12.524 |

Table E.31: Expected Time Spent by an Order at a Production Facility $(\lambda_1^r,\ \ \lambda_2^r) = (1,\ 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T^p]$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(S, R_1, R_2) = (2, 2, 4)$ | | | | $(S, R_1, R_2) = (4, 4, 6)$ | | | |
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.7 | $(0.25, 0.25)$ | 2.021 | 2.019 | 2.013 | 2.001 | 3.999 | 3.999 | 3.999 | 4.000 |
| | $(0.25, 1.00)$ | 2.179 | 2.160 | 2.139 | 2.120 | 4.026 | 4.022 | 4.012 | 4.024 |
| | $(0.25, 4.00)$ | 3.209 | 3.052 | 3.000 | 2.922 | 4.682 | 4.626 | 4.530 | 4.554 |
| | $(1.00, 0.25)$ | 2.203 | 2.181 | 2.166 | 2.129 | 4.031 | 4.028 | 4.017 | 4.023 |
| | $(1.00, 1.00)$ | 2.468 | 2.414 | 2.388 | 2.330 | 4.137 | 4.125 | 4.094 | 4.079 |
| | $(1.00, 4.00)$ | 3.627 | 3.414 | 3.377 | 3.150 | 4.970 | 4.879 | 4.781 | 4.699 |
| | $(4.00, 0.25)$ | 2.941 | 2.871 | 2.836 | 2.957 | 4.437 | 4.422 | 4.368 | 4.342 |
| | $(4.00, 1.00)$ | 3.371 | 3.242 | 3.206 | 3.217 | 4.742 | 4.695 | 4.620 | 4.544 |
| | $(4.00, 4.00)$ | 4.845 | 4.454 | 4.410 | 4.220 | 5.960 | 5.767 | 5.660 | 5.549 |
| 0.8 | $(0.25, 0.25)$ | 2.098 | 2.087 | 2.073 | 2.076 | 4.006 | 4.005 | 4.000 | 4.019 |
| | $(0.25, 1.00)$ | 2.556 | 2.498 | 2.459 | 2.394 | 4.166 | 4.156 | 4.108 | 4.116 |
| | $(0.25, 4.00)$ | 4.893 | 4.494 | 4.431 | 4.267 | 5.912 | 5.807 | 5.634 | 5.574 |
| | $(1.00, 0.25)$ | 2.574 | 2.519 | 2.491 | 2.424 | 4.172 | 4.163 | 4.123 | 4.114 |
| | $(1.00, 1.00)$ | 3.197 | 3.054 | 3.015 | 2.910 | 4.531 | 4.498 | 4.416 | 4.373 |
| | $(1.00, 4.00)$ | 5.701 | 5.123 | 5.099 | 4.803 | 6.601 | 6.363 | 6.204 | 6.009 |
| | $(4.00, 0.25)$ | 4.448 | 4.294 | 4.250 | 4.403 | 5.585 | 5.522 | 5.401 | 5.399 |
| | $(4.00, 1.00)$ | 5.325 | 4.974 | 4.942 | 4.949 | 6.271 | 6.105 | 6.019 | 5.904 |
| | $(4.00, 4.00)$ | 8.192 | 7.448 | 7.334 | 7.068 | 8.788 | 8.443 | 8.217 | 8.051 |
| 0.9 | $(0.25, 0.25)$ | 2.531 | 2.491 | 2.453 | 2.392 | 4.148 | 4.141 | 4.084 | 4.123 |
| | $(0.25, 1.00)$ | 4.064 | 3.845 | 3.784 | 3.623 | 5.124 | 5.082 | 4.908 | 4.845 |
| | $(0.25, 4.00)$ | 10.539 | 9.525 | 9.232 | 9.093 | 10.793 | 10.426 | 10.068 | 9.864 |
| | $(1.00, 0.25)$ | 4.031 | 3.855 | 3.806 | 3.702 | 5.100 | 5.076 | 4.932 | 4.866 |
| | $(1.00, 1.00)$ | 5.807 | 5.357 | 5.307 | 5.108 | 6.467 | 6.365 | 6.178 | 6.039 |
| | $(1.00, 4.00)$ | 12.699 | 11.204 | 10.987 | 10.572 | 12.465 | 11.784 | 11.683 | 11.269 |
| | $(4.00, 0.25)$ | 9.854 | 9.105 | 9.208 | 9.377 | 10.291 | 10.085 | 9.889 | 9.906 |
| | $(4.00, 1.00)$ | 12.015 | 10.988 | 11.050 | 10.828 | 12.257 | 11.711 | 11.332 | 11.340 |
| | $(4.00, 4.00)$ | 19.423 | 17.477 | 16.702 | 16.551 | 18.615 | 17.408 | 17.287 | 17.094 |

Table E.32: Expected Time Spent by an Order at a Production Facility
$(\lambda_1^r, \quad \lambda_2^r) = (2, 1)$

| $\rho$ | $(cp^2, ca_i^2)$ | $E[T^p]$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(S, R_1, R_2) = (2, 2, 4)$ | | | | $(S, R_1, R_2) = (4, 4, 6)$ | | | |
| | | LoVis | MedVis | HiVis | GI/G/2 | LoVis | MedVis | HiVis | GI/G/2 |
| 0.7 | $(0.25, 0.25)$ | 1.347 | 1.347 | 1.341 | 1.334 | 2.667 | 2.667 | 2.667 | 2.667 |
| | $(0.25, 1.00)$ | 1.452 | 1.440 | 1.426 | 1.414 | 2.683 | 2.681 | 2.675 | 2.683 |
| | $(0.25, 4.00)$ | 2.139 | 2.028 | 2.000 | 1.951 | 3.115 | 3.077 | 3.013 | 3.038 |
| | $(1.00, 0.25)$ | 1.469 | 1.457 | 1.446 | 1.420 | 2.688 | 2.688 | 2.680 | 2.682 |
| | $(1.00, 1.00)$ | 1.645 | 1.612 | 1.593 | 1.554 | 2.756 | 2.751 | 2.730 | 2.720 |
| | $(1.00, 4.00)$ | 2.434 | 2.270 | 2.237 | 2.103 | 3.297 | 3.245 | 3.181 | 3.135 |
| | $(4.00, 0.25)$ | 1.968 | 1.930 | 1.907 | 1.974 | 2.959 | 2.954 | 2.917 | 2.896 |
| | $(4.00, 1.00)$ | 2.257 | 2.165 | 2.148 | 2.148 | 3.161 | 3.133 | 3.084 | 3.031 |
| | $(4.00, 4.00)$ | 3.238 | 2.961 | 2.943 | 2.819 | 3.966 | 3.819 | 3.776 | 3.705 |
| 0.8 | $(0.25, 0.25)$ | 1.398 | 1.389 | 1.380 | 1.384 | 2.671 | 2.670 | 2.667 | 2.679 |
| | $(0.25, 1.00)$ | 1.701 | 1.662 | 1.636 | 1.594 | 2.777 | 2.770 | 2.736 | 2.744 |
| | $(0.25, 4.00)$ | 3.237 | 2.985 | 2.945 | 2.838 | 3.946 | 3.855 | 3.742 | 3.710 |
| | $(1.00, 0.25)$ | 1.714 | 1.673 | 1.653 | 1.614 | 2.782 | 2.772 | 2.746 | 2.742 |
| | $(1.00, 1.00)$ | 2.124 | 2.033 | 2.005 | 1.937 | 3.021 | 2.999 | 2.944 | 2.913 |
| | $(1.00, 4.00)$ | 3.818 | 3.434 | 3.403 | 3.193 | 4.362 | 4.218 | 4.118 | 3.999 |
| | $(4.00, 0.25)$ | 2.991 | 2.846 | 2.812 | 2.928 | 3.716 | 3.652 | 3.573 | 3.593 |
| | $(4.00, 1.00)$ | 3.546 | 3.331 | 3.288 | 3.290 | 4.153 | 4.085 | 4.002 | 3.928 |
| | $(4.00, 4.00)$ | 5.510 | 4.919 | 4.859 | 4.697 | 5.866 | 5.505 | 5.478 | 5.354 |
| 0.9 | $(0.25, 0.25)$ | 1.689 | 1.660 | 1.633 | 1.595 | 2.766 | 2.763 | 2.722 | 2.749 |
| | $(0.25, 1.00)$ | 2.694 | 2.555 | 2.514 | 2.416 | 3.407 | 3.381 | 3.268 | 3.230 |
| | $(0.25, 4.00)$ | 7.050 | 6.259 | 6.171 | 6.062 | 7.175 | 6.853 | 6.691 | 6.576 |
| | $(1.00, 0.25)$ | 2.680 | 2.566 | 2.536 | 2.468 | 3.403 | 3.380 | 3.287 | 3.244 |
| | $(1.00, 1.00)$ | 3.846 | 3.563 | 3.534 | 3.405 | 4.301 | 4.237 | 4.118 | 4.026 |
| | $(1.00, 4.00)$ | 8.358 | 7.308 | 7.344 | 7.048 | 8.363 | 7.859 | 7.778 | 7.512 |
| | $(4.00, 0.25)$ | 6.499 | 6.181 | 6.119 | 6.251 | 6.780 | 6.682 | 6.535 | 6.604 |
| | $(4.00, 1.00)$ | 7.925 | 7.267 | 7.199 | 7.218 | 8.033 | 7.745 | 7.543 | 7.560 |
| | $(4.00, 4.00)$ | 12.913 | 11.301 | 11.038 | 11.034 | 12.679 | 11.585 | 11.550 | 11.396 |

# APPENDIX F

# QUEUEING RESULTS FOR SCN CONFIGURATIONS WITH TRANSIT TIME

This appendix presents the results for the analytical models developed in Chapter 9 for the 1R/2P and 2R/2P SCN configurations with HiVis in the presence of transit delays. Sections F.1 and F.2 present the results for validating the analytical models developed for the 1R/2P and 2R/2P SCN configurations, respectively.

## F.1 Validation of the Analytical Model for the 1R/2P SCN Configuration with Transit Time

In this section, we present the results for the validation of the analytical model developed for the 1R/2P SCN configuration with transit delays. Tables F.1 and F.2 present the results at the retail stores, while Tables F.3 and F.4 present the results at a production facility.

Table F.1: Performance Measures at the Retail Store
(Base-stock settings - (2, 4) and (4, 2))

| | | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | | | | | $(S,R) = (2,4)$ | | | | | | |
| 3 | (0.25, 0.25) | 0.142 | 0.189 | 1.249 | 1.482 | 0.085 | 0.236 | 1.454 | 1.828 | 3.248 | 3.482 |
| | (0.25, 1.00) | 0.105 | 0.152 | 2.896 | 2.878 | 0.063 | 0.189 | 3.235 | 3.393 | 4.893 | 4.878 |
| | (0.25, 4.00) | 0.061 | 0.094 | 13.210 | 12.845 | 0.036 | 0.117 | 14.059 | 14.174 | 15.202 | 14.845 |
| | (1.00, 0.25) | 0.146 | 0.149 | 3.021 | 2.918 | 0.182 | 0.185 | 3.536 | 3.427 | 5.020 | 4.918 |
| | (1.00, 1.00) | 0.113 | 0.120 | 5.496 | 5.228 | 0.141 | 0.149 | 6.197 | 5.940 | 7.495 | 7.228 |
| | (1.00, 4.00) | 0.073 | 0.085 | 16.710 | 15.643 | 0.091 | 0.106 | 18.027 | 17.097 | 18.706 | 17.643 |
| | (4.00, 0.25) | 0.064 | 0.072 | 13.456 | 12.901 | 0.253 | 0.090 | 14.371 | 13.900 | 15.459 | 14.900 |
| | (4.00, 1.00) | 0.055 | 0.067 | 16.887 | 15.765 | 0.217 | 0.084 | 17.868 | 16.905 | 18.895 | 17.765 |
| | (4.00, 4.00) | 0.042 | 0.065 | 28.882 | 27.269 | 0.164 | 0.081 | 30.129 | 29.174 | 30.879 | 29.269 |
| 6 | (0.25, 0.25) | 0.000 | 0.016 | 4.166 | 4.265 | 0.000 | 0.019 | 4.165 | 4.336 | 6.164 | 6.265 |
| | (0.25, 1.00) | 0.000 | 0.013 | 5.835 | 5.704 | 0.000 | 0.015 | 5.833 | 5.780 | 7.831 | 7.704 |
| | (0.25, 4.00) | 0.000 | 0.008 | 16.176 | 15.737 | 0.000 | 0.009 | 16.169 | 15.866 | 18.166 | 17.737 |
| | (1.00, 0.25) | 0.013 | 0.013 | 5.854 | 5.747 | 0.014 | 0.015 | 5.927 | 5.822 | 7.853 | 7.747 |
| | (1.00, 1.00) | 0.010 | 0.010 | 8.367 | 8.090 | 0.011 | 0.012 | 8.448 | 8.175 | 10.366 | 10.090 |
| | (1.00, 4.00) | 0.006 | 0.007 | 19.626 | 18.546 | 0.007 | 0.008 | 19.746 | 18.683 | 21.622 | 20.546 |
| | (4.00, 0.25) | 0.028 | 0.006 | 16.331 | 15.818 | 0.129 | 0.007 | 16.801 | 15.917 | 18.335 | 17.818 |
| | (4.00, 1.00) | 0.024 | 0.006 | 19.779 | 18.688 | 0.111 | 0.007 | 20.272 | 18.797 | 21.789 | 20.688 |
| | (4.00, 4.00) | 0.018 | 0.006 | 31.801 | 30.194 | 0.084 | 0.006 | 32.381 | 30.365 | 33.798 | 32.194 |
| | | | | | $(S,R) = (4,2)$ | | | | | | |
| 3 | (0.25, 0.25) | 0.469 | 0.416 | 0.460 | 0.757 | 0.427 | 0.660 | 0.865 | 1.295 | 3.459 | 3.757 |
| | (0.25, 1.00) | 0.405 | 0.374 | 1.310 | 1.495 | 0.367 | 0.592 | 2.198 | 2.387 | 4.308 | 4.495 |
| | (0.25, 4.00) | 0.248 | 0.245 | 10.204 | 1.131 | 0.224 | 0.387 | 13.563 | 13.418 | 13.197 | 13.131 |
| | (1.00, 0.25) | 0.368 | 0.372 | 1.525 | 1.490 | 0.582 | 0.588 | 2.412 | 2.371 | 4.525 | 4.490 |
| | (1.00, 1.00) | 0.308 | 0.316 | 3.353 | 3.176 | 0.485 | 0.500 | 4.843 | 4.646 | 6.353 | 6.176 |
| | (1.00, 4.00) | 0.201 | 0.224 | 13.181 | 12.725 | 0.316 | 0.354 | 16.484 | 16.402 | 16.180 | 15.725 |
| | (4.00, 0.25) | 0.150 | 0.211 | 10.469 | 9.755 | 0.570 | 0.332 | 12.303 | 12.359 | 13.463 | 12.755 |
| | (4.00, 1.00) | 0.132 | 0.194 | 13.697 | 12.485 | 0.503 | 0.306 | 15.775 | 15.488 | 16.703 | 15.485 |
| | (4.00, 4.00) | 0.102 | 0.172 | 24.017 | 23.867 | 0.386 | 0.272 | 26.753 | 28.841 | 27.028 | 26.867 |
| 6 | (0.25, 0.25) | 0.007 | 0.061 | 3.037 | 3.177 | 0.003 | 0.080 | 3.057 | 3.382 | 6.035 | 6.177 |
| | (0.25, 1.00) | 0.006 | 0.054 | 3.947 | 3.975 | 0.003 | 0.072 | 3.968 | 4.203 | 6.944 | 6.975 |
| | (0.25, 4.00) | 0.004 | 0.036 | 12.983 | 12.791 | 0.002 | 0.047 | 13.023 | 13.262 | 15.975 | 15.791 |
| | (1.00, 0.25) | 0.053 | 0.054 | 4.014 | 3.973 | 0.070 | 0.071 | 4.240 | 4.200 | 7.013 | 6.973 |
| | (1.00, 1.00) | 0.045 | 0.046 | 5.927 | 5.736 | 0.058 | 0.061 | 6.201 | 6.012 | 8.926 | 8.736 |
| | (1.00, 4.00) | 0.029 | 0.033 | 15.903 | 15.413 | 0.038 | 0.043 | 16.374 | 15.931 | 18.902 | 18.413 |
| | (4.00, 0.25) | 0.061 | 0.030 | 13.192 | 12.463 | 0.291 | 0.040 | 14.038 | 12.853 | 16.184 | 15.463 |
| | (4.00, 1.00) | 0.054 | 0.028 | 16.451 | 15.216 | 0.257 | 0.037 | 17.385 | 15.654 | 19.458 | 18.216 |
| | (4.00, 4.00) | 0.041 | 0.025 | 26.827 | 26.627 | 0.197 | 0.033 | 27.986 | 27.309 | 29.839 | 29.627 |

Table F.2: Performance Measures at the Retail Store
(Base-stock settings - (3, 6) and (6, 3))

| | | $f^r$ | | $E[B^r]$ | | $E[I^r]$ | | $E[W^{br}]$ | | $E[T^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | | | | | $(S, R) = (3, 6)$ | | | | | | |
| 3 | (0.25, 0.25) | 0.613 | 0.546 | 0.688 | 0.874 | 0.821 | 1.090 | 1.776 | 1.928 | 4.685 | 4.874 |
| | (0.25, 1.00) | 0.398 | 0.376 | 3.080 | 2.984 | 0.518 | 0.737 | 5.110 | 4.780 | 7.077 | 6.984 |
| | (0.25, 4.00) | 0.246 | 0.240 | 13.897 | 13.661 | 0.325 | 0.475 | 18.422 | 17.966 | 17.889 | 17.661 |
| | (1.00, 0.25) | 0.345 | 0.356 | 3.260 | 3.097 | 0.675 | 0.694 | 4.973 | 4.806 | 7.259 | 7.097 |
| | (1.00, 1.00) | 0.258 | 0.285 | 6.114 | 5.773 | 0.499 | 0.557 | 8.240 | 8.073 | 10.113 | 9.773 |
| | (1.00, 4.00) | 0.179 | 0.216 | 17.531 | 16.573 | 0.349 | 0.429 | 21.356 | 21.148 | 21.528 | 20.573 |
| | (4.00, 0.25) | 0.126 | 0.161 | 14.676 | 14.067 | 0.521 | 0.313 | 16.794 | 16.758 | 18.676 | 18.067 |
| | (4.00, 1.00) | 0.109 | 0.157 | 18.155 | 16.964 | 0.447 | 0.309 | 20.364 | 20.126 | 22.140 | 20.964 |
| | (4.00, 4.00) | 0.093 | 0.169 | 28.971 | 28.439 | 0.370 | 0.336 | 31.992 | 34.227 | 33.026 | 32.439 |
| 6 | (0.25, 0.25) | 0.039 | 0.123 | 2.892 | 2.972 | 0.024 | 0.188 | 3.008 | 3.390 | 6.889 | 6.972 |
| | (0.25, 1.00) | 0.024 | 0.082 | 5.578 | 5.372 | 0.015 | 0.125 | 5.710 | 5.850 | 9.574 | 9.372 |
| | (0.25, 4.00) | 0.015 | 0.053 | 16.583 | 16.268 | 0.009 | 0.082 | 16.829 | 17.186 | 20.573 | 20.267 |
| | (1.00, 0.25) | 0.075 | 0.077 | 5.699 | 5.520 | 0.114 | 0.117 | 6.162 | 5.982 | 9.698 | 9.520 |
| | (1.00, 1.00) | 0.055 | 0.062 | 8.699 | 8.310 | 0.083 | 0.094 | 9.206 | 8.861 | 12.697 | 12.310 |
| | (1.00, 4.00) | 0.039 | 0.048 | 20.241 | 19.218 | 0.059 | 0.074 | 21.054 | 20.192 | 24.237 | 23.218 |
| | (4.00, 0.25) | 0.055 | 0.035 | 17.429 | 16.807 | 0.274 | 0.053 | 18.439 | 17.415 | 21.429 | 20.807 |
| | (4.00, 1.00) | 0.047 | 0.035 | 20.944 | 19.708 | 0.233 | 0.053 | 21.963 | 20.413 | 24.927 | 23.708 |
| | (4.00, 4.00) | 0.039 | 0.038 | 31.788 | 31.160 | 0.193 | 0.058 | 33.131 | 32.387 | 35.848 | 35.160 |
| | | | | | $(S, R) = (6, 3)$ | | | | | | |
| 3 | (0.25, 0.25) | 0.943 | 0.867 | 0.119 | 0.259 | 2.743 | 2.843 | 2.070 | 1.953 | 6.116 | 6.259 |
| | (0.25, 1.00) | 0.730 | 0.695 | 1.553 | 1.531 | 1.998 | 2.198 | 5.737 | 5.015 | 7.549 | 7.531 |
| | (0.25, 4.00) | 0.435 | 0.432 | 11.516 | 11.252 | 1.181 | 1.362 | 20.348 | 19.815 | 17.507 | 17.252 |
| | (1.00, 0.25) | 0.664 | 0.681 | 1.654 | 1.563 | 2.086 | 2.141 | 4.911 | 4.895 | 7.654 | 7.563 |
| | (1.00, 1.00) | 0.527 | 0.551 | 3.989 | 3.755 | 1.620 | 1.720 | 8.422 | 8.371 | 9.988 | 9.755 |
| | (1.00, 4.00) | 0.349 | 0.392 | 14.497 | 14.014 | 1.072 | 1.234 | 22.251 | 23.050 | 20.495 | 20.014 |
| | (4.00, 0.25) | 0.277 | 0.333 | 11.699 | 11.227 | 1.260 | 1.015 | 16.169 | 16.824 | 17.704 | 17.227 |
| | (4.00, 1.00) | 0.244 | 0.312 | 15.297 | 14.068 | 1.103 | 0.959 | 20.236 | 20.445 | 21.307 | 20.068 |
| | (4.00, 4.00) | 0.197 | 0.301 | 27.163 | 25.552 | 0.872 | 0.951 | 33.805 | 36.578 | 33.152 | 31.553 |
| 6 | (0.25, 0.25) | 0.426 | 0.413 | 0.864 | 1.304 | 0.487 | 0.888 | 1.503 | 2.223 | 6.861 | 7.304 |
| | (0.25, 1.00) | 0.298 | 0.316 | 2.892 | 3.003 | 0.336 | 0.670 | 4.115 | 4.388 | 8.888 | 9.003 |
| | (0.25, 4.00) | 0.175 | 0.195 | 13.534 | 13.305 | 0.198 | 0.414 | 16.400 | 16.533 | 19.524 | 19.305 |
| | (1.00, 0.25) | 0.299 | 0.307 | 3.202 | 3.071 | 0.634 | 0.650 | 4.566 | 4.430 | 9.201 | 9.071 |
| | (1.00, 1.00) | 0.231 | 0.246 | 5.853 | 5.554 | 0.483 | 0.519 | 7.604 | 7.364 | 11.851 | 11.554 |
| | (1.00, 4.00) | 0.153 | 0.177 | 16.745 | 16.155 | 0.320 | 0.375 | 19.752 | 19.624 | 22.743 | 22.155 |
| | (4.00, 0.25) | 0.130 | 0.144 | 14.121 | 13.514 | 0.684 | 0.302 | 16.226 | 15.784 | 20.127 | 19.514 |
| | (4.00, 1.00) | 0.113 | 0.136 | 17.790 | 16.396 | 0.597 | 0.287 | 20.071 | 18.983 | 23.801 | 22.396 |
| | (4.00, 4.00) | 0.089 | 0.136 | 29.761 | 27.891 | 0.469 | 0.289 | 32.666 | 32.294 | 35.748 | 33.891 |

Table F.3: Performance Measures at a Production Facility
(Base-stock settings - (2, 4) and (4, 2))

| | | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | | | | | | $(S, R) = (2, 4)$ | | | | | |
| | (0.25, 0.25) | 0.936 | 0.929 | 0.082 | 0.123 | 2.204 | 2.326 | 2.540 | 3.459 | 8.159 | 8.246 |
| | (0.25, 1.00) | 0.713 | 0.705 | 0.916 | 0.845 | 1.538 | 1.670 | 6.363 | 5.735 | 9.822 | 9.689 |
| | (0.25, 4.00) | 0.442 | 0.434 | 6.086 | 5.864 | 1.034 | 1.148 | 21.737 | 20.735 | 20.136 | 19.728 |
| | (1.00, 0.25) | 0.678 | 0.686 | 0.919 | 0.866 | 1.527 | 1.561 | 5.709 | 5.513 | 9.838 | 9.732 |
| 3 | (1.00, 1.00) | 0.544 | 0.547 | 2.178 | 2.039 | 1.209 | 1.302 | 9.542 | 9.000 | 12.352 | 12.078 |
| | (1.00, 4.00) | 0.385 | 0.393 | 7.811 | 7.269 | 0.917 | 1.047 | 25.388 | 23.949 | 23.628 | 22.537 |
| | (4.00, 0.25) | 0.304 | 0.317 | 6.102 | 5.906 | 0.856 | 0.804 | 17.511 | 17.302 | 20.207 | 19.811 |
| | (4.00, 1.00) | 0.280 | 0.301 | 7.836 | 7.340 | 0.788 | 0.795 | 21.768 | 21.007 | 23.676 | 22.681 |
| | (4.00, 4.00) | 0.250 | 0.303 | 13.858 | 13.094 | 0.703 | 0.846 | 36.914 | 37.571 | 35.695 | 34.187 |
| | (0.25, 0.25) | 0.936 | 0.929 | 0.082 | 0.123 | 2.204 | 2.326 | 2.540 | 3.459 | 8.159 | 8.246 |
| | (0.25, 1.00) | 0.713 | 0.705 | 0.916 | 0.845 | 1.538 | 1.670 | 6.363 | 5.735 | 9.822 | 9.689 |
| | (0.25, 4.00) | 0.442 | 0.434 | 6.086 | 5.864 | 1.034 | 1.148 | 21.737 | 20.735 | 20.136 | 19.728 |
| | (1.00, 0.25) | 0.678 | 0.686 | 0.919 | 0.866 | 1.527 | 1.561 | 5.709 | 5.513 | 9.838 | 9.732 |
| 6 | (1.00, 1.00) | 0.544 | 0.547 | 2.178 | 2.039 | 1.209 | 1.302 | 9.542 | 9.000 | 12.352 | 12.078 |
| | (1.00, 4.00) | 0.385 | 0.393 | 7.811 | 7.269 | 0.917 | 1.047 | 25.388 | 23.949 | 23.628 | 22.537 |
| | (4.00, 0.25) | 0.304 | 0.317 | 6.102 | 5.906 | 0.856 | 0.804 | 17.511 | 17.302 | 20.207 | 19.811 |
| | (4.00, 1.00) | 0.280 | 0.301 | 7.836 | 7.340 | 0.788 | 0.795 | 21.768 | 21.007 | 23.676 | 22.681 |
| | (4.00, 4.00) | 0.250 | 0.303 | 13.858 | 13.094 | 0.703 | 0.846 | 36.914 | 37.571 | 35.695 | 34.187 |
| | | | | | | $(S, R) = (4, 2)$ | | | | | |
| | (0.25, 0.25) | 0.988 | 0.975 | 0.016 | 0.048 | 4.138 | 4.251 | 2.555 | 3.905 | 12.025 | 12.096 |
| | (0.25, 1.00) | 0.852 | 0.849 | 0.470 | 0.451 | 3.093 | 3.277 | 6.363 | 5.999 | 12.934 | 12.903 |
| | (0.25, 4.00) | 0.544 | 0.543 | 4.990 | 4.872 | 1.992 | 2.156 | 21.907 | 21.310 | 22.004 | 21.744 |
| | (1.00, 0.25) | 0.835 | 0.842 | 0.472 | 0.451 | 3.079 | 3.145 | 5.699 | 5.709 | 12.939 | 12.901 |
| 3 | (1.00, 1.00) | 0.700 | 0.703 | 1.435 | 1.338 | 2.466 | 2.601 | 9.563 | 9.000 | 14.867 | 14.676 |
| | (1.00, 4.00) | 0.480 | 0.495 | 6.431 | 6.185 | 1.754 | 1.963 | 24.678 | 24.492 | 24.843 | 24.371 |
| | (4.00, 0.25) | 0.434 | 0.452 | 4.949 | 4.711 | 1.700 | 1.609 | 17.480 | 17.179 | 21.887 | 21.423 |
| | (4.00, 1.00) | 0.396 | 0.417 | 6.599 | 6.090 | 1.548 | 1.544 | 21.826 | 20.891 | 25.211 | 24.180 |
| | (4.00, 4.00) | 0.337 | 0.380 | 11.814 | 11.797 | 1.347 | 1.550 | 35.601 | 38.043 | 35.611 | 35.594 |
| | (0.25, 0.25) | 0.988 | 0.975 | 0.016 | 0.048 | 4.138 | 4.251 | 2.555 | 3.905 | 12.025 | 12.096 |
| | (0.25, 1.00) | 0.852 | 0.849 | 0.470 | 0.451 | 3.093 | 3.277 | 6.363 | 5.999 | 12.934 | 12.903 |
| | (0.25, 4.00) | 0.544 | 0.543 | 4.990 | 4.872 | 1.992 | 2.156 | 21.907 | 21.310 | 22.004 | 21.744 |
| | (1.00, 0.25) | 0.835 | 0.842 | 0.472 | 0.451 | 3.079 | 3.145 | 5.699 | 5.709 | 12.939 | 12.901 |
| 6 | (1.00, 1.00) | 0.700 | 0.703 | 1.435 | 1.338 | 2.466 | 2.601 | 9.563 | 9.000 | 14.867 | 14.676 |
| | (1.00, 4.00) | 0.480 | 0.495 | 6.431 | 6.185 | 1.754 | 1.963 | 24.678 | 24.492 | 24.843 | 24.371 |
| | (4.00, 0.25) | 0.434 | 0.452 | 4.949 | 4.711 | 1.700 | 1.609 | 17.480 | 17.179 | 21.887 | 21.423 |
| | (4.00, 1.00) | 0.396 | 0.417 | 6.599 | 6.090 | 1.548 | 1.544 | 21.826 | 20.891 | 25.211 | 24.180 |
| | (4.00, 4.00) | 0.337 | 0.380 | 11.814 | 11.797 | 1.347 | 1.550 | 35.601 | 38.043 | 35.611 | 35.594 |

Table F.4: Performance Measures at a Production Facility
(Base-stock settings - (3, 6) and (6, 3))

| $\theta^{pr}$ | $(cs^2, ca_i^2)$ | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | | | | | | $(S, R) = (3, 6)$ | | | | | |
| | (0.25, 0.25) | 0.656 | 0.632 | 0.433 | 0.392 | 0.555 | 0.595 | 2.513 | 2.133 | 4.685 | 4.874 |
| | (0.25, 1.00) | 0.441 | 0.407 | 1.780 | 1.623 | 0.404 | 0.448 | 6.365 | 5.473 | 7.077 | 6.984 |
| | (0.25, 4.00) | 0.311 | 0.297 | 7.285 | 7.093 | 0.330 | 0.377 | 21.111 | 20.182 | 17.889 | 17.661 |
| | (1.00, 0.25) | 0.375 | 0.360 | 1.792 | 1.702 | 0.400 | 0.396 | 5.733 | 5.317 | 7.259 | 7.097 |
| 3 | (1.00, 1.00) | 0.309 | 0.309 | 3.308 | 3.108 | 0.339 | 0.371 | 9.572 | 9.000 | 10.113 | 9.773 |
| | (1.00, 4.00) | 0.254 | 0.268 | 9.090 | 8.572 | 0.299 | 0.350 | 24.340 | 23.424 | 21.528 | 20.573 |
| | (4.00, 0.25) | 0.143 | 0.184 | 7.578 | 7.377 | 0.274 | 0.275 | 17.690 | 18.088 | 18.676 | 18.067 |
| | (4.00, 1.00) | 0.146 | 0.190 | 9.353 | 8.828 | 0.262 | 0.282 | 21.882 | 21.784 | 22.140 | 20.964 |
| | (4.00, 4.00) | 0.156 | 0.216 | 14.804 | 14.551 | 0.262 | 0.304 | 35.135 | 37.109 | 33.026 | 32.439 |
| | (0.25, 0.25) | 0.656 | 0.632 | 0.433 | 0.392 | 0.555 | 0.595 | 2.513 | 2.133 | 6.889 | 6.972 |
| | (0.25, 1.00) | 0.441 | 0.407 | 1.780 | 1.623 | 0.404 | 0.448 | 6.365 | 5.473 | 9.574 | 9.372 |
| | (0.25, 4.00) | 0.311 | 0.297 | 7.285 | 7.093 | 0.330 | 0.377 | 21.111 | 20.182 | 20.573 | 20.267 |
| | (1.00, 0.25) | 0.375 | 0.360 | 1.792 | 1.702 | 0.400 | 0.396 | 5.733 | 5.317 | 9.698 | 9.520 |
| 6 | (1.00, 1.00) | 0.309 | 0.309 | 3.308 | 3.108 | 0.339 | 0.371 | 9.572 | 9.000 | 12.697 | 12.310 |
| | (1.00, 4.00) | 0.254 | 0.268 | 9.090 | 8.572 | 0.299 | 0.350 | 24.340 | 23.424 | 24.237 | 23.218 |
| | (4.00, 0.25) | 0.143 | 0.184 | 7.578 | 7.377 | 0.274 | 0.275 | 17.690 | 18.088 | 21.429 | 20.807 |
| | (4.00, 1.00) | 0.146 | 0.190 | 9.353 | 8.828 | 0.262 | 0.282 | 21.882 | 21.784 | 24.927 | 23.708 |
| | (4.00, 4.00) | 0.156 | 0.216 | 14.804 | 14.551 | 0.262 | 0.304 | 35.135 | 37.109 | 35.848 | 35.160 |
| | | | | | | $(S, R) = (6, 3)$ | | | | | |
| | (0.25, 0.25) | 0.851 | 0.855 | 0.188 | 0.208 | 1.310 | 1.411 | 2.516 | 2.879 | 6.116 | 6.259 |
| | (0.25, 1.00) | 0.599 | 0.584 | 1.277 | 1.167 | 0.900 | 0.992 | 6.370 | 5.603 | 7.549 | 7.531 |
| | (0.25, 4.00) | 0.384 | 0.370 | 6.666 | 6.445 | 0.649 | 0.729 | 21.592 | 20.455 | 17.507 | 17.252 |
| | (1.00, 0.25) | 0.551 | 0.553 | 1.284 | 1.211 | 0.892 | 0.905 | 5.719 | 5.414 | 7.654 | 7.563 |
| 3 | (1.00, 1.00) | 0.438 | 0.441 | 2.685 | 2.517 | 0.716 | 0.781 | 9.557 | 9.000 | 9.988 | 9.755 |
| | (1.00, 4.00) | 0.325 | 0.340 | 8.213 | 7.890 | 0.573 | 0.668 | 24.335 | 23.684 | 20.495 | 20.014 |
| | (4.00, 0.25) | 0.227 | 0.246 | 6.720 | 6.606 | 0.529 | 0.504 | 17.392 | 17.521 | 17.704 | 17.227 |
| | (4.00, 1.00) | 0.216 | 0.241 | 8.598 | 8.055 | 0.496 | 0.509 | 21.944 | 21.220 | 21.307 | 20.068 |
| | (4.00, 4.00) | 0.206 | 0.261 | 14.644 | 13.800 | 0.460 | 0.554 | 36.847 | 37.338 | 33.152 | 31.553 |
| | (0.25, 0.25) | 0.851 | 0.855 | 0.188 | 0.208 | 1.310 | 1.411 | 2.516 | 2.879 | 6.861 | 7.304 |
| | (0.25, 1.00) | 0.599 | 0.584 | 1.277 | 1.167 | 0.900 | 0.992 | 6.370 | 5.603 | 8.888 | 9.003 |
| | (0.25, 4.00) | 0.384 | 0.370 | 6.666 | 6.445 | 0.649 | 0.729 | 21.592 | 20.455 | 19.524 | 19.305 |
| | (1.00, 0.25) | 0.551 | 0.553 | 1.284 | 1.211 | 0.892 | 0.905 | 5.719 | 5.414 | 9.201 | 9.071 |
| 6 | (1.00, 1.00) | 0.438 | 0.441 | 2.685 | 2.517 | 0.716 | 0.781 | 9.557 | 9.000 | 11.851 | 11.554 |
| | (1.00, 4.00) | 0.325 | 0.334 | 8.213 | 7.890 | 0.573 | 0.668 | 24.335 | 23.684 | 22.743 | 22.155 |
| | (4.00, 0.25) | 0.227 | 0.246 | 6.720 | 6.606 | 0.529 | 0.504 | 17.392 | 17.521 | 20.127 | 19.514 |
| | (4.00, 1.00) | 0.216 | 0.241 | 8.598 | 8.055 | 0.496 | 0.509 | 21.944 | 21.220 | 23.801 | 22.396 |
| | (4.00, 4.00) | 0.206 | 0.261 | 14.644 | 13.801 | 0.460 | 0.554 | 36.847 | 37.338 | 35.748 | 33.891 |

## F.2 Validation of the Analytical Model for the 2R/2P SCN Configuration with Transit Time

In this section, we present the results for the validation of the analytical model developed for the 2R/2P SCN configuration with transit delays. Tables F.5 and F.6 present the performance measures at retail store 1 with base-stock settings (3, 6, 3) and (3, 3, 6), respectively. Tables F.7 and F.8 present the results at retail store 2 with base-stock settings (3, 6, 3) and (3, 3, 6), respectively. Table F.9 presents the results at a production facility at a base-stock setting of (3, 6, 3). The results at a production facility at a base-stock setting of (3, 3, 6) are not presented as they would be identical to the results for a base-stock setting of (3, 6, 3). This is due to the fact that the performance measures at a production facility are dependent only on its base-stock level (3 in this case).

Table F.5: Performance Measures at Retail Store 1 (Base-stock setting - (3, 6, 3))

| | | $f_1^r$ | | $E[B_1^r]$ | | $E[I_1^r]$ | | $E[W_1^{br}]$ | | $E[T_1^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta_i^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| 3 | (0.25, 0.25) | 0.439 | 0.4192 | 0.814 | 1.178 | 0.559 | 0.901 | 0.726 | 1.015 | 3.408 | 3.598 |
| | (0.25, 1.00) | 0.316 | 0.333 | 2.130 | 2.261 | 0.394 | 0.705 | 1.557 | 1.694 | 4.065 | 4.130 |
| | (0.25, 4.00) | 0.184 | 0.208 | 9.282 | 9.033 | 0.229 | 0.439 | 5.685 | 5.704 | 7.639 | 7.516 |
| | (1.00, 0.25) | 0.322 | 0.326 | 2.404 | 2.302 | 0.683 | 0.688 | 1.771 | 1.707 | 4.202 | 4.151 |
| | (1.00, 1.00) | 0.251 | 0.264 | 4.155 | 3.912 | 0.527 | 0.556 | 2.773 | 2.659 | 5.076 | 4.956 |
| | (1.00, 4.00) | 0.166 | 0.189 | 11.577 | 10.919 | 0.347 | 0.398 | 6.934 | 6.731 | 8.784 | 8.459 |
| | (4.00, 0.25) | 0.158 | 0.1597 | 9.288 | 9.140 | 0.803 | 0.332 | 5.517 | 5.439 | 7.643 | 7.570 |
| | (4.00, 1.00) | 0.139 | 0.150 | 11.965 | 11.052 | 0.706 | 0.313 | 6.940 | 6.502 | 8.976 | 8.526 |
| | (4.00, 4.00) | 0.107 | 0.146 | 19.341 | 18.708 | 0.544 | 0.307 | 10.824 | 10.947 | 12.665 | 12.354 |
| 6 | (0.25, 0.25) | 0.000 | 0.019 | 6.257 | 6.306 | 0.000 | 0.029 | 3.129 | 3.214 | 6.128 | 6.153 |
| | (0.25, 1.00) | 0.000 | 0.014 | 7.739 | 7.578 | 0.000 | 0.022 | 3.869 | 3.844 | 6.868 | 6.789 |
| | (0.25, 4.00) | 0.000 | 0.009 | 15.055 | 14.607 | 0.000 | 0.014 | 7.526 | 7.370 | 10.525 | 10.303 |
| | (1.00, 0.25) | 0.014 | 0.014 | 7.746 | 7.635 | 0.021 | 0.0212 | 3.925 | 3.872 | 6.871 | 6.818 |
| | (1.00, 1.00) | 0.011 | 0.011 | 9.647 | 9.373 | 0.016 | 0.017 | 4.873 | 4.74 | 7.820 | 7.687 |
| | (1.00, 4.00) | 0.007 | 0.008 | 17.244 | 16.532 | 0.011 | 0.012 | 8.677 | 8.334 | 11.616 | 11.266 |
| | (4.00, 0.25) | 0.049 | 0.007 | 14.746 | 14.818 | 0.260 | 0.010 | 7.748 | 7.459 | 10.371 | 10.409 |
| | (4.00, 1.00) | 0.043 | 0.006 | 17.491 | 16.749 | 0.229 | 0.010 | 9.128 | 8.427 | 11.737 | 11.374 |
| | (4.00, 4.00) | 0.033 | 0.006 | 24.973 | 24.411 | 0.176 | 0.009 | 12.908 | 12.282 | 15.481 | 15.205 |

Table F.6: Performance Measures at Retail Store 1 (Base-stock setting - (3, 3, 6))

| $\theta_i^{pr}$ | $(cs^2, ca_i^2)$ | $f_1^r$ Sim | $f_1^r$ Que | $E[B_1^r]$ Sim | $E[B_1^r]$ Que | $E[I_1^r]$ Sim | $E[I_1^r]$ Que | $E[W_1^{br}]$ Sim | $E[W_1^{br}]$ Que | $E[T_1^r]$ Sim | $E[T_1^r]$ Que |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | (0.25, 0.25) | 0.014 | 0.057 | 3.263 | 3.353 | 0.008 | 0.075 | 1.655 | 1.778 | 3.132 | 3.176 |
| | (0.25, 1.00) | 0.010 | 0.043 | 4.742 | 4.613 | 0.006 | 0.057 | 2.394 | 2.411 | 3.871 | 3.806 |
| | (0.25, 4.00) | 0.005 | 0.027 | 12.057 | 11.629 | 0.003 | 0.035 | 6.060 | 5.974 | 7.527 | 7.314 |
| | (1.00, 0.25) | 0.042 | 0.042 | 4.777 | 4.669 | 0.056 | 0.0551 | 2.493 | 2.437 | 3.888 | 3.835 |
| | (1.00, 1.00) | 0.032 | 0.036 | 6.670 | 6.401 | 0.042 | 0.0441 | 3.442 | 3.312 | 4.833 | 4.700 |
| | (1.00, 4.00) | 0.021 | 0.024 | 14.257 | 13.552 | 0.027 | 0.032 | 7.276 | 6.944 | 8.625 | 8.276 |
| | (4.00, 0.25) | 0.049 | 0.020 | 11.713 | 11.834 | 0.228 | 0.025 | 6.154 | 6.035 | 7.355 | 7.417 |
| | (4.00, 1.00) | 0.042 | 0.019 | 14.457 | 13.764 | 0.198 | 0.024 | 7.54 | 7.012 | 8.722 | 8.382 |
| | (4.00, 4.00) | 0.032 | 0.019 | 21.947 | 21.426 | 0.15 | 0.025 | 11.33 | 10.917 | 12.468 | 12.213 |
| 6 | (0.25, 0.25) | 0.000 | 0.000 | 9.257 | 9.278 | 0.000 | 0.001 | 4.627 | 4.641 | 6.128 | 6.139 |
| | (0.25, 1.00) | 0.000 | 0.000 | 10.738 | 10.556 | 0.000 | 0.000 | 5.368 | 5.280 | 6.868 | 6.778 |
| | (0.25, 4.00) | 0.000 | 0.000 | 18.055 | 17.594 | 0.000 | 0.000 | 9.025 | 8.799 | 10.525 | 10.297 |
| | (1.00, 0.25) | 0.000 | 0.000 | 10.725 | 10.615 | 0.000 | 0.000 | 5.361 | 5.209 | 6.860 | 6.807 |
| | (1.00, 1.00) | 0.000 | 0.000 | 12.631 | 12.357 | 0.000 | 0.000 | 6.314 | 6.180 | 7.812 | 7.678 |
| | (1.00, 4.00) | 0.000 | 0.000 | 20.233 | 19.520 | 0.000 | 0.000 | 10.113 | 9.762 | 11.611 | 11.260 |
| | (4.00, 0.25) | 0.014 | 0.000 | 17.551 | 17.808 | 0.065 | 0.000 | 8.895 | 8.906 | 10.273 | 10.404 |
| | (4.00, 1.00) | 0.012 | 0.000 | 20.319 | 19.740 | 0.057 | 0.000 | 10.273 | 9.870 | 11.651 | 11.370 |
| | (4.00, 4.00) | 0.009 | 0.000 | 27.840 | 27.402 | 0.043 | 0.000 | 14.041 | 13.703 | 15.414 | 15.201 |

Table F.7: Performance Measures at Retail Store 2 (Base-stock setting - (3, 6, 3))

| $\theta_i^{pr}$ | $(cs^2, ca_i^2)$ | $f_2^r$ Sim | $f_2^r$ Que | $E[B_2^r]$ Sim | $E[B_2^r]$ Que | $E[I_2^r]$ Sim | $E[I_2^r]$ Que | $E[W_2^r]$ Sim | $E[W_2^r]$ Que | $E[T_2^r]$ Sim | $E[T_2^r]$ Que |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | (0.25, 0.25) | 0.439 | 0.403 | 0.526 | 0.776 | 0.396 | 0.637 | 0.937 | 1.299 | 3.525 | 3.776 |
| | (0.25, 1.00) | 0.329 | 0.332 | 1.164 | 1.297 | 0.289 | 0.519 | 1.733 | 1.943 | 4.161 | 4.297 |
| | (0.25, 4.00) | 0.195 | 0.213 | 4.711 | 4.627 | 0.172 | 0.331 | 5.845 | 5.877 | 7.706 | 7.627 |
| | (1.00, 0.25) | 0.323 | 0.327 | 1.360 | 1.317 | 0.502 | 0.510 | 2.007 | 1.956 | 4.361 | 4.317 |
| | (1.00, 1.00) | 0.260 | 0.270 | 2.208 | 2.098 | 0.401 | 0.420 | 2.982 | 2.876 | 5.210 | 5.098 |
| | (1.00, 4.00) | 0.173 | 0.194 | 5.864 | 5.561 | 0.266 | 0.301 | 7.092 | 6.897 | 8.866 | 8.561 |
| | (4.00, 0.25) | 0.149 | 0.170 | 4.829 | 4.666 | 0.607 | 0.262 | 5.676 | 5.625 | 7.837 | 7.666 |
| | (4.00, 1.00) | 0.133 | 0.159 | 6.122 | 5.615 | 0.544 | 0.245 | 7.077 | 6.677 | 9.147 | 8.615 |
| | (4.00, 4.00) | 0.104 | 0.149 | 9.770 | 9.433 | 0.418 | 0.232 | 10.879 | 11.091 | 12.742 | 12.433 |
| 6 | (0.25, 0.25) | 0.006 | 0.059 | 3.135 | 3.216 | 0.003 | 0.077 | 3.153 | 3.416 | 6.132 | 6.216 |
| | (0.25, 1.00) | 0.004 | 0.047 | 3.880 | 3.840 | 0.002 | 0.062 | 3.894 | 4.030 | 6.874 | 6.840 |
| | (0.25, 4.00) | 0.003 | 0.030 | 7.543 | 7.336 | 0.001 | 0.039 | 7.558 | 7.562 | 10.535 | 10.336 |
| | (1.00, 0.25) | 0.046 | 0.046 | 3.916 | 3.868 | 0.059 | 0.061 | 4.104 | 4.056 | 6.917 | 6.868 |
| | (1.00, 1.00) | 0.036 | 0.038 | 4.853 | 4.728 | 0.047 | 0.050 | 5.034 | 4.915 | 7.856 | 7.728 |
| | (1.00, 4.00) | 0.024 | 0.027 | 8.627 | 8.296 | 0.030 | 0.036 | 8.839 | 8.528 | 11.631 | 11.296 |
| | (4.00, 0.25) | 0.065 | 0.023 | 7.536 | 7.435 | 0.317 | 0.031 | 8.069 | 7.613 | 10.546 | 10.435 |
| | (4.00, 1.00) | 0.059 | 0.022 | 8.853 | 8.398 | 0.284 | 0.029 | 9.430 | 8.587 | 11.886 | 11.398 |
| | (4.00, 4.00) | 0.045 | 0.021 | 12.574 | 12.228 | 0.216 | 0.028 | 13.139 | 12.490 | 15.540 | 15.228 |

Table F.8: Performance Measures at Retail Store 2 (Base-stock setting - (3, 3, 6))

| | | $f_2^r$ | | $E[B_2^r]$ | | $E[I_2^r]$ | | $E[W_2^{br}]$ | | $E[T_2^r]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta_i^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | (0.25, 0.25) | 0.982 | 0.897 | 0.011 | 0.086 | 2.881 | 2.947 | 0.621 | 0.833 | 6.008 | 6.086 |
| | (0.25, 1.00) | 0.878 | 0.810 | 0.235 | 0.329 | 2.360 | 2.551 | 1.919 | 1.725 | 6.230 | 6.329 |
| | (0.25, 4.00) | 0.570 | 0.558 | 2.999 | 2.994 | 1.459 | 1.698 | 6.955 | 6.780 | 8.992 | 8.994 |
| | (1.00, 0.25) | 0.796 | 0.805 | 0.347 | 0.330 | 2.489 | 2.523 | 1.701 | 1.689 | 6.348 | 6.330 |
| 3 | (1.00, 1.00) | 0.688 | 0.704 | 0.890 | 0.829 | 2.083 | 2.151 | 2.850 | 2.797 | 6.892 | 6.829 |
| | (1.00, 4.00) | 0.478 | 0.515 | 4.017 | 3.817 | 1.420 | 1.557 | 7.692 | 7.866 | 10.020 | 9.817 |
| | (4.00, 0.25) | 0.414 | 0.494 | 3.126 | 2.842 | 1.904 | 1.438 | 5.336 | 5.612 | 9.135 | 8.842 |
| | (4.00, 1.00) | 0.374 | 0.456 | 4.299 | 3.704 | 1.722 | 1.335 | 6.882 | 6.815 | 10.328 | 9.704 |
| | (4.00, 4.00) | 0.301 | 0.402 | 7.713 | 7.409 | 1.361 | 1.208 | 10.996 | 12.383 | 13.684 | 13.409 |
| | (0.25, 0.25) | 0.448 | 0.429 | 0.644 | 1.061 | 0.512 | 0.922 | 1.165 | 1.857 | 6.640 | 7.061 |
| | (0.25, 1.00) | 0.344 | 0.366 | 1.260 | 1.550 | 0.383 | 0.773 | 1.921 | 2.445 | 7.254 | 7.551 |
| | (0.25, 4.00) | 0.207 | 0.240 | 4.771 | 4.799 | 0.229 | 0.502 | 6.008 | 6.317 | 10.763 | 10.799 |
| | (1.00, 0.25) | 0.356 | 0.361 | 1.606 | 1.569 | 0.749 | 0.762 | 2.495 | 2.456 | 7.607 | 7.569 |
| 6 | (1.00, 1.00) | 0.295 | 0.305 | 2.418 | 2.316 | 0.612 | 0.638 | 3.427 | 3.332 | 8.421 | 8.316 |
| | (1.00, 4.00) | 0.199 | 0.220 | 6.007 | 5.719 | 0.410 | 0.459 | 7.503 | 7.330 | 12.011 | 11.719 |
| | (4.00, 0.25) | 0.195 | 0.200 | 5.262 | 4.815 | 1.043 | 0.411 | 6.542 | 6.021 | 11.273 | 10.815 |
| | (4.00, 1.00) | 0.177 | 0.186 | 6.513 | 5.752 | 0.945 | 0.383 | 7.932 | 7.068 | 12.549 | 11.753 |
| | (4.00, 4.00) | 0.140 | 0.170 | 10.096 | 9.555 | 0.738 | 0.355 | 11.715 | 11.516 | 16.061 | 15.555 |

Table F.9: Performance Measures at a Production Facility
(Base-stock setting - (3, 6, 3))

| | | $f^p$ | | $E[B^p]$ | | $E[I^p]$ | | $E[W^{bp}]$ | | $E[T^p]$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta_i^{pr}$ | $(cs^2, ca_i^2)$ | Sim | Que | Sim | Que | Sim | Que | Sim | Que | Sim | Que |
| | (0.25, 0.25) | 0.845 | 0.855 | 0.192 | 0.208 | 1.283 | 1.412 | 0.826 | 0.960 | 2.127 | 2.139 |
| | (0.25, 1.00) | 0.591 | 0.584 | 1.304 | 1.167 | 0.878 | 0.992 | 2.123 | 1.868 | 2.869 | 3.175 |
| | (0.25, 4.00) | 0.379 | 0.370 | 6.795 | 6.445 | 0.638 | 0.729 | 7.285 | 6.818 | 6.525 | 6.297 |
| | (1.00, 0.25) | 0.551 | 0.553 | 1.290 | 1.211 | 0.891 | 0.905 | 1.916 | 1.805 | 2.860 | 2.807 |
| 3 | (1.00, 1.00) | 0.436 | 0.441 | 2.718 | 2.517 | 0.712 | 0.781 | 3.208 | 3.000 | 3.809 | 3.678 |
| | (1.00, 4.00) | 0.325 | 0.334 | 8.413 | 7.890 | 0.576 | 0.668 | 8.297 | 7.895 | 7.596 | 7.260 |
| | (4.00, 0.25) | 0.258 | 0.246 | 6.354 | 6.606 | 0.577 | 0.504 | 5.707 | 5.840 | 6.236 | 6.404 |
| | (4.00, 1.00) | 0.240 | 0.241 | 8.421 | 8.055 | 0.529 | 0.509 | 7.388 | 7.073 | 7.619 | 7.370 |
| | (4.00, 4.00) | 0.221 | 0.261 | 14.066 | 13.801 | 0.481 | 0.554 | 12.002 | 12.446 | 11.349 | 11.201 |
| | (0.25, 0.25) | 0.845 | 0.855 | 0.192 | 0.208 | 1.283 | 1.412 | 0.826 | 0.960 | 2.127 | 2.139 |
| | (0.25, 1.00) | 0.591 | 0.584 | 1.304 | 1.167 | 0.878 | 0.992 | 2.123 | 1.868 | 2.869 | 3.175 |
| | (0.25, 4.00) | 0.379 | 0.370 | 6.795 | 6.445 | 0.638 | 0.729 | 7.285 | 6.818 | 6.525 | 6.297 |
| | (1.00, 0.25) | 0.551 | 0.553 | 1.290 | 1.211 | 0.891 | 0.905 | 1.916 | 1.805 | 2.860 | 2.807 |
| 6 | (1.00, 1.00) | 0.436 | 0.441 | 2.718 | 2.517 | 0.712 | 0.781 | 3.208 | 3.000 | 3.809 | 3.678 |
| | (1.00, 4.00) | 0.325 | 0.334 | 8.413 | 7.890 | 0.576 | 0.668 | 8.297 | 7.895 | 7.596 | 7.260 |
| | (4.00, 0.25) | 0.258 | 0.246 | 6.354 | 6.606 | 0.577 | 0.504 | 5.707 | 5.840 | 6.236 | 6.404 |
| | (4.00, 1.00) | 0.240 | 0.241 | 8.421 | 8.055 | 0.529 | 0.509 | 7.388 | 7.073 | 7.619 | 7.370 |
| | (4.00, 4.00) | 0.221 | 0.261 | 14.066 | 13.801 | 0.481 | 0.554 | 12.002 | 12.446 | 11.349 | 11.201 |

# GLOSSARY

The definitions of selected technical terms are included in this glossary to clarify their intended meaning and usage in the document.

**Net inventory level:** The net inventory level at a store is defined as the difference between the inventory level and the backorder level at that store. It takes a positive value when there is inventory in the store and a negative value when the store is backordered.

**Base-stock policy:** The base-stock policy is defined by the base-stock level at each stage, which represents the maximum planned inventory at that stage.

**One-for-one replenishment:** The consumption of an item triggers an immediate replenishment order for that item. This is a special case of the (s, S) inventory policy, namely, the (S-1, S) policy, where S is the base-stock level.

**Bernoulli routing policy:** The Bernoulli routing policy assigns an arriving order/customer to node/queue $k$ with a fixed probability $p_k$, such that $\sum_k p_k = 1$.

**Fill rate:** Fill rate is the probability that an order at a store is satisfied instantaneously by items in stock.

**Stock-out rate:** Stock-out rate is the probability that there are no items in stock when a demand realization occurs at a store (Stock-out rate = 1 - fill rate).

**Ready rate:** Ready rate is the probability that there is inventory in the store. It is to be noted that for Poisson arrivals, ready rate and fill rate will be the same because of the PASTA (Poisson Arrivals See Time Averages) principle [62].

**Squared coefficient of variation (SCV):** The SCV is defined as the ratio of the variance of a random variable to the square of its mean.

VITA

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Scope and Methods: The stock-out rate at a store has become a serious concern for firms in a supply chain network (SCN). This situation can lead to loss in customer good will and market share. Information sharing among the SCN partners is seen as a strategy to address this problem and we studied inventory visibility or the sharing of inventory information among the supply chain constituents. This research focused on developing performance evaluation models of two "building-block" type SCN configurations (one retail store and two production facilities, and two retail stores and two production facilities) under three levels of inventory visibility (LoVis, MedVis and HiVis). Our general approach was to first model the SCN under Poisson arrivals and exponential processing times. For this case, we initially used Continuous Time Markov Chain models. If this model did not yield a closed-form solution, then we developed approximate queueing models. We then extended these models to general inter-arrival and processing time distributions. We also studied the value of information sharing by comparing the results for the SCN configurations with and without information sharing.

Findings and Conclusions: In our research we have developed analytical models for two SCN configurations under three levels of information sharing. We solved the CTMC models numerically by fixing the maximum number of backorders. An M/M/2 based approximate queueing model for the SCN with HiVis was found to serve as a good approximation and provided useful bounds for the LoVis and MedVis cases. We then obtained correction factors based on analytical inferences and empirical observations to develop a modified M/M/2 model, which improved the performance prediction capability of the M/M/2 based model for the SCN with LoVis and MedVis. Our study on the value of information sharing showed that there is significance at lower base-stock levels and higher utilizations at the production facilities. The value was also found to be more sensitive to processing time variability than to inter-arrival time variability. We also developed analytical models for configurations with asymmetric production facilities under Poisson arrivals and exponential processing times. As an extension, we considered transit delays in the analytical models developed. Extensive numerical experiments indicated that the analytical models developed in our study yield accurate results (e.g., 90% of the results within a 15% error range) over a wide range of SCN parameter values when compared with simulation estimates.

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