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**UNIVERSITY OF OKLAHOMA**

**GRADUATE COLLEGE**

**SCAFFOLDED PROBLEM SOLVING,  
LEARNING APPROACHES AND  
UNDERSTANDING OF CONCEPTS  
IN AN INTRODUCTORY COLLEGE PHYSICS CLASS**

**A Dissertation**

**SUBMITTED TO THE GRADUATE FACULTY**

**In partial fulfillment of the requirements for the**

**Degree of**

**Doctor of Philosophy**

**By**

**CONSTANCE HAACK  
Norman, Oklahoma  
2000**

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**A Dissertation APPROVED FOR THE  
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP  
AND ACADEMIC CURRICULUM  
in  
SCIENCE EDUCATION**

**BY**

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## **Dedication**

**To my parents, Evelyn and Aldon Zink,  
who taught me the joy of learning and  
the value of thinking for myself.**

## Table of Contents

<b>Chapter I: Introduction</b> .....	<b>1</b>
<b>Problem Statement</b> .....	<b>5</b>
<b>Significance of the Study</b> .....	<b>5</b>
<b>Chapter II: Review of Related Literature</b> .....	<b>9</b>
<b>Knowledge and Understanding</b> .....	<b>9</b>
<b>Problem Solving</b> .....	<b>11</b>
<b>Problem Solving and Knowledge</b> .....	<b>12</b>
<b>Expert and Novice Problem Solvers</b> .....	<b>15</b>
<b>Scaffolded Problem Solving</b> .....	<b>21</b>
<b>Nature and Purpose of Scaffolding</b> .....	<b>21</b>
<b>Scaffolding Approaches</b> .....	<b>22</b>
<b>Meaningful Learning Orientation</b> .....	<b>26</b>
<b>Learning Orientation</b> .....	<b>27</b>
<b>Meaningful Problem Solving</b> .....	<b>29</b>
<b>Summary</b> .....	<b>30</b>
<b>Chapter III: Methodology</b> .....	<b>34</b>
<b>Sample and Site</b> .....	<b>34</b>
<b>Treatment Groups</b> .....	<b>35</b>
<b>Instrumentation</b> .....	<b>37</b>
<b>Mechanics Baseline Test</b> .....	<b>37</b>
<b>Learning Approach Questionnaire</b> .....	<b>38</b>
<b>Sound Physics Understanding</b> .....	<b>39</b>
<b>Test of Physics Understanding</b> .....	<b>40</b>
<b>Think Aloud Protocols</b> .....	<b>41</b>
<b>Time Frame</b> .....	<b>46</b>
<b>Chapter IV: Results</b> .....	<b>48</b>
<b>Statistical Analysis</b> .....	<b>48</b>
<b>Descriptive Statistics</b> .....	<b>48</b>
<b>Treatment Groups</b> .....	<b>48</b>
<b>Learning Orientation</b> .....	<b>49</b>
<b>Physics Understanding</b> .....	<b>51</b>
<b>Analysis of Covariance</b> .....	<b>52</b>
<b>Analysis of Think Aloud Protocols</b> .....	<b>54</b>
<b>The Protocol Problem Solvers</b> .....	<b>54</b>
<b>Transcript Codes</b> .....	<b>57</b>
<b>First Protocol Problem</b> .....	<b>57</b>
<b>Second Protocol Problem</b> .....	<b>59</b>

<b>Chapter V: Discussion and Conclusions</b> .....	<b>63</b>
<b>Discussion of Question 1</b> .....	<b>63</b>
<b>Between Group Differences</b> .....	<b>64</b>
<b>Scaffolded versus Non-scaffolded Problems</b> .....	<b>64</b>
<b>Protocol Problem Solvers' Understanding of Physics Concepts</b> ...	<b>66</b>
<b>Differences in Treatment Groups</b> .....	<b>68</b>
<b>Learning Approach</b> .....	<b>71</b>
<b>Main Effect Differences by Learning Approach</b> .....	<b>71</b>
<b>Interactions</b> .....	<b>73</b>
<b>Discussion of Question 2</b> .....	<b>74</b>
<b>Differences in Textbook Use</b> .....	<b>74</b>
<b>Contrast in Textbook Dependence</b> .....	<b>74</b>
<b>Textbook Use on a Difficult Problem</b> .....	<b>76</b>
<b>Influence of One Student on the Analysis</b> .....	<b>78</b>
<b>Possible Influences on Textbook Independence</b> .....	<b>78</b>
<b>Problem Checking</b> .....	<b>80</b>
<b>Number of Protocol Steps</b> .....	<b>83</b>
<b>Meaningful Problem Solving</b> .....	<b>84</b>
<b>Problem Representation</b> .....	<b>85</b>
<b>Approaches to Checking and Test Preparation</b> .....	<b>86</b>
<b>Cognitive Structure</b> .....	<b>87</b>
<b>Other Limitations</b> .....	<b>88</b>
<b>Conclusions</b> .....	<b>89</b>
<b>Question 1</b> .....	<b>89</b>
<b>Question 2</b> .....	<b>90</b>
<b>Implications for Practice</b> .....	<b>91</b>
<b>Further Research</b> .....	<b>92</b>
<b>References</b> .....	<b>95</b>

## List of Tables

Table 1:	Assignment of Learning Approach Categories .....	40
Table 2:	Assignment of Points on Test of Physics Understanding (TPU) .....	42
Table 3:	Summary of Coding Scheme for Problem Solving Knowledge .....	43
Table 4:	Comparison of Mechanics Baseline Test Scores .....	49
Table 5:	Learning Orientation Scores by Treatment Group .....	50
Table 6:	Distribution of Students in Learning Approach Categories According to Treatment .....	50
Table 7:	Descriptive Statistics for the Test of Physics Understanding According to Treatment and Learning Approach .....	51
Table 8:	Factorial Analysis of Covariance of the Test of Physics Understanding (TPU) .....	52
Table 9:	Questions Accompanying Protocol Collection .....	54
Table 10:	Information About Protocol Problem Solvers .....	56
Table 11:	Tally of Think-Aloud Protocol Codes for Problem 1 .....	58
Table 12:	Tally of Think-Aloud Protocol Codes for Problem 2 .....	61
Table 13:	Differences in Problem Solving Approach .....	62
Table G1:	Pearson Correlation Coefficients for Study Variables .....	127

## **List of Illustrations**

**Illustration 1: Comparison of TPU Means for Learning Approaches by Treatment . . 53**

## **Abstract**

**This study was an exploration of students' use of scaffolded problems as part of their homework in an introductory calculus-based physics class. The study included consideration of the possible relationship of students' meaningful and rote learning approaches. The sample was comprised of 48 students who had completed all study instruments. Of this number, 23 did homework assignments that included scaffolded problems that had been divided into multiple steps that simplify, highlight, and organize the knowledge associated with the problem solving process. The other 25 students did non-scaffolded homework assignments.**

**The Mechanics Baseline Test, given at the beginning of the study, measured students' prior knowledge of physics concepts. The Learning Approach Questionnaire, also given at the beginning of the study, measured students' meaningful and rote approaches to learning. Student responses to 6 qualitative physics problems and their selection of concepts associated with 4 quantitative physics problems was a gauge of their understanding of physics concepts. These 10 problems were distributed between 2 classroom examinations given during the study.**

**At the end of the study 4 students who had done scaffolded homework problems and 4 students who had done non-scaffolded homework problems participated in think aloud protocols. They verbalized their thoughts as they attempted to solve 2 physics problems. Characterizations of individual problem solving approaches emerged from the think aloud protocols.**

An analysis of statistical data showed that students who did scaffolded problems attained significantly greater understanding of physics concepts than students who did non-scaffolded assignments. There were no significant differences by learning approaches, and no significant interactions. This indicates that scaffolded homework problems may benefit students regardless of learning orientation.

Think aloud protocols revealed patterns of difference between students who had done scaffolded homework problems and students who had done non-scaffolded homework problems. These included a greater tendency among scaffolded students to include declarative knowledge and to perform problem checks. It also included a greater tendency among non-scaffolded students to rely on the textbook as a reference during problem representation. Overall, students who had done scaffolded problems appeared to solve problems in a manner closer to that seen in expert problem solvers. Additionally, they showed evidence of problem solving habits, for instance checking, that might have a long term benefit.



## **Chapter I: Introduction**

**Problem solving is the primary student-centered activity that teachers provide in college physics classrooms. A problem is commonly defined as a goal for which the path is unknown, thus problem solving involves a search for solution paths. It is typically a search that engages students with material presented in lectures and textbook readings. Problem solving is the means by which physics instructors expect their students to attain sound understandings of physics topics, that is, understandings that are in accord with those held by experts in physics.**

**However, there is ongoing concern about the extent to which the problem solving typically used in physics classes supports students' construction of sound understandings of physical science topics (Gabel, 1989; Hestenes, Wells, & Swackhamer, 1992; Maloney, 1993, 1997; Voska& Heikkinen, 2000). One concern may be traced to the form of the problems that instructors use with students. Typical homework problems are taken from the textbook and require the use of prescribed procedures in order to reach a specific quantitative answer. Thus, typical textbook problems usually require knowledge of steps and procedures to arrive at solutions. The following is an example of a quantitative problem.**

**Quantitative:           A projectile is fired with a speed of 5 meters per second at a  $45^\circ$  angle with the horizontal. If the landing area has the same elevation as the firing area and air resistance can be ignored, how far does the projectile travel horizontally before landing?**

**Qualitative problems require understanding of concepts, and they rarely use numbers. The following is example of a qualitative problem.**

**Qualitative:**               Where in the flight of the projectile is the direction of motion perpendicular to the direction of acceleration?

Research has shown that students who can successfully solve typical quantitative textbook problems, often fail to solve relatively simple qualitative or conceptual problems (Hake, 1996; Hestenes et al., 1992; Mason, Shell, & Crawley, 1997). Graduate students in physics often appear to lack conceptual understanding of fundamental physics topics (Wosilait, K., Heron, P. R. L., Shaffer, P. S., & McDermott, 1999). It is posed that the nature of understandings that facilitate the solution of qualitative problems may involve a different type of knowledge than that required for solution of quantitative problems. Such knowledge may be divided for analysis into declarative knowledge and procedural knowledge. A distinction between the two types of knowledge has been made by Lawson, Abraham, and Renner (1989), who describe declarative knowledge as "knowing that" and procedural knowledge as "knowing how". Anderson (1990) further develops the distinction between declarative and procedural knowledge with operational definitions: declarative knowledge can be stated in the form of propositions, and procedural knowledge can be stated in the form of conditional, if-then, statements.

The preponderance of knowledge needed for the quantitative problem in the preceding example may be procedural and can be expressed as conditional statements:

**If motion is projectile motion, then treat the horizontal and vertical components separately.**

**If the component is vertical, then apply equations for constant acceleration.**

**If the component is horizontal, then apply equations for constant velocity.**

The student who can locate and manipulate the proper equations will succeed in solving the quantitative problem. The mathematical task requires a number of steps including resolution of a vector into components and the solution of simultaneous equations. However, in spite of the complexity of the procedure, such equations and mathematical procedures may be used without consideration of their meaning.

The qualitative problem, on the other hand, specifically requires the declarative knowledge that, in some circumstances motion and acceleration have different directions. Projectile motion is a case in which the direction of motion is variable, but the acceleration is uniformly vertical. In addition, the student must know the procedure for orienting the velocity vector at various points in the trajectory. The knowledge needed for a qualitative problem therefore, is a mixture of procedural and declarative knowledge. Sound understanding implies that students can call upon and use interconnected units of procedural and declarative knowledge in solving such problems.

A number of researchers have concluded that students who are able to solve typical textbook problems, often lack associated declarative knowledge (Dickinson & Flick, 1998; Ferguson-Hessler, 1996; McMillan & Swadener, 1991). Students' command of declarative knowledge may also be unrelated to their command of procedural knowledge (Bryant, 1992). Many students who solve typical textbook problems apparently acquire procedural knowledge without building the connections with declarative knowledge that are necessary for sound understanding.

Given the available research, it is posed that typical textbook problems should be modified. The notion of structured guidance or "scaffolding" (Vygotsky, 1976; Rogoff, 1990) provides a basis for determining the form of such modifications.

Scaffolding is a technique in which an expert participates with a novice in an activity such as problem solving. The expert simplifies and directs the activity, often with clues and questions. The novice is thus allowed to participate at a level which would be impossible without assistance. Textbook problems can be designed to include embedded guidance that scaffolds students toward higher levels of understanding. The modified problems could guide students through efficient problem solving procedures and provide links between procedural knowledge and declarative knowledge. How may scaffolded problems be related to students' acquisition of procedural and declarative knowledge? Would scaffolded problems, compared to traditional problems, better help students construct sound understandings of physics concepts?

However, scaffolded problems alone may not be sufficient to bring about students' sound understanding of physics concepts. There may be differences in students' use of scaffolded problems, and in their problem solving approaches in general, that are of equal importance. Specifically, it is posed that students' orientations toward meaningful or rote learning may also be a factor in achieving understanding when given scaffolded problems. Ausubel, Novak, & Hanesian, (1978) describe meaningful learning as the purposeful formation of relationships among ideas, facts and information. Since scaffolded problems are designed to promote meaningful connections among units of declarative and procedural knowledge, working through such problems may facilitate meaningful understanding of physics topics. In what ways may rote or meaningful learners utilize scaffolded problems in achieving understanding? For instance, could scaffolded problems encourage rote learners to make meaningful connections in solving such problems? Might scaffolded problems

support the learning approach of meaningful learners and thus facilitate their attainment of sound understandings? The questions posed throughout this section were addressed in this study.

### **Problem Statement**

The purpose of this study was to explore the extent to which the use of scaffolded problems may impact students' attainment of sound understandings of related physics topics. The purpose was also to investigate possible differences in the attainment of sound understandings of physics concepts according to students' meaningful and rote learning orientation and interactions between scaffolded problem solving and learning orientation.

Two questions guided the study.

1. What are the differences, if any, in students' attainment of sound understandings relative to their use of scaffolded or non-scaffolded problems, meaningful learning orientation, and the interactions between these variables?
2. What are the differences, if any, in students' problem solving approaches based on use of scaffolded or non-scaffolded problems?

### **Significance of the Study**

The importance of the study is its implications for improving learning among students in introductory physics classrooms. The evidence gathered could be useful in addressing three important issues in physics education: 1) student lack of conceptual understanding, 2) student lack of satisfaction with the study of physics, and 3) a need

for teaching tools that fit into the context of large classes in which students are evaluated according to their ability to solve problems.

If scaffolded problem solving is successful, the use of scaffolded problems may assist students in constructing sound understandings of difficult and typically abstract topics in physics. Lack of conceptual understanding is likely to result in many students who remember physics as a set of disjointed rules and mathematical procedures. For students who choose not to pursue physics-related studies, the lack is highly unfortunate. These students lose an opportunity for insight into the natural world and possibly an opportunity to fully experience the nature of science in everyday life. For science and engineering majors the situation is more serious; introductory physics is often a foundation course for students in other areas of science and engineering, and understanding of physics concepts as well as experience with problem solving procedures would be most useful if it were transferable. However, it may be difficult or impossible to connect disjointed facts and procedures to subjects outside of physics if the underlying concepts have never been recognized.

Some students who do not understand the concepts associated with their studies become dissatisfied or frustrated (Hammer, 1989). Lack of conceptual understanding may be especially discouraging for students who attempt to learn meaningfully but cannot do so within the context of traditional problems. This difficulty may negatively influence their choice to pursue further studies in physics. The students who apparently reach graduate level studies without sound conceptual understanding of fundamental physics concepts, comprise a group that persevered in spite of this deficit in understanding. Other able students may avoid a domain in which the satisfaction of

meaningful learning seems to be unattainable. The guidance offered by scaffolding might lead to successful concept development for such students and thus, more meaningful learning. Satisfied students potentially add to the pool of successful physics students who pursue further study in physics or subjects related to physics.

The issues of understanding and course satisfaction are compounded for students with weak background knowledge. The guidance intended in the design of scaffolded problems may also have potential benefits for students who are poorly prepared to deal with the rigor of a college physics course that emphasizes problem solving. Physics problem solving is undeniably a complex process, and scaffolding is intended to make the process more explicit. Scaffolding can highlight the knowledge and connections that are most important. Thus, under-prepared students might grasp concepts that would otherwise be lost in a deluge of unfamiliar information. If scaffolding can bring more order and meaning to the study of physics, under-prepared students also seem more likely to attain conceptual understanding and ultimately, to succeed in physics.

Doubtless, most physics teachers want their students to succeed in attainment of sound understandings of physics concepts. They would also be pleased to know that students find their physics courses satisfying. However, in the real world, physics teachers work in a system that dictates large classes and values traditional problem solving as a primary means of assessing student achievement, so usable teaching tools must often fit within this system. Homework problem assignments are such teaching tools, and most introductory physics classes are designed to include such assignments. Yet, there has been little research done to test the value and potential of problems assigned as homework. Further, though the context of large classes and homework

problem assignments is very familiar, little research exists on problem solving done in this context. The bulk of problem solving studies have originated in isolated experimental situations. It is important to explore problem solving as it exists in typical classrooms.

The study of students who have used scaffolded or non-scaffolded problem assignments in a natural setting for the majority of a semester-long course, may also provide insight into the ways students learn to solve problems, or into the possibility of manipulating that learning process. It would be useful to examine the possible impact on students' problem solving approaches after they have used scaffolded problems over a substantial period of time.

Finally, the use of problem solving as a teaching tool and means of student assessment is not peculiar to introductory physics. If scaffolded problem solving is useful in introductory physics, it might be adapted for other situations. The study of scaffolded problem solving may suggest similar methods useful in upper level courses in physics and in domains such as chemistry and mathematics.



## **Chapter II: Review of Related Literature**

The study was an exploration of how scaffolded problems may impact students' understanding of physics concepts. This study also examined how meaningful learning orientation may interact with students' use of scaffolded problems in their attainment of sound understandings of physics concepts. Additionally, the study included an examination of student approaches to problem solving.

This chapter is a review of literature related to students' understandings and, in particular, the understandings associated with problem solving. It also includes a discussion of literature related to factors thought to influence student understanding, specifically, scaffolding (Vygotsky, 1976) and students' meaningful learning orientation (Ausubel, 1963).

### **Knowledge and Understanding**

Sound understandings, that is, understandings which are in accord with those held by experts in a domain such as physics, are comprised of interconnected units of knowledge. Individuals continuously construct understandings by assimilating data from the external environment (Piaget, 1964). In some cases transmitted instruction such as that found in lectures is well assimilated, but only when it is accompanied by the learner's internal construction process (Piaget, 1962, cited in Bybee & Sund, 1982). This construction process is essential, therefore research concerned with student understandings of physics concepts may benefit from an analysis of knowledge and the ways knowledge may be interconnected to form understandings.

The knowledge that learners construct may be analyzed in terms of declarative and procedural knowledge (Lawson et al., 1989). According to Anderson (1990), declarative knowledge is differentiated into perception based and meaning based categories. Perception based knowledge includes images such as a circle and orderings such as the alphabet. Meaning based knowledge is verbal and takes the form of propositions like, "Circles are round," and also extends into complex relationships among propositions.

Procedural knowledge subsumes knowledge associated with both motor activities and cognitive activities. In either case procedural knowledge can be stated in the form of production rules (Anderson, 1990; Lawson et al., 1989), which are conditional statements that specify the actions that will achieve a given goal. For instance, a production rule for drawing a circle might be stated, "If the goal is to draw a circle, then (1) select a center point, and (2) draw a closed line with all points equidistant from the center point." The procedural knowledge contained in production rules may be either domain-specific or domain-general. Domain-specific procedural knowledge applies to the knowledge that is more narrowly applicable to specific situations, and thus would be part of the knowledge within a subject area such as physics. Domain-general procedural knowledge applies across subject areas and pertains to generally applicable cognitive processes such as classification. Domain-general procedures are important to students as they learn a new subject (Anderson, 1990), before they have acquired the domain-specific knowledge that applies to specific situations within the subject.

Declarative and procedural knowledge are essential to effective problem solving. Yet, declarative and procedural knowledge deficits are commonly found among novice problem solvers including students in introductory physics courses (Chi et al, 1981; Dickenson, 1998; Ferguson-Hessler, 1996; Maloney, 1997; Zajchowski, 1993). Since problem solving is a heavily used student activity in physics courses, it is valuable to analyze the knowledge applied by novice problem solvers compared to the knowledge applied by more expert problem solvers. Research that has analyzed these differences sheds light on the construction of understandings associated with student problem solving. Problem solving and the knowledge applied during the problem solving process are examined in the following section.

### Problem Solving

Many science educators describe problem solving as a goal for which the path is unknown (Gabel & Bunce, 1994; Hayes, 1981; Maloney, 1994). Science educators debate the validity of the label "problem solving" for some typical textbook problems, and have proposed labeling such problems "exercises" rather than problems (Snider, 1989). However, Gabel (1989) offers a defense for the label "problem," because for most students the typical textbook problems meet the requirements of the general definition of problem. That is, the problem has a goal, and the path for reaching the goal is unknown. Though an expert in a given domain would almost automatically recognize the path to the goal, a novice must search for a path. The expert's "exercise" is the novice's "problem."

### **Problem Solving and Knowledge**

Relatively simple "exercises" remain problematic for students who have completed physics courses. Maloney (1993) used a set of twelve low difficulty quantitative problems dealing with momentum and kinetic energy in a test given to 40 college psychology students. The students had high aptitudes in mathematics and had completed varying numbers of courses in physics. Half of the test problems asked for numerical values explicitly related to momentum or kinetic energy. The other half of the problems also asked for numerical values, but momentum and kinetic energy were implicit rather than explicit in the problem statements. There was a high correlation between success on the explicit problems and the number of physics courses completed, but the correlation was very low on the implicit problems. Though the implicit problems were relatively simple, they remained problems with unknown paths of solution. The results indicate that students may remember and apply specific formulas associated with momentum and kinetic energy, but they fail to use the associated knowledge needed to identify situations where the formulas are applicable. Students did not typically connect the appropriate pieces of knowledge needed to exhibit sound understandings (Maloney, 1993).

Lack of connected information was also suggested by two studies in chemistry. Mason et al. (1997), compared student performance on paired quantitative and qualitative problems. They found that college chemistry students solved qualitative problems more quickly than quantitative problems, but answers to the quantitative problems were more likely to be correct. Even though the problems were paired to test the same concepts, many students did not possess the knowledge needed to answer

qualitative problems. Voska and Heikkinen (2000) found a similar lack of knowledge connections when they administered a two-tiered test to college chemistry students. The test asked first for an answer to a chemical equilibrium problem and then for the reason behind the answer. The first answer was correct in 53% of the cases, but only 33% of the students chose the correct reason for the first answer.

Reif and Allen (1992) investigated physics students' problem solving related to the concept of acceleration. They found that student could solve simple quantitative problems, and they demonstrated declarative knowledge of related concepts. However, when complexity was introduced into the problems, students lacked a basis for reasoning about the problems. Reif and Allen (1992) concluded that students' knowledge consisted of separate declarative and procedural knowledge elements that were not functional. Similarly, Bryant (1992) found that students' declarative and procedural knowledge of heat and temperature were statistically unrelated. Students often used domain-specific procedural knowledge to reach correct problem solutions, but coexisting declarative knowledge was faulty. In both studies, unconnected knowledge indicated lack of sound understandings.

The failure of introductory physics students to attain sound understandings of physics concepts is also documented by their poor performance on qualitative problems. The Force Concept Inventory, originated and tested by Hestenes et al. (1992), is a set of qualitative physics problems designed to assess understanding of basic concepts in mechanics. Physics professors were surprised to find that students who performed well in solving quantitative problems, were often unable to solve the qualitative problems in the Force Concept Inventory. Pretests and posttests of the Inventory given to university

students enrolled in calculus-based physics courses showed only modest gains in performance before and after traditional instruction with lectures and routine problem assignments. For instance, students enrolled in a calculus-based physics course at one major university had average pretest scores of 52% and a posttest average of 63%, an 11% gain. A score of 80% on the inventory is described as a "threshold of understanding" (Hestenes & Wells, 1992).

Hake (1998) confirmed the results of Hestenes et al. (1992) with analysis of the performances of over 6000 introductory college physics students on pretests and posttests of the Force Concept Inventory; students in traditional physics courses made only small gains with instruction. The qualitative problems required declarative knowledge in order to analyze and properly identify the physics concepts relevant to the problem situations. Procedural knowledge could only be applied after such analysis. Students apparently lacked the associated declarative and procedural knowledge required for the solution of qualitative problems. The small gains in test scores after a semester of study imply that such interconnected knowledge or understanding was not constructed during traditional instruction, that is instruction that consisted of lectures and typical textbook problem assignments.

McMillan & Swadener (1991) found that students lacked the ability to identify qualitative aspects of a problem, that is, the aspects associated with declarative knowledge. They asked six introductory college physics students, all of whom had just completed a section on electrostatics, to verbalize all thoughts occurring to them as they attempted to solve a routine quantitative electrostatics problem. Students were also interviewed and asked to elaborate on qualitative aspects of the problem. The

researchers had hypothesized that ability to arrive at a solution would be related to qualitative understanding, but found instead that none of the students "...demonstrated recognizable qualitative understanding of the problem situation (McMillan & Swadener, 1991, p. 667)." The students who were able to reach solutions either remembered or located relevant formulas. They solved quantitative problems by finding formulas and applying the procedural knowledge required to identify variables, but they did not use associated declarative knowledge. In similar studies, Ferguson-Hessler (1996) concluded that even students who were good physics problem solvers used almost no associated declarative knowledge, and Dickinson and Flick (1998) found the same pattern among students in a physics course for nonmajors.

Zajchowski & Martin (1993) also compared strong and weak students by asking them to verbalize their thoughts as they solved physics problems. They found that strong students made significantly more statements than weaker students indicating efforts to use declarative knowledge apart from formulas. The weaker students, however, relied almost entirely on memorized formulas.

#### Expert and Novice Problem Solvers

It is to be expected that if students have isolated units of procedural knowledge unlinked to related declarative knowledge, they will follow a problem solving process that differs from those with sound understandings. Such differences were found in research contrasting novices with experts (Chi, Feltovich, & Glaser, 1981; Dhillon, 1998; Larkin, 1983; Larkin, McDermott, Simon, & Simon, 1980). In these studies, novices were generally undergraduate students who recently completed the physics

course from which problems were drawn, and experts were generally professors or advanced graduate students in physics.

Chi et al. (1981) asked novices and experts to group 24 mechanics problems by problem type. Experts grouped the problems on the basis of underlying principles of physics, principles for which they presumably held sound understandings. Novices, however, grouped the problems according to surface features such as keywords or physical features mentioned in the problem statement. It is notable that the experts in the study took a longer time to group the problems, an indication that the problems were not so simple that the groupings were obvious. Concentration on the problem analysis was a distinct characteristic of experts.

Research on problem solving revealed instances in which students (novices) depended on formulas to solve problems (McMillan & Swadener, 1991) and were unable to solve problems when the required formulas were not explicitly identified (Maloney, 1993). In contrast, experts almost invariably concentrated their efforts on qualitative representations of a problem, that is, representations that incorporate declarative and procedural knowledge; and they evaluated their solutions (Chi et al., 1981; Larkin 1983, 1985). Experts dealt with formulas or equations only after the qualitative representation was satisfactory. For example, Larkin (1985) examined the performance of six experts on a difficult mechanics problem. All experts began by forming a representation of the problem, that is, a verbal problem was translated first into an "everyday" representation of the problem objects and their behavior, and then into a "physical" representation in which pertinent physics knowledge is identified (Larkin, 1985). Two of the experts quickly achieved an appropriate physical



representation and solved the problem by using an equation for virtual work, a physics concept recognized as very difficult (Larkin, 1985). Three of the experts worked on physical representations involving momentum, then rejected the momentum model in favor of virtual work, and finally produced an equation to generate an answer. The sixth expert worked on the representation but did not produce an equation and did not solve the problem. Thus, unlike novices who search first for equations or formulas to solve problems, the experts brought in equations near the end of the problem solving process.

The importance of representation in problem solving was also found in a set of studies by Chi et al. (1981) and a set of studies by Larkin (1983). In these studies, experts spent the bulk of their time on problem representation, and the physical representation of the problem, that is, the association of declarative knowledge, came before introduction of equations. In contrast, novices quickly wrote down equations, searched for formulas containing variables mentioned in the problem, and then used a means-ends approach. A means-ends approach is a domain-general procedure in which a goal or “end” is identified and then the difference between the goal and the information given in the problem is reduced (Anderson, 1990). The “means” for solving a problem is chosen because it produces the desired result, not because it makes sense in the light of declarative knowledge. For instance, in a quantitative projectile motion problem, if initial velocity ( $v_{ox}$  and  $v_{oy}$ ) is given in the problem and the desired goal is  $\Delta x$ , the horizontal distance from the launch point, a student pursuing a means-ends approach might first locate the formula  $\Delta x = v_{ox}t$ , which contains the goal, but also has a second unknown,  $t$ . The difference between the goal and the problem can be reduced, however, by using  $\Delta y = v_{oy}t - \frac{1}{2}gt^2$ , where  $t$  is the only unknown. When  $t$  is found, it is

inserted into the previous formula. By using the domain-general approach, the problem may be solved without reference to domain-specific, declarative knowledge.

Dhillon (1998) did a more detailed analysis of the problem solving process observed in four experts and nine novices. In this case, novices included three students who were beginning graduate work in physics. The results supported the conclusions of the earlier work (Chi et al., 1981; Larkin, 1983, 1985) and added more detailed information about the contrasts between novice and expert approaches. Most novices first tried to solve the problem with the one-step application of a formula, and most depended on textbook examples, often indiscriminately, to guide their problem solving. In contrast, experts did not use these strategies. Experts were characterized by their use of assessment or checking throughout the problem solving process. Novices tended to use more superficial checking, usually when they reached an impasse. All experts checked their final solutions, and only four novices checked their final solutions. Dhillon concluded that experts accessed well-organized knowledge, but novices used knowledge fragments with fewer connections. The lack of integrated knowledge resulted in novice use of problem solving strategies such as means-ends, that required less domain-specific knowledge.

The limitations of the means-ends approach as a learning vehicle were investigated in a series of studies on problem solving in science and mathematics described by Sweller (1989). In these studies subjects solved problems that stated either specific goals or general goals. For instance, in a geometry problem they might be given the specific goal of finding a particular angle in a geometry diagram, or they might be given the general goal of finding values for all of the angles in a diagram. Students who

were given specific goals generally used a means-ends approach, and they were more successful in obtaining correct answers than students who did problems with general goals. Thus, the means-ends approach was efficient. However, subjects who were given general goals attained greater understanding even though they had a lower percentage of correct answers. Sweller concluded that the means-ends approach is an inefficient means of learning and may be counterproductive because attention is diverted to inappropriate aspects of a problem. In physics the means-ends approach with its immediate search for formulas bypasses the need for qualitative, physical representation of problems that use declarative knowledge. Because students use a means-ends approach to problem solving, they rarely make a physical representation before problem solution. It is also unlikely that students engage in qualitative evaluation of the problem after solution. Thus, students may circumvent the use of most declarative knowledge in solving problems.

The lack of both physical representation and evaluation in the presence of formulas is evidenced in a study of preservice, secondary school physics teachers (Garrett, Satterly, Gil Perez, & Martinez-Torregrosa, 1990) in which subjects were asked to find the distance traveled in 5 seconds by an object with a trajectory described by  $e = 25 + 40t - 5t^2$ . Given the formula, most subjects inserted 5 seconds for  $t$  and accepted  $e = 100$  as the answer with no qualitative recognition that the initial position was not at zero. None of the subjects considered negative motion or the distinction between distance and displacement. They did not integrate the declarative knowledge with their procedural knowledge to construct an understanding of the situation. Subjects

also failed to use declarative knowledge to evaluate the validity of their answers (Garrett et al., 1990).

In summary, research shows that students generally have limited, novice-level understandings of physics concepts after introductory college-level instruction. In particular, they seem unable to identify qualitative aspects of problems, which is a possible indication that usable declarative knowledge is not associated with their procedural knowledge. Students often rely on a means-ends approach to problem solving, which is a domain general procedure that does not facilitate construction of sound understandings within a domain. Experts, on the other hand, approach problem solving by first performing a qualitative analysis, in which they use integrated, domain specific declarative knowledge and domain specific procedural knowledge to solve problems. Experts also routinely check their problem solving work.

Since introductory college physics students often fail to attain sound understandings of physics concepts, it would be valuable to explore a technique that encourages the connections between procedural and declarative knowledge, connections that may lead to such understandings. Students' use of means-ends approaches is also thought to inhibit attainment of such understandings, so if the technique prompted use of alternatives to means-ends approaches, it might further encourage students' attainment of sound understandings of physics concepts. It is posed that an instructional technique known as scaffolding may be used to promote students' connection of declarative and procedural knowledge in solving physics problems and promote desirable problem solving strategies.

## **Scaffolded Problem Solving**

### **Nature and Purpose of Scaffolding**

Scaffolding in problem solving may help students' avoid a means-ends approach and assist them in connecting declarative knowledge needed to construct sound understandings of physics. Scaffolding is described as guidance that helps the novice successfully complete a task, solve a problem, or otherwise reach a goal that would be too difficult without guidance (Rogoff, 1990; Vygotsky, 1976). The term scaffolding is appropriate because the type of support or guidance given the student is temporary (Wood, Bruner, & Ross, 1976). That is, scaffolding might encourage students' construction of sound understandings of physics concepts, and such understandings could then enable them to solve problems independently.

Scaffolding accompanies task performance. It is the assistance given by an adult or a more able peer during a novice's performance of a task (Vygotsky, 1976). However, scaffolding does not explicitly reveal solutions to the task. According to Anderson (1989), scaffolding supports novice efforts by simplifying and highlighting critical features of a task and by organizing information critical to the task. Wood et al. (1976) add that scaffolding must be structured so that the learner adheres to task requirements. In the task of physics problem solving, for instance, scaffolding might be structured to require that students include a final check of their solutions.

Wood et al. (1976) caution that the major risk in scaffolding is creation of student dependence on the teacher. Rogoff (1990) asserts that the scaffolding roles of the teacher and student should change over the course of an activity, so that teacher

guidance diminishes. The change of roles includes giving the student increasing responsibility for directing the task.

### Scaffolding Approaches

One scaffolding approach shown to improve students' understanding was a computer program that guided students through qualitative analysis of problems (Dufresne, Gerace, Hardiman, & Mestre, 1992; Mestre, Dufresne, Gerace, Hardiman, & Touger, 1993). The program was independent of any particular problem, and provided scaffolding that "forced" students to consider qualitative aspects of problems. The result of the scaffolding was that subjects improved their ability to identify the physics principles associated with each problem, an indication that their understandings had also improved as a result of scaffolding.

In a study by Heller & Reif (1984) students were orally guided or scaffolded through mechanics problems in an attempt to lead them to construct physical representations. The scaffolding elements of simplifying, highlighting, and organizing the task were all instantiated during the problem solving sessions. The researchers simplified the task by explicitly stating that the principle involved was Newton's second law,  $F = ma$ . Highlighting took the form of specifying relevant declarative knowledge, because it was discovered that unless the declarative knowledge was brought to students' attention, they did not use it. Furthermore, the researchers organized the task by requiring students to follow a series of steps involved in problem representation. When subsequent problem representations were compared with those of a control group, the scaffolded group had significantly better performance. Further, many students spontaneously assumed control at some point by adopting the steps that had been

introduced by the researchers. This happened regardless of the initial resistance of many students who were determined to solve the problems in a more familiar manner.

The scaffolding displayed in the Heller & Reif (1984) study was individualized to guide the problem solving performance of each subject, and the computer based scaffolding of Dufresne et al. (1992) and Mestre et al. (1993) was similarly individualized to a large extent. There is evidence that scaffolding can also be adapted to meet the needs of a group of students, though individualization is not possible. King (1994) studied children who worked in pairs and used a peer questioning technique. Some pairs invented their own questions about a lesson, and some pairs were scaffolded with prepared questions about knowledge relationships within the lesson. The relationships were highlighted with questions designed for the whole class, and children who were guided by the scaffolded questions evidenced greater comprehension than children whose questioning was not scaffolded.

Scaffolding for a general audience was also examined in a study by Huffman (1997). High school physics students were introduced to a generic problem solving approach in which they followed explicit steps that emphasized both quantitative and qualitative aspects during the problem solving process. The students subsequently made more complete problem representations than students who had been taught a typical textbook approach that provides only a general outline and emphasizes quantitative aspects. There are also anecdotal reports of success with a approach in physics that simplifies and organizes the problem solving task by providing labeled spaces on a worksheet (Maloney, 1997). Students were thus guided through an set of steps chosen by an expert.

A study by Catrambone (1995) in the field of statistics also supports the notion that generalized, textual scaffolding is also useful. Subjects in the study, college psychology students, were given worked examples to guide them while they did statistics problems. All of the worked examples had the same organization, but labels were added to highlight the steps in the examples given to the experimental group. In a posttest all subjects did well on problems that were similar to the example, and the experimental group performed significantly better on a novel problem. In a similar study, Quilici & Mayer (1996) found that highlighting the structural features of problems was particularly helpful for students of low ability. Highlighting may have assisted students in attaching the appropriate meaning or declarative knowledge to procedural steps.

Leonard, Dufresne, and Mestre (1996) also highlighted key elements of problems by using labels. Their labels, however, were applied to posted solutions to students' homework assignments in an introductory college physics course. Additionally, "strategy statements" were presented with the posted homework solutions and in class. The strategy statements described the organization of problem solutions and highlighted associated declarative knowledge. Although this was not step-by-step scaffolding that would occur during the problem solving process, the approach highlighted declarative knowledge and important the steps in problem solving, and by making problem solving components explicit, it may have simplified the process. Students in a class that used this approach were compared with students in a similar class that did not use the approach, and they scored significantly higher on a task requiring identification of concepts needed to solve various physics problems.



Scaffolding could also organize problem solving steps to include checking or assessment of the solution, and highlight possible checking methods. Checking is another phase of problem solving that involves declarative knowledge, and it is commonly found among expert problem solvers (Chi et al., 1981; Larkin, 1983, 1985). However, checking is weak or absent in many students (Garrett, Satterly, Gil Perez, & Martinez-Torregrosa, 1990). Romer (1993) divides problem checking into two parts that could be highlighted by scaffolding: a quantitative check of the units and a qualitative check achieved by consideration of limiting conditions. Prompting students to make qualitative checks of problem solutions could be used to further encourage their use of declarative as well as procedural knowledge in the problem solving process.

In summary, it appears that scaffolding, in a form that simplifies, highlights, and organizes the problem solving task, can be helpful to students learning to solve physics problems. Observations show that students assume control by adopting steps that were initially scaffolded, which suggests that scaffolding may influence their approach to problem solving, and also indicates that their understandings may be improved. Scaffolding may also be a useful part of textual materials designed to meet the needs of a variety of students. In this context scaffolding has been employed to simplify, highlight, and organize textual tasks.

However, a strategy for scaffolding within textual materials that provides integration of declarative and procedural knowledge has not been clearly defined. The strategies that have succeeded in individualized scaffolding situations, as in the Heller & Reif (1984) study, might also succeed in textual scaffolding. That is, students could be guided through the problem representation phase of problem solving by making the

relevant declarative knowledge more explicit. The problem could also be organized into a series of questions (King, 1994) or steps that prompt students to consider qualitative elements that are part of a physical representation and checking, and subsequently part of a sound understanding of related concepts. By highlighting the steps, as in the Catrambone (1995) study, students might also be more inclined to organize their problem solving tasks in a manner closer to that of an expert. As noted earlier, expert problem solving begins with representation, both qualitative and mathematical, and problem solving ends with checking, both qualitative and mathematical. This study will explore the use of homework problems that have been scaffolded in a way that incorporates the strategies used by expert problem solvers..

Scaffolding could be an important factor in promoting students' successful construction of sound understandings associated with problem solving. However, educators recognize that other factors are also important. The following section is a discussion of meaningful learning orientation, which describes students' approaches to learning, and is thought to mediate students' problem solving success and the construction of sound understandings of concepts. Students' meaningful learning orientation could also influence the way in which they use scaffolding.

### **Meaningful Learning Orientation**

Meaningful learning is defined as the formation of non-arbitrary and substantive (not verbatim or memorized) relationships between new knowledge and existing cognitive structures (Ausubel, 1963). According to Ausubel (1963), the three requirements for meaningful learning are that students must 1) have relevant extant

cognitive structures or interconnected knowledge, 2) be given a task or problem that is potentially meaningful, and 3) use a meaningful learning set or orientation. Relevant cognitive structure refers to the existence of prior knowledge to which new knowledge may be logically related. A meaningful task or problem is one which contains material that can be sensibly and non-arbitrarily related to an appropriate cognitive structure. A meaningful learning orientation is the inclination of the individual learner to make non-arbitrary connections among ideas rather than learning by rote. A meaningful learning orientation might be expected to characterize students who actively seek connections between declarative and procedural knowledge.

### Learning Orientation

Cavallo (1996) and Cavallo and Shaffer (1994) measured individual differences in students' meaningful learning and understanding of genetics topics. In both instances the researchers found a score that represented students' learning orientation on a scale that ranged from rote to meaningful. A high score indicated a meaningful learning orientation, and was positively correlated to students' sound understandings of genetics concepts (Cavallo & Shafer, 1994) and to students understandings of genetics interrelationships (Cavallo, 1996). Understandings of genetics interrelationships indicated that students had the procedural knowledge to do genetics problem solving, the declarative knowledge of genetics facts, and that they could relate the procedural and declarative knowledge. Thus, meaningful learning orientation was shown to be important to learning in the domain of genetics.

A number of researchers have noted the significance of meaningful learning with respect to understandings of physics concepts (Ferguson-Hessler and deJong, 1993;

Halloun, 1996). However, measures of individual differences in meaningful learning orientation have not generally been considered. An exception is a study by Williams and Cavallo (1995) in which meaningful learning orientation was found to predict understanding of physics concepts. They found that rote learners had more physics misconceptions, and suggested that this was the result of compartmentalized knowledge among rote learners. Thus, meaningful learning orientation also appears to be important in the domain of physics.

Later work in the measurement of individual differences in learning orientation showed that learning orientation might be measured on two independent scales of rote learning orientation and meaningful learning orientation (Cavallo, Miller, and Blackburn, 1996). This suggests that an individual could have a strong tendency to use meaningful learning approaches, and also a strong tendency to use rote learning approaches. Saunders (1998) found examples of such individuals among college chemistry students. Presumably, in any given instance students who have high tendencies toward both meaningful and rote approaches might choose either approach.

Further evidence that some students may choose either meaningful or rote approaches was found by Elby (1999). The researcher asked students in introductory physics courses to recommend tactics appropriate to a variety of situations that might arise in physics courses. Many students clearly differentiated approaches based on the situations, and they chose rote approaches in some situations and meaningful approaches in others. It is probable that among individuals there are varying tendencies to choose meaningful learning approaches, such tendencies might be indicated in a measurement of meaningful learning orientation. It is also probable that they also have

varying tendencies to choose rote learning approaches, and that these tendencies are independent of their meaningful learning tendencies.

There is also evidence that meaningful approaches to learning may be encouraged by scaffolding. Chin and Brown (2000) studied qualitative differences among eighth graders who had been selected on the basis of LAQ scores. Meaningful learners were characterized by predicting and self-explaining, which the researchers labeled “deep thinking processes.” However, these thinking processes generally occurred as the result of another person’s scaffolding or prompting. If students have the ability to choose the learning approaches they use, it may be possible to guide their choices through scaffolding.

### Meaningful Problem Solving

Meaningful problem solving is an extension of meaningful learning in which trial-and error approaches contrast with deliberate (meaningful) attempts to discover relationships underlying a problem solution (Ausubel et al., 1978). The requirements for meaningful problem solving are the same as for meaningful learning: relevant prior knowledge, a task or problem which is potentially meaningful, and a meaningful learning set or orientation. According to Ausubel et al. (1976), a meaningful problem solver can be expected to make sense of the problem-setting proposition (problem statement) by relating it to prior knowledge. The meaningful problem solver can also be expected to try to make sense of the answer, whereas a trial-and-error problem solver tends to accept an answer without considering its connections to other knowledge. The meaningful problem solver uses a meaningful learning approach focused on connecting

knowledge, and trial-and error problem solver uses a rote learning approach that does not include knowledge connections.

There is little specific research on meaningful problem solving, but the existence of problem solvers who make meaningful connections is implied in the genetics problem solving work of Cavallo (1996) and Cavallo & Shafer(1994). The low incidence of declarative knowledge associated with problem solving in physics (Dickenson & Flick, 1998; Ferguson-Hessler, 1996; McMillan & Swadener, 1990) suggests that meaningful problem solving in this domain may be rare.

In summary, meaningful learning approaches have been linked to students' attainment of sound understandings of science concepts. Students' use of rote learning approaches has been identified as a separate variable, and there has been very little investigation of a possible relationship to learning. Both meaningful and rote learning approaches may be important to some students, and the choice of approaches might be influenced by scaffolding. Further investigation of both meaningful and rote learning orientations and their relationship to students' attainment of sound understandings of physics concepts is warranted. Further investigation of the poorly researched relationship between meaningful problem solving and students' meaningful and rote approaches to learning is also warranted, since meaningful problem solving is at once desirable and rare among introductory physics students.

### Summary

Students' understandings within a domain may be analyzed by identifying declarative and procedural knowledge and associations between the two types of

knowledge (Anderson, 1990). Many students lack sound understandings of physics concepts after introductory college physics instruction (Hake, 1998; Hestenes et al., 1992; Hestenes & Wells, 1992; Maloney, 1993). In particular, students show inability to solve qualitative problems (Hake, 1998; Hestenes et al., 1992; Hestenes & Wells, 1992), where links between procedural and declarative knowledge are necessary, nor can they identify qualitative aspects of problems (McMillan & Swadener, 1991). Relevant declarative knowledge appears to be unrelated to the problem solving process (Bryant, 1992; Ferguson-Hessler, 1996).

Experts who solve physics problems rely heavily on use of domain specific knowledge for qualitative analysis of problems to identify problem types (Chi et al., 1981) and to make a physical representation of the problem and evaluate problem solutions (Chi et al., 1981; Dhillon, 1998; Larkin, 1983; 1985). Novices, such as the students typically found in introductory physics courses, tend to ignore qualitative aspects of problems (Dhillon, 1998; Garrett et al., 1990) and search for formulas with appropriate variables (McMillan & Swadener, 1991; Maloney, 1993). The novices generally employ a domain general, means-ends approach (Anderson, 1990; Chi et al., 1981; Dhillon, 1998; Larkin, 1983), which is thought to be a poor approach for achieving sound understandings (Sweller, 1989).

Scaffolding (Anderson, 1989; Rogoff, 1990; Vygotsky, 1976; Wood et al., 1976) could be used to modify typical textbook problems and encourage students to use both procedural and declarative knowledge in problem solving. Such an approach has been used to lead physics students to consider qualitative aspects of problems (Defresne et al., 1992; Mestre et al., 1993) and to highlight and organize the steps toward problem

solutions (Heller & Reif, 1984; Huffman, 1997; Maloney, 1997). Such an approach might also apply to problem checking, where student weaknesses in use of declarative knowledge have also been noted (Dhillon, 1998; Garrett et al., 1990; Romer, 1993). It is plausible that scaffolding embedded within the text of problems could affect the quality of students' understandings of physics concepts and students' approaches to problem solving. Since meaningful learning orientation is associated with students' acquisition of sound understandings (Williams & Cavallo, 1995; Cavallo, 1996), and scaffolding may encourage meaningful approaches (Chin & Brown, 2000), it is important to consider the how meaningful learning orientation may impact students' utilization of scaffolded problem solving.

One purpose of this study is the exploration of how students' use of scaffolded problems impacts their attainment of sound understandings of physics concepts. If problem solving homework assignments incorporate the techniques of scaffolding, are students more likely to attain such understandings? What differences, if any, might be related to highlighting, simplifying, and organizing the steps of problem solving tasks?

Another purpose of the study is the investigation of meaningful learning orientation according to students' use of scaffolded or non-scaffolded homework problems and their attainment of sound understandings of physics concepts. Does the outcome differ for meaningful learners and rote learners? Does use of scaffolded and non-scaffolded problems relate to differences in outcomes for meaningful learners and rote learners? Is there evidence, as Ausubel et al. (1978) suggest, that meaningful learners are also meaningful problem solvers who try to make sense of their answers?



**A final purpose of the study is to explore possible differences in the approach to solving problems according to students' use of scaffolded or non-scaffolded problem solving. For instance, are there differences in problem representation or in the association of declarative knowledge with the problem solving process? Is there evidence of expert behaviors such as qualitative analysis and checking? All of these questions will be addressed in the following chapters.**

## Chapter III: Methodology

### Sample and Site

The subjects of the study were approximately 110 students attending a major Midwestern university. Over 80% of the students were male. The students were enrolled in an introductory undergraduate physics course that dealt primarily with mechanics. Most students taking the course were freshman or sophomore engineering or physical science majors, and all had completed at least one semester of calculus.

One hundred four of the students completed the first examination and 84 completed the second examination. Fifteen of the 84 were eliminated from the study because they had not completed enough of the homework assignments. An additional 16 students were eliminated because they had changed treatments during the study. Finally, 5 students were eliminated because they had not completed one or more of the instruments used in the study.  $N$  for the final sample was 48.

The students attended three lectures each week, which were delivered by a single professor. Each student was also assigned to one of three recitation sections which met for one hour each week. The recitation sections, consisting of approximately 40 students each, were led by a single graduate teaching assistant. Students had the opportunity to request one of the three recitation times when they enrolled for the course. Thus, there was a chance that self-selection occurred. A fourth recitation section, consisting of approximately 20 students, was led by the professor for the course and was designated as an honors section; this section was not used in the study.

The professor had taught the course twice prior to this study, and he was a relatively new faculty member. He appeared to have a good rapport with his students, and he had received high student evaluations in previous semesters. His course organization included printed notes with many examples of mechanics problems. The notes were available to the students at the beginning of the semester.

Since this study depended heavily on the completion of homework assignments, a tally of completed homework assignments was made at the end of the study. Nine weekly homework assignments were made during the study, four assignments before the first examination and five additional assignments before the second examination. Students who had not completed at least two of the homework assignments that preceded each of two examinations were dropped as subjects.

### Treatment Groups

The study had a quasi-experimental design in which students were assigned to two treatment groups based on membership in recitation sections. Both treatment groups received weekly printed homework assignments.

Two of the recitation sections were assigned to the scaffolded treatment. The homework assignments for the scaffolded treatment group consisted of seven problems. (See Appendix A.) Scaffolded problems were written as a sequence of steps or questions that simplified, highlighted, and organized pertinent knowledge as suggested by the scaffolding guidelines of Wood et al. (1976) and Anderson (1989). These problems included labeled phases for representation, solution, quantitative cases, and checking. Scaffolded problems also made reference to pertinent declarative knowledge

and incorporated declarative knowledge in the sequence of questions. Selection of content details included in the scaffolded problems was informed by the researcher's experience in teaching a similar community college physics course for engineering majors and by the work of Arons (1990, 1997), a physics educator who has addressed common problems experienced by students in introductory physics courses.

The first three problems of each seven-problem assignment were fully scaffolded; each phase was comprised of detailed steps. However, since scaffolding should be gradually withdrawn as the student becomes more experienced, the fourth and fifth problems were partially scaffolded. Partially scaffolded problems indicated the necessity for representation and checking and provided limited guidance in associating declarative knowledge with the problem solving procedures. The sixth and seventh problems were non-scaffolded, typical textbook problems. The withdrawal of scaffolding was a response to the concern expressed by Wood et al. (1976) that students may become dependent on scaffolding.

The third recitation section was assigned to the non-scaffolded treatment. For each assignment this group received non-scaffolded, typical textbook problem assignments which consisted of ten problems chosen from among introductory physics textbooks that had not been used in the course during the last five years. The first four problems of the non-scaffolded assignment were comparatively simple, that is, problems that are generally designated as exercises (Snyder, 1989). These problems were chosen to familiarize students with the variables that they would confront in the remaining problems. The other six problems of the non-scaffolded assignment were

more difficult, and two of them were identical to the non-scaffolded problems assigned to the treatment group. (See Appendix B.)

Since an attempt to use the same problems in scaffolded and non-scaffolded forms seemed likely to contaminate the study by encouraging students from the non-scaffolded group to seek helpful information from the scaffolded problems, different problems were chosen to represent the same concepts. Each week three major concepts were identified by the professor for the course, and the three fully scaffolded problems instantiated the three concepts. The professor monitored the homework assignments to assure that the two treatment groups were offered similar exposure to the concepts.

All students were asked to report the amount of time spent on each problem within an assignment. Thus, the number of problems on the assignments could have been adjusted to ensure that the time devoted to solving problems remained approximately equal for the two treatments. Students were also asked to report work done on problems outside those assigned to their group. After the treatment began students were allowed to change treatment groups with the permission of the professor; this meant that they were no longer a part of the study.

### Instrumentation

#### Mechanics Baseline Test

The Mechanics Baseline Test (MBT) (Hestenes & Wells, 1992) is a set of 26 multiple choice problems that emphasizes concepts that cannot be grasped without formal knowledge of mechanics. It was designed to assess qualitative understanding by inclusion of distracters that are based on typical mistakes made by students with

deficient understandings (Hestenes & Wells, 1992). The test does not include problems that can be solved by simple reference to formulas. The MBT was administered at the beginning of the study in order to determine whether the treatment groups were initially equivalent in understanding of concepts in mechanics. The Cronbach alpha estimation of reliability was  $r = .62$  in this instance. The Mechanics Baseline Test is included in Appendix C.

### Learning Approach Questionnaire

Approaches to learning were measured by administering the Learning Approach Questionnaire (LAQ) at the beginning of the study. The LAQ is a 24-item Likert scale instrument that was modified (Cavallo, 1996; Saunders, 1998) from an earlier 50-item instrument designed to measure students' approach to learning and their beliefs about science (Donn, 1989). The items retained in the 24-item instrument comprise two subscales: 1) 11 questions that address rote learning approaches (LAQR) and 2) 13 questions that address meaningful learning approaches (LAQM). Cronbach alpha internal consistency coefficients for the subscales have been reported as  $r = .65$  for the LAQM and  $r = .80$  for the LAQR (Saunders, 1998). The Cronbach alpha statistics for this study were  $r = .68$  for the LAQM and  $r = .71$  for the LAQR.

Sample items from the LAQ include:

5. I find I have to concentrate on memorizing a good deal of what I have to learn.
15. I try to relate what I have learned in one subject to that in another.

Students respond to each statement by indicating their agreement, ranging from A (always true) to E (never true). Item 5 above is part of the LAQR (rote) subscale, and a

response of "always true" on the this item would suggest a rote learning approach. Item 15 is part of the LAQM subscale, and a response of "always true" would suggest a meaningful learning approach.

The LAQR and the LAQM were scored separately. Each response was given a value from zero to four; a response of E (never true) received zero points, and a response of A (always true) received four points. Thus, a mid-range score of two for each item would yield a score of 22 on the rote learning subscale and a score of 26 on the meaningful learning subscale. Scores that were at least 20% above the mid-range score were considered indicative of rote or meaningful approaches to learning. The high level was chosen to assure that students placed in the rote and meaningful categories were likely to be strongly representative of the learning approaches associated with each category.

Students were assigned to groups based on the combination of LAQR and LAQM scores. Scores at or below 26 on the LAQR as well as at or below 31 on the LAQM were assigned the label "low." Those above 26 on the LAQR and at or below 31 on the LAQM were assigned the label "rote." Students who scored above 31 on the LAQM and at or below 26 on the LAQR were assigned the label "meaningful." (See Table 1.) Students who scored above the chosen thresholds on both the LAQM and LAQR were assigned the label "high." The Learning Approach Questionnaire is included in Appendix D.

### **Sound Physics Understanding**

Two measures of sound understanding were included in the study. The first measure, called the Test of Physics Understanding (TPU), was incorporated into the first

and second examinations for the course. The second measure of sound understanding took the form of think aloud protocols collected in interviews with individual students.

Table 1

Assignment of Learning Approach Categories

		<u>LAOR</u>	<u>LAOM</u>
Category	Low	$\leq 26$	$\leq 31$
	Rote	$> 26$	$\leq 31$
	Meaningful	$\leq 26$	$> 31$
	High	$> 26$	$> 31$

Test of physics understanding. The Test of Physics Understanding incorporates two major gauges of student understanding that appear in the literature. The first gauge is ability to solve qualitative problems. Such problems are usually written in a multiple choice format (Hestenes et al., 1992). The second gauge is ability to identify major conceptual features of problems (Chi et al., 1981; Defresne et al., 1992; Leonard et al., 1996; Mestre et al., 1993).

The TPU consisted of a set of six qualitative problems and four quantitative problems and yielded a maximum score of 12 points. The qualitative problems were patterned after those on the Mechanics Baseline Test. That is, they were multiple choice problems, and the distracters represented responses expected from students who do not have sound understandings. In order to represent a broad range of mechanics concepts in



the TPU, the problems were distributed between the two class examinations given during the study. Problems appearing on each examination were based on the material recently addressed in lectures and homework assignments. Thus, two of the qualitative problems appeared on the first examination and four appeared on the second examination. Each correct answer to the qualitative problems was given a score of one point.

The quantitative problems were used to determine student ability to identify major conceptual features of problems. As with the qualitative problems, the quantitative problems were based on the material recently addressed in lecture and homework assignments. Two of the quantitative problems appeared on each examination. One of the quantitative problems on each examination was based on a single concept. Problem solutions that indicated use of the concept were given one point. A second quantitative problem on each test was based on two concepts. Problem solutions that indicated use of the concepts were given one point for each concept. (See Table 2 for a summary of the scoring.) The problems for the TPU were written by the professor for the course and reviewed by a second professor of physics for content validity. The Cronbach alpha estimate of reliability for the TPU was  $r = .71$ . The items from the TPU are included in Appendix E. A description of the concepts associated with each of the quantitative problems is also included in Appendix E.

Think aloud protocols. Think aloud protocols were collected in individual one-hour sessions at the end of the study. Protocols were obtained from eight volunteers, four from each treatment group. Each of the eight protocol problem solvers was paid twenty dollars.

The researcher asked preliminary and follow-up questions, but as suggested in the literature (Ferguson-Hessler, 1996; Larkin, 1985; McMillan & Swadener, 1991; Zajchowski & Martin, 1993), the researcher's role during problem solving was limited

Table 2

Assignment of Points on Test of Physics Understanding (TPU)

		<u>Points</u>
Test item	Qualitative problems	
	Examination 1	2
	Examination 2	4
	Single-concept quantitative problems	
	Examination 1	1
	Examination 2	1
	Two-concept quantitative problems	
	Examination 1	2
	Examination 2	2
TOTAL POINTS		12

to encouraging the students to describe their actions as they wrote problem solutions and to verbalize their thoughts. Feedback and answers were not provided. (See Appendix F for problems and preliminary questions.) There were two protocol problems, but in order to introduce the procedure, students solved a simpler problem first. This allowed them to become familiar with the process of verbalizing their thoughts, and it was an opportunity for them to ask questions. The simple introductory problem was also intended to increase their comfort. Students were told that they were free to use a textbook or a calculator at any time, and these items were placed in close proximity.

The entire one-hour sessions were audiotaped, and the audiotaped recordings were transcribed. Each transcript was then divided into units or “lines” that represented distinct pieces of knowledge. Students’ written problem solutions were then divided into sections and matched with lines in the transcripts.

Individual lines of each transcript were then assigned three codes. The codes describe the purpose, type, and quality of the knowledge found in each line. A summary of the codes is presented in Table 3.

Table 3

Summary of Coding Scheme for Problem Solving Knowledge

<u>Knowledge Purpose</u>	<u>Knowledge Type</u>	<u>Knowledge Quality</u>
Representation	Procedural	Appropriate
Solution	Declarative	Inappropriate
Checking	Textbook (specific, non-specific)	Faulty

The first code was assigned to describe the purpose: representation, solution, or checking. Lines coded as representation lines were those used to describe the problem situation. Such description included statements made in reference to diagrams, statements which named concepts to be applied, and statements of mathematical expressions or equations used to describe the situation. Lines coded as solution lines indicated manipulations that lead or could have lead to a solution or answer to the problem. The solution coding was also applied to final statements of a solution or answer to the problem. Lines coded as checks contained a question about a solution or a

solution step, followed by some indication of action. For instance, a problem solver might question an answer and then take action with a review of preceding algebra or a review of the applicable reasoning.

After each line was coded for its purpose (procedure, solution, checking), it was given a second code to indicate a knowledge category. The knowledge categories were procedural, declarative, and textbook. The procedural code was assigned to lines that indicated knowledge of how to perform an operation or where to apply an operation. Procedural knowledge was also identifiable, because it could be translated as conditional statements. The protocol data required a modification of Anderson's (1990) form for conditional statements. Instead of , "If the goal is A, then take action B," protocol statements took the form, "If condition A exists, then take action B." For instance, "If an object is falling, then indicate the influence of gravity." Generally, the condition associated with a unit of procedural knowledge had been indicated in the problem statement or in a previous line. In cases where the previous line contained the condition, the action taken in one line supplied the condition for the next line. For instance, in the line above, the "object falling" condition might be information found in the problem statement. The action for this statement is "indicate the influence of gravity." A subsequent statement of procedural knowledge might be, "If the influence of gravity is indicated, treat acceleration as a constant." The action from one conditional statement is used as the condition of the next statement.

The declarative knowledge code was used when there was a clear indication that physical ("physical" meaning "related to physics") facts had been associated with the problem situation. The declarative knowledge had to be a significant addition to the

procedural knowledge used in problem solution. Simple rephrasing of a problem statement, for example, counted as an appropriate procedure for representation, not an instance of declarative knowledge. The following statements illustrate closely related examples of procedural and declarative knowledge.

You have  $v_0$ , which is zero. (procedural)

Then  $v_0$  is zero, because it stops here. (declarative)

The normal force balances the gravitational force. (procedural)

The normal force equals the gravitational force, since nothing is moving in that direction. (declarative)

Some of the knowledge used in problem solving was gleaned directly from the textbook, so it was not counted as student knowledge, but instead given the designation “textbook.” Knowledge found by reference to the textbook was further classified as specific or nonspecific. In the “specific” cases students sought specific equations or used specific examples. In the “nonspecific” cases students looked through the book in search of any equation or example that might be useful.

Finally, with the third code all knowledge was further classified as appropriate, inappropriate, or faulty. Appropriate knowledge is knowledge that has the potential to lead to a correct solution. Inappropriate knowledge does not fit the problem situation either because it is not useful to the particular situation or because it is a misstatement of accepted scientific fact. The designation “faulty” was reserved for instances that were basically appropriate but included incorrect detail. For instance, knowledge was classified “faulty” when a student wrote an equation incorrectly or dropped a term as they worked through the steps of a solution.

Transcripts of the problem solving protocols were coded by the researcher and by a veteran professor of physics from another university, who had no knowledge of students' associations with treatments. In order to reach agreement on how the coding was to be done and to achieve consistency among the transcribed cases, they studied and reached agreement on specific examples from one of the transcripts. Most coding disagreements involved differentiation of procedural and declarative knowledge, Consistency in assigning the declarative code for type of knowledge was achieved by agreement that only clear, explicit indication of such factual knowledge constituted sufficient evidence of declarative knowledge. Questions were also raised with regard to coding of textbook use. It was agreed that textbook searches would be coded "appropriate" when the search appeared to be directed at appropriate information even if the search did not result in the recovery of such information. Inappropriate textbook information occurred only when students misused specific examples, though the "inappropriate" designation would have included misguided searches for inappropriate concepts had such searches occurred. Textbook information was only deemed faulty if it was recorded improperly.

These considerations were applied to coding of the remaining protocols. When they were compared, disagreement remained in less than 10 percent of the cases. These cases were discussed, and agreement was reached in each case.

### **Time Frame**

Total duration of the study was approximately eleven weeks during the spring semester. Administration of the treatment began during the second week of the course

and ended after the second examination, which was administered during the twelfth week of the course. The examinations were a regularly scheduled part of the course and were given to all students at the same time during a regular class period. Students completed four of the homework assignments before the first examination, and five additional homework assignments before the second examination.

Think aloud protocols were obtained from eight students during the two weeks after the completion of the second examination. An hour was scheduled for each protocol.

## **Chapter IV: Results**

The two research questions were addressed by analyses of physics understanding (TPU) scores, questionnaire (LAQ) data, and think-aloud protocols. This chapter is a presentation of the analytical results beginning with a summary of the descriptive statistics for each variable included in Research Question 1 and the presentation of the specific statistics used to answer Question 1..

**Question 1: What are the differences, if any, in students' attainment of sound understandings relative to their use of scaffolded or non-scaffolded problems, meaningful learning orientation, and the interactions between these variables?**

Following these analyses, the qualitative data from think-aloud protocols are described. To a limited extent the qualitative data also apply to Question 1, but the main function of the qualitative data is in response to Question 2.

**Question 2: What are the differences, if any, in students' problem solving approaches based on use of scaffolded or non-scaffolded problems?**

### **Statistical Analysis**

#### **Descriptive Statistics**

**Treatment groups.** The original sample of approximately 110 students was divided into scaffolded and non-scaffolded treatment groups according to recitation



section assignments. The final number of students in the scaffolded treatment group was 23. The final number in the non-scaffolded treatment group was 25.

The Mechanics Baseline Test (MBT) was administered at the beginning of the study to gauge students' prior knowledge of formal physics concepts. Scores on the 26 point MBT ranged from three to 18 with a mean of 9.3. A t-test, performed to determine whether the groups differed, revealed no statistically significant difference ( $p = .065$ ), as shown in Table 4. However, the result approached significance at the  $p < .05$  level, so the MBT was used in further analysis as a covariant.

Table 4

Comparison of Mechanics Baseline Test (MBT) Scores

<u>Treatment</u>	<u>Mean</u>	<u>Std.Error</u>	<u>t</u>	<u>p</u>
Scaffolded	10.26	.67	1.89	.065
Non-scaffolded	8.40	.71		

Learning orientation. The Learning Approach Questionnaire (LAQ) was the Likert-scale instrument used to measure learning orientation. Thirteen items on the questionnaire comprised the meaningful orientation subscale (LAQM) and 11 items on the questionnaire comprised the rote orientation subscale (LAQR). The LAQM scores ranged from 23 to 43 with a sample mean of 32.4 ( $SD = .8$ ). The LAQR scores ranged from 16 to 38 with a sample mean of 23.6 ( $SD = .8$ ). The scores according to treatment group are described in Table 5. All means for scaffolded and non-scaffolded groups were within the range of standard error for the total sample mean, thus there is no difference between groups on either the LAQM or the LAQR.

Table 5

Learning Orientation Scores by Treatment Group

<u>Treatment</u>	<u>Min</u>	<u>Max</u>	<u>Mean</u>	<u>Std.Error</u>
<b>LAQM Scores</b>				
Scaffolded	23	43	32.7	1.2
Non-scaffolded	23	42	32.1	1.0
<b>LAQR Scores</b>				
Scaffolded	16	33	23.1	0.9
Non-Scaffolded	16	38	24.0	1.2

Students were assigned to learning approach categories of high, meaningful, rote, or low based on LAQM and LAQR scores. Table 6 shows the numbers in various categories.

Table 6

Distribution of Students in Learning Approach Categories According to Treatment

	<u>High</u>	<u>Meaningful</u>	<u>Rote</u>	<u>Low</u>	<u>Total</u>
<b>Treatment</b>					
Scaffolded	3	8	2	10	23
Non-scaffolded	4	9	4	8	25
Total sample	7	17	6	18	48

**Physics understanding.** The Test of Physics Understanding (TPU) is a 12 point instrument intended to measure students' sound understanding of physics topics. Scores on the TPU ranged from one to ten with a mean of 5.65 (SD = 2.46). The range was identical for the treatment groups. The means according to treatment and learning approach are shown in Table 7. The means displayed are the treatment means adjusted by covariance with the MBT.

**Table 7**

**Descriptive Statistics for the Test of Physics Understanding According to Treatment and Learning Approach**

	Covaried	Std	Std	95%	
	Mean	Error	Dev	Confidence Interval	
				Low	High
Scaffolded	7.14	.56	2.15	6.02	8.27
High	7.03	1.24	1.00	4.53	7.59
Meaningful	7.45	.81	1.98	5.82	9.08
Rote	7.87	1.52	2.83	4.80	10.94
Low	6.22	.68	2.44	4.85	7.59
Non-scaffolded	4.35	.47	2.06	3.39	5.30
High	4.36	1.07	1.26	2.19	6.53
Meaningful	4.55	.71	3.09	3.11	5.99
Rote	3.47	1.09	1.26	1.27	5.67
Low	5.01	.77	1.13	3.46	6.57
Total sample					
High	5.70	.81	1.72	4.04	7.35
Meaningful	6.00	.54	3.03	4.91	7.09
Rote	5.67	.93	1.26	3.79	7.54
Low	5.62	.51	1.13	4.58	6.65

The continuous data for LAQM, LAQR, TPU, and MBT scores was further examined, and details are contained in Appendix G.

### Analysis of Covariance

A factorial analysis of covariance compared independent variables of treatment (scaffolded or non-scaffolded) and learning approach (high, meaningful, rote, or low) and the interaction of these variables on the dependent variable, TPU score, which was covaried with MBT score. Initial tests for homogeneity of variance indicated that a common slope for MBT between treatments ( $F = .069$ ,  $p = .996$ ) and among learning approach categories ( $F = .396$ ,  $p = .902$ ) could be assumed. Results of the analysis of covariance are displayed in Table 8.

Table 8

#### Factorial Analysis of Covariance of the Test of Physics Understanding (TPU)

<u>Source</u>	<u>df</u>	<u>mean square</u>	<u>F</u>	<u>sig</u>
Corrected model	8	13.320	2.911	.012
Intercept	1	109.460	23.926	.000
MBT	1	5.274	1.153	.290
Treatment	1	64.940	14.195	.001
Learning approach	3	1.429	0.094	.963
Treat * L'approach	3	4.227	0.924	.438
Error	39	4.575		
Total	48			
Corrected Total	47			

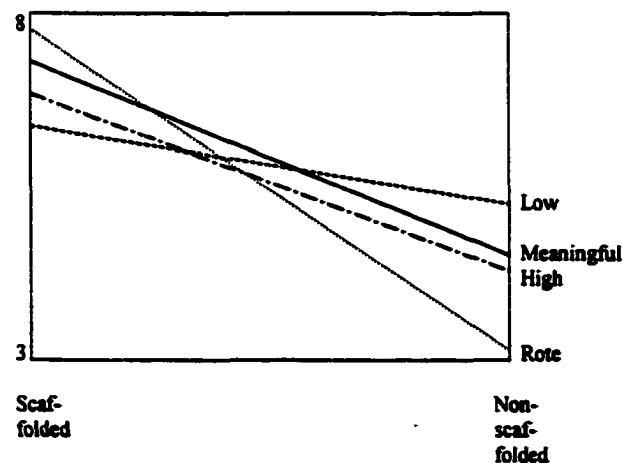
A main effect difference in TPU scores was found between the scaffolded and non-scaffolded treatment groups ( $p = .001$ ) The students who used scaffolded problem assignments evidenced significantly greater understanding of physics concepts than students who had done the non-scaffolded assignments.

There was no main effect difference among learning approach categories ( $p = .963$ ). No significant differences were indicated among students who tended to use rote approaches, those who tended to use meaningful approaches, those who used both approaches, and those who showed no strong use of either rote or meaningful approaches to learning.

There was also no significant interaction between learning approach and treatment ( $p = .438$ ). The relationship of the treatment to student TPU scores was independent of the learning approach. The similar effect of treatment on meaningful and rote approaches is evidenced by the similarity of slope in Illustration 1.

Illustration 1

Comparison of TPU Means for Learning Approaches by Treatment



### Analysis of Think-Aloud Protocols

Differences in students' problem solving approaches were revealed by analysis of think-aloud protocols and responses to accompanying interview questions. A description of the eight students who participated precedes the analysis of the protocols for each problem.

#### The Protocol Problem Solvers

In order to obtain a more complete picture of the problem solvers, the interviewer asked several questions before and after the protocol was taken. These questions are listed in Table 9.

Table 9

#### Questions Accompanying Protocol Collection

##### Before

What is your class standing and your major?

What is your background in science courses, both high school and college?

How did you prepare for the last exam?

##### After

If you'd been working on homework problems, how would it have been different than what you just did here?

What do you normally do to check your homework answers?

Do you have any other comments about homework?

---

Seven of the students were engineering majors and one was majoring in meteorology. Their science backgrounds were all similar with the exception of N2, who was retaking the physics course after making a grade of "D" in the previous semester.

Analysis of responses to the question about preparation for the most recent examination revealed five categories. Students identified the following strategies:

1. Worked on sample test problems
2. Reviewed homework problems
3. Went to review session
4. Reviewed textbook
5. Prepared "cheat sheet"

The "cheat sheet" was a one page list of equations or other information that each student was allowed to prepare for use during each examination.

There were six different basic responses to the question about checking answers.

Students suggested the following strategies:

1. Work the problem a different way
2. Decide whether signs make sense
3. Decide if the magnitude is reasonable
4. Check for appropriate units
5. Repeat the problem using the same method
6. Compare answers with other students

Students who had done scaffolded problems named almost twice as many checking strategies as students from the non-scaffolded group.

Table 10 is a summary of information from the questions and from the instruments administered at the beginning of the study. Sample means for the instruments are given in parentheses. The labels N1, N2, N3, and N4 were assigned to problem solvers whose homework assignments were comprised of non-scaffolded

Table 10

Information about Protocol Problem Solvers

	N1	N2	N3	N4	S1	S2	S3	S4
<u>Test Preparation</u>								
Sample problems	X			X	X	X	X	X
Homework problems			X			X		X
Review session				X				
Textbook		X					X	X
"Cheat sheet"	X		X					
<u>Checking</u>								
Different way							X	X
Signs					X	X		X
Magnitude						X		
Units		X			X			
Repeat problem			X		X			
Compare answers	X		X	X			X	
<u>Scores (mean)</u>								
MBT (9.3)	8	9	15	11	16	6	13	6
TPU (5.6)	7	4	5	5	9	7	10	7
LAQM (32)	34	33	34	29	39	30	39	37
LAQR (24)	24	27	29	17	19	22	16	30



problems, and the labels S1, S2, S3, and S4 were assigned to problem solvers whose homework assignments included scaffolded problems.

### Transcript Codes

Transcripts of the problem solving protocols were coded with a series of three codes. The results of the coding are shown in Tables 10 and 11. Within each table, three letters represent the coding for individual statements. The first of the three letters is the coding of purpose: R for representation, S for solution, and C for check. The second letter is the coding for knowledge description: P for procedural, D for declarative, and B for textbook reference. Subscripts were added to the textbook code to denote searches for specific ( $B_S$ ) or nonspecific ( $B_N$ ) information. The third letter is the further coding of knowledge: A for appropriate, F for faulty, and I for inappropriate.

### First Protocol Problem

The first protocol problem concerned an object moving against frictional force on an inclined plane. The complete problem is found in Appendix F. The most direct solution of both parts of the problem involves use of energy considerations, though the problem can also be solved with Newtonian equations. Examples of each of these solutions is given in Appendix H. Appendix I provides a summary of student solutions.

As seen in Table 11, the purpose of the majority of steps in the protocols was representation followed closely by solution steps. The proportions were approximately equal for students from both treatment groups. However, differences between the groups occurred in the total number of steps: 100 for the non-scaffolded students and 165 for the scaffolded students.

Table 11

Tally of Think-Aloud Protocol Codes for Problem 1

Codes:	Representation	R	Procedural		P	Accurate		A
	Solution	S			D			F
	Checking	C	Textbook, specific		B <sub>S</sub>	Inappropriate		I
			Textbook, nonspecific		B <sub>N</sub>			
	N1	N2	N3	N4	S1	S2	S3	S4
RPA	8	3	13	5	17	15	13	15
RPF	0	1	4	3	4	1	3	1
RPI	0	1	2	2	0	0	0	0
RDA	1	0	0	0	2	1	2	2
RB <sub>S</sub> A	0	1	3	2	1	0	0	0
RB <sub>S</sub> F	0	0	1	0	0	0	0	0
RB <sub>S</sub> I	0	0	0	1	0	0	0	0
RB <sub>N</sub> A	0	4	1	0	0	0	0	0
RB <sub>N</sub> F	0	1	1	0	0	0	0	0
RB <sub>N</sub> I	0	1	0	0	0	0	0	0
SPA	10	3	7	4	17	17	11	14
SPF	2	0	2	6	0	2	1	2
SPI	1	0	0	4	0	0	0	0
SDA	0	0	0	1	1	0	1	1
CPA	0	0	0	0	3	3	2	1
CPF	0	0	0	0	0	1	2	1
CDA	0	0	0	0	3	1	1	0
CDF	0	0	0	0	1	0	0	0
CB <sub>S</sub> A	1	0	0	0	0	2	0	0
Total	23	15	34	28	49	43	36	37

Other differences occurred in the use of the book and in the checking phase of problem solving. Non-scaffolded students referred to the book 16 times for representation and once for checking. Scaffolded students referred to the book once during representation and twice during checking. The single use of the textbook for checking was the only example of checking by a non-scaffolded student, whereas there were 21 examples of checking among scaffolded students.

The problem was completed successfully by S1, S2, and S3. S3 used energy considerations and S1 used energy considerations for the second part of the problem. The first part of the problem was also completed successfully by N3 and S4, who used Newtonian equations. N2 attempted to use energy considerations, but was unable to find enough information in the book to complete the representation. N4 was successful in completing the representation using Newtonian equations, but then brought in other equations and ignored the original representation.

N1 found a value for time in the first part of the problem and stopped without realizing that he had not completed the solution. S4 made a similar error in the second part of the problem.

### Second Protocol Problem

The second protocol problem involved three objects in motion: one in a straight line, two in rotation. As in the first problem, the solution was reached most directly by applying energy considerations, but Newtonian equations of motion were also applicable. Examples of these solutions are given in Appendix H. The problem is given in Appendix F.

As seen in Table 12, representation was the purpose of the great majority of steps. Non-scaffolded students used a total of 51 steps, including 41 representation steps. Scaffolded students used a total of 79 steps, 56 of them for representation. Non-scaffolded students referred to the textbook 14 times during representation, compared to nine times by scaffolded students.

S3, who used energy considerations, was the only student who completed a successful solution of the problem. All others used a Newtonian approach. Among the other problem solvers, N1, N3, and S4 arrived at solutions without considering the influence of the sphere. Among these three problem solvers, N3 alone wrote an expression for the inertia of a sphere but did not make further use of the expression. Instead, N3 found an example of a pulley and falling mass in the textbook, from which he copied an inappropriate expression for acceleration. N1 and S4 simply ignored the existence of the sphere.

S1 and N4 included the sphere in their representations, but did not reach solutions. Several pieces of S1's representation were correct, but there was a failure to treat the three bodies separately when adding forces or torques. N4, however, considered only forces, not the torques which were needed for dealing with rotating bodies.

Finally, N2 and S2 failed to complete their representations. Both spent much time in nonspecific book searches for helpful examples as did N3.

Many examples of differences appeared between non-scaffolded and scaffolded problem solvers in their approaches to problem solving. The differences became less pronounced when students dealt with the second, more difficult problem. A summary of the clearest differences is given in Table 13.

Table 12

Tally of Think-Aloud Protocol Codes for Problem 2

Codes:	Representation	R	Procedural		P	Accurate		A
	Solution	S	Declarative		D	Faulty		F
	Checking	C	Textbook, specific		B <sub>S</sub>	Inappropriate		I
			Textbook, nonspecific		B <sub>N</sub>			
	N1	N2	N3	N4	S1	S2	S3	S4
RPA	7	4	2	4	11	9	5	7
RPF	2	1	0	1	1	1	1	1
RPI	0	0	0	2	3	1	0	1
RDA	1	0	0	0	0	1	2	2
RDF	0	0	0	0	1	0	0	0
RB <sub>S</sub> A	1	3	2	1	3	2	1	0
RB <sub>S</sub> F	0	1	1	0	0	0	0	0
RB <sub>S</sub> I	0	0	2	0	0	0	0	0
RB <sub>N</sub> A	1	3	2	0	1	2	0	0
SPA	2	2	2	0	7	1	5	4
SPF	0	0	0	0	2	0	2	1
SPI	2	0	1	0	0	1	0	1
SDI	0	1	0	0	0	0	0	0
CPA	0	0	0	0	1	0	1	0
CPF	0	0	0	0	0	0	1	0
Total	16	15	12	9	30	18	18	17

**Table 13**

**Differences in Problem Solving Approach**

<b>Group</b>		<b><u>Non-scaffolded</u></b>	<b><u>Scaffolded</u></b>
<b>Representation Steps:</b>	<b>Problem 1</b>	100	165
	<b>Problem 2</b>	41	56
<b>Checking Steps:</b>	<b>Problem 1</b>	1	21
	<b>Problem 2</b>	0	3
<b>Textbook References:</b>	<b>Problem 1</b>	17	3
	<b>Problem 2</b>	14	9
<b>Declarative Knowledge:</b>	<b>Problem 1</b>	2	16
	<b>Problem 2</b>	2	6

---

## **Chapter V: Discussion and Conclusions**

**The study was an exploration of student use of scaffolded problems. One group of students received nine homework assignments with scaffolded problems and another group received nine homework assignments with routine textbook problems that addressed the same physics concepts. The exploration probed subsequent student understanding of physics concepts and their subsequent approaches to problem solving. It was hypothesized that student learning orientation, either meaningful or rote, might be differentially related to student attainment of understanding of physics concepts or interact with the scaffolded and non-scaffolded treatments.**

### **Discussion of Question 1**

**What are the differences, if any, in students' attainment of sound understandings relative to their use of scaffolded or non-scaffolded problems, meaningful learning orientation, and the interactions between these variables?**

**Two sources of data supplied evidence related to Question 1. Statistical analysis of students' scores on the Test of Physics Understanding (TPU) revealed a significant main effect difference between scaffolded and non-scaffolded treatment groups. There were no main effect differences and no interactions related to learning approach categories.**

Students' problem solving protocols were the second source of data. Compared to non-scaffolded students, protocols obtained from scaffolded students contained many more examples of declarative knowledge associated with the problem solving process.

### Between Group Differences

Scaffolded versus non-scaffolded problems. The most important finding of the study was the difference between understanding of physics concepts by scaffolded and non-scaffolded students. Students who solved scaffolded homework problems evidenced significantly ( $p < .001$ ) greater understanding of physics concepts as measured by the TPU. TPU scores were adjusted by taking into account student's initial understanding as measured by the MBT, and still the mean scores between the scaffolded and non-scaffolded groups were separated by more than one standard deviation with an effect size of 1.3. This result was somewhat surprising considering that homework is only one component of the course. A leveling effect might be expected from other course components: lecture, participation in recitation sections, sample test problems and outside assistance. The strength of the scaffolding treatment appeared to outweigh such a leveling effect.

Scaffolded problems may have contributed to students' understanding of physics concepts by making knowledge connections explicit. The students in this study were often prompted to connect declarative knowledge by answering questions that required declarative knowledge answers. For instance, after an applicable formula had been identified by a student, the next step in a scaffolded problem often asked for factual information connected with the formula. Highlighting declarative information with



questions seemed to work well in the step-by-step format of scaffolded problems. The results were similar to those of Leonard et al. (1996), who highlighted declarative knowledge within descriptive paragraphs about problem solving strategies presented after student problem solving assignments were complete; their students were better able to identify the concepts associated with physics problems. It also echoes the results of Reif and Heller (1984), who highlighted declarative knowledge in their oral questioning of individual students and found increased student ability to do problem representation, a probable indication of increased understanding of the relevant concepts. The scaffolded problems used in this study likewise highlighted the declarative knowledge, but the highlighting became part of the problem solving process and was adapted to meet the needs of a large class rather than particular individuals.

Another possible benefit of scaffolded problem solving was the emphasis on representation and checking, phases in problem solving that call on qualitative assessment that, in turn, involves connected declarative and procedural knowledge. Instead of following a means-ends search for applicable formulas to manipulate, scaffolded students were prompted to make a systematic representation of problems. They were also prompted to use a variety of methods for evaluating or checking their answers. The practice in problem representation and checking may have led to increased ability to identify the physics concepts related to the TPU problems that appeared on their examinations. The physical representation of a problem, that is the interpretation of the problem situation in terms of physics concepts, provided concrete practice in attaching concepts to procedures. Qualitative checking, an important feature of scaffolded problems, often involves a similar interpretation in terms of physics

concepts. The hypothesis that scaffolded representation and checking would produce such a result, is supported by evidence from the think aloud protocols; scaffolded students were much more likely than non-scaffolded students to produce a problem representation without resorting to a textbook search for formulas. They were also far more likely to engage in checking their work. Other evidence from the think aloud protocols is considered in the following section.

Protocol problem solvers' understandings of physics concepts. Another indication of students' sound understanding of physics concepts appeared in the expressions of declarative knowledge in association with problem solving procedures during the think aloud protocols. The expression of declarative knowledge by protocol problem solvers occurred predominately among scaffolded problem solvers, who overtly linked declarative knowledge to problems. The difference was particularly pronounced during problem representation, where there were thirteen instances of declarative statements by scaffolded problem solvers compared to two instances by the non-scaffolded problem solvers. The evidence supports Zajchowsk and Martin's (1993) claim that strong students attempt to use declarative knowledge. The strong students in this study were the scaffolded students as indicated by the TPU scores shown in Table 10. The group differences in TPU scores that appeared in the statistical analysis (Analysis of Covariance) predicted the same trend found among the protocol problem solvers. The MBT scores that measured physics knowledge at the beginning of the study were almost identical for non-scaffolded problem solvers (43 total MBT points) and the scaffolded problem solvers (41 total MBT points). The TPU scores, however, show a wide difference; the four scaffolded problem solvers scored about 60% higher than the

four non-scaffolded problem solvers. Thus, students' coincident expression of declarative knowledge may be an alternate measurement for gauging their understanding of physics concepts.

This is not to say that unstated declarative knowledge did not exist among all of the protocol problem solvers, but it is doubtful that such unstated knowledge would have existed to a greater degree among non-scaffolded students. It is likely, for instance, that all of the protocol problem solvers knew that the final velocity was zero in the first protocol problem because the jar came to a stop, but only N1 and S4 made overt statements of that declarative knowledge. It is less likely, however, that all students possessed S4's declarative knowledge that normal force and the parallel component of gravity must balance, "...because the jar does not leave the incline." Most of the problem solvers displayed the procedural knowledge that the normal force and the component of gravity in question should be written with an equality sign between them, but the declarative knowledge of S4 cannot be assumed, and though the declarative knowledge could exist as unstated knowledge, unstated knowledge was probably no more prevalent among non-scaffolded problem solvers than among scaffolded problem solvers. The tentative conclusion is that declarative knowledge was expressed more often by scaffolded problem solvers because they had linked the declarative and procedural knowledge in sound understandings of the physics concepts involved.

It is also possible that at least part of the inclusion of declarative knowledge statements was a result of habit. Scaffolded students may have become accustomed to seeing declarative knowledge as part of problem solution and thus included such statements more naturally in their solutions. If this is the case, the appearance of

declarative knowledge may indicate the formation of a habit of mind. It became clear from the analysis of protocols that other habits had been established, a subject which will be treated later in this chapter in the discussion of the Question 2. Perhaps statement of declarative knowledge is a similar case of habit formation, and such a habit could lead to continued association of declarative knowledge during problem solving in other contexts.

In summary there are definite differences between the treatment groups in students' understanding of physics concepts, and the differences are supported by evidence from the think aloud protocols. Other possible between group differences should also be considered, but an undetected difference seems unlikely. The possibilities are discussed in the following section.

Differences in treatment groups. It is possible that the treatment groups were different in some respect other than initial understanding, which was statistically controlled by covariance with MBT scores. One circumstance that might have generated a group difference was the change of students from one treatment group to the other. Such switches were expected on the basis of the student resistance to scaffolding methods reported by Heller & Reif (1984), and students were free to switch treatments. The characteristics of students who switched treatments were examined to assure that the students involved did not share a particular variable that contributed to treatment group switching.

Fifteen students assigned to the scaffolded treatment changed to the non-scaffolded treatment. Only one student changed from non-scaffolded to scaffolded homework. Thus, there was a possibility that self-selection somehow changed the nature

of the scaffolded treatment group. However, the fifteen students who changed to non-scaffolded homework problems were checked for differences in MBT and LAQ scores, and no differences were found between their scores and those of the students who did not switch. Thus it can be assumed that initial understanding and learning orientation were not major reasons for the switches and that the differences, if they existed, lay elsewhere. Other possibilities must be considered.

Time spent on homework was a potential difference between groups, so self-reported times spent on individual homework problems were tallied and compared after the second homework set. No differences were found, although the professor for the course felt that, on the basis of anecdotal evidence, non-scaffolded students might be spending more time on homework. This was supported by evidence on several time reporting forms where non-scaffolded students did not fill in times for the more difficult problems; instead they wrote comments such as “way too much.” Such remarks may indicate inefficient use of time by non-scaffolded students. The time spent by protocol problem solvers in nonspecific textbook searches may well mirror time consuming homework situations.

It can also be surmised from the statements of protocol problem solvers that time demands did not differ greatly. In response to the interview question which asked for further comments on homework problems, the considerable time demands of physics homework were spontaneously mentioned by N1, N2, N4, S1, and S2. Students from both treatment groups asserted that homework required a major time commitment.

Though real time demands were probably similar between groups, perceived time demands may have been different. Student comments during the treatment period

suggest that time demands of the homework problems were a factor in some of the switches in treatment. A written comment on the homework submitted by one of the scaffolded students stated that the non-scaffolded assignments were rumored to be much shorter. A likely explanation is that the honors group, a separate recitation section not included in the study, was also assigned non-scaffolded homework. Within the honors group there was a subgroup of seven students who reported very low homework times. These students may have influenced others by their comments, and it may have appeared to some students in the scaffolded group that non-scaffolded problems carried much lower time demands.

Students might also have been influenced to switch from scaffolded to non-scaffolded treatments because help was sometimes available from the honors students, who were doing non-scaffolded homework. In addition, the teaching assistant, who had received no special preparation for use of scaffolded problems, was reportedly more comfortable with the non-scaffolded problems and may have inadvertently encouraged students to switch treatments, because it was easier to assist students with non-scaffolded homework problems. If students changed treatments because help was more readily available, there is a chance that these tended to be weaker students. However, MBT scores, which measure prior knowledge, were not lower for this groups, and prior knowledge would presumably have been low among very weak students.

In conclusion, it seems likely that those who switched treatments did so for a variety of reasons and did not comprise a special group. Thus, the remaining scaffolded group was probably not different from the non-scaffolded group in general characteristics or in time spent on homework. There is a high probability that the

differences in physics understanding were the result of the differences in treatment. Evidence from the analysis of problem solving protocols supports this conclusion.

### **Learning Approach**

**Main effect differences by learning approach.** The students in this study were divided into four categories for analysis of their learning approaches. High or low labels were assigned to each score on the LAQM and LAQR. Scores were designated high only if they were at least 20% above a neutral score that indicated neither agreement nor disagreement on a Likert scale. Thus a “high” score on the LAQM or the LAQR meant that the student had strong responses to items designed to assess meaningful learning and rote learning approaches respectively. Though the scoring criterion for meaningful learning was high, half of the students in the study had “high” scores on the LAQM, and the average LAQM score for the sample (32.4) was above the threshold for the meaningful learning category. These introductory physics students as a group had a strong tendency to choose meaningful approaches to learning.

By comparison, only six of the 48 students in the final sample were rote learners. Thus, their tendency to choose rote learning approaches was quite pronounced. There may be several reasons for the low number of rote learners in this study. Perhaps rote learners are in general more rare than meaningful learners, or rote learners may be rare among those who choose higher education or among students who have been successful in the science and mathematics courses that might lead them to choose careers that are science oriented. In any case, the average LAQR score for the sample (23.6) was well below the threshold for the rote learning category. The introductory physics students as a group did not tend to choose rote learning approaches.

Students in the meaningful learning category had high LAQM scores, and students in the rote learning category had high LAQR scores. There was a third category of students that had high scores on both scales. These “high” students may have been flexible in their choice of approaches. They may also have been highly motivated to use any approach that might lead to mastery of the content.

The other learning approach category was comprised of students who scored below the designated levels on both the LAQM and the LAQR. These were termed “low” students, because their learning approach scores were low in comparison to the rote and meaningful learners. However, the threshold scores had been set at high levels, primarily to produce meaningful and rote categories that were distinctly different. Thus many of those assigned the low learning approach category because they did not meet the criteria set to produce the distinct group difference, actually had LAQM scores that were higher than a neutral level. Among the protocol problem solvers, for instance, N4 and S2 had scores below the threshold levels, yet both had LAQM scores that were above the neutral level. Probably the students in this category had no strong learning preferences, but it is also possible that they were simply less exuberant in their LAQ responses. Because the low group was made up of students who did not meet high meaningful or rote scale criteria, rather than because they showed some lack of learning approach, they should not be treated as a group that might be expected to have distinct group characteristics. However, a distinctive group might emerge if only students with markedly low scores were included.

There was no significant difference in the sound understanding of physics concepts among meaningful learners, rote learners, high scoring learners, and low



scoring learners. The averages by learning orientation category (Table 7) were highest for meaningful learners, which might have been predicted on the basis that meaningful learners seek understanding to a greater extent than other students. However, the statistics were weakened by small numbers, especially in the rote learning category, and there were large standard deviations for the various categories. This variability may indicate that learning approach is unrelated to attainment of sound understandings, but it seems more likely on the basis of previous research (Cavallo, 1996; Williams & Cavallo, 1995), that the relationship between learning approach and sound understanding exists, but it is overshadowed by other factors. It is, for instance, plausible that learners from any category might or might not be motivated to expend the effort needed to learn, and that such effort might be more important than learning approach. A hypothesis that learning approach might be overshadowed by other factors is supported by Elby's (1999) finding that students who indicated that meaningful approaches were useful for attaining understanding of subjects, nevertheless chose more rote approaches when such approaches seemed expedient in the pursuit of good grades.

Interactions. There were also no interactions between learning approaches and treatments. That is, the treatment effect was the same regardless of learning approach. Illustration I shows this result graphically; it is obvious that the slopes for all categories are very similar, indicating that differences that existed between treatments were nearly identical for all categories of learners. The scaffolded treatment was equally effective for all learning approach categories, but it is possible that the reason for the effect may vary among the learning approach categories. Rote learners, for instance, may have benefited from the scaffolded treatment because they were introduced to new and

helpful non-rote approaches. Meaningful learners, may have benefited from the same scaffolded problems because they received assistance that made meaningful learning possible for them as predicted by the Chin and Brown (2000) results indicating that meaningful learning approaches can be prompted in meaningful learners. They received assistance and avoided the frustration noted by Hammer (1989 ) in students who wish to learn meaningfully but are placed in situations where meaningful learning is not fostered.

### Discussion of Question 2

What are the differences, if any, in students' problem solving approaches based on use of scaffolded or non-scaffolded problems?

Distinct differences in approach to problem solving emerged from analysis of the problem solving protocols. One difference, the tendency of scaffolded problem solvers to state declarative knowledge associated with problem solving procedures, was discussed in connection with the first research question. The following sections are devoted to discussion of two other outstanding differences: textbook use and problem checking. A difference in the number of steps used in problem solving is also addressed. The discussion of Question 2 concludes with consideration of meaningful problem solving as evidenced in the protocols.

### Differences in Textbook Use

Contrast in textbook dependence. The most unexpected result in this study was the difference in students' use of the textbook during the problem solving protocols.

Before each problem solving protocol began, the researcher directed the student to use the textbook as needed, and to explain what they were doing so that textbook use would be a clear part of the audiotape. The textbook was easily accessible to all problem solvers, yet the four scaffolded students only referred to it three times during the solution of the first problem: once during representation and twice during checking. (See Table 11.) In contrast, the four non-scaffolded problem solvers referred to the textbook 17 times during solution of the first problem. They made 16 of the references during problem representation and one reference during checking. The differences were not as pronounced during the second problem solution; non-scaffolded students used the textbook 17 times versus 10 textbook references by scaffolded students. (See Table 12.) During solution of both problems, non-scaffolded problem solvers were much more dependent on the textbook.

One reason for a number of textbook references in the second problem was the need to find an expression for the rotational inertia of a sphere. N1 and S4, who did not consider the sphere in their representations, did not use the textbook for this purpose, but the mundane search for the sphere expression accounts for three references among non-scaffolded students and three among scaffolded students. Eliminating these references, non-scaffolded students made 14 textbook references compared to seven by scaffolded students, a stark contrast in students' dependence on the textbook for knowledge that was not at their command. However, the numbers alone do not adequately reflect the difference in textbook references during solution of the second problem.

**Textbook use on a difficult problem.** The complete coding scheme that represents problem solving steps (Tables 11 and 12) illuminates the quality of the textbook references. It was plain that some references to the textbook were much different than others, and that the knowledge sought in the textbook fell into two subcategories: specific and nonspecific. Specific textbook knowledge was targeted by problem solvers when they initiated textbook searches. For instance, in specific textbook references students stated that they were looking for the equations for motion or for a particular recalled example. A specific search indicates that the student had at least partial knowledge of a target. The assumption of partial knowledge does not apply when the student sought inappropriate knowledge such as an equation not applicable to the problem, but faulty textbook knowledge is included in this discussion as partial knowledge. In cases of faulty knowledge, the information was applicable to the problem, but was simply not recorded properly by the problem solver.

The scaffolded students showed partial knowledge in seven of the ten textbook references made during solution of the second problem; that is, seven of the references were specific and appropriate. Non-scaffolded students also made seven specific appropriate textbook references during representation of the second problem, and they made two more specific references that were faulty. Thus, there is not a great deal of difference between the scaffolded and non-scaffolded students in the evidence of partial knowledge with reference to the second problem. They were similar in their use of the textbook to augment the knowledge they possessed or partially possessed. The difference occurred in textbook references that did not indicate partial knowledge. There were eight such instances among non-scaffolded students compared to three instances

among scaffolded students. In two of the instances, N3 sought specific but inappropriate knowledge. All of the other cases were nonspecific searches for any information that might help in solving the problem. Nonspecific searches were made by N1, N2, N3, S1, and S2. N2 and N3 had also made such searches during solution of the first problem.

Nonspecific textbook searches were generally very time consuming operations that involved perusal of examples and equations. They were rarely productive and often seemed to be the path of last resort. As S2 concluded, "I'm just pulling things out of the air." It is reasonable then, that more students would engage in nonspecific textbook searches on a difficult problem, where they had less command of applicable knowledge. This explains the appearance of nonspecific searches among scaffolded students during the second problem solution, though it was absent during the solution of the first problem. Scaffolded students were capable of doing the representation of the first problem with only minor reference to the textbook, and three of the four scaffolded students successfully completed the first problem. The fourth scaffolded student, S4, stopped short of finishing the second part of the problem, but had done a complete representation without reference to the textbook. The demands of the second problem prompted greater textbook use among all students.

Textbook use for non-scaffolded students also rose with problem difficulty, although among these students it may also be a routine part of solving any problem. Non-scaffolded students referred to the textbook an equal number of times for both protocol problems, but the proportion of textbook use was greater for the second problem. They used a total of 80 steps for the first problem, but only 52 steps for the second problem. So, it appears that both scaffolded and non-scaffolded students had to

make greater use of the textbook for a more difficult problem. The difference was in the extent and character of textbook use.

Influence of one student on the analysis. The differences between scaffolded and non-scaffolded problem solvers are obvious, but there is an anomaly to be considered in the number of textbook references. The pattern of textbook use, or at least the pattern of nonspecific searches, was somewhat skewed by one student, N2. Nine of the 14 nonspecific textbook searches initiated by non-scaffolded students are attributable to N2, who might have been at a disadvantage because he had failed to solve a very simple warm-up problem that was meant to put students at ease and allow discussion of the protocol process before the process began. It is impossible to know whether the extreme textbook use was the result of nervousness or the result of N2's approach to problem solving. N2 was taking the physics course for the second time and had achieved a fairly high score for prior knowledge (MBT = 9), so facility at problem solving could be expected on the basis of experience, and under different circumstances he might have performed better. Alternately, his low score of understanding (TPU = 4) supports the notion that his problem solving approach was weak, and that he could not cope with any of the problems, including the simple warm-up problem. Though textbook use by N2 was an extreme case, it fits the general pattern of greater textbook use among non-scaffolded students, and the pattern exists independently from N2's contribution.

Possible influences on textbook independence. Almost all textbook references were made during the representation phase of problem solving. Thus, the pattern of low textbook use among scaffolded students suggests that scaffolded students were more capable of independent problem representation than non-scaffolded students. There is

also evidence that the scaffolded treatment may have encouraged a different attitude toward textbook use.

Comparison of statements made by students before and after protocol problem solving suggests that the scaffolded treatment encouraged textbook independence and possibly textbook avoidance. In response to the question about preparing for tests, N1 and N3 mentioned creation of a “cheat sheet,” a summary of textbook information that the professor allowed during the test. This item was mentioned again by N3 and also by N4 during problem solving. It was never mentioned by a scaffolded student; their knowledge seemed to be less dependent on external authority such as a textbook or “cheat sheet,” although S1 suggested in a final comment about homework that a list of equations to accompany the homework would have made the textbook completely unnecessary. S1 viewed this version of the “cheat sheet” as a way to get started, but the three aforementioned non-scaffolded students viewed it as a long term tool. S1’s comment suggests textbook avoidance, a suggestion that was corroborated by observations during problem representation that scaffolded students made several faulty representation steps that could easily have been corrected by reference to the textbook. In fact, the two textbook references made by scaffolded students in checking the first problem, led to corrections of two equations that had been written without reference to the textbook. All of the scaffolded students seemed to avoid textbook use.

The difference in textbook use was unexpected and unintended. The scaffolded problems were written with the assumption that students would make full use of the textbook as a resource. Perhaps they were well enough guided in doing problem representations that they acquired a taste for freedom from shuffling textbook pages.

Non-scaffolded students, on the other hand, exhibited reliance on textbook authority rather than cultivating their own ability to do representation. Textbook reliance may have occurred because there was no guidance that would have led them to believe that independent representation was a desirable goal. Without scaffolding, experience in problem representation was more a matter of locating the right equations and following the right examples, a pattern noted by previous researchers (Chi et al., 1981; Dhillon, 1998; Larkin, 1983; McMillan & Swadener, 1991).

The guidance offered by scaffolded problem solving may also account for the differences in students' tendency to check their work. This possibility is addressed in the following discussion.

### **Problem Checking**

Problem checking was indicated when students questioned a solution or a step in a solution and then took action to answer the question. Evaluative remarks such as, "I don't think this looks right," were not considered checks unless they were followed by an action such as a review of the preceding algebra (procedural check) or a review of the physical ideas involved (declarative check). When checks occurred, students often made changes in their work, and the subsequent steps were not counted as part of the check, but instead comprised additional representation and solution steps. Thus, each checking step that appears in Table 11 or Table 12 indicates an instance of checking that may have initiated a number of related non-checking steps. The tables show that scaffolded students engaged in checking on 24 occasions during the protocols, and there was only one instance of checking among the non-scaffolded students. The non-scaffolded students displayed the same paucity of checking noted previously by Garret et al. ¶



(1990), and it was more pronounced than the weak checking noted among Dhillon's (1998) novices.

The scaffolded homework problems explicitly included a checking phase that prompted students to use means of checking problems that they would have been unlikely to use on their own. The first evidence of the impact is found in Table 10: in response to the question, "What do you normally do to check your homework answers?" Scaffolded students gave nine suggestions compared to only five suggestions by non-scaffolded students. The types of checking advocated by students were also much different. Non-scaffolded students most often suggested comparing answers with another student as a method of checking, and it could be argued that this is not a true check of the problem. For instance, in the protocols checking was defined by a question followed by action, and checking with another student only meets the criteria in very loose terms. Had comparing answers been eliminated, there would have been only two remaining suggestions by non-scaffolded students compared to eight suggestions by scaffolded students. Scaffolded students evidenced a much deeper sense of the possible ways of checking their work.

The remaining evidence of the impact of scaffolding on problem checking is delineated in Tables 10 and 11. The single instance of checking by a non-scaffolded student, N1, was a textbook reference during solution of the first problem in order to check an equation. All scaffolded students used checking steps on the first problem. In total, they made 21 checks on the first problem: two textbook checks of equations, 13 checks of algebraic procedures, and six checks in which they called on declarative knowledge. The stark contrast between students from the two treatments suggests that

problem checking does not occur among students unless they are given specific guidance. Checking is rarely part of textbook examples, and apparently the non-scaffolded students did not consider the possibility of problem checking. Scaffolded students, however, seemed to have acquired the habit of checking their work, a habit that might be associated with a more expert or mature problem solving approach. In particular, their incorporation of declarative knowledge indicates the kind of qualitative analysis that Chi et al. (1981) and Larkin (1983) described in expert problem solvers.

The pattern of checking by scaffolded students is much weaker in the second protocol problem. There are two possible reasons: failure to reach a solution and time constraints. S3 was the only problem solver who reached a correct solution, and S3 engaged in checking at two points. N1, N3, and S4 also reached solutions, but their solutions were far from correct. S4 expressed doubt about the answer, and it is possible that a check would have followed, but in order to allow any time to answer the final interview questions, the protocol was ended. N1 and N3 did not spend as much time reaching their solutions, but in spite of available time, they did not show any inclination to check the solutions. N2, N4, S1, and S2 did not reach solution on the second problem, so very little checking would have been expected, although S1 made a single check of algebra. Thus, the second problem is not inconsistent with the first in that the three instances of checking occurred among scaffolded students, but due to time constraints and incomplete solutions, the second problem adds very little information about checking.

Finally, though problem checking may indicate a mature approach to problem solving by scaffolded students, their use of checking was not necessarily efficient. For

instance, S2 spent a significant amount of time analyzing his answer to the first part of the first protocol problem. In so doing, S2 called on a variety of procedural and declarative knowledge. The mistake turned out to be a simple error in writing down an equation for motion. Had S2 referred to the textbook at the beginning, the error would not have occurred. In a test situation the extra time could have had a serious negative effect. Checking required extra time and accounted for extra steps in problem solving. However, just as Sweller (1989) found that efficiency in problem solving did not always correspond to attainment of understanding, the occasional inefficiency of checking is surely outweighed by the benefits gained by analysis of various answers and problem solving procedures.

#### Number of Protocol Steps

A less substantive difference between the problem solving protocols of scaffolded and non-scaffolded students lies in the number of steps they used in problem solving. The information found in Table 11 shows that the four non-scaffolded students used a total of 100 steps and the four scaffolded students used a total of 165 steps during solution of the first problem. Table 12 shows 52 total steps for non-scaffolded students and 85 total steps for scaffolded students during solution of the second problem. The differences are readily explained in terms of the differences already discussed. For instance, some of the extra steps, albeit a limited number, can be explained by noting that declarative knowledge statements were counted as separate steps in problem solving. Since these statements seldom occurred among non-scaffolded problem solvers, they added to the separation in the number of solution steps. Similarly, differences in textbook use and problem checking led to differences in the number of solution steps.

Textbook use by non-scaffolded students reduced the number of steps attributed to them, particularly in the case of nonspecific textbook searches employed during problem representation. Very lengthy textbook searches were counted as single steps, and textbook independent representation, found mostly among scaffolded students, tended to add steps as students worked through details instead of finding a textbook example or an alternate form of an equation that required less manipulation. Possibly the lengthy book searches could have been divided into several steps as students considered individual examples, but there was limited verbalization during most of the textbook searches, so it would have been difficult to analyze the details.

Problem checking resulted in further added steps. Obviously checking by scaffolded students and lack of checking by non-scaffolded students accounts outright for a number of steps. Other steps were added by scaffolded students as they followed checking steps with new representation and solution steps. In summary, the difference in the number of protocol steps resulted from additions of declarative knowledge, the method of counting textbook searches, and the variety of steps that resulted from checking. This difference in steps does not indicate a difference in approach, but is instead an artifact of other differences.

### Meaningful Problem Solving

The substantial differences in the tendencies of scaffolded students to associate declarative knowledge during problem solving, to independently represent problems, and to spontaneously check their solutions may indicate habits formed while doing scaffolded homework assignments, but the differences also seem to indicate a degree of meaningful problem solving that is lacking among non-scaffolded problem solvers.

Meaningful problem solving is characterized by deliberate or meaningful attempts to discover the underlying relationships in a problem (Ausubel et al., 1978). References to declarative knowledge are direct associations with the underlying relationships in a problem, and the independent problem representation and problem checking are also instances in which the problem solvers work with the underlying relationships implicit in a problem.

Problem representation. Scaffolded students evidenced meaningful problem solving during representation by associating declarative knowledge, but the greatest contrast appeared in the form of lack of meaning among non-scaffolded students. Non-scaffolded students seemed to attach less meaning to their representations. For instance, N1 incorrectly included velocity as a force when summing forces in the first protocol problem. However, the mistake was not carried forward during solution. Writing the sum of forces seemed to be a procedure without meaning and irrelevant to the solution. Likewise, N4 included velocity in the sum of forces and explicitly stated that velocity was a force. In N4's case it was even more obvious that the summing of forces lacked meaning, because after the equation was written, it was dropped from consideration. N4 realized that it was an appropriate initial step, but did not relate it to the solution steps that involved manipulation of equations. In these cases and in several other instances, non-scaffolded students seemed to view problem representation as a set of steps to be followed in an almost rote fashion.

Possibly the scaffolded students also viewed problem representation as a prescribed set of steps, but they were more able to attach meaning. S3, for instance, who was the only student who reached a successful solution of the second problem, stated

after reading the problem, “I don’t know how to do this.” When the researcher prompted, “What do you know about the problem?”, S3 proceeded with the representation. Apparently S3 counted facility with a set of prescribed steps as “knowing how,” but representation was still possible in the absence of such knowledge.

A meaningful approach to problem solving was clearly adopted by S2, whose low LAQ scores indicated neither rote or meaningful learning orientation. S2’s representation of the first problem culminated in writing down an equation for displacement. The equation was written from memory, and though it contained appropriate terms it was flawed in its details. Thus, the resulting answer was incorrect. S2, however, looked at the answer as a meaningful entity and realized that the magnitude was probably unrealistic. Both declarative and procedural checks ensued, and the problem was finally corrected by reference to the textbook.

Approaches to checking and test preparation. Meaningful approaches among the scaffolded students are also indicated by student responses to the questions about homework checking and test preparation displayed in Table 10. The answers in each of the two cases were listed in order of decreasing meaningfulness. In the case of checking, comparing answers with another student shows the least thought or meaningfulness; there is no attention to the relationships underlying the problem. At the other extreme, solving the problem in a different way demands use of the relationship between two methods. Answers by non-scaffolded students all fall in the less meaningful response categories. The majority of responses among scaffolded students fall in the more meaningful response categories.

Test preparation responses are also listed from in order of decreasing meaningfulness. The responses at the bottom of the list are those that might be associated with rote learning, particularly the “cheat sheet.” At the other extreme, working sample problems probably demands the most thoughtful attention, that is, the most meaningful attention. Again, most responses by scaffolded students fall among the more meaningful categories; all of the scaffolded students stated that they used sample problems for test preparation. Responses among non-scaffolded students were much more scattered, and in two cases included the “cheat sheet.”

Cognitive structure. An explanation for the tendency of scaffolded students to engage in meaningful problem solving may be found in the three requirements for meaningful problem solving stated by Ausubel et al. (1976). Meaningful problem solving, like meaningful learning, requires a meaningful learning orientation, a potentially meaningful task or problem, and relevant extant cognitive structures. If meaningful learning orientation is indicated by scores that were above the neutral point on the LAQM, then the great majority of students in this study had meaningful learning orientations, and since some students found meaning in the problems, they were potentially meaningful. This leaves relevant extant cognitive structure as a variable. It appears that the necessary cognitive structure among non-scaffolded students was too weak for meaningful problem solving. Scaffolded students had stronger cognitive structures.

The scaffolding provided in homework problems may have helped students develop the cognitive structure that made meaningful problem solving possible. The students were novices for whom the task of meaningful problem solving was limited by

weak cognitive structure, and the weak cognitive structure required scaffolding to make it useable. With scaffolding the cognitive structure was strengthened, and students adopted more meaningful approaches such as qualitative checking of problem solutions.

Lack of cognitive structure also explains the difference in performance on the first and second protocol problems. S1 and S2 as well as N1, N2, and N3 resorted to nonspecific book searches in their representation attempts for the second problem. Students generally launched these searches when they were unable to approach the problem in any other way, and the searches were the antithesis of meaningful problem solving. Meaningful problem solving was not possible for most students, because they lacked relevant cognitive structure to deal with the second problem. Among the scaffolded students, the homework scaffolding was apparently insufficient. In fact, several protocol problem solvers, scaffolded and non-scaffolded, mentioned that rotational motion, which was key to solution of the second problem, was inadequately treated in the homework.

#### Other Limitations

As with any study conducted in a natural classroom setting, there are limitations associated with the setting and with the quasiexperimental design. The possibility of influence by the honors group has been discussed, and it is possible that over the history of the eleven-week study other unknown variables may have impacted the participants. Treatments were assigned according to student recitation sections, so self-selection on the basis of factors such as ethnicity, gender, and social ties, may have occurred before the study began. The factors were not considered in the study and their influence, if any, is unknown.



**The limitations of the natural setting included the Test of Physics**

**Understanding, an instrument comprised of items that were part of two teacher-written examinations. Though the instrument had face validity and acceptable reliability, it does not have a history of use and criticism that under girds the validity and reliability of more formal instruments.**

**The protocols also had limitations. Although protocol students were paid for their participation, the protocols depended on student willingness to participate and may have selected for more able or self-assured students. In addition, there were no rote learners among the protocol students, so the protocols produced no information about problem solving among rote learners.**

**During protocol collection the researcher knew in several instances the treatment assignment of the student. Though every attempt was made to avoid bias, it can not be absolutely guaranteed that inadvertent bias did not occur.**

## **Conclusions**

### **Question 1**

**Students who solved scaffolded homework problems evidenced significantly greater understanding of physics concepts. The think-aloud protocols provided supporting evidence that more sound understandings occurred among scaffolded students, because they overtly associated declarative knowledge with procedural knowledge in all phases of problem solving. The protocol analysis also suggests that, without intervention, introductory physics students are likely to rely almost exclusively on procedural knowledge during problem solving.**

Further, there was no differential benefit from the scaffolded treatment. Meaningful learning may be so prevalent among college physics students that variations are inconsequential, and meaningful learning approaches, as suggested by Chin and Brown (2000) appear when students are prompted. In this study scaffolding provided the prompts.

## Question 2

There were three pronounced differences in students' approaches to problem solving associated with their use of scaffolded or non-scaffolded homework problems. The first difference, overt expression of declarative knowledge, has been acknowledged. The second difference was that non-scaffolded students used the textbook to a much greater extent than scaffolded students. They were especially inclined to use the textbook in inefficient, time consuming searches for nonspecific information that might fit given situations. Scaffolded students were much more independent in their representation of problems, probably the result of more sound understandings of physics concepts, greater command of procedural knowledge, and habits formed during solution of homework problems that often allowed students to circumvent use of the textbook.

The third pronounced difference in approaches to problem solving was the tendency to check work. Scaffolded students checked their work, and non-scaffolded students did not check. The scaffolded students had incorporated checking in their problem solving approaches, and they had built a varied repertoire of methods for checking. Together, checking and problem representation are areas in which scaffolded

problem solvers had become more like the experts who possess more integrated knowledge than novices.

Integration of declarative knowledge, independent representation, and problem checking all seem to indicate a more meaningful approach to problem solving among students who experienced scaffolded problem solving. Meaningful problem solving was prompted. In addition, meaningful problem solving and more expert approaches to problem solving continued in the problem solving protocols, when no scaffolding was available. Scaffolded students formed habits as the result of scaffolded problem solving, and they continued to express declarative knowledge, make textbook independent representations, and check their problem solving.

### **Implications for Practice**

Students in introductory physics classes can learn mature methods of problem solving such as independent representation and checking, but they need guidance. If students are expected to apply physics concepts, they should be taught how to apply the concepts. This can be accomplished with scaffolded problems. Without such guidance the problem solving process carries little meaning for many students. Meaningful problem solving cannot be assumed even among students who have a tendency to use meaningful learning approaches.

The design of scaffolded problems should be based on careful consideration of the knowledge needed to understand the details and implications of problem steps. This includes both procedural and declarative knowledge, and it is likely to include knowledge that is tacit for the expert who teaches an introductory course. Teachers

should make efforts to become more aware of their tacit knowledge and design problems that make the knowledge explicit for students.

This study, done in a natural setting, suggests that there is unrealized value in problem solving homework assignments when the assignments are adjusted to scaffold students in becoming competent problem solvers who connect procedures with declarative knowledge and thus attach meaning to the problem solving process. The highlighting, organizing, and simplifying characteristic in scaffolding and demonstrated on a limited basis by other research (Catrombone, 1995; Chin & Brown, 2000; Dufresne et al., 1992; Heller & Reif, 1984; Huffman, 1997; Leonard, et al., 1996; Maloney, 1997; Mestre et al., 1993; and Quilici & Mayer, 1996) may be added to homework assignments to make a positive contribution to students' attainment of sound understanding.

### **Further Research**

Further research should include replication of the study with a larger number of students. Although the difference between treatment groups was pronounced, the natural setting is unique, so it is important to repeat the study in other settings. Replication that precludes students changing treatments would be especially valuable. Replication with a larger sample would allow statistical consideration of additional variables and further consideration of the effect of learning orientation. A replication study could also be modified with addition of a proven instrument to measure understanding of physics concepts. The Mechanics Baseline Test, for instance, could be divided and half used as a pretest, the other half as a posttest. Research could include comparison of such an

instrument with a teacher written test designed to assess specific material addressed in problem sets.

The variables included in a new study should be expanded to include English language proficiency. One of the scaffolded protocol students, a student with limited English proficiency, expressed the opinion that the scaffolded homework problems had been very useful in building understanding. In that student's opinion, the scaffolded problems avoided the complexities of the textbook and unlike the lecture, allowed the student to determine the pace at which material was considered. In the present study, there was no way of knowing whether the treatment groups differed in English proficiency.

The longitudinal effects of scaffolded problem solving should also be investigated. Students who have used scaffolded problems in their introductory mechanics course may establish habits that transfer to work in later courses. They may also have a better understanding of basic concepts that serve as a foundation for understanding more advanced physics concepts.

Implementation of scaffolded problem solving for an entire class is also an important area for research. In order to keep the treatments as separate as possible, this study was done without support for scaffolded problem solving during the lecture portion of the course. It is probable that the effect could have been strengthened if the solution of scaffolded problems had been modeled during the lecture and if the teaching assistant for the course had been trained in the use of scaffolded problems.

Questions also remain about the nature of the scaffolded problem solving effect. What happens during scaffolded problem solving? To what extent is the effect the result

of habits formed during the treatment, and to what extent has declarative knowledge been associated with procedural knowledge? Protocols in this study provided evidence of associated declarative knowledge, but there were only occasional declarative knowledge statements. Interviews wherein protocol problem solvers review and elaborate on their problem solving process might be enlightening.

Finally, questions about homework arose incidental to the study. There were relatively large numbers of students who completed little or no homework. The reasons for and effects of this phenomena should be investigated.

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## **Appendix A**

### **Scaffolded Homework Assignment**

The homework assignment in this appendix is representative of the assignments given as part of the scaffolded treatment. The pages are slightly reduced from the original version.

1. You are building a ramp to be used for sliding 20 kg boxes from one level to another. You want to know the greatest angle with the horizontal that can be used so that a box can be left sitting on the ramp without slipping. You also need to analyze the forces needed to move the box. You already know the coefficient  $\mu_s$  and  $\mu_k$  between box and ramp.

**PART I: Box on the ramp getting ready to move**  
**GETTING THE PICTURE**

- a. Draw the ramp at angle  $\theta$  and show all of the forces on the box. Choose the downward direction of the incline to be the  $+x$  direction. Also draw a free body diagram, but instead of weight, resolve the weight force into  $x$  and  $y$  components (Remember: direction of friction force is always that which resists motion.)

**SOLUTIONS**

- b. Since frictional force depends on the normal force, find the normal force first (In the  $y$ -direction,  $\sum F_y = ma$ , but there's no acceleration.)
- c. Find an expression for the angle,  $\theta_{\max}$ , when the box starts to slip. ( $\sum F_x = ma$ , and  $a = 0$  at the moment just before slipping. Be aware that there is a particular expression for the force of static friction at maximum.)
- d. Suppose  $\theta < \theta_{\max}$ . What is  $F_f$ ? (The  $\sum F_x = ma$  equation still applies, but the maximum value for  $F_f$  doesn't.)

**CHECK**

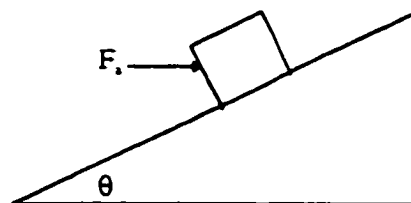
- e. According to the expression in part d, what is  $F_f$  when the box is setting on level ground with no push. (This should make sense.)

**SPECIFIC CASES**

- f. Suppose  $\mu_s = 0.40$ . At what angle will the box start to slip?
- g. Suppose  $\mu_k = 0.30$ . What is the acceleration after the box starts to slip? ( $\sum F_x = ? = ma$ .)
- h. How long will it take for the box to slip 8.0m down the incline? (Review of constant acceleration: begin with an equation relating displacement and time. You know the velocity at the beginning of the descent.)

**PART II: Pushing the box up the incline**  
**GETTING THE PICTURE**

- i. You decide to build the ramp with an angle,  $\theta$ , slightly less than  $\theta_{\max}$ . A force directed parallel with the ground is applied to the box. (Assume the box behaves like a point mass.) Draw diagrams, including free body diagram, showing the forces acting on the box. Change the  $+x$  direction to go with the direction of motion. Show appropriate components of the applied force and the weight.
- j. Why is the direction of  $F_x$  changed from Part I?
- k. Find a new expression for the normal force. (You are now summing 3 forces in  $\sum F_y = ma = 0$ .)



- l. Find an expression for the applied force,  $F_a$ , needed just to get the box moving. (At the moment just before movement, there's no acceleration.)

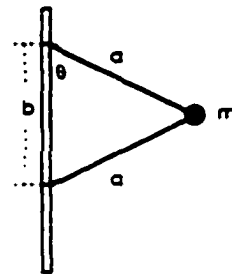
#### SPECIFIC CASE

- m. Going back to the case in part f, what force is needed to just get the box moving?

**CHECK:** The expression for applied force in PART I implies that for some angle  $\theta$ , the applied force would be negative. To see if this makes sense.

- n. In order for the expression to produce a negative number, does the angle become large or small?
- o. What happens to the x and y components of the applied force as the angle increases?
- p. What happens to the contribution of  $F_a$  to the normal force as the angle increases?
- q. In what physical sense is there a limit to the angle?

2. A mass,  $m$ , is attached to a vertical rod by means of 2 strings. When the system rotates about the axis of the rod, the strings are extended with the geometry shown in the diagram, and the tension in the top string is  $T_1$ .



#### GETTING THE PICTURE

- a. What provides the centripetal force in this situation? (I.e., what keeps the mass from flying off in a straight line?)
- b. Given the geometry,  $a$  and  $b$  constant, can you treat  $\theta$  as a known quantity?
- c. Draw a diagram of the system, showing the forces on the mass, then draw a free body diagram with the tension forces resolved into x and y components.

#### SOLUTION

- d. Write an expression for  $\theta$  using arccos. (Keep in mind that you've found  $\theta$ , but use the symbol  $\theta$  as you proceed with the problem.)
- e. Based on the free body diagram, write  $\sum F = ma$  expressions for the x direction (radial) and the y direction. Consider the nature of the acceleration in the x direction (uniform circular motion) and the lack of acceleration in the y direction.
- f. Solve the  $\sum F_y$  expression for the tension in the lower string ( $T_2$ ).
- g. Use the  $\sum F_x$  expression to derive an expression for speed in terms of  $T_1$ ,  $\theta$ ,  $m$  and length  $a$ . (You could eliminate  $T_2$  at the beginning, but it makes things unwieldy – wait until the end.)
- h. In general, what is the relationship of the period of rotation,  $\tau$ , with velocity,  $v$ , and radius  $r$ ?
- i. Find an expression for  $\tau$  in terms of  $T_1$ ,  $\theta$ ,  $m$  and length  $a$ .

#### CHECK

- j. According to the expression in part f, is the magnitude of  $T_2$  greater or less than  $T_1$ ? If the mass increased, would the difference between  $T_1$  and  $T_2$  increase or decrease?
- k. According to the expression in part f, what is happening to tension as period increases?

#### SPECIFIC CASE

- l. If  $a = 1.25$  m,  $b = 2.00$  m,  $m = 4.00$  kg, and  $T_1 = 85.0$  N, what is the tension in the lower string?
- m. What is the speed of the rotating mass?
- n. What is the period of rotation?
- o. How many revolutions per minute does the system make?



3. A carnival worker rides a motorcycle inside a hollow transparent sphere. After gaining sufficient speed, the rider travels in a vertical circle of radius  $r$ . Given the rider's mass of  $M$  and the motorcycle's mass of  $m$ , you will need to find the minimum speed needed to maintain contact with the sphere at the top and the normal force on the motorcycle at the top and bottom of the circle.

PART I: At the top

GETTING THE PICTURE

- Draw a diagram showing the forces on the motorcycle-rider combination at the top of the circle.
- What provides the centripetal force?
- What would the normal force be if the tires just lost contact with the sphere?

SOLUTION

- Write a  $\Sigma F = ma$  equation based on your diagram. Let the equation reflect your part c answer, and let it reflect the fact that the acceleration is centripetal acceleration.
- Solve for  $v$ , and state whether  $v$  would have to be greater or less than this value.

SPECIFIC CASE

- If  $r=20.0\text{m}$ ,  $m=60\text{kg}$ , and  $M=40.0\text{kg}$ , what is the minimum speed?
- If actual speed at the top is  $20.0\text{ m/s}$ , what is the normal force on the motorcycle by the sphere?
- What is the force exerted by the motorcycle on the sphere? (Newton's third law)

PART II: At the bottom

GETTING THE PICTURE

- Draw a diagram showing the situation at the bottom.

SOLUTION

- Write an  $\Sigma F = ma$  equation and solve for  $N$ . (Remember the alternate expression for acceleration in this case).

CHECK

- Given the same velocity, should the normal force be greater at the top or bottom? Your expression in part i should reflect this when compared to the expression in part j.

SPECIFIC CASE

- If the speed at the bottom is  $20.0\text{m/s}$  and other variables are those given in part e, what is the normal force at the bottom of the sphere.
- What force is exerted on the sphere by the tires?

4. A block is projected with an initial velocity  $v_0$ , up a inclined plane with a coefficient of static friction  $\mu_s$  and coefficient of kinetic friction  $\mu_k$ . You will need to find out whether or not the block slides back down the plane, and, if it slides down, its velocity when it returns to the bottom.

GETTING THE PICTURE: You will need separate diagrams for motion up the incline and down the incline, because the forces change somewhat. Picture under what circumstances the block will stick at the top of its path. Think about similarities and differences with constant acceleration problems you've seen before.

SOLUTION: You need to determine an angle  $\theta_{\text{min}}$  at which the block will slide back down rather than slip. Then in order to find a return velocity, you need an expression for distance traveled up the incline. Accelerations are a matter of  $\Sigma F = ma$ , which follows from your free body diagrams.

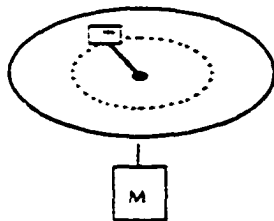
CHECK: The expression for  $\theta_{\text{min}}$  makes sense if it becomes larger as  $\mu_s$  increases. Also, you would expect the return velocity with friction at work to be less than the initial velocity, and the expression you found should reflect this.

5. Mr. And Mrs. Jones trade off driving as they take their RV to their favorite spot on the lake. Mr. J. wants to check the speed at which Mrs. J. rounds a particular unbanked curve on their route, so he suspends a 0.500 kg wrench by its center on a 1.80 m long string from the ceiling at the center of the RV. At equilibrium the string makes an angle of  $37.0^\circ$  with the vertical. Later he determines that the wrench was 50.4 m from the center of curvature of the unbanked curve. What was Mrs. J's speed.

GETTING THE PICTURE: Draw a diagram that includes the radius of curvature and both forces working on the wrench. Write the  $\Sigma F = ma$  equations for both directions.

CHECK : Mr. J. considers 25 mph the maximum safe speed for the curve. The results nearly caused a heart attack.

6. A 310-g paperback book rests on a 1.2 kg textbook. A force is applied to the textbook, and the two books accelerate together from rest to 96 cm/s in 0.42 s. The textbook is then brought to a stop in 0.33 s, during which time the paperback slides off. Within what range does the coefficient of static friction between the two books lie?
7. A mass  $m$  rest on a turntable, as shown in the diagram below. the coefficient of static friction between the mass and the table surface is  $\mu_s$ . Another mass,  $M$ , is attached to a string that passes through a hole at the center of the turntable and is then attached to  $m$ . What are the smallest and largest periods of revolution of the turntable,  $\tau_{\min}$  and  $\tau_{\max}$ , for which  $m$  remains fixed at a distance  $r$  from the center of the turntable?



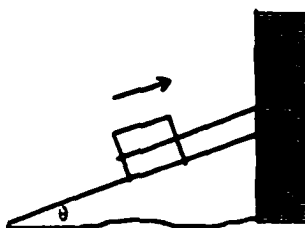
## Appendix B

### Non-Scaffolded Homework Assignment

The homework assignment in this appendix is representative of the assignments given as part of the non-scaffolded treatment. The pages are slightly reduced from the original version.

*Beginning exercises*

1. A boy of mass 35 kg runs into an ice-skating rink in street shoes and begins a smooth slide. He starts with a speed of 4.5 m/s and comes to a stop after sliding for 3.4 s. What is the coefficient of kinetic friction of his shoes on ice?
2. A rope is connected to a 25-kg cement block placed on a board leaning against a wall at an angle of  $25^\circ$  with respect to the horizontal (diagram below). The coefficient of kinetic friction between the cement block and board is  $\mu_k = 0.4$ . (a) What is the tension in the rope if it is pulled at constant speed straight up the board? (b) What is the tension if the rope is pulled up at constant speed at an angle  $40^\circ$  from the horizontal?



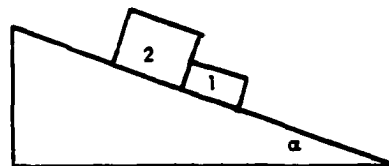
3. A rock swings in a nearly horizontal circle at the end of a string whose breaking tension is 12 N. The circular path is 0.25 m in radius, and the rock's mass is 150 g. What is the maximum speed the rock can have before the string breaks?
4. An accelerometer shows that an airplane flying at 850 km/h undergoes a vertical acceleration of  $0.17 \text{ "g"s}$  ( $1 \text{ g} = 9.8 \text{ m/s}^2$ ) at a certain moment. What is the radius of curvature of the airplane's (horizontal) path at that point?

*More difficult problems*

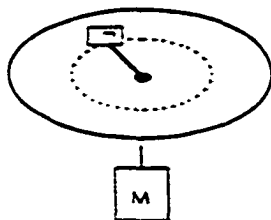
5. A bat crashes into the vertical front of an accelerating subway train. If the coefficient of static friction between bat and train is 0.86, what is the minimum acceleration of the train that will allow the bat to remain in place?
6. A 310-g paperback book rests on a 1.2 kg textbook. A force is applied to the textbook, and the two books accelerate together from rest to 96 cm/s in 0.42 s. The textbook is then brought to a stop in 0.33 s, during which time the paperback slides off. Within what range does the coefficient of static friction between the two books lie?

7. In the diagram below, block 1 has mass  $m_1$ . The coefficient of static friction between block 1 and the inclined plane is  $\mu_{1i}$ . Block 2 has mass  $m_2$ , and the coefficients of static friction between block 2 and the inclined plane is  $\mu_{2i} < \mu_{1i}$ . (a) Beginning at  $\alpha = 0$ , the incline angle is gradually increased. Show that the critical angle  $\alpha_c$  at which the blocks will start to slide down the plane is

$$\alpha_c = \tan^{-1} \frac{\mu_{1i} m_1 + \mu_{2i} m_2}{m_1 + m_2}$$



- (b) What would happen if  $\mu_{2i}$  were greater than  $\mu_{1i}$ ?
8. An Olympic hammer thrower whirls a 7.3 kg hammer on the end of a 120 cm chain. If the chain makes a  $10^\circ$  angle with the horizontal, what is the speed of the hammer?
9. In a popular amusement park ride, riders walk into the vertical cylinder through a door which is then closed. The riders stand with their backs against the wall. The cylinder starts to rotate on its axis: when it reaches full speed, the floor is lowered, leaving the riders "glued to the wall" by friction. (a) Express the minimum value of the coefficient of static friction  $\mu_s$  between the riders' clothing and the wall in terms of the radius  $r$  of the cylinder, the speed  $v$  of the wall, and any constants that you need. (b) If  $r = 2.5$  m and the minimum expected value of  $\mu_s$  is 0.1, find the maximum allowable value of the rotation period  $\tau$ .
10. A mass  $m$  rest on a turntable, as shown in the diagram below. the coefficient of static friction between the mass and the table surface is  $\mu_s$ . Another mass,  $M$ , is attached to a string that passes through a hole at the center of the turntable and is then attached to  $m$ . What are the smallest and largest periods of revolution of the turntable,  $\tau_{\min}$  and  $\tau_{\max}$ , for which  $m$  remains fixed at a distance  $r$  from the center of the turntable?



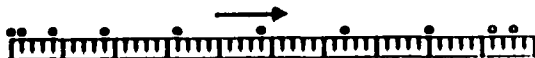
## **Appendix C**

### **Mechanics Baseline Test**

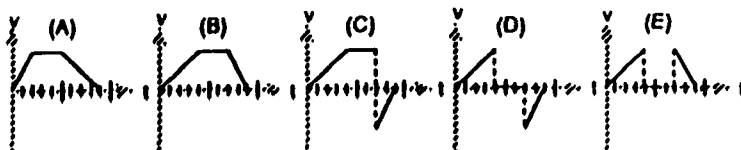
**The Mechanics Baseline Test is reduced slightly from the version administered in the study.**

## Mechanics Baseline Test

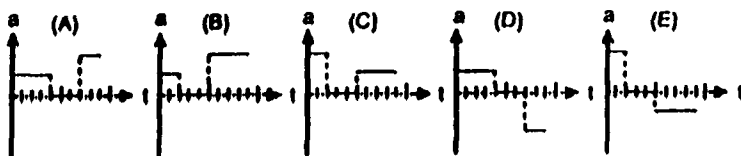
- Refer to the diagram below when answering the first two questions. This diagram represents a multiframe photograph of an object moving along a horizontal surface. The positions as indicated in the diagram are separated by equal time intervals. The first flash occurred just as the object started to move and the last just as it came to rest.



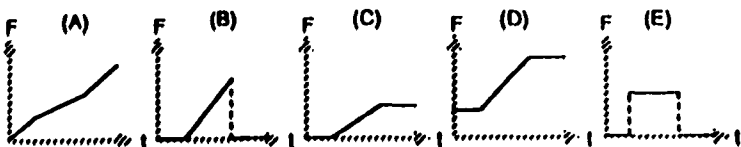
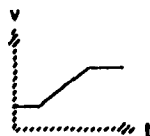
1. Which of the following graphs best represents the object's velocity as a function of time?



2. Which of the following graphs best represents the object's acceleration as a function of time?

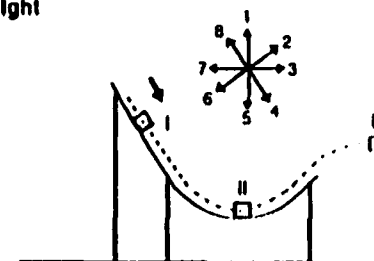


3. The velocity of an object as a function of time is shown in the graph at the right. Which graph below best represents the net force-vs.-time relationship for this object?



- Refer to the graph on the right when answering the next three questions.

This diagram depicts a block sliding along a frictionless ramp. The eight numbered arrows in the diagram represent directions to be referred to when answering the questions.



4. The direction of the acceleration of the block, when in position I, is best represented by which of the arrows in the diagram?

(A) 1 (B) 2 (C) 4 (D) 5  
(E) None of the arrows, the acceleration is zero.

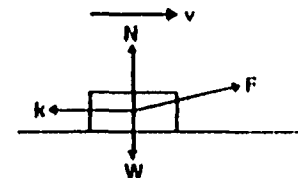
5. The direction of the acceleration of the block when in position II is best represented by which of the arrows in the diagram?

(A) 1 (B) 3 (C) 5 (D) 7  
(E) None of the arrows, the acceleration is zero.

6. The direction of the acceleration of the block (after leaving the ramp) at position III is best represented by which of the arrows in the diagram?

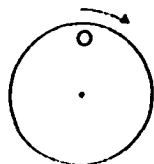
(A) 2 (B) 3 (C) 5 (D) 6  
(E) None of the arrows, the acceleration is zero.

7. A person pulls a block across a rough horizontal surface at a constant speed by applying a force  $F$ . The arrows in the diagram correctly indicate the directions, but not necessarily the magnitudes of the various forces on the block. Which of the following relations among the force magnitudes  $W$ ,  $k$ ,  $N$ , and  $F$  must be true?



(A)  $F = k$  and  $N = W$  (B)  $F = k$  and  $N > W$   
(C)  $F > k$  and  $N < W$  (D)  $F > k$  and  $N = W$   
(E) None of the above choices

8. A small metal cylinder rests on a circular turntable, rotating at a constant speed as illustrated in the diagram at the right. Which of the following sets of vectors best describes the velocity, acceleration, and net force acting on the cylinder at the point indicated in the diagram?

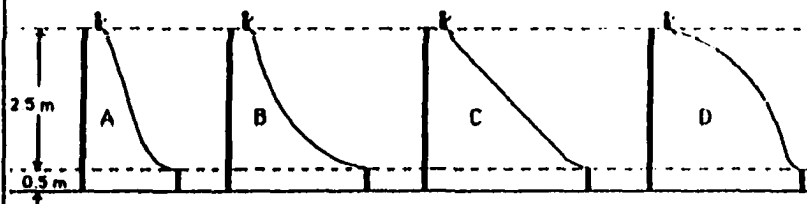


(A)	(B)	(C)	(D)	(E)

9. Suppose that the metal cylinder in the last problem has a mass of 0.10 kg and that the coefficient of static friction between the surface and the cylinder is 0.12. If the cylinder is 0.20 m from the center of the turntable, what is the maximum speed that the cylinder can move along its circular path without slipping off of the turntable?

- |                                    |                                    |
|------------------------------------|------------------------------------|
| (A) $0 < v \leq 0.5 \text{ m/s}$   | (B) $0.5 < v \leq 1.0 \text{ m/s}$ |
| (C) $1.0 < v \leq 1.5 \text{ m/s}$ | (D) $1.5 < v \leq 2.0 \text{ m/s}$ |
| (E) $2.0 < v \leq 2.5 \text{ m/s}$ |                                    |

10. A young girl wishes to select one of the frictionless playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide.

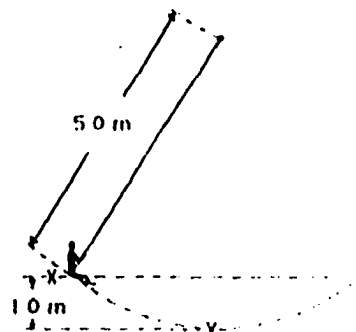


Which of the slides illustrated in the diagram above should she choose?

- (A) A      (B) B      (C) C      (D) D  
(E) It doesn't matter, her speed would be the same for each.

- Refer to the diagram below when answering the next two questions.

X and Z mark the highest and Y the lowest positions of a 50.0 kg boy swinging as illustrated in the diagram to the right.



11. What is the boy's speed at point Y?

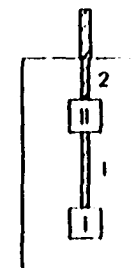
- (A) 2.5 m/s      (B) 7.5 m/s  
(C) 10. m/s      (D) 12.5 m/s  
(E) None of the above.

12. What is the tension in the rope at point Y?

- (A) 250 N      (B) 525 N      (C)  $7 \times 10^2 \text{ N}$       (D)  $1.1 \times 10^3 \text{ N}$   
(E) None of the above.

- Refer to the diagram below when answering the next two questions.

Blocks I and II, each with a mass of 1.0 kg are hung from the ceiling of an elevator by ropes 1 and 2.



13. What is the force exerted by rope 1 on block I when the elevator is travelling upward at a constant speed of 2.0 m/s?

- (A) 2 N      (B) 10 N      (C) 12 N  
(D) 20 N      (E) 22 N

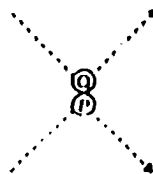
14. What is the force exerted by rope 1 on block II when the elevator is stationary?

- (A) 2 N      (B) 10 N      (C) 12 N      (D) 20 N      (E) 22 N

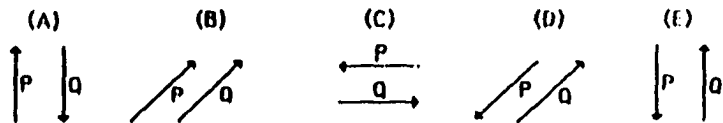


- Refer to the following diagram when answering the next two questions.

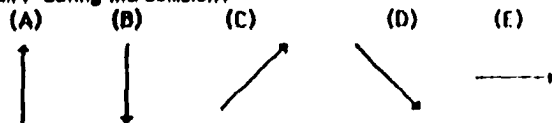
The diagram to the right depicts the paths of two colliding steel balls, P and Q.



15. Which set of arrows best represents the direction of the change in momentum of each ball?



16. Which arrow best represents the direction of the impulse applied to ball Q by ball P during the collision?



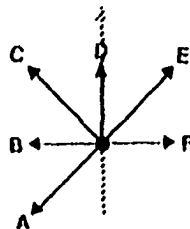
17. A car has a maximum acceleration of  $3.0 \text{ m/s}^2$ . What would its maximum acceleration be while towing a second car twice its mass?

(A)  $2.5 \text{ m/s}^2$  (B)  $2.0 \text{ m/s}^2$  (C)  $1.5 \text{ m/s}^2$   
(D)  $1.0 \text{ m/s}^2$  (E)  $0.5 \text{ m/s}^2$

18. A woman weighing  $6.0 \times 10^2 \text{ N}$  is riding an elevator from the 1<sup>st</sup> to the 6<sup>th</sup> floor. As the elevator approaches the 6<sup>th</sup> floor, it decreases its upward speed from  $8.0$  to  $2.0 \text{ m/s}$  in  $3.0 \text{ s}$ . What is the average force exerted by the elevator floor on the woman during this  $3.0 \text{ s}$  interval?

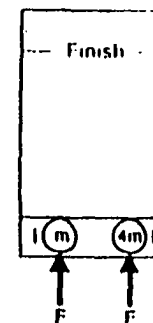
(A)  $120 \text{ N}$  (B)  $480 \text{ N}$  (C)  $600 \text{ N}$   
(D)  $720 \text{ N}$  (E)  $1200 \text{ N}$

19. The diagram at the right depicts a hockey puck moving across a horizontal, frictionless surface in the direction of the dashed arrow. A constant force  $F$ , shown in the diagram, is acting on the puck. For the puck to experience a net force in the direction of the dashed arrow, another force must be acting in which of the directions labeled A, B, C, D, E?



- Refer to the diagram below when answering the next three questions

The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.



20. Which puck will have the greater kinetic energy upon reaching the finish line?

(A) I (B) II  
(C) They both have the same amount.  
(D) Too little information to answer.

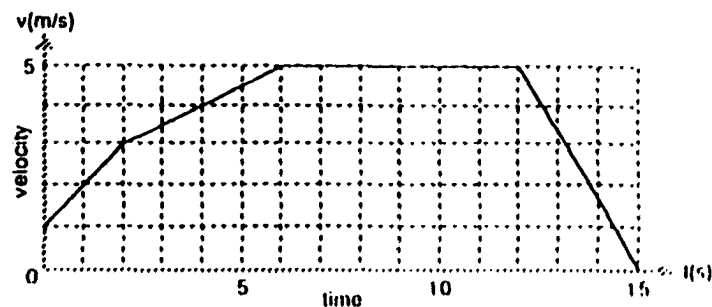
21. Which puck will reach the finish line first?

(A) I (B) II  
(C) They will both reach the finish line at the same time.  
(D) Too little information to answer.

22. Which puck will have the greater momentum upon reaching the finish line?

(A) I (B) II  
(C) They will both have the same momentum  
(D) Too little information to answer.

- Refer to the following kinematical graph when answering the next three questions.



The graph represents the motion of an object moving in one dimension

23. What was the object's average acceleration between  $t = 0$  s and  $t = 6.0$  s?

- (A)  $3.0 \text{ m/s}^2$  (B)  $1.5 \text{ m/s}^2$  (C)  $0.03 \text{ m/s}^2$  (D)  $0.67 \text{ m/s}^2$   
(E) None of the above.

24. How far did the object travel between  $t = 0$  and  $t = 6.0$  s?

- (A) 20 m (B) 8.0 m (C) 6.0 m (D) 1.5 m  
(E) None of the above.

25. What was the average speed of the object for the first 6.0 s?

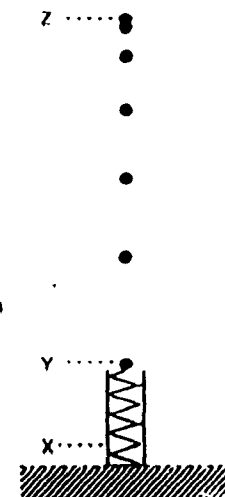
- (A) 3.3 m/s (B) 3.0 m/s (C) 1.8 m/s (D) 1.3 m/s  
(E) None of the above.

Refer to the diagram in the right margin to answer the following question.

The figure represents a multiframe photograph of a small ball being shot straight up by a spring. The spring, with the ball atop, was initially compressed to the point marked X and released. The ball left the spring at the point marked Y, reaches its highest point at the point marked Z.

26. Assuming that the air resistance was negligible:

- (A) The acceleration of the ball was greatest just before it reached point Y (still in contact with the spring).  
(B) The acceleration of the ball was decreasing on its way from point Y to point Z.  
(C) The acceleration of the ball was zero at point Z.  
(D) All of the above responses are correct.  
(E) The acceleration of the ball was the same for all points in its trajectory from points Y to Z.



## Appendix D

### Learning Approach Questionnaire

The Meaningful and Rote Learning Scales (LAQM and LAQR) are contained in the single instrument. The LAQM is comprised of items 1, 2, 3, 6, 9, 10, 11, 13, 15, 17, 20, 21, and 23. The LAQR is comprised of items 4, 5, 7, 8, 12, 14, 16, 18, 19, 22, 24.

## Learning Approach Questionnaire

The following questions refer to how you study and learn about science in this class. For each item there is a five point scale ranging from "Always True" to "Never True". Beside each question, choose the letter that best fits your immediate reaction. Do not spend a long time on each item; your first reaction is probably the best one.

Do not worry about projecting a good image. There are no "correct" answers. Your answers are confidential.

	Always True				Never True
1. I generally put a lot of effort into trying to understand things which initially seem difficult.	A	B	C	D	E
2. I try to relate new material, as I am learning it, to what I already know on that topic.	A	B	C	D	E
3. While I am studying, I often think of real life situations to which the material I am learning would be useful.	A	B	C	D	E
4. I find I tend to remember things best if I concentrate on the order in which the teacher presented them.	A	B	C	D	E
5. I find I have to concentrate on memorizing a good deal of what I have to learn.	A	B	C	D	E
6. I go over important topics until I understand them completely.	A	B	C	D	E
7. I find it best to accept the statements and ideas of my lectures and question them only under special circumstances.	A	B	C	D	E
8. Teachers shouldn't expect students to spend significant amounts of time studying material everyone knows won't be examined.	A	B	C	D	E
9. I often find myself questioning things that I hear in lectures or read in books.	A	B	C	D	E
10. I find it useful to get an overview of a new topic for myself, by seeing how the ideas fit together.	A	B	C	D	E
11. After a lecture, I reread my notes to make sure they are legible and that I understand them.	A	B	C	D	E
12. I am very aware that teachers know a lot more than I do, and so I concentrate on what they say as important rather than rely on my own judgment.	A	B	C	D	E
13. I set out to understand thoroughly the meaning of what I am asked to read.	A	B	C	D	E
14. I tend to like subjects with a lot of factual content rather than theoretical kinds of subjects.	A	B	C	D	E

	Always True				Never True
15. I try to relate what I have learned in one subject to that in another.	A	B	C	D	E
16. The best way for me to understand what technical terms mean is to remember the textbook definition.	A	B	C	D	E
17. Puzzles and problems fascinate me, particularly where you have to work through the material to reach a logical conclusion.	A	B	C	D	E
18. I usually don't think about the implications of what I learn in class or how it relates to my life.	A	B	C	D	E
19. I learn some things by rote, going over and over them until I know them by heart.	A	B	C	D	E
20. When I'm starting a new topic, I ask myself questions about it which the new information should answer.	A	B	C	D	E
21. I spend a lot of my free time finding out more about interesting topics which have been discussed in different classes.	A	B	C	D	E
22. Often I have to read things in science without really understanding them.	A	B	C	D	E
23. In trying to understand new topics, I explain them to myself in ways that other people don't seem to understand.	A	B	C	D	E
24. I generally restrict my study to what is specifically set as I think it is unnecessary to do anything extra.	A	B	C	D	E

## **Appendix E**

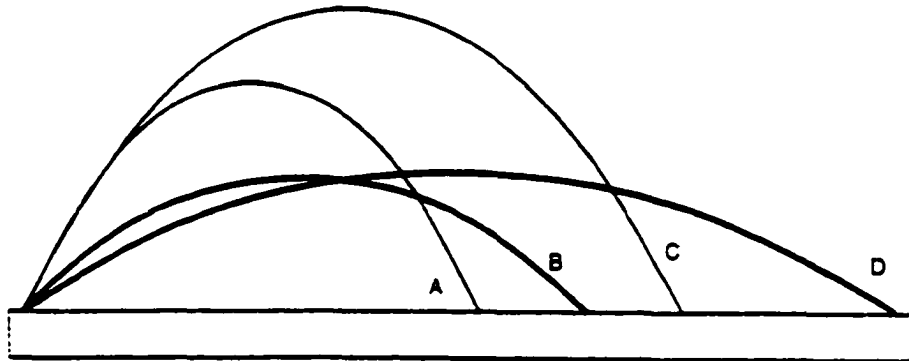
### **Test of Physics Understanding**

The items on the TPU were administered as parts of two examinations. They are reproduced individually in this appendix. The qualitative problems are listed first. The first two qualitative problems appeared on the first class examination, and the remaining four qualitative problems appeared on the second class examination. Correct answers are indicated at the beginning of each problem.

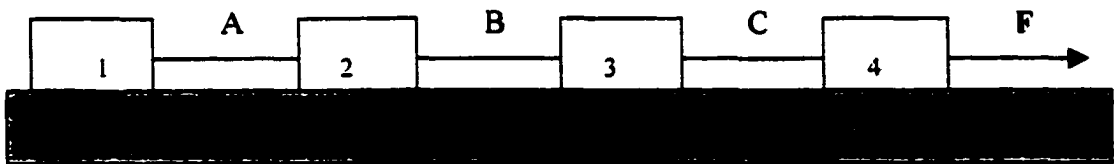
The first two quantitative problems appeared on the first examination, and the remaining two quantitative problems appear on the second examination. The concepts used in scoring each of these problem are listed in italics after each problem.

### Qualitative Problems

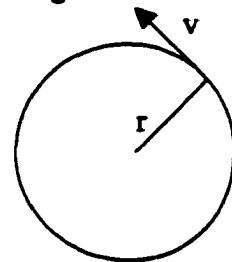
1. C The figure below shows four trajectories of a projectile. Which of the trajectories has the longest flight time?



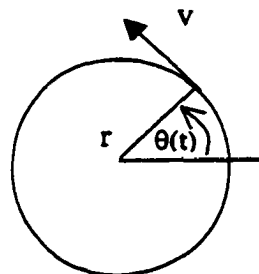
2. A The figure below shows four masses attached by sections of rope and pulled by a force  $F$  as shown. Assume that the mass of each block is identical and blocks 2, 3, and 4 slide without friction whereas block one slides with friction. For what section of rope is the magnitude of the tension smallest?
- A. Section A
  - B. Section B
  - C. Section C
  - D. The magnitude of the tension at A, B, and C are all equal.



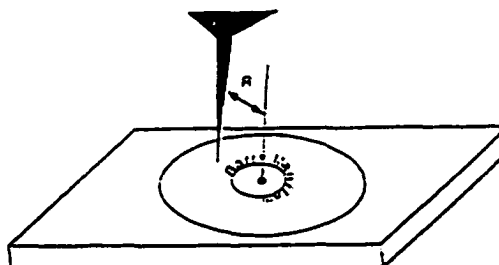
3. C Which of the following *must* be true if a force  $F$  is conservative?
- A.  $F$  must be a constant vector.
  - B. The direction of  $F$  must be constant.
  - C. The work done by  $F$  on a particle which starts and ends at the same point must be zero.
  - D. The work done by  $F$  on a particle that moves from a point A to a point B must be positive.
4. C An object rotates on a circle of radius  $r$  with non-uniform circular motion. At a certain instance in time, its speed is  $v$ , but it is slowing down. The magnitude of its acceleration is
- A. equal to  $v^2/r$
  - B. less than  $v^2/r$
  - C. greater than  $v^2/r$
  - D. not enough information is given to know



5. D An object rotates on a circle of radius  $r$ . The position of the object is described by  $\theta(t)$ . If at a given instant in time the angular acceleration  $\alpha = \frac{d^2\theta}{dt^2}$  is observed to be less than  $0.0 \text{ rad s}^{-2}$ , the object
- A. is traveling in the clockwise direction
  - B. is traveling in the counterclockwise direction
  - C. must not be stationary at that instant in time
  - D. Not enough information is given to make any of the above conclusions.



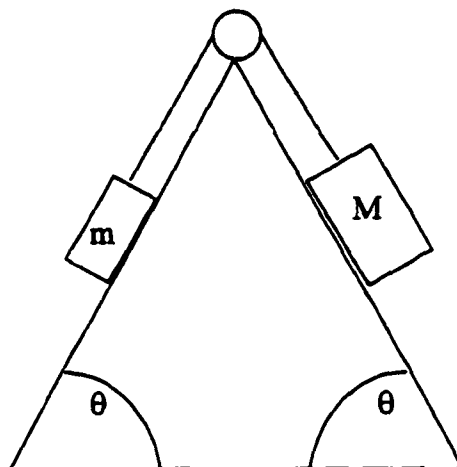
6. B You stop a spinning turntable in a time  $t_{\text{stop}}$  by applying a constant frictional force with a nail pushed against the record a distance  $\frac{1}{2} R$  from the spindle (center of rotation). Assuming the force of the nail is the only significant force acting to slow down or speed up the turntable, if you were to have placed the nail a distance  $R$  from the edge of the turntable you would have stopped it in a time
- A.  $t_{\text{stop}} / \sqrt{2}$
  - B.  $t_{\text{stop}} / 2$
  - C.  $t_{\text{stop}} / 4$





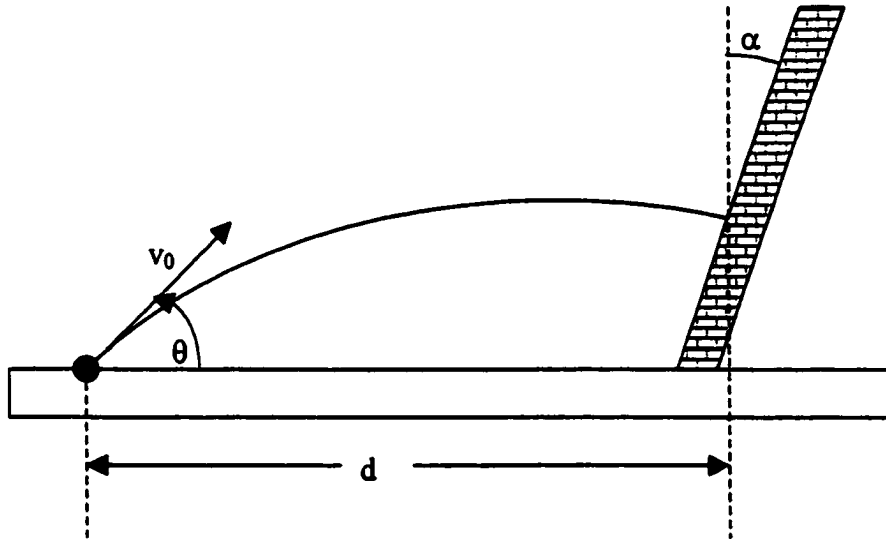
### Quantitative Problems

1. Two blocks attached by a rope that passes over a (frictionless-massless-) pulley slide on opposite sides of an isosceles triangle as shown in the figure. Both blocks slide without friction. Derive an expression for the tension in the rope in terms of  $\theta$ ,  $g$ ,  $m$ , and  $M$ .



**CONCEPT:** *Newton's second law applies (separately) to the 2 bodies,  $m$  and  $M$ , which move with the same acceleration. (Some indication of equal accelerations was necessary.)*

2. A rubber ball is thrown with an initial velocity  $v_0 = 3.5 \text{ ms}^{-1}$  at an angle of  $\theta = 30^\circ$  from the horizontal. The ball is observed to hit a tilted wall inclined at an angle  $15^\circ$  from vertical and bounce directly back along its initial trajectory. What is the horizontal distance  $d$  that the ball travels before hitting the wall? (You may assume the ball bounces, changes direction, but not speed when it hits the wall. First determine the direction of the velocity just before the hit in order that the ball will simply reverse its direction upon striking the wall.)

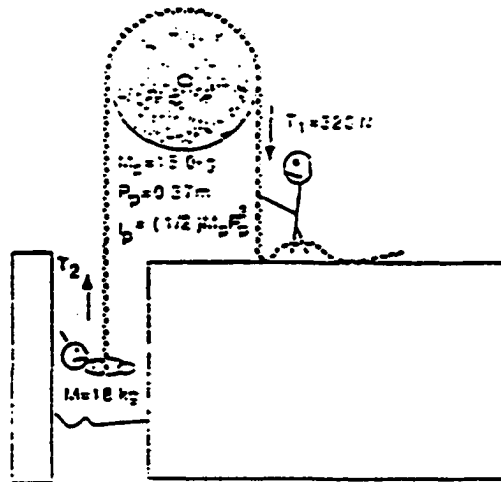


**CONCEPTS:**

*Horizontal velocity is constant, so distance is the product of horizontal velocity and time.*

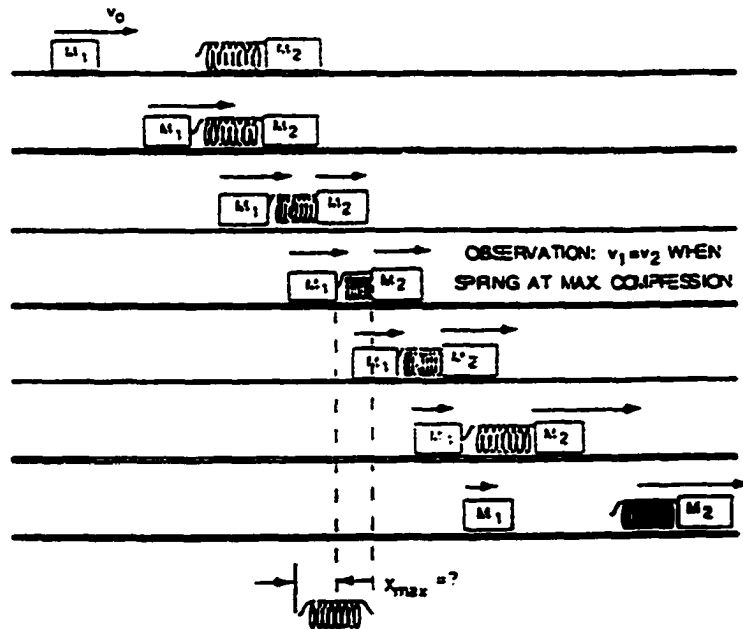
*Velocity at any point in the trajectory is tangent to the path, so velocity components are related to the angle. (The geometry gives  $\tan \alpha = v_y/v_x$  at the point of impact.)*

3. Your youngest son rushes in to tell you that your eldest son has fallen down the well. Fortunately, the fallen child is grabbing tightly to the well rope so you may pull him up. If you pull on the rope with a constant force of  $T_1 = 320\text{ N}$ , what is the upward acceleration of your child? (Use the data given in the figure and assume that frictional forces and the mass of the rope can be neglected. Note  $T_2 \neq T_1$ .)



**CONCEPT:** Newton's second law applies in its linear form to the lifted mass and in rotational form to the pulley. (Both must be indicated.)

4. The figure shows the collision of two blocks that slide without friction along a single axis. The first block of mass  $M_1$  has an initial velocity  $v_0$  whereas the second block of mass  $M_2$  starts at rest. The collision is buffered by a spring. Use conservation of energy and momentum ( and the observation given in the figure) to determine the maximum compression  $x_{\max}$  of the spring in terms of  $v_0$ ,  $M_1$ ,  $M_2$  and the spring constant  $k$ .



#### CONCEPTS:

*The conserved momentum and energy is initially equivalent to the kinetic energy of the first mass.*

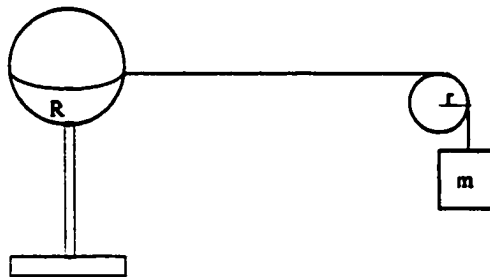
*The final situation (+<sup>th</sup> frame) has both potential (compressed spring) and kinetic energy components.*

## **Appendix F**

### **Problems and Questions for Think Aloud Protocols**

### Protocol Problems

1. A 1.10 kg cookie jar is moving up a  $40^\circ$  incline. At a point 0.85 m from the bottom (measured along the incline), it has a speed of 2.1 m/s. The coefficient of kinetic friction between the jar and the incline is 0.15.
  - a) How much further up the incline will the jar move?
  - b) How fast will it be going when it slides back to the bottom of the incline?
  
2. A uniform spherical shell of mass  $M$  and radius  $R$  rotates about a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia  $I$  and radius  $r$ , and is attached to a small object of mass  $m$ . There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it has fallen a distance  $h$  from rest?



### Questions Accompanying Protocol Collection

- |               |                                                                                                                                                                                                                        |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <u>Before</u> | What is your class standing and your major?<br>What is your background in science courses, both high school and college?<br>How did you prepare for the last exam?                                                     |
| <u>After</u>  | If you'd been working on homework problems, how would it have been different than what you just did here?<br>What do you normally do to check your homework answers?<br>Do you have any other comments about homework? |

## Appendix G

### Analysis of Continuous Data

LAQM, LAQR, TPU, and MBT scores were further examined by producing the Pearson Correlation Coefficients displayed in Table G1. The scaffolded treatment was coded as “1” and the non-scaffolded treatment was coded as “2”. There was no significant correlation between the treatment groups and any of the pre-treatment measurements. However, there was a significant correlation between the MBT scores and the LAQM and TPU scores. There was also a high correlation between treatment and TPU scores.

Table G1

#### Pearson Correlation Coefficients for Study Variables

	<u>MBT</u>	<u>LAQM</u>	<u>LAQR</u>	<u>TPU</u>
Treatment	-.269	-.055	.092	-.533**
MBT		.318*	.164	.331*
LAQM			-.125	.138
LAQR				-.247

\*  $p < .05$   
\*\*  $p < .01$

## Appendix H

### Solutions for Protocol Problems

Each of the problems may be solved using either energy considerations or Newton's laws of motion. An example of each type of solution is given, and the reasoning is presented in italics. Several other legitimate solution paths are possible.



## First Protocol Problem Solved Using Energy Considerations

Part a)

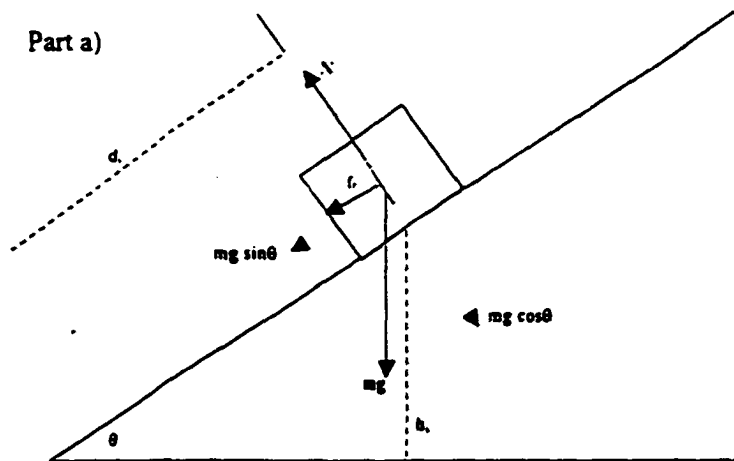


Diagram shows initial conditions. Dotted lines show resolution of the weight,  $mg$ , into components  $mg \sin \theta$  and  $mg \cos \theta$ . The force of friction acts against upward motion. Initial distance up the incline is given by  $d$ , and initial height by  $h_i$ . Force due to friction,  $f_f$ , is parallel to the incline and its direction is opposed to motion. The normal force is shown by  $N$ .

(Numeric values will be substituted after an algebraic solution.)

$$KE_i = \frac{1}{2} mv^2$$

$$U_i = mgh_i$$

Expressions of initial kinetic and potential energy

$$KE_f = 0$$

$$U_f = mgh_f$$

Expressions of final kinetic and potential energy where  $h_f$  is the final height.

$$W_f = f_f d, \text{ where } f_f = \mu N \text{ and } N = mg \cos \theta$$

Work done against friction is the product of frictional force and the distance traveled,  $d$ . The frictional force, in turn, is the product of the normal force,  $N$ , and the coefficient of friction,  $\mu$ . Further, the normal force is equal to the component of weight perpendicular to the incline,  $mg \cos \theta$ .

$$W_f = \mu mgd \cos \theta \quad \text{Substitution}$$

Initial energy equals the final potential energy plus the work done against friction.

$$\frac{1}{2} mv^2 + mgh_i = mgh_f + \mu mgd \cos \theta$$

Simplified by eliminating the factor  $m$ .  
Trigonometric conversion

$$\frac{1}{2} v^2 + gh_i = gh_f + \mu g d \cos \theta$$

$$\text{But } h_i = d_i \sin \theta \quad \text{and} \quad h_f = (d_i + d) \sin \theta$$

Substitution

$$\text{So } \frac{1}{2} v^2 + g d_i \sin \theta = g(d_i + d) \sin \theta + \mu g d \cos \theta$$

Algebraic simplification

$$\text{And } \frac{1}{2} v^2 = g d \sin \theta + \mu g d \cos \theta$$

Algebraic manipulation

$$\frac{1}{2} v^2 = d(g \sin \theta + \mu g \cos \theta)$$

$$d = \frac{v^2}{2(g \sin \theta + \mu g \cos \theta)}$$

$$d = \frac{(2.1 \text{ m/s})^2}{2[(9.8 \text{ m/s}^2)(\sin 40^\circ) + (.15)(9.8 \text{ m/s}^2)(\cos 40^\circ)]}$$

Substitution of numerical values

Calculation

$$d = 0.30 \text{ m}$$

Part b)

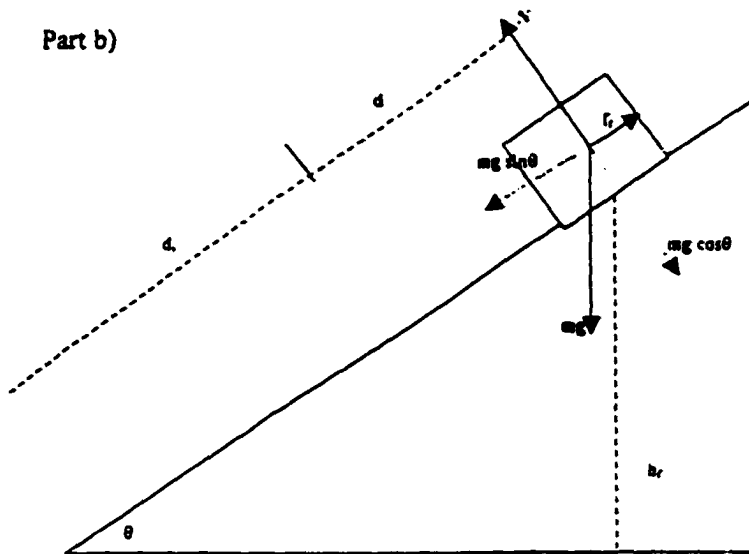


Diagram shows initial conditions similar to Part a, but position is different and the force of friction acts against downward motion. Initial distance up the incline is given by  $d_i + d$  and initial height by  $h_f$ . Expressions originated in Part a.

$$KE_i = 0$$

$$U_i = mgh_f$$

Expressions of initial kinetic and potential energy

$$KE_f = \frac{1}{2} mv^2$$

$$U_f = 0$$

Expressions of final kinetic and potential energy where  $v$  is the final speed.

$$W_f = f_f(d + d_i) = \mu mg(d + d_i) \cos \theta$$

Same as Part a, except for distance

$$mgh_f = \frac{1}{2} mv^2 - \mu mg(d + d_i) \cos \theta$$

Initial(potential) energy equals the final (kinetic) energy plus the work done against friction.

$$gh_f = \frac{1}{2} v^2 - \mu g(d + d_i) \cos \theta$$

Simplified by eliminating the factor  $m$ .

$$\frac{1}{2} v^2 = gh_f - \mu g(d + d_i) \cos \theta$$

Algebraic manipulation

$$v = (2[gh_f - \mu g(d + d_i) \cos \theta])^{1/2}$$

where  $h_f = (d_i + d) \sin \theta$

From Part a

$$v = (2[(9.8\text{m/s}^2)(.85\text{m} + .30\text{m})(\sin 40^\circ) - (.15)(9.8\text{m/s}^2)(.85\text{m} + .30\text{m})(\cos 40^\circ)])^{1/2}$$

$$v = 3.4\text{m/s}$$

Substitution of numerical values  
Calculation

# First Protocol Problem: Solved Using Newton's Laws of Motion

Part a)

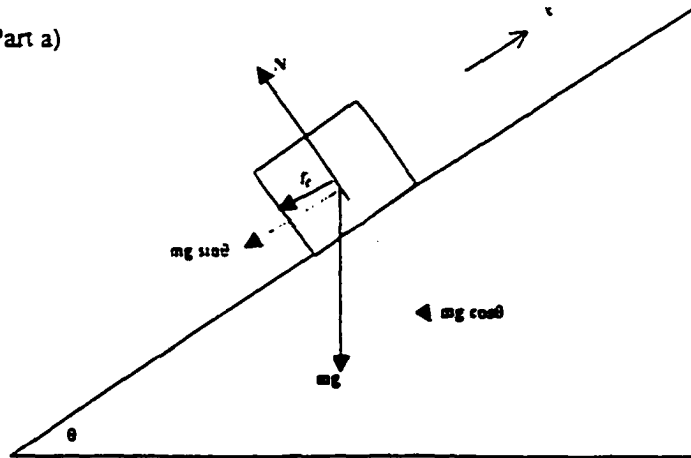


Diagram shows initial conditions. Dotted lines show resolution of the weight,  $mg$ , into components  $mg \sin \theta$  and  $mg \cos \theta$ . The force of friction acts against upward motion. Force due to friction,  $f_f$ , is parallel to the incline and its direction is opposed to motion. The normal force is shown by  $N$ . The  $x$ -direction is parallel to the incline.

(Numeric values will be substituted after an algebraic solution.)

$$\Sigma F_x = -mg \sin \theta - f_f = ma$$

Sum of forces in the  $x$ -direction equals the product of mass and acceleration (Newton's 2<sup>nd</sup> Law).

$$-mg \sin \theta - \mu N = ma$$

Substituting definition of frictional force

$$\Sigma F_y = N - mg \cos \theta = ma = 0$$

Sum of forces in the  $y$ -direction is zero because there's no acceleration in that direction.

$$N = mg \cos \theta$$

Algebraic manipulation

$$-mg \sin \theta - \mu mg \cos \theta = ma$$

Substitution

$$-g \sin \theta - \mu g \cos \theta = a$$

Elimination of factor  $m$  gives expression for acceleration

$$v_f^2 = v_0^2 + 2a(\Delta x)$$

Equation for relationship between velocity and change in position given constant acceleration

$$\Delta x = \frac{-v_0^2}{2a}$$

Algebraic manipulation. In this case  $v_f$  can be eliminated, because the final velocity is 0

$$\Delta x = \frac{-v_0^2}{2(-g \sin \theta - \mu g \cos \theta)}$$

Substitution of expression above for acceleration. Negative signs will cancel

$$\Delta x = \frac{(2.1 \text{ m/s})^2}{2[(9.8 \text{ m/s}^2) \sin 40^\circ + (.15)(9.8 \text{ m/s}^2) (\cos 40^\circ)]}$$

Substitution of numerical values

$$\Delta x = 0.30 \text{ m}$$

Calculation

Part b)

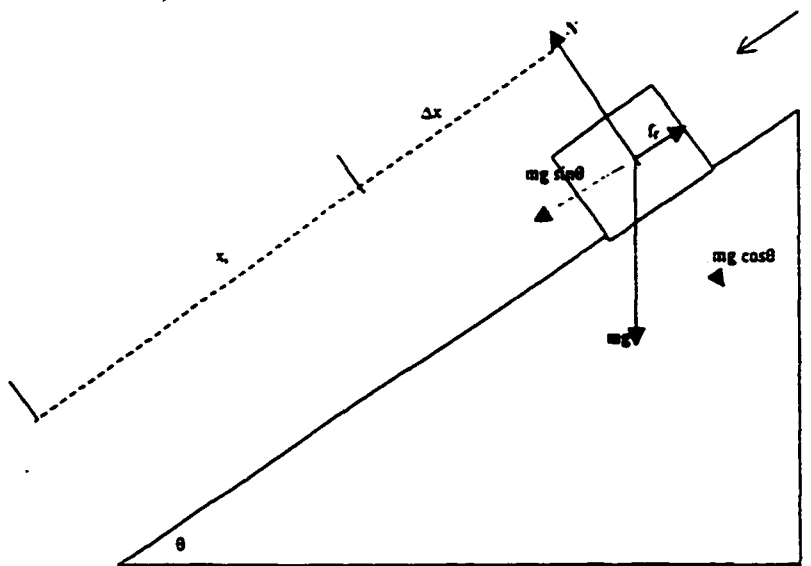


Diagram shows initial conditions similar to Part a, but position is given by  $x_i + \Delta x$ , where  $\Delta x$  was found in Part a. The force of friction acts against downward motion.

The  $x$ -direction parallels the incline and is positive in the direction of motion.

$$\Sigma F_x = mg \sin \theta - f_r = ma$$

Sum of forces in the  $x$ -direction equals the product of mass and acceleration.

$$mg \sin \theta - \mu N = ma$$

Substituting definition of frictional force

$$mg \sin \theta - \mu mg \cos \theta = ma$$

The expression for normal force found in Part a is substituted.

$$g \sin \theta - \mu g \cos \theta = a$$

Elimination of factor  $m$  gives expression for acceleration.

$$v_f^2 = v_0^2 + 2a(x_i + \Delta x)$$

Equation for relationship between velocity and change in position given constant acceleration

$$v_f = [2a(x_i + \Delta x)]^{1/2}$$

Algebraic manipulation In this case  $v_0$  can be eliminated, because the initial velocity is 0.

$$v_f = [2(g \sin \theta - \mu g \cos \theta)(x_i + \Delta x)]^{1/2}$$

Substitution of expression above for acceleration

$$v_f = (2[(9.8\text{m/s}^2)(\sin 40^\circ) - (.15)(9.8\text{m/s}^2)(\cos 40^\circ)] [.85\text{m} + .30\text{m}])^{1/2}$$

Substitution of numerical values

$$v_f = 3.4 \text{ m/s}$$

Calculation

## Second Protocol Problem Solved Using Energy Considerations

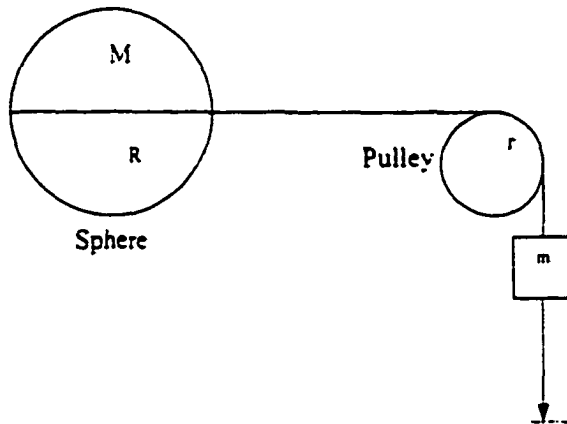


Diagram shows the mass connected by a string to both sphere and pulley. The mass will descend a distance  $h$ .

The sphere has mass  $M$  and radius  $R$ , these dimensions will be used to find a moment of inertia

The pulley has a radius of  $r$  and moment of inertia  $I_p$

The descending object has mass  $m$ .

$$U_i = mgh \quad KE_i = 0$$

$$U_f = 0 \quad KE_f = \frac{1}{2} I_s \omega_s^2 + \frac{1}{2} I_p \omega_p^2 + \frac{1}{2} m v^2$$

$$I_s = \frac{2}{3} MR^2 \quad \text{known conversions}$$

$$\omega_s = v/R$$

$$I_p \text{ is given}$$

$$\omega_p = v/r$$

$$U_i + KE_i = U_f + KE_f$$

$$mgh = \frac{1}{2} I_s \omega_s^2 + \frac{1}{2} I_p \omega_p^2 + \frac{1}{2} m v^2$$

$$mgh = \frac{1}{2} \left( \frac{2}{3} MR^2 \right) (v/R)^2 + \frac{1}{2} I_p (v/r)^2 + \frac{1}{2} m v^2$$

$$mgh = \frac{1}{3} M v^2 + \frac{1}{2} I_p (v^2/r^2) + \frac{1}{2} m v^2$$

$$v = \left( \frac{6mghr^2}{2Mr^2 + 3I_p + 3mr^2} \right)^{1/2}$$

The initial and final potential and kinetic energies ( $U_i$ ,  $U_f$ ,  $KE_i$ ,  $KE_f$ ) are identified. Potential energy is associated only with the descending mass, and is assigned a value of 0 at the bottom of its descent. There is no initial kinetic energy, but final kinetic energy is associated with the sphere, the pulley, and the mass.  $I_s$  and  $I_p$  are the moments of inertia for the sphere and pulley, and  $\omega_s$  and  $\omega_p$  are the respective angular velocities. The speed of the descending mass as well as any point on the string are given by  $v$ .

Conservation of energy

Substitution

Substitution

Algebraic manipulation

Algebraic manipulation

## Second Protocol Problem: Solved Using Laws of Motion

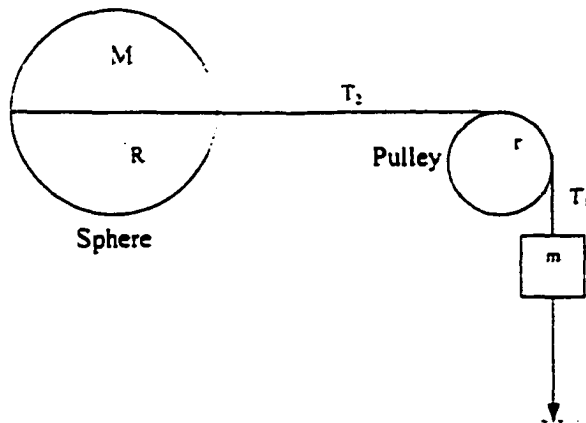


Diagram shows the mass connected by a string to both sphere and pulley. The mass will descend a distance  $h$ .

The sphere has mass  $M$  and radius  $R$ ; these dimensions will be used to find a moment of inertia.

The pulley has a radius of  $r$  and moment of inertia  $I_p$ .

The descending object has mass  $m$ .

Since the pulley has inertia, the tension will differ on either side of the pulley, thus  $T_1$  and  $T_2$ .

Sphere:

$$\sum \tau = I_s \alpha_s = T_2 R$$

where  $\alpha_s = a/R$   
and  $I_s = \frac{2}{3} MR^2$

Pulley:

$$\sum \tau = I_p \alpha_p = T_1 r - T_2 r$$

where  $\alpha_p = a/r$

Mass:

$$\sum F = ma = mg - T_1$$

The sum of torques acting on the sphere is equal to the product of the sphere's inertia and its angular acceleration. The only torque on the sphere is produced by  $T_2$ . The direction of motion is chosen to be positive.

The sum of torques acting on the pulley is equal to the product of the pulley's inertia and its angular acceleration. The torque on the sphere produced by  $T_1$  is positive and that produced by  $T_2$  is negative.

The sum of forces acting on the descending mass is equal to the product of its mass and acceleration.

$$T_2 R = \left(\frac{2}{3} MR^2\right)(a/R)$$

Substitution for the sphere

$$T_2 R = \frac{2}{3} MaR \quad \text{so } T_2 = \frac{2}{3} Ma$$

Algebraic manipulations

$$(T_1 - T_2)r = I_p a/r$$

Substitution for the pulley

$$T_1 = m(g-a)$$

Algebraic manipulation of equation for the mass

$$[m(g-a) - \frac{2}{3} Ma]r = I_p a/r$$

Substitution of tension expressions into the last pulley equation

$$mgr^2 - mar^2 - \frac{2}{3} Mar^2 = I_p a$$

Algebraic manipulation

$$a = \frac{3mgr^2}{3I_p + 3mr^2 + 2Mr^2}$$

*Substitution of tension expressions into the last pulley equation*

$$v^2 = v_0^2 + 2ah$$

*Equation relating velocity and distance (h) for a constant acceleration (a).  $v_0$  is 0 in this case.*

$$v = (2ah)^{1/2}$$

*Algebraic manipulation*

$$v = \left( \frac{6mghr^2}{3I_p + 3mr^2 + 2Mr^2} \right)^{1/2}$$

*Substitution of acceleration expression*

## **Appendix I**

### **Summaries of Student Solutions**

**This appendix contains summaries of the problem solving approaches of the eight students who did think aloud protocols.**



## N1

Problem 1 – Begins a diagram as he reads, then completes it with x, y-axes. He describes the diagram but does very little labeling. This includes “frictional force, which is resisting motion,” but this force isn’t given an arrow on the diagram.

He then puts down coordinates for force due to gravity and cancels mass to convert to acceleration coordinates. He gives a solution plan, then proceeds with an expression for normal force and an expression for frictional force. He then writes  $v = v_0 + at$  where  $a$  is  $a_g + a_f$ . He then calculated numerical values for  $a_f$ , but instead of  $a_g$  finds  $F_g$ , then adds  $F_g$  and  $a_f$ , which he has labeled  $f_f$ , to get  $F$ .

He says he thinks he did it right, but proceeds to check in the book for an example using sine and cosine in relation to force. When he is satisfied on this point, he divides the total force by mass and uses this to get a value for time. However, this is where he stops, confident that .29 s is the answer. He did not check back to the problem to find that it did not answer the stated problem. He also skipped part b.

Problem 2 – After reading the problem he states “you know radius and you know inertia, so you can find torque.” He then draws a diagram showing the radius of the pulley but not the radius of the sphere. He also includes the mass with descent of “h” indicated.

He goes to the book to find rotational inertia equations. Before doing anything with the equations he goes back to the diagram and indicates mass  $M$ . He also puts a “K” at the top of his indication of the descent, “h”, saying “and you use kinetic.” No further reference is made to kinetic energy.

As he visualizes the problem, he mentions the mass, “the friction on the pulley axle” presumably meaning tension. (The pulley axle was given as frictionless in the problem statement.”

He writes a torque equation considering only the  $mg$  force, not the tension in the cord on either side of the pulley. He seems to assume the tension is  $mg$ , as if it were stationary. From other simple relationships he derives an expression for acceleration, or at least such an expression can be extrapolated from what he has written.

He affirms that the mass will be accelerating for a distance “ $h$ ” and goes to the book to find Newtonian equations. He writes down a time dependent equation first, then the velocity equation independent of time. This he solves for  $v$  without ever considering the previous expression for acceleration. Naturally, he gets a velocity expression appropriate to free fall.

To focus his attention on the sphere, which he had totally neglected, I asked what would happen if the sphere’s radius increased. He quickly answered that the torque would increase (torque equals cross product of force and radius) and so would angular and linear acceleration, which made sense in the light of the equations he had written for the pulley, but not considering moment of inertia change. It never seemed to occur to him that the sphere was not part of his solution.

## N2

He became very confused on the warmup problem, and it is possible that he never really recovered. Alternatively, maybe he is just floundering through the course and will get enough the second time through to go on.

**Problem 1 – He reads the problem and draws a diagram with .85 m indicated but not associated with a specific point. He reads and rereads the problem and finally says he is stuck, unable to interpret the .85 m. He redraws the incline with .85 m near the bottom. I explain that it had started at the bottom and was still moving at .85 m. He redraws with .85 m where the block is labeled with correct velocity.**

**He finishes another rereading and immediately says, “Where is it in the book?” Because there is no acceleration given, only velocity, he looks for kinetic energy information. This takes a very long time.**

**He draws a force diagram, finds the normal force, then the frictional force (numerical values). He goes back to the book, perusing one example after another. He finally concludes that kinetic energy is inappropriate, because everything that has to do with inclines deals with forces. He concludes that he is “pretty much lost.”**

**Problem 2 – He reads and rereads the problem, then asks a clarifying question regarding the problem diagram. Again he rereads, saying that he is trying to visualize the situation.**

**He decides to “just work with the smaller pulley” and draws a diagram with the pulley. He indicates the weight force downward. He evidently ignores the motion and the effect of the pulley’s inertia when he indicates a horizontal force (?tension) equal to the weight force. He adds the rotating sphere to this diagram, and goes to the book to find “inertia of a sphere.”**

**The book search is lengthy with long pauses. He looks for a relationship between velocity and rotation and then for examples dealing with torque and with pulleys. He finally finds an example he thinks is appropriate and he finds and writes**

down I for the spherical shell. In the example he notes the equation  $Tr = I\alpha$ , but does not know what “r” represents. This example is directly adapted and correctly used as far as it goes. All this took extreme amounts of time.

### N3

Problem 1 – He begins to diagram as he reads and includes labels that apply to both a and b, specifically for velocity. He says he is putting in forces on the second diagram and then includes velocity and the coefficient of kinetic friction. He also treats these as forces in finding the sum of forces. He writes the x- and y-components of each “force” and finds numerical values for the gravity components. (These are never used.)

Then he writes the x-“forces” and y-“forces” in an equation,  $mg \sin\theta + v + \mu_k = N + mg \cos\theta$ , though he gives no indication either before or after that he really thinks the sums are equal.

A lot of page turning follows while he retrieves equations for the frictional force and for velocity in terms of acceleration and distance. He writes down the equation for frictional force,  $F_k = \mu_k N$ , but he never uses the equation, instead reverting to use of  $\mu_k$  alone in finding the sum of forces. When he writes down the velocity/acceleration/distance equation, he realizes that he does not know the acceleration.

After much more page turning, he concentrates on some examples in Chapter 6, in particular an example with friction on an inclined plane. It shows that the sum of the forces in the x-direction can be equated to  $ma$ . So he takes the sum of forces on the left

side of the equation above and divides by mass to get acceleration. A numerical value is calculated and plugged into the velocity equation.

The same velocity equation is appropriately used for part b, but he assumes that the acceleration is the same as part a instead of recognizing that the frictional force has reversed direction.

Problem 2 – After reading the problem, he draws a diagram and does to the book, where he quickly finds a chapter on rotation and says, “you figure there’s no friction, so it falls at an acceleration of  $\alpha$ ” as if lack of friction led to this conclusion.

This is followed by a long session of page turning and looking through examples. During this time he writes down 2 other rotational relationships that are never used. He also finds moment of inertia for the sphere, though he chooses the solid sphere expression. This, too, is never used.

He finally finds an example with a pulley and a falling mass. Here the variables  $m$  and  $M$  apply to the falling mass and the pulley. He takes the expression for acceleration directly from the example then divides it by the radius of the sphere to change it to angular acceleration. He indicates the division, but not the conversion from  $a$  to  $\alpha$ . (Later, he uses the divided expression for  $a$ .)

He finally gets back to the same velocity equation he used for the first problem, substituting  $h$  for  $x - x_0$  and his angular acceleration expression for  $a$ .

#### N4

Problem 1 – She reviews the givens, then draws and labels diagrams of the incline and the forces. She questions whether velocity is a force and concludes that it is. However,

when she did the sum of forces, she did not include velocity. (Could it be because summing forces is a procedure without meaning?)

She goes through the process of getting acceleration in terms of  $g$ ,  $\theta$ , and  $\mu_k$ . But then she goes to the book for a position formula and chooses one for projectile motion, where motion in the  $x$ -direction implies constant velocity. She follows the procedure for projectile motion, finding time from  $v_y = v_0 \sin \theta - gt$ , and plugging  $t$  into the constant velocity expression for  $x$ .

She totally ignored the expression she had derived for acceleration until part b. There she used it in the equation  $y = \frac{1}{2} at^2 + v_{0y}t + y_0$  along with the part a expression for time and the projectile motion value  $v_{0y} = v_0 \sin \theta$ . (Solution and representation were not strongly related.)

## S1

Problem 1 – He reads the problem and draws a single diagram that includes the forces parallel to the incline, showing  $F_k$  as  $\mu_k N$ .

He first considers using conservation of energy with “dissipating energy of the friction,” but decides it is easier to use forces. He sums the forces, finds an expression for acceleration, then writes down equations for distance,  $r$  (trajectory equation) and velocity. Both equations have incorrect constants. He uses both to get an expression for  $t$  in terms of known quantities.

He chooses  $r$  to be zero at .85 meters up the incline since the problem asks “how much further.” He substitutes numerical values and produces an answer that would be correct if the equations had the correct constants.

He looks at part b, starts to use the acceleration from part a, but corrects himself. He then tries the force approach from part a, but feels that a time expression is problematic, so he changes to conservation of energy.

He looks in the book, because he can not remember how to write an equation for conservation of energy with a non-conservative force. Since the total change must be zero, he adds all individual energy changes and equates them to zero.

He notices that the expression for velocity has been increased by incorporation of a frictional force term rather than decreased, so concludes it must be wrong. He looks back for a sign problem, but does not immediately find it. (We go on to the next problem.)

Problem 2 – He reads the problem and goes immediately to the book to locate the rotational inertia of a spherical shell. He also gets the rotational inertia of a disk, which he considers to be like the pulley.

He surmises that he will have to “add up tensions on the string to find acceleration of the object.” At this point he draws a diagram and labels tensions in the 2 sections of the string.

He looks in the book to find information on torque and determines torques on the sphere and pulley – unfortunately using  $mg$  instead of  $T_1$  for the pulley. These 3 torques are all added together and equated to  $I\alpha$ , with a positive sign denoting anything that makes the hanging object descend.

He does a sum of forces for the hanging object and gets an expression for  $T_1$ .

He decides to use the sphere to find  $T_2$  and gets a correct expression, but then tries to put it back into the total sum of torques. At that point he feels he has made an early mistake and may have too many terms.

At this point, in the interest of time, we move on. Note that in response to follow-up questions, he felt he should have applied conservation of energy.

## S2

Problem 1 – After reading the problem he draws a simple diagram of the situation and then a force diagram showing normal and frictional forces and the components of the gravitational force. He then writes down his “real” equation,  $x = x_0 + v_0t + at^2$  (leaving out the  $\frac{1}{2}$ ).

He rereads the question in part a, then sums the forces to equal  $ma$  in the  $x$ -direction and zero in the  $y$ -direction.

Back under his “real” equation he writes  $v = v_0 + at$ , and concludes that he needs an expression for acceleration, which he finds from the force equations. He also finds time,  $t$ , from the velocity equation.

Working on the “real” equation, he chooses to find total distance up the incline and uses .85 m as  $x_0$ . When he substitutes his time and acceleration expressions, he neglects to square time in the last term. Numerical values are substituted without units.

He finds that his answer is negative and begins checking his equations. He discovers the missing time square and works through the figures again. This time he is dissatisfied because the second and third terms cancel, leaving the final distance the same as the initial distance. At this point he goes to the book and finds the equation and



finds he needs  $\frac{1}{2}$  in the last term. This time he reaches a correct answer though he still doubts it, because it is smaller than he expected.

For part b, the approach is the same including diagrams and force summing. This time he gets a time expression by solving  $0 = x_0 + v_0t + \frac{1}{2}at^2$  and substituting into  $v = at$ . At this point we moved on to the next problem.

Problem 2 – He reads the problem twice and then draws a basic diagram of the situation. He goes to the book to find the linear/rotational relationships. He identifies tangential acceleration as an important idea and writes  $a_t = \alpha r$ . He goes through a series of equations that seem to describe parts of the situation. This includes a sum of forces for the hanging mass which he concludes must be at rest “at that moment” so  $T = mg$ .

He does a lot of searching through the book, but never finds a common thread to pull the problem together. Finally, he asks, “Do you want me to keep going?” and admits, “I’m just pulling whatever I can think of out of the air.”

### S3

Problem 1 – He reads the problem and concludes quickly that he can apply conservation. He states that conservation alone will not work, but he can use the work-energy theorem.

His diagram of the incline shows only the two dimensions given and a vertical line to show height at .85 m. He writes expressions for height, potential energy, and kinetic energy – initial and final. He sets up the equation that equates initial energy to final energy and the work done by friction; it is done in such a way that the derived length will be the total distance up the incline, a situation that he recognizes a little later.

His only mistake is in subtracting the work done by friction instead of equating the initial energy to the sum of the final (potential) energy and the energy lost to friction.

When he reaches a numerical answer, he does not trust it because it is negative. [Even with the sign mistake for work done by friction, the answer should have been positive, so the problem might have been an unfamiliar calculator.] He checks his equation carefully, changes it to produce the distance that the object moves beyond .85 m, and produces a new numerical answer that is more satisfactory to him, though it contains an arithmetic error..

He finishes part b, where he makes the same error in subtracting the work done by the frictional force.

Problem 2 – He reads the problem and immediately decides to use conservation of energy again. He describes the initial energy situation and surmises that the final situation with sphere, pulley, and object in motion will be “a mess.”

The solution is reached directly and is correct except for an inadvertent change when writing a number. It is notable, however, that when he first reached the answer, he had a negative sign for the expression equated with  $v_0^2$  because he had originally assigned a value of zero for initial potential energy. He did not consider that the height was also negative. At this point he drew a sketch of the hanging object showing “h” first with an arrow down, then up. He changed the origin to solve the dilemma.

#### S4

Problem 1 – After reading the problem she draws a diagram of the situation and then a free body diagram. Originally, she had the frictional force in the direction of motion, bt

soon changed it, explaining that “ the coefficient is the opposite respect to movement.” [This use of coefficient is probably a language anomaly, because she shows  $F_k$  on the diagrams.]

She thinks through the system she has drawn as a “positive system” as she diagrams and before writing down sums of forces for the x- and y-directions. She notes that the weight (y-direction) and normal are opposite because “nothing leaves the incline.” She substitutes the appropriate expression for  $F_k$ .

At this point she decides to use Newton’s laws. She identifies the known variables including  $v = 0$  “since it stops.” Note that she uses a notation ( $u$  and  $s$  instead of  $v_0$  and  $d$  or  $x$ ) probably learned in high school, and she recalls the  $v^2$  equation.

She works out a correct expression for the distance, but when she begins to substitute numeric quantities, she finds that the answer is negative. Instead of going back to her frame of reference to interpret the meaning of negative distance, she goes back and adds negative signs to the expressions for the sum of forces in the x-direction , acceleration, and distance. The final answer is correct.

She rereads part b, but concludes that she is looking for “ $t$ ” [part b asks for “ $v$ ”], and this is what she finds. Her first remark is that she thinks “the solution has to be the same.” But then she draws a free body diagram and sees that “everything will change” because force of friction acts upwards.

To find “ $t$ ” she applies the quadratic equation to  $\frac{1}{2}gt^2 + ut - s = 0$ . She recognizes later that “ $g$ ” should be changed to “ $a$ ” and makes the appropriate substitutions. However, she does not catch the fact that she had dropped the “ $-s$ ” in applying the quadratic equation and would have had the wrong units. She might have

seen this if I had let her go ahead with numerical values, but in the interest of time, we stopped.

Problem 2—She reads the problem, rereads the end, then draws a free body diagram of the isolated mass and associates it with the section of rope acting on the pulley. She concludes that tension throughout the rope must be the same. However, she correctly concludes that the sum of forces in the y-direction equals  $ma$ , “because the weight is falling down.”

She sets up a torque equation for the pulley, but she only considers the torque from the vertical rope and equates the answer with  $I\alpha$ . This gives her an expression for tension that she substitutes to solve for  $\alpha$  in terms of mass, acceleration, and  $I$  and  $r$  for the pulley.

At this point she looks over the variables she has and notes again that she needs to find velocity. She does not have “time” so she falls back on the  $v^2$  equation for uniform acceleration and uses the  $\alpha$  expression instead of acceleration, not noting that “ $a$ ” is contained within the expression. She never at any time considers the effect of the sphere.