NODAL DISTRIBUTION STRATEGIES FOR DESIGNING AN OVERLAY NETWORK FOR LONG-TERM GROWTH

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Chapter One Introduction

Growth of telecommunications during the 1990's was driven by the explosive development of the Internet. During this time, many totally new networks were built, but in today's economic climate, new investments will be focused towards getting the maximum from existing infrastructure. Also being able to plan from the beginning network designs that can cost-effectively adjust to changing traffic is important (Birman, 2001). The ability of network architecture to adjust and grow to handle increased volumes of traffic and diverse Quality of Service, QoS, or be scalable, is an important feature in network design. One factor affecting the scalability of a network is the design of the physical network topology. The number, size, and arrangement of the nodes and links in the directed graph that represents the physical network describe the physical topology. Much research has been done to create methods to help design the optimal or best network graph with respect to cost (for example; Grover and Doucette 2001, Gendron, et.al. 1999, Chang and Gavish 1993) but in general the research has focused on one design period, not long term growth, and finding one "best" design. Also in the area of large wide area networks (WANs), optimization research has focused on minimizing the inter-nodal cost function, which is the set of costs associated with the links and the size of the links connecting the given nodes that will deliver the required traffic demands.

While the nodal distribution strategy, called the node placement or facilities location problem, has received much attention in many disciplines, it has received somewhat less attention for WAN design. Nodal distribution strategies affect the number, size and location of nodes in the network. Usually these problems require heuristic algorithms for sub-optimal solutions because this problem has been shown to be NP-hard and intractable (Yeung and Yum 1998 and Banjeree, Mukherjee and Sarkar 1994). Most of the previous nodal distribution research has focused on determining the "optimal" distribution of nodes for any given network state and thus examined only one nodal distribution pattern. Scalability, or the long-term growth potential, of nodal distribution strategies has not been examined extensively nor have comparison studies of different distribution strategies been done. Additionally, examining the nodal distribution strategies of an overlay on top of existing underlying legacy network have not been extensively examined other than as hierarchical or multi-layered networks. The research project presented in this paper will examine the impact of the nodal distribution strategy of a new service overlay on a legacy network and will examine the impact of changes to the optimal design. Specifically, this study will examine and compare long term costs of nodal distribution strategies that deploy a service using switches in numerous locations with minimal backhaul of traffic [the distributed approach] to the strategy of using switches in a limited number of locations resulting in more backhaul of traffic [the centralized approach]. The key question for this research is the impact of the number of nodes in a new service overlay. In other words, "Which is better, a service overlay with fewer larger switches or smaller switches distributed throughout the network? Examples of scenarios that this problem describes include deploying a VoIP service over an IP network, deploying an ISP backbone over an ATM network, deploying an ATM backbone over a SONET network, or more generally, deploying a new layer of the some type of service over an older existing network.

Typically, in network design optimization a methodology is created to define the "optimal" or best solution depending upon the constraint parameters to be considered. There is a broad breath and depth of research available to assist in the decision of the optimal design. All too often after the optimal design has been defined, operational considerations that were not known during the optimization process pop up and force the final network design to be changed. One way to understand the impact of change to the final design and cost structure is to create an efficient or production frontier for that network problem (Fare, Grosskopf, and Knox Lovell, 1994). The efficient frontier or production frontier process defines the optimal mix of parameters that will create the most efficient use of resources for the given problem. By examining the efficient frontier for any given legacy network topology the impact of change can be examined from many perspectives.

The remainder of this dissertation is structured as follows. In Chapter Two, a review of previously published research of nodal distribution, efficient or production frontier, and problem solution methods is presented. Chapter Three will be a review and analysis of the mathematical model presented in this study. The experimental methods used to analyze the case studies will be reviewed in Chapter Four. Chapter Five will be a presentation of results and interpretation. Chapter Six will be a summary including future research.

Chapter Two Literature Review

Chapter two of this paper will review published literature relating to designing of telecommunications networks and long-term growth of networks. Network design is the process of describing the form and function of the network so that day-to-day functionality happens. Robert S. Cahn (1998) in his book Wide Area Network Design: Concepts and Tools for Optimization makes the statement *"In network design there are no clear winners, only clear losers. The design process is at its heart the solution to an ill-defined problem"* (p. 2). Network design optimization problems related to this study are grouped into four categories, I. facilities (or node) location, II. link design including location and capacity, III. multi-level or hierarchical network design and IV. multi-period design approaches that include growth. The appropriate network design optimization literature grouped by the above-described categories will be presented and reviewed. In each group a representative LP formulation for each problem will be presented. Then the efficient frontier or production frontier process, a concept borrowed from finance, agriculture economics, and operations research literature will be reviewed. Another topic that is important to designing and modeling networks is traffic modeling and it will be discussed last.

Optimization and Network Design

This section of this paper will review some methods used in published research relating to network design optimization as it relates to network scalability and design. While this is a lengthy review section, it is by no means an exhaustive review of optimization and the techniques relating to network design. The intent is to give an overview of the general area presenting first a general overview of the techniques used to find "optimal" solutions for network design problems and then second present the specific linear programming problems that will be used in this study. Mathematical modeling and optimization are well-developed and mature areas of research with a variety of articles available for the interested reader who is referred to Bertsekas (1998), Sanso and Soriano (1999), and Grover and Doucette (2001) for broader overviews.

To find the optimized design, an exhaustive analysis must examine every possible combination of all parameters thereby proving the best fit to meet the specified design characteristics. Very quickly, these types of problems become NP-hard and intractable especially when examining communication networks. Finding near-optimal solutions requires faster heuristic algorithms, establishing acceptable constraints or bounds and then "relaxing" these constraints so the solutions can be found with reasonable resources and time. The bulk of published research and literature deals with ways of making this very complex problem simpler and easier to solve.

General Linear Programming Problem Statement

General linear programming (LP) techniques are widely used to define optimal telecommunication network designs to meet one or more parameters. One way to state this problem is as a general topology design and capacity expansion problem. The formulation presented by Chang and Gavish (1993) and many others since will be the format of this paper.

Given number of switches and their locations,
- traffic requirements for each period,
- cost structures as functions of time

Minimize net present worth of total cost;

With respect to

- when and where to install (network topology and design and expansion),
- when and where to expand line capacities (network capacity expansion),
- how to route network traffic (routing decisions);

Subject to Reliability constraints,
- QoS constraints of delay constraints,
- Flow conservation constraints,
- Capacity constraints,
- Other types of side constraints.

I. Facilities Location or Node Placement

Most of the network design and optimization work focuses on the placement and capacity of the links that carry traffic flow from node to node. Most likely because node location is usually decided well in advance and link costs, both installation and transport, tend to dominate total network costs, especially in large WANs. Node location falls into the general problem of facilities location, a combinatorial optimization problem, like link design, that quickly becomes NP-hard as the number of node locations grows. The node location problem in this research is "how to choose which nodes of an existing legacy topology to use in the backbone of a new service overlay". This problem is related to the well-studied terminal-concentrator problem introduced in the 1960's to connect switching centers of the public switched telephone network (PSTN). Gourdin, Labbe, and Yaman (2002) as part of an overview article present the uncapacitated facility location problem (UFLP) that solves the following three design phases in an iterative manner:

1. the number and locations of concentrators and the assignment of terminals to these concentrators
2. the access networks
3. the backbone network.

Designing large-scale extensive computer networks very quickly becomes an intractable problem and must be subdivided into multiple steps. Many methods to subdivide the problem have been presented. Pirkul and Nagarajan (1992) presented a design for a tree/star network using a two-phase algorithm. A first step sweep phase divides the set of nodes into regions. The second step formulates a path for each node within a region to the central node via point-to-point links. Thus, a star design is created within each region. The central node becomes a node on the backbone and the backbone nodes are connected. Gavsih (1982) presented formulations of the terminal/concentrator problem using multidrop capacitated links and expanded the formulation in 1992 so that no apriori knowledge of the network topology was needed. These two works are often used as the basis for additional advanced formulations.

Balakrishnan, Magnati, and Wong (1995) developed a formulation that installed concentrators and expanded the size of links with minimum cost based on a decomposition method using Lagrangian Relaxation and dynamic programming. Yeung and Yum (1998) examined node placement using a ShuffleNet graph structure. They proposed a "gradient algorithm" that minimizes the average internodal hop distance. Other work examining node placement presented by Ali (2002) examined the optimization of multicast node in wavelength-routed all-optical networks. This heuristic method examined the location of nodes and developed a near-optimal design using blocking probability as the performance metric. Murkherjee et al (1996) using virtual topologies combined subsets of all the nodes and links in the physical topology to develop optimal or near optimal WDM network design. Chamberland, Sanso, and Marcotte (2000) using a dual-based heuristic that yielded near-optimal designs, proposed a solution to the design problem of the appropriate switches for core network nodes. They proposed a mixed 0-1 linear programming model that includes the location of switches, the configuration of the switches, ports, and multiplexers, the design of a star topology access network and a backbone network of a fixed ring or a tree topology. Using a greedy heuristic to provide a good starting point and a Tabu search heuristic to improve the solution, a final solution was proposed that would minimize the total cost of the network. The problem involves selecting the switch sites, the type of ports to be used, selecting the multiplexers, connecting the users to the switches through OC-3 links, and interconnecting the switches through OC-192 links in a specified topology. For a more comprehensive overview of the facilities location problem the reader is referred to a text by Drezner and Hamacher (2002) which contains a compilation of mainstream facilities location topics relating to many disciplines with extensive up-to-date reference lists at the end of each chapter

Summary

Much work has been done, as highlighted in the previous section, in terminal concentrator (node or switch) design and node connection to centralized concentrators that designs optimal or near-optimal connections of all the nodes in the physical topology. The limitation as far as this research is concerned is that the location of the concentrators is usually decided in advance. Once the location or identification of the set of concentrators has been accomplished there are many formulations for solving this problem.. The following discussion presents an example chosen because either it was the most recent example of that problem or it was often cited in published literature.

I. Facilities (or Node) Location

Using the uncapacitated facilities location problem (UFLP) presented by Gourdin, Labbe, and Yaman (2002) the problem is to determine the number and location of concentrators and assign the terminals to these concentrators. The goal of the problem is to minimize the cost of installing the concentrators and the cost of serving terminals via the concentrators. Concentrators are the backbone switches and terminals represent all of the node locations in the legacy network. The UFLP is stated as follows:

N = {set of terminal locations}
M = {set of concentrator locations}
C_{ij} = is the cost of assigning terminal i to concentrator at location j
F_j = cost of installing a concentrator at location j

$$\text{Min} \sum_{i\in N} \sum_{j\in M} C_{ij}x_{ij} + \sum_{j\in M} F_j y_j \qquad 2.1$$

subject to:

$$\sum_{j\in M} x_{ij} = 1 \quad \text{for all } i\in N; \qquad 2.2$$

$$x_{ij} \le y_j \text{ for all } i\in N, j\in M; \qquad 2.3$$

$$x_{ij} \in \{0,1\} \text{ for all } i\in N, j\in M; \qquad 2.4$$

$$y_j \in \{0,1\} \text{ for all } j\in M. \qquad 2.5$$

$$y_j = \begin{cases} 1 & \text{if concentrator is installed at location } j \\ 0 & \text{otherwise; for all } j\in M. \end{cases} \qquad 2.6$$

$$x_{ij} = \begin{cases} 1 & \text{if terminal I is assigned to the concentrator at location } j \\ 0 & \text{otherwise; for all } i\in N \text{ and } j\in M. \end{cases} \qquad 2.7$$

Equation 2.1 is the objective statement to minimize cost. Constraints 2.2 and 2.4 state that each terminal should be assigned to exactly one concentrator and constraint 2.3 is so that a terminal can be assigned to a concentrator only if this concentrator is installed. UFLP is the first phase of an iterative design approach that would feed into the next phase of the access and backbone design.

II. Network Design (link location and capacity)

This section will examine the area of designing link configuration of the core or backbone networks that provide transport. Not being constrained by existing ring topologies, the design of new mesh optical networks took advantage of state-of-the-art optical switches that pushed optical network technology development. Since this area is so broad, the literature reviews will be grouped into the following sections; design principles, multi-commodity flow problems, and heuristic adaptations.

Design Principles

A common network design scheme defines networks around several parameters grouped into three categories, cost, QoS, and reliability (Cahn 1998). Cost includes implementation and maintenance. QoS groupings describe the type of services that are offered by this network and therefore a potential revenue metric. Reliability refers to the network's ability to recover from a failure. All three of these parameters while being distinctly different do interact with each other and can negatively influence each other so therefore must be balanced against each other. QoS and reliability impact the cost of the network because when those factors are measured or put into the design criteria the cost of the network increases.

Seven characteristics are often used to guide network design; capacity, scalability, modularity, upgradability, flexibility, reliability, universality, and transparency (Dumortier, Masetti, and Sotom 1995). High capacity in any new design accommodates not only the known traffic but also future needs. Broadband applications will require increasing amounts of bandwidth and more and more users will demand more bandwidth. Scalability of a design requires that the network be able to grow gracefully to accommodate increasing demands. Modularity demands that the design be simple enough so that the network is constructed of a relatively small number of elements that can be used to deploy nodes and links in a large size range. Upgradability characteristics are those that will allow the network to evolve without frequent substantial investments due to incompatibility of new versions with previously installed network base. Changing traffic demands are a reality of network life and network design must show flexibility to accommodate these inevitable changes. Reliability of a network means among other things that the network can recover from failures (in other words, has built-in protection) with a minimum amount of delay (has speedy restoration capabilities). Any good network design requires that the network be capable of supporting a wide range of services, both current and future, supporting the universality of digitized information flow. Transparency, in and of itself, is not a design requirement but is necessary to support universality and other modularity requirements. Networks, ideally, should be able to accommodate a variety of applications without each application being impacted by the other. In other words, applications using the network should be transparent to each other. All of these factors impact the cost of the network. Increasing any factor will increase the cost.

The MENTOR algorithms presented by Cahn (1998) and Kershenbaum et al (1991) are a heuristic approach using three parameters, weight, radius, cost, that can be adjusted to define and design, backbone and access to the backbone. By directing the design development with three principles, first, the shortest path is usually the lowest cost, second, links should have high utilization, and third, use long high capacity links when ever possible, the MENTOR algorithms develop a near-optimal or very good design solutions. Design optimization routines are usually complex and require simplification modifications to allow the solution to be found with reasonable computing times. Heuristic approaches such as MENTOR are commonly used in practice to develop network designs due to the relative simplicity and ease of use. The solutions, while not necessarily optimal, are usually "good enough" especially when realistically constrained. MENTOR can also be used for a multi-layer hierarchical design problem.

Grover and Doucette (2001) presented a 1-0 mixed integer formulation of the complete mesh-restorable topology design with a three stage process for topology planning and growth of optical mesh networks called mesh topology routing and sparing, MTRS. Their heuristic solves *"three problems (W1, S2 and J3) of reduced complexity to approximate an optimal single-stage solution to MTRS. W1 finds a fixed charged plus routing, FCR, type minimal topology and capacity solution as justified by working flows alone. S2 finds a min-cost topology augmentation as justified by restorability considerations alone. J3 revises the working flows of W1 to exploit the augmented* *topology of S2 and coordinates them with the assignment of restoration capacity and selection of edges to minimize the total cost of realization"* (Grover and Doucette 2001). The union of the three edge sets allows the high quality approximation of MTRS. Each of these three problems are NP-hard in themselves and combined would be even more difficult to solve but creating a 3 stage approach allows the problem to be solved. The union of the three sets creates an effective topology space to solve a restricted instance of the full problem.

Multi-commodity Flow

The multi-commodity flow problem involves "*a collection of several networks whose flows must independently satisfy conservation of flow constraints*" (Bertsekas 1998, p. 349). Associated with the directed graph of the network topology will be a collection of flow vectors of different traffic values. The sum of traffic flows on each arc (or link) of the graph is used to define capacity. Saniee (1996) reported a multi-commodity flow formulation for the routing of traffic problem that achieved maximum network throughput with minimum blocking loss due to a single switch failure. Girard and Sanso (1998) reported a multi-commodity flow model applied to the design of circuit-switched networks with reliability constraints. The results showed that this approach compared favorably with other exact dimensioning algorithms in use at the time, especially when failures were considered. Hadjiat, Maurras, and Vaxes (2000) presented a primal partitioning technique for single and non-simultaneous multicommodity flow problems. Their use of a simplex-based algorithm modified by a refined primal partitioning to speed it up, presented a cost effective solution to the design of the French national telecommunications network. Mateus and Franqueria (1998) considered an integer programming formulation with a partial multicommodity structure to model and define a generalized access network design problem that connects every remote unit to its central node in a telecommunications network.

Bienstock and Saniee (2001) updated the multi-commodity flow approach to propose a methodology for designing ATM networks with the relatively newly proposed Brownian motion model to define data traffic flows. The multi-commodity problem can be difficult due to three aspects, 1) the large number of different but interrelated capacity decisions with rapidly changing cost profiles; 2) the complicated nature of the paths used for routing, and the potentially very large size of the formulation; and 3) the complexity of defining the nature of data traffic. This problem is also often plagued with a very large duality gap and can be presented in large, difficult and ill conditioned linear programs. The heuristics defined by this formulation were generally good in that solutions were within 10% of the lower bound or optimal solution for 75% of the test cases. The solutions were generally independent of the numerical values of the input data, ran in the magnitude order of tens of seconds, were more dependent on node variables than link variables and the path generation step had greater impact in constraining the solutions than previously thought. In general, Bienstock and Saniee (2001) found that the addition of statistical multiplexing could significantly reduce networking costs in the range of 10-40% over other approaches.

Heuristic Techniques

Many researchers have proposed using heuristic local search techniques as an approach to solving these NP-hard network design problems. The following section will review Tabu search, general genetic algorithms, Lagrangian heuristics, and other approaches.

Tabu Search

To avoid being trapped in a less than optimal local minimum, the Tabu search approach allows accepting a worse or even infeasible solution from within the current neighborhood to continue the search for the better solution. A list of recently obtained solutions is maintained in a forbidden (Tabu) list (Bertsekas 1998).

Lee et al (2000) proposed a methodology to find an optimal capacity allocation so that the total cost of ATM switch modules is minimized. First, they formulated the integer-programming model as a bin-packing problem with capacity constraints. Then they developed a Tabu search heuristic that was restricted by tight lower bounds. Their results show that this type of approach provides good structure for configuring an ATM switch. Shyur, Lu, and Wen (1999) also presented a formulation of the spare capacity planning for network restoration using Tabu search. The results from their uphill and downhill procedures in the neighborhood structure exhibited better performance than other approaches they compared. Their results showed similar or better spare capacity/working capacity ratios than random problem experiments.

Genetic Algorithms

A group of techniques inspired by real-life processes of genetics and evolution called genetic algorithms can define neighborhood generation mechanisms (Bertsekas 1998). An existing solution is modified by "splicing and mutation" to obtain neighboring solutions. Initially, the methodology solved the traveling salesman problem that attempts to define the "best" method for traversing a collection of nodes. These approaches are problem-dependent and require a lot of trial and error but can be quite easy to implement according to Bertsekas (1998).

The literature regarding the application of genetic algorithms to telecommunication networks is rich and abundant and the following are a few of the reports using genetic algorithms. These were chosen to reflect the variation in use of this technique as well as the development of the application of this technique. Celli et al (1995) developed genetic algorithms to help optimize the design of the Italian national telephone system to develop B-ISDN services. Kumar et al (1995) applied genetic algorithms to the solution of various problems in the design of computer local area networks as compared to centralized systems. Dengiz et al (1997) developed genetic evolutionary algorithms to aid in the design of computer networks but added reliability as a design constraint. Garcia, Mahey, and LeBlanc (1998) presented a new generic auto-calibrating local search algorithm combined with a genetic algorithm to address multiperiod network expansions. Cheng (1998) used genetic algorithms to aid Kerbache and MacGregor in the design of backbone network layouts to define a more cost effective or reliable layout. Sayoud et al (2001) presented the development of a variation called steady state. This application minimizes the total installation cost of a telecommunications network by designing an optimal topology and assigning needed capacities. This approach included the option to terminate the procedure early with a reasonable good solution that satisfied feasibility requirements. Kumar et al (2002) put forth a multi-objective genetic algorithm procedure to define a network set-up while minimizing network delay and installation cost that were subject to reliability and flow constraints. To add QoS constraints to the development of an Industrial Ethernet network, Krommenacker et al (2002) proposed a genetic algorithm approach for the optimization and design of industrial control networks.

Lagrangian heuristics

Another area of great interest to researchers in optimization is defining the heuristics used to solve the NP-hard problem. Lagrangian heuristics or relaxations are an approach for obtaining the lower bounds to use in the branch-and-bound method (Bertsekas 1998). *"A key idea of Lagrangian relaxation is the minimization of the Lagrangian L (x, μ) over the set of remaining constraints that yields a lower bound to the optimal cost of the original problem"* (Bertsekas 1998, p. 493).

Pirkul and Gupta (1997) presented a topological design of centralized computer networks using a Lagrangian heuristic that solved the problem with gaps of 2.7% to 10.4% of the lower bound using a predefined number of concentrators. This type of approach may be applied to the design of access layers of networks. Holmberg and Yuan (1998) presented a common solution approach to solve fixed charged network design models, capacitated or uncapacitated, directed or undirected. They proposed a Lagrangian heuristic using Lagrangian relaxation, subgradient optimization, and primal heuristics. This approach easily solved small, constrained problems to a near optimal conclusion but the solution to larger more difficult problems needed more modifications.

Other Approaches

A novel use of knowledge management approaches presented by Dutta and Mitra (1993) was to integrate heuristic knowledge and optimization models to develop designs for communication networks. Suggestions from optimization models as well as heuristic knowledge interacting through an electronic blackboard developed a network design. A truth maintenance system records the justification for design choices and a dependency directed backtracking mechanism continues to choose other alternatives as warranted. This hybrid approach for tool development allows for the integration of many types of knowledge management resources used in decision-making.

Kerbache and MacGregor Smith (2000) presented a combination of approaches from other operations research areas. They presented combined optimization and analytical queuing network models to provide design methodologies. Using this approach, alternative designs were compared for average delay times and maximum throughputs. They developed an approximate analytical decomposition technique for modeling finite queuing networks called the Generalized Expansion Method, GEM, and used a mathematical optimization procedure to determine optimal routes using multi-objective parameters. Guha, Meyerson, and Managala (2000) reported first constant approximations for designing minimum cost hierarchical networks. First, they modeled hierarchical caching with caches are placed in layers. Each layer satisfied a fixed percentage of the demand. Then using the caching balance, traffic demands are routed.

Lakamraju, Koren, and Krishna (2000) presented another approach developing a series of filters relating to specified design requirements. Randomly generated network designs are passed sequentially through the filters and those that pass are on the short list of "good" designs. Rosenberg (2001) developed a dual ascent method that solves a sequence of dual uncapacitated facility location problems. A Steiner tree based heuristic was the basis for this method that provides a primal feasible design. This work improves upon the research presented by Kim and Tcha (1992).

Medova (1998) and Gurkan, Ozge and Robinson (1999) proposed stochastic programming optimization approaches. Medova (1998) developed a chance-constrained stochastic programming model for integrating multiple services in an ATM network. The model described was a prototype software system for network design and management. With the network topology as a given a chance-constrained stochastic program for network dimensioning and traffic management to support multiple classes of service is proposed. Gurkan, Ozge, and Robinson (1999) described a stochastic optimization problem with stochastic constraints to solve a network design problem. They find link capacities for a stochastic network with random demand and supply at each node, minimize the sum of the capacity cost and measure the expected blocking rate.

Summary

As described above the network design problem has been well investigated using many different approaches. Each approach added something to the specific focus chosen but there is still not one overwhelmingly better approach than another. The following is a generally well-accepted network design problem formulation adopted from Grover and Doucette (2001).

Network Design Problem Formulation

The link capacity design of the topology can be calculated using a modification of the fixed-charge plus routing (FCR) problem statement of Grover and Doucette (2001). The capacitated version will have existing edge capacities and/or edge capacity limitations to be respected.

- n is the number of modes, **N** is the set of such nodes
- **A** is the set of (n(n-1)/2) possible bi-directional edges in the graph on the set of nodes N.
- **D** is the set of all non-zero demand quantities exchanged between nodes, indexed by *r*.
- d^r is the amount of demand associated with the *r*th demand pair in **D**. Demands are treated as being unidirectional but the unidirectional information implies the bi-directional capacity design corresponding to a real transport network.
- O[*r*] is the node that is the origin for the *r*th demand pair in D. T[r] is the corresponding target or destination.
- c_{ij} (= c_{ji}) is the incremental cost of adding one unit of capacity to edge (i,j).
- F_{ij} is the fixed cost for establishment of an edge in the graph (directionally) from node i to node j. (The full fixed charge for the bi-directional edge is effected through asserting symmetry of the edge decision variables below.)
- w^r_{ij} is the amount of working flow routed over the edge between nodes (i,j) in the direction from i to j for relation *r*.
- w_{ij} is the working capacity assigned to the edge between nodes (i,j) to support all working flows routed over that edge in the (i,j) direction.
- $\partial_{ij} = \partial_{ji}$ is the I/O decision variable indicating whether an edge in the graph is to exist between nodes (i,j) in the design, Equals 1 if the edge is selected, zero otherwise.
- K is an arbitrary but large positive constant, larger than any expected accumulation of working capacity on any one edge in the solution.

FCR: Min $$\sum_{i,j \in A} \{c_{ij} * w_{ij} + F_{ij} * \partial_{ij}\} \qquad 2.8$$

s.t.

$$\Sigma\, w^r_{nj} = d^r \text{ for all } r \in D,\ n = O[r]. \qquad 2.9$$

$$\sum_{nj \in A}$$

$$\sum_{jn \in A} w^r_{jn} = d^r \quad \text{for all } r \in D,\ n = T[r]. \qquad 2.10$$

$$\sum_{in \in A} w^r_{in} \quad - \quad \sum_{nj \in A} w^r_{nj} = 0 \quad \text{for all } r \in D, \text{ for all } n \notin \{O([r],T[r])\} \qquad 2.11$$

$$w_{ij} = \sum w^r_{ij} \quad \text{for all } i,j \in A \qquad 2.12$$

$$w_{ij} \ <= K * \partial_{ij},\ \partial_{ij} = \partial_{ji},\ \partial_{ij} \in \{0,1\}, \quad w_{ij} \text{ integer for all } ij \in A. \qquad 2.13$$

2.8 objective statement, minimize cost of network while routing all traffic demands between node pairs.
2.9 2.10 and 2.11 are the flow balance constraints of the node-arc transportation problem. They assert that the total source flow equals the demand and that the total sink flow also equals the demand, and that no net sourcing or sinking of flow for the given O-D pair occurs at any other node (i.e., "trans-shipment").
2.12 Definition of required edge capacity in terms of the simultaneous flows over the edge
2.13 set of constraints that establish the boundary on w_{ij}, the 0,1 values for ∂_{ij}, and the integer constraint on the working capacity of the link.

III. Multi-level or Hierarchical network design

Most physical networks today are a mesh of nodes and links with logical topologies overlaid on the mesh. Each logical overlay operates as a separate network independent of other networks. Increasingly the need is to merge these distinct networks into one unit operating as one network with several layers. This section of this paper will review some of the relevant literature relating to combining topologies and using logical topology design, LTD, to overlay logical topologies on physical topologies to expand network functionality as well as the general problem of network link design. There has been a wide variety of research published covering applying optimization techniques to multi-level telecommunication networks. Some of the classic contributions in the field are Cahn (1998), Balakrishnan et al (1995), Kershenbaum (1993), Chang and Gavish (1993), and Gavish (1991).

The design process to define the optimal topology or near optimal topology for a network should take into account the diverse nature of the traffic carried. Each traffic type has its own special characteristics such as tolerance for delay, restoration needs and tolerance for packet loss. The legacy optical core networks for the most part are ring topologies that are optimized to give the best performance for voice traffic. While rings provide fast restoration needed to support circuit switched voice traffic, mesh networks provide greater efficiency in the use of network resources and can be more economical to deploy. With the advent of optical switches and DWDM, mesh topologies were optimally designed to carry data traffic.

An early approach to this concept was to design a hierarchical network with two physical topologies. Lee, Ro, and Tcha (1993) present a two-level hierarchical network structure with the upper level as a hub-ring and the lower level access network with star-type connections. By partitioning the whole problem into two easy problems, a dual-base approach can be used to formulate the design problem into a mixed 0-1 integer-programming model. A heuristic procedure is used on the dual-based lower-bounding solution to construct a primal feasible solution from the dual procedure.

Brown et al (1994) presented a comparison of two architectures, mesh/ring, and mesh/arc for survivable self-healing transport networks. In mesh/arc networks, the core consisted entirely of mesh connections and the access portion of the network is either incomplete rings or “arcs” of add-drop multiplexers. Mesh/ring networks are mesh core networks with ring topologies for access. They presented the case that mesh/arc architecture topologies could recover from failure relatively quickly and were cheaper to deploy than mesh/rings. Mesh/arc were also more flexible in reacting to traffic demand changes.

Chang and Gavish (1993) presented a formulation using a primal heuristic and a dual-based lower-bounding procedure for subproblems of the larger overall problem. Lagrangian relaxation was used to decompose the problem into two independent optimization problems; a continuous routing, capacity expansion problem, and a minimal spanning tree problem. Combining these subproblems with a lower bound for the main problem, a branch-and-bound procedure to do a global search using a heuristic was described to solve the problem.

Yoon, Baek, and Tcha (1998) presented a design methodology for a distributed fiber transport network using hubbing technology. This formulation of the complex network design problem redefined commodity flows using a dual-based heuristic that yielded near-optimal designs. Mukherjee, et al (1996) presented the concept of an arbitrary virtual topology embedded on a given physical fiber network to exploit the advantages of wavelength multiplexers and optical switches in wavelength routing. They introduced the concept of “all-optical lightpaths” that are set up to carry packets as far as possible over the stream of wavelengths in the optical domain only converting back to electronic domain when necessary. Their approach was to formulate an optimization problem that optimally selected a virtual topology subject to transceiver and wavelength constraints using two functions, first to minimize the network average packet delay and second, to maximize the scale factor by which the traffic matrix can be scaled up. Since these types of problems quickly become NP-hard they used a heuristic approach to solve the problem. It was an iterative approach that combined simulated annealing algorithms to search for a good topology and flow deviation approaches to optimally route the traffic on the virtual topology.

Guo, Acampora, and Zhang (1997) described a hyper-cluster solution for scalable and reconfigurable wide-area lightwave network architecture. A hyper-cluster approach uses a logical hierarchy for addressing but insures that all nodes have the same number of transceivers. Hyper-clusters are a cluster of regular graphs with a clustering structure that follows traffic distribution. Prathombutr and Park (2002), as a way to design a multi-layer optical network, using logical topologies presented clustering to create subdivisions. The logical topology, a set of lightpaths formed to serve traffic demands, was created by analyzing traffic demand and the physical topology to classify the nodes into either Optical layer or Electrical layer. The clustering method uses the multivariate analysis to cluster the data by a combination of characteristics of the network nodes. These characteristics can include cost of equipment, location, policy, or other factors deemed important for the design.

Tran and Beling (1998) presented a heuristic approach to design the topology of a two-tiered network by integrating access area and backbone design problems into a single mathematical program. Since this type of problem is quite difficult to solve, usually the problem is subdivided. While enhancing the solvability of the problem, subdividing can produce inferior results. Using simple probability models on link costs also simplifies the procedure.

Banerjee and Mukherjee (2000) defined a solution to the LTD using an exact integer linear programming formulation that minimized the average packet hop distance. This approach was equivalent to maximizing the total network throughput under balanced flows using lightpaths. Balancing resource tradeoffs between transceivers and switch sizes can create a well-balanced network with good utilization rates. Additionally, their problem formulation provided a reconfiguration methodology to allow the virtual topology to adapt to changing traffic conditions.

LTD defines logical topologies that will minimize congestion (Krishnaswamy and Sivarajan 2001). The authors present a general linear formulation that considered routing traffic demands by routing and assigning wavelengths to lightpaths as a combined optimization problem. Their solution worked well for small examples but for large networks, the integer constraints were relaxed and a lower bound on congestion was established. Another approach to LTD presented by Lee et al (2000) used a multi-commodity flow approach to define the problem. They created a general cost function that covered all system components and presented two solutions, one based on integer programming and the other on heuristics developed to solve this problem. The integer-programming approach yielded the network configuration with the minimum implementation cost but the problem was of immense size. The heuristic based on a minimum variance algorithm performed considerably better than other presently used algorithms such as shortest path.

Sen, Bandyopadhyay, and Sinha (2001) presented an alternative method for examining the structure of the LTD problem. This work examined the problem from a graph theory perspective. While previous graph theory work presented the overlay as a regular structure such as hypercube, de Bruijn graph, Kautz graph, and Cayley graph, this paper proposed a generalized multimesh (GM), a semi regular structure. By developing a new metric, flow numbers can be used to evaluate topologies. Flow number is the minimum threshold capacity on the links in that network that is able to sustain a traffic flow. Much work has been done in the graph theory examining how to connect or create overlays but very little has been applied to telecommunication network design.

LTD problems can focus on different parameters such as reliability. Arakawa, Katou, and Murata (2003) present a new concept called "Quality of Reliability (QoR), a realization of QoS with respect to the reliability needed in a WDM network. QoR was defined in terms of the recovery time from when a failure occurs to when traffic on the affected primary lightpath is switched to the backup lightpath." A heuristic algorithm was proposed that designed a logical topology that satisfies the QoR requirement set forth for every node pair. Their objective was to minimize the number of wavelengths needed in the logical topology to carry the traffic required QoR. Initial results from this approach indicate that 25% fewer wavelengths are needed than with other algorithms.

Grover and Doucette (2002) developed a methodology using a meta-mesh chain of subnetworks to increase the capacity efficiency on spare facility graphs. A loop-back-type space capacity is provided only for the working demands that begin or end in a chain and not for the entire flow that crosses a chain. The express flows (those that begin or end elsewhere) are entirely mesh-protected within the meta-mesh graph that is of higher average degree of nodal connectivity. This approach creates a new class of restoration that is intermediate between span and path restoration with most of the efficiency of path restoration and nearing the localized nature and speed of span restoration.

Cruz, MacGregor Smith, and Mateus (1999) developed a solution to solve to optimality the uncapacitated fixed-charge network flow problem (FCN) using a Lagrangian relaxation to define boundaries. Their approach was to develop a solution to the multi-level network optimization (MLNO) problem that integrates into the same model location, topology and dimensioning of a network. While the initial application of this work was for the design of electrical power systems, interconnecting powerstations and load centers of a national power grid, the multi-characteristic nature of a telecommunications network is another area that this approach might prove powerful.

Dahl, Martin, and Stoer (1998) presented a routing solution through virtual paths in a layered network. Their solution was developed using an integer linear programming model where 0-1 variables represented different paths. A cutting plane approach produced reasonable results for solving real world pipe selection and routing paths. Peusch, Kuri, and Gagnaire (2002) proposed an approach to the multi-commodity flow problem used to formulate the LTD problem and the lightpath routing (LR) problem using mixed integer linear programming techniques. By tackling the two problems, LTD and LR, with separate models the problem becomes realistically solvable. By modularizing the approach, different combinations of the optimization models and the objective functions are developed.

Grosso et al (2001) used Tabu search optimization meta-heuristics to develop a logical topology over a WDM wavelength routed network. They formulated the LTD problem for traffic affected by a degree of uncertainty using a stochastic description of the traffic pattern, an existing topology, and a multi-hop routing strategy. Their results suggest that local search techniques such as Tabu are promising and worthy of further investigation. Shyur and Wen (2001) also presented a methodology for solving a similar problem of virtual paths in an ATM system. Their approach seemed to show better performance than the existing random path algorithm especially as the problem size grows larger.

Marsan et al (2002) presented a mixed integer linear programming, MILP, formulization of the optimal logical topology, LTD, with multicast traffic under deterministic and stochastic traffic patterns. Using greedy and metaheuristic (Tabu) algorithms, an optimal design to the NP-hard problem was found. Lower bounds and numerical results showed that their proposed metaheuristic Tabu-based formulation outperformed other greedy approaches. Extending the problem to handle changing traffic patterns their proposed methodology found no degradation in the solution.

III. Multi-level (or Hierarchical) Network Design Problem

The multi-level or hierarchical network optimization (MLNO) problem is formulated using a similar approach to that of the basic network design problem but uses different cost functions for each level in the design. Cruz, MacGregor Smith, and Mateus (1999) present the MLNO as follows:

L = {set of all levels in design $l = 1,2,\ldots m$}
R^l = {set of lth level candidate supply nodes}
c^l_{ij} = non-negative per unit cost for the lth level flow on arc (i,j)
x^l_{ij} = lth level flow through arc (i,j)

f^{l}_{ij} = non-negative fixed cost for using arc (i,j) to support lth level flow
y^{l}_{ij} = Boolean variable which assumes the value 1 or 0 depending on whether or not the arc (i,j) is being used to support lth level flow
f_i = non-negative allocation for the lth level candidate supply node i
z_i = Boolean variable which is set to 1 or 0 depending on whether or not the node i is being selected to provide lth level flow

$$\text{Min} \sum_{l \in L} \left[\sum_{(i,j) \in A} (c_{ij}^{l} x_{ij}^{l} + f_{ij}^{l} y_{ij}^{l}) + \sum_{i \in R^{l}} f_i z_i \right] \qquad 2.14$$

with similar constraints to those listed in section II Network Topology Design.

Summary

The multi-level network design has received much attention so that combining designs can improve the performance of the network. One of the basic premises is that there will be different costs for each level of the network. This study will use the same cost function for each link regardless whether it is functioning as a backbone link connecting concentrators (backbone nodes) or access link connecting terminals (end nodes) to concentrators. Using the same cost function for each level, the multi-level concept therefore is not a part of this problem.

IV. Multi-period Design Approaches that include Growth

Adding the multi-period component to the problem enlarges the problem of network topology and capacity design to include the concept of expansion. Most approaches are iterative techniques that compare the formulation results for each planning phase to determine the optimal solution by either sequential single period formulations or dynamic formulations. Sequential single period formulations require the output of period t be the input of the t+1 period.

Chang and Gavish (1993) present an LP formulation of the design and capacity expansion problem with a family of heuristics and a dual-based lower bounding procedure using Lagrangian relation and a global search strategy. Garcia, Mahey, and LeBlanc (1998) formulated a model with discrete characteristics that have changing monthly (but not minute-by-minute) point-to-point traffic requirements and budget constraints. This formulation does not include congestion and capacity considerations. It uses a generic self-calibrating local improvement template algorithm purported to improve the performance and flexibility of classical approaches that solve the design of the network with changing traffic requirements.

Pickavet and Demeester (1999) introduced a mathematical model of the multi-period reliable network-planning (MPRNP) problem to compare single-period planning verses multi-period planning. They used two different techniques, a sequential single-period approach, and an integrated multi-period approach. The multi-period approach puts more emphasis on scheduling the right investments at the right time. Extensive simulations on a wide range of problem instances showed that the integrated multi-period approach leads to a cost savings (average 4.4%) on the problem investments over the more traditional sequential single-period planning approach. The relative differences were rather small but when comparing the three cost model approaches the choice of algorithmic model used was more important than the cost model. No clear influence of network size, relative growth demand over planning horizon, or presence of an initial network was detected.

Ouorou, Luna, and Mahey (2001) looked at the multicommodity network expansion under changing demands problem. They applied a generalized decomposition method to a mixed integer nonlinear formulation of the integrated problem of network design and decomposition. Their two-step procedure incorporates a master program level that proposed to expand capacities on some arcs and a convex cost multicommodity flow subproblem including price sensitive demands. This topology-tuning approach combines the allocation of bandwidth with the routing of traffic to develop an effective solution.

IV. Multi-period Design Problem

Modified after Chang and Gavish (1993) the following is a formulation of the network design problem with multiple periods.

Definitions:
A = set of all links $_{ij}$.
c_{ij} = cost function for each link from i to j per capacity unit.

w_{ij} = capacity needed on link i to j.

T = { set of planning periods, t = 1, 2,…n}.

$$\text{Min} \sum_{t \in T} \sum_{i,j \in A} \{c_{ij}^t * w_{ij}^t\} \qquad 2.15$$

subject to:

similar constraints as above plus

$c_{ij}^t - c_{ij}^{t-1} >= 0$, for all $i,j \in A$ and for all $t \in T$.

Summary

A multi-period design problem allows the optimization study to examine changes in multiple periods and define by time period, monetary investments or growth. Pickavet and Demeester, 1999, showed that the only important factor between sequential single period investment and multiperiod dynamic was the timing of investments. Since this study is examining the impact of nodal distribution strategies the sequential single period process will be used because in this study there will be no real time associated with the growth periods only the amount of growth.

Efficient or Production Frontier

Using the Efficient Frontier, a concept borrowed from finance, agriculture economics, and operations research literature, (Markowitz, 1959 and Farrell, 1957) the design process creates a set of designs that are efficient combination of chosen parameters. The efficient frontier represents a suite of efficient combinations of nodes and links for the problem and the cost of any design can be related to the frontier thereby essentially measuring the efficiency of that design. A brief discussion of the Efficient Frontier concept will be presented but interested readers are referred to Fare et.al (1994) and Copeland et.al (2005) for more comprehensive discussions.

By developing a set of designs by doing sensitivity analyses, the designer creates an envelope of cost functions. The lower boundary of this envelope is the efficient frontier or suite of "best" designs as far as the parameters used for the optimization. The efficiency of any design is the relationship of the final cost of a design to the frontier. Evaluating the distance any point is from the efficient frontier gives a measure of the efficiency of the design as related to the parameters used in the optimization. The closer the design cost to the efficient frontier the more efficient the design. By understanding the impact of design changes to the final cost of the network, well-informed design changes can be implemented with regard to final or long term cost.

Traffic models and projections

There are several previously published approaches to developing the traffic matrix (Cahn 1998). The first method is to assume an equal level of traffic between all node pairs. While this is the simplest approach, it is the least realistic. It is often used in the initial proof-of-concept testing of the methodology as well as exploring the effects of other parameters outside of traffic modeling. Second, population density of a city is used as the size factor for determining the type and amount of traffic between city pairs. Larger population centers would exchange more traffic than smaller population centers and may grow at different rates. This approach is more realistic but is more complex.

For the most part, network-planning models in use today were designed for voice traffic on the plain old telephone system networks (POTS). The growing impact of data traffic associated with the explosive growth of the Internet and multimedia applications such as KaZaA that deliver MP3 music files has changed the focus of network traffic models. Some reports estimate that more than 60% of traffic carried on networks today is data and the growth of data related traffic is not expected to slow. While the amount of data traffic carried on networks is growing, the main revenue source for network carriers is still voice traffic (Maesschalck et al 2003). Thus, traffic models that emulate connection oriented circuit switched traffic still dominate the network-planning field.

New traffic models are needed to accurately emulate the changing nature of today's traffic but also carefully adjust for voice traffic, the major source of revenue for carriers. Dwivedi and Wagner (2000) presented a model that differentiated between three traffic types: voice traffic, transaction data traffic (mainly business generated modem and IP traffic), and Internet traffic (IP traffic not related to business environment, mainly downloading of WWW pages). This traffic model was modified and used by Maesschalck et al (2003) for a topology comparison of the Pan-European carrier networks. Generally, network planners when developing new traffic models either use historical trends based on internal data or on various models that relate population density/size for a given area and distance between city pairs as predictors of volumes of traffic (Cahn 1998). These models usually make growth assumptions based on the population census data for a given geographic area that work very well for predicting voice traffic change but do not differentiate traffic types. In today's Internet environment the basis has changed and new models are needed. Dwivedi and Wagner (2000) presented a traffic analysis with a generalization of Internet traffic in1999 captured by the following: "*for voice traffic assume 14 minutes of long-distance traffic person per day, 5 minutes of transaction modem use per non-production employee per day, and 25 minutes of continuous modem access to the internet per host per day*" (Dwivedi and Wagner 2000). Using this data to develop proportionality constraints, the total traffic pattern is best modeled using the following equation:

$$\text{Voice traffic (i, j)} = K_v * P_i * P_j / D_{ij}$$

$$\text{Transaction data traffic (i,j)} = K_T * E_i * E_j / \sqrt{D_{ij}}$$

Internet traffic (i,j) $= K_I * H_i * H_j$

The traffic between cities i and j depends on the total population P_i, the non-production business employees E_i, and the number of Internet host H_i, in each city as well as the distance D_{ij} between the two cities of interest. Growth rates based on US census data were calculated and average growth rate for each traffic type was predicted. Voice traffic was extrapolated to grow at 8% per year, transaction data traffic at 34% per year and Internet traffic at 157% per year. This analysis was done during 1998-1999, the peak of the e-commerce dotcom boom times, and estimations seemed good for the times. Since then, published reports have indicated that the number of Internet users has grown at about 40% per year (Maesschalck et al 2003). Other recent reports have indicated that Internet traffic is expected to double every year (Legard 2003). Even with the uncertainty in Internet traffic growth, breaking traffic-growth projections into individual components is certainly a valid approach, although more complex than using one type of traffic.

Summary

In summary, the scope and depth of the efforts to define methods that develop the optimal or near optimal network design are significant. Much work has been done to develop techniques for network design to define optimal or near optimal results but still there is no clear best method or solution technique. Logical overlays or virtual topologies and multiple-growth periods have had some attention but no studies were found that compared nodal distribution strategies. The development of traffic models is still very subjective and much work needs to be done. A traffic matrix using population-based values is the most commonly used process most likely due to the ease of data access, usually national population surveys. The research effort presented in this paper will focus the impact of nodal placement strategy upon long term cost effectiveness of a new service overlay on a legacy network topology. The next chapter will discuss the mathematical model used in this study.

CHAPTER 3 MATHEMATICAL MODEL

This study set out to answer three questions. First, what is the most cost effective switch distribution strategy for a new service overlay on an existing network for long term growth; overlay switches distributed over many legacy nodes, or overlay switches distributed over one or a very few legacy nodes? Second, what does the efficient frontier of overlay switch designs and costs look like as overlay network designs deviate from the best? And lastly, are there heuristics that can be defined to help point the way towards the least cost design? This section will present and analyze the cost model used to answer these three questions.

Mathematical Model

For this study the cost for each growth period of the new service overlay is defined by the following equation (Equation 3.1)

$$\text{Overlay System Cost} = \text{Overlay Switch Cost} + \text{Overlay Link Cost.} \qquad 3.1$$

The overlay switch cost consists of two parts; the cost for the switch chassis and the cost for each of the ports or connections needed to accommodate the traffic flow through the overlay switches. Overlay link cost is related to the length of the links and the capacity of the links used in the overlay.

Overlay Switch Cost = Overlay Chassis Cost + Overlay Port Cost

An overlay network consists of backbone nodes and access nodes. An access node will not require an overlay switch, while a backbone node will require an overlay switch. Overlay backbone switches will have a minimum of two overlay logical connections, and overlay access nodes will attach to a backbone switch via a single logical connection utilizing assets of the underlying legacy network.

Overlay Chassis Cost

The chassis cost for each overlay backbone node is captured by a function that is dependent upon the number of connections or flows that must move through the switch. As the number of traffic flows increases beyond a certain point, a larger more expensive switch may be required. The actual cost function of the chassis increases in a step-like manner based on the number of connections needed (Figure 1). An alternative method would be to model the chassis cost with a smooth function that shows a similar reduction in unit cost with growth in the size of the switch chassis (Figure 1). There are some economies of scale that can be achieved with larger switches. Usually there will be a reduction in cost per unit connection over smaller switches.

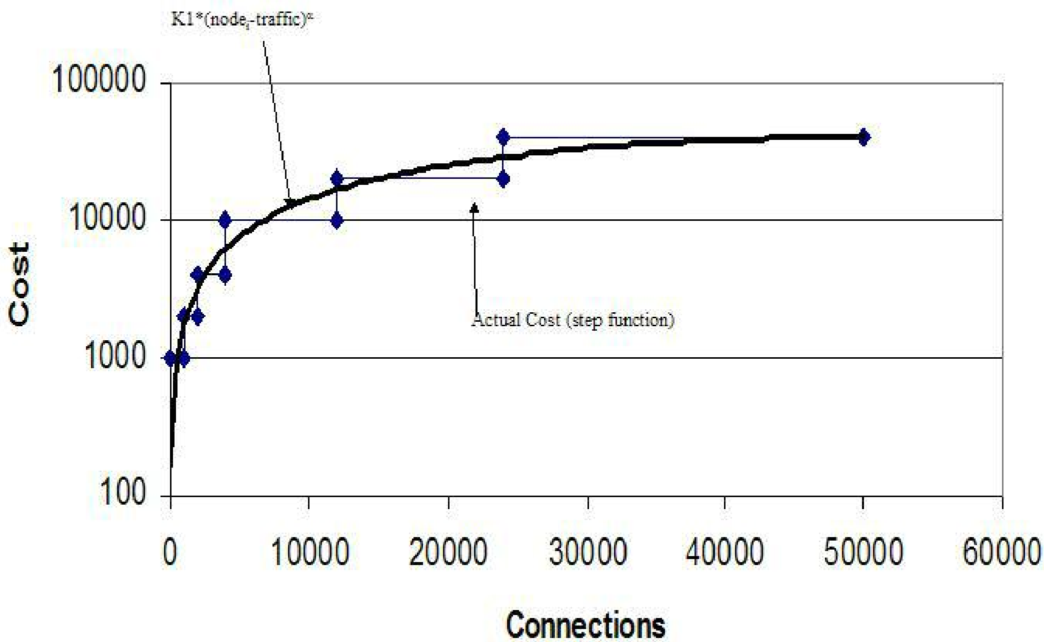


Figure 1 Cost function for the overlay switch chassis. While the actual function is step-like, it can be modeled with a function that shows the reduction in unit cost with growth in size of the switch chassis. This is a log-normal plot.

Individual Overlay Chassis Cost

The experimental method discussed in chapter 4 uses the actual step function to model chassis costs. The approximation of the step function is used in this chapter, as it is easier to visualize changes in switch costs as traffic flow increases with the smooth approximation function. The individual overlay switch chassis cost is defined by the following equation (Equation 3.2).

$$\text{Individual Overlay Chassis Cost} = x_i \ (C_1 + K_1 * (node_i\_traffic)^{\alpha}), \qquad 3.2$$

x_i = binary constant, (0 or 1) indicating whether or not an overlay switch is located at node i.

$node_i\_traffic$ = the amount of traffic flow that will move through the node i. Also called the number of connections needed at a switch.

C_1 = initial cost of chassis

K_1 = arbitrary unit cost of a chassis

α = Exponential factor that modifies the impact of traffic flow upon the size of the switch chassis. $\alpha \leq 1$. The closer α is to 1, the closer the function is to linear. A value of α less than 1 allows economies of scale to be captured. $\alpha = 1$ in this study.

Total Overlay Chassis Cost

The total overlay chassis cost for the network is modeled by summing the chassis costs over the entire network overlay (Equation 3.3).

$$\text{Total Overlay Chassis Cost} = \sum_{i=1}^{n} [x_i (C_1 + K_1 * node_i\_traffic^{\alpha})] \qquad 3.3$$

N = set of nodes in the network overlay, {1, …, n}.

Overlay Port Costs

Port/connection costs are the second part of the overlay switch cost. For each connection that is needed to and from an overlay switch there is a per port/connection cost. Generally there is a flat per port or connection cost as modeled by Figure 2. Although, economy of scale effects can be seen when purchasing large volumes, so β (as defined below) can be < 1.

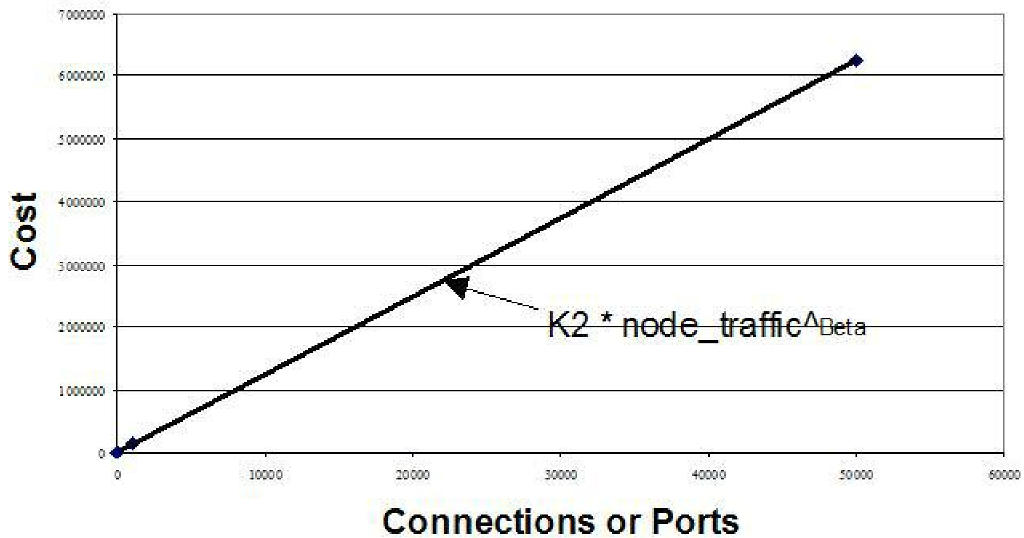


Figure 2 Connection or port cost model. This is modeled by a simple linear cost per unit function. Although with different cost functions β could be less than 1 reflecting a cost structure that allows for economies of scale.

Individual Overlay Switch Port Costs

Port cost for an individual overlay switch is modeled by the following equation (Equation 3.4).

$$\text{Port Cost}_i = K_2 * (\text{node}_i\text{\_traffic})^{\beta} \qquad 3.4$$

K_2 = an arbitrary unit cost assigned to each connection.

β = an exponential factor that will describe the shape of the curve, $\beta \leq 1$. For this study $\beta = 1$ but with different cost functions β could be less than 1 reflecting economies of scale.

Total Overlay Port Cost

Total port cost is found by summing the individual overlay switch port costs together over the entire overlay network design (Equation 3.5).

$$\text{Total Port Cost} = \sum_{i=1}^{n} x_i \text{ Port Cost}_i = K_2 \sum_{i=1}^{n} x_i \, (\text{node}_i\text{\_traffic})^{\beta} \qquad 3.5$$

Overlay Link Costs

Overlay link costs are a function related to the length of the link and the capacity or size of link used in the overlay to deliver the traffic flow (Figure 3).

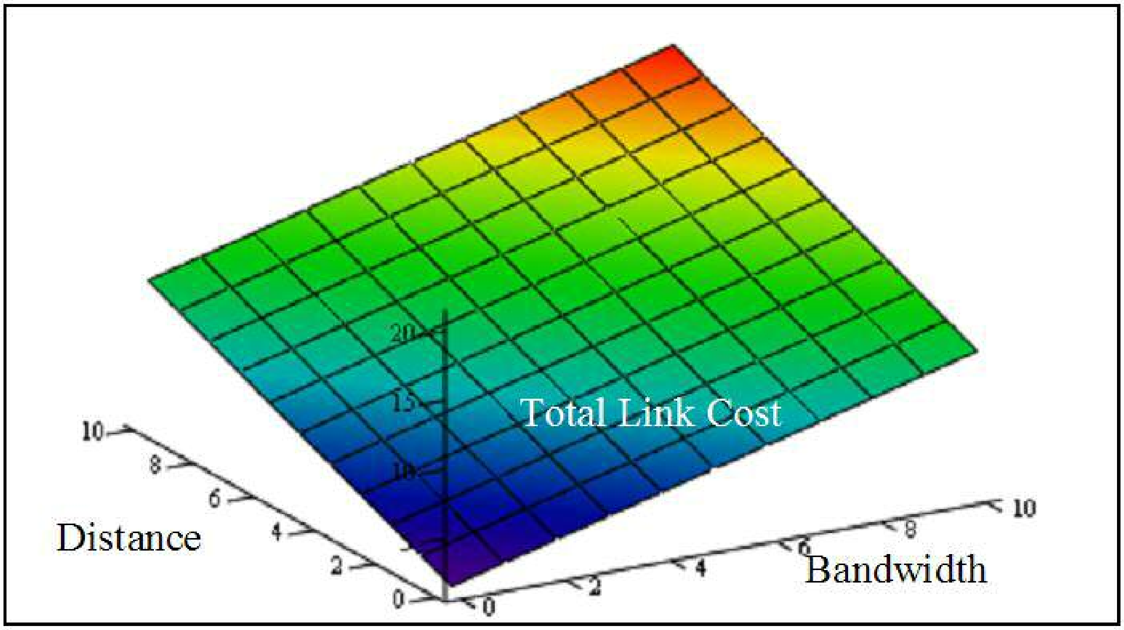


Figure 3 Overlay Link Cost - Cost for overlay link cost is based on both the amount of bandwidth or capacity needed and the length of the link. The shape of the cost function will depend upon the relationship between distance and bandwidth cost. The relative relationship between bandwidth and distance will determine the shape of the curve. For this figure the cost of distance and the cost of bandwidth are approximately the same.

Individual Overlay Link Costs

Overlay link costs for an individual link are modeled by the following equation (Equation 3.6).

$$\text{Overlay Link Costs} = K_L * \text{Capacity}_{ij}^{\chi} * \text{Distance}_{ij}^{\delta} \qquad 3.6$$

K_L = an arbitrary unit cost factor

Capacity_{ij} = link capacity from node i to node j

$Distance_{ij}$ = route miles from node i to node j

χ and δ; exponential factor for traffic and distance respectively. For this study χ and $\delta = 1$ but other cost models could have these values less than one. For instance, some Frame Relay service providers do not price connectivity in terms of distance so δ could be equal to zero. Other cost models with $\chi < 1$ could show economies of scale related to capacity.

Total Overlay Link Costs

Total overlay link costs for overlay designs are modeled by summing the cost for each over the entire overlay network (Equation 3.7).

$$\text{Total Overlay Link Costs} = K_L * \sum_{i=1}^{n} \sum_{j=1}^{n} (Capacity_{ij})^{\chi} * (Distance_{ij})^{\delta} \qquad 3.7$$

Total Cost of a New Service Overlay on a Legacy Network

The total cost of a new service overlay would be the sum of the above stated equations, (Equations 3.3, 3.5, and 3.7) represented by Equation 3.8.

Total Overlay System Cost = total Overlay Chassis Cost + total Overlay Port Cost + total Overlay Link Cost

$$= \sum_{i=1}^{n} [x_i \; (C_1 + K_1 * (node_i\_traffic)^{\alpha})]$$

$$+ \quad K_2 \sum_{i=1}^{n} x_i \, (node_i\_traffic)^{\beta}$$

$$+ \quad K_L * \sum_{i=1}^{n} \sum_{j=1}^{n} (traffic_{ij})^{\chi} * (distance_{ij})^{\delta} \qquad 3.8$$

$_i$ = indicates which node from the set N= {1,2,…,n} where n is the total number of nodes in the overlay design. The set of i = the set of j.

To understand how to minimize the total cost of the overlay network as defined in Equation 3.8 above, the components of overlay switch and overlay link are best examined separately. Minimizing overlay switch costs, including both chassis and port costs, requires that the number of overlay switches be minimized as well as the sum of the traffic flows per node be minimized so that at each node there will be the fewest ports and smallest chassis. To accomplish the fewest number of ports, generally, each traffic flow should be routed through the minimal number of overlay switches. Each flow should be moved along the most direct path, i.e. the path with the fewest hops in the overlay network. Minimizing the number of hops would be easiest in overlay designs with high connectivity. Designs with minimal number of paths, such as a ring topology, would force all traffic flows to follow the same paths thereby creating the need for larger switches that handle more traffic. Note though, that if α and/or $\beta < 1$, economies of scale may make it cheaper to route certain traffic flows through a larger number of switches.

To minimize overlay link costs, traffic flows should generally be routed along the shortest path. In this study where the full impact of the cost of distance is a part of the cost model, i.e. $\delta = 1$, the shortest path is least costly. If χ or δ are < 1, link costs might very well be minimized by aggregating lightly used direct links onto two or more indirect links that are more heavily loaded. Examining the impact of link utilization, when χ or δ are < 1, upon the question of overlay switch distribution was not considered in the case studies presented in following chapters. This is a subject for future research.

Minimizing overlay switch costs and overlay links costs can be but are not necessarily mutually compatible. The shortest path is not always the one with the fewest hops.

Additionally, whether switch or link costs will dominate the process depends also upon the relative relationship of the values of the unit cost functions. The next section of this chapter will examine this aspect.

Cost Relationships

The relative relationship between the unit cost of overlay links and the unit cost of overlay switches will tend to determine which factor in the total cost equation (Equation 3.8) will have the most impact. This discussion focuses entirely on the overlay cost functions and not the legacy network. Figures 4, 5, and 6 show the impact of varying the relationship between the unit overlay link costs, K_L, and unit overlay switch costs, K_S. K_S is a generalized sum of K_1 and K_2 defined above as the unit cost of overlay switch ports and overlay switch chassis. When the unit cost of an overlay link, K_L, is significantly greater than the unit cost of an overlay switch, K_S, or $K_L >> K_S$, the driving factor towards the total cost will tend to be the cost of links (Figure 4).

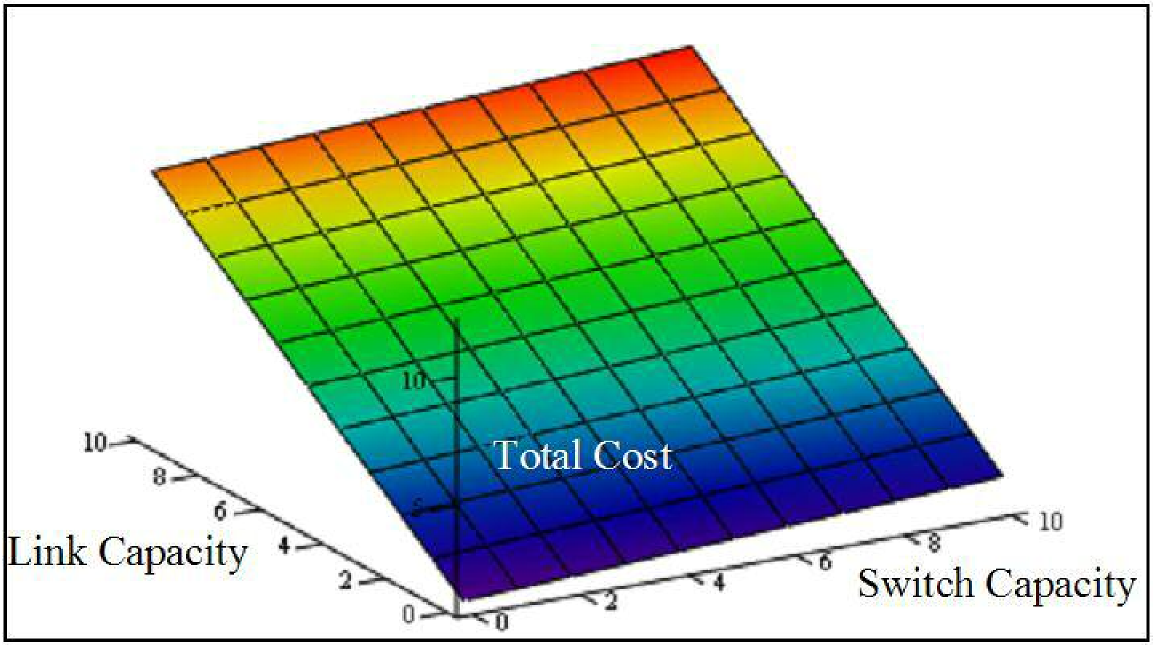


Figure 4 Cost functions where Overlay Link Unit, K_L, Costs >> Overlay Switch Unit, K_S, Costs show that in this scenario total Link costs are the driving factor in the total cost of the network because unit link costs are much greater than unit switch costs.

Unit link costs, K_L, are driven by both the cost of adding additional capacity to the legacy link and the cost of that capacity per distance unit. When overlay link costs, K_L, are significantly greater than the overlay switch costs it is the impact of link costs that likely drives the total cost of the network. Conversely, when $K_S >> K_L$, overlay switch costs are the probable driving factor (Figure 5). When the two factors are similar or equal, $K_S \approx K_L$, then both will have an approximately equal impact (Figure 6). Predictably the larger of the two unit cost functions, links or switches, will tend to drive the shape of the total overlay cost function. When the two functions are similar other factors such as legacy network characteristics will likely drive the impact of each function.

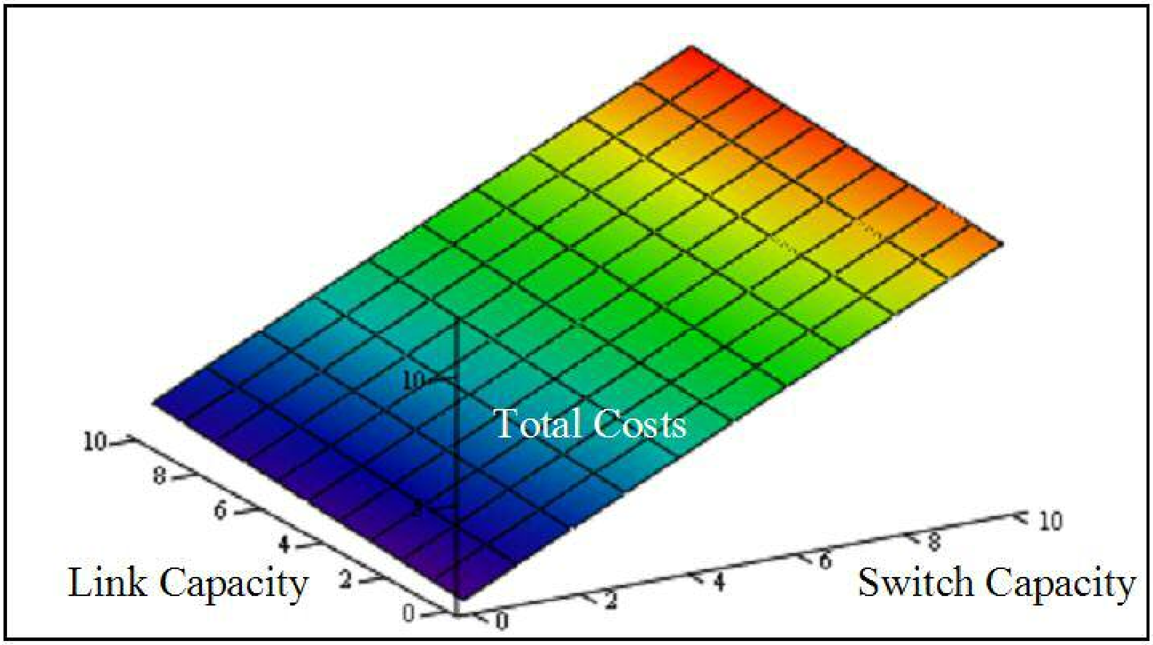


Figure 5 Overlay Link Unit K_L costs << Overlay Switch Unit K_S costs. In this scenario switch/port cost is the driving factor.

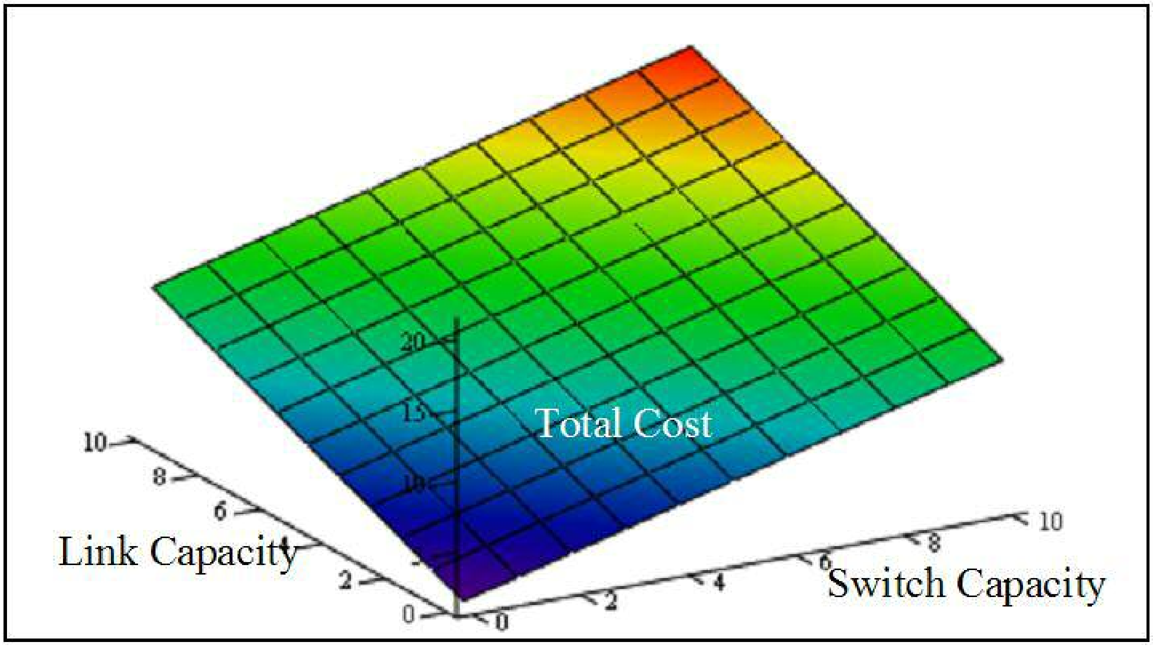


Figure 6 Overlay Link Unit K_L Cost ≈ Overlay Switch Unit K_S Cost. When each factor, link and switch costs are about equal, each will have about the same impact to the total cost function.

This discussion while relatively straightforward shows the impact on the final overlay total cost of changing the relative relationship of the parts of the total function. The previous discussion is specific to α, β, χ and $\delta = 1$. It becomes more complicated when economies of scale come into play and α, β, χ and $\delta \neq 1$. In the telecommunications industry, there are many approaches to determining cost functions and the impact of each factor on the total function. Future research could vary the values of α, β, χ, and δ and examine the impact of economies of scale.

Overlay link unit costs can dominate when constructing a brand new overlay network requires laying cable, building all the physical support hardware for the cable links, and all of the associated installation costs. When a network operator decides to put additional functionality on its legacy network, additional link unit costs might be limited to those associated with reassigning link capacity that could be less than the cost of the overlay switches and other switch related costs. Another example of a cost function where the impact of link costs might be very low would be a Frame Relay revenue function. In this case, the user might pay for just the bandwidth committed and not the distance of the links used. While simplified, the examples presented above do represent real world cost functions.

The purpose of this research was to investigate the impact of the number and location of overlay switches on the total overlay cost function. In some overlay designs, the cost of adding link capacity would dominate the cost function, while in others the switch cost function could dominate. The North American, NSFNet and Pan-European test cases examined in this study used overlay switch and overlay link costs of similar relative value, though K_S was greater than K_L. For this study and α, β, χ, and δ, = 1. Other relationships of α, β, χ, δ, and K would be the subject of future research.

Network Design Heuristics For Choosing Locations Of New Service Overlay Switches On A Legacy Topology

The following discussion outlines three design heuristics that when applied to the network design process can help lower the total cost of a new service overlay on an existing legacy network. Given that this is an intractable NP-hard problem the heuristics as applied may have a limited impact but should help guide towards a low solution. The basic concepts of the three heuristics, connectivity, location and traffic volume, are well-known network design concepts discussed in many references including Cahn, 1998 and Drezner and Hamacher 2002. This research applies these concepts to designing a new service overlay on a legacy topology.

The cost model used to develop the application of these heuristics was Equation 3.8 where α, β, χ and $\delta = 1$. The heuristics stated below should be applicable with other choices of α, β, χ and δ, but to a varying extent and are the subject of future research.

The three heuristics are as follows; locate overlay switches at nodes in a centralized location of the legacy network as opposed to the periphery, locate overlay switches at legacy node locations with high connectivity, and lastly, locate overlay switches at legacy nodes with high traffic flow demand. While each heuristic describes a different concept these ideas tend to work in concert and the impact of each will not always individually be definable. Each heuristic will be discussed individually in the following part of this chapter.

Heuristic 1 - Locate overlay switches at nodes in a centralized location of the legacy network

Locating overlay switches at centralized locations within the legacy topology can help reduce total link costs because traffic flow paths should tend to be shorter. Backhaul distances of traffic flows from source node to the nearest overlay switch to reach the overlay backbone will be shorter when overlay switches are located at a central location. Reducing total link cost is important when link cost dominates the total cost function and when the full impact of link distance is a part of the total link cost equation, i.e. Equation

3.7, $\delta = 1$. When distance is less important to the total cost, i.e. $\delta < 1$, minimizing link costs by keeping total link distance at a minimum will have less of an impact upon total cost and therefore the impact of this heuristic will be less. As δ decreases to 0, total link distance will have less and less of an impact on the total cost of the overlay and the centralized core location of an overlay switch will be less important. Knowing the form of the cost function prior to designing the overlay will be important for minimizing costs.

Heuristic 2 - Locate overlay switches at legacy node locations with high connectivity

Choosing to locate overlay switches at legacy node locations with high connectivity can help minimize link costs as well as possibly switch costs. With more link connections available, traffic flows will have more routing options creating a higher probability of each traffic flow being routed via the shortest route. This will reduce link costs because route path distances will be less. Choosing switch locations with higher connectivity on the legacy topology can reduce backhaul distance costs because traffic flows can likely be routed on a more direct path from access nodes to the nearest overlay switch. Switch costs can be impacted because with more path choices traffic flows can potentially be routed through fewer switches creating the need for fewer total connections and possibly smaller switch chassis.

Heuristic 3 - Locate overlay switches at legacy nodes with high traffic flow demands

Locating overlay switches at legacy nodes with high traffic flow demand should help reduce total overlay costs by reducing link costs. If a large amount of traffic flow is generated at a node site without an overlay switch, traffic flow path distances will be increased because the entire traffic flow will have to be carried to a single overlay switch before entering the overlay backbone. As a result, some of that traffic will likely initially travel in the wrong direction resulting in large traffic flows being inefficiently routed, potentially increasing the cost for links. Hence, placing switches at nodes with high traffic demands will offer the potential to reduce the distance a large amount of traffic might have to be backhauled thereby reducing link costs.

Switch costs might also be reduced. For example, suppose a node Edge1 has U units of traffic, and an overlay switch is not placed there. The node is then an edge node and will have a single overlay connection, moving U units of traffic, to an overlay backbone switch at node Backbone1. This backbone switch will now have to handle U – U2 units of traffic coming in, and U – U2 units of traffic going out, that it would otherwise not touch, U2 being traffic from node Edge1 that would normally hit the backbone switch at node Backbone1 even if the node Edge1 had a backbone switch. Money is saved as a switch with capacity U at the edge node is not needed but the capacity of the backbone overlay switch must be increased by 2(U – U2) units. Depending on the values of U and U2, and the cost function, this may or may not result in switch cost savings.

Backhaul of Traffic Flow

Backhaul is the movement of traffic flow from an access node that is not on the overlay backbone to an overlay switch on the backbone. Figure 7 illustrates the backhaul concept. Overlay backbone switches are located at nodes 1 and 3 but not at node 2. Node 2 is an access node whose traffic flow needs to be hauled to a switch on the overlay backbone. Traffic flow that must move from node 2 to node 1 will first be carried to an overlay switch, in this case to the switch at node 3 and then on the overlay backbone via the logical link 1-3 to the overlay switch nearest its destination. In this example, logical link 1-3 traverses through node 2 but utilizes resources of the underlying network. Note in Figure 7 traffic flow from node 1 to node 2 would travel over the legacy network link between nodes 2 and 3 twice, once as it is moved from overlay switch 1 to overlay switch 3, and again as it is hauled back to access node 2. Backhaul of traffic flows increases link costs because inefficient routing (going out and back on the legacy network link between nodes 2 and 3) necessitates the use of larger capacity links from the underlying legacy network. As far as the overlay switches at nodes 1 and 3 are concerned they have a direct connection with each other and are likely not aware that traffic actually passes though node 2. Were an overlay backbone switch also located at node 2, switch costs would increase due to the addition of this switch, but link costs would decrease as traffic flow from nodes 1 and 2 could travel directly between the two locations and not via the route 2-to-3-to-1.

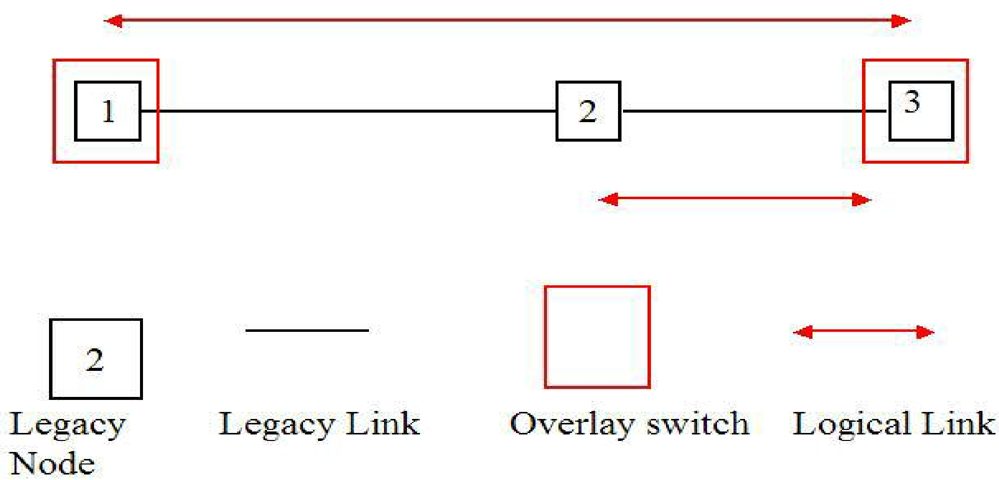
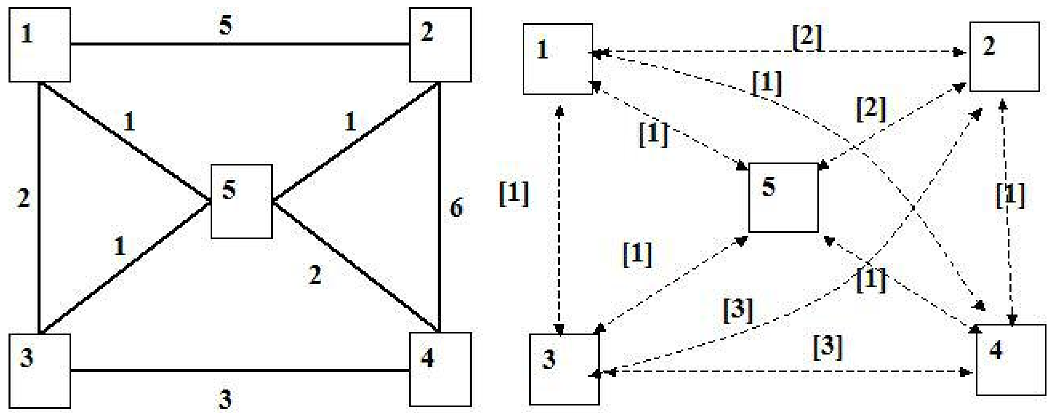


Figure 7 Backhaul of traffic flow to an overlay switch. For this example, a logical backbone link between nodes 1 and 3 is created because the overlay backbone does not have a switch at node 2. The traffic flow to and from node 2 will be carried to the nearest switch, in this case at node 3. The flow to node 1 would be carried along link 2-3 first to node 3 and then back across logical link 1-3 to node 1. It will be carried on the legacy link between nodes 2 & 3 twice

Examples of Heuristics

A series of simplified examples illustrating the impact of the heuristics are described in the following section. Figure 8 illustrates an example of the impact of placing an overlay switch in a centralized location of the legacy network versus a perimeter location as well as at a node with high connectivity. A 5-node legacy network with one central node is presented to illustrate this concept. The link distances (costs) are indicated on each link and the amount of bi-directional traffic flow associated with each node pair is indicated by [ ].



Legacy Network – distances are indicated by each link

Traffic flow matrix between all node pairs. Amount of traffic is indicated by []

Figure 8a **Centralized versus periphery overlay switch location**. Legacy network with 1 centralized node and 4 periphery nodes. Link costs are indicated in the diagram on the left and traffic flow amounts on the right.

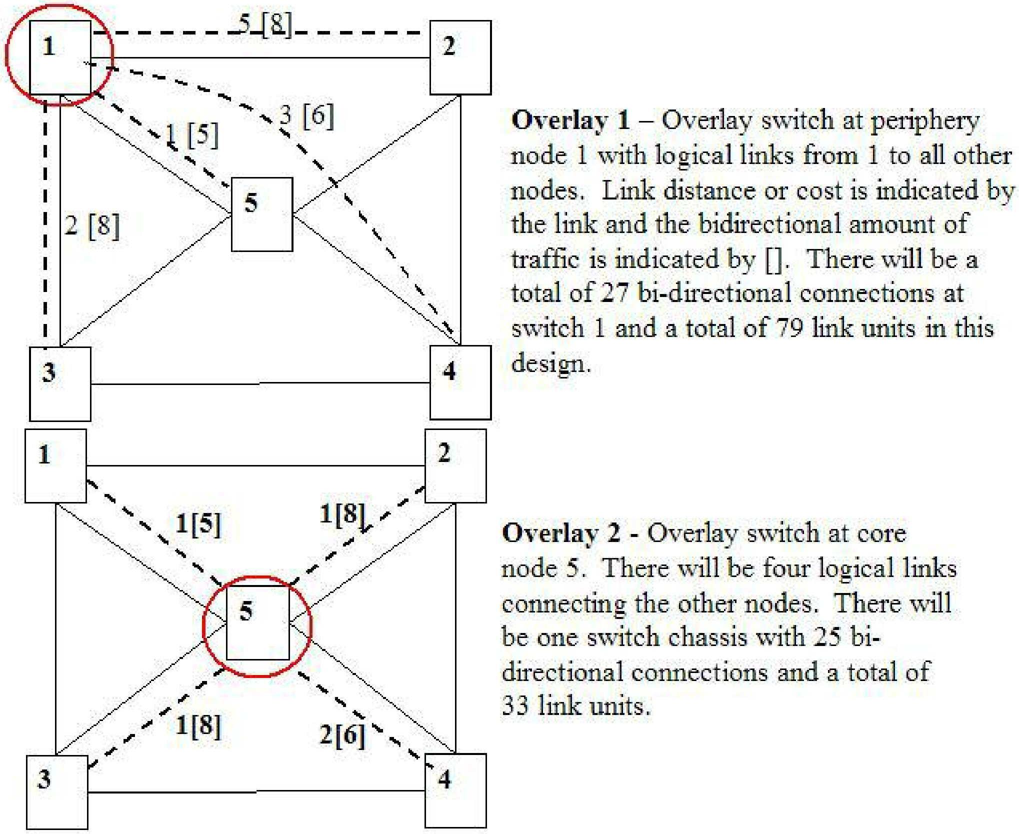


Figure 8b **Centralized versus periphery overlay switch location**. The total link cost for Overlay 2 is 33 link units, which is less than that of Overlay 1 of 79 where the switch is located at a periphery node. In Overlay 2 the traffic from node 4 is carried over one legacy link while in Overlay 1 it is carried over two links. Also in Overlay 1, all traffic between nodes 4 and 5 must be backhauled to the switch at node 1 which increases the path distance.

With a single overlay switch at node 1, four logical links are in Overlay 1, 1-2, 1-4, 1-5 and 1-3 (Figure 8). All traffic flow to and from node 2, 8 units, will travel on logical link 1-2 for a total cost of 8 * 5 = 40. On logical link 1-3, all traffic flow to and from node 3, 8 units, will travel on link 1-3 for a total cost 8*2 = 16. On logical link 1-5, all traffic flow to and from node 5, 5 units, will travel on link 1-5 for a total cost 5*1 = 5. From node 4 on logical link 1-4, 6 units of traffic flow will travel over a distance of 3 for 6*3=18 cost. The total link costs are 79 link units for Overlay 1. The single switch chassis at node 1 would have all the traffic flow moving through it for a total of 27 bi-directional connections. For this design there would be one switch chassis, 27 ports or connections and a link cost of 79. The route distance for the overlay's links is 11.

Overlay 2 has the overlay switch at node 5 with logical links, 1-5, 2-5, 3-5, and 4-5 (Figure 8). Bi-directional traffic from node 1 of 5 units will be carried over logical link 1-5 for a link distance of 1 with a total link cost of 5. From node 2, 8 bi-directional traffic flow units will be carried over logical link 2-5 a link distance of 1 for a total link cost of 8. Node 3 bi-directional traffic of 8 units will be carried over logical link 3-5 a link distance of 1 for a total of 8. From node 4, 6 bi-directional traffic flow units will be carried over logical link 4-5 a link distance of 2 for a total of 12. The total link cost for this overlay design is 33 with 27 bi-directional connections associated with the switch at node 5. The total route distance for the overlay's links has decreased to 5 as compared to Overlay 1.

Placing an overlay switch at node 1, which is on the network periphery at a location with lower that average legacy connectivity and a lower than average amount of traffic originating and terminating locally, costs more than placing an overlay switch at node 5. The latter choice supports a centralized location, higher than average legacy connectivity, but an even lower amount of originating and terminating traffic. Overlay 2 is better in two of three heuristics, and has a lower cost. The dominant reason Overlay 1 has greater costs as compared to Overlay 2 is because link costs are greater due to increased link distances, and this is mostly due to the decentralized location of the backbone overlay switch in Overlay 1.

The next series of diagrams, Figures 9a-d, provide a second example of the effects of the heuristics. The legacy network (Figure 9a) consists of four nodes in a star connection with three links (bold dark lines) connecting the central node, 1, to the other three nodes. Each node pair exchanges traffic in the amounts indicated in the [] brackets along the dashed arrow.

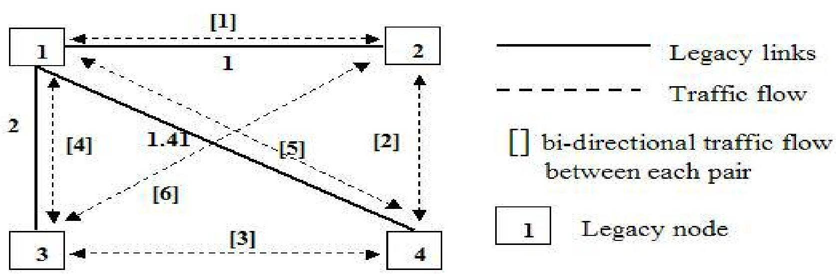


Figure 9a Legacy network, links between physical nodes 1-2, 1-3 and 1-4. Link lengths are indicated be each link. Dashed arrows indicate traffic flow between each node pair. Bi-directional traffic flow amounts between each node pair is indicated with [ ].

The cost of each link is indicated along side each link. Overlay 1 (Figure 9b) has an overlay switch located on legacy node 1. This design has the costs of one switch chassis, 32 bi-directional connections, and 49.1 link costs. Locating the overlay switch at legacy node 3 (Figure 9c) changes the total bi-directional connections to 29, with one switch chassis and a link cost of 81.1. In this example, Heuristic 2 favors Overlay 1, while Heuristic 3 favors Overlay 2.

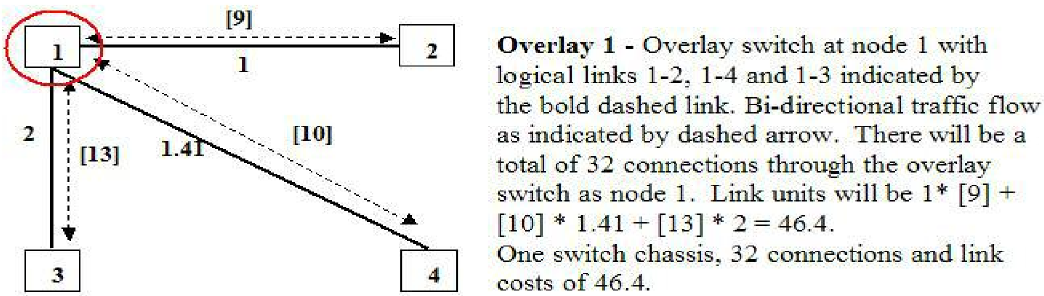


Figure 9b **Overlay 1** - Overlay switch at node 1 with logical links 1-2, 1-4 and 1-3 indicated by the bold dashed link. Bi-directional traffic flow as indicated by dashed arrow. There will be a total of 32 connections through the overlay switch as node 1. Link units will be 1* [9] + [10] * 1.41 + [13] * 2 = 49.1. One switch chassis, 32 connections and link costs of 49.1.

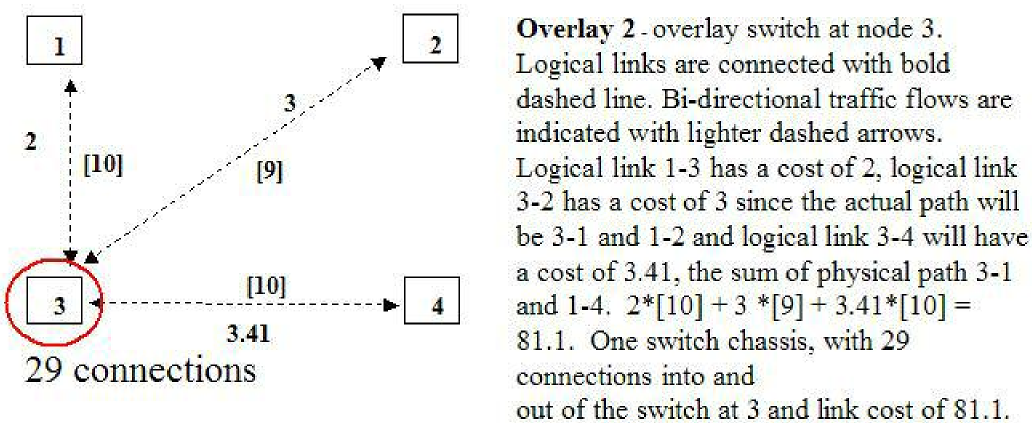


Figure 9c **Overlay 2** - overlay switch at node 3. Logical links are connected with bold dashed line. Bi-directional traffic flows are indicated with lighter dashed arrows. Logical link 1-3 has a cost of 2, logical link 3-2 has a cost of 3 since the actual path will be 3-1 and 1-2 and logical link 3-4 will have a cost of 3.41, the sum of physical path 3-1 and 1-4. 2*[10] + 3 *[9] + 3.41*[10] = 81.1. One switch chassis, with 29 connections into and out of the switch at 3 and link cost of 81.1.

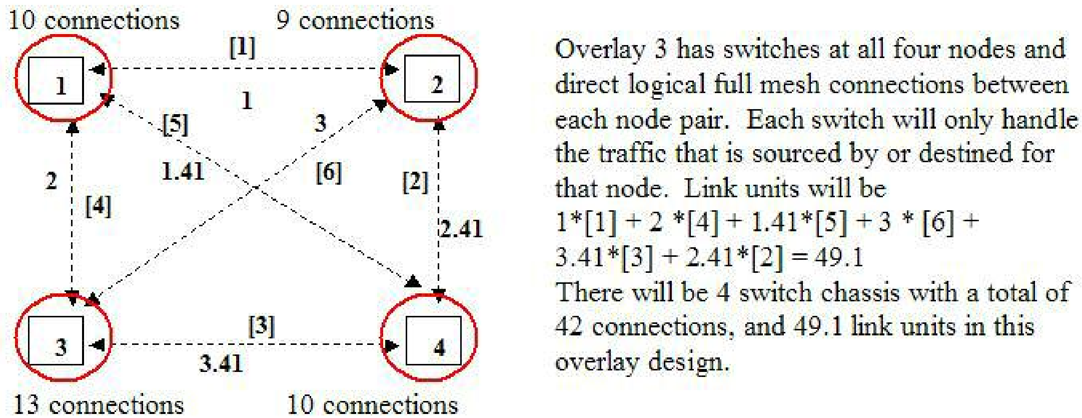


Figure 9d **Overlay 3** - Overlay 3 has switches at all four nodes and direct logical full mesh connections between each node pair. Each switch will only handle the traffic that is sourced by or destined for that node.

In Overlay1, the overlay switch was placed at node 1 with three links connecting directly to it where in Overlay 2 the switch was at node 3 where only one legacy link connected directly to it. This latter design, Overlay 2, created longer path distances for the traffic flows. Route distances increased from 4.41 to 8.41 units and the link cost requirements went from 49.1 link units in Overlay 1 to 81.1 link units in Overlay 2. This increase in link units is largely due to the requirement that all traffic flow move through the overlay switch at node 3 in Overlay 2 and the one legacy link connecting node 3 to the network. The violation of heuristic 2 resulted in higher link distances that caused a greater cost. All traffic must travel over the 1-3 legacy link creating inefficient routing and thus the greater link cost than for Overlay 1. Overlay 2 (Figure 9c) illustrates a reduction in the traffic flow from 32 to 29 connections needed at the switch because the overlay switch is located at the node with high traffic flow demands. The traffic flow demand associated with the node location of the overlay switch is routed only once through the switch. Given a different link cost structure, legacy network configuration, or traffic matrix, locating the overlay switch at the legacy node with higher traffic demands could at times be the better choice.

Finally, Figure 9d shows an example of a full logical link mesh overlay, Overlay 3. There will be a total of 42 connections at the switches, four switch chassis (one at each node) and 49.1 link units in this overlay design, thus creating a more costly design than Overlay 1 (Figure 9b). The full logical link mesh overlay creates the lowest link costs when the logical links are along the shortest path between each communicating node pair. While link costs of this overlay design are the lowest for this legacy topology, each node will have an overlay switch and each switch will have connection charges. With this particular legacy design, a 4-node star, with one link connecting each of the three access nodes to a single core switch, the full logical link mesh overlay will not be the lowest cost design even though it will have the lowest link costs, equal to that in Overlay 1.

Summary

A mathematical model for the cost of a new service overlay was presented and analyzed. The relationship between the parts of the model, link and switch costs, was discussed and arguments as to when each part might dominate the total cost function were presented. Finally, three design heuristics were defined that can tend to help drive down the total overlay cost. The three heuristics are first, locate overlay switches at nodes in the center of the legacy network as opposed to the periphery; second, locate overlay switches at legacy nodes with high connectivity; and third, locate overlay switches at legacy nodes with high traffic flow demands. The design heuristics presented in this chapter are guidelines to making choices of where to locate overlay switches for the overlay backbone. Understanding the legacy network characteristics of topology and traffic matrix will be required to successfully implant a low cost overlay solution.

Chapter 4 Experimental Methods

This research investigates the impact of nodal distribution strategy upon the long-term cost of a new service overlay installed on an existing legacy telecommunications network. A byproduct of examining different overlay design strategies is the development of efficient cost frontiers comparison of network designs. In this chapter an outline of a set of experiments conducted with a linear programming (LP) problem formulation of the service overlay with growth problem (SOGP) is presented. Using this formulation the optimal solution for the overlay was described for several different network legacy topologies. By manually controlling the node input to the problem, various nodal distribution strategies for the overlay were examined for four case study legacy networks. The basic assumption in this investigation for this problem was that an underlying legacy network topology (nodes and connections between nodes) exists. A new service overlay will be built on top of the existing network structure with no additional physical links or nodes added to the legacy physical topology. Thus allowing traffic flows of the new service to be transmitted over the existing topology.

Service Overlay with Growth Problem [SOGP] Formulation

Definitions:

- n is the number of nodes, **N** is the set of such nodes. **N** = {1, 2, 3, … , n} = i = j.
- **A** is the set of possible bi-directional edges in the graph on the set of nodes **N**.
- **T** is the set of time periods, t, in the study. **T** = {1, 2, … , t}.

- $\mathbf{D^t}$ is the set of all non-zero flow demand quantities exchanged between nodes, indexed by *r*. D will be different for each time period t. The elements of $\mathbf{D^t}$ represent the traffic to be carried by the network for each city pair.
- d^{rt} is the amount of flow demand associated with the *r*th demand pair in **D** for period, t.
- O[*r*] is the node that is the origin for the *r*th flow demand pair in **D**.
- TR[r] is the corresponding target or destination.
- cl_{ij} (= cl_{ji}) is the incremental cost of adding one unit of capacity to edge (i,j). These incremental steps can be the same or different for each time period.
- w^{rt}_{ij} is the amount of working flow of the rth demand routed between nodes (i,j) on link i,j or j,i for period t.
- lw_{ij}^t is the working capacity assigned to the edge between nodes (i,j) to support all working flows routed over that edge for period t.
- S_{jk} is the fixed cost of a switch at node j. There are incremental steps, k, in this function determined by the number of ports needed to carry all the traffic flow coming through and into the switch at node j. These incremental steps can be the same or different for each time period. The value of S_{jk} at each increment of k will be arbitrarily defined.
- cp_j is the incremental cost of adding one unit of capacity to switch at node j. These incremental costs can be the same or different for each time period.
- P_j^t = the number of connections needed in switch at node j to accommodate all the traffic flow that will enter or leave this switch. It is the sum of the demand flow routed to/from and through node j.
- CS_{jk} is the incremental cost of the switch chassis. CS_{jk} is a constant value set before the model is analyzed. These incremental costs can be the same or different for each time period. This formulation has them the same for each node.
- K is the set of incremental costs, k corresponding to the number of different sizes of chassis used in the model, b.. $\mathbf{K} = \{0, 1, 2, \ldots, k\}$.
- CN_{jk} is the maximum number of connections that any switch size will allow. For each k there will be a predefined constant such as 1000, depending on the size of the switch chassis.
- M is an arbitrarily large constant that is greater than any capacity needed to accommodate all the flow through a switch.
- z_{jk} is a binary variable (0,1) used to facilitate the step cost function of the switch chassis. The number of different sizes of the chassis, b, is the number of different values for the subscript k. K ={0,1,2, …, k). z_{jk} will be 1 for the appropriate size of P_j and zero for the rest. Only one chassis will be installed at each node. When k = 0 there are no (0) connections for that nodes and therefore no switch at that nodes.
- x_j is a binary variable (0,1) used to control the nodes that are allowed to have a switch in the design. This variable can be manually controlled.

Objective Statement:

Min

$$\sum_{t\in T} \left\{ \sum_{j\in N} x_j S_j^t + \sum_{j\in N} x_j \{cp_j^t * P_j^t\} + \sum_{i,j\in A} \{cl_{ij}^t * lw_{ij}^t\} \right\} \quad 4.1$$

Subject to:

$$\sum_{j\in A} w^{rt}_{ij} - \sum_{j\in A} w^{rt}_{ji} = d^{rt} \text{ for all } r \in \mathbf{D},\ i = O[r] \text{ and for all } t\in T. \quad 4.2$$

$$\sum_{j\in A} w^{rt}_{ij} - \sum_{j\in A} w^{rt}_{ji} = -\,d^{rt} \text{ for all } r \in \mathbf{D},\ i = TR[r] \text{ and for all } t\in T. \quad 4.3$$

$$\sum_{j\in A} w^{rt}_{ij} - \sum_{j\in A} w^{rt}_{ji} = 0 \text{ for all } r \in \mathbf{D}, \text{ for all } i \notin \{O[r] \text{ or } TR[r]\} \text{ and for all } t\in T. \quad 4.4$$

$$lw_{ij}^t = \sum_{i,j\in A} w^{rt}_{ij} + \sum_{j,i\in A} w^{rt}_{ji} \text{ for all } i,j\in \mathbf{A} \text{ and for all } t \in \mathbf{T}. \quad 4.5$$

$$P_j^t = lw_{ij}^t + lw_{ji}^t \text{ for all } i,j\in \mathbf{A} \text{ and for all } t \in \mathbf{T}. \quad 4.6$$

$$P_j^t <= M * (z_{j0} + z_{j1} + z_{j2} + \ldots + z_{jk}) \quad 4.7$$

$$z_{jk}^t \in \{0,1\} \text{ for all } j\in \mathbf{N}, \text{ for all } i,j \in \mathbf{A} \text{ and for all } t\in T. \quad 4.8$$

$$P_j^t <= 0z_{j0} + CN_1\, z_{j1} + CN_2\, z_{j2} + \ldots + CN_{jk}\, z_{jk} \quad 4.9$$

$$S_j^t = 0z_{j0} * CS_{j0}^t + z_{j1}^t * CS_{j1}^t + \ldots + z_{jk}^t * CS_{jk}^t \quad 4.10$$

$$\sum z_{jk}^t = 1, \text{ for all } j \text{ and for all } k \text{ and for all } t \in \mathbf{T}. \quad 4.11$$

$$lw_{ij}^t - lw_{ij}^{t-1} >= 0, \text{ for all } i,j \in \mathbf{A} \text{ and for all } t \in \mathbf{T}. \quad 4.12$$

$$w^{rt}_{ij},\ w^{rt}_{ji} >= 0. \quad 4.13$$

4.1 - The objective statement seeks to minimize the cost of the network overlay based on three factors, the cost of the node/switch chassis, the cost of each connection to the switch chassis and the cost of additional capacity on a link to support the required flow.

4.2, 4.3, 4.4 – These are standard flow balance constraints that require that all the flow leaves the source, all the flow arrives at the destination and that at all transshipment nodes the flow is balanced so that all the flow that enters the nodes leaves the node.

4.5 – This constraint, the link capacity constraint, sums all working flow, $w^{rt}_{ij,}$ and $w^{rt}_{ji,}$ that traverse this link to determine the amount of capacity needed on the link.

4.6 – This constraint is the switch size constraint that sums all working flows that go though this link to determine the number of connections needed for the switch at node j.

4.7 – This constraint says that the value of P_j^t will be either zero if there is not a switch at this location or it will be less than M that is a number sufficiently larger than any connection capacity needed at any switch.

4.8 – This statement defines the binary choice variable, z_{jk}^t, defined for each node, j, as to which size of switch chassis is installed at the node.

4.9 - This constraint determine the correct size of switch to be placed at a node based on the number of connections required to deliver the traffic flow.

4.10 – This constraint calculates the cost of the switch based on the previous constraint and that since this is a minimization problem the cheapest size will be selected based on the correct value of z_{jk} from constraint 4.9.

4.11 – This constraint requires that there be only one switch size for each node if any at all.

4.12 – This constraint requires that the input to the next period, t, be the output of the pervious period.

4.13 – This constraint states that the working flow, $w^{rt}_{ij,}$ and w^{rt}_{ji}, will be either zero or a positive number.

This formulation modeled after Grover and Doucette 2001 and Chang and Gavish 1993 creates a sum of shortest paths for the traffic flow solution for the input design criteria of nodes, links and traffic flow values. The implementation of this problem was done using single sequential periods with the output of the first period being the input to the next.

Plan of Analysis

Step 1: Legacy topology, T_0

The initial legacy topology, T_0, for proof of concept testing was a 9-node model (Figure 10) (see Appendix A for details).

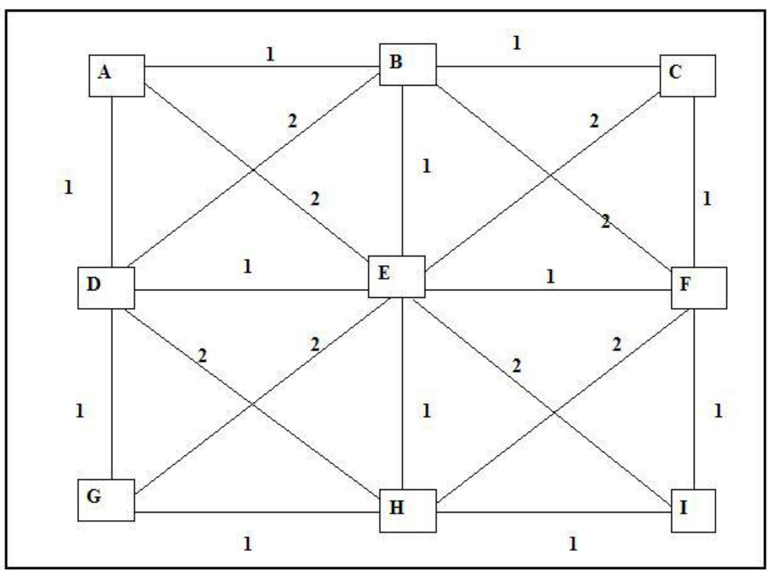


Figure 10 – Initial 9 node proof of concept model

Three case studies then were conducted using a generic North American network based on that of WilTel (Figure 11), the NSFNet from the US research network, the beginnings of the Internet (Figure 12), and a generic Pan-European model based on that used in Maesschalck et al, 2003 (Figure 13). The North American network has 27 nodes and 43 links. The Pan-European network has 28 nodes and 43 links. The NSFNet network model has 15 nodes and 22 links. The location and populations details for each of the case studies are included in Appendix A.

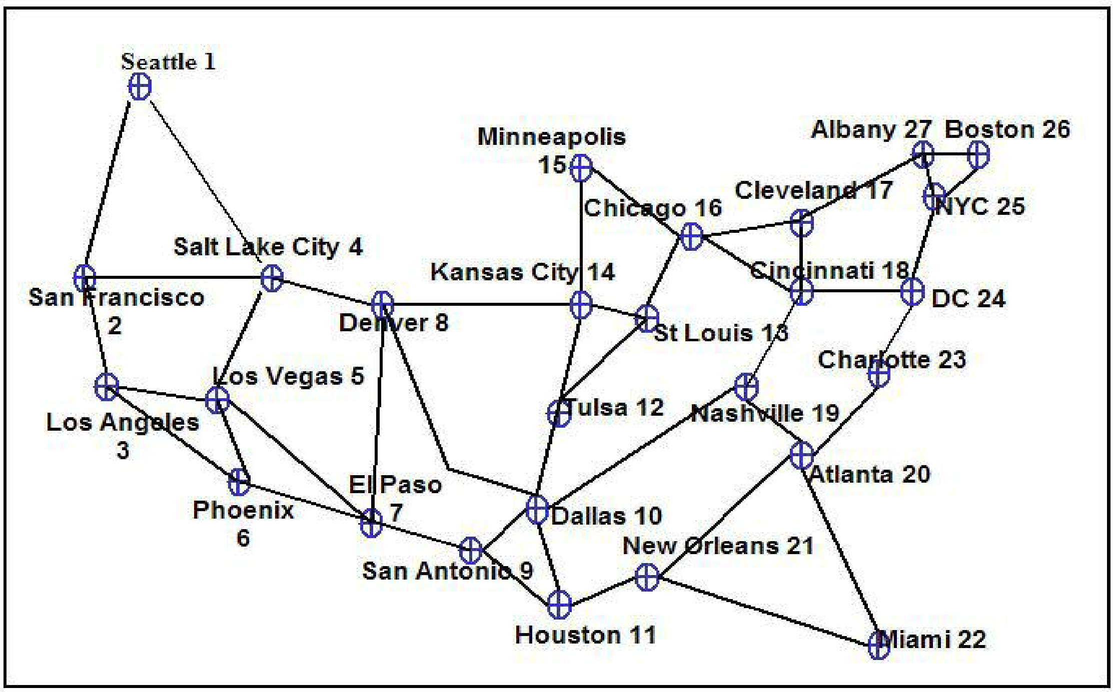


Figure 11 The North American legacy topology based on the WilTel North American network. The number by each city name is an index. The details of population, location, and link length are in Appendix A.

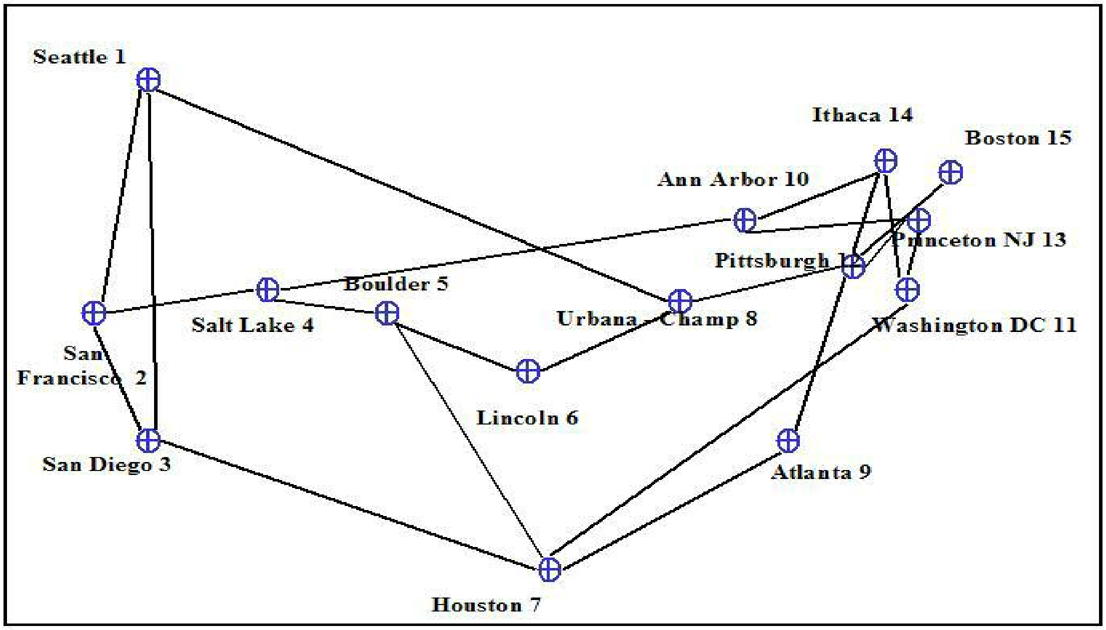


Figure 12 The NSFNet legacy topology. The number by each city is an index. The details of population, location, and link length are in Appendix A.

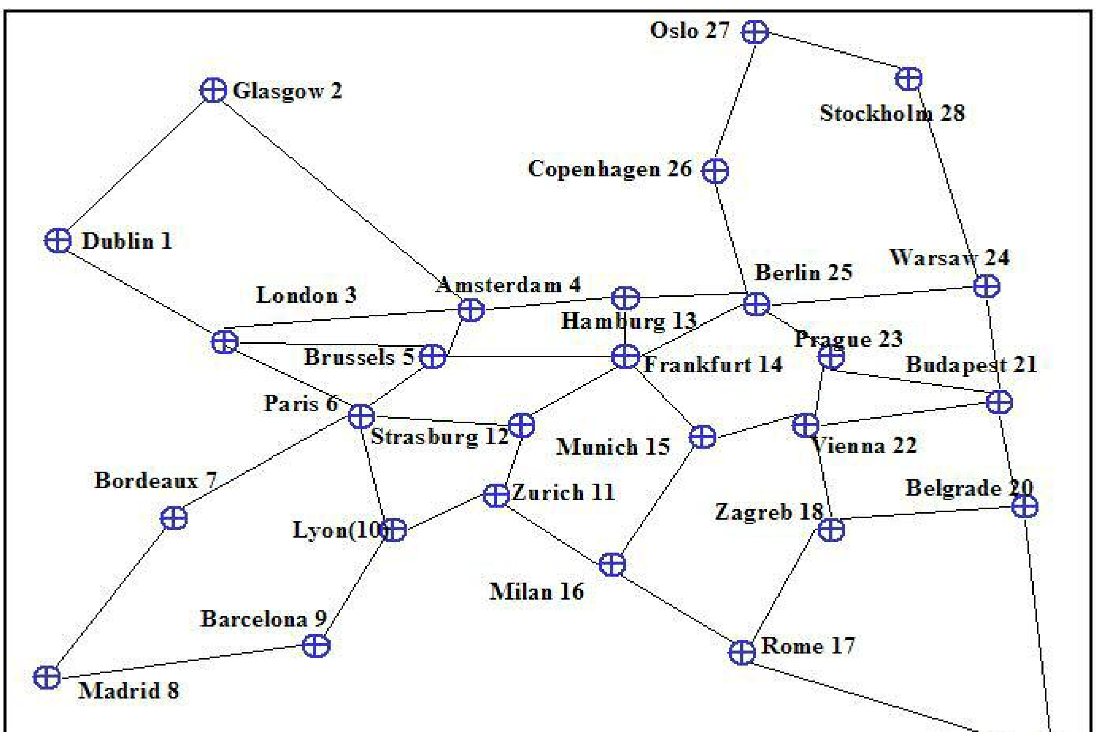


Figure 13 Pan-European legacy topology. The number by each city is an index. The details of population, location, and link length are in Appendix A.

Step 2: Traffic Matrix, TM_t

The TM_1 was developed using the population-based function as described by Cahn (1998, P. 107),

$$Traf_\phi(i,j) = \frac{\phi \text{ x } ((Pop_i \text{ x } Pop_j)/Pop\_max^2 + Pop_{off})^{Pop\_Power}}{((dist(i,j)/dist\_max) + Dist_{off})^{Dist\_Power}} \quad 4.14$$

$Traf_\phi$ (i,j) = the amount of traffic flow to be carried between city node pair, (i,j).

ϕ = a scaling factor needed to adjust the value to the appropriate level.

Pop_i = the population of the city i.

Pop_max = a population normalization factor that of the largest population.

Dist_max = a distance normalization factor that of the longest distance between city pairs.

$Dist_{off}$ = small real positive number for the purpose of avoiding division by 0

Pop_{off} = small real positive number for the purpose of avoiding traffic to and from small nodes set to 0.

Pop_Power and Dist_Power = factors that allow for controlling the importance of distance and population in establishing the amount of traffic. Most voice traffic models have distance as an important factor in creating traffic but in data models traffic distance becomes irrelevant (Cahn 1998). This model will include distance but minimize its importance by using a small value for Dist_power.

A traffic matrix, 1X, was calculated using population data from the US Census database for the North American and the NSFNet case studies. For the Pan-European topology the traffic matrix was calculated using the populations of the metropolitan city areas from data supplied by European city mayors (http://www.citymayors.com/features/euro_cities.html). Distance between each city pair was calculated using the Microsoft VirtualGlobe, 1998 edition. An ϕ of 1000 was used to create numbers within the range needed for appropriate transmission rates. These values were then translated from bits per sec (bps) into the number of Digital Signal 3, DS3, needed. For example, in the North American case study, from Seattle (node 1) to San Francisco (node 2), have populations of 3.6 million and 7 million respectively. The distance between the two is 1084 miles. Using the formula presented above a value of .1623 is calculated, with α=1000, scaled value of 162.3 is created. This number is 162.3 million bps. Dividing 162.3 million bps by 45 million bps, the approximate value of a DS3, gives 3.6 DS3 or rounded to 4 DS3. The amount of traffic flow to be carried between node 1 and node 2 on the North American case study for the 1X growth stage would be 4 DS3s. Once the input parameters were established a full traffic flow matrix was calculated and the traffic values are included in Appendix B.

Step 3: Cost Models

The cost models for the overlay were developed using generic cost functions based on data from previously published data from WilTel and other public data. These functions were built from the carrier perspective, essentially, the cost required to implement the new service overlay. Other cost functions can be developed to emphasize other perspectives such as that of expected revenue or cost to the customer. Different cost structures can produce different results from those seen in this study.

There were three cost functions used in this study, the cost of capacity on a link, the cost of the switch chassis and the cost of each flow connection in and out of the switch chassis. Each unit of connection in the switch is a constant cost which in this study was set to 125. The actual switch chassis costs are a step function that is not generally linear and were set to \$1000, \$2000, \$4000, \$10000, and \$20000 with switch sizes set to 1000 ports, 2000 ports, 4000 ports, 12000 ports, and 24000 ports. Larger switches while overall more costly show a decrease in cost per unit.

Each unit of link cost is a function of the distance of the link and the link capacity times a dollar amount. The distance was determined by summing the linear distances between the end legacy nodes traversed by the overlay link. For this study to keep link costs simple, the dollar amount was arbitrarily set to $1 pr distance unit. Capacity multiplied by distance and by the unit link cost created the total link cost for each link.

Step 4: Develop "Best" network topology

MPL, Maximal, Inc. software was used to implement the linear programming (LP) formulation of the SOGP and the single "best" design for the input parameters was calculated. This model was implemented in single sequential periods with the topology output, including number, location and size of switches, the links and the capacity of links of one growth period as the input for the next growth period. The difference between each period was the increase in the amount of traffic. The actual MPL implementation of the SOGP is presented in Appendix C. In the MPL implementation, chassis and ports were allowed at all nodes including all the access nodes. These access node costs, chassis and ports, were manually deducted from the final costs to determine the final total cost for the overlay design.

Step 5: Analyze the impact of nodal distribution – create efficient frontiers

To understand the impact of overlay switch/nodal distribution upon long-term cost, the number and distribution of switches in the service overlay were manually varied using different design strategies developed around both number of switches, node population size and location of the nodes in the backbone. To approximate network growth a five level traffic matrix was developed. 1X growth stage represented the traffic matrix calculated as described in Step 2 of this chapter. Four additional growth stages were calculated by multiplying the initial amount of traffic demand, 1X growth stage, by 1.5, 3.5 and 10 creating five growth stages 1X, 1.5X, 3X, 5X, and 10X. For this study each design strategy was modeled using all five-growth stages.

This study was to examine not only the impact of number of switches in the service overlay for long term growth potential but the location and size of the nodes as well. For each case study several one and two switch designs were calculated. The location of the backbone design for each scenario was established using the shortest path connections between each backbone node. From these analyses, a series of total costs for each traffic growth stage were developed. Other scenarios were developed having more switches included in the overlay backbone. The variations were based on the design heuristics of Chapter 3 and included not only the number of overlay switches but location, periphery versus central locations and population of the node. To control the population variable three groups of nodes were established based on population size of the city at the node, small, medium, and large, for each real world case study. The details of the three groups are presented in Appendix A.

Summary

In summary, a LP formulation of the SOGP using a combination of relevant portions of several previously defined LP models was used. A single sequential period approach was used by manually controlling the input to sequential periods. This model was validated using a proof-of-concept 9-node model (Figure 10). Three case studies were developed, a North American topology (Figure 11), the NSFNet topology (Figure 12), and a Pan-European topology (Figure 13). A series of traffic models based on initial data modified by population density heuristics were calculated. These inputs were used in the SOGP and the optimal network designs were calculated. The inclusion of nodes in the backbone of the overlay was manually controlled to create scenarios of different node distributions in the service overlay. Nodes were chosen based on population of the city, location (near the center of the network or near the periphery), and legacy network connectivity at the node site. A series of total costs for each configuration for each growth stage was calculated.

Chapter 5 Results of Case Studies Analyses

Using the mathematical model described in Chapter 3 and as implemented by the SOGP formulation described in Chapter 4, four separate case studies were evaluated, first, a small proof-of-concept 9-node legacy network (Figure 10) and then three larger closer-to-real-world legacy models, a North American, NA, model (Figure 11), the NSFNet (Figure 12), and a Pan-European, PE, model (Figure 13). This chapter presents the results of the experimental analyses and compares the results to the design heuristics discussed in Chapter 3.

The primary backbone design strategy used to develop test network configurations used the philosophy that once a legacy node was in the overlay backbone the overlay switch handled all traffic flows that went through this node. Hereafter this will be referred to as design strategy 1. A secondary strategy 2 created a mesh topology with logical links between each node pair in the overlay backbone. The only traffic flow handled by an overlay switch in the full logical link mesh is traffic flow that originates and terminates at that switch location. Those nodes not in the overlay backbone will have access to the nearest overlay backbone switch and that switch will handle (originate and terminate) their traffic flow. Thru traffic flow that passes through a node with an overlay switch will be processed by the assets of the underlying legacy network. Hereafter this will be referred to as design strategy 2. A limited number of mixed topologies, using a combination of legacy and logical links, were also evaluated.

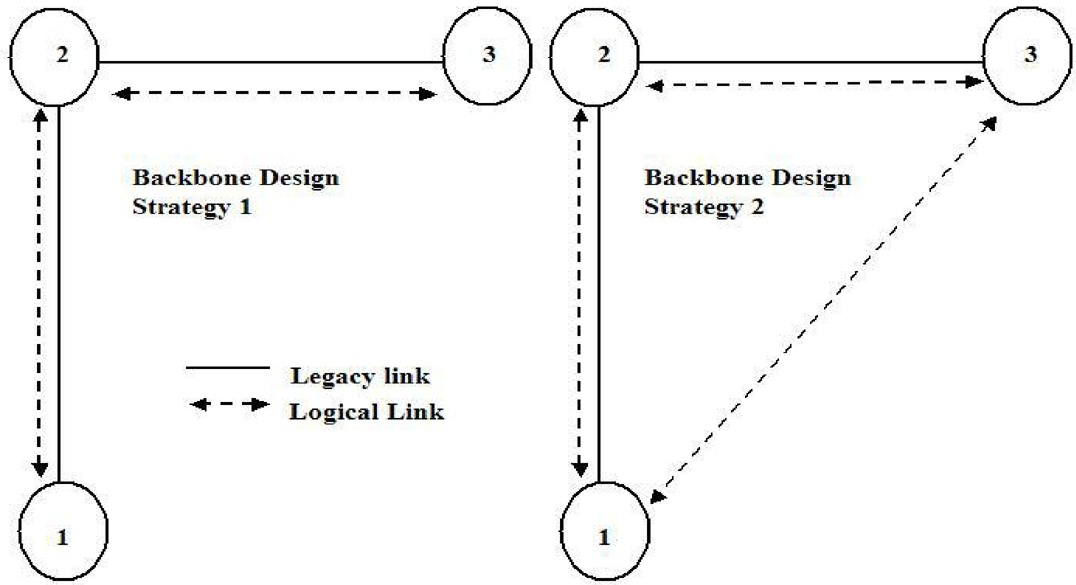


Figure 14 – Backbone design strategies using logical links or legacy links. Backbone Design Strategy 1 – Legacy Links- once a node is in the overlay all traffic flows that move through that node are seen by the overlay switch. In this design there are two logical links that carry traffic flows and all traffic flows will move through the overlay switch at node 2 Backbone Design Strategy 2 – Logical Links - while there are switches at all three nodes, there are three logical links, one between each legacy node pair. The overlay switch at node 2 only handles the traffic flows that start or end at node 2.

Growth Studies - Scalability

One of the original goals of this study was to examine design strategies for long-term growth and the impact of growth on the most cost effective design. A series of traffic flow models were chosen to approximate long-term growth. The 1X growth stage represented initial traffic as calculated by the formula presented in Chapter 4, equation 4.14. Sequential growth stages were a multiplication of that initial traffic flow matrix by a scale factor, 1.5X, 3X, 5X and 10X. Using the linear programming formulation, SOGP, defined in Chapter 4, the most cost effective overlay design for the initial 1X growth stage was the most cost effective design for all of the sequential growth stages of designs evaluated. The price structure used in this set of experiments did not feature economy of scale regarding link costs. Doubling the required link bandwidth doubled the costs. Overlay switch costs did show economy of scale savings, but only when heavy traffic loads were processed. This cost structure did not warrant any radical overlay redesigns as the network traffic increased. Future research could be done using link economies of scale that might show different behaviors than those seen in this study as far as the impact of growth on the total cost of an overlay.

The 9-node Case Study

The 9-node network topology presented in Chapter 4 (Figure 10) is well connected with an average degree of connectivity of 4.4 and has a high skewness of connectivity of 1.17 (Table 1). Skewness is a measure of symmetry, or more precisely, the lack of symmetry of a data set (see Appendix C for details of skewness calculation). The distribution of a data set is symmetric if it looks the same to the left and right of the center point. The high skewness of the distribution of the degree of nodal connectivity indicates that there are more nodes with a high degree of connectivity than those of average or lower degree of connectivity. Link distances for this model are a value of 1 or 2 with an average of 1.4 and the switch costs are an arbitrary 10, 20, 30, 40 and 50 for sizes of 50, 100, 200, and 500 connections respectively (See Appendix A for details). The unit cost constants of switch costs, K_S, are much greater than link costs, K_L, $K_S >> K_L$.

Network	Nodes	C	C	Pop	Links	Link length
	#	Average	Skewness	Ave in M	#	(Mean)
9 node	9	4.44	1.17	NA	20	1.40
PE	28	3.04	0.73	1.42	43	248.21
NA	27	2.93	0.14	3.79	43	436.95
NSFNet	15	3.19	0.28	3.92	22	1085.45

Table 1. Network characteristics of case study legacy networks. Network characteristics of number of nodes, connectivity of nodes, population of nodes, number of links, link length are presented in this table. Statistical values were calculated using the descriptive analysis package for data analysis in Microsoft Excel. C is the connectivity of the nodes in the network.

The most cost effective overlay design evaluated for this model is the one switch centralized in a star pattern (Figure 15). For this case study two different one-switch scenarios were evaluated. The location of the one switch is very important. First, the switch was located at the most central node of the legacy network and second at a periphery node. The central node location was the most cost effective design evaluated. Locating the overlay switch at a periphery node increased the link costs due to increased backhaul distances. An intermediate distribution of nearly 45% (4 out of 9) of the legacy nodes in the service overlay is the next most cost effective long-term approach evaluated.

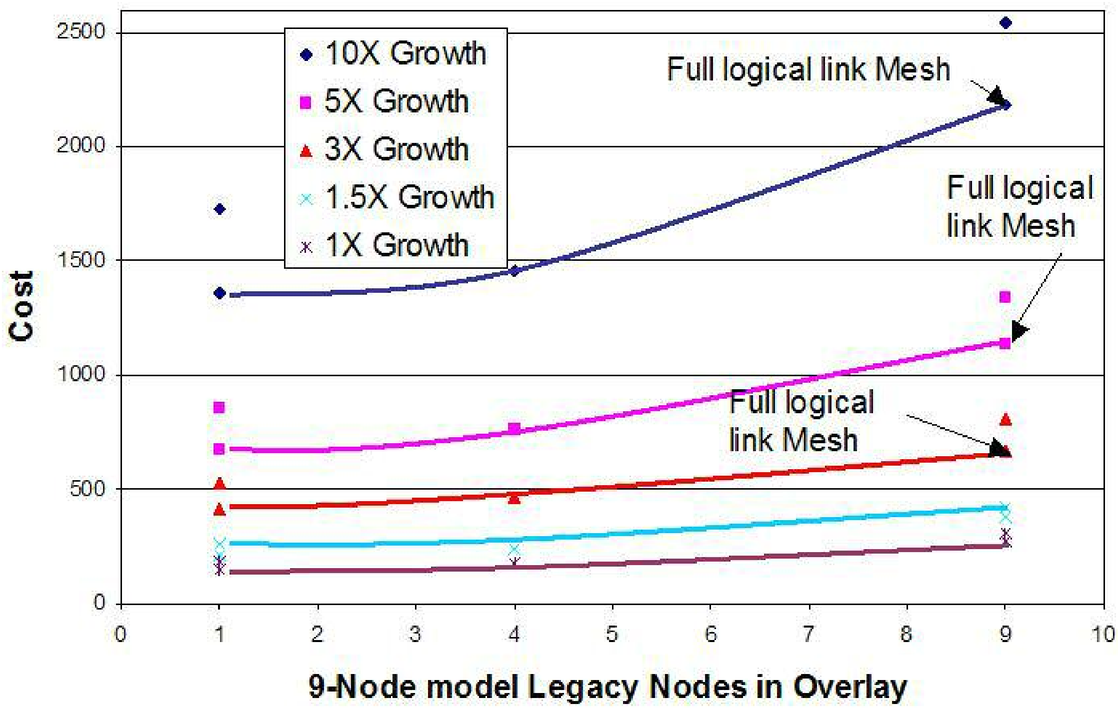


Figure 15 Experimental total cost results for the 9-node legacy network. One switch on a central location was the lowest cost solution evaluated by this study while one node switch located at a periphery location was more costly.

For this 9-node legacy topology, two overlay designs with switches at all nodes were evaluated using both design strategies. There were switches at all nodes with logical links between all nodes, as per design strategy 2, and the other design used the links of the legacy topology, as per design strategy 1. These two designs have minimized link costs because all traffic flow paths are available so the sum of traffic flow paths will be minimal. Each design will have the same number of overlay switch chassis but the number of connections per switch will be different. The design with logical links will have the minimal number of connections because the overlay switches will handle only local traffic flows and no pass through traffic flow, so the switch chassis will be of the smallest size possible. For this 9-node legacy network the backbone design strategy creating a full logical link mesh becomes the lower cost design evaluated for overlay switches distributed over all legacy nodes.

The cost function for this legacy network is dominated by the cost of switches including both chassis and port costs, as well as the unit cost of switches being much greater than the unit costs of links, $K_S >> K_L$. Link distances, as well, were minimal and therefore had limited impact upon the total cost. For this type of network design where total switch costs are dominant, total network costs tend to be impacted first by switch costs. Once switch costs are minimized then link costs need to be addressed. A centralized node location with one switch that has high connectivity is the lowest cost design evaluated for this legacy network (Figure 15). As the overlay design strategy includes switches at more legacy nodes, the total switch cost increases and therefore the total design cost increases. The efficient frontier for this legacy topology connects the low cost one-switch scenario, which for this study is the lowest cost design; to the intermediate distribution design and then to the full mesh logical link design. For this legacy topology, a centralized distribution will tend to be the lowest cost solution with increasing costs as more nodes are added.

Two of the design heuristics, place overlay switches at legacy node locations with high connectivity and at central rather than periphery legacy locations, are reflected in the results of this case study. The centralized location of the one switch in the low cost design is at a location of high connectivity, which also reduces backhaul of traffic flows and minimizes link costs. Also, most traffic flow is carried directly to the switch because the central node with the overlay switch is directly connected to all of the other nodes. So traffic flow is routed efficiently and backhaul distance of traffic flow is minimal. The centralized location of the one switch in the low cost design is at a location of high connectivity, which also reduces backhaul of traffic flows and minimizes link costs.

North American and NSFNet Case Studies

The North American, NA, and the NSFNet models have network characteristics of longer link distances, average of 437 miles and 1085 respectively and lower skewness of connectivity of nodes, 0.14 and 0.28 respectively (Table 1) (see Appendix A for details). For the implementation of these network designs the unit cost of link capacity was set to 1, $K_L = 1$. While the unit cost of switches, K_S, is much greater than the unit cost of links, K_L, the long link distances result in the dominant factor being the cost of links.

The total link costs for the NA overlay designs evaluated range from $30 Million to less than $80 Million while total switch costs were in the range of $5 Million to less than $30 Million. For the NSFNet case study the link cost range was between $10 and $40 Million and switch cost range was between $1 and $6 Million. As mentioned previously, link costs tend to dominate the total cost for these two legacy topologies (Figure 16). The key to minimizing costs for these case studies is to focus on minimizing link costs.

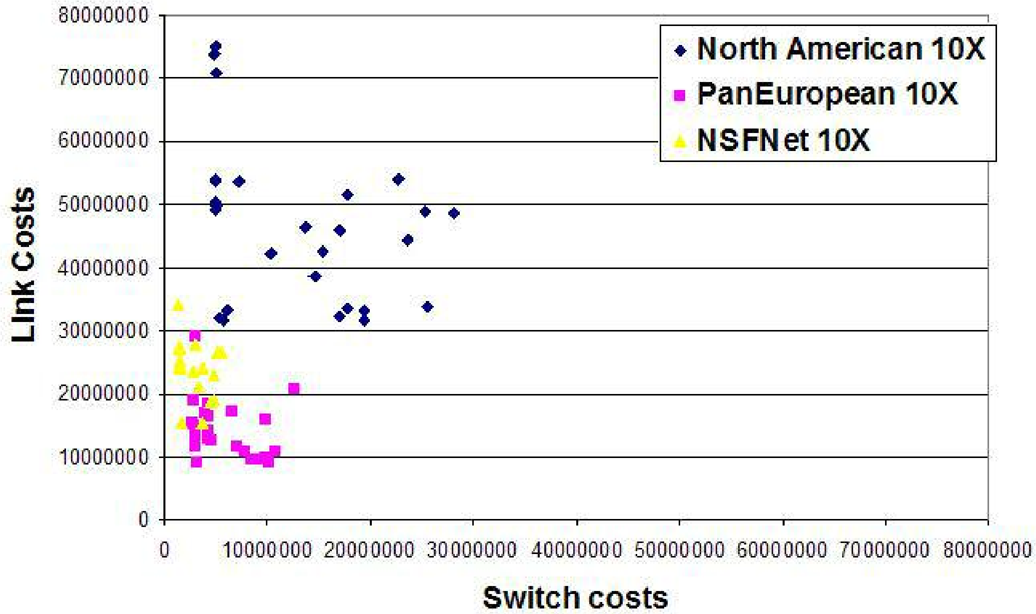


Figure 16 Switch verses Link Costs. Switch costs were plotted against link costs for the three real-world case study networks.

For both these network case studies, the fully distributed switch approach with a full logical link mesh overlay and a switch at every node following backbone design strategy 2 is the most cost effective design evaluated (Figure 17 and 19). Link cost functions due to the long link lengths dominate the total cost function for these case study legacy topologies. The cost structure used in these experiments, coupled with the long link distances, tilts the low cost advantage towards highly connected overlay networks. As noted in Chapter 3, the cost structure used does not offer any economy of scale benefits for high capacity links. Doubling a link capacity doubles the cost. Hence, aggregating flows onto a reduced number of high capacity but possibly more roundabout overlay trunk paths offers no potential savings.

A full mesh overlay that uses the shortest path connections between each node pair has the smallest possible total link capacity requirement of all possible network designs [Cahn, 1998]. For example, if a direct logical link carrying B units of traffic between two nodes is removed from a full mesh overlay, the traffic carried by the network decreases by B units. But this traffic must still be carried between the two end nodes. If rerouted over a two hop path, B additional units of traffic will now have to be carried over *two* pre-existing links instead of one direct link. The total amount of traffic carried by the overlay network will increase by B units- B units were saved by eliminating the direct link, but 2B units were added. Note also that the total switch ports required by the overlay network will also increase, by 2B units. A relay switch will need an increased capacity of B units on two links.

With this cost structure, traffic flow paths need to be minimal for link costs to be minimized. In these experiments, all logical links on a full mesh overlay network are routed over the shortest path. Depending on the configuration of the underlying legacy network, in the above example, the total distance that any rerouted traffic must be hauled may increase. Rerouted traffic will never travel a shorter distance than direct traffic. At best, it will remain the same. The overall impact of this is that a full mesh overlay network will have the minimum possible link cost for the cost structure used in these experiments. At best, an alternative overlay network can have the same link cost of a full mesh, but not less.

A full mesh overlay will not necessarily have the smallest total switch cost. Though if an overlay switch is placed at every legacy node, the full logical link mesh switch cost will be the lowest because the number of connections in each switch will be minimal since each switch will only handle the traffic flow from the legacy node. A partial overlay mesh will require an increased number of ports to handle relay traffic. Completely eliminating overlay switches except for one core switch has the potential to reduce switch cost dramatically. Figure 9 in chapter 3 shows some examples. With the cost structure used in these experiments, full logical mesh designs will not always be the lowest cost for all legacy network topologies. For example, in the design of Figure 9d the full logical mesh is not the lowest cost design.

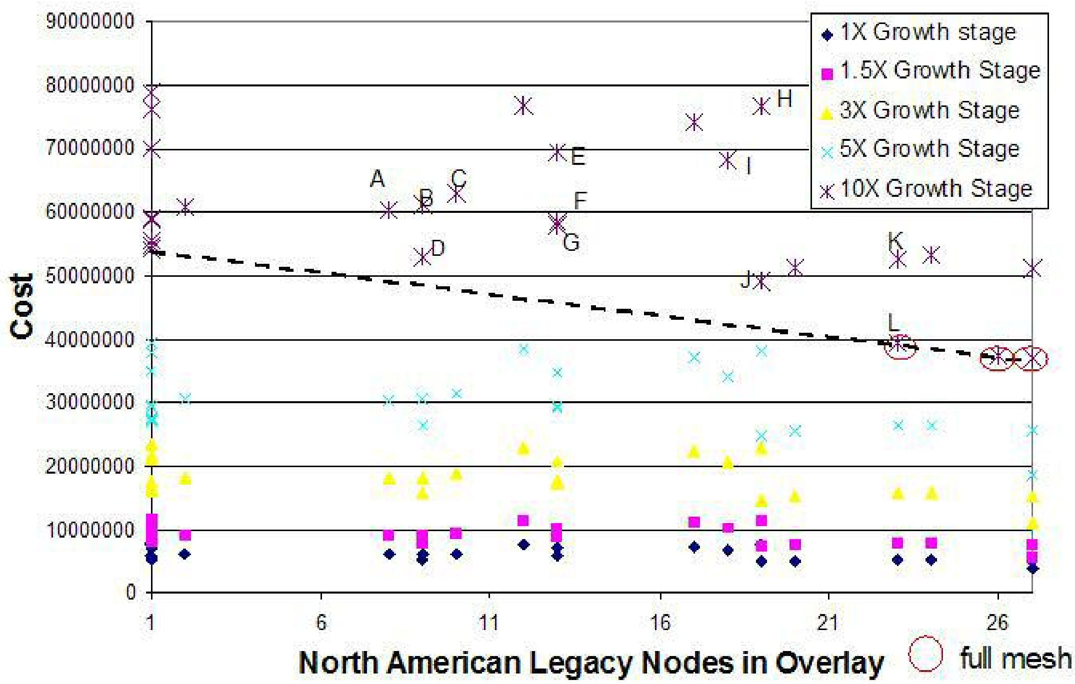


Figure 17 North American Legacy Overlay Design studies. All of the five growth stages are presented but the efficient frontier is drawn only for the 10X growth period. Lowest cost design strategies evaluated for this study are the distributed overlays with switches at all or most nodes.

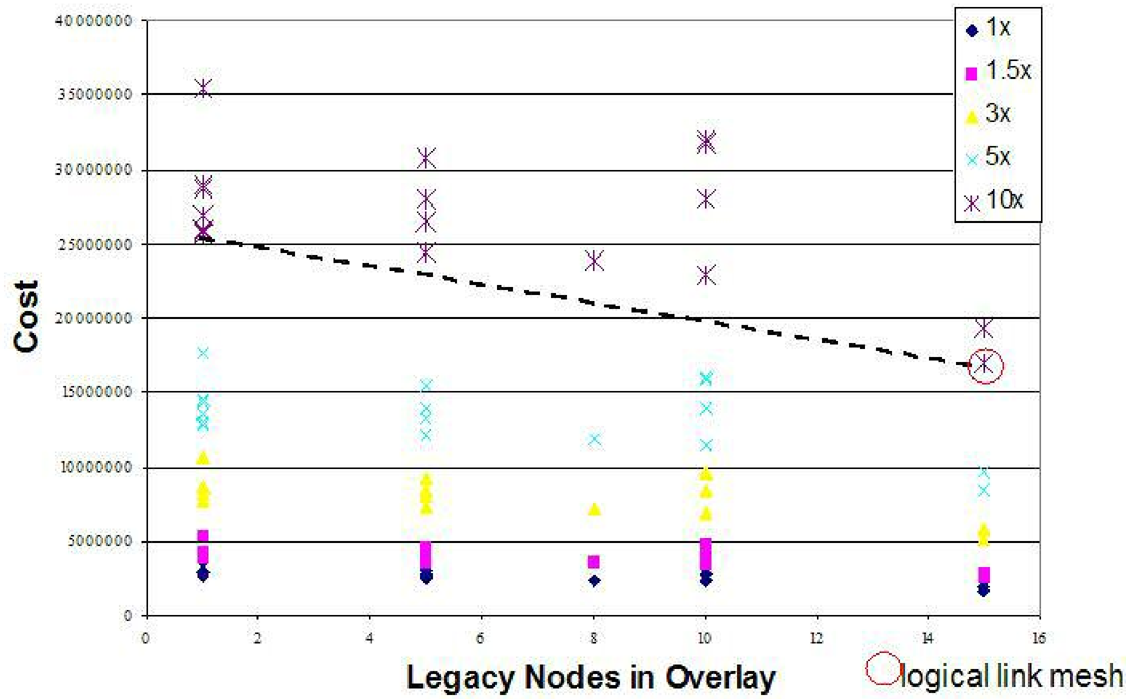


Figure 18 NSFNet Legacy Overlay Design studies. All of the five growth stages are presented but the efficient frontier is drawn only for the 10X growth period. Lowest cost design strategies evaluated are the distributed with switches at all or most nodes.

Some combinations of traffic flows over adjacent routes and elimination of certain switches may result in lower total cost designs those evaluated in Figures 18 and 19. Careful application of the heuristics described in Chapter 3 should point the way to a lower cost design. Certain nodes would be made access only and not a part of the overlay backbone based on choices made due to the design heuristics. The increase in traffic flow paths and therefore the increase in link cost could possibly be offset by the reduction in switch costs. However, as this is an intractable NP-hard problem, the only sure way to find a lower cost design is via exhaustive testing of alternate overlay configurations, guided by the design heuristics.

Two additional logical link designs for the NA case study with fewer switches in the overlay backbone were analyzed. A full logical link mesh design with only 26 nodes in the overlay backbone was constructed. Node 27 (Albany) was removed from the overlay backbone with its only access to the overlay the link to node 25 (New York, see Figure 11). Node 27 has one of the smallest traffic flow demands for this case study, is a periphery node and has a connectivity of 3. Analysis of this design showed that while the total switch costs of the 26-node overlay with full logical mesh connectivity were less than that of the 27-node design, link costs increased more than enough to offset the decrease in switch costs. The total cost of the 26-node overlay was larger than that of the 27-node overlay design. The second logical link design removed four nodes from the overlay backbone that were 2-degree connectivity nodes, specifically nodes 1 (Seattle), 15 (Minneapolis), 22 (Miami), and 26 (Boston). These four nodes are in the middle to large population groups (Appendix A). This design had 23 switches in the overlay backbone and resulted in a larger total cost than the fully distributed approach due to increases in link costs that offset decreases in switch costs.

For the NA designs evaluated in this study, the designs with switches at most or all nodes tend to be the lower cost. The key to this trend may be both the long link lengths and the low connectivity of the nodes of the legacy NA and NSFNet designs. The low nodal connectivity does not allow many path choices thus total path lengths between overlay designs may be similar. The dominant link costs in this model due to the longer link lengths must be minimized for total cost to be minimized. While the lowest total cost designs evaluated for this study for both the NA case study and the NSFNet were the lowest link cost they were not the lowest switch cost (Figure 16) indicating that link costs drove total cost with this legacy topology and the factors used in this study.

The efficient frontiers, suites of best evaluated in this study, for both of these designs are approximately linear decreasing from one switch to switches fully distributed at all nodes in the legacy network for the NA and NSFNet case studies (Figure 17 and 19). As previously mentioned, in the NA and NSFNet case studies link costs tend to dominate the cost function. For the NA and NSFNet topologies increases in switch costs are offset by decreases in link costs. Increased link costs in more centralized one-switch overlay designs are due to increased amounts of backhaul of traffic flows. These increased link costs offset the decrease in switch costs with fewer overlay switches.

The impact of the design heuristics can most easily be seen in the total costs of one-switch overlay designs in all three case studies. In Figure 19 the costs of all the one-switch overlay designs for each case study evaluated were plotted against the connectivity of the underlying legacy node. For each case study, the lowest cost one-switch design evaluated was located at a central and well-connected legacy location in the set of designs evaluated. Population or the level of traffic flow demands originating and terminating at the switch location was not the deciding factor in cost for the designs evaluated in this study. Central location along with connectivity seem to be more important factors in total cost, for the designs studied, than the amount of traffic flow demand associated with the legacy node. While the number of designs for each case study was limited, the interoperations between the heuristics are consistent.

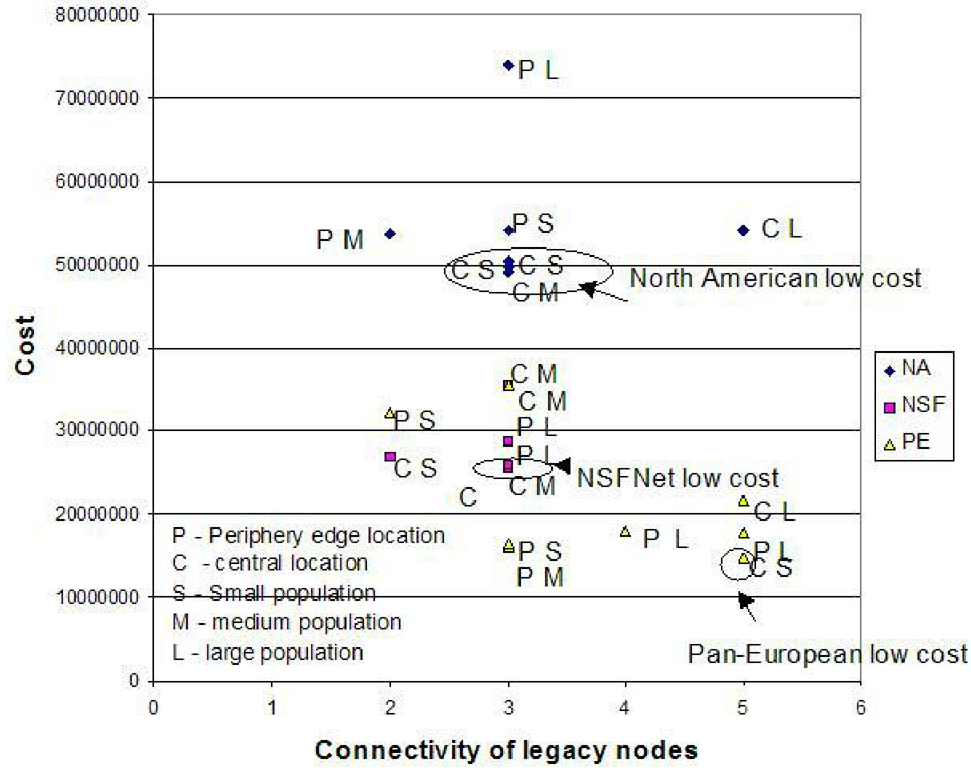


Figure 19 One-switch one node design total costs verses connectivity of legacy node. All lowest cost designs evaluated for this study were at central and fairly well connected locations. At least for these designs evaluated population or traffic demands were not a major deciding factor.

The impact of the design heuristics is also seen in the multimode designs of in the NA case study, albeit more subtly. Points labeled A-D in Figure 17 have approximately 1/3 of the total legacy nodes in the overlay backbone. The nodes of point A are the large population nodes. The nodes of point B are from the medium population group and the nodes of point C are the small population group (See Appendix A for details). The overlay backbones for these three designs were relatively minimally connected with mostly 2-degree connectivity between each overlay backbone node. These designs were constructed to essentially indicate the impact of design heuristic 3, place overlay switches at legacy nodes with large traffic demands. For the purposes of this study, population of the legacy node was assumed to indicate traffic demand, i.e. large population generates large traffic demands. While the total cost of these three designs was very similar there were some differences, the design with the high population was the lowest cost of the three designs and the design with the smallest population was the highest cost. Considering only traffic flow demands, at least with these three scenarios, the traffic demand of legacy nodes used in the overlay backbone has a limited impact upon the total cost of the overlay.

Point D has similar number of switches in the overlay backbone as the previous discussion but control factor for this design was that the legacy nodes in the overlay backbone had to have a high connectivity. Each legacy node included in the overlay backbone had a 4 or 5-degree connectivity. This design created a backbone with much higher connectivity than designs A-C and the total cost of design D was much less than that for designs A-C. The strategy associated with design D was to evaluate heuristic 2, locate overlay switches at legacy nodes with high connectivity. Comparing the designs A – D, the number of nodes in the backbone does not seem to be a dominant factor in the total cost of the overlay but the connectivity of the overlay backbone is important. Population of the legacy node seems to have some, albeit limited, impact so design heuristic 3 is also supported.

Designs associated with points E, F, and G had 13 nodes in the overlay, approximately ½ of the legacy nodes in the NA case study. Point E had overlay backbone switches at 8 large population nodes and 5 small nodes. Points F and G are associated with two backbone variations using the same set of legacy nodes, 8 large population nodes and 5 medium population nodes. The difference between the two designs is the connectivity of the overlay backbone. The design associated with point G had nodes located at legacy locations with higher connectivity than that associated with point F. The impact of design heuristic 2, locating overlay backbone nodes at locations of high connectivity, is seen with this scenario. Design heuristic 3, locate overlay switches at legacy nodes with high traffic flow demands, is also seen in this analysis as the design with the small population nodes, point E, as compared to the other two, has the higher cost.

Designs H, I, and J have switches at approximately 2/3 of the NA case study legacy nodes. These designs were constructed to compare the impact of number of overlay switches in the overlay backbone, traffic demands, and backbone connectivity. In design H, all large population nodes were eliminated so switches were placed at only the 9 small population nodes and 10 medium population nodes. This design had relatively poor connectivity with an average degree of connectivity of 1.8. Design I had overlay switches at the 10 medium population nodes and the 8 large population nodes. The connectivity of this design was an average of 1.7. Finally, design J had a mix of population sizes and all 2-degree legacy locations were eliminated. This design was better connected with an average degree of connectivity of 3. The backbone design strategy used to connect the nodes with switches was that of design strategy 1 explained at the start of this chapter. Comparing these three designs shows that population of legacy node or traffic demands generated did have an impact in creating total cost. Design H with small and medium population nodes in the overlay backbone was much more costly than design I or J. Design J, a mix of population sizes, was much better connected and had the lowest cost of the three designs. Design heuristics 2 and 3 are supported by this analysis.

To compare the impact of the two-backbone design strategies designs K and L were constructed with overlay switches at 23 legacy nodes. The four legacy nodes removed from the backbone had 2-degree connectivity and were periphery nodes. Design K is associated with design strategy 1 that included only legacy links in the backbone while design L was constructed using design strategy 2 of a full logical link mesh. The 23 nodes in the overlay backbone have logical links between each communicating pair and 4 nodes had only access connections to the nearest overlay backbone node. Both designs have the same link costs, which would be the sum of shortest path for each traffic demand, but different switch costs. Design L switch costs are less than those of design K because design L has logical mesh links between all nodes on the overlay backbone. The difference between designs K and L is due only to the different design strategy.

Multi-node designs defined for the NSFNet case study show the impact of the design heuristics in a similar manner. The size of the population of the nodes included in the overlay backbone had some impact in total cost comparison. Having nodes with the larger populations in the overlay backbone did produce somewhat lower cost total designs as compared to the designs with nodes of smaller population in the overlay. For the designs evaluated connectivity of the overlay backbone was a more important factor. The higher the connectivity of the nodes in the overlay backbone the lower the total cost of the design. Both design heuristics 2 and 3 are supported by the multi-node cases of the NSFNet case study.

Pan-European Model Case Study

Key topology characteristics of the Pan-European, PE, network are shorter link distances than NA or NSFNet and more nodes with high connectivity as shown by the high skewness of connectivity of nodes (Table 1). For the PE model network designs, the unit cost of link capacity was set to 1, $K_L = 1$. The unit cost for switches, K_S, was the same as defined for the NA and the NSFNet models, so $K_S >> K_L$. For the designs evaluated in this case study, the range of switch costs was from \$3 to 15 Million while the range for link costs was \$10-30 Million (Figure 16). The differences between switch and link costs of the designs evaluated in this study are much less than the other two larger case studies and in some instances actually are very similar. Among the designs evaluated for this research, the full logical mesh overlay was again the lowest cost design (Figure 19).

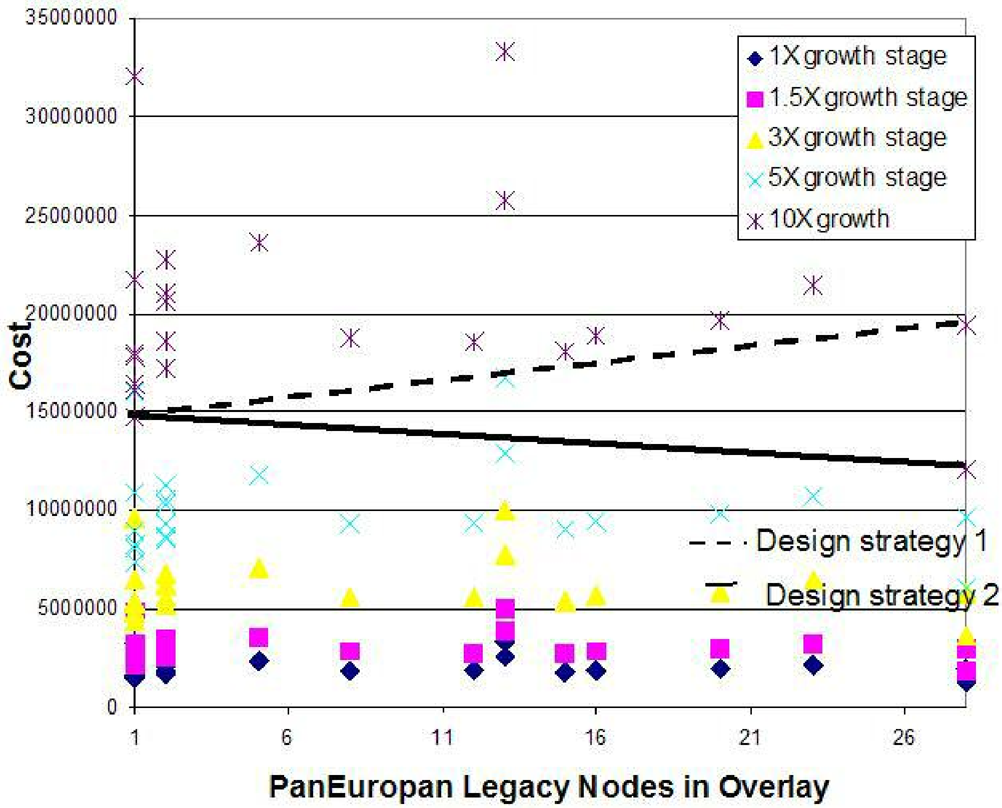


Figure 20 Pan-European Legacy Network overlay design studies. Five traffic demand growth periods for each design scenario were modeled. The efficient frontier is drawn only on the 10X data. The solid line indicates the efficient frontier for the designs with full logical link mesh overlays, design strategy 2. For design strategy 1 that utilized the direct links of the legacy topology the efficient frontier is a dashed line.

The one-switch overlay approach was the next most cost effective design evaluated in this study but the location of the overlay switch was critical (Figure 20). A central location of high connectivity was important in lowering the total cost of the overlay. There are several possible reasons for this. First, the location of the switch at a centralized node in the legacy network is important because link distances tend to be minimized which in turn minimizes backhaul costs. Periphery locations for a switch require longer flow paths thus increasing the distance traffic flow must be backhauled. The degree of connectivity for the periphery location also has an effect upon the total cost, the higher the connectivity of the switch location, the lower the total cost of the overlay.

For the lower cost one-switch overlay designs evaluated in this study, traffic volume from the node location was not an important factor in influencing total cost. The lowest cost one-switch overlay design had the switch located at node 14 (Frankfurt, see Figure 9). This location ranked 21 out of 28 in population size but was a centralized node location with a degree of connectivity of 5. Periphery locations of similar connectivity to that of node 14 but with higher traffic flow demands showed a somewhat higher total cost. This implies that the volume of traffic flow from any given node, while important to the total overall cost, at least for designs evaluated, was less important than minimizing backhaul distances.

The efficient frontier for this case study when drawn to include the full logical link mesh shows the lower cost designs to have more switches distributed over the full PE legacy network (Figure 20). When using design strategy 1 with all switches on the backbone handling all traffic moving through the node to draw the efficient frontier, the opposite relationship is seen. Centralized switch designs tend to be less costly when using design strategy 1 for the PE legacy network evaluated. Were the design strategy 2 applied to overlay backbones consisting of a smaller number of overlay switches, designs with costs lower than that of the single node designs, but in most cases higher than that of the full mesh, are to be expected. Patiently applying the design heuristics could show other designs that may have lower costs than any of the examined configurations, but since this is an intractable NP-hard problem it might very well require extensive analyses. The network designer using the design heuristics, knowledge of the traffic flow matrix, and other undefined model constraints could develop a set of designs that define a more continuous efficient frontier for the problem.

The lowest cost one-switch overlay design among those studied in this research has the switch at a central node of fairly high connectivity, degree 5 (Figure 20). This is consistent with the heuristics that recommend locating overlay switches at nodes of high connectivity on the legacy layer and locating the overlay switch in the central part of the legacy network. This design strategy also reduced backhaul distance of traffic flow by having the switch located at a centrally located node. Locating switches at nodes of high traffic flow can still impact total cost but minimizing backhaul distance in this case was an over riding factor.

Comparison of multi-node designs for the PE case study using 13 nodes in the overlay backbone showed similar impact of the design heuristics, as did the multi-node overlay designs for the NA and NSFNet case studies. For the multi-node designs studies population size of the node again had less of an apparent impact than the connectivity of the node in effecting the total cost of the overlay.

Summary

Defining the lowest cost overlay for a legacy network will depend upon the relative value of link cost to switch cost, the traffic flow demands, and the connectivity of the network. When switch costs dominate, minimizing switch costs is the first key to keeping the total cost down. For other topologies where link costs dominate the number of switches becomes less of a factor in the total cost function than minimizing link costs.

Determining the dominant influence in the cost function will be the key to minimizing costs and predicting the shape of the efficient frontier. The results of the four case studies analyzed for this dissertation are consistent with the three heuristics presented in Chapter 3 with the cost structure tested except that traffic flow demands had less of an impact upon reducing costs as compared to centralized location and connectivity.

Efficient frontiers for each case study, based on the designs evaluated, were drawn indicating that for the NA and NSFNet case studies the distributed approach with switches at more nodes tend to be more cost efficient, regardless of whether design strategy 1 or 2 is used. For the PE topology case study, based on the designs evaluated for this study, the two design strategies produce different results. Using design strategy 2 with a full logical link mesh design switches at every legacy node was the lowest cost design. If that design philosophy is not used, then a single switch that is located at a central highly connected node is the more cost effective.

Chapter 6 Summary and Future Research

Summary

This research looked at design issues associated with building an overlay network on top of an existing legacy network with overlay network switches and links not necessarily matching the switch and link locations of the underlying network. While there are many studies that present methods to define low cost network designs, there are few studies that define an overlay network that dos not necessarily match the topology of the underlying network. Since this is an intractable NP-hard problem and finding the optimal solution is not always feasible, three design heuristics were presented that can help guide the network designer to developing low cost solutions. Also to examine the impact in changing network designs due to real world constraints the concept of efficient frontier was applied to this problem.

The answer to the question of which design philosophy is better for the service overlay, that of centralized versus distributed overlay switches, depends upon the characteristics of the legacy topology and the cost function defined for the overlay. This study developed a mathematical model that has two basic components, switch costs and link costs, for defining the total cost of a network overlay. The three heuristics presented can be used to help point to the direction of keeping costs under control when design changes are required. The three heuristics are first, locate overlay switches at nodes in the center of the legacy network as opposed to the periphery; second, locate overlay switches at legacy nodes with high connectivity; and third, locate overlay switches at legacy nodes with high traffic flow demands. Applying the concept of efficient frontiers to the world of network design and building a suite of best designs gives the network designer greater insight into how to design the best network in the face of changing real-world constraints.

The nature of the underlying legacy topology determines the dominant factor, link or switch costs to the total cost function as well as the unit cost for switches and links. For the cost model and the case studies evaluated using the design strategies in this study, distributed approaches generally tend to be a good choice when the link costs dominate the total cost function because total path distances and therefore link costs need to be minimized in preference over switch costs. A distributed overlay tends to have lower link costs because there is usually a greater probability that total path distances can be minimized because of greater connectivity. More connections set up the potential for more traffic flow path choices allowing each traffic flow to be sent along shorter paths. The results of the NA and NSFNet case studies evaluated in this study support this assertion.

In legacy network topology designs that have many nodes with high connectivity, the overlay link costs can be relatively similar between designs and the switch costs can have a large impact upon total cost. The results of the designs evaluated in this study for the 9-node and PE case studies tend to support this assertion. Although, the overlay design strategy of using a full logical link mesh overlay is the lower cost for the PE case study because both link and switch costs were at minimum.

By building a suite of design strategies, network designers can understand the impact of changes in designs due to the number of nodes in an overlay and which nodes to include in the network overlay. As unforeseen constraints develop the designer will understand how to manage changes to the final design to continually produce a cost effective design.

Future research

This study used essentially linear expansion of both switch and link capacity and cost. Incorporating economies of scale concepts into the general pricing structure for both number of ports and the amount of capacity added to a link would provide other cost models. Also, multiplexing concepts where smaller units of traffic flow are added together for transport could be added to provide additional costs models. This study set the link cost function to increase linearly with distance. In other overlay strategies, distance can be much less important. Future research would be to define different cost models and further refine heuristics for those models. Many different cost functions can be defined and future research could be to use different relationships of the exponent factors of the mathematical model, α, β, χ and δ and different link and switch unit costs.

One of the basic assumptions of this study is that best is least cost. Modeling other parameters such as utilization of resources or minimizing flow delay gives different insights into network design. Another limitation of this study is that link capacity only included working capacity in calculations. Networks today must have guaranteed deliverability so including restoration and protection capacity in the study of the "best" design would be an important factor. Another factor to consider is that this study used only the shortest path concept to structure the routing of traffic flows. Future research could use other routing philosophies than shortest path.

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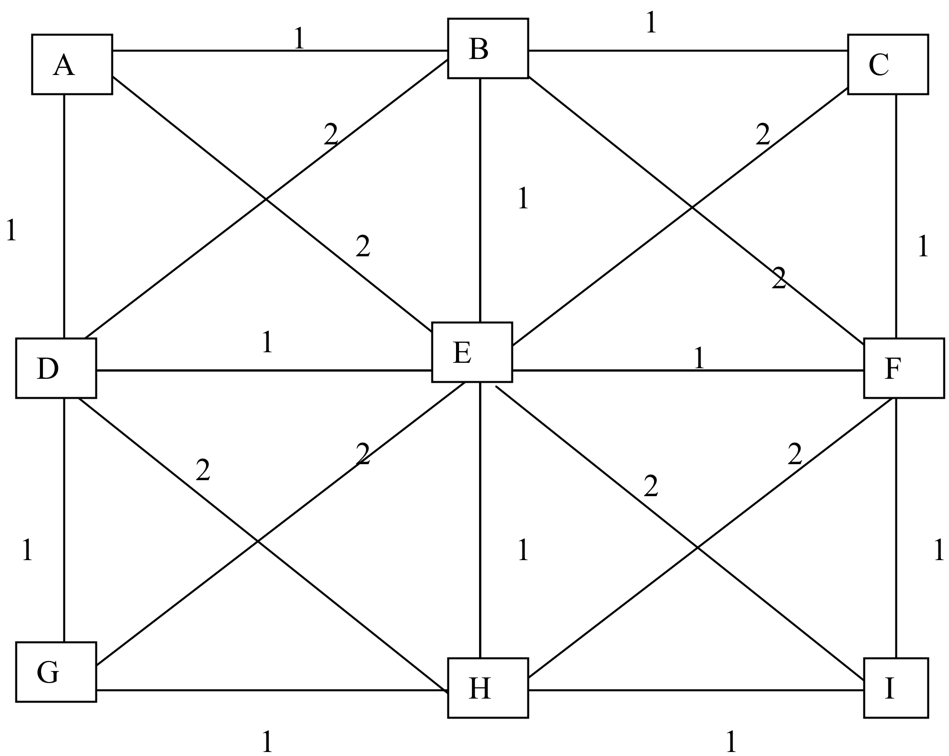
APPENDICES

Appendix A Network Details

9-node test case network



Numbers by each link indicate a distance value for that link.

Linear Cost Function for Links 1 size unit = 1 traffic unit = 1 cost unit

Switch Cost

size	cost
50	10
100	20
200	30
500	40

North American Network

Node #	City	area pop	Size group	C (degree of connectivity)
1	Seattle	3.6 M	Medium	2
2	San Francisco	7.0 M	Large	3
3	Los Angeles	16.4 M	Large	3
4	Salt Lake City	1.3 M	Small	4
5	Los Vegas	1.6 M	Small	4
6	Phoenix	2.3 M	Medium	3
7	El Paso	.680 M	Small	3
8	Denver	2.6 M	Medium	4
9	San Antonio	1.6 M	Small	3
10	Dallas	5.2 M	Large	5
11	Houston	4.7 M	Large	3
12	Tulsa	.803 M	Small	3
13	St Louis	2.6 M	Medium	3
14	Kansas City	1.8 M	Medium	4
15	Minneapolis	3.0 M	Medium	2
16	Chicago	9.2 M	Large	4
17	Cleveland	3.0 M	Medium	3
18	Cincinnati	2.0 M	Medium	4
19	Nashville	1.2 M	Small	3
20	Atlanta	4.1 M	Medium	4
21	New Orleans	1.3 M	Small	3
22	Miami	3.9 M	Large	2
23	Charlotte	1.6 M	Small	2
24	DC	4.9 M	Large	3
25	NYC	9.3 M	Large	3
26	Boston	5.8 M	Large	2
27	Albany	.876 M	Small	3

population data from US census 2000/ factfinder.census.gov

Links I	Set of A j		length km
1	1SEA	4SLKC	842
	4SLKC	1SEA	842
2	2San Fran	4SLKC	744
	4SLKC	2San Fran	744
3	2San Fran	1SEA	820
	1SEA	2San Fran	820
4	2San Fran	3Los Angeles	422
	3Los Angeles	2San Fran	422
5	4SLKC	5Los Angeles	486
	5Los Vegas	4SLKC	486
6	5Los Vegas	3Los Angeles	275
	3Los Angeles	5Los Vegas	275
7	4SLKC	8Denver	293
	8Denver	4SLKC	293
8	5Los Vegas	6Phoenix	287
	6Phoenix	5Los Vegas	287
9	3Los Angeles	6Phoenix	367
	6Phoenix	3Los Angeles	367
10	5Los Vegas	7El Paso	817
	7El Paso	5Los Vegas	817
11	6Phoenix	7El Paso	440
	7El Paso	6Phoenix	440
12	7El Paso	8Denver	715
	8Denver	7El Paso	715
13	7El Paso	9San Anton	557
	9San Anton	7El Paso	557
14	8Denver	10Dallas	899
	10Dallas	8Denver	899
15	8Denver	14Kan City	612
	14Kan City	8Denver	612
16	9San Anton	10Dallas	280
	10Dallas	9San Anton	280
17	9San Anton	11Houston	197
	11Houston	9San Anton	197
18	10Dallas	11Houston	244
	11Houston	10Dallas	244
19	10Dallas	12Tulsa	263
	12Tulsa	10Dallas	263
20	12Tulsa	14Kan City	251
	14Kan City	12Tulsa	251
21	12Tulsa	13St Louis	391
	13St Louis	12Tulsa	391

22	14Kan City	13St Louis	253
	13St Louis	14Kan City	253
23	15Minneapolis	16Chicago	446
	16Chicago	15Minneapolis	446
24	10Dallas	19Nashville	668
	19Nashville	10Dallas	668
25	11Houston	21New Orleans	352
	21New Orleans	11Houston	352
26	21New Orleans	22Miami	945
	22Miami	21New Orleans	945
27	21New Orleans	20Atlanta	532
	20Atlanta	21New Orleans	532
28	19Nashville	20Atlanta	243
	20Atlanta	19Nashville	243
29	22Miami	20Atlanta	705
	20Atlanta	22Miami	705
30	20Atlanta	23Charlotte	240
	23Charlotte	20Atlanta	240
31	23Charlotte	24DC	422
	24DC	23Charlotte	422
32	19Nashville	18Cincinnati	279
	18Cincinnati	19Nashville	279
33	16Chicago	17Cleveland	363
	17Cleveland	16Chicago	363
34	16Chicago	18Cincinnati	300
	18Cincinnati	16Chicago	300
35	18Cincinnati	24DC	407
	24DC	18Cincinnati	407
36	24DC	25NYC	248
	25NYC	24DC	248
37	17Cleveland	27Albany	495
	27Albany	17Cleveland	495
38	27Albany	26Boston	166
	26Boston	27Albany	166
39	26Boston	25NYC	248
	25NYC	26Boston	248
40	25NYC	27Albany	159
	27Albany	25NYC	159
41	15Minneapolis	14Kan City	440
	14Kan City	15Minneapolis	440
42	18Cincinnati	17Cleveland	252
	17Cleveland	18Cincinnati	252
43	16Chicago	13St Louis	424
	13St Louis	16Chicago	424

source for distances
The Road Atlas, 2002, Rand McNally, Skokie, Ill

Pan European Network

Nodes				c	Pop Grouping	Pop (M)
4	Amsterdam, Netherlands	52 21 N	4 54 E	4	Small	0.729
19	Athens, Greece	37 58 N	23 43 E	2	Small	0.772
9	Barcelona, Spain	41 23 N	2 11 E	2	Medium	1.5
20	Belgrade, Serbia	44 50 N	20 30 E	3	Medium	1.6
25	Berlin, Germany	52 32 N	13 25 E	5	Large	3.4
7	Bordeaux, France	44 50 N	0 34 W	2	Small	0.21
5	Brussels, Belgium	50 50 N	4 20 E	4	Medium	1
21	Budapest, Hungary	47 30 N	19 05 E	3	Medium	1.8
26	Copenhagen, Denmark	55 43 N	12 34 E	2	Small	0.499
1	Dublin, Ireland	53 20 N	6 15 W	2	Small	0.482
14	Frankfurt, Germany	50 06 N	8 41 E	5	Small	0.644
2	Glasgow, Scotland	55 53 N	4 15 W	2	Small	0.612
13	Hamburg, Germany	53 33 N	9 59 E	3	Medium	1.7
3	London, England	51 30 N	0 10 W	4	Large	7.1
10	Lyon, France	45 46 N	4 50 E	3	Small	0.415
8	Madrid, Spain	40 24 N	3 41 W	2	Large	2.8
16	Milan, Italy	45 27 N	9 17 E	3	Medium	1.3
15	Munich, Germany	48 08 N	11 35 E	3	Medium	1.2
27	Oslo, Norway	59 55 N	10 45 E	2	Small	0.505
6	Paris, France	48 52 N	2 20 E	5	Large	2.2
23	Prague, Czech Republic	50 06 N	14 26 E	3	Medium	1.2
17	Rome, Italy	41 48 N	12 36 E	3	Large	2.7
28	Stockholm, Sweden	59 20 N	18 03 E	2	Small	0.744
12	Strasbourg, France	48 35 N	7 45 E	3	Small	0.252
22	Vienna, Austria	48 13 N	16 22 E	4	Medium	1.5
24	Warsaw, Poland	52 15 N	21 00 E	3	Medium	1.6
18	Zagreb, Croatia	45 48 N	15 58 E	3	Small	0.868
11	Zurich, Switzerland	47 23 N	8 33 E	3	Small	0.36

source for pop data
http://www.citymayors.com/features/euro_cities.html
source for Lat and Long
http://www.getty.edu/vow/TGN

Pan European Links

Links						Distance mi
1Dublin	53 20 N	6 15 W	Glasgow	55 53 N	4 15 W	193
2Dublin	53 20 N	6 15 W	London	51 30 N	0 10 W	286
3Glasgow	55 53 N	4 15 W	Amsterdam	52 21 N	4 54 E	245
4London	51 30 N	0 10 W	Paris	48 52 N	2 20 E	213
5Paris	48 52 N	2 20 E	Strasbourg	48 35 N	7 45 E	247
6Paris	48 52 N	2 20 E	Lyon	45 46 N	4 50 E	244
7Paris	48 52 N	2 20 E	Bordeaux	44 50 N	0 34 W	310
8London	51 30 N	0 10 W	Amsterdam	52 21 N	4 54 E	224
9Bordeaux	44 50 N	0 34 W	Madrid	40 24 N	3 41 W	345
10Madrid	40 24 N	3 41 W	Barcelona	41 23 N	2 11 E	316
11Barcelona	41 23 N	2 11 E	Lyon	45 46 N	4 50 E	330
12Lyon	45 46 N	4 50 E	Zurich	47 23 N	8 33 E	208
13Paris	48 52 N	2 20 E	Brussels	50 50 N	4 20 E	162
14Brussels	50 50 N	4 20 E	Amsterdam	52 21 N	4 54 E	108
15Amsterdam	52 21 N	4 54 E	Hamburg	53 33 N	9 59 E	227
16Brussels	50 50 N	4 20 E	Frankfurt	50 06 N	8 41 E	198
17Hamburg	53 33 N	9 59 E	Frankfurt	50 06 N	8 41 E	245
18Frankfurt	50 06 N	8 41 E	Strasbourg	48 35 N	7 45 E	113
19Strasbourg	48 35 N	7 45 E	Zurich	47 23 N	8 33 E	91
20Zurich	47 23 N	8 33 E	Milan	45 27 N	9 17 E	138
21Milan	45 27 N	9 17 E	Munich	48 08 N	11 35 E	150
22Hamburg	53 33 N	9 59 E	Berlin	52 32 N	13 25 E	159
23Berlin	52 32 N	13 25 E	Copenhagen	55 43 N	12 34 E	222
24Copenhagen	55 43 N	12 34 E	Oslo	59 55 N	10 45 E	298
25Oslo	59 55 N	10 45 E	Stockholm	59 20 N	18 03 E	258
26Stockholm	59 20 N	18 03 E	Warsaw	52 15 N	21 00 E	502
27Berlin	52 32 N	13 25 E	Prague	50 06 N	14 26 E	174
28Prague	50 06 N	14 26 E	Vienna	48 13 N	16 22 E	157
29Vienna	48 13 N	16 22 E	Munich	48 08 N	11 35 E	220
30Munich	48 08 N	11 35 E	Frankfurt	50 06 N	8 41 E	189
31Milan	45 27 N	9 17 E	Rome	41 48 N	12 36 E	302
32Vienna	48 13 N	16 22 E	Zagreb	45 48 N	15 58 E	168
33Warsaw	52 15 N	21 00 E	Budapest	47 30 N	19 05 E	339
34Prague	50 06 N	14 26 E	Budapest	47 30 N	19 05 E	277
35Zagreb	45 48 N	15 58 E	Rome	41 48 N	12 36 E	323
36Zagreb	45 48 N	15 58 E	Belgrade	44 50 N	20 30 E	230
37Belgrade	44 50 N	20 30 E	Budapest	47 30 N	19 05 E	196
38Belgrade	44 50 N	20 30 E	Athens	37 58 N	23 43 E	502
39Rome	41 48 N	12 36 E	Athens	37 58 N	23 43 E	645
40Warsaw	52 15 N	21 00 E	Berlin	52 32 N	13 25 E	320
41Brussels	50 50 N	4 20 E	London	51 30 N	0 10 W	200
42Berlin	52 32 N	13 25 E	Frankfurt	50 06 N	8 41 E	264

43	Vienna	48 13 N	16 22 E	Budapest	47 30 N	19 05 E	135

source for distances
http://www.csgnetwork.com/longlatdistance.html

NSFNet Node #	City	Pop Grouping	area pop	c
1	Seattle	Medium	3.6 M	3
2	San Francisco	Large	7.0 M	3
3	Los Angeles/San Diego	Large	16.4 M	3
4	Salt Lake City	Medium	1.3 M	3
5	Denver/Boulder	Medium	2.6 M	3
6	Lincoln	Small	0.25 M	2
7	Houston	Large	4.7 M	4
8	Urbana-Champaign	Small	.180 M	3
9	Atlanta	Large	4.1 M	2
10	Ann Arbor/Detroit	Large	5.46 M	3
11	DC	Large	4.9 M	3
12	Pittsburgh	Medium	2.37 M	5
13	Princeton	Small	.036 M	3
14	Ithaca	Small	.048 M	3
15	Boston	Large	5.8 M	1

population data from US census 2000/ factfinder.census.gov

NSFNet Links

links

I	j	distance
1	8	2856
1	3	1716
1	2	1094
2	4	961
2	3	738
3	7	2101
4	5	600
4	10	2311
5	6	713
5	7	1423
7	11	1961
6	8	716
8	12	705
9	12	1190
10	14	595
10	13	798
11	13	270
11	14	403
12	13	462
12	15	775
12	14	367
7	9	1125

Appendix B Traffic Matrix

	to city	pop		from city	pop	distance	traf	Alpha	/45	DS-3 Traffic units used in study
					M	miles		1000		
1	Seattle	3.6	2	San Francisco	7	1084	0.1623	162.28308	3.6062907	4
1	Seattle	3.6	3	Los Angeles	16	1560	0.2951	295.05229	6.5567175	7
1	Seattle	3.6	4	Salt Lake City	1.3	1135	0.0758	75.828081	1.6850685	2
1	Seattle	3.6	5	Los Vegas	1.6	1420	0.0788	78.826532	1.7517007	2
1	Seattle	3.6	6	Pheonix	2.3	1799	0.0873	87.333583	1.9407463	2
1	Seattle	3.6	7	El Paso	0.7	2188	0.0628	62.776588	1.3950353	2
1	Seattle	3.6	8	Denver	2.6	1644	0.0924	92.409707	2.053549	3
1	Seattle	3.6	9	San Antonio	1.6	2876	0.074	73.973805	1.6438623	2
1	Seattle	3.6	10	Dallas	5.2	2710	0.1246	124.5699	2.76822	3
1	Seattle	3.6	11	Houston	4.7	3051	0.1163	116.31189	2.5847087	3
1	Seattle	3.6	12	Tulsa	0.8	2513	0.0637	63.713566	1.415857	2
1	Seattle	3.6	13	St Louis	2.6	2775	0.0881	88.129921	1.9584427	2
1	Seattle	3.6	14	Kansas City	1.8	2423	0.078	77.970382	1.7326752	2
1	Seattle	3.6	15	Minneapolis	3	2248	0.0955	95.52498	2.1227773	3
1	Seattle	3.6	16	Chicago	9.2	2794	0.1798	179.82461	3.9961025	4
1	Seattle	3.6	17	Cleveland	3	3260	0.0923	92.298376	2.051075	3
1	Seattle	3.6	18	Cincinnati	2	3174	0.0788	78.792129	1.7509362	2
1	Seattle	3.6	19	Nashville	1.2	3178	0.0678	67.794153	1.5065367	2
1	Seattle	3.6	20	Atlanta	4.1	3506	0.1066	106.64099	2.3697997	3
1	Seattle	3.6	21	New Orleans	1.3	3384	0.0688	68.761844	1.528041	2
1	Seattle	3.6	22	Miami	3.9	4402	0.1017	101.70364	2.260081	3
1	Seattle	3.6	23	Charlotte	1.6	3672	0.0723	72.300459	1.6066769	2
1	Seattle	3.6	24	DC	4.9	3742	0.1168	116.80933	2.595763	3
1	Seattle	3.6	25	NYC	9.3	3871	0.1758	175.76214	3.9058254	4
1	Seattle	3.6	26	Boston	5.8	4008	0.1281	128.14784	2.8477297	3
1	Seattle	3.6	27	Albany	0.9	3793	0.0623	62.298739	1.3844164	2
2	San Francisco	7	1	Seattle	3.6	1084	0.1623	162.28308	3.6062907	4
2	San Francisco	7	3	Los Angeles	16	558	0.5671	567.05393	12.601199	13
2	San Francisco	7	4	Salt Lake City	1.3	959	0.0956	95.637953	2.1252878	3
2	San Francisco	7	5	Los Vegas	1.6	664	0.1076	107.59915	2.3910923	3
2	San Francisco	7	6	Pheonix	2.3	1045	0.1244	124.44943	2.7655429	3
2	San Francisco	7	7	El Paso	0.7	1593	0.074	73.977113	1.6439358	2
2	San Francisco	7	8	Denver	2.6	1521	0.1291	129.10445	2.8689877	3
2	San Francisco	7	9	San Antonio	1.6	2389	0.0966	96.562816	2.1458403	2
2	San Francisco	7	10	Dallas	5.2	2378	0.1954	195.37071	4.3415714	5
2	San Francisco	7	11	Houston	4.7	2414	0.1814	181.40326	4.0311835	5
2	San Francisco	7	12	Tulsa	0.8	2346	0.0748	74.830366	1.662897	2
2	San Francisco	7	13	St Louis	2.6	2797	0.1222	122.19814	2.7155143	3

2	San Francisco	7	14	Kansas City	1.8	2417	0.1019	101.93869	2.2653041	3
2	San Francisco	7	15	Minneapolis	3	2552	0.1341	134.14066	2.9809035	3
2	San Francisco	7	16	Chicago	9.2	2980	0.2988	298.81914	6.6404253	7
2	San Francisco	7	17	Cleveland	3	3480	0.1303	130.32267	2.8960594	3
2	San Francisco	7	18	Cincinnati	2	3280	0.1044	104.41911	2.3204246	3
2	San Francisco	7	19	Nashville	1.2	3153	0.0834	83.423052	1.8538456	2
2	San Francisco	7	20	Atlanta	4.1	3435	0.1596	159.64832	3.5477404	4
2	San Francisco	7	21	New Orleans	1.3	3088	0.0863	86.263579	1.9169684	2
2	San Francisco	7	22	Miami	3.9	4165	0.1516	151.56037	3.3680082	4
2	San Francisco	7	23	Charlotte	1.6	3696	0.0927	92.720041	2.0604454	3
2	San Francisco	7	24	DC	4.9	3918	0.1786	178.62918	3.9695372	4
2	San Francisco	7	25	NYC	9.3	4127	0.2924	292.41017	6.4980038	7
2	San Francisco	7	26	Boston	5.8	4330	0.2003	200.28796	4.4508435	5
2	San Francisco	7	27	Albany	0.9	4110	0.0729	72.918916	1.6204204	2
3	Los Angeles	16.4	1	Seattle	3.6	1135	0.3032	303.21193	6.738043	7
3	Los Angeles	16.4	2	San Francisco	7	558	0.5671	567.05393	12.601199	13
3	Los Angeles	16.4	4	Salt Lake City	1.3	934	0.1478	147.78579	3.2841286	4
3	Los Angeles	16.4	5	Los Vegas	1.6	367	0.1805	180.49506	4.0110013	4
3	Los Angeles	16.4	6	Pheonix	2.3	576	0.2257	225.72465	5.0161033	5
3	Los Angeles	16.4	7	El Paso	0.7	1131	0.1029	102.9304	2.2873422	3
3	Los Angeles	16.4	8	Denver	2.6	1340	0.2313	231.33003	5.1406674	6
3	Los Angeles	16.4	9	San Antonio	1.6	1939	0.1585	158.4562	3.521249	4
3	Los Angeles	16.4	10	Dallas	5.2	1997	0.3931	393.13265	8.7362812	9
3	Los Angeles	16.4	11	Houston	4.7	2213	0.3571	357.14472	7.9365494	8
3	Los Angeles	16.4	12	Tulsa	0.8	2040	0.1058	105.78569	2.350793	3
3	Los Angeles	16.4	13	St Louis	2.6	2556	0.2184	218.37542	4.8527871	5
3	Los Angeles	16.4	14	Kansas City	1.8	2187	0.1697	169.69656	3.7710346	4
3	Los Angeles	16.4	15	Minneapolis	3	2461	0.2448	244.7672	5.4392711	6
3	Los Angeles	16.4	16	Chicago	9.2	2807	0.6343	634.28757	14.095279	15
3	Los Angeles	16.4	17	Cleveland	3	3302	0.2382	238.17948	5.2928774	6
3	Los Angeles	16.4	18	Cincinnati	2	3048	0.1771	177.14983	3.9366628	4
3	Los Angeles	16.4	19	Nashville	1.2	2860	0.1276	127.6486	2.8366355	3
3	Los Angeles	16.4	20	Atlanta	4.1	3120	0.3084	308.39712	6.8532693	7
3	Los Angeles	16.4	21	New Orleans	1.3	2686	0.1347	134.74913	2.9944251	3
3	Los Angeles	16.4	22	Miami	3.9	3760	0.2907	290.71923	6.4604273	7
3	Los Angeles	16.4	23	Charlotte	1.6	3410	0.1504	150.43354	3.3429675	4
3	Los Angeles	16.4	24	DC	4.9	3698	0.3529	352.86585	7.8414634	8
3	Los Angeles	16.4	25	NYC	9.3	3942	0.6205	620.54091	13.789798	14
3	Los Angeles	16.4	26	Boston	5.8	4173	0.4037	403.74024	8.9720053	9
3	Los Angeles	16.4	27	Albany	0.9	3952	0.104	103.97084	2.310463	3
4	Salt Lake City	1.3	1	Seattle	3.6	1135	0.0758	75.828081	1.6850685	2
4	Salt Lake City	1.3	2	San Francisco	7	959	0.0956	95.637953	2.1252878	3

4	Salt Lake City	1.3	3	Los Angeles	16	934	0.1478	147.78579	3.2841286	4
4	Salt Lake City	1.3	5	Los Vegas	1.6	587	0.0684	68.406929	1.520154	2
4	Salt Lake City	1.3	6	Pheonix	2.3	812	0.0707	70.656823	1.5701516	2
4	Salt Lake City	1.3	7	El Paso	0.7	1110	0.0601	60.061431	1.3346985	2
4	Salt Lake City	1.3	8	Denver	2.6	597	0.074	74.042707	1.6453935	2
4	Salt Lake City	1.3	9	San Antonio	1.6	1748	0.0626	62.573164	1.3905148	2
4	Salt Lake City	1.3	10	Dallas	5.2	1605	0.082	82.048751	1.8233056	2
4	Salt Lake City	1.3	11	Houston	4.7	1930	0.0781	78.118858	1.7359746	2
4	Salt Lake City	1.3	12	Tulsa	0.8	1475	0.0593	59.276575	1.3172572	2
4	Salt Lake City	1.3	13	St Louis	2.6	1866	0.0674	67.418067	1.4981793	2
4	Salt Lake City	1.3	14	Kansas City	1.8	1487	0.0645	64.532506	1.4340557	2
4	Salt Lake City	1.3	15	Minneapolis	3	1585	0.0705	70.514202	1.5669823	2
4	Salt Lake City	1.3	16	Chicago	9.2	2025	0.101	101.04719	2.245493	3
4	Salt Lake City	1.3	17	Cleveland	3	2522	0.0676	67.626731	1.5028162	2
4	Salt Lake City	1.3	18	Cincinnati	2	2333	0.063	63.007383	1.4001641	2
4	Salt Lake City	1.3	19	Nashville	1.2	2239	0.0591	59.145609	1.3143469	2
4	Salt Lake City	1.3	20	Atlanta	4.1	2545	0.0731	73.140101	1.6253356	2
4	Salt Lake City	1.3	21	New Orleans	1.3	2306	0.0595	59.497779	1.3221729	2
4	Salt Lake City	1.3	22	Miami	3.9	3358	0.0703	70.292039	1.5620453	2
4	Salt Lake City	1.3	23	Charlotte	1.6	2776	0.06	59.998059	1.3332902	2
4	Salt Lake City	1.3	24	DC	4.9	2970	0.0761	76.095102	1.6910023	2
4	Salt Lake City	1.3	25	NYC	9.3	3168	0.0975	97.469462	2.165988	3
4	Salt Lake City	1.3	26	Boston	5.8	3371	0.0796	79.638966	1.7697548	2
4	Salt Lake City	1.3	27	Albany	0.9	3146	0.0557	55.708907	1.2379757	2
5	Los Vegas	1.6	1	Seattle	3.6	1799	0.0772	77.204688	1.7156597	2
5	Los Vegas	1.6	2	San Francisco	7	664	0.1076	107.59915	2.3910923	3
5	Los Vegas	1.6	3	Los Angeles	16	367	0.1805	180.49506	4.0110013	5
5	Los Vegas	1.6	4	Salt Lake City	1.3	587	0.0684	68.406929	1.520154	2
5	Los Vegas	1.6	6	Pheonix	2.3	413	0.0773	77.310188	1.7180042	2
5	Los Vegas	1.6	7	El Paso	0.7	924	0.0618	61.840903	1.3742423	2
5	Los Vegas	1.6	8	Denver	2.6	976	0.0746	74.577975	1.6572883	2
5	Los Vegas	1.6	9	San Antonio	1.6	1729	0.0646	64.570027	1.4348895	2
5	Los Vegas	1.6	10	Dallas	5.2	1724	0.0878	87.826177	1.9516928	2
5	Los Vegas	1.6	11	Houston	4.7	1975	0.0836	83.577405	1.8572757	2
5	Los Vegas	1.6	12	Tulsa	0.8	1721	0.0595	59.450815	1.3211292	2
5	Los Vegas	1.6	13	St Louis	2.6	2218	0.0695	69.451589	1.5433686	2
5	Los Vegas	1.6	14	Kansas City	1.8	1840	0.0655	65.497009	1.4554891	2
5	Los Vegas	1.6	15	Minneapolis	3	2085	0.0724	72.380837	1.6084631	2

5	Los Vegas	1.6	16	Chicago	9.2	2453	0.1101	110.08593	2.4463539	3
5	Los Vegas	1.6	17	Cleveland	3	2945	0.0701	70.121926	1.558265	2
5	Los Vegas	1.6	18	Cincinnati	2	2708	0.0645	64.473431	1.4327429	2
5	Los Vegas	1.6	19	Nashville	1.2	2540	0.0599	59.869024	1.3304228	2
5	Los Vegas	1.6	20	Atlanta	4.1	2807	0.0772	77.228626	1.7161917	2
5	Los Vegas	1.6	21	New Orleans	1.3	2434	0.0607	60.729695	1.3495488	2
5	Los Vegas	1.6	22	Miami	3.9	3508	0.0744	74.426938	1.653932	2
5	Los Vegas	1.6	23	Charlotte	1.6	3090	0.0612	61.239308	1.3608735	2
5	Los Vegas	1.6	24	DC	4.9	3355	0.0808	80.813395	1.7958532	2
5	Los Vegas	1.6	25	NYC	9.3	3590	0.1069	106.85617	2.3745814	3
5	Los Vegas	1.6	26	Boston	5.8	3818	0.0852	85.235561	1.8941236	2
5	Los Vegas	1.6	27	Albany	0.9	3594	0.056	56.00836	1.2446302	2
6	Pheonix	2.3	1	Seattle	3.6	1799	0.0873	87.333583	1.9407463	2
6	Pheonix	2.3	2	San Francisco	7	1084	0.1241	124.07215	2.7571589	3
6	Pheonix	2.3	3	Los Angeles	16	576	0.2257	225.72465	5.0161033	6
6	Pheonix	2.3	4	Salt Lake City	1.3	812	0.0707	70.656823	1.5701516	2
6	Pheonix	2.3	5	Los Vegas	1.6	413	0.0773	77.310188	1.7180042	2
6	Pheonix	2.3	7	El Paso	2.3	545	0.083	82.990565	1.8442348	2
6	Pheonix	2.3	8	Denver	0.7	943	0.0638	63.760859	1.416908	2
6	Pheonix	2.3	9	San Antonio	2.6	1365	0.08	80.001796	1.7778177	2
6	Pheonix	2.3	10	Dallas	1.6	1427	0.0703	70.260618	1.5613471	2
6	Pheonix	2.3	11	Houston	5.2	1635	0.103	102.99379	2.288751	3
6	Pheonix	2.3	12	Tulsa	4.7	1502	0.0991	99.066419	2.201476	3
6	Pheonix	2.3	13	St Louis	0.8	2046	0.0608	60.770952	1.3504656	2
6	Pheonix	2.3	14	Kansas City	2.6	1687	0.0785	78.535869	1.7452415	2
6	Pheonix	2.3	15	Minneapolis	1.8	2055	0.0699	69.854392	1.5523198	2
6	Pheonix	2.3	16	Chicago	3	2340	0.0799	79.868098	1.7748466	2
6	Pheonix	2.3	17	Cleveland	9.2	2815	0.1335	133.54773	2.9677273	3
6	Pheonix	2.3	18	Cincinnati	3	2540	0.0793	79.26958	1.7615462	2
6	Pheonix	2.3	19	Nashville	2	2325	0.0709	70.881991	1.5751553	2
6	Pheonix	2.3	20	Atlanta	1.2	2560	0.0631	63.095835	1.4021297	2
6	Pheonix	2.3	21	New Orleans	4.1	2116	0.0906	90.624625	2.0138805	3
6	Pheonix	2.3	22	Miami	1.3	3189	0.0627	62.699245	1.3933166	2
6	Pheonix	2.3	23	Charlotte	3.9	2868	0.0864	86.358489	1.9190775	2
6	Pheonix	2.3	24	DC	1.6	3190	0.0653	65.329187	1.4517597	2
6	Pheonix	2.3	25	NYC	4.9	3447	0.0936	93.596231	2.0799162	3
6	Pheonix	2.3	26	Boston	9.3	3698	0.131	131.04576	2.912128	3
6	Pheonix	2.3	27	Albany	5.8	3478	0.1013	101.34906	2.2522014	3
					0.9					
7	El Paso	0.68	1	Seattle	3.6	2188	0.0628	62.776588	1.3950353	2
7	El Paso	0.68	2	San Francisco	7	1593	0.074	73.977113	1.6439358	2
7	El Paso	0.68	3	Los Angeles	16	1131	0.1029	102.9304	2.2873422	3
7	El Paso	0.68	4	Salt Lake City	1.3	1110	0.0601	60.061431	1.3346985	2
7	El Paso	0.68	5	Los Vegas	1.6	924	0.0618	61.840903	1.3742423	2
7	El Paso	0.68	6	Pheonix	2.3	545	0.0665	66.488118	1.4775137	2
7	El Paso	0.68	8	Denver	2.6	893	0.0649	64.91191	1.4424869	2
7	El Paso	0.68	9	San Antonio	1.6	826	0.0624	62.397171	1.3866038	2
7	El Paso	0.68	10	Dallas	5.2	931	0.0722	72.211436	1.6046986	2
7	El Paso	0.68	11	Houston	4.7	1102	0.0698	69.792473	1.5509438	2
7	El Paso	0.68	12	Tulsa	0.8	1095	0.0587	58.711684	1.3047041	2

7	El Paso	0.68	13	St Louis	2.6	1669	0.0616	61.567612	1.3681692	2
7	El Paso	0.68	14	Kansas City	1.8	1357	0.0604	60.447828	1.3432851	2
7	El Paso	0.68	15	Minneapolis	3	1864	0.0621	62.055573	1.3790127	2
7	El Paso	0.68	16	Chicago	9.2	2025	0.0784	78.362474	1.7413883	2
7	El Paso	0.68	17	Cleveland	3	2465	0.0605	60.502957	1.3445102	2
7	El Paso	0.68	18	Cincinnati	2	2158	0.0586	58.553209	1.3011824	2
7	El Paso	0.68	19	Nashville	1.2	1890	0.0571	57.080574	1.2684572	2
7	El Paso	0.68	20	Atlanta	4.1	2086	0.0644	64.39743	1.431054	2
7	El Paso	0.68	21	New Orleans	1.3	1595	0.0582	58.222929	1.2938429	2
7	El Paso	0.68	22	Miami	3.9	2660	0.0625	62.454312	1.3878736	2
7	El Paso	0.68	23	Charlotte	1.6	2416	0.0569	56.888703	1.2641934	2
7	El Paso	0.68	24	DC	4.9	2788	0.0648	64.809704	1.4402156	2
7	El Paso	0.68	25	NYC	9.3	3071	0.0757	75.682209	1.6818269	2
7	El Paso	0.68	26	Boston	5.8	3340	0.0661	66.051185	1.4678041	2
7	El Paso	0.68	27	Albany	0.9	3130	0.0537	53.660221	1.1924494	2
8	Denver	2.6	1	Seattle	3.6	1644	0.0924	92.409707	2.053549	3
8	Denver	2.6	2	San Francisco	7	1521	0.1291	129.10445	2.8689877	3
8	Denver	2.6	3	Los Angeles	16	1340	0.2313	231.33003	5.1406674	6
8	Denver	2.6	4	Salt Lake City	1.3	597	0.074	74.042707	1.6453935	2
8	Denver	2.6	5	Los Vegas	1.6	976	0.0746	74.577975	1.6572883	2
8	Denver	2.6	6	Pheonix	2.3	943	0.0825	82.517051	1.8337122	2
8	Denver	2.6	7	El Paso	0.7	893	0.0649	64.91191	1.4424869	2
8	Denver	2.6	9	San Antonio	1.6	1295	0.0728	72.835517	1.618567	2
8	Denver	2.6	10	Dallas	5.2	1067	0.1134	113.38741	2.5197201	3
8	Denver	2.6	11	Houston	4.7	1419	0.1053	105.34375	2.3409723	3
8	Denver	2.6	12	Tulsa	0.8	886	0.0663	66.318011	1.4737336	2
8	Denver	2.6	13	St Louis	2.6	1278	0.0837	83.684796	1.8596621	2
8	Denver	2.6	14	Kansas City	1.8	898	0.0773	77.299936	1.7177764	2
8	Denver	2.6	15	Minneapolis	3	1122	0.089	88.964528	1.9769895	2
8	Denver	2.6	16	Chicago	9.2	1479	0.1528	152.81123	3.3958051	4
8	Denver	2.6	17	Cleveland	3	1975	0.0847	84.693471	1.8820771	2
8	Denver	2.6	18	Cincinnati	2	1759	0.0751	75.103953	1.6689767	2
8	Denver	2.6	19	Nashville	1.2	1645	0.0671	67.1239	1.4916422	2
8	Denver	2.6	20	Atlanta	4.1	1949	0.0962	96.207855	2.1379523	3
8	Denver	2.6	21	New Orleans	1.3	1741	0.0678	67.835938	1.5074653	2
8	Denver	2.6	22	Miami	3.9	2777	0.0911	91.13775	2.0252833	3
8	Denver	2.6	23	Charlotte	1.6	2184	0.0695	69.549191	1.5455376	2
8	Denver	2.6	24	DC	4.9	2400	0.1026	102.55288	2.2789528	3
8	Denver	2.6	25	NYC	9.3	2619	0.1462	146.17408	3.2483129	4
8	Denver	2.6	26	Boston	5.8	2844	0.11	109.98117	2.444026	3
8	Denver	2.6	27	Albany	0.9	2620	0.0611	61.087334	1.3574963	2
9	San Antonio	1.6	1	Seattle	3.6	2876	0.074	73.973805	1.6438623	2
9	San Antonio	1.6	2	San Francisco	7	2389	0.0966	96.562816	2.1458403	3
9	San Antonio	1.6	3	Los Angeles	16	1939	0.1585	158.4562	3.521249	4
9	San Antonio	1.6	4	Salt Lake City	1.3	1748	0.0626	62.573164	1.3905148	2
9	San Antonio	1.6	5	Los Vegas	1.6	1729	0.0646	64.570027	1.4348895	2
9	San Antonio	1.6	6	Pheonix	2.3	1365	0.0705	70.530716	1.5673493	2
9	San Antonio	1.6	7	El Paso	0.7	826	0.0624	62.397171	1.3866038	2
9	San Antonio	1.6	8	Denver	2.6	1295	0.0728	72.835517	1.618567	2

9	San Antonio	1.6	10	Dallas	5.2	411	0.0983	98.284723	2.184105	3
9	San Antonio	1.6	11	Houston	4.7	304	0.0964	96.447927	2.1432873	3
9	San Antonio	1.6	12	Tulsa	0.8	787	0.0635	63.482693	1.4107265	2
9	San Antonio	1.6	13	St Louis	2.6	1278	0.0729	72.91775	1.6203944	2
9	San Antonio	1.6	14	Kansas City	1.8	1139	0.0683	68.278687	1.5173041	2
9	San Antonio	1.6	15	Minneapolis	3	1791	0.0734	73.375186	1.6305597	2
9	San Antonio	1.6	16	Chicago	9.2	1700	0.1138	113.78917	2.5286483	3
9	San Antonio	1.6	17	Cleveland	3	2027	0.0726	72.565525	1.6125672	2
9	San Antonio	1.6	18	Cincinnati	2	1674	0.0673	67.344027	1.4965339	2
9	San Antonio	1.6	19	Nashville	1.2	1328	0.0634	63.432928	1.4096206	2
9	San Antonio	1.6	20	Atlanta	4.1	1421	0.0821	82.104599	1.8245466	2
9	San Antonio	1.6	21	New Orleans	1.3	818	0.0667	66.706616	1.4823692	2
9	San Antonio	1.6	22	Miami	3.9	1848	0.0789	78.944478	1.7543217	2
9	San Antonio	1.6	23	Charlotte	1.6	1782	0.0644	64.397025	1.431045	2
9	San Antonio	1.6	24	DC	4.9	2235	0.0839	83.908363	1.8646303	2
9	San Antonio	1.6	25	NYC	9.3	2549	0.1103	110.3213	2.4515844	3
9	San Antonio	1.6	26	Boston	5.8	2845	0.0876	87.618147	1.9470699	2
9	San Antonio	1.6	27	Albany	0.9	2655	0.0576	57.613833	1.2803074	2
10	Dallas	5.2	1	Seattle	3.6	2710	0.1246	124.5699	2.76822	3
10	Dallas	5.2	2	San Francisco	7	2378	0.1954	195.37071	4.3415714	5
10	Dallas	5.2	3	Los Angeles	16	1997	0.3931	393.13265	8.7362812	9
10	Dallas	5.2	4	Salt Lake City	1.3	1605	0.082	82.048751	1.8233056	2
10	Dallas	5.2	5	Los Vegas	1.6	1724	0.0878	87.826177	1.9516928	2
10	Dallas	5.2	6	Pheonix	2.3	1427	0.1042	104.22593	2.3161318	3
10	Dallas	5.2	7	El Paso	0.7	931	0.0722	72.211436	1.6046986	2
10	Dallas	5.2	8	Denver	2.6	1067	0.1134	113.38741	2.5197201	3
10	Dallas	5.2	9	San Antonio	1.6	411	0.0983	98.284723	2.184105	3
10	Dallas	5.2	11	Houston	4.7	362	0.1725	172.45637	3.8323637	4
10	Dallas	5.2	12	Tulsa	0.8	380	0.08	79.974139	1.7772031	2
10	Dallas	5.2	13	St Louis	2.6	879	0.1152	115.192	2.5598223	3
10	Dallas	5.2	14	Kansas City	1.8	732	0.0988	98.831698	2.1962599	3
10	Dallas	5.2	15	Minneapolis	3	1390	0.1194	119.42858	2.6539684	3
10	Dallas	5.2	16	Chicago	9.2	1296	0.2535	253.50117	5.6333594	6
10	Dallas	5.2	17	Cleveland	3	1650	0.1177	117.6541	2.6145355	3
10	Dallas	5.2	18	Cincinnati	2	1308	0.0986	98.563041	2.1902898	3
10	Dallas	5.2	19	Nashville	1.2	991	0.0833	83.283872	1.8507527	2
10	Dallas	5.2	20	Atlanta	4.1	1158	0.1452	145.18741	3.2263868	4
10	Dallas	5.2	21	New Orleans	1.3	710	0.0878	87.770319	1.9504515	2
10	Dallas	5.2	22	Miami	3.9	1785	0.1357	135.66077	3.0146839	4
10	Dallas	5.2	23	Charlotte	1.6	1495	0.0889	88.933809	1.9763069	2
10	Dallas	5.2	24	DC	4.9	1903	0.1557	155.68342	3.4596315	4
10	Dallas	5.2	25	NYC	9.3	2205	0.2439	243.92091	5.4204647	6
10	Dallas	5.2	26	Boston	5.8	2495	0.1702	170.16291	3.7813979	4
10	Dallas	5.2	27	Albany	0.9	2293	0.0708	70.795563	1.5732347	2
11	Houston	4.7	1	Seattle	3.6	3051	0.1163	116.31189	2.5847087	3
11	Houston	4.7	2	San Francisco	7	2414	0.1814	181.40326	4.0311835	5
11	Houston	4.7	3	Los Angeles	16	2213	0.3571	357.14472	7.9365494	8
11	Houston	4.7	4	Salt Lake City	1.3	1930	0.0781	78.118858	1.7359746	2
11	Houston	4.7	5	Los Vegas	1.6	1975	0.0836	83.577405	1.8572757	2

11	Houston	4.7	6	Pheonix	2.3	1635	0.0983	98.332157	2.185159	3
11	Houston	4.7	7	El Paso	0.7	1102	0.0698	69.792473	1.5509438	2
11	Houston	4.7	8	Denver	2.6	1419	0.1053	105.34375	2.3409723	3
11	Houston	4.7	9	San Antonio	1.6	304	0.0964	96.447927	2.1432873	3
11	Houston	4.7	10	Dallas	5.2	362	0.1725	172.45637	3.8323637	4
11	Houston	4.7	12	Tulsa	0.8	713	0.0748	74.77744	1.6617209	2
11	Houston	4.7	13	St Louis	2.6	1093	0.1077	107.70592	2.3934648	3
11	Houston	4.7	14	Kansas City	1.8	1042	0.0923	92.293404	2.0509645	3
11	Houston	4.7	15	Minneapolis	3	1703	0.1113	111.26721	2.4726046	3
11	Houston	4.7	16	Chicago	9.2	1515	0.2313	231.33215	5.1407144	6
11	Houston	4.7	17	Cleveland	3	1796	0.1107	110.74297	2.4609549	3
11	Houston	4.7	18	Cincinnati	2	1437	0.0937	93.667866	2.0815081	3
11	Houston	4.7	19	Nashville	1.2	1071	0.0802	80.230915	1.7829092	2
11	Houston	4.7	20	Atlanta	4.1	1130	0.1369	136.90757	3.0423904	4
11	Houston	4.7	21	New Orleans	1.3	514	0.087	86.981367	1.9329193	2
11	Houston	4.7	22	Miami	3.9	1556	0.1294	129.37702	2.8750448	3
11	Houston	4.7	23	Charlotte	1.6	1492	0.0857	85.680384	1.9040085	2
11	Houston	4.7	24	DC	4.9	1963	0.1455	145.47922	3.2328716	4
11	Houston	4.7	25	NYC	9.3	2285	0.2248	224.83909	4.9964243	5
11	Houston	4.7	26	Boston	5.8	2587	0.1583	158.31836	3.5181858	4
11	Houston	4.7	27	Albany	0.9	2404	0.0688	68.77528	1.5283396	2
12	Tulsa	0.8	1	Seattle	3.6	2513	0.0637	63.713566	1.415857	2
12	Tulsa	0.8	2	San Francisco	7	2346	0.0748	74.830366	1.662897	2
12	Tulsa	0.8	3	Los Angeles	16	2040	0.1058	105.78569	2.350793	3
12	Tulsa	0.8	4	Salt Lake City	1.3	1475	0.0593	59.276575	1.3172572	2
12	Tulsa	0.8	5	Los Vegas	1.6	1721	0.0595	59.450815	1.3211292	2
12	Tulsa	0.8	6	Pheonix	2.3	1502	0.0625	62.46232	1.3880516	2
12	Tulsa	0.8	7	El Paso	0.7	1095	0.0587	58.711684	1.3047041	2
12	Tulsa	0.8	8	Denver	2.6	886	0.0663	66.318011	1.4737336	2
12	Tulsa	0.8	9	San Antonio	1.6	787	0.0635	63.482693	1.4107265	2
12	Tulsa	0.8	10	Dallas	5.2	380	0.08	79.974139	1.7772031	2
12	Tulsa	0.8	11	Houston	4.7	713	0.0748	74.77744	1.6617209	2
12	Tulsa	0.8	13	St Louis	2.6	577	0.0685	68.526673	1.522815	2
12	Tulsa	0.8	14	Kansas City	1.8	352	0.0679	67.908473	1.5090772	2
12	Tulsa	0.8	15	Minneapolis	3	1007	0.067	66.990056	1.4886679	2
12	Tulsa	0.8	16	Chicago	9.2	965	0.0883	88.329916	1.962887	2
12	Tulsa	0.8	17	Cleveland	3	1372	0.0653	65.26815	1.4504033	2
12	Tulsa	0.8	18	Cincinnati	2	1059	0.0633	63.33427	1.4074282	2
12	Tulsa	0.8	19	Nashville	1.2	824	0.0619	61.875012	1.3750003	2
12	Tulsa	0.8	20	Atlanta	4.1	1087	0.0703	70.276425	1.5616983	2
12	Tulsa	0.8	21	New Orleans	1.3	879	0.0619	61.901178	1.3755817	2
12	Tulsa	0.8	22	Miami	3.9	1890	0.0663	66.347364	1.4743859	2
12	Tulsa	0.8	23	Charlotte	1.6	1371	0.0606	60.6447	1.34766	2
12	Tulsa	0.8	24	DC	4.9	1696	0.0702	70.234861	1.5607747	2
12	Tulsa	0.8	25	NYC	9.3	1974	0.0834	83.373536	1.8527453	2
12	Tulsa	0.8	26	Boston	5.8	2245	0.0713	71.334887	1.5852197	2
12	Tulsa	0.8	27	Albany	0.9	2036	0.0563	56.252488	1.2500553	2
13	St Louis	2.6	1	Seattle	3.6	2775	0.0881	88.129921	1.9584427	2
13	St Louis	2.6	2	San Francisco	7	2797	0.1222	122.19814	2.7155143	3
13	St Louis	2.6	3	Los	16	2556	0.2184	218.37542	4.8527871	5

				Angeles						
13	St Louis	2.6	4	Salt Lake City	1.3	1866	0.0674	67.418067	1.4981793	2
13	St Louis	2.6	5	Los Vegas	1.6	2218	0.0695	69.451589	1.5433686	2
13	St Louis	2.6	6	Pheonix	2.3	2046	0.0772	77.192925	1.7153983	2
13	St Louis	2.6	7	El Paso	0.7	1669	0.0616	61.567612	1.3681692	2
13	St Louis	2.6	8	Denver	2.6	1278	0.0837	83.684796	1.8596621	2
13	St Louis	2.6	9	San Antonio	1.6	1278	0.0729	72.91775	1.6203944	2
13	St Louis	2.6	10	Dallas	5.2	879	0.1152	115.192	2.5598223	3
13	St Louis	2.6	11	Houston	4.7	1093	0.1077	107.70592	2.3934648	3
13	St Louis	2.6	12	Tulsa	0.8	577	0.0685	68.526673	1.522815	2
13	St Louis	2.6	14	Kansas City	1.8	384	0.0822	82.208423	1.8268538	2
13	St Louis	2.6	15	Minneapolis	3	754	0.0919	91.861813	2.0413736	3
13	St Louis	2.6	16	Chicago	9.2	427	0.1683	168.29841	3.7399647	4
13	St Louis	2.6	17	Cleveland	3	796	0.0915	91.474853	2.0327745	3
13	St Louis	2.6	18	Cincinnati	2	496	0.0831	83.140417	1.8475648	2
13	St Louis	2.6	19	Nashville	1.2	407	0.0749	74.853776	1.6634172	2
13	St Louis	2.6	20	Atlanta	4.1	755	0.1042	104.2158	2.3159067	3
13	St Louis	2.6	21	New Orleans	1.3	963	0.0714	71.352249	1.5856055	2
13	St Louis	2.6	22	Miami	3.9	1710	0.0952	95.238039	2.1164009	3
13	St Louis	2.6	23	Charlotte	1.6	915	0.075	74.969381	1.6659862	2
13	St Louis	2.6	24	DC	4.9	1145	0.1095	109.46195	2.4324878	3
13	St Louis	2.6	25	NYC	9.3	1405	0.1546	154.56118	3.434693	4
13	St Louis	2.6	26	Boston	5.8	1673	0.1154	115.40677	2.5645948	3
13	St Louis	2.6	27	Albany	0.9	1458	0.0644	64.387676	1.4308372	2
14	Kansas City	1.8	1	Seattle	3.6	2423	0.078	77.970382	1.7326752	2
14	Kansas City	1.8	2	San Francisco	7	2417	0.1019	101.93869	2.2653041	3
14	Kansas City	1.8	3	Los Angeles	16	2187	0.1697	169.69656	3.7710346	4
14	Kansas City	1.8	4	Salt Lake City	1.3	1487	0.0645	64.532506	1.4340557	2
14	Kansas City	1.8	5	Los Vegas	1.6	1840	0.0655	65.497009	1.4554891	2
14	Kansas City	1.8	6	Pheonix	2.3	1687	0.0711	71.097842	1.579952	2
14	Kansas City	1.8	7	El Paso	0.7	1357	0.0604	60.447828	1.3432851	2
14	Kansas City	1.8	8	Denver	2.6	898	0.0773	77.299936	1.7177764	2
14	Kansas City	1.8	9	San Antonio	1.6	1139	0.0683	68.278687	1.5173041	2
14	Kansas City	1.8	10	Dallas	5.2	732	0.0988	98.831698	2.1962599	3
14	Kansas City	1.8	11	Houston	4.7	1042	0.0923	92.293404	2.0509645	3
14	Kansas City	1.8	12	Tulsa	0.8	352	0.0679	67.908473	1.5090772	2
14	Kansas City	1.8	13	St Louis	2.6	384	0.0822	82.208423	1.8268538	2
14	Kansas City	1.8	15	Minneapolis	3	660	0.0823	82.31697	1.829266	2
14	Kansas City	1.8	16	Chicago	9.2	664	0.131	130.99787	2.9110637	3
14	Kansas City	1.8	17	Cleveland	3	1129	0.0789	78.874781	1.7527729	2
14	Kansas City	1.8	18	Cincinnati	2	869	0.0729	72.885975	1.6196883	2
14	Kansas City	1.8	19	Nashville	1.2	763	0.0674	67.416282	1.4981396	2
14	Kansas City	1.8	20	Atlanta	4.1	1090	0.0874	87.416705	1.9425934	2
14	Kansas City	1.8	21	New Orleans	1.3	1099	0.0662	66.218091	1.4715131	2
14	Kansas City	1.8	22	Miami	3.9	2001	0.0815	81.488421	1.8108538	2
14	Kansas City	1.8	23	Charlotte	1.6	1292	0.0676	67.554183	1.5012041	2
14	Kansas City	1.8	24	DC	4.9	1520	0.0908	90.845015	2.0187781	3
14	Kansas City	1.8	25	NYC	9.3	1764	0.1216	121.55009	2.7011132	3
14	Kansas City	1.8	26	Boston	5.8	2012	0.0951	95.057357	2.1123857	3
14	Kansas City	1.8	27	Albany	0.9	1791	0.0604	60.414702	1.3425489	2

15	Minneapolis	3	1	Seattle	3.6	2248	0.0955	95.52498	2.1227773	3
15	Minneapolis	3	2	San Francisco	7	2552	0.1341	134.14066	2.9809035	3
15	Minneapolis	3	3	Los Angeles	16	2461	0.2448	244.7672	5.4392711	6
15	Minneapolis	3	4	Salt Lake City	1.3	1585	0.0705	70.514202	1.5669823	2
15	Minneapolis	3	5	Los Vegas	1.6	2085	0.0724	72.380837	1.6084631	2
15	Minneapolis	3	6	Pheonix	2.3	2055	0.0808	80.816305	1.7959179	2
15	Minneapolis	3	7	El Paso	0.7	1864	0.0621	62.055573	1.3790127	2
15	Minneapolis	3	8	Denver	2.6	1122	0.089	88.964528	1.9769895	2
15	Minneapolis	3	9	San Antonio	1.6	1791	0.0734	73.375186	1.6305597	2
15	Minneapolis	3	10	Dallas	5.2	1390	0.1194	119.42858	2.6539684	3
15	Minneapolis	3	11	Houston	4.7	1703	0.1113	111.26721	2.4726046	3
15	Minneapolis	3	12	Tulsa	0.8	1007	0.067	66.990056	1.4886679	2
15	Minneapolis	3	13	St Louis	2.6	754	0.0919	91.861813	2.0413736	3
15	Minneapolis	3	14	Kansas City	1.8	660	0.0823	82.31697	1.829266	2
15	Minneapolis	3	16	Chicago	9.2	572	0.1812	181.1721	4.0260466	5
15	Minneapolis	3	17	Cleveland	3	1017	0.0948	94.757693	2.1057265	3
15	Minneapolis	3	18	Cincinnati	2	972	0.0824	82.398819	1.8310849	2
15	Minneapolis	3	19	Nashville	1.2	1124	0.0714	71.368667	1.5859704	2
15	Minneapolis	3	20	Atlanta	4.1	1463	0.1054	105.39253	2.3420561	3
15	Minneapolis	3	21	New Orleans	1.3	1695	0.0701	70.098315	1.5577403	2
15	Minneapolis	3	22	Miami	3.9	2436	0.0983	98.345913	2.1854647	3
15	Minneapolis	3	23	Charlotte	1.6	1513	0.0745	74.47495	1.6549989	2
15	Minneapolis	3	24	DC	4.9	1504	0.1149	114.93932	2.554207	3
15	Minneapolis	3	25	NYC	9.3	1637	0.1676	167.59007	3.7242238	4
15	Minneapolis	3	26	Boston	5.8	1808	0.1239	123.9354	2.7541201	3
15	Minneapolis	3	27	Albany	0.9	1584	0.0653	65.34754	1.4521676	2
16	Chicago	9.2	1	Seattle	3.6	2794	0.1798	179.82461	3.9961025	4
16	Chicago	9.2	2	San Francisco	7	2980	0.2988	298.81914	6.6404253	7
16	Chicago	9.2	3	Los Angeles	16	2807	0.6343	634.28757	14.095279	15
16	Chicago	9.2	4	Salt Lake City	1.3	2025	0.101	101.04719	2.245493	3
16	Chicago	9.2	5	Los Vegas	1.6	2453	0.1101	110.08593	2.4463539	3
16	Chicago	9.2	6	Pheonix	2.3	2340	0.1358	135.84014	3.0186698	4
16	Chicago	9.2	7	El Paso	0.7	2025	0.0784	78.362474	1.7413883	2
16	Chicago	9.2	8	Denver	2.6	1479	0.1528	152.81123	3.3958051	4
16	Chicago	9.2	9	San Antonio	1.6	1700	0.1138	113.78917	2.5286483	3
16	Chicago	9.2	10	Dallas	5.2	1296	0.2535	253.50117	5.6333594	6
16	Chicago	9.2	11	Houston	4.7	1515	0.2313	231.33215	5.1407144	6
16	Chicago	9.2	12	Tulsa	0.8	965	0.0883	88.329916	1.962887	2
16	Chicago	9.2	13	St Louis	2.6	427	0.1683	168.29841	3.7399647	4
16	Chicago	9.2	14	Kansas City	1.8	664	0.131	130.99787	2.9110637	3
16	Chicago	9.2	15	Minneapolis	3	572	0.1812	181.1721	4.0260466	5
16	Chicago	9.2	17	Cleveland	3	496	0.183	183.00879	4.0668619	5
16	Chicago	9.2	18	Cincinnati	2	407	0.1439	143.8884	3.1975201	4
16	Chicago	9.2	19	Nashville	1.2	643	0.1072	107.15798	2.3812884	3
16	Chicago	9.2	20	Atlanta	4.1	948	0.2172	217.23397	4.8274216	5
16	Chicago	9.2	21	New Orleans	1.3	1344	0.1048	104.76623	2.3281386	3
16	Chicago	9.2	22	Miami	3.9	1918	0.1971	197.13694	4.3808209	5
16	Chicago	9.2	23	Charlotte	1.6	949	0.1196	119.57714	2.6572697	3
16	Chicago	9.2	24	DC	4.9	960	0.2482	248.22705	5.5161567	6
16	Chicago	9.2	25	NYC	9.3	1145	0.4138	413.83875	9.1964166	10

16	Chicago	9.2	26	Boston	5.8	1367	0.2751	275.07134	6.1126965	7
16	Chicago	9.2	27	Albany	0.9	1145	0.0899	89.896904	1.997709	2
17	Cleveland	3	1	Seattle	3.6	3260	0.0923	92.298376	2.051075	3
17	Cleveland	3	2	San Francisco	7	3480	0.1303	130.32267	2.8960594	3
17	Cleveland	3	3	Los Angeles	16	3302	0.2382	238.17948	5.2928774	6
17	Cleveland	3	4	Salt Lake City	1.3	2522	0.0676	67.626731	1.5028162	2
17	Cleveland	3	5	Los Vegas	1.6	2945	0.0701	70.121926	1.558265	2
17	Cleveland	3	6	Pheonix	2.3	2815	0.0785	78.520261	1.7448947	2
17	Cleveland	3	7	El Paso	0.7	2465	0.0605	60.502957	1.3445102	2
17	Cleveland	3	8	Denver	2.6	1975	0.0847	84.693471	1.8820771	2
17	Cleveland	3	9	San Antonio	1.6	2027	0.0726	72.565525	1.6125672	2
17	Cleveland	3	10	Dallas	5.2	1650	0.1177	117.6541	2.6145355	3
17	Cleveland	3	11	Houston	4.7	1796	0.1107	110.74297	2.4609549	3
17	Cleveland	3	12	Tulsa	0.8	1372	0.0653	65.26815	1.4504033	2
17	Cleveland	3	13	St Louis	2.6	796	0.0915	91.474853	2.0327745	3
17	Cleveland	3	14	Kansas City	1.8	1129	0.0789	78.874781	1.7527729	2
17	Cleveland	3	15	Minneapolis	3	1017	0.0948	94.757693	2.1057265	3
17	Cleveland	3	16	Chicago	9.2	496	0.183	183.00879	4.0668619	5
17	Cleveland	3	18	Cincinnati	2	358	0.0886	88.583321	1.9685182	2
17	Cleveland	3	19	Nashville	1.2	741	0.0738	73.802978	1.6400662	2
17	Cleveland	3	20	Atlanta	4.1	894	0.1098	109.83187	2.4407083	3
17	Cleveland	3	21	New Orleans	1.3	1489	0.0709	70.90061	1.5755691	2
17	Cleveland	3	22	Miami	3.9	1769	0.1012	101.23137	2.2495859	3
17	Cleveland	3	23	Charlotte	1.6	706	0.0793	79.291529	1.762034	2
17	Cleveland	3	24	DC	4.9	492	0.1256	125.56562	2.7903472	3
17	Cleveland	3	25	NYC	9.3	649	0.1808	180.811	4.0180222	5
17	Cleveland	3	26	Boston	5.8	884	0.1317	131.70537	2.926786	3
17	Cleveland	3	27	Albany	0.9	668	0.0701	70.147003	1.5588223	2
18	Cincinnati	2	1	Seattle	3.6	3174	0.0788	78.792129	1.7509362	2
18	Cincinnati	2	2	San Francisco	7	3280	0.1044	104.41911	2.3204246	3
18	Cincinnati	2	3	Los Angeles	16	3048	0.1771	177.14983	3.9366628	4
18	Cincinnati	2	4	Salt Lake City	1.3	2333	0.063	63.007383	1.4001641	2
18	Cincinnati	2	5	Los Vegas	1.6	2708	0.0645	64.473431	1.4327429	2
18	Cincinnati	2	6	Pheonix	2.3	2540	0.0703	70.309484	1.562433	2
18	Cincinnati	2	7	El Paso	0.7	2158	0.0586	58.553209	1.3011824	2
18	Cincinnati	2	8	Denver	2.6	1759	0.0751	75.103953	1.6689767	2
18	Cincinnati	2	9	San Antonio	1.6	1674	0.0673	67.344027	1.4965339	2
18	Cincinnati	2	10	Dallas	5.2	1308	0.0986	98.563041	2.1902898	3
18	Cincinnati	2	11	Houston	4.7	1437	0.0937	93.667866	2.0815081	3
18	Cincinnati	2	12	Tulsa	0.8	1059	0.0633	63.33427	1.4074282	2
18	Cincinnati	2	13	St Louis	2.6	496	0.0831	83.140417	1.8475648	2
18	Cincinnati	2	14	Kansas City	1.8	869	0.0729	72.885975	1.6196883	2
18	Cincinnati	2	15	Minneapolis	3	972	0.0824	82.398819	1.8310849	2
18	Cincinnati	2	16	Chicago	9.2	407	0.1439	143.8884	3.1975201	4
18	Cincinnati	2	17	Cleveland	3	358	0.0886	88.583321	1.9685182	2
18	Cincinnati	2	19	Nashville	1.2	387	0.0718	71.833288	1.5962953	2
18	Cincinnati	2	20	Atlanta	4.1	600	0.0952	95.215665	2.1159037	3
18	Cincinnati	2	21	New Orleans	1.3	1131	0.0671	67.147442	1.4921654	2
18	Cincinnati	2	22	Miami	3.9	1537	0.0866	86.599566	1.9244348	2

18	Cincinnati	2	23	Charlotte	1.6	541	0.0738	73.772619	1.6393915	2
18	Cincinnati	2	24	DC	4.9	648	0.1017	101.67307	2.2594015	3
18	Cincinnati	2	25	NYC	9.3	912	0.1365	136.48652	3.0330338	4
18	Cincinnati	2	26	Boston	5.8	1190	0.1044	104.35791	2.3190647	3
18	Cincinnati	2	27	Albany	0.9	984	0.0643	64.336087	1.4296908	2
19	Nashville	1.2	1	Seattle	3.6	3178	0.0678	67.794153	1.5065367	2
19	Nashville	1.2	2	San Francisco	7	3153	0.0834	83.423052	1.8538456	2
19	Nashville	1.2	3	Los Angeles	16	2860	0.1276	127.6486	2.8366355	3
19	Nashville	1.2	4	Salt Lake City	1.3	2239	0.0591	59.145609	1.3143469	2
19	Nashville	1.2	5	Los Vegas	1.6	2540	0.0599	59.869024	1.3304228	2
19	Nashville	1.2	6	Pheonix	2.3	2325	0.0637	63.655546	1.4145677	2
19	Nashville	1.2	7	El Paso	0.7	1890	0.0571	57.080574	1.2684572	2
19	Nashville	1.2	8	Denver	2.6	1645	0.0671	67.1239	1.4916422	2
19	Nashville	1.2	9	San Antonio	1.6	1328	0.0634	63.432928	1.4096206	2
19	Nashville	1.2	10	Dallas	5.2	991	0.0833	83.283872	1.8507527	2
19	Nashville	1.2	11	Houston	4.7	1071	0.0802	80.230915	1.7829092	2
19	Nashville	1.2	12	Tulsa	0.8	824	0.0619	61.875012	1.3750003	2
19	Nashville	1.2	13	St Louis	2.6	407	0.0749	74.853776	1.6634172	2
19	Nashville	1.2	14	Kansas City	1.8	763	0.0674	67.416282	1.4981396	2
19	Nashville	1.2	15	Minneapolis	3	1124	0.0714	71.368667	1.5859704	2
19	Nashville	1.2	16	Chicago	9.2	643	0.1072	107.15798	2.3812884	3
19	Nashville	1.2	17	Cleveland	3	741	0.0738	73.802978	1.6400662	2
19	Nashville	1.2	18	Cincinnati	2	387	0.0718	71.833288	1.5962953	2
19	Nashville	1.2	20	Atlanta	4.1	345	0.0839	83.854553	1.8634345	2
19	Nashville	1.2	21	New Orleans	1.3	754	0.0649	64.884323	1.4418738	2
19	Nashville	1.2	22	Miami	3.9	1314	0.0749	74.893104	1.6642912	2
19	Nashville	1.2	23	Charlotte	1.6	550	0.068	68.02052	1.5115671	2
19	Nashville	1.2	24	DC	4.9	914	0.0823	82.299859	1.8288857	2
19	Nashville	1.2	25	NYC	9.3	1224	0.1023	102.28075	2.2729055	3
19	Nashville	1.2	26	Boston	5.8	1521	0.0833	83.252135	1.8500475	2
19	Nashville	1.2	27	Albany	0.9	1332	0.0598	59.831429	1.3295873	2
20	Atlanta	4.1	1	Seattle	3.6	3506	0.1066	106.64099	2.3697997	3
20	Atlanta	4.1	2	San Francisco	7	3435	0.1596	159.64832	3.5477404	4
20	Atlanta	4.1	3	Los Angeles	16	3120	0.3084	308.39712	6.8532693	7
20	Atlanta	4.1	4	Salt Lake City	1.3	2545	0.0731	73.140101	1.6253356	2
20	Atlanta	4.1	5	Los Vegas	1.6	2807	0.0772	77.228626	1.7161917	2
20	Atlanta	4.1	6	Pheonix	2.3	2560	0.0891	89.06136	1.9791413	2
20	Atlanta	4.1	7	El Paso	0.7	2086	0.0644	64.39743	1.431054	2
20	Atlanta	4.1	8	Denver	2.6	1949	0.0962	96.207855	2.1379523	3
20	Atlanta	4.1	9	San Antonio	1.6	1421	0.0821	82.104599	1.8245466	2
20	Atlanta	4.1	10	Dallas	5.2	1158	0.1452	145.18741	3.2263868	4
20	Atlanta	4.1	11	Houston	4.7	1130	0.1369	136.90757	3.0423904	4
20	Atlanta	4.1	12	Tulsa	0.8	1087	0.0703	70.276425	1.5616983	2
20	Atlanta	4.1	13	St Louis	2.6	755	0.1042	104.2158	2.3159067	3
20	Atlanta	4.1	14	Kansas City	1.8	1090	0.0874	87.416705	1.9425934	2
20	Atlanta	4.1	15	Minneapolis	3	1463	0.1054	105.39253	2.3420561	3
20	Atlanta	4.1	16	Chicago	9.2	948	0.2172	217.23397	4.8274216	5
20	Atlanta	4.1	17	Cleveland	3	894	0.1098	109.83187	2.4407083	3
20	Atlanta	4.1	18	Cincinnati	2	600	0.0952	95.215665	2.1159037	3

20	Atlanta	4.1	19	Nashville	1.2	345	0.0839	83.854553	1.8634345	2
20	Atlanta	4.1	21	New Orleans	1.3	680	0.0818	81.827177	1.8183817	2
20	Atlanta	4.1	22	Miami	3.9	975	0.1247	124.69389	2.7709754	3
20	Atlanta	4.1	23	Charlotte	1.6	364	0.091	91.040004	2.0231112	3
20	Atlanta	4.1	24	DC	4.9	875	0.1433	143.30756	3.1846124	4
20	Atlanta	4.1	25	NYC	9.3	1205	0.2147	214.66304	4.7702897	5
20	Atlanta	4.1	26	Boston	5.8	1510	0.152	151.96371	3.3769714	4
20	Atlanta	4.1	27	Albany	0.9	1358	0.0702	70.19776	1.5599502	2
21	New Orleans	1.3	1	Seattle	3.6	3384	0.0688	68.761844	1.528041	2
21	New Orleans	1.3	2	San Francisco	7	3088	0.0863	86.263579	1.9169684	2
21	New Orleans	1.3	3	Los Angeles	16	2686	0.1347	134.74913	2.9944251	3
21	New Orleans	1.3	4	Salt Lake City	1.3	2306	0.0595	59.497779	1.3221729	2
21	New Orleans	1.3	5	Los Vegas	1.6	2434	0.0607	60.729695	1.3495488	2
21	New Orleans	1.3	6	Pheonix	2.3	2116	0.0651	65.114415	1.446987	2
21	New Orleans	1.3	7	El Paso	0.7	1595	0.0582	58.222929	1.2938429	2
21	New Orleans	1.3	8	Denver	2.6	1741	0.0678	67.835938	1.5074653	2
21	New Orleans	1.3	9	San Antonio	1.6	818	0.0667	66.706616	1.4823692	2
21	New Orleans	1.3	10	Dallas	5.2	710	0.0878	87.770319	1.9504515	2
21	New Orleans	1.3	11	Houston	4.7	514	0.087	86.981367	1.9329193	2
21	New Orleans	1.3	12	Tulsa	0.8	879	0.0619	61.901178	1.3755817	2
21	New Orleans	1.3	13	St Louis	2.6	963	0.0714	71.352249	1.5856055	2
21	New Orleans	1.3	14	Kansas City	1.8	1099	0.0662	66.218091	1.4715131	2
21	New Orleans	1.3	15	Minneapolis	3	1695	0.0701	70.098315	1.5577403	2
21	New Orleans	1.3	16	Chicago	9.2	1344	0.1048	104.76623	2.3281386	3
21	New Orleans	1.3	17	Cleveland	3	1489	0.0709	70.90061	1.5755691	2
21	New Orleans	1.3	18	Cincinnati	2	1131	0.0671	67.147442	1.4921654	2
21	New Orleans	1.3	19	Nashville	1.2	754	0.0649	64.884323	1.4418738	2
21	New Orleans	1.3	20	Atlanta	4.1	680	0.0818	81.827177	1.8183817	2
21	New Orleans	1.3	22	Miami	3.9	1076	0.0778	77.805005	1.7290001	2
21	New Orleans	1.3	23	Charlotte	1.6	1045	0.0654	65.400391	1.453342	2
21	New Orleans	1.3	24	DC	4.9	1553	0.0807	80.698193	1.7932932	2
21	New Orleans	1.3	25	NYC	9.3	1882	0.1022	102.23482	2.271885	3
21	New Orleans	1.3	26	Boston	5.8	2188	0.0829	82.885948	1.84191	2
21	New Orleans	1.3	27	Albany	0.9	2027	0.058	58.006291	1.2890287	2
22	Miami	3.9	1	Seattle	3.6	4402	0.1017	101.70364	2.260081	3
22	Miami	3.9	2	San Francisco	7	4165	0.1516	151.56037	3.3680082	4
22	Miami	3.9	3	Los Angeles	16	2760	0.2993	299.25407	6.6500905	7

22	Miami	3.9	4	Salt Lake City	1.3	3358	0.0703	70.292039	1.5620453	2
22	Miami	3.9	5	Los Vegas	1.6	3508	0.0744	74.426938	1.653932	2
22	Miami	3.9	6	Pheonix	2.3	3189	0.0855	85.508679	1.9001929	2
22	Miami	3.9	7	El Paso	0.7	2660	0.0625	62.454312	1.3878736	2
22	Miami	3.9	8	Denver	2.6	2777	0.0911	91.13775	2.0252833	3
22	Miami	3.9	9	San Antonio	1.6	1848	0.0789	78.944478	1.7543217	2
22	Miami	3.9	10	Dallas	5.2	1785	0.1357	135.66077	3.0146839	4
22	Miami	3.9	11	Houston	4.7	1556	0.1294	129.37702	2.8750448	3
22	Miami	3.9	12	Tulsa	0.8	1890	0.0663	66.347364	1.4743859	2
22	Miami	3.9	13	St Louis	2.6	1710	0.0952	95.238039	2.1164009	3
22	Miami	3.9	14	Kansas City	1.8	2001	0.0815	81.488421	1.8108538	2
22	Miami	3.9	15	Minneapolis	3	2436	0.0983	98.345913	2.1854647	3
22	Miami	3.9	16	Chicago	9.2	1918	0.1971	197.13694	4.3808209	5
22	Miami	3.9	17	Cleveland	3	1769	0.1012	101.23137	2.2495859	3
22	Miami	3.9	18	Cincinnati	2	1537	0.0866	86.599566	1.9244348	2
22	Miami	3.9	19	Nashville	1.2	1314	0.0749	74.893104	1.6642912	2
22	Miami	3.9	20	Atlanta	4.1	975	0.1247	124.69389	2.7709754	3
22	Miami	3.9	21	New Orleans	1.3	1076	0.0778	77.805005	1.7290001	2
22	Miami	3.9	23	Charlotte	1.6	1051	0.0829	82.882136	1.8418253	2
22	Miami	3.9	24	DC	4.9	1488	0.1331	133.07108	2.9571352	3
22	Miami	3.9	25	NYC	9.3	1758	0.2002	200.24705	4.4499345	5
22	Miami	3.9	26	Boston	5.8	2025	0.1434	143.44182	3.1875961	4
22	Miami	3.9	27	Albany	0.9	1968	0.0672	67.242134	1.4942696	2
23	Charlotte	1.6	1	Seattle	3.6	3672	0.0723	72.300459	1.6066769	2
23	Charlotte	1.6	2	San Francisco	7	3696	0.0927	92.720041	2.0604454	3
23	Charlotte	1.6	3	Los Angeles	16	3410	0.1504	150.43354	3.3429675	4
23	Charlotte	1.6	4	Salt Lake City	1.3	2776	0.06	59.998059	1.3332902	2
23	Charlotte	1.6	5	Los Vegas	1.6	3090	0.0612	61.239308	1.3608735	2
23	Charlotte	1.6	6	Pheonix	2.3	2868	0.066	65.980382	1.4662307	2
23	Charlotte	1.6	7	El Paso	0.7	2416	0.0569	56.888703	1.2641934	2
23	Charlotte	1.6	8	Denver	2.6	2184	0.0695	69.549191	1.5455376	2
23	Charlotte	1.6	9	San Antonio	1.6	1782	0.0644	64.397025	1.431045	2
23	Charlotte	1.6	10	Dallas	5.2	1495	0.0889	88.933809	1.9763069	2
23	Charlotte	1.6	11	Houston	4.7	1492	0.0857	85.680384	1.9040085	2
23	Charlotte	1.6	12	Tulsa	0.8	1371	0.0606	60.6447	1.34766	2
23	Charlotte	1.6	13	St Louis	2.6	915	0.075	74.969381	1.6659862	2
23	Charlotte	1.6	14	Kansas City	1.8	1292	0.0676	67.554183	1.5012041	2
23	Charlotte	1.6	15	Minneapolis	3	1513	0.0745	74.47495	1.6549989	2
23	Charlotte	1.6	16	Chicago	9.2	949	0.1196	119.57714	2.6572697	3
23	Charlotte	1.6	17	Cleveland	3	706	0.0793	79.291529	1.762034	2
23	Charlotte	1.6	18	Cincinnati	2	541	0.0738	73.772619	1.6393915	2
23	Charlotte	1.6	19	Nashville	1.2	550	0.068	68.02052	1.5115671	2
23	Charlotte	1.6	20	Atlanta	4.1	364	0.091	91.040004	2.0231112	3
23	Charlotte	1.6	21	New Orleans	1.3	1045	0.0654	65.400391	1.453342	2
23	Charlotte	1.6	22	Miami	3.9	1051	0.0829	82.882136	1.8418253	2
23	Charlotte	1.6	24	DC	4.9	329	0.0975	97.464627	2.1658806	3
23	Charlotte	1.6	25	NYC	9.3	857	0.1212	121.24606	2.6943568	3
23	Charlotte	1.6	26	Boston	5.8	1160	0.0949	94.895905	2.1087979	3
23	Charlotte	1.6	27	Albany	0.9	1029	0.0626	62.62278	1.3916173	2
24	DC	4.9	1	Seattle	3.6	3742	0.1168	116.80933	2.595763	3

24	DC	4.9	2	San Francisco	7	3918	0.1786	178.62918	3.9695372	4
24	DC	4.9	3	Los Angeles	16	3698	0.3529	352.86585	7.8414634	8
24	DC	4.9	4	Salt Lake City	1.3	2970	0.0761	76.095102	1.6910023	2
24	DC	4.9	5	Los Vegas	1.6	3355	0.0808	80.813395	1.7958532	2
24	DC	4.9	6	Pheonix	2.3	3190	0.0943	94.278774	2.0950839	3
24	DC	4.9	7	El Paso	0.7	2788	0.0648	64.809704	1.4402156	2
24	DC	4.9	8	Denver	2.6	2400	0.1026	102.55288	2.2789528	3
24	DC	4.9	9	San Antonio	1.6	2235	0.0839	83.908363	1.8646303	2
24	DC	4.9	10	Dallas	5.2	1903	0.1557	155.68342	3.4596315	4
24	DC	4.9	11	Houston	4.7	1963	0.1455	145.47922	3.2328716	4
24	DC	4.9	12	Tulsa	0.8	1696	0.0702	70.234861	1.5607747	2
24	DC	4.9	13	St Louis	2.6	1145	0.1095	109.46195	2.4324878	3
24	DC	4.9	14	Kansas City	1.8	1520	0.0908	90.845015	2.0187781	3
24	DC	4.9	15	Minneapolis	3	1504	0.1149	114.93932	2.554207	3
24	DC	4.9	16	Chicago	9.2	960	0.2482	248.22705	5.5161567	6
24	DC	4.9	17	Cleveland	3	492	0.1256	125.56562	2.7903472	3
24	DC	4.9	18	Cincinnati	2	648	0.1017	101.67307	2.2594015	3
24	DC	4.9	19	Nashville	1.2	914	0.0823	82.299859	1.8288857	2
24	DC	4.9	20	Atlanta	4.1	875	0.1433	143.30756	3.1846124	4
24	DC	4.9	21	New Orleans	1.3	1553	0.0807	80.698193	1.7932932	2
24	DC	4.9	22	Miami	3.9	1488	0.1331	133.07108	2.9571352	3
24	DC	4.9	23	Charlotte	1.6	329	0.0975	97.464627	2.1658806	3
24	DC	4.9	25	NYC	9.3	329	0.2702	270.20703	6.0046008	7
24	DC	4.9	26	Boston	5.8	635	0.1834	183.38248	4.0751661	5
24	DC	4.9	27	Albany	0.9	510	0.0789	78.940974	1.7542439	2
25	NYC	9.3	1	Seattle	3.6	3871	0.1758	175.76214	3.9058254	4
25	NYC	9.3	2	San Francisco	7	4127	0.2924	292.41017	6.4980038	7
25	NYC	9.3	3	Los Angeles	16	3942	0.6205	620.54091	13.789798	14
25	NYC	9.3	4	Salt Lake City	1.3	3168	0.0975	97.469462	2.165988	3
25	NYC	9.3	5	Los Vegas	1.6	3590	0.1069	106.85617	2.3745814	3
25	NYC	9.3	6	Pheonix	2.3	3447	0.1319	131.91623	2.9314719	3
25	NYC	9.3	7	El Paso	0.7	3071	0.0757	75.682209	1.6818269	2
25	NYC	9.3	8	Denver	2.6	2619	0.1462	146.17408	3.2483129	4
25	NYC	9.3	9	San Antonio	1.6	2549	0.1103	110.3213	2.4515844	3
25	NYC	9.3	10	Dallas	5.2	2205	0.2439	243.92091	5.4204647	6
25	NYC	9.3	11	Houston	4.7	2285	0.2248	224.83909	4.9964243	5
25	NYC	9.3	12	Tulsa	0.8	1974	0.0834	83.373536	1.8527453	2
25	NYC	9.3	13	St Louis	2.6	1405	0.1546	154.56118	3.434693	4
25	NYC	9.3	14	Kansas City	1.8	1764	0.1216	121.55009	2.7011132	3
25	NYC	9.3	15	Minneapolis	3	1637	0.1676	167.59007	3.7242238	4
25	NYC	9.3	16	Chicago	9.2	1145	0.4138	413.83875	9.1964166	10
25	NYC	9.3	17	Cleveland	3	649	0.1808	180.811	4.0180222	5
25	NYC	9.3	18	Cincinnati	2	912	0.1365	136.48652	3.0330338	4
25	NYC	9.3	19	Nashville	1.2	1224	0.1023	102.28075	2.2729055	3
25	NYC	9.3	20	Atlanta	4.1	1205	0.2147	214.66304	4.7702897	5
25	NYC	9.3	21	New Orleans	1.3	1882	0.1022	102.23482	2.271885	3
25	NYC	9.3	22	Miami	3.9	1758	0.2002	200.24705	4.4499345	5
25	NYC	9.3	23	Charlotte	1.6	857	0.1212	121.24606	2.6943568	4
25	NYC	9.3	24	DC	4.9	329	0.2702	270.20703	6.0046008	7
25	NYC	9.3	26	Boston	5.8	306	0.3099	309.8511	6.88558	7

25	NYC	9.3	27	Albany	0.9	216	0.1012	101.17376	2.2483058	3
26	Boston	5.8	1	Seattle	3.6	4008	0.1281	128.14784	2.8477297	3
26	Boston	5.8	2	San Francisco	7	4330	0.2003	200.28796	4.4508435	5
26	Boston	5.8	3	Los Angeles	16	4173	0.4037	403.74024	8.9720053	9
26	Boston	5.8	4	Salt Lake City	1.3	3371	0.0796	79.638966	1.7697548	2
26	Boston	5.8	5	Los Vegas	1.6	3818	0.0852	85.235561	1.8941236	2
26	Boston	5.8	6	Pheonix	2.3	3698	0.1008	100.76508	2.239224	3
26	Boston	5.8	7	El Paso	0.7	3340	0.0661	66.051185	1.4678041	2
26	Boston	5.8	8	Denver	2.6	2844	0.11	109.98117	2.444026	3
26	Boston	5.8	9	San Antonio	1.6	2845	0.0876	87.618147	1.9470699	2
26	Boston	5.8	10	Dallas	5.2	2495	0.1702	170.16291	3.7813979	4
26	Boston	5.8	11	Houston	4.7	2587	0.1583	158.31836	3.5181858	4
26	Boston	5.8	12	Tulsa	0.8	2245	0.0713	71.334887	1.5852197	2
26	Boston	5.8	13	St Louis	2.6	1673	0.1154	115.40677	2.5645948	3
26	Boston	5.8	14	Kansas City	1.8	2012	0.0951	95.057357	2.1123857	3
26	Boston	5.8	15	Minneapolis	3	1808	0.1239	123.9354	2.7541201	3
26	Boston	5.8	16	Chicago	9.2	1367	0.2751	275.07134	6.1126965	7
26	Boston	5.8	17	Cleveland	3	884	0.1317	131.70537	2.926786	3
26	Boston	5.8	18	Cincinnati	2	1190	0.1044	104.35791	2.3190647	3
26	Boston	5.8	19	Nashville	1.2	1521	0.0833	83.252135	1.8500475	2
26	Boston	5.8	20	Atlanta	4.1	1510	0.152	151.96371	3.3769714	4
26	Boston	5.8	21	New Orleans	1.3	2188	0.0829	82.885948	1.84191	2
26	Boston	5.8	22	Miami	3.9	2025	0.1434	143.44182	3.1875961	4
26	Boston	5.8	23	Charlotte	1.6	1160	0.0949	94.895905	2.1087979	3
26	Boston	5.8	24	DC	4.9	635	0.1834	183.38248	4.0751661	5
26	Boston	5.8	25	NYC	9.3	306	0.3099	309.8511	6.88558	7
26	Boston	5.8	27	Albany	0.9	225	0.0866	86.6321	1.9251578	2
27	Albany	0.88	1	Seattle	3.6	3793	0.0623	62.298739	1.3844164	2
27	Albany	0.88	2	San Francisco	7	4110	0.0729	72.918916	1.6204204	2
27	Albany	0.88	3	Los Angeles	16	3952	0.104	103.97084	2.310463	3
27	Albany	0.88	4	Salt Lake City	1.3	3146	0.0557	55.708907	1.2379757	2
27	Albany	0.88	5	Los Vegas	1.6	3594	0.056	56.00836	1.2446302	2
27	Albany	0.88	6	Pheonix	2.3	3478	0.0585	58.501578	1.3000351	2
27	Albany	0.88	7	El Paso	0.7	3130	0.0537	53.660221	1.1924494	2
27	Albany	0.88	8	Denver	2.6	2620	0.0611	61.087334	1.3574963	2
27	Albany	0.88	9	San Antonio	1.6	2655	0.0576	57.613833	1.2803074	2
27	Albany	0.88	10	Dallas	5.2	2293	0.0708	70.795563	1.5732347	2
27	Albany	0.88	11	Houston	4.7	2404	0.0688	68.77528	1.5283396	2
27	Albany	0.88	12	Tulsa	0.8	2036	0.0563	56.252488	1.2500553	2
27	Albany	0.88	13	St Louis	2.6	1458	0.0644	64.387676	1.4308372	2
27	Albany	0.88	14	Kansas City	1.8	1791	0.0604	60.414702	1.3425489	2
27	Albany	0.88	15	Minneapolis	3	1584	0.0653	65.34754	1.4521676	2
27	Albany	0.88	16	Chicago	9.2	1145	0.0899	89.896904	1.997709	2
27	Albany	0.88	17	Cleveland	3	668	0.0701	70.147003	1.5588223	2
27	Albany	0.88	18	Cincinnati	2	984	0.0643	64.336087	1.4296908	2
27	Albany	0.88	19	Nashville	1.2	1332	0.0598	59.831429	1.3295873	2
27	Albany	0.88	20	Atlanta	4.1	1358	0.0702	70.19776	1.5599502	2
27	Albany	0.88	21	New Orleans	1.3	2027	0.058	58.006291	1.2890287	2
27	Albany	0.88	22	Miami	3.9	1968	0.0672	67.242134	1.4942696	2

27	Albany	0.88	23	Charlotte	1.6	1029	0.0626	62.62278	1.3916173	2
27	Albany	0.88	24	DC	4.9	510	0.0789	78.940974	1.7542439	2
27	Albany	0.88	25	NYC	9.3	216	0.1012	101.17376	2.2483058	3
27	Albany	0.88	26	Boston	5.8	225	0.0866	86.6321	1.9251578	2

Pan-European Traffic Matrix

I	pop	j	pop	I	j	dist (km)	traf	*1000		
1	0.482	2	0.612	1	2	320	0.007017	7.016961	0.155932	1
1	0.482	3	7.1	1	3	465	0.079222	79.22225	1.760494	2
1	0.482	4	0.729	1	4	757	0.007822	7.821801	0.173818	1
1	0.482	5	1	1	5	770	0.010714	10.71414	0.238092	1
1	0.482	6	2.2	1	6	782	0.02354	23.54037	0.523119	1
1	0.482	7	0.21	1	7	1032	0.002194	2.193971	0.048755	1
1	0.482	8	2.8	1	8	1452	0.028373	28.37315	0.630514	1
1	0.482	9	1.5	1	9	1477	0.015176	15.17629	0.337251	1
1	0.482	10	0.415	1	10	1163	0.00429	4.290155	0.095337	1
1	0.482	11	0.36	1	11	1237	0.003701	3.701141	0.082248	1
1	0.482	12	0.252	1	12	1115	0.002615	2.614872	0.058108	1
1	0.482	13	1.7	1	13	1075	0.017697	17.69705	0.393268	1
1	0.482	14	0.644	1	14	1085	0.006699	6.698584	0.148857	1
1	0.482	15	1.2	1	15	1378	0.012218	12.21778	0.271506	1
1	0.482	16	1.3	1	16	1418	0.013202	13.20164	0.29337	1
1	0.482	17	2.7	1	17	1883	0.026714	26.71354	0.593634	1
1	0.482	18	0.868	1	18	1799	0.008624	8.624332	0.191652	1
1	0.482	19	0.772	1	19	2855	0.007345	7.344683	0.163215	1
1	0.482	20	1.6	1	20	2146	0.015638	15.6383	0.347518	1
1	0.482	21	1.8	1	21	1894	0.017799	17.79939	0.395542	1
1	0.482	22	1.5	1	22	1680	0.014998	14.99829	0.333295	1
1	0.482	23	1.2	1	23	1467	0.012149	12.14855	0.269968	1
1	0.482	24	1.6	1	24	1820	0.01588	15.88031	0.352896	1
1	0.482	25	3.4	1	25	1316	0.034761	34.76132	0.772474	1
1	0.482	26	0.499	1	26	1244	0.005128	5.1276	0.113947	1
1	0.482	27	0.505	1	27	1266	0.005181	5.181101	0.115136	1
1	0.482	28	0.744	1	28	1616	0.007466	7.465769	0.165906	1
								#VALUE!		
2	0.612	1	0.482	2	1	320	0.007017	7.016961	0.155932	1
2	0.612	3	7.1	2	3	559	0.099156	99.15582	2.203463	3
2	0.612	4	0.729	2	4	713	0.009981	9.981261	0.221806	1
2	0.612	5	1	2	5	798	0.013563	13.56285	0.301397	1
2	0.612	6	2.2	2	6	898	0.029539	29.53876	0.656417	1
2	0.612	7	0.21	2	7	1258	0.002737	2.737168	0.060826	1
2	0.612	8	2.8	2	8	1717	0.035476	35.4765	0.788367	1
2	0.612	9	1.5	2	9	1683	0.01904	19.04034	0.423119	1
2	0.612	10	0.415	2	10	1297	0.005394	5.394345	0.119874	1
2	0.612	11	0.36	2	11	1290	0.004682	4.681711	0.104038	1
2	0.612	12	0.252	2	12	1152	0.003311	3.310525	0.073567	1
2	0.612	13	1.7	2	13	948	0.022719	22.71868	0.50486	1
2	0.612	14	0.644	2	14	1077	0.008511	8.510813	0.189129	1
2	0.612	15	1.2	2	15	1383	0.015508	15.50795	0.344621	1
2	0.612	16	1.3	2	16	1492	0.016685	16.68485	0.370774	1
2	0.612	17	2.7	2	17	1972	0.033773	33.773	0.750511	1
2	0.612	18	0.868	2	18	1801	0.010949	10.94927	0.243317	1
2	0.612	19	0.772	2	19	2881	0.009318	9.317567	0.207057	1

2	0.612	20	1.6	2	20	2130	0.01987	19.87003	0.441556	1
2	0.612	21	1.8	2	21	1848	0.022652	22.65171	0.503371	1
2	0.612	22	1.5	2	22	1640	0.019086	19.08574	0.424128	1
2	0.612	23	1.2	2	23	1405	0.015486	15.48578	0.344129	1
2	0.612	24	1.6	2	24	1689	0.020303	20.30304	0.451179	1
2	0.612	25	3.4	2	25	1205	0.044487	44.48715	0.988603	1
2	0.612	26	0.499	2	26	1052	0.006608	6.608214	0.146849	1
2	0.612	27	0.505	2	27	992	0.006722	6.72218	0.149382	1
2	0.612	28	0.744	2	28	1378	0.009618	9.618084	0.213735	1
3	7.1	1	0.482	3	1	465	0.079222	79.22225	1.760494	2
3	7.1	2	0.612	3	2	559	0.099156	99.15582	2.203463	3
3	7.1	4	0.729	3	4	354	0.122255	122.2547	2.716772	3
3	7.1	5	1	3	5	319	0.168928	168.9284	3.753964	4
3	7.1	6	2.2	3	6	344	0.369693	369.6934	8.215409	9
3	7.1	7	0.21	3	7	743	0.033242	33.24221	0.738716	1
3	7.1	8	2.8	3	8	1265	0.423186	423.1855	9.404123	10
3	7.1	9	1.5	3	9	1144	0.228751	228.7512	5.08336	6
3	7.1	10	0.415	3	10	737	0.065738	65.73754	1.460834	2
3	7.1	11	0.36	3	11	775	0.056785	56.78505	1.26189	2
3	7.1	12	0.252	3	12	650	0.040335	40.33492	0.896332	1
3	7.1	13	1.7	3	13	721	0.26978	269.7803	5.995118	6
3	7.1	14	0.644	3	14	636	0.103262	103.2616	2.294703	3
3	7.1	15	1.2	3	15	919	0.186548	186.5483	4.145518	5
3	7.1	16	1.3	3	16	958	0.201367	201.3672	4.474826	5
3	7.1	17	2.7	3	17	1432	0.403527	403.5266	8.967258	9
3	7.1	18	0.868	3	18	1337	0.130535	130.5354	2.900786	3
3	7.1	19	0.772	3	19	2394	0.11001	110.0101	2.44467	3
3	7.1	20	1.6	3	20	1692	0.235503	235.5033	5.233406	6
3	7.1	21	1.8	3	21	1448	0.268746	268.7462	5.972137	6
3	7.1	22	1.5	3	22	1232	0.227244	227.2444	5.049876	6
3	7.1	23	1.2	3	23	1031	0.184689	184.689	4.1042	5
3	7.1	24	1.6	3	24	1446	0.238916	238.9155	5.309233	6
3	7.1	25	3.4	3	25	930	0.52801	528.0097	11.73355	12
3	7.1	26	0.499	3	26	953	0.077329	77.32917	1.718426	2
3	7.1	27	0.505	3	27	1152	0.076965	76.96524	1.710339	2
3	7.1	28	0.744	3	28	1431	0.111201	111.2011	2.471135	3
4	0.729	1	0.482	4	1	757	0.007822	7.821801	0.173818	1
4	0.729	2	0.612	4	2	713	0.009981	9.981261	0.221806	1
4	0.729	3	7.1	4	3	354	0.122255	122.2547	2.716772	3
4	0.729	5	1	4	5	174	0.018009	18.00881	0.400196	1
4	0.729	6	2.2	4	6	431	0.037341	37.34107	0.829801	1
4	0.729	7	0.21	4	7	928	0.003349	3.349133	0.074425	1
4	0.729	8	2.8	4	8	1481	0.042836	42.83571	0.951905	1
4	0.729	9	1.5	4	9	1242	0.023316	23.31567	0.518126	1
4	0.729	10	0.415	4	10	739	0.006748	6.748141	0.149959	1
4	0.729	11	0.36	4	11	614	0.005944	5.943843	0.132085	1
4	0.729	12	0.252	4	12	469	0.00425	4.249967	0.094444	1
4	0.729	13	1.7	4	13	369	0.029186	29.18553	0.648567	1
4	0.729	14	0.644	4	14	363	0.011069	11.06918	0.245982	1
4	0.729	15	1.2	4	15	669	0.019675	19.67452	0.437212	1

4	0.729	16	1.3	4	16	830	0.020932	20.93244	0.465165	1
4	0.729	17	2.7	4	17	1298	0.041802	41.80232	0.92894	1
4	0.729	18	0.868	4	18	1088	0.013652	13.65185	0.303374	1
4	0.729	19	0.772	4	19	2166	0.011402	11.40223	0.253383	1
4	0.729	20	1.6	4	20	1419	0.024573	24.57295	0.546066	1
4	0.729	21	1.8	4	21	1150	0.028172	28.17163	0.626036	1
4	0.729	22	1.5	4	22	936	0.023905	23.9046	0.531213	1
4	0.729	23	1.2	4	23	712	0.019573	19.57339	0.434964	1
4	0.729	24	1.6	4	24	1094	0.025152	25.15248	0.558944	1
4	0.729	25	3.4	4	25	577	0.056418	56.41791	1.253731	2
4	0.729	26	0.499	4	26	620	0.008232	8.232334	0.182941	1
4	0.729	27	0.505	4	27	914	0.008064	8.064461	0.17921	1
4	0.729	28	0.744	4	28	1125	0.011667	11.66701	0.259267	1
5	1	1	0.482	5	1	770	0.010714	10.71414	0.238092	1
5	1	2	0.612	5	2	798	0.013563	13.56285	0.301397	1
5	1	3	7.1	5	3	319	0.168928	168.9284	3.753964	4
5	1	4	0.729	5	4	174	0.018009	18.00881	0.400196	1
5	1	6	2.2	5	6	264	0.05301	53.01008	1.178002	2
5	1	7	0.21	5	7	761	0.004673	4.672606	0.103836	1
5	1	8	2.8	5	8	1315	0.059396	59.39612	1.319914	2
5	1	9	1.5	5	9	1065	0.032423	32.42305	0.720512	1
5	1	10	0.415	5	10	569	0.009457	9.456778	0.210151	1
5	1	11	0.36	5	11	491	0.008299	8.299058	0.184424	1
5	1	12	0.252	5	12	351	0.005956	5.955836	0.132352	1
5	1	13	1.7	5	13	491	0.03919	39.19	0.870889	1
5	1	14	0.644	5	14	315	0.015336	15.33582	0.340796	1
5	1	15	1.2	5	15	604	0.027214	27.21418	0.60476	1
5	1	16	1.3	5	16	696	0.029142	29.14204	0.647601	1
5	1	17	2.7	5	17	1174	0.05786	57.8599	1.285775	2
5	1	18	0.868	5	18	1024	0.018827	18.82698	0.418377	1
5	1	19	0.772	5	19	2091	0.015693	15.69263	0.348725	1
5	1	20	1.6	5	20	1367	0.033822	33.82199	0.7516	1
5	1	21	1.8	5	21	1130	0.038704	38.70445	0.860099	1
5	1	22	1.5	5	22	912	0.032865	32.86472	0.730327	1
5	1	23	1.2	5	23	719	0.026828	26.82778	0.596173	1
5	1	24	1.6	5	24	1095	0.0345	34.49993	0.766665	1
5	1	25	3.4	5	25	578	0.07738	77.3801	1.719558	2
5	1	26	0.499	5	26	621	0.011291	11.29116	0.250915	1
5	1	27	0.505	5	27	912	0.011064	11.06445	0.245877	1
5	1	28	0.744	5	28	1126	0.016003	16.00287	0.355619	1
6	2.2	1	0.482	6	1	782	0.02354	23.54037	0.523119	1
6	2.2	2	0.612	6	2	898	0.029539	29.53876	0.656417	1
6	2.2	3	7.1	6	3	344	0.369693	369.6934	8.215409	9
6	2.2	4	0.729	6	4	431	0.037341	37.34107	0.829801	1
6	2.2	5	1	6	5	264	0.05301	53.01008	1.178002	2
6	2.2	7	0.21	6	7	496	0.010642	10.64211	0.236491	1
6	2.2	8	2.8	6	8	1051	0.133306	133.3058	2.962352	3
6	2.2	9	1.5	6	9	831	0.072882	72.88167	1.619593	2
6	2.2	10	0.415	6	10	393	0.021403	21.40309	0.475624	1
6	2.2	11	0.36	6	11	487	0.018269	18.26947	0.405988	1

6	2.2	12	0.252	6	12	397	0.012987	12.98693	0.288598	1
6	2.2	13	1.7	6	13	749	0.083328	83.32803	1.851734	2
6	2.2	14	0.644	6	14	478	0.032729	32.72903	0.727312	1
6	2.2	15	1.2	6	15	685	0.059259	59.25881	1.316862	2
6	2.2	16	1.3	6	16	639	0.064565	64.56456	1.434768	2
6	2.2	17	2.7	6	17	1104	0.127988	127.9883	2.844185	3
6	2.2	18	0.868	6	18	1081	0.041222	41.22248	0.916055	1
6	2.2	19	0.772	6	19	2096	0.034516	34.51607	0.767024	1
6	2.2	20	1.6	6	20	1449	0.074016	74.0162	1.644805	2
6	2.2	21	1.8	6	21	1245	0.084417	84.41711	1.875936	2
6	2.2	22	1.5	6	22	1031	0.071534	71.53447	1.589655	2
6	2.2	23	1.2	6	23	884	0.057998	57.99754	1.288834	2
6	2.2	24	1.6	6	24	1370	0.074394	74.39363	1.653192	2
6	2.2	25	3.4	6	25	879	0.164407	164.4065	3.653478	4
6	2.2	26	0.499	6	26	1028	0.023803	23.80322	0.52896	1
6	2.2	27	0.505	6	27	1345	0.02352	23.51962	0.522658	1
6	2.2	28	0.744	6	28	1544	0.034219	34.21875	0.760417	1
7	0.21	1	0.482	7	1	1032	0.002194	2.193971	0.048755	1
7	0.21	2	0.612	7	2	1258	0.002737	2.737168	0.060826	1
7	0.21	3	7.1	7	3	743	0.033242	33.24221	0.738716	1
7	0.21	4	0.729	7	4	928	0.003349	3.349133	0.074425	1
7	0.21	5	1	7	5	761	0.004673	4.672606	0.103836	1
7	0.21	6	2.2	7	6	496	0.010642	10.64211	0.236491	1
7	0.21	8	1.5	7	8	556	0.007191	7.191256	0.159806	1
7	0.21	9	0.415	7	9	449	0.002023	2.022858	0.044952	1
7	0.21	10	0.36	7	10	436	0.001759	1.758662	0.039081	1
7	0.21	11	0.252	7	11	761	0.001177	1.177497	0.026167	1
7	0.21	12	1.7	7	12	758	0.007946	7.946067	0.176579	1
7	0.21	13	0.644	7	13	1236	0.002885	2.884847	0.064108	1
7	0.21	14	1.2	7	14	907	0.005524	5.523893	0.122753	1
7	0.21	15	1.3	7	15	1001	0.005933	5.933177	0.131848	1
7	0.21	16	2.7	7	16	771	0.012602	12.60218	0.280048	1
7	0.21	17	0.868	7	17	1107	0.003927	3.926618	0.087258	1
7	0.21	18	0.772	7	18	1297	0.003443	3.443307	0.076518	1
7	0.21	19	1.6	7	19	2159	0.00681	6.809512	0.151322	1
7	0.21	20	1.8	7	20	1658	0.007851	7.85095	0.174466	1
7	0.21	21	1.5	7	21	1538	0.006588	6.587699	0.146393	1
7	0.21	22	1.2	7	22	1346	0.005334	5.334424	0.118543	1
7	0.21	23	1.6	7	23	1267	0.007151	7.151417	0.15892	1
7	0.21	24	3.4	7	24	1783	0.01473	14.73045	0.327343	1
7	0.21	25	0.499	7	25	1332	0.00222	2.220326	0.049341	1
7	0.21	26	0.505	7	26	1520	0.00222	2.220246	0.049339	1
7	0.21	27	0.744	7	27	1839	0.003214	3.214149	0.071426	1
7	0.21	28	0.482	7	28	2039	0.002062	2.062372	0.04583	1
8	2.8	1	0.482	8	1	1452	0.028373	28.37315	0.630514	1
8	2.8	2	0.612	8	2	1717	0.035476	35.4765	0.788367	1
8	2.8	3	7.1	8	3	1265	0.423186	423.1855	9.404123	10
8	2.8	4	0.729	8	4	1481	0.042836	42.83571	0.951905	1
8	2.8	5	1	8	5	1315	0.059396	59.39612	1.319914	2
8	2.8	6	2.2	8	6	1051	0.133306	133.3058	2.962352	3

8	2.8	7	0.21	8	7	556	0.013424	13.42368	0.298304	1
8	2.8	9	1.5	8	9	503	0.096641	96.64126	2.147584	3
8	2.8	10	0.415	8	10	910	0.025464	25.46403	0.565867	1
8	2.8	11	0.36	8	11	1248	0.021483	21.48336	0.477408	1
8	2.8	12	0.252	8	12	1284	0.015	15	0.333333	1
8	2.8	13	1.7	8	13	1789	0.098172	98.17247	2.18161	3
8	2.8	14	0.644	8	14	1446	0.037924	37.92363	0.842747	1
8	2.8	15	1.2	8	15	1485	0.070494	70.49412	1.566536	2
8	2.8	16	1.3	8	16	1186	0.077933	77.93305	1.731846	2
8	2.8	17	2.7	8	17	1362	0.159862	159.8619	3.552486	3
8	2.8	18	0.868	8	18	1702	0.050357	50.35707	1.119046	2
8	2.8	19	0.772	8	19	2372	0.043422	43.42207	0.964935	1
8	2.8	20	1.6	8	20	2032	0.09131	91.30997	2.02911	3
8	2.8	21	1.8	8	21	1976	0.102992	102.9919	2.288709	3
8	2.8	22	1.5	8	22	1809	0.086534	86.53365	1.92297	2
8	2.8	23	1.2	8	23	1775	0.069349	69.3486	1.54108	2
8	2.8	24	1.6	8	24	2292	0.090285	90.28518	2.006337	3
8	2.8	25	3.4	8	25	1869	0.19555	195.5503	4.345563	4
8	2.8	26	0.499	8	26	2072	0.028425	28.42547	0.631677	1
8	2.8	27	0.505	8	27	2387	0.028387	28.38745	0.630832	1
8	2.8	28	0.744	8	28	2592	0.041498	41.49785	0.922175	1
9	1.5	1	0.482	9	1	1477	0.015176	15.17629	0.337251	1
9	1.5	2	0.612	9	2	1683	0.01904	19.04034	0.423119	1
9	1.5	3	7.1	9	3	1144	0.228751	228.7512	5.08336	6
9	1.5	4	0.729	9	4	1242	0.023316	23.31567	0.518126	1
9	1.5	5	1	9	5	1065	0.032423	32.42305	0.720512	1
9	1.5	6	2.2	9	6	831	0.072882	72.88167	1.619593	2
9	1.5	7	0.21	9	7	449	0.007312	7.311534	0.162479	1
9	1.5	8	2.8	9	8	503	0.096641	96.64126	2.147584	3
9	1.5	10	0.415	9	10	533	0.014259	14.25885	0.316863	1
9	1.5	11	0.36	9	11	838	0.011918	11.91756	0.264835	1
9	1.5	12	0.252	9	12	915	0.00828	8.27956	0.18399	1
9	1.5	13	1.7	9	13	1476	0.05353	53.52965	1.189548	2
9	1.5	14	0.644	9	14	1096	0.020828	20.82765	0.462837	1
9	1.5	15	1.2	9	15	1055	0.03894	38.93999	0.865333	1
9	1.5	16	1.3	9	16	728	0.04355	43.54994	0.967776	1
9	1.5	17	2.7	9	17	858	0.089202	89.20171	1.98226	2
9	1.5	18	0.868	9	18	1216	0.027814	27.81394	0.618088	1
9	1.5	19	0.772	9	19	1878	0.023776	23.77587	0.528353	1
9	1.5	20	1.6	9	20	1532	0.05021	50.20993	1.115776	2
9	1.5	21	1.8	9	21	1501	0.056592	56.59173	1.257594	2
9	1.5	22	1.5	9	22	1349	0.047619	47.61921	1.058205	2
9	1.5	23	1.2	9	23	1355	0.03808	38.08009	0.846224	1
9	1.5	24	1.6	9	24	1869	0.049298	49.2984	1.09552	2
9	1.5	25	3.4	9	25	1501	0.106895	106.8955	2.375455	3
9	1.5	26	0.499	9	26	1765	0.015457	15.45671	0.343483	1
9	1.5	27	0.505	9	27	2148	0.015359	15.35915	0.341315	1
9	1.5	28	0.744	9	28	2280	0.022502	22.50179	0.50004	1
10	0.415	1	0.482	10	1	1163	0.00429	4.290155	0.095337	1
10	0.415	2	0.612	10	2	1297	0.005394	5.394345	0.119874	1

10	0.415	3	7.1	10	3	737	0.065738	65.73754	1.460834	2
10	0.415	4	0.729	10	4	739	0.006748	6.748141	0.149959	1
10	0.415	5	1	10	5	569	0.009457	9.456778	0.210151	1
10	0.415	6	2.2	10	6	393	0.021403	21.40309	0.475624	1
10	0.415	7	0.21	10	7	436	0.002027	2.027346	0.045052	1
10	0.415	8	2.8	10	8	910	0.025464	25.46403	0.565867	1
10	0.415	9	1.5	10	9	533	0.014259	14.25885	0.316863	1
10	0.415	11	0.36	10	11	339	0.00354	3.539635	0.078659	1
10	0.415	12	0.252	10	12	386	0.002455	2.454845	0.054552	1
10	0.415	13	1.7	10	13	946	0.015408	15.40847	0.34241	1
10	0.415	14	0.644	10	14	566	0.006093	6.092733	0.135394	1
10	0.415	15	1.2	10	15	579	0.011332	11.33234	0.25183	1
10	0.415	16	1.3	10	16	341	0.012777	12.77673	0.283927	1
10	0.415	17	2.7	10	17	748	0.024968	24.96779	0.55484	1
10	0.415	18	0.868	10	18	863	0.00793	7.929938	0.176221	1
10	0.415	19	0.772	10	19	1784	0.006609	6.609379	0.146875	1
10	0.415	20	1.6	10	20	1228	0.014172	14.17223	0.314938	1
10	0.415	21	1.8	10	21	1104	0.016096	16.0955	0.357678	1
10	0.415	22	1.5	10	22	917	0.013632	13.63242	0.302943	1
10	0.415	23	1.2	10	23	864	0.010962	10.96196	0.243599	1
10	0.415	24	1.6	10	24	1384	0.01402	14.02043	0.311565	1
10	0.415	25	3.4	10	25	979	0.030725	30.72516	0.682781	1
10	0.415	26	0.499	10	26	1231	0.004419	4.418999	0.0982	1
10	0.415	27	0.505	10	27	1626	0.004361	4.360618	0.096903	1
10	0.415	28	0.744	10	28	1749	0.006381	6.38134	0.141808	1
11	0.36	1	0.482	11	1	1237	0.003701	3.701141	0.082248	1
11	0.36	2	0.612	11	2	1290	0.004682	4.681711	0.104038	1
11	0.36	3	7.1	11	3	775	0.056785	56.78505	1.26189	2
11	0.36	4	0.729	11	4	614	0.005944	5.943843	0.132085	1
11	0.36	5	1	11	5	491	0.008299	8.299058	0.184424	1
11	0.36	6	2.2	11	6	487	0.018269	18.26947	0.405988	1
11	0.36	7	0.21	11	7	761	0.001682	1.682138	0.037381	1
11	0.36	8	2.8	11	8	1248	0.021483	21.48336	0.477408	1
11	0.36	9	1.5	11	9	838	0.011918	11.91756	0.264835	1
11	0.36	10	0.415	11	10	339	0.00354	3.539635	0.078659	1
11	0.36	12	0.252	11	12	145	0.002263	2.262626	0.050281	1
11	0.36	13	1.7	11	13	694	0.013722	13.72245	0.304943	1
11	0.36	14	0.644	11	14	307	0.005531	5.530609	0.122902	1
11	0.36	15	1.2	11	15	242	0.010467	10.4671	0.232602	1
11	0.36	16	1.3	11	16	215	0.011422	11.42188	0.253819	1
11	0.36	17	2.7	11	17	685	0.021818	21.81802	0.484845	1
11	0.36	18	0.868	11	18	590	0.0071	7.099974	0.157777	1
11	0.36	19	0.772	11	19	1619	0.005785	5.784961	0.128555	1
11	0.36	20	1.6	11	20	963	0.012561	12.56067	0.279126	1
11	0.36	21	1.8	11	21	791	0.014371	14.37138	0.319364	1
11	0.36	22	1.5	11	22	591	0.012268	12.26787	0.272619	1
11	0.36	23	1.2	11	23	525	0.009907	9.907064	0.220157	1
11	0.36	24	1.6	11	24	1043	0.012473	12.47334	0.277185	1
11	0.36	25	3.4	11	25	669	0.027528	27.52813	0.611736	1
11	0.36	26	0.499	11	26	964	0.003917	3.917006	0.087045	1
11	0.36	27	0.505	11	27	1404	0.003834	3.833738	0.085194	1

11	0.36	28	0.744	11	28	1470	0.005625	5.624589	0.124991	1
12	0.252	1	0.482	12	1	1115	0.002615	2.614872	0.058108	1
12	0.252	2	0.612	12	2	1152	0.003311	3.310525	0.073567	1
12	0.252	3	7.1	12	3	650	0.040335	40.33492	0.896332	1
12	0.252	4	0.729	12	4	469	0.00425	4.249967	0.094444	1
12	0.252	5	1	12	5	351	0.005956	5.955836	0.132352	1
12	0.252	6	2.2	12	6	397	0.012987	12.98693	0.288598	1
12	0.252	7	0.21	12	7	758	0.001178	1.177888	0.026175	1
12	0.252	8	2.8	12	8	1284	0.015	15	0.333333	1
12	0.252	9	1.5	12	9	915	0.00828	8.27956	0.18399	1
12	0.252	10	0.415	12	10	386	0.002455	2.454845	0.054552	1
12	0.252	11	0.36	12	11	145	0.002263	2.262626	0.050281	1
12	0.252	13	1.7	12	13	577	0.009751	9.751244	0.216694	1
12	0.252	14	0.644	12	14	186	0.003994	3.994252	0.088761	1
12	0.252	15	1.2	12	15	288	0.007245	7.244951	0.160999	1
12	0.252	16	1.3	12	16	362	0.007726	7.72559	0.17168	1
12	0.252	17	2.7	12	17	829	0.01503	15.02996	0.333999	1
12	0.252	18	0.868	12	18	693	0.004905	4.905149	0.109003	1
12	0.252	19	0.772	12	19	1746	0.004021	4.021403	0.089365	1
12	0.252	20	1.6	12	20	1054	0.008723	8.723286	0.193851	1
12	0.252	21	1.8	12	21	848	0.010001	10.00062	0.222236	1
12	0.252	22	1.5	12	22	635	0.008538	8.537729	0.189727	1
12	0.252	23	1.2	12	23	512	0.006949	6.948566	0.154413	1
12	0.252	24	1.6	12	24	1022	0.008747	8.746935	0.194376	1
12	0.252	25	3.4	12	25	590	0.019468	19.46767	0.432615	1
12	0.252	26	0.499	12	26	855	0.00277	2.770444	0.061565	1
12	0.252	27	0.505	12	27	1276	0.002707	2.706878	0.060153	1
12	0.252	28	0.744	12	28	1370	0.003962	3.962475	0.088055	1
13	1.7	1	0.482	13	1	1075	0.017697	17.69705	0.393268	1
13	1.7	2	0.612	13	2	948	0.022719	22.71868	0.50486	1
13	1.7	3	7.1	13	3	721	0.26978	269.7803	5.995118	6
13	1.7	4	0.729	13	4	369	0.029186	29.18553	0.648567	1
13	1.7	5	1	13	5	491	0.03919	39.19	0.870889	1
13	1.7	6	2.2	13	6	749	0.083328	83.32803	1.851734	2
13	1.7	7	0.21	13	7	1236	0.007615	7.615279	0.169228	1
13	1.7	8	2.8	13	8	1789	0.098172	98.17247	2.18161	3
13	1.7	9	1.5	13	9	1476	0.05353	53.52965	1.189548	2
13	1.7	10	0.415	13	10	946	0.015408	15.40847	0.34241	1
13	1.7	11	0.36	13	11	694	0.013722	13.72245	0.304943	1
13	1.7	12	0.252	13	12	577	0.009751	9.751244	0.216694	1
13	1.7	14	0.644	13	14	392	0.02567	25.66973	0.570438	1
13	1.7	15	1.2	13	15	613	0.046209	46.20881	1.026862	2
13	1.7	16	1.3	13	16	898	0.048485	48.48535	1.077452	2
13	1.7	17	2.7	13	17	1307	0.097421	97.42076	2.164906	3
13	1.7	18	0.868	13	18	960	0.032187	32.18673	0.715261	1
13	1.7	19	0.772	13	19	2027	0.026755	26.75508	0.594557	1
13	1.7	20	1.6	13	20	1230	0.058046	58.04646	1.289921	2
13	1.7	21	1.8	13	21	925	0.066962	66.96204	1.488045	2
13	1.7	22	1.5	13	22	744	0.056846	56.84647	1.263255	2
13	1.7	23	1.2	13	23	489	0.047043	47.04283	1.045396	2

13	1.7	24	1.6	13	24	752	0.060582	60.58188	1.346264	2
13	1.7	25	3.4	13	25	252	0.139688	139.6883	3.104184	4
13	1.7	26	0.499	13	26	289	0.020319	20.31902	0.451534	1
13	1.7	27	0.505	13	27	710	0.019213	19.21317	0.426959	1
13	1.7	28	0.744	13	28	809	0.027997	27.99738	0.622164	1
14	0.644	1	0.482	14	1	1085	0.006699	6.698584	0.148857	1
14	0.644	2	0.612	14	2	1077	0.008511	8.510813	0.189129	1
14	0.644	3	7.1	14	3	636	0.103262	103.2616	2.294703	3
14	0.644	4	0.729	14	4	363	0.011069	11.06918	0.245982	1
14	0.644	5	1	14	5	315	0.015336	15.33582	0.340796	1
14	0.644	6	2.2	14	6	478	0.032729	32.72903	0.727312	1
14	0.644	7	0.21	14	7	907	0.002964	2.964489	0.065878	1
14	0.644	8	2.8	14	8	1446	0.037924	37.92363	0.842747	1
14	0.644	9	1.5	14	9	1096	0.020828	20.82765	0.462837	1
14	0.644	10	0.415	14	10	566	0.006093	6.092733	0.135394	1
14	0.644	11	0.36	14	11	307	0.005531	5.530609	0.122902	1
14	0.644	12	0.252	14	12	186	0.003994	3.994252	0.088761	1
14	0.644	13	1.7	14	13	392	0.02567	25.66973	0.570438	1
14	0.644	15	1.2	14	15	305	0.018444	18.44356	0.409857	1
14	0.644	16	1.3	14	16	517	0.019223	19.22263	0.42717	1
14	0.644	17	2.7	14	17	961	0.037924	37.92438	0.842764	1
14	0.644	18	0.868	14	18	725	0.012488	12.48844	0.277521	1
14	0.644	19	0.772	14	19	1802	0.010247	10.24696	0.22771	1
14	0.644	20	1.6	14	20	1064	0.022274	22.27432	0.494985	1
14	0.644	21	1.8	14	21	814	0.025646	25.64641	0.56992	1
14	0.644	22	1.5	14	22	595	0.021934	21.93395	0.487421	1
14	0.644	23	1.2	14	23	413	0.01805	18.05031	0.401118	1
14	0.644	24	1.6	14	24	891	0.022621	22.62126	0.502695	1
14	0.644	25	3.4	14	25	423	0.051052	51.05164	1.134481	2
14	0.644	26	0.499	14	26	670	0.007227	7.226506	0.160589	1
14	0.644	27	0.505	14	27	1100	0.00701	7.009718	0.155772	1
14	0.644	28	0.744	14	28	1185	0.010259	10.25916	0.227981	1
15	1.2	1	0.482	15	1	1378	0.012218	12.21778	0.271506	1
15	1.2	2	0.612	15	2	1383	0.015508	15.50795	0.344621	1
15	1.2	3	7.1	15	3	919	0.186548	186.5483	4.145518	5
15	1.2	4	0.729	15	4	669	0.019675	19.67452	0.437212	1
15	1.2	5	1	15	5	604	0.027214	27.21418	0.60476	1
15	1.2	6	2.2	15	6	685	0.059259	59.25881	1.316862	2
15	1.2	7	0.21	15	7	1001	0.005477	5.476778	0.121706	1
15	1.2	8	2.8	15	8	1485	0.070494	70.49412	1.566536	12
15	1.2	9	1.5	15	9	1055	0.03894	38.93999	0.865333	1
15	1.2	10	0.415	15	10	579	0.011332	11.33234	0.25183	1
15	1.2	11	0.36	15	11	242	0.010467	10.4671	0.232602	1
15	1.2	12	0.252	15	12	288	0.007245	7.244951	0.160999	1
15	1.2	13	1.7	15	13	613	0.046209	46.20881	1.026862	2
15	1.2	14	0.644	15	14	305	0.018444	18.44356	0.409857	1
15	1.2	16	1.3	15	16	348	0.036892	36.89188	0.819819	1
15	1.2	17	2.7	15	17	698	0.072614	72.61366	1.613637	2
15	1.2	18	0.868	15	18	421	0.024294	24.29402	0.539867	1
15	1.2	19	0.772	15	19	1499	0.01942	19.41961	0.431547	1

15	1.2	20	1.6	15	20	773	0.042665	42.66473	0.948105	1
15	1.2	21	1.8	15	21	564	0.049255	49.25547	1.094566	2
15	1.2	22	1.5	15	22	356	0.042499	42.49895	0.944421	1
15	1.2	23	1.2	15	23	300	0.034405	34.40538	0.764564	1
15	1.2	24	1.6	15	24	812	0.042487	42.48741	0.944165	1
15	1.2	25	3.4	15	25	504	0.093866	93.86559	2.085902	3
15	1.2	26	0.499	15	26	843	0.013209	13.20854	0.293523	1
15	1.2	27	0.505	15	27	1311	0.012859	12.85855	0.285745	1
15	1.2	28	0.744	15	28	1316	0.018938	18.93758	0.420835	1
16	1.3	1	0.482	16	1	1418	0.013202	13.20164	0.29337	1
16	1.3	2	0.612	16	2	1492	0.016685	16.68485	0.370774	1
16	1.3	3	7.1	16	3	958	0.201367	201.3672	4.474826	5
16	1.3	4	0.729	16	4	830	0.020932	20.93244	0.465165	1
16	1.3	5	1	16	5	696	0.029142	29.14204	0.647601	1
16	1.3	6	2.2	16	6	639	0.064565	64.56456	1.434768	2
16	1.3	7	0.21	16	7	771	0.006068	6.067716	0.134838	1
16	1.3	8	2.8	16	8	1186	0.077933	77.93305	1.731846	2
16	1.3	9	1.5	16	9	728	0.04355	43.54994	0.967776	1
16	1.3	10	0.415	16	10	341	0.012777	12.77673	0.283927	1
16	1.3	11	0.36	16	11	215	0.011422	11.42188	0.253819	1
16	1.3	12	0.252	16	12	362	0.007726	7.72559	0.17168	1
16	1.3	13	1.7	16	13	898	0.048485	48.48535	1.077452	2
16	1.3	14	0.644	16	14	517	0.019223	19.22263	0.42717	1
16	1.3	15	1.2	16	15	348	0.036892	36.89188	0.819819	1
16	1.3	17	2.7	16	17	478	0.081083	81.08335	1.801852	2
16	1.3	18	0.868	16	18	526	0.025874	25.87375	0.574972	1
16	1.3	19	0.772	16	19	1463	0.021085	21.08457	0.468546	1
16	1.3	20	1.6	16	20	887	0.045682	45.68172	1.015149	2
16	1.3	21	1.8	16	21	789	0.051908	51.90778	1.153506	2
16	1.3	22	1.5	16	22	623	0.044112	44.11226	0.980272	1
16	1.3	23	1.2	16	23	645	0.03519	35.19016	0.782004	1
16	1.3	24	1.6	16	24	1146	0.044669	44.66936	0.992653	1
16	1.3	25	3.4	16	25	841	0.097518	97.51772	2.16706	3
16	1.3	26	0.499	16	26	1162	0.013914	13.9141	0.309202	1
16	1.3	27	0.505	16	27	1609	0.013673	13.67295	0.303843	1
16	1.3	28	0.744	16	28	1649	0.020099	20.09852	0.446634	1
17	2.7	1	0.482	17	1	1883	0.026714	26.71354	0.593634	1
17	2.7	2	0.612	17	2	1972	0.033773	33.773	0.750511	1
17	2.7	3	7.1	17	3	1432	0.403527	403.5266	8.967258	9
17	2.7	4	0.729	17	4	1298	0.041802	41.80232	0.92894	1
17	2.7	5	1	17	5	1174	0.05786	57.8599	1.285775	2
17	2.7	6	2.2	17	6	1104	0.127988	127.9883	2.844185	3
17	2.7	7	0.21	17	7	1107	0.012214	12.21413	0.271425	1
17	2.7	8	2.8	17	8	1362	0.159862	159.8619	3.552486	4
17	2.7	9	1.5	17	9	858	0.089202	89.20171	1.98226	2
17	2.7	10	0.415	17	10	748	0.024968	24.96779	0.55484	1
17	2.7	11	0.36	17	11	685	0.021818	21.81802	0.484845	1
17	2.7	12	0.252	17	12	829	0.01503	15.02996	0.333999	1
17	2.7	13	1.7	17	13	1307	0.097421	97.42076	2.164906	3
17	2.7	14	0.644	17	14	961	0.037924	37.92438	0.842764	1

17	2.7	15	1.2	17	15	698	0.072614	72.61366	1.613637	2
17	2.7	16	1.3	17	16	478	0.081083	81.08335	1.801852	2
17	2.7	17	2.7	17	18	515	0.167434	167.4337	3.720748	4
17	2.7	18	0.868	17	19	1055	0.0507	50.69987	1.126664	2
17	2.7	20	1.6	17	20	721	0.096558	96.55765	2.145726	3
17	2.7	21	1.8	17	21	811	0.107558	107.5575	2.390167	3
17	2.7	22	1.5	17	22	765	0.090075	90.0748	2.001662	3
17	2.7	23	1.2	17	23	921	0.070928	70.92757	1.576168	2
17	2.7	24	1.6	17	24	1320	0.091608	91.60839	2.035742	3
17	2.7	25	3.4	17	25	1184	0.196575	196.575	4.368334	5
17	2.7	26	0.499	17	26	1533	0.028185	28.18492	0.626332	1
17	2.7	27	0.505	17	27	2008	0.027821	27.82127	0.61825	1
17	2.7	28	0.744	17	28	1977	0.041048	41.04769	0.912171	1
18	0.868	1	0.482	18	1	1799	0.008624	8.624332	0.191652	1
18	0.868	2	0.612	18	2	1801	0.010949	10.94927	0.243317	1
18	0.868	3	7.1	18	3	1337	0.130535	130.5354	2.900786	3
18	0.868	4	0.729	18	4	1088	0.013652	13.65185	0.303374	1
18	0.868	5	1	18	5	1024	0.018827	18.82698	0.418377	1
18	0.868	6	2.2	18	6	1081	0.041222	41.22248	0.916055	1
18	0.868	7	0.21	18	7	1297	0.003871	3.87149	0.086033	1
18	0.868	8	2.8	18	8	1702	0.050357	50.35707	1.119046	2
18	0.868	9	1.5	18	9	1216	0.027814	27.81394	0.618088	1
18	0.868	10	0.415	18	10	863	0.00793	7.929938	0.176221	1
18	0.868	11	0.36	18	11	590	0.0071	7.099974	0.157777	1
18	0.868	12	0.252	18	12	693	0.004905	4.905149	0.109003	1
18	0.868	13	1.7	18	13	960	0.032187	32.18673	0.715261	1
18	0.868	14	0.644	18	14	725	0.012488	12.48844	0.277521	1
18	0.868	15	1.2	18	15	421	0.024294	24.29402	0.539867	1
18	0.868	16	1.3	18	16	526	0.025874	25.87375	0.574972	1
18	0.868	17	2.7	18	17	515	0.053827	53.82682	1.196152	2
18	0.868	19	0.772	18	19	1084	0.014462	14.4618	0.321373	1
18	0.868	20	1.6	18	20	374	0.032675	32.67455	0.726101	1
18	0.868	21	1.8	18	21	304	0.037296	37.29638	0.828808	1
18	0.868	22	1.5	18	22	269	0.031334	31.33414	0.696314	1
18	0.868	23	1.2	18	23	492	0.024008	24.00816	0.533515	1
18	0.868	24	1.6	18	24	807	0.030749	30.74868	0.683304	1
18	0.868	25	3.4	18	25	770	0.065601	65.60074	1.457794	2
18	0.868	26	0.499	18	26	1125	0.009317	9.317073	0.207046	1
18	0.868	27	0.505	18	27	1609	0.009129	9.129327	0.202874	1
18	0.868	28	0.744	18	28	1511	0.013528	13.52753	0.300612	1
19	0.772	1	0.482	19	1	2855	0.007345	7.344683	0.163215	1
19	0.772	2	0.612	19	2	2881	0.009318	9.317567	0.207057	1
19	0.772	3	7.1	19	3	2394	0.11001	110.0101	2.44467	3
19	0.772	4	0.729	19	4	2166	0.011402	11.40223	0.253383	1
19	0.772	5	1	19	5	2091	0.015693	15.69263	0.348725	1
19	0.772	6	2.2	19	6	2096	0.034516	34.51607	0.767024	1
19	0.772	7	0.21	19	7	2159	0.003286	3.28559	0.073013	1
19	0.772	8	2.8	19	8	2372	0.043422	43.42207	0.964935	1
19	0.772	9	1.5	19	9	1878	0.023776	23.77587	0.528353	1
19	0.772	10	0.415	19	10	1784	0.006609	6.609379	0.146875	1

19	0.772	11	0.36	19	11	1619	0.005785	5.784961	0.128555	1
19	0.772	12	0.252	19	12	1746	0.004021	4.021403	0.089365	1
19	0.772	13	1.7	19	13	2027	0.026755	26.75508	0.594557	1
19	0.772	14	0.644	19	14	1802	0.010247	10.24696	0.22771	1
19	0.772	15	1.2	19	15	1499	0.01942	19.41961	0.431547	1
19	0.772	16	1.3	19	16	1463	0.021085	21.08457	0.468546	1
19	0.772	17	2.7	19	17	1055	0.045093	45.09251	1.002056	2
19	0.772	18	0.868	19	18	1084	0.014462	14.4618	0.321373	1
19	0.772	20	1.6	19	20	808	0.027345	27.34503	0.607667	1
19	0.772	21	1.8	19	21	1125	0.029892	29.89159	0.664258	1
19	0.772	22	1.5	19	22	1285	0.024616	24.61563	0.547014	1
19	0.772	23	1.2	19	23	1539	0.019373	19.37296	0.43051	1
19	0.772	24	1.6	19	24	1604	0.025733	25.73291	0.571843	1
19	0.772	25	3.4	19	25	1805	0.054091	54.0905	1.202011	2
19	0.772	26	0.499	19	26	2139	0.007814	7.814	0.173644	1
19	0.772	27	0.505	19	27	2610	0.007761	7.761016	0.172467	1
19	0.772	28	0.744	19	28	2409	0.011521	11.52103	0.256023	1
20	1.6	1	0.482	20	1	2146	0.015638	15.6383	0.347518	1
20	1.6	2	0.612	20	2	2130	0.01987	19.87003	0.441556	1
20	1.6	3	7.1	20	3	1692	0.235503	235.5033	5.233406	6
20	1.6	4	0.729	20	4	1419	0.024573	24.57295	0.546066	1
20	1.6	5	1	20	5	1367	0.033822	33.82199	0.7516	1
20	1.6	6	2.2	20	6	1449	0.074016	74.0162	1.644805	2
20	1.6	7	0.21	20	7	1658	0.006979	6.978622	0.15508	1
20	1.6	8	2.8	20	8	2032	0.09131	91.30997	2.02911	3
20	1.6	9	1.5	20	9	1532	0.05021	50.20993	1.115776	2
20	1.6	10	0.415	20	10	1228	0.014172	14.17223	0.314938	1
20	1.6	11	0.36	20	11	963	0.012561	12.56067	0.279126	1
20	1.6	12	0.252	20	12	1054	0.008723	8.723286	0.193851	1
20	1.6	13	1.7	20	13	1230	0.058046	58.04646	1.289921	2
20	1.6	14	0.644	20	14	1064	0.022274	22.27432	0.494985	1
20	1.6	15	1.2	20	15	773	0.042665	42.66473	0.948105	1
20	1.6	16	1.3	20	16	887	0.045682	45.68172	1.015149	2
20	1.6	17	2.7	20	17	721	0.096558	96.55765	2.145726	3
20	1.6	18	0.868	20	18	374	0.032675	32.67455	0.726101	1
20	1.6	19	0.772	20	19	808	0.027345	27.34503	0.607667	1
20	1.6	21	1.8	20	21	318	0.068538	68.53788	1.523064	2
20	1.6	22	1.5	20	22	493	0.05531	55.30966	1.229104	2
20	1.6	23	1.2	20	23	745	0.042797	42.79723	0.95105	1
20	1.6	24	1.6	20	24	827	0.056562	56.56176	1.256928	2
20	1.6	25	3.4	20	25	1004	0.118198	118.1979	2.626621	3
20	1.6	26	0.499	20	26	1334	0.016914	16.91448	0.375877	1
20	1.6	27	0.505	20	27	1802	0.016653	16.6534	0.370076	1
20	1.6	28	0.744	20	28	1624	0.024771	24.7714	0.550476	1
21	1.8	1	0.482	21	1	1894	0.017799	17.79939	0.395542	1
21	1.8	2	0.612	21	2	1848	0.022652	22.65171	0.503371	1
21	1.8	3	7.1	21	3	1448	0.268746	268.7462	5.972137	6
21	1.8	4	0.729	21	4	1150	0.028172	28.17163	0.626036	1
21	1.8	5	1	21	5	1130	0.038704	38.70445	0.860099	1
21	1.8	6	2.2	21	6	1245	0.084417	84.41711	1.875936	2

21	1.8	7	0.21	21	7	1538	0.007905	7.905239	0.175672	1
21	1.8	8	2.8	21	8	1976	0.102992	102.9919	2.288709	3
21	1.8	9	1.5	21	9	1501	0.056592	56.59173	1.257594	2
21	1.8	10	0.415	21	10	1104	0.016096	16.0955	0.357678	1
21	1.8	11	0.36	21	11	791	0.014371	14.37138	0.319364	1
21	1.8	12	0.252	21	12	848	0.010001	10.00062	0.222236	1
21	1.8	13	1.7	21	13	925	0.066962	66.96204	1.488045	2
21	1.8	14	0.644	21	14	814	0.025646	25.64641	0.56992	1
21	1.8	15	1.2	21	15	564	0.049255	49.25547	1.094566	2
21	1.8	16	1.3	21	16	789	0.051908	51.90778	1.153506	2
21	1.8	17	2.7	21	17	811	0.107558	107.5575	2.390167	3
21	1.8	18	0.868	21	18	304	0.037296	37.29638	0.828808	1
21	1.8	19	0.772	21	19	1125	0.029892	29.89159	0.664258	1
21	1.8	20	1.6	21	20	318	0.068538	68.53788	1.523064	2
21	1.8	22	1.5	21	22	216	0.065877	65.87711	1.463936	2
21	1.8	23	1.2	21	23	446	0.050162	50.16167	1.114704	2
21	1.8	24	1.6	21	24	549	0.065815	65.81473	1.46255	2
21	1.8	25	3.4	21	25	689	0.137307	137.3066	3.051258	4
21	1.8	26	0.499	21	26	1014	0.019499	19.49878	0.433306	1
21	1.8	27	0.505	21	27	1483	0.019074	19.07352	0.423856	1
21	1.8	28	0.744	21	28	1319	0.028401	28.40054	0.631123	1
22	1.5	1	0.482	22	1	1680	0.014998	14.99829	0.333295	1
22	1.5	2	0.612	22	2	1640	0.019086	19.08574	0.424128	1
22	1.5	3	7.1	22	3	1232	0.227244	227.2444	5.049876	6
22	1.5	4	0.729	22	4	936	0.023905	23.9046	0.531213	1
22	1.5	5	1	22	5	912	0.032865	32.86472	0.730327	1
22	1.5	6	2.2	22	6	1031	0.071534	71.53447	1.589655	2
22	1.5	7	0.21	22	7	1346	0.006668	6.66803	0.148178	1
22	1.5	8	2.8	22	8	1809	0.086534	86.53365	1.92297	2
22	1.5	9	1.5	22	9	1349	0.047619	47.61921	1.058205	2
22	1.5	10	0.415	22	10	917	0.013632	13.63242	0.302943	1
22	1.5	11	0.36	22	11	591	0.012268	12.26787	0.272619	1
22	1.5	12	0.252	22	12	635	0.008538	8.537729	0.189727	1
22	1.5	13	1.7	22	13	744	0.056846	56.84647	1.263255	2
22	1.5	14	0.644	22	14	595	0.021934	21.93395	0.487421	1
22	1.5	15	1.2	22	15	356	0.042499	42.49895	0.944421	1
22	1.5	16	1.3	22	16	623	0.044112	44.11226	0.980272	1
22	1.5	17	2.7	22	17	765	0.090075	90.0748	2.001662	3
22	1.5	18	0.868	22	18	269	0.031334	31.33414	0.696314	1
22	1.5	19	0.772	22	19	1285	0.024616	24.61563	0.547014	1
22	1.5	20	1.6	22	20	493	0.05531	55.30966	1.229104	2
22	1.5	21	1.8	22	21	216	0.065877	65.87711	1.463936	2
22	1.5	23	1.2	22	23	252	0.043502	43.50154	0.966701	1
22	1.5	24	1.6	22	24	560	0.054759	54.75931	1.216874	2
22	1.5	25	3.4	22	25	524	0.116976	116.9759	2.599464	3
22	1.5	26	0.499	22	26	872	0.016463	16.46294	0.365843	1
22	1.5	27	0.505	22	27	1353	0.016028	16.02751	0.356167	1
22	1.5	28	0.744	22	28	1242	0.023795	23.79542	0.528787	1
23	1.2	1	0.482	23	1	1467	0.012149	12.14855	0.269968	1
23	1.2	2	0.612	23	2	1405	0.015486	15.48578	0.344129	1

23	1.2	3	7.1	23	3	1031	0.184689	184.689	4.1042	5
23	1.2	4	0.729	23	4	712	0.019573	19.57339	0.434964	1
23	1.2	5	1	23	5	719	0.026828	26.82778	0.596173	1
23	1.2	6	2.2	23	6	884	0.057998	57.99754	1.288834	2
23	1.2	7	0.21	23	7	1267	0.005364	5.363563	0.11919	1
23	1.2	8	2.8	23	8	1775	0.069349	69.3486	1.54108	2
23	1.2	9	1.5	23	9	1355	0.03808	38.08009	0.846224	1
23	1.2	10	0.415	23	10	864	0.010962	10.96196	0.243599	1
23	1.2	11	0.36	23	11	525	0.009907	9.907064	0.220157	1
23	1.2	12	0.252	23	12	512	0.006949	6.948566	0.154413	1
23	1.2	13	1.7	23	13	489	0.047043	47.04283	1.045396	2
23	1.2	14	0.644	23	14	413	0.01805	18.05031	0.401118	1
23	1.2	15	1.2	23	15	300	0.034405	34.40538	0.764564	1
23	1.2	16	1.3	23	16	645	0.03519	35.19016	0.782004	1
23	1.2	17	2.7	23	17	921	0.070928	70.92757	1.576168	2
23	1.2	18	0.868	23	18	492	0.024008	24.00816	0.533515	1
23	1.2	19	0.772	23	19	1539	0.019373	19.37296	0.43051	1
23	1.2	20	1.6	23	20	745	0.042797	42.79723	0.95105	1
23	1.2	21	1.8	23	21	446	0.050162	50.16167	1.114704	2
23	1.2	22	1.5	23	22	252	0.043502	43.50154	0.966701	1
23	1.2	24	1.6	23	24	518	0.044078	44.07773	0.979505	1
23	1.2	25	3.4	23	25	277	0.098002	98.00176	2.177817	3
23	1.2	26	0.499	23	26	633	0.013528	13.52832	0.300629	1
23	1.2	27	0.505	23	27	1114	0.013047	13.04698	0.289933	1
23	1.2	28	0.744	23	28	1052	0.019319	19.31907	0.429313	1
24	1.6	1	0.482	24	1	1820	0.01588	15.88031	0.352896	1
24	1.6	2	0.612	24	2	1689	0.020303	20.30304	0.451179	1
24	1.6	3	7.1	24	3	1446	0.238916	238.9155	5.309233	6
24	1.6	4	0.729	24	4	1094	0.025152	25.15248	0.558944	1
24	1.6	5	1	24	5	1095	0.0345	34.49993	0.766665	1
24	1.6	6	2.2	24	6	1370	0.074394	74.39363	1.653192	2
24	1.6	7	0.21	24	7	1783	0.006932	6.931976	0.154044	1
24	1.6	8	2.8	24	8	2292	0.090285	90.28518	2.006337	3
24	1.6	9	1.5	24	9	1869	0.049298	49.2984	1.09552	2
24	1.6	10	0.415	24	10	1384	0.01402	14.02043	0.311565	1
24	1.6	11	0.36	24	11	1043	0.012473	12.47334	0.277185	1
24	1.6	12	0.252	24	12	1022	0.008747	8.746935	0.194376	1
24	1.6	13	1.7	24	13	752	0.060582	60.58188	1.346264	2
24	1.6	14	0.644	24	14	891	0.022621	22.62126	0.502695	1
24	1.6	15	1.2	24	15	812	0.042487	42.48741	0.944165	1
24	1.6	16	1.3	24	16	1146	0.044669	44.66936	0.992653	1
24	1.6	17	2.7	24	17	1320	0.091608	91.60839	2.035742	3
24	1.6	18	0.868	24	18	807	0.030749	30.74868	0.683304	1
24	1.6	19	0.772	24	19	1604	0.025733	25.73291	0.571843	1
24	1.6	20	1.6	24	20	827	0.056562	56.56176	1.256928	2
24	1.6	21	1.8	24	21	549	0.065815	65.81473	1.46255	2
24	1.6	22	1.5	24	22	560	0.054759	54.75931	1.216874	2
24	1.6	23	1.2	24	23	518	0.044078	44.07773	0.979505	1
24	1.6	25	3.4	24	25	522	0.124812	124.8117	2.773593	3
24	1.6	26	0.499	24	26	672	0.01795	17.94965	0.398881	1
24	1.6	27	0.505	24	27	1062	0.01747	17.46956	0.388212	1

24	1.6	28	0.744	24	28	809	0.02635	26.35047	0.585566	1
25	3.4	1	0.482	25	1	1316	0.034761	34.76132	0.772474	1
25	3.4	2	0.612	25	2	1205	0.044487	44.48715	0.988603	1
25	3.4	3	7.1	25	3	930	0.52801	528.0097	11.73355	12
25	3.4	4	0.729	25	4	577	0.056418	56.41791	1.253731	2
25	3.4	5	1	25	5	578	0.07738	77.3801	1.719558	2
25	3.4	6	2.2	25	6	879	0.164407	164.4065	3.653478	4
25	3.4	7	0.21	25	7	1332	0.015128	15.12848	0.336188	1
25	3.4	8	2.8	25	8	1869	0.19555	195.5503	4.345563	5
25	3.4	9	1.5	25	9	1501	0.106895	106.8955	2.375455	3
25	3.4	10	0.415	25	10	979	0.030725	30.72516	0.682781	1
25	3.4	11	0.36	25	11	669	0.027528	27.52813	0.611736	1
25	3.4	12	0.252	25	12	590	0.019468	19.46767	0.432615	1
25	3.4	13	1.7	25	13	252	0.139688	139.6883	3.104184	4
25	3.4	14	0.644	25	14	423	0.051052	51.05164	1.134481	2
25	3.4	15	1.2	25	15	504	0.093866	93.86559	2.085902	3
25	3.4	16	1.3	25	16	841	0.097518	97.51772	2.16706	3
25	3.4	17	2.7	25	17	1184	0.196575	196.575	4.368334	5
25	3.4	18	0.868	25	18	770	0.065601	65.60074	1.457794	2
25	3.4	19	0.772	25	19	1805	0.054091	54.0905	1.202011	2
25	3.4	20	1.6	25	20	1004	0.118198	118.1979	2.626621	3
25	3.4	21	1.8	25	21	689	0.137307	137.3066	3.051258	4
25	3.4	22	1.5	25	22	524	0.116976	116.9759	2.599464	3
25	3.4	23	1.2	25	23	277	0.098002	98.00176	2.177817	3
25	3.4	24	1.6	25	24	522	0.124812	124.8117	2.773593	3
25	3.4	26	0.499	25	26	359	0.040034	40.03367	0.889637	1
25	3.4	27	0.505	25	27	840	0.037886	37.88573	0.841905	1
25	3.4	28	0.744	25	28	814	0.055965	55.96546	1.243677	2
26	0.499	1	0.482	26	1	1244	0.005128	5.1276	0.113947	1
26	0.499	2	0.612	26	2	1052	0.006608	6.608214	0.146849	1
26	0.499	3	7.1	26	3	953	0.077329	77.32917	1.718426	2
26	0.499	4	0.729	26	4	620	0.008232	8.232334	0.182941	1
26	0.499	5	1	26	5	621	0.011291	11.29116	0.250915	1
26	0.499	6	2.2	26	6	1028	0.023803	23.80322	0.52896	1
26	0.499	7	0.21	26	7	1520	0.002194	2.193867	0.048753	1
26	0.499	8	2.8	26	8	2072	0.028425	28.42547	0.631677	1
26	0.499	9	1.5	26	9	1765	0.015457	15.45671	0.343483	1
26	0.499	10	0.415	26	10	1231	0.004419	4.418999	0.0982	1
26	0.499	11	0.36	26	11	964	0.003917	3.917006	0.087045	1
26	0.499	12	0.252	26	12	855	0.00277	2.770444	0.061565	1
26	0.499	13	1.7	26	13	289	0.020319	20.31902	0.451534	1
26	0.499	14	0.644	26	14	670	0.007227	7.226506	0.160589	1
26	0.499	15	1.2	26	15	843	0.013209	13.20854	0.293523	1
26	0.499	16	1.3	26	16	1162	0.013914	13.9141	0.309202	1
26	0.499	17	2.7	26	17	1533	0.028185	28.18492	0.626332	1
26	0.499	18	0.868	26	18	1125	0.009317	9.317073	0.207046	1
26	0.499	19	0.772	26	19	2139	0.007814	7.814	0.173644	1
26	0.499	20	1.6	26	20	1334	0.016914	16.91448	0.375877	1
26	0.499	21	1.8	26	21	1014	0.019499	19.49878	0.433306	1
26	0.499	22	1.5	26	22	872	0.016463	16.46294	0.365843	1

26	0.499	23	1.2	26	23	633	0.013528	13.52832	0.300629	1
26	0.499	24	1.6	26	24	672	0.01795	17.94965	0.398881	1
26	0.499	25	3.4	26	25	359	0.040034	40.03367	0.889637	1
26	0.499	27	0.505	26	27	484	0.005816	5.815668	0.129237	1
26	0.499	28	0.744	26	28	520	0.00852	8.52041	0.189342	1
27	0.505	1	0.482	27	1	1266	0.005181	5.181101	0.115136	1
27	0.505	2	0.612	27	2	992	0.006722	6.72218	0.149382	1
27	0.505	3	7.1	27	3	1152	0.076965	76.96524	1.710339	2
27	0.505	4	0.729	27	4	914	0.008064	8.064461	0.17921	1
27	0.505	5	1	27	5	912	0.011064	11.06445	0.245877	1
27	0.505	6	2.2	27	6	1345	0.02352	23.51962	0.522658	1
27	0.505	7	0.21	27	7	1839	0.002182	2.181647	0.048481	1
27	0.505	8	2.8	27	8	2387	0.028387	28.38745	0.630832	1
27	0.505	9	1.5	27	9	2148	0.015359	15.35915	0.341315	1
27	0.505	10	0.415	27	10	1626	0.004361	4.360618	0.096903	1
27	0.505	11	0.36	27	11	1404	0.003834	3.833738	0.085194	1
27	0.505	12	0.252	27	12	1276	0.002707	2.706878	0.060153	1
27	0.505	13	1.7	27	13	710	0.019213	19.21317	0.426959	1
27	0.505	14	0.644	27	14	1100	0.00701	7.009718	0.155772	1
27	0.505	15	1.2	27	15	1311	0.012859	12.85855	0.285745	1
27	0.505	16	1.3	27	16	1609	0.013673	13.67295	0.303843	1
27	0.505	17	2.7	27	17	2008	0.027821	27.82127	0.61825	1
27	0.505	18	0.868	27	18	1609	0.009129	9.129327	0.202874	1
27	0.505	19	0.772	27	19	2610	0.007761	7.761016	0.172467	1
27	0.505	20	1.6	27	20	1802	0.016653	16.6534	0.370076	1
27	0.505	21	1.8	27	21	1483	0.019074	19.07352	0.423856	1
27	0.505	22	1.5	27	22	1353	0.016028	16.02751	0.356167	1
27	0.505	23	1.2	27	23	1114	0.013047	13.04698	0.289933	1
27	0.505	24	1.6	27	24	1062	0.01747	17.46956	0.388212	1
27	0.505	25	3.4	27	25	840	0.037886	37.88573	0.841905	1
27	0.505	26	0.499	27	26	484	0.005816	5.815668	0.129237	1
27	0.505	28	0.744	27	28	416	0.008771	8.770992	0.194911	1
28	0.744	1	0.482	28	1	1616	0.007466	7.465769	0.165906	1
28	0.744	2	0.612	28	2	1378	0.009618	9.618084	0.213735	1
28	0.744	3	7.1	28	3	1431	0.111201	111.2011	2.471135	4
28	0.744	4	0.729	28	4	1125	0.011667	11.66701	0.259267	1
28	0.744	5	1	28	5	1126	0.016003	16.00287	0.355619	1
28	0.744	6	2.2	28	6	1544	0.034219	34.21875	0.760417	1
28	0.744	7	0.21	28	7	2039	0.003183	3.183413	0.070743	1
28	0.744	8	2.8	28	8	2592	0.041498	41.49785	0.922175	1
28	0.744	9	1.5	28	9	2280	0.022502	22.50179	0.50004	1
28	0.744	10	0.415	28	10	1749	0.006381	6.38134	0.141808	1
28	0.744	11	0.36	28	11	1470	0.005625	5.624589	0.124991	1
28	0.744	12	0.252	28	12	1370	0.003962	3.962475	0.088055	1
28	0.744	13	1.7	28	13	809	0.027997	27.99738	0.622164	1
28	0.744	14	0.644	28	14	1185	0.010259	10.25916	0.227981	1
28	0.744	15	1.2	28	15	1316	0.018938	18.93758	0.420835	1
28	0.744	16	1.3	28	16	1649	0.020099	20.09852	0.446634	1
28	0.744	17	2.7	28	17	1977	0.041048	41.04769	0.912171	1
28	0.744	18	0.868	28	18	1511	0.013528	13.52753	0.300612	1

28	0.744	19	0.772	28	19	2409	0.011521	11.52103	0.256023	1
28	0.744	20	1.6	28	20	1624	0.024771	24.7714	0.550476	1
28	0.744	21	1.8	28	21	1319	0.028401	28.40054	0.631123	1
28	0.744	22	1.5	28	22	1242	0.023795	23.79542	0.528787	1
28	0.744	23	1.2	28	23	1052	0.019319	19.31907	0.429313	1
28	0.744	24	1.6	28	24	809	0.02635	26.35047	0.585566	1
28	0.744	25	3.4	28	25	814	0.055965	55.96546	1.243677	2
28	0.744	26	0.499	28	26	520	0.00852	8.52041	0.189342	1
28	0.744	27	0.505	28	27	416	0.008771	8.770992	0.194911	1

NSFNet Traffic matrix　　pop maxc=16.4　　dist max= 4151

city pairs		dist	pop	pop	dist				traff alpha	TU	DS-3
I	j	km	I	j	I	j	dist	traffic	1000	\45	
1	2	1094	3.6	7	1	2	1094	0.14537	145.3705	3.230455	4
1	3	1713	3.6	16.4	1	3	1713	0.271597	271.5975	6.035499	7
1	4	1128	3.6	1.3	1	4	1128	0.068169	68.16903	1.514867	2
1	5	1629	3.6	2.6	1	5	1629	0.085495	85.49506	1.89989	2
1	6	2163	3.6	0.25	1	6	2163	0.053646	53.64592	1.192132	2
1	7	3042	3.6	4.7	1	7	3042	0.113186	113.1858	2.515239	3
1	8	2828	3.6	0.18	1	8	2828	0.052574	52.57356	1.168301	2
1	9	3504	3.6	4.1	1	9	3504	0.104995	104.9955	2.333233	3
1	10	3061	3.6	5.46	1	10	3061	0.123376	123.376	2.741689	3
1	11	3728	3.6	4.9	1	11	3728	0.115648	115.6476	2.569946	3
1	12	3431	3.6	2.37	1	12	3431	0.08183	81.82994	1.818443	2
1	13	3840	3.6	0.04	1	13	3840	0.050495	50.4946	1.122102	2
1	14	3599	3.6	0.05	1	14	3599	0.050686	50.68636	1.126364	2
1	15	4006	3.6	5.8	1	15	4006	0.127613	127.6133	2.835851	3
2	1	1094	7	3.6	2	1	1094	0.14537	145.3705	3.230455	4
2	3	740	7	16.4	2	3	740	0.483925	483.9254	10.7539	11
2	4	965	7	1.3	2	4	965	0.084901	84.90056	1.886679	2
2	5	1526	7	2.6	2	5	1526	0.1187	118.7	2.637778	3
2	6	2233	7	0.25	2	6	2233	0.056807	56.80746	1.262388	2
2	7	2645	7	4.7	2	7	2645	0.172971	172.9707	3.843793	4
2	8	3066	7	0.18	2	8	3066	0.054815	54.81473	1.218105	2
2	9	3440	7	4.1	2	9	3440	0.15691	156.9101	3.48689	4
2	10	3307	7	5.46	2	10	3307	0.192423	192.4229	4.276065	5
2	11	3925	7	4.9	2	11	3925	0.177536	177.5362	3.945248	4
2	12	3643	7	2.37	2	12	3643	0.111766	111.766	2.483688	3
2	13	3643	7	0.04	2	13	3643	0.050975	50.97523	1.132783	2
2	14	4092	7	0.05	2	14	4092	0.051231	51.23124	1.138472	2
2	15	3896	7	5.8	2	15	3896	0.200975	200.9749	4.466109	5
3	1	1713	16.4	3.6	3	1	1713	0.271597	271.5975	6.035499	7
3	2	740	16.4	7	3	2	740	0.483925	483.9254	10.7539	11
3	4	1009	16.4	1.3	3	4	1009	0.130865	130.8646	2.908103	3
3	5	1341	16.4	2.6	3	5	1341	0.210603	210.603	4.680066	5
3	6	2026	16.4	0.25	3	6	2026	0.06565	65.64951	1.458878	2
3	7	2093	16.4	4.7	3	7	2093	0.338578	338.5778	7.52395	8
3	8	2655	16.4	0.18	3	8	2655	0.061203	61.20264	1.360059	2
3	9	3038	16.4	4.1	3	9	3038	0.300739	300.7391	6.683091	7
3	10	3112	16.4	5.46	3	10	3112	0.383784	383.7837	8.528527	9
3	11	3645	16.4	4.9	3	11	3645	0.349041	349.0408	7.756463	8
3	12	3405	16.4	2.37	3	12	3405	0.194783	194.7826	4.328503	5
3	13	3885	16.4	0.04	3	13	3885	0.052203	52.20252	1.160056	2
3	14	3706	16.4	0.05	3	14	3706	0.052958	52.95801	1.176845	2
3	15	4151	16.4	5.8	3	15	4151	0.403462	403.4616	8.965814	9
4	1	1128	1.3	3.6	4	1	1128	0.068169	68.16903	1.514867	2
4	2	965	1.3	7	4	2	965	0.084901	84.90056	1.886679	2
4	3	1009	1.3	16.4	4	3	1009	0.130865	130.8646	2.908103	3

4	5	597	1.3	2.6	4	5	597	0.063602	63.602	1.413378	2
4	6	1281	1.3	0.25	4	6	1281	0.051736	51.73623	1.149694	2
4	7	1933	1.3	4.7	4	7	1933	0.0732	73.20033	1.626674	2
4	8	1997	1.3	0.18	4	8	1997	0.051193	51.19296	1.137621	2
4	9	2546	1.3	4.1	4	9	2546	0.070104	70.10423	1.557872	2
4	10	2344	1.3	5.46	4	10	2344	0.076763	76.7632	1.705849	2
4	11	2969	1.3	4.9	4	11	2969	0.073881	73.88123	1.641805	2
4	12	2683	1.3	2.37	4	12	2683	0.061678	61.67805	1.370623	2
4	13	3138	1.3	0.04	4	13	3138	0.050282	50.28235	1.117386	2
4	14	2931	1.3	0.05	4	14	2931	0.050373	50.37264	1.119392	2
4	15	3373	1.3	5.8	4	15	3373	0.078149	78.14935	1.736652	2
5	1	1629	2.6	3.6	5	1	1629	0.085495	85.49506	1.89989	2
5	2	1526	2.6	7	5	2	1526	0.1187	118.7	2.637778	3
5	3	1341	2.6	16.4	5	3	1341	0.210603	210.603	4.680066	5
5	4	597	2.6	1.3	5	4	597	0.063602	63.602	1.413378	2
5	6	734	2.6	0.25	5	6	734	0.0532	53.20016	1.182226	2
5	7	1457	2.6	4.7	5	7	1457	0.09631	96.31033	2.14023	3
5	8	1448	2.6	0.18	5	8	1448	0.052218	52.21783	1.160396	2
5	9	1982	2.6	4.1	5	9	1982	0.090209	90.20933	2.004652	3
5	10	1820	2.6	5.46	5	10	1820	0.10352	103.5202	2.300448	3
5	11	2423	2.6	4.9	5	11	2423	0.097813	97.8128	2.173618	3
5	12	2143	2.6	2.37	5	12	2143	0.073326	73.32629	1.629473	2
5	13	2614	2.6	0.04	5	13	2614	0.050543	50.54276	1.123172	2
5	14	2413	2.6	0.05	5	14	2413	0.050697	50.69666	1.126592	2
5	15	2859	2.6	5.8	5	15	2859	0.106389	106.3895	2.364211	3
6	1	2163	0.25	3.6	6	1	2163	0.053646	53.64592	1.192132	2
6	2	2233	0.25	7	6	2	2233	0.056807	56.80746	1.262388	2
6	3	2026	0.25	16.4	6	3	2026	0.06565	65.64951	1.458878	2
6	4	1281	0.25	1.3	6	4	1281	0.051736	51.73623	1.149694	2
6	5	734	0.25	2.6	6	5	734	0.0532	53.20016	1.182226	2
6	7	1237	0.25	4.7	6	7	1237	0.054946	54.94561	1.221014	2
6	8	715	0.25	0.18	6	8	715	0.050928	50.92751	1.131722	2
6	9	1341	0.25	4.1	6	9	1341	0.054344	54.34419	1.207649	2
6	10	1091	0.25	5.46	6	10	1091	0.055719	55.71887	1.238197	2
6	11	1690	0.25	4.9	6	11	1690	0.054983	54.9833	1.221851	2
6	12	1409	0.25	2.37	6	12	1409	0.052698	52.69756	1.171057	2
6	13	1877	0.25	0.04	6	13	1877	0.050379	50.37928	1.11954	2
6	14	2413	0.25	0.05	6	14	2413	0.050275	50.27533	1.11723	2
6	15	2859	0.25	5.8	6	15	2859	0.055559	55.55911	1.234647	2
7	1	3042	4.7	3.6	7	1	3042	0.113186	113.1858	2.515239	3
7	2	2645	4.7	7	7	2	2645	0.172971	172.9707	3.843793	4
7	3	2093	4.7	16.4	7	3	2093	0.338578	338.5778	7.52395	8
7	4	1933	4.7	1.3	7	4	1933	0.0732	73.20033	1.626674	2
7	5	1457	4.7	2.6	7	5	1457	0.09631	96.31033	2.14023	3
7	6	1237	4.7	0.25	7	6	1237	0.054946	54.94561	1.221014	2
7	8	1321	4.7	0.18	7	8	1321	0.053679	53.67905	1.192868	2
7	9	1129	4.7	4.1	7	9	1129	0.123033	123.0327	2.734061	3
7	10	1739	4.7	5.46	7	10	1739	0.146517	146.5173	3.255941	4
7	11	1963	4.7	4.9	7	11	1963	0.136508	136.5083	3.033518	4
7	12	1833	4.7	2.37	7	12	1833	0.092067	92.06656	2.045923	3

7	13	2245	4.7	0.04	7	13	2245	0.050896	50.89621	1.131027	2
7	14	2200	4.7	0.05	7	14	2200	0.051116	51.11648	1.135922	2
7	15	2585	4.7	5.8	7	15	2585	0.151955	151.9545	3.376767	4
8	1	2828	0.18	3.6	8	1	2828	0.052574	52.57356	1.168301	2
8	2	3066	0.18	7	8	2	3066	0.054815	54.81473	1.218105	2
8	3	2655	0.18	16.4	8	3	2655	0.061203	61.20264	1.360059	2
8	4	1997	0.18	1.3	8	4	1997	0.051193	51.19296	1.137621	2
8	5	1448	0.18	2.6	8	5	1448	0.052218	52.21783	1.160396	2
8	6	715	0.18	0.25	8	6	715	0.050928	50.92751	1.131722	2
8	7	1321	0.18	4.7	8	7	1321	0.053679	53.67905	1.192868	2
8	9	786	0.18	4.1	8	9	786	0.053503	53.50346	1.188966	2
8	10	452	0.18	5.46	8	10	452	0.05465	54.6502	1.214449	2
8	11	975	0.18	4.9	8	11	975	0.053953	53.95253	1.198945	2
8	12	707	0.18	2.37	8	12	707	0.052372	52.37237	1.16383	2
8	13	1168	0.18	0.04	8	13	1168	0.05058	50.57967	1.123993	2
8	14	1018	0.18	0.05	8	14	1018	0.050646	50.64623	1.125472	2
8	15	1461	0.18	5.8	8	15	1461	0.054375	54.37494	1.208332	2
9	1	3504	4.1	3.6	9	1	3504	0.104995	104.9955	2.333233	3
9	2	3440	4.1	7	9	2	3440	0.15691	156.9101	3.48689	4
9	3	3038	4.1	16.4	9	3	3038	0.300739	300.7391	6.683091	7
9	4	2546	4.1	1.3	9	4	2546	0.070104	70.10423	1.557872	2
9	5	1982	4.1	2.6	9	5	1982	0.090209	90.20933	2.004652	3
9	6	1341	4.1	0.25	9	6	1341	0.054344	54.34419	1.207649	2
9	7	1129	4.1	4.7	9	7	1129	0.123033	123.0327	2.734061	3
9	8	786	4.1	0.18	9	8	786	0.053503	53.50346	1.188966	2
9	10	951	4.1	5.46	9	10	951	0.134943	134.9429	2.998731	3
9	11	874	4.1	4.9	9	11	874	0.126384	126.3835	2.808523	3
9	12	842	4.1	2.37	9	12	842	0.08732	87.32048	1.940455	2
9	13	1145	4.1	0.04	9	13	1145	0.051119	51.1188	1.135973	2
9	14	1191	4.1	0.05	9	14	1191	0.051287	51.28664	1.139703	2
9	15	1509	4.1	5.8	9	15	1509	0.139642	139.6423	3.103162	4
10	1	3061	5.46	3.6	10	1	3061	0.123376	123.376	2.741689	3
10	2	3307	5.46	7	10	2	3307	0.192423	192.4229	4.276065	5
10	3	3112	5.46	16.4	10	3	3112	0.383784	383.7837	8.528527	9
10	4	2344	5.46	1.3	10	4	2344	0.076763	76.7632	1.705849	2
10	5	1820	5.46	2.6	10	5	1820	0.10352	103.5202	2.300448	3
10	6	1091	5.46	0.25	10	6	1091	0.055719	55.71887	1.238197	2
10	7	1739	5.46	4.7	10	7	1739	0.146517	146.5173	3.255941	4
10	8	452	5.46	0.18	10	8	452	0.05465	54.6502	1.214449	2
10	9	951	5.46	4.1	10	9	951	0.134943	134.9429	2.998731	3
10	11	681	5.46	4.9	10	11	681	0.151794	151.794	3.373201	4
10	12	374	5.46	2.37	10	12	374	0.100059	100.0594	2.223541	3
10	13	796	5.46	0.04	10	13	796	0.051456	51.45623	1.143472	2
10	14	571	5.46	0.05	10	14	571	0.051835	51.83474	1.151883	2
10	15	1041	5.46	5.8	10	15	1041	0.16977	169.7698	3.772662	4
11	1	3728	4.9	3.6	11	1	3728	0.115648	115.6476	2.569946	3
11	2	3925	4.9	7	11	2	3925	0.177536	177.5362	3.945248	4
11	3	3645	4.9	16.4	11	3	3645	0.349041	349.0408	7.756463	8
11	4	2969	4.9	1.3	11	4	2969	0.073881	73.88123	1.641805	2
11	5	2423	4.9	2.6	11	5	2423	0.097813	97.8128	2.173618	3

11	6	1690	4.9	0.25	11	6	1690	0.054983	54.9833	1.221851	2
11	7	1963	4.9	4.7	11	7	1963	0.136508	136.5083	3.033518	4
11	8	975	4.9	0.18	11	8	975	0.053953	53.95253	1.198945	2
11	9	874	4.9	4.1	11	9	874	0.126384	126.3835	2.808523	3
11	10	681	4.9	5.46	11	10	681	0.151794	151.794	3.373201	4
11	12	306	4.9	2.37	11	12	306	0.095145	95.14509	2.114335	3
11	13	269	4.9	0.04	11	13	269	0.051764	51.76427	1.150317	2
11	14	399	4.9	0.05	11	14	399	0.051862	51.86243	1.152498	2
11	15	634	4.9	5.8	11	15	634	0.15817	158.1704	3.514898	4
12	1	3431	2.37	3.6	12	1	3431	0.08183	81.82994	1.818443	2
12	2	3643	2.37	7	12	2	3643	0.111766	111.766	2.483688	3
12	3	3405	2.37	16.4	12	3	3405	0.194783	194.7826	4.328503	5
12	4	2683	2.37	1.3	12	4	2683	0.061678	61.67805	1.370623	2
12	5	2143	2.37	2.6	12	5	2143	0.073326	73.32629	1.629473	2
12	6	1409	2.37	0.25	12	6	1409	0.052698	52.69756	1.171057	2
12	7	1833	2.37	4.7	12	7	1833	0.092067	92.06656	2.045923	3
12	8	707	2.37	0.18	12	8	707	0.052372	52.37237	1.16383	2
12	9	842	2.37	4.1	12	9	842	0.08732	87.32048	1.940455	2
12	10	374	2.37	5.46	12	10	374	0.100059	100.0594	2.223541	3
12	11	306	2.37	4.9	12	11	306	0.095145	95.14509	2.114335	3
12	13	460	2.37	0.04	12	13	460	0.051245	51.24522	1.138783	2
12	14	367	2.37	0.05	12	14	367	0.05143	51.43001	1.142889	2
12	15	775	2.37	5.8	12	15	775	0.102575	102.5754	2.279454	3
13	1	3840	0.04	3.6	13	1	3840	0.050495	50.4946	1.122102	2
13	2	3643	0.04	7	13	2	3643	0.050975	50.97523	1.132783	2
13	3	3885	0.04	16.4	13	3	3885	0.052203	52.20252	1.160056	2
13	4	3138	0.04	1.3	13	4	3138	0.050282	50.28235	1.117386	2
13	5	2614	0.04	2.6	13	5	2614	0.050543	50.54276	1.123172	2
13	6	1877	0.04	0.25	13	6	1877	0.050379	50.37928	1.11954	2
13	7	2245	0.04	4.7	13	7	2245	0.050896	50.89621	1.131027	2
13	8	1168	0.04	0.18	13	8	1168	0.05058	50.57967	1.123993	2
13	9	1145	0.04	4.1	13	9	1145	0.051119	51.1188	1.135973	2
13	10	796	0.04	5.46	13	10	796	0.051456	51.45623	1.143472	2
13	11	269	0.04	4.9	13	11	269	0.051764	51.76427	1.150317	2
13	12	460	0.04	2.37	13	12	460	0.051245	51.24522	1.138783	2
13	14	283	0.04	0.05	13	14	283	0.051086	51.08584	1.135241	2
13	15	366	0.04	5.8	13	15	366	0.051791	51.79133	1.150918	2
14	1	3599	0.05	3.6	14	1	3599	0.050686	50.68636	1.126364	2
14	2	4092	0.05	7	14	2	4092	0.051231	51.23124	1.138472	2
14	3	3706	0.05	16.4	14	3	3706	0.052958	52.95801	1.176845	2
14	4	2931	0.05	1.3	14	4	2931	0.050373	50.37264	1.119392	2
14	5	2413	0.05	2.6	14	5	2413	0.050697	50.69666	1.126592	2
14	6	2413	0.05	0.25	14	6	2413	0.050275	50.27533	1.11723	2
14	7	2200	0.05	4.7	14	7	2200	0.051116	51.11648	1.135922	2
14	8	1018	0.05	0.18	14	8	1018	0.050646	50.64623	1.125472	2
14	9	1191	0.05	4.1	14	9	1191	0.051287	51.28664	1.139703	2
14	10	571	0.05	5.46	14	10	571	0.051835	51.83474	1.151883	2
14	11	399	0.05	4.9	14	11	399	0.051862	51.86243	1.152498	2
14	12	367	0.05	2.37	14	12	367	0.05143	51.43001	1.142889	2
14	13	283	0.05	0.04	14	13	283	0.051086	51.08584	1.135241	2

14	15	447	0.05	5.8	14	15	447	0.051987	51.98656	1.155257	2
15	1	4006	5.8	3.6	15	1	4006	0.127613	127.6133	2.835851	3
15	2	3896	5.8	7	15	2	3896	0.200975	200.9749	4.466109	5
15	3	4151	5.8	16.4	15	3	4151	0.403462	403.4616	8.965814	9
15	4	3373	5.8	1.3	15	4	3373	0.078149	78.14935	1.736652	2
15	5	2859	5.8	2.6	15	5	2859	0.106389	106.3895	2.364211	3
15	6	2859	5.8	0.25	15	6	2859	0.055559	55.55911	1.234647	2
15	7	2585	5.8	4.7	15	7	2585	0.151955	151.9545	3.376767	4
15	8	1461	5.8	0.18	15	8	1461	0.054375	54.37494	1.208332	2
15	9	1509	5.8	4.1	15	9	1509	0.139642	139.6423	3.103162	4
15	10	1041	5.8	5.46	15	10	1041	0.16977	169.7698	3.772662	4
15	11	634	5.8	4.9	15	11	634	0.15817	158.1704	3.514898	4
15	12	775	5.8	2.37	15	12	775	0.102575	102.5754	2.279454	3
15	13	366	5.8	0.04	15	13	366	0.051791	51.79133	1.150918	2
15	14	447	5.8	0.05	15	14	447	0.051987	51.98656	1.155257	2

Appendix C MPL SOGP Implementations

North American network model – maximum growth 10x

```
TITLE SOGP
{11-Jun-05}
INDEX
        n       := 1..27;       {nodes in network}
        i       := n;
        j       := n;
        r       := 1..702;      {index of demand pairs (traffic),
                                each will have origination O[r] and destination T[r]
                                as well as a demand value d}
        k       := 0..6; {index of different switch sizes}

DATA
        CS[k]:=  (0,1000,2000,4000, 10000, 20000,40000); {cost of switch chassis for
        each level of   k}

        cp[j]   := DATAFILE("ConnectionCOSTcp.csv")   ;  {fixed cost of adding one
more            connection to a switch j}

        O[r]    := SPARSEFILE("NAfulltraffic.csv", 2);      { origin for rth demand}

        T[r]    := SPARSEFILE("NAfulltraffic.csv",3);       { destination (termination)
for rth         demand}

        D[r]    := SPARSEFILE("NAfulltraffic.csv", 8);      { Demand value or amount of
traffic to              be exchanged between origin, O, and destination, T, indexed by r}

        cl[i,j] := SPARSEFILE("LINKCOST.csv");
                        { cost function for adding one unit of demand to each link in the
model}

        M       := 10000000;            {constant, larger than any switch capacity}

VARIABLES

        w[r,i,j] WHERE (cl);            { working flow for each rth demand pair }
        lw[i,j]  WHERE (cl);            { total working flow one each i,j link }
        P[j];                           { the number of connections needed in switch at
                                        node j to accommodate the traffic flow
                                        that will enter or leave this switch; switch capacity}
        S[j];           {cost of switch chassis at node j at level k through node j }

BINARY VARIABLE
```

```
    z[j,k];          {binary variable to match fixed cost of switch to correct size,k, if j
is in                model}

MODEL

    MIN   TotalCost = SUM(j: (S[j])) + SUM(j: (cp[j] * P[j])) +  SUM(i,j: (cl[i,j] *
lw[i,j] ));

SUBJECT TO
    { balance constraints - for each demand pair the total source flow equals
    the demand, the total sink flow equals the demand, and that no net sourcing
    or sinking of flow for the given O-D pair occurs at any other node (ie
transshipment) }

        Supply[r,i] where i=O[r]: SUM(j:  w[r, i, j]) - SUM(j:  w[r, i:=j, j:=i]) =
D[r];

        Demand[r,i] where i=T[r]: SUM(j:  w[r, i, j]) - SUM(j:  w[r, i:=j, j:=i]) =  -
D[r];

        Balance[r,i] where i<>O[r] and i<>T[r]: SUM(j:  w[r, i, j]) = SUM(j:  w[r,
i:=j, j:=i]) ;

    { link capacity constraint - defines total working capacity needed to deliver all
traffic flow}

        Capacity[i,j]:   lw[i,j]  =        SUM(r:          w[r,i,j])  ;


    {total number of connections needed for switch at node j,port capacity}

        PortCap[j]:     P[j] >= SUM(i: lw[i,j]) + SUM(i: lw[i:=j,j:=i] );

    {requires that if switch is not installed at j then there will be no connection cost}

        Swconst[j]: P[j] <= M * (z[j,1] + z[j,2] + z[j,3] + z[j,4] + z[j,5] +z[j,6]);


    {calculates the size of the switch chassis, at node j}

        swstep[j]:      P[j] <= 0*z[j,0] + 1000*z[j,1] + 2000*z[j,2] + 4000*z[j,3]+
                        12000*z[j,4] + 24000*z[j,5] + 50000*z[j,6];


    {calculates the cost of the switch, at node j}
```

```
swcost[j]:        S[j] = (z[j,0]*CS[0] + z[j,1]*CS[1] + z[j,2]*CS[2] +
z[j,3]*CS[3] + z[j,4]*CS[4]  + z[j,5]*CS[5] +z[j,6]*CS[6]);

{constrains the size of switch chassis to no more than one size per node, if
any at all}

swlimit[j]:       SUM(k: z[j,k]) <= 1;

END
```

Appendix D – Skewness calculations

"Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

Definition of Skewness For univariate data $Y_1, Y_2, ..., Y_N$, the formula for skewness is:

$$skewness = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3}{(N-1)s^3}$$

where $\bar{Y}$ is the mean, s is the standard deviation, and N is the number of data points. The skewness for a normal distribution is zero, and any symmetric data should have a skewness near zero. Negative values for the skewness indicate data that are skewed left and positive values for the skewness indicate data that are skewed right. By skewed left, we mean that the left tail is heavier than the right tail. Similarly, skewed right means that the right tail is heavier than the left tail. Some measurements have a lower bound and are skewed right. For example, in reliability studies, failure times cannot be negative."

http://www.itl.nist.gov/div898/handbook/eda/section3/eda35b.htm

Another variation on the formula for skewness that is used by Microsoft Excel© is

Skewness = $[n/ (n-1)(n-2)] * \Sigma[(Y_i - Y)/s]^3$ where n is the number of data points, Y is the mean, and s is the standard deviation.

VITA

Susan Jean Chinburg

Candidate for the Degree of

Doctor of Philosophy

Thesis: NODAL DISTRIBUTION STRATEGIES FOR DESIGNING AN OVERLAY NETWORK FOR LONG-TERM GROWTH

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Title of Study: NODAL DISTRIBUTION STRATEGIES FOR DESIGNING AN OVERLAY NETWORK FOR LONG-TERM GROWTH

Pages in Study: 156 Candidate for the Degree of Doctor of Philosophy

Major Field: Business Administration

Scope and Method of Study:

This research looked at nodal distribution design issues associated with building an overlay network on top of an existing legacy network with overlay network switches and links not necessarily matching the switch and link locations of the underlying network. A mathematical model with two basic components, switch costs and link costs, was developed for defining the total cost of a network overlay. The nature of the underlying legacy topology determines the dominant factor, link or switch costs to the total cost function as well as the unit cost for switches and links.

Findings and Conclusions:

The three design heuristics presented first, locate overlay switches at nodes in the center of the legacy network as opposed to the periphery; second, locate overlay switches at legacy nodes with high connectivity; and third, locate overlay switches at legacy nodes with high traffic flow demands, can be used to help point to the direction of keeping costs under control when design changes are required. Applying the concept of efficient frontiers to the world of network design and building a suite of best designs gives the network designer greater insight into how to design the best network in the face of changing real-world constraints. For the cost model and the case studies evaluated using the design strategies in this study, distributed approaches generally tend to be a good choice when the link costs dominate the total cost function because total path distances and therefore link costs need to be minimized in preference over switch costs. A distributed overlay tends to have lower link costs because there is usually a greater probability that total path distances can be minimized because of greater connectivity. More connections set up the potential for more traffic flow path choices allowing each traffic flow to be sent along shorter paths. In legacy network topology designs that have many nodes with high connectivity, the overlay link costs can be relatively similar between designs and the switch costs can have a large impact upon total cost.

ADVISER'S APPROVAL: Dr. George Scheets