UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

USING HERMENEUTIC PHENOMENOLOGY TO INVESTIGATE PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' BELIEFS ABOUT MATHEMATICS AND THE TEACHING AND LEARNING OF MATHEMATICS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

DOCTOR OF PHILOSOPHY

By

PAUL HOWARD
Norman, Oklahoma
2013

USING HERMENEUTIC PHENOMENOLOGY TO INVESTIGATE PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' BELIEFS ABOUT MATHEMATICS AND THE TEACHING AND LEARNING OF MATHEMATICS

A DISSERTATION APPROVED FOR THE
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC CURRICULUM

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Acknowledgements

I thank Christ Jesus my Lord, that he considered me faithful
1Timothy 1:12, NIV

Numerous people have blessed me throughout this journey. First, thank you to my committee members. You have all inspired me to become a better person. Dr. Vaughn, you taught me to be faithful to the research process and to enjoy the winding roads that seem endless. Dr. Houser, you are a deep thinker and this has inspired me to never stop challenging myself to grow as an educator and as a person. Dr. Ruan, you have taught me to enjoy the moment. Thank you for your wonderful insight about life. Dr. Chiodo, thank you for instilling in me a lifelong commitment to research. Dr. Reeder, you have been an amazing advisor and mentor. Thank you for your support, encouragement and mostly for your friendship. My committee members have helped make my time at the University of Oklahoma a wonderful experience. Thank you.

To all of my friends who have encouraged me to persevere, thank you. Shawn, you always expressed great confidence in me. You are a wonderful friend and mentor. Ann, your friendship means the world to me. Thank you for all of the great conversations we have had over the years.

Seth and Josh, you are the best sons a father could ever expect. Your love, faithfulness and commitment to enjoying life are inspirational. Thank you for being you. To the love of my life, my friend and my greatest supporter, Bonnie I love you and thank you for walking alongside me. You have blessed me beyond measure. Thank you!

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Abstract

This research used an interpretive paradigm and a hermeneutic phenomenology methodology to investigate mathematical belief systems of pre-service secondary mathematics teachers enrolled in a small private mid-western university. It seemed likely that pre-service teachers would have varied beliefs regarding the nature of mathematics. A goal of this research was to investigate the different types of beliefs regarding the nature of mathematics that existed among these pre-service teachers. This research also investigated how these beliefs regarding the nature of mathematics interacted with participants' beliefs regarding teaching and learning of mathematics.

All participants completed a likert-type survey and created a personal metaphor for the teaching and learning of mathematics. Three participants were selected to participate in a series of three interviews. The findings indicated that the pre-service mathematics teachers who participated in this study held a variety of philosophical beliefs about mathematics and the teaching and learning of mathematics. Likert-type survey findings revealed that traditional views about mathematics were held by most participants. All three participants who were selected for the interview process held at least some degree of alignment with traditional perspective of mathematics as a set of rules and algorithms to be mastered. The findings provided insight that may be useful to anyone interested the teaching and learning of mathematics.

CHAPTER ONE

INTRODUCTION

If the artist does not perfect a new vision in his process of doing, he acts mechanically and repeats some old model fixed like a blueprint in his mind.
John Dewey

As a profession, mathematics educators are well aware of the importance of the relationship between what teachers of mathematics believe about the nature of mathematics and the teaching and learning of mathematics (Cooney, Shealy, & Arvold, 1998; Thompson, 1992; Ernst, 2010). Unfortunately, the views of mathematics suggested by current reform efforts are often in conflict with the views held by pre-service teachers (Cooney, Shealy & Arvold, 1998; Ernst, 2010). A current trend in philosophy of mathematics is that mathematics is a fallible science based on human creation (Davis & Hersch, 1981; Hersch, 2006; Ernst, 2010). Mathematics is viewed less as a subject of structured certainty and more as a subject that constructs meanings that are open to revision and transformation (Davis & Hersch, 1986; Bradley, 2000; Ernst, 2010). This perspective aligns with the ideas expressed by Dewey (1897, 1916) that learning is a reflective activity and is consistent with views of mathematics emphasized in current reform efforts in mathematics education (NCTM, 1989, 2000). This consistency between philosophers of mathematics and mathematics educators that mathematics is a fallible, human creation open to revision is not a perspective held by many pre-service teachers (Cooney, Shealy & Arvold, 1998). This is not surprising since studies have shown that pre-service teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics are influenced significantly by their experiences with mathematics prior to entering a formal pre-service program (Cooney, 1999). The following anecdote illustrates the creative, transforming nature of mathematics and exposes the contrasting ways that mathematics education developed in the United States.

The crisis struck four days before Christmas 1807

These words are the opening words of an analysis textbook, “A Radical Approach to Real Analysis” (Bressoud, 2007). The crisis was initiated by the submission at the Institut de France of a paper, Theory of the Propagation of Heat in Solid Bodies, by the French mathematician Joseph Fourier. The mathematical procedures (series of functions) used by Fourier were suspect, yet no one could explain why they accurately worked in describing heat flow across metal plates. Fourier’s use of series of functions was viewed with suspicion because it challenged the prevailing understanding of the nature of mathematical functions, specifically from a computational perspective. This crisis forced mathematicians to critically examine all of the logical underpinnings of both calculus and arithmetic (Bressoud, 2007). It is important to note that, motivated by a physical problem, mathematicians sought to place mathematical computations on firm logical foundations resulting in the development of new mathematical ideas. Thus the landscape of mathematics changed drastically through the 19th century. “Few of those who witnessed the incident of 1807 would have recognized mathematics as it stood one hundred years later” (Bressoud, 2007, p. 1).

While this crisis raced through the mathematical community of the 19th century, a very young America was establishing its educational system which included the teaching and learning of mathematics (Jones & Coxford, 2002). The act of problem solving had a deep and direct influence on the development of new mathematical computational methods in the 19th century (Jones & Coxford, 2002; DeVault & Weaver, 2002) but this connection was mostly missing in the early teaching practices in the United States where the emphasis in mathematics classrooms was primarily on computational literacy and algorithmic mastery (Jones & Coxford, 2002). In the 19th century the prevailing view of how students learned mathematics was based on a philosophy of mental discipline (Senk & Thompson, 2003). Doing mathematics was considered a good mental exercise and a crucial component to improving a student's mental capacity (Grouws & Cebulla, 2000). If problem solving was approached at all it was after procedures were mastered (DeVault & Weaver, 2002). This is a critical area of concern today as researchers in mathematics education claim that "students at all grade levels need greater exposure to problem situations which promote the generation of important mathematical ideas, not just the application of previously taught rules and procedures" (English & Sriraman, 2010, p. 267).

Mathematics Education Reform

In 1980 the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* calling for a change in mathematics curriculum placing more emphasis on problem solving skills. By the late 1980's studies began to reveal that the United States was performing poorly on international assessments (Senk & Thompson, 2003). McKnight et al. (1987) suggested that the mathematics curriculum being used in the United States negatively impacted U.S. student's performance on international assessments. This motivated a call for a change from teaching with a rote computational emphasis to an approach that placed emphasis on problem solving and development of critical thinking skills (NCTM, 1989, 2000).

Mathematics education reform for the past several decades has been led by the NCTM. Reform as envisioned by the NCTM (2000, 1989) recognizes the need to change not only the content, but also how mathematics is taught. In 1989, continuing its educational influence, the NCTM published the *Curriculum and Education Standards for School Mathematics.* This document made specific recommendations about the goals and content of school mathematics in the United States. *Curriculum and Education Standards for School Mathematics* identified four standards that should be at the center of reform efforts (NCTM, 1989). Problem solving, mathematical communication, mathematical reasoning and being able to make mathematical connections were called to be critical components of mathematics curriculum (NCTM, 1989). The NCTM challenged the mathematics education world with their suggestion that all children should construct their own mathematical knowledge from their personal experiences with mathematics (NCTM, 2000). The NCTM recognized that what mathematics students learn is intimately connected with how they learn mathematics. Teaching mathematics from a perspective that involves problem solving and active learning demands a philosophical position of teaching and learning that is in direct contrast to a position that considers learning mathematics as mastering arithmetic computation and algebraic manipulation (Thompson, 1984; Lester, 2010; Goldin 2010; Cai, 2010). This leads directly to philosophical issues about the very nature of mathematics which has motivated research in the area of teacher beliefs about the teaching and learning of mathematics (Pajares, 1992; Ambrose, 2004; Beswick, 2006; Cross, 2009).

The call to reform has not fallen on deaf ears. Today, in 2013, a majority of the states, the District of Columbia and the Department of Education have formally adopted what is known as the Common Core State Standards Initiative (CCSSI, 2013). The adoption process will have a direct influence on mathematics education in the United States. The Common Core State Standards Initiative (CCSSI) is a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). These standards were developed in collaboration with teachers, school administrators, and experts, to provide a clear and consistent framework to prepare our children for college and the workforce (CCSSI, 2013).

> “These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, *why* a particular mathematical

statement is true or where a mathematical rule comes from" (CCSSI, 2013, p. 4).

The CSSI describes a variety of expertise that should be developed by students interested in a career of teaching mathematics. Heavily influenced by the NCTM calls to reform, problem solving, reasoning and proof, communication of mathematical ideas, representation and connections are included in the CCSSI and remain important areas of expertise that future teachers should strive for (CCSI, 2013). Productive dispositions and perseverance are important characteristics that should be developed. Students should have a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (CCSSI, 2013, p. 6).

Effect of Reform on Teacher Beliefs

The characteristics of mathematics education reform during the past few decades brought forth a philosophical shift about the very nature of mathematics and mathematical knowledge (McLeod & McLeod, 2002; Ernst, 2010). The envisioned mathematics classroom looks much different from traditional mathematics classrooms that most current teachers experienced in their own education (Goldsmith & Schifter, 1997; Boaler, 2002; Beswick, 2007; Cross, 2009). "For many teachers, these changes involve reconstituting fundamental notions of teaching, learning, mathematics in addition to inventing different kinds of classroom opportunities for learning" (Goldsmith & Schifter, 1997, p. 20). Thompson (1992) notes that a teacher's conception of the nature of mathematics can be viewed as conscious or sub-consciously held beliefs, views and preferences about mathematics that forms a personal philosophy of mathematics, though in some teachers this might not be developed and articulated into a formal philosophy of mathematics. A critical role of teacher education programs should be to provide opportunities for pre-service teachers to examine their beliefs about the nature of mathematics as well as their beliefs about the teaching and learning of mathematics (Thompson, 1992). Thus it seems important that mathematics educators examine what is known about the philosophical belief systems of pre-service teachers.

Thompson (1984) conducted a case study to investigate the relationship between teachers' beliefs about mathematics and instructional practice. The study concludes that "there is strong reason to believe that in mathematics, teachers' conceptions (their beliefs, views and preferences) about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners" (Thompson, 1984, p. 105). The feelings and beliefs teachers experienced as learners carry forward to their adult lives and these beliefs are important factors that influence their pedagogical decisions in the classroom (Lerman, 2002; Lester, 2002; Philipp, 2007). Therefore, it is not surprising that there has been extensive research in the area of changing teacher beliefs about the teaching and learning of mathematics (Lerman, 2002; Ambrose, Philipp, Chauvot & Clement, 2007; Charlambous, Panaoura & Philippou, 2009; Chval, Lanin, Arbaugh & Bowzer, 2009). This ought to bring forth a call to those involved with teacher education programs to examine program effects on teacher beliefs about mathematics. If teacher educators do not change what they develop, then there will be a continued need to change what has been developed. Thompson recognized this in 1984 when she wrote:

> "As more is learned about teacher conceptions of mathematics and mathematics teaching, it becomes important to understand how these conceptions are formed and modified. Only then will the findings be of use to those involved in the professional preparation of teachers, attempting to improve the quality of mathematics education in the classroom" (p. 280).

Students entering mathematics teacher education programs bring with them preconceptions about mathematics and the teaching and learning of mathematics based on their own prior experiences as students of mathematics (Thompson, 1992; Carter & Doyle, 1996; Cady & Reardon, 2007). Richardson (1996) identified teachers' beliefs as their own conceptual frameworks and as connecting mathematical knowledge with the act of teaching. Researchers have shown that mathematics teachers' knowledge about mathematics is translated into practice through the filter of their own belief systems (Richardson, 1996; Philippou & Christou, 2002; Swafford, 2003). "Likewise, the teachers' mathematical beliefs and views influence their general pedagogical outlook, the learning climate they will contribute to, and specifically their choices of teaching strategies and learning activities" (Philippou & Christou, 2002, p. 212).

A concern for teacher educators stems from the fact that many students enter mathematics pre-service programs lacking the requisite conceptual understanding relative to the nature of mathematics and teachers' mental organization of mathematical knowledge (Philippou & Christou, 2002; Swafford, 2003; Cady & Reardon, 2007). Calls to reform have placed increased demand on teachers who are expected to engage students with meaningful mathematical tasks that lead to mathematical discourse and seek to strengthen students' mathematical understanding (Philippou & Christou, 2002). Researchers acknowledge the integration of content knowledge and pedagogical knowledge with beliefs as critical in the education of teachers of mathematics (Shulman, 1986; Theule-Lubienski, 1994; Cooney, 1994; Philippou & Christou, 2002). Cooney (1994) recognized that content knowledge, pedagogical content knowledge and one's beliefs towards these types of knowledge are distinctly different entities. This is important for teacher educators because it sheds light on the idea that teaching mathematics is fundamentally connected to the beliefs that the teacher holds about the nature of mathematics and about the teaching and learning of mathematics.

In light of the above findings and discussion, mathematics teacher pre-service programs may want to consider the belief systems students bring to teacher education programs and provide experiences that may help students overcome myths and misconceptions about mathematics and the teaching and learning of mathematics. Students may benefit from pre-service programs that are designed to allow students to examine their belief systems in a way that fosters significant mathematical experiences such that it allows new perspectives about the nature of mathematics and the teaching and learning of mathematics. With this in mind, this study examined secondary pre-service mathematics teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics. This seemed important since mathematics education research suggests that teachers' beliefs about mathematics and the teaching and learning of mathematics, have a direct effect on the teaching practices adopted in the classroom (Thompson, 1984; Cooney, 1985; Thompson, 1992; Wilson & Cooney, 2002).

With a view toward improving teacher education it seemed important to investigate philosophical beliefs systems of pre-service secondary mathematics teachers in order to better understand their beliefs about the nature of mathematics and the teaching and learning of mathematics. This is not a new idea, as nearly forty years ago Scheffler (1970) argued for the inclusion of philosophy of subject matter in the preparation of future teachers. Matthews (1999) notes that that this call for philosophers of different disciplines to engage themselves in the pedagogical concerns of developing high school teachers largely fell on deaf ears. Teachers' views of mathematics teaching are likely to reflect their perceptions of students' mathematical knowledge, how mathematics is learned, and what their role as a teacher should be. Thus the following questions are the focus of this research:

Research Questions

1. What are the philosophical beliefs of pre-service secondary mathematics students at a small private university regarding the nature of mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with pre-service secondary mathematics students' beliefs regarding the teaching and learning of mathematics?

Organization of the Dissertation

This dissertation is comprised of five chapters. Chapter One provides background information and details the research questions addressed by this study. Chapter Two offers a literature review of issues related to this research study. Chapter Three describes the research methodology employed to conduct the study. Chapter Four offers an analysis of the research findings and provides a summary of these findings. Chapter Five presents a discussion of the research findings including the implications of the findings and directions for future research.

CHAPTER TWO

REVIEW OF RELATED LITERATURE AND BACKGROUND

Introduction

This chapter provides background information for this study of study pre-service teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics. The research questions under consideration are:

1. What are the philosophical beliefs of pre-service secondary mathematics students' at a small private university regarding the nature of mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with pre-service secondary mathematics students' beliefs regarding the teaching and learning of mathematics?

A brief historical background sets the stage for the examination of these questions. Establishing that indeed beliefs matter is followed by a discussion of reform efforts in mathematics education and the implications regarding teacher belief systems. Background information concerning constructivism is included because it is currently the dominant theory of learning in mathematics education (Noddings, 1990; Wheatley, 1991; Sriraman & English, 2010; Ernst, 2010; Goodchild, 2010) and can therefore be an appropriate lens to assist in data analysis. Four distinct perspectives on constructivism in mathematics education are discussed detailing how these perspectives are each based on different belief systems (Ernst, 1988). Research has shown that teacher identity is intimately connected to belief systems, so it is natural to include a brief discussion concerning teacher identity (Gresalfi & Cobb, 2011; Battey & Franke, 2008). This includes pre-service teachers' identity as a mathematical thinker and their identity as a future teacher of mathematics. Distinguishing between beliefs and knowledge is then discussed as a precursor to a section regarding defining what is meant by the term "beliefs". This chapter concludes with a discussion concerning methods used in research to study belief systems.

Historical Background

"Meaningless! Meaningless!" says the Teacher.
"Utterly meaningless! Everything is meaningless."
King Solomon

The first half of the 20th century was dominated by arithmetic drills (Jones & Coxford, 2002; Senk & Thompson, 2003). The 1930's and 1940's brought forth calls to teach mathematics with meaning (Jones & Coxford, 2002). "Meaningful arithmetic, in contrast to 'meaningless' arithmetic, refers to instruction which is deliberately planned to teach arithmetical meanings and to make arithmetic sensible to children through its mathematical relationships" (Brownell, 1947, p. 266). Brownell notes that there can hardly be wholly meaningless arithmetic. Early in the 20th century educational psychologists led by Thorndyke, Judd and Dewey conducted research on how children learn arithmetic (Mayer, 2001; Schoenfeld, 2001).

Heavily influenced by the work of Thorndyke, textbooks during the 1920's emphasized drill and computation with little emphasis on the child (Jones

& Coxford, 2002). Dewey and Judd took the opposing view and argued that the concept of number develops out of a child's activity (Osborne & Crosswhite, 2002). In particular, Dewey argued that the main goal of education was to develop the child's ability to think (Senk & Thompson, 2003). Dewey also strongly believed that true education comes through stimulation of the child's mind through social interactions (Dewey, 1897, 1916). Advocates of child-centered learning recommended that curriculum include engaging activities to help students reflect on fundamental concepts (Carpenter, Fennema, Franke, Levi & Empson, 1997; Kajander, 2010). Thus during the 1920's "Thorndyke and Dewey in this early period can be seen as pitted against the same foes – Dewey providing penetrating analyses, Thorndyke his own analyses and overwhelming mounds of data" (Osbourne & Crosswhite, 2002, p. 215).

The education battles went to the backburners with America's involvement in World War II. Society, including education, shifted from a focus on the well-being of society to a concern for existence of the American way of life (Osbourne & Crosswhite, 2002). During the years of war and immediately following, it became clear that American schools were not providing the quality education it had desired. "That service recruits in such large numbers failed to exhibit minimal competence in many subjects including mathematics pointed the finger directly at the giant that is the American elementary school" (DeVault & Weaver, 2002, p. 135). The winds of change calling for teaching mathematics with meaning was gaining momentum when, in 1957 Russia launched Sputnik, the first successfully launched satellite (Devault & Weaver, 2002). This fostered an outcry among the American public who began demanding for changes in the education system that would enable the United States to compete with global advances in technology. These were some of the forces that resulted in the "New Math" reforms of the 1960's. Curriculum was developed that placed heavy emphasis on conceptual understandings of mathematical concepts. Instructional practices emphasizing the nature of set theory, manipulatives and number bases quickly brought controversy (Osborne & Crosswhite, 2002). Teachers were ill-prepared to teach in new ways and parents were extremely impatient with how their children were being taught (Senk & Thompson, 2003).

This frustration fueled an extreme backlash that resulted in the "Back to the Basics" movement of the 1970's (Kline, 1973) "Back to the Basics" curriculum often involved students working at their own pace through programmed instructional books and procedural competency was assessed through mastery tests students would take when they felt ready (Senk & Thompson, 2002). The problem was that the pendulum had swung away from any consideration of conceptual understanding (Senk & Thompson, 2003). In 1980 the NCTM published *An Agenda for Action* which called for changes in mathematics curriculum with problem solving as an explicit goal. The National Council of Supervisors of Mathematics (NCSM) wrote a report advocating problem solving and applying mathematics to real life contexts (NCMS, 1977).

During the 1970's there was some effort on the part of mathematics education researchers to challenge current educational practices which also challenged the belief systems of teachers who were expected to teach in new and different ways (Green, 1971; Thompson, 1984). Swafford & Kepner (1980) examined the implementation of an application orientated algebra program that challenged traditional first year algebra methods and beliefs about teaching algebraic methods and then applying the concepts to word problems. The National Science Foundation (NSF) funded this study in 1974 and data were collected during the 1976-77 school year. The goals included evaluation of application orientated materials in a broad spectrum of schools, evaluation of students' understanding of concepts and evaluation of students' attitudes toward mathematics (Swafford & Kepner, 1980). The study concluded that there was an apparent weakness in the experimental materials in the area of developing traditional algebraic skills. Experimental and control groups showed comparable results on items of translating algebraic expressions and solving linear equations, yet there was no significant difference in attitude towards mathematics. This study provided evidence that consumer related problem solving skills would be improved with a wider attention to real life applications (Swafford & Kepner, 1980, p. 207). These results were challenging for teachers of mathematics who held the traditional belief that algebraic methods should be taught and *then* applied to word problems.

Where are we today?

> "It is sometimes asserted that the best way to teach mathematical ideas is to start with an interesting problem whose solution requires the use of ideas. The usual instructional procedure, of course, moves in the opposite

direction. The mathematics is developed first and then is applied to problems. . . Problems play an essential role in helping students to learn concepts. Details of this role and the role of problems in learning other kinds of mathematical objects are much needed" (Begle, 1979, p. 72).

These words did not present new ideas in 1979 and they are certainly not new ideas in 2013. Yet the problem (pun intended!) continued to persist. For decades now the field of mathematics education has witnessed cycles of pendulum swings between a focus on computational efficiency and a focus on problem solving (Senk & Thompson, 2002; English & Sriraman, 2010). Though researchers continue to call for mathematics instruction in which problem solving generates new mathematical ideas, the actual instruction in classrooms across the country seem to be stuck in traditional methods (Schoenfeld, 1992; Hamilton, 2007; Lesh & Zawojewski, 2007; Cai, 2010). Mathematics teaching continues to be characterized by the belief that,

> "problem-solving abilities are assumed to develop through the initial learning of basic concepts and procedures that are then practiced in solving word problems. Exposure to a range of problem-solving strategies and applications of these strategies to novel or non-routine problems usually follows. As we discuss later, when taught this way, problem solving is seen as independent of and isolated from the development of core mathematical ideas, understandings and processes" (English & Sriraman, 2010, p. 265).

The call to use problem solving as an instructional tool to generate new mathematics challenges not only teachers' instructional practices, but also their philosophical beliefs about the very nature of mathematics (Lampert, 1990; Nathan & Koedinger, 2000). Beginning in the 1980's, researchers began to understand that if teaching mathematics was to shift to an emphasis on problem solving to generate mathematical thinking then an examination of teachers' beliefs about the nature of mathematics must be at least a part of the solution (Thompson, 1984; Grootenboer, 2010).

Beliefs Matter

Has it ever occurred to you, Winston, that by the year 2050, at the very latest, not a single human being will be alive who could understand such a conversation as we are having now? . . . The whole climate of thought will be different. In fact, there will be no thought, as we understand it now.

These words spoken to Winston were written in 1949 and are found in George Orwell's classic, 1984. The future had no hope but would be a time of doom and gloom. Fortunately 1984 came to pass and these words were not fulfilled. Thompson conducted a research study published in 1984 that provided the future of mathematics education a sense of hope that is now beginning to be realized. Prior to 1984 mathematics education researchers paid little attention to teachers' beliefs about mathematics (Thompson, 1984). "Most research on the relationship between the effectiveness of mathematics teachers and their knowledge has focused on the teachers' knowledge of mathematics" (Thompson, 1984, p. 105). And, quite surprisingly, research at the time indicated that there was very little connection between teacher effectiveness and teacher subject matter knowledge (Shulman, 1986; Schoenfeld, 1983). Thompson's study shifted the focus of research in a few significant ways. First, she shed light on new dimensions of teaching and learning mathematics. In this way, Thompson was visionary as she considered the possibility that teachers' views and beliefs about mathematics mattered in the classroom. Secondly, Thompson focused specifically on mathematics teaching and not teaching in general. This was a profound shift in thought that has implications today, nearly thirty years later. Thompson recognized that if progress was going to be made in understanding the teaching and learning of mathematics, then new perspectives would need to be explored and that beliefs should be a part of the discussion (Davies, 2010; Devlin, 1997; Howell, 2001).

Sensing that good teaching requires more than just sound subject matter knowledge, Thompson used case studies to investigate the belief systems of three junior high school mathematics teachers. The three mathematics teachers were observed daily for a period of four weeks. Only observation data were collected during the first two weeks. During the final two weeks the observations were followed by daily interviews. These case studies revealed sharp differences among the three teachers in the conceptions they held regarding mathematics and the teaching and learning of mathematics. "They differed in their awareness of the relationships between their beliefs and their practice, the effect of their actions on students, and the difficulties and subtleties of the subject matter" (Thompson, 1984, p. 124). These differences seemed to be related to differences in the teachers' abilities to reflect on their teaching actions in relation to their beliefs about the nature of mathematics and the teaching and learning of mathematics.

Thompson concluded that the three teachers' pedagogical decisions were deliberately based on their conceptions about the nature of mathematics as well as their beliefs about what counts as evidence of mathematics learning. The relationship between beliefs and teaching practice is complex and "teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers characteristic patterns of instructional behavior" (Thompson, 1984, p. 126).

The implications of Thompson's study are profound and far reaching. During the early 1990's the National Council of Teachers of Mathematics (NCTM) began organizing a national reform, calling for rich, engaging mathematics instruction. The vision calls for a classroom where "Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers" (NCTM, 2000, p. 3). Many teachers of mathematics have beliefs about mathematics and the teaching and learning of mathematics that are linked to their past experiences with traditional ways of teaching that are not consistent with calls to reform (Cooney, Shealy & Arvold, 1998; Charlambous, Philippou & Kryiakides, 2002; Charlambous & Philippou, 2010).

Teaching mathematics for rich, conceptual understanding seemed to require that teachers have a richly developed mathematical belief system (Thompson, 1992). Thus since Thompson's research there has been significant effort focused on examining how rich belief systems are formed and more importantly how they can change (Nesbit & Bright, 1999; Cooney, 1999; Philipp et al., 2007). Fortunately, those interested in pre-service education have entered this discussion (Wilson & Cooney, 2002; Lloyd, 2002). This point is acknowledged by Philippou & Constantinos (2002) who write "that a pre-service program should consider the structure of beliefs the students bring to teacher education and provide experiences that help students overcome common myths and misconceptions about mathematics, its teaching and learning" (p. 216).

Reform Efforts in Mathematics Education

Reform in mathematics education recognized that teaching mathematics required a deep understanding of mathematics and how students learn mathematics (NCTM, 1989, 2000). The National Council of Teachers of Mathematics (NCTM) has been at the center of reform efforts. There was wide recognition that curriculum must become more than a collection of activities and that it be coherent, focused on important mathematics and well-articulated across all grade levels (NCTM, 1989, 2000). Ongoing recommendations by the National Council of Teachers of Mathematics call for changes in secondary school mathematics curricula and teaching (NCTM, 2000). The Core-Plus Mathematics Project (CPMP), funded by the National Science Foundation (NSF) in 1992, was a reform curricula developed in response to this NCTM call. Huntley (2000) conducted a study to "test the vision of Standards-based mathematics education, using the CPMP treatment of algebra and functions as a specific case" (p. 329). Huntley compared the effects of the CPMP curriculum to more traditional curriculum on growth of student understanding, skill, and problem solving ability in algebra.

The Core-Plus Mathematics Project was a 3-year mathematics curriculum with a 4th year available to students preparing to go to college. A key idea of the curriculum was that algebraic ideas were developed through problem-solving contexts. "Each unit is comprised of several multi-day lessons in which major ideas are developed through investigations of applied problems" (Huntley, 2000, p. 330). Graphing calculators and computers were important components of the curriculum. This was in direct contrast to traditional curricula in which algebraic ideas were developed in written form through algebraic manipulations. Traditional approaches emphasized routine performance of algebraic manipulation and not application of mathematical knowledge to significant problem solving contexts (Huntley, 2000).

This raised the interesting question facing mathematics educators: What mathematics is most important for students to learn? Huntley (2000) acknowledged that this question remains unanswered.

> "Our study does not provide information needed to answer the question about what mathematics is most worth learning, but it does suggest the kinds of trade-offs that might be expected when one allocates time to

topics an ways that differ from allocations in the typical U.S. high school curriculum" (p. 354).

Huntley (2000) did not settle the dispute about how much emphasis should be placed on symbolic manipulation. The results concluded that students with "modest manipulation skills could outperform more symbolically capable students on tasks that required formulation of mathematical representations for problem situations" (p. 357). The study also concluded that students with access to technology could overcome their limited symbol manipulation skills.

Huntley (2000) closed by acknowledging that both reform and traditional curricula needed improvement in order to attain widely agreed upon goals and that teacher beliefs were at the center of reform efforts. The question remains open as to what understanding and skill in algebra was most important. "Some aspects of both reform and traditional curricula need to be studied in more depth with methods other than those used in this study" (Huntley, 2000, p. 361). The point of this discussion was that embracing curricular changes will likely challenge the philosophical belief systems of teachers who are expected to make changes in their classroom instruction. Traditional teaching methods have been challenged which has also challenged teachers to evaluate their philosophical beliefs about the very nature of mathematics. Whereas the roots of reform reach far beyond the 1970's and 1980's, it was then that researchers began to look more closely at the roles of teacher beliefs about mathematics as new curriculum was being developed.

Current reform in mathematics education calls for teaching from the perspective that students construct new mathematical knowledge (Ernst, 1999; Ernst, 2010; Cai & Wang, 2010). The concept of constructivism is certainly not a new idea, yet is at the center of current reform in mathematics education (Ernst, 1999; Goodchild, 2010). To teach in a way that aligns with a constructivist's perspective is to acknowledge not only assimilation of things to be learned, but also changes in the learner's existing cognitive structure (Herscovics, 1989). Research has shown that construction of knowledge is not trivial. Herscovics (1989) examined past research that has succeeded in discovering the existence of cognitive obstacles in the learning of algebra by looking at "the learning of higher-order algebraic concepts, namely, equations in two variables, their graphs, and the notion of function" (Herscovics, 1989, p. 63).

Several researchers found that students' prior arithmetic knowledge can be an obstacle in their construction of meaning for algebraic expressions (Herscovics, 1989; Collis, 1974; Davis, 1975). In particular, concatenation (juxtaposition of symbols) denotes implicit addition for the arithmetic student, but in algebra concatenation denotes multiplication. This illustrates how difficult it is to overcome existing frameworks in order to construct new ones, and even how the old and new frameworks may conflict with one another. Herscovics (1989) writes that "three distinct sources have been described in this survey: obstacles induced by instruction, obstacles of an epistemological nature and obstacles associated with the learner's process of accommodation" (p. 82). Obstacles occur when new mathematical ideas come into conflict with previously constructed knowledge.

That prior mathematical knowledge creates a stumbling block to learning new mathematical ideas is significant. As students construct new mathematical objects, teachers need to be very careful with their language as they discuss these new concepts. Not surprisingly, teachers' perceptions about how students construct knowledge and how teachers can teach from a constructivist perspective are manifestations of their beliefs about the nature of mathematics and the teaching and learning of mathematics. For this reason, constructivism can be a useful lens to help gain an understanding of mathematical belief systems of pre-service teachers.

Constructivism in Mathematics Education

The reform movement in mathematics education has called for a move to teaching mathematics through problem solving and real world applications (Herscovics, 1989; Senk & Thompson, 2003; Swafford, 2003; Santos-Trigo & Camacho-Machin, 2009). This has resulted in extensive research on the learning of mathematics and how students use mathematics to solve problems (Boston & Smith, 2009; Herbst, 2006; Hollerbrand, Conner & Smith, 2010). Not surprisingly, this research has been led by cognitive psychologists seeking to develop and understand theories of learning (Sriraman & English, 2010). A major theme that has emerged is that learning is a constructive process (vonGlasersfeld, 1995). Constructivist theories of learning are well received today by many educators not just in cognitive psychology, but also in the field of mathematics education (Sriraman & English, 2010; Ernst, 2010). In mathematics education constructivism can be characterized as a cognitive position and a philosophical perspective (Ernst, 2010). As a cognitive position, a constructivist would hold that all knowledge is constructed by the learner.

The roots of constructivism can be traced back to Emmanuel Kant who is generally acknowledged with first describing an epistemological subject (Noddings, 1990). Kant sought to explain how a thinking subject generates knowledge. In doing so, Kant was the first to distinguish between empirical knowledge and logical mathematical knowledge. Building upon Kant's ideas Piaget generated his own explanation of the development of mathematical knowledge (von Glasersfeld, 1990). Prior to Piaget, classical abstraction was an extension of empirical observations. Piaget then developed the concept of reflective abstraction that was distinct from the concept of classical abstraction (Noddings, 1990). Reflective abstraction is the idea of internalizing our physical operations on events that we observe. Thus Piaget claimed that personal activities were directly linked to cognitive development. The following quote articulates Piaget's connection to constructivism nicely.

> "Piaget's theories are, in the important sense just described, thoroughly constructivist. Not only are intellectual processes themselves constructive, but cognitive structures themselves are products of continued construction. Constructivism is rooted in the idea of an epistemological subject, an

active knowing mechanism that knows through continued construction" (Noddings, 1990, p. 9).

The theory of learning that is now commonly referred to as "constructivism" has been the basis for plenty of research in mathematics education in the United States (Ernst, 2010). Constructivism then, is a theory about human knowing characterized by a belief that knowledge is always a product of our cognitive experiences.

It is important to note that in mathematics education reflective practices are an essential component of constructivism. This is because mathematics is primarily a human activity. "Reflection as the objectification of a construct functions as the bootstrap by which the mathematician pulls her/himself up in order to stabilize the current construction and to obtain the position from which the next construct can be created" (Confrey, 1994, p. 109). This statement illustrates precisely why mathematics educators have embraced various forms of constructivism. There is a critical philosophical issue that arises when one considers the idea of creating mathematical concepts. There are also different philosophical positions about what exactly is created and this has resulted in the acceptance of various forms of constructivism within the mathematics education community.

Although constructivism in mathematics education has taken on various forms they all hold to the following principles: (1) All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction. (2) There exist cognitive structures that are activated in the process of construction. These structures account for construction: that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer. (3) Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt. (4) Acknowledgement of constructivism as a cognitive position leads to the adaptation of methodological constructivism (Noddings, 1990). Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism.

In mathematics education the acceptance of constructivist principles about knowledge and how knowledge is developed implies a way of teaching that is consistent with cognitive constructivism (Klanderman, 2001; Raymond, 1997). Following Ernst (1989), the four views of constructivism discussed here are:

1. Radical constructivism
2. Simple (weak) constructivism
3. Enactivism
4. Social constructivism

Radical Constructivism

At the Eleventh annual International Conference on the Psychology of Mathematics Education (PME) in Montreal during the summer of 1983 Ernst von Glasersfeld delivered a controversial plenary panel presentation on radical constructivism (Ernst, 1994). The attacks on radical constructivism at that conference actually served to launch von Glasersfeld's flavor of constructivism into widespread international acceptance. Radical constructivism is "an unconventional approach to the problems of knowledge and knowing" (von Glasersfeld, 1990, p.1). To von Glasersfeld, Piaget's model of constructivism seemed somewhat trivial. Thus von Glasersfeld labeled his model "radical" to distinguish it from the ideas of Piaget. The two basic principles of radical constructivism are: (1) Knowledge is not passively received but built up by the cognizing subject. (2) The function of cognition is adaptive and serves the organization of the experimental world, not the discovery of an ontological reality outside the mind of the knower (von Glasersfeld, 1995). The controversy at the PME conference sat squarely on the second principle that replaces the notion of truth with the notion of viability (Ernst, 1994).

Quite naturally, the certainty of mathematical truths has been the catalyst for arguments against radical constructivism. In response, von Glasersfeld (1990) argues that "the theoretical infallibility of mathematical operations cannot be claimed as proof that these operations give access to an ontological reality" (p. 25). The thrust of von Glasersfeld's argument can be traced to the ideas of Kant who distinguished between a priori and a posteriori judgments. For Kant, the judgment that three angles of a triangle in a plane add up to 180 degrees is a synthetic a priori judgment which is possible because the human mind shapes the condition for the knowledge to exist. The mathematical truths of a triangle hold because of the nature of space and time. Yet the human mind is able to impose a particular concept of space and time that make the mathematical judgments to be true (Russell, 2007). For von Glasersfeld it is important to note that a triangle in its pure form is only something that can be conceptualized, and not something that can be constructed in reality. A triangle is constructed by connecting three points by straight line segments. Yet, a line segment conceptually has no width or breadth and a point is defined as having no width or breadth. These ideas are only mental conceptions since it is impossible to draw a point without drawing a dot that has dimension.

Philosophically there seemed to be a need to distinguish between mathematics that the human mind “does” and mathematics that “exists”. This is not a distinction that von Glasersfeld makes, but making this distinction would be consistent with radical constructivism. This distinction would then allow the possibility of conceptually discovering mathematical truths that do exist even though they are not representations of an exact reality. Thus when a person “does” mathematics, they are trying to make sense of physical realities by applying mathematical conceptions. The point would not be to discover an ontological reality, but to use mathematical conceptions to gain some understanding of physical reality. This seems to be consistent with the second principle of radical constructivism. Making this distinction between the mathematics the mind does with the mathematics that exists leaves allows me as a researcher to accept von Glasersfeld’s second principle, which many mathematics educators seem to struggle with. Due to this inner struggle, there are some constructivists who have not accepted the second principle resulting in what is commonly referred to as simple (weak) constructivism (Ernst, 2010).

Simple Constructivism

Beliefs about whether or not absolute knowledge is attainable distinguish different views of constructivism. Simple constructivism accepts the idea that true representations of empirical and experiential worlds are possible (Ernst, 2010). Thus there are many mathematics educators who hold the general principles of constructivism (including von Glasersfeld's first principle) but reject von Glasersfeld's controversial second principle. Both simple and radical constructivism are theories about learning that can be traced to the ideas of Piaget (Noddings, 1990). Lerman (1989) writes of von Glasersfeld's controversial second principle,

> "the second is more controversial and perhaps worrying, since it appears to lead us immediately into problems on two levels: firstly, whether it is ever possible to understand what anyone else is saying or meaning, that is problems of private languages, and secondly, what kind of meaning can thus be given to what we all accept as known, that is, the nature of knowledge in general and of mathematical knowledge in particular" (p. 211).

Thus the temptation might be to accept the first principle and be content with simple constructivism leaving the second principle to philosophers and conference discussions. Though many have taken this easy way out, Lerman (1989) rightfully argues that this is not a very wise approach. The second principle should be examined if for no other reason than because of its significance with the very nature of mathematics.

Enactivism

Enactivism is another theory of learning that is consistent with the basic principles of constructivism (Ernst, 2010). Enactivism was described in the influential book The Embodied Mind written by Francisco Varela, Evan Thompson and Eleanor Rosch (1991). Fundamental to enactivism is the idea that knowledge comes from a process of self-creation. “Enactivism is a theory of cognition as the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs” (Varela et al., 1991, p. 9). Thus an individual knower is not just an observer of physical reality, but is also physically entrenched in the world and is shaped physically and cognitively through interactions in the world. Enactivism reflects the importance of the individual in the construction of reality, but acknowledges the importance that the individual co-exists and is co-developed with the world.

Cognitive scientists often assume a sharp distinction between independently existing, external objects and their internal representations in the mind of a knower. Varela et al. (1991) suggested replacing this distinction with an “enactive” description of what it means to know. The authors explained the fundamental differences by illustrating with the answers from three questions:

> “Question 1: What is cognition?
>
> Cognitivist Answer: Information processing as symbolic computation – rule based manipulation of symbols.
>
> Enactivist Answer: Enaction. A history of structural coupling that brings forth a world.
>
> Question 2: How does it work?

Cognitivist Answer: Through any device that can support and manipulate discrete functional elements – the symbols. The system interacts only with the form of the symbols, not their meaning.

Enactivist Answer: Through a network consisting of multiple levels of interconnected, sensorimotor sub networks.

Question Three: How do I know when a cognitive system is functioning? properly?

Cognitivist Answer: When the symbols appropriately represent some aspect of the real world, and the information processing leads to successful solution to the problem given to the system.

Enactivist Answer: When it becomes part of an ongoing existing world or shapes a new one" (Varela et al, 1991, p. 42-3)

Enactivism is not significantly different than Piaget's theory of knowing and the radical constructivism to which Piaget's ideas are central. However, Ernst does point out that an important distinction needs to be made between radical constructivism and enactivism. For enactivism it is not "a matter of an individual having a cognitive structure, which determines how the individual can think, or of there being conceptual structures which determine what new concepts develop. The organism as a whole is its continually changing structure which determines its own actions on itself and its world" (Ernst, 2010, p. 43). For enactivism and radical constructivism the emphasis is on the individual as a cognitive knower and the social dimension is external to the primary sources of knowledge. This naturally provides motivation for another view of constructivism.

Social Constructivism

Social constructivism views individual learners and the social environment as intimately linked in a way that the link cannot be broken (Ernst, 2010). People are formed through social interactions as well as by their individual internal experiences. These ideas originated with Vygotsky's (1978) work on the origins of language in the individual as something that is internalized from social experiences. Conversation provides a powerful basis for learning and cognitive development. The Zone of Proximal Development (Vygotsky, 1978) describes a process in which both individual and private meanings and collective and public expressions are mutually shaped through conversation (Ernst, 2010).

Cobb & Yackel (1996) develop a theoretical framework that coordinates psychological constructivism with social interactionism. They were confronted with classroom social norms that could not be characterized as "psychological entities that can be attributed to any particular individual" (p. 212). This initiated an interest in classroom social norms as they sought to account for students' mathematical development as it occurred in a social setting which is an issue not explicitly addressed in radical constructivism.

To encourage meaningful learning of mathematics, teachers should understand how to craft their instructional practices so that they are in sync with how students learn mathematics (Polya, 1945; Green, 1971; Mason, 1982). For over 30 years now cognitive psychologists have made significant progress in understanding students' mathematical understandings (Ernst, 2010). The real value of this progress is found in how cognitive theories can influence classroom instructional practices. Constructivist theories of cognitive development indicate that it is essential to distinguish between meaningful mathematical learning and rote mathematical learning (Schoenfeld, 1983). Students ought to be afforded the opportunity to engage in mathematical thinking that encourages construction of mathematical meanings (Cai, 2010). This requires that instructional practices be structured upon an understanding of the theories of cognitive development (Ernst, 2010). This is not to say that any pedagogic practices are completely determined by one particular theory. Learning theories do not necessarily imply particular pedagogical approaches. "Nevertheless, certain emphases are foregrounded by different learning theories, even if they are not logical consequences of them" (Ernst, 2010, p.45). Ernst goes on to offer implications for educational practice:

> Simple constructivism suggests the need and value for:
>
> 1. Sensitivity towards and attentiveness to the learner's previous learning and constructions.
>
> 2. Identification of learner errors and misconceptions and the use of diagnostic teaching and cognitive conflict techniques in attempting to overcome them.
>
> Radical constructivism suggests attention to:
>
> 1. Learner perceptions as a whole, i.e., of their overall experiential world.
>
> 2. The problematic nature of mathematical knowledge as a whole, not just the learner's subjective knowledge, as well as the fragility of all research methodologies.
>
> Enactivism suggests that we attend to:
>
> 1. Bodily movements and learning, including the gestures that people make.
>
> 2. The role of root metaphors (cultural) as the basal grounds of learner's meanings and understandings.
>
> Social constructivism places emphasis on:
>
> 1. The importance of all aspects of the social context and of inter-personal relations, especially teacher-teacher and learner-

learner interactions in learning situations including negotiation, collaboration, and discussion.

2. The role of language, texts and signs in the teaching and learning of mathematics (Ernst, 2010, p. 45).

Ernst (2010) points out the importance of understanding “these eight focuses in the teaching and learning of mathematics could be legitimately attended to by teachers drawing on any of the learning theories for their pedagogy” (p. 46). Social constructivism is at the very center of reform methods. Teacher beliefs about the nature of mathematical truths should be of interest to mathematics educators as they seek to equip teachers to teach in ways consistent with constructivist principles. This study provided important insight into where teacher beliefs about the nature of mathematical truths fit into the picture as teachers go through the process of changing from traditional ways to constructivist methods consistent with the current reform. A social constructivist’s view of mathematical learning focuses on the student being engaged in an active learning process (Cobb & Yackel, 1996). The teacher becomes the facilitator, posing well designed problems that challenge the students to construct their own mathematical knowledge. This is in direct contrast to traditional views of learning that most teachers experienced when they were in elementary school.

Laying a foundation for research on teacher change, Goldsmith & Schifter (1997) argue that if the change of teaching from a practice based on a passive view of learning to a practice based on a constructivist's view of learning were to be a developmental process, then the changes in knowledge, belief and practice could be described by: (a) qualitative reorganizations of understanding, (b) orderly progressions of stages, and (c) transitions mechanisms (p. 21). Goldsmith & Schifter (1997) present the thoughts of a particular teacher who experienced struggles as he came to question many of his fundamental beliefs about the teaching of mathematics. The teacher "wrote of needing to rethink what it means to learn mathematics and what kinds of mathematics his students should be learning" (p. 25). The change required involves more than just acquiring new instructional techniques. The changes go to the very root of a teacher's beliefs about mathematics and how mathematics is learned. These beliefs are embedded in the identity of teachers.

Teacher Identity

Pre-service secondary mathematics teachers sense of who they are as a future teacher of mathematics is at least partially shaped by their beliefs about the nature of mathematics and the teaching and learning of mathematics (Wilson, 2010; Gresalfi & Cobb, 2011). Thus a discussion regarding teacher identity seems appropriate to help explore the concept of teacher beliefs. Teacher identity brings together personal knowledge, beliefs, values and classroom practices (Battey & Franke, 2008). There is continuing debate about teacher identity and whether or not a teacher has one or multiple identities (Wilson, 2010; Gresfali & Cobb, 2011). Finding theoretical rigor concerning the concept of identity may be important, but is not the focus of this research. This research was far more interested in how teachers view and define their own identities and what this reveals about their beliefs about the nature of mathematics and the teaching and learning of mathematics. This line of thought seems appropriate for teacher educators who design and implement teacher education programs.

When teachers teach mathematics they teach much more than just factual knowledge and skills (Lester, 2002; Llinares, 2002). They also communicate beliefs, values and emotional responses about mathematics, all based on their mathematical belief systems (Grootenboer, 2006). Gaining insight into the construct of teacher identity seemed natural when seeking to explore pre-service teachers' mathematical belief systems. Understanding how pre-service teachers develop their teacher identity may shed light on their belief systems.

Owens (2008) conducted a study that explored the ways in which pre-service teachers develop their identity as mathematical thinkers. The basis for the study was an understanding that pre-service mathematics teachers do not develop their identity as a teacher in isolation to their identity as a mathematical thinker. Teachers of mathematics must understand first-hand what it means to engage in mathematical problem solving activities if they are to be effective teachers (English & Sriraman, 2010). For Owen's study, data were collected from students enrolled in their first year of a primary teacher education program. The students were in their first mathematics course for primary teachers. The course was equivalent to a final high school mathematics course. The course placed emphasis on real-life problem solving skills designed to develop skills while exploring patterns, relationships in numbers, spatial reasoning and measurement and was based on a constructivists' approach to mathematics teaching and learning. The course also placed heavy emphasis on the idea that the classroom was a community of learners all working together. The students began at different positions in their development as mathematical thinkers. They were heavily influenced by their experiences with high school mathematics. High level of success in high school mathematics translated to a confidence to try new problems, though some students' past success was restricted to success with algorithms which lead to a floundering when they could not remember a particular formula. There were very clear instances of working together in ways that "was soon seen as a community helping each other so that the pre-service teachers' confidence grew and some of them really flourished with their community projects" (Owens, 2008, p. 44). Thus one conclusion was that the development of identity as a mathematical thinker was linked to social identity. Owens (2008) concluded that learning experiences rooted in problem solving, social interactions and engaging with technology all had a positive effect on the development of identity as a mathematical thinker.

Acknowledging that the practice of teaching mathematics was rooted and shaped by teachers' sense of teacher identity Grootenboer & Ballantyne (2011) conducted a study to explore "the nexus" of these identities. In the mathematics classroom, teacher identity consisted of the ways the teacher perceived themselves as a teacher as well as how they perceived themselves as mathematicians. The study used qualitative methods to explore the professional and discipline-based aspects of high school teachers' identities and the relationships between these aspects and their teaching practice. Data consisted of interviews, classroom observations and document analysis. The eight participants were high school mathematics teachers from South-East Queensland, Australia. The researchers were surprised at the vast differences between the teaching styles of the eight teachers, ranging from strict, highly structured classrooms, to loose informal classroom settings. All eight teachers were acknowledged as being effective teachers, illustrating that there was more to effective teaching than teaching methods. All eight participants identified themselves firstly as teachers, all highly engaged in caring for their students. Surprisingly, when asked if they considered themselves as mathematicians, all eight participants responded no. However, they did want the students to see themselves as mathematicians. Though they did not perceive of themselves as mathematicians, they all reported a strong mathematical sense of self.

Grootenboer & Ballantyne (2011) acknowledged that their research was only preliminary and that they were continuing the project, yet they did discuss three conclusions. First, each participant had coherent beliefs about how children learn mathematics and their teaching was consistent with those beliefs. This was significant because it illustrates that practice was rooted in beliefs. Secondly, all participants did not identify themselves as mathematicians, though each was confident working with mathematics. Lastly, all participants' classroom practice was characterized by quick decisions seemingly taken with limited thought process. "The sense of immediacy about these decisions and actions that are the fabric of classroom teaching, indicated to us that they were made from the teachers' identity or sense of self" (Grootenboer & Ballantyne, 2011, p. 310).

Mathematical identity flows from dispositions and relationships with mathematics (Leatham & Hill, 2010). For this study, teacher identity was conceptualized as an individual's relationship with teaching and their relationship with mathematics. Some aspects of a teacher's identity may be easily observable, however underlying a teacher's identity are dispositions about mathematics that are not easily observable (Leatham & Hill, 2010). Understanding the construct of teacher identity helped to tell the story of dispositions held about the nature of mathematics and how those dispositions might interact with beliefs regarding the teaching and learning of mathematics.

Distinguishing Between Knowledge and Beliefs

The essential quality of proof is to compel belief
Fermat

As the introductory quote from Fermat indicates, epistemological questions about the nature of knowledge and belief systems are not easily answered. Yet researchers interested in researching belief systems are called to at least address their position regarding this philosophical issue (Southerland, Sinatra & Matthews, 2001). The history of Western philosophy was significantly shaped by the thoughts of Plato. The basic philosophical questions regarding knowledge and beliefs central to discussions today were addressed by Plato 2500 years ago (Russell, 2007). Plato argued that for a proposition to become knowledge it had to meet three criteria: truth, belief and evidence. Thus for Plato, knowledge was justified, true belief. Since the time of Plato, much philosophical debate has centered on the question of what constitutes justification (Russell, 2007).

Understanding the difficulty in establishing what constitutes justification, Binmore (2005) wrote an interesting paper discussing why the distinction between knowledge and beliefs might matter and began by recognizing that the English language recognizes that to know something is not quite the same as to believe something. Many philosophers today join Plato and claim that the distinction should begin with the idea that knowledge can be thought of as justified true belief (Ernst, 2010). Yet the construct of justification has been difficult to nail down. "Knowledge should be regarded as justified true belief, but such an attempt at a definition has had little influence in rational choice theory, presumably because the question of what should count as a justifying argument is left hanging in the air" (Binmore, 2005, p. 97).

To illustrate the difficulty in grasping the idea of justification Binmore (2005) gave an example of a woman standing on a curb who knows that bad things would happen if she were to step in front of a car. She has knowledge, not necessarily because she has experienced stepping in front of a moving car, or even that she has experienced other people stepping in front of moving cars. Her knowledge is based on her ability to imagine another world in which bad things would happen if she stepped in front of a moving car. Binmore argued that this idea of imagining things happening in another world compels us to make a distinction between beliefs and knowledge. Thus,

> “if we analysts were in the habit of saying that the players in a game have a common belief that they are all rational – instead of common knowledge – we would then be forced to commit ourselves to an expanded world in which different types of players might be more or less clever when deciding how to play (Binmore, 2005, p. 102).

Ozakpinar (2011) argued that all types of knowledge are beliefs with different degrees of certainty and the only justification that can lead to certainty is faith. He wrote that “only knowledge known through professing faith is absolutely certain” (Ozakpinar, 2011, p. 287). On the other hand, knowledge obtained empirically, with a conscious understanding of the necessity of objective evidence, is probable knowledge due to the limitations of sensory and mental capabilities of the human race. Thus, for Ozakpinar, empirical knowledge is always probable knowledge and the strength of the probability determines a person’s degree of belief in the truth of it. Ozakpinar concluded by stating that all types of knowledge have in their nature different degrees of belief.

Seeking clarity in discussing the distinctions between knowledge and beliefs in educational research, Southerland, Sinatra & Matthews (2001) note that various definitions employed in science education have been shaped by philosophy and educational psychology, two distinct fields of study. They examined distinctions made between knowledge and beliefs by both philosophers and by educational psychologists and how these distinctions have influenced science education. The influence of philosophy was clear. Philosophers have long held that a lack of solid empirical evidence to support a true opinion meant that opinion could not be considered knowledge. This was a problem that Plato was well aware of. The philosophical response to this problem has taken various forms. Empiricists such as David Hume and Bertrand Russell claimed that scientific claims could not be said to be knowledge unless they were grounded in sense data. For rationalists the foundation of knowledge was some form of clear and distinct ideas (as with Descartes' cognito) or synthetic a-priori claim (as with Kant) (Southerland, Sinatra & Matthews, 2001). Both the empiricist and the rationalist approach knowledge from the Platonic tradition; there exists a thinking individual, some claim to knowledge and a form of reasoning that grounds the claim as knowledge.

Karl Popper (1934) instigated a challenge to the traditional view of a cognizant observer having claims of knowledge. In The Logic of Scientific Discovery Popper formulated his objectivist account of knowledge without a "knowing" subject. Popper laid out three distinct worlds. (1) The objective, material world. (2) The subjective world of an individual's mental operations. (3) The scientific world of observations and theories (Popper, 1934). Using this theory there was a marked distinction between scientific knowledge and personal knowledge. Thus for an objectivist there are many states of mind ranging from wishes, hopes and fears as well as beliefs. Some beliefs may be true and some may be false, but all are cognitive states. The understanding here was that justified true beliefs were knowledge states of mind. Clearly the idea of adequate justification was still problematic as justification could be just that my mother told me my belief was true.

Educational psychology began weighing in on the matter of knowledge in the latter part of the 20th century by acknowledging that the constructs of knowledge and beliefs had an influential place in educational research. Educational psychologists were concerned with how knowledge and beliefs were developed (Southerland, Sinatra & Matthews, 2001). Alexander & Dochy (1995) acknowledged that most educational psychologists understood knowledge as needing justification, or empirical evidence of support but viewed beliefs as less accountable to justification. Some educational psychologists held that knowledge was the over-arching construct with beliefs being one piece of an individual's knowledge base (Southerland, Sinatra & Matthews, 2001). It should be noted that this idea was in stark contrast to philosophical definitions of knowledge as justified, true belief because it did not require justification and truth to have knowledge. However, Alexander & Dochy (1995) found that most current educational psychologists in the United States and the Netherlands held the same view as philosophers that knowledge required some form of external justification. Alexander & Duchy (1995) concluded that in general, educational psychologists perceived the concepts of knowledge and beliefs as overlapping, both flowing from experiences. The distinction stemmed from the thought that knowledge arose from school or formal educational experiences whereas beliefs arose from every day experiences of life.

Characteristics of Belief Systems

Perhaps a more constructive direction would be to avoid the thorny issue of attempting to pin down justification and turn to a discussion of the characteristics of belief systems that are markedly different from the construct of knowledge. This was the approach taken by Green (1971) and Furinghetti & Pehkonen (2002). Green identified three dimensions characterizing belief systems: quasi-logicalness, psychological centrality and cluster structure. Furinghetti & Pehkonen discussed these dimensions explicitly drawing distinctions with knowledge systems.

Quasi-Logicalness

Knowledge was generally understood to be formed from premises and conclusions deduced from them. Beliefs on the other hand generally had a relationship that may not be as logical. Within a belief system, beliefs may not be held in consensus with other beliefs. Thus it makes perfect sense that a person could have beliefs that were in direct contradiction with other beliefs held by the same person. This could not be the case in a knowledge system.

Psychological Centrality

Most people held their beliefs in varying degree of importance. The most important beliefs were psychologically more important when compared with other beliefs. Furinghetti & Pehkonen (2002) stated that, generally speaking, knowledge lacks this dimension. Perhaps this might be, but it also seemed that knowledge could be held with varying degrees of conviction, which seemed to be a similar construct to levels of importance.

Cluster Structure

"Nobody holds beliefs in total independence of all other beliefs. Beliefs always occur in sets or groups' (Green, 1971, p. 41). This clustering structure helped to explain some of the inconsistencies that could be found in an individual's belief system. Furinghetti & Pehkonen illustrated this with an example from mathematics research (Hasemann, 1987). Children would add fractions using the rule of adding the numerators and adding the denominators then also correctly perform diagrammatic solutions and not be troubled with believing that both answers were correct. Both algorithms were included in the students' belief system but seemed to be held in different clusters since two answers were acceptable for the same task.

Additionally, belief clusters may have an evaluative component that was absent in knowledge systems.

> "A belief system typically has extensive categories of judgments, which are grouped into "good" and "bad". As a typical example, those who support so-called "green values" also usually believe that nuclear power is bad, materialism and waste are bad, natural alternative energy sources are good, recycling is good" (Furinghetti & Pehkonen, 2002, p. 45).

The point here was not to have an exhaustive discussion about the distinctions between beliefs and knowledge, but rather to acknowledge the thorny issues involved and bring a little clarity to the relationship between knowledge and beliefs. This brief discussion about the distinctions between knowledge and beliefs precedes a discussion about how the concept of belief systems was defined for this study.

Defining Beliefs

Research in the area of beliefs about the nature of mathematics certainly had areas of difficulty, perhaps the greatest being to characterize the meaning of the term “beliefs”. Work in the area of beliefs was found in many disciplines ranging from psychology to history and the sciences. Torner (2002) discussed this issue by examining literature in mathematics education to shed light on how mathematics educators defined the term beliefs. Characteristics of common definitions were discussed and a four-point definition of beliefs was outlined. “The model focuses on belief object, range and content of mental associations, activation level or strength of each association and some associated evaluation maps” (Torner, 2002, p. 73). The object, range and strength levels of beliefs were fairly straight forward concepts. Torner (2002) noted that beliefs relied quite heavily on evaluative and affective components. A teacher’s belief system about mathematics was shaped by her emotional response and sense of identity in relation to mathematics. Evaluation maps were used as a language scale to express likes and dislikes, approval and disapproval. Torner’s purpose was to

> “search for common ground in definitions of beliefs. The proposed four-components-model motivates one to specify the belief object, reflect the

> breadth of the content set of beliefs, to trace possibly interacting membership degree functions as attributes of beliefs, and to identify evaluation maps in question" (Torner, 2002, p. 90).

Clearly this did not result in a precise definition of all components, but it offered a framework to give consistency to the term "beliefs".

Furinghetti & Pehkonen (2002) also searched mathematics education literature in an effort to clarify the understanding of beliefs among mathematics education researchers. The study sought to investigate what researchers in the field of mathematics education intended when they conducted research regarding mathematical beliefs and conceptions. Surprisingly, they began by noting that often researchers used the concept of belief yet left it undefined, or worse gave contradictory definitions. Seeking to understand the concept of beliefs among mathematics educators a survey was developed and sent to a panel of mathematics education researchers. The survey consisted of nine characterizations related to beliefs purposively selected from research literature. The authors explicitly stated that the list of characterizations was not intended to be comprehensive. During the spring of 1999 a questionnaire was sent to 22 mathematics education specialists in the field of beliefs who had been invited to the international meeting of Mathematical Beliefs and their Impact on Teaching and Learning Mathematics held in the fall of 1999. There were 18 responses to the questionnaire. The results were presented at the international meeting which allowed the specialists in attendance to give comments.

Table 1

Nine Characterizations of Beliefs Included in Survey

Characterization 1 (Hart, 1989)	"We use the word belief to reflect certain types of judgments about a set of objects"
Characterization 2 (Lester, 1989)	"Beliefs constitute the individual's subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements."
Characterization 3 (Lloyd & Wilson 1998)	"We use the word conceptions to refer to a person's general mental structures that encompass knowledge, beliefs, understandings, preferences and views."
Characterization 4 (Nespor, 1987)	"Belief systems often include affective feelings and evaluations, vivid memories of personal experiences and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge are."
Characterization 5 (Ponte, 1994)	"Beliefs and conceptions are regarded as part of knowledge. Beliefs are the incontrovertible personal "truths" held by everyone, deriving from experience or from fantasy, with strong effective and evaluative component."
Characterization 6 (Pehkonen, 1998)	"We understand beliefs as one's stable subjective knowledge (which also include feelings) of a certain object or concern to which tenable grounds may not always be found in objective considerations."
Characterization 7 (Schoenfeld, 1992)	"Beliefs – to be interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior."
Characterization 8 (Thompson, 1992)	"A teacher's conceptions of the nature of mathematics may be viewed as that teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics."
Characterization 9 (Torner & Grigutsch, 1999)	"Attitude is a stable, long lasting, learned predisposition to respond to certain things in a certain way. The concept has a cognitive (belief) aspect, an affective (feeling) aspect and a conative (action) aspect."

Table 2

Degree of Agreement/Disagreement of Respondents

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Yes	7	7 *	9	4 *	2	7 *	11 *	11 *	7
Partly Yes	4 *	1	3	-	1 *	1	1	1	4 *
Partly	2	7	4 *	4	-	1	3	2	2
Partly No	1	-	-	2	-	-	-	-	2
No	4	3	2	8	15	9	3	4	3

Those who participated in the survey were asked to express their level of agreement or disagreement to each of the nine characterizations of beliefs and to explain why they agreed or disagreed. Each participant was also asked to give their own characterization to the concept of belief, yet only half of the respondents gave their own characterization of beliefs. Table 1 lists the nine characterizations gathered from the literature including the author of each characterization. The respondents were not given the author's name associated with each characterization. Table 2 lists the distribution of the levels of agreement respondents had for each of the characterizations. The asterisks in Table 2 represent how I rated my own level of agreement to the characterizations.

Furinghetti & Pehkonen (2002) noted the results showed no clear pattern to be observed, though there were a few points of consistency. Most notably the respondents were unified in their "No" response to characterization number five. Two reasons were identified for this negative response. (1) Most respondents hesitated to regard beliefs as a part of knowledge and (2) most respondents had difficulty with the word “incontrovertible” which is synonymous with “undeniable”. The authors also noted that they were not surprised that most respondents agreed with characterizations 7 and 8. The authors published multiple papers that were widely used as referenced literature in research on mathematical belief systems.

Furinghetti & Pehkonen (2002) were explicitly aware of the fact that complete agreement was highly unlikely when it comes to characterizing the term “beliefs”.

> “Nevertheless, it can be asked that the authors of studies on beliefs reduce the terms and the concepts involved in their work to a minimum. Additionally, researchers are challenged to make clear their assumptions, the meaning they give to basic words and the relationship between the concepts involved” (Furinghetti & Pehkonen, 2002, p. 55).

To help guide future research in the area of beliefs about mathematics the authors made several suggestions that can bring consistency to the research community:

1. To consider two types of knowledge (objective and subjective).
2. To consider beliefs as belonging to subjective knowledge.
3. To include affective factors in belief systems and distinguish between affective and cognitive beliefs if needed.
4. To consider degrees of stability, and to acknowledge that beliefs are open to change.
5. To take note of the context (population, subject, etc.) and the research goal within which beliefs are considered (Furinghetti & Pehkonen, 2002, p. 55).

For this research these five suggestions are accepted and form the basis for how the term "beliefs" is used.

Measuring Beliefs

Finding ways to infer belief systems has continued to be a challenge to researchers (Leder & Forgaz, 2002). Schoenfeld (1992) pleaded for instruments to measure belief systems in the context of mathematical activities in order to integrate cognition and affect. Ernst (1988) developed a three dimensional model to examine philosophical beliefs of mathematics teachers. The Platonist view considers mathematics as a static, objective body of knowledge that exists and is waiting to be discovered by inquisitive minds. Mathematics is not invented or created. The Instrumentalist view regards mathematics as a set of rules and facts to be learned while the Problem Solving views mathematics as a dynamic field of knowledge that is created by human beings and is continually changing. Charlambous (2002) conducted a study to examine the efficiency of Ernst's model in describing teachers' philosophical beliefs. The study also examined the factors that influence the development of philosophical beliefs as well as the consistencies between these beliefs and actual teacher practices in the context of mathematics teaching and learning.

Charlambous (2002) used mixed methods to conduct the study and a three part questionnaire was developed. The first part contained a likert-type survey reflecting philosophical beliefs following the three dimensional model developed by Ernst. The second part contained questions concerning the teaching and learning of mathematics. Each question included six statements and the teacher was asked to put the statements into an order that corresponded to their own beliefs about the teaching and learning of mathematics. The last portion of the questionnaire had open ended questions regarding teacher practices. The questionnaire was sent to 345 teachers with 229 responses. Using an intentional sampling, five teachers were selected for semi-structured interviews.

Charlambous concluded that those interviewed did not seem to have thought deeply about their beliefs about the teaching and learning of mathematics. Though the study provided evidence that philosophical beliefs influenced teaching practice, the connection was complex. This was most notable in the inconsistencies among the teachers' beliefs and their practices. The study "underlies the importance of prompting teachers to reflect upon and examine their own belief systems" (Charlambous, 2009, p. 9)

Likert-type questionnaires have been frequently used when seeking to understand the mathematical beliefs of teachers (Raymond, 1997; Nesbitt & Bright, 1999). Ambrose et al. (2003) noted three deficiencies with likert-type questionnaires. First, it is often difficult (impossible) to know exactly how respondents interpret the words in the questions. Secondly, likert-type surveys usually do not provide useful contexts. Lastly, likert-type questions do not have a mechanism to determine how important issues are to the respondent. Recognizing these deficiencies Ambrose et al. (2003) conducted a study to test the effectiveness of an alternative instrument for assessing teachers' beliefs about mathematics. The instrument was developed over a two year period. The web-based instrument was made up of seven segments, each including multiple questions concerning a specific situation. Each segment was associated with two or three beliefs and two segments contained video clips of children doing mathematics. The respondents were asked to respond to questions referring to the video clips. The authors provided a complete description of how they assigned belief scores based on data collected from the instrument. Ambrose (2003) noted the effectiveness of using video clips to create contexts allowed users to respond in their own words rather than choosing from a list of pre-determined responses. An obvious limitation was the effort required to code the response into a meaningful format. “Although our instrument measured change between the beginning and end of the treatment, it seemed neither too easy, nor too difficult; that is, it measured neither a floor effect nor a ceiling effect” (Ambrose et al., 2003, p. 39).

Metaphors have also been used to examine belief systems of teachers. Reeder, Utely & Cassel (2009) used metaphors to examine pre-service teachers’ beliefs concerning their conceptualizations of the roles of the teacher and learner of mathematics. 200 pre-service elementary and early childhood teachers enrolled in a large Midwestern public university participated in the study. Participants were supplied with paper, markers and colored pencils and were asked to construct a visual metaphor for the teaching and learning of mathematics. The data were analyzed based on three categories as used by Schubert (1986). These categories were production, journey and growth which will be discussed in more detail in chapter three where the research methods for this study are outlined.

Reeder et al. (2009) concluded that despite opportunities to actively reflect on their beliefs about mathematics and the teaching and learning of mathematics, the pre-service teachers' beliefs remained mostly static. "The reflective practices and experiences in this teacher education program had minimal impact in its efforts to bring about a shift in these pre-service teachers' beliefs about mathematics teaching and learning" (Reeder et al., 2009, p. 296).

Summary

This chapter began with historical background to set the stage that pre-service teacher's beliefs about the nature of mathematics and the teaching and learning of mathematics is an important issue today in mathematics education. Reform efforts have played a significant role in mathematics education for the past few decades and in the midst of these reform efforts, constructivism has emerged as the dominant theory of learning. Beliefs about the nature of mathematics and the teaching and learning of mathematics have been shown to have a direct influence on how constructivism is perceived by teachers of mathematics. Thus examining beliefs through the lens of constructivism made sense. A working definition of the term "beliefs" was given and lastly, several methods for examining beliefs were discussed.

CHAPTER THREE

RESEARCH PARADIGM

Introduction

Discovery has been the aim of scientific research since the time of Rene Descartes. Yet how those discoveries have been made has varied with the nature of the things being studied (Strauss & Corbin, 1998). In this research the goal was not to discover explanations about mathematical belief systems. Rather the goal of this study was to explore, interpret and understand the phenomena of mathematical belief systems of pre-service secondary mathematics teachers. Thus a qualitative research design lent itself to the goals of this study. The research questions considered were:

1. What are the philosophical beliefs of pre-service secondary mathematics students' at a small private university regarding the nature of mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with pre-service secondary mathematics students' beliefs regarding the teaching and learning of mathematics?

Cresswell (2009) points out that conducting qualitative research involves the intersection of three components. First, it is important that researchers explicate the philosophical worldview that influences their research. Secondly, a strategy of inquiry should be chosen that is consistent with the stated philosophical world view. Thirdly, specific research methods should then flow directly from the chosen strategy of inquiry. At first glance these components seem to have a linearity, from philosophical world view, to strategy of inquiry and lastly to specific research methods. In reality it should be understood that designing a research plan is a reflexive exercise in which all three components interact to strengthen each other. These three components should flow freely from a well-articulated research paradigm.

Interpretivism was chosen as the research paradigm because a primary goal of the study was to interpret the belief systems of pre-service secondary mathematics teachers in an effort to gain understanding. The researcher explored what pre-service secondary mathematics teachers believed about the nature of mathematics and the teaching and learning of mathematics. A goal of this research was to understand a human phenomenon and pre-service teachers' experiences of this phenomenon. This goal fits nicely with the philosophical position and strategies of an interpretive paradigm. The goal was to access understanding and meanings as opposed to explaining. According to the interpretive paradigm, meanings are constructed by human beings as they engage in the world around them (Crotty, 1998). Findings emerged from interactions between the researcher and the participants throughout the research process. Subjectivity was valued because human beings are situated in a world constructed by experiences. The interpretive paradigm was chosen as suitable because of its potential to shed light on new understandings of the complex human phenomenon of belief systems.

Social constructivism as a philosophical world view and as a theory of learning influenced this research. Chapter two discussed social constructivism as a theory of learning within mathematics education. However, social constructivism is also widely viewed as a more general world view (Berger, 1966; Schwandt, 2007; Cresswell, 2009). Hermeneutic phenomenology was the chosen strategy of inquiry because it is consistent with both the paradigm of interpretivism and social constructivism as a philosophical world view (Dowling, 2007). The research methods presented in Chapter Three were chosen to be consistent with the interpretive paradigm.

Interpretivism

Qualitative research involves inquiry which seeks to construct a natural description of social issues in ways that may deepen a researcher's understanding of particular phenomena (Cresswell, 1998; Crotty, 1998; Patton, 2002; Ajjawi & Higgs, 2007). This process seems to demand a research process that is much different from the logical positivism that has dominated traditional research since the days of Rene Descartes (Jones, Torres & Armino, 2006). Interpretivism shifts the emphasis of research away from the perspective that there exists an objective reality that needs to be understood (Ajjawi & Higgs, 2007). Within the research paradigm of interpretivism the focus is often on an attempt to access the meanings of a phenomena from participants' experiences rather than attempting to explain or predict their behavior as is the case with a positivist research paradigm (Ajjawi & Higgs, 2007).

The purpose of conducting research under the paradigm of interpretivism was to gather information that might allow the researcher to gain understanding and meaning from the participants' perspectives. "Central to interpretivism is the idea that all human activity is fundamentally a social and meaning making experience, that significant research about human life is an attempt to reconstruct that experience" (Eisenhart, 1988, p. 102). Research findings emerged as the researcher attempted to enter into and make sense of the world of the participants (Laverty, 2003). An interpretive paradigm fit nicely for this particular research because of the potential to gain new understanding of mathematical belief systems that were a complex and multi-dimensional phenomena (Ajjawi & Higgs, 2007).

The components presented by Cresswell align well with the paradigm of interpretivism. Social constructivism is an appropriate philosophical world view. Social constructivism already discussed as a philosophy of learning, is also useful in a broader sense as a world view. Hermeneutic phenomenology is a research method that is consistent with interpretivism. Figure 1 illustrates an overview of the research approach.

Many of the tenets of Interpretivism flow from a philosophical stance that is quite different from the logical underpinnings of positivism common in traditional research approaches (Jones, Torres & Armino, 2006). Interpretivism contains the idea that human activity is a social and meaning making experience. Research about human life is an attempt to reconstruct life experiences. Methods to investigate experiences should be modeled after those experiences (Eisenhart, 1988). Knowledge and meanings exist only because they are constructed socially.

An overview of the methodology for this study is illustrated in Figure 1

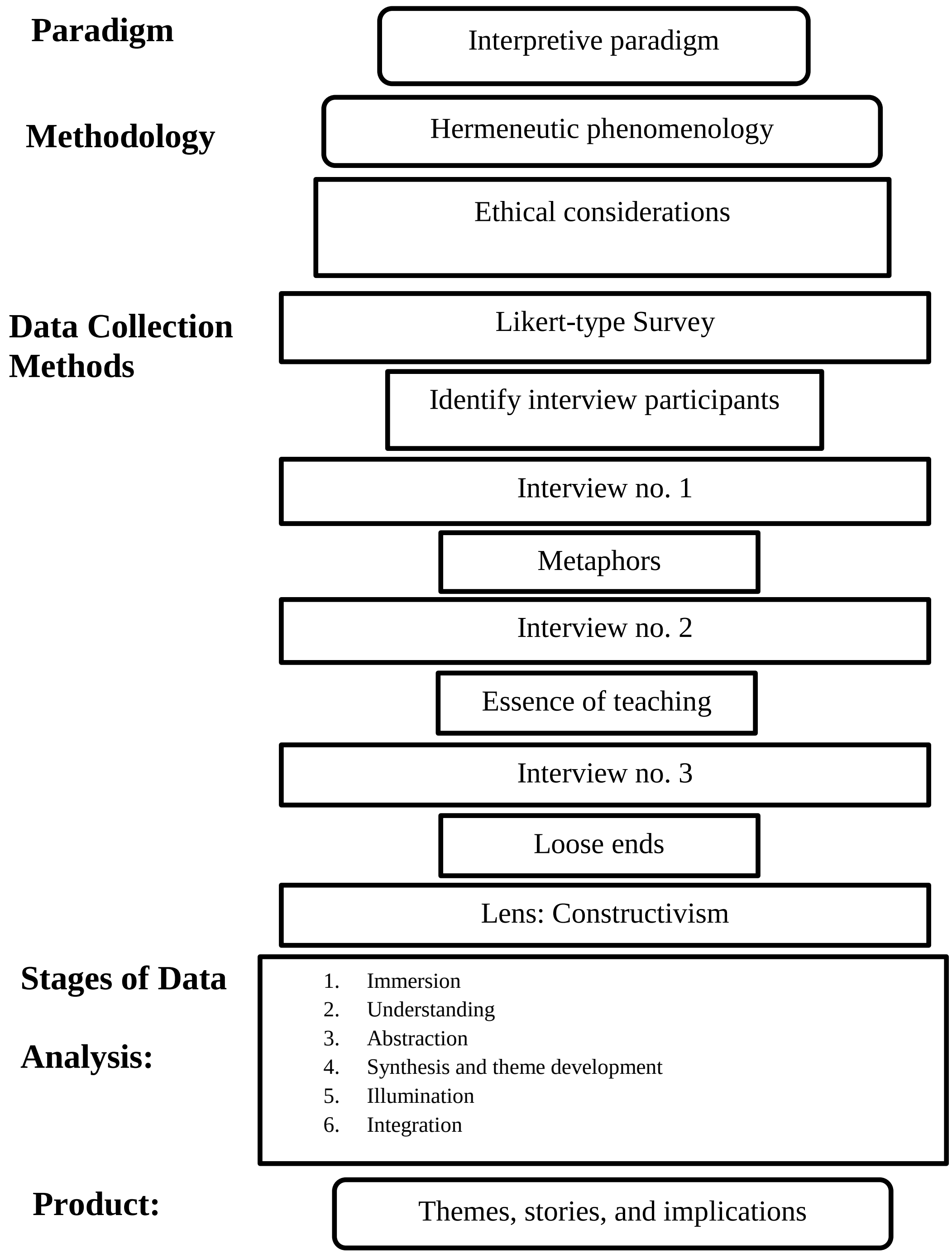

Figure 1. An overview of the research approach.

Phenomenology

Often referred to as the father of phenomenology, Edmund Husserl (1859 – 1938) was critical of researchers who applied methods of the natural sciences to human issues. Applying positivistic approaches to human issues ignored the fact that psychology deals with human beings who do not simply react to the world around them, but rather were developing internal meanings to their lived experiences (Moustakas, 1994). Husserl was convinced that researchers who focused on external physical responses to experiences were missing the lived experiences of those being studied. For Husserl, the phenomenological method opened exciting new areas of research to examine *the lived experience of people* (Dowling, 2007). This was a radical departure from the Cartesian dualism of reality existing apart from the lived experiences of the individual.

Hermeneutical Phenomenology

Martin Heidegger (1889-1976) studied under Husserl. Eventually, the work of Heidegger departed from Husserl's approach and became what today we call *Hermeneutic Phenomenology* (Dowling, 2007). Researchers often refer to phenomenology and hermeneutic phenomenology interchangeably without making distinctions between them (Laverty, 2003). Both are concerned with the lived experience of individuals in the world. The goals of research were to shine a light on aspects within the lived experience in order to find some sense of understanding and meaning from the individual's perspective. The way the exploration of lived experiences proceeds was where the two methodologies departed from each other.

Phenomenology focuses on individuals as understanding beings (Moustakas, 1994). Understanding is the way individuals know the world in which they live. This is distinctly different from hermeneutic phenomenology which focuses on the situated meaning of an individual in the world. Understanding is a basic form of human existence as an individual understands the way they are in the world (Laverty, 2003). “Husserl was interested in acts of attending, perceiving, recalling, and thinking about the world and human beings were understood primarily as knowers. Heidegger, in contrast, viewed humans as beings primarily concerned creatures with an emphasis on their fate in an alien world” (Laverty, 2003, p. 7). Consciousness cannot be separated from the world, but should be considered as constructed through lived experiences in the world.

Central to this idea is that understanding is not a way that we know the world around us, but rather the way we *are* in the world. This seemed particularly important when seeking to investigate secondary pre-service teachers’ beliefs about the nature of mathematics and the teaching and learning of mathematics. For this study, investigating pre-service teachers’ beliefs about mathematics should be intimately linked to *who they are as future teachers of mathematics.* Philosophical beliefs about the nature of mathematics as well as beliefs about how students learn mathematics are concepts that are linked to beliefs about teaching mathematics (Beswick, 2007; Charlambous & Philippou, 2010). Mathematics is not something that is external to the pre-service teachers, but rather is a part of who they are as teachers of mathematics.

A central idea for Heidegger was that meanings were found as human beings were experiencing the world while at the same time meanings were constructed from background experiences of the past (Laverty, 2003). Thus, for Heidegger, pre-understandings were a structure for being in the world. This was evident for pre-service teacher beliefs about mathematics which could not be separated from past experiences with mathematics. For pre-service teachers these experiences were mostly school experiences of learning mathematics. These past experiences were important components of hermeneutical phenomenology which was an interpretive process of coming to gain understanding of phenomena. Researchers then must enter not only the lived experiences of the participants, but also the past experiences of the participants. This interpretive process was achieved through what has become known as a hermeneutical circle which moves back and forth through the parts of experiences and the whole of experiences (Ajjawi & Higgs, 2007).

The Hermeneutic Process

When a researcher decides to take a phenomenology approach to research, the first step of the research process should be self-reflection. The purpose of this self-reflection might be to become aware of personal biases in order to bracket them, or put them aside that they might not interfere with the research process. The idea would be to protect from imposing personal assumptions of the researcher on the study. This is distinctively different from the hermeneutical approach. Self-reflection was still a critical component, but biases and assumptions are not set aside, rather they are embedded and essential to the interpretive process. “The researcher is called, on an ongoing basis, to give considerable thought to their own experience and to explicitly claim the ways in which their position or experiences relates to the issues being researched” (Laverty, 2003, p. 17). A researcher choosing a hermeneutical phenomenological approach becomes involved in a process of co-construction of the data with the participant as together they engage in what was called a hermeneutic circle. The researcher and the participants work together to bring to life the experiences being investigated. Through multiple interactions the researcher and participant work together to make sense of themes that presented themselves. “This going into, a going on and back which gives forth making present a theme. It is possible to hear that which is said in a description and which is unsaid” (Kidd & Kidd, 1981). This hermeneutic cyclical process was an appropriate approach to this study of mathematical belief systems of pre-service secondary teachers keeping in mind that the participants *were pre-service teachers of mathematics*. Rather than characterizing beliefs as if they are external to the researcher and the participants, trust was built that allowed the researcher and the participants to collaborate and work *together* to understand the mathematical belief systems of the participants and what it meant to the participants that they will be teachers of mathematics. Thus a researcher positionality statement seemed appropriate.

Researcher Positionality

Researcher positionality is important for all qualitative research, but particularly for hermeneutical phenomenology research where the researcher is expected to make interpretations of the data. I received my MS degree in mathematics with no formal education in teaching and learning. As a faithful Christian I have a firm belief in the existence of an eternal God. This perspective definitely influenced how I thought about the nature of mathematics. I perceived mathematics as a set of eternal truths that human beings sought to understand. Three years after completing my MS degree I entered a PhD program in mathematics education offered in the Mathematics department of a large Northwest university. It was in my first course in the program that my beliefs were deeply challenged. I was assigned to read and summarize a chapter on beliefs in a research handbook. Early in the chapter I read "that mathematics is humanly created in response to social views of the world and not to a platonic set of artifacts uncovered over time" (Romberg, 1992, p. 752). When I read this I had visceral antagonistic feelings. I did not see any way that this position could align with my faith in an eternal God. As I dialoged with colleagues and thought more deeply about things, I slowly began to see that indeed mathematics is a human activity and that somehow humans do create mathematical knowledge. As my beliefs changed, I also noted that my view towards teaching was changing as well. By the end of my first year in the PhD program I realized the influence that my beliefs had on my teaching. I also began to wonder if Christian educators who did not examine their beliefs would hold similar belief systems that I once had. Thus my interest in studying belief systems was rooted in personal experience and was driven by my desire to help Christian mathematics educators become better mathematics teachers. Not only was this study about pre-service mathematics teachers, but the participants were enrolled in a Christian university. My goal was to offer research that might help improve the pre-service program where I teach.

Data Collection Methods

This study sought to understand pre-service secondary mathematics teachers' belief systems about the nature of mathematics and the teaching and learning of mathematics. Data were collected during the spring 2012 and fall 2012 semesters. A likert-type beliefs survey was given to 15 pre-service secondary mathematics teachers enrolled in a mathematics pre-service teacher degree program at a private Midwestern Christian university with an enrollment of approximately 2,200 students. Survey results determined which participants were asked to participate in a series of three interviews. Three participants would be intentionally selected to participate in interviews based upon the three philosophies outlined by Ernst. Central to the hermeneutic process was the interactions between the researcher and the participants. This was accomplished through a series of three interviews. The first interview included a segment where the participants were asked to create a drawing that is a metaphor for the teaching and learning of mathematics (Reeder et al., 2009). This metaphor was then discussed during the interview. At the conclusion of the interview the participants were asked to continue reflecting on their metaphor. The participants were also asked to think about a particular mathematical topic they would be able to effectively teach. They were asked to also think about the essence of teaching this concept. This “essence of teaching” assignment was the main topic of discussion during the second interview. The purpose of the third interview was to have a conversation about personal reflections while talking about their mathematical belief systems.

Methods of Data Analysis

Data analysis methods were developed from phenomenological and hermeneutical principles in literature about useful ways to interpret data (Ajjawai & Higgs, 2007; Moustakas, 1994). Six stages of analysis have been identified as a useful framework for hermeneutical research, namely immersion, understanding, abstraction, synthesis and theme development, illumination of phenomena and integration (Ajjawi & Higgs, 2007).

Immersion

Immersion was the first stage of the hermeneutic process and involves constructing texts from the data. Phenomenological data analysis was a useful way to transform lived experience into a textual expression of its essence (Ajjawai & Higgs, 2007, p. 622). In this case, artwork created during the metaphor portion of the interviews was considered as textual data to be interpreted. Texts came from the interviews as well as from researcher journal notes. These texts were read, and re-read until the researcher became very familiar with the data. This process was referred to as immersion (van Manen, 1997). The aim of this step was to get a well-developed sense of the preliminary interpretations of the texts. Important thoughts were documented as memos, attached to specific pieces of text.

Understanding

The second stage involved identification of first order constructs, which were the ideas expressed in the participants own words (Ajjawi & Higgs, 2007). These were statements or pictures that captured the essence of what the participants were trying to communicate. First order constructs were identified for all participants. These constructs were to be central to understanding interactions between the researcher and the participants. The researcher's understandings of first order constructs were checked at each stage by feeding back to the participants ideas discussed during previous stages. Gadamer (1998) saw understanding as the basic structure of human experience from which new considerations could be created.

Abstraction

The purpose of the abstraction stage was to identify second order constructs. These were abstractions from the first order constructs using the researcher's theoretical and professional knowledge (Ajjawi & Higgs, 2007). The purpose of second order constructs was to give a deeper understanding to first order constructs, but expressed in the words of the researcher rather than the participants. Second order constructs were linked to first order constructs, and collectively informed understanding of the entire transcript.

Synthesis and Theme Development

Themes emerged from the first three stages of the data analysis process. Themes and sub-themes were categorized and elaborated. Relationships between first and second order constructs were clarified. It was imperative at this stage that the researcher moved between the data and the literature, in a hermeneutical circle, understanding the parts to understand the whole and back. This was to be a very reflective process.

Illumination of Phenomena

Illuminating and illustrating the themes involved a process that attempted to find links between research literature and the themes that were developed (Ajjawi & Higgs, 2007). Participants specific experiences during the research process were examined to maintain faithfulness to participants overall experiences.

Integration

The last stage involved a critique by the researcher. Themes and interpretations were articulated and communicated. This also involved a critical reflection concerning the implications of the findings (Ajjawi & Higgs, 2007). Lastly, it was important to note that in practice, the hermeneutical cycle should not be understood as a linear progression through the stages. When the researcher enters the hermeneutic cycle it may well be at one particular stage, but the researcher was always aware of how all of the stages interacted as one whole process. Table 3 summarizes the six stages of the hermeneutic process.

Table 3

Stages of Data Analysis Developed for this Research

Stage	Tasks Completed
1. Immersion	• Organizing the data-set into texts • Iterative reading of texts • Preliminary interpretations of texts
2. Understanding	• Identifying first order (participants) constructs • Coding of data
3. Abstraction	• Identifying second order (researcher) constructs • Grouping second order constructs into sub-themes
4. Synthesis and theme development	• Grouping sub-themes into themes • Further elaboration of themes • Comparing themes across sub-discipline groups
5. Illumination of phenomena	• Linking the literature to the themes identified above • Reconstructing interpretations into stories
6. Integration	• Critique of the themes by the researcher • Reporting final interpretation of research findings • Implications

Framework

Teachers of mathematics have three main components to their belief systems. (1) Their views about the nature of mathematics. (2) Their view about the nature of teaching mathematics. (3) Their view of the nature of learning mathematics. The framework outlined by Ernst (1994) has been used by multiple researchers interested in belief systems and the impact on teaching practice (Schilling, 2010).

Ernst (1989) distinguishes between three philosophies of mathematics that have been observed in the teaching of mathematics. (1) The Instrumentalist view that mathematics is a collection of unrelated facts and procedures. Skills mastery and correct performance is of primary importance. (2) The Platonist view that mathematics is a static body of knowledge with an external, objective reality. Mathematics is discovered, not created. (3) The Problem Solving view that mathematics is a dynamic body of knowledge that is under constant revision. Mathematics is a human creation influenced by social norms and cultural settings. Mathematics is a process of inquiry and is not a finished product (Ernst, 1989).

Teachers' views of the nature of mathematics cannot be separated from their teaching practices. A teacher's view about the nature of mathematics provides a basis for the teacher's mental images of teacher practice. The relationship between teachers' views of the nature of mathematics and their models of its teaching and learning are illustrated in Figure 2 (Ernst, 1989).

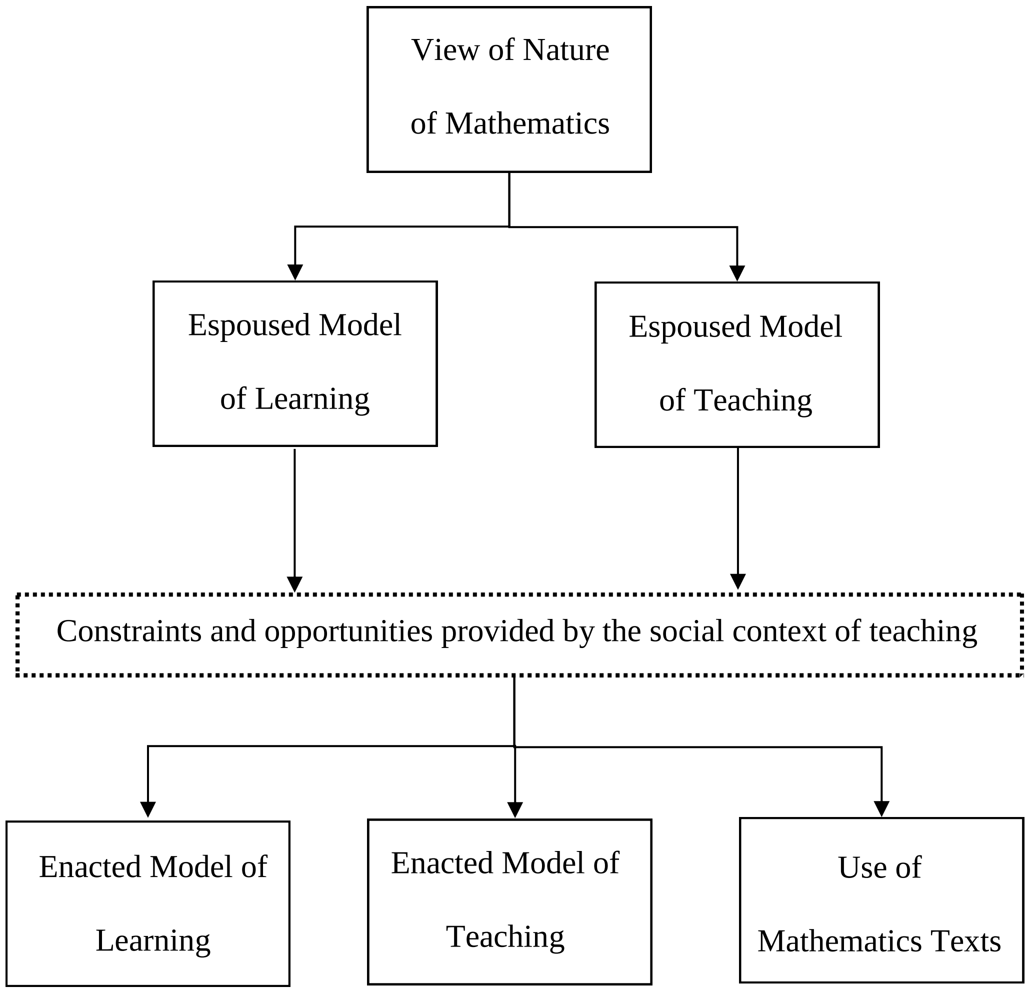

Figure 2. Relationships between beliefs and their impact on practice.

Teachers' roles, intended outcomes and use of curriculum are all influenced by the teacher's view of mathematics and impact teaching practice. For the Instrumentalist's view the teacher is an instructor with skills mastery the typical goal. The teacher will likely adhere strictly to the textbook. Teachers with a Platonist's view become an explainer, guiding the students to new discovery with the goal of discovery of interrelated mathematical concepts. Modification of the textbook becomes useful in the discovery process. A teacher who holds a Problem Solving view of mathematics is a facilitator, helping students create new mathematical ideas through problem posing and solving. The curriculum is constructed as needed to facilitate the creation of new ideas (Ernst, 1994).

Data Description

Surveys can be a useful tool for studying belief systems. The beliefs survey used in this study was modified from a version used and validated by Kajander (2007). The survey was also used by Shilling (2010) in her exploration of pre-service mathematics teachers' belief systems. A likert-type survey was used because it provided initial insight into the belief systems of pre-service secondary mathematics teachers. For this study the likert-type survey was the initial survey given to all 15 participants.

The likert-type survey for this study consisted of 16 items with the participants rating each statement as (1) very true (2) somewhat true (3) somewhat not true (4) not true at all. The survey used by Schilling and adopted for this study items aligned with the three philosophies described by Ernst (1994). These statements and their alignments are given in Table 3. These items were designed to probe the pre-service teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics. Some items included in the survey provided information about general beliefs about mathematics such as the item "I like doing mathematics" however only the items aligned with the three philosophies outlined by Ernst were to be analyzed.

Based on results of likert-type survey three participants were purposively selected for interviews and creation of metaphors based on the three philosophies outlined by Ernst (1988). Creation of metaphors can reveal conceptualizations about the nature of mathematics and the teaching and learning of mathematics (2000; Reeder et al., 2009). Analysis of metaphor data was based on three root metaphors for education as identified by Schubert (1986). These root metaphors are production, journey and growth. Reeder et al. (2009) presented useful descriptions of these three root metaphors. The production metaphor describes mathematics and the teaching and learning of mathematics as an industrial process where mathematics is a raw material is refined by the teacher and instilled in the students. The journey metaphor views the teacher as a guide, taking students on a journey of discovery. Lastly the growth metaphor characterizes the teacher as helping students grow into insightful learners. Examining metaphors based on the root metaphors alongside Ernst's three philosophies gave valuable insight into the belief systems of pre-service teachers.

Example of Data Analysis

Chapter three concludes with a brief example using data from an unpublished research project (Howard, 2011) to illustrate the data analysis process used in chapter four of this dissertation. This example is not intended to be a thorough analysis of data, but rather a short glimpse at how hermeneutical phenomenological methods will be used in chapter four. Participants of the study were freshmen mathematics education majors at a small, private Mid-western university. Each participant was engaged in a short interview during which they were asked to draw a picture of a metaphor for the teaching and learning of mathematics. They were asked to verbally explain the details of their metaphor. Each participant was also asked various question related to the nature of mathematics as well as the nature of teaching and learning of mathematics. Data from two participants, one male and one female, was used here to provide a short illustration of the six stages of the hermeneutical phenomenology process.

Immersion

The first stage of the hermeneutic process allowed the researcher to become familiar with the data set (Ajjawi & Higgs, 2007). Data was organized into texts and read multiple times allowing the researcher to meditate on the data. Mike’s metaphor consisted of a picture of spaceship travelling to the moon (Figure 3). The picture was very simple and had a flag posted on the surface of the moon.

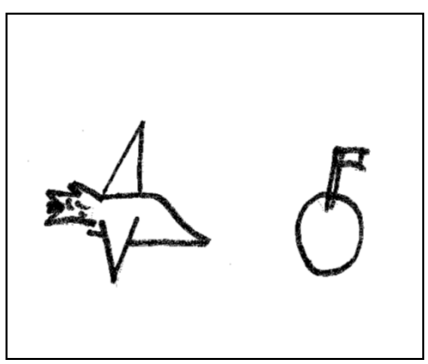

Figure 3. Mike’s metaphor for teaching and learning mathematics.

The initial perception was of travel to a new location with unknown features.

Researcher: Can you describe your metaphor?

Mike: Yes. Mathematics is like travelling to the moon, where you have never been.

Researcher: Where is the mathematics? The teacher? The students?

Mike: The math is the moon, the posting of the flag. That is the goal. The teacher is the pilot of ship, at the controls. The students are the passengers, along for a fun ride.

This revealed that Mike viewed mathematics as the teacher taking students somewhere new. In Mike's conception, the teacher controls the method and direction of travel. Mike's view about the nature of mathematics and the teaching and learning of mathematics were revealed in the following dialogue.

Researcher: So, what *is* mathematics?

Mike: Math is finding new patterns in numbers and things.

Researcher: What does it mean to *teach* mathematics?

Mike: It means to guide students to find the patterns.

Initial interpretation of the data revealed that Mike viewed teaching as guiding students to new things, specifically to patterns that can be discovered. The teacher was in control and the students were buckled in their seat. Finding and exploring new patterns was analogous to a frontier explorer. Once new patterns were discovered and learned the area is conquered, a flag is posted and the teacher will take the students looking for a new frontier to conquer.

Beth's metaphor depicts a Wal-Mart truck going from one building to another to deliver products to people who leave with new things (Figure 4). The

Wal-Mart truck was leaving a Wal-Mart warehouse labeled as the source of mathematics. The truck was approaching one side of a Wal-Mart store with students leaving the store from the opposite side of the building. The impression was that mathematics was being delivered to the back of the store while the students were customers who entered and leave through the front of the store. Beth's metaphor shows the students leaving the store, apparently having obtained the mathematics that was delivered.

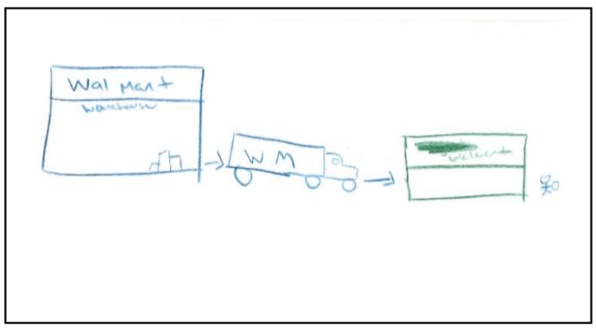

Figure 4. Beth's metaphor for teaching and learning mathematics.

> Researcher: Can you describe your metaphor?
>
> Beth: Well, the big building is the warehouse of mathematics. The teacher drives the truck and delivers the math to the school where students leave with the math.

Beth's view was that the teacher has access to the warehouse of mathematical knowledge and has the capabilities to deliver the knowledge to the students. The transfer of knowledge happened at the school. The students leave the school with knowledge.

> Researcher: What *is* mathematics?
>
> Beth: Tools and algorithms that come from the warehouse.

> Researcher: How is mathematics taught?
>
> Beth: The teacher knows the answers and gives students the ways to do things correctly to get the right answers.

Beth viewed mathematics as rules and algorithms to be learned in order to have the ability to get correct answers.

Understanding

The understanding stage was the beginning of the coding process. The root metaphors described and used by Reeder et al (2009) helped guide this initial process. Mike's metaphor appeared to be a strong journey metaphor. Mike understood that the teacher was in control of the journey and the students were going where she was taking them. Beth's metaphor was classified as a production metaphor, complete with warehouse and distribution truck. The teacher had access to the mathematics and delivered it in precise ways to the students. The product was the students leaving the school with mathematical abilities. For Beth, these abilities were limited to knowing the rules and algorithms and how to use them to get the right answers.

Abstraction

During the abstraction stage of the hermeneutical cycle was when data was aligned with the framework developed by Ernst. Mike seemed to hold a Platonist's view about the nature of mathematics. The teacher knew a very specific course of travel for the students. Students were guided to discovering specific mathematical concepts that the teacher had in mind. Beth seemed to hold a strong instrumentalist view of mathematics. For Beth, skill mastery of algorithms was central to learning mathematics. The teacher delivered the methods and understanding came from repetition of algorithms. There was not much room for students to approach mathematics in different and unique ways.

Synthesis and Theme Development

Themes were brought together by exploring the interaction of beliefs regarding the nature of mathematics with beliefs regarding the teaching and learning of mathematics. Mike's beliefs regarding the nature of mathematics were strongly linked to his view that mathematics was a discovery process. Mike viewed himself as one who would guide students through the process of learning mathematics. Consistent with this discovery process, Mike believed that the discovery process occurred best in a social setting. Thus Mike viewed himself as a teacher who would lead *a group* of students on a journey. For Mike, not only was learning a social endeavor, but *teaching* was also very social. Mike looked forward to teaching in a school that with multiple teachers of mathematics and expressed a strong belief that he would learn how to teach mathematics by interacting with other mathematics teachers. Mike also had a strong belief that mathematics was a fixed body of knowledge and his confidence in himself as a teacher stemmed from the fact that he viewed himself as having a solid understanding of the mathematics needed to teach high school level mathematics courses.

Beth's views about teaching flowed primarily from her belief that she had personally mastered the mathematics she would be teaching. Her relationship with mathematics was primarily based on mastery of algorithms with a firm grasp of multiple applications. Beth's relationship with her students would likely be centered on her authority as a mathematics teacher. Beth's belief that mathematics was a set of rules and algorithms which she had mastered was the basis for her confidence as a teacher. Beth was confident in her mathematical abilities both as a learner and as a teacher.

Illumination of Phenomena

The illumination stage was where themes in the hermeneutical cycle were brought together (Ajjiwa & Higgs, 2007). For this study this was accomplished through telling a story of the interactions between beliefs about the nature of mathematics and beliefs about the teaching and learning of mathematics. Mike's classroom will likely have a distinct feel that the students are on a journey. Mike will be seen as very much in control of the class as he leads them through a discovery process. Mike expects that his students will be involved in dialogue with other students as they seek to find mathematical concepts. Mike will have a clear idea of the concepts that he wants the students to discover.

Beth's classroom will look quite different than Mike's. Beth will have her students seated in the traditional row format, with all students facing forward. Beth will be standing in front of the class offering a detailed explanation of algorithms with completely worked out examples. Students will then be working individually on worksheets while Beth roams around the room seeking to answer questions students may have. Discussions between students will not only be absent but be deemed as problematic, getting in the way of focused, individual attention on the worksheets.

Integration

Mike's beliefs regarding the teaching of mathematics were characterized by a discovery teaching method. Mike seemed to be the type of teacher who will seek to refine his abilities to help students discover new mathematics. Mike's use of problem solving in the classroom will be limited only to problems in which he knows specific mathematical concepts he wants the students to learn. Open-ended problem solving where students are allowed to create their own mathematics is not likely to occur in Mike's classroom. This could create a conflict for Mike if he is to teach in a school that is actively implementing Common Core Mathematics, however, Mike does seem genuinely interested in engaging in discussions with teachers who would offer to mentor him.

Beth's view of teaching was characterized as traditional which aligned with her beliefs about mathematics. Implications for her will be that she will struggle to come to grips with the idea that there are other methods to both teaching and learning of mathematics. Beth also seemed likely to pass down to her students the perspective that mathematics is a list of rules and manipulations that are disconnected from any practical use. Beth's beliefs regarding the learning of mathematics were also traditional. Students need to be shown how to do mathematics before they can work mathematics on their own.

Summary

Chapter three outlined the methodological paradigm implemented for this research. Interpretivism was a natural paradigm for research involving philosophical belief systems. The purpose of conducting research under the Interpretivism paradigm was to collect information that allowed the researcher to gain understanding and meaning from the participants' perspectives. Hermeneutical phenomenology fits nicely into this paradigm. The hermeneutic cycle allowed the researcher to become engaged with the data on a personal level, seeking to understand the lived experiences of the participants. This seemed appropriate when considering belief systems.

The hermeneutic cycle also provided a nice way to integrate the theoretical framework with the process of interpretation. Chapter three concluded with an example of how the theoretical framework will be intertwined with the hermeneutical cycle. The immersion stage allowed the researcher to become familiar with the data apart from a theoretical framework. The emphasis was on getting to know the data. Analysis of metaphors using the framework developed by Schubert (1986) occurred during the understanding stage of the hermeneutic cycle. This allows the voice of the participants to be central. The abstract stage allowed the researcher's voce to be heard through the three philosophies developed by Ernst (1988). The metaphor analysis and philosophy analysis were brought together in the fourth stage, synthesis and theme development. Beliefs about the teaching and learning of mathematics were introduced in the fifth stage through which to tell the story of how beliefs systems might impact teacher practice of the participants. Lastly the integration stage was the final critique by the researcher.

CHAPTER FOUR

RESULTS AND ANALYSIS OF FINDINGS

The research questions under consideration are:

1. What are the philosophical beliefs of pre-service secondary mathematics students' at a small private university regarding the nature mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with pre-service secondary mathematics students' beliefs regarding the teaching and learning of mathematics?

The goal of this chapter was to describe the results and analysis of the study with a goal of answering the research questions. The results section first gave an overview of the likert-type survey data from all 15 participants who completed the survey. This was followed by a detailed analysis of interview and metaphor data collected from three of the participants. The hermeneutic cycle was employed in the data analysis process, and the results were presented with respect to the components of this cycle (Moustakas, 1994). The wholeness of the hermeneutic cycle was evident throughout the process.

The likert-type survey consisting of 16 statements was modified by a version utilized and validated by Kajander (2007). As discussed in chapter two, the items in the likert-type survey aligned with the three distinct philosophies recognized by Ernst (1988). Each likert-type item corresponded to a single statement about mathematics addressing the nature of mathematics, the teaching of mathematics or the learning of mathematics. A four point rating scale was utilized, 1 being very true, 2 sort of true, 3 not very true and 4 very true. Items 1 through 4 align with the Platonist view of mathematics, items 5 through 10 align with the Instrumentalist view of mathematics and items 11 through 16 align with the Problem Solving view of mathematics. Tables 4 – 6 listed items according to alignment and included mean ratings with standard deviations.

Table 4

Likert-type Survey Items Related to the Platonist's View (n = 15)

Scale:1 = very true, 2 = sort of true, 3 = not very true, 4 = not at all true	Mean	SD
Survey item		
1. Some people cannot be good at math no matter how hard they try.	3.27	0.70
2. Mathematical facts exist independent of human activity.	2.67	0.90
3. The mathematical body of mathematics is fixed, and always has been.	3.07	0.80
4. I think that all mathematical knowledge is interconnected.	1.87	0.34
Note: Agreement with these statements indicates alignment with the Platonist's view.		

Table 5

Likert-type Survey Items Related to the Instrumentalist's View (n = 15)

Scale:1 = very true, 2 = sort of true, 3 = not very true, 4 = not at all true	Mean	SD
Survey item		
5. Mathematics is a collection of facts, formulas and procedures.	1.73	0.70
6. To do well in mathematics I have to memorize all the formulas that are relevant.	2.33	0.87
7. To do well in math I have to be taught the right procedure.	1.93	0.77
8. It is the teacher's job to teach the right steps in each new method to the students before they have to use it.	1.73	0.68
9. Mathematics is a useful tool primarily used for calculations.	2.33	0.87
10. Doing mathematics means memorizing particular rules and procedures	1.93	0.77
Note: Agreement with these statements indicates alignment with the Instrumentalist's view.		

Table 6

Likert-type Survey Items Related to the Problem Solving View (n = 15)

Scale:1 = very true, 2 = sort of true, 3 = not very true, 4 = not at all true	Mean	SD
Survey item		
11. Mathematics is a creative human activity. [a]	1.27	0.44
12. In mathematics you can be creative and discover by yourself things you did not already know. [a]	2.33	0.87
13. Math problems can be done correctly in only one way. [b]	3.40	0.88
14. There are many different equivalent ways to define correctly a mathematical concept. [a]	2.27	1.00
15. There is usually one best way to write the steps in a solution to a math question. [b]	2.4	0.95
16. I think that mathematics as a discipline can be revised. [a]	2.33	1.07
Note: Agreement with these statements designated with “a” indicates alignment with the Problem Solving view. Disagreement with statements designated with “b” indicates this alignment.		

Findings Related to Likert-type Survey

Frequencies and relative frequencies for each statement were presented in Tables 7 – 9. There was little evidence that any of the participants were in strong agreement with the Platonist view of mathematics. No respondent rated all four statements as very true. All the respondents who rated at least one statement as very true also rated at least two other statements as not very true. However, all respondents rated as true or very true the statement that all mathematical knowledge is inter-connected.

Table 7

Statistics for Statements Aligned with Platonist View

Question 1			Question 2			Question 3			Question 4		
	F	rel. *f*		*f*	rel. *f*		*f*	rel. *f*		*F*	rel. *f*
1	0	0.00	1	1	0.07	1	1	0.07	1	2	0.13
2	2	0.13	2	6	0.40	2	1	0.07	2	13	0.87
3	7	0.47	3	5	0.33	3	9	0.60	3	0	0.00
4	6	0.40	4	3	0.20	4	4	0.26	4	0	0.00

Table 8

Statistics for Statements Aligned with Instrumentalist View

Question 5			Question 6			Question 7		
	F	rel. *f*		*F*	rel. *f*		*F*	rel. *f*
1	6	0.40	1	3	0.20	1	4	0.26
2	7	0.47	2	5	0.33	2	9	0.60
3	2	0.13	3	6	0.40	3	1	0.07
4	0	0.40	4	1	0.07	4	1	0.07
Question 8			Question 9			Question 10		
	F	rel. *f*		*F*	rel. *f*		*F*	rel. *f*
1	6	0.40	1	3	0.20	1	4	0.26
2	7	0.47	2	5	0.33	2	9	0.60
3	2	0.13	3	6	0.40	3	1	0.07
4	0	0.00	4	1	0.07	4	1	0.07

Table 9

Statistics for Statements Aligned with Problem Solving View

Question 11			Question 12			Question 13		
	F	rel. *f*		*F*	rel. *f*		*F*	rel. *f*
1	9	0.60	1	3	0.20	1	1	0.07
2	4	0.27	2	5	0.33	2	1	0.07
3	1	0.47	3	6	0.40	3	4	0.27
4	1	0.40	4	1	0.07	4	9	0.60
Question 14			Question 15			Question 16		
	F	rel. *f*		*F*	rel. *f*		*F*	rel. *f*
1	4	0.27	1	3	0.20	1	4	0.27
2	5	0..47	2	5	0.33	2	5	0.40
3	4	0.20	3	5	0.33	3	3	0.20
4	2	0.07	4	2	0.13	4	3	0.07

The means found in Table 5 showed an alignment with the Instrumentalist view of mathematics. The class means showed some agreement with all six items aligned with the Instrumentalist view of mathematics. This alignment was also seen in the frequency distributions. For each of items 5, 7, 8 and 10, at least 86% of the responses were either very true or sort of true. Results showed 53% of responses for each of items 6 and 9 were very true or sort of true with only two responses of not very true. The strongest agreement was with items 5 and 8 indicating that mathematics was a collection of facts, formulas and procedures and that it was the teacher's job to teach the steps in each new method to the students before they have to use the method.

Of the six statements aligned with the Problem Solving view of mathematics there was evidence that the 15 participants a group agreed with two of the statements. They agreed with the statement that mathematics is a creative human activity and reverse agreement was seen as they disagreed with the statement that mathematics can be done correctly in only one way.

Metaphors and Interviews

Three participants were intentionally selected to participate in the interviews. The initial research design called for three participants to be selected based on the three philosophies of mathematics as outlined by Ernst (1988). The expectation was that for each philosophy the participant with the strongest alignment based on the likert-type survey would be selected for the interview process. Of all 15 participants, the likert-type survey data seemed to indicate that

Sarah aligned most strongly with the Instrumentalist view of mathematics while Steve had the most alignment with the Problem Solving view. Thus these two participants were intentionally selected to represent their respective philosophical position. None of the participants strongly aligned with the Platonist's view. Concerning the Platonist view of mathematics, the original thought was to select the participant that was most strongly aligned as with the other views. However, the data suggested that none of the participants had a strong alignment with the Platonist view. Thus Mary was chosen to participate in the interviews because she seemed the most strongly aligned *against* the Platonist view of mathematics. The intention of purposeful selection of these participants was to identify three participants with distinct philosophical positions.

The three participants who were selected to participate in the interview process were first asked to create a metaphor for the teaching and learning of mathematics. Immediately after creating their metaphor, each participant participated in a short interview. Each participant was interviewed in weekly intervals for a total of three interviews for each participant. Analysis of the metaphor data as well as the interview data followed the same hermeneutic cycle discussed in chapter three.

Background Information for Interview Participants

Sarah was a sophomore majoring in mathematics education who excelled in mathematics courses during high school. Her public high school mathematics courses included algebra I & II, geometry, pre-calculus and mathematics decision making. She expressed her desire to teach mathematics at the junior high level. Mary was a junior mathematics education major. Her public high school mathematics courses included algebra, geometry, statistics and trigonometry. She has completed three semesters of college level calculus and one semester of statistics. She expressed her desire to teach high school mathematics. Steve was a junior, also majoring in mathematics education. His private high school mathematics courses included algebra I & II, geometry, trigonometry and pre-calculus. At the college level he has completed three semesters of calculus, differential equations, statistics and discrete mathematics. He expressed interest in teaching high school mathematics for a school that would offer him opportunities to coach basketball. Interestingly, none of the participants chosen for the interview process took calculus in high school, though all took college calculus during the first semester of their freshman year.

Hermeneutic Cycle

The hermeneutic process involves a circle through which understanding can occur (Moustakas, 1994). The texts provide descriptions of experiences. Interpretation is a reflexive process "that awaits fresh interpretations from different personal perspectives" (Moustakas, 1994, p. 1). Thus, the process is not linear, but rather a moving into and out of the texts in a fluid kind of way. Moustakas (1994) mentions the necessity to interpret the texts as a whole in a gestalt of interconnected meanings. Interpretation must be open to spontaneous shifts throughout the text. The hermeneutic circle can be thought of as a metaphor for interpreting and understanding texts through exploration of the parts and the whole simultaneously.

Immersion

The first stage of the hermeneutic process allowed the researcher to become familiar with the data set (Ajjawi & Higgs, 20070. Data were organized into texts that represented the participants' perspectives. These texts were read multiple times allowing the researcher to meditate on the data. Through iterative readings and meditation the researcher formed preliminary interpretations.

Sarah. Sarah's metaphor depicted a watering can pouring water onto what appeared to be seeds falling to the ground with an arrow pointing from the poured water towards flowers at the base of a tree. The initial impression was that water was being given for the use of the flowers and the tree. The watering can seemed to be floating in the air, not being held by a person. The can was also decorated with a flower (See Figure 5).

When asked to discuss her metaphor, Sarah spoke of the trees and flowers as the learning that occurs, as the students grow. She further indicated the seeds represented the students as they grow and learned new mathematics. Sarah shared that the students grew into bigger and better people. When asked about the pitcher Sarah said that demonstrated how "the teacher holds the mathematics in her brain and instills it into the students."

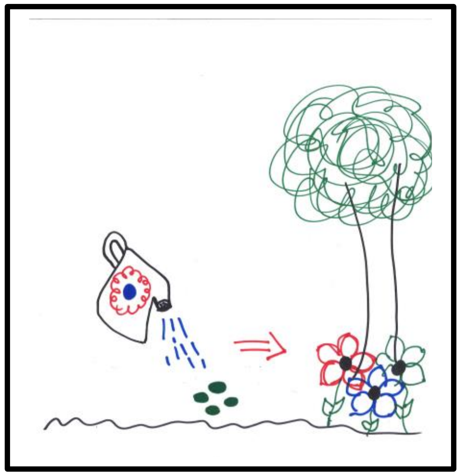

Figure 5. Sarah's metaphor for teaching and learning mathematics.

> Researcher: So the watering can is the teacher?
>
> Sarah: Yes.
>
> Researcher: Why is the can decorated?
>
> Sarah: Because the teacher needs to look appealing.
>
> Researcher: Explain what you mean by appealing.
>
> Sarah: Well, when I teach, I want to be appealing.
>
> Researcher: Describe yourself as an appealing teacher.
>
> Sarah: I have big curly hair, in some nice dress pants and a nice blouse. I just have the biggest smile on my face and I might say a few jokes here and there and the students might not laugh, but I will. I see my students saying 'take Miss Smith's class! She is hilarious.' I see myself explaining what is a function, you know, the x's and y's. But I will do it so people laugh. That is a big goal of mine that my students feel comfortable to talk to me about anything and about math too.

Sarah conveyed that she wanted her students to perceive her not just as a teacher but as a likeable person. The stories Sarah shared during her interviews also expressed that she viewed mathematics as a set of algorithms and rules.

Researcher: Describe a bad mathematics experience from high school.

Sarah: In 10th grade, in algebra two, we had to figure out percentages and such about Olympic sports events and it was super confusing. The teacher did not give us any formulas and it was super confusing and I was so flustered. I could not find an equation to solve and the teacher was no help at all.

Researcher: OK. So can you describe a good mathematics experience?

Sarah: In one class, the students got to pick a math problem they wanted to solve on the board and explain to the class how they solved it. You got a reward too, so that was pretty exciting and to show people how smart you are.

Researcher: How can people tell how smart you are?

Sarah: By watching you easily solve a problem, go through the steps without getting messed up. You know, uh, solving an equation the right way the first time.

Researcher: What about when it comes to teaching mathematics? How do you see yourself?

Sarah: I see myself as breaking it down as simple as possible so my students get it.

Sarah saw herself as an engaged teacher who had the ability to inspire students. She expressed confidence in her mathematics abilities and wanted to pass that confidence along, especially to her female students.

Researcher: Describe *how* you would teach mathematics.

Sarah: Too often, the girls don't think they can do it. I would teach so that they know they can do it too. Not just the girls, but the boys too. They all need to have it taught in a way that is encouraging.

Sarah tended to avoid details when asked *how* she would teach mathematics.

Researcher: Yes, but *how* would you teach mathematics?

Sarah: I would explain it carefully. You know, with easy steps so they would not get discouraged.

Researcher: What would be a particular topic you would like to teach?

Sarah: Solving equations, I guess.

Researcher: Any particular type of equations?

Sarah: I have always liked quadratic equations. They really are not very hard.

Researcher: Why are they not very hard?

Sarah: Well, it is just plugging numbers in. It is easy to know what goes where.

Researcher: How would you teach this? Can you describe how you would teach this?

Sarah: I would start by explaining the formula. Show them how it works.

Researcher: How would you do that?

Sarah: Write it on the board. Do some examples. Let them try some.

Though Sarah perceived herself as an explainer, she described how she would show students how to use rules and algorithms. Thus she viewed herself as one who was able to explain *algorithms* not necessarily *concepts;* steps and rules rather than meaning and understanding.

Mary. Her interviews made it very clear that Mary considered mathematics to be the use of formulas and equations, but teachers did not have to explicitly supply all the formulas needed. This was also evident in her metaphor (See Figure 6). Interestingly, from Mary's perspective, mathematics was not delivered to the students, but rather the students are *planted into* mathematics. This was distinctively different than Sarah's perspective, even though their metaphors were very similar.

Researcher: Where is the mathematics in your metaphor?

Mary: It is what the students are being planted into.

Researcher: The mathematics is the soil?

Mary: Yes. Students are planted into the soil, then the teacher, the sun, the water all help the flower grow. The students grow until they have an understanding of mathematics.

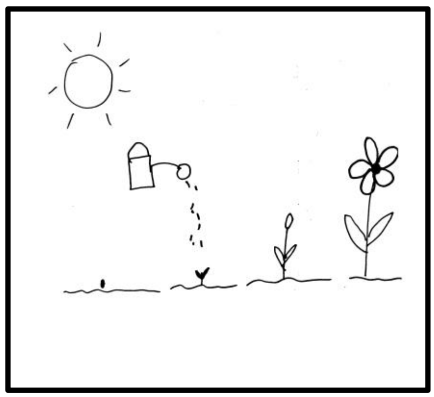

Figure 6. Mary's metaphor for teaching and learning mathematics.

Whereas formulas and equations were central to Mary's view of mathematics, it became clear that Mary did not view herself as a dispenser of information. Mary viewed herself as a "helper", one who encouraged students to teach formulas to themselves. As the researcher I was somewhat shocked by this discovery because Mary was chosen to participate in the interviews because the likert-type survey data indicated an anti-Platonist view of mathematics. Yet she viewed herself as a teacher helping students discover formulas through exploring patterns which was very much a Platonist perspective. Mary viewed herself as one who was capable of helping students see the real life applications of mathematics.

When asked to describe herself as a teacher, part of her response included the following:

> Mary: I would love my students to be able to see the real life applications of mathematics. I have always been able to see how math is applicable and would explain this to friends.
>
> Researcher: So which comes first, applications or formulas?
>
> Mary: I think that depends on the topic. Sometimes it is better to give the students an opportunity to solve a problem and develop a formula themselves and sometimes it is better to give them the formula and then show them the real life application.

Steve. Steve's metaphor depicted three people playing basketball. The tallest person was the farthest from the goal and seemed to be pointing toward the goal. The two smaller players were near the goal with one player shooting the basketball toward the goal. There was also a scoreboard with an arrow pointing to the right, toward the goal (See Figure 7).

> Researcher: Can you describe your metaphor?
>
> Steve: Sure. The teacher is the coach, teaching them basic math so they can score points.
>
> Researcher: So the goal is to score points?
>
> Steve: Yes. When students learn things they show success by scoring more points, doing things correctly gets you more points.

Steve described how the team worked together to get an end result of winning the game.

> Researcher: Where are the students in your metaphor?
>
> Steve: The team. The students work together hoping to win the game.

The image of a teacher being a coach was also revealed as Steve related past experiences with mathematics.

> Researcher: Can you share a story about a positive mathematics experience?
>
> Steve: Well, in high school I had one teacher who was really good at pointing me in the right direction. I remember once when I could not solve a problem. He came and pointed out a direction I had not seen. I ended up figuring it out by going in the direction he described.

> Researcher: Can you share a story about a positive mathematics experience?
>
> Steve: Well, in high school I had one teacher who was really good at pointing me in the right direction. I remember once when I could not solve a problem. He came and pointed out a direction I had not seen. I ended up figuring it out by going in the direction he described.

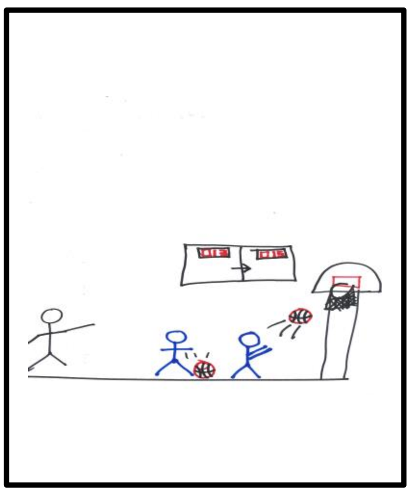

Figure 7. Steve's metaphor for teaching and learning mathematics.

Steve described the teacher as pointing him in the right direction, which was consistent with his metaphor having the teacher be a coach giving directions to the team

Discussing Steve's metaphor revealed some of his perspective of problem solving in mathematics. Though the coach did teach some things, Steve's metaphor revealed it was up to the students to engage with the mathematics together for a common goal. The main purpose of the *immersion* step of the hermeneutic cycle was to gain these initial impressions of the raw data (van Manen, 1997). Data was organized into texts that have been read over many times in order to get the initial impressions necessary to facilitate coding of the texts. Understanding was the next step of the hermeneutic process.

Understanding

Understanding human experience was a critical component of hermeneutical phenomenology (Gadamer, 1998). The purpose of the understanding stage was to identify first order constructs, those constructs that came from the participants' perspective (Ajjawi & Higgs, 2007). The metaphor data was central to this process. The framework used for the understanding stage was developed by Schubert (1986) and implemented by Reeder, et al (2009) and involved three root metaphors for education. These were production, journey and growth metaphors. As described by Reeder, et al (2009), the production metaphor involved seeing the students as raw material to be changed by a skilled teacher. The school was a factory producing knowledgeable students where the teacher talked and the students listened. The journey metaphor viewed the classroom as being led by the teacher on a field trip in which the students engaged in exploring new landscapes. Students were actively involved in the exploration process.

Lastly, the growth metaphor envisioned a caring teacher who helped grow the students into maturity in the subject matter where conversation and collaboration was highly valued (Reeder et al, 2009).

Sarah. The researcher's initial viewing of both Sarah and Mary's metaphors revealed strong growth metaphors. Both had watering cans pouring water with flowers growing. This was confirmed by Sarah as she described her metaphor.

> Researcher: Can you describe your metaphor?
>
> Sarah: Sure. Students grow as they learn new math. The seeds represent the students and they become flowers and trees as they learn.
>
> Researcher: Where is the mathematics in your metaphor?
>
> Sarah: The math is the water, being poured onto the students so that they can grow.

Sarah viewed herself as the one who had the mathematics to be supplied to the students.

The immersion stage revealed that Sarah was concerned with her appearances. Sarah explained that she wanted to be viewed as an appealing teacher, liked by her students. This was communicated by her as she explained why the watering pitcher was decorated with a flower. Sarah also shared how important it was for her to appear smart and confident. Sarah described herself as confident by relating how she would explain steps in a clear manner,

Mary. When asked to explain her metaphor, Mary also confirmed the growth nature of her metaphor.

> Researcher: Can you describe your metaphor?
>
> Mary: The students are planted into mathematics and grow into beautiful flowers.
>
> Researcher: Planted?
>
> Mary: Yes. The students are planted into the soil, which is the math. There they get nutrients to help them grow. The teacher, the sun, the water, they all help the students grow.

Though Mary and Sarah had extremely similar pictures, their beliefs were vastly different. Strikingly, each had a different perspective of mathematics in their metaphor, revealed when they described their meanings. For Sarah, the mathematics was the water being delivered to the students which allowed them to grow. For Mary, mathematics was the very soil they are *planted into*. This will be discussed in more detail in later stages of the hermeneutic cycle.

The immersion stage also revealed that Mary did not view herself as the dispenser of information, but rather as a helper. Mary explained how important it was for her to help students discover new mathematics. Real world examples were explained by Mary to be an important way for her students to gain mathematical understanding. Mary also described herself as a flexible teacher, acknowledging that she would let context determine whether or not she would give students the formulas first, or let them search through applications.

Steve. Steve's metaphor seemed to depict a combination of a journey and a production metaphor. As earlier discussed, Steve described his metaphor in terms of being a goal to score points. This tended towards a production metaphor where points were produced, hopefully delivered in a win. Steve even used this language as he described his metaphor.

> Researcher: Where is the mathematics in your metaphor?
>
> Steve: The mathematics is in the scoring points. That is the goal to learn how to score points. When you listen to the coach you learn things. When you listen to the teacher you learn how to be good at math.

However, Steve also used language that implied his metaphor was a journey metaphor.

> Researcher: So the students work together?
>
> Steve: Yes. As they go through the game they need to know that they have a common destination. The coach (teacher) leads them together to the destination which should be a victory for the whole team.

That Steve viewed learning as a journey was also revealed when he shared a story about a good mathematics teacher.

> Researcher: can you describe a good mathematics teacher?
>
> Steve: A good math teacher works well with the students. He teaches so the students understand what they are doing.

Using the metaphor data proved to be a very useful way to examine participant constructs. In their own words and pictures, each participant was able to provide insightful information about teaching and learning mathematics from their own perspective. The next stage of the hermeneutic cycle sought to gain insights from the *researcher perspective.*

Abstraction

The purpose of the abstraction stage was to generate constructs using the researcher's theoretical and personal knowledge. The guiding framework for the abstraction stage of the hermeneutic cycle was the three philosophies that were outlined by Ernst (1994). These were the philosophies discussed in chapter three. As outlined by Ernst, these three philosophies of mathematics observed in the teaching of mathematics were (1) the Instrumentalist view, (2) the Platonist's view and (3) the Problem Solving view. Reviewing, the Instrumentalist's view was that mathematics is a collection of unrelated facts and algorithms to be practiced by drill, seeking mastery. The Platonist's view was that mathematics was a static body of knowledge that had an existence apart from human effort. Mathematics was discovered as human beings observed an external reality. The Problem Solving view was that mathematics was a human endeavor. Mathematics was created, not discovered. Mathematics was a process of inquiry, not a finished product but open to revision.

As previously noted, 15 participants completed the likert-type survey. Based on the results, three participants were selected for the interview process. Originally, one participant representing each of the three philosophies was desired. Recapping, Sarah seemed to align most strongly with the Instrumentalist's view. Steve had the most alignment with the Problem Solving view. Since no one seemed to have strong alignment with the Platonist's view, Mary was chosen for the interesting reason that she seemed the most *anti-Platonist.*

Sarah. Sarah's likert-type survey revealed she believed that mathematics was a collection of facts and formulas and that doing mathematics meant memorizing all relevant formulas and being taught the right procedures. She responded as very true that it was the teacher's job to teach the steps in each new mathematics method before the students were expected to use them. Interviews with Sarah confirmed her strong Instrumentalist's view of mathematics. As already noted, when explaining her metaphor she spoke of how the mathematics was the water being delivered to the students. This expressed the Instrumentalist's view of mathematics where the teacher is the expert delivering mathematics to the students. When asked to describe a good mathematics teacher Sarah responded, "A good math teacher answers questions fully and guides you to the right answer. A follow up led to the following discussion.

> Researcher: Describe another favorite mathematics experience.
>
> Sarah: In third grade I was asked to teach the class how to solve for area, and it was really cool because I explained it with detail and the teacher was impressed.
>
> Researcher: What exactly did you explain in detail?
>
> Sarah: Which formula. How to use it. What went where.

When asked to describe herself as a teacher Sarah responded by saying "I listen and watch it". Then I practice it on my own. This confirmed a very strong Instrumentalist's view of mathematics.

Mary. Mary was initially selected to participate in the interviews because she had the most *anti-Platonist* view of mathematics. Mary disagreed that mathematical ideas existed independently of human activity. She also agreed with the statement that mathematics was a creative human activity, which conflicts with the Platonist's view that mathematics is discovered, not created. That Mary viewed mathematics as a creative, human endeavor seemed to place her in the Problem Solving view of mathematics. This was also supported by her metaphor in which students were *planted* into mathematics. Mary viewed the teacher as a helper rather than an explainer and the goal was to understand.

> Researcher: How do students learn mathematics?
>
> Mary: One of the best ways is by application. It is beneficial for students to see mathematics in their everyday lives.

Mary agreed that mathematics can be done in many ways, and that there were many equivalent ways to define mathematical concepts. On the surface it appeared that Mary held a problem solving view of mathematics that was opposed to the Instrumentalist's view of mathematics as a fixed entity. Yet for Mary there existed a conflict. Mary agreed with the statement that mathematics was a collection of facts and formulas. She also agreed with the statement that to do well in mathematics she needed to memorize all the relevant formulas. Mary also believed it was the teacher's job to teach the steps in each new method before the students had to use them. This was revealed in the likert-type survey and the interviews.

> Researcher: Share a story about a fantastic teacher.
>
> Mary: When we first started to learn about polar graphs, he started by showing us how they worked, and some cool tricks about polar graphs.

The conflict was evident as she explained her metaphor. She talked about her belief that the teacher was a helper rather than an explainer and that the goal was for students to understand. Yet she still believed the water and the sun were *given* by the teacher to the student to help them grow.

Steve. Steve was chosen to participate because he had a fairly strong alignment with the problem solving view of mathematics. He agreed that mathematics was a creative human activity and that students can be creative and discover things by themselves they did not already know. Steve's metaphor and interviews confirmed that he has a Problem Solving view of mathematics. When describing his metaphor Steve said that the goal of learning is for students to figure out how to score on their own. Steve's interview also revealed his Problem Solving perspective.

> Researcher: Share a story from a high school mathematics class.
>
> Steve: One teacher always came up with very odd problems we had never seen before. That were pretty hard, but he would not want to help us figure things out. He might point us in the right direction, but would not give direct help. That was fun.

Steve's metaphor involving a basketball game supported the idea that Steve had a Problem Solving view of mathematics. The teacher was the coach who did not do the work for the students. For Steve, the coach was definitely the expert, but did not give the players the points. Rather the coach *facilitated* the abilities of the students to score points.

Researcher: In your metaphor, what is the job of the coach?

Steve: To help them learn the addition of scoring. The teaching is the helping, and the learning is the students figuring out how to score on their own.

Ernst pointed out that the Problem Solving view of mathematics is at the highest level. "At the highest level, the problem solving view sees mathematics as a dynamically organized structure located in a social and cultural context" (Ernst, 1994, p. 2). Steve showed evidence of the social and cultural elements of the Problem Solving view of mathematics.

Researcher: Describe the players in your metaphor.

Steve: They are the students, working together to score points, that is to learn math.

Researcher: Describe how the game of basketball relates to teaching mathematics.

Steve: Well, every game is different. Different teams play differently, and it is up to the coach to tap into his specific players talents to win the game. A good coach can win games with completely different players by relying on their strengths. The same is true in teaching math.

The abstraction stage of the hermeneutic process was the initial stage of identifying second order (researcher) constructs and was guided by Ernst's framework (1994). The next stage, synthesis and theme development, was the process of comparing and developing themes with a view toward a cohesive story to be told in later stages.

Synthesis and Theme Development

The goal of this stage was to begin bringing together the participant constructs from the understanding stage and the researcher constructs from the abstraction stage (Ajjawi & Higgs, 2007). Once again, it was important to point out that this process was only reported in a linear fashion. As the researcher worked through the hermeneutic process, the stages were not approached in such a linear way, but rather re-entered previous stages at will and as needed. Data were brought together through the lens of constructivism in order to begin understanding participants' beliefs about the nature of mathematics and how they interacted with their beliefs regarding the teaching and learning of mathematics.

Pre-service teachers will eventually transition into careers where they will be expected to understand how to craft their instructional practices so they align best with how students learn mathematics (Charlambous & Philippou, 2010). This requires that instructional practices be structured upon an understanding of the theories of cognitive development (Schoenfeld, 1983). As detailed in chapter two, constructivism has emerged as a major theme in mathematics education. Constructivism as a theory of learning has been well received by many mathematics educators and was thus the lens through which beliefs will be brought together in an effort to tell the story of the participants as future teachers of mathematics (Ernst, 2010).

Sarah. Sarah's viewed about the nature of mathematics expresses a strong Instrumentalist's view of mathematics. Mathematics is viewed as an accumulation of facts and rules to be used and mastered. This view carries over to Sara's views about teaching mathematics. Her metaphor and interview data express that she believed that the teacher should deliver the mathematics to the students. Student outcome was measured by correct skills mastery and correct performance. On the surface, it appeared that Sarah's views about mathematics and the teaching and learning of mathematics were in complete conflict with the principles of constructivism. Whereas Sarah's views did conflict with constructivism, the conflict was not a complete rejection of constructivist's principles.

Enactivism as one constructivist position suggested that teachers attend to bodily movements and gestures. This characteristic of teaching was evident in Sarah. Her outward appearance was very important to her and she viewed this as a positive when it comes to teaching mathematics.

> Researcher: Why is your appearance so important?
>
> Sarah: Because a big goal of mine is for students to feel comfortable around me. I want my appearance to say that math is fun. I see my students saying "Take Miss Smith's class! She is hilarious." I think when I act funny students will be more willing to learn.

Sarah's view of teaching also has slight hints of the social nature of learning that was a cornerstone of social constructivism. Sarah viewed herself as a cheerleader, encouraging groups of students.

> Researcher: What made you decide to major in mathematics education?
>
> Sarah: I really want to be a teacher because I love being a cheerleader, leading groups of people toward something fun.

Granted, Sarah did not hold a strong sense that mathematics was social, but it would be a mistake to say that a social view of mathematics was non-existent in Sarah. Sarah most definitely had a deep sense of care for her students which was clearly a major cornerstone in the development of constructivism from the time of Dewey through the present day.

Mary. Mary did not have a developed sense of constructivism, but there were very definite beliefs consistent with the basic tenets of social constructivism. Mary rated as very true the statement that mathematics was a creative human endeavor and her interview demonstrated her view of the social perspective of the human endeavor. During the interview process she said, “ I have always been able to see how math applies to the real world and have always enjoyed explaining this to my friends.”

> Researcher: Where is the learning in your metaphor?
>
> Mary: The learning is the students growing together as they interact with the water and sun which are provided by the teacher. The learning is in that interaction with each other, both students and teacher.

This social aspect also was revealed when Mary answered questions about justification.

> Researcher: What does “justification” mean in mathematics.
>
> Mary: It means working together to verify an answer. One student may know the right answer, but if they cannot verify it to others then they may not really understand things. Justification is the process of verifying things to the whole class.

Mary also viewed the teacher as the facilitator of an active learning process.

> Researcher: What is mathematics?
>
> Mary: Kind of everything. The study of patterns in the world. By getting students to examine real applications the teacher engages students to find mathematics in the world around them.
>
> Researcher: How do students find mathematics in the world?
>
> Mary: By engaging in fun applications. A good teacher will set up a group of students to engage in something fun, finding math along the way.
> 2

Steve. Of all the interview participants, Steve seemed to have the most developed sense of constructivism, primarily social constructivism. Mathematics as a social environment was evident through the interview process.

> Researcher: What is mathematics?
>
> Steve: Mathematics is what we as a civilized society use to help us understand the world around us. Mathematics is not in some vacuum, but is what people get when they look around.
>
> Researcher: How?
>
> Steve: Well, like when the coach helps the players see something. Then the players analyze what is around them and together they play better.

Central to Steve's metaphor was the team aspect of learning. This was at the heart of social constructivism. Steve's metaphor also revealed some major tenets of simple constructivism. Simple constructivism calls for attentiveness to the students' previous learning and constructions (Ernst, 2010). This was revealed in Steve's view of both the coach and the players. Steve described the coach as one who knows the abilities and knowledge of the players. The coach pays close attention to what the players have learned. Steve also acknowledged that the players needed to be good at learning from their mistakes. For Steve, this was an important piece of learning mathematics, paying close attention to what has already been learned. That will make for a better team.

Simple constructivism also called for identification of learner errors and the use of diagnostic teaching and cognitive techniques in attempting to overcome them (Ernst, 2010). This is extremely made clear in Steve's basketball metaphor. Steve explained that both the coach and the players need to recognize and seek to improve upon errors in order to improve. For Steve, this was definitely a team thing.

The synthesis and theme development stage of data analysis had brought the data to the point that a story could now be told. The next stage was illumination of phenomena and was the process of linking that data to literature in such a way that a story was told (Ajjawi & Higgs, 2007).

Illumination of Phenomena

Prior stages have linked the data to corresponding frameworks in an attempt to lay the foundation for a story to be told. This stage will examine each participant by examining the interaction between participants' views regarding the nature of mathematics with their views regarding the teaching and learning of mathematics. Mathematics as a field was related to, yet distinctly different than the fields of teaching and learning mathematics.

Pre-service secondary mathematics teachers sense of who they are as a teacher of mathematics was at least partially shaped by their beliefs about mathematics and the teaching and learning of mathematics (Wilson, 2010; Gresalfi & Cobb, 2011). Teachers of mathematics teach much more than mathematical content, they also communicate beliefs, values and emotional responses to mathematics (Grootenboer, 2006). Teacher practice is rooted and shaped by a teacher's beliefs regarding teaching and learning (Grootenboer & Ballantyne, 2011).

These stories are the researcher's thoughts on what the future teacher practice might look like for the three participants. These stories are predictive in nature based on the previous discussion about the participants views about the nature of mathematics and the teaching and learning of mathematics. The stories are not intended to explain exactly what the future practices will look like, but will give insight to very possible pictures of what these future teacher's classroom practices might look like.

Sarah. There is little doubt that Sarah's future classroom will be an appealing, fun place to be. Sarah places strong emphasis on the idea that she be a fun teacher that the students will enjoy being around. Sarah is energetic and appealing and it is likely that this perspective of her future classroom will become a reality. She will be well dressed and definitely has the personality to inject humor into her classroom. She has strong intentions that her students will laugh and she has the personality to make that happen.

Research has shown that the practices of teachers of mathematics are linked to how the teachers learned mathematics (Charlambous, Philippou & Clement, 2002; Philipp et al, 2007; Ambrose et al., 2003). Sarah described good teaching as the ability to carefully explain the rules and algorithms. She described how she was encouraged to solve problems on the board and felt really good when she was successful. Thus she views learning mathematics as a process where students watch the teacher go through the steps, then they get the chance to try things themselves. This is very likely how her future classroom will look like.

She will be very kind and considerate and very encouraging. She will slowly explain the steps to the process at hand, then she will ask the students to try it themselves. She will offer strong praise to students who master the skill and encourage those who struggle. The classroom will be very teacher centered with Sarah doing all the explaining and encouraging. She will pour the “water” onto the students so that they can gain skill mastery.

Sarah described herself as understanding the social nature of mathematics and would even see her classroom as a social setting with students talking mathematics with each other. The reality will likely be that students will share mathematical work with each other, but their sharing will be limited to showing each other that they have mastered a given skill. When Sarah described how she learned mathematics she referred to explaining things to other students, but it was always in the context of explaining to other students that she had mastered the skill. Sarah never referred to the idea of students *explaining meaning* to each other. Just that a skill was mastered.

Sarah has a strong sense that students grow in their mathematical knowledge as evident by her growth metaphor. But her strong Instrumentalist view of the nature of mathematics and the teaching and learning of mathematics will likely result in very traditional teaching practices dominated by Sarah dispensing mathematical information to the students so they can master the material.

Mary. Mary had a growth metaphor that was very similar to Sarah's growth metaphor. As mentioned earlier, the meanings of the metaphors were vastly different which implies that their teaching practices will be vastly different. For Sarah, her metaphor contained water, which was the mathematics poured onto the students. Mary's metaphor also contained water being poured, but the water was *not* the mathematics. The mathematics for Mary was the *soil* the students were planted into. This distinction is mentioned again because it illustrates how vastly different their future classroom practices will be. Mary does not view herself as the dispenser of mathematics. Rather, her view of teaching was that the teacher immerses students into mathematics. Her classroom will likely be a place where she immerses students *into* mathematics in the form of real life applications. The classroom will likely be teacher centered in the sense that Mary will decide what mathematics to plant the students into and how best to help the students navigate the setting into which the students were placed. Mary expressed the belief that students learn mathematics through a discovery process. Thus Mary's classroom practice will have a distinct sense of discovery. This was very interesting because the likert-type survey revealed Mary as the most *anti-Platonist,* yet discovery is central to the Platonist's view. Formulas and algorithms are central to Mary's views about the nature of mathematics and this will be seen in her classroom practice. However, Mary will not *give* students the formulas, but rather students will be allowed to explore and discuss things as they seek to find the formulas needed to solve real world problems. Because formulas and algorithms are central to Mary's views about the nature of mathematics, it seems likely that her classroom is more of some kind of balance between her explaining formulas and algorithms and students seeking to find them.

The social aspect of Mary's classroom practice will be clearly seen. Students will actively discuss the mathematics they have been placed into. Their conversations will not be limited to showing each other that they have mastered a skill, but will also include discourse regarding how they understand the mathematics they are working in. Again, this social discourse will be in stark contrast to the social discourse that will be seen in Sarah's classroom.

Steve. Of the three participants, Steve had the weakest Instrumentalist's view of mathematics. Thus his classroom practice will be distinctly different. The only similarities will stem from the fact that both Mary and Steve held some sense of a Problem Solving view of mathematics. Steve's metaphor of a coach teaching basketball players will dominate his classroom practice. Steve very much sees himself as a good coach and this will be clearly evident in both the way he teaches mathematics and the way students learn mathematics in his classroom. Steve will not do the work for the students, but will rather expect a strong work ethic from his students. This will be seen as he challenges students to do things themselves. That is not to say that Steve will be unaware of his students' weaknesses. Like any good coach, Steve will have a keen eye open to how his students are progressing and he will offer advice and encouragement as he sees it is needed.

Central to Steve's metaphor is the idea of teamwork. His classroom practice will demonstrate his desire for the students to work together on mathematics. Steve will encourage his students to not only analyze the mathematics around them, but also their fellow students perceptions about the mathematics. Steve will expect his students to help each other succeed as a team. This will demand that the students be encouraged to see not only their own weaknesses, but also each other's weaknesses. Steve's classroom practice will be dominated by this social interaction.

Now that the stories have been told, it is time to answer to research questions that motivated this research. Thus the last stage of the hermeneutic cycle, integration involves a critique of the previous stages of the hermeneutic cycle. This will involve not only reporting on the research findings, but also offering initial implications which will be addressed more fully in chapter five.

Integration

This last stage of the hermeneutic process attempts to explain how the previous stages' offers insights towards answering the questions that have motivated this research. The research questions under consideration are:

1. What are the philosophical beliefs of pre-service secondary mathematics students at a small private university regarding the nature of mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with their beliefs regarding the teaching and learning of mathematics?

The answers to these questions cannot really be extended beyond the three participants who participated in the interview process. However, gaining insight to these three participants does give researchers plenty to think about regarding belief systems of secondary pre-service mathematics teachers.

Question 1. The first important observation was that of the fifteen participants who participated in the likert-type study, none showed strong inclination toward a Platonist's perspective of mathematics and the teaching and learning of mathematics. A majority aligned with an Instrumentalist view of mathematics with a few aligning with a problem solving view which illustrated, not surprisingly, that there was not one philosophical position that captured all participants. Sarah revealed what a pre-service secondary teacher who held a dominant Instrumentalist's view of mathematics looks like. Mary revealed what it looks like to hold a mixed problem solving view and an Instrumentalist's view while Steve gave insight into what it looks like to hold a primarily Problem Solving view of mathematics.

Sarah aligned strongly with all tenets of the Instrumentalist's view of mathematics. This was evident in her likert-type survey data and her metaphor and interview data. Her philosophical views were dominated by a rule, algorithm perspective of mathematics with skills mastery the goal of teaching. Mary aligned mostly with the Problem Solving view with some alignment with the Instrumentalist's view of mathematics. This blend definitely softened the Instrumentalist's views that Mary held regarding the nature of mathematics. Mary held the view of the nature of mathematics is a set of rules and algorithms consistent with an instrumentalist's view, but her views on teaching and learning were dominated by a Problem Solving view of mathematics. This unique blend was distinctly different than the views of Sarah that were Instrumentalist throughout. Steve's philosophical position on the nature of mathematics and the teaching and learning of mathematics was mostly consistent with the Problem Solving view.

Thus, the answer to question one was that there seemed to be mostly an Instrumentalist's view of mathematics with some holding a Problem Solving view. Mary offered evidence that it was possible for secondary pre-service teacher mathematics teachers to hold one view on the nature of mathematics with seemingly contradictory views on the teaching and learning of mathematics which leads directly to question two.

Question 2. Sarah offered evidence that traditional classroom practices were still likely to exist. Her strong Instrumentalist's view of the nature of mathematics carried over to her beliefs concerning the teaching and learning of mathematics. Sarah's metaphor revealed that the teacher is responsible for instilling mathematics into the minds of the students. This consistency between her beliefs about the nature of mathematics and her beliefs regarding the teaching and learning of mathematics will likely lead to a classroom setting that is very traditional, teacher centered and mastery focused. Since the Instrumentalist view is embedded in both Sarah's beliefs about the nature of mathematics and her beliefs about the teaching and learning of mathematics it seemed that these beliefs would be resistant to change.

Mary illustrated that an Instrumentalist's view of the nature of mathematics does not necessarily lead to traditional classroom practice. This seemed to be precisely because her beliefs regarding the teaching and learning of mathematics was *not* primarily Instrumentalist, but rather Problem Solving. Thus her beliefs about classroom practices seemed to be more shaped by her views about teaching and learning than by her views about the nature of mathematics. This was evident when she described her metaphor. Though she did view mathematics as a set of rules and algorithms, the goal of the teaching was to embed students into mathematics rather than instill mathematics into their mind. Also, her belief that learning was a social activity allowed her to believe that though mathematics might be algorithmic, students could work together to come up with different types of algorithms.

Steve's beliefs regarding teaching and learning of mathematics was shaped mostly by his Problem Solving views of the nature of mathematics while he held a blend of an Instrumentalist view with a problem solving view regarding the nature of mathematics. Thus, at least to some degree, Steve believed mathematics contained skills to be mastered. Yet, for Steve, those skills could be mastered through problem solving in a social context.

Implications

The implications for teacher education are deep. Traditional Instrumentalist's views about the nature of mathematics and the teaching and learning of mathematics still exist among future teachers. Thus we can expect some classroom practices in the future to remain fixed on traditional methods and philosophies. However, there was also some evidence that at least some future teachers are philosophically moving away from traditional ways of thinking, especially regarding the teaching and learning of mathematics. This will be discussed more fully in chapter five.

CHAPTER FIVE

CONCLUSION AND IMPLICATIONS

Pre-service teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics have a direct influence on classroom practice (Beswick, 2007; Philippou & Christou, 2002; Presmeg, 2002). The importance of examining pre-service teachers' beliefs is emphasized by the fact that many pre-service teachers hold beliefs that are in conflict with beliefs suggested by both how mathematics was historically developed and by current reform efforts within the field mathematics education (Cady & Reardon, 2007; Cai, 2010; Ernst, 2010). Many of those involved in mathematics education research recognize that mathematics is a human creation open to revision and transformation (Ernst, 2010; English, 2010). This perspective is not commonly held by secondary mathematics teachers (Lester, 2002; Ernst, 2010; English, 2010). Secondary mathematics teachers' beliefs about mathematics are heavily influenced by their interaction with mathematics during their education process (Barlow & Cates, 2006; Beswick, 2007). Mathematics teaching processes in the United States has a long history of being dominated by perspectives that mathematics is a collection of rules and algorithms that are useful as mental exercises and memorization, drill and mastery of algorithms dominate the mathematics classrooms today (Jones & Coxford, 2002; Senk & Thompson, 2003). Thus pre-service teacher programs are faced with the challenge of engaging students in mathematics in ways the will challenge and change deeply held beliefs about the nature of mathematics and the teaching and learning of mathematics.

Overview of Study

The purpose of this study was to examine mathematical belief systems of pre-service teachers at a particular small private Midwestern university. Despite decades of reform, secondary mathematics classrooms across the United States is still dominated by perspectives that mathematics consists of rules and algorithms that are to be mastered (Senk & Thompson, 2003). Very little effort is put forth to allow students to generate mathematics. This study examined beliefs regarding the nature of mathematics using a hermeneutic phenomenology approach to gain insight about how those beliefs interact with pre-service teachers' beliefs regarding the teaching and learning of mathematics. Results of this study could be very useful for mathematics educators who desire to plan pre-service programs that give students opportunities to examine their belief systems. Two research questions were addressed in this research.

1. What are the philosophical beliefs of pre-service secondary mathematics students at a small private university regarding the nature of mathematics?
2. How do these philosophical beliefs regarding the nature of mathematics interact with their beliefs regarding the teaching and learning of mathematics?

The participants of this study were 15 current mathematics education majors at a small, private Midwestern university. The 15 participants were given an initial likert-type survey assessment to gain insight into their beliefs about the nature of mathematics and the teaching and learning of mathematics. Three participants were purposively selected from the fifteen participants representing the three philosophies outlined by Ernst (1988), namely the Instrumentalist's view, the Platonist's view and the Problem Solving view. Ernst conjectured that these three philosophies form a hierarchy with the Instrumentalist's view at the lowest level followed by the Platonist's view with the Problem Solving view being the highest level, viewing mathematics as a dynamic structure embedded in social and cultural contexts (Ernst, 1988).

Summary of the Findings

The three purposively selected participants were interviewed multiple times. They also were asked to create a metaphor for the teaching and learning of mathematics. During the interview process the participants were asked to explain the details of their metaphor including telling where the mathematics, the teacher, the students, the teaching and the learning were in their metaphor. Not surprisingly, this study revealed that pre-service teachers hold a variety of beliefs relative to the nature of mathematics and the teaching and learning of mathematics. Also, not surprisingly, was the finding that none of the three participants held beliefs that were exclusive to one of the three philosophies outlined by Ernst. Thus it seemed that when examining pre-service teachers' beliefs systems it would be a mistake to think that a specific pre-service teacher would hold beliefs exclusive to one philosophy. Rather, this research implied that a pre-service teacher can hold all three philosophies in varying degrees, though all three participants were seen to hold one dominant view of the nature of mathematics and the teaching and learning of mathematics.

Mary was a good example of a pre-service teacher who held all three philosophies in varying degrees. Interestingly, her view of the nature of mathematics had a distinctive Instrumentalist's view but her view of *teaching* was a Problem Solving view which seemed opposed to the Instrumentalist's view. Mary has some degree of viewing mathematics as a set of rules and algorithms, but that mathematics should not be *taught* as a set of rules and algorithms. Thus it would be a mistake to think that pre-service teachers hold the same dominant view of the *nature* of mathematics as their view about *teaching* mathematics. Mary's dominant philosophy seems to be the Problem Solving view of the teaching and learning of mathematics, though she has a slight belief that mathematics is learned through a discovery process which is distinctively a Platonist's perspective of the teaching and learning of mathematics. That Mary holds all three views in varying degrees should not be surprising since the field of mathematics and the field of teaching and learning are distinctive, though related fields.

Sarah illustrates that the Instrumentalist's view of mathematics is still an active belief system and this should give rise to concern for mathematics educators and mathematics education programs. She holds strong views that mathematics is a set of rules and algorithms and that mathematics should be taught as a set of rules and algorithms. She will very likely continue the traditional methods of teaching and learning as she dispenses mathematical knowledge to what she perceives as inactive students. The glimmer of hope for Sarah is that she does view that mathematics is a social endeavor, though it is not clear how that will be evident in her teaching practice.

Steve presents the strongest evidence of the Problem Solving view of mathematics, though he also has a slight degree of seeing mathematics as a set of rules and algorithms to be mastered. Learning mathematics as a set of skills is a perspective that will likely be evident in his teaching, though student *understanding* of those skills will be just as important to Steve as student mastery of those skills. The social perspective was seen clearly in his metaphor involving teamwork as well as during discussions regarding the teaching and learning of mathematics.

Implications

The findings of this research provide useful information for anyone interested in the teaching and learning of mathematics. Pre-service teachers themselves should be interested in this research because they will be seeking employment teaching mathematics. Pre-service teachers should be aware of how their beliefs about mathematics and the teaching of mathematics will interact with curriculum and teaching practice. During an interview process a prospective teacher should be asking about particular curriculum and curricular material being used in the school in which they are interested in teaching. Further, they should be aware of how their particular views will interact with the curriculum they will be expected to use. At the school level, principals should also be interested in hiring prospective mathematics teachers who hold philosophical beliefs that are consistent with the philosophy and goals at their particular school.

Those interested in mathematics teacher preparation programs should also be interested in this research. Scheffler (1970) argued over forty years ago for those involved in teacher education programs to include “philosophy of…” for all disciplines. This is a call that has gone largely unnoticed, but perhaps it is time to understand that beliefs matter when it comes to teaching and learning. Pre-service mathematics teacher programs are interested in mathematical content knowledge, pedagogical knowledge, and even pedagogical content knowledge (Shulman, 1986). This is evident as all pre-service teacher programs have accreditation processes that include assessing pre-service teachers’ content knowledge and pedagogical knowledge. Pre-service programs are also expected to assess pedagogical dispositions. This is likely the closest programs get to assess their pre-service teachers’ philosophical views about the content they will be teaching. Research has provided evidence that belief systems impact classroom practice (Cady & Reardon, 2007; Barlow & Cates, 2006; Beswick, 2006; Cai, 2010; Torner, Rolka, Rosken & Sriraman, 2010).Thus teacher educators should be interested in philosophical belief systems. Perhaps now is the time for mathematics teacher educators to take seriously the call to include philosophical issues in mathematics teacher education programs. If this is indeed the case, then perhaps assessment models should be designed to assess the philosophical views of pre-service mathematics teachers.

This research has shown that hermeneutical phenomenology can be useful to examine the philosophical position of pre-service teachers. Metaphors have been effectively used as a tool to examine teacher belief systems (Mahlios & Maxson, 1995; Reeder et al., 2009). For this research, metaphors were appropriate to use as data for hermeneutic phenomenology. The three root metaphors of production, journey and growth, developed by Shubert (1986) provided a nice link to the philosophies outlined by Ernst. This framework could provide a useful tool for assessing mathematics teacher education programs. Critical to the process was the interview portion where participants were able to explain their ideas about mathematics teaching and learning as depicted in their metaphors. Without the interview process the researcher would not have been able to identify where the mathematics, the teacher, the students, the teaching and the learning were in the metaphors. This was particularly evident when examining Sarah and Mary's metaphors. The two metaphors were very similar visually, but the representation of the mathematics was completely different. This illustrates the time commitment needed to examine philosophical belief systems. However, the time commitment would likely be worth the effort since this may provide mathematics educators a way to assess development of philosophical belief systems within teacher education programs.

An assessment could be designed where students entering a pre-service mathematics teacher education program took a likert-type survey assessment, created a metaphor and answered a few questions about their metaphors. These data could be recorded by rating each of Ernst's three philosophies on a scale to indicate perceived level of alignments. Combined with a few comments, this could be used as an initial assessment of philosophical beliefs about mathematics and the teaching and learning of mathematics. This process could be completed upon graduation to assess views after completing a pre-service program. Comparison of the pre-test and post-test would reveal some information about the effectiveness of the teacher education program in developing philosophical belief systems of pre-service teachers.

Once pre-service mathematics teacher education programs begin assessing belief systems a next logical step might involve an examination of the ways a program might help pre-service teachers examine and develop their mathematical belief systems. Pre-service teachers' belief systems are influenced by their experiences as students of mathematics (Cady & Reardon, 2007; Philipp et al, 2007; Ambrose et al., 2003; Beswick, 2007). Thus an examination of pre-service teachers' belief systems might reveal a need to carefully examine how mathematics content is taught teacher education programs. In general, pre-service teachers complete their mathematics content courses separate from courses in which they study pedagogical ideas (Ambrose et al, 2003; Charlambous, Philippou, & Kryiakides, 2002; Wilson & Cooney, 2002). Perhaps this needs to be challenged. Ambrose et al (2003) conjecture that "until PST's begin to learn about mathematical thinking so that some of their beliefs about mathematics, teaching and learning change, they fail to recognize that their own mathematical understanding is insufficient" (p. 440).

Research has shown that the history of mathematics can be taught in ways that challenge belief systems (Brown, 2012). Pre-service teachers being exposed to how mathematics has been developed through time often change their perceptions of mathematics. Brown (2012) revealed that "lower division mathematics history and mathematics methods course performance did have a significant relationship with mathematics efficacy beliefs, as measured by Mathematics Teaching Efficacy Beliefs Instrument" (p. 191). Teacher education programs that do not offer a course in the history of mathematics may want to consider the value of such a course.

Recommendations for Future Research

The participants for this research project were particular students from a small, private, Christian, Midwestern University. A major part of the results of this study detailed a hypothetical story of what their future classrooms might look like based on viewing their beliefs about the nature of mathematics and the teaching and learning of mathematics. A very natural future study would be to observe these particular teachers 3 – 5 years into their teaching careers. A longitudinal study could consider both pre-service teachers' beliefs about mathematics and mathematics teaching and learning and their eventual classroom practices.

Along these same lines, future research is needed to determine if pre-service teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics is a good predictor for future teacher practices. Studies have shown that beliefs do impact classroom practice but there is no evidence of research that demonstrates whether or not pre-service teacher beliefs can accurately predict future classroom practices. Multiple longitudinal studies could help develop a theory on how pre-service teachers' beliefs can predict classroom practices.

Another possible area of research might concern the development of assessment models. The implementation of assessments of beliefs about the nature of mathematics and the teaching and learning of mathematics in pre-service teacher education programs should be done in ways that are backed by empirically grounded research. Ultimately this might involve longitudinal studies that track pre-service teachers through their teaching careers. Assessments of belief systems should be relevant to improving future teaching practice. This is where it might be important to observe graduates 3 – 5 years into their careers to see how their belief systems interact with their future teaching practice.

Central to this study was the three philosophies distinguished by Ernst (1988). This framework has been used in multiple research studies involving teachers' conceptions about mathematics and the teaching and learning of mathematics (Schilling, 2010). Now that this particular study has been conducted and results formulated a critical reflection reveals the possibility that a fourth philosophy may be needed. As mentioned in chapter one, the Common Core State Standards are influencing curricular changes on a state by state basis. One of the hallmarks of the Common Core State Standards with regards to mathematics is the concepts of justification and proof. This study relied heavily on the three philosophies outlined by Ernst, and looking back the concepts of justification and proof seems to be missing in the discussions. Perhaps a research based analysis of Ernst's three philosophies would reveal that the concepts of justification and proof lead a fourth distinct philosophical view of mathematics. There may be evidence that proof and justification are embedded in the Problem Solving view of mathematics, but these concepts did not arise in the analysis stage of this particular research study. A critical look at Ernst's framework seems to be worth the effort.

Research has also shown the pre-service teachers' beliefs systems regarding the mathematics are resistant to change (Ambrose, 2004; Barlow & Cates, 2006; Beswick, 2006). Perhaps one area of research could explore effects of allowing pre-service teachers opportunities to examine their own beliefs regarding the nature of mathematics and their beliefs regarding the teaching and learning of mathematics. The framework developed in this study could be extended to a heuristic approach where pre-service teachers examined their own metaphors. Perhaps an auto-biographical approach to examining their own beliefs could be a catalyst for change.

Concluding Thoughts

This study revealed that there is value in looking into pre-service mathematics teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics. This is an important area of research in todays' world where there is much interest in how mathematics is taught and learned. There are many implications for both those who teach mathematics and those involved in the preparation of teachers. Mathematics teacher education programs are clearly interested in developing future mathematics teachers who are well prepared to teach in our ever-changing educational system. Thus it seems evident that mathematical belief systems should be of some importance in pre-service programs.

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Appendix

IRB Approval Letters

The University of Oklahoma®

OFFICE OF HUMAN RESEARCH PARTICIPANT PROTECTION - IRB

IRB Number: **13695**
Approval Date: **January 05, 2012**

January 06, 2012

Paul Howard
College of Education ILAC
820 Van Vleet Oval Rm 100
Norman, OK 73013

RE: Using Hermeneutic Phenomenology To Investigate Pre-Service Secondary Mathematics Teachers' Beliefs About Mathematics and The Teaching and Learning Of Mathematics

Dear Mr. Howard:

On behalf of the Institutional Review Board (IRB), I have reviewed and granted expedited approval of the above-referenced research study. This study meets the criteria for expedited approval category 6 & 7. It is my judgment as Chairperson of the IRB that the rights and welfare of individuals who may be asked to participate in this study will be respected; that the proposed research, including the process of obtaining informed consent, will be conducted in a manner consistent with the requirements of 45 CFR 46 as amended; and that the research involves no more than minimal risk to participants.

This letter documents approval to conduct the research as described:

Survey Instrument Dated: January 04, 2012 Interview questions
Consent form - Subject Dated: January 04, 2012
Protocol Dated: January 04, 2012
IRB Application Dated: January 04, 2012
Other Dated: December 14, 2011 Research design
Survey Instrument Dated: December 14, 2011 Initial beliefs survey
Other Dated: December 14, 2011 Verbal recruitment script
Other Dated: December 14, 2011 Recruitment email
Letter Dated: December 12, 2011 OCU site support

As principal investigator of this protocol, it is your responsibility to make sure that this study is conducted as approved. Any modifications to the protocol or consent form, initiated by you or by the sponsor, will require prior approval, which you may request by completing a protocol modification form. All study records, including copies of signed consent forms, must be retained for three (3) years after termination of the study.

The approval granted expires on January 04, 2013. Should you wish to maintain this protocol in an active status beyond that date, you will need to provide the IRB with an IRB Application for Continuing Review (Progress Report) summarizing study results to date. The IRB will request an IRB Application for Continuing Review from you approximately two months before the anniversary date of your current approval.

If you have questions about these procedures, or need any additional assistance from the IRB, please call the IRB office at (405) 325-8110 or send an email to irb@ou.edu.

Cordially,

Aimee Franklin, Ph. D
Vice Chair, Institutional Review Board

1816 West Lindsey, Suite 150 Norman, Oklahoma 73069 PHONE: (405) 325-8110

Ltr_Prot_Fappv_Exp

OKLAHOMA CHRISTIAN UNIVERSITY

December 12, 2011

Mr. Paul Howard
3513 Meadow Lane
Edmond, OK 73013

RE: Request for Oklahoma Christian University to Serve as a Non-OU Site for Conducting the Study Entitled "*Using Hermeneutic Phenomenology to Investigate Pre-service Secondary Mathematics Teachers' Beliefs About Mathematics and the Teaching and Learning of Mathematics*" as part of your graduate work at the Jeanne Rainbolt College of Education, University of Oklahoma

Dear Mr. Howard,

Based upon the materials provided on December 6, 2011, the members of the University Research Board (URB) of Oklahoma Christian University (OC) have reviewed the study and approve of OC as a non-OU site. We request that you notify the URB when the study has been completed. If the study is not completed in one calendar year, you must provide an update of the study and request that the study be allowed to continue.

We wish you the best as you move forward with this research. If I can be of any assistance or if you have questions, please contact me at 405-370-3128 (cell) or email (bill.luttrell@oc.edu).

Sincerely,

William E. Luttrell

William E. Luttrell, PhD
Chair, University Research Board
Oklahoma Christian University

WEL:wel

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