# DIFFERENTIAL EFFECTS OF REINFORCEMENT RATE AND DELAY ON RESPONSE ALLOCATION: A SYSTEMATIC APPROACH UTILIZING THE 

## MATCHING LAW

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## CHAPTER I

## INTRODUCTION

Behavior analysis is the study of behavior and is concerned with describing, explaining, and applying the natural laws governing behavior in an objective manner. Within behavior analysis there are two identifiable groups: basic behavior analysts and applied behavior analysts. For more than a decade there has been discussion about the increasing separation between these two groups (Marr, 1991; Pierce \& Epling, 1980; Rider, 1991). The goal of basic behavior analysis has traditionally been a scientific endeavor which attempts to identify and describe regularities in the interaction of the behavior and the environment (Nevin, 1984; Skinner, 1953). Toward this end, the experimental analyst has tended to the development of mathematical or formal laws and theories of behavior (Baum, 1989; Marr, 1989; Nevin, 1984; Skinner, 1950). As the experimental analysis of behavior has attempted to better explain the principles governing behavior, applied behavior analysis attempts to utilize these principles to develop behavioral technology to assist people in applied settings and with behaviors of social importance.

Unfortunately, applied behavior analysis has not kept pace with the current research in the experimental area of behavior analysis. This lapse in knowledge results in applied behavior analysts who do not use current knowledge in basic behavior analysis to
maximize the effectiveness of the technologies they are developing in an attempt to help people (Davidson, 1992; Pierce \& Epling, 1980). Therefore, a gap remains between principles of experimental behavior analysis and their application in applied behavior analysis (Epling \& Pierce, 1983; Mace, 1994).

One area of study that may bridge the gap between applied and experimental behavior analysis is the matching law. The matching law (Herrnstein, 1961, 1970) is a behavioral phenomenon widely supported in the experimental analysis of behavior (Davidson \& McCarthy, 1988) and is a mathematical description of choice behavior as well as a formal representation of the organism-environment interaction. A central premise of matching theory is that, at any given moment, individuals have a variety of alternative behaviors in which to engage, and they select one behavior to the exclusion of others. Choices among behaviors occur continuously (e.g., the child may switch to a different activity at any time), and consequences (either programmed or naturally occurring) are associated with each selection (McDowell, 1988). More simply, the matching law states that an individual will distribute his or her behavior between alternatives in the same ratio that reinforcements have been obtained for these alternatives.

The matching law has implications for teacher and student behavior such as academic response allocation in classrooms (Martens, 1992; Martens, Halperin, Rummel \& Kilpatrick, 1990; Neef, Mace \& Shade, 1993; Neef, Mace, Shea \& Shade, 1992; Shriver \& Kramer, 1997), academic completion (Skinner \& Robinson, 1996), and even to the allocation of three-point shots with college basketball players (Vollmer \& Bourret, 2000). Specifically, within the classroom, the matching law indicates that it is necessary
to attend to concurrent schedules of reinforcement affecting student and teacher behavior when observing and intervening in the classrooms in order to develop and modify effective interventions and avoid unwanted side effects of interventions (McDowell, 1982). The matching law has been operationalized as:

$$
\mathrm{R} 1 / \mathrm{R} 1+\mathrm{R} 2=\mathrm{r} 1 / \mathrm{r} 1+\mathrm{r} 2
$$

where R1 equals the rate of one specifically define behavior, R2 equals the rate of another defined behavior, r 1 equals the rate of reinforcement obtained for R 1 , and r 2 equals the rate of reinforcement for R2 (Killeen, 1972) and is referred to as the strict matching law (SML). Myerson and Hale (1984) thoroughly discussed theoretical applications of strict matching across various types of competing reinforcement schedules. For example, when using on-task and off-task behavior as competing responses and both are reinforced on variable interval schedules, one can demonstrate the usefulness of the matching law. Suppose that on-task behavior is being reinforced on a variable-interval (VI) 10-min schedule and off-task behavior is being reinforced on a VI 4-min schedule. Knowing this, the obtained levels of reinforcement and time allocated to each behavior can be calculated utilizing the strict matching law formula. Thus, when the rate of reinforcement for off-task behavior remains the same, the reinforcement schedule for on-task behavior must be increased from a VI 10-min to a VI 2-min to increase time on-task. Examples like this show how a strict matching equation can predict response allocation based on changes in reinforcement rate for one or both response alternatives.

Another form of the matching law, termed the Generalized Matching Law (GML) has not received as much empirical attention for describing naturally occurring human behavior. The GML differs from the strict matching law in that it describes the
relationship between response rates (or duration) and reinforcement of one alternative, relative to all other alternatives. The generalized matching equation is often expressed in a logarithmic form to obtain a straight line:


R1 and R2 represent the rates of responding for Responses 1 and 2, and r1 and r2 are the rates of obtained reinforcement for those responses. The equation also includes two fitted parameters, $s$ and $b$, that permit the fitting of a straight line to the obtained data. These fitted parameters have been conceptualized as representing sensitivity $(s)$ to the schedules and bias (b) toward one of the responses (Baum, 1974). Bias and sensitivity account for two types of deviations commonly observed in matching research (McDowell, 1989) and the GML describes or accounts for these two systematic deviations from strict matching (refer to the first equation) (Baum, 1974). Bias occurs when the choice between behaviors and reinforcement attained is not symmetrical, believed to be the result of the difficulty or complexity of the response and/or the latency or type of reinforcement (Baum, 1974; Davison \& McCarthy, 1988; McDowell, 1989). The bias parameter ( $\log b$ ) measures the tendency of a subject to respond more consistently on one alternative, independent of reinforcer-rate differences. Sensitivity to schedule of reinforcement refers to an individual's alternation between responses more (overmatching) and less (undermatching) often than is predicted by strict matching (Baum, 1979, 1982). This can occur even when behavioral responses and/or reinforcing stimuli are similar or even the same. However; this parameter $(s)$ is a measure of sensitivity of behavioral responding to relative changes in reinforcement (rate, delay, and quality).

McDowell (1989) pointed out that deviations from matching are much more likely when concurrent arrangements are asymmetrical rather than symmetrical. He referred to concurrent arrangements as being symmetrical when identical response options (e.g., completing a math problem in stack A vs. stack B) produce qualitatively identical reinforcers (e.g., 2 tokens for each correct response). By contrast, asymmetrical concurrent arrangements are ones in which either the responses (e.g., communication vs. aggression) or the type of reinforcers (e.g., a toy vs. attention) are different. In the natural human environment, the response options available to an individual are often different (e.g., mow the lawn vs. watch television), as are the reinforcers associated with each response. As a result, individuals may often allocate responding in ways that seem to deviate from matching. Although deviations from matching may be more likely in asymmetrical concurrent arrangements, significant deviations from matching also may occur under symmetrical concurrent arrangements. In essence, in situations where symmetrical concurrent arrangement occur, behaviors (identical behavioral response options) and reinforcement (identical reinforcing stimuli contingent on either behavior) deviations from matching can also occur. However, empirical support utilizing a systematic approach to understanding sensitivity to the components of reinforcement on response allocation or combining two variables associated with a symmetrical and asymmetrical methodology has not been addressed (Neef, Shade, \& Miller, 1994). Additionally, its impact on the GML formula and whether one sensitivity variable has more impact over another warrant further investigation.

The bias and sensitivity parameters of the GML have important implications for observing and intervening with naturally occurring behavior, specifically regarding
problem behavior both in and out of the classroom (Neef, Mace, \& Shade, 1993; Neef, Mace, Shea, \& Shade, 1992; Symons, Hoch, Dahl, \& McComas, 2003). For example, Neef, Shade, and Miller (1994) examined the viability of an assessment methodology using a combination of reinforcement dimensions such as reinforcer rate, quality, and delay along with response effort to determine the effects of response allocation on two concurrent sets of math problems for six children with learning and behavior difficulties. These response and reinforcer variations differentially impacted the relative time students allocated to the two sets of problems. Students in this study extended previous investigations on the separate effects of reinforcer quality (Neef et al., 1992), reinforcer delay (Neef et al., 1992), reinforcer rate (Mace et al., 1994), and response effort (Neef et al., 1991) by systematically examining how each of these reinforcement dimensions combine to influence behavior allocation. This study (Neef et al., 1991) supports the impact of reinforcer dimensions, thus dimensions of both bias (response effort) and sensitivity (reinforcer rate, quality, and delay) on responding, confirming that choice is an orderly phenomenon governed by the reinforcement properties of response alternatives.

The GML has potential to assist in effectively utilizing observational data gathered in the classroom to develop more effective interventions, decrease potential side effects, and evaluate outcomes (Shriver \& Kramer, 1997). Studies using the generalized matching equation show how variables beyond rates of reinforcement can influence student choice behavior in the classroom. Additionally, these data indicate how different variables interact and affect choice behavior differently across subjects. Thus, GML studies utilized in the educational setting provide an example of how basic theory can be
applied in complex settings where many different causal variables interact and influence choice behavior in accordance with the generalized matching law.

The GML allows for the quantitative description of many student behaviors and respective relative reinforcement schedules by measuring factors that account for deviations from strict matching and account for choice behavior. The GML potentially allows for the measurement of the individual's sensitivity in contacting relevant concurrent reinforcement schedules by taking into account specific changes within reinforcement dimensions (rate and delay). In addition, the GML may have the ability for individual measurement of sensitivity in specific response and reinforcer parameters such as delay to reinforcer access and rate of reinforcement. However, little empirical work exits that evaluate the systematic measurement and resulting weight of these sensitivity variables to response allocation.

The majority of matching law studies has focused on relative rates of reinforcement. While some applied researchers have begun to investigate relative quality and immediacy of reinforcement and relative response effort (Neef et. al., 1994), these parameters have not been explored as much as have reinforcement rates in applied or basic experimental research. Future experimental, applied, and application research should focus on investigating the interaction among reinforcement rates, quality, immediacy, and response effort. Such research may allow for better prediction and control of behavior across organisms and environments, including educational environments.

As stated above, the matching law has received extensive empirical support in basic research. Matching law and matching theory could provide the quantifiable and
molar perspective required to more accurately describe, explain, predict and control human behavior in applied settings than has been achieved to date. Empirical support for this proposition is lacking, however, as the utility of the matching law has not been extensively tested in the applied settings.

The current study attempts to address the lack of empirical research using the matching law in applied settings through quantitative assessment of student response allocation in the presence of variables associated with the sensitivity parameter (reinforcement delay and rate of reinforcement). This study will utilize curriculum-based measurement of a mathematical calculation task as an efficient and effective progress monitoring tool for student academic behavioral responses and to incorporate behavioral response allocation to components of the matching law within an applied behavioral paradigm. The matching law will be applied to data from student response allocation to mathematical problems while manipulating rate and delay of reinforcement. If the matching law is determined to accurately describe human behavior in natural settings, future research may address the functional utility of the matching law for use in predicting and controlling human behavior in natural settings.

## Limitations of Internal and External Validity

Threats to internal validity include meeting statistical assumptions such as the groups being normally distributed, homogeneity of variance and covariance, correlations between the means and variances across groups and an ill-conditioned matrix because the variables are linear combinations of each other. Every effort will be made to address these assumptions before the study and while the data are being analyzed. Threats to external validity are as important to the outcome of this study. Generalizability of the
results is limited only in the sense that participants are being recruited from Stillwater, Oklahoma. Also, by limiting the students to only those pre-referred for excessive problem behaviors with no previous clinical diagnosis the interpretations must be made carefully as not to generalize beyond that population. Sample size is a threat to external validity. Overall, the limitations will be addressed as much as possible to lessen their influence over the results.

In the proceeding chapter an extensive overview of literature related to the development of matching law and matching law theory will be presented. In addition, a thorough examination of the development and applied relevance of the generalized matching law will be addressed. And lastly, the matching law theory and its application to human research in behavioral therapy and in the classroom setting will assist in exemplifying future direction of matching law research and its implications to the current study.

## CHAPTER II

## REVIEW OF LITERATURE

This chapter begins with a brief history and development of the matching law and matching law theory. Then the development of the generalized matching law will be discussed and the relation of its components with current human research. Next, a brief overview of curriculum-based measurement and its use in assessment and monitoring student academic achievement in mathematics will be addressed. And finally, a literature review of the empirical research conducted to date on the utility of the matching law with humans participants, specifically manipulating sensitivity variables and its impact on response allocation will be reviewed.

## Brief History of the Matching Law

In 1970, Herrnstein proposed the Quantitative Law of Effect, also called the "matching law." The matching law was based on over a decade's research on single, multiple, and concurrent schedules of reinforcement (e.g., Ferster \& Skinner, 1957) and mathematically described the contrast effects found between components of multiple schedules and the matching found between relative frequencies of responding and reinforcement in concurrent schedules. This discussion will focus primarily on the matching research literature which utilizes concurrent schedules varying components of reinforcement and response effort, as this research best facilitates description and discussion of matching theory and is most relevant to the current study.

Development of the Matching Law. Herrnstein's (1961) early formulation of the matching relation was demonstrated when pigeons were submitted to concurrent variableinterval (VI) schedules, the ratio of response rates on the two schedules equals the ratio of their reinforcement rates. The matching law was developed stemming from an experiment in which pigeons in an operant conditioning chamber had two keys to peck distributing food pellets at variable interval schedules (Herrnstein, 1961). The keys were available continuously during experimental sessions and pecking was reinforced with two variable-interval schedules, mutually independent and running simultaneously. The relative frequency is obtained by dividing the number of pecks on one key by the sum to both. In the context of operant conditioning this was a concurrent schedule and was clearly a version of the familiar "choice" experiment. This experiment was notable, however, in that is used continuous exposure to alternatives instead of discrete trials and reinforcements came on interval, instead of ratio, schedules. To further discriminate between the two interval schedules, a changeover delay (COD) between response/reinforcement on one schedule and subsequent response/reinforcement on the other schedule is usually employed. This manipulation enhances the relevancy of the concurrent schedules so that the pigeons would tend to alternate between each key after each response, effectively responding on one schedule of reinforcement, a combination of the two schedules is available (Skinner, 1950). The COD has been demonstrated not to be the controlling variable in the finding of matching (Herrnstein, 1970; Shull \& Pliskoff, 1967).

Thus, Herrnstein's first simple equation was biased and summarized the findings such that the distribution of responses on the keys was found to be proportional to the
distribution of reinforcement. This relationship was mathematically expressed as Equation 1.

$$
\begin{equation*}
\mathrm{R} 1 /(\mathrm{R} 1+\mathrm{R} 2)=\mathrm{r} 1 /(\mathrm{r} 1+\mathrm{r} 2) \tag{1.}
\end{equation*}
$$

Where R1 equals rate of responding on one key, R2 equals rate of responding on the other key, r1 equals the rate of reinforcement obtained for R1, and r2 equals the rate of reinforcement obtained for R2. This equations states that the proportion of behavior on a given key matches proportion of reinforcement for responding on that key. Thus, if $40 \%$ of the reinforcers are delivered to Key 1 then $40 \%$ of the organism's behavior will be distributed to that alternative. The matching relationship also can be extend to situations in which an organism responds on more than two schedules (Herrnstein, 1974). This equation describes a line, as diagrammed in Figure 1.


Figure 1. Plot of the matching equation when the proportional rate of behavior equals or matches the proportional rate of reinforcement.

Simple algebraic manipulation of this equation allows one to describe matching behavior in ratio form:

$$
\mathrm{R} 1 / \mathrm{R} 2=\mathrm{r} 1 / \mathrm{r} 2
$$

2. 

Herrnstein (1970) recognized that the quantitative law of effect should not only recognize behavior in which two (or more) responses were controlled by the experimenter, but also when only one response was controlled by the experimenter. For instance, in an operant conditioning chamber in which there is only one key for the pigeon to peck to receive experimenter controlled reinforcement, the pigeon may still "choose" not to peck that key, but to engage in other reinforcing behavior not controlled by the experimenter (i.e., grooming). This can be described by this equation:

$$
\mathrm{R} 1 / \mathrm{R} 1+\mathrm{R} 0=\mathrm{r} 1 / \mathrm{r} 1+\mathrm{r} 0
$$

$$
3 .
$$

where R1 equals rate of pecking on the key (the response manipulated by the experimenter), R 0 equals rate of all other behavior the pigeon could exhibit, rl equals rate of reinforcement obtained by R1, and r0 equals rate of reinforcement obtained for R0. Herrnstein (1970) assumed that $\mathrm{R} 1+\mathrm{R} 2$ was a constant $(\mathrm{k})$, describing the total rate or amount of behavior an organism can exhibit. When substituting $k$ for R1 + R2 and manipulating algebraically, this resulted in an equation that describes a hyperbola, often called Herrnstein's hyperbola:

$$
\begin{equation*}
\mathrm{R} 1=\mathrm{k}(\mathrm{r} 1) / \mathrm{r} 1+\mathrm{r} 0 \tag{4.}
\end{equation*}
$$

$k$ has been demonstrated to vary with respect to reinforcement parameters such as magnitude and immediacy, but this does not affect matching theory's conceptualization of behavior as choice (McDowell, 1986).

Other researchers (Baum, 1974; Baum \& Rachlin, 1969; Dallery, Soto, McDowell, 2005) have shown that the matching law can be expressed in terms of time spent on an alternative. The multiple alternative and single alternative forms of the
matching equation utilizing time as a measure of behavior can be expressed as Equation 5 and 6 respectively. .

$$
\mathrm{T} 1 /(\mathrm{T} 1+\mathrm{T} 2)=\mathrm{r} 1 /(\mathrm{r} 1+\mathrm{r} 2) 5 . \quad \text { and } \quad \mathrm{T} 1=\mathrm{k}(\mathrm{r} 1) / \mathrm{r} 1+\mathrm{r} 0 \quad 6 .
$$

Here proportion of time, $\mathrm{T}_{\mathrm{i}}$, spent on an alternative is equal to the proportional rate of reinforcement for that alternative. This equation permits a specification of choice behavior (human and non-human behavior) when responding is continuous rather than discrete reinforcement. For instance, behaviors like standing, looking at objects, and talking to others can be addressed in this formulation.

Both Staddon (1968) and Baum and Rachlin (1969) ran into deviations from matching using Equation 2. However the data appeared to have reliability that was ordered by considering the ration of responses to the ratio of the rates of reinforcement. Thus, Baum (1974a) reports the matching relationship in terms of ratios.

$$
\mathrm{B} 1 / \mathrm{B} 2=\mathrm{r} 1 / \mathrm{r} 2 \quad 7 .
$$

In this formulation $B_{i}$, represents behavior and $r_{i}$ rate of reinforcement. This equation is algebraically equivalent to Equation 2.

The basis of matching theory's constant-k requirement lies in Herrnstein's assumption that single-alternative responding entails the matching principle (deVilliers \& Herrnstein, 1976; Herrnstein, 1974). The matching principle holds that reinforcement alters the distribution of behavior between response alternatives while the total amount of behavior remains constant. To maintain a stable amount of behavior, increases in responding on one alternative must result in matching decreases in responding to other alternatives. To obtain the single alternative equation from the matching principle (Herrnstein, 1961) Herrnstein's equation related response rate, R, to reinforcement rate, r.

Thus, the specific behavior an individual exhibits at any given time relative to the behavior that the individual could exhibit at that time is proportional to the amount of reinforcement obtained for that specific behavior relative to the amount of reinforcement obtained for the other behavior the individual could exhibit. Behavior and consequences for that behavior are considered within the context in which the behavior occurs. The context may include the various environmental contexts (e.g., behavior the organism could perform, learning history, organism physiology, potential reinforcement in the environment, and contact with the reinforcement).

Matching law and human behavior. Since Herrnstein's classic study, considerable research with animals has confirmed the basic tenets of the matching law (de Villiers, 1977). Several applications with human subjects have further shown that matching theory has generality across species. Pierce and Epling (1983) reviewed the matching literature with human subjects and found that most studies confirmed that humans also distribute their behavior in relation to the rate of reinforcement for response alternatives. Researchers who have failed to observe matching with humans have speculated that some subjects formulate inaccurate rules regarding reinforcement contingencies that may interfere with matching (Lowe \& Horne, 1985).

Matching theory states that a person may exhibit several different behaviors at any given time. However, a person will exhibit the behavior that obtains the highest rate of reinforcement over time, relative to the total amount of reinforcement that is available in the environment at the given time (Herrnstein, 1961). For instance, at any given moment an elementary school student may complete worksheet problems, ask a question of the teacher, leave his or her seat, and talk to a peer, read a book, and so on. According
to matching theory, the student will engage more often in the alternatives whose relative rate of reinforcement is greater and less often in behavior that results in relatively less reinforcement. Thus, descriptive models that include concurrent schedules and matching provides a more comprehensive understanding of behavior in applied settings than the initial matching law formula (Dallery, Soto, McDowell, 2005; Myerson \& Hale, 1984; Pierce \& Epling, 1983). The initial matching theory simply describes relations between rates of reinforcement and responding on concurrent- and single-response alternative schedules. The modern matching law theory takes into account and incorporates deviations from strict matching (i.e., bias and degrees of sensitivity) into the concurrent response rate equations. Dallery, Soto, and McDowell (2005) utilized human participants and monetary reinforcers to find that the modern theory provided an excellent description of reinforced responding accounted for $95 \%$ to $99 \%$ of the total variance in choice responding.

Matching theory provides an interesting structure to applied researchers because it provides a theoretical framework and experimental paradigm that show promise for accommodating the multiple response alternatives that are available to humans in natural situations (Mace, 1994). Although the results of basic research generally support matching theory, its applied value continues to evolve. The overwhelming majority of human matching studies have been conducted under controlled laboratory conditions and with response topographies that are not applied in nature (e.g., lever or key pressing, contrived discussion). Nevertheless, Pierce and Epling (1983), McDowell (1988), and Myerson and Hale (1984), among others, argue that the matching theory may have practical implications that should not be overlooked by applied behavior analysts.

As indicated above, the vast majority of research on concurrent schedules has involved balanced choices (Davison \& McCarthy, 1988) between alternatives that differ only in the rate of reinforcement each alternative produces, while the reinforcers, response manipulation, and delays to reinforcement are held constant. Under these balanced choice conditions, the matching law provides a good description of human choice behavior in laboratory or controlled settings (Pierce \& Epling, 1983). In addition, a handful of applied studies have found socially relevant human behavior subject to concurrent variable-interval (VI) schedules (Conger \& Killeen, 1974; Martens \& Houk, 1989; Martens, Neef, Mace, \& Shade, 1993) and to concurrent variable-ratio (VR) schedules (Mace, McCurdy, \& Quigley, 1990) to be allocated in proportions predicted by the matching law.

In contrast, most choices in applied settings are unbalanced. Fuqua (1984) stated that the nonexistence of procedures to explain comparisons of mismatched behaviors in applied settings is a significant limitation to the matching law. Specifically, Fuqua suggested that laboratory studies often present topographically similar choices, while the potential practical application of the matching law would more likely consists of topographically dissimilar choices. Studies presenting topographically dissimilar choices are limited. Hence, more basic research is needed to determine how different response and reinforcement parameters interact to affect choice. Another problem with applying the matching law to the real world is identifying and measuring the existing schedule of reinforcement. Although researchers can retrospectively determine a schedule of reinforcement that occurred in the natural setting, they are unable to predict the forthcoming schedule of reinforcement. The inability to predict schedules of
reinforcement limits the ability of research and clinicians to alter responding. Thus, the matching law may be limited to describing responding in the natural setting and not used to control or manipulate responding. Similarly, the effects of reinforcement history may be dramatic influences on response behavior.

## The Generalized Matching Law

In the previous formulas, all other variables are assumed to be constant and choice behavior is accounted for based solely on relative rates of reinforcement. However, in applied settings, when given the choice of two behaviors, other variables may influence choice. Mace (1994) highlighted that in applied settings two behaviors may be reinforced with qualitatively different reinforcers. Furthermore, reinforcement for one behavior may be immediate, while reinforcement for competing behaviors may be delayed. The responses themselves may also vary with respect to effort required to complete competing responses.

The generalized matching equation takes those variables into account and represents the relations between the logarithms as ratios. It incorporates other variables beyond relative reinforcement rates that may influence choice behavior and indicates that deviations from strict matching may occur (Baum, 1974, b; Wearden \& Burgess, 1982). Strict matching describes data that conforms to the matching law (single or multiple alternatives) without significant deviation. The generalized matching equation can be expressed as follows:

$$
\log \mathrm{R} 1=\left(\begin{array}{c}
\log \mathrm{r} 1  \tag{7.}\\
-\cdots \\
\mathrm{R} 2
\end{array}\right)+\left(\begin{array}{c}
\log b \\
-\cdots-- \\
\mathrm{r} 2
\end{array}\right)
$$

R1 and R2 represent the rates of responding for Responses 1 and 2, and r1 and r2 are the rates of obtained reinforcement for those responses. The equation also includes two fitted parameters, $s$ and $b$, that permit the fitting of a straight line to the obtained data. These fitted parameters have been conceptualized as representing sensitivity (s) to the schedules and bias (b) toward one of the responses (Baum, 1974). Two types of deviations are commonly observed in matching research: sensitivity and bias (McDowell, 1989). Researchers often refer to the GML as the modern theory of matching, one difference between the formal and modern theories is that they make different assumptions concerning $k$ (Dallery, Soto, \& McDowell, 2005; McDowell, 2005). Specifically, the modern theory retains Herrnstein's assumption that total behavior should remain constant in a given environment. Thus one must start with the foundational equation (Equation 2), make the assumption that $k$ remains constant, and then one can simply derive the remaining equations algebraically. The modern theory of matching permits unbalance in choice and under control or over control of response allocation by reinforcer distribution (McDowell, 2005).

The sensitivity to reinforcement, or $s$, component can account for variations in reinforcement delay, amount, and quality within an experiment (e.g., Dallery, Soto, \& McDowell, 2005; McDowell, 2005; Neef, Marckel, Ferreri, Bicard, Sayaka, Aman, Miller, Jung, Nist, \& Armstrong, 2005; Neef \& Lutz, 2001; Neef, Mace, \& Shade, 1993; Neef, Shade, \& Miller, 1994). Research manipulating either one or two of these sensitivity variables has elicited a mixed outcome related to the impact on response allocation to two concurrent schedules of reinforcement (Dallery, McDowell, \& Soto,

2004; Dallery, Soto, \& McDowell, 2005; Martens, Lochner, \& Kelly, 1992; Neef, et al., 1992). Few attempts at manipulating three variables of sensitivity (delay, amount, and quality) on response allocation have been attempted (Neef et al., 2005; Neef \& Lutz, 2001; Neef, Mace, \& Shade, 1993; Neef, Shade, \& Miller, 1994). Outcome supports individual variations in distribution of behavior across concurrent schedules of reinforcement upon implementation of higher or lower, rate and quality of reinforcement and immediate or delayed access to the reinforcer. Current research has implemented levels of sensitivity variables identified as high or low based on arbitrary thresholds (Mace, Neef, Shade, \& Mauro, 1996; Neef \& Lutz, 2001; Neef, et al., 1993; Neef, et al., 1994). One study conducted by Neef and colleges (2005), a baseline was conducted prior to experimental phases to establish the participant's sensitivity to each dimension in isolation (high vs. lower dimension of sensitivity. This was done to confirm that the participant's responding was sensitive to the richer schedule or favorable level of the dimension. Based on these baseline results, responding was elicited exclusively to the richer schedule for the immediate versus delayed condition suggesting the levels implemented were not a minimal measure of change in response allocation. Other baseline results did not meet or elicit a discrepant change in percent time allocation (e.g., response allocation below an $80-20 \%$ response to one schedule over another). This was exhibited for some participants with reinforcement quality. Few studies have attempted to measure an individual threshold for sensitivity variables prior to manipulation of various sensitivity variables across individuals.

There are two types of sensitivity referred to as overmatching and undermatching. Overmatching refers to when behavior is distributed across relative response alternatives
proportional to relative obtained reinforcement, but to a greater degree than expected by strict matching (Symons et al., 2003). Thus, the organism responds to the richer schedule of reinforcement more than the leaner variable interval schedule, and even more than strict matching would predict.

Undermatching occurs most often when and refers to when behavior is not distributed across relative response alternative proportional to relative obtained reinforcement, but to a lesser degree than expected by strict matching (see Wearden \& Burgess, 1982 for a review). In other words, the organism responds to the leaner variable interval schedule more than the richer variable interval schedules that the matching law would predict. In addition, when responses by an organism alternate between concurrent schedules without regard to the identified schedules of reinforcement (i.e., seemingly random responses), the organism is described as responding with indifference, an extreme form of undermatching.

Bias (c) refers to a preference for one response alternative over another due to response requirements, such as response effort or number of responses (e.g., Mace et al., 1996). Bias is a deviation from strict matching that occurs when the choice between behaviors is not symmetrical (Baum, 1974; McDowell, 1989). At least four sources of bias can be documented: (1) response bias, (2) discrepancy between scheduled and obtained reinforcement, (3) qualitatively different reinforcers, and (4) qualitatively different schedules. Defining all sources of bias are beyond of the scope of this paper, however, in relation to response bias, when two operants appear similar differences may occur in a variety of ways. For instance, one response may require more effort than the other, one may be accompanied by stimuli inherently preferable to the organism (e.g.,
color preferences), or one may be more comfortable than the other, due to factors such as degree and kind of movement (Baum, 1974).

Uneven choice is the result of difference in either response or reinforcement parameters. Response parameters refer to the variables affecting the organism's behavior, such as the difficulty of a response (e.g., amount of effort required to push a lever or pushing a button in an operant chamber), complexity of response, and learning history of the response. Reinforcement parameters refer to variables affecting the value of the reinforcement for the organism, such as latency of reinforcement, amount, type, and duration. When an organism is presented a choice in which there is a difference in either response and/or reinforcement parameters, bias matching will result. The organism will tend to respond on the alternative that requires an easier and/or less complex behavior and/or results in attainment of a higher valued reinforcer (e.g., type, larger amount, smaller latency, longer duration) (Davison \& McCarthy, 1988).

Empirical Studies on Sources of Sensitivity. The Generalized Matching Law (GML) has not received as much empirical attention for describing naturally occurring human behavior. The sensitivity parameters of the GML have important implications for observing and intervening with naturally occurring behavior. For instance, although sensitivity has not been well explained (McDowell, 1989), the sensitivity parameter may describe an individual's sensitivity to differences in value between concurrent reinforcement schedules and/or changes that occur within the reinforcement schedules the individual contacts. For instance, previous research with possums (Trichosurus Vulpecula) has indicated that with possums exposed to three different types of feed response allocation matched changes in reinforcer rate (Bron, Sumpter, Foster, \&

Temple, 2003). Generalizing to humans, one might surmise that organismic differences in individual children (e.g., development, cognition, biology) will impact the child's sensitivity to reinforcement schedules. How sensitive an individual is to the different reinforcement schedules contacted will obviously impact the efficacy of introducing changes in reinforcement for intervention purposes in an applied setting. The GML provides a possible measure of sensitivity and subsequently may be used to design and evaluate more effective introduction of reinforcement-based intervention in the classroom.

Neef, Mace, and colleagues have conducted a series of investigations on the effects of response effort and reinforcement rate, quality, and delay on students' time allocation to concurrently available sets of math problems across clinical and non-clinical populations (Mace, Neef, Shade, \& Mauro, 1994, 1996; Neef, Mace, \& Shade, 1993; Neef, Mace, Shea, \& Shade, 1992; Neef et al., 2005; Neef et al., 1994). In the formal experimental phases, Neef et al. (1992) alternated between phases in which the quality of the reinforcers delivered were equal (either program money or nickels on both schedules) or unequal (program money on the VI 30-s schedule and nickels on the VI 120-s schedule). Time allocation closely matched reinforcement rates (e.g., approximately $80 \%$ of responding on the VI 30-s schedule) when the quality of reinforcement associated with each schedules was the same. However, time allocation shifted toward the leaner schedule when it was associated with a higher quality reinforcer (nickels). That is, each participant displayed a preference for nickels over program money, which resulted in a deviation from matching in which the effects of reinforcer quality overrode the effects of reinforcement rate (time allocation was biased toward the higher quality reinforcer). In
subsequent studies, these investigators have used similar methods to evaluate the effects of (a) reinforcement delay, which (consistent with basic investigations) shifted responding away from matching toward more immediate reinforcement (Neef et al., 1993); (b) problem difficulty, which did not result in a deviation from matching (i.e., time allocation matching rate of reinforcement independent of problem difficulty (Mace et al., 1996); and (c) a variety of adjunction procedures (changeover delays, demonstrations, limited holds, and timers), which were necessary to produce matching as the relative rates of reinforcement available from two concurrent VI schedules were systemically manipulated (Mace et al., 1994). These investigations are noteworthy in that the effects of response and reinforcement parameters on matching and deviations from matching were evaluated naturally, systemically, and across different clinical populations (e.g., students with severe emotion, learning, and behavioral disabilities) using a socially meaningful target response as the dependent variable (math problems).

## Matching Research Conducted with Humans

Research utilized in the development of the matching law theory is primarily based on animal behavior. Experimental research with human subjects can be difficult (Baron, Perone, \& Galizio, 1991; Branch, 1991), due to varying environmental confounds that manifest when translating basic research to applied research. Reliance on animal behavior in basic behavior analysis research is one of the reasons for separation between basic and applied behavior analysis. Some have noted a disconnect between experimental and applied areas of behavior analysis (Mace, 1994; Fuqua, 1984). Lack of an applied approach to research based on methodology and technology makes it difficult to control for the many variables interacting in the natural setting (e.g., sources of reinforcement,
individual behaviors of humans). This may contribute to the mixed results and empirical support across and within studies of the matching law in applied human research. With several studies involving experimental manipulations of reinforcement rates to evaluate matching with human behavior in applied settings (Fisher \& Mazur, 1997), the practical implications in the applied behavior analyst literature is undecided and debatable (Mace, McCurdy \& Quigley, 1990).

Experimental research with humans. Although the results of basic research generally support matching theory, its applied value has yet to be determined. The overwhelming majority of human matching studies have been conducted under controlled laboratory conditions and with response topographies that are not applied in nature (e.g., lever or key pressing, contrived discussion). Exploring a method for employing the matching law in natural, human setting was first attempted by Conger and Killeen (1974) utilizing a video taped discussion group manipulating time spent talking and number of verbal reinforcers given. Observations and results closely approximate the formulation for the matching law theory. Alternatives to the matching law theory have been proposed (Davison \& McCarthy, 1988; McDowell, 1980) but the generalized matching law equation is substantially utilized in the experimental research of applied human choice behavior. The generalized matching law may have practical implications that should not be overlooked by applied behavior analysts. In a review of the literature, Pierce and Epling (1983) evaluated 16 studies, related to human performance on concurrent interval schedules of reinforcement, with 13 supporting the statement that human performance is described by the matching law. However, three studies failed to confirm the matching law as a description of human behavior. These three studies were evaluated by Pierce and

Epling (1983) for limitations in methodology. Pierce and Epling (1983) also noted the profound implications of the matching law at both the individual and social interaction levels. Humans have been found to deviate from strict matching in terms of sensitivity (Dallery, Soto, \& McDowell, 2005; Martens, Lochner, \& Kelly, 1992; Neef, Mace, Shea, \& Shade, 1992; Symons, Hock, Dahl, \& McComas, 2003) and bias (Mace, Neef, Shade, \& Mauro, 1996; Neef, Shade, \& Miller, 1994) similar to deviations from strict matching in non-human research (Soto, McDowell, \& Dallery, 2005). Most recent experimental research has supported the matching law in describing human behavior under concurrent levels of reinforcement (Mace, McCurdy, \& Quigley, 1990).

In an experimental study with all sessions conducted in a classroom setting (Mace, McCurdy, \& Quigley, 1990) two variable ratio (VR) schedules of reinforcement were utilized and varied across two types of reinforcers obtained (e.g., chips or candy) and two academic response alternatives (e.g., division or multiplication) for two special education students (one subject age 16 years and one subject age 12 years) completing mathematic problems. The students exhibited higher rates of one response alternative (e.g., division or multiplication) completed and correct on a richer schedule of reinforcement than on the leaner schedule of reinforcement. This was described by the authors as consistent with the matching law. A more recent experimental study by Mace, Neef, Shade, and Mauro (1994) demonstrated undermatching and bias responding with three adolescent students (one female subject and two male subjects) within the special education classroom on completion and time allocated to math problems. Reinforcers were arranged systematically in separate experimental phases according to three different concurrent variable-interval schedules. Although substantial undermatching and bias
were observed by all subjects, only after adjunct procedures (e.g., changeover delays, limited holds, timers, and demonstrations) were introduced, correlations between relative proportion of time allocation and relative rates of obtained reinforcement accounted for $75-99 \%$ of the total variance. According to the authors these experimental findings extends the literature on human choice and provides a quantitative and practical application of the matching law in educational and clinical settings.

## Applied Research Using the Matching Law with Humans

Matching law in behavioral therapy. Myerson and Hale (1984) address the importance of utilizing the matching law in practical implications. It stated the relationship of matching theory to applied, choice situations. In other words, the efficacy of the intervention depends not only on the schedule value but also on the type of schedule chosen by the behavior analyst and, in addition, on the interaction of the schedule with the schedule that maintains the appropriate behavior. Noll (1995) stressed the applied relevance of the matching law and emphasized the efficacy of behavior therapy is a function of altering relative reinforcement rates. Thus, client behavior changes because behavior therapists alter relative rates of reinforcement such that the richer alternative is chosen. Thus, according to Noll (1995), the matching law affords the behavior therapist a method for quantitatively analyzing the impact of extraneous sources of reinforcement.

A direct application of this approach was utilized by McDowell $(1981,1988)$ on a case of a 10-year-old child exhibiting self-injurious scratching behavior. Researchers identified that self-injurious scratching was under stimulus control as it occurred principally while he and his family were watching television in the living of their home.

In addition, data from experimental assessment using a reversal design showed that the behavior was reinforced by verbal reprimands from family members. Herrnstein's hyperbola accounted for $99.67 \%$ of the variance in the data. This was one of the first cases in providing evidence of the utility of the matching law in describing human behavior in an uncontrolled environment where other factors that might have influenced the behavior had ample opportunity to do so.

A more recent study applying the matching law in a therapeutic setting was utilized by Borrero and Vollmer (2002) with 4 individuals with developmental disabilities a clinical laboratory environment (3 participants) and the participant's home (1 participant). Descriptive observations were completed to identify and record potential reinforcers, problem behaviors, and appropriate behaviors. Multiple sources of reinforcement were evaluated and alternated as test conditions in a multielement design. The attention condition utilized therapist attention diverted for occurrences of problem behavior and was utilized to test whether problem behavior was reinforced by adult attention. The tangible condition was established with the therapist interacting with the participant on a fixed-time (FT) schedule with occurrences of problem behavior resulting in 30 s of access to the preferred tangible item. The tangible condition was designed to test whether problem behavior was reinforced by preferred tangible items. The escape condition instituted removal of instructional materials contingent upon problem behavior to test whether problem behavior was reinforced by escape from the instructed activities. With all data for each subject and all sources of reinforcement were entered into the matching equation, the data points fell close to the matching line indicating adherence to the matching equation. Thus, the allocation of responding between problem behavior and
appropriate behavior matched the relative rate of reinforcement and relative rate of responding. This study demonstrates that when all sources of reinforcement were included in the analysis, the matching law provided an accurate description of response allocation for all participants.

Matching law in an applied school setting. Since the current research project will be utilizing human participants (e.g., elementary-aged children), it is pertinent to address past research conducted using the matching law theory with human participants within the school setting. Billington and DiTommaso (2003) provide a review of matching theory and research along with descriptions of strategies and procedures that can be used to alter students' behavior within classroom environments. This review addressed the generalized matching law with students based on situations beyond relative rate of reinforcement but reinforcement quality, immediacy of the reinforcer and response effort in the classroom. It was noted that results from these studies have important applied implications and that further experimental and applied research should focus on investigating the interaction among reinforcement rates, quality, immediacy, and response effort within the educational environment. Martens and Houck (1989) designed a study to assess the application of the single alternative form of the matching law as a description of students' classroom behavior. The student was an 18-year-old moderately mentally handicapped female in a public high school special education classroom. The student was observed during the morning in a classroom staffed by one teacher and three teaching assistants; a laptop computer was used to record real-time student and staff (teacher and an aide) behavior. Two mutually exclusive and exhaustive of subject behavior (on-task and disruption), and five categories of staff behaviors (instruction,
praise, reprimand, proximity, attend others) were used to represent contact with the subject by other individuals in the classroom. The aide behaviors of instruct, praise, and proximity were identified as reinforcers for disruptive subject behavior, and the aide behavior of instruct was identified as a reinforcer for on-task behavior. The reinforcer values for the student behavior of disruption were used as an estimate of extraneous reinforcement $\left(r^{\circ}\right)$ in the equation that utilized reinforcer values for on-task behavior as the target reinforcer value $\left(\mathrm{r}^{1}\right)$. This was reversed for the matching equation utilizing reinforcer values for disruptive behavior. Using Wilkinson's method (McDowell, 1981), hyperbolas were fitted to the data sets which varied in shapes as a function of extraneous reinforcement as predicted by Herrnstein's law of effect. The single alternative form of the matching equation accounted for $87 \%$ of the variance in the disruptive behavior of the student and $44 \%$ of the variance in the on-task behavior of the student.

In a similar study, Martens, Halperin, Rummel, and Kilpatrick (1990) assessed the application of the single alternative form of the matching equation as an effect of teacher attention contingent on regular classroom behavior. The subject was a 6-year-old male attending a remedial summer school program. On-task and off-task student behavior as well as eight categories of teacher attention were monitored over a two-week period using a computer-assisted observation system. Teacher behavior categories were based on time of behavior (group reading, group instruction, teacher alone) and type of behavior (praise, reprimand, interact, attend others, proximity). Reinforcement was defined as teacher attention contingent on student behavior. Using Wilkinson's method, Herrnstein's hyperbola was fit to the data. The single alternative form of the matching
law accounted for an average $51 \%$ of the variance in on-task student behavior and $47 \%$ of the variance in off-task student behavior.

A similar study utilizing the single alternative form of the matching law in the classroom setting by Martens, Halperin, Rummel, and Kilpatrick (1990) assessed this application as a description of the behavior of a 6-year-old boy in a regular summer school classroom. Observation and coding were completed using a laptop computer. The student behavior coded was mutually exclusive and exhaustive (on-task and off-task). Teacher behavior was coded based on several categories: group reading, group instruction, praise, reprimand, interact, attend others, proximity, and teacher alone. Reinforcement was defined as teacher attention contingent on student behavior. Using Wilkinson's method, Herrnstein's hyperbola was fit to the data. The single alternative form of the matching law accounted for an average $51 \%$ of the variance in on-task behavior of the student and $47 \%$ of the variance in off-task behavior of the student.

In another study by Martens, Lochner, and Kelly (1992), the single-alternative form of the matching law is utilized once again this time to assess variable-interval schedules of social reinforcement contingent on academic engagement with 2 fourthgrade boys (ages 9 and 10 year) in a regular classroom. In the first experiment, four concurrent variable-interval schedules of reinforcement (verbal praise) was delivered by the experimenter in the classroom contingent on the boys "engagement" or on-task behavior. Herrnstein's hyperbola was fit to the data using Wilkinson's method and accounted for $99.1 \%$ and $87.6 \%$ of the variance in academic engagement by the two boys. In a second experiment reported in the same article (Martens, et al., 1992), two 8-year-old male students were exposed to two concurrent variable-interval schedules of
reinforcement (praise) delivered by an experimenter in a classroom contingent on on-task or student academic engagement. An alternating treatments design was utilized to study the discriminability of the schedules and subsequent student behavior. Data obtained under the alternating treatment conditions indicated that the conditions were clearly discriminable for both subjects, suggesting that control over behavior had been established by the reinforcement procedures. It was shown with higher rates of academic engagement occurring for both boys under the richer variable interval schedule of reinforcement, and lower rates of academic engagement occurring under the leaner schedules of reinforcement.

A more recent study utilizing the simple and generalized matching equations were used by St. Peter, Vollmer, Bourret, Borrero, and Sloman (2005) to describe naturally occurring behavior-environment interaction and to assess the likelihood of obtaining spurious matching when relating attention and problem behavior. Participants were three students (one 16-year-old male, one 19-year-old male, and one 14-year-old female) all with varying degrees of developmental disabilities. Matching relations were evaluated from data collect during descriptive observations conducted on the playground and in each participant's classroom. Functional analyses were conducted and identified two behaviors (attention and escape) that were tested for reinforcing effects on inappropriate behavior. In addition, a no-consequence or alone session and control condition were utilized to test for behavioral maintenance related to automatic reinforcement or not providing any programmed consequence for problem behavior. The data for all participants were then analyzed using the simple matching equation and the generalized matching equation. Results demonstrated that spurious matching between problem
behavior and attention resulted in the higher the attention from caregivers the higher the rate of problem behavior. According to the authors, these results provide evidence for the possibility of spurious matching in descriptive observations.

Shriver and Kramer (1997) attempted to utilize the generalized matching law in the classroom setting to assess its utility for quantitatively describing student behavior based on teacher behavior relative to reinforcement allotment. Data was collected using computer-based observational software. This study explored the relative efficacy in the generalized matching law as a descriptive approach to student behavior relative to teacher attention. Participants included 12 first grade and 19 fourth grade students observed using computer-based observational systems occurring in reading group time. Two first and fourth grade students were randomly chosen from each classroom for observational coding of student behavior consisting of: reading aloud, reading silently, writing, listening, transition, waiting, verbal appropriate and inappropriate, task appropriate and inappropriate. Types of teacher attention that may reinforce student behavior were coded as: instruction, listening, approval and disapproval, business management, monitoring, independent work, and off camera. In addition, description of teacher attending was defined in terms of who the teacher was attending to such as: group, target student, peer student, and no attention. Results based on linear regression of the generalized matching law formula accounted for over $70 \%$ of the variance in student behavior in the classroom. This study extended previous matching law research by addressing specific sources of reinforcement and rule-governed contingencies and its ability to describe human performance accurately.

Neef, et al., (1992) took this research topic one step further to assess choice of academic engagement relative to two concurrently available tasks associated with unequal versus equal rates and qualities of reinforcement. Participants were three special education students (two subjects' age 18-years and one subject age 14-years) with experimental conditions within the classroom setting. Participants were asked to complete math problems from two alternative sets on concurrent variable-interval schedules of reinforcement. Both reinforcer rate and quality, manipulated using highquality (nickels) and low-quality items ("program money" in the school's token economy program), were alternated across sessions for both sets of problems. The students exhibited higher rates of math problems completed and completed accurately on the richer schedule of reinforcement than on the leaner schedule of reinforcement. This was addressed by the authors as consistent with an outcomes related to the application of the matching law. This experiment also demonstrated bias responding toward the higher quality reinforcer. This study is important in that it addressed two bias variables related to reinforcement and manipulated those across two concurrently available schedules and qualities of reinforcement.

A similar study by Neef, Shade, and Miller (1994) examined how three reinforcer variables (rate, quality, and delay) along with response effort combined to influence the choices of 6 youths (three males and three females ranging in age from 14 years to 18 years) with learning and behavior difficulties completing two concurrent sets of math problems. During experimental conditions, the two response alternatives varied along two of the four dimensions (reinforcer rate, quality, and delay, and response effort) depending on the condition in effect. Rate of reinforcement was introduced within two
concurrent schedules across three possible variable-interval schedules (VI 30 s , VI 60 s , and VI 90 s). Reinforcer quality was manipulated between two types of reinforcers identified previously as student's relative preference for the reinforcers associated with the two respective problem sets. Reinforcer delay varied based on access to reinforcers earned for the respective set of problems between immediate (at the end of the session) access and delayed (the next day) access. A replication phase was added, during this phase the experimental conditions that resulted in the greatest and lowest percentage of time allocation were reapplied. A result demonstrated that responding of each of the students was differentially affected by the reinforcer dimensions and confirms that choice is an orderly phenomenon governed by the specific sensitivity properties of the response alternatives. This study also provided evidence for the matching law as a description of student behavior in completing math problems. This study contributed to methodology derived from the matching theory for assessing individual responsiveness to variables that collectively affect the value of the reinforcers.

A more recent study by Neef and Lutz (2001), similar to the two studies just described, provided support for the matching law theory as a description of student behavior in completing math problems using a briefer assessment model that was computer based expanding to children diagnosed with emotional and behavioral problems. This study utilized three concurrent variable interval schedules of reinforcement, two different types of reinforcers, immediacy versus delayed access to reinforcement and high and low levels of response effort as determined by the rate and accuracy of samples problems completed during a pretest. Participants were 4 male and 7 female students, ages 9 to 13 years in an urban hospital setting. Results support the
previous study in that the choices of each student were differentially influenced by one or more reinforcer or response dimensions, and relative sensitivity to those dimensions varied across individuals. The study demonstrated the functional utility of the matching law in assessing bias in reinforcement parameters (e.g., quality of reinforcement, rate of reinforcement, delay of reinforcement) and a sensitivity parameter (e.g., response effort) for emotionally disturbed students utilizing a briefer, computer-based assessment.

Future direction for matching law research. Past research on the matching law along with new research surfacing regarding general concerns associated with reinforcing academic behavior can provide a framework for precisely predicting student behavior in the classroom. Learning requires academic engagement-students must be responding (Berliner, 1984; Greenwood, Delquadri, \& Hall, 1984). These responses are sometimes overt and measurable (e.g., homework or independent seatwork) and other times more covert (e.g., paying attention to a demonstration, thinking about a question to ask in class). Regardless, if students can perform these academic behaviors, then responding is a matter of choice (Skinner, Williams, Neddenriep, 2004; Skinner, Wallace, \& Neddenriep, 2002). When students choose to engage in competing behaviors, then educators should attempt to arrange the academic environment to increase the probability of their choosing to engage in assigned or desired academic behaviors (Skinner, 2004). Educators can increase the probability of students' choosing to engage in desired academic behaviors by enhancing the rates, quality, and immediacy of reinforcement for desired behaviors and/or reduce reinforcement for competing behaviors (Skinner, Williams, Neddenriep, 2004). Basic and applied research on choice behavior has provided a framework for precisely predicting student choice behavior in classroom settings. These studies have
shown that choice behavior is relative to reinforcement for desired behaviors (e.g., academic responding) versus reinforcement for competing behaviors (e.g., scribbling in a notebook). Educators who ignore these principles and fail to reinforce desired academic behaviors are less likely to have student choose to engage in desired behaviors. Future research is needed to address the reinforcement variables shown to influence the probability of students' choosing to engage in desired academic behaviors in the classroom setting. Specifically reinforcement variables related to rate and immediacy/delay have been shown to influence student academic behaviors with further research needed addressing these reinforcement variables and a systematic approach to assessing these variable across student academic output.

## Math-Curriculum Based Measurement (M-CBM)

Overview of Curriculum-Based Measurement (CBM). Over the past decade, CBM has gained widespread empirical support as a form of educational assessment (Kame'enui, Francis, Fuchs, Good, et al., 2002). CBM is a set of standardized and specific measurement procedures that can be used to index student performance in the basic academic skill areas of spelling, reading, written expression, and math calculation (Deno, 1985; Deno \& Fuchs, 1987; Fuchs \& Deno, 1991; Shinn, 1989). As a variant of curriculum-based assessment (CBA), CBM uses dynamic uses dynamic indicators in the basic skill areas for making educational decisions such as, screening, instructional planning, and program evaluation (Shinn \& Bamonoto, 1998). When used within a problem-solving model (Deno, 2002), the primary purposes of CBM are to: (a) obtain points of knowledge in basic skills areas and to identify potential areas of academic weakness, and (b) monitor student responsiveness to instruction in a structured manner.

When used to monitor student progress in a structured manner, CBM has demonstrated heightened sensitivity to student change over time (Fuchs, 1986, 1989, 1993) and has been highly accepted as a form of assessing academic skills (Eckert \& Shapiro, 1999; Eckert, Shapiro, \& Lutz, 1995; Shapiro \& Eckert, 1994)

CBM has several distinguishing features. First, CBM assesses student performance and progress towards long term goals. Thus, CBM evaluates general outcomes rather than mastery of skills is a primary distinction of CBM. Fuchs and Deno (1991) referred to CBM as general outcome measurement. For instance, with CBM, alternate forms of short tests were developed to that sample performance toward the longterm goal, not just the content or the skills the student is currently learning (Fuchs, 2004; Stecker, Fuchs, \& Fuchs, 2005). Performance on these measures illustrates what a student is able to do relative to the long-term goal or general outcome. For example, Fuchs and Deno $(1992,1994)$ found that monitoring student's performance on their own curriculum materials was not necessary for meeting technical adequacy or for utilizing CBM procedures appropriately and successfully. Teachers could use reading passages from outside the student's curriculum and still use CBM information effectively to monitor student progress and to make instructional decisions. Overall improvement in reading on a variety of grade-appropriate materials is noted as the general outcome, not just successful reading of other passages from the student's curriculum. Thus a major component of CBM procedures involves the determination of the pool of items or content that reflects the general outcome (Stecker, Fuchs, \& Fuchs, 2005).

A second important feature of CBM is frequent monitoring and graphical depiction of student scores for decision making: typically students are assessed once or
twice weekly with scores plotted on a time series, equal-interval graph (Stecker, Fuchs, \& Fuchs, 2005). Thus, CBM represents decisive assessment, as data reflect how a student performs over a period of time. Because the content or level of difficulty of measures and time allotted for assessment tasks remains constant, student change in performance can be compared across time. In other words, CBM can be used in a predictive fashion to estimate whether students are on target toward meeting long-term goals; however, data can also be used to judge current performance and to determine whether the most recent instructional program has been effective in producing student growth. This decision can be particularly important in special education as CBM can be influential in teacher instructional planning and individualized instruction (i.e., altering instruction to meet an individual student's needs).

A third, critical feature of CBM is its documented technical adequacy. In an assessment methodology to determine a student's academic performance, utilizing measures that are technically sound is important. Research has validated the use of CBM procedures for assessing ongoing student performance and for instructional decisionmaking (see Shinn, 1989). Procedures have been implemented for a variety of elementary-level content and been applied to several academic domains such as: reading, spelling, written expression, mathematical computation, and mathematical concepts and applications. General CBM assessment procedures remain the same even with the use of CBM procedures extending to secondary levels (Busch \& Espin, 2003; Espin \& Tindal, 1998) and early literacy skills (Kaminski \& Good, 1998) and even when the specific content or curriculum varies. Within each academic domain, equivalent forms are used to teachers can determine whether student performance changes over time. Using
cumulative data across multiple assessments can also reduce measurement error and can allow the teacher to judge whether the student appears to be on track toward attaining the long-term goal as well as to make decisions appropriately about the efficacy of the current instructional program.

CBM application in mathematics. Although mathematics was not utilized initially in CBM development, its importance in specifying the distinction as a method for sampling student performance as different as reading and students with disabilities frequently exhibit poor achievement in both reading and mathematics were major factors in applying CBM in mathematics (Stecker, Fuchs, \& Fuchs, 2005). Proficiency in the language of mathematics is becoming an increasingly vital skill for all individuals in today's society. Recent national studies indicated that the current performance of United States students may be such that students will not have the necessary skills to meet the changing demands of the United States workplace. Studies have addressed early mathematics and the use of CBM measures to target early mathematics skills and concepts (Clark \& Shinn, 2004). Shinn (1989) described general procedures for developing CBM assessments of basic computational facts for single-skill tests or simple sets of mixed computational skills by grade level. CBM assessments were developed that represented the most critical computational skills at each grade level. Twenty-five problems were generated for each measure, and problem types were represented in similar proportion to their importance in the state-level curriculum. Problems were assigned in random order on the page, and students were instructed to being with the first item and to complete it if possible and then move to the next item. Students were told they could attempt a problem even if they did not think they could get the entire answer
correct because they could be given partial credit for any part of the answer that was correct. Each digit in the answer was scored as correct as long as it was the correct numeral in the right place (i.e., consideration was given to place value). The total number of correct digits in answers was a more sensitive index of student change than number of problems correct, so digits correct became the datum plotted on the student's graph (L.S. Fuchs, Fuchs, Hamlett, \& Stecker, 1990). The data utilization-rules and decision-making processes were applied to mathematics as were used in reading.

Student progress monitoring can be monitored in mathematics concepts and applications (L.S. Fuchs, Fuchs, Hamlett, Thompson, et al., 1994). Assessments were developed in similar fashion to the computational measures with problems representing critical, grade-level skills in conceptual knowledge/understanding and applications. For example, depending on grade level, assessments may have included items pertaining to money, measurement, word problems, graphs/charts, and geometry. The same problem types were used for each alternate form, and student performance was depicted by the number of points correct in the student's answer. Points were used instead of digits because some items involved selection of the correct answer, such as choosing a line, ray, or line segment, rather than computing answers that contained digits; however, most problems required a numerical response, and one point was assigned to each digit correct in the answer. Despite the increasing popularity for CBM efficiency and accuracy in conducting assessment of academic skills and the pressing need for mathematical proficiency little empirical support has been documented implementing a choice paradigm and measuring its impact on response allocation utilizing M-CBM worksheets.

## Implications for Current Research

This study will attempt to manipulate and measure those reinforcement variables (i.e., reinforcement rate and delay) related to choice behavior. This study expands from traditional research utilizing the generalized matching law and student response allocation (Mace, et al., 1996; Neef, et al., 1994; Neef, et al., 1994; Neef, et al., 1992) such that experimental levels of reinforcement were predetermined and not based on individual-student responding. This study will address the sensitivity variables of reinforcement noted in the generalized matching law and attempt to elicit a sensitivity threshold for rate and immediacy of reinforcement. These two thresholds will then be implemented concurrently to assess the influence on student academic behavior and relative impact one variable (rate or immediacy) may have on the other.

## Statement of the problem and hypotheses

1. Can a threshold be determined for rate of reinforcement?
2. Can a threshold be determined for delay of reinforcement?
3. When placed on a concurrent schedule of reinforcement, does one threshold of sensitivity have more weight (impact) than another?

## Statement of the problem and hypotheses

4. Student response allocation to stimuli increased systematically by rate of reinforcement while maintaining baseline levels of other stimuli will demonstrate a behavioral threshold in which response allocation will be exclusive to one stimulus over the other.
5. Student response allocation to stimuli increased systematically by delay of reinforcement while maintaining baseline levels of other stimuli will demonstrate
a behavioral threshold in which response allocation will be exclusive to one stimulus over the other.
6. When both rate and delay of reinforcement threshold states are implemented concurrently, student response allocation to math problems will be exclusive to one sensitivity variable (i.e., rate or delay) over the other producing a parameter shift.
7. When both rate and delay of reinforcement threshold states are implemented concurrently, student response allocation to math problems will be exclusive to one sensitivity variable (i.e., rate or delay) over the other producing a parameter shift with one impacting response allocation more than the other.

## CHAPTER III

## METHODOLOGY

## Participants and Setting

Five participants (Jodi, James, Mitch, Kenny, and Allen) between the ages of 6 and 11 years served as participants in this study. Four males (James, Mitch, Kenny, and Allen) and one female (Jodi) completed the study. At the time of the study, one participant was in second grade (Kenny), three participants were in third grade (James, Allen, and Mitch), and one participant was in fourth grade (Jodi). Exclusionary criteria for this study were if the child had a previously diagnosed disability (i.e., emotional disturbance, learning disability, and pervasive developmental disorder) prior to the study, if the child was receiving special education services, or was currently being served on an Individual Education Plan. Parent consent and student assent for participation were obtained for all participants in the study. All participants were allowed to complete the research and were provided with all the benefits of participation. Participants were solicited from an after-school program at two different public elementary school sites in Oklahoma. All treatment sessions were conducted in the hallway or a separate classroom within the school building.

## Materials

Consent and Assent Forms. Parents were informed of the research procedures (see Appendix A) and written consent was required before child participation (see Appendix
B). Parents were provided a copy of the consent form (see Appendix C). After written parental consent was obtained, child participants were briefly provided with a description as to what their participation would entail (see Appendix D) and were read the assent form to offer them the opportunity to assent to participate (see Appendix E).

Math worksheets. Math worksheets were generated randomly using math problems based upon identified student instructional range on single calculation skills (viz., addition sums to 9 , multiplication, sums to 81 , or subtraction from 18). Math worksheets were created utilizing a Microsoft Excel spreadsheet specifically configured to generate random numbers for the given problems so that multiple, equivalent math worksheets could be created quickly and easily for repeated use. Problems were presented in vertical format with the number of problems per page dependant upon problem complexity.

Discriminative stimuli. Each individual math problem presented on the math worksheets was associated with one of two visual cues. These cues are designed to serve as a discriminative stimulus $\left(\mathrm{S}^{\mathrm{D}}\right)$ to assist in later discriminating associated contingencies. The visual cue associated with each math problem consisted of either a circle or square designating each problem. These visual cues alternated for each problem presented (viz., circle, square, circle, and square) with circles being presented first in half of the probes and squares being presented first on the other half. For each participant, one cue (either circles or squares) was associated with manipulations in rate while the other cue was associated with manipulations in amount. These were randomly assigned to participants prior to beginning the study. An equal number of circles and squares were present on each math sheet.

Identification of instructional materials. Prior to the initiation of experimental conditions, preliminary sessions were conducted to identify an instructional level mathematics skill for each participating student. During these sessions, an instructional level skill (Fuchs \& Deno, 1982) was determined for each student by calculating the digits correct per minute. Decision rules for participant math skill levels were determined by comparing the participants' performance on the math worksheet (digits correct per minute) to the criteria for direct assessment of math skill levels based on standards from Deno and Mirkin (1977) for frustration, instructional, and mastery levels in Table 1.

Table 1. Criteria for Identification of Instructional Math Material

|  |  | Criterion |
| :--- | :--- | :--- |
| Grade | Level | Median digits correct per minute |
| Grades 1-3 | Frustration | $0-9$ |
|  | Instructional | $10-19$ |
|  | Mastery | $20+$ |
|  |  |  |
| Grade 4+ | Frustration | $0-19$ |
|  | Instructional | $20-39$ |
|  | Mastery | $40+$ |

Standard instructions for administration of a math probe were read to the participant (Shinn, 1989). The participant was given two minutes to complete the math probe. The researcher calculated the digits correct per minute to determine the instructional level of the participant. If the participant skipped problems on the math worksheet or omitted any problems these problems were scored as errors. Obviously, this would inflate the number of incorrect digits per minute and deflate the number of correct digits per minute. It is important to note this deviation, however, because skipping problems usually indicated that a student has mastered only certain skills assessed on the worksheet (Shapiro, 1996). The researcher then compared the participants' number of digits correct to the table above to determine the participants' instructional levels.

Instructional level work was utilized during baseline and treatment phases. Instructional level work for James ( $3^{\text {rd }}$ grader) and Kenny ( $2^{\text {nd }}$ grader) was subtraction from 20. For Mitch ( $3^{\text {rd }}$ grader) and Jodi ( $4^{\text {th }}$ grader), instructional levels were 2 X 2 addition without regrouping. And for Allen ( $3^{\text {rd }}$ grader), instructional level work was identified as multiplication 0 to 9 .

Reinforcer delivery. Students were surveyed about reward preferences using a reward menu (Hishinuma, 2005). Typical items listed on the survey included pencils, pens, erasers, colors, candy, stickers and small toys. Parents had an opportunity to tell the researcher to eliminate any items that were unacceptable from the reward menu. Those items were then placed in a reward box. Upon completion of the math worksheet for all conditions, except baseline, the student was given an opportunity to randomly choose a prize from a reward box. Reward delivery consisted of allowing the student access to the reward box for the purpose of selecting a tangible reward. Items in the reward box varied with each participant and new rewards were added throughout the study to decrease satiation. Utilizing a variety of prizes in the reward box decreased the likelihood of participant satiation to one specific reinforcer. The intent of the reward box was to promote and maintain high levels of participant responding.

## Interobserver Agreement

Researchers totaled the number of problems completed correctly and incorrectly for each of the response alternatives at the end of each session. A second researcher collected interobserver agreement data across experimental conditions for all participants. An agreement for response allocation was defined as both observers recording the same
number of responses to each problem type upon completion of the math probe. The total agreement across all reliability sessions was $100 \%$ agreement.

## Experimental Conditions

Assignment to conditions. Participants were randomly assigned to one of the two conditions in order to counterbalance the order in which reinforcement was manipulated. During the threshold determination phase reinforcement conditions (rate and delay) were counterbalanced by participant with the half of the participants starting with rate threshold determination and the other half beginning with the delay threshold determination condition. One, two, or three 10-minute sessions, depending on how many worksheets the participant wanted to complete, were conducted per day, 3 to 5 days per week, for each participant. The total number of sessions for each participant varied ( $\mathrm{M}=$ 43). The number of sessions completed daily varied based on experimental condition (rate vs. delay). Thus with longer delay sessions ( 10 minutes or 30 minutes) one to two sessions were completed depending on the time allotted. However, during the rate threshold conditions two to three sessions were completed daily with each participant. The average number of problems completed correctly per two minutes from baseline to the final experimental phase varied across each participant (Kenny $=24$, Mitch $=30$, Allen $=33$, James $=28$, and Jodi $=22$ ). Initial $S^{D}$ presentation (circle or square) on each math worksheet was counterbalanced based on initial shape which designated each problem (circle, square, circle square, etc.). In essence the order in which the conditions were manipulated was randomly assigned, in addition to, the determination as to whether the circle was associated with the rate or the delay.

Administration of Math Probe. All conditions were administered consistent with administration of a classroom math probe. Students were read the following, modeled from Witt, Daly, and Noell (2000):

We want to see how many problems you can do correctly in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. (p. 183)

The researcher then gave the math worksheet to the student. The student was instructed to write his or her name on the back of the math probe. Once this was complete the researcher would say, "Start working!" Once the two minutes had elapsed, the researcher said, "Stop working, pencils down, and turn your paper over." The researcher then collected the worksheets and calculated number of problems completed accurately during treatment phases.

Baseline. Baseline phases consisted of the administration of the identified instructional-level single skill math probes with the alternating shape cues (viz., circle, square, circle, square, etc.) designating each problem. Contingencies associated with each cue were identical and consisted of the immediate delivery of reinforcement at a rate of one prize for every 10 correctly completed math problems, contingencies were calculated for circles and squares separately.

This phase was designed to measure response allocation in the presence of circle and square cues under identical conditions and to ensure no preference for circles or squares existed under baseline conditions. The baseline contingency was incorporated
into the math worksheet administration instructions described above and are detailed below. Instructions were adopted from Witt, Daly, and Noell (2000):

We want to see how many problems you can do correctly in 2 minutes. You will start working on the problems when I say ‘START WORKING!’ When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. (p. 183)

The researcher then gave the math worksheet to the student. The researcher then said, "We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others."

The student was instructed to write his or her name on the back of the math probe. Once this was completed the researcher would say, "Start working!" Once the two minutes have elapsed, the researcher said, "Stop working, pencils down, and turn your paper over." Upon completion of the math worksheet, the researcher calculated and recorded the number of circled and squared problems completed correctly for each participant. The total number of problems completed divided by 10 equaled the number of prizes earned. The participant then read the following: "Great job (name). You have earned (number of circled problems completed correctly divided by 10 rounded to the nearest prize) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10 rounded to the nearest prize) prizes for completing (number of squared problems completed correctly) squared math problems. You may
now choose a prize from the prize box." Once the participant had chosen a prize from the box, the researcher read the following: "You may now return to your activity." The participant was then returned to his or her activity.

Rate threshold identification. The point at which a shift in rate of reinforcement impacted choice responding was identified by systematically altering the rate of reinforcement for the rate $S^{D}$, until participant responding matched or shifted to the richer schedule. In this phase, the sensitivity to schedule change was evaluated by increasing the response requirement associated with the participant's rate $S^{D}$. During baseline sessions, the initial rate of reinforcement for both circled and squared problems started at one token for every 10 problems completed correctly. During rate threshold identification sessions, the number of completed problems required to receive reinforcement (prizes) was systematically increased for the rate $S^{D}$ (either circle or square). A base 2 logarithmic scale $(10,12,16,24$, and 40$)$ was used to increase performance requirements for reinforcement with the incremental rate changes occurring only after stable performance was achieved during the previous incremental condition. Problem completion requirement continued to increase until percentage of allocated response shifted to at least an $80 \%-20 \%$ split in favor of the richer schedule. During this condition response in the presence of the delay $S^{D}$ remained identical to that of the baseline condition (i.e. earn immediate access to reinforcement and one token for every ten correctly completed problems).

The purpose of this condition was to determine the minimum amount of change in rate of reinforcement needed to produce a substantial shift in response allocation. In essence, this phase was designed to identify participant sensitivity to changes in rate of
reinforcement. Once the math worksheet was given to the participant, the researcher said, "We want to see how many problems you can do correctly in 2 minutes. You will start working on the problems when I say ‘START WORKING!' When you hear me say ‘Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others."

The number of circled and squared problems completed correctly were calculated and recorded by the researcher. Upon completion of the math probe, participants were read the following: "Great job, (name). You have completed (number of circled/squared problems completed correctly) circled/squared problems so you have earned (number of circled/squared problems completed correctly divided by 10 and rounded to the nearest prize) prizes from the prize box. Also, you have completed (number of squared/circled problems completed correctly) squared/circled problems so you have earned (number of squared/circled problems completed correctly divided by the corresponding base 2 logarithmic scale for that session $(10,12,16,24,40)$ prizes from the prize box." After each participant was given the opportunity to choose their prizes he or she was told to return to their activity.

Delay threshold identification. The point at which a shift in delay of reinforcement impacted choice responding was identified by systematically increasing the delay to reinforcement for the delay $S^{D}$. In this phase, the sensitivity to schedule change was evaluated by increasing the reinforcer delivery delay associated with participant's delay $S^{D}$ until participant responding matched or shifted to the richer schedule. During delay threshold identification sessions, the number of completed problems required to receive reinforcement (prizes) remained the same for both $S^{D}$
conditions ( 10 correctly completed problems for 1 prize), however the delivery of prizes was delayed systematically increasing by 2 minutes, 5 minutes, 10 minutes, and 30 minutes for each session. Delay increments continued to increase until percentage of allocated response shifted to at least an $80 \%-20 \%$ split in favor of the richer schedule. During this condition response in the presence of the rate $S^{D}$ remained identical to that of the baseline condition (i.e. immediate access to reinforcement and one prize for every ten correctly completed problems).

The purpose of this condition was to determine the minimum amount of change in delay of reinforcement needed to produce a substantial shift in response allocation. In essence, this phase was designed to identify participant sensitivity to changes in delay of reinforcement. With implementation of small increments in delay and minimal response shifting, the researcher read the following prior to the administration of the math probe as only a verbal prompt regarding the condition contingencies. "We want to see how many problems you can do correctly in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. You will earn your prizes for (baseline $S^{D}$ condition [squared or circled]) problems now but will have to wait (delay increment condition) minutes to earn your prizes for the (manipulated $S^{D}$ condition [squared or circled]) problems completed." The researcher then calculated and recorded the number of circled and squared problems completed correctly.

Upon completion of the math worksheet participants were told: "Great job, (name). For completing (number of rate $S^{D}$ problems completed correctly) problems you have earned (number of correctly completed rate $S^{D}$ problems divided by 10 and rounded
to the nearest prize) prizes." Once the participant had received those prizes the researcher would begin the timer for the determined amount of delay. No attention was given to the participant. Once the designated delayed time had elapsed the researcher would read the following: "Since you completed (number of delay $S^{D}$ problems completed correctly) problems you have now earned (number of correctly completed delay $S^{D}$ problems divided by 10 and rounded to the nearest prize) prizes." During the shorter delay times (10 minutes or less) participants sat with the researcher with limited verbal exchange and minimal attention given. For the thirty minute delay conditions, the participant was allowed to return to the activity during the delay time. Once the delay time was met, the participant was asked to leave his or her activity to retrieve a prize from the reward box. After the specified delay period was complete, each participant was given the opportunity to choose their prizes from the prize box. He or she was then told to return to their activity.

Rate threshold vs. delay threshold. To determine which has a greater impact on response allocation to math problem the reinforcement thresholds for rate and delay were implemented on a concurrent schedule. Schedule of reinforcement for both variables were implemented based on response allocation and threshold levels determined in previous threshold conditions. For instance, the threshold levels determined for each sensitivity variable (rate and delay) were assigned to the associated $S^{D}$ (circle or square), to determine an impact on percentage of response allocation to math problems. The use of a concurrent schedule of reinforcement allowed for comparisons of two contingencies associated with each cue and its impact on response allocation.

## Experimental Design

An ABACAD reversal design was used to evaluate the impact of the threshold identification procedures as well as the comparisons of rate to delay on response allocation to math problems.

## CHAPTER IV

## FINDINGS

To examine the systematic increase in each sensitivity variable (rate and delay) and its impact on response allocation, the percentage of response allocation was derived after each math probe administration for each of the phase change. These percentages were then graphed with phase change completed after stable performance was achieved during the previous incremental condition. To clearly depict each participant's percentage of response allocation across each threshold determination phase and the comparison phase, Figure 2 represents a simple line graph display for each participant. Overall, based on information provided in Figure 2, threshold levels for response allocation were determined for both rate and delay for each participant. These threshold levels were then directly compared with a majority response allocation to one reinforcement variable over another attained for each participant.

## Research Question 1

Can a threshold be determined for rate of reinforcement?
It was hypothesized that rate threshold could be determined across participants. Threshold was defined as response allocation of eighty-percent or more to one discriminative stimulus when presented with two concurrent schedules of reinforcement rate. Rate thresholds were identified for all participants. Individualized schedules of rate
thresholds were determined. The rate of reinforcement or rate threshold (a response allocation of $80 \%$ or more to one schedule) was elicited and varied across participants. To most clearly depict each participant's percentage of response allocation across each condition, as well as individualized threshold levels for both rate and delay, Figure 2 represents a separate line graph display for each participant.

For James, after an initial baseline phase was completed, the first increment in reinforcement rate was implemented. A response percentage is displayed for each rate increase. Response percentage is represented as responses allocated with total responses divided by the number of manipulated items completed and total responses divided by the number of control items completed multiplied by 100. According to James' data represented in Figure 2, when rate of reinforcement was increased to one reinforcer for every 12 problems completed with the other discriminative stimulus at baseline levels (i.e., one reinforcer for every 10 problems completed) percentage of response allocation did not meet threshold levels with a range of 51 to $52 \%$.

Rate of reinforcement was then increased to 16 problems completed for access to one reinforcer for a specified discriminative stimulus (i.e., circle or square) while the other discriminative stimulus remained at baseline levels. Percentage of response allocation failed to reach threshold levels with response allocation ranging from 50 to $52 \%$. Rate was then increased to 24 problems completed correctly for access to one reinforcer. Response allocation during this phase failed to meet threshold levels with percentage of response allocation for the manipulated stimulus compared to the baseline level stimulus (i.e., one reinforcer for every 10 problems completed correctly) ranging from 50 to $55 \%$. Rate of reinforcement was increased again to one reinforcer for every 40
problems completed correctly. Initially, response allocation failed to shift to the richer schedule with five sessions of percentage response allocation ranging from 50 to $51 \%$. After six sessions within this phase, percentage response allocation shifted to the richer schedule (i.e., one reinforcer for every 10 problems completed correctly) with a range of 92 to $100 \%$ response allocation. Based on James' threshold response levels attained for this phase, a rate threshold of 40 was determined.

According to data for Mitch, the rate threshold condition was completed after the initial baseline phase was completed. Percentage responding is displayed in Figure 2. Response percentages were represented by responses allocated with total responses divided by the number of manipulated items completed and total responses divided by the number of control items completed multiplied by 100. Based on Mitch's data, rate of reinforcement for one discriminative stimulus was increased from 10 problems completed correctly for one reinforcer to 12 problems with failure to obtain threshold with response allocation ranging from 48 to $50 \%$ for the manipulated stimulus. Increase in rate of reinforcement was continued from 12 to 16 and 16 to 24 problems completed correctly before access to one reinforcer with no shift in response allocation and percentage of response allocation for both phase changes ranging from 49 to $50 \%$.

Rate of reinforcement was then increased to 40 problems completed correctly for one reinforcer. The other discriminative stimulus remained at baseline levels (i.e., one reinforcer for every 10 problems completed correctly). Initially, response did not shift to the richer schedule with a range of response allocation from 50 to $52 \%$. After the fourth session within this phase change (rate 40), response allocation shifted to the richer schedule (i.e., one reinforcer for every ten problems completed correctly) with a range of

81 to $96 \%$. Based on these results, the percentage of response allocation was not impacted after baseline and after rate was increased from 12, 16, and 24 problems completed for access to one reinforcer. However, a threshold for rate was established when the rate of responding to obtain a reinforcer was increased to 40 problems for one reinforcer. A threshold for rate was determined for Mitch as 40 problems completed for access to one reinforcer.

According to Allen's data, the rate threshold condition was completed after the initial baseline phase was completed. According to Figure 2, responses allocated with total responses divided by the number of one discriminative stimulus completed and total responses divided by the other discriminative stimulus completed multiplied by 100 and are displayed for each rate increase. Rate of reinforcement was increased to 12 problems completed correctly for access to one reinforcer. Percentage response allocation failed to attain threshold levels with a range of response from 50 to $52 \%$. Rate of reinforcement was increased from 12 to 16 problems completed correctly to obtain one reinforcer for one discriminative stimulus while the other remained at baseline levels. Percentage response allocation failed to meet threshold levels with percentage of response allocation ranging from 51 to $52 \%$. Rate of reinforcement was increased from 16 to 24 problems completed correctly for access to one reinforcer with failure to reach threshold levels with percentage of response allocation less than $80 \%$.

The rate of reinforcement was increased from 24 to 40 problems completed correctly for access to one reinforcer while the other discriminative stimulus remained at baseline levels (i.e., one reinforcer for 10 problems completed correctly). Percentage response allocation immediately shifted with $100 \%$ response allocation to the richer
schedule. Similar to James' and Mitch's results, the percentage of response allocation was not impacted after baseline with incremental increases in rate from rate 12 , rate 16 , and rate 24 . However, a threshold for rate was immediately established when the rate of responding to obtain one reinforcer was increased to 40 problems. Based on this information, threshold for rate was determined for Allen as 40 problems completed for access to one reinforcer.

According to the results analysis for Jodi, the rate threshold condition was completed after the second baseline phase was implemented. For each rate increase, response percentages are displayed as responses allocated with total responses divided by the number of one discriminative stimulus completed and total responses divided by the number of the other discriminative stimulus completed multiplied by 100. For Jodi, rate threshold was determined for Jodi after rate of response for access to the reinforcer was increased to 12. According to the date, rate of response allocation did not immediately shift during this phase with percentage response allocation for the first three sessions at $50 \%$ for the manipulated stimuli. In the fourth session for this phase, percentage response allocation for the richer schedule shifted to $88 \%$. Threshold levels were maintained with stable response allocation toward the richer schedule at 85 and $100 \%$ respectively. Thus, the rate threshold for Jodi was a rate of 12 problems completed for access to one reinforcer.

For Kenny, the rate threshold condition was completed after the second baseline phase was established. Response percentages for each phase change are displayed in figure two. Response percentages are responses allocated with total responses divided by the number of one discriminative stimulus (circles or squares) completed and total
responses divided by the number of the other discriminative stimulus (circles or squares) completed multiplied by 100 . Kenny's results illustrate that percentage of response allocation did not alter when rate was increased from a rate of 12 problems completed correctly for one reinforcer to rate 16 with range of $52 \%$ to $54 \%$ responding across both phases. Rate of reinforcement was then increased from 16 problems completed correctly for one reinforcer to 24 . Percent response allocation did not initially shift with the first session in this phase with responding reaching threshold levels (73\%). Threshold levels were then attained with a range of $95 \%$ to $96 \%$ response allocation to the richer schedule. Thus, threshold for rate was established, with response allocation of $80 \%$ or more to the richer schedule, when rate of response for access to one reinforcer was increased to 24 problems. A rate threshold was obtained for Kenny of 24 problems completed for access to one reinforcer.

## Research Question 2

Can a threshold be determined for delay of reinforcement?
Based on results presented in Figure 2, a delay threshold for James was determined. Once baseline was established, delay of reinforcement was increased with rate of reinforcement maintained for both discriminative stimuli at 10 problems completed correctly for one reinforcer. Initially, reinforcement delay was set at 600 seconds or 10 minutes from time when the contingency (i.e., number of reinforcers earned) was stated to when the reinforcer was administered. The other discriminative stimulus was remained at baseline levels (i.e., immediate access to the reinforcer). Percentage of response allocation for this phase ranged from $43 \%$ to $61 \%$ without threshold levels attained. Reinforcement delay was increased to 1800 seconds or 30
minutes from contingency stated to access to the reinforcer. Threshold was not immediately determined for this phase with percentage response allocation for the first three sessions in this phase ranging from $37 \%$ to $52 \%$. Threshold levels were then determined and maintained with stable response allocation exclusively to the richer schedule (i.e., $100 \%$ response allocation). Based on this data, delay threshold was determined for James as 1800 seconds or 30 minutes.

According to Mitch's data analysis in Figure 2, a threshold was established for delay of reinforcement. After a return to baseline was implemented after the rate threshold phases, the delay threshold condition was implemented with baseline levels (i.e., immediate access to the reinforcer) maintained for the other discriminative stimulus. After phase change was implemented, delay threshold was not immediately established. Delay was implemented at 600 seconds or 10 minutes from time contingency was stated to access to the reinforcer. For the first three sessions in this phase, percentage response allocation did not meet threshold levels and ranged from $49 \%$ to $74 \%$. In the fourth session in this phase, a shift in response to the richer schedule and a delay threshold (80\% or more response allocation) at 600 seconds or 10 minutes delay was established.

Based on information provided in Figure 2, delay threshold was established for Allen. After a baseline phase was implemented delay of access to the reinforcer was incrementally increased for one discriminative stimulus. Immediate access to the reinforcer was maintained for the other discriminative stimulus. After baseline, delay of reinforcement was increased from time contingency was given to time the reinforcer was given to 120 seconds or 2 minutes. Percentage of response allocation during this phase ranged from $48 \%$ to $50 \%$ and threshold levels were not met. Access to reinforcement was
increased to 600 seconds or 10 minutes for one discriminative stimulus with immediate access for the other. Percentage of response allocation did not initially meet threshold levels with the first session in this phase at $53 \%$. After that session, threshold levels were obtained with percent response allocation to the richer schedule or to the immediate access to the reinforcer with percentage in response allocation greater than $80 \%(91 \%$, $94 \%$, and $92 \%$ ). Thus, delay threshold was established with allocation of response to the more immediate reinforcement schedule when delay was increased to 600 seconds or 10 minutes. Response allocation to the immediate reinforcer met threshold levels with $80 \%$ responding or more to the richer schedule.

As represented in Figure 2, a threshold for delay of reinforcement was established for Jodi. Access to the reinforcer was incrementally increased for one discriminative stimulus while the other stimulus remained at baseline levels (i.e., immediate access to the reinforcer). The initial increment in delay of reinforcement was established as 600 seconds or 10 minutes. Percentage of response allocation for this phase did meet threshold criteria on two occasions; however, threshold levels were not maintained and did not represent stable responding. Percent response allocation for this phase ranged from $45 \%$ to $100 \%$. A threshold in responding was not established for this phase. Delay of reinforcement was then increased to 1800 seconds or 30 minutes. Responding to the richer reinforcement schedule (e.g., immediate access to the reinforcer) was immediately established. One-hundred percent response allocation was given to the immediate access stimulus and a delay threshold was obtained for Jodi as 1800 seconds or 30 minutes.

Based on data analysis and information presented in Figure 2, delay threshold was determined for Kenny. Delay to access the reinforcer was increased in increments starting
at 600 seconds or 10 minutes for one stimulus and immediate access (i.e., baseline levels) to the reinforcer for the other stimulus. Percentage of response did not meet threshold levels. Percent responding for this phase ranged from $49 \%$ to $52 \%$. Threshold for delay was not established for delay of reinforcer at 600 seconds or 10 minutes. After stable responding was established, delay of reinforcement was increased to 1800 seconds or 30 minutes. Upon phase change, percent response allocation failed to shift to the richer schedule. For the first four sessions in this phase, response allocation ranged from $43 \%$ to $53 \%$. Response allocation shifted for one session to the more immediate schedule with $73 \%$ of response to the more immediate schedule. For the next session, response then shifted to the delayed scheduled with $32 \%$ response allocation to the immediate access stimulus. Response then shifted, again to the more immediate schedule with $95 \%$ response allocation to the richer schedule (i.e., immediate access to the reinforcer). Response shifted again to the leaner schedule ( $11 \%$ response to the more immediate schedule) before response stabilized for three sessions with a range of response from 50\% to $52 \%$. Kenny then shifted and maintained response allocation exclusively to the richer schedule. Thus, a shift in response to the richer or more immediate reinforcer was established. A delay threshold was established for Kenny as 1800 seconds or 30 minutes.

## Research Question 3

When placed on a concurrent schedule of reinforcement, does one threshold of sensitivity have more weight (impact) than another?

To determine the impact of one sensitivity threshold (rate or delay) on response allocation, a final phase was conducted implementing the rate and delay thresholds determined previously on a concurrent schedule. This phase allowed for the direct
comparison of the two reinforcement variables on response allocation. In addition, when each sensitivity threshold (i.e., rate and delay) were implemented concurrently, this phase would determine if one variable would shift response allocation to at least an 80-20\% split in favor of one schedule over the other variable. According to data provided in Figure 2, a response threshold (i.e., response allocation at or above $80 \%$ ) was established for each participant in this phase. Four out of the 5 participants (James, Mitch, Allen and Jodi), chose to allocate responses towards the delay schedule rather than the rate schedule. Thus, these participants chose to complete fewer problems for an increased rate of reinforcement and wait for access to the reinforcer rather then be allowed immediate access to the reinforcer for completing more problems.

For James, responding immediately shifted to the delay threshold variable ( $100 \%$ ), with continued, stable responding of $80 \%$ or above to the delay variable $(97 \%$, $89 \%, 100 \%$, and $100 \%$ ). According to Mitch's data, responding did not immediately split to one variable over another with $51 \%$ responding to rate and $49 \%$ response allocation to the delay variable. Response allocation then split to $80 \%$ or above to the delay threshold variable ( $97 \%$ ). Responding was maintained towards the delay variable at $100 \%$ for two additional sessions. For Allen, response allocation did not immediately shift to one variable over another with response allocation with the first two sessions of this phase ranging from $49 \%$ to $51 \%$ for both rate and delay. Percentage response allocation then shifted and was maintained to the delay variable with a range of response allocation from $98 \%$ to $100 \%$ for the delay threshold variable. And lastly, according to Jodi's data, response allocation did not immediately shift to one variable or the other for seven sessions with percentage response allocation ranging from $48 \%$ to $52 \%$ for both
variables. Percentage response then shifted with $100 \%$ response allocation to the delay threshold variable.

One participant, Kenny, allotted responses to the rate threshold variable or completing more problems at a decreased rate of reinforcement with immediate access to the reinforcer, rather than responding to the delay variable or completing fewer problems for more reinforcement and waiting for access to the reinforcer. According to Kenny's data, responding did not immediately shift from one variable to the other. The following session, response allocation shifted to the delay variable (100\%). Stable responding was not obtained, response then shifted to the rate variable for two sessions ( $89 \%$ and $96 \%$ ). Responding then shifted to $56 \%$ responding to rate and $44 \%$ responding to delay for one session. Majority responding was then obtained and maintained for the rate variable with percent response allocation at $80 \%, 81 \%, 100 \%$, and $100 \%$.

Overall a majority response allocation ( $80 \%$ or above) was obtained and maintained for one threshold variable (rate or delay) over the other discriminative stimulus for each participant. In the comparison phase (i.e., rate vs. delay), four out of 5 participants allocated response to the delay variable with one participant choosing to respond to the rate variable. Thus, when reinforcement rate and delay were implemented on concurrent schedules, more participants chose to allocate responses to the schedule that allowed more reinforcement for fewer problems with delayed access to the reinforcer. One participant chose to receive less reinforcement for completing more problems with immediate access to the reinforcer.

## Figure Captions

Figure 2. Participants' percentage of response allocation across the rate and delay threshold and rate vs. delay conditions.






## CHAPTER V

## CONCLUSION

The primary purpose of this study was to determine if a threshold response level could be obtained by systematically increasing the rate of reinforcement for one variable while maintaining a baseline level for rate of reinforcement for the other. A secondary purpose was to systematically increase delay of reinforcement for one variable while maintaining an immediate access to reinforcement for the other while measuring its impact on response allocation (percentage of response to the richer schedule). Systematic increases in delay were continued to determine if a threshold level could be obtained. The purpose of the two threshold determination phases was to establish the minimum amount of change in reinforcement rate/delay to produce a shift in response allocation to one over the other. And finally this research sought to evaluate response allocation when presented with two concurrent threshold levels of reinforcement (delay and rate). When presented with these two choice schedules of reinforcement, it was assessed whether percentage of response allocation would shift to one variable schedule (rate or delay) over another. This final comparison phase was completed to determine the impact on response allocation when presented with concurrent schedules of individualized reinforcement parameters. In turn this would provide a more sensitive comparison of two reinforcement variables (rate and delay) at its weakest point in behavior change and its impact on behavior response.

## Summary of Findings

To address these research questions, five participants responded for access to a reward bag across two threshold phases and one comparison phase to determine percentage of response allocation to the manipulated versus control schedule. In addition, concurrent rate and delay schedules were implemented to determine whether responding would shift to one schedule over the other. Each threshold was determined by systematically increasing or manipulating one variable while maintaining baseline or control levels for the other variable. Thus, reinforcer magnitude (rate or delay) on one alternative was increased systematically across sessions, while the magnitude at the other alternative remained at baseline levels. It was increased for one variable until responding was at $80-20 \%$ response to the richer schedule. These thresholds were then introduced on a concurrent schedule to determine response allocation shifted to one sensitivity variable over the other.

This study provides evidence that sensitivity variables from the generalized matching law theory can be manipulated systematically with human participants to determine threshold levels of response allocation for both reinforcement rate and delay. The results also show that when these threshold levels are delivered concurrently a majority of participants chose the delay schedule over rate schedule or more reinforcement for less response with delay rather than immediate access to the reinforcer. Similar to other studies manipulating reinforcement rate and delay (Neef \& Lutz, 2001; Neef, et al., 1992; Neef, et al., 1994), percent response allocation or percent time allocated was used to measure participant responding when given a concurrent reinforcement schedule. However results are contradictory to other studies that
manipulate reinforcement delay and rate (Neef \& Lutz, 2001; Neef, et al., 1993; Neef, et al., 1992) and its impact on response allocation. However, previous studies did not utilize individualized threshold levels and implemented a predetermined level of reinforcement rate and delay.

Rate threshold was determined for each participant. A threshold was determined for rate of reinforcement by systematically increasing the number of problems to complete for access to one reinforcer for one discriminative stimulus, while the other discriminative stimulus remained at baseline levels throughout the threshold phase. Rate of reinforcement was increased according to the number of problems to complete for access to one reinforcer. Rate of reinforcement was increased according to a base 2 logarithmic scale (i.e., 10, 12, 16, 24, and 40) for one discriminative stimulus while rate of reinforcement for the other discriminative stimulus remained at one reinforcer for every 10 problems completed correctly. Rate of reinforcement was increased within the rate threshold phase once stable response allocation was achieved. A threshold level was determined once responding split to an $80-20 \%$ in response allocation to the richer schedule.

Threshold levels for the first rate increment (12 problems completed for one reinforcer) was obtained for one participant (Jodi). Although the threshold level was obtained for the first rate increment, threshold responding was not obtained immediately with the change in rate. Responding for this participant for the first three sessions within the rate threshold phase were at $50 \%$ for both the manipulated and control variables. After the third session, responding split to $88 \%$ response to the richer schedule. Response allocation was maintained at threshold levels for three sessions. For Kenny, rate threshold
levels were met when rate of reinforcement was increased to 24 problems complete for one reinforcer. For the previous reinforcement rate increments (12 and 16) response allocation was between $46 \%$ and $48 \%$ for the richer schedule. After stable responding, rate was increased to 24 problems complete for one reinforcer. Response allocation for the first session in this phase shift did not initially meet threshold levels (73\%). In the next session, threshold levels were met and maintained for four sessions with response allocation for the richer schedule above $90 \%$.

For the remaining three participants (Allen, James, and Allen) rate threshold levels were met when rate of reinforcement was increased to 40 problems completed for one reinforcer. For James and Mitch, threshold levels were not immediately obtained with the shift in reinforcement rate with 5 sessions for Mitch and 6 sessions for James prior to threshold levels obtained (i.e., response allocation greater than $80 \%$ ) and a shift in response to the richer schedule. For Allen, an immediate shift in response allocation ( $100 \%$ response to the richer schedule) was obtained and threshold levels were maintained for 3 sessions until rate of reinforcement was increased from 24 to 40 problems to complete for one reinforcer.

It should be noted that these findings demonstrate threshold levels can be obtained and do vary across participants. These reinforcement rate schedules were increased systematically and introduced concurrently with baseline levels of reinforcement rate creating a choice paradigm. For four out of the five participants, responding did not immediately shift to the richer schedule with the introduction of an increased rate schedule exemplifying that rate of reinforcement was increased in minimal increments to not elicit immediate shifts in response allocation. Given rate of reinforcement relies on
the number of problems completed correctly in two minutes, as rate increased the participant may have received reinforcement for the discriminative stimuli at baseline rate and no reinforcement for the other manipulated variable depending on number of problems completed. This point varied across participants depending upon the number of problems each participant completed in two minutes. Based on the average number of problems completed across conditions, Kenny (24) and Jodi (22) had the lowest average number of problems completed and the lower rate threshold levels (Kenny = Rate 24, Jodi = Rate 12). The other three participants (Mitch, Allen, and James) with higher average number of problems completed all had rate threshold of 40 .

In previous research that reinforcement delay was implemented with other sensitivity variables (reinforcement rate and quality) to determine its impact on response allocation (Neef \& Lutz, 2001a; Neef \& Lutz, 2001b; Neef, et al., 1992; Neef, Shade, \& Miller, 1994), arbitrary levels of immediate versus delayed levels of reinforcement were used. This study was able to determine a response allocation delay threshold for each participant. Delay of reinforcement was defined as the amount of time from when the contingency was given to when the participant was allowed access to the reinforcer. Delay to the reinforcer was increased systematically for one discriminative stimulus according to a base 5 logarithmic schedule ( $5 \mathrm{~min}, 10 \mathrm{~min}, 30 \mathrm{~min}, 1440 \mathrm{~min}$, etc.), while immediate access to reinforcement was maintained throughout the delay threshold phase for the other discriminative stimulus. Delay of reinforcement was increased once stable responding was obtained. A delay threshold was defined as response allocation of $80 \%$ or greater to the richer, more immediate schedule.

Two participants (Mitch and Allen) met threshold levels when delay to the reinforcer was increased to 10 minutes ( 600 seconds). Threshold responding was not immediately obtained for these participants with the increase in delay. For Mitch three sessions were below threshold response levels ( $51 \%, 49 \%$, and $74 \%$ ) and Allen's response when delay was increased to 10 minutes was at $53 \%$ for one session prior to shifting to threshold levels ( $80 \%$ or greater responding to the richer schedule). It is important to note that response levels did not immediately shift when delay of reinforcement was increased. Thus, it can be stated that increases in delay of reinforcement were not increased too much to obtain a true threshold in responding.

For the remaining three participants (James, Jodi, and Kenny), delay threshold was obtained when delay was increased to 30 minutes. For Kenny, an increase in delay of reinforcement allocated a shift in responding. After six sessions of non-threshold responding, threshold levels were met for one session and shifted the next session to the leaner schedule. Responding then returned for three sessions back to non-threshold levels ( $50 \%, 50 \%$, and $48 \%$ ) before shifting to the richer schedule with threshold levels met and maintained for 3 sessions before returning to the baseline phase. For James, upon implementing an increase in delay of reinforcement for one stimulus, threshold levels were not met for three sessions with percent responding at $48 \%, 52 \%$, and $63 \%$ to the richer schedule. Responding then shifted and threshold levels were maintained for three sessions prior to returning to baseline. Jodi did immediately shift upon increasing the delay of reinforcement from 10 minutes to 30 minutes for one discriminative stimulus. The increase in delay was implemented after threshold levels were not met or maintained at 10-minute delay. Upon increasing delay to 30 minutes, response allocation shifted
exclusively to the richer schedule. This responding was maintained for three sessions prior to returning to baseline.

A comparison phase using rate and delay threshold levels obtained in previous phases was completed. Although previous studies have attempted to measure response allocation when comparing two concurrent schedules of reinforcement rate and delay (Neef \& Lutz, 2001a, 2001b; Neef, et al., 1992; Neef, et al., 1994), they have failed to measure response allocation or percent time allocation using individualized schedules of rate and delay of reinforcement within a matching law paradigm. This phase was used to measure if one reinforcement variable (rate or delay) has greater impact on response allocation when threshold level reinforcement schedules are implemented concurrently. For four out of the five participants (Mitch, Allen, Jodi, and James), response was allocated to the delayed schedule. Thus, a majority of participants would wait for access to more reinforcement rather than obtaining less reinforcement immediately. One participant (Kenny) allocated responses to the immediate schedule obtaining less reinforcement.

## Implications for this study

The primary purpose for this study was to systematically manipulate reinforcement rate and delay to determine the minimum amount of change to produce a shift in response allocation to one sensitivity variable over another. This study expanded previous research (Neef, et al., 2005) that used a baseline phase manipulating reinforcement sensitivity variables using arbitrary levels of high/low reinforcement rate and immediate or delayed access to the reinforcer to confirm the participant's responding was sensitive to the richer, more immediate reinforcement schedule. A rate and delay
threshold was determined for each participant. Thus, an individualized threshold level of responding was determined for both reinforcement rate and delay. These rate and delay threshold levels were then implemented concurrently to conclude if response allocation shifted to one schedule over the other. This extends work by (Borrero \& Vollmer, 2002; Dallery, Soto, \& McDowell, 2005; Martens, Lochner, \& Kelly, 1992; Mace, McCurdy, \& Quigley, 1990; Mace, et al., 1994; Symons, Hoch, Dahl, \& McComas, 2003) and others who attempted to use the generalized matching law theory to describe human behavior using predetermined levels of high/low rate of reinforcement and arbitrary levels of immediate versus delayed reinforcement. This study utilizes previous research methodology conducted by ( Neef, et al., 2005; Neef \& Lutz, 2001a, 2001b; Neef, et al., 1992; Neef, et al., 1994) that have implemented sensitivity variables (reinforcement rate and delay) from the matching law theory concurrently to determine its impact on response allocation with human participants. Our results suggest that when reinforcement rate and delay are increased systematically and in small increments response thresholds can be determined. These thresholds offer individualized levels of sensitivity variables (rate and delay) to compare when implemented on a concurrent schedule. This is extends previous research (Neef, et al., 2005; Neef \& Lutz, 2001a, 2001b; Neef, et al., 1992; Neef, et al., 1994) who used arbitrary levels of reinforcement rate and delay to compare its impact on math problem completion as a measurable response allocation.

This study utilized a threshold determination component to determine the most sensitive incremental change in reinforcement rate and delay to produce a shift in response allocation. In addition it provides a direct comparison of two sensitivity variables at their most sensitive or weakest point in behavior change and measured its
impact on response allocation. These determined thresholds were then compared concurrently to determine the impact on response allocation to one variable over another. Results were consistent across participants with a majority of subjects choosing to allocate response to the delay schedule. Thus, immediacy was less important than the amount of reinforcement earned.

Implications for these findings may be based on a learned history of student participants to wait for access to reinforcement or a delayed reinforcement access within the classroom or school setting (i.e., grade on paper, praise by teachers or staff, being called on in class). Other implications for these findings may be related to the age of participants (elementary aged) and willingness to wait for access to the reinforcer rather than the amount of reinforcer earned. Results showed the youngest participant (Kenny) allocated responses to the more immediate schedule while the older participants (Allen, Jodi, Mitch, and James) responded to the delayed schedule with more reinforcement. Future research using pre-kindergarten or early elementary participants may produce different result outcomes.

This contradicts previous studies (Neef \& Lutz, 2001a, 2001b; Neef, et al., 1992; Neef, et al., 1994) not using a threshold level with results demonstrating variable response allocation to one reinforcement schedule over another across participants. This study has implications on completing experimental research with human subjects using the matching law theory.

## Limitations

Generalizability is one limitation to this study. Because all participants came from a rural school district in Oklahoma, caution should be taken when attempting to
generalize these results to all children in grades one through five. It should be stated that a second limitation to this study may be the generalizability of the results to other setting and subjects. Although five subjects in a single-case design can be considered adequate for interpretation, it is important to consider that further replication is needed. Therefore it would benefit researchers to collect additional data using this sample. However, consistent results between subjects used in this study would suggest reasonable generalizability across settings and subjects.

Related to reinforcer delivery, a preference assessment was not completed to determine and measure initial preference and change in preference for one reinforcer over another. However, a reward menu was used to determine and rate possible reinforcer options for each participant's reward bag. Failure to shift response allocation when changes in reinforcement rate and delay were implemented at initial threshold levels may have been impacted by a decrease in reinforcement potency and failure to continually assess preference for the reinforcer.

Although increments in reinforcement rate and delay were set a minimal level, a final limitation for this study may have been implementing delay of reinforcement at an elevated level. This may have produced an immediate shift in response allocation to the more immediate schedule and not a true threshold level was determined. This may have been an implication in response allocation for one participant (Mitch) producing an immediate shift in response allocation and not measuring an individualized threshold level. This places question in determination of threshold levels and whether a true threshold level was determined. Future studies should address this issue by implementing
delay and rate of reinforcement levels at smaller, systematic levels to determine if shifts in response allocation represent definable threshold levels.

Suggestions for Future Research

With these results, future research is needed to substantiate these findings using the matching law theory with human subjects and manipulating reinforcement rate and delay. Future research to address these limitations may include the following expanding participant demographic criteria to assess response allocation with clinical vs. nonclinical populations and older versus younger children. In addition, with any single subject research design additional studies are needed to replicate this design to determine if similar results can be reproduced. Use of a preference assessment component to determine preference for one reinforcer over the other may increase the potency of the reinforcer. This may impact response allocation and elicit a shift in responding to reinforcement rate or delay with more definable threshold levels implemented concurrently. In addition, the use of a preference assessment may be necessary if reinforcement quality is manipulated to measure its impact on response allocation and offer another comparison variable with reinforcement rate and delay. Reinforcement quality was not used in this study based on difficulties defining and measuring reinforcement quality across participants. Using a preference assessment may offer a quantitative and observable measure of defining reinforcement quality for research participants.

And lastly, future studies may address the bias variable or response effort and its impact on response allocation. Determining if a threshold level can be obtained and its impact on response allocation when presented concurrently with other sensitivity
variables (i.e., reinforcement rate and delay) may offer additional research using variable from the generalized matching law theory in an experimental design with human subjects.

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APPENDICES

## APPENDIX A

SITE PARTICIPATION LETTER


My name is Lezlee Greguson and I am a fourth year, doctoral candidate in the school psychology program at Oklahoma State University. I am currently collecting data for my doctoral dissertation and I am in need of participants. This study is intended for children ranging from second to fifth grade and will require a minimal amount of your child's time and effort to complete. In one session, your child will be asked to complete math problems on a worksheet for two minutes. Your child may be asked to complete several sessions at one time totaling no more than 10-15 minutes. Upon completion of each session, your child will be told how many problems correct he or she completed and may be given a prize for his or her work. The total amount of time your child will be asked to work with me will be about 10 to 15 minutes each day. You or your child may choose to not participate in this study at any time and a consent form will be given to you, in addition to, a child assent form read and signed by your child upon approval to participate in this study. Any identifying information will not be elicited nor distributed to unknown individuals. There is minimal risk for your participation in this study with several benefits for your child's participation such as increased exposure to basic math problems and possible increased fluency and accuracy with a targeted math skill.

I appreciate your time and consideration in your child's participation in this study. If you have any further questions or concerns about this study please contact me via email or phone.

Email: lezlee.greguson@ okstate.edu
Phone: 405-744-5474


Lezlee Greguson, M.A.

## APPENDIX B

CONSENT AND ASSENT FORMS

# PARENT PERMISSION FORM 



We appreciate your participation in this research study. Your child will be receiving a math worksheet and given two-minutes to complete as many problems as possible. Upon completion of the two minutes, your child may receive a small incentive for their participation. A list of possible incentives to be given will be given to you prior to starting the research project if there are items that you do not approve your child receive. For instance, please notify the researcher if your child is allergic to any types of small candies or snacks. We are interested in your child's math problem solving performance; however, this study is not intended as a math intervention but additional practice with math facts currently in your child's skill range. Thank you for your child's time and participation.

L, $\qquad$ hereby authorize or direct Lezlee Greguson and/or her research assistants, to perform the following treatment or procedure:

Administer a curriculum-based mathematical probe to my child. My child's participation should take approximately 15 to 20 minutes. My child will be completing three math worksheets, 1-3 times per week for about $8-10$ weeks. I understand that my child's participation is completely voluntary, there is no penalty for not choosing to participate, my child may withdraw from the study at any time with no penalty to me or my child, and that my child's participation and responses will be completely confidential. There is minimal risk or possible discomfort to my child for participating. I understand that only aggregate data are to be used and that my child's individual responses will not be identified. I understand that the researchers will utilize only my child's first name to be used only for the purposes of this study and only the researchers will have access to it. My child's responses will be kept confidential under lock and key in the primary investigator's office. All of my child's responses and math worksheets with my child's first name will be destroyed at the completion of the project. I understand that upon completion of the research study a write-up of results may be given to me upon request. I understand that this study may help applied behavioral analysts, educators, and other professionals who work in the educational and school psychology fields understand student behavior under various reinforcement parameters. I understand that if my child has been previously diagnosed with a psychological disorder (learning disability, emotional disturbance) or is currently receiving special educational services, based on my verbal disclosure of this disability, my child will not participate in this study.

This is done as part of an investigation entitled:

Differential effects of reinforcement rate and delay on response allocation: A systematic approach utilizing the matching law.
I may contact Lezlee Greguson at 744-5474 or lezlee.gregusonapokstate.edu or Gary Duhon, Ph.D. at 744-9436 or duhong@okstate.edu. If I have questions about the research and my rights as a research volunteer, I may contact Dr. Sue C Jacobs, IRB Chair, 219 Cordell North, Stillwater, OK 74078, 405-744-1676 or irb@okstate.edu.

Parental Signature for Minor
I have read and fully understand the consent form. As parent or guardian I authorize
_ (print name) to participate in the described research.

| Parent/Guardian Name (printed) | Date |
| :--- | :---: |
| Signature of Parent/Guardian |  |

I certify that I have personally explained this document before requesting that the participant sign it.
$\overline{\text { Signature of Researcher Date }}$
**Please sign and return this form to your child's after-school program coordinator.

## Child Assent Form

Research Project Title: Differential effects of reinforcement rate and delay on response allocation: A systematic approach utilizing the matching law.

## Read the following sections to the child:

What is the project about?
I want to see how many math problems you can finish on this math worksheet in two minutes.

What will I have to do?
You will have to complete as many math problems as you can in two minutes. You have to work for the entire two minutes and do your best work.

What are the risks of the project?
You will not be hurt in any way by this project. If at any point you don't want to do the project you don't have to and you can stop at any time.

What are the good things about the project?
Working on this project may help you be faster and more accurate with basic math skills in school.

Alternative Procedures:
You don't have to do this if you do not want to. You can stop whenever you want to. You do not have to do anything that makes you feel uncomfortable or sad. No one will be upset with you if you say "no" or if you say "yes" and then change your mind.

You have been told about what will happen during the project.
You have been told what you have to do for the project.
You have been told that you do not have to do this if you do not want to.
You have been told that you can stop whenever you want to.

Signature of Child
Date

Signature of Person Reading and Obtaining Consent

Date

Primary Investigator: Lezlee Greguson, M.A. 405-744-5474

## APPENDIX C

REWARD MENU

## Reward Menu

| Folders | Small Fruit Candies |
| :---: | :---: |
| Bracelets | Erasers |
| Suckers | Key chains |
| Pens | Stickers |
| Gummy Candies | Pencils |
| Small Bouncy Balls | Miniature Puzzles |
| Small Chocolate Candies | Small Notebooks |
| 3-Ring Binders |  |

**** Place an ' $X$ ' next to the items you do not want in the reward box. *** All children will be asked to put their rewards in their backpack until they are picked up to go home.


APPENDIX D
SCRIPTS

## CBA Administration Script

$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
$\qquad$ 3. Read the following instructions:

We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil.
$\qquad$ 4. Instruct the student to write his/her name on the back of the worksheet.
5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
$\qquad$ 7. Calculate the digits correct per minute (DCPM). Take the total number of digits correct and divide by 2.
__8. Record the digits correct per minute below.
$\qquad$ 9. Refer to the CBA cut-off scores below.
$\qquad$ 10. If student is not at instructional level state the following:

Great job (name). We are going to do a few more math problems for only for 2 minutes at a time.
_-9a. Place another math worksheet in front of the student.
_9b. State previous instructions \# 1-9; continue until instructional level is met.
__11. If the student is at instructional level state the following:
Great job (name). You really worked hard for me today. I will be back tomorrow (or next day) and we will do some more fun math problems, okay?
12. Tell the student: You can now return to your activity.

| GRADE/Skill | $\mathbf{1 - 3}$ | Errors | $\mathbf{4 - u p}$ | Errors |
| :---: | :---: | :---: | :---: | :---: |
| Frustration | $<10$ | $\mathbf{8 0 r}>$ | $\mathbf{0 - 1 9}$ | $\mathbf{8 ~ 0 r}>$ |
| Instruction | $10-19$ | $\mathbf{3 - 7}$ | $\mathbf{2 0 - 3 9}$ | $\mathbf{3 - 7}$ |
| Mastery | $>20$ | 20 r $<$ | $\mathbf{2 0}$ | $\mathbf{2 0 r}<$ |


| Skill Level <br> Grade | Skill | DCPM | Errors | Level |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Addition to 9 |  |  |  |
| 2 | Subtraction from 9 |  |  |  |
| 2 | Addition to 19 |  |  |  |
| 2.5 | Subtraction from 19 |  |  |  |
| 3 | 2 X 2 Addition |  |  |  |
| 3 | Multiplication 0 to 9 |  |  |  |
| 4 | Division 0 to 9 |  |  |  |
| 4.5 | $2 \times 2$ Multiplication |  |  |  |
| 5 | $3 \times 2$ Multiplication |  |  |  | Shape First: $\qquad$ Name: $\qquad$

Baseline \#: $\qquad$ ,
$\qquad$

## Baseline Condition Script

$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
$\qquad$ 3. Read the following instructions:

> We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say ‘Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. You will receive one prize for every 10 circled problems and one prize for every 10 squared problems. You will receive those prizes now.
$\qquad$ 4. Instruct the student to write his/her name on the back of the worksheet.
___5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
7. Calculate the number of circled problems completed correctly.
$\qquad$ 8. Record the number of circled problems completed correctly below. Divide by 10.
__9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes now.
___12. Tell the student: You can now return to your activity.
$\qquad$ Square

Name: $\qquad$
Baseline \#: $\qquad$

## Baseline Condition Script

$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
$\qquad$ 3. Read the following instructions:

> We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say ‘Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. For every 10 circled problems completed correctly you will earn one prize. For every 10 squared problems completed correctly you will earn one prize.
$\qquad$ 4. Instruct the student to write his/her name on the back of the worksheet.
___5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
7. Calculate the number of circled problems completed correctly.
$\qquad$ 8. Record the number of circled problems completed correctly below. Divide by 10.
-_9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes now.
__12. Tell the student: You can now return to your activity.

Shape First: $\qquad$ Name: $\qquad$
Rate 12 \#: $\qquad$
Rate 12 CF Condition Script
$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:

> We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say ‘START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.
__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
$\qquad$ 7. Calculate the number of circled problems completed correctly.
$\qquad$ 8. Record the number of circled problems completed correctly below. Divide by 12.
$\qquad$ 9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
$\qquad$ 11. Read the following to the student:

> Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 12 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box.
$\qquad$ 12. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 12=$ $\qquad$
(Round to nearest whole number $=$ number of prizes from the reward box)
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
(Round to nearest whole number = number of prizes from the reward box)
Total Number of Problems Completed Correctly $=\ldots \quad$ (Total)

| Number of Circled problems completed $=\ldots$ |
| :--- |
| prob. completed |


| Number of squared problems completed $=\ldots$ |
| :--- |
| prob. Completed |

$\qquad$ Name:
Rate 12 \#: $\qquad$

## Rate 12 SF Condition Script

$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:


#### Abstract

We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.


__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
_7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 10.
-19. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 12.
$\qquad$ 11. Read the following to the student:

> Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 10 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 12) prizes from the prize box.
__12. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
(Round to nearest whole number = number of prizes from the reward box)
\# of Squared problems completed correctly: $\qquad$ $\div 12=$ $\qquad$
(Round to nearest whole number = number of prizes from the reward box)

Total Number of Problems Completed Correctly = $\qquad$ (Total)

Number of Circled problems completed $=$ $\qquad$ $\div$
 $=$ $\qquad$ * $100=$ $\qquad$ \% of prob. completed

Number of squared problems completed $=$ $\qquad$ $\div \frac{\text { Total }}{\text { Total }}$ $=$ $\qquad$ * $100=$ $\qquad$ \% of prob. Completed
$\qquad$ Rate 16

Shape First: $\qquad$ Circle

Name: $\qquad$
Rate 16 \#: $\qquad$

## Rate 16 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___ 2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.
$\qquad$ 4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by $\mathbf{1 6 .}$
__9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 16 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box.

## __12. Tell the student: You can now return to your activity.

$$
\begin{aligned}
& \text { \# of Circled problems completed correctly: } \quad \div 10=\overline{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }} \\
& \text { \# of Squared problems completed correctly: } \quad \div 16=\overline{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}
\end{aligned}
$$

Total Number of Problems Completed Correctly $=\ldots$
Number of Circled problems completed $=\ldots$
Number of squared problems completed $=\ldots$
$\qquad$
$\qquad$ Square

Name: $\qquad$
Rate 16 \#: $\qquad$
Rate 16 SF Condition Script
$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.
$\qquad$ 4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 10.
__9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 16.
__11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 10 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 16) prizes from the prize box.

## ___12. Tell the student: You can now return to your activity.

\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
(Round to nearest whole number = number of prizes from the reward box)
\# of Squared problems completed correctly: $\qquad$ $\div 16=$
(Round to nearest whole number = number of prizes from the reward box)
Total Number of Problems Completed Correctly $=\ldots$
Number of Circled problems completed $=\ldots$

Number of squared problems completed $=\ldots$$\quad$| (Total) |
| :--- |

Condition: _ Rate 24
Rate 24 \#: $\qquad$
Shape First: __Circle

## Rate 24 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.
__4. Instruct the student to write his/her name on the back of the worksheet.
$\ldots \quad 5$. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 24.
__ 9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
$\qquad$ 11. Read the following to the student:

Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 24 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box.
___12. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 24=$ $\qquad$
\# of Squared problems completed correctly:
(Round to nearest whole number = number of prizes from the reward box)
$\qquad$ $\div 10=$ $\qquad$

Total Number of Problems Completed Correctly $=$ $\qquad$ (Total)


Condition: _ Rate 24
Rate 24 \#: $\qquad$
Shape First: $\qquad$ Square

Name: $\qquad$
Rate 24 SF Condition Script
$\qquad$ 1. Introduce yourself to the student.
$\qquad$ 2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:


#### Abstract

We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.


__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 10.
-19. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by $\mathbf{2 4}$.
$\qquad$ 11. Read the following to the student:

> Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 10 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 24) prizes from the prize box.
__12. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
(Round to nearest whole number = number of prizes from the reward box)
\# of Squared problems completed correctly: $\qquad$ $\div 24=$ $\qquad$
Total Number of Problems Completed Correctly $=\ldots$ (Total)


Condition: _Rate 40
Rate 40 \#: $\qquad$
Shape First: __Circle
Rate 40 CF Condition Script
$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. I have given you another sheet if you need to use it.
___4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 40.
__ 9. Calculate the number of squared problems completed correctly.
_10. Record the number of squared problems completed correctly. Divide by 10.
__11. Read the following to the student:

> Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 40 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box.
$\qquad$ 12. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 40=$
$=\overline{\text { est whole number }=\text { number of prizes from the reward box) }}$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$
$=\overline{\text { est whole number }=\text { number of prizes from the reward box) }}$

Total Number of Problems Completed Correctly $=\square$ (Total)


Condition: Rate 40
Rate 40 \#: $\qquad$
Shape First: Square $\qquad$
Rate 40 SF Condition Script
$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. I have given you another sheet if you need to use it.
___4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
__6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 10.
_-9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 40.
__11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 10 and rounded to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 40) prizes from the prize box.
___12. Tell the student: You can now return to your activity.

$$
\begin{aligned}
& \text { \# of Circled problems completed correctly: } \quad \div \div 10=\overline{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }} \\
& \text { \# of Squared problems completed correctly: } \quad \div 40=\overline{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}
\end{aligned}
$$

| Total Number of Problems Completed Correctly $=\ldots$ |  |  |
| :---: | :---: | :---: |
| Number of Circled problems completed $=$ |  | \% of prob. completed |
| Number of squared problems completed $=$ | Total | \% of prob. Completed |

$\qquad$
$\qquad$ Name:

## Delay 120 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil.
__4. Instruct the student to write his/her name on the back of the worksheet.
$\ldots 5$. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__ 7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 10.
_-9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student: HAND OUT PRIZES FOR SQUARE ONLY
Great job (name). You have earned (number of circled problems completed correctly divided by 10 ) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for SQUARED problems now. You will have to wait for the prizes earned from the CIRCLED problems.
___12. Begin timer for 120 seconds ( 2 minutes).
___ 13. Once 120 seconds has elapsed state the following:
Since you completed (number of CIRCLED problems completed correctly) problems you have now earned (number of correctly completed CIRCLED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.

$$
\begin{aligned}
& \text { \# of Circled problems completed correctly: } \quad \div \div 10=\overline{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }} \\
& \text { \# of Squared problems completed correctly: } \frac{\div 10=}{\text { (Round to nearest whole number = number of prizes from the reward box) }}
\end{aligned}
$$

Total Number of Problems Completed Correctly $=\ldots$ (Total)

| Number of Circled problems completed $=$ |  | * $100=$ | \% of prob. completed |
| :---: | :---: | :---: | :---: |
|  | Total |  |  |
| Number of squared problems completed $=$ |  | * $100=$ | \% of prob. Completed |
|  | Total |  |  |

Condition: __Delay 120 Delay 120
Delay 120 \#: $\qquad$ Shape First: $\qquad$
$\qquad$ Name: $\qquad$

## Delay 120 SF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil.
__4. Instruct the student to write his/her name on the back of the worksheet.
___5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 10.
_-9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
__11. Read the following to the student: HAND OUT PRIZES FOR CIRCLE ONLY
Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for CIRCLED problems now. You will have to wait to earn the prizes for completing the SQUARED problems.
$\qquad$ 12. Begin timer for 120 seconds (2 minutes).
$\qquad$ 13. Once 120 seconds has elapsed state the following:

Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) prizes and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
(Round to nearest whole number $=$ number of prizes from the reward box)
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$

Total Number of Problems Completed Correctly $=$ $\qquad$ (Total)
Number of Circled problems completed $=$ $\qquad$ $\div$
 $-$ $\qquad$ * $100=$ $\qquad$ \% of prob. completed

Number of squared problems completed $=$ $\qquad$ $\div \frac{\text { Total }}{\text { Total }}=$ $\qquad$ * $100=$ $\qquad$ \% of prob. Completed
$\qquad$
$\qquad$ Name:

## Delay 600 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil.
__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__ 7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 10.
-19. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student: HAND OUT PRIZES FOR SQUARE ONLY
Great job (name). You have earned (number of circled problems completed correctly divided by 10 ) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for SQUARED problems now. You will have to wait for the prizes earned from the CIRCLED problems.
__12. Begin timer for 600 seconds ( 10 minutes).
___ 13. Once 600 seconds has elapsed state the following:
Since you completed (number of CIRCLED problems completed correctly) problems you have now earned (number of correctly completed CIRCLED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.

$$
\begin{aligned}
& \text { \# of Circled problems completed correctly: } \frac{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}{\div 10=} \\
& \text { \# of Squared problems completed correctly: } \frac{\div 10=}{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}
\end{aligned}
$$


$\qquad$
$\qquad$ Name:

## Delay 600 SF Condition Script

$\qquad$ 1. Introduce yourself to the student.
___2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. You will earn the prizes for circled problems completed now, but you will have to wait for the prizes for completing squared problems.
_4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
_7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 10.
-_9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
__11. Read the following to the student: HAND OUT PRIZES FOR CIRCLE ONLY
Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for CIRCLED problems now. You will have to wait to earn the prizes for completing the SQUARED problems.
_12. Begin timer for 600 seconds ( 10 minutes).
__ 13. Once 600 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) prizes and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.

$$
\begin{array}{ll}
\text { \# of Circled problems completed correctly: } \quad \div \div 10= \\
\text { \# of Squared problems completed correctly: } & \div \div 10= \\
\text { (Round to nearest whole number }=\text { number of prizes from the rearest whole number }=\text { number of prizes from the reward box) }
\end{array}
$$


$\qquad$ Name:

## Delay 1800 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:

> We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. You will earn your prizes for squared problems now but will have to wait 30 minutes to earn your prizes for the circled problems completed.
_-4. Instruct the student to write his/her name on the back of the worksheet.
___ 5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
8. Record the number of circled problems completed correctly below. Divide by 10 .
__ 9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10 .
__11. Read the following to the student: HAND OUT PRIZES FOR SQUARE ONLY
Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for SQUARED problems now. You will have to wait for the prizes earned from the CIRCLED problems.
___12. Begin timer for 1800 seconds ( 30 minutes).
$\qquad$ 13. Once 1800 seconds has elapsed state the following:

Since you completed (number of CIRCLED problems completed correctly) problems you have now earned (number of correctly completed CIRCLED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.

$$
\begin{aligned}
& \text { \# of Circled problems completed correctly: } \quad \underset{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}{\div 10=} \\
& \text { \# of Squared problems completed correctly: } \quad \underset{\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }}{\div 10=}
\end{aligned}
$$


$\qquad$
$\qquad$ Name: $\qquad$

## Delay 1800 SF Condition Script

$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:


#### Abstract

We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. You will earn your prizes for circled problems now but will have to wait 30 minutes to earn your prizes for the squared problems completed.


__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
___7. Calculate the number of circled problems completed correctly.
___8. Record the number of circled problems completed correctly below. Divide by 10 .
-9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10 .
___11. Read the following to the student: HAND OUT PRIZES FOR CIRCLE ONLY

> Great job (name). You have earned (number of circled problems completed correctly divided by 10) prizes for completing (number of circled problems completed correctly) circled problems correctly. You have also earned (number of squared problems completed correctly divided by 10) prizes for completing (number of squared problems completed correctly) squared math problems. You may choose those prizes earned for CIRCLED problems now. You will have to wait to earn the prizes for completing the SQUARED problems.
$\qquad$ 12. Begin timer for 1800 seconds ( 30 minutes).
$\qquad$ 13. Once 1800 seconds has elapsed state the following:

Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) prizes and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\overline{\text { arest whole number }=\text { number of prizes from the reward box) }}$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$
(Round to nearest whole number = number of prizes from the reward box)

$\qquad$ Circle

Name: $\qquad$
Rate 12 vs. Delay 600 \#:

## Rate 12 vs. Delay 600 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will be getting one prize for every 12 circled problems and you will get to pick your prize immediately. You will get one prize for every 10 squared problems and will have to wait for 10 minutes to get your prize.
___4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
___ Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 12.
__9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
$\qquad$ 11. Read the following to the student:

Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 12 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
$\qquad$ 12. Begin timer for 600 seconds ( 10 minutes).
$\qquad$ 13. Once 600 seconds has elapsed state the following:

Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
$\qquad$ 14. Tell the student: You can now return to your activity.
problems completed correctly: $\qquad$ $\div 12=$ $\qquad$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$

$\qquad$ Name:
Rate 12 vs. Delay 600 \#:

## Rate 12 vs. Delay 600 SF Condition Script

$\qquad$ 1. Introduce yourself to the student.
_2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will be getting one prize for every 12 squared problems and you will get to pick your prize immediately. You will get one prize for every 10 circled problems and will have to wait for 10 minutes to get your prize.
__4. Instruct the student to write his/her name on the back of the worksheet.
__ 5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
___ Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 12.
__9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job, (name). You have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 12 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for squared problems now, but you will have to wait for your circle problems.
$\qquad$ 12. Begin timer for 600 seconds ( 10 minutes).
$\qquad$ 13. Once 600 seconds has elapsed state the following:

Since you completed (number of CIRCLED problems completed correctly) problems you have now earned (number of correctly completed CIRCLED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
___ 14. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 10=$ $\qquad$
\# of Squared problems completed correctly: $\qquad$ $\div 12=$ $\qquad$

Total Number of Problems Completed Correctly $=\ldots$ (Total)
Number of Circled problems completed $=$ $\qquad$ $-\frac{}{\text { Total }}=$ $\qquad$ * $100=$ $\qquad$ \% of prob. completed $\div \frac{\text { Total }}{\text { Total }}$ $\qquad$ * $100=$ $\qquad$ $\%$ of prob. Completed
$\qquad$ Circle

Name:
Rate 16 vs. Delay 600 \#: $\qquad$ -

## Rate 16 vs. Delay 600 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
__2. Place the math worksheet in front of the student face down.
_3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will be getting one prize for every 16 circled problems and you will get to pick your prize immediately. You will get one prize for every 10 squared problems and will have to wait for 10 minutes to get your prize.
__4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
_7. Calculate the number of circled problems completed correctly.
_8. Record the number of circled problems completed correctly below. Divide by 16.
-19. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:

> Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 16 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
_12. Begin timer for 600 seconds (10 minutes).
___ 13. Once 600 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.

## 14. Tell the student: You can now return to your activity.

\# of Circled problems completed correctly: $\qquad$ $\div 16=$ $\qquad$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$

$\qquad$ Date: $\qquad$

## Rate 24 vs. Delay 600 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will be getting one prize for every 24 circled problems and you will get to pick your prize immediately. You will get one prize for every 10 squared problems and will have to wait for 10 minutes to get your prize.
__4. Instruct the student to write his/her name on the back of the worksheet.
$\ldots \quad 5$. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
_7. Calculate the number of circled problems completed correctly.
8. Record the number of circled problems completed correctly below. Divide by 24 .
_-9. Calculate the number of squared problems completed correctly.
___10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 24 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
__12. Begin timer for 600 seconds ( 10 minutes).
___ 13. Once 600 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
14. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 24=$ $\qquad$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$


Condition: Rate 24 vs. Delay 1800
Shape First: __Circle
$\qquad$
$\qquad$ Name: Date: $\qquad$
$\qquad$
Rate 24 vs. Delay 1800 CF Condition Script
$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will be getting one prize for every 24 circled problems and you will get to pick your prize immediately. You will get one prize for every 10 squared problems and will have to wait for 30 minutes to get your prize.
__4. Instruct the student to write his/her name on the back of the worksheet.
___5. Get your stopwatch ready and say: "Start working."
6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
_7. Calculate the number of circled problems completed correctly.
8. Record the number of circled problems completed correctly below. Divide by 24 .
_-9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
___11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 24 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
__12. Begin timer for 1800 seconds (30 minutes).
___ 13. Once 1800 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
14. Tell the student: You can now return to your activity.
\# of Circled problems completed correctly: $\qquad$ $\div 24=$ $\qquad$
\# of Squared problems completed correctly: $\qquad$ $\div 10=$ $\qquad$


Condition: __Rate 40 vs. Delay 600 $\qquad$ Shape First: __Circle $\qquad$ —

Name: $\qquad$
Date: $\qquad$

## Rate 40 vs. Delay 600 CF Condition Script

$\qquad$ 1. Introduce yourself to the student.
2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:

> We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others.
_4. Instruct the student to write his/her name on the back of the worksheet.
___5. Get your stopwatch ready and say: "Start working."
___6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
7. Calculate the number of circled problems completed correctly.
8. Record the number of circled problems completed correctly below. Divide by 40.
_19. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
__11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 40 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
_12. Begin timer for 600 seconds (10 minutes).
___ 13. Once 600 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
___ 14. Tell the student: You can now return to your activity.

$$
\begin{array}{ll}
\text { \# of Circled problems completed correctly: } & \div 40= \\
\text { \# of Squared problems completed correctly: } & \text { (Round to nearest whole number }=\text { number of prizes from the reward box) } \\
& \div 10= \\
\text { (Round to nearest whole number }=\text { number of prizes from the reward box) }
\end{array}
$$



Condition: __Rate 40 vs. Delay 1800
Rate 40 vs. Delay 1800 \#:
$\qquad$
__ 1. Introduce yourself to the student.
__2. Place the math worksheet in front of the student face down.
__3. Read the following instructions:
We want to see how many problems you can do in 2 minutes. You will start working on the problems when I say 'START WORKING!' When you hear me say 'Stop working,' you will need to immediately turn your paper over and quietly put down your pencil. It's okay to skip around the paper and complete some problems and not others. You will get one prize for every 40 circled problems completed correctly immediately and you will get one prize for every 10 squared problems but you will have to wait 30 minutes to received those prizes.
__4. Instruct the student to write his/her name on the back of the worksheet.
__5. Get your stopwatch ready and say: "Start working."
_6. Begin timer. Once two minutes has elapsed say, "Stop working, please put your pencil down."
__7. Calculate the number of circled problems completed correctly.
__8. Record the number of circled problems completed correctly below. Divide by 40.
_-9. Calculate the number of squared problems completed correctly.
__10. Record the number of squared problems completed correctly. Divide by 10.
__11. Read the following to the student:
Great job, (name). You have completed (number of circled problems completed correctly) circled problems so you have earned (number of circled problems completed correctly divided by 40 DO NOT round to the nearest token) prizes from the prize box. Also, you have completed (number of squared problems completed correctly) squared problems so you have earned (number of squared problems completed correctly divided by 10) prizes from the prize box. You may choose your prizes for circled problems now, but you will have to wait for your square problems.
__12. Begin timer for 1800 seconds (30 minutes).
__ 13. Once 1800 seconds has elapsed state the following:
Since you completed (number of SQUARED problems completed correctly) problems you have now earned (number of correctly completed SQUARED problems divided by 10 and rounded to the nearest ones) and you may choose those prizes now.
14. Tell the student: You can now return to your activity.

| \# of Circled problems completed correctly: | $\div 40=7$ |
| :--- | :--- |
| \# of Squared problems completed correctly: | (Round to nearest whole number = number of prizes from the reward box) |
|  | $\div 10=$ |
| (Round to nearest whole number = number of prizes from the reward box) |  |



## APPENDIX E

## MATH WORKSHEETS








## APPENDIX F

 IRB APPROVAL LETTER
# Oklahoma State University Institutional Review Board 

| Date: | Wednesday, September 20,2006 |  |  |
| :---: | :---: | :---: | :---: |
| IRB Application No | ED06175 |  |  |
| Proposal Tite: | Differential Effects of Reinforcement Rate and Delay on Response Allocation: A Systematic Approach Utilizing the Matching Law |  |  |
| Reviewed and Processed as: | Exempt |  |  |
| Status Recommended by Reviewer(s): Approved |  | Protocol Expires: | 9/19/2007 |
| Principal Investigator(s |  |  |  |
| Lezlee Greguson 434 Willard | Gary J Duhon 423 Willard |  |  |
| Stillwater, OK 74078 | 8 Stillwater, OK 74078 |  |  |

The IRB application referenced above thas been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manher consistent with the IRB requirements as outlined In section 45 CFR 46.
(The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

As Princlpal Investigator, it is your respons|blility to do the following:

1. Conduct this study exactly as It has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval.
2. Submit a request for continuation if the study extends beyond the approval period of one calendar year. This continuation must recelve $\mathbb{R B}$ review and approval bafore the rasearch can continue.
3. Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of this research; and
4. Notify the IRB office in writing when your research project is complete

Please note that approved protocols are subject to monltoring by the IRB and that the IRB office has the authority to inspect research records associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact Beth McTernan in 219
Cordell North (phone: 405-744-5700, beth.moternan@okstate.edu).

Sue C. Jacobs, Enair
Sue C. Jacobs, Chair
Institutional Review Board

VITA

Lezlee Greguson
Candidate for the Degree of
Doctor of Philosophy

# Thesis: DIFFERENTIAL EFFECTS OF REINFORCEMENT RATE AND DELAY ON RESPONSE ALLOCATION: A SYSTEMATIC APPROACH UTILIZING THE MATCHING LAW 

Major Field: Educational Psychology
Option: School Psychology
Biographical:
Personal Data: Born in Sioux Falls, South Dakota on May 18, 1979, the daughter of Paul and Linda Greguson

Education: Graduated from Watertown Senior High School, Watertown, South Dakota in May 1997; received Bachelor of Arts degree in Psychology and a minor in Child Development from the University of Nebraska, Lincoln, Nebraska in 2001; received a Master of Arts degree in Clinical Psychology from Minnesota State University, Mankato, Minnesota in May 2003; Completed the requirements for the Doctorate of Philosophy degree with a major in Educational Psychology and option in School Psychology at Oklahoma State University in July, 2008.

Experience: Completed an independent research study at Minnesota State University that compared disabled and non-disabled children's self-concept and their parents' perception of their child's self-concept under the supervision of Dr. Nancy Fenrick; completed a clinical practicum placement at Munroe-Meyer Institute in Omaha, Nebraska; employed by Oklahoma State University, Department of Educational Psychology as a graduate teaching assistant and senior clinic supervisor; completed schoolbased and clinic-based practicum placements in intervention, consultation, and assessment at Oklahoma State University; completed a pre-doctoral internship at Munroe-Meyer Institute in Omaha, Nebraska as part of the Nebraska Internship Consortium in Professional Psychology.

Professional Memberships: American Psychological Association, National Association of School Psychologists, Association of Behavior Analysis International, Nebraska Psychology Association

# of Study: DIFFERENTIAL EFFECTS OF REINFORCEMENT RATE AND DELAY ON RESPONSE ALLOCATION: A SYSTEMATIC APPROACH UTILIZING THE MATCHING THEORY 

Pages in Study: 134
Candidate for the Degree of Doctor of Philosophy
Major Field: Educational Psychology
Scope and Method of Study: This study measured those reinforcement variables related to choice behavior and its impact on response allocation. This study addressed the two sensitivity variables of reinforcement (rate and delay) in matching law theory and attempted to elicit a sensitivity threshold for rate and immediacy of reinforcement. These two thresholds were then implemented concurrently to assess the influence on response allocation and the relative impact one variable (rate or immediacy) may have on the other. Five participants (four males and one female), between the ages of 6 and 11 years served as participants in this study. Percentage of math problems completed was used to measure response allocation. Each individual math problem presented on the math worksheets was associated with one of two visual cues. For each participant one cue (either circles or squares) was associated with a manipulation in rate or delay while the other cue was associated with no manipulation. Contingencies associated with each cue were randomly assigned to participants prior to beginning the study. Baseline phases were implemented in which contingencies associated with each cue were identical and consisted of the immediate delivery of reinforcement at a rate of one token for every 10 correctly completed math problems. For rate and delay threshold conditions, one cue or discriminative stimuli remained at baseline levels while the other systematically increased until percentage of allocated response shifted to at least an $80 \%-20 \%$ split in favor of the richer schedule. For the rate versus delay condition, both rate and delay thresholds were implemented concurrently to determine its impact on response allocation.

Findings and Conclusions: Results revealed that individualized rate and delay threshold could be determined and varied across participants. When comparing rate and delay thresholds, four to five participants chose to allocate response to the delay variable while one participant chose to allocate response to the rate variable. Thus, when reinforcement rate and delay were implemented on concurrent schedules, more participants chose to allocate responses to the schedule that allowed more reinforcement for fewer problems with delayed access to the reinforcer. One participant chose to receive less reinforcement for completing more problems with immediate access to the reinforcer.

ADVISER'S APPROVAL: Gary J. Duhon, Ph.D.

