COURSEWORK ON BASE NUMERATION SYSTEMS AND ITS INFLUENCE ON PRE-SERVICE ELEMENTARY TEACHERS' UNDERSTANDING OF PLACE VALUE CONCEPTS

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CHAPTER 1

INTRODUCTION

Degree plans for elementary education majors require coursework in many disciplines. Although there is controversy about what mathematics content courses should be included in the program, most colleges require at least twelve credit hours or a four-course equivalent. In these mathematics classes students learn about problem solving, fractions, decimals, geometry, integers, and other topics. Since many students struggle with the content, they often come to class not only afraid of mathematics, but also with deficiencies in their mathematics backgrounds. These students learn basic operations and are usually able to do simple computations, but they lack a depth of understanding.

Studies acknowledge that elementary teachers are not adequately prepared with an explicit understanding of mathematics. "It appears that prospective teachers may have mastered basic skills, but they lack the deeper conceptual understanding that is necessary when responding to student questions and extending lessons beyond the basics" (Center for Study of Teaching and Policy, 2001, p. 9). The National Research Council report in 2002 states elementary teachers' understanding of the mathematics they teach is inadequate. This lack of subject matter depth can lead to students' dependence on rote memorization of mathematical algorithms to solve problems rather than a development of reasoning skills. As college students they often try to generalize and memorize their way through required mathematics courses, rather than make connections among ideas

(National Research Council, 2002). In fact many equate understanding in mathematics with the ability to complete steps in an algorithm (Folk, 2006).

Too many future elementary teachers begin college with mathematics deficiencies, both in content and in understanding. Their college mathematics courses focus mainly on computational skills rather than the development of reasoning skills and mathematical ideas. To illustrate these points the Conference Board of Mathematical Sciences uses the place value structure which states that elementary teachers need an explicit understanding of place value to aid children in building a strong numeric foundation (CBMS, 2001). Understanding place value as part of number and number sense is crucial to arithmetic success. Place value is the basis for the representation of whole numbers. Ideas of place value are used to order numbers and to aid in estimation and approximation (CBMS, 2001).

Statement of the Problem

In the college mathematics classroom, place value is often studied in connection with numeration systems, but elementary pre-service teachers question the need to learn different numeration systems. The study of numeration systems includes practice with number-base problems as well as ancient numeration systems like the Mayan or Babylonian. Number base work typically includes learning to count in different bases, converting back and forth between base ten and other bases, and performing number base computations.

With the limited number of mathematics courses required for elementary pre-service teachers, every course must provide meaningful instruction and content. Mathematics courses need to adequately prepare these prospective elementary teachers to effectively educate their own future students; therefore, the curriculum for each course is important. Does the study of base numeration systems contribute to elementary pre-service teachers' understanding of place value?

Context and Need for the Study

Textbooks for mathematics courses for elementary pre-service teachers have traditionally included the study of bases and different place value systems in their curricula (Bassarear, 2005; Billstein, Libeskind, & Lott, 2007). Why? Bassarear says the pre-service elementary teacher should work with other bases "so that your understanding of the fundamental ideas of the base and place value become deeper and deeper" (2005, p. 115).

Many elementary school textbooks explore place value numeration systems. Greenes et al. (2005) includes work with base two and base five in the grade four Houghton Mifflin text. Fennell and Altreri (1998) study the Babylonian system in their edition of the Silver Burdett grade three book. Clements, Jones, Moseley, and Schulman (1998) invite students to construct a place value system in a McGraw-Hill third grade textbook.

Children's literature also reflects an interest in place value systems. D.A. Adler's book uses nickels and pennies to show *Base Five* (Adler, 1975). *How to Count Like a Martian* (St. John, 1975) has chapters on systems from the imaginary Martian to the ancient Mayan. Luce's book *Counting Systems, The Familiar and the Unusual* puts different number systems into practice, such as base nine for baseball teams and base two for lights on and off (Luce, 1969).

Are there other reasons to study different place value systems? Casebeer researched the study of numeration systems in school mathematics in the 1960s. He hypothesized three reasons for studying this topic: 1) as foundational information, 2) for greater understanding of our Hindu-Arabic system, and 3) to gain empathy for children's experiences in learning place value for base ten (Casebeer, 1967).

Writing in the 1970s, LeBlanc reiterated some of the same reasons as Casebeer for supporting the study of numeration systems. He suggested that prospective elementary teachers study other base systems to understand the characteristics of our base ten system. He wrote that work with other bases helps with the understanding of groupings and place value and provides elementary pre-service teachers with experiences that children may have in learning the Hindu-Arabic system. He drew a clear connection between teacher understanding of place value and children's development of numeric concepts (LeBlanc, 1976).

Research involving place value has primarily focused on elementary students and practicing elementary teachers. Jones' and Thornton's review of literature (1993) of the 1980s and early 1990s identified many studies showing children's deficiencies in place value understanding. Fuson's research (1990a, 1990b) used manipulatives, such as cubes and base-ten blocks, to solve addition and subtraction multidigit problems to help students learn and apply place value concepts. She found that children have "inadequate understanding of the base-ten place value system of written multidigit numbers, and consequently long-term errors in multidigit calculation procedures" (Fuson et al., 1997, p. 130).

Zazkis and Khoury studied place value, elementary pre-service teachers, and their understanding of non-integer numbers in bases other-than-ten. They concluded that their pre-service students did not fully understand our number system structure. The results suggested that these college students' partial understanding allowed them to accurately complete problems, but might interfere with conveying place value concepts to children (Zazkis & Khoury, 1994).

In 2003, McClain studied elementary pre-service teachers' understandings of place value and multidigit addition and subtraction. In the course of this study she used base eight materials. One conclusion of her research was that the future teachers realized that conceptual understanding was necessary for teaching and explaining place value to others. Her students shifted their goals from "a focus on correct procedures to an emphasis on students' [children's] understanding" (McClain, 2003, p. 304).

Two recent studies conducted at the University of Texas investigated instructional strategies, elementary pre-service teachers, and place value (Hannigan, 1998; Rusch, 1997). Farro-Lynd's work (2003) focused on elementary pre-service teachers' understanding and misconceptions of place value. There appears to be no recent research connecting pre-service teachers, numeration systems, and place value. The focus of this project will be the teaching of base numeration systems and its impact on elementary pre-service teachers' place value understanding.

Purpose

This study grew out of classroom practice. The researcher found that her elementary pre-service teachers encountered difficulties while studying numeration systems. These students did not see any application for the concepts and usually struggled to understand the content. This project questions the value of teaching numeration systems to elementary pre-service teachers. Are the reasons given by textbook authors and researchers valid? In particular, does the study of other-than-ten bases increase elementary education students' understanding of our base ten system? Does place value numeration systems instruction help elementary pre-service teachers generalize place value concepts? These are the questions to be researched in this study.

Research Questions

Specifically, four questions are addressed in this study:

1. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course?
2. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course?
3. Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester?
4. Is there a significant difference in gains made in explicit place value understanding between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)?

Elements of the Study

This research was a quantitative quasi-experimental/control group design study. The objective was to compare gains of explicit place value understanding possessed by two groups of elementary pre-service teachers taking a required mathematics course. The course curriculum included a section on base numeration systems. The independent variable in the study was the instruction, or withholding of instruction, of base numeration systems. The dependent variable was growth in explicit place value understanding as measured by the Assessment of Place Value Understanding instrument.

Two groups of pre-service elementary teachers participated in this study, a sample of convenience. One group of 35 was enrolled in a required mathematics content course at a medium size regional state university. A second group of 25 was enrolled in a similar course at a small private university. Both groups' curriculum contained a section which addressed place value concepts and numeration systems.

The instrument used in this research is the Assessment of Place Value Understanding. This pretest and posttest assessment was developed by two PhD. candidates at the University of Texas in 1997. In a review of literature for their separate dissertations, they found "no previously validated assessment instruments that would provide insight into participants' explicit understanding of place value" (Rusch, 1997, p. 12). They also found existing instruments were directed at children and not appropriate for assessing the knowledge of adults. A description of the Assessment of Place Value Understanding and the validation procedures are included in Chapter 3. The Appendix contains copies of the thirteen item pretest (only the pretest is used in this research), along with information regarding the objectives, rationale, and scoring strategies used.

Definition of Terms

Numeration System - "A numeration system is a collection of properties and symbols agreed upon to represent numbers systematically" (Billstein et al., 2004, p. 135).

Place Value - "Place value assigns a value of a digit depending on its placement in a numeral. To find the value of a digit in a whole number, we multiply the place value of the digit by its face value, where the face value is a digit" (Billstein et al., p. 136).

Base Numeration System - A true place value system has a base and a set of symbols, including a symbol for zero. The Hindu-Arabic system used in the United States is an example of a base ten numeration system.

Explicit Understanding - "The level of mathematical understanding that is characterized by a precise understanding of the concept being addressed which can be clearly articulated and convincingly justified" (Rusch, 1997, p. 16).

Procedural Understanding - "Knowledge of the formal language of mathematics, that is, symbols and syntax; and the rules, algorithms, or procedures used to solve mathematical tasks" (Post & Cramer, 1989, p. 222.).

Pre-service Teachers - "Person studying to teach mathematics as one of several subjects, future elementary school teachers" (Graeber, 1999, p. 191).

Limitations of the Study

This research represents a preliminary exploration into the relationship between understanding place value and understanding base numeration systems. This study updates earlier reports, reexamines, and investigates only one small construct of teacher knowledge. With its narrow focus, this study is meant to inform and raise questions regarding course content for elementary pre-service teachers in their undergraduate program. The researcher's hope is that others will take up the charge to investigate this and other related topics about curriculum and expertise in subject matter knowledge.

The Assessment of Place Value Understanding was administered during this study. Since this is a relatively new instrument to measure explicit understanding, there may be some hidden limitations on its use. Other assessments were researched for this study, but tests for this population of adult college students were normally of general arithmetic concepts, including topics such as geometry, number theory, fractions, sets, and the four basic operations on the set of whole numbers. None of the instruments focused solely on place value.

Due to time constraints, control of participant sampling was not possible. All classes used were chosen by convenience. Although the classes were at different colleges with different instructors, efforts were made to at least select classes with similar students in a comparable college program. All students in the two classes were invited to participate. Even so, the samples lead to major questions regarding generalizabilty.

Although teaching methodology was not considered in this research, the teachers have similar pedagogy. Both instructors have many years of university teaching experience, and both are friends and students in the same doctoral program. Texts and curricula for the classes were different, but content pertinent to this research was similar. Content comparisons are discussed in Chapter 3.

There was also no way to control for the participant's concentration and effort while taking the assessments. Although both instructors encouraged the participants, there was no sufficient incentive to ensure their best work.

Conclusion

Place value is a major component of the Hindu-Arabic numeration system. Although many students are able to gain some expertise in computation, many more find mathematics problems difficult to comprehend. “Knowledge of place value has great implications for success in arithmetic tasks…. Therefore, care should be taken that students develop a meaningful understanding of numbers” (Cathcart, Pothier, Vance, & Bezuk, 2000, p. 106).

One major reason for studying base numeration systems in the elementary education major curriculum is the connection to place value. This study examined the relationship: Does studying base numeration systems lead elementary pre-service teachers to a higher level of understanding of our base-ten system?

Summary of the Chapters

Chapter One contains the introduction and impetus for the problem. The introduction to the study in Chapter One defends the importance of the topic, states the purpose of the study and identifies the questions to be answered. Included in Chapter One are definitions of terms and an outline of the remainder of this dissertation.

Chapter Two begins with the progress of education during the past twenty years. The importance of teachers’ content knowledge is discussed as well as the connection between teaching and student understanding. Research on elementary pre-service teachers’ mathematical skills is then addressed. Finally the importance of place value and how children learn is presented.

Chapter Three describes the methodology used in this study. Development and piloting information about the instrument used – the Assessment of Place Value Understanding (APVU) – is detailed as well as a matrix of place value grouping schemes used in the instrument. The sample of students is identified and information about the instructors and course content. The five level scale of mathematical understanding employed in the assessment rubrics is defined. Scoring of the tests is discussed and the statistical tests used to analyze the data

Chapter Four details the findings of this research. Demographic information that was collected on the participants is summarized. Sample scoring data to check for rater reliability is given. Each questions in the study is answered by analyzing data in total, in each grouping, and in each rubric. Tables show data details and results.

Chapter Five summarizes this study and offers conclusions about the research. Finally, recommendations are offered for future work.

CHAPTER 2

LITERATURE REVIEW

Over the past thirty years national attention has focused on quality education in the United States. Federal legislation and government research set rigorous standards for curriculum, teacher quality, and assessment. The desire for highly qualified teachers became a priority. The National Council of Teachers of Mathematics (NCTM) began publishing standards for the mathematics community in the 1980s with *An Agenda for Action* (NCTM, 1980). In numerous publications this organization identified content appropriate for students by grade level and defined mathematics and pedagogical content to enhance teacher preparation. The NCTM *Professional Standards* (1991) emphasized the importance of teachers' knowledge and their understanding of the subject matter.

Since research shows a strong connection between teacher knowledge and student achievement, teachers need to know more content than the subject matter they teach. In order to present mathematics clearly and effectively, teachers must have explicit understanding of the subject. Elementary pre-service teachers should have this explicit understanding for all segments of mathematics they will be required to teach.

For elementary teachers, place value is a foundation concept. Yet many teachers, and thus their students, do not have an explicit understanding of this topic. As part of the college mathematics curriculum, base numeration systems are often taught to help these pre-service teachers understand our base ten system. This research examines the effectiveness of teaching base numeration systems and their effectiveness in improving place value understanding.

Specifically, four questions are addressed in this study:

1. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course?
2. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course?
3. Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester?
4. Is there a significant difference in gains made in explicit place value understanding between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)?

This chapter begins with a historical perspective on America's focus on education and the importance of mathematics. The public call for "highly qualified" teachers and the search for a clear definition of this new term causes all teacher educators to reexamine their instruction. Section two of this chapter sights literature assessing the subject matter qualifications of today's teachers and pre-service teachers. Many teachers and pre-service teachers can calculate properly but do not understand the logic behind the algorithms. In section three the researcher makes connections between teacher preparation and children's achievement in the classroom. Teachers need to have a positive attitude and an understanding of the mathematics they teach. Section four discusses elementary students' knowledge of place value and the essential nature of this concept. Research with elementary students shows that they do not have a mastery understanding of the place value system. The focus of this study, numeration systems, is discussed in section five. A number of studies are outlined dealing with elementary pre-service teachers. Yet there appears to be a void in the literature connecting place value understanding, elementary pre-service teachers, and numeration systems. This study is meant to begin the probe into rigorous place value course content for the elementary pre-service teacher.

Historical Perspective

Heading into the 21st century, Americans renewed their focus on quality education. The '80s and '90s were inundated with multiple reports and studies on schools, student achievement, and teacher effectiveness. Among the reports issued were *A Nation At Risk* (1983), completed by the National Commission on Excellence in Education, *A Nation Prepared: Teaching for the 21st Century* (1986) written by the Carnegie Forum on Education, *Action for Excellence* (1983), authored by the Task Force on Education and the Economy, Education Commission of the States, and *A Call for Change* (1985), reported by the National Commission for Excellence in Teacher Education. In *A Nation At Risk*, concern was expressed about "the widespread public perception that something is seriously remiss in our educational system" (National Commission on Excellence in Education, 1983, p 1). A call was issued to reform the Nation's schools and reverse this declining trend. Recommendations for teachers included higher standards for certification and a competence in an academic discipline.

In *A Nation Prepared: Teaching for the 21st Century,* the Carnegie Forum on Education declared that teachers needed a sense of mathematics if their students were to have mathematical ideas and learn reasoning skills. Teachers' content knowledge must include breadth and depth. "They [teachers] must be people whose knowledge is wide-ranging and whose understanding runs deep" (1986, p. 25).

After President George H.W. Bush convened an education summit in 1989 of the nation's governors, Congress passed the Goals 2000: Educate America Act. This set a national education agenda. Based on eight goals involving students, teachers, and communities, Goals 2000 required states to develop clear and rigorous standards. Goal Five, emphasizing mathematics and science, advocated more pre-service training to help these prospective teachers gain broader and deeper mathematics understanding (Grinstein & Lipsey, 2001).

In his second State of the Union address, President Clinton issued a " 'Call to Action ' that included as a priority improving the quality of teachers in every American classroom" (Lewis et al., 1999, p. iii). The next year a Harris Poll revealed that about 90% of Americans believed "the best way to raise student achievement is to ensure a qualified teacher in every classroom" (CBMS, 2001, p. 3). This interest led to the 1998 amendments to the Higher Education Act of 1965. Part of these provisions included teacher quality enhancement grants to improve the quality of the future teaching force by improving the preparation of prospective teachers (United States Department of Education, 2003).

Studies and reports were published examining school mathematics. These included *Everybody Counts* (1989) by the National Research Council and *Educating*

Americans for the 21st Century (1983) reported by the National Science Board Commission. *An Agenda for Action* (1980), *Priorities in School Mathematics* (1981), and *Guidelines for the Preparation of Teachers of Mathematics* (1981) were written by the National Council of Teachers of Mathematics (NCTM). These works made recommendations for school mathematics curricula, teacher training, and public support of the discipline. The National Research Council wrote that the main objective of the elementary school mathematics curriculum should be the concept of number sense. The National Science Board Commission (1983) stressed the importance of number sense and place value and called for high quality teachers. The NCTM encouraged improved pre-service teacher education, focusing on teacher skills necessary to help children learn more than computation.

The National Council of Teachers of Mathematics compiled their first *Curriculum and Evaluation Standards for School Mathematics* in 1989. This publication established high expectations for students of mathematics in kindergarten through grade twelve. The document provided a comprehensive "listing by grade-level bands of the mathematics that students should know about problem solving, communication, connections, and various content aspects of mathematics relevant to those grade levels" (Usiskin & Dossey, 2004, p. 7). *The Curriculum Standards* were written in response to the increased national interest in the teaching and learning of mathematics and the publication prompted states to develop their own standards to set the bar for student performance. The K – 4 standard six of number sense and numeration identified understanding place value as a "critical step in the development of children's comprehension of number concepts" (NCTM, 1989, p. 39). Sample learning activities were included. As part of the evaluation standard eight on mathematical concepts, the importance of place value was connected to calculating subtraction problems.

The National Council of Teachers of Mathematics followed up its *Standards* text with the *Professional Standards.* This publication was based on the assumption that teachers are key in changing the mathematics curriculum and pedagogy. The *Professional Standards* spelled out "what teachers need to know to teach toward new goals in mathematics education and how teaching should be evaluated for the purpose of improvement" (1991, p. viii). Professional development standard two contained a section on number systems and number sense which identified place value as important content for teachers to understand.

Mathematics understanding has also been studied on an international basis. The Second International Mathematics Assessment was completed in 1986, comparing United States student achievement with sixteen other countries (Grinstein & Lipsey, 2001). The Third International Mathematics and Science Study (now called the Trends in International Mathematics and Science Study or TIMSS) compiled data on United States students and students from other countries in 1995, 1999, and 2003 (National Center for Education Statistics, 2001). These studies "suggested that American practices in mathematics education are not yielding the kind of learning that is both desirable and possible" (Watanabe & Thompson, 2004, p. 11). TheUnited States' mathematics curriculum emphasized procedural skills and provided little depth in subject matter.

As the 21st century began, the focus on education continued. President George W. Bush worked to pass the No Child Left Behind Act (NCLB), enacted in 2001. This act, PL 107-110, mandated that all teachers must be deemed highly qualified. Although there is disagreement about what it means to be "well qualified," part of the NCLB definition of "high quality" included an ability to demonstrate expertise in the subject matter. States were charged to raise standards for teacher certification and monitor practices to prepare, train, and recruit high quality teachers (United States Department of Education, 2001).

With the higher level of mathematical sophistication needed in the 21st century, increased mathematical skills and a solid understanding of mathematical concepts were needed by educators at all levels. Media and further publications stressed the importance of the subject of mathematics. "School mathematics instruction and the mathematical preparation of teachers are in the spotlight; because after reading and writing, mathematics is widely viewed as the most important component of K-12 education" (CBMS, 2001, p. 3). *The Nation's Report Card: Mathematics 2000* claimed that "the ability to know and use mathematics is a necessity in daily life" (Braswell et al., 2001, p. 1). Even television reinforced the importance of mathematics in one of its programs. The series *Numb3rs* began "We all use math every day, to predict weather, to tell time, to handle money. Math is more than formulas and equations. It's logic. It's rationality. It's using your mind to solve the biggest mysteries we know" (Heuton, Falacci, & Zucker, 2005).

In 2000 the National Council of Teachers of Mathematics published an update of its 1989 *Standards* entitled *Principles and Standards for School Mathematics.* It stated

> The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. . . . In this changing world,

world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures (NCTM, 2000, p. 4-5).

Pre-Service Elementary Teachers' Knowledge of Mathematics

With the call for highly qualified teachers, articles and research from the academic community investigated elementary teachers' knowledge of the subjects they teach. Collias, Pajak and Rigden (2000) begin their article entitled "One Cannot Teach What One Does Not Know" with the quote, "Graduates of university teacher preparation programs in the United States, though well trained in teaching methods, often have insufficient knowledge of the subject matter they will teach" (p.1). The authors reported that a 2000 survey found thirty-three percent of United States teachers did not believe that a mastery of subject matter was important to teaching. An article by McDiarmid, Ball, and Anderson (1989) on subject-specific pedagogy used a literature review to defend the statement, "Unfortunately, considerable evidence suggests that many prospective [elementary] teachers do not understand their subjects in depth" (p. 199). Subject matter content is taken for granted, and teacher preparation programs focus on pedagogical skills.

Evidence of poor elementary teachers' knowledge of content was referenced in an article by Brown, Cooney, and Jones stating that "pre-service elementary teachers do not possess a level of mathematical understanding that is necessary to teach elementary school mathematics" (1990, p. 642). Other research by Post, Harel, Behr, and Lesh (1991) specifically found that twenty to thirty percent of teachers scored less than fifty percent on an instrument measuring conceptual knowledge of rational numbers. In fact almost half the teachers missed very fundamental items like ⅓ ÷ 3.

In their literature review, Brown and Borko found that "research also suggests that prospective elementary teachers often do not have adequate content knowledge when they begin student teaching" (1992, p. 220). Previous research had postulated that this lack of knowledge was due to a focus on procedural rather than conceptual understanding. Students entering mathematics courses associated mathematics with mechanical and abstract symbols and rules. Coursework seemed to help the elementary pre-service teachers develop more conceptual understanding about their own learning but did not enhance their thinking about their students' learning. Pre-service teachers should have " both knowledge of mathematics and knowledge about mathematics" (p. 212).

Schwartz and Riedesel (1994) reported on a number of studies that showed teachers and pre-service teachers had only a procedural knowledge of mathematics. Duckworth focused on teachers and long division. These teachers knew the algorithm for long division, but they did not know why the procedures worked. Duckworth concluded that teachers' understanding of arithmetic varied widely. Her second conclusion stressed the connection of understanding and classroom practice. As teachers' understanding deepened, their teaching methods changed. Wheeler studied pre-service teachers and division problems with zero. Sixty-two elementary pre-service teachers were tested and interviewed on problems dealing with a divisor or dividend of zero. Division by zero problems were most difficult for the future teachers. Only 23.1% made no errors. Even though many could do the problems accurately, "many could not adequately elaborate on the question, 'What is zero?'" (p. 4).

An article by Fosnot (1989) described a classroom scenario common for many education majors.

> They frequently sat in math classes and practiced procedures, proofs, and calculations. They learned to memorize and regurgitate these procedures and rarely were given concrete materials to make the abstract notation meaningful. The result is that many teachers do not understand the concepts they are expected to teach (p. 71).

Post and Cramer (1989) found that the mathematical background of new teachers included a concentration on drill and practice rather than an emphasis on understanding the concepts. These authors stated that this lack of conceptual knowledge "dominates school mathematics curricula at virtually all levels" (p. 222). To illustrate this focus on mechanistic skills, a survey from a Minnesota school district was sighted. Elementary junior high teachers reported over sixty percent of class time was spent working textbook problems and practicing speed and accuracy in computational algorithms. This type of instruction using just the printed page generally does not lead to higher level thinking or understanding.

McDiarmid and Wilson conducted teacher interviews and learned that many had mastery of mathematical rules but could not explain the reasons behind the rules. One teacher said, "I don't know why. My teacher never told me" (1992, p. 101). Ball (1990) also found this deficiency. In her interviews a future teacher said, "I absolutely do it [a mathematical calculation] by the rote process- I would have to think about it" (p. 458).

The students' idea of "doing" mathematics was to describe the steps of the algorithm, without any thought as to the meaning or reasons for the procedures.

In a 2001 report, The Center for the Study of Teaching and Policy concluded that "prospective elementary teachers had relatively sound procedural, or rule-dominated knowledge of basic mathematics, especially in arithmetic but had difficulty when pushed to explain why an algorithm or procedure works" (p. 9). They lacked sound conceptual understanding of the underlying concepts of the mathematics.

Teacher Knowledge and Student Learning

The quality of teachers is a concern because of the strong connection between teachers' knowledge and student achievement. In its *Professional Standards,* the National Council of Teachers of Mathematics stated,

> Knowledge of both the content and discourse of mathematics is an essential component of teachers' preparation for the profession. Teachers' comfort with, and confidence in, their own knowledge of mathematics affects both what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the kinds of learning environments they create, and the discourse in their classrooms (1991, p. 132).

Studies (Collias, Pajak, & Rigden, 2000; Darling-Hammond, 2000) stressed that teachers' knowledge is an important influence on student progress. The statistical report of Darling-Hammond based on the National Assessment of Educational Progress data concluded that "the proportion of well-qualified teachers is by far the most important determinant of student achievement" (2000, p. 30). The Collias, Pajak, and Rigden

(2000) article details the STEP program, an initiative to help university faculty in teacher preparation. The program guides universities to reorganize their teacher preparation programs to involve all faculty in an accountability system for graduates' knowledge. The authors stated that "What a teacher knows and can do makes the crucial difference in what children learn. In fact teacher expertise is the most important factor in student achievement" (p. 3). Bad teachers can adversely effect student learning and put children at an academic disadvantage.

A compilation of research report authored by the Center for the Study of Teaching and Policy found that "several studies showed a positive connection between teachers' subject matter preparation and … higher student achievement, … particularly in mathematics" (2001, p. 7). The report claimed that good teaching could not take place if teachers lacked conceptual understanding of the subject matter. In an annual report on teacher quality published in 2005, the United States Department of Education stated, "Ensuring that America's teachers are of the highest quality is an important national priority because they hold the key to student success. Simply put, teachers matter" (USDE & OPE, p. 5).

A 2001 government publication, *The Nation's Report Card: Mathematics 2000*, stressed the importance of teachers having adequate content knowledge. "To better serve the students they teach, teachers need preparation in the content areas of mathematics that are part of their students' curriculum" (Braswell, p. 135).

Reporting on states' progress toward meeting the NCLB Act requirement of highly qualified teachers, Walsh and Snyder (2004) concluded that understanding subject matter was a critical skill for teachers. They stated that there was no substitute for subject knowledge. If a teacher did not know a concept, then that concept would not be known to the teacher's students.

Grossman, Wilson, and Shulman (1989) reported on the Knowledge Growth in a Profession Project at Stanford University, investigating the role content knowledge plays in instruction preparation. The Project found that "teachers need to understand their subject matter in ways that promote learning" (p. 24). Subject matter knowledge also influenced content in the elementary classroom and classroom practice. The conclusion of the authors in this report was that beginning teachers are not prepared to transform their own learning into a form appropriate for their students. A solution was to have a subject-specific knowledge of pedagogy, a methods class. This course would unite pedagogy and content and reinforce teacher knowledge.

"Teachers have a better chance of being able to help their pupils develop flexible understandings of subject matter if they understand their subject matter well" (McDiarmid, Ball, & Anderson, 1989, p. 199). McLaughlin and Talbert (1993) suggested that if teachers had only cursory knowledge of their subject matter they would be able to teach only the facts or leave learning up to the students.

Liping Ma quoted an old Chinese saying, "Know how, and also know why" (Ma, 1999, p. 108). Teachers must not only know how to manipulate an algorithm but they must know why it makes sense mathematically; therefore, they must have conceptual understanding as well as procedural understanding. This higher-level understanding can improve teachers' knowledge of mathematics, thus helping to improve students' mathematical abilities. According to Ma, how well teachers understand the subject matter they teach directly impacts the level of student learning.

In 1989, Ball outlined strategies for change regarding teachers and adequate subject matter knowledge. “Teachers should not just be able to do mathematics; if they are to teach for understanding, they must also have a sense for the mathematical meanings underlying the concepts and procedures” (p. 89). Her article in 1990 on the mathematical understanding of prospective teachers stressed the importance of the depth of knowledge teachers must have to present the subject matter effectively to children. Teachers must not only be able to compute problems accurately, they must be able to explain steps, discuss reasoning, and offer alternative solutions. In later research with Bass, Ball wrote that teachers needed to know more than the subject matter knowledge that students learn “in order to have broad perspective on where their students are heading. Teachers’ own knowledge on the subject affects what they teach and how they teach” (Ball & Bass, 2000, p. 86).

Hungerford’s article, based on his classroom experiences with pre-service teachers, explained that elementary teachers’ attitude about mathematics influence their students’ attitudes. Teachers who are afraid of mathematics are likely to transfer that feeling to their students. It is unlikely that students will develop an appreciate for mathematics if their teachers have little interest in the subject (1991).

Place Value and Elementary Student Knowledge

In the first NCTM *Standards,* a strand in the K-4 grade group was number sense and numeration. The section began:

> Children must understand numbers if they are to make sense of the ways numbers are used in their everyday world. They need to use numbers to quantity, to identify locations, to identify a specific object

> in a collection, to name, and to measure. Furthermore, an understanding of place value is crucial for later work with numbers and computation. (1989, p. 38).

According to Dienes (1960), place value and using base ten is our way to communicate numbers, and it is imperative that students understand this concept. "The notation of the number using place value, with the base of ten is a method of communicating numbers, and it is essential that children should learn the meaning of such communication as effectively as possible" (Dienes, p. 51).

Place value concepts create a foundation for student number sense. According to Smith, Lambdin, Lindquist, and Reys,

> Many young children can count to one hundred or beyond, but actually have little or no sense of what [the] numbers … actually mean. Before students can make sense of addition or subtraction of multi-digit numbers, it is essential that they develop a good understanding of place value (2001, p. 48).

The *Principles and Standards for School Mathematics* set goals for thinking and reasoning. The authors recommended that "students develop a deep understanding of important concepts and proficiency with important related skills…This recommendation implied that our students deserve to learn more than the procedures of mathematics – they also need to make sense of mathematics" (Lappan, 2005, p. 1).

Tracy and Gibbons tested teaching materials and activities to help elementary students and pre-service teachers learn decimals and metrics. They used a number line, decimal squares, videos, the calculator, and the United States Metric Association website to develop classroom lessons. These two teachers found that "the problem lies in that the emphasis on place value tends to be on computation, not conceptual understanding. For students to be successful, they must first understand them [concepts]" (1999, p. 2).

Textbook authors Lanier and Taylor created an activities manual for preservice elementary students. In it they stated, "One reason many elementary and middle school students have trouble with mathematics is due in part to only being taught an algorithm with no meaning behind it" (2004, p. 11). These authors suggested that teachers should lay a strong foundation and explain to students why algorithms work.

Several research papers have shown that elementary students have difficulty learning place value concepts. Articles by Fuson (1990a, 1990b) began with the premise that children perform poorly on place value tasks. She then explored words used in place value (for example, eleven versus ten one) and proposed new textbook characteristics. Given the problem 527 + 435, more than half of the third graders asked were unable to solve it by "counting on with hundreds, tens, and ones from 527 (527, 627, 727, 827, 927, 937, 947, 957, 958, 959, 960, 961, 962)" (1990a, 368). Fuson indicated that, "Many elementary schoolchildren …. do not fully understand the base ten structure of multidigit number words" (1990a, p. 350).

Kamii (1985) conducted tests in classrooms in Chicago for place value understanding on students in grades 1 (n = 13), 4 (n = 35), 6 (n = 48), and 8 (n = 41). These tests were given at the same elementary school and at the junior high school that the students would normally attend afterwards. Children were asked to circle the number of objects that the one represented in the number 16. None of the first graders circled ten, the correct answer. Eighteen out of thirty-five (51%) fourth graders were correct, compared to twenty-nine out of forty-eight (60%) sixth graders and thirty-two out of forty-one (78%) eighth graders. When she discussed these results with two mathematics teachers, "they were not surprised. They said that some children never seem to get place value" (p. 63).

Ashlock (1990) found that children had difficulties with computational algorithms because they did not have an adequate understanding of place value. He observed that students could often write numerals correctly but could not explain why they were written that way. Children could identify and name place values, but they could not master the place value system as a whole.

Work by Ball included a study with nineteen pre-service elementary and secondary teachers. She stated that "Since place value is a fundamental idea and since pupils often find it difficult, it seemed a critical area of prospective teachers' knowledge to investigate" (1988, p. 90). These pre-service teachers were given a number of problems to 'debug' or find and explain the error in children's mathematics problems. Her conclusion was that these prospective teachers did not have an explicit understanding of place value. They were able to compute properly, but they were unable to articulate the underlying concepts of the problems.

Numeration Systems

Historical numeration systems and base number systems are traditionally part of the elementary education major curriculum. Textbooks provide information and practice for systems such as the Babylonian (base 60), Mayan (modified base 20), and other bases. The rationale for this topic being included in the elementary curriculum is typically that studying these systems help students better understand our base ten.

Authors Cathcart, Pothier, Vance, and Bezuk state that we study ancient numeration systems "with a view to developing a better understanding and appreciation of our own system" (2000, p. 124).

Research involving numeration systems has taken many forms. Hamilton's classroom work in the 1960s focused on college student understanding of a new 'Make-Believe Arithmetic.' This creative number system used inventive symbols for a base six system. He found that the concept of base was difficult for students to understand. The use of Arabic numerals in a number system other than base ten was confusing for the students (1961). Hamilton created a new system of base six using new symbols and new words for the numbers. The class worked on addition and multiplication tables and even constructed a ruler for measurement. With these new symbols students were successful in learning and were able to transfer their understanding to base ten.

Sawada and Atkinson (1981) replicated Hamilton's study with a nondecimal invented base five numeration system. They found that their pre-service teachers gained empathy for the difficulties children have in understanding base ten numeration. "Students think they know everything there is to know about the counting numbers, the names they say, and the symbols we write, when …they have little or no insight into the system" (p. 367). These researchers stated that many students learned our number system by rote. They concluded that an intensive study of a single base system using non-digit symbols had value in pre-service teacher education.

Casebeer examined teaching styles for introducing place value systems to pre-service teachers in 1967. Assuming a value to incorporating base numeration system content into the elementary education major mathematics curriculum, Casebeer tested two methods of instruction for two weeks in his mathematics classes for pre-service elementary teachers. He developed programmed materials to compare two sequences for introducing base numeration systems. One sequence was based on generalizing our base ten concept to other bases, the more traditional approach. The other sequence was based on sets and grouping of sets. Both sequences were used in all four classes studied, half the students receiving instruction in the sequence one method and then sequence two method and the other half receiving instruction in the sequence two method and then sequence one. He found no differences in student performance between the order of the two teaching techniques. Instruction for this research combined both methods, typically beginning with the traditional approach to establishing a connection to base ten and then using manipulatives to discuss sets.

Research conducted by Haukebo (1967) investigated sixty-two elementary pre-service teachers in two mathematics classes and their study of numeration systems. He used three different bases with these future teachers over a three week period. Haukebo's hypothesis was that no differences in base-ten understanding would exist between the group receiving instruction in base-ten and the group receiving instruction in base numeration systems as measured by his test of arithmetic understanding. A fifty-two problem test was constructed to test for arithmetic understanding. Problems involved simple computation, place value problems, and base problems. He concluded that although every group of students increased their arithmetic understanding, differences between the two groups were not significant.

In 1972 Skipper studied various teaching methods for the study of numeration systems by pre-service teachers. He used three groups of students; one employing two sets of manipulatives in the classroom, one using one set of manipulatives in the classroom, and a lecture only group. Approximately fifty students participated in each group. Tests for knowledge used Skipper's Test on Numeration System (TONS). This was strictly a base computation test using problems like 0.25 = ______ $_{twelve}$ (p. 158) and 103 $_{five}$ + 244 $_{five}$ = ______ (p. 153). His study was inconclusive on the connection of base numeration systems and better understanding of the decimal system. Even so, as a result of his research, Skipper wrote that he thought pre-service elementary teachers could learn base systems and recommended that pre-service teachers be exposed to more coursework in numeration systems (1972).

Hungerford experimented with base five instruction in his college classroom in 1992. He found that his students were frustrated and skeptical about this newly constructed number system, but he received many positive comments about the lessons at the end of the semester. "The new arithmetic forces students to come to grips with place value. Even though the mechanics look much the same as before, they must think about what's going on and understand how it really works" (p. 1).

Zazkis and Khoury (1994) worked with pre-service elementary teachers on place value and non-decimal fractions. Twenty students participated. Students were asked to calculate addition and subtraction problems in other bases and then convert non-decimals to base ten. An example of a problem and its incorrect answer follow:

$$\begin{array}{r} 2.23_{four} \\ \underline{+\ \ 3.33_{four}} \\ 21.31_{four} \end{array}$$

The correct answer should be 12.22 $_{four}$ (p. 204).

Students were interviewed about their answers to analyze their paper and pencil responses. The researchers concluded that "pre-service teachers' constructions of place value number systems are fragile and incomplete" (p. 222). A significant number of the pre-service teachers were able to perform addition and subtraction calculations but could not explain their work and identify the proper place value of the digits.

In 2003, a research study conducted by McClain was published. She developed a sequence of problems involving a fictional candy factory. Packaging for this factory used base eight mathematics. Candies were packaged eight to a roll and eight rolls to a box. Problems relating to this packaging were given to the class of twenty-four elementary pre-service teachers who served as subjects in the study. One problem asked students to determine how many candies were in one box, three rolls, and two pieces. Many pre-service teachers initially needed to draw and count individual pieces but later devised more simplified figures as their understanding increased. At the beginning of the experiment, McClain stated that "their [pre-service teachers] understandings of both place value and multidigit addition and subtraction were very superficial and grounded in rules for manipulating algorithms" (p. 289). At the end of the project, she reported relative proficiency in the tasks.

In Farro-Lynd's (2003) research she wrote that " number base work reinforces many of the concepts and procedures learned in base ten such as place value, regrouping, renaming, and computation algorithms" (p. 7). Her study involved 104 pre-service elementary teachers and place value misconceptions, using a twelve item written test and follow-up interviews. Students participated in the study while taking the second in a two course mathematics sequence where place value was taught in the first course. The test used a problem mixture of rounding whole numbers, base computations, comparing and ordering numbers, and renaming decimals. Sample problems include “Name a decimal between 0.1 and 0.11 (results were 45% correct), and round 99,721 to the nearest thousand (results were 93% correct)” (p. 124 & p. 126). The three base problem accuracies were 11%, 42%, and 17%. Overall, the 104 students averaged 37% correct on the test. She concluded that these pre-service teachers had weak conceptual understanding of place value structure, and she recommended further studies of this population and topic.

Research by Rusch (1997) involved 206 pre-service elementary teachers throughout the state of Texas. The focus of the study was on the influence of different teaching strategies on place value understanding. Three classes employed a constructivist approach to teaching, and six classes used the direct approach, or lecture. Constructivism employs the idea of students constructing their own learning. Teachers become a ‘guide on the side’ rather than a ‘sage on the stage.’ Rather than lecture, the instructors create situations in which students can discover mathematical concepts. Rusch and Hannigan developed the APVU to determine gains in students’ understanding of place value using the two different teaching approaches. Although gains were made by both groups, no significant difference in understanding between groups was found.

Hannigan (1998) followed Rusch’s work with a study of place value understanding and the effectiveness of using writing prompts to enhance that understanding. Four classes of approximately twenty-five students each participated. Two instructors each had a treatment class that received freewriting assignments developed by Hannigan and a control class that received no special assignments. Writing prompts invited students to define an algorithm, describe its logic, and give examples using the algorithm. Again, no difference in gains were found in place value understanding between the treatment group and the control group.

Conclusion

Pre-service elementary teachers are required to take a number of mathematics courses as part of their degree plan. In these courses textbooks have traditionally included bases and different place value systems in their curricula (Bassarear, 2005; Billstein, Libeskind, & Lott, 2007; Cathcart, Pothier,Vance, & Bezuk, 2000). The generally accepted reasons for including these topics are twofold: 1) to acquire an explicit understanding of place value, and 2) to gain perspective into elementary students' difficulties in learning our number system.

Does studying base numeration systems help pre-service teachers build on their understanding of place value? Place value is a major component of the Hindu-Arabic numeration system. In the early 1960s Dienes stressed the essential nature of place value in communicating the meaning of numbers. The National Council of Teachers of Mathematics promoted the importance of students' learning place value concepts in its first *Standards* (1989) within the strand of number sense and numeration, and it continues to emphasize this topic.

Research has shown a connection between place value understanding and mathematical competency. "Knowledge of place value has great implications for success in arithmetic tasks… Therefore, care should be taken that students develop a meaningful understanding of numbers" (Cathcart, Pothier,Vance, & Bezuk, 2000, p. 106). Yet elementary students do not have an adequate understanding of place value. Studies by

Fuson (1990a, 1990b), Kamii (1985), and Ashlock (1990) found primary student knowledge of place value to be lacking.

Based on the findings of previous studies, is this lack of understanding because elementary teachers have a poor understanding of the concepts themselves? Ball (1988, 1990), McDiarmid and Wilson (1992), and Zazkis and Khoury (1994) concluded that elementary and pre-service teachers knew facts and could memorize algorithms, but did not understand the underlying mathematical processes. This lack of place value knowledge disrupted the teaching and learning of number concept development and computation.

Research has been conducted on the place value understanding of elementary pre-service teachers (Hannigan, 1998; Rusch, 1997). Research has been conducted focusing on numeration systems and elementary pre-service teachers' understanding (Casebeer, 1967; Farro-Lynd, 2003; Skipper, 1972). Yet no research has been found that attempted to connect numeration systems and place value understanding of elementary pre-service teachers. The researcher found no studies utilizing the APVU other than the work by its two authors.

Zazkis (1999) wrote that pre-service teacher instruction should "provide them with experiences and challenges that re-examine and enrich their understanding of mathematics and its pedagogy" (p. 650). Currently, numeration systems are an accepted part of the elementary pre-service teacher mathematics curriculum. With today's call for highly qualified educators, all teacher educators should examine their curriculum to assure rigorous content. This study attempts to add to the dialogue on the mathematical preparation and required curriculum content established for elementary pre-service teachers by investigating the study of numeration systems and their influence on place value understanding.

CHAPTER 3

METHODOLOGY

This research was a quantitative quasi-experimental/control group design study. The objective was to compare gains of explicit place value understanding possessed by two classes of elementary pre-service teachers taking a required mathematics course. The course curriculum included a section on base-numeration systems. The independent variable in the study was the instruction, or withholding of instruction, of base-numeration systems. The dependent variable was growth in explicit place value understanding as measured by the Assessment of Place Value Understanding (APVU) instrument.

The four research questions guiding this study were:

1. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course?
2. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course?
3. Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester?
4. Is there a significant difference in gains made in explicit place value understanding

between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)?

Participants

All of the participants in the study were undergraduate college students pursuing a career in elementary education. Normally the students taking the courses used in this study are sophomores and juniors. The researcher presumed that all students had some pre-existing knowledge about the place value concepts and representation in our base-ten numeration system. The two classes used in the study were chosen by convenience.

Initially, a number of universities expressed interest in helping with this research. Many schools were eliminated due to the semester scheduling of the mathematics content courses. Other schools did not follow through on their first commitment. Only one instructor other than the researcher agreed to aid in this study. This challenge of research as outlined in an article by Heid et al. (2006) reinforces the concern that linking research and practice is a problem. Even with the need for more school-based research, teachers are reluctant to allow their classrooms to be laboratories for experimental, unproven content and changes in pedagogy. Conversely, control groups are difficult to assign if teachers perceive the intervention of the researcher to be impractical, unusable, or intrusive.

One class of approximately 35 students was taught by an instructor at a regional, medium- sized public university (experimental group). One class of approximately 25 students was taught by the researcher at a small private university (control group). The two colleges are in close geographic proximity to each other in the same suburban area. The instructors were both experienced teachers with many years of college teaching experience, and they were friends and fellow students in the same doctoral program. All students in the two classes were invited to participate in the research project.

Comparison of Classes

Both mathematics courses were components of the required degree plan at their respective universities and were designed specifically for elementary education majors with enrollments limited to that group of majors. The curriculum in both courses used in the study normally includes sections on numeration systems. At the private university, the course was usually taken as the second or third course in a four- course sequence. At the public university, the course could be taken second, third, or fourth in a similar four-course required sequence. Neither school had a specific mathematics methods course for elementary education majors.

Since two different texts were used in the courses at the two schools, content was compared. Table 1 gives the number of weeks that the major course content at the two schools was presented as well as the order in which the material was taught. Although material in the two courses was not identical, the basic place value topics were similar as well as the focus objectives on numeration systems. Table 2 details the specific numeration system content taught by each instruction during the semester.

The instructional style of the professors was not a component of this research. Each instructor was free to use any method of her choice, including lectures, modeling, and videos. However, it was known to the researcher that both instructors used a constructivist approach and employed many manipulatives in their classrooms. Hands-on activities include work with base-ten and multi-base blocks.

Table 1

Comparison of Course Content

	Numeration system instruction		
	Without (Control group)	With (Experimental group)	
Topic	Number of Weeks	Topic	Number of Weeks
Problem solving	One	Problem solving	One
Sets	One	Number theory	Two
Pre-number skills	One-half	Sets	One & one-half
Numeration systems	One & one-half	Pre-number skills	One-half
Egyptian, Roman, Hindu-Arabic, time			
4 basic operations	Two	Numeration systems	Two & one-half
Integers	One	4 basic operations	Three
Number theory	One & one-half		
Rationals	One & one-half		
Decimals	One		
Base numeration systems	One		

Table 2

Detail Numeration System Content

Numeration system instruction	
Without	With
Hindu-Arabic, base-ten	Egyptian
Egyptian	Roman
Roman	Chinese
Time	Greek
Time delay for study	Hindu-Arabic, base-ten
Babylonian	Babylonian
Mayan	Mayan
Bases (multi – 2, 3, 4, 5, others)	Bases (multi – 2, 3, 4, 5, others)

Note: Content in bold taught after the posttest for this research.

The researcher (control group) delayed teaching the base numeration system content section of the course curriculum while the study was in place. Only simple information about these numeration systems was introduced. In particular, notation of the base systems was introduced so that students' work on the second test would better reflect their understanding of place value concepts rather than their lack of knowledge about the problem structure. For example, the researcher explained to her students that 234_{eight} indicated a number in the base eight numeration system, not a Hindu-Arabic base ten number. Base system instruction was given at the end of the semester after the test was administered for the second time and the study had been completed. At that time the researcher taught the concepts of base numeration systems, including addition, subtraction, and multiplication. Sections from the syllabus delayed for the study were taught using textbook material.

The instructor (experimental group) taught base numeration systems as usual which involved extending the use of base-ten blocks for the Hindu-Arabic system to multi-base blocks for the different number bases. Babylonian and Mayan systems were also studied to show the variety of base numeration systems in other contexts. Class notes state:

> As early numeration systems began to use grouping they became conceptually harder, and at the same time easier to use on a mechanical basis. In our Hindu-Arabic system we group by tens. Children need experience grouping by twos, threes, . . . all the way to tens. The idea of an exchange point is the key to understanding place value. Also, grouping using smaller numbers provides more practice than waiting until ten objects have been collected. (Parrott, 2007, p. 89).

The following three pages were taken from the instructor's class notes for the experimental group to provide some insight as to the method of teaching of these numeration systems and the instructional approaches employed.

Numerals in the Mayan System are expressed vertically. The place value at the bottom of the column is 1.

Ex: Write the Mayan numeral as a Hindu-Arabic numeral.

Solution The given Mayan numeral has four places. From top to bottom, the place values are 7200. 360, 20, and 1. Represent the numeral in each row as a familiar Hindu-Arabic numeral using Table 4.2. Multiply each Hindu-Arabic numeral by its respective place value. Then find the sum of these products.

Mayan numeral		Hindu-Arabic numeral		Place value		
	=	14	×	7200	=	100.800
	=	0	×	360	=	0
	=	7	×	20	=	140
	=	12	×	1	=	12
						100.952

The sum on the right indicates that the given Mayan numeral is 100,952 as a Hindu-Arabic numeral.

Changing to Base Ten:

To change a numeral in a base other than ten to a base ten numeral:

1. Find the place value for each digit in the numeral.
2. Multiply each digit in the numeral by its respective place value.
3. Find the sum of the products in step 2.

Digit Symbols and Place Values in Various Bases

Base	Digit Symbols	Place Values
two	0, 1	$\ldots, 2^4, 2^3, 2^2, 2^1, 1$
three	0, 1, 2	$\ldots, 3^4, 3^3, 3^2, 3^1, 1$
four	0, 1, 2, 3	$\ldots, 4^4, 4^3, 4^2, 4^1, 1$
five	0, 1, 2, 3, 4	$\ldots, 5^4, 5^3, 5^2, 5^1, 1$
six	0, 1, 2, 3, 4, 5	$\ldots, 6^4, 6^3, 6^2, 6^1, 1$
seven	0, 1, 2, 3, 4, 5, 6	$\ldots, 7^4, 7^3, 7^2, 7^1, 1$
eight	0, 1, 2, 3, 4, 5, 6, 7	$\ldots, 8^4, 8^3, 8^2, 8^1, 1$
nine	0, 1, 2, 3, 4, 5, 6, 7, 8	$\ldots, 9^4, 9^3, 9^2, 9^1, 1$
ten	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	$\ldots, 10^4, 10^3, 10^2, 10^1, 1$

Base Ten	Base Five
0	0
1	1
2	2
3	3
4	4
5	10
6	11
7	12
8	13
9	14
10	20
11	21
12	22
13	23
14	24
15	30
16	31
17	32
18	33
19	34
20	40

Converting from Base Ten to Other Bases:

To convert a base ten numeral to a numeral in a base other than ten, we need to find how many groups of each place value are contained in the base ten numeral.

When the base ten numeral consists of one or two digits, we can do this mentally.

Example: Suppose we want to convert the base ten numeral, 6, to a base four numeral.

The place values in base four are:

$$\ldots 4^3, \; 4^2, \; 4, \; 1$$

The place values that are less than 6 are 4 and 1. We can express 6 as one group of four and two ones left over:

$$6_{\text{ten}} = (1 \times 4) + (2 \times 1) = 12_{\text{four}}$$

(Parrott, 2007, pp. 115, 118,119).

Both instructors taught traditional place value concepts, including expanded notation, and the Hindu-Arabic system (our base-ten system). "Place value assigns a value to a digit depending on its placement in a numeral" (Billstein, Libeskind, & Lott, 2007, p. 155). An example of a place value question from the control group text would be to give the place value of the underlined numeral: 827, <u>3</u>67 (Billstein, Libeskind, & Lott, 2007, p. 166). An example of a problem from the experimental group class: "Consider the numbers 40 and 400 in the Hindu-Arabic numeration system. What does the 4 represent in each number?" (Angel & Porter, 1997, p. 180). Expanded notation is illustrated by the number 1234 = (1•1000) + (2•100) + (3•10) + (4•1). Both texts ask students to write numerals in expanded form and the reverse. Both instructors used materials such as the base ten blocks, place value charts, chip trading, and Cuisenaire rods to reinforce base-ten concept development.

Data Collection

Data collection consisted of three parts:

1. Collection of demographic information on all participants.
2. Administration of Form A of the Assessment of Place Value Understanding (APVU) instrument at the beginning of the semester before numeration system concepts and place value were taught.
3. Administration of the same form of the APVU near the end of the semester after numeration system concepts and place value were/were not taught in the individual class sections.

The APVU instruments were given as part of the normal classroom requirements. Anonymous demographic information was collected, as were the average ACT scores of the two groups which were compared to check for equivalence between the groups. Consent to use the scores was included in consent forms acknowledging subjects' participation in the project.

Students were allowed approximately 45 minutes to take each one of the two assessments at each location. This time allotment was similar to the time given in the test developers' previous studies. Administration of both the first test and second test was scheduled as close together as possible at the two schools to maintain similar total class time at each location.

The first administration of the APVU test was given to each group at about the third week of the semester. This was later than planned due to inclement weather, but no place value concepts had been taught at that point in the semester. The second administration of the APVU test was given by the researcher during week fourteen of the semester, allowing enough time for the base numeration systems content to have been taught to the control group before the end of the semester. The instructor of the experimental group administered the second test during the last week of the semester.

Instrument

The Assessment of Place Value Understanding (APVU, pretest and parallel posttest) was developed by Rusch and Hannigan in 1997 as part of their Ph.D. work at the University of Texas. For this study only the pre-test form of the test was used. This choice was made due to the original authors' lack of confidence and documentation regarding the comparability of the initial two forms. After its introductory use, Hannigan (1998), one of the APVU authors, expressed concern in her dissertation about some problems which arose indicating that the two forms may not have been truly parallel.

One minor change was made in the original test by this researcher. For questions 3, 8, and 13, relating to number-base problems, the researcher asked the students to be more specific about describing their processes in calculating the solutions. No change was made in the rubrics for these problems. This assessment was based on a matrix of place value grouping schemes shown in Table 3 from Rusch.

The APVU was piloted multiple times as the authors continued to revise and refine the questions. According to Rusch, “It was necessary to pilot the assessment several times to ensure clarity in the communication of each question, validity of the questions, and to control for the time required to complete the assessment. Furthermore, rubrics for assessing the responses to the assessment questions were developed and carefully tested for clarity and inter-rater reliability” (1997, p. 51).

The instrument was first piloted orally with sixteen elementary pre-service teachers at the University of Texas. Content was analyzed, and a second handwritten test was given to the same sixteen participants to develop a time frame for giving the test in a paper-pencil format. A third pilot was conducted with nineteen students at Austin Community College and 265 University of Texas students. A stratified sample of 45 tests was selected for use in the development and analysis of the rubrics. Five outside evaluators scored the test responses to check the reliability of the assessment rubrics. During the course of their research, the authors, Rusch and Hannigan, used the APVU for thirteen classes and 312 students (Rusch, 1997).

Table 3

Matrix of Explicit Place Value Understanding

	Systematic	Non-Systematic
	Systematic grouping schemes are those in which the number of items required to create one group of the next place value remains constant.	Non-systematic grouping schemes are those in which the number of items required to create one group of the next place value varies from place to place.
Familiar A familiar grouping scheme is one which most preservice teachers have come across in both the traditional mathematics curriculum and in their daily living routines. …	The Base 10 System Metric Measurement	Time seconds, minutes, hours, days, weeks, . . Imperial Measurement inches, feet, yards, cups, pints, quarts, gallons, …
Unfamiliar An unfamiliar grouping scheme is one which uses a place value structure, but which preservice teachers may not have come across in either their school years or their daily life.	Base n Systems (other than base 10) Base 2, Base 5, Base 16	Foreign Coinage British pence, shilling, pound, …

(1997, p. 38)

The tests used in this study were graded using the Rusch - Hannigan rubrics. Rater reliability was checked by analyzing scoring differences between the two instructors on a small sample of tests. Twenty percent of the first test was scored by the instructors. Results of this analysis are discussed in Chapter 4.

The rubrics employ a 1 – 5 scale. The numbers in the scale indicate the level of student understanding based on the descriptions found.

1. Algorithmic: Knowledge of how to manipulate symbols to get an "answer." This level of mathematical knowledge does not necessarily imply understanding of why the algorithmic steps make sense.
2. Tacit: Algorithmic knowledge with some intuitive understanding of the logical foundation from which the concept or algorithm emerges. This understanding is, however, somewhat vague and as a result it is difficult for the individual to articulate the logic which brings meaning to the concept.
3. Explicit: Precise understanding of the concept being addressed which can be clearly articulated and convincingly justified. Explicit understanding includes a clear understanding of the connections between the concept being addressed and related concepts, as well as an ability to articulate the logical development of related algorithmic procedures and generalizations.
4. Pedagogical (Self and Peers): Knowledge of how they (self) and individuals of similar age and experience (peers) build mathematical understanding. Able to thoughtfully choose pedagogical tools and strategies and are able to use language effectively to teach peers.

5. Pedagogical(Children): Knowledge of how children build mathematical understanding. Able to thoughtfully choose pedagogical tools and strategies and are able to use language effectively to teach children (Rusch, 1997, p. 34).

Data Analysis

Using Rusch's and Hannigan's work as a guide, the researcher used simple means and standard deviations for a total score to partially answer questions one and two. Question one: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course? Question two: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course? These statistics were compared using an independent sample t-test. A univariate analysis of variance was used to compare means in the four cells of the matrix (familiar systematic, familiar non-systematic, unfamiliar systematic, and unfamiliar non-systematic). Boxplots were constructed to give more specific information about the data. Correlations were calculated to examine possible connections within the data.

To answer questions three and four, again total mean scores were compared – pre versus post -means in the four cells, and means for rubric scores were compared. Question three: Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester? Question four: Is there a significant difference in gains made in explicit place value understanding between the group that had numeration system instruction (experimental group) and the group that did not have the numeration system instruction (control group)? Analysis was done with a split-plot ANOVA for question three. Gains were calculated for question four, and then an independent t-test was utilized to discover significant differences. Of particular interest to the researcher were comparisons in the unfamiliar systematic grouping cell, as this in particular focused on non-decimal base systems, an important facet of this study. A modified Bonferroni correction was used to determine α. Normally used at the 0.05 level, due to the large number of dependent variables, α was recalculated to 0.05/11 = 0.0045.

No qualitative data was collected, although many test questions asked the elementary pre-service teachers to explain their answers. An example was question one. This question asked subjects to analyze an elementary student's error pattern. As part of the question, the elementary pre-service teacher is asked to "Briefly describe your assessment of what Caroline does not understand" (Rusch, 1997, p. 124). These comments were helpful as descriptive data to accompany the quantitative results.

Conclusion

This research was designed to assess place value understanding of pre-service elementary teachers. The APVU was developed in 1997 just for this population; thus, it was chosen as the instrument to be used. Due to the lack of teacher commitment, only two classes participated in the project. These classes were compared by teacher, by content, and by student, with the help of a demographic sheet. The researcher used one form of the APVU as a pretest and posttest during the semester. Timing for these pretests and posttests was coordinated between the instructors of both groups. The assessment tasks were scored and analyzed by the researcher in three categories: total, cell or grouping, and rubric. Comparisons were drawn on pretest and posttest means.

To validate earlier literature and research, the hypothesis of this study is that the class receiving normal instruction in non-base-ten numeration systems (experimental group) will show a significant gain in scores, first exam to second, when compared to the without instruction class of students (control group). Of particular interest to the researcher are the gains for both groups in the unfamiliar systematic cell which contained the base numeration systems problems. Chapter Four will use a number of statistical tests and graphs to analyze the pretest and posttest scores of the two groups. These results can then be used to answer the research questions. Conclusions and recommendations will be stated in Chapter Five.

CHAPTER 4

FINDINGS

This study investigated the influence of base numeration instruction on place value understanding for elementary pre-service education majors. Two university classes of elementary pre-service teachers were used to test the hypothesis that base numeration system instruction would increase the conceptual understanding of place value for the participants. Using a sample of convenience, the researcher and another instructor administered a pretest to each class near the beginning of the semester. During the semester the researcher's class (control group) did not receive base system instruction. This instruction was incorporated at the end of the semester after the study was completed. During the semester the other instructor taught her class (experimental group) following the usual syllabus, including base system content. At the end of the semester the same test was given to both groups.

The instrument used was the Assessment of Place Value Understanding (APVU). This thirteen item test was developed by Rusch (1997) and Hannigan (1998) at the University of Texas to specifically test place value understandingof pre-service elementary teachers. The assessment can be found in Appendix A of this study.

Four questions guided this research:

1. What is the level of explicit place value understanding that the elementary pre-service

teachers participating in the study bring with them to their required mathematics content course?

2. What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course?
3. Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester?
4. Is there a significant difference in gains made in explicit place value understanding between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)?

Demographic Information

During the semester of research, two classes of elementary education major students were invited to participate. One, at a private university, was taught by the researcher. One, at a public university, was taught by an instructor known to the researcher. Both universities are located in the same metropolitan area of a southwestern United States city.

Each student was required to take the Assessment of Place Value Understanding (APVU) twice, once at the beginning of the semester and again near the end of the semester. Students were then asked by the researcher to use their test results for this study. Only scores for students who completed these three components were used in this research. This reduced the sizes of the samples to 23 for the control group and 32 for the experimental group.

These 55 students then completed a demographic information sheet. The main purpose for this additional request was to give the researcher some perspective about the students and to check for similarity of groups. In particular, ACT scores were solicited. This self-reported data is summarized in Table 4 Due to the small sample sizes, equivalency testing for the classes was not attempted.

Table 4

ACT Mean Scores

	Numeration system instruction	
	Without	With
Number of respondents	8	14
Percent of respondents	35%	44%
Mean	22.75	19.71

Other demographic information revealed that all students were female. Only two – at the public university – were part-time students; the other 53 reported full-time student status. All students were taking their second or third college mathematics course. A summary of other demographic information is show in Table 5.

Rater Reliability

After data collection, a twenty percent sample of the pretests was scored by the researcher and the instructor. This was implemented to assess inter-rater reliability and measurement accuracy. Discussion of the scores between the raters then allowed for refinement of rubric meaning. The rubrics established by the authors of the APVU utilize

Table 5

Demographic Information

	Numeration system instruction	
	Without	With
Average age	19.6	29.4
Range	18 - 22	21 - 49
Average GPA	3.34	3.28
Range	2 - 4	1.8 - 3.86

a one to five point scale to rate student understanding with : (1) algorithmic understanding; (2) tacit understanding; (3) explicit understanding; (4) self and peer pedagogical understanding; and (5) child pedagogical understanding. The data for this sample is summarized in Table 6. These results paralleled results previously obtained by the authors of the APVU; thus, it was determined that there was sufficient consistency to allow only the researcher to score all tasks and tests.

Table 6

APVU Sample Scoring

	Number of items	Percent of items
Differ by 0	322	86.6
Differ by 1	37	9.9
Differ by 2	10	2.7
Differ by 3	3	0.8
Differ by 4	0	0

Question One

The first research question was: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course? This question was initially analyzed for all tasks for both the without instruction (control group) and instruction students (experimental group). A sample mean and standard deviation was calculated for each group's scores. This information is summarized in Table 7.

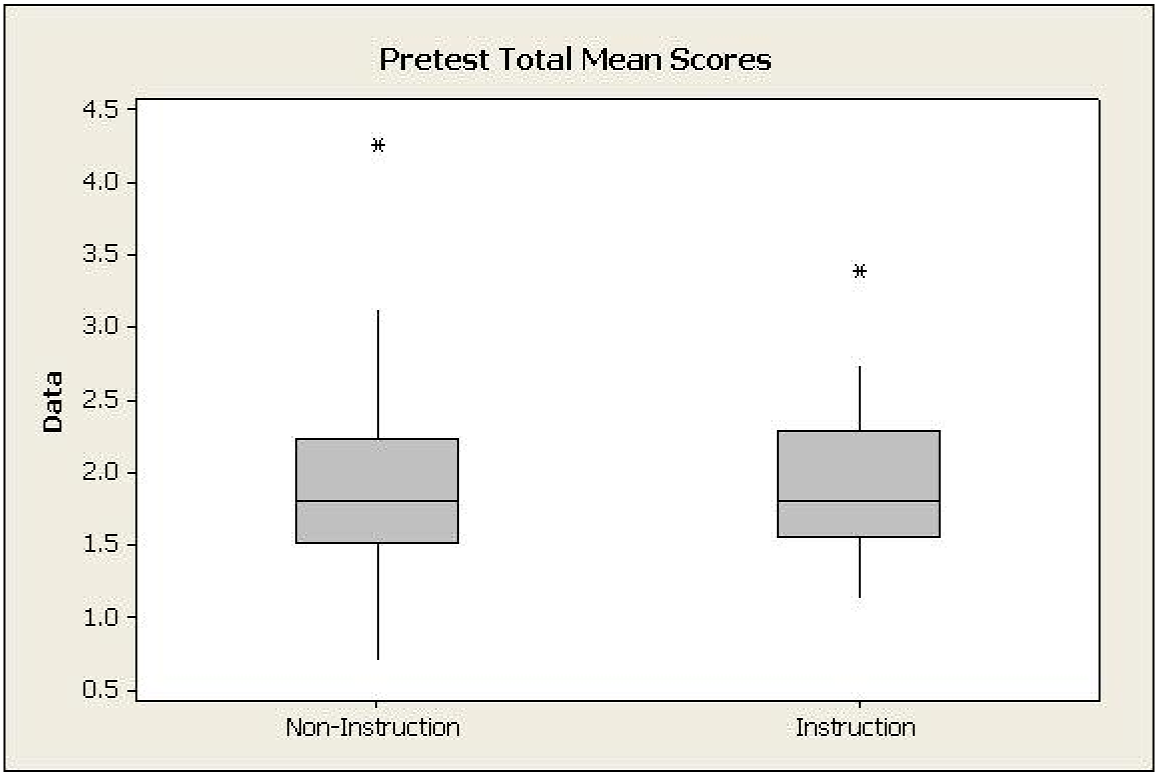
Table 7

All Tasks Pretest Item Mean Scores

	Numeration system instruction	
	Without	With
Number of participants	23	32
Mean	1.95	1.95
Standard deviation	0.73	0.49

These scores indicated that the students achieved the same baseline results. No significant difference was found in the means. The boxplots in Figure 1 show somewhat similar distributions with similar medians of 1.81 and 1.845. As the standard deviations indicate, the non-instruction students' scores varied more. A score close to two on the scale of mathematical understanding used in this research indicates some intuitive knowledge of the concept foundation. However, this understanding is somewhat vague, and the student may have difficulty articulating the logic behind their work.

Figure 1



To further define scores and place value understanding, the pretest scores were analyzed by cell. Matrix cell definitions were presented in Chapter 3. A table identifying the relationship of cell content and problems from the APVU is found in Appendix B.

Pretest cell scores for both sites are presented in Table 8. Means and standard deviations were calculated for each cell and analyzed using a univariate analysis of variance to determine significant differences in the baseline scores of the two groups. Best scores were achieved in the familiar systematic cell. These problems focused on the vocabulary of place value understanding and the ability of students to evaluate elementary student work. Scores approaching three on the rubric scale indicate more precise understanding of the concepts.

Table 8

Pretest Cell Means

	Numeration system instruction	
	Without	With
Familiar	2.60	2.43
systematic	(.87)	(.70)
Familiar	2.26	2.40
non-systematic	(1.20)	(1.16)
Unfamiliar	1.30	1.32
systematic	(.75)	(.52)
Unfamiliar	2.29	2.38
non-systematic	(1.34)	(1.00)

Note: Standard deviation in parentheses, * $p < .0045$

The familiar non-systematic cell problems dealt with time problems. Scores around two again indicated some intuitive knowledge; however, the large standard deviations showed a broad range of scores (and therefore knowledge) among the two groups. Lowest scores were discovered in the unfamiliar systematic cell. Tasks here required the students to apply place value concepts in different base numeration systems. Some problems were typical base computation problems, while others required creativity and sophistication to interpret the task. A score of one on the rubric indicates a basic knowledge of an algorithm, but does not imply further understanding. The unfamiliar non-systematic cell problems presented students with an unusual setting for place value concepts. Results here were similar to the familiar non-systematic cell, although higher standard deviations indicate a larger range of values. As with the "all tasks" scores, the cell analyses revealed similar baseline results. All cells produced no significant differences and homogeneity of variance.

To further explore the pretest scores, means and standard deviations were calculated for the tasks by rubric. The seven rubrics used are defined in Appendix C. A table showing the correlation of rubrics and problems can be found in Appendix D. Results for this rubric analysis are summarized in Table 9.

Highest scores were found with the error reproduction rubric. This rubric was utilized with question numbers 1 and 12 on the test – debugging elementary student work. A score of three, explicit understanding, indicates the students gave an inaccurate reproduction of the error, but there was some evidence of partial recognition of the error pattern.

No significant differences were found among the scores on any of the rubrics. Standard deviations were above one (or one level of understanding) for the without instruction students' Error Reproduction and both sets of students analysis of symbolic representation. This rubric was used with the foreign coinage problems, numbers 9 and 10. A level two, tacit understanding, for this rubric indicates the students used base ten techniques to arrive at an answer, not place value representation.

Thus, for question one of this research, students at both schools achieved similar baseline results. The overall averages of 1.95 showed almost a tacit level of understanding. The more specific numbers of the cells and rubrics showed a range of understanding of algorithmic (choice of correct representations) to explicit (error reproduction), with most cells and rubrics in the level 2 tacit range.

Table 9

Pretest Rubric Means

	Numeration system instruction	
Rubric	Without	With
Error reproduction	3.41	3.05
	(1.19)	(.92)
Depth of analysis	2.00	1.89
	(.83)	(.70)
Use of descriptive	1.73	1.66
language	(.68)	(.67)
Choice of correct	1.13	1.16
representations	(.34)	(.37)
Accurate	2.27	2.30
computation	(.78)	(.50)
Analysis of comp.	1.45	1.53
method	(.91)	(.59)
Analysis of sym.	2.04	2.31
representation	(1.42)	(1.18)

Note: Standard deviation in parentheses, * $p < .0045$

Question Two

The second research question was: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course? This question was initially analyzed for all tasks for the public and private university students. A sample mean and standard deviation was calculated for each school's student scores. This information is summarized in Table 10 .

Table 10

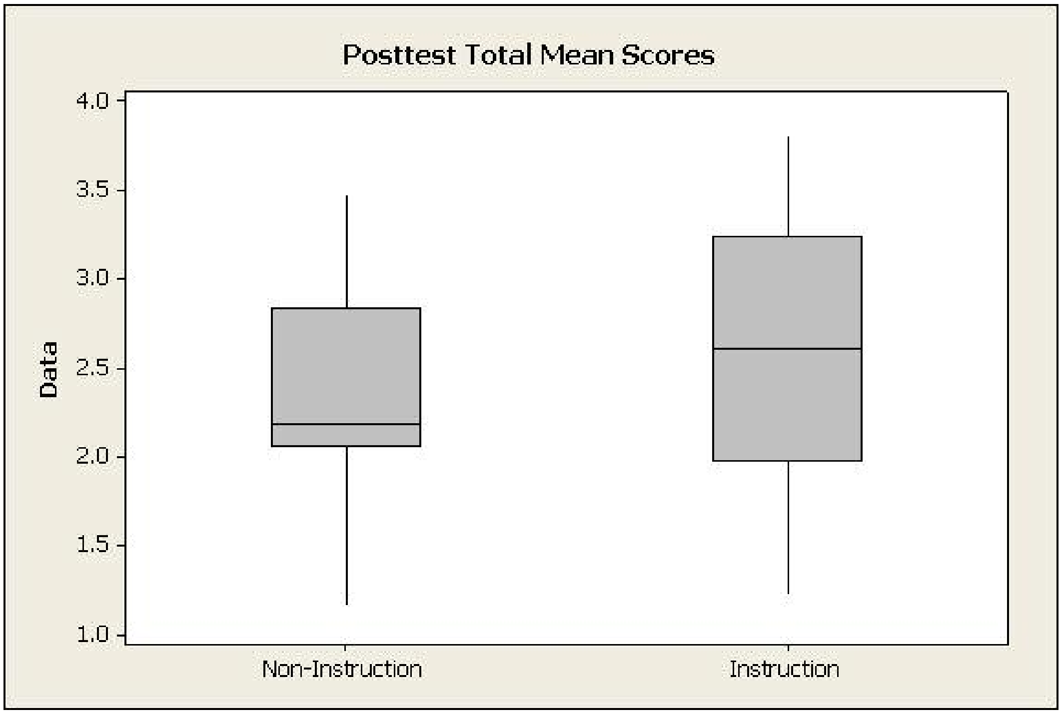
All Tasks Posttest Item Mean Scores

	Numeration system instruction	
	Without	With
Number of participants	23	32
Mean	2.35	2.59
Standard deviation	0.53	0.70

Similar analyses were conducted with the posttest data as with the pretest data. A comparison of the posttest means showed no significant difference ($p = 0.16$) in the level of explicit understanding of place value concepts. Again boxplots were constructed from the data to show median differences of 2.19 versus 2.61 in Figure 2. Scores were more evenly distributed within the instruction class of students.

To look for specific differences or levels of understanding, the posttest scores were analyzed by cell and by rubric, similar to the analyses of the pretest scores. The results are summarized in Tables 11 and 12.

Figure 2



Significant differences were found in the unfamiliar non-systematic cell. This grouping of problem topics included the other-than-base-ten base problems, numbers three, five, six, eight, eleven, and thirteen on the APVU. Some of these problems involved basic computation in the other bases; some involved a higher level of problem-solving as well as base place value knowledge to complete the questions.

The difference in scores was anticipated by the researcher. Prior to the posttest, the researcher's students (control group) received only instruction on base ten problems and place value concepts. The instructor's students (experimental group) had over two weeks of instruction with different base systems and had classroom experience with base computation.

Although analyses showed no significant differences in mean scores in the other three cells, the boxplots in Figures 3 - 6 show clear differences in distribution. In two cells, the familiar systematic cell dealing with children's error patterns and the familiar non-systemic cell of time problems, the without instruction students (control group) had a higher 25th, 50th, and 75th percentile.

Table 11

Posttest Cell Means

	Numeration system instruction: Without	Numeration system instruction: With
Familiar systematic	3.00 (.76)	2.80 (.80)
Familiar non-systematic	3.33 ** (1.02)	2.56 ** (1.05)
Unfamiliar systematic	1.53 * (.51)	2.47 * (.96)
Unfamiliar non-systematic	2.57 (.81)	2.60 (.88)

Note: Standard deviation in parentheses, * $p < .0045$, ** $p < .05$

Figure 3

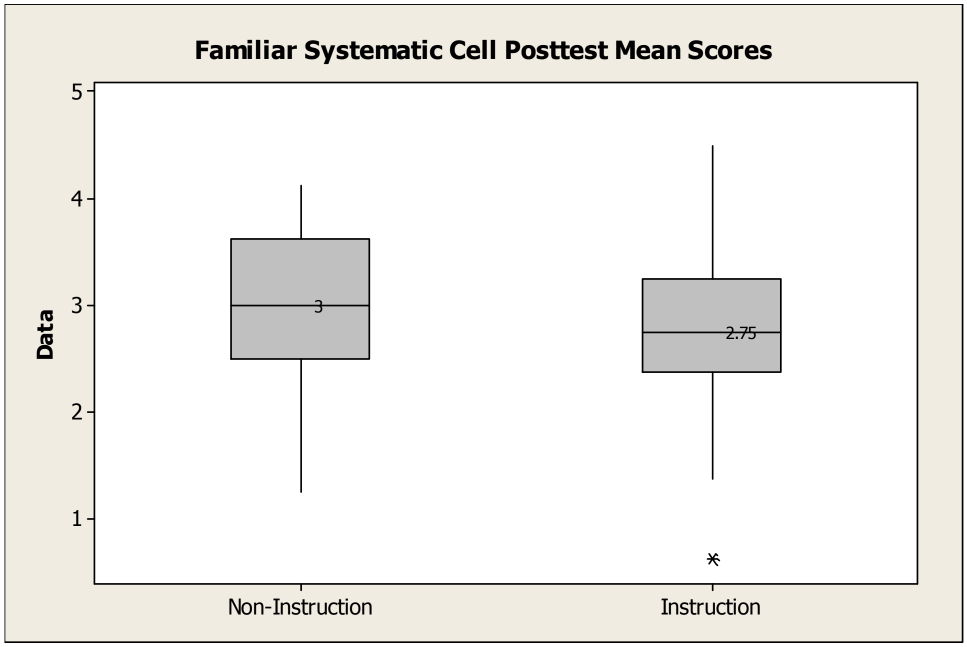


Figure 4

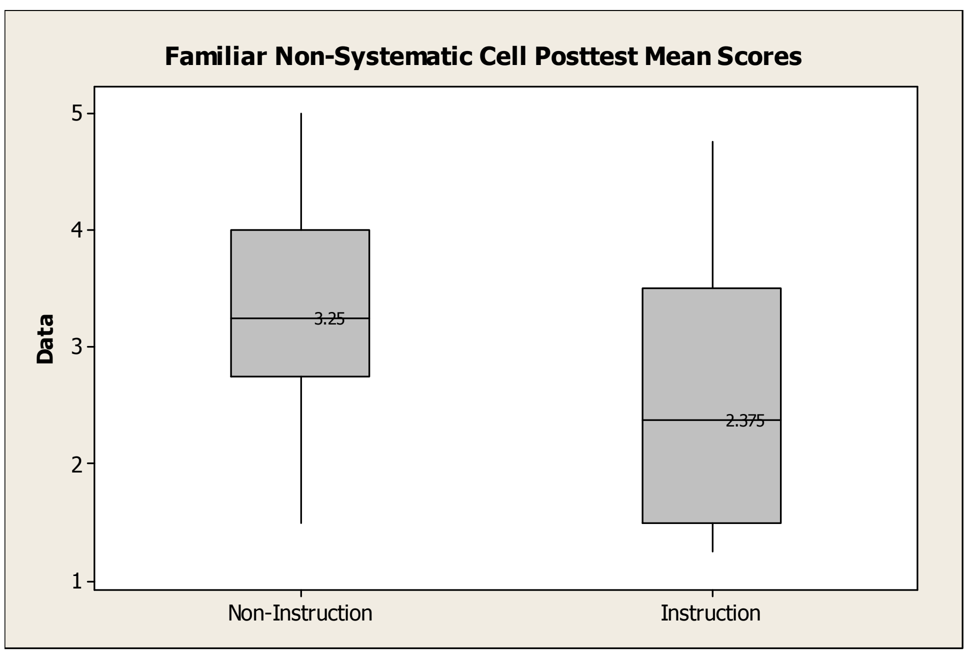


Figure 5

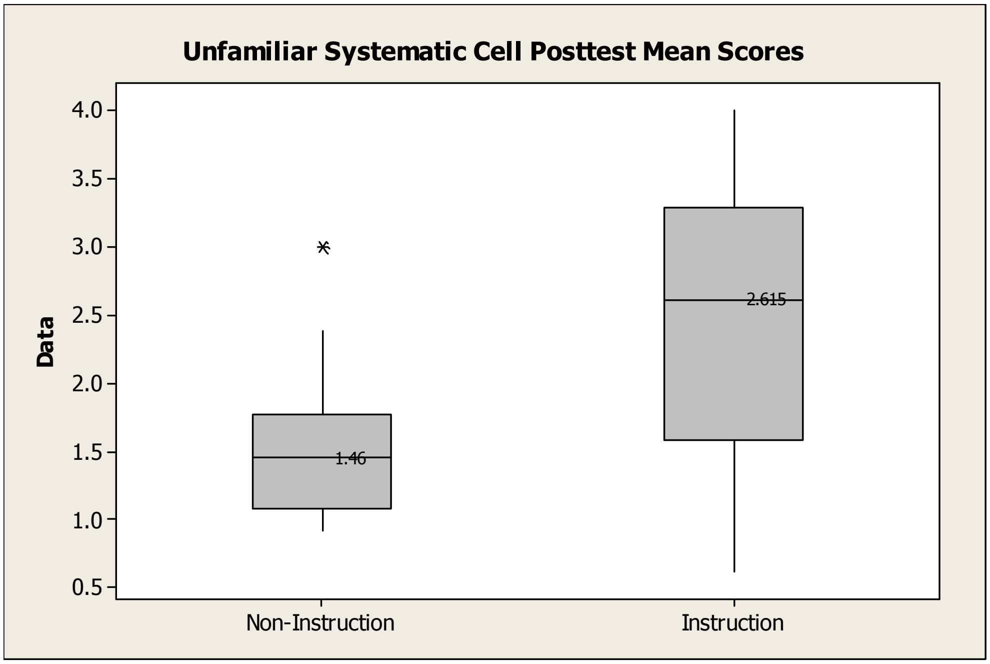
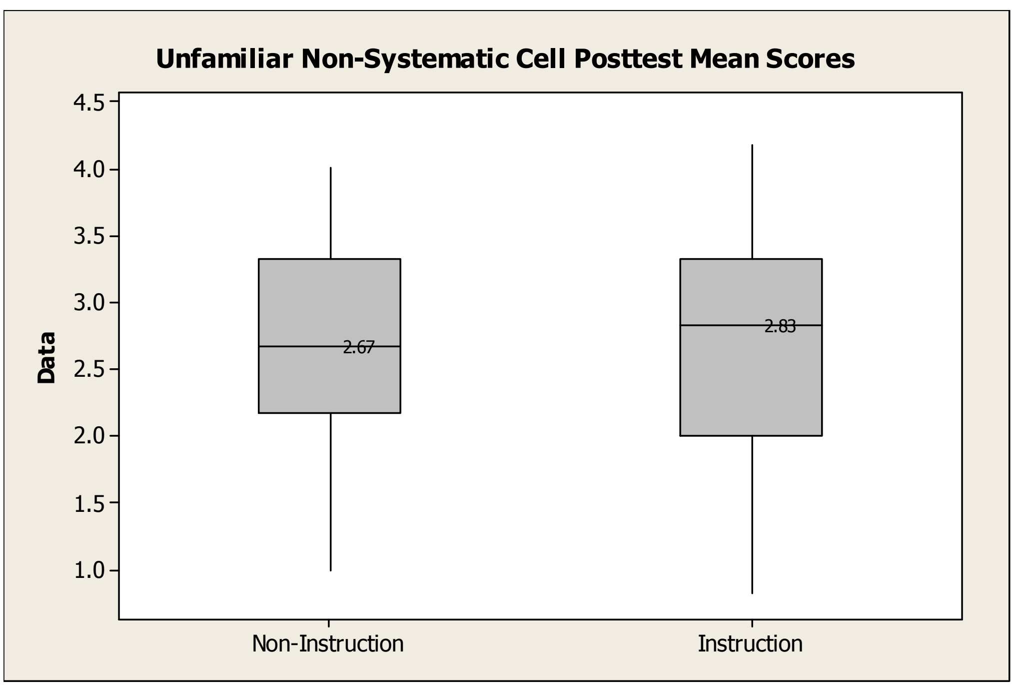


Figure 6



The analysis by rubric identified no areas of significant difference at the α = 0.0045 level. However, differences were apparent at the 0.05 level. These rubrics included: choice of correct representation, accurate computation, and analysis of computation method. Since all of these rubrics were primarily used for the unfamiliar systematic cell problems, it would follow that significance in these cells would bring significance in the rubrics. A table showing the correlation of rubrics and problems can be found in Appendix D.

Table 12

Posttest Rubric Means

	Numeration system instruction	
	Without	With
Error reproduction	3.87	3.33
	(1.11)	(.99)
Depth of analysis	2.32	2.11
	(.71)	(.64)
Use of descriptive	2.01	1.95
language	(.65)	(.69)
Choice of correct	1.04 **	1.25 **
representations	(.21)	(.44)
Accurate	2.64 **	3.10 **
computation	(.57)	(.86)
Analysis of comp.	2.00 **	2.58 **
method	(.63)	(.95)

	Within	With
Analysis of sym. representation	2.48 (.81)	2.53 (.98)

Note: Standard deviation in parentheses, * p < .0045, ** p < .05

Tables 13 and 14 show the correlations of posttest rubrics with each other. Significant correlations showing linear relationships exist between many of the rubrics. These results are consistent with the rubric/task correlation table found in Appendix D.

Table 13

Without Instruction Rubric Correlations

	DA	DL	AC	CM	SR	CR
ER	.558 **	.422 *	.357	.392	.175	.222
DA		.937 **	.530 **	.639 **	.538 **	.210
DL			.470 *	.589 **	.558 **	.081
AC				.888 **	.652 **	.222
CM					.581 **	.116
CR						.141

Note: * p < .05, ** p < .01

ER = Error Reproduction; DA = Depth of Analysis; DL = Use of Descriptive Language; AC = Accurate Computation; CM = Analysis of Computation Method; CR = Choice of Correct Representation; SR = Analysis of Symbolic Representation

Table 14

With Instruction Rubric Correlations

	DA	DL	AC	CM	SR	CR
ER	.730 **	.696**	.590**	.479**	.207	-.083
DA		.930 **	.531 **	.564 **	.040	.243
DL			.563**	.571 **	.105	.201
AC				.926 **	.531**	.055
CM					.433*	.104
CR						.244

Note: * p < .05, ** p < .01

ER = Error Reproduction; DA = Depth of Analysis; DL = Use of Descriptive Language; AC = Accurate Computation; CM = Analysis of Computation Method; CR = Choice of Correct Representation; SR = Analysis of Symbolic Representation

Question Three

The third research question was: Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester? To analyze the data, a repeated measure analysis of variance statistic was calculated. As with the previous two questions, data was compared by pretest and posttest across all tasks (in total), by cell, and then by rubric.

For the pretest and posttest all task means, no significant differences were found. This can be explained by re-examining the data discussion for question 2. Posttest averages for the without instruction students (control group) were higher in the cell for the time problems. Posttest averages for the instruction students (experimental group) were significantly higher in the cell for the base problems. Thus, these posttest differences contributed to no overall difference between pretest and posttest means. Table 15 summarizes these results.

An ANOVA performed on the pretest/posttest scores for each of the two classes did find differences in each class (p = 0.04 and p = 0.00, respectively). This showed there was some growth in the participant's place value understanding.

Table 15

Change in Understanding from Pretest to Posttest

	Numeration system instruction	
	Without	With
Pretest	1.95	1.95
Posttest	2.35	2.59

The repeated measures ANOVA indicated significant differences in the unfamiliar systematic cell. This demonstrated the significant effect of instruction on the pretest to posttest change of scores in the base numeration grouping of problems. These results are similar to the posttest analyses. In the unfamiliar cell, the instruction students increased their average scores by more than one level of understanding. Unfortunately, even their ending score of 2.47 indicates less than a precise understanding of the place value concepts. Table 16 provides the summary of analysis of cells.

Table 16

Change in Understanding from Pretest to Posttest by Matrix Cell

	Numeration system instruction	
	Without	With
Familiar systematic		
Pretest	2.60	2.43
Posttest	3.00	2.80
Familiar non-systematic		
Pretest	2.26	2.40
Posttest	3.33	2.56
Unfamiliar systematic		
Pretest	1.30	1.32
Posttest	1.53 *	2.47 *
Unfamiliar non-systematic		
Pretest	2.29	2.38
Posttest	2.57	2.60

Note: * $p < .0045$, ** $p < .05$

Analysis by rubric found no significant differences between the two groups. Growth was achieved on tasks in every rubric, but there was not sufficient evidence that instruction had an effect on the change in scores. Only the error reproduction rubric indicates a reasonably sophisticated level of understanding. A Level 3 score shows a partial recognition of the error pattern, while a Level 4 score gives evidence that the error is understood as only a computational error.

Correct answers to Problem one – Caroline's Addition Error – are 7110, 757, and 1112. The error pattern work for Problem one should look like:

$$\begin{array}{r} {}^{3} \\ 252 \\ \underline{+\ 585} \\ 7110 \end{array}$$

An example of a level 4 score is shown in "Mary's" work – 719, 757, and 1112. In this case Mary recognized the pattern correctly and then added incorrectly – 2 + 3 + 5 = 9. An example of a level 3 score is shown in "Cindy's" work – 739, 757, and 3011. She recognized Caroline's basic error of adding left to right, but didn't replicate the second error of carrying the wrong digit. For example:

$$\begin{array}{r} {}^{1} \\ 252 \\ \underline{+\ 585} \\ 739 \end{array}$$

Rubric analyses are summarized in Table 17.

Table 17

Change in Understanding from Pretest to Posttest by Rubric

	Numeration system instruction	
Rubric	Without	With
Error reproduction		
Pretest	3.41	3.05
Posttest	3.87	3.33
Depth of analysis		
Pretest	2.00	1.89
Posttest	2.32	2.11

	Without	With
Use of descriptive language		
Pretest	1.73	1.66
Posttest	2.01	1.95
Choice of correct representations		
Pretest	1.13	1.16
Posttest	1.14	1.25
Analysis of comp. method		
Pretest	1.45	1.53
Posttest	2.00	2.58
Analysis of sym.representation		
Pretest	2.04	2.31
Posttest	2.48	2.53

Note: * $p < .0045$, ** $p < .05$

Question Four

The fourth research question was: Is there a significant difference in gains made in explicit place value understanding between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)?

Independent sample t tests were calculated for the gains for each group in total, in cells, and in rubrics. Only the unfamiliar systematic cell revealed a significant difference with a mean gain of 0.2335 for the without instruction students (control group) and a mean gain of 1.1522 for the instruction students ($p = 0.00$) (experimental group). The scores in this cell reflect knowledge of problems relating to the research focus, base n problems where $n \neq 10$.

Final means in this cell were 1.53 for the without instruction control group students. On the scale of explicit understanding used for this research, this score indicates the students possessed an understanding between algorithmic meaning (process with no meaning) and tacit meaning (intuition of the meaning behind the process). The final mean of 2.47 in the base n cell for the instruction student scores' (experimental group) indicates an understanding between tacit meaning and explicit meaning (clear understandings of the connections).

Conclusion

Beginning scores for both groups of students on the APVU indicated a similar baseline level of understanding of place value concepts. The overall average of 1.95 for both groups showed almost a tacit level of understanding – some intuitive knowledge, but vague understanding and poor articulation of the mathematical logic. This similarity of scores carried through the cell analyses and rubric analyses. No significant differences were found between the two groups. Means for the pretest were generally in the level one and level two range. Level one indicates algorithmic understanding, computation without meaning.

Both groups showed an increase in scores in total, by cell, and by rubric. However, this increase still left many means in the algorithmic and tacit levels. Overall, both group means were in the two range, 2.35 and 2.59.

Cell and rubric means were also generally at the tacit level of understanding. For the without instruction students, only the familiar systematic cell (error pattern debugging) and the familiar non-systematic cell (time) indicated an explicit level of understanding. For the instruction students, none of the cell means were above the tacit level.

Rubric analyses indicated the best understanding for all subjects was in the area of error reproduction. Here all scores were in level 3, explicit understanding. However, on these debugging problems, students were not able to use descriptive language or show a depth of analysis.

Analysis of data leads to interpretation of this data and its implications. These numbers must be put in perspective and related to the research questions of this study. Chapter Five will provide a summary of these results along with implications of this research and recommendations for further study.

CHAPTER 5

CONCLUSION

Summary

As the No Child Left Behind Act is revamped and renewed, there is much debate about highly qualified teachers and improved student learning. New studies are investigating the mathematics content knowledge and the skills that teachers must demonstrate to improve student achievement. Questions are being raised about what mathematicians/mathematics educators can do to improve the mathematical preparation of teachers.

New work by Thames (2006) reports that many college courses today leave teachers unprepared, lacking sufficient mathematical knowledge needed for competent teaching. This, in turn, affects student learning. Perhaps more “rigorous content relevant to teaching and learning” needs to be explored (p. 2). That search for meaningful curriculum has been the focus of this research. At present, in most mathematics sequences for elementary education majors, the topic of base numeration systems is studied. Authors and researchers give rationale for this content, but is that rationale correct? Does this subject help students acquire an explicit understanding of place value?

This study investigated the role of base numeration system instruction as it relates to place value understanding in pre-service elementary teachers. To accomplish this task, the Assessment of Place Value Understanding (APVU) was administered to fifty-five pre-service elementary teachers at two universities in the southwest. These students were enrolled in a required mathematics course developed specifically for elementary education majors.

The APVU was chosen to ascertain knowledge because it was specifically developed for this population to measure place value understanding. This assessment instrument connected the population of elementary pre-service teachers and place value concepts and included a section of problems involving base numeration systems. These three elements were components of this research.

This chapter provides a brief summary of the numerical findings of this research and reviews the four guiding questions. Conclusions about pre-service elementary teacher place value understanding are discussed using the rubrics and the five levels of understanding associated with the assessment instrument, the APVU. These conclusions are then compared to previous studies. Limitations of the study are listed before improvements and changes lead to recommendations for future research.

During the course of a single semester, one version of the APVU was given twice to each of the two groups, once early in the semester (pretest) and once near or at the end of the semester (posttest). During the weeks of class, regular instruction was provided to the students at the public university by an instructor known to the researcher. No changes were made to the instructor's methods or syllabus. The conventional curriculum covered such topics as base ten place value and base numerations systems as part of the content. This group of subjects was identified as the experimental group. During the same weeks, the private university students received instruction from the researcher in base ten place value and all syllabus topics except base numeration systems. Once the posttest was given to the researcher's class, the missing topic of base numeration systems was taught at the end the semester. This group was identified as the control group.

The choice of the two classes in the sample was constructed by convenience. Although many schools were contacted about participating in the study, only one followed through on the commitment. The researcher used her own class as the non-instruction/delayed group because no other instructor contacted wanted to deviate from the normal sequence of topics.

The pretest scores provided baseline information on the place value understanding that the pre-service elementary teachers had prior to the semester of instruction. Question one asked: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study bring with them to their required mathematics content course? Although there was much concern about the sampling used in this study, pretest scores showed no significance differences between the two groups of students. The overall means for both groups was a 1.95 on a scale from 1 to 5 of understanding. Medians were 1.81 for the without instruction group (control group) versus 1.845 for the instruction or experimental group. Boxplots showed similar distributions. Based on this five level scale, these average scores were at less than a *tacit* level. This score level indicates that students possess the ability to use an algorithm but demonstrate vague understanding of its logic. The thirteen-item test was then analyzed by the four groupings (cells) of similar problems and by rubrics utilized on each problem. No significant differences between the two groups were found in any category. Thus, both groups entered the study with comparable levels of place value understanding.

Using the five level scale of understanding , the elementary pre-service teachers' average scores indicated almost a *tacit* level of understanding.

The posttest scores provided information on the place value understanding that the pre-service elementary teachers had after the semester of instruction. Question two asked: What is the level of explicit place value understanding that the elementary pre-service teachers participating in the study have at the end of their required mathematics content course? Results of the posttest scores showed a slightly higher total mean score for the instruction students – 2.35 for the without instruction (control) group versus the mean score of 2.59 for the instruction (experimental) group. However, this was not enough to be statistically significant. Individual cell analysis found a difference in means in only one of the areas – base problems. The instruction students did significantly better with the base problems, demonstrating a mean of 2.47 compared to the without instruction group (control group) mean of 1.53. This result was expected due to the study design. Thus, both groups of students demonstrated an average *tacit* level of understanding of place value as the semester ended.

Rubric analyses showed no significant differences between the two groups. However three rubrics, used for problems included in the unfamiliar systematic (base) cell, did show marked differences. Choice of correct representation, accurate computation, and analysis of computation method rubrics showed higher means for the instruction (experimental) group. A comparison of scores of the choice of correct representation, rubric used for Problem 11 of the APVU, found that its mean was 1.25 for the instruction group and 1.04 for the without instruction (control) group. The accurate computation rubric score means, used for Problems 2, 3, 5, 6, 7, 8, 9, 10, and13, were

3.10 for the instruction group and 2.64 for the without instruction group. Means of 2.58 for the instruction group and 2.00 for the without instruction group were found for the analysis of computation method rubric, used for Problems 2, 3, 5, 6, 7, 8, 9, 10, and 13.

Next the pretest and posttest for both groups of study are analyzed. Comparing scores across the groups answered question three. Question three asked: Is there any significant change in the participants' explicit place value understanding from the beginning of the semester to the end of the semester? Mean scores for the two groups were analyzed for significant changes in understanding from pretest to posttest. Each group, individually, showed significant growth, but the means across the groups were not significant. Only the unfamiliar systematic cell dealing with base numeration problems showed significance at the $\alpha = 0.0045$ level between the two groups. Again, this result was anticipated due to the study design. No rubric analyses produced significant differences. Thus, there were no significant changes in place value understanding across the groups from pretest to posttest.

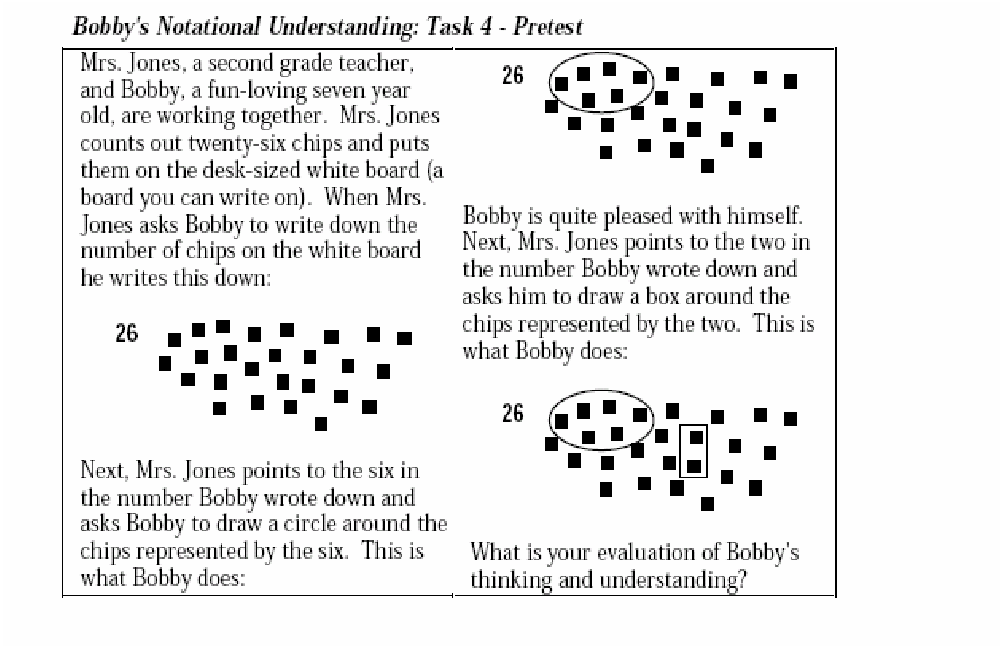
Lastly, question four explored the gains made by the students during the semester. Question four asked: Is there a significant difference in gains made in explicit place value understanding between the class that has numeration system instruction (experimental group) and the class that does not have the numeration system instruction (control group)? The differences in gains for both groups of students were calculated and analyzed. Total mean scores and rubric scores calculated for each of the two groups showed no differences in gains. Only the unfamiliar systematic cell problems indicated a significant difference, a gain on mean scores of 0.23 (1.30 to 1.53) for the non-instruction students and 1.15 (1.32 to 2.47) for the instruction students. This information correlated with other data previously obtained. Thus, there were no significant gains in understanding as measured by the APVU for the students in either group.

Conclusions

The Assessment of Place Value Understanding (APVU) was developed to indicate levels of mathematical understanding from algorithmic to pedagogical for pre-service elementary teachers. A rubric scale of one to five corresponds to the five levels of depth of understanding. Each question on the APVU used two to three rubrics to evaluate the understanding students demonstrated on each problem. The lowest level referred to as *algorithmic* understanding indicates a student has knowledge of computational procedures, but not necessarily an understanding of why the process works. The second level described as *tacit* understanding indicates the student has some intuitive reasoning ability, but an inability to articulate the logic of the concept. At level three, *explicit* understanding, the student demonstrates a precise understanding of the concept with good articulation of the logic and a clear sense of connections among ideas. Level four named *pedagogical*, (self and peers), denotes students who know how they and their peers build mathematical understanding. Lastly, level five, *pedagogical*, (children), asserts that students have a level of understanding needed for effective teaching. Students at this level know how others build knowledge and how to choose appropriate tools for teaching.

Using Problem 4 from the APVU as a guide for discussion, the following represent student responses that serve as examples of each level of understanding. Analyzing problem 4 required the use of the Depth of Analysis and Use of Descriptive

Language rubrics. These rubrics are described in Appendix C.


Bobby's Notational Understanding: Task 4 - Pretest

Mrs. Jones, a second grade teacher, and Bobby, a fun-loving seven year old, are working together. Mrs. Jones counts out twenty-six chips and puts them on the desk-sized white board (a board you can write on). When Mrs. Jones asks Bobby to write down the number of chips on the white board he writes this down:

26

Next, Mrs. Jones points to the six in the number Bobby wrote down and asks Bobby to draw a circle around the chips represented by the six. This is what Bobby does:

26

Bobby is quite pleased with himself. Next, Mrs. Jones points to the two in the number Bobby wrote down and asks him to draw a box around the chips represented by the two. This is what Bobby does:

26

What is your evaluation of Bobby's thinking and understanding?

Level 1 – Algorithmic "He thinks he can pick any chip to represent the number he wants."

Level 2 - Tacit "He does not fully understand place value. He thinks in terms of units."

Level 3 - Explicit "He is not understanding place values, ones and tens. He is counting by ones only."

Level 4 - Pedagogical (self/peers) "Bobby isn't using place value skills. He circled the six (1s) but when Mrs. Jones pointed to the two he just circles 2 not realizing that the 2 had a place value and should have been a 20."

Level 5 - Pedagogical (children) "Bobby doesn't understand the place value that the 2 represents. He is viewing it as a single digit not as the number of tens that the second digit represents. The two represents

> 2 groups of two or twenty, so he is not understanding the value of the position."

In this study it was disappointing to find that both groups of students began the semester with less than a *tacit* understanding of place value (1.95 mean score). Current research in workshops sponsored by the Mathematical Sciences Research Institute on critical issues in mathematical education promotes the importance of clear explanations, choosing useful examples, and evaluating students' ideas (Thames, 2006). Students were unable to provide explanations or accurately assess student work on the APVU. Also disappointing in the results was that after a semester of instruction, all the students' understanding had improved to only a mid-level *tacit* understanding.

These low mean scores and some large standard deviations within the groups (more than one level) may have been the result of varied abilities. These scores also may have been the result of student apathy in answering all the questions. Because there was no incentive for students to do their best work, a number of problems, or parts of problems, were left blank by each group of students. Approximately twenty to twenty-five percent of all students left at least one problem blank. The instruction (experimental) group left an average of 2.125 questions blank on the pretest and 1.67 problems blank on the posttest. The without instruction (control) group left an average of 2 problems blank on the pretest and 1.625 problems blank on the posttest. More blanks occurred for problems 11, 12, and 13, although all problems from 3 to 13 had at least one blank per group. These blanks were scored as a zero and had an affect on the calculation of mean scores and standard deviations. Interpretation of the blanks was difficult for the researcher to assess without follow-up. No answers to the problems may have indicated confusion about the question, no knowledge of how to answer the question, or no desire to answer the question. Earlier examination of the pretests could have allowed the researcher time to question students about the problems they did not complete. Unfortunately, all tests were examined at the end of the semester to help with the rater reliability regarding scoring. Due to already small sample sizes, it was not possible for the researcher to discard tests with blanks.

Analyses indicate that for these 55 students, the treatment of base numeration system instruction did not significantly impact understanding as measured by the APVU. There was, however, growth in each class' place value understanding. The change in the means from pretest to posttest for the total, in every cell, and in every rubric indicate increases in understanding for both groups of students. These results do suggest that instruction in this content area had a positive influence on the development of understanding in place value and numeration systems.

Student responses from Problem 1 show this increase in instruction. Problem 1 on the APVU asked students to replicate and explain Caroline's addition error pattern. One student on the pretest wrote, "She is adding from left to right instead of right to left." On the posttest the same student answered, "Caroline doesn't understand to add from right to left. She is adding as if she is reading a book. She also carries the wrong numbers to the next digits place" The posttest response is still not at a pedagogical level, but the student has begun to include place value language in her response.

Authors of the APVU, in their studies ten years ago, had similar findings. Rusch's research with place value understanding, constructivist, and direct instruction concluded that "the participants' failure to use place value concepts in even moderately sophisticated ways suggests that, by the end of the course dealing with place value concepts, they had not yet achieved the recommended ‘explicit’ level of understanding” (Rusch, 1997, p. 104). Gains made by the constructivist class of students were not significantly greater than the gains made by the direct instruction students. Hannigan’s study with place value understanding and writing prompts concluded that students did show some growth in their place value understanding. However, understanding was still not at the desired level for teachers, and the treatment of freewriting made no difference in the students’ scores (Hannigan, 1998). The current study’s findings support the results of these two research projects.

Recent research in elementary pre-service teachers’ place value understanding by Farro-Lynd found a “lack of foundational knowledge of the structure of place value” (Farro-Lynd, 2003, p. 104). Her study focused on misconceptions of place value concepts by pre-service elementary teachers. Although the current study’s focus was on numeration system instruction and used a different assessment instrument, this researcher found a similar lack of understanding. Skipper’s study in 1972 using manipulatives with numeration systems concluded instruction methods provided no significant difference in place value knowledge. The current research studied understanding, not just knowledge, and found no significant different in place value understanding. Haukebo’s research in 1967 was similar to this project in that the research focused on place value understanding and the instruction/non-instruction of base numeration systems. Haukebo found that “ although improvement in arithmetic understanding was apparent”, there was no significant difference in treatment groups (Haukebo, 1967, p. 109). This current study used a different instrument to arrive at the same conclusion.

This research found no evidence that place value understanding is substantially improved by teaching base numeration systems. Students tend to learn the algorithms for the problems and can do simple addition and subtraction problems in other bases, yet this manipulation does not constitute understanding of the place value concepts. This conclusion is consistent with other studies involving numeration systems and consistent with other studies involving place value.

Limitations

This study had a very limited scope and small sample sizes. Due to time constraints, control of participant sampling was not possible. All classes used were chosen by convenience. Although the classes were at different colleges with different instructors, efforts were made to at least select classes with similar students in a comparable college program. All students in the two classes were invited to participate. Even so, the samples lead to major questions regarding generalizabilty.

Although teaching methodology was not considered in this research, the teachers have similar pedagogy. Both instructors have many years of university teaching experience, and both are friends and students in the same doctoral program. Texts and curricula for the classes were different, but content pertinent to this research was similar. Content comparisons are discussed in Chapter 3.

Students received no compensation for participating in the study. Both professors encouraged students' diligence, concentration, and effort, but there was no way to control for these variables. Thus, there was no assurance of best work.

Recommendations

This research represents a preliminary exploration into the relationship between understanding place value and understanding base numeration systems. The researcher is hopeful that others will continue the study into this topic. Thus, recommendations and improvements are suggested for future projects.

One improvement involves sampling and the uncontrollable variables found in the present research. Due to time problems and the reluctance of instructors to participate in this study, certain elements of the research design were pre-determined. The researcher had no control over class enrollments, student demographics, differences in curriculum, or teacher teaching styles. All these variables are linked because of the course schedule of many universities. Ideally, larger sample sizes should be used. This would eliminate the possibility of a few extreme scores skewing the data results. Also larger sample sizes could insure normal distributions in the data to provide a better base for calculations. Ideally, one instructor should be used to teach the normal instruction students (experimental group) and the delayed instruction students (control group). This would eliminate curriculum and teacher style as confounding variables. However, these ideals may be difficult to achieve due to university schedules. At the researcher's university the course used in this study is offered as a single section per semester, yielding only approximately fifty students per year. Other universities offer the course only once a year, scheduling one or two sections as needed. Again, only small sample sizes are possible. Thus, the research design is a challenge to any researcher taking up the task. If the obstacles of sample size or instructor cannot be overcome, a more complete study might include student interviews. Student input, in addition to their paper and pencil work on the APVU, could add many details about their thinking and place value understanding and clarify their calculations and explanations on the APVU. The APVU could also be given in an oral exam interview format.

A second recommendation involves the improvement of the assessment tool. The authors created two forms of the original APVU, a pretest form and a posttest form. Problems were similar, but not identical. This researcher and her committee choose to give the pretest form twice and eliminate the posttest form. This was done because of a concern for the parallel nature of the two forms. Author of the APVU Hannigan noted parallel difficulties with the time problems and suggested modifications to make the tests truly parallel (Hannigan, 1998).

The original authors suggested other changes after using the instrument, basically in structure or wording of the problems. Both authors felt Problem 11, the notational structure task, offered little to the test. Students showed little or no improvement in their scores, and many students found the question confusing. Author Hannigan suggested giving the students more direction for answering the questions. "In trying not to 'give away the game' the authors left out references to place value and 'concepts.' The students might have a better idea of what is expected if a few references to 'conceptual understanding' were included in the APVU" (Hannigan,1998, p. 104). Author Rusch would eliminate some "bugs" in the problems. For example, in the pretest task one, Caroline's addition problem, a partial sum is eleven. This number does not help the rater discriminate between the tens digit and the ones digit. To aid in interpreting students' work, the sum should consist of two distinct numerals.

The researcher offers the following suggestions for the pretest form of the APVU: (1) Eliminate the self-constructed representation of Problem 11, and reword the problem. Such a challenging problem should be included in the APVU, but in its present form it contains too many elements and appears overwhelming to the students; (2) Change some of the tasks from Problems 1 and 12. Some problems do not illustrate the error pattern, and others as Rusch discussed involve numbers that are difficult to interpret; (3) Rewrite Problem 6. This is an excellent problem, but many students misread the problem and answered a different question; (4) Provide a template for Problem 2 similar to that given in Problem 7. This would eliminate any confusion in notation.

A third suggestion involves further studies. Much can be learned from the past to help the future. Hamilton's study (1961) and the follow-up research by Sawada and Atkinson (1981) on 'Make-Believe Arithmetic' suggested that the traditional notation of Hindu-Arabic numerals in the teaching of base numeration systems may present confusion for the pre-service teachers. "If exercises were in base ten, they knew it … well …, and if exercises were in another base, they got so confused over the different meaning attached to the Arabic symbols that they couldn't do anything" (Hamilton, 1961, p. 242). "The students are so thoroughly familiar with the [base ten] system that everything is automatic, so automatic that no thought needs to be expended" (Sawada & Atkinson, 1981, p. 367). Teaching a positional base system using invented symbols brought a greater understanding of the base ten system and an ability to transfer learning to problems using traditional notation.

Additional research supports this alternative instructional strategy for teaching base systems. An article by Dahlke (1982) gave directions for inventing symbols for various base systems. He also suggested that oral names for decimal numbers, such as the inconsistent pattern of fifteen versus twenty-six, may be confusing in the study of other bases using the traditional notation. Fuson (1990a, 1990b), in her studies with elementary children, also noted the confusing decimal names for some two digit numbers.

Confusion for pre-service elementary education teachers may be due to instruction in numerous base systems. Hungerford suggested an intense study of only one system. He concentrated on ‘fen arithmetic’, a base five systems using the standard digits zero through four. Counting avoided the base ten confusion of two digit names by using consistent names – one, two, three, four, fen, fenone, fentwo, fenthree, fenfour, twofen, twofenone, etc. He approached the system as a new arithmetic, not just “explaining place value by translating from base ten to other bases and back” (Hungerford, 1992, p. 1). After covering approximately two chapters of material in the textbook with fen mathematics, Hungerford revealed the base concept and connected the system to base ten. Student comments confirmed his contention that this process encouraged thinking about place value concepts.

These ideas from past research show promise. Although place value understanding was increased in this research, the level of student understanding was still low. Perhaps the content is not the issue; perhaps the issue is the instructional approach. Suggestions include inventing a new base system with new symbols and a focus on only one base system, not many. Since the studies mentioned are dated, future researchers could use the ideas to spark experimentation in college mathematics classrooms and continue the dialogue of critical issues in mathematics education.

A fourth suggestion is a call for universities to require a mathematics methods course in their curriculum for elementary pre-service teachers. Neither university involved in this study require such a course. Work by McDiarmid, Ball, and Anderson (1989) and Grossman, Wilson, and Shulman (1989) mentioned in Chapter Two of this study suggests that a mathematics methods course could help student come to understand that mathematics is more than computation. Field experiences as part of the course requirement could help these elementary pre-service teachers learn why elementary students make mistakes and could offer many opportunities for these future teachers to develop an ability to explain the mathematical logic behind the traditional algorithms.

The mathematical preparation of pre-service elementary teachers is critical to their future students' learning. In outlining a curriculum for these perspective teachers, the Conference Board of the Mathematical Sciences (2001) states, "Teachers must be able to call upon a richly integrated understanding of operations, place value, and computation in the domains of whole numbers, integers, and rationals" (p. 58). According to the National Council of Teachers of Mathematics, "Foundational ideas like place value… should have a prominent place in the mathematics curriculum because they enable students to understand other mathematical ideas and connect ideas"(2000, p. 15).

In Chapter One of this research, numerous reasons were sited for pre-service elementary teachers to study base numeration systems as part of their mathematical preparation. These reasons included placement of place value number systems in elementary school textbooks, use of place value number systems in children's literature, and acquisition of an explicit understanding of place value and the base ten system. Simply a brief introduction to numeration systems may be sufficient for pre-service teachers to handle elementary school texts or read children's books, but more rigorous content must be put forth to increase the pre-service elementary teachers' understanding of place value.

The goal of the researcher was to shed light on one aspect of the mathematical content presented to our pre-service elementary teachers. With the clear importance of mathematics in our society and the limited number of required college mathematics courses that elementary education majors need, each course must provide meaningful instruction and content. If not, classroom teachers will be under prepared to handle the academic curiosity of their pupils, and these teachers will allow algorithms and memorization to pass for understanding. At present there is a "vicious cycle: poor K – 12 mathematics instruction produces ill-prepared college students, and undergraduate education often does little to correct the problem" (CBMS, 2001, p. 55).

For this researcher, the real discovery in this study was not whether base numeration system instruction improved place value understanding. No, the real discovery was the low level of understanding that these pre-service elementary teachers had at the end of the study. The place value concept was taught during the semester, and this topic is unlikely to be covered again in their mathematics classes. These levels of understanding are alarming and indicate how unprepared these future teachers are to enter the elementary classroom and engage their students in meaningful learning. This study should alert mathematics teacher educators to inadequacies in the traditional curriculum. More research is needed into the understanding of concepts. Lessons and assessments should stress not just computational accuracy. Mathematics teacher educators need to broaden their concept of 'knowing' to include understanding. A quotation from Liping

Ma earlier in this work should guide all educators, “Know how, and also know why” (Ma, 1999, p. 108).

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APPENDIXES

APPENDIX A

ASSESSMENT OF PLACE VALUE UNDERSTANDING
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NAME ____

SCHOOL ______________________________ DATE ________________

Caroline's Addition Error Pattern: Task 1 - Pretest

Caroline is a conscientious student at Merrywillow Elementary School. She is having a bit of trouble with addition. Below is a sample of Caroline's addition work:

$$\begin{array}{r} 432 \\ +\ 265 \\ \hline 697 \end{array} \qquad \begin{array}{r} {}^{1} \\ 74 \\ +\ 43 \\ \hline 18 \end{array} \qquad \begin{array}{r} {}^{4} \\ 385 \\ +\ 667 \\ \hline 9116 \end{array} \qquad \begin{array}{r} {}^{0\,0} \\ 563 \\ +\ 545 \\ \hline 118 \end{array}$$

What answers would Caroline give for these three problems? Show all your work.

$$\begin{array}{r} 252 \\ +\ 585 \\ \hline \end{array} \qquad \begin{array}{r} 421 \\ +\ 336 \\ \hline \end{array} \qquad \begin{array}{r} 618 \\ +\ 782 \\ \hline \end{array}$$

Caroline's errors indicate that there is something she doesn't understand. Briefly describe your assessment of what Caroline does not understand.

Picabo's Race Times: Task 2 - Pretest

In a downhill skiing competition, every racer makes two runs. The times are totaled and the skier with the fastest combined times is declared the winner of the competition. Picabo Street (pronounced Peek-a-Boo Street -- she is real-life champion skier from California) has a time of 2 minutes 53.67 seconds (2:53.67) on her first run and a time of 2:50.54 on her second run.

What is her combined time for the competition? Show all your work.

Subtraction Base 7: Task 3 - Pretest

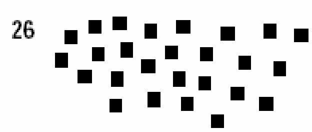
Find the difference:

$$\begin{array}{r} 614_{seven} \\ -\ 352_{seven} \\ \hline \end{array}$$

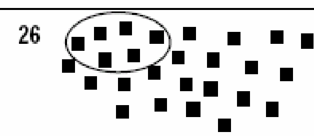
Briefly describe how you arrived at your answer.

Bobby's Notational Understanding: Task 4 - Pretest

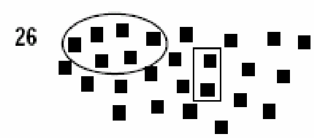
Mrs. Jones, a second grade teacher, and Bobby, a fun-loving seven year old, are working together. Mrs. Jones counts out twenty-six chips and puts them on the desk-sized white board (a board you can write on). When Mrs. Jones asks Bobby to write down the number of chips on the white board he writes this down:



Next, Mrs. Jones points to the six in the number Bobby wrote down and asks Bobby to draw a circle around the chips represented by the six. This is what Bobby does:



Bobby is quite pleased with himself. Next, Mrs. Jones points to the two in the number Bobby wrote down and asks him to draw a box around the chips represented by the two. This is what Bobby does:



What is your evaluation of Bobby's thinking and understanding?

Milk Cartons - Conversion: Task 5 - Pretest

In a Wisconsin dairy, the number six has special significance for the dairy workers. When they prepare the cartons of milk for shipping, they put six cartons in a box, six boxes in a crate, six crates in a flat, and six flats on a pallet.

6 cartons = 1 box
6 boxes = 1 crate
6 crates = 1 flat
6 flats = 1 pallet

So that they know how many more cartons of milk are needed to complete a pallet, the dairy workers have developed a system for describing the status of a pallet at any point during the packing process. Their notation is:

(# of flats)(# of crates)(# of boxes)(# of cartons)

for any partially filled pallet. For example, a 4312 is a partially filled pallet that has 4 flats, 3 crates, 1 box and 2 cartons on it. This notation could mean that there are exactly those containers on the pallet or the equivalent quantity of milk cartons waiting to be packed. Also, using this notation, a 10000 is a completed, ready-to-ship pallet.

While you were on a field trip to the dairy, you counted 321 cartons of milk that were ready to be placed on a pallet. How would you represent that amount of cartons in the dairy's notation? Show all your work.

Milk Cartons - Subtraction: Task 6 - Pretest

As you were walking out the door, you heard someone yell, "I have a 2045 that needs filling!" How many milk cartons, in dairy notation, do they need to complete the pallet? Show all your work.

Space Shuttle, Friday AM to Friday AM: Task 7 - Pretest

To ensure that all events in a space shuttle launch are perfectly coordinated, NASA and the space program keep track of the timing of events down to fractions of seconds. The way they keep track of the time of day looks like this:

Hours : Minutes : Seconds : Hundredths of a Second

The first time the shuttle fires its thrusters is at 8:44:35:16 on Friday morning and the next time it fires them is at 11:26:33:07 Friday morning.

How much time has elapsed between one firing and the next? Show all your work.

Multiplication Base 4: Task 8 - Pretest

Find the product:

$$\begin{array}{r} 12_{\text{four}} \\ \underline{\times\ 23_{\text{four}}} \end{array}$$

Briefly describe how you arrived at your answer.

Zorandria Situation

The Kingdom of Zorandria has developed a coinage system for transacting business. The Royal Yellow (Y) is the basic coin. The other coins, the Royal Green (G), the Royal Red (R), and the Royal Blue (B), are worth more. The system is the following:

(Y) is the basic coin,	(R) is worth (G) (G),
(G) is worth (Y) (Y) (Y) (Y),	(B) is worth (R) (R) (R).

Zorandria - Addition: Task 9 - Pretest

You are a Zorandrian rug merchant and you have two rugs for sale. The prices for the two rugs are:

Price 1 (G) (Y) (Y) (R) (B) (R) (Y) (R) (B) (R) (Y)

Price 2 (G) (Y) (B) (B) (G) (Y) (Y) (G) (B) (G) (Y)

A buyer wishes to buy both rugs and asks for your price using the fewest number of coins possible. What is the price of the pair?

The Zorandrians often use the counting tables below to help with calculations. Feel free to use the table if you think it will be helpful. Show all your work.

(Larger tables are provided to the students for each problem.)

◯	◯	◯	◯

◯	◯	◯	◯

Zorandria - Subtraction: Task 10 - Pretest

In another area of your shop, you have two rugs for sale at the following prices:

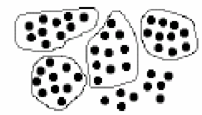
Turindale Rug: (R) (Y) (B) (B) (R) (G) (Y) (Y) (B) (G)

Brandicline Rug: (B) (B) (G) (Y) (G) (G) (R)

The Turindale rug is definitely more expensive than the Brandicline rug. Using the fewest number of coins possible, how much more expensive is the Turindale rug? Feel free to use one or both of the counting tables below if you think it will be helpful. Show all your work.

49 Buttons: Task 11 - Pretest

A young child and a teacher are working with buttons and loops of yarn. The teacher organizes the buttons into four equal size groups which are inside the loops with nine buttons left outside the loops:



The child says there are 49 buttons. The teacher is quite pleased.

Then the child adds a loop, rearranges the buttons (but doesn't add any new ones) and states that the quantity of buttons is represented by 54. The teacher is very surprised, but agrees that the child is correct.

Next, the child takes away two loops and again rearranges the buttons without adding or removing any buttons. This time the child says that the quantity is represented by 31 and again the teacher confirms that the child is correct.

Circle the numbers below which would be another correct way that this precocious child might represent the quantity of buttons in this set. (You are not limited to one choice.)

61 100 39 70 57

List two other ways to represent this quantity of buttons.

Briefly explain what the child appears to be thinking and why the teacher keeps confirming that the child is correct in that thinking.

James' Subtraction Error Pattern: Task 12 - Pretest

James is a bright and feisty little boy with a quick smile and mischief in his eyes. He loves to play kickball, but he's having some trouble with his arithmetic. Below is a sample of James' subtraction work:

$$\begin{array}{r} 6\,\overset{8}{\cancel{9}}\,{}^{1}3 \\ -\ 2\,4\,8 \\ \hline 4\,4\,5 \end{array} \qquad \begin{array}{r} \overset{5}{\cancel{7}}\,{}^{1}2\,{}^{1}6 \\ -\ 3\,4\,9 \\ \hline 2\,8\,7 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{4}}\,{}^{1}3\,{}^{1}4 \\ -\ 2\,7\,6 \\ \hline 6\,8 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}\,{}^{1}2\,5 \\ -\ 1\,5\,1 \\ \hline 1\,7\,4 \end{array}$$

What answers would James give for these three problems? Show all your work.

$$\begin{array}{r} 451 \\ -\ 239 \\ \hline \end{array} \qquad \begin{array}{r} 730 \\ -\ 284 \\ \hline \end{array} \qquad \begin{array}{r} 853 \\ -\ 497 \\ \hline \end{array}$$

James' errors indicate that there is something he doesn't understand. Briefly describe your assessment of what James does not understand.

Addition Base 6: Task 13 - Pretest

Find the sum:

$$\begin{array}{r} 341_{\text{six}} \\ +\ 235_{\text{six}} \\ \hline \end{array}$$

Briefly describe how you arrived at your answer.

APPENDIX B

APVU CELL CONTENT

Table A1

Cell	Content	Problems
Familiar Systematic	Base 10 System Metric Measurement	1, 4, 12
Familiar Non-Systematic	Time Imperial Measurement	2, 7
Unfamiliar Systematic	Base n Systems not Base 10	3, 5, 6, 8, 11, 13
Unfamiliar Non-Systematic	Foreign Coinage	9, 10

APPENDIX C

RUBRICS

Error Reproduction

Level 5	Accurate reproduction of error.
Level 4	Evidence that the error is understood but a computational error exists.
Level 3	Inaccurate reproduction of error with evidence that there is partial recognition of the error pattern.
Level 2	Inaccurate reproduction of error with no evidence that there is recognition of the error pattern. OR An incomplete attempt.
Level 1	No attempt at task.

Depth of Analysis

Level 5	Develops an accurate and elaborate analysis of the place value concepts not understood by the child.
Level 4	Develops an accurate analysis of the place value concepts not understood by the child.
Level 3	Mentions what is not understood by the child, but leaves it undeveloped.
Level 2	Provides an accurate description of some or all behaviors, but no analysis of understanding.
Level 1	Provides an analysis that is irrelevant, incorrect, or uninformative.

Use of Descriptive Language

Level 5 Uses accurate and highly-specific place value language. For example, uses the formal term "regrouping" in place of the informal terms "carrying" or "borrowing" and "groups of ten" in place of "ten" or "one."

Level 4 Uses accurate and specific place value language. For example, ses the informal terms "carrying" and "borrowing" but evidence suggests that those terms are being used as synonyms for the formal term "regrouping" and/or uses the word "ten" but evidence suggests that the term is being used as a synonym for the more specific phrase "one group of ten."

Level 3 Uses accurate but non-specific place value language to describe a place value concept. For example, uses the informal terms "carrying" and "borrowing" as synonyms for the formal term "regrouping" and/or the word "one" is being used as a synonym for the more specific word "ten" or phrase "group of ten."

Level 2 May use accurate but non-specific place value language; however, evidence suggests that the language is used to indicate an observed behavior rather than to describe a place value concept.

Level 1 No or inaccurate use of place value language. OR Analysis does not use place value language to describe behaviors.

Choice of Correct Representations

Level 5 Chosen representations in part A are precisely and technically correct; i.e., 1 group of n_2 is perceived as more correct than n groups of n_1. Accuracy of both self-constructed representations provides supporting evidence for this explicit understanding.

Level 4 Chosen representations in part A are correct; i.e., 1 group of n_2 and n groups of n_1 are perceived as equally correct. Accuracy of both self-constructed representations provides supporting evidence for this advanced tacit understanding.

Level 3 Chosen representations in part A are substantially correct; i.e., n groups of n_1 is perceived as more correct than 1 group of n_2. Both self-constructed representations are conceptually correct, though not necessarily technically correct, which provides supporting evidence for tacit understanding.

Level 2 Some chosen and/or self-constructed representations may be correct, but there is insufficient evidence to support a conclusion of tacit understanding.

Level 1 No attempt or both self-constructed representations are incorrect or absent.

Accurate Computation

Level 5 Correct computation.

Level 4 Incorrect computation caused by a computational error.

Level 3 Incorrect computation with evidence of a minor conceptual error.

Level 2 Incorrect computation with evidence of significant conceptual errors, incomplete computation, or total confusion.

Level 1 No attempt at task.

Analysis of Computation Method Version for Problems 2, 3, 5, 6, 7, 8, 13

Level 5 Complete, sophisticated, and insightful adaptation of the traditional algorithm within the mixed-grouping place value structure. For example, the computation process uses only an adaptation of the traditional algorithm; i.e., symbolic regrouping is used accurately across units (minutes, seconds, hundredths of seconds) as well as within units.

Level 4 Partial adaptation of the traditional algorithm within the mixed grouping place value structure. For example, symbolic regrouping may be used from hundredths of seconds to seconds, but not used across larger units; instead, an appropriate alternative regrouping strategy is used.

Level 3 No evidence of the adaptation of the traditional algorithm; however, alternative regrouping strategies are consistently applied to the mixed-grouping place value structure. OR A partial adaptation was utilized but the alternative regrouping strategy was left incomplete.

Level 2 No evidence of the adaptation of the traditional algorithm to the mixed-grouping place value structure. Alternative regrouping strategies may have been attempted but are disorganized and/or inaccurately applied.

Level 1 No evidence that the mixed-grouping place value structure is recognized or utilized. Base-ten strategies may have been consistently applied in inappropriate situations. OR Computation (task) is incomplete with insufficient evidence to determine a strategy.

Analysis of Computation Method Version for Problems 9 & 10

Level 5 Complete, sophisticated and insightful symbolic adaptation of the traditional base-ten algorithms to the mixed-grouping place value structure. For example, symbolic (digits rather than pictures) regrouping is part of the algorithm. There is no indication of a need to use illustrations.

Level 4 Partial symbolic adaptation of the traditional base-ten algorithms to the mixed-grouping place value structure. There may be illustrations used to clarify and/or support the computation.

Level 3 No evidence of any symbolic adaptation of the traditional base-ten algorithms; however, alternative symbolic and/or pictorial strategies were consistently applied to the mixed-grouping place value structure. For example, regrouping the quantity as all Ys, accurate computation (or with minor error), and regrouping as fewest number of coins is an appropriate strategy.

Level 2 No evidence of any symbolic adaptation of the traditional base-ten algorithms. Alternative symbolic and/or pictorial strategies may have been attempted but are disorganized and/or inaccurately applied to the mixed-grouping place value structure. For example, regrouping the quantity as all Ys that has major computational errors or is not regrouped as the fewest number of coins is a poor attempt at a strategy. OR The computation (task) is incomplete, through confusion or omission, but there is some clear evidence that alternative strategies were consistently applied to some elements of the task.

Level 1 No evidence that the mixed-grouping place value structure was recognized or utilized. Calculation strategies used are inappropriate. Computation (task) is incomplete with no evidence of consistent application of alternative strategies; or computation (task) is not attempted.

Analysis of Symbolic Representation

Level 5 Constructs a logical and consistent symbolic representation (i.e., digits only) using place value columns which are organized in either an increasing or decreasing order (i.e., Y, G, R, B or B, R, G, Y).

Level 4 Constructs a logical and consistent algebraic representation (i.e., digits and letters) using place value columns which are organized in either an increasing or decreasing order (i.e., Y, G, R, B or B, R, G, Y).

Level 3 Constructs a logical and consistent pictorial representation (i.e., circles, tally marks, or letters without digits) using place value columns which are organized in either an increasing or decreasing order (i.e., Y, G, R, B or B, R, G, Y). OR Constructs a logical and consistent symbolic or algebraic representation in which place value columns are utilized, but not in an increasing or decreasing order.

Level 2 Constructs reasonable representation which may have algebraic or symbolic elements, but does not utilize place value columns. For example, converts to all yellow coins and uses digits to compute in base ten, and then converts back to mixed coins.

Level 1 Attempts to construct a representation but the result of the attempt is inaccurate or incomplete.

(Rusch, 1997)

APPENDIX D

RUBRIC/TASK CORRELATION

Table A2

Tasks	ER	DA	DL	AC	CM	SR	CR
Error patterns							
1, 12	X	X	X				
4		X	X				
11		X	X				X
Time							
2, 7				X	X		
Base n							
3, 8, 13				X	X		
Carton conversion							
5, 6				X	X		
Foreign coinage							
9, 10				X	X	X	

Note: ER = Error Reproduction; DA = Depth of Analysis; DL = Use of Descriptive Language; AC = Accurate Computation; CM = Analysis of Computation Method;

CR = Choice of Correct Representation; SR = Analysis of Symbolic Representation

APPENDIX F

INSTITUTIONAL REVIEW BOARD LETTER

Oklahoma State University Institutional Review Board

Date:	Friday, March 30, 2007
IRB Application No	ED0733
Proposal Title:	Coursework on Base Numeration Systems and its Influence on Pre-service Elementary Teachers' Understanding of Place Value Concepts
Reviewed and Processed as:	Exempt

Status Recommended by Reviewer(s): Approved Protocol Expires: 3/29/2008

Principal
Investigator(s

Dorothy J. Radin
3713 W. Hartford St.
Broken Arrow, OK 74012

Patricia Lamphere- Jordan
247 Willard
Stillwater, OK 74078

The IRB application referenced above has been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as outlined in section 45 CFR 46.

☑ The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

As Principal Investigator, it is your responsibility to do the following:

1. Conduct this study exactly as it has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval.
2. Submit a request for continuation if the study extends beyond the approval period of one calendar year. This continuation must receive IRB review and approval before the research can continue.
3. Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of this research; and
4. Notify the IRB office in writing when your research project is complete.

Please note that approved protocols are subject to monitoring by the IRB and that the IRB office has the authority to inspect research records associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact Beth McTernan in 219 Cordell North (phone: 405-744-5700, beth.mcternan@okstate.edu).

Sincerely,

Sue C Jacobs

Sue C. Jacobs, Chair
Institutional Review Board

VITA

Dorothy Janice Radin

Candidate for the Degree of Doctor of Education

Dissertation: COURSEWORK ON BASE NUMERATION SYSTEMS AND ITS INFLUENCE ON PRESERVICE ELEMENTARY TEACHERS' UNDERSTANDING OF PLACE VALUE CONCEPTS

Major Field: Curriculum and Instruction - Secondary Mathematics Education

Biographical:

Education: Graduated from Peoria High School, Peoria, Illinois in June 1966; received Bachelor of Science degree in Secondary Education from Minot State College, Minot, North Dakota in March 1971; received a Master of Arts degree in Mathematics Education from DePaul University, Chicago, Illinois in March 1992. Completed the requirements for the Doctor of Education degree with a major in Secondary Mathematics Education at Oklahoma State University, Stillwater, Oklahoma in July 2007.

Experience: Employed as substitute and part-time mathematics instructor by Racine Public Schools, Racine, Wisconsin, 1972 – 1974; employed as an insurance clerk by Sentry Insurance, Appleton, Wisconsin, 1975 – 1977; employed as substitute and gifted mathematics instructor by DesMoines and West DesMoines Schools, Iowa, 1985 - 1989; employed as mathematics instructor by Grandview College, DesMoines, Iowa, 1988 – 1989; employed as high school paraprofessional by Lake Park High School, Roselle, Illinois, 1989 – 1990; employed as a computer lab coordinator by Willowbrook High School, Villa Park, Illinois, 1990 – 1992; employed as adjunct mathematics instructor by Moraine Valley Community College, Palos Hills, Illinois, 1990 – 1992 and College of DuPage, Glen Ellyn, Illinois, 1992 – 1995; employed as mathematics instructor/technology coordinator at Trinity High School, River Forest, Illinois, 1993 – 1995; employed as adjunct instructor at Tulsa Community College, Tulsa, Oklahoma, 1995 – 1997; employed as mathematics instructor by Oral Roberts University, Tulsa, Oklahoma, 1996 to present.

Professional Memberships: National Council of Teachers of Mathematics, Mathematical Association of America, Association of Mathematics Teacher Educators.

ABSTRACT

Name: Dorothy Radin Date of Degree: July, 2007

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: COURSE ON BASE NUMERATION SYSTEMS AND ITS INFLUENCE ON PRE-SERVICE ELEMENTARY TEACHERS' UNDERSTANDING ON PLACE VALUE CONCEPTS

Pages in Study: 124 Candidate for the Degree of Doctor of Education

Major Field: Curriculum and Instruction - Secondary Mathematics Education

Scope and Method of Study: The purpose of this study was to examine the effectiveness of base numeration system instruction on place value understanding for college elementary education majors. Participants in the study were students in two university mathematics classes, one class at a private college and one class at a public college. Both colleges were in the same southwestern United States city metropolitan area. Each participant completed the Assessment of Place Value Understanding twice, pretest and posttest. T-tests, analysis of variance, and repeated measures ANOVA were used on means of total scores, cell scores, and rubric scores to test the four research questions.

Findings and Conclusions: Only the mean scores of the cell of base n problems, $n \neq 10$, showed significance differences between the two classes ($p = .00$). This was expected due to the research design and treatment of base numeration system instruction. All averages by total, cell, and rubric showed gains from pretest to posttest, meaning students' understanding of place value concepts improved. However, final understanding was at a less than sophisticated level. Instruction led to higher scores in the base problems, but did not improve overall place value understanding. More research is needed to examine the connection between elementary teachers' classroom instruction and the foundational concept of place value understanding.

Advisor's Approval: Dr. Patricia L. Jordan