BARYON ASYMMETRY AND FLAVOR SYMMETRY BEYOND THE STANDARD MODEL

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TABLE OF CONTENTS

LIST OF TABLES

Table Page

LIST OF FIGURES

Figure Page

CHAPTER 1

INTRODUCTION

1.1 What is baryogenesis?

Symmetry of matter and anti-matter has been established in terrestrial colliders, but, at the same time, all the structures we have observed in the universe–stars, galaxies, clusters, consist of matters and not anti-matter. We are then led to the question how the universe came about, from a symmetric initial configuration.

Anti-matter can be observed in the form of anti-proton in cosmic rays. Present data shows thare are about 10^{-4} anti-protons for each proton [1]. This fraction is consistent with the collider process $P + P \to 3P + \bar{P}$. This shows there is no primordial anti-matter in our galaxy. If there were galaxies made of anti-matter (we don't talk about how matter and anti-matter are seperated), we would observe enhanced γ-radiation because of the annihilations within these galaxies and normal galaxies. This radiation has not been observed. We conclude therefore that the matter-anti-matter symmetry, or baryon asymmetry, is lost.

There are two ways to determine the baryon asymmetry of the universe [2]. The first is the observation of the abundance of light elements D, ^{3}He, ^{4}He and ^{7}Li. In the theory of big bang nucleosynthesis, the abundance of light elements depends sensitively on the number difference between baryon and anti-baryon. Another is the measurement of cosmic microwave background (CMB) anisotropies. The effect of baryons is to change the distribution of CMB through gravity. Through these measurements, we can infer the asymmetry of baryon and antibaryon.

There are two ways to define the baryon asymmetry of the universe [2]:

$$\eta \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (6.21 \pm 0.16)^{-10},$$
$$Y_{\Delta B} \equiv \left.\frac{n_B - n_{\bar{B}}}{s}\right|_0 = (8.75 \pm 0.23)^{-11}. \tag{1.1}$$

where n_B, $n_{\bar{B}}$ and s are respectively the density of baryon, antibaryon and entropy, the subscript 0 means "at present time". We will more often use $Y_{\Delta B}$ because the entropy is constant during the expansion of the universe.

Various considerations suggest that the baryon asymmetry is produced dynamically rather than through an initial condition of the universe. The first, based on observations, the asymmetry of quark and antiquark numbers is only around 10^{-7}, such a fine tuning is implausible. The second, based on the features of CMB, is that inflation has taken place in the history of the universe is highly possible. Any primordial asymmetry has been exponentially suppressed by inflation.

The scenario to address the dynamical production of baryon asymmetry is known as baryogenesis.

1.2 The Sakharov conditions

Three conditions are needed to realize baryon asymmetry, as given by Sakharov:

1. Baryon number violation. This condition is needed to evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.

2. C and CP violation. This condition makes sure that different numbers of baryon and antibaryon are involved in a process with any chirality.

3. Out of equilibrium dynamics. This condition guarantees that the asymmetry produced in a reaction process won't be canceled by the inverted reaction.

For the first condition, we can understand it from quantum mechanics.

Consider baryon number operator B, from quantum mechanics, $i\frac{d}{dt}B = [H, B]$, in an isotropic universe. We have $B(t) = B(t = 0) - i\int_0^t dt' \, [H, B(t')]$. If the universe has zero initial baryon number and $[H, B]$ (baryon number conservation), the baryon stays at zero at all times.

For the second condition, consider a process $X \to Y + B$ with decay with Γ. If C is conserved,

$$\Gamma(X \to Y + B) = \Gamma(\bar{X} \to \bar{Y} + \bar{B}), \tag{1.2}$$

The change rate of baryon number is

$$\frac{d}{dt} B \propto \Gamma(X \to Y + B) - \Gamma(\bar{X} \to \bar{Y} + \bar{B}) = 0, \tag{1.3}$$

so baryon number vanishes.

If C is violated, but CP is still conserved, we consider X decays into fermions of left-hand and right-hand,

$$X \to f_{1L} f_{2L}, \qquad X \to f_{1R} f_{2R}. \tag{1.4}$$

Under CP,

$$f_L \to \bar{f}_R, \qquad f_R \to \bar{f}_L, \tag{1.5}$$

so CP conservation implies

$$\Gamma(X \to f_{1L} f_{2L}) = \Gamma(\bar{X} \to \bar{f}_{1R} + \bar{f}_{2R}) \quad \text{and} \quad \Gamma(X \to f_{1R} f_{2R}) = \Gamma(\bar{X} \to \bar{f}_{1L} + \bar{f}_{2L}). \tag{1.6}$$

Thus we conclude

$$\Gamma(X \to f_{1L} f_{2L}) + \Gamma(X \to f_{1R} f_{2R}) = \Gamma(\bar{X} \to \bar{f}_{1R} + \bar{f}_{2R}) + \Gamma(\bar{X} \to \bar{f}_{1L} + \bar{f}_{2L}). \tag{1.7}$$

As long as the initial state has the same number of X and $\bar{X}$, the total baryon number of fermions stays at zero.

For the third condition, we consider the same process $X \to Y + B$. In equilibrium, $\Gamma(X \to Y + B) = \Gamma(\bar{Y} + \bar{B} \to \bar{X})$. The product baryon number is destroyed by the inverse decay.

When X decays out of equilibrium, at temperature T, $M_X > T$. The energy of decay products $E_{Y+B} \sim T$. The energy is not enough for the inverse process and the inverse process is Boltzmann suppressed, $\Gamma(\bar{Y} + \bar{B} \to \bar{X}) \sim e^{-M_X/T}$.

1.3 Models of baryogenesis

Several models have been invented to address the baryon asymmetry in the universe. We can only give a brief reviews of the models.

1.3.1 Planck-scale baryogenesis

It is generally believed that the quantum theory of gravitation doesn't conserve any global quantum numbers. When a star collapses, it can form a black hole. All quantum numbers are lost in the black hole, since black holes are only characterized by their masses, angular momentums and charges. Therefore virtual processes involving black holes are expected to violated baryon number.

We cannot reliably extract detailed predictions from quantum gravity for baryon number violation, we expect the baryon violating processes to be described by effective operators at low energies. The leading operator allowed by the standard model symmetries and violating baryon number have dimension 6. An example is

$$L_B = \frac{1}{M_P^2}\bar{e}\bar{d}^*\bar{d}^*\bar{d}^*. \tag{1.8}$$

The fermion fields d, $\bar{d}$, e, $\bar{e}$ etc. represent fermions with left chirality. The $*$ means charge conjugate. This kind of operators are tiny at low energies, they can only be important at extremely high temperatures, $T \sim M_P$. But we don't have knowledge how such a small baryon asymmetry $\sim 10^{-10}$ can arise at that high temperature. Furthermore, even if baryon number asymmetry was generated in the early era, this asymmetry was completely diluted by the inflation of the universe. So Planck scale baryogenesis is not likely the possible one.

1.3.2 GUT baryogenesis

GUT's satisfy all the three Sakharov's conditions. B-violating interactions can be from the gauge interactions. CP violation can be form fermion mixing, as in the standard model. Out-of-equilibrium can be associated with decays of gauge bosons or Higgs. At high temperature comparable with the masses of heavy particles, the production rates of these particles are lower than the decay rates. Generally this mechanism can generate baryon asymmetry compatible with the observation.

We show the mechanism of CP violation in the minimal $SU(5)$ GUT. Fermions with left chirality of each generation fall into two representations:

$$\bar{5}_i = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e \\ \nu \end{pmatrix}, \text{ and } \quad 10_i = \begin{pmatrix} 0 & \bar{u}_2 & -\bar{u}_1 & Q_1^1 & Q_1^2 \\ -\bar{u}_1 & 0 & \bar{u}_3 & Q_2^1 & Q_2^2 \\ \bar{u}_1 & -\bar{u}_3 & 0 & Q_3^1 & Q_3^2 \\ -Q_1^1 & -Q_2^1 & -Q_3^1 & 0 & \bar{e} \\ -Q_1^2 & -Q_2^2 & -Q_3^2 & -\bar{e} & 0 \end{pmatrix}, \tag{1.9}$$

where $Q_i^1 = u_i$ and $Q_i^2 = d_i$ which transform as doublets under $SU(2)$. A Higgs obeying 24 of $SU(5)$ representation acquire vacuum expectation value

$$\langle H \rangle = v \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}, \tag{1.10}$$

breaking $SU(5) \to SU(3) \times SU(2)_L \times U(1)_Y$ and the vector bosons corresponding to the broken generators acquire mass of order gv. We refer these massive gauge bosons as X. They are associated with the $SU(5)$ generators not commuting with $\langle H \rangle$ such as

$$T_4^1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{1.11}$$

there are in total 12 of them.

We can easily see that these gauge bosons X mediate interactions violating baryon number B and lepton number L, but conserve $B - L$. Although CPT theorem guarantees that the total decay width of X and its anti-particle $\bar{X}$ are the same, generally there can be slight difference of decay width in the partial width for

$$X \to dL; \quad X \to \bar{Q}\bar{u} \tag{1.12}$$

and

$$\bar{X} \to \bar{d}\bar{L}; \quad \bar{X} \to Qu. \tag{1.13}$$

Of course this demands CP violation. CP violation can be induced by the complex mixing of fermions of different generations.

However, GUT baryogenesis is not the origin of baryon asymmetry in the universe. There are convincing evidence that the universe underwent an era of inflation. Universe inflation will dilute the baryon asymmetry generated to close to zero if it happens after the spontaneous symmetry breaking of $SU(5)$. For inflation dilution not happening, inflation should happen at higher energy scale than 10^{16} GeV which is the scale of GUT symmetry breaking. This means the reheating temperature of the universe after inflation is of the order 10^{16} GeV. It is shown that in supersymmetry scenario reheating temperature higher than 10^9 GeV leads to cosmological problems, especially the over production of gravitinos [77].

1.3.3 electro-weak baryogenesis

The standard model satisfies all three conditions, but it fails to produce large enough baryon asymmetry. For more detail please see [60].

1. Baryon number is violated due to triangle anomaly. This anomaly leads to an non-perturbative sphaleron interaction involving nine left-handed quarks (three of each generation) and three left-handed leptons (one of each generation). A selection rule is obeyed,

$$\Delta B = \Delta L = \pm 3. \tag{1.14}$$

This process is suppressed at low temperature, however, it is unsuppressed at high temperature. We will need this interaction.

2. Weak interaction violates C maximally. CP is violated through CKM mixing of quarks. CP violation can be parameterized by Jarlskog invariant, whose magnitude is of order 10^{-20}.

3. Within the standard model, departure from equilibrium can occur at the electro-weak phase transition. The mass of Higgs, however, limits the strength of the phase transition.

We next discuss the three conditions in the electro-weak theory one by one:

Anomalous B violation

Baryon number and lepton number are conserved in the standard model. Experimentally proton decay has not been observed, $\tau_p \gtrsim 10^{33} yrs$. However, B violation can happen through the non-perturbative interaction in non-Abelian gauge theories because of chiral anomaly [11]. The standard model is a non-Abelian gauge theory. At low temperatures, these processes are ignorable. But in the early universe when the temperature was high, such an non-perturbative interaction can be strong. These configurations are known as "sphalerons".

Consider the lagrangian for a massless Dirac fermionψ with $U(1)$ gauge symmetry [12]:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\gamma^\mu(\partial_\mu - igA_\mu)\psi. \tag{1.15}$$

It is invariant under the local transformation,

$$\psi(x) \to e^{i\theta(x)}\psi, \qquad A_\mu(x) \to A_\mu(x) + i\partial_\mu\theta(x), \tag{1.16}$$

with

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{1.17}$$

It is also invariant under the global chiral transformation,

$$\psi(x) \to e^{i\gamma_5\theta(x)}\psi \tag{1.18}$$

with the current,

$$j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi, \tag{1.19}$$

which is conserved at tree level but not at loop level. At one loop,

$$\partial_\mu j_5^\mu = \frac{g^2}{(4\pi)^2}\tilde{F}_{\mu\nu}F^{\mu\nu} = \frac{g^2\epsilon_{\rho\sigma\mu\nu}}{(4\pi)^2}F^{\rho\sigma}F^{\mu\nu} \tag{1.20}$$

The origin of the right side is anomaly. It can be regarded as the total divergence involving gauge fields and it is related to their topological property: it is the the "winding number" or the "Chern-Simons number" of the gauge field configuration. In four dimension, the space-time integral of the right side vanishes for $U(1)$ gauge theory, but not necessarily for non-Abelian gauge theory.

The gauge group of the standard model is $SU(3)_c \times SU(2)_L \times U(1)_Y$. The $B+L$ currents arise only from the $SU(2)$ gauge interactions, which is chiral. We ignore all the other interactions including Yukawa couplings. The relevant fermion fields are the three generations of quark and lepton doublets $\psi_L^f = \{q_L^{i,\ \alpha}, l_L^f\}$, with f being the family index, α color index. The gauge interaction is

$$\mathcal{L} = \sum_i \bar{\psi}^f \gamma^\mu(\partial_\mu - i\frac{g}{2}\sigma^a W_\mu^a)\psi_L^f, \tag{1.21}$$

where σ is the Pauli matrix and a=1, 2, 3, is the $SU(2)$ index. It has global $U(1)$ symmetry for each of the 12 fields, corresponding to each of the 12 fermion numbers of $SU(2)$ doublets (3 families of quarks and leptons with quarks of 3 colors).

$$\psi_L^f(x) \to e^{i\beta}\psi_L^f(x). \tag{1.22}$$

The associated currents,

$$j_\mu^f = \bar{\psi}_L^f\gamma_\mu\psi_L^f, \tag{1.23}$$

are conserved at tree level, but not at loop level, by anomaly,

$$\partial^\mu j_\mu^f = \frac{g^2}{64\pi^2}\tilde{F}_{\mu\nu}^a F^{a\mu\nu} \tag{1.24}$$

We define $Q^f(t) = \int d^3x j_0^f$, $\Delta Q^f = Q^f(t=+\infty) - Q^f(t=-\infty)$. We have

$$\Delta Q^f = \frac{g^2}{64\pi^2}\int d^4x \tilde{F}_{\mu\nu}^a F^{a\mu\nu}. \tag{1.25}$$

From the topological point of view, this corresponds to maps from S^3 to the gauge group $SU(2)$. There are generally gauge field configurations such that this is a non-zero integer. This implies that fermion number is violated, although there is no perturbative interaction generating them. However, we have the selection rule (4.2).

Different vacua configurations of gauge field correspond to different integer numbers of (4.9). These integer numbers are the topological property of the vacua. Different vacua cannot be continuously deformed into each other without generating non-vacuum gauge fields, so these vacua are separated by a potential barrier. The gauge system is similar to a particle moving in periodical potential, as shown in Fig.1.1.

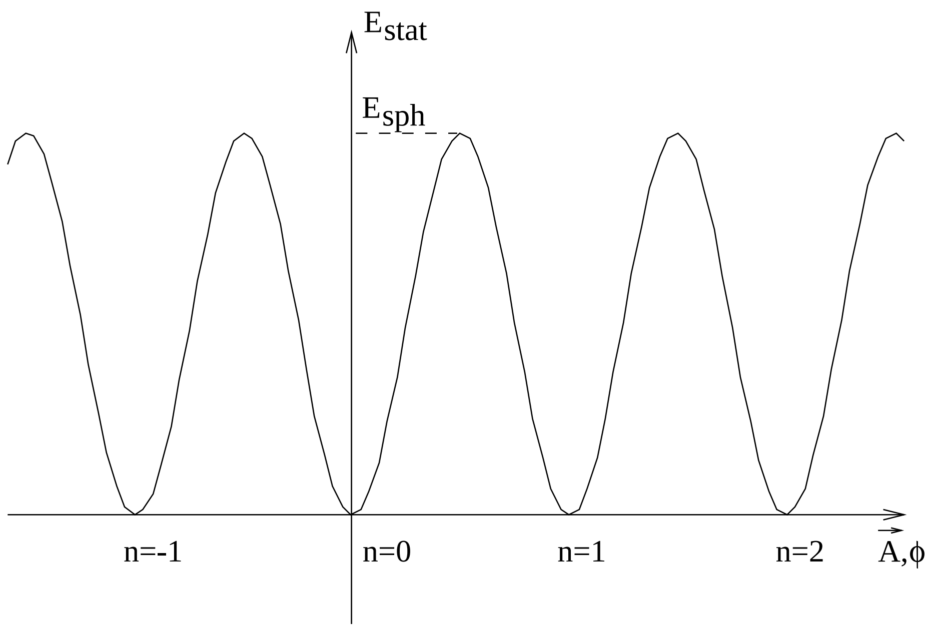

Figure 1.1: Schematic plot of the static energy as function of gauge fields. The minima correspond to the classical vacua

The change of vacua is through the quantum tunneling effect of the potential, described by instantons. If a instanton changes fermion number by an integer N, the action of instanton is [2]

$$\left| \int d^4x \tilde{F}^a_{\mu\nu} F^{a\mu\nu} \right| \geq \frac{64\pi^2 N}{4g^2}. \tag{1.26}$$

It is large because of the small value of gauge coupling constant g. The associated rate is highly suppressed,

$$\Gamma \propto e^{-(\text{instanton action})} \leq e^{-4\pi/\alpha_W} \tag{1.27}$$

and the baryon number violation is unobservable.

Vacuum can be viewed as the minima of of filed configurations. For the gauge field configurations, two minima may correspond to different fermion numbers. Sphlaerons are the thermal fluctuations tunneling between these states in the presence of Higgs vacuum expectation value. At high temperature, the sphaleron fluctuations are large and there is more possibility of $B+L$ violation. At temperature T the $B+L$ violating rate mediated by sphalerons is

$$\Gamma_{\rm sph} \propto e^{-E_{\rm sph}/T}, \tag{1.28}$$

where $E_{\rm sph} = 2Bm_W/\alpha_W$ is the potential energy at the top of field configuration between two such minima. $1.5 \leq B \leq 2.75$ for λ/g varying from 0 to ∞, where λ is the Yukawa coupling.

For leptogenesis, $B+L$ interactions occur at temperatures far above the electro-weak phase transition. The rate can be estimated as

$$\Gamma_{B+L \text{ violating}} \simeq 250\alpha_W^5 T. \tag{1.29}$$

This implies that at temperatures higher than the electro-weak phase transition and lower than 10^{12} GeV, sphaleron interactions are strong enough to be in equilibrium.

CP violation in electro-weak theory

CP violation in eletroweak theory is through the quark mixing of the quark sector. The gauge interaction in the electro-weak theory is through the coupling of currents and gauge fields. The charged current interaction of $W^{\pm}$ is

$$L_W = j_\mu^+ W^{+\mu} + j_\mu^- W^{-\mu}, \tag{1.30}$$

with

$$j^{+\mu} = \sum_i \bar{u}_L^i \gamma^\mu d_L^i, \tag{1.31}$$

and $j^{-\mu}$ is its charge conjugate, i is the flavor index. The flavor basis and the mass basis of quarks are different. They are different by unitary mixing transformations.

$$d_L^i = (U_d)_j^i D_L^j, \quad \text{and} \quad u_L^i = (U_u)_j^i U_L^j, \tag{1.32}$$

we use $U_i(D_i)$ to represent up (down) quarks in the mass basis. The mixing of quarks of mass eigenstates in the flavor space leads to the charged current in the mass basis:

$$j^{+\mu} = \sum_i \bar{U}_L^i \gamma^\mu V_{ij} D_L^j = \sum_i \bar{U}_L^i \gamma^\mu (U_u^\dagger)_i^l (U_d))_j^l D_L^j. \tag{1.33}$$

The matrix

$$V_{ij} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{1.34}$$

is the Kobayashi-Maskawa matrix, where $s_\alpha \equiv \sin\theta_\alpha,\ c_\alpha \equiv \cos\theta_\alpha$ $(\alpha = 1, 2, 3)$. It can be proved [5] that when the number of flavors is 3, the 3×3 matrix V_{ij} can be parameterized by 3 angles θ_α and a phase δ. With a complex mixing matrix, it can be proved that CP violation is non-zero. The magnitude of CP violation in the electro-weak theory is determined by

$$\epsilon_{CP} = 2FF'J, \tag{1.35}$$

with

$$F = (m_t - m_c)(m_t - m_u)(m_c - m_u)/m_t^3, \quad F' = (m_b - m_s)(m_b - m_d)(m_s - m_b)/m_b^3, \tag{1.36}$$

and

$$J = \mathrm{Im}(V_{11} V_{22} V_{12}^* V_{21}^*) = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta. \tag{1.37}$$

J is the normally called Jarlskog invariant. It looks asymmetric in the flavor indices. This is not true, actually the magnitude of Jarlskog invariant with any combination of two different flavor indices are equal. We can see CP violation is non-zero unless

$$\theta_i = 0,\ \theta_i = \frac{\pi}{2},\ \delta = 0,\ \delta = \pi. \tag{1.38}$$

Out of equilibrium in electro-weak theory

We have shown that two of the three Sakharov conditions are satisfied in the electro-weak theory. The third one, out-of-equilibrium is through the eletroweak phase transition in the universe. When phase transition happens, portions of the universe with broken phase form bubbles expanding in the universe. These bubbles collide and finally the whole universe becomes one phase. On the walls of the expanding phase bubbles, equilibrium is lost. Baryon number violating processes through sphalerons happen on the walls and generate baryon number asymmetry. After the phase bubbles swipe through the universe, if the sphaleron processes are fast enough, they washout the generated baryon number asymmetry. For this not happening, the sphaleron rate (1.28) in the bubbles should be less than the universe expanding rate H. We can realize the phase transition mechanism is closely related with brayogenesis. Next we give a brief review of the phase transition mechanism.

In the standard model, the minimal electro-weak theory, the Higgs potential is given by

$$U(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4. \tag{1.39}$$

The potential has a minimum at $\langle\Phi\rangle = \sqrt{\mu^2/\lambda}$, breaking the gauge symmetry and giving mass to the gauge bosons by the Higgs mechanism.

This is at zero temperature. At finite temperature, the $\langle\Phi\rangle$ depends on temperature, to determine it, we must compute the free energy as a function of Φ. The leading temperature-dependent corrections are obtained by noting that the masses of the various fields in the theory- W, Z bosons and the Higgs field, depend on Φ. The free energy at finite temperature is given by

$$F(\Phi, T) = \pm T \sum_i \int \frac{d^3p}{(2\pi)^3} \ln(1 \pm e^{-\sqrt{p^2+m_i^2(\Phi)}/T}), \tag{1.40}$$

where T is temperature, the sum is over all fields, the plus sign is for bosons and minus sign for fermions. In the standard model we can regard all fermions are massless except top quark at temperature over 100 GeV. The effective potential (1.40) depends on the top quark mass m_t, gauge boson masses M_W, M_Z and Higgs mass M_H. Performing the integral we will find the effective potential is [7]

$$F(\Phi, T) = D(T^2 - T_0^2)\Phi^2 - ET\Phi^3 + \frac{\lambda}{4}\Phi^4 + \dots, \tag{1.41}$$

with

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2) = \frac{1}{4D}(M_H^2 - 8Bv_0^2), \tag{1.42}$$

and

$$
\begin{aligned}
B &= \frac{3}{64\pi^2 v_0^4}(2M_W^4 + M_Z^4 - 4m_t^4), \\
D &= \frac{1}{8v_0^2}(2M_W^2 + M_Z^2 + 2m_t^2), \\
E &= \frac{1}{4\pi v_0^3}(2M_W^3 + M_Z^3) \sim 10^{-2}.
\end{aligned} \tag{1.43}
$$

If the cubic term is absent, we have a second order phase transition at temperature T_0, between a phase $\langle \Phi \rangle = 0$ and $\langle \Phi \rangle \neq 0$. This is a phase transition between massless gauge bosons and massive gauge bosons, since the gauge boson masses depend on Φ.

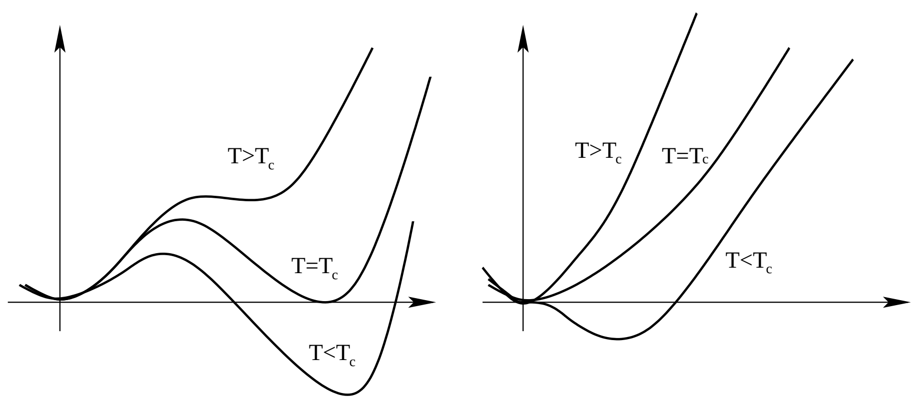

Figure 1.2: Generic free energy functions for first and second order phase transitions

The cubic term is not zero, although it is very small, $E \sim 10^{-2}$, hence the phase transition is of first order. This is indicated in Fig. 1.2. There are two distinct minimum at the critical temperature. A first order transition is not, in general, an adiabatic process.

To avoid washing out the baryon asymmetry generated, the sphaleron rate after the phase transition should be smaller than the universe expansion rate. This in turn, means that M_W and the vacuum expectation value of Higgs must be large immediately after phase transition. One can relate $\langle \Phi \rangle$ with Higgs mass at zero temperature. The condition for sphaleron not washing out baryon number asymmetry, it is required that the Higgs mass is small

$$
M_H \leq 45 \text{ GeV}. \tag{1.44}
$$

However, even for an unrealistically light Higgs, the actual production of baryon asymmetry in the standard model is highly suppressed. The standard model CP violation arising from quark mixing is proportional to the Jarlskog invariant (1.35). This leads only to baryon asymmetry of order 10^{-20} in the standard model.

Supersymmetric extensions of the standard model contain new sources of CP violation and more parameters, allowing possible larger strength of first order phase transition.

The new sources of CP violation may come from the chargino mass matrix:

$$\overline{\psi}_R M_\chi \psi_L = \begin{pmatrix} \overline{\tilde{w}^+} & \overline{\tilde{h}_2^+} \end{pmatrix}_R \begin{pmatrix} m_2 & gH_2(x) \\ gH_1(x) & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_1^+ \end{pmatrix}_L + \text{H.c.}, \tag{1.45}$$

where $\tilde{w}$ and $\tilde{h}$ are the superpartners of W boson wino and the charged Higgs Higgsino. The indices of the Higgsino indicate the two Higgs in the minimal supersymmetric standard model (MSSM). m_2 and μ are mass parameters. Other sources of CP violation include phases in scalar masses.

In spite the fact that supersymmetry introduces more sources of CP violation and parameter space for first order phase transition, it is hard to realize the observed baryon asymmetry. We know, smaller Higgs mass means easier realization of first order phase transition. A lighter right handed stop allows for first order phase transition for Higgs of 115 GeV. The LEP experiment puts a low bound for Higgs mass is 114 GeV. The constraint of electro-weak baryogenesis is mainly from the phase transition strength. Various extensions of MSSM can increase the phase transition strength, but they are very sensitive to other phenomenological constraints such as the electric dipole moment of neutron. Eletroweak baryogenesis is on the verge to be confirmed or ruled out by the upcoming accelerator experiments.

1.3.4 Affleck-Dine baryogenesis

The Affleck-Dine baryogenesis works in the supersymmetry scenario [6]. In the supersymmetry scenario, some scaler fields carry baryon number or lepton number can acquire large vacuum expectation value in some directions (flat directions). At high temperature when the expansion rate of the universe is large, the VEVs of these scaler fields are frozen. When temperature decreases and the universe expansion slows down, the scaler fields start to excite. Since these fields carrying baryon number or lepton number, it can be shown that a net baryon number or lepton number is generated in the universe.

We next illustrate this point. Consider a complex field ϕ carrying baryon number, the free Lagrangian is

$$L = |\partial_\mu \phi|^2 - m^2 |\phi|^2. \tag{1.46}$$

The current associate with a phase transformation is

$$j_B^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*). \tag{1.47}$$

There is also a "CP" symmetry:

$$\phi \leftrightarrow \phi^*. \tag{1.48}$$

If the field is spatial constant $\phi(x,t) = \phi(t)$, this system is equivalent to an isotopic harmonic oscillator in two dimensions. With supersymmetry in mind, we regard $m \sim M_W$ In supersymmetric models, some high order terms will break this symmetry. Since the universe expands, the movement of the oscillator damps, the effect of the symmetry breaking becomes less and less important.

We next include the quartic terms in the Lagrangian:

$$L_I = \lambda |\phi|^4 + \epsilon \phi^3 \phi^* + \delta \phi^4 + \text{c.c.}. \tag{1.49}$$

These terms obviously violate baryon number. For complex ϵ and δ they also violate CP. The coupling constants λ, ϵ, δ are extremely small, at the order of $(M_W/M_P)^2$ or $(M_W/M_{GUT})^2$.

Because of the small parameters, the complex field acquires a large VEV. We assume the VEV ϕ_0 is real for simplicity. At early times when the damping motion is not very small the equation of motion of the imaginary part is

$$\ddot{\phi}_i + 3H\dot{\phi}_i + m^2 \phi_i \approx \text{Im}(\epsilon + \delta)\phi_r^3, \tag{1.50}$$

Asymptotically we can think the right side falls as $t^{9/2}$ (the field is pressureless). We can get the solutions, with the universe being mainly made of radiation or matter

$$\begin{aligned} \phi_i &= a_r \frac{\mathrm{Im}(\epsilon+\delta)\phi_0^3}{m^2(mt)^{3/4}} \sin(mt+\delta_r) \quad \text{(radiation)}, \\ \phi_i &= a_m \frac{\mathrm{Im}(\epsilon+\delta)\phi_0^3}{m^3 t} \sin(mt+\delta_m) \quad \text{(matter)}, \end{aligned} \tag{1.51}$$

These constants can be numerically obtained:

$$a_r = 0.85,\ a_m = 0.85,\ \delta_r = -0.91,\ \delta_m = 1.54. \tag{1.52}$$

Substituting these solutions to (1.47), we find the baryon number is violated:

$$\begin{aligned} n_B &= 2a_r \mathrm{Im}(\epsilon+\delta)\frac{\phi_0^2}{m(mt)^2} \sin(\delta_r+\pi/8) \quad \text{(radiation)}, \\ n_B &= 2a_m \mathrm{Im}(\epsilon+\delta)\frac{\phi_0^2}{m(mt)^2} \sin(\delta_m) \quad \text{(matter)}. \end{aligned} \tag{1.53}$$

We can see that if the parameter ϵ and δ vanish or are real, n_B vanishes.

In MSSM, the D-term has many flat directions, with nonrenomalizable terms generated at Planck scale or GUT scale, baryon number can be easily broken. The Affleck-Dine mechanism of baryogenesis is a promising scenario. There is a broad range of parameters generating baryon asymmetry n_B/n_γ as large as observed. This baryon asymmetry is generated long after the inflation, so the reheating of the universe does not provide significant constraint.

1.4 Leptogenesis

Baryogenesis through leptogenesis is a promising scenario to address the baryon asymmetry of the universe. Since our later chapters involve leptogenesis, we give a little detailed discussion to the idea of leptogenesis. Experimentally it has been found neutrino are massive, unlike what the standard model assumed. An $SU(2)$ singlet Majorana right-handed (RH) neutrinos are introduced into the standard model, and they couple to left-handed neutrinos throgh Yukawa couplings. When electroweak symmetry is broken, neutrinos acquire masses through see-saw mechanism. In the early time, the universe was like a heat bath in thermal equilibrium. When temperature gets lower, the right handed neutrinos start to decay into neutrinos and Higgs. These decays of the RH neutrinos are CP violating in the background of expanding universe. If the expansion rate of the universe is larger than the decay rate of the right handed neutrinos, this mechanism provide a potential scenario satisfying the Sakharov conditions, except that it is lepton number L violation rather than baryon number B violation. We have shown that at temperature higher than the electro-weak phase transition, the baryon number violating electro-weak sphaleron processes are in equilibrium. Since these processes has the selection rule $\Delta B = \Delta L$, there processes convert lepton number asymmetry into baryon number asymmetry.

We give a presentation of the idea of leptogenesis, to be consistent with supersymmetry, we also introduce the up sector Higgs H_u. The lagrangian of neutrino sector is given by

$$\mathcal{L} = Y_{i,m} L_i H_u N_m - \frac{1}{2} M_{m,n} N_m N_n + \text{h.c.} \tag{1.54}$$

where $i = 1, 2, 3$, are family indices and $m, n = 1, 2, \ldots \mathcal{N}$ with $\mathcal{N}$ the number of RH neutrinos. The Majorana mass matrix of RH neutrinos $M_{m,n}$ is very heavy. Without losing generality, we choose it to be diagonal. The Yukawa matrix is generally complex which means CP violation.

When H_u acquires vacuum expectation value (VEV), we integrate out the RH neutrino fields and get the mass matrix of light neutrinos

$$\mathcal{L}_{eff} = \frac{1}{2} \sum_k \frac{v_u^2}{M_k} Y^T_{i,k} Y_{k,j} \nu_i \nu_j + \text{h.c.}. \tag{1.55}$$

v_u is the VEV of H_u, k runs over 1 to $\mathcal{N}$. If we suppose $\mathcal{N} = 3$, then the effective mass matrix of light neutrinos is symmetric, thus it has 6 phases. 3 of the 6 phases can be absorbed through rephasing the neutrinos.

The decay of RH neutrinos to light neutrinos and Higgs can produce lepton asymmetry at temperature $T \leq M$, which means that the decay is out of equilibrium. The lepton asymmetry is partially converted into baryon asymmetry through sphaleron interactions [8]. This is known as leptogenesis.

1.4.1 CP violation

Generally CP violation is from the interferences of particle decays at tree level and loop level. In the loop level decays, if the decaying particles are heavy enough and the intermediate states can be on shell, there is an absorptive part. We will see if this is the fact and the couplings of particle interactions are complex, the decays widths of CP conjugate are different. This is the origin of CP violation.

As an example, in this section we discuss the mechanism that produces CP violation in the decays of right handed neutrinos, which is important in leptogenesis. We should keep in mind this is not the only origin of CP violation of leptogenesis. We will show some new ideas of leptogenesis in the later chapters.

The CP asymmetry in the lepton flavor α, produced in the decay of RH neutrino N is defined as

$$\epsilon_{\alpha\alpha} \equiv \frac{\Gamma(N \to \phi l_\alpha) - \Gamma(N \to \bar{\phi}\bar{l}_\alpha)}{\Gamma(N \to \phi l_\alpha) + \Gamma(N \to \bar{\phi}\bar{l}_\alpha)}. \tag{1.56}$$

The CP violation is from the interference of tree level (subscript 0) and loop level (subscript 1) amplitudes. It is important to include all one loop diagrams. We can separate the amplitudes into coupling constants and amplitude parts,

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 = c_0\mathcal{A}_0 + c_1\mathcal{A}_1. \tag{1.57}$$

The matrix element for the CP conjugate process is

$$\bar{\mathcal{M}} = c_0^*\bar{\mathcal{A}}_0 + c_1^*\bar{\mathcal{A}}_1, \tag{1.58}$$

where$|\mathcal{A}_i|^2 = |\bar{\mathcal{A}}_i|^2$. So the CP symmetry is

$$\begin{aligned}\epsilon_{\alpha\alpha} = & \frac{\int d\Pi_{\phi,\, l}(2\pi)^4\delta^4(P_i - P_f)|c_0\mathcal{A}_0 + c_1\mathcal{A}_1|^2 - \int d\Pi_{\phi,\, l}(2\pi)^4\delta^4(P_i - P_f)|c_0^*\mathcal{A}_0 + c_1^*\mathcal{A}_1|^2}{2\sum_\beta \int d\Pi_{\phi,\, l}(2\pi)^4\delta^4(P_i - P_f)|c_0\mathcal{A}_0|^2} \\ = & \frac{\mathrm{Im}(c_0c_1^*)}{\sum_\alpha |c_0|^2}\frac{2\int d\Pi_{\phi,\, l}(2\pi)^4\delta^4(P_i - P_f)\mathrm{Im}(\mathcal{A}_0\mathcal{A}_1^*)}{\int d\Pi_{\phi,\, l}(2\pi)^4\delta^4(P_i - P_f)|\mathcal{A}_0|^2},\end{aligned} \tag{1.59}$$

where we use

$$d\Pi_{\phi,\, l} = \frac{d^3p_\phi}{(2\pi)^3 2E_\phi}\frac{d^3p_l}{(2\pi)^3 2E_l}. \tag{1.60}$$

β runs over the lepton flavors. We assume one-loop contributions are much smaller than tree-level ones and therefore negiligible.

We can always chose proper phase for c_0 to make $\mathcal{A}_0$ real, so the imaginary part of $\mathcal{A}_1$ is very important. The imaginary part or the absorptive part of a one-loop diagram is possible if the intermediate can be on-shell. The absorptive part is determined by the Cutkosky rule [12],

$$2\text{Im}(\mathcal{A}_0\mathcal{A}_1^*) = \mathcal{A}_0(N \to \phi l_\alpha) \sum_\beta \int d\Pi_{\phi',\ l'} (2\pi)^4 \delta'^4(P_i - P_f') \mathcal{A}_0^*(N \to \bar{\phi}' \bar{l}_\beta') \mathcal{A}_0^*(\bar{\phi}' \bar{l}_\beta' \to \bar{\phi}' \bar{l}_\beta'). \quad (1.61)$$

The ϕ' and l_β' refer to the on-shell intermediate states and $d\Pi_{\phi',\ l'}$ is the integral element in this phase.

In the model described by the Lagrangian Eq. (1.9), if we assume hierarchical RH neutrino masses $M_1 \ll M_2, M_3$, we get the CP violation parameter for N_1 [62]:

$$\epsilon_{11} \simeq \frac{3}{16\pi} \frac{M_1}{(Y^\dagger Y)_{11} v_u^2} \sum_{i=2,3} \text{Im}(Y^\dagger m_\nu Y)_{11}. \quad (1.62)$$

The upper bound of $epsilon_{11}$ is

$$|\epsilon_{11}| \leq \frac{3M_1 m_{\max}}{16\pi v_u^2} \sqrt{B_{\phi l_\alpha}^{N_1} + B_{\bar{\phi}\bar{l}_\alpha}^{N_1}}, \quad (1.63)$$

where $B_{\phi l_\alpha}^{N_1} \equiv \Gamma(N_1 \to \phi l_\alpha)/\Gamma_D$ and $m_{\max}$ is the largest light neutrino mass.

1.4.2 Out of equilibrium processes

The final baryon asymmetry is determined by the three conditions of Sakharov, where each condition gives a suppression factor,

$$Y_{\Delta B} \approx \frac{135\xi(3)}{4\pi^4 g_*} \sum_\alpha \epsilon_{\alpha\alpha} \cdot \eta_\alpha \cdot C\ , \quad (1.64)$$

with α being the lepton flavor index. The first factor is the equilibrium number of RH neutrino of each generation divided by the entropy density at $T \ll M$. Its magnitude is of order 4×10^{-3} when we choose the relativistic degrees of freedom g_* to be 106.75, as in the standard model. $\epsilon_{\alpha\alpha}$ is the CP violation parameter we previously mentioned. η is the washout factor. Its range is $0 < \eta_\alpha < 1$. C is the sphaleron conversion factor, and counts the lepton number of flavor α needed to generate a baryon.

The non-equilibrium is provided by the expansion of the universe. When the RH neutrino decay rate is slower than the expansion speed of the universe, which is represented by the inverse of Hubble constant, H^{-1}, the out-of-equilibrium decay guarantees that the lepton number produced in the CP violating RH neutrino decays is not canceled by the inverse decay, and the lepton asymmetry survives. we will consider a simple mechanism of the out-of-equilibrium decays of one RH neutrino N_1 with mass M_1, and the conclusion can be generalized to more N-s. The real process can be very complicated and needs to resolve the Boltzmann equation.

The universe experiences a state of inflation, after which everything is diluted to be nearly zero. The decaying RH neutrinos are produced when the universe reheats after inflation. A thermal number of N_1 can be produced if the reheating temperature $T_{\text{reheat}} > M_1/5$. The processes producing N_1 can be inverse decay $hl_\alpha \to N_1$, the $2 \to 2$ scattering including t quark $t_R + Q_L \to h \to l_\alpha N_1$ (s channel), $l_\alpha t_R \to h \to Q_L N_1$ (t channel), or gauge interaction processes. The inverse decays and gauge interactions are negligible.

The production rate of RH neutrino cab be estimated as

$$\Gamma_{\text{prod}} \sim \sum_\alpha \frac{Y_t^2 |Y_{\alpha 1}|^2}{4\pi} T. \tag{1.65}$$

If $\Gamma_{\text{prod}} > H$, since Y_t is of order 1, the N_1 decay is also in equilibrium, $\Gamma_D > H(T = M_1)$, where

$$\Gamma_D = \sum_\alpha \Gamma(N_1 \to l_\alpha h, \bar{l}_\alpha \bar{h}) = \frac{(Y^\dagger Y)_{11} M_1}{8\pi}. \tag{1.66}$$

The Hubble constant is

$$H(T = M_1) = 1.66 g_*^{1/2} \frac{T^2}{m_{\text{pl}}}\bigg|_{T=M_1}. \tag{1.67}$$

It is useful to introduce two dimensional parameters, $\tilde{m}$ and m_* to represent Γ_D and $H(T = M_1)$. They are of order of light neutrino mass,

$$\tilde{m} \equiv \sum_\alpha \tilde{m}_{\alpha\alpha} = \sum_\alpha \frac{|Y_{\alpha 1}|^2 v_u^2}{M_1} = 8\pi \frac{v_u^2}{M_1^2} \Gamma_D, \tag{1.68}$$

$$m_* \equiv 8\pi \frac{v_u^2}{M_1^2} H(T = M_1) \simeq 1.1 \times 10^{-3}, \tag{1.69}$$

where v_u is the VEV of h_u. The condition $\Gamma_D > H$ is equivalent to $\tilde{m} > m_*$.

Next we discuss the out-of-equilibrium effects under 3 conditions:

1. Strong washout, $\tilde{m} > m_*$ and $\tilde{m}_{\alpha\alpha} > m_*$.

At $T \sim M_1$, the number of N_1 in the heat bath is in the thermal equilibrium ($n_{N_1} \sim n_\gamma$) and the lepton number symmetry $Y_{\Delta l} \simeq 0$, because the inverse decay is fast enough to cancel the effect of CP violating decay, and totally wash out the lepton asymmetry. When the temperature decreases to $T < M_1$, the phase space of inverse decay is thermally suppressed, so the washout is not complete and lepton asymmetry survives. The condition is described as

$$\Gamma_{ID}(hl_\alpha \to N_1) \simeq \frac{1}{2}\Gamma_{\alpha\alpha} e^{-M_1/T} < H. \tag{1.70}$$

We assume the temperature at which the condition (1.68) is satisfied to be T_α. The number of N_1, $n_{N_1} \propto e^{-M_1/T_\alpha}$. So the efficient factor of flavor α

$$\eta_\alpha \simeq \frac{n_{N_1}(T_\alpha)}{n_{N_1}(T \ll M_1)} \simeq e^{-M_1/T_\alpha} \simeq \frac{\tilde{m}_*}{\tilde{m}_{\alpha\alpha}}. \tag{1.71}$$

2. Intermediate condition, $\tilde{m} > m_*,\ \tilde{m}_{\alpha\alpha} < m_*$.

Under this condition the number of N_1 is in equilibrium, but the lepton number flavor of α is not. This produces a lepton number asymmetry $-\epsilon_{\alpha\alpha} n_{N_1} \sim -\epsilon_{\alpha\alpha} n_\gamma$ in flavor α . As the temperature decreases and N_1 decay starts to be out of equilibrium, an asymmetry $\epsilon_{\alpha\alpha} n_{N_1}$ in flavor of α is produced. Thus at the lowest order of $\tilde{m}_{\alpha\alpha}$, the final lepton asymmetry vanishes. What really happens, however, is that part of the original lepton asymmetry $-\epsilon_{\alpha\alpha} n_{N_1}$ is washed out. This part is of the order

$$\eta_\alpha \sim \frac{\tilde{m}_{\alpha\alpha}}{m_*}. \tag{1.72}$$

3. Weak washout, $\tilde{m} < m_*$.

In this case the number of N_1 is not in thermal equilibrium. However

$$n_{N_1} \sim \Gamma_{\rm prod}\tau_U n_\gamma \sim \frac{1}{2}\Gamma_{\rm prod}\frac{1}{H(T \sim M_1)} n_\gamma < n_\gamma. \tag{1.73}$$

So that the efficiency factor is

$$\eta_\alpha \sim \frac{\tilde{m}\tilde{m}_{\alpha\alpha}}{m_*^2}. \tag{1.74}$$

The lepton number asymmetry is converted into baryon number asymmetry by sphaleron interactions by the factor C,

$$Y_{\Delta B} = C\sum_{\alpha} Y_{\Delta\alpha} = \begin{cases} \frac{12}{37}\sum_{\alpha} Y_{\Delta\alpha} & \text{standard model} \\ \\ \frac{10}{31}\sum_{\alpha} Y_{\Delta\alpha} & \text{supersymmetry} \end{cases} \tag{1.75}$$

Now we can estimate the baryon number asymmetry.

$$Y_{\Delta B} \sim 10^{-3} \times \sum_{\alpha} \epsilon_{\alpha\alpha} \times \eta_{\alpha}, \tag{1.76}$$

with η_α given above depending on the strength of washout.

1.4.3 CP violation from self-energy correction–resonant leptogenesis

In this section we discuss CP violation from self-energy corrections to the decaying particles. we will show that if the mass difference of the decaying particles are of the order of decay width, CP violation is resonantly enhanced. The enhanced CP violation helps decrease the RH neutrino masses and correspondingly the reheating temperature of the universe. We adopt this section from [14].

Field theoretical background

We first discuss briefly the theoretical description of mixing between stable particles in a simple scalar theory within the framework of the Lehmann-Symanzik-Zimmermann formulism (LSZ). We will then extend our consideration to unstable mixing particles. The effective theoretical method will be carried over to the case of unstable fermions, and finally we calculate CP violation from self-energy corrections to the fermions.

Consider a field theory with N real scalars S_i^0, with $i = 1, 2, \ldots, N$. We assume they are stable. The bare fields S_i^0 and their bare masses M_i^0 can be expressed with their renormalized counterparts,

$$\begin{aligned} S_i^0 &= Z_{ij}^{1/2} S_j = (\delta_{ij} + \frac{1}{2} Z_{ij}) S_j, \\ (M_i^0)^2 &= (M_i)^2 + \delta M_i^2, \end{aligned} \tag{1.77}$$

where summation is assumed in the repeated indices. The $Z_{ij}^{1/2}$ and δM_i^2 are the wave-function and mass renormalization constants, respectively. They can be determined from the two-point correlation functions, $\Pi_{ij}(p^2)$, under some renormalization conditions.

The two-point correlation functions, $\Pi_{ij}(p^2)$ has two poles at $p^2 = M_i^2$ and $p^2 = M_j^2$. The two pole parts are related through

$$\Delta_{ij}(p^2)\big|_{p^2\to M_i^2, M_j^2} = Z_{im}^{1/2}\frac{\delta_{mn}}{p^2 - M_n^2}Z_{nj}^{1/2T}. \tag{1.78}$$

Using LSZ formalism we can determine the renormalized $n-1$-non-amputated amplitudes, $S_{i\ldots}$ for a fixed given external line i, from the corresponding unrenormalized n-point Green function$G_{i,\ldots}$,

$$\begin{aligned} S_{i,\ldots} &= \lim_{p^2\to M_i^2} G_{j,\ldots}Z_{ji}^{-1/2T}(p^2 - M_i^2) \\ &= \lim_{p^2\to M_i^2} \mathcal{T}_{k,\ldots}^{\text{amp}} Z_{km}^{1/2}\frac{\delta_{mn}}{p^2 - M_n^2}Z_{ji}^{-1/2T}(p^2 - M_i^2) \\ &= \lim_{p^2\to M_i^2} \mathcal{T}_{k,\ldots}^{\text{amp}} Z_{km}^{1/2}Z_{ki}^{1/2}, \end{aligned} \tag{1.79}$$

where $\mathcal{T}_{k,\ldots}^{\text{amp}}$ is the amplitude amputated at the k-th leg. This procedure can be generalized to all external legs.

For simplicity we consider two unstable scalar particles, S_1, S_2, and the conclusion can be easily generalized to the situation with more particles. We are interested in the width effects of these particles. To do this, we calculate all the two point correlation functions Π_{ij}, summing up the geometry series of Π_{ij}, we obtain the inverse propagator matrix:

$$\Delta_{ij}^{-1}(p^2) = \begin{pmatrix} p^2 - (M_1^0)^2 + \Pi_{11}(p^2) & \Pi_{12}(p^2) \\ \Pi_{21}(p^2) & p^2 - (M_2^0)^2 + \Pi_{22}(p^2) \end{pmatrix}, \tag{1.80}$$

where $\Pi_{12}(p^2) = \Pi_{21}(p^2)$. The reverting matrix gives

$$\Delta_{11}(p^2) = \left(p^2 - (M_1^0)^2 + \Pi_{11}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_2^0)^2 + \Pi_{22}(p^2)}\right)^{-1}, \tag{1.81}$$

$$\Delta_{22}(p^2) = \left(p^2 - (M_2^0)^2 + \Pi_{22}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_1^0)^2 + \Pi_{11}(p^2)}\right)^{-1}, \tag{1.82}$$

$$\begin{aligned} \Delta_{12}(p^2) = \Delta_{21}(p^2) = -\Pi_{12}(p^2)\left[p^2 - (M_2^0)^2 + \Pi_{22}(p^2)(p^2 - (M_1^0)^2 + \Pi_{11}(p^2)))\right. \\ \left.-\Pi_{12}^2(p^2)\right]^{-1}. \end{aligned} \tag{1.83}$$

We can also get the off-diagonal ($i \neq j$) resummed scalar propagators

$$\Delta_{ij}(p^2) = -\Delta_{ii}(p^2)\frac{\Pi_{ij}(p^2)}{p^2-(M_j^0)^2+\Pi_{jj}(p^2)} = -\frac{\Pi_{ij}(p^2)}{p^2-(M_i^0)^2+\Pi_{ii}(p^2)}\Delta_{jj}(p^2). \tag{1.84}$$

They are related to the renormalized propagators $\hat{\Delta}_{ij}(p^2)$ through

$$\Delta_{ij}(p^2) = Z_{im}^{1/2}\hat{\Delta}_{mn}(p^2)Z_{nj}^{1/2T} \tag{1.85}$$

This also holds for renormalized scalar propagator, as long as we replace the unrenormalized quantities with the renormalized ones. For the renormalized S-matrix elements we get

$$\begin{aligned}
\hat{S}_{i,\ldots} &= \lim_{p^2\to M_i^2} \mathcal{T}_{k,\ldots}^{\mathrm{amp}} Z_{km}^{1/2}\hat{\Delta}_{mn}(p^2)Z_{nj}^{1/2T}Z_{ji}^{-1/2T}\hat{\Delta}_{ii}^{-1}(p^2) \\
&= \lim_{p^2\to M_i^2}\left(\mathcal{T}_{k,\ldots}^{\mathrm{amp}}Z_{ki}^{1/2} - \mathcal{T}_{k,\ldots}^{\mathrm{amp}}Z_{km}^{1/2}\frac{\hat{\Pi}_{mi}(p^2)(1-\delta_{mi})}{p^2-(M_m^0)^2+\hat{\Pi}_{mm}(p^2)}\right) \\
&= S_{i,\ldots} - S_{j,\ldots}\frac{\hat{\Pi}_{mi}(M_i^2)(1-\delta_{ij})}{M_i^2-M_j^2+\hat{\Pi}_{jj}(M_i^2)},
\end{aligned} \tag{1.86}$$

where $S_{i,\ldots}$ and $S_{j,\ldots}$ are the renormalized transition amplitudes in the stable particle approximation. When renormalize, the counter-terms don't absorb the imaginary part of self-energy $\hat{\Pi}_{ij}(M_i^2)$. This imaginary part is actually what we want to include into the calculation. Because of the imaginary part the decay amplitude stays analytic when the particle masses are strictly degenerate. Finally, the treatment to unstable particles is only effective. The decaying particle can not be an asymptotic state in the initial state. This resummed decay amplitude should be regarded as an effective part which can be embedded into a resummed S-matrix element.

Similar to the scalar situation, the resummed fermion propagator matrix is

$$S_{ij}(\not{p}) = \begin{pmatrix} \not{p} - M_1^0 + \Sigma_{11}(\not{p}) & \Sigma_{12}(\not{p}) \\ \Sigma_{21}(\not{p}) & \not{p} - M_2^0 + \Sigma_{22}(\not{p}) \end{pmatrix}, \tag{1.87}$$

inverting it we have

$$S_{11}(\not{p}) = \left(\not{p} - M_1^0 + \Sigma_{11}(\not{p}) - \Sigma_{12}(\not{p})\frac{1}{\not{p} - M_2^0 + \Sigma_{22}(\not{p})}\Sigma_{21}(\not{p})\right)^{-1}, \tag{1.88}$$

$$S_{22}(\not{p}) = \left(\not{p} - M_2^0 + \Sigma_{22}(\not{p}) - \Sigma_{21}(\not{p})\frac{1}{\not{p} - M_1^0 + \Sigma_{11}(\not{p})}\Sigma_{12}(\not{p})\right)^{-1}, \tag{1.89}$$

$$S_{12}(\not{p}) = -S_{11}(\not{p})\Sigma_{12}(\not{p})[\not{p} - M_2^0 + \Sigma_{22}(\not{p})]^{-1}$$
$$= -[\not{p} - M_1^0 + \Sigma_{11}(\not{p})]^{-1}\Sigma_{12}(\not{p})S_{22}(\not{p}) \quad (1.90)$$

$$S_{21}(\not{p}) = -S_{22}(\not{p})\Sigma_{21}(\not{p})[\not{p} - M_1^0 + \Sigma_{11}(\not{p})]^{-1}$$
$$= -[\not{p} - M_2^0 + \Sigma_{22}(\not{p})]^{-1}\Sigma_{21}(\not{p})S_{11}(\not{p}) \quad (1.91)$$

The analogous expression for the fermionic propagators is

$$S_{ij}\not{p} = (Z_{Lim}^{1/2}P_L + Z_{Rim}^{1/2}P_R)\hat{S}_{mn}(\not{p})(Z_{Lnj}^{1/2\dagger}P_R + Z_{Rnj}^{1/2\dagger}P_L), \quad (1.92)$$

where Z-s are the wave-function renormaization constants and $P_{L(R)}$ are chirality projectors.

For the resummed decay width for fermions we get, similarly with Eq. (1.43)

$$\hat{S}_{i,\ldots}u_i(p) = S_{i,\ldots}u_i(p) - (1-\delta_{ij})S_j\hat{\Sigma}_{ji}(\not{p})[\not{p} - M_j^0 + \Sigma_{jj}]^{-1}u_i(p). \quad (1.93)$$

CP violation in RH neutrino decays

We outline the calculation of CP violation from self-energy corrections in the decays of two RH neutrinos. we will show that when the their masses are nearly degenerate, CP violation is resonantly enhanced.

Let's consider the model with Lagrangian like Eq.(1.9), where the decay of RH neutrinos is to lepton and Higgs, $N \to l^- h^+$. The decay amplitude of N_1 can be written down as, with self-energy correction at one loop,

$$\mathcal{T}_{N_1} = Y_{l1}\bar{u}_l P_R u_{N_1} - iY_{l2}\bar{u}_l P_R[\not{p} - M_2 + i\Sigma_{22}^{\rm abs}(\not{p})]^{-1}\Sigma_{21}^{\rm abs}(\not{p})u_{N_1}. \quad (1.94)$$

The absorptive part of $\Sigma_{ij}^{\rm abs}$, at one loop, has the form

$$\Sigma_{ij}^{\rm abs} = A_{ij}(p^2)\not{p}P_L + A_{ij}^{ast}(p^2)\not{p}P_R, \quad (1.95)$$

with

$$A_{ij}(p^2) = \frac{Y_{l'i}Y_{l'j}^*}{32\pi}\left(\frac{3}{2} + \frac{1}{2}\left(1 - \frac{M_H^2}{p^2}\right)\right). \quad (1.96)$$

In a wide range of RH neutrino masses we can take M_H to be zero and

$$A_{ij}(p^2) = \frac{Y_{l'i}Y^*_{l'j}}{16\pi}. \tag{1.97}$$

The amplitude $\bar{\mathcal{T}}_{N_1}$ of the CP conjugate decay $N_1 \to l^+h^-$ can be written as

$$\begin{aligned}\bar{\mathcal{T}}_{N_1} &= Y^*_{l1}\bar{v}_{N_1}P_L v_l - iY^*_{l2}\bar{v}_{N_1}\Sigma^{\rm abs}_{12}(-\not{p})[-\not{p} - M_2 + i\Sigma^{\rm abs}_{22}(-\not{p})]^{-1}P_L v_l \\ &= Y^*_{l1}\bar{u}_l P_L u_{N_1} - iY^*_{l2}\bar{u}_l P_L[\not{p} - M_2 + i\bar{\Sigma}^{\rm abs}_{22}(\not{p})]^{-1}\bar{\Sigma}^{\rm abs}_{21}(\not{p})u_{N_1},\end{aligned} \tag{1.98}$$

where the charge-conjugate absorptive part is

$$\bar{\Sigma}_{ij}(\not{p}) = A_{ij}(p^2)\not{p}P_R + A^*_{ij}(p^2)\not{p}P_L. \tag{1.99}$$

In the last step we used these identities: $u(p,s) = C\bar{v}^T(p,s)$ and $C\gamma_\mu C^{-1} = -\gamma_\mu$.

With the Dirac equation of motion for the singlet neutrino, we can find the simple form of the decay amplitudes:

$$\mathcal{T}_{N_1} = \bar{u}_l P_R u_{N_1}\left(Y_{l1} - iY_{l2}\frac{M_1^2(1+iA_{22})A^*_{21} + M_1M_2A_{21}}{M_1^2(1+A_{22})^2 - M_2^2}\right) \tag{1.100}$$

$$\bar{\mathcal{T}}_{N_1} = \bar{u}_l P_L u_{N_1}\left(Y^*_{l1} - iY^*_{l2}\frac{M_1^2(1+iA_{22})A_{21} + M_1M_2A^*_{21}}{M_1^2(1+A_{22})^2 - M_2^2}\right). \tag{1.101}$$

For the decay of N_2, we only need to switch 1 and 2. The charge-conjugate amplitudes differ from each other by the complex conjugate of Yukawa coupling and the chirality operator, as expected.

The CP violation parameters are

$$\epsilon_{N_i} = \frac{|\mathcal{T}_{N_i}|^2 - |\bar{\mathcal{T}}_{N_i}|^2}{|\mathcal{T}_{N_i}|^2 + |\bar{\mathcal{T}}_{N_i}|^2}\ , \qquad (i = 1, 2) \tag{1.102}$$

Strictly speaking, we need to include both self-energy corrections and vertex corrections for CP violation. We didn't do that because we are mainly interested in the condition that the two RH masses are nearly degenerate. Under this condition, the contribution from vertex corrections is negligible compared with from self-energy corrections.

We can write the CP violation parameters in a more compact form. We assume the near degeneracy of the two RH neutrino masses: $\Delta M^2 = M_1^2 - M_2^2 \ll M_1^2 \sim M_2^2$, and define a small parameter

$r = \Delta M^2/(M_1 M_2)$. Ignoring high order Yukawa couplings of $\mathcal{O}(Y^4)$, we get

$$\epsilon_{N_1} \approx \frac{\Im(Y_{l1}^* Y_{l2} Y_{l'1}^* Y_{l'2})}{8\pi |Y_{l1}|^2} \frac{r}{r^2 + 4A_{22}^2}, \tag{1.103}$$

$$\epsilon_{N_2} \approx \frac{\Im(Y_{l1}^* Y_{l2} Y_{l'1}^* Y_{l'2})}{8\pi |Y_{21}|^2} \frac{r}{r^2 + 4A_{11}^2}. \tag{1.104}$$

In supersymmetric models, the decay width of N_i is $\Gamma = M_i(Y^\dagger Y)_{ii}/(4\pi) = 4A_{ii}$. We see that CP violation is resonantly enhanced when the mass splitting is of the order of decay width, or r is of the order of A_{ii}. We also see that the inclusion of the finite decay width of the RH neutrinos is important. They appear in the denominator of the expression of CP violation. When the mass split approaches zero this expression remains analytic. This makes physical sense. We will use this conclusion in the study of two particular models in Chapter 2 and Chapter 3.

1.5 Flavor changing neutral currents in SUSY multi-Higgs models

Although the standard model has offered a successful description of the strong and electro-weak interactions, it fails to address the problems such as the gauge group, the number of flavors, the dynamics of flavors and the mass generation of fermions. From a phenomenological point of view we have strong reasons to believe there is three flavors of quarks and leptons, but there is not obvious theoretical reason of three flavors. Anomaly cancelation requires the number of lepton flavor and quark flavor are equal, but not the total numbers of flavors. Extensions of the standard model, such as GUT and supersymmetry, although remedy some of the problems, none of them address the problem of flavor number.

Given the assumption of three flavors of leptons and quarks, we still have no idea the number of Higgs doublets. We may wonder if there is similar flavor structures among leptons and Higgs. Actually in some string motivated models, fermions and Higgs fall into the same group representations [15]. Other models that include a non-minimal Higgs content have also been extensively addressed. The consequences of extending the Higgs sector are abundant, and have implications which range from the theoretical to the experimental level. For instance, if the extra Higgses are light, the addition of these states in a minimal SUSY scenario will spoil the unification of interactions at energy scale

10^{16} GeV. In our phenomenological studies, we consider only the low energy physics of the extended Higgs sector. The most significant phenomenological implication might be the potential problem of the existence of the tree level flavor changing neutral currents (FCNC) mediated by the neutral Higgs. In the standard model and its minimal supersymmetry extension, these processes are absent since the Yukawa couplings of quarks and leptons to neutral Higgs are diagonal in the mass basis. The experimental data and the standard model prediction are in good agreement, thus these contributions of the tree level FCNC processes need to be suppressed. The suppression of the effects of the tree level exchange of neutral Higgs can be achieved in several ways:

(1) Discrete flavor symmetries. Imposing a flavor symmetry to the couplings of the fermions and Higgs ensures some internal relation among the contributions of neutral Higgs to the phenomenological contribution of particular fermion combinations. FCNC effects cancelation can be the result of these symmetries. In addition, the flavor symmetries can realize some special textures of the mass matrices of the fermions after symmetry breaking. These mass matrices are closely related with some of direct phenomenological measurements.

(2) Suppression of Yukawa couplings. Most stringent phenomenological constraints are from the mixing of neutral bosons such as $K-\bar{K}$, $B-\bar{B}$, etc., associated with quarks of down sector. Smallness of the Yukawa coupling of the down sector quarks and Higgs suppress the FCNC contribution through neutral Higgs exchange. The major shortcoming of Yukawa suppression is that we don't have physical reason to explain the smallness of the Yukawa couplings.

(3) Decoupling of extra Higgses. Since the contribution of Higgs exchange is inversely proportional to the square masses of the neutral Higgses, large Higges masses suppress the FCNC contributions. In some highly predictive models this is the only choice for FCNC suppression. This may lead to a fine-tuning scenario in association with electro-weak symmetry breaking. When the decoupling approach and the Yukawa coupling suppression are taken together, it is possible to attain Higgs spectra that is not very heavy.

We organize the article in 5 parts. In chapter 2 we show a model to address the neutrino masses and leptogenesis. Left handed neutrinos are coupled to right handed neutrinos and the mass of light neutrinos are generated through the see-saw mechanism. We determine some of the free parameters of in the model through neutrino oscillation data. With this parameter set, we calculated the CP violation of the right handed neutrino decays generated with the two loop correction of the right handed Majorana mass matrix. Because of the special texture of the right handed neutrino mass matrix, CP violation is resonantly enhanced. We then study the resonant leptogenesis and compare our study with the observed baryon asymmetry of the universe.

Chapter 3 is devoted to the study of leptogenesis in new ways with $B-L$ gauge symmetry in supersymmetry scenario. Through introduction a pair of Higgs one of which is coupled to right handed neutrinos, for the conservation of $B-L$ charge. The right handed neutrino Majorana masses are generated through Higgs mechanism when $B-L$ symmetry is broken. CP violation happens in the decays of the Higgs fields to right handed neutrinos when supersymmetry is softly broken. We studied the soft leptogenesis in this scenario.

In Chapter 4 we studied an extension of the model in chapter 3. We introduced two pairs of Higgs to break $B-L$ symmetry instead of 1 pair. CP violation can happen in the supersymmtry limit in the decays of the Higgses, since the Yukawa coupling is generically complex. We studied leptogenesis in this scenario and find wide parameter space for large enough baryon asymmetry as large as observed.

In Chapter 5 we studied a model with discrete flavor symmetry Q_6. We concentrate on the phenomenological FCNC implications. Q_6 symmetry exerts strong constraints on the form of mass matrices of leptons, quarks and their supersymmetry partners. We studied the phenomenological FCNC implications in the neutral boson mixing, μ rare decays and the electric dipole moment of electron and neutron through neutral Higgs exchange. With the help of precise experimental data we determined the range of the masses of heavy Higgses.

In Chapter 6 we summarized our study and made some remarks for future work.

CHAPTER 2

COMMON ORIGIN FOR CP VIOLATION IN COSMOLOGY AND IN NEUTRINO OSCILLATIONS

2.1 Introduction

While the standard model (SM) of strong and electro-weak interactions has been extremely successful in confronting experimental data, it leaves several questions unanswered. On the observational side, it does not provide a viable dark matter candidate, nor a dynamical mechanism to explain the observed baryon excess in the universe. Furthermore, the model needs to be extended, albeit in a minor way, to accommodate small neutrino masses as needed for atmospheric [16] and solar neutrino oscillation data [17]. On the theoretical side, the model suffers from the quadratic divergence problem, which destabilizes the Higgs boson mass.

An elegant synthesis of these issues is provided by low energy supersymmetry (SUSY) and the seesaw mechanism [56]. Low energy SUSY can cure the quadratic divergence problem for the Higgs boson mass. In its simplest form it also provides a natural dark matter candidate, the lightest SUSY particle (LSP). The seesaw mechanism assumes the existence of right–handed neutrinos (RHN) N which facilitates small neutrino masses. It also provides a dynamical mechanism for baryon asymmetry generation via the lepton number violating decays of the N [19]. The induced lepton asymmetry is converted into baryon asymmetry via the electro-weak sphalerons [20] (for reviews of leptogenesis see Ref. [21, 22]).

Attractive as it is, the SUSY seesaw framework is not without its problems. First, the generic leptogenesis mechanism is impossible to test experimentally. This is primarily because the dynamics occurs at a very high energy scale, beyond reach of foreseeable experiments. The parameters that are relevant for leptogenesis are not the same that appear in low energy neutrino oscillation experiments.

(The number of low energy observables in neutrino sector is nine, while leptogeneis in the general setting involves a total of eighteen parameters.) Second, in supergravity models, successful leptogenesis is in conflict with the gravitino abundance. This is because of the lower bound on the lightest RHN mass $M_{N_1} \gtrsim 10^9$ GeV, (assuming hierarchical masses for N) [23] which would suggest rather high reheat temperature, of order 10^9 GeV. This conflicts with reheat temperature suggested by gravitino abundance $T_{\rm reheat} < 10^8$ GeV [24, 25].

In this paper we suggest a scenario where the aforementioned problems of the SUSY seesaw framework are alleviated. The gravitino overproduction problem is avoided by resorting to resonant leptogenesis scenario [26–28] which assumes quasi–degenerate N fields. In this case the mass of the N fields can be as low as a TeV, consistent with successful leptogenesis, thus avoiding the gravitino problem. We supplement the resonant leptogenesis scenario with flavor symmetries which restrict the form of the neutrino Yukawa coupling matrices. Such flavor symmetries are anyway needed to guarantee the near degeneracy of the N states. We identify three possible textures for the Dirac Yukawa couplings of the neutrinos that yield two quasi–degenerate N fields and a sum rule for the neutrino oscillation angle θ_{13}. Interestingly, in all three models, there is a single phase that controls cosmological CP asymmetry and CP violation in neutrino oscillations. We are able to constrain the range of the CP violation parameter $|\delta|$ from cosmology. Somewhat similar classification of textures has been recently pursued in Ref. [29] and earlier in Ref. [30], [31]. Our emphasis is on the connection between cosmological CP asymmetry and CP violation in neutrino oscillations. It turns out that, in our framework, there is a lower limit on the SUSY parameter $\tan\beta > 12$. This arises since the mass splitting between the qusi–degenerate N fields is generated from renormalization group flow, which depends on $\tan\beta$.

In our analysis we use the results of a global fit to the neutrino oscilation data [32]:

$$|\Delta m^2_{\rm atm}| = (2.18 - 2.64) \times 10^{-3} {\rm eV}^2 \, (2\sigma) \, , \qquad \Delta m^2_{\rm sol} = (7.25 - 8.11) \times 10^{-5} {\rm eV}^2 \, (2\sigma) \, ,$$

$$\sin^2\theta_{23} = 0.39 - 0.63 \, (2\sigma) \, , \qquad \sin^2\theta_{12} = 0.27 - 0.35 \, (2\sigma) \, . \tag{2.1}$$

Currently we do not know the sign of $\Delta m^2_{\rm atm}$, i.e. whether neutrinos have normal mass hierarchy or inverted mass hierarchy. Also, the value of the third mixing angle θ_{13} is unknown. Only an upper bound [32]

$$\sin^2\theta_{13} \leq 0.04\,(2\sigma) \tag{2.2}$$

is available currently. Nothing is known about the CP violating phase δ (and also about two 'Majorana' phases) of the leptonic mixing matrix.

We will identify explicit models wherein these unknown mixing parameters are significantly constrained. It will be highly desirable to relate the CP violation parameters in the leptonic mixing matrix with the cosmological CP asymmetry. Such a strategy was pursued successfully in Ref. [33]. While in Ref. [33] a close connection between cosmological CP violation and neutrino CP violation was realized, since the setup used hierarchical RHN masses, straightforward SUSY extension of that scenario would lead to gravitino overproduction. Our texture models are tailor–made for resonant leptogenesis, which would avoid this problem.[1]

2.2 Texture Zeros for Predictive Models

Let us consider the lepton sector of MSSM augmented with two right–handed neutrinos (RHN) N_1 and N_2. The relevant Yukawa superpotential couplings are given by

$$W_{\rm lept} = W_e + W_\nu \ ,$$

$$\text{with} \quad W_e = l^T Y_e e^c h_d \ , \qquad W_\nu = l^T Y_\nu N h_u - \frac{1}{2} N^T M_N N \ , \tag{2.3}$$

where h_d and h_u are up and down type MSSM Higgs doublet superfields respectively. We will work in a basis in which the charged lepton Yukawa matrix is diagonal:

$$Y_e = {\rm Diag}\,(\lambda_e,\ \lambda_\mu,\ \lambda_\tau) \ . \tag{2.4}$$

[1]For a concrete demonstration within predictive model see [34].

As far as the RHN mass matrix M_N is concerned, we will assume that at high scale (identified with the GUT scale later on) it has the form

$$M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M \ . \tag{2.5}$$

This form of M_N is crucial for our studies. It has interesting implications for resonant leptogenesis and also, as we will see below, for building predictive neutrino scenarios. Specific neutrino models consistent with resonant leptogenesis with a texture similar to (2.5) was investigated in [34]. Here we attempt to classify all possible scenarios with degenerate RHNs which lead to predictions consistent with experiments. Thus, with a basis (2.4) and the texture (2.5) we can discuss possible texture zeros in the matrix Y_ν, which is of dimension 3×2. One can easily verify that two (and more) texture zeros in Y_ν do not lead to results compatible with the neutrino data. However, with only one texture zero, there are scenarios compatible with experiments and leading to interesting predictions.

The matrix Y_ν contains two columns. Since due to the form of M_N (2.5) there is exchange invariance $N_1 \to N_2$, $N_2 \to N_1$, it does not matter in which column of Y_ν we set one element to zero. We choose here the second column of Y_ν having one texture zero. This leads to the three following possible forms for Y_ν:

$$\text{Texture A}: \qquad Y_\nu = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} , \tag{2.6}$$

$$\text{Texture B}_1: \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ a_3 & b_3 \end{pmatrix} , \qquad \text{Texture B}_2: \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & 0 \end{pmatrix} . \tag{2.7}$$

A few words about the parametrization, used in (2.6) and (2.7), are in order. With the basis (2.4) and the form of M_N given in (2.5), the one texture zero 3×2 matrix Y_ν has only one physical phase. Other phases can be rotated away by proper phase redefinitions of the fields. Moreover, in Y_ν there are five real parameters $|a_{1,2,3}|$ and two absolute values of the b-entries. The mass parameter M in (2.5) is in general complex, but its phase is not relevant for the physics of neutrino oscillations. These systems lead to predictive scenarios with texture A corresponding to normal mass hierarchy and textures B_1 and B_2 corresponding to inverted mass hierarchy. We will study these cases in turn.

2.2.1 Texture A: Normal Hierarchical Case

We will discuss this case in details. With (2.5), (2.6) and using the seesaw formula for the light neutrino mass matrix $M_\nu = \langle h_u^0 \rangle^2 Y_\nu M_N^{-1} Y_\nu^T$, we arrive at

$$M_\nu = \begin{pmatrix} 0 & a_1 b_2 & a_1 b_3 \\ a_1 b_2 & 2a_2 b_2 & a_2 b_3 + a_3 b_2 \\ a_1 b_3 & a_2 b_3 + a_3 b_2 & 2a_3 b_3 \end{pmatrix} \frac{(v \sin\beta)^2}{M} , \tag{2.8}$$

where $v \simeq 174$ GeV. The matrix in (2.8) is rank two and leads to the one massless neutrino and two massive neutrinos labeled m_2 and m_3. This structure corresponds to the normal hierarchical case, i.e.

$$M_\nu^{\rm diag} = {\rm Diag}\,(0,\ m_2,\ m_3)\ , \tag{2.9}$$

with $m_3 \gg m_2$. From (2.8) we can see that the mixing θ_{12} and θ_{23} are generated. The absolute value of the overall factor $a_3 b_3 (v \sin\beta)^2/M$ determines one mass scale, say the value of m_3. Besides this overall factor the matrix has four parameters: one phase and three real parameters. Three of these parameters can be fixed from three observables θ_{12}, θ_{23} and $\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}$ (where $\Delta m^2_{\rm sol} = m_2^2$ and $\Delta m^2_{\rm atm} = m_3^2 - m_2^2$). Due to the condition $m_1 = 0$ we will still have one prediction (independent from the value of the phase), which determines the angle θ_{13}.

One physical phase remains undetermined. Indeed this *single* phase will be directly related to the CP violation in neutrino oscillations and in leptogenesis. We will discuss this connection in more details in Sect. 2.3.

Now, let us derive the prediction of this model. To achieve this and also get other useful relations we will use the equality

$$M_\nu = P U^* P' M_\nu^{\rm diag} U^\dagger P, \tag{2.10}$$

where U is the lepton mixing matrix, given in a standard parameterization by:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \tag{2.11}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. The P and P' are diagonal phase matrices $P = \mathrm{Diag}\,(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$, $P' = \mathrm{Diag}\,(1, e^{i\rho_1}, e^{i\rho_2})$. Phases in P can be removed by field redefinition, while P' is physical, and contains the two Majorana phases. The matrix equation (2.10) gives six relations. One of them, namely the relation for the $(1,1)$ elements of M_ν and the right hand side of (2.10) with the form of U given in Eq. (2.11), gives

$$\tan\theta_{13} \simeq \sin\theta_{12}\sqrt{\frac{m_2}{m_3}} . \tag{2.12}$$

Since this case corresponds to the normal hierarchical neutrino mass spectrum (with $m_1 = 0$), with the help of (2.1) we have at 2σ level $m_2 = \sqrt{\Delta m^2_{\mathrm{sol}}} \simeq (8.51 - 9.01)\cdot 10^{-3}$ eV and $m_3 = \sqrt{|\Delta m_{\mathrm{atm}}|^2 + \Delta m^2_{\mathrm{sol}}} \simeq (4.7 - 5.2)\cdot 10^{-2}$ eV. Using these values in (2.12), together with 2σ accuracy value of θ_{12}, we obtain range $\sin^2\theta_{13} \simeq 0.042 - 0.062$. This fits well with an upper bound, within 3σ, given in Ref. [32], while the low limit (0.042) is pretty close to the 2σ upper bound of θ_{13}. Future measurements of the θ_{13} will test the validity of this scenario. One more word about the neutrino sector: since the $(1,1)$ element of the light neutrino mass matrix vanishes, the neutrino–less double β-decay ($0\nu2\beta$) does not take place in this scenario. That is, $m_{\beta\beta} = |U^2_{e2}m_2e^{i\bar{\rho}} + U^2_{e3}m_3| = 0$. There is only one Majorana phase, since $m_1 = 0$, which is $\bar{\rho} = \rho_2 - \rho_1$. This is determined from the phase δ as follows

$$\bar{\rho} = \pi - 2\delta . \tag{2.13}$$

2.2.2 Textures B_1 and B_2 : Inverted Hierarchical Cases

The textures B_1 and B_2 both lead to the inverted hierarchical neutrino mass pattern. Using these textures (2.7), the form of M_N given in (2.5) and the seesaw formula for the light neutrino mass matrices we obtain:

$$\text{For Texture B}_1: \qquad M_\nu = \begin{pmatrix} 2a_1b_1 & a_2b_1 & a_1b_3+a_3b_1 \\ a_2b_1 & 0 & a_2b_3 \\ a_1b_3+a_3b_1 & a_2b_3 & 2a_3b_3 \end{pmatrix} \frac{(v\sin\beta)^2}{M} \ , \tag{2.14}$$

$$\text{For Texture B}_2: \qquad M_\nu = \begin{pmatrix} 2a_1b_1 & a_1b_2+a_2b_1 & a_3b_1 \\ a_1b_2+a_2b_1 & 2a_2b_2 & a_3b_2 \\ a_3b_1 & a_3b_2 & 0 \end{pmatrix} \frac{(v\sin\beta)^2}{M} \ . \tag{2.15}$$

In order to derive predictions for both cases, we can still use the relation (2.10), which is general, but for M_ν use the forms corresponding to the cases $B_{1,2}$, and for $M_\nu^{\rm diag}$ an inverted hierarchical form:

$$M_\nu^{\rm diag} = {\rm Diag}\,(m_1,\ m_2,\ 0)\ . \tag{2.16}$$

We use the same form as before for the phase matrix P, while for the P' we use $P' = {\rm Diag}\,(e^{i\rho_1}, e^{i\rho_2}, 1)$. For cases B_1 and B_2 the predictive relations emerge by equating the $(2,2)$ and $(3,3)$ elements respectively (which are zero) of the expressions at the both sides of Eq. (2.10). Doing so we arrive at:

$$\text{For texture B}_1: \qquad \sin^2\theta_{12} \simeq \frac{1}{2} - \frac{\sin\theta_{13}\tan\theta_{23}\cos\delta}{|\tan^2\theta_{23}\sin^2\theta_{13}+e^{2i\delta}|} + \frac{1}{8}\frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|} \ , \tag{2.17}$$

$$\text{For texture B}_2: \qquad \sin^2\theta_{12} \simeq \frac{1}{2} + \frac{\sin\theta_{13}\tan\theta_{23}\cos\delta}{|\tan^2\theta_{23}+\sin^2\theta_{13}e^{2i\delta}|} + \frac{1}{8}\frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|} \ . \tag{2.18}$$

As we see, for both cases, the deviation of $\sin^2\theta_{12}$ from 1/2 (i.e. deviation of θ_{12} from $\pi/4$) is due to the non–zero value of θ_{13} and it also depends on $\cos\delta^2$. In fact, the product $\sin\theta_{13}\cos\delta$ should not be too small, otherwise the angle θ_{12} will be close to $\pi/4$ which is excluded. Using the current experimental data (2.1) (within 2σ-deviations) we obtain the following constraints for θ_{13} and $\cos\delta$:

$$\text{For texture B}_1: \qquad \theta_{13} \gtrsim 0.12\ , \qquad \cos\delta \gtrsim 0.573 \quad (|\delta| \lesssim 0.96)\ ,$$

[2]Similar relation has been obtained in Ref. [34] within a specific model with $\theta_{23} \simeq \pi/4$. Here, since θ_{23} is not fixed from the model, we will have somewhat wider allowed ranges for θ_{13} and especially for δ. Cases of texture zeros giving these relations have been identified recently in Ref. [29]. Correlation similar to Eqs. (2.17) and (2.18) have been obtained within scenarios with 'quark-lepton complementarity' [35].

$$\text{For texture B}_2: \quad \theta_{13} \gtrsim 0.129 \ , \qquad \cos\delta \lesssim -0.614 \quad (|\pi-\delta| \lesssim 0.91) \ . \tag{2.19}$$

The last terms in Eqs. (2.17), (2.18) are practically unimportant for the neutrino sector, but as we will see in section 2.3.1 they become crucial for the leptogenesis CP violation. The leptonic asymmetry will be determined by a phase $\propto \frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|}\sin\delta$ which would vanish in the limit $\frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|} \to 0$.

By the fixed model parameters (see sect. 2.3.1 for relation between Yukawa couplings and the angles θ_{ij}, δ) we can compute one more observable. In contrast to the normal hierarchical neutrinos (corresponding to the texture A), cases B_1 and B_2 have non–zero $\beta\beta_{0\nu}$ amplitudes, for both cases given by

$$m_{\beta\beta} = |U_{e1}^2 m_1 e^{i\bar\rho} + U_{e2}^2 m_2| \ , \qquad \text{with} \qquad \bar\rho = \rho_2 - \rho_1 \ . \tag{2.20}$$

For $m_{\beta\beta}$ and $\bar\rho$ for scenarios B_1 and B_2 respectively we derive:

$$\text{For texture B}_1: \qquad m_{\beta\beta} = \sqrt{|\Delta m^2_{\rm atm}|}\, c_{13}^2 \frac{|{\rm tg}_{12}^2 - 1 + 2{\rm tg}_{12}{\rm tg}_{23} s_{13} e^{i\delta}|}{|{\rm tg}_{12} + {\rm tg}_{23} s_{13} e^{i\delta}|^2} \ ,$$

$$\cot\frac{\bar\rho}{2} = -\frac{{\rm tg}_{23}(1+{\rm tg}_{12}^2) s_{13}\sin\delta}{{\rm tg}_{12}(1-{\rm tg}_{23}^2 s_{13}^2) + {\rm tg}_{23}(1-{\rm tg}_{12}^2) s_{13}\cos\delta} \ , \tag{2.21}$$

$$\text{For texture B}_2: \qquad m_{\beta\beta} = \sqrt{|\Delta m^2_{\rm atm}|}\, c_{13}^2 \frac{|{\rm tg}_{12}^2 - 1 - 2{\rm tg}_{12}{\rm ctg}_{23} s_{13} e^{i\delta}|}{|{\rm tg}_{12} - {\rm ctg}_{23} s_{13} e^{i\delta}|^2} \ ,$$

$$\cot\frac{\bar\rho}{2} = \frac{{\rm tg}_{23}(1+{\rm tg}_{12}^2) s_{13}\sin\delta}{{\rm tg}_{12}({\rm tg}_{23}^2 - s_{13}^2) - {\rm tg}_{23}(1-{\rm tg}_{12}^2) s_{13}\cos\delta} \ , \tag{2.22}$$

where ${\rm tg_{ij}} \equiv \tan\theta_{ij}$ and ${\rm ctg_{ij}} \equiv \cot\theta_{ij}$. Applying allowed ranges for δ and θ_{13} given in Eq. (2.19) and the measured neutrino oscillation parameters (2.1) (within 2σ) for $m_{\beta\beta}$ we obtain:

$$\text{For textures B}_1 \ \& \ \text{B}_2: \qquad 0.013 \ {\rm eV} \lesssim m_{\beta\beta} \lesssim 0.023 \ {\rm eV} \ . \tag{2.23}$$

Upper bounds for $m_{\beta\beta}$ are obtained for $|\delta| = 0.96$ and $|\pi-\delta| = 0.91$ for cases B_1 and B_2 respectively, while lower limits correspond to $\delta = 0$ and $\delta = \pm\pi$. Planned experiments will certainly be able to test viability of these predictions. Note that the textures B_1 and B_2 in the neutrino sector give results which are practically indistinguishable (besides the allowed ranges for δ). However, as we will see in the next section the scenario B_2 fails to generate sufficient leptogenesis, while the texture B_1 (and also the texture A) will work very well for this purpose.

2.3 Resonant Leptogenesis

Within the scenarios considered in the previous section, we have assumed an off–diagonal form for the RHN mass matrix M_N. This gives the desired degeneracy between the two RHN states. The degeneracy will be lifted with small corrections to the $(1,1)$ and/or $(2,2)$ elements of M_N. Even in the unbroken SUSY limit, 1-loop corrections (corresponding to the wave function renormalization) will split the degeneracy. The SUSY breaking effects has dramatic impact on the degeneracy of the scalar components of $N_{1,2}$ superfields. This is discussed separately in the Appendix. As far as the fermionic RHN sector is concerned, the degeneracy there holds with pretty high accuracy. Therefore, this is an appealing framework for resonant leptogenesis, in which enhancement of the CP asymmetry happens because of quasi-degenerate RHN neutrinos [26–28]. One nice property of the resonant leptogenesis is that, it avoids the lower bound ($M_{N_1} \gtrsim 10^9$ GeV) for the lightest RHN mass. This bound, called as Davidson-Ibarra bound, emerges within most of the scenarios with hierarchical right–handed neutrinos [23]. Once this bound is avoided, the reheat temperature can be sufficiently low to avoid the gravitino problem, which is common for low scale SUSY models [24, 25] with the gravity mediated SUSY breaking.

Since our models of neutrino masses and mixing are predictive and involve very limited number of parameters, we expect that we will not have much freedom in the calculation of leptogenesis. As we have already mentioned, an important ingredient for the resonant leptogenesis is the form of M_N given in Eq. (2.5). Note that the mass matrix of the fermionic RHNs coincides with M_N of the superpotential mass term. First we will discuss radiative corrections to the superpotential mass matrix M_N, which directly can be applied to the fermionic RHNs. This structure can be justified by some symmetry at high scale. However, at low energies, due to the radiative corrections the $(1,1)$ and $(2,2)$ entries in M_N will receive non-zero corrections. These corrections are calculable thanks to the well defined neutrino models we have presented above. To be brief, eventually two RHNs are become quasi-degenerate, and the CP asymmetries ϵ_1 and ϵ_2 generated by out-of-equilibrium decays of the fermionic components of

N_1 and N_2 states respectively are given by [27, 28]

$$\epsilon_1 = \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(M_2^2 - M_1^2)\, M_1 \Gamma_2}{\left(M_2^2 - M_1^2\right)^2 + M_1^2 \Gamma_2^2} \ , \qquad \epsilon_2 = \epsilon_1 (1 \leftrightarrow 2) \ . \tag{2.24}$$

Here M_1 and M_2 (we will use the convention $M_2 > M_1$) are the mass eigenvalues of the RH neutrino mass eigenstates. $\hat{Y}_\nu = Y_\nu U_N$ is the Dirac Yukawa coupling matrix in the basis where RH neutrino mass matrix is diagonal and real: $U_N^T M_N U_N = \mathrm{Diag}\,(M_1,\ M_2)$. Γ_i is the tree level decay width of $\bar{N}_i$ (mass eigenstates of RHN) and is given by $\Gamma_i = \frac{M_i}{4\pi}(Y_\nu^\dagger Y_\nu)_{ii}$. From (2.24) we see that in order to have non–zero CP asymmetry two conditions need to be satisfied. First, the RHN masses should be split, and secondly the element $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ must be complex. To realize both of these conditions, we need to include radiative corrections into our study. As we will see shortly, the desired result can be obtained only at two-loop level. In our treatment we assume that the textures we have considered are realized at the GUT scale $M_G \simeq 2 \cdot 10^{16}$ GeV. At low scales, due to the renormalization group effects the zero entries in the flavor matrices will receive some corrections. To compute these corrections we set up the RG equation for the matrix M_N (only its renormalization has relevance for us), which at two–loop order is given by [36]:

$$16\pi^2 \frac{d}{dt} M_N = 2 M_N Y_\nu^\dagger Y_\nu + 2 Y_\nu^T Y_\nu^* M_N$$
$$-\frac{1}{8\pi^2} M_N \left(Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu + Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu (3\lambda_t^2 + \mathrm{tr}(Y_\nu^\dagger Y_\nu) \right)$$
$$-\frac{1}{8\pi^2} \left(Y_\nu^T Y_e^* Y_e^T Y_\nu^* + Y_\nu^T Y_\nu^* Y_\nu^T Y_\nu^* + Y_\nu^T Y_\nu^* (3\lambda_t^2 + \mathrm{tr}(Y_\nu^\dagger Y_\nu) \right) M_N + \frac{3}{20\pi^2} M_N \left(g_1^2 + 5 g_2^2 \right) \ , \tag{2.25}$$

where $t = \ln \mu$. The first line in (2.25) corresponds to the 1-loop correction and will be responsible for the mass splitting between RHNs. However, the two-loop correction, presented in a second line of Eq. (2.25), will be crucial for the CP phase of $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$. Since we intend to have $M_{1,2} \lesssim 10^7$ GeV, in order to get reasonable scale for the light neutrino mass, the matrix elements of Y_ν should be much less than unity. Thus, we can solve the RG equation analytically to a good approximation. One–loop correction to the M_N can be found from (2.25) to be

$$\delta M_N^{1-\mathrm{loop}} \simeq -\frac{1}{8\pi^2} \left(M_N Y_\nu^\dagger Y_\nu + Y_\nu^T Y_\nu^* M_N \right)_{\mu = M_G} \ln \frac{M_G}{M} \ . \tag{2.26}$$

From this we see that at scale $\mu = M$ the form of M_N will become

$$M_N = M \begin{pmatrix} -\delta_N & 1 \\ 1 & -\delta_N^* \end{pmatrix} . \tag{2.27}$$

Interestingly enough, this structure, of correlated phases of $(1,1)$ and (2,2) entries of M_N, persists also at two–loop order. What is more important, one can see that at the one–loop level the phase of δ_N is determined by the phase of $(Y_\nu^\dagger Y_\nu)_{12}$ and therefore $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ will be real at this level. This property can be easily seen also from different angle. Regardless of the form of Y_ν (including all possible radiative corrections to it), it can be written in the form

$$Y_\nu = \mathcal{U} \begin{pmatrix} 0 & 0 \\ \hat{a}_2 & 0 \\ \hat{a}_3 & \hat{b}_3 \end{pmatrix} \tilde{P} , \tag{2.28}$$

with $\mathcal{U}$ some unitary matrix, $\hat{a}_{2,3}, \hat{b}_3$ all real parameters and $\tilde{P} = \mathrm{Diag}\left(1, e^{i\xi}\right)$. Using now the form (2.28) in the first line of (2.25), one can show that $\mathcal{U}$ drops out and we remain with the non physical phase ξ which can be absorbed in N_2. With this, any complexity in $\delta M_N^{1-\mathrm{loop}}$ and $(Y_\nu^\dagger Y_\nu)_{12}$ disappears and we have no CP violation at the one–loop level. That is why it is important to include two–loop radiative corrections for the renormalization of M_N. Indeed, the term $M_N Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu$ in the second line of Eq. (2.25) is important. The appearance of the combination $Y_e Y_e^\dagger$ plays an important role. With the basis (2.28) we see that the matrix $\mathcal{U}$ does not disappear, and thus we expect to have CP violation (induced through two–loop correction). From (2.25), this correction can be approximated as follows:

$$\delta M_N^{2-\mathrm{loop}} \simeq \frac{2}{(16\pi^2)^2} \left(M_N Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu + Y_\nu^T Y_e^* Y_e^T Y_\nu^* M_N \right)_{\mu = M_G} R_\ell \ln \frac{M_G}{M} . \tag{2.29}$$

where we have suitably absorbed CP conserving and flavor universal corrections (coming from the entries $\mathrm{Tr}(Y_\nu^\dagger Y_\nu)$, g_i^2, λ_t^2 etc.) in the overall scale M. The RG factor R_ℓ ($\ell_i = (e, \mu, \tau)$) is for the running of Y_e Yukawa couplings and can strongly deviate from one. It is defined as:

$$R_{e,\mu,\tau} = \frac{\int_M^{M_G} \lambda_{e,\mu,\tau}^2(t) dt}{\lambda_{e,\mu,\tau}^2(M_Z) \ln \frac{M_G}{M}} . \tag{2.30}$$

In the approximation (2.29), the fourth powers of Y_ν have been neglected. Actually, for calculating the mass splitting $M_2^2 - M_1^2$ - the combination appearing in (2.24) - it is enough to keep only correction $\delta M_N^{1-\text{loop}}$ of (2.26). However, to deal with the CP violating effect, we need to include also two–loop effects. Thus, at the scale $\mu = M$ for M_N we use

$$M_N(M) = M_N(M_G) + \delta M_N^{1-\text{loop}} + \delta M_N^{2-\text{loop}} \ , \tag{2.31}$$

with $M_N(M_G)$, $\delta M_N^{1-\text{loop}}$ and $\delta M_N^{1-\text{loop}}$ given by Eqs. (2.5), (2.26) and (2.29) respectively. This completes the calculation of supersymmetric part, which will be useful for calculation of leptogenesis via fermionic RHN decays. However, inclusion of soft SUSY breaking terms, in general, may affect the leptogenesis induced through the right–handed sneutrino decays. In an Appendix we studied this case separately and shown that under plausible assumptions the right–handed sneutrino decays practically do not contribute to the net baryon asymmetry. Thus, we should relay on the fermionic RHN decays which, as we show below, generate sufficient baryon asymmetry.

2.3.1 Asymmetry Via Fermionic RHN Decays

Leptogenesis for Normal Hierarchical Case

For this case we will take the form of Y_ν given by Eq. (2.6). For leptogenesis study, it is convenient to parameterize this Yukawa matrix as follows:

$$\text{Texture A :} \qquad Y_\nu = \begin{pmatrix} x\alpha_1 & 0 \\ x\alpha_2 & b \\ xe^{i\phi} & 1 \end{pmatrix} \cdot \bar{\beta} \ , \tag{2.32}$$

where the couplings $\alpha_{1,2}, b, \overline{\beta}$ and x are real parameters. Only single phase ϕ appears. This has been achieved by suitable redefinition of phases of $l_{1,2,3}$ and $N_{1,2}$ superfields. First we will relate the parameters appearing in Y_ν to some observables. The relation (2.10) enables us to express α_1, α_2, b and $\bar{\beta}$ in terms of x, neutrino mass, the scale $|M|$ and lepton mixing matrix elements. Also ϕ can be determined by the phase δ and the leptonic mixing angles. Doing so, we arrive to the following relations

$$\alpha_1 = 2\left|\frac{\mathcal{A}_2}{\mathcal{A}_1}\right| \ , \quad \alpha_2 = \left|\frac{\mathcal{A}_2\mathcal{A}_4}{\mathcal{A}_1\mathcal{A}_3}\right| \ , \quad b = \left|\frac{\mathcal{A}_3}{\mathcal{A}_2}\right| \ , \quad \bar{\beta} = \frac{1}{v\sin\beta}\left(\frac{m_3}{2}\left|\frac{\mathcal{A}_1 M}{x}\right|\right)^{1/2} \ , \tag{2.33}$$

$$\phi = \mathrm{Arg}\left(\frac{\mathcal{A}_2^2\mathcal{A}_4}{\mathcal{A}_1\mathcal{A}_3^2}\right) \ , \tag{2.34}$$

where

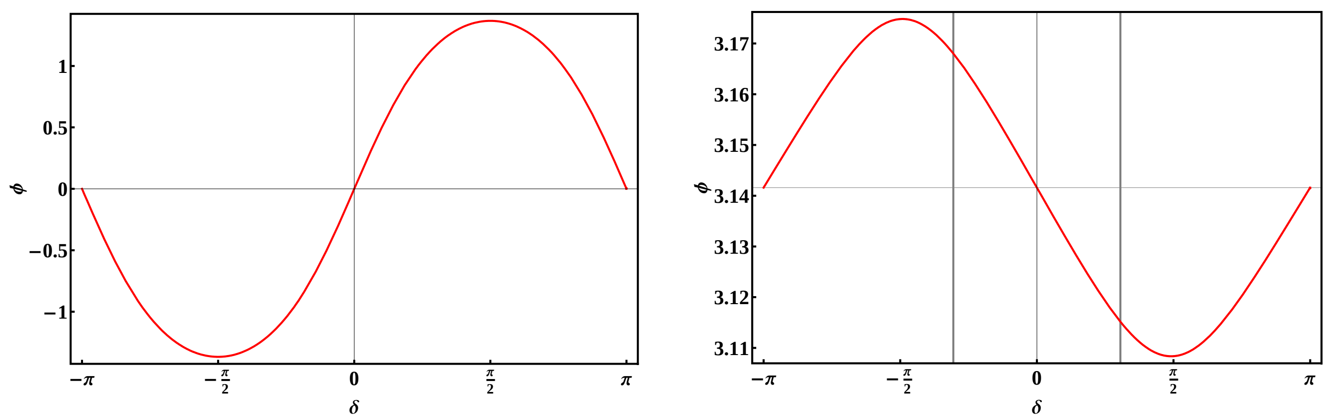

Figure 2.1: Correlation between ϕ and δ. Left side: normal hierarchical case (texture A). Right side: inverted hierarchical case (texture B_1). The vertical lines, for right panel, correspond to the maximal allowed value of $|\delta| = 0.96$.

$$\mathcal{A}_1 = U_{\tau 3}^2 - U_{\tau 2}^2\frac{U_{e3}^2}{U_{e2}^2} \ , \quad \mathcal{A}_2 = U_{e3}U_{\tau 3} - U_{\tau 2}\frac{U_{e3}^2}{U_{e2}} \ , \quad \mathcal{A}_3 = U_{e3}U_{\mu 3} - U_{\mu 2}\frac{U_{e3}^2}{U_{e2}} \ , \quad \mathcal{A}_4 = U_{\mu 3}^2 - U_{\mu 2}^2\frac{U_{e3}^2}{U_{e2}^2} \tag{2.35}$$

These will be useful upon studying the leptogenesis. As we have already mentioned, remarkable thing is the fact that there is a single CP violating phase ϕ which is related to the phase δ controlling the CP violation in the neutrino oscillations. The same phase will appear in the CP asymmetry of the resonant leptogenesis. In Fig. 2.1 we show correlation between ϕ and δ.

Furthermore, applying expressions (2.26), (2.29), for the splitting parameter δ_N of (2.27) we obtain

$$\delta_N \simeq \left(b\alpha_2 + e^{i\phi} - \frac{\lambda_\tau^2 R_\tau}{16\pi^2}e^{i\phi}\right)\frac{x\bar{\beta}^2}{4\pi^2}\ln\frac{M_G}{M} \ , \tag{2.36}$$

where we have ignored the couplings λ_e and λ_μ because the main effect is obtained by the tau Yukawa coupling. In Eq. (2.36), the coupling λ_τ is defined at M_Z scale, and therefore the quantity R_τ accounts for the renormalization effects mostly due to λ_τ running, and is given in Eq. (2.30). Now we can give the unitary matrix U_N diagonalizing M_N (by the transformation $U_N^T M_N U_N = M_N^{\rm Diag}$):

$$U_N \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\eta/2} & -ie^{-i\eta/2} \\ e^{i\eta/2} & ie^{i\eta/2} \end{pmatrix} , \qquad (2.37)$$

where

$$\eta = \mathrm{Arg}\left(b\alpha_2 + e^{i\phi} - \frac{\lambda_\tau^2 R_\tau}{16\pi^2} e^{i\phi} \right) . \qquad (2.38)$$

At the same time we have

$$(Y_\nu^\dagger Y_\nu)_{21} = \bar{\beta}^2 x \left| b\alpha_2 + e^{i\phi} \right| e^{i\eta'} , \qquad \text{with} \qquad \eta' = \mathrm{Arg}\left(b\alpha_2 + e^{i\phi} \right) . \qquad (2.39)$$

Therefore, the complex phase appearing in $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}$ will be proportional to the mismatch $\eta - \eta'$, which using (2.38) and (2.39) takes the form

$$\eta - \eta' \simeq -\frac{\lambda_\tau^2 R_\tau}{16\pi^2} \frac{b\alpha_2}{|b\alpha_2 + e^{i\phi}|^2} \sin\phi . \qquad (2.40)$$

Note once again that the phase $\eta - \eta'$, determining the lepton asymmetry, is proportional to $\sin\phi$, which itself is related to the phase δ of the lepton mixing matrix. The model gives the relation between them by Eq. (2.34). Also, it is rather impressive that other parameters, b and α_2, appearing in (2.40) can be calculated by the lepton mixing matrix elements through the relations (2.33), (2.35).

The masses of two right handed neutrinos are

$$M_1 = |M|(1-\kappa) , \quad M_2 = |M|(1+\kappa) , \quad \text{with} \quad \kappa \simeq \left| x(b\alpha_2 + e^{i\phi}) \right| \frac{\bar{\beta}^2}{4\pi^2} \ln\frac{M_G}{M} . \qquad (2.41)$$

Here, the unknown parameter x appears which is free and can be varied upon numerical calculations. Finally, we give expressions build from the elements of the matrix $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)$ appearing in the expressions of the CP asymmetries of Eq. (2.24). These are:

$$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} \simeq \frac{\bar{\beta}^2}{2} \left(x^2(1+\alpha_1^2+\alpha_2^2) + 1 + b^2 + 2x|b\alpha_2 + e^{i\phi}| \right) ,$$

$$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} \simeq \frac{\bar{\beta}^2}{2} \left(x^2(1+\alpha_1^2+\alpha_2^2) + 1 + b^2 - 2x|b\alpha_2 + e^{i\phi}| \right) ,$$

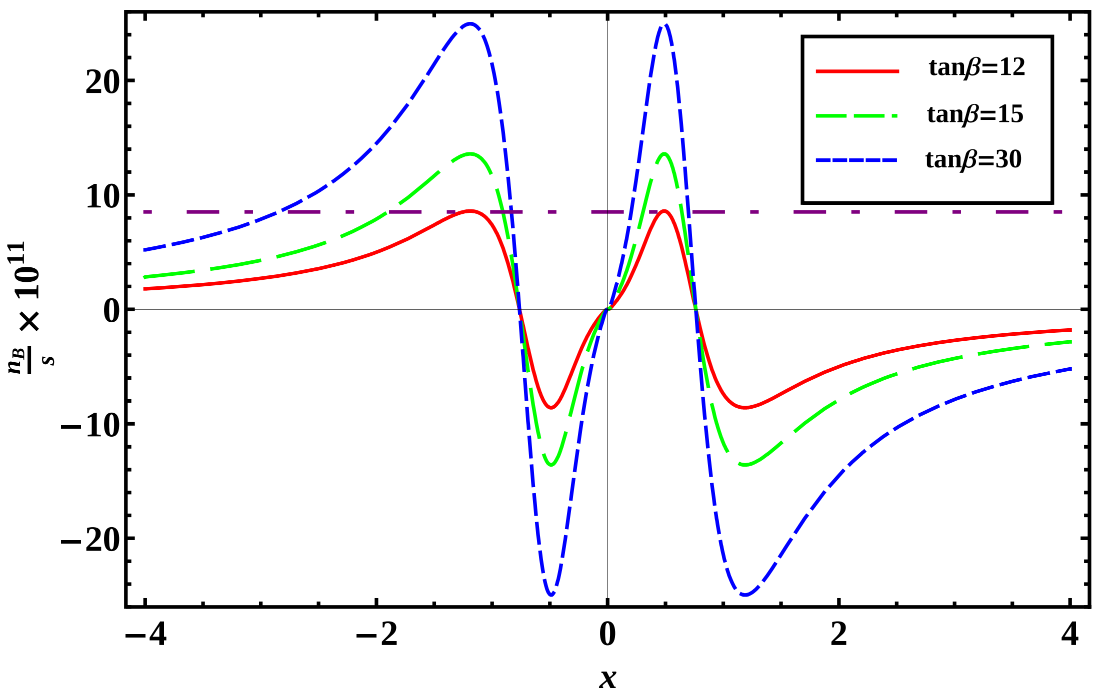

Figure 2.2: Baryon asymmetry for normal hierarchical case (texture A), for different values of $\tan\beta$ and $M = 10^7$ GeV, $\delta = 1.3$.

$$\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{21} \simeq \frac{\lambda_\tau^2 R_\tau}{16\pi^2}\bar{\beta}^4 x b \alpha_2 \left(x^2(1+\alpha_1^2+\alpha_2^2) - 1 - b^2\right)\frac{\sin\phi}{|b\alpha_2 + e^{i\phi}|} \; . \tag{2.42}$$

In order to compute generated baryon asymmetry of the Universe, recall that the lepton asymmetry is converted to the baryon asymmetry via sphaleron processes [20] and is given by $\frac{n_B}{s} \simeq -1.48 \times 10^{-3}({\kappa_f}^{(1)}\epsilon_1 + {\kappa_f}^{(2)}\epsilon_2)$, where the efficiency factors ${\kappa_f}^{(1,2)}$ are given by the extrapolating formula [21]:

$${\kappa_f}^{(1,2)} = \left(\frac{3.3\times 10^{-3}\mathrm{eV}}{\tilde{m}_{1,2}} + \left(\frac{\tilde{m}_{1,2}}{0.55\times 10^{-3}\mathrm{eV}}\right)^{1.16}\right)^{-1} ,$$

$$\text{with } \; \tilde{m}_1 = \frac{(v\sin\beta)^2}{M_1}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} \; , \quad \tilde{m}_2 = \frac{(v\sin\beta)^2}{M_2}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} \; . \tag{2.43}$$

Collecting all this, we can now calculate $\frac{n_B}{s}$. One can try the different values of M in a mass range which would not cause the gravitino problem. We can also try different values of the phase δ, relevant also for the CP violation into the neutrino oscillations. As we have already mentioned, there is one more free parameter x, which we will vary. It is quite interesting that this system, by requiring to have baryon asymmetry in the range of the observed amount $\left(\frac{n_B}{s}\right)_{\rm exp} = (8.75 \pm 0.23)\cdot 10^{-11}$, dictates the preferred range for the MSSM parameter $\tan\beta$. The reason for this is simple. The strength of the Yukawa coupling λ_τ, determining the amount of the CP violation [see Eq. (2.40)], depends on the value of $\tan\beta$: $\lambda_\tau = \frac{m_\tau}{v}\sqrt{1+\tan^2\beta}$. By simple but quite complete numerical simulation we obtain, in this model, the low bound on the $\tan\beta$. Upon calculations we take into account the renormalization effects. Namely, the running of λ_τ. Obtained low bound for $\tan\beta$ is: $\tan\beta \gtrsim 12$ (corresponds to $|\delta| \simeq 1.3$ and $M = 10^7$ GeV, $R_\tau = 0.617$). Smaller values of $\tan\beta$ do not give sufficient baryon asymmetry. This also indicates that the non SUSY version (i.e. SM augmented by two RHNs) of this scenario will fail to generate baryon asymmetry through the leptogenesis. The presented scenario also allows to derive the low bound for the absolute value of the phase δ. This comes out from the maximal allowed value of $\tan\beta \lesssim 58$ (from the requirement that $\lambda_{b,\tau} \lesssim 1$ all the way up to the GUT scale). With $\tan\beta = 58$, $M = 10^7$ GeV ($R_\tau = 2.17$) in order to have needed baryon asymmetry we should have $|\delta| \gtrsim 0.012$. It is interesting to note that for $\tan\beta \lesssim 35$, for generating the baryon asymmetry we need $|\delta| \gtrsim 0.1$. This limit for the CP violating phase is within the reach of future experiments. In Figs. 2.2 and 2.3 we plot $\frac{n_B}{s}$ for different choices of the model parameters.

Leptogenesis for Inverted Hierarchical Case

Now we study the leptogenesis for the inverted hierarchical case. We note right away that the scenario with texture B_2 of (2.7) does not work for the leptogenesis. The reason is following. Due to the zero in $(3,2)$ entry of this texture, it is easy to see from Eq. (2.29) that the λ_τ coupling do not contribute to the CP asymmetry induced at 2-loop level. The couplings λ_e and λ_μ do contribute, however they are small and can not induce needed asymmetry.

Thus, we focus here only on case with texture B_1. For this case, it is convenient to write Y_ν with the parameterization

$$\text{Texture B}_1: \quad Y_\nu = \begin{pmatrix} x\alpha_1 & b \\ x\alpha_2 & 0 \\ xe^{i\phi} & 1 \end{pmatrix} \cdot \bar{\beta}\ , \tag{2.44}$$

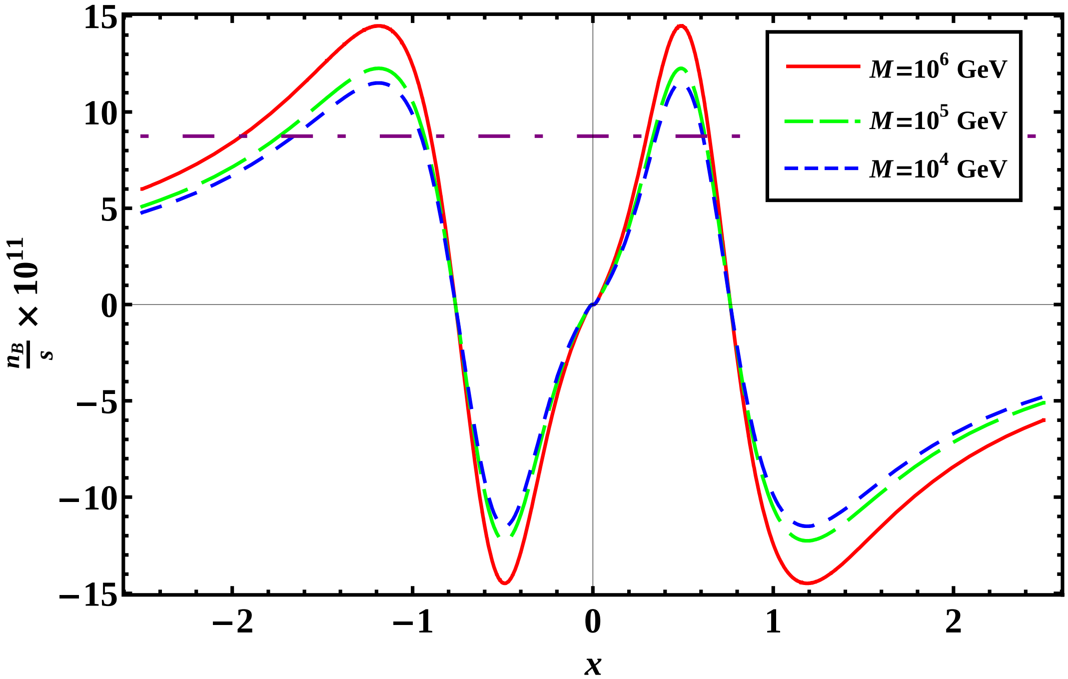

Figure 2.3: Baryon asymmetry for normal hierarchical case (texture A), for different values of M and $\tan\beta = 15$, $\delta = 1.3$.

where, as in case of texture A, by suitable phase redefinition of $l_{1,2,3}, N_{1,2}$ superfields we left with only single phase ϕ. Remaining parameters are real. First we will express the model parameters $\alpha_{1,2}, b, \bar{\beta}$ in terms of matrix elements of U, neutrino mass m_2, the M, and x. By solving the equations derived from the relation (2.10) we obtain

$$\text{For Texture B}_1\ :\quad \alpha_1 = \left|\frac{\mathcal{B}_2\mathcal{B}_4}{\mathcal{B}_1\mathcal{B}_3}\right|\ ,\quad \alpha_2 = 2\left|\frac{\mathcal{B}_2}{\mathcal{B}_1}\right|\ ,\quad b = \left|\frac{\mathcal{B}_3}{\mathcal{B}_2}\right|\ ,\quad \bar{\beta} = \frac{1}{v\sin\beta}\left(\frac{m_2}{2}\left|\frac{\mathcal{B}_1 M}{x}\right|\right)^{1/2}\ ,$$

$$\phi = \text{Arg}\left(\frac{\mathcal{B}_2^2\mathcal{B}_4}{\mathcal{B}_1\mathcal{B}_3^2}\right)\ , \tag{2.45}$$

where

$$\mathcal{B}_1 = U_{\tau 2}^2 - U_{\tau 1}^2\frac{U_{\mu 2}^2}{U_{\mu 1}^2}\ ,\quad \mathcal{B}_2 = U_{\mu 2}U_{\tau 2} - U_{\tau 1}\frac{U_{\mu 2}^2}{U_{\mu 1}}\ ,\quad \mathcal{B}_3 = U_{e2}U_{\mu 2} - U_{e1}\frac{U_{\mu 2}^2}{U_{\mu 1}}\ ,\quad \mathcal{B}_4 = U_{e2}^2 - U_{e1}^2\frac{U_{\mu 2}^2}{U_{\mu 1}^2} \tag{2.46}$$

As we see, also in this case the phase ϕ is related to the δ-phase of the leptonic mixing matrix U (see Eq. (2.11)). In particular using the relation (2.17) in (2.45) for ϕ and performing simple algebra we derive

$$\phi \simeq \text{Arg}\left(\frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|}\frac{e^{i\delta}}{4s_{13}\tan\theta_{23}} - 1\right) \quad\Longrightarrow\quad \phi \simeq \pi - \frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|}\frac{\cot\theta_{23}}{4\sin\theta_{13}}\sin\delta\ . \tag{2.47}$$

Since the phase ϕ will appear in the leptonic CP asymmetry, with relation (2.47) we will be able to make calculations in terms of measured neutrino oscillation parameters and the CP phase δ. In Fig. 2.1 the correlation between ϕ and δ is shown.

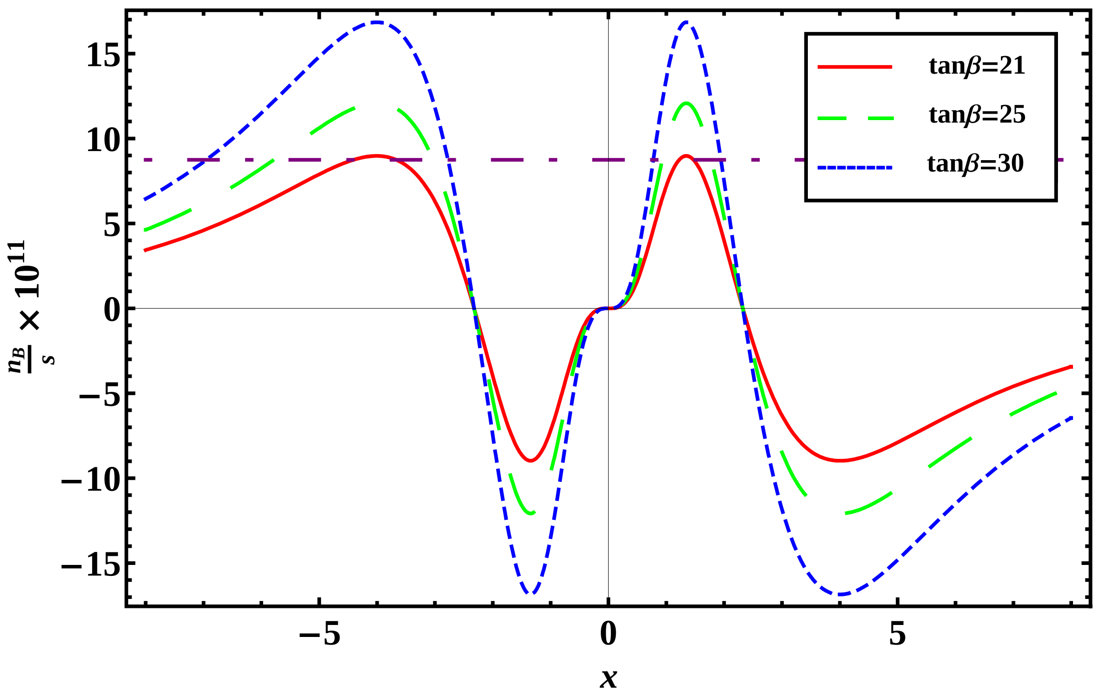

Figure 2.4: Baryon asymmetry for inverted hierarchical case (texture B_1), for different values of $\tan\beta$ and $M = 10^4$ GeV, $\delta = 0.96$.

Now we are ready to investigate the leptogenesis for the inverted hierarchical scenario (B_1). The way of calculation is same as was presented in the previous subsection, so we will keep discussion short and give only several expressions and final results. Using the expressions of Eqs. (2.26)-(2.31) and the form of the texture B_1 in (2.44), for the phase mismatch (in analogy of Eq. (2.40) we obtain

$$(\eta - \eta')^{(B_1)} \simeq -\frac{\lambda_\tau^2 R_\tau}{16\pi^2}\frac{b\alpha_1}{|b\alpha_1 + e^{i\phi}|^2}\sin\phi\ , \tag{2.48}$$

where here and below we will use superscript '(B_1)' in order to distinguish expressions corresponding to the scenario B_1 from those of the texture A. Moreover, for the splitting parameter (in analog to

Eq. (2.41)) we have

$$\kappa^{(B_1)} \simeq \left|x(b\alpha_1+e^{i\phi})\right| \frac{\bar{\beta}^2}{4\pi^2}\ln\frac{M_G}{M} \ . \tag{2.49}$$

We will also give the expression for $\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2$ which will help to understand some physics. We have

$$\left(\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2\right)^{(B_1)} \simeq \frac{\lambda_\tau^2 R_\tau}{16\pi^2}\bar{\beta}^4 x b\alpha_1 \left(x^2(1+\alpha_1^2+\alpha_2^2)-1-b^2\right)\frac{\sin\phi}{|b\alpha_1+e^{i\phi}|} \ . \tag{2.50}$$

Note, that according to (2.47) the phase ϕ is close to π and one may suspect that also final result for the CP violation should be suppressed by the factor$\sim \Delta m_{\mathrm{sol}}^2/(|\Delta m_{\mathrm{atm}}^2| 4\sin\theta_{13}) \approx 1/20$. However, such suppression do not takes place because the combination $|b\alpha_1+e^{i\phi}|$, appearing in the denominator of the last multiplier of (2.50), is suppressed by precisely same factor! Indeed, using the relations of Eqs. (2.17), (2.45)-(2.47) we derive

$$\left|b\alpha_1+e^{i\phi}\right| \simeq \frac{\Delta m_{\mathrm{sol}}^2}{|\Delta m_{\mathrm{atm}}^2|}\frac{\cot\theta_{23}}{4\sin\theta_{13}} \ . \tag{2.51}$$

With these for the combination appearing in (2.50) we get

$$\frac{\sin\phi}{|b\alpha_1+e^{i\phi}|} \simeq \sin\delta, \tag{2.52}$$

showing that suppression factors mentioned above drop out and it is maximized with $|\delta| \simeq 1.115$ (maximal allowed value which is acceptable for viable neutrino sector). Moreover, because of the suppression of the combination $\left|b\alpha_1+e^{i\phi}\right|$, also the RHN mass splitting parameter in (2.49) gets additional suppression, which makes two RHNs more degenerate. This also gives some enhancement of the (resonant) CP asymmetry factors $\epsilon_{1,2}$.

Without bothering to give other expressions, we will move to the presentation of the main results. In this scenario, from the requirement of needed baryon asymmetry, the $\tan\beta$ is bounded from below. Interesting thing is that the leptogenesis dictates $\tan\beta \gtrsim 21$ (lower values do not give sufficient baryon asymmetry) For obtaining this low bound we have taken $M = 10^4$ GeV ($R_\tau \simeq 0.71$) and maximal allowed value for the $\delta \simeq 0.96$. Note that within this scenario low values of M give larger lepton asymmetries. It is also possible to derive low bound for $|\delta|$. This is obtained by largest allowed (from requirement $\lambda_{b\tau} \lesssim 1$ up to the GUT scale) value of $\tan\beta$. Namely, with $\tan\beta \simeq 58$, $M = 10^4$ GeV (we

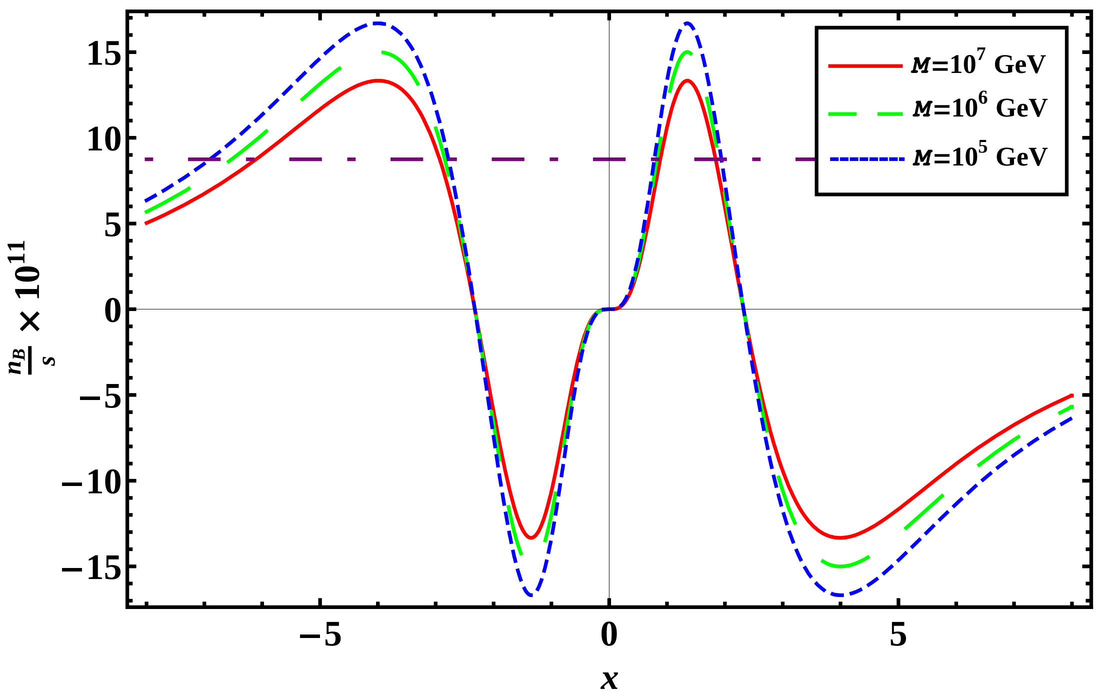

Figure 2.5: Baryon asymmetry for inverted hierarchical case (texture B_1), for different values of M and $\tan\beta = 30$, $\delta = 0.96$.

have for these choices $R_\tau \simeq 1.95$), needed baryon asymmetry can be generated with $|\delta| \gtrsim 0.021$ (note that for $|\delta| = 0.021$, for acceptable solar mixing angle we should choose $\sin^2\theta_{23} \simeq 0.6$ and $\sin^2\theta_{13} \simeq 0.04$). Worthwhile for noting that for $\tan\beta \lesssim 45$ for generating sufficient baryon asymmetry we need $|\delta| \gtrsim 0.1$. The latter value is within the reach of planned experiments. We have performed numerical calculations without approximations and made sure that our analytical expressions, presented above, are good approximations. In Figs. 2.4 and 2.5 we show baryon asymmetries for several different choices of the model parameters.

2.4 Summary

In this paper we have considered an extension of MSSM with two quasi-degenerate right–handed neutrinos. Our motivation was to realize resonant leptogenesis which avoids the gravitino problem generic for low scale SUSY scenarios. With this setup we have classified all viable texture zeros of the neutrino Dirac Yukawa matrices which lead to consitent predictions. We find three predictive scenarios, each with one texture zero. One model has normal hierarchical neutrino mass spectrum, while the remaining two have inverted hierarchical mass pattern. The predictive power of these models show up also in the resonant leptogenesis. The model with the normal mass hierarchy (texture A) and one of the inverted hierarchical scenario (with texture B_1) lead to the successful leptogenesis. In Appendix we have discussed the impact of the soft SUSY breaking terms on the CP asymmetry generated by RH sneutrino decays and concluded that with natural choice of the soft SUSY breaking terms, scalar RH neutrinos do not contribute sizably to the total baryon asymmetry. Thus, the baryon asymmetry is due to fermionic RHN decays and the leptonic CP phase is directly related to to CP violation in neutrino oscillation. Putting together the predictions from the neutrino sector and the results from leptogenesis calculations, we have obtained the following predictions:

For normal hierarchical case (texture A)

$$\sin^2\theta_{13} \gtrsim 0.05\ , \qquad |\delta| \gtrsim 0.012\ , \qquad m_{\beta\beta} = 0\ , \qquad \tan\beta \gtrsim 12\ ;$$

$$\text{with} \qquad \tan\beta \lesssim 35\ , \qquad |\delta| \gtrsim 0.1\ .$$

For the inverted hierarchical case corresponding to texture B_1:

$$\theta_{13} \gtrsim 0.12\ , \quad 0.021 \lesssim |\delta| \lesssim 0.96\ , \quad 0.013\ \text{eV} \lesssim m_{\beta\beta} \lesssim 0.023\ \text{eV}\ , \quad \tan\beta \gtrsim 21\ ;$$

$$\text{with} \qquad \tan\beta \lesssim 45\ , \qquad |\delta| \gtrsim 0.1\ .$$

The texture B_2 do not generate the baryon asymmetry within this scenario and other mechanism need to be invoked [38]. However, from the viewpoint of the neutrino sector the texture B_2 is viable and gives:

$$\theta_{13} \gtrsim 0.129\ , \qquad |\pi - \delta| \lesssim 0.91\ , \quad 0.013\ \text{eV} \lesssim m_{\beta\beta} \lesssim 0.023\ \text{eV}\ .$$

Future experiments will examine the viability of these scenarios.

Appendix: Asymmetry Via $\tilde{N}$ Decays

In this appendix we will discuss the contribution to the net baryon asymmetry from the out of equilibrium resonant decays of the right handed sneutrinos (RHS). With inclusion of the soft SUSY breaking terms, the RHS mass spectrum and couplings will be altered and one should expect result different from that corresponding to the fermionic RHN decays. Besides soft SUSY breaking couplings, there are other particularities, highlighted below, which distinguish cases of RHN and RHS decays. We are considering the system with two RHN superfields $N_{1,2}$ which have two complex scalar components $\tilde{N}_{1,2}$. With SUSY breaking term, the masses of RHS's will differ from their fermionic partners' masses. Thus we will have four real mass-eigenstate RHS's $\tilde{n}_{i=1,2,3,4}$ with masses $\tilde{M}_{i=1,2,3,4}$ respectively. Assuming that the SUSY scale is smaller (at least by factor of 10) than the scale M (the overall tree level mass for the RHN superfields) we expect that the states $\tilde{n}_i$ remain quasi-degenerate. To study the resonant $\tilde{n}$-decays we will apply ressumed effective amplitude technic [27]. An effective amplitudes for the real $\tilde{n}_i$ decay, say into the lepton l_α ($\alpha = 1, 2, 3$ is a generation index) and antilepton $\bar{l}_\alpha$ respectively are given by [27]

$$\hat{S}_{\alpha i} = S_{\alpha i} - \sum_j S_{\alpha j} \frac{\Pi_{ji}(\tilde{M}_i)(1-\delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \ , \quad \hat{\bar{S}}_{\alpha i} = S_{\alpha i}^* - \sum_j S_{\alpha j}^* \frac{\Pi_{ji}(\tilde{M}_i)(1-\delta_{ij})}{\tilde{M}_i^2 - \tilde{M}_j^2 + \Pi_{jj}(\tilde{M}_i)} \ , \qquad (2.53)$$

where $S_{\alpha i}$ is a tree level amplitude and Π_{ij} is a two point Green function's (polarization operator of $\tilde{n}_i - \tilde{n}_j$) absorptive part. The CP asymmetry is then given by

$$\epsilon_i^{sc} = \frac{\sum_\alpha \left(|\hat{S}_{\alpha i}|^2 - |\hat{\bar{S}}_{\alpha i}|^2 \right)}{\sum_\alpha \left(|\hat{S}_{\alpha i}|^2 + |\hat{\bar{S}}_{\alpha i}|^2 \right)} \ . \qquad (2.54)$$

We will apply (2.53) and (2.54) for our scenario, however, also derive general expressions applicable for different models.

Toegether with superpotential couplings (2.3) we include the following soft SUSY breaking terms

$$V_{\rm SB}^{\nu} = \tilde{l} A_\nu \tilde{N} h_u - \frac{1}{2} \tilde{N}^T B_N \tilde{N} + {\rm h.c.} \qquad (2.55)$$

We do not display here soft mass^2 terms, such as $\tilde{m}_{1,2}^2 |\tilde{N}_{1,2}|^2$, because B_N plays much more significant role in the splitting of RHS masses. For simplicity we will assume at GUT scale (M_G) the 'proportionality' $A_\nu \propto Y_\nu$ and degeneracy in $B_N \propto M_N$. Thus,

$$\text{at} \ \ \mu = M_G : \qquad A_\nu = m_A Y_\nu \ , \qquad B_N = m_B M_N \ . \tag{2.56}$$

Similarly, for the charged lepton sector we can assume $A_e = m_E Y_e$. Performing RG studies, similar way as we have done in section 2.3, we will have

$$\text{at} \ \ \mu = M : \qquad B_N \simeq m_B M \begin{pmatrix} -(1+2\frac{m_A}{m_B})\delta_N & 1 \\ 1 & -(1+2\frac{m_A}{m_B})\delta_N^* \end{pmatrix} \ . \tag{2.57}$$

Note that with $A_\nu = m_A Y_\nu$ and $A_e = m_E Y_e$ at high scale, the A_ν will remain well aligned with Y_ν also at low scales. With diagonalization of total mass matrix of the RHS's, for mass-eigenstate ($\tilde{n}_i$) masses we get

$$\tilde{M}_1^2 = |M|^2(1-|\delta_N|)^2 - |M||m_B - (m_B+2m_A)|\delta_N|| \ ,$$

$$\tilde{M}_2^2 = |M|^2(1-|\delta_N|)^2 + |M||m_B - (m_B+2m_A)|\delta_N|| \ ,$$

$$\tilde{M}_3^2 = |M|^2(1+|\delta_N|)^2 - |M||m_B + (m_B+2m_A)|\delta_N|| \ ,$$

$$\tilde{M}_4^2 = |M|^2(1+|\delta_N|)^2 + |M||m_B + (m_B+2m_A)|\delta_N|| \ . \tag{2.58}$$

Interaction of $\tilde{n}$ states with leptons and sleptons has the form

$$\tilde{h}_u l Y_F \tilde{n} + h_u \tilde{l} Y_B \tilde{n} + \text{h.c.} \tag{2.59}$$

where

$$Y_F = Y_\nu \tilde{V} \ , \quad Y_B = Y_\nu M_N^* \tilde{V}^* + A_\nu \tilde{V} \ , \text{where} \quad \tilde{V} = U_N \left(\rho_u e^{i\tilde{\theta}}, \ \rho_d \right) \ ,$$

$$\text{with} \quad \rho_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \ , \quad \rho_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 1 & i \end{pmatrix} \ , \quad \tilde{\theta} \simeq 2\text{Im}\left(\frac{m_A}{m_B}\right)|\delta_N| \ . \tag{2.60}$$

With these we can calculate the absorptive part of the polarization diagram with external legs $\tilde{n}_i$ and $\tilde{n}_j$. At 1-loop level it is given by

$$\Pi_{ij}(p) = \frac{i}{8\pi}\left(p^2 Y_F^\dagger Y_F + p^2 Y_F^T Y_F^* + Y_B^\dagger Y_B + Y_B^T Y_B^*\right)_{ij} \ , \tag{2.61}$$

where p denotes external momentum in the diagram.

Now we are ready to calculate the lepton asymmetry. Note that in unbroken SUSY limit, neglecting finite temperature effects ($T \to 0$), the $\tilde{N}$ decay does not produce lepton asymmetry. The reason for this is following. The decay of $\tilde{N}$ in two fermion is $\tilde{N} \to l\tilde{h}_u$, while in two scalars is $\tilde{N} \to \tilde{l}^* h_u^*$. Since the rates of these processes are same due to SUSY (at $T = 0$), the lepton asymmetries created from these decays cancel each other. However, with $T \neq 0$ the cancelation is partial and one has

$$\tilde{\epsilon}_i = \epsilon_i(\tilde{n}_i \to l\tilde{h}_u)\Delta_{BF} \ , \tag{2.62}$$

with temperature dependent factor Δ_{BF} given in [37]. We note that Eq. (3.35) is valid when we have the alignment $A_\nu = m_A Y_\nu$. Without this alignment other terms in r.h.s of (3.35) proportional to m_A/M will appear. Since we are assuming the alignment and $m_A/M \lesssim 0.1$, the SUSY breaking effects would not affect decay amplitudes significantly and we can apply (3.35) for our study. Thus, we just need to compute $\epsilon_i(\tilde{n}_i \to l\tilde{h}_u)$ - the asymmetry created by $\tilde{n}_i$ decays in two fermions. Using in (2.53) $S_{\alpha i} = (Y_F)_{\alpha i}$, with (2.54) after straightforward calculation we obtain

$$\epsilon_i(\tilde{n}_i \to l\tilde{h}_u) \simeq \frac{1}{(Y_F^\dagger Y_F)_{ii}} \left\{ 2\sum_j \frac{(\tilde{M}_i^2 - \tilde{M}_j^2)\mathrm{Im}(\Pi_{ji})}{(\tilde{M}_i^2 - \tilde{M}_j^2)^2 + |\Pi_{jj}|^2} \mathrm{Im}(Y_F^\dagger Y_F)_{ji} + \right.$$

$$\left. \sum_{j,k} \mathrm{Im}(Y_F^\dagger Y_F)_{kj} \frac{(\tilde{M}_j^2 - \tilde{M}_i^2)\mathrm{Im}(\Pi_{kk}) - (\tilde{M}_k^2 - \tilde{M}_i^2)\mathrm{Im}(\Pi_{jj})}{\left((\tilde{M}_i^2 - \tilde{M}_j^2)^2 + |\Pi_{jj}|^2\right)\left((\tilde{M}_i^2 - \tilde{M}_k^2)^2 + |\Pi_{kk}|^2\right)} \Pi_{ji}\Pi_{ki} \right\} \ . \tag{2.63}$$

In (2.63) for the absorptive part Π we should use (2.61) with $p = \tilde{M}_i$. Now, the baryon asymmetry created from the lepton asymmetry due to $\tilde{n}$ decays is:

$$\frac{\tilde{n}_B}{s} \simeq -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \frac{\tilde{\epsilon}_i}{\Delta_{BF}} \eta_i = -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \epsilon_i(\tilde{n}_i \to l\tilde{h}_u)\eta_i \ , \tag{2.64}$$

where we have taken into account that an effective number of degrees of freedom, including two RHN superfields, is $g_* = 228.75$. η_i are an efficiency factors which depend on $\tilde{m}_i \simeq \frac{(v\sin\beta)^2}{M} 2(Y_F^\dagger Y_F)_{ii}$, and take into account temperature effects by integrating the Boltzmann equations [37]. Before discussing this in more details, it is more instructive to see what are the effects of the soft SUSY breaking terms in the CP asymmetry given by Eq. (2.63). The parameter ϵ_i is controlled by the imaginary parts of the elements of the matrix $Y_F^\dagger Y_F$. First note that the phase $\tilde{\theta}$ appearing in this matrix (see Eq. (2.60)) for $M \lesssim 10^7$ GeV is $\tilde{\theta} \lesssim 10^{-10}$ and can be safely ignored. With this, the matrix $Y_F^\dagger Y_F$ has the form

$$Y_F^\dagger Y_F = \begin{pmatrix} \hat{\sigma}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} & \hat{\sigma}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12} \\ \hat{\sigma}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21} & \hat{\sigma}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} \end{pmatrix} , \quad \text{with} \quad \hat{\sigma} = \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} , \tag{2.65}$$

where $\hat{Y}_\nu = Y_\nu U_N$ is the same matrix appearing in the CP asymmetries (2.24) induced by fermionic RHN decays. Note that the matrix $\hat{\sigma}$ has purely imaginary entries and they can be new sources for the CP violation. For instance, the element $(Y_F^\dagger Y_F)_{12}$ has the large phase. This means that there happens the 'conversion' between $\tilde{n}_1$ and $\tilde{n}_2$ states. On the other hand, from Eq. (2.58) one can see that the degeneracy of $\tilde{M}_1^2$ and $\tilde{M}_2^2$ is split by the B-term and unless $m_B \lesssim 10$ MeV the resonant enhancement does not happen (similar to the case of soft leptogenesis [37]). Since the natural value of m_B is from few$\times$100 GeV to few TeV, we conclude that this channel does not give important contribution to the CP asymmetry. For those states amongst which degeneracy is not ruined (the 'pairs' $\tilde{n}_1 - \tilde{n}_3$ and $\tilde{n}_2 - \tilde{n}_4$) by the B-terms, the CP asymmetry is controlled not by imaginary components of $\hat{\sigma}$ but by $\text{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ (like to those corresponding to the fermionic RHN decays, Eq. (2.24)). Thus, the CP asymmetry via $\tilde{n}_i$ decays would not be larger than asymmetry generated due to their fermionic partners. Moreover, due to the efficiency factors η_i, the $\tilde{n}_B/s$ turns out to get additional suppression in comparison to the n_B/s (the total baryon asymmetry due fermionic RHNs). We have checked this on two examples corresponding to the textures of A and B_1. Namely, we have performed calculations for $(m_A,\ m_B) = (10^3 i,\ 10^3)$ GeV and for several choice of model parameters $(\tan\beta, M, \delta)$. For a given set of these parameters, for fixed x we can calculate the values of the masses $\tilde{m}_i = \frac{(w \sin\beta)^2}{M} 2(Y_F^\dagger Y_F)_{ii}$. With given values of $\tilde{m}_i$, according to Ref. [37] we picked up the corresponding values of η_i and with help of Eqs. (2.63), (3.36) calculated $\tilde{n}_B/s$. For the texture A we obtained $\frac{\tilde{n}_B}{n_B} < 4 \cdot 10^{-3}$, while for the texture B_1: $\frac{\tilde{n}_B}{n_B} < 10^{-2}$. These confirm that the baryon asymmetry via $\tilde{n}$ decays is a negligible effect. For completeness we also examined the case corresponding to texture B_2. The latter does not give relevant asymmetry also through $\tilde{n}$ decays.

CHAPTER 3

NEW WAYS TO LEPTOGENESIS WITH GAUGED $B - L$ SYMMETRY

3.1 Introduction

Baryon number minus lepton number ($B - L$) is a non–anomalous symmetry in the standard model. There is a perception that all non–anomalous symmetries may have a gauge origin. $B - L$ may then be a true gauge symmetry broken spontaneously at a high energy scale. Such a scenario fits well with the small neutrino masses observed in experiments. This is because gauging of $B - L$ requires the introduction of right–handed neutrinos N_i, one per family, for canceling the triangle anomaly associated with $[U(1)_{B-L}]^3$. These N_i fields facilitate the seesaw mechanism [56] to generate small neutrino masses. In this context one is able to relate the mass of the heavy right–handed neutrino to the scale of $B - L$ symmetry breaking. With just the standard model gauge symmetry the right–handed neutrinos are not compelling, and even if they are introduced, their bare Majorana masses are not protected and can take values as large as the Planck mass.

In the supersymmetric context there is yet another motivation for gauging $B - L$. It would lead to a natural understanding of R–parity [57,58]. This can be seen by writing the R–parity transformation as $R = (-1)^{3(B-L)+2S}$, which clearly shows the close relation between R parity and $B - L$. If the $B - L$ gauge symmetry is broken by Higgs fields carrying even number of $B - L$ charge, then a discrete Z_2 symmetry will remain unbroken, which would serve as R–parity. Such Higgs fields are just the ones needed for generating large Majorana neutrino masses for the right–handed neutrinos, which requires $B - L$ breaking by two units. R–parity is usually assumed in MSSM as an ad hoc symmetry, in order to avoid rapid proton decay and to identify the lightest SUSY particle as the cosmological dark matter. These are natural consequences of gauged $B - L$ symmetry. This symmetry also fits inside of $SO(10)$ grand unification, which is very well motivated because of the unification of quarks and leptons of a family into a single multiplet. It is well known that with or without supersymmetry, existence of right–handed neutrinos can explain the observed excess of baryons over antibaryons in the universe via leptogenesis [59]. The N field decays into leptons, generating an asymmetry in lepton number, which is converted to baryon asymmetry by electro-weak sphalerons [60]. (For reviews on leptogenesis see [61, 62].)

In this paper we investigate baryogenesis via leptogenesis in supersymmetric models with gauged $B-L$. We have identified a new source for leptogenesis in this context. The symmetry breaking sector that spontaneously breaks $B-L$ symmetry produces an excess of $\tilde{N}$ over $\tilde{N}^*$ in their decays, where $\tilde{N}$ stands for the scalar partner of the right-handed neutrino N. This asymmetry in $\tilde{N}$ is converted into ordinary lepton asymmetry when the $\tilde{N}$ decays into leptons and Higgs bosons. The electro-weak sphalerons convert this lepton asymmetry into baryon asymmetry.

In this scenario, one realizes resonant [63–65] and soft leptogenesis [66, 67]. Resonant leptogenesis assumes nearly degenerate states (fermions or scalars) that decay into leptons producing an asymmetry. Usually the needed degeneracy is achieved by postulating additional symmetries. In our context, supersymmetry guarantees near degeneracy of the Higgs states. This comes about since in the SUSY limit, the Higgs scalars responsible for $B-L$ symmetry breaking form partners of a Dirac fermion, leading to two complex (or four real) degenerate scalar states. Once SUSY breaking is turned on, this degeneracy is lifted, but by terms that are suppressed by a factor $M_{\rm susy}/M_\Delta$, where M_Δ denotes the mass of the decaying heavy Higgs particle. In the simplest model with gauged $B-L$ symmetry, CP violation needed for leptogenesis is provided by soft SUSY breaking effects. Thus the model realizes soft leptogenesis. We compute the baryon asymmetry generated through this $\tilde{N}$ asymmetry in a simple model with gauged $B-L$ symmetry. As in soft leptogeneis, we find that for a range of soft SUSY breaking parameters, reasonable values of baryon asymmetry can be generated. This mechanism works well when the mass of the decaying Higgs filed is less than about 10^8 GeV. The Davidson–Ibarra bound [68], which requires the decaying right–handed neutrino to be heavier than 10^9 GeV in conventional leptogeneis, is evaded in our framework because the source of CP violation resides in SUSY breaking couplings. Such a bound causes a problem with gravitino abundance [69, 70], which requires the reheat temperature after inflation to be $T_R < 10^7$ GeV. Our scenario does not have gravitino problem, since the mass of the heavy Higgs particle is $< 10^8$ GeV. Some of the soft SUSY parameters have to take unusually small values, a situation common with soft leptogenesis, although the parameters that are small in our models are different ones, associated with $B-L$ symmetry breaking.

We present the minimal gauged SUSY model in Sec. 2, work out the spectrum of the model after SUSY breaking in Sec. 3, and compute the cosmological lepton asymmetry in Sec. 4.

3.2 Minimal Supersymmetric Gauged $B-L$ Model

The minimal supersymmetric model with gauged $B-L$ symmetry extends the gauge group of MSSM to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The triangle anomaly associated with $[U(1)_{B-L}]^3$ is canceled by contributions from right–handed neutrinos N_i, which must exist, one per family. Since the N_i fields should be much heavier than the weak scale in order for the seesaw mechanism for small neutrino masses to be effective, we assume that $B-L$ symmetry is broken in the SUSY limit. The simplest set of scalar superfields that would achieve this – if one insists, as we do, on renormalizable coulings – is $\{\overline{\Delta},\ \Delta,\ S\}$, where the first two fields carry $B-L$ charges of ± 2, while S is neutral. All three fields are neutral under $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $B-L$ charge of the Δ field is chosen so that it has direct Yukawa couplings with the N fields, which would provide large Majorana masses for them upon spontaneous symmetry breaking. This choice also guarantees that R–parity of MSSM will remain unbroken even after spontaneous symmetry breaking, since $\langle\Delta\rangle \neq 0$ leaves an unbroken Z_2 symmetry, which functions as R–parity. Our normalization of $B-L$ charge is as follows. $(N,\ e^c)$ have charge $+1$, L has charge -1, Q has charge $1/3$ while $(u^c,\ d^c)$ fields carry charge $-1/3$. No other fields beyond MSSM fields are introduced.

The superpotential of the model consistent with the extended gauge symmetry is given by

$$\begin{aligned} W &= W_{\rm MSSM} + W^{(B-L)} , \\ W^{(B-L)} &= \lambda S(\Delta\bar{\Delta} - M^2) + \frac{1}{2} f_{ij} N_i N_j \Delta + Y_\nu^{\alpha i} L_\alpha N_i H_u \ . \end{aligned} \tag{3.1}$$

Here W_{MSSM} is the MSSM superpotential. L_α denotes the left–handed lepton doublets, H_u is the up–type Higgs doublet, and i, α are family indices. Note that all R–parity violating couplings are forbidden in the superpotential by the $B-L$ symmetry. The Majorana masses for the right–handed neutrinos arise only after spontaneous breaking of $B-L$ symmetry after $\langle\Delta\rangle \neq 0$ develops, via the couplings f_{ij}. The Dirac Yukawa couplings Y_ν will then generate small neutrino masses via the seesaw mechanism. Bare mass terms for S as well as for $\Delta\overline{\Delta}$ and an S^3 term have not been written in Eq. (3.1). This is for simplicity and their omission can be justified by invoking an R symmetry.

We minimize the potential, which contains F–terms resulting from Eq. (3.1) and a D–term corresponding to the $B-L$ symmetry, in the SUSY limit. Demanding the vanishing of F–terms, $F_S = F_\Delta = F_{\bar{\Delta}} = 0$, yields $\langle S\rangle = 0$ and $\langle\Delta\bar{\Delta}\rangle = M^2$. The vanishing of the D–term implies $|\Delta| = |\bar{\Delta}|$. Without loss of generality we choose $\langle\Delta\rangle = |M|$. Consequently we have $\langle\bar{\Delta}\rangle = |M|e^{i\phi_{M^2}}$, with the definition $\phi_{M^2} \equiv \arg(M^2)$. The spectrum of the model in the SUSY limit consists of a massive vector multiplet $\mathcal{V}_B$ and a pair of degenerate chiral multiplets $(\Delta_0,\ S)$ with masses given by

$$M_{\mathcal{V}_{\mathcal{B}}} = 2g_B|M|, \qquad M_\Delta = \sqrt{2}|\lambda||M| \ . \tag{3.2}$$

Here g_B denotes the $B-L$ gauge coupling. In this limit, the $B-L$ gaugino pairs up with a Higgsino (denoted Δ') which is a linear combination of Δ and $\overline{\Delta}$ fields. The orthogonal combination Δ_0 pairs up with the S–Higgsino to forma a Dirac fermion. Small SUSY breaking effects, to be discussed shortly, will split the masses of the two Weyl components in each of these Dirac fermions. The $(\Delta_0,\ S)$ system consists of two complex scalars as well – corresponding to four real nearly degenerate scalar states once small SUSY breaking effects are included, which are physical. It is these nearly degenerate scalar states that will be relevant for leptogenesis.

We will be interested in the limit where the physical Higgs multiplet $(\Delta_0,\ S)$ is somewhat lighter than the gauge supermultiplet, that is, in the limit $\sqrt{2}\lambda \ll 2g_B$. Precisely how much lighter will be quantified later, but we will not need a larger hierarchy in masses, $M_\Delta < 0.1\ M_{\mathcal{V}_{\mathcal{B}}}$ or so will suffice. With such a mild hierarchy in masses, the dominant decay of the $(\Delta,\ S)$ Higgs fields will be into right-handed neutrino fields. This will enable a new way of generating lepton asymmetry stored in $\tilde{N}$ fields. With $M_\Delta \ll M_{\mathcal{V}_{\mathcal{B}}}$, we can integrate out the vector supermultiplet to obtain an effective superpotential $W_{\rm eff}$ and an effective Kähler potential $K_{\rm eff}$ involving only the $(\Delta_0,\ S)$ fields and the MSSM superfields.

To obtain the effective Lagrangian of the theory after integrating out the vector superfield, we work in the unitary gauge and make supersymmetric transformations on the $(\Delta,\ \overline{\Delta})$ fields, the gauge vector multiplet $\mathcal{V}_B$, and all fields Φ_i carrying $B-L$ charge q_i to go to a new basis with $(\Delta',\ \Delta_0)$ fields and a shifted $\mathcal{V}_B$ gauge superfield:

$$\begin{aligned} \Delta &= (|M| + \frac{1}{\sqrt{2}}\Delta_0)e^{q_\Delta g_B \Delta'}\ , \qquad \bar{\Delta} = (|M| + \frac{1}{\sqrt{2}}\Delta_0)e^{-q_\Delta g_B \Delta' + i\phi_{M^2}}\ , \\ \mathcal{V}_B &= \mathcal{V}_B^0 - \Delta' - \Delta'^\dagger\ , \qquad \Phi_i \to e^{q_i g_B \Delta'}\Phi_i\ . \end{aligned} \tag{3.3}$$

We have kept the $B-L$ charge of $\Delta,\ \overline{\Delta}$ fields to be $(q_\Delta,\ -q_\Delta)$ to be more general.

With these redefinitions, the original Kähler Lagrangian, given by

$$\mathcal{L}_D^{(B-L)} = \int d^4\theta \left(\Delta^\dagger e^{q_\Delta g_B \mathcal{V}_B}\Delta + \bar{\Delta}^\dagger e^{-q_\Delta g_B \mathcal{V}_B}\bar{\Delta} + \sum_i \Phi_i^\dagger e^{q_i g_B \mathcal{V}_B}\Phi_i\right) \tag{3.4}$$

transform into

$$\mathcal{L}_D^{(B-L)} = \int d^4\theta \left(\left\{2|M|^2 + \sqrt{2}|M|(\Delta_0 + \Delta_0^\dagger) + \Delta_0^\dagger\Delta_0\right\}\text{Cosh}(q_\Delta g_B \mathcal{V}_B^0) + \sum_i \Phi_i^\dagger e^{q_i g_B \mathcal{V}_B^0}\Phi_i\right)\ . \tag{3.5}$$

Observe that the Δ' field has disappeared in Eq. (3.5), it has been eaten up by the gauge superfield $\mathcal{V}_B^0$. In the process the gauge field $\mathcal{V}_B^0$ becomes massive, all its components acquire a mass $M_{\mathcal{V_B}}^2 = q_\Delta^2 g^2 |M|^2$, as can be readily seen by expanding the Cosh function in Eq. (3.5).

Now we can integrate out the massive gauge superfiled $\mathcal{V}_B^0$. We obtain the following effective Kähler Lagrangian:

$$\begin{aligned} \mathcal{L}_{\rm D,eff}^{(B-L)} &= \int d^4\theta \left[\Delta_0^\dagger\Delta_0 + \sum_i \Phi_i^\dagger\Phi_i - \frac{1}{4q_\Delta^2|M|^2}\left(\sum_i q_i\Phi_i^\dagger\Phi_i\right)^2 \right. \\ &\left. + \frac{\Delta_0 + \Delta_0^\dagger}{4\sqrt{2}q_\Delta^2|M|^3}\left(\sum_i q_i\Phi_i^\dagger\Phi_i\right)^2 + \frac{1}{8q_\Delta^2|M|^4}(\Delta_0^\dagger\Delta_0 - \Delta_0^2 - \Delta_0^{\dagger 2})\left(\sum_i q_i\Phi_i^\dagger\Phi_i\right)^2 + \cdots\right], \end{aligned} \tag{3.6}$$

where the $\cdots$ indicate terms with higher powers of $1/|M|$. Eq. (3.6) describes the interactions of the light Δ_0 field with other light MSSM fields through the exchange of the gauge supermultiplet. Notice that these interactions are suppressed by $1/|M|^3$.

With the redefinition of fields given in Eq. (3.3), the superpotential of Eq. (3.1) becomes $W_{\rm eff} = W_{\rm MSSM} + W(\Delta_0, N)$ where $W(\Delta_0, N)$ involving Δ_0 and N states is given by:

$$W(\Delta_0, N) = \lambda S e^{i\phi_{M^2}} \left(|M|\sqrt{2}\Delta_0 + \frac{1}{2}\Delta_0^2 \right) + \frac{1}{2} f_{ij} (|M| + \frac{1}{\sqrt{2}}\Delta_0) N_i N_j + Y_\nu^{\alpha i} L_\alpha N_i H_u \; . \qquad (3.7)$$

Note that the Δ' field has disappeared in Eq. (3.7). Majorana masses for N have been generated with $M_{N_i} = |f_i||M|$, where $|f_i|$ are the real and diagonal eigenvalues of the matrix f_{ij}. It is also clear from Eq. (3.7) that $(\tilde{\Delta}_0,\ \tilde{S})$ fields pair up to form a Dirac fermion with a mass given by $M_\Delta = \sqrt{2}|\lambda||M|$. Their scalar partners $(\Delta_0,\ S)$ are of course degenerate with these fermions, since SUSY breaking has not yet been turned on.

We assume that at least one of the N_i fields is lighter than Δ_0. Such situation is quite natural, especially when the N_i fields have hierarchical masses. We denote this light N_i field simply as N (assuming for simplicity that only one such field is lighter than Δ_0) with its mass given by $M_N = |fM|$. The dominant decays of Δ_0 scalar will then be $\Delta_0 \to \tilde{N} + \tilde{N}$, $\Delta_0 \to \tilde{N}^* \tilde{N}^*$, and $\Delta_0 \to NN$. There is also a subdominant decay of Δ_0 into $\tilde{N}\tilde{N}^*$. Here N denotes the right–handed neutrino, while $\tilde{N}$ stands for its scalar partner. Supersymmetry will dictate that the decays of the fermionic partner of Δ_0, denoted as $\tilde{\Delta}_0$ will be to $\tilde{N}N$ and $\tilde{N}^* N$ final states with an identical width. The total width for the decays of the scalar Δ_0 is given by

$$\Gamma(\Delta_0 \to \tilde{N}\tilde{N} + \tilde{N}^*\tilde{N}^* + \tilde{N}\tilde{N}^* + NN) = \frac{|f|^2}{64\pi} M_\Delta \sqrt{1 - \frac{4M_N^2}{M_\Delta^2}} \; . \qquad (3.8)$$

Since in our scheme, lepton asymmetry is initially created as an asymmetry in $\tilde{N}$ versus $\tilde{N}^*$, we are interested in range of model parameters where these decays are essentially out-of-equilibrium at temperatures around the mass of Δ_0. For $M_\Delta \sim (10^6 - 10^8)$ GeV, this requirement implies that $|f|$ in Eq. (3.8) should obey $|f| \leq 10^(- 5)$. For such small values of $|f|$, it is important to check if the gauge boson mediated decays of Δ_0 will have a comparable rate. To check this, we have computed the total decay width of Δ_0 scalars into four MSSM fields. These could be four scalars, four fermions, or two scalars plus two fermions, all of the MSSM. The total width is given by

$$\Gamma(\Delta_0 \to \Phi_i^* \Phi_i \Phi_j^* \Phi_j) = \frac{256 \times 4 (g_B/2)^6}{360 \times (2\pi)^5} \left(\frac{M_\Delta^7}{M_{\mathcal{V}_{\mathcal{B}}}^6} \right) \; . \qquad (3.9)$$

In Eq. (3.9), Φ_i stands for any of the scalar or fermion fields of MSSM. The factor 256 arises as $[\mathrm{Tr}(q_i^2)]^2$, while the factor 4 is to account for the various types of final states stated above. We see that these decays are suppressed by phase space and inverse power of the $\mathcal{V}_\mathcal{B}$ mass. If we demand that the decays of Δ_0 given in Eq. (3.8) dominates over the ones in Eq. (3.9), we arrive at an inequality $(g_B/2)M_\Delta/M_{\mathcal{V}_\mathcal{B}} < 1.5|f|^{1/3}$, or using Eq. (3.2), $|\lambda| \leq 4.3|f|^{1/3}$. If $|f| = 10^{-5}$, this translates into a limit $|\lambda| \leq 0.09$. This a rather mild hierarchy, which is quite natural. We will henceforth assume that the two body decay of Δ_0 into $\tilde{N}\tilde{N}$ dominates over the four body decay, which would enable us to create lepton asymmetry in $\tilde{N}$.

3.3 Spectrum including SUSY breaking

In the supersymmetric limit we have seen that four real scalar fields belonging to the $(\Delta_0,\ S)$ superfileds are degenerate in mass. The corresponding fermions are also degenerate in mass. This degeneracy will be lifted once SUSY breaking interactions are taken into account. One would arrive at two quasi–degenerate Majorana fermions and four quasi–degenerate real scalar fields. Their mass splitting and coupling to the $(N,\ \tilde{N})$ fields are crucial for the estimation of the induced lepton asymmetry in $\tilde{N}$. Here we compute these splitting and couplings.

Soft supersymmetry breaking interactions are introduced in the usual way as in supergravity. For the $(\Delta_0,\ S,\ N)$ sector the relevant soft breaking terms are given by

$$V_{\rm soft} = \{A_\lambda \lambda S \Delta \overline{\Delta} - C_\lambda \lambda M^2 S + \frac{A_f f_{ij}}{2} \Delta \tilde{N}_i \tilde{N}_j + h.c.\} + m_i^2 \Phi_i^* \Phi_i \ . \tag{3.10}$$

The dimensional parameters $\{A_\lambda,\ A_f,\ C_\lambda\}$ will be taken to have values near the TeV scale. Mass–splitting within degenerate multiplets will be induced at order $M_{\rm susy} \sim$ TeV, so we will ignore terms of order $M_{\rm susy}^2$ and higher. The soft squared mass parameters m_i^2 in Eq. (3.10) can then be neglected.

We now minimize the potential including soft SUSY breaking, keeping linear terms in $M_{\rm susy}$. First we obtain the redefined soft breaking terms after the transformation of Eq. (3.3) is applied to Eq.

(3.10). This yields

$$V_{\text{soft}} = \lambda M^2(A_\lambda - C_\lambda)S + A_\lambda \lambda e^{i\phi_{M^2}} S(\sqrt{2}|M|\Delta_0 + \frac{1}{2}\Delta_0^2) + \frac{1}{2}A_f f_{ij}(|M| + \frac{1}{\sqrt{2}}\Delta_0)\tilde{N}_i\tilde{N}_j + \text{h.c.} + m_i^2|\Phi_i|^2 \ . \quad (3.11)$$

The full potential is given by $V = V_F + V_{\text{Soft}}$, with V_F obtained from Eq. (3.7) as

$$V_F = |\lambda\Delta_0|^2|\sqrt{2}|M| + \frac{1}{2}\Delta_0|^2 + |\lambda\sqrt{2}|M|S + \lambda S\Delta_0 + \frac{e^{-i\phi_{M^2}}}{2\sqrt{2}} f_{ij}\tilde{N}_i\tilde{N}_j|^2 + |f_{ij}(|M| + \frac{1}{\sqrt{2}}\Delta_0)\tilde{N}_j|^2 \ . \quad (3.12)$$

Minimization of V shows that the field S develops a vacuum expectation value (VEV) of order M_{susy} given by

$$\langle S^* \rangle = \frac{C_\lambda - A_\lambda}{2\lambda^*} e^{i\phi_{M^2}} \ . \quad (3.13)$$

The shift in the VEV of the Δ_0 field is of order M_{susy}^2 and thus negligible. As a consequence of $\langle S \rangle \neq 0$, the mass matrix in the fermion sector spanning $(\tilde{\Delta}_0, \ \tilde{S})$ gets modified. We now have this matrix given by

$$\mathcal{M}_{fermi} = e^{i(\phi_{M^2} + \phi_\lambda)} \begin{pmatrix} |\lambda| \langle S \rangle & M_\Delta \\ M_\Delta & 0 \end{pmatrix} . \quad (3.14)$$

Here we have denoted the phase of λ as ϕ_λ. Eq. (3.14) leads to two quasi–degenerate Majorana fermions with masses given by $M_{\psi_{1,2}} = M_\Delta \pm |\lambda \langle S \rangle|$.

In the bosonic sector, the squared mass matrix spanning $(\text{Re}(\Delta_0), \text{Re}(S), \text{Im}(\Delta_0), \text{Im}(S))$, is found to be (to order M_{susy})

$$\mathcal{M}^2_{boson} = M_\Delta^2 \begin{pmatrix} 1 & \kappa_R + \kappa_R' & 0 & \kappa_I - \kappa_I' \\ \kappa_R + \kappa_R' & 1 & -\kappa_I' & 0 \\ 0 & -\kappa_I' & 1 & -\kappa_R' \\ \kappa_I - \kappa_I' & 0 & -\kappa_R' & 1 \end{pmatrix} , \quad (3.15)$$

$$\text{with} \quad (\kappa_R, \kappa_I, \kappa'_R, \kappa'_I) = \frac{\sqrt{2}}{|M|}\left(\mathrm{Re}(\langle S\rangle),\ \mathrm{Im}(\langle S\rangle),\ \mathrm{Re}(\frac{A_\lambda e^{i\phi_{M^2}}}{2\lambda^*}),\ \mathrm{Im}(\frac{A_\lambda e^{i\phi_{M^2}}}{2\lambda^*})\right) . \tag{3.16}$$

The eigenvalues of the matrix in Eq. (3.15) are found to be:

$$M^2_{X_{1,2}} = M^2_\Delta\left(1 + \Delta_{12} \mp \Delta_{14}\right) , \quad M^2_{X_{3,4}} = M^2_\Delta\left(1 - \Delta_{12} \pm \Delta_{14}\right) \tag{3.17}$$

with the definitions

$$\Delta_{12} = \frac{|\lambda v_S|}{M_\Delta} , \qquad \Delta_{14} = \frac{\left||A_\lambda| + \lambda v_S e^{i\phi_{M^2}}\right|}{M_\Delta} . \tag{3.18}$$

Thus $\Delta_{12} = (M^2_{X_1} - M^2_{X_2})/M^2_\Delta$ parameterizes the fractional mass splitting in X_1 and X_2, and similarly Δ_{14} in X_1 and X_4. These two mass splitting will be relevant for leptogenesis calculation. We also note the identities $\Delta_{12} = \Delta_{43}$ and $\Delta_{14} = \Delta_{23}$. There are two other mass splitting which can be obtained in terms of Δ_{12} and Δ_{14}, but those two turn out to be not relevant for leptogenesis.

The mass eigenstates X_i are related to the original states as

$$\begin{aligned}
\mathrm{Re}(\Delta_0) &= \frac{c_\alpha}{\sqrt{2}}(X_1 + X_3) + \frac{s_\alpha}{\sqrt{2}}(X_2 + X_4) , \quad \mathrm{Re}(S) = \frac{c_\beta}{\sqrt{2}}(X_1 - X_3) + \frac{s_\beta}{\sqrt{2}}(X_2 - X_4) , \\
\mathrm{Im}(\Delta_0) &= -\frac{s_\alpha}{\sqrt{2}}(X_1 + X_3) + \frac{c_\alpha}{\sqrt{2}}(X_2 + X_4) , \ \mathrm{Im}(S) = -\frac{s_\beta}{\sqrt{2}}(X_1 - X_3) + \frac{c_\beta}{\sqrt{2}}(X_2 - X_4).
\end{aligned} \tag{3.19}$$

Here two mixing angles appear which we denote as (α, β). We use the notation $c_\alpha = \cos\alpha,\ s_\alpha = \sin\alpha$, etc. These two angles are given by

$$\tan 2\alpha = -\frac{|A_\lambda|/M_\Delta \sin\{\arg(C_\lambda - A_\lambda)\}}{|A_\lambda|/M_\Delta \cos\{\arg(C_\lambda - A_\lambda)\} + \Delta_{12}} , \quad \beta = \alpha - \phi_{M^2} - \phi_\lambda - \arg(C_\lambda - A_\lambda) . \tag{3.20}$$

We shall use these results in the next section where we compute the lepton asymmetry stored in $\tilde{N}$ arising from the decays of these scalar states.

3.4 Cosmological lepton asymmetry

In our scenario, cosmological lepton asymmetry is generated in the out of equilibrium decays of the X_i scalars into $\tilde{N}$ and $\tilde{N}^*$, the scalar partners of the right–handed neutrino. One loop corrections to the decay induces CP asymmetry, leading to an asymmetry in $\tilde{N}$ versus $\tilde{N}^*$. This induced asymmetry is converted to usual lepton asymmetry when $\tilde{N}$ and $\tilde{N}^*$ decay into leptons and a Higgs boson, which subsequently is converted to baryon asymmetry via electro-weak sphaleron processes.

As shown in Sec. 2, the dominant decay of the X_i scalars will be into final states with $\tilde{N}$ scalars and N fermions, with a smallish coupling $\lambda \leq 0.1$ and $|f| \sim 10^{-5}$. The tree level decay diagrams are shown in Fig.3.1. The total decay rate for these decays is given in Eq. (3.8). The decay of X_i, which are real scalars, into final states with opposite lepton number (-2 and $+2$) (see Fig. 3.1 (a) and (b)) raises the possibility that an asymmetry can be produced in $\tilde{N}$ number. For $M_\Delta = 10^6 - 10^8$ GeV and $|f| = 10^{-5} - 10^{-4}$, the lepton number violating decays of the X_i fields will be out of equilibrium. The efficiency factor in the production of $\tilde{N}$ asymmetry will then be nearly one.

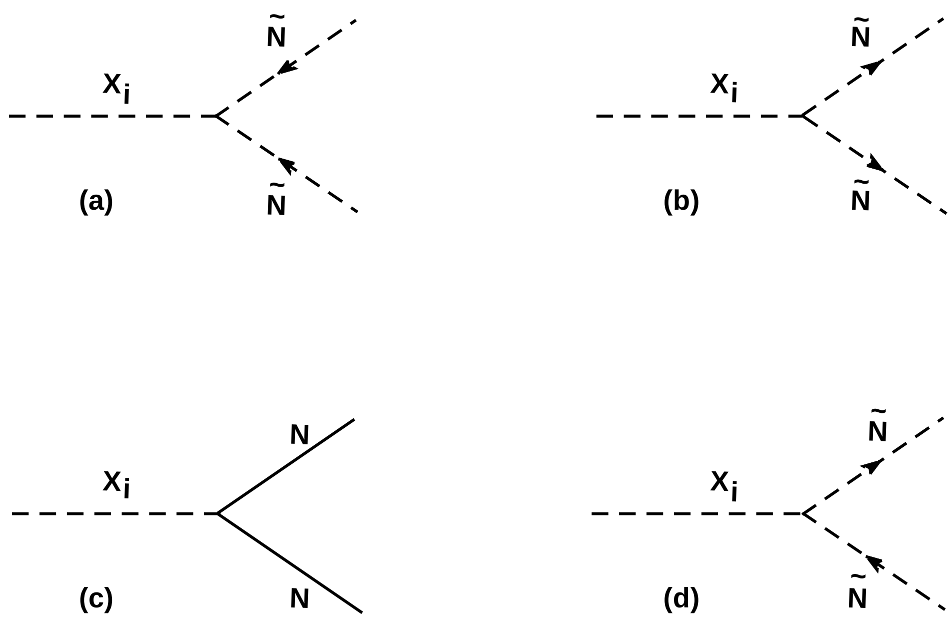

Figure 3.1: Tree level decays of X_i scalars into $\tilde{N}, \tilde{N}^*$ and N.

We now proceed to calculate the induced $\tilde{N}$ asymmetry. For this purpose we need to identify the interaction of the X_i fields with $\tilde{N}$. Since the X_i fields are quasi–degenerate, the dominant contribution to lepton asymmetry will arise from wave function corrections shown in Fig. 3.2. These corrections have a resonance enhancement, which is lacking in the vertex correction diagrams. SUSY provides the quasi–degeneracy of X_i fields, which enables us to realize resonant leptogenesis in $\tilde{N}$. The required CP violation arises in the model from soft SUSY breaking couplings. Thus this scenario is also soft leptogenesis, but with four X_i fields involved in the decay.

From the Lagrangian given in Eqs. (3.11) and (3.12), one can read off the cubic scalar interactions relevant for the wave function corrections of Fig. 2. The couplings of X_i to $\tilde{N}$ is found to be

$$V^{(3)} = \left(\tilde{N}\tilde{N}F_{\tilde{N}\tilde{N}i}X_i + \text{h.c.}\right) + |\tilde{N}|^2 F_{|\tilde{N}|i}X_i \ , \tag{3.21}$$

where we have defined

$$\begin{aligned} F_{\tilde{N}\tilde{N}i} &= \frac{f}{4}\left(a_1+a_2+M_\Delta e^{i\omega},\ -i(a_1-a_2+M_\Delta e^{i\omega}),\ a_1+a_2-M_\Delta e^{i\omega},\ -i(a_1-a_2-M_\Delta e^{i\omega})\right)_i \\ F_{|\tilde{N}|i} &= \frac{|f||M_N|}{\sqrt{2}}\left(c_\alpha,\ s_\alpha,\ c_\alpha,\ s_\alpha\right)_i \\ \text{with} \qquad & a_1 = \frac{C_\lambda - A_\lambda}{2}e^{i\alpha}\ , \quad a_2 = A_f e^{-i\alpha}\ , \quad \omega = \beta - \phi_\lambda - \phi_{M^2}\ . \end{aligned} \tag{3.22}$$

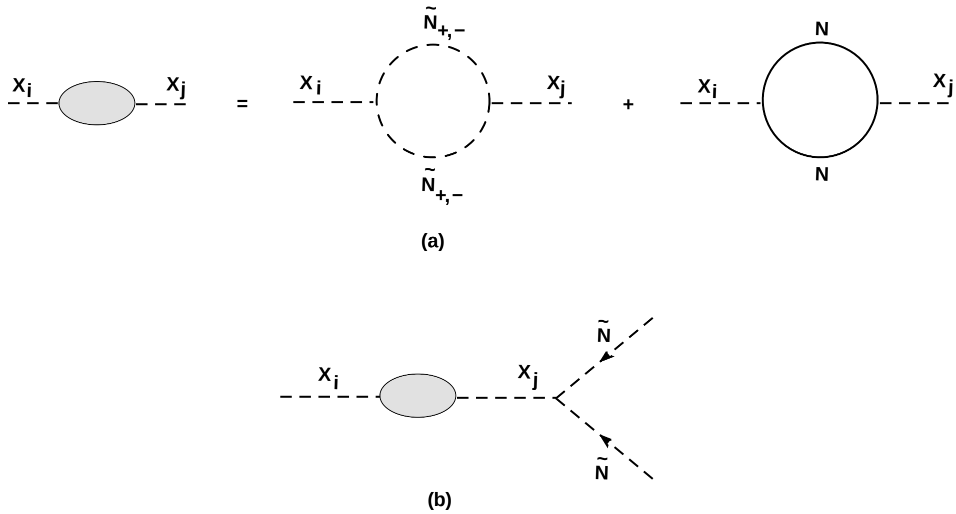

Figure 3.2: Loop diagrams generating CP asymmetry in the decay $X_i \to \tilde{N}^*\tilde{N}^*$. The blob in (b) corresponds to the resummed two point functions shown in (a).

The $\tilde{N}$ and $\tilde{N}^*$ states mix after SUSY breaking. This splitting effect will show up in the loops of Fig. 2. To take these effects into account, we go to the mass eigenbasis of these states $\tilde{N}_+$ and $\tilde{N}_-$ which are given by

$$\tilde{N}_+ = \frac{1}{\sqrt{2}}(e^{ix}\tilde{N} + e^{-ix}\tilde{N}^*)\ , \qquad \tilde{N}_- = \frac{1}{\sqrt{2}i}(e^{ix}\tilde{N} - e^{-ix}\tilde{N}^*)\ . \tag{3.23}$$

Note that $\tilde{N}_\pm$ are real fields with masses given by

$$M^2_{\tilde{N}_+} = |M_N|^2 + |M_N| \left| A_f + \frac{C_\lambda - A_\lambda}{2} \right| \ , \quad M^2_{\tilde{N}_-} = |M_N|^2 - |M_N| \left| A_f + \frac{C_\lambda - A_\lambda}{2} \right| \ . \tag{3.24}$$

Here we have defined the phase parameter x as $x = \frac{1}{2}\left(\phi_f + \arg(A_f + \frac{C_\lambda - A_\lambda}{2})\right)$.

In the $\tilde{N}_a = (\tilde{N}_+, \tilde{N}_-)$ $(a = \pm)$ basis, the cubic scalar interactions can be written as

$$V^{(3)} = \tilde{N}_a \tilde{N}_b F_{ab\,i} X_i \ , \tag{3.25}$$

where

$$\begin{aligned} F_{++i} &= \frac{1}{2}\left(e^{-2ix} F_{\tilde{N}\tilde{N}i} + e^{2ix} F^*_{\tilde{N}\tilde{N}i} + F_{|\tilde{N}|i}\right) \ , \quad F_{--i} = -\frac{1}{2}\left(e^{-2ix} F_{\tilde{N}\tilde{N}i} + e^{2ix} F^*_{\tilde{N}\tilde{N}i} - F_{|\tilde{N}|i}\right) \ , \\ F_{+-i} &= F_{-+i} = i\left(e^{-2ix} F_{\tilde{N}\tilde{N}i} - e^{2ix} F^*_{\tilde{N}\tilde{N}i}\right) \ . \end{aligned} \tag{3.26}$$

It is now straightforward to work out the absorptive part of the two point function arising from the $\tilde{N}$-loops. We find it to be

$$\begin{aligned} \Pi^B_{ij}(p^2) &= \frac{1}{32\pi}\left(2K_{++}F_{++\,i}F_{++\,j} + 2K_{--}F_{--\,i}F_{--\,j} + K_{+-}F_{+-\,i}F_{+-\,j}\right) \ , \\ \text{where} \qquad K_{ab} &= \left(1 - 2\frac{M^2_{\tilde{N}_a} + M^2_{\tilde{N}_b}}{p^2} + \frac{(M^2_{\tilde{N}_a} - M^2_{\tilde{N}_b})^2}{p^4}\right)^{1/2} \end{aligned} \tag{3.27}$$

When considering X_i-decay, one should set $p^2 = M^2_{X_i}$. We have, for example, for the absorptive part of Π_{12},

$$\Pi^B_{12} \simeq \frac{|f|^2}{32\pi} \hat{A}_1 M^2_\Delta \left(1 - 4\frac{M^2_N}{M^2_\Delta}\right)^{1/2} \ . \tag{3.28}$$

We will also need the Yukawa couplings of the X_i fields with the N fermions. It is given by

$$\begin{aligned} \mathcal{L}_{NNY} &= NNY_F X + \text{h.c.} \\ \text{with} \qquad & Y_F = \frac{f e^{-i\alpha}}{4\sqrt{2}} (1,\ i,\ 1,\ i) \ . \end{aligned} \tag{3.29}$$

The absorptive part arising through the fermionic loop in Fig. 2 is found to be

$$\Pi^F_{ij}(p^2) = \frac{1}{8\pi} p^2 (1 - 4M^2_N/p^2)^{1/2} \left(Y^\dagger_F Y_F + Y^T_F Y^*_F\right)_{ij} \ . \tag{3.30}$$

We now combine these results to compute $\epsilon_{\tilde{N}}$, the $\tilde{N}$ asymmetry parameter defined as

$$\epsilon_{\tilde{N}} = \sum_i \frac{\Gamma(X_i \to \tilde{N}\tilde{N}) - \Gamma(X_i \to \tilde{N}^*\tilde{N}^*)}{\Gamma(X_i \to \tilde{N}\tilde{N}) + \Gamma(X_i \to \tilde{N}^*\tilde{N}^*)} \ . \tag{3.31}$$

We find it to be

$$\epsilon_{\tilde{N}} = 4\left[\frac{2\Delta_{12}\Gamma/M_\Delta}{4\Delta_{12}^2 + (\Gamma/M_\Delta)^2} \cdot \frac{\hat{A}_1}{M_\Delta} + \frac{2\Delta_{14}\Gamma/M_\Delta}{4\Delta_{14}^2 + (\Gamma/M_\Delta)^2} \cdot \frac{\hat{A}_2}{M_\Delta}\right] \ . \tag{3.32}$$

Here we have defined two effective A–parameters as follows.

$$\begin{aligned}
\hat{A}_1 &= |A_f| \sin\phi_1 - 2\left|A_f + \frac{C_\lambda - A_\lambda}{2}\right| \frac{(M_N/M_\Delta)^2}{1 - 4(M_N/M_\Delta)^2} \sin\phi_3 \\
\hat{A}_2 &= \frac{1}{2}|c_\lambda - A_\lambda| \sin\phi_2 - 2\left|A_f + \frac{C_\lambda - A_\lambda}{2}\right| \frac{(M_N/M_\Delta)^2}{1 - 4(M_N/M_\Delta)^2} \sin\phi_4
\end{aligned} \tag{3.33}$$

The phases appearing in Eq. (3.32) are related to the original phases in the model through the relations

$$\begin{aligned}
\phi_1 &= 2\alpha - 2\phi_{M^2} - 2\phi_\lambda - \arg(A_f) - \arg(C_\lambda - A_\lambda) \ , \\
\phi_2 &= 2\phi_{M^2} + 2\phi_\lambda + 2\arg(C_\lambda - A_\lambda) \ , \\
\phi_3 &= 2\alpha - 2\phi_{M^2} - \arg(C_\lambda - A_\lambda) - \arg(A_f + \frac{C_\lambda - A_\lambda}{2}) \ , \\
\phi_4 &= 2\phi_{M^2} + 2\phi_\lambda + \arg(C_\lambda - A_\lambda) + \arg(A_f + \frac{C_\lambda - A_\lambda}{2}) \ .
\end{aligned} \tag{3.34}$$

It should be mentioned that the asymmetry given in Eq. (3.32) includes fermionic and bosonic loop contributions. It turns out that the fermionic loop is entirely canceled by the bosonic loop, the left-over piece from the bosonic loop is what is given in Eq. (3.32). This cancelation is not surprising, since the fermion loop corrections do not feel the effects of SUSY breaking. We also note that the off–diagonal Π_{ij} have one power of $M_{\rm susy}/M_X$ suppression, so the decay vertex has to be supersymmetric. This feature simplifies the calculations somewhat. In Eq. (3.32) we have added the asymmetry arising from all four of the X_i scalar fields.

In principle, the decays of the Higgsinos $(\tilde{\Delta}_0, \tilde{S})$ into $\tilde{N}$ and N can create an asymmetry in $\tilde{N}$. However, we find that there is not sufficient CP violation in these decays in the minimal model.

Now we are ready to estimate the lepton asymmetry created by $\tilde{N}$-decays at the second stage where $\tilde{N}$ decays into a lepton and a Higgs boson. Note that lepton asymmetry between $\tilde{N}$ and $\tilde{N}^*$ will be completely converted into lepton asymmetry in the MSSM sector. There is however one peculiarity related to SUSY. $\tilde{N}$ has two primary decay channels $\tilde{N} \to L\tilde{H}_u$ and $\tilde{N} \to \tilde{L}^* H_u^*$. Since the rates of these processes are the same due to SUSY (at zero temperature), the lepton asymmetries created from these decays cancel each other. However, with $T \neq 0$ the cancelation is only partial (due to temperature effects which explicitly break SUSY) and one has

$$\tilde{\epsilon} = \epsilon(\tilde{N} \to L\tilde{H}_u)\Delta_{BF} \ , \tag{3.35}$$

with the temperature dependent factor Δ_{BF} given in [61]. Now, the baryon asymmetry created from the lepton asymmetry due to $\tilde{N}$ decays is:

$$\frac{\tilde{n}_B}{s} \simeq -8.6 \cdot 10^{-4} \frac{\tilde{\epsilon}}{\Delta_{BF}} \eta = -8.6 \cdot 10^{-4} \epsilon_{\tilde{N}} \eta \ , \tag{3.36}$$

where we have taken into account an effective number of degrees of freedom, including one RHN superfield, to be $g_* = 236$. In the last stage of Eq. (3.36) we have substituted $\tilde{\epsilon}$ by $\epsilon_{\tilde{N}}$ - the $\tilde{N}$ asymmetry created at the first stage by X_i-decays. η is an efficiency factor which depends on $\tilde{m} \simeq \frac{(v \sin\beta)^2}{M} Y_\nu^2$, and which takes into account temperature effects by integrating the Boltzmann equations [61]. For instance, efficiency η reaches its maximal value, $\eta \approx 0.1$ for $\tilde{m} \approx 10^{-3}$ eV. Thus, in order to generate the experimentally observed asymmetry $\left(\frac{n_B}{s}\right)_{\rm exp} = (8.75 \pm 0.23) \cdot 10^{-11}$, we need to have $\epsilon_{\tilde{N}} \gtrsim 10^{-6}$. Going back to Eq. (3.32), we see that an enhancement of $\epsilon_{\tilde{N}}$ will happen for small values of Δ_{ij}. The natural values of these parameters are $\sim M_{susy}/M_\Delta$. However, some cancelation can make either of these parameters smaller. Assuming that this happens for Δ_{14}, with the parametrization $\Delta_{14} = \delta_{14} M_{susy}/M_\Delta$ and $\hat{A}_2 = \delta_2 M_{susy}$ we have $\epsilon_{\tilde{N}} \approx 2\delta_2 \Gamma/(\delta M_\Delta)$. On the other hand, out of equilibrium decay of X_i states requires $\Gamma \lesssim H = 1.7\sqrt{g_*} M_\Delta^2/M_{\rm Pl}$. Therefore, we have $\epsilon_{\tilde{N}} \lesssim 3.4\sqrt{g_*}\delta_2 M_\Delta/(\delta M_{\rm Pl})$. With the choice $\delta_2 \approx 3$ and $\delta \approx 1/300$ and $M_\Delta \simeq 10^8$ GeV, we obtain $\epsilon_{\tilde{N}} \simeq 10^{-6}$. This has be achieved by the suppressed value δ, which does not seems quite natural. Similar situation occurs in the soft leptogenesis scenario. However, note that within our setup we do not need to constrain the value of the Dirac Yukawa coupling Y_ν very much. Only thing which is needed to be fulfilled is the out of equilibrium decays of $\tilde{N}$. At the first stage, we have assumed $\Gamma \lesssim H$ which insures no additional dilution. One can also investigate the dilution effects within this scenario in the regime where there is significant departures from the condition $\Gamma \lesssim H$, but we do not attempt it here.

We conclude with a few remarks. We have kept corrections of order $M_{\rm susy}/M_\Delta$ in the computation of CP asymmetry, and not any higher power. It is known that if the mass of the decaying field is closer to the SUSY scale, second order vertex corrections can be important proportional to the mass of the MSSM gaugino [71]. In our scheme, these vertex corrections do not exist, since the $B - L$ gaugino has decoupled and since $\tilde{N}$ does not couple to MSSM gauginos. A natural question to ask is whether the soft SUSY breaking corrections that induce lepton asymmetry can also lead to excessive CP violation in electron and neutron dipole moments. With universal soft breaking mass parameters there is a potential problem. We note that if the theory is embedded in SUSY left–right model, then all the Dirac Yukawa couplings and A–terms are hemitian due to parity symmetry. That will make all EDM contributions vanishingly small [72]. On the other hand, parity symmetry implies that the Majorana–type couplings (such as A_f and f in our model) are complex symmetric, which can serve to induce the lepton asymmetry.

CHAPTER 4

LEPTOGENESIS OF $B-L$ GAUGE THEORY IN SUSY LIMIT

4.1 Introduction

An attractive way to explain the tiny neutrino masses is the seesaw mechanics which extends the standard model by introducing three right handed neutrinos which are standard model singlets. In this scenario, however, these right handed neutrinos can have masses as large as the Planck scale, which would be inconsistent with neutrino oscillation data. In $B-L$ gauge theories [73, 74], right handed neutrinos are naturally introduced, one for each family, for canceling the triangle anomaly associated with $[U(1)_{B-L}]^3$. By this right handed neutrino masses are no longer arbitrary, but related to the $B-L$ gauge symmetry breaking at high scales.

In addition, the R-parity [57,58], $R=(-1)^{3(B-l)+2S}$, is supposed to be conserved in supersymmetric models to suppress proton decay rates and to identify the lightest supersymmetric particle as the dark matter candidate. In $B-L$ gauge theories, the $B-L$ charge of the Higgs coupled to right handed neutrinos requires $B-L$ is broken by two. This gives a natural understanding of R-parity.

It is widely believed that the baryon asymmetry of the universe can be generated through leptogenesis [59]. In the leptogenesis scenario, the lepton number asymmetry is through the CP violating decay of right handed neutrinos to leptons and Higgs. In a foregoing work [74] we studied the new ways of leptogenesis by the decay to $B-L$ breaking Higgs to the super partners of right handed neutrinos, $\tilde{N}$ and $\tilde{N}^*$. CP violation necessary for leptogenesis is from the complex soft supersymmetry breaking terms. Although acceptable values of baryon asymmetry can be found without incurring the gravitino abundance problem, a 1/300 fine tuning is need to have large enough CP violation. In this work we have extended the study of the $B-L$ gauge theory leptogenesis to include two Higgs. With generally complex Yukawa coupling of the Higgs and right handed neutrino, we have showed that with one loop corrections to the Higgs wave function and the Yukawa coupling, large enough asymmetry of $\tilde{N}$ over $\tilde{N}^*$ can be acquired in the SUSY limit. Our evaluation works well when the decaying Higgs is less than 10^8 GeV. In supersymmetry scenario the abundance of gravitino consistent with nucleosynthesis requires the reheat temperature $T_R < 10^8$ GeV. The requirement with the so-called Davidson-Ibarra bound [68] requires the masses of decaying particles in the scenario of leptogenesis to be close less than 10^9 GeV. In our study, the masses of decaying Higgs lighter than 10^8 GeV are consistent with the gravitino abundance.

4.2 The supersymmetric gauged $B - L$ model

We extend the study of [74] by introducing two pairs of scalar fields Δ_i and $\bar{\Delta}_i$ $(i = 1,\ 2)$ with $B - L$ charge respectively 2 and -2 and a scalar field S with $B - L$ charge 0. They are all SM gauge group $SU(3) \times SU(2)_L \times U(1)_L$ singlets. We list the $B - L$ charges of all fields in this model in the table:

e^c	L	(u^c, d^c)	Q	N	Δ_i	$\overline{\Delta}_i$	S
1	-1	-1/3	1/3	1	-2	2	0

.

The general renormalizable superpotential with $B - L$ symmetry can be written down as

$$\begin{aligned} W &= W_{\text{MSSM}} + W_{(B-L)} \\ W_{(B-L)} &= M_{ij}\bar{\Delta}_i\Delta_j + \lambda_{ij}\bar{\Delta}_i\Delta_j S + \frac{1}{2}\mu_S S^2 + \frac{1}{3}\kappa S^3 + \frac{1}{2}f^i_{\alpha\beta}\Delta_i N_\alpha N_\beta + Y_\nu^{\alpha k} L_k N_\alpha H_u. \end{aligned} \tag{4.1}$$

Here the lepton generation indices k and right handed neutrino generation indices are from 1 to 3. When the $B - L$ symmetry is broken, the right handed neutrinos acquire their Majorana masses. Since B_l charge is broken by 2, R-parity is still conserved. The singlet field S guarantees that $B - L$ symmetry can be broken in the SUSY limit.

We need to minimize the potential. In the SUSY limit F-terms from (4.1) and the D-term of the $B - L$ gauge symmetry should vanish. We write down the minimum conditions:

$$\begin{aligned} F_S &= \lambda_{ij}\bar{v}_i v_j + \mu_S s + \kappa s^2 = 0, \\ F_{\Delta_i} &= M_{ji}\bar{v}_j + \lambda_{ji}\bar{v}_j s = 0, \\ F_{\bar{\Delta}_i} &= M_{ij} v_j + \lambda_{ij} v_j s = 0, \end{aligned} \tag{4.2}$$

where we have defined the vacuum expectation value of $v_i = \langle \Delta_1 \rangle$, $\bar{v}_i = \langle \bar{\Delta}_1 \rangle$ and $s = \langle S \rangle$.

Without losing generality, we assume diagonal M_{ij} with eigenvalues M_1, M_2, and the vacuum condition for v_i and $\bar{v}_i$ are

$$\begin{aligned} M_1\bar{v}_1 + \lambda_{11}\bar{v}_1 s + \lambda_{21}\bar{v}_2 s &= 0, \\ M_2\bar{v}_2 + \lambda_{12}\bar{v}_1 s + \lambda_{22}\bar{v}_2 s &= 0, \\ M_1 v_1 + \lambda_{11} v_1 s + \lambda_{12} v_2 s &= 0, \\ M_2 v_2 + \lambda_{21} v_1 s + \lambda_{22} v_2 s &= 0. \end{aligned} \tag{4.3}$$

For nonzero solutions for $v_1,\ v_2,\ \bar{v}_1,\ \bar{v}_2$, we need

$$(M_1 + \lambda_{11}s)(M_2 + \lambda_{22}s) - \lambda_{12}\lambda_{21}s^2 = 0, \tag{4.4}$$

which has the solutions

$$s = \frac{-M_2\lambda_{11} - M_1\lambda_{22} \pm \sqrt{(M_2\lambda_{11} + M_1\lambda_{22})^2 - 4M_1M_2(\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21})}}{2(\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21})} \tag{4.5}$$

and

$$v_2 = -\frac{M_1 + \lambda_{11}s}{\lambda_{12}s}v_1, \qquad \bar{v}_2 = -\frac{M_1 + \lambda_{11}s}{\lambda_{21}s}\bar{v}_1. \tag{4.6}$$

The vanishing D-term requires

$$|v_1|^2 + |v_2^2| = |\bar{v}_1|^2 + |\bar{v}_2|^2. \tag{4.7}$$

With the condition of (4.6), we have

$$v_1^2 \frac{|\lambda_{21}|\sqrt{|\lambda_{12}s|^2 + |M_1 + \lambda_{11}s|^2}}{|\lambda_{12}|\sqrt{|\lambda_{21}s|^2 + |M_1 + \lambda_{11}s|^2}} \left| \lambda_{11} - 2\frac{M_1 + \lambda_{11}s}{s} + \frac{\lambda_{22}(M_1 + \lambda_{11}s)^2}{\lambda_{12}\lambda_{21}s^2} \right| = |\mu_S s + \kappa s^2| e^{i\phi}, \tag{4.8}$$

where

$$\phi = \pi - \arg(\bar{v}_1/v_1) + \arg(\mu_S s + \kappa s^2) - \arg\left(\lambda_{11} - 2\frac{M_1 + \lambda_{11}s}{s} + \frac{\lambda_{22}(M_1 + \lambda_{11}s)^2}{\lambda_{12}\lambda_{21}s^2}\right), \tag{4.9}$$

is an arbitrary phase. It can be chosen so that v_1 is real. We follow this choice and we get

$$\begin{aligned} v_1 &= |\mu_S s + \kappa s^2|^{\frac{1}{2}} \left| \lambda_{11} - 2\frac{M_1 + \lambda_{11}s}{s} + \frac{\lambda_{22}(M_1 + \lambda_{11}s)^2}{\lambda_{12}\lambda_{21}s^2} \right|^{-\frac{1}{2}} \left(\frac{|\lambda_{21}|\sqrt{|\lambda_{12}s|^2 + |M_1 + \lambda_{11}s|^2}}{|\lambda_{12}|\sqrt{|\lambda_{21}s|^2 + |M_1 + \lambda_{11}s|^2}} \right)^{-\frac{1}{2}}, \\ \bar{v}_1 &= v_1 \frac{|\lambda_{21}|}{|\lambda_{12}|} \sqrt{\frac{|\lambda_{12}s|^2 + |M_1 + \lambda_{11}s|^2}{|\lambda_{21}s|^2 + |M_1 + \lambda_{11}s|^2}} e^{i\theta}, \end{aligned} \tag{4.10}$$

where $\theta = \arg(\bar{v}_1/v_1)$ can be determined from (4.9) by setting $\phi = 0$, v_2 and $\bar{v}_2$ are determined from (4.6).

We then redefine 4 orthogonal fields from Δ_i and $\bar{\Delta}_i$,

$$\begin{aligned}
\hat{\Delta}_1 &= \frac{v_1^*}{u}\Delta_1 + \frac{v_2^*}{u}\Delta_2, \\
\hat{\Delta}_2 &= \frac{\bar{v}_1^*}{\bar{u}}\bar{\Delta}_1 + \frac{\bar{v}_2^*}{\bar{u}}\bar{\Delta}_2, \\
\hat{\Delta}_3 &= \frac{v_2}{v}\Delta_1 - \frac{v_1}{v}\Delta_2 + \frac{\bar{v}_2}{v}\bar{\Delta}_1 - \frac{\bar{v}_1}{v}\bar{\Delta}_2, \\
\hat{\Delta}_4 &= \frac{\bar{u}|v_2|}{vu}\Delta_1 - \frac{\bar{u}v_1 v_2^*}{vu|v_2|}\Delta_2 + \frac{u\bar{v}_2 v_2^*}{v\bar{u}|v_2|}\bar{\Delta}_1 - \frac{u\bar{v}_1 v_2^*}{v\bar{u}|v_2|}\bar{\Delta}_2,
\end{aligned} \tag{4.11}$$

where

$$v = \sqrt{|v_1|^2 + |v_2|^2 + |\bar{v}_1|^2 + |\bar{v}_2|^2}, \quad u = \sqrt{|v_1|^2 + |v_2|^2}, \quad \bar{u} = \sqrt{|\bar{v}_1|^2 + |\bar{v}_2|^2}. \tag{4.12}$$

It can be shown that $\langle \hat{\Delta}_1 \rangle = u$, $\langle \hat{\Delta}_2 \rangle = \bar{u}$, $\langle \hat{\Delta}_3 \rangle = \langle \hat{\Delta}_4 \rangle = 0$. The Goldstone field should be a linear combination of $\hat{\Delta}_1$ and $\hat{\Delta}_2$. Rewriting the superpotential with newly defined fields

$$\hat{\Delta}_1 = (u + \hat{\Delta})e^{q_\Delta g_B \hat{\Delta}'}, \qquad \hat{\Delta}_2 = (\bar{u} + \hat{\Delta})e^{-q_\Delta g_B \hat{\Delta}'}, \tag{4.13}$$

the $\hat{\Delta}'$ doesn't appear in the superpotential. It is the Goldstone and is absorbed in the gauge field. 3 fields contribute to leptogenesis, $\hat{\Delta}$, $\hat{\Delta}_3$ and $\hat{\Delta}_4$ of (4.11).

4.3 The effective interactions

With these newly defined fields, the effective superpotential of the $\hat{\Delta}$, $\hat{\Delta}_3$ and $\hat{\Delta}_4$ is

$$W_{\text{eff}}(\hat{\Delta}) = \begin{pmatrix} \hat{\Delta} & \hat{\Delta}_3 & \hat{\Delta}_4 \end{pmatrix} \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{13} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix} \begin{pmatrix} \hat{\Delta} \\ \hat{\Delta}_3 \\ \hat{\Delta}_4 \end{pmatrix}, \tag{4.14}$$

where

$$
\begin{aligned}
\mu_{11} &= (M_1+\lambda_{11}s)V_{11}V_{32}+\lambda_{12}sV_{21}V_{32}+\lambda_{21}sV_{11}V_{42}+(M_2+\lambda_{22}s)V_{21}V_{42},\\
\mu_{22} &= (M_1+\lambda_{11}s)V_{13}V_{33}+\lambda_{12}sV_{23}V_{33}+\lambda_{21}sV_{13}V_{43}+(M_2+\lambda_{22}s)V_{23}V_{43},\\
\mu_{33} &= (M_1+\lambda_{11}s)V_{14}V_{34}+\lambda_{12}sV_{24}V_{34}+\lambda_{21}sV_{14}V_{44}+(M_2+\lambda_{22}s)V_{24}V_{44},\\
\mu_{12} &= \frac{1}{2}\left((M_1+\lambda_{11}s)(V_{13}V_{32}+V_{11}V_{33})+\lambda_{12}s(V_{23}V_{32}+V_{21}V_{33})\right.\\
&\qquad\left.+\lambda_{21}s(V_{13}V_{42}+V_{11}V_{43})+(M_2+\lambda_{22}s)(V_{23}V_{42}+V_{21}V_{43})\right),\\
\mu_{13} &= \frac{1}{2}\left((M_1+\lambda_{11}s)(V_{14}V_{32}+V_{11}V_{34})+\lambda_{12}s(V_{24}V_{32}+V_{21}V_{34})\right.\\
&\qquad\left.+\lambda_{21}s(V_{14}V_{42}+V_{11}V_{44})+(M_2+\lambda_{22}s)(V_{24}V_{42}+V_{21}V_{44})\right),\\
\mu_{23} &= \frac{1}{2}\left((M_1+\lambda_{11}s)(V_{14}V_{33}+V_{13}V_{34})+\lambda_{12}s(V_{24}V_{33}+V_{23}V_{34})\right.\\
&\qquad\left.+\lambda_{21}s(V_{14}V_{43}+V_{13}V_{44})+(M_2+\lambda_{22}s)(V_{24}V_{43}+V_{23}V_{44})\right),
\end{aligned}
\tag{4.15}
$$

and from (4.11)

$$
V_{ij} = \begin{pmatrix}
v_1/u & 0 & v_2^*/v & \bar{u}|v_2|/(vu)\\
v_2/u & 0 & -v_1^*/v & -\bar{u}v_2v_1^*/(vu|v_2|)\\
0 & \bar{v}_1/\bar{u} & \bar{v}_2^*/v & u\bar{v}_2^*v_2/(v\bar{u}|v_2|)\\
0 & \bar{v}_2/\bar{u} & -\bar{v}_1^*/v & -u\bar{v}_1^*v_2/(v\bar{u}|v_2|)
\end{pmatrix}. \tag{4.16}
$$

The symmetric matrix μ_{ij} can be diagonalized with an orthogonal matrix O,

$$
O_{ki}\mu_{kl}O_{lj} = \mu_i\delta_{ij} \tag{4.17}
$$

and we represent the mass eigenstates with Δ_i $(i = 1,\ 2,\ 3)$. This should not induce confusion.

In the mass eigenstates Δ_i, the effective superpotential can be given as

$$
W_{\text{eff}}(\Delta_i, N_\alpha) = \frac{1}{2}\mu_i\Delta_i\Delta_i + \frac{1}{2}F^i_{\alpha\beta}\Delta_i N_\alpha N_\beta + \frac{1}{2}m_\alpha N_\alpha N_\alpha, \tag{4.18}
$$

where

$$
F^i_{\alpha\beta} = (V_{11}O_{1i}+V_{13}O_{2i}+V_{14}O_{3i})f^1_{\alpha\beta} + (V_{21}O_{1i}+V_{23}O_{2i}+V_{24}O_{3i})f^2_{\alpha\beta}, \tag{4.19}
$$

we use Roman letters (i, j) to indicate the generation the Higgs and Greek letters (α, β) to indicate the generation of right handed neutrinos. Without losing generality we work at the mass basis of right handed neutrinos. The Yukawa coupling $F^i_{\alpha\beta}$ is generally complex. The F-terms derived are

$$F_{\Delta_i} = \mu_i \Delta_i + \frac{1}{2} F^i_{\alpha\beta} \tilde{N}_\alpha \tilde{N}_\beta$$
$$F_{\tilde{N}_\alpha} = F^i_{\alpha\beta} \Delta_i \tilde{N}_\beta + m_\alpha \tilde{N}_\alpha, \tag{4.20}$$

and we can read the Feynman rules from them.

The lepton number violation is from the CP violation decays of Δ_i and the super partners to right handed neutrinos and super partners. We assume the CP violation is only at this stage of decays. At the next stage of decays, lepton numbers transfer to leptons and sleptons completely. Since the ferminonic right handed neutrinos decay both to leptons and anti-leptons, the decays don't lead to lepton number violation, therefore we only consider the decays which have at least on bosonic right handed neutrino in the decay products. The tree level decays which can lead to lepton asymmetry are listed in Fig. refdecaymodes.

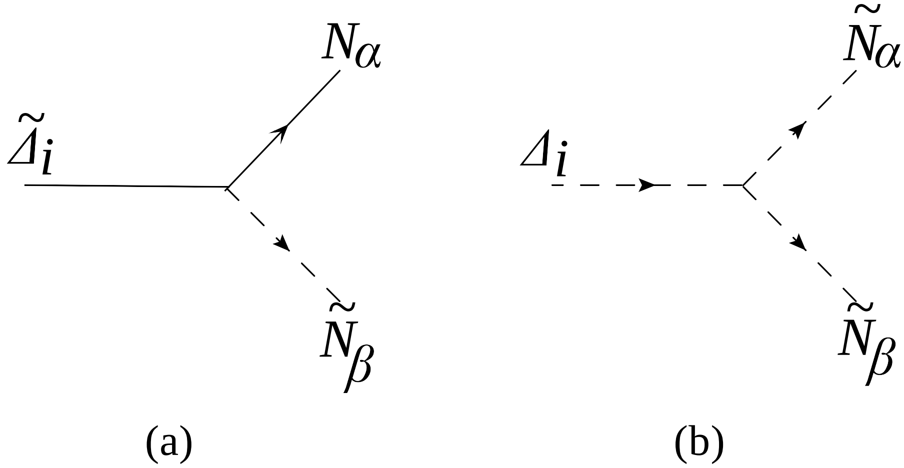

Figure 4.1: Tree level decays contributing lepton number asymmetry

CP violation necessary to the final lepton number asymmetry is from the interferences of tree level processes and loop level processes. At one loop there are two types of diagrams, self-energy and vertex diagrams. The CP violations from the interferences of these two types of diagrams with tree level diagrams are respectively ϵ type and ϵ' type. We list the self-energy diagrams in Fig. 4.2.

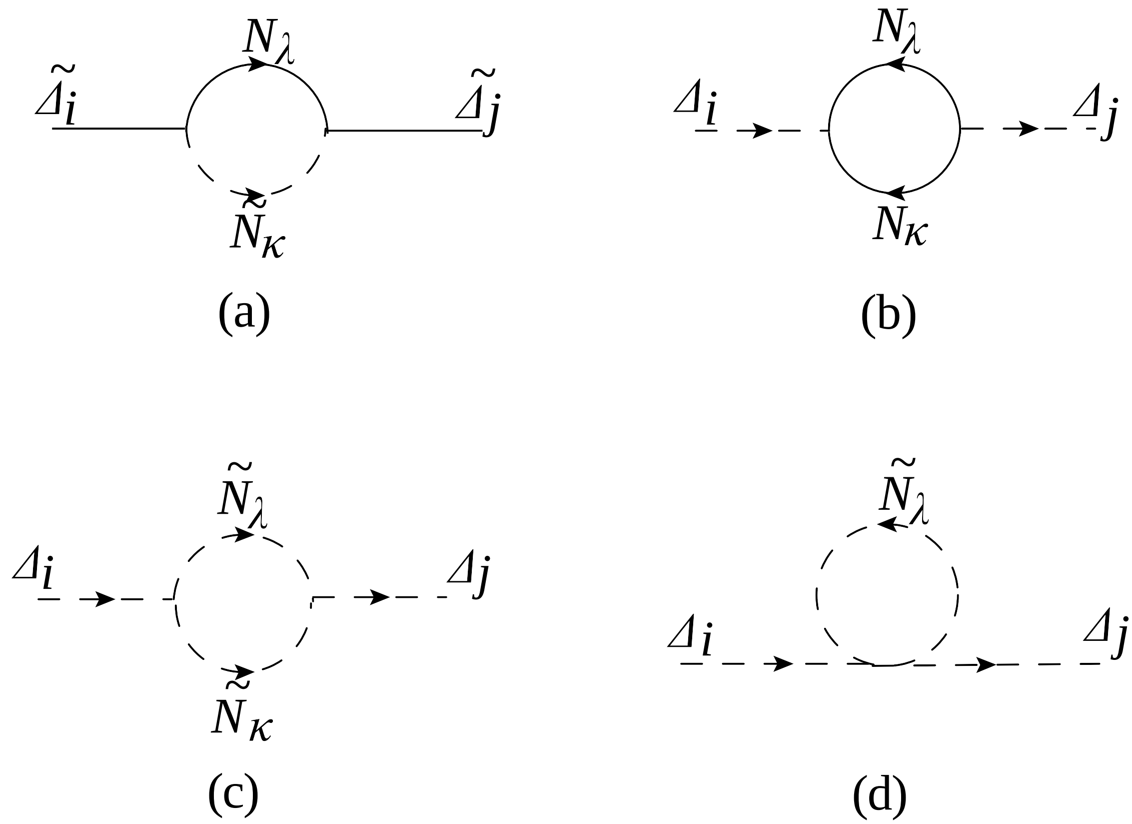

Figure 4.2: One loop self energy diagrams

If the masses of the decaying particles are nearly degenerate the ϵ type CP violation is much larger than the $\epsilon^{'}$ type. We don't make this assumption here, rather study the more general case by considering the contribution of vertex correction. We list the one loop vertex diagrams in Fig. 4.3.

4.4 CP violation

In this part we calculate the magnitude of CP violation necessary for leptogenesis. In the calculations we have freely used the results of [80]. In addition we have taken the approximation that $m_\alpha \ll \mu_i$. This is not an unreasonable choice, but simplifies the calculation. We have gained the results of the

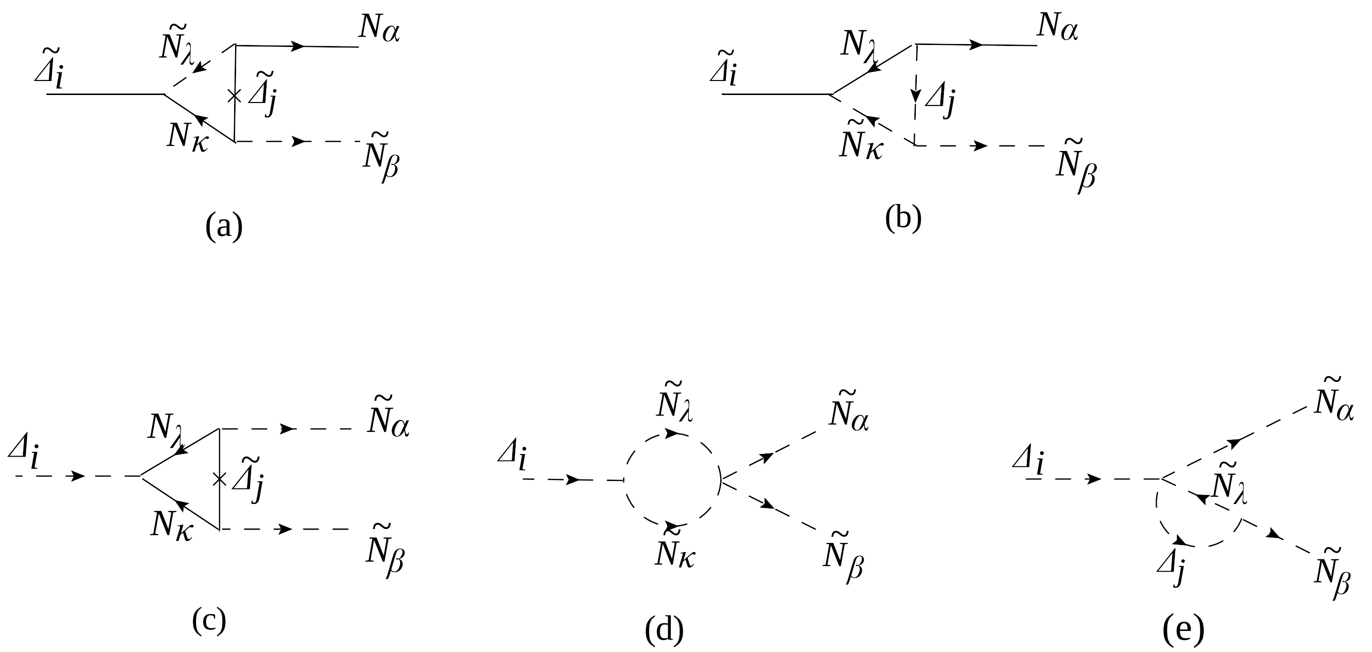

Figure 4.3: One loop vertex diagrams

absorptive parts.

$$\begin{aligned}
\Sigma^{\text{abs}}_{a\ \ ij}(\not p) &= -\not p\left[A_{ij}P_R + A^*_{ij}P_L\right]\cdot 2,\\
\Pi^{\text{abs}}_{b\ \ ij}(p^2) &= 2A^*_{ij}p^2\cdot 2,\\
\Pi^{\text{abs}}_{c\ \ ij}(p^2) &= 2A_{ij}\mu_i\mu_j\cdot 2,\\
\Pi^{\text{abs}}_{d\ \ ij}(p^2) &= 0,
\end{aligned} \tag{4.21}$$

with $A_{ij} = \frac{1}{128\pi}\text{tr}\left(F^{i\dagger}F^j\right)$ and $P_{L,R} = \frac{1\pm\gamma_5}{2}$. The number behind the "·" is the symmetry factor of the corresponding diagram. We suppose the lightest Δ field is much lighter than the heavier ones and is the one decays out of equilibrium. The ϵ type CP violation factor of the decay of fermionic $\tilde{\Delta}_1$ is generated by the interference of the tree level process Fig. 4.1-a and the loop process Fig. 4.2-1.

$$\epsilon^1_{\text{F}} = \sum_{\alpha,\beta}\frac{|T^{\alpha\beta}_{\tilde{\Delta}_1}| - |\bar{T}^{\alpha\beta}_{\tilde{\Delta}_1}|}{|T^{\alpha\beta}_{\tilde{\Delta}_1}| + |\bar{T}^{\alpha\beta}_{\tilde{\Delta}_1}|}, \tag{4.22}$$

with

$$T^{\alpha\beta}_{\tilde{\Delta}_1} = \frac{1}{2}F^{1*}_{\alpha\beta}\bar{u}_{N_\alpha}P_L u_{\tilde{\Delta}_1} - i\frac{1}{2}\sum_j F^{j*}_{\alpha\beta}\bar{u}_{N_\alpha}P_L \frac{\Sigma^{\rm abs}_{j1}(\not{p})}{\not{p} - \mu_j + i\Sigma^{\rm abs}_{jj}(\not{p})}u_{\tilde{\Delta}_1},$$
$$\bar{T}^{\alpha\beta}_{\tilde{\Delta}_1} = \frac{1}{2}F^{1}_{\alpha\beta}\bar{u}_{N_\alpha}P_L u_{\tilde{\Delta}_1} - i\frac{1}{2}\sum_j F^{j}_{\alpha\beta}\bar{u}_{N_\alpha}P_L \frac{\bar{\Sigma}^{\rm abs}_{j1}(\not{p})}{\not{p} - \mu_j + i\bar{\Sigma}^{\rm abs}_{jj}(\not{p})}u_{\tilde{\Delta}_1}, \tag{4.23}$$

where

$$\begin{aligned} \Sigma^{\rm abs}_{ij}(\not{p}) = \Sigma^{\rm abs}_{a\ \ ij}(\not{p}) &= A_{ij}\not{p}P_L + A^*_{ij}\not{p}P_R \\ \text{and}\ \ \bar{\Sigma}^{\rm abs}_{ij}(\not{p}) &= A_{ij}\not{p}P_R + A^*_{ij}\not{p}P_L, \end{aligned} \tag{4.24}$$

which are the charge conjugate self-energies, similar with [63–65]. Generally the masses of $\tilde{\Delta}$'s are not degenerate (no resonant CP violation), with good approximation, we get

$$\epsilon^1_{\rm F} = -\sum_{j\neq 1}\frac{{\rm Im}\left({\rm tr}(F^{1\dagger}F^j)^2\right)}{32\pi\cdot{\rm tr}(F^{1\dagger}F^1)}\frac{\mu_1\mu_j}{\mu_1^2-\mu_j^2} \tag{4.25}$$

The CP violation of bosonic Δ_1 decay is from the interference of Fig. 4.1-b and Fig. 4.2-2, 3.

$$\epsilon^1_{\rm B} = \sum_{\alpha,\beta}\frac{|T^{\alpha\beta}_{\Delta_1}| - |\bar{T}^{\alpha\beta}_{\Delta_1}|}{|T^{\alpha\beta}_{\Delta_1}| + |\bar{T}^{\alpha\beta}_{\Delta_1}|}, \tag{4.26}$$

with

$$T^1_{\alpha\beta} = \frac{1}{2}\mu_1 F^{1*}_{\alpha\beta} - i\frac{1}{2}\sum_j \frac{\Pi^{\rm abs}_{j1}}{\mu_1^2-\mu_j^2+i\Pi^{\rm abs}_{jj}}\mu_j F^{j*}_{\alpha\beta},$$
$$\bar{T}^1_{\alpha\beta} = \frac{1}{2}\mu_1 F^{1}_{\alpha\beta} - i\frac{1}{2}\sum_j \frac{\Pi^{\rm abs*}_{j1}}{\mu_1^2-\mu_j^2+i\Pi^{\rm abs}_{jj}}\mu_j F^{j}_{\alpha\beta}, \tag{4.27}$$

where

$$\Pi^{\rm abs}_{ij}(p^2) = \Pi^{\rm abs}_{b\ \ ij}(p^2) + \Pi^{\rm abs}_{c\ \ ij}(p^2). \tag{4.28}$$

We have

$$\epsilon^1_{\rm B} = \sum_{j\neq 1}\frac{{\rm Im}\left({\rm tr}(F^{1\dagger}F^j)^2\right)}{32\pi\cdot{\rm tr}(F^{1\dagger}F^1)}\frac{\mu_1\mu_j}{\mu_1^2-\mu_j^2} \tag{4.29}$$

under the approximation that the masses of decaying particles are far from degenerate.

The ϵ' type of CP violation is generated through the interference of tree level processes and vertex corrections. The diagrams 1 of Fig. 4.3 interfere with Fig. 4.1-a, b, generating CP violation in fermionic $\tilde{\Delta}$'s decays, diagrams Fig. 4.3-c, d, e interfere with Fig. 4.1-b, generating CP violation in bosonic Δ's decays. Under the approximation that $m_\alpha \ll \mu_1$, we have the absorptive parts of the diagrams in Fig. 4.3:

$$\begin{aligned}
\Lambda_a^{\rm abs}(p_1,p_2) &= \sum_j \frac{\mu_j}{16\pi}\cdot\frac{1}{8}(F^{j\dagger}F^iF^{j\dagger})_{\alpha\beta}\cdot\left[1-\left(1+\frac{\mu_j^2}{p_1^2}\right)\ln\left(1+\frac{p_1^2}{\mu_j^2}\right)\right]\frac{\not{p}_1}{p_1^2}P_L\cdot 2,\\
\Lambda_b^{\rm abs}(p_1,p_2) &= \sum_j \frac{\mu_j}{16\pi}\cdot\frac{1}{8}(F^{j\dagger}F^iF^{j\dagger})_{\alpha\beta}\cdot\left[1-\frac{\mu_j^2}{p_1^2}\ln\left(1+\frac{p_1^2}{\mu_j^2}\right)\right]\frac{\not{p}_1}{p_1^2}P_L\cdot 2,\\
\Lambda_c^{\rm abs}(p_1,p_2) &= -\sum_j \frac{\mu_j}{16\pi}\cdot\frac{1}{8}(F^{j\dagger}F^iF^{j\dagger})_{\alpha\beta}\cdot\ln\left(1+\frac{p_1^2}{\mu_j^2}\right)\cdot 2,\\
\Lambda_d^{\rm abs}(p_1,p_2) &= \sum_j \frac{1}{16\pi}\cdot\frac{1}{8}\mu_i{\rm tr}(F^{i\dagger}F^j)F^{j*}_{\alpha\beta}\cdot 2,\\
\Lambda_e^{\rm abs}(p_1,p_2) &= 0,
\end{aligned} \tag{4.30}$$

where p_1 and p_2 are respectively the momentums of $N_\alpha(\tilde{N}_\alpha)$ and the decaying $\Delta_i(\tilde{\Delta}_i)$.

The ϵ' type CP violation parameter of fermionic $\tilde{\Delta}_1$ decay is

$$\epsilon'^1_{\rm F} = \sum_{\alpha\beta}\frac{|T'^{\alpha\beta}_{\tilde{\Delta}_1}|^2-|\bar{T}'^{\alpha\beta}_{\tilde{\Delta}_1}|^2}{|T'^{\alpha\beta}_{\tilde{\Delta}_1}|^2+|\bar{T}'^{\alpha\beta}_{\tilde{\Delta}_1}|^2}, \tag{4.31}$$

with

$$T'^{\alpha\beta}_{\tilde{\Delta}_1} = \frac{1}{2}F^{1*}_{\alpha\beta}\bar{u}_{N_\alpha}P_L u_{\tilde{\Delta}_1}+\bar{u}_{N_\alpha}\Lambda_{\rm F}u_{\tilde{\Delta}_1}, \tag{4.32}$$

where $\Lambda_{\rm F}$ is the one loop vertex function, only the absorptive part has contribution to CP violation. The absorptive part of $\Lambda_{\rm F}$ is

$$\Lambda_{\rm F}^{\rm abs}(p_1,p_2) = \Lambda_a^{\rm abs}(p_1,p_2). \tag{4.33}$$

We get

$$\epsilon'^1_{\rm F} = \sum_{j\neq 1}\frac{{\rm Im}\left({\rm tr}(F^{1\dagger}F^j)^2\right)}{32\pi\cdot{\rm tr}(F^{1\dagger}F^1)}\left[2-\left(1+2\frac{\mu_j^2}{\mu_1^2}\right)\ln\left(1+\frac{\mu_1^2}{\mu_j^2}\right)\right]\frac{\mu_j}{\mu_1}. \tag{4.34}$$

The ϵ' type CP violation parameter of bosonic Δ_1 decay is

$$\epsilon'^{1}_{\mathrm{B}} = \sum_{\alpha\beta} \frac{|T'^{\alpha\beta}_{\Delta_1}|^2 - |\bar{T}'^{\alpha\beta}_{\Delta_1}|^2}{|T'^{\alpha\beta}_{\Delta_1}|^2 + |\bar{T}'^{\alpha\beta}_{\Delta_1}|^2} \tag{4.35}$$

where

$$T'^{\alpha\beta}_{\Delta_1} = \frac{1}{2}\mu_1 F^{1*}_{\alpha\beta} + \Lambda_{\mathrm{B}}, \tag{4.36}$$

as the fermionic decay, only the absorptive part of Λ_{B} has contribution to CP violation, and we have

$$\Lambda^{\mathrm{abs}}_{\mathrm{B}}(p_1, p_2) = \Lambda^{\mathrm{abs}}_{b}(p_1, p_2) + \Lambda^{\mathrm{abs}}_{c}(p_1, p_2). \tag{4.37}$$

We get

$$\epsilon'^{1}_{\mathrm{B}} = -\sum_{j\neq 1} \frac{\mathrm{Im}\left(\mathrm{tr}(F^{1\dagger}F^{j})^2\right)}{32\pi \cdot \mathrm{tr}(F^{1\dagger}F^{1})} \ln\left(1 + \frac{\mu_1^2}{\mu_j^2}\right)\frac{\mu_j}{\mu_1}. \tag{4.38}$$

For simplicity, we give an order of magnitude analysis of the CP violation parameter. To simplify the analysis, we choose

$$\lambda_{11}/\lambda_{12} \sim x, \qquad \lambda_{12} \sim \lambda_{21} \ll 1, \qquad \lambda_{11} M_2 \gg M_1. \tag{4.39}$$

From this choice of parameter and (4.5)(4.6) $s \sim x M_2 \lambda_{12}$, $v_2/v_1 \sim \bar{v}_2/\bar{v}_1 \sim x$. From (4.15)(4.16), we get,

$$\mu_{11} \sim M_2(\lambda_{12} + x)x, \qquad \mu_{22} \sim M_2, \qquad \mu_{33} \sim M_2, \tag{4.40}$$

$$\mu_{12} \sim M_2(\lambda_{12} + x), \quad \mu_{13} \sim M_2(\lambda_{12} + x), \quad \mu_{13} \sim M_2(\lambda_{12} + x). \tag{4.41}$$

Masses of right handed neutrinos are generated through Higgs mechanism. For satisfying the out of equilibrium condition of Δ_1 decay, $F^1_{\alpha\beta} \sim 10^{-5}$ for $\mu_1 \sim 10^6$ GeV, which means that the VEV of Δ_1 contributes very little to the right handed neutrino masses. The constraint to the Δ_2 Yukawa coupling $F^2_{\alpha\beta}$ is, from (4.5), (4.6), (4.10)

$$\sqrt{\frac{\mu_s s + \kappa s^2}{\lambda_{11}}}\, x F^2_{\alpha\beta} \sim m_\alpha. \tag{4.42}$$

Typically we choose $x \sim 10^{-2}$, $\lambda_{12} \sim 10^{-1}$, $M_2 \sim 10^8$ GeV, and furthermore we choose $\kappa \sim 10^{-2}-10^{-3}$, $\mu_s \sim 10^4 - 10^5$ GeV, so we can have $m_\alpha \sim 10^4 - 10^5$ GeV for $F^2_{\alpha\beta} \sim 10^{-1}$. Similar for $F^3_{\alpha\beta}$. With this set of parameters we find $\mu_1 \sim 10^6$ GeV and $\mu_2 \sim \mu_3 \sim 10^8$ GeV. CP violation doesn't distinguish the types of CP violation. The total CP violation parameter is

$$\epsilon^1_{\tilde{N}} = \epsilon^1_{\rm F} + 2\epsilon^1_{\rm B} + \epsilon'^1_{\rm F} + 2\epsilon'^1_{\rm B}, \tag{4.43}$$

we have counted the number of $\tilde{N}$ which is effective to generate the final lepton number asymmetry in the decay products. With the optimal choice of the phase of Yukawa couplings we get $\epsilon^1_{\tilde{N}} \sim 10^{-5}-10^{-6}$ with the mentioned parameters above.

4.5 Departure from Equilibrium

In the last section we estimated the amount of CP violation in the decays of the $B - L$ symmetry breaking scalars Δ_i and $\bar{\Delta}_i$ $(i = 1, 2)$ into $\tilde{N}$ and $\tilde{N}^*$. If the decays of the lightest physical Δ_1 satisfy the out-of-equilibrium condition,

$$\Gamma^\Delta_1 = \frac{1}{8\pi} \sum_{\alpha,\beta} \left(F^{1*}_{\alpha\beta} F^1_{\alpha\beta} \right) \mu_1 \ < \ H(T) = 1.66\sqrt{g_*}\frac{T^2}{M_{Pl}} \,, \qquad \text{at } T \sim \mu_1 \tag{4.44}$$

then the decays of Δ_1 would generate an asymmetry in $\tilde{N}$ and $\tilde{N}^*$. As mentioned in the previous section, this is satisfied for $F^1_{\alpha\beta} \sim 10^{-5}$ and $\mu_1 \sim 10^6$ GeV. Here $H(T)$ is the Hubble constant at the temperature T, M_{Pl} is the Planck scale and g_* is the effective degrees of freedom.

Below the temperature, $T \sim \mu_1 \sim 10^6$ GeV, the number density of Δ_1 will fall off exponentially. Since supersymmetry is unbroken at this scale, this will also mean that there is an asymmetry in the number density of the particles and antiparticles of the N-s [74], before they decay. In other words, the physical Majorana particles will have unequal combination of N and N^c, reflected by Majorana phases and complex decay widths. If the lepton number violating interactions, produced by the Majorana masses are not in equilibrium, a nonvanishing chemical potential μ_N will be generated for N during the decays of Δ_1.

We shall now assume that both the particles N-s decays very weakly, so that there is no fast lepton number violation after the decays of Δ_1, which can erase the asymmetry in $\tilde{N}$-s and $\tilde{N}^*$-s. In other words, we require that N-s satisfy the out-of-equilibrium condition during the period $\mu_1 > T > m_\alpha$. This can be stated as

$$\Gamma_\alpha^N = \frac{1}{16\pi} \left(Y^{\alpha *} Y^\alpha \right) \; m_\alpha \; < \; 1.66 \sqrt{g_*} \frac{T^2}{M_{Pl}} . \qquad \text{at } T \sim m_\alpha \tag{4.45}$$

with $\alpha = 1, \; 2, \; 3$. This condition is satisfied with the Yukawa couplings $Y^\alpha \sim 10^{-6}$.

The smallness of the Yukawa couplings Y^α will imply that there cannot be enough CP violation in the decays of N-s, which can contribute towards the lepton asymmetry of the universe. So, around the temperature $T \sim m_\alpha$, the only lepton asymmetry available is the asymmetry stored in $\tilde{N}$-s and $\tilde{N}^*$-s.

Below the temperature $T \sim m_1$, say at $T \sim 0.1 \; m_1 \sim 10^3$ GeV, although most of the $\tilde{N}$-s and $\tilde{N}^*$-s have decayed away and their number densities have fallen exponentially, the difference in the number densities of $\tilde{N}$-s and $\tilde{N}^*$-s will remain unchanged as there are no fast lepton number violation at this stage. The lepton number violating decays of N_α is suppressed by T/m_α, where m_α is also the scale of lepton number violation, and still satisfies the out-of-equilibrium condition

$$\Gamma_\alpha^N = \frac{1}{16\pi} \left(Y^{\alpha *} Y^\alpha \right) \; m_\alpha \; \left(\frac{T}{m_\alpha} \right) \; < \; \sqrt{1.66 g_*} \frac{T^2}{M_{Pl}} . \qquad \text{at } T \sim 0.1 \; m_\alpha \tag{4.46}$$

But the lepton number conserving decays of N_α into light leptons and the usual standard model Higgs doublet is not suppressed by the lepton number violating scale. Thus the equilibrium decays of $\tilde{N}$-s and $\tilde{N}^*$-s will convert the lepton asymmetry stored in the difference in number densities of $\tilde{N}_\alpha$ and $\tilde{N}_\alpha^*$ into a lepton asymmetry of the light left-handed leptons, which in turn, will generate the baryon asymmetry of the universe in the presence of the sphalerons.

Thus the asymmetry in $\tilde{N}$-s and $\tilde{N}^*$-s, created during the decays of the Δ_1, will generate the baryon asymmetry of the universe. The amount of asymmetry thus depends only on the couplings F^1, and not on the Yukawa couplings Y^α, which enters in the light neutrino mass matrices. Thus the amount of generated baryon asymmetry is not related to the light neutrino masses, and hence, the DI bound is absent in this model.

To estimate the amount of lepton asymmetry at the different stages, we need to solve the Boltzmann equations. The number density n_Δ of the scalar Δ_1 will evolve following the Boltzmann equation

$$\frac{dn_\Delta}{dt} + 3Hn_\Delta = -\Gamma_1^\Delta(n_\Delta - n_\Delta^{eq}), \tag{4.47}$$

where n_Δ^{eq} is the equilibrium distribution of Δ_1. The first term in the left represents the time evolution of the number density of Δ_1, while the second term appears for the expansion of the universe. The right-hand side gives the departure from equilibrium distribution during the decays and inverse decays of Δ_1. Given the number densities of Δ_1, we can now determine the time evolution of the asymmetry in $\tilde{N}$-s and $\tilde{N}^*$-s, which we write as $n_{\tilde{N}}$. The corresponding Boltzmann equation will be given by

$$\frac{dn_{\tilde{N}}}{dt} + 3Hn_{\tilde{N}} = \epsilon^1\Gamma_1^\Delta(n_\Delta - n_\Delta^{eq}) - \left(\frac{n_{\tilde{N}}}{n_\gamma}\right) n_\Delta^{eq}\Gamma_1^\Delta - 2n_\gamma n_{\tilde{N}}\langle\sigma_v\rangle, \tag{4.48}$$

where the first term in the right corresponds to generation of an asymmetry in $\tilde{N}$-s and $\tilde{N}^*$-s due to the CP violation in the decays and inverse decays of Δ_1, while the other terms deplete the generated asymmetry due to lepton number violating interactions.

The lepton asymmetry will come from the decays of the right-handed neutrinos δ (which is negligible in this case because Y^α is too small) and also from the asymmetry stored in $\tilde{N}_\alpha$. We can thus write down the Boltzmann equation for the generation of lepton asymmetry as

$$\frac{dn_l}{dt} + 3Hn_l = \epsilon^1\Gamma_1^\Delta(n_\Delta - n_\Delta^{eq}) + \delta\Gamma_\alpha^N(n_N - n_N^{eq}) - \left(\frac{n_l}{n_\gamma}\right) n_\Delta^{eq}\Gamma_1^\Delta - 2n_\gamma n_{\tilde{N}}\langle\sigma_v\rangle. \tag{4.49}$$

Since the out-of-equilibrium condition is satisfied by the decays of Δ_1 and $\tilde{N}$-s, the amount of asymmetry will be given by the amount of CP violation ϵ^1.

4.6 Leptogenesis

The CP violating decays of Δ and $\tilde{\Delta}$ generate an asymmetry of $\tilde{N}$ over $\tilde{N}^*$. Following the arguments in the previous section, this asymmetry will be converted into a baryon asymmetry before the electroweak phase transition. This may be understood from the following arguments also. At the next stage of decay, $\tilde{N}$ decays to $L\tilde{H}_u$ or $\tilde{L}^*H_u^*$. At zero temperature, these decays have the same width because of SUSY, and the lepton asymmetries from the decays of $\tilde{N}$ and $\tilde{N}^*$ cancel each other. When $T \neq 0$, the cancelation is partial due to the thermal effects [81]. This is because of the effective mass acquired by particle excitations inside the early universe plasma and because of final-state Fermi blocking and Bose stimulation. We assume the decays of $\tilde{N}$ and $\tilde{N}^*$ are CP conserved (this is because the Yukawa couplings Y^i are extremely small), the number asymmetry of Δ and $\tilde{\Delta}$ is fully converted to lepton number asymmetry through the temperature effects. Then we have

$$\frac{\tilde{n}_B}{s} \simeq -8.05 \cdot 10^{-4} \epsilon^1_{\tilde{N}} \eta, \tag{4.50}$$

where we have taken into account an effective number of degree of freedom $g_* = 240$ with 3 right handed neutrinos and number of Higgs doublet 2. η is the efficiency factor, it includes the nonequilibrium sneutrino density, the partial wash-out, and the temperature effects. The value of η depends on the $\tilde{m} \simeq v_u^2 (Y_\nu Y_\nu^\dagger)/m_\alpha$ and the right handed neutrino mass m_α. The efficiency factor reaches its maximal value $\eta \approx 0.1$ for $\tilde{m} \approx 10^{-3}$ eV. With the choice of parameters realizing $\epsilon^1_{\tilde{N}} \sim 10^{-5} - 10^{-6}$, the baryon asymmetry $\frac{n_B}{s}$ can be as large as the observed value of $(8.75 \pm 0.23) \cdot 10^{-11}$.

For completeness we shall now relate the amount of CP asymmetry in $\tilde{N}$ and $\tilde{N}^*$ to the amount of baryon asymmetry before the electro-weak phase transition. We consider all the particles to be ultra-relativistic, which is the case above the electro-weak scale, and express the difference between the number of particles (n_+) and antiparticles (n_-) in terms of their chemical potential μ as

$$n_+ - n_- = n_d \frac{gT^3}{6} \left(\frac{\mu}{T}\right), \tag{4.51}$$

where $n_d = 2$ for bosons and $n_d = 1$ for fermions. For all the interactions in equilibrium, the chemical potentials of all the fields appearing in the interaction will get related. Using the interactions allowed by the standard model, we can relate the chemical potentials of the quarks, leptons, gauge bosons and the Higgs scalar and express them in terms of four independent quantities

$$\mu_0 = \mu_0^H; \quad \mu_W; \quad \mu_u = \mu_{uL}; \quad \mu_\nu = \mu_{\nu e L} + \mu_{\nu \mu L} + \mu_{\nu \tau L}. \tag{4.52}$$

In our model we also include the chemical potential of the right-handed neutrinos μ_N, which corresponds to the difference in the number densities of $\tilde{N}$ and $\tilde{N}^*$. Since the lepton number violating interactions of N, given by the Majorana masses, are not in equilibrium, and an asymmetry in $\tilde{N}$ over

$\tilde{N}^*$ has been created in the decays of Δ_1, μ_N is nonvanishing. However, the lepton number conserving Yukawa interactions of N are in equilibrium before the electro-weak phase transition, implying

$$\mu_N = \mu_\nu + \mu_0 \,. \tag{4.53}$$

We shall now use the sphaleron condition for three generations,

$$9\mu_u + 6\mu_W + \mu_\nu = 0. \tag{4.54}$$

and express the baryon number (B), lepton numbers (L), electric charge (Q) and the hypercharge (Y) number densities in terms of these independent chemical potentials,

$$B = 12\mu_u + 6\mu_W \tag{4.55}$$

$$L = 3\mu_\nu + 6\mu_W - 3\mu_0 \tag{4.56}$$

$$Q = 24\mu_u + (12 + 2m)\mu_0 - (4 + 2m)\mu_W \tag{4.57}$$

$$Q_3 = -(10 + m)\mu_W \tag{4.58}$$

where m is the number of Higgs doublets ϕ. We regard $m = 2$, as we counted the effective degrees of freedom of the system when leptogenesis happens.

At temperatures above the electro-weak phase transition, $T > T_c$, both Q and Q_3 vanish, giving us the relations $\mu_0 = -12/(6+m)\mu_u$ and $\mu_W = 0$, whereas the sphaleron transition gives $\mu_\nu = -9\mu_u$. Including the interactions of the right-handed neutrinos, we can now express the amount of baryon asymmetry in terms of the $B - L$ asymmetry, which in turn can be expressed in terms of the chemical potential of the right-handed neutrinos as

$$B = \frac{24 + 4m}{66 + 13m}(B - L) \tag{4.59}$$

$$(B - L) = -\frac{66 + 13m}{22 + 3m}\mu_N \tag{4.60}$$

Thus the amount of baryon asymmetry is directly related to the amount of N asymmetry, given by the chemical potential μ_N, which is generated in the decays of Δ_1 and stored as an asymmetry in $\tilde{N}$ over $\tilde{N}^*$.

4.7 Summary

In summary, we studied the question of leptogenesis in a supersymmetric extension of the standard model, in which $B-L$ is a gauge symmetry. Two pairs of Higgs scalar fields that break the $B-L$ symmetry can produce a nonvanishing chemical potential for the right-handed neutrinos, given by an asymmetry in $\tilde{N}$ and $\tilde{N}^*$. Before the electro-weak phase transition, this asymmetry can be transferred to a baryon asymmetry of the universe. Since the final baryon asymmetry depends only on the couplings of the Higgs scalars, and not on the Yukawa couplings that give the Dirac masses to the neutrinos, the amount of generated baryon asymmetry is now independent of the neutrino masses. As a result, the asymmetry can be created in this model at a much lower scale without conflicting with the gravitino problem and thus evading the DI bound.

CHAPTER 5

FlAVOR VIOLATION IN SUPERSYMMETRIC Q_6 MODEL

5.1 Introduction

The gauge interactions of the standard model (SM) fermions are invariant under separate $U(3)_L \times U(3)_R$ transformations. This global symmetry is broken explicitly by the fermion Yukawa couplings. For the light fermions violation of this symmetry is small, being proportional to their masses. This feature has played a crucial role in the success of the SM in the flavor sector. In extensions of the SM this property is generally lost, often leading to excessive flavor changing neutral current (FCNC) processes.

A case in point is the supersymmetric standard model which is the subject of this paper. While the gauge interactions of the SUSY SM respect the $U(3)_L \times U(3)_R$ global symmetry, there are new sources of violation of this symmetry, in the soft SUSY breaking sector. Indeed, generic soft SUSY breaking scenarios lead to excessive FCNC in processes such ad $K^0 - \overline{K^0}$ mixing, $B^0 - \overline{B^0}$ mixing, $D^0 - \overline{D^0}$ mixing, and flavor changing leptonic decays such as $\mu \to e\gamma$ [83]. This problem is most severe in the $K^0 - \overline{K^0}$ system. SUSY box diagrams involving gluino and squarks modify the successful SM prediction for ΔM_K and ϵ_K, leading to the following constraints for the real and imaginary parts of the amplitude [84]:

$$\left|(\text{Re},\ \text{Im})(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}\right|^{1/2} \leq (9.6 \cdot 10^{-4},\ 1.3 \cdot 10^{-4}) \left(\frac{\tilde{m}}{500\ \text{GeV}}\right) . \tag{5.1}$$

Here $(\delta_{AB})_{ij} = (m^2_{AB})_{ij}/\tilde{m}^2$ is a flavor violating squark mass insertion parameter, for $(A, B) = (L, R)$, with $\tilde{m}$ being the average mass of the relevant squarks ($\tilde{d}$ and $\tilde{s}$ in this case). For this estimate the gluino mass was assumed to equal the average squark mass. Now, the natural magnitude of the mixing parameters $(\delta^d_{LL})_{12}$ and $(\delta^d_{RR})_{12}$, in the absence of additional symmetries, should be of order the Cabibbo angle, ~ 0.2. Since the parameters $(\delta_{AB})_{ij}$ split the masses of the squarks, one sees from Eq. (5.1) that a high degree of squark mass degeneracy is needed for consistency.

Analogous limits from $B_d^0 - \overline{B_d^0}$ mixing are less severe, as given by [85]:

$$\left|(\text{Re},\ \text{Im})(\delta^d_{LL})_{13}(\delta^d_{RR})_{13}\right|^{1/2} \leq (2.1 \cdot 10^{-2},\ 9.0 \cdot 10^{-3}) \left(\frac{\tilde{m}}{500\ \text{GeV}}\right) . \tag{5.2}$$

Note that the natural value of this mixing parameter, in the absence of other symmetries, is $V_{ub} \sim 3 \times 10^{-3}$. The constraints from Eq. (5.2) are well within limits. $B_s - \overline{B}_s$ mixing provides even weaker constraints.

It can be argued that a natural explanation for solving this problem is to enhance the symmetry of the SUSY SM by assuming a non–Abelian symmetry G (a subgroup of the $U(3)_L \times U(3)_R$) that pairs the first two families into a doublet, with the third family transforming trivially [87].[1] Invariance under G will then lead to degeneracy of squarks, as needed for phenomenology. A variety of such models have been proposed in the literature [87], [88], [89], [90], [91]. In Ref. [87], $SU(2)$ family symmetry and its variants were proposed to solve the SUSY FCNC problem. If the symmetry is global, one has to deal with the Goldstone bosons associated with its spontaneous breaking. Global symmetries are susceptible to violations from quantum gravity. Local gauge symmetries are more natural, but in the SUSY context there would be new FCNC processes arising from the family $SU(2)$ D–terms [92]. Exceptions to this generic problem are known to exist [88].

A more natural solution to the problem is perhaps to choose G to be a non–Abelian discrete symmetry group [89]. In this case there would be no D–term problem, since there are no gauge bosons associated with G. Spontaneous breaking of such symmetries will not lead to Goldstone bosons. If the symmetry breaking occurs before the inflationary era, such models should also be safe from potential cosmological domain wall problems. Such non–Abelian discrete symmetries have found application in understanding the various puzzles associated with the quark and lepton masses and mixing angles with or without supersymmetry [93], more recently for understanding the tri-bimaximal neutrino mixing

[1] Grouping all three families into an irreducible triplet representation of G is also possible. The large top quark mass however reduces the original $U(3)_L \times U(3)_R$ symmetry to $U(2)_L \times U(2)_R$, so we find it is easier to work with $(2+1)$ assignment.

pattern [94]. It would be desirable to find a symmetry that sheds light on the fermion mass and mixing puzzle, and at the same time solves the SUSY FCNC problem.

The supersymmetric standard model has another problem. In the flavor conserving sector CP violation is generically too large. Neutron and electron electric dipole moments (EDM) receive new contributions from SUSY loops. Unless the new phases in the SUSY breaking sector are small or conspire to be small, experimental limits on the EDM of the neutron (d_n), electron (d_e), and atoms will be violated by two to three orders of magnitude (depending on the squark and slepton masses) [95], [96]. The imaginary parts of the left–right squark mixing parameters must satisfy the constraints (from the experimental constraints $d_n < 6.3 \times 10^{-26}$ e-cm, $d_e < 4.3 \times 10^{-27}$ e-cm) [97]

$$\mathrm{Im}[(\delta^d_{LR})_{11}] \leq 1.9 \times 10^{-6} \left(\frac{\tilde{m}}{500\ \mathrm{GeV}}\right), \quad \mathrm{Im}[(\delta^e_{LR})_{11}] \leq 1.7 \times 10^{-7} \left(\frac{\tilde{m}}{100\ \mathrm{GeV}}\right), \tag{5.3}$$

assuming that the gluino/Bino has the same mass as the squark/slepton. Now, since these mixing parameters are expected to be suppressed by fermion helicity factors (but enhanced by the MSSM parameter $\tan\beta$) the natural values for these mixing parameters are of order $(1 \times 10^{-4},\ 3 \times 10^{-6})$ respectively, (for $\tan\beta = 10$ and assuming order one phases). This implies that the CP violating phases arising from the soft SUSY breaking sector must satisfy $\theta_d \leq 1/53$, $\theta_e \leq 1/63$ (for gluino (Bino) mass of 500 GeV (100 GeV). Why this is so, while the Kobayashi-Maskawa phase takes order one value, is the SUSY CP puzzle. It would be desirable to resolve this puzzle based on a symmetry principle in the same context where the SUSY FCNC problem is solved.

The purpose of this paper is to study a recently proposed SUSY model based on the non–Abelian symmetry group Q_6 [90] which addresses these issues. Q_6 is a finite subgroup of $SU(2)$ with twelve elements. Apart from providing a solution to the SUSY flavor problem, this class of models can also constrain the quark masses and mixing. It was shown in Ref. [90] that with the assumption of spontaneous (or soft) CP violation, there is a non-trivial relation between quark masses and mixing in this model. This sum rule was found to be consistent with experimental data.

A crucial aspect of the Q_6 model relevant for the quark mixing sum rule is that CP violation occurs either spontaneously or softly. This can help ameliorate the SUSY CP problem mentioned above. CP invariance requires that the gaugino masses, the μ terms and the trilinear A terms be all real. In the

Q_6 model of Ref. [90] it was found that there is a phase alignment mechanism that makes the phases of the sfermion mixing terms arising from the A–terms to align with the phases of the fermion masses. So SUSY CP violation is suppressed to a large extent. However, spontaneously induced complex VEVs do lead to non-zero contributions to EDM. Here we analyze these contributions. Since these complex VEVs are accompanied by the Higgsino μ terms, a simple solution to the problem is found by making the Higgsinos to be lighter than the squarks. Adequate suppression of EDM is obtained for $\mu \sim 100$ GeV, while squark masses are of order 500 GeV. This suggestion obviously has testable implications for physics that will be probed at the LHC.

The fermion mass matrices that allow for a non-trivial prediction and the phase alignment is a generalization of well studied models [98]. The mass matrices for up and down quarks and the charged leptons take the following form:

$$\mathbf{M} = \begin{pmatrix} 0 & C & 0 \\ \pm C & 0 & B \\ 0 & B' & A \end{pmatrix} . \tag{5.4}$$

The main feature of such mass matrices is that the phases can be factorized, i.e., $M = P \cdot M^0 \cdot Q$, with M^0 being real and P, Q being diagonal phase matrices. This feature, when combined with the Q_6 symmetry, has an the interesting consequence that CP violation induced by SUSY loops are suppressed. This will be discussed in more detail in Sec. 4.

The form of Eq. (5.4) can be obtained in renormalizable theories based on Q_6 symmetry. This requires the introduction of three families of Higgs doublets, which fall into $2+1$ representations of the Q_6 group, very much like the quarks and leptons. With multiple Higgs fields coupling to fermions, invariably there will be tree-level FCNC mediated by the Higgs bosons. The flavor changing Higgs couplings are not arbitrary, but can be computed in terms of the fermion masses and mixing. We will show that these FCNC processes are within acceptable range, provided that the Higgs boson masses lie in the $(1-5)$ TeV range (except of course for the standard model–like Higgs boson, which has a mass in the $(100-130)$ GeV range). While Higgsinos are naturally light in this scenario, in the bosonic sector only the lightest SM–like Higgs will be accessible to LHC experiments.

One of our major results is that non–standard CP violation is highly suppressed in this class of models. The phase factorizability of the fermion mass matrices implies that much of the SUSY induced CP violation is small. The structure of the Yukawa couplings in the model implies that the amplitudes for tree–level FCNC induced by neutral Higgs bosons are nearly real (see discussions in Sec. 5). While there can be significant new contributions to meson–anti-meson mixing, there is very little CP violation beyond the standard model.

Our analysis is similar in spirit to that of Ref. [91]. Our approach is slightly different, with some differences in analytical results, fits, spectrum, and conclusions. In particular, we have presented complete analytical results for the Higgs boson spectrum, and we have a new proposal to solve the SUSY EDM problem, which requires light Higgsinos. We have also derived generalized constraints on SUSY FCNC parameters for the $B_{d,s} - \overline{B}_{d,s}$ system appropriate for a (2+1) mass spectrum.

The plan of the paper is as follows. In Sec. 2 we describe the SUSY Q_6 model, lay out the parameter choice, and summarize the prediction for the quark sector. In Sec. 3 we analyze the Higgs potential involving the three pairs of Higgs doublets. We provide analytic expressions for the mass spectrum of Higgs bosons as well as numerical fits. Consistency of symmetry breaking and spontaneous CP violation will be established here. In Sec. 4 we address tree–level FCNC processes mediated by the heavy Higgs bosons. Sec. 5 is devoted to analysis of the SUSY flavor violation and EDM within the model. In Sec. 6 we conclude.

5.2 Supersymmetric Q_6 Model

Q_6 is the binary dihedral group, a subgroup of $SU(2)$, of order 12. It has the presentation

$$\{A, B; A^6 = E, B^2 = A^3, B^{-1}AB = A^{-1}\} \ . \tag{5.5}$$

The 12 elements of Q_6 can be represented as

$$\{E, A, A^2, ..., A^5, B, BA, BA^2, ..., BA^5\}. \tag{5.6}$$

In the two dimensional representation the generators are given in a certain basis by

$$\mathbf{A} = \begin{pmatrix} \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\,. \tag{5.7}$$

The irreducible representation of Q_6 fall into $2, 2^{'}, 1, 1^{'}, 1^{''}, 1^{'''}$, where the 2 is complex–valued but pseudo-real, while the $2^{'}$ is real valued. (Q_6 is the simplest group with two distinct doublet representations, which is very useful for model building.) The 1 and $1^{'}$ are real representations, while $1^{''}$ and $1^{'''}$ are complex conjugates to each other. The group multiplication rules are given as

$$1' \times 1' = 1,\ 1'' \times 1'' = 1,\ 1''' \times 1''' = 1',\ 1'' \times 1''' = 1,\ 1' \times 1''' = 1'',\ 1 \times 1'' = 1''' \tag{5.8}$$

$$2 \times 1' = 2,\ 2 \times 1'' = 2',\ 2 \times 1''' = 2',\ 2' \times 1' = 2',\ 2' \times 1'' = 2,\ 2' \times 1''' = 2 \tag{5.9}$$

$$2 \times 2 = 1 + 1' + 2',\ 2' \times 2' = 1 + 1' + 2',\ 2 \times 2' = 1'' + 1''' + 2 \tag{5.10}$$

The Clebsch–Gordon coefficients for these multiplication can be found in Ref. [90].

The fermions of all sectors (up–quark, down–quark, charged leptons) are assigned to $2+1$ representations of Q_6. The model assumes three families of Higgs bosons, which are also assigned to $2+1$ under Q_6. Their transformation properties are given by

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = 2, \quad \psi^c = \begin{pmatrix} -\psi_1^c \\ \psi_2^c \end{pmatrix} = 2', \quad \psi_3 = 1' \quad \psi_3^c = 1''', \tag{5.11}$$

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = 2', \quad H_3 = 1'''. \tag{5.12}$$

Here ψ generically denotes the fermion fields, and H denotes the up–type and the down–type Higgs fields which are doublets of $SU(2)_L$. Due to the constraints of supersymmetry, H^u and H_3^u couple only to up quarks, while H^d and H_3^d couple to down–type quarks and leptons. The Yukawa couplings of the model in the down quark sector arise from the superpotential

$$W = \alpha_d \psi_3 \psi_3^c H_3 + \beta_d \psi^T \tau_1 \psi_3^c H - \beta_d' \psi_3 \psi^{cT} i\tau_2 H + \delta_d \psi^T \tau_1 \psi^c H_3 + \text{h.c.} \tag{5.13}$$

with similar results for up–type quarks and charged leptons. This leads to the mass matrix for the down quarks given by

$$\mathbf{M_d} = \begin{pmatrix} 0 & \delta_d v_{d3} & \beta_d v_{d2} \\ -\delta_d v_{d3} & 0 & \beta_d v_{d1} \\ \beta'_d v_{d2} & \beta'_d v_{d1} & \alpha_d v_{d3} \end{pmatrix} . \tag{5.14}$$

Here $v_{d1},\ v_{d2},\ v_{d3}$ are the vacuum expectation values of $H^d_{1,2,3}$ fields, which break the Q_6 symmetry.

Now, the potential of the Q_6 model admits an unbroken S_2 symmetry which interchanges $H_1^{u,d} \leftrightarrow H_2^{u,d}$. This unbroken symmetry allows us to choose a VEV pattern

$$v_{u1} = v_{u2}, \quad v_{d1} = v_{d2}, \quad . \tag{5.15}$$

Consequently, a 45^0 rotation of the matrix in Eq. (5.14) in the 1-2 plane can be done both in the up and the down quark sectors without inducing CKM mixing. This will bring the mass matrices to the desired form of Eq. (5.4). By using the unbroken S_2 symmetry, we make a 45^0 rotation on the Higgs fields, $\hat{H}_{1,2} = (H_1 \pm H_2)/\sqrt{2}$, so that $\hat{H}_1$ acquires a VEV, while $\langle \hat{H}_2 \rangle = 0$. We shall drop the hat on these redefined fields, and simply denote the VEV of the redefined H_1 as v_1.

We assume that CP is a good symmetry of the Lagrangian, and that it is broken spontaneously by the VEVs of scalar fields. If the full theory contains SM singlet Higgs fields, spontaneous CP violation in the singlet sector will show up as soft CP violation in the Higgs doublet sector. Explicit examples of this sort have been given in Ref. [90]. For now we simply assume that the Yukawa couplings in Eq. (5.13) are real, and the CKM CP violation has a spontaneous origin, via complex VEVs of the Higgs doublet fields. We denote the phase of these (redefined) VEVs as

$$\Delta\theta_u = \arg(v_{u3}) - \arg(v_{u1}), \qquad \Delta\theta_d = \arg(v_{d3}) - \arg(v_{d1}). \tag{5.16}$$

We make an overall 45° rotation on the Q_6 doublets, Q, D^c and U^c, and then a phase rotations on these fields:

$$U \to P_u U, \qquad U^c \to P_{u^c} U^c \tag{5.17}$$

and similarly for D and D^c fields, where

$$P_{u,\ d} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta_{u,\ d}) & 0 \\ 0 & 0 & \exp(i\Delta\theta_{u,\ d}) \end{pmatrix},$$

$$P_{u^c,\ d^c} = \begin{pmatrix} \exp(-i2\Delta\theta_{u,\ d}) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(-i\Delta\theta_{u,\ d}) \end{pmatrix}. \tag{5.18}$$

This will make the originally complex mass matrices of Eq. (5.4) real, which we parameterize as

$$M_{u,\ d} = m^0_{t,\ b} \begin{pmatrix} 0 & q_{u,\ d}/y_{u,\ d} & 0 \\ -q_{u,\ d}/y_{u,\ d} & 0 & b_{u,\ d} \\ 0 & b'_{u,\ d} & y^2_{u,\ d} \end{pmatrix}. \tag{5.19}$$

These real mass matrices can be diagonalized by the following orthogonal transformations:

$$O^T_{u,\ d} M_{u,\ d} M^T_{u,\ d} O_{u,\ d} = \begin{pmatrix} m^2_{u,\ d} & 0 & 0 \\ 0 & m^2_{c,\ s} & 0 \\ 0 & 0 & m^2_{t,\ b} \end{pmatrix},$$

$$O^T_{u^c,\ d^c} M^T_{u,\ d} M_{u^c,\ d^c} O_{u^c,\ d^c} = \begin{pmatrix} m^2_{u,\ d} & 0 & 0 \\ 0 & m^2_{c,\ s} & 0 \\ 0 & 0 & m^2_{t,\ b} \end{pmatrix}. \tag{5.20}$$

The CKM matrix $V_{\rm CKM}$ is then given by

$$V_{\rm CKM} = O^T_u P_q O_d, \tag{5.21}$$

where

$$P_q = P^\dagger_u P_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\theta_q} & 0 \\ 0 & 0 & e^{i\theta_q} \end{pmatrix} \tag{5.22}$$

with $\theta_q = \Delta\theta_d - \Delta\theta_u$.

Now it is clear how the Q_6 setup reduces the number of parameters in the quark sector. The total number of parameters in the quark sector is nine (four real parameters each in M_u and M_d, plus a single phase θ_q), which should fit ten observables. Spontaneous CP violation is crucial for this reduction of parameters. With explicit CP violation, there would have been one more phase parameter. The single prediction of this model was numerically studied in Ref. [90], and shown to be fully consistent with data. Here we present a numerical fit to all the quark sector observables, which deviates somewhat from the fit given in Ref. [90]. The difference arises since here we have attempted to be consistent with the recent lattice determination of light quark masses. An excellent fit to the quark masses and mixing, including CKM CP violation, is obtained with the following choice of parameters at a momentum scale of $\mu = 1$ TeV.

$$m_t^0 = 150.7 \text{ GeV}, \; m_b^0 = 2.5515 \text{ GeV}, \quad \theta_q = \Delta\theta_d - \Delta\theta_u = -1.40,$$

$$\begin{aligned} q_u = 1.5142 \cdot 10^{-4}, \quad b_u = 0.0395, \quad b_u^{'} = 0.0770474, \quad y_u = 0.99746, \\ q_d = 0.0043435, \quad b_d = 0.02609, \quad b_d^{'} = 0.69138, \quad y_d = 0.8100, \end{aligned} \tag{5.23}$$

This choice yields at $\mu = 1$ TeV, the following masses and mixing for the quarks:

$$\begin{aligned} m_u &= 1.13 \text{ MeV}, \quad m_c = 0.461 \text{ GeV}, \quad m_t = 150.50 \text{ GeV}, \\ m_d &= 2.53 \text{ MeV}, \quad m_s = 50.99 \text{MeV}, \quad m_b = 2.43 \text{ GeV}, \\ |V_{CKM}| &= \begin{pmatrix} 0.9745 & 0.2244 & 0.0033 \\ 0.2242 & 0.9737 & 0.0408 \\ 0.0093 & 0.0399 & 0.9991 \end{pmatrix}, \\ \eta_W &= 0.3465, \end{aligned} \tag{5.24}$$

where η_W is the CP violation parameter in the Wolfenstein parametrization. These values, when extrapolated to lower energy scales, give extremely good agreement with data [100].

We have computed the orthogonal matrices that diagonalize M_u and M_d. These rotation matrices will be relevant for our discussion of Higgs–induced flavor violation, as well as FCNC arising via SUSY loop diagrams. We find

$$O_d = \begin{pmatrix} 0.9840 & -0.1782 & 0.0041 \\ 0.1781 & 0.9838 & 0.0188 \\ -0.0074 & -0.0178 & 0.9998 \end{pmatrix}, \quad O_{d^c} = \begin{pmatrix} 0.9645 & -0.2640 & -0.0001 \\ -0.1817 & 0.6642 & 0.7251 \\ 0.1915 & -0.6994 & 0.6886 \end{pmatrix},$$

$$O_u = \begin{pmatrix} 0.9988 & -0.0495 & 1.17 \cdot 10^{-5} \\ 0.0494 & 0.9980 & 0.0395 \\ -0.0020 & -0.0394 & 0.9992 \end{pmatrix}, \quad O_{u^c} = \begin{pmatrix} 0.9988 & 0.0496 & -6.00 \cdot 10^{-6} \\ -0.0494 & 0.9958 & 0.0771 \\ 0.0038 & -0.0770 & 0.9970 \end{pmatrix} \tag{5.25}$$

In the case of charged leptons, there is some arbitrariness in the values of $(A,\ B,\ B',\ C)_\ell$ of Eq. (5.4), since we have three observables (charged lepton masses) and four parameters (without including the neutrino sector). We shall present a fit with a simplifying assumption $B'_\ell = B_\ell$. At $\mu = 1$ TeV, a consistent fit for all the lepton masses is found with the following input values:

$$A_\ell = 1.67536 \text{ GeV}, \quad B_\ell = B'_\ell = 0.430588 \text{ GeV}, \quad C_\ell = 0.00742877 \text{ GeV} . \tag{5.26}$$

These yield the following eigenvalues at $\mu = 1$ TeV:

$$m_e = 0.4963 \text{ MeV}, \quad m_\mu = 104.686 \text{ MeV}, \quad m_\tau = 1779.5 \text{ MeV}. \tag{5.27}$$

These values correspond to the central values of charged lepton masses when extrapolated down to their respective mass scales [100]. The orthogonal matrix that diagonalizes M_e is given by

$$O_e = \begin{pmatrix} 0.9976 & 0.0688 & 9.81 \cdot 10^{-4} \\ 0.0664 & -0.9697 & 0.2352 \\ -0.0171 & 0.2346 & 0.9720 \end{pmatrix}, \tag{5.28}$$

with O_{e^c} obtained from the above by flipping the signs in the first row and column.

5.3 Symmetry breaking and the Higgs boson spectrum

We now turn to the discussion of symmetry breaking and the Higgs boson spectrum in the model. We shall confine here to the case of having three pairs of Higgs doublets, and no Higgs singlets in the low energy theory. It is however, assumed that singlet fields are present in the full theory, so that spontaneous Q_6 breaking in the singlet sector appears as soft breaking in the doublet sector. As shown in Ref. [90], it is possible to realize such a scenario while preserving the $1 \leftrightarrow 2$ interchange symmetry for members $(1,\ 2)$ inside Q_6 doublets. We seek a consistent picture where CP violating phases are generated in the Higgs doublet VEVs. As it turns out, CP also has to be softly broken in the bilinear soft SUSY breaking terms, or else there would be no CP phases in the VEVs.

The superpotential that we consider is the most general one consistent with softly broken Q_6 symmetry, but preserving the S_2 interchange symmetry:

$$\begin{aligned} W_{\rm eff} &= \mu_1(H_1^u H_1^d + H_2^u H_2^d) + \mu_3 H_3^u H_3^d + \mu_{13}(H_1^u + H_2^u)H_3^d \\ &\quad + \mu_{31} H_3^u (H_1^d + H_2^d) + \mu_{12}(H_1^u H_2^d + H_1^d H_2^u). \end{aligned} \tag{5.29}$$

As mentioned earlier, we make a 45° rotations in H_1^d, H_2^d and H_1^u, H_2^u space, with $\hat{H}^u_{1,\ 2} = \frac{H_1^u \pm H_2^u}{\sqrt{2}}$ and $\hat{H}^d_{1,\ 2} = \frac{H_1^d \pm H_2^d}{\sqrt{2}}$, so that the superpotential becomes

$$W_{\rm eff} = (\mu_1 + \mu_{12})\hat{H}_1^u \hat{H}_1^d + (\mu_1 - \mu_{12})\hat{H}_2^u \hat{H}_2^d + \mu_3 \hat{H}_3^u \hat{H}_3^d + \sqrt{2}\mu_{13}\hat{H}_1^u \hat{H}_3^d + \sqrt{2}\mu_{31}\hat{H}_3^u \hat{H}_1^d\ . \tag{5.30}$$

The redefined fields have $\langle \hat{H}_2^u \rangle = \langle \hat{H}_2^d \rangle = 0$. We work in the hatted basis from now on, and drop the hat on the new fields.

The soft SUSY breaking Lagrangian is given, in the rotated basis, as

$$\begin{aligned} V_{\rm soft} &= (b_1 + b_{12})H_1^u \epsilon H_1^d + (b_1 - b_{12})H_2^u \epsilon H_2^d + b_3 H_3^u \epsilon H_3^d \\ &+ \sqrt{2} b_{13} H_1^u \epsilon H_3^d + \sqrt{2} b_{31} H_3^u \epsilon H_1^d + \text{h.c.} \\ &+ m_{d1}^2(|H_1^d|^2 + |H_2^d|^2) + m_{d3}^2 |H_3^d|^2 + m_{u1}^2(|H_1^u|^2 + |H_2^u|^2) + m_{u3}^2 |H_3^u|^2, \end{aligned} \tag{5.31}$$

where $\epsilon = i\sigma_2$.

The full scalar potential including the soft terms, the F terms and the D terms has the form

$$
\begin{aligned}
V \;=\;& M_{d1}^2(|H_1^{d^0}|^2+|H_1^{d^-}|^2)+M_{d3}^2(|H_3^{d^0}|^2+|H_3^{d^-}|^2) \\
&+M_{u1}^2(|H_1^{u0}|^2+|H_1^{u+}|^2)+M_{u3}^2(|H_3^{u0}|^2+|H_3^{u+}|^2) \\
&+\{M_{13}^2(H_1^{d^{0*}}H_3^{d^0}+H_1^{d^{-*}}H_3^{d^-})+M_{31}^2(H_3^{u0*}H_1^{u0}+H_3^{u+*}H_1^{u+})+\text{h.c.}\} \\
&+M_{d2}^2(|H_2^{d^0}|^2+|H_2^{d^-}|^2)+M_{u2}^2(|H_2^{u0}|^2+|H_2^{u+}|^2) \\
&+\{b_1'(H_1^{u+}H_1^{d^-}-H_1^{u0}H_1^{d^0})+b_3(H_3^{u+}H_3^{d^-}-H_3^{u0}H_3^{d^0}) \\
&+\sqrt{2}b_{13}(H_1^{u+}H_3^{d^-}-H_1^{u0}H_3^{d^0})+\sqrt{2}b_{31}(H_3^{u+}H_1^{d^-}-H_3^{u0}H_1^{d^0}) \\
&+b_2'(H_2^{u+}H_2^{d^-}-H_2^{u0}H_2^{d^0})+\text{h.c.}\} \\
&+\frac{1}{8}(g_1^2+g_2^2)(|H_1^{u+}|^2+|H_1^{u0}|^2+|H_3^{u+}|^2+|H_3^{u0}|^2+|H_2^{u+}|^2+|H_2^{u0}|^2 \\
&\qquad -|H_1^{d^-}|^2-|H_1^{d^0}|^2-|H_3^{d^-}|^2-|H_3^{d^0}|^2-|H_2^{d^-}|^2-|H_2^{d^0}|^2)^2 \\
&+\frac{1}{2}g_2^2|H_1^{u+}H_1^{d^{0*}}+H_1^{u0}H_1^{d^{-*}}+H_3^{u+}H_3^{d^{0*}}+H_3^{u0}H_3^{d^{-*}} \\
&\qquad +H_2^{u+}H_2^{d^{0*}}+H_2^{u0}H_2^{d^{-*}}|^2.
\end{aligned}
\tag{5.32}
$$

Here we have redefined new effective parameters for convenience as

$$
\begin{aligned}
M_{d1}^2 &= |\mu_1+\mu_{12}|^2+2|\mu_{31}|^2+m_{d1}^2, & M_{d3}^2 &= |\mu_3|^2+2|\mu_{13}|^2+m_{d3}^2, \\
M_{u1}^2 &= |\mu_1+\mu_{12}|^2+2|\mu_{13}|^2+m_{u1}^2, & M_{u3}^2 &= |\mu_3|^2+2|\mu_{31}|^2+m_{u3}^2, \\
M_{d2}^2 &= |\mu_1-\mu_{12}|^2+m_{d1}^2, & M_{u2}^2 &= |\mu_1-\mu_{12}|^2+m_{u1}^2, \\
M_{13}^2 &= \sqrt{2}(\mu_1+\mu_{12})^*\mu_{13}+\sqrt{2}\mu_3\mu_{31}^*, & M_{31}^2 &= \sqrt{2}(\mu_1+\mu_{12})\mu_{31}^*+\sqrt{2}\mu_3^*\mu_{13}, \\
b_1' &= b_1+b_{12}, & b_2' &= b_1-b_{12}.
\end{aligned}
\tag{5.33}
$$

Before analyzing the spectrum, let us note that the potential should be bounded from below along all D–flat directions. The following conditions should be satisfied:

$$
\begin{aligned}
&M_{d1}^2+M_{u1}^2-2|b_1'|>0,\quad M_{d1}^2+M_{u2}^2>0,\quad M_{d1}^2+M_{u3}^2-2\sqrt{2}|b_{31}|>0 \\
&M_{d2}^2+M_{u1}^2>0,\quad M_{d2}^2+M_{u2}^2-2|b_2'|>0,\quad M_{d2}^2+M_{u3}^2>0, \\
&M_{d3}^2+M_{u1}^2-2\sqrt{2}|b_{13}|>0,\quad M_{d3}^2+M_{u2}^2>0,\quad M_{d3}^2+M_{u3}^2-2|b_3|>0.
\end{aligned}
\tag{5.34}
$$

In our numerical analysis, we shall verify that these conditions are indeed met.

We parameterize the VEVs of the four neutral Higgs fields as

$$v_{u1} = v \sin\beta \sin\gamma_u \, e^{i\theta_{u1}}, \qquad v_{u3} = v \sin\beta \cos\gamma_u \, e^{i\theta_{u3}},$$

$$v_{d1} = v \cos\beta \sin\gamma_d \, e^{i\theta_{d1}}, \qquad v_{d3} = v \cos\beta \cos\gamma_d \, e^{i\theta_{d3}}. \tag{5.35}$$

Thus we have $|v_{u1}|^2 + |v_{u3}|^2 + |v_{d1}|^2 + |v_{d3}|^2 = v^2 = (174 \text{ GeV})^2$. $\gamma_{u(d)}$ reflect the orientation of the VEVs in the $H_{u(d)1} - H_{u(d)3}$ space, while $\tan\beta$ is analogous to the up/down VEV ratio of MSSM.

We can rewrite the potential of the $H_1 - H_3$ sector of the neutral Higgs fields which acquire VEVs in a compact form:

$$\begin{aligned} V_N^{(1-3)} &= \begin{pmatrix} H_1^{u0*} & H_3^{u0*} \end{pmatrix} \begin{pmatrix} M_{u1}^2 & M_{31}^2 \\ M_{31}^{2\;*} & M_{u3}^2 \end{pmatrix} \begin{pmatrix} H_1^{u0} \\ H_3^{u0} \end{pmatrix} + \begin{pmatrix} H_1^{d0*} & H_3^{d0*} \end{pmatrix} \begin{pmatrix} M_{d1}^2 & M_{13}^2 \\ M_{13}^{2\;*} & M_{d3}^2 \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} \\ &+ \left[\begin{pmatrix} H_1^{u0} & H_3^{u0} \end{pmatrix} \begin{pmatrix} -b_1' & -\sqrt{2} b_{13} \\ -\sqrt{2} b_{31} & -b_3 \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} + \text{h.c.} \right] \\ &+ \frac{1}{8}(g_1^2 + g_2^2) \left[\begin{pmatrix} H_1^{u0*} & H_3^{u0*} \end{pmatrix} \begin{pmatrix} H_1^{u0} \\ H_3^{u0} \end{pmatrix} - \begin{pmatrix} H_1^{d0*} & H_3^{d0*} \end{pmatrix} \begin{pmatrix} H_1^{d0} \\ H_3^{d0} \end{pmatrix} \right]^2 . \end{aligned} \tag{5.36}$$

This suggests a unitary transformation that would diagonalize the first two matrices in Eq. (5.36), while leaving the D–term unaffected. With such a rotation we have

$$\begin{aligned} V_N^{(1-3)} &= \begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} h_3^* & h_4^* \end{pmatrix} \begin{pmatrix} m_3^2 & 0 \\ 0 & m_4^2 \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} \\ &+ \left[\begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} m_{13}^2 & m_{14}^2 \\ m_{23}^2 & m_{24}^2 \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} + \text{h.c.} \right] \\ &+ \frac{1}{8}(g_1^2 + g_2^2) \left[\begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} - \begin{pmatrix} h_3^* & h_4^* \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} \right]^2 . \end{aligned} \tag{5.37}$$

The unitary transformations to go from Eq. (5.36) to Eq. (5.37) are defined as

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = U_U \begin{pmatrix} H_1^u \\ H_3^u \end{pmatrix} = Q_u \begin{pmatrix} \cos\omega_u & -\sin\omega_u \\ \sin\omega_u & \cos\omega_u \end{pmatrix} \begin{pmatrix} e^{i\phi_u} & 0 \\ 0 & e^{i(\phi_u+\theta_{M_{31}})} \end{pmatrix} \begin{pmatrix} H_1^u \\ H_3^u \end{pmatrix},$$
$$\begin{pmatrix} h_3 \\ h_4 \end{pmatrix} = U_D \begin{pmatrix} H_1^d \\ H_3^d \end{pmatrix} = Q_d \begin{pmatrix} \cos\omega_d & -\sin\omega_d \\ \sin\omega_d & \cos\omega_d \end{pmatrix} \begin{pmatrix} e^{i\phi_d} & 0 \\ 0 & e^{i(\phi_d+\theta_{M_{13}})} \end{pmatrix} \begin{pmatrix} H_1^d \\ H_3^d \end{pmatrix}, \quad (5.38)$$

with $\theta_{M_{31}} = \arg(M_{31}^2)$, $\theta_{M_{13}} = \arg(M_{13}^2)$ and

$$\omega_u = \frac{1}{2}\tan^{-1}\left(\frac{2|M_{31}^2|}{M_{u3}^2 - M_{u1}^2}\right), \qquad \omega_d = \frac{1}{2}\tan^{-1}\left(\frac{2|M_{13}^2|}{M_{d3}^2 - M_{d1}^2}\right). \quad (5.39)$$

The two phases ϕ_u and ϕ_d here are arbitrary. $\phi_u - \phi_d$ does not appear in the potential (being proportional to $U(1)_Y$ charges). $\phi_u + \phi_d$ can be used to remove one phase of the bilinear terms in the potential. $Q_{u,d}$ are arbitrary diagonal phase matrices. If desired, one can take advantage of these phases to remove all but one phase from the parameters of the potential. Since we are interested in going back to the original basis from this rotated basis, we find it convenient to set $Q_{u,d}$ to be identity.

The other parameters of this transformation are

$$m_{1,2}^2 = \frac{1}{2}\left[M_{u3}^2 + M_{u1}^2 \pm \sqrt{(M_{u3}^2 - M_{u1}^2)^2 + 4|M_{31}^2|^2}\right],$$
$$m_{3,4}^2 = \frac{1}{2}\left[M_{d3}^2 + M_{d1}^2 \pm \sqrt{(M_{d3}^2 - M_{d1}^2)^2 + 4|M_{13}^2|^2}\right]. \quad (5.40)$$

and

$$\begin{pmatrix} m_{13}^2 & m_{14}^2 \\ m_{23}^2 & m_{24}^2 \end{pmatrix} = U_U^* \begin{pmatrix} -b_1' & -\sqrt{2}b_{13} \\ -\sqrt{2}b_{31} & -b_3 \end{pmatrix} U_D^\dagger. \quad (5.41)$$

If we choose

$$\phi_u + \phi_d = \pi + \arg\Big[b_1' \sin\omega_u \sin\omega_d + \sqrt{2}b_{31}\cos\omega_u \sin\omega_d e^{-i\theta_{M_{31}}}$$
$$+\sqrt{2}b_{13}\sin\omega_u\cos\omega_d e^{-i\theta_{M_{13}}} + b_3\cos\omega_u\cos\omega_d e^{-i(\theta_{M_{31}}+\theta_{M_{13}})}\Big], \quad (5.42)$$

m_{24}^2 is real and positive (with $Q_{u,d}$ set to identity). We shall adopt this phase convention in our numerical study. However, we shall present analytical results that hold in an arbitrary phase convention.

The task at hand is somewhat simplified, since Eq. (5.37) is relatively simple to analyze. The eight real neutral Higgs bosons in $H_{1,3}^{u,d}$ can be conveniently parameterized as

$$
\begin{aligned}
h_1 &= e^{i\delta_1}\left[v_1 + \frac{1}{\sqrt{2}}(\phi_1 + ie\phi_5 + ia\phi_7 + i\frac{v_1}{v}G)\right], \\
h_2 &= e^{i\delta_2}\left[v_2 + \frac{1}{\sqrt{2}}(\phi_2 + if\phi_6 + ib\phi_7 + i\frac{v_2}{v}G)\right], \\
h_3 &= e^{i\delta_3}\left[v_3 + \frac{1}{\sqrt{2}}(\phi_3 + ig\phi_5 + ic\phi_7 - i\frac{v_3}{v}G)\right], \\
h_4 &= v_4 + \frac{1}{\sqrt{2}}(\phi_4 + ih\phi_6 + id\phi_7 - i\frac{v_4}{v}G).
\end{aligned}
\tag{5.43}
$$

Here v_i ($i = 1,\ 2,\ 3,\ 4$) are the magnitudes of the VEVs of the redefined fields h_i, and δ_i are their phases. Without loss of generality we have taken v_4 to be real. G in Eq. (5.43) is the Goldstone field eaten up by the Z gauge boson. We shall work in the unitary gauge and set $G = 0$. We have checked explicitly that the G field does not mix with other scalar fields, and that its mass is exactly zero. The coefficients of various fields in Eq. (5.43) are functions of the v_i's:

$$
\begin{aligned}
a &= \frac{v_1\sqrt{v_2^2+v_4^2}}{v\sqrt{v_1^2+v_3^2}}, \quad b = -\frac{v_2\sqrt{v_1^2+v_3^2}}{v\sqrt{v_2^2+v_4^2}}, \quad c = -\frac{v_3\sqrt{v_2^2+v_4^2}}{v\sqrt{v_1^2+v_3^2}}, \quad d = \frac{v_4\sqrt{v_1^2+v_3^2}}{v\sqrt{v_2^2+v_4^2}}, \\
e &= \frac{v_3}{\sqrt{v_1^2+v_3^2}}, \qquad f = \frac{v_4}{\sqrt{v_2^2+v_4^2}}, \qquad g = \frac{v_1}{\sqrt{v_1^2+v_3^2}}, \qquad h = \frac{v_2}{\sqrt{v_2^2+v_4^2}}.
\end{aligned}
\tag{5.44}
$$

We shall allow for the soft SUSY breaking parameters (b_i) in the Higgs potential to be complex. Phase rotations cannot remove all phases from the potential, one phase is unremovable. Without this phase, the model cannot induce complex VEVs to the doublets, as shown in Ref. [101] by a geometric argument. For the case when all parameters in the Higgs potential are real, we have numerically verified that the CP violating extremum would generate two massless modes, signalling inconsistency with symmetry breaking [101].

We take the soft bilinear terms $m_{13}^2,\ m_{14}^2,\ m_{23}^2,\ m_{24}^2$ of Eq. (5.37) to be complex, and denote the phase of m_{ij}^2 as θ_{ij}. The minimization conditions then read as

$$\begin{aligned}
&m_1^2 v_1 + |m_{13}^2| v_3 \cos(\theta_{13}+\delta_1+\delta_3) + |m_{14}^2| v_4 \cos(\theta_{14}+\delta_1) + \frac{1}{4}(g_1^2+g_2^2) v_1 (v_1^2+v_2^2-v_3^2-v_4^2) = 0,\\
&m_2^2 v_2 + |m_{23}^2| v_3 \cos(\theta_{23}+\delta_2+\delta_3) + |m_{24}^2| v_4 \cos(\delta_2+\theta_{24}) + \frac{1}{4}(g_1^2+g_2^2) v_2 (v_1^2+v_2^2-v_3^2-v_4^2) = 0,\\
&m_3^2 v_3 + |m_{13}^2| v_1 \cos(\theta_{13}+\delta_1+\delta_3) + |m_{23}^2| v_2 \cos(\theta_{23}+\delta_2+\delta_3) - \frac{1}{4}(g_1^2+g_2^2) v_3 (v_1^2+v_2^2-v_3^2-v_4^2) = 0,\\
&m_4^2 v_4 + |m_{14}^2| v_1 \cos(\theta_{14}+\delta_1) + |m_{24}^2| v_2 \cos(\delta_2+\theta_{24}) - \frac{1}{4}(g_1^2+g_2^2) v_4 (v_1^2+v_2^2-v_3^2-v_4^2) = 0,\\
&|m_{13}^2|(v_1^2+v_3^2)\sin(\theta_{13}+\delta_1+\delta_3) + |m_{23}^2| v_1 v_2 \sin(\theta_{23}+\delta_2+\delta_3) + |m_{14}^2| v_3 v_4 \sin(\theta_{14}+\delta_1) = 0,\\
&|m_{24}^2|(v_2^2+v_4^2)\sin(\theta_{24}+\delta_2) + |m_{14}^2| v_1 v_2 \sin(\theta_{14}+\delta_1) + |m_{23}^2| v_3 v_4 \sin(\theta_{23}+\delta_2+\delta_3) = 0,\\
&|m_{14}^2| v_1 v_4 \sin(\theta_{14}+\delta_1) - |m_{23}^2| v_2 v_3 \sin(\theta_{23}+\delta_2+\delta_3) = 0.
\end{aligned} \tag{5.45}$$

Denoting the squared matrix for ϕ_i, $i = 1, 2, \ldots 7$ from the $H_1 - H_3$ sector as

$$\mathcal{M}^2_{0,(1-3)} = \mathcal{M}^2_{ij}, \tag{5.46}$$

we obtain

$$
\begin{aligned}
\mathcal{M}_{11}^2 &= \lambda v_1^2 + \kappa\frac{v_2 v_4}{v_1^2}[\cot(\theta_{14}+\delta_1) - \cot(\theta_{13}+\delta_1+\delta_3)],\\
\mathcal{M}_{22}^2 &= \lambda v_2^2 + \kappa\frac{v_4}{v_2}[\cot(\theta_{23}+\delta_2+\delta_3) - \cot(\theta_{24}+\delta_2)],\\
\mathcal{M}_{33}^2 &= \lambda v_3^2 + \kappa\frac{v_2 v_4}{v_3^2}[\cot(\theta_{23}+\delta_2+\delta_3) - \cot(\theta_{13}+\delta_1+\delta_3)],\\
\mathcal{M}_{44}^2 &= \lambda v_4^2 + \kappa\frac{v_2}{v_4}[\cot(\theta_{14}+\delta_1) - \cot(\theta_{24}+\delta_2)],\\
\mathcal{M}_{55}^2 &= \kappa\frac{v_2 v_4}{v_1^2+v_3^2}[\frac{v_3^2}{v_1^2}\cot(\theta_{14}+\delta_1) + \frac{v_1^2}{v_3^2}\cot(\theta_{23}+\delta_2+\delta_3) - \frac{(v_1^2+v_3^2)^2}{v_1^2 v_3^2}\cot(\theta_{13}+\delta_1+\delta_3)],\\
\mathcal{M}_{66}^2 &= \kappa\frac{1}{v_2 v_4(v_2^2+v_4^2)}[v_2^4\cot(\theta_{14}+\delta_1) + v_4^4\cot(\theta_{23}+\delta_2+\delta_3) - (v_2^2+v_4^2)^2\cot(\theta_{24}+\delta_2)],\\
\mathcal{M}_{77}^2 &= \kappa\frac{v_2 v_4(v_1^2+v_2^2+v_3^2+v_4^2)}{(v_1^2+v_3^2)(v_2^2+v_4^2)}[\cot(\theta_{14}+\delta_1) + \cot(\theta_{23}+\delta_2+\delta_3)],\\
\mathcal{M}_{12}^2 &= \lambda v_1 v_2, \qquad \mathcal{M}_{13}^2 = -\lambda v_1 v_3 + \kappa\frac{v_2 v_4}{v_1 v_3}\cot(\theta_{13}+\delta_1+\delta_3),\\
\mathcal{M}_{14}^2 &= -\lambda v_1 v_4 - \kappa\frac{v_2}{v_1}\cot(\theta_{14}+\delta_1), \qquad \mathcal{M}_{15}^2 = -\kappa\frac{v_2 v_4}{v_3\sqrt{v_1^2+v_3^2}}, \qquad \mathcal{M}_{16}^2 = \kappa\frac{v_2^2}{v_1\sqrt{v_2^2+v_4^2}},\\
\mathcal{M}_{17}^2 &= \kappa\frac{v_2 v_4}{v_1}\frac{\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{\sqrt{v_1^2+v_3^2}\sqrt{v_2^2+v_4^2}}, \qquad \mathcal{M}_{23}^2 = -\lambda v_2 v_3 - \kappa\frac{v_4}{v_3}\cot(\theta_{23}+\delta_2+\delta_3),\\
\mathcal{M}_{24}^2 &= -\lambda v_2 v_4 + \kappa\cot(\theta_{24}+\delta_2), \qquad \mathcal{M}_{25}^2 = \kappa\frac{v_1 v_4}{v_3\sqrt{v_1^2+v_3^2}}, \qquad \mathcal{M}_{26}^2 = -\kappa\frac{v_2}{\sqrt{v_2^2+v_4^2}},\\
\mathcal{M}_{27}^2 &= -\kappa\frac{v_4\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{\sqrt{v_1^2+v_3^2}\sqrt{v_2^2+v_4^2}}, \qquad \mathcal{M}_{34}^2 = \lambda v_3 v_4, \qquad \mathcal{M}_{35}^2 = -\kappa\frac{v_2 v_4}{v_1\sqrt{v_1^2+v_3^2}},\\
\mathcal{M}_{36}^2 &= \kappa\frac{v_4^2}{v_3\sqrt{v_2^2+v_4^2}}, \qquad \mathcal{M}_{37}^2 = -\kappa\frac{v_2 v_4}{v_3}\frac{\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{\sqrt{v_1^2+v_3^2}\sqrt{v_2^2+v_4^2}}, \qquad \mathcal{M}_{45}^2 = \kappa\frac{v_2 v_3}{v_1\sqrt{v_1^2+v_3^2}},\\
\mathcal{M}_{46}^2 &= -\kappa\frac{v_4}{\sqrt{v_2^2+v_4^2}}, \qquad \mathcal{M}_{47}^2 = \kappa\frac{v_2\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{\sqrt{v_1^2+v_3^2}\sqrt{v_2^2+v_4^2}},\\
\mathcal{M}_{56}^2 &= \kappa\frac{1}{v_1 v_3\sqrt{v_1^2+v_3^2}\sqrt{v_2^2+v_4^2}}[v_2^2 v_3^2\cot(\theta_{14}+\delta_1) + v_1^2 v_4^2\cot(\theta_{23}+\delta_2+\delta_3)],\\
\mathcal{M}_{57}^2 &= \kappa\frac{v_2 v_4}{v_1 v_3}\frac{\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{(v_1^2+v_3^2)\sqrt{v_2^2+v_4^2}}[v_3^2\cot(\theta_{14}+\delta_1) - v_1^2\cot(\theta_{23}+\delta_2+\delta_3)],\\
\mathcal{M}_{67}^2 &= \kappa\frac{\sqrt{v_1^2+v_2^2+v_3^2+v_4^2}}{\sqrt{v_1^2+v_3^2}(v_2^2+v_4^2)}[v_2^2\cot(\theta_{14}+\delta_1) - v_4^2\cot(\theta_{23}+\delta_2+\delta_3)]. \qquad (5.47)
\end{aligned}
$$

Here we have defined $\lambda = (g_1^2+g_2^2)/2 = M_Z^2/v^2$ and $\kappa = m_{24}^2\sin(\theta_{24}+\delta_2)$.

The potential of the $H_2^u - H_2^d$ fields which do not acquire VEVs is

$$\begin{aligned} V_N^{(2)} &= M_{u2}^2|H_2^u|^2 + M_{d2}^2|H_2^d|^2 - \{b_2' H_2^u H_2^d + \text{h.c.}\} \\ &+ \frac{g_1^2+g_2^2}{8}(|H_2^u|^2 - |H_2^d|^2 + |v_{u1}|^2 + |v_{u3}|^2 - |v_{d1}|^2 - |v_{d3}|^2)^2 \ . \end{aligned} \tag{5.48}$$

The corresponding squared mass matrix for the scalars in the basis $(\text{Re}H_2^u,\ \text{Im}H_2^u,\ \text{Re}H_2^d,\ \text{Im}H_2^d)$ is

$$\mathcal{M}_{0(2)}^2 = \begin{pmatrix} M_{u2}^2 - \frac{m_Z^2}{2}\cos 2\beta & 0 & \text{Re}b_2' & -\text{Im}b_2' \\ 0 & M_{u2}^2 - \frac{m_Z^2}{2}\cos 2\beta & -\text{Im}b_2' & -\text{Re}b_2' \\ \text{Re}b_2' & -\text{Im}b_2' & M_{d2}^2 + \frac{m_Z^2}{2}\cos 2\beta & 0 \\ -\text{Im}b_2' & -\text{Re}b_2' & 0 & M_{d2}^2 + \frac{m_Z^2}{2}\cos 2\beta \end{pmatrix} . \tag{5.49}$$

This matrix has two pairs of degenerate eigenstates, owing to an unbroken $U(1)$ symmetry.

The $H_1^{u,d} - H_3^{u,d}$ sector charged Higgs boson mass matrix is, in the basis $\{H_1^{u+},\ H_3^{u+},\ H_1^{d-*},\ H_3^{d-*}\}$,

$$\mathcal{M}_{\pm(1-3)}^2 = (\mathcal{M}^2)_{ij},$$

with

$$\begin{aligned} &\mathcal{M}_{11}^2 = M_{u1}^2 - \frac{1}{2}m_Z^2\cos 2\beta + \frac{1}{2}g_2^2|v_{d1}|^2, && \mathcal{M}_{22}^2 = M_{u3}^2 - \frac{1}{2}m_Z^2\cos 2\beta + \frac{1}{2}g_2^2|v_{d3}|^2, \\ &\mathcal{M}_{33}^2 = M_{d1}^2 + \frac{1}{2}m_Z^2\cos 2\beta + \frac{1}{2}g_2^2|v_{u1}|^2, && \mathcal{M}_{44}^2 = M_{d3}^2 + \frac{1}{2}m_Z^2\cos 2\beta + \frac{1}{2}g_2^2|v_{u3}|^2, \\ &\mathcal{M}_{12}^2 = \mathcal{M}_{21}^{2\ *} = M_{31}^2 + \frac{1}{2}g_2^2 v_{d1}^* v_{d3}, && \mathcal{M}_{13}^2 = \mathcal{M}_{31}^{2\ *} = b_1' + \frac{1}{2}g_2^2 v_{u1}^* v_{d1}^*, \\ &\mathcal{M}_{14}^2 = \mathcal{M}_{41}^{2\ *} = \sqrt{2}b_{13} + \frac{1}{2}g_2^2 v_{u3}^* v_{d1}^*, && \mathcal{M}_{23}^2 = \mathcal{M}_{32}^{2\ *} = \sqrt{2}b_{31} + \frac{1}{2}g_2^2 v_{u1}^* v_{d3}^*, \\ &\mathcal{M}_{24}^2 = \mathcal{M}_{42}^{2\ *} = b_3 + \frac{1}{2}g_2^2 v_{u3}^* v_{d3}^*, && \mathcal{M}_{34}^2 = \mathcal{M}_{43}^{2\ *} = M_{13}^2 + \frac{1}{2}g_2^2 v_{u1} v_{u3}^*. \end{aligned} \tag{5.50}$$

Finally, the $H_2^u - H_2^d$ sector charged Higgs mass matrix is, in the basis $\{H_2^{u+},\ H_2^{d-*}\}$,

$$\mathcal{M}_{\pm(2)}^2 = \begin{pmatrix} M_{u2}^2 - \frac{1}{2}m_Z^2\cos 2\beta & b_2' \\ b_2'^{\ *} & M_{d2}^2 + \frac{1}{2}m_Z^2\cos 2\beta \end{pmatrix} \tag{5.51}$$

Now we present two sets of numerical fits (cases (1) and (2)) which show the consistency of symmetry breaking. We are interested in choosing the SUSY breaking parameters (including the μ terms) around the TeV scale, guided by arguments of naturalness. At the same time we wish the spectrum to be consistent with FCNC constraints arising from meson–anti-meson mixing. We have explored parameter space of the Higgs potential where both these constraints are met. For the FCNC constraint, we allow the new Higgs exchange contribution to ΔM be not more than the experimentally measured values.

Case (1)

The parameters in the original Higgs potential of Eq. (5.36) are taken to have the following values.

$$\begin{aligned}
&M_{d1} = 3.754 \text{ TeV}, \quad M_{d3} = 3.586 \text{ TeV}, \quad M_{u1} = 4.782 \text{ TeV}, \quad M_{u3} = 2.152 \text{ TeV}, \\
&M_{31} = 2.336\ e^{i0.792} \text{ TeV}, \quad M_{13} = 1.346\ e^{-i1.205} \text{ TeV}, \quad b_1' = 3.144\ e^{i2.963} \text{ TeV}^2, \\
&b_3 = 3.196\ e^{i2.064} \text{ TeV}^2, \quad b_{31} = 4.052\ e^{i2.186} \text{ TeV}^2, \quad b_{13} = 3.438\ e^{i3.109} \text{ TeV}^2, \\
&M_{u2} = 4.550 \text{ TeV}, \qquad M_{d2} = 4.850 \text{ TeV} \qquad b_2' = 0.000 \text{ TeV}^2,
\end{aligned} \tag{5.52}$$

In the representation of Eq. (5.37) this choice corresponds to

$$\begin{aligned}
&m_1 = 4.937 \text{ TeV}, \quad m_2 = 1.767 \text{ TeV}, \quad m_3 = 3.923 \text{ TeV}, m_4 = 3.401 \text{ TeV}, \\
&m_{13} = 1.851 e^{-i1.437} \text{ TeV}, \quad m_{14} = 2.736 e^{-i0.732} \text{ TeV}, \quad m_{23} = 2.442\ e^{i1.347} \text{ TeV}, \\
&m_{24} = 2.104 \text{ TeV}\ .
\end{aligned} \tag{5.53}$$

For completeness we also give values of other parameters, $\omega_u = 0.70, \quad \omega_d = 0.622, \quad \phi_u + \phi_d = 1.005$.

We obtain numerically the VEV parameters to be

$$\tan\beta = 2.00, \quad \Delta\theta_d = -0.03, \quad \Delta\theta_u = 1.37, \quad \tan\gamma_d = 2.50, \quad \tan\gamma_u = 0.33. \tag{5.54}$$

The mass eigenvalues of the Higgs bosons in the $H_1 - H_3$ sector are found to be

$$\begin{aligned}
M_{h0} = (99.4,\ 115.1) \text{ GeV}, \quad M_1 = 3.299 \text{ TeV}, \quad M_2 - M_1 = 0.226 \text{ GeV}, \\
M_3 = 4.161 \text{ TeV}, \quad M_4 - M_3 = 0.411 \text{ GeV}, \quad M_5 = 5.124 \text{ TeV}, \quad M_6 - M_5 = 0.040 \text{ GeV}.
\end{aligned} \tag{5.55}$$

Note the appearance of nearly degenerate states $(M_1,\ M_2)$ etc, with their mass splitting being proportional to $m_Z^2/4$. The Higgs bosons from the H_2 sector have degenerate masses given by

$$M_7 = M_8 = 4.850 \text{ TeV} \qquad M_9 = M_{10} = 4.550 \text{ TeV}. \tag{5.56}$$

The charged Higgs bosons are nearly degenerate with its neutral partner, so we list the mass splitting:

$$M_{\pm 1} - M_1 = -0.532 \text{ GeV}, \quad M_{\pm 2} - M_3 = -0.156 \text{ GeV}, \quad M_{\pm 3} - M_5 = 0.032 \text{ GeV} . \qquad (5.57)$$

In the $(H_2^u - H_2^d)$ sector, the two charged Higgs bosons are degenerate with the neutral ones given in Eq. (5.56).

The mass eigenstates H_i are mixtures of h_i, $i = 1, 2, \ldots 7$ states in the (1-3) sector. The orthogonal transformation that diagonalizes the mass matrix of Eq. (5.46) is

$$H^k = \begin{pmatrix} 0.0662 & 0.8919 & 0.2708 & 0.3562 & 8.60 \cdot 10^{-7} & 1.23 \cdot 10^{-6} & 2.15 \cdot 10^{-6} \\ 0.0314 & -0.0023 & 0.0427 & -0.0324 & -0.4002 & 0.8800 & 0.2482 \\ 0.3322 & -0.2620 & -0.3269 & 0.8428 & 0.0204 & 0.0293 & 0.0509 \\ -0.0357 & 0.0026 & -0.0484 & 0.0368 & 0.1514 & 0.3354 & -0.9272 \\ -0.0644 & 0.3645 & -0.9010 & -0.2159 & 0.0231 & 0.0332 & 0.0578 \\ 0.0430 & -0.0032 & 0.0584 & -0.0444 & 0.9029 & 0.3311 & 0.2607 \\ -0.9365 & -0.0553 & -0.0289 & 0.3345 & 0.0279 & 0.0401 & 0.0697 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{pmatrix},$$

(5.58)

with $k = 0, \cdots 6$. Since $b_2' = 0$ in this case, the H_2^0 mass matrix is diagonal, and thus the mass eigenstates are the original state.

Case (2)

Here we take the input parameters corresponding to Eq. (5.36) to be

$$\begin{aligned} &M_{d1} = 3.980 \text{ TeV}, \quad M_{d3} = 5.412 \text{ TeV}, \quad M_{u1} = 2.765 \text{ TeV}, \quad M_{u3} = 3.692 \text{ TeV}, \\ &M_{31} = 2.825\, e^{i0.781} \text{ TeV}, \quad M_{13} = 1.693\, e^{-i0.949} \text{ TeV}, \quad b_1' = 3.698\, e^{i1.495} \text{ TeV}^2, \\ &b_3 = 3.097\, e^{i1.522} \text{ TeV}^2, \quad b_{31} = 7.420\, e^{i2.428} \text{ TeV}^2, \quad b_{13} = 1.840\, e^{-i2.772} \text{ TeV}^2, \\ &M_{u2} = 3.550 \text{ TeV}, \qquad M_{d2} = 5.850 \text{ TeV}, \qquad b_2' = 1.234 e^{i1.56} \text{ TeV}^2. \end{aligned} \qquad (5.59)$$

This choice corresponds to parameters in Eq. (5.37) to be

$$m_1 = 4.377 \text{ TeV}, \quad m_2 = 1.154 \text{ TeV}, \quad m_3 = 5.466 \text{ TeV}, m_4 = 3.906 \text{ TeV},$$
$$m_{13} = 3.281 e^{i1.271} \text{ TeV}, \quad m_{14} = 1.702 e^{i0.974} \text{ TeV}, \quad m_{23} = 3.190\ e^{-i0.501} \text{ TeV},$$
$$m_{24} = 2.326 \text{ TeV}, \tag{5.60}$$

with $\omega_u = -0.501, \quad \omega_d = -0.606, \quad \phi_u + \phi_d = 4.786$.

The Higgs VEV parameters are found for this input to be

$$\tan\beta = 2.40, \quad \Delta\theta_d = -0.06, \quad \Delta\theta_u = 1.34, \quad \tan\gamma_d = 1.80, \quad \tan\gamma_u = 1.00. \tag{5.61}$$

The mass spectrum of Higgs boson in the $H_1 - H_3$ sector is

$$M_{h0} = (104.1,\ 119.2) \text{ GeV}, \quad M_1 = 2.869 \text{ TeV}, \quad M_2 - M_1 = 0.325 \text{ GeV}$$
$$M_3 = 5.114 \text{ TeV}, \quad M_4 - M_3 = 0.132 \text{ GeV}, \quad M_5 = 5.658 \text{ TeV}, \quad M_6 - M_5 = 0.087 \text{ GeV}, \tag{5.62}$$

while the mass eigenvalues of Eq. (5.49) are

$$M_7 = M_8 = 5.856 \text{ TeV} \qquad M_9 = M_{10} = 3.541 \text{ TeV}. \tag{5.63}$$

The charged Higgs boson masses are given by

$$M_{\pm 1} - M_1 = 0.225 \text{ GeV}, \quad M_{\pm 2} - M_3 = 0.182 \text{ GeV}, \quad M_{\pm 3} - M_5 = -0.064 \text{ GeV}\ . \tag{5.64}$$

with the remaining two charged Higgs bosons being degenerate with the neutral ones given in Eq. (5.63).

The orthogonal matrix diagonalizing Eq. (5.46) is

$$H^k = \begin{pmatrix} 0.3919 & 0.8356 & 0.2620 & 0.2819 & 1.02\cdot 10^{-4} & -5.99\cdot 10^{-5} & 7.35\cdot 10^{-5} \\ 0.4761 & -0.2234 & -0.1898 & 0.1764 & 0.6693 & 0.4070 & 0.2065 \\ -0.5233 & -0.0380 & 0.4553 & 0.4166 & 0.4223 & -0.2386 & 0.3295 \\ 0.5065 & -0.2376 & 0.1014 & -0.0942 & 0.0903 & -0.8105 & 0.0535 \\ -0.0893 & 0.2828 & 0.0490 & -0.7599 & 0.1969 & 0.0134 & 0.5415 \\ -0.0625 & 0.0293 & 0.3803 & -0.3534 & 0.4512 & 0.0437 & -0.7212 \\ 0.2783 & -0.3363 & 0.7285 & -0.0671 & -0.3510 & 0.3422 & 0.1805 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{pmatrix} \tag{5.65}$$

The matrix diagonalizing Eq. (5.49) is

$$H^k = \begin{pmatrix} -0.0568 & 0.000 & 0.9984 & 0.0000 \\ 0.0000 & -0.0568 & 0.0000 & 0.9984 \\ 0.0000 & 0.9984 & 0.0000 & 0.0568 \\ 0.9984 & 0.0000 & 0.0568 & 0.0000 \end{pmatrix} \begin{pmatrix} \mathrm{Re}(H_2^u) \\ \mathrm{Im}(H_2^u) \\ \mathrm{Re}(H_2^d) \\ \mathrm{Im}(H_2^d) \end{pmatrix}, \qquad (5.66)$$

with $k = 7, \cdots 10$.

In these fits, M_{h_0} is the light standard model–like Higgs boson mass, for which radiative corrections are significant. In our computation we have included known two loop corrections. The two values listed for M_{h_0} correspond to zero and maximal left–right stop mixing ($X_t = 0$ or 6). We have taken $m_t = 174$ GeV, $M_{\rm SUSY} = 1.5$ TeV and $\alpha_s(m_t) = 0.108$ for these evaluations and used the analytic approximation given in Ref. [102].

An interesting feature of these two fits is that the diagonal entries of the quadratic mass matrix of the potential of Eq. (5.36) are all positive. This of course does not preclude some soft squared masses turning negative as in the MSSM via large top quark Yukawa coupling (since the diagonal entries also receive μ term contributions), however, this is not necessary for symmetry breaking to be triggered. Yet, one of the eigenvalues of this matrix is negative, which facilitates symmetry breaking. For the two cases we find these eigenvalues to be

$$\begin{aligned} &\text{Case } (\mathbf{1}) : \{(5.123 \text{ TeV})^2, \quad (4.161 \text{ TeV})^2, \quad (3.300 \text{ TeV})^2, \quad -(38.682 \text{ GeV})^2\} , \\ &\text{Case } (\mathbf{2}) : \{(5.658 \text{ TeV})^2, \quad (5.115 \text{ TeV})^2, \quad (2.869 \text{ TeV})^2, \quad -(45.40 \text{ GeV})^2\} . \end{aligned} \qquad (5.67)$$

The conditions for boundedness of the potential listed in Eq. (5.34) are found to be satisfied for both cases.

5.3.1 Neutralino and Chargino masses

The symmetry breaking parameters do not fully determine the masses of the neutralinos and the charginos. Here we present analytical results for their mass matrices.

The mass matrix of $\tilde{H}_1 - \tilde{H}_3$ sector neutralino in the basis of $\{\widetilde{B},\ \widetilde{W}^0,\ \widetilde{H_1^u}^0,\ \widetilde{H_3^u}^0,\ \widetilde{H_1^d}^0,\ \widetilde{H_3^d}^0\}$ is

$$\mathcal{M}_{\chi^0(13)} = \begin{pmatrix} M_{\tilde{B}} & 0 & \frac{g_2 v_{u1}}{\sqrt{2}} & \frac{g_2 v_{u3}}{\sqrt{2}} & -\frac{g_2 v_{d1}}{\sqrt{2}} & -\frac{g_2 v_{d3}}{\sqrt{2}} \\ 0 & M_{\tilde{W}} & -\frac{g_1 v_{u1}}{\sqrt{2}} & -\frac{g_1 v_{u3}}{\sqrt{2}} & \frac{g_1 v_{d1}}{\sqrt{2}} & \frac{g_1 v_{d3}}{\sqrt{2}} \\ \frac{g_2 v_{u1}}{\sqrt{2}} & -\frac{g_1 v_{u1}}{\sqrt{2}} & 0 & 0 & -(\mu_1 + \mu_{12}) & -\sqrt{2}\mu_{13} \\ \frac{g_2 v_{u3}}{\sqrt{2}} & -\frac{g_1 v_{u3}}{\sqrt{2}} & 0 & 0 & -\sqrt{2}\mu_{31} & -\mu_3 \\ -\frac{g_2 v_{d1}}{\sqrt{2}} & \frac{g_1 v_{d1}}{\sqrt{2}} & -(\mu_1 + \mu_{12}) & -\sqrt{2}\mu_{31} & 0 & 0 \\ -\frac{g_2 v_{d3}}{\sqrt{2}} & \frac{g_1 v_{d3}}{\sqrt{2}} & -\sqrt{2}\mu_{13} & -\mu_3 & 0 & 0 \end{pmatrix} . \tag{5.68}$$

The mass matrix of the $\tilde{H}_2$ sector in the basis $\{\ \widetilde{H_2^u}^0,\ \widetilde{H_2^d}^0\}$ is

$$\mathcal{M}_{\chi^0(2)} \begin{pmatrix} 0 & -(\mu_1 - \mu_{12}) \\ -(\mu_1 - \mu_{12}) & 0 \end{pmatrix} . \tag{5.69}$$

The mass matrix of charginos of the $\tilde{H}_1 - \tilde{H}_3$ sector in the basis $\{\widetilde{W}^+,\ \widetilde{H_1^u}^+,\ \widetilde{H_3^u}^+,\ \widetilde{W}^-,\ \widetilde{H_1^d}^-,\ \widetilde{H_3^d}^-\}$ has a block–diagonal form:

$$\mathcal{M}_{\chi^\pm(13)} = \begin{pmatrix} 0 & \mathbf{X^T} \\ \mathbf{X} & 0 \end{pmatrix} \tag{5.70}$$

with

$$\mathbf{X} = \begin{pmatrix} M_{\tilde{W}} & g_1 v_{u1} & g_1 v_{u3} \\ g_1 v_{d1} & \mu_1 + \mu_{12} & \sqrt{2}\mu_{31} \\ g_1 v_{d3} & \sqrt{2}\mu_{13} & \mu_3 \end{pmatrix} . \tag{5.71}$$

The chargino mass matrix in the $\tilde{H}_2^u - \tilde{H}_2^d$ sector in the basis of $\{\widetilde{H_2^u}^+,\ \widetilde{H_2^d}^-\}$ is

$$\mathcal{M}_{\chi^\pm(2)} = \begin{pmatrix} 0 & \mu_1 - \mu_{12} \\ \mu_1 - \mu_{12} & 0 \end{pmatrix} \tag{5.72}$$

5.4 Tree level Higgs induced FCNC processes

In this section we discuss various FCNC processes mediated by tree–level neutral Higgs boson exchange.

5.4.1 Neutral meson mixing via Higgs exchange

Accurate measurements exist [103] for neutral meson–anti-meson mixing in the $K^0 - \overline{K^0}$, $B_d^0 - \overline{B_d^0}$, $B_s^0 - \overline{B_s^0}$ and in $D^0 - \overline{D^0}$ sectors. In the Q_6 model there are new contributions to these mixing arising through tree–level Higgs exchange. These new contributions will modify the SM predictions, which are all in good agreement with data. Here we compute these new contributions, following the analysis of Ref. [104], with updated QCD corrections and hadronic matrix elements.

The Yukawa coupling $\alpha_{u,\ d}$, $\beta_{u,\ d}$, $\beta'_{u,\ d}$, $\delta_{u,\ d}$ of Eq. (5.14) can be determined from the mass matrix Eq. (5.19):

$$\alpha_{u,\ d} = \frac{m^0_{t,\ b} y^2_{u,\ d}}{|v_{u,\ d3}|}, \qquad \beta_{u,\ d} = \frac{m^0_{t,\ b} b_{u,\ d}}{|v_{u,\ d1}|}$$

$$\beta'_{u,\ d} = \frac{m^0_{t,\ b} b'_{u,\ d}}{|v_{u,\ d1}|}, \qquad \delta_{y,\ d} = \frac{m^0_{t,\ b} q_{u,\ d}/y_{u,\ d}}{|v_{u,\ d3}|}, \tag{5.73}$$

Using the input values given in Eq. (5.23) we get for the two cases

Case (1)

$$\alpha_d = 0.0409, \qquad \beta_d = 6.51\cdot 10^{-4}, \qquad \beta'_d = 0.0173, \qquad \delta_d = 3.35\cdot 10^{-4},$$
$$\alpha_u = 0.7195, \qquad \beta_u = 0.0858, \qquad \beta'_u = 0.1672, \qquad \delta_u = 1.10\cdot 10^{-4}.$$

Case (2)

$$\alpha_d = 0.0526, \qquad \beta_d = 7.46\cdot 10^{-4}, \qquad \beta'_d = 0.0198, \qquad \delta_d = 4.30\cdot 10^{-4},$$
$$\alpha_u = 0.9354, \qquad \beta_u = 0.0372, \qquad \beta'_u = 0.0724, \qquad \delta_u = 1.43\cdot 10^{-4}.$$

After 45° rotation in the Q_6 doublet space, the Yukawa coupling matrices in the down sector are

$$Y_{d1} = O_d^T P_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta_d \\ 0 & \beta'_d & 0 \end{pmatrix} P_{d^c} O_{d^c}, \quad Y_{d2} = O_d^T P_d \begin{pmatrix} 0 & 0 & \beta_d \\ 0 & 0 & 0 \\ \beta'_d & 0 & 0 \end{pmatrix} P_{d^c} O_{d^c},$$

$$Y_{d3} = O_d^T P_d \begin{pmatrix} 0 & \delta_d & 0 \\ -\delta_d & 0 & 0 \\ 0 & 0 & \alpha_d \end{pmatrix} P_{d^c} O_{d^c}, \tag{5.74}$$

where $P_d,\ P_{d^c}$ are defined in Eq. (5.18). The Yukawa couplings in the up–quark sector and the charged lepton sector are similar.

The new Higgs–mediated contributions to $\Delta F = 2$ Hamiltonian, responsible for the neutral meson–antimeson mixing has the form [104]

$$H_{\rm eff} = -\frac{1}{2{M_k}^2}\left(\bar{q}_i\left[Y^k_{ij}\frac{1+\gamma_5}{2} + Y^{k*}_{ji}\frac{1-\gamma_5}{2}\right]q_j\right)^2. \tag{5.75}$$

Here $q_{i,j}$ are the relevant quark fields contained in the meson. Y^k_{ij} are the Yukawa couplings of q_i, q_j with Higgs mass eigenstates H^k mediating FCNC interactions, $k = 1,\ 2,\ \ldots 10$ in our model, 6 from the $(H_1 - H_3)$ sector and 4 from the H_2 sector. (The light standard model–like Higgs boson has practically no FCNC couplings.) Y^k_{ij} can be obtained via inverse transformations, Eq. (5.38), (5.43) and (5.58) or (5.65).

We obtain

$$\begin{aligned} M^{\phi}_{12} = \langle\phi|H_{\rm eff}|\bar{\phi}\rangle = -\frac{{f_\phi}^2 m_\phi}{2{M_k}^2}\Big[-\frac{5}{24}\frac{m_\phi^2}{(m_{q_i}+m_{q_j})^2}\left(Y^{k2}_{ij} + Y^{k*2}_{ji}\right)\cdot B_2\cdot\eta_2(\mu) \\ +Y^k_{ij}Y^{k*}_{ji}\left(\frac{1}{12} + \frac{1}{2}\frac{{m_\phi}^2}{(m_{q_i}+m_{q_j})^2}\right)\cdot B_4\cdot\eta_4(\mu)\Big]. \end{aligned} \tag{5.76}$$

Here ϕ is the neutral meson $(K^0,\ B^0_d,\ B^0_s,\ D^0)$. For our numerical study we use the modified vacuum saturation and factorization approximation results for the matrix elements [84, 85]

$$\begin{aligned} \langle\phi|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\mp\gamma_5)f_j|\bar{\phi}\rangle &= {f_\phi}^2 m_\phi\left(\frac{1}{6} + \frac{{m_\phi}^2}{(m_{q_i}+m_{q_j})^2}\right)\cdot B_4, \\ \langle\phi|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\pm\gamma_5)f_j|\bar{\phi}\rangle &= -\frac{5}{6}{f_\phi}^2 m_\phi\frac{{m_\phi}^2}{(m_{q_i}+m_{q_j})^2}\cdot B_2. \end{aligned} \tag{5.77}$$

B_2 and B_4 are equal to one in the vacuum saturation approximation, but are found to be slightly different from one in lattice simulations. We use $(B_2,\ B_4) = (0.66,\ 1.03)$ for the K^0 system, $(0.82,\ 1.16)$ for the B^0_d and B^0_s systems, and $(0.82,\ 1.08)$ for the D^0 system [84]. In Eq. (5.76) $\eta_2(\mu)$, $\eta_4(\mu)$ are QCD correction factors of the Wilson coefficients C_2 and C_4 of the effective $\Delta F = 2$ Hamiltonian in going from the SUSY scale M_s to the hadronic scale μ. These factors are computed as follows. The $\Delta F = 2$ effective Hamiltonian has the general form

$$\mathcal{H}^{\Delta F=2}_{\rm eff} = \sum_{i=1}^{5} C_i\ Q_i + \sum_{i=1}^{3}\tilde{C}_i\ \tilde{Q}, \tag{5.78}$$

where

$$Q_1 = \bar{q}_{iL}^{\alpha}\gamma_\mu q_{jL}^{\alpha}\bar{q}_{iL}^{\beta}\gamma^\nu q_{jL}^{\beta}, \quad Q_2 = \bar{q}_{iR}^{\alpha} q_{jL}^{\alpha}\bar{q}_{iR}^{\beta} q_{jR}^{\beta}, \quad Q_3 = \bar{q}_{iR}^{\alpha} q_{jL}^{\beta}\bar{q}_{iR}^{\beta} q_{jL}^{\alpha},$$
$$Q_4 = \bar{q}_{iR}^{\alpha} q_{jL}^{\alpha}\bar{q}_{iL}^{\beta} q_{jR}^{\beta}, \qquad Q_5 = \bar{q}_{iR}^{\alpha} q_{jL}^{\beta}\bar{q}_{iL}^{\beta} q_{jR}^{\alpha}, \tag{5.79}$$

with $\tilde{Q}_{1,2,3}$ obtained from $Q_{1,2,3}$ by the interchange $L \leftrightarrow R$.

For computing $\eta_{2,4}$ we take the SUSY scale M_s to be 1 TeV. All the supersymmetric particles and heavy Higgs bosons are integrated out at 1 TeV. The Wilson coefficients evolve from M_s down to the hadron scale μ according to the equations

$$C_r(\mu) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)})\eta^{a_i} C_s(M_s), \tag{5.80}$$

Here η is defined as $\eta = \alpha_s(M_s)/\alpha_s(m_t)$. The magic numbers a_i, $b_i^{(r,s)}$ and $c_i^{(r,s)}$ can be found in Ref. [84] for the K system, in Ref. [85] for the $B_{d,s}$ system and in Ref. [86] for the D system. With $M_s = 1$ TeV and $\alpha_s(m_Z) = 0.118$, and $m_t(m_t) = 163.6$ GeV we find $\eta = \alpha_s(1\text{ TeV})/\alpha_s(m_t) = 0.0882/0.108 = 0.8167$.

At the SUSY scale, the neutral Higgs bosons in our model generate only operators Q_2 and Q_4. Consequently, at the hadron scale, for the K^0 system, we find

$$C_2(\mu) = C_2(M_s)\cdot(2.54), \qquad C_4(\mu) = C_4(M_s)\cdot(4.81),$$
$$C_3(\mu) = C_2(M_s)\cdot(-1.8\times10^{-3}), \qquad C_5(\mu) = C_4(M_s)\cdot(0.186), \tag{5.81}$$

leading to $\eta_2(\mu) = 2.54$, $\eta_4(\mu) = 4.81$. Although operator mixing induce non-zero C_3 and C_5 at the hadronic scale, their coefficients are found to be rather small.

For the $B^0_{d,s}$ system, following the same procedure, we find

$$C_2(\mu) = C_2(M_s)\cdot(2.00), \qquad C_4(\mu) = C_4(M_s)\cdot(3.12),$$
$$C_3(\mu) = C_2(M_s)\cdot(-2.44\times10^{-2}), \qquad C_5(\mu) = C_4(M_s)\cdot(0.0874). \tag{5.82}$$

And for the D^0 system we have

$$C_2(\mu) = C_2(M_s)\cdot(2.31), \qquad C_4(\mu) = C_4(M_s)\cdot(3.99),$$
$$C_3(\mu) = C_2(M_s)\cdot(-1.30\times10^{-2}), \qquad C_5(\mu) = C_4(M_s)\cdot(0.144). \tag{5.83}$$

In all cases we see that the induced operators C_3 and C_5 are negligible.

$K^0 - \overline{K^0}$ **mixing constraint**:

In the K^0 system, tree–level neutral Higgs boson exchange contributes to $K_L - K_S$ mass difference, as well as to the indirect CP violation parameter, modifying the successful SM predictions. The mass difference is computed from $\Delta m_K = 2\mathrm{Re}M_{12}^K$, while the CP violation parameter is $|\epsilon_K| \simeq \frac{\mathrm{Im}M_{12}^K}{\sqrt{2}\Delta m_K}$. We seek consistency with the precisely measured experimental values $\Delta m_K/m_K \simeq (7.1 \pm 0.014) \times 10^{-15}$ and $|\epsilon_K| \simeq 2.3 \times 10^{-3}$. In our calculation, we choose $m_K = 498$ MeV and $f_K = 160$ MeV. For the two numerical fits we find the new contributions to be

$$\begin{aligned} \text{Case } (\mathbf{1}): &\quad (\Delta m_K/m_K)^{\text{new}} = 7.361 \times 10^{-15}, \quad \epsilon_K^{\text{new}} = 2.00 \times 10^{-4}, \\ \text{Case } (\mathbf{2}): &\quad (\Delta m_K/m_K)^{\text{new}} = 5.721 \times 10^{-15}, \quad \epsilon_K^{\text{new}} = 2.28 \times 10^{-5}. \end{aligned} \tag{5.84}$$

The contributions from $H_1^0 - H_3^0$ sector and H_2^0 sector to $\mathrm{Re}(M_{12}^K)$ are respectively (3.033×10^{-15}, -1.200×10^{-15}) GeV for case (1) and (2.512×10^{-15}, -1.088×10^{-15}) GeV for case (2). We see that the new contributions to the mass difference is significant, but consistent with data. New contributions to CP violation is suppressed, which is a generic feature of Higgs exchange in this class of models. We elaborate on this issue later in this section.

$B_d^0 - \overline{B_d^0}$ **mixing constraint**:

For the $B_d^0 - \bar{B}_d^0$ system We use as input $m_{B_d} = 5.281$ GeV, $f_{B_d} = 240$ MeV and seek consistency with the experimental value $\Delta m_{B_d} = 3.12 \times 10^{-13}$ GeV. We find for the Higgs induced contribution

$$\begin{aligned} \text{Case } (\mathbf{1}): &\quad (\Delta m_{B_d})^{\text{new}} = 2.997 \times 10^{-13} \text{ GeV}, \\ \text{Case } (\mathbf{2}): &\quad (\Delta m_{B_d})^{\text{new}} = 2.728 \times 10^{-13} \text{ GeV}. \end{aligned} \tag{5.85}$$

The contributions from $H_1^0 - H_3^0$ sector and H_2^0 sector to $M_{12}^{b_d}$ are (2.298×10^{-14}, 1.269×10^{-13}) GeV for case (1) and (2.137×10^{-14}, 1.150×10^{-13}) GeV. Again, we see consistency with experimental values. CP violation parameter is found to be extremely tiny, $\sim 10^{-5}$, from Higgs boson exchange.

$B_s^0 - \overline{B_s^0}$ **mixing constraint**:

For the $B_s^0 - \overline{B_s^0}$ system, we use $m_{B_s} = 5.37$ GeV, $f_{B_s} = 295$ MeV and compare the new contributions with $\Delta m_{B_s} = 1.067 \times 10^{-11}$ GeV.

$$\begin{aligned} \text{Case } (\mathbf{1}): \quad (\Delta m_{B_s})^{\text{new}} &= 1.688 \times 10^{-12} \text{ GeV}, \\ \text{Case } (\mathbf{2}): \quad (\Delta m_{B_s})^{\text{new}} &= 1.396 \times 10^{-12} \text{ GeV}, \end{aligned} \tag{5.86}$$

The $H_1^0 - H_3^0$ sector and the H_2^0 sector contribute to $M_{12}^{B_s^0}$ given by $(8.532 \times 10^{-13}, -9.460 \times 10^{-15})$ GeV for case (1) and $(7.067 \times 10^{-13}, -3.835 \times 10^{-15})$ GeV for case (2). These new contributions are within experimentally allowed range. Higgs mediated CP violation is again found to be highly suppressed.

$D^0 - \overline{D^0}$ **mixing constraint**:

For the $D^0 - \overline{D^0}$ mixing we use $m_D = 1.864$ GeV, $f_D = 200$ MeV and compare the new contribution with $\Delta m_D = 1.27 \times 10^{-12}$ GeV.

$$\begin{aligned} \text{Case } (\mathbf{1}): \quad (\Delta m_D)^{\text{new}} &= 8.620 \times 10^{-13} \text{ GeV}, \\ \text{Case } (\mathbf{2}): \quad (\Delta m_D)^{\text{new}} &= 2.645 \times 10^{-13} \text{ GeV}, \end{aligned} \tag{5.87}$$

The $H_1^0 - H_3^0$ sector contribution has different sign from that of the H_2^0 sector. We find for $M_{12}^{D^0}$ these contributions to be $(4.402 \times 10^{-15}, -4.354 \times 10^{-13})$ GeV for case (1) and $(2.568 \times 10^{-15}, -1.348 \times 10^{-13})$ GeV for case (2). Again these limits are within experimental range.

We have found that new sources of CP violation through tree–level Higgs is very small in meson–anti-meson mixing with typical values $\text{Im}(M_{12}) \sim 10^{-4}\ \text{Re}(M_{12})$. This can be understood heuristically as follows. There are two types of contributions to the meson mixing as given in Eq. (5.76). The first term, proportional to B_2 respects a global $U(1)$ symmetry (strangeness in the K^0 system), which is only broken by the mass–splitting in the neutral Higgs boson spectrum between a pair of particles. However, this splitting is very small, of order m_Z^2 in the squared mass, see Eqs. (5.55) . The couplings of the nearly degenerate Higgs in each pair differ by a factor i, owing to the $U(1)$ symmetry, and the two contributions cancel, in the limit of exact degeneracy. For both the real and imaginary parts of M_{12} the contributions from the first term is suppressed by a factor $m_Z^2/(4M_k^2)$. Such a suppression is absent in the second term of Eq. (5.76), since the operator Q_4 explicitly breaks the $U(1)$ symmetry. Thus, although the first term has CP violation, in relation to the CP–conserving second term, it is suppressed by a factor $m_Z^2/(4M_k^2) \sim 10^{-4}$. Now, the second term, while it has no suppression factor, it is purely real. This can be seen from the following observation. In the mass basis of fermions in the original basis we have the relation (owing to the vanishing of off–diagonal mass terms in the mass eigenbasis)

$$(Y_{d3})_{ij} \left\langle H_3^d \right\rangle = -(Y_{d1})_{ij} \left\langle H_1^d \right\rangle \tag{5.88}$$

for $i \neq j$. The couplings of mass eigenstates of Higgs boson to down–type quarks are simply linear combinations of H_1^d and H_3^d. Since we assume CP to be spontaneously broken, all components of $(Y^k)_{ij}$ with $i \neq j$ have the same phase. As a result the second term of Eq. 5.76 becomes real. The constraint imposed by SUSY, that H_u^* fields do not couple to down–type quarks, and the fact that only two of the down–type Higgs bosons acquire VEVs is very crucial for this result.

5.4.2 Neutron electric diploe moment from Higgs exchange

Higgs boson exchange can generate non-zero electric dipole moments for the fermions. These diagrams are however suppressed by the light fermion Yukawa couplings. For the d quark EDM arising from neutral Higgs boson exchange at the one–loop level we find [104]

$$d_d = \frac{Q_d e}{16\pi^2} \mathrm{Im}(Y_{dq}^k Y_{qd}^k) \frac{m_q}{M_k^2} \left[\frac{3}{2} - \ln\left(\frac{M_k^2}{m_q^2} \right) \right] \xi_d, \tag{5.89}$$

where $\xi_d = (\alpha_s(M_k)/\alpha_s(\mu))^{16/23} \approx 0.12$, and q is summed over d, s and b. The neutron EDM is determined using the quark model via

$$D_n = 4d_d/3 - d_u/3. \tag{5.90}$$

We find

$$\begin{aligned} \text{Case (1)}: &\quad D_n = 1.809 \times 10^{-31} \ \ \mathrm{e-cm}, \\ \text{Case (2)}: &\quad D_n = 6.091 \times 10^{-31} \ \ \mathrm{e-cm}, \end{aligned} \tag{5.91}$$

which are well within experimental limits. The EDM of the electron is similarly found to be extremely small from the Higgs boson exchange diagrams.

5.4.3 $\mu \to 3e$ and $\tau \to 3\mu$ decays

Tree–level Higgs boson exchange can lead to flavor violating leptonic decays such as $\tau \to 3\mu$ and $\mu \to 3e$. The effective weak interaction mediating such decays can be parameterized as

$$G_{\rm eff} = \left| \sum_k (Y_e)^k_{11} (Y_e)^k_{12} \frac{1}{M_k^2} \right| \tag{5.92}$$

The effective couplings are found for $\mu \to 3e$ for the two cases to be

$$\begin{aligned} \text{Case (1)}: \quad & G_{\rm eff} = 4.432 \times 10^{-13}\ G_F, \\ \text{Case (2)}: \quad & G_{\rm eff} = 4.191 \times 10^{-13}\ G_F\ . \end{aligned} \tag{5.93}$$

And the couplings for $\tau \to 3\mu$ decay are

$$\begin{aligned} \text{Case (1)}: \quad & G_{\rm eff} = 45.721 \cdot 10^{-8}\ G_F, \\ \text{Case (2)}: \quad & G_{\rm eff} = 6.977 \cdot 10^{-8}\ G_F\ . \end{aligned} \tag{5.94}$$

Such small effective couplings will lead to negligible contributions to the decay branching ratios. For example, the branching ratio for $\tau \to 3\mu$ is of order 10^{-15}, well below the experimental sensitivity. We conclude that Higgs mediated FCNC in the lepton sector are all safe.

5.5 FCNC mediated by SUSY particles

In this section we turn attention to the flavor changing processes mediated by the supersymmetric particles. The main motivation for the non–Abelian Q_6 model was to bring such processes under control by a symmetry reason. Here we analyze meson–antimeson mixing, flavor violating leptonic decays, and the EDM of the neutron and the electron. We present our proposal to suppress SUSY contributions to the EDM by making the Higgsinos of the model light, with masses of order 100 GeV.

Owing to the Q_6 symmetry, the first two family squarks (and similarly sleptons) are degenerate in mass, while the third family, which is a Q_6 singlet has a different mass. In the fermion sector

Q_6 symmetry is broken, which means that there will be SUSY loop induced flavor violation in the model. Constraints on such flavor violation has been listed in Ref. [84–86] assuming all three families of squarks are degenerate. While these results are applicable for the K^0 and D^0 system in our model, they do not work well for the $B^0_{d,s}$ system. This is because the masses of the $\tilde{b}$ and $\tilde{d,s}$ masses are not the same.

5.5.1 Generalized constraints for B_d system

We have generalized the results of Ref. [85] by allowing for $\tilde{b}$ mass to be different from the masses of $\tilde{d,s}$. We define new parameters

$$y^d_{A,B} = \frac{(\tilde{m_b}^2)_{A,B}}{\tilde{m_d}^2_{A,B}} \tag{5.95}$$

for $A, B = L, R$. We expect these y parameters to be of order one, but not very close to one. Taking account of $y \neq 1$ we have generalized the constraints on the squark mixing parameters from B^0_d system as follows.

The effective $\Delta F = 2$ Hamiltonian for $B_{d,s}$ system can be written as

$$\begin{aligned}
\mathcal{H}_{\text{eff}} &= \sum_{i=1}^{5} C_i\, Q_i + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q} \\
&= -\frac{\alpha_s}{216 m^2_{\tilde{d}}} \{(\delta^d_{13})^2_{LL}(24 Q_1 x f_6(x,y) + 66 Q_1 \tilde{f}_6(x,y)) + (\delta^d_{13})^2_{RR}(24\tilde{Q}_1 x f_6(x,y) + 66\tilde{Q}_1 \tilde{f}_6(x,y)) \\
&+ (\delta^d_{13})_{LL}(\delta^d_{13})_{RR}(504 Q_4 x f_6(x,y) - 72 Q_4 \tilde{f}_6(x,y) + 24 Q_5 x f_6(x,y) + 120 Q_5 \tilde{f}_6(x,y)) \\
&+ (\delta^d_{13})^2_{RL}(204 Q_2 x f_6(x,y) - 36 Q_3 x f_6(x,y)) + (\delta^d_{13})^2_{LR}(204\tilde{Q}_2 x f_6(x,y) - 36\tilde{Q}_3 x f_6(x,y)) \\
&+ (\delta^d_{13})_{LR}(\delta^d_{13})^2_{RL}(-132 Q_4 \tilde{f}(x,y) - 180 Q_5 \tilde{f}_6(x,y))\},
\end{aligned} \tag{5.96}$$

The functions $f_6(x,y)$ and $\tilde{f}_6(x,y)$ are

$$\begin{aligned}
f_6(x,y) = & \frac{1}{(x-1)^3(y-1)^3(z-1)^3}\Big[-\ln x(x+y+xy-3x^3)(y-1)^3 \\
& +\ln y(x+y+xy-3y^2)(x-1)^3 \\
& +2(x-1)(y-1)(-x+y+x^2-y^2-x^3+y^3+2x^2y-2xy^2)\Big] \\
\tilde{f}_6(x,y) = & \frac{1}{(x-1)^3(y-1)^3(z-1)^3}\Big[2\ln x\cdot x(x^2-y)(y-1)^3 \\
& +2\ln y\cdot y(x-y^2)(x-1)^3 \\
& +(x-1)(y-1)(x^2-y^2+x^3-y^3-7x^2y+7xy^2+x^3y-xy^3)\Big].
\end{aligned} \tag{5.97}$$

Generalizing the results of Ref. [85] we obtain the squark mixing coefficients $(\delta^d_{13})_{AB}$ with $A,\ B = (L,\ R)$ as shown in Table 1. Here we have used the same input as in Ref. [85], so that for $y=1$ our results coincide. We have used the next-to-leading order lattice calculation results for the matrix elements. For some of the mixing parameters we made a simplifying assumption that y^d_L and y^d_R are equal.

5.5.2 SUSY flavor change in Q_6 model

In the Q_6 model the mass matrices of squarks in the flavor basis can be written as

$$(m_{\tilde{q}})^2_{AA} = m^2_{\tilde{q}A}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & y \end{pmatrix}, \tag{5.98}$$

q can be u or d, A can be L or R. Making the same unitary transformation on the squark fields as the ones on the quarks which diagonalize the quark mass matrices, we find the mass matrices of squarks in the SUSY basis (where the gluino coupling matrix is identity in the flavor space) to be:

$$(\tilde{m}_{\tilde{d}})^2_{LL} = O_d^T P_d^* (m_{\tilde{d}})^2_{LL} P_d O_d = m^2_{\tilde{d}L} \left[I + (y_l^d - 1) O_d^T P_d^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_d O_d \right]$$
$$= m^2_{\tilde{d}L} \left[I + (y_L^d - 1) \begin{pmatrix} 5.43 \cdot 10^{-5} & 1.31 \cdot 10^{-4} & -0.0074 \\ 1.31 \cdot 10^{-4} & 3.17 \cdot 10^{-4} & -0.0178 \\ -0.0074 & -0.0178 & 0.9996 \end{pmatrix} \right], \tag{5.99}$$

Note that this matrix is real, a consequence of the phase factorization of the fermion mass matrix. Similarly,

$$(\tilde{m}_{\tilde{d}})^2_{RR} = m^2_{\tilde{d}R} \left[I + (y_R^d - 1) \begin{pmatrix} 0.0367 & -0.1339 & 0.1318 \\ -0.1339 & 0.4891 & -0.4816 \\ 0.1319 & -0.4816 & 0.4742 \end{pmatrix} \right], \tag{5.100}$$

$$(\tilde{m}_{\tilde{u}})^2_{LL} = m^2_{\tilde{u}L} \left[I + (y_L^u - 1) \begin{pmatrix} 3.85 \cdot 10^{-6} & 7.74 \cdot 10^{-5} & -0.0020 \\ 7.74 \cdot 10^{-5} & 0.0016 & -0.0394 \\ -0.0020 & -0.0394 & 0.9984 \end{pmatrix} \right], \tag{5.101}$$

$$(\tilde{m}_{\tilde{u}})^2_{RR} = m^2_{\tilde{u}R} \left[I + (y_R^u - 1) \begin{pmatrix} 1.46 \cdot 10^{-5} & 2.94 \cdot 10^{-4} & 0.0038 \\ 2.94 \cdot 10^{-4} & 0.0059 & -0.0768 \\ 0.0038 & -0.0768 & 0.9941 \end{pmatrix} \right]. \tag{5.102}$$

$K^0 - \bar{K}^0$ mixing via squark–gluino loops have several contributions. The most stringent limit arises from the $(LL) - (RR)$ mixing, which requires [84]

$$\frac{|(y^d - 1)|}{(0.51 + 0.49\, y^d)^{1/4}} \; < \; 0.23 \left(\frac{\tilde{m}}{500 \text{ GeV}} \right) . \tag{5.103}$$

Here we have assumed $y_L^d = y_R^d = y^d$, and took the gluino mass to be equal to the first two family squark mass. For first two family squark mass of 500 GeV, this translates to the limit $0.77 \leq y^d \leq 1.24$.

For 1 TeV squarks, this limit is relaxed to $0.58 \leq y^d \leq 1.48$. We see that for y^d order one, the most stringent limit on squark mediated FCNC is satisfied.

The Q_6 model also generates significant $(RR)(RR)$ contributions to the $K^0 - \overline{K^0}$ mixing. We find

$$0.68 \leq y^d \leq 1.37 \tag{5.104}$$

for squark and gluino mass of 500 GeV. This constraint is also easily satisfied in the model.

In the B_d^0 system, the analogous constraints are (from the $(LL)(RR)$ operator)

$$\frac{|(y^d - 1)|}{(0.53 + 0.47\ y^d)^{1/4}} \ < \ 0.69 \left(\frac{\tilde{m}}{500\ \text{GeV}}\right) \ . \tag{5.105}$$

This limit leads to $0.48 \leq y^d \leq 1.85$ for squark-gluino mass of 500 GeV. The $(RR)(RR)$ squark mixing gives no constraint from the B_d system. Similarly, there are no constraints arising from the D^0 system, nor from other type of operators in the model.

In the leptonic sector, we find the (LL) slepton mixing (which is the same for the (RR) slepton mixing) to be

$$(\tilde{m}_{\tilde{e}})^2_{LL} = m^2_{\tilde{e}L} \left[I + (y^e_L - 1) \begin{pmatrix} 2.93 \cdot 10^{-4} & -4.02 \cdot 10^{-3} & -0.0167 \\ -4.02 \cdot 10^{-3} & 0.0550 & 0.2280 \\ -0.0167 & 0.2280 & 0.9447 \end{pmatrix} \right] . \tag{5.106}$$

There are stringent constraints on the mixing parameter $((\delta^e)_{LL})_{12}$ from the decay $\mu \to e\gamma$ [105]. On the face of it, the mixing presented above would appear to be in mild conflict with data by a factor of few. However, since such a constraint is very week for the $((\delta^e)_{RR})_{12}$ mixing, we point out that the flexibility in the lepton sector mass matrix can be used to make the (LL) contribution small in exchange for larger (RR) contributions. That is, assume $B \ll B'$ in Eq. (5.4).

5.5.3 Left–Right squark mixing and a solution to the EDM problem

So far we have ignored SUSY flavor violation arising from the left–right squark mixing. It turns out that these operators do not give significant contributions to meson–antimeson mixing, since such mixing have fermion chirality suppression. However, these mixing can generate new contributions to the neutron (and electron) electric dipole moments. Here we analyze constraints from the EDM and suggest a simple solution to the SUSY EDM problem.

First, as shown in Ref. [90], the trilinear A–term induced phases align with the phases of the fermion mass matrices, even without assuming proportionality of the A–terms with the respective Yukawa couplings. This feature arises due to the phase factorization of the fermion mass matrix. Left–right squark mixing also receive contributions from the superpotential μ-terms. We derive the mass matrix for the down squark sector to be:

$$(m_{\tilde{d}})^2_{LR} = F_1^{d*} \begin{pmatrix} 0 & \delta_d & 0 \\ -\delta_d & 0 & 0 \\ 0 & 0 & \alpha_d \end{pmatrix} + \sqrt{2} F_2^{d*} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta_d \\ 0 & \beta_d^{'} & 0 \end{pmatrix}, \tag{5.107}$$

with

$$F_1^d = \mu_3 v_{u3} + \mu_{13} v_{u1}, \qquad F_2^d = \mu_{31} v_{u3} + \frac{\mu_1 + \mu_{12}}{2} v_{u1}. \tag{5.108}$$

After the unitary transformations to the left and the right squarks, corresponding to case (1), we have the (LR) mixing matrix in the flavor basis as

$$(\tilde{m}_{\tilde{d}})^2_{LR} = O_d^T P_d (m_{\tilde{d}})^2_{LR} P_{d^c} O_{d^c} = F_1^{d*} \begin{pmatrix} -1.75 \cdot 10^{-4} & 4.14 \cdot 10^{-4} & 3.09 \cdot 10^{-5} \\ -4.46 \cdot 10^{-4} & 3.84 \cdot 10^{-4} & -5.45 \cdot 10^{-4} \\ 0.0078 & -0.0286 & 0.0282 \end{pmatrix}$$
$$+ \sqrt{2} F_2^{d*} e^{i\ \Delta\theta_d} \begin{pmatrix} 4.53 \cdot 10^{-5} & -1.66 \cdot 10^{-4} & -1.24 \cdot 10^{-5} \\ 1.79 \cdot 10^{-4} & -6.52 \cdot 10^{-4} & 2.18 \cdot 10^{-4} \\ -0.0031 & 0.0115 & 0.0125 \end{pmatrix}, \tag{5.109}$$

$$(\tilde{m}_{\tilde{u}})^2_{LR} = O_u^T P_u (m_{\tilde{u}})^2_{LR} P_{u^c} O_{u^c} = F_1^{u*} \begin{pmatrix} -1.62 \cdot 10^{-5} & 2.18 \cdot 10^{-4} & -0.0014 \\ -2.18 \cdot 10^{-4} & 0.0022 & -0.0283 \\ 0.0027 & -0.0554 & 0.7168 \end{pmatrix}$$
$$+ \sqrt{2} F_2^{u*} e^{i\ \Delta\theta_u} \begin{pmatrix} 3.24 \cdot 10^{-5} & -6.53 \cdot 10^{-4} & 0.0042 \\ 6.53 \cdot 10^{-4} & -0.0131 & 0.0848 \\ -0.0082 & 0.1661 & 0.0162 \end{pmatrix}. \tag{5.110}$$

with

$$F_1^u = \mu_3 v_{d3} + \mu_{31} v_{d1}, \qquad F_2^u = \mu_{13} v_{d3} + \frac{\mu_1 + \mu_{12}}{2} v_{d1}. \tag{5.111}$$

$$(\tilde{m}_{\tilde{e}})^2_{LR} = O_e^T P_e (m_{\tilde{e}})^2_{LR} P_{e^c} O_{e^c} = F_1^{d*} \begin{pmatrix} -1.10 \cdot 10^{-5} & 2.85 \cdot 10^{-4} & 6.72 \cdot 10^{-4} \\ -2.75 \cdot 10^{-4} & 0.0039 & 0.0257 \\ -4.89 \cdot 10^{-5} & 0.0047 & 0.0313 \end{pmatrix}$$
$$+\sqrt{2} F_2^{d*} e^{i\ \Delta\theta_d} \begin{pmatrix} 2.93 \cdot 10^{-6} & -1.14 \cdot 10^{-4} & -2.69 \cdot 10^{-4} \\ 1.10 \cdot 10^{-4} & -0.0043 & -0.0103 \\ 1.96 \cdot 10^{-5} & -0.0019 & 0.0093 \end{pmatrix}, \tag{5.112}$$

Corresponding to case (2) these matrices are:

$$(\tilde{m}_{\tilde{d}})^2_{LR} = O_d^T P_d (m_{\tilde{d}})^2_{LR} P_{d^c} O_{d^c} = F_1^{d*} \begin{pmatrix} -2.25 \cdot 10^{-4} & 5.32 \cdot 10^{-4} & 3.97 \cdot 10^{-5} \\ -5.73 \cdot 10^{-4} & 4.93 \cdot 10^{-4} & -7.00 \cdot 10^{-4} \\ 0.0100 & -0.0368 & 0.0362 \end{pmatrix}$$
$$+\sqrt{2} F_2^{d*} e^{i\ \Delta\theta_d} \begin{pmatrix} 5.20 \cdot 10^{-5} & -1.90 \cdot 10^{-4} & -1.42 \cdot 10^{-5} \\ 2.05 \cdot 10^{-4} & -7.48 \cdot 10^{-4} & 2.50 \cdot 10^{-4} \\ -0.0036 & 0.0131 & 0.0144 \end{pmatrix}, \tag{5.113}$$

$$(\tilde{m}_{\tilde{u}})^2_{LR} = O_u^T P_u (m_{\tilde{u}})^2_{LR} P_{u^c} O_{u^c} = F_1^{u*} \begin{pmatrix} -2.11 \cdot 10^{-5} & 2.83 \cdot 10^{-4} & -0.0018 \\ -2.83 \cdot 10^{-4} & 0.0028 & -0.0368 \\ 0.0036 & -0.0720 & 0.9319 \end{pmatrix}$$
$$+\sqrt{2} F_2^{u*} e^{i\ \Delta\theta_u} \begin{pmatrix} 1.41 \cdot 10^{-5} & -2.83 \cdot 10^{-4} & 0.0018 \\ 2.83 \cdot 10^{-4} & -0.0057 & 0.368 \\ -0.0036 & 0.0720 & 0.0070 \end{pmatrix}. \tag{5.114}$$

$$(\tilde{m}_{\tilde{e}})^2_{LR} = O_e^T P_e (m_{\tilde{e}})^2_{LR} P_{e^c} O_{e^c} = F_1^{d*} \begin{pmatrix} -1.41\cdot 10^{-5} & 3.66\cdot 10^{-4} & 8.62\cdot 10^{-4} \\ -3.53\cdot 10^{-4} & 0.0050 & 0.0330 \\ -6.28\cdot 10^{-5} & 0.0061 & 0.0402 \end{pmatrix}$$

$$+\sqrt{2} F_2^{d*} e^{i\ \Delta\theta_d} \begin{pmatrix} 3.36\cdot 10^{-6} & -1.31\cdot 10^{-4} & -3.08\cdot 10^{-4} \\ 1.26\cdot 10^{-4} & -0.0049 & -0.0118 \\ 2.24\cdot 10^{-5} & -0.0022 & 0.0106 \end{pmatrix}, \qquad (5.115)$$

Note that these matrices are in general complex, since $F_i^{u,d}$ are complex because of the spontaneously induced phases of the VEVs. This means that these matrices will contribute to neutron and electron EDM. Since these complex coefficients are proportional to $\mu v/\tilde{m}^2$, we find a simple solution to the SUSY EDM problem: Let the μ terms be of order 100 GeV, in which case one finds a suppression factor of 10^{-2} for the effective phase that enters the EDM expression. With this suppression factor, from the $(1,1)$ elements of these (LR) mixing matrices, we see that neutron and electron EDM constraints can be satisfied, even with the spontaneously induced phases in the VEVs being of order one.

The proposed solution to the SUSY EDM problem has direct experimental consequences for LHC. We predict that the Higgsinos should be light, and three such pairs of doublet Higgsinos should be observable at the LHC. Their scalar partners, however, are inaccessible, since their masses lie in the few TeV range.

$y \diagdown x$	0.25	1.0	4.0	0.25	1.0	4.0
	$\lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{LL}}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{LL}}\rvert$		
0.25	3.4×10^{-2}	1.6×10^{-1}	2.5×10^{-1}	7.2×10^{-2}	3.4×10^{-1}	1.2×10^{-1}
1.0	6.2×10^{-2}	1.4×10^{-1}	7.0×10^{-1}	1.3×10^{-1}	3.0×10^{-1}	3.4×10^{-1}
4.0	1.6×10^{-1}	2.7×10^{-1}	—	3.3×10^{-1}	5.8×10^{-1}	—
	$\lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{RR}}\rvert = \lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{LL}}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{RR}}\rvert = \lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{LL}}\rvert$		
0.25	1.4×10^{-3}	2.4×10^{-2}	1.0×10^{-2}	4.4×10^{-3}	1.0×10^{-2}	4.3×10^{-3}
1.0	1.9×10^{-2}	2.1×10^{-2}	2.8×10^{-2}	8.0×10^{-3}	9.0×10^{-1}	1.2×10^{-2}
4.0	4.8×10^{-2}	4.0×10^{-2}	1×10^{-1}	2×10^{-2}	1.7×10^{-2}	4.6×10^{-2}
	$\lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{LR}}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{LR}}\rvert$		
0.25	1.7×10^{-2}	3.7×10^{-2}	1.6×10^{-2}	3.6×10^{-2}	8.4×10^{-2}	3.6×10^{-2}
1.0	3.0×10^{-2}	3.3×10^{-2}	4.5×10^{-2}	6.6×10^{-2}	7.4×10^{-2}	1.0×10^{-1}
4.0	7.5×10^{-2}	6.4×10^{-2}	1.7×10^{-1}	1.7×10^{-1}	1.4×10^{-1}	3.9×10^{-1}
	$\lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{LR}}\rvert = \lvert\mathrm{Re}(\delta^d_{13})_{\mathrm{RL}}\rvert$			$\lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{LR}}\rvert = \lvert\mathrm{Im}(\delta^d_{13})_{\mathrm{RL}}\rvert$		
0.25	1.4×10^{-2}	5.9×10^{-2}	—	2.3×10^{-2}	4.4×10^{-1}	—
1.0	2.6×10^{-2}	5.2×10^{-2}	—	9.0×10^{-3}	2.3×10^{-2}	—
4.0	6.5×10^{-2}	1.0×10^{-1}	—	2.3×10^{-2}	4.4×10^{-2}	—

Table 5.1: Maximum allowed values for $|\mathrm{Re}(\delta^d_{13})_{\mathrm{AB}}|$ and $|\mathrm{Im}(\delta^d_{13})_{\mathrm{AB}}|$, with $A, B = (L, R)$. A new parameter y is introduced, with $y = m^2_{\tilde{b}}/m^2_{\tilde{d}}$. The definition of other parameters and their values follow Ref. [85].

CHAPTER 6

CONCLUSIONS

Leptogenesis provides a promising mechanism for baryogenesis. Combined with supersymmetry, leptogenesis offers more space for the problem of baryon asymmetry in the universe. In the supersymmetric models the supersymmetry breaking B term is one source of CP violation. When B term is complex, we realized soft leptogenesis. We considered the scenario in a different way from the traditional soft leptogenesis in Chpater 3. A problem of soft leptogenesis is the extremely small B term to produce the observed baryon asymmetry. The reason is because the mixing of decaying particles is only between the real part and the imaginary part of the only scalar RH neutrino. Fine tuning is needed. One way to avoid this problem is to introduce more fields, so that the generically complex Yukawa can introduce new source of CP violation, as we did in Chapter 4.

As we did in Chapter 2, resonant leptogenesis is an attractive way for addressing baryogenesis problem. Usually difficulty in the bayogenesis models is the magnitude of CP violation is very small. The reason is that CP violation is a loop effect and furthermore, the Yukawa coupling in the neutrino sector is tiny. In resonant leptogenesis scenario, the loop suppression is resonantly enhanced. This opens large space for leptogenesis model building.

Different types of seesaw mechanism can be constructed. The masses of light neutrinos can be generated through the exchange of tree-level $SU(2)$-triplet fermions. In this scenario, there is no constraint of the upper limit of neutrino masses from leptogenesis, because neutrino mass constraints don't induce asymmetry washout effects. Resonant leptogenesis can still play its role in this scenario. There will more complicated mixing between the triplet with constraints of symmetry. Ignoring flavor effects and assuming strong hierarchy between the heavy fermions, the lower bound to the lightest triplet fermion is

$$M_1 \geq 1.5 \times 10^{10} \text{ GeV}. \tag{6.1}$$

This large mass is in contradiction to the abundance of gravitino in supersymmetric models. Resonant leptogenesis can help decrease this mass limit.

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Major Field: THEORETICAL PHYSICS

We have studied various models of baryon asymmetry of the universe and flavor symmetry beyond the standard model. In the study of baryon asymmetry models, we concentrate on the mechanism of leptogenesis, baryon asymmetry through lepton asymmetry. In the first model of baryon asymmetry, CP violation necessary for baryon asymmetry has the same origin with CP violation of neutrino oscillations. We have shown that the observed baryon asymmetry is realizable within the bound of the current neutrino data. Baryogenesis is via resonant leptogenesis, which is free of the gravitino problem. We have also made predictions for the upcoming neutrino experiments. In the other models of baryon asymmetry we have studied some new ways of leptogenesis in the supersymmetry scenario through the decay of the right handed sneutrinos.. We have shown that the new ways of leptogenesis are viable to generate the observed baryon asymmetry without inducing the gravitino abundance problem existing in supersymmetry models. In the study of flavor symmetry we have made phenomenological studies of a multi-Higgs model based on the Q_6 flavor symmetry. Through fitting experimental data of neutral meson mixing, we have determined the mass spectrum of the heavy Higgs. We then made other phenomenological studies with these Higgs masses. We have found the contributions through heavy Higgs exchange are sufficiently suppressed by flavor symmetry.

ADVISOR'S APPROVAL: ____________________