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ESSAYS ON IMPLIED VOLATILITY

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Abstract

This dissertation consists of three empirical essays regarding the estimate of future market volatility implied by an option price. According to the theory of market efficiency, this implied volatility should represent the market participants' consensus expectation of the volatility over the remaining life of the option. But this assertion has been hotly debated. The first essay presents a new explanation for the previous finding that implied volatility is biased and inefficient. I find that implied volatility varies around the market's true volatility expectation due to measurement error and that this measurement error biases tests towards rejecting the informational efficiency of the implied volatility. When I control for the measurement error utilizing instrumental variables estimation, the results in most cases no longer reject the hypothesis that implied volatility is unbiased and informationally efficient.

The second essay relates to the implied volatility "smile". While previous explanations for the smile have focused on possible errors in the pricing formula, I argue that the smile may be caused by investors' preferences for certain strike prices for hedging purposes. I find that market inefficiency is partly responsible for the smile since abnormal returns can be made over time based on the implied volatility differences. The third essay compares the relative forecasting efficiency of implied volatilities across different strike prices. I find, contrary to the general belief and practice, that the implied volatility calculated from an at-the-money option is less informative than those calculated from options with relatively higher strike prices and that an average measure may not be effective.

ESSAYS ON IMPLIED VOLATILITY

Chapter 1

Introduction

Since market volatility is a critical factor in pricing derivatives and assets with derivative characteristics, understanding volatility is of vital interest to traders, investors, risk managers and finance researchers. This dissertation explores the implied volatility calculated from the observed option prices. While many aspects of implied volatility such as its information content, predictive power, time-series properties, and volatility smile have been studied extensively, there are still a number of unresolved issues. The aim of this dissertation, which contains three empirical essays examining options on S&P 500 futures, is to augment and strengthen the understanding of implied volatility in terms of three issues, thereby facilitating more efficient use of this measure. The empirical evidence presented in this dissertation should be informative to both practitioners and academics who are interested in forecasting market volatility.

The first essay concerns the predictive power of implied volatility. According to the efficient market hypothesis, all information is quickly and correctly incorporated into asset prices. Hence, if the option pricing model is correct, the implied volatility calculated from an observed option price should represent the market's forecast of the underlying asset's volatility over the remaining life of the option. Therefore, it should be both unbiased and informationally efficient. However, previous studies have found that implied volatility is a biased estimator of subsequent realized volatility and is not efficient in that it fails to incorporate all available information, including historical returns. Some studies even report that implied volatility has lower predictive power than historical volatility which is calculated from historical returns. Is implied volatility really a biased and inefficient estimator or are the previous findings due to some other reasons? I argue and show that implied volatility varies around the market's true volatility expectation due to such factors as bid-ask spreads, non-synchronous prices, and possible deficiencies in the pricing formula and that this measurement error biases tests toward rejecting the informational efficiency of implied volatility. No previous work has systematically examined the effect of measurement error on the predictive power and efficiency tests.

The second essay relates to the implied volatility smile which has gained considerable attention. The smile refers to the cross-sectional variation in implied volatility across options with different strike prices but the same maturity. In other words, on a given day, one obtains different implied volatilities over the same period from options with different strike prices. If the option pricing model is correct and the market is efficient, there should be no smile since, as noted above, all implied volatilities calculated from options with the same expiration date should represent the same market forecast. The prevailing explanation for the smile is that the Black-Scholes (1973) (BS hereafter) option pricing model is incorrect leading researchers to propose more complicated option pricing models. However, none of the new models has successfully explained the smile motivating me to search for an alternative explanation. In the second essay, I test whether the smile represents a market inefficiency rather than deficiencies in the BS formula. If the BS formula is correct and the smile in fact represents a market inefficiency, then a trading strategy in which one buys options with low implied volatility and simultaneously sells options with high implied volatility should make money over time. I find that it is quite profitable for put options even after accounting for the transaction costs.

The third essay investigates the relative forecasting efficiency of implied volatility estimates calculated from options with different strike prices. On a given day for a given expiration date, many different options are traded which differ by strike price and whether they are calls or puts. As just noted, each of these contracts provides its own implied volatility estimate and these implied volatilities differ across different strike prices. Which of these implied volatilities should be utilized to forecast future volatility? Reflecting the general belief that at-the-money options are the most informative because they are the most sensitive to volatility changes, one popular procedure is to calculate the implied volatility from at-the-money options only. Another common procedure is to average together several, often two or four, different implied volatilities calculated from near-the-money options thereby averaging out measurement error. Are the implied volatilities calculated from at-the-money options more efficient? Do call options or put options yield better forecasts? Do we miss some important information by ignoring all other calls or puts which are either in- or out-of-the-money? Does an average necessarily reduce measurement error and improve the forecast?

These questions are important to both researchers and practitioners. Several previous studies compared the relative performance of different *average* implied volatility estimators as well as sometimes a couple of individual implied volatility estimators. Although their results were quite mixed, they leaned toward favoring at-the-money options. The problem is that these studies generally suffered from limited data sets which may not give them enough degrees of freedom. Also as far as I know, no one has systematically investigated the relative forecasting ability of implied volatility across different strike prices and between calls and puts. The third essay systematically examines the forecasting efficiency of these different implied volatility estimators, i.e., estimators from individual call or put options with different strike prices as well as a widely utilized average implied volatility calculated from the four nearest-the-money options.

In the first essay, I find that implied volatility has considerable predictive ability for the future realized volatility in the S&P 500 futures option market. I further find that measurement error does exist, does affect the predictive power and efficiency tests of the implied volatility, and is at least partially responsible for the apparent inefficiency of implied volatility. The measurement error effect is, in general, more severe for an individual implied volatility calculated from a randomly chosen call or put than it is for the average implied volatility calculated from the four at-the-money options.

In the second essay, I find that market inefficiency is partly responsible for the implied volatility smile in the options on S&P 500 futures market since abnormal returns can be made over time by buying put options with low implied volatility and simultaneously selling put options with high implied volatility in a delta neutral ratio. It, however, remains a puzzle why the profits from this trading strategy are significant for puts but not for calls and why the standard deviations of the profits are large despite the fact that the positions are delta neutral.

In the third essay, I find, contrary to the general belief, that at-the-money options do not yield the best forecast of future volatility in the market for options on S&P 500 futures. Implied volatilities calculated from options with relatively high strike prices (out-of-the-money calls and in-the-money puts) among all calls and puts with the same maturity seem to be more informative and more efficient. Actually, up to a certain level of strike price, the forecasting ability of implied volatility increases as strike price increases. The results are robust across options with different time-to-maturities and hold for both the samples of including and excluding the 1987 market crash. The results are also robust for different measures of relative efficiency: an OLS regression measure, root mean squared error, mean absolute error, and mean absolute percentage error.

The remainder of this dissertation is organized as follows. Chapter 2 discusses the data and the calculation of implied, historical, and realized volatilities. Each of the three empirical essays will then be presented. Chapter 3 examines the effect of measurement error on the tests of predictive power of implied volatility. Chapter 4 offers and tests an alternative explanation for the implied volatility smile. Chapter 5 explores the best estimators among the implied volatilities calculated from options with different strike prices. Finally, Chapter 6 summarizes the dissertation.

Chapter 2

Data and Volatility Measures

2.1 Data

The data for this dissertation consist of daily settlement prices of options on S&P 500 futures traded on the Chicago Mercantile Exchange from January 28, 1983 to April 30, 1998. S&P 500 futures prices over the same period are also utilized to calculate implied, realized, and historical volatilities. Today, S&P 500 futures and options are the most widely recognized stock index contracts in the world and have become an indispensable risk management tool of mutual funds and institutional investors. The volatility of the S&P 500 futures can therefore be interpreted as a good measure of aggregate stock market volatility.

An advantage of using options on index futures instead of options on the index itself is that you do not need to consider dividends in calculating implied volatility on the futures thereby reducing calculation error. Harvey and Whaley (1992) showed that ignoring dividends or employing other ad hoc dividend-adjusted valuation procedures can produce large errors in pricing options on S&P 100 index. Another advantage is that arbitrage between options and their underlying futures is easier and much cheaper than arbitrage involving all stocks in the index.

The third advantage of the data set is that the options and their underlying futures are traded side by side on the Chicago Mercantile Exchange and close at the same time alleviating the problem of non-synchronous quotes. Furthermore, I utilize a much longer data set in this dissertation than previous studies (more that 15 years of daily data). This is important because the overlap in realized volatility periods sharply reduces the effective degrees of freedom. Consider options which mature on March 20. On a given day, a number of different implied volatilities calculated from calls or puts with different strike prices but with this same expiration date are predicting the volatility of the underlying futures over the same future period. In addition, the implied volatilities calculated from the option prices on consecutive days, such as January 1, January 2, January 3, etc., are predicting the volatility over virtually identical periods so are not independent. For example, in more than 15 years of daily data of options on S&P 500 futures with the underlying futures contracts maturing every three months, there are 217,226 observations but only about 60 truly independent subsets.

Options on S&P 500 futures are American type options and are daily cash settled or "marked to the market". One contract of the underlying futures has a value of the S&P 500 index times 500. The minimum pricing increment, or "tick size", is 0.05 points (equivalent to $25). However, the Chicago Mercantile Exchange reduced the multiplier from 500 to 250 and increased the minimum pricing increment to 0.10 points (but still equivalent to $25) on October 31, 1997.

On a given day, options differ in three dimensions: expiration date, call or put, and strike price. Before July 1987, options on S&P 500 futures were traded only for contracts expiring in March, June, September, and December. These options expire on the same day as their underlying futures contracts and are traded starting about nine months or a year before expiration. For example, the September 1986 options were traded from December

23, 1985 through September 18, 1986, or 187 trading days. In August 1987, short-term serial contracts were introduced. These are contracts maturing in the next three months in which there is not already a quarterly contract. For instance, April 1993 options were traded from January 18, 1993 to April 16, 1993, or 63 trading days. The serial options do not expire on the same day as their underlying futures but share the same underlying futures with the next nearest March, June, September or December options. For example, the underlying futures of October 1993 options expires in December 1993 and that of the February 1997 options expires in March 1997.

As an example, Figure 2.1 shows that on February 25, 1997, one could trade options with six different expiration dates. The options expiring in March, June, September and December are quarterly cycle contracts while the options expiring in April and May are serial contracts. But, before July 1987, one could only trade the quarterly cycle options.

In this dissertation, I stratify the options into four groups based on time to expiration. Group 1 contains options maturing first, e.g., the March contracts in Figure 2.1. Group 2 contains the options maturing next, e.g., the April contracts in Figure 2.1. Since as an expiration date approaches the option prices become small relative to transaction costs, observations on options with fewer than ten business days to expiration are excluded.[1] On these days, options with the second nearest expiration date move into Group 1 and options

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[1] To minimize data errors, observations which violate boundary conditions are also eliminated from the data set, i.e., call prices should be greater than the present value of current futures price minus the present value of the strike price.

with the third nearest expiration date move into Group 2, etc.[2] As a result, options' time-to-maturity ranges from 10 to 36 trading days in Group 1, from 28 to 57 trading days in Group 2, from 47 to 78 trading days in Group 3, and from 67 to 99 in Group 4.[3]

On a given day, for the same time to maturity, there are both calls and puts which further differ by strike price. As we know, a call confers the right, but not the obligation, to buy a futures contract and a put confers the right, but not the obligation, to sell a futures contract. A call (put) with a strike price less than (greater than) the underlying futures price is referred to as *in the money*. On the other hand, a call (put) whose strike price is greater than (less than) the underlying futures price is referred to as *out of the money*. An option (a call or a put) with a strike price equal to the underlying futures price is referred to as *at the money*. Since the strike prices are in the increments of 5 points for options on S&P 500 futures, such as 825, 830, and 835 etc., there are seldom situations when a futures price is exactly equal to an option's strike price. In practice, therefore, the options with strike prices nearest to the underlying futures price (i.e. the two nearest-the-money calls and the two nearest-the-money puts) are often taken as the at-the-money options. Figure 2.2 illustrates the different calls and puts on S&P 500 futures with the same time-to-maturity on a given

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2 Note that for dates before July 1987, there may not be observations each date for each group.

3 As will be discussed in Subsections 2.2.2 and 3.3.3, there is a considerable overlap in the realized volatility periods in the options data. The overlap within each expiration group could exist in two dimensions: different options (calls or puts with different strike prices) observed on the same day and the same (or different) options observed on different days. Within each expiration group, on a given day, implied volatilities calculated from calls or puts with different strike prices are forecasting exactly the same future realized volatility. On the other hand, realized volatilities for the same (or different) options with the same underlying futures observed on different days are partially overlapped. For example, realized volatilities for options with the same underlying futures contract observed on day t and day t+1 only differ because the former is calculated from the daily return series from day t+1 through the option expiration date while the latter from the same daily return series but from day t+2 through the option expiration date.

day. In summary, options on S&P 500 futures differ in three dimensions: expiration date, option type (call or put), and strike price. Within each expiration group, the observations are both time series and cross-sectional.

Trading activity of options differs by time-to-maturity and strike price and also differs between calls and puts. In general, options with shorter time-to-maturity are traded more heavily. For the same maturity, out-of-the-money options (both calls and puts) are more actively traded than in-the-money options while near-the-money options are usually traded more often than far-from-the-money options. As a result, prices of far-from-the-money options are often not observable or "stale" which tends to introduce more measurement error. Figure 2.3 shows how the average trading volume of the options differs by the "moneyness" which is defined as the ratio of strike price over the current underlying futures price minus one. Within each expiration group, the closer the strike price is to the futures price, the more frequently an option is traded. Also observe that out-of-the-money calls (puts) are consistently traded more heavily than the corresponding in-the-money calls (puts).

Options at every strike price may not be traded every day. Figure 2.4 illustrates the total number of daily observations available for options with the same ranking in terms moneyness in the data set. As shown in Figure 2.4, within each group, the number of observations displays a pattern similar to that of the average trading volume shown in Figure 2.3. In general, out-of-the-money calls (puts) have more observations than in-the-money calls (puts). In the analysis, I utilize eight nearest-in-the-money calls (puts) and eight nearest-out-of-the-money calls (puts) as shown in Figure 2.2.

2.2 Methodology

2.2.1 Implied Volatility

As outlined above, all the three essays concern implied volatility. These implied volatilities are calculated utilizing Black's (1976) model for European options on futures[4]:

$$C=e^{-RT}\,[F\,N(d_1) - K\,N(d_2)] \tag{2.1}$$

$$P=e^{-RT}\,[K(1-N(d_2)) - F(1-N(d_1))] \tag{2.2}$$

Where
$$d_1=\frac{\ln(\frac{F}{K}) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$$

$$d_2=d_1 - \sigma\sqrt{T}$$

C -- price of a call option

P -- price of a put option

T -- an option's time to expiration

R -- risk-free interest rate [5]

F -- price of the underlying futures

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4 Since it assumes European options while the options on S&P 500 futures are American, using Black's model introduces a small upward bias in the implied volatility. Jorion (1995) shows that this difference is quite small relative to the typical bid-ask spread, e.g., using a European model overestimates a 12 % true volatility as 12.02 %. Also since stock index futures, unlike individual stocks, pay no dividends, dividends do not appear in the Black's model for options on futures.

5 For the risk-free interest rate, I extract daily observations of yields to maturity of 3-month, 6-month, and 1-year Treasury bills. A simple convex combination is used to interpolate interest rates according to the option's time to expiration. The interest rate choice has little impact on the results because the option pricing formula is quite insensitive to the interest rate (See Sheikh(1993)).

K -- strike price

σ -- annualized standard deviation of the continuously compounded return on the futures.

The implied volatility for an option with strike price i on day t, $\sigma_{ISD,i,t}$, is calculated by substituting the settlement price of the option with strike price i on day t for $C_{i,t}$ in Equation 2.1 or $P_{i,t}$ in Equation 2.2 and solving for $\sigma_{ISD,i,t}$ using an iterative procedure. The implied volatilities (also the realized and historical volatilities) are annualized by multiplying by the square root of 252 - the approximate number of trading days in one year. In Chapter 3 and Chapter 5, I also utilize an average implied volatility ($\sigma_{ISD4,t}$) which is the average of the implied volatilities calculated from the two nearest-the-money calls and the two nearest-the-money puts [6] since this measure is often utilized by practitioners as well as some researchers.

2.2.2 Realized Volatility

According to the BS model and the market efficiency hypothesis, implied volatility, $\sigma_{ISD,i,t}$, should represent the market's forecast on day t of actual volatility over the remaining life of the option. To judge how well it forecasts, we must measure actual ex-post volatility over this same period. The realized volatility, $\sigma_{RLZ,t}$, over the period from day t through the option expiration date, N, is calculated as the standard deviation of returns over this period,

6 When utilizing $\sigma_{ISD4,t}$, I actually ignore all the other options with the same expiration date observed on the same day. As a result, in each expiration group, there is only one observation for this average implied volatility per day and therefore, I have much fewer observations in each of the four groups. For example, there are 3,212 observations of $\sigma_{ISD4,t}$ for group 1 observed for 3,212 trading days.

i.e.,

$$\sigma_{RLZ,t} = \sqrt{252 \times [\frac{1}{N-t-1} \sum_{s=t+1}^{N} R_s^2 - \frac{1}{(N-t)(N-t-1)} (\sum_{s=t+1}^{N} R_s)^2]} \qquad (2.3)$$

where $R_s = \ln(F_s / F_{s-1})$ and F_s is the futures settlement price on day s and where $t \leq N-10$.[7] This period is chosen to match that covered by the implied volatility calculated from the option price.

Note that there is a considerable overlap in realized volatility in the data set. First, options observed on the same day with different expiration dates (i.e., from different groups) have overlaps in their corresponding realized volatilities if they share the same underlying futures contract. In terms of the example shown in Figure 2.1, the options in Groups 2, 3 and 4 observed on February 25, 1997 have overlapped realized volatilities because they all share the same underlying futures contract which matured on June 19, 1997. For instance, realized volatilities for options from Group 2 and Group 3 observed on February 25, 1997 have an overlap period from May 16, 1997 through June 19, 1997. Second, options with the same time to maturity (i.e., within the same expiration group) on a given day have exactly the same realized volatility. In other words, on a given day, implied volatilities calculated from calls or puts with different strike prices but the same time to maturity are

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7 As discussed in Section 2.1, observations on options with fewer than ten business days are excluded since as an option date approaches the option prices become small relative to transaction costs. In addition, the number of observations utilized to calculate a realized volatility is determined by the number of trading days in the option's remaining life. Realized volatilities calculated by utilizing fewer than ten observations are more likely subject to small sample problems. Accordingly, the last observation on realized volatility for a given futures contract is at N-10 and the realized volatility, $\sigma_{RLZ,t}$, is always calculated over at least 10 trading days in this dissertation.

forecasting exactly the same realized volatility. Third, there are overlaps in observations on different days for the same option. As shown in Equation 2.3, realized volatility, $\sigma_{RLZ,t}$, calculated from the return series of a given futures contract on day t and that calculated from the same return series on day t+1 only differ because one return, R_{t+1}, is dropped from the set covered by the summation sign. $\sigma_{RLZ,t}$ and $\sigma_{RLZ,t+1}$ both include the returns: R_{t+2} through R_N. All these overlaps seriously bias the *standard errors* of OLS estimates downward although the OLS parameter estimates are still unbiased and efficient. In Chapter 3, I will discuss and utilize a technique for correcting for the heteroskedasticity and serial correlation caused by these overlaps.

2.2.3 Historical Volatility

Besides implied volatility, historical volatility is often utilized to forecast future realized volatility even though the majority of empirical evidence shows that historical volatility has a lower predictive power than implied volatility. For comparison, I also utilize a measure of historical volatility in Chapter 3 and Chapter 5. This measure is the standard deviation of returns of the underlying futures over the last M trading days:

$$\sigma_{HIS,t} = \sqrt{252 \times [\frac{1}{M-1} \sum_{s=t-M+1}^{t} R_s^2 - \frac{1}{M(M-1)} (\sum_{s=t-M+1}^{t} R_s)^2]} \qquad (2.4)$$

In Chapters 3 and 5, M equals forty trading days since this most closely matches the sixty calendar day period utilized by Canina and Figlewski (1993) and some other studies. However, the results in this dissertation are not sensitive to this choice.

In summary, realized volatility is measured from day t through the option's expiration day, N, and the two forecasts for this realized volatility are calculated: implied volatilities calculated from option prices observed on day t and the standard deviation of returns over the last 40 trading days before day t. Figure 2.5 shows the respective data periods utilized to calculate these three volatility measures.

2.2.4 Summary Statistics of Implied, Realized, and Historical Volatilities

Since the data set covers the 1987 market crash, all realized, historical and implied volatilities involving returns or prices in this period are extremely high and sometimes tend to dominate the results. Accordingly, the analyses are conducted separately for the samples ***excluding*** and ***including*** observations affected by the 1987 stock market crash. Table 2.1 reports summary statistics for realized, implied and historical volatilities for the samples including and excluding 1987 market crash in terms of each of the four expiration groups as well as the overall sample. For example, the first row in Panel A shows the statistics for all the options in the four expiration groups together including the 1987 crash while that in Panel B reports the same statistics excluding the 1987 crash.

As shown in Table 2.1, implied volatility generally exceeds the subsequent realized volatility. For instance, the mean of the implied volatility is 0.1686 for the overall sample including the 1987 crash while that of subsequent realized volatility is 0.1420. This difference is significant at the 0.01 level. Note that observations are both time series and cross-sectional. There are 3,212 trading days but 77,123 observations in Group 1 because on each day, implied volatility is calculated separately from up to 16 calls and 16 puts.

Chapter 3

Essay I: Measurement Error and the Predictive Power of Implied Volatility [8]

3.1 Introduction

As well known, if the option market is efficient and the option pricing model is correct, the implied volatility calculated from an observed option price should represent the market's best forecast of the underlying asset's volatility over the remaining life of the option. As such, it should be both unbiased and informationally efficient, that is, it should correctly impound all available information, including the asset's price history. Consequently, measures of historical volatility (or other measures based on past returns, such as a GARCH measure) should add no additional predictive power. However, the evidence to date on this issue has been mixed. While most of the studies find that implied volatility outperforms historical volatility in forecasting future volatility, they normally also find that implied volatility fails to incorporate all available information, including historical volatility. Moreover, some studies even find that implied volatility's predictive power is quite low. For example, using two years of transaction data for ten individual stocks, Lamoureux and Lastrapes (1993) rejected the Hull and White (1987) class of stochastic volatility models in favor of a GARCH model. Separately, Canina and Figlewski (1993)

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8 This chapter builds on and expands my 1996 summer research project and the joint working paper with Professor Louis Ederington "Is implied volatility an informationally efficient and effective predictor of future volatility?".

found that when ex-post volatility of the S&P 100 index is regressed on the implied volatility calculated from individual S&P 100 options, the coefficient of the implied standard deviation is insignificant, and often negative, while the coefficient of the historical volatility is positive and significant.

In this first essay, I examine whether the implied volatilities calculated from options on S&P 500 futures are informationally efficient and effective predictors of actual realized volatilities. Specifically, I explore whether the evidence against the informational efficiency of implied volatility is due to "measurement error", that is to deviations of implied volatility from the market's true volatility expectation due to bid-ask spreads, non-synchronous prices, minimum price increments, errors in the option pricing formula, or arbitrage restrictions. While econometrics texts commonly show that measurement error in an independent variable tends to bias its coefficient toward zero, no previous work has systematically investigated the effect of measurement error regarding implied volatility.

By examining more than 15 years of daily data, I find that the implied volatility calculated from options on S&P 500 futures has considerable predictive ability. In other words, when realized volatility is regressed on both implied volatility and historical volatility, the slope coefficient of the implied volatility is always significantly different from zero and larger than that of the historical volatility. At the same time, the coefficient of historical volatility is not always significantly different from zero and the adjusted R^2 is much higher than observed in previous studies on options on stock index. However, I also find that implied volatility from options on S&P 500 futures differs from the market's true volatility expectation and that this measurement error is at least partially responsible for the apparent inefficiency of implied volatility. When I control for measurement error utilizing an instrumental variables estimation, the predictive power of implied volatility rises while that of historical volatility falls and in most cases implied volatility turns out to be unbiased and efficient. Finally, I find that the results differ depending on the forecasting horizon and whether the sample includes the 1987 stock market crash.

This essay differs from previous studies in that it tests and corrects for measurement error and in that it considers how sensitive the results are to the forecast horizon and whether the data set includes the 1987 market crash. It also differs in that it utilizes a much longer data period, more than 15 years of daily data for options on S&P 500 futures, than previous studies. This is important because the overlap in realized volatility periods sharply reduces the effective degrees of freedom and biases the standard error estimates.

The rest of this essay is organized as follows. The next section reviews the literature on the relative predictive power of implied and historical volatilities and describes the traditional test of informational efficiency. Section 3.3 describes the measurement error hypothesis and the procedure for correcting heteroskedasticity and serial correlation caused by the overlaps of the realized volatility periods. Section 3.4 reports the OLS results on informational efficiency of the individual implied volatilities versus an implied volatility average, and examines how these results depend on forecast horizon and whether the 1987 crash is included. In the meantime, the measurement error is tested and corrected through an instrumental variables estimation. Section 3.5 summarizes and concludes this essay.

3.2 Literature Review

Most early studies of implied volatility, such as Latané and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981), and Park and Sears (1985), were content to test whether implied volatility contained *any* information regarding subsequent realized volatility. Whether implied volatility was informationally efficient was not an issue. Most found that implied volatility had some predictive ability and interpreted this finding as confirmation of Black-Scholes option pricing theory.[9]

More recent studies have compared the relative forecasting ability of implied and historical volatilities and have examined how efficiently implied volatility incorporates the information available to market participants. Most follow basically the same approach. Actual realized volatility (usually the standard deviation), $\sigma_{RLZ,t}$, from day t through the remaining life of the option is regressed on either implied volatility calculated from the observed price of an option i, $\sigma_{ISD,i,t}$, or some measure based on historical time series data, $\sigma_{TS,t}$, (either the standard deviation of returns over some past period or a GARCH estimate derived from historical data), or both. That is they estimate:

$$\sigma_{RLZ,t} = \alpha_0 + \alpha_1 \sigma_{ISD,i,t} + u_{i,t} \quad \text{and} \quad \sigma_{RLZ,t} = \alpha_0^* + \alpha_1^* \sigma_{TS,t} + u_t \tag{3.1}$$

or they estimate:

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9 One problem in most of these early studies was that implied and actual volatility periods were not matched precisely. For instance, Chiras and Manaster (1978) measured actual realized volatility over a 20 month period from month t through t+20 regardless of the time period covered by the implied volatility, that is, regardless of whether the options contract matured in 20 months or 5. In addition, all of these early studies suffered from limited data sets and, therefore, small degrees of freedom, for instance, 23 monthly observations in the case of Chiras and Manaster (1978) and five months of daily data in the case of Park and Sears(1985).

$$\sigma_{RLZ,t} = \beta_0 + \beta_1 \sigma_{ISD,i,t} + \beta_2 \sigma_{TS,t} + u_{i,t} \tag{3.2}$$

The hypothesis that $\sigma_{ISD,i,t}$ is unbiased and informationally efficient implies that α_0=0 and α_1=1 in the $\sigma_{ISD,i,t}$ version of Equation 3.1 and implies that β_0=0, β_1=1, and β_2=0 in Equation 3.2.

Estimates of Equation 3.2 from several previous studies are reported in Table 3.1 where, to obtain a representative summary regression for each study, I have averaged together the estimates for different markets and data periods when a study reports more than one. While results differ, a couple of consistencies stand out. With the single exception of Canina and Figlewski (1993), all studies find that $\beta_1 > 0$ and most find that it is significant, implying that implied volatility does have informational content. On the other hand, in almost all studies, $\beta_0 > 0$ and $\beta_1 < 1$ (although β_1 is very close to one in Day and Lewis's crude oil regression) implying that implied volatility is not an unbiased and informationally efficient estimator.

The studies differ on whether historical time series measures add incremental information, i.e., on the sign and significance of β_2. As measured by the R^2, predictability also differs widely. It appears fairly high for crude oil, moderate for individual stocks, and relatively low for stock indices and foreign exchange rates. In contrast to the other studies, Canina & Figlewski (1993) (C&F hereafter) find that in the S&P 100 index option market, implied volatility is actually a poorer forecaster of subsequent realized volatility than historical volatility and that implied volatility adds no incremental information to that contained in the historical volatility.

Utilizing over seven years of daily data on 25 stock options, Bartunek and Chowdhury (1995) compared forecasts of implied volatility, GARCH volatility and historical volatility. Their implied volatility measure was calculated from several options with different strike prices by a minimized-squared-pricing-error method.[10] They found no obvious superiority of any one forecast over any of the others. However, their realized volatility period and the implied volatility period are not matched exactly as discussed in the footnote 9. Most recently, Christensen & Prabhala (1998) compared the predictive power of implied volatility and historical volatility by utilizing monthly data on options on S&P 100 index. They found that implied volatility outperforms historical volatility in forecasting future volatility and even subsumes the information content of past volatility in some of their specifications. Their major contribution is that they utilized an instrumental variables analysis similar to the one introduced in this essay.[11] However, while correcting the problems of overlapping in realized volatility, their nonoverlapping sampling procedure, similar to that of Feinstein (1989), threw away most of the available observations and in effect threw away a lot of valuable information. In addition, in their monthly data set, they kept only at-the-money call options and ignored all the other call options and all the put options.

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10 This method was developed by Beckers (1981) and Whaley (1982) which will be further discussed in Section 5.2.

11 The instrumental variables analysis introduced in this essay was developed independently before the publication of Christensen & Prabhala (1998).

3.3 Methodology

3.3.1 Measurement Error and its Impact

A finding that implied volatility is not informationally efficient could mean either that investors are not rational or that the implied volatility, $\sigma_{ISD,t}$, [12] does not represent investors' true volatility expectation, $\sigma_{TE,t}$. C&F (and Figlewski, 1997) lean toward the latter arguing that factors not in the option pricing formula influence option prices and that market frictions impede the arbitrage and speculation which would ensure $\sigma_{ISD,t} = \sigma_{TE,t}$ in perfect markets. This essay explores and tests this argument, which is that whether the results rejecting the informational efficiency of $\sigma_{ISD,t}$ are due to these deviations of $\sigma_{ISD,t}$ from $\sigma_{TE,t}$.

As also discussed by Figlewski (1997), there are several reasons, why the implied volatility might differ from the market's true expectation. One, $\sigma_{ISD,t}$ is calculated from both the option price and the underlying asset's price and these two prices may reflect trades at different times. Two, bid-ask spreads in both the option and the underlying asset will introduce measurement error as will the fact that option prices are not continuous, instead trading in discrete minimum increments. Three, the option pricing model used to calculate implied volatility may be in error. For instance, implied volatility is often calculated using the Black-Scholes model which assumes (among other things) that (1) the option is European, (2) volatility is constant (or at least deterministic), (3) returns are log-normally distributed. Violations of any would cause the implied volatility calculated from option

12 To simplify the notation in this section, the subscript *i* which designates a call or a put with a different strike price has been dropped from $\sigma_{ISD,i,t}$.

prices using the BS formula to differ somewhat from the market's true volatility forecast.[13]

Four, hedging activities, or other factors not in the option pricing model, may push option prices away from the prices consistent with the market's volatility expectation and market imperfections may prevent the arbitrage and speculation which would equate $\sigma_{ISD,t}$ with $\sigma_{TE,t}$. For example, holders of stock portfolios may buy out-of-the-money puts (pushing up their prices and implied volatilities) in order to protect against a possible market decline even if they think the puts are somewhat overpriced. If these hedging trades push $\sigma_{ISD,t}$ above $\sigma_{TE,t}$, then, in perfect markets, arbitrage and speculation should occur which would push put prices and their implied volatilities back down. For instance, if put prices rise relative to call prices, then put-call parity arbitrage should occur. Of course, while put-call parity arbitrage would lower the implied volatility in put prices, it would simultaneously tend to raise call prices above those consistent with the market's volatility expectation - leaving both somewhat above the market's true expectation. If both the call and the put prices exceed those consistent with volatility expectations, then in a perfect market, speculators should sell both in a delta neutral ratio or sell a call (a put) option and buy (sell) the underlying asset. Either would push option prices and implied volatilities down. However, continually re-balancing these positions would entail large transaction costs and the speculator would lose if actual volatility turned out to be larger than anticipated. In summary, arbitrage and speculation may not fully return option prices to levels fully

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A number of studies argue that the mis-pricing caused by these is likely to be minor, at least for at-the-money options. For instance, Fleming (1998) and Jorion (1995) find that stochastic and deterministic models yield virtually the same option prices for at-the-money options, and Jorion (1995) also reports that the difference between American and European option prices is much smaller than normal bid-ask spreads.

consistent with the market's volatility expectations. The evidence on volatility smiles and smirks indicates that some deviations exist due to reasons three and/or four.

If $\sigma_{ISD,t}$ varies around $\sigma_{TE,t}$ for any of these four reasons, then in ordinary least squares estimations of Equations 3.1 and 3.2, the estimates of α_1 and β_1 will be biased downward and those of α_0 and β_0 will be biased upward. If historical time series volatility ($\sigma_{TS,t}$) is an important determinant of the market's true volatility expectation, then the OLS estimate of β_2 will be biased upward as well. Hence, the evidence in Table 3.1 against the informational efficiency of implied volatility could be explained by this measurement error.

To see this, suppose both actual and implied volatilities revolve around the market's true expectation, $\sigma_{TE,t}$, i.e.,

$$\sigma_{RLZ,t} = \sigma_{TE,t} + \epsilon_t \quad \text{and} \quad \sigma_{ISD,t} = \sigma_{TE,t} + \eta_t \tag{3.3}$$

Note that ϵ_t is an expectational error and η_t is a measurement error. If investors are rational, then the expectational error, ϵ_t, should be independent of all time t variables, including $\sigma_{TE,t}$ and η_t. Suppose now that Equation 3.1 is estimated using ordinary least squares and $\sigma_{ISD,t}$. The OLS estimator is $\hat{\alpha}_1 = \sum \sigma_{RLZ,t}\,\sigma_{ISD,t} \div \sum \sigma^2_{ISD,t}$. Substituting the expressions in (3.3) for $\sigma_{RLZ,t}$ and $\sigma_{ISD,t}$,

$$\hat{\alpha}_1 = \frac{\sum (\sigma_{TE,t} + \epsilon_t)(\sigma_{TE,t} + \eta_t)}{\sum (\sigma_{TE,t} + \eta_t)^2} \tag{3.4}$$

Since market efficiency implies that ϵ_t is independent of $\sigma_{TE,t}$ and η_t, if η_t is independent of $\sigma_{TE,t}$, then in the probability limit,

$$\text{plim}(\alpha_1) = \frac{\text{Var}(\sigma_{TE})}{\text{Var}(\sigma_{TE}) + \text{Var}(\eta)} \tag{3.5}$$

Note that if there is no measurement error, i.e., $\text{Var}(\eta) = 0$, then the expected value of $\alpha_1 = 1$ but, if there is measurement error, i.e., $\text{Var}(\eta) > 0$, then $E(\alpha_1) < 1$. Since $\alpha_0 = \bar{\sigma}_{RLZ} - \alpha_1 \bar{\sigma}_{ISD} = (1 - \alpha_1)\bar{\sigma}_{ISD} + \bar{\epsilon} - \bar{\eta}$ and $E(\bar{\epsilon}) = E(\bar{\eta}) = 0$, if $E(\alpha_1) < 1$, then $E(\alpha_0) > 0$. While I have examined Equation 3.1 for simplicity, it can be shown that measurement error will cause OLS estimates of β_1 in Equation 3.2 to be biased downward and of β_0 and β_2 to be biased upward.

For several reasons, $\text{Var}(\eta)$ is probably particularly large in Canina and Figlewski (1993) which may explain why their estimate of β_1 is smaller than in other studies. While other studies obtain a single implied volatility for each day by averaging together the implied volatilities calculated from individual options with the same maturity but differing strike prices, C&F treat each strike price as a separate observation. So, while other studies utilize the average of implied volatilities calculated from several nearest-the-money options, C&F do not. In addition, S&P 100 index options are particularly susceptible to the non-synchronous data problem discussed above because S&P 100 Index options market and the NYSE do not close at the same time. Finally, as C&F point out, because of the transaction costs involved in buying and selling 100 stocks, it is difficult to arbitrage between the option and the underlying portfolio.

Although OLS estimates of Equations 3.1 and 3.2 are biased by measurement error, if there exists a variable, Z, which is correlated with the market's true volatility expectation,

$\sigma_{TE,t}$, but not with the measurement error η_t, then it is possible to obtain unbiased estimates using instrumental variables. The instrumental variables estimator of α_1 in Equation 3.1 is:

$$\alpha_1' = \frac{\sum \sigma_{RLZ,t} Z_t}{\sum \sigma_{ISD,t} Z_t} \tag{3.6}$$

substituting again for $\sigma_{RLZ,t}$ and $\sigma_{ISD,t}$ yields:

$$\alpha_1' = \frac{\sum (\sigma_{TE,t} + \epsilon_t) Z_t}{\sum (\sigma_{TE,t} + \eta_t) Z_t} \tag{3.7}$$

If η_t and Z_t are uncorrelated and markets are efficient in the sense that ϵ_t and Z_t are independent, then $\text{plim}(\alpha_1') = E(\sum \sigma_{TE,t} Z_t) / E(\sum \sigma_{TE,t} Z_t) = 1$. Likewise, it can be shown that instrumental variables estimates of the β parameters in Equation 3.2 are unbiased.

3.3.2 Instrumental Variable Selection

Consider the implied volatilities at time t and j days before: $\sigma_{ISD,t} = \sigma_{TE,t} + \eta_t$ and $\sigma_{ISD,t-j} = \sigma_{TE,t-j} + \eta_{t-j}$ where η is the measurement error in each. Since the market's two true volatility expectations should only differ due to new information received between t-j and t and to the fact that $\sigma_{TE,t}$ covers j fewer days, $\sigma_{TE,t-j}$ should be strongly correlated with $\sigma_{TE,t}$. Whether or not the two measurement errors η_t and η_{t-j} are correlated depends on whether the deviations of σ_{ISD} from σ_{TE} are short- or long-lived and depends on the cause. If the deviations are due to non-synchronous prices, bid-ask spreads, or minimum tick sizes, then η_t and η_{t-j} should be independent. However, if the deviations of σ_{ISD} from σ_{TE} are due to errors in the option pricing formula, e.g., stochastic volatility or non-lognormal returns, or to market imperfections, then the deviations may be more long-lived so that part of the same error is contained in both η_t and η_{t-j}. If the time t and time t-j measurement errors are correlated, some bias will remain in the instrumental variables estimates of Equation 3.2.

In summary, using $\sigma_{ISD,t-j}$, as an instrumental variable one can test and correct for measurement error due to clearly temporary factors, like non-synchronous quotes and bid-ask spreads, but we may not be fully able to correct for more long-lived errors. Christensen and Probhala (1998) utilized one period lagged implied volatility as their instrumental variable since they utilized a monthly data set.

Employing different instrumental variables, I find that the results are robust across different choices of instrumental variables. I tested several instrumental variables such as an average implied volatility from the two nearest-the-money calls and the two nearest-the-money puts or an average implied volatility calculated from all the 32 different options which are observed one, five, or ten trading days before day t. However, to save space, I only report the estimation using the average implied volatility calculated from the two nearest-the-money calls and the two nearest-the-money puts observed ten trading days before day t.

3.3.3 Correction for Heteroskedasticity and Serial Correlation

As mentioned in Chapter 2, there is a considerable overlap in realized volatility periods in the data set. Options on S&P 500 futures differ by expiration date, between calls and puts, and by strike price. Options observed on a given day with the same time to maturity (i.e., within the same expiration group) overlap in their realized volatilities. Actually, on a given day, implied volatilities calculated from *calls or puts* with different *strike prices* but the same maturity date are forecasting exactly the same realized volatility. In addition, there are overlaps for options observed on different days. As shown in Equation 2.3, the $\sigma_{RLZ,t}$ calculated for a given futures contract on day t and that calculated for the same futures contract on day t+1 only differ because one return, R_{t+1}, is dropped from the set covered by the summation sign. $\sigma_{RLZ,t}$ and $\sigma_{RLZ,t+1}$ both include the returns: R_{t+2} , ... , R_N. These extensive overlaps mean that even data sets with several thousand observations effectively have far fewer degrees of freedom. More importantly, although OLS estimates of the α's and β's in Equations 3.1 and 3.2 are still unbiased and efficient in the presence of heteroskedasticity and serial correlation caused by these overlaps, the OLS estimates of their standard errors are biased downward.

To avoid the heteroskedasticity and serial correlation problems caused by these kind of overlaps, Feinstein (1989) and Christensen and Prabhala (1998) utilized non-overlapping monthly data sets, which, however, threw a lot of valuable information. Hansen (1982) provides a method to correct the heteroskedasticity and serial correlation caused by such overlaps, which has been used by Jorion (1995) and C&F (1993) among others. Let X_i represent a row vector of the *i*th observations on (K-1) independent variables and let X represent a M×K matrix of the M observations on (K-1) independent variables with 1's in the first column. Representing the OLS regression error for the observation *i* with ε_i, the Hansen variance-covariance matrix of the estimated coefficients is

$$(X'X)^{-1}\,\Omega\,(X'X)^{-1} \tag{3.8}$$

where Ω can be consistently estimated by

$$\hat{\Omega} = \sum_{i=1}^{M} (\hat{\varepsilon}_i)^2 X_i' X_i + \sum_{i=1}^{M} \sum_{j=i+1}^{M} Q(i,j)\, \hat{\varepsilon}_i \hat{\varepsilon}_j \,(X_i' X_j + X_j' X_i) \tag{3.9}$$

where Q(i, j) is an indicator which is equal to one if observations i and j overlap and is equal to zero otherwise.

3.4 Empirical Results

3.4.1. OLS Multiple Regressions and Informational Efficiency of Implied volatility

OLS multiple regressions of realized volatility on individual implied volatilities and historical volatility using Equation 3.2 are shown in Table 3.2 where the results for the sample including the 1987 crash are shown in Panel A and those for the sample excluding the 1987 crash are reported in panel B. The latter is presented because the σ_{RLZ}s involving the October 1987 crash are five to seven times higher than in other periods and tend to dominate the regression results. Reflecting this, the adjusted R^2s in Panel B are generally much higher than those in Panel A. Each observation on the implied volatility in the regressions is calculated from a single option price. This structure of the data set is similar to that of C&F in that each observation of their implied volatilities is also calculated from an individual option. As mentioned before, on each trading day, there are many different options which differ by maturity, option type (call or put), and strike price. Also in the data set, on each day there are options with up to four maturity dates which are stratified into four expiration groups as explained in Chapter 2. Hence, within each expiration group, all options observed on a given day have the same time to maturity. Group 1 contains options with time to expiration between 10 to 36 trading days while group 2 consists of options expiring in 28 to 57 trading days, etc. Within each group, call (put) options with up to 16 different strike prices around the underlying futures price are utilized: eight in-the-money calls, eight out-of-the-money calls, eight in-the-money puts and eight out-of-the-money puts.

In Tables 3.2, both the OLS t statistic (t_{OLS}), and the corrected t statistic using Hansen's procedure (t_H) for the null hypothesis that $\beta_1 = 0$ are reported. t_H is consistently much lower than t_{OLS} confirming that the OLS estimates of the standard errors are seriously biased. For instance, in the first regression in Table 3.2, t_H is 4.200 while OLS t is 111.571 for the hypothesis that $\beta_1 = 0$. In addition, the t_H for the null hypothesis that $\beta_1 = 1$ is also presented.

In sharp contrast to C&F's finding of low predictive power for implied volatility, I find that implied volatility has much high predictive power than historical volatility and subsumes the information contained in historical volatility especially for options with longer time-to-expiration. As also shown in this table, the OLS results are quite sensitive to whether the 1987 crash period is included or excluded and to the forecast horizon. While implied volatility is significant in all regressions, historical volatility is significant for group 1 when the crash is included and all groups when the crash period is excluded.

However, as in previous studies, the results in Table 3.2 are inconsistent with the hypothesis that implied volatility is an unbiased and informationally efficient estimator of future realized volatility. If implied volatility is an unbiased and efficient predictor of actual realized volatility and there is no measurement error, then we should observe that $\beta_0 = \beta_2 = 0$ and that $\beta_1 = 1$. In contradiction of the efficiency hypothesis, the estimated coefficients of $\sigma_{ISD,i,t}$, β_1, are less than 1.0 in all of the regressions. They are all significantly less than one except for groups 3 and 4 in Panel A. In further contradiction of the efficiency hypothesis, the estimates of β_0 are positive and significantly different from zero in all but two regressions. Results on whether or not historical volatility provides incremental information, that is whether or not $\beta_2 = 0$, depend on the horizon and whether or not the 1987 crash period is included. In the four regressions including the 1987 crash, the estimates of β_2 are not significantly different from zero, indicating that implied volatility subsumes the information of historical volatility. For all the regressions excluding the 1987 crash, estimates of β_2 are significantly greater than zero indicating that historical volatilities do contain some incremental information which is not contained in implied volatility in these cases. However, the coefficients of β_2 are still smaller than those of implied volatility.

Table 3.3 reports the OLS results when realized volatility ($\sigma_{RLZ,t}$) is regressed on an average implied volatility ($\sigma_{ISD4,t}$) and a historical volatility measured over the last 40 trading days. The implied volatility measure [14], $\sigma_{ISD4,t}$, is the average of the four implied volatilities calculated from the two nearest-the-money calls and the two nearest-the-money

[14] As also mentioned in the footnote 6 in Chapter 2, when the average implied volatility ($\sigma_{ISD4,D}$) is utilized all the other options with the same expiration date observed on the same day are, in fact, ignored. As a result, in each expiration group, there is only one observation for this implied volatility measure per day and therefore, there are fewer observations in each of the four groups. For example, there are only 3,212 observations in group 1.

puts. This measure is often used by practitioners as well as some researchers.[15] The only difference between Tables 3.2 and 3.3 is that the latter utilizes this average implied volatility measure instead of all the individual implied volatilities with each serving as a separate observation. Note that the *i* subscript is dropped in Table 3.3 because there is only one observation per day in this case while there are up to 32 observations per day in each group in Table 3.2. The coefficients of implied volatility (β_1) and the adjusted R^2s are much higher in Table 3.3 than the corresponding figures in Table 3.2. For example, for options maturing in 10 to 36 trading days when the 1987 market crash is included, β_1 is 0.8056 and the adjusted R^2 is 0.2447 in Table 3.3 while the β_1 is 0.3713 and the adjusted R^2 is 0.2049 in Table 3.2 for the same forecasting horizon.

A comparison of Tables 3.3 with Table 3.2 suggests that, in general, an individual implied volatility calculated from a randomly chosen option contains more measurement error than the average implied volatility calculated from several *near-the-money* options and that this measurement error is largely responsible for the low β_1 values and R^2s in Table 3.2. The significant improvement in the predictive ability of this average implied volatility is probably due to the fact that this average measure is calculated from the four nearest-the-money options which are traded most actively and should therefore contain less measurement error caused by "stale" prices. Chapter 6 will thoroughly investigate the relative predictive power of different implied volatility estimators across different strike

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15 For example, the implied volatility, which Knight-Ridder Financial Company sells to the market, is actually calculated using the two nearest-the-money calls and the two nearest-the-money puts. Jorion (1995), however, utilized an average implied volatility calculated from one at-the-money call and one at-the-money put.

prices as well as this average measure. As will be shown in Chapter 6, the four components of this average implied volatility yield much better forecast than implied volatilities calculated from options with low strike prices but worse than some of the implied volatilities calculated from options with relatively higher strike prices. This may partially explain why C&F obtains much lower measures of the predictive power of implied volatility while Jorion (1995) finds the higher predictive power. Note also that β_2 is often significantly different from zero in Table 3.2 but not in Table 3.3 indicating more severe measurement error effect in Table 3.2.

3.4.2. Measurement Error Test

Next, whether there is measurement error in the data set is tested by utilizing a procedure proposed by Hausman (1978). In the test, the first step is to regress implied volatility on one or more instrumental variables, then the residual from this first regression, call it U_t, is inserted as a third independent variable in the OLS estimation of Equation 3.2. The null hypothesis of zero measurement error implies a zero coefficient for U_t. If there is measurement error in implied volatility measure, which is not repeated in the instrumental variable(s), then a negative coefficient is expected.

The results are robust across different instrumental variables or different combination of instrumental variables. To save space, I only report the results when utilizing a single instrumental variable which is the average implied volatility calculated from the two nearest-the-money calls and the two nearest-the-money puts observed ten trading days before day t ($\sigma_{ISD4,\,t-10}$). Table 3.4 shows the estimation of the second-step regressions based on observations on individual implied volatilities while Table 3.5 shows that based on an average implied volatility. In both tables, the instrumental variable is $\sigma_{ISD4,\,t-10}$. As Table 3.4 shows, coefficients of U_t are negative in all the regressions and significantly different from zero in all but group 4, rejecting the no-measurement-error null. Note that this test not only rejects the null that the implied volatility is free of measurement error but also indicates that the measurement errors in $\sigma_{ISD,\,i,\,t}$ and the instrumental variable ($\sigma_{ISD4,\,t-10}$) are at least partially independent. In other words, while it is still possible that η_t, and η_{t-10} are correlated due to a long-lived measurement error, it is clear that at least part of the measurement error is short-lived so that its effects can be eliminated using instrumental variables estimation. However, in Table 3.5, although the coefficients of U_t are all negative, they are only significant for group 1, group 2 and the overall sample including 1987 market crash, implying that the average implied volatility contains relatively less measurement error than individual implied volatilities.

3.4.3 Instrumental Variables Estimation

As shown in Tables 3.2 and 3.3 and confirmed by the measurement error tests reported in Tables 3.4 and 3.5, in general, there is less measurement error in the average implied volatility calculated from the two nearest-the-money calls and the two nearest-the-money puts than a randomly chosen individual implied volatility. Compared with C&F's data, the measurement error here should be less since the futures and options cease trading at roughly the same time and arbitrage is easier. Consistent with this view, I find that implied volatility has considerable forecasting ability (and that historical volatility has little or less incremental predictive power) while C&F do not. Nonetheless, it remains the case that I consistently find in Tables 3.2 & 3.3 that $\beta_0>0$, $\beta_1<1$ and $\beta_2>0$, and that often these differences are significant. I will next examine whether the measurement error which remains in the data is responsible for these deviations of the estimated β parameters from their theoretical values.

To correct for the bias caused by this measurement error, I estimate Equation 3.2 (without the residual term) by the instrumental variables estimation with $\sigma_{ISD4,\ t-10}$ as the instrumental variable. The results are presented in Tables 3.6 and 3.7 where Table 3.6 is based on all the individual implied volatilities and Table 3.7 is based on the average implied volatility. Again the analyses are stratified by option expiration groups and are reported with and without the 1987 crash. Comparing the coefficients in Table 3.6 with those in Table 3.2, one observes that in all ten regressions, the instrumental variables estimates of β_1 are higher than the OLS estimates, and the instrumental variables estimates of β_0 and β_2 are smaller. In fact, in seven out of the ten regressions in Table 3.6, the coefficients of implied volatility are not significantly different from *one*. Consider, for instance, the OLS and instrumental variables estimates of Equation 3.2 for group 1, i.e., a forecast horizon of 10 to 36 trading days in Panel A in both Tables 3.2 and 3.6. While the OLS estimate of β_1 is 0.3713, the instrumental variables estimate of β_1 is 0.8535, a little bit smaller than one but not significantly different from one. While the OLS estimate of β_0 is 0.0502, positive and significantly different from zero, the instrumental variables estimate of β_0 is -0.0059, smaller and not significantly different from zero. Finally, the estimate of β_2 is reduced from 0.1589 to -0.0271 with the former significantly different from zero and the latter not.

All the three changes are what one would expect if $\sigma_{ISD,i,t}$ measures the market's true expectation with error and the three changes occur to different degrees in all ten regressions. If the market's true expectation is an unbiased and informationally efficient estimator of the future actual volatility, in all of the regressions reported in Panel A of Table 3.6, β_0 should be insignificantly different from zero and β_1 should be insignificantly different from one. With just a few exceptions these predictions are met. The exceptions occur in the overall sample and group 2 in Panel A and the overall sample, group 3, and group 4 in Panel B but again estimates of β_0 and β_2 fall and those of β_1 rise. A comparison of Table 3.7 with Table 3.3 shows a similar pattern. Note that in terms of the instrumental variable estimations, the results of individual implied volatilities in Table 3.6 are quite similar to those of the average implied volatility in Tables 3.7.

In summary, the results in this essay confirm the hypothesis that implied volatility is only a rough measure of the market's true expectation and accordingly implied volatility would sometimes appear to be an informationally inefficient predictor of actual volatility. While it is possible that more long-lived differences exist, many of the differences between implied volatility and the market's true volatility expectation appear to be due to short-lived measurement error attributable to such factors as bid-ask spreads, minimum price increments, and non-synchronous prices. When I correct for these short-lived measurement errors by utilizing the average implied volatility observed ten trading days before day t as the instrumental variable, the evidence in most cases no longer rejects the hypothesis that implied volatility represents the market's best forecast of future volatility.

3.5 Conclusions

While this chapter contains a number of findings regarding implied volatility and historical volatility forecasts of the market volatility, the most important findings are these. One, in contrast to some of the previous studies, I find that implied volatility contains considerable information regarding future realized market volatility. Certainly, implied volatility is a much better estimator of realized volatility than historical volatility. I argue that Canina & Figlewski's (1993) (C&F) finding to the contrary was probably due to two facts: (1) their measures of implied volatility contained considerable measurement error (as they themselves hypothesize) because of the attributes of S&P 100 index options and C&F's using of individual options (rather than an average), and (2) their data period was quite short (given the many overlapping observations) and excluded the 1987 crash.

Two, results are sensitive to (1) the forecast horizon, (2) whether or not the data set includes the 1987 crash, (3) the presence of measurement error, and (4) whether or not one controls for the overlaps in realized volatility observations. Measurement error in implied volatility estimates seriously biases the parameter estimates in ordinary least squares regressions while the overlap in realized volatility observations seriously biases the standard error estimates. Furthermore, the measurement error effect is generally more severe in an individual implied volatility calculated from a randomly chosen option than in the average implied volatility calculated from the four nearest-the-money options.

Three, there is no evidence that the market's forecasts of future volatility are irrational or fail to correctly impound all available information. Like virtually all previous studies I find that when ex-post realized volatility is regressed on implied volatility utilizing ordinary least squares, the coefficient of implied volatility is less than one and the intercept is positive - rejecting the theory that implied volatility represents an unbiased and efficient forecast of actual volatility. However, I find that this is due to deviations of implied volatility from the market's true volatility expectations - deviations possibly caused by non-synchronous price observations, bid-ask spreads, and minimum price increments. When I control for these deviations or measurement errors by instrumental variables estimation, the parameter estimates are, in most cases, consistent with the hypothesis that implied volatility represents the market's best forecast of future volatility.

Chapter 4

Essay II: The Implied Volatility Smile and Option Market Efficiency

4.1 Introduction

Numerous studies have documented the phenomenon known as the implied volatility smile which refers to the cross-sectional variation in implied volatility across options with the same expiration date but different strike prices. That is, at a given time, different strike prices yield different implied volatilities. Note that if the option pricing model is correct and the market is efficient, then, for options with the same maturity date observed at the same time, implied volatilities should be the same regardless of different strike prices and therefore there should be no smile. The prevailing explanation for the smile is that the Black-Scholes option pricing model is incorrect leading researchers to develop more complicated option pricing models. But none of the new models has successfully explained the smile motivating me to search for an alternative explanation.

In this second essay, I test an alternative explanation for the smile which is that the smile exists because investors prefer to be long or short the options with certain strike prices. For instance, stock market investors may prefer to buy far out-of-the-money puts to protect against a market decline. This would tend to drive up the implied volatilities for these out-of-the-money puts. In a perfect market, arbitrage would tend to eliminate these differences in implied volatilities. But the market may not be perfect. For example, Canina and Figlewski (1993) attribute their finding of low predictive power of implied volatility calculated from OEX options to, among other factors, arbitrage restrictions.

If the smile is caused by investors' preferences for certain strike prices and these differences are not eliminated by arbitrage, a trading strategy of selling those options with high implied volatilities and simultaneously buying those options with low implied volatilities in a delta-neutral ratio should make money over time. In other words, the existence of a smile may suggest a market inefficiency. In this essay, I explore the profits to a strategy of buying calls (puts) with low implied volatility and writing calls (puts) with high implied volatility in a ratio to make the position delta neutral. If the strategy does not make excess profits over time, one cannot reject the market efficiency hypothesis and the smile may reflect a misspecification of the option pricing model. But if the strategy makes excess profits, then one cannot reject Black-Scholes model and the smile would likely represent some market inefficiency, such as arbitrage restrictions.

The evidence in this essay shows that the strategy based on implied volatility differences can make significant profits over time especially for positions involving put options, suggesting that market inefficiency may be the cause of the implied volatility smile. In addition, the results in this essay suggest further tests to isolate the exact causes of the smile in the S&P 500 futures option market.

The rest of this essay is organized as follows. The next section briefly reviews the literature on the implied volatility smile. Section 4.3 discusses the methodology and the hypothesis. Empirical results are presented in Section 4.4. Section 4.5 summarizes the findings and suggests the further study for this issue.

4.2 Literature Review

Several previous studies have documented the implied volatility smile in different markets. They generally find that implied volatility is greater for out-of-the-money and in-the-money options than for at-the-money options, i.e., a symmetric U shaped implied volatility across different strike prices. However, Rubinstein (1994) and Dumas, Fleming and Whaley (1998) indicated that 'smirk' is a more appropriate description of the pricing bias in equity index options since 1987, i.e., implied volatility decreases monotonically as the strike price increases. In this essay, I find a similar pattern in the implied volatility for options on S&P 500 futures as will be discussed in Subsection 4.3.1.

One prevailing explanation for the smile is that the Black-Scholes (1973) (BS hereafter) model is wrong due to some incorrect assumptions. For example, BS assumes a constant or a deterministic volatility while it might be stochastic, or BS assumes log-normal returns and they might not be log-normal, or BS assumes European options and they might be American options, or BS assumes no (or continuous) dividend payments and the underlying asset may pay discrete dividends. If one of these is the cause, then, if implied volatility were calculated using the correct formula, there would be no smile or smirk, that is implied volatility would be the same for all strike prices. Also there would be no consistent profits to a trading strategy of selling the options with high (BS) implied volatilities and simultaneously buying options with low (BS) implied volatilities.

Several previous empirical studies, such as Merton (1980), French and Roll (1986), and Schwert and Seguin (1990), claimed that the BS model's constant volatility and normality assumptions were not supported by the evidence. Utilizing transaction prices from options on S&P 500 futures from January 28, 1983 through December 30, 1983, Whaley (1986) tested the American option pricing model and found that both a moneyness bias and a maturity bias exist and that a riskless hedging strategy using the American (also European) futures option pricing models generates abnormal risk-adjusted rates of return after the transaction costs paid by floor traders. The results rejected the joint hypothesis that the American futures option pricing models are correctly specified and that the S&P 500 futures option market is efficient for the one year sample.

Attempts at reconciling the option pricing theory with the implied volatility smile have mostly centered around two approaches. One consists of jump-diffusion models, for example, Jarrow and Rosenfeld (1984), Amin (1993), and Bates (1996), which augment the BS return distribution with a Poisson-driven jump process. The other consists of the stochastic volatility models which extend the BS model by allowing the volatility of the return process to evolve randomly over time, such as Hull and White (1987), Wiggins (1987), Amin and Ng (1993), and Heston (1993). However, Heynen (1994) finds that the observed smile pattern is inconsistent with various stochastic volatility models while Das and Sundaram (1999) find that neither jump-diffusion models nor stochastic volatility models constitute an adequate explanation for the empirical evidence. The fact that these new models can not explain the implied volatility smile motivates me to search for an alternative way to explain the volatility smile.

4.3 Methodology

4.3.1 The Implied Volatility Smile for Options on S&P 500 Futures

Figure 4.1a illustrates the implied volatility smile or smirk for options on S&P 500 futures with 10 to 99 trading days to maturity for the sample including the 1987 market crash, while Figure 4.1b shows the same graphs for the sample excluding the 1987 market crash. Both figures display a cross-sectional pattern of implied volatility similar to that reported in Rubinstein (1994) and Dumas, Fleming and Whaley (1998). As discussed in Section 2.1, on each day t, up to 32 different options (calls and puts) with the same expiration date are observed and utilized, i.e., eight in-the-money calls, eight out-of-the money calls, eight in-the-money puts, and eight out-of-the-money puts. Each dot (for a put) or diamond (for a call) in Figures 4.1a & 4.1b represents the mean "moneyness" and the mean implied volatility for one of these 32 different options over the period from January 28, 1983 to April 30, 1998. The "moneyness" is defined as (K/F - 1) where K stands for the strike price and F stands for the underlying futures price. The strike prices (K) for options on S&P 500 futures are set up in increments of 5 points. The mean implied volatility is reported on the Y axis while the mean moneyness (K/F -1) is reported on the X axis. For example, the farthest right diamond shows the mean moneyness and the mean implied volatility for the eighth from-the-money, but out-of-the-money, call. Similarly, the farthest left dot shows the mean moneyness and the mean implied volatility for the eighth from-the-money, but out-of-the-money, put. Both figures show that as strike price (moneyness) increases, implied volatilities for both calls and puts, on average, decline monotonically up to a certain level of moneyness and then go back up. Note that implied volatilities for puts and calls with the same strike price are virtually the same except for high strike prices where those for calls are lower.

4.3.2. Hypothesis and Methodology

As discussed in Sections 4.1 and 4.2, the implied volatility smile may reflect an incorrect option pricing model because of wrong assumptions, or it may reflect a market inefficiency. This essay tests a trading strategy in which one sells the option with the highest implied volatility and buys the option with the lowest implied volatility among the 16 near-the-money calls (or the 16 near-the-money puts) with the same time to expiration on a given day. The hypothesis is that if this strategy consistently makes abnormal profits over time, the smile may reflect a market inefficiency - not a misspecification of the option pricing model. On the other hand, if the strategy does not make money over time, then the smile is not a reflection of the market inefficiency and may occur because the option pricing model is incorrect.

Under the hypothesis that the smile may reflect a market inefficiency, options with some strike prices may be overvalued while options with some other strike prices may be undervalued. Since the price of a call (or a put) is positively related to the volatility of the underlying asset's return, an option with a high implied volatility may be overpriced while an option with a low implied volatility may be underpriced. Following the simple rule of "buying low and selling high", one could buy the option with the lower implied volatility and sell the option with the higher implied volatility. For instance, in terms of options on S&P 500 futures as displayed in Figures 4.1a and 4.1b, one would (on average) sell the farthest out-of-the-money put or the farthest in-the-money call and (on average) buy the relatively far in-the-money put or the relatively far out-of-the-money call among the 32 options considered.[16] Here, two specific trading strategies are plausible.[17] One involves buying the call with the lowest implied volatility and simultaneously selling the call with the highest implied volatility. Similarly, the other involves buying the put with the lowest implied volatility and simultaneously selling the put with the highest implied volatility.

Many option traders use a delta-neutral trading strategy in an attempt to make their positions relatively immune to changes in the underlying asset price.[18] I also employ such a strategy in order to minimize the risk. The delta of a portfolio of derivative securities is defined as the sensitivity of the portfolio value to the underlying asset price or the theoretical dollar change in the portfolio value for a one dollar change in the underlying asset price. If the delta of a portfolio is zero, the portfolio is referred to as *delta neutral*. As we know, the value of a derivative security such as a call or a put depends on the value of its underlying asset. Since the price of a financial asset such as the S&P 500 futures changes constantly, the price of a derivative on this financial asset will also change. But the value of a delta neutral portfolio should not change with a small change in the price of its

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16 Of course, Figures 4.1a and 4.1b represent the means over more than 15 years and the smiles on a given day may not look exactly like this. In implementing the strategy, I only consider the positions which sell in-the-money call (out-of-the-money put) and buy out-of-the-money call (in-the-money put).

17 One might suggest that there are four strategies with each combination of buying and selling puts or calls. But actually, only two combinations will work because of the requirement for the delta neutrality of the positions. For example, buying a call with the lower implied volatility and selling a put with the higher implied volatility will never make the position delta neutral.

18 Practitioners also use gamma-hedging against changes in delta and vega-hedging against changes in volatility or a combination of them. However, in most cases, the benefits of the gamma-hedging and the vega-hedging are small. Hull and White (1987b) compares the relative performance of various hedging schemes. They find that the Delta-gamma hedging performs well when the traded option has a constant implied volatility and a short time to maturity, but it can perform far worse than Delta-hedging in other situations.

underlying asset. It is important to realize that a position only remains delta neutral for relatively small changes in the underlying asset's price and for a relatively short period of time since delta changes with both a change in the underlying asset price and passage of time. Consequently, when delta hedging is implemented, the hedged position should be adjusted, or rebalanced periodically. However, since rebalancing positions could be very expensive because of the transaction costs required and because practices vary, I do not rebalance the positions in the analysis.

For options on S&P 500 futures, deltas are calculated as follows:

$$\begin{aligned} \text{call delta} &= \frac{\partial C}{\partial F} = e^{-RT} N(d_1) > 0 \\ \text{put delta} &= \frac{\partial P}{\partial F} = e^{-RT} [N(d_1) - 1] < 0 \end{aligned} \tag{4.1}$$

where

$$d_1 = \frac{\ln(\frac{F}{K}) + \frac{\sigma^2}{2} T}{\sigma \sqrt{T}}$$

and the variables are as defined in Subsection 2.2.1.

Specifically, the delta-neutral strategy proceeds as follows. On a given day, first identify the call with the highest implied volatility (C_H) and the call with the lowest implied volatility (C_L) among the 16 near-the-money calls [19] with different strike prices but the same time to expiration. If the difference between the highest and the lowest implied volatilities

19 To reduce measurement error, I actually only search among those options whose strike prices are within 10% from the underlying futures price, i.e. the absolute value of the moneyness less or equal to 10%.

is greater than a chosen threshold,[20] 3% , buy <u>one</u> contract of the call with the lowest implied volatility and simultaneously sell a number of contracts of the call with the highest implied volatility. The number of the contracts sold is N_{SC} = call delta_L /call delta_H , where L stands for the low implied volatility option and H stands for the high implied volatility option. Note that N_{SC} is actually equal to $N(d_L)/N(d_H)$ as defined in Equation 4.1. Since the value of this call position is $W_C = C_L - N_{SC}\,C_H$, the delta of the position is

$$\begin{aligned} \text{call position delta} &= \frac{\partial W_C}{\partial F} = \frac{\partial C_L}{\partial F} - N_{SC}\frac{\partial C_H}{\partial F} \\ &= \text{call delta}_L - \left(\frac{\text{call delta}_L}{\text{call delta}_H}\right)\text{call delta}_H \\ &= 0 \end{aligned} \tag{4.2}$$

Similarly, on a given day, buy one contract of the put with the lowest implied volatility and simultaneously sell a number of contracts of the put with the highest implied volatility with the number sold N_{SP} = put delta_L /put delta_H . The delta of the put position ($P_L - N_{SP}\,P_H$) is,

$$\begin{aligned} \text{put position delta} &= \frac{\partial W_P}{\partial F} = \frac{\partial P_L}{\partial F} - N_{SP}\frac{\partial P_H}{\partial F} \\ &= \text{put delta}_L - \left(\frac{\text{put delta}_L}{\text{put delta}_H}\right)\text{put delta}_H \\ &= 0 \end{aligned} \tag{4.3}$$

Again, note that N_{SP} is actually equal to $(N(d_L)-1) / (N(d_H)-1)$.

For the data on options on S&P 500 futures, more than 95% of the delta neutral positions involving calls entail a *negative* net investment. That is, one receives more money

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20 I analyze only the positions in which the difference between the high implied volatility and the low implied volatility is less or equal to 50%.

selling the high implied volatility call than one pays to buy the low implied volatility call. Conversely, more than 95% of the delta neutral positions involving puts incur a *positive* net investment. To facilitate the exposition, I only analyze call positions with a negative net investment and put positions with a positive net investment.[21] For ease of interpretation, I also standardize the net investments to -$1 for call delta neutral positions and $1 for put delta neutral positions. The hedged positions are held for one, five, or ten trading days or until the options' maturity and then are closed out. This process is repeated for each of the four expiration dates (in each of the four groups) on every trading day from January 28, 1983 through April 30, 1998.

For a bench mark, I also calculate the profits to a simple strategy of randomly buying calls (or puts) in the data set. A comparison of profits (losses) to this randomly buying strategy with those of the strategy based on implied volatility differences will further illustrate the issue. For example, if the simple randomly buying strategy makes similar or more profits than the strategy based on volatility differences, one could not make any conclusions about the hypothesis on implied volatility. These "benchmark" positions are also held for one, five, or ten trading days or until the option's expiration and are also standardized to $1.

4.4 Empirical Results

4.4.1 The Simple Strategy of Randomly Buying Options

21 It would be difficult to interpret the results if one includes both the positive and the negative net investments. Suppose a $1 investment makes 5% and a -$1 investment makes 2%. Since the net investment is zero, the return would be infinitive.

To better understand the effectiveness of the trading strategy based on implied volatility differences, let us first consider the profits (losses) to a simple strategy of randomly buying calls (puts) in the S&P 500 futures option market. One would expect that there are two factors affecting the results. First, implied volatility was generally above realized volatility as shown in Table 2.1 as well as Figures 4.1a and 4.1b. This implies that both calls and puts are generally overpriced and that if holding an option to expiration, one would generally makes money by writing either a call or a put. Secondly, the sample period of 1983 to 1998 was generally a long bull market. In 1983, the S&P 500 was around 200 and it gradually increased to around 1200 in 1998. This would implied that one would generally make money <u>buying</u> calls and <u>writing</u> puts.

The results of investing $1 and holding for one, five or ten trading days or to the option's maturity are presented in Table 4.1 where Panel A is for calls and Panel B is for puts for both samples of including and excluding the 1987 market crash. Group 1 contains options maturing in 10 to 36 trading days and group 2 contains the options which mature next. As Table 4.1 shows, on average, over the 1983-1998 data period, one could profit by buying calls and/or writing puts. This holds for all four option groups, all four holding periods, and both the samples of including and excluding the 1987 market crash. In general, the average profits increase as the holding period gets longer. For example, for the nearest-to-expiration calls including the 1987 market crash, the average profits are $0.1758 (or 17.58%) on an investment of $1 over a holding period of ten trading days while the average profits are $0.0152 (or 1.52%) for a one trading day holding period.

For puts, since the figures in Table 4.1 report the profits or losses on a $1 ***purchase***, it is clear that profits could have been made by ***writing*** puts. As the table shows, it was, on average, profitable to randomly write puts over the 1983-1998 period. The profits increase as the holding period gets longer. For example, randomly writing $1 worth of the nearest-to-expiration puts with the sample including the 1987 market crash and holding for 10 trading days make, on average, $0.2156 (or 21.56%) while writing $1 worth of the nearest-to-expiration puts and holding for one trading day make, on average, $0.0233 (or 2.33%). Note that the twenty percent profits in ten trading days by writing $1 worth of puts is a pretty good return, though, of course, the standard deviations are quite large.[22]

In comparison, in most cases, the profits to writing a put exceed the profits to buying a call. Two underlying factors mentioned earlier in this subsection may explain the results. First, a call has a positive delta and a put has a negative delta. Since the data period, 1983-1998, roughly coincides with the great bull market, the S&P 500 futures, in general, increases over time which causes call price increasing and put price decreasing over time. This factor makes both buying a call and writing a put profitable. Second, according to Table 2.1, implied volatility was, on average, greater than realized volatility over the data period which means that in general both calls and puts were overpriced. If $\Delta F = 0$ (no change in the underlying futures price), then, on average, one should have been able to make money by writing both puts and calls. Note that for puts both factors work in the same direction, implying larger profits for writing puts. However, for calls these two forces offset. This would explain why the profits to writing puts exceed the profits to buying calls.

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22 Note that the standard deviation generally declines as the holding period gets shorter and follows the pattern expected if daily profits are independent, i.e. standard deviation for a ten trading day holding period is roughly the square root of 10 times as large as that for a one trading day holding period.

4.4.2 Delta Neutral Strategy Based on Implied Volatility Differences

Table 4.1 demonstrates that, in general, calls were underpriced and puts were overpriced. This, however, is not the purpose of this essay and serves only as a benchmark for the following analysis. What I am interested in is to show that some calls (puts) are overpriced relative to other calls (puts) with different strike prices but the same time to maturity implying that the smile may reflect a market inefficiency.

Table 4.2 reports the profits to the strategy of buying options with the lowest implied volatility and simultaneously selling options with the highest implied volatility in a delta neutral ratio. Panel A reports results for call positions and Panel B for put positions. In both panels, I report separately the results including and excluding the 1987 market crash. The holding periods considered are one, five, or ten trading days, or to the options' maturity respectively. As explained above, the net investment is -$1 for a position involving calls and $1 for a position involving puts. For calls, the results are mixed. Significant positive mean profits are observed for group 1 with a holding period of one trading day and for group 2 with holding periods of one and five trading days. For example, the average profits is $0.0194 for a -$1 investment in a position involving the second nearest-to-expiration calls with a five-trading-day holding period. The profits of this call position are quite impressive when compared with those in Table 4.1. For example, the average profit to randomly investing $1 in call options in group 2 with a five-trading-day holding period are $0.0542. That means if one randomly writes these calls with a net investment of -$1 and hold for five trading days, he / she will, on average, ***lose*** $0.0542. In contrast, on average, the strategy based on volatility differences would ***make*** $0.0194. However, the profits for many of the delta neutral positions involving calls are either negative or are not significant especially for the longer holding periods.

Looking next at the results of the trading strategy based on volatility differences when applied to puts, I observe that the average profits are all positive and significantly different from zero ranging from $0.0135 to $0.8847 with a $1 investment for different groups and different holding periods as well as for both the samples of including and excluding the 1987 market crash. For instance, the average profits are $0.5624 to investing $1 in a delta neutral position involving the nearest-to-expiration puts with a holding period of 10 trading days for the sample including the 1987 market crash. These profits are even more impressive when compared with those of the simple strategy of randomly buying a put. From Table 4.1, we know that if we randomly invest $1 in a nearest-to-expiration put and hold for 10 trading days, we would have, on average, ***lost*** $0.2156 while, on average, the trading strategy based on volatility differences ***makes*** $0.5624. The same pattern holds for all the other expiration groups and all the other holding periods as well as for both the samples including and excluding the 1987 market crash.

To further examine the performance of the delta neutral strategy based on volatility difference relative to that of the randomly buying strategy, I reports the abnormal profits to the delta neutral strategy in Table 4.3 in which Panel A is for positions involving calls and Panel B for positions involving puts. The abnormal profits are defined as the difference between the profits from investing -$1 ($1) in a delta neutral call (put) position as reported in Table 4.2 and the average profits from investing -$1 ($1) in each of the available calls (puts) with the same expiration date observed on the same day. All the figures in Table 4.3

are greatly improved in comparison with those in Table 4.2, i.e., the profits are bigger and losses are smaller. For instance, the profits for positions involving calls in groups 2 with a holding period of five trading days are 0.0803 while those in Table 4.2 are 0.0194. Actually, more significantly positive profits are observed for call positions in Table 4.3 than in Table 4.2. However, the losses for groups 1 and 2 for holding until expiration are still significantly different from zero. The abnormal profits to the delta neutral strategy involving puts are all significantly positive and are much larger than the corresponding ones in Table 4.2.

Up to now, the analyses in this essay have ignored the transaction costs. For options on S&P 500 futures, the transaction costs include the clearing fee for floor traders and include commission and bid-ask spreads for other investors. Table 4.4 reports the profits or losses for the delta neutral positions for the sample including the 1987 market crash after transaction costs of $10 per contract as utilized in Whaley (1986).[23] The total transaction costs are restricted to between $50 and $100. For all the call positions, the strategy based on implied volatility difference lose money. However, for the put positions with holding periods of 5, or 10 trading days or until expiration, abnormal profits are still large and significant. For example, the profits from the hedged position involving puts in group 1

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23 Whaley (1986) assumed $10 transaction costs per contract for a floor trader. Actually, the clearing fee paid by floor traders is on an order of $1.50 per contract. The transaction costs in this market are quoted on a "round-turn" basis. Since commission rates are negotiated between each customer and his or her broker, it is difficult to assess the exact representative commission charges. Brokers usually charge a lump sum plus per contract fee. For example, E-trade charges its Internet customers $20 plus $1.75 per contract with minimum commission charge of $29.95. In this essay, the average number of contracts for the call positions is 1.2 and that of put positions is 8. Therefore, the $10 per contract overestimates the transaction costs incurred by floor traders or large institutional investors but may underestimate those incurred by individual investors.

with a holding period of ten trading days are $0.5110 or 51.10%. But for positions with a holding period of one trading day, transaction costs wipe out all the potential profits. In summary, in the options on S&P 500 futures market, large average profits could be made by floor traders or large institutional investors with a strategy of buying options with low implied volatility and simultaneously selling options with high implied volatility, suggesting that the implied volatility smile may reflect a market inefficiency not a misspecification of option pricing formula.

4.4.3 Reasons for the High Risk of Delta Neutral Positions

Since the option positions in Tables 4.2, 4.3 and 4.4 are delta neutral while those in Table 4.1 are not, one would expect the standard deviations of profits to be consistently smaller. While the standard deviations of positions involving calls in Table 4.2 are smaller than those in Table 4.1, the standard deviations of most of the positions involving puts are larger. For example, the standard deviation of profits when investing in the nearest-to-expiration puts including the 1987 market crash with a holding period of five trading days reported in Table 4.2 is $1.9096 while that based on randomly writing puts reported in Table 4.1 is $1.4837. Why is the risk of the delta neutral strategy sometimes higher than that for the random investment especially for the put positions? One possibility is that, while the positions are hedged against small changes in the underlying futures, larger price changes may occur - particularly over the longer horizons.

A second possibility is that this reflects noise in the option prices which will be larger for the hedged position since it involves more options and also options with extreme implied volatilities which are usually quite far away from the money. Noise will lead to a higher variance for the spread position since more options are involved for each position. Suppose, for instance, a spread position involves buying a contract of one option and selling a contract of another. The variance of the noise on this position should be roughly twice that on a contract of a single option. Since the delta neutral strategy involves buying one contract of the option with the lowest implied volatility and simultaneously selling a number of contracts of the option with the highest implied volatility, the variance of this hedged position should be larger than that of a contract of a single option and should also depend on the hedged ratio (N_{SC} for a call position and N_{SP} for a put position). Specifically, for example, the variance of a hedged call position would be roughly $\{1+(N_{SC})^2\}$ times that of a single option. The ratios are, on average, around 0.2 for call positions and around 7 for put positions - about 35 times different. Therefore, both variances of the profits (losses) for the hedged call position and the hedged put position should be larger than the variance of the profits (losses) for a single option. In addition, the variance of the profits (losses) for hedged put position should be much higher than for hedged call position. That may explain why the standard deviations are much higher for the hedged put positions than for the hedged call positions reported in Table 4.2.

4.5 Summary

This essay shows that the smile or smirk exists in options on S&P 500 futures market. In particular, the implied volatility decreases monotonically and eventually goes back up as strike price increases. The prevailing explanation for the smile is that the Black-

Scholes option pricing model is incorrect because of wrong assumptions leading researchers to develop more complicated models. But, none of these new models has successfully explained the smile motivating me to search for an alternative explanation.

This second essay tests an alternative explanation for the smile that the smile exists because of the inefficiency of the option market. The results utilizing options on S&P 500 futures show that consistent profits can be made from a delta neutral trading strategy of buying low implied volatility put options and simultaneously selling high implied volatility put options at least for floor traders and large institutional investors, suggesting that option market inefficiency is at least partially responsible for the implied volatility smile. However, I do not observe the high profitability for trades involving calls.

The results suggest further tests to identify the sources of the profits to the trading strategy based on implied volatility difference and to explain the difference in profits between calls and puts and the large variance of the profits. This may also help isolate the exact causes of the smile. According to the BS model, the profits / losses on an option position can be attributed to (1) changes in the underlying futures price, (2) changes in volatility, (3) passage of time, (4) interest rate change, and (5) residuals (noise). Based on the total differentiation, one may explore what proportion of the variance of profits / losses on the trades can be attributed to each of these.

Chapter 5

Essay III: Which Implied Volatilities are the Most Informative?

5.1 Introduction

If the option pricing model is correct and the market is efficient, the implied volatility at time t should represent the market's best forecast of future volatility over the remaining life of the option. However, as discussed in Chapter 4, implied volatility varies between calls and puts and across different strike prices. That is at time t there are, not one, but a number of implied volatilities. Which, then, of these implied volatilities should one utilize in forecasting future volatility? One popular procedure is to use the implied volatility calculated from a *single* at-the-money option reflecting the wide spread belief that at-the-money option is the most sensitive to volatility change, and is therefore the most informative. Another procedure is to utilize an *average* implied volatility calculated from several, often two or four, near-the-money options which is supposed to reduce measurement error.

Several previous studies, such as Beckers (1981), Gemmill (1986), Feinstein (1989) and Turvey (1990), compared forecasting ability of different *average* implied volatilities along with the implied volatility calculated from an at-the-money option. But no one has systematically compared the forecasting ability of individual implied volatilities across different options (calls or puts with different strike prices). In addition, these previous studies generally suffered from limited data sets and therefore lacked degrees of freedom.

In these previous studies, the average implied volatilities were usually calculated from only a few near-the-money options. Although their results were mixed, most of them leaned towards the implied volatility calculated from an at-the-money option. Is the implied volatility calculated from an at-the-money option really the most informative? Do we miss some important information by ignoring all other calls or puts? Can we obtain a better volatility forecast by utilizing other in- or out-of-the money options? Does an average scheme effectively reduce measurement error?

This third essay examines which of the individual implied volatilities (calculated from calls or puts with different strike prices) yields the best forecast for the future realized volatilities of the S&P 500 futures. I find, contrary to the general belief and practice, that the implied volatility calculated from an at-the-money option is not the most informative among all individual implied volatilities. Up to a certain level of strike price, the implied volatilities calculated from calls and puts with relatively higher strike prices, i.e. out-of-the-money calls and in-the-money-puts, seem to have more predictive power.

The remainder of this essay is organized as follows. Section 5.2 reviews the previous studies on comparing the relative predictive power of different implied volatility averages. Section 5.3 discusses the data and methodology. The preliminary empirical results are reported in Section 5.4. A brief summary in Section 5.5 concludes the chapter.

5.2 Literature Review

As illustrated in Chapter 4, implied volatility differs between calls and puts and across different strike prices. Which one then should we utilize in forecasting future volatility? In practice, people use either a single at-the-money option or several near-the-money options to generate an implied volatility forecast. For instance, Lamoureux and Lastrapes (1993), Kleidon and Whaley (1992), and Xu and Taylor (1994) utilized a single at-the-money option to obtain an implied volatility estimate for their studies. On the other hand, Schmalensee and Trippi (1978), Jorion (1995), and Weber (1996) employed an equally-weighted average of implied volatilities calculated from several near-the-money options. Practitioners also tend to use an average implied volatility. For example, the implied volatility, which Knight-Ridder Financial Company sells to investors, is actually the mean of four implied volatilities calculated from the two nearest-the-money calls and the two nearest-the-money puts. Bloomberg L. P. reports a weighted average call (put) implied volatility calculated from the two "at-the-money" calls (puts).

Researchers have also suggested several weighting schemes other than equal weighting even though they have not been used very often in practice. The reasons for giving different weights to different options are (1) that implied volatilities calculated from different options are not equally sensitive to a volatility change, and (2) that different options are not traded in the same frequency. As a result, the measurement error may vary across different options. Latané and Rendleman (1976) suggested weighting the implied volatilities of different options by the vega of the option, i.e., the first derivative of the option's price with respect to the standard deviation of the returns. Chiras and Manaster (1978) argued that Latané-Rendleman estimator does not use proper weights since the weights do not add to one and suggested weighting the implied standard deviation by the price elasticity of the option with respect to its implied standard deviation. Beckers (1981) and Whaley (1982) suggested a quadratic loss function estimator which minimizes squared errors of Black-Scholes option prices.

Several studies have compared the relative forecasting ability of different averaging schemes as well as a couple of individual implied volatilities calculated from single options. However, no previous studies have systematically compared the forecasting performance of different individual implied volatilities between calls and puts and across different strike prices. For example, using daily closing prices of equity *call* options over a 75 trading day period from October 13, 1975 to January 23, 1976, Beckers (1981) compared the forecasting ability of three measures of implied volatility: (1) an average in which the weights are based on the option's vega, (2) the minimized-squared-pricing-error implied volatility scheme, and (3) the implied volatility calculated from the *call* with the highest vega. He found that the implied volatility calculated from the *call* with the highest vega, which is usually at-the-money, outperforms the other two measures. Note, however, that the data period was very short.

Gemmill (1986) looked at 13 equity *call* options traded on the London Traded Options Market. Utilizing monthly closing prices from May 1978 to July 1983, he compared six different weighting schemes: equally weighted average of individual implied volatilities, elasticity weights, minimized-squared-pricing-error measure, at-the-money call, the furthest-from-the-money in-the-money call, and the furthest from-the-money out-of-the-money call. He found that the implied volatility calculated from the furthest from-the-money in-the-money call yields the best forecast of subsequent realized volatility but only marginally better than historical volatility and that the furthest from-the-money out-of-the-money calls contained no information relevant to forecasting future volatility. As also mentioned in his paper, the London traded option market was thin, the option and the underlying stock prices were not simultaneously observed, and no account was taken of dividends, leaving one skeptical of his results.

Feinstein (1989) conducted both a theoretical and an empirical studies on the predictive power of implied volatility utilizing monthly data for options on S&P 500 futures from June 1983 through December 1988. He compared seven measures: the nearest-out-of-the-money call, an equally weighted average of individual implied volatilities, vega weights, elasticity weights, the nearest-out-of-the-money put, an average of the nearest-out-of-the-money call and the nearest-out-of-the-money put, and an intertemporal average of nearest-out-of-the-money calls over five days. He pointed out that the BS model for at-the-money options is well approximated by a linear function of volatility and found that the implied volatility from the single nearest-out-of-the-money call was the most efficient and dominated.

Using daily observations of *put* options on soybean and live cattle futures from January 9, 1987, through December 30, 1988, Turvey (1990) evaluated four weighting methods: an equally weighted average of individual implied volatilities calculated from puts, an at-the-money put only, vega weights, and elasticity weights. He found that weighting implied volatilities by vega, which usually puts the highest weight on at-the-money options, provides better forecasts than the other three measures.

Corrado and Miller (1996) presented an econometric analysis of several efficient methods utilized to estimate option implied volatilities. They found that simultaneous equation estimators and weighted average estimators have the same attainable variance bound, and are equally efficient when used with appropriate weights. They further showed, through a simulation example, that the average of implied volatilities calculated from an at-the-money call and an at-the-money put is relatively more efficient.

In summary, these previous studies compared the relative forecasting ability of different averaging schemes (including a couple of implied volatilities with each estimated from a single option). Although their results were quite mixed, they leaned towards favoring at-the-money options. But these previous studies generally suffered from limited data sets because they utilized either monthly data or cross-sectional data over a short period of time which did not give them enough degrees of freedom. Moreover, no one has yet systematically compared the forecasting performance of implied volatility between calls and puts and across different strike prices.

5.3 Data and Methodology

5.3.1 Data

In the options on S&P 500 futures market, as mentioned in Chapter 2, I utilize up to 32 options, i.e., eight in-the-money calls, eight out-of-the-money calls, eight in-the-money puts and eight out-of-the-money puts for each expiration date observed on the same day. Options further away from the money are very thinly traded and therefore have fewer observations and may contain considerable measurement error.

Table 5.1 reports the summary statistics for different implied volatility estimators, the realized volatility, and a historical volatility measure for both the samples including and excluding the 1987 market crash over the period from January 28, 1983 through April 30, 1998. The means of these volatility measures are also graphed in Figures 4.1a and 4.1b. In this table as well as other tables and figures, RLZSD represents the realized volatility over the remaining life of an option on S&P 500 futures and HIS40 stands for the historical volatility measured over the last 40 trading days. For an individual implied volatility, the first three letters (ISD) stand for implied standard deviation, the fourth letter C (P) represents a call (put) option, the fifth letter I (O) indicates an in-the-money (out-of-the-money) option, and the last digit refers to the relative position of the option from the money. For example, ISDCI3 represents the implied volatility calculated from a call option which is the third from the money and in the money. Note that the strike prices are designated in an incremental of five points.

The individual implied volatilities are listed according to their options' strike prices relative to the observed underlying futures prices with the lowest strike price listed first followed by the second lowest strike price, etc. However, ISD4 is the equally weighted average of the implied volatilities calculated from the two nearest-the-money calls and the two nearest-the-money puts. As shown in Table 5.1 as well as Figures 4.1a and 4.1b, on average, almost all the implied volatilities for both the samples including and excluding the 1987 market crash are higher than the subsequent realized volatility, indicating that implied volatilities generally overestimate the realized volatility over the 1983-1998 period. For instance, the mean of all the implied volatilities calculated from the eighth from-the-money in-the-money calls in the overall sample including the 1987 market crash is 0.2114 while that of subsequent realized volatility is 0.1444.

On the other hand, the 40-trading-day historical volatility is, on average, only slightly higher than the realized volatility but lower than the majority of the implied volatility measures.[24] Figures 4.1a and 4.1b illustrate the means of various implied volatility estimators and the mean moneyness for both the overall samples including and excluding the 1987 market crash. Moneyness is defined as (K/F - 1) where K represents a strike price and F stands for the underlying futures price. In fact, as Figures 4.1a and 4.1b show, on average, implied volatility declines monotonically up to a certain strike price and then goes back up as the strike price increases, a pattern similar to what described in Rubinstein (1994) and Dumas, Fleming and Whaley (1998).

5.3.2 Simple Criteria for Comparing Forecasting Efficiency

To compare the forecasting efficiency of different implied volatility estimators, I employ three commonly utilized criteria: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and mean absolute percentage error (MAPE).[25] RMSE is simply square root of the mean of the squared forecast errors:

$$\mathrm{RMSE}_i = \sqrt{\frac{1}{N_i}\sum_{t=1}^{N_i}(\sigma_{RLZ,t} - \sigma_{EST,i,t})^2} \tag{5.1}$$

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[24] Note that the historical volatility is not necessarily the lag of the realized volatility since the historical volatility always covers the last 40 trading days while the period covered by realized volatility varies from 14 through 99 trading days. From Table 2.1, we can see that the means of the historical volatility are higher than those of the realized volatility for Groups 1, 2 and 4 but lower for Group 3.

[25] See Brailsford and Faff (1996).

where $\sigma_{EST,i,t}$ indicates one of the 34 different volatility estimators (32 individual implied volatilities, one average implied volatility, and one historical volatility) and N_i represents the number of observations available for the volatility estimator i. The MAE is computed as the mean of the absolute values of forecast errors:

$$\text{MAE}_i = \frac{1}{N_i} \sum_{t=1}^{N_i} |\sigma_{RLZ,t} - \sigma_{EST,i,t}| \tag{5.2}$$

Lastly, the MAPE is calculated as

$$\text{MAPE}_i = \frac{1}{N_i} \sum_{t=1}^{N_i} \frac{|\sigma_{RLZ,t} - \sigma_{EST,i,t}|}{\sigma_{RLZ,t}} \tag{5.3}$$

Among the three criteria, RMSE penalizes the forecast with large but infrequent errors. I expect that RMSE is more sensitive to the 1987 market crash than MAE while MAPE is least affected by the crash.

5.4 Empirical Results

5.4.1 Results Based on RMSE, MAE and MAPE

To compare the forecast efficiency of various implied volatility estimators, I first evaluate them based on the three commonly used criteria defined above: the root mean squared error (RMSE), the mean absolute error (MAE), and the mean absolute percentage error (MAPE). The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration from January 28, 1983 through April 30, 1998. ISD4 is the average of the volatilities calculated from the two nearest-the-money calls and the two nearest-the-money puts and is a commonly used measure for implied volatility because its four components are, in practice, referred to as "at-the-money options". As a benchmark for comparison, I also include a historical volatility measured over the last 40 trading days, labeled as HIS40.

In total, there are 32 individual implied volatilities from calls or puts with different strike prices, one average implied volatility (ISD4) and one historical volatility (HIS40). RMSEs, MAEs and MAPEs along with their relative ranks for the 34 volatility measures are reported in Table 5.2. Panel A reports the results for the sample including the 1987 market crash while Panel B reports those for the sample excluding the 1987 market crash. The RMSE, MAE, and MAPE for each estimator are calculated by utilizing all available observations in the data set. The number of observations available for each estimator is reported in Table 5.1. The estimator with the lowest forecast error, therefore the most efficient one, is given the rank of 1 while the estimator with the highest forecast error is given a rank of 34.

Notice that all the RMSEs and MAEs in Panel A are higher than their counterparts in Panel B while there is not much difference in MAPEs between Panel A and Panel B, indicating the dominating effect of the 1987 market crash on RMSE or MAE. As mentioned earlier, among the three criteria, RMSE penalizes the forecast with large but infrequent errors. Since the volatilities are five to seven times higher during the 1987 market crash period than in the other periods, it is not surprising that RMSEs are very sensitive to whether including or excluding the 1987 market crash and MAEs and MAPEs are not. The results in Table 5.2 show just that. While the rankings for the sample including the 1987 market crash based on MAE and MAPE are similar to those for the sample excluding the 1987 market crash, the rankings between the two samples including and excluding the 1987 market crash based on RMSE are quite different especially for calls.

For a better illustration, the results in Table 5.2 are put into several graphs. Figures 5.1a and 5.1b illustrate the forecast efficiency based on RMSE for both the samples including and excluding the 1987 market crash. The MAEs and MAPEs for only the sample including the 1987 market crash are graphed in Figures 5.2 and 5.3 since the 1987 market crash does not affect these two measures very much. In Figures 5.1a, 5.1b, 5.2, and 5.3, the X axis represents the mean moneyness and the Y axis measures either RMSE or MAE or MAPE. There are several interesting points revealed in these figures. First, the forecasting efficiency, in general, increases to a certain level and then eventually declines as the strike price increases with the exception of Figure 5.1a. In other words, implied volatilities calculated from out-of-the-money calls and in-the-money puts which are not very far from the money have the lowest forecasting error based on all the three criteria: RMSE, MAE, and MAPE. Second, the patterns of the forecast errors based on all three criteria match precisely the smiles shown in Figures 4.1a and 4.1b, a reverse J shape. Third, the historical volatility (HIS40) calculated over the last 40 trading days yields better forecasts than some options with very low strike prices but worse than other options especially those with relatively higher strike prices except for RMSEs including 1987 market crash. Fourth, there is no significant difference between calls and puts except for the high strike prices with the forecast errors larger for puts again except for Figure 5.1a.

5.4.2 Can an Average Implied Volatility Improve the Forecasting Performance?

As shown in Table 5.2 as well as in Figures 5.1a, 5.1b, 5.2 and 5.3, the implied volatilities calculated from the four at-the-money options (ISDCI1, ISDCO1, ISDPI1 and ISDPO1) clearly do not yield the best forecasts. They are better than the options with relatively lower strike prices but worse than some of the options with relatively higher strike prices. The average of these four implied volatility (ISD4) is also worse than some options with higher strike prices and are not better than its components. This finding is important since practitioners as well as finance researchers often obtain implied volatility estimate from one of these four options or from the average of them. The results suggest that an averaging scheme may not necessarily reduce the measurement error and improve the forecasting efficiency.

Because the number of observations available varies across different volatility estimators, the RMSEs (also the MAEs and the MAPEs) for different estimators are not perfectly comparable. Note that ISD4 has more observations than any of its four components since ISD4 is the average of the non-missing values of the four components (ISDCI1, ISDCO1, ISDPIi and ISDPO1). Suppose that ISD4 has an observation on a day during the 1987 market crash but ISDCI1 does not. The measure for ISDCI1 would tend to be better for this reason even if ISD4 may actually be better. To further test whether the average implied volatility (ISD4) yields better forecast than its individual components, i.e., whether averaging reduces measurement error, I recalculated their RMSEs, MAEs, and MAPEs when all the four at-the-money options with the same expiration date are observed on a given day. That is the average measure and its components all have the same number of observations. The results are reported in Table 5.3. Based on all the three criteria: RMSE, MAE, and MAPE and for both the samples including and excluding the 1987 market crash, the average implied volatility (ISD4) does not improve the forecast. The two components with the higher strike price (the nearest-from-the-money, out-of-the-money call and the nearest-from-the-money, in-the-money put) consistently demonstrate more predictive power than the average implied volatility as well as the other two components with the lower strike price.

However, there is no difference at all in the predictive power between a call and a put with the same strike price, i.e. between ISDCI1 and ISDPO1 or between ISDCO1 and ISDPI1. For instance, for the sample including the 1987 market crash, the RMSE for ISD4 is 0.0767 while the RMSEs for ISDCI1 and ISDPO1 with the lower strike price are both 0.0778 and the RMSEs for ISDCO1 and ISDPI1 are both 0.0757. In addition, I have tested five more different average measures.[26] Although I do not report them here for the sake of saving space, the results for these five average measures and their individual components reveal identical pattern as shown in Table 5.3. This suggests that averaging several implied volatilities may not be effective in reducing measurement error and the choice of which strike price to utilize is more important.

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26 Specifically, I have tested the following average measures when all of their components are observed on a given day:
ISD16 = the average of ISDCI1-ISDCI4, ISDCO1-ISDCO4, ISDPI1-ISDPI4, and ISDPO1-ISDPO4;
ISD8 = the average of ISDCI1, ISDCI2, ISDCO1, ISDCO2, ISDPI1, ISDPI2, ISDPO1, and ISDPO2;
ISDH8 = the average of ISDCO1-ISDCO4 and ISDPI1-ISDPI4;
ISDH4A = the average of ISDCO1, ISDCO2, ISDPI1, and ISDPI2; and
ISDH4B = the average of ISDCO3, ISDCO4, ISDPI3, and ISDPI4.
For the meaning of the symbols, please refer to the notes in Table 5.1.

5.4.3 Are the Results Due Solely to the Bias of the Implied Volatility?

As shown in Chapter 3, implied volatilities are generally biased estimators of the future realized volatility that is also confirmed in Table 5.1 as well as in Figures 4.1a and 4.1b. All the means of the 32 different implied volatility estimators are higher than that of realized volatility (0.1444). It is also clear that the biases increase as the strike price decreases. Consequently, an obvious question is whether the results in Table 5.2 merely reflect the biases already documented in Table 5.1. In other words, if the results are just due to the bias and one corrects for the bias by subtracting an appropriate amount from a implied volatility estimator, then there would be no clear ranking.

In fact, the mean squared error (MSE) can be expressed in terms of the variance and the squared bias of the forecast error. Specifically, one can decompose the MSE as follows:

$$\begin{aligned} \text{MSE} &= \frac{1}{N}\Sigma(\sigma_{RLZ} - \sigma_{ISD})^2 \\ &= \frac{1}{N}\Sigma[\,(\sigma_{RLZ} - \bar{\sigma}_{RLZ}) - (\sigma_{ISD} - \bar{\sigma}_{ISD}) + (\bar{\sigma}_{RLZ} - \bar{\sigma}_{ISD})\,]^2 \\ &= \frac{1}{N}\Sigma[\,(\sigma_{RLZ} - \bar{\sigma}_{RLZ}) - (\sigma_{ISD} - \bar{\sigma}_{ISD})]^2 + \frac{1}{N}\Sigma\,[\,(\bar{\sigma}_{RLZ} - \bar{\sigma}_{ISD})\,]^2 \\ &= \frac{1}{N}\Sigma[\,(\sigma_{RLZ} - \sigma_{ISD}) - (\overline{\sigma_{RLZ} - \sigma_{ISD}})\,]^2 + [\,(\bar{\sigma}_{RLZ} - \bar{\sigma}_{ISD})\,]^2 \\ &= \text{Var}(\,\sigma_{RLZ} - \sigma_{ISD}) + (\text{bias})^2 \end{aligned} \tag{5.4}$$

The results of this decomposition of the MSE into: (1) the variance of the forecast error $\text{Var}(\sigma_{RLZ} - \sigma_{ISD})$ and (2) the squared bias as well as their rankings are reported in Table 5.4 where Panel A and Panel B report the results for the samples including and excluding the 1987 market crash respectively. Note that RMSE is the square root of MSE so both of them will give exactly the same ranking. The decomposition for the sample including the 1987 market crash is also illustrated in Figure 5.4a and 5.4b where 5.4a is for calls and 5.4b for puts. As shown in Figures 5.4a and 5.4b as well as in Table 5.4, the bias does exits and decreases as the strike price increases but is relatively inconsequential except for very low strike price options. MSE depends mostly on the variance of the forecast error. The ranking by variance of the forecast error more or less resembles that by MSE. For example, in the sample including the 1987 market crash, the implied volatility calculated from the eighth in-the-money call (ISDCI8) has one of the largest biases, is ranked 33th in terms of MSE before correcting the bias, and is still ranked the 33th after correcting the bias, i.e., in terms of Var(σ_{RLZ} - σ_{ISD}). Notice that the implied volatilities calculated from the options with low strike prices are both biased and inefficient.

As shown in Table 5.4, the historical volatility has very small bias but one of the largest variance of the forecast error. This means that even though the historical volatility is the least biased estimator, it contains considerable noise and yields a much less efficient forecast compared with the implied volatility in general as also reported in Chapter 3. Table 5.4 clearly shows that the findings in Table 5.2 are not solely due to the biases and the variance of the forecast error actually plays a bigger role. That means even if one corrects the biases, the implied volatilities calculated from the options with lower strike prices still tend to have lower forecasting ability.

5.4.4 *A Comparison Based on OLS regressions*

Another possible approach for comparing forecast effectiveness is to estimate

Equations 3.1 or 3.2 and then use the estimated equation to generate a volatility forecast, which will provide a measure equivalent to the RMSE in Table 5.2. For instance, estimating Equation 3.1 for the sample excluding the 1987 market crash in terms of ISDCO3, I obtain that $\beta_0 = 0.0283$ and $\beta_1 = 0.7116$. Suppose I then use these parameter estimates to generate volatility forecasts, e.g., $\hat{\sigma}_{ISD,t} = 0.0283 + 0.7116\,\sigma_{ISDCO3,t}$.

The RMSE of the OLS regression is equal to

$$\sqrt{\frac{1}{N}\sum_{t=1}^{N}(\sigma_{RLZ,t} - \hat{\sigma}_{ISD,t})^2}$$

which is very similar to RMSE in Equation 5.1. Actually, the RMSE in Table 5.2 can be viewed as the RMSE from a regression where $\beta_0 = 0$ and $\beta_1 = 1$ while the $Var(\sigma_{RLZ} - \sigma_{ISD})$ in Table 5.4 can be viewed as the MSE from a regression where $\beta_0 = \bar{\sigma}_{RLZ} - \bar{\sigma}_{ISD}$ and $\beta_1 = 1$.

Table 5.5 reports the results of OLS regressions of realized volatility on various implied volatility estimators and for comparison also on historical volatility using Equation 3.1. Figures 5.5a, 5.5b, 5.5c and 5.5d illustrate the regression's RMSE, the adjusted R^2, the intercept, and the slope coefficient respectively for the sample including the 1987 market crash. Based on the OLS regression's root mean squared error (RMSE), the forecasting ability for calls including the 1987 market crash decreases and then goes back up as strike price increases although the rankings for puts are not quite clear. However, for the sample excluding the 1987 market crash, the results based on the OLS regressions' RMSE show a similar pattern which is that up to a certain level, the forecast ability increases as strike price increases.

Further comparisons can be made in terms of adjusted R^2s and the parameters of the regressions. The adjusted R^2 in Table 5.5 and in Figure 5.5b, however, displays rather consistent pattern which is that the adjusted R^2 increases as strike price increases for both the calls and the puts. Since $R^2 = 1$- Regression's MSE / Var(σ_{RLZ}). Regression's RMSE and the adjusted R^2 should give the same ranking. But they do not, probably because the number of observations differs across different implied volatility estimators so that Var(σ_{RLZ}) differ. However, the adjusted R^2 should be more important than regression's RMSE.

As mentioned in Chapter 3, if the option market is efficient and the option pricing model is correct, the implied volatility calculated from an observed option price should represent the market's best forecast of the underlying asset's volatility over the remaining life of the option. As such, it should be both unbiased and informationally efficient. Therefore we should observe that β_0=0 and β_1=1 in Equation 3.1 and β_0=0, β_1=1, and β_2=0 in Equation 3.2. As discussed in Chapter 3, however, the evidence to date on this issue has been mixed. While most of the previous studies find that implied volatility outperforms historical volatility in forecasting future volatility, they normally also find that implied volatility is biased and fails to incorporate all available information. That is they find that β_0>0, β_1<1, and β_2>0. Moreover, some studies even find that implied volatility's predictive power is quite low.

For the sample including the 1987 market crash as shown in Figure 5.5c and 5.5d as well as in Panel A of Table 5.5, the intercepts are positive and significantly different from zero for relatively lower strike prices. The intercept decreases monotonically and become insignificantly different from zero as strike price increases to a certain point (the fifth out-of-the-money call or the sixth in-the-money put). The slope coefficient of the implied volatilities increases monotonically as the strike price increases up to a certain level (the sixth out-of-the-money call or the sixth in-the-money put). In addition, for the implied volatilities calculated from each of the eight out-of-the-money calls as well as ISDPI3 and ISDPI4, the intercepts are not significantly different from zero and the slope coefficients are not significantly different from one, indicating that these estimators are unbiased.

In contrast, the historical volatility has one of the lowest adjusted R^2s (0.1143), the largest intercept of 0.1008 and the smallest slope coefficient of 0.2936 for the sample including the 1987 market crash. Both the intercept and the slope coefficient are significantly different from zero indicating that historical volatility in general has lower predictive power than most of the implied volatility estimators. For the sample excluding the 1987 market crash shown in Panel B of Table 5.5, although the slope coefficients of the implied volatilities are never significantly different from one but the patterns are similar to those in Panel A. Historical volatility outperforms only several out-of-the money puts and several in-the-money calls.

In both the samples of including and excluding the 1987 crash, the coefficients and the adjusted R^2s for the four "at-the-money" implied volatilities are ranked in the middle, indicating again that they are not the most efficient. Further more, the average of these four nearest-the-money implied volatilities could not improve the forecasting performance. In both the Panel A and Panel B in Table 5.5, forecasting efficiency increases as strike price increases. In addition, there is not significant difference in regression's RMSE, adjusted R^2, intercept or slope coefficient between calls and puts with the same strike price.

Next, realized volatility is regressed on both an implied volatility estimator and the historical volatility utilizing Equation 3.2 and the results are reported in Table 5.6. The results reveal almost the same patterns as those reported in Table 5.5. The intercept and the slope coefficient of the historical volatility (β_2) decrease while the slope coefficient of implied volatility increases as strike price increases. Clearly, the relative efficiency of implied volatility increases as strike price increases based on the coefficient of implied volatility, the adjusted R^2 and the regression's RMSE. However, several implied volatility estimators calculated from the calls (puts) with low strike prices are less efficient than historical volatility (HIS40) in that the coefficient of the historical volatility is significantly greater than zero and is greater than that of the implied volatility.

Table 5.2 through Table 5.6 report the results for the overall sample with the four nearest-to-expiration options each day. To save space, I do not report the results for the four subgroups stratified by options' time to expiration. As a matter of fact, the results for the four subgroups are almost the same as those reported here for the overall sample.

5.4.5. An Interpretation of the Results

The results discussed above suggest that up to a certain level, forecasting efficiency of implied volatilities increases as the strike price increases, i.e. implied volatilities calculated from out-of-the money calls and in-the-money puts have relatively higher forecasting ability than those calculated from in-the-money calls and out-of-the-money puts.

The predictive power of the implied volatility estimators calculated from the four at-the-money options as well as their average are ranked somewhere in the middle. How should one interpret these results?

As well known, if the option pricing model is correct and the market is efficient, then implied volatilities calculated from options with different strike prices but with the same expiration date should be the same. However as shown in Table 5.1 as well as in Figures 4.1 and 4.2, the implied volatility actually varies across different strike prices. Chapter 4 suggests that the smile or "smirk" may reflect investors' preferences for options with certain strike prices and a market inefficiency. In consistent with this argument, an explanation for the results reported in this chapter is that the demand for out-of-the-money puts is driven by hedgers hedging against market declines and therefore these puts may be overpriced. If the market were perfect, arbitrage would tend to drive these prices down. Because of the put-call parity, in-the-money calls may also be overpriced. Figures 4.1a and 4.1b actually show that there is no difference in implied volatility between calls and puts with low strike prices. Therefore, the implied volatilities calculated from these options may not reflect market expectations. Fluctuations in the implied volatilities of the low strike price options due to fluctuations in hedging pressure will then create relatively more measurement error between the implied volatility and the market's true expectation.

However, options with high strike prices are demanded primarily by speculators, so these implied volatilities may better reflect the market's true expectation. As a result, the options with lower strike prices may contain more measurement error than options with higher strike prices. This can be further seen from the relationship of the measurement error and the slope coefficient of the OLS regressions. As discussed in Chapter 3, measurement error tends to bias the OLS estimate of the slope coefficient downward. Table 5.5 shows that options with higher strike prices have larger slope coefficients which is consistent with the hypothesis that these options contain less measurement error. Similarly, the slope coefficients of options with lower strike prices are smaller which is what one would expect if these options contain more measurement error. In fact, as strike price increases, the slope coefficients increase monotonically up to a certain level of the moneyness. However, the forecasting efficiency eventually declines as strike price further increases since options further away from the money are less actively traded and therefore contain relatively more measurement error.

5.5 Summary

On a given day, for the same forecasting horizon, one can obtain a number of different implied volatility estimates calculated from calls or puts with different strike prices. Which of them is the most informative and should be utilized to forecast the future volatility? One popular procedure is to utilize the implied volatility calculated from a single at-the-money option which reflects the popular belief that at-the-money options are the most informative and contain the least measurement error. Another common procedure is to obtain an average over several implied volatilities calculated from near-the-money options and ignore all the others. Although several previous studies have compared the forecasting ability of different *averaging* schemes, no one has systematically investigated the relative forecasting efficiency of different individual implied volatilities calculated from *calls or* *puts with different strike prices*. In addition, these previous studies generally suffered from limited data sets and therefore lacked the degrees of freedom.

By utilizing over 15 years of daily data on options on S&P 500 futures, I find, contrary to the general belief and practice, that implied volatility calculated from an at-the-money option is not the most efficient compared with those calculated from out-of-the-money calls and in-the-money puts. Specifically, I find that implied volatilities calculated from calls and puts with relatively higher strike prices have more predictive power in forecasting future volatility. In fact, up to a certain level of strike price, the predictive power increases as an option's strike price increases. The empirical evidence also suggests that averaging over several implied volatilities may not be effective, implying that the choice of which strike price to use is more important than the weighting scheme which has received much more attention in the literature.

The results in this chapter have also successfully explained why the regression results for the average implied volatility calculated from the two nearest-the-money calls and the two nearest-the-money puts presented in Chapter 3 are better than those utilizing all the individual implied volatilities. The difference between these two regressions is apparently not due to the averaging effect but that all the four components of the average implied volatility actually are better estimators although not the best among all the individual implied volatilities. The evidence helps us to identify the different levels of measurement error for options with different strike prices. In the regression of realized volatility on all the individual implied volatilities in Chapter 3, the majority of the measurement error actually comes from the out-of-the-money puts and in-the-money calls.

In addition, there is no difference in predictive power between a call and a put with the same strike price. In comparison, historical volatility yields better forecast than some of the options with low strike prices but worse than most of the other implied volatilities especially those with high strike prices. The results presented in this chapter hold for several different criteria and for both the samples including and excluding the 1987 market crash. The results also hold for different expiration groups. This is an interesting and useful finding in that both researchers and option traders could use the results to improve their forecasting performance.

Chapter 6

Dissertation Summary

Since volatility is a critical factor in pricing financial options, understanding the estimate of future market volatility implied by an observed option price is vital interest to practitioners and finance researchers. As well known, if the option pricing model is correct and the option market is efficient, the implied volatility calculated from an observed option price should represent the market participants' consensus expectation of the volatility over the remaining life of the option. While this assertion has been hotly debated and implied volatility has been extensively studied, there are still a lot of unresolved issues. Utilizing more than fifteen years of daily data on options on S&P 500 futures, this dissertation examines implied volatility in terms of three issues, thereby facilitating more efficient use of this measure in forecasting future volatility.

The first essay (Chapter 3) investigates the predictive power of implied volatility. Previous studies found that implied volatility calculated from an observed option price is a biased estimator of subsequent realized volatility and is not efficient in that it fails to incorporate all available information, including historical returns. In contrast to some of the previous studies, I find that implied volatility contains considerable information regarding the future realized volatility. In general, implied volatility has more predictive power than historical volatility. However, the results are quite sensitive to (1) the forecast horizon, (2) whether or not the data set includes the 1987 market crash, and (3) whether or not one corrects for heteroskedasticity and serial correlation caused by the overlap in realized volatility. The overlap in realized volatility estimates seriously biases the standard error estimates. Most importantly, I find that implied volatility varies around the market's true volatility expectation due to measurement error caused by bid-ask spread, non-synchronous prices, and possible deficiencies in the option pricing formula and that this measurement error biases tests towards rejecting the informational efficiency of implied volatility. When I control for the measurement error by utilizing instrumental variables estimation, the results in most cases no longer reject the hypothesis that implied volatility is unbiased and informationally efficient. No previous work has systematically examined this measurement error effect.

The second essay (Chapter 4) relates to the implied volatility "smile" which refers to the cross-sectional variation in implied volatility across options with different strike prices but the same maturity date. In this essay, I document the smile or "smirk" in options on S&P 500 futures market from 1983 to 1998. In particular, the implied volatility decreases monotonically and eventually goes back up as strike price increases, i.e., a reverse J shape. Previous explanations for the smile have mostly focused on possible errors in the option pricing formula and, as a result, researchers have developed more complicated option pricing models such as stochastic volatility models and jump-diffusion models. But none of these new models has successfully explained the smile, motivating me to search of an alternative explanation.

I argue and test that the smile may be caused by investors' preferences for certain strike prices for hedging purposes and these differences are not eliminated by arbitrage because the option market may not be efficient. In other words, the smile may actually represent a market inefficiency. I find that market inefficiency is at least partly responsible for the implied volatility smile in the options on S&P 500 futures market since abnormal returns can be made over time by buying put options with low implied volatility and simultaneously selling put options with high implied volatility in a delta neutral ratio. However, the standard deviations of the profits are quite large even though the positions are delta neutral and the delta neutral strategy involving calls does not generate as much profits. The results suggests further tests to identify the exact sources of the profits and to explain the difference in profits between calls and puts as well as the large variance of the profits.

On a given day, many different options with the same expiration date are traded which differ by strike price and whether they are calls or puts. The implied volatilities calculated from these different options are actually forecasting the same future volatility over the same period. Then which of the implied volatilities or which average measure is the most informative and should be utilized in forecasting future volatility? Previous studies have investigated the relative forecasting performance of several *averaging schemes* as well as a couple of individual implied volatilities calculated from a single option. Although their results are mixed, they tend to favor at-the-money options. But these previous studies generally suffered from limited data sets and therefore lack the degrees of freedom.

In practice, investors and finance researchers often utilize the implied volatility calculated from a single at-the-money option or an average over several, often two or four, implied volatilities calculated from the nearest-the-money options and totally ignore all the other options. Does an at-the-money option yield the best forecast? Does an average necessarily reduce measurement error? While no previous study has done this before, the third essay (Chapter 5) systematically examines the relative forecasting efficiency of implied volatilities calculated from calls and puts with different strike prices but the same expiration date.

I find, contrary to the general belief and practice, that the implied volatility calculated from an at-the-money option is less informative than those calculated from some out-of-the money calls or some in-the-money puts. Up to a certain level, the predictive ability of implied volatility increases as strike price increases. However, there is not much difference in forecasting ability between a call and a put with the same strike price.

In contract, historical volatility generally yields better forecasts than the implied volatilities calculated from the options with very low strike prices but worse than the implied volatilities calculated from most other options especially from those options with high strike prices. This suggests that in testing relative predictive power of implied volatility and historical volatility, the choice of which implied volatility to utilize makes a big difference. Canina and Figlewski (1993) utilized all the individual implied volatilities and obtained a lower predictive power for implied volatility than for historical volatility. On the other hand, Jorion (1995) employed an average implied volatility calculated from one nearest-the-money call and one nearest-the-money put while Christensen and Prabhala (1998) obtained their implied volatility measure from a single at-the-money call option. According to the evidence presented in Chapter 5, it is not surprising that both Jorion (1995) and Christensen & Prabhala (1998) conclude that implied volatility has much more predictive power than historical volatility.

The results are robust across options in different expiration groups and hold for both the samples of including and excluding the 1987 market crash. The results are also quite consistent based on several different criteria for measuring relative efficiency: root mean squared error, mean absolute error, mean absolute percentage error, and OLS regression measures. The evidence suggests that an average is not necessarily more informative than its components. The choice of which strike price to utilize is more important than the weighting schemes which have received much more attention in the literature. The findings presented in this dissertation should be informative to both investors and finance researcher who are interested in more efficient use of implied volatility.

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Table 2.1
Summary Statistics of Implied Volatility and its Corresponding Realized and Historical Volatilities for Options on S&P 500 Futures

	Group	Trading Days to Expiration	Obs	Realized Volatility RLZSD				Implied Volatility ISD				Historical Volatility HIS40			
				Mean	Std	Min	Max	Mean	Std	Min	Max	Mean	Std	Min	Max
Panel A: Include 1987 Crash	all	10-99	217,226	0.1420	0.0913	0.0378	1.3533	0.1686	0.0581	0.0633	1.0571	0.1462	0.1044	0.0511	1.0806
	1	10-36	77,123	0.1383	0.0827	0.0378	1.3533	0.1734	0.0620	0.0662	0.9528	0.1495	0.1107	0.0511	1.0806
	2	28-57	65,629	0.1411	0.0930	0.0562	1.1517	0.1670	0.0625	0.0675	1.0571	0.1421	0.0958	0.0511	1.0768
	3	47-78	40,472	0.1478	0.1027	0.0619	0.9516	0.1638	0.0491	0.0633	0.5213	0.1432	0.1061	0.0511	1.0786
	4	67-99	34,002	0.1452	0.0914	0.0687	0.8108	0.1668	0.0484	0.0772	0.7927	0.1505	0.1030	0.0564	1.0709
Panel B: Exclude 1987 Crash	all	10-99	212,002	0.1323	0.0481	0.0378	0.5540	0.1651	0.0494	0.0633	0.7078	0.1367	0.0530	0.0511	0.3807
	1	10-36	75,478	0.1315	0.0543	0.0378	0.5540	0.1696	0.0538	0.0662	0.7078	0.1385	0.0540	0.0511	0.3807
	2	28-57	64,316	0.1319	0.0469	0.0562	0.3277	0.1633	0.0499	0.0675	0.4732	0.1346	0.0525	0.0511	0.3807
	3	47-78	39,099	0.1339	0.0440	0.0619	0.2804	0.1599	0.0430	0.0633	0.3878	0.1315	0.0471	0.0511	0.3609
	4	67-99	33,109	0.1329	0.0393	0.0687	0.2518	0.1645	0.0438	0.0772	0.3719	0.1429	0.0570	0.0564	0.3621

The data set covers the period from Jan. 28, 1983 through April 30, 1998. The implied volatility (ISD) is calculated utilizing Black's (1976) model from an observed option price. The realized volatility (RLZSD) is the standard deviation of daily returns of the underlying futures over the remaining life of the option. The historical volatility (HIS40) is the standard deviation of daily returns of the underlying futures over the last 40 trading days. Group 1 contains the nearest-to-expiration options, and Group 2 contains the second nearest-to-expiration options, etc.

Table 3.1
Evidence from Previous Studies on the Rationality of Implied Volatility as a Predictor of Future Realized Volatility

Results of the estimation of $\sigma_{RLZ,t} = \beta_0 + \beta_1 \sigma_{ISD,i,t} + \beta_2 \sigma_{TS,t} + u_{i,t}$ are reported where $\sigma_{RLZ,t}$ is the realized volatility, $\sigma_{ISD,i,t}$ is an implied volatility, and $\sigma_{TS,t}$ is a time series measure, either past volatility and/or a GARCH measure. Standard errors are shown in parentheses (some inferred from reported t values). Some equations represent averages of several regressions.

Study	Market	Obs	Forecast horizon	Intercept (β_0)	Implied vol (β_1)	GARCH vol (β_2)	Historic vol (β_2)	R^2	Notes
Canina & Figlewski (1993)	S&P 100 Index	daily, calls 3/83-3/87	7-127 calendar days	0.0827 (0.0284)	-0.0641 (0.0853)		0.4866 (0.2201)	0.1695	avg of 32 equations. HIS: over 60 calendar days.
Day & Lewis (1992)	S&P 100 Index	weekly, calls 3/83-12/89	nearby contract and: > 7 days	-0.0001 (0.0011)	0.601 (0.583)	0.298 (0.710)		0.027	HIS: previous week. Realized and implied volatility periods do not match.
				0.00018 (0.00016)	0.632 (0.620)	-0.243 (0.868)	0.123 (0.018)	0.038	
Lamoureux & Lastrapes (1993)	10 non-dividend paying stocks	daily, calls 4/82-3/84	90-180 days	1627.258 (387.765)	0.6652 (0.3726)	-1.7516 (0.5588)		0.2918	averge of 10 stock regressions.
				3451.775 (882.035)	0.4705 (0.2175)	0.1658 (0.7577)	-3.9105 (1.5260)	0.4741	
Day & Lewis (1993)	crude oil futures	daily, calls 11/86-3/91	nearby and second nearby contracts	0.004 (0.0035)	0.970 (0.157)	-0.006 (0.3915)		0.607	HIS: same as the forecast horizon.
				0.004 (0.0045)	0.901 (0.169)	0.053 (0.854)	-0.061 (0.203)	0.608	
Jorion (1995)	foreign exchange futures	daily, calls & puts 1/85-2/92	3-100 calendar days	0.3317 (0.1250)	0.6313 (0.1853)		-0.0897 (0.1007)	0.1386	averge of 3 currencies regressions. HIS: 20 trading days.
				0.3230 (0.2170)	0.5507 (0.1817)	0.0203 (0.2873)		0.1380	
Guo (1996)	foreign exchange rates	daily: 1/86-2/93	30-90 trading days	0.0065 (0.0034)	0.2925 (0.1327)	0.1539 (0.3366)		0.0687	avg of two currencies HIS: 60 trading days. Realized and implied volatility periods do not match.
				0.0057 (0.0034)	0.4558 (0.1890)	0.2974 (0.3264)	-0.2120 (0.1275)	0.0907	

Table 3.2
The Predictive Properties of Individual Implied Volatilities - OLS Results
$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD,i,t} + \beta_2\sigma_{HIS40,t} + u_{i,t}$$

The realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an option on S&P 500 futures is regressed on an individual implied volatility ($\sigma_{ISD,i,t}$) and historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days using daily observations from Jan. 28, 1983 through April 30, 1998. On a given day, for each expiration date, eight in-the-money calls (puts) and eight out-of-the-money calls (puts) are utilized. The t_{OLS} and t_H are OLS t statistic and t statistic by Hansen (1982) procedure respectively. The second column represents the option's trading days to expiration.

	Trading Days to Maturity		β_0	β_1	β_2	Adj R^2	Obs
Panel A: Including the 1987 market crash	Overall 10-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0448** (77.506) (5.027)	0.4937**†† (111.571) (4.200) (-4.307)	0.0953 (38.694) (1.690)	0.1596	217,226
	Group 1 10-36	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0502** (61.434) (6.773)	0.3713**†† (62.651) (5.462) (-9.250)	0.1589** (47.864) (3.381)	0.2049	77,123
	Group 2 28-57	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0498** (49.575) (5.515)	0.4339**†† (50.377) (3.926) (-5.122)	0.1327 (23.634) (1.917)	0.1663	65,629
	Group 3 47-78	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0123 (7.118) (0.460)	0.8228** (64.106) (2.907) (-0.626)	0.0054 (0.909) (0.070)	0.1574	40,472
	Group 4 67-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0287 (15.806) (1.059)	0.7472* (52.998) (2.125) (-0.719)	-0.0546 (-8.250) (-0.355)	0.1244	34,002
Panel B: Excluding the 1987 market crash	Overall 10-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0480** (156.418) (4.584)	0.2946**†† (116.813) (6.039) (-14.462)	0.2603** (110.665) (3.501)	0.2963	212,002
	Group 1 10-36	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0517** (87.943) (4.316)	0.2592**†† (59.493) (5.549) (-15.863)	0.2585** (59.587) (2.979)	0.2197	75,478
	Group 2 28-57	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0479** (92.113) (3.750)	0.2995**†† (65.668) (5.661) (-13.241)	0.2606** (60.112) (2.850)	0.3258	64,316
	Group 3 47-78	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0302* (46.300) (2.260)	0.4150**†† (75.550) (5.506) (-7.761)	0.2837** (56.650) (2.780)	0.4277	39,099
	Group 4 67-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0490** (75.042) (3.749)	0.3114**†† (51.159) (2.728) (-6.032)	0.2287** (48.818) (2.600)	0.4112	33,109

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 3.3
The Predictive Properties of the Average Implied Volatility - OLS Results

$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD4,t} + \beta_2\sigma_{HIS40,t} + u_t$$

Realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an option on S&P 500 futures is regressed on the average implied volatility ($\sigma_{ISD4,t}$) (calculated from the two nearest-the-money calls and the two nearest-the-money puts) and the historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days using daily observations from Jan. 28, 1983 through April 30, 1998. The t_{OLS} and t_H are OLS t statistic and t statistic for the Hansen (1982) procedure respectively.

	Trading Days to Maturity		β_0	β_1	β_2	Adj R^2	Obs
Panel A: Including the 1987 market crash	Overall 10-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0115 (3.806) (0.705)	0.8719** (33.295) (3.898) (-0.573)	-0.0483 (-3.677) (-0.508)	0.2024	9,764
	Group 1 10-36	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0135 (2.892) (1.125)	0.8056** (19.464) (5.534) (-1.336)	-0.0060 (-0.292) (-0.097)	0.2447	3,212
	Group 2 28-57	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0207 (3.771) (1.667)	0.7871** (15.618) (3.760) (-1.017)	-0.0123 (-0.433) (-0.106)	0.1914	3,029
	Group 3 47-78	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	-0.0114 (-1.568) (-0.339)	1.0565** (17.770) (3.024) (0.162)	-0.0763 (-2.778) (-0.785)	0.2048	2,021
	Group 4 67-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0086 (0.947) (0.213)	0.9758* (13.109) (2.022) (-0.050)	-0.1382 (-0.395) (-0.705)	0.1511	1,502
Panel B: Excluding the 1987 market crash	Overall 10-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0303** (20.376) (2.954)	0.6340**†† (39.116) (7.807) (-4.507)	0.0283 (2.122) (0.387)	0.3570	9,505
	Group 1 10-36	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0303** (10.431) (2.658)	0.6999**†† (22.146) (7.989) (-3.426)	-0.0453 (-1.688) (-0.483)	0.2969	3,136
	Group 2 28-57	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0310** (12.213) (2.613)	0.6156**†† (21.918) (7.605) (-4.749)	0.0461 (1.942) (0.584)	0.3813	2,946
	Group 3 47-78	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0198 (6.579) (1.456)	0.6533**†† (20.643) (6.105) (-3.240)	0.0951 (3.627) (1.111)	0.4463	1,964
	Group 4 67-99	coefficient $t_{OLS}(H_0: \beta=0)$ $t_H(H_0: \beta=0)$ $t_H(H_0: \beta=1)$	0.0449** (14.048) (3.001)	0.4387**†† (12.414) (2.588) (-3.311)	0.1304 (5.025) (1.162)	0.4263	1,459

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 3.4
Measurement Error Tests for Individual Implied Volatilities

$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD,i,t} + \beta_2\sigma_{HIS40,t} + \beta_3 U_{i,t} + u_{i,t}$$

Realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an S&P 500 futures option is regressed on an individual implied volatility ($\sigma_{ISD,i,t}$), the historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days, and the residual from the first step regression of implied volatility on the instrumental variable ($\sigma_{ISD4,t-10}$) and the historical volatility. Regressions use daily observations from Jan. 28, 1983 through April 30, 1998. The t_H is the corrected t statistic from the Hansen (1982) procedure.

	Group	Trading Days to Expiration		β_0	β_1	β_2	β_3	Adj R^2	Obs
Panel A: Including the 1987 market crash	Overall	10-99	coefficient	-0.0192	1.0744**	-0.1395*	-0.7598**	0.1786	207,021
			$t_H(H_0: \beta=0)$	(-1.067)	(5.906)	(-2.194)	(-6.886)		
	1	10-36	coefficient	-0.0059	0.8535**	-0.0271	-0.6130**	0.2277	76,797
			$t_H(H_0: \beta=0)$	(-0.368)	(7.861)	(-0.727)	(-4.125)		
	2	28-57	coefficient	-0.0259	1.2651**	-0.3205*	-0.9629**	0.1861	62,498
			$t_H(H_0: \beta=0)$	(-0.906)	(4.284)	(-2.379)	(-3.455)		
	3	47-78	coefficient	-0.0493	1.3466**	-0.1512	-1.0620*	0.1814	33,866
			$t_H(H_0: \beta=0)$	(-0.925)	(2.661)	(-1.150)	(-2.469)		
	4	67-99	coefficient	-0.0015	1.0244*	-0.1617	-0.4572	0.1257	33,860
			$t_H(H_0: \beta=0)$	(-0.037)	(2.088)	(-0.733)	(-1.952)		
Panel B: Excluding the 1987 market crash	Overall	10-99	coefficient	0.0106	0.8004**	-0.0775	-0.6488**	0.3269	202,224
			$t_H(H_0: \beta=0)$	(0.734)	(6.012)	(-0.785)	(-5.312)		
	1	10-36	coefficient	0.0055	0.8568**	-0.1398	-0.7026**	0.2525	75,224
			$t_H(H_0: \beta=0)$	(0.324)	(5.139)	(-1.080)	(-4.333)		
	2	28-57	coefficient	0.0102	0.8613**	-0.1439	-0.7504**	0.3685	61,398
			$t_H(H_0: \beta=0)$	(0.536)	(4.900)	(-1.165)	(-3.943)		
	3	47-78	coefficient	0.0005	0.7995**	0.0471	-0.6012**	0.4704	32,543
			$t_H(H_0: \beta=0)$	(0.027)	(5.405)	(0.431)	(-4.096)		
	4	67-99	coefficient	0.0367*	0.4698*	0.1328	-0.2327	0.4161	33,059
			$t_H(H_0: \beta=0)$	(1.981)	(2.260)	(0.970)	(-1.396)		

* significantly different from zero at the 0.05 level.

** significantly different from zero at the 0.01 level.

Table 3.5
Measurement Error Tests for the Average Implied Volatility

$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD4,t} + \beta_2\sigma_{HIS40,t} + \beta_3 U_t + u_t$$

The realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an option on S&P 500 futures is regressed on the average implied volatility ($\sigma_{ISD4,t}$) calculated from the two nearest-the-money calls and the two nearest-the-money puts, the historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days, and the residual from the first step regression of implied volatility on the instrumental variable ($\sigma_{ISD4,t-10}$) and the historical volatility. Regressions use daily observations from Jan. 28, 1983 through April 30, 1998. The t_H is the corrected t statistic for the Hansen (1982) procedure .

	Group	Trading Days to Expiration		β_0	β_1	β_2	β_3	Adj R^2	Obs
Panel A: Including the 1987 market crash	all	10-99	coefficient	-0.0045	1.0380**	-0.1217	-0.2988*	0.2006	8,863
			$t_H(H_0: \beta=0)$	(-0.184)	(3.450)	(-1.002)	(-2.481)		
	1	10-36	coefficient	0.0032	0.9052**	-0.0448	-0.1816*	0.2501	3,184
			$t_H(H_0: \beta=0)$	(0.266)	(6.597)	(-0.792)	(-2.001)		
	2	28-57	coefficient	-0.0157	1.2405**	-0.2829	-0.6237*	0.1929	2,783
			$t_H(H_0: \beta=0)$	(-0.493)	(2.958)	(-1.361)	(-2.343)		
	3	47-78	coefficient	-0.0266	1.2182**	-0.1174	-0.2177	0.1954	1,410
			$t_H(H_0: \beta=0)$	(-0.605)	(2.732)	(-1.005)	(-1.265)		
	4	67-99	coefficient	0.0153	0.9277*	-0.1325	0.1594	0.1437	1,486
			$t_H(H_0: \beta=0)$	(0.427)	(2.058)	(-0.654)	(0.645)		
Panel B: Excluding the 1987 market crash	all	10-99	coefficient	0.0252*	0.7352**	-0.0488	-0.1935	0.3495	8,653
			$t_H(H_0: \beta=0)$	(2.218)	(5.646)	(-0.458)	(-1.600)		
	1	10-36	coefficient	0.0225	0.8439**	-0.1504	-0.2290	0.2971	3,112
			$t_H(H_0: \beta=0)$	(1.784)	(5.601)	(-1.157)	(-1.657)		
	2	28-57	coefficient	0.0220	0.7849**	-0.0873	-0.3419	0.3732	2,730
			$t_H(H_0: \beta=0)$	(1.463)	(4.441)	(-0.635)	(-1.546)		
	3	47-78	coefficient	0.0161	0.7115**	0.0709	-0.1197	0.4674	1,362
			$t_H(H_0: \beta=0)$	(1.046)	(5.208)	(0.692)	(-0.600)		
	4	67-99	coefficient	0.0475**	0.4026*	0.1516	0.0855	0.4226	1,449
			$t_H(H_0: \beta=0)$	(3.122)	(2.067)	(1.093)	(0.427)		

* significantly different from zero at the 0.05 level.

** significantly different from zero at the 0.01 level.

Table 3.6
Instrumental Variables Estimates of the Predictive Properties of Individual Implied Volatilities and Historical Volatility

$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD,i,t} + \beta_2\sigma_{HIS40,t} + u_{i,t}$$

The realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an option on S&P 500 futures is regressed on an individual implied volatility ($\sigma_{ISD,i,t}$) and the historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days using $\sigma_{ISD4,t-10}$ as an instrumental variable. Regressions use daily observations from Jan. 28, 1983 through April 30, 1998. The t_{OLS} and t_H are OLS t statistic and t statistic by Hansen (1982) procedure respectively.

	Trading Days to Maturity		β_0	β_1	β_2	Adj R^2	Obs
Panel A: Including the 1987 market crash	Overall 10-99	coefficient	-0.0192	1.0744**	-0.1395*	0.0886	207,021
		$t_{OLS}(H_0: \beta=0)$	(-16.90)	(108.49)	(-31.50)		
		$t_H(H_0: \beta=0)$	(-1.337)	(7.359)	(-2.291)		
		$t_H(H_0: \beta=1)$		(0.510)			
	Group 1 10-36	coefficient	-0.0059	0.8535**	-0.0271	0.1374	76,797
		$t_{OLS}(H_0: \beta=0)$	(-3.68)	(64.24)	(-4.69)		
		$t_H(H_0: \beta=0)$	(-0.502)	(9.226)	(-0.788)		
		$t_H(H_0: \beta=1)$		(-1.584)			
	Group 2 28-57	coefficient	-0.0259	1.2651**	-0.3205**	0.0419	62,498
		$t_{OLS}(H_0: \beta=0)$	(-9.99)	(45.41)	(-20.60)		
		$t_H(H_0: \beta=0)$	(-1.404)	(7.031)	(-3.447)		
		$t_H(H_0: \beta=1)$		(1.473)			
	Group 3 47-78	coefficient	-0.0493	1.3466**	-0.1512	0.1226	33,866
		$t_{OLS}(H_0: \beta=0)$	(-18.46)	(65.90)	(-19.57)		
		$t_H(H_0: \beta=0)$	(-1.479)	(3.954)	(-1.778)		
		$t_H(H_0: \beta=1)$		(1.018)			
	Group 4 67-99	coefficient	-0.0015	1.0244**	-0.1617	0.1101	33,860
		$t_{OLS}(H_0: \beta=0)$	(-0.58)	(45.58)	(-16.58)		
		$t_H(H_0: \beta=0)$	(-0.054)	(2.708)	(-0.867)		
		$t_H(H_0: \beta=1)$		(0.064)			
Panel B: Excluding the 1987 market crash	Overall 10-99	coefficient	0.0106	0.8004**†	-0.0775	0.1544	202,224
		$t_{OLS}(H_0: \beta=0)$	(19.79)	(130.42)	(-17.29)		
		$t_H(H_0: \beta=0)$	(0.833)	(8.725)	(-0.954)		
		$t_H(H_0: \beta=1)$		(-2.176)			
	Group 1 10-36	coefficient	0.0055	0.8568**	-0.1398	0.0243	75,224
		$t_{OLS}(H_0: \beta=0)$	(4.89)	(67.72)	(-15.23)		
		$t_H(H_0: \beta=0)$	(0.380)	(8.799)	(-1.565)		
		$t_H(H_0: \beta=1)$		(-1.471)			
	Group 2 28-57	coefficient	0.0102	0.8613**	-0.1439	0.1565	61,398
		$t_{OLS}(H_0: \beta=0)$	(11.77)	(78.88)	(-16.81)		
		$t_H(H_0: \beta=0)$	(0.660)	(7.646)	(-1.304)		
		$t_H(H_0: \beta=1)$		(-1.232)			
	Group 3 47-78	coefficient	0.0005	0.7995**†	0.0471	0.3591	32,543
		$t_{OLS}(H_0: \beta=0)$	(0.49)	(78.73)	(6.27)		
		$t_H(H_0: \beta=0)$	(0.033)	(8.072)	(0.506)		
		$t_H(H_0: \beta=1)$		(-2.024)			
	Group 4 67-99	coefficient	0.0367**	0.4698**††	0.1328	0.3985	33,059
		$t_{OLS}(H_0: \beta=0)$	(38.16)	(43.25)	(18.44)		
		$t_H(H_0: \beta=0)$	(2.717)	(3.981)	(1.539)		
		$t_H(H_0: \beta=1)$		(-4.494)			

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 3.7
Instrumental Variables Estimates of the Predictive Properties of the Average Implied Volatility and Historical Volatility

$$\sigma_{RLZ,t} = \beta_0 + \beta_1\sigma_{ISD4,t} + \beta_2\sigma_{HIS40,t} + u_t$$

The realized volatility ($\sigma_{RLZ,t}$) over the remaining life of an option on S&P 500 futures is regressed on the average implied volatility ($\sigma_{ISD4,t}$ calculated from the two nearest-the-money calls and the two nearest-the-money puts) and the historical volatility ($\sigma_{HIS40,t}$) measured over the last 40 trading days using $\sigma_{ISD4,t-10}$ as an instrumental variable. Regressions use daily observations from Jan. 28, 1983 through April 30, 1998. The t_{OLS} and t_H are OLS t statistic and t statistic by Hansen (1982) procedure respectively. The second column shows the option's trading days to maturity.

			β_0	β_1	β_2	Adj R^2	Obs
Panel A: Including the 1987 market crash	Overall 10-99	coefficient	-0.0045	1.0380**	-0.1217	0.1941	8,863
		$t_{OLS}(H_0: \beta=0)$	(-0.98)	(23.78)	(-6.10)		
		$t_H(H_0: \beta=0)$	(-0.291)	(4.939)	(-1.305)		
		$t_H(H_0: \beta=1)$		(0.181)			
	Group 1 10-36	coefficient	0.0032	0.9052**	-0.0448	0.2476	3,184
		$t_{OLS}(H_0: \beta=0)$	(0.49)	(14.23)	(-1.56)		
		$t_H(H_0: \beta=0)$	(0.276)	(6.756)	(-0.822)		
		$t_H(H_0: \beta=1)$		(-0.708)			
	Group 2 28-57	coefficient	-0.0157	1.2405**	-0.2829*	0.1605	2,783
		$t_{OLS}(H_0: \beta=0)$	(-1.51)	(10.35)	(-4.23)		
		$t_H(H_0: \beta=0)$	(-1.015)	(6.200)	(-2.530)		
		$t_H(H_0: \beta=1)$		(1.202)			
	Group 3 47-78	coefficient	-0.0266	1.2182**	-0.1174	0.1950	1,410
		$t_{OLS}(H_0: \beta=0)$	(-2.44)	(13.84)	(-3.39)		
		$t_H(H_0: \beta=0)$	(-0.614)	(2.768)	(-1.019)		
		$t_H(H_0: \beta=1)$		(0.496)			
	Group 4 67-99	coefficient	0.0153	0.9277	-0.1325	0.1433	1,486
		$t_{OLS}(H_0: \beta=0)$	(1.34)	(9.32)	(-3.13)		
		$t_H(H_0: \beta=0)$	(0.385)	(1.843)	(-0.591)		
		$t_H(H_0: \beta=1)$		(-0.144)			
Panel B: Excluding the 1987 market crash	Overall 10-99	coefficient	0.0252*	0.7352**††	-0.0488	0.3447	8,653
		$t_{OLS}(H_0: \beta=0)$	(13.16)	(29.61)	(-2.58)		
		$t_H(H_0: \beta=0)$	(2.417)	(9.087)	(-0.654)		
		$t_H(H_0: \beta=1)$		(-3.273)			
	Group 1 10-36	coefficient	0.0225*	0.8439**	-0.1504	0.2904	3,112
		$t_{OLS}(H_0: \beta=0)$	(6.10)	(16.67)	(-3.88)		
		$t_H(H_0: \beta=0)$	(1.989)	(10.119)	(-1.699)		
		$t_H(H_0: \beta=1)$		(-1.872)			
	Group 2 28-57	coefficient	0.0220	0.7849**†	-0.0873	0.3586	2,730
		$t_{OLS}(H_0: \beta=0)$	(6.80)	(17.60)	(-2.44)		
		$t_H(H_0: \beta=0)$	(1.787)	(8.009)	(-0.917)		
		$t_H(H_0: \beta=1)$		(-2.194)			
	Group 3 47-78	coefficient	0.0161	0.7115**††	0.0709	0.4664	1,362
		$t_{OLS}(H_0: \beta=0)$	(4.12)	(16.80)	(2.19)		
		$t_H(H_0: \beta=0)$	(1.128)	(7.453)	(0.888)		
		$t_H(H_0: \beta=1)$		(-3.022)			
	Group 4 67-99	coefficient	0.0475**	0.4026*††	0.1516	0.4219	1,449
		$t_{OLS}(H_0: \beta=0)$	(12.07)	(8.24)	(4.47)		
		$t_H(H_0: \beta=0)$	(3.190)	(2.352)	(1.310)		
		$t_H(H_0: \beta=1)$		(-3.490)			

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 4.1
Profits (Losses) to a Random $1 Investment in Calls or Puts on S&P 500 Futures

Panel A: for Calls

Group	Trading Days to Expiration	Holding Period (trading days)	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$0.0871**	$2.1301	7.535	33,961	$0.0978**	$2.1382	8.390	33,619
		10	$0.1758**	$2.7354	10.671	27,576	$0.1802**	$2.7442	10.866	27,375
		5	$0.0913**	$1.2345	14.021	35,977	$0.0945**	$1.2374	14.446	35,748
		1	$0.0152**	$0.3468	8.420	36,995	$0.0158**	$0.3469	8.749	36,794
2	28-57	Expiration	$0.2256**	$2.1109	18.783	30,901	$0.2472**	$2.1239	20.277	30,343
		10	$0.1135**	$1.1115	18.065	31,288	$0.1203**	$1.1149	18.977	30,932
		5	$0.0542**	$0.5940	16.184	31,475	$0.0579**	$0.5949	17.184	31,151
		1	$0.0082**	$0.2239	6.548	31,613	$0.0091**	$0.2234	7.203	31,324
3	47-78	Expiration	$0.2864**	$1.9874	20.067	19,384	$0.3071**	$1.9965	21.248	19,077
		10	$0.0317**	$0.6376	6.916	19,392	$0.0334**	$0.6378	7.263	19,226
		5	$0.0198**	$0.4198	6.578	19,406	$0.0206**	$0.4201	6.804	19,303
		1	$0.0064**	$0.1612	5.542	19,424	$0.0063**	$0.1612	5.425	19,372
4	67-99	Expiration	$0.5667**	$2.4718	29.564	16,630	$0.5988**	$2.4896	30.675	16,263
		10	$0.0753**	$0.4274	22.721	16,627	$0.0774**	$0.4270	23.326	16,553
		5	$0.0507**	$0.3066	21.343	16,630	$0.0520**	$0.3065	21.814	16,556
		1	$0.0109**	$0.1301	10.829	16,630	$0.0111**	$0.1301	11.016	16,556

The data sets contain daily observations of the options from January 28, 1983 through April 30, 1998. Group 1 contains the nearest-to-maturity options on a given day and Group 2 contains the second nearest-to-maturity options, etc. The investment (initial cash outflow) is standardized to $1 for a call or a put. The profits (losses) do not account for transaction costs. The holding periods are either a number of trading days or until the option's expiration.

* significantly different from zero at the 0.05 level.
** significantly different from zero at the 0.01 level.

Table 4.1 (continued)
Profits (Losses) to a Random $1 Investment in Calls or Puts on S&P 500 Futures

Panel B: for Puts

Group	Trading Days to Expiration	Holding Period (trading days)	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$-0.5079**	$1.9151	-49.215	34,445	$-0.6065**	$0.9926	-112.777	34,069
		10	$-0.2156**	$1.7150	-20.670	27,036	$-0.2661**	$1.0190	-42.753	26,810
		5	$-0.1062**	$1.4837	-13.376	34,925	$-0.1294**	$1.2153	-19.817	34,659
		1	$-0.0233**	$0.3452	-12.652	35,258	$-0.0260**	$0.3193	-15.243	35,026
2	28-57	Expiration	$-0.6337**	$1.4692	-74.671	29,969	$-0.7201**	$0.8441	-146.740	29,586
		10	$-0.1389**	$0.9751	-24.610	29,867	$-0.1798**	$0.5101	-60.614	29,578
		5	$-0.0566**	$0.6939	-14.123	29,944	$-0.0767**	$0.4308	-30.663	29,657
		1	$-0.0093**	$0.2647	-6.075	30,012	$-0.0131**	$0.2096	-10.734	29,735
3	47-78	Expiration	$-0.6179**	$1.3825	-60.043	18,051	$-0.7332**	$0.8313	-117.644	17,795
		10	$-0.0741**	$0.4472	-22.246	18,036	$-0.0796**	$0.4199	-25.350	17,891
		5	$-0.0286**	$0.3421	-11.237	18,030	$-0.0295**	$0.3403	-11.621	17,943
		1	$-0.0083**	$0.1465	-7.601	18,043	$-0.0081**	$0.1465	-7.431	18,000
4	67-99	Expiration	$-0.6567**	$1.3964	-57.639	15,021	$-0.8000**	$0.6539	-148.634	14,761
		10	$-0.1055**	$0.3455	-37.394	15,000	$-0.1054**	$0.3456	-37.303	14,973
		5	$-0.0576**	$0.2469	-28.593	15,002	$-0.0575**	$0.2470	-28.475	14,975
		1	$-0.0115**	$0.1195	-11.807	15,013	$-0.0114**	$0.1195	-11.706	14,986

The data sets contain daily observations of the options from January 28, 1983 through April 30, 1998. Group 1 contains the nearest-to-maturity options on a given day and Group 2 contains the second nearest-to-maturity options, etc. The investment (initial cash outflow) is standardized to $1 for a call or a put. The profits (losses) do not account for transaction costs. The holding periods are either a number of trading days or until the option's expiration.

* significantly different from zero at the 0.05 level.
** significantly different from zero at the 0.01 level.

Table 4.2
Profits (Losses) to a Delta Neutral Strategy of Buying Low Implied Volatility Options and Selling High Implied Volatility Options with No Transaction Costs

Panel A: for Calls with a Net Investment of -$1

Group	Trading Days to Expiration	Holding Period	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$-0.0985**	$1.3483	-3.812	2,723	$-0.0981**	$1.3495	-3.791	2,718
		10	$-0.0031	$1.0840	-0.123	1,801	$-0.0026	$1.0853	-0.102	1,796
		5	$0.0009	$0.3282	0.144	2,571	$0.0013	$0.3284	0.196	2,566
		1	$0.0066**	$0.0857	4.011	2,705	$0.0067**	$0.0856	4.044	2,700
2	28-57	Expiration	$-0.2608**	$1.0656	-11.347	2,150	$-0.2586**	$1.0646	-11.228	2,136
		10	$0.0169	$0.5265	1.457	2,059	$0.0199	$0.5254	1.710	2,045
		5	$0.0194**	$0.2401	3.708	2,109	$0.0210**	$0.2389	4.018	2,095
		1	$0.0048**	$0.0734	3.032	2,143	$0.0051**	$0.0712	3.312	2,129
3	47-78	Expiration	$-0.2893**	$0.8784	-10.948	1,105	$-0.2893**	$0.8784	-10.948	1,105
		10	$-0.0032	$0.2812	-0.379	1,091	$-0.0032	$0.2812	-0.379	1,091
		5	$0.0060	$0.1500	1.329	1,098	$0.0060	$0.1500	1.329	1,098
		1	$0.0017	$0.0502	1.154	1,103	$0.0017	$0.0502	1.154	1,103
4	67-99	Expiration	$-0.2708**	$1.2982	-6.516	976	$-0.2502**	$1.2864	-6.049	967
		10	$-0.0244**	$0.1514	-5.036	974	$-0.0230**	$0.1508	-4.731	965
		5	$-0.0077*	$0.1015	-2.370	976	$-0.0064*	$0.1002	-1.990	967
		1	$-0.0002	$0.0456	-0.135	976	$-0.0001	$0.0447	-0.081	967

The data sets contain daily observations of options from January 28, 1983 through April 30, 1998. The delta neutral positions are formed by buying the call (put) with the lowest implied volatility and simultaneously selling the call (put) with the highest implied volatility among the 16 calls (16 puts) with different strike prices but the same expiration date on a given day. The number of contracts sold is adjusted to make the position delta neutral. A position is held only when the difference between the high and the low implied volatilities is greater than 3% and less than 50%. The net investment for positions involving the two calls is standardized to negative $1, i.e., an initial cash *inflow* of $1. However the net investment for positions involving the two puts is standardized to positive $1, i.e., an initial cash *outflow* of $1. The profits (losses) do not account for transaction costs.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.

Table 4.2 (continued)

Panel B: for Puts with a Net Investment of $1

Group	Trading Days to Expiration	Holding Period	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$0.7669**	$4.1461	9.652	2,723	$0.8847**	$2.6707	17.229	2,705
		10	$0.5624**	$1.0916	22.492	1,906	$0.5582**	$1.0874	22.364	1,898
		5	$0.2398**	$1.9096	6.384	2,584	$0.2378**	$1.9139	6.295	2,568
		1	$0.0355**	$0.5176	3.568	2,711	$0.0343**	$0.5172	3.443	2,695
2	28-57	Expiration	$0.8059**	$3.1514	11.951	2,184	$0.7838**	$3.1336	11.657	2,172
		10	$0.3343**	$0.7042	22.117	2,171	$0.3260**	$0.6883	22.005	2,159
		5	$0.1252**	$0.6275	9.299	2,172	$0.1212**	$0.6220	9.059	2,160
		1	$0.0224**	$0.2676	3.903	2,181	$0.0219**	$0.2595	3.926	2,169
3	47-78	Expiration	$0.4855**	$2.7121	6.036	1,137	$0.4855**	$2.7121	6.036	1,137
		10	$0.1665**	$0.4557	12.321	1,137	$0.1665**	$0.4557	12.321	1,137
		5	$0.0598**	$0.4129	4.879	1,134	$0.0598**	$0.4129	4.879	1,134
		1	$0.0135**	$0.1375	3.321	1,136	$0.0135**	$0.1375	3.321	1,136
4	67-99	Expiration	$0.5920**	$2.0468	8.729	911	$0.5941**	$2.0483	8.745	909
		10	$0.1510**	$0.4123	11.007	903	$0.1497**	$0.4118	10.911	901
		5	$0.0692**	$0.3026	6.885	907	$0.0682**	$0.3020	6.797	905
		1	$0.0184**	$0.1800	3.082	910	$0.0182**	$0.1798	3.045	908

The data sets contain daily observations of options from January 28, 1983 through April 30, 1998. The delta neutral positions are formed by buying the call (put) with the lowest implied volatility and simultaneously selling the call (put) with the highest implied volatility among the 16 calls (16 puts) with different strike prices but the same expiration date on a given day. The number of contracts sold is adjusted to make the position delta neutral. A position is held only when the difference between the high and the low implied volatilities is greater than 3% and less than 50%. The net investment for positions involving the two calls is standardized to negative $1, i.e., an initial cash *inflow* of $1. However the net investment for positions involving the two puts is standardized to positive $1, i.e., an initial cash *outflow* of $1. The profits (losses) do not account for transaction costs.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.

Table 4.3
Abnormal Profits (Losses) to a Delta Neutral Strategy of Buying Low Implied Volatility Options and Selling High Implied Volatility Options with No Transaction Costs

Panel A: for Calls with a Net Investment of -$1

Group	Trading Days to Expiration	Holding Period	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$-0.0187	$2.6867	-0.363	2,723	$-0.0168	$2.6888	-0.325	2,718
		10	$0.2422**	$2.8096	3.658	1,801	$0.2448**	$2.8131	3.689	1,796
		5	$0.1068**	$1.1130	4.867	2,571	$0.1078**	$1.1139	4.904	2,566
		1	$0.0196**	$0.3157	3.230	2,705	$0.0197**	$0.3159	3.243	2,700
2	28-57	Expiration	$-0.1142**	$2.0426	-2.593	2,150	$-0.1062*	$2.0455	-2.398	2,136
		10	$0.1428**	$1.2640	5.126	2,059	$0.1487**	$1.2661	5.312	2,045
		5	$0.0803**	$0.6645	5.552	2,109	$0.0832**	$0.6656	5.724	2,095
		1	$0.0152**	$0.2196	3.202	2,143	$0.0154**	$0.2191	3.252	2,129
3	47-78	Expiration	$-0.0814	$1.8102	-1.494	1,105	$-0.0814	$1.8102	-1.494	1,105
		10	$0.0230	$0.7465	1.020	1,091	$0.0230	$0.7465	1.020	1,091
		5	$0.0210	$0.4485	1.554	1,098	$0.0210	$0.4485	1.554	1,098
		1	$0.0069	$0.1564	1.473	1,103	$0.0069	$0.1564	1.473	1,103
4	67-99	Expiration	$0.1386	$2.7424	1.579	976	$0.1664	$2.7399	1.888	967
		10	$0.0266	$0.4558	1.820	974	$0.0327*	$0.4533	2.242	965
		5	$0.0297**	$0.3208	2.894	976	$0.0339**	$0.3192	3.300	967
		1	$0.0069	$0.1249	1.728	976	$0.0077	$0.1245	1.935	967

The data sets contain daily observations of the options from January 28, 1983 through April 30, 1998. Group 1 contains the nearest-to-maturity options on a given day and Group 2 contains the second nearest-to-maturity options, etc. The abnormal profits are defined as the difference between the profits from investing -$1 ($1) in a delta neutral call (put) position as reported in Table 4.2 and the average profits from investing -$1 ($1) in each of the available calls (puts) with the same expiration date on the same day. The profits (losses) do not account for transaction costs.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.

Table 4.3 (continued)
Abnormal Profits (Losses) to a Delta Neutral Strategy of Buying Low Implied Volatility Options and Selling High Implied Volatility Options with No Transaction Costs

Panel B: for Puts with a Net Investment of $1

Group	Trading Days to Expiration	Holding Period	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$1.3502**	$4.8306	14.585	2,723	$1.5136**	$2.5853	30.450	2,705
		10	$0.8129**	$1.2659	28.034	1,906	$0.8089**	$1.2643	27.873	1,898
		5	$0.3479**	$2.8301	6.249	2,584	$0.3492**	$2.8345	6.244	2,568
		1	$0.0620**	$0.7032	4.589	2,711	$0.0605**	$0.7042	4.461	2,695
2	28-57	Expiration	$1.5428**	$3.1301	23.035	2,184	$1.5214**	$3.1140	22.770	2,172
		10	$0.5222**	$0.8288	29.359	2,171	$0.5137**	$0.8167	29.225	2,159
		5	$0.2039**	$0.8929	10.644	2,172	$0.1998**	$0.8906	10.427	2,160
		1	$0.0370**	$0.4048	4.272	2,181	$0.0365**	$0.3980	4.273	2,169
3	47-78	Expiration	$1.1854**	$2.9079	13.746	1,137	$1.1854**	$2.9079	13.746	1,137
		10	$0.2436**	$0.6573	12.495	1,137	$0.2436**	$0.6573	12.495	1,137
		5	$0.0924**	$0.6419	4.847	1,134	$0.0924**	$0.6419	4.847	1,134
		1	$0.0223**	$0.2247	3.348	1,136	$0.0223**	$0.2247	3.348	1,136
4	67-99	Expiration	$1.3789**	$1.9372	21.484	911	$1.3807**	$1.9388	21.471	909
		10	$0.2601**	$0.5144	15.195	903	$0.2594**	$0.5147	15.126	901
		5	$0.1252**	$0.3989	9.455	907	$0.1244**	$0.3987	9.382	905
		1	$0.0299**	$0.2259	3.989	910	$0.0297**	$0.2256	3.967	908

The data sets contain daily observations of the options from January 28, 1983 through April 30, 1998. Group 1 contains the nearest-to-maturity options on a given day and Group 2 contains the second nearest-to-maturity options, etc. The abnormal profits are defined as the difference between the profits from investing -$1 ($1) in a delta neutral call (put) position as reported in Table 4.2 and the average profits from investing -$1 ($1) in each of the available calls (puts) with the same expiration date on the same day. The profits (losses) do not account for transaction costs.

Table 4.4
Profits (Losses) to a Delta Neutral Strategy Based on Implied Volatility Differences with Transaction Costs for the Sample Including the 1987 Crash

Group	Trading Days to Expiration	Holding Period (trading days)	Call Positions with a Net Investment of -$1				Put Positions with a Net Investment of $1			
			Mean	Std Dev	T: Mean=0	Obs	Mean	Std Dev	T: Mean=0	Obs
1	10-36	Expiration	$-0.1125**	$1.3830	-4.241	2,716	$0.6873**	$3.8811	9.243	2,724
		10	$-0.0137	$1.1342	-0.513	1,798	$0.5110**	$1.0281	21.705	1,907
		5	$-0.0124	$0.3363	-1.862	2,564	$0.1981**	$1.8217	5.530	2,585
		1	$-0.0075**	$0.0866	-4.504	2,698	$0.0035	$0.4419	0.413	2,712
2	28-57	Expiration	$-0.2792**	$1.0792	-11.989	2,148	$0.7296**	$2.8647	11.903	2,184
		10	$0.0037	$0.5373	0.315	2,057	$0.2908**	$0.6563	20.644	2,171
		5	$0.0061	$0.2441	1.145	2,108	$0.0916**	$0.5881	7.260	2,172
		1	$-0.0094**	$0.0759	-5.755	2,141	$-0.0072	$0.2518	-1.337	2,181
3	47-78	Expiration	$-0.3003**	$0.8851	-11.279	1,105	$0.4452**	$2.5494	5.891	1,138
		10	$-0.0119	$0.2842	-1.383	1,091	$0.1490**	$0.4932	10.189	1,138
		5	$-0.0025	$0.1514	-0.545	1,098	$0.0447**	$0.4464	3.370	1,135
		1	$-0.0069**	$0.0508	-4.478	1,103	$0.0010	$0.2817	0.124	1,137
4	67-99	Expiration	$-0.2777**	$1.3065	-6.641	976	$0.5670**	$1.9881	8.608	911
		10	$-0.0301**	$0.1524	-6.162	974	$0.1333**	$0.3858	10.382	903
		5	$-0.0133**	$0.1021	-4.056	976	$0.0533**	$0.2840	5.656	907
		1	$-0.0057**	$0.0460	-3.875	976	$0.0036	$0.1609	0.670	910

The data set contains daily observations of the options from January 28, 1983 through April 30, 1998. The delta neutral positions are formed by buying the call (put) with the lowest implied volatility and simultaneously selling the call (put) with the highest implied volatility among the 16 calls (16 puts) with different strike prices but the same time to expiration on a given day. The number of contracts sold is adjusted to make the position delta neutral. A delta neutral position is held only when the difference between the high and the low implied volatilities is greater than 3% and less than 50%. The net investment for the delta neutral positions involving the two calls is standardized to negative $1 (a $1cash *inflow*). However, the net investment for the delta neutral positions involving the two puts is standardized to positive $1 (a $1 cash *outflow*). Transaction costs are assumed to be $10 per contract as suggested by Whaley (1986) and the total costs for each position are restricted to between $50 and $100.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.

Table 5.1
Summary Statistics of Implied Volatility by Strike Price

Volatility		Symbol	Including the 1987 Crash				Excluding the 1987 Crash			
			Mean	Std	Mean K/F -1	Obs	Mean	Std	Mean K/F -1	Obs
calls	in-the-money	ISDCI8	0.2114	0.0574	-0.0843	3,891	0.2086	0.0495	-0.0830	3,795
		ISDCI7	0.2039	0.0577	-0.0751	4,449	0.2007	0.0477	-0.0742	4,343
		ISDCI6	0.1962	0.0549	-0.0659	5,107	0.1934	0.0461	-0.0652	4,993
		ISDCI5	0.1903	0.0552	-0.0567	5,860	0.1873	0.0455	-0.0562	5,739
		ISDCI4	0.1817	0.0521	-0.0460	6,828	0.1790	0.0448	-0.0457	6,686
		ISDCI3	0.1726	0.0508	-0.0345	7,849	0.1700	0.0437	-0.0343	7,686
		ISDCI2	0.1653	0.0511	-0.0213	8,643	0.1625	0.0432	-0.0212	8,455
		ISDCI1	0.1592	0.0508	-0.0073	9,124	0.1562	0.0427	-0.0073	8,918
	out-of-the-money	ISDCO1	0.1543	0.0494	0.0069	9,297	0.1514	0.0421	0.0068	9,081
		ISDCO2	0.1505	0.0490	0.0210	9,251	0.1474	0.0415	0.0208	9,032
		ISDCO3	0.1477	0.0496	0.0342	9,001	0.1442	0.0413	0.0339	8,774
		ISDCO4	0.1455	0.0510	0.0456	8,453	0.1418	0.0414	0.0452	8,232
		ISDCO5	0.1438	0.0512	0.0548	7,511	0.1398	0.0416	0.0541	7,310
		ISDCO6	0.1442	0.0540	0.0636	6,469	0.1393	0.0420	0.0625	6,268
		ISDCO7	0.1465	0.0567	0.0707	5,183	0.1407	0.0430	0.0689	5,002
		ISDCO8	0.1500	0.0604	0.0788	4,216	0.1431	0.0432	0.0763	4,052
puts	out-of-the-money	ISDPO8	0.2129	0.0635	-0.0891	7,566	0.2091	0.0522	-0.0880	7,386
		ISDPO7	0.2066	0.0615	-0.0800	7,977	0.2033	0.0544	-0.0792	7,798
		ISDPO6	0.1993	0.0598	-0.0702	8,363	0.1958	0.0497	-0.0695	8,162
		ISDPO5	0.1906	0.0569	-0.0597	8,741	0.1873	0.0475	-0.0592	8,538
		ISDPO4	0.1825	0.0557	-0.0477	9,021	0.1791	0.0460	-0.0474	8,812
		ISDPO3	0.1737	0.0532	-0.0353	9,222	0.1706	0.0450	-0.0350	9,014
		ISDPO2	0.1663	0.0524	-0.0215	9,288	0.1632	0.0442	-0.0214	9,089
		ISDPO1	0.1599	0.0516	-0.0074	9,179	0.1567	0.0430	-0.0074	8,987
	in-the-money	ISDPI1	0.1548	0.0504	0.0067	8,561	0.1518	0.0422	0.0067	8,388
		ISDPI2	0.1510	0.0501	0.0204	7,591	0.1480	0.0420	0.0203	7,439
		ISDPI3	0.1505	0.0520	0.0325	6,064	0.1472	0.0425	0.0322	5,942
		ISDPI4	0.1523	0.0533	0.0431	4,633	0.1486	0.0432	0.0427	4,528
		ISDPI5	0.1533	0.0558	0.0516	3,465	0.1486	0.0427	0.0507	3,379
		ISDPI6	0.1565	0.0585	0.0623	2,699	0.1504	0.0427	0.0609	2,619
		ISDPI7	0.1641	0.0648	0.0721	2,063	0.1557	0.0457	0.0697	1,978
		ISDPI8	0.1720	0.0717	0.0818	1,661	0.1610	0.0466	0.0780	1,577
Average		ISD4	0.1588	0.0525	———	9,764	0.1550	0.0427	———	9,505
Historical		HIS40	0.1485	0.1107	———	9,988	0.1373	0.0527	———	9,699
Realized		RLZSD	0.1444	0.0961	———	9,988	0.1330	0.0473	———	9,699

The notes are on the next page.

Table 5.1 (continued)
Summary Statistics of Implied Volatility by Strike Price

Notes:

The data set contains daily observations of options on S&P 500 futures with one of the four nearest to expiration dates (10-99 trading days to expiration) from January 28, 1983 through April 30, 1998. On a given day and for a given expiration date, I analyze up to eight in-the-money calls (puts) and up to eight out-of-the money calls (puts).

In the symbol for an implied volatility estimator calculated from a single option, the first three letters (ISD) represent Implied Standard Deviation. The fourth letter (C or P) stands for a Call or a Put. The fifth letter (I or O) refers to In-the-money or Out-of-the-money. The last digit indicates the relative position of an option from the money where 1 indicates that the option is the nearest from-the-money and 2 indicates that the option is the second nearest from-the-money etc. For example, ISDCI2 stands for the implied volatility calculated from the second from-the-money, in-the-money call. See Figures 2.2, 4.1a, and 4.1b for the graphical illustration of the various implied volatility estimators.

ISD4 is the average of the four implied volatilities: ISDCI1, ISDCO1, ISDPI1, and ISDPO1.

RLZSD stands for the realized volatility over the remaining life of an option on S&P 500 futures.

HIS40 represents the historical volatility measured over the last 40 trading days.

The sixth and the tenth columns report the mean of an option's moneyness which defined as (K/F -1),
where K -- strike price.
F -- underlying futures price.

Table 5.2
Forecasting Efficiency of Different Implied Volatility Estimators
Based on RMSE, MAE and MAPE

Symbol	RMSE	RMSE's Rank	MAE	MAE's Rank	MAPE	MAPE's Rank	Obs
Panel A: Including the 1987 Crash							
ISDCI8	0.1194	33	0.0875	33	0.7183	33	3,891
ISDCI7	0.1129	32	0.0804	31	0.6527	31	4,449
ISDCI6	0.1039	29	0.0731	29	0.5971	29	5,107
ISDCI5	0.0969	27	0.0669	27	0.5342	27	5,860
ISDCI4	0.0915	21	0.0605	25	0.4820	25	6,828
ISDCI3	0.0856	14	0.0531	23	0.4163	23	7,849
ISDCI2	0.0828	9	0.0472	20	0.3601	21	8,643
ISDCI1	0.0819	7	0.0433	18	0.3197	19	9,124
ISDCO1	0.0841	12	0.0409	14	0.2893	14	9,297
ISDCO2	0.0859	16	0.0393	11	0.2681	9	9,251
ISDCO3	0.0861	17	0.0381	5	0.2541	6	9,001
ISDCO4	0.0876	18	0.0379	4	0.2472	5	8,453
ISDCO5	0.0893	20	0.0376	1	0.2448	4	7,511
ISDCO6	0.0925	22	0.0377	3	0.2412	3	6,469
ISDCO7	0.0945	24	0.0383	6	0.2384	2	5,183
ISDCO8	0.0957	25	0.0392	10	0.2362	1	4,216
ISDPO8	0.1127	31	0.0880	34	0.7463	34	7,566
ISDPO7	0.1082	30	0.0823	32	0.6875	32	7,977
ISDPO6	0.1021	28	0.0752	30	0.6205	30	8,363
ISDPO5	0.0965	26	0.0677	28	0.5505	28	8,741
ISDPO4	0.0927	23	0.0608	26	0.4825	26	9,021
ISDPO3	0.0888	19	0.0538	24	0.4178	24	9,222
ISDPO2	0.0838	11	0.0478	21	0.3660	22	9,288
ISDPO1	0.0833	10	0.0438	19	0.3238	20	9,179
ISDPI1	0.0783	3	0.0399	13	0.2927	15	8,561
ISDPI2	0.0791	4	0.0386	8	0.2756	12	7,591
ISDPI3	0.0793	5	0.0383	7	0.2703	11	6,064
ISDPI4	0.0827	8	0.0395	12	0.2677	8	4,633
ISDPI5	0.0776	2	0.0377	2	0.2584	7	3,465
ISDPI6	0.0807	6	0.0388	9	0.2682	10	2,699
ISDPI7	0.0856	15	0.0427	16	0.2883	13	2,063
ISDPI8	0.0757	1	0.0422	15	0.2975	17	1,661
ISD4	0.0854	13	0.0428	17	0.3050	18	9,764
HIS40	0.1197	34	0.0509	22	0.2948	16	9,988

Notes: The data set contains daily observations of options on S&P 500 futures with 10-99 trading days to expiration from January 28, 1983 through April 30, 1998. For the meaning of the symbols, please refer to the notes in Table 5.1. RMSE, MAE and MAPE are calculated by utilizing Equations 5.1, 5.2 and 5.3 respectively.

Table 5.2 (continued) Forecasting Efficiency of Different Implied Volatility Estimators Based on RMSE, MAE and MAPE							
Symbol	RMSE	RMSE's Rank	MAE	MAE's Rank	MAPE	MAPE's Rank	Obs
Panel B: Excluding the 1987 Crash							
ISDCI8	0.0937	33	0.0793	33	0.7196	33	3,795
ISDCI7	0.0855	31	0.0721	31	0.6522	31	4,343
ISDCI6	0.0779	29	0.0659	29	0.5964	29	4,993
ISDCI5	0.0720	27	0.0603	27	0.5322	27	5,739
ISDCI4	0.0655	26	0.0544	25	0.4806	25	6,686
ISDCI3	0.0579	23	0.0472	23	0.4140	23	7,686
ISDCI2	0.0519	21	0.0412	21	0.3573	21	8,455
ISDCI1	0.0477	17	0.0369	18	0.3162	19	8,918
ISDCO1	0.0447	13	0.0338	13	0.2847	14	9,081
ISDCO2	0.0425	9	0.0316	7	0.2626	8	9,032
ISDCO3	0.0409	6	0.0302	6	0.2483	6	8,774
ISDCO4	0.0405	5	0.0297	5	0.2412	5	8,232
ISDCO5	0.0395	3	0.0291	4	0.2388	4	7,310
ISDCO6	0.0379	1	0.0283	1	0.2347	3	6,268
ISDCO7	0.0383	2	0.0284	2	0.2319	2	5,002
ISDCO8	0.0396	4	0.0291	3	0.2295	1	4,052
ISDPO8	0.0944	34	0.0825	34	0.7509	34	7,386
ISDPO7	0.0882	32	0.0767	32	0.6907	32	7,798
ISDPO6	0.0810	30	0.0697	30	0.6230	30	8,162
ISDPO5	0.0728	28	0.0620	28	0.5514	28	8,538
ISDPO4	0.0652	25	0.0547	26	0.4821	26	8,812
ISDPO3	0.0582	24	0.0476	24	0.4161	24	9,014
ISDPO2	0.0528	22	0.0419	22	0.3635	22	9,089
ISDPO1	0.0483	18	0.0374	20	0.3204	20	8,987
ISDPI1	0.0448	14	0.0341	14	0.2891	16	8,388
ISDPI2	0.0432	12	0.0326	11	0.2717	13	7,439
ISDPI3	0.0428	10	0.0323	9	0.2665	11	5,942
ISDPI4	0.0430	11	0.0327	12	0.2633	9	4,528
ISDPI5	0.0413	7	0.0317	8	0.2547	7	3,379
ISDPI6	0.0423	8	0.0324	10	0.2650	10	2,619
ISDPI7	0.0468	16	0.0356	16	0.2866	15	1,978
ISDPI8	0.0499	19	0.0373	19	0.2993	17	1,577
ISD4	0.0464	15	0.0357	17	0.3011	18	9,505
HIS40	0.0502	20	0.0355	15	0.2666	12	9,699

Notes: The data set contains daily observations of options on S&P 500 futures with 10-99 trading days to expiration from January 28, 1983 through April 30, 1998. For the meaning of the symbols, please refer to the notes in Table 5.1. RMSE, MAE and MAPE are calculated by utilizing Equations 5.1, 5.2 and 5.3 respectively.

Table 5.3
Forecasting Efficiency of the Average Implied Volatility and its Four Components when they are all Observed Based on RMSE, MAE and MAPE

Symbol	RMSE	MAE	MAPE	Mean K/F -1	Obs	Note
Panel A: Including the 1987 Crash						
ISDCI1	0.0778	0.0428	0.3265	-0.0075	8,038	In-the-money call
ISDPO1	0.0778	0.0428	0.3265	-0.0075	8,038	Out-of-the-money put
ISD4	0.0767	0.0410	0.3088	-0.0003	8,038	The average of the four
ISDCO1	0.0757	0.0393	0.2923	0.0069	8,038	Out-of-the-money call
ISDPI1	0.0757	0.0394	0.2927	0.0069	8,038	In-the-money put
Panel B: Excluding the 1987 Crash						
ISDCI1	0.0479	0.0372	0.3230	-0.0075	7,900	In-the-money call
ISDPO1	0.0479	0.0372	0.3230	-0.0075	7,900	Out-of-the-money put
ISD4	0.0461	0.0354	0.3052	-0.0003	7,900	The average of the four
ISDCO1	0.0444	0.0338	0.2885	0.0068	7,900	Out-of-the-money call
ISDPI1	0.0445	0.0338	0.2888	0.0068	7,900	In-the-money put

Notes: RMSE, MAE and MAPE are calculated by utilizing Equations 5.1, 5.2 and 5.3 respectively when all the four nearest-from-the-money options with the same expiration date are observed on each day. The data set contains daily observations of options on S&P 500 futures with 10-99 trading days to expiration from January 28, 1983 through April 30, 1998. For the meaning of the symbols, please refer to the notes in Table 5.1.

Table 5.4
Decomposition of Mean Squared Error (MSE)
into Variance and Squared Bias of Forecasting Error

	Estimator	MSE	Rank by MSE	Var of $(\sigma_{RLZ} - \sigma_{ISD})$	Rank by Var	Bias $(\bar{\sigma}_{RLZ} - \bar{\sigma}_{ISD})^2$	Rank by Bias	Obs
Panel A: Including the 1987 Crash								
calls	ISDCI8	0.014250	33	0.009925	33	0.004325	32	3,891
	ISDCI7	0.012748	32	0.009242	32	0.003506	31	4,449
	ISDCI6	0.010791	29	0.008006	28	0.002785	29	5,107
	ISDCI5	0.009385	27	0.007172	20	0.002213	27	5,860
	ISDCI4	0.008378	21	0.006737	12	0.001641	25	6,828
	ISDCI3	0.007324	14	0.006272	7	0.001052	23	7,849
	ISDCI2	0.006853	9	0.006233	6	0.000619	21	8,643
	ISDCI1	0.006709	7	0.006363	9	0.000347	19	9,124
	ISDCO1	0.007068	12	0.006901	17	0.000167	15	9,297
	ISDCO2	0.007376	16	0.007298	22	0.000078	10	9,251
	ISDCO3	0.007419	17	0.007385	24	0.000034	7	9,001
	ISDCO4	0.007677	18	0.007668	26	0.000009	5	8,453
	ISDCO5	0.007971	20	0.007969	27	0.000003	4	7,511
	ISDCO6	0.008562	22	0.008562	29	0.000000	2	6,469
	ISDCO7	0.008933	24	0.008933	30	0.000000	1	5,183
	ISDCO8	0.009158	25	0.009157	31	0.000001	3	4,216
puts	ISDPO8	0.012709	31	0.007415	25	0.005295	34	7,566
	ISDPO7	0.011705	30	0.007330	23	0.004375	33	7,977
	ISDPO6	0.010422	28	0.007005	18	0.003417	30	8,363
	ISDPO5	0.009309	26	0.006815	15	0.002494	28	8,741
	ISDPO4	0.008591	23	0.006874	16	0.001717	26	9,021
	ISDPO3	0.007888	19	0.006807	14	0.001082	24	9,222
	ISDPO2	0.007018	11	0.006337	8	0.000681	22	9,288
	ISDPO1	0.006934	10	0.006564	11	0.000370	20	9,179
	ISDPI1	0.006125	3	0.005900	2	0.000225	16	8,561
	ISDPI2	0.006250	4	0.006128	4	0.000122	13	7,591
	ISDPI3	0.006289	5	0.006192	5	0.000097	11	6,064
	ISDPI4	0.006831	8	0.006761	13	0.000070	9	4,633
	ISDPI5	0.006022	2	0.005955	3	0.000067	8	3,465
	ISDPI6	0.006520	6	0.006422	10	0.000098	12	2,699
	ISDPI7	0.007330	15	0.007208	21	0.000122	14	2,063
	ISDPI8	0.005723	1	0.005447	1	0.000276	18	1,661
Average	ISD4	0.007287	13	0.007033	19	0.000254	17	9,764
Historical	HIS40	0.014319	34	0.014302	34	0.000017	6	9,988

The notes are on the next page.

	Estimator	MSE	Rank	Var	Rank	Bias	Rank	Obs
Table 5.4 (continued)								
Panel B: Excluding the 1987 Crash								
calls	ISDCI8	0.008773	35	0.003320	36	0.005454	33	3,795
	ISDCI7	0.007308	33	0.002867	34	0.004441	31	4,343
	ISDCI6	0.006065	31	0.002543	32	0.003522	29	4,993
	ISDCI5	0.005179	29	0.002421	29	0.002758	27	5,739
	ISDCI4	0.004289	28	0.002142	26	0.002147	25	6,686
	ISDCI3	0.003349	25	0.001901	23	0.001448	23	7,686
	ISDCI2	0.002696	23	0.001760	20	0.000936	21	8,455
	ISDCI1	0.002275	17	0.001672	18	0.000603	19	8,918
	ISDCO1	0.001996	13	0.001606	13	0.000389	16	9,081
	ISDCO2	0.001802	9	0.001546	7	0.000257	13	9,032
	ISDCO3	0.001675	6	0.001499	4	0.000176	8	8,774
	ISDCO4	0.001638	5	0.001525	6	0.000114	6	8,232
	ISDCO5	0.001561	3	0.001469	3	0.000091	5	7,310
	ISDCO6	0.001434	1	0.001351	1	0.000083	4	6,268
	ISDCO7	0.001467	2	0.001389	2	0.000078	3	5,002
	ISDCO8	0.001572	4	0.001519	5	0.000053	2	4,052
puts	ISDPO8	0.008919	36	0.002870	35	0.006048	34	7,386
	ISDPO7	0.007784	34	0.002687	33	0.005098	32	7,798
	ISDPO6	0.006554	32	0.002492	30	0.004062	30	8,162
	ISDPO5	0.005302	30	0.002242	28	0.003060	28	8,538
	ISDPO4	0.004256	27	0.002036	25	0.002219	26	8,812
	ISDPO3	0.003385	26	0.001880	22	0.001505	24	9,014
	ISDPO2	0.002791	24	0.001793	21	0.000997	22	9,089
	ISDPO1	0.002329	19	0.001698	19	0.000631	20	8,987
	ISDPI1	0.002009	14	0.001596	10	0.000413	17	8,388
	ISDPI2	0.001870	12	0.001598	11	0.000272	14	7,439
	ISDPI3	0.001832	10	0.001606	12	0.000226	11	5,942
	ISDPI4	0.001853	11	0.001650	17	0.000203	9	4,528
	ISDPI5	0.001709	7	0.001546	8	0.000163	7	3,379
	ISDPI6	0.001788	8	0.001582	9	0.000206	10	2,619
	ISDPI7	0.002189	16	0.001940	24	0.000249	12	1,978
	ISDPI8	0.002491	21	0.002170	27	0.000321	15	1,577
Average	ISD4	0.002155	15	0.001647	16	0.000508	18	9,505
Historical	HIS40	0.002518	22	0.002499	31	0.000019	1	9,699

Notes: The data set contains daily observations of options on S&P 500 futures with 10-99 trading days to expiration from January 28, 1983 through April 30, 1998. For the meaning of the symbols, please refer to the notes in Table 5.1. MSE is the mean squared error which is the square of RMSE and gives exactly the same ranking as RMSE. As shown in Equation 5.4, MSE can be decomposed into two parts: (1) the variance of the forecasting error and (2) the squared bias of the average forecasting error. Var denotes the variance of the forecasting error and Bias denotes the squared average forecasting error.

Table 5.5 Predictive Power of Different Implied Volatility Estimators Based on a Single Variable Regression ($RLZSD_t = \beta_0 + \beta_1 \sigma_{i,t} + u_{i,t}$)							
	Estimator	**Reg's RMSE**	**Rank by RMSE**	β_0	β_1	**Adj. R^2**	**Obs**
Panel A: Including the 1987 Crash							
calls	ISDCI8	0.0925	32	0.0706**	0.3551**††	0.0461	3,891
	ISDCI7	0.0903	29	0.0578**	0.4261**††	0.0689	4,449
	ISDCI6	0.0851	24	0.0463**	0.4951**††	0.0923	5,107
	ISDCI5	0.0805	17	0.0438**	0.5227**††	0.1138	5,860
	ISDCI4	0.0795	13	0.0302*	0.6108**††	0.1380	6,828
	ISDCI3	0.0775	4	0.0238*	0.6743**††	0.1635	7,849
	ISDCI2	0.0777	5	0.0215*	0.7197**††	0.1832	8,643
	ISDCI1	0.0789	9	0.0183	0.7680**††	0.1962	9,124
	ISDCO1	0.0827	21	0.0135	0.8290**	0.1971	9,297
	ISDCO2	0.0851	25	0.0128	0.8560**	0.1955	9,251
	ISDCO3	0.0858	26	0.0103	0.8907**	0.2097	9,001
	ISDCO4	0.0874	27	0.0107	0.9057**	0.2182	8,453
	ISDCO5	0.0892	28	0.0074	0.9372**	0.2242	7,511
	ISDCO6	0.0925	31	0.0075	0.9441**	0.2327	6,469
	ISDCO7	0.0945	33	0.0090	0.9383**	0.2407	5,183
	ISDCO8	0.0956	34	0.0147	0.9092**	0.2482	4,216
puts	ISDPO8	0.0793	12	0.0398**	0.4713**††	0.1244	7,566
	ISDPO7	0.0799	15	0.0374**	0.4986**††	0.1286	7,977
	ISDPO6	0.0789	10	0.0344**	0.5339**††	0.1407	8,363
	ISDPO5	0.0791	11	0.0295**	0.5833**††	0.1495	8,741
	ISDPO4	0.0805	18	0.0234*	0.6445**††	0.1659	9,021
	ISDPO3	0.0808	19	0.0215*	0.6869**††	0.1696	9,222
	ISDPO2	0.0782	8	0.0210*	0.7169**††	0.1874	9,288
	ISDPO1	0.0801	16	0.0176*	0.7695**††	0.1970	9,179
	ISDPI1	0.0761	2	0.0166	0.7960**††	0.2174	8,561
	ISDPI2	0.0778	6	0.0167	0.8164**†	0.2165	7,591
	ISDPI3	0.0782	7	0.0155	0.8316**	0.2339	6,064
	ISDPI4	0.0817	20	0.0176	0.8300**	0.2265	4,633
	ISDPI5	0.0767	3	0.0161*	0.8418**†	0.2725	3,465
	ISDPI6	0.0797	14	0.0135	0.8507**†	0.2807	2,699
	ISDPI7	0.0840	23	0.0201*	0.8102**††	0.2802	2,063
	ISDPI8	0.0716	1	0.0269**	0.7472**††	0.3589	1,661
Average	ISD4	0.0832	22	0.0165	0.7958**††	0.2014	9,764
Historical	HIS40	0.0905	30	0.1008**	0.2936**††	0.1143	9,988
The notes are on the next page.							

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 5.5 (continued)							
	Estimator	Reg's RMSE	Rank by RMSE	β_0	β_1	Adj. R^2	Obs
Panel B: Excluding the 1987 Crash							
	ISDCI8	0.0471	34	0.0660**	0.3297**††	0.1067	3,795
	ISDCI7	0.0449	33	0.0562**	0.3880**††	0.1452	4,343
	ISDCI6	0.0439	30	0.0446**	0.4626**††	0.1902	4,993
	ISDCI5	0.0434	28	0.0429**	0.4903**††	0.2084	5,739
	ISDCI4	0.0412	24	0.0377**	0.5307**††	0.2493	6,686
	ISDCI3	0.0399	21	0.0306*	0.5963**††	0.2994	7,686
	ISDCI2	0.0387	19	0.0300**	0.6268**††	0.3286	8,455
calls	ISDCI1	0.0381	13	0.0297**	0.6528**††	0.3480	8,918
	ISDCO1	0.0377	9	0.0297**	0.6737**††	0.3619	9,081
	ISDCO2	0.0372	5	0.0293**	0.6921**††	0.3733	9,032
	ISDCO3	0.0368	4	0.0283**	0.7116**††	0.3892	8,774
	ISDCO4	0.0373	6	0.0285**	0.7238**††	0.3917	8,232
	ISDCO5	0.0367	3	0.0273**	0.7366**††	0.4100	7,310
	ISDCO6	0.0352	1	0.0259*	0.7485**††	0.4436	6,268
	ISDCO7	0.0357	2	0.0262*	0.7513**††	0.4501	5,002
	ISDCO8	0.0375	7	0.0280	0.7538**†	0.4300	4,052
	ISDPO8	0.0443	32	0.0431**	0.4218**††	0.1981	7,386
	ISDPO7	0.0436	29	0.0393**	0.4555**††	0.2239	7,798
	ISDPO6	0.0427	27	0.0385**	0.4782**††	0.2367	8,162
	ISDPO5	0.0415	25	0.0343*	0.5214**††	0.2619	8,538
	ISDPO4	0.0404	22	0.0315*	0.5614**††	0.2900	8,812
	ISDPO3	0.0392	20	0.0312**	0.5896**††	0.3138	9,014
	ISDPO2	0.0386	17	0.0325**	0.6074**††	0.3260	9,089
	ISDPO1	0.0383	15	0.0305**	0.6449**††	0.3446	8,987
puts	ISDPI1	0.0375	8	0.0290**	0.6749**††	0.3651	8,388
	ISDPI2	0.0379	10	0.0288**	0.6940**††	0.3722	7,439
	ISDPI3	0.0382	14	0.0269*	0.7152**††	0.3880	5,942
	ISDPI4	0.0384	16	0.0311**	0.6944**††	0.3788	4,528
	ISDPI5	0.0380	12	0.0224	0.7636**††	0.4238	3,379
	ISDPI6	0.0387	18	0.0185	0.7818**†	0.4270	2,619
	ISDPI7	0.0422	26	0.0273	0.7233**†	0.3793	1,978
	ISDPI8	0.0441	31	0.0346*	0.6737**††	0.3359	1,577
Average	ISD4	0.0379	11	0.0298**	0.6624**††	0.3567	9,505
Historical	HIS40	0.0409	23	0.0707**	0.4534**††	0.2550	9,699

Notes: The realized volatility ($RLZSD_t$) over the remaining life of an option on S&P 500 futures is regressed on a volatility estimator ($\sigma_{i,t}$) which is either one of the 32 individual implied volatilities, or the average implied volatility, or the historical volatility (HIS40) measured over the last 40 trading days. For the meaning of the symbols, please refer to the notes in Table 5.1. The significance of the coefficients is based on the t statistics from the Hansen (1982) procedure which corrects for the heteroskedasticity and serial correlation caused by the overlaps in realized volatility.

Table 5.6
Predictive Power of Different Implied Volatility Estimators
Based on a Two Variable Regression
($RLZSD_t = \beta_0 + \beta_1\sigma_{ISD,i,t} + \beta_2 HIS40_t + u_{i,t}$)

	Estimator	Reg's RMSE	Rank by RMSE	β_0	β_1	β_2	Adj. R^2	Obs
Panel A: Including the 1987 Crash								
calls	ISDCI8	0.0912	30	0.0992**	0.0664††	0.2204**	0.0733	3,891
	ISDCI7	0.0895	29	0.0818**	0.1779*††	0.1831**	0.0856	4,449
	ISDCI6	0.0846	24	0.0639**	0.3006**††	0.1428**	0.1021	5,107
	ISDCI5	0.0802	17	0.0566**	0.3732**††	0.1078*	0.1196	5,860
	ISDCI4	0.0794	14	0.0384**	0.5062**††	0.0750	0.1407	6,828
	ISDCI3	0.0775	4	0.0251*	0.6547**†	0.0142	0.1635	7,849
	ISDCI2	0.0776	5	0.0200	0.7428**	-0.0167	0.1833	8,643
	ISDCI1	0.0789	11	0.0140	0.8402**	-0.0502	0.1972	9,124
	ISDCO1	0.0825	21	0.0056	0.9640**	-0.0907	0.2002	9,297
	ISDCO2	0.0849	25	0.0024	1.0368**	-0.1173	0.2008	9,251
	ISDCO3	0.0854	26	-0.0014	1.0929**	-0.1265	0.2162	9,001
	ISDCO4	0.0871	27	-0.0002	1.0993**	-0.1196	0.2243	8,453
	ISDCO5	0.0887	28	-0.0056	1.1693**	-0.1400	0.2325	7,511
	ISDCO6	0.0920	31	-0.0067	1.1913**	-0.1426	0.2417	6,469
	ISDCO7	0.0939	32	-0.0054	1.1786**	-0.1331	0.2495	5,183
	ISDCO8	0.0949	33	-0.0008	1.1599**	-0.1349	0.2581	4,216
puts	ISDPO8	0.0784	9	0.0609**	0.2538**††	0.1755**	0.1453	7,566
	ISDPO7	0.0792	13	0.0556**	0.2974**††	0.1634**	0.1436	7,977
	ISDPO6	0.0785	10	0.0491**	0.3638**††	0.0138**	0.1509	8,363
	ISDPO5	0.0789	12	0.0404**	0.4517**††	0.0988*	0.1545	8,741
	ISDPO4	0.0804	18	0.0300**	0.5606**††	0.0604	0.1676	9,021
	ISDPO3	0.0808	19	0.0250*	0.6384**††	0.0339	0.1701	9,222
	ISDPO2	0.0782	8	0.0225	0.6946**	0.0152	0.1874	9,288
	ISDPO1	0.0801	16	0.0160	0.7952**	-0.0175	0.1971	9,179
	ISDPI1	0.0761	2	0.0130	0.8537**	-0.0375	0.2180	8,561
	ISDPI2	0.0777	6	0.0116	0.9034**	-0.0564	0.2179	7,591
	ISDPI3	0.0781	7	0.0098	0.9291**	-0.0608	0.2358	6,064
	ISDPI4	0.0817	20	0.0122	0.9205**	-0.0550	0.2281	4,633
	ISDPI5	0.0765	3	0.0074	0.9791**	-0.0765	0.2766	3,465
	ISDPI6	0.0795	15	0.0027	1.0039**	-0.0778	0.2849	2,699
	ISDPI7	0.0840	23	0.0140	0.8909**	-0.0394	0.2813	2,063
	ISDPI8	0.0715	1	0.0320**	0.6832**††	0.0300	0.3599	1,661
Average	ISD4	0.0831	22	0.0115	0.8719**	-0.0483	0.2024	9,764

The notes are on the next page.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Table 5.6 (continued)

	Estimator	Reg's RMSE	Rank by RMSE	β_0	β_1	β_2	Adj. R^2	Obs
Panel B: Excluding the 1987 Crash								
calls	ISDCI8	0.0445	33	0.0754**	0.0179††	0.4030**	0.2053	3,795
	ISDCI7	0.0430	31	0.0649**	0.1014††	0.3574**	0.2168	4,343
	ISDCI6	0.0423	28	0.0539**	0.1731††	0.3424**	0.2497	4,993
	ISDCI5	0.0423	27	0.0505**	0.2351*††	0.2937**	0.2503	5,739
	ISDCI4	0.0404	23	0.0445**	0.2968**††	0.2572**	0.2800	6,686
	ISDCI3	0.0395	21	0.0348**	0.4361**††	0.1693*	0.3113	7,686
	ISDCI2	0.0386	19	0.0321**	0.5288**††	0.1019	0.3328	8,455
	ISDCI1	0.0381	13	0.0307**	0.5984**††	0.0549	0.3491	8,918
	ISDCO1	0.0377	9	0.0299**	0.6591**††	0.0143	0.3619	9,081
	ISDCO2	0.0372	5	0.0289**	0.7162**††	-0.0232	0.3734	9,032
	ISDCO3	0.0368	4	0.0275**	0.7651*††	-0.0511	0.3900	8,774
	ISDCO4	0.0373	6	0.0276**	0.7850**†	-0.0578	0.3928	8,232
	ISDCO5	0.0367	3	0.0263**	0.8046**	-0.0627	0.4115	7,310
	ISDCO6	0.0352	1	0.0248*	0.8222**	-0.0666	0.4454	6,268
	ISDCO7	0.0357	2	0.0250*	0.8294**	-0.0697	0.4521	5,002
	ISDCO8	0.0375	7	0.0268	0.8260**	-0.0633	0.4316	4,052
puts	ISDPO8	0.0425	30	0.0506**	0.1734*††	0.3331**	0.2603	7,386
	ISDPO7	0.0423	29	0.0463**	0.2238**††	0.2976**	0.2697	7,798
	ISDPO6	0.0416	25	0.0439**	0.2597**††	0.2760**	0.2744	8,162
	ISDPO5	0.0408	24	0.0395**	0.3259**††	0.2317*	0.2869	8,538
	ISDPO4	0.0400	22	0.0357**	0.4033**††	0.1774*	0.3036	8,812
	ISDPO3	0.0390	20	0.0341**	0.4644**††	0.1362	0.3213	9,014
	ISDPO2	0.0385	17	0.0345**	0.5050**††	0.1087	0.3306	9,089
	ISDPO1	0.0382	15	0.0315**	0.5904**††	0.0557	0.3457	8,987
	ISDPI1	0.0375	8	0.0293**	0.6558**††	0.0190	0.3651	8,388
	ISDPI2	0.0379	11	0.0284**	0.7215**††	-0.0269	0.3724	7,439
	ISDPI3	0.0382	14	0.0261*	0.7731**†	-0.0567	0.3891	5,942
	ISDPI4	0.0384	16	0.0311**	0.7087**††	-0.0143	0.3788	4,528
	ISDPI5	0.0379	10	0.0211	0.8772**	-0.1062	0.4285	3,379
	ISDPI6	0.0386	18	0.0170	0.8751**	-0.0836	0.4299	2,619
	ISDPI7	0.0422	26	0.0273	0.7254**	-0.0020	0.3790	1,978
	ISDPI8	0.0439	32	0.0370*	0.5530**††	0.1086	0.3410	1,577
Average	ISD4	0.0379	12	0.0303**	0.6340**††	0.0283	0.3570	9,505

Notes: The data set contains daily observations for options with 10-99 trading days to expiration from January 28, 1983 through April 30, 1998. The realized volatility ($RLZSD_t$) over the remaining life of an option on S&P 500 futures is regressed on an implied volatility estimator and the historical volatility ($HIS40_t$) measured over the last 40 trading days. For the meaning of the symbols, please refer to the notes in Table 5.1. The significance of the coefficients is based on the t statistics from the Hansen (1982) procedure which corrects for the heteroskedasticity and serial correlation caused by the overlaps in realized volatility.

* significantly different from zero at the 0.05 level. ** significantly different from zero at the 0.01 level.
† significantly different from one at the 0.05 level. †† significantly different from one at the 0.01 level.

Figure 2.1
Options on S&P 500 Futures with Different Expiration Dates
Traded on February 25, 1997

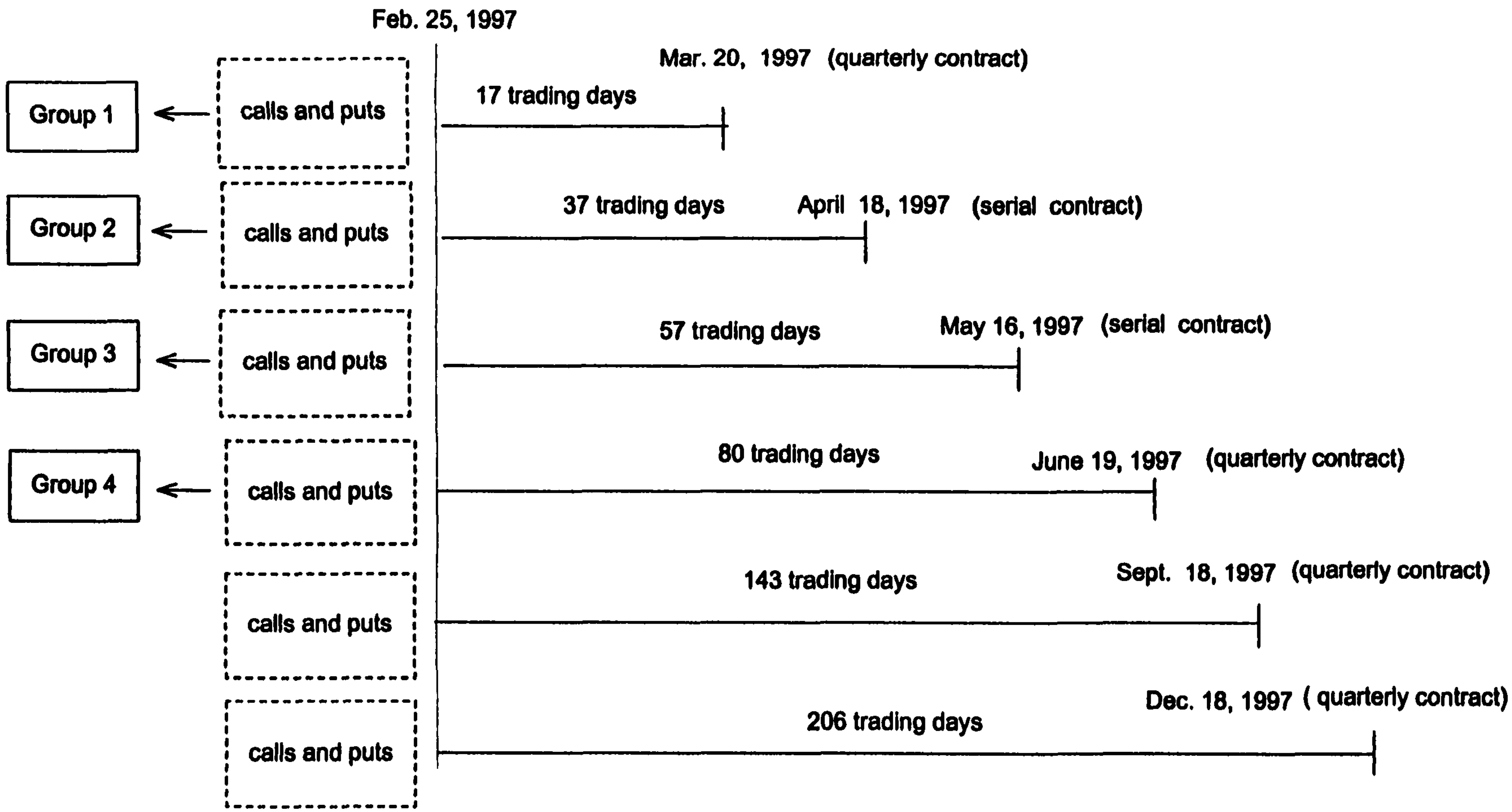
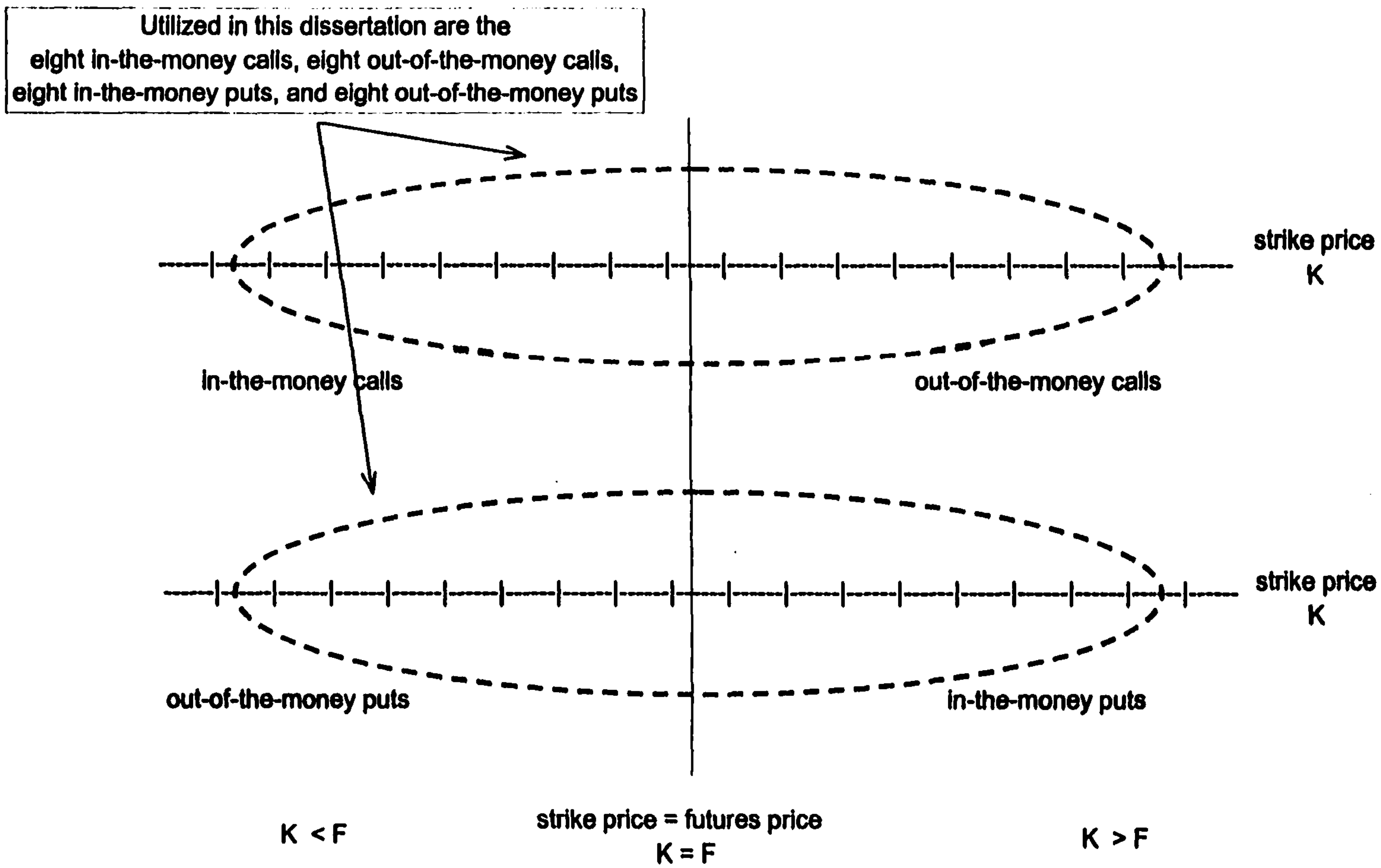


Figure 2.2
Calls and Puts with the Same Expiration Date but Different Strike Prices



Note: The calls and puts on S&P 500 futures shown here have the same expiration date and are observed on the same day.
Each hash mark represents a strike price tick.

Figure 2.3
Average Trading Volume of Options on S&P 500 Futures by Strike Price

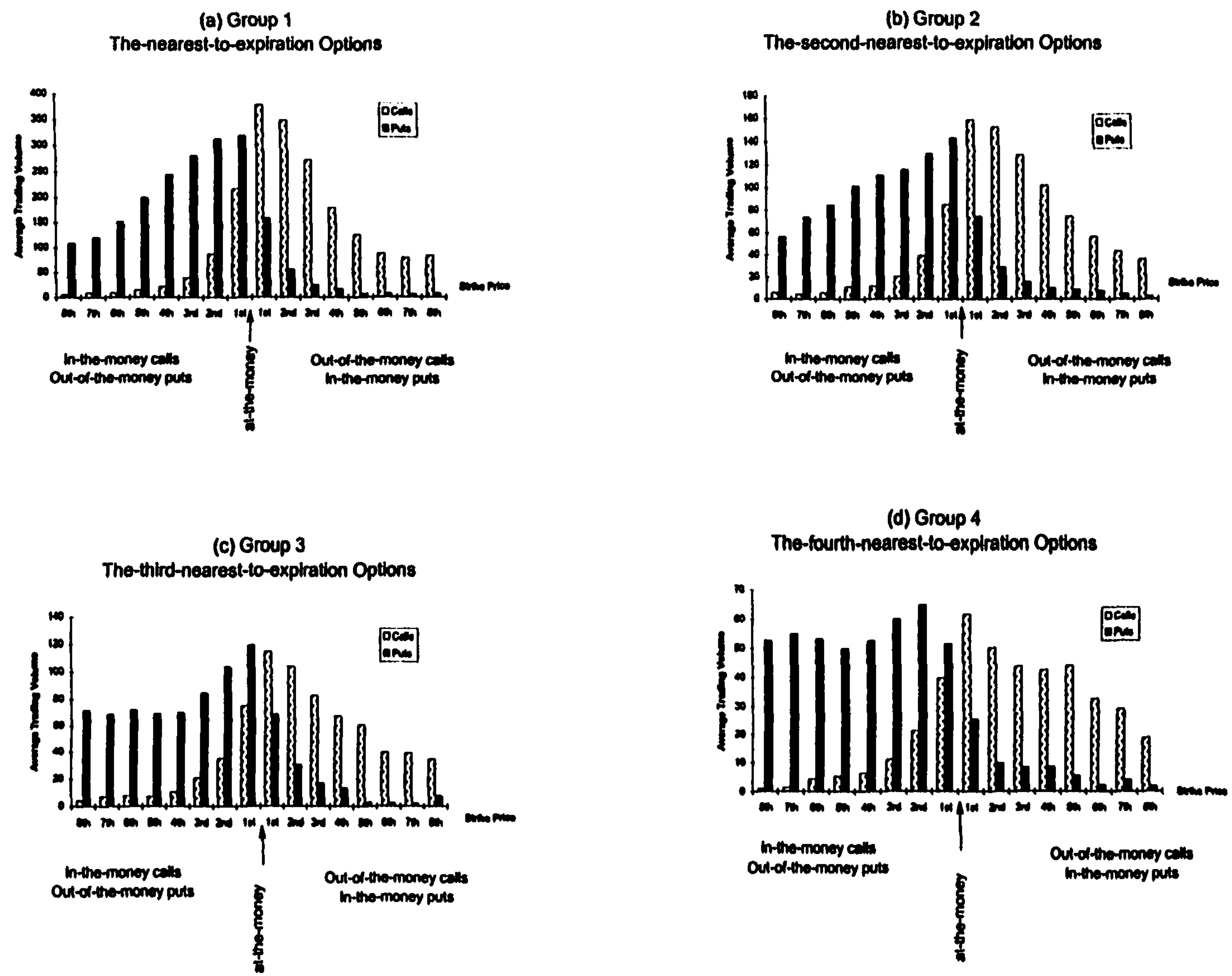


Figure 2.4
Number of Observations of Options on S&P 500 Futures by Strike Price

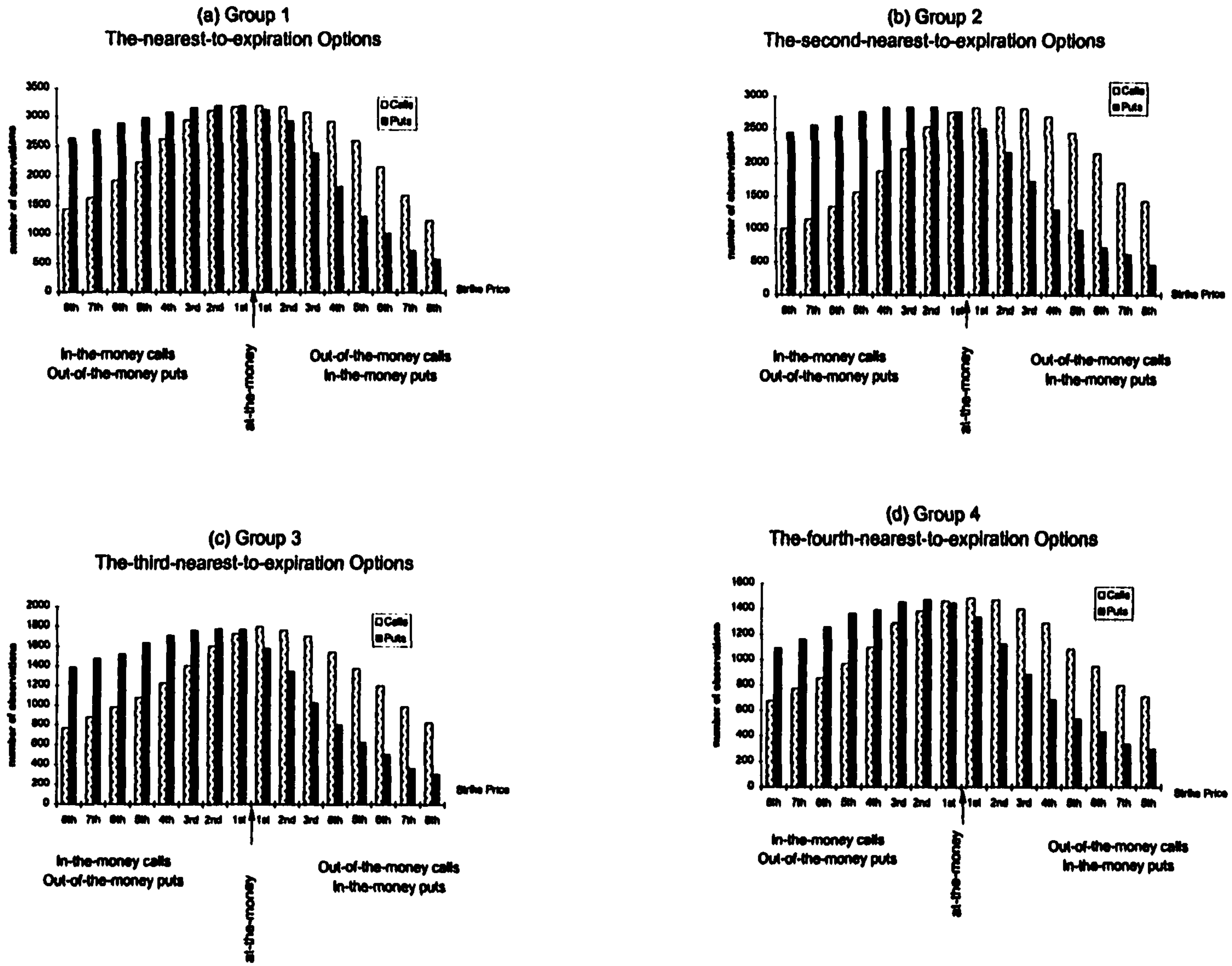


Figure 2.5
Data Periods Utilized to Calculate Realized Volatility, Implied Volatility and Historical Volatility

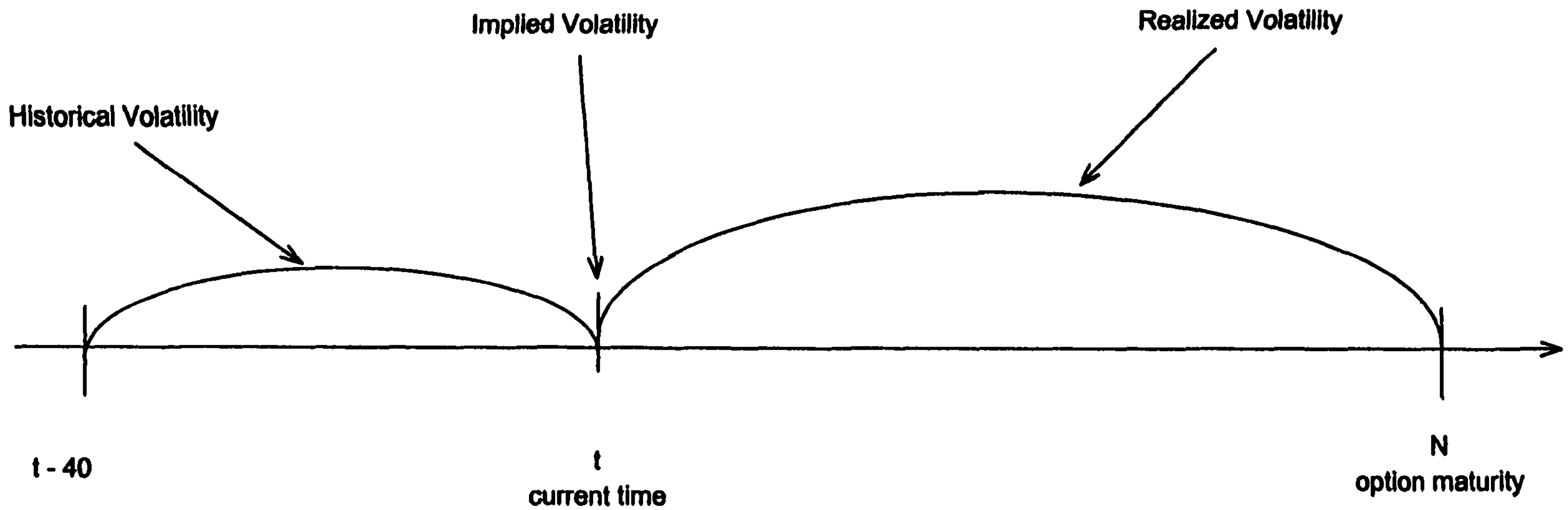
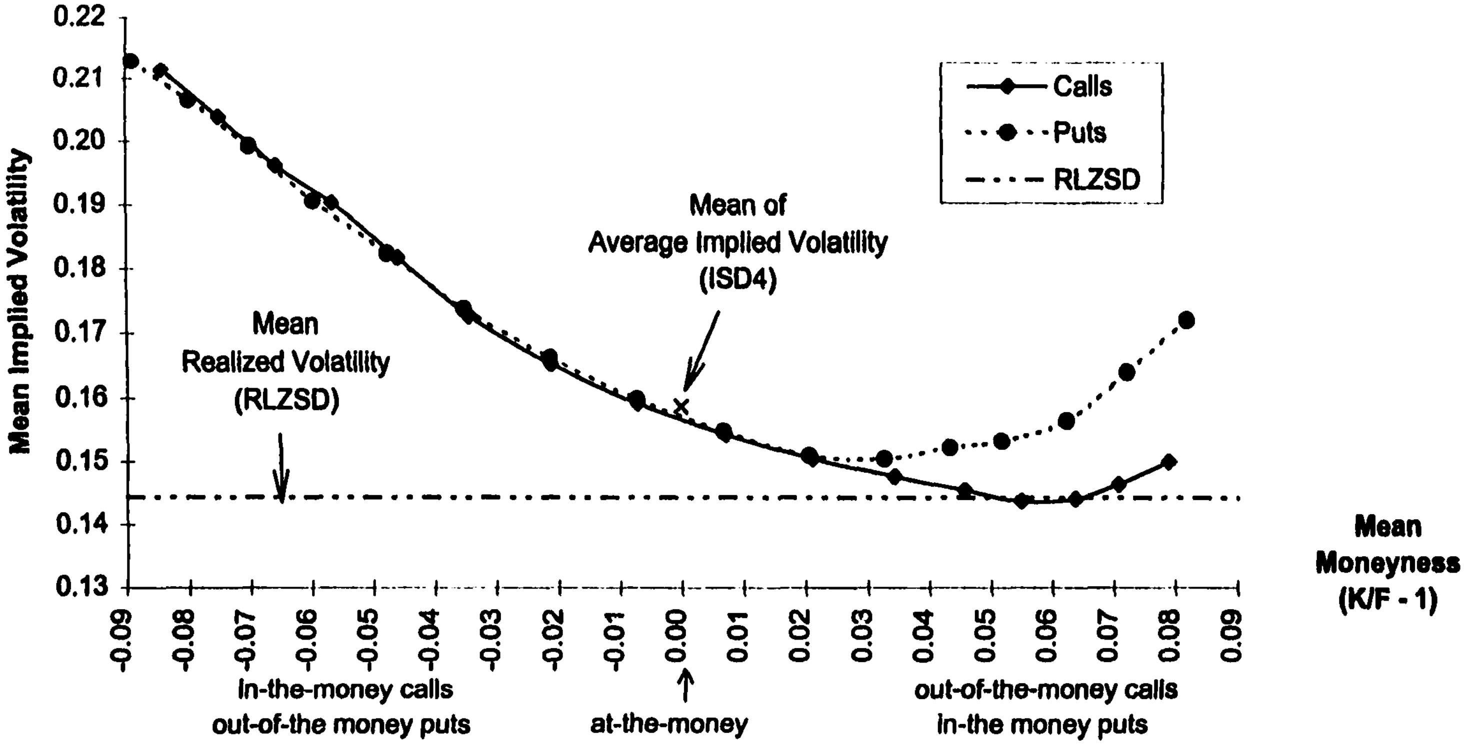
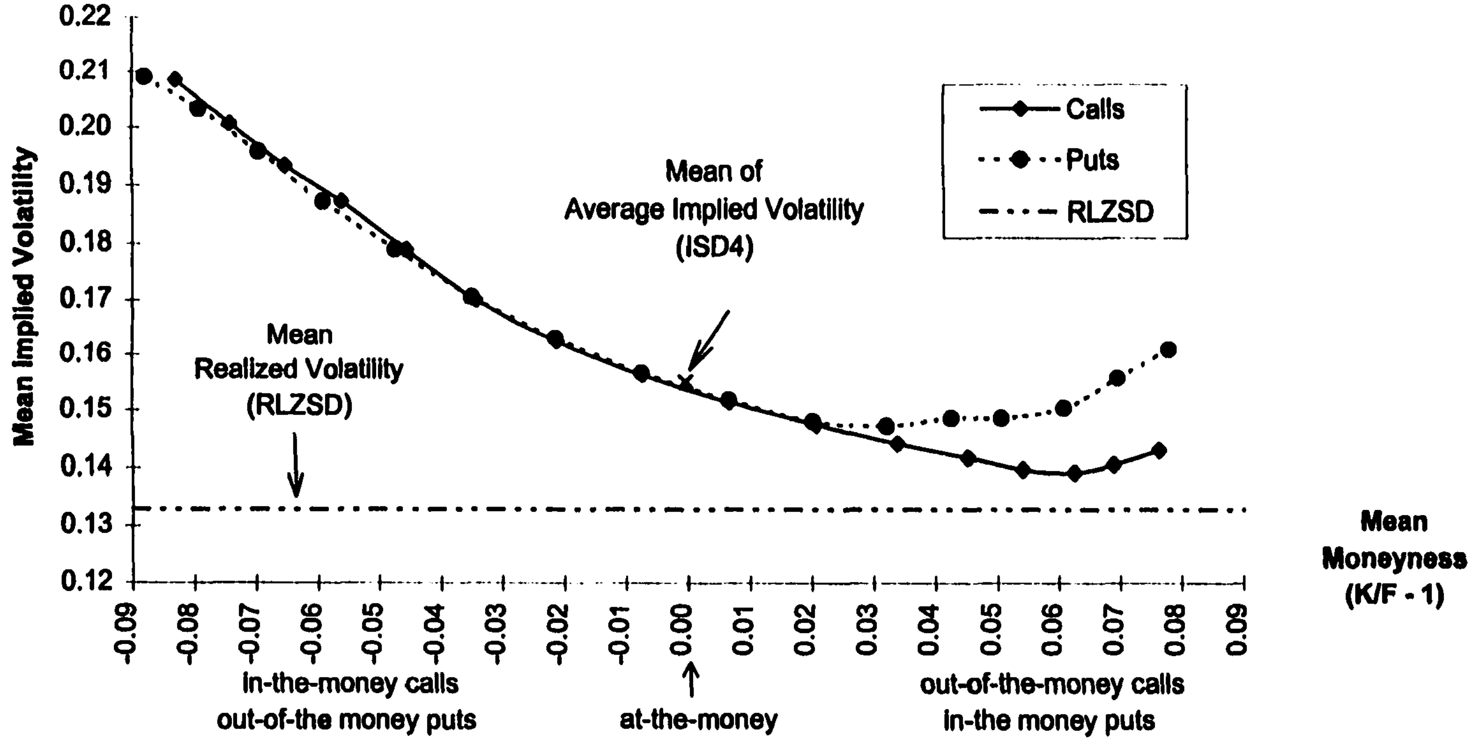


Figure 4.1a
Implied Volatility Smile for Options on S&P 500 Futures
with 10 to 99 Trading Days to Expiration including the 1987 Market Crash



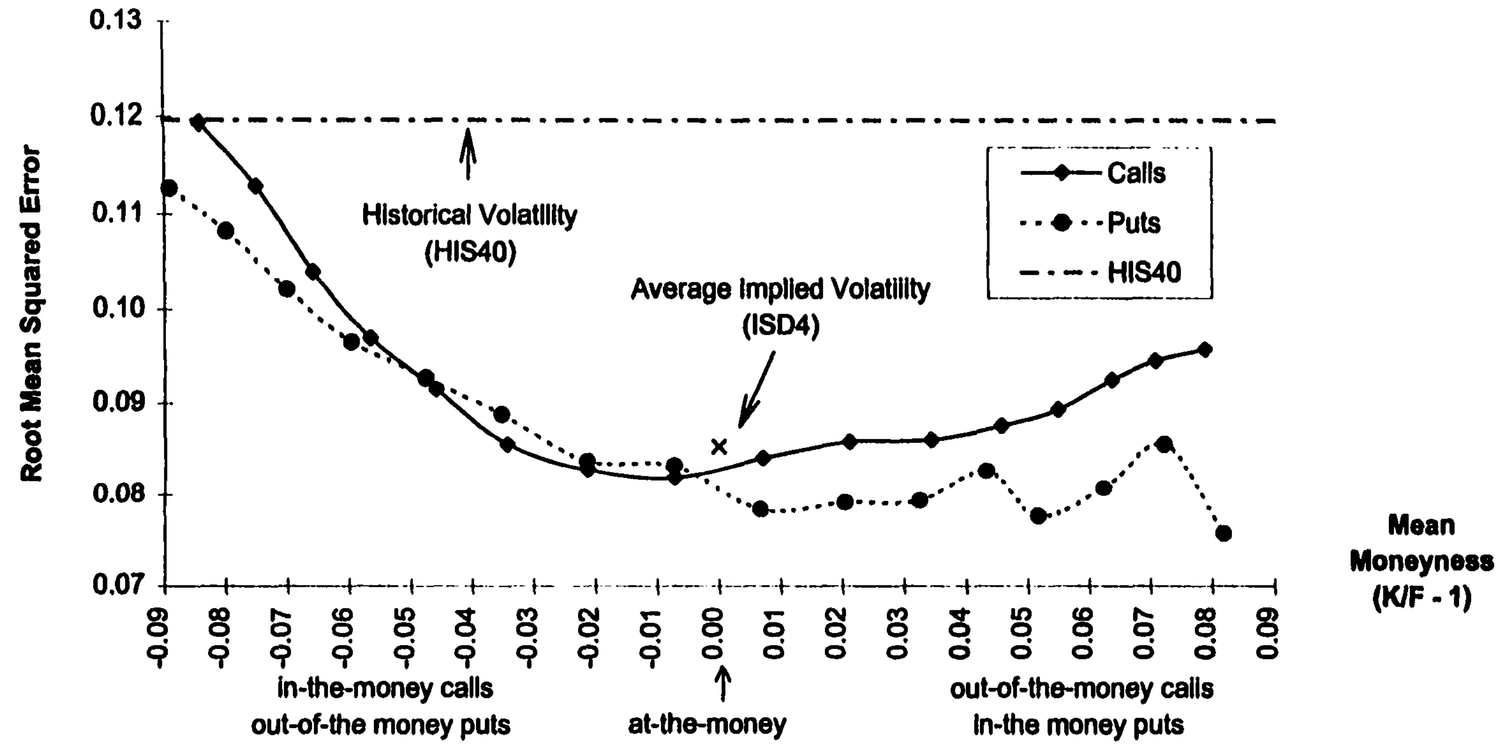
Note: The data set contains daily observations of the four nearest-to-expiration options. The X axis represents the mean moneyness of the options over all the available observations. The moneyness is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the mean implied volatility over all the available observations. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the mean implied volatility for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the mean implied volatility for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the money puts.

Figure 4.1b
Implied Volatility Smile for Options on S&P 500 Futures
with 10 to 99 Trading Days to Expiration Excluding the 1987 Market Crash



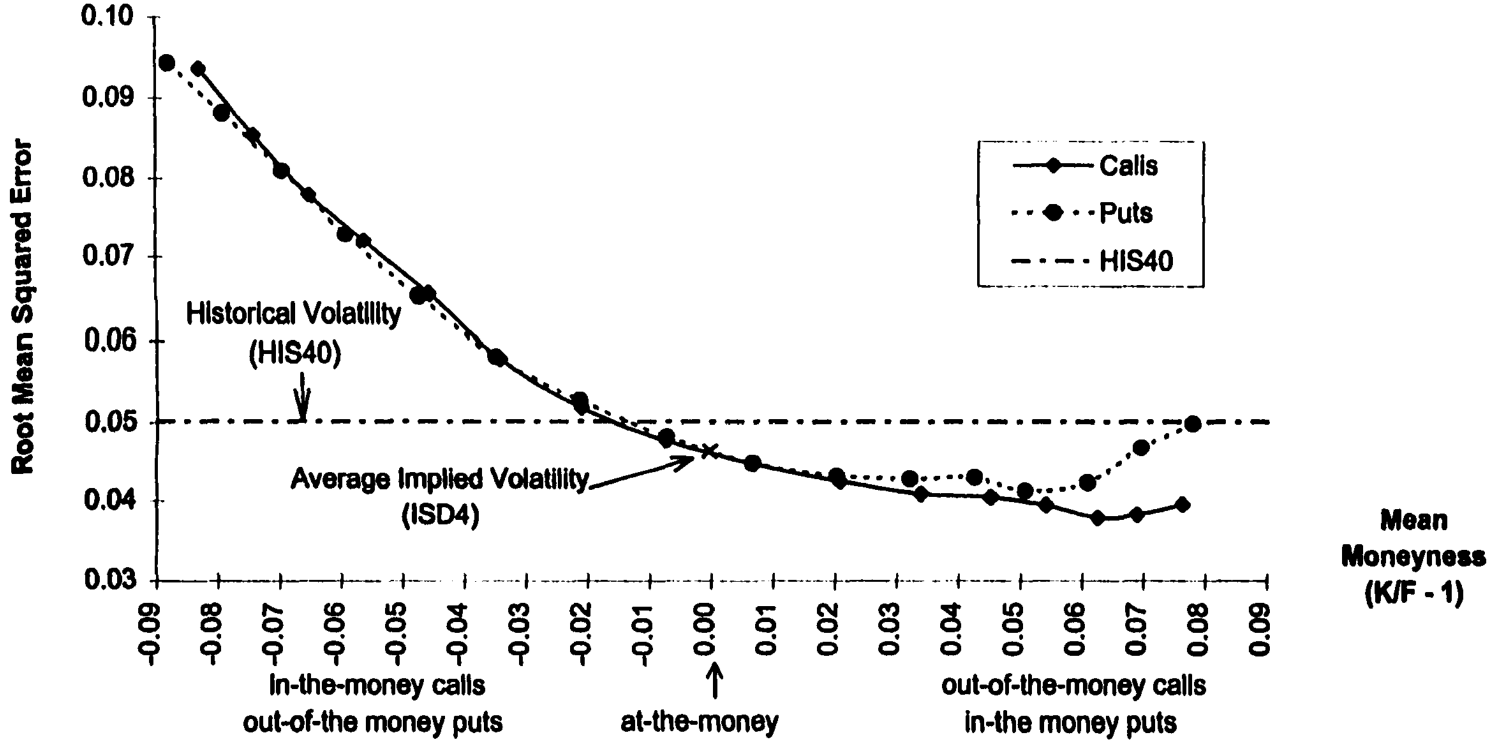
Note: The data set contains daily observations of the four nearest-to-expiration options. The X axis represents the mean moneyness of the options over all the available observations. The moneyness is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the mean implied volatility over all the available observations. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the mean implied volatility for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the mean implied volatility for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the money puts.

Figure 5.1a
Root Mean Squared Errors (RMSEs) of Different Implied Volatility Estimators Including the 1987 Market Crash



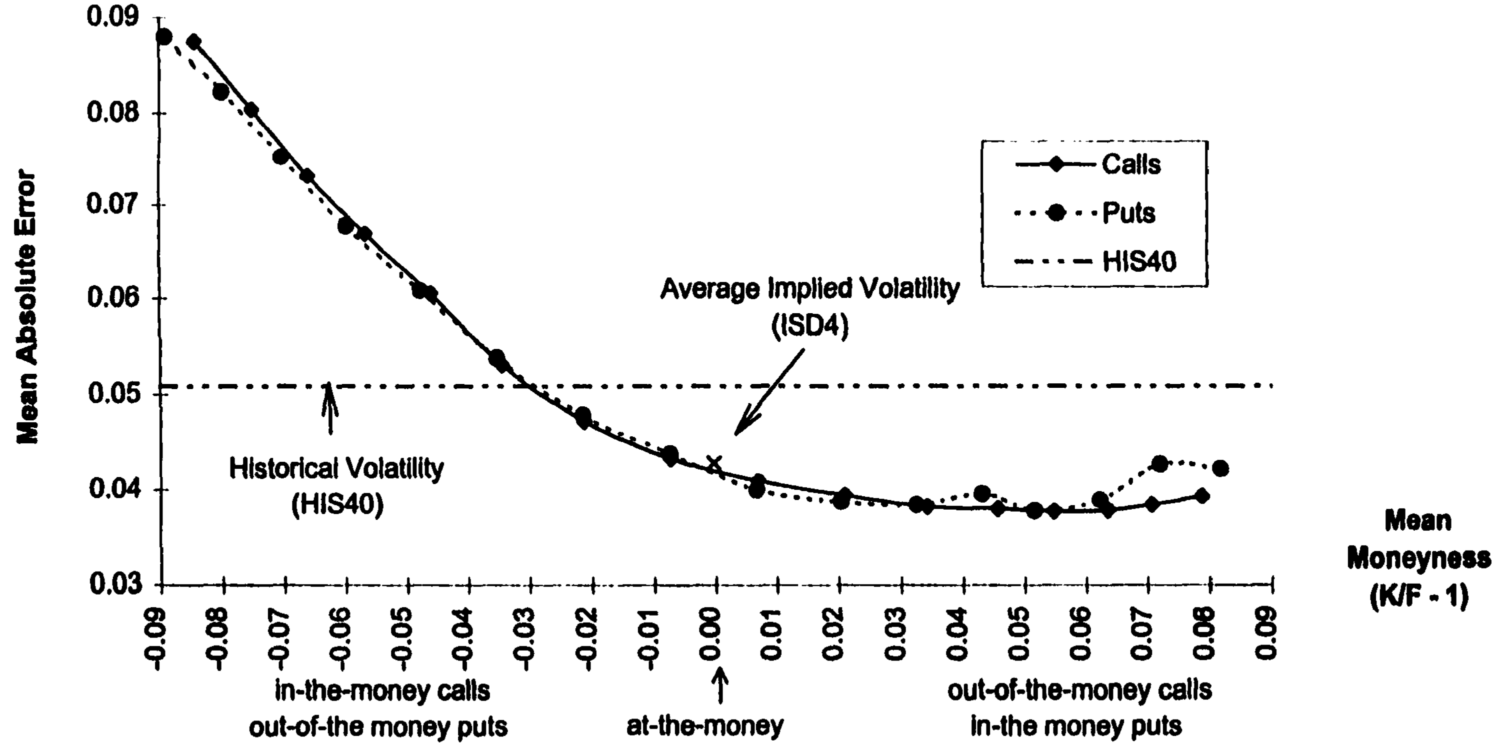
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the root mean squared error (RMSE). The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the RMSE for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the RMSE for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.1b
Root Mean Squared Errors (RMSEs) of Different Implied Volatility Estimators
Excluding the 1987 Market Crash



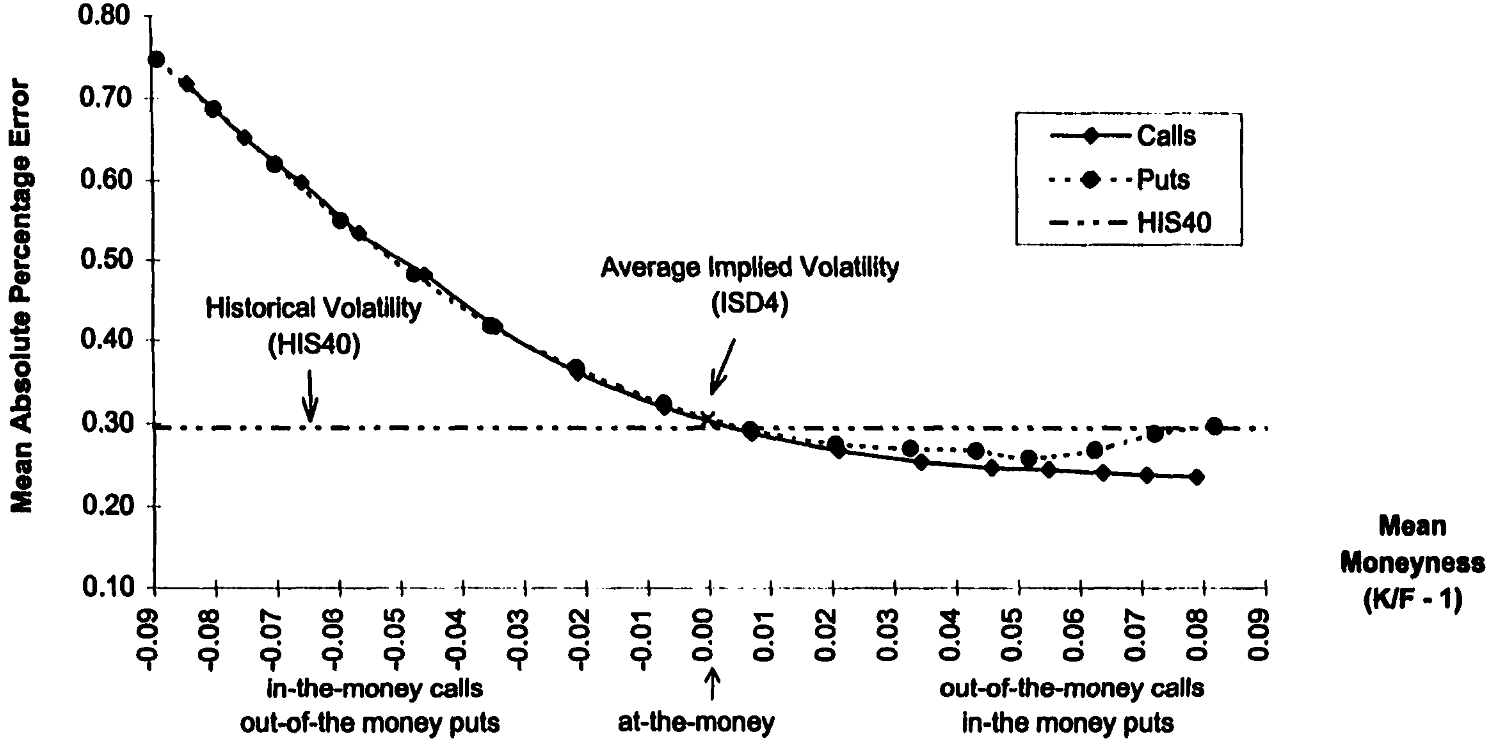
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the root mean squared error (RMSE). The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the RMSE for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the RMSE for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.2
Mean Absolute Errors (MAEs) of Different Implied Volatility Estimators Including the 1987 Market Crash



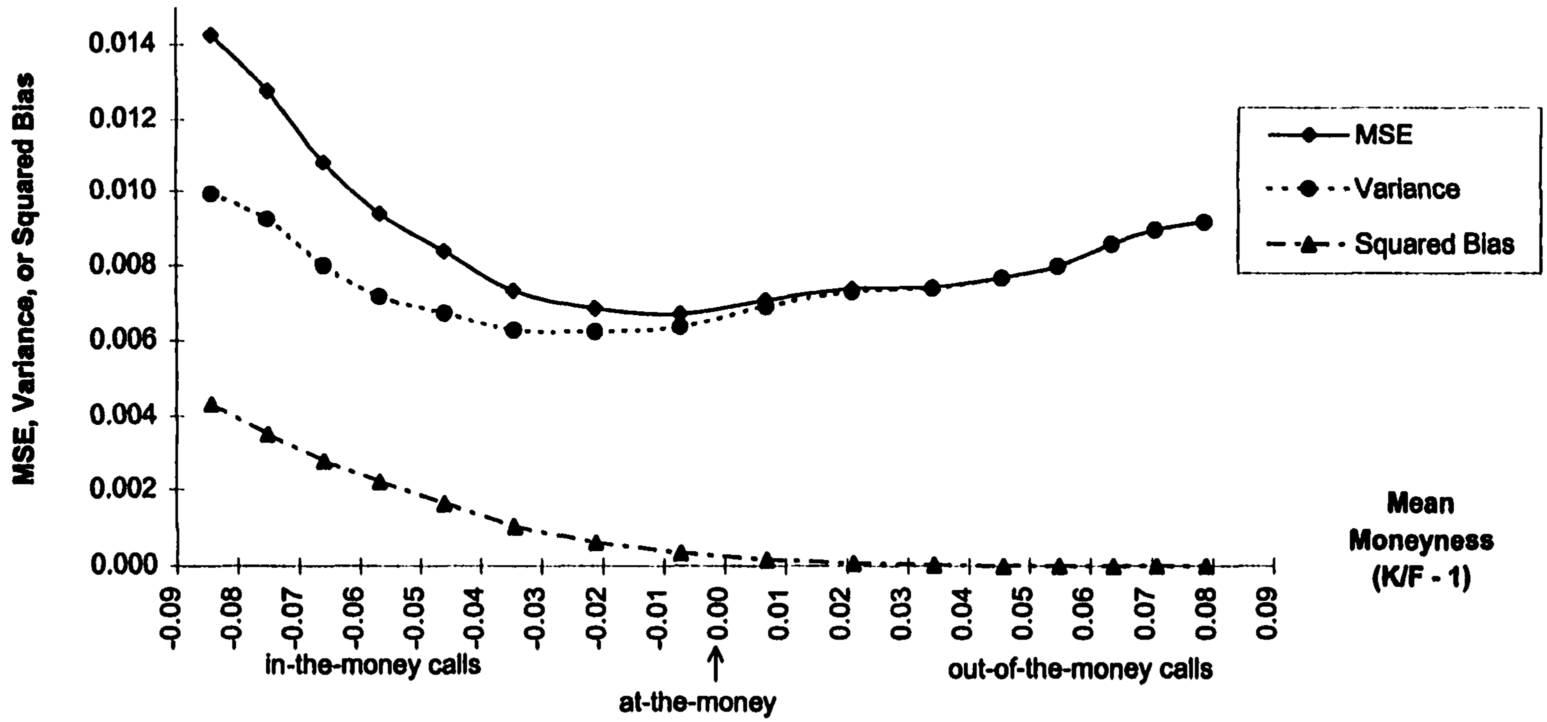
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 Futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the mean absolute error (MAE). The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the MAE for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the MAE for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.3
Mean Absolute Percentage Errors (MAPEs) of Different Implied Volatility Estimators Including the 1987 Market Crash



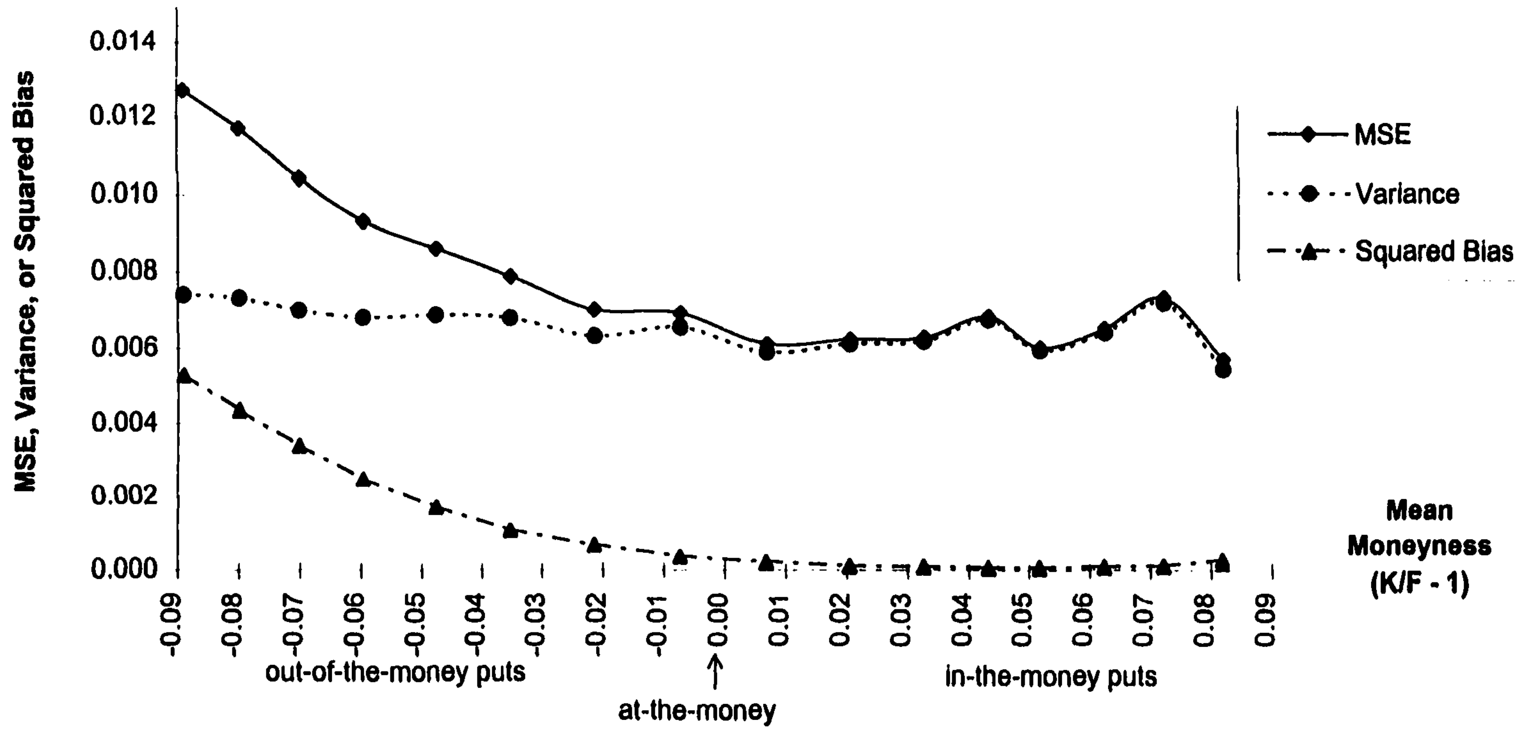
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 Futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the Mean Absolute Percentage Error (MAPE). The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the MAPE for the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the MAPE for the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.4a
Decomposition of Mean Squared Error (MSE) into Variance and Squared Bias for Calls Including the 1987 Market Crash



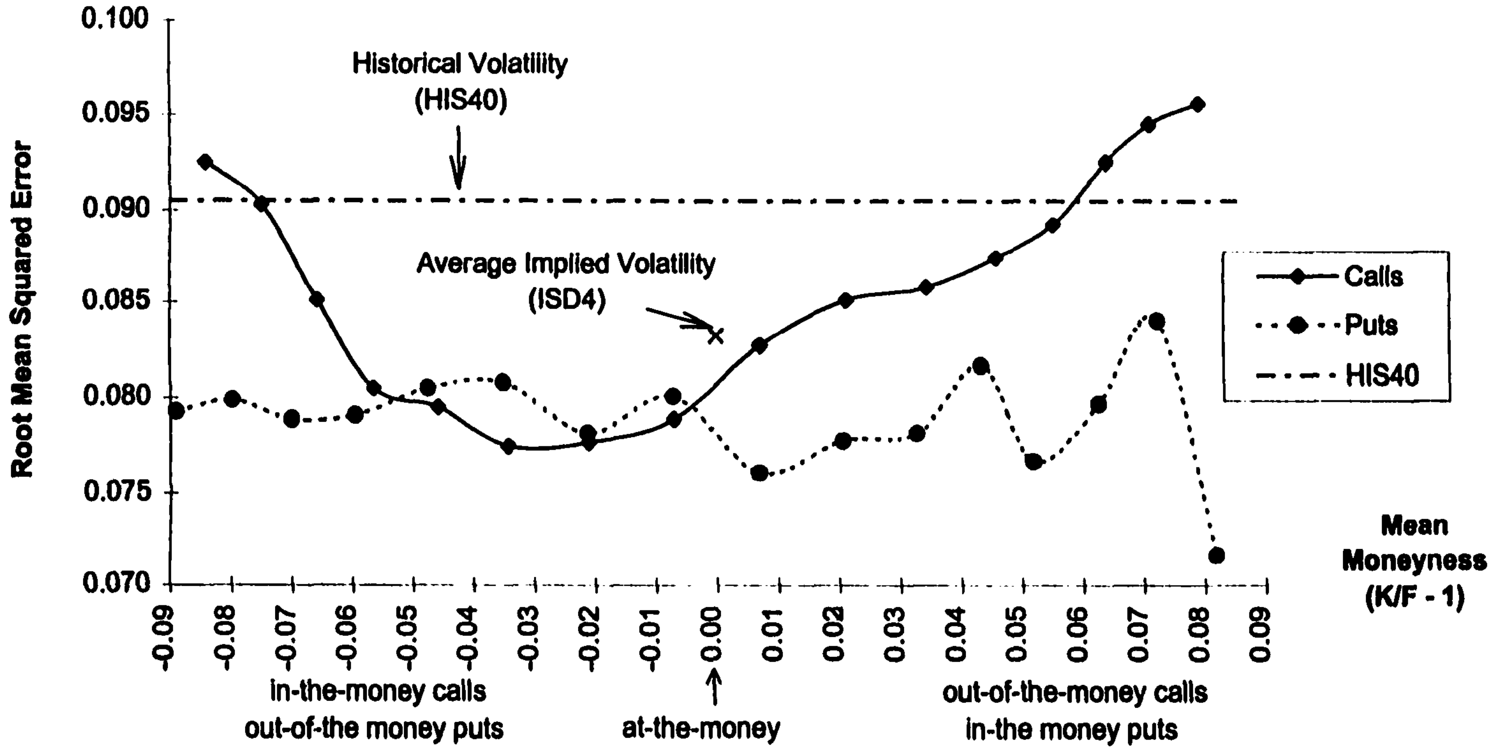
Note: The data set contains daily observations of the four nearest-to-expiration call options on S&P 500 Futures with 10 to 99 trading days to expiration. Equation 5.4 shows that mean squared error (MSE) can be decomposed into the variance of the forecast error and the squared bias. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures either Mean Squared Error (MSE), or the Variance of the forecast Error, or the squared Bias.

Figure 5.4b
Decomposition of Mean Squared Error (MSE) into Variance and Squared Bias for Puts Including the 1987 Market Crash



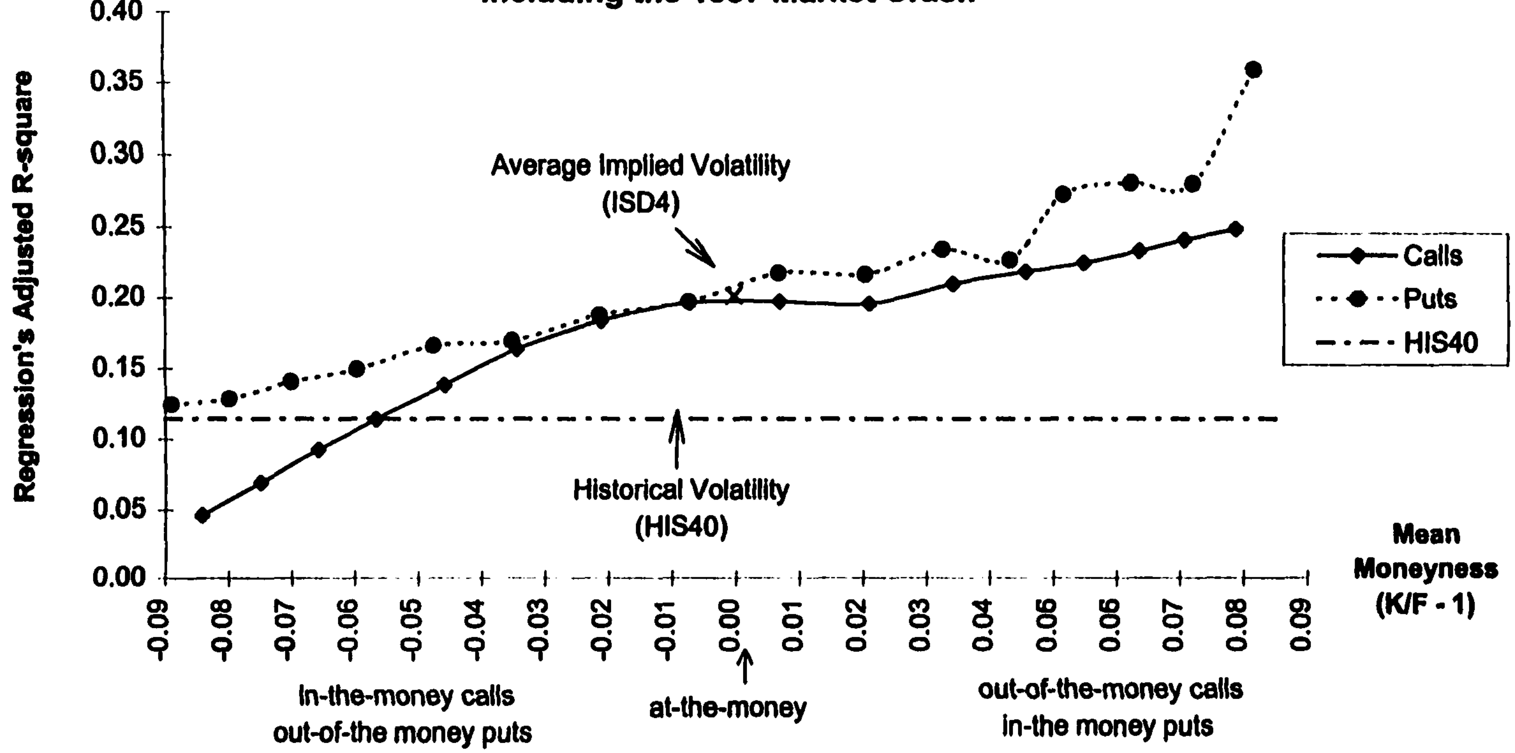
Note: The data set contains daily observations of the four nearest-to-expiration put options on S&P 500 Futures with 10 to 99 trading days to expiration. Equation 5.4 shows that mean squared error (MSE) can be decomposed into the variance of the forecast error and the squared bias. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures either Mean Squared Error (MSE), or the Variance of the forecast Error, or the squared Bias.

Figure 5.5a
Regression's Root Mean Squared Errors (RMSEs) of Different Estimators
Including the 1987 Market Crash



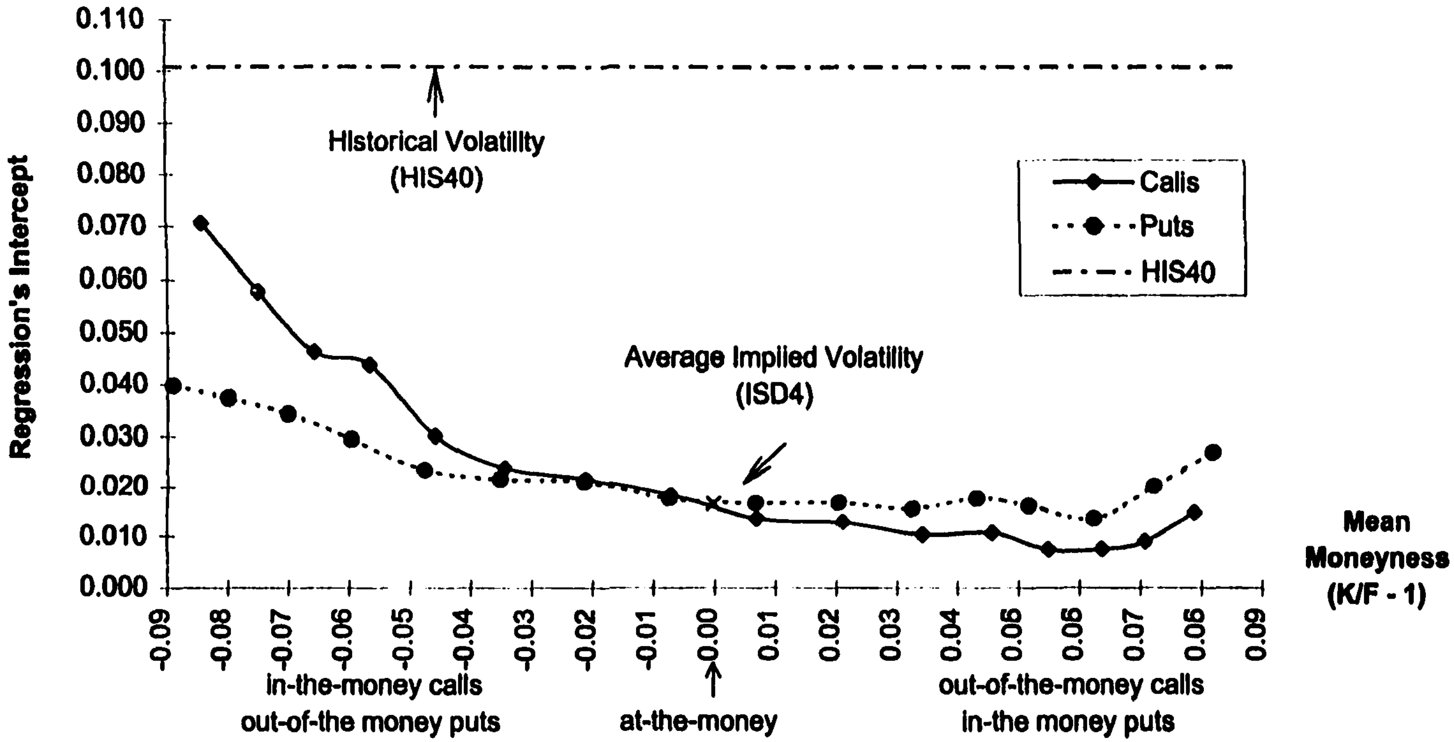
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the OLS regression's RMSE reported in Table 5.5. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the RMSE of the OLS regression of the realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the RMSE of the OLS regression of realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the money puts.

Figure 5.5b
Regression's Adjusted R-squares of Different Estimators
Including the 1987 Market Crash



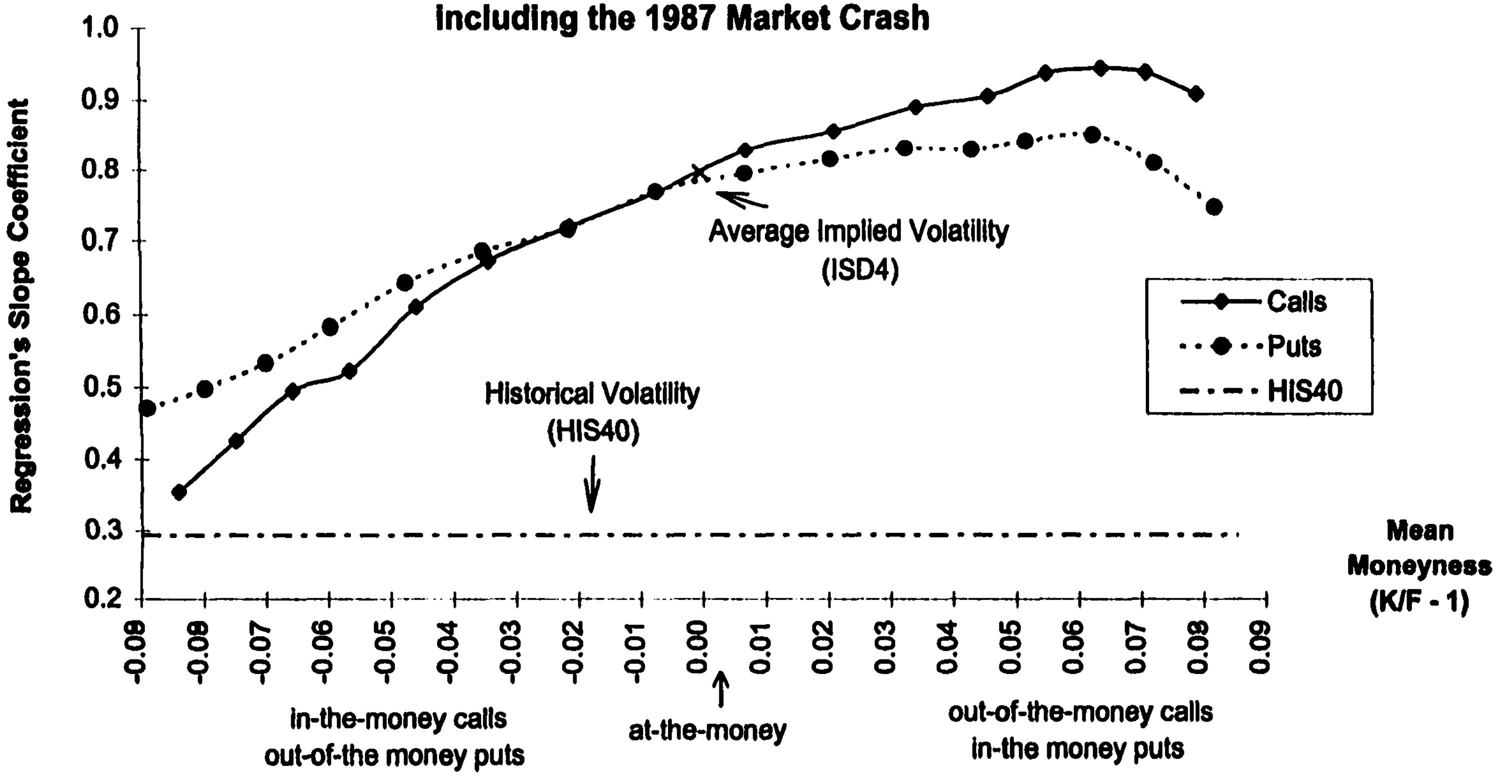
Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the OLS regression's adjusted R^2 reported in Table 5.5. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the adjusted R^2 of the OLS regression of the realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the adjusted R^2 of the OLS regression of realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.5c
Regression's Intercepts of Different Estimators
Including the 1987 Market Crash



Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the OLS regression's intercept reported in Table 5.5. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the intercept of the OLS regression of the realized volatility on the implied volatility calculated from the eighth from-the-money, out of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the intercept of the OLS regression of realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

Figure 5.5d
Regression's Slope Coefficients of Different Estimators Including the 1987 Market Crash



Note: The data set contains daily observations of the four nearest-to-expiration options on S&P 500 futures with 10 to 99 trading days to expiration. The X axis represents the mean moneyness of the options which is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. The Y axis measures the OLS regression's slope coefficient reported in Table 5.5. The diamonds represent calls and the dots represent puts. For example, the farthest right diamond shows the mean moneyness and the slope coefficient of the OLS regression of the realized volatility on the implied volatility calculated from the eighth from-the money, out-of-the-money call. Similarly, the farthest left dot shows the mean moneyness and the slope coefficient of the OLS regression of realized volatility on the implied volatility calculated from the eighth from-the-money, out-of-the-money put. ISD4 represents the average implied volatility calculated from the two-nearest-the-money calls and the two nearest-the-money puts.

IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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