METACOGNITIVE DEVELOPMENT IN A COLLEGE-LEVEL GEOMETRY COURSE FOR PRE-SERVICE ELEMENTARY TEACHERS: A CASE STUDY

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CHAPTER I

INTRODUCTION

Metacognitive thinking is most succinctly defined as thinking about one's thinking and better metacognitive thinking tends to lead to more in-depth mathematics learning (Martinez, 2006). Many educators have commonly accepted the notion that both students' metacognitive thinking and their awareness of their thinking can be improved through carefully selected instructional activities. This mixed-methods case study explored the use of metacognitive instructional activities in a college-level mathematics course designed for pre-service elementary teachers. Developing metacognitive thinking skills helps students think more critically and effectively, and thus become better mathematics learners (Costa, 2008). Teachers tend to identify a lack of metacognitive processing in their students' thinking and it is fairly common to hear teachers from around the world make the following statements about students' behaviors in the classroom:

- They just blurt out answers. They should think before they respond.
- They depend on me for their answers. I wish they would think for themselves.
- They give up so easily on difficult tasks. I'd like them to hang in there.
- They can't seem to work in groups. They must learn to cooperate and work together.

- They don't apply their knowledge. I want them to use what they know in other situations.
- They are afraid to take risks. I'd like them to be more creative, more adventuresome (Costa, 2008, p. 20).

With the general consensus that students need to learn how to think, it is important for teachers to consider how to help students reach this goal. Through the development of their metacognition, students become more aware of their thinking and share it with others. These shared thoughts then become objects that are critiqued, analyzed, and discussed to provide new understandings. Helping students become more aware of their thoughts, more able to share their thoughts, and better able to evaluate their thoughts and those of others leads to better metacognitive thinking, which in turn, leads to better mathematics learning.

Background and Context

Effective learning requires more than taking in new information and fitting it into one's existing knowledge scheme; it also involves thinking directly about new information, understanding its relationship to what is already known, understanding how one goes about learning this information, and actually being aware that one is learning (Fisher, 1998). This type of learning requires a shift away from an emphasis on merely knowing, to an emphasis on what students can do with their knowledge and how they use that knowledge at the appropriate time (Schoenfeld, 2007). Schools must begin to move beyond targeting only content and discrete skills and provide students with opportunities to learn how to make and evaluate their own decisions, much like they will be expected to do in adulthood (Gourgey, 1998; Kamii, 1991). Instruction that emphasizes using appropriate knowledge at the appropriate time requires an approach different from traditional lecture-driven instruction and many schools have found sociocultural theory promotes this change (Mercer, 2008).

The focus on sociocultural learning through strategies such as communication and scaffolding allow a person to become familiar with both new ideas and discourse as part of a new community of learners. The students are able to test thoughts and ideas that are being brought into the new community and must become aware of the questions they have and the information they already understand so that they can actively participate in the dialogue. Participating in the dialogue requires the students to be more engaged with their own thinking than if they were simply sitting and passively receiving information being given to them by the teacher. Students must be able to analyze and evaluate the shared thinking of others while comparing other's thoughts with their own thinking. Mathematics classrooms that embrace sociocultural learning theories promote the sharing of thoughts and discussions about thinking that will in turn promote an understanding of mathematics that goes deeper than regurgitating and thoughtlessly mimicking mindless procedures.

Mathematics is an investigation of pure logic and reason. Mathematical thinking exposes one's thought processes and makes obvious the learner's ability to reason and think logically, hence the reason many people find the subject of mathematics challenging (Carson & Rowlands, 2007). Mathematical thinking requires problem solvers to be flexible and resourceful with their thinking (Schoenfeld, 2007). Likewise, problem solvers must not only be aware of multiple paths, but must also recognize and give up on a misguided path when they are headed in a wrong direction or butting up against roadblocks. Problem solving requires students to use their metacognition to make inferences, draw conclusions, synthesize ideas, generate hypothesis, compare and contrast, find and articulate problems, analyze and evaluate alternatives, and monitor thinking (King, 2002). The use of metacognition can help students identify what they know and decide what to do with that knowledge. The learning of mathematics is empowering and the process of learning mathematics can help students develop flexible and analytic thinking strategies while preparing them to work through arguments put forth by others to arrive at shared solutions. The mathematics classroom must provide an environment rich with sense-making activities that will allow students to understand mathematics in meaningful ways (Schoenfeld, 1992).

Unfortunately, many mathematics classrooms tend to be void of rich sense-making activities and instead provide only shallow opportunities for students to demonstrate their reasoning and mathematical application skills. The problems used in these classrooms are thinly veiled in real-world contexts and only serve to provide additional practice for the procedure of the day (Schoenfeld, 1992). This type of curriculum does not allow students to abstract their own understandings and make sense of choosing appropriate strategies. These students come to believe mathematics is something arbitrary, inaccessible, and quickly solved within only a few seconds (Carson & Rowlands, 2007; Schoenfeld, 1992). Unfortunately this student perception may be tied to teachers' beliefs that the teaching of mathematics is only about delivering the facts and procedures, while failing to realize how the teachers' beliefs are causing their students to perceive mathematics as irrelevant, boring, unpopular, and overly cerebral (Carson & Rowlands, 2007; B. Patton, Fry, & Klages, 2008). A classroom rich in discussions that encourage metacognitive thinking promotes an understanding of mathematics and develops students' thinking skills that reach beyond the four walls of the classroom.

A New Direction For Mathematics Instruction

Rote memorization of procedures and procedures disguised in application problems must be replaced by an emphasis on critical thinking so students can learn to develop a personal viewpoint, make decisions, and take actions (Crawford, 2007; Kamii, 1991). Helping students learn to think in this way was important to both the National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC) when they outlined what mathematics education should encompass. The NRC (2001) described that teaching for mathematical proficiency requires the integration of procedural fluency, strategic fluency, adaptive reasoning, and a productive disposition. Thus students should be given opportunities to not only practice procedures, but also to justify, explain, and reflect on both their actual solution and their route to arrive at that solution while developing a good disposition towards the overall mathematics process. Justifying, explaining, and reflecting on a solution and a route to a solution requires an awareness of both what was done and why it was done. This is metacognitive thinking at its richest.

The NCTM (2000) also emphasized the need for a change away from the traditional practice of rote, memorized, and meaningless procedures by devoting five of their ten standards for school mathematics to mathematical processes. The process standards "highlight ways of acquiring and using content knowledge" (p. 29). The five process standards consist of problem solving, reasoning and proof, communication, connections, and representation all require students to talk about, think about, visualize, and connect content in ways that are new to many people and important for the changing needs of society. The changes called for by both NCTM and NRC require a significant and challenging process of transforming the teaching and learning of mathematics. Rather than being forced to solve a mathematics problem using one specific "right" method, students should do their own thinking and choose what they consider to be the most efficient strategy, then have the opportunity to reflect, refine, discuss, and amend their thinking and reasoning through the process of communication which allows the student to build meaning and permanence of their ideas (Kamii, 1991; Steele, 2001). Mathematics education must provide opportunities for students to develop their ability to think and reason critically, and this should be done through sharing and reflecting on how students think through and reason about a problem. By talking about and discussing students' thoughts, other students can refine their own thinking. Thus, through this process students become more effective and efficient thinkers. Authentic tasks and opportunities to help students move beyond rote memorization of facts and procedures provide an environment rich in thinking and doing real mathematics in a social and collaborative setting. When students are engaged in the activities, they are more likely to take interest in and be more successful with mathematics (Schoenfeld, 1992).

Metacognition

With a new emphasis on the development of thinking and reasoning skills, it is important to understand how students can improve their thinking, or *cognition*. Improving one's cognition requires thinking about thinking, also known as *metacognition*. The term metacognition was first used in the late 1970s and became more fully developed in both reading and mathematics literature throughout the 1980s. Within mathematics, theories about metacognition developed alongside problem solving and the two "are perhaps the two most overworked and least understood buzzwords of the 1980s" (Schoenfeld, 1992, p. 9). Further, because of this great firestorm of research, both are poorly defined, poorly grounded, and commonly misunderstood (Schoenfeld, 1992; Wilson & Clarke, 2002). Metacognition is more specifically defined as the monitoring and control of thought and is considered improvable through carefully designed instructional strategies (Martinez, 2006; Schraw, 1998). "Cognitive strategies are invoked to make cognitive progress, metacognitive strategies to monitor it" (Flavell, 1979, p. 909). Carson and Rowlands (2007) use the analogy of a suspension bridge to clarify the meaning of metacognition describing the anchors at either end of the bridge as concrete learning on one end and abstract learning on the other. The size and stability of the bridge are determined by the learning experiences, and the use of metacognition and meta-narrative are the supporting cables that are key to the solid structure.

Many educators recognize that problem solving involves cognition, but successful problem solvers are also constantly using their metacognition to take a step back monitoring and evaluating their overall progress, reworking their thinking when not moving towards a solution, constantly searching for new and more efficient pathways, and asking themselves if they are accomplishing what they set out to do (Martinez, 2006). This process of planning, using available tools, monitoring progress, and reflecting on the overall process is the metacognitive component of problem solving (Cardelle-Elawar, 1995; Cohors-Fresenborg, Kramer, Pundsack, Sjuts, & Sommer, 2010; Martinez, 2006). While learning problem solving in schools is often linked to the use of Polya's heuristic strategies, the common drilling of Polya's strategies to solve specifically designed contrived problems does not develop the intended metacognitive thinking skills because little time is spent on how to *manage* and *regulate* the use of the skills (Garofalo & Lester, 1985; Schoenfeld, 1992). Students should instead view problem solving from a more holistic approach when solving any task that has been set in front of them. This means using their metacognitive skills to identify the relevant information, sort out what they do and don't know about the problem, and then determine how this problem should be handled (Schoenfeld, 1992). More simply stated, students need to know how to recognize what they do and don't know (knowledge of cognition) and then determine what to do with this information (regulation of cognition). Knowledge of cognition refers to what students know about their cognition, such as what skills, resources, and abilities are available, how to implement procedures, and when and why to use a particular procedure. Regulation of cognition refers to determining what to do with that information, such as planning a strategy, managing information, monitoring progress, debugging errors, and evaluating overall work and efficiency (Schraw & Dennison, 1994). Having both an awareness and control over one's knowledge and regulation; hence having an awareness of one's metacognition, is critical to his or her ability to successfully solve problems (Garofalo & Lester, 1985).

Importance of Metacognition

Metacognition is commonly recognized as important, but a better understanding of how metacognition is developed and the role it plays in mathematical understandings is necessary (Schraw & Moshman, 1995; Wilson & Clarke, 2002). Costa (1984) identifies metacognition as an indicator of the educated intellect; therefore, metacognitive instruction needs to move beyond just problem solving and become a part of general mathematics instruction (Garofalo & Lester, 1985). Further, since teachers that are more metacognitively aware are more successful developing students' metacognition, it is important to help teachers become more aware of metacognition so it is better incorporated into classroom instruction (Garofalo & Lester, 1985; van der Walt & Maree, 2007).

Metacognitive thinking must be explicitly taught and recognized within specific content domains, such as in mathematics content (Desoete, 2007; Lin, 2001). Metacognition is a habit of mind that develops cognitively and socially competent learners. Teaching metacognitive thinking should be integrated and embedded in mathematics instruction rather than being seen as separate and additional content that must be added to an already full curriculum. The development of metacognitive thinking can best be achieved by coordinating strategy training and sociocultural support into everyday classroom activities (Lin, 2001). Further, the classroom environment must support metacognitive thinking or it will not occur. Providing only knowledge without experience or experience without knowledge will do little to enhance students' metacognitive development (Livingston, 2003).

Metacognition is important for school success and while any individual study can be picked apart, when the collection is taken as a whole, the research presents a strong case that metacognitive instruction should be included in the classroom (Sternberg, 1998). Several studies have identified positive correlations between students' ability to think metacognitively and their mathematical success (Kramarski, Weisse, & Kololshi-Minsker, 2010; Schraw & Moshman, 1995; Sternberg, 1998; van der Stel & Veenman, 2010). Additionally, metacognition correlates with other aspects believed to impact student success such as motivation (Crawford, 2007), anxiety (Kramarski, et al., 2010), self-efficacy (Gourgey, 1998), and intellectual ability (van der Stel & Veenman, 2010) while also contributing more to problem-solving success than IQ (Swanson, 1990). Overall, students with metacognitive awareness are more likely to understand how, when, and why to use cognitive strategies (Cardelle-Elawar, 1995). Researchers also suggested that success in mathematics is more closely related to students' metacognition than their ability (Swanson, 1990; van der Stel & Veenman, 2008; Veenman, van Hout-Wolters, & Afflerbach, 2006).

Metacognitive development allows students to transfer knowledge to new situations while also impacting the students' acquisition, comprehension, retention, and application of what was learned, making students more efficient learners, critical thinkers, and problem solvers (Hartman, 1998; Leat & Lin, 2003; Mevarech, Terkieltaub, Vinberger, & Nevet, 2010). As an example of how metacognitive development enhances learning, consider a child that overhears an argument, the child will internalize the structure of an overheard argument and later employ similar strategies when left to solve a similar problem alone in the future (Martinez, 2006). This is an example of the external dialogue becoming the inner dialogue of metacognition, is a goal in developing student metacognition, and is a goal of instruction promoting metacognitive development (Holton & Clarke, 2006).

Improving Metacognitive Thinking

Metacognition is important for mathematical success and research has suggested that metacognition can be increased through carefully selected and practiced activities such as scaffolding, group and class discussions, effective questioning, and making thinking visible (Kramarski & Zoldan, 2008; Lin, 2001; Schoenfeld, 1992; Schraw, 1998). Metacognitive development is not always an automatic process and the use of these instructional strategies promote a more complete and deeper metacognitive development among all students than can be expected to happen naturally over time; this is especially true for low performing students (Cardelle-Elawar, 1995; Mevarech, et al., 2010; Pintrich, 2002; van der Stel & Veenman, 2010). Using carefully planned activities to promote metacognitive development allows for greater transfer and generalizations of knowledge in new contexts (Fisher, 1998), richer mathematical discourse (Gillies, 2004), and a greater ability to construct and integrate new knowledge into what students already know (King, 2002).

Since students can learn to think metacognitively and metacognition is important for mathematical success, it is important that researchers continue to develop a better understanding of both how metacognition effects student learning and how future teachers can be prepared to develop metacognitive thinking in their own students. Lin (2001) suggests that future research should explore the type of support teachers need to mediate instructional activities that promote metacognitive development. To better understand how students develop their metacognition, researchers must first look where metacognition is already being developed so that they better understand what kinds of teacher knowledge, behaviors, and beliefs are necessary to create and implement a curriculum that supports metacognitive development (Schoenfeld, 1992). To better understand what is actually happening in the classroom, researchers must understand how teachers are putting metacognitive theory into practice in their own classrooms (Steele, 2001; Yimer & Ellerton, 2010).

Statement of the Problem and Research Questions

Current literature lacks description and exploration of currently embedded instructional strategies that promote metacognitive development. Instead, a majority of the current literature is focused on explaining the relationship between metacognition and mathematics success or determining if metacognition is something that can be improved by testing targeted strategy interventions or attempts to change the classroom culture for the sake of metacognition. Because of the close relationship between metacognition and the NCTM process standards, it may be possible for a teacher to promote metacognitive development in a classroom that is focused on implementing standards-based instruction as called for by the *Principles and Standards for School Mathematics* (National Council of

Teachers of Mathematics, 2000). The purpose of this mixed-methods case study was to investigate a mathematics teacher's embedded metacognitive development in her college-level geometry course while also attempting to determine the impact the course had on the students' overall metacognitive awareness. The specific research questions were:

1. What types of metacognitive thoughts are being modeled in the classroom?
2. How does a college mathematics instructor promote metacognitive thinking in her students?
3. Does a college-level course change the metacognitive awareness of pre-service elementary teachers?

This mixed-methods study was conducted with elementary pre-service teachers in an inquiry-based sophomore-level geometry content course. Data consisted of instructor interviews, observations of instruction, student demographic information, student surveys, textbook analysis, and student metacognitive awareness inventories. Interviews and select observations were transcribed verbatim and qualitatively analyzed for moments of metacognitive activity. The pre/post Metacognitive Awareness Inventory measured change in students' metacognitive awareness during the time of the study.

Limitations

This study relied heavily on self-reported data, from students and the instructor, to describe specific metacognitive instances they experienced and their reactions to these experiences. Due to the covert nature of metacognition, it was very difficult to observe when students were thinking about their own thinking. However, it was possible to observe when the teacher explicitly did something intended to prompt students to think about their thinking. A second limitation was the time frame for data collection. Due to scheduling difficulties, data was collected over a four-week period rather than the entire semester. The delay of the start of data collection allowed for the creation of class norms prior to the gathering of the preliminary data including the Metacognitive Awareness Inventory scores and the initial student survey. Lastly, this study was conducted with only one instructor and thus results are not generalizable beyond her classrooms. The intent, however, was to provide a glimpse of what was currently happening in this classroom to determine the next step for supporting students' metacognitive development.

Summary

There has been a call for important changes in mathematics instruction that help students learn to think more effectively and efficiently. Metacognition is thinking about one's own thinking and has been identified as indicative of better mathematics learning. Students can improve their metacognition by hearing the thinking of others and then comparing it to their own through carefully designed instructional strategies that promote effective discussions and questioning in an effective learning community. The next chapter discusses what metacognition is, describes metacognition within mathematics, explores some of the specific interventions that have been implemented in classrooms, synthesizes what the literature suggests is the ideal classroom environment for promoting metacognitive development, and closes with a description of the role of the teacher in this ideal metacognitive classroom. The third chapter outlines the research methods for collecting and analyzing the data, followed by a presentation of the findings in the fourth chapter. Lastly, the fifth chapter provides a discussion about what this study means for teacher education, mathematics education, and continued research in metacognitive development.

CHAPTER II

REVIEW OF LITERATURE

This study explored ways a college mathematics instructor embedded strategies to improve metacognitive thinking through her instruction and determined whether a college course could change students' metacognitive awareness. Therefore, it was important for the researcher to understand what the literature suggests about metacognition and the development of metacognition in mathematics learning. This chapter begins with current definitions and facets of metacognition, discusses the relationship between metacognition and mathematics, and then reviews examples of specific intervention strategies that have been implemented with the goal of improving students' metacognition. The chapter then describes characteristics of a classroom that are ideal for developing students' metacognition and closes by describing the role the teacher plays in creating the ideal classroom to promote metacognitive development.

What is metacognition?

Metacognition is most simply defined as thinking about thinking and allows students to make their thinking conscious and overt so that it can be an object for learning (Fisher, 2007). The domain of metacognition is diverse and has been recognized as not only important for student learning, but also tends to be inconsistent and lack coherence

(Sternberg, 1998; Veenman, et al., 2006). Throughout the years, metacognition has been given multiple, disjointed, or contradictory meanings or has been used in over-inclusive ways (Desoete, 2007; Kahwagi-Tarabay, 2010; Livingston, 2003; Schneider & Artelt, 2010). The definitions in Table 1 provide an overview of several definitions of metacognition. A possible explanation for the inconsistent definitions may be a result of Flavell's classic 1976 "kitchen sink" definition of metacognition (Schoenfeld, 1992). While this early definition lumped many categories of metacognitive thought together, these have since been sorted into more functional categories such as declarative knowledge about procedures and cognitive processes, self-regulatory procedures such as monitoring and decision making, and beliefs and effects on cognitive performance. Adding to the confusion of multiple definitions for metacognition is the use of several other terms that are very closely related to, but distinct from metacognition itself, such as meta-memory, self-regulation, executive control, metacognitive beliefs, metacognitive awareness, metacognitive experiences, metacognitive knowledge, feeling of knowledge, judgment of learning, theory of mind, metamemory, metacognitive skills, executive skills, higher-order skills, metacomponents, comprehension monitoring, learning strategies, heuristic strategies, and self-regulation (Livingston, 2003; Veenman, et al., 2006). For the purpose of this study we will borrow the most common two-component definition of metacognition as the knowledge of cognition and the regulation of cognition (Schraw & Dennison, 1994).

While many researchers take pieces from multiple definitions to create their own, there does seem to be an evolving trend based on the use of the word *meta* which refers to something that transcends the subject to which it is related (Fisher, 1998). Veenman, et

Table 1:
Sample Definitions of Metacognition

Author(s)	Date	Definition of Metacognition
Flavell	(1976)	The monitoring and control people have over their own cognition
Costa	(1984)	Our ability to plan a strategy, be conscious of the steps to reach the solution, and to reflect and evaluate on the process of getting to the solution
Garofalo & Lester	(1985)	An examining of one's own thoughts and knowledge
Swanson	(1990)	Knowledge and control one has over one's thinking and learning
Schraw & Dennison	(1994)	Knowledge and regulation of cognition
Sternberg	(1998)	Includes understanding of control and processes which are also complex
Lin	(2001)	Ability to understand and monitor one's own thoughts, assumptions, and implications of one's activities
Wilson & Clarke	(2002)	Awareness, evaluation, and regulation that individuals have of their own thinking
Holton & Clarke	(2006)	Any act that operates on a cognitive thought to assist in solving a problem
Holton & Clarke	(2006)	The monitoring, control, and evaluation of thought
Pyon	(2008)	The ability of students to explain their thinking through written words
Schneider & Artelt	(2010)	Knowledge or cognitive activity that takes or regulates a cognitive enterprise as its object

al. (2006) explains "most conceptualizations of metacognition have in common that they take the perspective of "higher-order cognition about cognition" thus students are monitoring and regulating their thinking while also doing the thinking, looking through a window to see and think about one's own thoughts (p. 5). Thus a key component in defining metacognition is that the object of the thinking must be *thinking*. Holton and

Clarke (2006) suggest that beliefs, intuition, and knowledge of one's own thought processes may influence metacognition, but are not in themselves metacognition because they are not thinking acts. While beliefs and intuition are commonly identified as impacting metacognitive thinking, the literature agrees that these are not in themselves metacognitive acts.

Cognition Versus Metacognition

Delineating what is cognitive (thinking) and what is metacognitive (thinking about thinking), can sometimes be difficult, but ultimately the interpretation depends on the actual use of the information (Flavell, 1979). Garofalo and Lester (1985) clarify the difference by explaining that cognition is *doing*, metacognition is *choosing*, *planning*, and/or *monitoring* what to do while Holton and Clarke (2006) explained cognition as the way a learner's mind works on the world, whereas metacognition is the way a person's mind acts on their cognition. Further, metacognition includes cognitive elements but cognition does not necessarily include metacognitive elements (Fisher, 1998). Examples of metacognition include recognizing that one is having more trouble learning concept A than concept B, having the thought that something should be double checked before accepting it as fact, looking over and thinking about all the answer choices of a multiple choice question before selecting an answer, having the sense that a note should be made about something so it is not forgotten, or deciding if one should be asked a question about something because he/she does not understand (Flavell, 1976). Metacognition does not, however, involve the use of rote strategies (Garofalo & Lester, 1985).

While the use of metacognitive thoughts is important for thinking and solving problems, these thoughts or the use of these thoughts does not guaranty accuracy. A given metacognitive act may, in fact, lead to a dead end rather than to a solution (Holton & Clarke, 2006). One's metacognition may be either accurate, inaccurate, or fail altogether (Flavell, 1979; Schoenfeld, 1992; Wilson & Clarke, 2002). Metacognitive thinking can be practiced to the point that it becomes automatic, which does reduce cognitive load; however, this automatization makes it difficult for students to report and share their thinking with others (Martinez, 2006; Schraw & Moshman, 1995; Sternberg, 1998).

While some people have argued that cognition and metacognition are the same, Veenman et al., (2006) explained that the two are unique because the object of thought differs between cognition and metacognition. Cognitive knowledge does not guaranty metacognitive knowledge (Schraw, 1998). Metacognition is not just about what knowledge and skills you have, but what you actually do with those skills. Metacognition is about having an awareness of one's cognition and regulating one's actions based on what one knows and the decisions one makes based on what he or she knows. Trying to categorize the wealth of knowledge one knows and what he or she does with this knowledge is quite a daunting feat and many different schemas for categorizing metacognition have evolved.

Knowledge and Regulation of Cognition: A Common Taxonomy

Researchers have provided a variety of taxonomies for classifying components of metacognition. Common categories for organizing metacognitive thoughts include components such as metacognitive awareness, metacognitive experiences, prediction, planning, monitoring, evaluation, self-regulation, and metacognitive beliefs. Holton and Clarke (2006) cite Schoenfeld's suggestion that metacognition is comprised of self-regulation, knowledge of one's own thought processes, and beliefs and intuitions. While many researchers agree that a more precise taxonomy of metacognition is needed, most research tends to use the two component approach of knowledge of cognition and regulation of cognition (Veenman, et al., 2006). This taxonomy consists of a split between knowledge of cognition (what one knows) and regulation of cognition (what one does with what they know) (Schraw & Moshman, 1995). While these two components are indeed distinct from each other, they are also highly inter-correlated (Schraw & Dennison, 1994; Schraw & Moshman, 1995). See Figure 1 for a diagram the highlights the components of metacognition.

Knowledge of cognition and regulation of cognition each make a unique but strongly inter-correlated contribution to metacognitive activity. Some researchers suggest that the two components work in unison to help students self-regulate (Schraw & Dennison, 1994). While there is a great deal of speculation regarding the development of each area, there is a significant difference in the regulation of cognition between undergraduate and graduate students, but not a difference between knowledge of cognition (Young & Fry, 2008). This finding seems theoretically sound since students must first have the knowledge of what they are thinking before they are able to regulate their thinking.

Knowledge of Cognition

Knowledge of cognition refers to what students know about their cognition, how to use strategies or procedures, and why or when to use a particular strategy (Kramarski & Zoldan, 2008; Schraw, 1998). Within cognitive knowledge, *declarative* knowledge refers to knowledge of one's self, skills, and resources; *procedural* knowledge refers to knowledge about how to do and implement a learning procedure; and *conditional* knowledge refers to when and why to use declarative and procedural knowledge (Schraw, 1998; Schraw & Dennison, 1994).

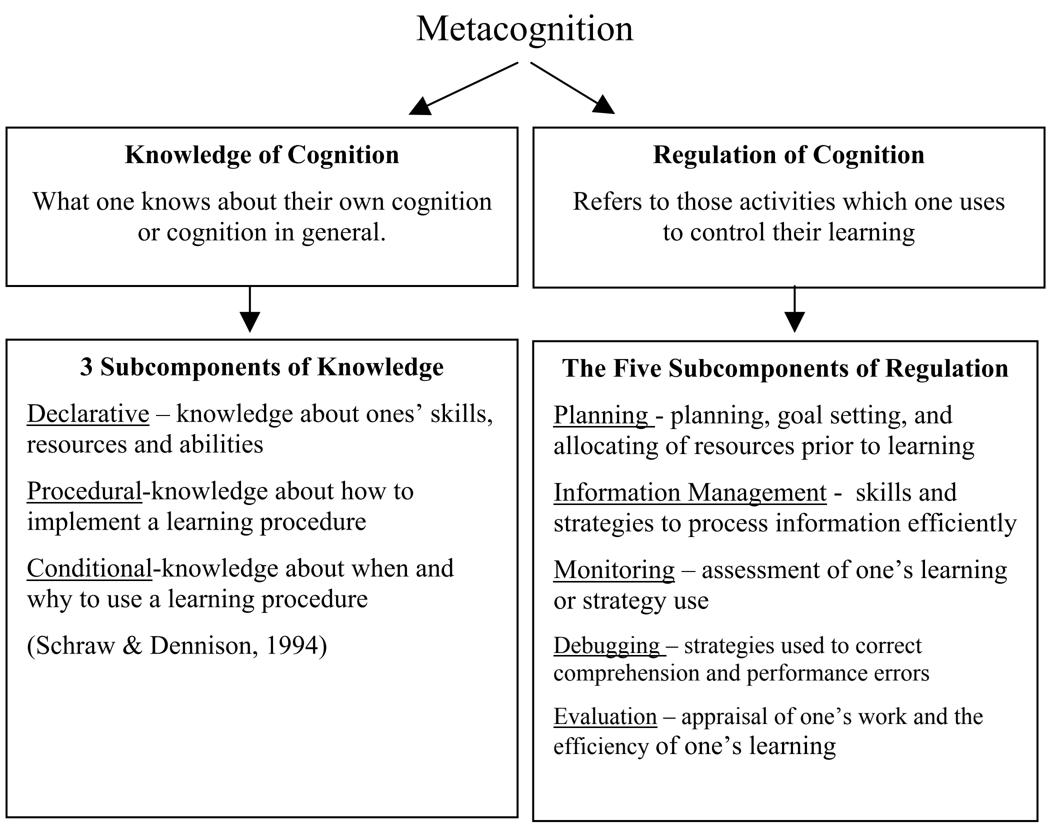

Figure 1. Components and Subcomponents of Metacognition. Adapted from, "Assessing Metacognitive Awareness," by G. Schraw and R. Dennison, 1994, *Contemporary Educational Psychology, 19*(4), p. 460-475.

When sorting knowledge of cognition into one of these three categories, it is important to remember that an "act" usually encompasses more than one category, thus sorting "acts" into only one of the three categories is extremely challenging (Garofalo & Lester, 1985). Metacognitive knowledge refers only to the knowledge about cognition. Metacognitive knowledge leads you to plan, monitor, evaluate, and revise cognitive progress and can thus lead to a wide variety of metacognitive experiences (Flavell, 1979). Simply recognizing how well you know something isn't necessarily the end task, rather one must decide what to do with the information and how strongly to believe what one thinks he or she knows (Flavell, 1979). To be considered metacognitive knowledge, rather than just general knowledge, one must be *doing* something with the knowledge. To simply identify a strength or weakness is not in itself a metacognitive act, but to choose one strategy over another because of a perceived strength or weakness of that thought now makes the knowledge or act metacognitive (Livingston, 2003).

Regulation of Cognition

Regulation of cognition commonly refers to planning, monitoring, and evaluating one's cognition (Sternberg, 1998), but may sometimes also be described as including information management and debugging (Schraw & Dennison, 1994). Planning consists of activities such as goal setting and resource allocation prior to learning. Information management refers to skills and strategies used to process information. Monitoring consists of the ongoing assessment of one's learning or strategy use. Debugging consists of strategies to correct comprehension or performance. Lastly, evaluation consists of appraising one's work and/or efficiency completing the work (Garofalo & Lester, 1985; Kramarski & Zoldan, 2008; Schraw & Dennison, 1994). Setting sub goals, self-questioning, checking answers, rereading, and finding computational errors are further examples of regulatory activities (Pintrich, 2002). An important distinction between knowledge of cognition and regulation of cognition is that while knowledge of cognition refers to actually knowing these strategies, the category of regulation refers to the actual use of these strategies (Pintrich, 2002). Metacognitive regulation consists of activities that improve performance through better use of attentional resources and existing strategies along with a greater awareness of comprehension breakdowns (Schraw, 1998).

A person may be stronger in one regulatory category than in another (Sternberg, 1998), but researchers hypothesized that improving one aspect of regulation will also improve other aspects (Schraw, 1998).

Metacognitive Awareness Inventory

Metacognitive awareness is an awareness of one's metacognition, it helps students build autonomy and thus is important for helping students see themselves as agents of their own thinking (Fisher, 2007). The Metacognitive Awareness Inventory was created by Schraw and Dennison (1994) to provide a quick and more reliable way to identify a person's metacognitive awareness than the commonly used interview/observation procedures. The authors had three main purposes in creating the inventory: (a) to test the idea of the two main processes of metacognition, knowledge of cognition and regulation of cognition; (b) to address the statistical relationship between knowledge of cognition and regulation of cognition since they are commonly accepted to be mutually correlated and compensatory; and (c) to validate the instrument by comparing the measures from the instrument with monitoring ability, test performance, and accuracy of monitoring one's test performance. Two experiments were conducted to address the three goals. Experiment 1 established reliability by addressing the first two purposes and Experiment 2 established validity by addressing the third.

Experiment 1 was conducted with 197 undergraduates in an introduction to education psychology course. Initially there were 120 items created, but only 52 remained on the final version of the instrument after items were dropped based on extreme mean scores, high levels of inter-correlation, and high variability in pilot studies. The 52 remaining items provided at least four items for each of the eight scales of declarative knowledge, procedural knowledge, conditional knowledge, planning, information management strategies, monitoring, debugging strategies, and evaluation of learning. A 100mm bipolar scale was used for experiment 1. The unrestricted factor analysis suggested six factors rather than eight and did not have highly reliable factors. Each pair of factors was correlated in excess of r = .30, hence the researchers conducted a restricted factor analysis.

The restricted factor analysis conducted as a second part to experiment 1 consisted of both an oblique and an orthogonal two-factor solution, both of which provided similar results. Twenty-five items from the questions in the knowledge of cognition category loaded unambiguously onto factor 1, 19 items from the question in the regulation of cognition category loaded unambiguously onto factor 2, six items had a loading factor greater than .30 on both factors, and two items loaded onto neither factor. The six questions that loaded onto both factors spanned across five of the eight theoretical subcomponents and on both areas of knowledge and regulation. The two items that failed to load on either factor were both regulation items, but varied within the eight subcomponents. The coefficient for items loading on each factor reached .91 while the coefficient for the entire instrument reached .95. Thus Schraw and Dennison concluded that the MAI does reliably measure knowledge of cognition and regulation of cognition as predicted in the literature. Further, knowledge of cognition did indeed consist of declarative, procedural, and condition knowledge while regulation of cognition consists of planning, information management, monitoring, debugging, and evaluation. Additionally, since the factors are highly inter-correlated it may be suggested that knowledge of cognition and regulation work together.

Experiment 2 was used to both support experiment 1 and to validate the instrument as a whole. While there were some slight differences in how the items loaded, overall, similar results were obtained that support the two factors of metacognition, knowledge of cognition and regulation of cognition. To establish validity, the researchers compared scores from the MAI with a pre-test judgment of monitoring ability, test performance score, and monitoring accuracy score. For each comparison, the participants were sorted into high/medium/low categories for each of the independent variables while knowledge of cognition and regulation of cognition scores were used as the dependent variables throughout. Based on the findings, Schraw and Dennison (1994) concluded that knowledge and regulation are related, but each uniquely contributes to cognitive performance.

While Flavel (1979) is commonly credited as the earliest to define, discuss, and theorize metacognition, Schraw and Dennison (1994) provided an empirical argument for the two-component theory of metacognition that is referenced in nearly every discussion regarding metacognition. Upon its development, the MAI was intended to provide an accurate and efficient method for identifying a person's metacognitive awareness. The next few paragraphs provide an overview of how this instrument has been used in research since its development.

Abd-El-Khalick and Akerson (2009) conducted a pre/post quasi-experimental study with 49 pre-service elementary teachers in a science methods course that employed metacognitive strategy instruction including concept mapping, researching the development of ideas of peers, and responding to case studies as the intervention for the control group. The researchers used the MAI with a 10-point likert scale, the View of the

Nature of Science Questionnaire, and follow-up interviews to suggest strategy intervention did increase the students' metacognitive awareness, knowledge of cognition, and regulation of cognition. Additionally, the researchers suggested a relationship existed between improved metacognitive awareness and enhanced views of the nature of science, meaning the teachers' ideas were more in line with reforms occurring in science education. The findings suggested that metacognitive strategies should be included in instruction to improve science teachers' views of the nature of science and suggested further studies to determine the long-term retention of the enhanced views and to understand the causality in the relationship between metacognitive awareness and the teachers' views of the nature of science.

Sperling, Howard, Staley, and Dubois (2004) reported two separate studies within their publication. The first study consisted of 109 first-year students in an academic strategies class while the second study consisted of 40 juniors and seniors in an educational psychology course. These participants completed the 5-point likert scale MAI, the Learning Strategies Survey, and the Motivated Strategies for Learning Questionnaire. Other data included the SAT math and verbal scores, and high school grade point averages. Results suggested a significant correlation between knowledge of cognition and regulation of cognition, positive and significant correlations between metacognition and strategy use, a negative correlation between SAT math and the MAI. Sperling et al. (2004) also suggested future studies should include the development of effective interventions.

The MAI was developed to provide a more efficient means for obtaining a level of metacognitive awareness for students so these skills are targeted and receive appropriate instructional interventions. Researchers have remained true to this idea and continue to seek ideas for helping students improve their achievement based on their metacognitive awareness.

Metacognition and Mathematics

Metacognition has had a long history with mathematics education and is now commonly recognized as important to problem solving (Yimer & Ellerton, 2010). Flavell argues in his later works that by helping students bring their thinking to a conscious level, students become more aware of their own thought processes and attain greater control over their thinking, which, in turn, helps them to gain control over the organization of their learning (Fisher, 2007). However, the skills necessary to make thinking conscious need time to develop (Garofalo & Lester, 1985). These skills become especially important as students begin to move beyond basic computations to clarify goals, to lessen confusion, to understand and monitor important concepts, to make predictions, and to choose appropriate actions (Gourgey, 1998).

Metacognition and Mathematics Success

Metacognition about specific strategies, knowing when and why to use a strategy, rather than simply identifying the strategies themselves are critical for students' learning and understanding (Kramarski, et al., 2010; Lin, 2001; Sternberg, 1998). Students are more likely to use a variety of strategies when they possess the understanding of the *when* and *why* knowledge, than when they only have the knowledge of *how* (Pintrich, 2002). Therefore, effective instruction should be explicit about the use of generalizations as well as the naming of strategies and thoughts about when to use and why to use specific thinking strategies (Kramarski, et al., 2010; Schraw, 1998).

To further the argument for the importance of developing student's metacognitive thinking, success in mathematics is ultimately tied to ability, motivation, and metacognition and is a greater predictor of mathematics performance than intelligence (Bobby Hoffman & Spatariu, 2008; Swanson, 1990; Veenman & Spaans, 2005). Swanson's (1990) foundational study used extreme case sampling with fourth- and fifth-grade students grouping them by metacognition and intelligence. The study uses metacognitive questionnaires and problem-solving think alouds, where students verbalize their internal thoughts, to suggest that individuals with high levels of metacognition outperformed students with low levels of metacognition regardless of their aptitude while also requiring fewer steps reported to solve the problem. These results suggest that high levels of metacognition can compensate for overall ability. The author suggested, as a possible explanation, that high metacognition students were more likely to generate and test hypothesis and transition better from one hypothesis to another once the initial test failed. Further, high aptitude/high metacognition students tend to have a richer variety of heuristics and strategy subroutines which suggests a greater advantage for if-then thinking and prioritizing strategies which leads to a need to understand how metacognition is studied in mathematics.

How Metacognition is Studied in Mathematics

There are two distinct areas of research that exist under the metacognition umbrella. The first is understanding metacognition itself and the second is understanding how to develop students' metacognition. During the early years of studying metacognition, researchers often suggested incorporating metacognition into an adopted framework or model that already existed. One of the most commonly adopted frameworks was Polya's problem-solving steps, but some researchers warned that Polya's conceptualization only implicitly considered metacognition (Garofalo & Lester, 1985). Commonly accepted methods for assessing metacognition include self-report questionnaires, think-aloud protocols, systematic observations of behavior, interviews, stimulated recall, online computer log file registrations, overt behavior or utterances from metacognitive activities, and group discussions (Desoete, 2009; van der Stel & Veenman, 2010; Veenman, et al., 2006; Wilson & Clarke, 2002).

Two of the greatest challenges to studying metacognition is getting the thoughts out of a person's head so these thoughts can be studied and determining what to look for in observations. Flavell provides some of the earliest guidance for studying metacognition in his advice to Resnick and Glaser which suggests that they follow their own understandings for how to solve problems; find out what is going on in a child's brain versus relying solely on observed behaviors; recognize that much of human thought is multi-directional, erratic, hard to follow, and difficult to model; and devise naturalistic versus academic tasks to provide rich data (Flavell, 1976).

The second major body of metacognitive research consists of how to develop a students' metacognition, metacognitive thinking, and metacognitive awareness and is the focus of the remainder of this chapter, as well as the focus of this current study. Research within this body of literature tends to adopt either strategy training or the creation of a supportive social environment as the basic approach for promoting metacognitive growth and either specific domain content or knowledge of self as the content within the basic approach (Lin, 2001). While all aspects and combinations of aspects are important and provide a balanced approach to metacognitive development, most research tends to focus on one approach and one content (Lin, 2001).

Problem Solving and Think Alouds

Problem solving requires a cyclical process between cognition and metacognition, but the importance of metacognition is often not recognized until the tasks are more challenging and no longer automatic. The use of metacognition does not, however, automatically guaranty problem-solving success and experienced problem solvers tend to spend a great deal of time creating a plan and deciding how to monitor its effectiveness before delving into a new task (Schoenfeld, 1992; Schraw & Moshman, 1995; Wilson & Clarke, 2002). Students, however, do the opposite. When faced with a new problem they tend to immediately start performing operations on numbers rather than thinking carefully about the problem, devising a plan, and determining how to monitor their progress. Students also rarely recognize and reflect on answers that do not make sense in the context of the problem (Gourgey, 1998; Schoenfeld, 1992). Effective problem-solvers clarify goals, seek to understand relationships, monitor their understanding, and determine if they are making progress toward the desired goal (Gourgey, 1998), all of which are metacognitive activities. During problem solving, students' knowledge of mathematics concepts interacts with their control, understandings, and use of solution strategies to work towards a final solution, hence students' cognition about mathematics continuously interacts with their metacognition about their progress towards a solution (Gourgey, 1998).

Students of all ages tend to be deficient in the necessary skills of monitoring and assessing, both of which are skills for general mathematical and problem-solving success

(Garofalo & Lester, 1985). This deficiency may be related to the lack of metacognitive instruction occurring in classrooms where instruction tends to focus on mathematical procedures and techniques, but neglects the role of metacognition in situations students need to determine when and why to use a particular technique (Depaepe, De Corte, & Verschaffel, 2010; Gourgey, 1998). Interestingly, teachers do tend to say they support strategies commonly accepted as promoting metacognitive development, such as think alouds and problem solving, and teachers do commonly implement the desired classroom structure, such as group work and discussions. However, teachers fail to facilitate or explicitly discuss the metacognitive aspects of the thinking or focus on metacognitive activities occurring in their classrooms (van der Walt & Maree, 2007). This is unfortunate since students can internalize the questioning process and begin to recognize the struggle of working on metacognitive processes as worthwhile while becoming more efficient at metacognitive thinking through repeated practice and ongoing encouragement (Gourgey, 1998).

Designing instruction which requires students' debriefing of problem solutions and sharing of thinking is often difficult to create since students avoid the risk of exposing their thoughts and ideas when they feel their solutions are not correct or their thinking is not valued (Leat & Lin, 2003). This struggle to create an effective classroom environment may provide a possible explanation for why, even though the foundational structures are present and the beliefs are present, teaching students to monitor and control their thought processes is often neglected in mathematics classrooms. The sharing of and reflecting on thinking is foundational for promoting metacognitive development.

Students' explanations of their thoughts serves multiple purposes. First, students' explanations of their thoughts make public their own reasoning and justification (Steele, 2001). Second, students' explanation of their thoughts can serve as an assessment of their awareness of their own thinking (Costa & Marzano, 1987). Through their descriptions, students think about the processes they need, data they need, potential plans, and the consequences of those plans, thus learning to think about their thinking and, with practice, increase their metacognitive awareness. The increase in metacognitive awareness starts a chain reaction that helps students become able to trace their problem solving path from the start to finish, including dead ends and justifications while also becoming more perseverant, prideful, self-correcting, and autonomous in their work (Costa, 1984; Costa & Marzano, 1987). Through the process of formulating, sharing, and comparing, students also become more aware of and understand the perspective of the listener and correct themselves as they explain their reasoning. In this way, students learn to think critically which commonly leads to a higher level of reasoning (Goos, Galbraith, & Renshaw, 2002; Kamii, 1991). The monitoring and critiquing that occurs during student interactions provides students opportunities to internalize a variety of strategies that are both cognitive and metacognitive and thus promote metacognitive development.

Socially Mediated Metacognition

Promoting metacognitive development does not end with merely getting students' thinking out into the discussion and available for analysis. Facilitating and navigating a discussion about or taking action because of the shared thinking is the critical next step. These discussions can create a zone of proximal development for those involved, supporting students as they move from what they know to new knowledge. Monitoring and regulation, the two prongs of metacognition, are commonly recognized as in-the-head processes and likened to internalized self-instruction. Some researchers have extended this view and consider collaborative conversations between like ability students to be an overt representation of the internalized metacognitive monitoring and regulation. This idea of socially shared metacognition (Hurme, Merenluoto, & Jarvela, 2009) or socially mediated metacognition (Goos, et al., 2002) can provide a model of the metacognitive thinking that students should ultimately internalize, make their own, and use in new problem contexts. Students must challenge, clarify, and endorse new ideas that may be helpful. Students must be accountable for explaining, justifying and challenging ideas, strategies, and solutions that do not make sense or are incomplete (Goos, et al., 2002).

Socially shared metacognition requires that a group member contribute a metacognitive message about how to process or carry out a task and then, other members acknowledge and further develop the message (Hurme, et al., 2009). Students participating in a group setting where socially shared metacognition is present experience reduced feelings of difficulty when working alone and may be attributed to the presence and use of regulation messages provided by the group members that contribute to the direction of how to proceed through the task. These regulatory metacognitive messages are intended to regulate, interrupt, change, promote, and steer group processes as the students work together for the solution (Hurme, et al., 2009). In promoting metacognitive development, an ultimate goal would be for a student to then reflect on and internalize the group solution process so that they can repeat the process alone if faced with a similar task. The role of socially mediated metacognition is to scaffold the student across the zone of proximal development.

On the other hand, students participating in a group setting that is lacking socially shared metacognition tend to report that the task is more difficult than if working alone and this may be attributed to the surface level cognitive and social interaction that lacks domain and/or metacognitive knowledge. These conversations tend to compare only answers from individual work and do not utilize each other's thinking to work through the task, but instead when only the results are shared, students feel the task has been accomplished when they have fulfilled the requirements of the problem (Hurme, et al., 2009). Creating and supporting group collaborations rich with socially shared metacognition provide a model of the metacognitive processes students can and should internalize to further develop their own metacognitive thinking and mathematical success.

Attitudes Toward Metacognition and Metacognitive Instruction

Metacognitive skills develop slowly throughout the school years, but there is always room for improvement through adolescence and adulthood (Schneider & Artelt, 2010). One of a teacher's most difficult challenges is to motivate students that have consistently been rewarded for mindlessly learning facts and procedures to start thinking and developing metacognitively (Gourgey, 1998; Sternberg, 1998). Learning to think and develop metacognitively is difficult for students because they do not know how to or why they should be more active in their learning and are often uncomfortable with the extra effort required (Gourgey, 1998). Teachers must be patient and persistent and find the delicate balance between building confidence, leveraging availability and producing strategies from within the student, all while helping students internalize the metacognitive processes (Gourgey, 1998; Bobby Hoffman & Spatariu, 2008). Sternberg (1998) critiques that "the fact that students often do not welcome metacognitive training shows, in [his] opinion, a failure in our schooling" and continues by expressing the importance of metacognitive skills for learning and using information and knowledge (Sternberg, 1998, p. 130). Luckily researchers and classroom teachers have been working together to design and implement strategies to make this process easier.

Interventions Used in Classrooms for Research

The first part of this chapter discussed the variety of definitions of metacognition, the relationship between metacognition and mathematics, and the importance of working to improve one's metacognitive thinking. Since metacognition is important for student success in mathematics and can be developed, researchers have designed and tested a plethora of ideas to promote metacognitive development in students. The next part of this chapter provides a few examples of specific interventions, such as problem solving, modeling, prompting, IMPROVE, and cooperative learning that have been used in mathematics classrooms to improve metacognitive thinking and/or metacognitive awareness.

Problem-solving Context

One of the most common methods for providing metacognitive instruction is to combine it with another already well-developed framework such as a problem-solving framework. Cardelle-Elawar (1995) provided teacher training for 12 teachers to implement metacognitive strategies in conjunction with problem-solving processes in their third through eighth grade classrooms. The purpose of the study was to determine the impact of the training on student achievement and attitudes towards mathematics. Student achievement and attitudes were contrasted within six control classrooms and six experimental classrooms. In the experimental classrooms, teachers implemented traditional, lecture-based mathematics instruction where students were primarily passive learners and were focused on obtaining the right answer. The students in these classrooms were mostly low-achieving, Hispanic, and low-socioeconomic students. They completed a mathematics achievement instrument and a mathematics attitude survey. On the other hand, students in the experimental classrooms were presented more open-ended lessons and commonly asked questions such as:

- Do I understand the question?
- Why do you think this strategy is most appropriate for solving this problem?
- What will happen if I do this instead of that?
- What made you think that was an error?
- Is there another way to do this problem?
- Do you have all the information you need?
- Do you know how to organize the information you have?, and What operations do I struggle with?

The questions were part of the Mayer's method for solving problems and guided students in their problem-solving venture while also providing information to the teacher (Cardelle-Elawar, 1995, p. 84).

Lessons in the experimental classrooms consisted of three main sections (a) an introductory discussion used to model the self-questioning process, (b) independent work to allow students to practice while the teacher roamed the classroom providing metacognitive feedback to help students focus on errors and provide direction for self-correction, and (c) the final stage which allowed students the opportunity to summarize their learning for the day.

The integration of Mayer's Problem Solving strategies as a method to promote metacognitive instruction was considered a success by the researchers because students became more independent thinkers and provided more metacognitive control (Cardelle-Elawar, 1995). In this setting, errors were considered a source of learning and students' improvement in their own ability to self-correct was considered an accomplishment. This ability to self-correct did not seem to develop in the traditional classrooms as students continued to rely on the teacher for the right answer. Lastly, metacognitive instructional training provided a structure that helped low-achievers learn to think for themselves and promotes problem-solving success. Researchers suggested this change may be a result of learning to think reflectively; determining whether the information they have is what they need, and deciding what to do with that information once it has been identified.

A study by Depaepe, et. al (2010) contrasted the instructional practices of two teachers selected from ten originally engaged in a pilot study. The resulting study provided a comparison between these two teachers who reflected differing values toward the use of metacognition and a heuristic approach to teaching problem solving. Conducted with sixth grade Flemish students and consisting of weekly observations for seven months, this study provided data from teacher interviews, student group interviews, and a word problem test taken by the students. While teachers in this study did not receive specific intervention training, the purposeful selection of these two teachers, based on their extreme models of problem-solving instruction, provides a naturalistic look at classroom procedures for the differing instructional approaches.

Findings from this study suggest that the teacher, rather than the student, provided most references to metacognition or heuristics. Neither teacher discussed creating a plan, but dialogues about how and why to use particular strategies were discussed more frequently in the classroom of the teacher that valued the problem-solving approach. Despite the type of instruction, most students valued their classroom learning experiences. The researchers concluded that the teacher who placed greater value on the problem-solving heuristics approach provided a more metacognitive instructional environment for her students, although there was not a significant difference between the two sets of students scores on their word problem test. This finding is in contradiction to the now commonly accepted notion that metacognitive instruction will improve a student's ability to solve math problems and thus warrants further investigation (Depaepe, et al., 2010).

While the previous study discussed differences in the use of metacognition between two teachers who incorporated varying levels of problem solving in their classrooms, Lazakidou, Paraskeva, and Retalis (2007) focused on the use of Sternberg's metacognitive development theory with 48 fourth-grade students in the classrooms of three experienced teachers. Sternberg's theory uses questions before, during, and after solving a problem as prompts for student thinking which integrates metacognitive questioning and prompting into problem-solving. Data was collected over an entire year, but the targeted instruction lasted only two months. Findings suggested that metacognitive instruction is most effective when integrated throughout the problem-solving experiences and students with a medium problem-solving ability tend to experience the most growth in metacognitive skills when they are given the opportunity to work collaboratively with their classmates.

Problem solving is a common context for integrating metacognition and this final study has become a classic example of research in this area. Yimer and Ellerton (2010) focused on the use of cognitive and metacognitive activities that occurred while pre-service elementary teachers specializing in middle school mathematics solved problems throughout their mathematics course. Data for this study included semi-structured task-oriented interviews, small-group and whole-class observations, stimulated recall interviews, and students' reflective journal writings. The goals of the study were to identify and describe change in students' metacognitive functioning over the course the semester while also describing how students valued metacognition and problem solving.

Findings suggested there is a symbiotic relationship between cognition and metacognition and students tend to move effortlessly between the two levels, supporting the idea that actions that may be cognitive for some students are metacognitive for others and vice versa. This study confirmed the importance of engagement, transformation, implementation, and evaluation as important elements while also emphasizing the important role of a final reflection over the entire problem-solving process for helping students internalize what they have learned. This final phase of reflection requires students to move beyond a mere reflection on the overall process for solving problems and assimilate those paths and strategies used for solving that particular problem into their overall knowledge system.

Each of these studies used a problem-solving context to help students improve their metacognition. The emphasis was on problem-solving instruction and it was through the problem-solving experiences that the researchers worked to test, extend, and/or elaborate on current understandings of metacognitive development in the classroom. Cardelle-Elaware (1995) trained the teachers who then trained the students to internalize metacognitive thinking through problem solving. Depaepe, DeCorte, and Verschaffel (2010) compared the metacognitive thinking of students in classrooms with differing levels of active problem solving integrated in the instruction. Lazakidou, Paraskeva, and Retalis (2007) taught Sternberg's integration of problem solving, cognition, and metacognition to students and then described their metacognitive changes over the year. Lastly, Yimer and Ellerton (2010) described changes in pre-service teacher's metacognition through an entire problem-solving course. While the methods of each study were different, each supports the idea that metacognition can be developed and that it can be developed through problem-solving experiences.

Metacognitive Prompting

Unlike the previously discussed idea of teaching problem solving to promote metacognitive development, the use of metacognitive prompting is a strategy designed to serve as an external stimulus that prompts the participant to reflect or to evoke a strategy with the intent of enhancing the learning objective (Bobby Hoffman & Spatariu, 2008). The idea relates to strategies such as self-monitoring, self-questioning, self-reflection, and self-explanations and shares a similar intent of helping the participant learn to internalize the need and ability to invoke metacognitive thinking when facing a challenging task. As previously discussed, the knowledge of the strategy does not guaranty its use or the successfulness of its use. Metacognitive prompting is designed to help students learn when and why to implement a particular strategy and how to justify its use to help the student internalize when and why to use that particular strategy in the future. Examples of metacognitive prompts include questions related to similar previous problems, identifying steps that can be used to solve the problem, determining the validity of the solution, judging efficiency, identifying more efficient strategies, and determining the best method for solving a problem (Bobby Hoffman & Spatariu, 2008). Metacognitive prompting moves beyond the use of feedback, which only provides knowledge of results and or corrective information, to stimulating reflection that supports metacognitive monitoring.

Hoffman and Spatariu (2008) used a regression design to test the effectiveness of metacognitive prompting on the correctness of the multiplication responses, the time required to find the solution, and the efficiency of the problem-solving process while controlling for mathematics ability and self-efficacy. Findings from the study suggested that metacognitive prompting does promote problem-solving success in willing and able participants while also helping students become more efficient at solving problems. Already successful problem solvers and those who are unwilling or unmotivated to devote resources to problem solving may find metacognitive prompting unnecessary or intrusive. There is a need to control for students' background knowledge to determine the role self-efficacy plays on problem solving between the metacognitive prompting and no prompting groups. As a last note, metacognitive prompting initially leads to an awareness of multiple strategies and thus may lead to longer, on-task problem-solving time and initially more inefficient routes to the solution, but the researchers found that as metacognitive prompting becomes an automatized and internalized process the overall time and efficiency was positively impacted (Bobby Hoffman & Spatariu, 2008). At first this last finding may seem contradictory; however, it is somewhat like the phrase "it will get worse before it gets better." Learning a new skill almost always requires more time initially than will be required once the skill is mastered. This is true when using metacognitive thinking as well.

IMPROVE

Regarding interventions designed to promote metacognitive development in students, perhaps the most proliferous lines of research stem around the IMRPOVE method of instruction introduced by Kramarski and Mevarech in Israel. IMPROVE is an acronym for Introducing new concepts, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment. Since introducing this method of instruction in 1997, Kramarski and Mevarech have created a cohesive body of knowledge that suggests that IMPROVE is a highly effective method for promoting metacognitive development and tends to be even more effective when combined with other effective instructional methods such as cooperative learning, error analysis, classroom discourse, problem solving, heterogeneous classrooms, and prompting.

The IMPROVE method, first tested with seventh graders, integrates social cognition and metacognition theories, and consists of three independent components, metacognitive activities, peer interaction, and systematic provision of feedback-corrective-enrichment (Mevarech & Kramarski, 1997). Each of the words in the acronym serve as a stage in the overall lesson where the instructional strategy was designed to make thinking more explicit by talking about knowing what to do, looking at the big picture, finding when, why, and how to do particular strategies and thinking, and focusing on things to think about (Kramarski & Zoldan, 2008). The initial studies for this method confirmed that IMPROVE has a strong impact on mathematical learning and reasoning and can be implemented throughout the year (Mevarech & Kramarski, 1997).

One of the more recent studies using the IMPROVE method was conducted with third and sixth grade Israeli students. The findings suggest that IMPVROVE does benefit students at both grade levels, but had more impact on third grade students. Further, the IMPROVE method of instruction has more impact on word problems that require students to think through the problem-solving process rather than relying on keywords, thus emphasizing the importance of metacognitive instruction in math content instruction (Mevarech, et al., 2010). This study consisted of a pre/post 16-item word problem test, with IMPROVE instruction being provided to the entire class during the single month between the pre- and post- test.

Another study that used this instructional approach combined IMPROVE with diagnosing errors to investigate the effects of combining the approaches on students' reasoning skills on linear functions, the effects on reducing conceptual errors, and the impact of the approaches on the students' metacognitive knowledge (Kramarski & Zoldan, 2008). The diagnostic errors approach required students to evaluate other students' answers, identify a wrong answer, reflect on the reasons why the wrong answer occurred, and how students knew the answer was wrong. Diagnosing errors along with the IMPROVE method was most effective at increasing metacognitive knowledge, reducing the number of conceptual errors, improving the number of mathematical explanations, and increasing the students' ability to use mathematical procedures and problem solve. Students receiving each strategy alone, diagnosing errors and IMPROVE, experienced statistically significant improvement across all dependent variables. The control group did not experience any improvement. Lastly, the students that received training in both diagnosing errors and the IMPROVE method experienced the greatest changes in all categories measured. Results from this study also supported the importance for generating questions before, during, and after task performance because these questions help students understand what is most important and thus where to focus their attention. Based on their findings, Kramarski and Zoldan (2008) called for a metacognitive classroom culture that values making connections, encourages students to formulate plans about how they learned and how they will remember the new material, encourages self-questioning and error analysis, and supports the use of a social environment as the key element to the metacognitive classroom culture.

Cooperative Learning

The IMPROVE method integrates theories from both social cognition and metacognition with the intent of promoting metacognitive thinking in students. An important element in the IMPROVE environment is a classroom culture that promotes thinking and discussion about thinking. Because of the important role of discourse in both cooperative learning and metacognitive development, Kramarski (2004) chose to investigate the effects of the two instructional strategies on students who were working to make sense of graphs and whether there were exhibited differences in the discourse between the two groups. One hundred ninety-six eighth graders, across six Israeli classrooms, at two different schools were randomly assigned by class to the cooperative learning only group or the cooperative learning and metacognition group. The collected data came from a graph interpretation task, a graph construction task, and interactions from recorded group discussions.

Findings suggest that mathematical discourse was promoted under both conditions, but cooperative learning paired with metacognition had a greater positive impact on learning. This finding could possibly be attributed to the elaborate explanations that were part of the discussions in the combination groups whereas the cooperative students' discussions tended to involve only technical help. Further, it was also suggested that metacognitive instruction promoted transfer of graph interpretation knowledge to the graph construction task. This study supports the idea that asking students "why" questions helps them elaborate and retain information as they explain, clarify, expand on, and justify their thinking especially regarding the thinking of others. The discourse in both contexts included metacognitive talk and opportunities for students to discover the relevant information, lead to cognitive conflicts that they had to work to resolve, required mutual reasoning to work through, and engaged students in reflective discourse, all of which are critical for metacognitive development (Kramarski, 2004).

Given the importance of discourse and the need to share thinking so that it can be thought about by others, it is important to give all students the opportunity to both speak and listen. This environment requires the instructor to play a different role in the classroom by facilitating effective conversations and discussions rather than maintaining silence while the teacher lectures. Gillies (2004) identified the important skills teachers needed to facilitate effective discussions and provided professional development training specifically designed to help teachers embed cognitive and metacognitive questioning within a cooperative group learning structure. Gillies' study focused on 25 fifth through seventh grade teachers from 10 schools in Australia who taught a total of 772 students. Data from this study included both teacher and student discourse analyzed according to time spent doing different tasks. The results suggest that students of teachers that were trained for both metacognitive questioning and cooperative learning obtained higher learning outcome scores and created a richer classroom discourse than those from teachers trained only in cooperative learning (Gillies, 2004; Gillies & Khan, 2009). Further, the teachers trained for both instructional approaches demonstrated more mediating behaviors than their peers who received only cooperative learning training. The researchers theorized these results were due to a greater awareness and ability to mediate student thinking and teacher learning during the teacher training. When students are taught how to ask questions and challenge each other, they become aware of the importance of providing complete and elaborate responses (Gillies & Khan, 2009).

The previous section focused specifically on interventions that were used by researchers to promote metacognitive development in students. Some studies focused on training the teacher while others focused directly on the student. Even though the studies in the last section did begin to touch on the importance of classroom culture, all of the studies discussed here emphasized the use of strategy training with the intention that students would internalize the strategy and apply the strategy in future contexts. This is consistent with Lin's (2001) findings which explain that conducting research for creating supportive learning environments, an important element in metacognitive development, is very difficult to accomplish within the research setting and very difficult to measure given the large number of uncontrollable variables and restrictions present in the classroom setting. Discussions and information about supportive social learning environments are often found when describing an idealistic environment that will promote metacognitive development rather than in actual research studies that have been conducted in classrooms. The next section will discuss researchers' theories and ideas about the creation of an ideal environment for promoting metacognitive development.

Idealistic Setting for Promoting the Development of Metacognitive Thinking

The previous section discussed the use of strategies for promoting metacognitive development. While strategies are much easier to teach and control for in the research setting, the creation of a safe and collaborative learning environment is also important for promoting metacognitive development. This section synthesizes ideas presented in the research as being an ideal setting for developing metacognitive thinking. First we will discuss an overview of general ideas and frameworks presented by researchers, then we will briefly discuss the development of and the need for language, followed by the idealistic social learning environment, and lastly a focus on the metacognitive strategies that should be present in mathematics instruction. The following explanation paints a picture of what a classroom primed for developing metacognitive thinking in students should look like, sound like, and be like.

When faced with a difficult task, skillful thinkers participate in an internal dialogue to determine the most appropriate course of action. An environment that promotes student metacognitive development prompts students into the desired internal dialogue by teaching them to use questioning to determine what is already known about the problem, identify available resources, look at the problem from a new perspective, break the problem down into manageable parts, identify emotions that might block progress, and as a group, work together to determine a solution (Costa, 2008). These metacognitive activities should be an integral part of the mathematics classroom. Student learning lies in the students' own thoughts and actions so teachers must have the ultimate goal of helping students organize their thinking and helping students create a habit of asking how to improve their thinking (Cardelle-Elawar, 1995; Crawford, 2007). Students should be given opportunities to reflect on and evaluate their productivity of learning, but not to the extent these activities create a greater burden on the student (Cardelle-Elawar, 1995; Costa, 2008).

Adolescents need practice with being in charge of their own thinking and in recognizing how and when their thinking improves. This requires that teachers move away from a traditional lecture-driven style of instruction to a shared ownership of the classroom allowing students opportunities to make choices and decisions so students will learn how to manage and monitor their own learning (Crawford, 2007). Students must be given opportunities to experience problems similar to those they will face in the future that require them to face complexity, reason logically, make decisions, and evaluate their own thinking (Crawford, 2007). These metacognitive skills will not develop on their own simply through experiences, they must be explicitly taught as part of the learning environment and should focus on the strengths and weaknesses of students as problem solvers (Desoete, 2007; Lin, 2001; Schneider & Artelt, 2010). Further, this instruction is more effective when embedded within subject area content, and should be prolonged and encouraged for the students to develop metacognitive thinking (Pintrich, 2002; Veenman, et al., 2006).

Metacognitive instructional approaches are most effective when they integrate both the knowledge and regulation of the processes and strategies through experiences allowing the students to work both cognitively and metacognitively in an environment rich with mathematical sense-making, habits, and dispositions reflected in students' work (Livingston, 2003; Schoenfeld, 1992). These classroom communities must provide opportunities for students to apply these skills in a meaningful context, observe others using the skills, and have an "expert" reflect on what and how well students incorporate the metacognitive thinking model (Schraw, 1998). Additionally, effective learning communities require the integration of cultural learning, individual construction, and peer interaction so students are lead toward self-regulation (Schraw & Moshman, 1995). Teachers can encourage students to share their thinking as they monitor their progress, evaluate their strategies, and generate alternative ideas, while also developing classroom norms that require explanations and justifications forcing students to think deeply about and informally prove mathematical ideas (Costa, 2008; Brittany Hoffman, Breyfogle, & Dressler, 2009).

Frameworks for Increasing Metacognition in the Classroom

Two frameworks for promoting metacognitive development have been discussed in the literature. Like many other elements of metacognition, the frameworks emphasize similar ideas, but organize those ideas using different hierarchies. The first framework discussed in this section requires an understanding of metacognition and metacognitive theory while the second framework emphasizes basic classroom instructional design principles that promote metacognitive development. The framework provides four suggestions to increase metacognition in classroom. The suggestions are to promote general awareness, improve knowledge of cognition, improve regulation of cognition, and foster conducive learning environments (Schraw, 1998). Promoting general awareness consists of helping students understand and distinguish between cognition and metacognition, modeling of metacognition by both the teacher and the student, and allowing students to work within the student's zone of proximal development. According to Schraw (1998) this is best accomplished through whole class and group discussions, modeling, and reflecting on the process. Improving knowledge of cognition consists of promoting strategy use and being explicit about metacognitive awareness while allowing students to construct knowledge about how, when, and where to use the strategies. This is best accomplished through the use of the Strategy Evaluation Matrix, which consists of a four-column chart where each row is devoted to its own strategy and outlines how, when, and why to use that strategy (Schraw, 1998).

Improving regulation of cognition is Schraw's third way to increase metacognition in the classroom and consists of using a regulatory checklist to assist with planning, monitoring, and evaluating progress. This checklist provides prompts to help students at each phase. The planning prompts ask questions such as what is the nature of the task, what is my goal, what kind of information and strategies do I need, and how much time and resources do I need for monitoring my thinking. The monitoring prompts ask questions such as do I have a clear understanding of what I am doing, does the task make sense, am I reaching my goals, and do I need to make changes. Lastly, the evaluating prompts ask questions such as have I reached my goal, what worked, what didn't work, and would I do things differently next time? Schraw's final suggestion for increasing metacognition in the classroom is by fostering conducive learning environments and suggests that these learning environments promote mastery over performance. As previously stated, this framework although theory laden does provide suggestions in line with the major strands of metacognitive theory, but it does not provide a clear picture of what this would actually look like in the classroom.

A second framework presented in the literature was developed from a synthesis of metacognitive literature. Lin (2001) synthesized several studies surrounding the development of metacognitive thinking in students and created a framework consisting of four underlying principles for balancing metacognitive instructional approaches with content. This framework does provide a clearer picture of characteristics that should be embraced in a classroom promoting metacognitive development.

The first principle suggests that students must be provided with frequent opportunities to self-assess what they know and do not know. This principle is one of the most commonly addressed and consists of strategies such as modeling to help teach students how to monitor their learning. The second principle suggests that students must be helped in articulating their thinking. Students must be provided encouragement to share thinking, make their thinking explicit, and be expected to explain and justify thinking. Lin suggests that this can be accomplished through activities such as guided questions, prompts, and social modeling. The third principle suggests a need to foster a shared understanding of the goals for metacognitive activities. If students are aware of and value metacognitive activities, they are more likely to engage in them. This requires the students and instructor to be explicit about why and when metacognitive activities are needed, an awareness of where one is in the learning cycle, and a shared understanding of the purpose of metacognitive activities. Interestingly, there is little discussion of sample methods for achieving this principle.

The fourth principle suggests students must develop knowledge of self-as-learner with respect to one's culture or the role he or she is playing in that environment. This consists of creating supportive social environments where students identify strengths and weaknesses as well as beliefs and assumptions that impact learning so they develop a better understanding of their ability to learn. Lin (2001) emphasized that helping students better understand themselves as a learner may increase confidence and motivation that, in turn, impacts learning. Creating choices of roles for students to practice in the classroom or teaching students with different personalities in a virtual learning environment were examples provided to demonstrate how students can acquire this knowledge.

These two very different frameworks both discuss the importance of sharing thinking, questioning, prompting, reflecting, learning about self, and the need for a social environment promoting metacognitive thinking. However, neither framework provides a clear picture of how to get students talking and sharing their thinking, how to create the social environment necessary for sharing thinking, or incorporating additional strategies beyond the basic modeling and prompting. Table 2 provides a list of design elements that are discussed throughout the literature, followed by a more in-depth discussion about how to develop the language necessary for effective discourse in the classroom and the creation of a social environment that encourages student learning.

Table 2:
Instructional Design Elements for Promoting Metacognitive Development in Classrooms

Planning	Student generated questions	Non-judgmental feedback
Reflecting	Identify needs	Paraphrase student ideas
Labeling behavior	Clarify terminology	Role play and simulations
Journaling	Modeling	Think alouds
Discussing multiple strategies	Problem solving	Reciprocal teaching
Cognitive Coaching	Direct explanation	Elaboration
Self-questioning	Self-assessment	Reflection
Self-evaluation	Stop n Write	Meta-teaching
Error detection	Effort and attention allocation	Constructing visuals
Revising	Rereading	Activate prior knowledge
Community discourse	Make thinking visible	Identify emotions
Thinking together	Regulatory checklist	Card sorting

Developing the Language and Dialogue Needed to Share Thinking

The sharing of thinking through some form of communication is a critical element in making the implementation of any of these design elements listed in the above table an actual success. According to Blanton and Stylianou (2003), Vygotsky's theory of the relationship between thought and language and the development of language through social interactions plays a critical role in metacognitive development. The social construction of meaning is facilitated through talk and it is hoped that the discourse patterns that are used to create learning are internalized over time to promote continued learning (Blanton, Stylianou, & David, 2003; Leat & Lin, 2003; Martinez, 2006). Students should be provided opportunities for word meanings to surface so they can combine the words they hear with the thoughts in their head to construct new meanings, and then practice those meanings while providing explanations and justifications to complete a cycle of learning (Steele, 2001). This focused, reasoned, and sustained dialogue promotes students' conceptual learning and problem solving abilities (Mercer, 2008).

The ability to participate in this in-depth dialogue is necessary for cognitive and metacognitive development. Students need to internalize the ability to participate in dialogue by first learning to listen to each other, then learning to respond to each other's ideas, and finally being willing to change their own ideas based on what they have heard. With these skills, the conversation changes from simply stating what one thinks to creating a shared space of actual dialogue which ultimately creates a space for shared thinking (Fisher, 2007). Students must be taught how to ask and answer questions as part of the dialogue so they can participate effectively in reasoned argumentation, problem solving, and learning (Gillies & Khan, 2009). Students must also learn to ask questions of themselves and their peers that will assess and build deep understandings of mathematics (Oakes & Star, 2008). Students often do not realize their lack of understanding until they are unable to question or explain that concept (Lin, 2001). Once students have learned and internalized their ability to participate in dialogue, it becomes easier and more natural to get their thinking out of their head to be analyzed, evaluated, and revised which helps them continue the cycle of learning.

The clarification, elaboration, justification, and critique that will occur when students are actively creating the dialogue described above provide an opportunity for students to engage with metacognitive utterances (Blanton, et al., 2003). Multiple opportunities with these metacognitive utterances are needed and once students describe what is going on in their heads, they develop flexibility of thought as they compare what they are hearing with what they are thinking, make judgments about the utility of the various strategies, and critique the ideas presented (Costa & Marzano, 1987; Crawford, 2007; Goos, et al., 2002; Pintrich, 2002). Also, by responding to ideas presented, students have an opportunity to reflect on what they do and do not understand, while holding each other accountable for clear and complete explanations of both why and how students chose their strategy (Goos, et al., 2002; Lin, 2001).

Developing language skills helps students learn how to and then participate in dialogue and promotes both cognitive and metacognitive growth. If students do not have the words or do not know how to effectively participate in classroom dialogue, then carrying out the design strategies presented in the literature for promoting metacognitive development becomes impossible. Nearly all of the design elements revolve around students being able to share their own thinking and then build upon the shared thinking within the entire classroom. Students are expected to internalize a discourse that surrounds the initial shared thought so they can repeat similar discourse when faced with completing a new task on their own. Through the process of internalization, the students make the ideas their own and learn that new concept. Once students have developed the vocabulary and knowledge of effective dialogue, they must be willing to participate in that dialogue. If students are not willing to share their thinking, then there will be nothing to discuss and the learning cycle is broken. The need for a learning community where students feel safe and comfortable sharing unpolished and incomplete thinking about a solution that may or may not be correct is critical to the success of metacognitive thinking.

Creating the Necessary Social Environment

Collaborative work in a social environment is important for metacognitive development. Children working together tend to act above their normal development level and are better able to regulate their own and their partner's thinking which creates opportunities for students to move beyond what one student could do alone (Goos, et al., 2002). Successful collaboration requires more than just social and cognitive interaction in a group that has been told to work together (Hurme, et al., 2009). For groups to attain socially shared metacognitive thinking, members of the group must make their thinking and feelings visible and base their work on argumentation and explanation of processes and not just getting the answer. When this happens, students report a decrease in feelings of difficulty and are able to explain why they think their answer and solution process is correct (Hurme, et al., 2009).

Working collaboratively allows students to elaborate and make connections between information because they are engaged in clarifying their own thinking while also becoming aware of what they actually know, thus building rich knowledge networks (Kramarski, 2004). This process can be deepened even further by training students to ask metacognitive questions of themselves and others, which forces students to move beyond merely coming to a consensus about a right answer, but to instead explore each other's reasoning and perspectives to arrive at a shared understanding (Kramarski, 2004). This process, known as mutuality, is an important feature for effective collaboration and requires students to propose and defend their ideas while they are clarifying, justifying, and questioning what they do and don't understand (Goos, et al., 2002). The resolutions of conflicts that arise in these situations make issues meaningful to students and prepares then for future theoretical discussions (Schoenfeld, 1992). Creating group processes through collaborative work enhances the learning for all students because students must monitor both their own and their group's thinking processes (Carson & Rowlands, 2007; Hurme, et al., 2009). The process becomes difficult because students must suspend what they think they know, be open to and entertain the ideas of others, and support the conclusion reached by the group through the collaborative work. Group members must shift away from a "me" to an "us" (Costa, 2008). Ultimately, learning depends on conversation and on the negotiations of meanings that occur as part of either an internal dialogue or dialogue with another (Fisher, 1998). Cooperative/collaborative learning promotes this type of dialogue, which in turn promotes metacognitive development.

Creating this type of social environment for metacognitive learning requires the teacher builds a supportive learning culture and knows how to create a classroom culture where students feel safe and comfortable (Costa, 2006; Lin, 2001). This environment typically requires a change in classroom culture as it tends to be more dynamic and much less predictable than a strategy-training classroom, but these changes are imperative for metacognitive development since the classroom environment tends to have a greater impact on student thinking than strategies alone (Lin, 2001). Learning is shaped and defined through classroom culture and gaining membership into the learning community leads to greater mathematical thinking and knowing (Schoenfeld, 1992). When authority is shifted away from the instructor and focused on the class to determine reasonableness, students take on a more active role in the overall dialogue and learn to hold each other accountable for explanations and justifications as they develop a need and appreciation for mathematical argumentation creating a learning culture and community within their classrooms (Brittany Hoffman, et al., 2009; Schoenfeld, 1992). While the teacher may be playing a different role in this classroom, their role is critical as they are the ones that provide the guidance, instruction, and opportunities to mold and shape the student interactions so that students will begin to take on this new role.

Role of the Teacher in Developing Metacognitive Thinking

Through the questions they ask and instructional activities they design, teachers tell students what to do, how to do it, and how to act while they are doing it (Costa & Marzano, 1987). In the ideal classroom, gone are the days when the teacher stands at the board or the overhead going through procedures before asking students to mimic those procedures on their homework assignment. The teacher no longer scurries around the room while the students are completing independent seatwork, trying to answer all of their questions, and providing moral support for students who lack confidence in their abilities. This "new" classroom setting promotes metacognitive development by requiring a very different role for the teacher that specifically encourages metacognition by helping students understand and articulate what is going on in their heads (Costa, 1984). Teachers need to model and discuss how they are thinking as examples for their students so that students will begin to recognize their own thinking skills and eventually take over planning and regulation on their own (van der Walt & Maree, 2007). As a starting point, teachers should identify important skills and strategies, determine how they were constructed within their own understandings, and model how they can effectively share this information with others (Schraw, 1998).

The teachers' modeling of their own thinking is a first step; however, there are many more designs that teachers can implement in their instruction to promote metacognitive thinking. Teachers must plan the context and encourage students' active engagement within the social learning community (Steele, 2001) as well as provide opportunities for students to practice their thinking rather than merely applying rote procedures to repetitive problems (Cardelle-Elawar, 1995). These activities may include think alouds, reciprocal teaching, and cooperative learning along with deep questioning and probing for deeper thinking beyond the initial sharing of thinking; and activities for reflecting through talk and/or writing (Costa, 2006; Oakes & Star, 2008; Whitebread, et al., 2009). Teachers must provide students with opportunities to engage, ask questions, share thinking, and reflect on their own thinking and the thinking of others. This can be accomplished through appropriately selected activities that require students to track and provide rationale for their decisions. The students should also be reminded of important overlooked ideas and opportunities created for reflection and self-evaluation. Teachers also need to give feedback and support students self-evaluation through multiple learning opportunities (Cardelle-Elawar, 1995; Crawford, 2007).

Teachers must "model and coach, probe and challenge, guide and monitor, motivate and encourage, expect and hold accountable, and assess and prompt" so that students will "grow intellectually, socially, and personally" (Crawford, 2007, p. 131). In the conclusions or discussion sections of most studies, researchers emphasize the general characteristics discussed in the previous paragraph, but they do little to acknowledge that each student is different and each group situation is unique. Sometimes the direction provided by a teacher may cause more confusion than clarification or the direction of discussions may deny the students of the important, but difficult task of resolving their own struggles. Thus, the challenge for the teacher is to learn to negotiate when, why, and how to intervene in group discussions so that they promote true collaborative group work and scaffold students' thinking through selection of strategies, identification of errors, and evaluation of answers (Goos, et al., 2002). Creating the "right" context with appropriate modeling and guidance is a subtle but critical challenge for the teacher (Schoenfeld, 1992).

From the Students' Perspective

While many researchers have made proposals of what teachers should do in the classroom to promote metacognitive development, little work has been done to describe a big picture of what metacognitive instruction actually looks like and sounds like. Information that does exist is in the form of generalizations and insights from research, but teachers do not have time to translate these ideas into actual practice (Leat & Lin, 2003). The purpose of Leat and Lin's (2003) research was to provide a language that teachers could use to describe what classroom instruction should look like and sound like from the students' perspective. This study consisted of teacher researchers observing each other's classes immediately followed by group interviews of 3-6 students. The student groups were representative of the ability levels present in the class and the students were asked what they learned and how the teacher helped them learn the information. Transcripts from the group interviews were then triangulated with video data and researcher observations to create a framework for understanding and identifying ten roles a teacher could follow to enhance student understanding. Although the framework was not created specifically for promoting metacognition, the researchers do argue that it could form the foundation for developing metacognitive thinking in students.

The researchers have successfully implemented this framework into many professional development opportunities for in-service teachers.

Teacher roles that help pupils learn metacognitive strategies consist of first using stimulating strategies and second attending to groups and individuals. These first two roles help students recognize that the activity is about both thinking and the sharing of thinking which shows that their reasoning is being valued which in turn builds their confidence. The third role of the teacher is to encourage students to ask questions to develop an understanding of what makes a good question. This allows the teacher to cede control and authority over to whom is generating the knowledge. The first three roles could be considered foundational for the development of metacognitive thinking in the classroom.

The next two roles, four and five, increase the cognitive resources available to the students, so they begin to demonstrate the choices they have for how students undertake a task. Fourth, teachers should collate idea and strategies, which helps students appreciate that they have a choice in how to complete a task and builds on the idea that groups can do more together than individuals can do alone. Fifth, teachers should provide and promote a variety of heuristics and alternative representations. The researchers discuss heuristics such as identifying what, when, where, why, and how along with fact or opinion, concept mats, and Venn diagrams. The use of these strategies builds students' confidence in being able to handle information.

The researchers identified the next three roles as the most critical for creating the appropriate talk and overall environment for metacognitive development, and viewed this as reflecting Vygotsky's description of social talk as a precursor to inner speech. The sixth role of the teacher is to promote and manage discourse. Students identified that the discourse must be more than simply sharing ideas to be constructive emphasizing that it was the interactions of their own ideas with others that accelerated thinking. Seventh, teachers should make pupils explain themselves, especially beyond their initial statement. When expected to provide more than a short response, students are able to clarify their thinking through their explanation, which leads to a deeper cognitive process. Eighth, teachers must provide feedback to both the whole class and to individuals about their thoughts and explanations. Students want to receive praise, but they also want critical feedback.

The final two roles are very closely related and place heavy demands on teachers' subject knowledge to promote transfer and develop a broad understanding of why the particular knowledge is important. The ninth role, suggests that teachers should make connections to context where students might use their new learning. Lastly, the tenth role of the teachers is to communicate the purpose of the lesson by helping students make sense of their new knowledge in regards to a "bigger picture." This framework was created from the students' responses about how the teacher helped them learn. Interestingly, the student responses mirror and extend the ideas researchers hypothesized and suggested as important for promoting metacognitive development. Unfortunately, there is still little understanding of how to actually make these things happen in the classroom.

There is a great need for continued research regarding the role of the teacher and how to support teachers as they take on the challenge of changing the classroom environment to promote metacognitive development. There is currently only a cursory understanding of how to help both teachers and students adapt to a new environment (Lin, 2001). Further, understanding students' metacognition and knowing how to translate the students' understanding of their metacognition into effective learning are two separate elements that a teacher must negotiate and must negotiate in tandem with understanding of aspects of the students' overall learning process to design the most appropriate instruction (Sternberg, 1998). Teachers must accept this challenge of helping students develop an awareness of their own metacognition (Fisher, 1998) and desire to embed metacognitive instruction in their lessons (Veenman, et al., 2006). Teachers do, however, also acknowledge that they lack the necessary knowledge and tools to know how to accomplish this task and to know how to help students understand its importance (Veenman, et al., 2006). Teachers are the key players in fostering student engagement. Teachers contribute significantly by creating and mediating various design features that afford students opportunities to develop knowledge about the self-as-learner, to identify learning goals, and to pursue their personal interests in meaningful ways. How best to help teachers learn to support student knowledge about the self-as-learner remains a challenge (Lin, 2001).

Summary

This literature review provided a glimpse of metacognition, it's relationship to mathematics, reasons students should develop metacognition, how researchers have studied metacognition in mathematics, what an ideal metacognitive classroom should look like, and the role of the teacher in creating this idealistic setting. This was but a glimpse of the research and in no way was intended to be exhaustive. Metacognition is most simply the thinking about thinking and has a positive correlation with students' success in mathematics. Developing metacognitive thinking is possible through carefully designed instructional settings that promote the sharing, critiquing, reflecting on, and revising of thinking; and metacognitive thinking should not be expected to fully develop on its own. In this environment, the role of the teacher changes to a more facilitative role rather than as a provider of knowledge through lecture and telling. A classroom that promotes metacognitive development will look and sound very different from a traditional lecture-based classroom and further research is needed to fully understand how to support both teachers and students in creating this change. To create a plan for helping teachers promote metacognitive development, it is first necessary to understand what is already occurring in the classroom. This is the purpose of the research that follows. The following chapters describe the study of instructional practices promoting metacognitive development that are already embedded in an inquiry-based college classroom as well as the implications of these findings. Information from this study provides a better understanding of what is already occurring so that future studies can address the needs and avoid recreating the wheel that is effective metacognitive instruction.

CHAPTER III

METHODS

Metacognition is most succinctly defined as thinking about thinking. Research has suggested that higher levels of metacognitive thinking tend to lead to greater mathematics success, and metacognition is teachable through carefully designed instructional strategies that encourage student thinking. A problem, however, is that a majority of the current literature is focused on either explaining the relationship between metacognition and mathematics learning or on testing targeted interventions specifically designed by the researcher to promote metacognitive thinking.

The purpose of this mixed methods case study was to describe metacognitive instructional strategies embedded in the current instructional practices by an instructor in a college-level geometry content course designed for per-service elementary teachers and to determine if the course impacted students' overall metacognitive awareness. This study employed a variation of the embedded mixed methods design where the quantitative data played a supportive secondary role to the qualitative data (Creswell & Plano Clark, 2007). The primary purpose of this study used qualitative data to describe embedded activities that promoted metacognitive development in the classroom while the quantitative data was used to determine if these methods impacted the students' metacognitive awareness. More specifically, the research questions for this study were:

1. What types of metacognitive thoughts are being modeled in the classroom?
2. How does a college mathematics instructor promote metacognitive thinking in her students?
3. Does a college-level course change the metacognitive awareness of pre-service elementary teachers?

Design

Answering these research questions required both qualitative and quantitative information to provide an overall description of embedded metacognitive activities in the classroom as well as a quantitative analysis suggesting the usefulness of these methods. Thus, this study employed the use of mixed methods research. As a methodology, mixed methods involve philosophical assumptions that guide qualitative and quantitative approaches throughout the overall study. Further, as a method, mixed methods focuses on collecting, analyzing, and mixing both types of data in a single study or series of studies (Creswell & Plano Clark, 2007). In most general terms, the main idea was to use both qualitative and quantitative data to provide a better understanding of the problem than either type of data could do alone (Creswell & Plano Clark, 2007). Through the use of the pragmatist view, the mixing of both qualitative and quantitative data made it possible to use the most appropriate method to address the purpose of this study by offsetting the weaknesses posed by each type of data and allowing for exploration at multiple levels including the teacher's perspective, the students' perspective, the researcher's perspective, and the textbook's perspective (Creswell & Plano Clark, 2007).

Describing the overall mixed methods design consists of four talking points: (a) the implementation of the design, (b) the type of data which received the higher priority within the study design, (c) how the two types of data were integrated, and (d) what theories were used to guide the study (Creswell, 2003, p. 212). This study used a variation of the embedded mixed methods design and consisted of three main pieces to the overall study design (Creswell & Plano Clark, 2007). First, qualitative interviews with the instructor both before and after the researcher's time spent in the classroom. Second, pre- and post- mixed methods information collected from the students during the course. Finally, qualitative information collected through observations collected by the researcher during the time in the classroom and analysis of course documents.

To look at the design from the inside out, the study was anchored in the researcher's qualitative observations of the classroom. Both qualitative and quantitative data were collected from the students just before and directly following the researcher's observations. Lastly, qualitative interviews with the course instructor bookended data collection on the whole. Since the main purpose in this study was to describe metacognitive development that was already occurring in the classroom, the emphasis was on the qualitative data in this study while the quantitative data played a supportive role of identifying whether these embedded strategies had an impact on students' metacognitive awareness.

The third talking point relates to the mixing of the two types of data. In this study, the mixing of the qualitative and quantitative data occurred during the interpretation of the findings to provide a better overall understanding of the embedded metacognitive development strategies in this classroom. This means each data source, instructor interviews, student surveys, classroom observations, and the Metacognitive Awareness

Inventory (MAI) administered to the students, were each analyzed separately then brought together to provide a final holistic interpretation.

Lastly, it is important to understand that the underlying theory of this study was that it is possible to increase students' ability to think and use their metacognition through the use of carefully designed instructional activities. This study explored the use of these specific activities, such as the explicit modeling of metacognition, questioning, error analysis, and reflection in a safe and collaborative environment, to determine if these activities impacted the students' thinking in this non-traditional classroom. See Figure 2.

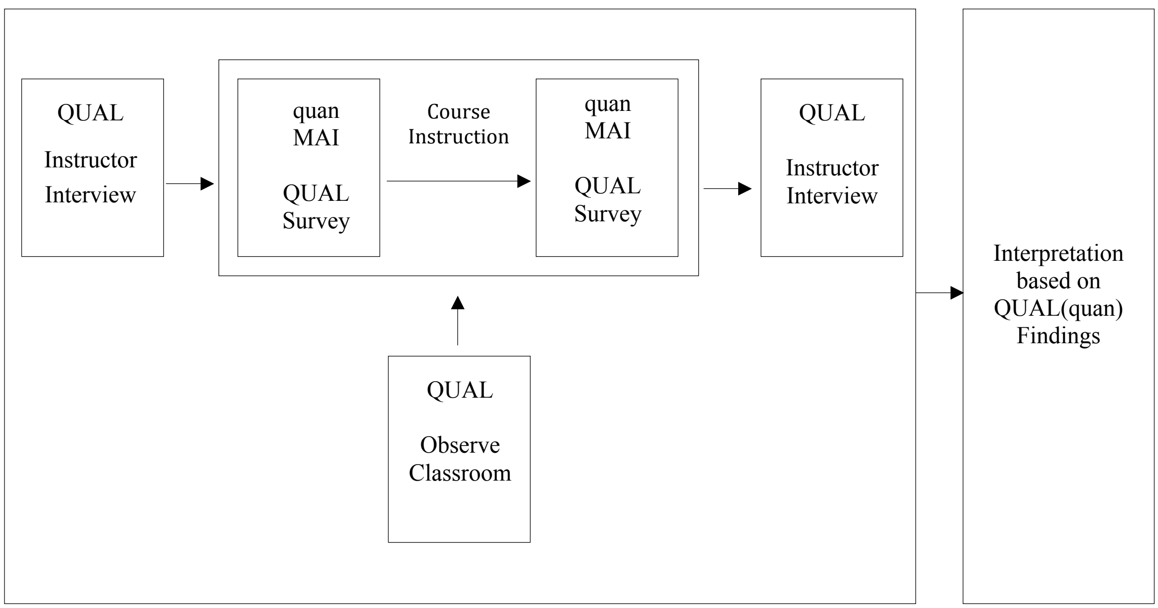

Figure 2. Variation of Embedded Mixed-Methods Design. QUAL indicates the primary role of the qualitative data. quan indicates the supporting role of the quantitative data. QAUL(quan) indicates the embedded interpretation with an emphasis on the qualitative data.

The use of qualitative and quantitative data together provided insight into not only what was happening, but also how well it was working. The use of only quantitative data or only qualitative data separate from the other would not have provided a complete understanding of what was occurring in this classroom and might have led to inaccurate conclusions. This study first explored qualitatively how the instructor promoted metacognitive development in the classroom and then used the quantitative data to enhance the description of the embedded design by providing evidence of the impact of the instruction on the students' thinking. If the study had only considered quantitative changes in the students, what may have caused those changes would not have been as clear. Similarly, if only a description of the embedded metacognitive development had been provided, it would be unclear if the students' thinking had been impacted. These data worked in tandem to provide a better understanding of the overall picture.

Overall, this case study was about understanding how an instructor promoted metacognitive development in her classroom. This study was descriptive in nature and provided insight to what actually occurred in this classroom where the students and instructor work together as a system to create an environment for student learning. The study of this case of metacognitive development with this instructor and her students provides a concrete description of the context so that the reader can bring their experience and understanding to the interpretation and determine its generalizability (Merriam, 1998).

As final note about the general design of this study, the researcher in this study did not manipulate or control any element within this study, but instead studied the real occurrences as they evolved naturally within the classroom. This type of study is known as *naturalistic inquiry* and is in contrast to the experimental studies that are prevalent throughout metacognitive development literature. According to Patton (2002), naturalistic inquiry takes place in a real-world setting where the researcher does not manipulate the phenomenon of interest, but instead allows the phenomenon to unfold naturally with no predetermined course. This design contrasts with experimental studies where the researcher manipulates, changes, or holds an external influence constant and where only a very limited set of outcome variables are measured. Because this study focused on describing the embedded instructional strategies of the instructor, it was necessary to see and talk about those strategies as they naturally occurred.

Setting

Participants for this study consisted of one instructor and the students from two sections of a course taught by this instructor at a south-central, mid-size regional suburban university. This course and these students were purposively selected based on conversations with faculty members at the university about the student-centered teaching philosophy implemented in the delivery of this particular course. This sample was selected from a sophomore level geometry content course designed for pre-service elementary teachers that served as an intensity sample because it maintained a student-centered approach to teaching rather than a more traditional lecture-driven method of instruction (M. Patton, 2002). The emphasis in this course was for students to revisit content they should have learned previously in high school and to fill in any holes in their understanding. Mathematics department faculty members, such as current and previous course instructors, department heads, and other department stakeholders that have been involved with this course, explained during informal conversations that students in this course were expected to do a majority of the talking, explaining, and describing of solutions and mathematical understandings. Students worked with their peers, asked each other questions about content they do not understand, shared explanations, and worked together to create a better understanding for all students. A majority of the responsibility for learning was put in the hands of the students, as they must attempt their homework before class and arrive at class ready to ask questions about what they do not understand. In addition to the description above, many faculty members who teach this course commonly identify and discuss the necessary group interactions, importance of student talk, and how this course is very different from other mathematics courses taught in the department.

This course was selected over other courses in the mathematics department because of the perceptions the researcher gained through conversations with mathematics department faculty members regarding the instructional intent that students model and share their thinking and develop understanding while questioning each other. The instructor prompted students' thinking rather than providing direct instructional responses to students' questions attempting to create a community of learners. The intent was to encourage students to work together to learn and to evaluate new material rather than rely on the teacher as the sole presenter of new knowledge. The characteristics of both student and teacher expectations mirrored those identified in previous research as critical to the success of developing students' metacognitive thinking in the classroom.

Participants

At the time of data collection, two instructors were teaching five sections of the course. The instructor selected for this study was chosen based on both her previous experience teaching the course and because she was the only instructor teaching multiple sections of the course during the semester of data collection. To better understand the teacher's embedded instructional practices, it was important to see the same instructor with more than one group of students as a means to provide a greater understanding of her common practices.

The instructor and researcher selected the specific sections of students together. One section was eliminated from the sample due to a time conflict with the researcher's teaching schedule and another section was eliminated because the instructor did not consider the students and the culture they had developed in that class to be representative of what commonly occurred in the course. All students from each of the two remaining sections were solicited for participation in the study based on their enrollment in the course. Recruitment occurred during a regularly scheduled class time just prior to the initial data collection. In the Monday-Wednesday-Friday morning section scheduled from 9-9:50 am (MWF), 28 students were enrolled, of which 82% provided consent to participate in the study, 61% provided demographic information, and 46% were present for both the pre- and post- administration of the Metacognitive Awareness Inventory (MAI). Only students with both a pre- and post- MAI score were included in the MAI analysis. For the section of the course meeting Tuesday and Thursday evenings from 7:30 to 9:00pm (TR), 29 students were enrolled, of which 97% provided consent to participate in the study, 86% provided demographic information, and 83% were present for both the pre- and post- administration of the MAI. See Table 3 for a summary of student participation.

At the time the sections were selected, the researcher anticipated that the two sections would provide an interesting contrast in student demographics since one was held during a more traditional morning class time, and the other was held as a late evening course. However, from the demographics in Table 4, it appears this was not necessarily the case. Both sections had very similar student populations. Most of the students were female and identified themselves as either white or Caucasian. The MWF section did have three males while the TR had only one male student. While both sections were predominately white, both sections had at least one Hispanic student and one Native American or American Indian student. The TR section also had a student that self-identified as black and white as well as two students who chose not to identify their ethnicity. More than half of the students in each section identified their major as either early childhood or elementary education (53% for MWF and 64% for TR).

Table 3
Student participation by section

	Number of Students	
	MWF 9-9:50am	TR 7:30-9:00pm
Enrolled in course	28	29
Consented to study	23	28
Provided demographic information	17	25
Pre and Post MAI	13	23

While the previous demographics were fairly consistent with the researcher's expectations, differences between the sections regarding traditional and non-traditional students were present but not significant. The TR section did have a higher percentage of students that graduated more than 5 years prior to taking the sophomore level course, approximately the same percentage of students were single; however, more of the students in the evening class had at least one child at home.

Table 4
Participant Demographics by section

	Number of Students	
Characteristics	MWF 9-950am	TR 730-900pm
Number of students providing demographic information	17	25
Gender		
Female	14	24
Male	3	1
Ethnicity		
Caucasian/white	15	19
Hispanic	1	2
Am Indian/Native Am	1	1
Black/White	0	1
Blank	0	2
Major		
Early Childhood Educ	4	6
Elementary Education	9	16
Special Education	3	3
Education	1	0
Graduated before 2005 (more than 5 years before data collection)	7	13
Marital Status		
Single	10	14
Married	5	10
Divorced	2	1
At least one child at home	5	11

Researcher Background

Merriam (1998) explained that the researcher is the primary instrument in qualitative research and thus observations and analyses are “filtered through that human beings’ worldview, values, and perspectives (p. 22).” To better understand the researcher, this section describes how the researcher became interested in the topic of developing metacognitive thinking, the researcher's experience within mathematics education, and an explanation of the changing teaching roles of the researcher over the course of the study.

Initial Interest in Metacognition

The researcher's initial interest in the topic of metacognition arose out of frequent frustration in students' lack of understanding and ability to think through and persevere in their efforts to solve mathematics problems. Through continued research and attempts to help students make connections between their current understandings and new understandings, it became apparent that there was a need for explicit classroom discussion about how one is thinking versus simply talking only about specific problems. She also became aware that teaching students how to think about mathematics would require more than continued repetition of shared procedures, but instead involved encouraging student conversations about when and why to use those particular procedures along with discussions about ways to monitor progress throughout the overall process. The type of instruction the researcher was exploring in her own classroom was very different from what students had experienced in their previous mathematics classrooms and thus she wrestled with the difficulty in understanding how to create this type of learning and discussion as part of her normal classroom environment. The return to the research literature was not as fruitful as she hoped since at that time only limited information could be found about what this type of classroom would actually look like. How to create this type of environment spurred her research from being an experimental study that tests a specifically designed strategy to a naturalistic mixed methods study looking at the metacognitive impact on the student learners.

Researcher's Experience in Mathematics Education

The teaching and learning of mathematics has been a life-long passion of the researcher. While in elementary school, she used to teach her stuffed animals everything she had learned in school and in high school. When she was in high school, she made a commitment to herself and her future students that she would find a better and more exciting way to teach mathematics than what she was experiencing. After completing a traditional path to teacher certification through a university teacher education program, the researcher began teaching seventh grade mathematics and was praised for how she got her students involved during class. During her second year as a mathematics teacher, the researcher was given the opportunity to attend a mandatory district-wide training for second-year teachers focused on implementing structured cooperative learning in the classroom. She learned how to integrate a framework that provided classroom management skills so students could break out of traditional rows and learn to embrace the students' natural desire to talk to each other. While the cooperative learning training provided the researcher with greater confidence in classroom management, the researcher was also completing coursework for a master's degree in mathematics. This master's degree was specifically designed to enhance the mathematics content knowledge of teachers that primarily taught mathematics. This combination of content and pedagogy helped the researcher envision a new way to teach mathematics and promote student learning.

Implementing the new vision for teaching of mathematics did, however, prove to be a struggle. After two years teaching seventh grade, the researcher moved to the high school and taught a variety of courses to students of all abilities in grades nine through twelve. During this time, the researcher began to feel very isolated in the ongoing struggle to implement her vision of her students talking and working together in a collaborative environment to develop a deeper understanding and appreciation for the mathematics the State wanted them to learn. This struggle lead the researcher back to the formal learning environment of the university, where she continued her quest to understand how to break free from the traditional classroom instructional practice and implement a new vision of mathematics teaching and learning that was consistent with current reform efforts. Explorations of the phenomenon of implementing standards-based instruction pointed to a variety of stakeholders, but understanding their perspectives was ultimately tied to understanding their thought process about mathematics and the teaching of mathematics.

After leaving the public school classroom, the researcher continued to stay in touch with colleagues and explored additional teaching opportunities. During the time spent completing doctoral requirements, the researcher was given the opportunity to teach both a pedagogy course focusing on the teaching of mathematics in grades 5-8 for pre-service elementary teachers and an opportunity to teach mathematics content courses for pre-service teachers. While teaching these courses the researcher maintained her promise to continue learning how to teach mathematics in a different way so that all students could develop a deeper understanding. Through these combined opportunities to continue her teaching experiences, the researcher was led to study metacognition at the university level with this specific geometry content course.

Changing Teacher Roles of the Researcher

While designing this study, the researcher had very little knowledge or understanding of the course or its students. What she did know came from casual conversations with other faculty members in the mathematics department. During the design phase of the study, the researcher had only taught College Algebra and a general mathematics course intended for non-majors. During the semester of data collection, however, the researcher did begin teaching the course intended to come first in the sequence of four mathematics content courses required for students working toward teacher certification. The course used for this study is intended to be the second course in the four-course sequence. Data collection and preliminary analysis occurred during the spring and summer semesters while final analysis and interpretations occurred during the fall and spring of the next academic year.

During the fall semester, the researcher began teaching a section of the same geometry course as used in this study. While effort was made to separate her "two hats" of teacher and researcher, it was not possible to say that one role had no impact on the other. It is possible, however, that teaching the course provided better understanding of the course in general, especially in regards to the role of the text and the students' reactions to the text. By looking through and comparing researcher notes from the data collection phase of the study to the thoughts and feelings experienced by the researcher during the analysis and interpretation phases, there is awareness of a change in understanding about why the instructor may have made particular instructional decisions.

One of the clearest examples of how the researcher "hat" and teacher "hat" became intertwined came when a particular assignment had been the focus of analysis on an evening prior to the discussion of that very assignment in class the following day. The analysis of the data was pointing to a missed opportunity in the discussion for making metacognitive thinking explicit and when the class discussion the following day was headed in the same direction, the researcher wanted to try what she thought the instructor should have done in her class the previous semester. During the class discussion the researcher extended the discussion about the construction of a line parallel to a given line through a given point, by asking students to discuss and explain why the construction worked and when they might use a similar method for a construction in the future. Interestingly, most of the researcher's students responded in one of three ways. The first reaction was a silent pondering of the question that was posed. Second, a few students reacted by answering the question and showed excitement for being asked the challenging question and commented that they were excited to answer these type of questions in their future classrooms. Lastly, a few students responded with disbelief that asking for justification was important and believed that all that was important was that they could do the problem, not that they understood why or when to try a similar process. Further, the comments made by the students in the last category frustrated those students who fell into the other two categories and intimidated students in the first. These thoughts and feelings were expressed in both personal conversations the researcher had with many of the students who shared what they were thinking and through written anonymous reflections that were collected during the class period following the incident. Further, this one event also made an appearance in the open-ended responses on the student evaluations of the instructor and course at the end of the semester demonstrating the impact this incident clearly had on the students.

Because of the reported impact this event had on the students just past the mid-point of the semester, the researcher-as-instructor took great care pushing the students to move beyond procedural explanations. Since the students always worked with a group of trusted friends, the researcher tended to focus and facilitate the questions specific to that group rather than on the class as a whole. The researcher-as-instructor commonly used information and thoughts gathered from one group as the stem for the question used with another group. For example, the other group said that ______ and your group said _______, why do you think the solutions are different? Whose idea or solution is correct? Why might they/you have thought about it that way?

In summary, all of the researcher's experience in a variety of settings impacted her understanding of student thinking and how to encourage student discussions. The frustrations she has experienced and the promise that she made to make mathematics meaningful and relevant continued to motivate her to better understand student thinking and ways to help students learn to think. After finding only limited information about what thinking looks like and sounds in the classroom, her goal became to help students develop their metacognition in a real-world setting, as opposed to an experimentally contrived environment. The researcher realized it was important to provide this information to others and thus began a search to find a teacher who implemented "metacognitive development" in his or her classroom. After struggling to find a teacher that knew of and about metacognition, the decision was made to focus on a non-traditional classroom with the belief that developing students' metacognition should be a natural by-product of standards-based instruction since underlying ideas of the NCTM

Process Standards are similar to experimental interventions that have been successful in promoting metacognitive development.

Data

To better understand how a teacher embeds metacognitive development as a regular part of instruction and to determine if a single course can actually change a student's level of metacognitive awareness, the researcher looked at classroom instruction from a variety of viewpoints including the instructor's perspective, the students' perspective, and the researcher's perspective. Interviews were used to explore the teacher's perspective, while open-ended surveys were used to explore the students' perspective. The researcher also observed the classroom to counter or support the self-reported information from the interviews and surveys. The textbook provided additional information about both the course and the expected content of the discussions. Lastly, the Metacognitive Awareness Inventory measured student's metacognitive awareness (Schraw & Dennison, 1994). A description of the qualitative procedures will be provided first followed by a description of the quantitative procedures. See Figure 2 for a model of the overall order of the data collection or Table 5 for an overview of the data collection.

Qualitative Data

Textbook. Course documents communicate important ideas and information between students and the instructor and thus provide additional information about how the instructor embeds metacognitive development in the course. Documents collected for this study included the syllabus, course schedule, textbook, supplemental notes and activities, exams, and copies of the graded exam that followed the observed portion of the course. While the syllabus and course schedule provided information about the overall

Table 5
Summary of Data Collected for This Study

When collected	Data Collected	Format	Completed by	Analysis
Week 10	Early Semester Instructor Interview	Interview Guide	Instructor	Qualitative Content Analysis
Week 11	Demographic	MC and open ended	Students	Descriptive
	MAI	52 question 5 point Likert Scale		Quantitative
	Early Semester Survey	Open-ended		Both Content Analysis Frequency Count
Weeks 12-14	Classroom Observations	Non-participant	Students and Instructor	Both Content Analysis Frequency Count
Ongoing	Documents	Syllabus, course outline, textbook	Instructor	Qualitative Descriptive
Week 14	MAI	52 question 5 point Likert Scale	Students	Quantitative
	Late Semester Survey	Open-ended		Both Content Analysis Frequency Count
April 2011	Follow-up instructor interview	Interview Guide	Instructor	Qualitative Content Analysis

course design and expectations of the student, the single most important document used in this study was the textbook. The textbook was a driving force for classroom discussions and, considering the information and guidance it provided for both the student and instructor for developing metacognitive thinking, was very important.

Observations. Fourteen classroom observations were conducted to help triangulate the self-reported data from both the instructor interviews and student surveys. Eight sessions were observed for the MWF section and six class sessions were observed during the TR section. Observations were conducted of every class meeting between Exam 2 and Exam 3 with the exception of two MWF meetings. The observations were prearranged with the instructor, all were videotaped, and field notes were taken during the observation (Emerson, Fretz, & Shaw, 1995). Because all class sessions followed similar patterns three MWF meetings were selected at random, using Random.org™, to be transcribed. Corresponding activities were then transcribed from the TR videotapes. Saturation of data was reached through the use of this subset of classroom observations. Actual observations of the classroom were critical to understanding the established mathematical norms and learning environment created by the instructor. Relying solely on the self-reported data would have led to overlooking embedded practices that might not have been recognized as important by either the instructor or the students and could not have been gathered through questioning due to the subconscious nature of some metacognitive activities. The observations provided a glimpse at instructional practices and instructional language that supports metacognitive development.

During the observations the researcher played the role of the observer participant (Merriam, 1998). In this role, the activities of the researcher are known to the group being observed with the primary role of the researcher being to gather information. Further, the group members being investigated controlled the level of information provided. Prior to conducting the observations, the researcher informed students of the purpose and intent of the study through the use of informed consent letters. Students were provided a written copy of the informed consent before it was discussed in class and they were asked to discuss any concerns with the researcher at any time during the research. Students were given the option to participate in all study activities, participate in all study activities but not appear on video, or not participate at all. Not all students provided their consent; however, those that did not give consent tended to be silent participants in the class discussions rather than participating verbally.

The use of the role of the observer participant provided an outsider's view of the classroom, meaning the researcher only saw what was happening, but did not have access to know what it was like to be a part of the setting or program (M. Patton, 2002). However, since a challenge in qualitative research is to do justice to both the insider and outsider perspectives, it was important to also include self-report data to balance the outsider perspective of the researcher (M. Patton, 2002).

Instructor Interviews. Qualitative interviews allow the researcher to find out details that cannot be directly observed (M. Patton, 2002). This study used an interview with the instructor at the beginning and the end of the study to capture the instructor's view of her world, an understanding of the terminology she used, judgments she made and the complexities of her actions through the use of questions that explored experiences, behaviors, values, feelings, knowledge, and sensory information (M. Patton, 2002). Both interviews used an interview guide that listed questions to be explored during the interview. The use of the interview guide, rather than an informal conversation interview or standardized open-ended interview, helped the researcher to maximize the time available by prioritizing topics and establishing a conversational tone that allowed the researcher to explore, probe, and ask questions that expanded topics that were not anticipated (M. Patton, 2002). Each interview was conducted in the instructor's office with the door closed and audio recorder between the researcher and instructor at a time selected by the instructor. Each interview lasted approximately 50 minutes and immediately preceded other obligations.

More specifically, the first instructor interview was conducted during week 10 of the semester and focused on the instructor's beliefs about mathematics, mathematics education, the development of thinking in students, and specific course-related information regarding course objectives, class norms, socio-mathematical norms that have been developed, and instructional practices. Questions for this interview are found in Appendix A and formed the foundation for understanding more about the instructor and her perspective about teaching mathematics.

The follow-up interview with the instructor was the final piece of data collected as a part of this study and was conducted after preliminary data analysis had occurred. Unfortunately, due to unforeseeable circumstances the interview occurred approximately one year after the completion of prior data collection, but the delay allowed for feedback on specific patterns that had been identified in the data that could not have otherwise been captured. Questions were reflective in nature and the researcher attempted to delve deeper into patterns that were identified from the previously collected data. Because of the long delay, the interview did not allow for reflection on specific occurrences from the observations. The questions used for the final interview were more general, but provided deeper, philosophical reflection based on the findings from the other data sources. The interview was audio taped and transcribed verbatim.

Student Surveys. Due to limited resources, pre- and post- open-ended surveys served as a written form of standardized open-ended interviews and provided all the students an opportunity to describe their thoughts in their own words regarding metacognition and its development in the class (Johnson & Turner, 2003; M. Patton, 2002). This data provided information about the student perspective of classroom activities along with their beliefs and experiences with instructional activities that are described in the literature as effective for promoting metacognitive development. The student surveys provided information that could not have been collected through observations or closed-ended surveys. Great care was taken during the construction of the surveys to:

- Assure the questionnaire matched the research objectives
- Understand the researcher participants
- Use natural and familiar language
- Use simple, clear, and precise writing
- Make sure the questions were not leading
- Avoid double-barreled and double negative questions, and
- Consider the specific use of open-ended versus closed-ended questions (Johnson & Turner, 2003).

The Early Semester Survey was an open-ended survey completed by students during the second class meeting during week 11 and focused on students' beliefs towards mathematics, mathematics education, their own cognitive and metacognitive development, and the cognitive and metacognitive development of others. A sample question from this survey is "Describe a time when you were presented with a solution to a problem and asked to find the mistake in the person's thinking." The Late Semester Survey was administered during week 14 of the semester in conjunction with the MAI during the last class meeting prior to their last exam in Week 14. Both surveys can be found in Appendix B. Questions on this survey asked students to reflect on their experiences that occurred during this specific course and to describe activities that promoted/did not promote the development of metacognitive thinking. Questions from this survey include, "Describe how this course has effected the way you think about solving mathematical problems" and "When you had a question about a problem, how did you know it was time to ask a question rather than continue to try and work on it?"

Quantitative Data

Demographics. Demographic information was collected from students by the researcher during the regularly scheduled class time on the first class meeting of week 11. This was the first class meeting following the second exam. Demographic information provided details about students' ethnicity, year of high school graduation, current classification, major, intended use of their major, mathematics course history, study habits, and family information and was used to paint a picture of the students in the section. The Demographic Profile can be found in Appendix C. This information helped the researcher understand who the students were and what they brought into the classroom since this may have impacted their perceptions about metacognition.

Metacognitive Awareness Inventory. The Metacognitive Awareness Inventory (MAI) was administered by the researcher at both the beginning and the end of the study. The data from the MAI was collected to provide direct evidence of the impact of instruction on students' metacognitive awareness. At the beginning of the study, the MAI was administered at the same time as the demographic profile. At the end of the study it was administered at the same time as the late semester survey. The MAI was designed by Schraw and Dennison (1994) to measure a person's awareness of his or her own metacognition. The MAI is a 52-item instrument created to provide a method that was more efficient for identifying a person's metacognitive awareness than the currently accepted practice of think alouds. The MAI can be found in Appendix D. The inventory measures two types of metacognitive knowledge: (a) knowledge of cognition and (b) regulation of cognition. The original inventory used a 100mm scale where one end of the scale was labeled as 0% confidence and the other 100% confidence. The participant was asked to place a slash on the scale representative of their confidence on each item. Validation experiments conducted by Schraw and Dennison (1994) provide support for the two-component view of metacognition with an internal consistency ranging from .93 to .88, which suggests that each of the two components does make a unique contribution to a person's cognitive performance, and thus the MAI can be predictive of future performance. Further, the MAI was written without regard for specific content, allowing the original instrument to be used in a variety of contexts without modification. Despite the lack of studies to check for internal consistency and reliability, the MAI has been used in metacognitive studies in a variety of contexts and has commonly been adapted to a five-point Likert scale (Young & Fry, 2008).

Overcoming Challenges to Design Implementation

Initially this study was designed to allow the students to provide a pre- and post-analysis of the impact the embedded instruction within this course had on their metacognition and metacognitive awareness. Unfortunately, the applications for the Institutional Review Boards at both universities requiring approval were misplaced during the initial signature-gathering phase of the overall review process. This delay greatly impacted the timing of the pre/post portion of the study design since approval was not granted until Week 9 of the semester, the week prior to the university being dismissed for the week of Spring Break. After contemplating options which included delaying the study to the fall semester, redesigning the overall study, or continuing on with the study as planned; the researcher decided last option was best. The greatest impact of this delay was in measuring change in students' metacognitive awareness before and after the entire course to determine if the course had an impact on students' metacognitive awareness. This challenge was addressed though the analysis of the data in exploring possible reasons changes might occur.

Data Analysis

One of the main goals in developing metacognition in the classroom is to make metacognitive thinking explicit. Metacognitive thinking must be brought out of a person's head so that it can be shared, discussed, evaluated, and revised. Researchers have used a variety of strategies such as think alouds, modeling, and questioning to help make metacognitive thinking visible, but have also realized there are necessary environmental characteristics in which this type of sharing should occur. The qualitative data from this study was analyzed in light of three key main ideas (a) making metacognitive thinking explicit (b) using strategies to promote metacognitive development and (c) developing a classroom environment that promotes the sharing of metacognitive thinking. More specifically, two different frameworks for developing metacognitive thinking were used as lenses to help understand what was occurring in the classroom. The two focal points were ideas promoted by Lin (2001) and Schraw (1998).

Two Lenses for Promoting Metacognitive Development

Lin's Four Principles. The first framework suggests improving students' metacognitive thinking by providing opportunities for students to self-assess, to help students articulate their thinking, to foster a shared understanding of metacognitive goals, and to help the students develop knowledge of self-as-learner (Lin, 2001). Opportunities for students to self-assess can be provided through strategy training, questioning and prompts. Requiring students to explain and justify their thinking along with explaining where they are in the learning process were recommended to help students articulate their thinking. Lin also explained that by fostering a shared understanding of metacognitive goals, students would be more willing to participate in the discussion. Lastly, to develop students' knowledge of self-as-learner the instructor should help students learn to specify their role in their culture, identify learner characteristics, and help students understand dimensions of their own personality (Lin, 2001).

Schraw's Promoting Metacognitive Awareness. The second framework requires greater understanding of metacognition and suggests that students' metacognition can be developed by promoting general awareness of one's metacognition, improving knowledge of cognition, improving regulation of cognition, and by fostering an environment that promotes metacognitive awareness (Schraw, 1998). Promoting a general awareness of metacognition requires discussion about the role of metacognition in learning, that teachers model their own metacognition, and that time is allotted for discussion and reflection. Improving knowledge of cognition requires making all types of metacognitive knowledge explicit. Improving regulation of cognition can be accomplished through the use of prompting to promote the act of regulation in students. Fostering an environment that promotes metacognitive awareness requires an emphasis on mastery over performance. Lastly, additional ideas for promoting metacognitive development include applying metacognitive skills in meaningful contexts, observing the

Table 5
Two Frameworks for Developing Metacognition

Promoting General Metacognitive Awareness (Schraw, 1998)	Designing Metacognitive Activities (Lin, 2001)
Promote General Awareness	Opportunities to self-assess
Discuss mc role in learning	Strategy training, questioning, prompts
Teachers model their own mc	
Allot time for discussion and reflection	Help students articulate their thinking
	Support explaining and justifying thinking
Improve Knowledge of Cognition	Explain where they are in learning process
Make all types of mc knowledge explicit	
Improve Regulation of Cognition	Foster Shared Understanding of MC Goals
Prompt to promote regulation	Greater awareness means more willing
Foster an Environment Promoting MC Awareness	Develop Knowledge of Self-As-Learner
	Specify role in current culture
Mastery over performance	Learner characteristics
	Personality Dimensions
Additional Ideas	How these impact learning
Apply mc skills in meaningful context	
Observe experts using mc skills	
Have experts reflect on one's work	

Note: mc = metacognition. Framework in left column is adapted from "Promoting General Metacognitive Awareness ," by G. Schraw, 1998, Instructional Science, 26(1-2), p. 113-125. Framework in the right column is adapted from, "Designing Metacognitive Activities," by X. Lin, 2001, 49(2), p. 23-40.

use of metacognition by experts, and having experts reflect on one's work (Schraw, 1998). Refer to Table 6 for a summary of these two frameworks.

Defining Metacognition. Understanding metacognition and its components is critical for implementing Schraw's framework for promoting metacognitive development. Schraw and Denison's (1994) definition of metacognition and it's components was used to identify explicit examples of metacognitive thinking in the observation data. This framework separates knowledge of cognition from the regulation of cognition and is commonly cited in the literature (Kramarski & Zoldan, 2008; Mevarech, et al., 2010; Schneider & Artelt, 2010). Knowledge of cognition refers to what students know about their own thinking and regulation refers to the activities students use to control their thinking. Metacognitive knowledge is declarative, procedural, or conditional where declarative knowledge refers to knowledge about one's skills, resources, and abilities. Procedural knowledge refers to one's knowledge about how to implement a procedure, while conditional knowledge refers to knowledge about when and why to use a particular procedure. Figure 3 describes the two components of metacognition.

Knowledge of cognition is a matter of knowing something about one's cognition whereas regulation of cognition refers to what you do with that knowledge. Regulation of cognition refers to activities such as planning, information management, monitoring, debugging, and evaluation. Planning refers to planning, goal setting, and allocating appropriate resources based on one's knowledge. Information management refers to the use of skills and strategies to efficiently process information. Monitoring consists of assessing one's progress in learning or strategy use. Debugging refers to the use of strategies to correct errors. Lastly, evaluation consists of appraising and accuracy and efficiency of one's work. This study looked for examples where a person's metacognitive knowledge was made explicit and share's what this looked like and how the thinking was made explicit in the classroom setting.

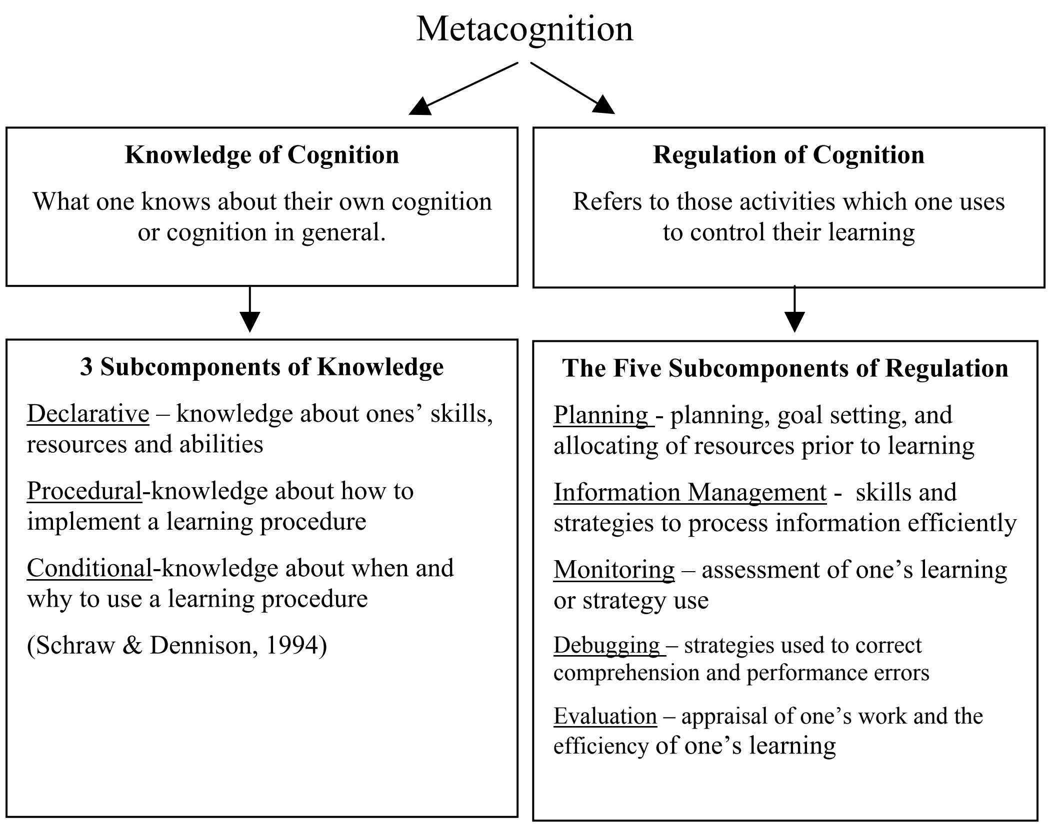

Figure 3. Components and Subcomponents of Metacognition. Adapted from, "Assessing Metacognitive Awareness," by G. Schraw and R. Dennison, 1994, *Contemporary Educational Psychology, 19*(4), p. 460-475.

Lin's (2001) and Schraw's (1998) frameworks along with Schraw and Dennison's (1994) framework for defining metacognition and its components provide a lens for understanding the qualitative data collected in this study. Analyzing the data in this study consisted of searching for the elements presented in these frameworks and making sense of what was present in the data to describe the embedded metacognitive development occurring in this classroom.

Qualitative Data Analysis

Qualitative data used in this study included documents, classroom observations, interviews with the instructor using an interview guide, and open-ended surveys with the students. The text of each of these data sources was searched for recurring words and themes. Patton (2002) refers to this type of analysis as content analysis and explains that content analysis is more generally referred to as, "Any qualitative data reduction and sense-making effort that takes a volume of qualitative material and attempts to identify core consistencies and meanings" (p. 453). The remainder of this section will discuss the preparation of the data, a more specific description of content analysis with each type of data, and a brief explanation of why this method was selected for the data analysis.

Course Syllabus and Schedule. The course syllabus and schedule were analyzed to provide understanding of the overall expectations of the students in this course. The syllabus was read thoroughly and notes were made about the content of the document. These ideas were then consolidated into themes and presented in the findings. This method of analysis was used to better understand the instructor's written expectations of the students and the overall structure of the course.

Course Textbook. *Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers* (Aichelle & Wolfe, 2008) was the required textbook for this course. This text provided an underlying foundation for discussions and activities that occurred in within this classroom and thus was analyzed to gain an understanding of the written curriculum including a description of the teaching philosophy and the expectations of the students prior to the classroom discussions. The textbook data was prepared by collecting two separate parts from the text. The first part consisted of all information written by the authors to the student about the intent of the course and how to be successful learners in the course. The second part prepared for analysis consisted of the pages assigned to students during the period of data collection, including all assigned pages studied between Exam 2 and Exam 3. Patterns were identified in the text by reading through each section, making notes about the content, organizing the notes by common themes, and then coding the overall data according to the identified themes. This analysis provided the researcher an understanding of the textbook authors' expectations and explanations of how students could become successful learners in this course. This analysis of the textbook sections also became a description of the intended curriculum to then be compared with the enacted curriculum.

Classroom Observations. A subset of classroom observations were randomly selected to be transcribed and subjected to content analysis to counterbalance the self-report data provided by the instructor and the students. Video vignettes from the classroom observations were transcribed with an emphasis on the classroom dialogue. Tone and gestures were also included in the transcript when they offered depth and meaning to the overall data. The transcripts were organized by textbook page references so that the MWF Friday discussions of a page were paired with the TR discussions of the same page. The observation transcripts then underwent two separate analyses. While both were content analyses, the first was an inductive analysis which involved discovering patterns from the data in light of the lenses previously described (M. Patton, 2002). For this analysis, the researcher read through all data making notes about what was occurring in the classroom. These notes were then organized into themes. A final pass was made through the data to confirm the consistency of the coding. This analysis provided insight into what was occurring in the classroom and provided a balance to the self-reported data.

The second method of content analysis used for the classroom observations was a deductive analysis where the data was analyzed using Schraw and Denison's (1994) framework defining metacognition and its sub-components (M. Patton, 2002). See Figure 3 on page 92 for a model of this framework. To complete this analysis, the researcher made a first pass through the data to identify all explicit episodes of metacognitive thinking. A metacognitive episode typically began with a question posed by a student on the homework assignment and consisted of multiple interactions between the student and instructor. The episode typically ended when the conversation moved on to a new topic.

After identifying episodes of shared metacognition, several passes were made through the observation transcripts such that each shared episode was coded for the appropriate subcomponent of metacognition as described by Schraw and Dennison (1994). Each pass through the data focused on a single subcomponent and each subsequent pass was used to code for the next subcomponent while also verifying previous codes. A final pass through the data was made to verify all codes and create a tally for each subcomponent. Episodes could have been coded in both a knowledge subcomponent and a regulation subcomponent since knowledge is a process of knowing what you know while regulation is doing something with that knowledge. In addition to coding for the subcomponent, each episode was also coded for who provided the thought. An episode was coded as *student* if the student provided the thought, *teacher* if the teacher provided the thought, or both *student and teacher* if the episode relied on both the student's and instructor's interaction. This process of coding for both type and supplier of the thought allowed the researcher to describe who was thinking and what type of thinking was being shared in the classroom. Appendix E contains coding examples.

Schraw and Denison's (1994) framework for defining metacognition and its components was selected predominantly because of its common use within metacognition literature and its ability to capture the metacognitive thoughts that were both present and absent in the classroom. The use of a single coding framework built solely from the collected data would not have provided a complete picture of what was absent or not being modeled and discussed in the classroom. According to Schraw's (1998) framework for promoting metacognitive awareness, identifying the types of metacognitive thoughts that were both shared and not shared is important because it is important for all types of metacognitive thoughts to be made explicit.

Instructor Interviews. Interviews conducted with the instructor using an interview guide allowed the conversation to evolve and focused on thoughts, feelings, and beliefs about mathematics, mathematics education, the development of metacognitive thinking, as well as the instructor's perceptions of the classroom environment. All interviews were transcribed verbatim and subject to content analysis as described in previous discussions. Multiple passes were made through the interview data and notes were made about how the data could be used to describe the embedded instruction in this classroom. These notes were then organized into themes and a few final passes were made through the transcripts to complete the coding process according to the themes that were identified. This analysis provided details related to the main ideas of promoting metacognitive development while sorting and grouping the instructor's ideas presented across both interviews could be compared with themes from other collected data.

Student Surveys. Open-ended surveys were administered to the students to provide insight into their thoughts, feelings, and beliefs about mathematics, mathematics education, and the development of metacognition. This data also underwent content analysis and major themes were identified. These themes were then grouped into categories for further analyses. This data was first organized by sorting student responses to each question. Each student response for the question was then read and assigned a one or two word description. These descriptions were then organized into categories and a final pass was made through all responses, keeping a tally of the student responses in each category. This type of analysis provided a voice for the students by reporting their feelings using their own words rather than words created from the literature or by the researcher. Tallies also identified how many students experienced those feelings within the popular categories. Words used by students that were either in extreme contrast to the more prevalent opinions or words used by students that provided interesting perspectives were also reported in the findings.

Quantitative Data Analysis

Metacognitive Awareness Inventory. Schraw and Dennison's (1994) framework provides categories for actual metacognitive thoughts, but many students automatize their metacognitive thinking and thus have varying levels of metacognitive awareness. The Metacognitive Awareness Inventory (MAI) measures students' awareness of their own metacognitive thinking. A maximum score of 260 indicates that the participant identified every example of metacognitive awareness as "very true of me."

A score of 52 indicates that the participant identified every example of metacognitive awareness as "not true of me at all." This study used the MAI as a pre/post measure to look for change in the students' awareness levels. The sum of the responses from the MAI was calculated to provide an MAI score and only the scores from students that had both a pre- and post- score were used in this analysis. The scores for students with both pre- and post- data were then entered into SPSS. A t-test for dependent samples was used to identify changes in metacognitive awareness over the course of the semester. For this test alpha was set to 0.05 and H_1 suggested there was a difference between the pre- and post- scores and H_0 suggested there was no difference between the pre- and post- scores. This method of analysis was selected to identify changes in students' metacognitive awareness over the course of the study.

Summary

The analysis of both qualitative and quantitative data described how an instructor promoted metacognitive thinking in her students and to determine if the instruction that was observed during the study changed students' metacognitive awareness. Understanding the types of shared metacognitive thoughts and describing the embedded instruction was accomplished through the analysis of actual shared metacognitive thoughts, in conjunction with the use of specific strategies embedded in classroom instruction, while also considering the overall development of the learning community. Lastly, the analysis of student scores on the Metacognitive Awareness Inventory was used to identify changes in students' metacognitive awareness.

Triangulation of the Data

Triangulation of different types of data and methods of data analysis helps to build coherent and credible themes (Creswell, 2003; M. Patton, 2002). Further, the use of a variety of strategies reduces bias and limits distortions that would occur through the use of only a single method, source, or analyst. M. Patton (2002) identifies the triangulation of methods, sources, analysts, and theories as four different kinds of triangulation for improving qualitative data analysis.

Triangulation of Methods

Methods triangulation requires checking the consistency of the findings that are generated through different types of data collection methods. The comparison of the qualitative and quantitative data serves as a form of comparative analysis where convergent findings increase confidence and divergent findings provide insight for better understanding the complexity of the phenomenon (M. Patton, 2002). More specific to this study, the quantitative data was used to support the effectiveness of the embedded metacognitive instruction on students' metacognitive awareness. This provided depth in understanding the impact of the instruction rather than on simply drawing conclusions that the embedded methods identified as suggestions in the literature impacted students' awareness.

Triangulation of Sources

Triangulation of sources requires checking for consistency from different data sources within the same methods. When differences do occur, it is not necessarily because one method is better than the other, but instead that the different data types are capturing different things. The analysis should attempt to understand those differences that become apparent. The comparison of the different sources may occur by cross-checking the data collected at different times or by comparing information gathered through different means (M. Patton, 2002). This study consisted of many layers of cross-checking within the qualitative data. Most explicit because of the organization of the findings is the comparison between the textbook, the observations, the instructor interviews, and the student surveys. This comparison was important since each source provided a different perspective about what was occurring or was expected to occur in the class. A second comparison of sources was checked for consistency of what the instructor said over time. Several of the questions and statements from the first interview were also discussed in the second interview. Convergent responses served as confirmation of the instructor's response while divergent responses typically lead to more discussion during the interview to understand those differences. A third comparison within the qualitative sources was the ability to compare what people, both the students and the instructor, said in public versus what they say in private via the student surveys and instructor interviews. Like other comparisons, there tended to be both convergent and divergent responses within this comparison and the final instructor interview provided an opportunity for the researcher to explore these ideas further.

Triangulation with Multiple Analysts

Triangulation with multiple analysts can be conducted using two or more independent analysts, a review of findings by the inquiry participants, or by having an expert audit review (M. Patton, 2002). This type of triangulation was the most lacking in regards to this study. While the use of independent analysts and the use of inquiry participants to review the findings were both completely absent from this study, the doctoral committee did consist of experts that assessed the overall process of the study. The committee consisted of experts in quantitative research, qualitative research, mixed-methods research, and metacognition.

Triangulation of Theory/Perspective

Triangulation of the theory or perspective consists of using multiple perspectives to interpret the data. This type of triangulation allows for explorations of how different assumptions affect the findings and interpretations (M. Patton, 2002). This study used two differing theories of how to promote metacognitive development as lenses for analyzing and interpreting the data in this study. The first lens, developed by Schraw (1998), seemingly requires an understanding of metacognition to ensure proper implementation of the theory in the classroom while the second focuses on more general ideas of instruction rather than on specific ideas in metacognition (Lin, 2001). The use of these two separate theories forced a look at the data in two separate and unique ways and the divergence of the findings from the two separate theories presented strong support for the interpretation of the findings.

Issues of Trustworthiness Within the Qualitative Data

Issues of validity and reliability are important for the overall credibility of the study and allow the researcher to create meaningful and accurate conclusions from the study's data (Creswell & Plano Clark, 2007). Trust is built through addressing issues regarding internal validity, reliability, and external validity (Merriam, 1998).

Internal Validity

Internal validity deals with the question of how the research findings align with reality and was addressed through triangulation, identification of researcher bias member checks, long-term observations, and presentation of negative or discrepant information (Creswell, 2003; Merriam, 1998). Both triangulation and identification of researcher biases were addressed previously in this chapter. A thick description is a literal description of the entity being investigated and should take the reader into the experience of the classroom (Merriam, 1998; M. Patton, 2002). A thick description of the classroom activities was presented in the findings to help the reader feel present in the classroom. Rather than observing only one or two days of the classroom the researcher observed four of the sixteen weeks of instruction. After a few weeks of observations the researcher had a good understanding of what was occurring in the classroom and reached saturation shortly thereafter. At this point, the researcher did not believe there was new or different information that would come out of the observations.

Reliability

Reliability of qualitative data refers to the extent to which the researcher's findings can be replicated and requires a description of the researcher's position, triangulation, and an audit trail (Merriam, 1998). Both the researcher's position and triangulation have been discussed at length, and this chapter provides an audit trail by providing a record of design, implementation, and analysis decisions that were made and should provide the details necessary to replicate the study (Merriam, 1998).

External Validity

External validity refers to the extent to which the findings of this study can be applied to other contexts and is achieved through thick descriptions, a description of how typical this classroom is in comparison to other college-level mathematics classrooms, and the use of multiple sites, cases, or situations. Between this chapter and the report of the findings, it should be possible to determine how closely a situation matches this research context and how typical this course is in comparison to others. Lastly, this study used two separate sections of the course taught by the same instructor to expand the findings across the instructor rather than only one section.

Summary

In summary, this naturalistic, embedded, mixed method design sought to describe the metacognitive instructional strategies used in a college-level geometry content course for pre-service elementary teachers and to determine if the instruction observed during the study had an impact on the students' metacognitive awareness. Data collected included interviews with the instructor, classroom observations, measures of metacognitive awareness, and open-ended surveys asking students to reflect on mathematics and the teaching of mathematics. The predominant qualitative data was analyzed for examples of explicit metacognitive thinking, use of strategies suggested in the literature as effective for promoting metacognitive development, and information regarding the development of an effective learning community. The variety of the qualitative data allowed for a collection of information from differing perspectives thus providing a more balanced look at how the instructional design impacts student learning from a variety of perspectives. Lastly, the Metacognitive Awareness Inventory was used to identify quantitative changes in students' metacognitive awareness during the study. The next chapter presents details of the data collected and begins with descriptions of explicit metacognitive thinking, a description of the strategy use for making metacognitive thinking visible, the results from the Metacognitive Awareness Inventory, and then closes with a discussion of the development of the learning community in the classroom. The final chapter presents an analysis of the data along with implications for the findings of the study.

CHAPTER IV

FINDINGS

This mixed methods, naturalistic case study sought to understand the development of metacognitive thinking in pre-service elementary teachers by specifically answering three research questions. The first question asked what types of metacognitive thoughts are being shared in the classroom and was answered by analyzing episodes of metacognitive thoughts that were shared during a series of classroom lessons observed by the researcher. The second question asked how a college mathematics instructor promoted metacognitive thinking in her students. Data to answer this question was collected from student surveys, instructor interviews, classroom observations, and a review of the textbook. The third and final question posed was does a college-level mathematics course change the metacognitive awareness of pre-service elementary teachers and was answered with the results from the pre-post- Metacognitive Awareness Inventory (MAI). The MAI was analyzed for change in metacognitive awareness over the course of the study and the responses on the surveys were analyzed to identify themes. Two interviews were conducted with the instructor using an interview guide, transcribed, and subjected to content analysis. To balance the self-reported data provided by the students and instructor, data for this study also included classroom observations, which were transcribed and analyzed for both the explicit sharing of metacognition and identifying who shared their thinking. After spending time in the classroom, it became clear to the researcher that the textbook also greatly impacted classroom discussions. A content analysis was conducted with the text to determine what expectations were placed on the students to be prepared for classroom discussions.

Patterns emerging from the analysis of the data painted a picture of the overall classroom norms, the explicit modeling of cognition in the classroom, the use of strategies in the classroom, and led to a description of the learning community in which the students actively participated. The first section of this chapter discusses the overall classroom culture and class norms. This section provides descriptions of the classroom from the perspective of the observations, the text, the teacher, and the students. The second section of the chapter uses Schraw and Dennison's (1994) categorization of metacognitive thoughts as a framework for identifying episodes of shared thinking. This analysis provides a description of the type of thinking that is being shared in the class as well as who is doing the sharing.

Since it is often difficult for students to share their thoughts, researchers have identified strategies such as modeling, prompting, questioning, and reflecting as methods for helping students become more aware of and identify their thoughts. Researchers have also expressed the importance of using those thoughts as launching points for discussions with the hopes that students will internalize the discussion for use when faced with a similar problem at a future date. Findings in the third section will discuss how the text uses these strategies, the actual use of these strategies in the classroom, the instructor's beliefs about these strategies, and the students' opinions about the usefulness of these strategies as a part of instruction.

A learning community has the potential for providing a safe and supportive environment for students. This learning community allows students to not only share their thinking, but also to analyze and critique thinking that is shared during class discussions. The final section of this chapter describes the learning community in this classroom from the perspectives of both the instructor and the students before summarizing overall patterns in the data.

The classroom and class norms

Before starting an in-depth description of the type of metacognitive thinking that is being made explicit, the strategies that are embedded in instruction, or the learning community present in the class, it is first important to have a basic understanding of the static classroom setting, underlying class norms, and students' general perspectives about the course. This classroom is physically constructed in a way similar to what many think of as a "traditional" mathematics classroom. There are chalk boards on two of the four walls, a teacher desk in the front of the room, a computer and document camera on the teacher desk, an interactive whiteboard that also serves as the "screen" for the computer and document camera projector, and 19 tables aligned in four rows with an aisle going down the middle of the classroom. All of the tables are facing the front with two student chairs at each table. This means there can be up to four students in all but one of the rows. The room is small, allowing just enough room between each row for people to move in and out of the rows and down the center aisle. Figure 4 provides a floor plan of the room. Just outside the classroom doors are two tables with approximately 10 chairs where students occasionally gather before class to discuss homework or general life happenings. Conversations from outside the classroom tend to move inside until the students are settled and, if they haven't already, they will begin discussing their homework due for the day.

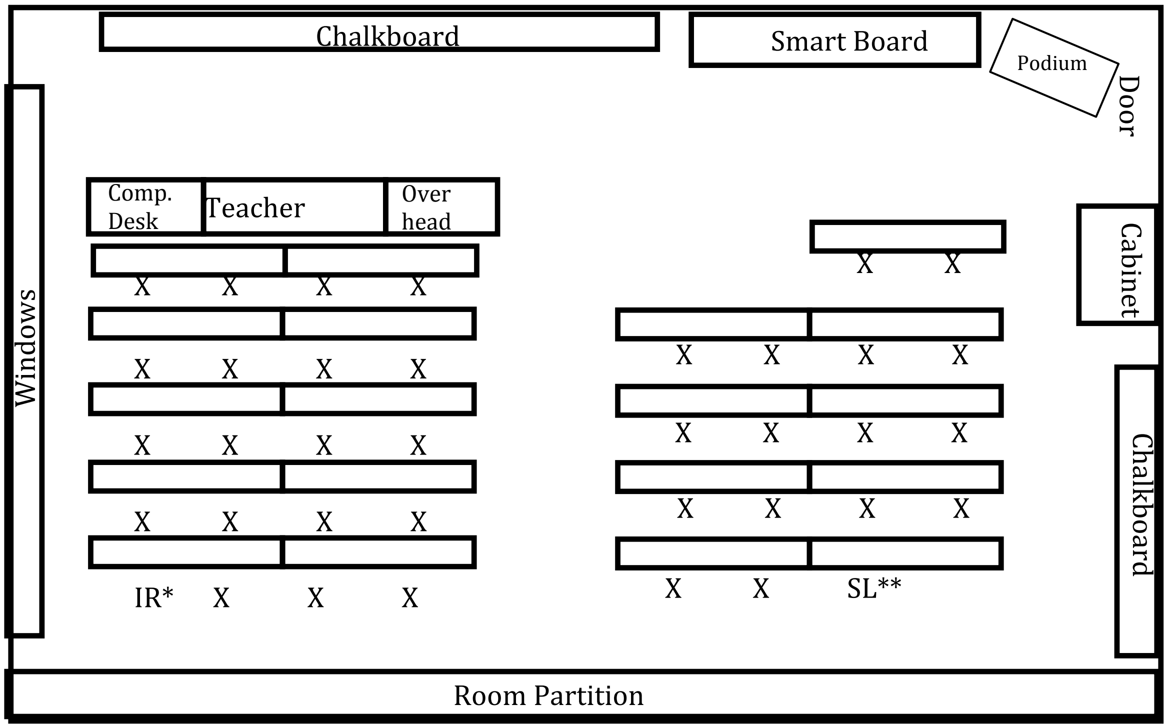

Figure 4. Floor plan of the classroom. *IR indicates the initial location of the researcher and video camera. **SL indicates the location of the researcher and video camera after the first week of observations.

During the time of the observations students were learning about the Pythagorean Theorem, finding perimeters of compound figures, working with the geometry of circles, recognizing symmetry of figures, and completing constructions with miras. Because of the inquiry-based philosophy and the speed at which the course moved, students often completed assignments over multiple topics on the same day. For example, approximately two thirds of the way through the observations, students were applying the knowledge they had gained about the Pythagorean theorem, starting to solidify their understandings of perimeters of compound figures, testing hypothesis about the geometry of circles, and just beginning to explore symmetry. Figure 5 provides a diagram of the overall unit structure between Exam 2 and Exam 3.

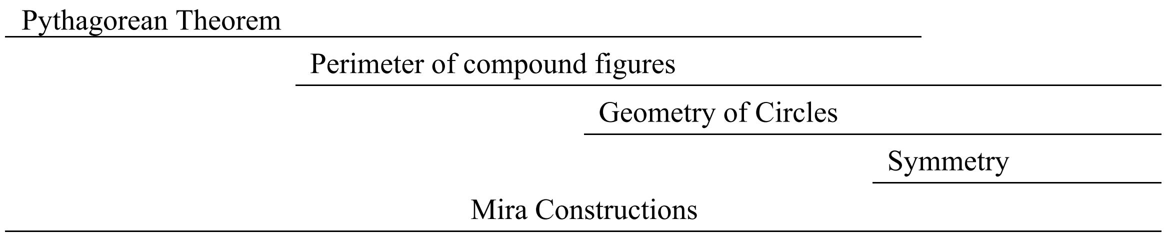

Figure 5. Content covered during the classroom observations. The far left side of the line indicates exploratory ideas intended to be connected to students' prior knowledge. The far right side of the line represents activities and problems intended to apply new knowledge students have gained as they progressed across the spectrum from exploration to application. Mira constructions occurred throughout the observation time. A vertical line passing through the figure would indicate a single day of instruction and the type of activities that would be completed during that day.

From the analysis of the observations, it was clear that the flow of class activities throughout the class time was similar and consistent. Students talked with each other for approximately the first five minutes of class while the instructor distributed the graded assignments from the previous class meeting. Then she almost always said "Okay, let's go ahead and get started. What questions do you have on page __?" At this time students yelled out problem numbers that they did not understand or were not able to complete on that particular page of their assigned homework. Next, the instructor asked, "Who would like to come show us how to do this one," and a student volunteer approached the document camera in the front of the room and worked through that particular problem as if she were the teacher providing an example. Once the student finished, the instructor asked if there are any questions for the presenting student and asked the student to answer any questions or re-explain any material as requested by other students. After the problem presentation, the instructor thanked the student, the student returned to her seat, and the process was repeated for the next problem on that page. Once all of the questions had been answered for that particular page, the instructor asked if there were are any further questions that needed to be answered over that page and, if not, they moved on to the next page in the assignment. This process was repeated until all of the homework problems were discussed. Students then stapled all of their homework pages together and submitted the assignment to be spot checked for an accuracy grade by a separate grader assigned for the course. By this time, it is typically the end of class.

The instructor's expectations of the students were made clear through the analysis of the syllabus and through the instructor interview. The grading for the course consisted of several homework assignments, one quiz, three exams, and a comprehensive final. Students received a homework schedule for the entire semester on the first day of class and were expected to look over each page of the assigned homework, complete as much as they could, and come to class with questions for the class discussion. The homework schedule was originally designed by the textbook authors, but has been revised by the instructor of the course at this university to meet the specific needs of their students. The quiz was created by the instructor and reflects the types of questions that students can expect on the exams. Exams are also created by the instructor and assess the content the students have studied using similar questions as those found in the textbook. Only the final is comprehensive.

One of the most frequent explanations of how this particular course is different from other mathematics content courses is that the students are expected to do most of the talking and explaining while the instructor serves as a facilitator or guide. The second most common description, this one typically provided by the students, is that students are expected to complete their homework assignments before being taught how to do the problems. These two ideas are the underlying foundation of the inquiry-based setting that is the heart of this classroom.

The text is a driving force for the course itself and the discussions that occur in this classroom. The instructor relies heavily on the questions that are asked on the assignment pages and the questions the students ask in response to the questions given on the page; and, thus, it is important to understand the intent of the text as described by the textbook authors. The Aichele and Wolfe *Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers* (2008) textbook was selected by the course instructors to serve as the foundation for implementing inquiry-based learning in a course on geometry content. Throughout the introductory material of the book and the appendices written for the student, the authors emphasize that their goal is to "provide an inquiry-based experience" (p. xi) for students. The authors explain that:

> The topics presented in this inquiry-based curriculum are "developed" by letting students explore evidence that can lend both credibility and meaning to basic relationships. Often, students are asked to observe, describe, and conjecture about relationships that appear in models and examples explored in activities. Also, activities are designed to stimulate group or whole-class discussion of students observations to arrive at a social consensus about the truth and meaning of geometric ideas (p. xi).

Furthermore, the authors want the geometry the students are learning in this course to connect with examples, previous experiences, and well-known ideas rather than to more abstract mathematics courses the student may take in the future. The sharing of student thinking and the methods used to work out problems "affords students insight into how and why different methods work in solving problems" (p. xii), and the classroom discussions are an essential resource for motivating students to make sense of the material. The instructor's "ability to manage the discussions and activities without traditional lecturing is a critical factor in the success of this approach, which [the authors] have found enables students to arrive at a deeper, more robust, understanding and appreciation for the power, beauty, and meaning of geometry" (p. xii).

In the preface written directly to the students, the authors of the textbook explain that their goal is support a richer and deeper understanding of elementary geometry that the students will need for teaching. More specifically, the activities in the text support "inquiry-based" curriculum where whole-class discussions and group work replace traditional lectures. In particular, this curriculum is explicitly developed to provide future elementary and middle school teachers with experience:

- Recalling and appropriately using standard geometric ideas
- Learning and making sense of new geometric ideas
- Discussing geometry with peers
- Asking questions about geometry
- Listening and understanding as others talk about geometry
- Gaining meaning from reading geometry
- Expressing geometric ideas through writing
- Thinking about geometry and
- Doing geometry (p. xiv).

The authors further explain in the appendix that the activities are designed to stimulate class discussion. They also emphasize the personal responsibility for learning that is placed on the student by explaining, "We as individuals are the only ones who can really know if we understand something" (p. 641) and provide a list of questions for the student to self-assess how well the material has been learned. Also in the appendix is an explanation to the student that the curriculum is designed around the idea that the "student tries first" rather than the more traditional "template problem approach" (p. 642) which consists of sample problems delivered through traditional lecture that the students then practice on their own at home.

Throughout the information written to the students there is an emphasis on the professional judgment of the teacher to determine the balance between lecture, group work, and inquiry-based discussions, emphasizing the important role the instructor plays in course development and the authors' definition of inquiry-based learning. A curriculum provides a guiding document and underlying foundation for a particular style of instruction; however, the instructor has the important role of deciding how to bring that curriculum to life. The instructor in this study explained that an inquiry-based classroom involved:

> Active participation where the teacher is gently prodding and guiding the students as they discover and question themselves and peer learning where students are discussing with one another and helping one another out… the teacher doesn't just stand up and talk but instead steps back, closes their mouth and only asks questions…Inquiry-based learning is about leading students to discover the material…. rather than directly telling them what they need to learn.

The instructor explained that in this inquiry-based classroom she envisions herself as a facilitator or guide. There is more student talk than teacher talk in the inquiry-based classroom, and the students should be actively engaged throughout the entire class. The teacher's role is to make sure everyone is involved. The instructor identified "actively engaged" as the most critical element in making the inquiry-based classroom effective and explained that active learning entails going over the homework before class and then asking questions of the whole class or small groups during class to make sure students understand the material.

She also explained that students were expected to complete their homework so "they can have meaningful discussions" and, while she knew the students believed they were expected to complete their homework before being taught the material, the expectation was actually just that they should have looked over the assigned pages, tried and completed as many as possible, and then asked questions in class about anything they didn't understand. She explained that she does not require students to get up and present problems to the class, but she strongly encourages them to do so and struggles with knowing how to get more of them up to present.   Lastly, regarding the overall design of the course, the instructor wants the students to leave the class with a "better knowledge of geometry and second with the ability to think for themselves and the attitude willing to dig a little deeper and try on their own and not be scared to make mistakes, and then if they make a mistake just continue to pursue the solution."

In this college setting, the students provide the final dimension for understanding the overall course design. When students were asked about two things they really liked about how the course was taught in the post-survey, they reported they "liked the group discussions" (22/44), while several also reported they "liked going over homework in class" (5/42) and that the "teacher explained things clearly" and "in more than one way" (5/42). Additionally, one student commented that "The course caused her to think on her own" and another student reported that she "Realized she could figure out a problem she was struggling with and the course made her try." In contrast to the above responses, one student reported there was "nothing she liked about the way the course was taught" and another expressed that "inquiry-based teaching should never be used," but these students did not elaborate on their responses.

When asked how the way the course was taught impacted their learning, also on the survey, several students reported some type of direct positive impact on their thinking (8/44) including that they "now trusted their thinking more," "were more likely to think outside the box," "thought in new ways," "thought harder," "thought more," "thought differently," and "thought on their own." Several students reported that the mode of instruction positively impacted their confidence (5/44) by explaining the course "made [them] trust more in [their] own thought" and "[the instruction] gave me more confidence in myself." Students also reported that the way the course was taught helped them "learn to provide better explanations about their work" (5/44). Again, two students reported that the way the course was taught "did not impact their learning;" however, these were not the same two students that did not like inquiry-based learning reported above. One student also reported that the mode of instruction "helped [her] question [herself] and work to the answer."

Lastly, when asked on the survey about two things they would change about how the course was taught, about a third (15/44) of the students suggested "provide more direct instruction" either through lecture or through the text, about a fifth (8/44) of the students suggested "less homework," and about a fourth (12/44) commented that "nothing" should be changed. Other notable responses included a student comment that there should be “less busy work and more discovery,” while another commented, “every student should be required to get up at some point.”

This first section has provided an explanation of what the classroom looks like and what is expected of the students and the instructor. The students must attempt their assignments and come to class prepared to discuss any homework problems they did not understand, while the instructor is responsible for creating effective classroom discussions for students so their questions will be answered. The text serves an important role of helping students identify what they do and don’t know about the current material so they know what to ask during class and provided an explanation to the students about how this course would be different than other mathematics courses they may have taken. The next part of this chapter presents examples of explicit sharing of metacognitive knowledge and regulation, followed with examples of strategy use, then provides a description of the learning community created within the classroom.

Explicit Modeling of Knowledge and Regulation of Cognition

The explicit modeling of one student’s metacognition helps other students develop their own metacognitive thinking. Hearing and/or seeing the thinking of another student, then comparing it with one’s own thoughts, evaluating the shared thinking, and determining what to do with the thought is critical for metacognitive development. Metacognition is commonly divided into two separate, but related components. The first component is knowledge of cognition and encompasses declarative, procedural, and conditional knowledge. Declarative knowledge is knowledge about one’s skills, resources, and abilities. Procedural knowledge is knowledge about how to implement a learning procedure. Conditional knowledge is knowledge about when and why to use a particular learning procedure. Knowledge of cognition, in contrast to regulation, is simply what one knows about their own thinking or thinking in general. What one does with that information is the second category, regulation of cognition. This category encompasses the activities used to control one's thinking such as planning, information management, monitoring, debugging, and evaluation. Planning consists of goal setting and resource allocation while information management refers to skills and strategies used to process information efficiently. Monitoring is an ongoing assessment of one's learning or strategy use. Debugging strategies are used to correct comprehension and performance errors and lastly, evaluation is an appraisal of the accuracy and efficiency of one's learning.

Identifying both types of cognitive thought along with who is sharing that cognitive thought provides an initial outline of what types of metacognition are being modeled in this classroom. A table containing the content and number of problems on each page can be found in Appendix F. Appendix E provides sample chunks of dialogue, coding, and a complete description of each category. The observation data presented below provided insight into what types of metacognitive thoughts were being shared in the classroom and describes the modeling/prompting that was done by the instructor. Thus, analysis of the observations provided data to answer what types of metacognitive thoughts are being shared in the classroom and how the instructor is promoting metacognitive thinking in her students.

Knowledge of Cognition

Of the 99 total metacognitive episodes, 78% were episodes of shared Knowledge of Cognition. Declarative statements of knowledge, such as "I am confused if you are supposed to draw the parallel line or the perpendicular line," were made only by the students and consisted of only 4% of all knowledge of cognition episodes. Procedural knowledge episodes, such as "I started by finding the perimeter of the field…so that would be 200 feet cause this is 550 and 350 so that makes this 200 and…" consist of knowledge about how to implement a procedure and provided 74% of all shared knowledge episodes. The student-only thoughts provided 54% of the procedural episodes, 32% were provided through integrated student-and-teacher thoughts, and 14% were provided by the teacher-only thoughts. Conditional knowledge episodes consist of knowledge about when and why to implement a procedure, such as, "the diagonals of a kite are perpendicular and when bisected, so that is why this one works" and consisted of 22% of all shared knowledge episodes. There were no student-only thoughts, 76% were provided through integrated student-and-teacher thoughts, and 24% were provided through teacher-only thoughts. A summary of this data is provided in Table 7. Sample episodes and descriptors for each category are reported in Appendix E. The data presented here suggests that most of the knowledge of cognition that is shared is procedurally oriented and is provided by the student; however, the sharing of conditional knowledge always involved the teacher.

Table 6
Frequency of Shared Knowledge of Cognition

Section	Declarative			Procedural			Conditional			Totals		
	SO*	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO
MWF	2	-	-	16	10	5	-	7	2	18	17	7
TR	1	-	-	15	8	3	-	6	2	16	14	5
Both	3	-	-	31	18	8	-	13	4	34	31	12
Grand Totals		3			57			17			77	

*SO: student only
ST: integrated student and teacher thoughts
TO: teacher only thought

Regulation of Cognition

Regulation of cognition is the second category of metacognitive thought and refers to activities of controlling one's learning. Observation episodes were coded for planning, information management skills, monitoring, debugging, and evaluation. Of the 99 total metacognitive episodes, only 52% were episodes of shared regulation of cognition in the classroom. Note the fewer instances of regulation of cognition episodes (52%) in comparison to knowledge of cognition episodes (78%). Planning episodes consist of planning, goal setting, and allocating resources prior to beginning the task and consisted of only 10% of the total regulation episodes. Student-only thoughts and integrated student-and-teacher thoughts each provided 20% of all planning episodes while 60% were provided by teacher-only thoughts. Table 8 presents a summary of the number of episodes relating to sharing of regulation of cognition.

Table 7
Frequency of Shared Regulation of Cognition

	Planning			Info. Mgmt.			Monitoring			Debugging			Evaluation			Totals		
	SO*	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO
MWF	1	1	2	1	3	2	-	3	1	-	6	1	-	5	-	2	18	6
TR	-	-	1	3	5	1	2	1	-	1	5	-	-	6	-	6	17	2
Both	1	1	3	4	8	3	2	4	1	1	11	1	-	11	-	8	35	8
Grand Totals		5			15			7			13			11			51	

*SO: student only
ST: integrated student and teacher thoughts
TO: teacher only thought

The next three subcomponents of regulation of cognition consist of thoughts occurring during the use of skills and strategies while solving problems. Information management consists of activities such as organizing, elaborating, summarizing or selective focusing that occur during the activity and consisted of 29% of the regulation of cognition thoughts. Student-only thoughts provided 27% of the information management skills, 53% were provided by integrated student-and-teacher thoughts, and 20% were provided by teacher only regulatory thoughts. Monitoring episodes consists of ongoing assessment of one's learning and strategies used while working and provided 14% of all regulation episodes. Student-only thoughts provided 29% of the monitoring episodes, 57% were provided by integrated student-and-teacher thoughts, and 14% were provided by teacher-only thoughts. Debugging is the last subcomponent that occurs during the activity and consists of strategies used to correct understanding and errors. This subcomponent provided 25% of all regulation episodes. Student-only thoughts provided 7% of these episodes, 85% were provided by integrated student-and-teacher thoughts, and 7% were provided by teacher-only regulatory thoughts.

The final subcomponent of regulation of cognition occurs after the learning experience. Evaluation consists of analyzing performance and effectiveness after the completion of the learning activity and consisted of 22% of all regulation episodes. There were no student-only or teacher-only evaluation episodes, meaning 100% of evaluation episodes were provided through integrated student-and-teacher thoughts. There is very little discussion of planning before students began working on a problem and most evaluation occurred through integrated student-and-teacher thinking. Further, most regulatory thoughts occurred during the process of solving the problem and most thoughts were shared through integrated student-and-teacher thoughts.

Regulation is commonly described as what students are doing with their cognitive knowledge. Hence it makes sense that episodes could be coded for both a knowledge category and a regulation category. For example, a person could be planning how to implement a procedure. In this case the episode would be coded as procedural knowledge and planning regulation. While it is possible for a person to be aware of her knowledge and not do anything with it, it does not seem possible to regulate cognition in the absence of an awareness of some type of knowledge. An analysis of the observation data indicated there was a total of 99 episodes of shared thinking. Of those 99, 33 episodes were coded with both a knowledge and a regulation subcomponent leaving 44 statements coded as only knowledge and 18 coded as only regulatory. At first glance, it seems odd to have 18 statements coded as regulation without any knowledge; however, it is possible that the knowledge statement was not made explicit during the episode. Table 9 displays a summary of the statements that received both a knowledge and regulation code.

Table 8
Episodes Coded for both Knowledge and Regulation of Cognition

		Knowledge of Cognition			
		Declarative	Procedural	Conditional	Total
Regulation	Planning	-	1*	1	2
	Info Mng	1	10	-	11
	Monitoring	-	4	-	4
	Debugging	-	5	2	7
	Evaluation	-	1	8	9
	Total	1	21	11	33

*Number of Occurrences

From the observation data, it can be concluded that the sharing of metacognitive knowledge is more frequent than the sharing of regulatory knowledge. Further, procedural knowledge far exceeds any other type of metacognitive thought shared in this classroom and is commonly done either by the students themselves or in conjunction with the teacher. There are very few episodes which represent declarative thinking. Regulatory thinking is typically shared through episodes that consisted of integrated thinking by the student and by the instructor. While this section has described the types of metacognitive thoughts that are being shared in the classroom, the following section further explores episodes of integrated thinking and describes how a college mathematics instructor promotes metacognitive thinking in her students.

Metacognitive Strategies

The previous section of this chapter described the actual sharing of thinking within the classroom. This section denotes the strategies being used by the instructor to promote metacognition. The findings from the observations show an emphasis on how students modeled their procedural understandings of the work they were doing. Through the integrated student-and-teacher thoughts, these findings also suggest the important role of the instructor in helping students become more aware of their thinking. Using prompting and questioning to help students get their thinking out of their head and into the discussion assists in the development of a community where students feel comfortable and thus their own thoughts become critical elements in the development of metacognitive thinking. Three main ideas, the use of strategies, the development of dialogue, and creating social supports have been used in research to promote the sharing of thinking and thus promote metacognitive development. Information from the observations, the textbook analysis, the instructor interviews, and the student surveys were explored for each of these ideas as a way to paint a picture of how the instructor used metacognitive strategies, the development of effective dialogue, and the creation of social supports to promote metacognitive thinking in her students.

Strategies Identified in the Observations

Creating a learning environment that promotes metacognitive development hinges on the sharing of thinking and common strategies to help students become more aware of their own thinking. These activities include modeling, prompting, questioning, analyzing errors, and reflecting on experiences. From the observation data, there are 99 episodes of shared thinking, indicating thinking is being made visible and, more importantly, although at differing frequencies, both the students and the teacher are modeling their thinking. Students frequently modeled their procedural knowledge, while the instructor rarely modeled only her thinking, but instead more commonly integrated her modeling thoughts with those of the students.

A second popular strategy for helping students share their thinking was through the use of prompts and questions such as, "What do you need to do next?" or "How do you do that?" The use of prompts and questions was evident through the integrated student-and-teacher thoughts representing 40% of the total knowledge of cognition episodes and 69% of the total regulation of cognition episodes. These results show the students tend to have an initial thought, but are not able to work through the entire idea completely on their own and thus need to be prompted or questioned through the development of the remaining thought.

A third strategy common in the literature is analyzing errors. This activity can be present in either debugging while solving or through the evaluation after solving the problem. In either case nearly all of the episodes of debugging and evaluation, 85% debugging and 100% evaluation, were integrated student-and-teacher thinking, with only a single episode being completed by a student. Although procedural thinking is being modeled, students are not sharing their own thoughts related to their regulation of cognition.

Strategies Identified in the Textbook

The textbook for this course provided the launching point for the class discussions. During the first few minutes, class typically started with either a brief overview of content or by the instructor asking the students to identify questions from the homework assignment that they did not understand. Class time focused on answering questions about the problems presented on the assigned pages, thus the textbook played a critical role in shaping the discussion. For this reason, a description of how the book supports scaffolding, modeling, prompting, analyzing errors and reflection was a focus of this research and is discussed below.

The inquiry-based philosophy of the text ideally provides students with the opportunity to learn how to use self-questioning for learning new information from already internalized knowledge. The early pages of a topic, such as the Pythagorean theorem, give students the opportunity to use knowledge they have already developed in the class, specifically finding the area of objects and measuring lengths, to start building their ideas about the Pythagorean theorem. After determining the areas of the squares built from each side of the right triangle, students are asked to "describe any relationship you see between the areas of the three squares" (p. 319). The next problem asked the students to test this hypothesis with a triangle that is not a right triangle, then they must use previous knowledge of perpendicular lines to explain why their hypothesis was or was not true. Lastly, on the first page of the assignment the students are asked to identify the triangles for which the Pythagorean Theorem relationship would work. Through the next few pages the students are asked to continue using the relationship in new situations and to compare this strategy with another for finding slant lengths on the geoboard. The chapter is then brought to a close with a variety of problems requiring students to use their new knowledge of the Pythagorean theorem and perimeter. This process of exploring the new ideas with previous knowledge, formulating ideas/hypothesis, testing the hypothesis, providing direct instruction or describing the procedure, and providing additional practice is a common structure throughout the text and allows students to build on their knowledge from their previous understandings.

The main goal of the text is to provide students the opportunity to try a problem first using the knowledge they already have rather than having the instructor provide a set of example problems students must go home and practice. While the text does provide the problems for students to try before any formal discussion takes place, it also almost always provides a summary near the end of the chapter of the information they should have discovered by working the problems. While this information offers students an opportunity to confirm what they have learned, it also may serve as the initial source of information for students who look ahead, circumventing the inquiry activities in the early pages of the chapter.

Assuming that they work through the pages as intended, students are given the opportunity to develop their own thoughts and ideas before coming to class for the discussions. Some of the problems ask the students to record their work or to describe how they completed their construction, but with the exception of the Construct and Describe (CD) problems, students are not asked to describe the process they used to complete problems. Construct and Describe problems require students to complete a geometric construction and then provide a step-by-step description of how they completed the construction. Students are never asked to justify why their construction is valid. Once in class, students are expected to share their solution process, but since they have only recorded an example of their final work, they may not remember all of the steps that were a part of their unpolished process of trying to work from the beginning to end of the problem. Since the textbook is the foundation for class discussions, it is important to note that the textbook does not ask students to record their complete solution processes, and students present the work that is recorded on their paper. Unfortunately only polished final thinking is prepared, modeled, and discussed in class.

The philosophy of the textbook that the students try the problems first before class discussion takes place also allows nearly every problem to serve as a declarative knowledge prompt. The students must identify if they have the knowledge, skills, and resources to complete the problem. If they do, they complete the problem and if they don't they must decide if they will proceed by waiting and taking the question to class or entering the regulatory process of making a plan, managing information they know, monitoring their progress, debugging any issues that arise, and evaluating their solution when they finish. This, however, is the essence of problem solving so it is important to look beyond just the question for any additional opportunities the text provides students for modeling their metacognitive thinking.

The text provides very few prompts that specifically address conditional knowledge or regulation of cognition. Thus, students are asked to do, but rarely asked to explain how, when, why or to justify their work. One example does occur in a place following the section where students use both the *square root* and *area of a square* method for finding the slant length of a segment on a geoboard. The text asks the student to "Describe in your own words, the way you like best to calculate the length of a slanted line on the geoboard" (p. 328). This problem asks students to reflect on the two processes and choose a favorite, but then only asks the students to explain the procedure. Students are not asked to explain why one of the methods is their favorite or when it may be more expedient to use one method over the other. Further, while students did talk about this question a bit on their own before class, there was no whole class discussion focused on this problem during class time. Students missed out on opportunities to not only share their reflection, but also to share their conditional knowledge of when or why a particular strategy should be chosen over another.

Nearing the end of the chapters, after several exploratory problems, an example of the appropriate procedure and an opportunity to find the glide-reflection line, the task asks students to "check to see if the glide-reflection line works" (p. 544). Students are given the instructions to check if the process works and if it doesn't, they are instructed to "work through the steps again or discuss the problem with somebody in your group" (p. 544). First, this was the only example of students being specifically prompted to check, debug if necessary, and to talk with someone if the process did not work. The idea of checking with a neighbor is an implied understanding of the course at this time, but if all of the students in the group complete the procedure correctly then those students never have the opportunity to debug. Without this opportunity to practice debugging, students are not given the opportunity to hear and internalize the metacognitive process of debugging.

Reflection allows the student to think back through the process of solving a problem and determie more effective or efficient ways of learning and doing the problems. While this text does ask students to identify relationships in specific problems, these questions tend to be more information management thinking rather than reflective thinking since students are doing regulation of thought while solving the problem. In the pages used during the data collection phase, the text never asked students to identify if there is another way to solve a problem, identify an easier/better way to solve a problem, summarize their learning, or prompt them to go back and check that they have actually answered the original question. Students are also never asked to describe their overall complete problem-solving process that includes dead ends and wrong turns. From these examples, it can be seen that the text provides the initial opportunities for students to think about what they can do and try what they can when solving the problems. Most of the facilitation of the sharing of thinking is left to the instructor, so the development of metacognitive thinking is in the hands of the instructor.

Teacher Perceptions of the Strategies

Modeling, prompting, analyzing errors, and reflecting are all strategies that promote metacognitive development, but are not prevalent in the text and must be facilitated by the instructor in the classroom. Through the observations it was evident that modeling of student procedural thinking and prompting students when they were in need of assistance was important to the instructor, but analyzing errors and reflecting on the process were not as important. The interview with the instructor provided the researcher with the opportunity to discuss these ideas in greater depth. First, student modeling of thinking is important and is the backbone for the instruction in this course and modeling multiple methods for completing a single problem is often left in the hands of the students. The instructor explained:

> If we have multiple ways I encourage the students to share them. I have had a student get up before and do a problem one way and then somebody else goes well I got the same answer, but I did it another way and I ask oh okay would you be willing to share that with the class?

However, when asked if she would bring out another typical way of solving a problem that didn't come out through class discussion, the instructor explained:

> If I know probably half the class has done it this other way I will say has anybody done this problem using whatever and if somebody says yes then I say, would you be willing to explain that cause sometimes students could benefit from those other methods. If it's just another cool method, then not necessarily.

This suggests that the instructor does not view multiple strategies as an important aspect in this course. Further, alternative solutions are typically only shown when a solution is presented that is not consistent with the way most students solved the problem.

Analyzing errors and debugging are approaches that have been suggested for promoting metacognitive development, but based on the observation data, the debugging that is occurring in the classroom consists of integrated student-and-teacher thoughts, rather than students accomplishing this task on their own. From the observations, it was more common for correct answers and solution strategies to be presented in class rather than strategies that were in need of the debugging process. The instructor explains that she does tend to rely on the students to identify and ask questions about errors, but she is:

> Not going to let them leave the math classroom having something that is completely incorrect so [she] will keep prodding and pushing until somebody brings something up or it gets to the point where we have been on it for a while

and they're getting tired, I may go ahead and step in and mention, well in mathematics society we typically accept it this way.

Thus, the students are encouraged and supported through the debugging process, but when they are getting tired or the end of class is near, they are provided with a correct solution that is typically consistent with the most commonly accepted procedure for completing the problem. Analyzing errors and debugging are regulatory processes that the students are not showing on their own and, when students show that they are getting tired or time is running out, the instructor will give them the information they were trying to understand rather than summarizing and continuing to work on their understandings.

Student Perceptions of the Strategies

To attain a better understanding of how the students felt about sharing and modeling their thinking, analyzing errors, and reflecting, they were asked on the survey to consider a scenario where they were given a problem, asked to work on it individually, and then discuss it in a small group by comparing their answers and strategies for solving the problem. The scenario explained that while discussing in their groups, students realized there were major differences in their group member's responses so the teacher had them share their work and thinking on the board so other students could discuss the solution and ask each other questions. When asked on the survey what should happen after this whole class discussion, slightly less than half of the students responded that the "*instructor* should give the answer" and the "*instructor* should provide a correct explanation" or description of how to solve the problem (17/43). Slightly fewer students expressed that the "class should come to an agreement" and the "class should decide on the correct approach" to the problem (13/43). Other interesting responses included the instructor should "reveal" or "make apparent" the answer and the correct solution strategy or allow the class to "vote on the correct strategy."

In the follow-up question about what should happen next, the students were asked if this type of discussion was an effective use of class time. Nearly all of the students who responded on the survey reported that the discussion was an effective use of time (30/43), mostly because it "gave students the opportunity to work together" (5/43), to share and to discuss "multiple strategies" (16/43), "causing them to think differently" and "explain what they were doing." A few students, however, reported on the survey that they did not believe this was an effective use of time (5/43) citing that this class procedure was a "waste of time" (4/43) that caused "confusion" and "frustration" (3/43). Finally, a small number of students (3/43) responded that this type of class instruction could be effective if the "teacher found a balance" (1/43) and the "teacher explained the differences" between the strategies presented (1/43).

Many students reported having experienced discussion around multiple strategies and solutions (36/43), many reporting it had actually occurred in this class. There were a couple of students that reported they had not experienced a situation similar to this one. However, an interesting follow up is that only slightly more than a third of the students reported positive feelings towards this experience and described it as something they "loved," "liked," being "fun," or "positive" (15/43). Students further explained that the experience helped them "better understand the material," "see how others solved the same problem," "helped them find the easiest method for them to work the problem," and "feel more confident that they could find the right answer." A handful reported that they were "fine" with the experience (3/43) and did not provide any further details while less than a fourth of the students reported negative feelings towards the experience (7/43) citing feelings of “apprehension,” “confusion,” “frustration,” and “just [not] liking the experience.”

In summary, the large number of responses suggesting that the teacher should provide the right answer and explanation suggests that many view the teacher as the only expert with the right answer and the knowledge. Several students did express that the class should “come to a conclusion,” but did not clarify how that conclusion or agreement should be reached. Further, while most students did believe the class procedure presented in the scenario would be an effective use of time, they cited that it allowed students to “show multiple methods” and “work together,” but did not mention any cognitive activity related to regulation activities such as evaluation or debugging. Lastly, while many students did have a positive similar experience to the scenario presented, a significant number of students expressed concerns such as “confusing students” or “taking too much class time,” which could have an overall negative impact on the likeliness the students will use a similar procedure in the future.

Developing the Learning Community

Observations of the Learning Community

As described at the beginning of this chapter, a typical discussion in this class began with a brief one or two sentence overview of the material covered on the assigned page, followed by the instructor asking the students “okay guys what do we need to see on this page” or something very similar, student volunteers presented their work on each of the problems. After the completion of their explanation, the instructor asked if there were any questions of that student. At this point, either there were no questions and the student returned to her seat or another student asked her to repeat part of her explanation. The instructor occasionally asked questions of the presenting student if a more complete explanation was needed or stepped in at the conclusion of an explanation to summarize an important element or piece of the student's explanation. The instructor's questions tended to focus on how to do the procedures through questions such as "What do you do next?" or "How did you do it?" Once the student completed her explanation the instructor asked the class, "Do you agree?" or "Is it right?" The class discussions that stemmed from these questions rarely emphasized when or why to use a particular procedure or how to think through a procedure when the students got stuck or were unsure of what to do next.

After looking more closely at the discussions, it was noted that nearly every student question or comment was followed by a teacher question or comment. Further, there were only a few instances in which a student comment followed a comment or question from another student, and even fewer of these instances contained a comment that was in direct response to or built on the previous student comment. Also important, was that when multiple students responded to a question posed by the instructor, the instructor would continue the class discussion using only the "correct" student offered idea. In the following example, notice there are two separate instances where there are student comments back-to-back; however, in neither example were the comments related to each other. Instead, the individual comments were each unique responses to the question posed by the instructor.

1. Student A: Are we expected to know to extend the lines?
2. Instructor: Yeah let's talk about that. It's a good question there. How did she know to extend the lines?
3. Student B: The directions say to
4. Student C: Obtuse angle
5. Instructor: Say that again
6. Student C: Any time the triangle is obtuse, the line
7. Instructor: How do you know which lines to extend?
8. Student C: Uh the ones of the leg, uh the shortest sides, you extend
9. Student D: If you had it as the floor and dropped a string from the vertex
10. Instructor: Anybody view it differently? Okay are we done with this page?

In addition to the exchange pattern above, it was common for the instructor to repeat responses given by a student as part of her next statement and sometimes consolidate or rephrase the student's response to clarify the explanation. Another common practice was to ask students a series of questions to "poke and prod" until they have given parts of the entire process, and then the instructor put the pieces together and provided a complete beginning-to-end explanation of the procedure being discussed. The next example shows an instance where the instructor asked a series of questions, repeated the responses provided by the students, and wrapped the smaller steps of solving the problem into a single brief explanation. Notice there is also no summary of the entire process provided by either the students or by the instructor after completing this multistep problem. Lastly, while the teacher asked questions to clarify the explanation, most of the questions tended to be leading questions that had a single right answer.

1. Instructor: Eight asks you how long it is going to take to walk around this trapezoidal field. Well in order to find out how long it is going to take you to walk around, what do you gotta know?
2. Student E: The whole length.
3. Instructor: The whole length, which is called?
4. Students: Perimeter.
5. Instructor: Perimeter, so we need to know this length right? What did you do to find that length?
6. Student F: Pythagorean theorem.
7. Instructor: Pythagorean theorem, you don't say (chuckling). How did you do Pythagorean theorem?
8. Student F: Subtracted 350 from 550 and you get like… uh…. well…draw a line.
9. Instructor: Draw a line here.
10. Student F: And then to get that bottom part you subtract 350 from 550 its 200 and then?
11. Instructor: So you have 200 and 200 and as <student> said, you do Pythagorean theorem yet again… what do you get for the question mark side? What is that slanted length, as a decimal?
12. Student G: 200 square roots of 2.
13. Instructor: I want a decimal.

14. Student H: 282.84.
15. Instructor: Does that sound good for that slanted length? Now we are not by any means done with the problem. So we need to know the full perimeter so what are we gonna do with this 282.84?
16. Student I: Add it up.
17. Instructor: Add it up with all the rest of it so we have a 350 we need to add in there. We have a 200 we need to add in there a 550, what's the full perimeter? (working the rest of the problem on the board) 1382.84. Are we done yet?
18. Student J: No.
19. Instructor: No? what do we need to do last?
20. Student J: <inaudiable>
21. Instructor: Yes you did.
22. Student J: 4.6.
23. Instructor: (writing on the board) Now you're done (circled final answer of 4.6), give you a rate. Let's be done with that page.

This example highlights the use of closed, convergent questions with the instructor's summaries at the end of the various stages of solving the problems. At other times, the questions from the instructor prompted the student to explain part of a procedure again, before the instructor summarized the overall class discussion and collapsed the exploration into a nice neat package of important information.

A second element for creating an effective learning community for students is to help them learn to evaluate thoughts and explanations from other students when shared as part of the class discussions. This means letting students determine if the ideas presented are complete and correct or if the thoughts are in need of revising. Nearly every problem presented during the observations had both a correct solution strategy and a final answer. There were only a few occasions when the overall strategy that was presented needed to be examined further. In each of these instances, the instructor asked the class if they had any questions for the student presenting the problem, just as she always does. When the student was incorrect, the instructor used her tone of voice, body language, or repeated the question to signal to the students that they needed to look closer and rethink their explanation. One example is as follows:

1. Instructor: Who wants to come explain the equilateral triangle?
2. Student L: Uhm, first I found the bisector of the line, perpendicular bisector and just called that *l* and then I found the angle bisector of this line (angle bisector of the right angle). So… uhm…. I reflected on this line up here and just label that c and then I connected (drew c to a)
3. Instructor: Questions? Comments?
4. Student K: How did you get c again? I'm sorry
5. Student L: Yeah, uhm, the line *l* that I made and line a right here. I just found the angle bisector by adjusting the mira until A reflected on line *l*.
6. Instructor: Everybody's fine with this? (tone was consistent as if the solution had been correct)
7. Student M: Is there another way to find it?
8. Instructor: I think we need to talk about this way a little bit
9. Student N: I was gonna say, aren't you supposed to find from point A.

Like aren't you supposed to lay the mira on point A and reflect B up onto the line *l*?

10. Instructor: It doesn't matter. Tell me what the goal of our CD problem was.
11. Students: To create an equilateral triangle.
12. Instructor: Uh-huh we are supposed to create an equilateral triangle, look closely
13. Student O: Oh she had it right, but didn't quite get there. You still need to find the perpendicular bisector of the line.
14. Student P: But it's not equilateral.
15. Student O: But you need to copy, use the mira on point A or on point B and reflect the dot onto that line.
16. Instructor: And that is what Student N was saying. First of all we need to understand why we have an error here. So Student P is saying it's not equilateral?
17. Student P: It doesn't look like it.
18. Student O: AC and BC are the same size, but AB is not.
19. Instructor: Okay, AC and BC are the same length, but this one is not. So what kind of triangle has she created?
20. Students: Isosceles.
21. Instructor: It's actually an isosceles.
22. Student O: It's actually an isosceles right.
23. Instructor: I think it is too.

24. Student O: A very well done isosceles right.
25. Instructor: Yeah that one is one that comes up later on just so you know.
26. Student L: (the one that did the problem) So I should feel better about myself?
27. Instructor: So let's see this again. Actually I am gonna just let somebody else come up. I'm not gonna talk. Who's doing it? You guys are correct. She did start the correct way. The perpendicular bisector is a good place to start. We just need to do that second step a little bit differently. Student N is gonna come explain.
28. Student N: Started out finding the perpendicular bisector and then lay the mira on point A and reflect point B onto the line and mark the point then just go through and connect, like that to make the equilateral triangle
29. Instructor: Does that look like an equilateral triangle? (said in an affirming tone).
30. Student O: Excellent job.
31. Instructor: Did you see? There was really just a slight difference between what Student L did. She had the right idea. Just went through the wrong point. Did you guys catch that? Is everybody okay with what you need to do to get this? This is where Student N laid his mira. Guys when you get done with a CD problem, look at what it asks you to do and make sure that what you did is what you were supposed to have done. Just check it. It was so close (to student that presented the wrong answer). Now you probably are never gonna get up here again are ya?

32. Student L: Probably (chuckles)

33. Instructor: Doubt that.

These are two episodes reveal how the instructor handled "rightness" in the classroom discussions. The first example occurs in lines 7 and 8 when student M asked if there is another way to find the equilateral triangle and the instructor responded by specifying that the first method needs to be discussed before another is presented. After the instructor indicated that the first method may not be correct, another student offered a possible correction to which the instructor responded that it doesn't matter and redirected the class to the initial instructions asking if the construction matched the intentions. The instructor and students then worked together to identify what was wrong with the initial construction, but they never discussed where in the original construction the error occurred. The class did not discuss why the error occurred, but instead focused on identifying why the final construction was incorrect before the instructor asked another student to explain the construction from the beginning. Thus this sequence involved a presentation, the instructor identifying it was not correct, an instructor-lead explanation of how to identify that it was wrong, and then the instructor asking for another student to try again. There was not a comparison between the original incorrect construction and the second construction that was correct. The next student then presented her solution and, again in line 29, it is the instructor who confirmed the correctness of the method rather than the students.

The episode presented above was similar to other episodes that involved debugging and also serves as a representative sample of previously discussed occurrences. Lines 13, 14, and 15 provide an additional example of how back-to-back student comments do not build on each other. Each statement added its own independent thought to the conversation, rather tying together student comments about the problem. There is an overall pattern of interaction alternating between a single student and the instructor rather than with students interacting with other students.

Repeating student comments was also a prevalent action taken by the instructor as demonstrated throughout this episode. Line 12 provides an example of the instructor repeating a student's comment and redirecting the discussion after another student made a comment, while line 19 provides an example of a slight clarification made by the instructor for the use of more accurate terminology. Within this episode there is also an example of the instructor providing direction for the student to reflect on a final answer. In Line 31, she did not ask the *students* to actually do the reflection, but instead she did it herself.

A last comment about this episode before moving on addresses the last, but biggest, element of creating a learning community to promote metacognitive development, the creation of a safe learning environment. The sharing of one's thoughts is a very personal and risky task. A student must feel that he or she will not be judged or belittled if his/her thoughts are not correct, implying that a safe environment must be created in which students feel free to discuss their thinking. Line 26 provides an example of how important this environment is when the student specifically states, "so I should feel better about myself?" after finding out she had actually completed a construction for a future problem that has not yet been discussed. A second example comes in lines 31-33 when the instructor interacts with the same student by asking "now you're probably never gonna get up here again" and the student responds "probably." While on the surface this may seem playful joking and an example of good rapport between the student and instructor, other students who observed this interaction may feel that the instructor did not appreciate the sharing of a wrong explanation, but only wants students to share correct responses. Interpreting student feelings and reactions to statements was beyond this study, but it is important to note the impact statements such as these may have on a students' feeling of safety and support in the learning community.

Although the students presented most of the procedures by explaining problems they worked through their homework, the instructor did most of the regulatory thinking and guided the overall discussion by questioning the students about the procedures and then summarizing incomplete, "messy" thoughts that had been presented. Students rarely built discussions based on other students' comments, but instead tended to interact only with the instructor. The few times when one student interacted with another student, these interactions tended to be questions about the procedure just presented by the student serving in the "teacher" role while presenting her solution to a problem. Also, discussions revolved around getting the correct answer to the question on the homework page, rather than using the questions as a launching point for discussions. The instructor also tended to be the only person responsible for validating ideas presented in class and did so through the questions she asked and the body language or tone of voice she used when asking questions. She did "poke and prod" until students arrived at the correct response, but this process was more of guess and check until the instructor agreed with a response that had been thrown out by the students rather than thinking about or discussing the validity of the ideas that have been presented. While there did not seem to be a shortage of volunteers, the same students repeatedly volunteered, and most students did seem concerned about having a correct and polished explanation before volunteering to present a solution. During these observations, the student talk was evident; however, most of the talk concerned sharing and repeating procedures rather than thinking together to build concepts or solutions.

The Instructor's Perceptions of the Learning Community

While the observations tend to suggest procedurally driven discussions between the students and the teacher in a whole class setting, the instructor wanted students to talk with and help each other understand the material. During the interview, when describing an ideal interaction between students, the instructor explained that she would want the students to be "asking questions and guiding them to it, rather than just what I have seen throughout the years of, oh, copy down this problem." She also admitted that one of her biggest struggles was deciding:

> How much do you say and how much do you keep your mouth shut and let them say…trying to hold back that part of me that wants to, like a traditional lecture, just spill it all out so we can move on but that may be quicker but they don't always keep it, they don't always own that understanding.

She also often reminded herself to:

> Never say something that a student could so I always went, ok, now wait a second now they probably discovered that, just pull it out of them, who's got it and you just gotta keep asking them these questions and trying to form them in different ways and word them differently and so they finally get to the point and if along

> the way we discover different thing then so be it but in the worksheets there is typically something you know there is a goal in mind.

Throughout the interview, student "talk" in this setting was typically associated with getting up and presenting a problem. When students were at their seats, they were expected to compare the presented work with their own and then ask questions about anything they did not understand. During the interview, the instructor explained that she:

> Encourages them to get up to talk, they eventually realize [they] do learn more and understand more if [they] have looked at [their homework] beforehand and if [they] are actively paying attention and talking…it doesn't have to be a whole class discussion you know if they are talking within their tables I just hang out a few minutes and let them discuss cause there is learning going on there.

A possible student interaction consisted of a student offering "that one is an equilateral triangle [and the other student responds] well are you sure, I thought it was isosceles and then they'll just kind of bounce back and forth among themselves so being very actively participating in the course…"

While the instructor identified the ideal questions and talk she would like to hear from her students, she also recognized that the students have to learn "how to play the game, you know to answer questions, to ask questions, to interact, and some of them are just, they've been taught to sit in class quietly" so the teacher must "walk around and say things like 'hey Sally you want to share what you did on #3?' and start making them comfortable with you sharing the ones that [they] know." Another way the instructor explained that she helps students become less dependent on her as the instructor is to "Always ask if they have any questions for the student that is presenting, so that puts that person in the role so I try to keep the role of teacher off me and always put it back onto the students." When a student does ask her a specific question she reported that she will explain:

> This is a class question, so let's have the class try to answer it. Other students try to answer it and there is always a time where if they are struggling with an answer I will, they know that, they trust me as a teacher 'cause they know I'm not gonna let them sit there and flounder on something. If no one is getting it I am gonna pull them back and step into that role a little less of a facilitator and say ok guys this is how it is and then we go on.

Encouraging and helping the students gain confidence in their math ability is important to the instructor, but she struggles with how to question and prompt students through inaccurate or incomplete work in front of the whole class without embarrassing the student. For example, the instructor explained during the interview that when trying to help students become more comfortable sharing their work, she will encourage a student to share a solution she knows is correct rather than encouraging multiple students to share their works-in-progress as discussion starters. She also mentioned during the interview that her students know that she "won't pick on you if [she] sees that it's wrong, but if [she] sees that you are on the right track [she] might say hey so and so do you mind getting up [to share your work]?"

The instructor recognizes the importance the students place on being right and explained that:

> They don't want to get up unless they know they are right. I've seen a couple get up and go oh wait let me check it with so and so okay then get up or could you

> look at this real quick and then I'll get up and you know then if that's what they need the first time or whatever that's fine. But I love whenever a student goes I don't know if it's right, but I'm gonna get up anyway. And I always try to respond with well that's fine, the class will help you out you now cause there's always somebody sitting down who has it correct and will raise that question if necessary.

From this explanation it became clear that while the instructor likes the idea of students presenting work that needs to be checked for accuracy, she values *rightness* and expects the students at their seats that have it correct to question the student presenting the problem so that the work that is shared is the solution.

The emphasis seemed to be on sharing the correct process rather than on learning how to solve the problem as reflected in the talk surrounding the debugging process. This approach may discourage students who lack confidence from sharing their unfinished or incorrect work. As an example from the instructor interview shows:

> Just last week I had a weak student that got up and she thought it was 100% right…and she got up and it was completely wrong and students were copying down what she said because they didn't know what they were doing and that looked good and there were a handful that were just like wait no, what about this. And she was explaining to them why her way was right and their way was wrong and she was like ooooh yea, and I asked her whenever she walked away, do you think that you understood that better? Do you think that you will ever make that mistake again? and she was like, nope, cause she learned right there in front of the class in front of everybody.

The instructor also explained that when a student presented a wrong solution or wrong strategy "there will be students who haven't even attempted it as they walked into the class and are just blindly copying down what that first student is doing, and so maybe they don't realize the mistake, and then the other students are talking with that one student up there, so I know for sure that the student who is up there has learned from what just happened." Both of these examples taken from the interview, were not directly observed within the classroom context, and thus are stripped of non-verbal cues as to the emotions that may have been reflected in the speech patterns or the body language of the instructor. However, these statements also suggest the importance of "right" answers and how the instructor perceives potential benefits for the students learning from incorrect problem presentations.

One of the instructor's main goals for the course includes breaking the cycle of students' fear of mathematics:

> If they are scared of it and they shy away from it, then they are just passing it on to their students and that is just a viscous cycle of people being scared of math….[this course] helps [students] whenever they are getting up and coming and actually realizing that they can make mistakes in front of the class and other people will help correct them and that's ok cause they may be the ones correcting on the next questions…but there are always a handful of students that may get up only once or twice a semester or none at all that I feel like it may be perpetuating in them.

To help break the cycle and help students feel comfortable the instructor explained:

> I try to do a lot of encouraging, a lot of, okay who wants to get up, thank you so much… I have had students complain about professors in the past, they ask a question and the teacher makes the comment of you should already know that or why don't you know that or more or less belittles them in front of the class and that student shuts up and never asks a question again and that is completely opposite of what we are trying to do in an inquiry-based classroom.

By placing emphasis on the positive encouragement, and not calling students out, she works quickly to correct errors, rather than helping students learn to navigate through the regulatory processes themselves, thus cutting short any opportunities to promote metacognitive development in the classroom.

From the interviews it seems the instructor encouraged the students to present problems. Though she believed the problem presentations are where the learning occurs, she struggled when deciding how much to say and how to get the students to say it. Further, students needed to learn how to participate and interact in the class discussions since the structure is very different from a traditional lecture-driven course. The instructor gave information to the students if they get stuck or run short on class time. She also recognized the value students place on having their work complete and correct before sharing it with others and thus encourages students to become comfortable presenting problems she has already determined to be correct. While the instructor expressed a desire to create an effective learning community, the focus on being "right" and building confidence contradicted the incorporation of necessary characteristics that led to the creation of a learning community that promotes metacognitive development.

Student Perceptions of the Learning Community

Based on the observation data, there is an emphasis on sharing procedural knowledge where the instructor is responsible for most of the regulatory activity that occurs throughout the classroom. Most of the presentations involved the sharing of complete and polished work with the occasional incorrect answer that was immediately discussed and quickly followed by the presentation of another correct solution. The instructor of this course valued the sharing of student thinking and believes student talk and active engagement with that talk is pivotal to the overall success of student learning. The instructor recognized the value the students place on having complete and correct work before sharing how they worked the problem, but liked it when students presented something that they are not 100% confident is correct. The sharing of incomplete and unpolished thinking rather than polished final products is important in developing metacognitive thinking and promoting a learning community in which students felt comfortable sharing unpolished thoughts. When students were asked how they decided if they were willing to share their work or thoughts about a problem in class, about three fourths of the students (31/43) identified "knowing they have the right answer" and "feeling confident in their work" as the deciding factor. The second most common response for how students decided to present a solution was that "no one else would" (6/43).

While sharing student work with the rest of the class was an important element of the course, identifying when to ask for help is also an important step in developing metacognitive skills. Students reported they determined it was time to ask for help when they "realized their final answer was different from someone else's" final answer.

Further, in regards to whom they would ask for help, about half (22/44) of the students responded that they would ask a “neighbor” while the about one-third (17/44) responded that they would ask the “instructor.” About one-third (15/44) reported they would ask this person (whoever they identified as the person from whom they would get help) because “they knew that the person knew what they were doing” or “was good at math.” Approximately one-fourth (10/44) of the students responded that they asked this person because they were “close by” or “convenient.” Also, about one- fourth (9/44) of the students identified they would ask this person because they were “comfortable with them” and “could trust them.” Several also commented, that it is “what she does,” in reference to the instructor (5/44).

This survey data provided the students’ perspective of how the instructor implemented the strategies and developed a learning community that is necessary for metacognitive development. Students felt it was important to have complete and correct work to present to the class which limits opportunities for including the modeling of regulation of cognition. From the students’ perspective, the teacher holds the knowledge and should share what she knows after a class discussion which suggests that the students tended to rely on the instructor to determine what is right or wrong and provide final answers and explanations rather than developing these ideas together through socially mediated metacognition. Further, knowing that the person they are seeking help from understands the material was important to the students. Lastly, while students did appreciate being exposed to multiple strategies for solving the problem, the emphasis on hearing these multiple strategies was to choose which method they liked best or to find out how to do the problem, but the students also cautioned that presenting multiple strategies for a solution did not always create positive learning opportunities.

Impact of Instruction on Students' Metacognitive Awareness

The Metacognitive Awareness Inventory (MAI) is an instrument designed by Schraw and Dennison (1994) to measure how aware a person is of his or her metacognition. In this study the MAI was used to determine if there was a change in the students' awareness through the course. If there was a significant change in the students' metacognitive awareness, then it could be suggested that this course could serve as an example of how to promote metacognitive development in the classroom. Further, the maximum score of 260 represents a student with a high level of metacognitive awareness whereas a score of zero on the MAI represents a complete absence of metacognitive awareness. Also, a standard categorization of a "high metacognitively aware student" or "low metacognitively aware student" has not yet been found in the literature. Students in each section were given the MAI at the beginning of data collection period that occurred in the 11th week of instruction, just after their second exam. The MAI was repeated again at the end of data collection, just before their third and final regular course exam during week 14.

First, to identify any differences between the two sections, a two-sample t-test assuming equal variances using a pooled estimate of the variance was performed to test the hypothesis that the scores on the pre-MAI between the two sections were the same. The Levene Test for Homogeneity of Variance where $p < .05$ was used and found equal variances for both the pre-comparison and post-comparison ($p=.692$; $p=.633$). The mean scores of the pre-MAI for the MWF section ($M=185.92$, $SD=23.666$, $n=13$) was not significantly different (p=.077) from the mean score of the pre-MAI for the TR section (M=202.30, SD=27.075, n=23). Similar results were also found for the post-MAI where the MWF section (M=190.38, SD=23.768, n=13) was not significantly different (p=.129) from the mean score for the post-TR section (M=204.48, SD=27.319, n=23). Thus there was no difference in the students' metacognitive awareness between these two sections. Table 10 provides a summary of the MAI data for both sections.

Table 9
Pre and Post MAI for MWF and TR Sections

		Pre	Post	p
MWF (n=13)				
	M	185.92	190.38	.406
	SD	23.66	23.768	
TR (n=23)				
	M	202.3	204.48	.546
	SD	27.075	27.319	

Note. 95% confidence interval.

Second, and more importantly, a paired t-test was performed to determine if there was a change in the students' metacognitive awareness between the beginning of data collection in week 11 and the conclusion of data collection in week 14. The mean change in MAI scores for the MWF section (M=-4.462, SD=18.684, n=13) was not significantly different from zero (t=-.861, p=.406, n=13), thus failing to reject H_0 and suggesting that the observed instruction had no impact on the students' metacognitive awareness in the MWF section. Similar results were found in the TR section (M=-2.174, SD=16.991, n=23) where the differences in scores were also not significantly different from zero (t=-.614, p=.546, n=23) thus also failing to reject H_0 and suggesting the course had no impact on the students' metacognitive awareness in this section either. Thus, this data suggests that the metacognitive awareness of the students in each section was statistically the same during weeks 11 and 14, and the observed instruction did not impact the metacognitive awareness of the students during the 4 week timeframe.

Summary of Findings

Metacognition consists of both knowledge of cognition and regulation of cognition. Knowledge of cognition encompasses declarative, procedural, and conditional thoughts while regulation of cognition consists of doing something with those thoughts such as planning, managing information, monitoring, debugging, and evaluating. Students' metacognitive thinking can be improved through opportunities for them to hear the thoughts of others so that they can compare these ideas and make adjustments in their own thinking. The use of strategies such as modeling, prompting, questioning, and reflecting are effective ways to help students share their thoughts so that the thoughts may become objects of the discussion. The sharing of procedural knowledge by the students was prevalent in these two sections; however, conditional knowledge required heavy prompting by the instructor. A similar statement can also be made about regulation in general. Most regulatory thoughts required the assistance of the instructor, very few were made by only the student. Regarding the type of cognition that was modeled, knowledge of cognition episodes were much more prevalent than regulation of cognition episodes.

In class, students presented their complete and believed to be correct solutions to problems from the homework the other students were not able to complete on their own. When these presentations were incomplete, inaccurate, or confusing the course instructor expected students to ask questions of the student who was presenting. When this did not happen, the instructor would step in and ask questions of all the students to help them work through the problem. In this classroom, students tended to observe the instructor's reaction to a presentation to confirm or refute the student's explanation rather than evaluating and questioning the solution themselves. Reliance on the instructor was also evident in modeling multiple solution strategies. Additional strategies were presented only when the instructor questioned the first strategy presented or specifically prompted students to provide an additional strategy. This prompting was only done if the instructor knew most of the students had used a different method or if a different method would be required for success on a future assignment. Because of the limited sharing of multiple strategies, there was even less discussion about when and why to use those strategies or how to regulate knowledge about those strategies.

The textbook for this course was specifically designed to promote inquiry-based instruction, requiring students to try problems first based on their previous knowledge, rather than receiving direct instruction before practicing similar problems at home. While the text does encourage students to talk with others about these questions, the authors do not describe what this "talk" should sound like. Thus, in this setting, the instructor relied heavily on the text to provide the content for the class discussions. Unfortunately, the problems from the text rarely required students to think beyond finding the answer to the problem. Very few questions asked students to describe their overall solution process, to consider alternative strategies for solving the problem, or to identify why they chose a particular strategy. Because these questions are left out, students were not given the opportunity to consider this other type of thinking about how they would respond and organize their thoughts in preparation for sharing in the class discussion.

Building an effective learning community is the final piece to the puzzle when promoting metacognitive development in the classroom. A learning community requires collaborative thinking and a feeling of safety. Students must feel confident in exposing their incomplete works in progress to be analyzed and evaluated by their peers. Students in both sections identified on their surveys that being confident they had the right answer and solution strategy was how they decided to present a problem. Most solutions that were presented were correct and most students agreed with the presentation. If a student did ask a question, it tended to require nothing more than repeating part of the procedure that had been shared. Further, when questions did arise, the discussion tended to alternate between the student and instructor rather than between the students themselves, suggesting a lack of collaborative discourse occurring during the discussions.

Based on the interviews and observations, this college-level mathematics instructor tried to promote metacognitive thinking in her students by encouraging students to share their thinking, encouraging them to build a learning community by turning to their classmates for help when needed, and prompting students to do their own thinking when applying their knowledge to a new situation. However, despite her intentions, she was still perceived as the "more knowing other" and students heavily relied on her for confirming ideas and solutions. These interactions limited effective discussions between the students. While a few of the students may have had ideas to share, it was ultimately up to the instructor to validate student thoughts. Lastly, in determining if embedded instructional strategies can increase the metacognitive awareness of pre-service elementary teachers, the findings of this study are inconclusive since this data itself does not suggest that the instruction had an impact on students' metacognitive awareness. However, it was apparent that the underlying foundation for metacognitive development was present in this course.

This chapter identified types of shared metacognitive thoughts and painted a picture of both the embedded strategies and learning community that was developed in this classroom. Further, the MAI suggested the instruction described in the results had no impact on metacognitive awareness during the four weeks of the study. It is now important to look at what these results mean for mathematics students, teachers and teacher educators.

CHAPTER V

CONCLUSION

The purpose of this study was to describe how a college-level geometry content course promoted metacognitive development in pre-service elementary teachers. With the underlying theoretical idea that carefully designed instructional strategies can improve students' metacognitive thinking, this study employed naturalistic inquiry techniques to better understand what was actually happening in a real classroom setting. The use of both qualitative and quantitative data within an embedded mixed-methods design provided an opportunity for multiple types of triangulation and thus promoted a greater understanding of the elements involved in this case study. More specifically, this study sought to answer the following three research questions. First, what types of metacognitive thoughts are being modeled in the classroom? Second, how does a college mathematics instructor promote metacognitive thinking in her students? Third, does a college-level course change the metacognitive awareness of pre-service elementary teachers as measured by the Metacognitive Awareness Inventory?

This study took place at a mid-size regional university in a suburban setting. At the university, one mathematics instructor was selected because of her experience teaching an inquiry-based geometry content course designed specifically for pre-service.

elementary teachers. This course was unique in that it is commonly described as a course in which students do the talking and teach each other in a non-traditional setting. The study also collected data from the students (n=23 and n=28) in two of the four sections the instructor was teaching during the semester in which the study took place.

This study used both qualitative and quantitative data to gain an understanding of the embedded metacognitive instruction occurring in the class. Qualitative data included semi-structured interviews with the instructor, open-ended surveys with the students, non-participant observations of the classroom, the course textbook, and additional documents such as the course syllabus and calendar. Quantitative data included demographic information and the Metacognitive Awareness Inventory. A pre- and post-semi-structured interview with the instructor bookended the data collection. The interviews were transcribed then subjected to content analysis to identify patterns and themes from the data. A demographic survey gathered descriptive student data and was given at the same time as the first administration of the Metacognitive Awareness Inventory (MAI). The MAI measured change in students' metacognitive awareness.

Students also completed early and a late semester open-ended surveys given in the same weeks as the pre- and post- administration of the MAI. The Early Semester Survey (Appendix B) consisted of five open-ended questions designed to understand student beliefs and perceptions towards mathematics and mathematics education while the Late Semester Survey (Appendix B) consisted of nine different open-ended questions designed to reflect on how the course has helped students learn mathematics in relation to metacognition. Both surveys underwent content analysis to identify patterns and themes in the students' responses followed by a frequency count of student responses within the major themes.

In addition to the self-report data described above, non-participant observations were conducted between Exam 2 and Exam 3 in both sections. These observations were focused on how the instructor was promoting metacognitive development such as what types of metacognition were being shared, how the discussions were being facilitated, and what type of environment was being created. The observations consisted of a total of 14 visits, eight in the MWF section and six in the TR section. Field notes were taken during all observations. After the completion of data collection, three class meetings were randomly selected as a representative sample from early, middle, and late observations of the MWF class to be transcribed along with the matching homework discussions from the TR section. Episodes of metacognitive thinking that were shared in class were classified by both the type of cognition, as described by Schraw and Dennison (1994), and by who was providing the shared thought, the teacher, the student, or integrated student and teacher thought. These episodes were then tallied and summarized in a frequency table.

The final data source for this study was a variety of course documents. The textbook played a critical role in classroom discussions and thus also underwent content analysis to identify themes and patterns as they relate to metacognitive development. Other documents included the syllabus and assignment schedule. These were analyzed to describe the overall class structure and design.

Discussion

What Types of Metacognitive Thoughts are Being Modeled in the Classroom?

From the observation data, it can be concluded that there was more modeling of knowledge of cognition than of regulation of cognition. Further, while the students did provide about half of the knowledge episodes, nearly all of the regulation episodes were integrated student-and-teacher thoughts. Nearly all of the knowledge episodes shared by students were procedural while there were no student episodes of conditional knowledge. Nearly all of the regulation thoughts required prompting from the instructor to make them explicit during discussions. Further, explicit examples of planning and evaluation were absent from the observations altogether. The instructor did occasionally model her own thinking, but usually prompted students through an explanation rather than by sharing her own thoughts.

Making thinking visible and modeling metacognition is one of the most important elements in developing metacognitive thinking. Once students hear or see the thinking of another person they are able to compare it with their own thoughts and make necessary adjustments in their own thinking based on this comparison (Goos, et al., 2002; Brittany Hoffman, et al., 2009). When students are not explicitly sharing their conditional and regulation thoughts they may still be thinking metacognitively, but the metacognitive thought is not being shared as an object for learning within the community. Mathematics instruction has traditionally focused on procedures and only in the last few decades has there been a push to include more thinking and reasoning skills as a regular part of classroom activity. Because of traditional instructor driven lecture that has been prevalent in mathematics education, it may be possible that both the students and the instructor are not fully aware of the different types of thoughts they have and use throughout their problem solving processes. A better understanding of why there is such a great focus on procedural knowledge in comparison to other types of metacognition may be beneficial in understanding how to help teachers implement standards-based instruction that focuses on the process standards.

How Does a College Mathematics Instructor Promote Metacognitive Thinking in Her Students?

Two frameworks have been discussed in the literature for improving metacognitive thinking in students. The first relies heavily on an understanding of metacognition for its implementation and suggests that to improve student metacognition an instructor should promote general awareness, improve knowledge of cognition, improve regulation of cognition, and foster a conducive learning environment (Schraw, 1998). Improving awareness, knowledge, and regulation all require explicit modeling and collaborative discussions. Thus in this classroom, the instructor provided opportunities for both modeling and collaborative discussion; however, the modeling was mostly procedural and the discussions were not typically collaborative.

The last suggestion in this framework is to foster a learning environment conducive for metacognitive thinking. Researchers recommend that this be accomplished by promoting mastery over performance goals (Schraw, 1998). While this instructor expressed beliefs during the interview that were focused on a mastery-through-explanations environment, meaning she wanted students to persevere through a task and attain deeper learning, class discussions during the observations tended to focus on getting the right answer, making sure everyone knew a process to get that right answer, and moving on to the next question, which seemed to emphasize a performance-oriented classroom.

The second framework discussed in the literature is less reliant on understanding metacognition itself and suggests four principles for promoting metacognitive development. The four principles suggested by Lin (2001) include providing frequent opportunities for students to self-assess what they do and don't know, helping students articulate their thinking, fostering a shared understanding of the goals for metacognitive activities, and developing knowledge of self-as-learner. While the homework problems provided students the opportunity to try a problem and determine whether or not they could do it, there was little discussion to help students identify what parts of the problem they did know and how to find information about the parts they didn't know. From the high frequency of integrated student-and-teacher thoughts, it was clear that the instructor helped students to articulate their thinking. An interesting note here was that it was the instructor that was helping to articulate thinking rather than one of the students. Also, there was limited balance of the type of thoughts that were shared since most thoughts represented procedural knowledge. Fostering a shared understanding of metacognitive activities requires explicit discussion about metacognition and why metacognition is important (Lin, 2001). The absence of this principle from this classroom can most likely be attributed to the instructor's lack of thorough understanding of metacognition and its pedagogical procedures. Lastly, Lin (2001) suggests that helping students develop knowledge of self-as-learner requires students to identify strengths and weaknesses as well as beliefs and assumptions about how they learn. While the instructor explained in the interview that she wanted students to become familiar with a new way of learning, the researchers observed no discussion or explicit prompts directing the students to think about and consider themselves as a learners or thinkers.

To promote metacognitive thinking in her students, the instructor relied heavily on the textbook for the focus of the classroom discussions. The primary goal during class discussions was for all students to get an answer and be able to explain how they arrived at their answer, but there was limited discussion about when or why to use certain procedures. There was also limited discussion on how to regulate thinking in relation to the problems discussed which could have been attributed to the lack conditional knowledge or regulation of knowledge questions being asked in the homework problems.

While there was significant modeling and prompting occurring, there was also a significant amount of thinking being shared. The students believed they must have a correct and final answer before they presented their problem to the class. Further, most of the interactions alternated between a single student and the instructor rather than among groups of students as they talked with and built upon on each other's shared thoughts. Although students were doing most of the talking and sharing of their procedural thoughts, they were not working together as a learning community to promote metacognitive development.

While the instructor provided the opportunities for students to talk with each other, she did not facilitate discussions that moved students beyond the procedural knowledge needed to complete the problem from the textbook by looking for generalizations that would inform future experiences. Rich discussions require focus beyond simply showing a procedure and then asking questions until the procedure is clear. Since the presentations nearly always consisted of complete, final, polished correct thinking it was difficult for students to learn how to use metacognition to help monitor and evaluate progress in their own thinking.

Discussion which center on multiple strategies is also content for effective metacognitive development, but unless a student specifically asked about a second strategy, it was unlikely that another strategy would be presented in class. There were few comparative discussions providing opportunities for students to judge the worth of multiple strategies. This made it nearly impossible to share conditional knowledge since students were not engaged in discussions about when or why they used or might use one particular method over another.

There remain a variety of reasons why the class discussions may not have moved beyond surface level discussions of a procedure. One possible explanation is that the teacher simply did not know how to create and facilitate this type of discussion. A second reason may be that she did not value this type of discussion as a learning tool. A third possible explanation is the lack of metacognitive prompts in the textbook for discussing important aspects of the content. A final possible explanation is that students had been trained through their previous school experiences to get the right answer and did not have the patience or motivation to discuss the process of thinking about a solution other than the most direct path to the final answer. Talking, communicating, and sharing thoughts in mathematics class is a relatively new process so the instructor truly may not have recognized that most of the time when the students were talking, they were serving as "the expert" and thus the classroom still followed a predominantly traditional approach. Oftentimes when people are successful in mathematics they have automatized many of the regulatory processes and thus may see them as a natural part of learning that should just happen. Perhaps this instructor does not believe that these are skills that can be taught and learned in the classroom environment.

A last possible explanation for why the discussions did not move beyond procedures was due to the instructor's reliance on the text to provide the content for the discussions. Because the text does not present questions that cause students to think metacognitively, the responsibility falls on the instructor to facilitate the discussions that lead to metacognitive thinking. Since this instructor relied on the text to provide discussion content, discussions about metacognitive thinking tended to be omitted from class discussions. The facilitation of a discussion that promotes metacognitive development is different from prompting and using questions to help students think aloud. Helping students learn to listen to each other and value the comments contributed to the discussion while simultaneously removing one's self from the discussion is a delicate but necessary skill. Students do not automatically know how to talk with and question each other, especially about their own personal thoughts. The sharing of thoughts and the development of reasoning and argumentation skills require a safe environment where students are comfortable exposing their unpolished thinking (Leat & Lin, 2003). Students must somehow be made aware of and come to accept that not only are reasoning and argument acceptable, but both of these should be expected as part of the learning process. While each of these sections included a handful of students that seemed to embrace this type of community and thus were willing to share their thinking and question others, there were many other students that did not feel safe and comfortable sharing their ideas and arguing with other students. A better understanding of how to create a learning community, where all students are comfortable and confident sharing both polished final thinking as well as unpolished works in progress, is important for further implementation of instruction that promotes metacognitive development.

Does the Observed Instruction Change the Metacognitive Awareness of Pre-service Elementary Teachers?

The statistical analysis of the pre- and post- Metacognitive Awareness Inventory suggests that students did have an awareness of their metacognition, but the instruction that was observed during the study did not seem to impact the students' metacognitive awareness. The null hypothesis of research question 3 that states there is no difference between the pre and post MAI scores cannot be rejected at the 0.05 level. These results were not surprising since there were only three weeks between the pre- and post-administration. At the time the initial data was collected, students had already settled in to the classroom routine and norms and, thus, may have become more metacognitively aware prior to the beginning of the study. Since the pre-MAI was not collected until Week 11 of the semester, it is not possible to determine if the course had an impact on students' metacognitive awareness. Because of this limitation in the study, it is important to look beyond the numbers and compare what happened in the classroom to ideas from the literature about ways to improve metacognitive thinking.

Missed Opportunities for Promoting Metacognitive Growth

The instructor in this course did provide opportunities for her students to share their thinking and cognitive modeling did occur. When students were unable to provide a cognitive thought on their own, the instructor prompted students as they worked through their thinking until they arrived at an answer. Further, the instructor did encourage the students to ask each other questions and to evaluate methods that were presented to the class. Interestingly, even though she did include think alouds, modeling, prompting, and questioning as a major focus for instruction (all strategies for promoting metacognitive development), the instructor did not facilitate these activities in a way that promoted the development of metacognition beyond students' procedural knowledge. Throughout the observations, there were many times where the perfect set-up occurred for a rich metacognitive discussion, but the instructor did not seize the opportunity. Before talking specifically about a few of these missed opportunities, it is important to look at an entire strategy for metacognitive development that was largely absent from the classroom observations.

Reflection is perhaps one of the most talked about strategies in the literature, aside from think alouds, to develop students' metacognitive thinking (Cardelle-Elawar, 1995; Cohors-Fresenborg, et al., 2010). The process of reflection can be completed through writing, group discussions, or through interpersonal communications. Within the framework used for the classroom observations, reflection falls under evaluation (Schraw & Dennison, 1994). A reflective activity requires a student to look back and consider the effectiveness and efficiency of a particular process rather than simply recalling the process (Yimer & Ellerton, 2010). This was perhaps one of the least used strategies in the course. There were no direct instructions within the textbook problems that required the students to reflect on their work. Beyond asking the students if they understood or if they had questions for the student that had just presented a problem, the instructor did not prompt the students to reflect on their own work. There were no explicit episodes of reflection observed, and reflection was not discussed during the instructor interviews or on the student surveys, though perhaps the instructor perceived reflection to be a natural automatized process and thus overlooked ways to help students develop their ability to reflect on their own work. Because of the importance of reflection in the overall process of developing metacognitive thinking (Cardelle-Elawar, 1995; Yimer & Ellerton, 2010), it is important to develop a better understanding of why reflection was overlooked and how to support the instructor's awareness of the need to teach students how to reflect and to learn from those reflections.

The lack of reflection in the class was not the only way the instructor missed an opportunity for promoting metacognitive development. There were several instances during the observations where the discussion lead up to a point where the instructor had a choice about which direction to steer the conversation and she chose, consciously or subconsciously, to steer away from discussion that would promote metacognitive development. The instructor likely did not realize this was occurring; however, reflecting on and identifying where the conversation could have moved in a different direction is important for future growth. For example, when selecting a single representative problem from a page, the instructor often made a suggestion to a student based on the perceived difficulty level of the problem, but did not facilitate a discussion as to why that problem seemed more difficult or how students could create a plan for solving the problem. As another example, when an incorrect solution was presented, the instructor was very concerned about students' feelings. She made sure they were not embarrassed and tried to limit confusion among other students by working quickly to identify the error and have another student provide a correct explanation. Rather than quickly glossing over the incorrect explanation, the instructor could have questioned and prompted students through the debugging process (Schoenfeld, 1992). This instructor also could have had other students provide possible strategies then promoted and facilitated a discussion using regulation of cognition to compare and evaluated the multiple strategies that were presented, without first identifying the accuracy of possible methods proposed by students (Kramarski & Zoldan, 2008).

During the observations, when a student identified a need for help on a problem, the instructor asked for a volunteer who came to the front of the room and explained how he/she worked the problem. Further, when a student didn't understand something in the explanation, the instructor would have the student who was presenting repeat that part of their explanation. Rather than jump straight into a procedural explanation of the problem, students could have been asked questions about what they did or did not know and how to develop a plan for getting to the solution. This process would have modeled regulation of cognition by helping the student identify what they did and did not understand so that they could enter the regulatory process (Cardelle-Elawar, 1995). There were several examples where the students' questions about a problem did not require the full procedural explanation, but instead a quick prompt to help them sort through information they already knew.

A final example of missed opportunities occurred when the instructor chose not to discuss and present multiple strategies for solving a problem. When multiple strategies were presented, the students were left to choose whichever method they liked best without any discussion. Having an explicit discussion, about when a particular procedure would be better than another procedure, is important in developing conditional knowledge, one type of metacognitive thinking. This type of discussion also provides insight into regulatory processes as well. While this list is in no way intended to be exhaustive, it does show a variety of examples of times when the instructor lead right up to, but failed to open the door for explicit metacognitive development. A better understanding of why these choices were made is important for promoting metacognitive development in the future.

Based on the evidence presented here, there are underlying elements embedded within the course to promote metacognitive development; therefore, it would be appropriate to repeat the pre/post MAI at the very beginning and very end of the semester. With a few minor extensions to instructional practices that are already a part of the class norm, the instructor could create the environment suggested in the literature (Lin, 2001; Schraw, 1998). The underlying belief that students should model their thinking, internalize a prompted thinking process, and participate in a learning community are already present, but a conscious effort to extend beyond procedural knowledge is necessary to promote other areas of metacognitive development. So, while the actual data shows no change in students' metacognitive awareness during the time of the study, based on the presence of these underlying beliefs and characteristics, it still seems that it is possible to engage students' metacognitive awareness and warrants further study over the course of the entire semester.

Limitations

Limitations in this study included the small sample size for the case study, the collection of data over only four weeks, the heavy use of self-report data by both the instructor and the students, and the use of the video camera during observations. First, using a single participant for this case-study limits generalizability of the findings because other instructors may be very different from this one in their use of metacognitive awareness in the classroom and thus very different results may have occurred. However, the decision to focus on just one instructor provided the opportunity to explore the classroom in greater depth under the limited resources and time constraints. For this study, the instructor was selected because of her experience teaching the course and her beliefs towards incorporating inquiry-based learning in the classroom.

The short time span between the pre- and post-assessments greatly limited this study. Since these are college-level students, they have had to attain some pre-determined level of mathematical understanding to gain acceptance into the university. Students have been learning how to learn mathematics from their previous mathematics class experiences for over a decade prior to taking this course and thus may have developed perceptions of how to be effective learners of mathematics. This course turned those perceptions upside down and required the students to step away from those perceptions to be more successful in the course. Because of this new perspective on how to be successful, students were forced to make rapid adjustments at the beginning of the semester or risk not being successful in the course. The changes necessary for student success most likely occurred at the beginning of the semester and thus were not captured during this study.

The current study relied on many types of self-report data including the MAI, the student surveys, and the instructor interviews. Understanding students' metacognition and what they think about certain activities is not something that can be observed by an outsider. Knowing what a person is thinking and feeling requires some type of prompt to let students know that the instructor is interested in what they have to say. Without the self-reported data used in this study, the data would lack connections to the students' and instructor's perceptions of what was occurring in the class and the perceived benefits of the various class activities and discussions.

A final possible limitation to the study was the video camera used to capture the discussions and behaviors during the observations. Though the camera was relatively small, it did not enter the classroom until week 11 and was positioned on a tripod in the back of the room. Students were aware of the camera and when they presented a problem they were directly across from the camera. After conversations with the instructor about changes in students' behavior that may have been attributed to the location of the camera, the decision was made during the second week to move the camera further away from where students were presenting. This move also had a positive impact on a students participation that had been more reserved than usual when the camera was near him. Although according to the instructor, class participation was closer to normal after the move, the presence of the camera still may have impacted class discussions. While the camera may have impacted behavior, it was critical to have the video taped episodes for understanding the shared thinking that was occurring. Much time was spent comparing the observation video and the MAI descriptors to identify episodes of metacognitive thinking that occurred and was shared. Capturing that information with field notes or a tally record sheet while sitting in the classroom would have been very limiting in understanding the big picture of what was occurring in the classroom.

Future Studies

Future studies should address the limitations presented and use the findings from this study as a foundation for further exploration of metacognitive instruction in mathematics classrooms. Conducting a true pre/post analysis of changes in students' metacognitive awareness will provide great insight regarding the effectiveness of embedded strategies for metacognitive development that teachers are already using in their classrooms. Additionally, scaling up this study to include more instructors across a wider variety of settings would provide greater insight into the overall current metacognitive development of students in college mathematics courses. Future studies should not only include similar types of data, but might also include interviews with students and stimulated video recall with both the student and instructor to provide an even greater and more intimate understanding of how the actions of teachers impact students' metacognitive development. As a last suggestion from the limitations, the use of smaller and more advanced technology would minimize the impact of the video camera on student behaviors during the observations. Future studies should not only address the issues that arise from the limitations, such as expanding to multiple instructors, using stimulated video recall and more discrete video equipment, and a true pre/post given over the entire semester of the course, but also should include studies to better understand the roles of the instructor, the roles of the students, as well as the role of metacognition in the classroom in general.

Future Studies with Instructors

The instructor plays a critical role in classroom instructional design (Costa & Marzano, 1987). More complete and better understandings about how and why instructors make particular decisions will provide a stronger foundation for professional development content. Future studies should develop a better understanding of instructors' perceptions about metacognition and address the impact professional development has on the instructors' ability to promote metacognitive development in the college-level classroom. Also important is a better understanding of what an instructor does know about the different types of metacognitive thoughts, creating discourse, and creating a learning community as well as how to embed these in the classroom. This information will be important as researchers continue to work to develop better understanding of how metacognition can be developed and supported in the college-level classroom. Personalized long-term professional development and support could be provided to instructors to help them become more aware of and reflect on their practices so enhanced discussion occurs in the learning environment.

A better understanding of *how* to create the necessary safe and collaborative learning community is also imperative for designing instruction for mathematics classrooms that promotes metacognitive development (Lin, 2001). This second and possibly much more critical and understudied element means understanding what encourages students to share their most early, beginning, unpolished, and unrefined thoughts so these thoughts can be analyzed, critiqued, and revised by their peers. In this study the instructor believed she had created this type of atmosphere and often cut short opportunities for metacognitive development so as to not embarrass or call a student out in front of the class. The students, however, did not necessarily mirror her image of this safe learning community and continued to look to the instructor for guidance and only shared their final, polished thinking. Further explorations into this discrepancy will be important and critical for understanding the development of effective learning communities that are imperative to the success of metacognitive development (Costa, 2006; Lin, 2001).

Differences between the students' and instructor's perceptions of the learning community in their classroom was not the only discrepancy that arose in the data. Differences also arose between what the instructor said she does, what she identified as important, and what the researcher observed in the classroom. To better understand how to bring instructor beliefs and the enacted curriculum closer together, it is important to know if the instructor is aware of the differences and understand why these differences occured. Interviewing the instructor using video recall and the instructor's reflections may provide insight into this discrepancy.

Perhaps one of the most important aspects is determining the impact embedded metacognitive instruction during teacher preparation courses has on future instruction once the pre-service teachers are responsible for their own classrooms. In essence this question is closely related to the question of how do teachers make instructional decisions? Does a teacher's lack of awareness of his or her own metacognition or lack understanding of the variety of metacognitive thoughts impact her instructional decisions. If it does, then it seems that metacognitive instruction during teacher preparation courses would help new teachers become more aware of both their own metacognition and awareness of the variety of types of metacognitive thoughts, which should in turn encourage them to include metacognition in their own instruction.

Future Studies with Students

Students play an important role in the classroom. Unfortunately, it seems that researchers often overlook how students' perceptions and identities as learners of mathematics impact the overall classroom environment. For example, a classroom that promotes metacognitive thinking requires the sharing and discussion of thoughts;

however, if students have not been trained to share and learn from their thinking or the thinking of others, they will not know how to do this. Thus it is important to develop an understanding of how to support students as they work to change their identity as a learner of mathematics. Students will no longer be expected to come in and listen to the instructor explain the problems then go home and practice, but instead students will be expected to think about what they do and don't know then work within their classroom community to develop understandings. New studies might focus on how students can be taught to participate in collaborative classroom environments that require the sharing of unpolished and unrefined thinking and how do the students' identities as a mathematics learners impact their beliefs about and participation in that community?

To continue exploring the creation of classroom discourse, researchers should know if there is a relationship and, if so, describe the relationship between the types and frequency of metacognitive thoughts modeled by students with high metacognitive awareness in comparison to students with low metacognitive awareness. Students with a high level of metacognitive awareness would have more thoughts to contribute to the class discussion than students with lower levels of metacognitive awareness. A study could be conducted to identify and analyze statements made by students selected through the use of extreme case sampling based on scores from the Metacognitive Awareness Inventory.

A safe, collaborative learning community promotes the sharing of thinking; a future study should examine how teacher comments, tone-of-voice, and revoicing impact students' feelings about the learning community. Exploring how interaction between a student and the instructor impacts the thoughts, feelings, and beliefs of another student not involved in that conversation is very important. Also, further explorations are necessary to better understand how the instructor's use of revoicing student comments and explanations impacts students and their desire to listen to each other. Using stimulated video recall interviews after classroom observations would allow the researcher to obtain student feelings and perceptions based on the instructor's actions during class. This understanding may provide insight to what instructors do during class, not realizing the impact these actions have on students' feelings, perceptions, and metacognitive interactions.

Additional Future Studies

While the previously described future studies relate specifically to the instructor or the student, there are opportunities to research the classroom as a whole as well. For example, does standards-based mathematics instruction in itself promote metacognitive development or should metacognition be considered as a separate and additional instructional focus? If a focus on NCTM's (2000) process standards promotes metacognitive development, then could professional development and research continue to focus on improving implementation of the NCTM Standards? However, if metacognitive development requires something extra or in addition to an emphasis on the process standards, researchers and teacher educators should explore ways to balance these aspects in an already full curriculum.

While there has been an emphasis over the last thirty years on what metacognition is, there is still much to be learned about how to develop and implement classroom instruction that develops metacognitive thinking in mathematics learners. Regarding further understanding the development of metacognition, it is important to explore how long it takes for students to develop effective metacognition skills. Is it possible to expect change in metacognitive awareness over a sixteen week course or should metacognitive development be expected to develop over a series of semester long courses? What is the long-term impact of including metacognitive development in teacher education programs? What should these programs look like? How early in a student's education should metacognitive instruction begin? Is there a point in time where it is likely that a person's metacognitive awareness has been developed to its full potential? Finding the answers to these questions should help mathematics educators focus efforts on developing students' metacognition.

Closing

Current research indicates that metacognition can be developed with students and higher levels of metacognition tend to lead to greater success in mathematics (Kramarksi, 2004; Swanson, 1990). The students that participated in this study are pre-service teachers and will soon be responsible for helping their own students learn mathematics. So it seems that helping pre-service teachers become more aware of their own metacognition should in turn help them develop their future students' metacognition. Through carefully designed classroom instruction, pre-service teachers could become more comfortable and familiar with an environment that promotes metacognitive thinking, and, thus, they will be more likely to create a metacognitive environment within their own classrooms. While there may be metacognitive thinking occurring in an instructional activity, students cannot learn from the thinking if it is not shared. Learning to think metacognitively is not easy, but the process can be reinforced when students are provided opportunities to not only share their thinking, but also to take the thinking that has been shared, compare it with their own, and monitor and adjust their own thinking based on their comparison with others. Students must then be encouraged to internalize this process so they can identify their own knowledge and know what to do with that knowledge.

Once students have internalized the process, they will no longer have to rely on the community to work through the material, but can instead rely on their own expertise. Through continued work to understand both the internalization process and the creation of effective learning communities, researchers will learn more about how to create environments that promote metacognitive development, which in turn will promote more effective learning. Students today must be able to do more than mindlessly repeat procedures; they must be able to identify, to think, and to reason through a plethora of information. Developing metacognitive thinking helps the students learn to do this and thus better prepares them for their life beyond the four walls of the mathematics classroom. Learning to think metacognitively is an important skill that greatly impacts a student's overall ability to become a successful, productive individual and thus is a facet of learning that we must continue to study and to understand. Metacognition is not a way of developing knowledge and skills for working with others, but instead it is a way of thinking that improves the student's ability to think and problem solve without the assistance of others. Continued research on metacognition and the development of metacognition in the mathematics classroom will help to tease out instructional strategies that fully enable student success.

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APPPENDICES

Appendix A
Instructor Interview Guide

Early Semester Instructor Interview

This interview should take approximately 30-60 minutes

1. Tell me about this course
    a. Classroom norms / general structure
        i. Structure of a typical class session
        ii. Communication
            1. Describe the types of communication (talking, written, other)
            2. Who does the most of each type
            3. Who validates
        iii. Questioning
            1. Types of questions
            2. Who asks / answers
            3. Who validates
    b. HW / Quiz / Exams
        i. General description
    c. Student reactions to this courses
        i. Likes / Dislikes
        ii. What they would change if they could
    d. What makes this course unique?
2. Tell me about your learning experience with the course
    a. How long have you taught it?
    b. Biggest challenges in your first semester
    c. How does this course challenge student thinking?
    d. What do you consider the biggest challenge now?
    e. Describe how teaching this course has impacted your beliefs about mathematics
    f. Describe how teaching this course has impacted your beliefs about the teaching of mathematics
    g. What makes this course unique
        i. Tell me more about inquiry based learning
3. In your opinion, what is mathematics?
4. In your opinion, what is the purpose of mathematics education?
5. How do you believe student's best learn mathematics?
    a. Do you think students agree with your belief? Tell me more.
6. What suggestions would you make to another instructor that is trying to get their students to do more thinking?
    a. What are some ways you help your students learn to think
    b. How can you help them change their way of thinking about a particular problem
    c. Do you believe a person can model their thinking for others?
        i. How?
        ii. How do you model your thinking for your students?
        iii. How do you have your students model their thinking for each other?
7. As an instructor what do you do when multiple solutions/ideas are presented during classtime?
    a. Who determines correctness / completeness of the solution
    b. Who explains why something is/is not correct?
8. As an instructor what do you do when you are presented with an idea that you are not 100% sure about?

9. Some people have suggested activities such as error detection, revising, modeling of thinking, using question stems or prompts, and developing a social environment for learning as effective for promoting student thinking about thinking. What experiences have you had with any of these?
    a. Talk about their place / value in the classroom
    b. Do you use any of these?
    c. Thought about using them, but haven't? Why?

Follow-up interview questions

1. What do you hope students to get out of the geometry course?
2. What do you consider the defining characteristics of an inquiry-based classroom.
    a. How do you feel your class aligns with this definition
    b. Are there times when you feel it does not align
    c. Most important characteristics that make it work
        i. Classroom
        ii. Students
        iii. Instructor
    d. What characteristic would you identify as the most important make/break for overall student learning?
        i. How do you ensure this happens
3. What is the role of teacher in an inquiry based classroom
    a. During/after student presentation of problems
    b. During/after group work
    c. Modeling/questioning/think alouds
4. Describe an ideal presentation of a problem
    a. How the person decides to present
    b. More than one presentation/multiple strategies
    c. What are the other students doing?
        i. What types of questions would you like students to ask?
        ii. Who should they be directed to?
    d. How would the idea differ if wrong/right?
    e. How often do you feel student presentations align with this idea
    f. During times that they don't what do you think is the reason?
5. How do you encourage student
    a. sharing of thinking
    b. discussions
    c. What is the role of the question "why?" in a mathematics classroom
        i. How does it fit in your classroom?
6. There were several times during the observation you were reading student body language and it was mentioned a few times in the first interview, what are some of the things you are looking for in their body language?
    a. Do you think students try to read your body language? Tell me more

7. When you were a math student, what did you like about math?
    a. Do you still feel the same way about math?
8. What is mathematics?
    a. Drill and practice are common terms thrown around in math, what do you think about those terms personally and in regards to the class?
    b. What are your feelings about a math problem that has multiple solutions?

        i. What about a textbook problem with multiple solutions?
    c. Is there a right answer?
    d. Is there a right process?
9. Tell me about the flow of the content…
    a. Who organized the calendar
    b. How do you decide which pages to include and which to leave out
10. CD problems
    a. What do you want students to get from them
    b. Intended purpose
    c. How do they connect with other content?
11. Tell me about the supplemental worksheets
    a. Purpose?
    b. Created by?
    c. How do they compare to text
    d. How do they fit in the overall curriculum
12. Many students commented on having to do the homework before they were taught how to do it… what are your thoughts/how would you respond?
13. You mentioned to the students a couple of times that they should just memorize something…. tell me more about your thinking in regards to memorizing in a math class
14. In summary, how would you describe to someone outside of education how this course is different from other mathematics courses
15. What is question/questions do you feel I should have asked and how would you answer it/them?

Appendix B
Student Surveys

Early Semester Survey

Please complete the questions below with as much information as you can.
This survey should take approximately 15 minutes.

1. Complete the sentence: The teaching of mathematics should…..

2. Describe a time when you revised/changed your solution to a mathematics problem. How did you determine the revision was necessary? How did you determine the revision was correct? Did other people help you decide? Please provide as many details and as much information as possible.

3. Describe a time when you were presented with a solution to a problem and asked to find the mistake in the persons thinking.

4. Consider this scenario: Students in a math class were given a problem to work on individually. Once they had completed the problem they were told to talk it over in a small group. The students compared their answers and strategies for solving the problem and realized there were major differences.

   The teacher had four of the students from the class share their solutions and strategies on the board. Among the four students there were <u>four different strategies</u> and <u>two different solutions</u>. The instructor encouraged discussion in the whole class and instead of telling the students who was right and wrong, the instructor let the students discuss the solutions and ask each other questions about the different strategies.

   a. What do you believe should happen after the whole class discussion?

   b. Please discuss why this was/was not an effective use of classtime.

   c. Please discuss why you do/don't believe the activity described would be beneficial in helping you better understand mathematics?

   d. As a student, have you experienced a situation similar to the one described?

   e. If so, please describe this situation.

   f. How did you feel about it?

5. Consider this scenario: During your mathematics class you were presented with two different explanations for the same problem and then asked you to compare the different solutions and determine if one, both, or none were correct.

   a. How would you react to this situation?

   b. Why do you think the instructor did this?

   c. What would you tell a friend about this situation when you are talking about it later?

6. Do you have any additional thoughts or comments you would like to share?

Late Semester Survey

Please complete the questions below with as much information as you can.
This survey should take approximately 15 minutes.

1. Describe how this course has affected the way you think about solving mathematical problems?

2. Thinking about the opportunities to show your work / discuss how you solved problems in class
   a. How often did you share your work / discuss how you worked the problem in front of the entire class? (several times a day, a couple of times a week, every once in a while, etc… you can also list the exact number of times if you know)

   b. How did you decide if you were willing/not willing to share your work or thoughts about a problem with the class?

   c. If you feel that you *frequently shared* how you answered questions please complete the following two sentences
      i. I share my work frequently because…
      ii. I think others don't share their work because...

   d. If you feel that you *rarely shared* how you answered questions please complete the following two sentences
      i. I never/rarely shared my work to a problem because…
      ii. I think others frequently share their work because…

3. When you had a question about a problem
   a. How did you know it was time to ask a question rather than continue trying to work on it?
   b. Who did you tend to ask?
   c. Why did you choose that person?
   d. Did you choose a different person each time? Explain.

4. Describe an opportunity you had during this semester to help another person understand a problem you were completing for class.
   a. How did this person ask you for help?
   b. How did you react when they asked for help?
   c. Why do you think this person asked you instead of someone else?
   d. How did you help the person?
   e. After your discussion with this person, how did you know the person understood the material and would be able to work the problem without additional help?

5. What are two things you really liked about the way this course was taught?

6. How did the way this course was taught impact your learning?

7. What are two things you would change about how this course was taught?

8. Choosing one of the two things you mentioned in the previous question, why did you not like this aspect of the course and why do you think it should be changed?

9. Is there anything else you would like to share with me about your experience with this course and how it has helped you learn?

Appendix C
Demographic Profile

Name______________________ Date_______ Year Graduated High School______

Classification (circle one)

Freshman Sophomore Junior Senior Other__________

Major__________________________________ Ethnicity____________ Gender_________

Are you under the age of 18? Yes No

What are your plans after you graduate with your degree:

What math courses did you take in high school? What was your approximate letter grade in the course?

What math courses have you taken in college? What was your approximate letter grade in the course?

Please also list any other courses you have taken that you believe are related to mathematics.

How many hours per week do you study for this course? Describe how you study.

Approximate # hours ________

How:

Family Information (Please circle)

Single Married Divorced Separated

Do you have children living with you No Yes, How many ____

Work/Military Experience: If you served in the military or worked fulltime for one year or more before entering college, please describe your experience (kind of work and number of years).

# of years ________ Kind of work:

Appendix D
Metacognitive Awareness Inventory

The following questions ask about the strategies you use when approaching your coursework.
Remember there are no right or wrong answers, just answer as accurately as possible.
If you think the statement is very true of you, circle 5; if a statement is not at all true of you, circle 1. If the statement is more or less true of you, find the number between 1 and 5 that best describes you.

1	2	3	4	5
Not true of me at all				Very true of me

1.	I ask myself periodically if I am meeting my goals.	1	2	3	4	5
2.	I consider several alternative to a problem before I answer.	1	2	3	4	5
3.	I try to use strategies that have worked in the past.	1	2	3	4	5
4.	I pace myself while learning in order to have enough time.	1	2	3	4	5
5.	I understand my intellectual strengths and weaknesses.	1	2	3	4	5
6.	I think about what I really need to learn before I begin a task.	1	2	3	4	5
7.	I know how well I did once I finish a test.	1	2	3	4	5
8.	I set specific goals before I begin a task.	1	2	3	4	5
9.	I slow down when I encounter important information.	1	2	3	4	5
10.	I know what kind of information is most important to learn.	1	2	3	4	5
11.	I ask myself if I have considered all options when solving a problem.	1	2	3	4	5
12.	I am good at organizing information.	1	2	3	4	5
13.	I consciously focus my attention on important information.	1	2	3	4	5
14.	I have a specific purpose for each strategy I use.	1	2	3	4	5
15.	I learn best when I know something about the topic.	1	2	3	4	5
16.	I know what the teacher expects me to learn.	1	2	3	4	5
17.	I am good at remembering information.	1	2	3	4	5
18.	I use different learning strategies depending on the situation.	1	2	3	4	5
19.	I ask myself if there was an easier way to do things after I finish a task.	1	2	3	4	5
20.	I have control over how well I learn.	1	2	3	4	5
21.	I periodically review to help me understand important relationships.	1	2	3	4	5
22.	I ask myself questions about the material before I begin.	1	2	3	4	5

23.	I think of several ways to solve a problem and choose the best one.	1	2	3	4	5
24.	I summarize what I've learned after I finish.	1	2	3	4	5
25.	I ask others for help when I don't understand something.	1	2	3	4	5
26.	I can motivate myself to learn when I need to.	1	2	3	4	5
27.	I am aware of what strategies I use when I study.	1	2	3	4	5
28.	I find myself analyzing the usefulness of strategies while I study.	1	2	3	4	5
29.	I use my intellectual strengths to compensate for my weaknesses.	1	2	3	4	5
30.	I focus on the meaning and significance of new information.	1	2	3	4	5
31.	I create my own examples to make information more meaningful.	1	2	3	4	5
32.	I am a good judge of how well I understand something.	1	2	3	4	5
33.	I find myself using helpful learning strategies automatically.	1	2	3	4	5
34.	I find myself pausing regularly to check my comprehension.	1	2	3	4	5
35.	I know when each strategy I use will be most effective.	1	2	3	4	5
36.	I ask myself how well I accomplished my goals once I'm finished.	1	2	3	4	5
37.	I draw pictures or diagrams to help me understand while learning.	1	2	3	4	5
38.	I ask myself if I have considered all options after I solve a problem.	1	2	3	4	5
39.	I try to translate new information into my own words.	1	2	3	4	5
40.	I change strategies when I fail to understand.	1	2	3	4	5
41.	I use the organizational structure of the text to help me learn.	1	2	3	4	5
42.	I read instruction carefully before I begin a task.	1	2	3	4	5
43.	I ask myself if what I'm reading is relation to what I already know.	1	2	3	4	5
44.	I reevaluate my assumptions when I get confused.	1	2	3	4	5
45.	I organize my time to best accomplish my goals.	1	2	3	4	5
46.	I learn more when I am interested in the topic.	1	2	3	4	5
47.	I try to break studying down into smaller steps.	1	2	3	4	5
48.	I focus on overall meaning rather than specifics.	1	2	3	4	5
49.	I ask myself questions about how well I am doing while I am learning something new.	1	2	3	4	5
50.	I ask myself if I learned as much as I could have once I finish a task.	1	2	3	4	5
51.	I stop and go back over new information what is not clear.	1	2	3	4	5
52.	I stop and reread when I get confused.	1	2	3	4	5

Appendix E
Descriptors and Examples of Subcomponent Coding

Subcomponent of Metacognition	MAI Questions Related to Subcomponent (question #)	Sample Episode for Subcomponent
	Knowledge of Cognition	
Declarative Knowledge knowledge about one's skills, intellectual resources, and abilities as a learner	I understand my intellectual strengths and weaknesses (5) I know what kind of information is most important to learn (10) I am good at organizing information (12) I know what the teacher expects me to learn (16) I am good at remembering information (17) I have control over how well I learn 20) I am a good judge of how well I understand something (32) I learn more when I am interested in the topic (46)	Student: I am confused on if you are supposed to draw the parallel line or the perpendicular line
Procedural Knowledge knowledge about how to implement learning procedures (eg strategies)	I try to use strategies that have worked in the past (3) I have a specific purpose for each strategy I use (14) I am aware of what strategies I use when I study (27) I find myself using helping learning strategies automatically (33)	Instructor: number eight… we are waiting patiently for somebody to get up the courage to come attack this problem Student: alright, how long would it take Joanna to walk around the trapezoidal field pictured here if she walks at a rate of 300 feet per minute, I started by finding the perimeter of the field, what would you call that?, altitude? Instructor: (shakes her head yes) absolutely, which we will talk about next Student: so that would be 200 feet cause this is 550 and 350 so that makes this 200 and because this is 200 this one would also be 200 so I am gonna find the hypotenuse of that triangle 200 squared plus 200 squared equals c squared, 200 squared is forty thousand plus forty thousand, will you stop me if I am doing this wrong? Instructor: you are doing fine, I'm sure they'll let you know (pointing to the class)

Conditional Knowledge knowledge about when and why to use learning procedures	I learn best when I know something about the topic (15) I use different learning strategies depending on the situation (18) I can motivate myself to learn when I need to (26) I use my intellectual strengths to compensate for my weaknesses (29) I know when each strategy I use will be most effective (35)	Instructor: this way if you connect the dots, you find what type of quadrilateral? Student: kite Instructor: yeah, a kite and the diagonals of a kite are perpendicular and when bisected, so that is why this one works, but if you have something that works, there are other ways, yes [there is another way to do this construction of reflecting a point over a line with a compass and straightedge]
	Regulation of Cognition	
Planning planning, goal setting, and allocating resources prior to learning	I pace myself while learning in order to have enough time (4) I think about what I really need to learn before I begin a task (6) I set specific goals before I begin a task (8) I ask myself questions about the material before I begin (22) I think of several ways to solve a problem and choose the best one (23) I read instructions carefully before I begin a task (42) I organize my time to best accomplish my goals (45)	Student: so I guess we just use Pythagorean Theorem on all of these? We are trying to find which ones are the point, the distance right? Okay (student draws in right triangle and finds length)
Information Management Skills skills and strategy sequences used on-line to process information more efficiently (eg organizing, elaborating, summarizing, selective focusing)	I slow down when I encounter important information (9) I consciously focus my attention on important information (13) I focus on the meaning and significance of new information (30) I create my own examples to make information more meaningful (31) I draw pictures or diagrams to help me understand while learning (37) I try to translate new information into my own words (39) I use the organizational structure of	Instructor: let's summarize a and b, what are you trying to find for a? the circumcenter of the circumscribing circle, in order to find the circumcenter… that's the point where what meets? Student: all the bisectors come together Instructor: be more specific for me, all the bisectors…. Student: perpendicular? Instructor: perpendicular bisectors, so in other words you need all three perpendicular bisectors, how do you find perpendicular bisectors? Student: you fold the paper and touch the points describing paper folding Instructor: so you do what you did back on number one, you have an example of perpendicular

	the text to help me learn (41) I ask myself if what I'm reading is related to what I already know (43) I try to break studying down into smaller steps (47) I focus on overall meaning rather than specifics (48)	bisectors from number 1, you just did it, so do what you did on number one three times…b, what are you trying to find on b?
Monitoring assessments of one's learning or strategy use	I ask myself periodically if I am meeting my goals (1) I consider several alternatives to a problem before I answer (2) I ask myself if I have considered all options when solving a problem (11) I periodically review to help me understand important relationships (21) I find myself analyzing the usefulness of strategies while I study (28) I find myself pausing regularly to check my comprehension (34) I ask myself questions about how will I am doing while I am learning something new (49)	Student: instead of uhm, her choosing two different points, can I just choose the one point and draw the arc from point p through the line and then use it where the line crosses as the second point?
Debugging strategies used to correct comprehension and performance errors	I ask others for help when I don't understand something (25) I change strategies when I fail to understand (40) I reevaluate my assumptions when I get confused (44) I stop and go back over new information that is not clear (51) I stop and reread when I get confused (52)	Student: are we supposed to know to extend the lines Instructor: yeah lets talk about that, it's a good question there, how did <student> know to extend the lines? Student: the directions say to Student: obtuse angle Instructor: say that again Student: any time the triangle is obtuse, the line Instructor: how do you know which lines to extend? Student: uh the ones of the leg, uh the shortest sides you extend Student: if you had it as the floor and dropped a strong from the vertex Instructor: any body view it differently? Okay are we done with this page?

Evaluation analysis of performance and strategy effectiveness after a learning episode	I know how well I did once I finish a test (7) I ask myself if there was an easier way to do things after I finish a task (19) I summarize what I've learned after I finish (24) I ask myself how well I accomplished my goals once I'm finished (36) I ask myself if I have considered all options after I solve a problem (38) I ask myself if I learned as much as I could have once I finish a task (50)	Instructor: will it work? (students wait and look to her for answer) you tell me? Did it? (students discussing amongst their neighbors) how did you check? How did you check to see if you construction worked? Student: mira? Instructor: yeah, whip out your mira, just like number 1 and check it

Appendix F
Content and Number of Problems on Each Page From the Observations

Page #	# Problems	Topic
329	4	Geoboard Perimeters
336	8	Perimeters and Right Triangles
489	4	Reflection Lines and Point-Image Segments
491	5	Constructions with the Mira
497	2	Altitude Constructions with the Mira
501	7	Finding Circumcenters of All Types of Triangles
545	8	Finding center of rotations and glide-reflection lines
547	2	Finding the center of rotation
654#6	1	CD problem using a mira: Find the third point to make an equilateral triangle
655#12	1	CD problem using a mira: Construct and isosceles right triangle so that the given side is one of the legs
WS 4 cen	20	Complete a grid with the headings Center, What do I construct?, Sketch, Location, Circle, Which manipulative?

Appendix G
Frequency of Shared Knowledge of Cognition

Page/#Problems	Section	Declarative			Procedural			Conditional			Totals		
	Section	SO*	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO
329/4	MWF	-	-	-	3**	-	2	-	-	-	3	-	2
	TR	-	-	-	2	-	2	-	-	1	2	-	3
336/8	MWF	1	-	-	3	2	-	-	1	-	4	3	-
	TR	-	-	-	7	-	-	-	-	-	7	-	-
489/4	MWF	1	-	-	2	2	2	-	-	1	3	2	3
	TR	-	-	-	-	-	-	-	-	-	-	-	-
491/5	MWF	-	-	-	-	3	1	-	-	-	-	3	1
	TR	-	-	-	-	4	-	-	-	-	-	4	-
497/2	MWF	-	-	-	2	-	-	-	1	-	2	1	-
	TR	1	-	-	1	-	-	-	-	-	2	-	-
501/7	MWF	-	-	-	3	-	-	-	-	-	3	-	-
	TR	-	-	-	1	-	-	-	-	-	1	-	-
545/8	MWF	-	-	-	1	2	-	-	1	-	1	3	-
	TR	-	-	-	3	4	1	-	2	-	3	6	1
547/2	MWF	-	-	-	-	-	-	-	-	-	-	-	-
	TR	-	-	-	-	-	-	-	-	1	-	-	1
654#6/1	MWF	-	-	-	1	1	-	-	-	1	1	1	1
	TR	-	-	-	-	-	-	-	-	-	-	-	-
655#12/1	MWF	-	-	-	1	-	-	-	-	-	1	-	-
	TR	-	-	-	1	-	-	-	-	-	1	-	-
WS 4 cen/20	MWF	-	-	-	-	-	-	-	4	-	-	4	-
	TR	-	-	-	-	-	-	-	4	-	-	4	-
Totals	MWF	2	-	-	16	10	5	-	7	2	18	17	7
	TR	1	-	-	15	8	3	-	6	2	16	14	5
	Both	3	-	-	31	18	8	-	13	4	34	31	12
Grand Totals			3			57			17			77	

*SO: student only thought

ST: integrated student and teacher thoughts

TO: teacher only thought

Appendix H
Frequency of Shared Regulation of Cognition

Page/#Problems	Section	Planning			Info. Mgmt.			Monitoring			Debugging			Evaluation			Totals		
	Section	SO*	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO	SO	ST	TO
329/4	MWF	-	-	-	-	-	-	-	2**	-	-	1	-	-	-	-	-	3	-
	TR	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	1	1
336/8	MWF	1	1	-	-	1	-	-	1	1	-	-	1	-	-	-	1	3	2
	TR	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-	2	-	-
489/4	MWF	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	1
	TR	-	-	-	-	-	-	-	1	-	-	-	-	-	1	-	-	2	-
491/5	MWF	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	1	1
	TR	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	1	-
497/2	MWF	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	1	-
	TR	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	1	-	-
501/7	MWF	-	-	1	1	-	-	-	-	-	-	-	-	-	1	-	1	1	1
	TR	-	-	1	-	-	-	-	-	-	1	-	-	-	1	-	1	1	1
545/8	MWF	-	-	1	-	1	-	-	-	-	-	3	-	-	-	-	-	4	1
	TR	-	-	-	2	3	-	-	-	-	-	4	-	-	-	-	2	7	-
547/2	MWF	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	TR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
654#6/1	MWF	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	1	-
	TR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
655#12/1	MWF	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	TR	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	1	-
WS 4 cen/20	MWF	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	4	-
	TR	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	4	-
Totals	MWF	1	1	2	1	3	2	-	3	1	-	6	1	-	5	-	2	18	6
	TR	-	-	1	3	5	1	2	1	-	1	5	-	-	6	-	6	17	2
	Both	1	1	3	4	8	3	2	4	1	1	11	1	-	11	-	8	35	8
Grand Totals			5			15			7			13			11			51	

*SO: student only

ST: integrated student and teacher thoughts

TO: teacher only thought

Appendix I
Informed Consent

Informed Consent - Students

Project Title: Instructional Practices for the Development of Metacognitive Thinking in Mathematics Classrooms

Investigator: Kansas Conrady Pope, MA
120 MCS
Edmond, OK 73003
405-974-5381

Purpose: The purpose of this research is to develop a better understanding of how instructors help students learn to think about their own thinking and the thinking of others. More specifically I am hoping to better understand specific activities that are being used to develop thinking about thinking, determine if pre-service teachers recognize and value these activities, and determine if awareness of thinking about thinking changes over the semester when participating in these activities. I am asking you to participate in this study since you are a pre-service teacher and enrolled in this course.

Procedures: As a participant in this study you will be asked to complete demographic information, The Metacognitive Awareness Inventory at both the beginning and end of the semester, an Early Semester Open Ended Survey, and a Late Semester Open Ended Survey. These open ended surveys will focus on your perceptions of mathematics and mathematics education, how you feel you learn to think, and how you believe your students will best learn mathematics. Each of these surveys should take approximately 10 minutes to complete. I will also look at comments that are returned to you on some of your graded homework, quizzes, and/or exams.

Based on the information you provide on your survey, you may also be asked to participate in an interview lasting approximately 30 minutes to better understand how you think about your thinking in mathematics. These interviews will be audio taped. The interview is optional. You may participate in other parts of the study and decline the interview if you choose.

Lastly, 10-15 class meetings will be videotaped during the semester and portions related to thinking about your own thinking or that of others will be used as part of this study. The video will be of the whole class and not focused on any individual or small group. After collecting the video I will choose particular sections that pertain directly to the purpose of this study. These portions of the video will be transcribed. If you choose not to participate in the study, anything you say or do during this video will not be transcribed or used. You may also choose to sit out of view of the camera. Please note that this video will not be seen by anyone other than myself, my supervising professor, and the research oversight staff at UCO and OSU.

Participation in this research is not a requirement of the course and will not have any bearing on your course grade. You have the right to refuse to answer any question. There is no extra credit for participation in this study. All information from this study will remain will remain confidential and thus will not be discussed with your instructor.

Risks of Participation:
There are no known risks associated with this project which are greater than those ordinarily encountered in daily life.

Confidentiality: As previously mentioned, all data will remain confidential. Only the researchers (investigator listed above and supervising professor) will have access to the data. No information will be shared with your course instructor and thus will have no bearing on your course grade. All data will be stored in a locked cabinet in my office. Paper copies of the data will be shredded after 1 year. Digital data will be destroyed after 5 years. All data will be reported using pseudonyms.

The records of this study will be kept private. Any written results will discuss group findings and will not include information that will identify you. Research records will be stored securely and only researchers and individuals responsible for research oversight will have access to the records. It is possible that the consent process and data collection will be observed by research oversight staff responsible for safeguarding the rights and wellbeing of people who participate in research.

Compensation:

There is no compensation for participation in this study.

Contacts:

Kansas Conrady Pope, MA	Patricia Jordan, Ed.D.
Kansas.pope@okstate.edu	Patricia.jordan@okstate.edu
120 MCS	245 Willard Hall
Edmond, OK 73003	Stillwater, OK 74078
405-974-5381	405-744-8142

If you have questions about your rights as a research volunteer, you may contact Dr. Shelia Kennison, IRB Chair, 219 Cordell North, Stillwater, OK 74078, 405-744-3377 or irb@okstate.edu.
Or
Dr. Jill A. Devenport, Chair of UCO Institutional Review Board, 405-974-5479, UCO-IRB Office, ADM 216, Office of Research and Grants, Campus Box 159, Edmond, OK 73034 or irb@uco.edu.

Participant Rights:

Participation in this study is voluntary. You may choose to discontinue this research activity at anytime without reprisal or penalty.

Signatures:

I have read and fully understand the consent form. I sign it freely and voluntarily. A copy of this form has been given to me.

____

Signature of Participant Date

I agree to be being audio taped and/or videotaped as part of this project.

____

Signature of Participant Date

I certify that I have personally explained this document before requesting that the participant sign it.

____

Signature of Researcher Date

Informed Consent – Course Instructor

Project Title: Instructional Practices for the Development of Metacognitive Thinking in Mathematics Classrooms

Investigators: Kansas Conrady Pope, MA
120 MCS
Edmond, OK 73003
405-974-5381

Purpose: The purpose of this research is to develop a better understanding of how instructors help students learn to think about their own thinking and the thinking of others. More specifically I am hoping to better understand specific activities that are being used to develop thinking about thinking, determine if pre-service teachers recognize and value these activities, and determine if awareness of thinking about thinking changes over the semester when participating in these activities. I am asking you to participate in this study since you are a pre-service teacher and enrolled in this course.

Procedures: As a participant in this study you will be asked to complete two interviews that will each last approximately one hour. These interviews will focus on your beliefs about developing student thinking about thinking as well as more general beliefs about mathematics and mathematics education. I will also collect and or talk with you about information such as lesson plans, course materials, syllabus, and exams as they relate to metacognitive activities, including written comments on graded student work. Lastly, 10-15 class meetings of each section will be videotaped during the semester and portions related to thinking about thinking will be used as part of this study.

Risks of Participation:

There are no known risks associated with this project which are greater than those ordinarily encountered in daily life.

Confidentiality:

As previously mentioned, all data will remain confidential. Only the researchers (investigator listed above and my supervising professor at OSU) will have access to the data. No information will be shared with your course instructor and thus will have no bearing on your course grade. All data will be stored in a locked cabinet in my office. Paper copies of the data will be shredded after 1 year. Digital data will be destroyed after 5 years. All data will be reported using pseudonyms.

The records of this study will be kept private. Any written results will discuss group findings and will not include information that will identify you. Research records will be stored securely and only researchers and individuals responsible for research oversight will have access to the records. It is possible that the consent process and data collection will be observed by research oversight staff responsible for safeguarding the rights and wellbeing of people who participate in research.

Compensation:

There is no compensation for participation in this study.

Contacts:

Kansas Conrady Pope, MA
Kansas.pope@okstate.edu
120 MCS
Edmond, OK 73003
405-974-5381

Patricia Jordan, Ed.D.
Patricia.jordan@okstate.edu
245 Willard Hall
Stillwater, OK 74078
405-744-8142

If you have questions about your rights as a research volunteer, you may contact Dr. Shelia Kennison, IRB Chair, 219 Cordell North, Stillwater, OK 74078, 405-744-3377 or irb@okstate.edu.

Or

Dr. Jill A. Devenport, Chair of UCO Institutional Review Board, 405-974-5479, UCO-IRB Office, ADM 216, Office of Research and Grants, Campus Box 159, Edmond, OK 73034 or irb@uco.edu.

Participant Rights:

Participation in this study is voluntary. You may choose to discontinue this research activity at anytime without reprisal or penalty.

Signatures:

I have read and fully understand the consent form. I sign it freely and voluntarily. A copy of this form has been given to me.

____

Signature of Participant Date

I agree to be being audio taped and/or videotaped as part of this project.

____

Signature of Participant Date

I certify that I have personally explained this document before requesting that the participant sign it.

____

Signature of Researcher Date

VITA

Kansas Conrady Pope

Candidate for the Degree of

Doctor of Philosophy

Thesis: METACOGNITIVE DEVELOPMENT IN A COLLEGE-LEVEL GOEMETRY COURSE FOR PRE-SERVICE ELEMENTARY TEACHERS: A CASE STUDY

Major Field: Education

Biographical:

Education:

Completed the requirements for the Doctor of Philosophy in Education at Oklahoma State University, Stillwater, Oklahoma in July, 2011.

Completed the requirements for the Master of Arts in Mathematics at The University of Texas at Arlington, Arlington, Texas in 2005.

Completed the requirements for the Bachelor of Science in Secondary Education at Oklahoma State University, Stillwater, Oklahoma in 2001.

Experience:

University of Central Oklahoma (Fall 2009 – present) Lecturer

Oklahoma State University (Fall 2007 – Summer 2009) Graduate Teaching Associate

Professional Memberships:

National Council of Teachers of Mathematics
Research Council for Mathematics Learning
School Science and Mathematics Association
Mathematical Association of America
American Education Research Association

ADVISER'S APPROVAL: Dr. Patricia Lamphere-Jordan

Name: Kansas Conrady Pope Date of Degree: July, 2011

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: METACOGNITIVE DEVELOPMENT IN A COLLEGE-LEVEL GEOMETRY COURSE FOR PRE-SERVICE ELEMENTARY TEACHERS: A CASE STUDY

Pages in Study: 208 Candidate for the Degree of Doctor of Education

Major Field: Education

Scope and Method of Study: The purpose of this mixed methods naturalistic inquiry study was to develop a better understanding of how metacognitive development is embedded in an inquiry-based college level geometry content course for pre-service elementary teachers. Data included demographic information, pre- and post- instructor interviews with the one course instructor using an interview guide, pre and post open-ended student surveys with the students from her two sections, non-participant classroom observations of both course sections, documents such as the textbook and course syllabus, and a pre/post administration the Metacognitive Awareness Inventory.

Findings and Conclusions: The analysis of the qualitative data suggests the presence of an underlying classroom structure that promotes the sharing of thinking and importance of class discussions to encourage metacognitive development, but there is a need to help students refine their thinking to make discussions more effective. In this classroom, more metacognitive knowledge is modeled than metacognitive regulation. Students do provide many of the modeling episodes, especially models of procedural knowledge, however; the instructor prompts the students through almost all episodes of modeled conditional knowledge and regulation episodes. Explicit modeling of each subcomponent of metacognition is needed and the development of an effective learning community where students freely share and collaborate would make this classroom more effective at promoting metacognitive development. Due to a delay in the administration of the pre-MAI, it was not possible to determine if the course impacted students' metacognitive awareness. Further exploration of metacognitive development is warranted and ideas for future studies are presented.

ADVISER'S APPROVAL: Dr. Patricia Lamphere-Jordan