BARYON ASYMMETRY OF THE UNIVERSE AND NEUTRINO PHYSICS

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INTRODUCTION

1.1 Matter Versus Anti-Matter

One of the great mysteries of modern particle physics and cosmology is the lack of antimatter in our surroundings. This is somewhat puzzling given that the relativistic field theories which underlie modern particle physics have built into them a fundamental symmetry; which states that for every particle there is an antiparticle degenerate in mass and with quantum numbers and charges of the opposite sign. In the old language of Dirac, for example, the relativistic wave equation for the electron has both positive and negative energy solutions, which led Dirac to predict the necessary existence of the positron. In more modern quantum field theory language, we think of creation or annihilation operators acting on a field that respectively create particles and destroy antiparticles or destroy particles and create antiparticles associated with the field. The properties and dynamics of the particles and antiparticles are fundamentally related. If we consider local, Lorentz-invariant field theory equations such as currently used for the "Standard Model" of particle physics, and we flip the signs of all charges that appear in them, effectively turning particles into antiparticles, and then perform a space reversal ($\overrightarrow{x} \rightarrow -\overrightarrow{x}$) followed by a time reversal ($t \rightarrow -t$), we recover the same equations. This symmetry of the equations, the so-called CPT (Charge-Parity-Time reversal) symmetry, implies, for example, that particle and antiparticles should have exactly the same mass. Similarly, if the protons exist, then anti-protons with the same characteristics should exist too (and in fact, is being made now at CERN and FermiLab). Stretching our imagination further, the Universe could then be filled with antimatter stars and galaxies that are indistinguishable from ordinary stars and galaxies if one studies them solely via their light emission or their gravitational attraction on neighboring bodies. This of course assumes that antimatter stars are spatially separated from matter stars, or else the two will annihilate each other. The fact is that anti-matter on earth is very rare, in fact, the only anti-protons ever observed were the ones produced at CERN or Fermilab. Cosmic probes into planets conclude they are made out of matter. With confidence, we can say that our entire solar system consists of matter only. One can argue that there could be patches or regions at the larger scale containing antimatter, but experiments [1] showed otherwise. There would be a strong detectable γ radiation originating from nucleons-antinucleons reactions if there was a cluster out there that contains one or many galaxies with both matter and antimatter. Furthermore, the well-tested Standard Model also implies that total charge as well as quantum numbers like baryon number and lepton number should be conserved in particle interactions, excluding the notion that there could be a region elsewhere that does not contain equal amounts of matter and antimatter. The most fundamental observation we can make about the observed universe is that it is dominantly made out of matter (no-antimatter). Baryogenesis, or Baryon Asymmetry (BA), (matter-antimatter asymmetry), explaining BA is one of the most challenging open questions in particle physics as well as in cosmology. The subject has been of concern to particle physicists since the discovery of microscopic CP violation, which encouraged the construction of concrete Baryogenesis scenarios. The subject became a standard part of modern cosmology with the introduction of grand unified theories (GUTs), introduced in the 1970s, which establish a possible source for baryon number violation, an essential component of Baryogenesis. More recent ideas have attempted to link the baryon asymmetry with details of models of electroweak symmetry breaking, and offer the possibility of testing models of Baryogenesis in future colliders such as the LHC. In this dissertation however, we concentrate on three of the most recent and popular mechanisms; realized in different ways: *Baryogenesis via Leptogenesis*[2], *Soft Leptogenesis*[3,4] and *resonant Leptogenesis*[5,6]. The results of our study are reported in [7–9].

In the second chapter, we calculate [7] lepton asymmetry induced in the decay of right–handed neutrinos in a class of minimal left–right symmetric models [10]. In these models, which assume low energy supersymmetry, the Dirac neutrino mass matrix has a determined structure. As a result, lepton asymmetry is calculable in terms of measurable low energy neutrino parameters. By solving the Boltzmann equations numerically we show that adequate baryon asymmetry is generated in these models in complete agreement with constrains by Big Bang Nucleosynthesis and the recent high precision measurement by the NASA satellite mission WMAP experiment [11]:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}, \tag{1.1}$$

where η_B is the baryon to photon ratio. Furthermore, we make predictions on the light neutrino oscillation parameters, which can be tested in next generation neutrino experiments.

In the third chapter of this thesis, we discuss a more recent idea, Soft Leptogenesis, which is an alternative and attractive mechanism to explain the baryon asymmetry we are after. This time, we study the effect of the interactions of the $SU(2)_R$ gauge boson W_R on the generation of the primordial lepton asymmetry[8]. $B-L$ violation occurs when Left–Right symmetry is broken by the vacuum expectation value (VEV) v_R of the $B-L=-2$ triplet scalar field, which gives Majorana masses to the r.h sneutrino, and lepton number is violated in their decay $\tilde{\nu}_{R1} \to \tilde{e}_R u \bar{d}$ as well as $\tilde{\nu}_{R1} \to \tilde{e}^*_R \bar{u} d$, these decays are mediated by the right handed gauge boson W_R, and can dominate the traditional $\nu_R \to L\phi^\dagger$, frequently used decay to explain BA. Furthermore, by Renormalization Group Equations (RGE) analysis, we show that the requirement of unconventionally small $B-$term is no longer needed. In addition, we use RGE running and SUSY breaking effect to naturally account for the complex $O(1)$ phase as dictated by the success of the scenario. The mass of r.h sneutrino can be $M_{\tilde{\nu}} \sim M_{W_R} \sim (10^9 \; - \; 10^{10})$ GeV.

In chapter 4 we present a new realization of inverted neutrino mass hierarchy based on $S_3 \times U(1)$ flavor symmetry[9]. In this scenario, the deviation of the solar oscillation angle from $\pi/4$ is correlated with the value of θ_{13}, as they are both induced by a common mixing angle in the charged lepton sector. We find several interesting predictions: $\theta_{13} \geq 0.13$, $\sin^2\theta_{12} \geq 0.31$, $\sin^2\theta_{23} \simeq 0.5$, $0 \leq \cos\delta \leq 0.7$ for the neutrino oscillation parameters and $0.01\ \mathrm{e}V \lesssim m_{\beta\beta} \lesssim 0.02\ \mathrm{e}V$ for the effective neutrino mass in neutrinoless double β-decay. We show that the same scenario can naturally explain the observed baryon asymmetry of the universe via resonant leptogenesis. The masses of the decaying right–handed neutrinos can be in the range $(10^3 - 10^7)$ GeV, which would avoid the generic gravitino problem of supersymmetric models.

In the appendix section, we briefly review the basic thermodynamics of the expanding universe, set up Boltzmann equations, review the formalism of CP violation in the kaon system, and make some comments about the numerical methods.

1.2 Sakharov criteria

The Standard Model of Cosmology provides a very satisfactory picture that accounts for variety of observational data, in particular, the observed 2.7^oK background black-body radiation is in total agreement with the nucleosynthesis calculation of the primordial helium abundance. On the downside, the Standard Model with only baryon-number conserving interactions does not fix baryon-number asymmetry ratio as indicated earlier. It is desirable that, independent of any initial conditions, such an asymmetry could be generated by underlying physical interactions. To achieve this, we must postulate new particles interactions, beyond those of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ Standard Model.

In 1967, Sakharov[12] proposed a radical alternative: our physics is wrong! More precisely, there is new physics beyond the Standard Model which, at higher energies than can currently be tested with accelerators, allows for baryon number violation. Assuming a highly symmetric state in the early Universe, a matter-antimatter asymmetry can be dynamically generated in an expanding Universe if the particle interactions and the cosmological evolution satisfy the so called Sakharov conditions, which we enumerate below

(i) Underlying theory must have processes that violate B number

$$\Delta B \neq 0$$

where B is the baryon number. If the baryon number B was conserved by the interactions, it would mean that the baryon number commutes with the Hamiltonian of the system H: $[B, H] = 0$. Hence, if $B(t_0) = 0$, we would have $B(t) \propto \int_{t_0}^{t} [B, H]\, dt' = 0$ at any subsequent time and no baryon number production would take place.

(ii) Both Charge Conjugation, and CP symmetry must be violated; otherwise, one can never establish baryon-antibaryon asymmetry (since the action of C and CP would transforms $n_B \to n_{\bar{B}}$). To see this, we define the following baryon number operator,

$$\hat{B} = \frac{1}{3} \sum_q \int d^3x : q^\dagger(x,t) q(x,t) : \quad ,$$

which is C-odd and CP-odd. This is evident from the action of P, C and T on the quark fields:

$$\begin{aligned} Pq(x,t)P^{-1} &= \gamma^0 q(-x,t), \quad Pq^\dagger(x,t)P^{-1} = q^\dagger(-x,t)\gamma^0, \\ Cq(x,t)C^{-1} &= \imath\gamma^2 q^\dagger(x,t), \quad Cq^\dagger(x,t)C^{-1} = \imath q^\dagger(x,t)\gamma^2, \\ Tq(x,t)T^{-1} &= -\imath q(x,-t)\gamma_5\gamma^0\gamma^2, \quad Tq^\dagger(x,t)T^{-1} = -\imath\gamma^2\gamma^0\gamma_5 q^\dagger(x,-t)\gamma^0. \end{aligned} \tag{1.2}$$

Then

$$\begin{aligned} P : q^\dagger(x,t)q(x,t) : P^{-1} &=: q^\dagger(-x,t)q(-x,t) :, \\ C : q^\dagger(x,t)q(x,t) : C^{-1} &= - : q^\dagger(x,t)q(x,t) :, \\ T : q^\dagger(x,t)q(x,t) : T^{-1} &=: q^\dagger(x,-t)q(x,-t) :, \end{aligned} \tag{1.3}$$

so that

$$P\hat{B}P^{-1} = \hat{B}, \quad C\hat{B}C^{-1} = -\hat{B}, \quad (CP)\hat{B}(CP)^{-1} = -\hat{B}.$$

A non-zero expectation value $< B\hat{B} >$ requires that the Hamiltonian violates C and CP. More intuitively, C symmetry would guarantee that $\Gamma(i \to f) = \Gamma(i^\dagger \to f^\dagger)$, while CP symmetry would guarantee that $\Gamma(i \to f) = \Gamma(\bar{i} \to \bar{f})^*$. With CP alone it might be possible to create baryon asymmetry in certain localized region of the phase space, but integrating over all momenta and summing over all spins would leave a vanishing asymmetry.

(iii) Departure from thermal equilibrium of X-particles mediating $\Delta B \neq 0$ processes is necessary. This is because if all processes, including those which violate baryon number, are in thermal equilibrium, the baryon asymmetry vanishes. This is a direct consequence of the CPT invariance. To see this, define $CPT \equiv \theta$, and the density matrix at time t for a system in thermal equilibrium as $\rho(t) = e^{-\beta(t)H(t)}$, then from Eq (1.3) we obtain the equilibrium average of B,

$$\begin{aligned} \langle B \rangle_T &= Tr\left(e^{-\beta H}\hat{B}\right) \\ &= Tr\left(\theta^{-1}\theta e^{-\beta H}\hat{B}\right) \end{aligned}$$

*$x^\dagger$ has opposite charge but same chirality as x. $\bar{x}$ has both opposite charge and chirality.

$$\begin{aligned}
&= Tr\left(\theta e^{-\beta H}\hat{B}\theta^{-1}\right) \\
&= Tr\left(\theta e^{-\beta H}\theta^{-1}\theta\hat{B}\theta^{-1}\right) \\
&= Tr\left(e^{-\beta H}\left(-\hat{B}\right)\right) \\
&= -Tr\left(e^{-\beta H}\hat{B}\right) \\
&= -\langle B\rangle_T
\end{aligned} \tag{1.4}$$

where $\beta = \frac{1}{k_B T}$, and we have used the fact that H commutes with the operator CPT that we called θ above. Thus $\langle B\rangle_T = 0$. Whence, to establish asymmetry dynamically, B violating processes must be out of equilibrium in the Universe. This can be seen as follows:

$$\frac{d\Delta n_B}{dt} = -\left[\gamma_{\not B} e^{-\left(\frac{m-\mu}{k_B T}\right)} - \gamma_{\not B} e^{-\left(\frac{\bar{m}-\bar{\mu}}{k_B T}\right)}\right] \tag{1.5}$$

where $\gamma_{\not B}$ denotes the rate for $\not B$ and μ is the chemical potential, and $\bar{\mu} = -\mu$. Since $m = \bar{m}$ by CPT theorem, $e^{-\frac{m}{k_B T}}$ is not relevant and we omit it. Then for $k_B T \gg \mu$,

$$\frac{d\Delta n_B}{dt} = \frac{-2\mu}{k_B T}\gamma_{\not B}. \tag{1.6}$$

On the other hand

$$\begin{aligned}
\Delta n_B &= \frac{2\zeta(3)}{\pi^2} g'(k_B T)^3 \left[e^{\frac{\mu}{k_B T}} - e^{\frac{-\mu}{k_B T}}\right] \\
&\simeq \frac{2}{\pi^2} g'(k_B T)^3 \frac{2\mu}{k_B T}.
\end{aligned} \tag{1.7}$$

Thus eliminating $\frac{2\mu}{k_B T}$

$$\begin{aligned}
\frac{d\Delta n_B}{dt} &= -\frac{\pi^2}{2}\frac{\gamma_{\not B}}{g'(k_B T)^3}\Delta n_B \\
&= -\frac{\pi^2}{2}\Gamma_{\not B}\Delta n_B
\end{aligned} \tag{1.8}$$

where $\Gamma_{\not B} = \frac{\gamma_{\not B}}{g'(k_B T)^3} = \frac{\gamma_{\not B}}{n_B}$ gives the rate for $\not B$. The solution of above equation gives

$$\Delta n_B = (\Delta n_B)_{\text{initial}}\, e^{-\frac{\pi^2}{2}\Gamma_{\not B} t}. \tag{1.9}$$

What we learn from this result is that if B-violating processes are ever in equilibrium, then these processes actually washes out any initial condition for $\Gamma_{\not B} t \geq 1$.

1.3 Boltzmann Equations

The processes of interest are active at high temperature while the universe is expanding, when the system is far from thermodynamic equilibrium, and one needs to follow evolution of a density while the particle species produces and collides with many different species. Boltzmann equations (BE) allow us to follow the effect of different interactions, in fact, all important calculations in Cosmology are done by means of BE. In this section, we introduce the basic elements for setting up BE.

It is usually a good approximation to assume Maxwell-Boltzmann statistics, so that the equilibrium number density of a particle i is given by

$$n_i^{\rm eq}(T) = \frac{g_i}{(2\pi)^3}\int d^3p_i\, f_i^{\rm eq} \qquad \text{with} \qquad f_i^{\rm eq}\,(E_i,T) = {\rm e}^{-E_i/T}\ . \tag{1.10}$$

For a massive non relativistic particle one finds

$$n_i^{\rm eq}(T) = \frac{g_i T m_i^2}{2\pi^2}{\rm K}_2\left(\frac{m_i}{T}\right)\ , \tag{1.11}$$

where K_2 is bessel function of the second type. For a massless particle one gets

$$n_i^{\rm eq}(T) = \frac{g_i T^3}{\pi^2}\ . \tag{1.12}$$

The universe expansion and different interations modify the particle densities. Since we are only interested in the effect of interactions, it is useful to scale out the expansion. This is done by taking the number of particles per comoving volume element, i.e. the ratio of the particle density n_i to the entropy density s,

$$Y_i = \frac{n_i}{s}\ , \tag{1.13}$$

as independent variable instead of the number density. In a radiation dominated universe the entropy density reads

$$s = g_*\frac{2\pi^2}{45}T^3\ . \tag{1.14}$$

In our case, elastic scatterings, which can only change the phase space distributions but not the particle densities, occur at a much higher rate than inelastic processes. Therefore, we can assume kinetic equilibrium, so that the phase space densities are given by

$$f_i(E_i, T) = \frac{n_i}{n_i^{\mathrm{eq}}} \mathrm{e}^{-E_i/T} \, . \tag{1.15}$$

In this framework the Boltzmann equation describing the evolution of a particle number Y_{J} in an isentropically expanding universe reads

$$\begin{aligned} \frac{\mathrm{d}Y_{\mathrm{J}}}{\mathrm{d}z} \;=\; & -\frac{z}{sH\left(m_{\mathrm{J}}\right)} \sum_{a,i,j,\dots} \left[\frac{Y_{\mathrm{J}} Y_a \dots}{Y_{\mathrm{J}}^{\mathrm{eq}} Y_a^{\mathrm{eq}} \dots} \gamma^{\mathrm{eq}} \left(\mathrm{J} + a + \dots \to i + j + \dots\right) \right. \\ & \left. - \frac{Y_i Y_j \dots}{Y_i^{\mathrm{eq}} Y_j^{\mathrm{eq}} \dots} \gamma^{\mathrm{eq}} \left(i + j + \dots \to \mathrm{J} + a + \dots\right) \right] \, , \end{aligned} \tag{1.16}$$

where $z = m_{\mathrm{J}}/T$ and $H\left(m_{\mathrm{J}}\right)$ is the Hubble parameter at $T = m_{\mathrm{J}}$. The γ^{eq} are space time densities of scatterings for the different processes. For a decay one finds

$$\gamma_D := \gamma^{\mathrm{eq}}(\mathrm{J} \to i + j + \dots) = n_{\mathrm{J}}^{\mathrm{eq}} \frac{\mathrm{K}_1(z)}{\mathrm{K}_2(z)} \Gamma \, , \tag{1.17}$$

where K_1 and K_2 are modified Bessel functions and Γ is the tree level decay width in the rest system of the decaying particle. Neglecting a possible CP violation, one finds the same reaction density for the inverse decay.

Calculation of Lepton Asymmetry will involve 2 body scattering. The reaction density for a two body scattering is given by,

$$\gamma^{\mathrm{eq}}(\mathrm{J} + a \leftrightarrow i + j + \dots) = \frac{T}{64\pi^4} \int\limits_{(m_{\mathrm{J}}+m_a)^2}^{\infty} ds \, \hat{\sigma}(s) \, \sqrt{s} \, \mathrm{K}_1\left(\frac{\sqrt{s}}{T}\right) \, , \tag{1.18}$$

where s is the squared center of mass energy and the reduced cross section $\hat{\sigma}(s)$ for the process $\mathrm{J} + a \to i + j + \dots$ is related to the usual total cross section $\sigma(s)$ by

$$\hat{\sigma}(s) = \frac{2\lambda(s, m_{\mathrm{J}}^2, m_a^2)}{s} \sigma(s), \tag{1.19}$$

where λ is the usual kinematical function

$$\lambda(s, m_{\mathrm{J}}^2, m_a^2) \equiv \left[s - (m_{\mathrm{J}} + m_a)^2\right] \left[s - (m_{\mathrm{J}} - m_a)^2\right] \, . \tag{1.20}$$

In order to compute the Baryon Asymmetry we will have to employ numerical solution to the coupled Boltzmann Equation for the Lepton Asymmetry density and the abundance of right handed neutrinos. We will shortly come back to this analysis and discuss it in detail.

1.4 Chemical potential, asymmetries relations and Sphalerons

In the standard model, baryon number violating processes convert three baryons to three antileptons. This violates conservation of baryon number and lepton number, but the difference $B - L$ is conserved. This is because $B - L$ has no anomalies in the Standard Model, while B (or L) has electroweak anomalies. A sphaleron is a static (time independent) solution to the electroweak field equations of the Standard Model, and it is involved in processes that violate baryon and lepton number. Such processes cannot be represented by Feynman diagrams, and are therefore called non-perturbative. This means that under normal conditions sphalerons are unobservably rare. However, they would have been more common at the higher temperatures of the early universe. In almost all theories of baryogenesis an imbalance of the number of leptons and antileptons is formed first, and sphaleron transitions then recycle this to an imbalance in the numbers of baryons and antibaryons. Below, we derive some of the relations between various asymmetry densities, establishing the connection between lepton asymmetry and baryon asymmetry.

As we will see, Sphaleron transitions lead to the baryon asymmetry by recycling a lepton asymmetry. Further $B + L$ asymmetry generated before EW transition i.e. at $T > T_{\mathrm{EW}}$, will be washed out. However, since only left handed fields couple to sphalerons, a non zero value of $B + L$ can persist in the high temperature symmetric phase if there exist a non vanishing $B - L$ asymmetry [see below]. In weakly coupled plasma, one can assign a chemical potential μ_i to each of the quark, lepton and Higgs field.

$$n_i - \bar{n}_i = \frac{2}{\pi^2} g' T^3 \left(\frac{2\mu_i}{T} \right),$$

where g' is the particle species effective degree of freedom, T is the temperature at any given time. This also implies

$$\begin{aligned} n_B &= B \left(\frac{4}{\pi^2} g' T^2 \right) \\ n_L &= L \left(\frac{4}{\pi^2} g' T^2 \right) \end{aligned} \tag{1.21}$$

where B and L are baryon and lepton asymmetries respectively. Note that in SM

$$\begin{aligned} q_{Li} &= \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \qquad B = \frac{1}{3},\ L = 0 \\ & u_{Ri}, d_{Ri} \\ \ell_{Li} &= \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \qquad B = 0,\ L = 1 \\ & \nu_{Ri}, e_{Ri} \end{aligned}$$

Thus in Eq. (1.21)

$$\begin{aligned} B &= 3 \times \frac{1}{3} \sum_i \left(2\mu_{qi} + 2\mu_{ui} + 2\mu_{di}\right) \\ L &= \sum_i \left(2\mu_{li} + 2\mu_{ei}\right) \end{aligned} \tag{1.22}$$

In high temperature plasma, quarks, leptons and Higgs interact via Yukawa and gauge couplings and in addition, via the non perturbative sphaleron processes. In thermal equilibrium all these processes yield constraints between various chemical potentials. The effective interaction

$$O_{B+L} = \Pi_i \left(q_{Li} q_{Li} q_{Li} \ell_{Li}\right)$$

yields

$$\sum_i \left(3\mu_{qi} + \mu_{li}\right) = 0. \tag{1.23}$$

Another constraint is provided by vanishing of total charge of plasma

$$\sum_i \left[\begin{array}{c} 3\tfrac{1}{3} 2\mu_{qi} + 3\tfrac{4}{3}\mu_{ui} \\ +3\left(-\tfrac{2}{3}\right)\mu_{di} + (-1)\, 2\mu_{li} + (-2)\,\mu_{ei} + \tfrac{1}{N}\,(1)\,\mu_\phi \end{array} \right] = 0$$

where we have used

$$Y_q = \frac{1}{3},\ Y_u = \frac{4}{3},\ Y_d = -\frac{2}{3},\ Y_l = -1,\ Y_{e^-} = -2,\ Y_\phi = 1$$

The above equation can be written as

$$\sum_i \left(\mu_{qi} + 2\mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{2}{N}\mu_\phi\right) = 0. \tag{1.24}$$

Furthermore, invariance of Yukawa couplings $\bar{q}_{Li}\phi d_{Ri}$, etc gives

$$\begin{aligned}
\mu_{qi} - \mu_\phi - \mu_{dj} &= 0 \\
\mu_{qi} - \mu_\phi - \mu_{uj} &= 0 \\
\mu_{li} - \mu_\phi - \mu_{ej} &= 0
\end{aligned} \tag{1.25}$$

When all Yukawa interactions are in equilibrium, these interactions establish equilibrium in different generations

$$\mu_{li} = \mu_l,\ \mu_{qi} = \mu_q \text{ etc.}$$

Thus we obtain from Eqs (1.23) and (1.24)

$$\begin{aligned}
\mu_q &= -\frac{1}{3}\mu_l \\
\mu_q + 2\mu_u - \mu_d - \mu_l - \mu_e + \frac{2}{N}\mu_\phi &= 0
\end{aligned}$$

giving

$$-\frac{4}{3}\mu_l + 2\mu_u - \mu_d - \mu_e + \frac{2}{N}\mu_\phi = 0. \tag{1.26}$$

Furthermore, Eqs. (1.25) implies

$$\begin{aligned}
-\frac{1}{3}\mu_l - \mu_\phi - \mu_d &= 0 \\
-\frac{1}{3}\mu_l - \mu_\phi - \mu_u &= 0 \\
\mu_l - \mu_\phi - \mu_e &= 0
\end{aligned} \tag{1.27}$$

Using the above equations, we can write (1.26) as

$$-\frac{4}{3}\mu_l + 2\left(-\frac{1}{3}\mu_l + \mu_\phi\right) - \left(-\frac{1}{3}\mu_l - \mu_\phi\right) - (-\mu_l - \mu_\phi) + \frac{2}{N}\mu_\phi = 0.$$

Thus finally we can express μ_q, μ_u, μ_d, μ_e, and μ_ϕ in terims of μ_l.

$$\begin{aligned}
\mu_\phi &= \frac{8}{3}N\frac{1}{4N+2}\mu_l = \frac{4N}{6N+3}\mu_l \\
\mu_d &= -\frac{1}{3}\mu_l - \mu_\phi \\
&= -\frac{1}{3}\mu_l - \frac{4N}{6N+3}\mu_l
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{6N+1}{6N+3}\mu_l \\
\mu_u &= -\frac{1}{3}\mu_l + \mu_\phi \\
&= -\frac{1}{3}\mu_l + \frac{4N}{6N+3}\mu_l \\
&= \frac{2N-1}{6N+3}\mu_l \\
\mu_e &= \mu_l - \mu_\phi \\
&= \mu_l - \frac{4N}{6N+3}\mu_l \\
&= \frac{2N+3}{6N+3}\mu_l \qquad (1.28)
\end{aligned}
$$

Hence from Eqs. (1.22)

$$
\begin{aligned}
B &= N\left\{-\frac{2}{3}\mu_l + \frac{2N-1}{6N+3}\mu_l - \frac{6N+1}{6N+3}\mu_l\right\} \\
&= \left[-4N-2+2N-1-6N-1\right]\frac{\mu_l}{6N+3} \\
&= -N\frac{(8N+4)}{3\,(2N+1)}\mu_l \\
&= -\frac{4N}{3}\mu_l \qquad (1.29) \\
L &= N\left(2\mu_l + \frac{2N+3}{6N+3}\mu_l\right) \\
&= \frac{14N^2+9N}{6N+3}\mu_l \qquad (1.30) \\
B-L &= -\frac{8N^2+4N+14N^2+9N}{6N+3}\mu_l \\
&= -\frac{22N^2+13N}{6N+3}\mu_l \qquad (1.31) \\
\frac{B}{B-L} &= \frac{8N^2+4N}{22N^2+13N} \\
&= \frac{8N+4}{22N+13} \\
&= \frac{8N_g+4n_H}{22N_g+13n_H} \equiv a \qquad (1.32)
\end{aligned}
$$

These relations hold for $T \gg v$. In general $B/(B-L)$ is a function of v/T. For SM, $N_g = 3$, $n_H = 1$ so that $a = 28/79$.

Thus finally we obtain

$$
Y_B(\equiv \frac{n_B - n_{\bar{B}}}{s})
$$

$$= \quad aY_{B-L} = \frac{a}{a-1} Y_L \tag{1.33}$$

From the relation between entropy density and photon number density, $s \simeq \eta_\gamma/7$, we find

$$\begin{aligned} Y_B &= \eta \left(\frac{\eta_\gamma}{s}\right) \simeq \frac{1}{7}\eta \\ &\simeq \frac{1}{7}(6 \pm 3) \times 10^{-10}. \end{aligned}$$

It is this number we try to explain via underlying physical process and in the context or realistic physical model. As mentioned earlier, there are several Baryon Asymmetry mechanisms that undertake the task of explaining this number, we concentrate on the 3 most popular Leptogenesis ideas. In specific frameworks, we analyze the mechanisms in details and derive interesting correlation with Leptogenesis and the physics of neutrinos.

CHAPTER 2

LEPTOGENESIS IN MINIMAL LEFT-RIGHT SYMMETRIC MODELS

2.1 **Introduction**

The discovery of neutrino flavor oscillations in solar, atmospheric, and reactor neutrino experiments [13] may have a profound impact on our understanding of the dynamics of the early universe. This is because such oscillations are feasible only if the neutrinos have small (sub–eV) masses, most naturally explained by the seesaw mechanism [14]. This assumes the existence of super-heavy right–handed neutrinos N_i (one per lepton family) with masses of order $(10^8 - 10^{14})$ GeV. The light neutrino masses are obtained from the matrix $M_\nu \simeq M_D M_R^{-1} {M_D}^T$ where M_D and M_R are respectively the Dirac and the heavy Majorana right-handed neutrino (r.h.n) mass matrices. The decay of the lightest right–handed neutrino N_1 can generate naturally an excess of baryons over anti-baryons in the universe [2] consistent with cosmological observations. The baryon asymmetry parameter is an important cosmological observable constrained by Big Bang Nucleosynthesis and determined recently with high precision by the WMAP experiment [11]:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}. \tag{2.1}$$

The decay of N_1 can satisfy all three of the Sakharov conditions [12] needed for successful generation of η_B – it can occur out of thermal equilibrium, there is sufficient C and CP violation, and there is also baryon number violation. The last condition is met by combining lepton number violation in the Majorana masses of the right–handed neutrinos with $B + L$ violating interactions of the Standard Model arising through the electroweak sphaleron processes [15]. A compelling picture emerges, with the same mechanism explaining the small neutrino masses and the observed baryon asymmetry of the universe. η_B appears to be intimately connected to the observed neutrino masses and mixings.

A more careful examination of the seesaw structure would reveal that, although there is an underlying connection, the light neutrino mass and mixing parameters cannot determine the cosmological baryon asymmetry, when the seesaw mechanism is implemented in the context of the Standard Model (SM) gauge symmetry. It is easy to see this as follows. Without loss of generality one can work in a basis where the charged lepton mass matrix and the heavy right–handed neutrino Majorana mass matrix M_R are diagonal with real eigenvalues. The Dirac neutrino mass matrix would then be an arbitrary complex 3×3 matrix with 18 parameters (9 magnitudes and 9 phases). Three of the phase parameters can be removed by field redefinitions of the left–handed lepton doublets and the right–handed charged lepton singlets. The neutrino sector will then have 18 $(= 15 + 3)$ parameters. 9 combinations of these will determine the low energy observables (3 masses, 3 mixing angles and 3 phases), while the lepton asymmetry (and thus η_B) would depend on all 18 parameters, leaving it arbitrary.

In this section of the thesis we show that it is possible to quantitatively relate η_B to light neutrino mass and mixing parameters by implementing the seesaw mechanism in the context of a class of supersymmetric left–right models [10]. We note that unlike in the SM where the right–handed neutrinos appear as rather ad hoc additions, in the left–right symmetric models they are more natural as gauge invariance requires their existence. Supersymmetry has the well–known merit of solving the gauge hierarchy problem. With the assumption of a minimal Higgs sector, it turns out that these models predict the relation for the Dirac neutrino mass matrix, in a basis where the charged lepton mass matrix is diagonal;

$$M_D = c \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{2.2}$$

where $c \simeq m_t/m_b$ is determined from the quark sector, leaving only the Majorana mass matrix M_R to be arbitrary. 3 phases in M_R can be removed, leaving a total of 9 parameters which determine both the low energy neutrino masses and mixings as well as the baryon asymmetry. It then becomes apparent that η_B is calculable in terms of the neutrino observables. There have been other attempts in the literature to relate leptogenesis with low energy observables [16,17]. Such attempts often make additional assumptions such as $M_D = M_{\mathrm{u}p}$ (which may not be fully realistic), or specific textures for lepton mass matrices.

While a lot has been learned from experiments about the light neutrino masses and mixings, a lot remains to be learned. Our analysis shows that cosmology puts significant restrictions on the light neutrino parameters. Successful baryogenesis requires within our model that three conditions be satisfied: $\tan^2\theta_{12} \simeq m_1/m_2$, $\beta \simeq \alpha + \pi/2$ and $\theta_{13} = (0.01 - 0.07)$. Here θ_{12} and θ_{13} are elements of the neutrino mixing matrix, m_i are the light neutrino mass eigenvalues and α, β are the Majorana phases entering in the amplitude for neutrinoless double beta decay. Future neutrino experiments will be able to either confirm or refute these predictions.

The rest of the chapter is organized as follows. In Sec. 2.2 we review briefly the minimal left–right symmetric model. In Sec. 2.3 we analyze leptogenesis in this model. Here we derive constraints imposed on the model from the requirement of successful leptogenesis. In Sec. 2.4 we calculate the lepton asymmetry parameter ε_1 generated in the model in N_1 decay. Sec. 2.5 summarizes the relevant Boltzmann equations needed for computing the baryon asymmetry parameter. Sec. 2.6 provides our numerical results for η_B. We devote Sec. 2.7 for Gravition discussion Finally, in Sec. 2.7 we conclude.

2.2 Brief review of the minimal left-right symmetric model

Let us briefly review the basic structure of the minimal SUSY left–right symmetric model developed in Ref. [10]. The gauge group of the model is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The quarks and leptons are assigned to the gauge group as follows. Left–handed quarks and leptons (Q, L) transform as doublets of $SU(2)_L$ $[Q(3,2,1,1/3)$ and $L(1,2,1,-1)]$, while the right–handed ones (Q^c, L^c) are doublets of $SU(2)_R$ $[Q^c(3^*,1,2,-1/3)$ and $L^c(1,1,2,1)]$. The Dirac masses of fermions arise through their Yukawa couplings to a Higgs bidoublet $\Phi(1,2,2,0)$. The $SU(2)_R \times U(1)_{B-L}$ symmetry is broken to $U(1)_Y$ by the VEV (v_R) of a $B-L=-2$ triplet scalar field $\Delta^c(1,1,3,-2)$. This triplet is accompanied by a left–handed triplet $\Delta(1,3,1,2)$ (along with $\bar{\Delta}$ and $\bar{\Delta}^c$ fields, their conjugates to cancel anomalies). These fields also couple to the leptons and are responsible for inducing large Majorana masses for the ν_R. An alternative to these triplet Higgs fields is to use $B-L=\pm 1$ doublets $\chi(1,2,1,-1)$ and $\chi^c(1,1,2,1)$, along with their conjugates $\bar{\chi}$ and $\bar{\chi}^c$. In this case non–renormalizable operators will have to be invoked to generate large ν_R Majorana masses. For definiteness we shall adopt the triplet option, although our formalism allows for the addition of any number of doublet Higgs fields as well. The superpotential invariant under the gauge symmetry involving the quark and lepton fields is

$$W = \mathbf{Y}_q Q^T \tau_2 \Phi \tau_2 Q^c + \mathbf{Y}_l L^T \tau_2 \Phi \tau_2 L^c + (\mathbf{f} L^T i\tau_2 \Delta L + \mathbf{f}_c L^{cT} i\tau_2 \Delta^c L^c) \ . \qquad (2.3)$$

Under left–right parity symmetry, $Q \leftrightarrow Q^{c*}, L \leftrightarrow L^{c*}, \Phi \leftrightarrow \Phi^\dagger$, $\Delta \leftrightarrow \Delta^{c*}$, along with $W_{SU(2)_L} \leftrightarrow W^*_{SU(2)_R}$, $W_{B-L} \leftrightarrow W^*_{B-L}$ and $\theta \leftrightarrow \bar{\theta}$. As a consequence, $\mathbf{Y}_q = \mathbf{Y}_q^\dagger$, $\mathbf{Y}_l = \mathbf{Y}_l^\dagger$, and $\mathbf{f} = \mathbf{f}_c^*$ in Eq. (3.7).* It has been shown in Ref. [10] that the hermiticity of the Yukawa matrices (along with the parity constraints on the soft SUSY breaking parameters) helps to solve the supersymmetric CP problem that haunts the MSSM.

Below v_R, the effective theory is the MSSM with its H_u and H_d Higgs multiplets.† These are contained in the bidoublet Φ of the SUSY left-right model, but in general they can also reside partially in other multiplets having identical quantum numbers under the MSSM symmetry (such as the χ, $\overline{\chi}$ doublet Higgs fields alluded to earlier). Allowing for such a possibility, the superpotential of Eq. (3.7) leads to the relations for the MSSM Yukawa coupling matrices

$$\mathbf{Y}_u = \gamma \mathbf{Y}_d, \qquad \mathbf{Y}_\ell = \gamma \mathbf{Y}_{\nu^D} \ . \qquad (2.4)$$

*We do not explicitly use these relations.

†The right-handed gauge bosons have masses of order $v_R \sim 10^{14}$ GeV and thus play no significant role in cosmology at $T \sim M_1 \ll v_R$.

These relations have been called up–down unification [10]. Here, the first relation of Eq. (2.4) implies $\frac{m_t}{m_b} \simeq \gamma \tan\beta \equiv c$ where γ is a parameter characterizing how much of H_u and H_d of MSSM are in the bidoublet Φ. The case of $H_{u,d}$ entirely in Φ will correspond to $\gamma = 1$ and $\tan\beta = m_t/m_b$. At first sight the first of the relations in Eq. (2.4) might appear phenomenologically disastrous since it leads to vanishing quark mixings and unacceptable quark mass ratios. It was shown in the first paper of Ref. [10] that including the one–loop diagrams involving the gluino and the chargino and allowing for a flavor structure for the soft SUSY breaking A terms, there exists a large range of parameters (though not the entire range possible in the usual MSSM) where correct quark mixings as well as masses can be obtained consistent with flavor changing constraints.

It is the second of Eq. (2.4) that concerns us here. This relation would lead to $M_D = cM_l$, with $c \simeq m_t/m_b$. The supersymmetric loop corrections for the leptonic mass matrices are numerically small compared to similar corrections in the quark sector, since no strongly interacting particles take part in these loops. Furthermore, leptonic mixing angles are induced at tree level through the structure in the Majorana neutrinos mass matrix, and any loop corrections to these will be subdominant. This is especially true since two of the leptonic mixing angles are large to begin with. We therefore ignore SUSY loop corrections to the lepton mass matrices.

One can thus go to a basis where the charged lepton and the Dirac neutrino mass matrices are simultaneously diagonal. The heavy Majorana mass matrix $M_R = \mathbf{f}v_R$ will then be a generic complex symmetric matrix. After removing three phases in M_R by field redefinitions, we are left with 9 parameters (6 magnitudes and 3 phases) which determine the light neutrino spectrum as well as the heavy neutrino spectrum. This in turns fixes the lepton asymmetry. The consequences of such a constrained system for leptogenesis will be analyzed in the next section.

In principle the $\Delta(1,3,1,+2)$ Higgs field can also acquire a small VEV of order eV [18]. In this case the seesaw formula would be modified, as will the calculation of the lepton asymmetry [18]. We will assume such type II seesaw contributions proportional to $\langle\Delta\rangle$ are zero in our analysis. This is consistent with the models of Ref. [10].

Leptogenesis in the context of more general left-right symmetric models has been analyzed in Ref. [19].

2.3 Leptogenesis in left-right symmetric framework

The $SU(2)_R \times U(1)_{B-L}$ symmetry is broken down to $U(1)_Y$ by the VEV $\langle \Delta^c \rangle = v_R \sim 10^{14}$ GeV. At least some of the right-handed neutrinos have masses below v_R. We thus focus on the neutrino Yukawa coupling in the context of MSSM. The $SU(2)_L \times U(1)_Y$ invariant Yukawa interactions are contained in the MSSM superpotential

$$W = lH_d\mathbf{Y}_\ell \; e^c + lH_u\mathbf{Y}_{\nu^D} \; \nu^c + \frac{1}{2}\underbrace{\nu^{cT}CM_R\nu^c}, \tag{2.5}$$

where l stands for the left-handed lepton doublet, and (e^c, ν^c) denote the conjugates of the right-handed charged lepton and the right–handed neutrino fields respectively. H_u, H_d are the MSSM Higgs fields with VEVs v_u, v_d. $M_l = \mathbf{Y}_\ell \; v_d$, $M_D = \mathbf{Y}_{\nu^D} \; v_u$ and M_R are respectively the charged lepton, the Dirac neutrino, and the Majorana r.h.n mass matrices. Then one can generate light neutrino masses by the seesaw mechanism [14]

$$M_\nu = -M_D M_R^{-1} {M_D}^T. \tag{2.6}$$

There is mixing among generations in both M_R and M_D, the light neutrino mixing angles will depend on both of these mixings. Within the SM or MSSM where M_D is an arbitrary matrix, the structure of the right-handed neutrino mass matrix can not be fully determined even if the light matrix M_ν were to be completely known from experiments. As noted in Sec. 2, in the minimal version of the left-right symmetric model one has

$$M_D = cM_l = c \; diag(m_e, m_\mu, m_\tau) \tag{2.7}$$

where $c \simeq \frac{m_t}{m_b}$. Here we have already gone to a basis where the charged lepton mass matrix is diagonalized. In the three family scenario, the relations between the flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ and the mass eigenstates (ν_1, ν_2, ν_3) can be expressed in terms of observables as

$$M_\nu = U^* M_\nu^{diag} U^\dagger, \tag{2.8}$$

where $M_\nu^{diag} \equiv diag(m_1, m_2, m_3)$, with m_i being the light neutrinos masses and U being the 3×3 mixing matrix which we write as $U = U_{PMNS}.P$. We parameterize U_{PMNS} [20] as

$$\begin{aligned} U_{PMNS} &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{\imath\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{\imath\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{\imath\delta} & c_{13}c_{23} \end{pmatrix} \end{aligned} \tag{2.9}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and δ is the Dirac CP violating phase which appears in neutrino oscillations. The matrix P contains two Majorana phases unobservable in neutrino oscillation, but relevant to neutrinoless double beta decay [21]:

$$P = \begin{pmatrix} e^{\imath\alpha} & 0 & 0 \\ 0 & e^{\imath\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.10}$$

Combining Eq. (3.21) with the seesaw formula of Eq. (3.18) and solving for the right-handed neutrino mass matrix we find

$$\begin{aligned} M_R &= c^2 M_l\, M_\nu^{-1} M_l \\ &= \frac{c^2 m_\tau^2}{m_1} \begin{pmatrix} \frac{m_e}{m_\tau} & 0 & 0 \\ 0 & \frac{m_\mu}{m_\tau} & 0 \\ 0 & 0 & 1 \end{pmatrix} U_{PMNS} P^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{m_1}{m_2} & 0 \\ 0 & 0 & \frac{m_1}{m_3} \end{pmatrix} U_{PMNS}^T \begin{pmatrix} \frac{m_e}{m_\tau} & 0 & 0 \\ 0 & \frac{m_\mu}{m_\tau} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{2.11}$$

This enables us to establish a link between high scale parameters and low scale observables.

We define a small expansion parameter ϵ as

$$\epsilon = \frac{m_\mu}{m_\tau} \simeq 0.059,$$

in terms of which we have

$$m_e = a_e \epsilon^3 m_\tau,\ \frac{m_1}{m_3} = a_{13}\epsilon,\ \theta_{13} = t_{13}\epsilon,\ \theta_{23} = \frac{\pi}{4} + t_{23}\epsilon. \tag{2.12}$$

Here a_e, a_{13}, t_{13} and t_{23} are $\lesssim \theta(1)$ parameters with $a_e = 1.400$. These expansions follow from low energy data assuming the picture of hierarchical neutrino masses.

We find that the requirement of generating adequate baryon asymmetry places significant constraints on the neutrino mixing parameters. Specifically, the following expansions

$$\frac{m_1}{m_2} = \tan^2\theta_{12} + a_{12}\epsilon \text{ and } \beta = \alpha + \frac{\pi}{2} + b\epsilon, \tag{2.13}$$

where a_{12} and b are $\lesssim \theta(1)$ parameters are required. To see this, we note that the CP asymmetry parameter ϵ_1 generated in the decay of N_1 is too small, of order $\varepsilon_1 \sim \frac{\epsilon^6}{8\pi} \sim 2\times 10^{-9}$ if a_{12} or b are much greater than 1. This is because the heavy neutrino masses would be strongly hierarchical in this case, $M_1 : M_2 : M_3 \sim \epsilon^6 : \epsilon^2 : 1$. This can be altered to a weak hierarchy $M_1 : M_2 : M_3 \sim \epsilon^4 : \epsilon^2 : 1$ by observing that the elements of the 2-3 block of M_R of Eq. (2.11) are all proportional to $\{\frac{m_1}{m_2}e^{2i\beta}\cos^2\theta_{12} + e^{2i\alpha}\sin^2\theta_{12}\}$ and by demanding this quantity to be of order ϵ. Eq. (2.13) is just this condition. $\varepsilon_1 \sim \frac{\epsilon^4}{8\pi} \sim 10^{-6}$ in this case, which can lead to acceptable baryon asymmetry, as we show.

An immediate consequence of Eq. (2.13) is that neutrinoless double beta decay is suppressed in the model. The effective mass relevant for this decay is found to be

$$m_{\beta\beta} = |\sum_i U_{ei}^2 m_i| \simeq |m_2 e^{2i\alpha}\epsilon(a_{12}c_{12}^2 - 2ibs_{12}^2) + m_3 s_{13}^2 e^{-2i\delta}|. \tag{2.14}$$

This is of the order $m_3\epsilon^2 \sim 10^{-4}$ eV, which would be difficult to measure. This amplitude is small because of a cancelation between the leading contributions proportional to m_1 and m_2 (see Eq. (2.13)).

In terms of these expansions, the r.h.n mass matrix becomes

$$M_R = \begin{pmatrix} A_{11}\epsilon^5 & A_{12}\epsilon^3 & A_{13}\epsilon^2 \\ A_{12}\epsilon^3 & A_{22}\epsilon^2 & A_{23}\epsilon \\ A_{13}\epsilon^2 & A_{23}\epsilon & A_{33} \end{pmatrix}, \tag{2.15}$$

where

$$A_{11} = \frac{M_\circ\epsilon\, a_e^2 e^{2\imath\alpha}\cos 2\theta_{12}}{\cos^2\theta_{12}}$$

$$\begin{aligned}
A_{12} &= -\frac{M_\circ \epsilon \, a_e e^{2\imath\alpha} \tan\theta_{12}}{\sqrt{2}} \\
A_{13} &= \frac{M_\circ \epsilon \, a_e e^{2\imath\alpha} \tan\theta_{12}}{\sqrt{2}} \\
A_{22} &= \frac{M_\circ \epsilon}{2} \left\{ a_{13} - a_{12} e^{2\imath\alpha} \cos^2\theta_{12} - 2\imath b e^{2\imath\alpha} \sin^2\theta_{12} + 2 e^{\imath(2\alpha+\delta)} t_{13} \tan\theta_{12} \right\} \\
A_{23} &= \frac{M_\circ \epsilon}{2} \left\{ a_{13} + a_{12} e^{2\imath\alpha} \cos^2\theta_{12} + 2\imath b e^{2\imath\alpha} \sin^2\theta_{12} \right\} \\
A_{33} &= -M_\circ \epsilon \, e^{2\imath\alpha} \left\{ t_{13} e^{\imath\delta} \tan\theta_{12} + \imath b \sin^2\theta_{12} - \frac{a_{13} e^{-2\imath\alpha}}{2} + \frac{a_{12}\cos^2\theta_{12}}{2} \right\}. \quad (2.16)
\end{aligned}$$

Here we defined $M_\circ = \frac{c^2 m_\tau^2}{m_1}$. This hierarchical mass matrix is diagonalized by a series of rotations U_1, U_2 and U_3 such that;

$$(KU_3U_2U_1)M_R(KU_3U_2U_1)^T = \begin{pmatrix} |M_1| & 0 & 0 \\ 0 & |M_2| & 0 \\ 0 & 0 & |M_3| \end{pmatrix} \qquad (2.17)$$

where $K = diag(k_1, k_2, k_3)$ with $k_i = e^{-\imath\phi_i/2}$ being phase factors which make each r.h.n masses M_i real, $M_i = |M_i| e^{\phi_i}$. $V = (KU_3U_2U_1)^T$ is the matrix that diagonalizes M_R. The unitary matrix U_1 is given by

$$U_1 = \begin{pmatrix} 1 & 0 & -\frac{A_{13}}{A_{33}}\epsilon^2 \\ 0 & 1 & 0 \\ \frac{A_{13}^\star}{A_{33}^\star}\epsilon^2 & 0 & 1 \end{pmatrix}. \qquad (2.18)$$

Similarly, U_2 and U_3 are unitary matrices with off-diagonal entries given by

$$(U_2)_{23} = -\frac{A_{23}}{A_{33}}\epsilon \quad , \qquad (U_3)_{12} = -\frac{\left(A_{12} - \frac{A_{13}A_{23}}{A_{33}}\right)\epsilon}{A_{22} - \frac{A_{23}^2}{A_{33}}}. \qquad (2.19)$$

The mass eigenvalues are found to be

$$\begin{aligned}
M_1 &= M_\circ k_1^2 \epsilon^5 \left(2 a_{13} a_e^2 e^{2\imath\alpha} \sin^2\theta_{12}\right) \\
&\times \left(2 t_{13}^2 e^{2\imath(\alpha+\delta)} \sin^2\theta_{12} + (a_{12} + 2\imath b + (a_{12} - 2\imath b)\cos 2\theta_{12}) a_{13} \cos^2\theta_{12}\right)^{-1} \\
\\
M_2 &= M_\circ k_2^2 \epsilon^3 e^{2\imath\alpha} \left(a_{13}(a_{12} + 2\imath b + (a_{12} - 2\imath b)\cos 2\theta_{12}) + 2 t_{13}^2 e^{\imath(\delta+\alpha)} \tan^2\theta_{12}\right) \\
&\times \left(-a_{13} + \imath b e^{2\imath\alpha} + e^{2\imath\alpha}(a_{12}\cos^2\theta_{12} - \imath b \cos 2\theta_{12}) + 2 e^{\imath\delta} t_{13} \tan\theta_{12}\right)^{-1} \\
\\
M_3 &= \frac{M_\circ k_3^2 \epsilon}{2} \left(a_{13} - \imath b e^{2\imath\alpha} - e^{2\imath\alpha}(a_{12}\cos^2\theta_{12} - \imath b \cos 2\theta_{12} + 2 t_{13} e^{\imath\delta} \tan\theta_{12}\right) \quad (2.20)
\end{aligned}$$

We use these results in the next section to determine ε_1.

2.4 CP violation and lepton asymmetry

Now that we have developed our framework, we can turn attention to the evaluation of the CP asymmetry ε_1 generated in the decay of the lightest r.h.n N_1. This arises from the interference between the tree-level and one-loop level decay amplitudes.* In a basis where the r.h.n mass matrix is diagonal and real, the asymmetry in the decay of N_i is given by [22]

$$\varepsilon_i = -\frac{1}{8\pi v^2\,(M_D^\dagger M_D)_{ii}} \sum_{j=2,3} \mathrm{Im}[(M_D^\dagger M_D)_{ij}]^2 \left[f\left(\frac{M_j^2}{M_i^2}\right) + g\left(\frac{M_j^2}{M_i^2}\right)\right] \tag{2.21}$$

where $f(x)$ and $g(x)$ represent the contributions from vertex and self energy corrections respectively. For the case of the non-supersymmetric standard model with right-handed neutrinos, these functions are given by [22]

$$f_{\mathrm{n}on-SUSY}(x) = \sqrt{x}\left[-1 + (x+1)\ln\left(1+\frac{1}{x}\right)\right], \qquad g_{\mathrm{n}on-SUSY}(x) = \frac{\sqrt{x}}{x-1}, \tag{2.22}$$

while for the case of MSSM plus right-handed neutrinos, they are given by

$$f_{\mathrm{S}USY}(x) = \sqrt{x}\,\ln\left(1+\frac{1}{x}\right), \qquad g_{\mathrm{S}USY}(x) = \frac{2\sqrt{x}}{x-1}. \tag{2.23}$$

Here v is the SM Higgs doublet VEV, $v \simeq 174$ GeV. For the case of MSSM, v in Eq. (2.21) is replaced by $v\sin\beta$. Hereafter, for definiteness in the numerical evaluation of the Boltzmann equations, we assume the SM scenario. However, our result should be approximately valid for the MSSM case as well.* Assuming a mass hierarchy $M_1 \ll M_2 < M_3$ in the right-handed neutrino sector i.e., ($x \gg 1$), which is realized in our model, see Eq. (2.15), the above formula is simplified to the following one:

$$\varepsilon_1 = -\frac{3}{16\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{k=2,3} \mathrm{Im}[\,(M_D^\dagger M_D)^2_{1k}\,]\,\frac{M_1}{M_k}. \tag{2.24}$$

---

*We will assume $M_1 \ll M_2 < M_3$. In this case, even if the heavier right-handed neutrinos N_2 and N_3 produce lepton asymmetry, it is usually erased before the decay of N_1.

*The function $f+g$ in MSSM is twice as big compared to the SM. However this is compensated by the factor $\frac{1}{g_*}$ that appears in η_B which in MSSM is half of the SM value.

ε_1 depends on the (1,1), (1,2) and (1,3) entries of $M_D^\dagger M_D$. These quantities can be related to the light neutrino mass and mixing parameters measurable in low energy experiments. In the basis where M_R is diagonal, these elements are

$$\begin{aligned}(M_D^\dagger M_D)_{11} &= (c\,m_\tau)^2\left(V_{31}V_{31}^* + V_{21}V_{21}^*\epsilon^2 + a_e^2 V_{11}V_{11}^*\epsilon^6\right)\\ (M_D^\dagger M_D)_{12} &= (c\,m_\tau)^2\left(V_{31}V_{32}^* + V_{21}V_{22}^*\epsilon^2 + a_e^2 V_{11}V_{12}^*\epsilon^6\right)\\ (M_D^\dagger M_D)_{13} &= (c\,m_\tau)^2\left(V_{31}V_{33}^* + V_{21}V_{23}^*\epsilon^2 + a_e^2 V_{11}V_{13}^*\epsilon^6\right),\end{aligned} \tag{2.25}$$

where $V = KU_3U_2U_1$ is the unitary matrix diagonalizing M_R. Straightforward calculations give, to leading order in ϵ,

$$\begin{aligned}(M_D^\dagger M_D)_{11} &= 8a_e^2c^2m_\tau^2\epsilon^4\,\cos^2\theta_{12}\sin^2\theta_{12}(a_{13}^2 + t_{13}^2\tan^2\theta_{12})\\ &\times\ 1/\left\{8t_{13}^4\sin^4\theta_{12} + 32a_{13}t_{13}^2 b\cos^2\theta_{12}\sin^4\theta_{12}\sin 2(\alpha+\delta)\right.\\ &+\ a_{13}\cos^4\theta_{12}[4a_{13}(a_{12}^2 - b^2)\cos 2\theta_{12} + a_{13}(a_{12}^2 + 4b^2)(3 + \cos 4\theta_{12})\\ &+\ \left.16a_{12}t_{13}^2\sin^2\theta_{12}\cos 2(\alpha+\delta)]\right\}\end{aligned} \tag{2.26}$$

$$\begin{aligned}(M_D^\dagger M_D)_{12}^2 &= 2a_e^2c^4m_\tau^4\epsilon^6\tan^2\theta_{12}e^{-\imath(\phi_1-\phi_2)}e^{-2\imath(2\alpha+\delta)}\left\{4(a_{13}^2 - t_{13}^2)\cos 2\theta_{12} - 2t_{13}\sin 2\theta_{12}\right.\\ &\quad\left.(2a_{13}e^{\imath(2\alpha+\delta)} - (a_{12}+2\imath b)e^{-\imath\delta}) + 4(a_{13}^2 + t_{13}^2) + t_{13}\sin 4\theta_{12}e^{-\imath\delta}(a_{12} - 2\imath b)\right\}^2\\ &\times\ 1/\left\{[\imath b e^{\imath\delta} - a_{13}e^{-\imath(2\alpha+\delta)} + a_{12}e^{-\imath\delta}\cos^2\theta_{12} - \imath b e^{-\imath\delta}\cos 2\theta_{12} + 2t_{13}\tan\theta_{12}]^2\right.\\ &\times\ [3a_{12}a_{13} - 2\imath a_{13}b + 4t_{13}^2e^{-2\imath(\alpha+\delta)} + 4\cos 2\theta_{12}(a_{12}a_{13} - t_{13}^2e^{-2\imath(\alpha+\delta)}) +\\ &\quad\left.a_{13}(a_{12}+2\imath b)\cos 4\theta_{12}]^2\right\}\end{aligned} \tag{2.27}$$

$$\begin{aligned}(M_D^\dagger M_D)_{13}^2 &= 2a_e^2c^4m_\tau^4\epsilon^4\sin^2\theta_{12}e^{-\imath(\phi_1-\phi_3)}(a_{13}\cos\theta_{12} + e^{-\imath(2\alpha+\delta)}t_{13}\sin\theta_{12})^2\\ &\times\ 1/\left\{a_{13}\cos^2\theta_{12}(a_{12} - 2\imath b + (a_{12}+2\imath b)\cos 2\theta_{12}) + 2t_{13}^2\sin^2\theta_{12}e^{-2\imath(\alpha+\delta)}\right\}^2\end{aligned} \tag{2.28}$$

These analytical expressions have been checked numerically. In Figure (1) we have plotted $|\varepsilon_1|$ as function of θ_{13} for fixed values of other observables. The solid line in Fig (1) which corresponds to the exact numerical evaluation agrees very well with the dashed line corresponding to the analytical expressions.

From Figure (1), it is apparent that θ_{13} is constrained in the model from cosmology. If $\varepsilon_1 < 1.3\times10^{-7}$, the induced baryon asymmetry would be too small to explain observations. As can be seen from Figure (1), θ_{13} should lie in the range $0.01 - 0.07$ for an acceptable value of ε_1. This result does not change very much with variations in the other input parameters. Electroweak sphaleron processes [15] will convert the induced lepton asymmetry to baryon asymmetry. The ratio of baryon asymmetry to entropy Y_B is related to the lepton asymmetry through the relation [23]:

$$Y_B = C\, Y_{B-L} = \frac{C}{C-1} Y_L \tag{2.29}$$

where $C = \frac{8N_f + 4N_\varphi}{22N_f + 13N_\varphi}$, $N_f = 3$ and $N_\varphi = 1, 2$ in the case of the SM and MSSM respectively. In either case $C \sim \frac{1}{3}$. In Eq. (2.29), $Y_B = \frac{n_B}{s}$ with $s = 7.04\, n_\gamma$.

There has been considerable interest in obtaining approximate analytical expression for baryon asymmetry [24,25]. In order to estimate this, the dilution factor, often referred to as the efficiency factor κ that takes into account the washout processes (inverse decays and lepton number violating scattering) has to be known. As an example, $\kappa = (2 \pm 1) \times 10^{-2} \left(\frac{0.01\ \mathrm{eV}}{\tilde{m}_1}\right)^{1.1\pm0.1}$ has been suggested in Ref. [24] from which $\eta_B \simeq 0.96 \times 10^{-2} \varepsilon_{N_1} \kappa$ has been calculated. In our work we solve the coupled Boltzmann equations numerically to estimate the baryon asymmetry without referring to the efficiency factor.

2.5 Numerical Boltzmann equations

In this section we set up the Boltzmann equations for computing the baryon asymmetry η_B generated through the out of equilibrium decay of N_1. In our model the right-handed neutrino masses are not independent of the CP asymmetry parameter ε_1. Therefore a self consistent analysis within the model is required.

In the early universe, at temperature of order N_1 mass, the main thermal processes which enter in the production of the lepton asymmetry are the decay of the lightest r.h. neutrino,* its inverse decay, and the lepton number violation scattering, $\Delta L = 1$ Higgs exchange plus $\Delta L = 2$ r.h.n exchange [26]. The production of the lepton asymmetry via the decay of the r.h.n is an out-of-equilibrium process which is most efficiently treated by means of the Boltzmann equations (BE).

*In our analysis we stick to the case where the asymmetry is due only to the decay of the lightest r.h. neutrino N_1.

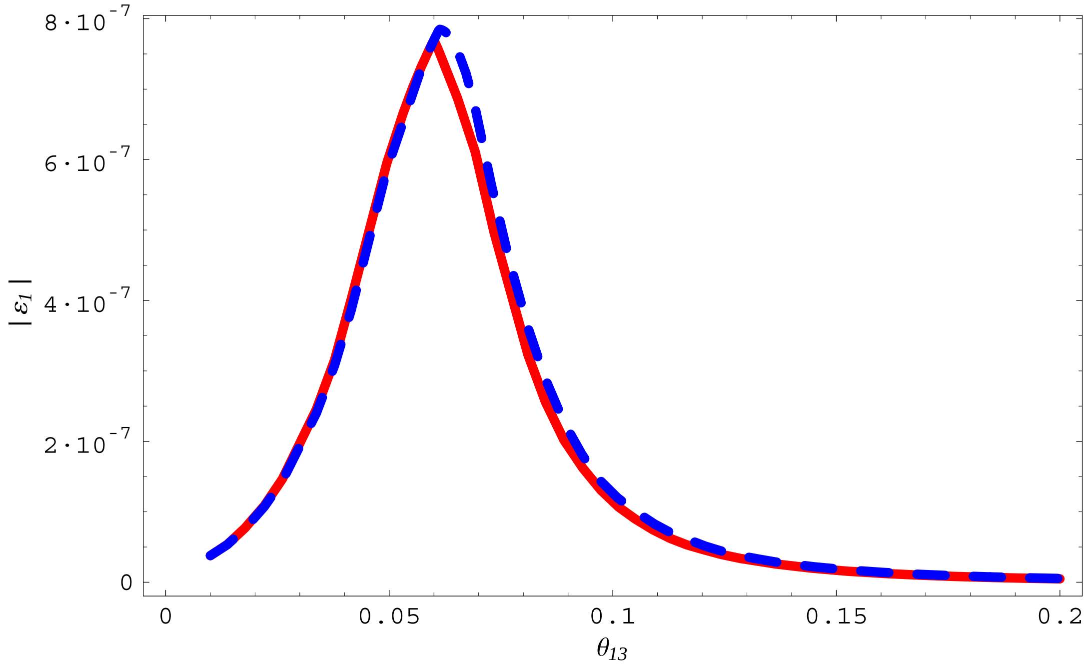

Figure 2.1. Plots for CP asymmetry parameter ε_1 using analytical (dotted) and numerical (solid) results as a function of the neutrino oscillation angle θ_{13}. The input parameters used are $a_{12} = 1$, $b = 1$, $\Delta m_{\odot}^2 = 2.5 \times 10^{-5}\,\mathrm{e}V^2$, $\Delta m_a^2 = 5.54 \times 10^{-3}\,\mathrm{e}V^2$ and $\{\delta, \alpha\} = \{\pi/4, \pi/4\}$. Our model requires $|\varepsilon_1| \gtrsim 1.3 \times 10^{-7}$ to successfully generate an adequate number for the BA. This criterium happens to be satisfied only in the region for which $0.01 \lesssim \theta_{13} \lesssim 0.07$, this interval is not too sensitive to variations in the input parameters.

The first BE which describes the evolution of the abundance of the r.h. neutrino and which corresponds to the source of the asymmetry is given by†

$$\frac{dY_{N_1}}{dz} = -\frac{z}{Hs(z)}\left(\frac{Y_{N_1}}{Y_N^{eq}} - 1\right)\left(\gamma_{D_1} + \gamma_{S_1}\right), \tag{2.30}$$

where $z = \frac{M_1}{T}$. Here $s(z)$ is the entropy density and $\gamma_{D_1}, \gamma_{S_1}$ are the interaction rates for the decay and $\Delta L = 1$ scattering contributions, respectively.

The second BE relevant to the lepton asymmetry is given by

$$\frac{dY_{B-L}}{dz} = -\frac{z}{s(z)H(M_1)}\left[\varepsilon_1\gamma_{D_1}\left(\frac{Y_{N_1}}{Y_N^{eq}} - 1\right) + \gamma_W\frac{Y_{B-L}}{Y_L^{eq}}\right], \tag{2.31}$$

where ε_1 is the CP violation parameter given by Eq. (2.21) and γ_W is the washout factor which is responsible for damping of the produced asymmetry, see Eq. (2.49) below. In Eqs. (2.30) and (2.31), Y_i^{eq} is the equilibrium number density of a particle species i, which has a mass m_i, given by

$$Y_i^{eq}(z) = \frac{45}{4\pi^4}\frac{g_i}{g_*}\left(\frac{m_i}{M_1}\right)^2 z^2 K_2\left(\frac{m_i z}{M_1}\right), \tag{2.32}$$

where g_i is the particle internal degree of freedom ($g_{N_i} = 2$, $g_\ell = 4$). At temperatures far above the electroweak scale one has $g_* \simeq 106.75$ in the standard model, and $g_* \simeq 228.75$ in MSSM. H, the Hubble parameter evaluated at $z = 1$, and $s(z)$, the entropy density, are given by

$$H = \sqrt{\frac{4\pi^3 g_*}{45}}\frac{M_1^2}{M_P} \quad , \quad s(z) = \frac{2\pi^2 g_*}{45}\frac{M_1^3}{z^3}, \tag{2.33}$$

where $M_P = 1.22 \times 10^{19}$ GeV. We also have

$$\gamma_{S_j} = 2\gamma_{t_j}^{(1)} + 4\gamma_{t_j}^{(2)}. \tag{2.34}$$

The decay reaction density γ_{D_j} has the following expression:

$$\gamma_{D_j} = n_{N_j}^{eq}\frac{K_1(z)}{K_2(z)}\Gamma_{N_j}, \tag{2.35}$$

†In this section we follow the notation of the first paper of Ref. [16] to which we refer the reader for further details.

where $K_n(z)$ are the modified Bessel functions. Γ_{N_j} of the r.h.n N_j is the tree level total decay rate defined as

$$\Gamma_{N_j} = \frac{(\lambda^\dagger \lambda)_{jj}}{8\pi} M_j, \tag{2.36}$$

where

$$n_{N_i}^{eq}(T) = \frac{g_i T m_i}{2\pi^2} K_2\left(\frac{m_i}{T}\right). \tag{2.37}$$

We used the definition $\lambda = M_D/v$. We define the reaction density $\gamma^{(i)}$ of any process $a + b \to c + d$ by

$$\gamma^{(i)} = \frac{M_1^4}{64\pi^4}\frac{1}{z}\int_{\frac{(M_a+M_b)^2}{M_1^2}}^{\infty} dx\ \hat{\sigma}^{(i)}(x)\sqrt{x}\ K_1\left(\sqrt{x}z\right), \tag{2.38}$$

where $\hat{\sigma}^{(j)}(x)$ are the reduced cross sections for the different processes which contribute to the Boltzmann equations. For the $\Delta L = 1$ processes involving the quarks, we have

$$\hat{\sigma}_{t_j}^{(1)} = 3\alpha_u \sum_{\alpha=1}^{3} \left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right)\left(\frac{x - a_j}{x}\right)^2, \tag{2.39}$$

$$\hat{\sigma}_{t_j}^{(2)} = 3\alpha_u \sum_{\alpha=1}^{3} \left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right)\left(\frac{x - a_j}{x}\right)\left[\frac{x - 2a_j + 2a_h}{x - a_j + a_h} + \frac{a_j - 2a_h}{x - a_j}\ln\left(\frac{x - a_j + a_h}{a_h}\right)\right] \tag{2.40}$$

where

$$\alpha_u = \frac{Tr(\lambda_u^\dagger \lambda_u)}{4\pi} \simeq \frac{m_t^2}{4\pi v^2}, \quad a_j = \left(\frac{M_j}{M_1}\right)^2, \quad a_h = \left(\frac{\mu}{M_1}\right)^2, \tag{2.41}$$

μ is the infrared cutoff which we set to 800 GeV [26,27]. For the $\Delta L = 2$ r.h.n exchange processes, we have

$$\hat{\sigma}_N^{(1)} = \sum_{\alpha=1}^{3}\sum_{j=1}^{3} \left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right)\left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right) A_{jj}^{(1)} + \sum_{\alpha=1}^{3}\sum_{n<j,j=1}^{3} Re\left(\lambda_{\alpha n}^* \lambda_{\alpha j}\right)\left(\lambda_{\alpha n}^* \lambda_{\alpha j}\right) B_{nj}^{(1)} \tag{2.42}$$

$$\hat{\sigma}_N^{(2)} = \sum_{\alpha=1}^{3}\sum_{j=1}^{3} \left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right)\left(\lambda_{\alpha j}^* \lambda_{\alpha j}\right) A_{jj}^{(2)} + \sum_{\alpha=1}^{3}\sum_{n<j,j=1}^{3} Re\left(\lambda_{\alpha n}^* \lambda_{\alpha j}\right)\left(\lambda_{\alpha n}^* \lambda_{\alpha j}\right) B_{nj}^{(2)} \tag{2.43}$$

where

$$A_{jj}^{(1)} = \frac{1}{2\pi}\left[1 + \frac{a_j}{D_j} + \frac{a_j x}{2D_j^2} - \frac{a_j}{x}\left(1 + \frac{x + a_j}{D_j}\right)\ln\left(\frac{x + a_j}{a_j}\right)\right], \tag{2.44}$$

$$A_{jj}^{(2)} = \frac{1}{2\pi}\left[\frac{x}{x + a_j} + \frac{a_j}{x + 2a_j}\ln\left(\frac{x + a_j}{a_j}\right)\right], \tag{2.45}$$

$$B_{nj}^{(1)} = \frac{\sqrt{a_n a_j}}{2\pi}\left[\frac{1}{D_j}+\frac{1}{D_n}+\frac{x}{D_j D_n}+\left(1+\frac{a_j}{x}\right)\left(\frac{2}{a_n-a_j}-\frac{1}{D_n}\right)\ln\left(\frac{x+a_j}{a_j}\right)\right. \tag{2.46}$$
$$\left. + \left(1+\frac{a_n}{x}\right)\left(\frac{2}{a_j-a_n}-\frac{1}{D_j}\right)\ln\left(\frac{x+a_n}{a_n}\right)\right],$$
$$B_{nj}^{(2)} = \frac{\sqrt{a_n a_j}}{2\pi}\left\{\frac{1}{x+a_n+a_j}\ln\left[\frac{(x+a_j)(x+a_n)}{a_j a_n}\right]+\frac{2}{a_n-a_j}\ln\left(\frac{a_n(x+a_j)}{a_j(x+a_n)}\right)\right\}, \tag{2.47}$$

and

$$D_j = \frac{(x-a_j)^2+a_j c_j}{x-a_j}, \qquad c_j = a_j\sum_{\alpha=1}^{3}\frac{\left(\lambda_{\alpha j}^{*}\lambda_{\alpha j}\lambda_{\alpha j}^{*}\lambda_{\alpha j}\right)}{64\pi^2}. \tag{2.48}$$

Finally, γ_W that accounts for the washout processes in the Boltzmann equations is

$$\gamma_W = \sum_{j=1}^{3}\left(\frac{1}{2}\gamma_{D_j}+\frac{Y_{N_j}}{Y_{N_j}^{eq}}\gamma_{t_j}^{(1)}+2\gamma_{t_j}^{(2)}-\frac{\gamma_{D_j}}{8}\right)+2\gamma_N^{(1)}+2\gamma_N^{(2)}\ . \tag{2.49}$$

Here, we emphasize the so-called RIS (real intermediate states) in the $\Delta L = 2$ interactions which have to be carefully subtracted to avoid double counting in the Boltzmann equations. This corresponds to the term $-\frac{1}{8}\gamma_{D_j}$ in Eq. (2.49). For more details see Refs. [24,28] and the first paper of Ref. [29].

2.6 Results and discussion

We are now ready to present our numerical results. First we make several important remarks. Even though our model is supersymmetric, we have considered in our BE analysis only the SM particle interactions. This is a good approximation (see footnote 7). The authors in Ref. [27] have demonstrated that SUSY interactions do not significantly change the final baryon asymmetry. Furthermore, we have not included in our analysis the effects of renormalization group on the running masses and couplings. The first paper of Ref. [29] has studied these effects. This paper has also included finite temperature effects and $\Delta L = 1$ scattering processes involving SM gauge bosons, which we have ignored in our analysis. This should be a good approximation since it is believed that these effects are significant in the weak washout regime and our model parameters seem to favor the strong washout regime with $\widetilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1} \simeq 0.1$ eV. Scattering processes involving gange bosons have also been studied in Ref. [28] in the context of resonant leptogenesis where they have been shown to be significant.

Our next step is to put this model to the test and check its predictions. In order to compute the value of the baryon asymmetry we proceed to numerically solve the Boltzmann equations. We scan the parameter space corresponding to the parameters a_{12}, b, the oscillation angle θ_{13}, the CP phase δ and the Majorana phase α. In order to automatically satisfy the oscillation data, we input the following light neutrino parameters:

$$\Delta m^2_{\odot} = 2.5 \times 10^{-5} \mathrm{e}V^2, \ \Delta m^2_a = 5.54 \times 10^{-3} \mathrm{e}V^2, \ \sin\theta_{12} = 0.52. \tag{2.50}$$

Using hierarchical spectrum, we see that the masses m_1, m_2 and m_3 are fixed. On the other hand we consider maximal mixing in the 2-3 sector of the leptonic mixing matrix, i.e $\theta_{23} = \frac{\pi}{4} + t_{23}\epsilon$ with t_{23} being zero ($t_{23} \sim \theta(1)$ has minimal impact on η_B). The CP phase δ and the Majorana phase α are allowed to vary in the intervals $[0, 2\pi]$ and $[0, \pi]$ respectively. We remind the reader that the second Majorana phase β is related to α through $\beta \simeq \alpha + \frac{\pi}{2} + b\epsilon$. θ_{13} will be allowed to vary in the interval [0; 0.2] as it is bounded from above by reactor neutrino experiments.

In Figure (2), for a given set of input parameters, we illustrate the different thermally averaged reaction rates $\Gamma_X = \frac{\gamma_X}{n^{eq}_{N_1}}$ contributing to BE as a function of $z = \frac{M_1}{T}$.

All rates at $z = 1$ fulfill the out of equilibrium condition (i.e. $\Gamma_X \lesssim H(z = 1)$), and so the expected washout effect due to the $\Delta L = 2$ processes will be small. The parameters chosen for this illustration are: $\delta = \pi/2$, $\alpha = \pi/2$, $a_{12} = 0.01$, $b = 0.9$, $cm_\tau = m_t \left(\frac{m_\tau}{m_b}\right) = 135$ GeV and $\theta_{13} = 0.02$. Eq. (2.50) fixes the light neutrino masses to be: $m_1 = 0.00271292$ eV, $m_2 = 0.00688186$ eV and $m_3 = 0.0380442$ eV. For this choice we obtain $\mid \epsilon_1 \mid \simeq 2 \times 10^{-7}$. The calculated r.h.n masses in this case are

$$M_1 = 9 \times 10^9 \ GeV, \quad M_2 = 8.7 \times 10^{11} \ GeV, \quad M_3 = 2.6 \times 10^{14} \ GeV. \tag{2.51}$$

The mass of the lightest r.h.n is consistent with lower bound derived in Ref. [29], $M_1 \geq 2.4 \times 10^9$ GeV, for hierarchical neutrino masses assuming that one starts with zero N_1 initial abundance (which is what we assumed in our calculation). This mass is also in accordance with the upper bound found in Ref. [30] following a model independent study of the CP asymmetry, and the bound derived in Ref. [24] based on the estimation of ν_R production and the study of the asymmetry washout.

Figure (3) represents the solution of the BE, N_1 abundance and the baryon asymmetry both as functions of z for the same set of parameters mentioned above. The final baryon asymmetry, in terms of the baryon to photon ratio, is (see dark, solid curve in Fig. (3) for $z \gg 1$)

$$\eta_B \simeq 6.03 \times 10^{-10}. \tag{2.52}$$

This number is inside the observational range of Eq. (2.1). Our codes were tested to reproduce the results in the first paper of Ref. [16] before being applied to this model.

2.7 **Gravitino Problem**

Leptogenesis scenario assumes the existence of heavy right handed neutrinos which are thermally generated with sufficiently adequate abundance, during the reheating phase occurring right after inflation. Therefore, the reheating temperature T_{RH} can not be much lower than 10^9 GeV, a bound on the right handed neutrino mass [30] necessary for the success of thermal Leptogenesis. This is already in conflict with a stringent upper bound on T_{RH} , which may be as low as $10^6 - 10^7$ GeV, required to avoid large Gravitino abundance which would upset the good predictions of BBN [31]. In Supersymmetry, the Gravitino is the superpartner of the Graviton; with mass of order natural SUSY scale; 1 TeV, therefore, the Gravitino is expected to be in the range of 100GeV $\leq m_{3/2} \leq$ 10 TeV. A combination of data and calculations of several light elements abundance leads to the following recent upper bound [32]

$$T_{RH} \leq (1.9 - 7.5)10^7 \text{ GeV},$$

which has been derived for $m_{3/2} \sim 100$ GeV. The standard thermal Leptogenesis with normal hierarchical r.h neutrino seems to be at odds with the constraint above; one has to invoke the BA in such way that these tensions are avoided. Thus $M_1 < T_{RH}$ is required, which for gravitino mass in the range 300 GeV to 3 TeV is in conflict with the predictions of Eq. (2.51).

There are several ways around this problem. (i) In gauge mediated SUSY breaking scenario the gravitino is the lightest SUSY particle with mass in the range 10^{-4} e$V < m_{3/2} < 100$ GeV. For $m_{\tilde{g}} < 100$ MeV, there are no cosmological or astrophysical problems. In such a scenario the axion can serve as the dark matter. (ii) In anomaly mediated SUSY breaking scenario, the gravitino mass is enhanced by a loop factor compared to the squark masses and is naturally of order 100 TeV. Such a gravitino would decay with a shorter lifetime without affecting big bang nucleosynthesis. The gaugino is a natural dark matter candidate in this case. (iii) The gravitino itself can be the LSP and dark matter with a mass of order 100 GeV, in which case it does not decay [33]. Other solutions include changing the dynamics of the leptogenesis process by invoking (iii) non–thermal leptogenesis [34], (iv) resonant leptogenesis [28,35], or (v) soft leptogenesis [36]. In the following two chapters we invoke Baryon Asymmetry via Resonant and Soft Leptogenesis. Especially, Our predictive inverted neutrino hierarchy involving two nearly degenerate r.h.n, allows for the self-energy contribution to the CP asymmetry to be resonantly enhanced, while the r.h.n masses are low enough to be compatible with the reheating temperature bound. It will be shown that baryon asymmetry can be maximized as long as M $(4-7)10^6$ GeV or above[9].

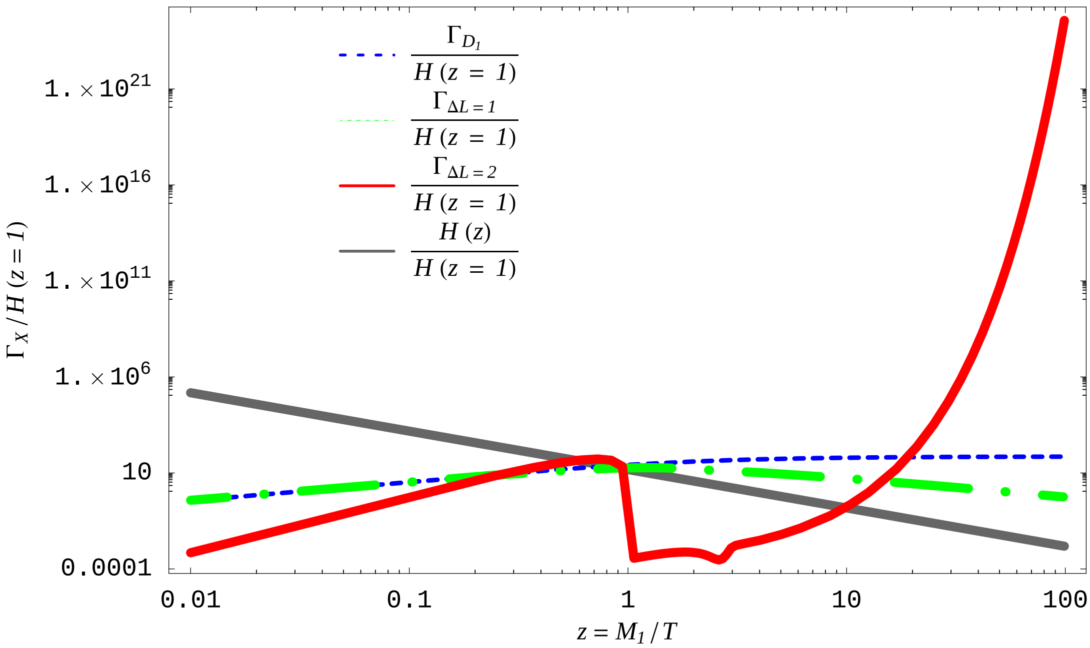

Figure 2.2. Various thermally averaged reaction rates Γ_X contributing to BE normalized to the expansion rate of the Universe $H(z = 1)$. The straight greyed line represents $H(z)/H(z = 1)$, the dashed line is for $\Gamma_{D_1}/H(z = 1)$, the dotted-dashed line represents $\Gamma_{\Delta L=1}/H(z = 1)$ processes and the red curve represents $\Gamma_{\Delta L=2}/H(z = 1)$.

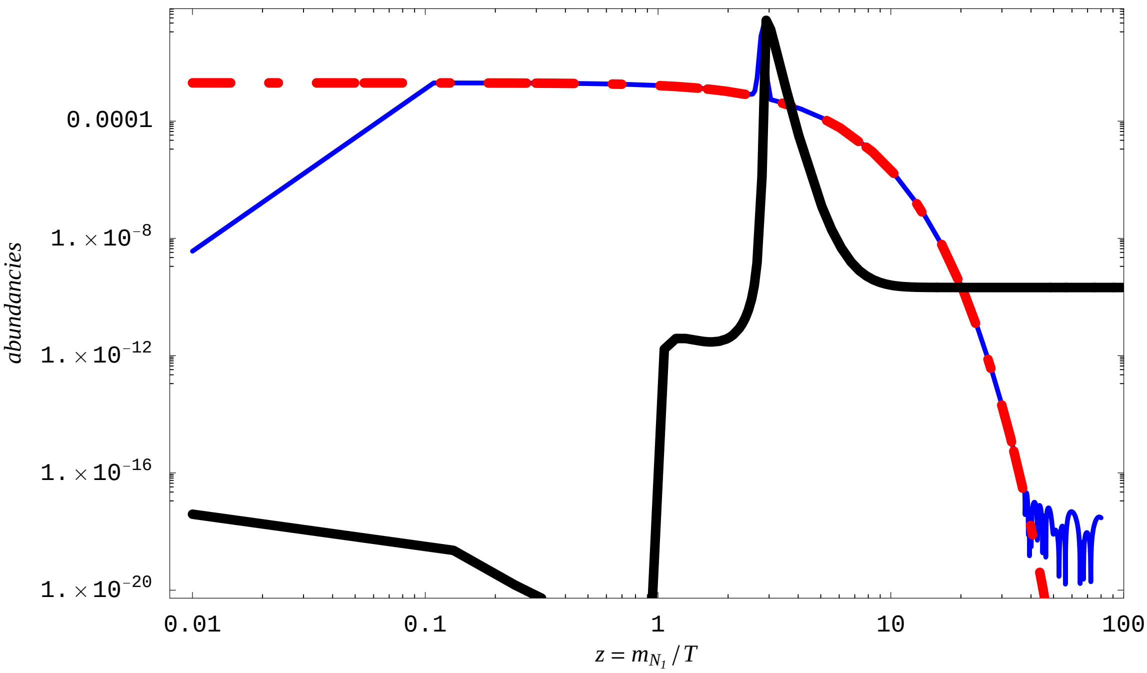

Figure 2.3. Evolution of Y_{N_1} (solid blue), Y_{N1}^{eq} (dot-dash) and the baryon asymmetry η_B (dark solid line) in terms of z in the model. The estimated value for the baryon asymmetry is $\eta_B \simeq 6.03 \times 10^{-10}$, with $Y_{N_1}^{ini} = 0$ and assuming no pre-existing $B - L$ asymmetry.

2.8 Conclusion

An attractive feature of the seesaw mechanism is that it can explain the origin of small neutrino masses and at the same time account for the observed baryon asymmetry in the universe by the out of equilibrium decay of the super-heavy right handed neutrinos. It is then very tempting to seek a link between the baryon asymmetry parameter η_B induced at high temperature and neutrino mass and mixing parameters observable in low energy experiments. No quantitative connection can be found between them in the SM. There have been several attempts in the literature [16,37,38] to establish a relationship between the two. In this paper we have addressed this question in the context of a class of minimal left–right symmetric models.

In the models under consideration the minimality of the Higgs sector implies that M_l and M_D (charged lepton and Dirac neutrino mass matrices) are proportional. As a result, the entire seesaw sector (including the heavy right–handed neutrinos and the light neutrinos) has only 9 parameters. This is the same number as low energy neutrino observables (3 masses, 3 mixing angles and 3 phases). As a result we are able to link the baryon asymmetry of the universe to low energy neutrino observables. This feature is unlike the SM seesaw which has too many arbitrary parameters. Our numerical solution to the coupled Boltzmann equations shows that this constrained system with $M_l \propto M_D$ leads to an acceptable baryon asymmetry. The requirement of an acceptable baryon asymmetry restricts some of the light neutrino observables. We find that $\tan^2\theta_{12} \simeq m_1/m_2$, $0.01 \lesssim \theta_{13} \lesssim 0.07$ and $\beta \simeq \alpha + \pi/2$ are needed for successful baryogenesis. Future neutrino oscillation experiments can directly probe into the dynamics of the universe in its early stages.

CHAPTER 3

BARYON ASYMMETRY VIA SOFT LEPTOGENESIS

3.1 Introduction

In this chapter we analyze lepton asymmetry induced in the right handed sneutrino $\tilde{\nu}_{R1} - \tilde{\nu}^{\dagger}_{R1}$ mixing and decay through W_R exchange in a class of SUSYLR models. Usual soft leptogenesis scenario requires small $B-$term and relatively low heavy neutrino mass. We include the effect of SUSY breaking contribution on the breaking parameters; and compute r.h.n soft parameters to show that Soft Leptogenesis mechanism implemented in SUSYLR framework leads to adequate baryon number asymmetry in the universe. We employ Renormalization Group Equations analysis and show that one achieve this result with natural values of Soft breaking parameters; $B \sim 100$ GeV. In this class of models; $M_{\tilde{\nu}_{R1}} \sim M_{W_R} \sim (10^9 \ - \ 10^{10})$ GeV, is not required to be small as originally proposed. There is no excessive CP violation in these models even when we assume universality of parameters.

3.1.1 Soft Leptogenesis, a brief review

Recently, soft Supersymmetry breaking effects have been utilized to explain the Baryon Asymmetry via the "Soft leptogenesis" mechanism [39,40]. In these models; lepton number violation occurs in the decay of the heavy right handed neutrino and sneutrino, $\nu^c \to L\phi^{\dagger}$, $\tilde{\nu}^c \to L\tilde{\phi}^{\dagger}$, etc. CP asymmetry needed for Leptogenesis the mixing of $\tilde{\nu}^c - \tilde{\nu}^{c\dagger}$ trough soft supersymmetric breaking terms. The relevant superpotential is given by;

$$W = Y_D \ell \nu^c H_u + \frac{1}{2} M_R \nu^c \nu^c \tag{3.1}$$

which generates small neutrino masses via the seesaw mechanism. Here, the light neutrino masses are obtained from the matrix $M_\nu \simeq M_D M_R^{-1} {M_D}^T$ where $M_D = Y_D \langle H_u \rangle$ and M_R are respectively the Dirac and the heavy Majorana right-handed neutrino (r.h.n) mass matrices. In supersymmetric models with seesaw mechanism, Soft SUSY breaking effect involving $\tilde{\nu}^c$, should be taken into account for the study of Leptogenesis. The corresponding soft SUSY breaking Lagrangian is;

$$-\mathcal{L}_{soft} = \tilde{m}^2 \tilde{\nu^c}^\dagger \tilde{\nu}^c + (\frac{1}{2} B M_R \tilde{\nu}^c \tilde{\nu}^c + A Y_D \tilde{\ell} \tilde{\nu}^c H + h.c.) \tag{3.2}$$

The parameters A and B in Eq. (3) are complex in general. Their presence will introduce mixing and $CP-$violation in the $\tilde{\nu}^c - \tilde{\nu}^{c\dagger}$ system, analogous to the well known $K^o - \overline{K}^o$ system (see appendix **A.2** for details). Successful Soft Leptogenesis can occurs even with one family of neutrinos, so we focus on that case. The mass matrix of the $\tilde{\nu}^c - \tilde{\nu}^{c\dagger}$ system is given by,

$$m^2_{\tilde{\nu}^c - \tilde{\nu}^{c\dagger}} = \begin{pmatrix} |M_R|^2 & B M_R \\ B^* M_R^* & |M_R|^2 \end{pmatrix} \tag{3.3}$$

Since the r.h.n mass M_R is much larger than the SUSY breaking scale B, diagonlization of the mass matrix of Eq. (4) will lead to the mass eigenstates $\widetilde{N}_\pm = \frac{1}{2}(\tilde{\nu}^c \pm \tilde{\nu}^{c\dagger})$ with masses eigenvalues,

$$M_\pm \simeq M_1 (1 \pm \frac{|B|}{2M_1}) \ , \tag{3.4}$$

The mass and width difference of the two sneutrino mass eigenstates are given by

$$\Delta m = |B|, \qquad \Delta\Gamma = \frac{2|A|\Gamma}{M_N}. \tag{3.5}$$

After Sphaleron effect takes place the final Baryon asymmetry (BA) is determined to be;

$$\frac{n_B}{s} \simeq -10^{-3} \ d \left[\frac{4\Gamma|B|}{4|B|^2 + \Gamma^2} \right] \frac{|A|}{M_1} \sin\phi \ . \tag{3.6}$$

ϕ is a CP inducing phase desired to be of order 1; $O(1)$, it would in general be contained in the trilinear or bilinear couplings of r.h.n. d is an efficiency parameter, often referred to as dilution factor. In general, it depends on the production mechanism for the r.h. sneutrino. Soft leptogenesis can be successful for rather low $\tilde{\nu}_{R1}$

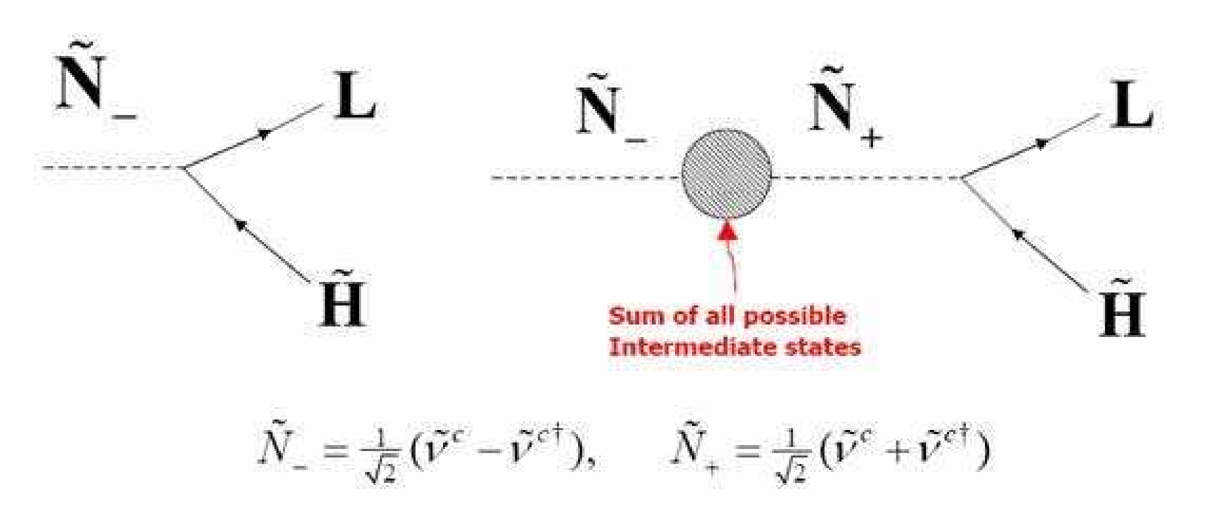

Figure 3.1. Interfering $\tilde{N}_-$ decay amplitudes for the fermionic final states. The blob in the diagram contains a sum of all possible intermediate states. The mixing between the two states $\tilde{N}_-$ and $\tilde{N}^+$ leads to CP violation.

masses which is favored from the Gravitino point of view, however, unconventionally suppressed $B-$term of order $\theta(1)$ GeV is required for this picture to succeed.

Supersymmetric left–right (SUSYLR) models based on the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group naturally includes r.h.n and implements seesaw for neutrino masses. In left–right models, parity symmetry imposes hermiticity on the Yukawa matrices and constrains the Soft breaking parameters in a way that helps solve the supersymmetric CP problem that hunts MSSM, leading to vanishing EDM, while allowing sufficient CP violation in $\tilde{\nu}_R$ mixing. It is therefore interesting to analyze the idea of Soft leptogenesis in the context of Left–Right symmetry. Here, we study the effect of the interactions of the $SU(2)_R$ gauge boson W_R on the generation of the the primordial lepton asymmetry via the Soft leptogenesis mechanism. $B-L$ violation occurs when Left–Right symmetry is broken by the VEV v_R of the $B-L=-2$ triplet scalar field $\Delta^c(1,1,3,-2)$, which gives Majorana masses to the r.h sneutrino and, lepton number is violated in their decays: $\nu_R \to L\phi^\dagger$, $\nu_R \to L^c\phi$ and $\tilde{\nu}_{R1} \to \tilde{e}_R u\bar{d}$ as well as $\tilde{\nu}_{R1} \to \tilde{e}^*_R \bar{u}d$, where this later is mediated by the right handed gauge boson W_R. We show that $\tilde{\nu}_{R1}$ decay through W_R exchange can dominate the traditional $\nu_R \to L\phi^\dagger$ frequently used decay to explain BA. Further more, by RGE analysis we show that the requirement of unconventionally small $B-$term is no longer needed, in addition, we use RGE running and SUSY breaking effect to naturally account for the complex $O(1)$ phase as dictated by the scenario success. The mass of r.h sneutrino can be $\sim M_{W_R} \sim (10^9 \ - \ 10^{10})$ GeV.

The rest of the chapter is organized as follows. In Sec. 3.2 we review the minimal left–right symmetric model. In Sec. 3.3 we analyze leptogenesis in this model. Here we review RGE and discuss their running effect of the soft breaking parameters in the model from the requirement of successful soft leptogenesis. In Sec. 3.4 we calculate the main two loop amplitude responsible for the mixing of $\tilde{\nu}^c - \tilde{\nu^c}^\dagger$. In Sec. 3.5 we analyze SUSY breaking effect on these parameters. we calculate the lepton asymmetry parameter ε_1 generated in the model in $\tilde{\nu}_{R1}$ decay. Sec. 3.6 provides our numerical results for η_B. Finally, in Sec. 3.7 we conclude.

3.2 The minimal left-right symmetric model

Let us briefly review the basic structure of the minimal SUSY left–right symmetric model developed in Ref. [10]. The gauge group of the model is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The quarks and leptons are assigned to the gauge group are listed in the table.

Left–handed quarks and leptons (Q, L) transform as doublets of $SU(2)_L$ $[Q(3,2,1,1/3)$ and $L(1,2,1,-1)]$, while the right–handed ones (Q^c, L^c) are doublets of $SU(2)_R$ $[Q^c(3^*,1,2,-1/3)$ and $L^c(1,1,2,1)]$. The Dirac masses of fermions arise through their Yukawa couplings to a Higgs bidoublet $\Phi(1,2,2,0)$. The $SU(2)_R \times U(1)_{B-L}$ symmetry is broken to $U(1)_Y$ by the VEV (v_R) of a $B-L=-2$ triplet scalar field $\Delta^c(1,1,3,-2)$. This triplet is accompanied by a left–handed triplet $\Delta(1,3,1,2)$ (along with $\bar{\Delta}$ and $\bar{\Delta}^c$ fields, their conjugates to cancel anomalies). These fields also couple to the leptons and are responsible for inducing large Majorana masses for the $\tilde{\nu}_R$. An alternative to these triplet Higgs fields is to use $B-L = \pm 1$ doublets $\chi(1,2,1,-1)$ and $\chi^c(1,1,2,1)$, along with their conjugates $\bar{\chi}$ and $\bar{\chi^c}$. In this case non–renormalizable operators will have to be invoked to generate large neutrino Majorana masses. For definiteness we shall adopt the triplet option, although our formalism allows for the

TABLE 3.1. Particle assignment in SUSYLR gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
Q	3	2	1	$-\frac{1}{3}$
L	1	2	1	$-\frac{1}{3}$
Q^c	3	1	2	-1
L^c	1	1	2	+1
Δ	1	3	1	+2
$\bar{\Delta}$	1	3	1	-2
Δ^c	1	1	3	-2
$\bar{\Delta}^c$	1	1	3	+2
Φ	1	2	2	0

addition of any number of doublet Higgs fields as well. Also, in order to keep the model general one has to allow for a number of singlet fields $S(1,1,1,0)$, for simplicity we only we assume one singlet. The most general superpotential and soft breaking terms invariant under the gauge symmetry are

$$\begin{aligned} W &= ih_Q(Q^T\tau_2\Phi_a Q^c) + ih_L(L^T\tau_2\Phi_a L^c) + if(L^T\tau_2\Delta L) + if_c(L^{cT}\tau_2\Delta^c L^c) \\ &+ M_\Delta \operatorname{Tr}\left(\Delta\bar{\Delta}\right) + M_{\Delta^c}\operatorname{Tr}\left(\Delta^c\bar{\Delta}^c\right) + M_{\Phi_a}\operatorname{Tr}\left(\Phi_a^T\tau_2\Phi_a\tau_2\right) \\ &+ \mu_\Delta S \operatorname{Tr}\left(\Delta\bar{\Delta}\right) + \mu_{\Delta^c} S\operatorname{Tr}\left(\Delta^c\bar{\Delta}^c\right) + \mu_{\Phi_a} S\operatorname{Tr}\left(\Phi_a^T\tau_2\Phi_a\tau_2\right) \\ &+ \frac{1}{6}Y_S S^3 + \tfrac{1}{2}M_S S^2 + L_S S\ , \end{aligned} \tag{3.7}$$

and the corresponding soft breaking terms;

$$\begin{aligned} -L_{SB} &= \tfrac{1}{2}\left(M_3^G\tilde{g}\tilde{g} + M_L^G\tilde{W}_L\tilde{W}_L + M_R^G\tilde{W}_R\tilde{W}_R + M_1^G\tilde{B}\tilde{B} + \text{h.c.}\right) \\ &+ \Big[iA_Q\tilde{Q}^T\tau_2\Phi_a\tilde{Q}^c + iA_L\tilde{L}^T\tau_2\Phi_a\tilde{L}^c + iA_f\tilde{L}^T\tau_2\Delta\tilde{L} \\ &\quad + iA_{f^c}\tilde{L^c}^T\tau_2\Delta^c\tilde{L}^c + A_\Delta S\operatorname{Tr}\left(\Delta\bar{\Delta}\right) + A_{\Delta^c}S\operatorname{Tr}\left(\Delta^c\bar{\Delta}^c\right) \\ &\quad + A_{\Phi_a}S\operatorname{Tr}\left(\Phi_a^T\tau_2\Phi_a\tau_2\right) + \frac{1}{6}A_S S + \text{h.c.}\Big] \end{aligned}$$

$$+\left[B_\Delta \operatorname{Tr}\left(\Delta\bar{\Delta}\right) + B_{\Delta^c} \operatorname{Tr}\left(\Delta^c\bar{\Delta}^c\right) + B_{\Phi_a} \operatorname{Tr}\left(\Phi_a^T\tau_2\Phi_a\tau_2\right) + \tfrac{1}{2}B_S S^2 + \text{h.c.}\right]$$
$$+\Big[m_Q^2\tilde{Q}^T\tilde{Q}^* + m_{Q^c}^2\tilde{Q^c}^\dagger\tilde{Q}^c + m_L^2\tilde{L}^T\tilde{L}^* + m_{L^c}^2\tilde{L^c}^\dagger\tilde{L}^c$$
$$+ m_\Delta^2 \operatorname{Tr}\left(\Delta^\dagger\Delta\right) + m_{\bar{\Delta}}^2 \operatorname{Tr}\left(\bar{\Delta}^\dagger\bar{\Delta}\right) + m_{\Delta^c}^2 \operatorname{Tr}\left(\Delta^{c\dagger}\Delta^c\right) + m_{\bar{\Delta}^c}^2 \operatorname{Tr}\left(\bar{\Delta}^{c\dagger}\bar{\Delta}^c\right)$$
$$+ m_{\Phi_a}^2 \operatorname{Tr}\left(\Phi_a^\dagger\Phi_a\right) + m_S^2 \mid S \mid^2 \Big], \tag{3.8}$$

Under left–right parity symmetry,

$$Q \leftrightarrow Q^{c*},\ L \leftrightarrow L^{c*},\ \Phi_a \leftrightarrow \Phi_a^\dagger,\ \Delta \leftrightarrow \Delta^{c*} \tag{3.9}$$

$$W_{SU(2)_L} \leftrightarrow W^*_{SU(2)_R},\ W_{B-L} \leftrightarrow W^*_{B-L},\ \text{and } \theta \leftrightarrow \bar{\theta} \tag{3.10}$$

By demanding parity invariance from this theory, we also find the following relations among the parameters [41,42]:

$$\begin{array}{cccc}
\mu_{\Phi_a} = \mu^*_{\Phi_a} & M_\Delta = M^*_{\Delta^c} & M_{\Phi_a} = M^*_{\Phi_a} & M_S = M_S^* \\
h_Q = h_Q^\dagger & h_L = h_L^\dagger & f = f_c^* & \mu_\Delta = \mu^*_{\Delta^c} \\
L_S = L_S^* & M_1^G = M_1^{G*} & M_L^G = M_R^{G*} & M_3^G = M_3^{G*} \\
g_L = g_R & B_\Delta = B^*_{\Delta^c} & B_{\Phi_a} = B^*_{\Phi_a} & B_S = B_S^*,
\end{array}$$

where g_L and g_R are the $SU(2)_L$ and $SU(2)_R$ coupling constants, respectively, and M_i^G are the gauge group masses. The correspondences, $h_Q = h_Q^\dagger$, $h_L = h_L^\dagger$, and $f = f_c^*$ in the above relations are very important feature of Left–Right symmetry. It has been shown in Ref. [10] that the hermiticity of the Yukawa matrices (along with the parity constraints on the soft SUSY breaking parameters) helps solve the supersymmetric CP problem that haunts the MSSM. These constraints also lead to zero EDM at the ν_R scale. In fact, EDM for the neutron and electron is only induced by RGE, but remains close to the current experimental limit. Notice that our $B-$term for r.h.n is contained in the term $A_{f^c}(\tilde{L^c}^T\tau_2\Delta^c\tilde{L}^c)$, so in general A_{f^c} would induce r.h.n $B-$term. We will discuss this in great detail in the following section.

Below v_R, the effective theory is the MSSM + r.h.n with its H_u and H_d Higgs multiplets. These are contained in the bidoublet Φ_a of the SUSY left-right model, but in general they can also reside partially in other multiplets having identical quantum numbers under the MSSM symmetry (such as the χ, $\overline{\chi}$ doublet Higgs fields alluded to earlier) *.

3.3 $\tilde{\nu}_R$ decay mediated by $SU(2)_R$ gauge boson W_R

The left-right supersymmetric potential $SU(2)_R \times U(1)_{B-L}$ symmetry is broken down to $U(1)_Y$ by the VEV $\langle \Delta^c \rangle = v_R \sim M_{W_R}$. We assume the right-handed neutrino ν_{R1} has masse below v_R. We focus on a single generation sneutrino and discuss the effect of RGE running on the soft leptogenesis mechanism. With SM gauge symmetry, the effective superpotential involving r.h.n below v_R is;

$$\begin{aligned} W &= (f_d^{ij} h_1 \tilde{d}^*_{Ri} \tilde{q}_{Lj} + f_u^{ij} h_2 \tilde{u}^*_{Ri} \tilde{q}_{Lj} + f_l^{ij} h_1 \tilde{e}^*_{Ri} \tilde{l}_{Lj} + f_\nu^{ij} h_2 \tilde{\nu}^*_{Ri} \tilde{l}_{Lj} + ... + h.c.). \\ & \quad +(\mu h_1 h_2 + \frac{1}{2} M_\nu^{ij} \tilde{\nu}^*_{Ri} \tilde{\nu}^*_{Rj} + h.c.) \end{aligned} \tag{3.11}$$

and the analogous soft breaking Lagrangian

$$\begin{aligned} -L_{soft} &= (A_d^{ij} h_1 \tilde{d}^*_{Ri} \tilde{q}_{Lj} + A_u^{ij} h_2 \tilde{u}^*_{Ri} \tilde{q}_{Lj} + A_l^{ij} h_1 \tilde{e}^*_{Ri} \tilde{l}_{Lj} + A_\nu^{ij} h_2 \tilde{\nu}^*_{Ri} \tilde{l}_{Lj} + ... + h.c.). \\ & \quad +(B\mu h_1 h_2 + \frac{1}{2} B_\nu^{ij} M_\nu^{ij} \tilde{\nu}^*_{Ri} \tilde{\nu}^*_{Rj} + h.c.) \end{aligned} \tag{3.12}$$

These parameters satisfy the boundary condition at v_R, $A_d = A_u = A_Q$. Mixing between the sneutrino $\tilde{\nu}_{R1}$ and anti-sneutrino $\tilde{\nu}^\dagger_{R1}$ in the Soft Lagrangian is introduced via the soft SUSY breaking terms, giving a source for the CP violation in the $\tilde{\nu}_{R1} - \tilde{\nu}^\dagger_{R1}$ system in a similar way it happens in the $K_0 - \overline{K}_0$ system. $\theta(1)$ non-vanishing CP inducing phase ϕ would in general be contained in $A-$term or $B-$term of r.h.n. It is this CP violation that is considered to be source of lepton number asymmetry. After sphaleron effect take place, the final baryon number to entropy ratio is determined to be

$$\frac{n_B}{s} = -10^{-3}\ d \left[\frac{4\Gamma |B_\nu|}{4|B_\nu|^2 + \Gamma^2} \right] \frac{|A_\nu|}{M_1} \sin\phi\ . \tag{3.13}$$

---

*Allowing for such a possibility, the superpotential of Eq. (3.7) leads to the relations for the MSSM Yukawa coupling matrices $\mathbf{f}_u = \gamma \mathbf{f}_d$, and, $\mathbf{f}_\ell = \gamma \mathbf{f}_{\nu^D}$ These relations have been called up–down unification [10]. Here, the first relation implies $\frac{m_t}{m_b} \simeq \gamma \tan\beta \equiv c$ where γ is a parameter characterizing how much of H_u and H_d of MSSM are in the bidoublet Φ. The case of $H_{u,d}$ entirely in Φ will correspond to $\gamma = 1$ and $\tan\beta = m_t/m_b$. The consequences of such relations on Baryon asymmetry have been analyzed in the context of thermal Leptogenesis [7]. Leptogenesis in the context of more general left-right symmetric models has been analyzed in Ref. [19]

M_1 is the lightest r.h.n mass and the decay width $\Gamma = \frac{(M_D^\dagger M_D)_{11}}{4\pi v^2} M_1$. d is efficiency factor; often referred to as a dilution factor, which takes into account the washout processes (inverse decays and lepton number violating scattering).* The determination of the dilution factor involves the integration of the full set of Boltzmann equations. A simple approximated solution which has been frequently used is given by [43]

$$d = \begin{cases} \sqrt{0.1\,\kappa}\,\exp\left(-\frac{4}{3}\sqrt[4]{0.1\,\kappa}\right) \quad , \quad \kappa \gtrsim 10^6 \\ 0.24(\kappa\,\ln\kappa)^{-3/5} \quad , \quad 10 \lesssim \kappa \lesssim 10^6 \\ 1/(2\kappa) \quad , \quad 1 \lesssim \kappa \lesssim 10 \\ 1 \quad , \quad 0 \lesssim \kappa \lesssim 1 \end{cases} \tag{3.14}$$

where the parameter κ, which measures the efficiency in producing the asymmetry, characterizes the wash-out effects due to the inverse decays and lepton number violating scattering processes together with the time evolution of the system, is defined as the ratio of the thermal average of the ν_{R_1} decay rate and the Hubble parameter at the temperature $T = M_1$,

$$\kappa = \frac{\Gamma}{H}, \text{ where } H = \sqrt{\frac{4\pi^3 g_*}{45}\frac{M_1^2}{M_{pl}}} \tag{3.15}$$

$M_{pl} \simeq 1.22 \times 10^{19}$ GeV is the Planck mass and, g_* is the effective degree of freedom.

$$\hat{\nu}_R \to \tilde{e}^+ u\bar{d}(\tilde{e}^- \bar{u}d)$$

As we pointed out before; once one considers Left–Right symmetry, a lepton number violating decay arises via the $SU(2)_R$ gauge boson W_R as it is indicated in the figure (1). If we call Γ_2 the decay width of the process $\hat{\nu}_R \to \tilde{e}^+ u\bar{d}(\tilde{e}^- \bar{u}d)$ and, $\Gamma_1 = \frac{(Y_\nu^\dagger Y_\nu)_{11}}{8\pi} M_1$, being the decay width of $\nu_R \to L\phi^\dagger(L^c\phi)$, then Γ_2 become the leading lepton violating decay and Γ_2 dominates if $\Gamma_2 \geq \Gamma_1$. In this case, BA will mainly be driven by decays such as $\hat{\nu}_R \to \tilde{e}^+ u\bar{d}$. Given that

$$\Gamma_2 \simeq \frac{9 G_F^2 M_{w_L}^4}{192\pi^3}\frac{M_1^5}{M_{w_R}^4}, \tag{3.16}$$

*Recently there has been considerable effort in obtaining semi analytical expressions for the efficiency so one does not have to solve Boltzmann equations every time. For e.g; see [24,25]. Rigorous derivations, however, have to include flavor effects on leptogenesis [43].

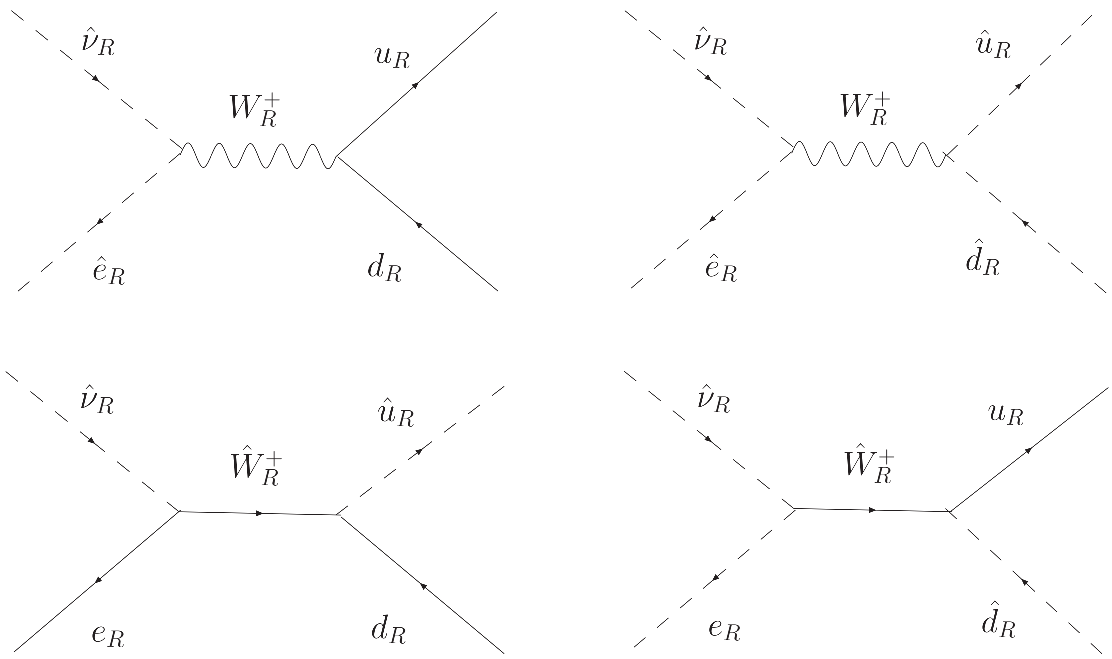

Figure 3.2. Diagrams Contributing to Leptogenesis: The lightest $\hat{\nu}_R$ decay diagrams via $SU(2)_R$ gauge boson exchange that appear in Left–Right models, corresponding to $\hat{\nu}_R \to \tilde{e}^+ u\bar{d}(\tilde{e}^- \bar{u}d)$. The lepton asymmetry can arise through $\tilde{\nu}_{R1} - \tilde{\nu}_{R1}^\dagger$ mixing and decay.

the condition translate into

$$(Y_\nu^\dagger Y_\nu)_{11} \lesssim 1.55 \times 10^{-4} \left(\frac{M_1}{M_{w_R}}\right)^4 \tag{3.17}$$

On the other hand, if $0 \lesssim (\Gamma_2/H) \lesssim 1$, the dilution parameter d can enhanced to equal 1, which puts a constraint on the mass M_1. A natural value for Γ_2 follows from SUSY breaking scale and preferred to be $\Gamma_2 \sim 100$ GeV. For optimal efficiency, i.e, $\Gamma_2 \sim H \sim 100$ GeV, we find $M_1 \simeq 6.92 \times 10^9$ GeV. From Eq. (3.16) we then compute $M_{w_R} \sim 4.45 \times 10^{10}$ GeV. The condition on the Dirac Yukawa coupling in Eq. (3.17) can be easily realized in Left–Right symmetry in way that is not in conflict with light neutrinos masses. Working a basis where the charged lepton mass matrix is diagonal $M_\ell = D_\ell$, there is mixing among generations in both M_R and M_D, where $M_D = vY_\nu$, the light neutrino mixing angles will depend on both of these mixings. While there is some arbitrariness in the forms for M_D and M_R, one simple possibility consistent with Soft Leptogenesis is as follows. As noted before, due left-right symmetry and assuming the existence of two or more bidoublet Φ_a, the dirac mass matrix is hermitian and can be diagonlized as $M_D = UDU^\dagger$ and r.h.n mass matrix as $M_R = VD_RV^T$, where U and V are unitary matrices. One can then generate light neutrino masses via the seesaw mechanism * [14]

$$M_\nu = M_D M_R^{-1} {M_D}^T. \tag{3.18}$$

Employing Eq. (3.18) to solve for M_ν,

$$M_\nu = UDU^\dagger V^* D_R^{-1} V^\dagger U^* DU^T \tag{3.19}$$

we explicitly make the simple choice $U = V^*$, so that M_ν becomes $M_\nu = UDD_R^{-1}DU^T$, where $D \equiv diag(d_1, d_2, d_3)$ and $D_R \equiv diag(M_1, M_2, M_3)$. M_ν is then found to be,

$$M_\nu = U \begin{pmatrix} d_1^2/M_1 & 0 & 0 \\ 0 & d_2^2/M_2 & 0 \\ 0 & 0 & d_3^2/M_3 \end{pmatrix} U^T \tag{3.20}$$

*In principle the $\Delta(1,3,1,+2)$ Higgs field can also acquire a small VEV $\lesssim \theta(eV)$. In this case the seesaw formula would be modified [18], as will the calculation of the lepton asymmetry. We will assume such type II seesaw contributions proportional to $\langle\Delta\rangle$ are zero in our analysis. This is consistent with the models of Ref. [10].

In the three family scenario, the relations between the flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ and the mass eigenstates (ν_1, ν_2, ν_3) can be expressed in terms of observables as

$$M_\nu = U^*_{PMNS} M_\nu^{diag} U^\dagger_{PMNS}, \tag{3.21}$$

where $M_\nu^{diag} \equiv diag(m_1, m_2, m_3)$, with m_i being the light neutrinos masses and U_{PMNS} being the 3×3 mixing matrix, we simply chosen U such that $U^* = U_{MNPS}$. We get the following identity;

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_3 \end{pmatrix} = \begin{pmatrix} d_1^2/M_1 & 0 & 0 \\ 0 & d_2^2/M_2 & 0 \\ 0 & 0 & d_3^2/M_3 \end{pmatrix} \tag{3.22}$$

In a basis where the r.h.n mass matrix is diagonal, the Dirac mass matrix is $\hat{M}_D = M_D V^* = M_D U = M_D U^*_{PMNS}$. The condition of Eq. (3.17) then reads

$$(\hat{M}_D^\dagger \hat{M}_D)_{11} = D_{11}^2 = m_1 M_1 \lesssim 1.16 \times 10^{-2} \text{ GeV}^2, \tag{3.23}$$

therefore, the lepton number violating right handed sneutrino decay via the $SU(2)_R$ gauge boson dominance can be easily realized, as long as $m_1 \lesssim 1.67 \times 10^{-3}$ eV, which is consistent with neutrino experiments.

It was concluded before that in order for Soft leptogenesis to succeed, the value of M_1 has to be very small; much smaller than the value naturally predicted by seesaw of $(10^9 \ - 10^{10})$GeV. Seesaw scale is also favorable by the traditional thermal leptogenesis, however, it makes M_1 borderline compatible with bounds derived on reheating temperature as imposed by Gravitino production, but not conclusively excluded [31]. Furthermore, it is believed that the Soft bilinear coupling has to be significantly below the M_{SUSY} for this mechanism to provide viable leptonic asymmetry. In the following we show that the above requirements do not hold in Left–Right symmetry. In fact, Soft leptogenesis can proceed in Left-Right model with natural values of $M_1 \sim (10^9 \ - 10^{10})$GeV and natural scale for the bilinear coupling $B \sim \Gamma \sim 100$ GeV. Also, by employing SUSY breaking effects on the running of RGE, we are able to naturally generate the $\theta(1)$ complex phase that drives leptogenesis.

3.4 Computing the two loop amplitude leading to $\tilde{\nu}^c - \tilde{\nu}^{c\dagger}$ mixing

In our analysis in the previous section, we have left out an important detail, the A-term appearing in Eq (3.13) was conveniently assumed to have the right order of magnitude for our estimate of Baryon Asymmetry to have the right order. Since we are introducing a new decay; $\tilde{\nu}^c \to e^c\tilde{u}^c d^c (e^c u^c \tilde{d}^c)$, to be the potentially dominant decay, leading to adequate baryon asymmetry, such statement has to enforced by computing the corresponding decay amplitude exactly. Our idea is that the mixing $\tilde{\nu}^c - \tilde{\nu}^{c\dagger}$ in Left-Right symmetry is introduced and mediated by the $SU(2)_R$ gauge boson (W_R). The Feynman Diagram leading to this picture has been depicted in Fig (3.4).

$$\begin{aligned}
A &= -g_R^3 m_{1/2} M_\lambda \frac{f}{\sqrt{2}}(3\times 3)\int\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4} \\
&\times\ Tr\left[\frac{\not{k}\left(\frac{1+\gamma_5}{2}\right)}{k^2-m_\ell^2+\imath\epsilon}\,\frac{(\not{k}+\not{p}+M_\lambda)}{(k+p)^2-M_\lambda^2+\imath\epsilon}\,\frac{\not{q}\left(\frac{1-\gamma_5}{2}\right)}{q^2-m_d^2+\imath\epsilon}\,\frac{(\not{k}+\not{p}+M_\lambda)\left(\frac{1+\gamma_5}{2}\right)}{(k+p)^2-M_\lambda^2+\imath\epsilon}\right. \\
&\times\ \left.\frac{M_\lambda\left(\frac{1-\gamma_5}{2}\right)}{(k+p)^2-M_\lambda^2+\imath\epsilon}\right]\frac{1}{(k+p-q)^2-m_{\tilde{u}^c}^2+\imath\epsilon} \qquad (3.24)\\
&= M_{1/2}N_f\int\int\frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4} \\
&\times\ \frac{k(k+p).q(k+p)-(k.q)(k+p)^2}{(k^2-m_e^2+\imath\epsilon)(q^2-m_d^2+\imath\epsilon)\left[(k+p)^2-M_\lambda^2+\imath\epsilon\right]^3\left[(k+p-q)^2-m_{\tilde{u}^c}^2+\imath\epsilon\right]}.
\end{aligned}$$

The 8-dimensional integral has to be done, notice the topology of our denominator with 6 propagators. The small masses can be set to zero, i.e, $m_e^2 = m_d^2 = m_{\tilde{u}^c}^2$, without introducing any infrared divergence. The denominator then takes the form;

$$Den = (k^2+\imath\epsilon)\left[(k+p-q)^2+\imath\epsilon\right]\left[(k+p)^2-M_\lambda^2+\imath\epsilon\right]^3(q^2+\imath\epsilon), \qquad (3.25)$$

we carry out the d^4q-integral first, but to do this, the denominator has to be simplified to become of the form $Den = q^n - f(k,p,M_\lambda,\imath\epsilon)$. First we employ Feynman parametrization on the denominator;

$$\frac{1}{a\ b\ c\ d^3} = \frac{\Gamma(6)}{2}\int_0^1 dX_1\int_0^1 dX_2\int_0^1 dX_3\int_0^1 dX_4\frac{X_4^2\delta\left(1-X_1-X_2-X_3-X_4\right)}{\left(aX_1+bX_2+cX_3+dX_4\right)^6}, \qquad (3.26)$$

where $a = (k^2 + \imath\epsilon),\ b = (q^2 + \imath\epsilon),\ c = ((k+p-q)^2 + \imath\epsilon)\,,\ d = ((k+p)^2 - M_\lambda^2 + \imath\epsilon)^3$ and $p^2 = M_{\tilde{\nu}^c}^2$. After manipulating a shift on the momentum q, the d^4q part of Eq (3.24) becomes of regular form;

$$J = \int_{-\infty}^{+\infty} \frac{d^4q}{(2\pi)^4} \frac{1}{\left[q^4 + f\left(k, p, M_\lambda, \imath\epsilon\right)\right]^6}, \tag{3.27}$$

for which we can use the known dimensional regularization formulas. We are able to perform the d^4k integral part of Eq (3.24) in similar fashion. The resulting quantity after integrating out q and k takes the form;

$$\begin{aligned} A &= \frac{6}{\sqrt{2}(16\pi^2)^2} f g_R^3 m_{1/2} M_\lambda \int_0^{1-X_3-X_4} dX_1 \int_0^{1-X_4} dX_3 \int_0^1 dX_4 \frac{X_3 X_4^2}{\left(1-X_1-X_4\right)^7} \frac{1}{A^4} \\ &\times \left[\ln\Lambda^2 - \ln\left(\frac{B^2p^2}{A^2} - \frac{C}{A}\right) - \frac{-\frac{p^2}{2}\left(1-\frac{B}{A}\right)\left(1-\frac{2B}{A}\right)}{\left(\frac{B^2p^2}{A^2} - \frac{C}{A}\right)} - \frac{-\frac{p^4}{6}\frac{B}{A}\left(1-\frac{B}{A}\right)^3}{\left(\frac{B^2p^2}{A^2} - \frac{C}{A}\right)^2} - \frac{11}{6} \right] \end{aligned} \tag{3.28}$$

where $A,\ B,\ C$ are functions of X_i's, M_λ and $\imath\epsilon$ as follows;

$$\begin{aligned} A &= 1 + \frac{X_3^2}{\left(1-X_1-X_4\right)^2} - \frac{1+X_3}{1-X_1-X_4} \\ B &= \frac{X_3^2}{\left(1-X_1-X_4\right)^2} - \frac{X_3+X_4}{1-X_1-X_4} \\ C &= BM_{\tilde{\nu}^c}^2 + M_\lambda^2 \frac{X_4}{1-X_1-X_4} - \frac{\imath\epsilon}{1-X_1-X_4} \end{aligned} \tag{3.29}$$

dX_i integrals are carried our numerically and $A \propto \chi m_{1/2}$, where χ is order one parameter. In table 3.2 we give an estimate of BA based on this numerical integration and the running of soft parameters as we discuss below.

3.5 SUSYLR RGEs effect on Soft Leptogenesis

In order to generate a baryon asymmetry consistent with the observed number of Eq. (2.1) one obtains the following constraints from Eq. (3.13):

$$A \sim 1\ \mathrm{Te}V,\quad M_1 \sim (10^9\ - 10^{10})\ \mathrm{GeV},\quad B \sim \Gamma \sim 100\ \mathrm{Ge}V,\ \text{and}\ \ \phi \sim 1, \tag{3.30}$$

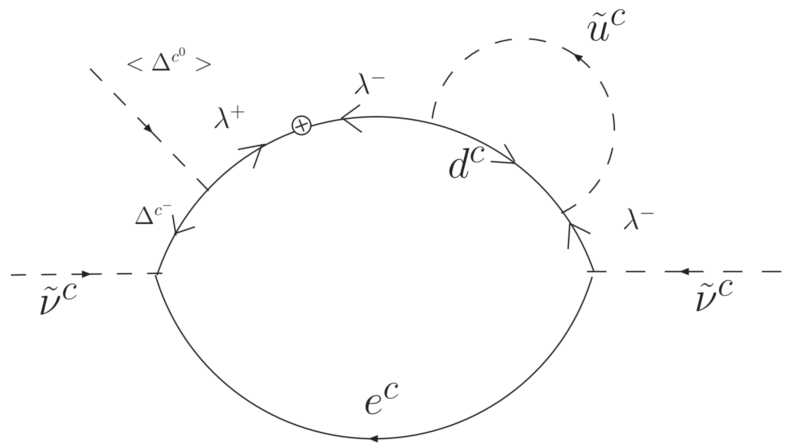

Figure 3.3. Two Loop Diagram Contributing to Leptogenesis: Feynman diagram arising from $\tilde{\nu}^c \to e^c \tilde{u}^c d^c$ decay, mediated by $SU(2)_R$ gaugino (labeled λ). Our results are based on the computation of the corresponding decay amplitude. The lepton asymmetry arises through the mixing of $\tilde{\nu}^c - \tilde{\nu}^{c^\dagger}$.

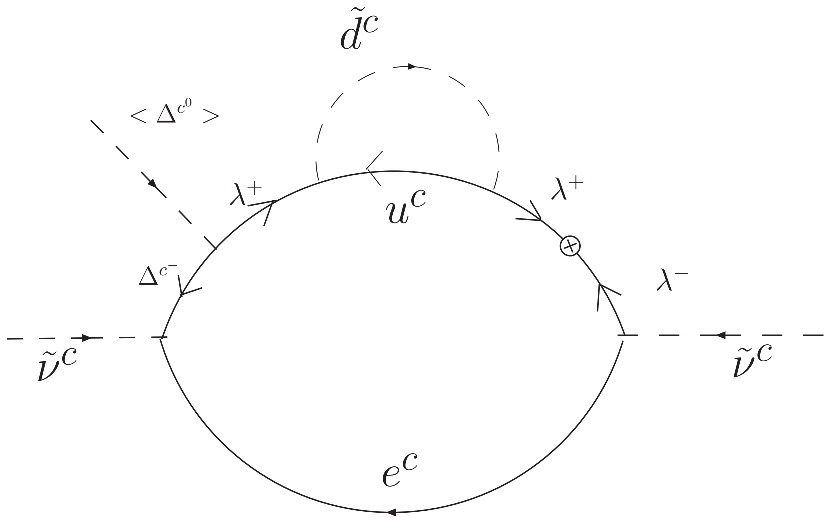

Figure 3.4. Two Loop Diagram Contributing to Leptogenesis: Feynman diagram arising from $\tilde{\nu}^c \to e^c \tilde{d}^c u^c$ decay, mediated by $SU(2)_R$ gauge boson, it is simply the supersymmetric correspondent of the previous Feynman amplitude. The lepton asymmetry arises through the mixing of $\tilde{\nu}^c - \tilde{\nu}^{c^\dagger}$

assuming optimal efficiency from Eq. (3.14). It is our purpose in this paper to accommodate for these constraints in a Left-Right symmetric framework. It is also desirable to have sufficient BA where the Soft parameters and the r.h.n mass assume their natural values.

Above υ_R, the breaking scale of $B-L$, the spectrum is that of Left-Right Symmetry and the gauge group is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The full set of one loop RGE Corresponding to the parameters

$$\left\{ \begin{array}{cccc} A_Q & A_L & A_f & A_{f^c} \\ A_\Delta & A_{\Delta^c} & A_\Phi & A_S \\ h_Q & h_L & f & f_c \\ B_\Delta & B_\Delta^c & B_\Phi & B_S \\ \mu_\Delta & \mu_\Delta^c & \mu_\Phi & Y_S \\ g_1 & g_L & g_R & g_3 \\ M_1^G & M_L^G & M_R^G & M_3^G \\ M_\Delta & M_\Delta^c & M_\Phi & M_S \\ L_S & C_S & & \end{array} \right. \tag{3.31}$$

Most of the RGEs for these parameters can be found in [41]. In SUGRA, it is allowed to set all $A-$terms to zero at M_{pl}, then soft breaking trilinear coupling like A_{f^c} would be induced at υ_R. The evolution of A_{f^c} is given by

$$\begin{aligned} 16\pi^2 \frac{d}{dt} A_{f^c} &= A_{f^c} \left[6 f_c^\dagger f_c + 2 h_L^\dagger h_L + 2\,\mathrm{Tr}\,\left(f_c^\dagger f_c\right) + \mu_{\Delta^c}^* \mu_{\Delta^c} - \frac{9}{2} g_1^2 - 7 g_R^2 \right] \\ &+ f_c \left[12 f_c^\dagger A_{f^c} + 4 h_L^\dagger A_L + 4\,\mathrm{Tr}\,\left(f_c^\dagger A_{f^c}\right) + \ 2\mu_{\Delta^c}^* A_{\Delta^c} + 9 g_1^2 M_1 + 14 g_R^2 M_R \right] \\ &+ \left[6 f_c f_c^\dagger + 2 h_L^T h_L^* \right] A_{f^c} + \left[12 A_{f^c} f_c^\dagger + 4 A_L^T h_L^* \right] f_c \end{aligned} \tag{3.32}$$

Above υ_R there is no $B-$term for r.h.n, but it will be induced by $A-$terms like A_{f^c}. In SUSYLR there is a proportionality between $A-$ and $B-$terms. In fact we can approximately estimate B_{ind}. The relevant term in this case is

$$-\text{Ł}_{SB} = i A_{f^c} \tilde{L^c}^T \tau_2 \Delta^c \tilde{L^c} + ... \tag{3.33}$$

when Δ^c acquires VEV we get the following term

$$\begin{aligned} \text{Ł} &= A_{f^c} \tilde{\nu^c} \tilde{\nu^c} < \Delta^c > \\ &\equiv B_{ind} \tilde{\nu^c} \tilde{\nu^c} \end{aligned} \tag{3.34}$$

$B-$term is then estimated to be, $B_{ind} \simeq (A_{f^c}\upsilon_R)/M_1 = A_{f^c}/f^c$. From Eq (3.32), setting $A_i = 0$ and analytically solving for A_{f^c} and finding its value at υ_R then estimate B_{ind} induced at υ_R;

$$B_{ind} \simeq -\frac{(f_c)_{11}\upsilon_R}{16\pi^2 M_1}\left\{9g_1^2 M_1^G + 14g_R^2 M_R^G\right\} Log\left(\frac{M_{pl}}{\upsilon_R}\right), \tag{3.35}$$

with $\upsilon_R = \frac{M_{w_R}}{g_R} \simeq 6.35 \times 10^{10}$ GeV, $M_{pl} \sim 10^{18}$ GeV where $g_1,\ g_R,\ M_1$ and M_R have natural values, it is possible to generate the right order of magnitude for the r.h. sneutrino $B-$term of $\theta(50\ -100)$ GeV. It is not possible however to explain the complex phase necessary for the Soft Leptogenesis, for that we employ supersymmetry breaking effect which has to be included anyways, otherwise, the result would be misleading. In the result section we numerically compute the $B-$term by including all the RGEs that enter in the calculation of the soft breaking parameters in addition to implementing SUSY breaking affect.

3.6 Symmetry breaking contribution to r.h.n $B-$term

In this section we analyze the effects of supersymmetry breaking on the bilinear coupling B which have to be included to get the correct magnitude. It turns out that the $\theta(1)$ phase needed has it's origin from the $F-$term of Δ^c. From Eq. (3.7), the part of the superpotential of interest to us;

$$\begin{aligned} W &= M_{\Delta^c}\,\mathrm{Tr}\left(\Delta^c\bar{\Delta}^c\right) + \mu_{\Delta^c} S\,\mathrm{Tr}\left(\Delta^c\bar{\Delta}^c\right) \\ &\quad + \frac{1}{6}Y_S S^3 + \tfrac{1}{2}M_S S^2 + L_S S \end{aligned} \tag{3.36}$$

For simplicity we denote $X = S$, $a = L_S$, $b = \frac{1}{2}M_S$, $c = \frac{1}{6}Y_S$ and $d = \mu_{\Delta^c}$. Then the corresponding soft potential is

$$\begin{aligned} V_{soft} &= \tilde{a}X + \tilde{b}X^2 + \tilde{c}X^3 + \tilde{d}X\Delta^c\bar{\Delta}^c + \tilde{M}_{\Delta^c}\Delta^c\bar{\Delta}^c \\ &\quad + m_X^2 \mid X \mid^2 + m_{\Delta^c}^2 \mid \Delta^c \mid^2 + m_{\bar{\Delta}^c}^2 \mid \bar{\Delta}^c \mid^2 \end{aligned} \tag{3.37}$$

where one can write the $D-$term

$$V_D = \frac{1}{4}g_B^2\left(\mid \Delta^c \mid^2 - \mid \bar{\Delta}^c \mid^2\right) \tag{3.38}$$

and

$$
\begin{aligned}
V_F &= \left|\sum_i \frac{\partial W}{\partial \phi_i}\right|^2 \\
&= \left|a + 2bX + 3cX^2 + d\Delta^c\bar{\Delta}^c\right|^2 + |dX + M_{\Delta^c}|^2 \left(|\ \Delta^c\ |^2 +\ |\ \bar{\Delta}^c\ |^2\ \right) \\
&= |d|^2 \mid \hat{X} \mid^2 \left(|\ \Delta^c \mid^2 + \mid \bar{\Delta}^c \mid^2\right) + \left|-a' + b'\hat{X} + 3c\hat{X}^2 + d\Delta^c\bar{\Delta}^c\right|^2 , \qquad (3.39)
\end{aligned}
$$

where in the last step we shifted X by $X = \hat{X} - M_{\Delta^c}/d$ and defined $a' = -\left(a - \frac{2b}{d}M_{\Delta^c} + \frac{3c}{d^2}M^2_{\Delta^c}\right)$, $b' = \left(2b - \frac{6c}{d}M_{\Delta^c}\right)$. In the supersymmetric limit;

$$
< \hat{X} > \ = 0 \ \text{ and } \ \left\langle \Delta^c\bar{\Delta}^c \right\rangle = a'/d \qquad (3.40)
$$

Δ^c is of order the breaking scale of Left-Right symmetry; $< \Delta^c >= \upsilon_R$, then $< \bar{\Delta}^c >= \upsilon_R e^{\imath\phi}$ where $\phi = arg(a'/d)$ and $|\upsilon_R| = \left|\frac{a'}{d}\right|^{1/2}$. Now if one includes SUSY breaking that we parameterize by small ϵ_X, ϵ and $\bar{\epsilon}$ as follow

$$
\begin{aligned}
< \hat{X} > &= \epsilon_X \qquad (3.41)\\
\Delta^c &= \left|\frac{a'}{d}\right|^{1/2} + \epsilon \\
\bar{\Delta}^c &= \left|\frac{a'}{d}\right|^{1/2} e^{\imath\phi} + \bar{\epsilon}\ e^{\imath\phi} ,
\end{aligned}
$$

with this, after computing $D-term$ and $F-$term Eq. (3.37) and the potential become;

$$
V_{soft} = \tilde{a}'\hat{X} + \left|\frac{a'}{d}\right|^{\frac{1}{2}} e^{\imath\phi}\left(\tilde{M}_{\Delta^c} - M_{\Delta^c}\frac{\tilde{d}}{d}\right)(\epsilon + \bar{\epsilon}) + \text{ h.c.} \qquad (3.42)
$$

$$
\begin{aligned}
V &= 2\,|a'd|\,|\epsilon_X|^2 + \left|b'\epsilon_X + \left|\frac{a'}{d}\right|^{\frac{1}{2}} e^{\imath\phi}\,(\epsilon + \bar{\epsilon})\right|^2 \\
&+ g_B^2 \left|\frac{a'}{d}\right| (Re(\epsilon - \bar{\epsilon}))^2 \\
&+ \left[\tilde{a}'\epsilon_X + \left|\frac{a'}{d}\right|^{\frac{1}{2}} e^{\imath\phi}\left(\tilde{M}_{\Delta^c} - M_{\Delta^c}\frac{\tilde{d}}{d}\right)(\epsilon + \bar{\epsilon}) + \text{ h.c.}\right] \qquad (3.43)
\end{aligned}
$$

where $\tilde{a}' = \left(\tilde{a} - 2\tilde{b}\frac{M_{\Delta^c}}{d} + 3\tilde{c}\,(\frac{M_{\Delta^c}}{d})^2 + \tilde{d}\left|\frac{a'}{d}\right|^{\frac{1}{2}} e^{\imath\phi}\right)$. Minimizing this potential with respect to ϵ_X and $(\epsilon + \bar{\epsilon})$, i.e, solving for $\frac{\partial V}{\partial \epsilon_X} = \frac{\partial V}{\partial(\epsilon+\bar{\epsilon})} = 0$ we find;

$$
\epsilon_X^* = \epsilon_X^1 + \epsilon_X^2 + \epsilon_X^3 \qquad (3.44)
$$

where upon expressing everything in term of the notation of Eq. (3.36);

$$\epsilon_X^1 = \left(M_S - \frac{Y_S M_{\Delta^c}}{\mu_{\Delta^c}}\right)\left(B_{\Delta^c} - \frac{M_{\Delta^c} A_{\Delta^c}}{\mu_{\Delta^c}}\right) \times \left\{2\left|\mu_{\Delta^c}\right|\left|L_S - \frac{M_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S M^2_{\Delta^c}}{2\mu^2_{\Delta^c}}\right|\right\}^{-1} \quad (3.45)$$

$$\epsilon_X^2 = -\left(C_S - \frac{B_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S M^2_{\Delta^c}}{2\mu^2_{\Delta^c}}\right) \times \left\{2\left|\mu_{\Delta^c}\right|\left|L_S - \frac{M_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S M^2_{\Delta^c}}{2\mu^2_{\Delta^c}}\right|\right\}^{-1} \quad (3.46)$$

$$\epsilon_X^3 = \frac{A_{\Delta^c}}{2\mu_{\Delta^c}\left|\mu_{\Delta^c}\right|} \times exp\left\{\imath\ arg\left(L_S - \frac{M_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S}{2}\frac{M^2_{\Delta^c}}{\mu^2_{\Delta^c}}\right)\right\} \quad (3.47)$$

$\epsilon + \bar{\epsilon}$ is not of interest to this calculation of the $B-$term contribution coming from SUSY breaking; therefore we do not write its solution. It turns out, however, that ϵ_X^* which is a complex quantity, enters the contribution of $F_{\Delta^c}-$term to the r.h. sneutrino $B-$term at υ_R. We therefore compute ϵ_X^* at υ_R from the running of RGEs. We find the $F-$ term for Δ^c to be;

$$\begin{aligned} |F_{\Delta^c}|^2 &= -\frac{f}{2}\tilde{\nu}_R\tilde{\nu}_R\frac{\mu^*_{\Delta^c}\epsilon^*_X}{|\mu_{\Delta^c}|^{\frac{1}{2}}}\left|L_S - \frac{M_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S}{2}\frac{M^2_{\Delta^c}}{\mu^2_{\Delta^c}}\right|^{\frac{1}{2}} \\ &\times\ exp\left\{\imath\ arg\left(-\frac{L_S}{\mu_{\Delta^c}} + \frac{M_S M_{\Delta^c}}{\mu^2_{\Delta^c}} - \frac{Y_S}{2}\frac{M^2_{\Delta^c}}{\mu^3_{\Delta^c}}\right)\right\} \end{aligned} \quad (3.48)$$

and so finally

$$B = \frac{f}{2}\frac{\mu^*_{\Delta^c}\epsilon^*_X}{|\mu_{\Delta^c}|^{\frac{1}{2}}}\left|L_S - \frac{M_S M_{\Delta^c}}{\mu_{\Delta^c}} + \frac{Y_S}{2}\frac{M^2_{\Delta^c}}{\mu^2_{\Delta^c}}\right|^{\frac{1}{2}} e^{\left\{-\imath\ arg\left(-\frac{L_S}{\mu_{\Delta^c}} + \frac{M_S M_{\Delta^c}}{\mu^2_{\Delta^c}} - \frac{Y_S}{2}\frac{M^2_{\Delta^c}}{\mu^3_{\Delta^c}}\right)\right\}} \quad (3.49)$$

The ϵ_X^* parameter appearing in B carries just the right order of the complex phase alluded to earlier as required for the soft leptogenesis. In the next section we show the result of numerical computation of RGEs and the effect of SUSY breaking discussed in this section.

3.7 Numerical result and estimation of BA

In This section, we report the result of our analysis in the table 3.2 and display a particular case in the figure 3.5, where the A-term is found to be between 700 GeV − − 1 TeV. We have taken into account all RGEs that enter the calculation of the $B-$term and $A-$terms for the r.h. sneutrino. Below, we write down some of the RGEs not available in the literature.

•Soft breaking terms below v_R

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} A_l &= A_l \left\{ -\frac{9}{5} g_1^2 - 3g_2^2 + 3\mathrm{Tr}(f_d^\dagger f_d) + \mathrm{Tr}(f_l^\dagger f_l) \right\} \\
&+ 2f_l \left\{ -\frac{9}{5} g_1^2 M_1 - 3g_2^2 M_2 + 3\mathrm{Tr}(f_d^\dagger A_d) + \mathrm{Tr}(f_l^\dagger A_l) \right\} \\
&+ 4(f_l f_l^\dagger A_l) + 5(A_l f_l^\dagger f_l) + 2(f_l f_\nu^\dagger A_\nu) + (A_l f_\nu^\dagger f_\nu), \qquad (3.50) \\
16\pi^2 \frac{d}{dt} A_\nu &= \left[A_\nu \left\{ -\frac{3}{5} g_1^2 - 3g_2^2 + 3\mathrm{Tr}(f_u^\dagger f_u) + \mathrm{Tr}(f_\nu^\dagger f_\nu) \right\} \right. \\
&+ 2f_\nu \left\{ -\frac{3}{5} g_1^2 M_1 - 3g_2^2 M_2 + 3\mathrm{Tr}(f_u^\dagger A_u) + \mathrm{Tr}(f_\nu^\dagger A_\nu) \right\} \\
&\left. + 4(f_\nu f_\nu^\dagger A_\nu) + 5(A_\nu f_\nu^\dagger f_\nu) + 2(f_\nu f_l^\dagger A_l) + (A_\nu f_l^\dagger f_l) \right], \qquad (3.51) \\
16\pi^2 \frac{d}{dt} B_\nu &= \left[2(B_\nu f_\nu^* f_\nu^T) + 2(B_\nu f_\nu f_\nu^\dagger) + 4(M_\nu f_\nu^* A_\nu^T) + 4(A_\nu f_\nu^\dagger M_\nu^T) \right] \qquad (3.52)
\end{aligned}
$$

•The soft parameter corresponding the the linear term in singlet field S

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} C_S &= \left[C_S \left\{ \tfrac{1}{2} Y_S Y_S^* + 3\mu_\Delta \mu_\Delta^* + 3\mu_{\Delta^c} \mu_{\Delta^c}^* + 8\mu_\Phi \mu_\Phi^* \right\} \right. \\
&+ L_S \left\{ 6\mu_\Delta^* A_\Delta + 6\mu_{\Delta^c}^* A_{\Delta^c} + 2Y_S A_S + 16\mu_\Phi^* A_\Phi \right\} \qquad (3.53) \\
&\left. + M_S \left\{ 2(Y_S M_S B_S) + 6(\mu_\Delta^* M_\Delta B_\Delta) + 6(\mu_{\Delta^c}^* M_{\Delta^c} B_{\Delta^c}) + 16(\mu_\Phi^* M_\Phi B_\Phi) \right\} \right]
\end{aligned}
$$

•Yukawa Couplings

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} f_l &= \left[f_l \left\{ -\frac{9}{5} g_1^2 - 3g_2^2 + 3\mathrm{Tr}(f_d f_d^\dagger) + \mathrm{Tr}(f_l f_l^\dagger) \right\} \right. \\
&\left. + 3(f_l f_l^\dagger f_l) + (f_l f_\nu^\dagger f_\nu) \right], \qquad (3.54) \\
16\pi^2 \frac{d}{dt} f_\nu &= \left[f_\nu \left\{ -\frac{3}{5} g_1^2 - 3g_2^2 + 3\mathrm{Tr}(f_u f_u^\dagger) + \mathrm{Tr}(f_\nu f_\nu^\dagger) \right\} \right. \\
&\left. + 3(f_\nu f_\nu^\dagger f_\nu) + (f_\nu f_l^\dagger f_l) \right]. \qquad (3.55)
\end{aligned}
$$

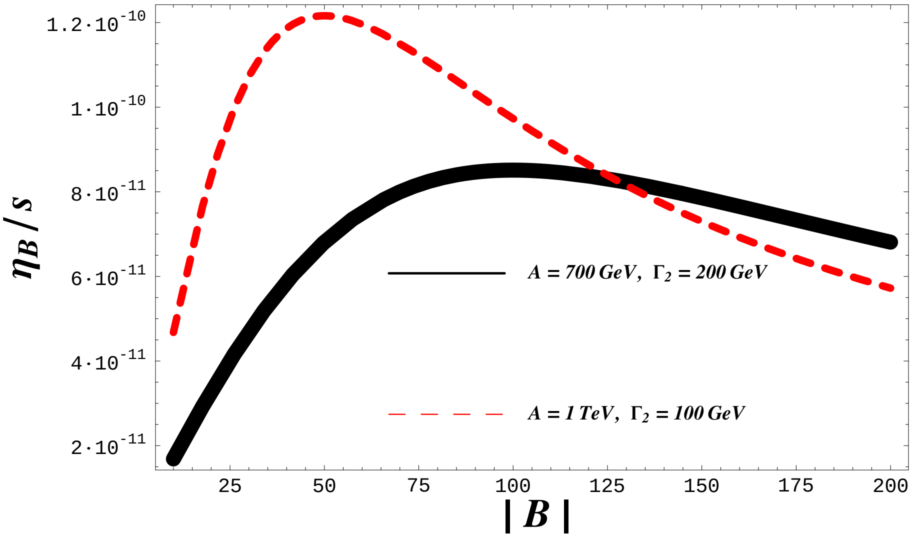

Figure 3.5. Dependence of BA on $B-$term: Two cases are shown above, depending on the choice of $A-$term and Γ_2. In both cases $M_1 = 6.9 \times 10^9$GeV, for which the dilution is enhanced ($d = 1$).

3.8 Conclusion

Soft Leptogenesis is an attractive mechanism to explain the baryon asymmetry. In this paper we have addressed this question in the context of a class of minimal left–right symmetric models. We analyze lepton asymmetry induced in the right handed sneutrino $\tilde{\nu}_{R1} - \tilde{\nu}^{\dagger}_{R1}$ mixing and decay due to soft SUSY breaking parameters in a class of minimal left–right symmetric models (SUSYLR). Successful soft leptogenesis scenario requires small $B-$term and relatively low heavy neutrino mass. We study the effect of full RGE running on the breaking parameters; this combined with the contribution SUSY breaking we compute r.h.n soft parameters and show that Soft Leptogenesis mechanism can indeed be fully implemented in SUSYLR framework leading to adequate baryon number asymmetry in the universe. We also discuss the benefits of working in the context of Left-Right Symmetry.

TABLE 3.2. Result: The left column of the table gives input values of the parameters at the Gut scale, where the right column shows the result of the Soft parameters at v_R following RGE running. The final estimation for the BA is also given.

Input of model at $M_{Gut} = 10^{18}$GeV	Output at $v_R = \frac{M_{w_R}}{g_R} = 6.35\ 10^{10}$GeV
$g_1 = g_2 = g_R = g_L \simeq 0.7$	
$M_1^G = M_2^G = M_L^G = M_R^G = 300$GeV	$\mid B_{ind}^{A_f} \mid = \frac{(A_f)_{11} v_R}{M_1} \sim 50$GeV, $\phi = 0$
$M_\Delta \sim M_{\Delta^c} \sim M_S \sim M_\Phi \simeq v_R$	
$\mu_\Delta = \mu^*_{\Delta^c} \sim \mu_\Phi \sim Y_S \simeq \theta(1)$	$\epsilon^*_X = -211.804 - 355.125\imath$
$(h_L)_{ii} \simeq m_{l_i}/(v \times Cos\beta)$, $(h_L)_{ij,i\neq j} \sim 0$	
$(h_Q)_{33} \simeq m_t/(v \times Sin\beta)$, $(h_Q)_{ij,i,j\neq 3} \sim 0$	$\mid B_{ind}^{F_{\Delta^c}} \mid \sim 100$GeV, $\phi = \theta(1)$
$tan(\beta) = 20$, $f_{ij} = (f_c^*)_{ij} = \frac{M_{R_{ij}}}{v_R}$	
$(A_L)_{ij} = A_0 (h_L)_{ij}$, $(A_Q)_{ij} = A_0 (h_Q)_{ij}$	$M_1 = 6.92 \times 10^9$GeV
$(A_f)_{ij} = A_0 f_{ij}$, $(A_{f^c})_{ij} = A_0 f^c_{ij}$	$d \sim 1$ for $0 \lesssim (\Gamma_2/H) \lesssim 1$
$A_\Delta = A^*_{\Delta^c} = A_0 \mu_\Delta$, $A_\Phi = A_0 \mu_\Phi$, $A_S = A_0 Y_S$	$\Gamma_2 \sim 100$ GeV
$B_\Delta = B_0 M_\Delta$, $B_{\Delta^c} = B_0 M_{\Delta^c}$	$A \sim 1$ TeV
$B_\Phi = B_0 M_\Phi$, $B_S = B_0 M_S$, $L_S \sim v_R^2$, $C_S = C_0 L_S$	
$A_0 \sim (300 - 500)$GeV, $B_0 = C_0 \sim 100$GeV	$n_B/s \simeq 1 \times 10^{-10}$
(universality condition)	
$m_{e,\mu,\tau,t} = \{0.35\ 10^{-3};\ 75.67\ 10^{-3};\ 1.22;\ 82.43\}$GeV	

CHAPTER 4

PREDICTIVE MODEL OF INVERTED NEUTRINO MASS HIERARCHY AND RESONANT LEPTOGENESIS

4.1 Introduction

A lot has been learned about the pattern of neutrino masses and mixings over the past decade from atmospheric [44] and solar [45] neutrino oscillation experiments. When these impressive results are supplemented by results from reactor [46], [47] and accelerator [48] neutrino oscillation experiments, a comprehensive picture for neutrino masses begins to emerge. A global analysis of these results gives rather precise determination of some of the oscillation parameters [49], [50]:

$$|\Delta m^2_{\text{a}tm}| = 2.4 \cdot \left(1^{+0.21}_{-0.26}\right) \times 10^{-3} \text{ e}V^2 , \quad \sin^2\theta_{23} = 0.44 \cdot \left(1^{+0.41}_{-0.22}\right) ,$$
$$\Delta m^2_{\text{S}ol} = 7.92 \cdot (1 \pm 0.09) \times 10^{-5} \text{ e}V^2 , \quad \sin^2\theta_{12} = 0.314 \cdot \left(1^{+0.18}_{-0.15}\right) ,$$
$$\theta_{13} \lesssim 0.2 . \tag{4.1}$$

While these results are impressive, there are still many important unanswered questions. One issue is the sign of $\Delta m^2_{\text{a}tm} = m_3^2 - m_2^2$ which is presently unknown. This is directly linked to nature of neutrino mass hierarchy. A positive sign of $\Delta m^2_{\text{a}tm}$ would indicate normal hierarchy ($m_1 < m_2 < m_3$) while a negative sign would correspond to an inverted mass hierarchy ($m_2 \gtrsim m_1 > m_3$). Another issue is the value of the leptonic mixing angle θ_{13}, which currently is only bounded from above. A third issue is whether CP is violated in neutrino oscillations, which is possible if the phase angle δ in the MNS matrix is nonzero. Forthcoming long baseline experiments [48], NOνA [51], T2K [52] and reactor experiments double CHOOZ and DaiBay will explore some or all these fundamental questions. Answers to these have the potential for revealing the underlying symmetries of nature.

While there exists in the literature a large number of theoretical models for normal neutrino mass hierarchy, such is not the case with inverted hierarchy. A large number of models for inverted hierarchy based on symmetries [53–55] that were proposed a few years ago are now excluded by the solar and Kamland data, which proved that θ_{12} is significantly away from the maximal value of $\pi/4$ predicted by most of these models. As a result, there is a dearth of viable inverted neutrino mass hierarchy models. In this chapter, we attempt to take a step towards remedying this situation.

Here we suggest a class of models for inverted neutrino mass hierarchy based on $S_3 \times U(1)$ symmetry. S_3 is the non-Abelian group generated by the permutation of three objects, while the $U(1)$ is used for explaining the mass hierarchy of the leptons. This $U(1)$ symmetry is naturally identified with the anomalous $U(1)$ of string origin. In our construction, the S_3 permutation symmetry is broken down to an Abelian S_2 in the neutrino sector, whereas it is broken completely in the charged lepton sector. Such a setup enables us to realize effectively a $\nu_\mu \leftrightarrow \nu_\tau$ interchange symmetry in the neutrino sector (desirable for an inverted hierarchical spectrum), while having non-degenerate charged leptons. The $U(1)$ symmetry acts as leptonic $L_e - L_\mu - L_\tau$ symmetry, which is also desirable for an inverted neutrino mass spectrum. The breaking of S_2 symmetry in the charged lepton sector enables us to obtain θ_{12} significantly different from $\pi/4$.

Interestingly, we find that the amount of deviation of θ_{12} from $\frac{\pi}{4}$ is determined by θ_{13} through the relation

$$\sin^2\theta_{12} \simeq \frac{1}{2} - \tan\theta_{13}\cos\delta \ . \tag{4.2}$$

When compared with the neutrino data, the relation (4.2) implies the constraints (see Fig. 1):

$$\theta_{13} \geq 0.13 \ , \qquad 0 \leq \delta \leq 43^o \ . \tag{4.3}$$

At the same time, the model gives

$$\sin^2\theta_{23} \simeq \frac{1}{2}(1 - \tan^2\theta_{13}) \ , \tag{4.4}$$

which is very close to 1/2. These predictions will be tested in forthcoming experiments.

Our models have the right ingredients to generate the observed baryon asymmetry of the universe via resonant leptogenesis. The $U(1)$ symmetry which acts on leptons as $L_e - L_\mu - L_\tau$ symmetry guarantee that two right–handed neutrinos are quasi-degenerate. This feature leads to a resonant enhancement in the leptonic CP asymmetry, which in turn admits low right–handed neutrino masses, as low as few TeV. With such light right-handed neutrinos (RHN) generating lepton asymmetry, there is no cosmological gravitino problem when these models are supersymmetrized.

The class of neutrino mass models and leptogenesis scenario that we present here will work well in both supersymmetric and non-supersymmetric contexts. However, since low energy SUSY has strong phenomenological and theoretical motivations, we stick here to the supersymmetric framework for our explicit constructions.

4.2 Predictive Framework for Neutrino Masses and Mixings

In order to build inverted hierarchical neutrino mass matrices which are predictive and which lead to successful neutrino oscillations, it is enough to introduce two right–handed neutrino states $N_{1,2}$. Then the superpotential relevant for neutrino masses is

$$W_\nu = l^T Y_\nu N h_u - \frac{1}{2} N^T M_N N \ , \tag{4.5}$$

where h_u denotes the up–type Higgs doublet superfield, while Y_ν and M_N are 3×2 Dirac Yukawa matrix and 2×2 Majorana mass matrix respectively. Their structures can be completely determined by flavor symmetries. In order to have predictive models of inverted hierarchy, the $L_e - L_\mu - L_\tau \equiv \mathbf{L}$ symmetry can be used [53–56]. This symmetry naturally gives rise to large θ_{23} and maximal θ_{12} angles. At the same time, the mixing angle θ_{13} will be zero. In order to accommodate the correct solar neutrino mixing angle, the **L**-symmetry must be broken. The pattern of **L**-symmetry breaking will determine the relations and predictions for neutrino masses and mixings. As a starting point, in the neutrino sector let us impose $\mu - \tau$ symmetry S_2: $l_2 \to l_3, l_3 \to l_2$, which will lead to maximal $\nu_\mu - \nu_\tau$ mixing, consistent with atmospheric neutrino data.

The leptonic mixing angles receive contributions from both the neutrino sector and the charged lepton sector. As an initial attempt let us assume that the charged lepton mass matrix is diagonal. We will elaborate on altering this assumption in the next subsection.

For completeness, we will start with general couplings respecting the S_2 symmetry. Therefore, we have

$$Y_\nu = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \!\!\begin{array}{c} \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{pmatrix} \alpha & 0 \\ \beta' & \beta \\ \beta' & \beta \end{pmatrix} \end{array} , \qquad M_N = \begin{array}{c} \\ N_1 \\ N_2 \end{array} \!\!\begin{array}{c} \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{pmatrix} -\delta_N & 1 \\ 1 & -\delta_N^{'} \end{pmatrix} \end{array} M \ . \tag{4.6}$$

Note that setting $(1,2)$ element of Y_ν to zero can be done without loss of generality. This can be achieved by proper redefinition of $N_{1,2}$ states. The couplings α, β and $(1,2), (2,1)$ entries in M_N respect $\mathbf{L}$ symmetry, while the couplings β', δ_N and $\delta_N^{'}$ violate it. Therefore, it is natural to expect that $|\beta'| \ll |\alpha|, |\beta|$, $|\delta_N|, |\delta_N^{'}| \ll 1$. Furthermore, by proper field redefinitions all couplings in Y_ν can be taken to be real. Upon these redefinitions δ_N and $\delta_N^{'}$ entries in M_N will be complex.

Integration of the heavy $N_{1,2}$ states leads to the following 3×3 light neutrino mass matrix:

$$m_\nu = \begin{pmatrix} 2\delta_\nu^{'} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & \delta_\nu & \delta_\nu \\ \sqrt{2} & \delta_\nu & \delta_\nu \end{pmatrix} \tfrac{m}{2} \ , \tag{4.7}$$

where

$$m = \frac{\langle h_u^0 \rangle^2}{M(1 - \delta_N \delta_N^{'})} \sqrt{2}\alpha \left(\beta + \beta' \delta_N^{'} \right) \ ,$$

$$\delta_\nu = \frac{\sqrt{2}}{\alpha} \frac{2\beta\beta' + \beta^2 \delta_N + (\beta')^2 \delta_N^{'}}{\beta + \beta' \delta_N^{'}} \ , \qquad \delta_\nu^{'} = \frac{\alpha}{\sqrt{2}} \frac{\delta_N^{'}}{\beta + \beta' \delta_N^{'}} \ . \tag{4.8}$$

The entries δ_ν, $\delta_\nu^{'}$ in (4.7) are proportional to the $\mathbf{L}$-symmetry breaking couplings and therefore one naturally expects $|\delta_\nu|, |\delta_\nu^{'}| \ll 1$. These small entries are responsible for $\Delta m^2_{\mathrm{Sol}} \neq 0$, i.e. for the solar neutrino oscillation. The neutrino mass matrix is diagonalized by unitary transformation $U_\nu^T m_\nu U_\nu = \mathrm{D}iag\,(m_1, m_2, 0)$, were $U_\nu = U_{23}U_{12}$ with

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \qquad U_{12} \simeq \begin{pmatrix} \bar{c} & -\bar{s}e^{\mathrm{i}\rho} & 0 \\ \bar{s}e^{-\mathrm{i}\rho} & \bar{c} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{4.9}$$

where $\bar{c} = \cos\bar{\theta}$, $\bar{s} = \sin\bar{\theta}$ and

$$\tan\bar{\theta} \simeq 1 \pm \frac{1}{2}\kappa \ , \qquad \kappa = \frac{|\delta_\nu|^2 - |\delta_\nu^{'}|^2}{|\delta_\nu^* + \delta_\nu^{'}|} \ . \tag{4.10}$$

The phase ρ is determined from the equation

$$|\delta_\nu|\sin(\omega_\nu - \rho) = |\delta_\nu^{'}|\sin(\omega_\nu^{'} + \rho) \ , \quad \omega_\nu = \mathrm{A}rg(\delta_\nu) \ , \quad \omega_\nu^{'} = \mathrm{A}rg(\delta_\nu^{'}) \ , \tag{4.11}$$

and should be taken such that

$$|\delta_\nu|\cos(\omega_\nu - \rho) + |\delta_\nu^{'}|\cos(\omega_\nu^{'} + \rho) < 0 \ . \tag{4.12}$$

This condition ensures $\Delta m^2_{\mathrm{S}ol} = m_2^2 - m_1^2 > 0$ needed for solar neutrino oscillations. For $\Delta m^2_{\mathrm{a}tm}$ and the ratio $\Delta m^2_{\mathrm{S}ol}/|\Delta m^2_{\mathrm{a}tm}|$ we get

$$|\Delta m^2_{\mathrm{a}tm}| \simeq |m|^2 \ , \quad \frac{\Delta m^2_{\mathrm{S}ol}}{|\Delta m^2_{\mathrm{a}tm}|} \simeq -2\left(|\delta_\nu|\cos(\omega_\nu - \rho) + |\delta_\nu^{'}|\cos(\omega_\nu^{'} + \rho)\right) = 2\left|\delta_\nu^* + \delta_\nu^{'}\right| \ . \tag{4.13}$$

With no contribution from the charged lepton sector, the leptonic mixing matrix is U_ν. From (4.9), (4.10) for the solar mixing angle we will have $\sin^2\theta_{12} = \frac{1}{2} \pm \frac{\kappa}{4}$. In order to be compatible with experimental data one needs $\kappa \approx 0.7$. On the other hand with $|\delta_\nu| \sim |\delta_\nu^{'}|$ and no specific phase alignment from (4.13) we estimate $|\delta_\nu| \sim |\delta_\nu^{'}| \sim 10^{-2}$. Thus we get the expected value $\kappa \sim 10^{-2}$, but with the θ_{12} mixing angle nearly maximal, which is incompatible with experiments. This picture remains unchanged with the inclusion of renormalization group effects. Therefore, we learn that it is hard to accommodate the neutrino data in simple minded inverted hierarchical neutrino mass scenario. In order for the scenario be compatible with the experimental data we need simultaneously

$$\left|\delta_\nu^* + \delta_\nu^{'}\right| = \frac{\Delta m^2_{\mathrm{S}ol}}{2|\Delta m^2_{\mathrm{a}tm}|} \simeq 0.016 \ , \qquad \frac{|\delta_\nu|^2 - |\delta_\nu^{'}|^2}{|\delta_\nu^* + \delta_\nu^{'}|} = \mp(0.52 - 0.92) \ . \tag{4.14}$$

Therefore, one combination of δ_ν and $\delta_\nu^{'}$ must be $\sim$ 50-times larger than the other. This is indeed unnatural and no explanation for these conditions is provided at this stage. To make this point more clear let's consider the case with $\delta_\nu = 0$*. In this case from (4.13) we have $|\delta_\nu^{'}| \simeq 0.016$. Using this in (4.10) we obtain $\sin^2\theta_{12} \geq 0.496$, which is excluded by the neutrino data (4.1).

Summarizing, although the conditions in (4.14) can be satisfied, it remains a challenge to have a natural explanation of these hierarchies. This is a shortcoming of the scenario. Below we present a possible solution to this conundrum which looks attractive and maintains predictive power without fine tuning.

4.2.1 Improved θ_{12} with $\theta_{13} \neq 0$

Let us now include the charged lepton sector in our studies. The relevant superpotential is

$$W_e = l^T Y_E e^c h_d \ , \tag{4.15}$$

where Y_E is 3×3 matrix in the family space. In general, Y_E has off–diagonal entries. Being so, Y_E will induce contributions to the leptonic mixing matrix. We will use this contribution in order to fix the value of θ_{12} mixing angle. It is desirable to do this in such a way that some predictivity is maintained. As it turns out, the texture

$$Y_E = \begin{pmatrix} 0 & a' & 0 \\ a & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} , \tag{4.16}$$

gives interesting predictions. In the structure (4.16) there is only one unremovable complex phase and we leave it in (1,2) entry. Thus, we make the parametrization $a' = \lambda_\mu \theta_e e^{\mathrm{i}\omega}$, while all the remaining entries can be taken to be real. In order to get the correct value of the electron mass for $\theta_e \ll 1$, we should take the coupling $a = \lambda_e/\theta_e$. For finding the unitary matrix which rotates the left–handed charged

*This case is realized within the model with $S_3 \times U(1)$ flavor symmetry presented in section 4.4.

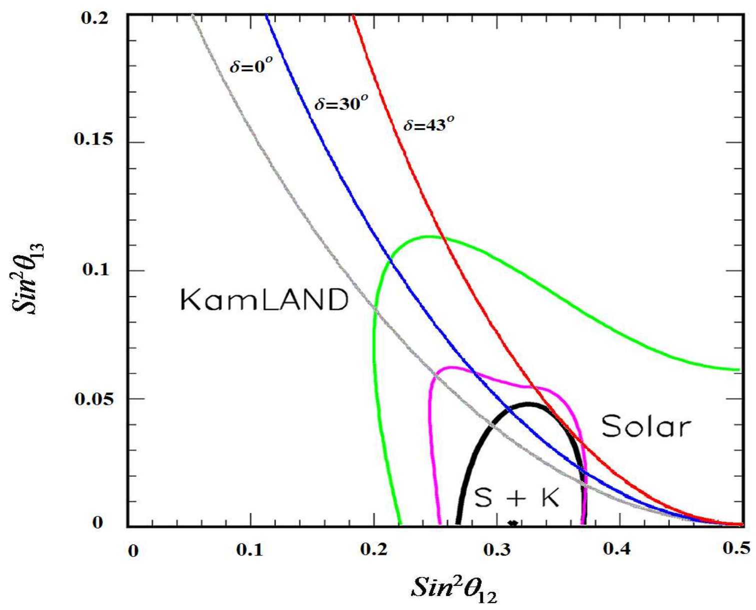

Figure 4.1. Correlation between θ_{12} and θ_{13} taken from Fogli *et al.* Three sloped curves correspond to $\theta_{12} - \theta_{13}$ dependance (for three different values of CP phase δ) obtained from our model according to Eq. (4.22)

lepton states, upon diagonalization of Y_E, we need to diagonalize the product $Y_E Y_E^\dagger$. Namely, with $U_e Y_E Y_E^\dagger U_e^\dagger = \left(Y_E^{\mathrm{d}iag}\right)^2$, it is easy to see that

$$U_e = \begin{pmatrix} c & se^{\mathrm{i}\omega} & 0 \\ -se^{-\mathrm{i}\omega} & c & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{4.17}$$

where $c \equiv \cos t$, $s \equiv \sin t$ and $\tan t = -\theta_e$. Finally, the leptonic mixing matrix takes the form

$$U^l = U_e^* U_\nu \ , \tag{4.18}$$

where $U_\nu = U_{23} U_{12}$ can be derived from Eq. (4.9). Therefore, for the corresponding mixing elements we get

$$U_{e3}^l = -\frac{s}{\sqrt{2}} e^{-\mathrm{i}(\omega+\rho)} \ , \quad |U_{e2}^l| = \frac{1}{\sqrt{2}} \left| c - \frac{s}{\sqrt{2}} e^{-\mathrm{i}(\omega+\rho)} \right| \ , \quad |U_{\mu 3}^l| = \frac{c}{\sqrt{2}} \ . \tag{4.19}$$

Comparing these with those written in the standard parametrization we obtain the relations

$$s_{13} = -\frac{s}{\sqrt{2}} \ , \qquad \omega + \rho = \delta + \pi \ , \tag{4.20}$$

$$s_{12}c_{13} = |U^l_{e2}| \ , \quad s_{23}c_{13} = |U^l_{\mu 3}| \ . \tag{4.21}$$

Using (4.20) and (4.19) in (4.21) leads to the prediction:

$$\sin^2\theta_{12} = \frac{1}{2} - \sqrt{1-\tan^2\theta_{13}}\,\tan\theta_{13}\cos\delta \ ,$$

$$\sin^2\theta_{23} = \frac{1}{2}\left(1-\tan^2\theta_{13}\right) \ . \tag{4.22}$$

Since the CHOOZ bound is $s_{13} \lesssim 0.2$, the first relation in (4.22), with the help of the solar neutrino data provides an upper bound for the CP violating phase: $\delta \lesssim \delta_{\mathrm{m}ax} \approx 48^o$. However, this estimate ignores the dependence of θ_{12} on the value of θ_{13} in the neutrino oscillation data. Having $\theta_{13} \neq 0$, this dependence shows up because one deals with three flavor oscillations. This has been analyzed in Ref. [50] and is shown in and Fig. 1 (borrowed from Ref. [50]) along with the constraints arising from our model. We have shown three curves corresponding to (4.22) for different values of δ. Now we see that maximal allowed value for δ is $\delta_{\mathrm{m}ax} \simeq 43^o$. Moreover, for a given δ we predict the allowed range for θ_{13}. In all cases the values are such that these relations can be tested in the near future. An interesting result from our scenario is that we obtain lower and upper bounds for θ_{13} and δ respectively

$$\theta_{13} \geq 0.13 \ , \qquad 0 \leq \delta \leq 43^o \ . \tag{4.23}$$

Finally, the neutrino-less double β-decay parameter in this scenario is given by

$$m_{\beta\beta} \simeq 2\sqrt{\Delta m^2_{\mathrm{a}tm}}\,\tan\theta_{13}\frac{\sqrt{1-\tan^2\theta_{13}}}{\sqrt{1+\tan^2\theta_{13}}} \ . \tag{4.24}$$

We have neglected the small contribution (of order $\Delta m^2_{\mathrm{S}olar}/\Delta m^2_{\mathrm{a}tm}$) arising from the neutrino mass matrix diagonalization. Since the value of θ_{13} is experimentally constrained ($\lesssim$ 0.2), to a good approximation we have $m_{\beta\beta} \approx 2\sqrt{\Delta m^2_{\mathrm{a}tm}}\tan\theta_{13}$. Using this and the atmospheric neutrino data (4.1) we find $m_{\beta\beta} \lesssim 0.02$ eV. Knowledge of θ_{13}-dependence on δ (see Fig. 1) allows us to make more accurate estimates for the

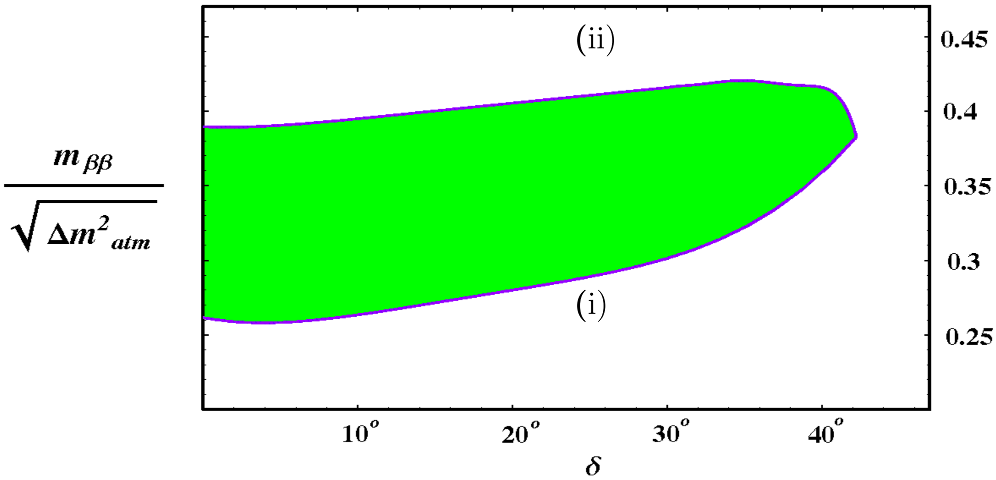

Figure 4.2. Curves (**i**) and (**ii**) respectively show the dependence of $\frac{m_{\beta\beta}}{\sqrt{\Delta m^2_{atm}}}$'s low and upper bounds on CP violating phase δ. The shaded region corresponds to values of $m_{\beta\beta}$ and δ realized within our model.

range of $m_{\beta\beta}$ for each given value of δ. The dependence of $m_{\beta\beta}$ on δ is given in Fig. 2. We have produced this graph with the predictive relations (4.22), (4.24) using the neutrino data [50]. Combining these results we arrive at

$$0.011 \text{ e}V \lesssim m_{\beta\beta} \lesssim 0.022 \text{ e}V. \tag{4.25}$$

As we see the predicted range, depending on the value of δ, is quite narrow. Future measurements of CP violating phase δ together with a discovery of the neutrino-less double β-decay will be another test for the inverted hierarchical scenario presented here.

4.3 Resonant Leptogenesis

Neutrino mass models with heavy right–handed neutrinos provide an attractive and natural framework for explaining the observed baryon asymmetry of the universe through thermal leptogenesis [57]. This mechanism takes advantage of the out-of-equilibrium decay of lightest right-handed neutrino(s) into leptons and the Higgs boson. In the scenario with hierarchical RHNs, a lower bound on the mass of decaying RHN has been derived: $M_{N_1} \geq 10^9$ GeV [58]. The reheating temperature can not be much below the mass of N_1. In low energy SUSY models (with $m_{3/2} \sim 1$ TeV) this is in conflict with the upper bound on reheating temperature obtained from the gravitino problem [59]. This conflict can be naturally avoided in the scenario of 'resonant leptogenesis' [5], [60]. Due to the quasi-degeneracy in mass of the RHN states, the needed CP asymmetry can be generated even if the right–handed neutrino mass is lower than 10^9 GeV.

Our model of inverted hierarchical neutrinos involves two quasi-degenerate RHN states and has all the needed ingredients for successful resonant leptogenesis. This makes the scenario attractive from a cosmological viewpoint as well. Now we present a detailed study of the resonant leptogenesis phenomenon in our scenario.

The CP asymmetry is created by resonant out of equilibrium decays of N_1, N_2 and is given by [60]

$$\epsilon_1 = \frac{\mathrm{I}m(\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{21}}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(M_2^2 - M_1^2)\, M_1 \Gamma_2}{(M_2^2 - M_1^2)^2 + M_1^2 \Gamma_2^2} \ , \tag{4.26}$$

with a similar expression for ϵ_2. The asymmetries ϵ_1 and ϵ_2 correspond to the decays of N_1 and N_2 respectively. Here M_1, M_2 are mass the eigenvalues of the matrix M_N in (4.6), while $\hat{Y}_\nu = Y_\nu U_N$ is the Dirac Yukawa matrix in a basis where RHN mass matrix is diagonal. The tree–level decay width of N_i is given as $\Gamma_i = (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii} M_i/(8\pi)$. The expression (4.26) deals with the regime $M_2 - M_1 \sim \Gamma_{1,2}/2$ (relevant for our studies) consistently and has the correct behavior in the limit $M_1 \to M_2$ [60]. From (4.6) we have

$$U_N^T M_N U_N = \mathrm{D}iag\,(M_1, M_2) \ , \qquad U_N \simeq \tfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\,e^{\mathrm{i}r} \\ e^{-\mathrm{i}r} & 1 \end{pmatrix} , \tag{4.27}$$

with

$$M_2^2 - M_1^2 = 2M^2 \left| \delta_N^* + \delta_N^{'} \right| \ , \qquad \tan r = \frac{\mathrm{I}m\left(\delta_N - \delta_N^{'}\right)}{\mathrm{R}e\left(\delta_N + \delta_N^{'}\right)} \ . \tag{4.28}$$

Introducing the notations

$$\frac{\alpha}{\beta} = x \ , \qquad \frac{\beta'}{\beta} = x' \ , \tag{4.29}$$

we can write down the appropriate matrix elements needed for the calculation of leptonic asymmetry:

$$\begin{aligned}
(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} &= \frac{1}{2}\beta^2 \left(2 + x^2 + 2(x')^2 + 4xx' \cos r\right) \ , \\
(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} &= \frac{1}{2}\beta^2 \left(2 + x^2 + 2(x')^2 - 4xx' \cos r\right) \ , \\
\mathrm{I}m(\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{21} &= -\frac{1}{4}\beta^4 \left(2 - x^2 - 2(x')^2 + 4xx' \cos r\right)^2 \sin 2r \ .
\end{aligned} \tag{4.30}$$

In terms of these entries the CP asymmetries are give by

$$\epsilon_1 = \frac{\mathrm{I}m(\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{21}}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}} \frac{|\delta_N^* + \delta_N^{'}|}{16\pi |\delta_N^* + \delta_N^{'}|^2 + (\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{22}/(16\pi)} \ , \qquad \epsilon_2 = -\epsilon_1 (1 \leftrightarrow 2) \ . \tag{4.31}$$

We have five independent parameters and in general one should evaluate the lepton asymmetry as a function of $x, x', |\delta_N|, |\delta_N'|$ and r. Below we will demonstrate that resonant decays of $N_{1,2}$ can generate the needed CP asymmetry.

It turns out that for our purposes we will need $|\delta_N^* + \delta_N'| \ll 1$. This, barring precise cancelation, implies $|\delta_N|, |\delta_N'| \ll 1$. From the symmetry viewpoint and also from further studies, it turns out that $\left|\frac{x'}{x}\right| \ll 1$ is a self consistent condition. Taking these and the results from the neutrino sector, to a good approximation we have

$$\beta^2 = \frac{\sqrt{\Delta m^2_{\mathrm{a}tm}}M}{\sqrt{2}x\langle h_u^0\rangle^2} \ , \qquad \mathrm{a}tm^2| \simeq 6 \cdot 10^{-3} \tag{4.32}$$

and

$$\epsilon_1 \simeq \epsilon_2 \simeq \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}} \frac{|\delta_N^* + \delta_N'|}{16\pi|\delta_N^* + \delta_N'|^2 + (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}^2/(16\pi)} \simeq$$
$$-\frac{(2-x^2)^2}{2(2+x^2)}\beta^2 \frac{|\delta_N^* + \delta_N'|}{16\pi|\delta_N^* + \delta_N'|^2 + (2+x^2)^2\beta^4/(64\pi)} \sin 2r \ , \tag{4.33}$$

where in the last expression we have ignored x' contributions. This approximation is good for all practical purposes. The combination $|\delta_N^* + \delta_N'|$ is a free parameter and since we are looking for a resonant regime, let us maximize the expression in (4.33) with respect to this variable. The maximum CP asymmetry is achieved with $|\delta_N^* + \delta_N'| = (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}/(16\pi)$. Plugging this value back in (4.33) and taking into account (4.30), (4.32) we arrive at

$$\bar{\epsilon}_1 \simeq \bar{\epsilon}_2 \simeq -\frac{(2-x^2)^2}{2(2+x^2)^2} \sin 2r \ , \tag{4.34}$$

where $\bar{\epsilon}_{1,2}$ indicate the maximized expressions, which do not depend on the scale of right–handed neutrinos. We can take these masses as low as TeV! The expression in (4.34) reaches the maximal values for $x \ll 1$ and $x \gg 1$. However, the final value of x will be fixed from the observed baryon asymmetry. The lepton asymmetry is converted to the baryon asymmetry via sphaleron effects [61] and is given by $\frac{n_B}{s} \simeq -1.48 \cdot 10^{-3}(\kappa_f^{(1)}\epsilon_1 + \kappa_f^{(2)}\epsilon_2)$, where $\kappa_f^{(1,2)}$ are efficiency factors given approximately by [62]

$$\kappa_f^{(1,2)} = \left(\frac{3.3 \cdot 10^{-3}\ \mathrm{eV}}{\tilde{m}_{1,2}} + \left(\frac{\tilde{m}_{1,2}}{0.55 \cdot 10^{-3}\ \mathrm{eV}} \right)^{1.16} \right)^{-1} \ ,$$

$$\text{with} \quad \tilde{m}_1 = \frac{\langle h_u^0 \rangle^2}{M_1} (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} \ , \quad \tilde{m}_2 = \frac{\langle h_u^0 \rangle^2}{M_2} (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} \ . \tag{4.35}$$

In our model, with $\left|\frac{x'}{x}\right| \ll 1$ we have

$$\tilde{m}_1 \simeq \tilde{m}_2 \simeq \frac{\sqrt{\Delta m^2_{\text{atm}}}}{2\sqrt{2}x}(2+x^2) \simeq 0.017 \text{ eV} \times \frac{2+x^2}{x} \ . \tag{4.36}$$

This also gives $\kappa_f^{(1)} \simeq \kappa_f^{(2)} \equiv \kappa_f$ and as a result we obtain

$$\frac{n_B}{s}\Big|_{\epsilon=\bar{\epsilon}} \simeq 1.48 \cdot 10^{-3} \kappa_f(x) \frac{(2-x^2)^2}{(2+x^2)^2} \sin 2r \ . \tag{4.37}$$

With $\sin 2r = 1$ in order to reproduce the experimentally observed value $\left(\frac{n_B}{s}\right)^{\text{exp}} = 9 \cdot 10^{-11}$ we need to take $x = 3.8 \cdot 10^{-5}$, $x = 5.3 \cdot 10^4$, $x = \sqrt{2} - 0.0047$ or $x = \sqrt{2} + 0.0047$. For these values of x we have respectively

$$\left|\delta_N^* + \delta_N^{'}\right|_{\epsilon=\bar{\epsilon}} \simeq \frac{2+x^2}{32\sqrt{2}\pi x} \frac{\sqrt{\Delta m^2_{\text{atm}}} M}{\langle h_u^0 \rangle^2} \simeq$$

$$\left(6 \cdot 10^{-7} \ , \ 6 \cdot 10^{-7} \ , \ 3.2 \cdot 10^{-11} \ , \ 3.2 \cdot 10^{-11}\right) \times \frac{1+\tan^2\beta}{\tan^2\beta} \frac{M}{10^6 \text{GeV}} \tag{4.38}$$

(fixed from the condition of maximization). The MSSM parameter $\tan\beta$ should not be confused with Yukawa coupling in (4.32)). Note that these results are obtained at the resonant regime $|M_2 - M_1| = \Gamma_{1,2}/2$. If we are away from this point, then the baryon asymmetry will be more suppressed and we will need to take different values of x. In Fig. 4.3 we show $\left|\delta_N^* + \delta_N^{'}\right| - x$ dependence corresponding to baryon asymmetry of $9 \cdot 10^{-11}$. The curves are constructed with Eqs. (4.32), (4.33). We display different cases for different values of the mass M and for two values of CP violating phase r. For smaller values of r the 'ovals' shrink indicating that there is less room in $\left|\delta_N^* + \delta_N^{'}\right| - x$ plane for generating the needed baryon asymmetry. We have limited ourselves to $\left|\delta_N^* + \delta_N^{'}\right| \lesssim 0.1$. Above this value the degeneracy disappears and the validity of our expression (4.26) breaks down*. Also, in this regime the inverted mass hierarchical neutrino scenario becomes unnatural. The dashed horizontal line in Fig. 4.3 corresponds to this 'cut–off'. This limits cases with larger masses [case (d) in Fig. 4.3, of $M = 10^{11}$ GeV]. The sloped dashed cut–off lines appear due to

*There will be another contributions to the CP asymmetry, the vertex diagram, which would be significant in the non-resonant case.

the requirement that the Yukawa couplings be perturbative ($\alpha, \beta \lesssim 1$). As one can see from (4.32), for sufficiently large values of M, with $x \gg 1$ or $x \ll 1$, one of the Yukawa couplings becomes non-perturbative.

As we see, in some cases (especially for suppressed values of r) the degeneracy in mass between N_1 and N_2 states is required to be very accurate, i.e. $\left|\delta_N^* + \delta_N'\right| \ll 1$. In section 4.4 we discuss the possibility for explaining this based on symmetries.

4.4 Model with $S_3 \times U(1)$ Symmetry

In this section we present a concrete model which generates the needed textures for the charged lepton and the neutrino mass matrices. It also blends well with the leptogenesis scenario investigated in the previous section. We wish to have an understanding of the appropriate hierarchies and the needed zero entries in the Dirac and Majorana neutrino couplings. Also, the values of masses $M_{N_{1,2}} \simeq M \lesssim 10^8$ GeV and their tiny splitting must be explained. Note that one can replace $\mathbf{L} = L_e - L_\mu - L_\tau$ symmetry by other symmetry, which will give approximate $\mathbf{L}$. For this purpose the anomalous $U(1)$ symmetry of string origin is a good candidate [63]. However, in our scenario the charged lepton sector also plays an important role. In particular, the structure (4.16) is crucial for the predictions presented above. We wish to understand this structure also by symmetry principles. For this a non Abelian discrete flavor symmetries can be very useful [64], [65], [66]. Therefore, in addition, we introduce S_3 permutation symmetry. The S_3 will be broken by two steps $S_3 \to S_2 \to 1$. Since in the neutrino sector we were using S_2 symmetry, we will arrange for that sector to feel only the first stage of breaking, i.e. S_2 will be unbroken in the neutral lepton sector.

Thus, the model we present here is based on $S_3 \times U(1)$ flavor symmetry. The S_3 permutation group has three irreducible representations $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$, where $\mathbf{1}'$ is an odd singlet while $\mathbf{1}$ and $\mathbf{2}$ are true singlet and doublet respectively. With doublets denoted by two component vectors, it is useful to give the product rule

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1y_1 + x_2y_2)_{\mathbf{1}} \;\oplus\; (x_1y_2 - x_2y_1)_{\mathbf{1}'} \;\oplus\; \begin{pmatrix} x_1y_2 + x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix}_{\mathbf{2}} \tag{4.39}$$

where subscripts denote the representation of the corresponding combination. The other products are very simple. For instance $\mathbf{1} \times \mathbf{1} = \mathbf{1}$, $\mathbf{1}' \times \mathbf{1} = \mathbf{1}'$, etc.

As far as the $U(1)$ symmetry is concerned, a superfield ϕ_i transforms as

$$U(1): \quad \phi_i \to e^{\mathrm{i}Q_i}\phi_i \ , \tag{4.40}$$

where Q_i is the $U(1)$ charge of ϕ_i. The $U(1)$ symmetry will turn out to be anomalous. The anomalous $U(1)$ factors can appear in effective field theories from string theory upon compactification to four dimensions. The apparent anomaly in this $U(1)$ is canceled through the Green-Schwarz mechanism [67]. Due to the anomaly, a Fayet-Iliopoulos term $-\xi \int d^4\theta V_A$ is always generated [68] and the corresponding D_A-term has the form [69]

$$\frac{g_A^2}{8}D_A^2 = \frac{g_A^2}{8}\left(-\xi + \sum Q_i|\phi_i|^2\right)^2 \ , \quad \xi = \frac{g_A^2 M_P^2}{192\pi^2}\mathrm{Tr}Q \ . \tag{4.41}$$

In SUSY limit one of the VEVs should set D_A-term to be zero.

For $S_3 \times U(1)$ breaking we introduce the MSSM singlet scalar superfields $\vec{S}, \vec{T}, X$, where vector symbols will denote S_3 doublets. The transformation properties - the S_3 'membership' and $U(1)$ charges - of these and other fields are given in Table 4.1. In the table we do not display MSSM pair of higgs doublet superfields h_u, h_d, noting that they are invariant under $S_3 \times U(1)$.

Further we will use the following VEV configuration:

$$\langle \vec{S} \rangle = (0,\ V) \ , \quad \langle \vec{T} \rangle = \tilde{V} \cdot (1,\ \mathrm{i}) \ , \quad \langle X \rangle = V_X \ . \tag{4.42}$$

These structures can be obtained in verious simple ways. With $\xi, Q_X < 0$, in Eq. (4.41) the VEV of the scalar component of X is fixed as $V_X = \sqrt{\xi/Q_X}$. The direction for $\vec{T}$ can be obtained from its bi-linear coupling with some neutral singlet Y [65]. Namely with superpotential coupling $Y\vec{T}^2$, the F-flatness condition gives the solution

TABLE 4.1. Transformation properties under $S_3 \times U(1)$.

	$\vec{S}$	$\vec{T}$	X	e_1^c	$\vec{e}^{\,c}$	l_1	$\vec{l}$	N_1	N_2
S_3	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)$	0	0	-1	$4-n$	$-n$	$n+2$	n	$-(n{+}1)$	$2m{-}(n+1)$

in (4.42) and $\langle Y\rangle = 0$. Similarly with couplings $Y'\left(\vec{S}^2 - V^2\right)$ we get VEV solution for $\vec{S}$ given in (4.42) and $\langle Y'\rangle = 0$. We just mentioned this simple minded examples in order to demonstrate that desirable VEVs can be obtained self-consistently (of course many other possibilities can be discussed).

Further we will use the following parametrization

$$\frac{V_X}{M_{\mathrm{P}_l}} \sim \frac{V}{M_{\mathrm{P}_l}} \equiv \epsilon \ . \tag{4.43}$$

All non renormalizable operators that we consider below will be cut off by appropriate powers of the Planck scale M_{P_l} and therefore in those operators the powers of ϵ will appear. The operators cut off with a different scale cut off can be obtained by integration of some vector like states and should be discussed separately.

Let is start with charged fermion sector. We will use the following operators:

$$\frac{1}{M_*^2}\left(\vec{l}\cdot\vec{S}\right)_{\mathbf{1}}\left(\vec{e}^{\,c}\cdot\vec{S}\right)_{\mathbf{1}} h_d + \frac{1}{M_{\mathrm{P}_l}^2}\vec{l}\cdot\vec{e}^{\,c}\cdot\vec{S}^2 h_d + \frac{X^2}{M_{\mathrm{P}_l}^3} l_1 \vec{e}^{\,c}\cdot\vec{S} h_d + \frac{X^4}{M_{\mathrm{P}_l}^3} e_1^c \vec{l}\cdot\vec{S} h_d + \frac{X^6}{M_{\mathrm{P}_l}^6} l_1 e_1 h_d \ , \tag{4.44}$$

where in the first operator the singlet $\mathbf{1}$-channel is indicated. This is crucial for our construction. Also, it is important that in first two terms $\vec{S}$ appears quadratically and not linearly. This can be easily insured by the reflection symmetry: $\vec{S} \to -\vec{S}, e_1^c \to -e_1^c, l_1 \to -l_1, X \to -X$ (this will be compatible also with the neutrino sector). With this, all couplings are invariant. Moreover, the first operator is not cut off by the Planck scale. This needs some justification. In Fig. 4.4 is shown one possibility how this coupling can be obtained. Indeed with L, E^c states in $\mathbf{1}$ representation of S_3 and masses$\sim M_*$ we get first coupling of (4.44).

Substituting appropriate VEVs in (4.44) and taking into account that $\vec{l} = (l_2,\ l_3)$, $\vec{e}^{\ c} = (e_2^c,\ e_3^c)$, for the charged lepton mass matrix we obtain

$$\begin{array}{c c} & \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} & \begin{pmatrix} \epsilon^6 & \epsilon^3 & 0 \\ \epsilon^4 & \epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} , \tag{4.45}$$

which nearly has the structure of (4.16). Only difference is the (1,1) entry which does not change any of our analysis. From (4.45) we get $\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^6 : \epsilon^2 : 1$, which is compatible with the observed hierarchies for $\epsilon \sim 0.2$. It is remarkable that with this value we get the (1,2) mixing $s \sim \epsilon \sim 0.2$ needed for accommodating neutrino data and have robust predictions discussed in sect. 4.2.1.

Now we turn to the neutrino sector. With transformation properties given in Table 4.1, and for

$$m, n: \ Integer \quad and \quad m > 0\ , \quad m > n + 1\ , \tag{4.46}$$

the Yukawa couplings have the form

$$\begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} l_1 \\ \vec{l} \end{array} & \begin{pmatrix} \epsilon & \epsilon^{2m+1} \\ 0 & \frac{\vec{T}}{M'}\epsilon^{2m-1} \end{pmatrix} \end{array} h_u\ , \qquad \begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} N_1 \\ N_2 \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & \epsilon^{2m} \end{pmatrix} \end{array} \epsilon^{2(m-n-1)} M_R\ . \tag{4.47}$$

Note that since ϵ is coming from the VEV of X -the odd S_3 singlet, the product $\vec{l}\cdot\vec{T}$ in (4.47) should be taken in $\mathbf{1}'$ channel. Using this fact and the VEV configuration for $\vec{T}$ given in (4.42) for the Dirac Yukawa matrix we obtain

$$Y_\nu = \begin{pmatrix} \epsilon & \epsilon^{2m+1} \\ 0 & \tilde{\epsilon}\epsilon^{2m-1} \\ 0 & -\,\mathrm{i}\tilde{\epsilon}\epsilon^{2m-1} \end{pmatrix} , \tag{4.48}$$

where $\tilde{\epsilon} \sim \tilde{V}/M'$. Making proper rotation of $N_{1,2}$ states to set (1,2) entry of matrix (4.48) to zero and at the same time performing phase redefinitions we will arrive to the form of (4.6) with

$$M = M_R \epsilon^{2(m-n-1)} \ , \qquad \alpha \sim \epsilon \ , \quad \beta \sim \tilde{\epsilon}\epsilon^{2m-1} \ , \quad \left|\delta_N^* + \delta_N^{'}\right| \sim \epsilon^{2m} \ . \tag{4.49}$$

Moreover we have

$$\left|\delta_\nu^* + \delta_\nu^{'}\right| \sim \frac{\epsilon^2}{\sqrt{2}\tilde{\epsilon}} \ . \tag{4.50}$$

For $\tilde{\epsilon} \sim 1$ (indicating that the cut off M' is not too large) and $\epsilon \sim 0.2$ we get the right magnitude for $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$. Furthermore, by proper selection of the integers m and n, from (4.49) we can get desirable mass for the right handed neutrinos and the desired degeneracy as well. All these will insure the success of resonant leptogenesis.

4.5 Conclusions

In this chapter we have presented a new class of models which realize an inverted spectrum for neutrino masses. These models predict definite correlation between neutrino mixing angles θ_{12} and θ_{13}. Deviation of θ_{12} from $\pi/4$ is controlled by value of θ_{13}. Our results are given in Eqs. (4.22)-(4.25) and plotted in Figs. 4.2.1, 4.2.

We have presented concrete models based on an S_3 permutation symmetry augmented with a $U(1)$ symmetry acting on the three flavors.

Our models can naturally lead to resonant leptogenesis since two right-handed neutrinos are quasi degenerate. The predictions of our model are testable in forthcoming experiments.

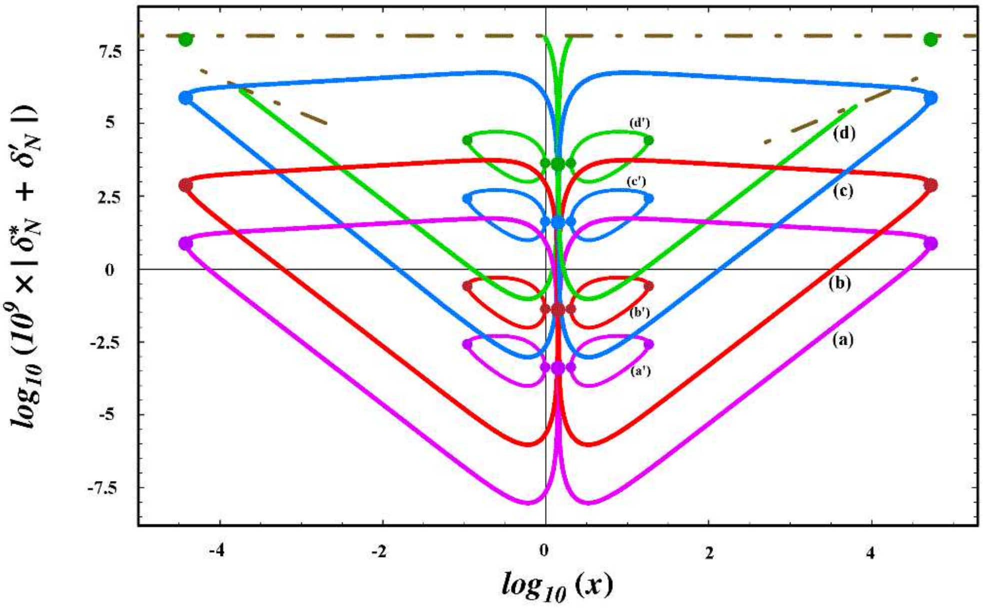

Figure 4.3. Resonant leptogenesis for inverted mass hierarchical neutrino scenario. In all cases $\frac{n_B}{s} = 9 \times 10^{-11}$ and $\tan\beta \simeq 2$. Curves (a), (b), (c), (d) correspond respectively to the cases with $M = (10^4, 10^6, 10^9, 10^{11})$ GeV and $r = \pi/4$. The curves with primed labels correspond to same values of M, but with CP phase $r = 5 \cdot 10^{-5}$. Bold dots stand for a maximized values of CP asymmetry [see Eq. (4.38)]. The 'cut off' with horizontal dashed line reflects the requirement $\left|\delta_N^* + \delta_N'\right| \lesssim 0.1$. Two sloped dashed lines restrict low parts of the 'ovals' of $M = 10^{11}$ GeV, insuring the Yukawa coupling perturbativity.

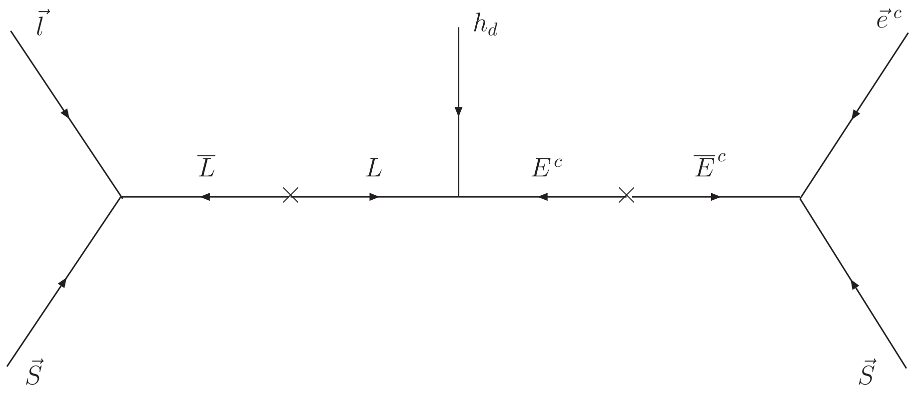

Figure 4.4. Diagram generating the first operator of Eq. (4.44)

CHAPTER 5

SUMMARY AND CONCLUSIONS

In this thesis I have presented the findings of research topics I have pursued during my Ph. D. study which primarily consist of explaining the Baryon Asymmetry in the universe. The Baryon Asymmetry problem in physics refers to the apparent fact that matter in the universe which have been observed are overwhelmingly matter as opposed to anti-matter, no helium atom (or larger atom) made of anti-matter, either in nature, or created synthetically, has ever been scientifically observed. Neither the standard model of particle physics, nor the theory of general relativity provide an obvious explanation for why this should be so. The challenges to the physics theories are then to explain how to produce this preference of matter over antimatter, and also the size of this asymmetry. An important quantifier is the asymmetry parameter,

$$\eta_B \equiv \frac{n_B}{n_\gamma} = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}. \tag{5.1}$$

There are competing theories to explain this phenomena, various Baryogenesis via Leptogenesis scenarios have been presented in the course of the last 40 years. We concentrate on three of the most popular mechanisms; realized in different ways: *Baryogenesis via Leptogenesis*, *Soft Leptogenesis*, and *Resonant Leptogenesis.* In the second chapter, we compute the Baryon Asymmetry induced in the decay of right–handed neutrinos in a class of minimal left–right symmetric models (LRSUSY). In these models, which assume low energy supersymmetry, the Dirac neutrino mass matrix has a determined structure, namely;

$$M_D = c \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{5.2}$$

where $c \simeq m_t/m_b$ is determined from the quark sector. As a result, lepton asymmetry is calculable in terms of measurable low energy neutrino parameters. By numerically solving the Boltzmann equations we show that adequate Baryon Asymmetry is generated in these models in complete agreement with recent NASA high precision measurements. Furthermore, we make predictions on the light neutrino oscillation parameters, which can be tested in next generation neutrino experiments.

In the third chapter, we discuss a more recent idea; Baryon Asymmetry via Soft Leptogenesis. We introduce the effect of the interactions of the $SU(2)_R$ gauge boson W_R on the generation of the BA. $B-L$ violation occurs when LRSUSY is broken by the VEV v_R of the $B-L=-2$ triplet scalar field $\Delta^c(1,1,3,-2)$, which gives Majorana masses to the r.h sneutrino, and lepton number is violated in their decay $\tilde{\nu}_{R1} \to e^c d\tilde{u}^c$, mediated by the right handed gauge boson W_R. We show that this decay dominate the traditional process $\nu_R \to L\phi^\dagger$ which drives Leptogenesis. We conclude that the requirement of unconventionally small $B-$term is no longer needed. In addition, we employ RGE and SUSY breaking effect to naturally account for the complex order 1 phase as dictated by the success of the scenario. The mass of r.h sneutrino can be $M_{\tilde{\nu}} \sim M_{W_R} \sim (10^9\ -\ 10^{10})$ GeV.

In the fourth chapter we turn our attention to Neutrino Physics, and its possible connection to Leptogenesis. We present a predictive model of inverted neutrino mass hierarchy based on $L_e - L_\mu - L_\tau$ combined with an S_2 permutation symmetry in the neutrino sector. Our analysis shows an interesting correlation between the mixing angles: $\sin^2\theta_{12} = \frac{1}{2} - \sqrt{1-\tan^2\theta_{13}}\tan\theta_{13}\cos\delta$, predicting $\theta_{13} \geq 0.13$, and $0 \leq \delta \leq 45^o$. Since the model involves two quasi-degenerate right handed neutrinos, it is natural to question its vitality with respect to Resonant Leptogenesis. We conclude that our predictive model of inverted neutrino hierarchy has all the ingredients to account for a viable Baryon Asymmetry, which makes the model even more attractive from Cosmology point of view as well. In this case, the degeneracy in mass between N_1 and N_2 states is required to be very accurate, i.e. $\left|\delta_N^* + \delta_N^{'}\right| \ll 1$. In section 4.4, it is discussed how this possibility is realized based on symmetries.

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APPENDICES

APPENDIX A

A.1 Basic Thermodynamics of The Expanding Universe

Many important calculations in Cosmology are done via the Boltzman equations. To employ this tool for the purpose of computing the Baryon Asymmetry, it is necessary to understand the thermodynamics of the early universe. In this appendix, we briefly review some basic thermodynamics of the early expanding universe, in which many particles are relativistic and in thermal equilibrium. First of all, the equilibrium density of particles of type i with momenta in a range d^3p centered on $\mathbf{p}$ is given by

$$g_i \frac{d^3p}{2\pi^3} f_i(\mathbf{p}) \,, \tag{A.1}$$

where g_i is the number of degrees of freedom and $f_i(\mathbf{p})$ is the Fermi-Dirac or Bose-Einstein distribution function:

$$f_i(\mathbf{p}) = \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) \pm 1} \,. \tag{A.2}$$

Here, E_i is the energy $E_i \equiv \sqrt{\mathbf{p}^2 + m_i^2}$, μ_i is the chemical potential of the particle i, and the plus (minus) sign is for fermions (bosons). The number density n_i, energy density ρ_i and pressure p_i of particle i are then given by the following equations:

$$n_i = \frac{g_i}{2\pi^3} \int f_i(\mathbf{p}) d^3p \,, \tag{A.3}$$

$$\rho_i = \frac{g_i}{2\pi^3} \int E_i f_i(\mathbf{p}) d^3p \,, \tag{A.4}$$

$$p_i = \frac{g_i}{2\pi^3} \int \frac{\mathbf{p}^2}{3E_i} f_i(\mathbf{p}) d^3p \,. \tag{A.5}$$

$T \gg m_i$		$T \ll m_i$
fermion	boson	
$n_i = \frac{3}{4} g_i \left(\frac{\zeta(3)}{\pi^2}\right) T^3$	$n_i = g_i \left(\frac{\zeta(3)}{\pi^2}\right) T^3$	$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_i}{T}\right)$
$\rho_i = \frac{7}{8} g_i \left(\frac{\pi^2}{30}\right) T^4$	$\rho_i = g_i \left(\frac{\pi^2}{30}\right) T^4$	$\rho_i = m_i n_i$
$p_i = \frac{1}{3}\rho_i$	$p_i = \frac{1}{3}\rho_i$	$p_i = T n_i \ (\ll \rho_i)$

TABLE A.1. The number density n_i, energy density ρ_i and pressure p_i of the particle i, which is thermal equilibrium, in the limits of $T \gg m_i$ and $T \ll m_i$. Where the following assumptions have been made: $|\mu_i| \ll T$ and $|\mu_i| < m_i$ (no Bose-Einstein condensation).

In Table A.1, shows these important cosmological quantities for the relativistic ($T \gg m_i$) and non-relativistic ($T \ll m_i$) limits.

Because the energy density of a non-relativistic particle is exponentially suppressed compared with the relativistic one, the total energy density of the radiation $\rho_{\mathrm{r}ad}$ is given by the following simple form:

$$\rho_{\mathrm{r}ad} = \frac{\pi^2}{30} g_*(T) T^4 \,, \tag{A.6}$$

where

$$g_*(T) \equiv \sum_{\substack{m_i \ll T \\ i = \mathrm{b}oson}} g_i + \frac{7}{8} \sum_{\substack{m_j \ll T \\ j = \mathrm{f}ermion}} g_j \,. \tag{A.7}$$

If there are particles which have different temperatures from that of the photon T, another factor $(T_i/T)^4$ should be multiplied in the above expression. (For example, at $T \ll$ MeV, neutrinos have temperature $T_\nu = (4/11)^{1/3} T$ for $m_\nu \ll T_\nu$.)

The mechanisms of leptogenesis discussed in this thesis work at temperatures far above the electroweak scale $T \gg 100$ GeV, where all the MSSM particles are expected to be in thermal equilibrium. In this case, we obtain

$$g_* = 228.75 \qquad for \quad \mathrm{M}SSM \,. \tag{A.8}$$

In the expanding universe (comoving volume) , it is useful to scale the densities by the entropy to account for the expansion. We introduce the entropy density s, which is defined by

$$\begin{aligned} s &\equiv \frac{\rho + p}{T} \\ &= \frac{4}{3T}\rho = \frac{2\pi^2}{45} g_*(T) T^3 \,. \end{aligned} \tag{A.9}$$

The entropy per comoving volume sR^3 is conserved as long as no entropy production takes place. Thus it is quite useful to take the ratio n_X/s when we calculate some number density of the particle species X. For example, if some X-number is conserved, the ratio of the X-number density to the entropy density remains a constant value

$$\frac{n_X}{s} = const \,, \tag{A.10}$$

as long as there is no entropy production, since both n_X and s scales as R^{-3} as the universe expands. As another example, if the X-particle is in thermal equilibrium and relativistic ($T \gg m_X$), the ratio is given by

$$\frac{n_X^{\mathrm{e}q}}{s} = \frac{45\zeta(3)}{2\pi^4} \frac{g_X}{g_*(T)} \quad \left(\times \frac{3}{4} \quad \text{for } fermion \right) \,, \tag{A.11}$$

where the temperature (or time) dependence only comes from $g_*(T)$. For a massless thermal photon γ, the density distribution, given by equation (A.1), is

$$n_\gamma = \frac{2}{\pi^2} T^3. \tag{A.12}$$

Far above their mass scales the massive particles are in thermal equilibrium. Therefore, the phase space distribution function is given by

$$f_i^{eq} = e^{-\frac{E_i}{T}} \,, \tag{A.13}$$

where again $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$, and the equilibrium density distribution becomes

$$n_i^{eq} = g_i \int \frac{d^3 p_i}{(2\pi)^3} f_i^{eq} \tag{A.14}$$

In terms of the dimensionless variables $x = \frac{\sqrt{\mathbf{p}^2+m_i^2}}{T}$ and $Z = \frac{m_i}{T}$, equation (A.14) can be rewritten as

$$\begin{aligned} n_i^{eq} &= g_i \frac{T^3}{2\pi^2} \int_Z^\infty x e^{-x} \sqrt{x^2 - Z^2} dx \\ &= g_i \frac{T^3}{2\pi^2} Z^2 K_2(Z) \end{aligned} \tag{A.15}$$

where $K_2(Z)$ is modified Bessel function of the second type. As $Z \to 0$, $z^2 K_2(Z) \to 2$. In this limit, the density distribution of massive particles is similar to that of with relativistic particles, and the approximation that at a temperature above their mass scale all particles are in thermally equilibrium is a valid assumption. As the Universe expands the temperature drops (Z increases). Therefore, the density distribution of all particles get diluted and is governed by the Boltzmann transport equations. K^0

A.2 FRW Universe and Boltzmann transport equations

The expansion of the Universe dilutes the number densities of all types of particles even in the absence of interactions at a rate

$$\frac{dn_i}{dt} = -3\frac{\dot{R}}{R}n_i = -3Hn_i \tag{A.16}$$

where $R(t)$ is scale factor in Freedman Robertson and Walker (FRW) Universe and $\dot{R}$ is derivative with respect to time. H is Hubble expansion factor. Thus in the absence of any interaction the Boltzmann transport equation for the given particle species i of density n_i is

$$\frac{dn_i}{dt} + 3Hn_i = 0. \tag{A.17}$$

Now we scale out the effect of the expansion of the Universe by considering the evolution of the number of particles in a comoving volume. This can be done by dividing the number density of the particle species i with its entropy density, i.e.

$$Y_i = \frac{n_i}{s}. \tag{A.18}$$

Using the conservation of entropy per comoving volume (sR^3 = constant), equation (A.18) can be written as

$$\frac{dn_i}{dt} + 3Hn_i = s\dot{Y} = 0. \tag{A.19}$$

As the Universe expands the momentum p_i of the particle species i falls as $1/R$ and thus also the temperature T. Under rescaling the momenta of massless particles remain unchanged. So they keep themselves in equilibrium with the thermal plasma. Above the mass scale of any massive particle it will behave as a massless one. Below its mass scale the interaction rate decreases in comparison to the Hubble expansion rate and hence it falls out of equilibrium because it needs several collision times to keep it in equilibrium with the thermal photons. The departure of the density of any species i from its thermal equilibrium value can be predicted by solving the Boltzmann transport equations.

For simplicity, let's consider the decay a massive species i to a set of particles Y. As a result the equation (A.19 becomes

$$\frac{dn_i}{dt} + 3Hn_i = -\sum_{i\leftrightarrow Y}\left[\frac{n_i}{n_i^{eq}}\gamma(i\to Y) - \frac{n_Y}{n_Y^{eq}}\gamma(Y\to i)\right],$$

where

$$\gamma(i\to Y) = \int d\Pi_i d\Pi_Y (2\pi)^4\delta^4(p_i - p_Y) f_i^{eq}|\mathcal{A}(i\to Y)|^2.$$

In equation (A.2), $d\Pi = \frac{1}{2E}\frac{d^3p}{(2\pi)^3}$. If we neglect CP-violation then $|\mathcal{A}(i\to Y)|^2 = |\mathcal{A}(Y\to i)|^2$. Using (A.14) the above equation (A.2) simplifies to

$$\frac{dn_i}{dt} + 3Hn_i = -\Gamma_D(n_i - n_i^{eq}),$$

where we have used

$$\Gamma_D = \frac{1}{2E_i}\int \frac{d^3p_Y}{(2\pi)^3 2E_Y}(2\pi)^4\delta^4(p_i - p_Y)|\mathcal{A}|^2.$$

Note that in the above simplification we have assumed $n_Y = n_i^{eq}$ and it is true because the decay products Y are massless till the later epochs of our interest. Substituting $Z = M_i/T$ and $Y_i = n_i/s$ in equation (A.2) we get

$$\begin{aligned}\frac{dY_i}{dZ} &= -\frac{\Gamma_D}{ZH(Z)}(Y_i - Y_i^{eq})\\ &= -D(Y_i - Y_i^{eq}).\end{aligned} \tag{A.20}$$

Considering the $2\leftrightarrow 2$ scatterings involving the species i equation (A.20) can be extended to

$$\frac{dY_i}{dZ} = -(D+S)(Y_i - Y_i^{eq}),$$

where $S = \Gamma_s/ZH$. This is the final Boltzmann equation for the evolution of any species i due to its decay and scatterings.

A.3 CP Violation in Neutral K-Meson System

Let $|K^0\rangle$ and $|\overline{K}^0\rangle$ be the stationary states of the K^0-meson and its antiparticle $\overline{K}^0$, respectively. Both states are eigenstates of the strong and electromagnetic interaction Hamiltonian, i.e.

$$(H_{\text{St}} + H_{\text{em}})\,|K^0\rangle = m_0|K^0\rangle \quad \text{and} \quad (H_{\text{St}} + H_{\text{em}})\,|\overline{K}^0\rangle = \overline{m}_0|\overline{K}^0\rangle \tag{A.21}$$

where m_0 and $\overline{m}_0$ are the rest masses of K^0 and $\overline{K}^0$, respectively. The K^0 and $\overline{K}^0$ states are connected through CP transformations. For stationary states, T, which is time reversal operator, does not alter them with the exception of an arbitrary phase. In summary, one gets

$$\begin{aligned} CP\,|K^0\rangle = e^{i\theta_{CP}}|\overline{K}^0\rangle \;\; \text{and} \;\; CP\,|\overline{K}^0\rangle = e^{-i\,\theta_{CP}}|K^0\rangle \\ T\,|K^0\rangle = e^{i\,\theta_T}|K^0\rangle \;\; \text{and} \;\; T\,|\overline{K}^0\rangle = e^{i\,\overline{\theta}_T}|\overline{K}^0\rangle \end{aligned} \tag{A.22}$$

where θ's are arbitrary phases and it follows that

$$2\,\theta_{CP} = \overline{\theta}_T \,-\, \theta_T \ .$$

by assuming $CPT\,|K^0\rangle = TCP\,|\overline{K}^0\rangle$.

If strong and electromagnetic interactions are invariant under CPT transformation, which is assumed here (see section 2 of chapter 1), it follows that $m_0 = \overline{m}_0$.

Next, we introduce a new interaction, V, violates strangeness conservation. Through such interactions, the K-mesons can decay into final states with no strangeness ($|\Delta S| = 1$) and K^0 and $\overline{K}^0$ can oscillate to each other ($|\Delta S| = 2$). Thus, a general state $|\psi(t)\rangle$ which is a solution of the Schrödinger equation

$$i\,\frac{\partial}{\partial t}|\psi(t)\rangle = (H_{\text{St}} + H_{\text{em}} + V)\,|\psi(t)\rangle \tag{A.23}$$

can be written as

$$|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle + \sum_{\text{f}} c_{\text{f}}(t)|\text{f}\rangle$$

where $a(t)$, $b(t)$ and $c_{\mathrm{f}}(t)$ are time dependent functions. For a new interaction which is much weaker than strong and electromagnetic interactions, perturbation theory and the Wigner-Weisskopf method can be applied to solve equation the the Schrödinger equation above. We obtain

$$i\,\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{\Lambda}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(\boldsymbol{M} - i\,\frac{\mathbf{\Gamma}}{2}\right)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{A.24}$$

where the 2×2 matrices $\boldsymbol{M}$ and $\mathbf{\Gamma}$ are often referred to as the mass and decay matrices.

The elements of the mass matrix are given as

$$M_{ij} = m_0\,\delta_{ij} + \langle i|V|j\rangle + \sum_{\mathrm{f}} P\left(\frac{\langle i|V|\mathrm{f}\rangle\langle \mathrm{f}|V|j\rangle}{m_0 - E_{\mathrm{f}}}\right) \tag{A.25}$$

where P stands for the principal part and $i = 1, 2$ denotes $K^0(\overline{K}^0)$. Let us split the Hamiltonian V into the known weak interaction part $H_{\mathrm{W}eak}$ and a hypothetical superweak interaction, $H_{\mathrm{S}w}$, i.e. $V = H_{\mathrm{W}eak} + H_{\mathrm{S}w}$. Since ordinary weak interactions do not produce a direct K^0-$\overline{K}^0$ transition, the second term of equation A.25 applies only for the superweak interaction for $i \neq j$. The third term is dominated by the weak interaction since the second order superweak interaction must be negligible. It follows that

$$M_{ij} = m_0\,\delta_{ij} + \langle i|H_{\mathrm{S}w}|j\rangle + \sum_{\mathrm{f}} P\left(\frac{\langle i|H_{\mathrm{W}eak}|\mathrm{f}\rangle\langle \mathrm{f}|H_{\mathrm{W}eak}|j\rangle}{m_0 - E_{\mathrm{f}}}\right)\ . \tag{A.26}$$

Note that the sum is taken over *a*ll possible intermediate states common to K^0 and $\overline{K}^0$ for $i \neq j$.

The elements of the decay matrix are given by

$$\Gamma_{ij} = 2\,\pi \sum_{f} \langle i|H_{weak}|\mathrm{f}\rangle\langle \mathrm{f}|H_{weak}|j\rangle\delta(m_0 - E_f) \tag{A.27}$$

The sum is taken over only *r*eal final states common to K^0 and $\overline{K}^0$ for $i \neq j$. Since Γ_{ij} starts from second order, the superweak Hamiltonian can be neglected. If

Hamiltonians are not Hermitian, transition probabilities are not conserved in decays or oscillations, i.e. the number of initial particles is not identical to the number of final particles. This is also referred as break down of unitarity. From here on, the hermiticity of all Hamiltonians will be assumed.

If V is Hermitian and invariant under T, CPT or CP transformations, the mass and decay matrices must satisfy the following conditions;

$$\begin{aligned}
\mathrm{T} \quad &: \quad \left| M_{12} - i\,\frac{\Gamma_{12}}{2} \right| = \left| M^*_{12} - i\,\frac{\Gamma^*_{12}}{2} \right| \\
CPT \quad &: \quad M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22} \\
\mathrm{C}P \quad &: \quad \left| M_{12} - i\,\frac{\Gamma_{12}}{2} \right| = \left| M^*_{12} - i\,\frac{\Gamma^*_{12}}{2} \right|,\ M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22}
\end{aligned}$$

where equations A.22, A.26 and A.27 are used. It follows that

$$\begin{aligned}
&\bullet\ \mathrm{i}f\ \ M_{11} \neq M_{22}\ or\ \Gamma_{11} \neq \Gamma_{22}\ \ : \\
&\quad \textbf{CPT and CP are violated} \\
&\bullet\ \mathrm{i}f\ \ \sin\left(\varphi_\Gamma - \varphi_M\right) \neq 0\ \ : \\
&\quad \textbf{T (or unitarity) \ and CP are violated}\ .
\end{aligned} \tag{A.28}$$

where $\varphi_M = \arg\left(M_{12}\right)$ $\ and\ $ $\varphi_\Gamma = \arg\left(\Gamma_{12}\right)$.

Note that CP is not conserved in both above cases; i.e. CP violation in the mass and decay matrices cannot be separated from CPT violation or T violation.

Solutions of equation A.24 for initially pure K^0 and $\overline{K}^0$ states are given by

$$|\mathrm{K}^0(t)\rangle = \left[\, f_+(t) - 2\,\varepsilon_{CPT}\, f_-(t)\,\right] |K^0\rangle + \left(1 - 2\,\varepsilon_{\mathrm{T}}\right)\, e^{-i\,\varphi_\Gamma} f_-(t)\, |\overline{K}^0\rangle, \tag{A.29}$$

leading to

$$|\mathrm{K}^0(t)\rangle = \frac{1}{\sqrt{2}} \left(|K_S\rangle\, e^{-i\,\lambda_{\mathrm{S}}\, t} + |K_L\rangle\, e^{-i\,\lambda_{\mathrm{L}}\, t} \right) \tag{A.30}$$

and

$$\begin{aligned}
|\overline{\mathrm{K}}^0(t)\rangle &= \left(1 + 2\,\varepsilon_{\mathrm{T}}\right) e^{i\,\varphi_\Gamma}\, f_-(t)\, |\mathrm{K}^0\rangle + \left[\, f_+(t) + 2\,\varepsilon_{\mathrm{C}PT}\, f_-(t)\,\right] |\overline{K}^0\rangle \qquad \text{(A.31)} \\
&= \frac{1 + 2\,\varepsilon_{\mathrm{T}}}{\sqrt{2}}\, e^{i\,\varphi_\Gamma} \\
&\quad \times \left[\, \left(1 + 2\,\varepsilon_{\mathrm{C}PT}\right) |\mathrm{K}_S\rangle\, e^{-i\,\lambda_S\, t} - \left(1 - 2\,\varepsilon_{\mathrm{C}PT}\right) |\mathrm{K}_L\rangle\, e^{-i\,\lambda_L\, t} \,\right]
\end{aligned}$$

where

$$f_{\pm}(t) = \frac{1}{2}\left(e^{-i\lambda_S t} \pm e^{-i\lambda_L t}\right) \; .$$

The parameters λ_S and λ_L are eigenvalues of $\mathbf{\Lambda}$, and K_S and K_L are the corresponding eigenstates given by

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2}}\left[(1 - 2\varepsilon_{CPT})|K^0\rangle + (1 - 2\varepsilon_T)\, e^{-i\varphi_\Gamma}|\overline{K}^0\rangle\right] \\ |K_L\rangle &= \frac{1}{\sqrt{2}}\left[(1 + 2\varepsilon_{CPT})|K^0\rangle - (1 - 2\varepsilon_T)\, e^{-i\varphi_\Gamma}|\overline{K}^0\rangle\right] \; . \end{aligned} \tag{A.32}$$

They have definite masses and decay widths given by λ_S and λ_L as

$$\lambda_{S(L)} = m_{S(L)} - i\,\frac{\Gamma_{S(L)}}{2}$$

with

$$\begin{aligned} m_{S(L)} &= \frac{M_{11} + M_{22}}{2} + (-)\,\Re\left(\sqrt{\Lambda_{12}\Lambda_{21}}\right) \\ &= \frac{M_{11} + M_{22}}{2} - (+)\;|M_{12}| \end{aligned}$$

and

$$\begin{aligned} \Gamma_{S(L)} &= \frac{\Gamma_{11} + \Gamma_{22}}{2} - (+)\,2\,\Im\left(\sqrt{\Lambda_{12}\Lambda_{21}}\right) \\ &= \frac{\Gamma_{11} + \Gamma_{22}}{2} + (-)\;|\Gamma_{12}| \end{aligned}$$

where we used

$$\varphi_\Gamma - \varphi_M = \pi - \delta\varphi, \quad |\delta\varphi| \ll 1$$

and

$$|\Lambda_{22} - \Lambda_{11}| \ll 1$$

which are derived from empirical facts, $m_L > m_S$, $\Gamma_S > \Gamma_L$ and small CP violation.

The two CP violation parameters ε_T and ε_{CPT} are given by

$$\begin{aligned} \varepsilon_T &= \frac{\Delta m\,\Delta\Gamma}{4\,\Delta m^2 + \Delta\Gamma^2}\left(1 + i\,\frac{2\,\Delta m}{\Delta\Gamma}\right)\delta\varphi \\ \varepsilon_{CPT} &= \frac{i\,2\,\Delta\Gamma}{4\,\Delta m^2 + \Delta\Gamma^2}\left(1 + i\,\frac{2\,\Delta m}{\Delta\Gamma}\right)(\Lambda_{22} - \Lambda_{11}) \; . \end{aligned}$$

As seen from the statements A.28, $\varepsilon_T \neq 0$ implies CP and T violation, and $\varepsilon_{CPT} \neq 0$ means CP and CPT violation. It should be noted that both ε_T and ε_{CPT} **d**o not depend on any phase convention. The phase of ε_T is given by the K_S-K_L mass and decay width differences which are not related to CP violation. This phase is often referred to as "superweak" phase:

$$\phi_{sw} = \arg(\varepsilon_T) = \tan^{-1}\left(\frac{2\,\Delta m}{\Delta\Gamma}\right) \ .$$

If we assume that ordinary weak interactions conserve CPT, i.e. $\Gamma_{11} = \Gamma_{22}$, the phase of the CP and CPT violation parameter ε_{CPT} is given by

$$\arg(\varepsilon_{CPT}) = \phi_{sw} + \frac{\pi}{2} \ .$$

A.4 Bessel Functions

When evaluating reaction densities according to Eq (A.6), one encounters the modified Bessel functions, it will be useful to summarize some formulae for Bessel functions. Modified Bessel functions with different indices are related via recursion relations,

$$x\mathrm{K}_{\nu-1}(x) + x\mathrm{K}_{\nu+1}(x) = 2\nu\mathrm{K}_{\nu}(x) \;, \tag{A.33}$$

$$\mathrm{K}_{\nu-1}(x) - \mathrm{K}_{\nu+1}(x) = 2\frac{d}{dx}\mathrm{K}_{\nu}(x) \;. \tag{A.34}$$

For integer index Bessel functions have the following series representation

$$\begin{aligned}\mathrm{K}_n(x) \;&=\; \frac{1}{2}\sum_{k=0}^{n-1}(-1)^k \frac{(n-k-1)!}{k!\left(\frac{z}{2}\right)^{n-2k}} \;+ \\ &+ (-1)^{n+1}\sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^{n+2k}}{k!(n+k)!}\left[\ln\left(\frac{x}{2}\right) - \frac{1}{2}\psi(k+1) - \frac{1}{2}\psi(n+k+1)\right] \;,\end{aligned} \tag{A.35}$$

where ψ denotes the derivative of the logarithm of the Gamma function

$$\psi(x) = \frac{d}{dx}\ln\Gamma(x) \;. \tag{A.36}$$

For integer argument it reads

$$\psi(n) = -\gamma_E + \sum_{k=1}^{n-1}\frac{1}{k} \;, \tag{A.37}$$

where $\gamma_E = 0.577216$ is Euler's constant. Hence, the leading terms of the series are given by

$$\mathrm{K}_0(x) \;=\; \ln\left(\frac{2}{x}\right) - \gamma_E + \ldots \;, \tag{A.38}$$

$$\mathrm{K}_n(x) \;=\; \frac{(n-1)!}{2}\left(\frac{2}{x}\right)^n + \ldots \;, \qquad \text{for } n \geq 1 \;. \tag{A.39}$$

The asymptotic expansion of modified Bessel functions reads

$$\mathrm{K}_\nu(x) = \sqrt{\frac{\pi}{2x}}\;\mathrm{e}^{-x}\sum_{k=0}^{\infty}\frac{1}{k!(2x)^k}\,\frac{\Gamma\left(\nu+k+\frac{1}{2}\right)}{\Gamma\left(\nu-k+\frac{1}{2}\right)} \;, \tag{A.40}$$

i.e. to leading order all Bessel functions have the same asymptotic behaviour,

$$\mathrm{K}_\nu(x) = \sqrt{\frac{\pi}{2x}}\,\mathrm{e}^{-x} + \dots\ .$$

A.5 Loop Integrals

A trick commonly used to combine propagators denominator is the Feynman formula that introduces the Feynman parameters;

$$\frac{1}{A_1 A_2 \ldots A_n} = \int_0^1 dX_1 \ldots dX_n \delta\left(\sum X_i - 1\right) \frac{(n-1)!}{[X_1 + A_1 X_2 A_2 + \ldots + X_n A_n]^n} \tag{A.41}$$

The most simplest case is that of two denominators, which is straightforward to check;

$$\frac{1}{A_1 A_2} = \int_0^1 dX \frac{1}{[XA_1 + (1-X)A_2]^2} \tag{A.42}$$

A more general formula in which some of the denominators have powers, such as encountered in Eq (3.24), is as follows;

$$\frac{1}{A_1^{m_1} A_2^{m_2} \ldots A_n^{m_n}} = \int_0^1 dX_1 \ldots dX_n \delta\left(\sum X_i - 1\right) \frac{\prod X_i^{m_i - 1}}{[\sum X_i A_i]^{\sum m_i}} \frac{\Gamma(m_1 + \ldots + m_n)}{\Gamma(m_1) \ldots \Gamma(m_n)} \tag{A.43}$$

This formula is true even when the m_i are not integers. In the following we give a list of $d-$dimensional integral, some of which have been used in carrying out the integration in Eq (3.24):

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - Q)^n} = \frac{(-1)^n \imath}{(4\pi)^{d/2}} \frac{\Gamma\left(n - \frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{Q}\right)^{n - \frac{d}{2}} \tag{A.44}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - Q)^n} = \frac{(-1)^{n-1} \imath}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma\left(n - \frac{d}{2} - 1\right)}{\Gamma(n)} \left(\frac{1}{Q}\right)^{n - \frac{d}{2} - 1} \tag{A.45}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{(k^2)^2}{(k^2 - Q)^n} = \frac{(-1)^{n-1} \imath}{(4\pi)^{d/2}} \frac{d(d+2)}{4} \frac{\Gamma\left(n - \frac{d}{2} - 1\right)}{\Gamma(n)} \left(\frac{1}{Q}\right)^{n - \frac{d}{2} - 2} \tag{A.46}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^\nu k^\nu}{(k^2 - Q)^n} = \frac{(-1)^{n-1} \imath}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma\left(n - \frac{d}{2} - 1\right)}{\Gamma(n)} \left(\frac{1}{Q}\right)^{n - \frac{d}{2} - 1} \tag{A.47}$$

A.6 Mathematica Code

In this appendix we would like to highlight some aspects of numerical solution to Boltzmann Equation (BE). Remind the first and second BE from Eq (2.30, 2.31) as follows;

$$\frac{dY_{N_1}}{dz} = -\frac{z}{Hs(z)}\left(\frac{Y_{N_1}}{Y_N^{eq}} - 1\right)\left(\gamma_{D_1} + \gamma_{S_1}\right), \tag{A.48}$$

$$\frac{dY_{B-L}}{dz} = -\frac{z}{s(z)H(M_1)}\left[\varepsilon_1\gamma_{D_1}\left(\frac{Y_{N_1}}{Y_N^{eq}} - 1\right) + \gamma_W\frac{Y_{B-L}}{Y_L^{eq}}\right], \tag{A.49}$$

which have to be simultaneously solved for z where $z = \frac{M_1}{T}$. The difficulty here is in integrating over x the reaction densities $\gamma^{(i)}$;

$$\gamma^{(i)} = \frac{M_1^4}{64\pi^4}\frac{1}{z}\int_{\frac{(M_a+M_b)^2}{M_1^2}}^{\infty} dx\ \hat{\sigma}^{(i)}(x)\sqrt{x}\ K_1\left(\sqrt{x}z\right), \tag{A.50}$$

where $\hat{\sigma}^{(i)}(x)$ is in general a complicated function of x, see Eq (2.40, 2.42, 2.43). Notice the dependence of the Bessel function over the parameter z. Carrying out the integration analytically is very difficult. However, the integration can be done numerically and interpolate the $\gamma^{(i)}$'s to obtain a smooth function of z, then proceed to solve $A.48,\ A.49$ by using the interpolated functions of the $\gamma^{(i)}(z)$. The integrand is highly oscillatory due to the decayed oscillation feature of Bessel functions, but *NIntegrate* command of *Mathematica 5.2* can very well handle the singularities if the correct settings for *AccuracyGoal*, integration *Method* and maximum number of recursive subdivisions (*MaxRecursion*) are properly chosen. Usually, specifying a high value for the *WorkingPrecision*, i.e, how many digits of precision should be maintained in internal computations, by itself sets the other options to optimum values. The code originally written is *Mathematica* was independently checked by a second *Maple 9* code to ensure the validity of the final solution. Below we make available the *Mathematica* notebook we constructed, it includes helpful comments at every step toward the final *NDSolve* execution.

VITA

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Abstract: In this thesis I have presented the findings of my research pursued during my Ph. D. study. The purpose of this thesis was to study different theoretical ideas in high energy physics model building addressed primarily towards understanding of one of the great mysteries of modern particle physics and cosmology; the Baryon Asymmetry (BA) of the universe (matter-antimatter asymmetry), and the close connection of this latter to Neutrino Physics. We concentrate on three of the most popular mechanisms to generate BA; realized in different ways: *Baryogenesis via Leptogenesis*, *Soft Leptogenesis*, and *Resonant Leptogenesis*. In the first chapter, we calculate BA induced in the decay of right–handed neutrinos in a class of minimal left–right symmetric models (LRSUSY). In these models, which assume low energy supersymmetry, the Dirac neutrino mass matrix has a determined structure. As a result, lepton asymmetry is calculable in terms of measurable low energy neutrino parameters. By numerically solving the Boltzmann equations we show that adequate BA is generated in these models in complete agreement with recent NASA high precision measurements. Furthermore, we make predictions on the light neutrino oscillation parameters, which can be tested in next generation neutrino experiments. In the second chapter, we discuss a more recent idea; Soft Leptogenesis. This time, we study the effect of the interactions of the $SU(2)_R$ gauge boson W_R on the generation of the BA. $B-L$ violation occurs when LRSUSY is broken by the VEV v_R of the $B-L=-2$ triplet scalar field $\Delta^c(1,1,3,-2)$, which gives Majorana masses to the r.h sneutrino, and lepton number is violated in their decay $\tilde{\nu}_{R1} \rightarrow \tilde{e}_R u\bar{d}$, mediated by the right handed gauge boson W_R, can dominate the traditional process $\nu_R \rightarrow L\phi^\dagger$ which drives Leptogenesis. We show that the requirement of unconventionally small $B-$term is no longer needed. In addition, we include RGE and SUSY breaking effect to naturally account for the complex order 1 phase as dictated by the success of the scenario. The mass of r.h sneutrino can be $M_{\tilde{\nu}} \sim M_{W_R} \sim (10^9\ -\ 10^{10})\ GeV$. In the third chapter we turn our attention to Neutrino Physics. We present a predictive model of inverted neutrino mass hierarchy based on $L_e - L_\mu - L_\tau$ combined with an S_2 permutation symmetry in the neutrino sector. Our analysis shows an interesting correlation between the mixing angles: $\sin^2\theta_{12} = \frac{1}{2} - \sqrt{1-\tan^2\theta_{13}}\tan\theta_{13}\cos\delta$, predicting $\theta_{13} \geq 0.13$, and $0 \leq \delta \leq 45^o$. Resonant Leptogenesis is discussed since the model involves two quasi-degenerate r.h.n, successfully generating adequate BA.

ADVISOR'S APPROVAL ______________________________