SIMPLIFYING ACTIVITY-BASED COSTING

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CHAPTER 1

INTRODUCTION

According to Argyris and Kaplan (1994), activity-based costing (ABC) is a costing model created in the mid-1980s that provides more accurate information to managers about the cost and profitability of their business processes, products, services, and customers. ABC provides more accurate cost information by exploiting causal relationships. This is made possible by recognizing that activities consume resources while cost objects (products, customers, etc.) consume activities. Thus, the cost of resources must be first assigned to activities (Stage 1 cost assignment), and then the cost of activities is assigned to cost objects (Stage 2 cost assignment).

While ABC is simple in concept, it is complex and costly to implement and operate. An organization must identify and find information for all resources, activities, and their associated drivers, which can number into the hundreds. Consequently, although ABC provides greater accuracy, ABC systems are not as widely adopted as might be expected because of their size, complexity, and cost (Krumwiede 1998a, 1998b; Kaplan and Anderson 2007a). Early attempts to simplify ABC focused on reducing the number of activities and drivers used while attempting to minimize the loss in accuracy (Babad and Balachandran 1993; Homburg 2001). In effect, size and some complexity issues were reduced at the expense of accuracy. These simplified systems also considered the costs to gather information for each activity/driver. However, these attempts required a full implementation of ABC before the simplification could occur. This meant that all activities and drivers had to be identified before the simplification could be done (after-the-fact simplification). If a full implementation must take place, the value of the simplification is questionable.

The next major simplification effort is more recent and is a before-the-fact simplification. Kaplan and Anderson (2004, 2007a) detail the complexities and costs of ABC. In general, they observe that ABC systems are expensive to build, complex to sustain, and difficult to modify or update. Specifically, they identify the following problems associated with ABC: (1) a time-consuming and costly interviewing and surveying process is required to identify activities and the resource drivers needed to assign resource costs; (2) since subjectivity is involved in assessing the time spent on various activities, it is difficult to validate the Stage 1 cost assignments; (3) data are expensive to store, process, and report; (4) it is difficult to update the ABC model to accommodate changing circumstances; and (5) the ABC model ignores the potential for unused activity capacity.

To address these problems, Kaplan and Anderson (2004, 2007a, 2007b) developed a simplified ABC system called Time-Driven ABC (TDABC). TDABC simplifies Stage 1 by devising a simpler and less time-consuming approach to assigning resource costs to activities. TDABC provides an easy way to update the ABC model as circumstances change and only assigns the cost of used activity capacity to cost objects. Moreover, it allows an integrated view and approach to cost determination. Thus, TDABC offers a number of significant advantages. However, an examination of its disadvantages and limitations has not been formally addressed.

Although the usage of process time equations may reduce the number of activities relative to a fully-implemented ABC system, TDABC ignores Stage 2 simplification. TDABC calculates activity costs and assigns these costs to cost objects similarly to that of ABC. Since TDABC does not simplify the Stage 2 cost assignment, the size and complexity of TDABC remains considerable because managing the costs and consumption ratios of hundreds of activities is cumbersome for product costing. Hence, under TDABC, Stage 1 is simplified whereas Stage 2 remains complex. Moreover, the accuracy loss of TDABC is another issue that needs to be explored. It is unlikely that TDABC can preserve the same level of accuracy of ABC in all circumstances.

The purpose of this study is to extend and expand the before-the-fact simplification of ABC. Additional simplification, while overcoming identified limitations of TDABC, should enhance the viability of ABC systems and, thus, represent a significant contribution to ABC literature and actual practice. Hence, the study will first explore the accuracy of TDABC relative to an *ideally* implemented duration-based ABC system (the benchmark). This will be shown in Chapter 3. Second, as will also be shown in Chapter 3, the study will attempt to specify the conditions that must exist for TDABC to match the ABC assignments (equivalency conditions). Assuming accuracy loss is potentially a significant problem, ways or means of modifying Stage 1 simplification to reduce the accuracy loss will be investigated in Chapter 4. Any such modifications will attempt to preserve the resolution of the problems mentioned by Kaplan and Anderson (2004, 2007a) referred to above. Third, as will be shown also in

Chapter 4, the study will provide a new simplified system along with the conditions of equivalency between the new system and ABC to reduce the complexity and, therefore, the cost of Stage 2. Reducing the overall cost and complexity of ABC systems should increase the likelihood of adoption. Fourth, the maximum absolute dollar error between TDABC and ABC systems will be assessed in Chapter 5, with the maximum absolute dollar error between the new simplified system, TDABC2, and ABC in Chapter 6. Finally, in Chapter 7, case studies will be used to explore the validity of the equivalency conditions using a particular company's data. The next chapter reviews the literature regarding ABC and TDABC, which provides the background for the motivation of this study.

CHAPTER 2

LITERATURE REVIEW

2.1. Development of ABC

Kaplan (1994) stated that in the early years of ABC, the description of ABC systems was based on an "inner logic" that claims that ABC systems are more accurate than the functional-based (or, traditional) systems. However, this "inner logic" was not enough to cause a breakthrough for ABC. The academicians, especially Kaplan and Cooper, tried to increase the acceptance of ABC by developing two theories concerning (1) the cost (and activity) hierarchy of factory costs (indirect and support expenses) and (2) what type of resource cost ABC measures.

Cooper developed the first ABC theory concerning the cost/activity hierarchy (Cooper 1990). A taxonomy (activity hierarchy) for the activity cost drivers was developed in which activities are classified as (from lowest to highest) unit-level, batch-level, product-level, or facility-sustaining-level based on the cause and effect relationships between the organizational expense and the level of the organization. Kaplan (1994) states this cost/activity hierarchy provides four advantages. First, all organizational expenses can be mapped to a particular organizational level where cause and effect relationships can be established. Second, the cost/activity hierarchy has provided "a much richer set of drivers of cost variability" (Kaplan 1994, 251). Third, there is a connection between activity levels (unit, batch, product, and facility) and modern developments in operations management. Finally, the activity hierarchy is beneficial for continuous improvement and lean production. Kaplan (1994) states that this activity hierarchy theory helps managers analyze each component of overhead costs to help reduce those costs.

Kaplan (1994) developed the second theory in which not all organizational expenses should be assigned to cost objects. ABC systems measure the costs of using resources, not the cost of supplying resources that financial systems measure. The cost of unused capacity is the difference between the cost of resources used and the cost of resources supplied. Once the cost of resources used is found using the ABC system, the cost of unused capacity can be determined. Thus, ABC systems do not directly measure the cost of unused capacity.

Additionally, for ABC to provide relevant data, Noreen (1991) found that the cost system must be well-specified in which the underlying cost function must satisfy three necessary and sufficient conditions. The first condition states that the total overhead cost can be partitioned into cost pools, with each cost pool depending on one activity. The second condition states that there must be a linear relationship between the cost in each cost pool and the level of activity in that cost pool. The third condition eliminates any dependency between products and eliminates joint processes, which means that the production of a product is not dependent on the production of another product. Because of these conditions and the basic intuition behind ABC, there has been some success in implementing ABC as the next section discusses.

2.2. Success of ABC

The main reason for the success of ABC systems in the firms that adopted and implemented them is the widespread support for ABC within the firm, adequate training, and managers who understand and know ABC information (Al-Omiri and Drury 2007). Additionally, research has found that ABC is adopted if 1) there is a current significant risk of cost distortions within the firm, 2) the firm is large, 3) the firm has continuous manufacturing processes as opposed to job shops, and 4) there is product diversity (Krumwiede 1998b). Furthermore, if there is a significant top management support of ABC, then ABC will most likely become integrated within the firm (Krumwiede 1998b).

However, the adoption and implementation rates for ABC are low. For instance, one research study stated that the adoption rate is 29 percent (Al-Omiri and Drury 2007). Another study stated that the rate is 24 percent (Krumwiede 1998b). Additionally, Gosselin (1997) gave a more informative study and divided the implementation rate from the adoption rate. He found that the adoption rate is 47.8 percent but the implementation rate is only 30.4 percent. Shields (1995) found that 75 percent of the firms that used ABC received a financial benefit. Finally, 85 percent of firms who routinely use ABC feel that it is worth it, whereas 15 percent do not think it is worth the cost (Krumwiede 1998b). The next section discusses the implementation issues and problems of ABC.

2.3. Implementation Issues and Limitations of ABC

The last section discussed what drives successful ABC implementation. However, there are reasons that ABC is not successfully adopted. For instance, Krumwiede (1998b) found a strong IT system can prevent ABC adoption or the continuation of implementing it. The reason is that firms with strong IT perceive that they already have enough information for decision making; thus, ABC is not worth the cost to implement it. Additionally, he found that weak top management support and insufficient training in ABC hinders implementation. Insufficient training causes employees to not understand and respect the benefits of an ABC system. Finally, some firms do not have enough patience to wait for the full benefits of implementation and that small firm size and job shops hinder ABC implementation (Krumwiede 1998a, 1998b).

Along with these implementation issues, ABC poses some limitations within the system. One limitation of ABC is that the linear approach of activity-based costing provides poor estimates of actual expenditures when there is a nonlinear or discontinuous relation between the demand for and provision of resources (e.g. the resources are provided on a joint and indivisible basis) (Maher and Marais 1998). A second limitation is that an ABC system is expensive, complex, and difficult to modify/update (Krumwiede 1998a; Kaplan and Anderson 2007a). A third limitation is that ABC systems also ignore unused capacity. A fourth limitation is that workers give subjective estimates of their time spent on various activities for Stage 1 cost assignments (Kaplan and Anderson 2007a). In spite of these limitations, the main reason that firms do not implement ABC is that they feel that the perceived benefits do not outweigh the implementation costs and that ABC will not enhance the control of costs (Al-Omiri and Drury 2007). Consequently, there is a trade-off between cost and accuracy. The next two sections focus on the published research that alleviates some of these limitations.

2.4. After-the-Fact Simplification

Simplification research that focuses on Stage 2 simplification (activity/driver reduction) includes the research by Babad and Balachandran (1993) and Homburg

(2001). Babad and Balachandran (1993) developed a model to identify an optimal subset of drivers from the fully specified ABC system that takes into consideration information costs of production and accuracy. Their model allows the decision maker to specify, as a constraint, the maximum number of drivers allowed in the simplified system. This approach combines the costs of the activities corresponding to the eliminated drivers with the activity costs associated with the selected drivers, defining a new, aggregated cost pool for each selected driver. In building more aggregate cost pools, all of the associated activity costs of an eliminated driver are given to the cost pool of a corresponding selected driver.

Homburg (2001) extends the Babad and Balachandran (1993) model by allowing the activity costs of the eliminated drivers to be allocated to multiple selected drivers, rather than one corresponding driver. The optimal subset of drivers is selected that minimizes accuracy loss with information costs expressed as a constraint in the model (drivers are selected that do not exceed a pre-specified level of information costs). The cost pool for a selected driver is the cost of the selected driver's associated activity plus a share of the costs of the eliminated activities. He then shows that his approach creates a simplified system with the same level of complexity as the Babad and Balachandran approach but with more accurate product costs compared to a benchmark system. The fact that Homburg's model produces a more accurate system with no greater information cost illustrates that the Babad and Balachandran model did not identify the optimal simplified system. However, both models assume that a simplified system must sacrifice accuracy.

If the system has to be fully specified before it can be simplified, then there is no benefit of simplification since the firm already has a fully specified ABC system. Additionally, whenever the system has to be updated, the fully-specified system must be updated and then simplified, which seems to be more costly and time consuming in the long run. The next section discusses some research providing a better approach: before-the-fact simplification.

2.5. Before-the-Fact Simplification

Kaplan and Anderson (2007a) identified a new system called Time-Driven ABC (TDABC) to alleviate some of the complexity of ABC. TDABC skips the stage of driving resource costs to activities and introduces process time equations to take care of diverse and complex transactions (Kaplan and Anderson 2007b). These time equations summarize the time it takes to perform each activity within a process. Hence, TDABC focuses on processes instead of activities, which makes the system more manageable. Kaplan and Anderson (2007a) state

> The TDABC model simulates the actual processes used to perform work through-out an enterprise. It can therefore capture far more variation and complexity than a conventional ABC model, without creating an exploding demand for data estimates, storage, or processing capabilities. Using TDABC, a company can embrace complexity rather than being forced to use simplified, inaccurate ABC models… (p. 8).

Anderson, et al. (2007) claim that TDABC is more accurate since actual transaction data are used instead of estimates. In addition, when the process time equations are built, it is easy to determine which step within the process time equation is consuming too much time. Kaplan and Anderson (2007a) provide other benefits of TDABC over ABC. First, employees do not need to be interviewed or surveyed to allocate resource costs to activities. Second, Stage 1 cost assignment is reduced because resource costs are assigned to the activities using two sets of estimates: 1) the cost of supplying resource capacity for the department (capacity cost rate) and 2) the demand for resource capacity (capacity usage rate, typically time) by each transaction processed in the department. These rates are used to allocate resource costs to activities. Third, TDABC simulates the actual processes, thus capturing more variation and complexity than does ABC without creating greater need for data estimating, storage, or processing capabilities.

Fourth, the TDABC model can be updated easier. In contrast, Kaplan and Anderson (2007a, 12) mention that "ABC requires a geometric expansion to capture the increase in complexity." Additionally, when a new activity is identified, the unit time required only needs to be estimated. The system is updated based on events instead of the calendar. Fifth, it takes only a couple of days instead of weeks to load, calculate, validate, and report findings. Finally, research has found that TDABC can incorporate unused capacity within the TDABC system (Kaplan and Anderson 2007a). Previously, researchers did not understand that unused capacity is vital in ABC systems.

However, there are disadvantages. Although TDABC is simpler and cheaper than ABC, TDABC does not reduce the number of activities/drivers that a company has to keep track of for the Stage 2 cost assignments. Additionally, TDABC will not work if the time to perform the activities cannot be reliably clocked or if the activities are not performed in a repetitive manner (Sherratt 2005).

2.6. Motivation

In conclusion, TDABC is a better simplification approach as opposed to the after-the-fact simplification models. In TDABC, Stage 1 cost assignment is simplified, but

Stage 2 remains complex and similar to the ABC system since all activity costs and their corresponding consumption ratios have to be known. The contribution that this paper will make is to prove that there is a way to simplify the ABC system considerably while maintaining accuracy when compared to the benchmark ABC system. With the simplification method, Stage 1 cost assignment is eliminated with the additional fact that the individual activity costs do not have to be known. If the individual activity costs do not have to be known, then Stage 2 cost assignment is somewhat simplified. To simplify Stage 2 further, TDABC will be modified and applied to Stage 2 as shown in Chapter 4. This simplification will eliminate the need to know the individual activity consumption ratios. The main purpose of this study is to show the limitations of TDABC and provide a simpler and cheaper before-the-fact simplified system.

It is possible that there are more limitations to TDABC since research has not shown the conditions in which TDABC matches a fully-specified benchmark ABC system (the benchmark). This study will mathematically analyze those conditions in the next chapter.

CHAPTER 3

EQUIVALENCY ANALYSIS OF TDABC

3.1. Model Definitions

In this section, the mathematical models for the ABC and TDABC are shown and used to compare the differences in cost assignments. The original models of Kaplan and Anderson are used to explore potential accuracy differences. In this study, the Stage 1 and Stage 2 models for ABC will incorporate duration drivers (time-based drivers) for easier comparison with TDABC. Assuming *m* activities and *n* resources, the Stage 1 cost assignment for ABC is modeled as follows:

$$C_a^{\alpha} = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$$

$$= \sum_{j=1}^{n} \rho_{aj} C_j \,,\ a = 1,\ldots, m, \qquad (1)$$

Where

C_a^{α} = cost assigned to activity *a* under ABC;

t_{aj} = activity *a*'s consumption of time for resource *j*;

t_j = total time used to supply resource *j* ($\sum_{a=1}^{m} t_{aj}$);

ρ_{aj} = relative frequency of use of resource *j* by activity *a* (the resource consumption ratio); and

C_j = total cost of resource *j*.

Equation 1 states that the total cost of an activity under the ABC system is the sum of the resource consumption ratios, ρ_{aj} , multiplied by the corresponding resource costs, C_j.

Assuming *k* cost objects and *m* activities, the model for ABC for Stage 2 cost assignment is as follows:

$$
\begin{aligned}
D_i^{\alpha} &= \sum_{a=1}^{m} \frac{\Im_{ia}}{\Im_a} C_a^{\alpha} \\
&= \sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha} \text{ , } i = 1, \ldots, k, \qquad (2)
\end{aligned}
$$

Where

D_i^{α} = cost assigned to cost object *i* under ABC;

C_a^{α} = total cost of activity *a*;

$\Im_{ia}$ = volume or actual absolute frequency of use of activity *a* by cost object *i*;

$\Im_a$ = total usage of activity *a* ($\sum_{i=1}^{k} \Im_{ia}$); and

υ_{ia} = relative frequency of use of activity *a* by cost object *i*.

Equation 2 states that the total cost of a cost object under the ABC is the sum of the activity consumption ratios, υ_{ia} , multiplied by the corresponding activity costs, C_a^{α} .

The model for TDABC Stage 1 cost assignment is given below (for simplicity only one resource pool is assumed[1]):

$$
\begin{aligned}
C_a^{\tau} &= c \sum_{j=1}^{n} t_{aj} \\
&= c \Im_a \text{ , } a = 1, \ldots, m, \qquad (3)
\end{aligned}
$$

Where

C_a^{τ} = cost of activity *a* under TDABC;

[1] The analysis can be easily generalized to more than one resource pool.

c = cost per unit of resource time; and

$\Im_a$ = total resource time for activity *a*.

Equation 3 states that the total cost assigned to activity *a* is the sum of the total resource time used by this activity multiplied by the cost per unit of time. The cost per unit of time, *c*, is simply the total resource cost for the pool divided by the total resource time used by all activities:

$$c = \frac{\sum_{j=1}^{n} C_j}{\sum_{j=1}^{n} t_j} = \frac{C_T}{t_T}, \tag{4}$$

Where

t_j = total time used to supply resource *j*;

C_T = total cost of resources; and

t_T = total resource time ($\sum_{j=1}^{n} t_j$).

The model for TDABC for Stage 2 cost assignment is

$$D_i^{\tau} = \sum_{a=1}^{m} C_a^{\tau} \upsilon_{ia},\ i = 1,\ldots, k \tag{5}$$

Equation 5 states that the total cost of cost object *i* (D_i^{τ}) under TDABC is the sum of each activity cost C_a^{τ} multiplied by the corresponding activity consumption ratio υ_{ia}. Equations (2) and (5) for the Stage 2 model for both ABC and TDABC are identical. Any differences in cost assignment between the two models are attributable to differences between C_a^{α} and C_a^{τ}. Thus, any potential accuracy loss must occur in Stage 1. Before any equivalency analysis is shown, the assumptions behind the analysis are first discussed in the next section.

3.2. Assumptions

Two major assumptions are needed to perform the equivalency analysis to find the necessary conditions for equivalency between TDABC and the fully-specified, benchmark ABC. The first assumption requires a linear relationship between the cost in each cost pool and the level of activity in that cost pool (Noreen 1991). Although Maher and Marais (1998) found that a linear relationship is a limitation of ABC due to poor estimates when there is a nonlinear or discontinuous relation between the demand for and provision of resources, this assumption is fundamental to ABC and will be used for the analysis.

TDABC assumes that resources are time driven; thus, the second assumption initially requires that all resources in the ABC system are assigned using duration drivers (time-based drivers). This assumption facilitates the equivalency analysis between TDABC and the benchmark ABC for Stage 1. This assumption is relaxed in Section 3.5 so that the effect of resource diversity on the equivalency conditions can be assessed.

3.3. Equivalency Analysis

Differences between C_a^{α} and C_a^{τ} are highlighted by differences in the information required to calculate each value. The information set for calculating C_a^{α} is $\{t_{aj}, C_j\}$. Detailed individual resource driver information and resource cost information are needed. Much effort and cost must be expended to gather this information through surveys, interviews, and unbundling the general ledger. The information set needed for calculating C_a^{τ} is $\{t_T, \Im_a, C_T\}$. Total time and total resource cost are readily available within an existing traditional cost system. TDABC avoids the need to collect detailed information for $\Im_a$ by 1) determining the time to perform one unit of activity; 2) determining the number of times the activity will be performed (usually defined by practical capacity); and 3) multiplying the time to perform one unit of activity by the number of times the activity will be performed.

Thus, TDABC allows activity costs to be calculated without knowing individual resource drivers or individual resource costs (only total resource time and total resource cost are needed). Whether the activity cost determined by TDABC is the same as that of ABC is a critical question. It is initially assumed that all resources are time driven. Later this assumption is relaxed.

First, an intermediate ABC (IABC) costing system is developed and analyzed that requires knowledge of total resource cost and individual resource drivers. Accordingly, the information set is $\{t_{aj}, C_T\}$. The development of the IABC system helps identify the conditions required for equivalency between ABC and TDABC. In the IABC system, an activity's cost is calculated by multiplying the activity's average resource consumption ratio by the total resource cost:

$$C_a^I = \bar{\rho}_a C_T, \tag{6}$$

Where

C_a^I = cost of activity *a* for the IABC system; and

$\bar{\rho}_a = \frac{\sum_{j=1}^{n} \rho_{aj}}{n}$, the average resource consumption ratio.

Equivalency between the Stage 1 cost assignments of ABC and IABC is established by the following reasoning. If a resource costs more (less), it does not mean that an activity has to consume a higher (lower) proportion of that resource's time. If this state of no linear correlation between resource consumption ratios and individual resource costs exists for every activity, then ABC and IABC are equivalent.[2] This equivalency is stated by the following proposition:

Proposition I: $C_a^{\alpha} = C_a^{I}$, $a = 1,\ldots, m$, if and only if there is no correlation between ρ_{aj} and C_j, $j = 1,\ldots, n$.

Proof: First, assume there is no correlation between ρ_{aj} and C_j for *each* activity *a* ($a = 1,\ldots, m$). The correlation between ρ_{aj} and C_j, $r_{c\rho}$, is defined as

$$r_{c\rho} = \frac{\sum_{j=1}^{n} \rho_{aj} C_j - \frac{\sum_{j=1}^{n} \rho_{aj}}{n} \sum_{j=1}^{n} C_j}{\sigma_\rho \sigma_c}$$, where $\sigma_\rho = \sqrt{\sum_{j=1}^{n} \rho_{aj}^2 - \frac{\left(\sum_{j=1}^{n} \rho_{aj}\right)^2}{n}}$ and

$\sigma_c = \sqrt{\sum_{j=1}^{n} C_j^2 - \frac{\left(\sum_{j=1}^{n} C_j\right)^2}{n}}$. If $r_{c\rho} = 0$, then $\sum_{j=1}^{n} \rho_{aj} C_j = \frac{\sum_{j=1}^{n} \rho_{aj}}{n} \sum_{j=1}^{n} C_j$, which implies that $C_a^{\alpha} = C_a^{I}$.

[2] Based on the linearity assumption from Section 3.2, all correlations discussed in this dissertation are linear.

Next, assume that $C_a^\alpha = C_a^I$. Since $C_a^\alpha = \sum_{j=1}^{n} \rho_{aj} C_j$ and $C_a^I = \frac{\sum_{j=1}^{n} \rho_{aj}}{n} \sum_{j=1}^{n} C_j$, then from the definition of $r_{c\rho}$, this immediately implies that $r_{c\rho} = 0$. **QED**

Table 1 provides a simple illustrative example of Proposition I, using two activities. Note that when the correlation between ρ_{aj} and C_j is zero, multiplying the average consumption ratios by the total cost produces the ABC cost assignments. As shown in Table 1, for Activity 1 (A1) and Activity (A2), IABC Stage 1 cost assignments are identical to those under ABC (C_1^α = C_1^I = $615 and C_2^α = C_2^I = $585). Hence, under IABC, there is no need to know the individual resource costs.

TABLE 1
Example Illustrating Proposition I

	Resource							
	R1	**R2**	**R3**	**R4**	**ABC Cost Assignment**[a]	$\bar{\rho}_a$ [b]	**IABC Cost Assignment**[c]	$r_{c\rho}$
A1	0.60	0.20	0.30	0.95	$615	0.513	$615	0
A2	0.40	0.80	0.70	0.05	$585	0.488	$585	0
	$95	$335	$370	$400	$1,200		$1,200	

[a] $C_a^\alpha = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$

[b] $\bar{\rho}_a = \frac{\sum_{j=1}^{n} \rho_{aj}}{4}$

[c] $C_a^I = \bar{\rho}_a C_T$

The information set for IABC is $\{t_{aj}, C_T\}$. IABC eliminates the need to know the individual resource costs C_j required for ABC; however, the detailed resource consumption ratios must be known. The correlation between ρ_{aj} and C_j is exploited to reduce the fineness of the ABC information set. This suggests the possibility of exploiting correlation relationships to establish equivalency between ABC and TDABC. Note that the information set for TDABC is $\{t_T, \Im_a, C_T\}$, which eliminates the need to know both C_j and t_{aj} of ABC. For TDABC, $r_{t\rho}$ (the correlation between ρ_{aj} and t_j) and $r_{c\rho}$ are both needed as shown by the following proposition.

Proposition II: $C_a^\alpha = C_a^\tau$, $a = 1,\ldots, m$, if and only if $cr_{t\rho}\sigma_t = r_{c\rho}\sigma_c$.

Proof: First assume that $cr_{t\rho}\sigma_t = r_{c\rho}\sigma_c$. By definition, $r_{t\rho} = \dfrac{\sum_{j=1}^{n}\rho_{aj}t_j - \bar{\rho}_a\sum_{j=1}^{n}t_j}{\sigma_\rho\sigma_t}$, where

$\sigma_t = \sqrt{\sum_{j=1}^{n}t_j^2 - \dfrac{\left(\sum_{j=1}^{n}t_j\right)^2}{n}}$. Substitute $r_{t\rho}$, $r_{c\rho}$, and c with their corresponding formulas and

simplify to obtain $\dfrac{\sum_{j=1}^{n}C_j}{\sum_{j=1}^{n}t_j}\sum_{j=1}^{n}\rho_{aj}t_j = \sum_{j=1}^{n}\rho_{aj}C_j$. Note that $\sum_{j=1}^{n}\rho_{aj}t_j$ is equivalent to $\sum_{j=1}^{n}t_{aj}$.

Hence, $\dfrac{\sum_{j=1}^{n}C_j}{\sum_{j=1}^{n}t_j}\sum_{j=1}^{n}t_{aj} = \sum_{j=1}^{n}\rho_{aj}C_j \Rightarrow C_a^\tau = C_a^\alpha$.

Next, assume that $C_a^\alpha = C_a^\tau$. From the definition of $r_{t\rho}$, $\sum_{j=1}^{n}\rho_{aj}t_j - \bar{\rho}_a\sum_{j=1}^{n}t_j = r_{t\rho}\sigma_\rho\sigma_t$

$$\Rightarrow \sum_{j=1}^{n} t_{aj} - \bar{\rho}_a \sum_{j=1}^{n} t_j = r_{t\rho}\sigma_\rho\sigma_t.$$ Since $C_a^\tau = \frac{\sum_{j=1}^{n} C_j}{\sum_{j=1}^{n} t_j} \sum_{j=1}^{n} t_{aj}$, then $\sum_{j=1}^{n} t_{aj} = \frac{\sum_{j=1}^{n} t_j}{\sum_{j=1}^{n} C_j} C_a^\tau$.

Substituting for $\sum_{j=1}^{n} t_{aj}$ provides $\frac{\sum_{j=1}^{n} t_j}{\sum_{j=1}^{n} C_j} C_a^\tau - \bar{\rho}_a \sum_{j=1}^{n} t_j = r_{t\rho}\sigma_\rho\sigma_t \Rightarrow$

$C_a^\tau = \bar{\rho}_a \sum_{j=1}^{n} C_j + r_{t\rho}\sigma_\rho\sigma_t \frac{\sum_{j=1}^{n} C_j}{\sum_{j=1}^{n} t_j} \Rightarrow C_a^\tau = \bar{\rho}_a \sum_{j=1}^{n} C_j + cr_{t\rho}\sigma_\rho\sigma_t$. From the $r_{c\rho}$ equation,

$\sum_{j=1}^{n} \rho_{aj} C_j - \bar{\rho}_a \sum_{j=1}^{n} C_j = r_{c\rho}\sigma_\rho\sigma_c \Rightarrow C_a^\alpha - \bar{\rho}_a \sum_{j=1}^{n} C_j = r_{c\rho}\sigma_\rho\sigma_c \Rightarrow$

$C_a^\alpha = \bar{\rho}_a \sum_{j=1}^{n} C_j + r_{c\rho}\sigma_\rho\sigma_c$. Accordingly, $C_a^\alpha - C_a^\tau = \sigma_\rho[r_{c\rho}\sigma_c - cr_{t\rho}\sigma_t]$. Thus, if

$C_a^\alpha = C_a^\tau$, then $cr_{t\rho}\sigma_t = r_{c\rho}\sigma_c$. **QED**

In the proof of the above proposition, it is shown that $C_a^\alpha - C_a^\tau = \sigma_\rho[r_{c\rho}\sigma_c - cr_{t\rho}\sigma_t]$. Interestingly, since the dollar value of the error between the two systems equals $C_a^\alpha - C_a^\tau$, then the dollar value of the error can be expressed as follows:

$$\varepsilon_a = \sigma_\rho[r_{c\rho}\sigma_c - cr_{t\rho}\sigma_t],\ a = 1,\ldots, m \tag{7}$$

When the error for each activity *a* is zero, there is equivalency, which implies that $r_{c\rho}\sigma_c - cr_{t\rho}\sigma_t = 0$. This expression implies that if $r_{c\rho}$ = 0, then $r_{t\rho}$ = 0. Thus, the following corollary to the Proposition II has been proved:

Corollary IIa: If $r_{c\rho}$ = 0 and $r_{t\rho}$ = 0, then $C_a^{\alpha} = C_a^{\tau}$, *a* = 1,…, *m*.

In the event that both $r_{c\rho}$ and $r_{t\rho}$ are nonzero, then it is also possible to establish an equivalency condition based on a required value for *c*. When $\varepsilon_a = \sigma_\rho[r_{c\rho}\sigma_c - cr_{t\rho}\sigma_t]$ = 0, solving for *c* and simplifying yields the following equivalency condition:

$$c = \frac{\sum_{j=1}^{n}\rho_{aj}C_j - \bar{\rho}_a\sum_{j=1}^{n}C_j}{\sum_{j=1}^{n}\rho_{aj}t_j - \bar{\rho}_a\sum_{j=1}^{n}t_j} = \frac{C_a^{\alpha} - C_a^{I}}{\sum_{j=1}^{n}\rho_{aj}t_j - \bar{\rho}_a\sum_{j=1}^{n}t_j}$$

This then establishes a second corollary:

Corollary IIb: If $c = \frac{C_a^{\alpha} - C_a^{I}}{\sum_{j=1}^{n}\rho_{aj}t_j - \bar{\rho}_a\sum_{j=1}^{n}t_j} = \frac{C_T}{t_T}$, then $C_a^{\alpha} = C_a^{\tau}$, *a* = 1,…, *m*.

According to Corollary IIb, if the rationale for zero correlation is not valid, it is still possible to obtain equivalency. However, a very special relationship must exist. The numerator $C_a^{\alpha} - C_a^{I}$ is the dollar error between ABC and IABC for activity *a*. The denominator $\sum_{j=1}^{n} \rho_{aj} t_j - \bar{\rho}_a \sum_{j=1}^{n} t_j$ is the unit time error between time allocated to activity *a* using ABC and the time allocated to activity *a* using IABC. Consequently,

$$\frac{C_a^{\alpha} - C_a^{I}}{\sum_{j=1}^{n} \rho_{aj} t_j - \bar{\rho}_a \sum_{j=1}^{n} t_j} = c_{\varepsilon}$$ represents the absolute dollar error per unit of error time. Note

that c_{ε} must be written in absolute form since the IABC cost assignment for activity *a* could be greater than that of ABC.

Table 2 provides an example illustrating Corollary IIa of Proposition II. Table 2 compares TDABC and ABC when there is no correlation between t_j and ρ_{aj} and between ρ_{aj} and C_j for each activity *a*. When $r_{c\rho}$ = 0 and $r_{t\rho}$ = 0 for each activity, the activity costs under TDABC (C_1^{τ} = \$615 and C_2^{τ} = \$585) are equal to those under ABC.

TABLE 2
Example Illustrating Corollary IIa

ABC

	Resource				
	R1	**R2**	**R3**	**R4**	**ABC Cost Assignment**[a]
A1	0.60	0.20	0.30	0.95	\$615
A2	0.40	0.80	0.70	0.05	\$585
Cost	\$95	\$335	\$370	\$400	\$1,200
Time	101	300	321	350	1,072

TDABC

	Unit Time	**Total Units of Activity**	$\mathfrak{I}_a$	***c***	**TDABC Cost Assignment**[b]	$r_{c\rho}$	$r_{t\rho}$
A1	36.63	15	549	\$1.12	\$615	0	0
A2	26.15	20	523		\$585	0	0
		Total Time	1,072		\$1,200		

[a] $C_a^{\alpha} = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$ [b] $C_a^{\tau} = c\mathfrak{I}_a$

Corollary IIb of Proposition II is demonstrated in the following example shown in Table 3. According to Table 3, $r_{c\rho}$ and $r_{t\rho}$ are nonzero and the cost per unit of time, *c*, and dollar error per unit of time, c_{ε}, are both equal to \$4. This satisfies Corollary IIb so that $C_a^{\alpha} = C_a^{\tau}$, where $C_1^{\tau} = C_1^{\alpha}$ = \$216 and $C_2^{\tau} = C_2^{\alpha}$ = \$264.

TABLE 3
Example Illustrating Corollary IIb

ABC

	Resource						
	R1	**R2**	**R3**	**R4**	**ABC Cost Assignment**[a]	$\bar{\rho}_a$	**IABC Cost Assignment**[b]
A1	0.20	0.40	0.60	0.80	\$216	0.50	\$240
A2	0.80	0.60	0.40	0.20	\$264	0.50	\$240
Cost	\$180	\$60	\$180	\$60	\$480		\$480
Time	35	40	25	20	120		

TDABC

	Unit Time	**Total Units of Activity**	$\Im_a$	c_{ε}[c]	***c***	**TDABC Cost Assignment**[d]	$r_{c\rho}$	$r_{t\rho}$
A1	9	6	54	\$4	\$4	\$216	-0.45	-0.85
A2	3	22	66			\$264	0.45	0.85
		Total Time	120			\$480		

[a] $C_a^{\alpha} = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$

[b] $C_a^{I} = \bar{\rho}_a C_T$

[c] $c_{\varepsilon} = \frac{C_a^{\alpha} - C_a^{I}}{\sum_{j=1}^{n} \rho_{aj} t_j - \bar{\rho}_a \sum_{j=1}^{n} t_j} = c = \frac{C_T}{t_T}$

[d] $C_a^{\tau} = c\Im_a$

If Proposition II does not hold, then there is a difference in the cost assigned to TDABC relative to that of ABC. Table 4 shows that when there is perfect correlation between t_j and ρ_{aj} and between ρ_{aj} and C_j (where $cr_{t\rho}\sigma_t \neq r_{c\rho}\sigma_c$), the activity costs under TDABC (C_1^τ = \$151 and C_2^τ = \$384) are not equal to those under ABC (C_1^α = \$249 and C_2^α = \$286). The average absolute percentage error of TDABC is 36.7 percent, with dollar error (ε_a) is \$97.93 for A1 and -\$97.93 for A2 (c = \$0.4977 and c_ε = \$0.488).

TABLE 4
Illustration Not Satisfying Proposition II

ABC

	Resource						
	R1	**R2**	**R3**	**R4**	**ABC Cost Assignment**[a]	$\bar{\rho}_a$	**IABC Cost Assignment**[b]
A1	0.80	0.25	0.30	0.15	\$249	0.375	\$201
A2	0.20	0.75	0.70	0.85	\$286	0.625	\$334
Cost	\$215	\$110	\$120	\$90	\$535		\$535
Time	101	318	299	357	1,075		

TDABC

	Unit Time	**Total Units of Activity**	$\Im_a$	c	**TDABC Cost Assignment**[c]	$r_{c\rho}$	$r_{t\rho}$
A1	19	16	304	\$0.4977	\$151	1	-1
A2	25.7	30	771		\$384	-1	1
		Total Time	1,075		\$535		

	ABC	**TDABC**	ε_a	**%** ε_a [d]	c_ε [e]
A1	\$249	\$151	\$98	39.24%	\$0.488
A2	\$286	\$384	(\$98)	-34.16%	
			Avg $\%\lvert\varepsilon_a\rvert$ [f]	36.7%	

[a] $C_a^\alpha = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$

[b] $C_a^I = \bar{\rho}_a C_T$

[c] $C_a^\tau = c\Im_a$

[d] $\%\,\varepsilon_a = \frac{C_a^\alpha - C_a^\tau}{C_a^\alpha}$

[e] $c_\varepsilon = \frac{C_a^\alpha - C_a^I}{\sum_{j=1}^{n} \rho_{aj} t_j - \bar{\rho}_a \sum_{j=1}^{n} t_j} \neq c = \frac{C_T}{t_T}$

[f] $\frac{39.24\% + \lvert -34.16\% \rvert}{2}$

If $r_{c\rho}$ and $r_{t\rho}$ are nonzero and TDABC and ABC are not equivalent, then the cost per unit of time would not be equal to the dollar error per unit of time. Therefore, $\varepsilon_a > 0$ for each *a*. From Equation 7, it is possible to analyze the effects of various variables on the magnitude of the error. For instance, the error will be larger in absolute magnitude if $r_{c\rho}$ and $r_{t\rho}$ are opposite in sign, which makes the two terms on the right hand side of Equation 7 additive. The magnitude of the error is also affected by variability in ρ_{aj}, t_j, and C_j. Additional analysis of the dollar error is needed in which the maximum absolute dollar error is identified and will be shown in Chapter 5.

3.4. Time Equations and Unused Capacity

Kaplan and Anderson (2007a, 2007b) stated that a process can be expressed in a process time equation that consists of all of the individual activities that make up the process. Time equations summarize the TDABC time information. Using time equations is a way of obtaining granularity (the level of detail) without having a separate activity for each event. If TDABC and ABC have the same granularity, then Proposition II holds. Time equations are based on the unit time for each activity and the number of times it is actually performed (or, actual activity used). The difference between the time equation based on practical activity and the time equation based on actual activity used is unused capacity. When unused capacity exists and the equivalency conditions are satisfied, the cost of activity *a* under ABC is equal to the cost of activity *a* under TDABC plus the cost of unused capacity for activity *a*. This only means that the cost of unused capacity for activity *a* is separated from the actual cost of the activity *a* used. Thus, there is no significant effect on the equivalency conditions. The next table is similar to Table 2 but has been modified to incorporate unused capacity. Time equations are then developed to illustrate the summarization of the TDABC time information.

Table 5 shows the same illustration as in Table 2 except that unused capacity now exists. For TDABC, the activity used represents the number of times the activity is performed. Practical activity represents the number of times the activity should be performed under normal operating conditions. Notice that, for equivalency, the ABC cost for an activity must equal the TDABC cost for an activity plus the cost of unused portion of the activity. Therefore, Table 5 illustrates that unused capacity has no significant effect on the conditions for equivalency.

To develop the process time equations, assume that the illustration in Table 5 concerns an ordering department that has two activities: number of repeat orders (A1) and the number of new orders (A2). The time equation that represents the total order processing time based on actual activity used is

Actual time used = 36.60(# of repeat orders used) + 26.15(# of new orders used)
= 36.60(10) + 26.15(15)
= 758 minutes

The time equation that represents the total order processing time based on practical activity is

Practical time = 36.60(# of repeat orders) + 26.15(# of new orders)
= 36.60(15) + 26.15(20)
= 1,072 minutes

The unused capacity time is the difference between the practical time and the actual time used, which is 314 minutes (1,072 minus 758). The total cost of unused capacity is $351 (314 x $1.12). To find the cost of unused capacity for each activity, the activities would have to be separated out of the time equation, and the results would be identical to those displayed in Table 5.

TABLE 5
Corollary IIa with Unused Capacity

	Resource							
	R1	**R2**	**R3**	**R4**	**ABC Cost**	$\mathbf{r_{c\rho}}$	$\mathbf{r_{t\rho}}$	**c**
A1	0.60	0.20	0.30	0.95	$615	0	0	$1.12
A2	0.40	0.80	0.70	0.05	$585	0	0	
Cost	$95	$335	$370	$400	$1,200			
Time	101	300	321	350	1,072			

	Unit Time	**Activity Used**	**Time Used**	**Practical Activity**	$\mathfrak{I}_a$	**TDABC Cost**	**Unused Cost**	**Total Cost**
A1	36.60	10	366	15	549	$410	$205	$615
A2	26.15	15	392	20	523	$439	$146	$585
			758		1,072	$849	$351	$1,200

Only Corollary IIa is shown for this analysis because if unused capacity is applied to Corollary IIb, the results are similar to the illustration in Table 3 of Corollary IIb and follow the same process as above for unused capacity. Consequently, it has been illustrated that time equations summarize the information of the TDABC system and have no bearing on the equivalency conditions since they are developed after the TDABC system has been implemented. Therefore, both unused capacity and time equations do not affect the equivalency conditions.

In Section 3.3, the conditions for equivalency between ABC and TDABC assume that all resources are time-driven. However, there are, in general, resources that are not time driven (e.g. some forms of capital, materials, and some forms of energy). In TDABC, the costs of these non-time-driven resources are pooled with the costs of resources that are time driven. This resource diversity can produce inaccurate activity costs. This inaccuracy can pose a major problem for TDABC if the costs of the non-time-driven resources are significant. When non-time-driven resources are significant, pooling can cause inaccurate cost assignments since there would be a lack of causal relationships for non-time-driven resources. In Section 3.5, resource diversity is examined and examples are used to illustrate this problem.

3.5. Resource Diversity

Resource diversity exists when there are a significant proportion of non-time-driven resources that are consumed in a different pattern from time-driven resources. In Stage 1 cost assignment, TDABC assigns the cost of all resources to the activities using time-based drivers, which means that time-based drivers are used to assign the costs of both time-based and non-time-based resources to activities. Let the set of all resources be $R = \{1,\ldots, n\}$. Next, partition R into a set of time-driven resources, $TD = \{1,\ldots, l\}$, and a set of non-time-driven resources, $NTD = \{l+1,\ldots, n\}$, where $R = TD \cup NTD$. If, on average, activities consume non-time-driven resources in the same pattern as time-driven resources, then equivalence between ABC and TDABC remains possible. As a result,

$\overline{\rho}_a^{\tau} = \frac{\sum_{j=1}^{l} t_{aj}}{l} = \overline{\rho}_a = \frac{\sum_{j=1}^{n} t_{aj}}{n}$, where $\overline{\rho}_a^{\tau}$ is the average consumption ratio for time-driven resources for activity a, $a = 1,..., m$. When $\overline{\rho}_a^{\tau} = \overline{\rho}_a$, there is no resource diversity and Proposition II applies. However, if $\overline{\rho}_a^{\tau} \neq \overline{\rho}_a$, then resource diversity (RD) exists and can be measured as follows:

$$RD = \overline{\rho}_a^{\tau} - \overline{\rho}_a \text{, } a = 1,\ldots, m \qquad (8)$$

This suggests the possibility that as *RD* increases, then the potential difference between ABC and TDABC also increases. Table 6 provides an illustration of resource diversity that shows the potential inaccuracy of TDABC. There are two activities, four time-driven resources (j = 1,…, l), and four non-time-driven resources (j = l+1,…, n). In the example, the time-driven resources are the labor resources (L1 – L4), and the non-time-driven resources are the materials resources (M1 and M2) and energy resources (E1 and E2). Additionally, $r_{c\rho}$ = 0 and $r_{t\rho}$ = 0 so that $cr_{t\rho}\sigma_t = r_{c\rho}\sigma_c$. Although the conditions for $C_a^{\alpha} = C_a^{\tau}$ are satisfied, $C_a^{\alpha} \neq C_a^{\tau}$ because of the effect of non-time-based resources.

TABLE 6
Resource Diversity I

ABC

	L1	L2	L3	L4	M1	M2	E1	E2
	Resources							
A1	0.25	0.70	0.15	0.10	0.80	0.40	0.30	0.50
A2	0.75	0.30	0.85	0.90	0.20	0.60	0.70	0.50
Cost	$3,565	$3,400	$2,900	$1,000	$1,500	$7,000	$3,000	$2,400
Time	3,800	1,500	1,400	1,000				

	ABC Cost[a]	$\overline{\rho}_a$	$r_{c\rho}$	$r_{t\rho}$
A1	$9,906	0.40	0	0
A2	$14,859	0.60	0	0
Cost	$24,765			

TDABC

	$\Im_a$	c	TDABC Cost[b]	$\overline{\rho}_a^{\tau}$
A1	2,310	$3.22	$7,430	0.30
A2	5,390		$17,336	0.70
	7,700		$24,765	

	ABC	TDABC	ε_a	% ε_a [c]
A1	$9,906	$7,430	$2,477	25.0%
A2	$14,859	$17,336	($2,477)	-16.7%
			Avg $\%\lvert\varepsilon_a\rvert$	20.8%

[a] $C_a^{\alpha} = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$

[b] $C_a^{\tau} = c\Im_a$

[c] $\%\varepsilon_a = \frac{C_a^{\alpha} - C_a^{\tau}}{C_a^{\alpha}}$

According to Table 6, $\overline{\rho}_a^{\tau}$ for A1 and A2 are 0.3 and 0.7, respectively, whereas $\overline{\rho}_a$ for A1 and A2 are 0.4 and 0.6, respectively. Thus, *RD* = -0.1 for A1 and 0.1 for A2. The dollar value of the error ε_a is $2,477 for A1 and -$2,477 for A2. The average absolute percentage error of TDABC is 20.8 percent. However, if *RD* increases, then the average absolute percentage error increases as shown in Table 7. Compared to Table 6, Table 7 shows that as *RD* doubles, the average absolute percentage error almost doubles. Hence, the illustration supports the claim that as *RD* increases, error increases.

TABLE 7
Resource Diversity II

ABC

	Resources							
	L1	**L2**	**L3**	**L4**	**M1**	**M2**	**E1**	**E2**
A1	0.25	0.70	0.15	0.10	0.60	0.85	0.40	0.95
A2	0.75	0.30	0.85	0.90	0.40	0.15	0.60	0.05
Cost	\$3,565	\$3,400	\$2,900	\$1,000	\$3,000	\$1,500	\$6,234	\$3,166
Time	3,800	1,500	1,400	1,000				

	ABC Cost[a]	$\overline{\rho}_a$	$r_{c\rho}$	$r_{t\rho}$
A1	\$12,383	0.50	0	0
A2	\$12,382	0.50	0	0
Cost	\$24,765			

TDABC

	$\Im_a$	c	**TDABC Cost**[b]	$\overline{\rho}_a^{\tau}$
A1	2,310	\$3.22	\$7,430	0.30
A2	5,390		\$17,336	0.70
Total Time	7,700		\$24,765	

	ABC	**TDABC**	ε_a	**%** ε_a [c]
A1	\$12,383	\$7,430	\$4,953	40.0%
A2	\$12,382	\$17,336	(\$4,953)	-40.0%
			Avg $\%\lvert\varepsilon_a\rvert$	40.0%

[a] $C_a^{\alpha} = \sum_{j=1}^{n} \frac{t_{aj}}{t_j} C_j$

[b] $C_a^{\tau} = c\Im_a$

[c] $\%\,\varepsilon_a = \frac{C_a^{\alpha} - C_a^{\tau}}{C_a^{\alpha}}$

If resource diversity is significant, then TDABC may be significantly less accurate than ABC. One possible resolution to this problem is discussed in Chapter 4.

3.6. Implications

The major implication that the equivalency conditions for TDABC have on research and practice is to show when the TDABC system will replicate the ABC system. The equivalency holds when the underlying conditions as outlined in Propositions I and II are satisfied and when all resources are time driven. However, when there is resource diversity, the assumption of all resources being time driven is violated. When this one assumption is violated, there is no equivalency although the conditions in Proposition I and Proposition II and its corollaries are met. This issue needs to be resolved. The next chapter provides a resolution by analyzing a Stage 2 simplification procedure in order to eliminate Stage 1 cost assignments while maintaining accurate costing of the cost objects.

CHAPTER 4

STAGE 2 SIMPLIFICATION

4.1. IABC Applied to Stage 2 and Model Definition

The previous chapter showed the conditions for accuracy for the TDABC system. Because of the potential inaccuracy of the TDABC system when there is resource diversity, this section will look at a way to simplify ABC while resolving the potential inaccuracy issue of TDABC. One resolution method is to extend Proposition I to Stage 2. Recall that Proposition I states that when the resource costs and resource consumption ratios are not linearly correlated, then the cost of a particular activity is basically its average resource consumption ratio multiplied by the total resource cost for all resources. This can be applied to other cost objects as well. The Stage 2 Intermediate system (IABC2) uses the IABC model to resolve the TDABC resource diversity issue and simultaneously offers some simplification for Stage 2. Assuming *k* cost objects and *m* activities, the IABC2 model is described as follows:

$$D_i^I = \overline{\upsilon}_i \sum_{a=1}^{m} C_a^{\alpha} = \overline{\upsilon}_i C_T \text{, } i = 1,\ldots,k, \qquad (9)$$

Where

D_i^I = cost assigned to cost object *i* under IABC2; and

$\overline{\upsilon}_i = \frac{\sum_{a=1}^{m} \upsilon_{ia}}{m} = \frac{\sum_{a=1}^{m} \mathfrak{I}_{ia} / \mathfrak{I}_a}{m}$, the average activity consumption ratio of cost object *i*.

Equation 9 states that the cost of cost object *i* is the average activity consumption ratio multiplied by the total cost. Accordingly, the information set for IABC2 is $\{\mathfrak{I}_{ia}, C_T\}$. For IABC2, the individual activity costs do not have to be known; only the total cost needs to be known and the individual activity consumption ratios. Since the individual activity costs do not have to be known, Stage 1 cost allocation is eliminated, which is a significant simplification and the issue of resource diversity is resolved. Equivalency between D_i^{α} and D_i^{I} is established by the following proposition:

Proposition III: $D_i^{\alpha} = \sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha} = \overline{\upsilon}_i C_T = D_i^{I}$, if and only if there is no correlation between υ_{ia} and C_a^{α} for each cost object *i*, *a* = 1,…, *m* and *i* = 1,…, *k*.

The proof of Proposition III parallels that of Proposition I and is, therefore, omitted.

Let $r_{C\upsilon}$ represent the correlation between υ_{ia} and C_a^{α} for cost object *i*. Parallel to the definition of $r_{c\rho}$, $r_{C\upsilon} = \frac{\sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha} - \overline{\upsilon}_i \sum_{a=1}^{m} C_a^{\alpha}}{\sigma_{\upsilon} \sigma_C}$, where $\sigma_C = \sqrt{\sum_{a=1}^{m} \left(C_a^{\alpha}\right)^2 - \frac{\left(\sum_{a=1}^{m} C_a^{\alpha}\right)^2}{m}}$ and $\sigma_{\upsilon} = \sqrt{\sum_{a=1}^{m} \upsilon_{ia}^2 - \frac{\left(\sum_{a=1}^{m} \upsilon_{ia}\right)^2}{m}}$. Table 8 shows an illustration of Proposition III. The illustration contains two cost objects and four activities. Since $r_{C\upsilon} = 0$, the costs assigned to the cost objects under IABC2 are identical to those under ABC.

TABLE 8
The Accuracy of the IABC2 System

Stage 1

	A1	A2	A3	A4	Resource Cost
Labor 1	0.16	0.26	0.20	0.38	$300,000
Labor 2	0.38	0.53	0.00	0.09	$650,000
Energy	0.20	0.29	0.10	0.41	$750,000
Materials	0.10	0.20	0.30	0.40	$800,000
Cost	$525,000	$800,000	$375,000	$800,000	$2,500,000

Stage 2

	A1	A2	A3	A4	ABC Cost[a]	$r_{C\upsilon}$
CO1	0.20	0.30	0.69	0.80	$1,243,750	0
CO2	0.80	0.70	0.31	0.20	$1,256,250	0
Cost	$525,000	$800,000	$375,000	$800,000	$2,500,000	

	IABC2 Cost[b]	$\overline{\upsilon}_i$
CO1	$1,243,750	0.4975
CO2	$1,256,250	0.5025
Cost	$2,500,000	

[a] $D_i^{\alpha} = \sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha}$

[b] $D_i^{I} = \overline{\upsilon}_i C_T$

TDABC simplifies Stage 1 cost assignment by eliminating the need to know the resource consumption ratios. However, TDABC must calculate the individual activity costs needed for Stage 2 calculations. As shown in Table 8 (which illustrates Proposition III), IABC2 eliminates the need to know activity costs for Stage 2, thus eliminating the potential problem of resource diversity introduced by TDABC. In addition, since IABC2 does not need activity costs, Stage 2 is also simplified. However, the activity consumption ratios have to be found for all activities to calculate the average activity consumption ratios for each cost object. Gathering this information is time-consuming and costly. Thus, further simplification is desirable. The next section analyzes a more desirable method in which TDABC is applied to Stage 2.

4.2. TDABC Applied to Stage 2 and Model Definition

A more desirable method is to develop a simplification of Stage 2 that avoids the need to gather all of the information necessary to calculate the average activity consumption ratios. One approach is to extend TDABC concepts found in Stage 1 to Stage 2. The presence of the IABC2 model suggests the possibility that a TDABC2 model is feasible. The TDABC2 model builds on IABC2 by eliminating the need to know all of the activities and their associated consumption ratios. If TDABC concepts are transferred to Stage 2, then TDABC2 would only require knowledge of the total cost, total time, the unit cycle time, and the number of units of the cost object that will be produced. Thus, TDABC2 is performed by 1) determining the cycle time for one unit of product (e.g. from the time the sales order is received until the finished good goes to the warehouse); 2) determining the number of units that will be produced; and 3) multiplying the cycle time by the number of units that will be produced.

The TDABC2 cost assignment model is as follows:

$$D_i^Z = \frac{\sum_{a=1}^{m} C_a^{\alpha}}{\sum_{a=1}^{m} \Im_a} \sum_{a=1}^{m} \Im_{ia} = c^Z \beta_i \theta_i, \; i = 1, \ldots, k, \qquad (10)$$

Where

D_i^Z = cost of cost object *i* under TDABC2;

$\Im_{ia}$ = time consumed of activity *a* by cost object *i*;

$\mathfrak{I}_a$ = total time of activity *a*;
β_i = unit cycle time for cost object *i*;
θ_i = number of units produced for cost object *i* at practical capacity; and
c^Z = cost per unit of activity time, where Z stands for TDABC2.

Equation 10 states that the total cost assigned to cost object *i* is the unit cycle time multiplied by the number of units produced and then multiplied by the cost per unit of time. The cost per unit of time, c^Z, is simply the total activity cost for the pool divided by the total activity time used by all cost objects:

$$c^Z = \frac{\sum_{a=1}^{m} C_a^{\alpha}}{\sum_{a=1}^{m} \mathfrak{I}_a} = \frac{C_T}{t_T}, \qquad (11)$$

Where
C_T = total overhead cost; and
t_T = total time in the system ($\sum_{a=1}^{m} \mathfrak{I}_a$).

Additionally, from Equation 10, the cycle time multiplied by the number of units produced is the sum of the time consumed of activity *a* by cost object *i* across all *a* (*a* = 1,…, *m*):

$$\sum_{a=1}^{m} \mathfrak{I}_{ia} = \beta_i \theta_i,\ i = 1,\ldots, k \qquad (12)$$

Notice also from Equation 10 that the cost for one unit of a cost object is the unit cycle time multiplied by c^Z, or $c^Z \beta_i$. Hence, the information set for TDABC2 is $\{C_T, t_T, \beta_i, \theta_i\}$. The total overhead cost (C_T), the total time in the system (t_T), and the number of units produced for cost object *i* at practical capacity (θ_i) can be found from the accounting records. The unit cycle time for cost object *i* (β_i) is found by clocking how long it takes from the time the sales order is received until the finished good goes to the warehouse.

4.3. Equivalency Analysis

Similar to the analysis of TDABC with the benchmark ABC for Stage 1, the assumptions behind the necessary equivalency conditions are linearity and that the Stage 2 cost assignments are duration based in benchmark ABC system. Conditions needed to establish equivalency between TDABC2 and ABC are derived from extending Proposition II and its corollaries to Stage 2. Like in Proposition II for TDABC, for TDABC2 $r_{C\upsilon}$ (the linear correlation between υ_{ia} and C_a^{α} for cost object *i*) and $r_{\Im\upsilon}$ (the linear correlation between υ_{ia} and $\Im_a$ for cost object *i*) are both needed. Parallel to the definition of $r_{t\rho}$, $r_{\Im\upsilon} = \dfrac{\sum_{a=1}^{m}\upsilon_{ia}\Im_a - \bar{\upsilon}_i\sum_{a=1}^{m}\Im_a}{\sigma_\upsilon\sigma_\Im}$, where $\sigma_\Im = \sqrt{\sum_{a=1}^{m}\Im_a^2 - \dfrac{\left(\sum_{a=1}^{m}\Im_a\right)^2}{m}}$ and σ_υ is defined as before. The extension of Proposition II to Stage 2 is shown in the following proposition.

Proposition IV: $D_i^{\alpha} = D_i^{Z}$ if and only if $r_{C\upsilon}\sigma_C = c^Z r_{\Im\upsilon}\sigma_\Im$, $i = 1,\ldots, k$.

The proof is parallel to that of Proposition II and is, therefore, omitted.

From the proof of the above proposition, $D_i^{\alpha} - D_i^{Z} = \sigma_{\upsilon}[r_{C\upsilon}\sigma_C - c^{Z} r_{\Im\upsilon}\sigma_{\Im}]$. Since the dollar value of the error between the two systems equals $D_i^{\alpha} - D_i^{Z}$, then the dollar value of the error can be expressed as follows:

$$\varepsilon_i = \sigma_{\upsilon}[r_{C\upsilon}\sigma_C - c^{Z} r_{\Im\upsilon}\sigma_{\Im}],\ i = 1,\ldots, k \qquad (13)$$

When the error for cost object *i* is zero, there is equivalency, which implies that $r_{C\upsilon}\sigma_C - c^{Z} r_{\Im\upsilon}\sigma_{\Im} = 0$. This expression shows that if $r_{C\upsilon}$ = 0, then $r_{\Im\upsilon}$ = 0. Using the same rationale that establishes equivalency between ABC and TDABC, $r_{\Im\upsilon}$ should also equal zero since this implies that a cost object does not need to consume a higher (lower) proportion of that activity's time if an activity has more (less) time available. Thus, the following corollary to Proposition IV has been proved:

Corollary IVa: If $r_{C\upsilon}$ = 0 and $r_{\Im\upsilon}$ = 0, then $D_i^{\alpha} = D_i^{Z}$, $i = 1,\ldots, k$.

In the event that both $r_{C\upsilon}$ and $r_{\Im\upsilon}$ are nonzero, then it is also possible to establish an equivalency condition based on a required value for c^{Z}. When $\varepsilon_i = \sigma_{\upsilon}[r_{C\upsilon}\sigma_C - c^{Z} r_{\Im\upsilon}\sigma_{\Im}]$ = 0, solving for c^{Z} and simplifying yields the following equivalency condition:

$$c^{Z} = \frac{\sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha} - \overline{\upsilon}_i \sum_{a=1}^{m} C_a^{\alpha}}{\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a} = \frac{D_i^{\alpha} - D_i^{I}}{\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a}$$

This then establishes a second corollary:

Corollary IVb: If $c^{Z} = \dfrac{\sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha} - \overline{\upsilon}_i \sum_{a=1}^{m} C_a^{\alpha}}{\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a} = \dfrac{C_T}{t_T}$, then $D_i^{\alpha} = D_i^{Z}$, i = 1,…, k.

According to Corollary IVb, if the rationale for zero correlation is not valid, it is still possible to obtain equivalency. However, a very special relationship must exist. The numerator $D_i^{\alpha} - D_i^{I}$ is the dollar error between Stage 2 ABC and IABC2 for cost object *i*. The denominator $\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a$ is the unit time error between time allocated to cost object *i* using ABC and the time allocated to cost object *i* using IABC2. Accordingly,

$\dfrac{D_i^{\alpha} - D_i^{I}}{\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a} = c_{\varepsilon}^{Z}$ represents the dollar error per unit of error time for Stage 2.

Again, parallel to c_{ε} in Chapter 3, c_{ε}^{Z} must be written in absolute form since IABC2 cost for cost object *i* could be greater than that of ABC Stage 2.

Table 9 provides an illustration of Corollary IVa in which the cost objects are product lines (P1 and P2). Only Stage 2 is shown of ABC. Table 9 compares TDABC2 and ABC when there is no correlation between υ_{ia} and C_a^{α} and between υ_{ia} and $\Im_a$ for cost object *i*. Notice from Table 9 that once c^{Z} and β_i (the unit cycle time for cost object *i*) are known, the cost per unit of product can be found. Again, β_i is an observed value. If the ABC system is duration-based, β_i must equal the cycle time calculated from the duration-based benchmark ABC system. The cycle time from the duration-based ABC is calculated by dividing the total hours for cost object *i* by the number of units produced at *practical* capacity (e.g. 400/800 = 0.5 unit cycle time for P1). To find the cost of the entire product line, the cost per unit of product is multiplied by the number of units produced (θ_i). Table 9 shows that the cost of each product under TDABC2 (D_1^Z = \$185 and D_2^Z = \$185) are equal to those under ABC since $r_{C\upsilon}$ = 0 and $r_{\Im\upsilon}$ = 0.

TABLE 9
Product Example Illustrating Corollary IVa

ABC

	Activities				
	A1	**A2**	**A3**	**A4**	**Hours**
P1	20	120	180	80	400
P2	80	180	120	20	400
	100	300	300	100	800

	A1	**A2**	**A3**	**A4**	**ABC Cost Assignment**[a]	$\overline{\upsilon}_i$	**IABC2 Cost Assignment**[b]
P1	0.20	0.40	0.60	0.80	\$185	0.50	\$185
P2	0.80	0.60	0.40	0.20	\$185	0.50	\$185
Cost	\$105	\$60	\$120	\$85	\$370		\$370

TDABC2

Total Cost	\$370
Total Hours	800
c^Z	\$0.46

	β_i	**Cost per Unit**	θ_i	**TDABC2 Cost Assignment**[c]	$r_{C\upsilon}$	$r_{\Im\upsilon}$
P1	0.5	\$0.23	800	\$185	0	0
P2	2	\$0.93	200	\$185	0	0
				\$370		

[a] $D_i^a = \sum_{a=1}^{m} \upsilon_{ia} C_a^{\alpha}$

[b] $D_i^I = \overline{\upsilon}_i C_T$

[c] $D_i^Z = c^Z \beta_i \theta_i$

To illustrate that TDABC2 can be applied to something other than products, Table 10 provides another illustration of Corollary IVa in which the cost objects are customers. This is the only customer example that will be shown since the next few product illustrations in this chapter can easily be adapted and applied to customers. Here, β_i represents the order cycle time for cost object *i* (the time from which the order is made to the time the payment is received), and θ_i represents the number of orders for cost object *i*. Table 10 shows that the cost of each customer under TDABC2 (D_1^Z = \$5,200 and D_2^Z = \$6,800) are equal to those under ABC since $r_{C\upsilon}$ = 0 and $r_{\Im\upsilon}$ = 0 for each customer.

TABLE 10
Customer Example Illustrating Corollary IVa

ABC

	Activities			
	A1	**A2**	**A3**	**Days**
Customer1	210	30	280	520
Customer2	490	120	70	680
	700	150	350	1,200

	A1	**A2**	**A3**	**ABC Cost**[a]	$\overline{\upsilon}_i$	**IABC2 Cost**[b]
Customer1	0.30	0.20	0.80	$5,200	0.43	$5,200
Customer2	0.70	0.80	0.20	$6,800	0.57	$6,800
Cost	$7,000	$1,500	$3,500	$12,000		$12,000

TDABC2

Total Cost	$12,000
Total Days	1,200
c^Z	$10

	β_i	**Cost per Order**	θ_i	**TDABC2 Cost**[c]	$r_{C\upsilon}$	$r_{\Im\upsilon}$
Customer1	26	$260	20	$5,200	0	0
Customer2	34	$340	20	$6,800	0	0
				$12,000		

[a] $D_i^a = \sum_{a=1}^{m} \upsilon_{ia} C_a^\alpha$

[b] $D_i^I = \overline{\upsilon}_i C_T$

[c] $D_i^Z = c^Z \beta_i \theta_i$

Corollary IVb of Proposition IV is demonstrated in the following example shown in Table 11. According to Table 11, $r_{C\upsilon}$ and $r_{\Im\upsilon}$ are nonzero and c^Z and the dollar error per unit of time, c_ε^Z, are both equal to $1.17. This satisfies Corollary IVb so that $D_i^\alpha = D_i^Z$, where $D_1^Z = D_1^\alpha$ = $640 and $D_1^Z = D_1^\alpha$ = $760. However, notice that IABC2 provides inaccurate results because Proposition III is violated.

TABLE 11
Example Illustrating Corollary IVb

ABC

	Activities				
	A1	**A2**	**A3**	**A4**	**Hours**
P1	60	183	146	160	549
P2	240	274	97	40	651
	300	457	243	200	1,200

	A1	**A2**	**A3**	**A4**	**ABC Cost Assignment**[a]	$\overline{\upsilon}_i$	**IABC2 Cost Assignment**[b]
P1	0.20	0.40	0.60	0.80	$640	0.50	$700
P2	0.80	0.60	0.40	0.20	$760	0.50	$700
Cost	$500	$200	$500	$200	$1,400		$1,400

TDABC2

Total Cost	$1,400	c^Z	$1.17
Total Hours	1,200	c_ε^Z [c]	$1.17

	β_i	**Cost per Unit**	θ_i	**TDABC2 Cost Assignment**[d]	$r_{C\upsilon}$	$r_{\Im\upsilon}$
P1	0.686	$0.80	800	$640	-0.45	-0.59
P2	3.257	$3.80	200	$760	0.45	0.59
				$1,400		

[a] $D_i^a = \sum_{a=1}^{m} \upsilon_{ia} C_a^\alpha$

[b] $D_i^I = \overline{\upsilon}_i C_T$

[c] $c_\varepsilon^Z = \dfrac{D_i^\alpha - D_i^I}{\sum_{a=1}^{m} \upsilon_{ia}\Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a}$

[d] $D_i^Z = c^Z \beta_i \theta_i$

If Proposition IV does not hold, then there is a difference in the cost assigned to TDABC2 relative to that of ABC Stage 2. Table 12 shows that when there is a nonzero correlation for $r_{C\upsilon}$ and $r_{\Im\upsilon}$ (where $r_{C\upsilon}\sigma_C \neq c^Z r_{\Im\upsilon}\sigma_\Im$), the cost of the cost objects under TDABC2 (D_1^Z = $643 and D_2^Z = $657) are not equal to those under ABC Stage 2 (D_1^α = $465 and D_2^α = $835). The average absolute percentage error of TDABC2 for the illustration is 29.76 percent, with dollar error (ε_i) is \$177.78 for P1 and -\$177.78 for P2 (c^Z = \$0.72 and c_ε^Z = \$1.50).

TABLE 12
Illustration Not Satisfying Proposition IV

ABC

	Activities				
	A1	**A2**	**A3**	**A4**	**Hours**
P1	75	160	175	480	890
P2	225	240	325	120	910
	300	400	500	600	1,800

	A1	**A2**	**A3**	**A4**	**ABC Cost Assignment**[a]	$\overline{\upsilon}_i$	**IABC2 Cost Assignment**[b]
P1	0.25	0.40	0.35	0.80	\$465	0.45	\$585
P2	0.75	0.60	0.65	0.20	\$835	0.55	\$715
Cost	\$500	\$300	\$400	\$100	\$1,300		\$1,300

TDABC2

Total Cost	\$1,300	c^Z	\$0.72
Total Hours	1,800	c_ε^Z [c]	\$1.50

	β_i	**Cost per Unit**	θ_i	**TDABC2 Cost Assignment**[d]	$r_{C\upsilon}$	$r_{\Im\upsilon}$
P1	1.1125	\$0.80	800	\$643	-0.97	0.86
P2	4.55	\$3.29	200	\$657	0.97	-0.86
				\$1,300		

	ABC	**TDABC2**	ε_i	**%** ε_i [e]
P1	\$465	\$643	\$178	38.23%
P2	\$835	\$657	\$178	-21.29%
			Avg $\%\lvert\varepsilon_i\rvert$ [f]	29.76%

[a] $D_i^a = \sum_{a=1}^{m} \upsilon_{ia} C_a^\alpha$

[b] $D_i^I = \overline{\upsilon}_i C_T$

[c] $c_\varepsilon^Z = \dfrac{D_i^\alpha - D_i^I}{\sum_{a=1}^{m} \upsilon_{ia} \Im_a - \overline{\upsilon}_i \sum_{a=1}^{m} \Im_a}$

[d] $D_i^Z = c^Z \beta_i \theta_i$

[e] $\% \varepsilon_i = \dfrac{D_i^Z - D_i^\alpha}{D_i^\alpha}$

[f] $\dfrac{38.23\% + \lvert -21.29\% \rvert}{2}$

If $r_{C\upsilon}$ and $r_{\Im\upsilon}$ are nonzero and TDABC2 and ABC are not equivalent, then the cost per unit of time would not be equal to the dollar error per unit of time. Therefore, $|\varepsilon_i| > 0$. The analysis of the effects of the various variables on the magnitude of the error in Equation 13 is similar to that for Equation 7 under Stage 1. Additional analysis of the dollar error in which the maximum error is identified will be shown in Chapter 6.

4.4. Unused Capacity in TDABC2

Parallel to Stage 1 analysis, unused capacity does not affect the necessary equivalency conditions for TDABC2. For equivalency, the cost of cost object *i* under ABC must equal that under TDABC plus its unused cost. Table 13 shows an example illustrating unused capacity. Table 13 is similar to Table 9 except that unused capacity is included. Notice that Table 13 verifies that the ABC cost for P1 = TDABC cost for P1 + cost of unused capacity ($185 = $173 + $12) and the cost for P2 = TDABC cost for P2 + cost of unused capacity ($185 = $176 + $9). The cost of unused capacity is equal to the cost per unit ($c^Z \beta_i$) multiplied by the difference between the number of units produced at practical capacity (θ_i) and the number of units actually produced (θ_i^A). The rationale is that the unit cycle time β_i is an observed value, so it must remain constant for each unit that is produced. Hence, the unused time is the difference between the total time available at practical capacity ($\beta_i \theta_i$) and the total time actually used ($\beta_i \theta_i^A$). Accordingly, the cost of unused capacity is $c^Z(\beta_i \theta_i - \beta_i \theta_i^A) = c^Z \beta_i (\theta_i - \theta_i^A)$.

TABLE 13
Corollary IVa with Unused Capacity

ABC

	Activities						
	A1	**A2**	**A3**	**A4**	**Hours**	$r_{C\upsilon}$	$r_{\mathfrak{I}\upsilon}$
P1	20	120	180	80	400	0	0
P2	80	180	120	20	400	0	0
	100	300	300	100	800		

	A1	**A2**	**A3**	**A4**	**ABC Cost Assignment**	$\overline{\upsilon}_i$	**IABC2 Cost Assignment**
P1	0.20	0.40	0.60	0.80	$185	0.50	$185
P2	0.80	0.60	0.40	0.20	$185	0.50	$185
Cost	$105	$60	$120	$85	$370		$370

TDABC2

Total Cost	$370
Total Hours	800
c^{Z}	$0.46

	β_i	**Cost per Unit**	θ_i	**TDABC2 Cost Assignment**	θ_i^A	**Unused Cost**	**Total Cost**
P1	0.5	$0.23	750	$173	800	$12	$185
P2	2	$0.93	190	$176	200	$9	$185
				$349		$21	$370

4.5. Implications

As shown analytically, under certain conditions, TDABC2 is equivalent to ABC assignments. TDABC2 has the benefit of IABC2 in which Stage 1 is eliminated and, thus, the problem of resource diversity is eliminated. Additionally, the linear relationship limitation due to poor estimates if there is a nonlinear or discontinuous relation between the demand for and provision of resources (Maher and Marais 1998) has also been resolved since Stage 1 has been eliminated.

TDABC2 is a feasible system in which only the unit cycle time, total time, total cost, and number of units produced need to be known. Accordingly, TDABC2 is as simple as a functional-based costing system but with the accuracy of an ABC system. This will have significant practical relevance. However, if the equivalency conditions are not satisfied, then error will exist. The maximum absolute dollar error for TDABC2 relative to ABC must be identified, but before doing so, the maximum absolute dollar error for TDABC relative to ABC must first be identified as shown in the next chapter and then extended to TDABC2 as shown in Chapter 6.

CHAPTER 5

STAGE 1 ERROR ANALYSIS

5.1. Analysis of the Maximum Error of TDABC

This chapter shows what the maximum absolute dollar error is for TDABC relative to ABC when the conditions in Proposition II and its corollaries are not met. From Chapter 3, the error for activity *a*, which is derived from Proposition II and shown in Equation 7, is $\varepsilon_a = \sigma_\rho[r_{c\rho}\sigma_c - c r_{t\rho}\sigma_t]$. If $r_{c\rho}$ and $r_{t\rho}$ are substituted in Equation 7 (let $\rho_{aj} = \frac{t_{aj}}{t_j}$, $\sum_{j=1}^{n} C_j = C_T$, and $\sum_{j=1}^{n} t_j = t_T$), then $\varepsilon_a = \sum_{j=1}^{n} \frac{C_j}{t_j} t_{aj} - \frac{C_T}{n}\left(\sum_{j=1}^{n} \frac{t_{aj}}{t_j}\right) - c\left[\sum_{j=1}^{n} t_{aj} - \frac{t_T}{n}\left(\sum_{j=1}^{n} \frac{t_{aj}}{t_j}\right)\right]$. Simplifying further yields

$$\varepsilon_a = \sum_{j=1}^{n}\left(\frac{C_j}{t_j} - c\right)t_{aj},\ a = 1,\ldots,m \tag{14}$$

The Stage 1 error analysis is based on one assumption that in any given instance in time, the total cost of resource *j* (C_j) and the total time available for resource *j* (t_j) are likely to be fixed, but the consumption of the resources may vary depending on the activity usage. If C_j and t_j are treated as constants in the system and the consumption of resource *j* by activity *a*, t_{aj}, is allowed to vary, then the following proposition shows that the maximum absolute dollar error of the system is $\sum_{j=1}^{n}\left|\delta_j\right| = \sum_{j=1}^{n}\left|C_j - ct_j\right|$, based on the second assumption that all resources are time driven, where $\delta_j \equiv C_j - ct_j$, which is basically the total dollar error contribution of resource *j*. $\frac{C_j}{t_j} - c$ is the dollar error contribution per unit of time of resource *j* ("unit dollar error contribution of *j*").

Proposition V: Given $\varepsilon_a = \sum_{j=1}^{n}\left(\frac{C_j}{t_j} - c\right)t_{aj}$ and $c \neq \frac{C_j}{t_j}$, the maximum absolute dollar error for the system is $\sum_{j=1}^{n}\left|\delta_j\right| = \sum_{j=1}^{n}\left|C_j - ct_j\right|$.

Proof: The dollar error for activity *a* is $\varepsilon_a = \sum_{j=1}^{n}\left(\frac{C_j}{t_j} - c\right)t_{aj} = \sum_{j=1}^{n}\left(C_j - ct_j\right)\frac{t_{aj}}{t_j}$.

Summing over all *a* yields the dollar error contribution of resource *j*:

$\delta_j = \sum_{a=1}^{m}\left(C_j - ct_j\right)\frac{t_{aj}}{t_j} = \frac{C_j - ct_j}{t_j}\sum_{a=1}^{m} t_{aj} = C_j - ct_j$. The total dollar error contribution of all resources is the sum of all δ_j:

$$\sum_{j=1}^{n}\delta_j = \sum_{j=1}^{n}\left(C_j - ct_j\right) = \sum_{j=1}^{n}C_j - c\sum_{j=1}^{n}t_j = \sum_{j=1}^{n}C_j - \frac{\sum_{j=1}^{n}C_j}{\sum_{j=1}^{n}t_j}\sum_{j=1}^{n}t_j = 0.$$

The total dollar error for all resources is zero implying that some resources provide a positive dollar error contribution and others a negative error contribution, such that

$C_j - ct_j > 0$ and $\frac{C_j}{t_j} - c > 0$ for $j = 1,\ldots, s$ and $C_j - ct_j < 0$ and $\frac{C_j}{t_j} - c < 0$ for $j = s+1,\ldots, n$. Hence, the resources can be partitioned into two sets: one to represent the resources that provide the positive dollar error contribution, $R^+ = \{R_1, R_2, ..., R_s\}$, and the other to represent resources that provide a negative dollar error contribution, $R^- = \{R_{s+1}, R_{s+2}, ..., R_n\}$, where $C_j - ct_j > 0$ ($\frac{C_j}{t_j} - c > 0$) with $j = 1,\ldots, s \in R^+$ and $C_j - ct_j < 0$ ($\frac{C_j}{t_j} - c < 0$) with $j = s+1,\ldots, n \in R^-$. Then, $\sum_{j=1}^{n} \delta_j$ can be rewritten as $\sum_{j=1}^{n} \delta_j = \sum_{j=1}^{s} (C_j - ct_j) + \sum_{j=s+1}^{n} (C_j - ct_j) = 0$. Thus, the total maximum absolute dollar error of the system which is essentially the total absolute dollar error contribution from all resources $j = 1,\ldots, n$ is $\sum_{j=1}^{n} |\delta_j| = \sum_{j=1}^{s} (C_j - ct_j) + \left| \sum_{j=s+1}^{n} (C_j - ct_j) \right| = \sum_{j=1}^{n} |C_j - ct_j|$. **QED**

Notice that the maximum absolute dollar error for the system, $\sum_{j=1}^{n} |\delta_j| = \sum_{j=1}^{n} |C_j - ct_j|$, does not depend on the t_{aj}'s. Maximizing the dollar error based on the t_{aj}'s involves finding a corner solution that is part of the maximum set of corner solutions. To find the set of t_{aj}'s that produces the maximum error, the resources must be ordered from largest to smallest $\frac{C_j}{t_j} - c$, where $\frac{C_1}{t_1} - c > \frac{C_2}{t_2} - c > \ldots > \frac{C_n}{t_n} - c$ in addition to partitioning the resources into R^+ and R^- sets. The "unit dollar error contribution" is a better measure of magnitude than $C_j - ct_j$ (total dollar error contribution of resource *j*) since the amount of time *could* vary from one resource to the next causing the corresponding $C_j - ct_j$ to not be of the same magnitude as the corresponding $\frac{C_j}{t_j} - c$. Next, partition the activities into two sets in which some activities consume resources that provide a positive dollar error contribution (A^+ set) and the rest of the activities consume resources that provide a negative dollar error contribution (A^- set):

$A^+ = \{a \mid a = 1,\ldots, q;\ \varepsilon_{aj} = (C_j - ct_j)\frac{t_{aj}}{t_j},\ j \in R^+$ and $\varepsilon_{aj} = (C_j - ct_j)\frac{t_{aj}}{t_j} = 0$ where $t_{aj} = 0$ if $j \in R^-\}$; and

$A^- = \{a \mid a = q+1,\ldots, m;\ \varepsilon_{aj} = (C_j - ct_j)\frac{t_{aj}}{t_j},\ j \in R^-$ and $\varepsilon_{aj} = (C_j - ct_j)\frac{t_{aj}}{t_j} = 0$ where $t_{aj} = 0$ if $j \in R^+\}$, where $A^+ \cup A^- = A$.

The above expressions for A^+ and A^- sets state that to maximize the dollar error using t_{aj} 's, activities in A^+ must only consume resources in R^+ and activities in A^- only consume resources in R^-. As a result, the t_{aj} 's that result in $\sum_{j=1}^{n}\left|\delta_j\right| = \sum_{j=1}^{n}\left|C_j - ct_j\right|$ is identified in the following corollary to Proposition V.

Corollary Va: The t_{aj} 's that produce the total maximum absolute dollar error of the system, $\sum_{j=1}^{n}\left|\delta_j\right| = \sum_{j=1}^{n}\left|C_j - ct_j\right|$, is $\sum_{a=1}^{m}\left|\varepsilon_a\right| = \sum_{a=1}^{m}\sum_{j=1}^{n}\left|(C_j - ct_j)\frac{t_{aj}}{t_j}\right|$.

Proof: Choose t_{aj} with $j = 1,\ldots, s \in R^{+}$, such that $\sum_{a=1}^{q} t_{aj} = t_j$ and t_{aj} with

$j = s+1,\ldots, n \in R^{-}$, such that $\sum_{a=q+1}^{m} t_{aj} = t_j$. As a result, the dollar error from the A^{+} set is

$\varepsilon_a^{+} = \sum_{j=1}^{s} \varepsilon_{aj} = \sum_{j=1}^{s} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j} > 0$ and from the A^{-} set is

$\varepsilon_a^{-} = \sum_{j=s+1}^{n} \varepsilon_{aj} = \sum_{j=s+1}^{n} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j} < 0$. Hence, $\sum_{a=1}^{m} \varepsilon_a = \sum_{a=1}^{q} \varepsilon_a^{+} + \sum_{a=q+1}^{m} \varepsilon_a^{-} = 0$. This means that

$\sum_{a=1}^{q} \varepsilon_a^{+} = \sum_{a=1}^{q}\sum_{j=1}^{s} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j} = \sum_{j=1}^{s} \left(C_j - ct_j\right) > 0$ using $\sum_{a=1}^{q} t_{aj} = t_j$ and

$\sum_{a=q+1}^{m} \varepsilon_a^{-} = \sum_{a=q+1}^{m}\sum_{j=s+1}^{n} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j} = \sum_{j=s+1}^{n} \left(C_j - ct_j\right) < 0$ using $\sum_{a=q+1}^{m} t_{aj} = t_j$. Thus, using these

expressions for $\sum_{a=1}^{q} \varepsilon_a^{+}$ and $\sum_{a=q+1}^{m} \varepsilon_a^{-}$,

$\sum_{a=1}^{m} \left|\varepsilon_a\right| = \sum_{a=1}^{q}\sum_{j=1}^{s} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j} + \left|\sum_{a=q+1}^{m}\sum_{j=s+1}^{n} \left(C_j - ct_j\right)\frac{t_{aj}}{t_j}\right| = \sum_{a=1}^{m}\sum_{j=1}^{n} \left|(C_j - ct_j)\frac{t_{aj}}{t_j}\right|$, which can be

rewritten as $\sum_{a=1}^{m} \left|\varepsilon_a\right| = \sum_{j=1}^{s} \left(C_j - ct_j\right) + \left|\sum_{j=s+1}^{n} \left(C_j - ct_j\right)\right| = \sum_{j=1}^{n} \left|C_j - ct_j\right| = \sum_{j=1}^{n} \left|\delta_j\right|$.

Since there are no cancellation effects of positive resource dollar error contributions with negative resource dollar error contributions for any a,

$\sum_{a=1}^{m} \left|\varepsilon_a\right| = \sum_{a=1}^{m}\sum_{j=1}^{n} \left|(C_j - ct_j)\frac{t_{aj}}{t_j}\right| = \sum_{j=1}^{n} \left|C_j - ct_j\right| = \sum_{j=1}^{n} \left|\delta_j\right|$. **QED**

Although $\sum_{a=1}^{m}|\varepsilon_a| = \sum_{j=1}^{n}|\delta_j|$, a program is identified that provides the maximum percentage error of each activity *a* that maximizes the average absolute percentage error of the system.[3] The percentage error for any given activity *a* is $\%\varepsilon_a = \frac{\varepsilon_a}{C_a^\alpha}$, which is the dollar error for activity *a* divided by the ABC cost for activity *a*. To find the maximum average absolute percentage error, one program is used for the positive sets, A^+ and R^+, and another program for the negative sets, A^- and R^-. From Proposition V and Corollary Va, let m^+ represent the number of activities in A^+ ($a = 1,\ldots, q$) and n^+ represent the number of resources in R^+ ($j = 1,\ldots, s$). After ordering all resources from largest positive to smallest positive $\frac{C_j}{t_j} - c$ and labeling them as $R_1, R_2,\ldots, R_s$, where $\frac{C_1}{t_1} - c > \frac{C_2}{t_2} - c > \ldots > \frac{C_s}{t_s} - c$, the most positive resource, R_1, must be the only resource consumed by $m^+ - (n^+ - 1)$ activities. The program for the maximization of the percentage errors for each activity in the A^+ set is as follows:

$$\text{Max} \sum_{a=1}^{q}\left(\frac{\sum_{j=1}^{s}\left(\frac{C_j}{t_j} - c\right)t_{aj}}{\sum_{j=1}^{s}\frac{t_{aj}}{t_j}C_j}\right) \qquad \text{(P1)}$$

s.t.

$$t_{a,1} \geq t_1\gamma,\ a = 1,\ldots, q\text{-}1 \qquad \text{(P2)}$$

$$t_{aj} \geq t_j\gamma,\ a = 1 + m^+ - (n^+ - 1),\ldots, q\text{-}1,\ j = 2,\ldots, s\text{-}1 \qquad \text{(P3)}$$

[3] An optimization software program such as LINGO can be used.

All other $t_{aj} \geq 0$, $j \in R^{+}$ and $a \in A^{+}$ (P4)

$$\sum_{a=1}^{q} t_{aj} = t_j \qquad \text{(P5)}$$

The objective function in (P1) provides the maximum percentage error in magnitude across all activities in A^{+}. (P2) is the first constraint that ensures that for R_1, at least the amount of $t_1\gamma$ is assigned to q-1 activities in A^{+}. The materiality and uniqueness parameter γ (e.g. it can be set as 0.1 to ensure that 10 percent of the time for resource j is assigned to the appropriate activities) is only assigned to q-1 activities instead of q to not over restrict the program and allow it to choose the optimal set of t_{aj}'s. (P3) is the second constraint that ensures uniqueness among resource vectors without over-restricting the program to allow for some t_{aj} in A^{+} and R^{+} to be zero or for some activities in A^{+} to consume all of a single resource in R^{+} (this is represented by (P4)). (P5) ensures that the sum of the activity consumptions in A^{+} of a particular resource in R^{+} is equal to the total time available for that particular resource j.

From Proposition V and its Corollary Va, let m^{-} represent the number of activities in A^{-} ($a = q$+1,…, m) and n^{-} represent the number of resources in R^{-} ($j = s$+1,…, n). First order all resources from least negative to most negative $\frac{C_j}{t_j} - c$ and label them as R_{s+1}, R_{s+2},…, R_n, where $\frac{C_{s+1}}{t_{s+1}} - c > \frac{C_{s+2}}{t_{s+2}} - c > \ldots > \frac{C_n}{t_n} - c$. The most negative R_n must be the only resource consumed by $m^{-} - (n^{-} - 1)$ activities. The program for the maximization the *magnitude* of the percentage errors in the R^- set is as follows (this involves minimizing because of dealing with negative values for $\frac{C_j}{t_j} - c$):

$$\text{Min} \sum_{a=q+1}^{m} \left(\frac{\sum_{j=s+1}^{n} \left(\frac{C_j}{t_j} - c \right) t_{aj}}{\sum_{j=s+1}^{n} \frac{t_{aj}}{t_j} C_j} \right) \quad \text{(P6)}$$

s.t.

$$t_{a,n} \geq t_n \gamma \text{ , } a = q+2,\ldots, m \quad \text{(P7)}$$

$$t_{aj} \geq t_j \gamma \text{ , } a = q+1,\ldots, m - m^- - (n^- - 1), \; j = s+2,\ldots, n-1 \quad \text{(P8)}$$

$$\text{All other } t_{aj} \geq 0 \text{ in } j \in R^- \text{ and } a \in A^- \quad \text{(P9)}$$

$$\sum_{a=q+1}^{m} t_{aj} = t_j \quad \text{(P10)}$$

The objective function in (P6) provides the maximum percentage error in magnitude across all activities in A^-. (P7) is the first constraint that ensures that for R_n, at least the amount of $t_n\gamma$ is assigned to $a = q+2,\ldots, m$ activities in A^- without over restricting the program and allow it to choose the optimal set of t_{aj}'s. (P8) is the second constraint that ensures uniqueness among resource vectors without over-restricting the program to allow for some t_{aj} in A^- and R^- to be zero or for some activities in A^- to consume all of a single resource in R^- (this is represented by (P9)). (P10) ensures that the sum of the activity consumptions in A^- of a particular resource in R^- is equal to the total time available for that particular resource *j*.

Typically, an activity will consume resources from both R^+ and R^-, thus allowing for some cancellation effects from the positive dollar error contribution from resources in R^+ and negative dollar error contribution from resources in R^-. Since

$\varepsilon_a^+ = \sum_{j=1}^{s}(C_j - ct_j)\frac{t_{aj}}{t_j} = \sum_{j=1}^{s}\delta_j \frac{t_{aj}}{t_j}$ and $\sum_{a=1}^{q}\varepsilon_a^+ = \sum_{a=1}^{q}\sum_{j=1}^{s}(C_j - ct_j)\frac{t_{aj}}{t_j} = \sum_{a=1}^{q}\sum_{j=1}^{s}\delta_j \frac{t_{aj}}{t_j} = \sum_{j=1}^{s}\delta_j$

with $C_j - ct_j > 0$ ($\frac{C_j}{t_j} - c > 0$) and $\varepsilon_a^- = \sum_{j=s+1}^{n}(C_j - ct_j)\frac{t_{aj}}{t_j} = \sum_{j=s+1}^{n}\delta_j \frac{t_{aj}}{t_j}$ and

$\sum_{a=q+1}^{m}\varepsilon_a^- = \sum_{a=q+1}^{m}\sum_{j=s+1}^{n}(C_j - ct_j)\frac{t_{aj}}{t_j} = \sum_{a=q+1}^{m}\sum_{j=s+1}^{n}\delta_j \frac{t_{aj}}{t_j} = \sum_{j=s+1}^{n}\delta_j$ with $C_j - ct_j < 0$ ($\frac{C_j}{t_j} - c < 0$),

then if any *a* in A^+ ($a = 1,\ldots, q$) consumes any resource *j* in R^- ($j = s+1,\ldots, n$), then there is a cancellation effect of negative resource error contributions with positive resource error contributions; thus, $\sum_{a=1}^{q}\varepsilon_a < \sum_{a=1}^{q}\varepsilon_a^+$. Likewise, if any *a* in A^- ($a = q+1,\ldots, m$) consumes any resource *j* in R^+ ($j = 1,\ldots, s$), then there is a cancellation effect of positive resource error contributions with negative resource error contributions; consequently, $\sum_{a=q+1}^{m}|\varepsilon_a| < \sum_{a=q+1}^{m}|\varepsilon_a^+|$. As a result, when there are any cancellation effects, $\sum_{a=1}^{m}|\varepsilon_a| < \sum_{j=1}^{n}|\delta_j|$, which implies that, $|\varepsilon_a| = \left|\sum_{j=1}^{n}\delta_j \frac{t_{aj}}{t_j}\right| \le \sum_{j=1}^{n}\left|\delta_j \frac{t_{aj}}{t_j}\right| = \sum_{j=1}^{n}\frac{t_{aj}}{t_j}|\delta_j|$. Hence, $\sum_{a=1}^{m}|\varepsilon_a|$ should be less than $\sum_{j=1}^{n}|\delta_j|$ as shown in the following corollary to Proposition V.

Corollary Vb: As already derived, if activities in A^+ only consume resources in R^+ and activities in A^- only consume resources in R^-, then $\sum_{a=1}^{m}|\varepsilon_a| = \sum_{j=1}^{n}|\delta_j|$. If any or all activities consume resources from both R^+ and R^-, then $\sum_{a=1}^{m}|\varepsilon_a| < \sum_{j=1}^{n}|\delta_j|$. Therefore,

$\sum_{a=1}^{m}|\varepsilon_a|$ can never exceed $\sum_{j=1}^{n}|\delta_j|$, implying $\sum_{a=1}^{m}|\varepsilon_a| \le \sum_{j=1}^{n}|\delta_j|$.

Proof: By using the triangle inequality $|\varepsilon_1| + |\varepsilon_2| + ... + |\varepsilon_m| \le |\delta_1| + |\delta_2| + ... + |\delta_n|$,

$\sum_{a=1}^{m}|\varepsilon_a| \le \sum_{a=1}^{m}\sum_{j=1}^{n}\frac{t_{aj}}{t_j}|\delta_j| = \sum_{j=1}^{n}|\delta_j| \Rightarrow \sum_{a=1}^{m}|\varepsilon_a| \le \sum_{j=1}^{n}|\delta_j|$. **QED**

If there are no cancellation effects (activities in A^+ only consume resources in R^+ and activities in A^- only consume resources in R^-), then $\sum_{a=1}^{m}|\varepsilon_a| = \sum_{j=1}^{n}|\delta_j|$. Since $C_j - ct_j > 0$ ($\frac{C_j}{t_j} - c > 0$) for $j \in R^+$ and $C_j - ct_j < 0$ ($\frac{C_j}{t_j} - c < 0$) for $j \in R^-$, if any or all activities consume resources from both R^+ and R^-, then there will be some cancellation effects within each of those ε_a's, and thus, the actual absolute dollar error across all activities will be less than the maximum absolute dollar error of the system, $\sum_{a=1}^{m}|\varepsilon_a| < \sum_{j=1}^{n}|\delta_j|$. Thus, $\sum_{a=1}^{m}|\varepsilon_a|$ can never exceed $\sum_{j=1}^{n}|\delta_j|$.

5.2. Examples Demonstrating Proposition V and Its Corollaries

Examples demonstrating Proposition V and its corollaries are shown in this section. Corollary Va will be shown first. Tables 14, 15, and 16 demonstrate examples of Corollary Va for *m* = *n*, *m* < *n*, and *m* > *n*, respectively. All three cases are shown to demonstrate that Proposition V and Corollary Va as well as program (P1) through (P10) are viable in each situation. The resources are ranked from largest to smallest $\frac{C_j}{t_j} - c$ and the resources are partitioned accordingly. For each table, the first three activities (A1 through A3) are in the A^+ set, and the rest of the activities are in the A^- set. In each table, $\sum_{a=1}^{m} |\varepsilon_a| = \sum_{j=1}^{n} |\delta_j|$ = \$1,700, thus satisfying Corollary Va. Program (P1) through (P10) is used to maximize each $\%\varepsilon_a$, which maximizes average $\% \sum_{a=1}^{m} |\varepsilon_a|$.

Table 14
Corollary Va for $m = n$

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0	0	0	0	0	$200	-0.61	-0.61
A2	0.42	0.10	0	0	0	0	$1,069	-0.70	-0.70
A3	0.48	0.90	1	0	0	0	$5,231	-0.70	-0.70
A4	0	0	0	1	0.90	0.56	$6,598	0.68	0.68
A5	0	0	0	0	0.10	0.34	$1,302	0.86	0.86
A6	0	0	0	0	0	0.10	$300	0.74	0.74
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	0	0	0	0	0	77.5	$2	$155
t_{2j}	328.96	97.5	0	0	0	0	426.46		$853
t_{3j}	368.5	877.5	1,075	0	0	0	2,321		$4,642
t_{4j}	0	0	0	1,275	1,327.5	986.7	3,589.2		$7,178
t_{5j}	0	0	0	0	147.5	610.8	758.3		$1,517
t_{6j}	0	0	0	0	0	177.5	177.5		$355
									$14,700

Table 14 (continued)

	Resources					
	R1	**R2**	**R3**	**R4**	**R5**	**R6**
C_j/t_j	$2.58	$2.26	$2.14	$1.96	$1.83	$1.69
$C_j/t_j - c$	$0.581	$0.256	$0.140	-$0.039	-$0.169	-$0.310
$C_j - ct_j$	$450	$250	$150	-$50	-$250	-$550
$\sum_{j=1}^{n} \lvert \delta_j \rvert$	$1,700					

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	$\lvert \varepsilon_a \rvert$	$\% \lvert \varepsilon_a \rvert$
$(C_j / t_j - c_j)t_{1j}$	$45	$0	$0	$0	$0	$0	$45	$45	22.50%
$(C_j / t_j - c_j)t_{2j}$	$191	$25	$0	$0	$0	$0	$216	$216	20.21%
$(C_j / t_j - c_j)t_{3j}$	$214	$225	$150	$0	$0	$0	$589	$589	11.26%
$(C_j / t_j - c_j)t_{4j}$	$0	$0	$0	-$50	-$225	-$306	-$581	$581	8.80%
$(C_j / t_j - c_j)t_{5j}$	$0	$0	$0	$0	-$25	-$189	-$214	$214	16.45%
$(C_j / t_j - c_j)t_{6j}$	$0	$0	$0	$0	$0	-$55	-$55	$55	18.33%
	$450	$250	$150	-$50	-$250	-$550	$0	$1,700	
								Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	16.26%

Table 15
Corollary Va for *m* < *n*

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0	0	0	0	0	$200	-0.61	-0.61
A2	0.42	0.10	0	0	0	0	$1,069	-0.70	-0.70
A3	0.48	0.90	1	0	0	0	$5,231	-0.70	-0.70
A4	0	0	0	1	0.90	0.64	$6,848	0.72	0.72
A5	0	0	0	0	0.10	0.36	$1,352	0.85	0.85
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	0	0	0	0	0	77.5	$2	$155
t_{2j}	328.96	97.5	0	0	0	0	426.46		$853
t_{3j}	368.5	877.5	1,075	0	0	0	2,321		$4,642
t_{4j}	0	0	0	1,275	1,327.5	1,134.96	3,737.46		$7,475
t_{5j}	0	0	0	0	147.5	640	787.5		$1,575
									$14,700
C_j/t_j	$2.58	$2.26	$2.14	$1.96	$1.83	$1.69			
$C_j/t_j - c$	$0.581	$0.256	$0.140	-$0.039	-$0.169	-$0.310			
$C_j - ct_j$	$450	$250	$150	-$50	-$250	-$550			

Table 15 (continued)

$\sum_{j=1}^{n} |\delta_j|$ $1,700

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	$\lvert\varepsilon_a\rvert$	$\%\lvert\varepsilon_a\rvert$
$(C_j / t_j - c_j)t_{1j}$	$45	$0	$0	$0	$0	$0	$45	$45	22.50%
$(C_j / t_j - c_j)t_{2j}$	$191	$25	$0	$0	$0	$0	$216	$216	20.21%
$(C_j / t_j - c_j)t_{3j}$	$214	$225	$150	$0	$0	$0	$589	$589	11.26%
$(C_j / t_j - c_j)t_{4j}$	$0	$0	$0	-$50	-$225	-$352	-$627	$627	9.15%
$(C_j / t_j - c_j)t_{5j}$	$0	$0	$0	$0	-$25	-$198	-$223	$223	16.52%
	$450	$250	$150	-$50	-$250	-$550	$0	$1,700	
								Avg $\sum_{a=1}^{m} \lvert\%\varepsilon_a\rvert$	15.93%

Table 16
Corollary Va for $m > n$

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0	0	0	0	0	$200	-0.61	-0.61
A2	0.42	0.10	0	0	0	0	$1,069	-0.70	-0.70
A3	0.48	0.90	1	0	0	0	$5,231	-0.70	-0.70
A4	0	0	0	1	0.90	0.47	$6,347	0.63	0.63
A5	0	0	0	0	0.10	0.33	$1,253	0.86	0.86
A6	0	0	0	0	0	0.10	$300	0.74	0.74
A7	0	0	0	0	0	0.10	$300	0.74	0.74
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	0	0	0	0	0	77.5	$2	$155
t_{2j}	328.96	97.5	0	0	0	0	426.46		$853
t_{3j}	368.5	877.5	1,075	0	0	0	2,321		$4,642
t_{4j}	0	0	0	1,275	1,327.5	838.5	3,441		$6,882
t_{5j}	0	0	0	0	147.5	581.5	729		$1,458
t_{6j}	0	0	0	0	0	177.5	177.5		$355
t_{7j}	0	0	0	0	0	177.5	177.5		$355
									$14,700

Table 16 (continued)

	Resources					
	R1	**R2**	**R3**	**R4**	**R5**	**R6**
C_j / t_j	$2.58	$2.26	$2.14	$1.96	$1.83	$1.69
$C_j / t_j - c$	$0.581	$0.256	$0.140	-$0.039	-$0.169	-$0.310
$C_j - ct_j$	$450	$250	$150	-$50	-$250	-$550
$\sum_{j=1}^{n} \lvert \delta_j \rvert$	$1,700					

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	$\lvert \varepsilon_a \rvert$	$\% \lvert \varepsilon_a \rvert$
$(C_j / t_j - c_j)t_{1j}$	$45	$0	$0	$0	$0	$0	$45	$45	22.50%
$(C_j / t_j - c_j)t_{2j}$	$191	$25	$0	$0	$0	$0	$216	$216	20.21%
$(C_j / t_j - c_j)t_{3j}$	$214	$225	$150	$0	$0	$0	$589	$589	11.26%
$(C_j / t_j - c_j)t_{4j}$	$0	$0	$0	-$50	-$225	-$260	-$535	$535	8.43%
$(C_j / t_j - c_j)t_{5j}$	$0	$0	$0	$0	-$25	-$180	-$205	$205	16.38%
$(C_j / t_j - c_j)t_{6j}$	$0	$0	$0	$0	$0	-$55	-$55	$55	18.33%
$(C_j / t_j - c_j)t_{7j}$	$0	$0	$0	$0	$0	-$55	-$55	$55	18.33%
	$450	$250	$150	-$50	-$250	-$550	$0	$1,700	
								Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	16.49%

Illustrations of Corollary Vb are shown in Tables 17, 18, and 19 for $m = n$, $m < n$, and $m > n$, respectively. The resources are ranked from largest to smallest $\frac{C_j}{t_j} - c$ and the resources are partitioned accordingly. As previously, for each table, the first three activities (A1 through A3) are in the A^+ set, and the rest of the activities are in the A^- set. In each table, all activities share all of the resources so that $\sum_{a=1}^{m} |\varepsilon_a| < \sum_{j=1}^{n} |\delta_j|$ = \$1,700. Table 17 shows that the average $\sum_{a=1}^{m} |\%\varepsilon_a|$ = 1.90 percent compared to 16.26 percent in Table 14. Compared to Table 15 with an average $\sum_{a=1}^{m} |\%\varepsilon_a|$ = 15.93 percent, Table 18 shows that the average $\sum_{a=1}^{m} |\%\varepsilon_a|$ = 2.18 percent. Table 19 shows that the average $\sum_{a=1}^{m} |\%\varepsilon_a|$ = 1.77 percent compared to 16.49 percent in Table 16. Notice that in each table, each $\%|\varepsilon_a|$ is less than 5 percent, which demonstrates that the actual maximum error is small since activities will typically consume resources in both R^+ and R^- sets.

Table 17
Corollary Vb for $m = n$

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0.20	0.10	0.30	0.20	0.30	$3,060	0.74	0.74
A2	0.30	0.10	0.10	0.10	0.20	0.10	$2,140	-0.43	-0.43
A3	0.20	0.30	0.10	0.10	0.10	0.20	$2,410	-0.27	-0.27
A4	0.10	0.20	0.10	0.10	0.30	0.10	$2,230	0.17	0.17
A5	0.10	0.10	0.40	0.20	0.10	0.10	$2,410	-0.18	-0.18
A6	0.20	0.10	0.20	0.20	0.10	0.20	$2,450	0.00	0.00
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	195	107.5	382.5	295	532.5	1,590	$2	$3,180
t_{2j}	232.5	97.5	107.5	127.5	295	177.5	1,037.5		$2,075
t_{3j}	155	292.5	107.5	127.5	147.5	355	1,185		$2,370
t_{4j}	77.5	195	107.5	127.5	442.5	177.5	1,127.5		$2,255
t_{5j}	77.5	97.5	430	255	147.5	177.5	1,185		$2,370
t_{6j}	155	97.5	215	255	147.5	355	1,225		$2,450
									$14,700

Table 17 (continued)

	Resources					
	R1	**R2**	**R3**	**R4**	**R5**	**R6**
C_j / t_j	\$2.58	\$2.26	\$2.14	\$1.96	\$1.83	\$1.69
$C_j / t_j - c$	\$0.581	\$0.256	\$0.140	-\$0.039	-\$0.169	-\$0.310
$C_j - ct_j$	\$450	\$250	\$150	-\$50	-\$250	-\$550
$\sum_{j=1}^{n} \lvert \delta_j \rvert$	\$1,700					

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	$\lvert \varepsilon_a \rvert$	$\% \lvert \varepsilon_a \rvert$
$(C_j / t_j - c_j)t_{1j}$	\$45	\$50	\$15	-\$15	-\$50	-\$165	-\$120	\$120	3.92%
$(C_j / t_j - c_j)t_{2j}$	\$135	\$25	\$15	-\$5	-\$50	-\$55	\$65	\$65	3.04%
$(C_j / t_j - c_j)t_{3j}$	\$90	\$75	\$15	-\$5	-\$25	-\$110	\$40	\$40	1.66%
$(C_j / t_j - c_j)t_{4j}$	\$45	\$50	\$15	-\$5	-\$75	-\$55	-\$25	\$25	1.12%
$(C_j / t_j - c_j)t_{5j}$	\$45	\$25	\$60	-\$10	-\$25	-\$55	\$40	\$40	1.66%
$(C_j / t_j - c_j)t_{6j}$	\$90	\$25	\$30	-\$10	-\$25	-\$110	\$0	\$0	0.00%
	\$450	\$250	\$150	-\$50	-\$250	-\$550	\$0	\$290	
							Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$		1.90%

Table 18
Corollary Vb for $m < n$

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0.30	0.10	0.30	0.40	0.20	$3,520	0.41	0.41
A2	0.30	0.10	0.10	0.20	0.10	0.40	$3,020	0.35	0.35
A3	0.20	0.30	0.10	0.10	0.30	0.10	$2,650	-0.25	-0.25
A4	0.10	0.20	0.10	0.30	0.10	0.10	$2,190	-0.10	-0.10
A5	0.30	0.10	0.60	0.10	0.10	0.20	$3,320	-0.31	-0.31
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	292.5	107.5	382.5	590	355	1,805	$2	$3,610
t_{2j}	232.5	97.5	107.5	255	147.5	710	1,550		$3,100
t_{3j}	155	292.5	107.5	127.5	442.5	177.5	1,302.5		$2,605
t_{4j}	77.5	195	107.5	382.5	147.5	177.5	1,087.5		$2,175
t_{5j}	232.5	97.5	645	127.5	147.5	355	1,605		$3,210
									$14,700
C_j/t_j	$2.58	$2.26	$2.14	$1.96	$1.83	$1.69			
$C_j/t_j - c$	$0.581	$0.256	$0.140	-$0.039	-$0.169	-$0.310			
$C_j - ct_j$	$450	$250	$150	-$50	-$250	-$550			

Table 18 (continued)

$\sum_{j=1}^{n} \left| \delta_j \right|$ $1,700

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	$\left\| \varepsilon_a \right\|$	$\% \left\| \varepsilon_a \right\|$
$(C_j / t_j - c_j) t_{1j}$	$45	$75	$15	-$15	-$100	-$110	-$90	$90	2.56%
$(C_j / t_j - c_j) t_{2j}$	$135	$25	$15	-$10	-$25	-$220	-$80	$80	2.65%
$(C_j / t_j - c_j) t_{3j}$	$90	$75	$15	-$5	-$75	-$55	$45	$45	1.70%
$(C_j / t_j - c_j) t_{4j}$	$45	$50	$15	-$15	-$25	-$55	$15	$15	0.68%
$(C_j / t_j - c_j) t_{5j}$	$135	$25	$90	-$5	-$25	-$110	$110	$110	3.31%
	$450	$250	$150	-$50	-$250	-$550	$0	$340	
							Avg $\sum_{a=1}^{m} \left\| \% \varepsilon_a \right\|$		2.18%

Table 19
Corollary Vb for *m* > *n*

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**ABC**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0.10	0.40	0.10	0.10	0.10	$2,160	-0.20	-0.20
A2	0.30	0.10	0.10	0.10	0.10	0.10	$1,870	-0.61	-0.61
A3	0.20	0.20	0.10	0.10	0.10	0.30	$2,490	0.27	0.27
A4	0.10	0.10	0.10	0.40	0.30	0.10	$2,760	0.27	0.27
A5	0.10	0.20	0.10	0.10	0.20	0.10	$1,960	0.00	0.00
A6	0.10	0.10	0.10	0.10	0.10	0.10	$1,470	0.00	0.00
A7	0.10	0.20	0.10	0.10	0.10	0.20	$1,990	0.32	0.32
Cost	$2,000	$2,200	$2,300	$2,500	$2,700	$3,000	$14,700		
Time	775	975	1,075	1,275	1,475	1,775	7,350		

	Resources								
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**Total**	**c**	**TDABC**
t_{1j}	77.5	97.5	430	127.5	147.5	177.5	1,057.5	$2	$2,115
t_{2j}	232.5	97.5	107.5	127.5	147.5	177.5	890		$1,780
t_{3j}	155	195	107.5	127.5	147.5	532.5	1,265		$2,530
t_{4j}	77.5	97.5	107.5	510	442.5	177.5	1,412.5		$2,825
t_{5j}	77.5	195	107.5	127.5	295	177.5	980		$1,960
t_{6j}	77.5	97.5	107.5	127.5	147.5	177.5	730		$1,470
t_{7j}	77.5	195	107.5	127.5	147.5	355	1,010		$2,020
									$14,700

Table 19 (continued)

	Resources					
	R1	R2	R3	R4	R5	R6
C_j / t_j	$2.58	$2.26	$2.14	$1.96	$1.83	$1.69
$C_j / t_j - c$	$0.581	$0.256	$0.140	-$0.039	-$0.169	-$0.310
$C_j - ct_j$	$450	$250	$150	-$50	-$250	-$550
$\sum_{j=1}^{n} \lvert \delta_j \rvert$	$1,700					

	Resources								
	R1	R2	R3	R4	R5	R6	Total	$\lvert \varepsilon_a \rvert$	$\% \lvert \varepsilon_a \rvert$
$(C_j / t_j - c_j) t_{1j}$	$45	$25	$60	-$5	-$25	-$55	$45	$45	2.08%
$(C_j / t_j - c_j) t_{2j}$	$135	$25	$15	-$5	-$25	-$55	$90	$90	4.81%
$(C_j / t_j - c_j) t_{3j}$	$90	$50	$15	-$5	-$25	-$165	-$40	$40	1.61%
$(C_j / t_j - c_j) t_{4j}$	$45	$25	$15	-$20	-$75	-$55	-$65	$65	2.36%
$(C_j / t_j - c_j) t_{5j}$	$45	$50	$15	-$5	-$50	-$55	$0	$0	0.00%
$(C_j / t_j - c_j) t_{6j}$	$45	$25	$15	-$5	-$25	-$55	$0	$0	0.00%
$(C_j / t_j - c_j) t_{7j}$	$45	$50	$15	-$5	-$25	-$110	-$30	$30	1.51%
	$450	$250	$150	-$50	-$250	-$550	$0	$1,700	
								Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	1.77%

5.3. Implications

This chapter analyzes and demonstrates that if activities consume resources from both R^+ and R^- sets (and they most likely will), then the average $\sum_{a=1}^{m} \left| \%\varepsilon_a \right|$ is significantly lower than if activities from A^+ only consume resources in R^+ and activities in A^- only consume resources in R^-. Additionally, the percentage error for each activity is not significant (e.g. less than 5 percent in Tables 17, 18, and 19). Hence, for Stage 1, TDABC is not significantly different from ABC provided that there is no resource diversity. However, previous discussion has shown that there could be a potentially significant error when resource diversity exists. TDABC2 eliminates this resource diversity issue and significantly reduces the complexity of Stage 2 cost assignments. The error analysis in this chapter is extended to Stage 2 in the next chapter to show the maximum absolute dollar error for TDABC2 relative to Stage 2 of ABC.

CHAPTER 6

STAGE 2 ERROR ANALYSIS

6.1. Analysis of the Maximum Error of TDABC2

If the equivalency conditions of Proposition IV and its corollaries are not satisfied, then error is introduced, and it is necessary to determine the maximum error possible. This chapter identifies the maximum absolute dollar error of TDABC2 relative to ABC and all analytics are parallel to those of Chapter 5. From Chapter 4, the error for activity *a*, which is derived from Proposition IV and shown in Equation 13, is $\varepsilon_i = \sigma_\upsilon [r_{C\upsilon}\sigma_C - c^Z r_{\Im\upsilon}\sigma_\Im]$. Parallel to the Stage 1 error analysis, if $r_{C\upsilon}$ and $r_{\Im\upsilon}$ are substituted in Equation 13 (let $\upsilon_{ia} = \frac{\Im_{ia}}{\Im_a}$, $\sum_{a=1}^{m} C_a = C_T$, and $\sum_{a=1}^{m} \Im_a = t_T$), then simplifying further yields

$$\varepsilon_i = \sum_{a=1}^{m} \left(\frac{C_a^\alpha}{\Im_a} - c^Z \right) \Im_{ia}, \; i = 1, \ldots, k \qquad (15)$$

The assumptions for Stage 2 concerning C_a^α and $\Im_a$ being fixed and that all activities are time driven are similar in rationale to those in the Stage 1 analysis. If C_a^α and $\Im_a$ are treated as constants in the system and the consumption of activity *a* by cost object *i*, $\Im_{ia}$, is allowed to vary, then the following proposition shows that the maximum absolute dollar error of the Stage 2 system is $\sum_{a=1}^{m}\left|\delta_a\right| = \sum_{a=1}^{m}\left|C_a^{\alpha} - c^{Z}\Im_a\right|$, where

$\delta_a \equiv C_a^{\alpha} - c^{Z}\Im_a$, which is the total dollar error contribution of activity *a*. $\frac{C_a^{\alpha}}{\Im_a} - c^{Z}$ is the dollar error contribution per unit of time of activity *a* ("unit dollar error contribution of *a*").

Proposition VI: Given $\varepsilon_i = \sum_{a=1}^{m}\left(\frac{C_a^{\alpha}}{\Im_a} - c^{Z}\right)\Im_{ia}$ and $c^{Z} \neq \frac{C_a^{\alpha}}{\Im_a}$, the maximum absolute dollar error for the system is $\sum_{a=1}^{m}\left|\delta_a\right| = \sum_{a=1}^{m}\left|C_a^{\alpha} - c^{Z}\Im_a\right|$.

The proof is parallel to that of Proposition V and is, therefore, omitted.

Based on the proof from Proposition VI, the activities can be partitioned into two sets: one to represent the activities that provide the positive dollar error contribution, $A^{+} = \{A_1, A_2, ..., A_q\}$ and the other to represent activities that provide a negative dollar error contribution, $A^{-} = \{A_{q+1}, A_{q+2}, ..., A_m\}$, where $C_a^{\alpha} - c^{Z}\Im_a > 0$ ($\frac{C_a^{\alpha}}{\Im_a} - c^{Z} > 0$) with $a = 1, \ldots, q \in A^{+}$ and $C_a^{\alpha} - c^{Z}\Im_a < 0$ ($\frac{C_a^{\alpha}}{\Im_a} - c^{Z} < 0$) with $a = q+1, \ldots, m \in A^{-}$.

Parallel to the Stage 1 error analysis, $\sum_{a=1}^{m}\left|\delta_a\right| = \sum_{a=1}^{m}\left|C_a^{\alpha} - c^{Z}\Im_a\right|$ does not depend on the $\Im_{ia}$'s. To find the set of $\Im_{ia}$'s that produces the maximum error, first (along with the partition of activities into A^{+} and A^{-} sets) the activities are ordered from largest to smallest $\frac{C_a^\alpha}{\Im_a} - c^Z$ (similar rational to that in Stage 1 error analysis). Second, partition the cost objects into two sets in which some cost objects consume activities that provide a positive dollar error contribution (I^+) and the rest of the cost objects consume activities that provide a negative dollar error contribution (I^-):

$I^+ = \{i \mid i = 1,\ldots, w;\ \varepsilon_{ia} = (C_a^\alpha - c^Z \Im_a)\frac{\Im_{ia}}{\Im_a}, a \in A^+$ and $\varepsilon_{ia} = (C_a^\alpha - c^Z \Im_a)\frac{\Im_{ia}}{\Im_a} = 0$

where $\Im_{ia} = 0$ if $a \in A^-\}$; and

$I^- = \{i \mid i = w+1,\ldots, k;\ \varepsilon_{ia} = (C_a^\alpha - c^Z \Im_a)\frac{\Im_{ia}}{\Im_a}, a \in A^-$ and $\varepsilon_{ia} = (C_a^\alpha - c^Z \Im_a)\frac{\Im_{ia}}{\Im_a} = 0$

where $\Im_{ia} = 0$ if $a \in A^+\}$, where $I^+ \cup I^- = I$.

To maximize the dollar error using $\Im_{ia}$'s, cost objects in I^+ only consume activities in A^+ and cost objects in I^- only consume activities in A^-. The following corollary to Proposition VI shows the $\Im_{ia}$'s that result in $\sum_{a=1}^{m}\left|\delta_a\right| = \sum_{a=1}^{m}\left|C_a^\alpha - c^Z \Im_a\right|$.

Corollary VIa: The $\Im_{ia}$'s that produce the total maximum absolute dollar error of the system, $\sum_{a=1}^{m}\left|\delta_a\right| = \sum_{a=1}^{m}\left|C_a^\alpha - c^Z \Im_a\right|$, is $\sum_{i=1}^{k}\left|\varepsilon_i\right| = \sum_{i=1}^{k}\sum_{a=1}^{m}\left|(C_a^\alpha - c^Z \Im_a)\frac{\Im_{ia}}{\Im_a}\right|$.

The proof parallels that of Corollary Va and is therefore omitted.

Although $\sum_{i=1}^{k}|\varepsilon_i| = \sum_{a=1}^{m}|\delta_a|$, a program is identified that provides the maximum percentage error of each cost object *i* that maximizes the average absolute percentage error of the system for Stage 2.[4] The percentage error for any given cost object is $\%\varepsilon_i = \frac{\varepsilon_i}{D_i^{\alpha}}$, which is the dollar error for cost object *i* divided by the ABC cost for cost object *i*. Parallel to the Stage 2 error analysis, to find the maximum average absolute percentage error, one program is used for the positive sets, I^+ and A^+, and another program for the negative sets, I^- and A^-. Let m^+ represent the number of activities in A^+ (a = 1,…, q) and k^+ represent the number of resources in I^+ (i = 1,…, w). First order all activities from largest positive to smallest positive $\frac{C_a^{\alpha}}{\Im_a} - c^Z$ and label them as $A_1, A_2, \ldots, A_q$. The most positive activity, A_1, must be the only activity consumed by $k^+ - (m^+ - 1)$ cost objects. The program for the maximization of the percentage errors for each cost object in the I^+ set is as follows:

$$\text{Max} \sum_{i=1}^{w} \left(\frac{\sum_{a=1}^{q} \left(\frac{C_a^{\alpha}}{\Im_a} - c^Z \right) \Im_{ia}}{\sum_{a=1}^{q} \frac{\Im_{ia}}{\Im_a} C_a^{\alpha}} \right) \qquad \text{(P11)}$$

s.t.

$$\Im_{i,1} \geq \Im_1 \gamma, \ i = 1,\ldots, w\text{-}1 \qquad \text{(P12)}$$

$$\Im_{ia} \geq \Im_a \gamma, \ i = 1 + k^+ - (m^+ - 1), \ldots, w\text{-}1, \ a = 2,\ldots, q\text{-}1 \qquad \text{(P13)}$$

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[4] An optimization software program such as LINGO can be used.

All other $\Im_{ia} \geq 0$, $a \in A^{+}$ and $i \in I^{+}$ (P14)

$$\sum_{i=1}^{w} \Im_{ia} = \Im_{a} \qquad \text{(P15)}$$

The objective function in (P11) provides the maximum percentage error in magnitude across all cost objects in I^{+}. (P12) is the first constraint that ensures that for A_1, at least the amount of $\Im_1 \gamma$ is assigned to *w*-1 cost objects in I^{+}. The materiality and uniqueness parameter γ (e.g. it can be set as 0.1 to ensure that 10 percent of the time for activity *a* is assigned to the appropriate cost objects) is only assigned to *w*-1 cost objects instead of *w* to not over restrict the program and allow it to choose the optimal set of $\Im_{ia}$'s. (P13) is the second constraint that ensures uniqueness among activity vectors without over-restricting the program to allow for some $\Im_{ia}$ in I^{+} and A^{+} to be zero or for some cost objects in I^{+} to consume all of a single activity in A^{+} (this is represented by (P14)). (P15) ensures that the sum of the cost object consumptions in I^{+} of a particular activity in A^{+} is equal to the total time available for that particular activity *a*.

Now, let k^{-} represent the number of cost objects in I^{-} ($i = w+1, \ldots, k$) and m^{-} represent the number of activities in A^{-} ($a = q+1, \ldots, m$). After ordering all activities from least negative to most negative $\frac{C_a^{\alpha}}{\Im_a} - c^{Z}$ and labeling them as $A_{q+1}, A_{q+2}, \ldots, A_m$, the most negative A_m must be the only activity consumed by $k^{-} - (m^{-} - 1)$ cost objects. The program for the maximization the *magnitude* of the percentage errors in the A^{-} set is as follows:

$$\text{Min} \sum_{i=w+1}^{k} \left(\frac{\sum_{a=q+1}^{m} \left(\frac{C_a^{\alpha}}{\Im_a} - c^{Z} \right) \Im_{ia}}{\sum_{a=q+1}^{m} \frac{\Im_{ia}}{\Im_a} C_a^{\alpha}} \right) \qquad \text{(P16)}$$

s.t.

$$\Im_{i,m} \geq \Im_m \gamma,\ i = w+2,\ldots,\ k \qquad \text{(P17)}$$

$$\Im_{ia} \geq \Im_a \gamma,\ i = w+1,\ldots,\ k - k^{-} - (m^{-} - 1),\ a = q+2,\ldots,\ m\text{-}1 \qquad \text{(P18)}$$

All other $\Im_{ia} \geq 0$ in $a \in A^{-}$ and $i \in I^{-}$ (P19)

$$\sum_{i=w+1}^{k} \Im_{ia} = \Im_a \qquad \text{(P20)}$$

The objective function in (P16) provides the maximum percentage error in magnitude across all cost objects in I^{-}. (P17) is the first constraint that ensures that for A_m, at least the amount of $\Im_m \gamma$ is assigned to $i = w+2,\ldots, k$ cost objects in I^{-} without over restricting the program. (P18) is the second constraint that ensures uniqueness among activity vectors without over-restricting the program to allow for some $\Im_{ia}$ in I^{-} and A^{-} to be zero or for some cost objects in I^{-} to consume all of a single activity in A^{-} (this is represented by (P19)). (P20) ensures that the sum of the cost object consumptions in I^{-} of a particular activity in A^{-} is equal to the total time available for that particular activity *a*.

However, parallel to that in Stage 1 error analysis, a cost object can consume activities from both A^{+} and A^{-}, thus allowing for some cancellation effects and a reduced error. Hence, the maximum error cannot exceed the dollar error contribution from all activities ($\sum_{i=1}^{k}|\varepsilon_i| \le \sum_{a=1}^{m}|\delta_a|$) and is stated in the following corollary.

Corollary VIb: If cost objects in I^+ only consume activities in A^+ and cost objects in I^- only consume activities in A^-, then $\sum_{i=1}^{k}|\varepsilon_i| = \sum_{a=1}^{m}|\delta_a|$. If any or all cost objects consume activities from both A^+ and A^-, then $\sum_{i=1}^{k}|\varepsilon_i| < \sum_{a=1}^{m}|\delta_a|$. Therefore, $\sum_{i=1}^{k}|\varepsilon_i|$ can never exceed $\sum_{a=1}^{m}|\delta_a|$, implying $\sum_{i=1}^{k}|\varepsilon_i| \le \sum_{a=1}^{m}|\delta_a|$.

The proof parallels that of Corollary Vb and is, therefore, omitted.

In summary, if there are no cancellation effects, then $\sum_{i=1}^{k}|\varepsilon_i| = \sum_{a=1}^{m}|\delta_a|$. If any or all cost objects consume activities from both A^+ and A^-, then there will be some cancellation effects within each of those ε_i's, and thus, the actual absolute dollar error across all cost objects will be less than the maximum absolute dollar error of the system, $\sum_{i=1}^{k}|\varepsilon_i| < \sum_{a=1}^{m}|\delta_a|$. As a result, $\sum_{a=1}^{m}|\varepsilon_a| \le \sum_{j=1}^{n}|\delta_j|$.

6.2. Examples Demonstrating Proposition VI and Its Corollaries

This section provides examples illustrating Proposition VI and its corollaries and using program (P11) through (P20). First, Corollary VIa will be shown in Tables 20, 21, and 22 for *k* = *m*, *k* < *m*, and *k* > *m*, respectively. The activities in each table are ranked from largest to smallest $\frac{C_a^\alpha}{\Im_a} - c^Z$ and the activities are partitioned accordingly. For each table, the first three cost objects (CO1 through CO3) are in the I^+ set, with the rest of the cost objects being in the I^- set. Table 20 is a continuation of Table 14, in which the activity costs from Stage 1 are assigned to the cost objects in Stage 2. Table 21 is a continuation of Table 15, and Table 22 is a continuation of Table 16. In each of the following tables, $\sum_{i=1}^{k} |\varepsilon_i| = \sum_{a=1}^{m} |\delta_a|$ = \$1,700, thus satisfying Corollary VIa. Program (P1) through (P10) is used to maximize each $|\%\varepsilon_i|$, which maximizes average $\sum_{i=1}^{k} |\%\varepsilon_i|$.

Table 20
Corollary VIa for $k = m$

	Activities								
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**ABC Cost**	$r_{C\upsilon}$	$r_{\mathfrak{I}\upsilon}$
CO1	0.10	0	0	0	0	0	$20	-0.40	-0.40
CO2	0.90	0.10	0	0	0	0	$287	-0.44	-0.43
CO3	0	0.90	1	0	0	0	$6,193	0.23	0.11
CO4	0	0	0	1	0.90	0	$7,770	0.46	0.56
CO5	0	0	0	0	0.10	0.90	$400	-0.41	-0.39
CO6	0	0	0	0	0	0.10	$30	-0.38	-0.36
Cost	$200	$1,069	$5,231	$6,598	$2,700	$300	$14,700		
Time	77.5	426.46	2,321.04	3,589.22	758.28	177.5	7,350		

	Activities						
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**Total**
$\mathfrak{I}_{1a}$	7.75	0	0	0	0	0	7.75
$\mathfrak{I}_{2a}$	69.75	42.65	0	0	0	0	112.40
$\mathfrak{I}_{3a}$	0	383.81	2,321.04	0	0	0	2,704.85
$\mathfrak{I}_{4a}$	0	0	0	3,589.22	682.45	0	4,271.67
$\mathfrak{I}_{5a}$	0	0	0	0	75.83	159.75	235.58
$\mathfrak{I}_{6a}$	0	0	0	0	0	17.75	17.75
Total	77.5	426.46	2,321.04	3,589.22	758.28	177.5	7,350

Table 20 (continued)

c^{Z}	\$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	7.750	1	\$16
CO2	28.100	4	\$225
CO3	54.097	50	\$5,410
CO4	85.433	50	\$8,543
CO5	2.356	100	\$471
CO6	8.875	2	\$35
			\$14,700

	Activities					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**
$C_a^{\alpha} / \mathfrak{I}_a$	\$2.58	\$2.51	\$2.25	\$1.84	\$1.72	\$1.69
$C_a^{\alpha} / \mathfrak{I}_a - c^{Z}$	\$0.581	\$0.507	\$0.254	-\$0.162	-\$0.283	-\$0.310
$C_a^{\alpha} - c^{Z} \mathfrak{I}_a$	\$45	\$216	\$589	-\$581	-\$214	-\$55
$\sum_{a=1}^{m} \lvert \delta_a \rvert$	\$1,700					

Table 20 (continued)

	Activities								
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**Total**	$\lvert\boldsymbol{\varepsilon_i}\rvert$	$\%\lvert\boldsymbol{\varepsilon_i}\rvert$
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{1a}$	\$4.50	\$0	\$0	\$0	\$0	\$0	\$4.50	\$4.50	22.50%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{2a}$	\$40.50	\$22	\$0	\$0	\$0	\$0	\$61.50	\$61.50	21.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{3a}$	\$0	\$194	\$589	\$0	\$0	\$0	\$783	\$783	12.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{4a}$	\$0	\$0	\$0	-\$581	-\$193	\$0	-\$774	\$774	9.96%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{5a}$	\$0	\$0	\$0	\$0	-\$21	-\$49.50	-\$70.50	\$70.50	17.72%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{6a}$	\$0	\$0	\$0	\$0	\$0	-\$5.50	-\$5.50	\$5.50	18.33%
	\$45	\$216	\$589	-\$581	-\$214	-\$55	\$0	\$1,700	
								Avg $\sum_{a=1}^{m}\lvert\%\varepsilon_a\rvert$	17.13%

Table 21
Corollary VIa for *k* < *m*

	Activities									
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**ABC Cost**	$r_{C\upsilon}$	$r_{\mathfrak{I}\upsilon}$
CO1	0.10	0	0	0	0	0	0	$20	-0.33	-0.33
CO2	0.90	0.10	0	0	0	0	0	$287	-0.35	-0.35
CO3	0	0.90	1	0	0	0	0	$6,193	0.31	0.19
CO4	0	0	0	1	0.90	0.47	0.17	$7,666	0.35	0.45
CO5	0	0	0	0	0.10	0.43	0.73	$474	-0.49	-0.46
CO6	0	0	0	0	0	0.10	0.10	$60	-0.48	-0.46
Cost	$200	$1,069	$5,231	$6,598	$2,700	$300	$300	$14,700		
Time	77.5	426.46	2,321.04	3,440.98	729.02	177.5	177.5	7,350		

	Activities							
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**Total**
$\mathfrak{I}_{1a}$	7.75	0	0	0	0	0	0	7.75
$\mathfrak{I}_{2a}$	69.75	42.65	0	0	0	0	0	112.40
$\mathfrak{I}_{3a}$	0	383.81	2,321.04	0	0	0	0	2,704.85
$\mathfrak{I}_{4a}$	0	0	0	3,440.98	656.12	83.16	30.14	4,210.40
$\mathfrak{I}_{5a}$	0	0	0	0	72.9	76.59	129.61	279.1
$\mathfrak{I}_{6a}$	0	0	0	0	0	17.75	17.75	35.5
Total	77.5	426.46	2,321.04	3,440.98	729.02	177.5	177.5	7,350

Table 21 (continued)

c^Z	\$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	7.750	1	\$16
CO2	28.100	4	\$225
CO3	54.097	50	\$5,410
CO4	84.208	50	\$8,421
CO5	2.791	100	\$558
CO6	17.75	2	\$71
			\$14,700

	Activities						
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**
$C_a^\alpha / \mathfrak{I}_a$	\$2.58	\$2.51	\$2.25	\$1.84	\$1.72	\$1.69	\$1.69
$C_a^\alpha / \mathfrak{I}_a - c^Z$	\$0.581	\$0.507	\$0.254	-\$0.155	-\$0.281	-\$0.310	-\$0.310
$C_a^\alpha - c^Z \mathfrak{I}_a$	\$45	\$216	\$589	-\$535	-\$205	-\$55	-\$55

$\sum_{a=1}^{m} \lvert \delta_a \rvert$	\$1,700

Table 21 (continued)

	Activities									
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**Total**	$\lvert \varepsilon_i \rvert$	$\% \lvert \varepsilon_i \rvert$
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{1a}$	\$4.50	\$0	\$0	\$0	\$0	\$0	\$0	\$4.50	\$4.50	22.50%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{2a}$	\$40.50	\$22	\$0	\$0	\$0	\$0	\$0	\$61.50	\$62	21.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{3a}$	\$0	\$194	\$589	\$0	\$0	\$0	\$0	\$783	\$783	12.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{4a}$	\$0	\$0	\$0	-\$535	-\$185	-\$26	-\$9	-\$755	\$755	9.84%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{5a}$	\$0	\$0	\$0	\$0	-\$20	-\$24	-\$40	-\$84	\$84	17.82%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{6a}$	\$0	\$0	\$0	\$0	\$0	-\$5	-\$5	-\$5	\$5	18.34%
	\$45	\$216	\$589	-\$535	-\$205	-\$55	-\$55	\$0	\$1,700	
									Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	17.13%

Table 22
Corollary VIa for *k* > *m*

	Activities							
	A1	**A2**	**A3**	**A4**	**A5**	**ABC Cost**	$r_{C\upsilon}$	$r_{\Im\upsilon}$
CO1	0.10	0	0	0	0	$20	-0.52	-0.51
CO2	0.90	0.10	0	0	0	$287	-0.58	-0.56
CO3	0	0.90	1	0	0	$6,193	0.10	0.03
CO4	0	0	0	1	0.80	$7,770	0.45	0.56
CO5	0	0	0	0	0.10	$400	-0.30	-0.25
CO6	0	0	0	0	0.10	$30	-0.30	-0.25
Cost	$200	$1,069	$5,231	$6,848	$1,352	$14,700		
Time	77.5	426.46	2,321.04	3,737.46	787.54	7,350		

	Activities					
	A1	**A2**	**A3**	**A4**	**A5**	**Total**
$\Im_{1a}$	7.75	0	0	0	0	7.75
$\Im_{2a}$	69.75	42.65	0	0	0	112.40
$\Im_{3a}$	0	383.81	2,321.04	0	0	2,704.85
$\Im_{4a}$	0	0	0	3,737.46	630.03	4,367.49
$\Im_{5a}$	0	0	0	0	78.75	78.75
$\Im_{6a}$	0	0	0	0	78.75	78.75
Total	77.5	426.46	2,321.04	3,737.46	758.28	7,350

Table 22 (continued)

c^{Z}	$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	7.750	1	$16
CO2	28.100	4	$225
CO3	54.097	50	$5,410
CO4	87.350	50	$8,735
CO5	0.788	100	$158
CO6	39.375	2	$158
			$14,700

	Activities				
	A1	**A2**	**A3**	**A4**	**A5**
$C_a^{\alpha} / \mathfrak{I}_a$	$2.58	$2.51	$2.25	$1.83	$1.72
$C_a^{\alpha} / \mathfrak{I}_a - c^{Z}$	$0.581	$0.507	$0.254	-$0.168	-$0.284
$C_a^{\alpha} - c^{Z} \mathfrak{I}_a$	$45	$216	$589	-$627	-$223

$\sum_{a=1}^{m} \left| \delta_a \right|$ $1,700

Table 22 (continued)

	Activities							
	A1	**A2**	**A3**	**A4**	**A5**	**Total**	$\lvert\varepsilon_i\rvert$	$\%\lvert\varepsilon_i\rvert$
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{1a}$	$4.50	$0	$0	$0	$0	$4.50	$4.50	22.50%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{2a}$	$40.50	$22	$0	$0	$0	$61.50	$61.50	21.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{3a}$	$0	$194	$589	$0	$0	$783	$783	12.65%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{4a}$	$0	$0	$0	-$627	-$179	-$805	$805	10.16%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{5a}$	$0	$0	$0	$0	-$22	-$22	$22	16.52%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{6a}$	$0	$0	$0	$0	-$22	-$22	$22	16.52%
	$45	$216	$589	-$627	-$223	$0	$1,700	
							Avg $\sum_{a=1}^{m}\lvert\%\varepsilon_a\rvert$	16.67%

Illustrations of Corollary VIb are shown in Tables 23, 24, and 25 for $k = m$, $k < m$, and $k > m$, respectively. As in the previous three tables, the activities in each table are ranked from largest to smallest $\frac{C_a^{\alpha}}{\Im_a} - c^{Z}$ and the activities are partitioned accordingly. For each table, the first three cost objects (CO1 through CO3) are in the I^+ set, with the rest of the cost objects being in the I^- set. In each table, all cost objects share all of the activities so that $\sum_{i=1}^{k} \left|\varepsilon_i\right| < \sum_{a=1}^{m} \left|\delta_a\right|$ = \$1,700. Table 23 shows that the average $\sum_{i=1}^{k} \left|\%\varepsilon_i\right|$ = 0.97 percent compared to 17.13 percent in Table 20. Compared to Table 21 with an average $\sum_{i=1}^{k} \left|\%\varepsilon_i\right|$ = 17.13 percent, Table 24 shows that the average $\sum_{i=1}^{k} \left|\%\varepsilon_i\right|$ = 0.84 percent. Table 25 shows that the average $\sum_{i=1}^{k} \left|\%\varepsilon_i\right|$ = 0.42 percent compared to 16.67 percent in Table 20. In each table, notice that each $\%\left|\varepsilon_i\right|$ is less than 5 percent, which demonstrates that the actual maximum error is very small since cost objects will typically consume activities in both A^+ and A^- sets.

Table 23
Corollary VIb for *k* = *m*

	Activities								
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**ABC Cost**	$r_{C\upsilon}$	$r_{\Im\upsilon}$
CO1	0.10	0.30	0.10	0.10	0.20	0.10	$1,814	-0.34	-0.35
CO2	0.10	0.20	0.20	0.20	0.10	0.10	$2,760	0.74	0.69
CO3	0.20	0.10	0.20	0.20	0.10	0.10	$2,673	0.62	0.60
CO4	0.30	0.20	0.10	0.10	0.30	0.10	$1,877	-0.60	-0.58
CO5	0.20	0.10	0.10	0.20	0.10	0.10	$2,150	0.27	0.33
CO6	0.10	0.10	0.30	0.20	0.20	0.50	$3,426	-0.002	-0.01
Cost	$200	$1,069	$5,231	$6,598	$2,700	$300	$14,700		
Time	77.5	426.46	2,321.04	3,589.22	758.28	177.5	7,350		

	Activities						
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**Total**
$\Im_{1a}$	7.75	127.94	232.1	358.92	151.66	17.75	896.12
$\Im_{2a}$	7.75	85.29	464.21	717.84	75.83	17.75	1,368.67
$\Im_{3a}$	15.5	42.65	464.21	717.84	75.83	17.75	1,333.78
$\Im_{4a}$	23.25	85.29	232.1	358.92	227.48	17.75	944.8
$\Im_{5a}$	15.5	42.65	232.1	717.84	78.83	17.75	1,101.67
$\Im_{6a}$	7.75	42.65	696.31	717.84	151.66	88.75	1,704.96
Total	77.5	426.46	2,321.04	3,589.22	758.28	177.5	7,350

Table 23 (continued)

c^{Z}	$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	896.120	1	$1,792
CO2	342.168	4	$2,737
CO3	26.676	50	$2,668
CO4	18.896	50	$1,890
CO5	11.017	100	$2,203
CO6	852.48	2	$3,410
			$14,700

	Activities					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**
$C_a^{\alpha} / \mathfrak{I}_a$	$2.58	$2.51	$2.25	$1.84	$1.72	$1.69
$C_a^{\alpha} / \mathfrak{I}_a - c^{Z}$	$0.581	$0.507	$0.254	-$0.162	-$0.283	-$0.310
$C_a^{\alpha} - c^{Z} \mathfrak{I}_a$	$45	$216	$589	-$581	-$214	-$55

$\sum_{a=1}^{m} \lvert \delta_a \rvert$	$1,700

Table 23 (continued)

	Activities								
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**Total**	$\lvert \varepsilon_i \rvert$	$\% \lvert \varepsilon_i \rvert$
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{1a}$	$4.50	$64.80	$58.90	-$58.10	-$42.90	-$5.50	$22	$22	1.20%
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{2a}$	$4.50	$43.20	$117.80	-$116.10	-$21.40	-$5.50	$22	$22	0.81%
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{3a}$	$9	$21.60	$117.80	-$116.10	-$21.40	-$5.50	$5	$5	0.20%
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{4a}$	$13.50	$43.20	$58.90	-$58.10	-$64.30	-$5.50	-$12	$12	0.65%
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{5a}$	$9	$21.60	$58.90	-$116.10	-$21.40	-$5.50	-$54	$54	2.49%
$(C_a^{\alpha} - c^{Z}\mathfrak{I}_a)\mathfrak{I}_{6a}$	$4.50	$21.60	$176.70	-$116.10	-$42.90	-$27.5	$16	$16	0.48%
	$45	$216	$589	-$581	-$214	-$55	$0	$132	
							Avg $\sum_{a=1}^{m} \lvert \%\varepsilon_a \rvert$		0.97%

Table 24
Corollary VIb for *k* < *m*

	Activities									
	A1	A2	A3	A4	A5	A6	A7	ABC Cost	$r_{C\upsilon}$	$r_{\mathfrak{I}\upsilon}$
CO1	0.10	0.30	0.10	0.10	0.20	0.10	0.40	$1,899	-0.44	-0.44
CO2	0.10	0.20	0.20	0.20	0.10	0.10	0.10	$2,735	0.77	-0.72
CO3	0.20	0.10	0.20	0.20	0.10	0.10	0.10	$2,648	0.66	0.64
CO4	0.30	0.20	0.10	0.10	0.30	0.10	0.10	$1,867	-0.44	-0.43
CO5	0.20	0.10	0.10	0.20	0.10	0.10	0.10	$2,125	0.31	0.37
CO6	0.10	0.10	0.30	0.20	0.20	0.50	0.20	$3,426	0.03	0.02
Cost	$200	$1,069	$5,231	$6,598	$2,700	$300	$300	$14,700		
Time	77.5	426.46	2,321.04	3,440.98	729.02	177.5	177.5	7,350		

	Activities							
	A1	A2	A3	A4	A5	A6	A7	Total
$\mathfrak{I}_{1a}$	7.75	127.94	232.1	344.1	145.8	17.75	71	946.44
$\mathfrak{I}_{2a}$	7.75	85.29	464.21	688.2	72.9	17.75	17.75	1,353.85
$\mathfrak{I}_{3a}$	15.5	42.65	464.21	688.2	72.9	17.75	17.75	1,318.95
$\mathfrak{I}_{4a}$	23.25	85.29	232.1	344.1	218.71	17.75	17.75	938.95
$\mathfrak{I}_{5a}$	15.5	42.65	232.1	688.2	72.9	17.75	17.75	1,086.85
$\mathfrak{I}_{6a}$	7.75	42.65	696.31	688.2	145.8	88.75	35.5	1,704.96
Total	77.5	426.46	2,321.04	3,440.98	729.02	177.5	177.5	7,350

Table 24 (continued)

c^{Z}	$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	946.442	1	$1,893
CO2	338.463	4	$2,708
CO3	26.379	50	$2,638
CO4	18.779	50	$1,878
CO5	10.869	100	$2,174
CO6	852.48	2	$3,410
			$14,700

	Activities						
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**
$C_a^{\alpha}/\Im_a$	$2.58	$2.51	$2.25	$1.84	$1.72	$1.69	$1.69
$C_a^{\alpha}/\Im_a - c^{Z}$	$0.581	$0.507	$0.254	-$0.155	-$0.281	-$0.310	-$0.310
$C_a^{\alpha} - c^{Z}\Im_a$	$45	$216	$589	-$535	-$205	-$55	-$55

$\sum_{a=1}^{m} \lvert \delta_a \rvert$ $1,700

Table 24 (continued)

	Activities									
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**Total**	$\lvert \varepsilon_i \rvert$	$\% \lvert \varepsilon_i \rvert$
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{1a}$	$4.50	$64.80	$58.90	-$53.48	-$41.04	-$5.50	-$22	$6.20	$6.20	0.33%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{2a}$	$4.50	$43.20	$117.80	-$106.96	-$20.52	-$5.50	-$5.50	$27	$27	0.99%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{3a}$	$9	$21.60	$117.80	-$106.96	-$20.52	-$5.50	-$5.50	$9.90	$9.90	0.37%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{4a}$	$13.50	$43.20	$58.90	-$53.48	-$61.56	-$5.50	-$5.50	-$10.40	$10.40	0.56%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{5a}$	$9	$21.60	$58.90	-$106.96	-$20.52	-$5.50	-$5.50	-$49	$49	2.31%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{6a}$	$4.50	$21.60	$176.70	-$106.96	-$41.04	-$27.50	-$11	$16.30	$16.30	0.48%
	$45	$216	$589	-$535	-$205	-$55	-$55	$0	$119	
									Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	0.84%

Table 25
Corollary VIb for $k > m$

	Activities							
	A1	**A2**	**A3**	**A4**	**A5**	**ABC Cost**	$r_{C\upsilon}$	$r_{\Im\upsilon}$
CO1	0.10	0.30	0.10	0.10	0.30	$1,954	-0.54	-0.52
CO2	0.30	0.30	0.10	0.20	0.10	$2,409	-0.46	-0.43
CO3	0.10	0.10	0.10	0.10	0.10	$1,470	0.00	0.00
CO4	0.30	0.10	0.10	0.10	0.20	$1,645	-0.68	-0.63
CO5	0.10	0.10	0.30	0.20	0.20	$3,336	0.71	0.64
CO6	0.10	0.10	0.30	0.30	0.10	$3,886	0.97	0.93
Cost	$200	$1,069	$5,231	$6,848	$1,352	$14,700		
Time	77.5	426.46	2,321.04	3,737.46	787.54	7,350		

	Activities					
	A1	**A2**	**A3**	**A4**	**A5**	**Total**
$\Im_{1a}$	7.75	127.94	232.10	373.75	236.26	977.80
$\Im_{2a}$	23.25	127.94	232.10	747.49	78.75	1,209.54
$\Im_{3a}$	7.75	42.65	232.10	373.75	78.75	735
$\Im_{4a}$	23.25	42.65	232.10	373.75	157.51	829.25
$\Im_{5a}$	7.75	42.65	696.31	747.49	157.51	1,651.71
$\Im_{6a}$	7.75	42.65	696.31	1,121.24	78.75	1,946.70
Total	77.5	426.46	2,321.04	3,737.46	758.28	7,350

Table 25 (continued)

c^{Z}	$2		
	β_i	θ_i	**TDABC2 Cost**
CO1	977.800	1	$1,956
CO2	302.385	4	$2,419
CO3	14.700	50	$1,470
CO4	16.585	50	$1,659
CO5	16.517	100	$3,303
CO6	973.350	2	$3,893
			$14,700

	Activities				
	A1	**A2**	**A3**	**A4**	**A5**
$C_a^{\alpha} / \mathfrak{I}_a$	$2.58	$2.51	$2.25	$1.83	$1.72
$C_a^{\alpha} / \mathfrak{I}_a - c^{Z}$	$0.581	$0.507	$0.254	-$0.168	-$0.284
$C_a^{\alpha} - c^{Z} \mathfrak{I}_a$	$45	$216	$589	-$627	-$223

$\sum_{a=1}^{m} \left| \delta_a \right|$ $1,700

Table 25 (continued)

	Activities							
	A1	**A2**	**A3**	**A4**	**A5**	**Total**	$\lvert \varepsilon_i \rvert$	$\% \lvert \varepsilon_i \rvert$
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{1a}$	$4.50	$64.80	$58.90	-$62.67	-$67.00	-$1.46	$1.46	0.07%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{2a}$	$13.50	$64.80	$58.90	-$125.34	-$22.33	-$10.47	$10.47	0.43%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{3a}$	$4.50	$21.60	$58.90	-$62.67	-$22.33	$0.00	$0.00	0.00%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{4a}$	$13.50	$21.60	$58.90	-$62.67	-$44.66	-$13.33	$13.33	0.81%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{5a}$	$4.50	$21.60	$176.70	-$125.34	-$44.66	$32.80	$32.80	0.98%
$(C_a^{\alpha} - c^{Z}\Im_a)\Im_{6a}$	$4.50	$21.60	$176.70	-$188.00	-$22.33	-$7.54	$7.54	0.19%
	$45	$216	$589	-$627	-$223	$0	$66	
							Avg $\sum_{a=1}^{m} \lvert \% \varepsilon_a \rvert$	0.42%

6.3. Implications

This chapter demonstrates that cost objects will typically consume activities from both A^{+} and A^{-} sets. As a result, the average $\sum_{i=1}^{k} |\%\varepsilon_i|$ is significantly lower than if cost objects from set I^{+} only consume resources in A^{+} and activities in I^{-} only consume resources in A^{-}. This chapter shows that TDABC2 is not significantly different from the fully-specified ABC system (e.g. the percentage errors are less than 2.5 percent in Tables 23, 24, and 25). Hence, TDABC2 should replicate the accuracy of the ABC system with the benefit of eliminating Stage 1 cost assignments and significantly reducing the Stage 2 cost assignments. However, some empirical analyses are needed to determine whether the equivalency conditions that are analytically proven actually hold. As an initial empirical analysis, the next chapter provides case studies based on data from an actual company to explore the validity of the equivalency conditions.

CHAPTER 7

CASE ANALYSES OF THE EQUIVALENCY CONDITIONS

7.1. Introduction to the Case Analyses and Assumptions

This chapter presents seven case studies using data from a particular company to provide anecdotal evidence that explores the validity of the equivalency conditions in Propositions I through IV.[5] Case studies are initially useful to identify and explore the validity of hypothesized relationships and, thus, serve as an important forerunner and input for other types of empirical testing (Lillis and Mundy 2005; Kaplan 1986). Since the case studies in this chapter contain data from only one company, any evidence of the validity of the equivalency conditions is anecdotal, which is, consequently, the limitation of this chapter. Therefore, more empirical analyses using data from a broad range of companies are needed beyond these case studies to verify the equivalency conditions further.

The data used in the case analyses are yearly company data. For the first four cases, enough data are available to perform Stage 1 and Stage 2 cost assignments for ABC, IABC, IABC2, and TDABC. Only Stage 1 data to perform ABC, IABC, and TDABC are available for the fifth case study, and only Stage 2 data to perform ABC and IABC2 are available for the sixth and seventh case studies. An overview of the names

[5] The name of the company is withheld for reasons of confidentiality.

of the resources, activities, and products/services (cost objects) for each case study are given in Table 26.

Table 26
Names of Resources, Activities, and Cost Objects for Each Case Study

Case 1					
R1	Salaries and Benefits	A1	Repair and maintain fixed equipment	CO1	Unit 1
R2	Travel	A2	Repair and maintain rotating equipment	CO2	Unit 2
R3	Communication	A3	Prepare equipment	CO3	Waste water treatment plant
R4	Special studies	A4	Fabricate piping and welding	CO4	Unit 3 - Gas Treater
R5	Depreciation	A5	Repair electrical equipment	CO5	Boiler & steam system
R6	Materials	A6	Receive and inventory materials	CO6	Tanks & Pipelines
R7	External contract services	A7	Dock and sail ships	CO7	Docks
R8	Outside Contractors	A8	Perform instrument calibration/repairs	CO8	Loading Racks
R9	Parts Inventory	A9	Equipment reliability	CO9	General Administration
R10	Rent	A10	Plan and schedule work activities	CO10	Dock and sail ships
R11	Internal Labor	A11	Manage/supervise departments		
R12	Training	A12	Manage internal contractors		
R13	Procurement cards	A13	Perform housekeeping & administrative		
R14	Other expenses	A14	Maintain pipelines and valves		
		A15	Monitor SAP work orders		

Table 26 (Continued)

Case 2

R1	Wages & Salaries	A1	Receive Product/Invoices Disputes	CO1	National Accounts / Buybacks
R2	Labor Burden	A2	Research Dispute	CO2	OEM
R3	Materials - General	A3	Write-up and Log RFC	CO3	Base Oils / White oils
R4	Memberships, Dues, Assessments	A4	Faxing/Re-Faxing of Invoices to Customers	CO4	GEO
R5	Postage & Freight	A5	Research & Coordinate Return Product	CO5	Wax
R6	Employee Development	A6	Support Sales Force & Field and Ad-hoc Requests	CO6	Retail / Private Label
R7	Meals & Entertainment	A7	Research and Process Credit Issues	CO7	SE
R8	Meals - Meetings & Relations	A8	New Customer Presentations and Customer Visits	CO8	NE
R9	Travel Expense	A9	Attend Meetings	CO9	MW
R10	Miscellaneous Expense	A10	Research & Manage Customer Issues	CO10	SW
R11	Management Costs	A11	Research & Evaluate Demurrage Claims		
R12	Services - General	A12	Research and Managing Sourcing and Allocations		
		A13	Manage Department		

Table 26 (Continued)

Case 3					
R1	Salary & Burden	A1	Enter/Maintain Customer Master Data	CO1	Branded Gasoline
R2	Travel	A2	Enter/Maintain Unbranded Customer Master Data	CO2	Unbranded Gasoline
R3	Education/Training	A3	Enter/Maintain Exchange Customer Master Data	CO3	7/11
		A4	Enter/Maintain Commercial Customer Master Data	CO4	Bulk
		A5	Enter/Maintain Asphalt Customer Master Data	CO5	Petrochemical
		A6	Enter/Maintain Lubes Customer Master Data	CO6	Industrial Products
		A7	Enter/Maintain ZV21 & Plant Maintenance Info.	CO7	Aviation
		A8	Enter/Maintain Other Info	CO8	Racing Fuel
		A9	Enter/Maintain Carrier Master Data Ins. Info.	CO9	Lubes
		A10	Coordinate Information (internal/external)	CO10	Asphalt
		A11	Issue & Execute Gasoline Contracts	CO11	Commercial
		A12	Issue & Execute Diesel Contracts	CO12	Other
		A13	Issue & Execute Customer Access Agreements		
		A14	Manage Department		
		A15	Execute Lubes Contracts		
		A16	Training		
		A17	Process Lubes Label Orders		

Table 26 (Continued)

Case 4					
R1	Wages & Salaries	A1	Receive Product/Invoices Disputes	CO1	National Accounts / Buybacks
R2	Labor Burden	A2	Research Dispute	CO2	OEM
R3	Materials - General	A3	Write-up and Log RFC	CO3	Base Oils / White oils
R4	Memberships, Dues, Assessments	A4	Faxing/Re-Faxing of Invoices to Customers	CO4	GEO
R5	Postage & Freight	A5	Research & Coordinate Return Product	CO5	Wax
R6	Employee Development	A6	Support Sales Force & Field and Ad-hoc Requests	CO6	Retail / Private Label
R7	Meals & Entertainment	A7	Research and Process Credit Issues	CO7	SE
R8	Meals - Meetings & Relations	A8	New Customer Presentations and Customer Visits	CO8	NE
R9	Travel Expense	A9	Attend Meetings	CO9	MW
R10	Miscellaneous Expense	A10	Research & Manage Customer Issues	CO10	SW
R11	Management Costs	A11	Research & Evaluate Demurrage Claims		
R12	Services - General	A12	Research and Managing Sourcing and Allocations		
		A13	Manage Department		

Table 26 (Continued)

Case 5				
R1	Wages & Salaries	A1	Set schedule/develop guidelines Operating Budget	**No Stage 2 Data**
R2	Labor Burden	A2	Develop/compile detail data for Operating Budget	
R3	Materials-General	A3	Develop Pro forma (Budget) Balance sheet/Cashflow	
R4	Services-General	A4	Prepare monthly Business Unit budget	
R5	Memberships	A5	Create Budget Presentation for the Board	
R6	Postage and Freight	A6	Load Budgeted Expenses in SAP	
R7	M & E	A7	Corporate Allocations	
R8	Travel	A8	Prepare Business Unit analysis	
R9	Miscellaneous	A9	Prepare Monthly Operating Report	
R10	Procurement	A10	Report out financial balance scorecard measures	
		A11	Prepare Actual Earnings Detail (Incl.Grimshaw Rpt)	
		A12	Compile Forecasting Data	
		A13	Prepare Results of Operations Presentation	
		A14	Prepare PDVMR Presentation	
		A15	Prepare Competitor Analysis	
		A16	Perform Special Projects	

Table 26 (Continued)

		Case 6		
No Stage 1 Data	A1	Attend IS/IT Training	CO1	Light Oil Marketing
	A2	Manage Projects & Contractors	CO2	Lubes Marketing
	A3	Analyze Requirements	CO3	Supply
	A4	Answer Customer Problems/Issues	CO4	Terminals
	A5	Maintain & Monitor Applications	CO5	Credit Card
	A6	New Systems/Project Development	CO6	Pricing
	A7	Implement Program (Roll-Out) & Train Clients		
	A8	Provide Data Statistics & Project Status to Mgmt		
	A9	Manage & Develop Vendor Relationships		
	A10	Define & Monitor Data Exchanges		
	A11	Perform Consulting & Special Projects		
	A12	Perform Admin & Mgmt/Internal		

Table 26 (Continued)

Case 7				
No Stage 1 Data	A1	**Activity names are unavailable**	CO1	Commercial
	A2		CO2	Aviation
	A3		CO3	Midatlantic
	A4		CO4	SE
	A5		CO5	NE
	A6		CO6	MW
	A7		CO7	SW
	A8			
	A9			
	A10			
	A11			
	A12			
	A13			

Not enough data are available to calculate the unit cycle time for TDABC2. Any validity of the equivalency conditions for TDABC2 will have to be inferred from the validity of the other equivalency conditions. If the case studies validate all of the equivalency conditions for Stage 1 (Proposition I and Proposition II and its corollaries) as well as the equivalency conditions for Stage 2 for IABC2 (Proposition III), then it can be inferred that the equivalency conditions for TDABC2 (Proposition IV and its corollaries) are also valid for this particular company.

Additionally, an assumption has to be made concerning the resource times for Stage 1 since the resource times are unavailable. Employees represent the only time-driven resource for this company, while the rest of the resources are non-time-driven. It is assumed that each employee works 2,000 hours per year. The company has the employees divided into labor resource groups (e.g. salaries, travel, etc.), and based on the data, each employee is in each of those groups. Furthermore, each activity consumes an equal amount of each of the labor resources. Thus, another assumption is that the employee time is divided evenly into each of the resource groups.

Finally, if any activity has zero consumption across all resources, it is eliminated from the data (the same rule applies for any cost object). For the average absolute percentage errors, assume that 20 percent or less is low error, 21 percent to 40 percent is moderate to low error, 41 percent to 60 percent is moderate error, and 61 percent and above is high error. The cases are presented in the following sections.

7.2. Case Study 1

For Stage 1, Case Study 1 has 14 resources and 15 activities. Only four of the resources are time driven (four labor groups: R1, R2, R3, and R14). Case 1 will be useful in demonstrating resource diversity, which causes inaccuracy of TDABC cost assignments regardless of whether Proposition II holds. There are 22.9 employees, with each working 2,000 hours per year for a total of 45,800 hours. The time is divided evenly among the four labor groups since the drivers are equal across each group. This means that the total time per employee per labor group is 500 hours (2,000 hours / 4 labor resource groups), and the total time per resource is 11,450 hours (45,800 hours / 4 labor resource groups).

Table 27 shows the resource consumption ratios and the Stage 1 cost assignments for ABC, IABC, and TDABC (dollars and time amounts in thousands). Panel A of Table 27 provides the resource consumption ratios, the correlation data ($r_{c\rho}$ and $r_{t\rho}$), and the ABC cost assignments. All of the $r_{c\rho}$'s across all activities are less than 40 percent in magnitude, with 10 of the 15 activities having $r_{c\rho}$'s of less than 20 percent. Overall, the $r_{c\rho}$'s are not significant. All of the $r_{t\rho}$'s are zero and are not significant either.

Panel B of Table 27 provides the IABC cost assignments. The average absolute percentage error (Avg $|\%\varepsilon_a|$) is 47.02 percent. A12 has the highest absolute percentage error of 264.58 percent, A3 the next (74.25 percent), and A13 the third (68.38 percent) with the rest of the activities having absolute percentage errors of less than or equal to 48.37 percent. A3, A12, and A13 could be considered outliers, thus overstating the average $|\%\varepsilon_a|$ of IABC. By looking at Case 1 by itself, this seems to be a significantly large error (Proposition I is violated) but in Section 7.9 this error will be compared to those in the other three cases to determine the overall average percentage error across all cases since each case represents one section of the company.

Panel C of Table 27 provides the TDABC cost assignments. Since only four of the 14 resources are time driven, there is a large average $|\%\varepsilon_a|$ of 36.12 percent due to resource diversity. For TDABC, this case purely represents the resource diversity issue and is not good in evaluating the equivalency conditions of Proposition II. However, the errors of both IABC and TDABC will be compared to those in the other cases to determine the overall average percentage error.

TABLE 27
Case Study 1, Stage 1 (Dollars and Time in Thousands)

Panel A:

	Resources																
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**	**R10**	**R11**	**R12**	**R13**	**R14**	**ABC Cost**	$r_{c\rho}$	$r_{t\rho}$
A1	0.13	0.13	0.13		0.18	0.13		0.20	0.18	0.16		0.18	0.24	0.13	$369	-0.33	0.00
A2	0.16	0.16	0.16		0.24	0.26		0.15	0.24	0.12		0.24	0.18	0.16	$488	-0.01	0.00
A3	0.05	0.05	0.05		0.08	0.06	0.60	0.08	0.08	0.05	0.60	0.08	0.08	0.05	$213	-0.19	0.00
A4	0.05	0.05	0.05		0.08	0.04		0.07	0.08	0.05		0.08	0.08	0.05	$142	-0.34	0.00
A5	0.04	0.04	0.04		0.06	0.15		0.07	0.06	0.04		0.06	0.06	0.04	$164	0.09	0.00
A6	0.04	0.04	0.04											0.04	$63	0.00	0.00
A7	0.01	0.01	0.01											0.01	$10	0.00	0.00
A8	0.06	0.06	0.06		0.10	0.06		0.21	0.10	0.07		0.10	0.10	0.06	$194	-0.20	0.00
A9	0.05	0.05	0.05		0.07	0.04		0.06	0.07	0.05		0.07	0.07	0.05	$130	-0.33	0.00
A10	0.14	0.14	0.14											0.14	$212	0.00	0.00
A11	0.08	0.08	0.08							0.32				0.08	$156	-0.18	0.00
A12	0.03	0.03	0.03	1										0.03	$58	-0.24	0.00
A13	0.04	0.04	0.04		0.07	0.04	0.40	0.06	0.07	0.05	0.40	0.07	0.07	0.04	$160	-0.20	0.00
A14	0.08	0.08	0.08		0.12	0.21		0.11	0.12	0.08		0.12	0.12	0.08	$291	0.05	0.00
A15	0.03	0.03	0.03											0.03	$46	0.36	0.00
Cost	$1,462	$11	$6	$19	$4	$514	$30	$154	$180	$111	$70	$76	$45	$13	$2,695		
Time	11.45	11.45	11.45											11.45	45.8		

TABLE 27 (Continued)

Panel B:

	$\bar{\rho}_a$	**IABC Cost**	$\lvert\%\varepsilon_a\rvert$
A1	0.13	$346	6.24%
A2	0.15	$403	17.39%
A3	0.14	$371	74.25%
A4	0.05	$134	5.60%
A5	0.05	$124	24.50%
A6	0.01	$33	48.37%
A7	0.00	$5	48.37%
A8	0.07	$190	2.16%
A9	0.04	$120	7.53%
A10	0.04	$110	48.37%
A11	0.05	$123	20.86%
A12	0.08	$213	264.58%
A13	0.10	$269	68.38%
A14	0.09	$232	20.39%
A15	0.01	$24	48.37%
Cost	$2,695	$2,695	
		Avg $\lvert\%\varepsilon_a\rvert$	47.02%

Panel C:

	TDABC Cost	$\lvert\%\varepsilon_a\rvert$
A1	$350	5.20%
A2	$443	9.28%
A3	$142	33.19%
A4	$141	0.39%
A5	$95	41.91%
A6	$114	80.71%
A7	$18	80.71%
A8	$173	10.97%
A9	$134	3.26%
A10	$384	80.71%
A11	$218	39.94%
A12	$71	21.05%
A13	$115	27.74%
A14	$215	25.96%
A15	$82	80.71%
Cost	$2,695	
	Avg $\lvert\%\varepsilon_a\rvert$	36.12%

c = $58.84 (not in thousands)

Table 28 provides the Stage 2 cost assignments for 10 cost objects under ABC and IABC2. The costs for the 15 activities come from Table 27, Panel A. Panel A of Table 28 provides the activity consumption ratios for each cost object and the ABC cost assignments. Notice that Cost Object 10 (CO10) consumes only one activity (A7) at 100 percent consumption. Thus, direct tracing is used for CO10, in which the activity cost for A7 is directly traced to CO10 and thus driver tracing is excluded. Consequently, A7 is extracted out of the computation of the average consumption ratios. This means that the sum of the consumption ratios across all activities for a single cost object is divided by 14 activities instead of 15. Consequently, no average resource consumption ratio is given to CO10 as shown in Panel B (DT represents direct tracing).

Panel B shows the correlation $r_{C\upsilon}$. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 37 percent in magnitude. Recall that IABC2 eliminates the need for Stage 1 cost assignments. Although IABC provided a large average percentage error (47.02 percent) in Table 27, Panel B, notice that the average percentage error (Avg $|\%\varepsilon_i|$) for IABC2 is 6.38 percent in Table 28, Panel B. CO9 has the largest absolute percentage error of 38.28 percent, with the next highest being CO8 of 14.64 percent. The rest of the cost objects have absolute percentage errors of less than or equal to 5.20 percent. It can be concluded that CO9 might be an outlier, thus overstating average $|\%\varepsilon_i|$ of IABC2.

TABLE 28
Case Study 1, Stage 2 (Dollars and Time in Thousands)

Panel A:

	Activities															
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**A13**	**A14**	**A15**	**ABC Cost**
CO1	0.25	0.18	0.19	0.10	0.20	0.19		0.26	0.24	0.19	0.19	0.19	0.19	0.13	0.19	$519
CO2	0.21	0.25	0.23	0.13	0.23	0.23		0.30	0.28	0.23	0.23	0.22	0.23	0.20	0.23	$614
CO3	0.21	0.11	0.13	0.17	0.13	0.13		0.09	0.08	0.13	0.13	0.16	0.13	0.13	0.13	$363
CO4	0.08	0.11	0.08	0.07	0.07	0.08		0.17	0.04	0.08	0.08	0.06	0.08	0.03	0.08	$221
CO5	0.13	0.11	0.13	0.23	0.10	0.13		0.09	0.08	0.13	0.13	0.12	0.13	0.17	0.13	$344
CO6	0.04	0.21	0.15	0.13	0.17	0.15		0.04	0.20	0.15	0.15	0.14	0.15	0.23	0.15	$406
CO7	0.04	0.04	0.04	0.03	0.03	0.04		0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	$100
CO8	0.04		0.04	0.10	0.03	0.04			0.04	0.04	0.04	0.06	0.04	0.07	0.04	$98
CO9			0.01	0.03	0.03	0.01				0.01	0.01	0.01	0.01		0.01	$20
CO10							1									$10
Cost	$369	$488	$213	$142	$164	$63	$10	$194	$130	$212	$156	$58	$160	$291	$46	$2,695

TABLE 28 (Continued)

Panel B:

	$\bar{\upsilon}_i$	**IABC2 Cost**	$\lvert\%\varepsilon_i\rvert$	$r_{C\upsilon}$
CO1	0.18	$483	0.59%	0.31
CO2	0.21	$572	0.57%	0.34
CO3	0.12	$336	1.10%	0.33
CO4	0.07	$197	5.20%	0.37
CO5	0.12	$327	1.49%	0.19
CO6	0.14	$375	1.39%	0.27
CO7	0.04	$94	0.55%	0.32
CO8	0.04	$105	14.64%	-0.16
CO9	0.01	$27	38.28%	-0.31
Cost	$2,685	$2,685		
CO10	DT[a]	$10	–	-0.37
Cost		$2,695		
		Avg $\lvert\%\varepsilon_i\rvert$	6.38%	

[a] DT stands for directly traced.

Case Study 1 has shown that the equivalency conditions for TDABC and IABC may or may not be valid since the average absolute percentage error is on the low to moderate side for TDABC (36.12 percent) due to resource diversity and on the moderate side for IABC (47.02 percent). However, IABC2 has a low average absolute percentage error of 6.38 percent, which means that the equivalency conditions for IABC2 may be valid for this company. The average absolute percentage errors from this case study will be compared to those of the other cases in Section 7.9. The next section illustrates Case Study 2.

7.3. Case Study 2

The Stage 1 data for the second case, Case Study 2, contain 12 resources and 13 activities, in which 11 of the resources are time driven. These 11 labor resource groups use 9 employees, which provide a total of 18,000 hours per year. The one resource (R11) is not a time-driven resource, and it is consumed exclusively by one activity (A13). Additionally, R11 is the only resource that A13 consumes. Hence, the resource cost for R11 is directly traced to A13. The time available for each of the time-driven resources is 1,636.4 hours (18,000 / 11 labor resource groups).

Table 29 provides the Stage 1 case information. Notice in Panel A that each of the time-driven resource vectors are linearly dependent (all of them are identical). Since all but one of the resources are time based and each of the *time-driven* resource vectors are linearly dependent, it follows that $r_{c\rho}$ and $r_{t\rho}$ for each activity should be zero (or, undefined) and they are as shown in Panel A. Panel B of Table 29 provides the IABC cost assignments. Additionally, R11 is extracted out of the computation of the average consumption ratios since it is directly traced A13 and since A13 only consumes this one resource, thus implying that the sum of the consumption ratios across all resources for a single activity is divided by 11 resources instead of 12. As a result, no average resource consumption ratio is given to A13. The average $|\%\varepsilon_a|$ of IABC cost assignments when compared to the benchmark ABC cost assignments is Panel A is zero, thus Proposition I holds for this case.

Panel C shows the TDABC cost assignments. Notice that nothing is assigned to A13 since it does not consume a time-driven resource. Under TDABC, the cost of R11, the non-time-driven resource, is pooled in with the other resource costs when calculating the capacity cost rate c (or the cost per hour). For TDABC cost assignments, the average $|\%\varepsilon_a|$ is 16.66 percent, but it includes the absolute percentage error of 100 percent for A13, which can be considered an outlier. It is interesting that if an activity does not consume a time-driven resource, then the cost of that activity would be zero. This shows that TDABC can introduce error when there are non-time-driven resources.

TABLE 29
Case Study 2, Stage 1 (Dollars and Time in Thousands)

Panel A:

	Resources														
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**	**R10**	**R11**	**R12**	**ABC Cost**	$r_{c\rho}$	$r_{t\rho}$
A1	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		0.11	$70	0.00	0.00
A2	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41		0.41	$260	0.00	0.00
A3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		0.11	$73	0.00	0.00
A4	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06		0.06	$40	0.00	0.00
A5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		0.10	$66	0.00	0.00
A6	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04		0.04	$27	0.00	0.00
A7	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08		0.08	$52	0.00	0.00
A8	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01		0.01	$7	0.00	0.00
A9	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$10	0.00	0.00
A10	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$12	0.00	0.00
A11	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$13	0.00	0.00
A12	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004		0.004	$3	0.00	0.00
A13											1		$62	0.00	0.00
Cost	$357	$192	$3	$0.15	$1	$2.5	$3.6	$0.7	$29	$0.85	$62	$44	$696		
Time	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6		1.6	18		

TABLE 29 (Continued)

Panel B:

	$\bar{\rho}_a$ [a]	IABC Cost[b]	$\lvert\%\varepsilon_a\rvert$
A1	0.11	$70	0.00%
A2	0.41	$260	0.00%
A3	0.11	$73	0.00%
A4	0.06	$40	0.00%
A5	0.10	$66	0.00%
A6	0.04	$27	0.00%
A7	0.08	$52	0.00%
A8	0.01	$7	0.00%
A9	0.02	$10	0.00%
A10	0.02	$12	0.00%
A11	0.02	$13	0.00%
A12	0.004	$3	0.00%
Cost	$634	$634	
A13	DT[b]	$62	–
		$696	
		Avg $\lvert\%\varepsilon_a\rvert$	0.00%

Panel C:

	TDABC Cost	$\lvert\%\varepsilon_a\rvert$
A1	$77	9.72%
A2	$285	9.72%
A3	$80	9.72%
A4	$44	9.72%
A5	$73	9.72%
A6	$30	9.72%
A7	$58	9.72%
A8	$8	9.72%
A9	$11	9.72%
A10	$13	9.72%
A11	$14	9.72%
A12	$3	9.72%
A13	$0[c]	100%
Cost	$696	
	Avg $\lvert\%\varepsilon_a\rvert$	16.66%

c **=** $38.65 (not in thousands)

[a] Since R11 is non-time-driven and is only consumed by A13, it is treated separately from the other resources. Hence, to find the average consumption ratios, divide by 11 resources instead of 12.
[b] DT stands for directly traced.
[c] The resource cost associated with R11 under ABC is pooled in with the other resources under TDABC because the total resource cost (including R11) is divided by the total hours available to find the cost per hour *c*. Since A13 has no time attached to it, then it receives a zero cost under TDABC.

Table 30 provides the Case Study 2 data for Stage 2 cost assignments under ABC and IABC2. There are 13 activities and 10 cost objects. Panel A provides the activity consumption ratios for each cost object and the benchmark ABC cost assignments. Panel B shows the correlation $r_{C\upsilon}$ and the IABC2 cost assignments. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 47 percent in magnitude. Although IABC provided a zero average absolute percentage error in Panel B of Table 29, the average $|\%\varepsilon_i|$ for IABC2 in Table 30, Panel B is 19.47 percent. CO7 has the largest absolute percentage error of 39.81 percent, with the rest of the cost objects having absolute percentage errors of less than or equal to 30.99 percent, with the smallest being 1.98 percent for CO8. However, the average absolute percentage error for IABC2 of 19.47 percent (less than 20 percent) is low.

TABLE 30
Case Study 2, Stage 2 (Dollars and Time in Thousands)

Panel A:

	Activities													
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**A13**	**ABC Cost**
CO1	0.188	0.196	0.188	0.183	0.109	0.193	0.181			0.200			0.100	$116
CO2	0.014	0.013	0.015	0.041	0.013								0.100	$14
CO3	0.116	0.108	0.114	0.105	0.054	0.124	0.125		0.167	0.100			0.100	$71
CO4	0.007	0.001	0.001	0.009									0.100	$7
CO5	0.080	0.091	0.090	0.066	0.101	0.116	0.101		0.083	0.116			0.100	$62
CO6	0.140	0.134	0.207	0.131	0.130	0.144	0.126	0.200	0.167	0.144			0.100	$95
CO7	0.075	0.063	0.063	0.096	0.139	0.086	0.063	0.200	0.167	0.079		0.333	0.100	$56
CO8	0.075	0.071	0.045	0.067	0.121	0.049	0.089	0.200	0.083	0.073			0.100	$53
CO9	0.159	0.184	0.145	0.162	0.200	0.144	0.189	0.200	0.167	0.144	1	0.333	0.100	$128
CO10	0.146	0.140	0.132	0.140	0.134	0.144	0.126	0.200	0.167	0.144		0.333	0.100	$93
Cost	$70	$260	$73	$40	$66	$27	$52	$7	$10	$12	$13	$3	$62	$696

TABLE 30 (Continued)

Panel B:

	$\overline{\upsilon}_i$	**IABC2 Cost**	$\lvert\%\varepsilon_i\rvert$	$r_{C\upsilon}$
CO1	0.12	$82	28.89%	0.47
CO2	0.01	$10	25.26%	0.16
CO3	0.09	$60	16.37%	0.26
CO4	0.01	$6	14.78%	0.05
CO5	0.07	$51	18.62%	0.33
CO6	0.12	$87	8.69%	0.16
CO7	0.11	$78	39.81%	-0.33
CO8	0.07	$52	1.98%	0.03
CO9	0.24	$167	30.99%	-0.21
CO10	0.15	$102	9.35%	-0.15
Cost	$696	$696		
		Avg $\lvert\%\varepsilon_i\rvert$	19.47%	

Case Study 2 has shown that the equivalency conditions for TDABC and IABC are valid for this company since the average absolute percentage errors are low (less than 10 percent for TDABC and zero for IABC). However, IABC2 provided a higher average absolute percentage error, but it can still be a valid cost assignment method since the average absolute percentage error did not exceed 20 percent. This error will be compared to those of the other cases in Section 7.9. The next section illustrates the third case study.

7.4. Case Study 3

The Stage 1 data for Case Study 3 contain 3 resources and 17 activities, in which all resources are time driven and use 12 employees who provide a total of 24,000 hours per year. The time available for each of the time-driven resources is 8,000 hours (24,000 hours / 3 labor resource groups).

Table 31 provides the Stage 1 case information. Notice in Panel A that each of the time-driven resource vectors are linearly dependent (all of them are identical). Since each of the time-driven resource vectors are linearly dependent, it follows that $r_{c\rho}$ and $r_{t\rho}$ for each activity should be zero (or, undefined) and they are as shown in Panel A. Panel B of Table 31 provides the IABC cost assignments. The average $|\%\varepsilon_a|$ of IABC cost assignments is zero due to all of the resource vectors being linearly dependent (causing no correlations). Hence, IABC replicates the ABC system. Panel C shows the same results for TDABC. Thus, both Propositions I and II hold for this case study.

TABLE 31
Case Study 3, Stage 1 (Dollars and Time in Thousands)

Panel A:

	Resources					
	R1	**R2**	**R3**	**ABC Cost**	$r_{c\rho}$	$r_{t\rho}$
A1	0.10	0.10	0.10	$64	0.00	0.00
A2	0.08	0.08	0.08	$49	0.00	0.00
A3	0.03	0.03	0.03	$16	0.00	0.00
A4	0.02	0.02	0.02	$13	0.00	0.00
A5	0.04	0.04	0.04	$21	0.00	0.00
A6	0.08	0.08	0.08	$46	0.00	0.00
A7	0.10	0.10	0.10	$61	0.00	0.00
A8	0.04	0.04	0.04	$25	0.00	0.00
A9	0.14	0.14	0.14	$84	0.00	0.00
A10	0.12	0.12	0.12	$76	0.00	0.00
A11	0.03	0.03	0.03	$21	0.00	0.00
A12	0.02	0.02	0.02	$13	0.00	0.00
A13	0.01	0.01	0.01	$6	0.00	0.00
A14	0.10	0.10	0.10	$60	0.00	0.00
A15	0.02	0.02	0.02	$10	0.00	0.00
A16	0.05	0.05	0.05	$29	0.00	0.00
A17	0.03	0.03	0.03	$19	0.00	0.00
Cost	$603	$5	$5	$613		
Time	8	8	8	24		

TABLE 31 (Continued)

Panel B:

	$\overline{\rho}_a$	**IABC Cost**	$\lvert\%\varepsilon_a\rvert$
A1	0.104	$64	0.00%
A2	0.080	$49	0.00%
A3	0.026	$16	0.00%
A4	0.022	$13	0.00%
A5	0.035	$21	0.00%
A6	0.075	$46	0.00%
A7	0.100	$61	0.00%
A8	0.040	$25	0.00%
A9	0.137	$84	0.00%
A10	0.124	$76	0.00%
A11	0.034	$21	0.00%
A12	0.021	$13	0.00%
A13	0.010	$6	0.00%
A14	0.098	$60	0.00%
A15	0.017	$10	0.00%
A16	0.047	$29	0.00%
A17	0.032	$19	0.00%
Cost	$613	$613	
		Avg $\lvert\%\varepsilon_a\rvert$	0.00%

Panel C:

	TDABC Cost	$\lvert\%\varepsilon_a\rvert$
A1	$64	0.00%
A2	$49	0.00%
A3	$16	0.00%
A4	$13	0.00%
A5	$21	0.00%
A6	$46	0.00%
A7	$61	0.00%
A8	$25	0.00%
A9	$84	0.00%
A10	$76	0.00%
A11	$21	0.00%
A12	$13	0.00%
A13	$6	0.00%
A14	$60	0.00%
A15	$10	0.00%
A16	$29	0.00%
A17	$19	0.00%
Cost	$613	
Avg $\lvert\%\varepsilon_a\rvert$		0.00%

c = $25.56 (not in thousands)

Table 32 provides the Case Study 3 data for Stage 2 cost assignments under ABC and IABC2. There are 17 activities and 12 cost objects. Panel A provides the activity consumption ratios for each cost object and the benchmark ABC cost assignments. Panel B shows the correlation $r_{C\upsilon}$ and the IABC2 cost assignments. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 49 percent in magnitude. Although IABC provided a zero average absolute percentage error in Table 31, Panel B, notice that the average $|\%\varepsilon_i|$ for IABC2 in Table 32, Panel B is 36.21 percent. CO11 has the largest absolute percentage error of 104.24 percent (a possible outlier), with the rest of the cost objects having absolute percentage errors of less than or equal to 50.32 percent, with the smallest being 2.79 percent for CO1. It seems that IABC2 did worse for this case study, but the average absolute percentage error of 36.21 percent is moderate to low.

TABLE 32
Case Study 3, Stage 2 (Dollars and Time in Thousands)

Panel A:

	Activities																	
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**A13**	**A14**	**A15**	**A16**	**A17**	**ABC Cost**
CO1	0.62						0.31	0.53		0.09	0.88	0.12	0.45	0.19		0.08		$115
CO2	0.09	0.98					0.17	0.19		0.12	0.05	0.88	0.55	0.21		0.20		$112
CO3	0.10	0.01					0.04	0.01		0.02				0.01		0.03		$12
CO4	0.13	0.01					0.01			0.01				0.01		0.02		$12
CO5	0.01													0.01		0.005		$1
CO6	0.01									0.06				0.01		0.002		$6
CO7	0.03						0.02			0.19				0.01		0.002		$18
CO8	0.01						0.01			0.005	0.07			0.01				$3
CO9						1		0.27		0.17				0.40	1	0.56	1	$136
CO10					0.97		0.03			0.03						0.01		$27
CO11				0.96			0.03			0.01				0.04		0.03		$19
CO12			1	0.04	0.03		0.38		1	0.27				0.12		0.05		$153
Cost	$64	$49	$16	$13	$21	$46	$61	$25	$84	$76	$21	$13	$6	$60	$10	$29	$19	$613

TABLE 32 (Continued)

Panel B:

	$\overline{\upsilon}_i$	**IABC2 Cost**	$\lvert\%\varepsilon_i\rvert$	$r_{C\upsilon}$
CO1	0.193	$118	2.79%	-0.03
CO2	0.201	$124	10.56%	-0.10
CO3	0.013	$8	36.34%	0.44
CO4	0.012	$7	38.73%	0.36
CO5	0.001	$7	35.87%	0.41
CO6	0.005	$0.8	50.02%	0.47
CO7	0.015	$3	50.32%	0.49
CO8	0.006	$9	8.90%	-0.04
CO9	0.259	$4	17.27%	-0.15
CO10	0.062	$159	47.43%	-0.13
CO11	0.064	$38	104.24%	-0.21
CO12	0.170	$104	32.08%	0.37
Cost	$613	$613		
		Avg $\lvert\%\varepsilon_i\rvert$	36.21%	

Case Study 3 has shown that, for this company, the equivalency conditions for TDABC and IABC are valid since the average absolute percentage errors are zero. However, IABC2 provided an accuracy loss of 36.21 percent, a moderate to low error. The next section presents the fourth case study.

7.5. Case Study 4

Stage 1 of Case Study 4 has 12 resources and 13 activities, in which 11 of the resources are time driven. These 11 labor resource groups use 9.4 employees, which provide a total of 18,800 hours per year. The one resource (R11) is not a time-driven resource, and it is consumed exclusively by one activity (A15). The time available for each of the time-driven resources is 1,709 hours (18,000 / 11 labor resource groups).

Table 33 provides the Stage 1 case information. Notice in Panel A that each of the time-driven resource vectors are linearly dependent (all of them are identical). Since each of the time-driven resource vectors are linearly dependent (and only one vector is non-time-driven and different), it follows that $r_{c\rho}$ and $r_{\iota\rho}$ for each activity should be zero (or, undefined), and they are as shown in Panel A. Panel B of Table 33 provides the IABC cost assignments. The average $|\%\varepsilon_a|$ of IABC cost assignments when compared to the benchmark ABC cost assignments in Panel A is 0.83 percent; thus, Proposition I holds since this error is very small. Panel C shows the TDABC cost assignments. For TDABC cost assignments, the average $|\%\varepsilon_a|$ is 14.09 percent, but it includes the absolute percentage error of 66.58 percent for A13 since it consumes 100 percent of the non-time-driven resource R11 and only 4 percent of each of the other resources. Since the absolute percentage error of 66.58 percent is much greater than the rest of the errors, the error for

A13 can be considered an outlier for TDABC. In spite of no correlation, TDABC can introduce error because of one non-time-driven resource.

TABLE 33
Case Study 4, Stage 1 (Dollars and Time in Thousands)

Panel A:

	Resources														
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**	**R10**	**R11**	**R12**	**ABC Cost**	$r_{c\rho}$	$r_{t\rho}$
A1	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		0.11	$70	0.00	0.00
A2	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39		0.39	$260	0.00	0.00
A3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		0.11	$73	0.00	0.00
A4	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06		0.06	$40	0.00	0.00
A5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		0.10	$66	0.00	0.00
A6	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04		0.04	$27	0.00	0.00
A7	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08		0.08	$52	0.00	0.00
A8	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01		0.01	$7	0.00	0.00
A9	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$10	0.00	0.00
A10	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$12	0.00	0.00
A11	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02		0.02	$13	0.00	0.00
A12	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004		0.004	$3	0.00	0.00
A13	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	1.00	0.04	$62	0.00	0.00
Cost	$357	$192	$3	$0.15	$1	$2.5	$3.6	$0.7	$29	$0.85	$62	$44	$696		
Time	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7		1.7	18.8		

TABLE 33 (Continued)

Panel B:

	$\bar{\rho}_a$	**IABC Cost**[a]	$\lvert\%\varepsilon_a\rvert$
A1	0.10	$68	0.57%
A2	0.36	$250	0.57%
A3	0.10	$70	0.57%
A4	0.06	$39	0.57%
A5	0.09	$64	0.57%
A6	0.04	$26	0.57%
A7	0.07	$51	0.57%
A8	0.01	$7	0.57%
A9	0.01	$10	0.57%
A10	0.02	$11	0.57%
A11	0.02	$12	0.57%
A12	0.004	$3	0.57%
A13	0.12	$85	3.92%
Cost	$696	$696	
		Avg $\lvert\%\varepsilon_a\rvert$	0.83%

Panel C:

	TDABC Cost	$\lvert\%\varepsilon_a\rvert$
A1	$74	9.72%
A2	$273	9.72%
A3	$76	9.72%
A4	$42	9.72%
A5	$70	9.72%
A6	$29	9.72%
A7	$55	9.72%
A8	$8	9.72%
A9	$11	9.72%
A10	$12	9.72%
A11	$13	9.72%
A12	$3	9.72%
A13	$27	66.58%
Cost	$696	
Avg $\lvert\%\varepsilon_a\rvert$		14.09%

c **=** $37 (not in thousands)

[a] Some of the costs for IABC only *seem* to be identical to ABC since the dollar amounts are rounded to the nearest thousand.

Table 34 provides the Case Study 3 data for Stage 2 cost assignments under ABC and IABC2. There are 13 activities and 10 cost objects. Panel A provides the activity consumption ratios for each cost object and the benchmark ABC cost assignments. Panel B shows the correlation $r_{C\upsilon}$ and the IABC2 cost assignments. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 46 percent in magnitude. For IABC2, the absolute percentage errors range from 3.26 percent to 38.38 percent. Although IABC provided a small average absolute percentage error (0.83 percent) in Table 32, Panel B, the average $\left|\%\varepsilon_i\right|$ for IABC2 in Table 34, Panel B is 22.96 percent, which is moderate to low.

TABLE 34
Case Study 4, Stage 2 (Dollars and Time in Thousands)

Panel A:

	Activities													
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**A13**	**ABC Cost**
CO1	0.188	0.196	0.188	0.183	0.109	0.193	0.181			0.200			0.100	$114
CO2	0.014	0.013	0.015	0.041	0.013								0.100	$16
CO3	0.116	0.108	0.114	0.105	0.054	0.124	0.125		0.167	0.100			0.100	$71
CO4	0.007	0.001	0.001	0.009									0.100	$10
CO5	0.080	0.091	0.090	0.066	0.101	0.116	0.101		0.083	0.116			0.100	$62
CO6	0.140	0.134	0.207	0.131	0.130	0.144	0.126	0.200	0.167	0.144			0.100	$94
CO7	0.075	0.063	0.063	0.096	0.139	0.086	0.063	0.200	0.167	0.079		0.333	0.100	$57
CO8	0.075	0.071	0.045	0.067	0.121	0.049	0.089	0.200	0.083	0.073			0.100	$54
CO9	0.159	0.184	0.145	0.162	0.200	0.144	0.189	0.200	0.167	0.144	1	0.333	0.100	$125
CO10	0.146	0.140	0.132	0.140	0.134	0.144	0.126	0.200	0.167	0.144		0.333	0.100	$92
Cost	$68	$249	$69	$38	$63	$26	$50	$7	$10	$11	$12	$3	$89	$696

TABLE 34 (Continued)

Panel B:

	$\overline{\upsilon}_i$	**IABC2 Cost**	$\lvert \%\varepsilon_i \rvert$	$r_{C\upsilon}$
CO1	0.118	$82	27.66%	0.46
CO2	0.015	$10	36.09%	0.27
CO3	0.086	$60	16.28%	0.27
CO4	0.009	$6	37.09%	0.17
CO5	0.073	$51	19.03%	0.35
CO6	0.125	$87	7.64%	0.15
CO7	0.112	$78	38.38%	-0.33
CO8	0.075	$52	3.26%	0.04
CO9	0.241	$167	33.58%	-0.23
CO10	0.147	$102	10.54%	-0.17
Cost	$696	$696		
		Avg $\lvert \%\varepsilon_i \rvert$	22.96%	

Case Study 4 has shown that the equivalency conditions for TDABC, IABC, and IABC2 are considered valid for this company since most of the average absolute percentage errors are low, with the one for IABC2 being a low to moderate amount of 22.96 percent.

7.6. Case Study 5

For Case Study 5, only the data for Stage 1 cost assignments are available for 10 resources and 16 activities. All of the resources are labor resources. There are seven employees, which provide a total of 14,000 hours per year. Table 35, Panel A provides the ABC Stage 1 cost assignments. All of the resource vectors are linearly dependent, thus providing zero correlations across all activities. Panels B and C shows that the cost assignments for IABC and TDABC are equivalent to those of ABC. Hence, the equivalency conditions in Propositions I and II hold perfectly for this case.

TABLE 35
Case Study 5, (Dollars and Time in Thousands)

Panel A:

	Resources												
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**	**R10**	**ABC Cost**	$r_{c\rho}$	$r_{t\rho}$
A1	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	$7.5	0.00	0.00
A2	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	$73.9	0.00	0.00
A3	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	$17.1	0.00	0.00
A4	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	$32.1	0.00	0.00
A5	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	$28.9	0.00	0.00
A6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	$5.4	0.00	0.00
A7	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	$7.5	0.00	0.00
A8	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	$172.5	0.00	0.00
A9	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	$33.2	0.00	0.00
A10	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	$15	0.00	0.00
A11	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	$102.8	0.00	0.00
A12	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	$54.6	0.00	0.00
A13	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	$53.6	0.00	0.00
A14	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	$28.9	0.00	0.00
A15	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	$18.2	0.00	0.00
A16	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	$98.6	0.00	0.00
Cost	$477.3	$257.8	$1.2	$1.2	$0.5	$0.6	$2.4	$6	$0.6	$2.4	$750		
Time	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	14		

TABLE 35 (Continued)

Panel B:

	$\bar{\rho}_a$	**IABC Cost**	$\lvert\%\varepsilon_a\rvert$
A1	0.01	$7.5	0.00%
A2	0.10	$73.9	0.00%
A3	0.02	$17.1	0.00%
A4	0.04	$32.1	0.00%
A5	0.04	$28.9	0.00%
A6	0.01	$5.4	0.00%
A7	0.01	$7.5	0.00%
A8	0.23	$172.5	0.00%
A9	0.04	$33.2	0.00%
A10	0.02	$15	0.00%
A11	0.14	$102.8	0.00%
A12	0.07	$54.6	0.00%
A13	0.07	$53.6	0.00%
A14	0.04	$28.9	0.00%
A15	0.02	$18.2	0.00%
A16	0.13	$98.6	0.00%
Cost	$750	$750	
		Avg $\lvert\%\varepsilon_a\rvert$	0.00%

Panel C:

	TDABC Cost	$\lvert\%\varepsilon_a\rvert$
A1	$7.5	0.00%
A2	$73.9	0.00%
A3	$17.1	0.00%
A4	$32.1	0.00%
A5	$28.9	0.00%
A6	$5.4	0.00%
A7	$7.5	0.00%
A8	$172.5	0.00%
A9	$33.2	0.00%
A10	$15	0.00%
A11	$102.8	0.00%
A12	$54.6	0.00%
A13	$53.6	0.00%
A14	$28.9	0.00%
A15	$18.2	0.00%
A16	$98.6	0.00%
Cost	$750	
Avg $\lvert\%\varepsilon_a\rvert$		0.00%

c = $37 (not in thousands)

Case Study 5 provides another illustration (similar to Stage 1 of Case Study 3) of Propositions I and II holding perfectly when resource vectors are linearly dependent. The final two case studies provide two illustrations for Stage 2 equivalency conditions.

7.7. Case Study 6

For Case Study 6, only Stage 2 data are available for 12 activities and 6 cost objects. Table 36 provides the Stage 2 cost assignments. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 43 percent. The average $|\%\varepsilon_i|$ for IABC2 is 13.93 percent, which is a low error (less than 20 percent). Therefore, this case validates Proposition III. The final case illustrating Stage 2, Case Study 7, is provided in the next section.

TABLE 36
Case Study 6 (Dollars and Time in Thousands)

Panel A:

	Activities												
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**ABC Cost**
CO1	0.4000	0.3234	0.1354	0.1292	0.2708	0.3462	0.2791	0.4377	0.4583	0.0859	0.2854	0.3917	$658.7
CO2	0.2333	0.1141	0.0909	0.5570	0.0903	0.0769	0.2713	0.1566	0.1528	0.0113	0.1293	0.2333	$515
CO3	0.0017	0.0025	0.0012	0.0013	0.0022	0.0015	0.0008	0.0004	0.0003	0.0004		0.0017	$3.6
CO4	0.1667	0.1563	0.6569	0.2103	0.1111	0.0769	0.3488	0.2830	0.0278	0.1599	0.4878	0.1667	$521.4
CO5	0.1167	0.2813	0.0584	0.0404	0.4167	0.4231	0.0620	0.1038	0.3472	0.7210	0.0976	0.1250	$596.6
CO6	0.0817	0.1225	0.0572	0.0618	0.1089	0.0754	0.0380	0.0185	0.0136	0.0215		0.0817	$176.6
Cost	$70.1	$135.1	$221.1	$497.9	$463.1	$509.5	$117.5	$89.8	$48.4	$63.6	$85.6	$170.2	$2,472

TABLE 36 (Continued)

Panel B:

	$\bar{\upsilon}_i$	**IABC2 Cost**	$\left|\%\varepsilon_i\right|$	$r_{C\upsilon}$
CO1	0.2953	$729.9	10.81%	-0.29
CO2	0.1764	$436.1	15.33%	0.28
CO3	0.0012	$2.9	20.59%	0.43
CO4	0.2377	$587.5	12.68%	-0.18
CO5	0.2328	$575.4	3.56%	0.05
CO6	0.0567	$140.2	20.59%	0.43
Cost	$2,472	$2,472		
		Avg $\left|\%\varepsilon_i\right|$	13.93%	

7.8. Case Study 7

For Case Study 7, there are 13 activities and 7 cost objects. Table 37 provides the Stage 2 cost assignments. The $r_{C\upsilon}$'s across all cost objects are less than or equal to 47 percent. The average $|\%\varepsilon_i|$ for IABC2 is 16.36 percent, which is a low error (less than 20 percent). Notice that CO2 has a very high percentage error of 49.03 percent compared to the errors of the rest of the cost objects. CO2 can be considered an outlier that overstates the actual average $|\%\varepsilon_i|$. Since the average $|\%\varepsilon_i|$ is considered low, this case validates Proposition III. The next section discusses and compares the results from all case studies.

TABLE 37
Case Study 7 (Dollars and Time in Thousands)

Panel A:

	Activities													
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**	**A13**	**ABC Cost**
CO1	0.1825	0.1788	0.1767	0.1808	0.1170	0.1808	0.1767	0.1875	0.1764	0.1170		0.4759	0.1432	$182.7
CO2	0.1183	0.1167	0.1167	0.1083		0.0834	0.0833		0.0832	0.0300	1		0.1428	$106.9
CO3	0.1447	0.1497	0.1480	0.1272	0.2066	0.1855	0.1813	0.0938	0.1814	0.1926		0.5039	0.1428	$174.7
CO4	0.1355	0.1355	0.1397	0.1397	0.1696	0.1314	0.1397	0.1875	0.1398	0.1576		0.0050	0.1428	$156
CO5	0.1321	0.1334	0.1313	0.1438	0.1736	0.1438	0.1329	0.2188	0.1331	0.1646		0.0050	0.1428	$156
CO6	0.1380	0.1355	0.1397	0.1397	0.1596	0.1314	0.1397	0.1875	0.1398	0.1576		0.0050	0.1428	$155.2
CO7	0.1488	0.1505	0.1480	0.1605	0.1736	0.1438	0.1463	0.1250	0.1464	0.1806		0.0050	0.1428	$168.1
Cost	$145.9	$226.4	$158.5	$135.8	$115.7	$56.2	$35.2	$13	$28.9	$80.5	$4.2	$5.9	$93.4	$1,099.6

TABLE 37 (Continued)

Panel B:

	$\overline{\upsilon}_i$	**IABC2 Cost**	$\lvert\%\varepsilon_i\rvert$	$r_{C\upsilon}$
CO1	0.1764	$194	6.17%	-0.38
CO2	0.1448	$159.2	49.03%	-0.42
CO3	0.1736	$190.9	9.30%	-0.40
CO4	0.1249	$137.4	11.97%	0.24
CO5	0.1273	$140	10.24%	0.13
CO6	0.1243	$136.7	11.93%	0.24
CO7	0.1286	$141.4	15.91%	0.47
Cost	$1,099.6	$1,099.6		
		Avg $\lvert\%\varepsilon_i\rvert$	16.36%	

7.9. Discussion of Case Study Results

To get a better overall picture of the average absolute percentage error, all average absolute percentage errors are averaged across the seven cases. Table 38 provides the summary of the average absolute percentage errors for IABC, TDABC, and IABC2. The averages of the average absolute percentage errors across all cases are 9.57 percent, 11.98 percent, and 19.22 percent for IABC, TDABC, and IABC2, respectively. All of the averages are relatively low (do not exceed 20 percent), and thus, the equivalency conditions presented in Propositions I, II, and III can be considered valid for this company.

Table 38
Comparison of Average Absolute Percentage Errors

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Average
IABC	47.02%	0.00%	0.00%	0.83%	0.00%	N/A	N/A	9.57%
TDABC	36.12%	9.72%	0.00%	14.09%	0.00%	N/A	N/A	11.98%
IABC2	6.38%	19.47%	36.21%	22.96%	N/A	13.93%	16.36	19.22%

For this particular company, it can be inferred that the equivalency conditions for TDABC2 (Proposition IV) are valid as well. Therefore, TDABC2 is a viable alternative to ABC, TDABC, IABC, and IABC2 along with the benefit of being accurate and easier to implement than the rest of the systems discussed in this study. The benefits of TDABC2 are that it eliminates 1) the resource diversity issue of TDABC, 2) the inaccuracy issue when resources are consumed by activities nonlinearly, 3) the high cost and complexity of implementation and updating of ABC, and 4) the need to find information for all resources, activities, and their associated drivers. Since the case studies in this chapter provide only anecdotal evidence, more empirical analysis is needed to validate the equivalency conditions further; hence, this is the limitation of this chapter.

Because of the limitation of these case studies, some practitioners might be concerned about whether TDABC2 would actually produce accurate cost assignments for their company. From Chapter 4, practitioners can see that TDABC2 works analytically in theory. Chapter 6 shows that the maximum average absolute percentage errors do not exceed 20 percent for each of the illustrations. From this chapter, practitioners can see that the equivalency conditions are valid (low errors) for the particular company demonstrated in the case studies. However, a question could remain about whether Proposition IV would be satisfied for their particular company. Three recommendations can be made depending on the current costing system in place.

First, if a company currently has the ABC system in place, then the TDABC2 system can be implemented parallel to the ABC system. For each cost object, the cost assignments from the TDABC2 system can then be compared to those of the ABC system. If the percentage errors of TDABC2 cost assignments compared to those of ABC are low (and most likely will be based on the analytics in Chapter 6), then TDABC2 will be proven to the practitioner to be a relatively accurate system. TDABC2 has an advantage over a current ABC system given that TDABC2 is easier and less costly to maintain and update than ABC.

Second, if a company currently has the TDABC system in place, the activity times, activity costs, and activity consumption ratios are already known. The practitioner can then use that information to determine if either Corollary IVa or Corollary IVb of Proposition IV reasonably holds. If either one reasonably holds, TDABC2 will be proven to the practitioner to be a fairly accurate system. TDABC2 has a couple advantages over a current TDABC system given that 1) TDABC2 resolves the resource diversity issue of

Stage 1, which provides the potential for more accurate cost assignments than those of a current TDABC system and 2) TDABC2 is easier and less costly to maintain and update than TDABC since Stage 2 is greatly simplified and Stage 1 is eliminated.

Third, if a company currently has a traditional (or, functional-based) costing system in place, it is recommended that the company go ahead and implement the TDABC2 system based on all of the analyses in Chapters 4, 6, and 7. Since it is already known in current research that ABC and TDABC are more accurate than the traditional costing systems, it can be implied that TDABC2 is also more accurate than the traditional systems. The reason is that, from the analytics, the equivalency conditions for TDABC2 are parallel to those of TDABC along with the fact that TDABC2 has the added benefits over TDABC in eliminating Stage 1 with its resource diversity issue and simplifying Stage 2.

The first two recommendations can provide an avenue for future research regarding finding empirical evidence of the value and accuracy of TDABC2 relative to ABC and TDABC for companies over a range of industries.

CHAPTER 8

SUMMARY OF FINDINGS

ABC was developed as a cost assignment method based on the logic of cause and effect relationships between resources and their associated cost drivers and between activities and their associated cost drivers (Cooper 1990; Kaplan 1994). These relationships between the costs and their drivers are assumed to be linear (Noreen 1991). Research has shown that ABC is adopted when 1) top management and employees support ABC, 2) there is adequate training, 3) managers understand the ABC information, 4) there is a significant risk of cost distortions within the firm, 5) the firm is large, 6) the firm has continuous manufacturing processes as opposed to job shops, and 7) there is product diversity (Krumwiede 1998b; Al-Omiri and Drury 2007). However, research has also shown that the adoption and implementation rates of ABC are less than 50 percent (Gosselin 1997; Krumwiede 1998b; Al-Omiri and Drury 2007). It has also been found that 85 percent of firms who routinely use ABC feel that it is worth it, whereas 15 percent do not think it is worth the cost (Krumwiede 1998b).

If any or all of the above seven reasons are not met, ABC adoption can be hindered**.** Additionally, strong IT can also hinder adoption because firms feel that the perceived benefits do not outweigh the implementation costs and that ABC will not enhance cost control (Al-Omiri and Drury 2007). Consequently, there is a trade-off between cost and accuracy. The overall reason for the low adoption rates is that ABC is a very complex and time-consuming system since all resources and their associated drivers as well as all activities and their associated drivers must be defined.

Babad and Balachandran (1993) and Homburg (2001) have attempted to simplify the ABC system, but concluded that their after-the-fact simplification methods provide a loss in accuracy and do not reduce the initial setup cost and complexity of the ABC system since the ABC system must be fully implemented before simplification can occur. The next simplification attempt is by Kaplan and Anderson (2007a, 2007b) who developed a before-the-fact simplification system called TDABC that simplifies the Stage 1 cost assignment by using process time equations, thus eliminating the need to survey and interview employees. Furthermore, Stage 1 cost assignment is reduced because resource costs are assigned to the activities using two sets of estimates: 1) the cost of supplying resource capacity for the department (capacity cost rate) and 2) the demand for resource capacity (capacity usage rate, typically time) by each transaction processed in the department (Kaplan and Anderson 2007a, 2007b). Additionally, TDABC has the advantage over ABC by incorporating unused capacity into the system (Kaplan and Anderson 2007a, 2007b).

For the purposes of the equivalency analyses, a duration-driver-based ABC system is used as a benchmark along with the assumption that there are linear relationships between costs and their associated drivers. When all resources are time-driven, TDABC is equivalent to ABC (Proposition II), but TDABC provides inaccurate activity costs when there is resource diversity. In addition to the potential inaccuracy, TDABC fails to simplify Stage 2. Additionally, unused capacity and time equations do not affect the equivalency conditions between ABC and TDABC.

If the conditions shown in Proposition II are violated, Corollary Va of Proposition V shows that the maximum absolute dollar error of TDABC is $\sum_{j=1}^{n}\left|\delta_j\right| = \sum_{j=1}^{n}\left|C_j - ct_j\right| =$ $\sum_{a=1}^{m}\left|\varepsilon_a\right| = \sum_{a=1}^{m}\sum_{j=1}^{n}\left|(C_j - ct_j)\frac{t_{aj}}{t_j}\right|$ if the resources and activities are partitioned into positive sets and negative sets. However, it was shown in Corollary Vb that if activities consume resources in both R^{+} and R^{-}, then the actual error of the system will be less than the maximum absolute dollar error of the system, which means that $\sum_{a=1}^{m}\left|\varepsilon_a\right|$ can never exceed $\sum_{j=1}^{n}\left|\delta_j\right|$, or $\sum_{a=1}^{m}\left|\varepsilon_a\right| \leq \sum_{j=1}^{n}\left|\delta_j\right|$.

Proposition I provides evidence that when there is no correlation between resource consumption ratios and activity costs, the cost assignment based on the average resource consumption ratios (IABC) matches the ABC Stage 1 cost assignments. Proposition III extends Proposition I to Stage 2 and shows that IABC2 cost assignments match ABC cost assignments when there is no correlation between activity consumption ratios and activity costs. IABC2 provides the advantage that the individual activity costs do not have to be known, thus eliminating Stage 1. However, the activity consumption ratios for all activities must be found, which retains most of the complexity of Stage 2. This complexity issue is a major limitation of IABC2.

Proposition II is extended to Proposition IV, which shows that TDABC2 cost assignments match ABC cost assignments when there is no correlation between activity consumption ratios and activity costs and between activity consumption ratios and activity time. TDABC2 has the benefit of IABC2 in which Stage 1 is eliminated and, thus, the problem of resource diversity is eliminated. The elimination of Stage 1 also resolves the findings by Maher and Marais (1998) concerning the poor estimates when there is a nonlinear or discontinuous relation between the demand for and provision of resources. Since TDABC2 only requires knowledge of the total cost, total time, the cycle time, and the number of units of the cost object that will be produced, it is as simple as a functional-based costing system and as accurate as an ABC system (as proven analytically). If the conditions shown in Proposition IV are violated, Proposition VI shows that the maximum absolute dollar error of TDABC2 is $\sum_{a=1}^{m}|\delta_a| = \sum_{a=1}^{m}\left|C_a^{\alpha} - c^{Z}\Im_a\right|$, where $\sum_{i=1}^{k}|\varepsilon_i| \le \sum_{a=1}^{m}|\delta_a|$.

Seven case studies containing data from a particular company are used to determine the validity of the equivalency conditions for IABC, TDABC, and IABC2. Overall, the case studies show that the equivalency conditions are relatively valid for this particular company. Hence, it can be inferred from the results that the equivalency conditions for TDABC2 hold as well. The major limitation of these case studies is that the data comes from one company; thus, any evidence of validity is purely anecdotal. More extensive empirical analyses are needed to verify the equivalency conditions further. Although a question remains about whether Proposition IV would be satisfied for other companies, three recommendations are given depending on the costing system currently in place for a given company.

In conclusion, this study has shown analytically that TDABC2 is a viable and simpler alternative to the ABC and TDABC systems currently in practice. The two major benefits of TDABC2 are that Stage 1 has been eliminated and Stage 2 has been greatly simplified. Since only the total cost, total time, the unit cycle time, and the number of units of the cost object that will be produced need to be known and are easy to gather, the cost to implement the system should be, obviously, significantly lower than that of ABC and TDABC. Since TDABC2 has been analytically proven to be as accurate as ABC under certain conditions, then there should be no significant tradeoff between the benefit of accuracy and the cost of the system. Thus, compared to the ABC and TDABC systems, TDABC2 is as accurate as ABC and more accurate than TDABC when there is resource diversity. TDABC2 should be of great benefit to practitioners who want a relatively accurate, easy to implement costing system.

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Major Field: Business Administration – Accounting

Scope and Method of Study: This study first analyzes the conditions under which Time-Driven Activity-Based Costing (TDABC) is equivalent to Activity-Based Costing (ABC). When these equivalency conditions do not hold, there is error. Accordingly, this study also analyzes the maximum absolute percentage error of TDABC that can occur when the equivalency conditions do not hold. However, even when the equivalency conditions are satisfied, TDABC can provide inaccurate costing results when resource diversity exists. Because of the potential resource diversity issue of TDABC, this study provides a new simplified ABC system (TDABC2) that overcomes the limitations of both ABC and TDABC. The equivalency conditions for TDABC2 relative to ABC are analyzed mathematically as well as the maximum absolute percentage error, which occurs when the equivalency conditions for TDABC2 do not hold. Finally, case studies containing data from a particular company are used as an initial analysis to provide anecdotal evidence of the validity of the equivalency conditions.

Findings and Conclusions: This study shows analytically that TDABC2 is a viable and simpler alternative to the ABC and TDABC systems currently in practice. The two major benefits of TDABC2 are that Stage 1 has been eliminated and Stage 2 has been greatly simplified. Since only the total cost, total time, the unit cycle time, and the number of units of the cost object that will be produced need to be known, the cost to implement the system should be, obviously, significantly lower than that of ABC, perhaps as low as that of the functional-based system. Since TDABC2 has been analytically proven to be as accurate as ABC under certain conditions, there should be typically no significant tradeoff between the benefit of accuracy and the cost of the system. Hence, TDABC2 should be of great benefit to practitioners who want a relatively accurate, low-cost, and easy to implement costing system.

ADVISER'S APPROVAL: Dr. Don R. Hansen