ECONOMIC APPROACH ON ALLOCATION OF IRRIGATION WATER UNDER SALINITY BASED ON DIFFERENT SOILS FOR POTENTIAL IRRIGATED AGRICULTURE USING EPIC CROP MODEL

By

JONGSAN CHOI

Bachelor of Science in Agricultural Economics
Kangwon National University
Chuncheon, South Korea
2000

Master of Science in Agricultural Economics
Kangwon National University
Chuncheon, South Korea
2002

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
In partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
December, 2011

ECONOMIC APPROACH ON ALLOCATION OF IRRIGATION WATER UNDER SALINITY BASED ON DIFFERENT SOILS FOR POTENTIAL IRRIGATED AGRICULTURE USING THE EPIC CROP MODEL

Dissertation Approved:

Dr. Arthur Stoecker

Dissertation Adviser

Dr. Francis Epplin

Dr. Jeffrey Vitale

Dr. Dan Storm

Outside Committee Member

Dr. Sheryl A. Tucker

Dean of the Graduate College

TABLE OF CONTENTS

LIST OF TABLES

Table Page

Table Page

Table Page

Table Page

Table Page

Table Page

Table Page

LIST OF FIGURES

Figure Page

Figure Page

Figure Page

Figure Page

CHAPTER I

INTRODUCTION

Problem Statement

The United States Army Corps of Engineers (USACE) has asked Oklahoma State University to estimate the net agricultural benefits from reducing the salt loading into the Elm Fork of the Red River just west of the highway 30 bridge in Harmon County. Saline soils and waters contain excessive amounts of soluble salts which preclude the practical and normal production of most agricultural crops. They have been a potential threat for agriculture in a study area. The study area is located along Elm and North Forks of the Red River in Greer, Harmon, Jackson and Tillman Counties of Oklahoma. A major source of the salt is a series of three canyons, which join the Elm Fork in Harmon County. The control point in this area contributes some 510 tons per day of chlorides in Elm and North Fork (Red River Chloride Control Project, 2010). If we use water from the Elm and/or the North Fork as irrigation water, it would quickly increase soil salinity and depress crop yield. Irrigated agriculture depends on adequate and high-quality water supplies. As the level of salinity increases in irrigation source, the quality of that water for plant growth decreases.

Currently, the USACE is investigating the potential benefits from irrigation if the source of chloride contamination were cut off at the control point. The specific area is defined by sections of land that either transverse or are adjacent to sections transverse by the Elm and North Forks of the Red River. This area is shown in Figure 1.

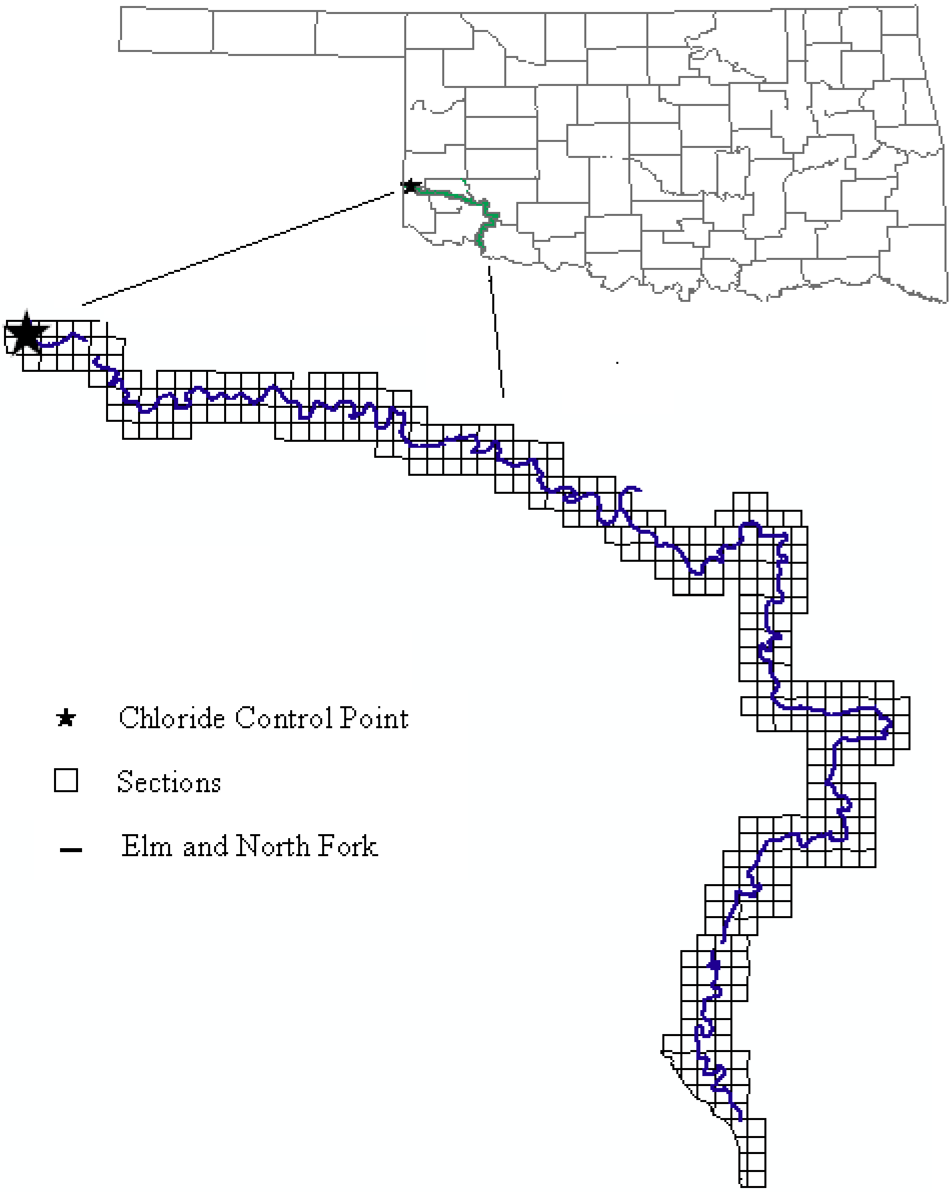


Figure 1. Study Area

Although salinity currently precludes irrigation, it is expected the irrigated area would increase rapidly in the study area. However, we do not know the relationship between yield, quality of irrigation water, soil containing salinity and the volume of irrigation water directly. Before applying irrigation in the study area, we need to determine how much of the shaded area in Figure 1 might be economically irrigated, how much salinity affects a cotton yield, and how much irrigation is required under salinity.

To assess the relationship between cotton yield, quantity and quality of irrigation water, and soil salinity, the Erosion Productivity Impact Calculator (Williams *et al*, 1990) crop model simulation will be used. The EPIC simulation model is a research tool usually that is commonly used to determine the response of crop yields to environmental factors. For the purpose of this study, the EPIC will be used to determine potential crop yields for cotton subject to the salinity of surface water and soil salinity with different levels of irrigation water for next 50 years.

Objectives

The development of irrigable land is one of the fundamental measures for increasing agricultural production. However, the study area is a non-irrigable because of a lack of sufficient ground water for irrigation and the salt load from the chloride control point. If irrigation is expanded along the alluvial plain the Elm and North Fork Rivers, it is important to understand the long term effect of using irrigation water with various levels of salinity on cotton yield based on the different soil types in the study area.

The objectives of this study are to 1) estimate the potential cotton response for each soil type to irrigation water and salinity content, 2) estimate the economic viability of establishing irrigation systems to irrigate potentially irrigable soils in the study area along the Elm and North Fork rivers, 3) estimate dynamic soil salinity changes in response to the amount of irrigation water, the salinity of irrigation water, and the soil salinity of the previous year, 4) determine that temporal use of water with the given levels of salt concentration that maximizes the Net Present Value (NPV) from irrigation for each soil type.

CHAPTER II

REVIEW OF LITERATURE

Crop simulation models have some ability to extend the results of crop experiments. The process of the actual experiment such as designing, building, and testing can be expensive and consequently is limited to select area. Simulation models are generally based on experiments covered over a broad geographical area and covers many years. However, crop simulation models can generate the level of detail that we cannot find in actual experiments. It also can be set to run for as many time steps we desire. After proper validation, it can be used to predict the crop yield under environmental changes and expand the results of actual experiments (Jame *et al*, 1996)

Crop simulation with salinity

Beginning in 1981, a mathematical model called the EPIC model was developed to determine the relation between soil erosion and soil productivity throughout the U.S.A (Williams, 1990). The EPIC is a field scale and daily time step model composed by soil and crop processes such as an erosion, nutrient balance, and related process. The EPIC crop model has been successfully applied in the study of erosion, water pollution, and crop growth and production. However, there is little literature on crop simulation with salinity.

Tayfur *et al* (1996) provides useful evidence on the salinity effect on decreasing crop yields. They extended the EPIC to consider the effects of root zone salinity in alfalfa production on a field scale under optimal and under water stress or limited irrigation conditions. The revised model was calibrated and validated with field data. The results suggest that an increase in salt concentration in applied irrigation water would dramatically decrease the total alfalfa yield under irrigation treatments.

Experiment with salinity

Salinity problems occur because irrigation water contains some amount of soluble salts. Evaporation and transpiration by plants leave these salts in the soil. These salts accumulate over time in soil and affect the crop yields. The matter of soil salinity and the use of irrigation water containing soluble salts is one of the major considerations when irrigation is used in the study area. The response function of the crop yield to salinity is an important factor in an economic model.

There is considerable literature available on crop yield response to irrigation water and salinity with experimental data. Yaron and Bresler (1970) determined the efficient combination of water quantity and quality in irrigation under specific field conditions. They used to a linear programming model to derive the optimal quantity-quality combinations under different levels of irrigation water and initial soil salinity. The authors used a leaching model to trace the salt distribution in the soil profile and restrictions on the chloride concentration in the soil solution. They compared the empirical estimates of the marginal rate of substitution of water salinity for quality with the cost of the water quantity and quality ratio. Unfortunately, information on the cost did not exist at that time. However, in the empirical estimates from the linear programming model, they found that as the quantity of irrigation water applied increases, the maximum permissible chloride concentration in irrigation water also increases.

Dinar and Knapp (1986) provide econometric estimates of yield response and salt accumulation in the soil under saline conditions with experimental data for alfalfa and cotton. They estimated to log and quadratic functions of yield and soil salinity. The dependent variables of crop yield and soil salinity at the end of the growing season were regressed on quantity of rainfall and applied irrigation water during the growing season, salt concentration of the irrigation water, soil salinity of the root zone at planting time, and pan evaporation during the growing season. The log yield response functions and the log soil salinity relations moved for alfalfa and cotton as they expected. The crop yield increases as water quantity increases, salt concentration decreases and soil salinity decreases. The quadratic yield function showed unexpected patterns. The crop yield generally increases as the quantity of water increases. However, when the quantity of water is held constant, the yield increases as initial soil salinity increases. The log soil salinity relations also exhibit for alfalfa and cotton as they expected. Ending soil salinity decreases as water quantity increases, salt concentration decreases. The quadratic soil salinity relations also did not behave as they expected. Ending salinity decreases as initial soil salinity increases, holding water quantity constant. They added the pan evaporation variable on the log and quadratic functions of yield and soil salinity. Its coefficient was a negative in yield response functions and a positive in soil salinity relations indicating the crop yield decreases and soil salinity increases as the pan evaporation decreases. In addition, they combined the estimated response functions and dynamic soil salt relations with an economic decision model to determine water applications for any give prices and initial soil salinity which maximize the net present value of profits. Profits increase as crop prices increase, decrease as irrigation water prices increase, and decrease as initial soil salinity increases. Contrary to their expectation, they found that profits increase as the initial soil salinity increase with a range of salinity EC levels from 4 to7 for alfalfa.

Dinar *et al* (1991) provided statistical estimates of crop-water response functions with various levels of salinity. They estimated the quadratic and log-log response function of yield, soil salinity and drainage volume for wheat, sorghum and wheatgrass in terms of the quantity and quality of the applied irrigation water and the initial level of root zone salinity at the beginning of the growing season. Their data came from a four-year lysimeter experiment. Coefficients from SAS Proc Mixed for the quadratic function were statistically significant and the function described the relative effects of input water quality and quantity on yield, soil salinity, and drainage volume for three crops. In case of the log-log response function, the estimated coefficients for water quantity were greater than or close to 1 for wheat and wheatgrass. This indicates that any increase in water quantity would increase yield with all other variables being constant. They found that final soil salinity increased with small amounts of irrigation water and then decreased with larger amounts of irrigation water. They also found that amount of and/or requirement for drainage increased as applied irrigation water increased, as the level of initial soil salinity increased, and as the salt concentration in irrigation water increased for three crops.

Feinerman (1994) estimates the response function to soil salinity of a given crop (potatoes) in a single-farm framework. He uses a switching regression to estimate a piecewise linear response function. Crop yield is dependent of average soil salinity below a certain critical threshold, and thereafter decreases linearly.

Datta *et al* (1998) estimate a set of production functions relating wheat yield to initial soil salinity and water quantity and quality. They used the functions to find optimal water application for given irrigation water quality, reuse of drainage water, reduction in income from using saline drainage waters mixed at various rates with good quality water. Crop yield response functions fitted to experimental data were quadratic, Cobb-Douglas and linear. They found that the quadratic function provided a better fit to the data for the response of cotton yield to selected variables than did the linear or Cobb-Douglas functions. They suggest that yield is not simply related to the average initial soil salinity but also to the salinity in irrigation water applied.

Kiani and Abbasi (2009) used experimental data to investigate crop response to both soil water content and soil salinity. They estimated linear, Cobb-Douglas, quadratic, and transcendental functions. They compared the various production functions in terms of their F-value, R^2, standard error (SE), and relative error (RE). They found the quadratic and transcendental functions predicted yield response very well. They also found that both soil water content and soil salinity affected the variation of yield. The effect of soil salinity on yield increases as soil water content is increased.

CHAPTER III

METHODOLOGY

Conceptual Framework

The response function of a crop yield to soil salinity is an important factor in an optimization model concerning irrigation or irrigation systems with water salinity (Feinerman, 1993). In this study, the specific yield response function will be estimated from the EPIC simulation results. The EPIC model will be used to simulate the yield of cotton on the soil types in the study area. The simulation will use different levels of irrigation, water salinity, and soil salinity. The results will indicate the changes in yield over time to soil salinity for each soil type in the study area. This approach has assumptions that the given crop was directly affected by irrigation water, water salinity, soil salinity and other possible factors (Datta, 1998). These functions were measured by Dinar and Knapp (1986), Dinar *et al* (1991), Datta (1998) and Kiani and Abbasi (2009). The general relationships of the factors for an individual soil type are specified as follows:

where *Y* is a crop yield per unit area, *Irr* is a quantity of irrigation water applied in acre-feet, *WS* is the dissolved salts in irrigation water, *SS* is the salt in the soil profile, *X* is a vector of all other factors affecting the crop yield and *t* is the simulation year.

The estimated crop response function and the dynamic soil salinity function can be incorporated into an economic decision model to determine optimal level of irrigation levels maximizing the net present value of profits. The dynamic programming optimization for individual soil types in the study area is constructed as follows:

subject to

where P_y is the price of cotton (\$/lb), Y_t is the cotton yield function (lbs/acre), is the quantity of irrigation water applied (acre-feet), is the irrigation cost (\$/acre-feet), and is total costs except for the irrigation cost.

Data and Procedure

In this study, it is necessary to complete the following steps to estimate the net agricultural benefits from reducing salt loading and expanding irrigation along the Elm and North Fork of the Red River. These steps include:

1) Determine the location and area of potentially irrigable soils along the Elm and North Forks
2) Establish soil parameters by depth for each of the irrigable soil types to be simulated
3) Establish crop management data and enterprise budgets for cotton
4) Use the EPIC model to simulate cotton yield and soil salinity for each of the major irrigable soil types identified in step 1
5) Calibrate the EPIC simulation model to conditions in Jackson County
6) Generate fifty years of daily maximum/minimum temperature, precipitation and solar radiation
7) Simulate and estimate the crop response functions and dynamic soil salinity functions for each soil type with randomly generated weather data
8) Set up and solve the necessary dynamic optimization models

Figure 2 represents the different implementation and solution steps graphically.

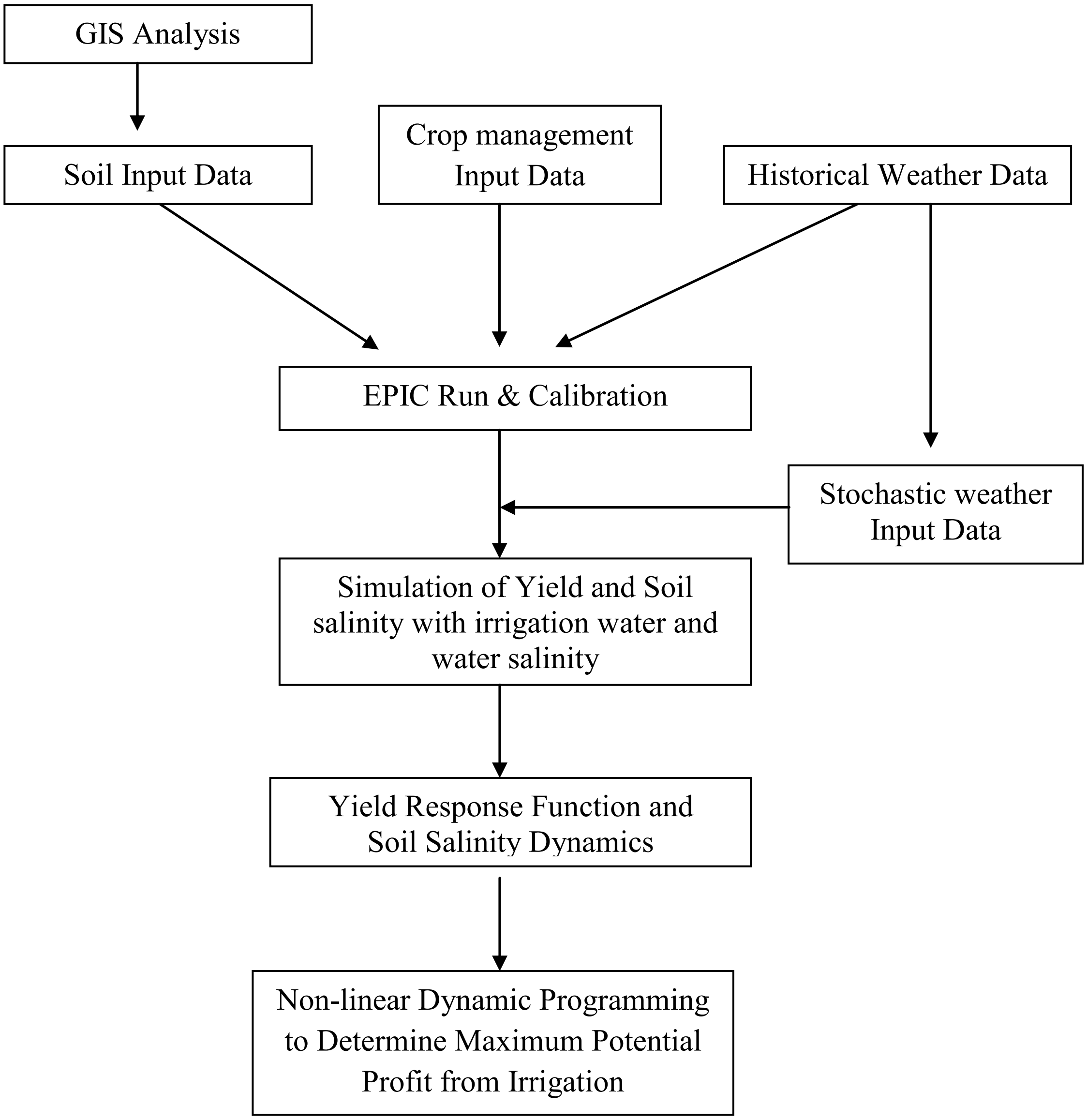


Figure 2. Study Procedure

The procedure consists of several different steps to achieve the academic purpose. It also includes applications of the Geographic Information System (GIS) technology and

Erosion Productivity Impact Calculator (EPIC) crop simulation model. GIS is used to capture the potentially irrigable soil types in the study area. It allows us to view, understand and visualize soil data. The EPIC model is able to utilize the soil data, plant parameters, and weather conditions to more accurately predict crop response yield to environmental factors in agriculture. This approach will offer the decision maker opportunities to have a crop management tool with economic considerations under the limitation of environmental conditions.

1. GIS Analysis

Irrigation is one of the major measures for increasing the production of agriculture. It can be seen that the development of irrigable land is one of the fundamental measures for increasing agricultural production, but not all soil types are suitable for irrigation. Finding the area of irrigable soil types will be the first step for making group of soil for their sustained use under irrigation.

GIS technology is a very useful tool to locate and determine the extension of irrigable soil in the study area. The study area consists of sections of land which are transversed by or are adjacent to sections that are transversed by the Elm and North Forks. The study area is made up of 339 640-acre sections. The approximate coordinates for latitude and longitude of Chloride Control Point are 35.0 N and 99.9 W respectively.

The original soil map of the study area contains various types of soils. Each soil type map has a land capability classification. To find irrigable soil types, we use the land capability classification obtained from SSURGO (Soil Survey Geographic database) soil data provided by the Natural Resource Conservation Service (NRCS). The land capability classification means the land categories according to the suitability of soil quality for the potential agricultural output. The National Soil Survey Handbook provides the definition of the land capability classification. Class codes I, II, III, IV, V, VI, VII, and VIII are used to represent land capability classes. Class codes I to VIII indicate progressively greater limitations and narrower choices for agriculture. Class I and Class II (2e and 2w) are chosen as irrigated land capability class for determining the most productive soils to irrigate. By definition, Class I soils have few limitations that restrict their use. Class II soils have moderate limitations that reduce the choice of plants or require moderate conservation practices. The land capability classification includes the capability subclass. The capability subclass is the second category in the land capability classification system. Class codes e, w, s, and c are used for land capability subclasses. Briefly, e, w, s and c are related with erosion problems, wetness problems, root zone limitations, and climatic limitations respectively. Subclass e and w are chosen for defining irrigable soil types. Land capability classification is made by adding the subclass e, w, s and c to class codes. I, IIe and IIw classes as the potential irrigated soil class are used in this study (National Soil Survey Handbook, USDA).

The irrigable soil areas that satisfy conditions of the land capability classification (I, IIe, and IIw) are found in Figure 5. Many types of irrigable soils still remain in the study area. The major irrigable soil types having the largest areas were selected to collect soil samples from actual fields. Potential major irrigable soil types found will be used as an individual soil input data for the EPIC simulation.

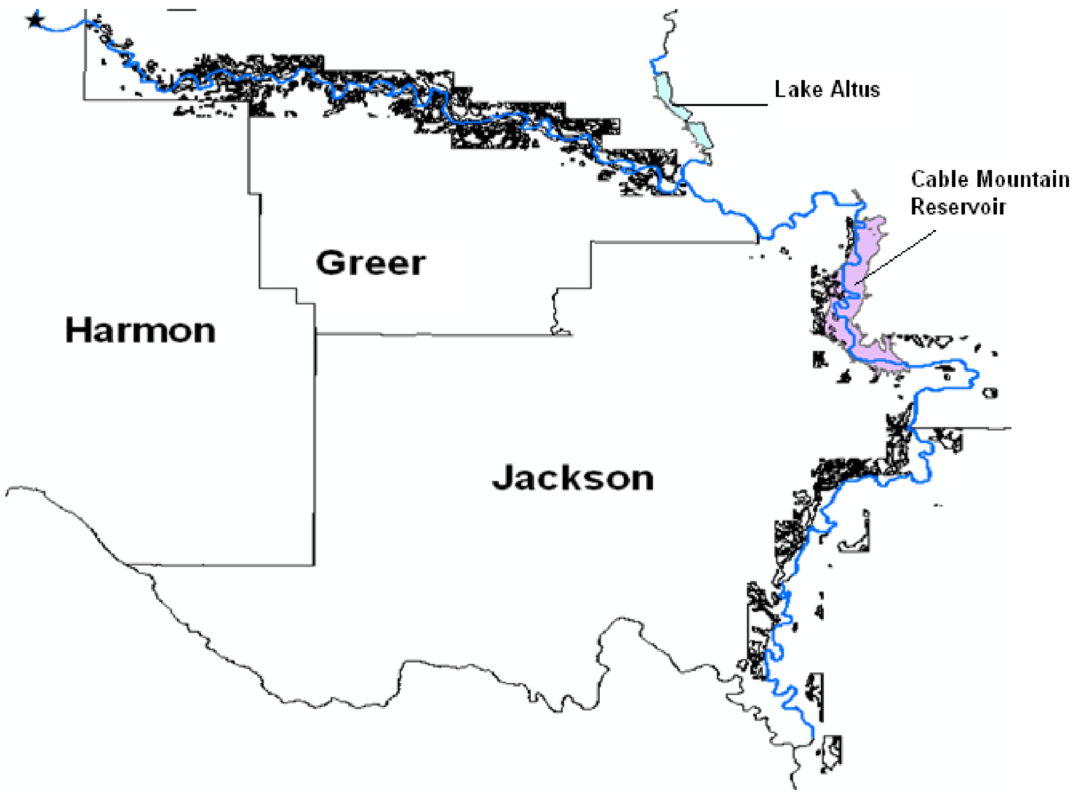


Figure 3. Irrigable Soil Area by Soil Type along the Elm and North Fork after Elimination of Soils with 10-meter Slopes Greater than Three Percent

2. EPIC Simulation

The Erosion Productivity Impact Calculator (EPIC) is a crop simulation model that can be used to assess the impact of weather, soil, water resources, and management strategies on agricultural production. It is useful as both a decision-making tool from the farm level to the national level and as a research tool. It can simulate alternative management strategies and develop, test and refine model components for simulating various physical and chemical processes (Williams *et al*, 1990).

The potential cotton yield in response to soil salinity, response to irrigation water, response to salinity in irrigation water will be simulated using the EPIC version 0509. EPIC simulations will be used to estimate cotton yields based on daily estimates of soil salinity, rainfall and temperature for next 50 years. Input data for the EPIC include weather, soil, crop management, and specific site information. It also includes parameter data files for major crops, fertilizers, and tillage practices (Cabelguenne *et al*, 1990).

Weather Data Generation

The EPIC simulation runs on a daily time step requiring the input of daily weather data. Minimum input requirements to set up weather data are daily precipitation and minimum and maximum temperature and latitude and longitude for the specific weather station.

Historical daily weather data can be directly used in the EPIC simulation when the length of historical daily weather is the same as the simulation period. It is also used to generate monthly weather statistics using the WXPM 3020 (Williams *et al*, 2006) weather simulator (ftp://ftp.brc.tamus.edu/pub/epic/wxparm/).

The EPIC program can simulate daily weather with the aid of a stochastic weather generator called the WXGEN (ftp://ftp.brc.tamus.edu/pub/epic/wxgen/) (Liu *et al*, 2009). The WXGEN can generate daily weather based on the monthly input statistics. A stochastic weather generator produces artificial daily time series of weather data based on the statistical characteristics of historical or observed weather at a specific location. Figure 4 represents the weather data generating process with the WXPM3020 program and stochastic weather generator the WXGEN.

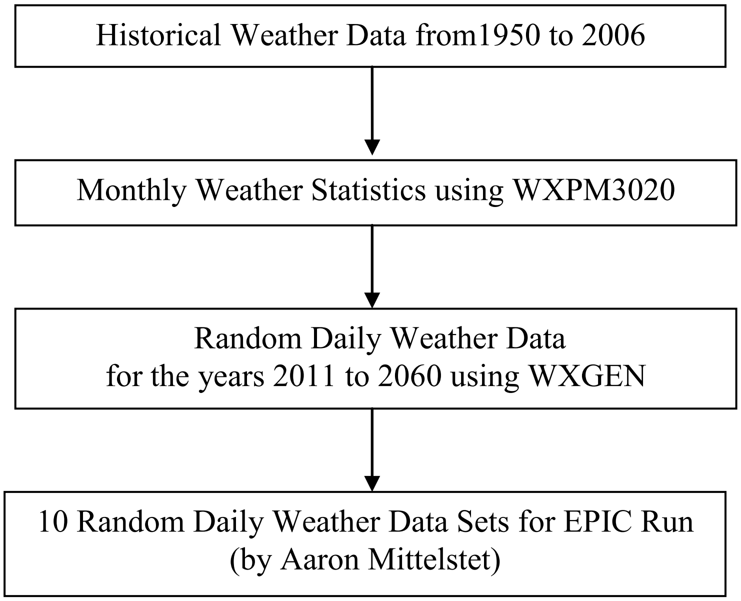


Figure 4. Weather Data Generating Process

The historical daily weather data for precipitation and minimum/max temperature from 1950 to 2006 at Jackson country obtained from National Climatic Data Center were used as the baseline weather data. The monthly weather statistics can be generated from the historical daily data by using the WXPM 3020 program. When the monthly weather statistics is available, the WXGEN is a useful tool in generating daily weather data (Liu *et al*, 2009). The WXGEN was used to randomly generate daily solar radiation, precipitation and minimum/max temperature for the years 2011 to 2060 based on the means, standard errors, and skew coefficients in the monthly weather statistics of the baseline weather data. 10 Random Daily Weather Data Sets for EPIC Run were generated by Aaron Mittelstet who is a research engineer of Biosystems and Agricultural Engineering in Oklahoma State University.

Table 1 shows the monthly statistics of the baseline weather data from years 1950 to 2006.

Table 1. Monthly Statistical Properties of the Daily Historical Weather Data at Altus Station, OK from years 1950 to 2006

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
TMX	12.04	15.55	20.13	25.92	30.40	34.62	36.21	34.13	29.77	23.93	16.21	12.21
TMN	-2.57	0.33	4.55	10.43	15.62	19.95	21.48	19.57	15.02	8.60	1.99	-1.83
SDMX	7.95	8.21	8.21	6.72	4.95	4.05	3.71	6.06	5.82	6.65	7.17	7.24
SDMN	5.25	5.45	5.41	5.26	4.36	3.11	2.22	3.61	5.25	5.74	5.51	4.90
PRCP	25.01	30.73	43.33	57.99	115.00	83.12	57.05	61.50	72.71	61.31	28.80	25.33
SDRF	9.43	10.93	11.42	13.16	17.64	19.25	13.15	17.70	18.04	19.43	9.02	9.16
SKRF	2.37	4.09	2.45	2.09	2.23	2.93	2.20	3.49	3.05	3.99	1.78	2.21
PW|D	0.08	0.10	0.12	0.13	0.20	0.14	0.12	0.13	0.12	0.11	0.08	0.08
PW|W	0.28	0.32	0.31	0.34	0.34	0.35	0.35	0.35	0.46	0.37	0.35	0.32
DAYP	3.05	3.70	4.54	4.95	7.32	5.18	4.82	5.18	5.35	4.61	3.35	3.21

Note: Variable definitions are as below.

TMX: Maximum daily air temperature (°C)
TMN: Minimum daily air temperature (°C)
SDMX: Monthly average standard deviation of daily maximum air temperature (°C)
SDMN: Monthly average standard deviation of daily minimum air temperature (°C)
PRCP: Precipitation (mm)
SDRF: Monthly standard deviation of daily precipitation (mm)
SKRF: Monthly skew coefficient for daily precipitation (mm)
PW|D: Monthly probability of wet day after dry day
PW|W: Monthly probability of wet day after wet day
DAYP: Number of days with precipitation

Soil Data

Soil is one of the important input components. Soil parameters should be prepared for the EPIC run. Soil data are composed of relevant physical and chemical parameters. Although up to ten soil layer parameters can be input into the EPIC, five or six soil layers were used to in this study set up soil input data. The following minimum parameter set was used on all soil types: soil albedo, soil hydrologic group, depth to bottom of layer, bulk density, percentage of sand, percentage of silt, soil pH, cation exchange capacity and electrical conductivity (EC). Table 2 shows the example of one of the irrigable soil types (Tipton Loam soil) used in the EPIC simulation as soil input data.

Table2. Tipton Loam Soil Input Data for EPIC Model

Tipton Loam Soil (Albedo =0.09, hydrologic group = B)

Soil layers	1	2	3	4	5	6
Depth(m)	0.15	0.3	0.6	0.9	1.2	1.5
Bulk Density(t/m^3)	1.43	1.43	1.5	1.5	1.5	1.5
Sand (%)	43.2	43.2	33.5	34.4	34.4	34.4
Silt (%)	38.8	38.8	36.5	37.6	37.6	37.6
Soil PH	6.7	7.5	7.8	7.9	8	8.1
Cation Exchange Capacity (cmol/Kg)	12.8	12.8	17	17	17	15.3
Electrical Conductivity (mmho/cm)	0.78	1.08	1.47	1.17	1.33	2

Values of soil pH and EC at different depths in the soil profile were obtained from the soil test conducted by the Oklahoma State University (OSU) Experiment Station

(Zhang *et al*, 2011). Other values are obtained from Soil Survey Geographic (SSURGO) Database in Natural Resource Conservation Service (NRCS).

The OSU Experiment Station collected samples of potentially irrigable soil affected by chloride loading at the control point along the Elm and North Fork of the Red river. The collected soil samples were located based on the result of GIS analysis. Figure 5 shows that 37 samples were collected along the Elm Fork across Greer County and 26 along the North Fork across Jackson, Kiowa and Tillman County. All 63 soil samples are classified into 15 soil types. Table 3 lists the soil samples along the Elm and North Fork rivers.

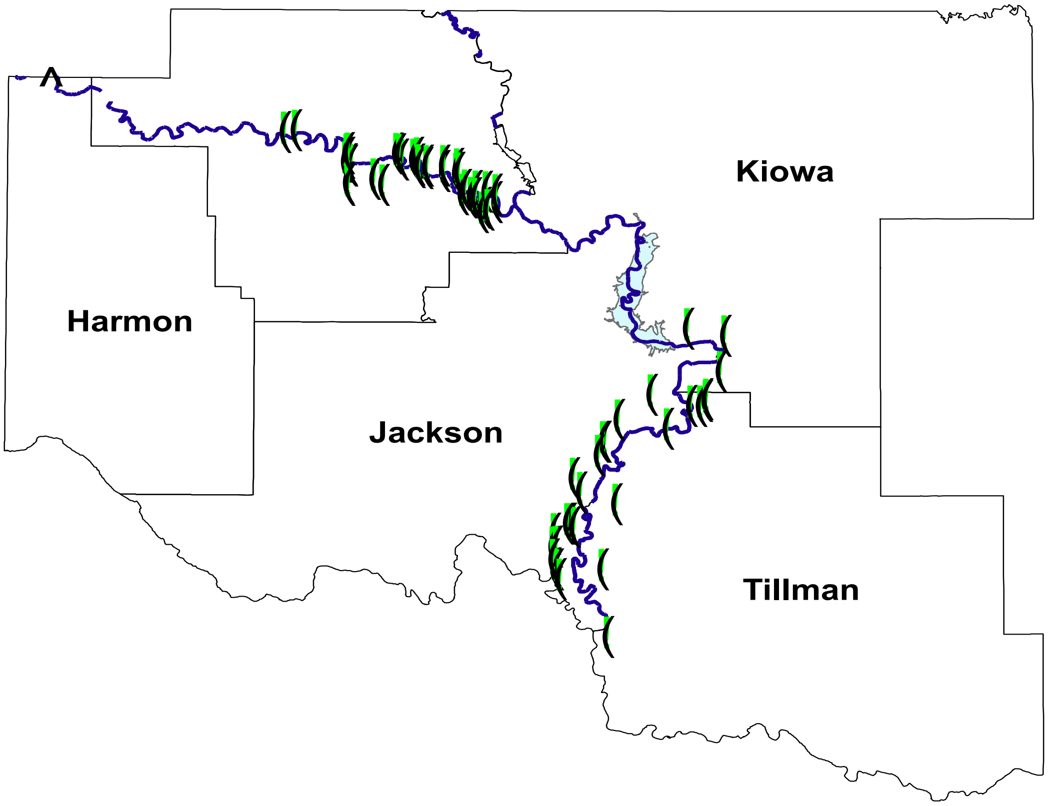


Figure 5. Soil Sample Points Collected in the Study Area (Source: Oklahoma State University Experiment Station, 2009)

Table 3. Tested and Collected Irrigable Soil Samples

Tested soil group	Collected soil group	County	Samples
Abilene loam	Abilene loam, 0-1% slopes	Tillman	1
Burford loam	Burford loam, 1-3% slopes	Tillman	1
Carey silt loam	Carey silt loam, 1-3% slopes	Kiowa	1
Frankirk loam	Frankirk loam, 1- 3 % slopes	Greer	2
Grandfield fine sandy loam	Grandfield fine sandy loam, 1-3% slopes	Jackson	1
Hardeman fine sandy loam	Hardeman fine sandy loam, 0-1% slopes	Jackson	1
	Hardeman fine sandy loam, 1-3% slopes	Jackson	2
Lawton loam	Lawton loam, 0-1% slopes	Greer	2
Madge loam and Madge fine sandy loam	Madge fine sandy loam, 1-3% slopes	Greer	3
	Madge loam, 1 -3% slopes	Greer	2
	Madge loam, 1-3% slopes	Jackson	1
Roark loam	Roark loam, 0 -1% slopes	Greer	6
	Roark loam, 0-1% slopes	Jackson	2
Spur clay loam	Spur clay loam, 0 -1% slopes, occasionally flooded	Greer	4
	Spur clay loam, 0-1% slopes, occasionally flooded	Jackson	1
	Spur clay loam, 0-1% slopes, rarely flooded	Greer	2
spur loam	Spur loam, 0 -1% slopes, occasionally flooded	Greer	6
Tillman clay loam	Tillman clay loam, 1-3% slopes	Kiowa	1
	Tillman clay loam, 1-3% slopes	Jackson	3
Tipton fine sandy loam	Tipton fine sandy loam, 0-1% slopes	Tillman	1
	Tipton fine sandy loam, 0-1% slopes	Jackson	1
Tipton loam	Tipton loam, 0 -1% slopes	Jackson	5
	Tipton loam, 0 -1% slopes	Tillman	3
	Tipton loam, 0-1% slopes	Greer	8
	Tipton loam, 1-3% slopes	Tillman	1
Westil clay loam	Westill clay loam, 1-3% slopes	Greer	2

Source: Oklahoma State University Experiment Station, 2009

Crop Management Data

The EPIC simulation program also requires data on the details of farm operations such as planting and harvesting timing, plant population, type and amounts of fertilizer and pesticides applied, potential heat units and others for the specific crop cultivating in the study area. Since the EPIC model simulates the potential cotton yield for next 50 years, actual information of crop operation schedule is not fully available. Most of the economic data were obtained from the cotton budget (Oklahoma State University Extension, 2011). We assume that the farmers in the study area follow this crop operation schedule.

Table 4. Summary of Crop Operation Data in EPIC Model

Month	**Cropping Practice**	
	Dryland	**Irrigation**
April	Bedder Tillage, Dry Fertilizer and Pesticide	Bedder Tillage, Dry Fertilizer and Pesticide
May	Planting and Row Cultivation	Planting and Row Cultivation
June	Pesticide	Pesticide and Irrigation
July		Irrigation
August	Pesticide	Pesticide and Irrigation
October	Harvest Ginning, Bagging and Ties	Pesticide, Harvest Ginning, Bagging and Ties
November	Kill and Shredder Tillage	Kill and Shredder Tillage
December	Field Cultivation	Field Cultivation

Source: OSU Enterprise Budget Software, 2011

Usual planting dates for cotton in Oklahoma are from May second until June eighteenth and harvesting dates are from October fourth through December twenty fourth (Usual Planting and Harvesting Dates for U.S. Field Crops, 2010). The duration of growing season used for the EPIC simulations was 160 days from May sixth to October twelfth. Dry fertilizer, Vydate LV, Pix, Roundup Max, Pix 8, Prep and Def 6 for pesticide were assumed to be applied in the study area during the crop growing season.

Model Evaluation

To validate the estimated crop response function, it is necessary that the EPIC simulation accurately predicts the observed yield. The evaluation is generally reported as a comparison of simulated and observed variables. It can be expected the simulated cotton yields will be overestimated because the EPIC model does not consider disease, insects and severe weather conditions such as hail. The parameters used to calibrate the EPIC model are shown in Table 5.

Table 5. Parameters related to Cotton Yield in EPIC Model

Crop Parameters	Symbol*	Parameters Used	Initial Parameters
Biomass-Energy Ratio	WA	20	20
Harvest Index	HI	0.5	0.55
Potential Heat Unit	PHU**	1760	1200 ~2400
Plant Population (plants/m^2)		8.5	7.41 ~ 12.35

Note : (*) Symbols of parameters are used in the EPIC model.
(**) Range of PHU for South and East Texas is from 1200 to 2400 which was defined by Ko *et al* (2009) and Wang *et al* (2005) respectively.

The parameters varied to calibrate the EPIC model were the Biomass Energy Ratio (WA), Harvest Index (HI), Potential Heat Unit (PHU) and Plant Population. The values used were based on literature and researcher's knowledge. According to the EPIC user guide for version 0509 (Williams *et al* 2006), the Biomass-Energy Ratio (WA) is the potential growth rate per unit of intercepted photosynthetically active radiation. Harvest Index (HI) is the percent of economic yield to the above ground biomass. The Potential Heat Unit (PHU) is the number of heat units expected for a typical growing season from plant to maturity. The optimal plant population for cotton has a wide range from 30,000 to 50,000 plants per acre, which can be converted into 7.41 to 12.35 plants per m^2 (Hake *et al*, 1996). These parameters were adjusted up or down until the simulated yield matched the 7-year observed yield of Jackson County.

The cotton yield at the county level was used to calibrate and validate the EPIC model. The EPIC model performance is evaluated by the paired t-tes for mean. It is used to investigate the relationship between two groups when each data point in one group corresponds to matching data point in the other group. It starts with comparing the means of each group of observations and simulations in this study. The observed variables for evaluation of the EPIC model are the dryland and irrigated cotton yields (lb/acre) of Jackson County obtained from National Agricultural Statistics Service (NASS) from years 2000 to 2006.

The Lugert-Altus Irrigation District in Jackson County covers approximately 48,000 acres and the annual irrigation delivery from Lake Altus for irrigation has varied from a low of 13,600 acre-feet in 1953 to a high of 106,542 acre-feet in 1998 (W.C.

Austin Project, 2005). The district supplies more than 85,000 ac-ft/acre of irrigation water to about 300 cotton farms in the area every year (Bimonthly Newsletter of OWRB, 2000). Salinity levels in the reservoir ranged from 1.8 to 2.4 EC levels (Oklahoma's Beneficial Use Monitoring Program-Lakes Sampling, 2009). For the model evaluation, it was assumed that approximately 1.64 ac-ft/acre per acre of irrigation water with an EC level of 2 from the Lugert-Altus Reservoir is applied for each simulation year.

The Holister Silty Clay Loam soil, which is a predominant soil type in the Lugert-Altus Irrigation District was used in Jackson County. Some of the more important properties of Holister Silty Clay Loam soil are shown Table 6.

Table 6. Holister Silty Clay Loam Soil Properties used in EPIC calibration for Jackson County Cotton Yield

Holister Silty Clay Loam (Albedo =0.16, hydrologic group = D)						
Soil layers	1	2	3	4	5	6
Depth(m)	0.15	0.3	0.6	0.9	1.2	1.5
Bulk Density(t/m^3)	1.4	1.48	1.48	1.48	1.48	1.48
Sand (%)	10	22.5	22.5	22.5	22.5	22.5
Silt (%)	56.5	32.5	32.5	32.5	32.5	36.5
Soil pH	7.74	7.65	7.66	7.82	7.9	7.88
Cation Exchange Capacity (cmol/Kg)	22.5	27.5	27.5	27.5	27.5	27.5
Electrical Conductivity (mmho/cm)	1.92	7.29	8.85	8.4	7.64	7.95

An Oklahoma State University (OSU) Experiment Station soil sampling study provided data on Soil pH and EC levels at different depths in the soil profile (Zhang *et al*, 2011). Other Data were obtained from the Soil Survey Geographic (SSURGO) Database in Natural Resource Conservation Service (NRCS).

Figure 6 shows the comparison between the observed and simulated cotton yield with irrigation levels of 1.64 ac-ft/acre with a salinity level EC of 2. The paired *t*-test for mean was used to test the null hypothesis of no difference between simulated and observed cotton yield. Figure 7 shows the difference two groups (Observed cotton yield - Simulated cotton yield) for irrigation.

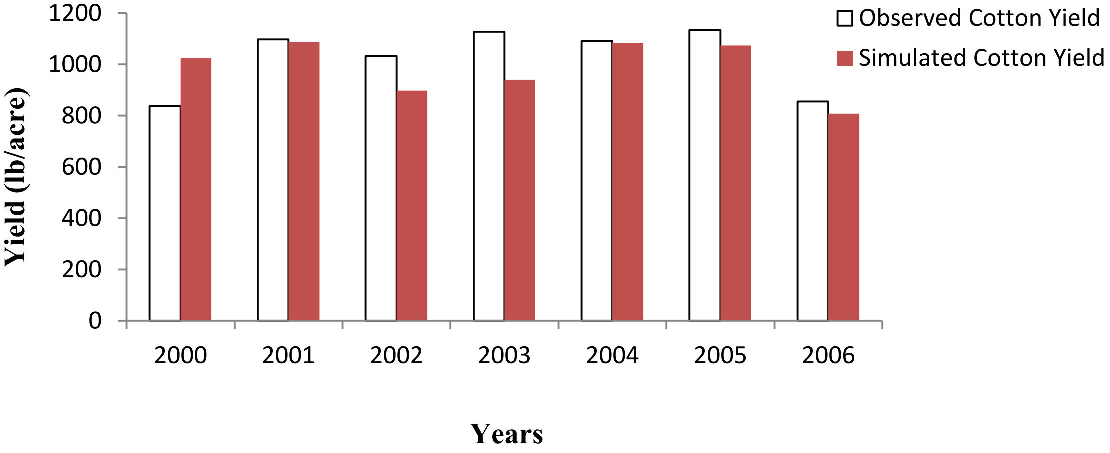


Figure 6. Comparison of the Simulated and Observed Irrigated Cotton Yields for Jackson County Obtained from National Agricultural Statistics Service (NASS)

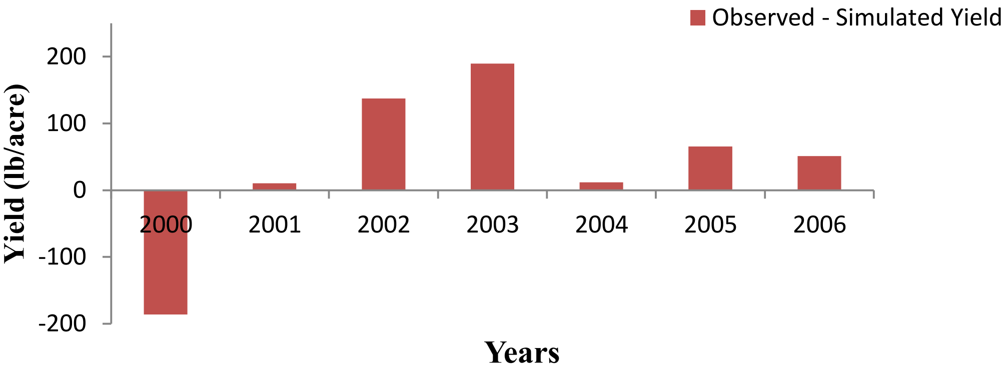


Figure 7. Difference between Observed and Simulated Irrigated Cotton Yield for Jackson County

If a statistical *t*-value is less than a critical *t*-value or *p*-value is larger than a significant level , we fail to reject the null hypothesis. Therefore, we conclude that there is no evidence of statistically significant difference between the two groups. Table 7 shows the results of the paired *t*- test using the SAS program.

Table 7. Results of Paired *t*-test for the mean of Observed and Simulated of Irrigated Cotton Yields in Jackson County

	Observed Yield (lb/acre)	**Simulated Yield (lb/acre)**
Mean of each group	1025	985
Observations	7	7
Mean Difference in Yields	40	
Standard Deviation of Difference	119	
Statistical *t*- value*	0.89	
p-value	0.41	
Critical *t*-value	2.45	

Note: (*) indicates statistical *t*-value is defined as —— where is a mean difference of yields of two group, is a standard deviation of difference and *n* is observations.

Since the statistical *t*-value (=) is less than the critical *t*-value (=2.45) and *p*-value (=0.41) is larger than the significant level , we fail to reject the null hypothesis. We conclude that there is no statistical difference at the 95% confidence level between the observed cotton yield group and simulated cotton yield group.

Table 8 shows the summary statistics of relative error, Nash-Sutcliffe efficiency and coefficient of determination (R^2) for observed and simulated irrigation cotton yield after calibration.

Table 8. Results of Yearly EPIC Model Calibration for Irrigation Cotton Yield in Jackson County for the Period from 2000 to 2006

Mean of Observed Yield (lb/acre)	**Mean of Simulated Yield (lb/acre)**	**Relative Error***	**Nash-Sutcliffe Efficiency****	**Coefficient of Determination (R^2)*****
1025	985	4%	-0.01	0.68

Note: (*) Relative Error is defined as R.E = ——————.

(**) Nash-Sutcliffe Efficiency is defined as E = ——— with *O* observed and *S* simulated Yield.

(***) Coefficient of Determination (R^2) is obtained from outputs of linear regression.

Relative Error is generally represented as percentage of the absolute error of simulated value minus observed value divided by observed value to assess the error between two models. Nash-Sutcliffe efficiency and coefficient of determination (R^2) are used to assess how well EPIC simulated cotton yield fits the observed cotton yield. Nash-Sutcliffe efficiency is defined as one minus of the absolute squared difference between the observed and simulated values divided by the variance of the observed values for 7- target year. The range of Nash-Sutcliffe efficiency is between - ∞ to 1. The efficiency of 1 means the simulated data perfectly fits the observed data. The efficiency of 0 indicates that the simulated model predictions are as accurate as the mean of the observed data, whereas the efficiency of lower than zero indicates that the mean of the observed data is a better predictor than the simulated model. Coefficient of determination (R^2) can be obtained from outcomes of linear regression of two data. R^2 is generally used as a measure of goodness-of-fit of linear regression which the range of R^2 is between 0 and 1. The R^2 values of 1 means observed data perfectly fits simulated data whereas the R2 values of 0 means observed data does not fit any simulated data. Generally, if

Relative Error is within 5%, Nash-Sutcliffe efficiency is larger than 0.4 and Coefficient of determination (R^2) is larger than 0.6, the simulated model well performed with prediction of observed cotton yield (Wang et al, 2006).

Relative Error between the observed and simulated mean of irrigation cotton yield is less than 5%. Nash-Sutcliffe Efficiency is close to zero indicating the simulated model predictions are as accurate as the mean of the observed data. Coefficient of Determination (R^2) is 0.68 indicating simulated model well explains the variation of observed data.

To ensure the reliability of the parameters used in the calibration for the irrigation system, the parameters were also applied to dry land (non-irrigation). Figures 8 and 9 show the comparison of the observed and the simulated dryland cotton yields between the years 2000 and 2006.

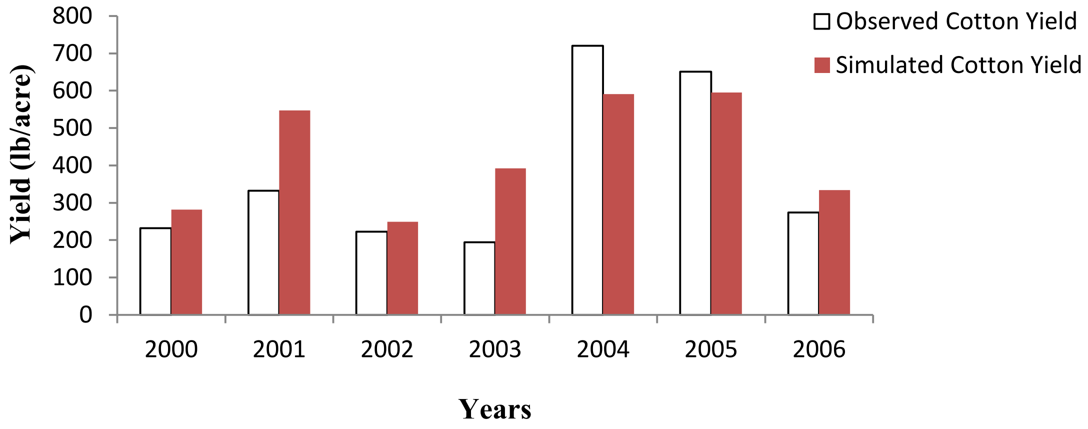


Figure 8. Comparison of the Simulated and Observed Dryland Cotton Yields for Jackson County Obtained from National Agricultural Statistics Service (NASS)

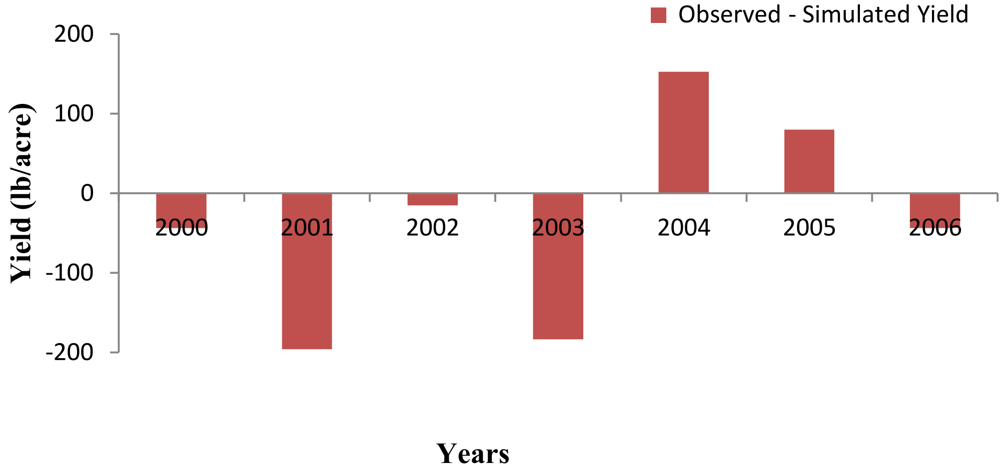


Figure 9. Difference between Observed and Simulated Dryland Cotton Yield for Jackson County

Results of paired t-test for the mean of observed and simulated dryland yield using SAS program are shown in Table 10.

Table 9. Results of Paired t-test for the mean of Observed and Simulated of Dryland Cotton Yields in Jackson County

	Observed Yield (lb/acre)	**Simulated Yield (lb/acre)**
Mean of each group	375	426
Observations	7	7
Mean Difference in Yields	-51	
Standard Deviation of Difference	125	
Statistical *t*- value	-1.07	
p-value	0.32	
Critical *t*-value	2.45	

Note: (*) indicates statistical *t*-value is defined as where is a mean difference of yields of two group, is a standard deviation of difference and *n* is observations.

Since the statistical *t*-value (=) is less than the critical *t*-value of 2.45 and the *p*-value of 0.32 is larger than significant level , we fail to reject the null hypothesis. We conclude that there is no statistical difference at the 95% confidence level between the observed cotton yield group and simulated cotton yield group.

Table 11 shows the summary statistics of relative error, Nash-Sutcliffe efficiency and coefficient of determination (R^2) for observed and simulated dryland cotton yield after calibration.

Table 10. Results of Yearly EPIC Model Validation for Dryland Cotton Yield in Jackson County for the Period from 2000 to 2006

Observed Yield (lb/acre)	**Simulated Yield (lb/acre)**	**Relative Error**	**Nash-Sutcliffe Efficiency**	**Coefficient of Determination (R^2)**
375	426	14%	0.61	0.69

Note: (*) Relative Error is defined as R.E = .

(**) Nash-Sutcliffe Efficiency is defined as E = with *O* observed and *S* simulated Yield.

(***) Coefficient of Determination (R^2) is obtained from outputs of linear regression.

Relative Error between the observed and simulated mean of irrigation cotton yield is larger than 5% indicating that there are some deviation between mean of observed and simulated dryland cotton yield. However, Nash-Sutcliffe Efficiency is 0.61 indicating the simulated data well fits the observed data. Coefficient of Determination (R^2) is 0.69 indicating simulated model well explains the variation of observed data.

The results of calibration and validation process indicate the EPIC simulated yields matched observed yields for the 7-target year. The calibrated and validated parameters related to cotton yield were used to simulate the potential cotton yield and soil salinity with different levels of irrigation water and water salinity for the next 50 years.

Simulation Design and process

After setting up the input data, the EPIC program was used to simulate the cotton yield, soil salinity and other variables for next 50 years. A simulation design is much like that of an agronomic field experiment. The designed simulation is applied with combinations of three different levels of plant water stress, three different levels of salt concentration of irrigation water and 10 stochastic weather scenarios over a 50-year period.

EPIC offers two options for irrigation. Sprinkler or furrow irrigation can be simulated by fixed or automatic option. The fixed option requires that application dates and amounts be specified in advance by the EPIC users. With the automatic option, the model decides when and how much water to apply. The user must input the plant water stress level to trigger automatic irrigation, the maximum volume applied per growing season, and the minimum time interval between applications (Williams, 1990). The automatic irrigation option was selected for use for this study to represent a more realistic irrigation practice. Plant water stress factors to trigger automatic irrigation were set at 0.1, 0.5 and 0.9. This factor ranges from zero (high stress) to one (no stress) and is computed as the ratio of actual plant water use to potential water use (Easterling *et al*, 1992). When plant water stress factor was set at 0.1, a total irrigation application is applied under 200 mm and the range of an amount of water for single application was limited to 50 mm. Similarly, when plant water stress factor was set at 0.5 and 0.9, a total irrigation application is applied under 800 mm and the range of an amount of water for single application was limited to 200 mm. The minimum interval between irrigations was set at 20 days during the growing season.

The three levels of salt concentration of irrigation water represent 0, 1,280, and 2,560 p.p.m (parts per millions). Salt concentration in p.p.m can be generally expressed in terms of Electrical Conductivity (EC). It is assumed that 1 EC (mmhos/cm) in irrigation water is equal to 640 p.p.m. These units of measurement can be converted to tons of salt per acre foot as follows (Agriculture Handbook No. 60, USDA):

$$640 \text{ p.p.m} = 1 \text{ EC mmhos/cm}$$

$$1 \text{ p.p.m} \times 0.00136 = \text{Tons per Acre-Foot}$$

For example, 0, 1,280 and 2,560 p.p.m of salt concentration can be converted to 0, 2 and 4 of EC and 0, 1.74 and 3.48 tons/ac-ft. In addition, 1,280 p.p.m of the salt concentration are equal to 2 mmhos/cm of EC or 1.74 tons of salt for every foot of irrigation water applied. If during the growing season, 400mm (1.3 ac-ft) of irrigation water is applied, the amount of salt in irrigation water is approximately 2.263 tons/acre (1,280 ppm × 0.00136 × 1.3). From this example, we can expect that the amount of salts in irrigation water can quickly increase the salinity level in the soil.

Based on the simulation design, a total of 90 simulations (3×3×10) were conducted for each soil type in the study area. The variables we need to estimate cotton yield and soil salinity response functions were taken from EPIC output.

Figure 10 illustrates how cotton yield and soil salinity are affected by environmental factors. During and before the growing season, cotton yield is affected by irrigation water applied, rainfall, soil salinity at planting, salinity in irrigation water. Total water used in the field is equal to irrigation water applied plus rainfall. Total salinity is equal to soil salinity at planting plus the amount of salt in irrigation water. From irrigation water, salts accumulate in the root zone. Soil salinity at harvest assumes to be affected by irrigation water, rainfall, soil salinity at planting and the amount of salt in irrigation water. Soil salinity at planting is assumed to be affected by non-growing season rainfall and soil salinity at harvest on the previous year.

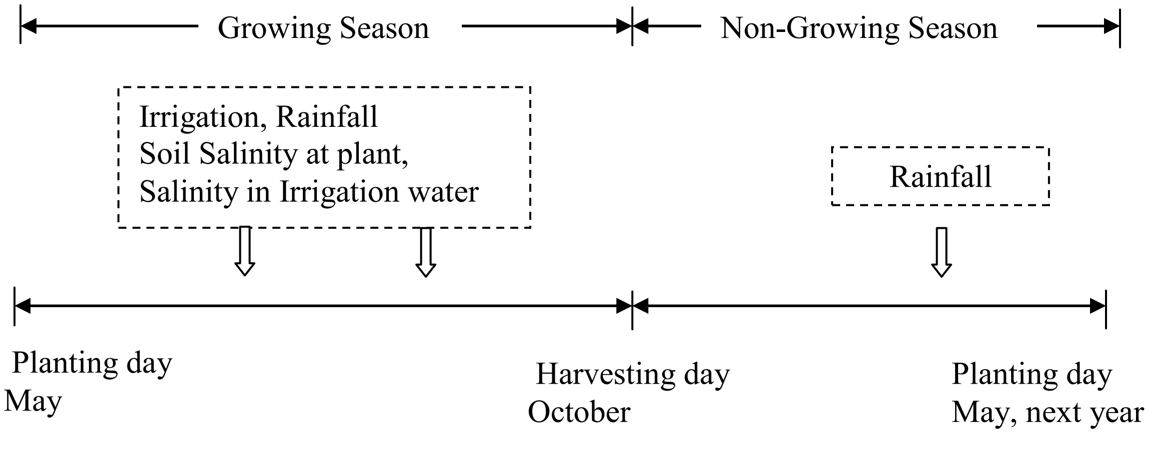


Figure 10. Environmental Factors Affecting Yield

The simulated cotton yields, irrigation water and growing season rainfall can be selected from the annual crop yield output file (*.ACY). In case of soil salinity levels at planting and harvest, they can be found in the Daily Soil Table output file (*.DSL) which is generated on a daily basis for each soil layer. Non-growing season rainfall was calculated by subtracting growing season rainfall from the sum of the monthly precipitation in Monthly Flipsim output file (*.MFS). The variables, descriptions and their unit conversions are shown in Table 9. Data selected from the EPIC output file are used to estimate cotton and dynamic soil salinity response function.

Table 11. EPIC Output File Variable Definition and Unit Conversion

EPIC Output File	Variable	Description	Unit Conversion
*.ACY	YLDG	Yield (Ton/Ha)	1 metric ton/Ha = 892 lbs/acre
*.ACY	IRGA	Irrigation Volume Applied (mm)	100mm = 0.328 feet
*.ACY	CRF	Growing season Rainfall (mm)	100mm = 0.328 feet
*.DSL	WLST	Salt Content in Soil (Kg/Ha)	1 kg/ha = 0.446×10^{-3} tons/acre
*.MFS	PRCP	Precipitation (mm)	100mm = 0.328 feet

Dynamic Optimization

The outputs of the EPIC simulations were used to estimate the cotton yield and soil salinity response functions. The estimated response functions for each soil type can be incorporated into an economic decision model to determine the optimal level of irrigation for any given level of salt concentration of irrigation water maximizing the net present value (NPV) of expected utility. Since crop yield and risk are generally influenced by fluctuations in weather conditions, uncertainty or risk exists in the agricultural production. The NPV of expected utility of profit instead of the NPV of profit is expressed as:

The von Neumann-Morgenstern utility function is used to maximize the expected value of profit. Mean-Variance (E-V) is incorporated to express expected utility (Hazell and Norton, 1986). Expected utility is represented as follows:

–

where is the von Neumann-Morgenstern utility function with and , is the absolute Arrow-Pratt risk aversion coefficient, defined as –

.

Expected utility can be transformed with respected to E-V of crop yield taking risk aversion as follows:

$$-$$

$$= \qquad -$$

where is the expected yield, is the variance of yield derived from the equation (Coyle, 1999). The level of risk of a producer is directly related to variances of crop yield. The variance of the crop yield is evaluated as the effect of risk factors in the agricultural production.

The final dynamic programming model maximizing the expected utility of profit for individual soil types in the study area is constructed as:

$$\overline{\qquad\qquad} \qquad -$$

subject to

where *P* is the price of cotton ($/lb), *E(Y)* is the expected cotton yield response function (lbs/acre) to quantity of total water applied and total salinity in soil, *TW* is the total quantity of water which is the sum of irrigation water and rainfall during the growing season. *Irr* is the quantity of irrigation water applied (ac-ft/acre), is the quantity of rainfall in feet, *TS* is the total quantity of salinity in soil which is the sum of total dissolved salt in irrigation water and soil salinity at planting, *WS* is the amount of salt in irrigation water (tons/ac-ft) which is the salt concentration (p.p.m) multiplied by the quantity of irrigation water, SS^{HA} and SS^{PL} is the quantity of soil salinity at harvest and planting (tons/acre) during the growing season respectively, $Rain^{G}$ is the growing season rainfall (ac-ft), is the quantity of soil salinity at harvest of the previous year, $Rain^{NG}$ is rainfall received during the non-growing season (ac-ft), is the irrigation cost ($/acre-feet), is the operation cost and is the fixed cost, *r* is discount rate.

The simulation design was conducted as a full factorial with three levels of irrigation water stress and three levels of irrigation water salinity, and 10 random weather data sets of 50-year. A modified quadratic yield response function of cotton for the individual soil type in the study area was specified as follows:

for weather scenarios
for water stress factor 0.1, 0.5 and 0.9 respectively
for salt concentration 0, 1280 and 2560 ppm respectively
for simulation years

where are the parameters to be estimated, is the simulated cotton yield for a soil with the level of a water stress factor and the level of salt concentration of irrigation water in year *t* under the weather scenario. is the total water from irrigation water applied (and the growing season rainfall (, is the non-growing season rainfall. is the total salinity which is the sum of the amount of salt in irrigation water ( ) and soil salinity at planting ( ). The interaction term, is the total salinity divided by total water, is a random effect of weather, and are assumed to be the independent and normal distributed error terms, and), respectively.

In crop yield response function, the specification of the interaction term does not follow the standard practice of being a product of the two linear variables. This term was formulated as a ratio because more water serves to increase the yield while more salt tends to decrease the yield. When specified as a ratio (total salt/total water), the two variables work in the same direction.

The soil salinity response functions at planting and harvest were also estimated for the individual soil type. The soil salinity function at harvest is assumed to be affected by irrigation water applied, dissolved salt in irrigation water and growing season rainfall. It can be constructed as follows:

where are the parameters to be estimated, is the soil salinity at harvest which is simulated from a set of combinations of the soil condition having the water stress factor and the level of salt concentration in year with a weather scenario .

is the quantity of irrigation water applied, is the amount of salt in irrigation water, is the soil salinity at planting, is the growing season rainfall in weather scenario and year , is a random effect of weather, and are assumed to be the independent and normal distributed error terms, and), respectively.

To estimate the dynamic soil salinity function at planting, we assumed that the amount of soil salinity at planting in the current year will be determined by soil salinity level at harvest in the previous year and non- growing season rainfall. The dynamic soil salinity function at planting is defined as:

where and are the parameters to be estimated, is the soil salinity at planting given the water stress factor, the level of salt concentration in year *t* with a weather scenario , is the soil salinity at harvest in the previous year,

is non-growing season rainfall in weather scenario and year *t*, is a random effect of weather, and are assumed to be the independent and normally distributed error terms, and (0, , respectively.

The yield variance function is expressed as the squared residuals of the estimated yield response function. It is expressed as the linear function of the irrigation and growing season rainfall which mainly affect crop yield and yield variability (risk), i.e., where , and are the parameters to be estimated, is a random effect of weather, and are assumed to be the independent and identical error terms, and (0, , respectively.

The coefficients of and represent the influence of irrigation water and growing season rainfall on yield variability (risk). The input variable is risk-reducing if and risk-increasing if , respectively (Finger and Schmid, 2007).

CHAPTER IV

RESULTS OF SIMULATION, REGRESSION AND OPTIMIZATION BY SOIL TYPE

SAS PROC MIXED is a powerful procedure for a wide variety of statistical analyses with both fixed and random effect in research situations. In this study, the fixed and random effects model was applied to EPIC data. Since we selected 10 random weather scenarios, weather is considered as the random effect in the model.

Since data selected from EPIC simulations with different inputs are in the form of panel data, autocorrelation and heteroskedasticity may occur in the model. Models to describe the variance as a function of independent variables in a regression model can be fitted to data where the variance increases or decreases as the values of the independent variables change. One of the great advantages of the likelihood-based estimation approach to mixed models is the ability to fit a variety of covariance structures (Littell *et al*, 2006). To fit a model with autocorrelation and heterogeneous variances, the model can be specified in PROC MIXED by using the REPEATED statement with the AR(1) for autocorrelation and GROUP = option for heterogeneous variances. The REPEATED statement specifies the covariance structures of the error term. The AR(1) models may adequately describe the autocorrelation and assumes a homogeneous variance and error correlations that decline exponentially with distance. Group = option defines an effect specifying heteroscedasticity in the covariance structure. Each new level of the GROUP effect produces a new set of covariance parameters with the same structures as the original group (SAS Institute Inc, 2008). In this study, GROUP = option specifies a different residual variance for each weather scenario.

The fitted models should be compared with model with an assumption without autocorrelation and heteroscedasticity to draw accurate conclusions from data. The Likelihood Ratio Test (LRT) is used to determine the better fitted model. PROC MIXED model is based on Maximum Likelihood Estimation (MLE) which maximizes the likelihood function with/without imposing any restrictions. The LR test requires estimating two models and comparing them. The LR test statistic is calculated in the following way (Johnston & DiNardo, 1997):

$$\mathrm{LR} = \qquad \frac{\quad}{\quad}$$

where *L* and *l* are the likelihood and log likelihood of the respective model.

Since the PROC MIXED model directly provides the -2 log-likelihood statistic, we can compare with the difference in the -2 log-likelihood of the restricted and unrestricted model for the LR test. The LR statistics follows a chi-square distribution with degrees of freedom equal to the difference in the number of degrees of freedom between the two models. By using the 1- PROBCHI function in SAS, which returns the value of the function of the chi-square distribution, SAS will compute the test statistic and its *p*-value from the -2 log-likelihood values (SAS Institute Inc, 2008). If the *p*-value is less than the critical value, we reject the null hypothesis of no difference between two models.

To determine if the estimated crop response function is concave with respect to variables we used in the regression, the second derivative test is examined by algebraically or numerically checking the signs of the second-order conditions of the variables (Beattie & Taylor, 1985). From the yield response function, the first order and second order conditions are derived. Given a modified quadratic functional form, $y = f(TW, TS, Rain^{NG})$ is represented as the equation below:

Given the functional form $y = f(TW, TS, Rain^{NG})$, this function can be extended with respect to the specified individual variables. The extended functional form $y = f(x_1, x_2, x_3, x_4, x_5)$ is represented as the equation below:

where x_1 is irrigation water applied (acre-feet), x_2 is the growing season rainfall (feet), x_3 is the salt concentration of irrigation water (tons/ac-ft), therefore, $x_1 \cdot x_3$ is the amount of salt in irrigation water (tons/acre) which is the product of salt concentration and irrigation water, x_4 is the salinity in the soil (tons/acre), x_5 is the non-growing season rainfall (feet).

The first-order conditions (F.O.C) with respect to the individual variable are

The second-order conditions (S.O.C) for variables except for are

The determinants derived from S.O.C is used to examine that the crop response function has a maximum yield with respect to irrigation water and salt concentration in irrigation water(, irrigation water (and soil salinity (, and salt concentration in irrigation water ( ) and soil salinity (, respectively. The Hessian matrix of second derivatives at the critical point is represented as the follows:

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -210.9, 200, -11.5 and 2.9. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the Hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

,

,

, and

All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. If the determinants of the respective orders have the indicated signs, , , , and , the matrix is negative definite. Therefore we conclude that the modified crop response function is concave and has a local maximum.

The dynamic optimization procedure for the economic decision model was performed by GAMS IDE. To solve the dynamic optimization problem, we need to know the cotton price, irrigation cost, operating cost, and fixed cost. The irrigated and dryland cotton budgets were revised from the OSU cotton budget for the surface-furrow irrigation system provided by Oklahoma State University Extension. We assumed that the farms in the study area follow this cotton budget. From APPENDIX- A for the irrigated cotton, the cost of irrigation water is 28.89$/ac-ft and total cost is 677.37 $/acre. From APPENDIX-B for the dryland cotton, total cost is 312.29$/ac-ft. We also suppose that 1) cotton price is fixed at 0.6 $/lbs for irrigation and dryland over 50 years planning horizon 2) discount rate is 4.125 percent for Federal water resources planning for fiscal year 2011 (http://www.economics.nrcs.usda.gov/cost/priceindexes/rates.html), 3) the available irrigation water is 2.62 ac-ft (800mm) or less, 4) one of risk neutrality and two levels of risk aversion coefficient are used in this analysis: 0, 0.025 and 0.05 which are used in the literature for irrigated producers (Johnson and Blackshear, 2004 and Wojciechowski *et al*, 2000), 5) dryland producers are risk-neutral since they are indifferent to the risk such as a big rainfall and drought and are concerned about expected profit, 6) the growing season and non-growing season rainfall is randomly generated based on the gamma distribution over the 50 years planning horizon.

Rainfall in the EPIC simulation is determined by generating from a skewed normal daily precipitation (Williams *et al*, 1992). The generated yearly rainfall and precipitation have a skewed distribution. The data that are skewed to the right are adequately modeled by a gamma density function (Wackerly *et al*, 2002). PROC UNIVARIATE with the HISTOGRAM statement is used to determine if the gamma distribution fits a data distribution used in the EPIC simulation. SAS output provides three goodness-of-fit tests which are the Kolmogrov-Smirnov, Cramer-von Mises and Anderson-Darling test (SAS Institute Inc., 2010). The *p*-values of all tests for growing and non-growing season rainfall are larger than 0.25. Since *p*-values are larger than significant value . We conclude that we fail to reject the null hypothesis of the gamma distribution and the fitted gamma distribution provides an appropriate model for distribution of generated growing and non-growing season rainfall. Figure 12 and 13 represent the fitted gamma distribution curve on the histogram and displays the mean, standard deviation, skewness and kurtosis of growing and non-growing season rainfall used in EPIC and dynamic optimization model, respectively.

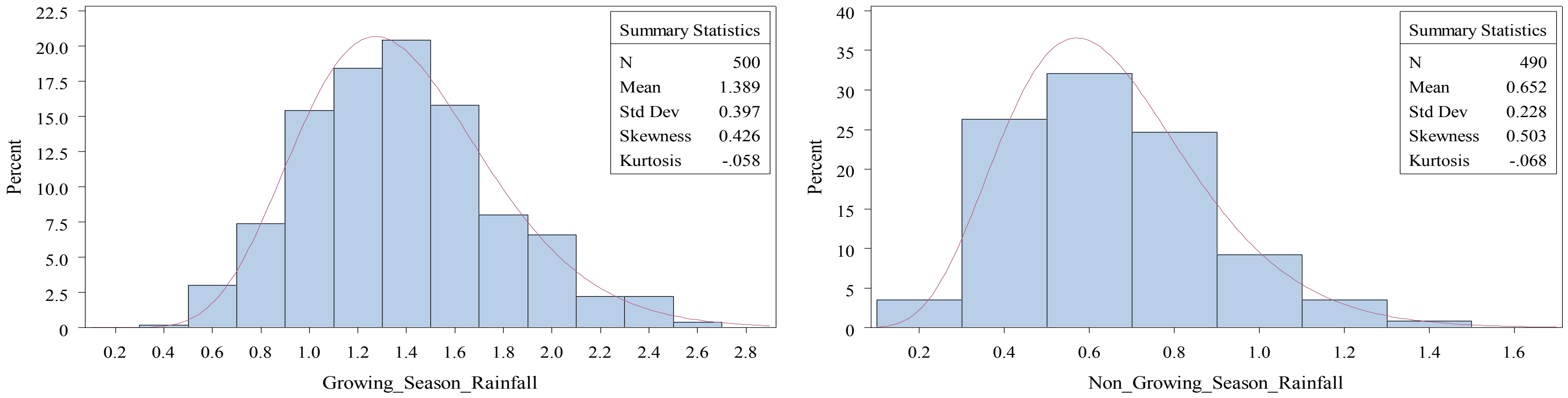


Figure 11. Statistics and Histogram with the Fitted Gamma Distribution for Growing and Non-Growing Season Rainfall used in EPIC

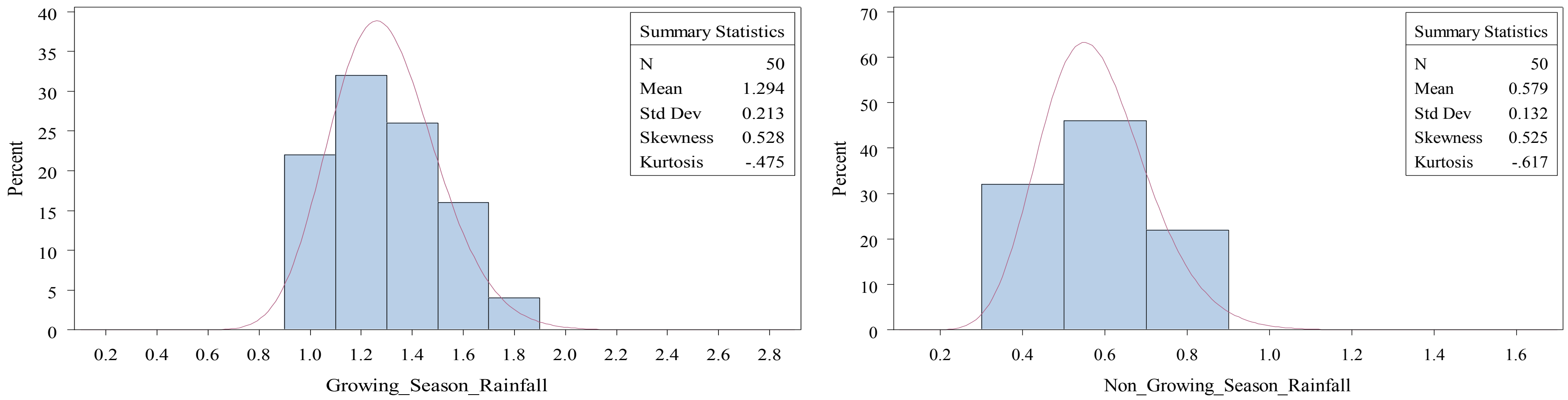


Figure 12. Statistics and Histogram with the Fitted Gamma Distribution for Growing and Non-Growing Season Rainfall randomly Generated based on Gamma distribution in Figure 12 for 50 years Planning Horizon of Dynamic Optimization Model

For the dynamic programming, the growing season rainfall and non-growing season rainfall are randomly generated for the 50 years planning horizon based on the gamma distribution. The *p*-values of Kolmogrov-Smirnov, Cramer-von Mises and Anderson-Darling test for rainfall are 0.068, 0.181 and 0.204, respectively. The *p*-values of their tests for non-growing season rainfall are 0.25, 0.191 and 0.18, respectively. Since their *p*-values are larger than significant value . Therefore, we conclude that we fail to reject the null hypothesis of the gamma distribution and the data are appropriately generated based on the gamma distribution in Figure 12. The generated random growing season rainfall and non-growing season rainfall are combined with the dynamic optimization model maximizing the net present value of expected utility for all soil types. Figure 13 shows the growing season and non-growing season rainfall based on the gamma distribution are distributed over 50 years.

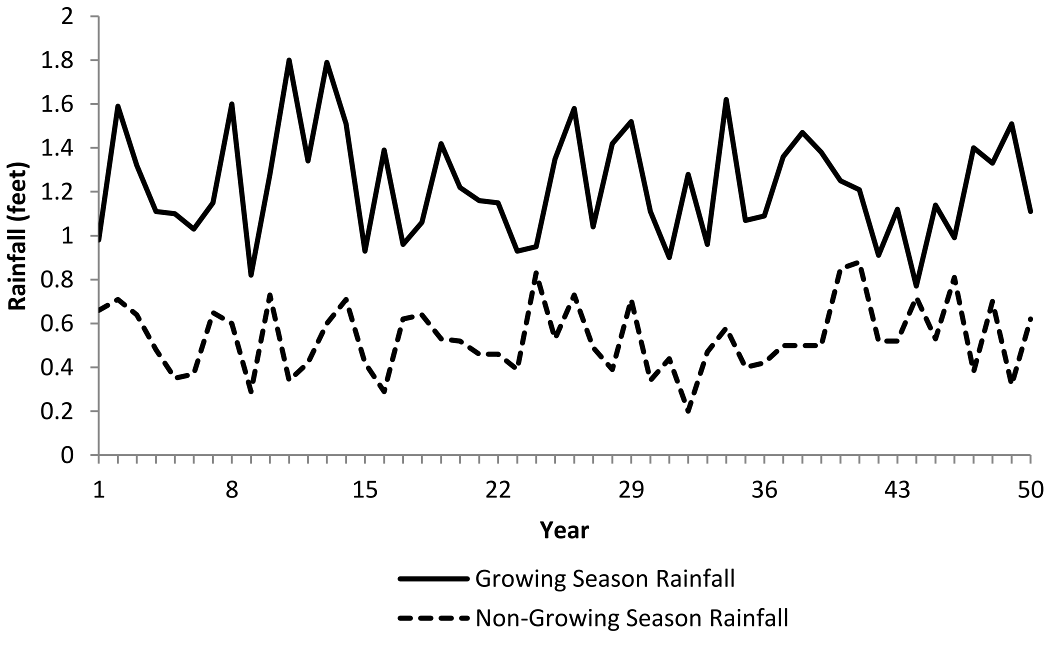


Figure 13. Distribution of Growing Season and Non- Growing Season Rainfall Generated based on Gamma Distribution over 50 years

Tipton Loam Soil, 0-1% Slope

EPIC Output Data

The quantity of salt in the soil at each depth in the EPIC *DSL output file is calculated by EPIC based on the initial EC values (mmho/cm) in the soil input file. Table 1-1 presents the calculated quantity of soil salinity based on the sampled data for the Tipton Loam soil at the start of the simulation. In EPIC, WSLT (Kg/ha) is automatically simulated at each depth on a daily basis for 50 years and the total value of them is also automatically calculated. It can be converted to tons per acre.

Table 1-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Tipton Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.07	0.78	1.08	1.47	1.17	1.33	2	
WSLT(kg/ha)*	9	103	153	587	444	504	756	2,555
Salinity(tons/acre)								1.14**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

The total salt in the 1.5 meter profile was calculated as 1.14 tons/acre on the first day of simulation. This will be used as the initial soil salinity in the dynamic programming. The level of soil salinity at the day of planting and harvest in each year can be selected from *DSL file. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall are also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 1-2.

Table 1-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Tipton Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	33 ~1,857	1,071
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.28
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 24.65	8.61
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 27.34	9.39
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

Ten sets of 50-year cotton yield and irrigation applications were simulated by EPIC given three levels of salt concentration of irrigation water and three levels of water stress to trigger irrigation from 50 mm to 800 mm. When we use irrigation water containing a high salt concentration on the crop land, the salts accumulate in the root zone. Saline soils have a very limited agricultural production. The range of data for the simulated yield and soil salinity at harvest with given levels of the salt concentration are shown on a box plot in Figure 1-1.

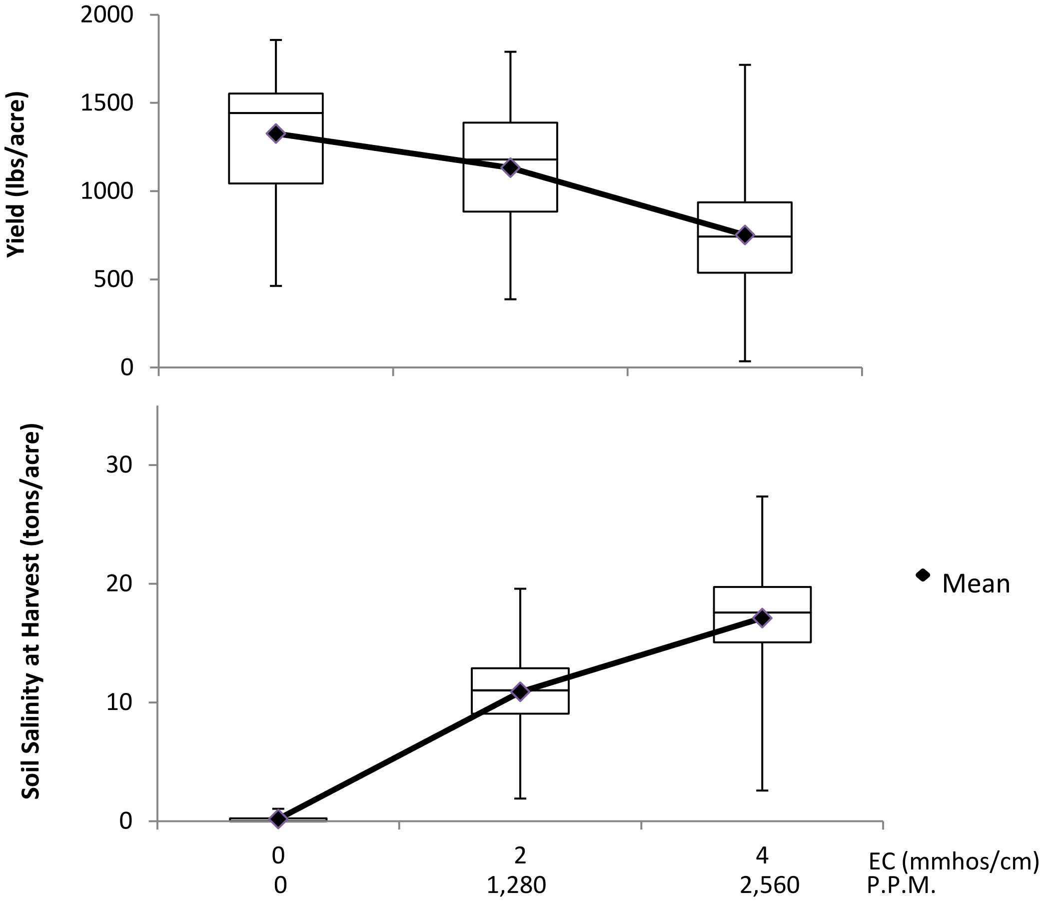


Figure 1-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying irrigation water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Tipton Loam Soil

A box plot visually provides a summary of simulated data. The box extends from the first quartile which is defined as the 25th percentile of the data to the third quartile which is defined as the 75th percentile of the data. The bottom and top are the minimum and maximum value of the data, respectively. The median is shown as a line across the box. The diamond sign is the average values of the simulated data at given levels of the salt concentration. As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. In addition, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. It is expected that the accumulated salts affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters of the modified quadratic yield function with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 1-3.

Table 1-3. Result of Likelihood Ratio Test for the Tipton Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56754	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56628		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 1-4.

Table 1-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Tipton Loam Soil

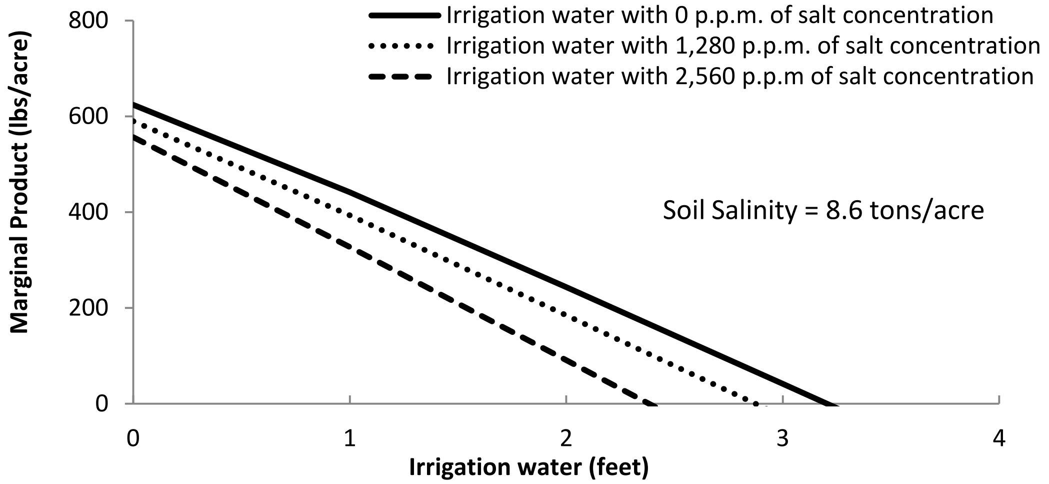
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-524.38*	42.1649
Total Water Applied		940.09*	30.0577
Total Salinity		1.6022	1.3225
Non-Growing Season Rainfall		112.39*	9.7781
(Total Water Applied)^2		-101.98*	5.3211
(Total Salinity)^2		-1.4344*	0.0393
(Total Salinity / Total Water Applied)		7.3683*	2.5073

Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

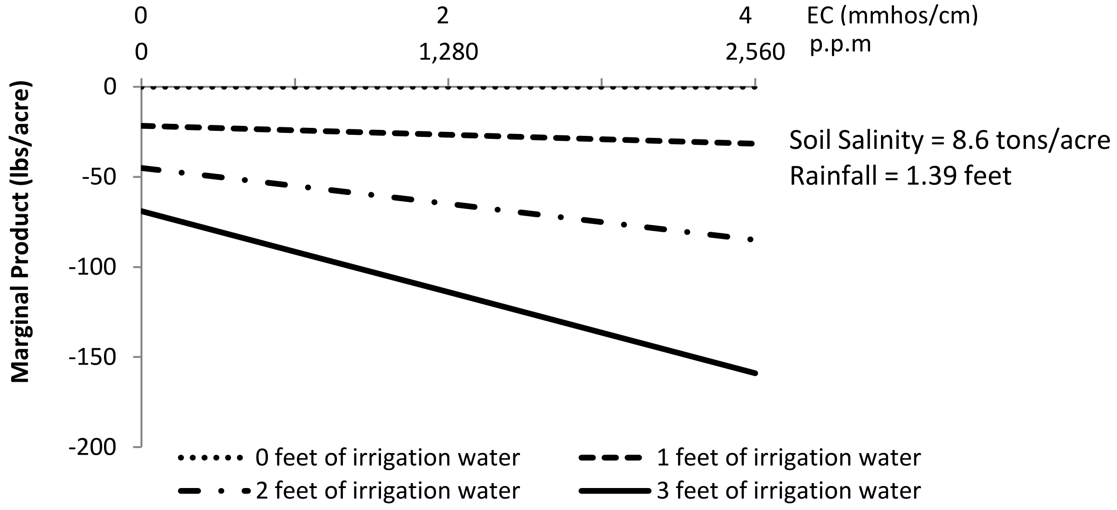
Except for the linear term, Total Salinity, all parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The first-order and second-order condition derived from this function are used to check the necessary tests.

The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 1-2.

Part (a) of Figure 1-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The individual irrigation water containing given salt concentration has a different point maximizing crop yield. The point of maximum yield with respect to irrigation declines as the salt concentration increased.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 1-2. Marginal Product of Irrigation Water and Salt Concentration for the Tipton Loam Soil

Part (a) of Figure 1-2 shows that the crop yield increases as long as the marginal product is positive as irrigation water increases. However, as the salt concentration of irrigation water increases, the marginal curve is reduced as expected. Each of marginal products has a different point maximizing crop yield. This point which is located on the horizontal axis is declining as the salt concentration increases. Part (b) of Figure 1-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines as the salt concentration increases in the irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 1-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases. When irrigation water with a high salt concentration is applied, salts will be rapidly accumulated in the soil and crop yield starts to decline. These results are consistent with Dinar's paper. They showed the crop yield is declining as initial levels of root zone soil salinity and average salt concentration of the applied irrigation water increase on their graph (Dinar *et al*, 1991).

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 ac-ft/acre, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -210.9, -200, -11.5 and -2.9. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

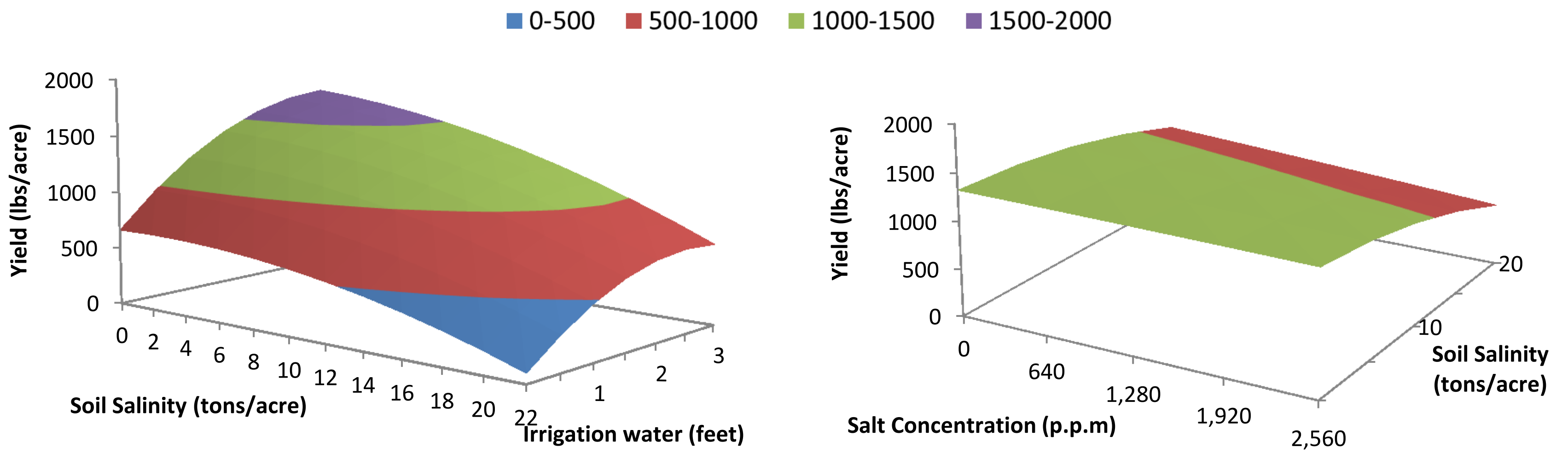
,

,

, and

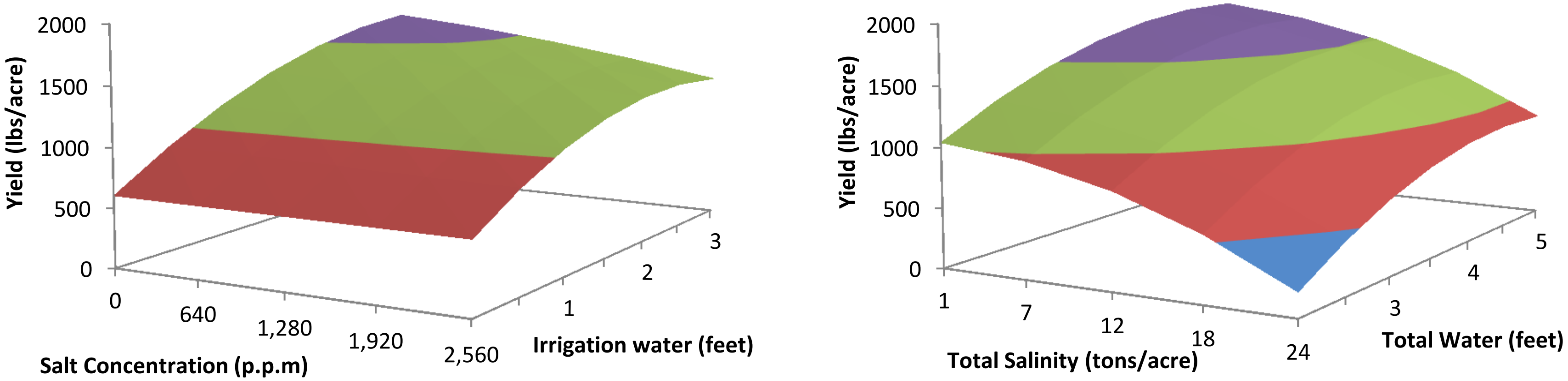
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Tipton Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. The function should be identified in another method to satisfy the function is globally concave. The three dimensional (3-D) surface is used to show the global concavity and illustrate the modified quadratic function with multi-variables. In this study, since more than 2 variables are used in the function, it is impossible to visualize all variables of the function. The 3-D surface has the crop yield to be plotted on the vertical axis and two responsible variables to be plotted on two horizontal axes. When two variables on horizontal axes are evaluated, other variables should be fixed at a certain value. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 1-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 1-3. 3-D Surface of Crop Response Function versus Independent Variables for the Tipton Loam Soil

In part (a) of Figure 1-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 1-3, the 3-D surface shows the crop response function on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.28 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 1-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 8.61 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 1-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. Meanwhile, the crop yield increases over the range of data as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 1-5, 1-6 and 1-7, respectively. The quantity of soil salinity at harvest is affected by irrigation water, amount of salt in irrigation water, soil salinity at planting and growing season rainfall during the growing season. Soil salinity at planting on the next year is also affected by soil salinity at harvest on the previous year and non-growing season rainfall.

Table 1-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Tipton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.6418*	0.0795
Irrigation Water		-0.4781*	0.0369
Amount of Salt in Irrigation Water		0.7049*	0.0140
Soil Salinity at Planting		0.8980*	0.0039
Growing Season Rainfall		-1.3373*	0.0387

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 1-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13017 and 12918, respectively.

As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate. The sign of the dependent variable responding to irrigation water and growing season rainfall is negative indicating the quantity of soil salinity decreases as irrigation water and growing season rainfall increase. Whereas, it's sign responding to the amount of salts and soil salinity at planting is positive indicating the quantity of soil salinity increases as the amount of salts in irrigation water and soil salinity at planting increase.

Table 1-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Tipton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.2914*	0.0521
Soil Salinity at Harvest on Previous Year		0.9149*	0.0023
Non-Growing Season Rainfall		-1.7457*	0.0656

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 1-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12746 and 12564, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate. The sign of the dependent variable responding to soil salinity at harvest on the previous year is positive, indicating the quantity of soil salinity at planting increases as the quantity of soil salinity at harvest in the previous year increases. Whereas, the sign of non-growing season rainfall is negative, indicating the quantity of soil salinity decreases as non-growing season rainfall increases.

Table 1-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Tipton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		9514.39*	2663.38
Irrigation Water		-7209.7*	977.52
Growing Season Rainfall		16071*	1368.75

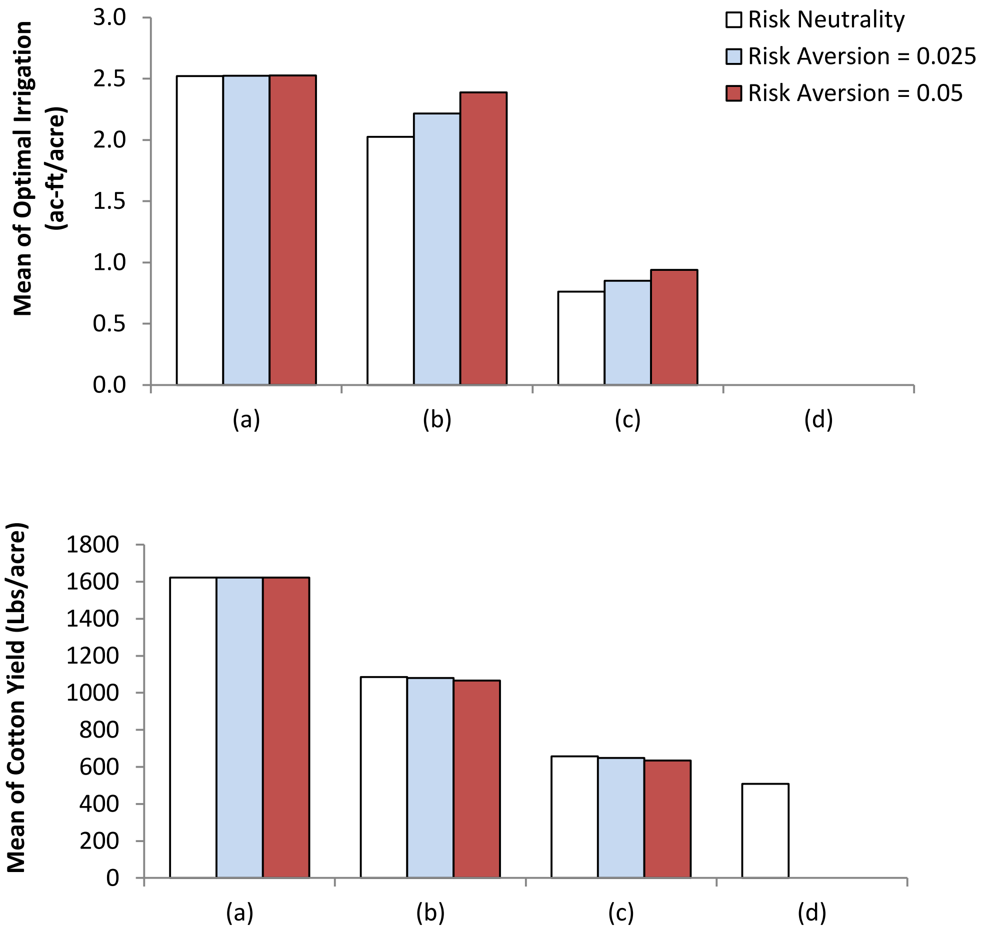
Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 1-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

The estimated crop yield response function, two soil salinity functions and yield variance function are incorporated in an economic decision model to find the optimal level of irrigation maximizing the net present value of the expected utility with different salt concentrations of irrigation water and three levels of risk. Under expected utility maximization, the producer will use irrigation water as an input that provides the maximum level of utility.

Figure 1-4 shows the 50-year average optimal level of irrigation that maximizes the net present value of expected utility and the average cotton yield. It shows that the optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 1-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers are willing to use more irrigation water than risk-neutral producers. It indicates that irrigation water is an effective risk management tool for risk-averse producers to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 1-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 1-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Tipton Loam Soil

The lower half of Figure 1-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 1-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 1-5.

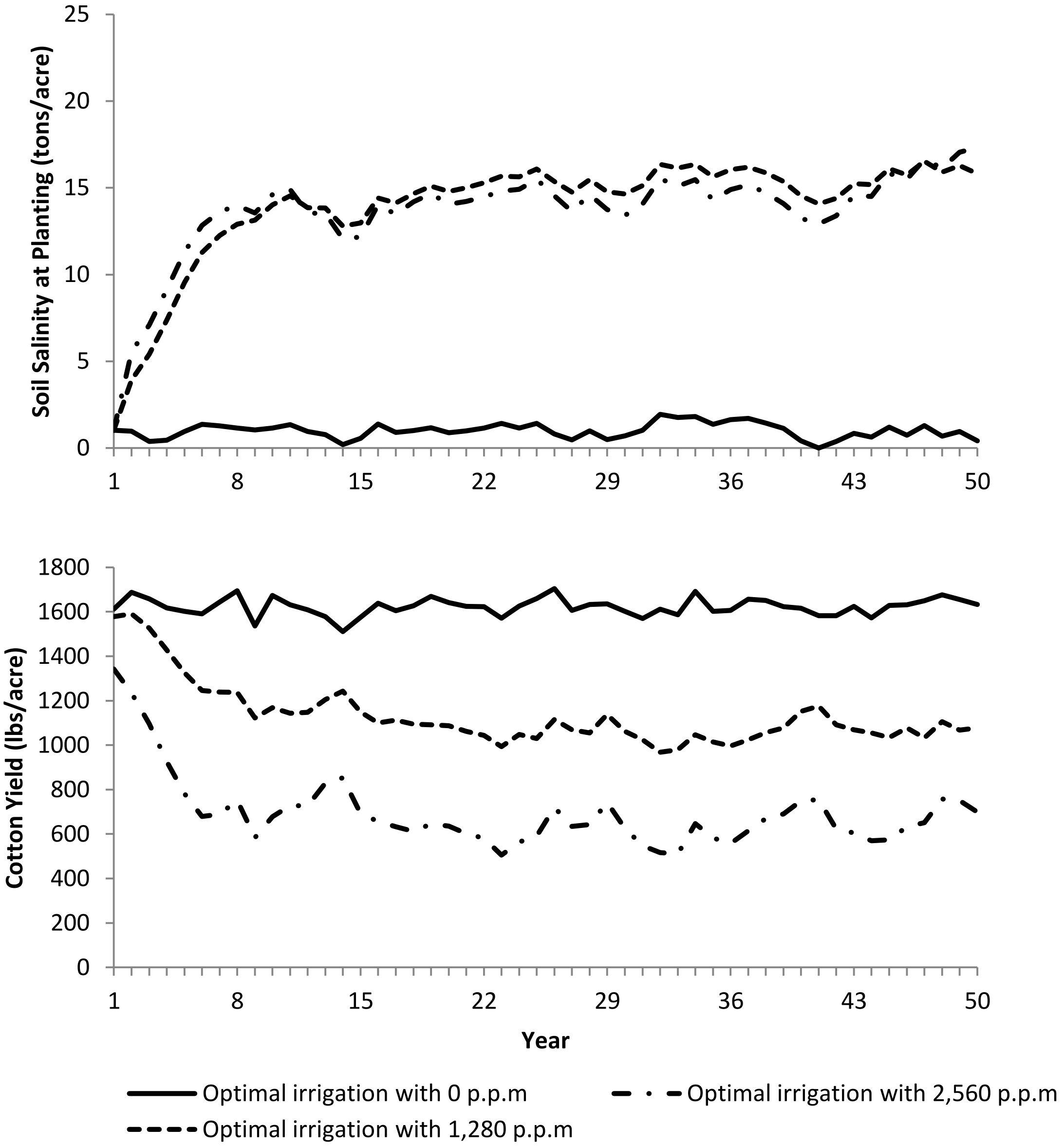
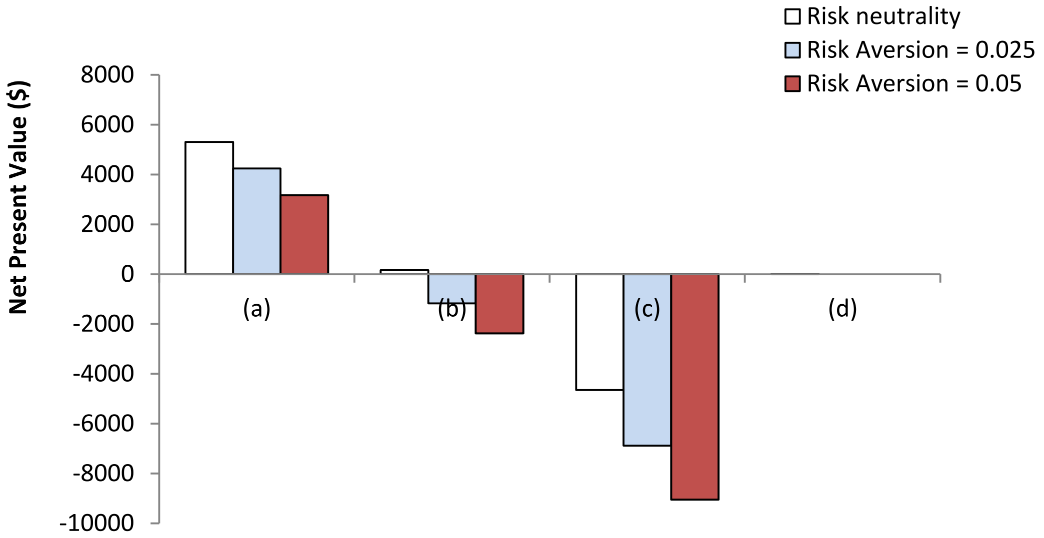


Figure 1-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Absolute Risk Aversion = 0.025 and Soil Depth = 1.5m for the Tipton Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 1-4 and 1-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 1-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 1-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for Tipton Loam Soil

In (b) and (c) of Figure 1-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly positive and larger than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 1-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 1-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Tipton Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5311	5307
1,280 p.p.m (ECw=2)	160	156
2.560 p.p.m (ECw=4)	-4656	-4660
Dryland	4	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 0 and 1,280 p.p.m is applied, the differences of their NPVs are positive. It indicates the producer can make profits from investment in irrigated production compared to dryland cotton production.

If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2 ~ 2.6 ac-ft/acre maximizing NPV of expected utility (see Figure 1-4).

Madge Fine Sandy Loam and Madge Loam, 2-3% Slope

EPIC Output Data

Table 2-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Madge Fine Sandy Loam and Madge Loam soil. The total salt in the 1.5 meter profile was calculated as 0.77 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 2-2.

Table 2-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Madge Fine Sandy Loam and Madge Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.05	0.77	0.9	0.73	0.89	1.35	1.19	
WSLT(kg/ha)*	8	101	128	304	369	486	334	1,730
Salinity(tons/acre)								0.77**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 2-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Madge Fine Sandy Loam and Madge Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	39 ~1,837	1,054
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.56	1.26
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 24.25	8.12
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 26.63	8.89
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest with given levels of the salt concentration are shown on a box plot in Figure 2-1.

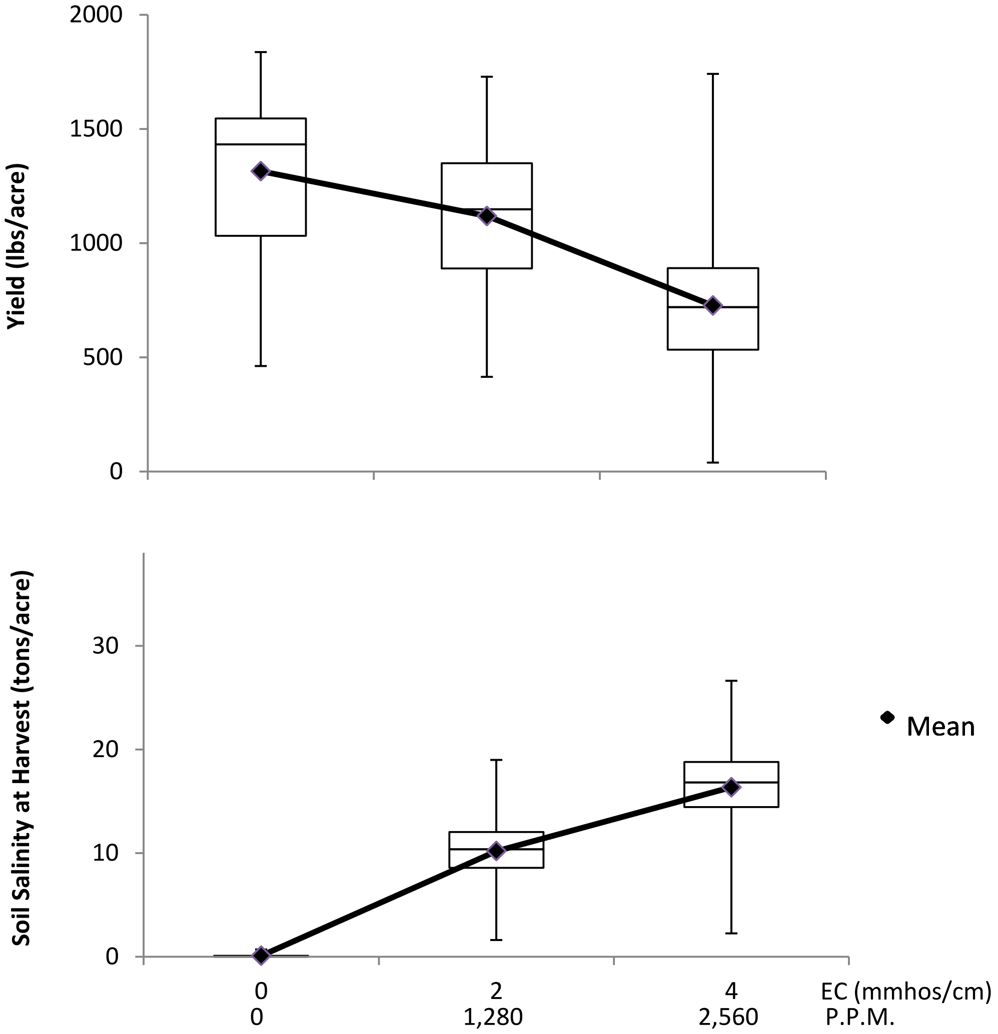


Figure 2-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Madge Fine Sandy Loam and Madge Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 2-3.

Table 2-3. Result of Likelihood Ratio Test for the Madge Fine Sandy Loam and Madge Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56481	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56324		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 2-4.

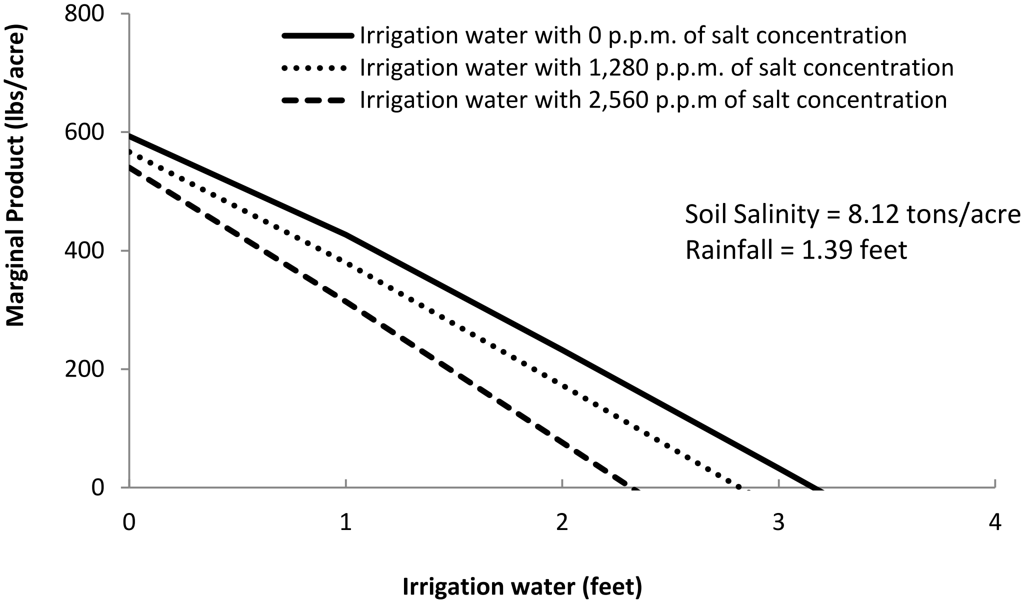
Table 2-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Madge Fine Sandy Loam and Madge Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-506.5*	41.6343
Total Water Applied		934.13*	29.8794
Total Salinity		-1.5346	1.3875
Non-Growing Season Rainfall		98.8585*	9.3584
(Total Water Applied)^2		-102.05*	5.3278
(Total Salinity)^2		-1.5414*	0.0414
(Total Salinity / Total Water Applied)		13.6835*	2.6717

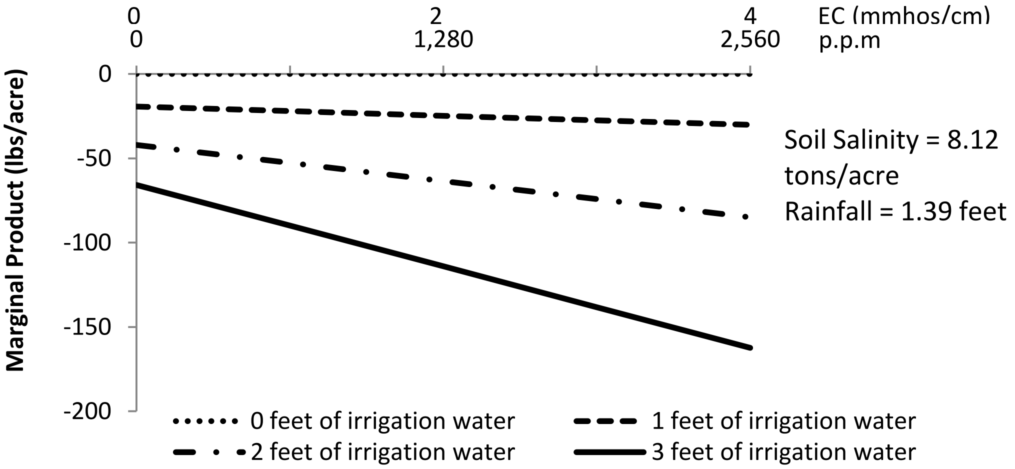
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

Except for the linear term, Total Salinity, all parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 2-2.

Part (a) of Figure 2-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The individual irrigation water containing given salt concentration has a different point maximizing crop yield. The point of maximum yield with respect to irrigation declines as the salt concentration increased.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 2-2. Marginal Product of Irrigation Water and Salt Concentration for the Madge Fine Sandy Loam and Madge Loam Soil

Part (b) of Figure 2-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 2-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -210.2, -196.7, -12.3 and -3.1. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

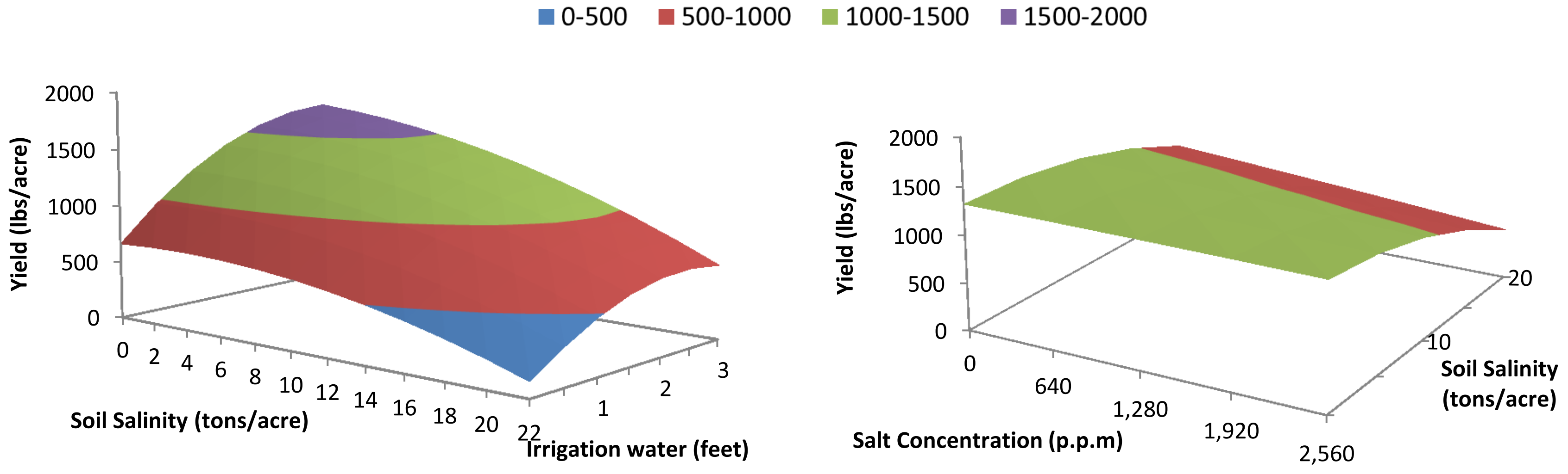
,

,

, and

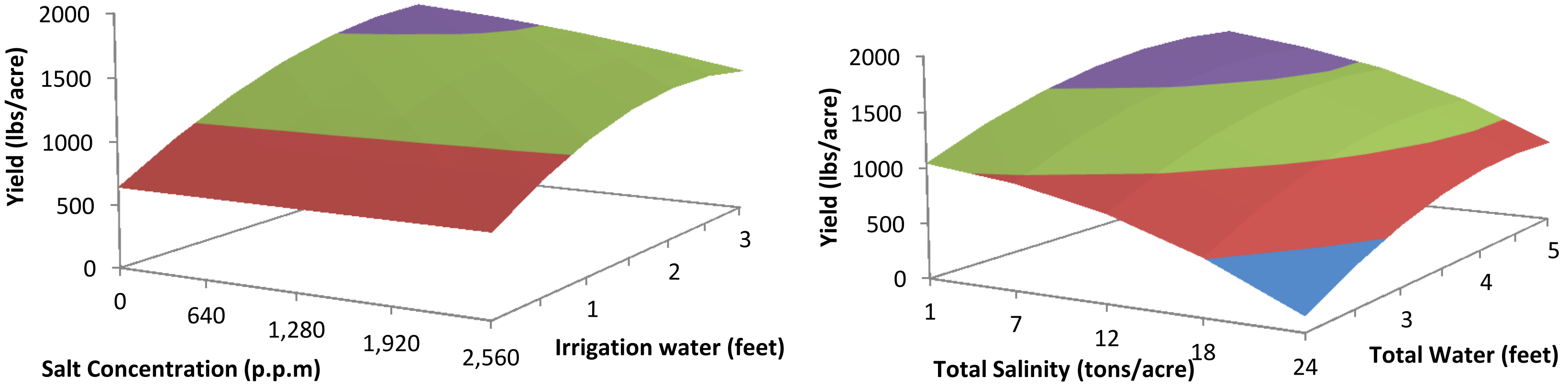
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Madge Fine Sandy and Madge Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 2-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 2-3. 3-D Surface of Crop Response Function versus Independent Variables for the Madge Fine Sandy and Madge Loam Soil

In part (a) of Figure 2-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 2-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.26 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 2-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 8.12 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 2-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 2-5, 2-6 and 2-7, respectively.

Table 2-5. Coefficients from SAS Proc for Dynamic Soil Salinity Levels at Harvest form EPIC Simulations for the Madge Fine Sandy Loam and Madge Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.4821*	0.0731
Irrigation Water		-0.4519*	0.0333
Amount of Salt in Irrigation Water		0.7292*	0.0137
Soil Salinity at Planting		0.8899*	0.0039
Growing Season Rainfall		-1.2609*	0.0354

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 2-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12207 and 12102, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 2-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Madge Fine Sandy Loam and Madge Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.247*	0.0490
Soil Salinity at Harvest on Previous Year		0.9139*	0.0022
Non-Growing Season Rainfall		-1.7148*	0.0601

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 2-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11958 and 11783, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 2-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Madge Fine Sandy Loam and Madge Loam Soil

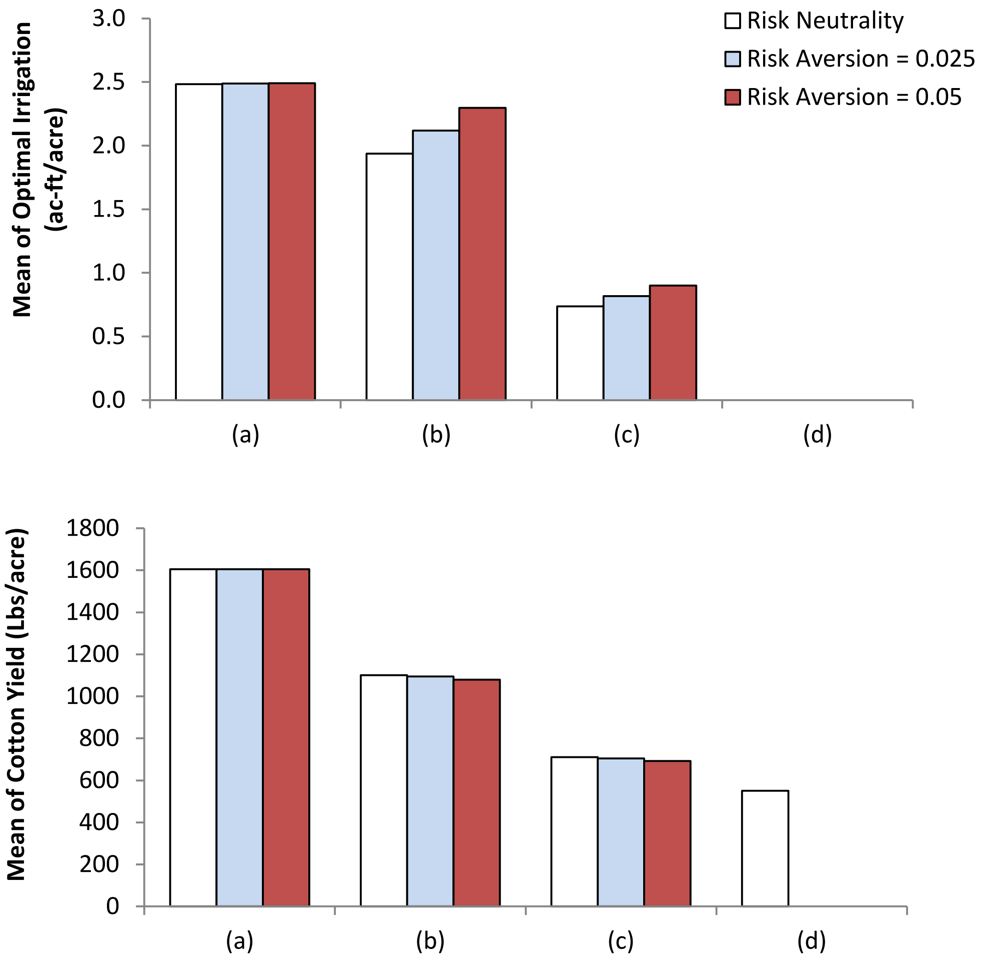
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		8287*	2538.86
Irrigation Water		-7163.81*	973.88
Growing Season Rainfall		15823*	1297.72

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 2-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 2-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 2-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 2-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 2-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Madge Fine Sandy Loam and Madge Loam Soil

The lower half of Figure 2-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 2-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 2-5.

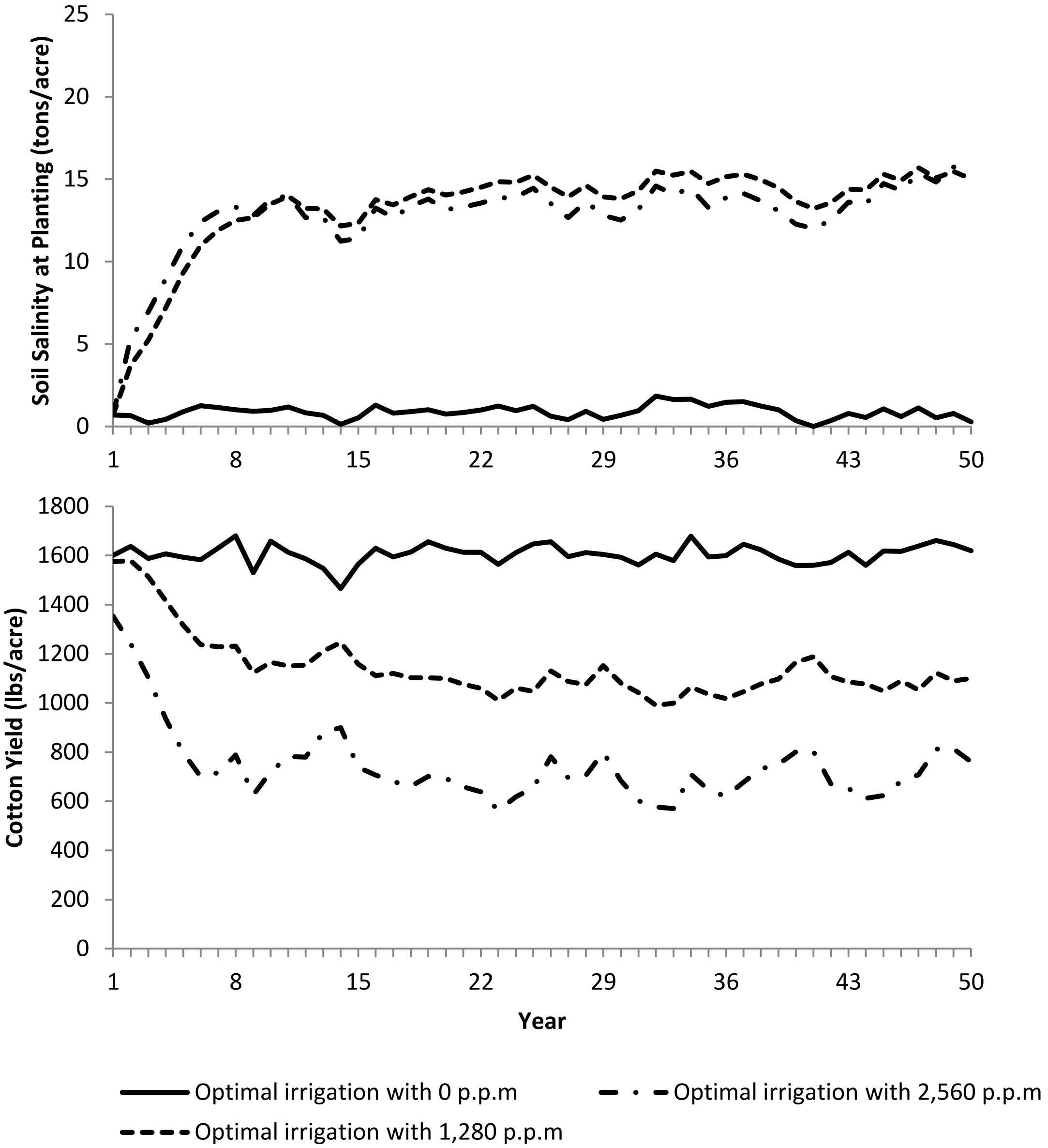


Figure 2-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Madge Fine Sandy Loam and Madge Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 2-4 and 2-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 2-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

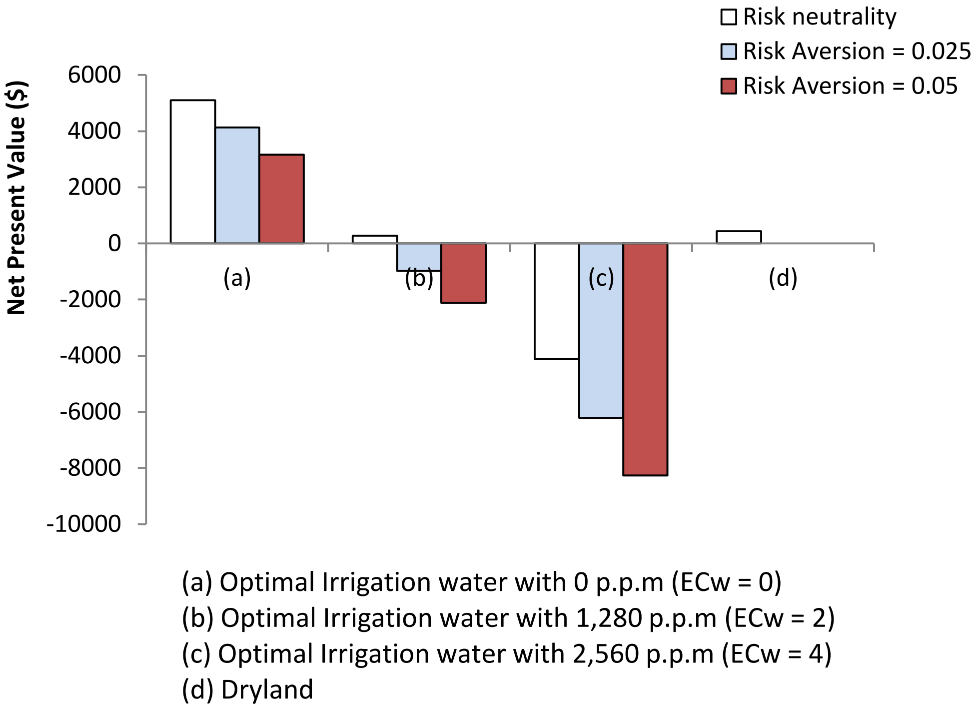


Figure 2-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Madge Fine Sandy Loam and Madge Loam Soil

In (b) and (c) of Figure 2-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly positive and similar to NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 2-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 2-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Madge Fine Sandy Loam and Madge Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5099	4662
1,280 p.p.m (ECw=2)	280	-157
2.560 p.p.m (ECw=4)	-4114	-4551
Dryland	437	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 2-4).

Roark Loam Soil, 0-1% Slope

EPIC Output Data

Table 3-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Roark Loam soil. The total salt in the 1.5 meter profile was calculated as 3.26 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 3-2.

Table 3-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Roark Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	0.9	0.69	1.2	1.35	2.16	5.48	7.71	
WSLT(kg/ha)*	9	110	206	461	1,172	2,969	2,374	7,302
Salinity(tons/acre)								3.26**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 3-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Roark Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	2 ~1,836	975
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 30.25	11.19
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 32.67	11.96
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

In case of cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 3-1.

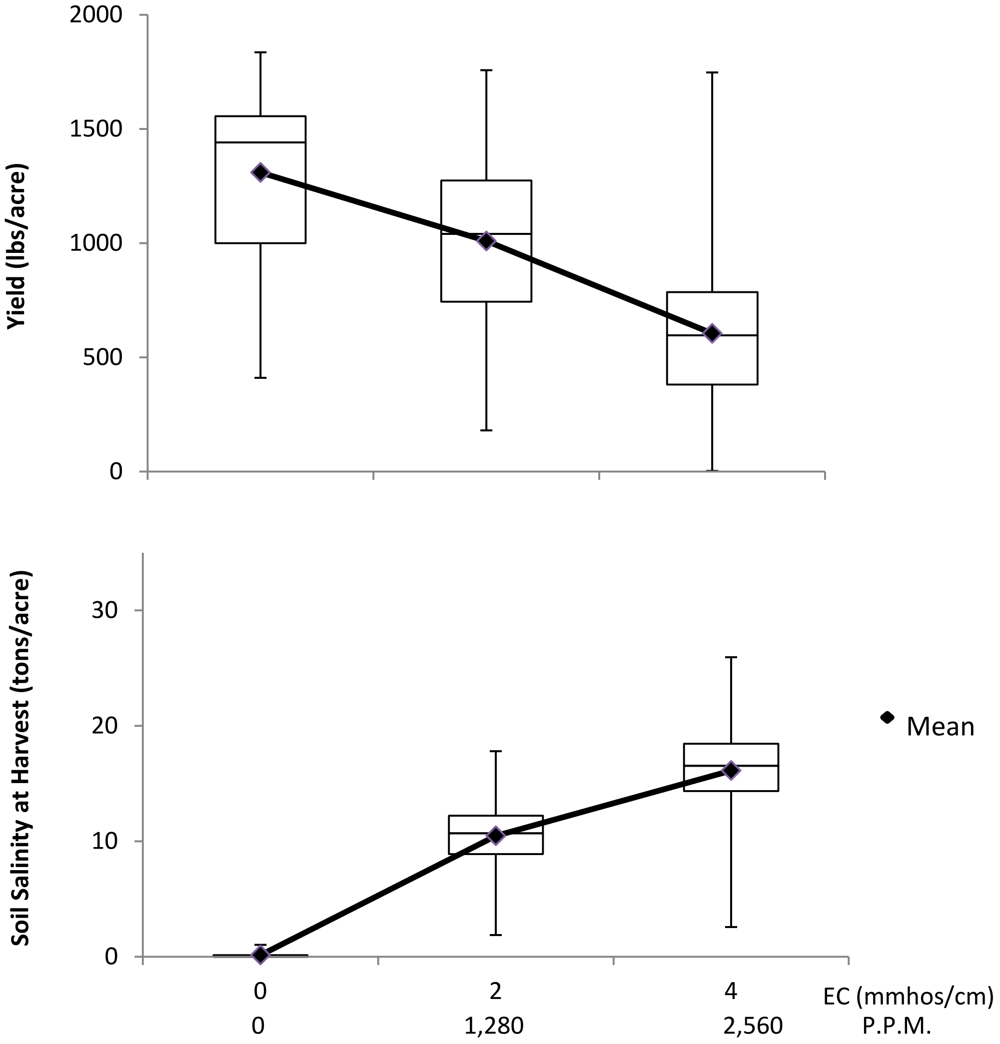


Figure 3-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Roark Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has a common error message of “Estimated G matrix is not positive definite.” which indicates that one variance component on the RANDOM statement to be zero. It should be removed from the model (http://support.sas.com/kb/22/614.html). Without the RANDOM statement which means that the model does not have the random effect part, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 3-3.

Table 3-3. Result of Likelihood Ratio Test without Random Effect for the Roark Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56656	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56505		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton function are shown in Table 3-4.

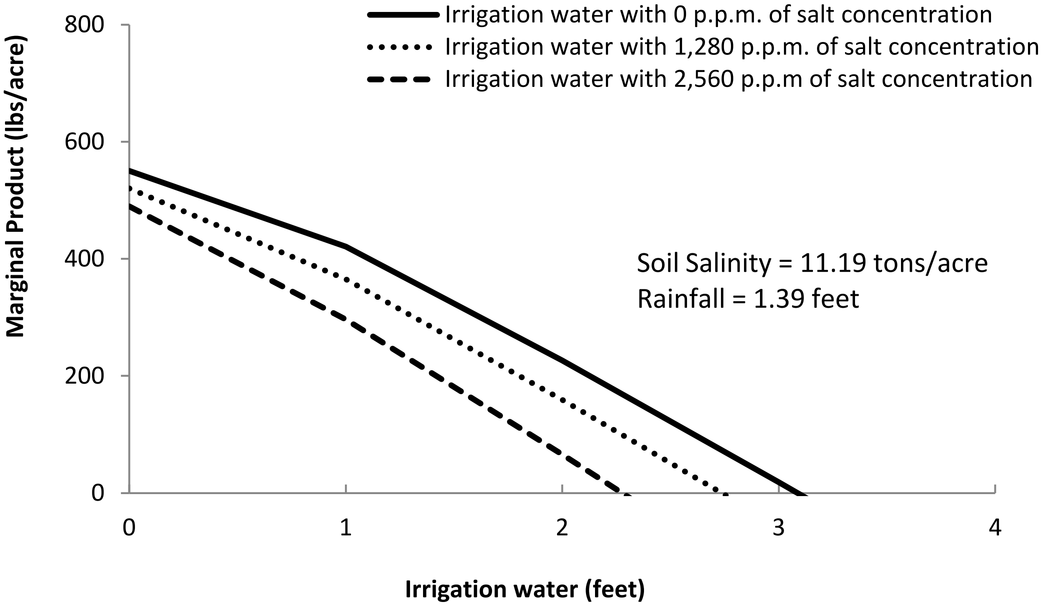
Table 3-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Roark Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-608.54*	45.1792
Total Water Applied		984.12*	31.2378
Total Salinity		-9.5391*	1.1490
Non-Growing Season Rainfall		109.09*	9.5289
(Total Water Applied)^2		-108.47*	5.3873
(Total Salinity)^2		-1.0853*	0.0321
(Total Salinity / Total Water Applied)		22.8326*	2.2366

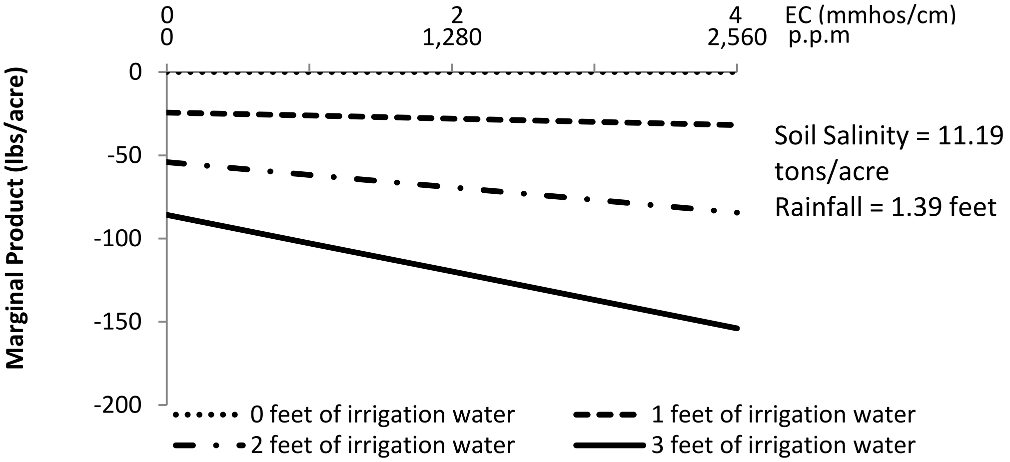
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 3-2.

Part (a) of Figure 3-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 3-2. Marginal Product of Irrigation Water and Salt Concentration for the Roark Loam Soil

Part (b) of Figure 3-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 3-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

= .

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -218.1, -204.7, -8.7 and -2.2. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

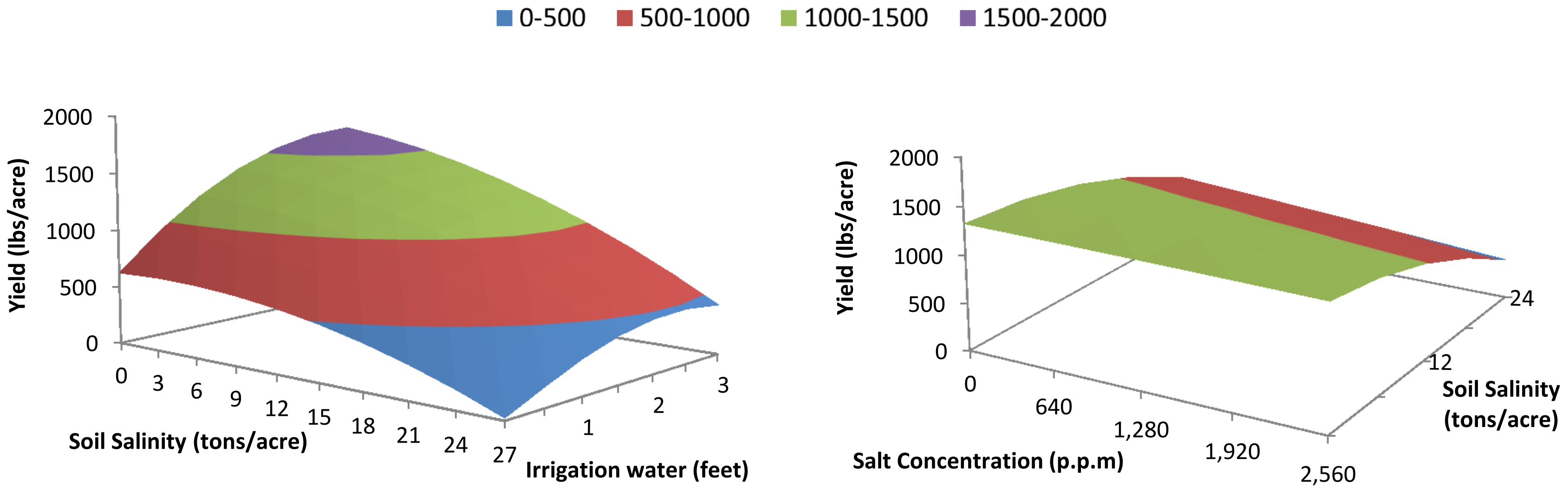
,

,

, and

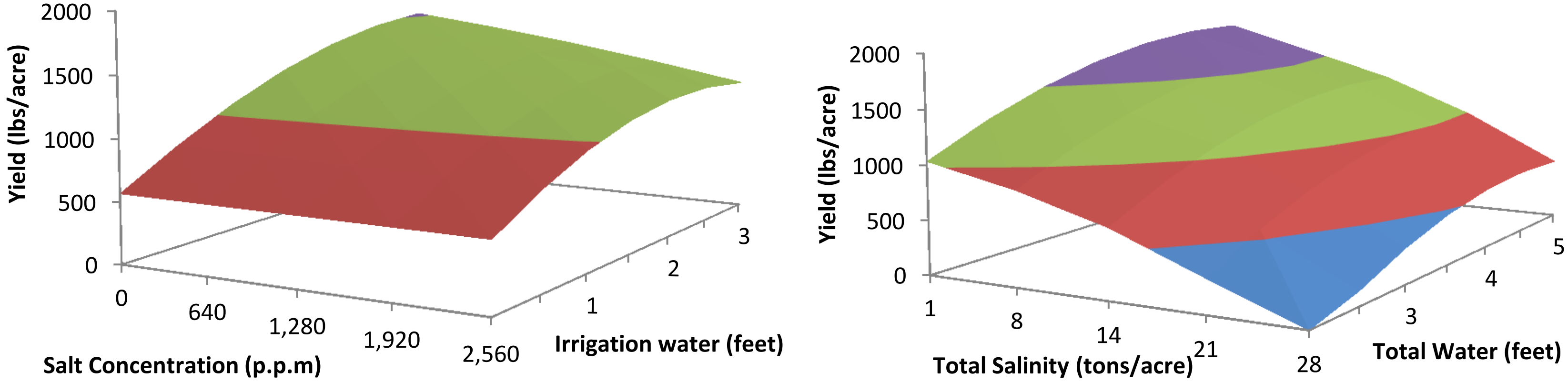
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Roark Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 3-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 3-3. 3-D Surface of Crop Response Function versus Independent Variables for the Roark Loam Soil

In part (a) of Figure 3-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 3-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 3-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 11.19 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 3-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 3-5, 3-6 and 3-7, respectively.

Table 3-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Roark Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.7933*	0.0884
Irrigation Water		-0.5622*	0.0399
Amount of Salt in Irrigation Water		0.7039*	0.0149
Soil Salinity at Planting		0.9164*	0.0035
Growing Season Rainfall		-1.3277*	0.0411

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 3-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13441 and 13365, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 3-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Roark Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.3664*	0.0556
Soil Salinity at Harvest on Previous Year		0.9339*	0.0021
Non-Growing Season Rainfall		-1.8512*	0.0676

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 3-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13008 and 12784, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 3-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Roark Loam Soil

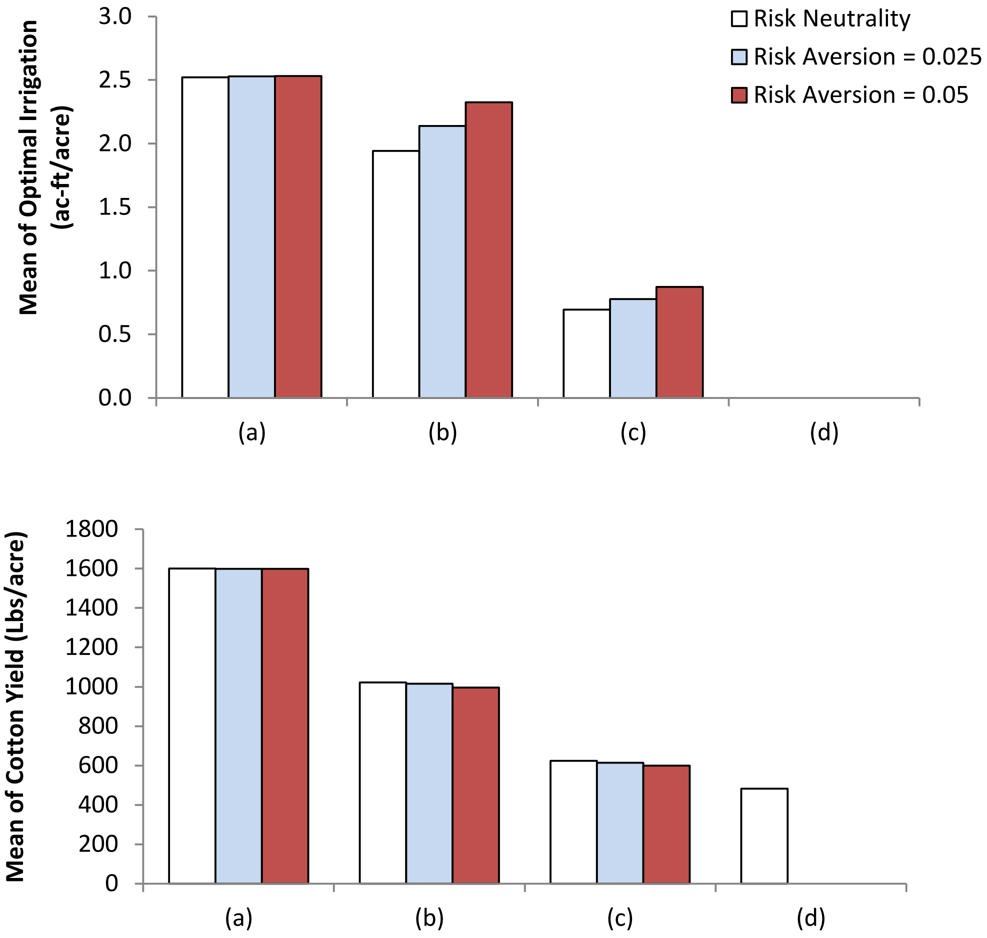
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		8475.29*	2585.17
Irrigation Water		-5592.14*	996.32
Growing Season Rainfall		15091*	1372.57

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 3-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 3-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 3-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 3-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 3-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Roark Loam Soil

The lower half of Figure 3-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. Although the level of irrigation water with 1,280 p.p.m contains some salts, it is slightly less used than the optimal level of irrigation with 0 p.p.m. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 3-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 3-5.

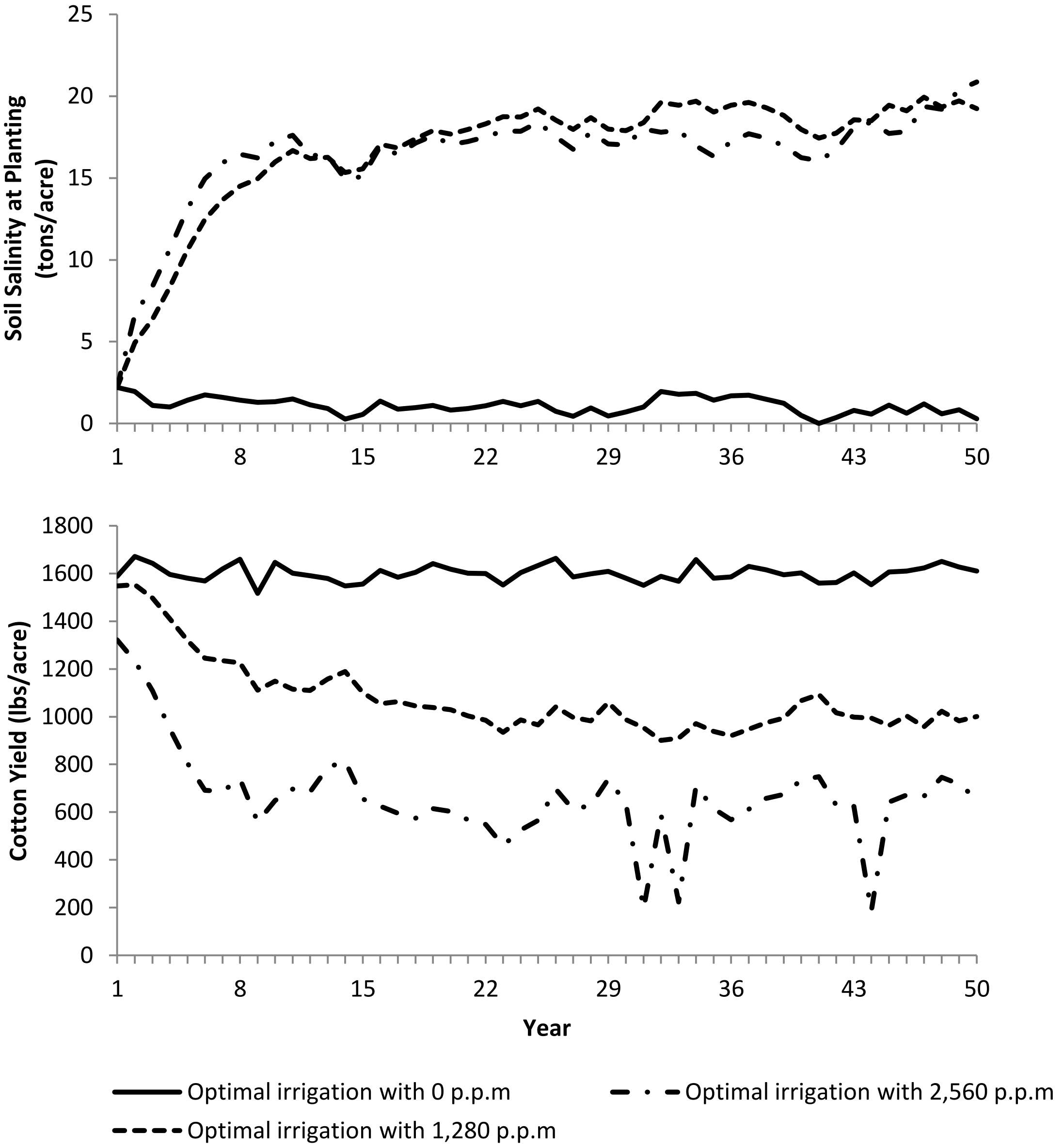
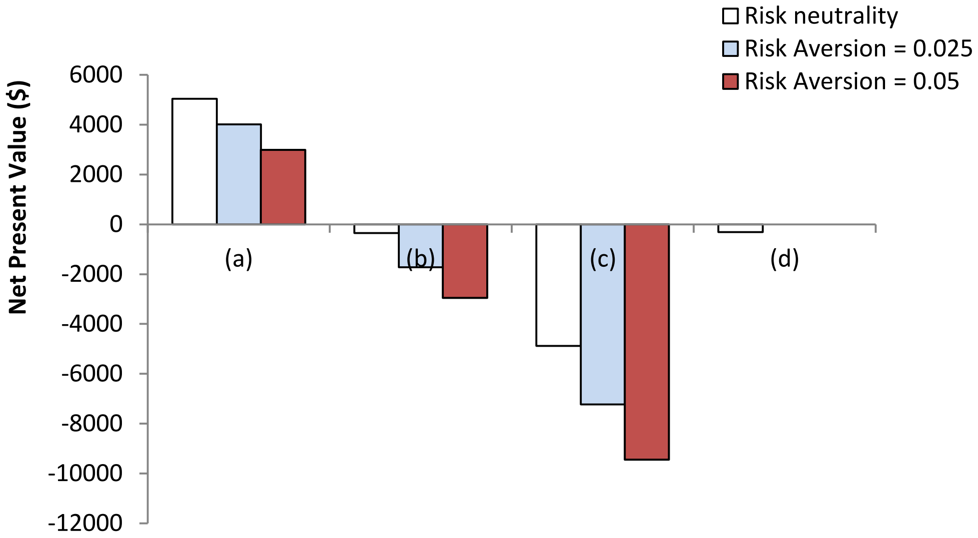


Figure 3-5. Changes of Quantity of Soil Salinity at planting over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth =1.5m for the Roark Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 3-4 and 3-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 3-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 3-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Roark Loam Soil

In (b) and (c) of Figure 3-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 3-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 3-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Roark Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50- year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5037	5343
1,280 p.p.m (ECw=2)	-351	-45
2.560 p.p.m (ECw=4)	-4879	-4572
Dryland	-306	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than 1,280 p.p.m, the optimal level of irrigation is approximately 1.9 ~ 2.5 ac-ft/acre maximizing NPV of expected utility (see Figure 3-4).

Spur Loam Soil, 0-1% Slope, occasionally flooded

EPIC Output Data

Table 4-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Spur Loam soil. The total salt in the 1.5 meter profile was calculated as 1.12 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 4-2.

Table 4-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Spur Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.51	1.07	0.74	1.01	1.77	2.1	2.36	
WSLT(kg/ha)*	11	127	130	353	617	731	549	2,518
Salinity(tons/acre)								1.12**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 4-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Spur Loam Soil

Variable	**Symbol (Unit)**	**Range**	**Mean**
Cotton Yield	Y (lbs/acre)	11 ~ 1818	998
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.55	1.25
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 23.76	8.16
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 25.94	8.92
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest with given levels of the salt concentration are shown on a box plot in Figure 4-1.

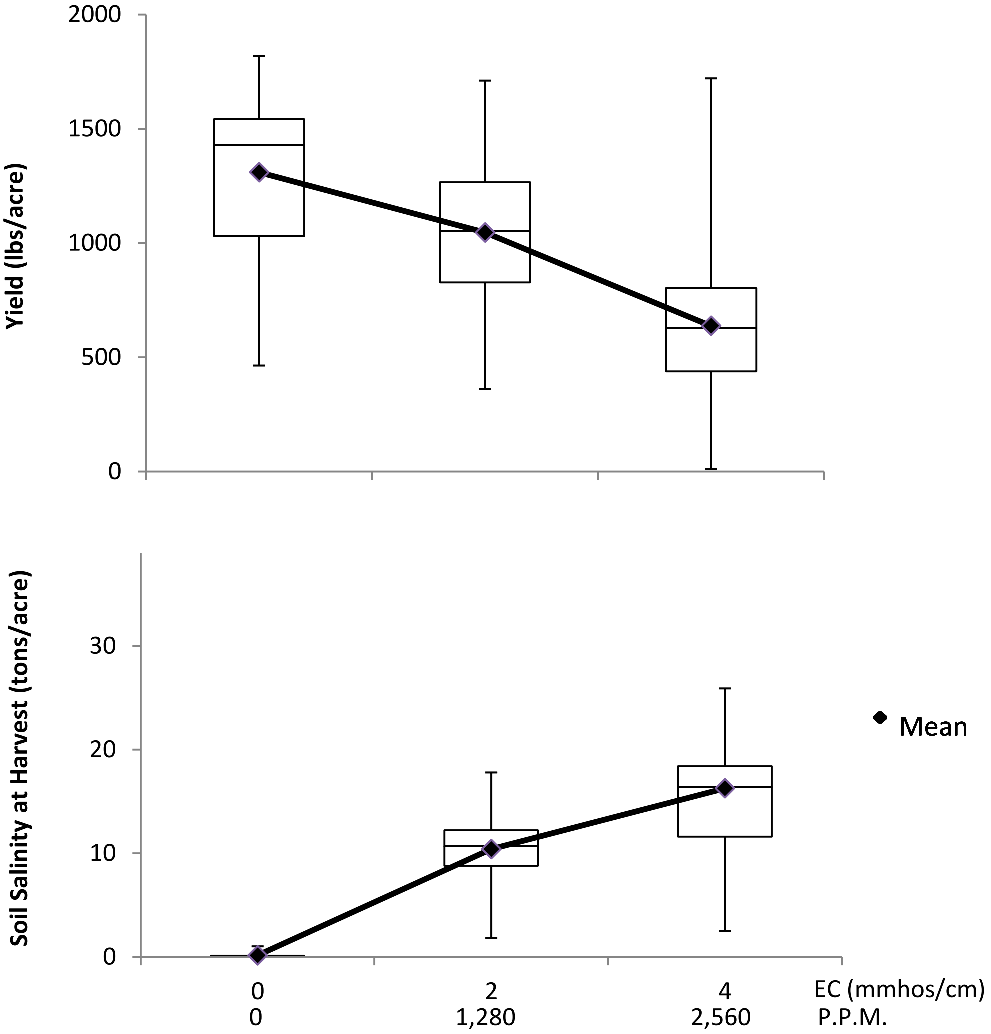


Figure 4-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Spur Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 4-3.

Table 4-3. Result of Likelihood Ratio Test for the Spur Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56375	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56222		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 4-4.

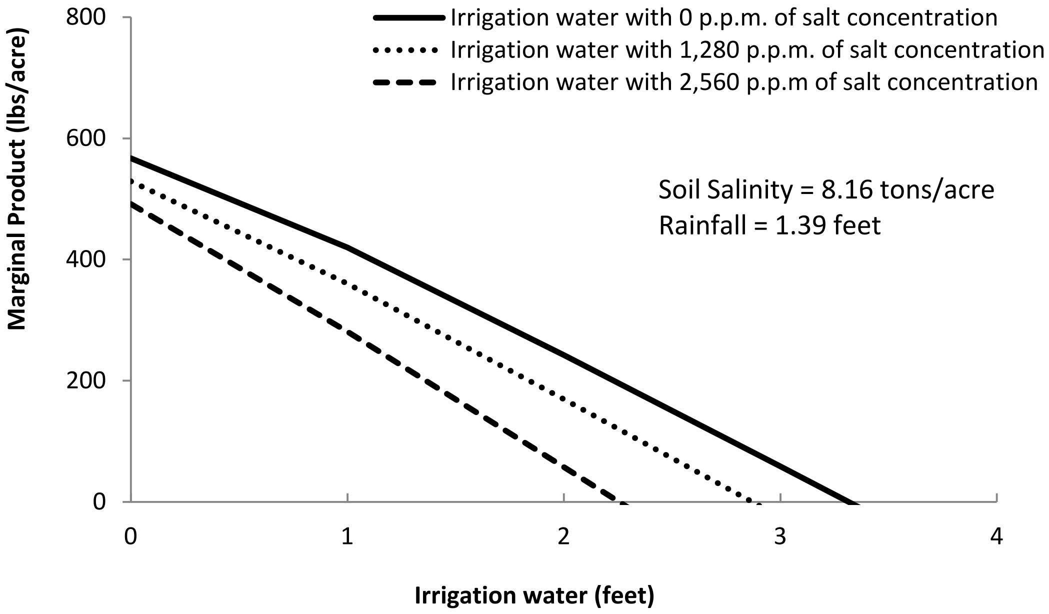
Table 4-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Spur Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-453.18*	41.9253
Total Water Applied		889.81*	30.0810
Total Salinity		-5.6511*	1.4055
Non-Growing Season Rainfall		102.58*	9.2648
(Total Water Applied)^2		-93.9975*	5.3650
(Total Salinity)^2		-1.6271*	0.0443
(Total Salinity / Total Water Applied)		14.5933*	2.6754

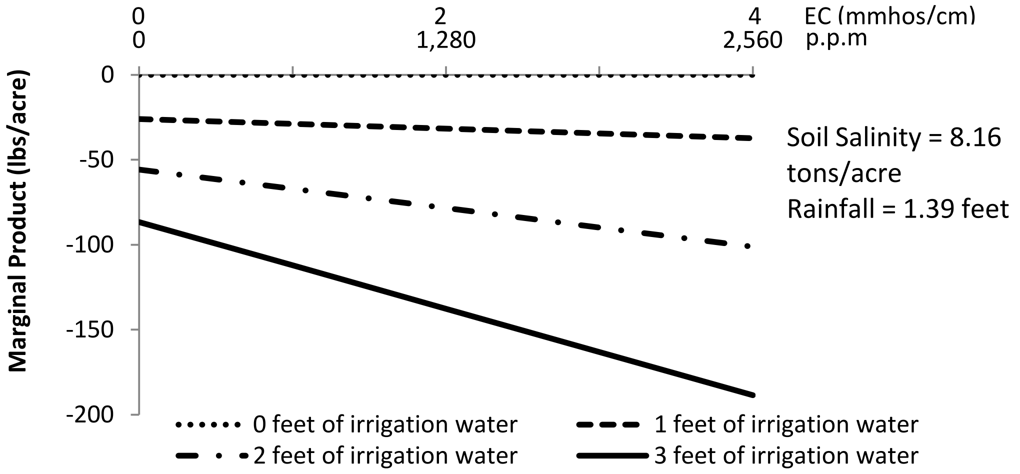
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 4-2.

Part (a) of Figure 4-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 4-2. Marginal Product of Irrigation Water and Salt Concentration for the Spur Loam Soil

Part (b) of Figure 4-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 4-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet,74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

= .

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -194.4, -180.1, -13 and -3.3. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

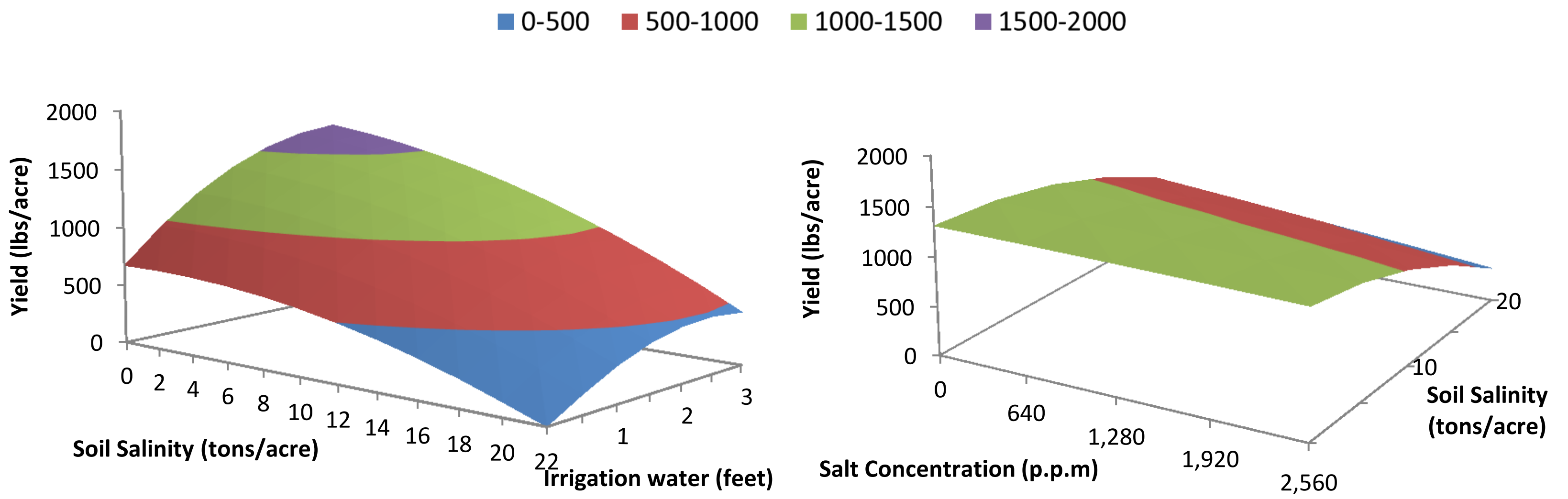
,

,

, and

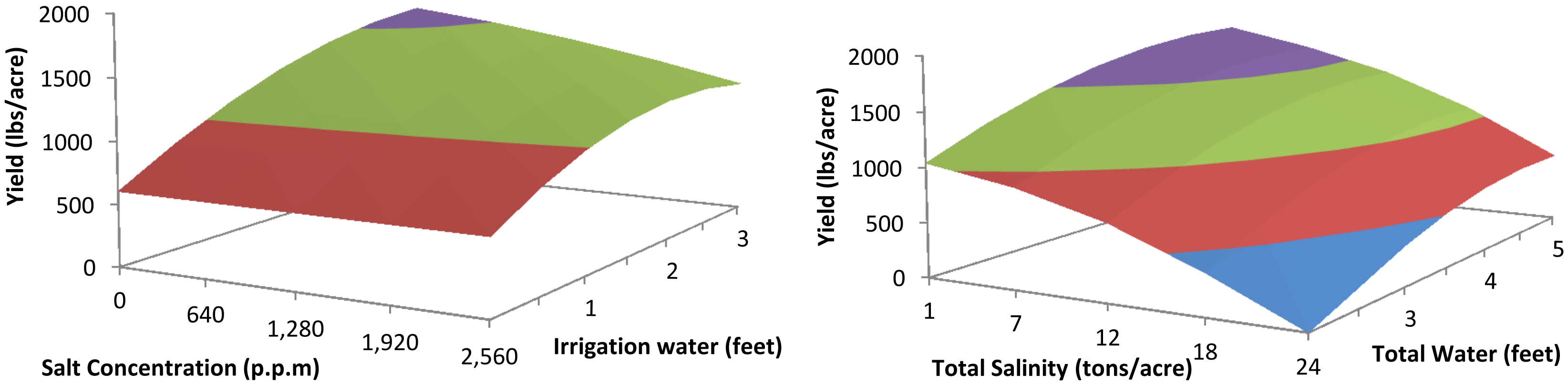
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Spur Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 4-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 4-3. 3-D Surface of Crop Response Function versus Independent Variables for the Spur Loam Soil

In part (a) of Figure 4-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 4-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.25 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 4-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 8.16 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 4-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 4-5, 4-6 and 4-7, respectively.

Table 4-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Spur Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.4821*	0.0731
Irrigation Water		-0.4519*	0.0333
Amount of Salt in Irrigation Water		0.7292*	0.0137
Soil Salinity at planting		0.8899*	0.0039
Growing Season Rainfall		-1.2609*	0.0354

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.
The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table -5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11862 and 11758, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 4-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Spur Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.247*	0.0490
Soil Salinity at Harvest on Previous Year		0.9139*	0.0022
Non-Growing Season Rainfall		-1.7148*	0.0601

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 4-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11695 and 11525, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 4-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Spur Loam Soil

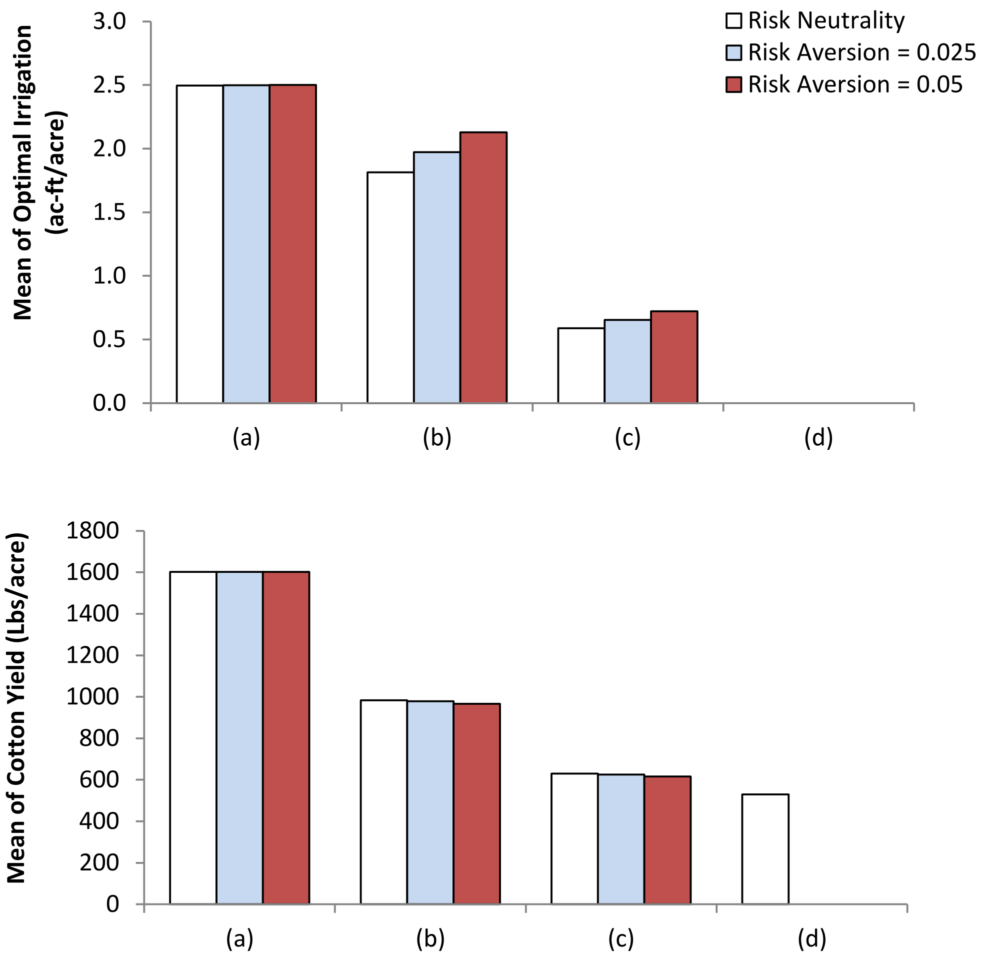
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		5082.37*	2454.59
Irrigation Water		-5874.23*	961.75
Growing Season Rainfall		16538*	1264.69

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 4-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 4-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 4-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 4-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 4-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Spur Loam Soil

The lower half of Figure 4-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 4-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 4-5.

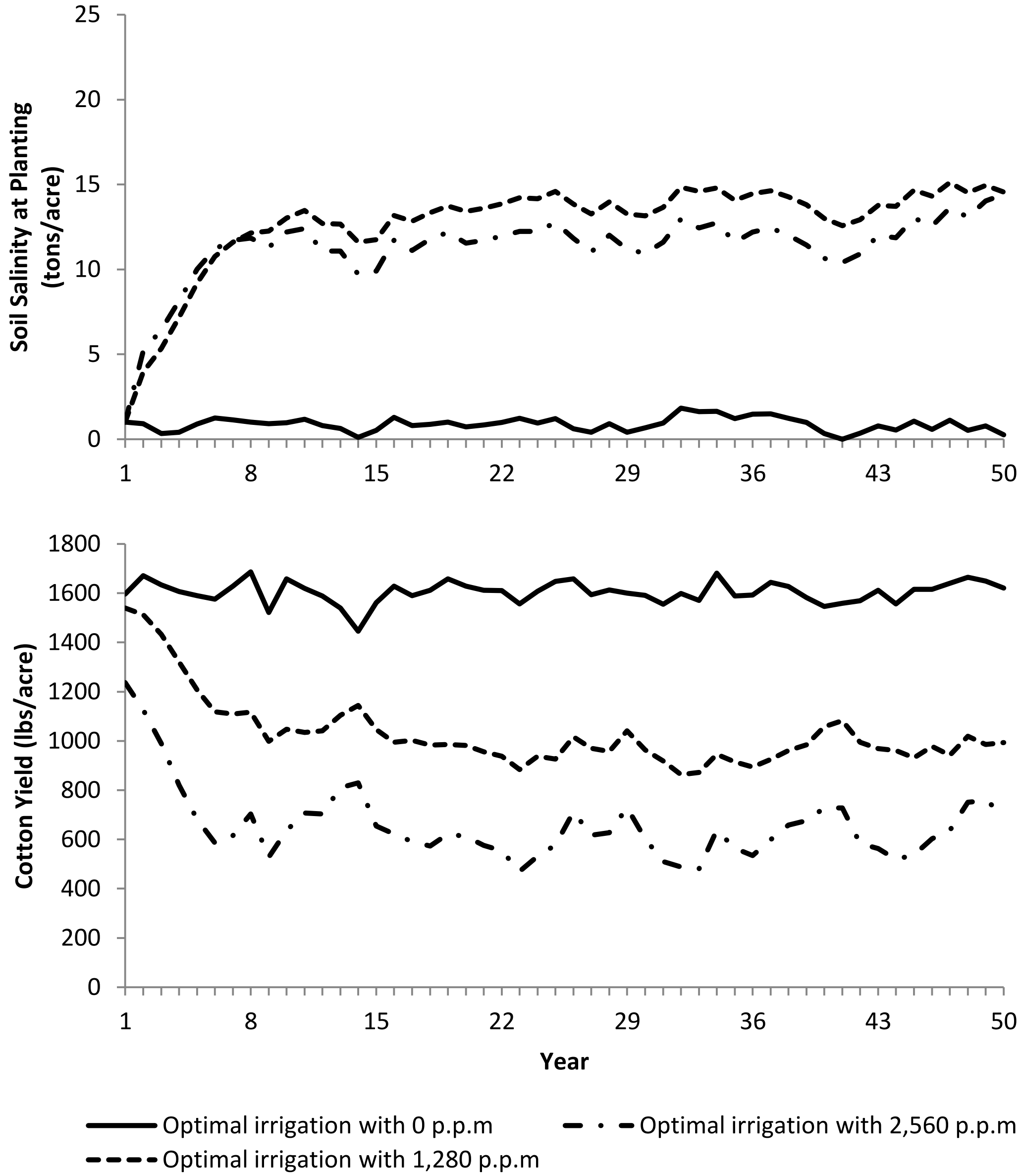


Figure 4-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Spur Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 4-4 and 4-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 4-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

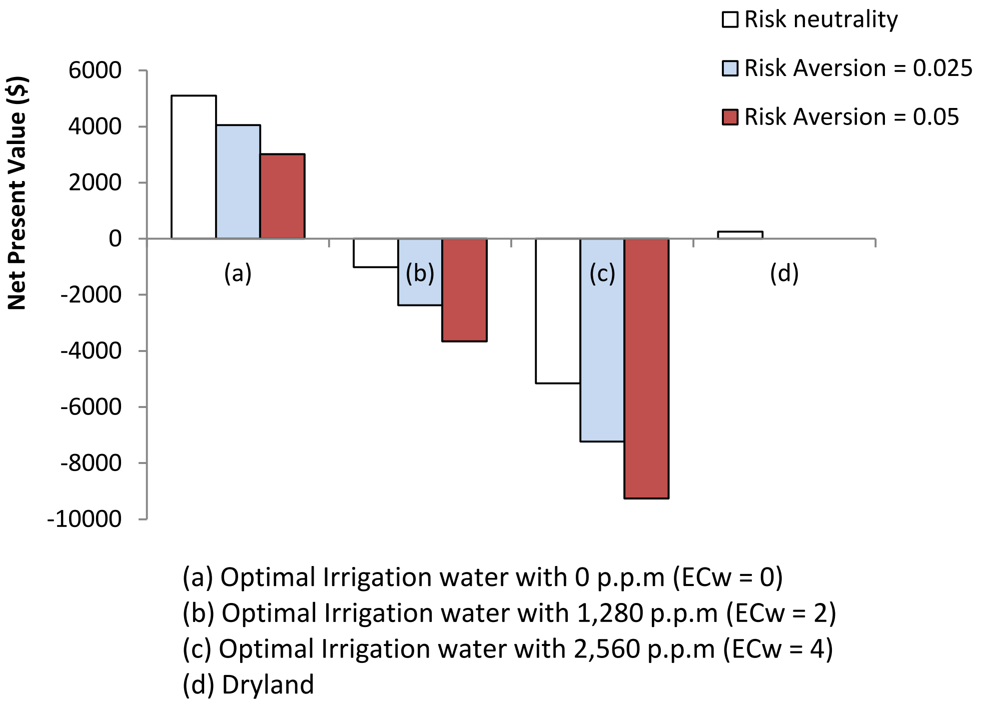


Figure 4-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Spur Loam Soil

In (b) and (c) of Figure 4-6, the net present values for the risk-averse and risk-neutral producer are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 4-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 4-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Spur Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5095	4845
1,280 p.p.m (ECw=2)	-1013	-1266
2.560 p.p.m (ECw=4)	-5158	-5409
Dryland	251	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative in Table 4-8. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than 1,280 p.p.m, the optimal level of irrigation is approximately 1.8 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 4-4).

Spur Clay Loam Soil, 0-1% Slope, occasionally and rarely flooded

EPIC Output Data

Table 5-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Spur Clay Loam soil. The total salt in the 1.5 meter profile was calculated as 1.39 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 5-2.

Table 5-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Spur Clay Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.38	1.11	1.03	1.4	1.43	1.38	2.94	
WSLT(kg/ha)*	16	206	206	559	569	548	1,021	3,127
Salinity(tons/acre)								1.39**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 5-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Spur Clay Clay Loam Soil

Variable	**Symbol (Unit)**	**Range**	**Mean**
Cotton Yield	Y (lbs/acre)	29 ~ 1848	1068
Irrigation Water	Irr (ac-ft/acre)	0.16 ~2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 26.3	9.11
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 28.76	9.88
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 5-1.

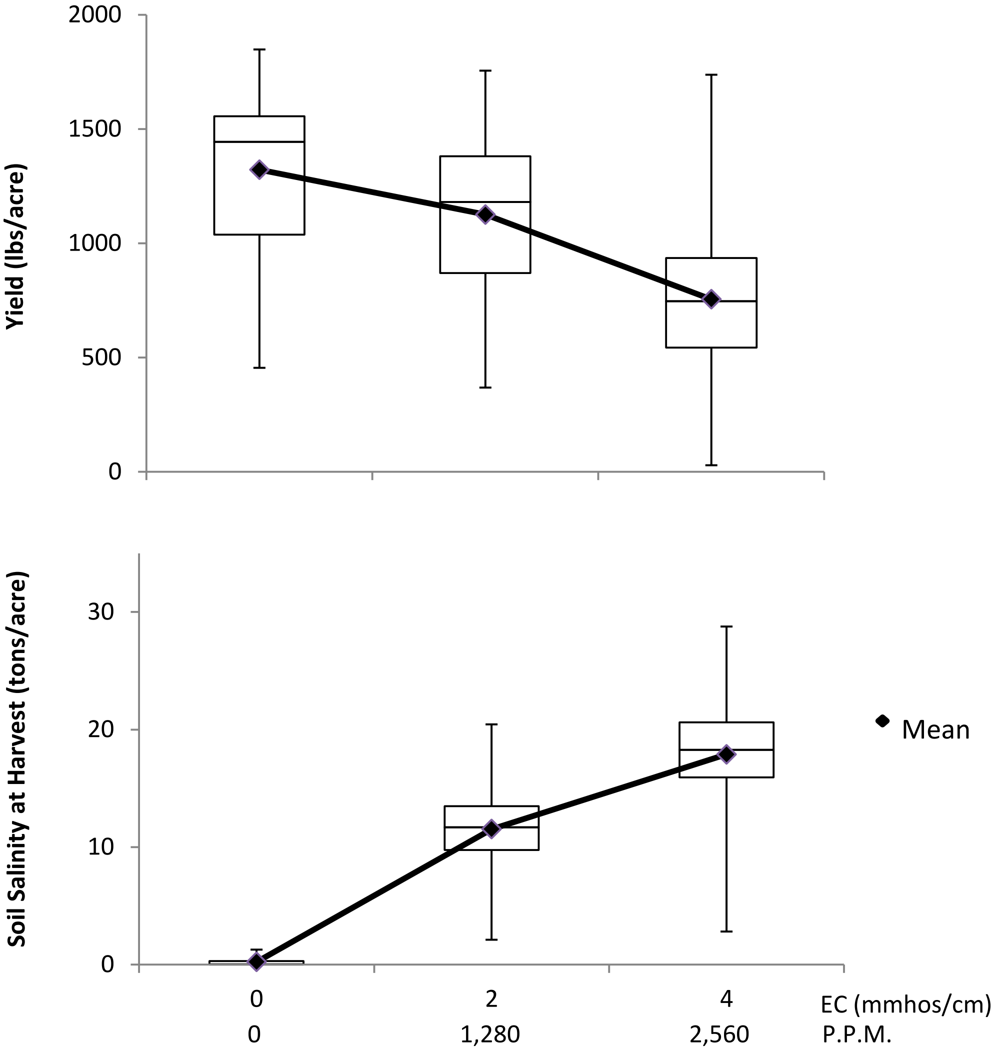


Figure 5-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Spur Clay Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 5-3.

Table 5-3. Result of Likelihood Ratio Test for the Spur Clay Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56696	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56566		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 5-4.

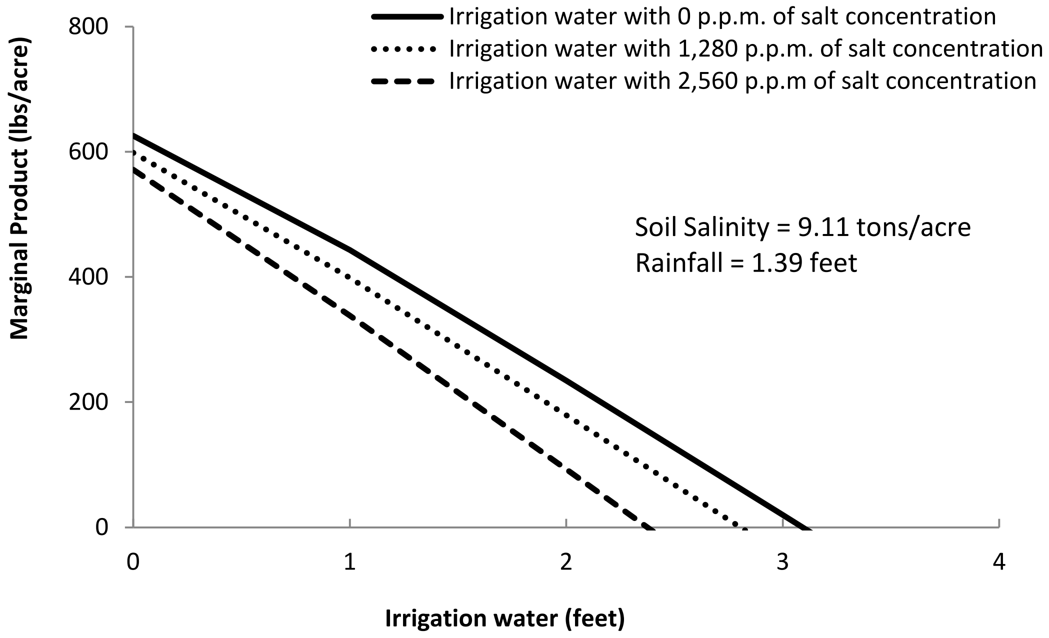
Table 5-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Spur Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-593.76*	42.0258
Total Water Applied		982.90*	29.8898
Total Salinity		-0.08595	1.2398
Non-Growing Season Rainfall		113.44*	9.6987
(Total Water Applied)^2		-109.05*	5.2725
(Total Salinity)^2		-1.3090*	0.0362
(Total Salinity / Total Water Applied)		11.4911*	2.3737

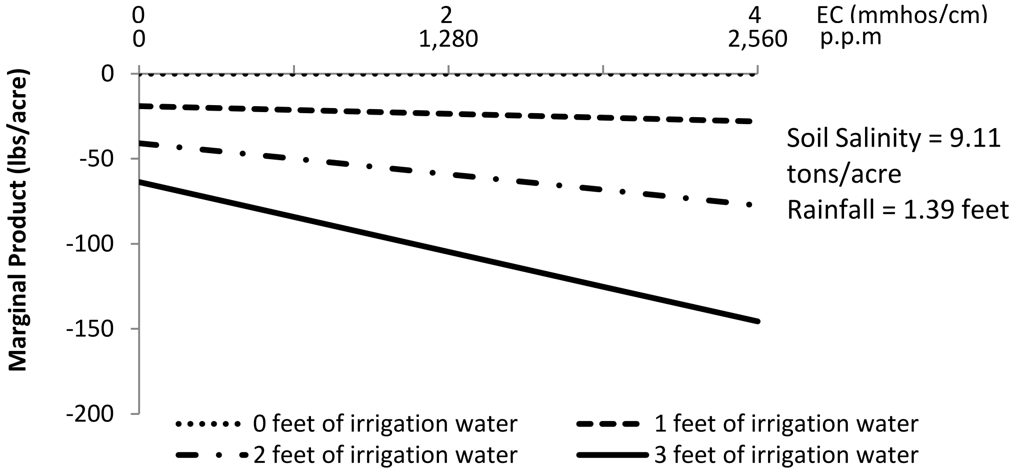
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

Except for a linear term of Total Salinity, all parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 5-2.

Part (a) of Figure 5-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 5-2. Marginal Product of Irrigation Water and Salt Concentration for the Spur Clay Loam Soil

Part (b) of Figure 5-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 5-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

= .

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -223.3, -211.9, -10.5 and -2.6. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

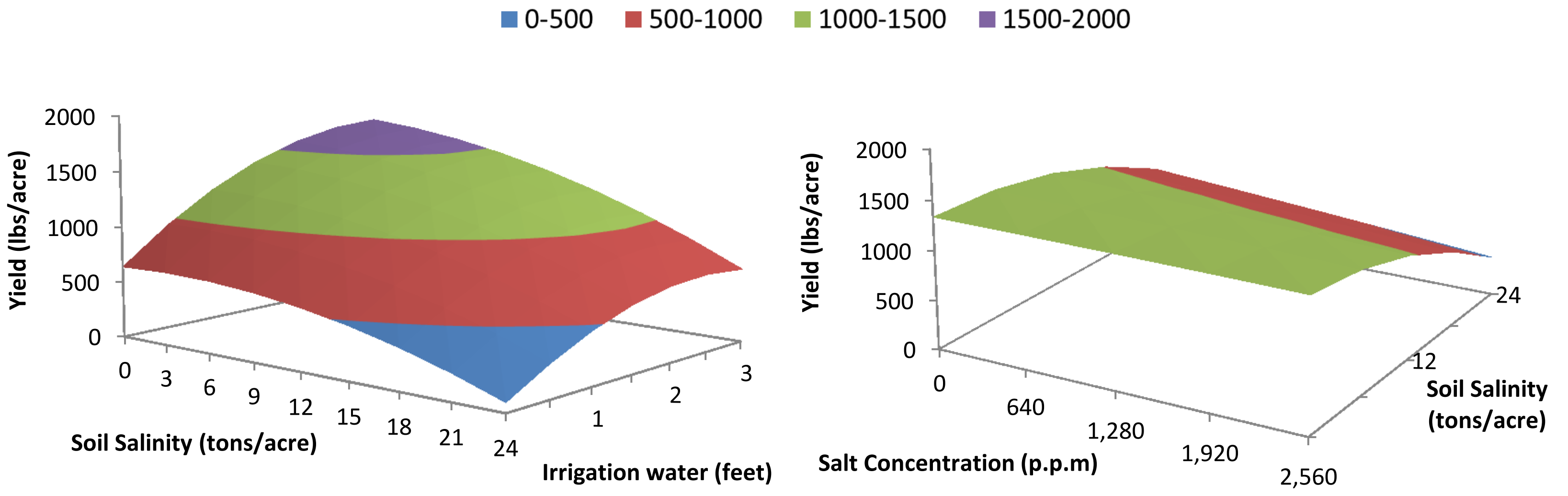
,

,

, and

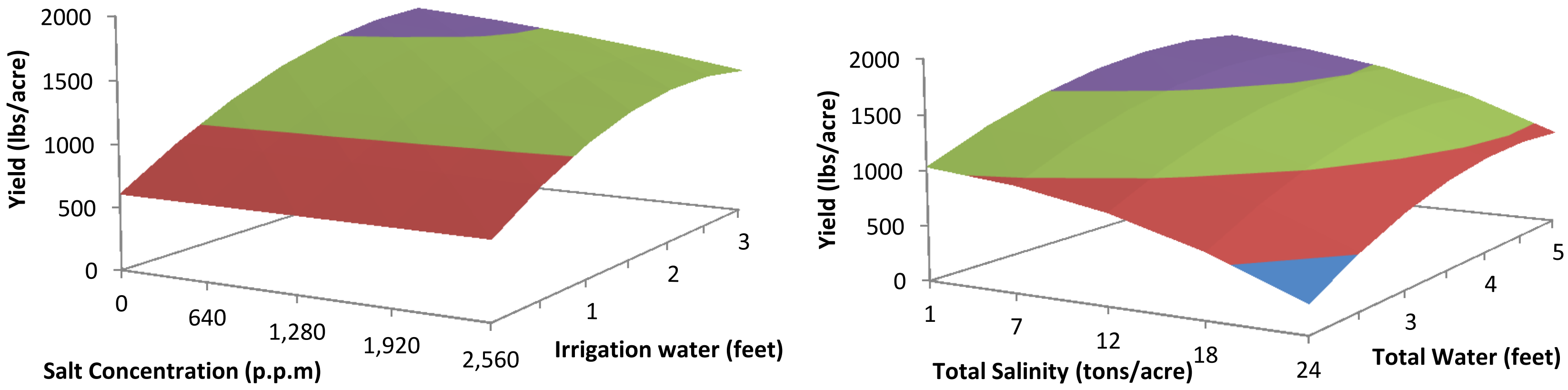
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Spur Clay Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 5-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 5-3. 3-D Surface of Crop Response Function versus Independent Variables for the Spur Clay Loam Soil

In part (a) of Figure 5-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 5-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure5 -3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 9.11 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 5-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 5-5, 5-6 and 5-7, respectively.

Table 5-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Spur Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.6866*	0.0811
Irrigation Water		-0.4853*	0.0364
Amount of Salt in Irrigation Water		0.7046*	0.0142
Soil Salinity at planting		0.9018*	0.0038
Growing Season Rainfall		-1.3533*	0.0394

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 5-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13183 and 13078, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 5-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Spur Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.2706*	0.0516
Soil Salinity at Harvest on Previous Year		0.9216*	0.0022
Non-Growing Season Rainfall		-1.7321*	0.0652

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 5-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12729 and 12550, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 5-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Spur Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		9959.74*	2619.52
Irrigation Water		-7177.03*	967.91
Growing Season Rainfall		15502*	1350.87

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 5-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 5-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 5-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 5-7.

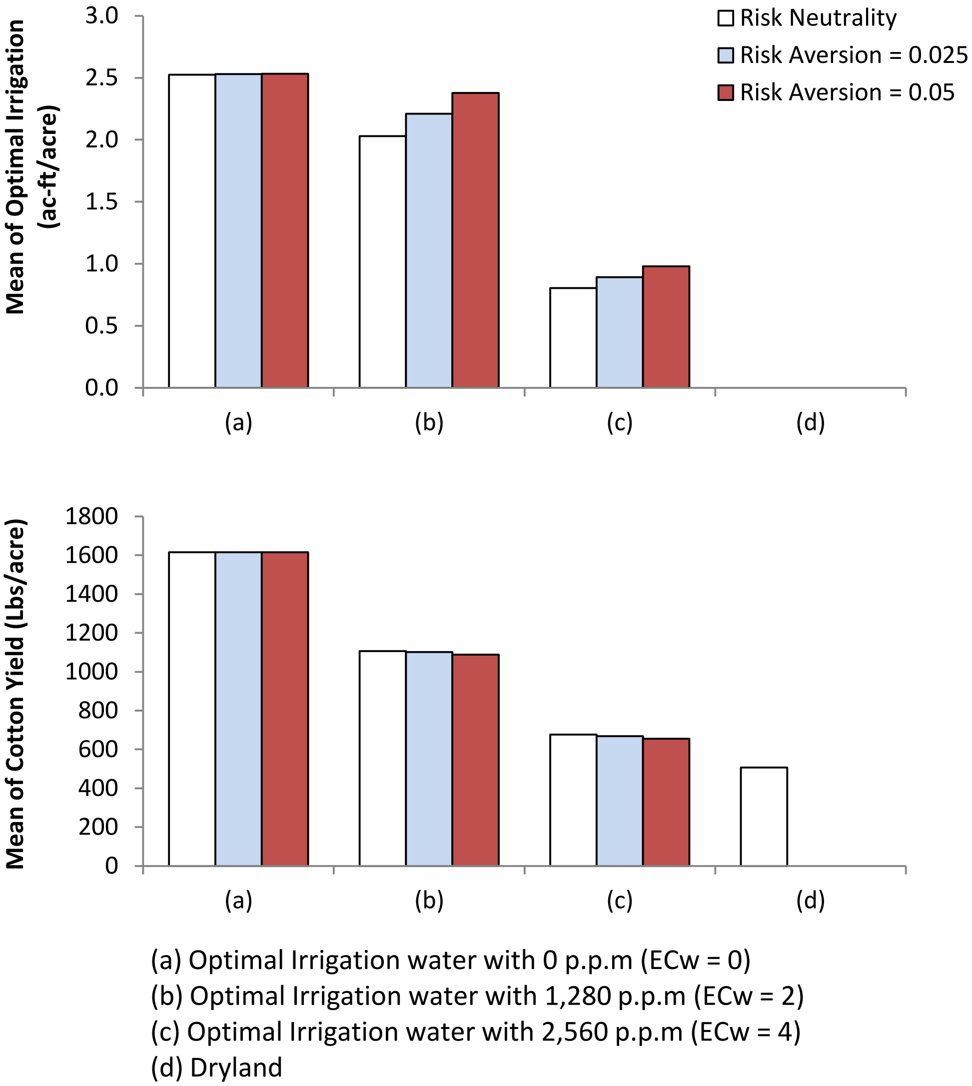


Figure 5-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Spur Clay Loam Soil

The lower half of Figure 5-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 5-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 5-5.

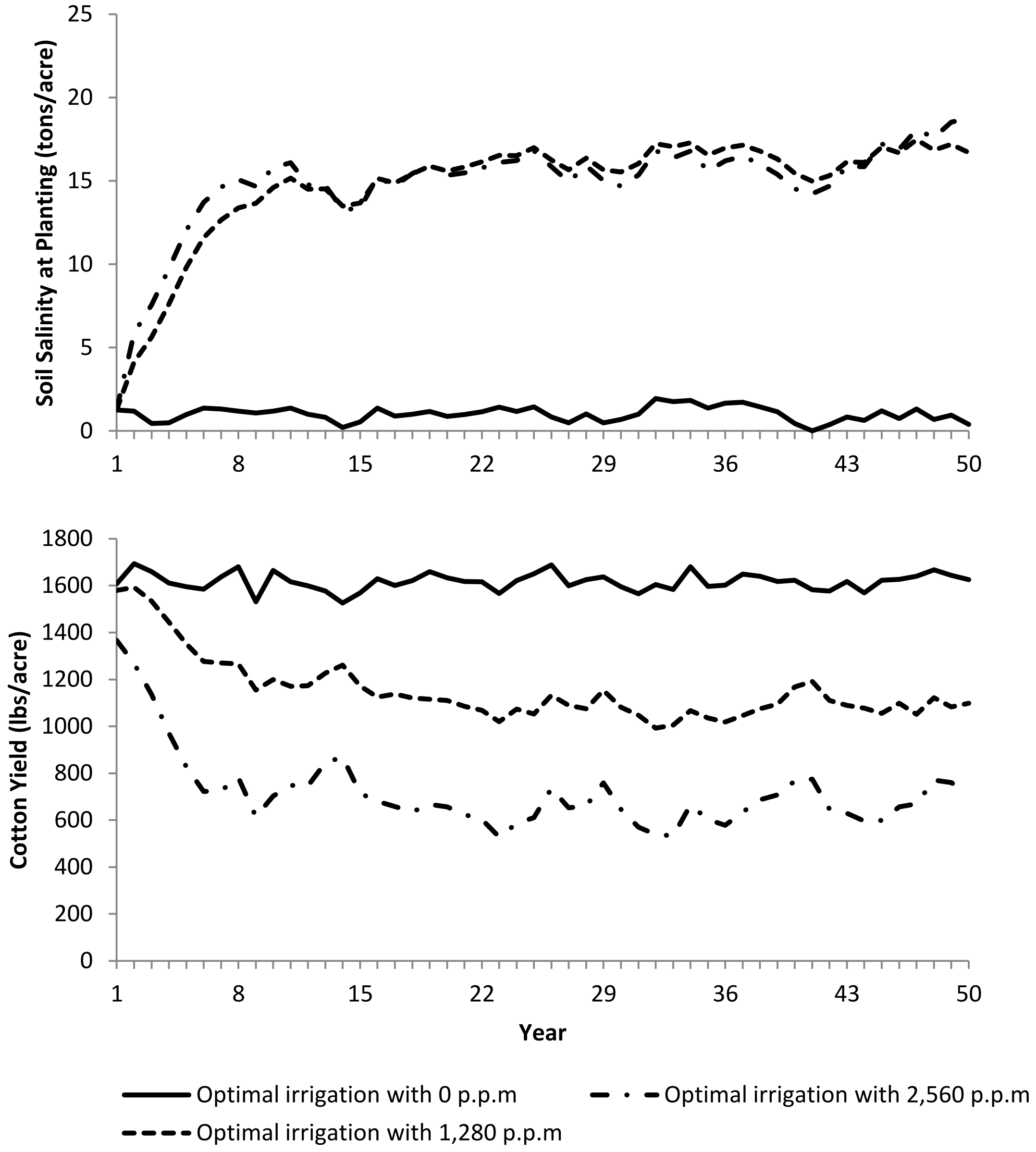
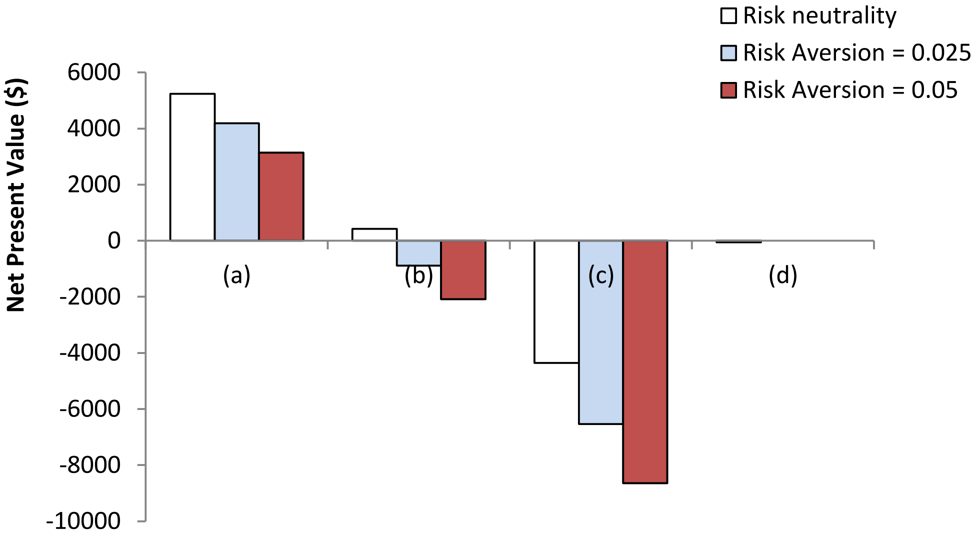


Figure 5-5. Changes of Quantity of Soil Salinity over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Spur Clay Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 5-4 and 5-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 5-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 5-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Spur Clay Loam Soil

In (b) and (c) of Figure 5-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly positive and larger than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 5-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 5-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Spur Clay Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5238	5297
1,280 p.p.m (ECw=2)	426	485
2.560 p.p.m (ECw=4)	-4354	-4295
Dryland	-59	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 0 and 1,280 p.p.m is applied, the differences of their NPVs are positive. It indicates the producer can make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 5-4).

Tillman Clay Loam Soil, 1-3% Slope

EPIC Output Data

Table 6-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Tillman Clay Loam soil. The total salt in the 1.5 meter profile was calculated as 5.66 tons/acre on the first day of simulation. The initial level of salinity in the Tillman Clay Loam soil is higher than other soil types. When irrigation water is applied to the crop land, it is infiltrated into the soil. Since the infiltration of water into the clay is slower than into the sand, Clay or Clay Loam soil types hold more water than Sandy soil types (Brouwer *et al*, 1985). These soil types also have higher irrigation efficiency than Sandy soil types (Sammis and Mexal, 1999).

Table 6-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Tillman Clay Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.38	1.11	1.03	1.4	1.43	1.38	2.94	
WSLT(kg/ha)*	19	231	435	1,471	1,942	3,903	4,695	12,696
Salinity(tons/acre)								5.66**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 6-2.

Table 6-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Tillman Clay Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	1 ~ 1,846	916
Irrigation Water	Irr (ac-ft/acre)	0.28 ~ 2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 35.81	14.07
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 36.4	14.84
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest with given levels of the salt concentration are shown on a box plot in Figure 6-1.

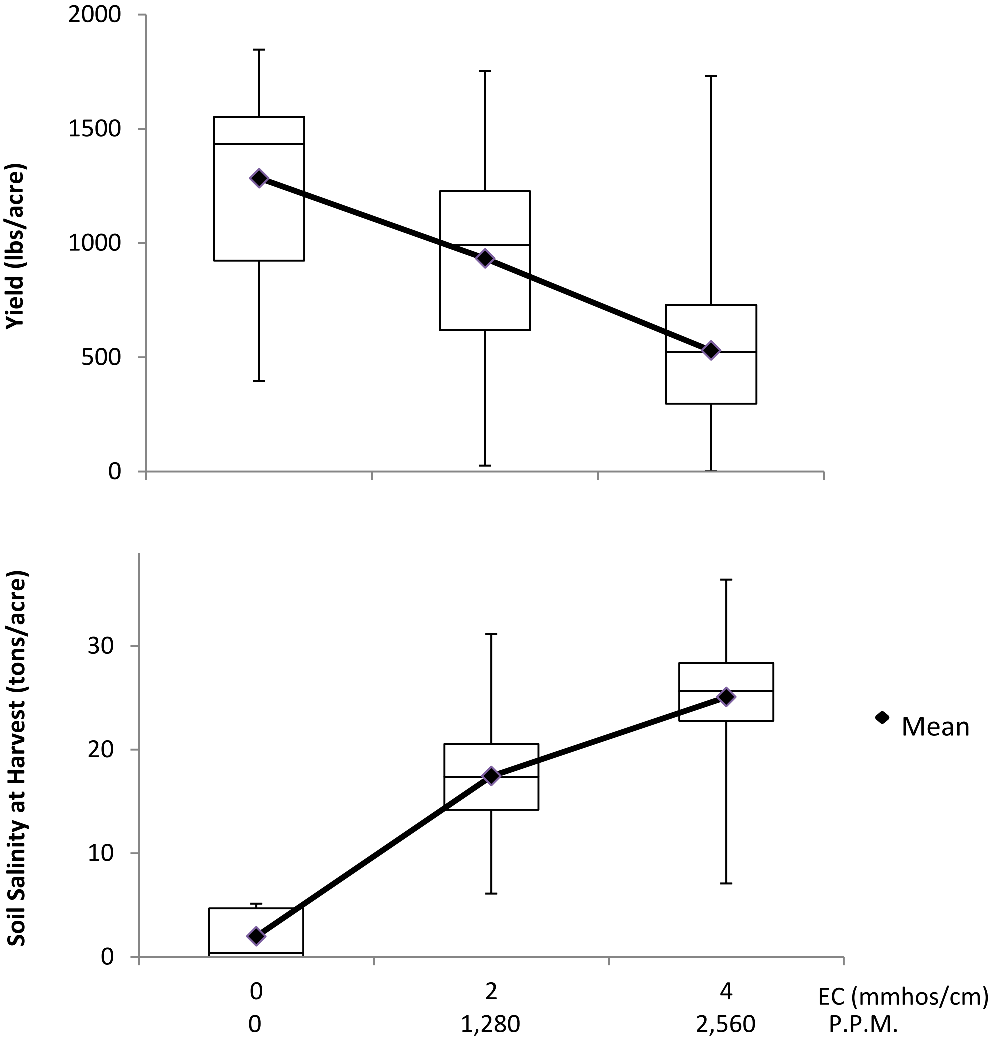


Figure 6-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Tillman Clay Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has a common error message of "Estimated G matrix is not positive definite." which indicates that one or more variance components on the RANDOM statement are estimated as being zero. It should be removed from the model (http://support.sas.com/kb/22/614.html). Without the RANDOM statement which means that the model does not have the random effect part, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 6-3.

Table 6-3. Result of Likelihood Ratio Test for the Tillman Clay Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56561	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56086		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 6-4.

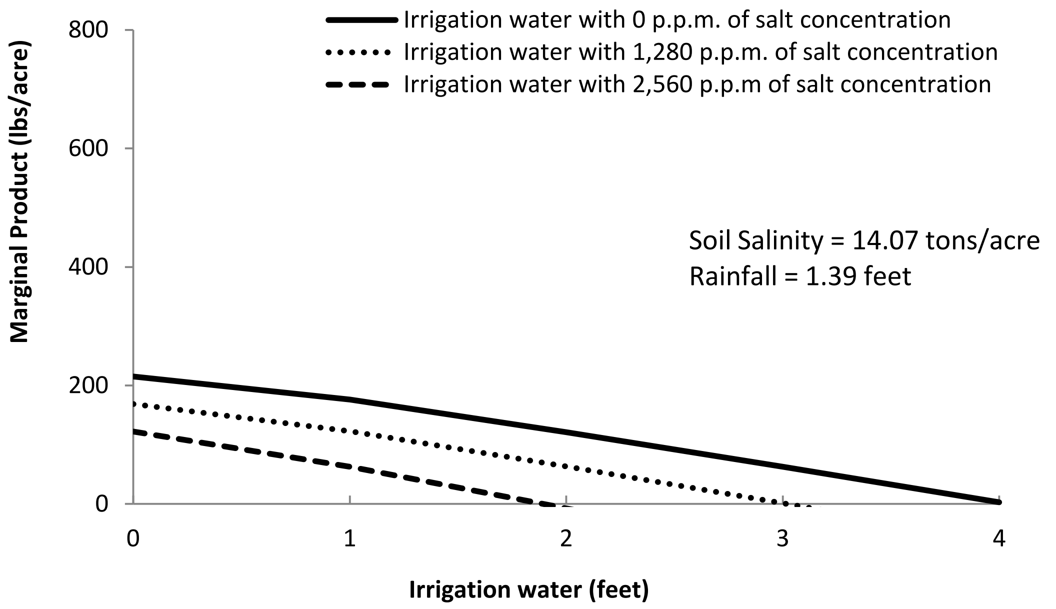
Table 6-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Tillman Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		625.82*	53.2979
Total Water Applied		333.04*	32.0535
Total Salinity		-15.1268*	2.0525
Non-Growing Season Rainfall		74.3944*	7.2338
(Total Water Applied)^2		-30.3992*	4.9898
(Total Salinity)^2		-0.5315*	0.0521
(Total Salinity / Total Water Applied)		4.5628*	2.2539

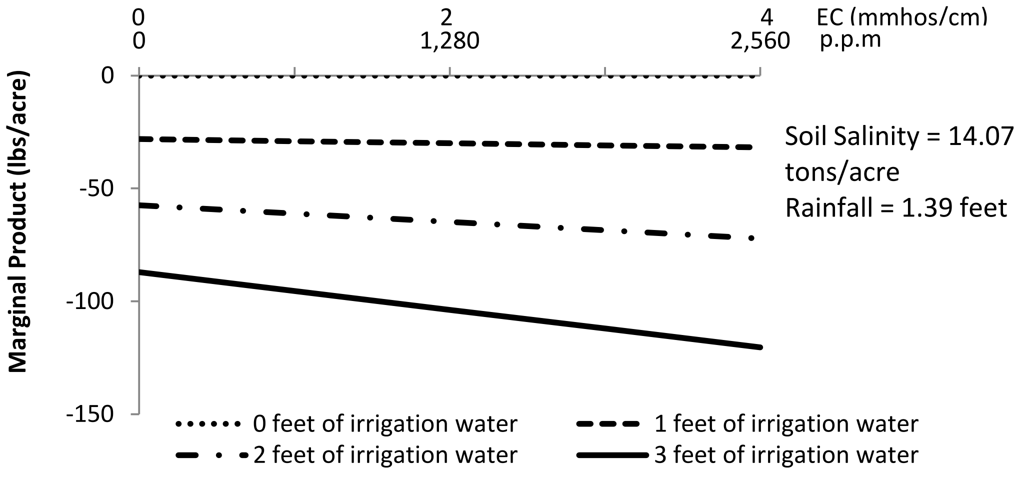
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 6-2.

Part (a) of Figure 6-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 6-2. Marginal Product of Irrigation Water and Salt Concentration for the Tillman Clay Loam Soil

Part (b) of Figure 6-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 6-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2.65 ac-ft/acre, 1.39 feet, 1 tons/ac-ft and 1 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -61.9, -60.3, -7.5 and -1.1. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

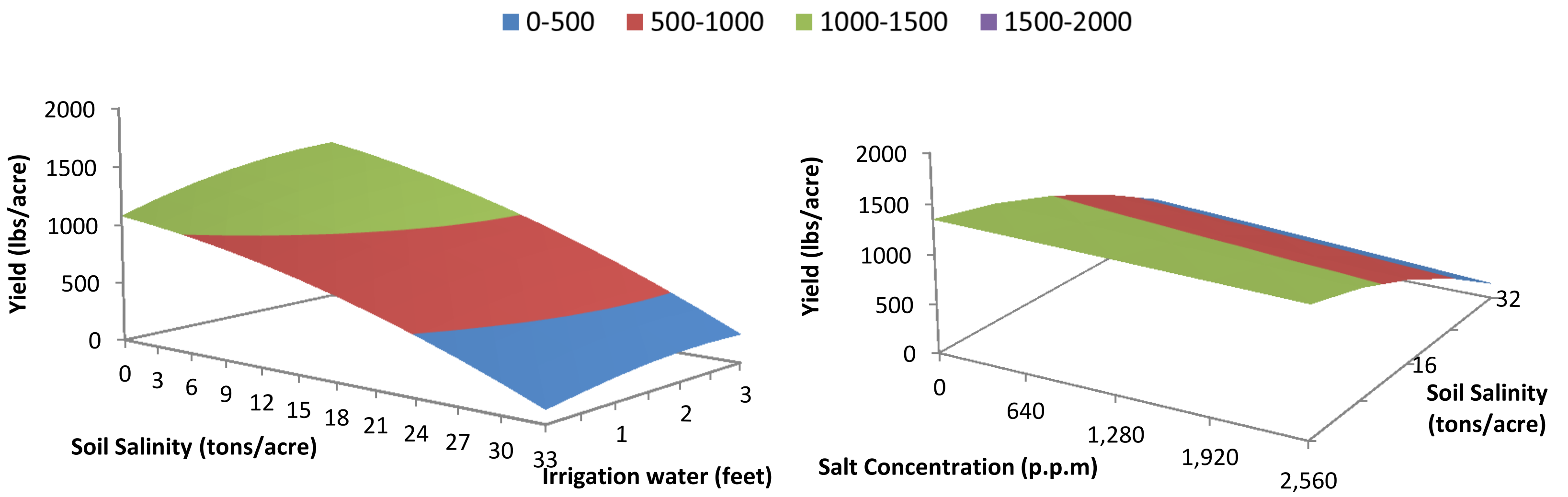
,

,

, and

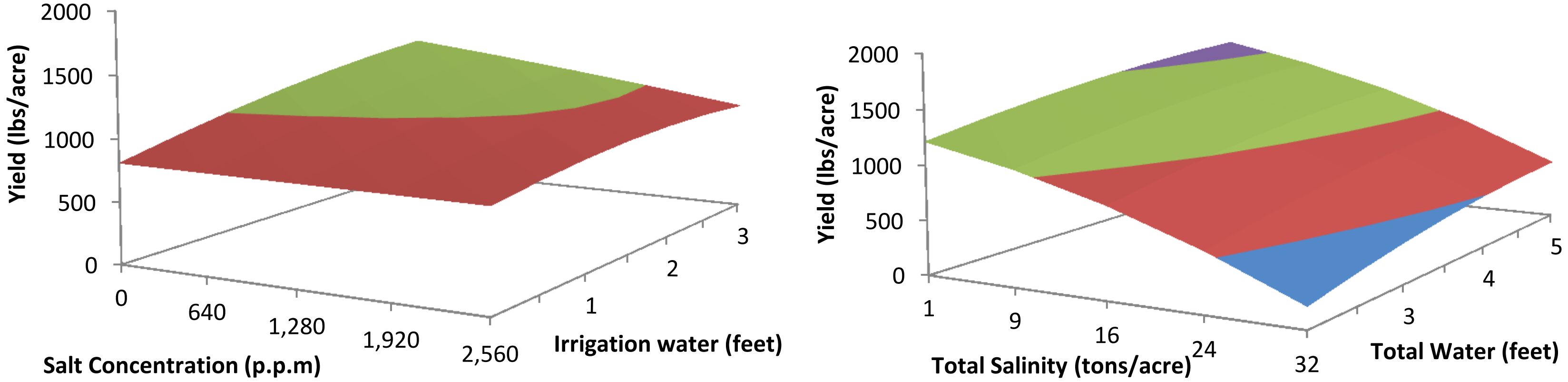
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Tillman Clay Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 6-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 6-3. 3-D Surface of Crop Response Function versus Independent Variables for the Tillman Clay Loam Soil

In part (a) of Figure 6-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. Although irrigation water increases, the crop yield slightly increases. Irrigation water barely affects cotton production. However, with dryland, the crop yield is higher than other soil types.

In part (b) of Figure 6-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 6-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 14.07 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 6-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 6-5, 6-6 and 6-7, respectively.

Table 6-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Tillman Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.9271*	0.0953
Irrigation Water		-0.6801*	0.0432
Amount of Salt in Irrigation Water		0.6960*	0.0151
Soil Salinity at Planting		0.9311*	0.0031
Growing Season Rainfall		-1.2572*	0.0407

Note: (*) indicates parameters significant at the 5% significant level. N=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 6-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11862 and 11758, respectively. As a result of LR test with a p-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 6-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Tillman Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.3701*	0.0573
Soil Salinity at Harvest on Previous Year		0.9494*	0.0018
Non-Growing Season Rainfall		-1.8865*	0.0697

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 6-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11695 and 11525, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 6-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Tillman Clay Loam Soil

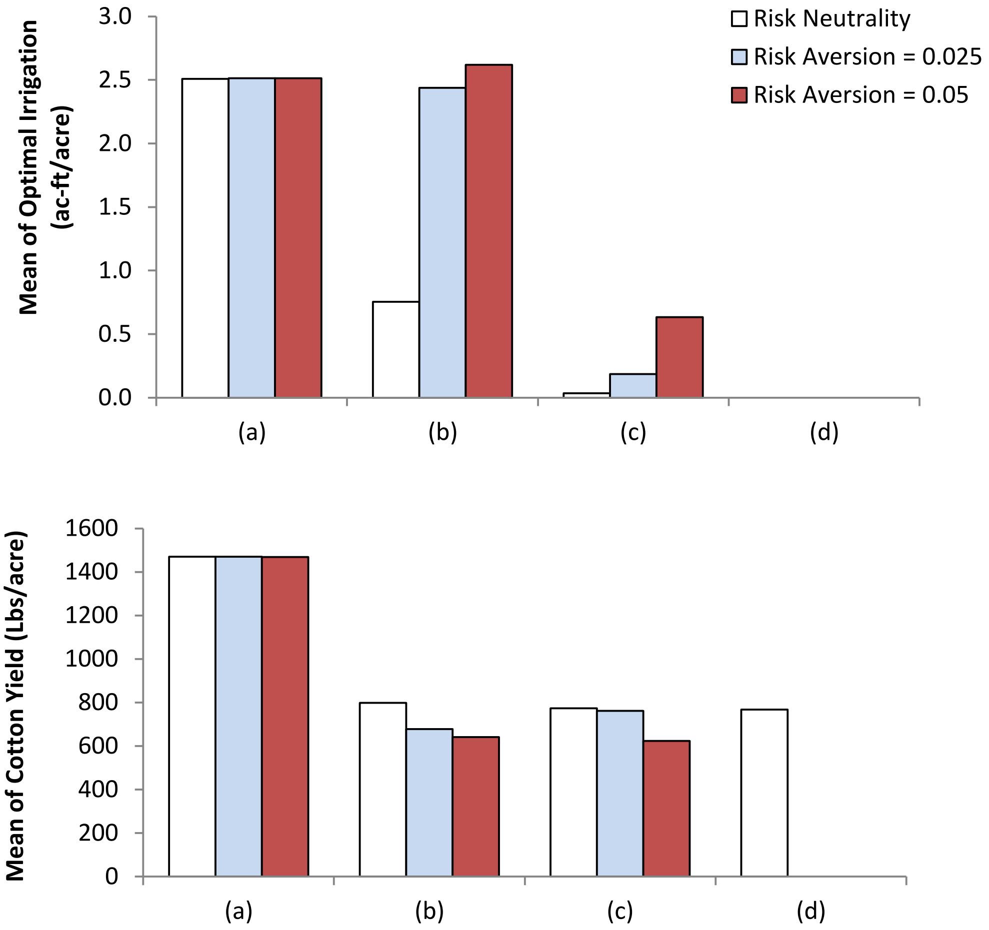
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		65985*	3507.68
Irrigation Water		-20408*	1427.99
Growing Season Rainfall		1012.42	1945.64

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 6-7. The parameter estimate of irrigation water for the yield variance function is significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., but the parameter estimates of the growing season rainfall is not significantly different from zero which means that it is no effect on the yield risk.

Dynamic Optimization

Figure 6-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 6-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use 2 ~ 3 times as irrigation water as the risk-neutral producers use to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 6-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 6-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Tillman Clay Loam Soil

The lower half of Figure 6-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. When irrigation water with the salt concentration is applied, the optimal level of irrigation water is very low. Although the risk-averse producers in (b) relatively used irrigation water as much as the optimal level of irrigation with 0 p.p.m, their cotton yield decreases. Generally, if plenty of irrigation water is used in the soil, the soil salinity is leached and positively affects the crop growth. In Tillman Clay Loam soil, we can expect that there is no leaching effect. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 6-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yields of (b) and (c) are similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 6-5.

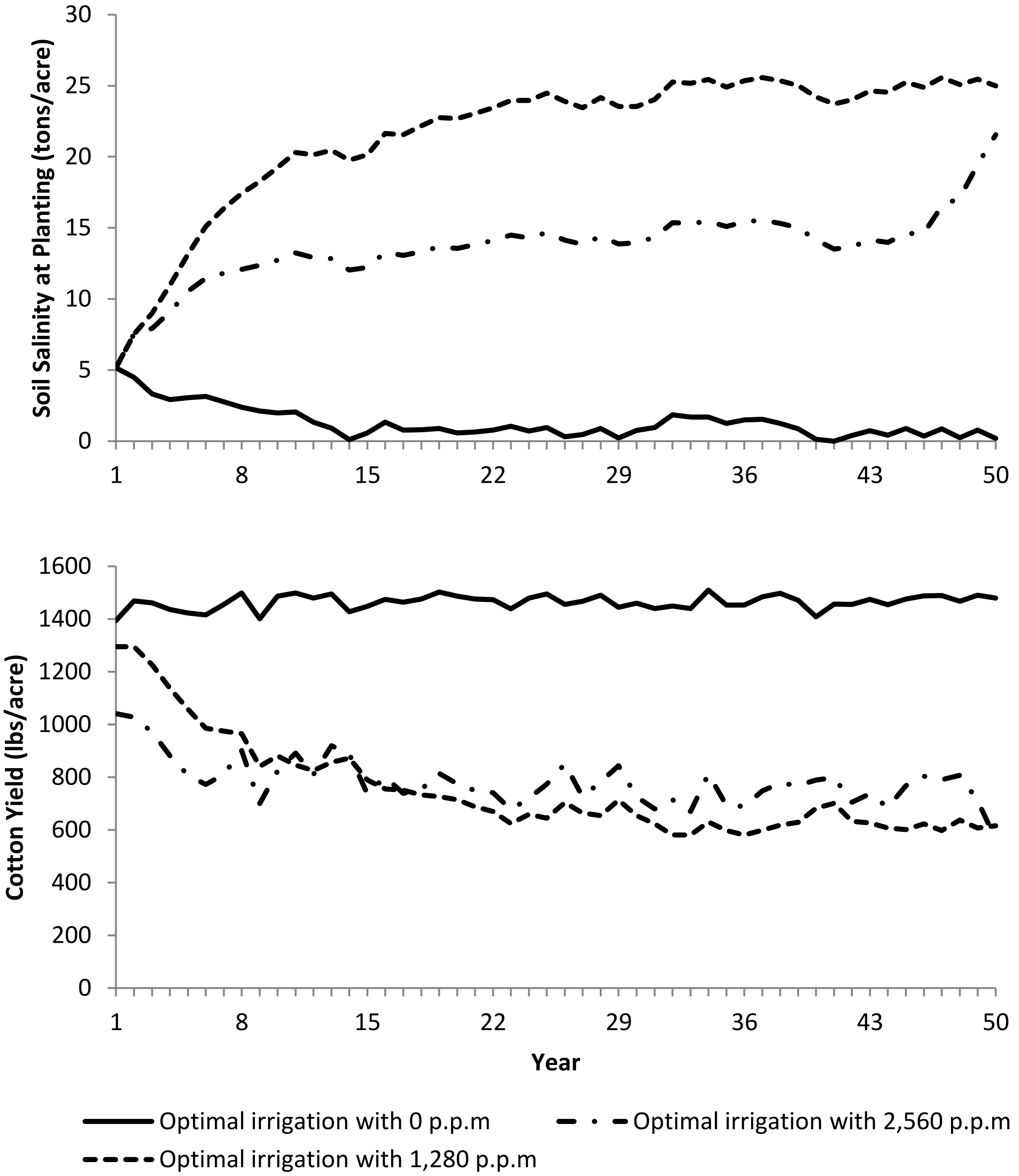


Figure 6-5. Changes of Quantity of Soil Salinity at Planting over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Tillman Clay Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 6-4 and 6-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 6-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

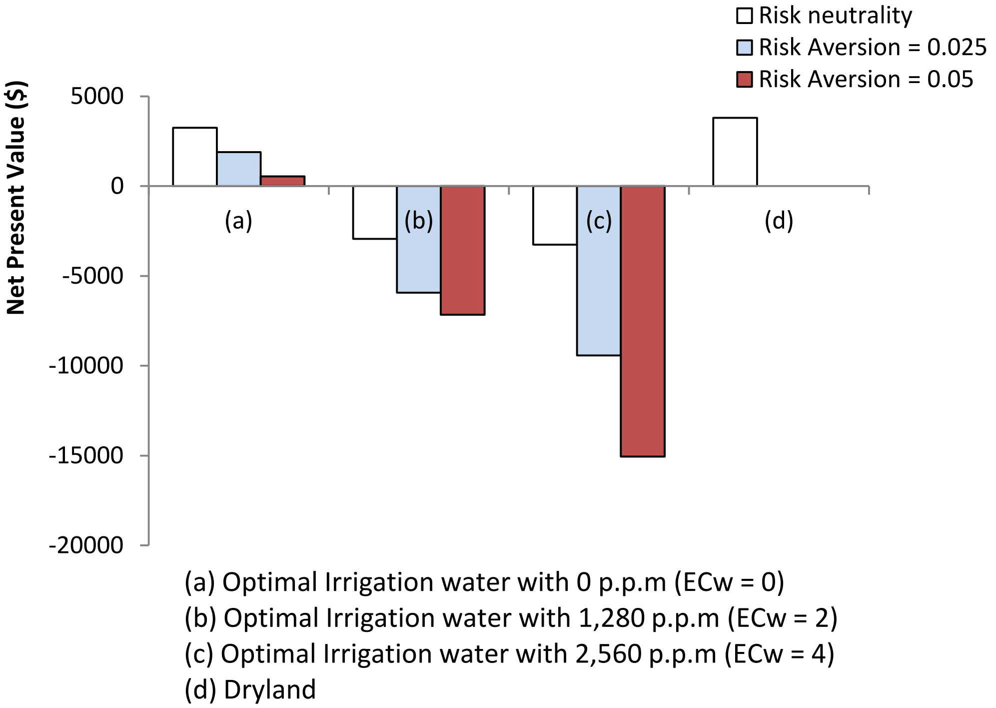


Figure 6-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Tillman Clay Loam Soil

In (b) and (c) of Figure 6-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To

implement the project, irrigation water should be used with 0 p.p.m or slightly more for the producers. Table 6-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 6-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Tillman Clay Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	3242	-562
1,280 p.p.m (ECw=2)	-2936	-6740
2.560 p.p.m (ECw=4)	-3262	-7066
Dryland	3804	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

All the differences between NPV of irrigated and dryland production are negative in Table 6-8. We can expect that the producer using irrigation water in the Tillman Clay Loam soil cannot make profit from investment in irrigated production compared to dryland cotton production. It means than the dryland producer has better profits than the producer using irrigation water.

Frankirk Loam Soil, 1-3% Slope

EPIC Output Data

Table 7-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Frankirk Loam soil. The total salt in the 1.5 meter profile was calculated as 2.22 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 7-2.

Table 7-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Frankirk Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.67	1.3	0.92	1.16	1.85	7.17	2.16	
WSLT(kg/ha)*	17	212	162	406	645	2,495	1,039	4,976
Salinity(tons/acre)								2.22**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 7-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Frankirk Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	4 ~ 1825	977
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.28
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 26.7	10.1
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 29.39	10.88
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 7-1.

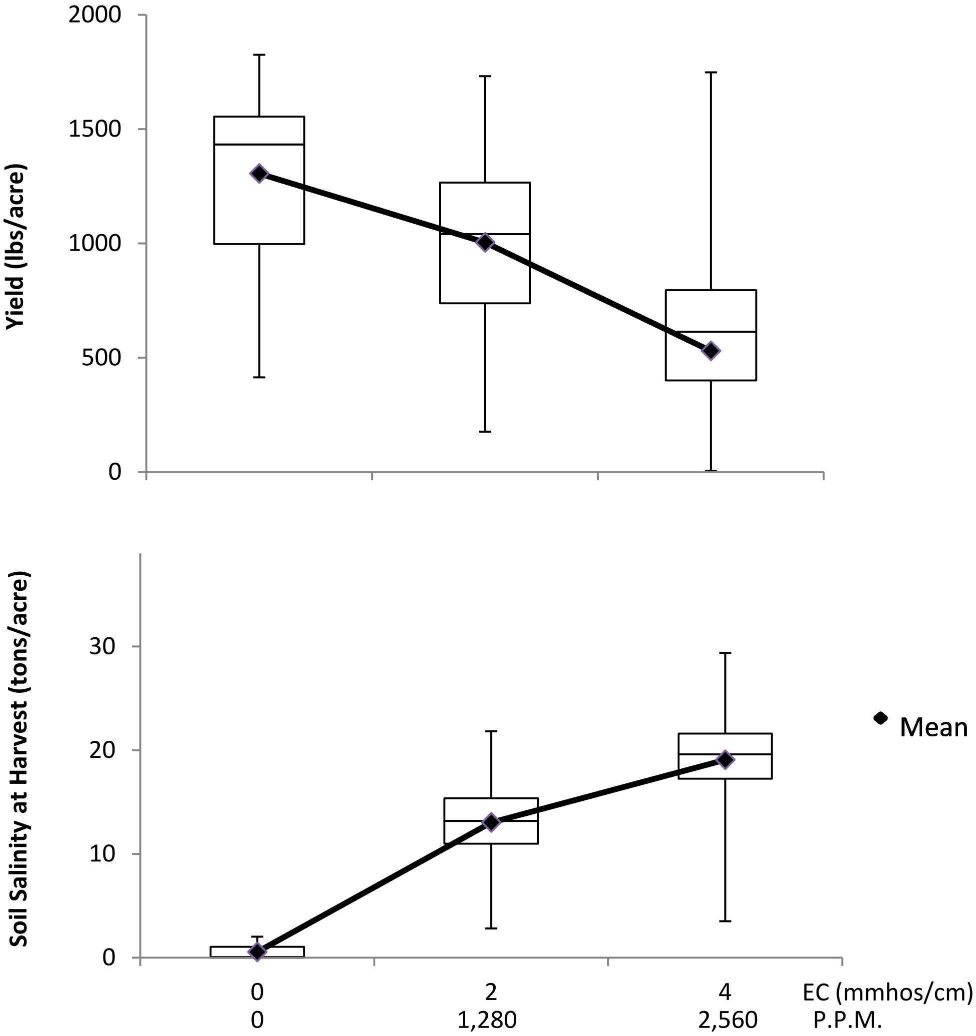


Figure 7-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Frankirk Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 7-3.

Table 7-3. Result of Likelihood Ratio Test for the Frankirk Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56684	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56569		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 7-4.

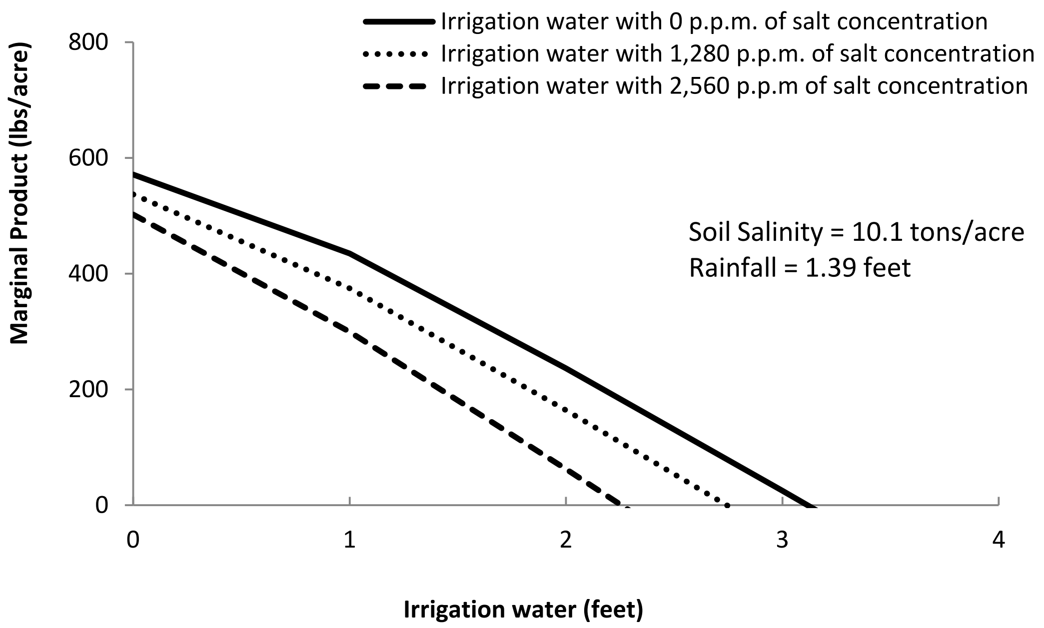
Table 7-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Frankirk Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-661.72*	44.6540
Total Water Applied		1003.36*	31.2836
Total Salinity		-9.8095*	1.2552
Non-Growing Season Rainfall		118.1*	9.6770
(Total Water Applied)^2		-109.98*	5.4464
(Total Salinity)^2		-1.1999*	0.0381
(Total Salinity / Total Water Applied)		22.427*	2.3361

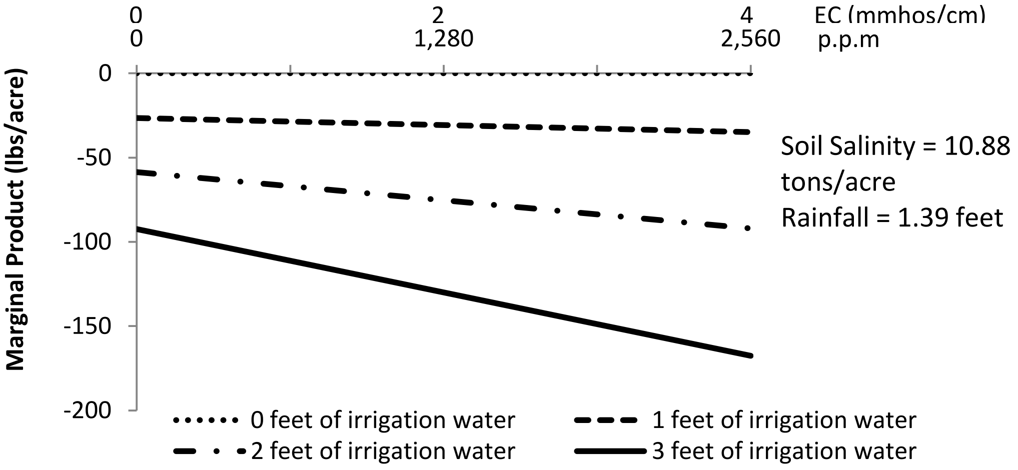
Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 7-2.

Part (a) of Figure 7-2 shows that the crop yield increases as irrigation water increases while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 7-2. Marginal Product of Irrigation Water and Salt Concentration for the Frankirk Loam Soil

Part (b) of Figure 7-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 7-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 4 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_{1,} x_2, x_{3,} x_{4,} x_5)$ except for x_5, the Hessian matrix at the critical point is

= .

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -222, -207.9, -9.6 and -2.4. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

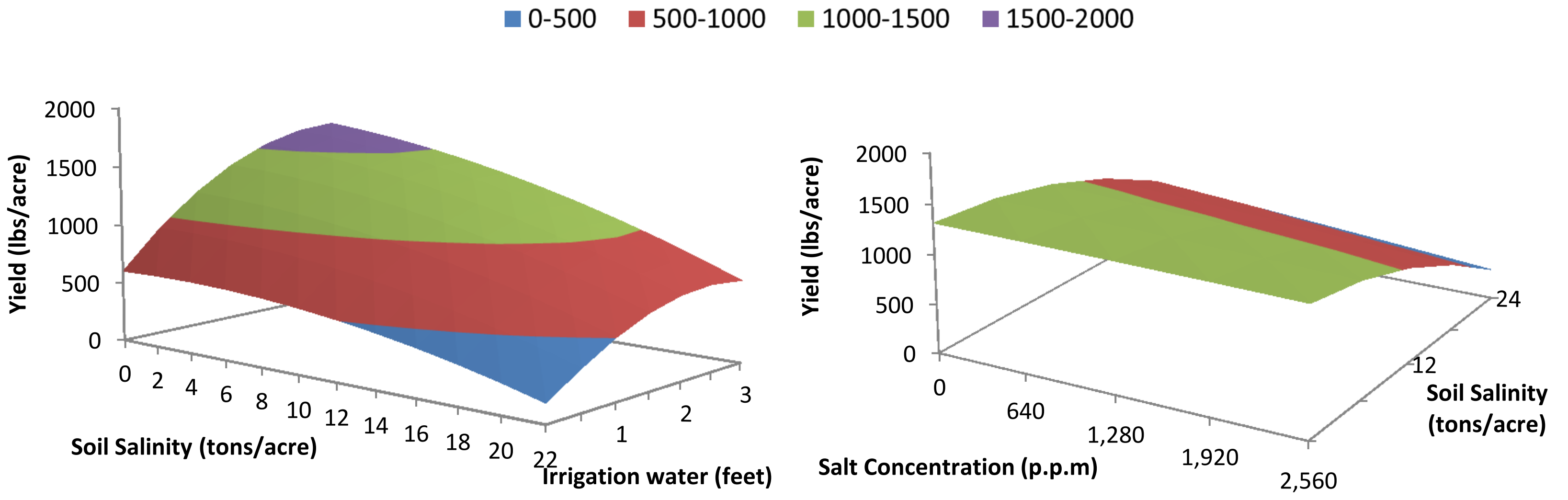
,

,

, and

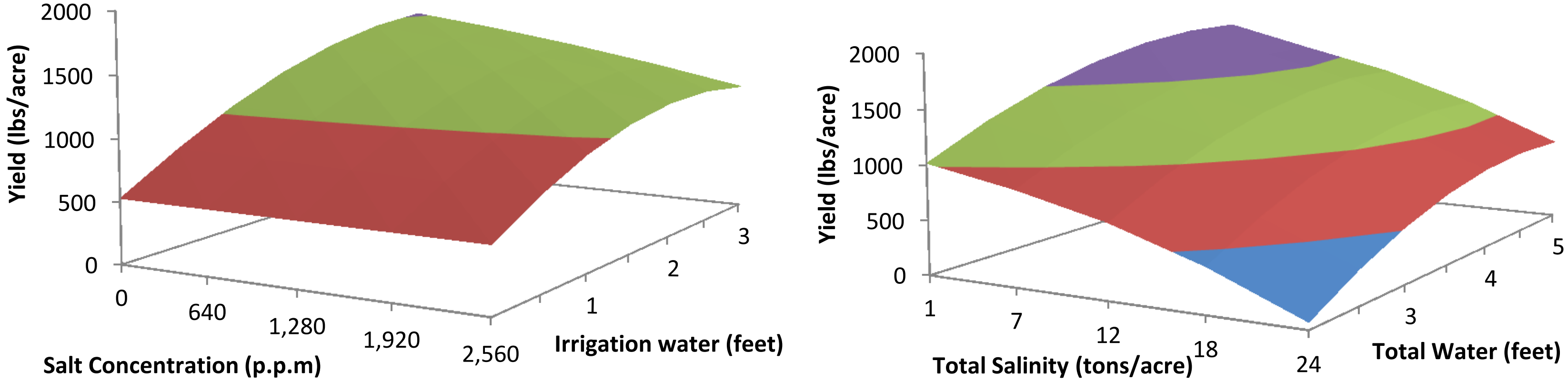
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Frankirk Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 7-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 7-3. 3-D Surface of Crop Response Function versus Independent Variables for the Frankirk Loam Soil

In part (a) of Figure 7-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 7-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.28 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 7-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes, holding 1.39 feet and 10.1 tons/acre of rainfall and soil salinity, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 7-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 7-5, 7-6 and 7-7, respectively.

Table 7-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Frankirk Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.4821*	0.0731
Irrigation Water		-0.4519*	0.0333
Amount of Salt in Irrigation Water		0.7292*	0.0137
Soil Salinity at Planting		0.8899*	0.0039
Growing Season Rainfall		-1.2609*	0.0354

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 7-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12602 and 12528, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 7-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Frankirk Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.247*	0.0490
Soil Salinity at Harvest on Previous Year		0.9139*	0.0022
Non-Growing Season Rainfall		-1.7148*	0.0601

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 7-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12649 and 12432, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 7-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for Frankirk Loam Soil

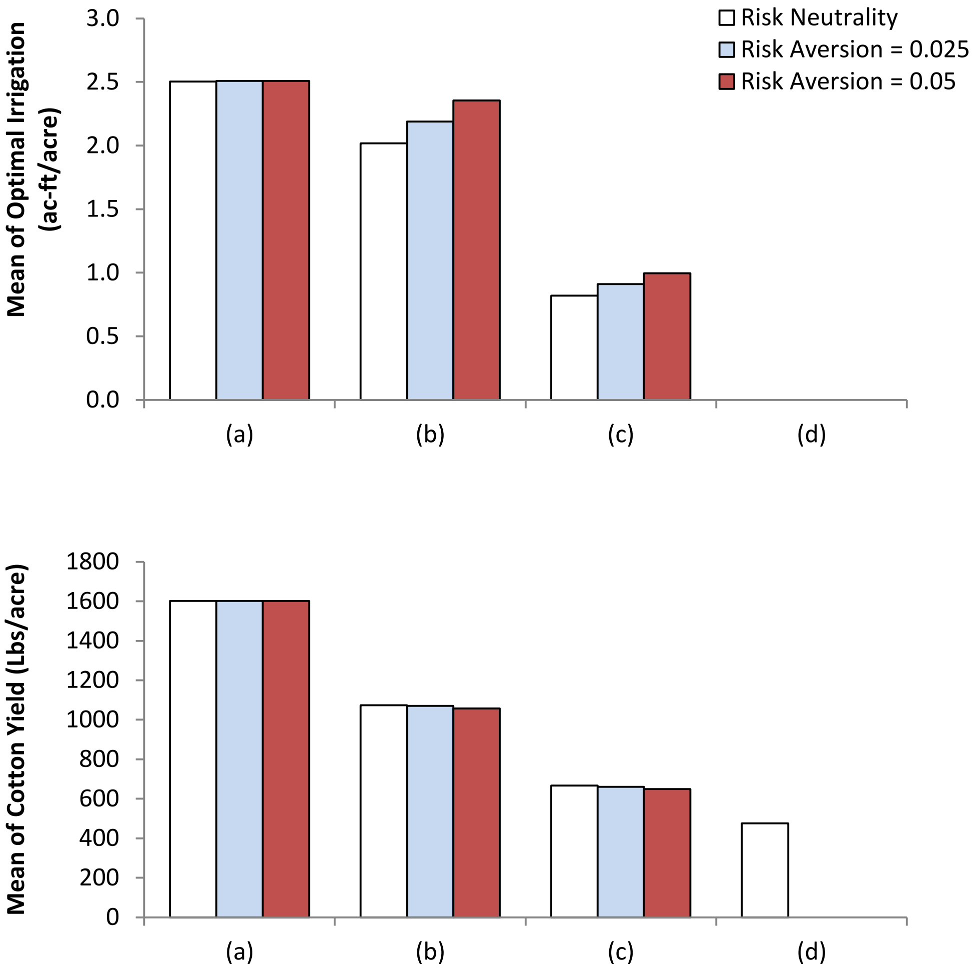
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		8529.4*	2548.51
Irrigation Water		-6721.06*	981.73
Growing Season Rainfall		16062*	1362.74

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 7-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 7-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 7-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 7-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 7-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Frankirk Loam Soil

The lower half of Figure 7-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 7-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 7-5.

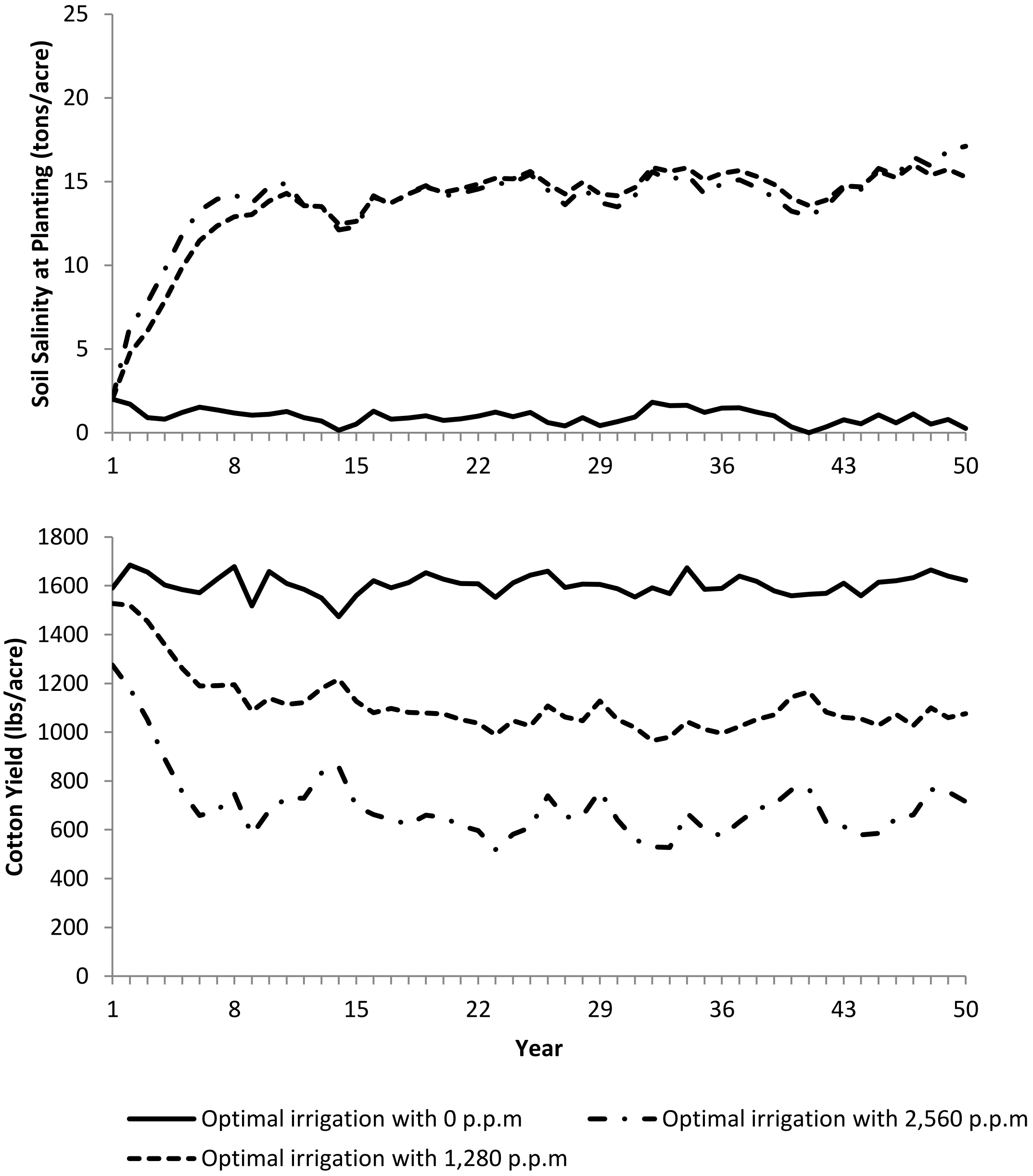
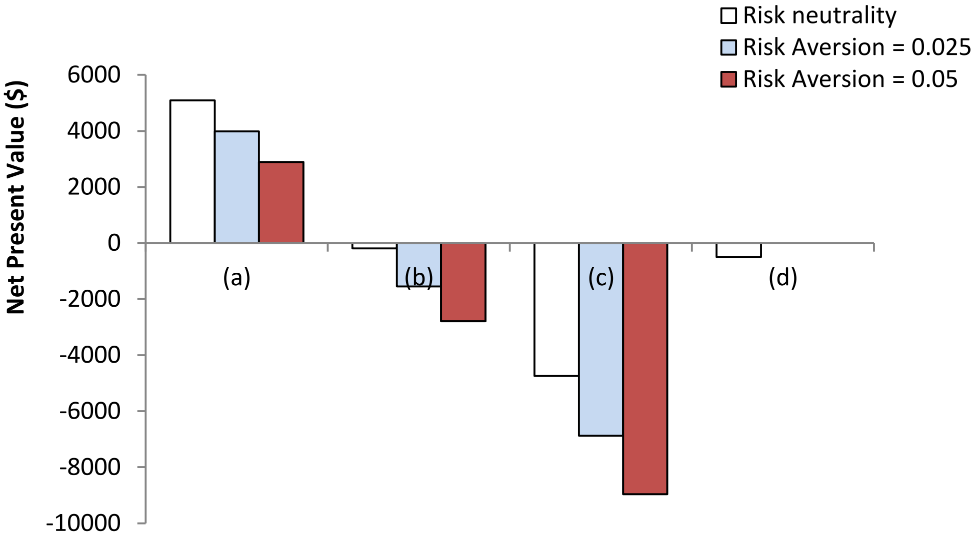


Figure 7-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Frankirk Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 7-4 and 7-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 7-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 7-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Frankirk Loam Soil

In (b) and (c) of Figure 7-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly larger than

NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 7-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 7-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Frankirk Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5088	5586
1,280 p.p.m (ECw=2)	-193	305
2.560 p.p.m (ECw=4)	-4749	-4251
Dryland	-498	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 0 and 1,280 p.p.m is applied, the differences of their NPVs are positive. It indicates the producer can make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 7-4).

Hardeman Fine Sandy Loam Soil, 0-1% Slope

EPIC Output Data

Table 8-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Hardeman Fine Sandy Loam soil. The total salt in the 1.5 meter profile was calculated as 0.31 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 8-2.

Table 8-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Hardeman Fine Sandy Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.42	0.96	0.88	0.5	0.54	0.49	0.58	
WSLT(kg/ha)*	9	101	101	118	126	114	135	704
Salinity(tons/acre)								0.31**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 8-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Hardeman Fine Sandy Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	25 ~ 1705	987
Irrigation Water	Irr (ac-ft/acre)	0.21 ~ 2.22	1.13
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 16.15	5.16
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 18.14	5.88
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 8-1.

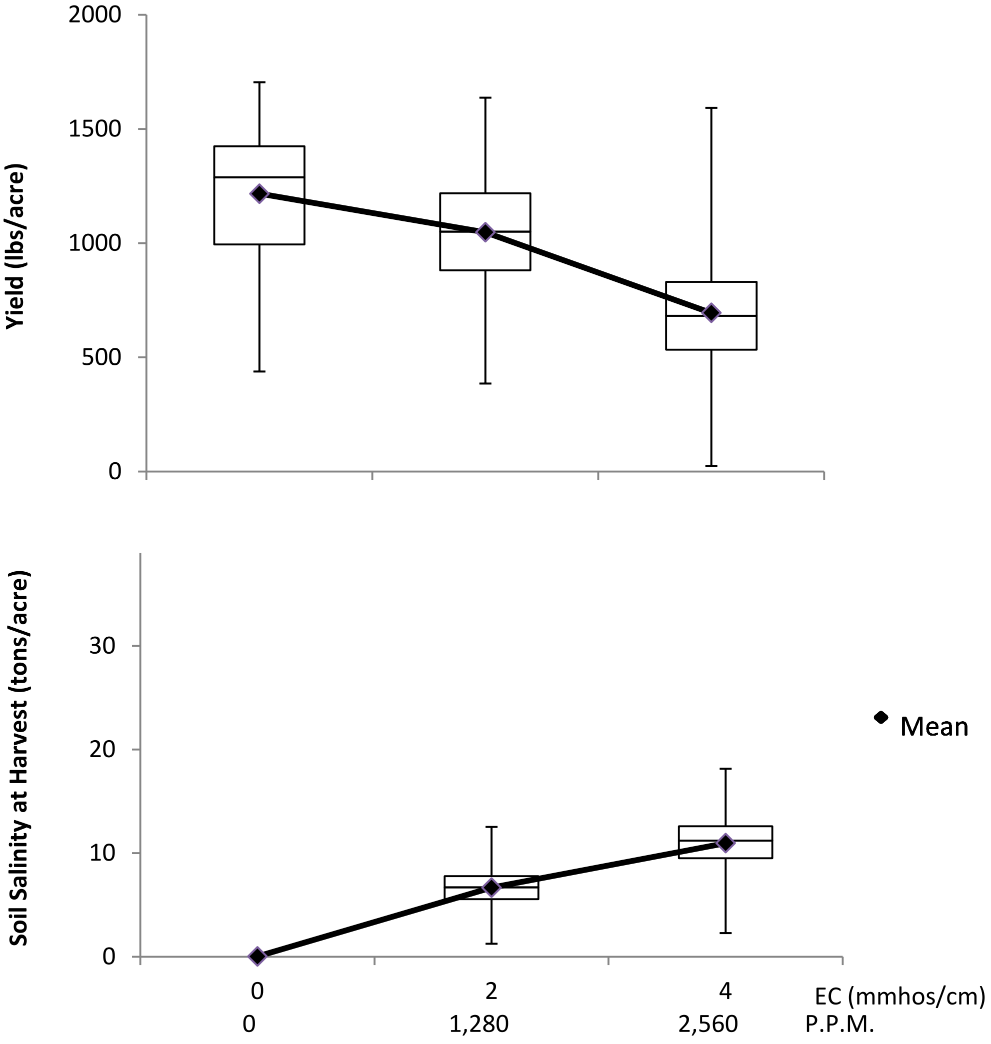


Figure 8-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Hardeman Fine Sandy Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 8-3.

Table 8-3. Result of Likelihood Ratio Test for the Hardeman Fine Sandy Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56068	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	55864		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 8-4.

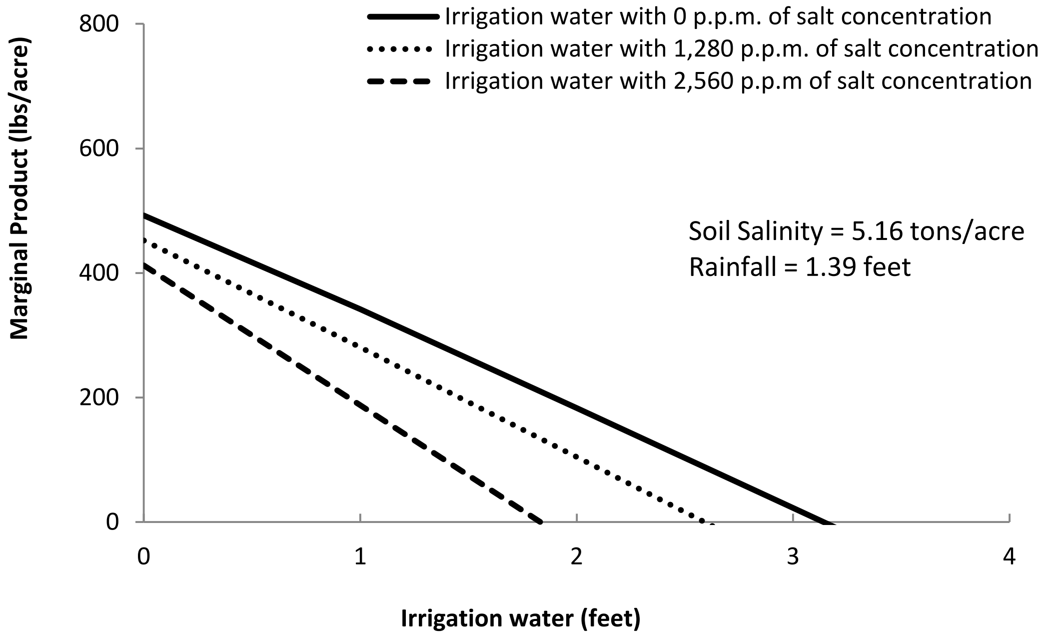
Table 8-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Hardeman Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-172.27*	44.6249
Total Water Applied		733.06*	32.7664
Total Salinity		0.5949	2.2251
Non-Growing Season Rainfall		62.9303*	8.7215
(Total Water Applied)^2		-80.7319*	6.0337
(Total Salinity)^2		-2.6511*	0.0946
(Total Salinity / Total Water Applied)		6.0391*	4.1848

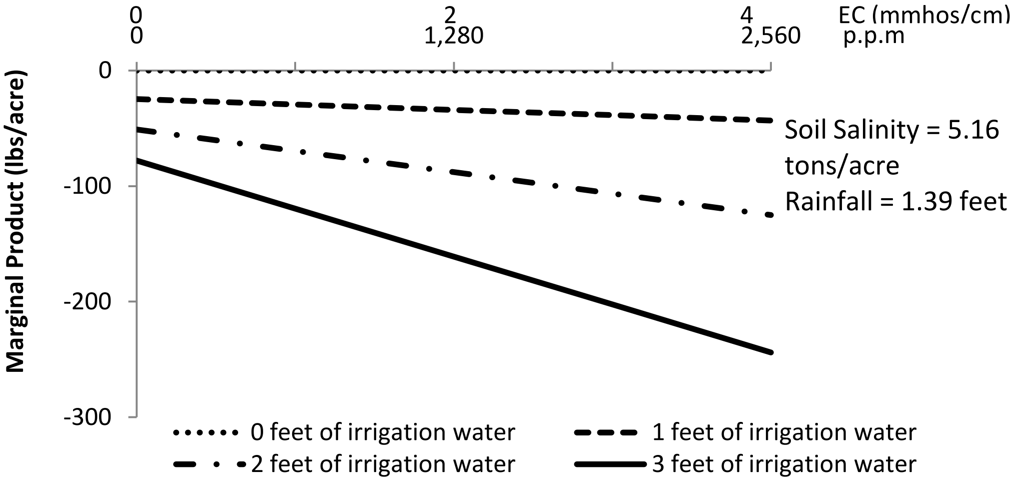
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

Except for the linear term, Total Salinity, all parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 8-2.

Part (a) of Figure 8-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 8-2. Marginal Product of Irrigation Water and Salt Concentration for the Hardeman Fine Sandy Loam Soil

Part (b) of Figure 8-2 verifies that that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 8-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 4 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -177, -159.1, -21.2 and -5.3. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3

sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

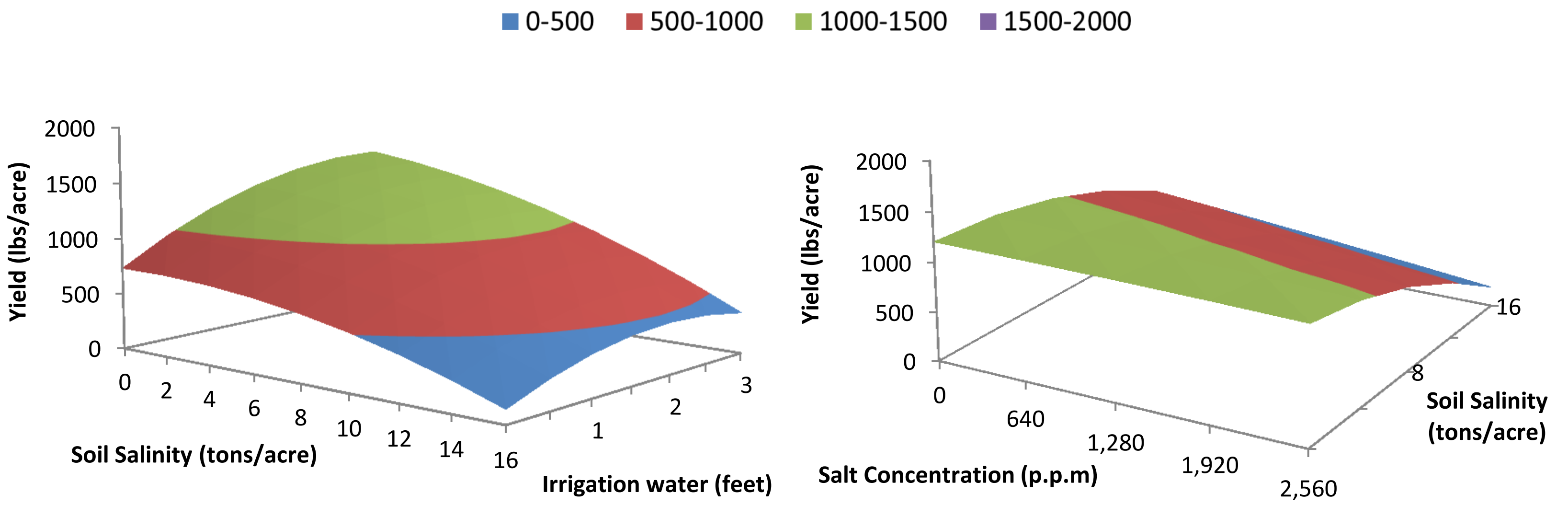
The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

,

, and

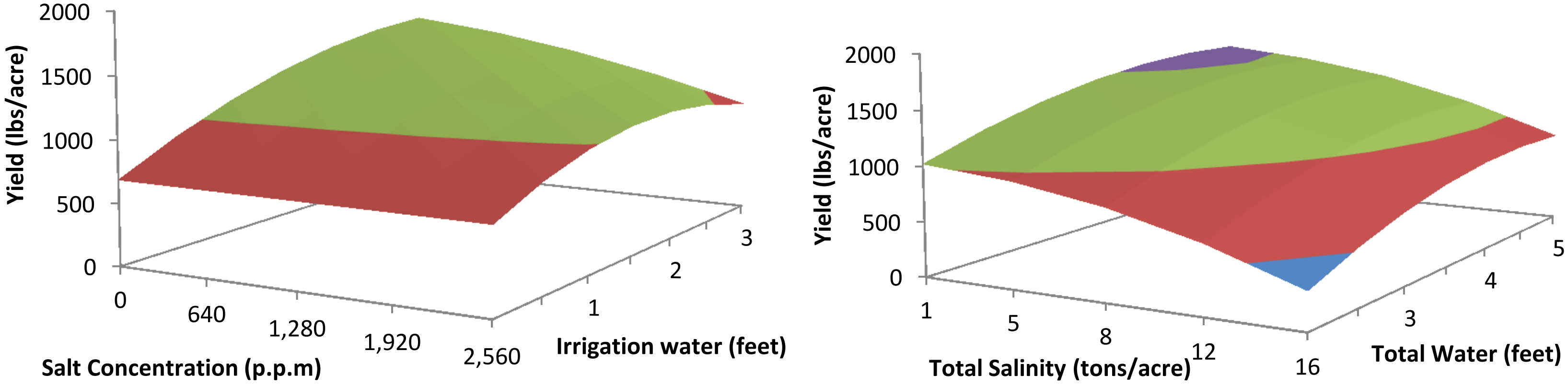
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Hardeman Fine Sandy Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 8-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 8-3. 3-D Surface of Crop Response Function versus Independent Variables for the Hardeman Fine Sandy Loam Soil

In part (a) of Figure 8-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 8-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, holding 1.39 feet and 1.13 feet of rainfall and irrigation water, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 8-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 5.16 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 8-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance

(functions are shown in Tables 8-5, 8-6 and 8-7, respectively.

Table 8-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Hardeman Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.8523*	0.0493
Irrigation Water		-0.3885*	0.0264
Amount of Salt in Irrigation Water		0.7539*	0.0111
Soil Salinity at Planting		0.8515*	0.0042
Growing Season Rainfall		-0.9316*	0.0230

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.
The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 8-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 8292 and 8205, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 8-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Hardeman Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.0541*	0.0349
Soil Salinity at Harvest on Previous Year		0.8711*	0.0023
Non-Growing Season Rainfall		-1.4912*	0.0433

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 8-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 8962 and 8835, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 8-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for Hardeman Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		7952.21*	2276.68
Irrigation Water		-5812.95*	1099.4
Growing Season Rainfall		13149*	1144.71

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 8-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 8-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 8-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 8-7.

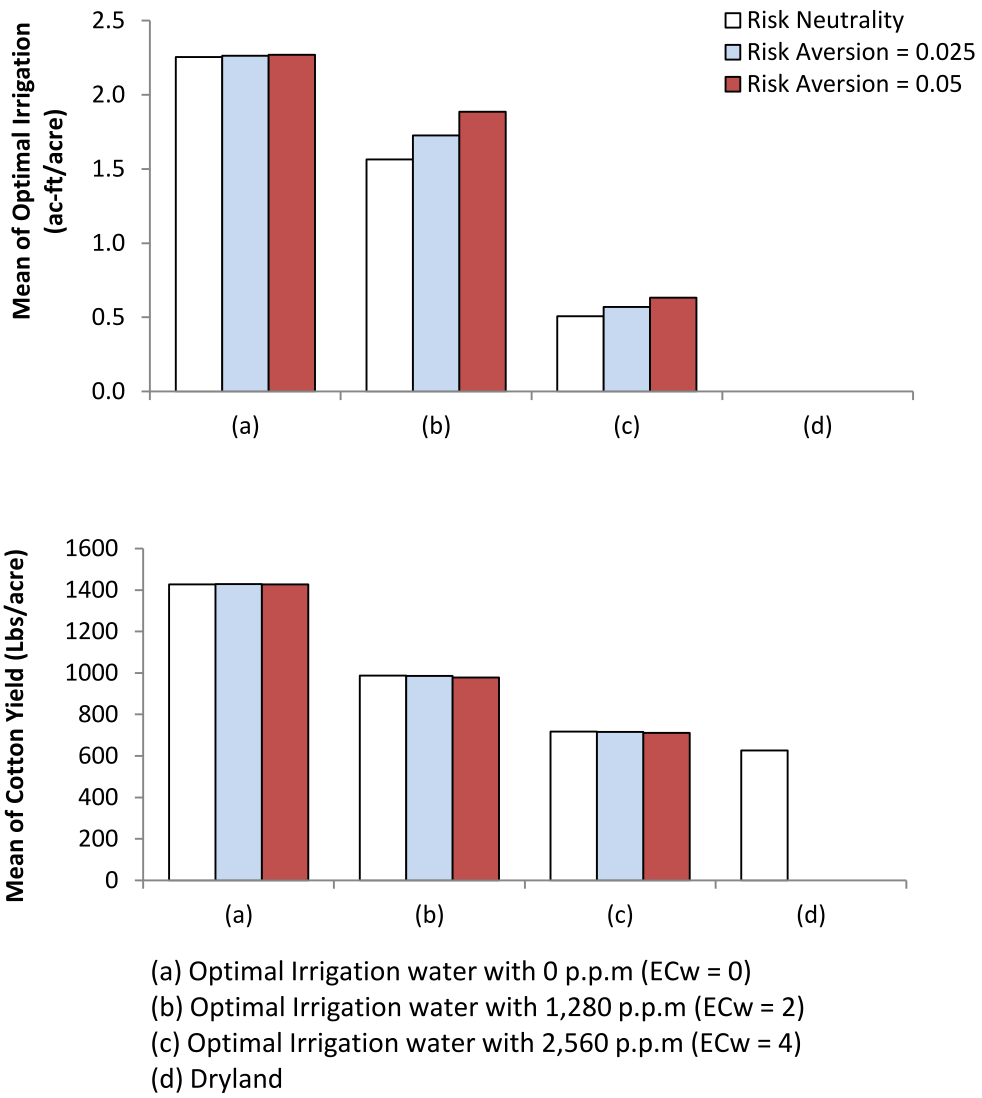


Figure 8-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Hardeman Fine Sandy Loam Soil

The lower half of Figure 8-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. Although the level of irrigation water with 1,280 p.p.m contains some salts, it is optimal to apply slightly less than the optimal level of irrigation with 0 p.p.m.

When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 8-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is slightly larger than (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 8-5.

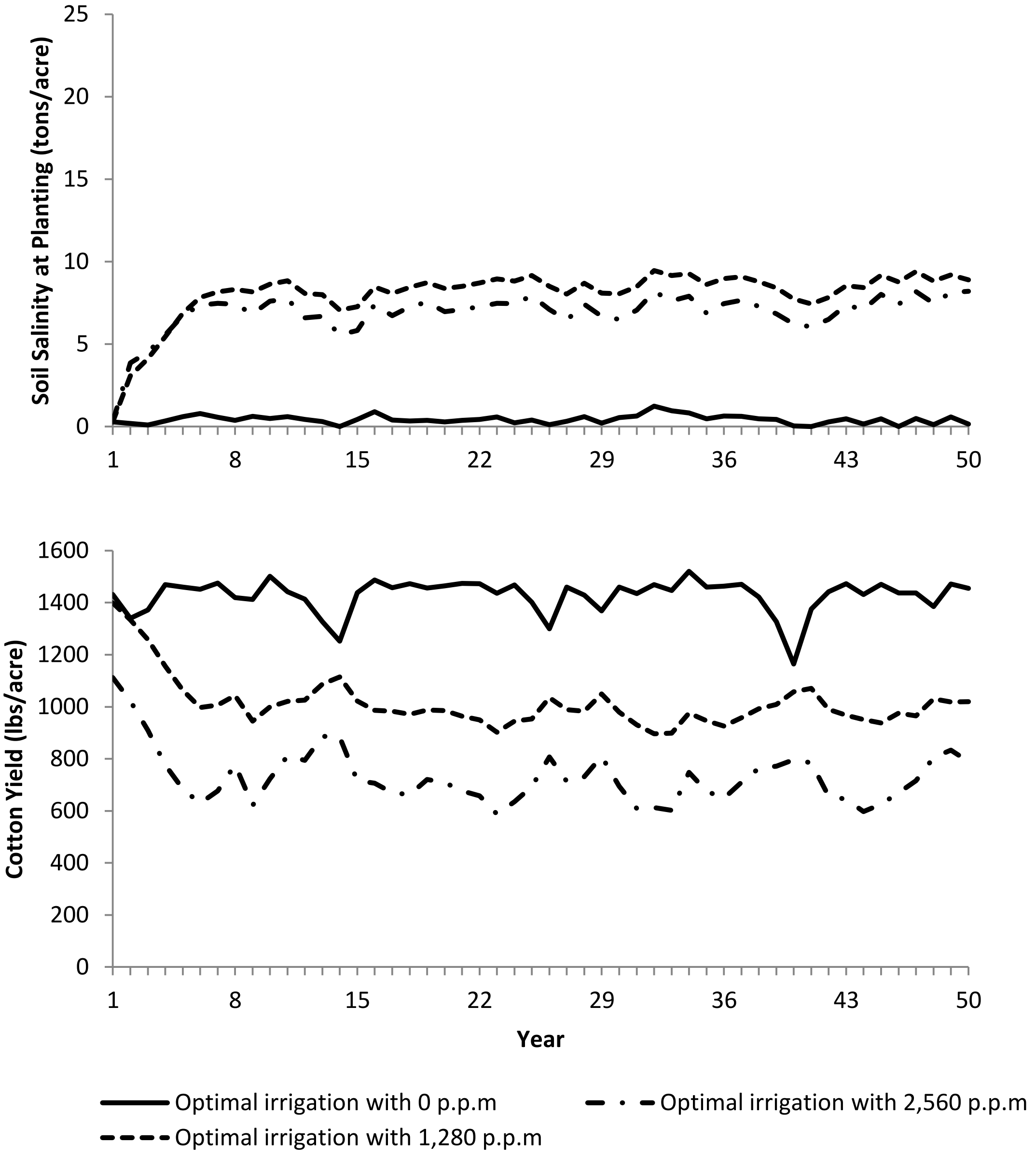
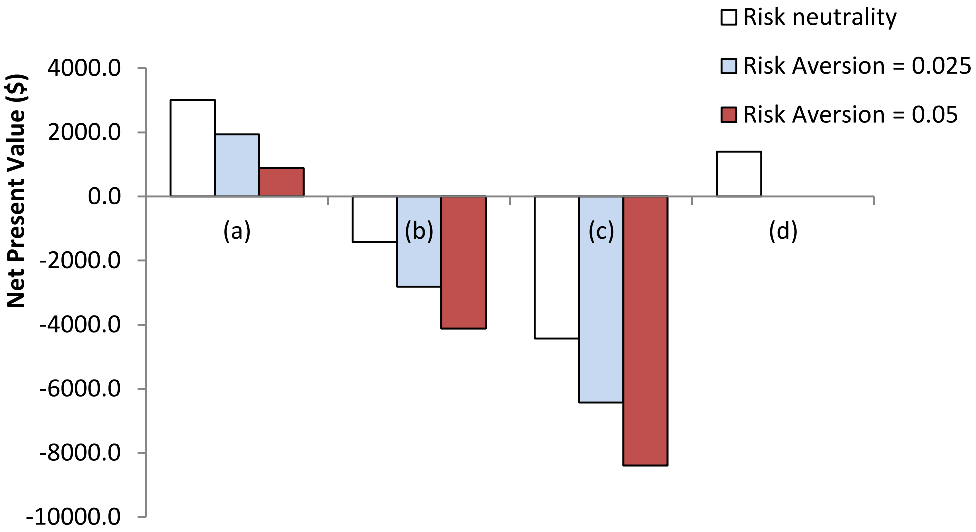


Figure 8-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth =1.5m for the Hardeman Fine Sandy Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 8-4 and 8-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 8-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 8-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for Hardeman Fine Sandy Loam Soil

In (b) and (c) of Figure 8-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To

implement the project, irrigation water should be used with 0 p.p.m or slightly more for the producers. Table 8-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 8-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for Hardeman Fine Sandy Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	3004	1604
1,280 p.p.m (ECw=2)	-1432	-2832
2.560 p.p.m (ECw=4)	-4432	-5832
Dryland	1400	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to slightly less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 1.56 ~ 2.25 ac-ft/acre to maximize NPV of expected utility (see Figure 8-4).

Lawton Loam Soil, 0-1% Slope

EPIC Output Data

Table 9-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Lawton Loam soil. The total salt in the 1.5 meter profile was calculated as 2.43 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 9-2.

Table 9-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Lawton Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	2.69	2.05	2.62	3.14	2.20	1.89	2.60	
WSLT(kg/ha)*	25	311	525	1470	1028	881	1211	5,452
Salinity(tons/acre)								2.43**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 9-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Lawton Loam Soil

Variable	**Symbol (Unit)**	**Range**	**Mean**
Cotton Yield	Y (lbs/acre)	12 ~ 1851	1025
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 29.9	10.8
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 32.49	11.58
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 9-1.

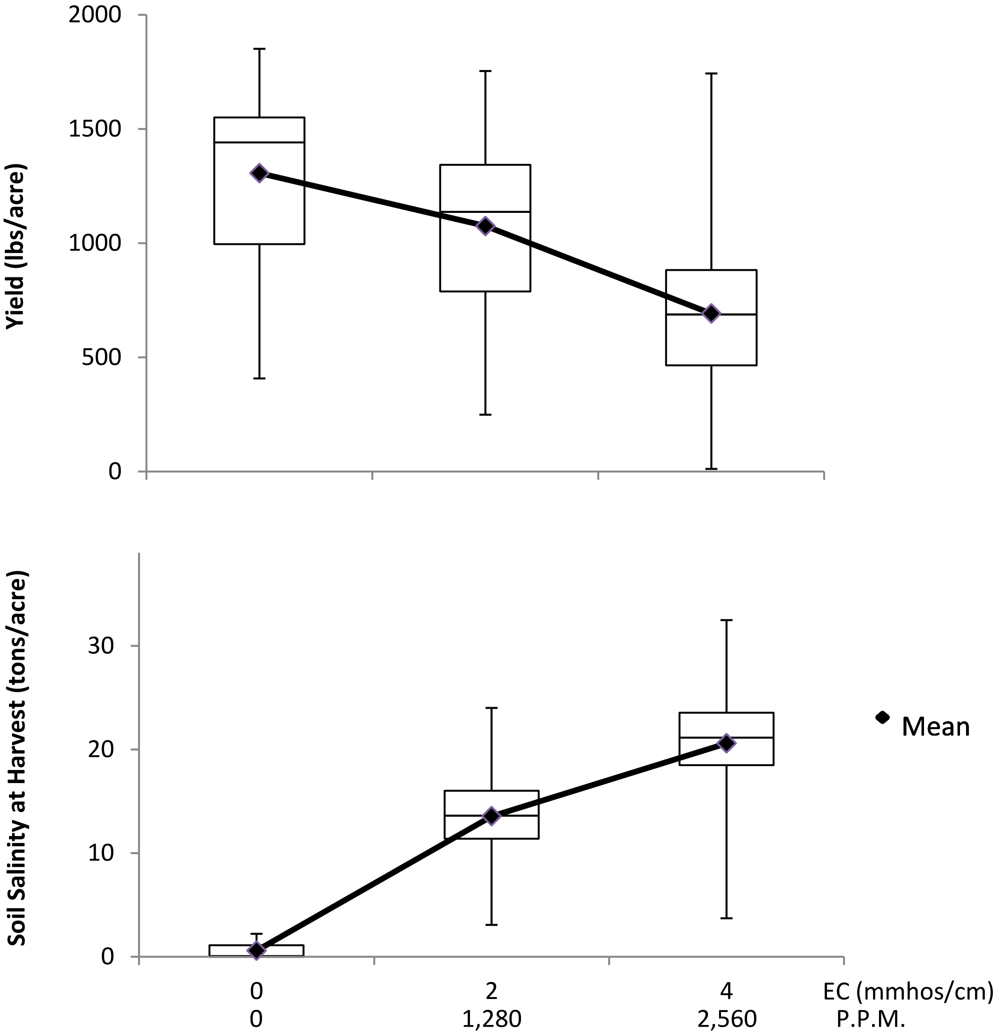


Figure 9-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Lawton Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has a common error message of “Estimated G matrix is not positive definite.” which indicates that one or more variance components on the RANDOM statement estimated as being zero. It should be removed from the model (http://support.sas.com/kb/22/614.html). Without the RANDOM statement which means that the model does not consider the random effect part, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 9-3.

Table 9-3. Result of Likelihood Ratio Test for the Lawton Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56769	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56613		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 9-4.

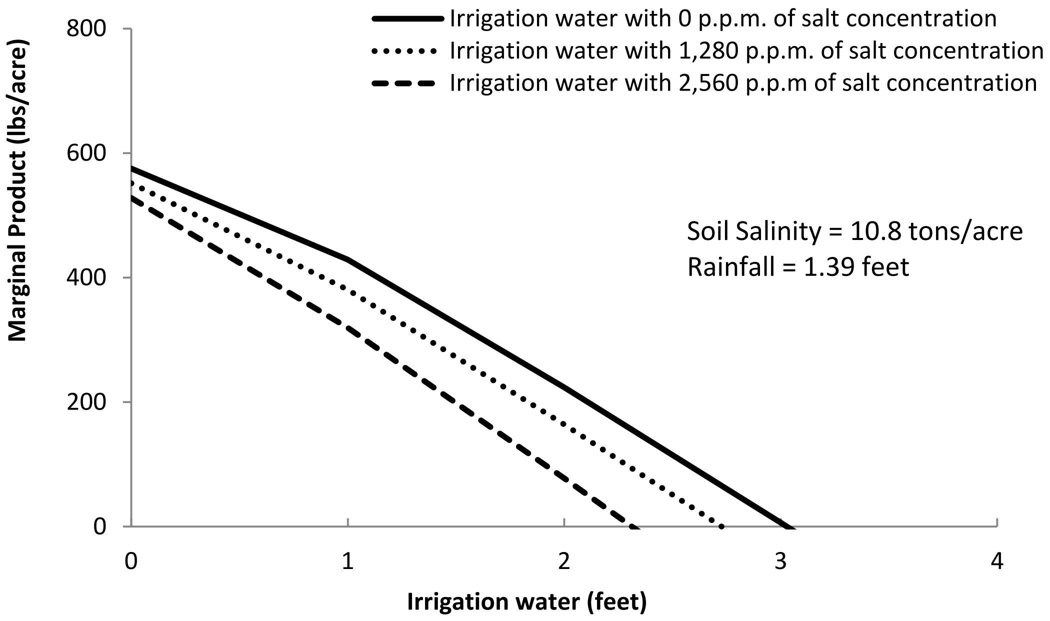
Table 9-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Lawton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-647.27*	45.1063
Total Water Applied		1007.96*	31.3420
Total Salinity		-4.7276*	1.1910
Non-Growing Season Rainfall		107.89*	9.5892
(Total Water Applied)^2		-112.73*	5.4200
(Total Salinity)^2		-1.1186*	0.0333
(Total Salinity / Total Water Applied)		21.3770*	2.2761

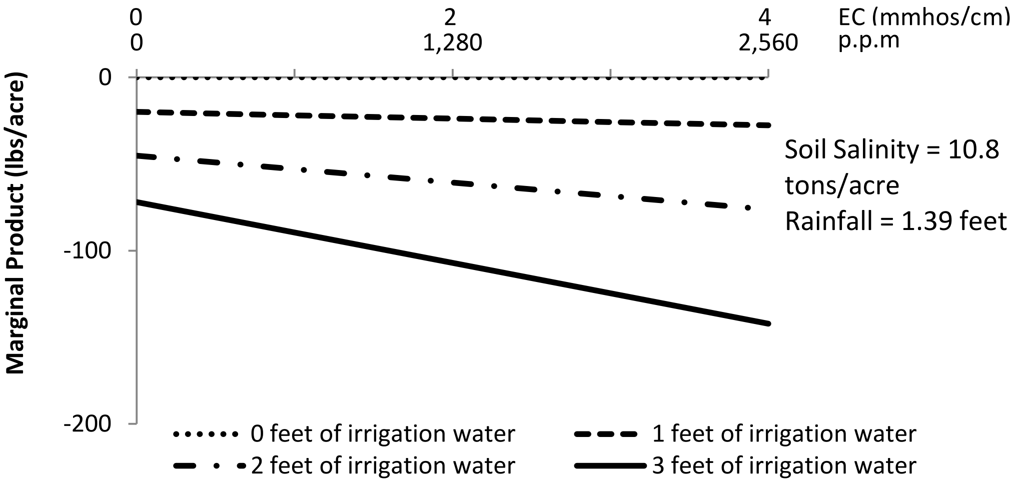
Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 9-2.

Part (a) of Figure 9-2 shows that the crop yield increases at decreasing rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 9-2. Marginal Product of Irrigation Water and Salt Concentration for the Lawton Loam Soil

Part (b) of Figure 9-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 9-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -227.2, -214, -8.9 and -2.2. The The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

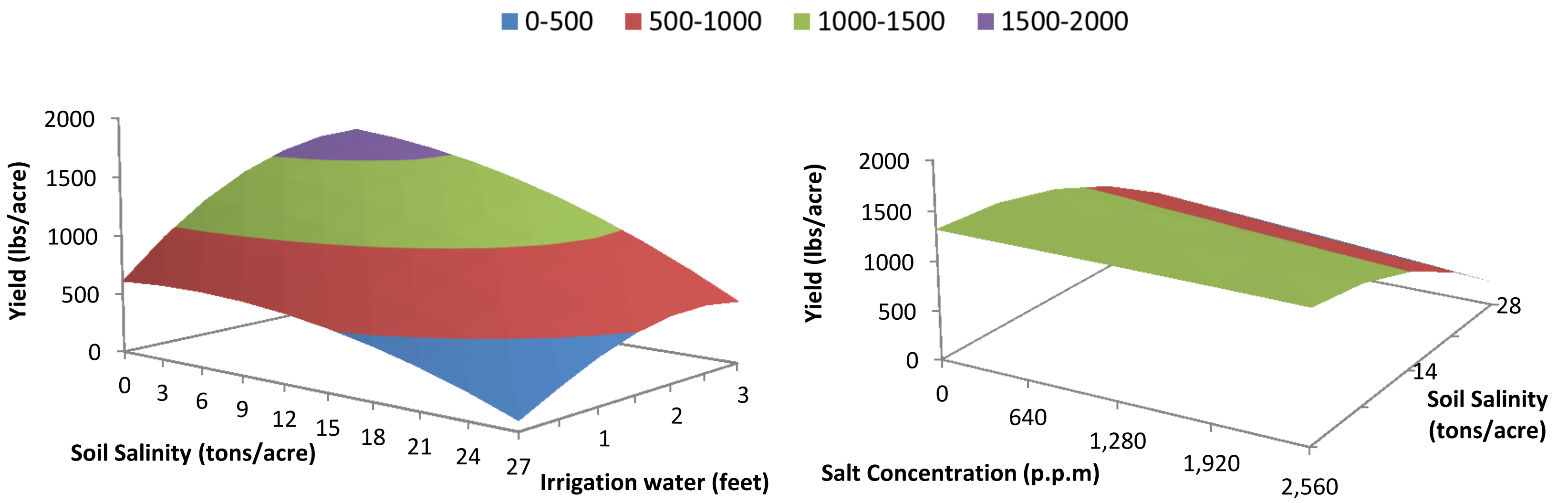
,

,

, and

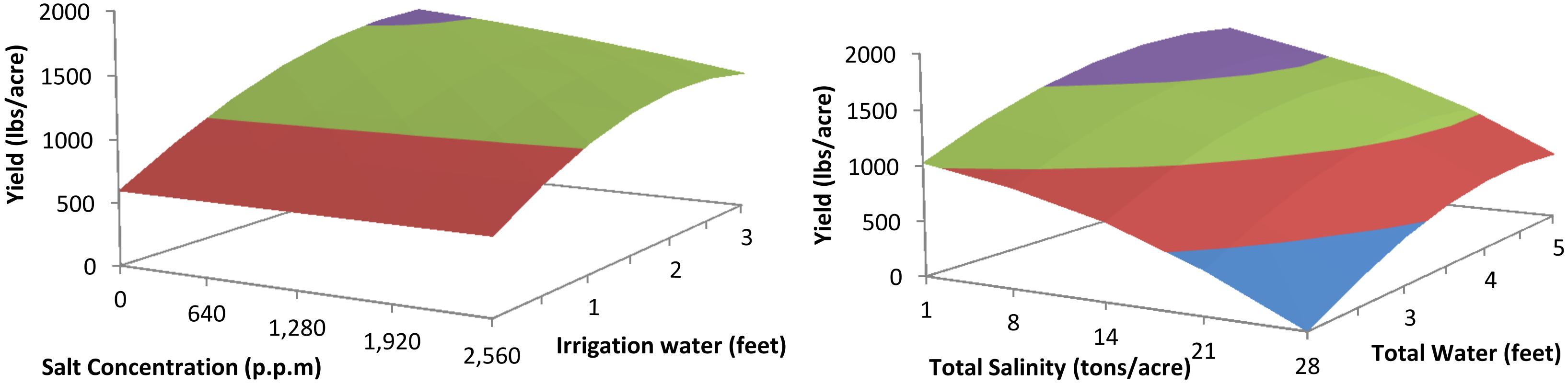
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Lawton Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 9-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 9-3. 3-D Surface of Crop Response Function versus Independent Variables for the Lawton Loam Soil

In part (a) of Figure 9-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 9-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 9-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 10.8 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 9-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance

(functions are shown in Tables 9-5, 9-6 and 9-7, respectively.

Table 9-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Lawton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.7261*	0.0868
Irrigation Water		-0.5362*	0.0390
Amount of Salt in Irrigation Water		0.6988*	0.0145
Soil Salinity at Planting		0.9169*	0.0034
Growing Season Rainfall		-1.3170*	0.0408

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.
The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 9-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13468 and 13383, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 9-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Lawton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.3195*	0.0550
Soil Salinity at Harvest on Previous Year		0.9340*	0.0021
Non-Growing Season Rainfall		-1.8106*	0.0676

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 9-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13114 and 12893, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 9-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Lawton Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		10042*	2631.46
Irrigation Water		-7763.05*	1006.33
Growing Season Rainfall		16441*	1393.56

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 9-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 9-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 9-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 9-7.

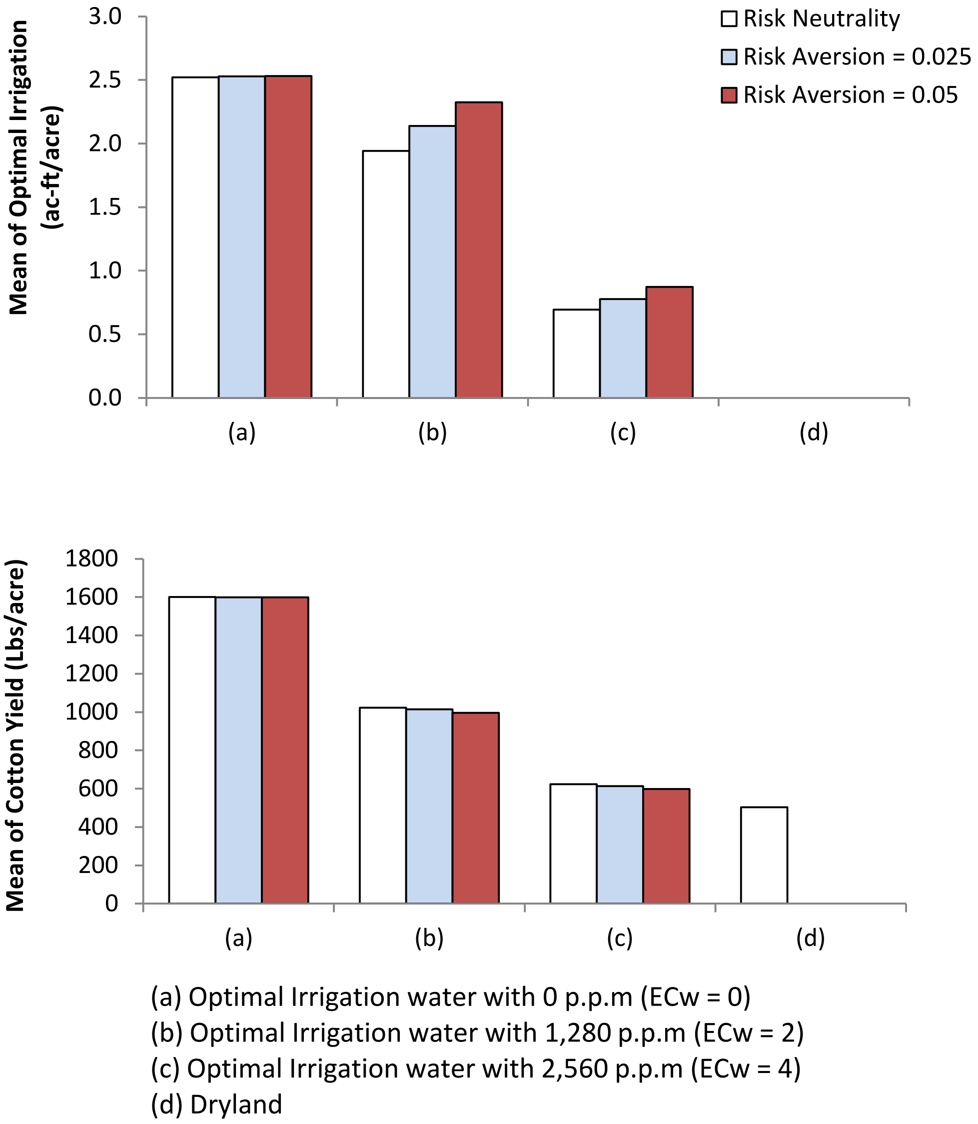


Figure 9-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Lawton Loam Soil

The lower half of Figure 9-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 9-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 9-5.

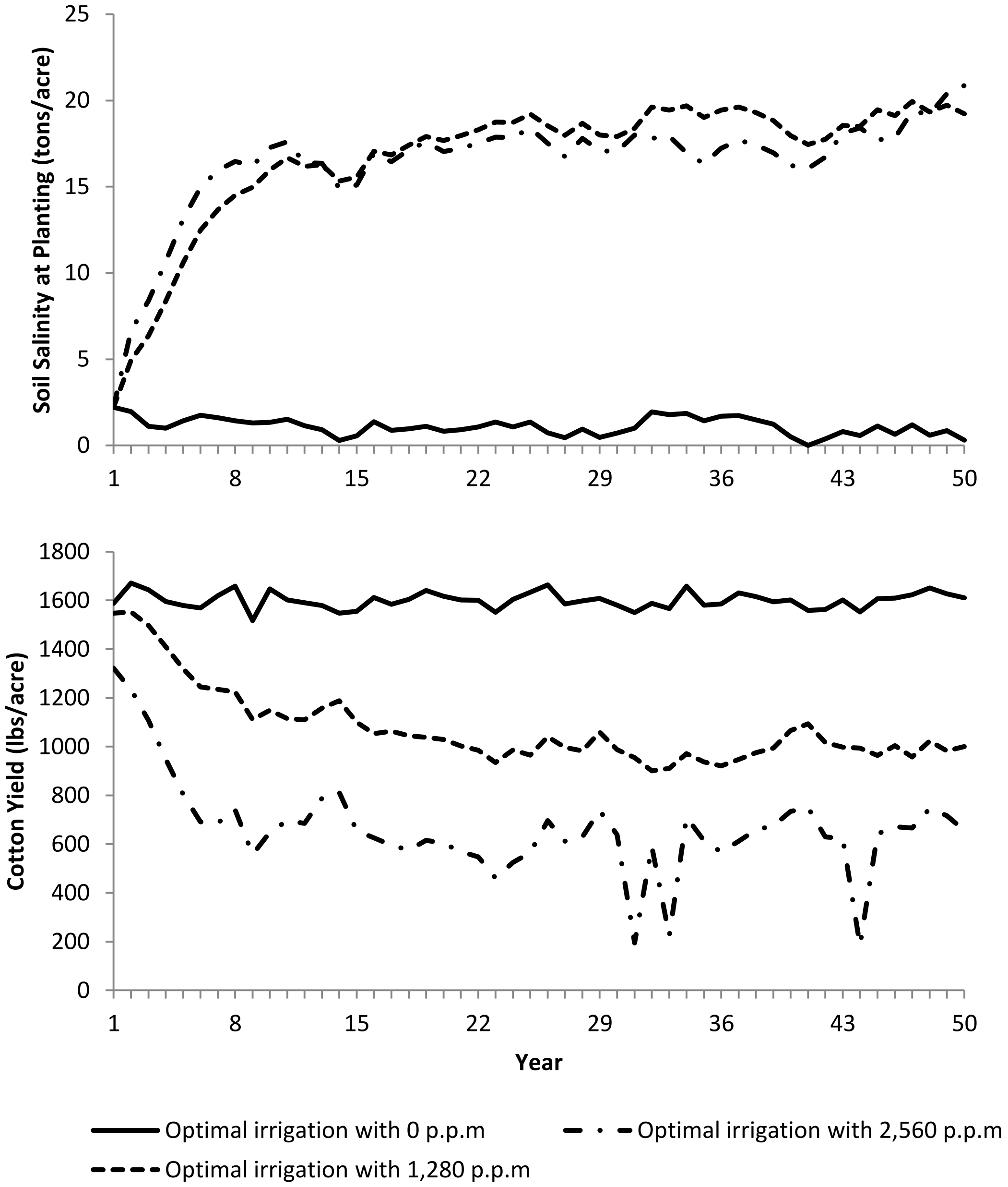
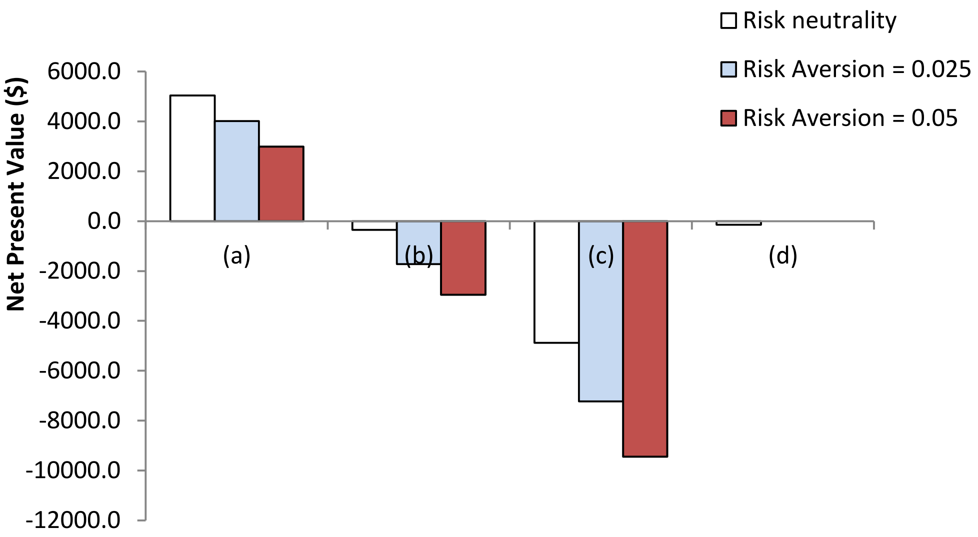


Figure 9-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Lawton Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied

over time, the quantity of soil salinity constantly increases and crop yield decreases as salts are accumulated in the soil. Figures 9-4 and 9-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 9-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 9-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Lawton Loam Soil

In (b) and (c) of Figure 9-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 9-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 9-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Lawton Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5037	5179
1,280 p.p.m (ECw=2)	-351	-209
2.560 p.p.m (ECw=4)	-4879	-4736
Dryland	-142	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 1.94 ~ 2.52 ac-ft/acre to maximize NPV of expected utility (see Figure 9-4).

Westill Clay Loam Soil, 1-3% Slope

EPIC Output Data

Table 10-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Westill Clay Loam soil. The total salt in the 1.5 meter profile was calculated as 1.17 tons/acre on the first day of simulation. The level of soil salinity at planting and harvest in each year can be selected from *DSL file. When irrigation water is applied to the crop land, it is infiltrated into the soil. Since the infiltration of water into the clay is slower than into the sand, Clay or Clay Loam soil types hold more water than Sandy soil types (Brouwer et al, 1985). These soil types also have higher irrigation efficiency than Sandy soil types (Sammis and Mexal, 1999).

Table 10-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Westill Clay Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.19	0.97	0.93	0.83	0.94	1.02	1.5	
WSLT(kg/ha)*	15	192	241	428	484	525	734	2,619
Salinity(tons/acre)								1.17**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 10-2.

Table 10-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Westill Clay Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	1 ~ 1831	940
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 35.37	12.86
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 36.57	13.67
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 10-1.

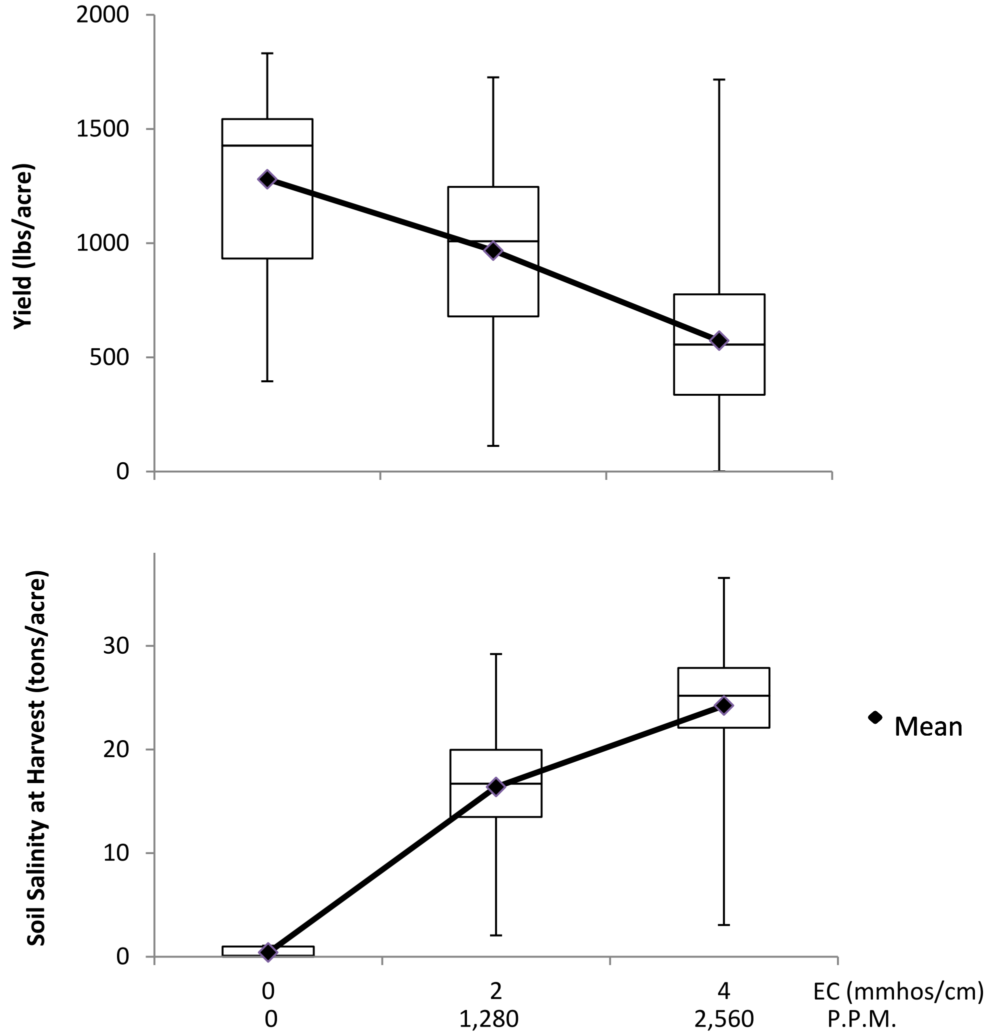


Figure 10-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Westill Clay Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has a common error message of "Estimated G matrix is not positive definite." which indicates that one or more variance components on the RANDOM statement are estimated as being zero. It should be removed from the model (http://support.sas.com/kb/22/614.html). Without the RANDOM statement which means that the model does not consider the random effect part, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 10-3.

Table 10-3. Result of Likelihood Ratio Test for the Westill Clay Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56762	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56234		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008). The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 10-4.

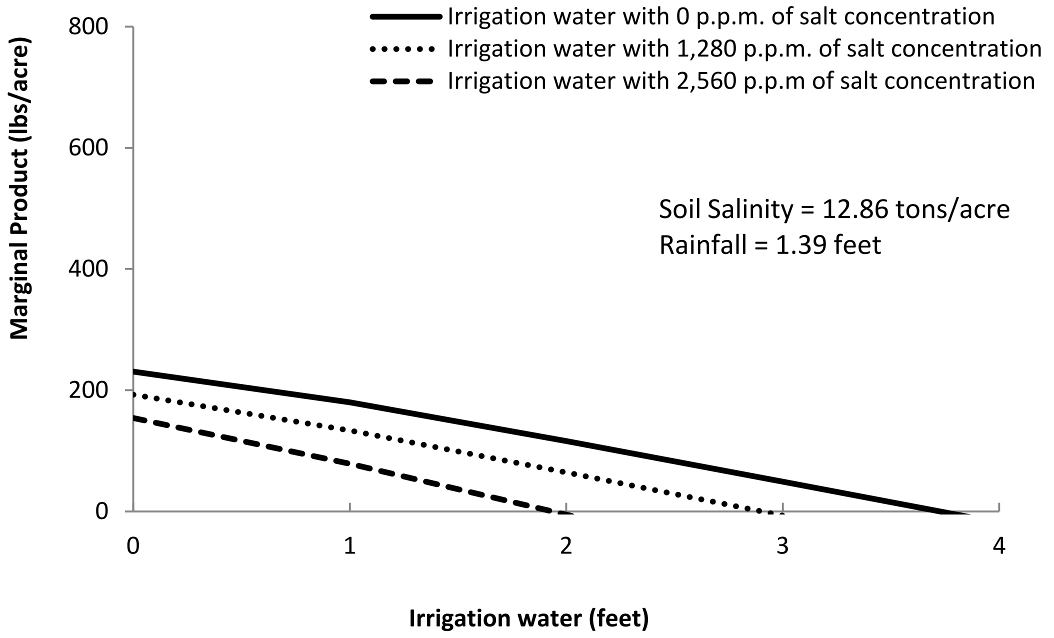
Table 10-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Westill Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		540.69*	47.1846
Total Water Applied		352.40*	29.1133
Total Salinity		-5.6377*	2.0373
Non-Growing Season Rainfall		68.9687*	7.3431
(Total Water Applied)^2		-34.2431*	4.6436
(Total Salinity)^2		-0.7469*	0.0535
(Total Salinity / Total Water Applied)		3.9715**	2.1131

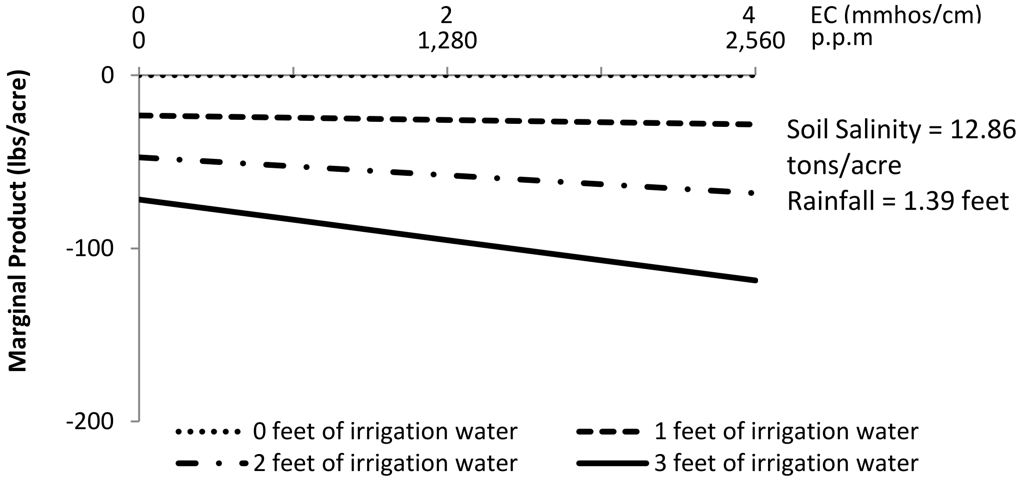
Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 10-2.

Part (a) of Figure 10-2 shows that the crop yield increases as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 10-2. Marginal Product of Irrigation Water and Salt Concentration for the Westill Clay Loam Soil

Part (b) of Figure 10-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 10-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2.5 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 5 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

= .

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -72.7, -67.2, -9.3 and -1.5. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

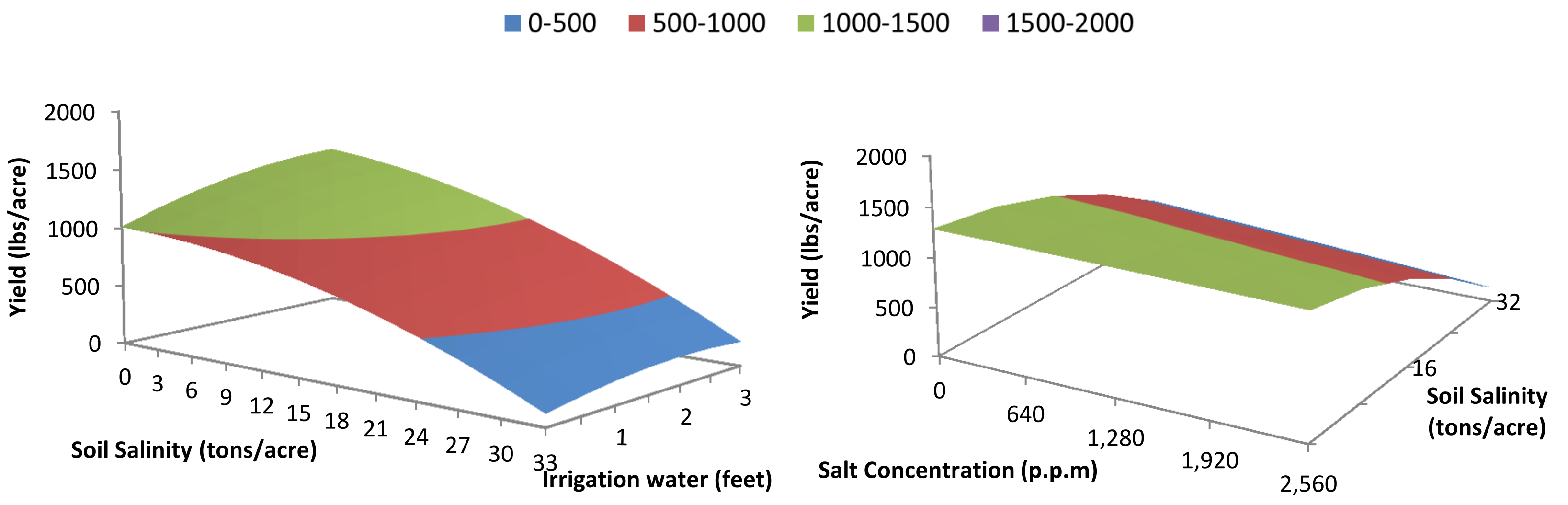
,

,

, and

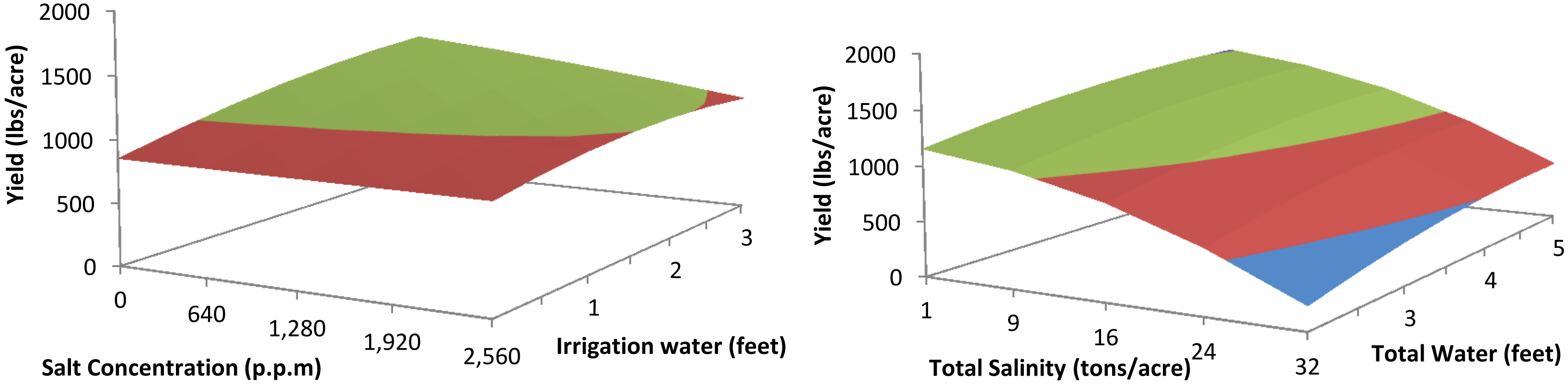
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Westill Clay Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 10-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 10-3. 3-D Surface of Crop Response Function versus Independent Variables for the Westill Clay Loam Soil

In part (a) of Figure 10-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. Although irrigation water increases, the crop yield slightly increases. Irrigation water barely affects cotton production. However, with dryland, the crop yield is higher than other soil types but very similar to Tillman Clay Loam soil.

In part (b) of Figure 10-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 10-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 12.86 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 10-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 10-5, 10-6 and 10-7, respectively.

Table 10-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Westill Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.5408*	0.0857
Irrigation Water		-0.5276*	0.0393
Amount of Salt in Irrigation Water		0.6854*	0.0141
Soil Salinity at Planting		0.9369*	0.0027
Growing Season Rainfall		-1.1868*	0.0400

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 10-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13334 and 13223, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 10-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Westill Clay Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.2526*	0.0540
Soil Salinity at Harvest on Previous Year		0.9499*	0.0017
Non-Growing Season Rainfall		-1.7693*	0.0677

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 10-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13056 and 12855, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 10-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Westill Clay Loam Soil

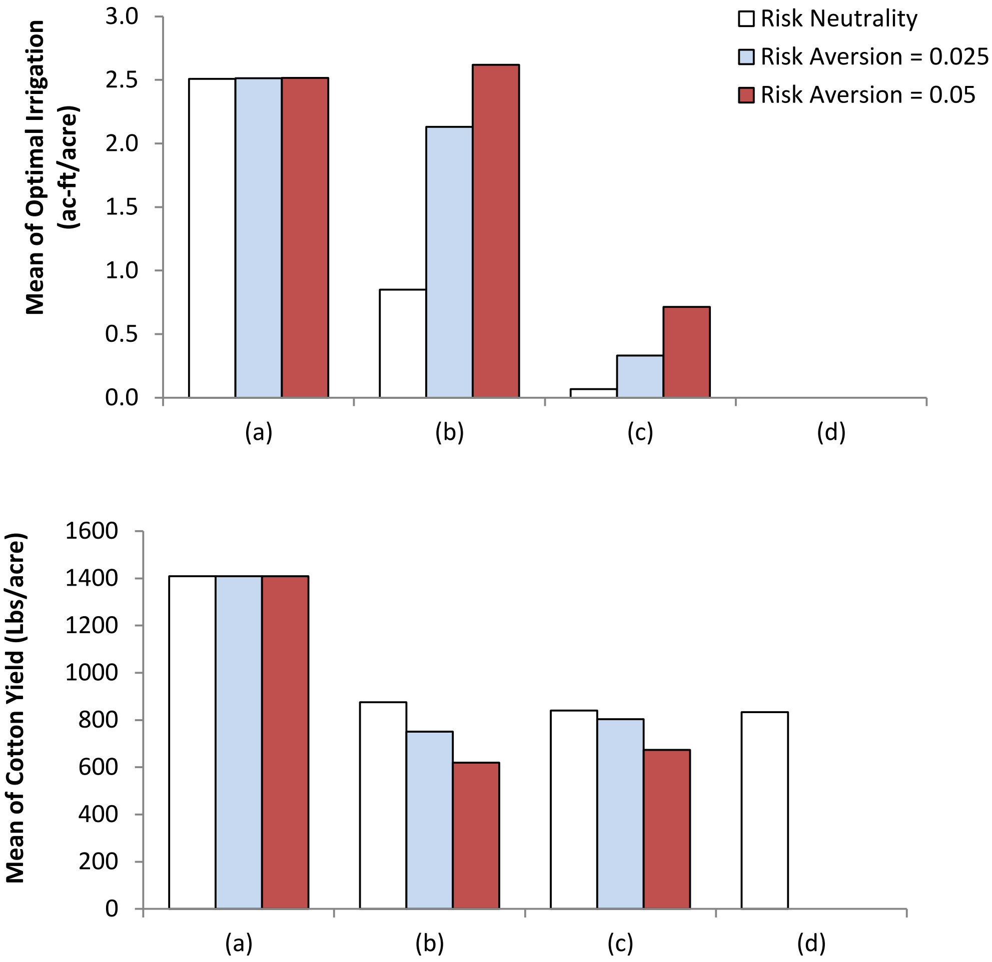
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		74025*	3646.02
Irrigation Water		-21943*	1491.59
Growing Season Rainfall		-1000.84	2020.82

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 10-7. The parameter estimate of irrigation water for the yield variance function is significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively. In the Westill Clay Loam soil, since the parameter estimates of the growing season rainfall is not significantly different from zero, there is no effect on the yield risk.

Dynamic Optimization

Figure 10-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 10-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use 2 ~ 3 times as irrigation water as the risk-neutral producers use to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 10-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 10-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Westill Clay Loam Soil

The lower half of Figure 10-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. When irrigation water with the salt concentration is applied, the optimal level of irrigation water is very low. Although the risk-averse producers in (b) relatively used irrigation water as much as the optimal level of irrigation with 0 p.p.m, their cotton yield decreases. Generally, if plenty of irrigation water is used in the soil, the soil salinity is leached and positively affects the crop growth. In the Westill Clay Loam soil, we can expect that there is no leaching effect. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 10-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yields of (b) and (c) are similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 10-5.

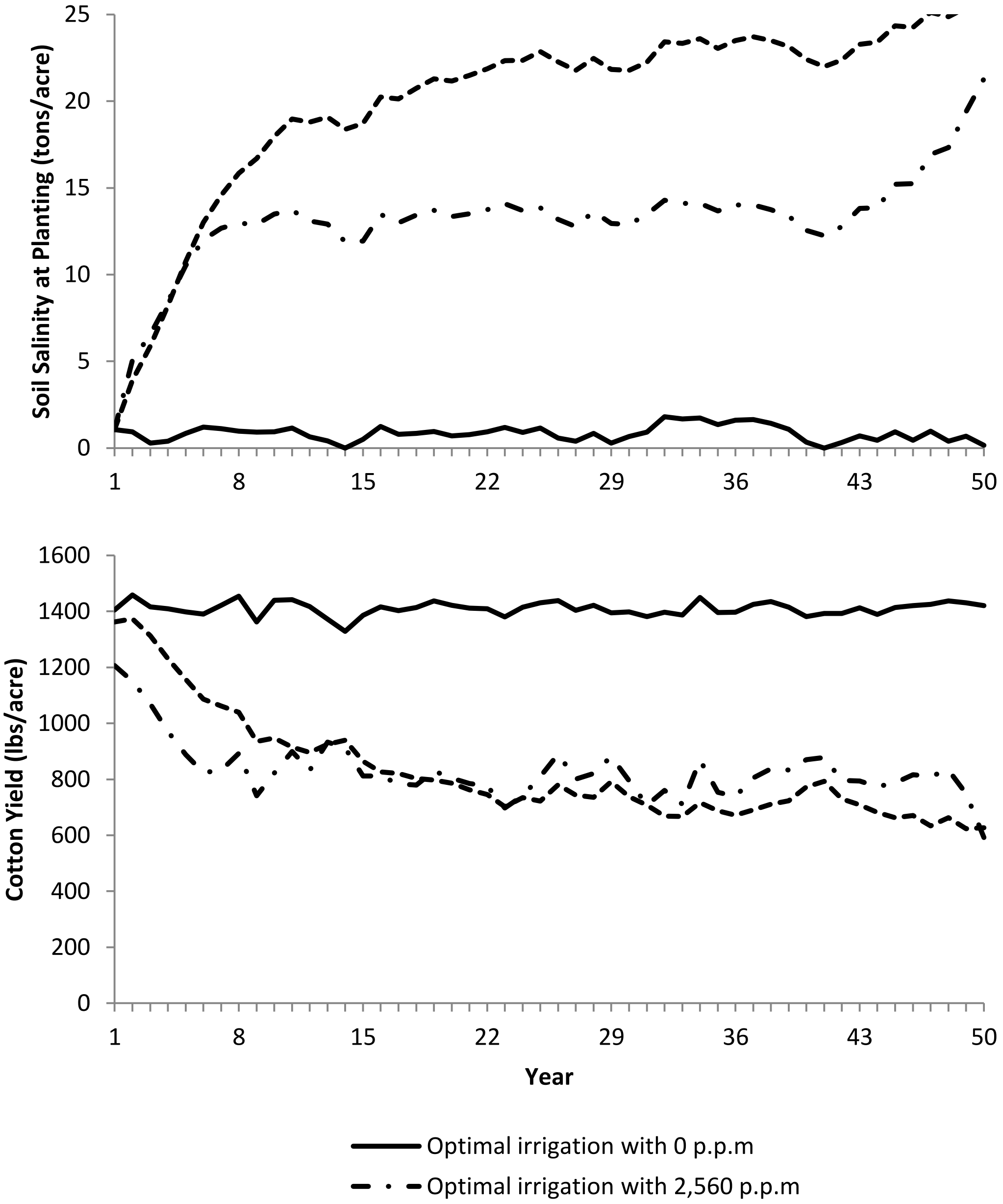


Figure 10-5. Changes of Quantity of Soil Salinity at Planting over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Westill Clay Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied

over time, the quantity of soil salinity constantly increases and crop yield decreases as salts are accumulated in the soil. Figures 10-4 and 10-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 10-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

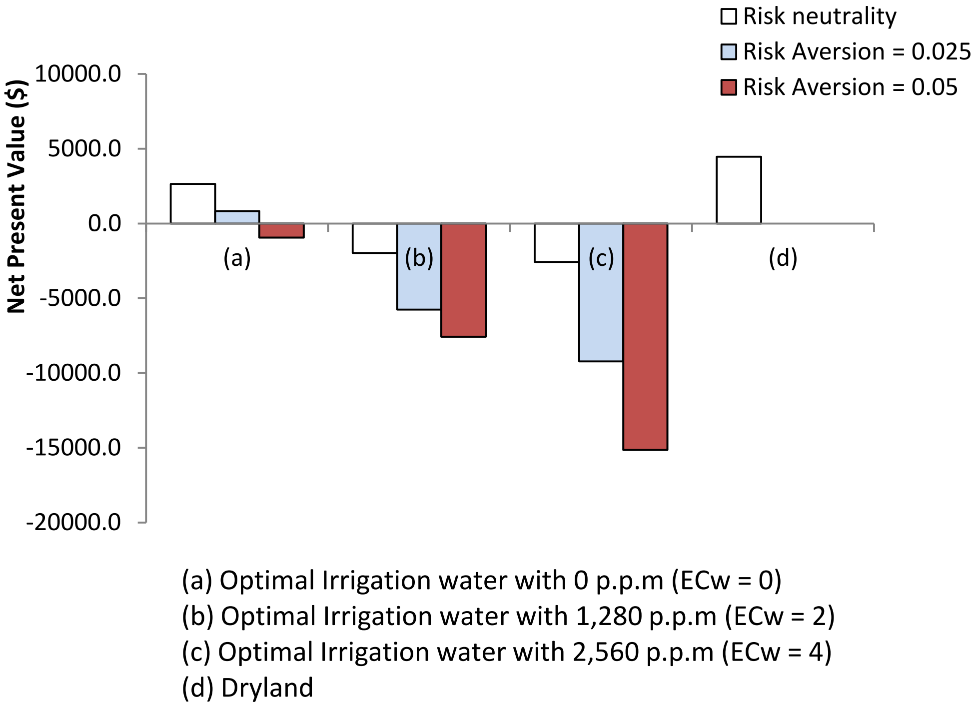


Figure 10-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Westill Clay Loam Soil

In (b) and (c) of Figure 10-6, the net present values are less than zero and also less than NPV of dryland. NPV of high risk-averse producers using (a) is even slightly less than zero. The negative NPV means that the project should be rejected. To implement the project, irrigation water should be used with 0 p.p.m or slightly more for the producers. Table 10-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 10-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Westill Clay Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	2640	-1830
1,280 p.p.m (ECw=2)	-1968	-6437
2.560 p.p.m (ECw=4)	-2571	-7041
Dryland	4469	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

All the differences between NPV of irrigated and dryland production are negative in Table 10-8. We can expect that the producer using irrigation water in Westill Clay Loam soil cannot make profit from investment in irrigated production compared to dryland cotton production. It means than the dryland producer has better profits than the producer using irrigation water.

Abilene Loam Soil, 0-1% Slope

EPIC Output Data

Table 11-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Abilene Loam soil. The total salt in the 1.5 meter profile was calculated as 7.79 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 11-2.

Table 11-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Abilene Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	9.79	2.64	3.37	1.66	3.52	11.64	14.31	
WSLT(kg/ha)*	53	396	659	903	1,910	6,307	7,234	17,462
Salinity(tons/acre)								7.79**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×11^{-3} tons/acre).

Table 11-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Abilene Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	8 ~ 1842	1043
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.31
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 33.33	12.48
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 34.28	13.21
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 11-1.

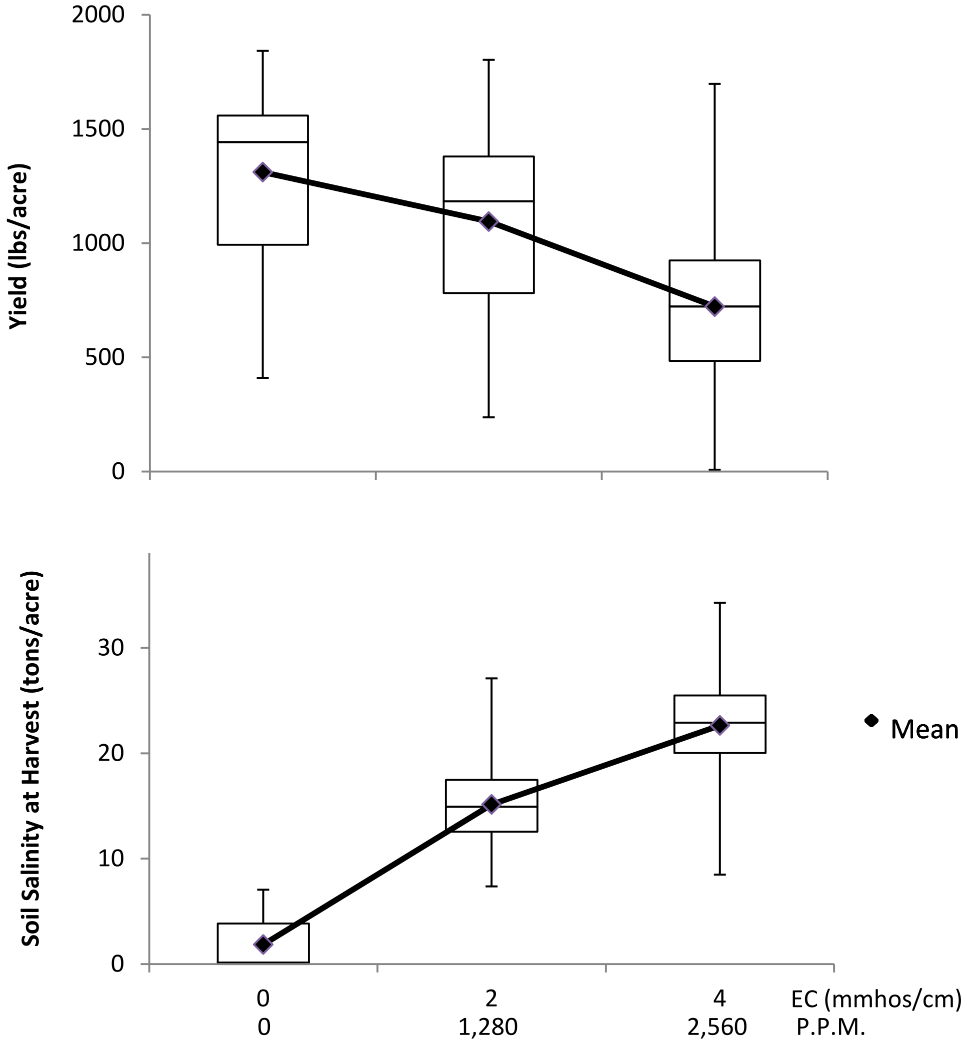


Figure 11-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Abilene Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 11-3.

Table 11-3. Result of Likelihood Ratio Test for the Abilene Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56734	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56595		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 11-4.

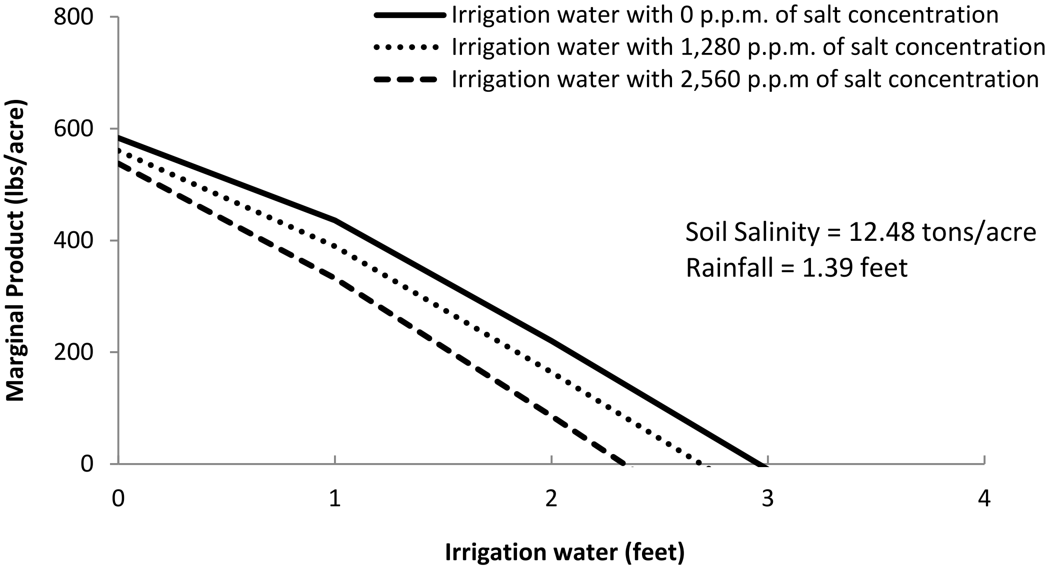
Table 11-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Abilene Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-701.52*	49.4081
Total Water Applied		1052.92*	32.9653
Total Salinity		-5.7208*	1.0687
Non-Growing Season Rainfall		107.77*	9.6346
(Total Water Applied)^2		-119.45*	5.5359
(Total Salinity)^2		-0.9133*	0.0287
(Total Salinity / Total Water Applied)		21.2446*	2.2770

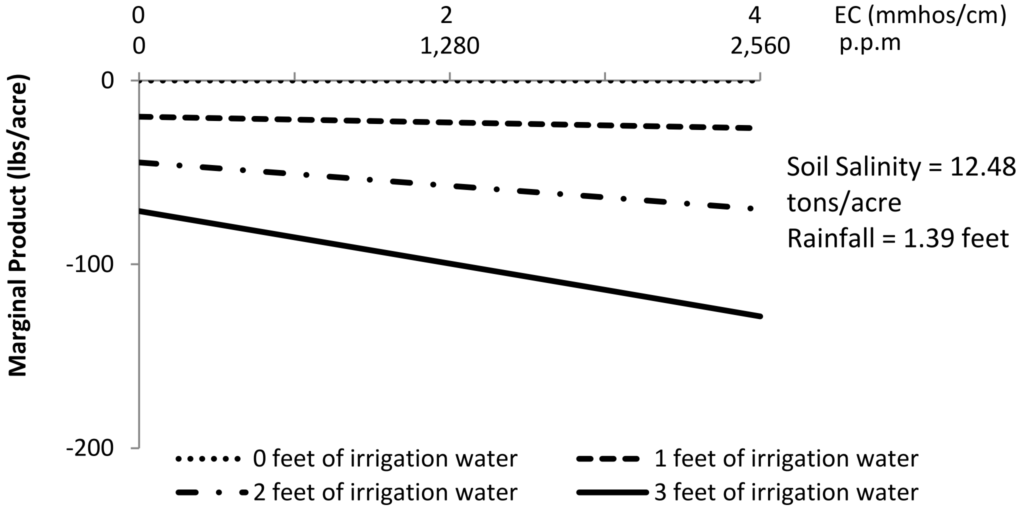
Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 11-2.

Part (a) of Figure 11-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 11-2. Marginal Product of Irrigation Water and Salt Concentration for the Abilene Loam Soil

Part (b) of Figure 11-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield

declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 11-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -239.4, -227.5, -7.3 and -1.8. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3

sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

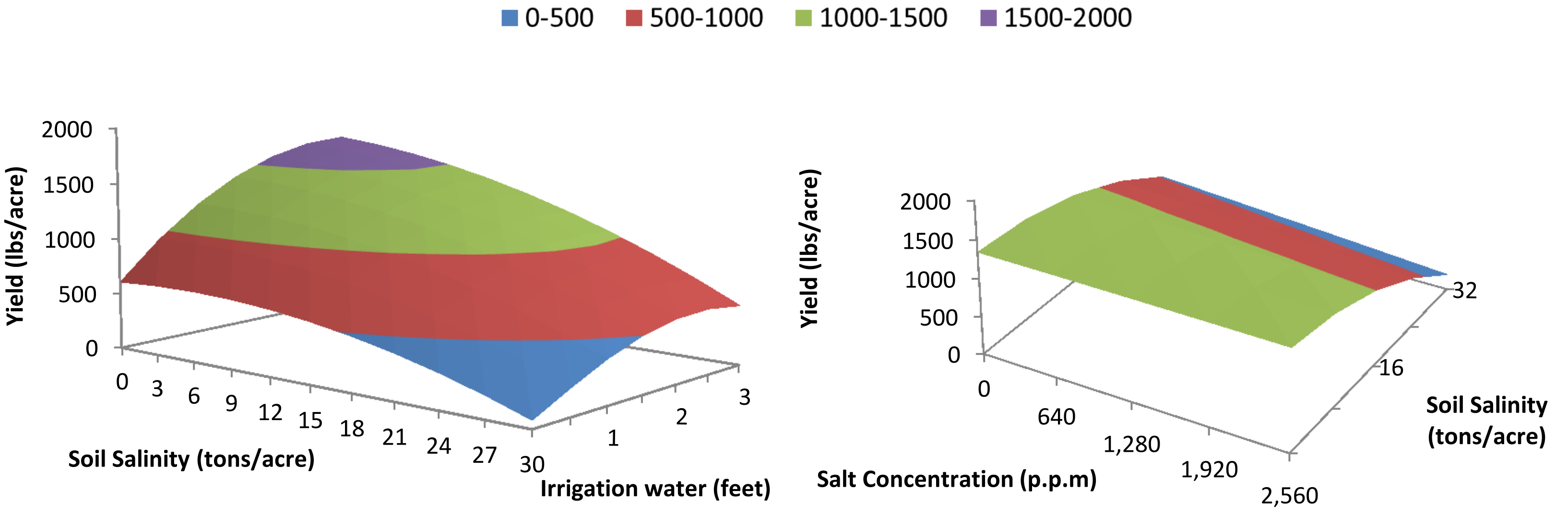
,

,

, and

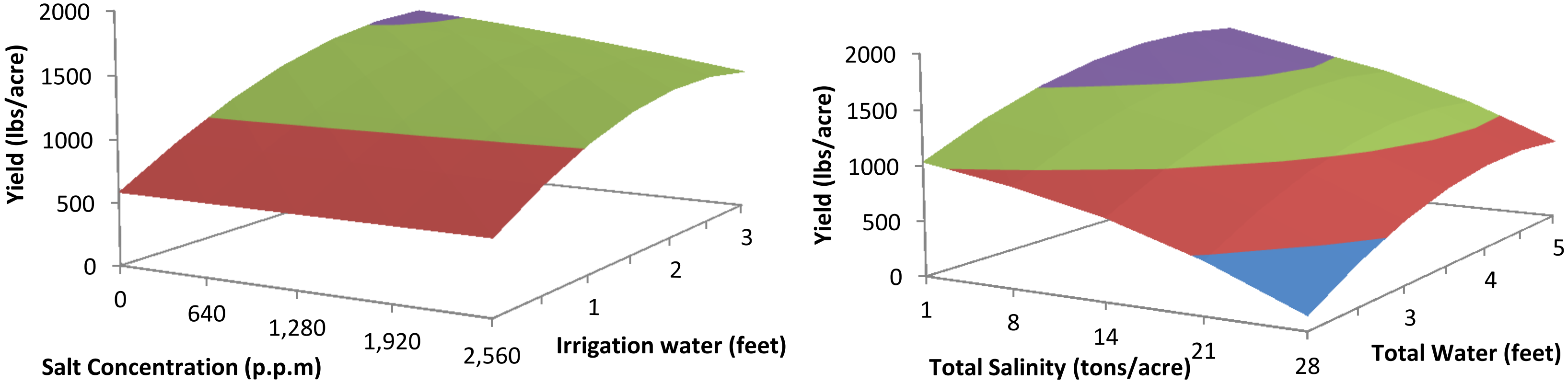
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Abilene Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 11-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 11-3. 3-D Surface of Crop Response Function versus Independent Variables for the Abilene Loam Soil

In part (a) of Figure 11-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 11-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, holding 1.39 feet and 1.31 feet of rainfall and irrigation water, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 11-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 12.48 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 11-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance

(functions are shown in Tables 11-5, 11-6 and 11-7, respectively.

Table 11-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Abilene Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		3.1533*	0.1031
Irrigation Water		-0.6580*	0.0466
Amount of Salt in Irrigation Water		0.6997*	0.0164
Soil Salinity at Planting		0.9182*	0.0039
Growing Season Rainfall		-1.4716*	0.0444

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.
The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table11 -5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 14258 and 14168, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 11-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Abilene Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.4122*	0.0596
Soil Salinity at Harvest on Previous Year		0.9378*	0.0022
Non-Growing Season Rainfall		-1.9137*	0.0730

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 11-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13802 and 13590, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 11-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Abilene Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		9615.97*	2585.94
Irrigation Water		-7476.88*	965.57
Growing Season Rainfall		16394*	1365.78

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 11-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 11-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 11-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 11-7.

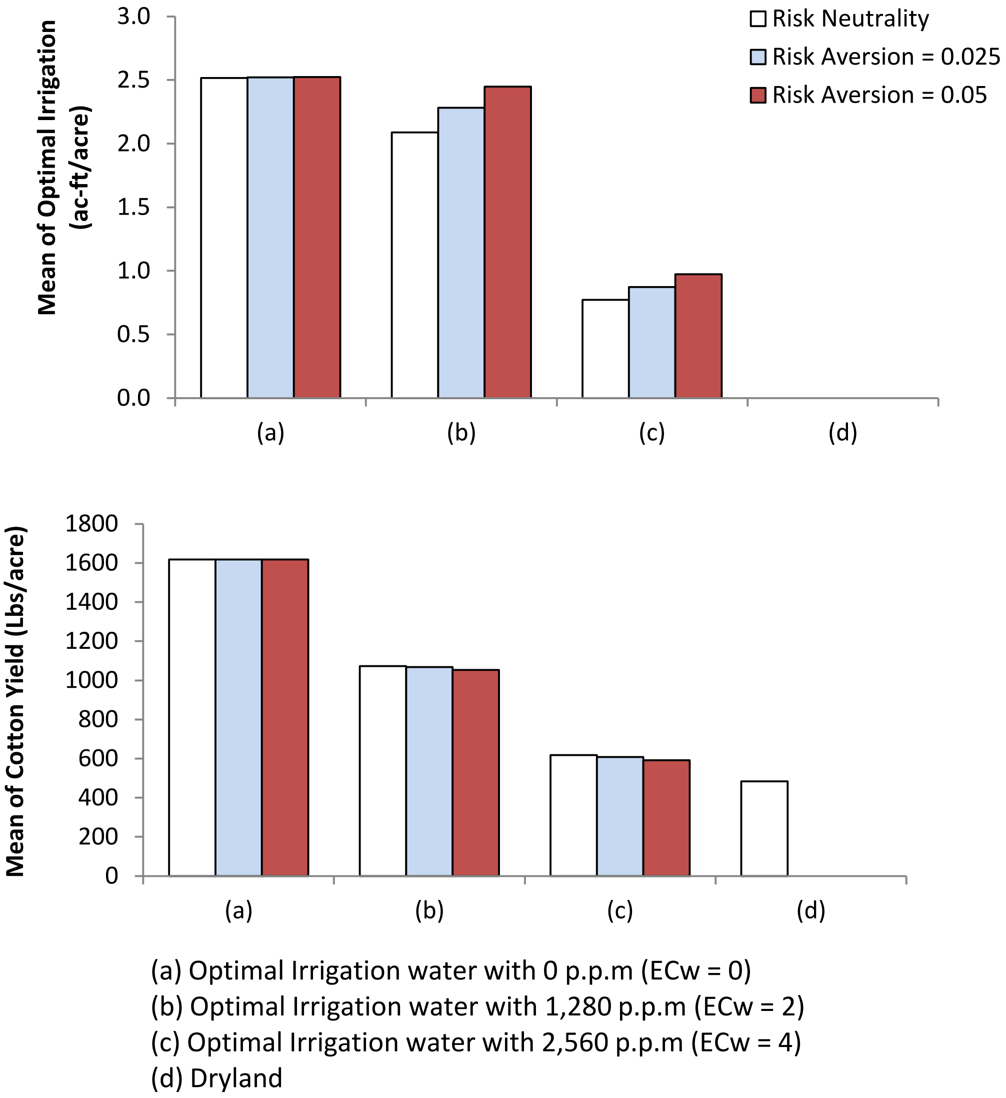


Figure 11-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Abilene Loam Soil

The lower half of Figure 11-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 11-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 11-5.

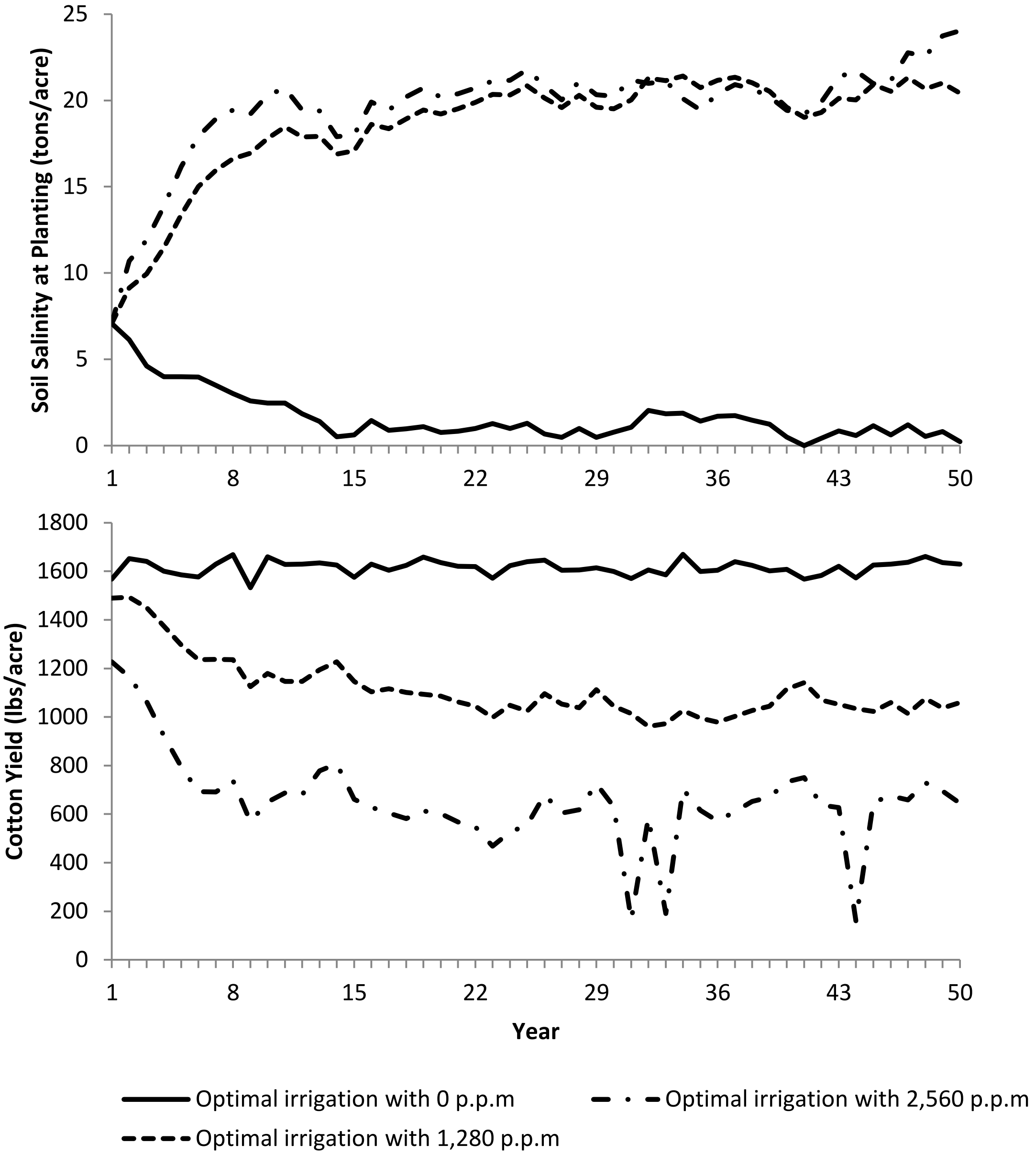
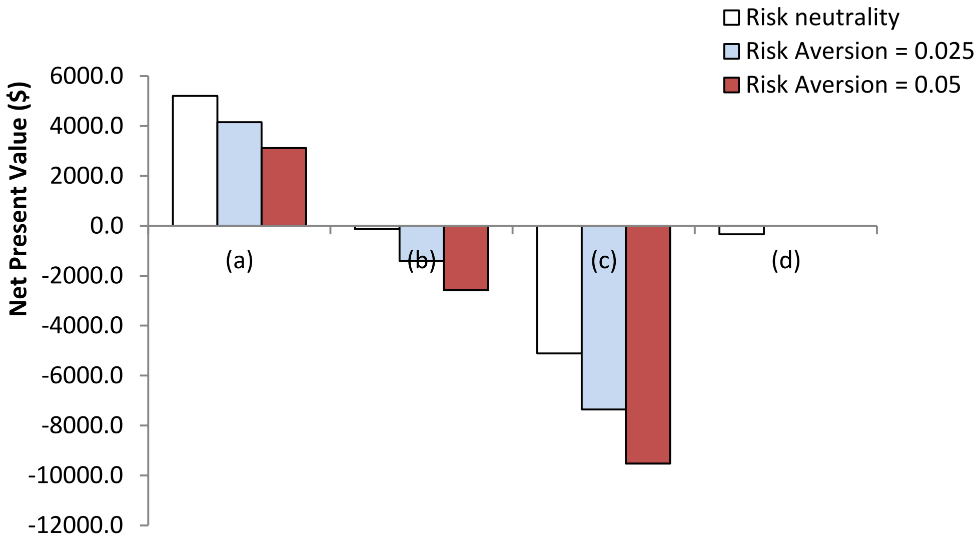


Figure 11-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Abilene Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied

over time, the quantity of soil salinity constantly increases and crop yield decreases as salts are accumulated in the soil. Figures 11-4 and 11-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 11-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 11-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Abilene Loam Soil

In (b) and (c) of Figure 11-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly larger than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 11-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 11-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Abilene Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5204	5536
1,280 p.p.m (ECw=2)	-130	202
2.560 p.p.m (ECw=4)	-5117	-4784
Dryland	-332	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 0 and 1,280 p.p.m is applied, the differences of their NPVs are positive. It indicates the producer can make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 11-4).

Burford Loam Soil, 1-3% Slope

EPIC Output Data

Table 12-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Burford Loam soil. The total salt in the 1.5 meter profile was calculated as 5.48 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 12-2.

Table 12-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Burford Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	0.95	0.73	0.81	1.41	3.85	10.23	9.03	
WSLT(kg/ha)*	9	116	180	597	1,894	5,037	4,449	12,283
Salinity(tons/acre)								5.48**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×12^{-3} tons/acre).

Table 12-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Burford Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	1 ~ 1854	930
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.32
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 38.05	14.40
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 38.09	15.17
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 12-1.

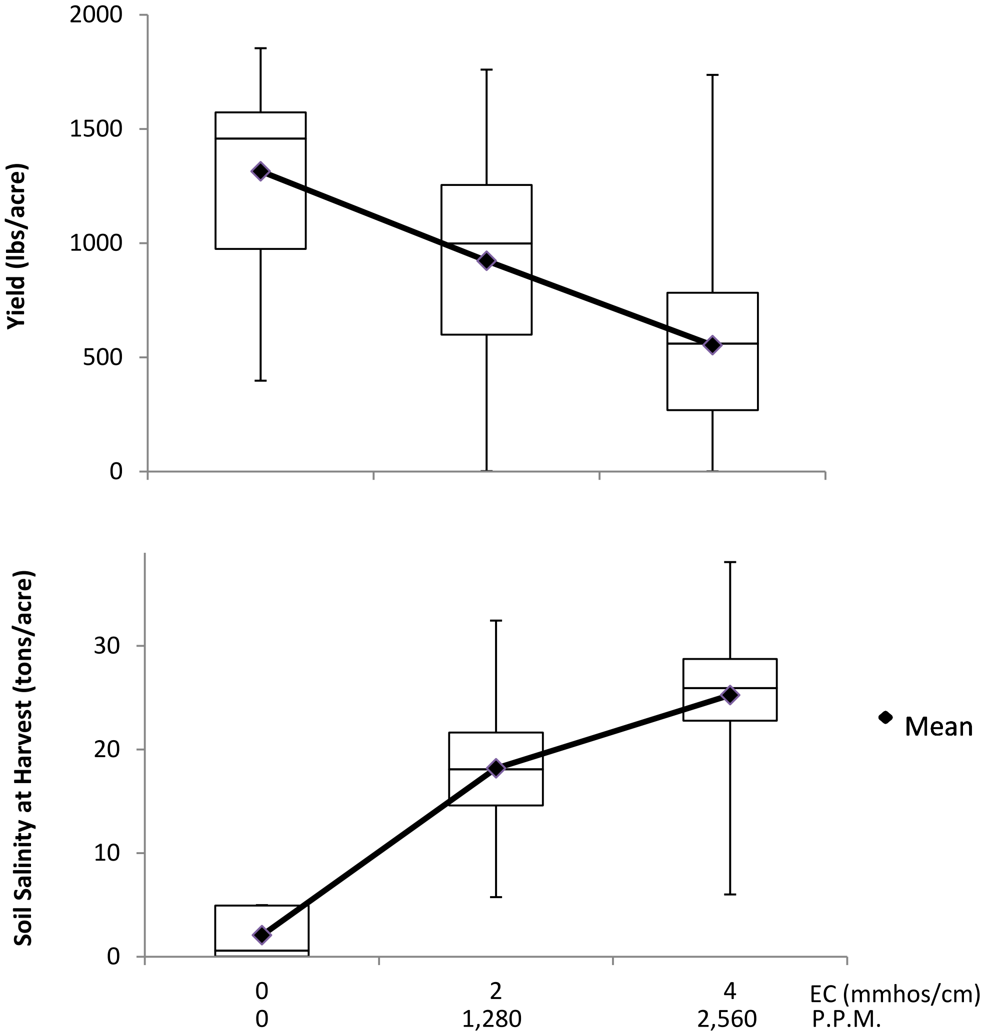


Figure 12-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Burford Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and/or GROUP = weather was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 12-3.

Table 12-3. Result of Likelihood Ratio Test for the Burford Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56998	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56860		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 12-4.

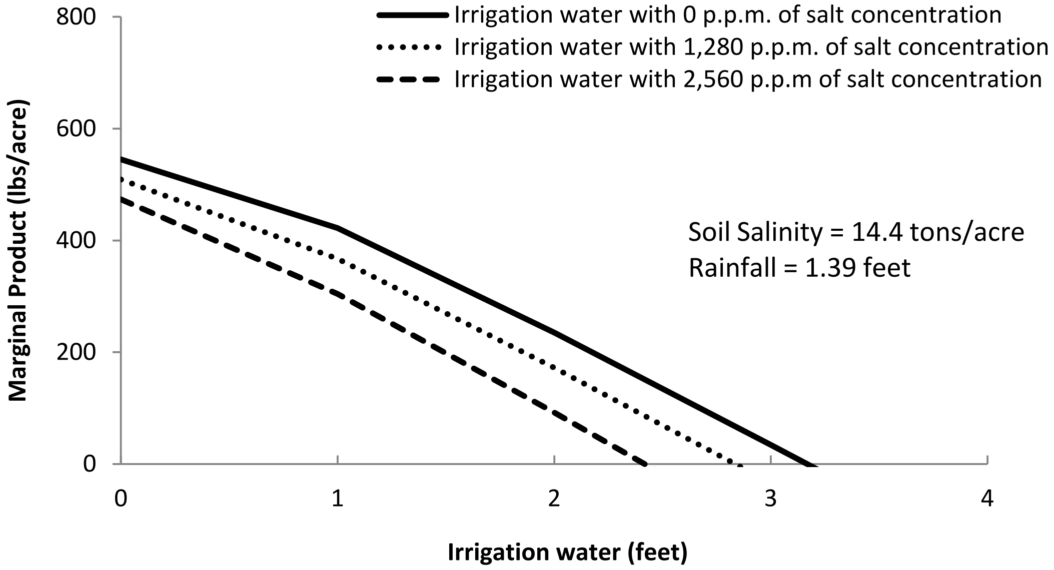
Table 12-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Burford Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-574.03*	51.3494
Total Water Applied		965.56*	34.1745
Total Salinity		-12.1268*	1.0150
Non-Growing Season Rainfall		127.51*	9.9122
(Total Water Applied)^2		-104.57*	5.6992
(Total Salinity)^2		-0.7261*	0.0276
(Total Salinity / Total Water Applied)		17.4225*	2.0409

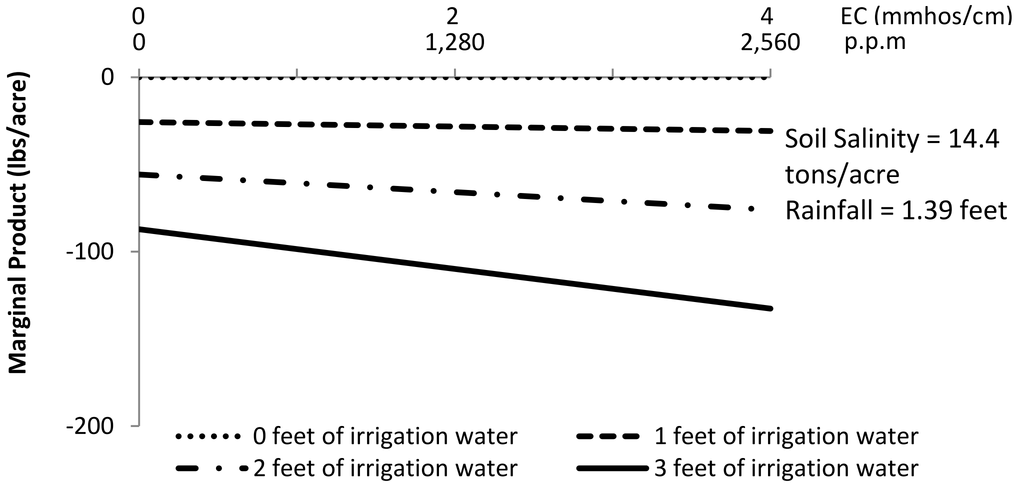
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 12-2.

Part (a) of Figure 12-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 12-2. Marginal Product of Irrigation Water and Salt Concentration for the Burford Loam Soil

Part (b) of Figure 12-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 12-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$= \qquad .$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -209.4, -199.8, -5.8 and -1.5. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

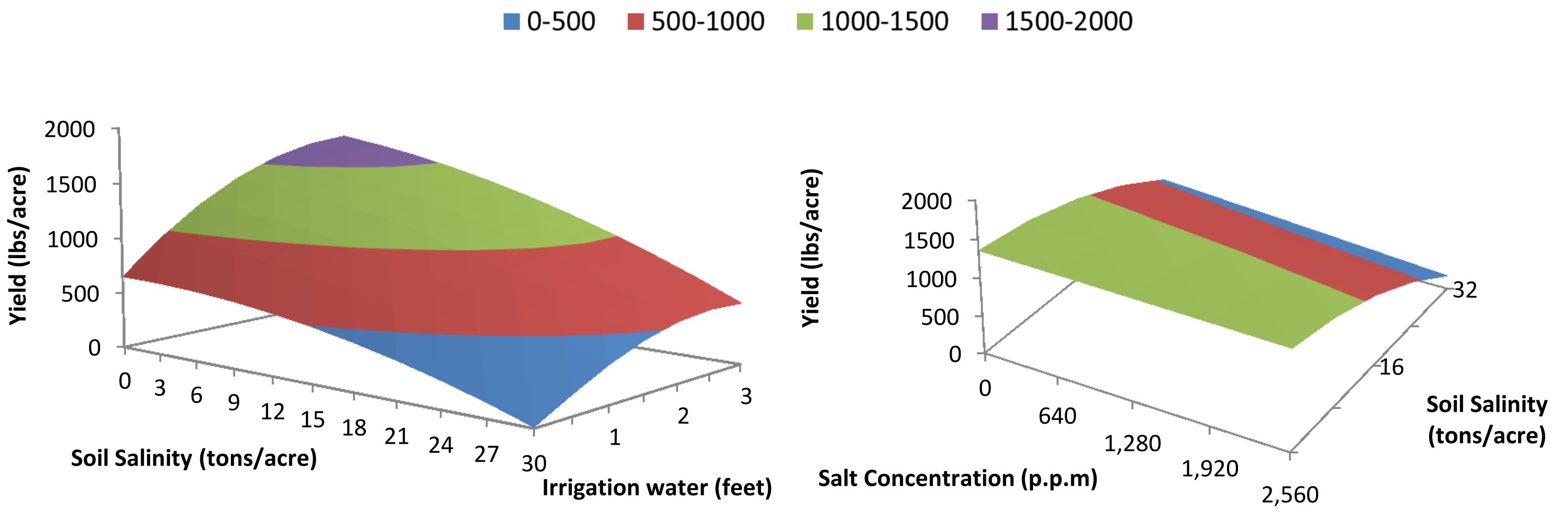
,

,

, and

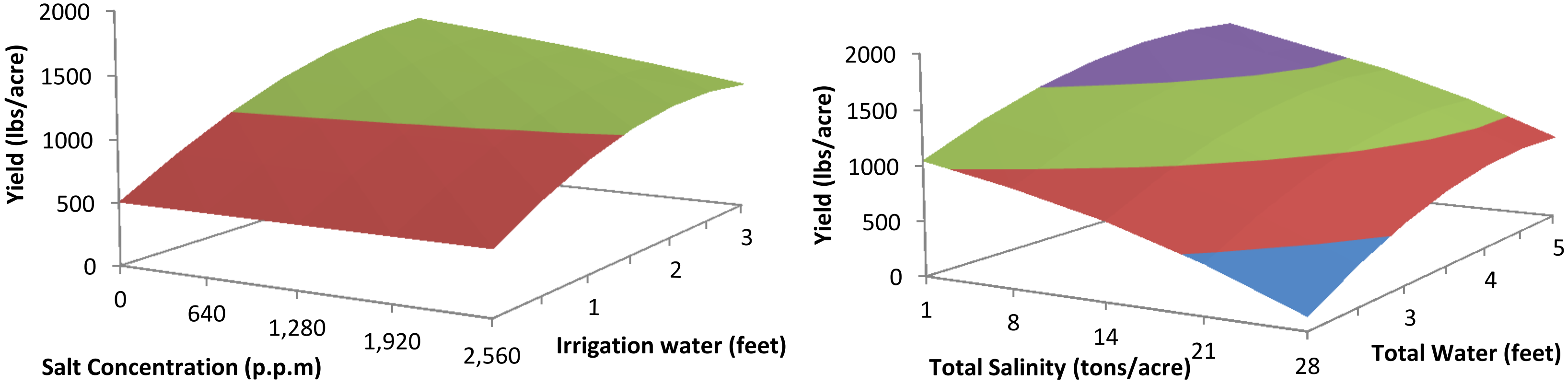
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Burford Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 12-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 12-3. 3-D Surface of Crop Response Function versus Independent Variables for the Burford Loam Soil

In part (a) of Figure 12-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 12-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, holding 1.39 feet and 1.32 feet of rainfall and irrigation water, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 12-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 14.4 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 12-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 12-5, 12-6 and 12-7, respectively.

Table 12-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Burford Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		3.2036*	0.1087
Irrigation Water		-0.6683*	0.0459
Amount of Salt in Irrigation Water		0.6540*	0.0156
Soil Salinity at Planting		0.9349*	0.0032
Growing Season Rainfall		-1.4610*	0.0492

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 12-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 15157 and 15007, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 12-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Burford Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.3692*	0.0636
Soil Salinity at Harvest on Previous Year		0.9503*	0.0020
Non-Growing Season Rainfall		-1.8581*	0.0778

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 12-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 14343 and 14122, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 12-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Burford Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		7391.17*	2695.14
Irrigation Water		-7678.80*	947.04
Growing Season Rainfall		19400*	1430.33

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 12-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 12-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer’s risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 12-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 12-7.

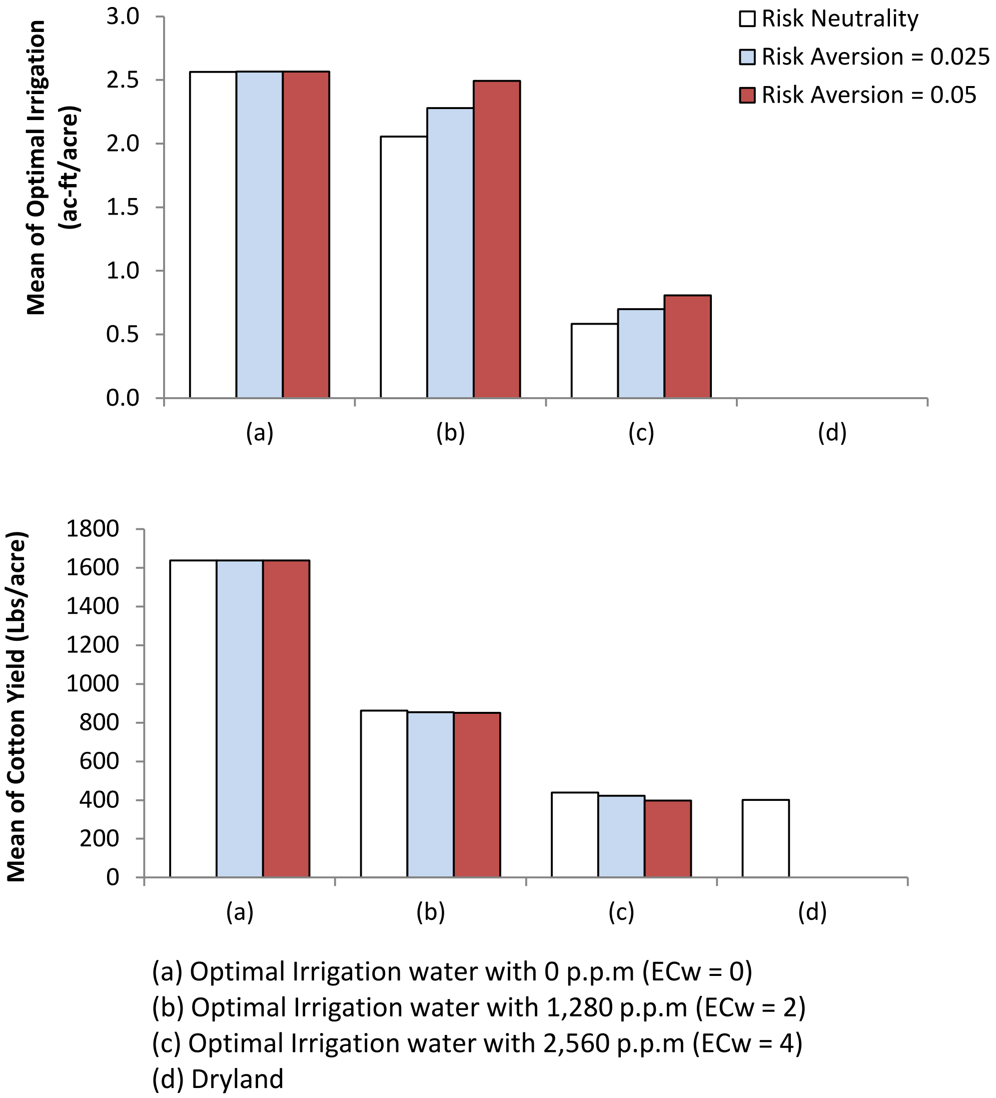


Figure 12-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Burford Loam Soil

The lower half of Figure 12-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 12-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 12-5.

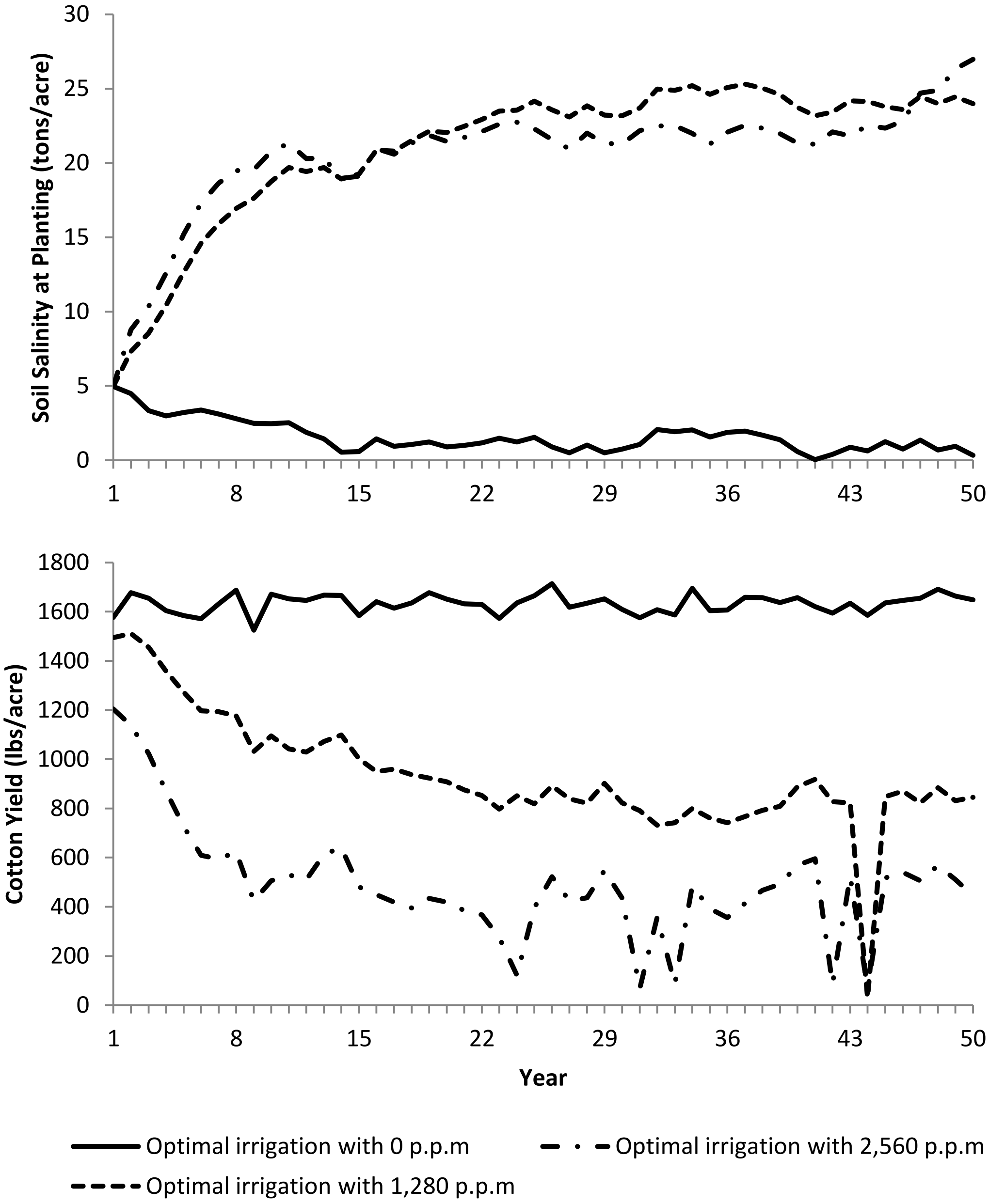


Figure 12-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth =1.5m for the Burford Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied

over time, the quantity of soil salinity constantly increases and crop yield decreases as salts are accumulated in the soil. Figures 12-4 and 12-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 12-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

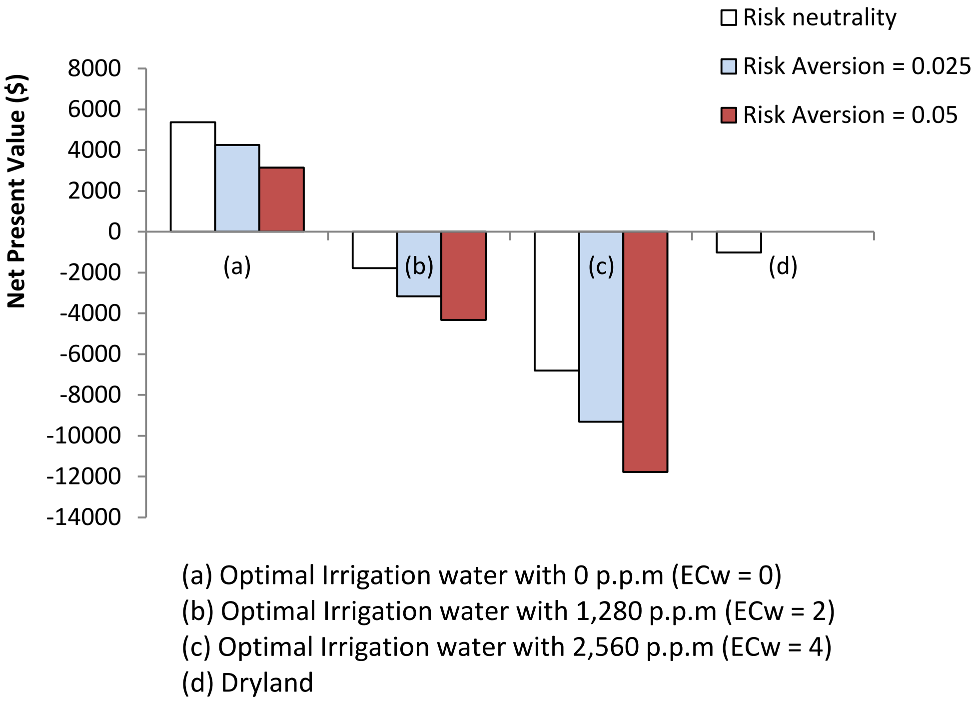


Figure 12-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Burford Loam Soil

In (b) and (c) of Figure 12-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than 1,280 p.p.m for the producers, respectively. Table 12-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 12-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Burford Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5370	6385
1,280 p.p.m (ECw=2)	-1788	-773
2.560 p.p.m (ECw=4)	-6806	-5791
Dryland	-1015	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m, the optimal level of irrigation is approximately 2.5 ac-ft/acre maximizing NPV of expected utility (see Figure 12-4).

Carey Silt Loam Soil, 1-3% Slope

EPIC Output Data

Table 13-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Carey Silt Loam soil. The total salt in the 1.2 meter profile was calculated as 0.53 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 13-2.

Table 13-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Carey Silt Loam Soil

	1*	1	2	3	4	5	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	
ECND(mmho/cm)	0.76	0.57	1.03	0.8	0.77	0.77	
WSLT(kg/ha)*	7	81	158	321	308	308	1,184
Salinity(tons/acre)							0.53**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 13-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Carey Silt Loam Soil

Variable	Symbol (Unit)	Range	Mean
Cotton Yield	Y (lbs/acre)	1 ~ 1885	982
Irrigation Water	Irr (ac-ft/acre)	0.16 ~ 2.62	1.29
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 26.15	9.02
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 26.6	9.77
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 13-1.

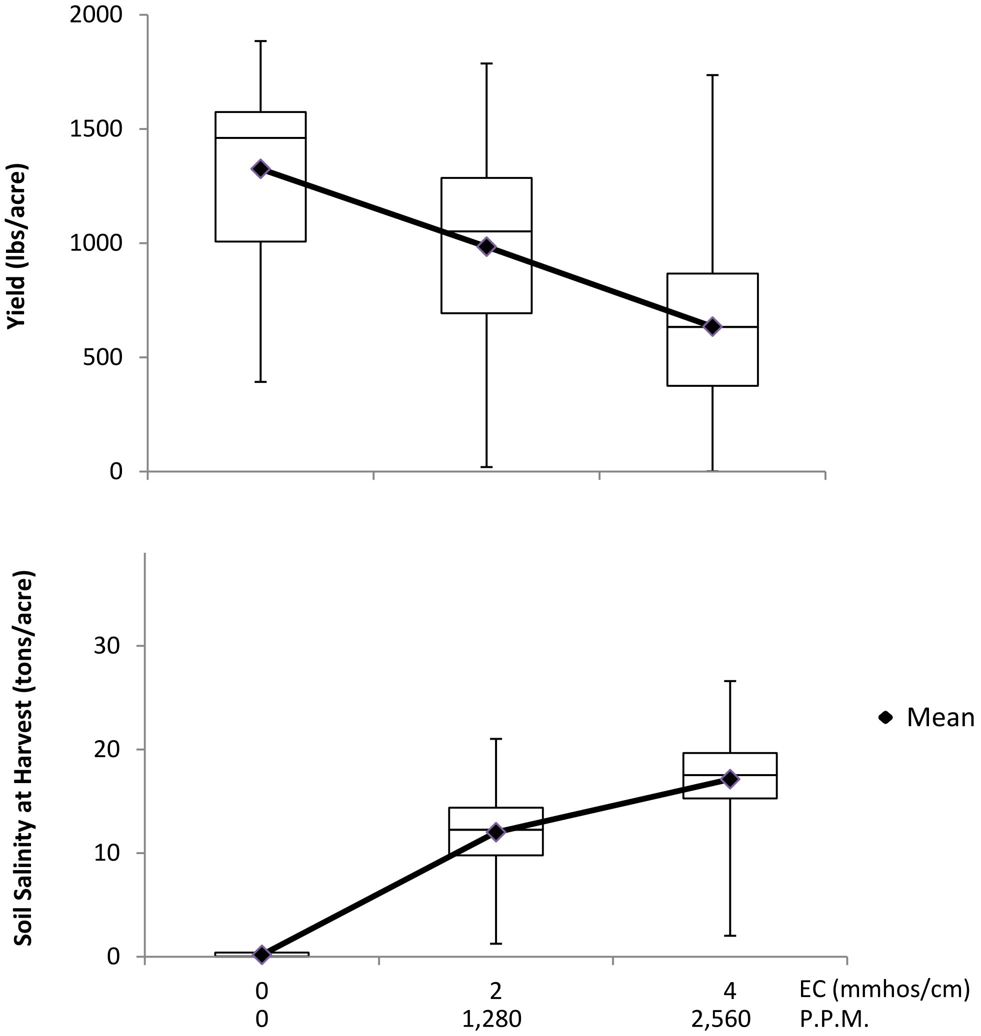


Figure 13-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) Salt Concentration of Irrigation Water for the Carey Silt Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with Type=AR(1) and/or GROUP = weather on the REPEATED statement was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has an error message of “Convergence criteria met but final hessian is not positive definite.” It is known that some parameters or variances in the model are estimated to be zero. Simplifying the model or removing variance components on the RANDOM statement is useful way to remedy this problem. Without RANDOM statement which means that the model does not consider the random effect part, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 13-3.

Table 13-3. Result of Likelihood Ratio Test for the Carey Silt Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	57313	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	57187		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 13-4.

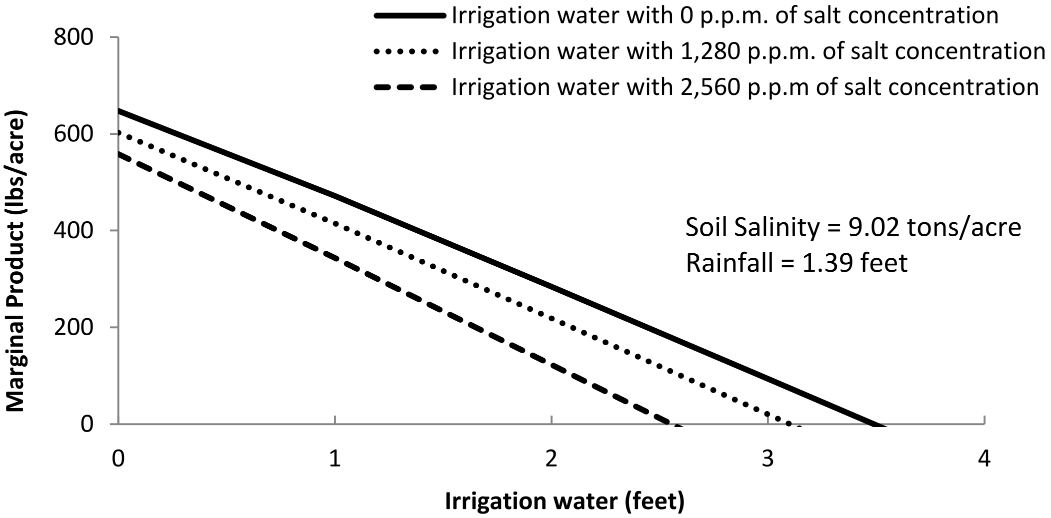
Table 13-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Carey Silt Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-593.59*	44.2519
Total Water Applied		938.83*	31.3391
Total Salinity		-6.8587*	1.3299
Non-Growing Season Rainfall		144.36*	10.4595
(Total Water Applied)^2		-96.0177*	5.4789
(Total Salinity)^2		-1.2517*	0.0439
(Total Salinity / Total Water Applied)		5.2844*	2.4329

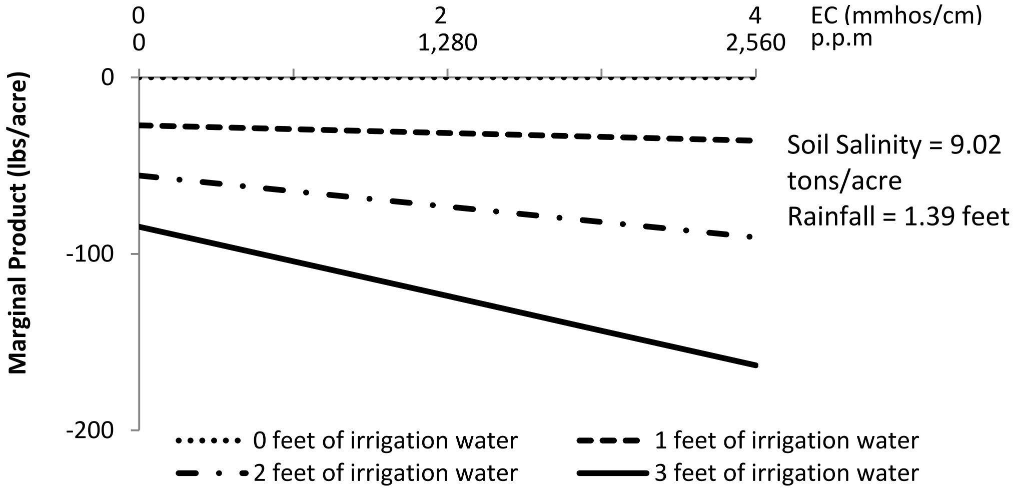
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 13-2.

Part (a) of Figure 13-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 13-2. Marginal Product of Irrigation Water and Salt Concentration for the Carey Silt Loam Soil

Part (b) of Figure 13-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield

declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 13-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$=$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -198.4, -189.2, -10 and -2.5. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3

sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

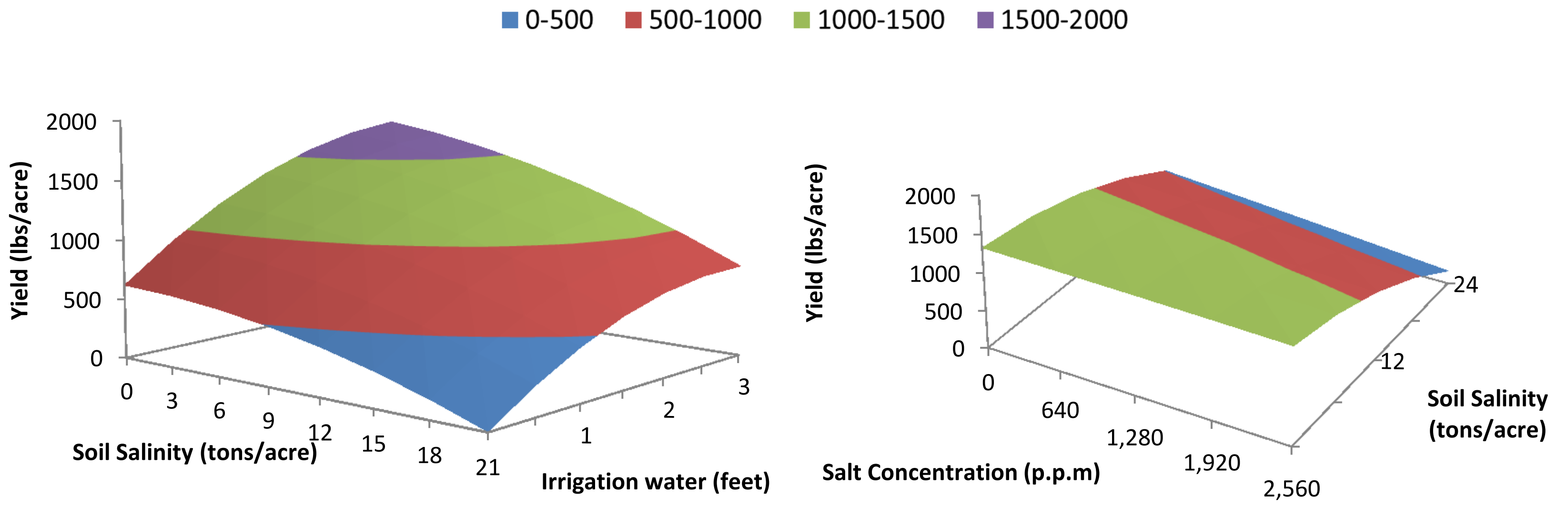
,

,

, and

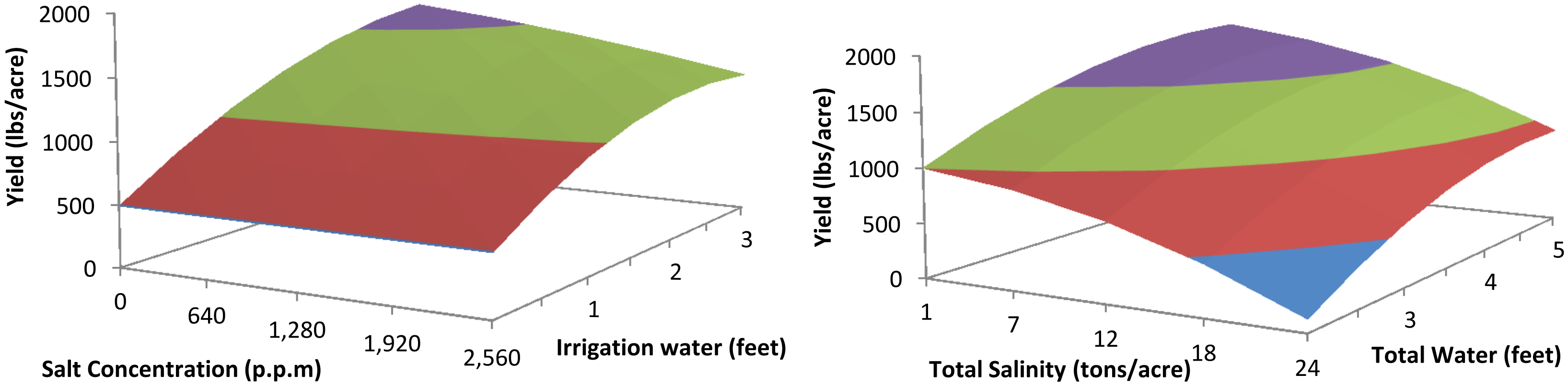
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Carey Silt Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 13-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 13-3. 3-D Surface of Crop Response Function versus Independent Variables for the Carey Silt Loam Soil

In part (a) of Figure 13-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 13-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.29 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 13-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 9.02 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. The crop yield as irrigation water increases.

In part (d) of Figure 13-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 13-5, 13-6 and 13-7, respectively.

Table 13-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Carey Silt Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.9288*	0.0894
Irrigation Water		-0.5360*	0.0378
Amount of Salt in Irrigation Water		0.6418*	0.0138
Soil Salinity at Planting		0.9072*	0.0038
Growing Season Rainfall		-1.4389*	0.0436

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.
The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 13-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 14039 and 13931, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 13-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Carey Silt Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.2368*	0.0556
Soil Salinity at Harvest on Previous Year		0.9260*	0.0025
Non-Growing Season Rainfall		-1.6985*	0.0701

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 13-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 13432 and 13201, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 13-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Carey Silt Loam Soil

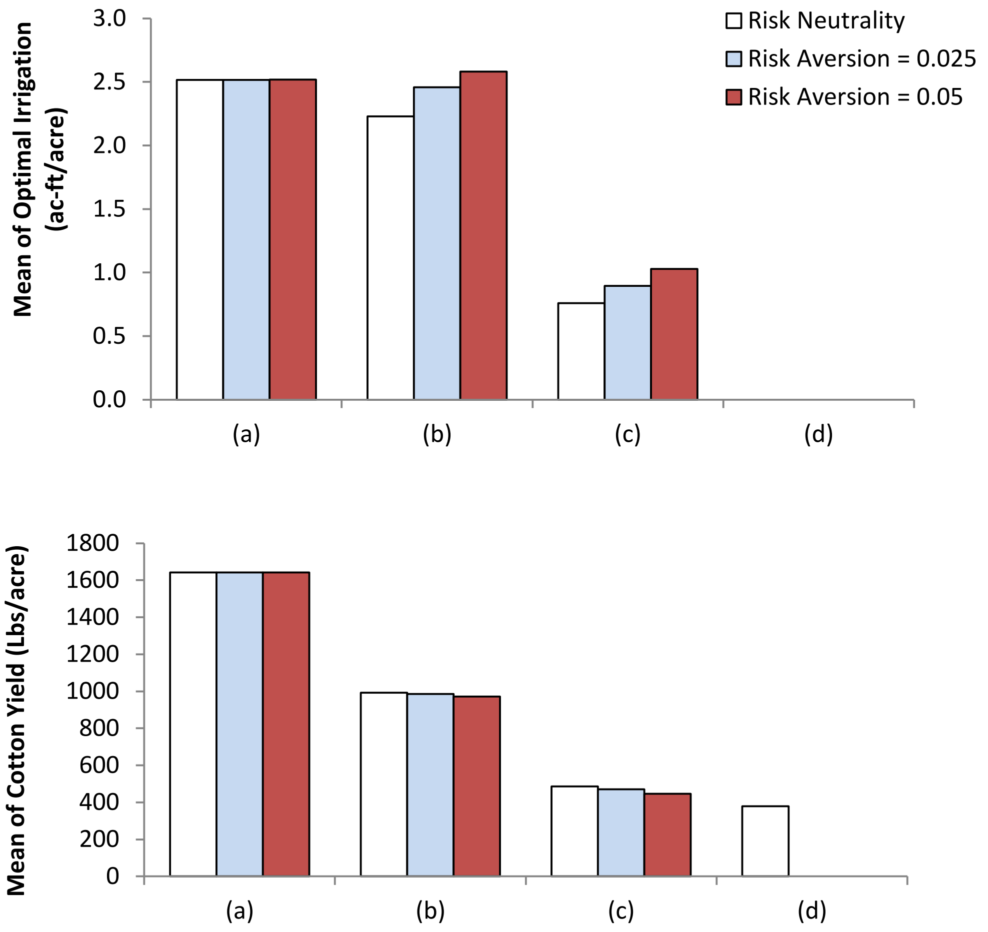
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		10540*	3045.92
Irrigation Water		-9201.64*	1060.71
Growing Season Rainfall		19412*	1599.71

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 13-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 13-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 13-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 13-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 13-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Carey Silt Loam Soil

The lower half of Figure 13-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 13-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 13-5.

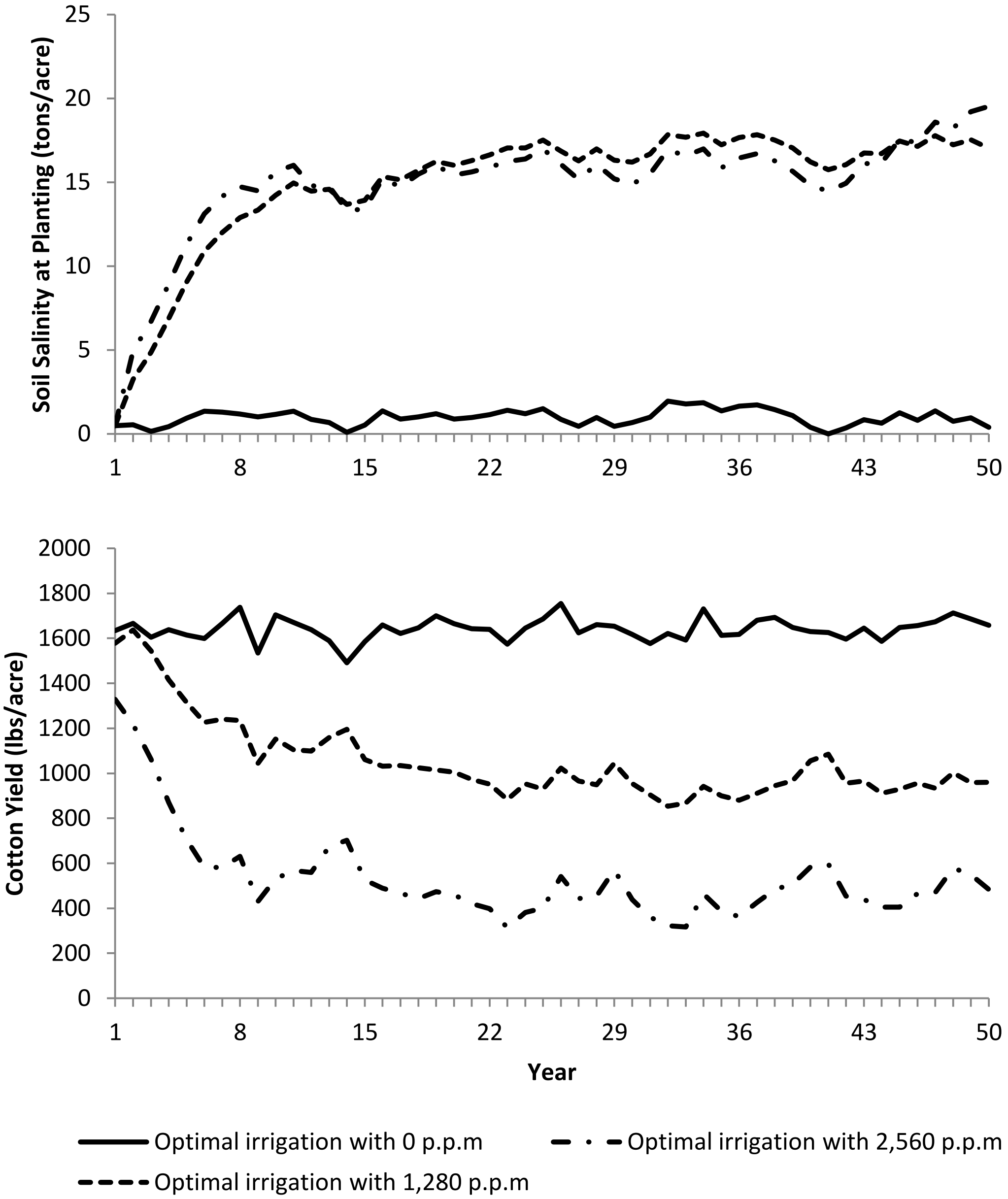


Figure 13-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.2m for the Carey Silt Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures -4 and -5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of Salt Concentration of Irrigation Water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 13-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

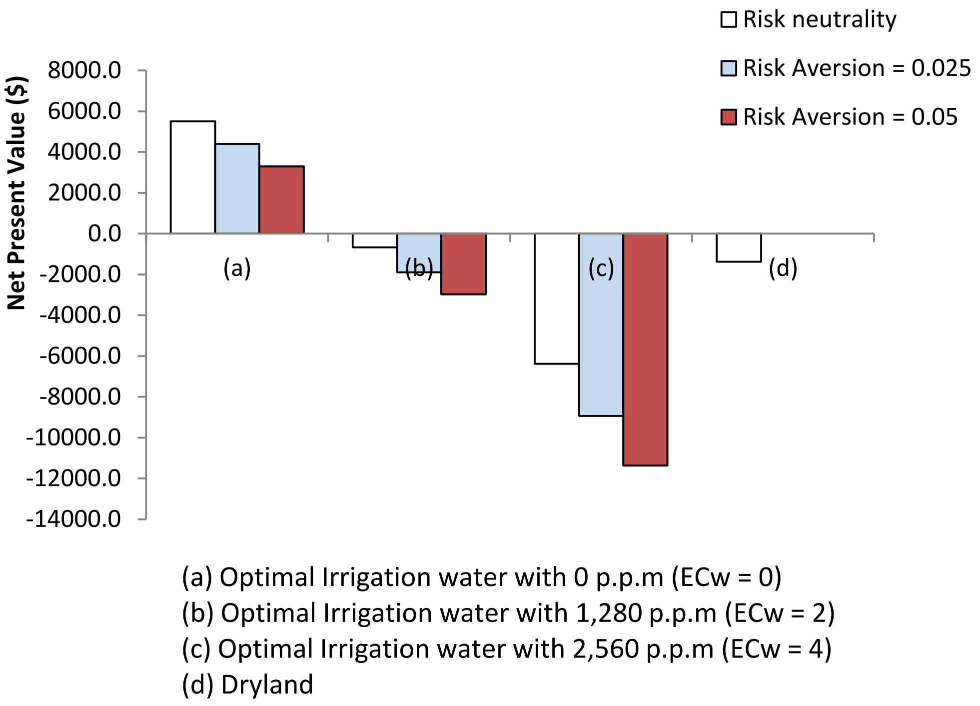


Figure 13-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Carey Silt Loam Soil

In (b) and (c) of Figure 13-6, the net present values are less than zero and also less than NPV of dryland but NPV of risk-neutral producers using (b) is slightly larger than

NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 13-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 13-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Carey Silt Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5514	6887
1,280 p.p.m (ECw=2)	-669	704
2.560 p.p.m (ECw=4)	-6388	-5015
Dryland	-1374	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 0 and 1,280 p.p.m is applied, the differences of their NPVs are positive. It indicates the producer can make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 2.2 ~ 2.5 ac-ft/acre to maximize NPV of expected utility (see Figure 13-4).

Grandfield Fine Sandy Loam Soil, 1-3% Slope

EPIC Output Data

Table 14-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Grandfield Fine Sandy Loam soil. The total salt in the 1.5 meter profile was calculated as 0.56 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 14-2.

Table 14-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Grandfield Fine Sandy Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.07	0.72	1.23	0.72	0.95	1.03	1.4	
WSLT(kg/ha)*	7	76	140	232	278	266	263	1,261
Salinity(tons/acre)								0.56**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.
Note: (*) indicates layer 1 and WSLT are simulated by EPIC.
(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 14-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Grandfield Fine Sandy Loam Soil

Variable	**Symbol (Unit)**	**Range**	**Mean**
Cotton Yield	Y (lbs/acre)	48 ~ 1730	1045
Irrigation Water	Irr (ac-ft/acre)	0.32 ~ 2.23	1.15
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 16.8	4.97
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 19.06	5.7
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 14-1.

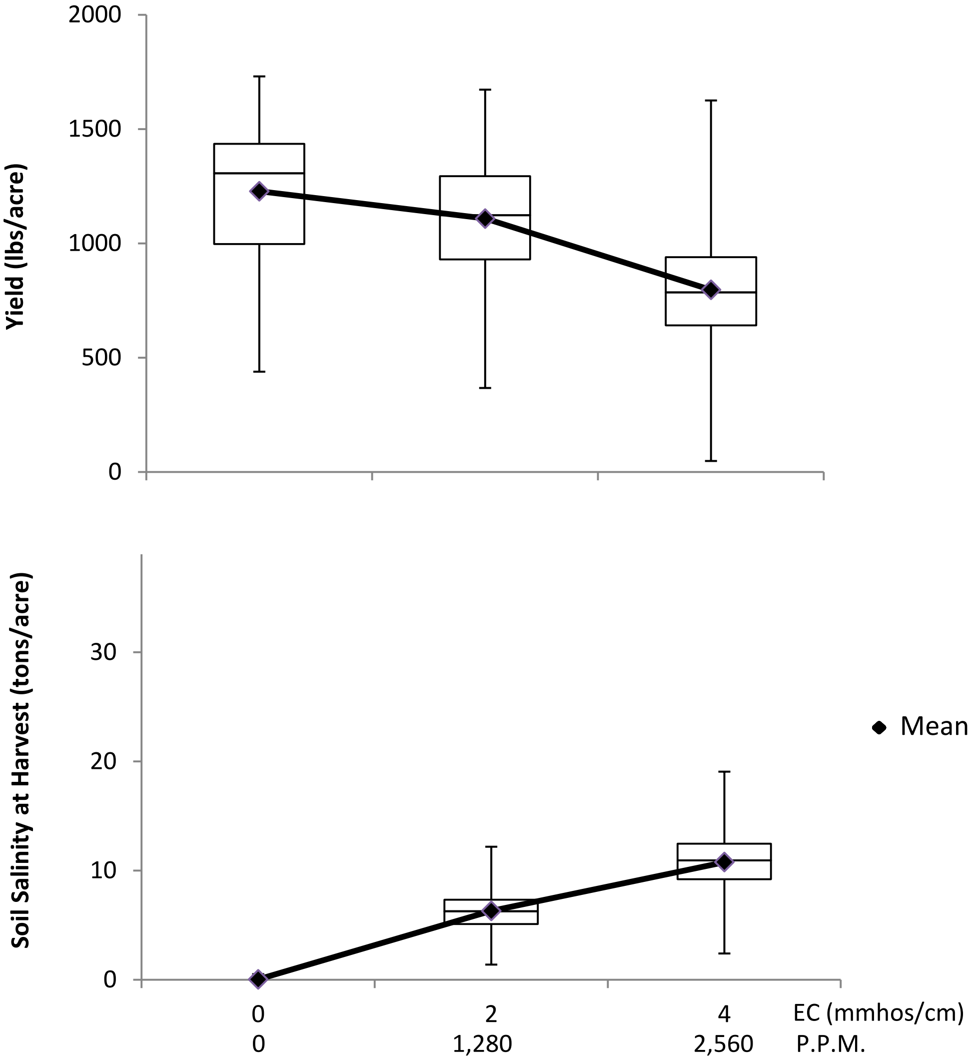


Figure 14-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Grandfield Fine Sandy Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with Type=AR(1) and GROUP = weather on the REPEATED statement was used to fit a model with autocorrelation and/or heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 14-3.

Table 14-3. Result of Likelihood Ratio Test for the Grandfield Fine Sandy Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56321	< 0.0001	Reject H_o
With Autocorrelation and Heteroscedasticity	56117		

Because the value of -2LogLikelihood with the distribution with 19 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008).

The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 14-4.

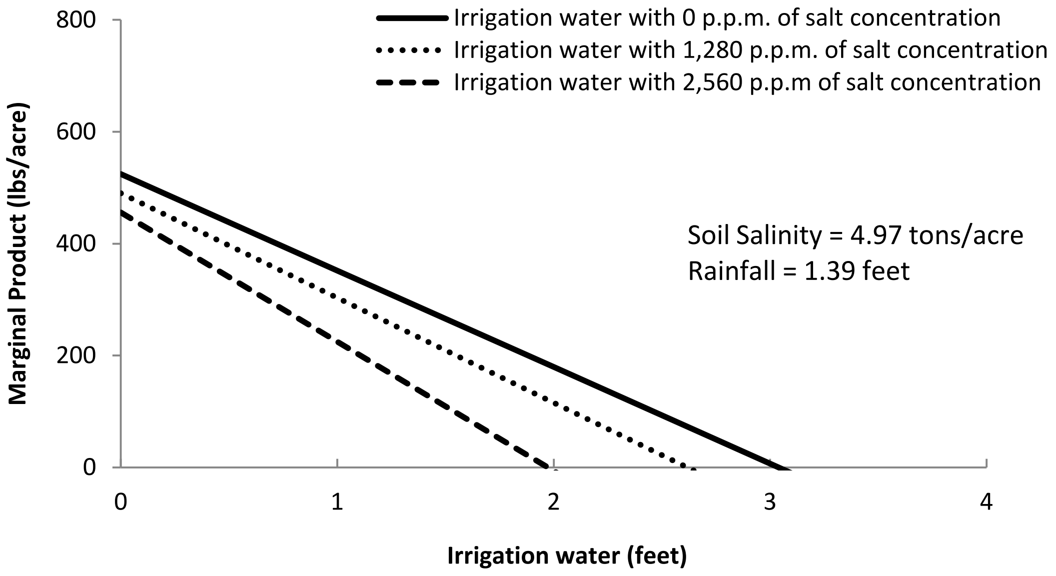
Table 14-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Grandfield Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-204.83*	44.9405
Total Water Applied		764.17*	32.8379
Total Salinity		4.4835**	2.3002
Non-Growing Season Rainfall		60.6276*	8.9573
(Total Water Applied)^2		-86.2759*	6.0242
(Total Salinity)^2		-2.4267*	0.0946
(Total Salinity / Total Water Applied)		4.1244	4.3884

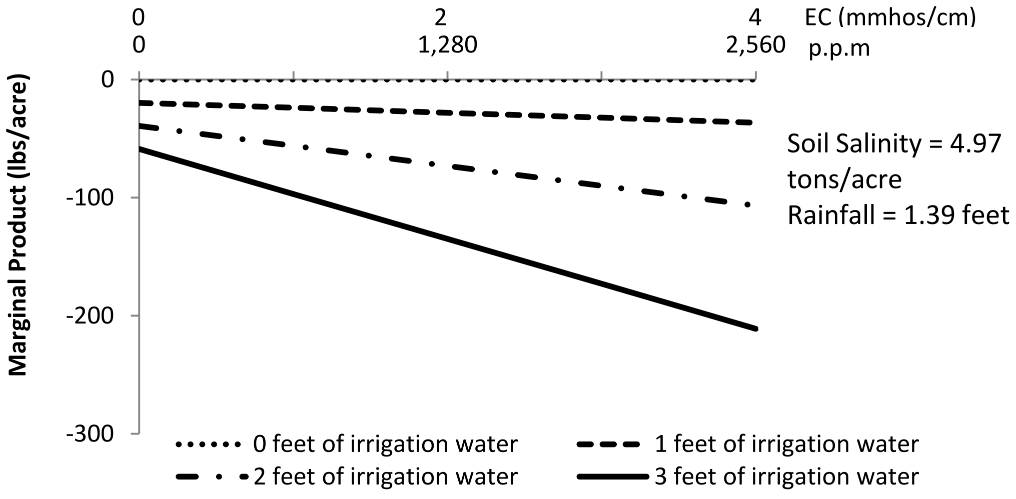
Note: Total Water Applied is the sum of irrigation water and growing season rainfall.
Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level.
(**) indicates parameters significant at the 10% significant level. *N*=4,500.

Except for (Total Salinity / Total Water Applied) of the interaction term, all parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 14-2.

Part (a) of Figure 14-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The point of maximum yield from irrigation occurs at lower levels as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 14-2. Marginal Product of Irrigation Water and Salt Concentration for the Grandfield Fine Sandy Loam Soil

Part (b) of Figure 14-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield declines when the salt concentration is increased in irrigation water, holding the rainfall

and soil salinity constant. From the (a) and (b) in Figure 14-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$=$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -187.2, -172.6, -19.4 and -4.9. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3 sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

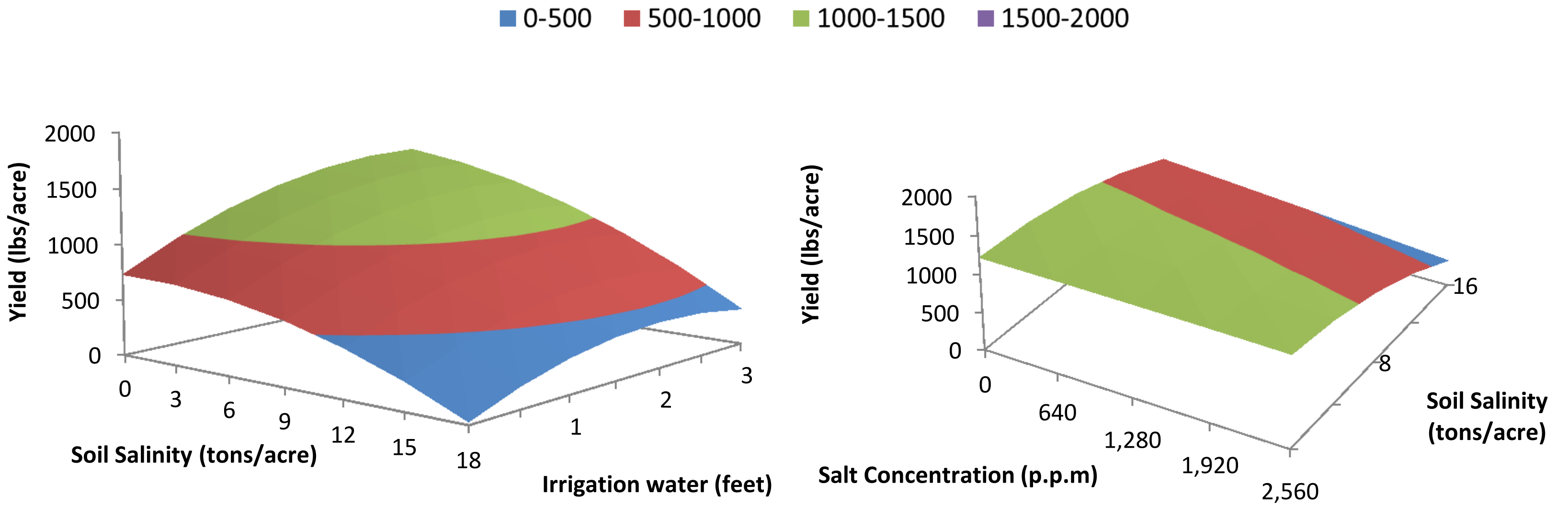
,

,

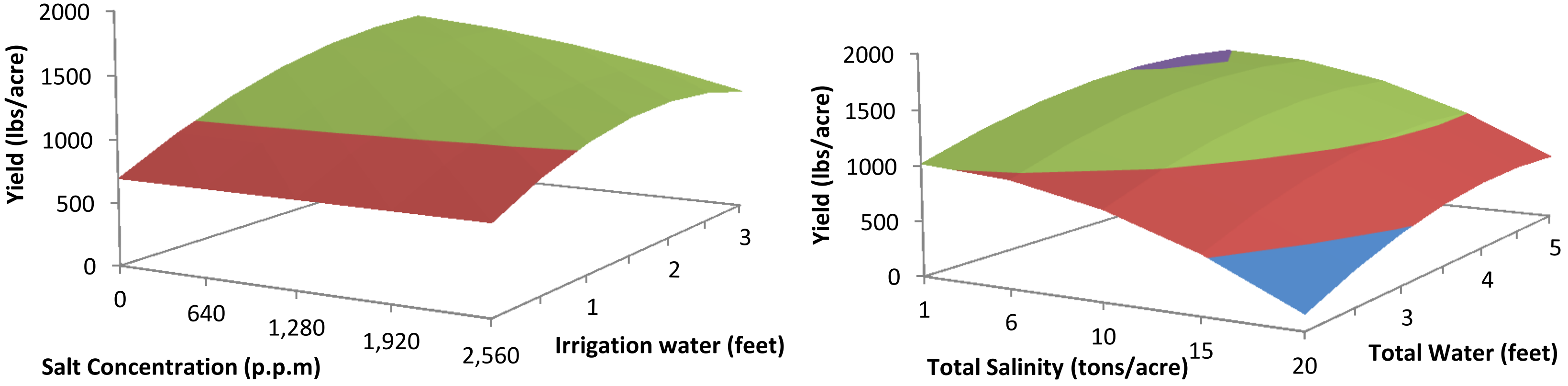
, and

All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Grandfield Fine Sandy Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 14-3.



(a) Yield versus Soil Salinity and Irrigation Water (b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water (d) Yield versus Total Salinity and Total Water

Figure 14-3. 3-D Surface of Crop Response Function versus Independent Variables for the Grandfield Fine Sandy Loam Soil

In part (a) of Figure 14-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 14-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, holding 1.39 feet and 1.15 feet of rainfall and irrigation water, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 14-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 4.97 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 14-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance

(functions are shown in Tables 14-5, 14-6 and 14-7, respectively.

Table 14-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Grandfield Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.9014*	0.0520
Irrigation Water		-0.3805*	0.0274
Amount of Salt in Irrigation Water		0.7578*	0.0115
Soil Salinity at Planting		0.8424*	0.0046
Growing Season Rainfall		-0.9770*	0.0246

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 14-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 8909 and 8827, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 14-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Grandfield Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.1058*	0.0369
Soil Salinity at Harvest on Previous Year		0.8705*	0.0025
Non-Growing Season Rainfall		-1.5774*	0.0463

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 14-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 9550 and 9427, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 14-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Grandfield Fine Sandy Loam Soil

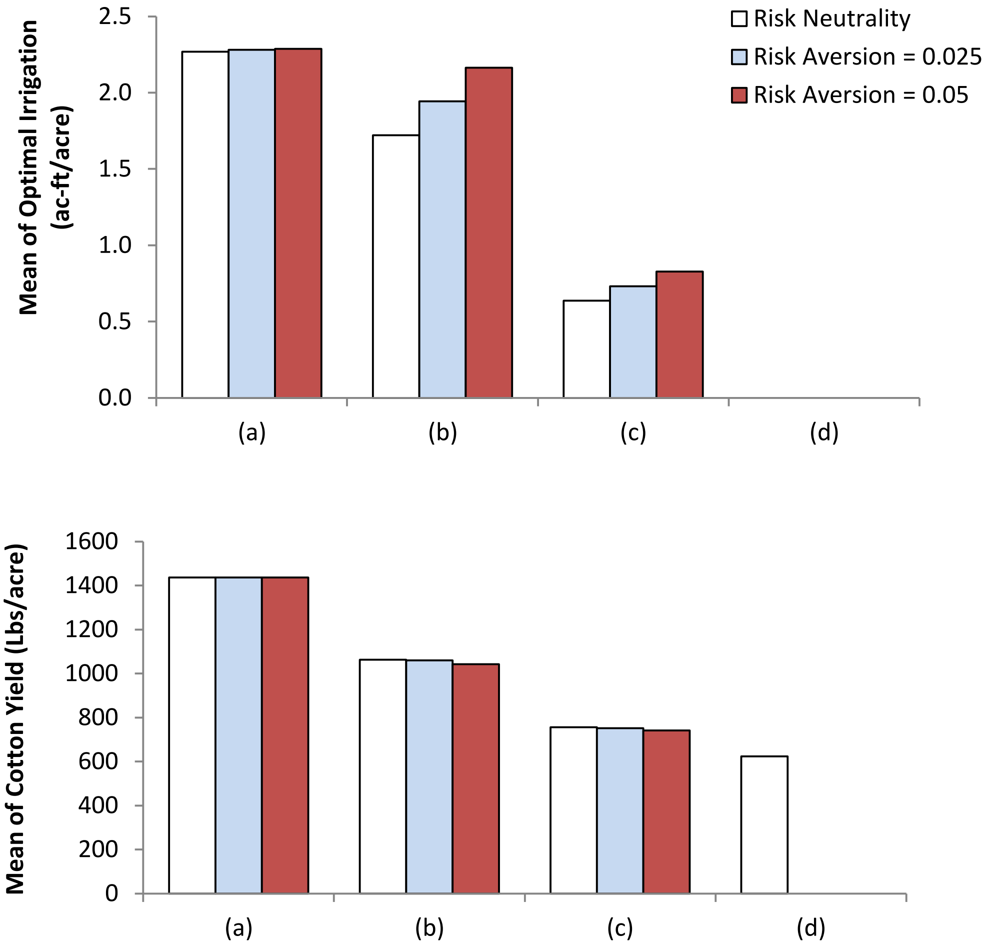
Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		11340*	2375.49
Irrigation Water		-8125.04*	1128.81
Growing Season Rainfall		13540*	1200.63

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 14-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 14-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 14-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 14-7.



(a) Optimal Irrigation water with 0 p.p.m (ECw = 0)
(b) Optimal Irrigation water with 1,280 p.p.m (ECw = 2)
(c) Optimal Irrigation water with 2,560 p.p.m (ECw = 4)
(d) Dryland

Figure 14-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Grandfield Fine Sandy Loam Soil

The lower half of Figure 14-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation

water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 14-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 14-5.

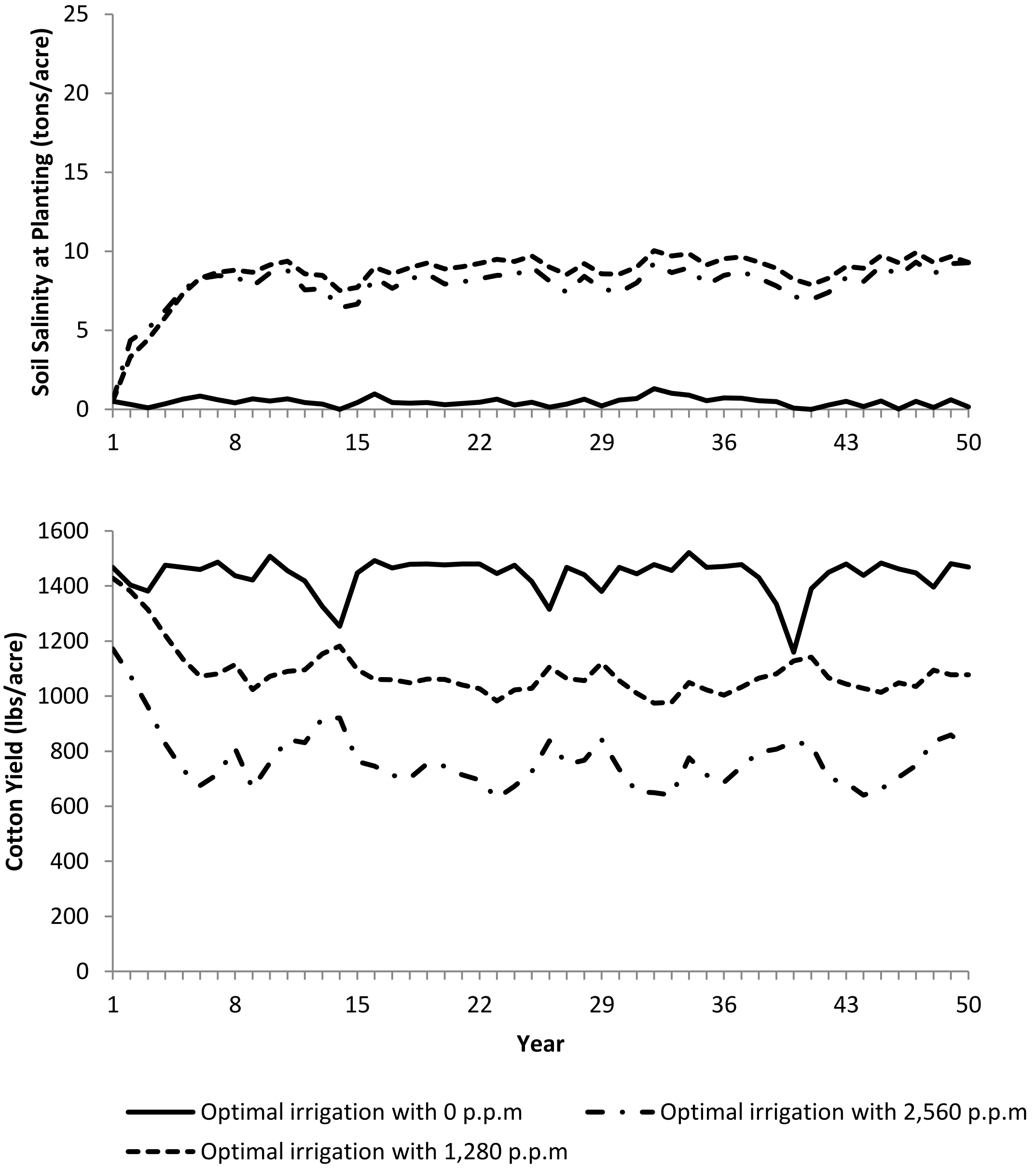


Figure 14-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth = 1.5m for the Grandfield Fine Sandy Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied over time, the quantity of soil salinity constantly increases and crop yield decreases as

salts are accumulated in the soil. Figures 14-4 and 14-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 14-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

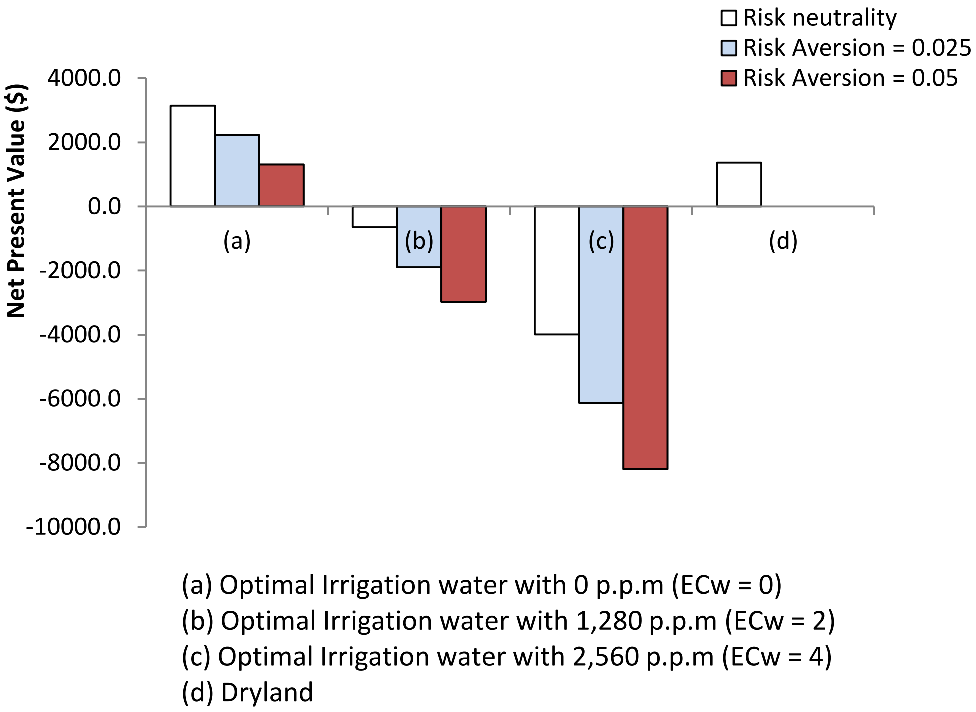


Figure 14-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Grandfield Fine Sandy Loam Soil

In (b) and (c) of Figure 14-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To

implement the project, the level of salt concentration should be limited to less than or equal to 1,280 p.p.m for the risk-averse and risk-neutral producer, respectively. Table 14-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 14-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Grandfield Fine Sandy Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	3147	1784
1,280 p.p.m (ECw=2)	-647	-2010
2.560 p.p.m (ECw=4)	-3998	-5361
Dryland	1363	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m by the decision maker, the optimal level of irrigation is approximately 1.72 ~ 2.27 ac-ft/acre to maximize NPV of expected utility (see Figure 14-4).

Tipton Fine Sandy Loam Soil, 0-1% Slope

EPIC Output Data

Table 15-1 is presented as the simulated quantity of soil salinity based on the sampled data for the Tipton Fine Sandy Loam soil. The total salt in the 1.5 meter profile was calculated as 2.27 tons/acre on the first day of simulation. The cotton yield, irrigation water, growing season rainfall and non-growing season rainfall were also obtained from the EPIC output file. The range of the simulated output variables which are used in the model are summarized in Table 15-2.

Table 15-1. Initial EPIC Soil Salinity Input Data based on Soil Samples of the Tipton Fine Sandy Loam Soil

	1*	1	2	3	4	5	6	TOTAL
DEPTH(m)	0.01	0.15	0.3	0.6	0.9	1.2	1.5	
ECND(mmho/cm)	1.25	0.84	1.09	1.03	2.22	6.15	5.16	
WSLT(kg/ha)*	8	88	124	311	884	2,444	2,048	5,096
Salinity(tons/acre)								2.27**

Source: Zhang *et al*, Oklahoma Soil Test Laboratory, 2011.

Note: (*) indicates layer 1 and WSLT are simulated by EPIC.

(**) indicates the value is calculated by the conversion (1 kg/ha = 0.446×10^{-3} tons/acre).

Table 15-2. Range and Mean of the Input and Output Variables Simulated from EPIC model for the Tipton Fine Sandy Loam Soil

Variable	**Symbol (Unit)**	**Range**	**Mean**
Cotton Yield	Y (lbs/acre)	5 ~ 1841	982
Irrigation Water	Irr (ac-ft/acre)	0.2 ~ 2.62	1.27
Soil Salinity at Planting	SS^{PL} (tons/acre)	0 ~ 24.87	9.23
Soil Salinity at Harvest	SS^{HA} (tons/acre)	0 ~ 27.7	10.03
Growing Season Rainfall	$Rain^{G}$ (feet)	0.47 ~ 2.61	1.39
Non-Growing Season Rainfall	$Rain^{NG}$ (feet)	0.06 ~ 1.64	0.66
Salt Concentration of Irrigation Water*	(tons/ac-ft)	0, 1.74 and 3.48	1.74

Note: (*) indicates the input variable to run EPIC.

To estimate the cotton yield, irrigation water and soil salinity at planting and harvest, these variables were simulated under the influence of the three salt concentration of irrigation water. The range of data for the simulated yield and soil salinity at harvest by given levels of the salt concentration are shown on a box plot in Figure 15-1.

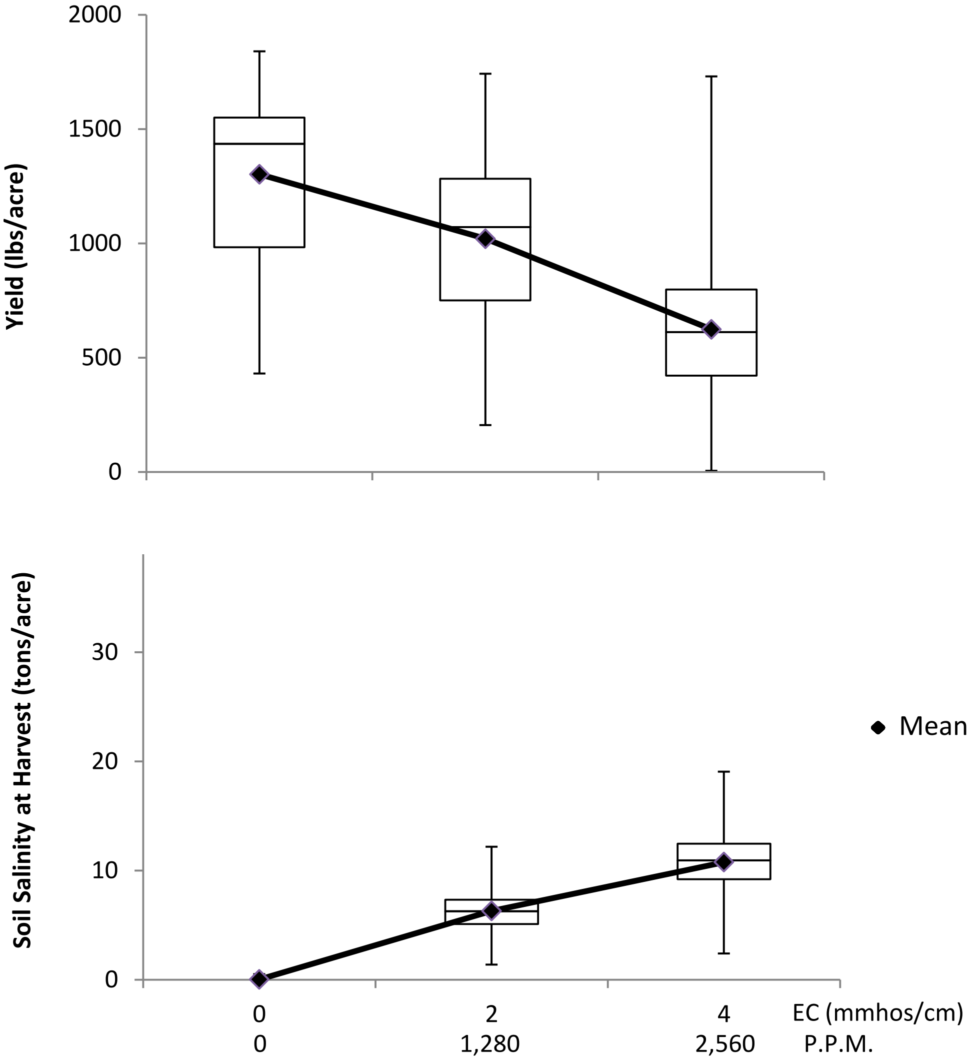


Figure 15-1. Fifty-year Average EPIC Simulated Yield and Soil Salinity after applying Irrigation Water with EC of 0, 2 and 4 (mmhos/cm) of Salt Concentration of Irrigation Water for the Tipton Fine Sandy Loam Soil

As the salt concentration of irrigation water increases, the mean of simulated yield data decreases and the mean of simulated soil salinity increases. Moreover, the mean of yield data decreases as the mean of soil salinity increases. The high level of salt concentration of irrigation water causes salts to accumulate in the soil. The accumulated salts will affect the reduction of crop yields.

Econometric Estimation

The SAS PROC MIXED procedure with Type=AR(1) and GROUP = weather on the REPEATED statement was used to fit a model with autocorrelation and/or heterogeneous variances. SAS output has two error messages of “Convergence criteria met but final hessian is not positive definite.” and “Estimated G matrix is not positive definite.” These indicate that parameters in the model or variance components on the RANDOM statement are estimated as being zero. Simplifying the model or removing variance components on the RANDOM and REPEATED statement is useful way to remedy these problems. Without RANDOM statement and Type=AR(1) on the REPEATED statement, new PROC MIXED statement is resubmitted and rerun. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The results of the LR test are shown in Table 15-3.

Table 15-3. Result of Likelihood Ratio Test for the Tipton Fine Sandy Loam Soil

Model	-2LogLikelihood	*p*-value	LR Test
Without Autocorrelation and Heteroscedasticity	56846	< 0.0001	Reject H_o
With Heteroscedasticity	56800		

Because the value of -2LogLikelihood with the distribution with 9 degrees of freedom has a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models. It indicates that the model fitted with autocorrelation and heteroscedasticity is more appropriate (SAS Institute Inc, 2008). The procedure of fitting a model with autocorrelation and heterogeneous variance reports parameter estimates along with standard errors. The results of cotton response function are shown in Table 15-4.

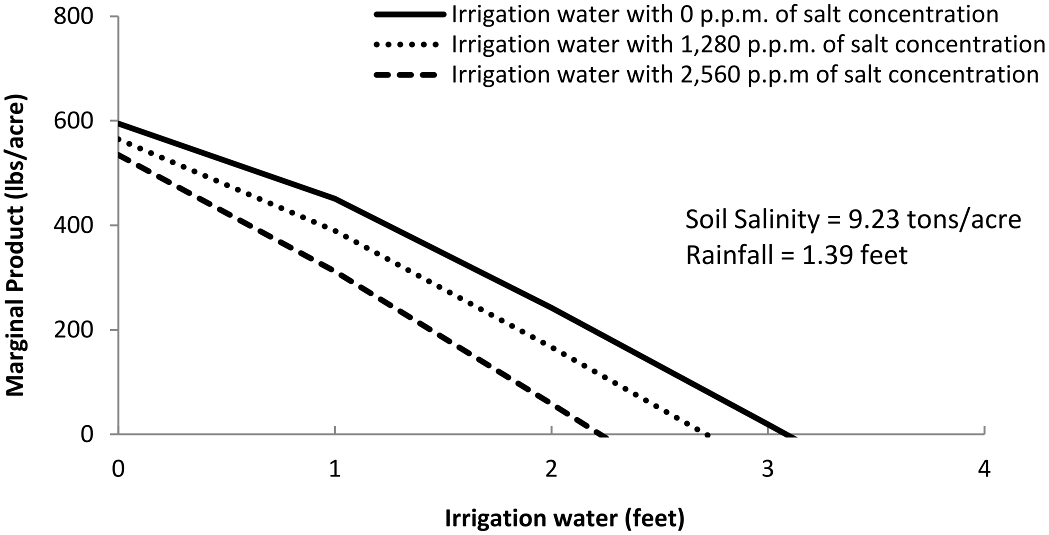
Table 15-4. Coefficient Estimates from SAS Proc Mixed for the Yield Response Function for the Tipton Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		-744.73*	46.2547
Total Water Applied		1049.98*	32.6681
Total Salinity		-12.2879*	1.3016
Non-Growing Season Rainfall		122.35*	10.1140
(Total Water Applied)^2		-115.86*	5.7307
(Total Salinity)^2		-1.3562*	0.0422
(Total Salinity / Total Water Applied)		27.9303*	2.5798

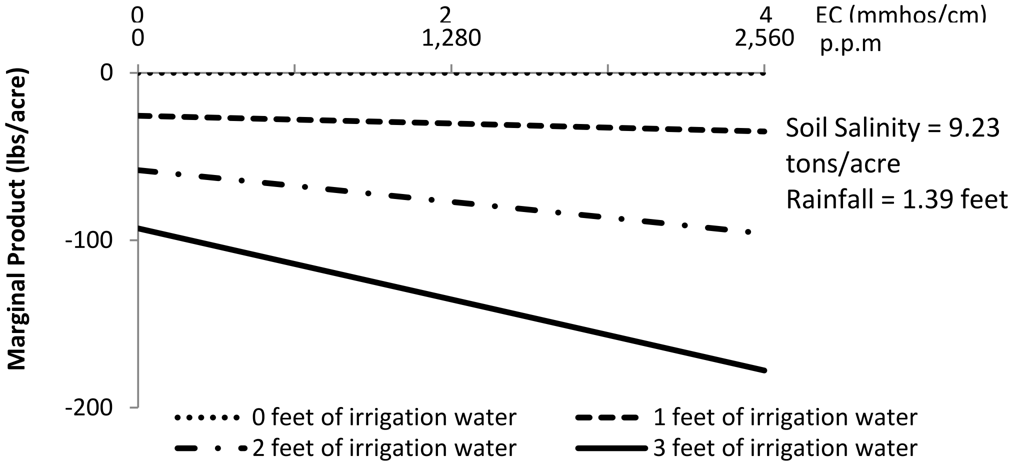
Note: Total Water Applied is the sum of irrigation water and growing season rainfall. Total Salinity is the sum of the amount of salt (irrigation water ×salt concentration of irrigation water) and soil salinity.
(*) indicates parameters significant at the 5% significant level. *N*=4,500.

All parameter estimates for the modified quadratic yield response function are significantly different from zero at the 5% significant level. The First Order Conditions (marginal product) show the effects of different amounts of irrigation water with different salt concentration on cotton yield in Figure 15-2.

Part (a) of Figure 15-2 shows that the crop yield increases rate as irrigation water increases in general while the rainfall and soil salinity are constant. An increase in the salt concentration of irrigation water reduces the marginal product of applied irrigation. The individual irrigation water containing given salt concentrations has a different point maximizing crop yield. This point is located on the horizontal axis. This is declining as the salt concentration increases.



(a) Marginal Product of Irrigation Water with Three Salt Concentrations



(b) Marginal Product of Three Salt Concentrations of Irrigation Water on Cotton Yield

Figure 15-2. Marginal Product of Irrigation Water and Salt Concentration for the Tipton Fine Sandy Loam Soil

Part (b) of Figure 15-2 verifies that the marginal product of the salt concentration of irrigation water is negative over the relevant data range. It means that the crop yield

declines when the salt concentration is increased in irrigation water, holding the rainfall and soil salinity constant. From the (a) and (b) in Figure 15-2, it can be concluded that irrigation water containing high salt concentration decreases crop yield and the point of maximum crop yield is reached at lower levels of irrigation as the salt concentration increases.

To compute the matrix of the second partial derivatives, we need the critical points. They are fixed at 2 ac-ft/acre, 1.39 feet, 1.74 tons/ac-ft and 7 tons/acre which are values of irrigation water (x_1), growing season rainfall (x_2), salt concentration of irrigation water (x_3) and soil salinity at planting (x_4), respectively. Given the detailed function $y = f(x_1, x_2, x_3, x_4, x_5)$ except for x_5, the Hessian matrix at the critical point is

$$=$$

For maximization problem, must be negative definite or negative semidefinite. If and only if or , is negative definite or negative semidefinite, respectively. Hence, the determinants should be all negative, the determinants should be positive, the determinat should be negative, and the determinant is positive.

The determinants are the diagonal elements of , , , , and which are -233.4, -216.7, -10.8, and -2.7. The order 4 determinant ( ) is formed by the 4 4 matrix as the above hessian matrix. The determinant is expanded into four 3 3

sub-matrices (and) along the diagonal elements in the hessian matrix as follows:

The determinants of and should be negative so that the determinant should be positive. The determinant of and can again be expanded into three 2×2 sub-matrices ( ), respectively, along their diagonal elements as follows:

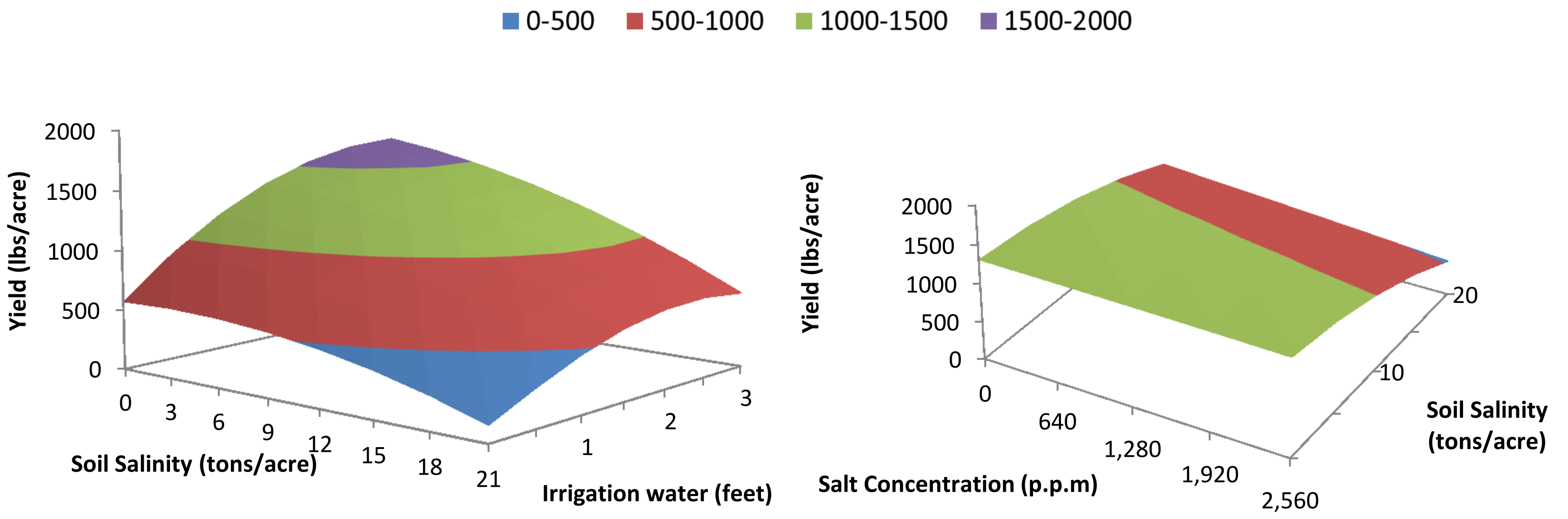
,

,

, and

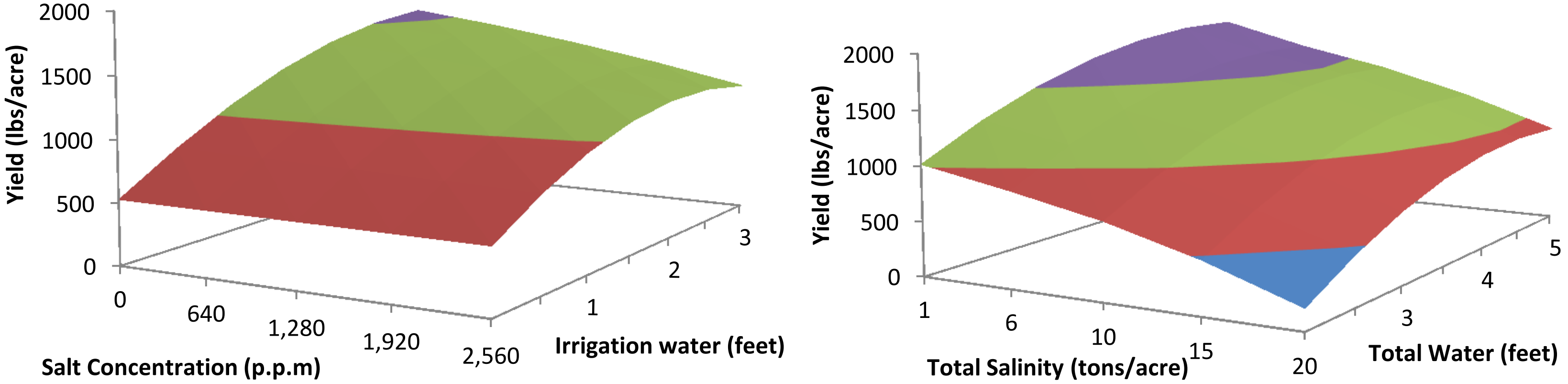
All determinants of the 2×2 matrix ( ) should be positive so that the determinants of and are negative. Since the values of determinants have the corrected signs, , , , and , the matrix is negative definite. We can conclude that the modified crop response function for the Tipton Fine Sandy Loam soil is concave and has a local maximum at the critical point.

The signs of second order derivative of this functional form only show that the function is locally concave at the point of evaluation. The values will be different at another point. It is identified that the function is globally concave with the three dimensional (3-D) surface. The 3-D surfaces of the modified crop response function with all combinations of two variables in the relevant range of data are plotted, holding 0.66 feet of non-growing season rainfall in Figure 15-3.



(a) Yield versus Soil Salinity and Irrigation Water

(b) Yield versus Salt Concentration and Soil Salinity



(c) Yield versus Salt Concentration and Irrigation Water

(d) Yield versus Total Salinity and Total Water

Figure 15-3. 3-D Surface of Crop Response Function versus Independent Variables for the Tipton Fine Sandy Loam Soil

In part (a) of Figure 15-3, the 3-D surface shows crop response function with yield on the vertical axis versus soil salinity and irrigation water on the horizontal axis, holding rainfall and salt concentration of irrigation water constant 1.39 feet and 1,280 p.p.m, respectively. The crop yield decreases as soil salinity increases with irrigation held constant. The crop yield first increases over the range of data as irrigation water increases. Beyond the level of irrigation water that maximizes crop yield, the crop yield starts to decrease.

In part (b) of Figure 15-3, the 3-D surface shows crop response function with yield on the vertical axis versus salt concentration and soil salinity on the horizontal axes, while rainfall and irrigation water are held constant at 1.39 feet and 1.27 feet, respectively. The crop yield decreases as soil salinity and salt concentration increase.

In part (c) of Figure 15-3, the crop yield is shown on the vertical axis versus salt concentration of irrigation water and irrigation water on the horizontal axes. Rainfall and soil salinity are held constant at 1.39 feet and 9.23 tons/acre, respectively. With dryland, the crop yield is constant although salt concentration increases. This is mathematically reasonable because salt is not being added without irrigation. When irrigation water is applied, the crop yield decreases as the salt concentration increases. Meanwhile, the crop yield increases as irrigation water increases.

In part (d) of Figure 15-3, the crop yield is shown on the vertical axis versus total salinity (the amount of salt in irrigation water plus soil salinity) and total water (irrigation water plus rainfall) on the horizontal axes. The crop yield decreases as total salinity increases. The crop yield increases as total water increases.

The soil salinity response functions at harvest and planting and yield variance (functions are shown in Tables 15-5, 15-6 and 15-7, respectively.

Table 15-5. Coefficients from SAS Proc Mixed for Dynamic Soil Salinity Levels at Harvest from EPIC Simulations for the Tipton Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		2.3090*	0.0734
Irrigation Water		-0.4691*	0.0340
Amount of Salt in Irrigation Water		0.7274*	0.0128
Soil Salinity at Planting		0.9048*	0.0035
Growing Season Rainfall		-1.1015*	0.0338

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500. The Amount of Salt in Irrigation Water is the product of irrigation water and salt concentration of irrigation water.

Table 15-5 shows that all parameter estimates for the soil salinity response function at harvest have the expected signs and are significantly different from zero at the 5% significant level. SAS PROC MIXED procedure with the REPEATED statement with Type=AR(1) and GROUP = weather was used to estimate the parameters with autocorrelation and heterogeneous variances. The Likelihood Ratio Test was used to determine the appropriate error function for the model. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 11687 and 11606, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 15-6. Coefficients from SAS Proc Mixed of Changes in Soil Salinity from Harvest to Planting from EPIC Simulations for the Tipton Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		1.3754*	0.0518
Soil Salinity at Harvest on Previous Year		0.9165*	0.0024
Non-Growing Season Rainfall		-1.8673*	0.0634

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

Table 15-6 shows that all parameter estimates for the dynamic soil salinity response function at planting have the expected signs and are significantly different from zero at the 5% significant level. The Likelihood Ratio Test was used to determine the appropriate error function for the model with autocorrelation and heterogeneous variances. The values of -2LogLikelihood without/with autocorrelation and heteroscedasticity are 12481 and 12267, respectively. As a result of LR test with a *p*-value of less than 0.01, we reject the null hypothesis of no difference between two models and conclude that the model fitted with autocorrelation and heteroscedasticity is more appropriate.

Table 15-7. Coefficients from SAS Proc Mixed for Yield Variance (Function for the Tipton Fine Sandy Loam Soil

Variable	Symbol	Parameter Estimates	Standard Errors
Intercept		10643*	2661.88
Irrigation Water		-8579.98*	1059.85
Growing Season Rainfall		16774*	1411.90

Note: (*) indicates parameters significant at the 5% significant level. *N*=4,500.

The coefficients relating the yield variance of irrigated cotton yield to random variables are shown in Table 15-7. All parameter estimates for the yield variance function are significantly different from zero at the 5% significant level. Irrigation water is risk-reducing (i.e., and growing season rainfall is risk-increasing (i.e., , respectively.

Dynamic Optimization

Figure 15-4 shows the 50-year average optimal level of irrigation and average cotton yield under different levels of irrigation water. The optimal level of irrigation declines as the salt concentration increases and also increases with more risk aversion. Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration in case of (a) is approximately constant in the upper half of Figure 15-4. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. However, in case of (b) and (c), the optimal level of irrigation water increases as the risk aversion coefficient increases. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers will use more water than risk-neutral producers. Irrigation water is an effective risk management tool to reduce crop yield variability (risk). This is consistent with negative sign on irrigation water in Table 15-7.

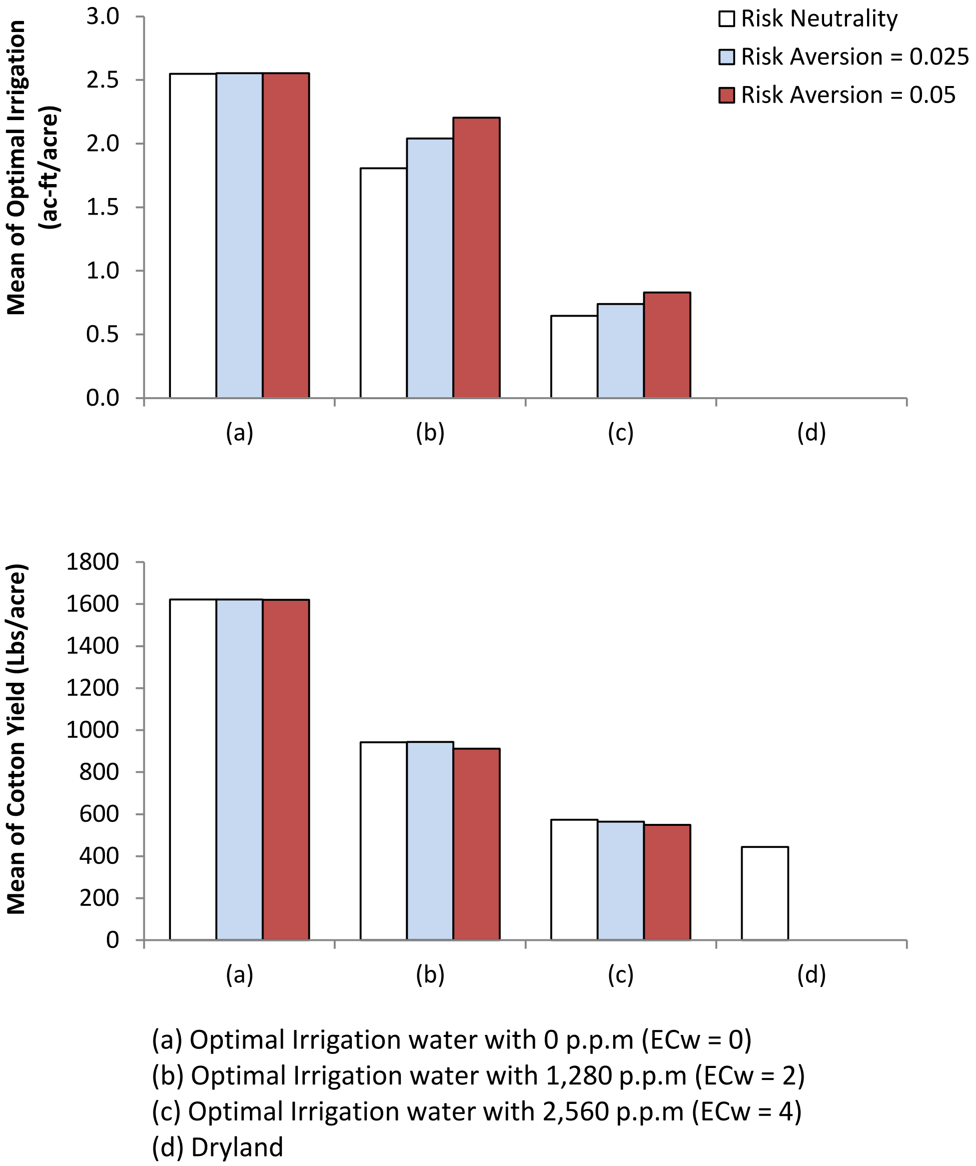


Figure 15-4. Average Optimal Application of Irrigation Water and Resulting Cotton Yield from 50-year Planning Horizon with Three Levels of Salinity in Irrigation Water and Risk Aversion for the Tipton Fine Sandy Loam Soil

The lower half of Figure 15-4 shows the 50 year average cotton yields with different levels of the salt concentration in irrigation water and degrees of risk aversion. Moreover, the dryland average yield was added on the graph to compare the irrigated average yield. It was optimized to apply nearly the same amounts of irrigation water in case of (a) and (b), although the yield declined with the increased salt in the irrigation water. When plenty of irrigation water is used in the soil, the soil salinity is leached. The leaching process will positively affect the crop growth. The comparisons of the average cotton yield of (a), (b), (c) and (d) in Figure 15-4 decrease as the salt concentration increases. If irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil and causes the crop yield to decrease. The average cotton yield of (c) is similar to (d). It indicates that irrigation water with a high salt concentration decreases cotton yield. However, the cotton yields are similar across the risk aversion coefficient. Using more irrigation water, the risk-averse producer decreases the crop yield variability (risk).

When the optimal level of irrigation with different levels of salt concentration is used in the crop land, it would affect changes of soil salinity over time. The quantity of soil salinity at planting is shown in Figure 15-5.

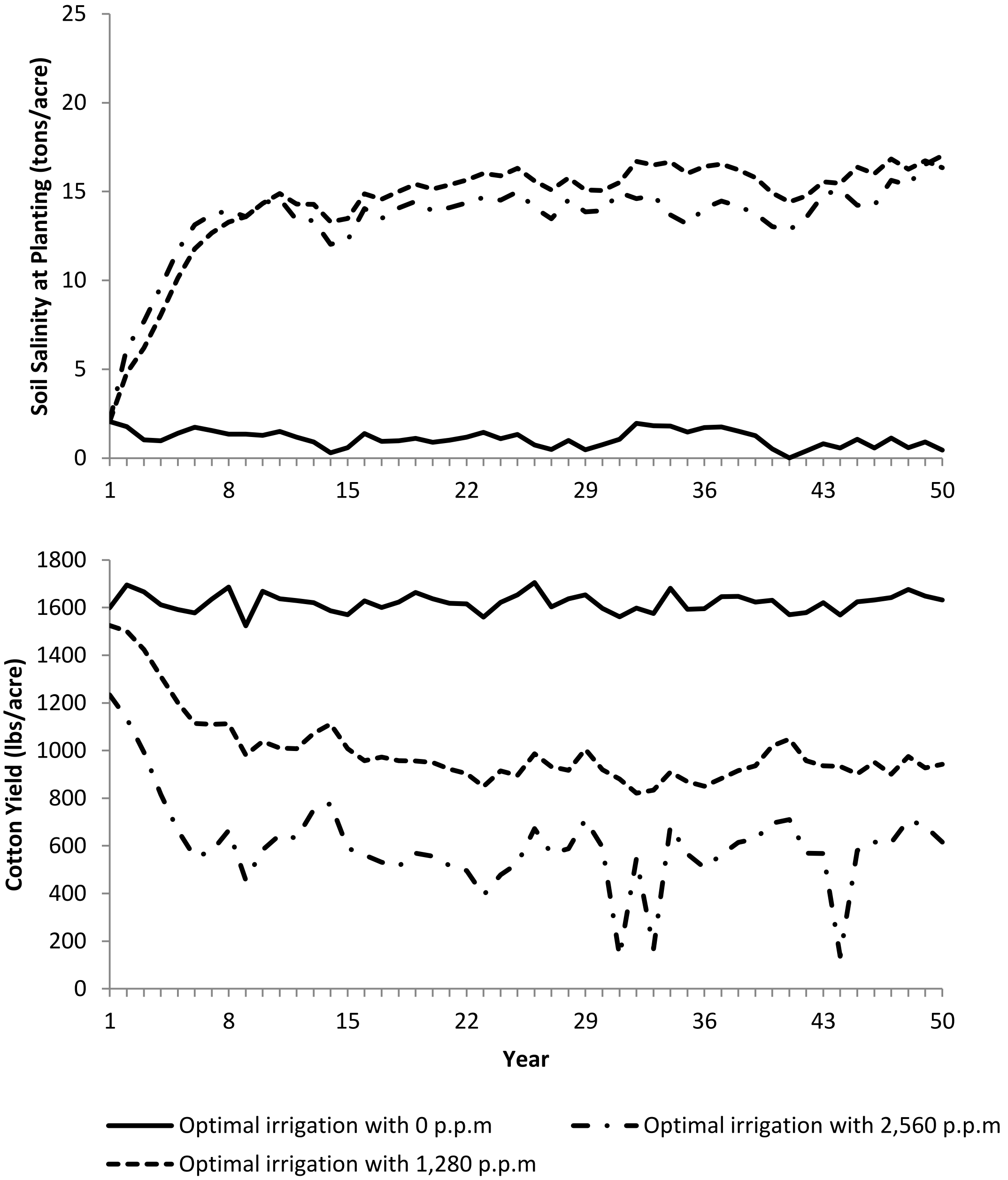


Figure 15-5. Changes of Quantity of Soil Salinity at Planting and Cotton Yield over 50 years Planning Horizon in case of Risk Aversion = 0.025 and Soil depth =1.5m for the Tipton Fine Sandy Loam Soil

The quantity of soil salinity fluctuates across the 50 year period because the change of salinity is linked to physical and chemical soil properties, irrigation water and rainfall. When the optimal level of irrigation with a given salt concentration is applied

over time, the quantity of soil salinity constantly increases and crop yield decreases as salts are accumulated in the soil. Figures 15-4 and 15-5 shows the cotton yield declined over time when the salt concentration in irrigation water is 1,280 p.p.m or more.

By comparing the net present value (NPV) of expected utility, we can determine the level of salt concentration of irrigation water for the sustainable irrigation and feasibility of the project. The NPV of expected utility is represented in Figure 15-6. NPV of expected utility decreases as the level of the salt concentration increases. NPV also decreases as the producer is more risk averse across the level of salt concentration because the more risk-averse producers are willing to receive reduced profits to decrease risk.

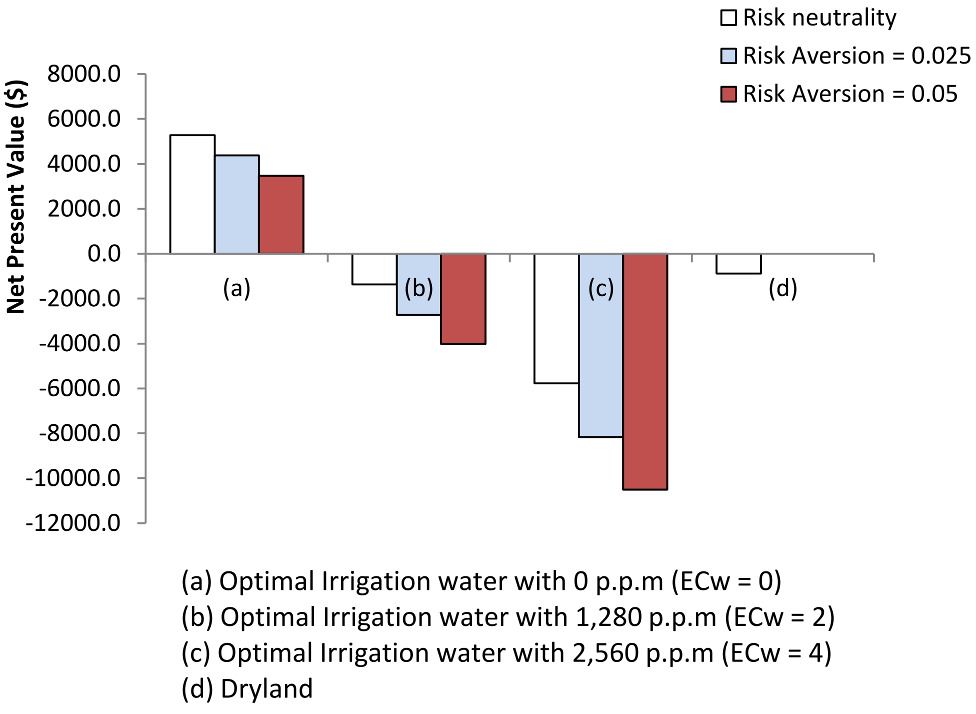


Figure 15-6. Net Present Value of Expected Utility of Optimal level of irrigation with Given Levels of Salt Concentration and Absolute Risk Aversion for the Tipton Fine Sandy Loam Soil

In (b) and (c) of Figure 15-6, the net present values are less than zero and also less than NPV of dryland. The negative NPV means that the project should be rejected. To implement the project, the level of salt concentration should be limited to less than 1,280 p.p.m for the producers. Table 15-8 shows the numerical NPV with different levels of salt concentration and difference between NPV of irrigated and dryland production in case of risk-neutrality.

Table 15-8. NPV of Given Salt Concentration and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer for the Tipton Fine Sandy Loam Soil

Salt Concentration	Net Present Value ($/acre)	Difference between 50-year NPV of Irrigated and Dryland Production ($/acre)*
0 p.p.m (ECw=0)	5286	6160
1,280 p.p.m (ECw=2)	-1365	-486
2.560 p.p.m (ECw=4)	-5773	-4894
Dryland	-879	

Note: (*) indicates the value of subtracting the NPV of dryland cotton from the NPV of the irrigated cotton.

When irrigation water with 1,280 and 2,560 p.p.m is applied, the differences of their NPVs are negative. It indicates the producer cannot make profits from investment in irrigated production compared to dryland cotton production. If the level of salt concentration is reduced to less than or equal to 1,280 p.p.m, the optimal level of irrigation is approximately 2.2 ~ 2.6 ac-ft/acre maximizing NPV of expected utility (see Figure 15-4).

CHAPTER V

CONCLUSIONS AND APPLICATIONS

The GIS technique was used to determine the area of potentially irrigable soils in the study area. The EPIC model was also used to estimate the expectation of the response yield to irrigation water, soil salinity under different salt concentrations of irrigation water and weather conditions. The crop and soil salinity response functions are estimated by SAS program. These response functions are incorporated in the dynamic optimization model which was implemented by GAMS program to determine the optimal level of irrigation and other optimal decision rules.

OSU Experiment Station collected and tested 15 samples of potentially irrigable soil types affected by chloride loading at the control point in the study area based on the results of GIS analysis. The results of soil test were used as one of input data for the EPIC model. Weather, soil, management, specific site information and parameter data files were used to operate the EPIC model.

The EPIC simulated and NASS observed cotton yields in Jackson County were used to to calibrate and validate the EPIC model over the 2001-2006 periods. The results indicated there are no statistical differences between simulated and observed cotton yield of irrigated and dryland.

It is well known that salinity has an adverse effect on the crop yield. The relationships between the crop yield and soil salinity are examined based on the simulation results of the EPIC model. The simulations of each soil type show that the crop yield decreases as the salt concentration and soil salinity increase. With irrigation water, the main cause increasing salinity of irrigable soil is the salt concentration of irrigation water. The amount of salts in irrigation water can rapidly increase the salinity level in the soil.

Results of EPIC, Regression and dynamic optimization are different by soil texture and types and weather conditions. We estimated 15 soil types. 15 soil types can be classified by 4 textures which are Loam, Silt Loam Clay Loam and Fine Sandy Loam texture. When irrigation water is applied to the crop land, it is infiltrated into the soil. The infiltration of water varies in soil textures. Generally the infiltration of water into the clay is slower than into the sand. Therefore, Clay Loam soil textures hold more water in the soil than Sandy Loam soil texture. In addition, the infiltration of Loam soil texture is between Clay loam and Fine Sandy Loam.

The results of crop response functions for the individual soil types indicate that the cotton yield increases as irrigation water and rainfall increase, while it decreases as the amount of salts in irrigation water and soil salinity increase. The cotton yield has a positive response to the non-growing season rainfall before planting cotton. In case of Total Water variable (irrigation water plus rainfall), beyond the point maximizing cotton yield, the curve of the response function turns downward indicating yield losses from excessive water use. The soil salinity response functions at planting and harvest have a negative sign on irrigation water, growing season rainfall and non-growing season rainfall indicating the level of soil salinity decreases as the variables related to water increase. When these variables provide sufficient of water to soil, the soil salinity is leached below the crop root zone and the soil would be less contaminated by salts. The washing process which is called leaching affects the crop growth positively. However, if irrigation water with a high salt concentration is applied to the crop land, it would allow salts to accumulate in the soil. On the contrary, irrigation water will be a detrimental factor to the crop growth. Before considering leaching effect to remove salts in the soil, the improvement of irrigation water quality should be preceded. In the yield variance function for irrigated cotton yield, the variance (risk) has a negative sign on irrigation water which is risk-reducing factor and a positive on the growing season rainfall which is risk-increasing.

The objective of using the dynamic optimization model is to empirically derive optimal level of irrigation with different levels of salt concentration and absolute risk aversion for irrigated cotton production when the irrigation system is applied to the study area. However, this study has several limitations in estimating the cotton yield response function and solving the dynamic optimization model. First, although there are many factors affecting the crop yield, we assumed that crop yield is directly affected by irrigation water, rainfall and salinity in water and soil. Second, the cotton price and cost vary in every week or every month. We fixed the cotton price and cost at a certain value for 50 years horizon planning in the dynamic optimization model. Last, the weather conditions for next 50 years were generated based on the historical data. Since the future weather has uncertainty, it cannot assure that the generated weather condition predicts the future weather conditions well.

The crop yield, optimal level of irrigation and soil dynamics are very sensitive to weather conditions (i.e., growing season rainfall and non-growing season rainfall). The quantity of soil salinity for all soil types fluctuates across the period of 50 years because the changes of salinity are linked to physical and chemical soil properties, irrigation water and rainfall. The optimal level of irrigation for all soil types also varies based on the salt concentration of irrigation water, producers' attitude (risk-aversion), properties of soil type and quantity of salinity in the soil. As the salt concentration of irrigation water increases, the optimal level of irrigation water decreases. Irrigation water with a high salt concentration allows salts to quickly accumulate in the soil and causes the cotton yield to decrease.

Regardless of the producer's risk attitude, the optimal level of irrigation with low salt concentration is approximately constant. It indicates that the optimal level of irrigation is independent of risk aversion coefficient at low salt levels. When irrigation water with 1,280 and 2,560 p.p.m of salt concentration is applied, the risk-averse producers are willing to use more irrigation water than risk-neutral producers.

The EPIC and dynamic optimization model can be used as a decision support tool to determine optimal irrigation water and control salts loading at the salt control point in the study area. Table 12 shows when optimal irrigation water containing 1,280 p.p.m of salt concentration is permitted from the salt control point in the study area, the difference between NPV of irrigated and dryland production for the risk-neutral producer by soil texture and types.

Table 16. Optimal Irrigation Water and Difference between NPV of Irrigated and Dryland Production for Risk-Neutral Producer with 1,280 p.p.m (ECw=2) of Salt Concentration by Soil Texture and Type

Soil Texture	Soil Type	Difference between NPV of Irrigated and Dryland Production ($/acre)	Optimal Irrigation water* (ac-ft/acre)
Loam	Tipton Loam	155.5	2 ~ 2.6
	Madge Fine Sand & Loam	-156.7	2 ~ 2.5
	Roark Loam	-45.2	1.9 ~ 2.5
	Spur Loam	-1263.5	1.8 ~ 2.5
	Frankirk Loam	305.4	2 ~ 2.5
	Lawton Loam	-209.3	1.9 ~ 2.5
	Abilene Loam	201.6	2 ~ 2.5
	Burford Loam	-772.9	2 ~ 2.5
Silt Loam	Carey Silt Loam	704.4	2.2 ~ 2.5
Clay Loam	Spur Clay Loam	484.9	2 ~ 2.5
	Tillman Clay Loam**	-6739.8	-
	Westill Clay Loam**	-6437.0	-
Sandy Loam	Hardeman Fine Sandy Loam	-2831.6	1.6 ~ 2.3
	Grandfield Fine Sandy Loam	-2009.9	1.7 ~ 2.3
	Tipton Fine Sandy Loam	-485.8	2.2 ~ 2.6

Note: (*) indicates the average optimal irrigation water when the level of salt concentration is controlled less than or equal to 1,280 p.p.m (ECw = 2).

(**) indicates the difference of NPV between irrigated and dryland productions for these soil types with 0 p.p.m (ECw = 0) is also negative.

The sign of the difference between NPV of irrigated and dryland production are positive indicating the producer can make profit from investment in irrigated production and negative indicating the dryland producer has better profits than the producer using irrigation water. In case of Tillman and Westil Clay Loam soil type, the difference of NPV between irrigated and dryland productions for these soil types with 0 p.p.m as well as 1,280 p.p.m is also negative. The producer using irrigation water in the Tillman and Westill Clay Loam soil type cannot make profit from investment in irrigated production.

If decision makers need to control the level of salt concentration less than or equal to 1, 280 p.p.m across all soil types except for Tillman and Westil Clay Loam soil type for the sustainable irrigation, the level of optimal irrigation water varies from1.6 to 2.6 ac-ft/acre depending soil texture and types. When , the drainage system is suggested to prevent the accumulated salts in the soil when irrigation water with a high salt concentration is used. Although we did not consider the drainage effect in this study, it may be a subject worthy of future study with the EPIC model.

The EPIC model was simulated with weather, soil properties and management as input data. It is also useful to simulate changes in planting date, modifying crop rotations, changing irrigation practices and tillage operations in input data. Through model calibration and validation, it is expected to perform proper simulations and produce reliable results. However, it requires technical skills to run the crop simulation model and an understanding of agronomic principles and terminologies (Jame and Cutforth, 1996).

By using the weather simulation program such as the WXGEN program, the EPIC can extend analyzing the response crop yield to the impact of climate changes such as global warming, flood and drought. The climate change will be a very important factor in changes of the crop yield in the future. The EPIC also can be used to justify decisions for the project implementation in the rural development sector in the developing countries. Many developed countries have carried out the project on making reservoirs or applying irrigation systems. However, it is difficult to expect the irrigated crop yield and analyze the economic effect of the project before implementing the project. With simulation, the EPIC can provide information that needs for justifying decisions whether the project is practicable or not.

REFERENCES

Arthurton, D. D., Moffitt, L. J., Allen, P. G., and Cox, D. A., 1995. “Do farm Businesses and Big Businesses Apply Different Capital Budgeting Procedures?” Agricultural and Resource Economics Review, 24(2): 147-148

Beattie, B. R. and Taylor, C. R. 1985. *The Economics of Production*, New York, John Wiley and Sons

Brouwer, C., Goffeau, A. and Heibloem, M. 1985. *Irrigation Water Management: Training Manual No.1 – Introduction to Irrigation*, Rome, Italy: Food and Agriculture Organization of the United Nations

Bimonthly Newsletter of the Oklahoma Water Resource Board. 2000. Available online at: http://www.owrb.ok.gov/news/news2/pdf_news2/WaterNews/WaterNews2000-2.pdf. accessed at January 30, 2011.

Cabelguenne, M., Jones, C. A., Marty, J. R., Dyke, P. T., and Williams, J. R. 1990. “Calibration and Validation of EPIC for Crop Rotations Southern France” *Agricultural Systems* 33: 153-171.

Coyle, B. T. 1999. “Risk Aversion and Yield Uncertainty in Duality Models of Production: a Mean Variance Approach” *American Journal of Agricultural Economics* 81: 553-567.

Datta, K. K., Sharma, V. P., and Sharma, D. P. 1998. “Estimation of a Production Function for Wheat under Saline Conditions” *Agricultural Water Management* 36: 85-94.

Dinar, A., and Knapp, K. C. 1986. “A Dynamic Analysis of Optimal Water Use under Saline Conditions” *Western Journal of Agricultural Economics*, 11(1): 58-66.

Dinar, A., Rhoades, J. D., Nash, P., and Waggoner, B. L. 1991. “Production functions relating crop yield, water quality and quantity, soil salinity and drainage volume” *Agricultural Water Management*, 19: 51-66.

Easterling, W. E., Rosenberg, M. J., Mckenney, M. S., Jones, C.A., Dyke, P.T, and Williams, J.R. 1992. “Preparing the erosion productivity impact calculator (EPIC) model to simulate crop response to climate change and the direct effects of CO_2" *Agricultural and Forest Meteorology*, 59:17-34.

Feinerman, E. 1994. "Value of information on crop response function to soil salinity in a farm-level optimization model" *Agricultural Economics* 10: 233-243.

Finger, R and Schmid, S. 2007. "The impact of Climate Change on Mean and Variability of Swiss Corn Production" Schriftenreihe der Gruppe Agrar-, Lebensmittel und Umweltokonomie, ETH Zurich: Nr.

Hake, S. J., Kerby, T. A., and Hake, K. D. 1996. "Cotton Production Manual" Publication 3352, University of California.

Hazell, P. B., and Norton, R., 1986. *Mathematical Programming for Economic Analysis in Agriculture*. Macmillan Publishing Company, New York.

Hillel, D. 2000. *Salinity Management for Sustainable Irrigation: Integrating Science, Environment, and Economics*. Washington, DC: World Bank.

Jame, Y. H., and Cutforth, H. W. 1996. "Crop Growth Models for Decision Support Systems" *Can. J. Plans Sci* 76: 9-19

Jonhson, P. N., and Blackshear, J. 2004, "Economic Analysis of Roundup Ready Versus Conventional Cotton Varieties in the Southern High Plains of Texas" *The Texas Journal of Agriculture and Natural Resource* 17:87-96

Johnston, J., and DiNardo, J. 1997, *Econometric Methods, Fourth Edition*, The McGraw-Hill Companies, Inc.

Kiani, A. R., and Abbasi, F. 2009. "Assessment of the Water-Salinity Crop Production Function of Wheat using Experimental Data of the Golestan Province, Iran" *Irrigation and Drainage* 58: 445-455.

Ko, J., Piccinni, G., and Steglich, E. 2009. "Using EPIC Model to Manage Irrigated Cotton and Maize" *Agricultural Water Management*, 96: 1323-1331

Littell, R. C., George A. M, Walter, W. S., Russell, D. W., and Oliver S. 2006, *SAS for Mixed Models, Second Edition*. Cary, NC: SAS Institute Inc.

Liu, J., Williams, J. R., Wang, X., and Yang, H. 2009. "Using MODAWEC to Generate Daily Weather Data for the EPIC Model" *Environmental Modeling & Software* 24: 655-664

Oklahoma's Beneficial Use Monitoring Program-Lakes Sampling. 2009. Available online at: http://www.owrb.ok.gov/quality/monitoring/bump.php. accessed January 30, 2011.

Oklahoma State University Extension. 2011, OSU Enterprise Budget Software.

Sammis, T. W., and Mexal, J. G. 1999. "Irrigation Water Management to Sustain Agriculture in the Desert" *New Mexico Journal of Science* 39: 301-309

SAS Institute Inc. 2008. *SAS/STAT® 9.2 User's Guide*. Cary, NC: SAS Institute Inc.

SAS Institute Inc. 2010. *Base SAS® 9.2 Procedures Guide: Statistical Procedures, Third Edition*. Cary, NC: SAS Institute Inc. Available online at: http://support.sas.com/documentation/cdl/en/procstat/63104/PDF/default/procstat.pdf. accessed June, 12, 2011.

SAS Institute Inc., SAS Campus Drive, Cary, North Carolina 27513, USA. Available online at: http://support.sas.com/kb/22/614.html. accessed June 3, 2011.

Tayfur, G., Tanji, K.K., House, B., Robinson, F., Teuber, L., and Kruse, G. 1995. "Modeling Deficit Irrigation in Alfalfa Production" *Journal of Irrigation and Drainage Engineering*: 442-451.

US Army Corps of Engineers, Red River Chloride Control Project Elm Fork, Area VI, Oklahoma, General Reevaluation Report. March, 2010. Available online at: *http://www.swt.usace.army.mil/library/libraryDetail.cfm?ID=469/*. accessed October 21, 2010.

U.S. Department of Agriculture, Natural Resources Conservation Service. National Soil Survey Handbook, title 430-VI. Available online at: *http://soils.usda.gov/technical/handbook/.* accessed October 1, 2010.

U.S. Department of Agriculture, National Agricultural Statistics Service. Agricultural Handbook, Number 628, Field Crops Usual Planting and Harvesting Dates. Available online at: http://usda.mannlib.cornell.edu/usda/current/plant/plant-10-29-2010.pdf/. accessed December 12, 2010.

U.S. Department of Agriculture, National Agricultural Statistics Service. Agricultural Handbook, Number 60, Diagnosis and Improvement of Saline and Alkali Soils. Available online at: http://ars.usda.gov/SP2UserFiles/Place/53102000/hb60_pdf/Hb60appe.pdf/. accessed May 20, 2011.

U.S. Department of the Interior Bureau of Reclamation, Oklahoma-Texas area office. W.C. Austin Project, Oklahoma. Available online at: http://www.usbr.gov/gp/otao/wcaustin2005.pdf. accessed January 1, 2011.

Wackerly, D. D., Mendenhaall, W. and Schaeffer, R. L. 2002. *Mathematical Statistics with Application, Six Edition*. Wadsworth Group.

Wang. X., He, X., Williams, J. R., Izaurralde. R.C., and Atwood, J. D. 2005. "Sensitivity and Uncertainty Analyses of Crop Yields and Soil Organic Carbon Simulated with EPIC" *American Society of Agricultural Engineers* 48(3): 1041-1054.

Williams, J. R., Richardson, C. W., and Griggs, R. H. 1992. "The Weather Factor: Incorporating Weather Variance into Computer Simulation" Weed Techology, Vol 6, No. 3: 731-735.

Williams, J. R., Wang, E., Meinardus, A., Harman, W. L., Siemers, M., and Atwood, J. D. 2006. "EPIC Users Guide v. 0509" Available online at: http://ftp.brc.tamus.edu/media/23015/epic0509usermanualupdated.pdf. accessed January 30, 2011.

Williams, J. R. 1990. "The Erosion-Productivity Impact Calculator (EPIC) Model: A Case History" *Philosophical Transactions: Biological Sciences* (329): 421-428.

Williams, J. R., Dyke, P. T., Fuchs, W. W., Benson, V. W., Rice, O. W., and Taylor, E.D. 1990. EPIC-Erosion/Productivity Impact Calculator: User Manual. *USDA Technical Bulletin 1768*.

Wojciechowski, J., Ames, G. C.W., Turner, S. S., and Miller, B. R. 2000. "Marketing of Cotton Fiber in the Presence of Yield and Price Risk" *Journal of Agricultural and Applied Economics* 32(3):521-529

Yaron, D., and Bresler, E. 1970. "A Model for The Economic Evaluation of Water Quality in Irrigation" *The Australian Journal of Agricultural Economics*, 14(June):53-62

Zhang. H., Banks, J. C., Godsey, C. B., and Stoecker, A. L. 2011. "Soil Samples along the Elm Fork and the North Fork of the Red River to Determine the Potential for Expanded Irrigation" Oklahoma State University Experiment Station.

APPPENDICES

APPENDIX A: Irrigated Cotton Budget of Surface-Furrow Irrigation System
(Pump Power Source: Electric)

	Units	Price	Quantity	$/Acre
PRODUCTION				
Cotton Lint	Lbs	$ 0.60	1032.00	$ 619.2
Cotton Seed	Cwt	$ 5.82	17.54	$ 102.08
Other Income	Dollars	$ -	0	$ -
OPERATING INPUTS				
Seed	Acre	$ 23.79	1	$ 23.79
Fertilizer	Acre	$ 78.32	1	$ 78.32
Custom Harvest	Acre	$ 144.48	1	$ 144.48
Pesticide	Acre	$ 47.01	1	$ 47.01
Growth Regulators/Harvest Aids	Acre	$ 25.88	1	$ 25.88
Crop Insurance	Acre	$ 9.91	1	$ 9.91
Annual Operating Capital	Dollars	7.00%	145.30	$ 10.17
Machinery Labor	Hrs.	$ 8.75	2.04	$ 17.85
Custom Hire	Acre	$ -	0	$ -
Machinery Fuel, Lube, Repairs	Acre	$ 53.77	1	$ 53.77
Rent	Acre	$ -	0	$ -
Ginning/Processing	Acre	$ 118.68	1	$ 118.68
Other Expense	Acre	$ 16.21	1	$ 16.21
Irrigation cost	Acre	$ 28.89	1	$ 28.89
Irrigation Labor	Hrs.	$ 8.75	1.68	$ 14.70
Total Operating Costs				**$ 589.66**
FIXED COSTS	**Units**	**Rate**		**$/Acre**
Machinery/Irrigation	$/value			
Interest at	Dollars	6.00%		$ 35.62
Taxes at	Dollars	1.00%		$ 6.88
Insurance	Dollars	0.60%		$ 3.56
Depreciation	Dollars			$ 41.65
Land	$/acre	$ -		
Interest at	Dollars	0.00%		$ -
Taxes at	Dollars	0.00%		$ -
Total Fixed Costs				**$ 87.71**
Total Costs (Operating + Fixed):				**$ 677.37**

Source: Oklahoma State University Extension

APPENDIX B: Dryland Cotton Budget of Surface-Furrow Irrigation System

	Units	Price	Quantity	$/Acre
PRODUCTION				
Cotton Lint	Lbs	$ 0.60	461.8	$ 277.08
Cotton Seed	Cwt	$ 5.82	7.85	$ 45.69
Other Income	Dollars	$ -	0	$ -
OPERATING INPUTS				
Seed	Acre	$ 14.3	1	$ 14.3
Fertilizer	Acre	$ 35.05	1	$ 35.05
Custom Harvest	Acre	$ 64.65	1	$ 64.65
Pesticide	Acre	$ 28.64	1	$ 28.64
Growth Regulators/Harvest Aids	Acre	$ 7.52	1	$ 7.52
Crop Insurance	Acre	$ 9.91	1	$ 9.91
Annual Operating Capital	Dollars	7.00%	86.42	$ 6.05
Machinery Labor	Hrs.	$ .75	1.488	$ 13.04
Irrigation Labor	Hrs.	$ -	0	$ -
Custom Hire	Acre	$ -	0	$ -
Machinery Fuel, Lube, Repairs	Acre	40.2	1	40.2
Irrigation Cost	Acre	$ -	0	$ -
Rent	Acre	$ -	0	$ -
Ginning/Processing	Acre	$ 3.1	1	$ 3.11
Other Expense	Acre	$ 12.5	1	$ 12.5
Total Operating Costs				**$ 284.97**
FIXED COSTS	**Units**	**Rate**		**$/Acre**
Machinery/Irrigation	$/value			
Interest at	Dollars	6.00%		$ 8.53
Taxes at	Dollars	1.00%		$ 2.21
Insurance	Dollars	0.60%		$ 0.85
Depreciation	Dollars			$ 15.73
Land	$/acre	$ -		
Interest at	Dollars	0.00%		$ -
Taxes at	Dollars	0.00%		$ -
Total Fixed Costs				**$ 27.32**
Total Costs (Operating + Fixed):				**$ 312.29**

Source: Oklahoma State University Extension

VITA

JONGSAN CHOI

Candidate for the Degree of

Doctor of Philosophy

Thesis: ECONOMIC APPROACH ON ALLOCATION OF IRRIGATION WATER UNDER SALINITY BASED ON DIFFERENT SOILS FOR POTENTIAL IRRIGATED AGRICUTLURE USING EPIC CROP MODEL

Major Field: Agricultural Economics

Biographical:

Education:

Completed the requirements for the Doctor of Philosophy in Agricultural Economics at Oklahoma State University, Stillwater, Oklahoma in December, 2011.

Completed the requirements for the Master of Science in Agricultural Economics at Kangwon National University, Chuncheon, Republic of Korea in 2002.

Completed the requirements for the Bachelor of Science in Agricultural Economics at Kangwon National University, Chuncheon, Republic of Korea in 2000.

Experience:
Graduate Research Assistant, Department of Agricultural Economics Oklahoma State University, August 2008 to July 2011.
Teaching Practicum in AGEC 5203 Advanced Price Analysis, Department of Agricultural Economics, Oklahoma State University, Spring, 2011.
Visiting Lecturer, Mongolia International University, Summer, 2008.
Volunteer, Korea International Cooperation Agency, Rural Development Project in Kampongcham Province, Cambodia, November 2004 to October 2006.
Assistant Teacher, Department of Agricultural & Resource Economics, Kangwon National University, March 2001 to October 2004.

Professional Memberships:

Name: JONGSAN CHOI Date of Degree: December 2011

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: ECONOMIC APPROACH ON ALLOCATION OF IRRIGATION WATER UNDER SALINITY BASED ON DIFFERENT SOILS FOR POTENTIAL IRRIGATED AGRICUTLURE USING EPIC CROP MODEL

Pages in Study: 351 Candidate for the Degree of Doctor of Philosophy

Major Field: Agricultural Economics

Scope and Method of Study: The objective of this study was to understand the long term effect of using irrigation water with water and soil salinity on cotton yield based on the 15 soil types along the Elm and North Fork of the Red River. The specific aims were to 1) estimate the potential cotton response for each soil type to irrigation water and salinity, 2) estimate the economic viability of establishing irrigation systems, 3) estimate dynamic soil salinity changes in response to irrigation water, the salinity of irrigation water, and the soil salinity of the previous year, 4) determine that temporal use of water with the given levels of salt concentration that maximizes the Net Present Value (NPV) of the expected utility from irrigation for each soil type. To assess the econometric relationships between cotton yield, quantity and quality of irrigation water, and soil salinity, the EPIC simulation model was used. The estimated crop yield response function, two soil salinity functions and yield variance function are incorporated in an economic decision model to find the optimal level of irrigation water maximizing NPV of the expected utility with different salt concentrations of irrigation water and three levels of risk. The dynamic optimization procedure for the economic decision model was performed by GAMS IDE.

Findings and Conclusions: The results of crop response functions for the individual soil types indicate that the cotton yield increases as irrigation water and rainfall increase, and it decreases as the amount of salts in irrigation water which is the product of irrigation and salt concentration and soil salinity increase. The soil salinity response functions at planting and harvest have a negative sign on irrigation water, growing season rainfall and non-growing season rainfall indicating the level of soil salinity decreases as the variables related with water increase. The yield variance function has a negative sign on irrigation water which is risk-reducing factor and a positive on the growing season rainfall which is risk-increasing. From the EPIC and dynamic optimization model, when irrigation water with a high salt concentration with or above 1280 p.p.m (ECw =2) is permitted from the salt control point in the study area, NPV of expected utility is negative and less than NPV of the dryland. The irrigation water containing salts should be controlled less than or equal to 1,280 p.p.m for sustainable irrigation.

ADVISER'S APPROVAL: Dr. Arthur Stoecker