UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

PROBLEM SOLVING DYNAMICS – STUDENTS NONROUTINE PROBLEM SOLVING ENGAGEMENT: A CASE STUDY OF FOUR NINTH-GRADE MATHEMATICS STUDENTS

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

In partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

REZA ROSS POURDAVOOD

Norman, Oklahoma

2003

PROBLEM SOLVING DYNAMICS – STUDENTS NONROUTINE PROBLEM SOLVING ENGAGEMENT: A CASE STUDY OF FOUR NINTH-GRADE MATHEMATICS STUDENTS

A Dissertation APPROVED FOR THE

DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC CURRICULUM

BY

Dr. Jayne Fleener, Committee Chair

Dr. Marilyn Breen

Dr. Raymond Miller

Dr. Anne Reynolds

Dr. Elizabeth Willner

ACKNOWLEDGMENTS

First and foremost, I would like to thank my Lord and Savior, Jesus Christ, the head of my life, and my only source to eternal life. Because of His love and mercy, I have successfully completed this dissertation. This process has truly exemplified by *Psalm 37:5*, which says, "Commit your way to the Lord, trust also in Him, and He shall bring *it* to pass." Although the journey has been tenacious, the destination proved more than worthwhile. For this reason, I must thank God for the tenacity and perseverance that was essential for my success.

Additionally, I dedicate this work to my beloved family—my wife, Leah, our three beautiful children, Crystal, Ross, and Michelle—all of whom displayed tremendous faith, patience, and trust. I am forever thankful to have all of them in my life. On a personal note, Leah, your encouraging words and prayers sustained me, which is why I am elated to be your husband. Thank you so much for being a blessing in my life.

To my Chairperson and a friend, Dr. Jayne Fleener, I am infinitely grateful for her mentorship, guidance, and most importantly, her patience during the past ten years. Dr. Fleener, you have always given me the appropriate advice to move forward with my Ph.D. course work and dissertation, and today, I am writing this acknowledgment because of you! I am also thankful for your tutelage during these years, and will always remember your kind words of encouragement and support.

To my committee members, Dr. Anne Reynolds, Dr. Ray Miller, Dr. Elizabeth Willner, and Dr. Marilyn Breen, I sincerely thank each of you for your trust and diligence. Most of you were present during the inception, and persevered with faith as I went on to complete this daunting project. As I now near the end of this life-work, I am continually humbled from your persistence and perseverance. To my editor, Ms. Holly Easttom, thank you for being a true grammarian. Being exposed to English as a second language is very challenging, and every one like myself prays for a person like you. Honestly, you have truly been a blessing to me. I hope you continue your own educational endeavors.

Again, I must offer Thanks to God for all of these people who have made fine footprints in my life. I shall always remember and cherish our moments together. You all have given me more than I deserve, which is why I believe God predestined your coming into my life. If I forgot anyone during these acknowledgements, please forgive me, for it was an only exclusion of the mind, not the heart. God bless all!

TABLE OF CONTENTS

LIST OF TABLES

ABSTRACT

The purpose of this case study was to explore the complex interplay among student beliefs, problem solving engagement, problem type, and mathematics understanding as well as dynamics within group discourse among four ninth-grade mathematics students. The study aimed to provide insight into the relationship that exist between student engagement and problem type they choose to solve and to understand how their problem solving discourse evolve as students participate in a collaborative problem solving environment. This case study, focusing on two dyads' problem posing, problem solving, and collaboration, sheds light on the envisioning of curriculum alternatives for mathematics education amidst the many constraints of current and traditional problem solving contexts. The analysis of both these dyad's 16-week long collaboration reveals that the role of conversation, prolonged problem solving interactions, and on-going negotiations and relationships is key in their transformation.

The results suggest, as students developed a culture within their dyads, of problem solving and problem posing, and collaboration, that engagement was increased. After evaluating the various data relating to problem type and participant engagement, it became evident that certain problem types engaged the students more than the others. While it was no surprise that routine problems were not engaging to them, it was also evident that their collaboration and discourse were very different under these circumstances. They were much less likely to question, challenge, argue, negotiate, or probe each others' thinking, and were much more likely to rely on and

accept the first answer. There were no efforts to modify, extend, or apply these routine problems to other contexts.

After examining the "big picture," it became evident that over the course of this 16-week period, significant transitional moments existed, during which collaborations among the dyads and the group seemed to change, and the quality of discourse improved for both groups. While not directly related to any specific problem type or context, these transitional moments seemed to be related to on-going negotiations and relationships, my role as a teacher/facilitator in the development of the dyads' effective listening (Davis, 1997) and to their beliefs and mathematical understandings. Prolonged problem solving and on-going negotiations and collaborations seemed to be related to students' experiences with productive interactions, shared authorities, and meaningful discourse as well as developing a supportive environment that was beneficial to its participants.

While this study did illuminate some limitations that a prolonged problem solving atmosphere might present in a traditional classroom setting, for the most part, the results of this work supported recent reform-based literature that advocates the use of nonroutine problems in a collaborative environment. This conclusion is equally supported by the enhanced level of mathematics conceptual understanding manifested in the participants at the close of this study, as well as changes that occurred in their pedagogical, collaborative, technical, and mathematics beliefs and assumptions.

CHAPTER I

INTRODUCTION

In today's technological and information-sensitive society, it is important that citizens be mathematically literate (Kenney & Silver, 1997; Paulos, 1989, 1991) and intellectually autonomous (Bauersfeld, 1992; Bishop, 1988; Cobb, 1989; National Council of Teachers of Mathematics [NCTM], 1991, 2000; Yackel & Cobb, 1996). Mathematical literacy and intellectual autonomy include the development of students' autonomy in and out of schools as life-long learners. That is, integrated learning should focus not on accumulation of information, but on reasoning about information with a strong emphasis on nonroutine problem solving, problem posing, and understanding as well as the representation and communication of solution findings (Brown, 2003; Doerr & English, 2003; English, et al., 2000; Fleener & Rogers, 1999; Greer, 2000; Hiebert, et al., 1996; Lajoie, 1998; Moore, 1998; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000; Shaughnessy, et al., 1998) and collective mathematical development (Davis & Simmt, 2003). Because students' problem solving, reasoning, and discussion (Brown, 2003; Steen, 1999) is the cornerstone of proficiency, mathematical literacy and technological competence must include learning opportunities that challenge students to be mindfully engaged (Langer, 1997), to think critically, to use technology collaboratively, and to work on tasks that are worthwhile (Davis, et al., 2000).

All too often, students in American schools are not challenged with problems or activities that they would value and engage their minds (Davis, et al., 2000; Langer, 1997; National Research Council (NRC), 2001). Historically, problem solving and communication in mathematics has been taught by rote, with very little mindful engagement by the teacher or students. The teacher simply explains the procedures needed to obtain the correct answers, and it makes no difference whether the students work individually or in groups (Langer, 1997; Lindquist, 1989, 1997; Lindquist, et al., 1995; Simon, 1995). This pervasive manner to problem solving and communication in mathematics undermines the process of meaningful learning; consequently, students regard mathematics as dull and far removed from reality and their own interests. The NCTM Standards (2000), suggests that mathematics problem solving "should be coherent ...[and]...should focus on mathematics content and processes that are worth the time and attention of students" (p.15). This problem-solving situation is inquiry-based, participatory, and mindful. Langer (1997) describes a mindful state as an interactive learning participation in which the nature of the interaction "is not a matter of fitting ourselves to an external norm; rather, it is a process by which we give form, meaning, and value to our world" (p. 137). Brown (2003) suggests that curricula and teachers ought to embrace problems, not just their solution. "The focus [should be] on human activity and dialogue involving mathematics" (Brown, 2003, p. 175). To improve understanding, Hiebert, et al., (1996) purport, students must take responsibility for sharing the results of their inquiries and for explaining and justifying their methods. Minimal participation in problem solving, reasoning, and communicative situations frequently results in lack of preparation, performance, and understanding of mathematics.

On the national level in 1994, Lester quoted from Dossey et al.'s telling report on the 1992 National Assessment for Educational Progress (NAEP):

> On extended constructed-response tasks, which required students to solve problems requiring a greater depth of understanding and then explain, at some length, specific features of their solutions, the average percentage of students producing satisfactory or better responses was 16 percent at grade 4, 8 percent at grade 8, and 9 percent at grade 12. (p. 660)

Dossey and Mullis later replicated the findings of their report, adding that "despite detailed instructions, substantial percentages of students appeared at a loss as to how to proceed in answering the extended constructed-response questions on the 1992 NAEP mathematics assessment" (Dossey & Mullis, 1997, p. 25).

NAEP mathematics assessment was first administered in the year 1990. A continuation of this assessment framework was used in the years 1992, 1996, and 2000 as well as a follow-up in the fall of 2003, which, thus far, the results reinforce these earlier findings. According to the National Center for Educational Statistics (NCES, 2003), in the year 2000, approximately 250,000 students were assessed in mathematical problem solving in national and state samples. National samples included grades 4, 8, and 12 while state samples included only grades 4 and 8. Questions were based on five content strands: (1) Number sense, properties and operations (2) Measurement (3) Geometry and spatial sense (4) Data analysis, statistics, and probability and (5) Algebra and functions. Students answered a combination of multiple-choice and constructed-response questions. Students in all three grades (4, 8, and 12) had higher average scores in 2000 than in 1990. According to NCES (2003), fourth- and eight-graders showed steady progress across the decade. Twelfth-graders made gains from 1990 to 1996, but their average score declined between 1996 and 2000. When compared with the international level, however, the results are quite astonishing. That is, according to Fuson, et al., (2000),

> … results from the recent Third International Mathematics and Science Study (TIMSS) indicate that the U.S. curriculum continues to be an "underachieving curriculum" compared to the mathematics curricula in higher achieving nations and that instruction in the United States is still more likely to focus on practice of skills than on understanding. (pp. 277-278).

International comparisons of American students' problem solving, reasoning, and communication, which are key outcomes of mathematics education, reveal that US students do not perform as well as their Asian counterparts. The Third International Mathematics and Science Study (TIMSS), which was conducted in 1995, tested 500,000 students in 41 countries, reported American eighth graders below the average in mathematics problem solving. The top 10 percent of Americans scored about the same as the average students from Singapore, the global leader.

In 1998, the TIMSS International Study Center conducted an in-depth analysis of the TIMSS 1995 mathematics and science achievement results for eight grades. One of the components of the project was to compare world-class mathematics and science achievement with U.S. national standards for these projects. The results were astonishing. Kelly, et al., (2000) reports,

> At eight grade, U.S. students performed below the international percentages for the three highest benchmarks, with only 5 percent reaching the top 10% benchmark, 18 percent reaching the Upper Quarter benchmark, 45 percent reaching the Median benchmark. Seventy-five percent of U.S. students reached the Lower Quarter benchmark, which matches the international percentage. (p. 9)

In 1999, a repeat of the TIMSS 1995 was conducted. The new study, known as the TIMSS-Repeat or TIMSS-R, provides the world with another snapshots of students' performance in eight-grade mathematics and science (NCTM, 2003). This time, only 38 countries participated in the TIMSS-R. Both the TIMSS (1995) and TIMSS-R (1999) mathematics tests for the eighth grade were designed and implemented to enable reporting in five content areas: (1) fractions and number sense (2) measurement (3) data representation, analysis, and probability (4) geometry (5) algebra. The United States performed above the international average in fractions and number sense in data representation, analysis, and probability and in algebra. However, it performed below the international average in measurement and geometry. Singapore was the global leader on all subscores. According to NCTM (2003): News & Media (nctm.org): What Can We Learn from TIMSS-Repeat?,

> Overall, U.S. students maintained their standing in the middle of the international ranking, Canadian students were among the few that showed significant gains in mathematics, and Asian countries held their berth above the international average… . Students in other countries reported working on mathematics projects during class more often than U.S. students (36 percent international average compared to the U.S. average of 29 percent). (p. 1)

TIMSS 1999 is only the second in what is expected to become, for every four years, a series of international surveys designed to reveal trends in achievement in mathematics and science.

Hiebert (1999) examines the current debates about the future of mathematics education and the role that research plays in that future—to resolve issues about *priorities* and *values*, which are often ignored—and suggests, therefore, by applying what we learn from research such as TIMSS, we can evaluate the current state of classroom teaching. The author states that we have a quite traditional way of teaching mathematics, which places the emphasis on teaching and computation procedures and places little attention to helping students develop conceptual ideas. Moreover, the researcher cites TIMSS (1995) as an example, and remarks that 78% of the topic were only demonstrated but not explained, and during 96% students were only doing *seatwork* that they had been shown how to do. This TIMMS data, Hiebert (1999) claims, shows that "the traditional U.S. curriculum is relatively repetitive, unfocused, and undemanding" (p. 11). Hiebert (1999) concludes that (1) students learn only what they have an opportunity to learn (2) traditional methods of mathematics teaching and learning are deficient (3) "we *can* design curriculum and pedagogy to help students meet the ambitious learning goals outlined by the NCTM Standards [and (4)] the question is whether we value these goals enough to invest in opportunities for teachers to learn to teach in the ways they require" (p. 16).

Similarly, Schmidt, et al., (2001), through in-depth analyses of information from TIMSS (1995)—an empirical data—set out to evaluate how curricular learning opportunities impact students' learning. The authors focus on curriculum and pedagogy by examining *artifacts* such as content standards, textbooks, teachers' goals, and the amount of time that teachers devoted to the topics. To ascertain the influence curriculum and textbooks have on achievement gains, the authors relied on data gleaned from TIMMS (1995) test results (which focused primarily on a cross-national comparison of 4th, 8th, and 12th grade students from approximately fifty countries, including the United States). Examining the relationship between achievement gains and the allocation of curriculum resources (both across countries and within countries), the authors argue that "national culture has an impact on learning" (Schmidt, et al., 2001, p. 10). To explore this notion, the authors examined the fundamental aspects of formal education in each country—aspects they believe are affected by social, political, and cultural contexts and are likely to shape student achievement. Schmidt, et al., (2001) identified countries such as Hong Kong, Korea, and Japan as "high performing," which contrasted with the comparatively low performance level of the United States (p. 33). Although all countries shared a relatively common core of curriculum, the higher performing countries seemed to utilize more in-depth textbooks and a curriculum that was more organized to take advantage of "the logic of subject matter disciplines" (e.g., mathematics) which "plays an important role in school learning" (Schmidt, et al., 2001, p. 356). With this in mind, the authors argue that a set of national priorities in content standards can be advantageous and not be equivalent to national control of a system, which is the case in the U.S. For that reason, they propose a reformed curriculum—one that equally addresses both the cognitive demand of tasks and the type of instructional activity. Schmidt, et al., (2001) assert: "In the end, schools matter … [and] … curriculum is related to learning" (p. 361).

Rather than memorizing inflexible procedures provided by a teacher or textbook, students seem to learn best by constructing their own mathematics. In fact, because construction of knowledge is an essential part of solving problems, the NCTM 1989, 1991, 1995, and 2000 placed problem solving at the core of the mathematics curriculum, stressing it in all aspects of mathematics instruction. Problem solving, therefore, should be a part of all mathematics activity, because being mathematically literate means being a good problem solver.

According to the NCTM (1989, 1991, 1995, & 2000), instruction in mathematics should use problem situations as a way of involving students in mathematical activities. In particular, the Standards advocate the use of problem solving in mathematics to provide opportunities for all students to engage in meaningful mathematics and to challenge their curiosities while increasing their confidence and value in doing mathematics.

A variety of sources (for example, AAAS, 1993; MSEB, 1989; and NCTM, 1989, 1991, 1995, 2000) agree there is a need for change in the school mathematics content and in the way that content is taught. The recent surge of technology in the work force similarly has a great influence on the use of mathematics and the need to reform how mathematics is taught in the schools. How technology is implemented and how mathematics is taught, however, may be determined by our beliefs about what mathematics is. Thus, the way teachers/learners view mathematics may contribute as a factor to how mathematical problem solving is approached in the classroom.

What is Mathematics?

Mathematics is the study of relationships and pattern recognition. It is a particular way of knowing, a part of human culture—a broad body of human knowledge (Devlin, 2000)—and a way of understanding different aspects of the world we live in. Mathematics is a discipline that is not static. It continues to expand and to grow. Schoenfeld (1992) echoes the National Research Council (NRC) (1989) as saying: "Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us" (p. 335). He views the study of mathematics as being exploratory and evolving and not as a rigid, absolute, closed body of facts that need to be memorized. Mathematics as a discipline should focus on seeking solutions, exploring patterns, and formulating conjectures, not just on memorizing rules and formulas and doing exercises. Mathematics is more than just memorizing computational procedures and equations. Mathematics education is a discipline that is two-tiered. It is comprised of *mathematics learning* which leads inevitably to *mathematical power*. These two dynamics are processes and ends in and of themselves.

Mathematics Learning

Learning to solve problems is the principal reason for studying mathematics. This learning proceeds through construction not absorption (Romberg & Carpenter, 1986) and does not occur in isolation but in collaboration with others in a learning community (Fleener, 1995). The NCTM Standards (1989, 1991, 1995, & 2000) are based on the assumption that learning is a constructive rather than a passive experience. Koehler and Grouws (1992) quote Cobb, et al., (1991) explaining: "'…mathematics learning is not a process of internalizing carefully packaged knowledge but is instead a matter of reorganizing activity'" (p. 118). Simon (1995) tells us that learning is the process by which human beings adapt to their experiential world. Building on previous learning, "[w]hen students are using prior knowledge to construct new mathematical knowledge, they are learning mathematics" (Lindquist, 1989, p. 3). Learning mathematics is not passive but deliberate. Students learn mathematics best when they are encouraged to become mindfully engaged (Langer, 1997) in their own learning environment to solve problems.

The learning of mathematics for students, according to the NCTM (2000), must be with understanding, "actively building new knowledge from experience and prior knowledge" (p.11). The NCTM (1995) lists five shifts for the mathematics problem-solving community to foster mathematics learning and understanding. They include: (1) a shift away from thinking of mathematics as arithmetic proficiency for most students to thinking of mathematics as power for all students to solve problems (2) a shift from memorizing and repetitive practice to investigating, speculating, and reasoning to find solutions to problem situations (3) a shift from concepts and skills in a linear order to the use of mathematical power in solving genuine problems.

Mathematical Power

Tsuruda's (1994) uses the following definition to describe the attributes of mathematical empowerment: "[M]athematically powerful students possess attitudes of appreciation, confidence, curiosity, inventiveness, persistence, reflection, and willingness" (p. 39). The NCTM Standards (1991) defines mathematical power to include the student's ability "to explore, conjecture, and reason logically; to solve nonroutine problems" (p. 1). Mathematical empowerment involves the development of a student's personal self-confidence and a disposition to seek, evaluate, and solve problems as well as to make sound decisions. Additionally, the learner's "flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power" (NCTM, 1991, p. 1).

Thus, implications for 21st century problem solving in mathematics include a shift from acquisition models of learning to student mathematical empowerment through life-long learning (Boyer, 1995). Empowering students mathematically includes providing opportunities for students to develop their "abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems" (NCTM, 1989, p. 5).

The NCTM Standards (1989) suggest five general goals for all students to reach in order to become empowered mathematically: (1) learn to value mathematics (2) become confident in their ability to do mathematics (3) become mathematical problem solvers (4) learn to communicate mathematically (5) learn to reason mathematically. According to the NCTM Standards (1989, 1991, 1995, & 2000), students who achieve these goals will become mathematically empowered.

Mathematical empowerment can also occur through supporting the learners' initiatives in coping with their complex and multiple realities, acceptance of multiple perspectives, and fostering a feeling of value for learning tasks in an authentic and collaborative curricular context.

Mathematics, says Devlin (2000), presents four faces to the world: (1) mathematics as computation, formal reasoning, and problem solving (2) mathematics as a way of knowing (3) mathematics as a creative medium (4) applications of mathematics. How can we create a sound pedagogical problem-solving context in the mathematics classroom where students are encouraged to link these four faces of mathematics and think critically and creatively, work on problems that are interesting to them and connected to the workplace, and use technology to search, question, and "know" mathematics? In order to better understand how we might create classroom contexts that support mathematical problem solving, we need to explore *pedagogical problem solving contexts*.

Pedagogical Mathematics Problem Solving Contexts

It is difficult to speak of mathematics context without referring to other aspects of mathematics learning which include the content and the processes. According to the NCTM (2000), in a *pedagogical problem-solving situation*, the content and curriculum as well as the context in which the mathematics is embedded should be engaging, meaningful, synergic, and should resemble tasks and models that are found in everyday life, i.e., in the workplace. "School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students" (NCTM, 2000, p.15). In a pedagogical problem-solving context, students are given opportunities to design, plan, evaluate, recommend, review, define, critique, explain, and make situations problematic. The challenges students face in these settings "are often nonroutine and open-ended, with solutions taking from minutes to days, and requiring diverse forms of presentation … some work is done alone and some in teams" (Forman & Steen, 2000, p. 139). These pedagogical problem-solving contexts may motivate students to link meaning with mathematics. Supporting mathematics problem solving contexts are approaches to teaching that encourage risk-taking in a supportive environment.

Problem Solving Pedagogy

Problem solving in mathematics includes the development of conceptual understanding. Kenney (1997) states that conceptual understanding is an essential part of problem solving. This understanding is developed best when students are mindfully engaged in solving problems that interest them and raise their curiosities (Langer, 1997). However, as Hiebert, et al., (1996) indicate, the history of problem solving in mathematics "has been infused with a distinction between acquiring knowledge and applying it" (p. 12). These researchers tell us that in order for students to develop mathematical problem solving reasoning, they need to be engaged in problem solving situations in which they are allowed to make the subject problematic. They quote Dewey (1929) as saying all reflective inquiry starts from a problematic situation. They describe reflective inquiry as having the following characteristics: "(1) problems are identified; (2) problems are studied through active engagement; (3) conclusions are reached as problems are (at least partially) resolved" (Hiebert, et al., 1996, p. 14).

Within the constructivists' pedagogical model of mathematics learning and doing, learners are to form a collaborative and conversational problem-solving partnership focused on sense making and reflection. Etchberger and Shaw (1992) provided the following example of such a problem-solving partnership in the solution to an inquiry in the process of knowledge construction:

> They [the learners] explain, clarify, elaborate, question, evaluate, justify, extend, and argue. The key to making sense is in this negotiation process. Meaning is being negotiated. Ways to solve problems are examined, tried, rejected, defended, justified, and explained. (p. 412)

Stein, et al., (1994) pointed out that what is learned through the constructivist dialogic problem-solving process as learners create their own tasks, "depends on the shared understandings that students negotiate with the teacher and with each other" (p.12). Problem-solving pedagogy in mathematics should enable all learners to experience mathematics as a dynamic engagement in solving problems. Thus, engaging students in problem solving, is a focus of problem solving pedagogy. Key to problem solving pedagogy is dialogical problem solving.

Dialogic Problem Solving

Dialogic problem solving can be defined as problem solving situations where two or more students, through reflective inquiry, critical thinking, and negotiation converse and communicate their ideas with one another and work together toward a shared meaning (Yackel & Cobb, 1996). When students are allowed to work together in pairs or in a group to negotiate problem-solving strategies and how to go about resolving their differences, they are given opportunities to collaborate, negotiate, and discuss mathematics as well as work toward the establishment of a supportive and synergistic environment of a dialogic pedagogy. Dialogical problem solving is supported by dialogical pedagogy.

Dialogic Pedagogy

There is a growing interest among educators to involve students in learning situations that are collaborative and to encourage dialogue at all grade levels. Dialogic pedagogy will be used to describe an approach to instruction focusing on conversation pattern and emerging collaboration. Dialogic pedagogy is related to collaborative learning environments.

Collaborative Learning Environments

Gokhale (1995) defines a collaborative learning environment as the grouping and pairing of students for the purpose of academic learning. The students are responsible for one another's learning as well as their own. Therefore, the development of one's understanding through mindful engaging, reflecting, and constructing as well as sharing and conversing should help other students develop their own understanding of the inquiry within the learning environment. Collaboration in education helps students to learn from each other's experiences, understandings, and reasoning. Effective collaboration and conversation are essential in establishing meaningful dialogic problem-solving pedagogy in mathematics. When students collaborate and converse with one another about mathematics, both by speech and by writing, they learn to clarify and defend their thinking, pool ideas, and share in decision-making. Moreover, these conversational activities among students shift the classroom environment from an environment being dominated by a researcher (teacher) to an environment negotiated by all members alike, teacher plus students. Tsuruda (1994) explains the importance of group work (collaboration) in learning this way: "Perhaps the most critical aspect of group work with regard to learning is the forum it creates for discussion. When students are placed in situations in which they must verbalize their thoughts and ideas, their thinking, of necessity, becomes clearer" (p. 100). Collaborative learning environments are supported by and perhaps inextricable from dialogic pedagogical practices. Synergy of shared vision not only is an important aspect of the establishment of a dialogic pedagogy, but also is an important part of generating enthusiasm and new energy for continuing dialogue to resolve more problems.

In what follows, I will discuss the theoretical framework of this study. Implicit in these ideas about creating mathematics learning environments where students become active problem solvers and empowered mathematically are perspectives of teaching and learning found in constructivist theories of knowing.

Socio-Autonomy

Carter and Fleener (2002) purport Piaget's (1973/1948) view that the central purpose of education is the development of autonomy. They cite Kamii and Dominick (1998) and Yackel and Cobb (1996) to claim that this view is consistent with current reform movements in education particularly in mathematics. That is, the goal of research in mathematics and mathematics education is "...to support teachers as they establish classroom environments that facilitate students' mathematical conceptual development" (Yackel & Cobb, 1996, p. 458).

All too often, this goal is "...lost when accomplishment is defined by grades and the successful completion of particular educational tasks (Fleener & Rogers, 1999, p. 9). Consequently, many students have experienced "education" as compliance with authority and the adoption of others' ideas without critical review (Fosnot, 1996). "They often tend to associate learning with the completion of a series of discreet tasks to receive good grades" (Rogers & Dunn, 1999, p. 271).

Individual development or autonomy "... does not imply 'freedom' as the term is often used; rather it is acting in accordance with one's core system of understandings, values and beliefs" (Fleener & Rogers, 1999, p. 14). Similarly, socio-autonomy may be defined as "*the potential for self-creation and self-production of social systems*, analogous to autonomy for the individual as an autopoietic, self-creating entity" (Fleener & Rogers, 1999, p. 15). Moreover, there exist a dialectical relationship between the individual and the environment, hence, communications "... that as a result of interaction, both are transformed or changed: the individual and the environment" (Fleener & Rogers, 1999, p. 14).

Fleener, et al., (2001) reiterates the importance of socioautonomy within social systems and focuses on the communicative process. The verbal exchange between student and teacher opens modes of exploration and encourages intellectual growth; this exchange is evident in language games utilized within the classroom. While some language games involve "playing at," or participants using appropriate language and operations to achieve meanings, these meanings are perceived as external and often obscured. "Playing in" language games also obscures meaning and communication because it assumes a script where the performance of the participants is measured against that pre-existing script. Individual autonomy may be achieved in both approaches, but true communication and shared knowledge is not. However, incorporating both types of language games, as well as "playing with" an approach that requires a teacher to act as an encourager, not an evaluator may be beneficial, but only if the role of listening is tantamount. Evaluative listening, or eliciting certain responses and interpretive listening, or listening for certain responses, do not encourage the generation of ideas (Davis, 1997) necessary for the development of understanding and autonomy which is vital for an individual's ability to communicate, respect others, function within a team, analyze information, make decisions, and evolve socially (Rogers & Long, 2002). Rather, understanding and autonomy is best developed when students are engaged in autonomous activities in supportive environments. These supportive environments are best provided through researchers/teachers who are prepared to operate in an autonomous fashion. To do this, educators must put theory into practice and constantly seek to understand the underlying principals and make decisions that are consistent with those principals.

Furthermore, it is necessary to implement an autonomous-supportive framework in order to nurture the development of autonomous educators (Rogers & Long, 2002).

This study is rooted in social and cognitive theories of constructivism with an emphasis on developing mathematical autonomy through socio-autonomous activity. Also embedded in this theoretical framework is the rejection of a focus on individualism and competition in learning environments, emphasizing, instead, interconnectedness and care as fundamental to learning and the curriculum (Fleener, 2002). Learning which takes place as a result of mindful engagement of the learner with his or her own environment, must be open and connected to other systems within the whole; it cannot be reduced to simple cause and effect. Learning, therefore, is adaptive and dynamic (Prigogine & Stengers, 1984; Wheatley, 1994). These perspectives are related to and compatible with some versions of constructivism. Constructivists view the learner as an active participant and sense maker of the daily experiences encountered both individually and socially (Bauersfeld, 1995; Cobb, 1989; Cobb & Bauersfeld, 1995; Cobb, Yackel, & Wood, 1989, 1992, 1993; Confrey, 1990, 1995; Davis, 1990, 1997, 2000; Noddings, 1990; Shapiro, 1989; Simon, 1995; Steffe & Gale, 1995; von Glasersfeld, 1984, 1987, 1991, 1995; Yackel, 1995). These perspectives emphasize the importance of social context, discourse, and experience. "Learning is the process by which human beings adapt to their experiential world" (Simon, 1995, p. 115).

This study includes consideration of not only problem situations that are nonroutine, engaging, and potentially meaningful to the students, but also harmonic to the participants' prior mathematical knowledge and experiences. "'The most basic responsibility of constructivist teachers is to learn the mathematical knowledge of their students and how to harmonize their teaching methods with the nature of that mathematical knowledge'" (Simon, 1995, p. 120, quoting Steffe & Wiegel, 1992). Focusing on the students' individual and collective mathematical autonomy through prolonged problem solving activity and not just on their tests achievement and rigid class-works is by far a greater challenge for both the teacher/researcher and the student.

Research Questions

This study will address the following questions:

1. What is the relationship between student engagement and problem type?
2. How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

In the next chapter, I provide essential background information on the importance of nonroutine problem solving activities, dialogical problem solving process, student collaboration, and student engagement in collaborative learning environments. This background information is relevant to my study because my focus is on student pairs and small group nonroutine problem-solving activities, individual and collective construction of mathematical knowledge, and attentive communication as well as working toward an evolving problem solving discourse as students participate in a supportive collaborative problem solving environment.

CHAPTER II

REVIEW OF RELATED LITERATURE

Most historical views of problem solving make a distinction between acquiring knowledge and applying that knowledge. Applying knowledge through problem solving in different contexts is a goal of mathematics education. Understanding how generalization to new contexts occurs is of central concern to learning theories. Interweaving learning theory, curriculum and instruction, mathematics education research and theory explores how to make mathematics meaningful for students.

Important to meaning-making efforts is the role of communications and social relations. Mathematical communications include classroom discourse as well as self-talk. Related to self-reflection and metacognition, some mathematics education researchers are exploring the role language plays in students' mathematics learning (Fleener, et al., 2003; Sfard, 2000) while others emphasize social discourse as a function of classroom norms (Yackel, 2000) or culture (Applebaum, 1995).

This chapter will explore the role communications may play in mathematical problem solving. The first section discusses the importance of nonroutine problem solving activities in mathematics for students' learning. The second section addresses the role of student discourse in problem solving, and reviews studies that explore the problem solving processes examining mathematical conversations. The third section discusses the research literature on student collaboration during problem solving, and the fourth section explores the literature on student engagement in collaborative learning environments.

Nonroutine Problem Solving

Problem solving, as a thinking process, implicates understanding that requires using prior knowledge, concepts, and understandings as well as newly constructed knowledge on the part of the student during his or her own mathematics problem solving experience. Learning mathematics means becoming a mathematical problem solver.

In order to understand what nonroutine problem solving is, it is important to understand how problem solving has been treated in the mathematics education literature. From a broader perspective, problem solving involves reaching a goal by providing an answer to a given state in which an answer or solution method is not initially known (Mayer, 1982, 1985; NCTM, 1989, 1991, 1995, 2000; Pugalee, 1995; Wilson, et al., 1993). This central view is also consistent with historical perspectives of the role of problem solving in the mathematics curriculum (Stanic & Kilpatrick, 1988). Pugalee (1995) defines nonroutine mathematical problems as "problems in which a possible solution or conclusion is not immediately evident" (p. 4). This definition is in contrast to the definition of routine problems: problems that are rigid and stress steps to be followed to reach a solution (e.g., Polya's 1957 and 1973 stages of problem solving). This method of problem solving is widely known as "a classic and clichéd four-step procedure: 'understanding the problem,' 'devising a plan,' 'carrying out the plan,' and 'looking back'" (Calvert, 2001, p. 14).

According to Calvert (2001), routine mathematics instruction has a long history in classrooms as teacher-directed communication with sequential skill-oriented practice. This method of instruction, for the most part, is founded on the formalist perspective as articulated by George Polya (Calvert, 2001). Many mathematics educators look to Polya's work when attempting to formulate methodology for teaching problem solving in the classroom. Characteristics of this approach include a linear, step-by-step method for problem solving, failing to take into consideration the dynamic nature of the problem solving process (Calvert, 2001; NCTM, 2000; Pugalee, 1995; Wilson, et al., 1993). Nonroutine problems can be used to encourage reflective approaches to problem solving, providing contexts for open-ended, dynamic methods and supporting student invention and creativity.

For example, Bransford, et al., (1996) discuss the issues of fostering mathematical thinking in middle school students who think they are "not good at mathematics" because all they have experienced has been routine, rule-following problem solving contexts (p. 203). Their research suggests it is necessary for educators to involve students with more than just routine problem solving, but also to utilize problem-based curricula including "class *projects* that are tailored to the interests and resources of students and their community" (Bransford, et al., 1996, p. 218).

Bransford, et al., (1996) contend by varying educational approaches (i.e. computation-based, worksheet-based, and student generated problem-based), it is easier to "foster the kinds of mathematical thinking that we [had] hoped to see in [the] classrooms" (p. 224). Although some of these approaches do have their drawbacks (i.e. collaborative learning became *hectic* at times, student-generated questions tend to tax a teacher's mathematics knowledge, and it is difficult at times to develop "on the spot" tools to enable students to conceptualize certain elements), such approaches allow students to question and learn for themselves to become more reliant on a learning community for aid in implementing some ideas (Bransford, et al., 1996).

To *flesh out* this concept, Bransford, et al., (1996) implemented "the Jasper *Woodbury Problem Solving Series*"—a series of 12 videodisc -based adventures. Studies of implementation efforts of the *Jasper* series show that by utilizing different mediums, students were able to conceptualize certain word-problems and develop new ways of solving these problems. This was true because the videos allowed the students to: identify and define their own word problems and to focus more directly on mathematical concepts, reasoning, and communication. Three key components emerged while conducting the research: students have variable views about mathematical thinking, changing these views is a daunting challenge, and our concept of mathematical thinking is still evolving; thus it is the educator's duty to strive to evolve with this change (Bransford, et al., 1996).

Such variable views on the part of the student are dependent on the presentation of nonroutine problems, which should involve "a situation in which an individual or a group is called upon to perform a task for which there is no readily accessible algorithm" (Lester, 1980, p. 287). This very definition indicates the nonroutine nature of problems as tasks that require creativity for their completion. However, traditional teaching methods still seem to overshadow more reform-based standards. According to Kilpatrick and Silver (2000), traditionally, teachers/educators have accepted their responsibility by offering their students clear explanations and instructional objectives within a classroom, prefacing complex knowledge with "hierarchical sequences of purported prerequisites," and tailoring activities to individual needs (p. 226). Although such approaches are beneficial, Kilpatrick and Silver propose a contingent model of mathematics education instead of a more anticipant model, which carefully follows a path that has been worked out in advance. The contingent approach follows a path that emerges during the lesson. In this aspect, educators "orchestrate the discourse so that these students in this class will function as an intellectual community" (p. 226). In turn, intellectual communities must begin at the level of the nonroutine problem solving process.

During the problem solving process, Beamish and Au (1995) suggested a number of key issues for educators to keep in mind while solving (nonroutine) problems with their students: (1) problem solving skills (concepts) are to be experienced (2) opportunities for students to verbalize their plans should be provided and (3) problems that require careful planning and evaluation after solving should be offered. According to this study, the problem solving pedagogy should have, as its goal, the development of students' individual intellectual autonomy as independent problem solvers.

Ernest (1991) states that mathematics is largely human problem posing and solving, mathematics education should be centrally concerned with this, and investigations should constitute a large part of the school mathematics curriculum. Although such an approach differs among educators and theorists, all share common distinctions between the object (or focus) of inquiry, the process of inquiry (the redefinition of focus as new questions are explored), and an inquiry based pedagogy (one that requires creativity for completion) that requires the teacher to relinquish some control over the process and allows students to find their own ways to solve the problem (Ernest, 1991). Such an emancipatory approach empowers learners epistemologically and encourages active learning.

The NCTM (1991) advocates such investigatory problem posing, problem solving, and generating activities by the learners and suggests students should be allowed to "formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). To this end, Silver and Cai (1996) set out to investigate the impact of open-ended problems for sixth and seventh graders. They developed an assessment instrument to measure students' mathematical thinking, reasoning, and understanding while engaged in problem solving. The students were asked to pose questions, provide answers, and justify their solution processes from the open-ended problems, which was then examined for solvability, complexity, and relationships within the sets of posed problems—large number of generated posed problems were categorized as being complex and solvable and nearly half were categorized as being sets of related problems. In addition to this study, there have been several studies which examine the impact of experience in formal problem solving. These studies suggest generative activity increases student interest and has a positive effect of students' conceptual understanding (English, 1998; English & Halford, 1995; Kilpatrick, 1987; Silver, 1994; Silver & Cai, 1996; Silver & Mamona, 1989; Silver, et al., 1990). Such studies illuminate the needs for educators to develop problem solving and problem posing skills if children are to experience "the diverse problem posing we desire," (English, 1998, p. 100).

Henningsen and Stein (1997) address limitations in students' thinking, reasoning, approaches to problem solving, and conceptual understandings in mathematics. To this end, the authors pose a question—what does it mean to be a mathematical doer and thinker?—and argue that answers to this question depend on one's view of the nature of mathematics. To further problematize this, Henningsen and Stein offer a view of mathematics that (1) is based on a dynamic and exploratory approach toward the discipline—citing Romberg (1994)—(2) requires a person to focus on the active and generative processes engaged in by doers and thinkers of mathematics—citing Schoenfeld (1992)—and (3) involves the use of tools to explore patterns, frame problems, and justify reasoning processes systematically—citing Burton (1984); National Research Council (1989); Romberg (1992); Schoenfeld (1992, 1994).

Additionally, Henningsen and Stein (1997) believe students' learning should be "seen as" the process of developing and gaining a "'mathematical disposition," as well as gaining "mathematical knowledge and tools" (p. 525). In order for such processes to occur, classrooms should become environments in which students may actively engage in mathematical activity that is rooted in rich, meaningful, and worthwhile mathematical tasks (Henningsen & Stein, 1997; NCTM, 1991, 2000; Resnick, 1987; Schoenfeld, 1994), which are central to students' thinking and doing of mathematics because tasks convey messages about what mathematics is, provide contexts for students' thinking, and may place differing cognitive demands on students (Henningsen & Stein, 1997, citing Doyle, 1983; Hiebert & Wearne, 1993; Marx & Walsh, 1988).

Similarly, Glasersfeld (1995) presents a learning theory that emerges from

Piaget's work and summarizes that "cognitive change and learning in a specific direction take place when a scheme, instead of producing the expected result, leads to perturbation, and perturbation, in turn, to an accommodation that maintains or establishes equilibrium" (p. 68). This theory of cognition involves two kinds of *viability*—a term borrowed from Piaget's work—and a twofold instrumentalism. Glasersfeld (1995) purports on the sensorimotor level, viable action schemes, as instruments, enable individuals to achieve goals as they experience in their interactions with the world. On the level of reflective abstraction, however, operative schemes enable an individual to develop a conceptual framework that reflects both thinking and acting, which, at their present point of experience, is viewed as viable (Glasersfeld, 1995). This first level might be called *utilitarian*—another term borrowed from Piaget—while the second may be referred to as strictly epistemic. Glasersfeld (1995) concludes that only "knowledge that results from inductive inferences and generalizations" allows these two levels to correspond "with ontological reality," and that "thought experiments constitute what is perhaps the most powerful learning procedure in the cognitive domain" (p. 69). Again, the implication here is that students be allowed to engage in cognitively demanding activities in order to develop a deeper mathematical understanding.

Henningsen and Stein (1997) cite Bennett and Desforges (1988), Doyle (1983, 1986, 1988), and Stein, et al, (1996) to concede that setting up instructional tasks to engage students in cognitively demanding activities has its difficulties. Specifically, they suggest problems with challenging students include (1) complex tasks often devolve into less demanding or sequential routines (2) a lack of alignment between tasks and students' prior knowledge restrict the potential benefits of providing cognitively challenging activities and (3) a higher personal risk occurs for students than is typically experienced in performing more routine activities. Yet they theorize that these difficulties may be due to classrooms' factors—teachers' lack of expectation for students to demonstrate understanding of the mathematics underlying the activities in which they are engaged, connections with students' prior knowing and understanding, appropriateness of cognitively demanding tasks with students' levels and kinds of prior experience, and allowing inappropriate amounts of time to be devoted to tasks.

In this study, Henningsen and Stein (1997) draw data from an earlier investigation—Stein, et al., (1996)—to explore the relationship between classroom environments and students' abilities to navigate problem solving with high-level mathematical thinking. Their study is "based on the premise that prior failures of poor and minority students are due to a lack of opportunities to participate in meaningful and challenging learning experiences rather than a lack of abilities or potential" (Henningsen & Stein, 1997, p. 527). They incorporate a previously constructed conceptual framework (e.g., Stein, et al., 1996) to define a mathematical task as a classroom activity with an aim to focus students' attention on a particular mathematical concept. They purport when students were allowed to participate in a more dynamic classroom environment, students' engagement and high-level problem solving increased, and were decreased when problems were simplified and/or taken over by the teacher, or when an insufficient amount of time was given the students to solve the problems.

Henningsen and Stein (1997) also noted that when students were instructed to refer to Polya's four-step processes as they solved problems, which "may have encouraged self-monitoring and regulation," found that "[d]uring task implementation, students' cognitive processes declined into procedural thinking that made little if any connection to understanding or meaning" (p. 541). The same conclusion was drawn in Reys, et al.'s, (2003), Riordan and Noyce, (2001), and TIMS-R (1999), suggesting the importance and value of nonroutine problem solving opportunities for cognitive engagement.

While addressing the centrality of the development of mathematical reasoning in mathematics education, Reid (2002) cites former research that addresses juvenile spontaneous reasoning in mathematics—(citing e.g., Artzt & Yaloz-Femia, 1999; Tang & Ginsburg, 1999)—, their approaches to argumentation—(citing e.g., Ball & Bass, 2000; Maher & Martino, 1996; Wood, 1999; Zack, 1999)—, teaching that supports reasoning—(citing e.g., Boldt & Levin, 1999; Fraivillig, Murphy, & Fuson, 1999; Lampert, 1990; Yackel & Cobb, 1996)—, assessing reasoning—(citing e.g., Carroll, 1999)—, and the very nature of mathematical reasoning—(citing e.g., Steen, 1999; Sternberg, 1999). Reid (2002) concludes that mathematical reasoning can further be examined in terms of "*ways* of reasoning (deductive, by analogy, etc.), *needs* to reason (to explain, explore, verify), and the degree of *formulation* or awareness of reason" (p. 6). The underlying theories of this perception are rooted in enactivism, which he divides into four major aspects: "structure determinism, cognition as bringing forth a world, coevolution of structures, and observer dependence" (p. 7). Mathematics, therefore, as embodied experience, must unfold for each individual as all students are provided experiences and opportunities for challenging their thinking and exploring their understandings.

McCaffrey, et al., (2001) suggest recent reform movements—(e.g., the National Science Foundation (NSF) and the National Council of Teachers of Mathematics (NCTM)—emphasize mathematics instruction that engages students' advances the development of their cognitive processes. This approach to mathematics curriculum implicates classrooms as communities instead of collections of individuals, uses logic to verify results instead of teacher authority, focuses on mathematics reasoning instead of procedures, emphasizing invention and problem solving rather than mere answer finding, and encourages students to make connections among these ideas (McCaffrey, et al., 2001).

Additionally, reform educators include cooperative learning groups and inquiry-based activities in the classroom—a practice that provides the application of mathematics to novel situations and encourages flexible thinking among students who are then able to outperform students who are taught through procedure and memorization (McCaffrey, et al., 2001). As an amendment to these reform efforts, McCaffrey et al. investigated professional development and student achievement by observing a group of tenth-grade mathematics students some of whom were enrolled in reform-based mathematics courses, while others were enrolled in traditional courses. The reform-based courses involved textbooks and activities that included cooperative projectswhile the tr aditional courses utilized traditional algebra, geometry, trigonometry, and precalculus course sequences (McCaffrey, et al., 2001). The reform-based courses challenged students to explore open-ended situations utilizing an exploratory approach, as well as collaborative projects. Of those that did incorporate reform curriculum in the classrooms, there were higher achievement test scores among the students (McCaffrey, et al., 2001). Such reform requires educators to reevaluate the features of mathematics, which is precisely what Devlin (2000) does in *the four faces of mathematics.*

The author asserts that there are four faces of mathematics: it is a form of computation and problem solving, a way of knowing, a creative medium, and an application. Devlin (2000) posits that mathematics itself is a crucial educational tool because it allows students to develop "*the ability to acquire specialized knowledge and skills*" which, in turn, allows them to adapt to changing circumstances during the course of their working lives (p. 16). To this end, the author contends mathematics should be taught "not as a utilitarian toolbox but as a part of human culture," much in the same manner that history or humanities is taught (Devlin, 2000, p. 17); he does not believe that the computative facet of mathematics should be ignored, but rather used in collaboration with other approaches. By realizing the many faces of mathematics, students are then able to make "*the invisible visible,*" or to "see" and hence to understand that mathematics concepts are processes of intellectual creativity, connectivity, and human ingenuity. All the four faces of mathematics, he argues, must be brought to the classroom with various applications so that students are able to recognize the scope and depth of the applications of mathematics in their world (Devlin, 2000).

Similarly, Kilpatrick and Silver (2000) challenge mathematics educators to focus on classroom pedagogy to prepare students for lifelong learning. They argue that despite the fact that most school mathematics curricula are now richer in topics and take these topics further than those of a century ago, still "[s]tudents aren't learning mathematics well enough; they leave school hating it.... The school mathematics curriculum is superficial, boring, and repetitious. It fails to prepare students to use mathematics in their lives outside of school" (p. 224). To promote students' understanding of mathematics, Kilpatrick and Silver (2000) call for better teaching. They purport John Dewey's observation of the past century—learning by doing and reflecting on what we do—as being extremely applicable to every day mathematics educators' critical reflection for classroom discourse. The teacher's role is to create a situation that allows students to make observations, seek clarification, and challenge them in order to explain or justify their thinking. The educator's role is to respond to his or her student and create an environment in which students may develop their own (and one another's) understanding.

On that note, Stevens (2000) echoes the sentiments of many mathematics educators and instructional designers including Paul Cobb, and advocates a type of educational process which he terms "project-based mathematics (PBM)," which involves the following basic structure: "students work on projects guided by the teacher, usually in groups, that are extended over weeks or months and are organized around fields of inquiry other than disciplinary mathematics [that] are intended to give shape and meaning to student uses and learning of mathematics" (p. 105). The author cites Battista (1999) to claim that PBM education is controversial because it challenges traditional assumptions about what "counts" as mathematics, but he attempts to extend its usefulness by analyzing four interactional events involving middle school students and a teacher.

The author observed four students (of mixed ethnicity and varying degrees of familiarity with one another) over the course of the project and focused on the contrast between emergent problems and ones that were assigned. After observing their interaction and conversations as they *tackled* various problems, Stevens (2000) concluded that while such a nontraditional approach to teaching and learning mathematics is not yet settled in our school system, students were able to associate mathematics with experience and therefore gain a greater understanding of the actual concepts. Stevens contends that traditional mathematics is problematic because "it seems neither to teach people to use mathematics as a generative resource in their out-of-school lives nor to enlist enthusiasts; ... helping *most* students learn to use mathematical tools and ideas to support arguments, to work together, to make things, and to resolve problematic situations from daily life" are more important objectives than merely raising standardized scores (p. 139). In his study, the emergent problems seemed to have fostered mathematical learning and became vehicles to important ends within the team.

Cobb (2000) also discusses the situated approach that he (and his colleagues in other settings) takes regarding mathematics education—an approach that involves the teacher as an active part of a team that supports student's autonomous learning. To explore this, he (and his colleagues—e.g., Cobb, et al., (2000))—implements varying problem solving activities into an experimental group, often incorporating computer-based tools. Each classroom event is then analyzed, discussed, and modified as the experiment continues. Two aspects of the experiment that is highly emphasized are the social perspective of the classroom (classroom social norms, sociomathematical norms, and classroom mathematical practices) and the psychological perspective (beliefs about individual role, others' role, and general nature of mathematical activity; mathematical beliefs and values; and mathematical interpretations and reasoning). According to Cobb (2000), the social perspective brings to the fore normative, whereas the psychological perspective brings to the fore the diversity in students' ways of participating in these taken-as-shared activities. Analysis of these aspects should then improve instructional designs to support students' mathematical learning. This process of planning, instruction, and analysis enables an educator to evaluate his or her fieldwork and nurture their students' mathematical understanding (Cobb, 2000).

Pertinent to fostering opportunities for students' mathematical learning is problem-centered learning (PCL) environment. Problem-centered inquiry is an approach that consists of tasks, collaborations, and presentations in order to encourage students to explore mathematical issues and to see mathematics itself as a system of meaning, patterns, and relationships—not just procedures and computation.

An environment which utilizes PCL consists of open dialogue and involved activity among all students. This is prompted when students are given opportunities to work on open-ended problems that have no known solution, work individually or in groups to find possible solutions, and then reflect upon the processes used to form that solution (Wheatley 1992). Successful approaches to PCL in students' mathematical problem solving involve tasks (which may take more than one hour to solve and involve more than one mathematical concepts), groups (or homogonous pairs, which differs from cooperative groups that usually involve assigned roles that limits mathematical understandings and creates inequalities among its members), and sharing (at which point each pair shares their solution to the entire class, and explain and justify their methodology) (Cassel, 2002). In this sense, PCL allows students to develop their own methods of problem solving, while accepting responsibility for their own learning. To this end, the focus on explanation and justification in this environment illuminates the necessity for argumentation in the classroom. Because students must actively listen to one another, question, comment, and share, argumentation fosters an interactive environment that "allows a collective consciousness to emerge [and] become aware as a mathematics community" (Cassel, 2002, p. 51).

Cassel (2002) notes that by explaining and justifying ideas, students are able to make new connections that form a mathematics community focused on the development of mathematics ideas (Reynolds & Wheatley, 1996). To ensure that mathematics discussion does not break down in the discourse, echoing Cobb, et al., (1997), Cassel (2002) stresses the need for "'reflective discourse,' [which] is a sociological construct by which a mathematical action becomes an entity, which can be manipulated" (p.39). By turning action into discussion, argumentation allows students to observe other perspectives within their dialogic community and form new ideas. To nurture such a community, then, teachers must act as facilitators, ask students to explain their thinking, listen, and respond by selecting activities which will further their mathematical thinking. Students, not teachers, should be the main contributor in this type of dialogue. By allowing students to discuss, present, and challenge each other's ideas, teachers provide rich argumentative opportunities, which may foster students' mathematical understanding. By attempting to convince others "through the use of certain modes of thought" (Woods, 1999, p. 170), students participate in discursive exchange, which allows them to make judgments, not just accept solutions; in short, they become autonomous, or "self-governing" (Kamii 2000, p. 71). This social interaction is a vital component in students' development of logic because it allows them to discard erroneous reasoning and come to an agreed solution. Cassel (2002) refers to this process as synergistic argumentation because the "sum total of the effectual learning" of the entire dynamic group is greater than a mere compilation of each individual's learning (p. 166), and create a dynamic dialogic community that motivates and interests students to become an active participant in their own learning.

Summary of Major Points

From both literature and application, it is safe to conclude that mathematics learning is increased with the use of nonroutine problem solving and the development of dialogic communities. Nonroutine problem solving is open-ended and allows multiple solutions, thus inviting greater conceptual understanding. In turn, this type of problem solving restructures the level (and focus) of communication between students and teachers, students and students, and students and self. Although such a shift in the classroom environment has its drawbacks, research suggests that these are outweighed by the benefits and will ultimately create more autonomous learners. Tied to nonroutine problems and the emergence of a dialogic community, is the literature on dialogical problem solving process in problem-solving instruction.

Communications and Problem Solving Process

Related to nonroutine problem solving, as delineated above, are issues of classroom curriculum, how children learn mathematics, and what constitutes learning. According to Sfard (2000), learning is inextricably linked to thinking. Learning is thinking and thinking is subordinated to, and informed by, the demands of communication. From this perspective, if having a better understanding of classroom discourse will offer a better understanding of "the dialogue one leads with oneself [and with others], then one must realize that investigating communication with others may be the best route to discovering the mechanisms of human thinking" (Sfard, 2000, p. 296). To this end, the author claims the best route to discovering human thinking is to investigate the nature of communication. That is, communication within a classroom is not merely helpful; it is integral. Before explaining the many facets of her new research, however, Sfard (2000) offers a rather complex aspect of the many components involved in communication. She challenges the common notion that communication is merely an exchange of information and argues that one must realize that the only way to define the relation of *sameness of meanings* is to say that this is the relation that enables successful communication, that is, that it is the relationship that conveys the meanings, not the information as conduits of information.

This theory is then utilized in her own goal, which is to study the implications of bringing mathematical objects into being when there is no ready-made discursive focus. In her study, she analyzed a classroom episode in which a group of seventh graders try to solve a problem that was intended for statistical thinking. "The students' exchange is analyzed in terms of the discursive processes that underlie mathematical problem solving and that occasionally bring about the emergence of a new mathematical object" (Sfard, 2000, p. 298). The episode in question comes from a lesson in which the students were asked to choose which brand of battery had the longer life-span based on limited information. The students were able to view a chart on a computer screen that reflected the life-span of several batteries. Sfard (2000) notices that the students use concrete visual terms when discussing the batteries (i.e. "the greens are the longest") and then compared two or more batteries, or colors, using spatial terms ("next to," "higher ones"). By using spatial metaphors and visual terminology, the students attempt to create a more pronounced focus. One student looks for consistency in battery life-spans, while another looks for intensity; this diversity exemplifies the process of establishing foci. "The process of focus-building is a result of an intricate negotiation between these complementing needs for *intuitive acceptability* and *operative rigor*" (Sfard, 2000, p. 314).

By examining such interactions, she concludes that the effectiveness of verbal communication is dependent on the quality of the focus; this discursive focus is three-fold: the pronounced element is public, the intended is private, and the attended mediates between the two. All three ingredients are discursive constructions that can only be realized through dialogue. Moreover, "[*m*]*athematical objects arise out of the needs of communication instead of being primary to communication*," which is a "reversal of the Platonic belief about the relation between 'mathematical reality' and discourse about this reality" (Sfard, 2000, p. 323). In short, one must communicate effectively with one self as well as with others to develop a logical conclusion. Part of this communication process evolves through the conceptualization process, or more specifically, as Sfard points out: "*Mathematical objects emerge through negotiations between metaphor and rigor*," and should be explored for mathematical creation (Sfard, 2000, p. 324).

Fleener, et al., (2002) examines Sfard's research, which focuses on communication and interaction between students and teachers as a means to explore various ways of emergent meaning and shared understandings. According to the authors, Sfard (2000, 2001) focuses on communications, not reified ideas or things, and, in the process, illustrates the integral relationships between dialectical process, cognition, inquiry, experience, and ideas. Fleener, et al., (2002) also discusses Wittgenstein's language games approach to meaning and problem solving. By considering both approaches, Fleener's study asserts social autopoiesis is an integral aspect of "shared understandings and emergent meaning" (p. 4). A key component to this notion is the idea of personal autonomy within social systems. Socio-autonomy, then, is the "creative potential for self-creation and self-production in social systems" (Fleener, et al., 2002, p. 5). In short, by engaging in a social system, individual participants also develop a sense of autonomy.

To continue their examination of the benefits of interactive dialogue in a classroom setting, Sfard and Kieren (2001) studied the conversations that took place between two thirteen-year old boys learning algebra. The authors focused on two main elements: focal analysis—(the mathematical content, or object level)and preoccupational analysis—(the participants' engagement in the conversations, or metalevel)—in order to examine the reasons why communication sometimes fails in the classroom.

In this process, the authors realized that thinking was an act of communication itself, and once this is realized, they believed they could bridge the gap between private and social communication. Through their observation, videotaping, transcription, and analysis, Sfard and Kieren (2001) realized that the students solved the problems individually, they could not agree upon an answer in a collaborative setting. This, then, led the researchers to the conclusion that students' mathematical problem-solving results may not be as important as their mathematics communication. Therefore, the role of conversation in learning is tantamount. In fact, Sfard and Kieren (2001) conclude that the notion of "communication [as] auxiliary to thinking and that mathematical *knowledge* and *thoughts*, are somehow primary to, or at least independent of, the acts of communication" (p. 47) is false. In short, thinking takes place within the activity of communication and simultaneous intra and interpersonal discourses shape one another. Furthermore the researchers define the nature of effective communication as an act that fulfills the participants' expectations and actually changes the way he or she might approach similar situations in the future.

In order to evaluate the effectiveness of communication, the authors offer an explanation of the different types of communicative responses involved in a dialogue: reactive and proactive (or inviting a response) which form channels between the participants. By examining these two factors in relation to the interaction between the boys in their study, the authors concluded that while individual problem solving might be an easy task for most of the participants, verbalizing their problem-solving strategies with a group in a productive manner seemed more difficult. Therefore, the participants' public channels of communication seemed dependant on their private channels of communication. In other words, an individual's knowledge of the subject matter had little impact on his or her ability to transfer this information to another. Yet the concept of learning by talking is integral to mathematics education and has proven to be beneficial to many students. Sfard and Kieren (2001) conclude, however, "[i]t is not necessarily true that two people who join forces can do more than the sum of what each one of them can do alone" (p. 70), and the reason this is true is because of the absence of productive communication. This dilemma, the authors contend, can be rectified by teachers who are able to facilitate perceptual mediation among their students; productive communication can be taught, and that should be the goal of every teacher (Sfard & Kieren, 2001).

The role of communication is vital in education and must also be examined in a pedagogical context. Menon's (1995) study, is an attempt, to "augment research on the role of context in problem posing … by studying student-constructed questions (SCQ) of some grade 5 and 6 students" (p. 25). More specifically, the author points out that individuals who are poor problem solvers in a classroom setting are not necessarily poor problem solvers in other contexts(Greeno, 1989; Menon, 1995; Stigler & Baranes, 1988). Within this frame of mind, Menon (1995) investigates his study with 5th and 6th grade students using student-constructed questions (SCQ), such as: "'[w]rite a word problem involving common and decimal fractions'" (p. 26). Students, then, worked in class, once a week, first in groups, and then individually, for about twenty minutes to construct and write the SCQ. According to Menon, results from this SCQ activity points out to the importance of using student experience and interest as the context of the SCQ, which, generally, reflects students' daily, out-of-school or in-school experiences. There is a call for motivating students "to take responsibility for and ownership of their learning" (Menon, 1995, p. 31). Koehler and Grouws (1992) echoing Yackel, et al., (1990) remind us that students' dialogical problem solving processes or peer-interaction during problem-solving activities is responsible for much of the learning that takes place.

Mathematics communication requires an understanding of symbolization in problem solving. In the introduction of their book Yackel and Cobb (2000) explore the role symbols play in students' mathematical learning and concludes that symbolizing and communicating is a much more dynamic approach to education than the previous static views of representation indicated; such an approach proactively supports mathematical learning instead of just merely analyzing activity. In this fashion, students are able to begin their experience using conventional mathematics symbols, but can eventually come to invent ways of symbolizing, which in turn allows the mathematics community in which they participate to explore new mathematical approaches. Because of this conclusion, Yackel (2000) focuses more on the activity of symbolizing rather than the symbols themselves. Similarly, Cobb (2000) concurs with the notion that symbolizing is integral to mathematical activity. However, as was concluded with Yackel, Cobb (2000) summarizes, "a student's use of symbols involves some type of meaning, and that development of meaning involves modifications in ways of symbolizing" (p. 19). In short, a student's experimental use of conventional symbols within a dynamic framework lends to the creation of new meaning. Such a process involves communicative relations between teachers and students, and to reinforce this notion, he cites Sfard and Dorfler's (2000) concern with both the subject and process of communication between the two parties. This process can be seen in the various ways students approach problem solving. One might graph information internally, while another might verbalize his calculations externally, but both students engage in an active discourse with conventional symbols in order to arrive at their own meaning (Cobb, 2000).

Moreover, in the same book, Lesh and Doer (2000) continue to support the notion of communicative engagement in the classroom and offer a definition of models and modeling as a research perspective. A model is a system that consists of "*elements*," "*relationships* among elements," "*operations* that describe how the elements interact," and "*patterns* or *rules*, such as symmetry, commutativity, or transitivity that apply to the preceding relationships and operations.... To be a model, a system must be used to describe some other system," and "to be a *mathematically* significant model, it must focus on underlying structural characteristics of the system being described" (p. 362). In an educational setting, this model then depends on a larger conceptual system such as language or symbols, thereby allowing students to utilize such tools to create new types of systems. Although some of this newly cognized ability or "representation system" cannot be shared with others, much of this information may be externalized, which allows other members of this model "community" to develop new internalized systems, thus enabling them with a greater problem solving ability (p. 364). In this way, models are formed not just by individuals, but by communities. Again, this is a dynamic activity, not static one; attention must be paid to the process, not the product (Lesh & Doer, 2000). This shared language of the classroom is a vital component to student collaboration.

Student Collaboration

Gokhale (1995) defined a collaborative problem-solving context as the grouping and pairing of students for the purpose of academic (mathematics) learning. The students are responsible for one another's learning as well as their own. In her own study she compared undergraduate students enrolled in Basic Electronics; one class was individualized while one was collaborative, and she concluded that student collaboration fosters the development of group critical thinking through discussion, clarification, and analysis. According to Gokhale (1995), student collaboration, in a dialogic problem-solving process, aids in the development of critical thinking through discussion, clarification of ideas, and evaluation of others' ideas. Gokhale (1995) posits the mathematics learning community should be a verbal community, where talking, listening, writing and reading are important areas of the activities.

Stein, et al., (1994) pointed out that students' engagement, collaboration, and negotiation through the problem solving process are essential elements of establishing a transformative pedagogy. For example, in a collaborative learning environment, students work together to solve problems just as teams of people work together in the workplace to solve problems.

Mathematics learning community, therefore, should focus on experience and relationships in order for students to realize their discursive potential; such a focus will allow students to take responsibility in their own learning process (Fleener, 1999). This approach is referred to as transformative education and involves collaborative critical inquiry, which will empower students and allow them to overcome anxiety about mathematics learning. The classroom can then be seen as an autopoietic social system where students can engage in a dialogic relationship with other individuals and with the environment itself. Such systems of communication and experience will allow students to develop mathematics potential and engage in meaningful mathematics discourse. In short, social meaning transforms those involved (Fleener, 1999).

The concept of students' collaborative problem solving has been addressed by others as well. For example, Maher and Martino (1996) began their longitudinal case study with the notion that when students are given opportunities to work in collaborative environments to initiate conversations with others and compare their ideas, they are afforded more momentums to build on concepts they have already learned. Thus, conversation, collaboration, and problem solving are combined to provide opportunities for meaning-making beyond what an individual typically may experience alone. When students are allowed to work collaboratively in groups and offer "proof and justification" for their answers, "disparate and distinct structures of knowledge interact and eventually become integrated" (Maher & Martino, 1996, p. 197). The authors observed one student over five years and evaluated her progress by choosing critical events from classroom activity, classroom discussion, individual interview, small group assessment, and written assessment. Through prolonged collaboration, the student was able to develop more and more sophisticated arguments to support her solutions.

Collaborative learning is not without its opponents, however. Lieken and Zaslavsky (1997) caution that although several studies have stressed the importance of students' active role in collaborative learning process, many educators have criticized collaborative learning because it allows some students to develop stronger problem-solving skills, but fails to enable the lower-end achieving students to have similar experiences because sometimes, for example, "highly competent students, by exhibiting far more active behavior, tend to dominate less competent students" (p. 334).

Drawing on Cummins and Sayers (1997), Fleener (1999) tells us that transformative pedagogy through collaborative critical inquiry is: (1) grounded in the lives of students (2) critical (3) multicultural, antiracist, pro-justice (4) participatory, experiential (5) hopeful, joyful, kind, visionary (6) activist (7) academically rigorous and (8) culturally sensitive. "Experiencing, embracing, and loving mathematics rather than mastering, controlling, or over-coming mathematics supports meaning-making and relationships with mathematics" (Fleener, 1999, pp. 102-103).

Further analysis of collaborative learning yields similar results. In her study, Dupree (1999) examines the relationships within a classroom of all-female students in an attempt to re-think traditional approaches of mathematics education, which may lack the personal level needed to foster mathematics understanding. The author stresses that students should not be mere recipients of lectures, but should actively participate in their own education. To do this, they should be encouraged to engage in problem-posing exercises (Dupree, 1999). Such an approach will lead students to becoming mindful problem-solvers instead of simple followers of pre-ordained rules.

In her study, collaborative problem solving was supported and facilitated by classroom conversations and opportunities for personal reflection. The classroom environment seemed crucial to supporting collaborative problem solving and reflective practice.

These environmental aspects include teachers who help students understand concepts instead of just teaching students how to do something (which seems to be the case in the United States), and collaborative activities that include both problem solving and problem posing. For this to occur, Dupree (1999) argues, there must be a sense of trust established within the classroom, which is established by listening and valuing one another's commentary.

Posing nonroutine problems and involving students in meaningful and challenging mathematics explorations and problem solving opportunities in a collaborative environment can support not only the development of individual understanding but may also contribute to individual and socioautonomy as key features of growth (Fleener, 2002; Yackel & Cobb, 1996). The next section supports the need for more research in the area of student engagement in collaborative/transformative learning environments.

Student Engagement in Collaborative Learning Environments

The previous sections have reviewed the literature on nonroutine problem solving, conversation as important to mathematical understandings, and collaboration as supportive of inquiry approaches to instruction that elicit and support mathematical discourse. Such a mathematical discourse is multi-faceted, but can be summarized with the following characteristics: Nonroutine problem solving, student discourse, collaboration, and student engagement.

Nonroutine problems or projects, as mentioned before, must be dynamic and creative in nature. This may involve student-generated problems and an implementation of technical (i.e. the Internet) aids. This type of problem solving is more of a contingent educational model, in that instructors do not anticipate certain responses from students, but allow them to come to their own conclusions. Such a model changes the teacher's role, in that he or she must relinquish some control and allow students to become more reliant on a community and themselves for their learning. The creation of a learning community must involve student discourse; students must actively engage in mathematics activity, perturbation, and argumentation. By creating a discourse, the educational emphasis is on invention, not procedure, which further enables the development of both the individual and the community. Verbalization is the key to better-thought and better education; therefore, students must actively listen to one another, but teachers should also listen "to" students and not "for" a particular answer, which may hinder mathematical growth (Davis, 1997). Once student discourse is established, collaborative learning becomes an integral component of the reform-based classroom. Collaboration allows students to connect in new ways, foster intellectual creativity, and utilize project-based mathematics—an approach that allows students to associate mathematics with experience and social perspectives. As stated earlier, problem-centered learning fosters mathematical and conceptual understanding among participants, which lends itself to mathematics learning in general. Further, collaborative projects are based on student engagement. Communication with others (and with self) is vital to thinking.

With communicative exchange, students are able to conceptualize and verbalize (both privately and publicly) to form a shared understanding and personal meaning. This development of autonomy in social systems improves individual potential for self-creation and production. For this to occur communication must be productive; teachers must facilitate perceptual mediation and create a trustworthy classroom environment that will perpetuate the creative and reflective process.

When students are offered opportunities to be inventive, collaborate with other students, and work on problem inquiries which are meaningful to them, they can build bridges to new and better understandings of and develop a deeper understanding and appreciation for mathematics. As understanding develops, so can confidence. Student confidence in solving mathematical problems has been found to be a significant predictor of their ability to effect mathematical learning (Pajares & Miller, 1995). Confidence, reflective thinking, imagination, and creativity may also be supported by social factors, including working in groups to solve difficult tasks. In addition, research on mathematical discourse supports a problem solving approach to mathematics learning. Historically, problem solving in mathematics has been viewed with a distinction between acquiring knowledge and applying that knowledge. This approach to problem solving is plagued by a push for correct answers and the practicing of skills. Correct answers with or without understanding are speedier for students to achieve, require less time on the part of teachers, and can be easily utilized to assess student performance. Therefore, the focus of instruction is on helping students to produce the correct answers. The benefits for this rigid-type of instructional procedure are questionable. Dewey (1926) warned educators about this by arguing that the quality of mental process, not the production of correct answers, should be the measure of educative growth.

This exploratory study focuses on students' nonroutine problem solving conversations in the area of mathematics. This dialogic problem-solving investigation will include learning opportunities in which students are challenged to think critically and to engage collaboratively to resolve their own problems and to understand and use mathematics. When students are allowed to work together in pairs or in a group to negotiate in choosing solution strategies and how to go about resolving their differences, they are given opportunities to collaborate, negotiate, and discuss mathematics as well as work toward the establishment of a supportive and synergistic context of a dialogic community.

CHAPTER III

THE RESEARCH DESIGN AND METHODOLOGY

In this chapter I present my research design and methodology, which are strongly influenced by Guba and Lincoln (1985, 1994) constructivist inquiry. The interpretive methodology, which I use in this study, is consistent with the current theories of learning mathematics. In summation, the research and application that follows is based upon the constructivist theory that learning varies from one individual to the next and cannot be effectively achieved with a blanket instructional method. Education, therefore, must not be treated as a factory that produces learned individuals, but as a multifaceted, contextual environment that fosters individual development within an interactive community.

In what follows, I outline the rationale for the use of the naturalistic (constructivist) inquiry approach in this study. Then, I restate the research questions. The context of the study is next which includes descriptions of the school and research settings and the participating students. Finally, I describe the procedures for this study, which are comprised of the data sources, the data collection, and the data analyses as well as the trustworthiness of the gathered data.

Rationale for Constructivist Inquiry Approach

Guba and Lincoln (1985, 1994) point out three crucial philosophical differences between positivistic and constructivist (post-positivistic) inquiry. These three differences are ontological, epistemological, and methodological in nature. Ontology refers to the nature of reality. Epistemology relates to ways of knowing.

Methodology defines how we do research.

Ontology. The nature of reality—the positivist views reality as being single, tangible, and "out there." The goals of the positivist researcher are for accurate prediction, discovery of underlying truths, and ultimate mastery over our environment. In education, therefore, positivist research attempts to identify the variables and discover the methods that "produce" the "best" learning. The constructivist (post-positivist) envisions reality to be multiple, contextual, and interactive constructed by the individual. Therefore, it cannot be predicted and/or controlled. Constructivist learning, therefore, challenges the positivist assumptions that learning is the same for different individuals and that learning can be "produced" by instructional methods. These ontological differences between the positivist and the constructivist perspectives also involve implications for epistemological differences.

Epistemology. Ways of knowing—the positivist version of the relationship of knower to the known is completely dualistic and objectivistic and suggests that the inquirer (the knower) can function in total independence from the object of inquiry (known) without influencing the "object." In contrast to this view, the constructivist envisions the relationship of knower and known as being inseparable and dialectical, and it suggests that the knower and known are interactively related and thus influence one another. Moreover, the naturalist (constructivist) believes that it is through this interaction that realities are shared and negotiated which help each knower to create his or her own meaning toward the development of his or her own construction of new knowledge. Ontological and epistemological differences between the positivist and constructivist (post-positivist) are directly connected with the way each does research, hence, the methodology.

Methodology. With implications for implicit values in inquiry—the positivist views research as being value-free and the goal of research is to generalize findings for the purpose of evaluation, measurement, or assessment. This method of measuring can be guaranteed by virtue of the objective methodology employed, which is based on the premise that by controlling "factors" within an environment the hypothesized effect will occur. That is, there exists a linear relationship between cause and effect (Guba, 1981; Guba & Lincoln, 1985, 1994). The constructivist, on the other hand, questions the generalizability of any research finding in positivistic terms and envisions all inquiry to be value-dependent and therefore impossible to separate causes from effects.

The positivist aims to produce research with human respondents as "subjects" and "...ignores their humanness, a fact that has not only ethical but also validity implications" (Guba & Lincoln, 1985, p. 27). That is, the positivist attempts to control and quantify our environment to produce desired results; therefore, his or her research is deterministic and reducing. The constructivist, however, views him or herself as well as other humans (participants) as the primary data-gathering sources and elects to carry out research in the natural setting "...to allow the study design to emerge (flow, cascade, unfold) rather than to construct it preordinately..." (Guba & Lincoln, 1985, p. 39) because what may emerge during the interaction between the knower and the phenomenon is largely unpredictable in advance. Moreover, it is through these interactions that meanings and interpretations of multiple realities are shared and negotiated which may give birth to new understandings. The relationship between social interaction and student learning is complex, as is its analysis. In order to provide a more concise definition of dialogic communication and establish guidelines for evaluating classroom discourse (i.e., between student to student, student to teacher, etc.) I relied upon criteria set forth by Cobb and Yackel in 1995.

Studying Dialogic Process

Cobb and Yackel (1995) offer an emergent design approach to study the complexity of social interaction and student learning. The emergent approach, which is viewed as a version of social constructivism, involves coordination of interactionism and psychological constructivism, which attempts to interpret the joint activity of the learners in the learning environment. The framework of this approach includes social norms, sociomathematical norms, and classroom mathematical practices. This approach is particularly relevant to the design of the current study, as described below.

Social Norms. Social norms, according to Cobb and Yackel (1995), account for an individual child's learning in the learning context by analyzing the conceptual reorganizations he or she has made while interacting with the other learners within the community. In such social norms, Cobb and Yackel (1995) concluded that (a) the most the facilitator can do is to initiate and guide the group discussions, allowing students to explain and justify solutions, ask questions, and make sense of any interpretations which may go on in the social context (b) neither the social norms nor the individual students' beliefs are given primacy over the other (c) social norms and beliefs are seen to be reflexively related and neither exists independently of the other

(d) social norms develop as students reorganize their beliefs and, conversely, this reorganization is aided by the social norms (e) this reflexive reorganization can be applied not to just mathematical development, but to almost any subject matter.

Sociomathematical Norms. What counts as an acceptable mathematical explanation and justification during a mindful problem-solving interaction? Yackel and Cobb (1996) attempt to answer this question by focusing on the joint activity of the learners with regard to their taken-as-shared basis for communication and discussion. This activity is termed as "*sociomathematical* norms, that is, normative aspects of mathematical discussions that are specific to students' mathematical activity" (Yackel & Cobb, 1996, p. 458). What constitutes as an acceptable mathematical reasoning is agreed upon by everyone within the learning community. That is, when students and not just their mentor(s) consider the adequacy of an explanation given to others, rather than just for themselves, the explanation itself becomes the object of discourse.

Sociomathematical norms are significant in developing students' intellectual autonomy. When a researcher asks the students if anyone has a different mathematical solution, the students must think about what constitutes a "different" solution from the solution already presented. This requires the student to explain his or her own thinking. Further, it requires the student to decide whether a different solution is an efficient and acceptable solution. The joint activity of the learners is very significant in the development of a student's responsibility to judge a solution as to its difference, its efficiency, and its appropriateness. This, then, is developing students' intellectual autonomy by drawing on the students' own intellectual abilities to make decisions and judgments. This is in contrast to intellectual heteronomy wherein students rely on an authority to know what action to take. Cobb and Yackel (1995) conclude: "These beliefs and values, it should be noted, are psychological constructs and constitute what the National Council of Teachers of Mathematics (1991) calls a mathematical disposition" (p.35).

Sociomathematical norms are important because they open wide the group activities, giving credence to students' contributions and judgments as to what constitutes an acceptable mathematical explanation. And, while engaging in such mathematical group activities, the students are becoming autonomous in mathematics. With these guiding principles established, I set out to create my own study of social interaction and its relationships to student learning using nonroutine problems as focal points for mathematics communication.

In this study, the students' nonroutine problem-solving discourse within pairs and in the group activities on searching, questioning, talking, listening, writing, negotiating, feeling, caring, sharing, and validating played a vital role for understanding and interpreting the research. This approach to research is called an emergent approach. The researcher, therefore, incorporated a combined research methodology of cognitive and social constructivism to best reflect and analyze the flow of this journey as the study unfolded. The implications of this methodology required asking questions, which were inherently unique to the context of the study as well as being complex, dynamic, and open-ended. These questions were nonroutine, demanded more opportunities for cognitive challenges, and required conversation with-self and with-others as well as having multiple possible solutions. By providing problems that could be approached in multiple ways and encourage communication among individuals and collaborative partners, it was possible to analyze how a constructivist, dialogic environment impacted student learning and problem solving. Therefore, a set of research questions was established to reflect this goal.

Research Questions

The purpose of this study was to examine the complex interplay among student beliefs, problem solving engagement, problem type, and mathematics understanding as well as the dynamics within group discourse among four ninth-grade mathematics students. The focus of this study was to explore the following questions:

1. What is the relationship between student engagement and problem type?
2. How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

Yet before these questions could be addressed in data collection, it was necessary to establish the criteria by which the reliability and trustworthiness of gathered data would be determined, which was gleaned from Guba and Lincoln's (1985, 1994) and Guba's (1981) proposal for establishing trustworthiness.

Trustworthiness

Guba and Lincoln (1985, 1994), based on Guba's (1981) proposal, give the techniques for establishing trustworthiness. These techniques focus on four criteria that best fit with constructivist epistemology, namely credibility, transferability, dependability, and confirmability. In what follows, borrowing from Guba and Lincoln (1985, 1994), I describe these four criteria in the context of nonroutine problem solving.

Credibility refers to those activities that increase the likelihood of authentic findings. According to Guba and Lincoln (1985, 1994), there are three such activities: prolonged engagement, persistent observation, and triangulation. Prolonged engagement refers to the investment of sufficient time to becoming oriented to the problem solving context. This orientation may take much needed time for the constructors of multiple realities to search, to question, to conjecture, to test, to validate, to share, to build trust, and to make sense of their own reality and the reality of the others. One validation of establishing credibility is acceptance of the findings by the members of the problem solving group being studied as well as by the critical consumer of the inquirer's report. Data, analytic categories, interpretations, and conclusions, which may result from the researcher's collective observations and triangulations, must be examined with the members from whom the data were originally collected. Guba and Lincoln (1985) call this member checking. Member checking is both formal and informal, and it occurs continuously throughout the data collection process. Member checking will enable the investigator to suggest that his or her representations are recognizable by the problem-solving members as appropriate understandings of their own (and multiple) realities. Member checking is an important component of credibility.

A second component of trustworthiness is the establishment of transferability. The establishment of transferability, in a precise sense, is impossible. For its validity to "hold in some other context, or even in the same context at some other time, is an empirical issue, the resolution of which depends upon the degree of similarity between sending and receiving (or earlier and later) contexts" (Guba & Lincoln, 1985, p. 316). Therefore, transferability of the constructivist findings can only be possible through the provision of thick description.

A third component of trustworthiness is dependability. Reflecting on their previous paper (Guba, 1981), Guba and Lincoln (1985) tell us that there can be no validity without reliability and therefore no credibility without dependability. One of the ways to establish dependability can be reached by the review of triangulation of data in relation to credibility. Guba and Lincoln (1985) characterize this as overlap methods and suggest that it should not be necessary to demonstrate a separate approach for dependability. Another approach for establishing dependability is through the inquiry audit, in a metaphorical sense, to authenticate the process by which the data were gathered and kept. An auditor examines the fairness and accuracy of the representation of the investigator's overall study with the participating students, which is closely connected to confirmability, the fourth component of trustworthiness.

The major approach for establishing confirmability is the audit as mentioned above. There are several stages in assessing confirmability in a research study. The auditor's first concern is to determine if the findings are grounded in the data. Formal and informal observations, interview notes, and clips from videotapes as well as document entries are examined. Next, the auditor is to reach a professional judgment about the appropriateness of inferences based on the gathered data, the appropriateness of emerging categories, and the quality of interpretations as well as the possibility of equally attractive alternatives. The auditor also looks at "the utility of the category structure: its clarity, explanatory power, and fit to the data" (Guba & Lincoln 1985, p. 323). Finally, the auditor reviews the inquirer bias to determine the extent to which the inquirer resisted early closure.

Because this study is grounded in social and cognitive constructivist methodology and focuses on students' nonroutine problem-solving dialogue, issues of trustworthiness—credibility, transferability, dependability, and confirmability—will be considered in the design and in the implementation of the project where the establishment of a dialogic problem-solving pedagogy is the goal of this research. This study attempted to establish credibility by electing activities, which allowed for the investment of sufficient time on the part of the volunteer students to becoming oriented to the problem-solving context, persistent observation and triangulation, as well as continuous formal and informal member checking throughout the data collection. The establishment of transferability, as mentioned above, is impossible in a precise sense. However, the summary of appropriate, relevant, thick, and meaningful description, which has emerged from this study, could be utilized as the bases for an index of transferability on the part of potential appliers. The establishment of dependability and credibility are closely related and inseparable. One of the ways to establish dependability for this study is done by the review of triangulation of gathered data. These data are gathered from transcriptions of videotaped and audiotaped discussions during problem solving, dyad interviews, and group problem-solving dialogue meetings, as well as students' verbal and written responses to questionnaires and problem-solving journal documentation and reflections, which are closely related to the establishment of the credibility for this research. Another approach for establishing dependability was to incorporate an inquiry auditor, as a "pump," to examine and authenticate the process of the inquiry. The role of the auditor is important for establishing confirmability in the constructivist research methodology. In this study, the auditor determined whether the findings were grounded in the gathered data by tracing back via the already established audit trail to the gathered data such as interview notes, videotaped clips, and document entries. The auditor also checked the appropriateness of inferences based on the gathered data and the efforts made by the researcher during the inquiry to ensure genuine and meaningful study. Once it was established how data would be deemed reliable and trustworthy, the exact methodology for collecting this data was determined.

Procedures

The procedures for this study include data sources, data collection, and data analysis. In what follows, I describe each of these.

Data Collection. The data collection process occurred in two phases. The purpose of the first phase of the study was to gather background information and provide team-building opportunities among and between the volunteer participants. The first phase lasted approximately six weeks and included the following components: (1) group discussion and negotiation of study procedures with the participants (2) individual participants' completion of questionnaires on Mathematics Learning Inventory (MLI) and one-on-one interviews (3) two, one-hour group meetings for further collection of background information (4) observations of participating students' preliminary pair and group activities starting the third week.

During the second phase, the researcher (1) observed and videotaped student dyads during nonroutine mathematics problem-solving interactions (2) followed each dyad videotaped interaction with pair interviews and (3) met with all participants every month for approximately two hours to discuss students' reflections on their nonroutine mathematics problem-solving interactions with peers, problem types, and group discourse. A final individual exit interview occurred with each of the participating partners at the concluding session of the study. Additionally, videotaped problem-solving sessions were made available for students to review and were used as prompts for follow-up interview sessions.

Data Sources. Data sources included the following: the students and parental consent forms (see Appendix A), the students' written responses to questionnaires (see Appendices B – D), the superintendent's responses to my questions regarding the information about the school, and transcripts of videotapes and audiotapes gathered from students' dyad problem-solving engagement and one-on-one interviews with the students. Data was also collected from transcripts of videotapes and audiotapes gathered from group discussions and dialogue occurring once every month for approximately two hours each time, my field notes from pairs problem-solving observations, and the students' journals.

Data Analysis. Data from transcriptions of videotaped and audiotaped discussions during problem solving, dyad interviews, and group problem-solving dialogue meetings, as well as students' verbal and written responses to questionnaires and problem-solving journal documentation and reflections were analyzed using a constant comparative method (Guba & Lincoln, 1985, 1994). The emerging categories from the multiple data sources were examined using a matrix of categories for comparing mathematics inquiry, inclusive use of technology, and mathematics understanding as well as changes within the dialogic discourse. This procedural framework was then applied to the selection of participants, research site, etc. The details that make up the context of this study are discussed below.

Context of the Study

This study investigated nonroutine collaborative problem-solving dynamics among four ninth-grade mathematics students. The number of participants provided ample opportunities to observe varying levels of interaction (i.e., one-to-one, within a dyad, and within a group), while still allowing for close observation and analysis permitted in a smaller group setting. The students attended Crossroad Christian School (CCS)*, which is a pre-kindergarten through twelfth-grade private school located in an urban area of about 500,000 people. There are several private and state-supported colleges and universities close to this school. While the participants were selected in cooperation with the school, all problem solving sessions took place away from the school either at my house or in public place. As mentioned above in "data collection," a booklet provided by CCS principle is the source for information regarding the school.

The School. Crossroad Christian School (CCS) was established in the fall of 1972 for grades one through eight. By 1976, the school was expanded to include pre-kindergarten through twelfth grade. CCS is a nonprofit, tax-exempt, private

---

* All names are pseudonym chosen by the participants.

educational institution that is separately incorporated and independently governed by a Board of Trustees. At the beginning of the 1988-89 school year, CCS moved to its present location.

The school's first senior class contained eight students who graduated in May 1977. After beginning with 200 students, enrollment declined to about 155 over the first three years. Then it began to increase, and growth has been relatively steady over the years. Currently, and since the 1986-1987 school year, each grade level contains two classes with a total school enrollment of approximately 1200. CCS is a protestant-based private Christian school. The majority of its students (about 98 percent) are middle-class American Caucasian, many of whom are female (about 35 percent). The school ranks as one of the best regional private Christian schools, whose graduates are known to be competitive in entering colleges and university.

CCS was chosen because of its academic record, and because I had an established acquaintance with Mr. David Mehlaff, who was a coach and a fundraiser for the school. After securing permission from the school's administration, I asked Mr. Mehlaff for help in selecting four ninth-grade mathematics students who would be willing to volunteer for this study. This particular age-level was chosen because of the stage of mathematics development generally associated with that grade. For example, I was confident that the students had had at least some exposure to multiple concepts (such as algebra, geometry, word and logic problems) at varying degrees of difficulty, yet they had yet to experience more involved mathematics problem solving discourse. Such a balance guaranteed that each participant had a basic mathematics understanding, had formed some opinions or assumptions about mathematics education and the use of technology in that educational process.

With that in mind, I asked Mr. Mehlaff's aid in selecting four students with analogous mathematics ability and social involvement who would be willing to participate in this study. Mr. Mehlaff then suggested four students he believed fit this criteria (i.e., each student had shared the same geometry teacher, were involved in some school-related sport, and took part in some church related musical activity). Once these students were identified, I decided to meet with the boys and their parents so that the exact nature of the study could be discussed and what would be required of the participants.

The First Meeting: Surveying the Landscape. With the school's permission and the aid of Mr. Mehlaff (who acted as a mediator during this first meeting), a two-hour, informational meeting was scheduled at the school for September 1, 1997. The purpose of the meeting was informative, but it was also intended to establish a sense of familiarity and trust between the four volunteer participants, their parents, and myself. After Mr. Mehlhaff introduced the participants and their families to me, I explained the purpose of the study, my intentions behind this research, and my professional and personal background (i.e., information regarding my immediate family, the quality of education and the relationship between students and teachers in my country, and how Iranian public schools used technological tools such as the Internet). In turn, I inquired about their individual families, their past and current mathematics experiences, and their level of familiarity with the Internet. These inquiries were intended not only to develop familiarity with the participants and their families, but also to evaluate their proximal prior technological and mathematics experience.

The parents wanted to know about my expectation of them and their children throughout this research process, which I explained would involve the students' commitment and consistent attendance because an absence would involve rescheduling. I also told the parents and their children that their names will be kept anonymously, and to that end, I asked the students to choose a name other than their birth names as their pseudonym, which they did. At that point, I discussed the possible setting for the research with the group; I explained that in order to implement nonroutine collaborative problem solving experiences that would enable authentic cognitive engagements among the students, the locale had to be one that provided a relatively comfortable environment in which their mathematics and communicative exchanges could be observed and recorded. Initially, I had considered the school as the site for the study, yet there was not a reliable computer available at CCS that had Internet access. Therefore, I suggested the study take place at my house because this option was the most convenient to myself and the boys, who would have access to facilities such as the Internet, snacks, soft drinks, etc.

Background Information and Team-Building

After conferring with the four participants and their parents, the next meeting was scheduled for September 8, 1997, one week later. The purpose of the second meeting was for the group to become better acquainted with one another, provide the parents an opportunity to view the research environment, and select dyads for collaborative projects. During this meeting, I learned that Jim and Bob were not only close friends, but were also neighbors, often challenging one another with friendly competition. I also learned that Paul, Clint, and their parents attend the same church as my family and I do, and had seen me before in that setting.

After the parents had toured the house and departed, the students engaged in a few moments of casual conversation, inquiring about one another's family, school, and church. These "icebreaking" questions were important so that the group could determine their commonalities and differences, which would then aid them in selecting a partner for the study.

The Participating Students. The four volunteers were then told that they would be grouped, by their own choosing, into two pairs of collaborative problem-solving dyads. I explained that the reasons that pairs are chosen in this way are as follows:

1. to accommodate self-selection of the four students who volunteered to participate in the research study,
2. to accommodate the expressed desire of student participants to work with closer friends toward problem-solving inquiries, and
3. to facilitate mathematics discourse among the participating students as well as problem solving dialogue.

With these factors in mind, Bob and Jim decided to work together as a team, because they already had established a friendship, and Paul and Clint chose to become partners. Because Bob and Jim had already worked together, there was more initial connection and corroboration between them than there was between Paul and Clint. Yet collaborative problem solving was a unique experience—an unfamiliar environment—to all of them because at school, the mathematics classroom environment involved students working individually to solve problems or to do homework. Therefore, although all four participants were in the same classroom, they did not have much opportunity to work with each other or with other students.

After choosing a peer and forming a team, the participants were asked to choose a research schedule that involved two weekly, two-hour meetings with me. Bob and Jim, members of team one, decided to meet every Monday and Wednesday, after school, from 4:00 to 6:00 P.M., and Paul and Clint, members of team two, decided to meet every Tuesday and Thursday, after school, between the hours of 4:00 to 6:00 P.M. Bob and Jim immediately worked out a car-pool arrangement, while Clint and Paul had to rely on their parents or myself and would arrive separately.

This concludes the discussion of research design, methodology, data collection, data analysis, and selection of research setting, participants, and dyads. The following chapter discusses the individual results of the MLI questionnaire, the one-on-one initial interview, and the first team-building project, which is referred to as the initial phase. Data gleaned from the questionnaires and interviews are then used as a starting point so that any changes or modifications to these beliefs and assumptions may be compared and noted in terms of individual or group development. The pairs' preliminary activity during this first stage of problem solving interactions will also be presented and discussed in terms of dialogic and constructivist importance. Also, the following chapter discusses results from phase two of this research study, which comprises of emerging patterns through dyads' engagement on problem types, problem solving discourse, and on-going negotiations and communications as well as evolving discourse through problem solving.

CHAPTER IV

THE RESEARCH RESULTS

Traditionally, mathematics education has largely been based on rote memorization and drill, placing the authoritative focus on the instructor and rendering the students as mere recipients of knowledge and information. In this type of environment, students behave as independent agents, and their mathematics success is rooted in their ability to remember formulas and reproduce results. Yet, research (as discussed in chapter two) indicates that this type of approach fails to utilize the many benefits available in a more reform-based curriculum. While many researchers may disagree about what the most important facets of a reform-based curriculum may be, they all seem to agree that it involves one or more of the following elements: active student discourse, technical innovation, and collaboration.

Recent studies show that an active student-teacher and student-student discourse creates a more open environment that allows students to explore mathematical concepts in a more meaningful way. In such a setting, the teacher surrenders some authority to allow students to generate their own problems and experiment with multiple methods for producing multiple solutions; the teacher acts as a facilitator to create opportunities for open discussions that are beneficial in developing mathematics understandings. In addition, classrooms that provide access to various technical aids (i.e. the Internet, instructional programs, etc.) reinforce the notion of vast mathematical possibilities within the students and inspire them to incorporate creativity in their problem solving. Finally, collaborative projects are integral to reform-based classrooms because they allow students to explore new ideas, challenge old ideas, share perspectives, propose possible solutions, and justify their methodologies. The very act of communication within a classroom leads to a greater opportunity for mathematical understanding for all those involved. However, such opportunities are largely missing from today's classrooms' discourse, a condition that is in direct relationship to problem type that is currently being offered. Specifically, the lack of nonroutine and open-ended problems in mathematics curriculum robs students of the opportunity to engage in problem solving projects over time; traditional methods of assessment such as test taking and homework assignments do not allow prolonged problem solving experiences to occur. Additionally, without an environment that fosters prolonged, conceptual problem solving, it is almost impossible for an active student to student or student to teacher discourse to evolve.

Because of the relationship between nonroutine problems and mathematics discourse, both factors must be considered when evaluating or modifying the current traditional curriculum. For that very reason, I propose that the following components must be incorporated in order to create a more productive environment: mathematics education settings that promote and keep into account the complex interplay and interactions among beliefs about what mathematics is, how engagement between teacher-student and student-student should take place in mathematical settings, what type of problems should be explored and experienced, and the nature of supportive community and discourse being promoted here in this study as catalysts for mathematics teaching and learning.

With these innovations in mind, I wanted to see first-hand how students who were engaged in a collaborative, dialogic community involving the use of technical support performed and learned as individuals and members of a larger community. To this end, I provided various nonroutine (open-ended) projects and problems, (see Appendix E), to be worked-on by four ninth-grade participants who had volunteered for a sixteen-week mathematics program to be conducted outside their school environment. These four students would then be divided into two pairs, or dyads, so that focused observations could be made of their problem solving, problem posing, communication, execution, collaboration, and overall success. The focus was, then, not only on their individual progress and use of technical aids, but also on their progression and development as collaborative pairs.

In order to conduct a thorough observation and analysis of their development and progression, however, the nature, content, and context of the study had to be multifaceted. For that reason, I concluded that each pair should meet and collaborate twice a week for sixteen weeks, and that the entire group should meet at least twice during that time—a schedule that provided ample opportunities to observe their interactions and communications. In addition, I believed that it was important to understand their initial beliefs and assumptions about mathematics, collaboration, technical incorporation, and educational responsibility to better interpret their interactions as the study progressed. Therefore, the purpose of this study was to examine the complex interplay among student beliefs, mathematics inquiry, and the use of technology as well as mathematics understanding and the evolution of a dialogic community among four ninth-grade mathematics students. The following sections present various problem solving situations in which the students were engaged in an effort to track their progression and proficiency in mathematics problem solving, problem posing, communication, use of technical tools, individual inquiry, and conceptual understanding. With that goal in mind, each project was then analyzed in terms of how it related to the research questions that drove this study. These questions are:

1. What is the relationship between student engagement and problem type?
2. How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

The Initial Phase

The first phase of this study began in the second week of September 1997 and lasted six weeks. Sessions were audiotaped and transcribed. The purposes of this initial phase of students' interactions and data collection were: (1) to engage participants in group discussion and negotiation of study procedures (2) to obtain background data by having individual participants complete MLI questionnaires and a one-on-one entry-interview and participate in two, one-hour group meetings (3) to make sure each participant knew how to use the Internet to log-on, browse through, and download information by providing two one-hour, pair-internet-workshop sessions and (4) to implement a preliminary (or ice-breaking) individual, pair, or group project (e.g., egg-drop project) designed specifically to provide team-building experiences.

The goals for implementing the initial phase of the study were to (1) establish an atmosphere of trust in which the participants could get to know each other, as pairs and as a group (2) examine each student's beliefs with respect to the nature of mathematics, problem solving discourse, use of technology, and mathematics understanding (3) provide a setting for discourse in which the participants could build their teams and have shared experiences and (4) negotiate days, times, and the lengths of each dyad's problem solving engagement for the second phase of the study.

To contribute to the collection of background information, each of the four volunteer participants was asked to complete a survey questionnaire "Math Learning Inventory or (MLI)"—A research-designed survey adapted from Fleener (1995). The survey was distributed to and completed by the participants in my home (see Appendices B – D). The participants' individual completion of questionnaires was then followed by one-on-one interviews that were designed to clarify responses generated in the MLI. Each individual student interview took about one hour to complete.

The following is a comparative analysis of the individual responses and how they relate to this study. A brief summary of the students' MLI survey results is in tables 1 – 4 at the end of this dissertation case study. What is perhaps most significant about these responses is that half the participants (2 out of 4) immediately expressed difficulty communicating their mathematics activity to another party, which is interesting because these students also express reluctance to participate in collaborative learning in the classroom even though they are not satisfied with the current traditional approaches to the curriculum. Similarly, each volunteer participant agreed that there are several possible solutions to every problem, but only half of the participants deemed creativity important in problem solving. Additionally, each student cautiously endorsed technology in the classroom. These initial findings are important because they emphasize the lack of technological, communicative and collaborative opportunities in their school classrooms and the students' lack of experiences with these approaches to mathematics learning. As these students engaged in collaborate, open-ended, technology supported, dialogic problem solving opportunities, it was hoped they would be able to better reflect upon the potential impact of these experiences. Results of the responses to subsections of MLI follow.

Basic Mathematics Assumptions

In order to get a sense of the four volunteer participants' initial assumptions and beliefs about mathematics, inventory items 1-4, 6, 11, 17, 29, 22-24, 27-28, 32-34, 36-37, 39, 45-45, 50-51, 60, and 68 were analyzed. These items specifically addressed basic mathematics assumptions as well as their personal feelings regarding their mathematics performance. Such information was vital because it allowed me to compare these initial beliefs with their exit interviews and determine if collaboration and technological innovation had changed their initial stances. Transcripts where students were specifically asked about these items or volunteered observation related to mathematical beliefs are summarized below.

For the most part, each participant shared similar assumptions and beliefs about mathematics, mathematical knowledge, and their own mathematical abilities. Each participant felt relatively confident about his mathematics ability; however, Bob and Clint expressed a weakness in geometry, and Paul and Jim said they had difficulty verbalizing their solutions to another party. All four participating students supposed that every problem had numerous possible solutions that should be explored to "widen understanding" as Clint remarked, but he and Bob both intimated that the "teacher's way was probably the best" way to approach problem solving, which indicates some caution in mathematics experimentation.

Similarly, Bob and Paul both believed mathematical problem solving requires more logic than creativity; in fact, Bob said there is little room for the creative process because "mathematics is just facts and truths. You just plug in the numbers [to a given formula]." He said he had never used creativity in problem solving. Again, they seemed willing, yet lacking experience with espousing creativity into their problem solving, problem posing, and open-ended problem solving.

All four participants stated that good grades do not necessarily reflect mathematics ability. However, Clint then said he believed grades might be a *slight* indication of ability and said that he knows he understands mathematics concepts when "the teachers give [him] good grades," and Bob said he expects good grades in subjects he knows well. They all shared the belief that some people are more mathematically inclined (yet Jim then said that people are not born with mathematics ability because "you have to study it and be taught it"), but that talent is not synonymous with enjoyment. Yet he also indicated that a student does not need "basic facts" to be an effective problem-solver, and later stated that a student can learn the "rest" if he "gets the basics."

It seems there is definitive confusion about the nature of mathematics understanding and how it is developed cognitively. Bob did say that only people who enjoyed mathematics were capable of inventing new mathematical truths. All four participants asserted that mathematics is applicable in everyday situations and is not just an abstraction, which reveals recognition that mathematics can be applied to outside classroom contexts. Likewise, all four participants agreed, to some extent, that there are absolute mathematical truths, but then seemed to reconsider that opinion during the interview process. For example, Bob defined a mathematical truth as something that consistently works in all formulas and are absolute; he then said that some truths could be proven wrong, while others can never be challenged (e.g. Einstein's equation of relativity). He also initially said that the invention of new truths was limited to mathematics professionals only, but then said they didn't have to "be clever" to invent mathematics knowledge—an interesting contrast.

On the same note, Jim strongly believed that mathematical truths are absolute, not relative because "two plus two is always four, never five." But he then said that truths are dependant on a person's perception of reality (i.e. a person who is color-blind perceives colors differently than others will), and he also strongly believed that truths could be proven wrong. To clarify this he offered, "when Newton was around, all the things he had were true in certain situations . . . but time has affected his theories." In essence, he believes a truth is true until it is proven otherwise, but that does not make it any less true before it was challenged—a concept that is consistent with post-modern perspectives. He also stated that the invention of mathematical truths is limited to mathematics professionals, yet of the three, only Jim labeled himself a mathematics inventor, while the other three considered themselves to be consumers of mathematics. Jim offered his definition of a consumer as one who accepts whatever is given to him/her, but an inventor asks "why it works, how it works, how to make it work, how to make more things with it, like how to make an equation to solve a certain problem." Bob said that he believed a consumer simply "consumes all the information, takes it, and learns it," while an inventor "invents problems, mathematical truths, and equations that will work every time . . . does something no one's done before." He said he only solves problems, but does not invent them.

Such contradictions about mathematical truths and inventions seems to indicate that the four participants have mixed understandings about what mathematical truths may be, and how they may have been invented or discovered. This, then, may explain why the boys seem uncertain about their own mathematical inventive abilities. From this analysis of their basic mathematics assumptions, it is plausible to see the four participating students' reliance on relatively traditional beliefs about mathematics (i.e. that it is a realm reserved for professionals and educators). Such reliance is also indicated in their wariness to experiment with mathematics and problem solving. This wariness is also evident in their views about technology, pedagogy, and collaboration as they approach working on mathematical problems.

Technological Assumptions

Information regarding their technological assumptions was gathered from the analysis of inventory items 5, 14, 35, 41, 61, and 69—questions that asked the participants how they viewed the use of computers and calculators in the classroom and whether or not their use should effect mathematics education. Again, this information is important to the study because (a) it enabled me to evaluate their level of technological proficiency and (b) I could then compare these technological assumptions with post-study responses.

Their collective technological background was almost nonexistent; of the four participants, only Jim expressed online proficiency—Clint had only utilized the Internet to send and receive e-mail. Bob said he had only "watched others" use the Internet, but was familiar with other programs, had used the computer since an early age, and had even tutored others in a Basic Computers class. Paul had never used the Internet. Three of the four participants stated unequivocally that they believed students should practice mathematics problems without the use of calculators, a tool they seemed to regard as convenient, but not necessary, yet two out of four admitted that calculators certainly "sped things up," which would provide more time for other activities; Paul even indicated that they should always be used on tests. All four participants agreed that technology and computer-use should not change current learning methodology, yet they all expressed some dissatisfaction with traditional methodologies. It is unclear whether they were hesitant to utilize technological aids because they believed using a calculator indicated a lack of conceptual knowledge, because they believed mathematics educators disapproved of them, or because they had had little experience using technology in problem solving.

Pedagogical Beliefs

Inventory items 8-10, 12-13, 18-19, 21, 25, 29-31, 38, 40, 42, 47-49, 52-59, 62-67, and 70 all pertained to pedagogical beliefs and beliefs about current approaches to mathematics education. The students' responses were valuable because they indicated a level of satisfaction (or dissatisfaction) with past educational experiences and views of the roles of educators within the curriculum. Based on their responses, it was evident that the students all had participated in traditional mathematics education. These factors were then compared to their pedagogical beliefs at the end of the study, which allowed me to understand how a nonroutine, collaborative, technologic environment may have effected their pedagogical assumptions.

All four participants expressed some degree of dissatisfaction with traditional methodologies; even though they were the only curriculum they had experienced and indicated confidence in their ability. All four strongly disagreed with the use of mathematics drilling and memorization as a teaching technique; in fact, Paul said that he believed that an individual who can "recognize patterns and relationships" has a much greater conceptual understanding than "someone who memorizes formulas." All four believed that tests would be more effective if they contained few longer, more involved problems instead of many shorter, less challenging problems and that teachers should provide unambiguous problems for general coursework. Paul defined ambiguous problems as either poorly worded or too advanced; he suggested teachers provide various problems to accommodate varying levels of ability.

Yet, only Clint said teachers should show students the correct way to solve a problem and, then, tell students whether or not their answers are correct. In fact, he emphasized that different approaches to a particular problem are beneficial because "they make it easier to do it the way the teacher told you to." Paul disagreed with Clint because he believed students have the ability to "pick up knowledge" without the teacher's intervention. Bob initially said the teacher should only provide the correct methodology if the student's methods were "inefficient," but later said that the correct way should be shown first, to everyone, to eliminate erroneous approaches. Jim initially said that students can learn without the help of teachers, but later said that it is the teacher's job to show the correct way to solve a problem, and that once a student understands "the basics," the teacher should only indicate when an error is being made, not what that error is—but that communication between a student and teacher should not be limited to these instances.

The four participants' views on personal educational responsibility varied. Paul and Bob both believed that teachers were ultimately responsible for students' learning; Bob even credited his past and current teachers with his own conceptual understanding and ability. Jim believed students were responsible for their own education, and Clint believed both parties were responsible. From these responses, it is apparent that each participant would welcome a change in their current mathematics education, but seem unsure about the role an educator should play in the classroom and how he or she should influence students' approaches to problem solving. They all seem to believe that the teacher is the ultimate authority in the classroom, but disagree about how that authority should be executed. The four participants seemed hesitant to embrace freedom in problem generation and methodologies, even though they advocated those very practices in mathematics education.

Collaborative Work

Because the very nature of my study focused on the collaboration that would take place between the participants, it was vital to understand what past collaborative experiences they had had, what opinion they had about those experiences, and what their beliefs were about cooperative projects in general. Inventory items 15-16, 26, and 43-44 addressed these issues, and their responses would then provide insight when evaluating their initial communication with one another, as well as how their beliefs changed over the course of the study.

When questioned about dialogic learning, Paul and Clint said they believed group work and collaborative learning were beneficial only if applied sparingly, yet Clint admitted he had tutored classmates in computer courses and said that sharing ideas in the classroom is helpful because "everyone could see, hear, and think what others were doing," likewise, Paul said his geometric understanding had improved with the help of another student. This is interesting because they both indicated definite caution with collaborative learning, yet both have had positive experiences in a group environment. This tension will become evident in their own exchanges.

Bob and Jim strongly believed group work was beneficial both to mathematics learning and application and referred to instances where they had worked with one another. This is equally interesting because their positive beliefs about collaboration will become evident in their exchanges. For the most part, it seems as if the group was open to collaborative projects, even though they had had little experience in that type of environment.

This background information and analyses on the four participating ninth-grade students was pertinent to this investigation to increase my understanding of the students' prior mathematics knowledge and beliefs, and the students' familiarity working with each other and with the Internet. To this end, we conducted three additional two-hour group meetings and two, two-hour Internet workshops at my house in order to triangulate data pertaining to initial beliefs and to start to build community and collaborations within the pairs and the group. During the group meetings students talked about several issues including their classroom experiences with their teacher(s), curriculum, and working with peers. During the Internet workshop sessions I asked each volunteer pair to search, negotiate, and design, an egg-drop project.

Team Building and Shared Experiences

In addition to the surveys, the initial phase of data collection included three additional two-hour group meetings. The purpose of these sessions was to provide opportunities for collaboration and shared problem-solving experiences. These experiences were essential to emerging individual reflective inquiry, group dynamics, approaches to problem solving, building trust, and collaborative communications.

These group meetings took place during the fourth week of September in my house. During these meetings, we (the students for the most part) negotiated which mathematics problems/projects from a list of suggestive activities to address, how to use the Internet to approach those problems.

As mentioned before, two of the four participating students never had used the Internet, and none of these students had access to a computer at home. Therefore, the students asked for an Internet-workshop at my house, which led to students' preliminary pair and group activities starting the first week of October.

As per the students' request, I conducted two, two-hour workshops on the use of the Internet prior to their preliminary pair and group activities. First, I worked with

Paul and Clint, and then with Jim and Bob. Because Jim had prior experiences with the Internet, he became a mentor to his partner, Bob, and later to the other pair, Paul and Clint. While continuing with the collection of background information, we decided, based on my suggestion, to tackle an egg-drop project. The main purpose for this project was to provide opportunities for team building and collaboration among the pairs. It was also hoped that the initial project would help build trust as the pairs designed and executed an open-ended project. We had negotiated and agreed to use limited materials, such as toothpicks, glue, cotton balls, paper, etc., to complete the project. Students were encouraged to seek help from their parents, neighbors, or high school peers as well as to search the Internet for finding useful information on similar projects. The objective for this particular task was to drop an egg from a significant height without breaking the egg itself.

By observing the exchanges that took place between myself and the participants, paired participants, and the group as a whole, the following categories had emerged from the first phase of the study, which pointed out the importance of group dynamics, individual, pair, or group problem solving approaches, and building trust as the four volunteer participants inquired into these collaborative and team-building efforts.

Group Dynamics. Students then searched the Internet for ideas that others had generated dealing with similar projects, and shared information as a group; they seemed to be forming a common bond and feeling of trust as a larger community (as opposed to a singular dyad). They exchanged ideas about possible materials, and approaches (i.e. using teepees made of straws, developing cushions for the eggs, creating parachutes), and locations for the execution of their projects. They were to conduct a "pilot drop" from the stadium at a university in a near-by town (approximately 35 feet high), and then move on to the University of Oklahoma stadium (a height of 192 feet) for the final drop. The group categorized this project as an open-ended problem/project where there were many possible solutions and many different ways to construct and execute it. Their group dynamic seemed to be active and productive. Following my suggestion, they all agreed to design and develop two different kind of egg-drop projects per dyad. Here, Jim and Bob discussed enlisting Jim's father to help them design an airplane-shaped egg-drop device with an engine to be ignited. The pair was establishing a timeline for completion of that project. They were the first dyad to engage in deeper communication about the specifics of their project.

Clint and Paul had expressed caution in collaborative environments, and their discourse reflected this reluctance. Granted, the two had had no familiarity with one another prior to the project (which is not uncommon in a collaborative environment), they did share similar mathematics ability and achievements, similar mathematics teachers and educational beliefs, as well as common social interests; they also belonged to the same church. Yet they seldom operated as a team; the two seemed to function, for the most part, as individuals who presented their solutions, independent of their partner's participation. Their relative lack of a collaborative discourse can be seen in the correlating table at the end of the book (see Tables 5 - 7), which graphs the communicative activity of both pairs through the course of the initial phase of this research study. These three tables exemplify both productive and nonproductive communications and the impact they may have on a collaborative learning environment. Clint and Paul seldom listened to one another. The dyad seldom felt compelled to cooperate with one another to reach a joint goal; this is painfully obvious in Clint's reaction to Paul's second attempt at the final execution of their egg-drop project. Their individuated responses to the collaboration was also apparent in their pronoun usage; each continually discussed his findings as "mine," or what "I" had accomplished. On the other hand, Bob and Jim consistently referred to their project as "ours" or what "we" have accomplished. Jim and Bob were also more inclined to ask for outside help to empower their group process, while the other two (Paul and Clint) relied predominantly on individual efforts. Jim and Bob seemed to develop and nurture a sense of trustworthiness within their dyad, whereas Clint and Paul never seemed to embrace this concept; they communicated to accomplish the tasks, but they never seemed to go beyond superficial communications to exchange meta-messages (Sfard & Kieran, 2001), that is, their individual mathematics understanding and cognition did not seem to change as a result of their communicative exchanges—a notion integral to the structure of collaborative learning.

Problem Solving Approaches. On October 3rd, 1997, Clint, Paul, Jim, and Bob conducted a pilot egg-drop project at a local regional university. Both projects consisted of a parachute model of some sort, and despite the high wind, their projects were successful and their eggs did not break, although one parachute was damaged; they immediately discussed possible modifications. What is interesting to note at this point is that pair two, Paul and Clint, seemed to realize how their project lacked in some areas, but were unwilling to discuss improvements; yet pair one, Jim and Bob, immediately began discussing possible modifications. Their contrasting approaches to collaborative communication and the impact this had on completing this project were quite evident.

On the afternoon of October 8^{th}, Clint, Paul, and I headed toward the University of Oklahoma football stadium to test their egg-drop project from a height of 192 feet. The two attempted their project (the same parachute projects) several times that evening, yet each time, there seemed to be some unexpected obstacle prohibiting them from becoming successful. Although they both participated in the project and seemed equally committed to its successful completion, the two had limited conversations.

The following week, Jim and Bob tested their first project, which consisted of a rocket engine in a carrying case and was unsuccessful. Their second attempt (a small teepee of sorts) was successful. The two discussed how to improve their results; again, these two communicated very effectively. In fact, it seemed as if their communicative exchanges were evolving to a certain point; they were able to exchange ideas using a shorthand of sorts because they could almost anticipate the other's thoughts in that community.

Each participant was asked to reflect on the problem, their initial approaches, and how those techniques fared in execution. After reflection and re-evaluation, the boys were then ready to refine their designs. This was the final stage of this project; the boys were asked to evaluate their projects and make any necessary modifications so that they could test their designs one last time. Jim and Bob had planned two different egg-drop projects, and although they worked together on both of their projects, each was mainly responsible for administrating and executing one project, both of which were successful. They were very familiar with each other's projects and had collaborated in their construction. The other pair, Clint and Paul, worked individually on their own project and had very little knowledge of each other's work; they seemed to be acting as individuals not as a collaborative pair. Although they were aware of the other's project, they seemed uninterested or in competition with one another. In fact, during one attempt, Clint broke Paul's egg as a "joke," an act which seemed almost hostile and indicated individuated approaches and goals and a lack of trust.

Bob and Jim, however, were very involved with each other's projects and Jim stated that they had asked a friend from geometry class because "a third set of hands was helpful." Bob said they consulted with a neighbor (who was a pilot) in order to make the bottle aerodynamic by adding wings and a tail fin; he believed this would reduce the force of the landing and thereby protect the egg. These two were communicating within their group, but were also involving the community in which they lived. It seemed that their collaborative effort produced more successful attempts than did their counterparts. For a brief overview of the results of these projects, see Table 7.

This initial phase provided the groundwork by which the remainder of the study was conducted. After this initial project was completed, the dyads then moved on to other types of problems, all of which were approached within a supportive, collaborative environment.

Students' Engagement on Problem Type and Problem Solving Discourse

In phase two of this research each participant and dyad were given a variety of nonroutine problems from which they chose and then worked collaboratively (see Appendix E). Again, their individual and group progression were evaluated in terms of inquiry, communication, technological support, problem solving development, and mathematics understanding.

During the second phase of the study, the researcher (1) observed and videotaped student dyads during their mathematics problem solving engagement (2) followed each dyad videotaped engagement with pair interviews (3) met with all participants once every two weeks for approximately two hours to discuss students' reflections on their problem solving engagement, problem type, and mathematics discourse and (4) negotiated meeting times (for example, whether or not to meet during school holidays such as Thanksgiving, Christmas, and etc.). A final individual exit-interview occurred with each participating partner at the concluding session of the study. The purposes of this interview were to (1) help the researcher to develop sound understandings of student dyads, by allowing the students to re-examine the process and justify their thoughts (2) provide opportunities for students to re-examine their own beliefs with respect to mathematics inquiry, problem type, and mathematics understanding by reflecting on their problem solving interactions and shared experiences. Videotaped problem solving sessions were available for students' review and were used as prompts for follow-up interview sessions.

The goals for implementing the second phase of this study were to (1) find what mathematics problems/projects students found interesting and problematic (2) explore the relationship between student engagement and problem type (3) identify emerging patterns with respect to student dyads during their mathematics problem solving engagement and (4) see how problem solving discourse evolve as students participated in a collaborative problem solving environment.

During the remaining course of the study, the participants were able to choose the problems that they would solve. These problems were either provided by me or extracted from various Internet sites (see Appendices E & F). From these sources, the students could then choose the problems that their dyad would address. The types of problems that were provided fell into one of the following four categories:

1. Routine problems or traditional algebra, geometry, or other formulaic problems that resembled the traditional problems found in most mathematics textbooks (in other words, these problems had relatively uniform approaches, as well as fixed solutions). Similarly, the boys engaged with one another and took an online IQ test (first as an individual then as a dyad), which is consistent with this type of routine problem solving.

2. Nonroutine problems that involved mathematical concepts such as fractions and ratios, but could be approached in several different ways in order to reach the same solution. These types of problems allowed the participants to modify the conditions on which a word, ratio, or fraction problem was based in order to reach a solution. Such an approach allowed them to develop varying methodologies to relatively traditional situations.

3. Open-ended problems were also provided for them, even though none of the participants chose to attempt one. Open-ended problems involved theory, logic, and/or experimental methodologies. One example of this type of problem would be a question such as "How much money does it take to make more money?"—a question which, as it is labeled, is open-ended in terms of approaches and possible solutions.

4. Projects that encouraged hands-on activity were also provided. The students eagerly embraced these types of problems, which involved trips to local educational facilities, online "treasure-hunting," and web-page designs.

Table 8 illustrates a summary of these four types of problems.

With these four problem types in mind, their interaction will be presented in terms of its type, the relationship between that type and student engagement, patterns that emerged through the dyad's prolonged mathematics problem solving engagement, evolving discourse through problem solving, and, finally, the production of meta-messages from this discourse through synergetic perturbations, argumentation, and on-going communications.

Data from transcriptions of videotaped and audiotaped discussions during problem solving, dyad interviews, and group problem-solving dialogue meetings, as well as students' verbal and written responses to questionnaires and problem-solving journal documentation and reflections, were analyzed using a constant comparative method (Guba & Lincoln, 1985, 1994). The emerging categories from the multiple data sources were analyzed by comparing student engagement, problem type, use of technology, and mathematics understanding as well as dynamics within the group discourse.

The problems, from which the boys chose, were provided by the researcher (see Appendices E & F). The focal analysis (Sfard & Kieran, 2001) was on the mathematics content within the problem type. The boys' exchanges and the ways in which they interacted with one another within their dyads then served as the preoccupational analysis—how they communicated with self and with their partner in problem solving and engaging with particular types of problems (Sfard & Kieran, 2001). Focal and preoccupational analyses are especially helpful when evaluating the effectiveness of collaborative communication and engagement. Thus, focal analysis was organized into the following categories:

1. **Problem Type**, or the categorization of problems by their type.
2. **Relationships between student engagement and problem type**, or the exploration of the possible relationship between the type of problem the dyads addressed and their level of energy, if any, as well as their level of trust, collaboration and discussion, creativity, productivity (or lack thereof), and etc.

After such possible relationships were identified, their preoccupational analysis was then examined within the structure of the focal analysis. This preoccupational analysis was organized and analyzed using the following categories:

1. **Evolving problem solving patterns through prolonged engagement.** Here, I focused on what the data revealed to me in terms

of students' mathematics problem solving approaches, use of technology, and their selection of problem types.

2. **Evolving Discourse Through Problem Solving.** Using a more analytical approach, the focus here was on the evolution of the discourse as the volunteer participants engaged in problem solving.

As noted before, such an organizational mathematical and discursive analysis is significantly linked to the type of problem utilized in problem solving. For this reason, it warrants further exploration.

Routine Problems. As stated before, these problems were generated from their own Internet search to which the links were provided. These selected routine problems were predominately at an eighth or ninth graded mathematics level (see Appendix F for a complete view of the links to these problems); they were consistent with problems that appeared in their textbooks at school (i.e. Pythagorean theorem, right triangles, simple linear equations, etc.), in that they called for the utilization of formulas to solve an unknown. In that manner, these problems were very consistent with traditional belief systems of viewing what mathematics may be and how students should approach solving these problems and, therefore, were very familiar to each participant.

It was important to note that the very fact that each participant used the Internet to inquire problems indicated an exploratory use of technology on their part. The majority of the participants expressed a hesitance about using computers and technology in the classroom, but each of them seemed intrigued with the notion of finding their own problems using the Internet; a concept that will be revisited in chapter five. However, when dealing with routine problems, their collective interest seemed to die once the Internet searching and problem collection was finished

Once the problems were gathered from the Internet, each participant tackled to find their answers, in a very traditional, linear fashion, moving very quickly on to the next problem; there seemed to be no challenge involved. One example of routine problems was the Die Problem, where the solver was asked to determine how many times the number two is expected to "come up" if a die is rolled 18 times. (*The die is rolled 18 times, how many times is the number 2 expected to come up? (Adapted from the Internet – NASA Page* http:\\www.nasa.gov*)*

Clint and Paul first approached this problem, and Paul did most of the talking, experimenting with rolling a die, and recording. He hypothesized that equal probability was independent of events, and then experimented with the die by rolling it 18 times to confirm this notion, but Clint seemed not interested to collaborate with Paul. He was simply watching his partner and seemed to be bored by the process. Their interaction was short and not productive, which is evident in the following exchange:

Paul: Clint, do you think the answer is 2 times, 3 times, 6 times, or 9 times?

Clint: I say number 2 will come up 2 times.

Paul: No, I say number 2 will come up 3 times.

Clint: Why?

Paul: Because the probability of rolling any number on the die is 1 out of 6. You then multiply that by 18 and reduce it by 6 and you get 3.

Clint: So 1 in every 6 sides multiply by 2.

Paul: Why multiply by 2? It asks how many times the number 2 comes up, you wouldn't multiply it by 2. Just don't worry about that number, it could be any number, it could be 5. It is the number that you want to come up.

Clint: I guess it's 3 times.

Paul: Do you see why?

Clint: Yeah.

Paul: We decided that the answer is 3 times on this question (speaking to the researcher).

Researcher: Clint, do you agree with Paul on this?

Clint: Well, he worked on this before at home so he already knows the answer.

Paul: I am just trying to show another way to do this. I was just trying to show you how I figured it out at home.

Researcher: How did you figure it Paul?

Paul: I took 18 times and the probability of it is...If you roll it six times you're saying each number will come up once and that means the probability to be 1 out of 6 and then multiply it by 18 which is 18/6. Then when you divide 18 by 6, it is 3 and so the answer is 3.

Clint: I understand.

This communication contrasted slightly with Jim and Bob, who, in turn, seemed to pick problems that were already solved and had first been explained by the other pair, or more particularly, by Paul. This approach was evident in their choice of the order of routine problems they solved, which mimicked the sequence of the preceding dyad. In short, Bob and Jim decided to solve the same routine problems that Paul and Clint had solved earlier that day. First, Jim read the problem aloud and quickly went on to solve the problem alone. I had to remind Jim to wait on his partner, Bob, whom I then asked to read the problem aloud to ensure that he understood the question.

Jim: If a die is rolled 18 times, how many times is the number 2 expected to come up?

Bob: A die is rolled 18 times, how many times is the number 2 expected to come up?

Jim: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,16, (background talking) 17, and 18.

Researcher: How many times the number 2 came up Jim?

Jim: 5 out of 18. 1, 2, 3, 4, 5, and 6. 6 out of 18.

This dialogue accurately illustrates the nature of their communicative experience with routine problems; Jim experimented rolling a die and recording the results, while Bob passively observed what Jim was doing. It seemed the two were rather bored by simple problems, and their interests waned. They did collaborate on an IQ test later that day, which they found on the Internet. First they took it individually and then collaboratively. They found out that they had better results working together as peers than either would have working individually (a conclusion at which Paul and Clint also arrived); this shared opinion showed a significant change in their assumptions about collaboration. Moreover, the two seemed much more excited with problem solving when they used the Internet to generate problems. In fact, Jim and Bob later collaborated on building a web page, a project the two both said they would not have attempted at the beginning of the study. Next, Bob and Jim worked on another routine problem, a surface area problem, which they downloaded from the Internet and read.

What is the surface area of a rectangle solid, when the length is 6 centimeters, the dept is 5 centimeters, and the height is 4 centimeters. (NASA Page http:\\www.nasa.gov*)*

Bob and Jim began to talk among each other very quietly, and I inquired what they were planning. Jim then offered:

Jim: Finding the area of each side and adding them all together.

Researcher: Bob, what are you doing there?

Bob: I'm surfing the Internet to look for a formula.

Researcher: What kind of formula? Is it for volume or area?

Bob: Surface area.

While Jim initially consulted Internet sources for aid (a fact which may be indicative of his prior online experiences), Bob tackled the problem solving in a much more traditional manner by attempting to find a formula on the Internet; yet they both seemed to agree that such an approach, as well as a delegation of tasks, was helpful and expedient.

Clint and Paul's approach to the same problem differed greatly, which was evident both immediately and later, when the dyad attempted to explain their methodologies to me that day. Their body language immediately indicated a communicative and collaborative problem. For example, while Clint was passively listening to Paul's explanations, his body movements (such as sighing and exaggerated repositioning) exemplified his severe boredom with the problem. At the same time, Paul read the problem aloud and tried to explain to me his already constructed solution to this problem.

Paul: I drew a rectangular prism. I drew it on this paper. See what you have to do is, all of these are rectangles and so you have the length is 6 centimeters so each side is 6. Do you see Clint?

Clint: I know. Surfaces are 148 centimeter square.

Researcher: Okay, how are you finding the surface area on that shape?

Paul: We added up the area of each side like the area of this side plus this side … plus this side. We took the area of each side and add them together.

Like Clint and Paul, Bob and Jim had very little discussion on these types of problems, and showed little interest in engaging in dialogue and collaboration while working on routine problems; in turn, their disinterest summarily ceased dyadic communication. Consequently, they quickly moved on to the next problem, which was another surface area problem and found from the same NASA-Page on the Internet. The pair independently used a formula (e.g. Area = $\pi r^2 + \pi rs$) to solve the problem without much discussion or elaboration. These problems were very similar to their textbook problems they were currently experiencing at school, and seemed to require no creative effort in their solution findings, which is evident from the following:

Researcher: Could you now explain to me how you solved it?

Jim: All we did was plug the numbers into the formula, side height is 6 and radius is 3 put pie in for that and you get the area and you get 7/8 square inches.

While working on routine problems, for the most part, none of the participants seemed challenged to think creatively or to create new mathematics. They associated answer finding with "doing" mathematics, and their engagement was short-lived and dull. They did not seem to comprehend that mathematics learning is more than answer finding.

When the dyads solved routine problems, the level of engagement was nil—a fact that was especially true with Clint and Paul—a pair who had failed to build a community as of yet. Paul seemed to know the answer already because he was more familiar with formulaic computation, and therefore, he did not engage in productive collaboration with Clint. Similarly, Bob and Jim were both familiar with these types of computations and therefore did not expend much energy or focused attentions in formulaic problem solving.

From this lack of collaboration, it seems safe to assume that routine problem solving may in fact rob students of opportunities to engage in critical thinking and negotiation. In short, there seemed to have been no synergy involved with these types of mathematical education, which is one of the main reasons reform-minded mathematics educators want to create a supportive environment by utilizing nonroutine problems. As discussed in Chapter Two, research indicates that a reform-based curriculum that engage students creates a richer atmosphere—one that invites students to pay attention and investigate solution-findings (or answers that are not immediately known such as those created by Pythagorean theorem, etc.). Nonroutine problems are much more multi-faceted, and by their nature have the potential to provide meaningful background that may involve association and communication between two or more students in the process of their solution finding. Therefore, the very type of problems used in mathematics curriculum may be relevant to the level of generative collaboration and communication that may lead to deeper conceptual understanding experienced by the students.

Nonroutine problems. Although the participants chose several nonroutine problems to solve collaboratively, the following two examples (the "marble problem," and the "fraction problem") will be discussed at length in this session and involved ample opportunities for students' collaboration, perturbation, and argumentation.

On Friday, November 21, Paul and Clint chose to solve a nonroutine problem (provided by the researcher) dealing with beads (see Appendix E). Their interests in this problem seemed to stem from the fact that it was nonroutine, or open-ended. Once the problem was selected, they decided to work with chocolates (red and green Reese's pieces) instead of beads. The original problem read like this:

> *Two jars are placed on the table. One contains 1000 blue beads and the other 500 yellow beads. Crystal took 20 beads out of the blue bead jar and put them into the yellow bead jar. After shaking that jar until the yellow and blue beads were thoroughly mixed, she randomly selected 20 beads from the mixed jar and put them into the jar of blue beads. After completing the task she asked were there more blue beads in the yellow bead jar than there were yellow beads in the blue bead jar? (Adapting from Maher & Alston, 1990)*

Clint and Paul decided to modify this problem to a more manageable size using chocolate Reese's, and I videotaped their problem solving process. After reading the problem a few times, the pair came up with this modified problem, or a new focal point:

> *We have two jars one of them has 100 red chocolate and the other one has 50 green chocolate. We're going to take out 20 of the green and put it into the red and shake them up randomly and we're going to take 20 of red and put it into green, we're try to figure it out which one have more that red in green or the green in the red so now we want to get started.*

To initiate the problem solving process, Clint took 20 green Reese's and put them inside the jar with the red Reese's, shook the mixed jar, and asked Paul to take 20 Reese's from the mixed jar and put them back into the jar of green Reese's. At first, the two communicated quite well with one another as they discussed possible approaches, questioned each other's theories, and exchanged ideas, which is seen in the following dialogue:

Paul: We need to figure out whether jar of red has more green in it or jar of green has more red in it.

Clint: Okay, first we started with 50 green and then took out 20.

Paul: So … that means we have 30 left.

Clint: 30 left, but the 20 is the 2/5 of that [of the 50].

Paul: Why would you say that? There are half as many greens as red, so …. we're trying to figure it out. Okay, now, how am I going to figure it out?

As the dyad continued working this problem, the level of collaboration seemed to wane a bit. Paul began talking to himself, as if he was working alone on this problem. It was quite evident that Paul did not view working with Clint to be valuable to him. In fact it appeared to me as if Paul preferred, at this time, to work alone and did not wish to exchange with Clint at all, which is illustrated by the following dialogic example:

Clint: Let's say this is jar one. Do you have the marker Paul?

Paul: I got it! [Shouting]

Paul acted as if he did not hear a word of what Clint had said to him. At this point, the dialogue seemed to end. Paul was excited about his result, but Clint acted indifferent and continued to draw pictures on a piece of paper using his marker.

When a dialogue resumed, Paul had taken on the role of teacher, often correcting Clint's methodologies and logics. This effectively silenced Clint, who began to concentrate on developing an algebraic formula to solve the problem while Paul made notations in his notebook, searching for a probability to solve the problem. Clint's body language indicated that he was bored working with Paul and lacked energy.

Paul then concluded that there would always be equal numbers of the opposite colors, but could not explain his findings to Clint, which exemplified his tacit understanding because he was unable to explain symbolism and representation to the researcher. Clint arrived at the same conclusion, but the two never seemed to "get" how the other had come to their solutions. There was effective communication between the two (they attempted to explain reasoning, challenge solutions, and extend understandings), but this informational exchange and perturbation failed to change their individual cognition or to involve meta-communication/messages.

The following day, Jim and Bob tackled the same problem, yet the pair was more interested in spending time on the Internet than doing anything else, which illustrated a growing desire to use technology in generating problems for their focal analyses. Eventually, the two began to work together on the "marble problem" (which was suggested by the researcher). They read the same problem and decided to use 150 Reese's chocolate candies, taking two out and putting two back in, instead of using 1500 beads.

After I suggested they repeat the process using 20 beads for exchange instead of only two, the pair then illustrated this process one more time so that I could understand their reasoning. They repeated the process and came to this conclusion:

"There's 13 red in the green jar and 13 green in the red jar, they came out evenly," Bob said. I asked them to explain their findings to me. "It depends how you pick them," Bob offered. "If you pick more green or red and mix them with another colored beads, then chances are that you are going to have more green beads than red or red than green because there's more green picked from after it's mixed."

They replaced the red/green Reese's, and tried the problem again. This time, Bob reversed the process by taking 10 red out of the green jar and mixing them; he then took the same amount out of the mixed jar and put them in the green jar. Bob asked Jim if there were more red in the green jar than there were green in the red jar, and Jim replied, "There's 7 red in here and 7 green in there." Jim separated the beads (Reese's) once again, read the problem aloud, and then suggested to Bob that they use 30 of each this time. Bob agreed, and after going through the process, Jim asked Bob how many red Reese's he had in the mixed jar.

Bob: I've got 19. How many did you take out from here?

Jim: 30.

Bob: 19 and 19 is more than 30! Somebody messed it up!

Bob suggested that, somehow, a mistake had been made by Jim while counting the Reese's. Jim shook his head in disagreement while smiling and uttering "Uh-Uhhh," suggesting that Bob was incorrect in his judgment of him and did not quite know what he was saying.

Bob perceived the marble problem as some sort of addition problem. This was evident when he added 19 and 19, the sum of which he knew was more than 30. Jim and Bob still seemed puzzled by the results of their experimentations with the

Reese's and tried several more times, repeating their experiments by drawing various numbers of Reese's from the jars; they concluded that there would be equal numbers of opposite color Reese's in each jar. The pair, then, tested this theory using drawn circles on a sheet of paper to farther investigate findings.

The two regarded one another with mixed expressions (curiosity, amusement, and confusion), and exchanged very few words to explain their thoughts processes to each other or to me. They finally agreed, with very little explanations, that their theory was correct, but could not explain why; yet I persisted. Bob and Jim reasoned that as long as the ratio of colored beads was 2 to 1, then they were always going to have the same number of beads in different colored jars. However, they were not satisfied with this explanation, and neither was I. Therefore, Jim tried the process one more time and came to the conclusion that it did not matter how many beads he started out with, he would still end up with the same amount of red and green beads at the end. "They are always equal," said Jim. Yet, he could not explain to himself, Bob, or to me why they were always equal.

Likewise, Bob was still not sure how this process worked either. He continued to struggle for understanding about the problem's mechanics by attempting to formulate an equation as a concrete proof solution to the problem. Bob attempted to explain what he was doing, and finally said, "I just don't know. I'm kind of trying to find out the reason why they are always equal. I know that it happens. I know it's always going to happen, but just don't yet know why?"

This relationship between prolonged engagement, conflict, perturbation, and argumentation provided potential opportunities for deeper conceptual understanding for Bob and Jim. The two began drawing with a few numbers of Reese's, and gradually increased these numbers before realizing they always ended up with the same amount of dissimilar candies in each jar. During their "marble problem" solving interactions, it was clear to me that the pair knew the answer to this problem intuitively, but was not able to verbalize their knowing to each other or to someone else. To farther problematize this, I asked Jim if he would care to share his reasoning with Bob, but Jim said he couldn't explain it very well. In turn, Bob still wanted to find an algebraic equation, which confused Jim. I asked Bob what he was doing and why. He offered:

Bob: I'm starting with an equation because it is easier for me to explain my thoughts this way. Okay, X is the red (Reese's) and Y is the green (Reese's). You subtract 10 greens or 10 reds, and add 10 right over here.

Researcher: Can you see that Jim?

Jim: Yeah, I can see it. But it's more confusing to use formula. It's easier for me to just look at the beads (Reese's), switch them around, and look and see what I have here than use an equation.

Bob: This is what you do if you want to solve it by writing it down. I have a ratio here, like Red to Green, and then, I'm gonna subtract 10Xs from here and add to there. So we got X – 10X and Y + 10X. If you got 100 red, now you have 90. 100X – 10X = 90X. Okay, say, take 3X's and 7 Y's from the mixed jar and put them back in here. Now, come back over here: 3X + 90X = 93X; this is my first equation. Then over here, you have 50Y – 7Y will be equal to 43Ys, and plus 7Xs.

Researcher: This is interesting to me Bob. Why did you use this method?

Bob: Just, if you didn't have all those beads (Reese's), this is how you do it on the paper.

Bob took some time to explain to me and to his partner, Jim, how he had arrived to his "equation" by showing and describing his solution strategies using his pen and notebook (a process both Jim and I paid considerable attention). This time, the pair seemed to be very confident about their reasoning because the team had formed an "equation" to back up their theory and knowing.

Bob and Jim also used self-talk and self-thinking as they attempted to solve the problem. Jim tried to formulize his approach at first, while Bob used a more hands-on approach—a process which was later "flipped" as Jim tried to experiment with hands-on and Bob attempted to create a formula. This approach contrasted with Paul and Clint, who never really experimented with formulas. Paul and Clint took less time to come to the same conclusion, but their approaches were not as developed as Jim and Bob's.

As with the second pair, Paul and Clint, their efforts to create or develop meta-communications/messages did not change their cognitions; they merely exchanged ideas. They did not communicate effectively. However, Bob and Jim seemed to have utilized parallel play more; they watched one another playing with the problem, paid attention, and listened to one another's explanations while extending their own solution approaches and strategies. This community seemed to develop more meta-messages, which in turn seemed to help their own individual and collective cognitions.

For the most part all participants agreed that the nature of this problem (marble problem) was nonroutine and challenging because it called for prolonged problem solving instead of limited problem solving, such as test taking or lecturing in traditional classrooms that are limited to 45-50 minute sessions (an issue which will also be addressed in the next chapter), there were many possible ways to solve the problem, and solutions to the posed problem were not immediately known to the solver(s). The dyads' nonroutine problem solving interactions resumed in February of the following year, after a long but needed Thanksgivings and Christmas breaks.

On February 5th, 1998, Paul and Clint worked on problems, which I provided for them to solve (see Appendix E). I videotaped their problem solving interactions. The two volunteer participants seemed interested and excited in resuming their problem solving efforts at my house. Their fraction problem was worded as such:

What proper fraction exceeds it's square by the greatest possible amount? (Adapted from Mathematics Teacher, September 1995, volume 88, #6.)

After reading the problem independently and silently first, Clint read the problem aloud to the pair, and then each peer, begun to solve the problem separately using a trail method. Paul and Clint each used a pen, a notebook, and a calculator to experiment with the problem and its possible solution(s). They seemed to have forgotten the meaning of a "proper" fraction. This was evident in Clint's first trail attempt to tackle with the number 5/3 as a starting fraction, which Paul objected to his assumption and quickly questioned the researcher about the very nature of a proper fraction (where the numerator is smaller than the denominator). They decided that 5/3 is an improper fraction. Their following exchanges clearly exemplify how their conversational interactions have begun to impact their cognitions. This became evident as Paul began his "self-aloud-talk" engagement with the problem:

Paul: Okay! What is the largest fraction that you can have that exceeds it's square by the largest amount?

Researcher: Is it any fraction Paul?

Paul: No. It has to be a proper fraction.

Researcher: Does it have to be the largest proper fraction Paul?

Paul: No, any fraction that you want it to be. As long as when you square it or multiple it by itself, it is larger than its product. For example, .1, that is 1/10, then when you square it 1/100 then you'r saying what is the most that you can have between fraction and its square?

Clint: It's the difference between the original fraction and what you get when you square it. It's going to be smaller, the larger the fraction is. Like 1/2 it's going to have a small difference.

Paul: Not necessarily!

Clint: Uh-hu, because if it comes out 1/4 it's just 1/36. The difference is larger the smaller the number is. So, the way you would find that is, find the smallest fraction you could get because, ... look, this is a greater fraction.

Paul: Not necessary! Look at .5! When you square it, (while talking to Clint, Paul is using his calculator to demonstrate his thinking at the same time), there is .25. That is a larger distance.

Clint: The largest proper fraction I could think of is 1/2.

Paul: No it isn't. You can have 2/3, 5/9, or 3/5. You could even have 3/3, which is 1.

Clint: No! Not 1! That's not even a fraction!

Paul: Yes it is! How about 1/4?

Researcher: Do you agree that all proper fractions once being squared are smaller than their original proper fraction? Now the question asks what proper fraction exceeds its square by the greatest possible amount.

Clint: Yeah. The answer is the largest proper fraction.

Paul: No it isn't! Because .9 is like .90, which is 90/100 but with .5 its 5/10.

Researcher: As you are checking these proper fractions, you may want to record your findings and look for a pattern.

Paul: I was doing that with .9 and .8 and so on ... .

Clint: It all came out in your head it is hard to explain how it comes out 9/10.

Researcher: Clint, do you think there is no answer for this problem?

Clint: I think the answer is 9/10, but still that wouldn't work because it would have a very small difference. The larger the proper fraction is, after squaring it the smaller the difference is, but we're looking for the greatest difference between a proper fraction and it's square.

Paul: I'm trying to see if there is a pattern or anything. I think "it" (the largest proper fraction) is .5 or 1/2 and the difference is .25 or 25/100, which is 1/4. Because hum ... when I squared 9/10, I got 81/100 and the difference was 9/100. I squared 8/10, I got 64/100 and the difference was 16/100. I squared .7, gave me the difference of 21/100, 6/10, gave me the difference of 24/100, and then I squared 5/10 which is 1/2, it gave me the difference of 25/100 that is the largest. Then I went .4 square and saw it's difference, which was .24 or 24/100. I tried .3 squared, the difference was .21 or 21/100, ... So, I found a pattern and the answer is .5 or 1/2.

Clint: I can't find a pattern.

Researcher: You can't find a pattern?

Paul: Oh yes, you can! Because when you take 4/10, for example, and square it it's .16 and find the difference subtract .4 and take off the negative because its the same answer its .24 its 24/100 then try other proper fractions. Okay, you increase like 9, 16, 21, 24, 25 then it goes down by the same distance as it was going up. It goes down by 25, 24, 21, 16, 9. So, the greatest possible amount is .25 or 25/100, which belongs to the proper fraction of .5 or 1/2. That's what I concluded: the proper fraction that exceeds by the greatest amount is 1/2 because it has the greatest distance between the two numbers when you square it.

Researcher: Paul, would you explain that with Clint?

Clint: It's hard to explain isn't it Paul?

Paul: Yeah, it is.

Clint: I understand it.

Paul: When you square 9/10, the difference is not very much only 9/100 and 8/10 is 16/100 and 7/10 is ... **wow I just saw something here ...** 9*1; 8*2; 7*3; 6*4; 5*5; 4*6; 3*7; 2*8; and 1*9. That's how it is going up and down. It's going up by 9, 16, 21, 24, and 25 is the highest it goes. Then it goes down by 24, 21, 16, and 9. Do you see Clint? When it reaches the middle it is at its peak. Therefore it is 1/2.

The two partners posed questions to one another, clarified definitions, tested cases by trial and error, organized data, and experimented with calculators and argumentation. They also engaged in parallel play and attempted to help each other recognize patterns. Such an exchange showed a definitive progression in their communication. This was apparent later when they tackled a logic problem, or the salary problem. The two partners discussed various approaches to solve the problem and both seemed to had benefited from their collaborative problem solving interactions.

Project-Based Problems. The "project" portion of the study involved a geometric scavenger hunt of sorts at the Omniplex, the egg-drop project, web design, and an Internet treasure hunt. The egg-drop project has already been discussed in detail and is only addressed at this point to make the following observations in terms of discourse evolution and problem solving patterns: During the initial phase, Paul dominated his dyad, and set the pace for their initial planning; he wanted his pair to experiment with Paintbrush and created a preliminary plan before searching the Internet for possible approaches, which contrasted slightly with the other pair. Perhaps because of his prior Internet experience, Jim opted instead to search the Internet and then used Paintbrush to draw a plan. From that point, the differences are much more obvious. Bob and Jim worked together on each project and paid attention to one another during each stage of development (making sure not to alienate the other during discussion or planning), while Clint and Paul were much more individuated. As mentioned before, there was almost a hint of hostility between the two. This may have been due to the fact that Paul's project was much more "polished," and Clint seemed to be jealous of this fact. Similarly, Paul designed several egg-drop executions, while Clint only designed one or two, which showed a difference in their level of engagement.

Even though each dyad approached this project differently, one thing was consistent. Both pairs were deeply involved with the project, and experienced a high level of engagement and interest. In fact, during the remainder of the study, they often asked if they could complete a similar project, although they did enjoy the treasure hunt at the Omniplex, where again, the two dyads approached their goal in different manners, although both were highly engaged in the activity.

Both pairs were given hints about the identity of a secret object within the complex, and were then asked to find that object (i.e. Bob and Jim were asked to find something that was brown and stood next to a fence, which turned out to be an elephant, and Clint and Paul were asked to find an object that glinted in the sun, which turned out to be a trashcan). Both pairs employed a notebook for initial thoughts, discussed possible locations with one another, and both were able to find their secret object relatively quickly. Again, Jim and Bob communicated well with one another, but seemed to employ non-verbal communication more so than their counterparts. Additionally, at this level, they seemed to experience more effective communication, but this may be due to the fact (again) of their pre-existing relationship.

Before the dyads split up to engage in an Internet treasure hunt, the group as a whole engaged in an Internet-inquiry into nonroutine problems. Interestingly, as a group, their productivity lagged. For example, Clint passively listened to Paul and nodded his head often (instead of offering verbal communication), while Paul again assumed the role of a teacher as he attempted to explain possible solutions to problems that were found on the Internet. One example of a problem they addressed was another probability problem, which asked the number of times a coin would land either heads or tales.

Paul actively experimented with the problem, actually flipping a coin in hopes of discovering a pattern. Yet Clint seemed bored and his level of engagement was nonexistent. Similarly, Jim was completely focused on searching the Internet for other problems instead of addressing the task at hand with his partner, Bob. He was focused on the computer, not the group discussion, as lacking as it was. Only Paul and Bob attempted to solve problems, independent of their partners, using trial and error method instead of a formulaic approach (which may indicate a lack of statistical knowledge, but a propensity for creative problem solving—reinventing statistics (Kamii, 2000)).

This episode may have been due to the fact that the group had already engaged in previous problem solving that day and had simply lost their enthusiasm for the matter, or it may be indicative of a larger issue. For example, Paul suggested that he was unable to focus in a larger group setting because he could not help but overhear other conversations and solution methods (some of which were incorrect). This suggestion is reminiscent of Sfard and Kieren's (2001) findings that a group setting is not always the most beneficial for students because peer-interactions may actually interfere with an individual's self-talk and the development of potential cognition;

that is, he or she may lose a "golden moment" of cognition as a peer attempts to converse while he or she is paying "attention" and is engaged in conversing with self. The other participants agreed that this might be true at times, but that peer-interaction was often helpful in that it allowed them to hear other approaches which had potential to perturb them to re-evaluate their own approaches and methodologies to problem solving and communication.

Interestingly, the project that followed this interaction, the Internet-search among dyads, was perhaps the most productive engagement for Clint and Paul. Here, the dyad searched for other problems that they would want to solve, and then attempted to solve various nonroutine problems as a pair. In this process of working on a common goal, the two sat close together, used a calculator, and equally engaged in conversation. They even shared the same piece of paper as they experimented with problem solving; both engaged in self-talk and dialogue. They seemed to have begun to build a community.

As Clint and Paul addressed one problem, which dealt with the center of gravity, each took turns drafting a graph for discussion, and each listened to the other as he explained his approaches to resolving the problem. The level of trust was evident at this point, and the two not only actively engaged in problem solving, but effectively deepened the other's cognitive understanding by their meaningful exchanges. As Clint gained self-esteem and self-respect, his communication with Paul began to become much more productive, as did his level of conceptual understanding on the problem and its solution. In fact, during a later project, it was he, not Paul, who solved a problem correctly and was able to verbalize these solutions to his partner. In short, he too was experiencing meta-communication/messages at the conclusion of the study.

The fact that Clint and Paul experienced productive communication and deepened conceptual understanding in the later stages of the study may be indicative of their heterogeneous mathematics levels. There was definitely a larger gap between their individual mathematics ability than there was between Jim and Bob—a fact that may have played an important role in their community building. Research does indicate that a closer level of cognitive ability is helpful in collaborative environments. Specifically, Leiken and Zaslavsky (1997) warned reform-based educators that heterogeneous collaborations failed to engage lower-end students and forced the more cognitively developed student to assume the role of a teacher. This was the case earlier in the study (i.e. when Clint expressed hostility to Paul's more sophisticated egg-drop projects and when the dyad engaged in problem solving interactions). But, Paul and Clint overcame this and did in fact connect with one another and affectively impacted one another's ability through their prolonged interactions.

Open-ended Problems. Each dyad was presented with several open-ended problems to which they did not attempt to solve. These problems (See appendix E) were different from any other problems they had seen before. In that, the nature of problems was open-ended. Their solutions required creativity and risks: issues that are seldom, if ever, catered in traditional mathematics education classrooms. Consequently, the pairs talked about these problems, but since they lacked having clear visions or ideas on how to begin or precede their solution strategies, they stopped working on them immediately. This may be indicative of their lack of experiences on such problems, an element which will be addressed in the following chapter. At this point, however, an entry/exit comparison combined with a brief summary of the nature of this study might provide some insight into the dyads' cognitive development, motivation, collaboration, and communication. For that reason, a discussion of the four volunteer participants' exit-interviews which took place at the end of this research study may be helpful.

Summary

While reflecting on this research in terms of the nature of the study, it is possible to see an evolution among the participants. For the most part, when they were given the opportunity to select their own problems and approach them from their own perspectives, the act of problem solving became more interesting to them and incited more meaningful communication. Bob and Jim displayed a much more "connected" environment within their dyad, and exhibited greater cognitive development early on. Paul and Clint's interactions may have been more indicative of the typical classroom where many students may be unfamiliar with one another and may have to work harder to establish a trustworthy community. Nevertheless, connections were made between Clint and Paul and they both seemed to have benefited from this.

There were several moments when Jim was able to "see" better because of Bob's explanations, and the very act of explaining proved to be beneficial to Bob. This was true with Clint and Paul as well, especially, toward the latter part of their collaborative problem solving experiences. Clint and Paul's willingness to engage in parallel play with the problems and attempt to help each other recognize patterns through conversations and exchanges showed a progression in their communications, even though their communications remained preoccupational, focusing on self-talk and personal understanding. The pair did not seem to come to meta-message communications during their problem solving processes; that is, while both individually seemed to use communication to further their understanding, the pair did not experience the synergy of collaboration.

Engaging and playing with nonroutine problems afforded both pairs expanded levels of energy. This is especially evident with the "marble problem." They were forced to depend on further investigation and experimentation which were seen when they played with the problem several times—they checked their solutions, modified their approaches, and attempted new solution methods. The notion of perturbation and argumentation is exemplified by these exchanges.

For example, early on, Clint seemed completely disengaged and disinterested; he was much more interested in checking his e-mail or searching for other problems on the Internet than he was in working with Paul. In turn, Paul also seemed individuated in terms of these problems. He often sought help at home from his parents before arriving at the sessions, and would therefore have several of the problems already worked out before ever conferring with his partner. This is evident in many sessions when he flipped back and forth in his notebook, looking for his finished solutions. So for Paul and Clint, the focal points (engaging with mathematics) were individuated. Paul would attempt to explain his answers to Clint, and Clint was uninterested. There were several times when Clint said that he understood a problem when in fact he did not. This pattern seemed to change by the end of the study; they communicated more effectively and seemed more "connected" when they tackled problems or projects, listening to each other and focusing on mathematical processes.

Even though Bob and Jim seemed to engage more in self-talk from the very beginning, their communications became more productive (Sfard, 2001) as the pairs experienced more prolonged collaborative problem solving. This was especially apparent at the end of the study, when the dyads presented problems they found from the Internet as they searched for "treasures." Their ownership of and interest in problems was evident in their conversations about them.

Exit interviews with all participants revealed a change in the way each participant viewed collaborative learning, technological educational tools, personal educational responsibility, and mathematics inquiry and problem solving. Tables Nine, Ten, Eleven, and Twelve summarize the changes in various technical, pedagogical, mathematical, and collaborative beliefs the students responded during their exit interview and compare exit interview data with entry data along these dimensions (see the end of the book). Problem solving collaboration and prolonged engagement seemed to benefit all students both mathematically and in their willingness to learn from each other.

In the next chapter, I will discuss the conclusions of the study. I will also discuss the limitations of this research study and related questions for further potential research explorations.

CHAPTER V

ANALYSIS AND DISCUSSION

In this research study I took a close look at two pairs of ninth-grade geometry students' problem posing, problem solving, and dyad interactions with types of problems including routine, nonroutine, or open-ended problems/projects. I also examined their discourse as each dyad participated in a sixteen-week-long collaborative problem solving environment. Prolonged research observations, data collection, and data analyses were continually utilized throughout the course of the study in light of two research questions as follows:

1. What is the relationship between student engagement and problem type?
2. How does problem solving discourse evolve as students participate in a collaborative problem solving environment?

The findings to this research inform a mathematics community interested in incorporating nonroutine problems/projects into current mathematics instruction. The results suggest that as students developed a culture within their dyads, became involved in problem solving and problem posing, and collaboration, their collective and individual engagement were increased. After evaluating the various data relating to problem type and participant engagement, it became evident that certain problem types engaged the students more than the others. While it was no surprise that routine problems were not engaging to them, it was interesting to find that their collaboration and discourse were similarly affected by these mathematics circumstances. The participants were much less likely to question, challenge, argue, negotiate, or probe each others' thinking, and were much more likely to rely on and accept the first answer at which they arrived. There were no efforts to modify, extend, or apply these routine problems to other contexts. This outcome proves by negation the point Kilpatrick and Silver (2000) made when they concluded that a positive mathematics experience will enable students to apply methodology and knowledge outside the classroom. In this case study, routine problems failed to engage the dyads' cognitive initiative outside a traditional environment and failed to keep them interested in the process of finding a solution. Similarly, these results affirm the thesis presented by Henningsen and Stein (1997): that students decline into procedural thinking when approaching a routine problem that fails to engage creative problem solving.

After examining the "big picture," it became evident that over the course of this 16-week period, significant transitional moments existed, during which collaborations among the dyads and the group seemed to change, and the quality of discourse improved for both groups. While not directly related to any specific problem type or context, these transitional moments seemed to be related to on-going negotiations and relationships, my role as a teacher/facilitator in the development of the dyads' effective listening (Davis, 1997), and to their beliefs and mathematical understandings. Prolonged problem solving and on-going negotiations and collaborations seemed to be related to students' experiences with productive interactions, shared authorities, and meaningful discourse as well as developing a supportive environment that was beneficial to its participants, a conclusion that reiterates the findings of McCaffrey, et al. (2001): a connection in a community setting helps individual cognition and improves mathematics ability and understandings.

These conclusions are supported by theorists/researchers who base their studies on the notion that mathematical literacy and understanding include the development of students' autonomy in and out of schools as life-long learners. That is, integrated learning should focus not on accumulation of information, but on mathematical reasoning with a strong emphasis on nonroutine problem solving, problem posing, and understanding, as well as representation and communication of solution findings (Brown, 2003; Davis & Simmt, 2003; Doerr & English, 2003; English, et al., 2000; Fleener & Rogers, 1999; Greer, 2000; Hiebert, et al., 1996; Lajoie, 1998; Moore, 1998; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000; Shaughnessy, et al., 1998). In addition, appropriate use of technology, as a problem solving tool, becomes important as students' mathematical competencies develop—a point specifically stressed by Cobb (2000), who purports the interest students take when allowed to utilize non-traditional, technical aids and innovations; an interest that is then taken as a shared phenomenon.

Likewise, participating in a discourse community, students' collaborative efforts and skills need to be supported in their mathematical experiences. Because problem solving, reasoning, and discussion (Brown, 2003) are the cornerstones of proficiency (Steen, 1999), mathematical literacy and technological competence must include learning opportunities that challenge students to be mindfully engaged (Langer, 1997), to think critically, to use technology collaboratively, and to work on tasks that are worthwhile (Davis, et al, 2000).

Relationship Between Student Engagement and Problem Type

After evaluating the various data relating to problem type and participant engagement, it became evident that certain problem types engaged the students more than the others. For example, routine problems such as those found in their textbooks that involved uniform approaches and fixed solutions failed to engage the participants. The participants approached these types of problems in a mechanical manner, addressing, formulizing, and solving them rather quickly before moving on to another problem.

As previously stated, while it was no surprise that routine problems were not engaging to them, it was also evident that their collaboration and discourse were very different under these circumstances. They were much less likely to question, challenge, argue, negotiate, or probe each others' thinking, and were much more likely to rely on and accept the first answer. There were no efforts to modify, extend, or apply these routine problems to other contexts.

Their interest was more piqued by the notion of finding their own problems on the Internet and working individually or in pairs solving nonroutine problems. Even though their interest may have waned as they began problem solving, they all enjoyed having the opportunity to select their own problems.

Nonroutine problems seemed to engage them more so than routine problems, and called for an active approach to problem solving. These types of problems allowed the participants to modify the conditions on which a word, ratio, or fraction was based in order to reach a correlating solution and seemed to hold their interests.

Perhaps a factor in their enthusiasm for the nonroutine problems was their ability to choose the problem that they would attempt to solve.

This freedom of inquiry into problem selection and solving was not only interesting in terms of their educational discourse, but also because of the implications of such methodologies in terms of classroom authority. When the participants were able to become actively engaged in their own education and to seek out the very problems they would be expected to "tackle," their interest, enthusiasm, and dedication to the project escalated. Routine problems (and problems that were mechanically "assigned" to them) initiated analogous responses; they mechanically approached problem solving and reasoning. However, when they were given an active voice in deciding what problems would be the focus of their investigation, they not only sought out challenging problems, they invested themselves much more in the actual solution process. Once given some authority in their own education, they took the necessary actions in becoming autonomous agents.

Although there were some differences in how the two dyads approached these nonroutine problems (i.e. Bob and Jim communicated with one another much more effectively using both verbal and nonverbal communication in order to emerge as a functioning dyad, while Paul and Clint were much more cautious in their communication with one another and did not emerge as a functioning, communicating dyad until much later in the study), both dyads expressed much more interest and energy in solving nonroutine problems that they themselves were responsible for generating. Even though there were several instances when one partner would communicate internally and one would communicate externally, their overall discourse eventually became harmonious. The process of working together also seemed to educate the four participants on the benefits of incorporating technology in their mathematics learning. In short, they realized how much was available to them in both problem posing and problem generating, not just in this study, but for their future mathematics endeavors.

Similarly, they all enjoyed project-based problem solving very much, and continually expressed a desire to return to such a "hands on" approach as they completed other components of the study. This is apparent in their problem selection; even though they were given the opportunity to explore open-ended problems, they expressed a desire to return to projects that involved trips to local educational facilities, online searching, and web-page designs. Again Bob and Jim acted as an effective, communicatively functioning dyad early in the study and expressed a desire to work together as a team. Their discourse was open and active; each listened to the other, shared his opinions, modified working "theorems" according to the addition of new information, and actively sought to communicate and compromise.

While this communicative relationship is probably due to the fact that the two had a prior relationship with one another and felt comfortable communicating with one another, their exchanges were highly successful nonetheless. This is evident in their successful compilation and completion of the egg-drop project (i.e. they actively engaged with one another during the egg-drop project and were consistently successful).

This exchange sharply contrasts with Paul and Clint; this dyad communicated begrudgingly with one another for the majority of the first half of the study. Their responses to one another were short, uninterested, and for the most part, completely individuated. Personal ownership of solution strategies seldom went beyond comments like, “my answer,” “my method,” or “my approach.” As a dyad, their initial discourse did not involve meaningful communication; that is, they shared ideas with one another sporadically, “tuned” the other participant out when he was sharing his ideas, and failed to incorporate this additional information into his own problem solving methodology. Again, this interaction may largely be due to the fact that the two had no previous relationship and had not established a trusting communicative relationship, but the results were startling nonetheless. Often, Paul would take on the role of a teacher with Clint, which may have been responsible for some of the hostility in the latter participant. This became evident during the egg-drop project when Clint destroyed his partner’s egg and then claimed to have been “joking.” However, they did begin to exhibit a more trustful relationship toward the latter half of the study, which is greatly due to the number of projects and problems that they worked on together. They too were much more communicative and engaged when dealing with nonroutine, self-generated problems.

From this study, one may conclude that a relationship exists between the kind of problems in which students are engaged and the nature of their mathematical conversations. Even in the case of Paul and Clint, they were much more likely to talk about and actively listen to each others approaches to nonroutine problems. Students were not as actively involved in problems that were centered on standard, formulaic approaches that required little or no creativity. In contrast, students were engaged in selecting their own problems for solving (particularly by using the Internet), and were more prone to engage with one another if a significant level of interest existed. Mathematical conversations were richer and more involved as problems were more open-ended, nonroutine, or ill-defined. Some of these findings may be of function of the level of trust that existed in this small community.

Building trust and the process of trust-building was important for these students' abilities to express their thinking and the quality of their discourse. When trust was established, then dyads were more likely to argue, conjecture, and share their own uncertainties with one another especially when engaged in nonroutine problem solving.

Evolving Beliefs about Mathematical Problem Solving and Collaboration

For the most part, at the beginning of the study, the participants' views on mathematical problem solving and collaboration were varied; Paul and Clint believed collaborative work in problem solving should be utilized sparingly, although Clint admitted that he had tutored other classmates in computer courses, and that sharing ideas in other courses was beneficial to the entire class. Paul confided that his geometric understanding had improved with help from another student.

Their counterparts, Jim and Bob, however, enthusiastically supported collaborative work from the very beginning of the study and seemed eager to explore further group work. It is clear from the initial interviews that these students had a very narrow view of mathematics collaborations. All viewed collaboration as helping someone find a correct answer or procedure.

Perhaps because of their initial openness to collaboration, Jim and Bob were the first dyad to engage in deeper communication about the specific nature of their project (the egg-drop project), while Clint and Paul failed to engage with one another meaningfully; they operated as independent agents for the most part. In fact, these two failed to "mesh" through most of the first half of the study. On many occasions, the two seemed disinterested in each other's findings, and, in turn, Paul's project seemed more successful than Clint's, which may have been a point of contention between the two. There was absolutely no hint at the formation of a community in the early stages of their interactions. This again reflects their narrow view of and lack of experience with collaborative problem solving opportunities.

When the dyads entered the second phase of the study, the nonroutine problem solving, the nature of their problem solving discourse seemed to mirror these initial differences. Clint and Paul seemed to be passive participants, while Bob and Jim actively engaged with one another. In other instances, Paul assumed the role of a teacher with Clint, which effectively stunted his level of involvement. Even with these opportunities to work collaboratively on nonroutine problems, Clint and Paul seemed unable to extend their ideas of mathematical collaboration beyond focusing on "right answers" and competition. Despite these differences, all four participants agreed that nonroutine problems were beneficial to their mathematics learning because they called for extended problem solving, not the 45-50 minute problem solving "sessions" offered at their school.

At one point during problem solving (later in the study) Paul and Clint began to communicate with one another on a more meaningful level. This suggests that the nature of communicative exchanges is a complex function of beliefs, experiences, and opportunities. Also a factor may have been the difference in mathematical abilities in the two groups. For example, Bob and Jim were much more homogenous with regard to their mathematical abilities and had already established a relationship prior to this study. Likewise, the heterogeneous relationship between Clint and Paul was of great importance. Because Paul seemed to have greater mathematical understanding and ability than his partner, Clint, Paul often adopted the role of a teacher as the two exchanged information—effectively silencing Clint and guaranteeing his passivity. In the end, the two did show promise of developing positive collaboration, but their communications paled in comparison to their counterparts.

After examining the "total context," it became evident that over the course of this 16-week period, significant transitional moments existed, during which collaborations among the dyads and the group seemed to change, and the quality of discourse improved for both groups. While not directly related to any specific problem type or context, these transitional moments seemed to be related to on-going negotiations and relationships, my role as a teacher/facilitator in the development of the dyads' effective listening (Davis, 1997), and to their beliefs and mathematical understandings. For example, Bob and Clint expressed initial eagerness in participating in group work and their mathematical ability was relatively homogenous; therefore, their transitional moments occurred more often and early on, allowing for a more generative communicative evolution. Likewise, Paul and Clint expressed relative hesitance in participating in a group project, and their mathematics ability was more heterogeneous; therefore, their transitional points occurred much later and produced a less profound communicative evolution.

Prolonged problem solving and on-going negotiations and collaborations seemed to be related to students' experiences with productive interactions, shared authorities, and meaningful discourse as well as developing a supportive environment that was beneficial to its participants. An important component to this case study was the role I played, as a teacher, to intervene and facilitate for the development of the dyads' effective listening (Davis, 1997) to accelerate mathematical discourse. Early on, when Clint and Paul were failing to listen to and expand on their thinking, I, as a teacher/facilitator, have encouraged them to listen to, respond to, and utilize each other's approaches.

In this case study, I, as a researcher, provided many experiences without time limitations and (according to the four ninth-grade participating students' interests) and, as a teacher, I intervened and facilitated for the development of the dyads' effective listening, which seemed to have been crucial to the development of their mathematics understandings. Moreover, all the four ninth-grade participating students seemed to have benefited from on-going negotiations and prolonged problem solving experiences—experiences which seemed to be beneficial in their establishment of more positive relationships, beliefs, and mathematical understandings within their small collaborative community.

Therefore, my role in this study was not only to set up contexts and observe problem solving and communicative patterns, but also, as a teacher, I intervened and facilitated the development of effective listening (Davis, 1997) and multiple approaches to solution strategies. The on-going progress and accelerated collaboration in Clint and Paul's problem solving interactions seemed to suggest the importance of my role as a teacher/facilitator in providing a supportive collaborative learning environment.

Limitations

This study was conducted outside a classroom environment. The tensions a teacher feels to "cover" the curriculum in the classroom makes it difficult to understand how effective students' choice of nonroutine problems could fit in that curriculum. Therefore, student autonomy related to student choice within a traditional classroom setting is yet untested.

Thus, while this study was incredibly illuminating in terms of developing a mathematical discourse and autonomy, it did have its limitations in terms of implementing the findings into a classroom setting. For example, most of the content addressed in this study (and most of the problems suggested for collaboration) involved prolonged problem solving—problems that cannot be solved in 45-50 minutes with meaningful experiences. Obviously, this type of problem solving would be problematic for a mathematics curriculum involving 50 minute to one hour class intervals that does not spouse the spirit of reform-based curriculum's dynamics cultures.

Additionally, the current systematic structure of mathematics education involves focus on measured results—testing that focuses solely on test results, not mathematical processes. This concentration on the production of "right answers" places the importance solely on correct results, not conceptual understanding. Results of this study that suggest the importance of prolonged collaborative engagement, nonroutine problem solving, and the development of mathematical communications in a supportive environment may have limited applicability in classroom contexts where teachers feel pressures to "perform." This is not to say students would not learn the mathematics anyway, but this has yet to be a part of our mathematical culture.

Similarly, the incorporation of nonroutine problem solving into mathematics curriculum may be resisted by the society as a whole. This is true because of the existing traditional culture of the schools. For so long, teachers, students, and parents have been "trained" to view mathematics education as the production of correct answers; therefore, any modification to such a curriculum may cause uneasiness in the educational community.

Therefore, in order to change the culture of the classroom into one that encourages problem solving and collaborations, mathematics educators need to provide more time and meaningful experiences to their students and not focus on routine problems. Educators must view mathematics as human activities, respect students for their mathematics capabilities, and value the culture of the classrooms. Teachers of mathematics should provide supportive collaborative learning environments in the classrooms that including other things, incorporate the use of technology, as a tool, and utilize the spirit of reform-based curriculum including nonroutine and open-ended problems/projects that encourage students to think, to investigate, and to share as well as to collaborate and communicate effectively beyond 50 minute to one hour class intervals.

Implications for Future Research

Future research needs to provide opportunities for classroom teachers to build on a context of nonroutine problem solving and research in the classroom.

Collaborative problem solving in mathematics classrooms are necessary, and will be beneficial when educators create and support opportunities for students' collaborative learning. Educators must seek to adapt ways to incorporate collaborative learning that involves nonroutine problem solving in the classroom. Additionally, this research calls for a greater need to study further the implications of utilizing technology, as a tool, in the classroom so that students may have more and richer opportunities and selections of pursuing their own problems. Such a supportive inquiry-based collaborative environment seems to engage students and is worthy of further experimentation.

REFERENCES

Applebaum, P. M. (1995). *Popular culture, educational discourse, and mathematics.* Albany, NY: State University of New York Press.

Bauersfeld, H. (1992). Integrating theories for mathematics education. *For the Learning of Mathematics 12 (2)*, pp. 19-28.

Bauersfeld, H. (1995). The structuring of the structures: Development and function of mathematizing as a social practice. *In L. Steffe & J. Gale (Eds.), Constructivism in education* (pp. 137-158). Hillsdale, NJ: Lawrence Erlbaum Associates.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education.* Dordrecht, Netherland: Kluwer.

Bransford, J., Zech, L., & et al. (1996). Fostering mathematical thinking in middle school students: lessons from research. *In R. Sternberg & B. Z. Talia (Eds), The nature of mathematical thinking*, pp. 203-250. Mahwah: Lawrence Erlbaum Associates, Publication.

Boyer, E. L. (1995). The educated person: Toward a coherent curriculum. *Yearbook of the Association for Supervision and Curriculum Development (ASCD)*, pp. 16-25. Alexandria, VA: ASCD.

Brown, T. (1996). Intention and Significance in the Teaching and Learning of Mathematics. *Journal for Research in Mathematics Education, 27(1),* pp. 52-66.

Brown, T. (2003). *Meeting the standards in primary mathematics.* New York: Routledge.

Calvert, L. (2001). *Mathematical conversations within the practice of mathematics.* New York: Peter Lang.

Carroll, W. (1999). Using short questions to develop and assess reasoning. *In L. V. Stiff (Ed.): Developing mathematical reasoning in grades K-12: 1999 Yearbook*, pp. 247-255. Reston, VA: National Council of Teachers of Mathematics.

Carter, A., & Fleener, M. J. (2002). Exploring the teacher's role in developing autonomy. *In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant, K. Nooney (Eds), Psychology of Mathematics Education-North*

American Chapter – Volume 2, pp. 819-829.

Cassel, D. G. (2002). *Synergistic argumentation in a problem-centered learning environment*. Unpublished doctoral dissertation, The University of Oklahoma, Norman.

Cobb, P. (1989). *A constructivist perspective on information-processing theories of mathematical activity.* Unpublished manuscript, Purdue University, West Lafayette, IN.

Cobb, P. (1989). Experiential, cognitive, and anthropological perspectives in mathematics education. *For the Learning of Mathematics*, 9, pp. 32-42.

Cobb, P. (1995). Cultural Tools and Mathematics Learning: A Case Study. *Journal for Research in Mathematics Education*, 26(4), pp. 362-385.

Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. *In B. Jo (Ed.), Multiple perspectives on mathematics teaching and learning*, pp. 44-82. Westport: Ablex Publishing.

Cobb, P. & Bauersfeld, H. (1995). Introduction: The coordination of psychological and sociological perspectives in mathematics education. *In P. Cobb & H. Bauersfeld (Eds.), Emergence of mathematical meaning: Interaction in classroom cultures*, Hillsdale, NJ: Lawrence Erlbaum Associates, pp. 1-16.

Cobb, P. & Yackel, E. (1995). Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research. Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. *ERIC Document Reproduction Service No. ED 389535.*

Cobb, P., Yackel, E., & McClain, K. (Eds.). (2000). *Symbolizing and communicating in mathematics classrooms: perspectives on discourse, tools, and instructional design.* New Jersey: Lawrence Erlbaum Associates, Pub.

Cobb, P., Yackel, E., & Wood, T. (1989). Young children's emotional acts while doing mathematical problem solving. *In D. McLeod & V. Adams (Eds.), Affect and mathematical problem solving: A new perspective.* New York: Springer-Verlag, pp. 117-148.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics*, 23, pp. 99-122.

Cobb, P., Yackel, E., & Wood, T. (1993). Learning mathematics: Multiple perspectives, theoretical orientation. In T. Wood, P. Cobb, E. Yackel, & D.

Dillon, (Eds.), Rethinking elementary school mathematics: Insights and issues. *Journal for Research in Mathematics Education (Monograph Series), 6,* pp. 21-32. Reston, VA: NCTM, Inc.

Commission on Standards for School Mathematics (CSSM) (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM, Inc.

Confrey, J. (1990). What constructivism implies for teaching? In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics. *Journal for Research in Mathematics Education Monograph Series (Monograph Number 4*), pp. 107-124. Reston, VA: NCTM, Inc.

Confrey, J. (1995). The relationship between radical constructivism and social constructivism. *In L. Steffe & J. Gale (Eds.), Constructivism in education*, pp. 185-226.. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cummins, J. & Sayers, D. (1997). *Brave new schools: Challenging cultural illiteracy through global learning networks.* New York: St. Martin's Press.

Davis, R. B. (1990). *Meaningful talk in the mathematics classroom.* Master's thesis. University of Alberta, Alberta, Edmonton, Canada.

Davis, R. B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education,* 28 (3), pp. 355-376.

Davis, R. B. & Simmt, E. (2003). Understanding Learning Systems: Mathematics Education and Complexity Science. *Journal for Research in Mathematics Education,* 34 (2), pp. 137-167.

Davis, R. B., Maher, C. A., & Noddings, N. (1990). Introduction: Constructivist view on the teaching and learning of mathematics. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics. *Journal for Research in Mathematics Education (Monograph Number 4),* pp. 1-3. Reston, VA: NCTM, Inc.

Davis, R. B., Sumara, D., & Luce-Kapler, R. (2000). *Engaging mind: Learning and teaching in a complex world.* Mahwah, NJ: Lawrence Erlbaum Associates.

Devlin, K. (2000). The four faces of mathematics. *In B. Maurice & C. Frances (Eds.), Learning mathematics for a new century*, pp.16-27. Reston, VA: NCTM, Inc.

Dewey, J. (1926). *Democracy and education.* New York: Macmillan.

Dossey, J. A. & Mullis, I. V. S. (1997). NAEP Mathematics—1990-1992: The national, trial state, and trend assessments. *In P. A. Kenney & E. A. Silver (Eds.), Results from the sixth mathematics assessment of the national assessment of educational progress*, pp. 17-32. Reston, VA: NCTM, Inc.

Dupree, G. N. D. (1999). *Mathematical empowerment: A case study of relational classroom learning.* Unpublished doctoral dissertation, The University of Oklahoma, Norman.

English, L. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, 29 (1), p. 107.

English, L. D., & Halford, G. (1995). *Mathematics education: Models and processes.* Mahwah, NJ: Lawrence Erlbaum Associates.

English, L. D., Charles, K. L., & Cudmore, D. H. (2000). Students' statistical reasoning during a data modeling program. *In T. Nakahara & M. Koyama (Eds.), Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, pp. 265-272. Hiroshima, Japan.

Ernest, P. (1991). *The philosophy of mathematics education.* New York: The Falmer Press.

Etchberger, M. & Shaw, K. (1992). Teacher Change as a Progression of Transitional Images: A Chronology of a Developing Constructivist Teacher. *School Science and Mathematics, 92 (8),* pp. 411-417.

Fleener, M. J. (1995). Dissipative structures and educational contexts: Transforming schooling for the 21st Century. *Paper presentation to the American Educational Research Association.* San Francisco, CA.

Fleener, M. J. (1999). Toward a poststructural mathematics curriculum: Expanding discursive possibilities. *Journal of Curriculum Theorizing, Summer*, pp. 100-105.

Fleener, M. J. (2002). Logical Foundations for an Organocentric Curriculum: Dewey's Logic and Complexity Sciences. *In W. Doll & N. Gough, (Eds.), Curriculum Visions*, pp. 152- 162, Peter Lang Publishers.

Fleener, M. J. (2002). *Curriculum dynamics: Recreating heart.* Peter Lang.

Fleener, M. J. & Rodgers, D. B. (1999). A systems theoretic approach to understanding transformation in learning communities. *Journal of Thought, 34(1),* pp. 9-22.

Fleener, M. J., et al. (2001). The challenges of collaboration: GEAR UP partnerships cutting across institutional and organizational barriers. *American Association of Colleges of Teacher Education Annual Meeting, March,* 1-4, Dallas, TX.

Fleener, M. J., et al. (2002). Tacit Knowledge, Heuristic Inquiry, and the Curriculum: Dancing with the Shadow of Curriculum Futures (symposium). Paper title: Heuristic Methodology and Tacit Understandings: Introducing the Shadow. *2002 Journal of Curriculum Theory Conference on Curriculum Theory and Classroom Practice*, October 24-26, Dayton, Ohio.

Fleener, M. J., et al. (2003). The K-12 Tapestry: Weaving Collaborative Efforts. *American Association of Colleges of Teacher Education*, January 25-27, New Orleans.

Fosnot, C. T. (1996). Constructivism, Theory, Perspectives, and Practice. *ERIC Document Reproduction Service No. ED 396998.* Teacher College Press. New York, NY.

Forman, S. L., & Steen, L. A. (2000). Making Authentic Mathematics Work for All Students. In A. Bessot & J. Ridgway (Eds.), *Education for Mathematics in the Workplace.* Dordrecht, Netherlands: Kluwer Academic Publishing.

Fuson, K. C., Carroll, W. C., & Drueck, J. V. (2000). Achievement results for second and third graders using the standards-based curriculum Everyday Mathematics. *Journal for Research in Mathematics Education, 31*, pp. 277-295.

Ge, X., & Land, S. (2003). Scaffolding students' problem-solving processes in an ill-structured task using question prompts and peer interactions. *ETR&D, 51 (1),* pp. 21-38.

Gokhale, A. A. (1995). Collaborative Learning Enhances Critical Thinking. *Journal of Technology Education, 7(1)*, pp. 1-7.

Greer, B. (2000). Statistical thinking and learning. *Mathematical Thinking and Learning, 2 (1 & 2)*, pp. 1-10.

Greeno, J. G. (1989). Situation models, mental models, and generative knowledge. *In D. Klahr & K. Kotovsky (Eds.), Complex information processing: The impact of Herbert Simon.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Guba, E. G. (1981). Criteria for Assessing the Trustworthiness of Naturalistic Inquiries. *Educational Communication and Technology Journal, 29 (2),* pp. 75-91.

Guba, E. G. (1985). The context of emergent paradigm research. *In Y. Lincoln (Ed.), Organizational theory and inquiry: The paradigm revolution*, pp. 79-94. Beverly Hills, CA: Sage.

Guba, E. G., & Lincoln, Y. S. (1985). *Naturalistic inquiry.* Beverly Hills, CA: Sage Publications.

Guba, E. G., & Lincoln, Y. S. (1994). Competing Paradigm in qualitative research. *In N. K. Denzin & Y. S. Lincoln (Eds.), Handbook of qualitative research.* Thousand Oaks, CA: Sage Publications.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28 (5),* PP. 524-549.

Hiebert, J. (1999). Relationships between research and the NCTM Standards. *Journal for Research in Mathematics Education, 30*, pp. 3-19.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996, May). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25 (4)*, pp.12-21.

Hiebert, J., & Wearne, D. (1993). Instructional Tasks, Classroom Discourse, and Students' Learning in Second Grade Arithmetic. *American Educational Research Journal, 30,* pp. 393-425.

Kahan, J. & Schoen, H. (2003). Visions of problems and problems of vision: Embracing the messiness of mathematics in the world. *Journal for Research in Mathematics Education, 34 (2)*, pp. 169-178.

Kamii, C. (1985). *Young children reinvent arithmetic: Implications of Piaget's theory.* Columbia, NY: Teacher College Press.

Kamii, C. (2000). *Young children reinvent arithmetic: Implications of Piaget's theory* (2nd ed.). New York : Teachers College Press.

Kelly, D. L., Mullis, I. V. S., & Martin, M. O., et al., (2000). Profiles of Student Achievement in Mathematics at the TIMSS International Benchmarks: U. S. Performance and Standards in an International Context. *TIMSS International Study Center BOSTON COLLEGE.* Chestnut Hill, Massachusetts, USA.

Kenney, P. A. (1997). Learning about NAEP: Information concerning the sixth mathematics assessment. Results from the Sixth Mathematics Assessment of

The National Assessment of Educational Progress. *In P. A. Kenney & E. A. Silver (Eds.),* pp. 1-15. Reston, VA: NCTM, Inc.

Kenney, P. A. & Silver, E. A. (1997). *Results from the sixth mathematics assessment of the national assessment of educational progress.* Reston, VA: NCTM, Inc.

Kieren, C. (2001). The Mathematical Discourses of a 13 year old Partnered Problem Solving and its Relation to the Mathematics that Emerges. *Educational studies in mathematics, 46 (1-3),* pp. 187-228.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? *In A. H. Schoenfeld (Ed.), Cognitive Science and Mathematics Education,* pp. 123-147. Hillsdale, NJ: Erlbaum.

Kilpatrick, J. & Silver, E. (2000). Unfinished business: challenges for mathematics educators in the next decades. *In B. Maurice & C. Frances (Eds.), Learning Mathematics for a New Century,* pp. 223-235. Reston, VA: NCTM, Inc.

Koehler, M. & Grouws, D. A. (1992). Mathematics teaching practices and their effects. *In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning,* pp. 115-126. New York: MacMillan.

Lajoie, S. P. (Ed.). (1998). *Reflections on statistics: Learning, teaching, and Assessment in grades K-12.* Mahwah, NJ: Lawrence Erlbaum Associates.

Langer, E. J. (1997). *The power of mindful learning.* Addison—Wesley Publishing Company, Inc.

Leiken, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education, 28 (3)*, pp. 331-354.

Lesh, R., & Doerr, H. (2000). Symbolizing, communicating, and mathematizing: key components of models and modeling. *In P. Cobb, et al. (Eds.), Symbolizing and communicating in mathematics classrooms*, pp. 361-383. Mahwah, New Jersey: Lawrence Erlbaum Associates, Publication.

Lester, F. K. (1980). A procedure for studying the cognitive processes used during problem solving. *Journal for Experimental Education, 48*, pp. 323-327.

Lester, F. K. (1994). Musings about Mathematical Problem-Solving Research: 1974-1994. *Journal for Research in Mathematics Education, 25 (6),* PP. 660-675.

Lindquist, M. M. (1989). It's Time to Change. *In P. R. Trafton (Ed.), New direction for elementary school mathematics*, pp. 1-13. Reston, VA: NCTM, Inc.

Lindquist, M. M. (1997). NAEP Findings Regarding the Preparation and Classroom Practices of Mathematics Teachers. *In P. A. Kenney & E. A. Silver (Eds.), Results from the sixth mathematics assessment of the national assessment of educational progress,* pp. 61-86. Reston, VA: NCTM, Inc.

Lindquist, M. M., Dossey, J. A. & Mullis, I. V. S. (1995). *Reaching standards: A progress reports on mathematics.* Princeton, N. J.: Policy Information Center, Educational Testing Service.

Maher, C. A. & Alston, A. (1990). Teacher Development in Mathematics in a Constructivist Framework. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist view on the teaching and learning of mathematics *Journal for Research in Mathematics Education, (Monograph Number 4),* pp. 147-165. Reston, VA: NCTM, Inc.

Maher, C., & Martino, A. (1996). The development of the idea of mathematical proof: a 5-year case study. *Journal for Research in Mathematics Education, 27 (2),* pp. 194-214.

Mayer, R. E. (1982). The psychology of mathematical problem solving. *In F. K. Lester & J. Garofalo (Eds.), Mathematical Problem Solving: Issues in Research,* pp. 1-13. Philadelphia: The Franklin Institute Press.

Mayer, R. E. (1985). Implications of cognitive psychology for instruction in mathematical problem solving. *In E. A. Silver (Ed.), Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives,* pp. 123-138. Hillsdale, NJ: Lawrence Erlbaum.

McCaffrey, D., et al., (2001). Interactions among Instructional Practices, Curriculum and Student Achievement: The Case of Standards-Based High School Mathematics. *Journal for Research in Mathematics Education, 32(5),* pp. 493-517.

Menon, R. (1995). The Role of Context in Student-Constructed Questions. *Focus on Learning Problems in Mathematics, 17 (1),* pp. 25-33.

Menon, R. (1996). Mathematical Communication through Student-Constructed Questions. *Teaching Children Mathematics, 2 (9),* pp. 530-532.

Moore, D. S. (1998). Statistics among the liberal arts. *Journal of the American Statistical Association, 93,* pp. 1253-1259.

National Council of Teachers of Mathematics (NCTM) (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, Inc.

National Council of Teachers of Mathematics (NCTM) (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM, Inc.

National Council of Teachers of Mathematics (NCTM) (1995). *Assessment Standards for School Mathematics.* Reston, VA: NCTM, Inc.

National Council of Teachers of Mathematics (NCTM) (1997, May). *Mathematics Teaching in the Middle School, 97 (2),* pp. 412-415.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, Inc.

National Research Council (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Reston, VA: NCTM, Inc.

National Research Council (2001). *No Child Left Behind: A Report to the Nation on the Future of Mathematics Education.* Reston, VA: NCTM, Inc.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist view on the Teaching and Learning of Mathematics. *Journal for Research in Mathematics Education (Monograph Number 4),* pp. 7-18. Reston, VA: NCTM, Inc.

Pajares, F. & Miller, M. D. (1995). Mathematics Self-Efficacy and Mathematics Performances: The Need for Specificity of Assessment. *Journal of Counseling Psychology, 42 (2),* pp. 190-198.

Paulos, J. A. (1989). *Innumeracy: Mathematical illiteracy and its consequences.* Hills and Wang. New York.

Paulos, J. A. (1991). *Beyond numeracy: Ruminations of a numbers man.* Alfred A. Knopf, Inc. New York.

Piaget, J. (1973). *To understand is to invent.* New York: Grossman.

Prigogine, I. & Stengers, I. (1984). *Order out of Chaos: Man's New Dialogue With Nature.* New York: Bantan Books.

Pugalee, D. K. (1995). *Using Journal Writing to Characterize Mathematical Problem Solving.* Doctoral dissertation. University of North Carolina at Chapel Hill.

Reid, D. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33(1),* pp. 5-29.

Resnick, L. B. (1980). The role of invention in the development of mathematical competence. *In R. H. Kluwe & H. Spada (Eds.), Developmental models of thinking,* pp. 213-244. New York: Academic Press.

Resnick, L. B. (1987). Learning in school and out. *Educational Researcher, 16*, pp. 13-20.

Reys, R., et al., (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34(1)*, pp.74-95.

Rogers, D. B. & Dunn, M. (1999). Struggling Toward Transformation: Developing Autonomy Through Teamwork. *Journal of Early Childhood Teacher Education, volume 20 (3)*, pp. 271-289.

Rogers, D. B., & Long, L. A., (2002). Tension, Struggle, Growth, Change: Autonomy in Education. *Childhood Education, volume 78 (5),* pp. 301-302.

Romberg, T. A. (1992). Further thoughts on the Standards: A reaction to Apple. *Journal for Research in Mathematics Education, 23*, pp. 432-437.

Romberg, T. A. & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. *In M. Wittrock (Ed.), Handbook of research on teaching, 3rd edition,* pp. 850-873. New York: Macmillan.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. C., Wiley, D. E., Cogan, L. S., et al., (2001). *Why school matter: A cross-national comparison of curriculum and learning.* San Francisco: Jossey-Bass.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning,* pp. 334-370. New York: MacMillan.

Schoenfeld, A. H. (1994). Reflections on Doing and Teaching Mathematics. *In A. H. Schoenfeld (Ed.): Mathematical Thinking and Problem Solving,* pp. 53-70. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schroeder, T. L. & Lester, F. K. (1989). Developing understanding via problem solving. *In P. R. Trafton, (Ed.), New Direction for Elementary School Mathematics,* pp. 31-42. Reston, VA: NCTM Yearbook.

Sfard, A. (2000). Steering (dis)course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for*

Research in Mathematics Education, 31 (3), pp. 296-327.

Sfard, A. (2001). Learning mathematics as developing a discourse. In R. Speiser, C. A. Maher, & C. N. Walter (Eds.), Proceedings of the Twenty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, pp. 23-43. Columbus, OH: *ERIC Clearinghouse for Science, Mathematics, and Environmental Education.*

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communication to learn more about mathematical learning. *Educational Studies in Mathematics, 46 (103)*, pp. 13-57.

Sfard, A., & Kieran, C. (2001). Cognition as Communication: Rethinking Learning-by-Talking through Multi-Faceted Analysis of Students' Mathematical Interactions. *Mind, Culture, and Activity 8 (1),* pp. 42-76.

Shapiro, B. L. (1989). What children bring to light: Giving high status to learners' view and actions in science. *Science Education, 73 (6),* pp. 711-733.

Shaughnessy, C. A., Nelson. J. E., & Norris, N. A. (1998). *NAEP 1996 Mathematics Cross-State Data Compendium for the Grade 4 and Grade 8 Assessment.* Washington, D. C.: National Center for Education Statistics.

Silver, E. A. (1994). On Mathematical Problem Solving. *For the Learning of Mathematics, 14 (1)*, pp. 19-28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27(5),* pp. 521-539.

Silver, E. A., & Mamona, J. (1989). Problem posing by middle school teachers. *In C. Maher, G. A. Goldin, & R. b. Davis (Eds.), Proceedings of the Eleventh Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* pp. 263-269. New Brunswick, NJ: Rutgers University.

Silver, E. A., Kilpatrick, J., & Schlesinger, B. (1990). *Thinking through mathematics.* New York: The College Entrance Examination Board.

Silver, E. A., Mamona-Downs, J., & Kenney, P. A. (1996). Posing Mathematical Problems: An exploratory study. *Journal for Research in Mathematics Education, 27(3)*, pp. 293-309.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26 (2),* pp.114-

145.

Stanic, G. M. A. & Kilpatrick, J. (1988). Historical perspectives on problem solving in the mathematics curriculum. *In R. Charles & E. A. Silver (Eds.), The Teaching and Assessing of Mathematical Problem Solving*, pp. 1-22. Reston, VA: NCTM, Inc.

Steen, L. (1999). Twenty questions about mathematical reasoning. *In L. V. Stiff (Ed.): Developing mathematical reasoning in grades K-12: 1999 Yearbook,* pp. 270-285. Reston, VA: NCTM, Inc.

Steffe, L. & Gale, J. (Eds.) (1995). *Constructivism in education.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Stein, M., Edwards, T., Norman, J., Roberts, S., Sales, J., Alec, R., & Chambers, J. (1994). A Constructivist Vision for Teaching, Learning and Staff Development. *ERIC Document Reproduction Service No. ED 383 557.*

Tsuruda, G. (1994). *Putting it together.* Portsmouth, NH: Heinemann.

von Glasersfeld, E. (1984). An introduction to radical constructivism. *In P. Walzlawick (Ed.), The invented reality,* pp. 17-40. New York: Norton.

von Glasersfeld, E. (1987). Learning as a constructive activity. *In C. Janvier (Ed.), Problems of Representation in the Teaching and Learning of Mathematics*, pp. 3-17. Hillsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1991). *Radical Constructivism in Mathematics Education.* Dordrecht, The Netherland: Kluwer.

von Glasersfeld, E. (1995). A Constructivist Approach to Teaching. *In L. Steffe & J. Gale (Eds.), Constructivism in education,* pp. 3-16. Hillsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1995). *Radical Constructivism: A Way of Knowing and Learning.* Washington: The Falmer Press.

Wheatley, G. H. (1992). The role of reflection in mathematics learning. *Educational Studies in Mathematics, 23,* pp. 527-541.

Wheatley, M. (1994*). Leadership and the new science: Learning about Organization from an Orderly Universe.* San Francisco, CA: Berrett-Koehler Publishers, Inc.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical Problem Solving. *In Wilson, P. S. (Ed.), Research Ideas for the Classroom: High*

School Mathematics (Chapter, 4), pp. 525-725. New York: MacMillan.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30,* pp. 171-191.

Yackel, E. (1995). Children's talk in inquiry mathematics classrooms. *In P. Cobb & H. Bauersfeld (Eds.), The emergence of mathematical meaning: Interaction in classroom cultures,* pp. 131-162. Hillsdale, NJ: Lawrence Erlbaum Associates.

Yackel, E. (2000). Creating a mathematics classroom environment that fosters the development of mathematical argumentation. *Working Group 1: Mathematics Education in Pre and Primary School, of the Ninth International Congress of Mathematical Education.* Tokyo/Makuhari, Japan.

Yackel, E. & Cobb, P. (1995). Classroom sociomathematical norms and intellectual autonomy. *Proceedings of the 19th conference of the International Society of the Psychology of Mathematics Education.* Recife, Brazil.

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education, 27 (4),* pp. 458-477.

Retrieval Information: Electronics Sources

Beamish, P. & Au, W. (1995, July). Learning with computers and instructional strategies. *Paper presented to the Australian Computers in Education Conference, 1995, July*, pp. 1-7. Retrieved April 15, 1997 from http://www.oltc.edu.au/cp/refs/wing.html.

Koschmann, T., Newman, D., Woodruff, E., Pea, R., & Rowley, P. (1993, October). Technology and Pedagogy for Collaborative Problem Solving as a Context for Learning: Report on a CSCW '92 Workshop. *ACM SIGCHI Bulletin, 25 (4),* pp. 57-60. Retrieved April 15, 1997 from http://www.covis.nwu.edu/Papers/CSCW_Workshop.html.

National Center for Educational Statistics (2003). *Using the third international mathematics and science study for making policy – policy analysis: What does TIMMS tell us about U. S. education?* Retrieved April 15, 2003 from http://www.nces.org/.

National Council of Teachers of Mathematics (NCTM) (2003). *News & Media: What Can We Learn from TIMSS-Repear?* Retrieved April 20, 2003 from http://www.nctm.org/.

Third International Mathematics and Science Study, (TIMSS) (1998). *International Study Finds the Netherlands and Sweden Best in Mathematics and Science Literacy.* Press release of the Third International Mathematics and Science Study. Retrieved June 10, 2000 from www.TIMSS.bc.edu/TIMSS1/presspop3.html.

APPENDIX A

INFORMED CONSENT

University of Oklahoma
Instructional Leadership and Academic Curriculum

Problem Solving Dynamics – Students Nonroutine Problem Solving Engagement: A Case Study of Four Ninth-Grade Mathematics Students

820 Van Vleet Oval
Norman, Ok 73019-0260

A Letter to the Parents

Date: 08-20-97

Ross Pourdavood
4704 S. Love Drive
Oklahoma City, OK 73135
e-mail: rosspour@flash.net

Dear Parents of 9th graders,

As an instructor of college mathematics and as a graduate student in mathematics education I am continually aware of the role problem solving has in today's American schools. Problem solving is the cornerstone of becoming literate in mathematics. Technological advancements such as graphing calculators, computer software, and the Internet (with supervision) are viewed as important components in the efforts of mathematics teaching and learning to facilitate active problem solving in mathematics. Administrators, teachers, researchers, and students at all grade levels across the United States and abroad are excited about the integration of such technology tools for doing problem solving in mathematics.

This Fall I plan to be working closely with your 9th grade mathematics student from Christian Heritage Academy (CHA) doing nonroutine collaborative problem solving using the Internet (with supervision) as part of my dissertation research. This study may last 16 weeks beginning mid-August. The participants will comprise of four 9th grade volunteers. The entire dissertation research will be directed by my chairperson Dr. Jayne Fleener, Associate Professor from the University of Oklahoma.

The study will be divided into two phases. Phase one will include background data collection on student beliefs and attitudes about mathematics and technology, instruction on computer use, and a group problem solving activity to build teamwork. Phase two will involve student pairs working every two weeks to solve mathematical problems that are nonroutine. Problems such as building a hyperbolic cube with cardboard, designing a skate board for all terrain, designing homepages (with supervision), or defining and solving their own problems will be explored.

These projects are tentative ones and may be modified or changed to better serve to the needs of the volunteer participants. These projects may be used not as an end-mean but only as a catalyst to improve the volunteers' critical thinking, reasoning, and problem solving process in mathematics. We will also welcome and incorporate ideas and suggestions related to this study, which may come, from the volunteer participants, their parents, and/or from their teachers at CHA.

I would like to ask you and your child to read the attached consent forms. If you have no objections, please sign the Parental Consent Form and have your child sign the Student Consent Form (I only need one parent signature and a student signature) and return them to me as soon as possible. You can use the enclosed envelope or have your child return the signed forms to Mr. David Mehlhaff at CHA. If you have any questions about the research I plan to be doing, please feel free to call me at home (732-3125) or e-mail me at rosspour@flash.net.

Thank you in advance for your help.

Sincerely,

Ross Pourdavood
Doctoral Student, Mathematics Education

PARENTAL CONSENT FORM

Title:	Problem Solving Dynamics – Students Nonroutine Problem Solving Engagement: A Case Study of Four Ninth-Grade Mathematics Students
Principal Investigator:	Mr. Ross Pourdavood Instructional Leadership and Academic Curriculum - Mathematics Education, University of Oklahoma Phone number: 359-5004 e-mail: rosspour@flash.net
Dissertation Director:	Dr. Jayne Fleener, Associate Professor, University of Oklahoma Phone number: 325-1498

This is to certify that I, ______________________________, hereby give permission to have my child or legal ward participate as a volunteer in a descriptive study as part of an authorized dissertation research project at the University of Oklahoma under the supervision of Dr. Jayne Fleener.

I understand that my child or legal ward will complete two questionnaires, spend time, out of school, working on the Internet (with the supervision of the researcher), using e-mail, and collaborating with the other volunteers in doing nonroutine problem solving in mathematics. In addition I understand my child will be audiotaped and videotaped with project personnel while working in pairs or in a group as well as individual or group interviews throughout the study. I understand that the materials and methods of this project have been approved by the administration at CHA and the Institutional Review Board at the University of Oklahoma for research on human subjects. I also understand that the result of this study may be used for academic presentations and publications.

I understand that confidentiality has been assured and my child's name will not appear in any publication or presentation associated with this project. I understand that I am free to withdraw my child or legal ward from the investigation at any time without prejudice. I understand that the above named researcher will answer any questions about the research procedures or my child's rights at any time.

I understand that by agreeing for my child or legal ward to participate in this research and signing this form I do not waive any of my legal rights.

______________________________ ______________

Parent/Guardian Signature Date

STUDENT CONSENT FORM

Title:	Problem Solving Dynamics – Students Nonroutine Problem Solving Engagement: A Case Study of Four Ninth-Grade Mathematics Students
Principal Investigator:	Mr. Ross Pourdavood Instructional Leadership and Academic Curriculum - Mathematics Education, University of Oklahoma Phone number: 732-3125 e-mail: rosspour@flash.net
Dissertation Director:	Dr. Jayne Fleener, Associate Professor, University of Oklahoma Phone number: 325-1498.

This is to certify that I, ______________________________, hereby agree to participate as a volunteer in a descriptive study as part of an authorized dissertation research project at the University of Oklahoma under the supervision of Dr. Jayne Fleener.

I understand that I will complete two questionnaires, spend time, out of school, working on the Internet (with the supervision of the researcher), using e-mail, and collaborating with the other volunteers in doing nonroutine problem solving in mathematics. In addition I understand that I will be audiotaped and videotaped with project personnel while working in pairs or in a group as well as individual or group interviews throughout the study. I understand that the materials and methods of this project have been approved by the administration at CHA and the Institutional Review Board at the University of Oklahoma for research on human subjects. I also understand that the result of this study may be used for academic presentations and publications.

I understand that confidentiality has been assured and my name will not appear in any publication or presentation associated with this project. I understand that I am free to withdraw from the investigation at any time without prejudice. I understand that the above named researcher will answer any questions about the research procedures or my rights at any time.

I understand that by agreeing to participate in this research and signing this form I do not waive any of my legal rights.

______________________________   ________________

Student Signature   Date

APPENDIX B

MATHEMATICS LEARNING INVENTORY

Please respond to each item by circling a 1-4 to correspond to the following:

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.

	SA	A	D	SD
1. Students don't have to like math to have mathematical power.	1	2	3	4
2. Only professional mathematicians can invent mathematical truths.	1	2	3	4
3. No two students can have the same understanding of mathematics.	1	2	3	4
4. Mathematics is pure intellect.	1	2	3	4
5. Computers and calculators should affect how mathematics is taught.	1	2	3	4
6. There is usually one best way to solve a math problem.	1	2	3	4
7. Being able to do the problems in the book is a good indication of math learning.	1	2	3	4
8. Math drills on mathematics facts are necessary for students to learn them.	1	2	3	4
9. Good mathematics teachers know how to control their students.	1	2	3	4
10. Teachers should provide unambiguous problems for students to solve.	1	2	3	4
11. Students don't have enough mathematical sophistication to make up their own mathematics problems.	1	2	3	4
12. Teachers should repeat student comments so all members of the class hear their ideas.	1	2	3	4
13. Students should be encouraged to try different solutions to problems.	1	2	3	4
14. Students should not use calculators until they know the basic operations.	1	2	3	4
15. Discussing solution strategies with peers helps facilitate math learning.	1	2	3	4
16. The teacher is ultimately responsible for the student's learning.	1	2	3	4
17. Mathematics is very abstract. (e.g., describing without physically seeing).	1	2	3	4
18. Students cannot learn mathematics well unless they pay attention to the teacher.	1	2	3	4

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.	SA	A	D	SD
19. Well organized lectures and carefully selected examples are important for effective mathematics teaching.	1	2	3	4
20. Even children who have not learned the basic facts can have effective methods for solving problems.	1	2	3	4
21. Children can learn mathematics effectively when the teacher does not tell them whether their answers are right or wrong.	1	2	3	4
22. Mathematical truths are relative.	1	2	3	4
23. Learning mathematics requires a good memory.	1	2	3	4
24. Mathematical truths are discovered rather than invented.	1	2	3	4
25. Students need repetition and practice in order to learn mathematics.	1	2	3	4
26. Students' explanations of their solutions to problems are good indicators of their mathematics learning.	1	2	3	4
27. Mathematics learning requires creativity.	1	2	3	4
28. Mathematics does not change.	1	2	3	4
29. It's best to let students find their own methods for solving problems.	1	2	3	4
30. The goals of math instruction are best achieved when students routinely produce correct answers to problems.	1	2	3	4
31. It's better to give students lots of practice with a variety of easy problems rather than a few more involved problems to solve.	1	2	3	4
32. A good indicator of learning in mathematics is if a student can get correct answers to problems.	1	2	3	4
33. There are some mathematical truths, which will never be proven wrong.	1	2	3	4
34. Children should be able to complete mathematics problems quickly.	1	2	3	4
35. The primary value of calculators in junior high school is to allow students to check their answers.	1	2	3	4
36. Confidence is not an important element of mathematics learning.	1	2	3	4

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.	SA	A	D	SD
37. Getting right answers is as good an indication of understanding as being able to explain reasons for the answers.	1	2	3	4
38. Student performance on standardized tests is a good indication of level of understanding.	1	2	3	4
39. Mathematical truths are dependent on our perceptions of reality.	1	2	3	4
40. Teachers should encourage students to solve problems in more than one way.	1	2	3	4
41. Computers and calculators should not affect what mathematics is taught.	1	2	3	4
42. Teaching short-cuts and more efficient mathematical procedures allows students to learn more material in less time.	1	2	3	4
43. Cooperative learning is not as efficient as direct teaching.	1	2	3	4
44. The teacher is the ultimate authority in the classroom.	1	2	3	4
45. Getting right answers is a better indication of understanding than being able to explain reasoning.	1	2	3	4
46. One must be clever in order to invent mathematical knowledge.	1	2	3	4
47. If a student is having trouble with a problem, it is the teacher's responsibility to tell the students how to do it.	1	2	3	4
48. Teachers should use a carefully structured skills guide when teaching mathematics to insure each skill is mastered.	1	2	3	4
49. It's inefficient to allow students to invent their own solution strategies before the teacher demonstrates the correct procedure.	1	2	3	4
50. Students don't understand fractions because fractions are hard.	1	2	3	4
51. The majority of students cannot figure mathematics out for themselves and must be explicitly taught.	1	2	3	4
52. Teachers should encourage students to develop their own solutions even if they are inefficient.	1	2	3	4
53. The teacher's job is to guide the students to discover the right answers.	1	2	3	4

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.	SA	A	D	SD
54. Effective teaching requires rewarding right answers.	1	2	3	4
55. Effective teaching requires correcting wrong answers.	1	2	3	4
56. If students really understand mathematics, they will do well on standardized tests.	1	2	3	4
57. It is more useful to allow children time to explore some tasks thoroughly than cover all of the curriculum material.	1	2	3	4
58. Students should master computation first; conceptual understanding comes later.	1	2	3	4
59. An effective mathematics teacher demonstrates the right way to do a problem.	1	2	3	4
60. Mathematics is abstract with little practical significance.	1	2	3	4
61. Computers and calculators have significantly changed how mathematics is done.	1	2	3	4
62. The role of the math teacher is to explain methods clearly and carefully.	1	2	3	4
63. When selecting the next topic to be taught, the teacher must consider the organization of the curriculum.	1	2	3	4
64. The math curriculum should be written by mathematicians.	1	2	3	4
65. Mathematics classrooms should be organized so that students can work quietly in their textbooks with as little distraction as possible.	1	2	3	4
66. The classroom should be arranged so the authority is centered on the teacher.	1	2	3	4
67. It is important to cover the math curriculum if students are to be successful next year.	1	2	3	4
68. It's not surprising that some students have difficulty with math; math is not for everyone.	1	2	3	4
69. Students shouldn't have to practice complicated long division problems because of calculators.	1	2	3	4

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.	SA	A	D	SD
70. On a math test, it's better to have a variety of many short problems than a few involved problems to assess the depth of mathematics understanding.	1	2	3	4

APPENDIX C

STUDENT PRIOR MATHEMATICS EXPERIENCES

Answer the following questions about your own mathematics experiences:

1 = Strongly Agree, 2 = Agree, 3 = Disagree, 4 = Strongly Disagree.	SA	A	D	SD
1. I'm not very confident about my ability to solve math problems.	1	2	3	4
2. I enjoy mathematical problem solving.	1	2	3	4
3. It's important to me to understand why math procedures for solving problems work.	1	2	3	4
4. Being a good mathematics problem solver is important to me.	1	2	3	4
5. I have been a consumer rather than inventor of mathematics.	1	2	3	4
6. I don't have a very good feel for how to use my mathematical knowledge.	1	2	3	4
7. I have trouble explaining my solutions to mathematics problems.	1	2	3	4
8. I've been satisfied to find only one right way to do a math problem.	1	2	3	4
9. Math has never been easy for me.	1	2	3	4
10. I don't like to work math problems just for fun.	1	2	3	4
11. I avoid math whenever I can.	1	2	3	4
12. Knowing more than one way to do a problem confuses me.	1	2	3	4
13. I received good grades in math.	1	2	3	4
14. My math grades were not a good indication of how well I understood mathematics.	1	2	3	4

15. Age: _________

16. Gender: _____male _____female

17. Ethnic group affiliation: ______________________________

(* Adapted from Fleener, 1995)

APPENDIX D

VOICES OF THE PARTICIPATING STUDENTS

1. Tell me about your school mathematics experiences.
2. Has mathematics always been easy for you? Please explain.
3. Tell me about your experiences with problem solving.
4. What do you find difficult about problem solving? Please explain.
5. If you could change one thing about the way your mathematics class is approached, what would it be?
6. How many times a day do you think you use mathematics? Please explain.
7. How would you respond to this: Problems are our friends; you cannot learn without them.
8. Why do I have to know about problem solving?
9. What does it mean to do problem solving?
10. What is your opinion of the usefulness of mathematics in school? In the workplace? In daily life?
11. What are your career goals?
12. To what do you attribute your success in school? At home? In life?
13. To what do you attribute your impediments in school? At home? In life?
14. How do you compare yourself to your classmates academically?

APPENDIX E

PROBLEM SOLVING TASKS

The following are some problems that were suggested and assigned by the researcher to the volunteer participants to use as bases to create their own mathematical problems to solve. These problems, for the most part, were mathematical in nature and some of these problems seemed to have been challenging and engaging. These problems were basically categorized as: (a) routine (b) nonroutine (c) open-ended and (d) project-oriented problems. In the following, I will give some of these categorical problems:

Routine Problems

1. The die is rolled 18 times, how many times is the #2 expected to come up? (Adapted from the Internet – NASA Page http:\\www.nasa)
2. What is the surface area of a rectangle solid, when the length is 6 centimeters, the dept is 5 centimeters, and the height is 4 centimeters. (NASA Page http:\\www.nasa)

Nonroutine Problems

1. Two jars are placed on the table. One contains 1000 blue beads and the other 500 yellow beads. Crystal took 20 beads out of the blue bead jar and put them into the yellow bead jar. After shaking that jar until the yellow and blue beads were thoroughly mixed, she randomly selected 20 beads from the mixed jar and put them into the jar of blue beads. After completing the task she asked were there more blue beads in the yellow bead jar than there

were yellow beads in the blue bead jar? (Adapting from Maher & Alston, 1990)

2. Leah bought a number of Christian song tapes in a store where a 8.695% sales tax is added to every purchase. If she did not have to pay the tax, she could have bought 2 more tapes for the same amount of money. How many tapes did she buy? (Round your answer to the nearest cents.)

   (Developed by the researcher, 1997)

3. Jayne read a book with more than 100 and fewer than 200 pages. The sum of the three digits in the number of pages is 10. The second digit is twice the last digit. How many pages did her book have?

   (Adapted from NCTM - MTMS, 1997)

4. The nth term of a sequence is 2n + 3 for all counting numbers n. What is the arithmetic mean (average) of the first ten terms of the sequence?

   (Adapted from NCTM - MTMS, 1997)

5. A rectangular polygon consists of five squares placed side by side. The perimeter of the rectangle is 60 cm. What is the area of the rectangle in square centimeters? (Adapted from NCTM - MTMS, 1997)

6. What proper fraction exceeds its square by the greatest possible amount?

   (Adapted from Mathematics Teacher Sept. 1995 Vol. 88, No. 6)

7. Your salary is to be raised 15 percent and then a month later reduced by 15 percent. There it will remain. However, you may elect to have the cut first followed by the raise one month later. Which is the better plan?

   (Adapted from Jamski, 1991)

8. Skydivers fall at 54 meters per second before their chutes open. They fall at 6 meters per second after their chutes open. If a skydiver jumped from a plane 1800 meters high and reached the ground in 2 minutes, how high was she when she opened her chute?

(Adapted from Pugalee, 1995)

Project-oriented Problems

1. From what height can you drop an egg without breaking it?
2. Do an Internet Treasure Hunt (with supervision).
3. Design a skate board for all terrain including the planet Mars.
4. Design a Web-Page.
5. Explore an Omniplex Geometrical Treasure Hunt (with Supervision).

Open-ended Projects

1. How much money does it take to make money?
2. Does time travel forever or does it die?

APPENDIX F

INTERNET SITES TO VISIT

The following are some Internet links that include K-12 science and mathematics problems that are routine, nonroutine, or open-ended problems and were used by the participating volunteers as potential sites to visit (with supervision), to explore, and to search as well as to solve problems.

1. Exploratorium: ExploraNet; Problems from Mathematics Teachers (http://www2.hawaii.edu/suremath/nctmjournal.html)
2. The Journal of Modern Problem Solving (http://www2.hawaii.edu/suremath/journal.html#algebra)
3. Science and Math Bookmarks #1 (http://206.76.136.3/resources/scimath1.html)
4. Mathematics Resource (http://www.dpi.state.nc.us/Internet.Resources/Math.rsrcs.html)
5. Activities for Pi Mathematics (http://www.ncsa.uiuc.edu)
6. Teaching Units - Integrating the Internet (http://www.indirect.com/www/dhixson/index3.html)
7. LFM - Kid's Corner - Student Stumpers (http://quest.arc.nasa.gov/mars/kids/stumpers.html)
8. Math and Science Gateway (http://www.tc.cornell.edu/)
9. CSC Mathematical Topics (http://www.csc.fi/math_topics/General.html)

TABLE ONE

Statements/Questions From The MLI Posed to Participants	Number of Participants who Disagreed with Statement	Clarifications and/or Explanations Provided during Interview
1. More than one answer is possible for math problem	0	Yet Clint stated that two students can learn in the same manner, and Bob believes two students can have same solution but different understanding (i.e. algebraically or geometrically)
2. Mathematics understanding is not synonymous with high standardized test scores	0	Both Bob and Paul specifically stated some students just don't "test well."
3. To some extent there are absolute mathematical truths	0	Jim stated mathematics truths are not relative and do not change, yet later said math truths can be proven wrong. In contrasts, Bob said some math can be proven wrong but are not relative.
4. Invention of new mathematics is not limited to mathematics professionals	1	Bob said only professionals can invent truths, but later said one didn't have to "be clever" to invent truths. Paul later intimated on other responses that it is limited to professionals
5. Some people are more mathematically inclined, but that talent is not synonymous with enjoyment	0	Paul said one can have math power w/out "liking it"; Clint said one must like it to do well, but later said he does well in English and does not like it; Bob said only people who like math can invent truths

Statements/Questions From The MLI Posed to Participants	**Number of Participants who Disagreed with Statement**	**Clarifications and/or Explanations Provided during Interview**
6. Mathematics is applicable in everyday situations and is not just an abstraction	0	Jim said one should use experience to generate problems and problem-solving relates to every-day life, yet there are "a bunch of things in math" that he does not use everyday.
7. Ability to do math problems Does not indicate math learning	1	Bob believes that ability does equate learning
8. Math learning requires creativity	1	Bob believed the process was rooted in logic b/c math is only "facts", but then said students should generate own problems; Paul intimated later that it requires more logic than creativity
9. I consider myself to be a math consumer, not an inventor	1	Only Jim labeled himself an inventor, yet Paul believed students should use own experience in generating and solving problems
10. I am relatively comfortable with my ability to solve problems	0	Bob considers himself good at math and "gets concepts" yet he and Clint express a weakness in geometry
11. I can verbalize my solutions to someone else	2	Paul and Jim expressed difficulty verbalizing mathematics solutions
12. Grades are not an indication of mathematics knowledge	1	Bob believed they were no indication, yet said *his* grades were an indication; Clint later stated they might be a slight indication

Table 1. Basic Mathematics Assumptions and Beliefs

TABLE TWO

Statements/Questions From The MLI Posed to Participants	**Number of Participants who Disagreed with Statement**	**Clarifications and/or Explanations Provided during Interview**
1. Computers should not effect how mathematics is taught in school	0	Yet all four expressed dissatisfaction with current curriculum
2. Students should practice math problems without use of calculators	1	Bob said they free more time to "learn the basics;" Jim agrees but believes they should be used only after basics are grasped, and Paul believes they are a detriment to mathematical understanding.
3. I have had prior experience with the Internet	3	Bob familiar with computer but had never used Internet; Clint had only used the Internet to send and receive e-mails. Jim was proficient.

Table 2. Technological Beliefs and Assumptions

TABLE THREE

Statements/Questions From The MLI Posed to Participants	Number of Participants who Disagreed with Statement	Clarifications and/or Explanations Provided during Interview
1. Math drills are ineffective educational tools	0	Yet, all four are products of the traditional classroom settings. Bob said that some students may need repetition to understand.
2. Teachers should provide unambiguous math problems	0	Paul said he believed multiple solutions should be encouraged; defined ambiguity as poorly worded problems or problems that are too advanced,
3. A few longer mathematics problems are better for a test than several short ones	0	All four stated that they wished to have experienced more with longer math problems
4. Teachers should not show students the "right" way to solve problems	1	Jim said it is the teacher's "job" to show correct way; Clint believed teachers should show correct way, but not confirm solutions. Bob believed teachers should only show method if students were "inefficient," but later said teachers should show correct way first
5. Teachers should not tell students if their answers are correct or incorrect	1	Bob believed teachers should provide correct answers
6. Teachers are not responsible for their students' learning	1	Paul and Bob believed teachers are ultimately responsible, Jim believed students were ultimately responsible, and Clint believed both parties were responsible.
7. Students must pay attention to teachers in order to learn	2	Jim and Paul believed students can learn without the help of their teachers, but Paul believed that it is more difficult.

Table 3. Views about Pedagogy and Role of Educators

TABLE FOUR

Statements/Questions From The MLI Posed to Participants	Number of Participants who Disagreed with Statement	Clarifications and/or Explanations Provided during Interview
1. Teachers should always control their classrooms	2	Jim and Bob disagreed, yet Bob later said that the teacher is the ultimate authority. Clint believed that order in the classroom reflected the teacher's ability to instruct
2. Group work should be limited in the classroom	2	Paul and Bob said it should be used sparingly, but Bob said he enjoyed hearing other students' solutions and methods. Clint had had positive experiences with group work

Table 4. Views on Collaborative Work

TABLE FIVE

(II= Individual Inquiry, GD= Group Dynamics, CD=Collaborative Discourse, C=Community)

Partici-Pant	Group Meeting One	Pilot Egg-drop Project	Main Execution	Reconstructed Project
Paul	**II:** Examined Jim's findings from the Internet search. **GD:** Helped to define project as an open-ended one with many solutions. **CD:** Discussed Internet usage with researcher and suggested use of a parachute in project to partner. Doesn't seem to be "bonding" with partner. **C:** No community as yet.	**GD:** Seemed interested in other pair's results. **CD:** Worked with partner to create project from toothpicks, glue, cotton, thread, and egg. *Successful*, yet still not "meshing" with Clint. **C:** No community as yet.	**CD:** Listened to researcher's advice to throw egg away from building; worked with partner to execute project; attempted several executions, all of which *failed.* **C:** No community as yet.	**II:** Makes parachute out of handkerchief; utilizes straws and fishing line in project. Attempts project three times and all three were *successful*. **CD:** Does not include partner in project, uses pronoun "I" exclusively. **C:** No comm-unity as yet.
Clint	**II**: Seemed excited by number of possible approaches. **GD**: Discussed various approaches with Paul **CD**: Seems disinterested in partner, yet showed enthusiasm for Paul's suggestions and asked researcher about possible materials and suggested sites for the "drop." **C:** No community as yet.	**GD:** Seemed interested in other pair's results. **CD:** Worked with partner to create project from toothpicks, glue, cotton, thread, and egg. *Successful*, yet still not "meshing" with Paul. **C:** No community as yet. 2	**CD:** Asked site employees about exact height of drop site; worked with partner to execute project; attempted several executions alone, all of which *failed.* **C:** No community as yet.	**II:** Makes parachute with pillows. First Attempt *successful.* **CD:** Does not include partner in project; purposefully breaks partner's egg during successful attempt. **C:** No comm.-unity as yet.

Table 5. Dyad One: Paul and Clint

TABLE SIX

(II= Individual Inquiry, GD= Group Dynamics, CD=Collaborative Discourse, C=Community)

Particip.	Group Meeting One	Pilot Egg-drop Project	Main Execution	Reconstructed Project
Bob	**II:** Searched Internet for ideas on project; became excited over concept of using a teepee and garbage bag. **GD:** Discussed possible approaches with Clint. **CD:** Discussed possible locations with researcher and was open to guidance from Jim on the Internet searches. (Note: Bob and Jim were already familiar with one another from school and church activities). **C:** Some community already.	**GD:** Supportive of other pair's projects. **CD:** Worked with partner to create cardboard airplane for project. *Successful* **C:** Some community already.	**II:** Formulated plan using rocket engine and parachute. **CD:** Helped build both his and partner's project, expressed to researcher enthusiasm of project. Both attempts to launch project *failed*, yet enthusiastic over partner's success. **C:** Some community already.	**CD:** Worked with partner to create project using cola bottle, wings, paper towel roll, and foam. Worked with partner to enlist outside help for project design and execution. *Successful* **C:** Some community already.
Jim	**II:** Planed to ask his father for help and establish a timeline for completion. He formulated an initial plan for project. **GD:** Helped group with the Internet usage and shared his search results with other pair. **CD:** Asked researcher several questions about possible materials, helps Bob one-on-one on computer, and adds to Bob's idea of using garbage bag as parachute. **C:** Some community already.	**GD:** Supportive of other pair's project. **CD:** Worked with partner to create cardboard airplane for project. *Successful* **C:** Some community already.	**II:** Formulated plan for teepee and parachute **CD:** Co-built both projects with partner, admitted the two had no time to "set up" before launch date, but did test engine. First attempt at project *successful*, discussed possible improvements with Bob. **C:** Some community already.	**CD:** Worked as a team with partner to create project, enlisted help from classmate, as well as from a neighbor-pilot. *Successful* **C:** Some community already.

Table 6. Dyad Two: Bob and Jim

TABLE SEVEN

Paired Students	Pilot Project	Main Project	Reconstructed Project
Clint and Paul	Collaborative Project: Did not exist; Parallel Play: Existed; Trust: Did not exist; Responses: Individuated; Communication: Superficial; Meta-Messages: Non-Productive	Collaborative Project: Existed some; Parallel Play: Existed; Trust: Did not exist; Responses: Individuated; Communication: Superficial; Meta-Messages: Non-Productive	Collaborative Project: Did not exist; Parallel Play: Existed; Trust: Did not exist; Responses: Individuated; Communication: Superficial; Meta-Messages: Non-Productive
Jim and Bob	Collaborative Project: Did exist; Parallel Play: Existed; Trust: Did exist; Responses: Collaborative; Sought outside help; Communication: Genuine; Meta-Messages: Productive	Collaborative Project: Did exist; Parallel Play: Existed; Trust: Did exist; Responses: Collaborative; Sought outside help; Communication: Genuine; Meta-Messages: Productive	Collaborative Project: Did exist; Parallel Play: Existed; Trust: Did exist; Responses: Collaborative; Sought outside help; Communication: Genuine; Meta-Messages: Productive

Table 7. A Summary of Egg-Drop Project Interactions

TABLE EIGHT

Routine Problems	Traditional Geometry	IQ-Tests	Traditional Probability
Nonroutine Problems	Marbles combinations	Salary increase or decrease	Skydiver
Project-Based Problems	Egg-Drop	Omniplex: Geometric Treasure- Hunting	Designing a web-page
Open-Ended Problems	How much money does it take to make money?	Does time travel forever or does it die-out?	How to build a hyperbolic cube with cardboard?

Table 8. The Four Categories (Types) of Problems

TABLE NINE

	Mathematics Conceptual Understanding	Mathematics Inquiry and Involvement	Techno-Ogical Matheatics Familiarity	Views on Group Work in P. Solving	Personal Educational Responsibility
Paul Initial Interview	-Felt confident, but cold not explain his solutions -Believed ability to do math problems does not indicate math learning -Believe grades are an indication of knowledge	-Labeled himself a consumer - Believed non-math professionals could invent new truths -Believed p. solving required creativity -Believes more than one solution exists per problem	-Never used the Internet -Strongly believed computers should not change how math is taught	-Believed limited amount of group work was beneficial in solving problems	-Believed teachers should not show "right" way to solve problems -Strongly believed teachers should not tell if correct or not. -Believed teachers are initially responsible for learning, but more so students
Exit Interview	-Believed peer interaction forced him to communicate, and can now explain solutions -Learned to solve nonroutine problems	-Believed peer interaction showed him to approach a problem in numerous ways -Learned to develop various ways to solve problems	-Learned to use the Internet -Believes technology can benefit math education	-Believed group work is very productive in class -Group-work improved communication skills	-Believed smaller classes would be more beneficial for him to work one-on-one with the teacher

Table 9. Paul's Entry-Exit Interviews

TABLE TEN

	Mathematics Conceptual Understanding	Mathematics Inquiry And Involvement	Technological Mathematics Familiarity	Views on Group Work in Problem Solving	Personal Educational Responsibility
Clint Initial Interview	-Felt confident, but experiencing problems with Geometry - Believed ability to do math problems does not indicate math learning -Believe grades are slight indication of knowledge	- Labeled himself a consumer - Believed non-math professionals could invent new truths -Believed problem-solving required creativity -Strongly believes more than one solution exists per problem	-Only used Internet for e-mail -Believed computers should not change how math is taught	-Strongly believed group work was beneficial in problem-solving	-Believed teachers should show "right" way to solve problems -Strongly believed teachers should not tell if problems are correct or not -Believed both students and teachers are equally responsible for learning
Exit Interview	-Improved Geometry grade from a D to B -Believed he had better mathematical conceptual understanding and thinking skills	-Realized he enjoyed choosing what problems to solve and how to approach them -Learned to invent approaches to which he relates	-Uses the Internet -Still believes technology may make problem-solving too easy	-Believes more classes should use group work. Collaboration taught him responsibility and teamwork -Learned to solve problems through group work	-Believes teachers who force a certain approach limit students -Realizes too much reliance on guidance from teachers limits his understanding Believes he is responsibility in his education

Table 10. Clint's Entry-Exit Interviews

TABLE ELEVEN

	Mathematics Conceptual Understanding	Mathe-Matics Inquiry and Involvement	Technological Mathematics Familiarity	Views on Group Work in Problem Solving	Personal Educational Responsibility
Jim Initial Interview	-Felt very confident, but could not explain his solutions - Believed ability to do math problems does not indicate math learning -Believe grades are some indication of knowledge	-Labeled himself an inventor - Believed non-math professionals could invent new truths - Believed problem-solving required creativity - Believes more than one solution exists per problem, but two students can learn in same way	-Relatively proficient on the Internet -Believed computers should not change how math is taught	-Strongly believed group work was beneficial in problem solving	-Believed teachers should not show students "right" way to solve problems -Strongly believed teachers should not tell students if problems are correct or not - Strongly believed teachers are not responsible for students learning,
Exit Interview	-Has better understanding of mathematics concepts -Group work improved his communication skills	-Still enjoys logic-based problems, but enjoys choosing which problems to solve	-Uses Internet often -Created a web-page with math links -Believes Internet access in the classroom would be beneficial	-Believes competitive nature of group work improved his problem-solving skills and kept him motivated -Believes group work teaches discipline	-Believes students must be more involved in education, and group collaboration would achieve this

Table 11. Jim's Entry-Exit Interviews

TABLE TWELVE

	Mathe-matics Conceptual Unders-tanding	Mathe-Matics Inquiry and Involv-ement	Techno-Logical Mathe-matics Familiarity	Views on Group Work in Problem Solving	Personal Educat-ional Respons-ibility
Bob Initial Interview	-Felt confident except with Geometry -Believed ability to do math problems indicates math learning -Believe grades are no indication of knowledge	-Labeled himself a consumer -Believed only math professionals could invent new truths - Believed problem-solving required logic, not creativity	-Never used the Internet -Believed computers should not change how math is taught	-Believed limited amount of group work was beneficial in problem solving	-Believed teachers should not show students "right" way to solve problems, should tell students if problems are correct, and were responsible for students learning
Exit Interview	-Believes he is a better problem-solver -Can better communicate problem-solving process	-Learned to develop different solutions to problems -Learned to incorporate outside assistance (friends, neighbors, etc.) -Being able to choose how to solve problems gave him a sense of self-reliance	-Uses the Internet, but still believes textbooks are more convenient for the classroom -Developing own website	-Said discussing problems improved his unders-tanding -Believes group work helps students become more productive -Believes sharing ideas helped him grasp concepts and ideas	-Believes students should be more involved in own education

Table 12. Bob's Entry-Exit Interviews