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GABRIEL T. MATNEY
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THE CLEARINGS OF AUTHENTIC LEARNING IN MATHEMATICS

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BY

Jayne Fleener

Teri Murphy

Neil Houser

Courtney Vaughn

Ann Reynolds

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ABSTRACT

This study is a hermeneutical phenomenological qualitative investigation of authenticity in mathematics learning. In the applicable literature the uses of the words “authentic” and “authenticity” are found to be problematic. Heidegger’s ideas on authenticity are employed to show the many shades of meaning for authenticity. Heuristic methodology was employed to describe the patterns of organization of the experience of authentic learning. Data sources include the researcher’s field notes, observations, classroom conversations, student reflections, and student interviews. Descriptions of the experiences of authentic learning were produced using the reflective guidelines associated with heuristic research: initial engagement, immersion, incubation, illumination, explication, and creative synthesis.

The themes found in this research where in strong relationship to one another and were explicated using Heideggerian terminology so as to show their distinctiveness without removing them from the context of the other themes. In the descriptions of the experience of authentic learning four interactive and inter-related themes presented themselves. These themes are called clearings, dynamic spaces where students reveal the patterns of organization for their learning. The four clearings were called: the clearings of mathematics, the clearings of relationship, the clearings of occupation, and the clearings of newness and wonder. The findings of the study make problematic the current use of the words “authentic” and “authenticity” as found in the literature. The findings were also found to relate to current research in curriculum theory. Researcher

recommendations for teachers and educational reformers are presented as suggested by this research.

Chapter 1

The Question of Authenticity

Every year, teachers prepare lessons, activities, and their rooms for the incoming students who will enter those doors. In secondary classrooms, these students will spend approximately one hour a day for 175 days with each teacher, engaged in the study of a subject whose importance has been pre-decided by a larger community with which the classroom is associated. It is well known and accepted that what is to take place in the classroom is called learning. To begin this paper then I would like to use my educational experience as a point by which a conversation about learning, authentic learning in particular, might be started, as well as, give insight into why the study came to be of interest to me.

Personal Background

Secondary school experiences.

Mrs. Simpson's seventh grade math class was always fun for me. She would tell stories and amusing anecdotes that made time fly. One day she told us a joke that went like this:

> A Hillbilly received word that he was going to have a son. He immediately set out to the fields to work extra hard. He took on work in town shoveling the manure off the streets. He cut back on his meals, and only ate one meal a day. He had his wife sew up the holes in his jeans and constantly sew the soles of his boots back to the hide so that he could save money. When his son was born, he was a proud father. He continued to work, but took on more jobs and less material luxuries. When his son was

of age, he said, “Son you are going to school today. You are going to learn about the vast secrets of life. It is important that you listen to your teachers and ask them every question you can think of. You will be the smartest person in our family some day.” The son went off to school and worked hard there while his mother and father toiled to make ends meet and save money. The son made it to graduation and as a present his dad gave him a college acceptance letter. The son said, “Dad, you know we cannot afford college.” The father proudly said, I have been saving since before you were born, and you will be the first person in our family to graduate from college.” The son went off to college and came back for fall break. At dinner the father enquired as to what the boy was learning at college. The son said, “Well we are learning about proper writing and about areas and πr^2.” The dad immediately stood up and began to chastise his son, the dad ripped at his shirt in despair and anger and said, “For years I have toiled and years I have saved. I went without for you so that you might go to college. Now you come back here and tell me *pie are square*? Son, *pie are not square*, pie are round and corn bread are square.”

This joke came after the proclamation that the area of a circle was πr^2. I raised my hand slowly and asked, “Why is the area of circle πr^2?” Mrs. Simpson smiled and poignantly said, “Because that is what gives us the correct answer.” At this point in my mathematics education we had been told that π was a never ending decimal whose value was approximately 3.14, but we had yet to discuss where π

had come from. While the mystery of this number was somewhat perplexing to me, it didn't seem to bother anyone else so I accepted that there was a number called pi and that it was irrational. However, in my mind I was not settled on this issue about the formula for the area of a circle and what this mysterious number had to do with it.

I did come to make better sense of π and why it held a special place and it was nice to know that it had a strong connection to the circle in which it could be seen as the number of times the length of the diameter goes into the circumference. Though I felt I understood π, I still could see no connection between the ratio of the circumference and diameter and the area of the circle. I continued to ask myself, "Why is π in this formula?"

We revisited the area of a circle formula in tenth grade geometry. In the midst of discussing why the length of the base multiplied by the height gave the area of all parallelograms, we were also given the formula for the area of a circle. So, I asked my question once again,

Me: Why is the area of a circle πr^2?

Teacher: Because when you square the radius Mr. Matney, that area will fit inside the circle about 3.14 times.

Me: Ok, but why 3.14 times? How do we know? Why not just 3 times or maybe 3.15 times? Why does the ratio of the circumference of the circle to its diameter have anything to do with the area?

Teacher: I'm not sure why π helps us find the area, but all you need to know for the homework and the test is that it does.

The question was of so much interest to me that I was insulted that she refused to engage with me in understanding it. I felt that my teacher really didn't understand why I was asking the question. Perhaps she thought I was being sarcastic (a trait I exhibited to her often), or perhaps she felt that I was not ready, mathematically, for such a discussion. While there is a plethora of possibilities for why my questions were dodged, my teacher was right about one thing, all I needed to know for her and everyone else was that πr^2 gives the area of a circle with radius length r. I was not only successful in that class, on that homework, and that test, I was successful in every math class I took where all that was required of me was to know what was asked over the time period I had to remember it. In spite of my success, that was not all I *needed* to know, because my questions about πr^2 remained. While the questions I was most interested in were usually in the same group as "why is πr^2 the area of a circle," these questions *seemed* to be generally unvalued by those guiding my formal education.

Undergraduate and graduate school experiences. Though I received many awards and honors in high school for my mathematical studiousness, I felt that my learning was fragmented, and that I didn't really know what geometry had to do with algebra, or trigonometry, or life for that matter. My first experiences with calculus were to be some of my most intense struggles with mathematics yet. As a freshman in college I had no prior calculus experiences. I dove straight into Differential Calculus and began to sink. Why was I studying limits? What did these things have anything to do with my previous learning? Where were we going with this stuff? Why does taking the derivative give the slopes of the tangent lines?

While working through my homework sets with my fellow classmates I would often concoct a way of arriving at right answers without any form of validation. My classmates would ask me, “How in the world did you get that?” More often than not I would have to reply, “I don’t know, it just seemed right.” I would at times put pieces of the fragmented knowledge together, without reason. I would create a system of solving math problems that would work for some but not for others. My affectionate classmates referred to this as my tendency to Gabrielatize mathematics. I was putting my own ideas into play instead of just following the convention. Why did I always do that? Standardized test were always so troublesome because I would put my own ideas into the question instead of simply considering the answer that was wanted.

It was getting harder and harder to memorize my way through math and I was beginning to despise it. It was during this period that my mathematical thinking as well as my thinking about mathematics reached a critical point of reorganization. By the time we reached sequences and series my thinking about mathematics had undergone a transformation in which mathematics was no longer about learning the formulas but it was about why they were thought to be true. It surprised me that my professors were open to my questions about why these things were true. Though I sometimes struggled to see their point, they were always willing to converse about the ideas and their connections. It was strange for me that they took such interest in my asking these kinds of questions. No teacher before seemed to really desire to pursue them with me. Even though I knocked on my professor's door every time there was an office hour, I was welcomed in despite the fact they already knew what I was going to ask. Why? Why? Why?

The amount of time I spent working on my mathematics doubled. With my new found inquiry into the "whys" of mathematics I was finally seeking the answers to the questions I felt I needed to ask. My love for the subject grew immensely. Though I received my Bachelors degree in Mathematics Education, I felt that I needed to continue the pursuit of my infatuation with pure mathematics.

As a first semester graduate student in an applied mathematics program I received a glimpse of where my formal education had taken me. Standing in my office looking out the 9 story high window I was recollecting the events of my life that had led me to be in this place. As an undergraduate I found mathematics to be a tough subject, and it was because of the challenge and relation of ideas that I felt drawn to it. Was that the only reason I was here? I was being pushed beyond what I thought were my mathematical limits. Only two out of every five proofs were of interest to me. "May be I am not going to be a mathematician after all," I thought to myself.

I was looking out the office window and I observed several cylindrical water towers spread throughout the county. I began trying to recall the volume and surface area formulas for cylinders. "Volume is $\pi r^2 h$. Or is it? That seems right." I continued to dialogue with myself until I decided that I would just have to prove it. At this point I was drawn back to the question that so perplexed me as a youth.

The volume was simply the area of the base times the height. As clear as this mathematical idea was to me, it rested on the assumption that the area of the base, πr^2 was right. I came to the realization that I was comfortable with *why* the volume of a cylinder was $\pi r^2 h$, but that I was still perplexed by *why* πr^2 was believed to be the area of a circle. Based on the traditional order of the curriculum it would be said that I understood these two things out of sequence, but so many of my mathematical meanderings fall into this "out of sequence" type that for me it was more the rule than the exception.

Once again my middle school question popped up. So I turned my attention to proving to myself this one simple idea about the relationship of the area of a circle, that circle's square radius, and the circle's circumference to radius ratio. A typical mathematical strategy when trying to work through a proof was to consider a simpler case. I consider a regular triangle and a square. Knowing the areas in advance I asked myself if half the perimeter (circumference) to the apothem (radius) ratio could contribute to finding the area of these figures. It of course did contribute. Within a matter of minutes it became clear to me π's connection to the formula for the area of a circle.

In less than a of couple hours I felt I had made sense of it in a formal way. I had for the first time made sense of the relationship between a circle's area, its radius, and the circles circumference to diameter ratio of about 3.14. I immediately went home to my wife with great enthusiasm and pronounced, "I have figured out why πr^2 gives the area of a circle!" After a glance from her that was questioning of my intelligence I explained my dilemma more fully and we sat down and conversed about my solution. It made sense to her as well and so now I felt that I could take it to my undergraduate students in college algebra. After class one day I wrote on the board, "If you ever wondered why πr^2 gives the area of a circle come to my office at 4:00pm."

To my great surprise, 10 of my 34 students showed up. They asked me what the secret was. We first talked about their ideas on the subject. They submitted no means for solution. I asked them to consider whether or not they had any understanding of where π came from and what it had to do with the area. I also suggested considering a simpler case for study, that of a square. We ended the meeting and agreed to meet in two weeks.

When we met again the students discussed the meaning of π but said that they could see no relationship between the area of a circle and this number. I mentioned that the reason I didn't understand the area of a circle formula was because the relationship among π, the radius, and the area were being expressed in a different way from other area formulas which look directly to some relationship involving side lengths and heights but never a perimeter to radius ratio. The other formulas were reified conventions to us, just as the area of a circle, and in that reification we had lost all hope of seeing a pattern that connected them.

I challenged the students to take a simple 2X2 square for which they knew the area and create a formula for it that was similar to the formula we had been told about for the area of a circle. I was trying to get them to construct a connection between the two and by this connection perhaps open more space for further connections. Two weeks later a student felt that she had made the connection. As she showed the rest of us how we could draw a radius to a square and that radius was always ¼ of ½ the perimeter, giving us a half the perimeter to the radius ratio of 4, no matter what size the square was. The others, having thought on the same problem understood this but when they were working on the problem got hung up on the fact that a square can't have a constant radius like a circle has so they wondered why she had chosen to take the radius that was perpendicular to a side of the square. Why did it work for a square but not a rectangle?

Why, indeed! The conversations blazed from that point on about how, why, and whether there could be any connection between these two formulas. By the end of the semester, 4 of the 10 students remained discussing the topic and eventually agreed upon a connection between the two formulas. The conversations about the relationships among shapes and their associated areas were exciting for us. Though I left my mathematical proof out of these conversations the students left the conversations feeling like they had made sense of why the area of a circle formula was true.

One student mentioned to me that she never knew how interrelated algebra and geometry were. Another student mentioned that the connection between a circle's area and a square's area was clear to him, but it was still unclear if it worked with other shapes containing more sides. While students had made better sense of the relations involved in the area of a circle they each left the conversations with something different.

As a masters student I came to realize that for me, when I explored these and many other mathematical relationships, mathematics was not some ethereal symbolic language that needed deciphering. Furthermore, I came to understand that the questions I had been asking all along were the same kinds of questions my professors, practicing mathematicians, were asking. It took me until graduate school to honestly realize what questions professional mathematicians were even interested in. For much of my childhood there was no distinction for me between mathematician, scientist, or physicist. Once I came to the realization that I had always asked youthful questions that pertained to the discourse community of professional mathematicians, I found new inspiration to make it through the very rigorous masters program.

My undergraduate degree and my heart were in mathematics education. I immediately turned my spare time into understanding *why* I had not made sense of the relationship between a circle's area and its radius length prior to this moment since it had been such an important question for me. I began asking people from across the nation in the areas of mathematics, engineering, and other equally technicalfields involving mathematics if they were familiar with *why* one square radius went into a circle about 3.14 times and if they cared about such questions. Most didn't know, and only a few said they even cared about such questions. That only few were interested in the question was not of concern to me, as I understood that everyone has varying interests. However, what did concern me was that those of us who were interested in these kinds of questions were never able to pursue them. These connections, the care for mathematical ideas (Noddings, 1992), is something that has always been apart of me. They are as real and important to me as the more practical balancing of the checkbook.

Why then, was it not possible during my earlier years of education to allow me to explore, or even help me nurture these ideas? Though I understood why allowing some students to pursue such questions would be very hard for the teacher, was memorizing all that stuff without any sense of why it makes sense or where it came from, actually helpful to me, or to anyone. In my study of mathematics I found that I could only prove something if I could see connections, if I could find through myself the patterns that connect. Memorizing never helped me make any connections; rather it was by inquiring about how things were related that enabled me to prove. Though I had been formally studying mathematics since I was 5, only a couple teachers encouraged my pursuit of the troublesome *why* questions. I should note that I do not personally fault the teachers who didn't help me explore these questions. It isn't that they were not trying to teach me things, in fact they were trying to teach me too many, too quickly, and too sporadically. There wasn't any time for my individual meanderings even if those meanderings were directed to the subject of study. They had other pressures afoot that disabled them from allowing the cognitive space and spacing that I needed in order to pursue the questions most pertinent to my being. While these pressures and ways of teaching are strong, I intuitively felt this answer was too simplistic a reason for not allowing a student to explore and nurture their interests. I sought to further explicate the pressures and ideas of education that had helped to mold the kind of organization it had taken.

Teaching experiences and PhD coursework. While I was finishing my masters and beginning my PhD in mathematics education I sought answers to why my education was the way it was. I researched technology, attitude, motivation, and math anxiety. None of these seemed to adequately address the issues most important to me. As I was now a secondary teacher I looked directly to my students interests as I taught them mathematics. I began to see the complexities involved in allowing students to explore their interests and nurture their aptitudes. It wasn't that it was a monumental feat to accomplish for me personally, but within the current educational system, with its modernistic way of seeing, this seemed close to impossible.

I was not interested in designing a classroom or a curriculum around the questions that *I* believed to be worthy mathematical investigations. What I desired was a classroom whose dynamics generated a disciplined inquiry and allowed the freedom and guidance to explore those mathematical inquiries as they related to my student's Being-in-the-world. Hence, my pedagogical questions became: How would I allow my students to engage in meaningful ways, with the subject of mathematics? How could I let them ask the questions most pertinent to their being and still teach them "mathematics" and follow some semblance of a required curriculum? Dare I challenge the traditional values of what mathematics was important or even more heretical, the very characterization of mathematics as the most absolute of all sciences. To engage in this kind of questioning, with my own students, could mean losing my job. After all, I was an agent of the state who was hired to teach what we as a society valued as "best" for our children. There are so many presumptions about learning in this view that it was hard to even have a conversation with others mathematics teachers about what, exactly, was being learned by our students. Working within these circumstances I sought to engage my students in mathematics in ways that might have meaning for them.

The problematic of creating authenticity. I joined an organization that operated under the banner of authentic teaching. The organization brought together secondary mathematics and science teachers, professors from the fields of mathematics, science, and education, and university students of varying levels. This cohort was to produce "authentic" lessons based on state and national standards that would then be funded, researched, and tried in the teachers' classrooms. As the organization was in its beginning stages many discussions revolved around the question, "what is authentic teaching and learning?" The answer concluded by most was that teaching by means of real-world examples and real-world problems is authentic teaching and hence, whatever was learned from those real-world examples was authentic learning. For me, this conclusion suffered from the same modernistic thinking as traditional education in that it separated teaching and learning from the student as well as other important factors, and put all its stock into the creation of activities. It apparently did not matter whether the student was interested in the "real-world problem." If it was a problem that involved mathematics by an applied approach within a real-world context, then it was authentic through and through, and it was believed that students would learn better.

Some of the students who might be considered as unsuccessful in the traditional mathematics classroom came to life and flourished when engaged in these real-world tasks. The questions that arose in the activities held something important for these students. They would spend hours investigating the problems and raise their own questions based on their findings. They would ask me everyday if we could talk to someone important about their ideas. As exciting as this was for me as a teacher, however, I recognized that most of my students were apathetic even to the real-world explorations and problems. The activities did not appeal to them any more than non-real world mathematics classroom activities. They did not find them valuable, meaningful, or purposeful. When I asked my class for their feelings on what we were doing one of the students replied, “None of it really makes a difference to me because it is all a bunch of obstacles I have to climb over to graduate.” A few other students agreed with this sentiment. What then, if anything, would perturb these students? If seeing how we, as a society, use these ideas does not appeal to them, then what would? I continued to rethink the curriculum.

I spoke again with the organization devoted to creating authentic learning experiences. I informed them of the difficulty and complexity involved in “creating” these authentic learning experiences. One member, a science teacher, said that if the activity related to real-world issues and used mathematics as a tool for the problem’s solution then we have created an authentic learning experience. Was this all that is required for mathematics to be authentic to students? It appeared to me that for some students the experiences might be highly meaningful but for others it was no better than the rote learning that had so epitomized my education experiences. They made no connections from the activities, and in fact felt disconnected and apathetic. I believed that these students had important questions to pursue, but these questions eluded them. There must be a way that such an important human way of thinking, like mathematics, could be "real" for them.

Some people in the organization told me they could see no other way for mathematics as a disciplined activity to be authentic. After all, Whitehead said that "Mathematics is thought moving in the sphere of complete abstraction from any particular instance of what it is talking about" (Whitehead, 1972). If what was real occurred only in concrete empirical manifestations, then how could such an intuitive and abstract discipline be authentic at all? It was during this time I came to believe that authenticity's meaning was being reduced to a mere pebble. That is, it was being seen as substance, specifically, the kind of materials and actions created and taken by teachers to their students. Furthermore I intuitively felt that genuine mathematics learning was more complicated than giving students hands-on and real-world activities. Also, if authentic teaching in mathematics meant an applied approach within a real-world context, then the questions I was most concerned with as a mathematician were not a part of authentic teaching and learning. Since I believed that mathematical endeavors were apart of human experience and human ways of seeing the world, I had trouble agreeing that this was indeed the meaning of "authentic" learning in mathematics since it eliminated the inquiries of those most likely to pursue a life in its discipline.

By reducing authentic learning to the classroom engagements involved in hands-on/real-world problems, questions about the relations of the squares of the sides of any right triangle seem less real or genuine than the questions about water pressure to water tower height, tensile strength of yarn versus steel, or the interrelatedness of biological systems? I felt that seeing authenticity anew could help us overcome the debates about mathematical formalism, mathematical basics, and practical mathematics. But as long as the meaning of authenticity was associated with purely hands-on/real-world questions, problems, and ways of teaching, it was doomed to fail to bring about dissolution of these long standing debates.

One student, one lesson, dynamic phenomenon. One day a student of mine helped me to see the possibility of authenticity more clearly and gave me another spark for a study that would seek to problematize authenticity. For the purpose of anonymity I will be giving all my students pseudonyms. Let's call this student Jose. He happens to be the same student who expressed the sentiment that the real-world activities we had been doing were of no interest to him and that they were just another hoop to jump through. Jose is one of my talented artists, the kind of artist that doodles instead of taking notes and spends more time drawing calligraphic numbers on the paper than answering the homework problems. Jose was also heavily involved in gang activity. He had been in multiple fights, had stolen cars, and had been in trouble for carrying weapons.

In geometry class we were studying the Pythagorean relationship. My students were looking at different sized right triangles, measuring the lengths of the sides, and constructing squares with those lengths. As they were engaged, Jose asked if we could square other things than lengths. I conversed with him about what he meant by this "squaring of things." He suggested a two dimensional kite, instead of a one dimensional line. I drew one, cut it out, and asked him to see if he could make a square of it.

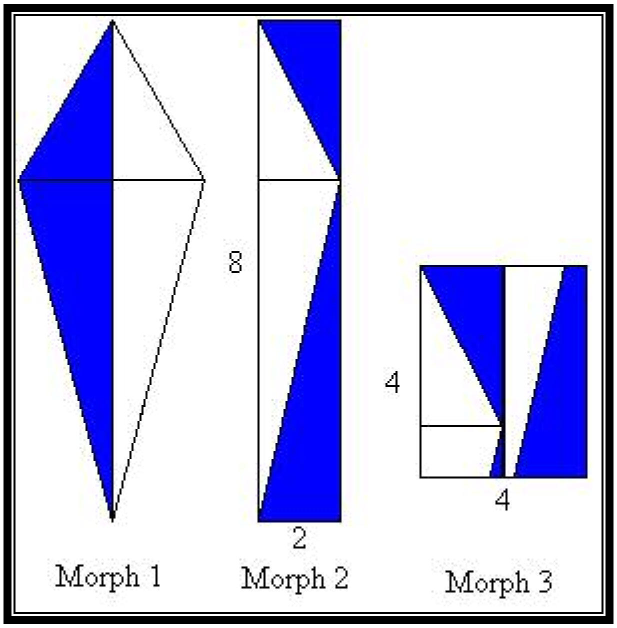


Figure 1. Jose worked on squaring a kite whose diagonals were 8 and 4. Then he chose to break up and reassemble the kite in this way to make a 4X4 square.

Upon his successful completion he further inquired about the possibility of squaring other shapes. This inquiry eventually led him to ask me, “Is it possible to make a square out of a circle?” I was completely taken aback by the question! I was fascinated that he was so intrigued about the relationships among these shapes. I allowed him some distance from what we were working on so that he might play with this question.

I felt that for this student the question was an authentic one. It is also one that has been a landmark in the history of mathematics, and has fascinated many people who are interested in mathematics (Burton, 1995). Since I did not prompt or lead this student to ask this question I was surprised the student asked the question at all. I had no idea such things were or could be of interest to him. I asked him if he had ever thought about it before. It turned out he was surprised he had an interest in it as well. I offered him an opportunity to forgo current out of class tasks and other homework assignments in favor of pursuing the problem he was interested in. When Jose was engaging in these questions about morphing one 2-dimensional shape into another, what he learned from their solutions was real and meaningful to him as much of our mathematics had not been. Having observed this I was intrigued. I felt I had begun to make sense of why my previous ideas of authentic learning in mathematics were troublesome. From these experiences and reflection on what was needed to be researched in order to get at why "authenticity" was so problematic for me, I came to ask the question, "What is the experience of authentic learning in mathematics?"

Chapter 2

Meanings of Authenticity

Continuing the conversation about how and why I chose to research the experience of authenticity, I will consider a meaning of authenticity through its use in academic literature and make the claim that in many academic articles it has become a word that creates the context by which we understand it. In a later chapter I will explore why such use is problematic for me based on the interpretations of my students' experiences of authentic learning. What follows in this chapter is not a search for the underlying meaning of authenticity in an abstract way but is rather a discussion about my interpretation of its use. After looking at the use of authentic and authenticity in literature I will turn to a discussion about communication and meaning, in order to better explain why these uses are problematic for me.

Literature and the Use of Authenticity

The current ways of using the words authentic and authenticity in education are problematic. These words are used through out academic literature as words which indicates "the quality of having correspondence to the real world" (Petraglia, 1998, p. 165). The constant association of everything authentic with the real-world is exactly the kind of abstraction that I have found problematic in my experiences as a student and a teacher. My critique of the literature is only towards the use of the words authentic and authenticity. In most cases I appreciate these articles for the ideas they discuss and in that I do not intend to debate their conclusions, but rather point to the difficulties that arise in the way the word authenticity is used and the implications for the meanings of authenticity base on that use.

One place “authenticity” can be found is in the discussion of some instructional designers. There are some instructional designers who are content with the idea that constructivism can never lead to prescriptive instruction. As such they have sought to provide proscriptive guidelines, which would direct the design of learning environments towards constructivist goals. It is in their guidelines for constructivist instruction that some designers have sought to talk about authenticity.

One of these important guidelines is that for the purpose of knowledge construction one should present students with “authentic” tasks that provide a context through real-world and case based learning environments (Duffy & Jonassen, 1992). The view that “authentic” tasks must necessarily be associated with real-worldliness is further elaborated in an article about creating a constructivist design model when it is specifically stated that a constructivist design methodology should work toward making environments that provide “a meaningful, authentic context for learning and using the knowledge they[students] construct, which should be: supported by case-based problems which have been derived from and situated in the real world with all its uncertainty and complexity and based upon authentic tasks(those likely to be encountered in real-life practice)”(Jonassen, 1994, p. 37).

This meaning of “authentic,” as real-world application supporting constructivist ideas about learning, is so entrenched in the way we talk about real learning that it is difficult for anyone seeking to design curriculum to see any other meaning for the word. According to Dunn (1994), instructional systems design with its stance towards prescription of curriculum and evaluation often took the blame for the problems in education. In this article Dunn explains that there are several constructivist recommendations for teaching and instruction. First and foremost he appeals to the statements of those involved in situated cognition and the idea that if reality is constructed within a specific context then teaching and instruction should take place in rich contexts that are associated with how the knowledge would be used in the real-world, thereby maximizing motivation and the transfer of knowledge (Brown, Collins, & Duguid, 1989). Immediately following this appeal to a constructivist recommendation he states, "In a word, they[instructional designs] need to be authentic" (Dunn, 1994, p. 84). Constructivist notions of enacting teaching and designing instruction seem to encourage an orientation toward "authenticity" as a deep real-world connectedness in learning.

The article goes on to indicate how instructional systems design might be used to inform those who wish to develop constructivist instruction. Dunn gives an example of a large instructional systems design program called Competency Based Teacher Education (CBTE). During this program pre-service teachers were expected to have considerable classroom experiences, review the literature, observe in classrooms, and attend personnel workshops. All of this was to give a pre-service teacher a real context with real students and assume that authenticity is the result. Though there were many technical and logistical problems with the

CBTE program, Dunn concludes that if the instructional systems design procedures had been followed exactly as prescribed, all the experiences of the pre-service teachers would have been “authentic.” The idea here is that instructional system designers can prescribe what is authentic. If “authenticity” is simply that which has association to the real-world then it would seem reasonable to say they can prescribe it.

The article by Dunn (1994) also brings into our purview situated cognition (situated learning), another area in which the academic literature expresses “authenticity” as one of its tenants. In an article entitled, *Learning with Media: Restructuring the Debate*,Jonassen et al. (1994) seek to reorg anize the debate about the role of media in learning by taking the emphasis off media as transmitter of messages and knowledge and putting the emphasis on examining the process of learning and the role of context and environment to support learning. For the consideration of the process of learning the authors present several contemporary theories of learning, such as situated learning(Brown et al., 1989), intentional learning (Scardamalia, Bereiter, McLean, Swallow, & Woodruff, 1989), and cognitive apprenticeship (Collins, Brown, & Newman, 1989), all of which claim that learning best takes place when it is done in the context of meaningful real-world tasks.

Jonassen et al. go on to specifically connect authenticity to situated learning. The charge here is that learning that is decontextualized leads to *inert* knowledge, that is, knowledge that the knower will not be able to use. Humans are always in the act of processing information so as to contextualize knowledge (Jonassen, Campbell, & Davidson, 1994). It is in this concern against inert knowledge that the authors go on to state that in situated learning students work on "'authentic tasks' whose execution takes place in a 'real-world' setting" (Winn, 1993, p. 16). This is more evidence of a strong association between what is thought to be real-world and what is believed to be "authentic."

The view of "authenticity" as a quality of corresponding to the real-world gets stronger when one begins to look at the literature involving apprenticeships, or instructional design that is focused on the training of those who will be in specific work contexts. In order to best apprentice a student, the learning and decisions the student must make should be mirrored as much as possible to the hard faced decisions that the student will encounter once the job begins. Collins (1994) suggest 5 principles that embody the kind of learning that needs to happen to best prepare workers to be skilled in some contexts but flexible enough to adapt to other methods and to be motivated and retain what they have learned so that they may apply it. One of these 5 principles is the "Authenticity Principle." The "authenticity principle" is the idea that "knowledge, skills, and attitudes to be learned should be embedded in tasks and settings that reflect the uses of these competencies in the world" (Collins, 1994, November-December, p. 30).

When speaking about the students who learn in these "authentic" contexts, Collins says,

> The relevance of what they are learning is very apparent to the learners, and they readily see how to apply what they learn when they go out on new jobs. The authenticity of the learning environment ensures that the knowledge

> gained will be readily available in the kinds of situations they will face in their work. (p. 30)

While we could debate over whether this proposed principle of “authenticity” is generalizable across all areas of education, my purpose for including it here is simply to highlight the intense connection that the meaning of authenticity has to the activities of humans in the real-world, as shown by the apprenticeship literature.

Evidence that a word’s meaning is unchallenged and unquestioned can be seen through the use of the word by those who use it yet refrain from seeking to explain their meaning. We must tread lightly when we use words in an unquestioning state as it is here that the words create the context around which discourse is structured. In our current discussion of the words “authentic” and “authenticity” it is perhaps nowhere more apparent that these words are used with specific and unquestioned connection to the real-world, than in the domain of curriculum research. The aforementioned literature seems to reveal a need to speak of a curriculum that is trying to model something humans do outside the classroom as “authentic” curriculum.

Without defining authenticity many articles make statements referring to authentic learning, curriculum, and instruction like, “The knowledge students acquire is expected to be related to everyday life,” (Roelofs & Terwel, 1999). Other articles advocate that authentic tasks are reflections of mathematics in life and work (Forman & Steen, 1999) or that authentic tasks “mirror real life experiences” (Nicaise, Gibney, & Crane, 2000). Another article discussed a project based on local school needs as the basis for authentic instruction. The project involved planning a beautification of the school grounds and was said to encourage student observation, description, and explanation of the natural world (Hopkins, 1999). The articles agreed that "A lesson gains in authenticity the more there is a connection to the larger social context within which students live" (Newmann & Wehlage, 1993).

The use of the word "authentic" in conjunction with the words task and activity are unproblematic when "authentic" is simply taken to mean that which is relevant to the real-world. But should we accept this meaning? Does it hold any reduction for us or limitations on how we talkabout curriculum and learning? To explore these questions further we will need to briefly discuss our assumptions about communication. By challenging the assumptions of communication, the uses of "authentic" and "authenticity" can be shown to be problematic.

Problematizing Our Ideas about Communication

Much of the way we go about doing our practice in education depends on our ideas about communication. Though not unchallenged, the view of those who see learning as measurable according to some set of variables and thus lending itself to methods of prescribed exactness, dominates the existing conversations about learning. From this view of learning is the assumption that external information can be carried to the subject successfully without consideration of what happens in between (Roy, 2004). The idea that one can transmit information from teacher to student became problematic for me well before I ever thought about educational issues. As students we often experience the struggling to make sense of what others are saying. This struggle is sometimes so intense that we seek any reduction of the intended message as a possible substitute for understanding the intended meaning. In mathematics classrooms this might be associated with the student responding, “Just show me what to do!” When one feels an absence of genuine understanding, “inert” operations and repetition seem an acceptable alternative. For many students, myself included, these “inert” operations and repetitions come to be understood as genuine mathematical understanding.

So what goes wrong in the assumption that information can be transmitted exactly? The separation of information from meaning has contributed to the sustainability of the assumption that learning is measurable according to some set of variables. As information informs it also contributes to making the structures that regulate thought and meaning, and as such prevent other possibilities. Information is not value-free but is given and interpreted in the context of some discourse among humans. The way we speak and write depends on the assumption that others have the same sense of the words we use in the context we believe we are using them. Roy (2004) explains further that

> Discourse is a restrictive and rarefying paradigm of knowledge guided along by rules of inclusion and exclusion; it determines what can be thought or uttered. In other words, reified, it appears as truth itself…What is necessary to challenge the masquerading of discourse as truth, and instead, to see it as an event or phenomena. To revert it to the status of an event and no more than an event is to dereify it by showing the processes and conditions of its arising. (p.6)

In order to challenge the masquerading of "discourse as truth" the assumption that within any discourse the intent of the sender is conserved by the receiver must be problematized. Though there is the experience of linguistic confusion in everyday conversation, it is rarely considered that the parts of our communication we feel we adequately understand in fact are built upon the same baseless bridges our experience of confusion is built upon.

Furthermore the parts of our discourse that go unchallenged might be the most dangerous, in that we limit our own possibilities by not understanding the demarcations we have established by our own use of language. As there is no discourse among humans that does not take place in a context and is not devoid of presuppositions we must consider that language isn't a transmitter of information but is rather an "impulsion" contingent upon the contexts of our socialization (Roy, 2004). Such impulsions are named, *order-words*, by Deleuze and Guattari (1987):

> We call order-words, not a particular category of explicit statements (for example the imperative), but the relation of every word or every statement to implicit presuppositions, in other words, to speech acts that are, and can only be, accomplished in the statement. Order-words do not concern commands only, but every act that is linked statements by a "social obligation." Every statement displays this link. The only possible definition of language is the set of all order-words, implicit presuppositions, or speech acts current in a given language at a given moment. (p. 79)

In any given discourse then, the order-words weuse create the context for us. So communication isn't the transmission of information but is rather the contextual connection of a string of order-words whose "impulsion" is revealed by what the word does or prevents from doing. Such a view of communication is hardly tenable to the idea that what the teacher intends to impart to the student is necessarily received by the student by way of a nice and neat isomorphism. The process of discourse is not as clean as the "instrumentalist" view would have us believe.

Since discourse is necessary, we must overcome the tendency to reduced meaning by challenging the order-words. Hence, we must find the words that create the contexts for our interpretations and dereify them. As order-words go unchallenged they assume hegemonic tendencies over our ability to see how they limit us. To challenge the order-words we must go directly to their use. Since there is no underlying meaning of language apart from its contextual use, it is only through the use of language that we can disclose the logic that both guides and limits our discourse (Fleener, 2002). Through the use of it we seek to reveal the presuppositions involved in its use.

Challenging Authenticity as Order-Word

As I alluded to in chapter 1, one of the reasons I became interested in authenticity was because I was involved with a group dedicated to designing authentic lessons for teachers. After several months into the program I finally asked what was meant by "authentic teaching." The standard response was that "authentic" teaching was teaching based on real-world or hands-on problems.

More than one of my colleagues in this organization went so far as to tell me that they couldn't fathom an "authentic" teaching of mathematics without a real-world context. When we stop problematizing a word and begin to treat it as though we all mean the same thing by it, it has become an order-word. These were my first clues that "authentic" and "authenticity" might be order-words. By problematizing the meaning of authenticity and by using it in a different way I am challenging its order-word status and hence seeking to ask for a serious conversation about the possibilities for education that lie beyond a reductive use of the word "authenticity."

Authenticity as order-word becomes further problematic when one considers the argument of Petraglia (1998). According to him, "authenticity," or "real-worldliness," is the desideratum of contemporary education. That is, "authenticity" as real-worldliness is desired by educators as something that is uncompromisingly essential. Petraglia covers briefly the historical development of education and considers the development of "authenticity" as real-worldliness. He argues that this form of "authenticity" has evolved from socio-political and philosophical view points associated with progressivism. Then more recently through constructivism "authenticity" has gone from a partial desideratum to a cornerstone of learning. As such a distinguished aspect in educational discourse it is important that we seriously consider what we mean when we speak of "authenticity," and that what we mean isn't being reduced.

Though Petraglia does not discuss Deleuze or communication he does recognize that "authenticity" is something of an order-word for us. He says,

> Perhaps part of the problem is that the language we use to talk about authenticity in education has been so comfortable and unacademic. Had we designated the desideratum, "scholastio-quotidian isomorphism" instead of the homely, "real-world problem solving," or "authentic learning" we would no doubt, think twice about what we were saying. Yet, the term authenticity and its synonyms lull us into the belief that we do not need to explain ourselves. (p. 13)

Petraglia is alluding to the complexity that resides in our use of the words "authentic" and "real-world" and how we have come to use those words nonchalantly to justify parts of our educational practice. To say, "I have developed an authentic lesson" is to presume its real-wordiness and furthermore that it will be of student interest.

The use of the words "authenticity" and "authentic" are shown in the literature to expose a presupposition toward real-worldliness. This presupposition reifies what is thought to be "authentic," regardless of the people involved in the context of that educational setting. In order to challenge this presupposition, or dereify it, we must go back to experience. Experience is always situated in a context and is experienced by someone, and as such it is interpretive. By exploring the experience of authentic learning I mean to look to the Heideggerian spirit of the original whole of being-in-the-world. The order-word status of authenticity indicates to me an over-looking of our being-in-the-world. Once being-in-the-world is neglected we find ourselves picking up the fragments and trying to piece them together into a coherent whole based on a reality of "substances" which have fallen away from the significance-whole of a world. It is in this way of being that we feel “obliged to construct ingenious theories to explain the commerce between a supposedly worldless subject and a cognized ‘world’” (King, 2001, p. 74). To problematize the order-word status of “authenticity” we must throw open the gates of meaning and reconsider it through a question of being, such as, “What is the experience of authentic learning in mathematics?”

Chapter 3

Shades of Meaning for Authenticity

In this chapter the ideas of Martin Heidegger will be considered as a problematization of the philosophical *and* pragmatic every-day dilemmas that helped establish the order-word of "authenticity." The works of Heidegger are complex and his particularly recursive and self-referential writing style seem to contribute to much confusion about his meaning. Yet his vision of the question of being can not so easily be articulated that we should hold him accountable for such misunderstandings. Indeed communication has already been discussed as a problematic issue. His many works are also quite expansive so to consider all of them is beyond the scope of this study. Therefore, for the most part, we will be discussing the ideas related to authenticity presented in his work, *Being and Time*. In viewing this one work of Heidegger I want to remind the reader, that while I find strong consideration to use the ideas I find in this work, we should not take the meanings found there as Heidegger's end-all way of viewing the issues. He wrote a great many works after *Being and Time* and his ideas of course evolved with his own being-in-the-world. The restriction of this study to *Being and Time* is due to Heidegger's extensive development of the idea of authenticity in this work.

Throughout *Being and Time*, Heidegger's terminology is in continual development as his many considerations bring new and deeper meanings to his terminology. We begin with a preparation for understanding one of Heidegger's key terms, *Da-sein*, which is important to his ideas about authenticity. The meaning of Da-sein is complex in its relation to Heidegger's other ideas and as such it will be continually developed throughout other sections, which will add dimension to the meaning of Da-sein. Secondly, Da-sein's constitution of being-in-the-world will be developed with a deliberation of the meaning of "world," the consideration of handy reality of things versus an objective reality, and a discussion of the self and others in connection with Da-sein's being-with-others-in-the-world. This will then have laid the ground for a discussion of the idea of authenticity as owned existence. In conclusion, I will recursively look at discourse and its meaning in light of this kind of Heideggerian thought and re-establish the importance of a study pointed to problematizing the order-word of "authenticity."

As we peruse the ideas of Heidegger we will look to what meanings there might be for "authenticity." Determining the new meanings of "authenticity" that Heidegger's ideas bring is difficult because much of what Heidegger discusses will be directly related to the idea of being-in-the-world and "everydayness." These words and their seeming connection to "authenticity" perhaps make for an easy reduction of "authenticity" to that of real-worldliness. However, Heidegger's ideas make it clear that "authenticity" can not mean what this facile reduction from our everydayness would have it to mean. Rather it is a mode of being of Da-sein operating from the complexities of being-in-the-world.

The Meaning of Da-sein

The full meaning of Da-sein cannot be given at the outset as a stand-alone definition apart from Heidegger's other ideas in *Being and Time*. In fact, the term

Da-sein seems to change meanings from Division One to Division Two of *Being and Time* as Heidegger moves to discuss an entirely different modality of human "being." As such, this section is meant only as a preparatory section, from which we might recursively re-seek the meaning of Da-sein throughout the other sections of this chapter.

Heidegger often used commonplace words in awkward ways so as to challenge their meanings. Da-sein is one of those words and as one reads through *Being and Time* the meaning of this word seems to be in constant development. A literal translation would note that in German, "Da" means "here," and that "-sein," means, "to be." So quite literally taken, Da-sein means "to be here." The ordinary German meaning of "to be here" is the existence of a thing (Zimmerman, 1981). Simple existence and "thing-ness" are not only reductions for what Heidegger has for the meaning of Da-sein, these very acts of reduction come from the unproblematic ways of thinking about human being. Heidegger will argue that these unproblematic ways of thinking are possible because of Da-sein's modality of inauthentic everydayness. We will further examine this in the section on "Authenticity and Inauthenticity as Modes of Da-sein."

Based on its more common meaning, one might consider Da-sein to mean "man" or "mankind." This would be a reduction of Heidegger's meaning for Da-sein because the use of these words typically reduces for us, the being of ourselves to substance, isolated existents, or a stagnant body of beings (noun). Heidegger's use of Da-sein in uncanny ways problematizes this common meaning. Heidegger sought with rigor the careful use and explanation of the meaning of his terminology.

For example, in dealing with the idea of the "self," Heidegger challenged the traditional neo-Platonic views of selfhood by rejecting the interpretation of the Greek word "ousia" as substance and replacing it with "the dynamic absence which lets a living being manifest its appearances" (Zimmerman, 1981). Heidegger saw that "ousia" as substance reduced for us, the meaning of self, to an object, as well as, reducing others to objects. Hence he sought to transform its meaning. Heidegger often attempts to re-appropriate the meanings of common vocabulary, such as Da-sein, without completely destroying its everyday sense (Heidegger, 1996, p. xiii). The word Da-sein must be understood as a re-appropriation of the meaning of human being and while its re-appropriation is not completely isolated from its usual sense, it is not to be thought about in our usual ways of man and mankind. So then, even though it seems to be immensely subtle I have come to prefer the use of "a human's being" for a person and "human being" for humankind. The use of the word "being" in both of these cases is meant in its verb sense, not the noun sense. It might be argued by some that these could even be misleading and that the meaning might better be related by "a human's becoming" and "human becoming." I would say however, and will purposely try to show, that the complexity of the meaning of "being" is suggestive of becoming and requires us to reorient our language to get-at the idea. Others who have looked at Heidegger's ideas about "being" have made similar observations (Doll, 1993; King, 2001; Zimmerman, 1981).

Important to any understanding of what Heidegger means by Da-sein is that he is not talking about “a human’s being” in one instance and “human being” in another. Through his use of Da-sein, Heidegger is simultaneously and conjunctively meaning “a human’s being,” and “human being.” Throughout *Being and Time* Da-sein is used in these ways precisely because Heidegger is not only examining the conditions by which it is possible to be a single and unique self, but also the existential-ontological elements of human being of which each single and unique self is an example (Zimmerman, 1981). The meaning of the question of being (human and otherwise) seems to unify our uniquely individual and multiplicitous ways of being. Heidegger (1996) expresses the terminology of Da-sein as related to the question of being in the following way:

> Thus to work out the question of being means to make a being—one who questions—transparent in its being. Asking this question, as a mode of *being* of a being, is itself essentially determined by what is asked about in it—being. This being which we ourselves in each case are and which includes inquiry among the possibilities of its being we formulate terminologically as Da-sein. (p. 7)

Given in this sense, Da-sein looks to the single and unique possession of our existence and the fact we more than exist as a rock would be said to exist, but that we have concern for, or questions about, the nature of being (Moran, 2000). Furthermore, we recognize others, as beings like ourselves who also have concern for the nature of being.

Da-sein is the embodied openness that discloses being. It is the clearing by which things, in their being, are made manifest. In his discussion about Da-sein, Mitchell (2001) explains that,

> Da-sein is like a space in which things let themselves be seen. If the phenomenal world is like a wood crowded with trees then Da-sein is the clearing in the forest, the space in which phenomena are made manifest. (p. 140)

As a self, Da-sein is not a transcendental ego but is rather itself the transcendence in its reaching-out or stretching-along. In other words, "human being" goes out beyond itself, as each Da-sein already is, and reaches to its possibilities of its being (King, 2001).

Heidegger is interested in analyzing our human way of being and he calls this his fundamental analysis of Da-sein. In doing this, he takes as "a priori" a fundamental structure of Da-sein called being-in-the-world. Any further illumination of the meaning of Da-sein must come from a discussion of Heidegger's claim that this structure of being-in-the-world is primordial and always whole. Through the foundation Heidegger builds on this fundamental structure he eventually concludes that, "The existential meaning is *Care*" (Heidegger, 1996, p. 41). That is to say, the meaning of Da-sein is care. With this analysis Heidegger sets out to show that Da-sein's fundamentally being-in-the-world is such that its whole existence is structured by "care" (Moran, 2000). The meaning of "being-in-the-world" needs to be further explicated before a developed sense of Da-sein as "care" can be explained.

The Meaning of Being-in-the-World

The meaning of world. To see the nature of Da-sein's being-in-the-world is to have a connected understanding of the interrelatedness of Da-sein and World. To have being-in-the-world means to be thrown into a context or a set of references in which Da-sein finds meaning. Each Da sein has his[1] own context from which he must either determine the way of his being, or let his being be determined by his everyday being-in-the-world-with-others. Each Da-sein's living context is first and foremost one of world *and* others. Furthermore, the world "is not something separate from Da-sein, but is Da-sein himself in the whole of his possibilities, which are essentially relational" (King, 2001, p. 60).

The world is not a thing, in and of itself, to be considered in its actuality and substantiality. "The world is the interrelated set of relationships which give form and content" to the experience of each Da-sein (Zimmerman, 1981, p. 27). World as these interrelated set of relationships is the reference-whole by which Da-sein *a priori* understands the whole of possibilities for its own being. This context gives Da-sein reference by which to understand his being-with-others, and that these others are recognized as fellow Da-sein in that they are in the-world in the same way (King, 2001).

With his idea of being-in-the-world Heidegger is attacking a strong philosophical disposition, attributed to Descartes and others who followed his line of reasoning. The reasoning of Descartes splits the self from the world, and then at once considers the self as an isolated identity not in the world. In this separated

[1] The use of "his" is not in any way meant to align or distill a gender bias. To avoid the awkwardness that comes from using gender neutral language I have elected to just use the masculine pronoun to generically speak of any human, male or female.

state Descartes and others then ask, “How can these two, self and world, be brought back together?” For Heidegger this line of reasoning is unable to see its own mistake. That is, “…if we can ask questions about Being, and ‘the world’ then this presumes that we have some sort of relation to the world, and that this is not separateness” (Mitchell, 2001, p. 125). Da-sein is this fundamental being-in-the-world and as such cannot be disclosed when considered in isolation from its context.

The primordial reality of handiness. Da-sein’s being is such that it discloses being. While rocks, plants, animals, and planets *exist* independent of Da-sein’s disclosure of them, their being and hence their *reality* is only comprehensible to Da-sein and is never separate from his being-in-the-world-with-them. There are many ways in which Da-sein discloses the being of being’s, but for Heidegger the way that is primary is the practical and everyday. The understanding and uncovering of the reality of things occurs by the for-seeing possibilities of Da-sein’s everyday being-in the-world. As Magda King (2001) aptly points out, Da-sein doesn’t experience things as mere substances that just by happen-stance show themselves in a universal space, but are revealed primarily in their handiness with the world.

According to King (2001), our tendency to objectify and make things substantial is a secondary mode of reality in which Da-sein separates things from the world. What is primary for Heidegger is the revealing of the handiness of things in the world, not the disjointing of things from their contexts. Typically the metaphysics and sciences are viewed as objectively truer in that they come closer to the "things-themselves." Heidegger flips this traditional notion on its head. King explains that

> We primarily understand that things are handily there, not for any accidental or arbitrary reasons, but because they can become accessible in their being only within a world. Everyday care understands the being of things from their relevance (Bewandtnis) to a world, and this is the way in which they can be discovered as they are "in themselves." It is quite erroneous to think that handiness is a "subjective coloring" we cast over things: it is a mode of being prescribed by the significance structure of the world, which enables us to understand things as they are "in themselves." (p. 72)

Only by revealing the being of things in their handiness is it even possible to approach things as substantial entities. To do so requires that Da-sein take a new, secondary look at things. This secondary looking breaks the reference-whole of Da-sein's being-in-the-world and seeing of things as for-the-sake-of some contextual projection. Instead, Da-sein's secondary revealing of the being of things shows them as things of certain qualities and properties.

By looking at things apart from their being-in-the-world with Da-sein, their whatness comes and covers over what they are for. In consideration of the 16th and 69th sections of Heidegger's *Being and Time*, King explains that it is only when

> Da-sein considers things out of context, in this secondary way that the mere whatness of things comes to the surface and hides what they are for. With this change, things are cut off from the *for the sake of* by reference to which they

> were originally understood as utensils. They "fall out of the world," they become unworlded (entweltlicht), and now present themselves as mere products of nature occurring in an indifferent universal space. (p. 73)

Hence, for Heidegger this traditional mode of reality, known as "substantial reality," or "objective presence," is secondary to the reality of the being of things in the world. As Heidegger himself puts it, "The being-in-itself of inner worldly beings is ontologically comprehensible only on the basis of the phenomenon of world" (Heidegger, 1996, p. 76).

Being-with others in the world. In our discussion about the meaning of world it was mentioned that each Da-sein has his own context from which he must either determine the way of his being, or let his being be determined by his everyday being-in-the-world-with-others. The world then, "is always already the one I share with the others. The world of Da-sein is a with world" (Heidegger, 1996, p. 118). Heidegger's consideration of being-in-the-world is far from a solitary viewing of each Da-sein. To be in a world, shared with others, where we relate to others is a primordial experience of being-in-the-world (Moran, 2000).

Being-with is disclosed to Da-sein in advance as a part of his own most being-in-the-world, and as such he relates himself to others. When Da-sein is alone and he is experiencing loneliness, it is because he has not ceased to be-with others. The being-with is so fundamental to Da-sein's self, that even when he believes he doesn't need the other and no longer seeks relevant association with them, this belief is only possible as a concealed mode of being-with. As King (2001) concisely put it, "He [Da-sein] understands them [others] in advance as the selves who are in the world in the same way as himself: their being has the same character of *for the sake of* as his own…His world is in advance a world he shares with others; his being-in-the-world is in itself a being-with-others-in-the-world" (p. 76). Here we see a deep connection between the self and other beings, in so much as Da-sein is being-in-the-world with them.

That Heidegger's idea of self is so intricately caught up with the being-with of others which is caught up with being-in-the-world should be no surprise at this point. The connectedness of his thought should be recognizable to us in that he is looking to set out the conditions for the possibility of experience of Da-sein (Mitchell, 2001). In this analyses Heidegger is further problematizing the Cartesian rationale by developing a meaning of 'self' that is not amiable to that of self as "substance," or "ego," or an identity "I." Da-sein's self is constituted by a fundamental structure of being-with, and this cannot be reduced, leaving Da-sein's self as an undifferentiated distinctiveness called "I" (King, 2001). How does the self then relate to experience?

The existential-ontological self as Heidegger seems to interpret it, is one that is active and creative, not susceptible to a reduction of thing-ness or mere presence. The possibility of experience comes from the interaction between this creative non-substantial self and a dynamic world. As was previously examined, based on the meaning of Da-sein's being-in-the-world, these two are inseparable for Heidegger and as such have no meaning apart from one another (Mitchell, 2001).

Da-sein finds himself inseparably thrown into a world in which he has care for things, care for others, and care for the possibilities that his being-in-the-world brings forth. The being of Da-sein in this way separates him from things, which neither care nor not care for their own being. Hence, it is in care, the fundamental constitution of Da-sein, that Da-sein is made whole.

Concern, being-toward, and possibilities. Da-sein has concern for things encountered in the world. This concern is not emotive but rather an orientation that Da-sein takes as a result of being-in-the-world (Mitchell, 2001). It is through concern that Da-sein's *being-toward* the entities in the world, is revealed. To understand the things encountered in the world we must have dealings with them, use them, and in that use be shown to have some particular concern for them in a way in which they are not considered in a world separated from us. Then by *being-toward* entities I simply mean our involvement in-the-world which characterizes these entities as useful or not in some particular way for us.

In Da-sein's being-toward he is projecting him-self forward to something like an aim. Da-sein conceives this aim as a possibility that may or may not be accomplished in the future. The aim lies already ahead of him and in conceiving of this aim as a possibility Da-sein "must be able to transcend, to go beyond himself as he already is to the possibilities of his being" (King, 2001, p. 32). In this way Da-sein exists primarily from the future, in being-towards his possibilities.

Da-sein is whole, in that his possible modalities of being are revealed in care. While Da-sein is the dynamic absence which let's things be made manifest, the manifestation of one's own possibilities is an existential choice of Da-sein. This is a choice that determines the way of Da-sein's being-a-self-in-the-world, and not the how he *is* in the world. Da-sein can choose to be undifferentiated everydayness, lostness in the "they," or one open to his finite temporality of being-in-the-world as care and as such modify his way of being-a-self-in-the-world to accept the possibilities that are uniquely his own.

Authenticity and Inauthenticity as Modes of Da-sein

The fundamental being-in-the-world-with-others allows Da-sein to be the clearing in which things are revealed. It is this being-with-the-others that creates averageness. This averageness is an existential of the "they." Heidegger (1996) says,

> The they maintains itself factically in the averageness of what is proper, what is allowed, and what is not. Of what is granted success and what is not. This averageness, which prescribes what can and may be ventured, watches over every exception which thrusts itself to the fore…everything gained by a struggle becomes something to be manipulated. Every mystery loses its power. (p. 127)

Da-sein's being-with others in the world tends to be leveled-off by the average everydayness of the "they." In Da-sein's everyday being in-the-world it most generally accepts what is given to it forth right, without stopping to consider what possibilities are his rather than what possibilities are the "they's."

In everydayness it is Da-sein's *tendency* to reduce things. With this reduction things are concealed in such a way that they are superficially taken by "understanding." In this way Da-sein understands the being of things at face value, accepting what the "they" has to say about things (Zimmerman, 1981). The "they" is everywhere in such a way that it presents every judgment as its own, taking responsibility away from Da sein. This disburdening of Da-sein's being "accommodates Da-sein in its tendency to take things easily and make them easy" (Heidegger, 1996, p. 128).

The "they" is not a definite group or thing that can be pinpointed and warned about. Each Da-sein's "they-self" is somewhat different. Da-sein's fundamental being-in-the-world-with-others opens him up to the "they," which is a kind of happening to which it can only be said that "no one did it" (Heidegger, 1996). The "they" is a way in which each Da-sein exists. King (2001) further elaborates,

> In his average everydayness, Da-sein finds the others in their care-taking being-in-the-world and finds himself among them as taking care with them…In this common absorption in the world, the "I myself" is not even clearly differentiated from all the other selves; the others are those among whom I also am, among whom I also find myself. In his self-forgetful everydayness, Da-sein is in the first place and for the most part not himself. (p. 79-80)

Da-sein then is most not himself when in his everydayness he is the "they-self." It is important to make the statement here that Heidegger is not saying that Da-sein ceases to be a self, but that his being is modified in a particular way, the way of the "they-self."

When Da-sein is in the particular mode of the "they-self" he is oriented toward things in their secondary reality, that is, things as "objective presence." This follows from everyday Da-sein's ready acceptance of what the "they-self" has established for the being *of* such things, instead of Da sein's concerned understanding of the being *for* such things. The ready acceptance of the "they" also conceals Da-sein's being. In this mode which conceals Da-sein's being he comes to see himself as object, substance, and identity. Zimmerman (1981) says that, "because I tend to conceal my Being, I tend to interpret myself as a thing. I encourage this tendency because I find it difficult to accept my finitude and shoulder the responsibility of freedom" (p. 52). To understand being in this way comes from the *inauthentic* mode of Da-sein.

On the other hand, as *authentic* Da-sein makes an existential modification of its everydayness, in that Da-sein freely takes over his being and allows it to be revealed as his own. As such, Da-sein doesn't exist in the world differently, but his orientation toward his being-in-the-world has been modified. Mitchell (2001) better articulates this idea in the following:

> When we encounter the world we have a choice; to stand out in the relation of Being-with, to impose our own possibilities on the world, and to give our own meanings to the entities we find as equipment, or to accept what we find and to attempt to dissolve distantiality and submerge our own Da-sein into that of "the they." (p. 132)

Mitchell goes on to discuss that Da-sein as inauthentic is disburdened in a way that makes him no longer need to create his own meanings or stand out in the world. Authentic Da-sein, on the other hand, stands out in the world of other Da-sein by revealing the world through his own being-in-the-world, with its projects that are uniquely his. Authenticity and Inauthenticity are ways of being, in that we may accept the meanings we find in the world, but as authentic we do so in an active and positive manner instead of the inauthentic way that would passively accept the meanings established by the "they."

For Heidegger, authenticity is not a thing, which can be captured and created. Authenticity is a mode of being which is caught up in our everyday being in-the-world in such a way that Da-sein actively approaches his own-most possibilities. This idea of authenticity is far removed from a meaning that is normally taken by the use of the word. The "authenticity" of the literature review can be interpreted as the "averaging" out of this meaning of authenticity. In its averaging out it no longer pertains to any particular Da-sein and as such pertains to everyone or precisely to "no one," otherwise known as the "they." When authentic is used in statements like, "I have created an authentic lesson," the average meaning is directly applied. In this case what is considered authentic pertains to everyone because of its implied inherent-ness to the "lesson" and hence it is applicable to "no one in particular" Authenticity is treated as a thing of quality within the lesson itself. Even the word "lesson" is problematic in that it too is a thing, devoid of those who enact it, or in Heideggerian terms, the word "lesson" seems to be absent of the Da-sein who would make its being manifest. Furthermore, the idea of "authentic," as that of any*thing* having correspondence to the real world, superficially establishes the being *of* what is authentic for Da-sein, instead of allowing Da-sein to see himself as the clearing *for* the being of such things in the world. In this way our own discourse about "authenticity" has become a discourse of the "they" which covers over Da-sein's being and hence, the more primordial way of understanding authenticity.

Recursive Consideration of the Need to Problematize Authenticity

Authenticity as a mode of being of Da-sein is not then what might be called a definite definition. In this sense of the word authenticity, there is a clear appeal to understand authenticity *as* and not authenticity *is*. For when it is said that authenticity *is* a mode of being which is caught up in our everyday being-in-the-world in such a way that Da-sein actively approaches his own-most possibilities, the *is*, is not meant to be a pointing to a definite quality or aspect of authenticity apart from Da-sein, but rather to reveal the complexity that resides with Da-sein's being *as* authentic. To continue a conversation about "authenticity" in a non-presumptive way will require that an approach to talking about "authenticity" that carefully plays this language game.

Heidegger (1996) considers discourse to be an "essential constitution" of our being. In Da-sein's inauthentic modality discourse is most susceptible to becoming what is called "idle talk." In idle talk Da-sein understands in an average way, without further inquiring into what the "talk" is all about, and then further speaks on what was heard. Idle talk is then a closing off of what is being talked about.

We interpret many things in this way, as our initial exposure to them is usually this average understanding. As it is not sensible to live without discourse, neither is it sensible to say we can live without idle talk. Heidegger (1996) demarcates Da-sein's discoursing in this way when he says, "All genuine understanding, interpreting and communication, rediscovery and new appropriation come about in it and out of it and against it" (p. 169). The "it" here is idle talk and Da-sein can not go "untouched and unseduced" by this way of interpreting, as this is how Da-sein himself initially came to know many things. Idle talk then is a "they" way of interpreting and as such it has already limited the ways or possibilities in which Da-sein can be affected by his being-in-the-world.

It is in this sense that idle talk is strongly related to the idea of order-words. Order-words carry with them explicit presuppositions towards certain acts of "social obligation" (Deleuze & Guattari, 1987). It is in idle talk that order-words become possible because each Da-sein that is discoursing is doing so always in-the-world-with-others. Since, according to Heidegger, idle talk is an inevitable way in which Da-sein discourses, order-words will always be a part of our discourse. So, as it was previously concluded, the words that create the contexts for "average" interpretations must be found, challenged, and de-reified. However, up to this point the order-word of "authenticity" has only been challenged by looking into its use in the literature, and by hermeneutically exploring another possible meaning by which it might be interpreted.

Through this hermeneutical exploration it was found that the everyday meaning of "authenticity" as found in the literature is itself, a leveling down and covering over the possibilities of authentic Da-sein. This analysis further reveals that what has been given thus far is not enough to challenge the order-word of authenticity. For even this chapter lays open to the everyday interpretations of idle talk.

What is needed is a method that secures "access to the phenomena themselves and the penetration through the prevailing disguises, the most dangerous of which, according to Heidegger, are those ossified concepts within a system that claim to be crystal clear, self-evident, and requiring no further justification" (King, 2001, p. 113). To avoid the reification of idle talk and its order-words the experience of "authenticity" should be considered. The discourse about "authenticity" should be grounded in that experience.

With a narrowing of focus to the learning of mathematics as it is experienced in a classroom by individual students I hope to find such ground so that an active and probing conversation about authenticity might emerge, and renew our discourse in a way that seeks to avoid its use as an order-word. In the next chapter the deliberation of such a method will be considered.

Chapter 4

A Research Methodology of Experiential Interpretation

The question of the experience of “authenticity” in learning must be wrested from the lived lives of students. Important to this end is a methodology that seeks to understand experience. Clark Moustakas' Heuristic Research: Design, Methodology, and Applications (1990) seemed to be the most appropriate approach to research the question “What is the experience of authentic learning in mathematics?” The qualitative research design of Moustakas accentuates the interaction of tacit knowledge and human experiences in order to illuminate further the meanings of those experiences. The model makes no distinction between researcher and participant nor does it attempt to isolate the experience from the researchers. The heuristic research methodology has six phases: initial engagement, immersion, incubation, illumination, explication, and creative synthesis. Through this method the inquiry of authentic learning will be engaged in a scientific way to find the meanings of this important human experience by self-inquiry, astute data collection through the being-with of students in mathematics class, and dialogue with students about their experiences of learning mathematics.

Heuristic Research as Hermeneutic Phenomenology

Heuristic research is a type of phenomenological research and as such this inquiry is intended to be one of description of the experience of authentic learning and not of explanation of what precise conditions can cause authentic learning. Phenomenology has itself gone through many manifestations due in large part to its different proponents like Husserl, Heidegger, Sartre, and Derrida. It challenged traditional epistemology in that it was grounded in neither rationalism nor empiricism. Indeed, phenomenology claimed to go beyond the subject-object dualism, as its proponents saw this dualism as a philosophical construction that imprecisely dealt with how humans experience the world (Moran, 2000). Heidegger took this further in showing that by his perspective this dualism was a secondary modification based on Da-sein's primordial being-in-the-world.

For the proponents of phenomenology coming after Husserl, starting with Heidegger, phenomenology becomes an interpretive endeavor. Phenomenology was a description of the way things appeared to consciousness, and for Heidegger *all* description was interpretive. To have description that was not interpretive required the subject-object dualism, in which the describer was no longer being-in-the-world. Of course, for Heidegger being-in-the-world is fundamental to our human being. As such, any attempt at wholesome description is only possible when situated within a historicized hermeneutics (Moran, 2000). Phenomena are our only access to the world because of our "own" being-in-the-world. Hence nothing can be said about phenomena apart from the situations in which they occur (Mitchell, 2001). Within this hermeneutic circle meaning is caught up with our own involvements and preoccupations with the world.

The interpretative nature of phenomena makes for a very diverse phenomenology. There is no one way to attempt to make manifest the things as they show themselves to be or said another way, "there is no such thing as *the one* phenomenology, and if there could be such a thing it would never become anything like philosophical technique" (Heidegger, 1982, p. 328). Moreover, Heidegger continues to say that the essential nature of method as a means of disclosing the being of things tends to organize in a way which it itself discloses. That is, every inquiry is guided before hand by what it is seeking and every answer in some part depends on the way the question is posed (Moran, 2000). It seems that for many proponents of phenomenology, it is more of a new way of seeing than a relation of philosophical propositions. Phenomenology is more of an approach to knowing than it is a theory of knowing.

The Heuristic research methodology is in many ways like a Heideggerian hermeneutical phenomenology. It is very much interpretive with its "emphasis on the investigator's internal frame of reference, self-searching, intuition, and indwelling" (Moustakas, 1990, p. 12). It is phenomenal in that it seeks to investigate some particular human experience. Though heuristic methodology seems to owe more to Husserl ideas of intuition than to Heidegger's recapitulation of phenomenology, it at the same time holds strong bonds to the hermeneutic and in that I feel is strongly adequate to approach the question of this study, "What is the experience of authentic learning in mathematics?"

Research Question

The research question emerged from the dynamic interplay of my lived experience as a teacher and the systematic searching that I was doing as a researcher. This is consistent with Moustakas' model of heuristic research. The suggested process for formulating a heuristic research question follows these five steps: (1) The researcher should freely list all areas of and aspects of interest. (2)

The list of related interests is then clustered into sub themes. (3) Discard the sub themes that imply causal relationships or contain intrinsic assumptions. (4) Mull over the remaining sub-themes until one, central question emerges that passionately awakens your interest. (5) Formulate the question in such a way that it specifies exactly what it is you want to know (Moustakas, 1990).

While in the process of formulating the research question, the experiences I had with my students played a pivotal role in the fleshing out of what I was most interested in researching. Experiences like the ones described in Chapter 1 helped guide and reveal the formulated research question. This is consistent with Moustakas' description of the formulation process:

> The question grows out of an intense interest in a particular problem or theme. The researcher's excitement and curiosity inspire the search; associations multiply as personal experiences bring the core of the problem into focus. As the fullness of the theme emerges, strands and tangents of it may complicate an articulation of a manageable and specific question. Yet this process of allowing all aspects to come into awareness is essential to the eventual formulation of a clear question. (p. 41)

By following the suggested steps by Moustakas for formulating a heuristic research question and being open to my lived experience as a teacher, I came to form the research question: "What is the experience of authentic learning in mathematics?" This question meets Moustakas' characteristics for heuristic research questions by: (1) Seeking to disclose more wholly the meaning of a human experience. (2) Seeking to discover qualitative rather than quantitative features of the phenomenon. (3) Engaging the whole of the researcher's being-in. (4) Not seeking to predict causal relationships. (5) Illuminating the phenomenon by cautious and intricate descriptions, illustrations, metaphors, and other creative rendering rather than by quantitative hierarchical structures of understanding phenomenon.

While maintaining these characteristics the question is simple and in concrete terms so that the study might reveal meanings of this particular human experience. Throughout the formulation process a number of embedded questions made it difficult to articulate this research question. For example: (1) What is the experience of authentic learning in mathematics from the teacher's point of view? (2) What is the experience of authentic learning in mathematics from a student's point of view? (3) What modifications in pedagogical practice might be suggested by the experience of authentic learning in mathematics? (4) What might the experience of authentic learning in mathematics mean for broader educational issues, such as school reform and national policy? These embedded questions only obfuscate the real potential lying in the clearing of the question: "What is the experience of authentic learning in mathematics?" The embedded questions not only employ a specific departure by which the research would seem suspect, but also limit the exploration of the possibilities that lie within its scope. For these reasons the broader question of the experience of authentic learning will be investigated.

Process of Heuristic Methodology

As a method heuristic research actively involves the researcher in self-processes and self-discoveries. In the words of Moustakas (1990) it is “a process of internal search through which one discovers the nature and meaning of experience and develops methods and procedures for further investigation and analysis” (p. 9). In this way heuristic research is unique, as other methods require little or no engagement on the part of the researcher into that which is being inquired about. Here, the researcher will throw themselves completely into the inquiry. It is even further required that the researcher have had a direct personal encounter with the phenomenon being investigated, and requires rigor, careful data collection, and a disciplined analysis. According to Moustakas, this immersion in the activity of the experience being investigated helps the researcher achieve an understanding of it. The six phases of heuristic research guide the researcher through the open-ended inquiry such that the self-directed search will enable the researcher to get inside the question.

These phases are initial engagement, immersion, incubation, illumination, explication, and creative synthesis. Moustakas (1990) describes the process involving these six phases of heuristic research as an “extremely demanding” one. Beginning with *Initial Engagement* the researcher works toward isolating the interests that are of “passionate concern” so as to wrest the question that resides within the researcher through explicit encounter with one’s autobiography and important “relationships within a social context” (p. 27). In *Immersion* the researcher engages the whole of their being around the question so as to be watchful to all possibilities for meaning. Anything connected with the question including intuitive clues or hunches become the raw material for immersion. *Incubation* allows a break from the intensity of the immersion process. The retreat from the intense focus allows tacit knowing and "intuition to continue to clarify and extend understanding on levels outside the immediate awareness" (p. 29). This incubation allows for a space in which an *Illumination* of spontaneous reorganization of significance and meaning may occur. When in the process of illumination, the researcher is in a receptive state of mind and without conscious awareness, an insight or modification occurs that reveals new constituents of the experience. From here the researcher is ready to fully examine the various layers of meaning. In this *Explication* phase, "the researcher explicates the major components of the phenomenon, in detail" and develops a depiction of the core themes (p. 31). Lastly, through *Creative Synthesis* the researcher moves "beyond any confined or constricted attention to the data itself and permit(s) an inward life on the question to grow, in such a way that a comprehensive expression of the essences of the phenomenon investigated is realized" (p. 32).

The validation process of heuristic research has the researcher constantly returning to the data to check the portrayal of the experience. As the primary investigator has collected and analyzed all the material and has been the one to sort through its relevance in elucidating the themes of that experience, the onus is on the primary investigator to achieve a valid depiction of the experience through the very process of heuristic research. Moustakas suggests that verification is enhanced by sharing with participants the themes and meanings derived from the research, and requesting their feedback as to the comprehensiveness and accuracy of these meanings.

By this process the study will seek to investigate with rigor the experience of authentic learning in mathematics, through the immersion in the being-with of students, as they pursue ideas of mathematics in and out of our classroom. This will be done with an “unwavering diligence to an understanding of both obvious and subtle elements of meaning and essence inherent in human issues, problems, questions, and concerns” (Moustakas, 1990, p. 37).

Context of the Study

Greater school context and dynamic structures. In the perspective of a heuristic research methodology the context of the study is a vast space of researcher interaction with and researcher reflection on being-in-the-world in a way that is focused on the question at hand. The range of large to small contexts in which this research took place cannot hope to be fully explicated but in what followsis provided an adequate amount of information by which to apply an understanding of what the students have to say about the experience of authentic learning in mathematics.

The research was conducted with secondary students, who are mostly upperclassmen, Juniors, at our high school. The school the research was conducted in was created in 2001 as an inner-city charter high school. It was chartered by a nonprofit ommunity service organization for the purpose of creating a safe and caring learning environment that fostered the all around success of the students in the local community. Students were drawn from a lottery of those who filled out application forms. There were no pre-requisite exams, grades, or benchmarks by which students were culled. All students who applied to come to the school were admitted to the lottery and selected in a blind and random process. The demographics of the student body correspond highly to the local community population.

The school began as only a 9th grade with 120 students. Each of the following years, the school added 120 more 9th grade students as the previous years students moved up to 10th grade. This would continue until each grade level had a total of 120 students with a total school population of 480. New faculty members were hired each year to accommodate the school growth.

Demographically the school has a population that is 78% minority; 60% Hispanic, 10% African American, 8% Native American, and 22% White. The school is located near the center of the largest metropolitan city in the state. The number of students who qualify for free and reduced lunch is just above 91%. The average class size is less than 20 students per class. There is one computer for every two students.

While the demographics are far from telling the story of a school, they do give us an *everyday* way of understanding it. Most of the students live in poverty and this brings with it a host of issues that change the dynamics of a school. In other words, the issues of poverty have changed and continue to change the way this school has organized itself.

After the first year the faculty and administration decided that they would establish a recursive transitioning with the grade level students they first taught.

Each group of core teachers transitioned with their students to the next grade level curriculum. Through much discussion and scheduling restructuring it was decided that this would best serve the needs of the students. Since the teachers already knew their parents, their influences from family and friends, and their academic experience, the faculty felt that it could start each successive year hitting the ground running, instead of relearning the history of a new set of students. This is particularly relevant to the study in that the research was with students that I had been teaching for a period of three years. As close as I got to students in only one year of teaching, my knowledge of them, academically and personally, grew exponentially as I taught them over this longer period of time. Due to the length of our time together I was able to go deeper into my students' experiences of authentic learning, and to see their perspectives more clearly.

The schedule of the school is also relevant to the context of the study. Each year the faculty and administration struggled to find a schedule that would address some of the issues we were facing as teachers of these students in the inner city. During the year of the study we decided on what might be understood as a half-block/half daily school schedule. As such the students attended school 5 days a week but only came to my class four days. One of those days was an extended time period of 85 minutes. The day of extended time allowed the space for some larger tasks to be given that would have been very difficult to do with half the time.

Before any reasonable pursuit of understanding the student's perspectives on "authentic" leaning in mathematics the philosophy and practices of our classroom should be described at some length. I feel that as a matter of validity the reader needs to understand, even if it is forced into "averageness" the spirit of our classroom dynamics before trying to make sense of what my students have to say about their experiences. In any sense, our classroom should not be thought of as one in which the mathematical ideas in the teacher and greater society are transmitted to the students. While saying this may be old-hat in the sense that most educators wouldn't say they believe in the transmission model, much of educational practice in the form of lesson design, implementation, and assessment reveals a strong hold on our underlying assumptions about learning from a transmission perspective. As an alternative I have sought out another way of approaching the learning of mathematics.

The spirit of problem centered learning. The way the curriculum unfolds in the classroom understudy can be called, in an "average" sense, problem centered learning. The curriculum is much more dynamic than that particular label might hold for some people. While some would quite possibly be tempted to call problem centered learning a teaching methodology, my experience with it has held too much serendipity for me to understand it as such. Before describing some of my experiences with problem centered learning let's turn to a more explicit description of it.

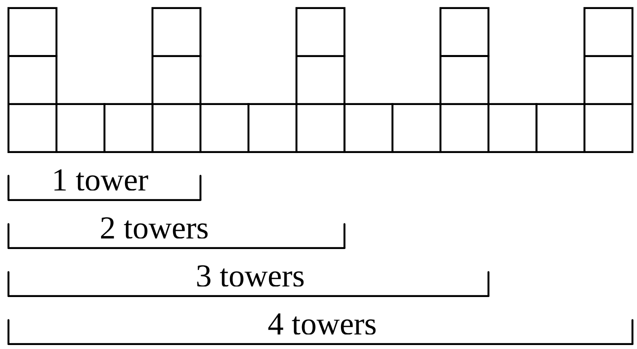
Much like the routine of a traditional classroom problem centered learning is not without its recommended routine. For example, the teacher starts the class with some kind of problem, task, or dilemma. The students then work on the problem with their partner for 30 minutes or so, and then students present their solutions to the class as a whole or have a whole-class discussion about the problem. Through the teaching of five classes of algebra II for the duration of this study, such a delineation of time was found be problematic. Some classes might work with this timeguideline near perfectly but others would need double the time, even a couple of days before they were ready to discuss it. On some occasions it was months before the classes in this study were ready to come back and revisit the problem that had been posed. On rare occasion the class needed more time to consider thethan is given by the scheduled school year.

Problem centered learning has its theoretical basis in constructivism. Constructivism is an epistemology that has two main principles. These principles state, that knowledge is not passively received but is actively constructed, and the function of cognition is adaptive and serves to organize the experiential world, not the discovery of ontological reality (von Glasersfeld, 1995). When these principles are applied to learning in the mathematics classroom the claim is that mathematics is learned best by experimenting, questioning, reflecting, discovering, inventing, and discussing (Ahmed, 1987). Through corroboration with others, students learn to organize their ideas. Corroboration in a constructivist-learning environment occurs through cooperation with other students *and* challenges offered by others. The role of the teacher is then to provide an environment and tasks that would generate student corroboration about constructs and allow students to make it their own reasonable knowledge.

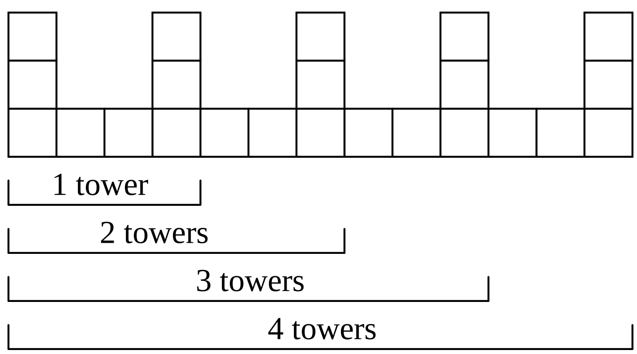
To accomplish this problem centered learning has three elements: tasks, groups, and sharing (Wheatley, 1991). First, the teacher carefully selects tasks that are accessible to all the students, embody the central ideas of the discipline, and are problematic for students. Task selection is one of the most difficult aspects of problem centered learning. It requires that the teacher not only pick the task, but do so in a mindful way so as to be sensitive to student's ways and patterns of thinking so as to choose task that might perturb these types of thinking. On more than one occasion a task was given too soon, or too late. The inappropriateness of the task's timing was revealed by the students as they either winced in the face of the task, with "we don't get it, what is the issue here?" or, "this is just the same old thing." On the occasions when students quickly recognized a parallel problem or idea the task was given to challenge students to recursively consider their previous ideas. The particular tasks given to challenge older ideas in new ways however failed for some classes, though not for all, and on that note, to say they failed for a class is not to mean that they failed for all students in that class, but only most of them.

Once a good task is selected the students work in groups, which in the classes under investigation, is usually restricted to pairs, to find a solution to the problem. Towards the end of the time together, students share their efforts and bounce ideas off of one another. Through the spirit of this engaged and meandering curriculum students make and revise their own conjectures based on teacher selected activities. The students have often times developed their own questions beyond the considered scope of the task. Upon this occurrence the activity becomes student selected. This sometimes occurs as an individual endeavor eventually shared with the rest of the classroom community, or it may develop into a full fledged class inquiry, unanticipated by the teacher.

As one example of some of the issues that these students have encountered in this way of teaching let me describe a particularly rich task with my algebra students. After playing with number patterns the algebra 1 students were given the challenge of finding a way to predict quantities based on the patterns they found. The task was given to perturb student's ways of approaching number patterns as very few of them had mentioned that they saw a connection between variable, formula, and pattern. Most of the students were still recognizing the pattern and then exhaustively repeating it to find a solution. As an attempt to problematize this process I gave my students the following task:



Use the tower pattern to find the ***number of blocks*** it would take to build **<u>10, 20, and 100</u>** towers. You may use centimeter cubes or any other objects. Explain any reasoning you used to figure out the problem.



Use the tower pattern to find the ***Perimeter*** of **<u>10, 20, and 100</u>** towers. You may use centimeter cubes or any other objects. Explain any reasoning you used to figure out the problem. Can you find the perimeter of 123,456 towers??

Figure 4.1. Task given to students to problematize their methods of patterning and predicting.

While there are multiplicitous ways in which one might approach solving these tasks as the discussion about student solutions will show, the most typical mathematical methods would have students identify a constant rate of change (slope) as the number of towers increases and then isolate from that rate of change an initial amount. That is to say, the pattern is linear and as such follows the basic pattern y = mx + b. So in the first task one might see that each time we add a tower, the number of blocks increases by 5. Since the first tower has eight, and the increase is 5 blocks per added tower, one must then have an initial amount of 3. So one would get the number of blocks (N) can be determined by the number of towers (T), when N = 5T + 3. Similarly, on the second task one might note the increase in the perimeter is 10 units for each tower added, and then the initial amount is 8. Hence the perimeter is determined by P = 10T + 8. These formulas can help find the number of blocks and the perimeter given only the number of towers in the pattern, as show in Table 4.1.

Table 4.1

T = Number of Towers	N = Number of Blocks	P = Perimeter
10	53	108
20	103	208
100	503	1008

The two tasks presented here helped to unfold a lot of discussion about how a formula can be connected to a number pattern. Students were given the option of using interlocking centimeter cubes to aid in the continued construction of the pattern. Some students chose to use the interlocking centimeter cubes, some chose to continue drawing the pattern, while still others made tables or claimed that they had all the information they needed to answer the questions. All of these various ways of approaching the problem were then discussed as each pair of students presented their ideas and ways of getting at a solution.

In working through the problems one might quickly see that both of these tasks present somewhat simple, linear formulas, and hence lend themselves to only a few different renderings of the formula by which a “correct” answer might be attained. While such things can perhaps be said of a “task” apart from those who are engaged in it, as will be described it was nothing of the sort. Through the dynamics of the classroom, the task being part of that interaction, a great deal of learning occurred.

The students did not readily see the connection between these patterns and the pieces of mathematics that they had learned prior to their time in our classroom. Some of the students had pre-algebra and had covered the commonly taught mathematical things like slope, $y = mx + b$, and simple functions. Though we had great discourse about the task and our working through it, absent from the discourse were words and phrases like, slope, rate of change (increase), linear equation (function), y-intercept (initial amount), and the relation between two sets of numbers. In earlier encounters with the spirit of problem centered learning I found it very curious that my students could be talking about these same ideas, without a “mathematical” vocabulary.

One such discussion took place during the presentation time on the first task about the number of blocks. Two very vocal groups went about solving the problem in a similar way, but with different amounts of towers being considered. The first group drew a fifth tower on the paper and counted 28 blocks for 5 towers. They then doubled this amount to get 56 total blocks for 10 towers. They were asked why they wanted to consider 5 towers instead the given 4 towers. They replied, “because 5 goes into 10, 20, and a 100. The number 4 doesn’t go into 10 and it doesn’t work out right.” I inquired further as to what was meant by "it doesn't work out right." They explained that they had first used 4 towers and found that it had 23 blocks. They then said, to make 10, we need to double four and add half of it. So 23+23 was 46 . This they thought would give them 8 towers worth of blocks. The problem for them came in the division of the four towers into half. Since 23 isn't divisible by 2, they concluded that 4 towers could not be used to solve the problem. They accused me of giving four towers on purpose, in that I knew ahead of time it wouldn't work and that it was a trick I was playing on them. Amused as I was that they felt I would purposefully trick them, in the back of my head I was thinking, "how does a teacher ask students to think about, that which they are not thinking about, without just giving them the answer?" The answer would come from where I would least expect it.

The second group was working on a similar strategy. However, they had used the interlocking centimeter cubes to extend the tower pattern out to 10 towers and then counted the number of blocks. Each group presented their findings and their way of finding the answer by the end of one 85 minute class period. Table 4.2 shows the findings of each group.

Table 4.2

GROUP 1		GROUP 2	
Number of Towers	Number of Blocks	Number of Towers	Number of Blocks
10	28 * 2 = 56	10	53
20	56 * 2 = 112	20	53 * 2 = 106
100	56 * 10 = 560	100	53 *10 = 530

These two groups used a multiplicative method to try and quickly get at the number of blocks for each number of towers. The base unit by which they were multiplying was however, different. Group 1 was using the number of blocks for

5 towers (28) and Group 2 was using the number of blocks for 10 towers (53). Group two was quick to point that Group 1 had not been consistent with what they multiplied by. To get 10 towers worth of blocks they took the number of blocks from 5 towers and doubled it. Once they got 10 towers Group 1 used the number of blocks for 10 to find the number of blocks in 20 and 100 towers.

After noticing and thinking about this inconsistency Group 1 replied that it only stood to reason that if you multiply the number of blocks in a certain number of towers to get the number blocks in a larger number of towers then it should work again. Somewhat confused the class began to discuss if this task could be solved without making the 100 towers and then just counting. Alma shouted, “There is a pattern right in front our faces, we can see it (referring to the picture), so it can’t be that hard.” Alma was part of a group who were trying to by pass the monotony of counting by first finding a formula and then applying it. While this group did not have anydefinite answers, they said that they didn’t think Group 1 or Group 2 had a correct result, but they couldn’t say why Groups 1 and 2 were not correct.

One of my students, Samuel, had come in late that day and since groups had already been arranged he chose to work by himself. It so happens thatSamuel was on an individual education plan (IEP) and had been in remedial mathematics classes most of his formal education. When he began working on the problem he started by making one tower out of interlocking centimeter cubes, and then two towers, and so on. He would place each new construction one behind the other and look at it intently for a few minutes. After a while I noticed he had switched to paper and was drawing towers. When the class came to the point in their conversation that they were debating about being forced to count, Samuel stood up and unfolded several papers that he had taped together. He said, “Here, I have already done it, I have drawn all 100 towers and I counted 503 blocks.” The class seemed to smile all at once and someone offered to recount as a double check for the right solution.

While the recount was going on the class began to discuss why their estimates were too high. I then asked Samuel why he felt he had to draw all of the towers to get the correct answer. Samuel replied, “Because when I doubled it I got the wrong answer.” This statement did not make sense to anyone. I inquired further by asking what was meant by “the wrong answer?” Samuel said, “It was too big because I had three more blocks that I didn’t have when I just counted it.” One student brought to the attention of the rest of the class that when you double Group 1’s 28 blocks for 5 towers you get 56 blocks for 10 towers, which is exactly 3 more than what Group 2 had found for the number of blocks in 10 towers. Alas, it was the end of class. Samuel approached me after class and said that even though he didn’t have a formula he knew why Group 1 and Group 2 were getting different answers. I asked him to show the class the next day and he presented a drawing to the class that looked like Figure 4.2.

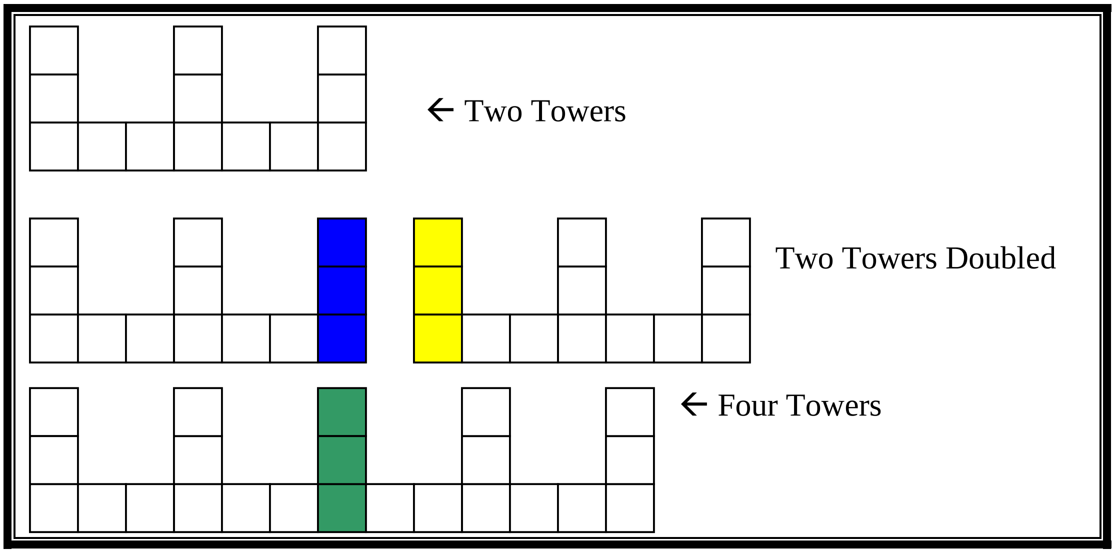


Figure 4.2. Samuel's explanation for why doubling gives three blocks too many.

When Samuel presented this at the first of class the next day a couple students spoke up and said that they had also noticed this overlapping. Convinced of his point, Groups 1 and 2 set out to create a formula that would compensate for this overlapping. I must in all honesty confess that at this point I felt that both of these groups would give something like an expression y = mx + b. I was however, gladly mistaken.

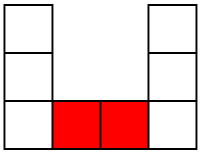
Using the unit of 28 blocks for 5 towers Group 1 sought to eliminate their overlap in a systematic way. As the problem only asked for the number of blocks for 10, 20, and 100 towers, the students found a way to get those answers, and in fact, the answers for any multiple of their unit of 5 towers. They noticed that to get 10 towers worth of blocks they could double the number of blocks in 5 towers and subtract the 3 blocks of overlap. Accounting for the overlap they generated the following pattern that led to their formula.

$$5T = 28(1) - 3(0) = 28 \ \ blocks$$
$$10T = 28(2) - 3(1) = 53 \ \ blocks$$
$$15T = 28(3) - 3(2) = 78 \ \ blocks$$
$$20T = 28(4) - 3(3) = 103 \ \ blocks$$
$$\vdots$$
$$100T = 28(20) - 3(19) = 503 \ \ blocks$$
$$General \ \ Number \ \ of \ \ Blocks \ \ (GB) \, for \ \ (T) \ \ Towers$$
$$GB = 28(\frac{T}{5}) - 3(\frac{T}{5} - 1)$$

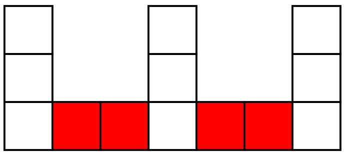
Figure 4.3. Group 1's pattern and formulaic solution to the tower task.

Group 2 found a similar result based on their unit of 10 towers. Each group made note of the difficulty in applying their formulas to any number of towers that was not a multiple of their base. As an extension the class was challenged to come up with a way to modify these formulas to account for any number of towers.

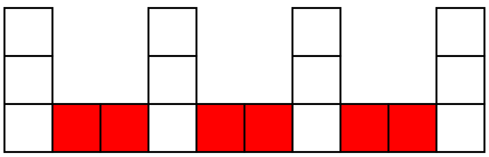
A couple of the other groups presented still different ways of figuring out the number of blocks for a given number of towers. By looking to partial patterns that applied to the whole, they were able to understand the problem in a different way. Group 3 saw two parts to the pattern: "vertical" blocks and "middle" blocks.



1 tower has 2 middle blocks

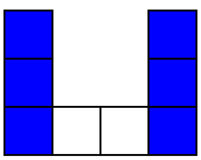


2 towers have 4 middle blocks

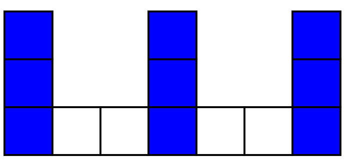


3 towers have 6 middle blocks

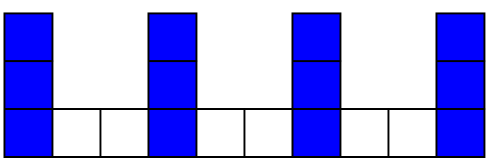
So we have the number of towers times two to get the number of middle blocks.



1 tower has 6 vertical blocks



2 towers have 9 vertical blocks



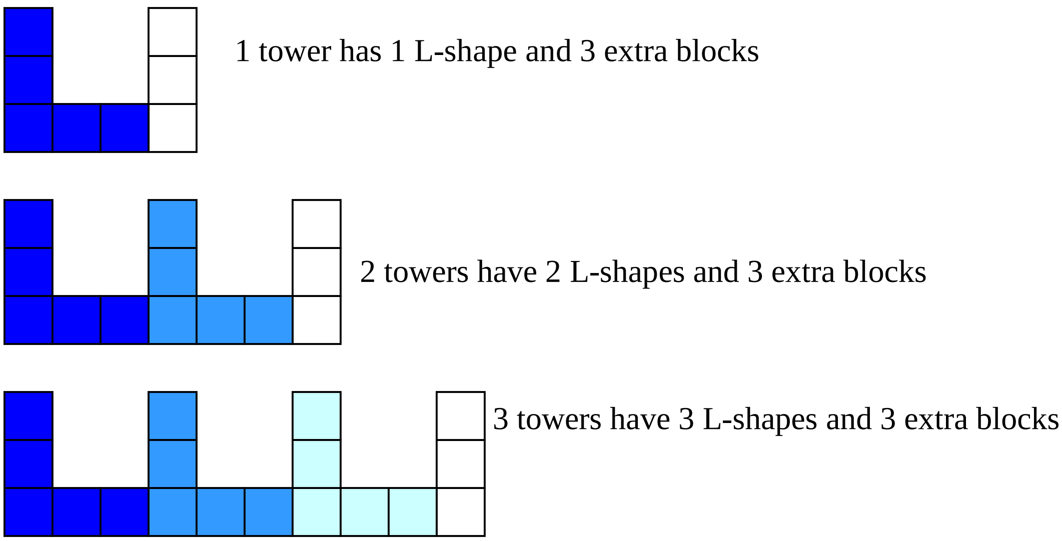
3 towers have 12 vertical blocks

Since the number of blocks in each column is 3 we have to multiply the number of towers times 3 and add 3 more each time. So we take the number of towers and add one tower times 3 to get the number of blocks. This gave us the formula:

$B = 2T + 3(T + 1)$. ***B is the blocks and T is the towers.***

Figure 4.4. The patterns Group 3 noticed and their final formula which combines these two patterns.

The class found this way of solving the task very intriguing and a student from Group 1 said, “This way is so easy that it’s genius. It is a lot simpler than dealing with multiplying groups of towers.” Group 4 presented another “easy” way of solving the task by recognizing an L-shaped pattern among the tower repetitions.



Each L-shape always has 5 blocks. So we have that the total number of blocks should be found by the formula: $B = 5T + 3$ ***when B is the # of blocks and T is the towers.***

Figure 4.5. Group 4’s presentation of the pattern they found to solve the tower task.

All the groups came to a consensus that each others ways of solving the problem made sense to them and that they had no further questions for one-another.

The second part of this task was to find the perimeter of the same tower pattern. Many similar discussions arose during our trials with the perimeter task. The development of the perimeter will not be considered in full because of its similarity to what was just described, but one discussion that came about because of the perimeter task that did not happen previously was interesting. Some students had previously constructed the tower pattern using the interlocking centimeter cubes. As these cubes are 3-dimensional it raised a question as to what was meant by perimeter. In my experience as a student I had often found it easy to over look the distinction between perimeter and surface area.

These students were now engaged in a conversation about perimeter. One of them touted off that it was the distance around an object. This was problematic for many of them because to count the sides around the 3-dimensional figure would give a different number than the 2-dimensional figure. As a class the students decided that surface area was the proper word for the area around the cubes, since they were flat and not just lengths which would be called distances. Such conversations happen frequently in our classroom. Though I admit that I am often tempted to squelch the debate about terminology it is to the student's benefit to really struggle with it, and the time spent in struggling can make all the difference in the way they approach learning mathematics.

While much more could be said about what has been presented here, it is not my purpose to do a full description of everything that happened during our trials to solve these tasks. The richness of the tasks and the interaction students had with them and each other, went far beyond what I thought might happen. This task was chosen because its discussion would be about happenings that occurred more than a year before this study on "authenticity" formally began. The example is rich enough to give a good idea about the happenings in our classroom. The purpose of this description is to help the reader have some kind of a feeling for what has gone on in the mathematics classroom that these students have engaged in over a three year period. Often when someone speaks of teaching a discipline such as mathematics it is easy to understand their teaching as a leveled-off form, in which certain routines and experiences have become the way of understanding that practice without ever “getting to the bottom” of what is meant. The description given here details the types of happenings that occur in the classes the research will focus on.

Procedure

Following the recommendations from Moustakas (1990) I entered *initial engagement* and sought a research question most pertinent to an engaged exploration of the happenings and experiences of learning the students were having. Through many experiences with students, some of which have been describe above, but also as a student I chose the question that connected my most endearing interests. It was chosen and worded in such a way so as to avoid the intention of finding causality and prediction. I ask the question, not in the hope of finding *the* prescriptive method by which all teachers might make learning authentic for their students, but to rather describe the phenomenon of authentic learning as it appears to me and is described by my students.

As I began the *Immersion* phase, I noted anything that seemed to be related to the inquiry, without giving any further significance to it. I listened to student explanations and sought their experience of learning mathematics. I carefully recorded any readings that I came across that seemed, even if remotely related to the experience of authentic learning in mathematics. Classroom incidents and student behavior sometimes caught my attention and were recorded. To go deeper into the phenomena the students involved were interviewed about their experience as well as asked to write about their experience of learning mathematics. After reviewing researcher notes and having informal discussions with students, or in some cases the entire class, the students were asked to write over a specific classroom interaction or happening. Students were also asked to reflect on their learning throughout the duration of this study and document their reflections in a portfolio.

From day to day I am a teacher, in that; I am in a classroom with students for 6 hours. So letting *incubation* occur was somewhat difficult for this inquiry. I did however get a chance for a couple weeks. During the time of this study I was working with a student teacher. When the appropriate time came I let go of my classroom involvement and let her be the lone teacher of my students. During these two weeks I took sometime to do some leisurely reading and help out around the school's office. It allowed me sufficient space by which *Illumination* could occur. Throughout the rest of the study there were brief culminating periods when it seemed as though immersion, incubation, and illumination would occur quite rapidly.

Through illumination the themes of authentic learning began to develop. During the *Explication* phase my thoughts turned to make explicit the emerging themes. I sought to understand the data more fully. While my readings of literature and philosophy were being done throughout the course of this study, it was during this phase that I came to realize the connection between the authentic experiences of learning my students had described and Heidegger's ideas about authenticity.

It was this connection that led me to consider how my *creative synthesis* should give form to the ideas and experiences of my students as these experiences related to experiences of authentically learning mathematics. Through the narrative accounts of the students the findings of the study will descriptively explicate the themes, as my students are themselves the clearings by which authentic learning is made manifest.

Chapter 5

The Dynamic and Interconnected Clearings of Authenticity

Clearings are the dynamic spaces in which the lived lives of students made manifest the relational patterns that I will be describing. In a most general sense each student is a clearing in which things are made manifest. Through our days together in and out of the classroom students revealed to me the diversity of their clearings and the many ways in which these clearings are modified by them. By modified I mean that the student's way of being the clearing changed, was in flux, and in that, the clearing often presented itself differently. As our classroom was one that sought to be engaged in the ideas of mathematics the discussion will begin there.

In this chapter the findings of my research will be explicated through the discussion of four areas; mathematics, relationship, occupation, and newness. These four aspects of their experiences with mathematics constitute the terrain of their understandings, the clearings of their experiences with mathematics, as described below. The discussion of each of these clearings must look to the particular contexts of students in their being-toward mathematics, their being-with others, their concern-for things in the world, and their stretching to the own-most possibilities of their being. Through the narratives described in this chapter and organized around these four themes the answer to the research question, "What is the experience of authentic learning in mathematics?" will be explicated.

The Clearings of Mathematics

Shades of meaning for mathematics. Mathematics is a clearing in that it is whatever student's reveal it to be in their being-toward it. It is easy for educators to operate under the assumption that the "average" and everyday understanding of mathematics is the view that students will naturally have. This assumption makes it easy for some educators to create what is called an "effective" lesson for learning a particular topic. What is neglected in this assumption is that student's experience can be vastly different from the teacher's and one another's experience. How students see a thing, like mathematics, is interpreted from their experience; not some transmitted idea from the teacher. Each, through their own walk with mathematics, creates an individual experiential clearing.

The clearing of mathematics is revealed by the students' being-toward it. In a student's being-toward there is a collage of interactions with the other clearings that will be discussed later, as well as the student's beliefs about mathematics and its association with them and the world. It became apparent to me in the course of this research that my students had vastly different beliefs about what mathematics was all about. Part of this was an erroneous assumption on my part. I assumed that since I had been their teacher for nearly three years they would have all begun to migrate toward a particular perspective about mathematics. Of course I was in error because of my lack of consideration for the many years of school and life experiences that happened prior to our meeting as well as my lack of consideration for how the students experience our classroom activities in different ways. When I realized this I asked the students to tell me what they thought mathematics was all about and why humans engage in it.

From this question came many good examples of my sobering realization that my students saw and experienced (the study of) mathematics differently. Their different ways of being-towards mathematics were wrapped up in the complexities of their being. Furthermore, in the students' being-toward mathematics changes modalities as it is related to the other clearings. The mode of the clearing of mathematics that the students are in at any particular time is wrapped up in and with, all the other clearings of their being. One of my biggest surprises camefro m Brian[2], a student who loves to engage in anything abstract and often comes in to just talk about theories of physics, artificial intelligence, properties of exponents, etcetera. From his reflection about what mathematics is and why humans engage in it Brian said,

> Math is the study of numbers and how they relate, not just to each other but to everything. Math relates to quite possible every single thing that happens in the universe. Disciples of Pythagoras actually worshipped numbers, because they knew they were the basis of all things, and they may have had a valid point. I mean think about it. Just thinking requires chemical responses and nerve responses in your brain. These all move at a certain rate of speed in relation to the actual completion of thought, so booya there's your math. Humans do math because to know the working of all numbers is to know the workings of the world. If you know that the sun has a distance of 93 million miles from the earth, you know that given the earth is in a constant elliptical

[2] All of the students' names presented in this research are pseudonyms.

> orbit, it is unlikely to crash into the sun, this gives you the edge over that little kid who seriously believes we're all going to be incinerated by the suns heat tomorrow because his brother told him so. Math is the knowledge to decipher things through numbers. If my family all wants equal slices of pie and we're not allowed to eat until someone figures out how much pie we each get, it is actually vital to my survival to understand fractions, other wise I may die of starvation, its just everything has a mathematical basis somewhere, you just have to look hard enough to find it.
>
> --Portfolio Entry

This kind of view of mathematics greatly surprised me because of some of the questions Brian had asked. Some of his questions really intuitively challenge the way mathematics is talked about and thought about. One of his most insightful questions occurred during the study of area. During the tasksthat challenged students to find the area of a circle he said,

> I just realized something. We are always trying to find the area of things based on the shape of a square. I mean our units even say it, duh! The area of this triangle is 12 *square units*. Why a square? How would our formulas for area be different if we asked, how many circular units went into a square instead of how many square units are in a circle?
>
> --Taken from researcher notes of a classroom conversation

With this kind of questioning of the foundations of how we talk and think about mathematics I had assumed Brian was coming to think about where our mathematics comes from instead of the idea that mathematics underlies everything. While Brian often exhibits this type of being-toward mathematics, what I have found is that Brian's being-toward mathematics changes and that it is dangerous to assume what his idea of mathematics will be for a given task.

Brian has a humorous personality and is a strong mathematics student. He likes to ask questions important to the discipline in his own unique way. During a classroom conversation about polynomial roots Brian asked,

> Why are they called 'roots' of the polynomial? Are we to believe that these polynomials have something to do with plants?
>
> --Classroom conversation

Through our classroom inquiries these questions eventually led to a metaphor of trees for polynomial roots. A tree cannot begin to grow with out its seed, which is like a root since it takes in nutrients and provides them to cells for their division. The roots take in more nutrients that allow for the further growth of the tree. Knowing the polynomial roots does the same thing, in that, if the roots are known, the entire polynomial can be reconstructed and graphed.

Brian rarely takes notes in class and loves to figure things out for himself. He furthermore enjoys arguing his point when he feels strongly about it. Brian's view that mathematics underlies everything really shows through in his everyday conversations about science, history, religion, and politics. He loves to appeal to statistics and theory to make his case. Atthe beginning of the year Brian's voice was very dominant in our class. The sophisticated way he presented his solutions was intimidating to some students. As the year went along and other students gained more confidence in their own ways of making sense of the problems we were working on, he found himself perturbed to rethink his ideas and approaches to the problems we were solving.

Unlike Brian, who has been my student for three years, Rae has only been with the school for one year. Rae's views of mathematics are different than Brian's views. Mathematics is not an underlying element of all things, but is rather a tool of ever increasing complexity to use throughout life. In her reflection about mathematics Rae mentions matrix codes and compounding money, two of our classroom activities. The latter of these will be discussed at length in the next chapter. During our study of matrices we took some time to play around with coding messages using different matrices and different matrix operations. Part of what I was interesting in seeing through these tasks was the students' ideas about decoding. Typically we use the inverse for this but students had no prior knowledge of finding inverses. For extra credit I created a bulletin board with a coded message in the main hallway for the entire school to see. Rae was the first in her class to break the code. So when she was asked to write about what mathematics was all about she wrote,

> Whenever I was little I thought math was all about adding and subtracting and I thought that it was just something they made you do at school. Now that I am older I have found out that it involves more complex things than just adding and subtracting and it is more useful in life than just using it to count things or to count money and not get cheated when you are buying something. Lately I have found out that it is useful in many other ways and in many different aspects of life and not only to count. I think that the number one

reason why humans engage in it is because it is useful like the government needs codes so to hide their top secrets and so they use Matrix codes. Or another example is the interest formulas that people use in business to find out how much money they will have in X amount of years. The second reason that I think humans engage in it is if you're like Mr. Matney, you just have a love for math, and those types of people make it a little more simple for us to learn.

--Excerpts from Rae's portfolio entry

Throughout the year, Rae began to realize the vast number of ways that humans use these abstract relationships in their various worldly "occupations." Rae also recognizes that her teacher is different in his being-toward mathematics, in that, he appears to love mathematics for its own study and not just for the wealth of application. Rae rarely asks me a question in class and when she does it is usually about checking whether her mechanisms for solving are correct.

Martin presents a somewhat different picture of mathematics than either Brian or Rae. For him it is a connected web of interlocking ideas. It seems to him that only by using this vast connected space can math be used to solve anything we want. He backs up this belief by pointing to many of our classroom tasks and activities. Like Rae he mentions matrix coding but also mentions finding someone who is lost in a forest and that computers have a connection to mathematics. In speaking about these experiences with mathematics he says,

To this day I have made a lot of mistakes too many to count. Thus, this wasn't in vain. Math has always been an experience to reveal my knowledge and

> even a challenge to create shortcuts for existing solving processes. I had already come to the conclusion that all math is connected no matter the size or difference there is between them. I also know that math usage can be applied to anything. It is in vain if you try to solve a new problem from the start, but to know how to recollect the ideas into a bigger picture, you can find ways to solve anything. Suppose you are trying to search for someone lost in a forest or trying to decode some message (such as the matrix code), you may not suspect math to be so obvious. You can use graphing, degrees, and scaling to pin point areas on a map, and during times of war, we probably had to use math to encode signals. Even the computer I am typing on is running on math. With that being said, I concluded that math isn't necessarily associated with numbers and the such, because it is an idea (or concept), it is what we associate the world with.
>
> --Except from in class reflection

Neither of his parents speaks any English. Although he has adapted well enough to speak and write in English his mother still struggles to find the words that she means in English. Martin's father left him, his mother, and his brothers for another woman shortly after Martin started school with us. Through that turmoil he has been in fights, expelled from school, and watched his brother go to jail for gang related activity. I have been Martin's teacher for three years minus a semester expulsion.

This past year I saw Martin fight back against his anger and really come to shine as a well mannered and gentle spirit. Everyday after school he goes to work with his mom to help her out at her very laborious janitorial job. His attentiveness in mathematics has grown a lot this past year. He is so focused on developing an understanding of things that he is a terrible group member. While he listens well to others and tries to help them out, his initial phase when a new problem is presented is to understand the problem as best he can, alone. When another student says something about the problem, I will often see him smile and say, “Oh ya, that’s right.” That is about all I can get out of him. He is the kind of student who is doesn’t speak unless spoken to. He is always thinking about something and when he is deep in thought you can see it written on his face.

Martin’s learning in our class is not only related to the tasks, his beliefs about mathematics, and his being-toward others, but the circumstances of his whole life. Through all the trials his family has been through he makes it clear that his relationship with his mother has a lot to do with his being-toward mathematics. When asked about his most recent success in mathematics, Martin says in reference to his mother, “Because if I can do good in math then she will know I am trying because math is the hardest class.” While his mother cannot help him in his mathematics studies her impact is still felt in the classroom through Martin. His deep thinking has contributed to our class in many ways.

Another student who likes to think a lot is Nancy. She enjoys reading, discussing issues, and helping out around the school. She will candidly tell you that mathematics is not her favorite subject. Nancy’s viewing of mathematics has still yet a different flavor. In response to the question, “What is mathematics?” she says,

> I think that mathematics is the study of numbers and how they relate. Humans find this very interesting because how they can relate numbers with real life situations. In the history of mankind the Aztec Indians used math in building and even in astronomy. Here [our classroom] we have discovered that there is more to math than just the quantity of things. Mathematics can relate to everything if you wanted it to.
>
> --In class writing assignment

Nancy's reference to the Aztec Indians and astronomy came from her in class investigation of the history of mathematics in Mexico, her parents' native country. For Nancy mathematics is quantitatively about numbers, but that is not where it ends. As she sees it, through the act of relating these abstract quantities to our real life situations mathematics takes on a more qualitative aspect. In my discussions with her she has made it clear that she feels very strongly that there is not just one application for mathematics. Mostly she means that the sciences are not the only place that mathematics can be applied. Most important to her is the application of mathematics to business.

An even more practical viewing of mathematics comes from Charlie a very matter of fact--how is it going to affect me kind of student. In his response to what mathematics is he mentions a project our class did with home design software. Each student group created a building for an organization that would address a need in our local community. This task will be mentioned by other students as well. Charlie says,

> Mathematics to me is an application of knowledge that can be used for things

such as building, designing, or even just measuring space for carpet. If humans didn't have math everything would take ten times the time and ten times the work, say you are putting siding on a house the house is actually about 6000 ft^2 and you make the estiment and you end up not having enough or you order too much not enough and you have just caused more time you need or you ordered enough but you didn't cut the siding the right length because you don't have math to measure it. If you order to much you're out of that much money. Humans to me engage in mathematics to save all the stress and aggravation it would cause if we didn't have it.

--In class writing assignment

For Charlie mathematics that only relates to itself is neat, but a waste. It has to get down to something that matters to his world, beyond the mathematics it self. He would very much agree with William James in that ideals should only be about changing reality. As a student who is going to enlist in the Army and currently participates in the reserves, he really took an interest in matrices. We will further explore this interest in the clearing of occupation and the clearing of newness and wonder.

A student that I feel best pulls all these views of mathematics together is Hali. In the first year she was in our class, she was known as the girl who didn't know much. The little mathematics she did know did not serve her well in finding solutions to the problems we were working on. Another student and I worked with her continually. Throughout the next year she appeared to be going through the most dramatic change I had ever seen in a student. After this change I asked her to write about how she saw the discipline of mathematics at this point in her life. According to Hali,

> At this point in my life, I think about mathematics as being very important to me. To me the meaning of mathematics is a huge part of your life in every way, out in the world, numbers aren't just numbers. Each and every number has a huge amount of information just waiting to come out. I'm pretty sure that with out mathematics life would not have changed much through out the centuries. I think humans engage in it because it's what you need for more technology and for the construction that goes on in the world, without it we would be no where. Mathematics not only helped in wars, technology, and construction, it helped, and helps, in lots of ways some people can't even imagine.
>
> --Written response to researcher question

She recognizes that in our being-in-the-world, numbers are not just numbers. Numbers are related and connected to a great many things. She sees a relationship between our mathematical ways of being and our history, even our future. This past year Hali has now moved on to assume the role of “the girl that knows almost everything.” In her class, when students wanted to check their thinking, they turned to Hali. When someone was gone and needed to know what was given, they turned to Hali. She became the single largest resource to her class, even larger than the teacher.

Critical points and transformation. By the stories of the students described above it may be hard to see over what contexts the students came to see the mathematics in their own particular ways. There have been a few students that I have been lucky enough to see a moment of transformation in their being-toward mathematics. Sometimes this seems to happen in an instance when the conditions cause a critical perturbation in a student's thinking and being. Other times it appears as a gradual transformation.

Small turbulences can possibly have a huge impact in a student's thinking and being-toward mathematics. For example, simple disruptions in a student's experience of what a mathematics classroom should be like can greatly alter the way they behave and think about mathematics. In the early days of teaching these students I had a few of them who could add, subtract, multiply, and divide fractions extremely well, but it occurred to me that this did not mean they had made sense of fractions.

One student in particular, Vicky, was very good at these operations and never made a mistake. When they were given fraction tasks she was not pleased. She said, "I know how to do fractions can't I do something else?" I asked her to participate with the class in these problems and if she found them too easy I would find something more challenging for her. One day, only a few weeks after school had begun, I gave the following problem:

> Five people want to split a pie. Four of them are adults and one of them is a child. All of the adults want the same size piece but the child's piece should be half the size of one adult piece. What fraction of the pie should the child get?

In the middle of working on this problem Vicky slammed her papers to the table, quickly stood up from the table in such a way that her chair went sliding across the room and with a long wavering finger pointed at me and said, “This is bullsh*t. You’re the teacher, now teach!” To disarm her I politely said, “And as your teacher I believe you will find a much more beneficial growth when you do not have to depend on me to figure things out.” Vicky was always hounding me about showing her exactly how it was done. She claimed that if I would only tell her she would remember. I would try to reason with her about what would happen if the problem was a little different, and I wasn’t around to show her the way, what then? “Is it not important that you understand it?” I asked. She did not respond.

I can remember with simple clarity the next day when Vicky figured out this problem for herself. She sat back, crossed her arms and smiled. I said, “Vicky would you like to talk about the problem with someone?” She said, “I ain’t given my answer to no one, not even you Mr. Matney until everyone is done and wants to talk about it.” She had arrived. While being-in-our-class and fighting me all the way her learning and being toward mathematics culminated into the solving of one problem, one small little problem. This helped her reorganize how she approached mathematics. The experience of having to solve that problem became a critical point of her transformation towards mathematics. She had begun her journey towards an active understanding of what was being talked about instead of wishing for the seemingly easier way of passively memorizing a pre-existing mathematics. Later Vicky and I discussed these events and in reflection Vicky said, “Mathematics is a lot funner when you understand it and aren’t just like moving numbers over here and letters there because that is what gives you the right answer."

While Vicky's transformation in modifying her being-towards mathematics seemed to happen very rapidly Lucy's seemed to come more slowly. Lucy just came to our school this past year from another state and as such has been my student for only one year. She immediately seemed to fit right in with her classmates. She accepted without confrontation the differences in the way our mathematics class did things and what she had been accustomed to. From a traditional perspective she was a strong student. She is one of those students that some teachers would say, "I wish I had a hundred more just like her!" While a strong student in the traditional sense of taking notes and memorizing the solution to problems and how they are worked, her test taking was atrocious. She scored well below her peers on the mathematics portion of the ACT. She came into our classroom saying, "I like math, I'm just not good at it."

I saw a gradual change in her approach to solving the problems in our mathematics class. While she accepted the way we went about learning she also would, from time to time, go and grab a textbook. I kept a number of subject area texts on the shelf for student reference. When I asked her what she was doing she said, "This book is huge, it has to have a formula that will solve this problem somewhere in it." Sometimes she was right in her assumption that a thorough reading of a part of a textbook might reveal the solution to a task or problem we were working on. A few months into the school year she retook the ACT and scored 3 points higher on her mathematics section. She was very happy to say the least.

For Lucy, the experiences of this year changed her whole viewing of things in mathematics. In one of their portfolio entries in the middle of the year I asked the students to reflect on what it was like to learn mathematics. Lucy wrote,

> I have never really learned anything in math until this year. In the previous years all my teachers done was to write the formulas on the board and tell us to memorize them. But once I entered this class I soon learned why thing are the mechanics of just being a machine. Mr. Matney always strives to make us learn the point of the problem so much; he actually gives us the problem and makes us find our own way of solving it. But once I ***actually did learn something*** in math, it is a whole new world; it makes mathematics turn from 2 dimensional to 3D in a moment.
>
> --Excerpt from Portfolio entry

It is interesting to note that she speaks of her previous experience with learning mathematics as that of non-learning. It isn't that she doesn't know things. In fact I was impressed at the number of mathematical things she had memorized. Amazingly (but not surprisingly) though, these rarely helped her solve any problems our class was working on. She knew the formulas but had trouble operating with them and recognizing their place within mathematics.

As one who interacted with this student all year I found her metaphors particularly revealing. She says that once she moved into our classroom she began to learn the "why's" of mathematics. With this change of aspect comes a change in her metaphor. No longer does she refer to mathematics as a machine to be picked apart and understood only through its pre-existing operations. Instead, she reflexively moves to describe her experience of “actually learning something” in mathematics, using a mathematical metaphor related to our in class discussion about dimensionality and the shape of space. The meaning of this mathematical metaphor is about a change of possibility, an opening of a new space for her, which is much different than the idea that mathematics is but a large group of cogs to be placed together and once they are all in place can run as a well oiled machine apart from the one who put the pieces into place.

Later on in the year I noticed Lucy approaching the task differently than she had before. After a brief discussion with her on what was different, I asked her to tell me about her view of mathematics and what it was like to really learn it. In this description she changed her metaphor and talks about meaning. Lucy writes,

> When I was younger I memorized formulas, I knew how to get the answer, but I did not know what it meant. So when I actually learn it is like knowing the instruments in a symphony but learning why is like hearing the music. Like all the math I learned was force fed, so therefore I did not like math because I did not understand it. Don’t get me wrong, I still don’t *love* math, but at least I understand why, which makes my brain work and forces it to think, not just to memorize. Which in turn makes me a better learner, because I can see something for more than what it is.
>
> --Response to researcher inquiry

Her ideas about mathematics are breath-taking. While both she and Martin share similar sentiments about the connections and relations of mathematics I very much enjoy the metaphorical way in which Lucy expresses what she thinks. Lucy compares memorizing different parts of mathematics to knowing a bunch of instruments, individually. She then says that understanding the "whys" of mathematics is like hearing the music of those instruments. In the hearing we understand what is meant by the instrument. In listening to a symphony we further understand the relation of those instruments to one another in that beautiful cacophony of sound. What a different view of mathematics this is from one in which the parts are isolated from the whole.

The students' experiences revealed to me the diversity of their ways of being-toward mathematics. However, the views about mathematics described above should not be taken as stand-alone or permanent beliefs isolated from the lived experience of these students. While these students have stated their beliefs as this or that, in my being-with them it appears to me that these beliefs not only continue to develop and change, but through interacting with others and new mathematical ideas the students change their being-toward mathematics for short periods of time during those interactions. Furthermore, sometimes from those interactions emerges a critical point from which a student's being-toward mathematics undergoes a dramatic transformation. These student's ideas and beliefs about mathematics move in the sphere of their being-in-the-world-with. For example, sometimes when other students are working with Brian they take on his view of mathematics in the way their approach becomes shaped by the interaction with him. Furthermore, sometimes Brian also takes on an entirely different view of mathematics when he is working with different people. To further examine this I now turn to descriptions of the clearings of relationship.

The Clearings of Relationship

Students are caught up in a world with others. In our being-in-the-world we do not cease to be-with others. The clearing of relationship is the space in which students in their experience of and with others are made manifest through being-with. While inter-classroom relationships are an easier phenomena for the research to describe as it is more handy in my being-there-with the students, the students have further revealed the importance of relationships that go beyond the classroom, the relationships a teacher might never know is intricatelyconnected to the concern of the student and their being-toward mathematics.

While students may sometimes tend to appear stable in their own beliefs and ways of being-toward mathematics, it is dangerous and inappropriate to assume that they are static. In the process of *incubation* during one week of this research a couple key events threw me back into immersion rather abruptly. It appeared to me that the relationships that students were in greatly interacted with their being-towards mathematics.

A student named Ross first brought my attention to this. Ross was always stand-offish when it came to learning mathematics and he rarely felt like contributing to our classroom discussion unless it was about music, sports, or humor. It wasn't part of his reputation to be good at anything academic. Although he always maintained a C or better in all of his classes, he rarely put his best foot forward. He was very content in his being this way. Athletics was what he enjoyed most. In the sports he played he went all out. Every coach would comment on how he left nothing for the end of the game and would readily sacrifice himself for a single play to go the team's way. Though Ross normally exhibited his greatest effort in the area of sports, during one lesson we did, he showed that same sportsman intensity towards his mathematics.

On a field experience to a state park the ranger there began to describe the park in all its beauty and diversity of plant and wildlife. In this description he told the students about how the park had been expanded through out the years. He told them that over 40,000 acres had been added to the park. Just then a student raised his hand and asked, "Is that a lot of land?" The student had never had any experience with what an acre was. I started asking my students if they had ever heard of various lengths or sizes, like a rod or a township. Square mile is about as much as they knew of any large areas.

So I decided that we would investigate the National Land Survey System through questions like, "How does one find an oil rig, out in the middle of a field, when it doesn't have a street address?" The primary intention of these tasks was to recursively consider geometric concepts such as length, area, and conversion, as well as give students the opportunity to make sense of the size of an acre. The National Land Survey System involves a lot of direction and fraction sense as well. Ross took a particularly fond interest in these tasks. He even contributed to discussion. I couldn't for the life of me understand why he was so interested. The class had done many practical tasks before this one. We had studied roller coasters, built our own models in class, as well as taken a trip to experience potential and kinetic energy at our local amusement park one Saturday. We had constructed geodesic domes, used software to design a building or a home, and studied coding theory. None of this seemed to appeal to Ross anymore than the purely mathematical relationships we had studied. After the study of Surveying was over Ross went back to his more usual way of being-toward mathematics. So I inquired further, “why the interest in this?” Ross replied,

When we were learning about acres and land measures that really got my attention because my grandfather has a family farm and he is always talking to my dad about this acre over here or that acre over there. My dad nods his head. I think we have a section of land out there, or at least part of a section. As we were learning about surveying I wondered what the directions to our land was. When I go and visit my grandfather this summer I am going to ask him if he knows and if he doesn’t I am going to show him I can find out for him. When we went outside and measured off an acre I paid extra attention to that task because I really want to know what my grandfather and dad are talking about. Then when you challenged us to see who could best estimate how many acres the school was on and I won, I knew I had ***learned it good***.

--Transcription from taped interview

The relationship Ross had with his father and grandfather worked to modify his usual stance towards mathematics when the task over this topic was presented. His being-in-the-world-with them was a critical relational part of the context by which he took such interest in this new topic. He engaged with the new topic in a way that made learning it his own-most possibility. This was something his family knew well and so he would know it well also. In this specific time of inquiry, Ross' familial relations were tied up with his learning in such a way that he modified his being-toward mathematics and sought his own possibility of knowing so that he "learned it good." A few days later another incident threw me back into a consideration of this same phenomenon.

Sometimes there are uncanny relationships that form within the classroom. In the interaction of those relationships there is the possibility that at some point a vital perturbation in a students learning emerges. A three year student of mine on an Individual Education Plan (IEP) named Gregory was having tremendous behavioral problems in his English class. He was at times quite violent. When he reached frustration he would punch walls, desks, doors, anything that would hurt him. He told me thateven though it hurt him to punch walls and things, it was better than hitting people. Though he was never a disruption to our classroom, other than the occasional use of blatant vulgar language, the counselor and English teacher requested a schedule change for him in which the English teacher and I would swap the classes in which we had him.

Gregory was then moved into a class with a student named Eliza who had at this point only had me as a teacher for half a year. It was not uncommon that I would spend twice as much time talking about the tasks with Gregory as I would other students. Gregory would often say in a frustrated voice, "this is so hard!" Eliza quickly picked up on his frustration and began to partner with him when I was elsewhere in the classroom. They began to make sense of the problems together. Gregory's verbal outbursts about the difficulty of the problems became less and less. His learning began to improve dramatically.

For Gregory the relationship that formed between him and Eliza had tremendous impact on his learning. He says, "It is like I can just understand her better than anyone else." In his collaborations with Eliza he showed marked improvement on all assessments. He became confident enough that even on the rare occasions that Eliza missed class he was able to work through many of the tasks we had. I once commented to him that I knew he could do it, with or without Eliza. To my surprise, Gregory retorted, "Actually I just have a conversation with her in my head. I can hear her say, DUH Gregory!" In his learning he hadn't stopped being-with Eliza, despite her absence. This is what Heidegger was saying about being-with. Gregory helped me to better make sense of being-with. His learning seemed intricately tied to that relationship just as Ross' learning about surveying was tied to his familial relationships. In my assumption that Gregory, himself, alone, was learning it when she was gone I was simultaneously reducing Gregory and learning in an "average everyday" way. He was not alone at all; his being-with was just modified to stretch out toward the possibilities of his learning in a different way than when she was physically present.

While Gregory's learning clearly appeared to have grown significantly, Eliza's work also grew in its detail of explanation. While Eliza's role of tutor to Gregory made it appear that Gregory was on the receiving end of the relationship, there was actually a much stronger dynamic that was at play. This was revealed to me by Eliza in her reflection on learning quadratics. Further discussion on that will be saved for the section about the clearing of newness and wonder.

Gregory and Eliza's being-with was very specific toward one another in this special way. They rarely asked for the opinion of others. It should not be presumed that this was purely a function of their being-with however, as the entire class they were in took on a similar being-towards others. No group really desired to interact with the others. They were like isolated pockets until we came together to discuss the problems they are working on. Other classes organized their being-with one another in completely different ways. Gregory and Eliza's class was small, about 11 students. Another of my classes of equal size organized differently. In this class the groups were much more dynamic and interactive with one another. Many times presentations generated whole class discussions and problem dissolution.

Rae was in this class where the groups moved in and out of each others ideas. Rae's being-with-others in our classroom was very different from Gregory and Eliza. Rae's being-with continually moved back and forth from specific group members to the more general classroom community. Rae worked quietly with her partners and discusses the situation with them. She would not commit to an answer until someone else, either in her group, or in the class, also said that they agree with the way she is thinking. She needed to make sure the tool she was using made sense to others in the context of that problem. Even when her group agreed on one solution to a problem, she sought others opinions on the matter.

Her being-with was in a continual state of self-reflection and corroboration with all the others around her. Rae said,

> I like working with others. Things make more sense to me that way. I don't like feeling that I have the right answer but no one understands me so I have to ask them cause if they understand it, it has a bigger chance of being right.
>
> --Classroom conversation about having to work in groups

Rae revealed that her being-with and its relationship to her being-toward mathematics were more expansive than just the classroom concerns. When asked to reflect on the importance of mathematics to her life she stated,

> I can use it (mathematics) in things at home, even if it's real simple things and things my parents didn't get to learn. To me it's (mathematics) important because of that but mostly because I can share things with my mother and she feels proud that I know.
>
> --Excerpt from in class reflection

Rae's relationship with her parents and especially her mother were in play with her classroom experiences. In knowing mathematics she said that her mother is proud of her. This same way of being proud was revealed through Rae's interaction with her class. When any one of her classmates figured something out, she complimented them on their successful attainment of a solution, thereby showing that she was proud and appreciative of them.

While Rae's class often exhibited the spirit of the freedom of ideas amongst each other in a way that Gregory and Eliza's class did not, it should not be assumed that every student in Rae's class had the same being-toward being-with-others as Rae did. Brian was also in this class and very rarely corroborated with others in this way. When he was working in his group, he and his partner didn't collaborate much on the problem but tended to work alone. Very seldom did they stop to talk about where they were in the problem, or to check one another's thinking. When one of them felt they had the answer then they would share that. While this was not always the case, it is important to note that this tended to take place no matter who was working with Brian. Brian said,

> I like working by myself for the most part. I guess I have just learned to trust my own ideas more than others, although I know I don't always have it right, I like to think I do. Lots of times people just confuse me so if I just stick to myself I can work it out.
>
> -- Classroom conversation about having to work in groups

This was not however Brian's permanent state of being-with. He did from time to time open up to his larger classroom community as will be shown in our further discussion on authentic learning. Just as Brian appeared to undergo modifications of his being-toward mathematics and being-with-others, so did every student. These modifications acted to clear the way for the possibilities the students had concern for.

The Clearings of Occupation

By the clearing of occupation I do not just mean the student's being-towards a particular average understanding of what constitutes an "occupation," such as pilot, fast food worker, herpetologist, or janitor. While this average understanding needs to not be absent from the meaning of occupation, I more generally take it to mean the relational space to which Da-sein *concern's* itself. So, the clearing of occupation is the clearing where students reveal what they have concern for. Student's lives occupy what they have concern for in their being-in-the-world. Sometimes this is a deep concern for an "occupation" that captures their heart and imagination. Other times it is a more focused concern for a particular dilemma at hand.

In the learning of matrices, Charlie is a good example of both of these modes of the clearing of occupation. It was mentioned earlier that Charlie often approached mathematics from a pragmatic perspective. When we first started matrices Charlie was all ears. He didn't initially see how any of it could be useful. Then he looked directly to the possibilities of storing tables of numbers so that they might be operated on to gain new information quickly. His concern in learning matrices was very intertwined with his being-toward mathematics, newness, and the connection of matrices usefulness to humans. In the clearing of newness and wonder we will further explore his general occupation with learning matrices.

There was however a specific concern he showed for matrices when we studied the use of matrices in coding. His interest in the military brought him in close to this. To find out more about his learning during this period I interviewed him. In response to the question, "What was fascinating about matrices?" he said,

> I have never seen anything like it before. It was just a whole new subject for me. It was a more difficult subject than I was used to and it was great to learn it even though it was difficult. At first I didn't see what it was used for. But,

> then, we started learning how it was put into military code and it could be used for systems of equations and stuff like that. As a member of the United States Army I found it cool that matrices could be used to hide messages. When we made our own codes by hand it took so long but with computers they can code and decode very quickly. Speed and safety are two things they talk a lot about at drill. I wanted to learn matrices better so I could know the mathematics of the codes we send.
>
> --Transcription from taped interview

Charlie's occupation with the Army had a strong relation to the possibilities he took as his own. In his day to day conversation I often heard him speak of what he learned in drill. He really loved to investigate how any abstraction can be useful to his or our being-in-the-world.

Another student whose being-toward mathematics is shown through his concern for a particular dilemma that happened at his "occupation" is Ricky. He sought to use his mathematics in a number of ways. Ricky came into class one day excited to tell us how he had helped some guys who work for is father. His father was a contractor. Ricky helped his father all his life and was sometimes given the responsibility to go check on a job to see how it was coming along.

One day in particular Ricky was excited because he was able to use his math to help a crew mark off a lot that needed to be square. Since they were marking off several lots on a patch of land it was important to start off correctly. While they knew the lengths of the sides of the square, they needed to ensure that the sides matched up perpendicularly. According to Ricky this is called squaring off the lot. Ricky described the event in what follows:

> One day I was at work (construction) and these guys working there could not square off the lot. In other words they couldn't set the property stakes to where all the sides would be equal and not be off. In order to square off the lot all the angles must be 90 degrees. So to check this, they set up stakes according to blue prints then measure from one corner to the counter corner like this:

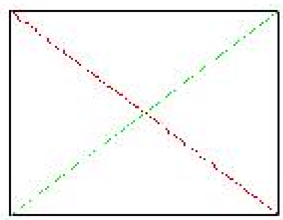


Figure 5.1. Ricky's drawing of the lot.

> --Excerpt from Ricky's written description of the event

As the boss' son it was his responsibility to make sure the job was done right. When Ricky came up to them they initially tried moving one stake around to get the fit they wanted. Then they showed him the blueprints. When he saw the blueprints he realized that he could use the Pythagorean Theorem to find the diagonal lengths and the square could be marked off easily knowing the length it was supposed to be. Ricky said,

> I used the Pythagorean Theorem to find what the exact measurement should be, and we got within like 2 or 3 inches off and they're like "that's fine, leave it!" so I did and I saved the day (and got a Happy Meal for lunch)! It is really cool when you get to use your mind to help out with an actual need to solve problem. When it was over I was like, wow, I guess I ***really learned*** the

> Pythagorean Theorem!
>
> --Excerpt from Ricky's written description of the event

After Ricky told this story I asked him, "If I saw you on the street and asked you what the Pythagorean Theorem was before this happened would you have known?" To which he answered laughingly, "Probably not!" An interesting note was that it didn't occur to him to use the Pythagorean Theorem until he saw the blue print. A solution to the problem lay hidden until that 2-dimensional context helped him realize what could be useful to solve the problem. When Ricky saw the problem in a 2-dimensional context it threw him into a different modality, one in which a solution to the dilemma presented itself in relation to Ricky's being. Through his concern for this particular problem in connection with his relationship to his father, the company, and the workers, the mathematical tool he needed emerged to help him solve the dilemma.

While students like Ricky and Charlie are currently engaged in an "occupation," other students aren't but are projecting themselves towards one as their own-most possibility. Sergio has his sights set on being an architect. In his being-toward this occupation, he understands it as his choice and seeks the means to ensure that this possibility comes about. In geometry , I gave the students some graphics design software and asked them to design a building that was related to a need in our local community. Students decided to build a number of things from homeless shelters to "All Girl" carwashes. (Guess which group thought this was a need)

Sergio's group decided to build a school that would not only teach students, but also parents, and as such would have daycare for both students and parents. It would also act as a place to stay for those who wanted to go back to school but whose job would not pay enough for them to devote that many hours and still pay the bills. With this as their social task, the building needed to be made affordably, as the idea would more than likely have to be supported with public funds. It was however a large undertaking in that many needs would have to be provided in a single location. When asked to reflect upon a time when he really learned something Sergio recalled that experience:

> An experience I remember that I **really learned something** in math is when in our 10th grade year with Mr. Matney we used a program called Home Architect. We used that program to build a school. I learned that we need a lot of geometry and math to be able to be an architect. It is more complicated now that I know what I need to become one. That school we were building was very complicated but fun to do. We had to figure out the area and dimensions to the buildings and such. It was a really great experience.
>
> --Portfolio entry

In the design of one of the buildings (a pentagon) the angles they wanted complicated the design. Sergio really had to think through the mathematical relationship between the sides and the angles, as well as deal with how the program organized the construction of those angles. As he pointed out, he came away having learned not only mathematics but also some of the ways of thinking involved in being an architect. His desire to be an architect went beyond himself. Sergio said,

> I want to design my parents the perfect house. If I know all about architecture I can do that. That is why I want to be one, because I want to help people. Just like in designing that school we were trying to help people who needed a good education but had problems.
>
> --In class discussion about why students wanted to enter into a certain profession

In his being-toward this "occupation" he said that his concern was related to others. These ways of being also interacted with Sergio's being-toward mathematics. He knew that architects need to know a lot of mathematics and as such he approached it with an ambition to learn it well. It should be noted however that there were some tasks and problems he was not able to make sense of. I carefully watched this for three years and I was amazed at how sometimes all the intentional effort put forth by Sergio seemed to profit little more than his exhaustion.

Each student has a being-towards some occupation. In their occupation with things the students are the clearings by which those concerns are made manifest. The students' occupations are intricately connected to being-with-others and their being-toward mathematics. In this interconnectedness when one or some of these clearings is modified, the student's being is modified. The clearings are all caught up in the context in which the students find themselves. The possibilities that present themselves to the students change as the context changes.

For example, Charlie had some very rough months during his study of geometry. His family was going through some very hard times. He wasn't getting along with either of his parents who had recently divorced from each other and both had remarried. Through this turmoil he did not look on mathematics in the same way as he had before. What he was occupied with changed drastically. He was more distant in his being-with. He no longer showed concern for the relation of mathematics to his occupation with the Army. He was only with us for half a year and then moved to two different schools. When I asked Charlie to describe the difference in his experiences with geometry versus algebra, he said,

> Algebra to me was a lot easier. Geometry I never really stayed at a school long enough to really grab hold of it. Things were going on I just wasn't interested in trying to learn it. It was too hard to try. I was having problems with parents, having to move, and not having a permanent place to live.
>
> --Transcription from taped interview

Later, during the year of this research study, Charlie was in his second year of algebra. Charlie stayed the entire year with one parent. He was on good terms with both of them. His being-towards mathematics was reconnected to his occupation with the Army. What was possible for him now seemed to change. No longer was it too hard. He retook geometry at the same time he took algebra and did very well.

The Clearing of Newness and Wonder

The clearing of newness and wonder is revealed through the student's being-toward their possibilities in-a-world of mystery. In this they seem to stretch out from a world of known into the unknown. The students concern themselves with the possibility of learning in the midst of their being-in-the-world-with. The new and the unknown are in relation with the rest of the student's being.

Sergio has already revealed his being-toward the particular "occupation" of Architect. In his fascination and desire to be one he furthermore revealed a connection between occupation and newness. In a reflection on what it is like to learn new things in mathematics Sergio says,

> The experience of learning something new makes me wonder how it can be used in the career I want to take. How does it relate and how is it used and what for. It also makes me wonder how important it is to this profession as an architect.
>
> --Excerpt from in class reflection

In his clearing of newness Sergio found wonder interacting with his being-toward occupation. In other words his wonderment was not some general form of abstraction but was directed to a specific concern of his being. In all of this his being-towards mathematics also took on a certain modification and was interacting with the newness and occupation.

Charlie said that during algebra he really learned about matrices. All year long Charlie seemed to breeze through the tasks and problems. While it took some of the other students days to recognize the connection between two seemingly unrelated problems, he quickly made the connection and applied whatever worked in that case to the problem. When it came to matrices he mentioned that it was more difficult, but the speed by which he made connections was still incredible. There was something different about this however. Charlie never really cared to talk about what he had figured out until we reached this subject of matrices. When I asked him for a reflection that described an experience when he really learned something he said,

> This year, when I came back to this school I told myself that I was going to make a change in my study habits and start doing better in school. I have been trying to make that happen, and this year I finally found out how it felt to be confident and have confidence in myself in math. One of these experiences would have to be when I learned how to do all of the mathematics involved in solving things dealing with matrixes. It was great to see something that I have never seen before and then find myself 10 minutes later teaching people how to do it. To feel that is probably the best feeling that you can get. I can remember it well, I walked in to math class and I saw a bunch of numbers in brackets and I was just lost thinking about trying to do it but I decided that I went this far in the year and there was no reason to make all that hard work go to waste just because there was a little bit of a challenge that I had to face. It was so difficult to even try to get my mind to think about the rows and columns and numbers it made me feel like I was about to explode. Then all of the sudden it clicked in my head how it worked all of the simple things that I had been trying to listen to but just didn't understand, suddenly it all seemed so easy. I was able to answer some aloud in class because I was confident enough to want to share my knowledge in the subject. I went around helping people figure it out and it just feels great to be able to do something like that.

--Portfolio entry

The interactions of two clearings appear to mefrom Charlie's story The first is in his appraisal that it was a great thing to see something completely different. Charlie reached out to this subject in a way that I had never seen before. In his being-toward this newness he sought to make learning about the matrices his own. The intrigue and wonder of these bracketed rows and columns seemed to captivate him beyond his normal way of being-toward mathematics as a practical manifestation. In other words his way of being-toward mathematics was modified. Our tasks had not dealt with anything real-world yet, so I was surprised to say the least that his interest was so strong from the start. While I don't know if the popularity of the movie *The Matrix* played a part in this fascination, I do know through discussions with Charlie that the movie was in his world.

The second interactive clearing that appeared to me was in Charlie's being-with-others. In his description of this experience he was quick to point out that he helped people. In his learning of matrices he underwent a modification of being-with. His way of interacting with peers completely changed, in the context of this lesson. Through Charlie's learning of matrices there was a dynamic interplay of all the clearings. His being-toward mathematics, his relationships, his occupations, and the newness and wonder interact in such a way that none of them were stable entities of Charlie's being. They appeared to be more like chaotic interactions of the clearings and from these interactions emerged a new way of being for Charlie. That is, he is learning in an active and transformative way; a way in which he recognized that what he had learned was real to him.

Sometimes newness and wonder take on a different feeling altogether. As was mentioned previously relationships play a serious part in the organization of authentic learning. Eliza felt she had to learn the lesson on quadratics because of her assumed role of tutor to Gregory. When I asked her to write about what it was like to learn quadratics Eliza described her experience with that lesson as one of anxiousness.

> The day I started learning quadratic equations was a very scary day. I walked into the classroom and looked at the board, as usual. On it was the topic we would be studying that day, it said quadratic equations. I started freaking out, surely this has to be a joke, Mr. Matney wouldn't do that to us, quadratics are serious, complicated problems, there's no way I'll be able to do this, might as well drop out now, there goes my grade, no way, this isn't happening, this is bad, all kinds of things were going through my head. Can you say spaz? I can. So I'm waiting for class to start, wild thoughts still running through my mind. I mean I was seriously freaking out; I'm talking sweaty palms, butterflies, the whole works. The lesson starts, I'm paying very close attention, practically hanging on Mr. Matney's every word when all of a sudden it hits me, like a bucket of water being dumped on your head in the middle of summer in the Sahara desert, this wasn't so bad, kinda fun, and really easy. My mind stopped reeling, I wasn't freaking out anymore, I was going to be okay, I could do this stuff, and I did. By the end of the class I was feeling like I had ***really learned something*** and it felt good.
>
> --Written response to researcher question

Eliza had missed the previous day of study on quadratics in which our task was simply to make connections between a particular type of number pattern we had occasionally run into and this formal name of that pattern. Eliza was great at recognizing this particular number pattern but we had not yet discussed its formal name. When she saw this word there was a strong anxiousness that gave her an increased incentive to be alert to what was being talked about. In noticing her distress and bemusement, I asked the class to discuss again that which we discussed the previous day. After that discussion, she didn't exactly settle down right away but engaged the task at hand. During the particular day described above the following task was given to the students.

> Quadratic Patterns: Is there a connection between the numbers a, b, and c in the general form of a quadratic equation and the graph of its parabola? Using your graphing calculator to experiment with different numbers and report your findings.

With a heightened state of awareness Eliza approached the day's task in a very unusual way. Her normal routine was laid back and nonchalant. The word she used most often is a humble, "whatever" accompanied by "that sounds good." This day however, she was intent and focused like I had never seen her. She went right to the task and worked straight on until it was time to discuss it. Eliza had never volunteered to give information before the class. Usually a classmate or I had to ask her what she found before she would make her contribution. This day however she freely contributed and had the most to say on the matter at hand.

Of course, by her testimony below of this experience, her attention to detail this day was not just about her learning it for a grade and being able to contribute in class but was intricately tied up with her relationship with Gregory. Earlier in the year, before Gregory was transferred into her class, Eliza's learning was not so bound by this incentive. She did some very serious inquiry into linear equations prior to Gregory's move into our classroom. During this particular period of time Gregory had been missing a lot of school for a variety of reasons, only part of which was related to his disability. In his absence however, Eliza had not ceased to be-with Gregory. After reading her reflection on learning quadratics I asked her what bothered her that she would have so much anxiety. She said,

> It seemed very hard because the word quadratic. Come on, that just isn't something you hear every day and I knew that if I couldn'tfigure it out then Gregory couldn't. When he comes back I know he will complain about how behind he is and to help him I have to be on top of it or he might never catch up.
>
> --Quoted in researcher notes from after class discussion

So while newness can be described as an anxious clearing for Eliza's experience to have "really learned something" this learning was also intricately tied up in being-with-others. It was also connected to her being-toward mathematics in that the nonchalant mode she was normally in was transformed into a different way of being-toward mathematics and interacted with her change in being-with Gregory. All of these clearings seemed to work to change each other as Eliza engaged in the learning of quadratics.

The narratives of these students have been arranged in such a way that the patterns of organization for learning by my students might be shown through their lived experiences. Through the dynamic interconnections of these clearings students have come to learn mathematics in a unique way. Some other stories of student learning will now be examined so as to better explicate the dynamic interconnections of the clearings in a more holistic way.

Patterns of Authentic Learning

When students have an experience where they really learn something the pattern of this learning seems to be organized by the interaction of these clearings. As the clearings are no particular things but are rather spaces in which students have differing modes of being, it should not be said that what is described is a particular method for the authentic learning of all students. On the contrary, the experiences of students "really" learning something greatly problematizes any way of talking about authentic learning that says it "is" this or that. To say such is to talk about authenticity in a "they" way.

To explore further the subtle complexities and differences that interact in such a way that the pattern of authentic learning becomes manifest more student experiences need to be considered. Grace is a student who at first appeared to have very little attraction to learning mathematics. Her understanding of fractions was vague and by rote standards she could not operate with them very well. In our study of numbers and number patterns she struggled to make connections. Grace is a very quiet person. So quiet, that when I heard her speak at all during the first year I taught her it was a surprise.

In her being-toward mathematics she is often modified in such a way that shapes make sense to her and furthermore it is her preference to make sense of other things through pictures and shapes. Show her that there is a double distributive aspect in which $(a+b)(c+d) = ac + ad + bc + bd$ and she will get a blank stare wondering how this can be. However, ask her to find the area of a rectangle of length $(x+3)$ by $(x+7)$ and watch the relation of shapes to the algebra of distribution become manifest through her.

What follows is an excerpt from Grace's mathematics autobiography, an assignment students do for their portfolio in our class. In this assignment students are given the freedom and latitude to discuss in whatever way they want their history with mathematics, what they think about mathematics, etcetera.

> In my life ever since I was little I always thought about numbers and shapes. My experiences of math has not only been held in school, but in my daily life. People tell me I know a lot about math, especially my mom. But I always feel like I don't know enough. My favorite part of math would have to be geometry or anything to do with shapes. I have the ability to think about shapes in such a perspective that it is easy for me to solve problems that deal with shapes or size.
>
> --Excerpt from Portfolio assignment on their mathematics autobiography

Grace's being-toward mathematics in this way is so strong that in class she is often occupied by shape. Her concern with and for shape allow her to quickly answer some problems in our mathematics class while relegating her to confusion on other problems and tasks that we consider.

I first noticed that Grace had real talent that had been hidden from me when I gave a task on number patterns that involved expanding shapes and their perimeters. Within minutes Grace stepped up to me and said here, "I am done with this." Shocked to have an answer so quickly, I looked over her work for a question that I could offer to her as a nurturing challenge. As I looked down I noticed that she more than had the pattern, she had written its algebraic representation. Grace had struggled with finding an algebraic way to represent the number patterns we had studied previously. So I was intrigued as to what was different. Had she now made a connection between number patterns and algebra? Not quite! She had made the connection as long as the context of shape was there, but in reverting back to issues of patterns that only involved numbers and not shapes she still struggled.

This was the first time I had given a task that involved shapes. She explained herself in full detail and interacted with others. The interaction with others in this way was also quite a shock to me. She usually just sat alone, quietly working on the day's tasks. If she finished early she rarely let me know but would just sit back in her chair and not say anything.

In Geometry she worked intensely all year long. She could not get enough of the problems. She would always ask for more. What took many of my classes days to connect and make sense of took her minutes. It wasn't that she had ever seen it before either. She had never investigated space, dimension, shapes, angles and their relations at least not formally. Furthermore, Grace had never seen these tasks before, I was sure of this because I made up a lot of them based on what ways of thinking I wanted to make problematic for students.

During one such task her being-toward mathematics revealed itself brightly. The events and quotes taken from this task were recorded in the researcher notes during the exercise. The task was simply to find the distance between two points (x_1,y_1) and (x_2,y_2) of the Cartesian coordinate system. While in a more traditional math setting one might have expected that I as the teacher would have previously given the commonly known distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. However, I felt the students needed to make a case for a formula if it was revealed by their concern. Students approached the problem in a number of ways. Some physically measured the distance in our common units of centim eters or inches and compared the lengths of these distances to one another, others found out how many units were between the points by marking a unit on the ruler or a piece of paper and then measuring.

While other students were making connections with this laborious method Grace and her partner raised their hands. Grace asked, “Why can’t we just use the Pythagorean Theorem on these problems?” I asked her, “doesn’t the Pythagorean Theorem involve triangles?” She answered that the Cartesian system was perfect for right triangles and that the Pythagorean Theorem involved right triangles. She then showed me that a triangle could be drawn from the two points in such way that the distance between the two points was the longer side (hypotenuse) of a right triangle. Grace hasa way of making a shape relate to the things she learns well.

It turns out that this task was given a couple weeks after Lucy had first come to our school. Lucy’s group was struggling to get the same answers as the groups beside them that were measuring. Lucy said, “Mr. Matney, isn’t this just the d equals the x’s subtracted and the y’s subtracted. I know I remember learning this sometime.” After Lucy had struggled to remember for a sufficient amount of time and had given up I asked her to work with Grace to see if she couldn’t shed some light on the matter. The two of them got together and began to explain their ways of understanding the problem.

As Grace went through her method she showed Lucy a series of steps.
1) Draw a right triangle. 2) Count the length of the sides. 3) Square the sides and add them to use the Pythagorean Theorem to find the square of the hypotenuse length. 4) Take the square root to find the hypotenuse length. Lucy showed her the formula she was working on. Grace said, “I don’t understand that at all.” Lucy said, “I know this will work but I must be forgetting something because my answers aren’t like everyone else’s.” As I was watching this exchange of ideas I smiled because Lucy was only missing the squaring part of her formula. She had written on her paper: $d = \sqrt{(x_1 - x_2) + (y_1 - y_2)}$.

Lucy inquired about how Grace did the last problem. The problem in question had very large numbers. It read “Find the distance between (-1322, 432) and (633, -2190).” Lucy said, “Surely you didn’t count how far apart they were.” Grace replied, “No, like for the x’s one point is over 633 from zero and the other is over the other way -1322 so you add the two distances and you get 633 + 1322 = 1955.” Lucy sat back for a moment, looking perplexed, then checked her paper and said, “Hey that’s what I got when I subtracted! So then you squared that number?” Grace answered affirmatively and Lucy went on to complete Grace’s steps for herself. At this point Lucy recognized the difference between her formula and Grace's steps.

Lucy added the squares outside each parenthesis on her written formula and commented that she thought that was it and it should work. She then proceeded to quickly work through a couple other problems and check her answer to Grace's and the other groups that had measured the units. When she was convinced Lucy said, "See that, my way works too." Grace just stared at all the symbols and mentioned that she still didn't know why but that she was glad for Lucy. Lucy began to explain by showing how her algebraic formula matched up with Grace's steps. Grace asked how her steps and this formula were the same. Not a second after she asked the question she saw the answer. "Oh the subtraction is finding the lengths of the legs, like on my picture, and then you square and add them cause of the Pythagorean Theorem and take the square root to find the distance instead of the square."

It seemed to me that Lucy had made this connection before Grace, but it was revealed to me that I was wrong by what Lucy said next with great excitement. "Ya, that's it! The formula only works because of the Pythagorean Theorem." Grace always sat by my desk so this conversation was just a few feet from me as I was taking notes. Being so close I could no longer stay out of the conversation and my excitement overtook me. I had to get in on it and talk about the connection with them. I should note that the groups who were only measuring played a huge role in their solutions. Only when Grace compared her answers to the measurements of other groups did she feel her approach was solid and furthermore, only when Lucy compared her answers to these measurements was her improperly remembered formula problematized. Grace and Lucy presented their findings to the class. I asked Grace to reflect on her learning that day, in particular the connection she made between the algebraic formula and her way of solving the problem. In her reflection on these events Grace wrote,

> It's like you look at something you've never seen before, just a bunch of letters and signs, and then you look at the meanings behind them all and it's like reading a book. Kind of like looking at Arab handwriting and thinking, what does this say anyways? Then look at its translation in English and then you understand. Especially when you can relate the hard stuff you really can't understand to the stuff you can do all day long.
>
> --Written reflection from researcher inquiry

Grace came to see that the algebraic formula was only understandable through its meaning, which was connected to the Pythagorean relationship. Through this event I saw a number of interactions bouncing and shaping one another. Both Grace and Lucy modified how they were towards mathematics through their interaction with one another. Both had a great appreciation for one another's perspectives. Through the interaction of their difference they were able to better understand their own way of doing things.

Grace and Lucy also underwent a modification in their being-with and being-toward their own-most possibilities. As Grace was *with* Lucy, a student who thought in terms of symbols and signs, she sought the meaning of the symbols and signs as her own-most possibility rather than just ignoring these "things" and sticking to her own step by step process based on a picture. Similarly Lucy was *with* Grace, a student who understood the relationships of mathematics generally through the connections of shapes, and so Lucy modified her being-toward her own-most possibility as she sought to understand how the relationship among the lengths of the sides of a right triangle related to her formula. Through all of this and in their being-in the newness and unknown, both in the problem and in each others way of approaching it, emerged a transformative understanding of the task at hand. For these particular students in this particular context I witnessed an experience of their authentic learning.

Disconnected in Their Being-Toward Mathematics

Throughout this research on the experience of authentic learning in mathematics, authenticity has appeared to be a far more complex phenomenon than even a general definition of "that which has correspondence to the real world" could begin to describe. As the findings have thus far described, real learning emerges from the complex interactions among the dynamic clearings of the student's being. However I have also seen the subtle complexities of their being interfere with their learning in our mathematics class. For every story of where someone like Eliza was inspired to learn something new despite its intimidation I have a story of where that intimidation and being-toward newness in the particular context of the student at that time interacted to destroy any learning that might have took place. For every story I have of how a relationship positively interacted with the student's being so that real learning could emerge, I have a story of how a relationship did the opposite.

Gregory had a positive relationship with Eliza, but this wasn't the only person in his life. She was not all of his being-with. One day Gregory came into class and said, "F#ck Mr. Joseph man! This is Bullsh*t. I hate this school!" I took Gregory over to my desk, sat him down and asked what was so wrong that he felt he needed to scream profanity in our classroom. Apparently he and Mr. Joseph got into an argument about a paper Gregory turned in. Mr. Joseph wanted it to be redone before he gave Gregory a passing grade. Gregory felt that the modifications Mr. Joseph wanted him to make were beyond the bounds of his IEP and as such he didn't feel he was obligated to make the modifications in order to receive credit for the class.

Any learning of mathematics was going to be a struggle for Gregory this day. He was emotionally distraught. I asked him to go to his seat and to have class with us. Gregory eventually calmed down and interacted with Eliza and the rest of us. However, his clearings were modified in such a way that learning mathematics was not a possibility of his being-toward. He took a few notes, listened politely to others and at the end of class came up to me and said, "Mr. Matney can I just come in and talk with you about this some other time because I don't understand it." Of course, the pattern of phenomena like these isn't isolated to Gregory.

While I don't feel it is necessary to go into this at great length I could tell stories about times when each of my students had similar problems whose dynamic complexities emerged as a way of being-against learning in general, much less mathematics. While I have any number of severe cases where students formal learning was affected by extreme environmental conditions like sleeping in a car during the winter for two weeks, or holding the slit wrist of your friend who just tried to commit suicide, or being so hungry that you would eat condiments for breakfast and lunch because your parents are too prideful to fill out a free and reduced lunch form, I also have stories of students who seem to have every advantage but through the complexities of their being become oriented toward mathematics in a way that eliminates the possibility of engaging it.

When I think of Charlie and the rough times he went through, it was torture for the both of us. He really wasn't enjoying life at all much less coming into our class to work on tasks that he could care less about at that point. His story is one of the extreme examples of how family distress and pressures can contribute to a student's ways of inactively existing in a classroom.

An example of a less extreme nature comes from Grace. She has a strong relationship with her family and they all exhibit a kind of contentedness. She always has clean clothes, good hygiene, food, and a compliment for everyone she knows. Grace's way of being-toward mathematics has at times been so rigid that she can't see a solution to a task. Many times it appears that through her being-with-others she is able to change her modality toward mathematics and find a solution. An example of this was given in Grace's interaction with Lucy about the distance formula. When her being-with was modified all the other clearings became modified and threw her into a new way of being-toward the possibilities that were hers. This does not happen with all activities or tasks however. It isn't about just putting her in a group with someone so that interaction can modify her clearings in a way that make the pattern of authentic learning emerge. All of the complexities of her being must be open or closed to the particular modifications of being that would have such a transformation occur. There is more to this phenomenon than just simply choosing a good task and grouping people appropriately.

The activity where we studied the National Land Survey System was perfect for a student like Ross whose being-with inspired a transformative learning experience. He sought to learn in such a way that he could have intelligent discourse with his family about things they were already engaged in. However, it is more complex than just saying Ross' being-with was involved. As Sergio's example showed, sometimes all the intentional energy a student can muster is not enough to "provide" the emergence of a transformative learning experience. Brian on the other hand exhibited a very different way of being-toward the task. Brian said,

> During this activity we also learned how to tell in what area of a section something was like if I was looking for something in the bottom corner of a section that was divided into 4 equal parts I would say I was looking for something in the SW of the S of section ## blah blah blaah. This activity would be useful if I ever went into land surveying but since I probably wont so it was not terribly interesting to me.

Brian's reflection about the activity revealed to me that he actually learned very little. Not only is his ambivalence toward the activity made obvious in word and tone, that his example was both incorrect and incomplete led me to further inquiry what he knew about the subject.

> Ok, I admit it, you caught me, I don't know anything about it (Land Survey System) because I wasn't paying attention. Dude, I don't care how big an acre is I mean come on! I did not find one question that was worth considering, but I guess I could have if I had tried.
>
> --After class discussion about Brian's reflection on Surveying

What we would call correspondence to the real-world did not at all ask Brian to modify his being in a way in which real learning could emerge. His occupations of concern lie elsewhere. There was a great disconnect between Brian's being-in-the-world and what is called real-worldliness.

It is also important to note that Ross himself did not modify his being-toward the topic of surveying because of its "real-world relevance" but rather the interactions of *his* being-in-the-world-with his family, specific to this activity his father and grandfather. Ross' being-in-the-world is not the average world of the "they." This is however what is typically meant by "real-world correspondence," in that, whatever is being talked about corresponds to everyone in the objective world, and then furthermore, it corresponds to no one in particular as it isn't about a person or their experiences; it is about an isolated world that exists apart from students' being-in-it. This average way of considering "real-world correspondence" fails to capture the complexity of correspondence in the lived lives of students. Ross' story reveals that he did not come to learn about the

National Land Survey System because of some abstract relation to a real-world, but rather in the complexities of his being in-the-world-with-others.

In researching the experience of authentic learning in mathematics I did not find a task, a textbook, a classroom organization style, or a way of explaining some mathematical construct that led to authentic learning in isolation from the complexes of the students' being-in-the-world. What was revealed by my students was that the experience of authentic learning is tied up with their being-in-the-world in such a way that from the interactions of the clearings of being-toward mathematics, being-in-relation-to-others, having concern and occupation, and projection-toward newness, their own-most possibilities of being emerged into learning that was real for them. This chapter was organized in such a way that the distinction of the clearings could be presented and yet the interconnectedness of these clearings would be preserved so as to not talk about these parts in isolation from the whole. The challenge of dealing with these complex phenomena in a classroom can not be adequately dealt with by any reductionistic teaching methodology. So to explore the possibilities of a better way of describing and dealing with these complexities I turn now to a discussion on the sciences of open systems and chaos and what they offer in light of my students' experiences of authentic learning.

Chapter 6

Understanding the Experience of Authentic Learning in Mathematics

The findings of this study make problematic the use of the words “authentic” and “authenticity” as found in the literature discussed in chapter 2. Using Moustakas’ heuristic research methodology of experience and Heideggerian ideas and terminology that apply to authenticity to explore the research question, “What is the experience of authentic learning in mathematics?” has revealed this experience to be very complex. The complexity that lies within this experience is reduced when “authenticity” is used in ways that only promote it as being that which has correspondence to the real world.

In these students’ experience of authentic learning some of the normal modalities of their clearings are thrown into such interaction that they are changing and in turmoil from the more stable state they usually appear in. From this chaotic change in modality of being and the interaction of these ways of being emerges a relevant transformation in the student, to say it in fewer words; the student has an experience of authentic learning. The clearings appear to be interacting in such a way that they are organizing one another rather than one of them causing the other to be in such and such a way.

To better understand this phenomenon the relevance of the theories of chaos and complexity will be considered. While the experiences of students have shown the need for a different way of approaching this phenomenon, I will turn to one last description of a classroom interaction among students whom I have previously discussed at length. From this final example further discussion about the theories of chaos and complexity will serve as ways of understanding the experience of authentic learning.

One day I began to take researcher notes on Rae and her interactions in a particular classroom task. On the outset I didn't know whether she would approach this task in a way that would reveal anything to me about the experience of authentic learning. I was really just documenting Rae's interaction with those around her so that I could make sense of these interactions. I soon realized that my focus on Rae soon had to turn to a more holistic emergence that was happening in my classroom. It was happening to Rae, in her being-with and being-toward, but it was also happening to some of my other students.

The task was a somewhat regular one; the topic of the question was one that might be heard in a traditional mathematics classroom. The question went as follows:

> You are given \$1000 from you grandparents when you are 6. Rather than spend it all on a bike and some video games you put half of it in a savings account that pays you 4% annual percentage rate. After the first year you have \$520 and after the second year you have \$540.80. Find a function that represents this number pattern for savings interest.

The students had not had any previous tasks concerning interest or savings, nor had they been given any formulas or tasks concerning the formal expressions of exponential formulas. They had however, recently been asked to consider exponents and their properties and months earlier were given two tasks that dealt with exponential growth, one being a simple number pattern and the other the famous, or in some circles infamous “Tower of Hanoi” problem.

Throughout the year I noticed that Rae always intently collaborated with others before committing to a particular way of doing things. Rae quietly goes about working toward a solution but never alone. She is in constant communication with her partners about where everyone is on the problem and what they are working on. For this problem, at first, she investigated it as a linear pattern. Sergio found a linear regression for the pattern but Rae convinced him that his regression was off just a bit because the “second jump is not the same as the first.” That is to say, the increase in money from year to year was not constant. After they were convinced that a linear model was inadequate for this problem they moved on to quadratic.

Rae’s other partner, Patricia, interjected at this time that she had figured out why the second “jump” was $0.80 more than the first. She told Rae, “4% of $520 is $20.80. I think its $20 for the 500 and $0.80 for the 20 and so when you add that you get the $540.80.” Rae and Sergio acknowledged that this made sense to them. Rae then responded, “Then isn’t it like a quadratic? I mean, doesn’t the quadratic pattern have increasing jumps like this each time?” Both of her partners began to look at some previous problems that involved quadratics. Patricia noted that with quadratic data the jumps were either increasing each time or decreasing each time. They agreed to proceed with finding a quadratic model.

About this time I interrupted the class and asked if they would like some additional information. It was apparent that the majority wanted more information. I wrote on the board that after 5 years your saving account statement has \$608.32. I asked the students to judge the bank statement's correctness based on the pattern or model they had established.

I walked over to Rae's group and asked where they were on the problem. Rae said, "We think we might have it but we need to use the new information as a check." After this they checked their quadratic model, which was, $y = 0.4x^2 + 19.6x + 500$ where y is the money and x is the number of years in the bank. It worked perfectly for the first three data points but the new piece of information was off by \$0.32. So they concluded that either the bank statement was off or they were off. While Sergio and Rae were off finding the values generated by their quadratic model Patricia went about her own task of finding the next growth amount of 4%. She said to Rae, "I got that after 3 years we should have \$562.432 in the bank. What do you get for 3 years?" Rae replied that the model said \$ 562.40--3 cents short. Sergio double checked Patricia's work and came to the same conclusion. Something was amiss, but not by much.

As the end of class was soon to roll around I asked if anyone felt they had a solution. Only one student, Lucy, felt she had a solution. Lucy said, "I don't have enough time to write it on transparency to present so can we just finish tomorrow?" The class agreed that they needed more time to work through it, but were excited to hear that a peer thought she had a solution.

Rae listened intently to Lucy the next day who discussed how she figured out that it wasn't quadratic due to the 5th year being off a little. So she tried a cubic function and it worked for all the data points. She wrote these on the board. Rae said, "Oh ya, cause part of a cubic increases each step like a parabola does."

Patricia wanted to double check her 3 year amount of $562.432 with Lucy's model. Lucy's model came up with the exact same solution.

Brian spoke up at this point. "I don't know if this means anything but when I take 4% of the previous amount 5 times I don't get exactly what the bank statement said. I get that the value should be $608.3264512. So if my calculations are correct Lucy's model is off by just a fraction of a penny after 5 years."

Rae now was going to get to the bottom of this 4% stuff herself. While the rest of the class debated the bank statements rounding and whether or not the bank had ripped them off, appealing to a cleverly funny movie called "Office Space," Rae was for the first time calculating the 4% growth after growth for herself. She began to write down these growths with all the decimals included. The class decided that the bank was only robbing you if you took out the money right then and even then it was only a little more than half a penny. One student laughed and said, "I mean really, what is the bank going to do, cut a penny in half?" They agreed that the bank should keep all the decimals for future calculations or they would be robbing people. Lucy argued however that her model was correct for the rounded version. At the end of class Rae approached me about the next function above cubic. I told her it was called a quartic, and it was of degree 4.

The next class period turned out to be a fascinating one for me. Rae came in and said, "We are ready to present Mr. Matney." I asked the rest of the class where they were and if they were ready to talk about Rae's group's solution. The class was ready except for Brian who wanted to work on it by himself some more as he felt that he too was close to a solution.

Rae began to explain that she had noticed that originally with 3 data points the quadratic equation worked perfectly but when the extra info was added, making it 4 data points, it took a cubic for it to work. Rae took the knowledge of the quartic function, whose main term is x^4, and noticed that it would work for 5 data points. Rae concluded that there was no solution to this problem. Every time you include another data point, it takes a polynomial of larger degree to satisfy it. Though I usually am able to control myself when my students stumble upon things that I find profound, this time my jaw hit the floor! I could hardly contain myself.

Rae's finding was of course not at all what I had expected! It is a well known result in numerical analysis that you can find a polynomial of degree n-1, which can model n points. To me, Rae had demonstrated that the polynomial models they had been trying were not going to work. After the class was sufficiently satisfied with what she had to say and had no more questions I asked if that meant there was no way to get the future values without doing it exhaustively.

Lucy said, "I have one that works but I'm not sure why so I didn't want to have to explain it cause I can't." I went over to her seat and looked on the calculator. Sure enough, she had found the exponential equation that worked, $y = 500(1.04)^x$ where y is the money and x is the number of years in the bank. She said that in her frustration she just started trying every regression model in the calculator until she found one that worked. Lucy was surprised about the numbers in the model. They seemed so simple. I asked her what she thought those two numbers meant. She said, “The 500 is how much we started with but…I don’t know why 1.04…maybe something to do with the 4% I guess.” Rae’s group tried the formula and agreed that it worked.

I wondered if any of them were curious about why it worked. After a few minutes Brian stepped up and said, “Of course it works it is just like exponents because we are just multiplying the same thing over and over.” He explained that when you multiply by .04 you get 4% of what you had, so you want to keep multiplying by 4% to keep it growing that way. I asked a question at this point. “So what does the 1 have to do with anything?” Sergio said, “Well 1 times a number is that number so it just gives you back what you had and the 4% will give you more added onto that.” Noticing that I had doubts that everyone saw this same way Sergio did Rae said, “Mr. Matney, we got this.” With that, we moved on to reflect upon the learning that developed from this task.

This last story was saved for this chapter because it descriptively shows the complexity, chaos, self-organization, and interrelatedness of authentic learning in light of the evidence about these student’s complexities shown in chapter 5. Through their interaction *and* working alone, through their histories *and* their concern for what they might possibly know, through their disciplinary concerns *and* their practical concerns for the task, through their classroom relationships *and* their out of classroom relationships, was organized an inquiry into the not-yet understood, out of which emerged a new way of seeing, a non-polynomial way of representing a certain pattern, as well as one students demonstration that the old way of seeing was not working.

Through my being-with these students in their experiences of authentic learning I have come to realize that our current ways of talking and understanding about what is happening in our attempts to educate does not adequately describe what is happening in my classroom. What is needed is a way of talking and understanding that deals with the complexity that is found in the classroom. For this a discussion of the ideas of systems and chaos theory will be considered.

The Theories of Systems and Chaos

The following discussion of open systems and chaos theory is not an attempt to cast them as a new meta-theory applicable to education in general. These theories are introduce here as a particular way of *creatively synthesizing* my findings. These theories offer the possibility of seeing the complexity of learning that has been presented in this study. While some would argue (Hunter & Benson, 1997) that we do not need open systems and chaos theory when one can get to the same end with the ideas of Whitehead, I feel that the meanings of the words used in these theories when re-sought through our trials with students may help us in getting past some of the rigidity of current educational practice which remains despite the ideas of Whitehead and Constructivism. Evidence of this can be seen through the trials and tribulations of the so called Math Wars. Besides the need for a new language by which the complexities found in this study can be described, the ways and means of mathematics education in general suffer from an either/or philosophical entrapment. Before showing how systems and chaos theory may act as a way of seeing that dissolves these either/or debates, I will discuss the entrenchment of the either/or philosophy within the discourse of mathematics education and how "authenticity" has been cast as a term that perpetuates this debate instead of helping to dissolve it.

Math Wars and the Need for a New Way of Seeing. Irrespective of what historical document you read about mathematics education in the 20th century, there permeates a struggle between a side that desires a practical mathematics curriculum, and a side that wants to focus more on rigorous abstraction or mental discipline in a mathematics curriculum. Looking back one sees that the conversations about curriculum in the 1890's were not so different than today's conversations. Contemporary issues resound with the same entrenched dichotomies that have plagued math education for over a century. Due to this either/or mentality the fires of the Math Wars are simply the culmination of a century of gathering wood in order to burn a deep rut of which it is difficult get out.

In 1892, the issues of mathematics utility and its ability to form mental discipline were beginning to arise. The Committee of Ten, while placing a tremendous amount of credibility in the training of mental faculties, did not claim that it was "the unique function of any particular part of the curriculum" (Osborne & Crosswhite, 1970, p. 164). With respect to mathematics they said that the arithmetic of commerce must be included in the curriculum. The Committee of Ten appointed a subcommittee, comprised of leading mathematicians, called the Conference on Mathematics. The purpose of the conference was to investigate the mathematics curriculum and make suggestions for its improvement. The conference made a different recommendation, not aligning themselves completely with the committee. The conference suggested the mathematics of bookkeeping and commerce not be taught until after a full course in algebra, if at all (Osborne & Crosswhite, 1970, p. 165). Thus, in a committee and its own subcommittee originated a debate that continues today.

Almost every single major committee established to deal with mathematics curriculum has had to deal with the question of utility versus pure abstraction. The National Committee of Fifteen on the Geometry Syllabus in its 1911, "Provisional Report," watered down the geometry curriculum and pleaded for more real and applied problems (Osborne & Crosswhite, 1970, p. 182). The pleas for more real and applied problems came out of a concern for motivation. The concern for utilitarian aims would continue to mount through the 1920's. Felix Kline, in one of his commentaries on Geometry, mentions the increased interest boys would take in the drawing of maps in the atlas. The teacher would be able to put more feeling into the lesson, than if he only pursued the abstract questions (Kline, 1932). The junior high texts of the twenties emphasized practical mathematics for the average person and the skill of computation with accuracy and speed.

In another report called The Reorganization of Mathematics in Secondary Education, more commonly known as the "1923 Report," by the National Committee on Mathematical Requirements, stated that the focus of Solid Geometry should be on developing the student's visual spatial relations. One of the main reasons for this focus was due to the "fundamental importance" of accurate spatial perception when dealing with practical applications (M.A.A., 1923). The utilitarian aims of mathematics in secondary education can be seen throughout *The 1923 Report* and in the conclusion is further stated, "The problems of real life frequently involve the ability to think correctly about the nature of the relationships which exist between related quantities" (M.A.A., 1923).

Further struggle for more utility in math came from two other sources. The first of these was "The Second Report of The Commission on Post-War Plans," was published in the May 1945 edition of the *Mathematics Teacher*. From this report, thesis eighteen states that "Simple and sensible applications to many fields must appear much more frequently in the sequential courses than they have in the past" (NCTM, 1945). The second source for more utility came as a reaction to The School Mathematics Study Group (SMSG) which had been developing materials for all grade levels during the late 1950's and 1960's. Their alternative geometry book used coordinates with the background ideas of set theory, symbolic logic, matrices, and Boolean algebra. Though most mathematicians and math educators viewed the changes positively, Morris Kline criticized the reforms as having little motivation, little use of intuition, no application, and little opportunity for student creation of material (Jones & Coxford Jr, 1970). And so yet again this common struggle of mathematics education arises.

In the present there is a similar pattern of struggle. To guide these curricular battles there has been the comparisons of international research reporting, like that of the Third International Mathematics and Science Study (TIMSS, 1995) that shows a decline in American students' math abilities as compared to similar nations. From the TIMSS it is gathered that United States elementary students do very well, but somewhere between there and middle school our students stop out performing the international average, and by the time they are seniors we only out performed two other countries.

The TIMSS study did little to resolve the existing debates in mathematics education and if it did anything it threw more fuel on an already well burning fire. What keeps these battles going is an either/or philosophy of education. Dewey in an article written in 1938 speaks of the either/or philosophy that drives us to think in terms of polar opposites until confronted by practical matters, which compel us to compromise while we still hold to the truth of the extreme (Dewey, 2001). According to Dewey the either/or philosophy that dominates education is whether or not education is a growth from within oneself or structured from the outside. When thought about at some length one realizes that these are silly dichotomies. An education is the interplay between both internal growth and maturity, and the complex structure of social formation. As such, what educators should be seeking is an understanding about the interplay between the two, and in doing so enable us to look beyond the belief of an absolute dichotomy. It is this dichotomous belief that has helped led to a bitter rivalry on the issue of what math is best to learn and how it is learned best.

In 1989, the *Curriculum and Evaluation Standards* from NCTM touted a new and balanced set of societal goals. The first of these new societal goals is t have a mathematically literate work force. This means that industrial employee’s should be able to work with others, be able to see how mathematics applies to complex problems, know many techniques to work on problems, and believe in the utility of mathematics (NCTM, 1989). In the 1989 *Standards* we see the down play of learning as a process of passive absorption. The *Standards* do not explicitly sanction a constructivist theory but do reflect the influence of it (Safford, 2000). Citing research, NCTM discusses learning as assimilation of new information and constructing personal meaning (NCTM, 1989, p. 10). This leaning towards a learning theory combined with either/or philosophy, and statements throughout the standards that re-emphasize the application of mathematics led to curricular developments like “Discovery Learning.” While statements about the role of applications are often made with balance of understanding and connecting, some would come to focus too much on the applied. This would happen as the *Standards*, the misguided constructivism thought to mean discovery learning, and the either/or philosophy intertwined during the 1990’s.

After the TIMSS (1995), more reform curricula’s were being implemented in the schools and others began developing. By 1998, backlash and the fires of the math wars were in full swing. In that year Linda Star (1998), in an article entitled “Math Wars,” noted the opposing views of those in California, a hot bed of the Math Wars.

> Back-to-basics advocates there (California) say that previous guidelines, based on NCTM standards, which emphasize problem-solving, has resulted in a watered-down curriculum and mediocre test scores for the state's students. Supporters of the NCTM standards say the problem-

solving approach motivates students and prepares them for the kind of math they'll use in real life and in the workplace. (p. 3)

The focus on this split between real life problem-solving and basic computation would continue despite U.S. Secretary of Education Richard W. Riley's attempt to call for "an end to the shortsighted, politicized, and harmful bickering over the teaching and learning of mathematics" (Riley, 1998).

The call for a "cease fire" by Riley would in turn throw more fuel on the math war fire. In October of 1999, the U.S. Department of Education endorsed ten curriculums, eight of which emphasize discovery-learning ("Math wars," 2000). The endorsement brought about a critical letter from a group of mathematicians and scientist known as Mathematically Correct to Secretary Riley that asked him "to withdraw the entire list of '*exemplary'* and '*promising'* mathematics curricula, for further consideration, and to announce that withdrawal to the public" (*Mathematically Correct*, 1999). In another statement Mathematically Correct, addresses its concerns about the curriculum's that have been developed following the NCTM guidelines. Specifically, they address that making mathematics a fun subject should not be the focus, and call for the inclusion of skill-building, abstraction, and practice (*Mathematically Correct*, 1998). Both of these issues revolve around the new curriculums over-emphatic stance on mathematics application as a way of understanding math.

Both sides continue to wage campaigns and incite parents, students, teachers, educators, scientist, and politicians to choose and wage war against the other side. In 2001 the National Research Council weighed in saying that students need to be comfortable enough with math to size up real-world problems and figure out what calculations they need. There is a variety of reasons proponents of real-world math have argued for its inclusion. NRC emphasizes the confidence and sense making real problems can give. Business often provides backing for the implementation of real-world problems since the problems help give mathematical training for future jobs and motivation. In November of 2000, Carus Corporation CEO, M. Blouke Carus, delivered an address to legislators and policymakers about work force development and the continuing “crisis” in education. With respect to mathematics he said that since it is abstract, students often get bored and lose interest. As well as calling for more applied problems Carus mentioned that many teachers do not know how math applies to the outside world, and so in addition to applied text book problems, teachers could take summer jobs in which they can apply their mathematics (Carus, 2001).

Further complicating the debate about real-world problems in math education is that some educators, who are genuinely concerned about mathematical understanding, do not necessarily advocate the tradition of transmission but note failures of real-world problems to bring about understanding. For example, some of these criticisms have been aimed toward application curriculums like *Everyday Math*, *Connected Math*, and *Contemporary Mathematics in Context* (CMIC) written by the Core Plus staff. Some have criticized parts of the latter for being lax on its mathematical reasoning. But this critique is far from being anti-application altogether. Those who are anti-application altogether generally attack directly the application because it is seen by them to be diametrically opposed to drill.

So, on the other side of the applications debate are those who are also entrenched in the either/or philosophy, out-right reject constructivism, and have alternative standards that itemize specific content skills that are measurable. From this perspective rote memorization of facts and algorithms is a basis for all mathematical knowing and learning (Quirk, 2000). Most of these “back-to-basics” advocates, who are against application curriculums, hold tight to the belief that education is strictly formed from outside the individual student. Their opponents, not all of which are strict advocates of application curriculums, call this method of teaching “drill-n-kill,” since “back-to-basics” promotes the idea that we need to drill students on math facts more (Rothstein, 2000). The drill method abdicates applied problems because of their inherent contextual nature which leads to a reduction of similarity and makes problematic the belief that one remembers by repetition. As such, common curriculums based on these views, such as Saxon Math, instantiate systematic repetition of the same kinds of problems each day. There is little or no exploration of how mathematical ideas are formed or how the skills are applied. With such curricula there are common complaints about a lack of student motivation and creativity.

Thus, there is a continual drawing of battle lines that deepens the rut of mathematics education. Though today’s debates echo the same issues as history, they have become much more complex and much more ingrained with the either/or philosophy. The issues of standardized testing as effective evaluation, heterogeneous versus homogeneous grouping, the use of algorithms, and the role of technology can only intensify the math war debate as long as arbitrary dichotomies are used to capture what it means to learn. Though some have given prescriptions for resolving the math wars, (Riley, 1998; Safford, 2000; Starr, 1998) they will only fuel further conflict as long as the ideas are conceptualized and received within the framework of the either/or philosophy that Dewey so aptly pointed to over a century ago.

What is needed is a new way of seeing the problem and dilemmas so that these divides can be dissolved (Fleener, Richardson, & Matney, 2003). The findings of this research have shown that from the experience of authentic learning it is not a question of whether to use real-world applications *or* not to use real-world applications. The learning of students was shown to be far more complex than either side can account for. It is for this very reason the ideas of systems and chaos are now discussed as a means of seeing anew these complexities of the experience of authentic learning.

Systems and Chaos. The theories of systems and chaos arose out of scientists reaching a frustrating dead end. In their discouragement they began to go out of the bounds of their traditional theories, crossing the breaches of various disciplines. There were some bumpy roads for these pioneers. In many cases, they were afraid to mention their meanderings to their colleagues. Graduate students were told that their careers could be endangered if they wrote their theses about events in experimental disciplines (Gleick, 1987). The dead ends that were being brought about by the limits of classical theories of physics and the expanding number of patterns being recognized in biology by researchers such as Maturana and Varela, allowed a few free thinkers to launch the revolutionary new science of systems and chaos.

Systems thinking is one of process, and chaos is “a science of the global nature of systems” (Gleick, 1987, p. 5). Systems approaches don’t focus on a foundation of things that constitute a given phenomena, but rather concentrate on the basic principles of organization. In this view everything is seen as a manifestation of relational processes. This dynamic view of systems initiated from the study of biology is described by Capra (1996) who says,

> According to the systems view, the essential properties of an organism, or living system, are properties of the whole, which none of the parts have. They arise from the interactions and relationships among the parts. These properties are destroyed when the system is dissected, either physically or theoretically into isolated elements. (p. 29)

In my experiences with learning I can identify with this kind of thinking. When I was required to know about things, in isolation from everything else, I felt like I was learning very little or at times, like I was learning nothing at all. Some of my students have experienced this as well. When a student like Lucy adamantly states that she did not learn anything when the teacher just put formulas up on the board and asked her to memorize them, she is casting a serious indictment about learning “things” in isolation.

Understanding the parts by their interaction with other parts throughout the whole allows for a different way of seeing student learning. For example, Charlie and Ross, taken in isolation from their family context, classroom and school interactions with others, previous learning experiences, their being-toward mathematics, and the interaction of these clearings in a particular classroom experience, might have appeared to have learned about the coding by Matrices and the National Land Survey System simply because they liked “authentic” activities. Looking at Charlie and Ross, not as things, but as people in a context allowed for a much richer description of their experience of learning which problematizes the idea that they learned precisely because the tasks had real-world relevance.

Systems science is not the search for the ultimate objective truths. According to Capra (1996), “No matter how many connections we take into account in our scientific description of a phenomenon, we will always be forced to leave others out” (p. 42). This is tough for a modernist mindset to accept. While there might not be an absolutely definable set of objectively observable reasons for why authentic learning happens that is not an excuse to throw in the towel. By looking at the possible patterns of organization we can have approximate knowledge about a phenomenon (Capra, 1996).

The pattern of organization that was found in this study of my students’ experiences involves the students’ modification of as well as the interaction of the four clearings that appeared to me. In these experiences students seemed to stretch-out in a way that was far from their “average” modalities and in this perturbation emerged a critical reorganization. For me this implies a new meaning on what the science of systems calls self-organization.

In the being-with my students throughout this study I kept noticing that not only could I not know and predict when authentic learning was going to occur (or for whom for that matter) I furthermore noticed that I couldn't determine what the transformation of learning was going to look like. In the findings of this study many surprises are mentioned. Sometimes students either made sense of things in a way that was unique, or they came to a conclusion that hadn't been anticipated, or the amount of time it took for them to make sense of it, sometimes days, sometimes minutes. These findings have a strong relation to the idea of dissipative structures. These structures are systems who are capable of spontaneous reorganizations when they approach a far from equilibrium state (Sawada & Caley, 1985). With dissipative structures we are unable to determine when the reorganization will occur or what the altered state will be. As Prigogine(1997) says,

> Once we have dissipative structures, we can speak of self-organization. Even if we know the initial values and boundary constraints, there are still too many states available to the system among which it "chooses" as a result of fluctuations. (p. 70)

Similarly, from the turbulences and complexities of being-in-the-world, I found there to be too many available modifications of the clearings of my students to see ahead of time what the result of this chaos would be.   From the experiences of my students self-organizing came to mean the "dynamic absence which let's their authentic being become manifest." This has some subtleties that need explanation. In self-organization students take an active role in their learning rather than suffering from a "they-organization." In their being-in-the-world the students come across a perturbation, often modifying the clearings of their being which then are chaotic in appearance, from which the students would seek to make sense of this perturbation and undergo a transformative process of self-organization, which I see as one of the possible meanings for authentic learning.

We normally think of chaos in the lives of students and in the classroom as an inhibitor to learning. From the findings of this study it seems to be a mistake to assume that learning has a way in which it always occurs best and furthermore that this best way comes from an orderly and sequential arrangement of topics. It appears to me that out of the dynamic process of self-organization, chaos emerges as order. This emergent order is not the clean predictable reversible order in the modernist sense. Deterministic rationality does not only fail to make sense in light of the findings of this study, in the sciences it is furthermore giving way to the idea that "chance, or probability, is no longer a convenient way of accepting ignorance, but rather part of a new, extended rationality" (Prigogine, 1997, p. 155).

Curriculum Dynamics

In education circles we can also find this new and extended rationality being discussed. Though the ideas of Whitehead and Dewey have been around for decades, through a post-modern lens their ideas seem more relevant than ever. Curricular theories are extending the ideas of Whitehead and Dewey and providing new visions for curriculum futures. In the ideas of many curricular theorists there is a way of viewing curriculum that is much more tenable to the findings of this study than is usuallyunderstood by the word “curriculum .” Curriculum as the set of books, supplies, and adopted teaching strategies that clear a particular path of learning falls well short of the experiences of learning that have been described here. To assume that these “things” are the arbitrators of learning is to decontextualize learning from those who engage in its process.

Curriculum theorist are challenging this usually unquestioned, reductionistic, “average everyday” way of understanding the curriculum. In his book *A Postmodern Perspective on Curriculum*, Doll (1993), proposes a vision of curriculum as a process. He says,

> Learning and understanding are made (not transmitted) as we dialogue with others and reflect on what we and they have said—as we “negotiate passages” between ourselves and others, between ourselves and our texts. Curriculum’s role, as process, is to help us negotiate these passages. (p. 156)

Curriculum as process is furthermore intricately tied to the idea of self-organization. When curriculum is approached from the idea of self-organization, challenge and perturbation are required ways in which learning is organized and re-organized, rather than the things of learning’s guaranteed destruction. This way of viewing curriculum sits well with the findings of this study. For example, when through the complex interactions of our class Rae finally became perturbed to investigate the 4% growth herself, she saw a pattern which re-organized her thinking toward the problem entirely. The finding of the pattern was her own-most possibility, not staged by the "they" but not absent from Rae's being-with and in-the-world so that the pattern emerged through Rae within the complexity. When Rae saw this pattern the problem was no longer about finding the polynomial model that worked, but the possibility presented to her was that no polynomial model would work. This re-organization then opened up other possible ways of being-toward that particular problem.

How are we to react to such appearances of chaos and order coming from a curriculum of process? Based on the findings of this study it does not make sense to seek out deterministic methods and teaching strategies that lock-in a prescriptive arrangement of order and chaos. The findings show that the learning of the students' in this study can be seen as dissipative, in that when the emergence occurs then what emerges is indeterminable. Curriculum theorists do however have ideas about how to deal with these phenomena. In order to work towards "design" and evaluation of the quality of a curriculum that is generated rather than predefined and is indeterminate yet bounded, he suggests the four R's; *Richness, Recursion, Relations, and Rigor* (Doll, 1993).

Necessary to a curriculum's *richness* is its amount of perturbing qualities. The amount of perturbation must be constantly negotiated among the students, teachers, and tasks. *Richness* is then the amount of depth and multiplicity of meanings of the curriculum. *Recursion* involves the reflective feedback of those engaged the curriculum. It is important that teachers and peers critique and respond to one another. Dialogue is thus essential in that through dialogue reflection occurs. *Relations* refer to the developing connections which give the curriculum its depth through the application of recursion. The connections are not those pre-existing in a text book. In fact, Doll comments that the text is seen as something to be revised rather than the end all source of authority on matters. In this sense the curriculum is self-organized by classroom communities. In the *richness* of curriculum there are many interpretations which are based on assumptions (often hidden) and ideas developing from the perturbations, recursion, and *relations*. *Rigor* is the attempt to deal with the multiplicity of interpretations and their hidden assumptions. *Rigor* means "to ferret out these assumptions , ones we or others hold dear, as well as negotiating passages between these assumptions, so the dialogue may be meaningful and transformative" (Doll, 1993, p. 183).

Throughout the process of immersion in this study there was a struggle with the language to talk about the complexity of the constantly changing and developing curriculum in our classroom without reducing it to this or that causation. Doll's four R's could help give educators the language to talk about their practice in non-reductionistic ways. The four R's are not rigid or decontextualized and for a teacher who is engaged with their students the four R's are not illusive. They seem to be a reasonable ground by which we can understand, loosely plan, and engage in the process of curriculum.

Another curricular theorist whose ideas about curriculum strongly relate to my findings is Jayne Fleener. For Fleener (2002), curriculum is creative, dynamic, emerging, and self-organizing. Building on the post-modern logics of relations, systems, and meaning she asks us to reconsider curriculum and its emergent patterns viewed in relationship. She challenges modernist assumptions that keep us holding to our view of curriculum as a "thing." She says,

> To change our very ideas about the meaning, purpose, and value of the curriculum and its relationship to schooling, we must change how we talk about the curriculum, including changing our metaphors and ways of seeing the curriculum…The logics of process, systems, and meaning offer the basis for a relational curriculum with self-creative, autonomous, and self-identity potentials. Such a curriculum becomes a meaning system with the potential to transform individuals as well as society. (p. 173-174)

In the findings of this study students were transformed through the process of our curriculum. Expanding their ideas and possibilities creates new ways of interacting and engaging in their being-in-the-world-with. The curriculum is then not some"thing," isolated from the lived experiences of those who make meanings in the first place.

These perspectives on the curriculum can help education re-organize its thinking so as to dissolve some long standing debates. They also give new language and new metaphors to describe the complexities that are encounter in the classroom. Furthermore, these curricular perspectives help re-organize how students are viewed. When my being-toward-students was in such a way that they were "things" or "products" to be made and shipped off (graduated), I was less open to their ways of being. In that closed off way of being-toward teaching I found it hard to modify my plans midstream. The curriculum was also a thing that had to be covered, not a process of meaning making. As long as I hoped to be at the quadratic formula by the end of the week, then my students learning was secondary to the flow of *my* curriculum-thing.

The "thing" thinking is pervasive. With the modernistic assumptions and metaphors about knowing and learning as sets of objects, discourse is relegated to being an "idle talk." As we cannot separate ourselves from this type of discourse we must find ways to emerge anew from it. Fleener (2002) suggests that we must change the way talk and the metaphors we use. The findings of this study have opened up new possibilities for understanding curriculum and established a new language for the discussion of curriculum and authentic learning. The findings greatly problematize the ways in which "thing" thinking has entrenched the mathematics education community for so long.

Engaging in a Conversation about Authenticity

With the pervasiveness of "thing" thinking it is hard to engage in a conversation about something like authenticity. If authenticity is seen as a "thing," such as an authentic task, isolated from those who would be giving the meaning in their engagement in that task, then it hardly seems that a conversation is even necessary. After all, when used in this isolated way it *is* simply a task that has correspondence to the real-world. By what other means could it be said to be anything different? In knowing what it *is*, it is further accepted that since it corresponds to the real-world then it must be of correspondence to the student's world and thereby be a motivating factor in their learning. The findings of this study make this assumption problematic.

Authentic learning is not a "thing" to be developed and evaluated in the modernist sense. However, the curricular theorists discussed above have provided some ways teachers and educators can be more open to its emergence. In my recommendations to both practicing mathematics teachers and pre-service teachers I would further stress a striving for hermeneutical listening, and an increased focus on the right amount of perturbation for students through tasks and a desirable and spirited learning community in the classroom. Often in my propensity to be a "teacher" I have failed to listen to my student's ideas in an interpretive way. Instead of trying to make sense of what they were saying I was evaluating their ideas by a comparison of what I heard them to be saying to what I considered a more mathematical way of explanation. In this evaluative mode of listening I squashed a lot of student thinking, which left to its own and further perturbed by me and others in a classroom discourse community might have developed into a sophisticated way of seeing and solving the problem.

Hermeneutical listening on the other hand re-orients the teacher's role in the classroom context. Teaching is no longer a matter of causing the learning of a particular construct in a prescriptive instructional sequence. Rather the teacher is engaged in a social process of learning by participating, interpreting, transforming, interrogating and hence, listening in a hermeneutic way (Davis, 1997). Teachers should see in what ways this perspective holds meaning for them and try out their own approaches in this same spirit.

Engaging a classroom in this way is interconnected to many other considerations, such as students' being-toward mathematics, personalities, moods, the task, et cetera, and the interaction of all these at that particular time work together to modify the being of the teacher. The possibility for the interaction of students about the task contributes greatly to the flow of mathematics (Davis, 1997). In the findings of this study it was shown that the students are an invaluable source of creativity and perturbation. In algebra classes students have approached problems from such perspectives that they are pursuing what the discipline of mathematics would call the ideas of calculus. The students just call the ideas their way of solving the problem. From these students a great many connections between the seemingly disconnected subject areas of mathematics has been shown.

It is not without reason that teachers will find a great deal more research has been done with elementary and middle school classes on issues such as hermeneutical listening and problem centered learning. Secondary mathematics has a heavy burden put on it in the form of standards, testing, and college admission and remediation rates. Such pressure has not been unfelt by me in my attempts to engage in the discipline of mathematics with my students to prepare them for these futures. Perhaps the descriptions of my students' developing understandings about secondary mathematical conceptions can serve as a basis by which this action is deemed probable. It is my further hope that more teachers might take away from this study their own ideas on perturbing their students and seeing their students, not as things or even systems, but in the complexities of their dynamic being.

The findings of this research show that I do not wish to abolish the "average everyday" understanding of authenticity, as real-world relatedness, in its entirety. After all, the lessons described in this study were not without connection to what we would call real-world activities for many students. From my perspective this "average everyday" way of understanding is just a leveling down of how we experience authentic learning primordially, that is, as human beings that are chaotically caught up in our concern in a with-world. As has been shown, this leveling down, or reduction, is dangerous when we view it as what authenticity "is," because it is considered apart from any particular Da-sein and hence we limit the possibilities for every Da-sein. Through this research authenticity has come to mean a resonant emergence bound up in the complexity of students' active negotiation of being-in-a-world-with-others. This conclusion is not a statement about what authenticity "is." Rather it descriptively shows the phenomena of authentic learning in its complexity and attempts to make sense of it by recognizing a pattern of its organization among the students of the study. From here future conversations about authenticity can develop.

The experiences of authentic learning in mathematics did not fit the typical ways of using the word authentic. Through my being-with the students in their sense making about mathematics the complex and often chaotic phenomena from which authentic learning somehow emerges was a self-organizing order of transformation of students' being. This is perhaps one of the most beautiful events one person could ever hope to be involved with. Such events are treasured by teachers because these events are the testimonies of their craft. In closing here is one last quote from Brian about authentic learning in mathematics.

> When I learn something in math, I have a sense of discovery. Like I just found something no one in the world knows about except me, and even though so far it's always been something that someone else found first. But it doesn't matter, because for a moment it's yours and yours alone.
>
> --Brian's reflection on what it is like to learn mathematics

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