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DEVELOPMENT OF A DIAGNOSTIC SURFACE ENERGY BUDGET MODEL

USING OKLAHOMA MESONET AND ARM DATA

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

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By

TODD MICHAEL CRAWFORD

Norman, Oklahoma

1998
DEVELOPMENT OF A DIAGNOSTIC SURFACE ENERGY BUDGET MODEL USING OKLAHOMA MESONET AND ARM DATA

A Dissertation APPROVED FOR THE SCHOOL OF METEOROLOGY

BY

[Signatures]
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ABSTRACT

In recent years, there has been a growing appreciation of the importance of land-atmosphere interactions in determining the state of the boundary layer. Both operational and research-oriented numerical models have been significantly improved by better parameterizations of the energy transfer that occurs at the earth's surface, but still exhibit significant problems in representing evapotranspiration. Recent studies have also shown that small horizontal variations in surface fluxes can dictate local alterations in the wind field and in the associated convergence maxima. These locations are then favored for the initiation of convection.

In order to begin to examine this phenomenon in more detail, a new technique has been developed to calculate the terms in the surface energy budget from standard meteorological observations. Using only 5- and 30-minute Oklahoma Mesonet data as input, the technique calculates net radiation ($R_n$), ground heat flux ($G$), and latent heat flux (LE). The sensible heat flux (H) is calculated as a residual. The $R_n$ term is calculated using observed values of downwelling shortwave radiation, along with original methods of estimating upwelling shortwave and downwelling longwave radiation. The modeled values of $R_n$ are unbiased and are consistently within 25 W m$^{-2}$ of the observed values. The $G$ term, which is the combination of a 5 cm soil flux term and a storage term, was difficult to verify. The magnitude of the observed values were qualitatively similar to those measured by FIFE, but much larger than those measured by ARM. The LE term is calculated by using the Penman-Monteith equation. As part of this equation, the surface resistance must be estimated. Using ARM data, simple parameterizations were developed (one each for eastern and western Oklahoma) for this term based on
observations of temperature, relative humidity, solar radiation, and soil moisture, along with estimates of leaf age.

The technique first calculates $R_n$ and $G$, then partitions their sum into $H$ and $LE$. Since there were no observations of $LE$ at Mesonet sites, it was decided to use pre-existing reliable estimates of $H$ to effectively verify the new estimates of both $H$ and $LE$. Due to problems with the soil moisture sensors at some of the sites, data from only two Mesonet sites were available to be used as verification. The estimates of $H$ were unbiased and within 60 W m^{-2} (RMSE) at the sites in both eastern and western Oklahoma.

The model was then used to calculate $H$ at 44 sites in Oklahoma during the daytime hours of July 1997. In general, $H$ was larger across western Oklahoma throughout the month. On 12 July, a strong meridional gradient in $H$ was noted across western Oklahoma. The location of the gradient corresponded to an area of enhanced southerly winds, which would be expected due to density differences between the warmer air to the north and the cooler air to the south. This alteration of the surface wind field by the flux gradient has implications for forecasts of convective initiation.
CHAPTER 1
INTRODUCTION AND BACKGROUND

Over the past few decades, improvements in numerical models have resulted in great strides in our ability to predict synoptic-scale motions in the atmosphere. In recent years mesoscale and storm-scale models have shown great promise in resolving and predicting atmospheric phenomena such as sea breezes, drylines, and lake effect snows. However, the current state of computing power and speed is still insufficient to model the more fundamental processes in our atmosphere. The first question many budding meteorologists may ask in their youth, while lounging lazily in a field of grass on a summer day, is "Why did that cloud develop where and when it did?" A less innocent (but similar) question, maybe asked by a significantly older meteorologist in graduate school, could be "What combination of atmospheric processes result in the development of convection in one location rather than another?"

Why convective storms form in certain locations, but not elsewhere, remains a poorly answered question. The idea that thunderstorm development is often a random process has led to the misnomer "air-mass thunderstorms." In large part, this is due to the lack of mesoscale resolution of conventional surface meteorological observations. Well-defined surface boundaries and boundary intersections often dictate preferred areas for initial convection. Even when a surface boundary is present, however, it is often difficult to predict exactly where and when storms will initiate along the boundary. In many cases, though, no detectable boundary is present, and the initiation of convection may be controlled by more subtle features. In the absence of any significant forcing aloft, small spatial variations of convergence at the surface may dictate the location of convection initiation.

Recent advances in observational capability provide an opportunity to examine these small-scale features in more detail. The Oklahoma Mesonetwork (Mesonet; Brock
et al., 1995) consists of 114 automated surface observing stations distributed across the state. These stations record incoming solar radiation, air temperature, relative humidity, wind speed and direction, soil temperature, barometric pressure and rainfall at five-minute intervals (Fig. 1.1). Sixty of the sites measure soil moisture at four different depths. The average station separation of ~30 km provides a unique real-time opportunity to detect surface features at much smaller spatial and temporal scales than was previously possible in Oklahoma. Unfortunately, the operational, high-resolution surface data provided by the Mesonet is not duplicated aloft, where radiosondes are launched only twice daily at a greatly reduced horizontal resolution. Thus, the use of this surface data set is of crucial importance in determining the characteristics of the planetary boundary layer at a given time. McGinley (1986) stated that "the ability to acquire information has outstripped the ability to assimilate it at the local level." The goal of this work is to create a diagnostic model which will calculate surface energy fluxes in real-time using Mesonet data as input.

This product can then be used in a variety of ways. Correlations between the sensible heat flux gradients and local wind perturbations could be calculated in order to better understand the "inland sea breeze" (Sun and Ogura, 1979). Calculated fluxes could be compared to those generated from numerical models. Variations in eastward dryline motion could be predicted based on the along-dryline variation of sensible heat flux. Predictions of evapotranspiration from hydrological models could be compared to the calculated values of latent heat flux. Finally, the gradients in surface fluxes could be correlated to the locations of convection initiation. Emanuel et al. (1995), in a report detailing "intellectual challenges of the day that may lead to improvements in weather observations, forecasting, and warning," stressed the importance of gaining a better understanding of land-atmosphere interactions for applications in numerical modeling and weather forecasting. The results from this research will assist in achieving this rather lofty goal.
1.1 History of Modeling Land-Atmosphere Interactions

Over the past 20 years, considerable work has been done in an effort to better understand the interaction between the atmosphere and the Earth's surface. Climate modelers were the first to attempt this rather imposing task. Deardorff (1978) filled each model grid box with a single vegetation type, or leaf, which completely covered the soil. Although this "big-leaf" model ignored the sub-grid variability, it did improve global precipitation forecasts. The non-linear interactions between these sub-grid variations and grid-averaged surface fluxes have become more apparent in recent years (Wetzel and Chang, 1987; Avissar, 1992) as regional and mesoscale modelers began to address the issue. Important field experiments such as HAPEX-MOBILHY (Hydrologic Atmospheric Experiment - Modélisation du Bilan Hydrique; André et al., 1986), the First ISLSCP (International Satellite Land Surface Climatology Project) Field Experiment (FIFE; Sellers et al., 1992) and the Boreal Ecosystem Atmosphere Study (BOREAS; Sellers et al., 1995) have provided important high-resolution data for improving current surface flux parameterizations. In FIFE, surface fluxes were measured at 20 locations in a 15 x 15-km area in east central Kansas. Betts and Ball (1995) used this data set and a boundary layer model to show that dry (wet) surfaces inhibit (promote) the development of convective precipitation, thus verifying the conclusions of Mintz (1984). The results from Rabin et al. (1990) supported this conclusion for air masses with relatively moist boundary layers. However, for drier boundary layers, convection developed first over drier surfaces. Using the same data set, Chen et al. (1996) concluded that the Oregon State University (OSU) land surface model (Pan and Mahrt, 1987) was the most effective parameterization currently available due to its quality and relative computational simplicity. The OSU model and the NMC mesoscale Eta model (Black, 1994) were recently coupled in order to improve the representation of the land-atmosphere interaction in an operational model.
1.2 Theory of Mesoscale Circulations

If two similar and adjacent patches of flat, bare soil, one moist and one dry, are irradiated evenly without wind, horizontal gradients in near-surface temperature and vapor pressure will develop due to relatively larger (smaller) magnitudes of evaporation over the moist (dry) patch of soil. More specifically, the incoming radiation will be partitioned into more sensible (latent) heat than latent (sensible) heat over the dry (moist) land. Jeffreys (1925) summarized the effects of this spatial variation of fluxes across the two patches of soil:

If a difference of temperature is maintained over any level surface within or in contact with a fluid, or if heat is supplied to or withdrawn from any region within the fluid, the fluid will move, and will continue to move until such difference of temperature or supply or removal of heat ceases.

Unfortunately, Jeffreys does not explain why this movement occurs. As the day progresses, the dry surface layer will warm relative to the moist surface layer. In a hydrostatic atmosphere, this will result in the dry column of air expanding vertically relative to the column over the moist surface. Because of this, horizontal pressure gradients develop near the top of the boundary layer and near the surface. As a result, air will flow from the "dry" ("wet") column to the "wet" ("dry") column near the top (bottom) of the boundary layer. Mass continuity ensures rising (sinking) motion over the dry (wet) land, completing a direct solenoidal circulation (Fig. 1.2). Observational (e.g., Segal et al., 1989) and modeling studies (e.g., Ookouchi et al., 1984) have confirmed this process, noting the development of an "inland sea breeze" (Sun and Ogura, 1979).

Small-scale spatial variations in surface fluxes can be established in many other ways. Spatial variations in antecedent rainfall, differences in vegetation, variations in cloud cover, and anthropomorphically-induced changes in land use (crop selection, burning, etc.) are examples. Physical intuition suggests that the scale of these surface
inhomogeneities is important in determining their effect on boundary layer development. Using a 2-D mesoscale model, Schädler (1990) found that the timing of the initiation of mesoscale vertical circulations is indeed dependent upon the horizontal extent of the inhomogeneities. The larger (smaller) the scale, the later (earlier) the vertical circulation developed, the longer (shorter) the circulation lasted and the weaker (stronger) the perturbation winds were. Dalu et al. (1991) and Dalu and Pielke (1993) theorized that the horizontal scale must be comparable to or larger than the local Rossby radius (~100 km) to affect the surface-layer wind field.

Smith et al. (1992), using FIFE data, demonstrated that variations in the cloud field provide the primary control on the magnitude of the flux terms by modulating the amount of solar radiation (and thus net radiation) reaching the surface. They also concluded that the partitioning of available energy at the surface into latent and sensible heat fluxes (Bowen ratio, see Eq. (1.7)) is mainly a function of surface moisture, and is not significantly affected by topography, vegetation, or cloudiness. Using the Colorado State University Regional Atmospheric Modeling System (RAMS), Lee (1992) found that thermal circulations will also be produced when two different surface types are situated next to one another. For example, a plot of bare wet soil next to unstressed grass results in near-surface flow from the grass to the soil, since more of the available energy is converted to sensible (latent) heat over the soil (grass). In a modeling study, Sun and Bosilovich (1996) demonstrated that the development of the boundary layer is most sensitive to the amount of soil water content and its spatial variability.

1.3 Prevalence of Mesoscale Circulations

On days when the synoptic-scale forcing is weak and surface sensible heat fluxes are large, mesoscale circulations tend to be predominant. Using a mesoscale model,
Pielke (1984) found that while a sensible heat flux gradient of 10 W m$^{-2}$/30 km has a minor effect on local winds, a gradient of 100 W m$^{-2}$/30 km has a minor but statistically observable effect, and a gradient of 1000 W m$^{-2}$/30 km creates a pronounced change. However, Pielke et al. (1997) found that these effects were important even with relatively strong surface winds (on the order of 10 m s$^{-1}$) in a simulated dryline case from May 1991. Conflicting evidence was presented by Hechtel et al. (1990), who concluded that airflow in the mixed layer was not necessarily affected by small-scale flux gradients. However, since the surface inhomogeneities were very small in scale in this case (450-900 m), it is likely that horizontal mixing could have smoothed out the gradients before the wind field would have had time to respond to the differential heating.

Mahrer and Pielke (1978) noted that if the surface winds oppose (parallel) the lower branch of the mesoscale circulation the mesoscale pressure gradient will tighten, intensifying (weakening) the circulation. Mahrt (1996) states that microscale (< 1 km) spatial variations of surface fluxes are too small to generate significant local vertical circulations. Using scaling analysis, he then proposes a minimum scale of variations (analogous to the Monin-Obukhov length) needed to produce these circulations:

$$\ell_{\text{min}} = \frac{\rho c_p T_s u^3}{g H},$$

(1.1)

where $\rho$ is the air density, $c_p$ the specific heat of air, $T_s$ the mean surface temperature, $u$ the measured wind speed, $g$ the gravitational constant, and $H$ the mean sensible heat flux.

It is apparent that weaker wind speeds are favorable for setting up local vertical circulations, since $\ell_{\text{min}}$ is then smaller, and spatial flux variations are more common at smaller scales than at larger scales. Stronger large-scale flows decrease perturbations through horizontal advection and mixing. Large values of sensible heat flux (highly unstable surface layer) tend to favor the establishment of these circulations as well. For typical warm-season values of $T_s = 300$ K, $u = 5$ m s$^{-1}$, and $H = 200$ W m$^{-2}$, $\ell_{\text{min}} \sim 20$ km, which is approximately one-half the average spacing of the Mesonet stations. This
implies that a significant difference in $H$ between two adjacent Mesonet stations is sufficient to alter the wind field locally.

1.4 Convective Initiation

Once it was shown that local dynamics could be affected by relatively small-scale variations in surface fluxes, one of the next logical steps was to examine how these perturbations can affect thunderstorm initiation. Sun and Ogura (1979) noted a case in which thunderstorms began to develop near an area of large temperature gradient. Using a linear model, Anthes (1984) concluded that planting regularly spaced bands of vegetation in semi-arid regions could result in an increase in convective precipitation due to the establishment of local convergence zones at the surface. Bluestein (1985) noted the possible importance of a synoptic-scale, thick cirrus band with a sharp edge in modulating severe convective activity along a cold front in the Southern Plains. Yan and Anthes (1988) used a two-dimensional numerical model to show that meso-beta scale (10-100 km) inhomogeneities in surface-moisture availability may directly affect convective initiation through the development of mesoscale vertical circulations between the dry and the moist land surfaces. The upward vertical motion and subsequent cooling produced by these circulations is important in reducing convection inhibition locally.

These effects are not limited to soil moisture differences. Rabin et al. (1990), through an observational study in Oklahoma, showed that convective clouds formed earliest over areas with high sensible (latent) heat flux in the presence of a relatively dry (moist) atmosphere. Zeng and Pielke (1995) concluded that the locally enhanced areas of horizontal convergence created by thermally induced vertical circulations could initiate or enhance thunderstorms. Segal et al. (1995) found that latent heat flux is a dominant forcing mechanism in the destabilization that leads to deep convection, while sensible heat flux is of secondary importance. They also emphasized the importance of accounting for moisture from surface evaporation in routine short-term forecasting of
deep convection. Avissar and Liu (1996) numerically simulated a domain with alternating strips of wet and dry land, resulting in sensible and latent heat flux gradients. They found that clouds and precipitation formed over the dry land, effectively eliminating these flux gradients through a negative feedback process. Hane et al. (1997) have presented observational evidence that horizontal gradients in surface energy fluxes along a dryline can induce mesoscale vertical circulations that may assist in storm initiation. Koch and Ray (1997), in an examination of 23 thunderstorm days in North Carolina, documented the "Piedmont trough." This feature was associated with convection on 13 of the 23 days. The authors speculated that this trough results from differential heating between the sandy soil of the eastern coastal plain and the clay soil of the Piedmont region to the west.

1.5 The Surface Energy Balance

The First Law of Thermodynamics states that energy can be neither created nor destroyed, but only converted into different forms. At the interface between the earth and its atmosphere, exchange of energy occurs due to radiative, convective, and conductive processes. Radiative energy is transferred by photons moving through the atmosphere at the speed of light. Convective energy is transferred from the surface to the lower atmosphere by the turbulent vertical motion of the air. Finally, conductive energy is transferred by molecular collision in the soil. This mode of transfer is much more effective in solids than in either liquids or gases (Oke, 1987), although conduction does occur in the atmosphere just above the surface.

The total energy input to the surface from the atmosphere must be either stored in the soil or given back to the atmosphere. The net radiation flux at the surface \( R_n \), that is, the difference between the sum of the downwelling shortwave \( SW_d \) and longwave radiation \( LW_d \) and the sum of the upwelling shortwave \( SW_u \) and longwave \( LW_u \) radiation, is partitioned into three dominant forms of "output" energy. Two forms are
characterized by convective exchange of heat (sensible heat flux - H) and moisture (latent heat flux - LE), while the third is characterized by conductive exchange (ground heat flux - G). Thus the surface energy balance is

\[ R_s = H + LE + G. \]  

(1.2)

This is illustrated in Fig. 1.3 for typical afternoon conditions. For this study, the effective "surface" is located at the aerodynamic roughness length \( z^{*} \) for momentum, \( z_{0h} \) for heat and moisture), which will be detailed later in section 1.7. For this study, positive values of the fluxes refer to energy transfer in the direction of the arrows in Fig. 1.3. This is consistent with the standard convention of denoting radiative (non-radiative) fluxes as positive when directed towards (away from) the surface (Garratt, 1992).

1.6 Typical Diurnal Cycle of Surface Energy Balance

On a typical clear day, the downwelling shortwave radiation \( (SW_d) \) increases steadily during the morning to a maximum at solar noon, before decreasing steadily throughout the afternoon. The upwelling shortwave radiation \( (SW_u) \) increases slowly in the early morning due to the decreasing albedo, then more rapidly, also peaking at solar noon. A steady decrease begins after that, with the decrease slowing towards sunset due to the late-afternoon increase in albedo. The albedo is larger during the early morning and late afternoon due to the low sun angle (see section 2.2 for more specific details).

The downwelling longwave radiation \( (LW_d) \) increases steadily throughout the morning and early afternoon before leveling off and then slowly decreasing in the late afternoon. Since \( LW_d \) is directly related to the temperature of the atmospheric layer near the surface, the afternoon maximum should be expected. The upwelling longwave radiation \( (LW_u) \)
follows the same pattern. A graph illustrating these cycles is shown in Fig. 1.4a. Here, positive (negative) values represent energy transfer towards (away from) the surface.

The daily cycles of LE and H on clear days are typically characterized by maxima around solar noon, and are similar in shape to the SW curve. In general, G is maximized in the early morning, when the winds are still relatively light, since the lack of turbulent transfer suppresses H and LE then. Because these fluxes are small, the energy represented by the increasing magnitude of Rs must be transferred into the soil as heat. Later in the day, as turbulent transfer is enhanced, H and LE comprise a more significant percentage of Rs, leaving less energy for the soil. The typical diurnal cycles of LE, H, and G are displayed in Fig. 1.4b.

1.7 Estimating Surface Sensible and Latent Heat Fluxes

Many different methods have been employed to estimate the turbulent fluxes (H and LE). Remote sensing is the most versatile and rigorous method, simply because satellites can detect radiance with a very high spatial resolution relative to current surface observing networks. However, this method has three major problems. First, the measured satellite radiance must be corrected for atmospheric and geometric effects before the surface fluxes can be extracted. Atmospheric effects generally refer to the transmissivity of the atmosphere at a normal range of terrestrial temperatures, which is inversely proportional to the amount of water vapor in an atmospheric column (Schmugge and Becker, 1991). Second, this method will only produce accurate readings during periods of clear skies, greatly limiting the utility of the flux calculations. Third, satellite grid points are averaged, and are not necessarily representative of any particular
Mesonet site. Because of this, it was decided that $H$ and $LE$ must be estimated based on near-surface observations alone.

There are three general approaches used in estimating sensible and latent heat fluxes at the surface: the eddy fluctuation method, the profile method, and the resistance method (Oke, 1987). The eddy fluctuation method (Businger et al., 1990) requires specialized, fast-response instruments that can detect transient perturbations in vertical velocity, temperature, and vapor pressure. These perturbations are associated with turbulent eddies that transport heat and moisture vertically. The formulations for $H$ and $LE$ are as follows:

$$H = \rho c_p w' T'$$  \hspace{1cm} (1.3)\\
$$LE = \rho L_w w' q' = \rho c_p R_d w' e' / \gamma R_v,$$  \hspace{1cm} (1.4)

where $w'$, $T'$, $q'$, and $e'$ are the perturbation vertical velocity, temperature, specific humidity, and vapor pressure, respectively; the overbar represents a short-term average; $L_v$ represents the latent heat of vaporization; $R_d$ and $R_v$ are the gas constants for dry air and water vapor, respectively; and $\gamma$ is the psychrometric constant, $pc_p/L_v$. The vapor pressure form of the latent heat flux will be used in this study because it is computationally simpler than the specific humidity form; there is a more direct conversion from relative humidity (measured by Mesonet) to vapor pressure than to specific humidity.

The magnitude of the individual perturbations is the difference between the instantaneous value and the average value. The determination of the proper averaging period is somewhat subjective. The goal is to average over a long enough period that a "representative" sample of the dominant turbulent structures is captured. For typical afternoon conditions with large sensible heat fluxes, the magnitude of the perturbations
(both wind and temperature) vary sharply in both space and time. In this case, an averaging period of as little as 15 minutes may be used. Averaging times will increase for more stable conditions. For any regime, the raw data must be subjectively examined before the proper averaging period can be determined (J. Schneider, National Severe Storms Laboratory, pers. comm.). The National Center for Atmospheric Research (NCAR) has developed the Atmosphere-Surface Turbulent Exchange Research (ASTER; Businger et al., 1990) facility, which provides state-of-the-art measurements of surface fluxes using the eddy fluctuation method for field projects. Unfortunately, the expense and sensitivity required to use this method render this method inapplicable in an operational environment.

The profile method is based on the assumption that the surface layer is a constant-flux layer, which means the gradients of temperature, moisture, and wind are linear. Calculation of surface sensible and latent heat fluxes requires measurement of air temperature, humidity, and wind on at least two levels:

\[
H = \rho c_p K_H \frac{dT}{dz} \tag{1.5}
\]

\[
LE = \rho c_p K_v \frac{de}{dz} / \gamma \tag{1.6}
\]

where \(K_H\) and \(K_v\) are the eddy conductivity (inversely proportional to the aerodynamic resistance, which is discussed in detail later in this section) of heat and water vapor, \(dT\) and \(de\) are the differences in temperature and vapor pressure between the two levels, and \(dz\) is the distance between the two levels (e.g., 7.5 m for the Mesonet temperature sensors). The eddy conductivity is directly related to the difference in wind speed between the two levels, and can be calculated explicitly (Halliwell and Rouse, 1989). The calculations for \(H\) and \(LE\) then follow rather easily. This is called the bulk aerodynamic profile method.

The two equations above are in flux-gradient form, whereby a flux of a quantity (\(H\) and \(LE\)) is the product of the conductivity of that quantity (\(\rho c_p K_H\) and \(\rho c_p K_v / \gamma\)) and the gradient of that quantity (\(dT/dz\) and \(de/dz\)). It is normally assumed that eddy conductivity
of heat and moisture are similar, based on observational results from micrometeorological experiments (Swinbank and Dyer, 1967). Employing this assumption ($K_h = K_v$), the Bowen ratio profile method (Bowen, 1926) can be used instead:

$$\beta = \frac{H}{LE} = \gamma \frac{dT}{de} \quad (1.7)$$

where $\beta$ is the Bowen ratio. If $\beta$ is known (through vertical profiles of moisture and temperature), $H$ and $LE$ can be calculated from the following two forms of the surface energy balance equation:

$$H = \frac{\beta}{1 + \beta} (R_a - G) \quad (1.8)$$

$$LE = \frac{1}{1 + \beta} (R_a - G) \quad (1.9)$$

However, more recent studies have concluded that $K_h$ and $K_v$ may not be equal, which would invalidate the assumptions made for the Bowen ratio method. About half of the Mesonet sites are equipped with temperature sensors at two levels, so the bulk aerodynamic method could be used to calculate sensible heat flux using $H$. However, the temperature sensors at the two levels are dissimilar. Even if the observational error associated with each sensor were negligible, a small bias in one of the sensors (e.g., due to different calibration standards) would render the flux calculations grossly inaccurate.

Another profile method used to estimate latent heat flux is referred to as the simple bucket method (Manabe, 1969):

$$LE = \omega LE_p = \omega \rho_s c_p K_v u (e_s(T_s) - e) / \gamma, \quad (1.10)$$

where $\omega$ is the soil moisture availability parameter, $LE_p$ is the potential latent heat flux, $\rho_s$ is the density of the air near the surface, $u$ is the observed wind speed, $e_s(T_s)$ is the saturation vapor pressure at the surface temperature, and $\gamma$ is the psychrometric constant. This formulation assumes that the surface is always saturated and thus overestimates latent heat flux in dry surface conditions.
The third method is referred to as the resistance or Penman-Monteith (PM) method (Penman, 1948; Monteith, 1965), or the Combination Model (Oke, 1987), since it combines a parameterization for the net radiation ($R_n$) and ground heat flux ($G$) in the surface energy balance with the PM approach of partitioning the available energy ($R_n - G$) into sensible and latent heat fluxes. This method only requires one level of data so that all 114 Mesonet sites can be utilized. Galinski and Thomson (1995) evaluated three different sensible heat flux calculation schemes employing the PM method. All three were designed to be applicable to mid-latitude, grass-covered surfaces, and were validated by directly measuring heat flux with a sonic anemometer (using the eddy fluctuation method) at Cardington in the United Kingdom. All three of the schemes tended to slightly underpredict the daytime sensible heat flux (Fig. 1.5), but the Berkowicz and Prahm (1982) scheme had the smallest mean bias (-8.0 W m$^{-2}$) and root mean square (38.5 W m$^{-2}$) errors (MBE and RMSE, respectively) with the highest correlation ($r = 0.81$). The BP scheme is likely superior because it indirectly accounts for soil moisture in the estimation of latent heat flux, while the Holtslag and Van Ulden (1983) and Smith (1990) schemes did not. All three schemes had larger errors in the summer months and when the humidity was particularly low.

Since it was developed as an analogue to Ohm’s law (current = electrical potential / resistance), this method is considered a resistance method. The transfer of a quantity can be viewed as an analogue to current in a circuit, whose flow is dependent upon the potential applied across the circuit and the resistance of the circuit. For meteorological applications, the "current" represents a flux, the "potential" is the gradient of the quantity, and the "resistance" is the suitability of the medium to transfer the quantity. An analogue between the electrical and the meteorological versions of Ohm’s law is illustrated in Fig.
1.6. For a given gradient, a larger (smaller) resistance results in a smaller (larger) flux.

Based on this logic, the following flux formulations can be made:

\[ u_z^2 = \frac{u(z_u) - u_x}{r_{sh}} = \frac{u(z_u)}{r_{sl}} \tag{1.11} \]

\[ H = \frac{T_s - T}{r_{sh}} \rho c_p \tag{1.12} \]

\[ LE = -\frac{e_s - e}{r_{sl}} \rho c_p \gamma \tag{1.13} \]

\[ G = \frac{T_s - T_s}{r_{soil}} \rho c_p + \text{storage term} \tag{1.14} \]

where \( u_z^2 \) is the momentum flux, or the friction velocity squared; \( u(z_u) \) is the measured wind speed at \( z_u \) (10 m); \( u_x \) is the surface wind (identically zero); \( T_s \) and \( e_s \) are the surface temperature and vapor pressure; \( T \) and \( e \) are the observed temperature and derived vapor pressure (from the observed relative humidity) at 1.5 m; \( r_{sh}, r_{sl}, \) and \( r_{soil} \) are the aerodynamic resistance to the momentum, sensible heat, and latent heat fluxes (units of s \(^2\) m \(^{-1}\)); \( T_s \) is the soil temperature at -10 cm; and \( r_{soil} \) is the conductive resistance of the soil.

The storage term is necessary in (1.14) to account for the temperature changes in the topsoil, which effectively reduces the transfer of heat through the soil. In this sense, the soil acts as a capacitor in the Ohm's law analogy. The formulation for \( G \) will be outlined further in section 2.3. Since momentum flux is always downward near the surface, the typical sign convention has been reversed in (1.11), with positive momentum fluxes representing downward transfer. If \( K_H = K_v \) as assumed, then \( r_{sh} = r_{sl} \) since (Berkowicz and Prahl, 1982)

\[ r_{sh} = \int_{1.5m}^{10m} \frac{dz}{K_H} \]

\[ r_{sl} = \int_{1.5m}^{10m} \frac{dz}{K_v} \tag{1.15} \]
To simplify the remainder of the analysis, we will define and use one representative aerodynamic resistance, \( r_a \), where \( r_s = r_{sh} = r_{sl} \).

Monteith (1965) showed that the latent heat flux could also be prescribed in another way:

\[
LE = \frac{e_s(T_s) - e_s \rho c_p}{r_s \gamma},
\]

where \( e_s(T_s) \) is the saturated vapor pressure at the surface temperature \( T_s \) and \( r_s \) represents the total resistance of the vegetation to evapotranspiration. This equation describes the process of transferring water vapor to the atmosphere through transpiration from the vegetation. This formulation assumes that the vapor pressure inside the stomata of the vegetation is saturated at the surface temperature, and that water vapor transfer occurs between the leaf and the drier air above. This assumption is a safe one in relatively steady atmospheric conditions, since the plant regulates its evaporation to maintain equilibrium with the overlying atmosphere (Rutter, 1975). The latent heat flux describes the total water vapor transfer from the surface to the overlying atmosphere.

This transfer occurs in two steps. The first is transpiration between the native vegetation and the adjacent air (described by (1.16)). This is mainly a biophysical process, controlled by the plant reaction to external and internal processes. The second process is turbulent transfer between the newly moistened air adjacent to the surface and the atmosphere above (described by (1.13)), which is controlled by the near-surface wind speed and atmospheric stability. If it is assumed that no water vapor is lost through condensation on the vegetation, then either (1.13) or (1.16) can be used to calculate LE. Fig. 1.7 illustrates these concepts (adapted from Lee, 1992). In the Ohm's law analogy, LE represents the current in this series circuit. By definition, the current remains constant throughout the circuit. Because of this, we can set the RHS of (1.13) and (1.16) equal to one another. In doing this, \( e_s \), which is not measured, can be eliminated from the system.
of equations. Now, the following formulations can be derived (see Appendix A) for use with data from only one level, using (1.2), (1.12), (1.13), and (1.16):

\[
LE = \frac{(R_s - G)\Delta r_s \gamma^{-1} + [\varepsilon_s(T) - \varepsilon] \rho c_p \gamma^{-1}}{r_i + (1 + \Delta \gamma^{-1})r_s} \quad (1.17)
\]

\[
H = \frac{(R_s - G)(r_s + r_i) - [\varepsilon_s(T) - \varepsilon] \rho c_p \gamma^{-1}}{r_i + (1 + \Delta \gamma^{-1})r_s} \quad (1.18)
\]

Here, \( \varepsilon_s(T) \) is the saturation vapor pressure at \( T \), and \( \Delta \) is well approximated from (A.4) as the local change in saturation vapor pressure with temperature \((\partial \varepsilon_s / \partial T)\) at \( T \). In order for the PM approach to work, it must be assumed that \( \Delta \) is constant between \( T_s \) and \( T \) (Fig. 1.8). It should also be noted here that the PM method was developed for agricultural surfaces with little exposed soil, and is not expected to be effective over bare soil. Most Mesonet sites are located in open fields of grass, however, which more closely resemble a field of crops than bare soil. Because the sites are fully vegetated, the method should be expected to work reasonably well for this study, and the \( r_i \) term will implicitly absorb any contribution from bare soil and standing water to the total magnitude of evapotranspiration.

The aerodynamic principles from which \( r_{lu} \) and \( r_s \) are derived are based on Monin-Obukhov similarity theory (Monin and Obukhov, 1954). This theory is based on the assumption that the magnitude of the vertical gradient of any conserved quantity is a function of \( z/L \), and is valid under assumptions of stationary and horizontally homogeneous conditions. According to Oke (1987), this condition is met when the surface layer is observed over time periods greater than ten minutes. Here, \( z \) is the distance above ground and \( L \) is the Monin-Obukhov length, a measure of the local stability as manifested by eddy size.
\[ L = \frac{T_u}{g k H} \rho c_p, \quad (1.19) \]

where \( k \) is the von Kármán constant (0.4). \( L \) is negative for \( H > 0 \) (unstable surface layer) and positive for \( H < 0 \) (stable surface layer). The Monin-Obukhov length can also be thought of as that height at which the magnitudes of the buoyancy and shear terms in the turbulent kinetic energy equation are equal (Garratt, 1992). If the terms are of opposite sign at that point, then \( L \) is designated as negative. In the region below (above) \( L \), mechanical (convective) turbulence is predominant (Emanuel, 1994). A more detailed examination of Monin-Obukhov similarity theory is provided in Appendix B.

Using similarity theory, wind and temperature profiles can be related to surface fluxes:

\[
\begin{align*}
  u(z) &= \frac{u_*}{k} \left[ \ln\left(\frac{z}{z_{om}}\right) - \psi_m(z/L) + \psi_m(z_{om}/L) \right] \\
  T(z) &= T_s + \frac{T_*}{k} \left[ \ln\left(\frac{z}{z_{oh}}\right) - \psi_h(z/L) + \psi_h(z_{oh}/L) \right].
\end{align*}
\quad (1.20, 1.21)
\]

Here, \( T_* \) is the flux temperature scale, which is related to the sensible heat and momentum fluxes:

\[ T_* = \frac{-H}{\rho c_p u_*}, \quad (1.22) \]

and where \( \psi_m \) and \( \psi_h \) are empirical stability correction functions (Paulson, 1970; Dyer, 1974):

\[
\psi_m(z/L) = \ln\left(\frac{1+x}{2}\right)\left(\frac{1+x^2}{2}\right) - 2 \tan^{-1} x + \frac{\pi}{2}, \quad \text{for } L < 0
\]

where \( x = (1 - 16z/L)^{1/4} \)

\[ = -5z/L \quad \text{for } L > 0 \]

\[ \text{for } L > 0 \]
\[ \psi_h(z/L) = \begin{cases} 2 \ln \left[ \frac{1 + x^2}{2} \right] & \text{for } L < 0 \\ \frac{-5z}{L} & \text{for } L > 0 \end{cases} \] (1.24)

For rougher natural surfaces \( z \) would be replaced by \( (z - d) \) in (1.20) and (1.21), where \( d \) is called the zero-plane displacement. This represents the level of the apparent momentum sink in the native vegetation (Oke, 1987), and is located at approximately two-thirds the height of the native vegetation. However, since the Mesonet sites are generally located at well-exposed sites with short vegetation, we will follow the common practice of omitting it from the equation (Priestley, 1959, p. 20).

The effect of the stability correction functions on the vertical wind and temperature profiles is illustrated in Fig. 1.9. For an extremely unstable surface layer \((-z/L < 3)\), Parlange and Katul (1995) found that the Paulson-Dyer stability correction functions were too large, and suggested an improved formulation. However, their formulation was developed based on only one data set, and even they stressed the need for further experiments in order to test the reliability of the new method. Chen et al. (1997) tested three surface layer parameterization schemes with the NCEP Eta model and found that there were no significant differences in model-simulated surface fluxes. Because of this, the more time-tested Paulson-Dyer functions will be used here. The stability correction functions are positive (negative) for unstable (stable) conditions. Including the stability correction functions insures well-behaved flux estimation in all stability regimes.

The aerodynamic resistances can be obtained from (1.11) and (1.20):

\[ r_{uU} = \frac{\left[ \ln \left( \frac{z_u}{z_{0m}} \right) - \psi_m(z_u/L) + \psi_m(z_{0m}/L) \right]^2}{(k^2u(z_u))} \] (1.25)

and (1.12), (1.21), and (1.22) (Berkowicz and Prahm, 1982):
where $z_i$ is the height of the temperature measurement (1.5 m). Momentum transfer is enhanced by small pressure fluctuations in the turbulent wakes behind the roughness elements, unlike heat or moisture transfer (Garratt, 1992). Because of this, $r_{*u} < r_*$ in most circumstances.

The stability correction functions (1.23) and (1.24) ensure that $r_{*u}$ and $r_*$ are larger (smaller) in stable (unstable) conditions. It is also apparent from (1.26) that larger (smaller) values of $u(z_i)$, $z_{om}$, and $z_{oh}$ will result in a smaller (larger) aerodynamic resistance. This can be easily explained: both higher wind speeds and taller and/or rougher vegetation result in more surface drag. Increased drag causes more frictionally generated eddies and thus more vertical mixing (less resistance to vertical transfer).

The aerodynamic roughness length for momentum $z_{om}$ is that height at which the wind speed goes to zero due to surface drag (Stull, 1986). It is proportional to both the height and density of the native vegetation. Rougher surfaces (larger $z_{om}$) are more strongly coupled to the atmosphere via turbulent transfer than are smooth surfaces. In fact, Lee (1992) noted that transpiration over grass-covered, moist surfaces has been found to be more significant than the evaporation over either wet soil or water, due to the increase in turbulent transfer associated with the rougher surfaces, as well as the increased surface area available for evaporation and the ability to tap deeper soil water.

Table 1.1 shows some typical values of $z_{om}$ (adapted from Pielke, 1984). Wieringa (1993) has developed a method of estimating $u_*$ and $z_{om}$ from gust observations during strong

<table>
<thead>
<tr>
<th>Vegetation height</th>
<th>$z_{om}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soils</td>
<td>0.1 - 1.0 cm</td>
</tr>
<tr>
<td>Short grass</td>
<td>0.3 - 1.0 cm</td>
</tr>
</tbody>
</table>
Long grass | 4 - 10 cm | 25 - 100 cm
---|---|---
Crops | 4 - 20 cm | 40 - 200 cm

Table 1.1 Typical values of $z_{om}$ and vegetation height (adapted from Pielke, 1984).

wind events using only a single wind observation. This method may be employed in the future to refine the estimates of $z_{om}$. For this study, however, we will assume that $z_{om} = 0.01 \text{ m}$ and $h = 0.1 \text{ m}$, which is typical for grasses (Wieringa, 1993). These values are at the upper end of the "short grass" category in Table 1.1. It is commonly assumed that the effective "surface" for heat and moisture ($z_{oh}$) is lower than $z_{om}$. However, Garratt et al. (1993) found that $z_{oh} \approx z_{om}$ for fully vegetated moist surfaces. Garratt (1992) had previously concluded that $z_{oh} = z_{om}/e^2$ for practical applications over a wide range of surfaces. This is the formulation that will be used in this study. Lee (1992) showed that small variations in roughness length have little effect on the calculated heat fluxes, since $H$ and $LE$ are both proportional to the natural logarithm of $z_{oh}$ (through the $r_e$ equation (1.26)). Changes of 10% from the reference value will produce only a 1 W m$^{-2}$ change in $H/LE$. To test this statement, $z_{oh}$ was varied between $z_{om}$ (Garratt et al. 1993) and $z_{om}/e^2$ (Garratt 1992) for a typical summer day at the Norman Mesonet site (NORM). Since this variation represents approximately a full order of magnitude, this will provide an estimate of the possible error involved in estimating $z_{oh}$. There is approximately a 50 W m$^{-2}$ difference (Fig. 1.10) between the two parameterizations of $z_{oh}$, refuting the Lee (1992) conclusion. Unfortunately, accurately specifying $z_{oh}$ requires knowledge of the vegetation height. Since these data are not available operationally, simple assumptions about $z_{oh}$ must be made.

To simplify the analysis, zero wind between $z_{om}$ and the soil and constant moisture and temperature profiles between $z_{oh}$ and the soil are typically assumed. This is illustrated with the hypothetical temperature profiles in Fig. 1.11.
CHAPTER 2
DATA AND METHODOLOGY

In this chapter, the data sets will be described and the specific details involved in calculating each of the terms in the surface energy budget will be outlined.

2.1 Data

2.1.1 Oklahoma Mesonet

Measurements of air temperature and relative humidity are made at 1.5 m at 115 sites with a Campbell Scientific/Vaisala HMP35C dual probe. Readings are taken every 3 seconds and are averaged at 5-minute intervals. The average reading is accurate to 0.35 °C and 3% relative humidity. Atmospheric pressure is measured with a Vaisala PTB 202 barometer at 12-second intervals, again averaging over 5 minutes. The readings are accurate to 0.4 mb.

The wind speed and direction are also measured at 10 m at 3-second intervals, with averaging every 5 minutes. An R. M. Young Model 5103 Wind Monitor is employed, which has a starting threshold of 1 m s⁻¹, and an accuracy of 2% of speed and 3° of direction. The LI-COR 200SZ pyranometer measures direct solar radiation at a height of 1.8 m to within 5% and with a resolution of 0.23 W m⁻². The 5-minute average is comprised of 100 three-second averages. Finally, soil temperature is measured at 10 cm below the surface using a Fenwal soil thermistor. Samples are taken at 30-second intervals, and are averaged over 15 minutes. The readings are accurate to 0.5° C. An overhead view of the instrumentation is presented in Fig. 2.1 (Oklahoma Climatological Survey, 1994).

Campbell Scientific, Inc. (CSI) 229-L heat dissipation matric potential sensors have been installed at 60 of the Mesonet sites (Fig. 2.2a) at depths of 5, 25, 60, and
75 cm below the ground. These sensors are comprised of a thermocouple and a heater wire enclosed in a hypodermic needle (Fig. 2.3). The needle is embedded in a cylindrical ceramic shield, whose porosity allows for equilibration with the surrounding soil (Reece, 1996). An electric current is passed through the heater wire for 20 seconds, resulting in the production of heat due to the resistance of the wire. This heat production is quantified as a rise in temperature within the sensor, as detected by the thermocouple. This temperature change can then be related to the matric (soil water) potential of the soil.

2.1.2 ARM

Observations from the Atmospheric and Radiation Measurements (ARM; Stokes and Schwartz, 1994) program were used to develop and test the parameterizations in this study. Net radiometers are used to measure net radiation, while soil heat flux plates and soil temperature sensors are used to measure the ground heat flux. At 11 of the 14 sites, sensible and latent heat fluxes are measured using a Bowen ratio system, while at the remaining three sites the more accurate eddy fluctuation technique is used.

Measurements of LW_a (SW_a) are made with an upward-looking hemispherical broadband Eppley pyrgeometer (pyranometer) at the Lamont Central Facility (CF) site (36.61°N, 97.49°W, altitude = 318 m). The instruments are mounted at a height of 1.5 m and employ a 1-second sampling interval. Surface energy fluxes are measured at 14 Energy Balance Bowen Ratio (EBBR) sites across Oklahoma (Fig. 2.2b), along with air temperature, vapor pressure, and wind speed. Two 13-minute averages of 30-second samples were then averaged to get 30-minute values.

Often, the most difficult part of getting an accurate measurement of downwelling longwave radiation is overcoming the problems associated with heating of the instrument dome, which can result in spuriously high LW_a. The upward-looking pyrgeometer at the CF site is shaded and ventilated to reduce these dome heating effects, and a subsequent correction is also made based on the measured dome and case temperatures (M. Splitt, personal communication).
2.1.3 FIFE

Data from the FIFE were acquired and used to verify the LW_d parameterization (section 2.2). Barometric pressure and the wet- and dry-bulb temperatures were measured at 2 m at 10 different locations within the FIFE domain (centered at 39.05°N, 96.53°W, altitude = 410 m). These values were averaged to obtain representative site values. The site-average mixing ratio was then calculated from the site-averaged barometric pressure and wet-bulb temperature data, and was then converted to vapor pressure. The LW_d and SW_d data were measured by Eppley pyranometers and pyrgeometers, respectively, at two different sites located 14 km apart. The data were again averaged to obtain representative site values for the area. All of the FIFE data have been averaged at half-hourly intervals (before site-averages were calculated).

2.2 Net Radiation (R_n)

The net radiation received at the surface is composed of four components:

\[ R_n = SW_d - SW_u + LW_d - LW_u, \]  

(2.1)

where SW and LW represent broadband shortwave (0.15-3.0 μm) and broadband longwave radiation (3.0-100.0 μm), respectively. The subscripts u and d refer to upwelling and downwelling, respectively. Of these four components, only SW_d is measured at the Mesonet sites.

2.2.1 Downwelling Shortwave Radiation (SW_d)

The pyranometer detects incoming direct shortwave radiation from the entire upward-looking hemisphere at wavelengths from 0.4-1.1 μm. The peak spectral response of the instrument is at 0.95 μm. The pyranometers have been calibrated against a higher quality sensor (one that measures radiation at all solar wavelengths) before being installed in the field. This calibration compensates for the wavelengths that the pyranometer
cannot detect. The spectral response of the pyranometer and the normalized blackbody
spectra of the sun and the earth are shown in Fig. 2.4.

2.2.2 Upwelling Shortwave Radiation (SW_u)

The upwelling shortwave radiation is parameterized as follows:

\[ SW_u = \alpha \cdot SW_d \]  \hspace{1cm} (2.2)

where \( \alpha \) is the surface albedo. Albedos vary both diurnally and annually. Assuming flat
terrain and constant soil moisture, the diurnal cycle is characterized by higher values in
the early morning and late afternoon, with the smallest values occurring around solar
noon. Figure 2.5a shows the diurnal variation at the ARM CF site. One clear day per
month for the period October 1995 - August 1996 was chosen. There is a rather obvious
relationship between the solar zenith angle and the albedo. This relationship is confirmed
by the observational data in Fig. 2.5b.

In the morning, the incoming solar radiation intercepts the surface at a small angle.
Since most natural vegetation (like grass) consists of vertically oriented "leaves", most of
the incoming solar radiation is intercepted before it reaches the soil. Since the albedo of
the vegetation is usually greater than that of the soil, higher surface albedos result near
sunrise and sunset. Conversely, at solar noon, the soil receives more of the direct solar
radiation, resulting in a significantly smaller total albedo. For this study, this diurnal
variation in albedo will be neglected. Since the biggest variations in albedo occur near
sunrise and sunset, when the magnitude of \( SW_d \) is small, the errors involved in assuming
a constant albedo throughout the day will be minimal. Betts and Ball (1998) showed that
the maximum errors found from this assumption were around 10 W m\(^{-2}\) near
sunrise/sunset.

The annual cycle of albedo is mainly a function of climatological soil wetness and
the color and state of the natural vegetation. Wetter soils are darker and have lower
albedos. In Oklahoma, soil wetness is maximized in late spring concurrent with the
rainfall maximum (Fig. 2.6). The color of the vegetation is also important. Active
vegetation (green) has a lower albedo than dormant vegetation (light brown or yellow). In general, Oklahoma is most "green" in late spring (Loveland et al., 1991), concurrent with the rainfall maximum. The combination of wet soil and the color of the vegetation produce a sinusoidal variation in albedo.

In order to parameterize the albedo more effectively, six years of radiation observations from a Norman experimental site were acquired (C. Duchon, University of Oklahoma (OU), pers. comm.) and averaged to produce a "climatological" annual albedo variation (Fig. 2.7). Individual days with missing data or with abnormally high albedos (due to snow cover) were not included in the averaging process, so some daily averages may be over less than six years. A Fourier analysis was performed and it was found that the annual cycle (harmonic 1) accounted for 69% of the total variance of the time series. Other harmonics accounted for no more than 5% of the total variance. Thus, it seems reasonable to model the annual albedo variation with a sine wave. The Fourier analysis shows that the minimum albedo at the Norman site occurs on May 13 and the maximum on November 11. The spring minimum occurs during the greenest and wettest time of year in Norman. The fall maximum occurs when vegetation is its lightest and before the hard freezes of winter. It is possible that the freezing, rotting, and subsequent darkening of dead vegetation could explain the winter drop in albedo. The range of albedos corresponds well to the 0.18-0.25 range specified by Oke (1987) for most crops and natural vegetation. Although this specification still does not take short-term variations in soil moisture into account, it is still more physically realistic than using a constant albedo at all times and locations.

However, it should be noted that using a spatially and temporally constant albedo will not result in large errors. For example, the parameterized Norman albedos (Fig. 2.7) range from 0.175-0.225. If a constant albedo of 0.2 was assumed instead, along with a typical SW4 value of 500 W m⁻², a maximum error of 12.5 W m⁻² will result. Also, mean yearly albedos range from approximately 0.18-0.22 from east to west across Oklahoma.
Again, using the constant albedo assumption would result in relatively small errors of around 10 W m$^{-2}$.

In order to confirm these results with an independent data source, data from four ARM sites were collected and analyzed. One clear day per month during the period September 1995 - August 1996 was chosen in order to resolve the annual albedo variation. On one of these days, snow covered the ground at two of the ARM sites, so data from this day were discarded. On each day, the albedos from the four sites were averaged, producing a "double-ramp" annual variation in albedo (Fig. 2.8a). This signal was also present in FIFE data taken from near Manhattan, Kansas. Fig. 2.8b from Betts and Ball (1998) displays a three-year (1987-1989) albedo average in 30-day bins. Again, as with the ARM data, the annual cycle is characterized by a relatively constant albedo through most of the late spring and summer. The periods of decreasing (days 50-110) and increasing albedo (days 260-360) likely correspond to the growth and senescence of the native vegetation, respectively. The FIFE data do not display the plateau throughout the winter months as the ARM data did. This is likely because the data from the days with snow cover were not removed from the FIFE data.

It is apparent that both the ARM and the FIFE data show some significant differences with the data from the Norman site, despite a comparable range of 0.18-0.24. Further examination of Figs. 2.7-2.8 reveals that the main difference in the data sets is the significant increase in albedo at the Norman site during the month of June (days 150-180). This may be representative of the rapid depletion of soil moisture that typically occurs before the hot, dry Oklahoma summer. This signal does not appear in the FIFE data from northeastern Kansas, possibly implying that soil moisture remains plentiful throughout the summer at that site. In fact, Richman and Lamb (1985) showed that the Norman site and the FIFE site are located in different climatic rainfall regions, so one would expect the albedo variations to be dissimilar as well. Even though the ARM data supports the FIFE data, the coarseness of the data set (one day per month) makes any firm
conclusions from this data set questionable as well. Because of this uncertainty, and since there is a significantly larger volume of data in the Norman site averages, the sinusoidal annual albedo variation found there will be used to model the albedo at all the Mesonet sites. The amplitude and phase of this variation will be left constant for all locations, while the mean albedo will vary across the state in order to represent the climatological variation in rainfall and greenness.

To specify the mean at each location, a monthly mean albedo chart from April 1996 will be used (Fig. 2.9). This 0.5° x 0.5° albedo product is derived from satellite estimates from the Global Energy and Water Experiment (GEWEX) Continental-Scale International Project (GCIP, Leese, 1993). While it is apparent that the absolute values of albedo provided by this product are too low, the albedo gradient across the state is still accurately represented as an east-to-west increase (R. Pinker, University of Maryland (UM), pers. comm.). To parameterize the albedo at a given time each of the Mesonet sites, the difference in the GCIP albedos between a given site and the Norman site is added to the calculated Norman albedo in Fig. 2.7.

2.2.3 Downwelling Longwave Radiation (LWd)

The amount of downwelling longwave radiation is determined by the bulk emissivity (\( \varepsilon_{am} \)) and effective temperature (\( T_{am} \)) of the overlying atmosphere according to

\[
LW_d = \varepsilon_{am} \sigma T_{am}^4,
\]

where \( \sigma \) is the Stefan-Boltzmann constant. Since it is difficult to specify \( \varepsilon_{am} \) or \( T_{am} \) for a vertical column of atmosphere, methods have been developed to parameterize \( LW_d \) from the measured temperature and/or vapor pressure near the surface during clear skies such that

\[
LW_d = \varepsilon_c(T,e)\sigma T^4,
\]

where \( \varepsilon_c \) is the effective clear-sky atmospheric emissivity, and \( T \) and \( e \) are the near-surface temperature and vapor pressure, respectively. In the past, empirical formulations for \( \varepsilon_c \) were developed based on least-squares regression of observed \( LW_d \) during periods
of clear skies. Many reasonably successful techniques using this method have been developed in recent decades.

Brunt (1932), on the basis of a perceived similarity between heat conduction and radiative transfer, theorized that $LW_d$ was related to the square root of $e$. Using monthly averages of $LW_d$ and $e$, he developed the first empirical relationship between the two quantities. Use of this technique resulted in a correlation coefficient of 0.97. Three decades later, Swinbank (1963) argued that $LW_d$ was not related to $e$ at all, but to the square of $T$ alone, and that the Brunt (1932) formula worked only due to the positive correlation between $e$ and $T$. Swinbank's new formula resulted in a correlation coefficient of 0.99 between observed and estimated $LW_d$ and an RMSE of less than 5 W m$^{-2}$.

Idso and Jackson (1969) developed an equation, also dependent on $T^2$, which was tested against a much wider range of temperatures than the previous formulations. This formula was touted as being "valid at any latitude and for any air temperature reached on earth," and had a correlation coefficient of 0.99 between measured and estimated $LW_d$. Staley and Jurica (1972) integrated the emissivity over the entire atmosphere using vertical profiles of vapor pressure, carbon dioxide and ozone, and related it to $e$. This formula was much more theoretical than previous efforts, and was successfully used by Deardorff (1978) in the development of a surface energy budget model. Finally, Satterlund (1979) found that previous formulations didn't perform well in temperatures below 0 °C, and developed a new formulation that claimed to be more accurate at extreme temperatures and of similar accuracy at moderate temperatures.

Brutsaert (1975) (hereafter B75) was the first to develop a more physically rigorous parameterization. B75 is based on Schwarzschild's equation (Liou, 1980, p. 22) and assumptions of standard atmospheric lapse rates of temperature and vapor pressure. Culf and Gash (1993) concluded that this method was superior to previous formulations, since it is easily adjusted for locally measured lapse rates. Like so many of the other
techniques, though, it was developed for clear skies alone. Since the presence of clouds significantly increases the total effective emissivity (ε) of the sky, modifications must be made to the existing clear-sky formulations. Deardorff (1978) developed a simple correction for cloudiness, and applied it to the Staley and Jurica (1972) clear-sky parameterization for emissivity.

For this study, an improved method has been developed which incorporates the B75 clear sky parameterization and the Deardorff (1978) cloudiness correction. A clear-sky model is used along with the observed magnitude of solar radiation to provide a proxy for fractional cloudiness. An annual sinusoidal modification has also been added to the clear-sky Brutsaert parameterization coefficient, representative of typical variations in vertical atmospheric profiles of water vapor pressure. This new parameterization performs better than previous methods and is accurate at any time of the year and in any sky condition, although its usefulness is restricted to daylight hours (Crawford and Duchon, 1998).

The cloudiness correction involves introducing a cloud fraction term (clf). In this study, clf is defined by

\[ clf = 1 - s \]  

in which \( s \) is the ratio of the measured solar irradiance to the clear sky irradiance. The clear-sky shortwave irradiance (I) at the ground is calculated using a previously developed model based on the results of Paltridge and Platt (1976) and Meyers and Dale (1983). This quantity is approximated by

\[ I = I_0 (\cos \gamma) T_R T_{pg} T_w T_a, \]  

where \( I_0 \) is the effective solar constant, \( \gamma \) is the solar zenith angle, and \( T_i \) the transmission coefficients for Rayleigh scattering (R), absorption by permanent gases (pg) and water vapor (w), and absorption and scattering by aerosols (a).

The effective solar constant (in W m\(^{-2}\)) is given by:

\[ I_0 = 1370(\tau/r)^2, \]  

30
where $r$ and $r$ are the average and daily distances between the sun and the earth, respectively. The cosine of the solar zenith angle ($Z$) is represented by
\[
\cos Z = \sin \gamma \sin \delta + \cos \gamma \cos \delta \cos H,
\]
where $\gamma$ is the latitude of the station, $\delta$ the solar declination, and $H$ the hour angle. The hour angle is
\[
H = \left(\frac{\pi}{12}\right)(t_{\text{noon}} - t),
\]
where $t_{\text{noon}}$ is local solar noon (~12.5 in central Oklahoma) and $t$ is the local solar time (e.g., $t = 12.5$ and $H = 0$ at local solar noon). The empirical expression for the product of the first two transmission coefficients is (Atwater and Brown, 1974)
\[
T_R T_{ps} = 1.021 - 0.084[m(0.000949p + 0.051)]^{1/2},
\]
where $p$ is the pressure in mb and $m$ is the optical air mass at $p = 1013$ mb given by $m = 35 \cos Z(1224 \cos^2 Z + 1)^{1/2}$. The third coefficient is (McDonald, 1960)
\[
T_w = 1 - 0.077(\text{um})^{0.5},
\]
where $u$ is the precipitable water given by $u = \exp[0.1133 - \ln(G + 1) + 0.0393T_d]$ (Smith, 1966), $T_d$ is the dewpoint ($^\circ$F), and $G$ is an empirical constant dependent upon time of year and latitude. The fourth transmission coefficient is (Houghton, 1954; Meyers and Dale, 1983)
\[
T_s = 0.935^n.
\]

Once $I$ was calculated from (2.6), direct observations of solar irradiance were used to calculate $s$ and $c_l$ in (2.5). Inclusion of the effects of clouds yields
\[
LW_d = [c_l + (1-c_l) \varepsilon_c] \sigma T^4 = \varepsilon \sigma T^4.
\]
As $c_l$ increases from 0 to 1, $\varepsilon$ proportionally increases between the clear-sky value ($\varepsilon = \varepsilon_c$) and the limiting (but unobserved) value ($\varepsilon = 1$). Calculated values of $c_l$ less than zero were adjusted back to zero so as to be physically realistic. This happens occasionally when the observed solar radiation is larger than the theoretical clear-sky solar radiation, due to instrument error or to large amounts of diffuse radiation reaching the sensor.
Six popular \( \varepsilon \) formulations, modified by the cloudiness correction and inserted into (2.13), were tested against data collected at the ARM CF:

\[
\varepsilon = \left[ \frac{\text{clf} + (1 - \text{clf})(0.68 + 0.036e^{1/2})}{\text{clf} + (1 - \text{clf})(0.67(1670q)^{0.08})} \right] \quad \text{Anderson (1954)} \quad (2.14)
\]

\[
\varepsilon = \left[ \frac{\text{clf} + (1 - \text{clf})(9.36 \times 10^{-6} T^2)}{\text{clf} + (1 - \text{clf})(1 - (0.261 \exp(-7.77 \times 10^4 (273.15-T)^2))} \right] \quad \text{Swinbank (1963)} \quad (2.15)
\]

\[
\varepsilon = \left[ \frac{\text{clf} + (1 - \text{clf})(1 - (0.261 \exp(-7.77 \times 10^4 (273.15-T)^2))}{\text{clf} - K_{l} - \text{clf}(1.24(e/T)^{1/7})} \right] \quad \text{Idso/Jackson (1969)} \quad (2.16)
\]

\[
\varepsilon = \left[ \frac{\text{clf} + (1 - \text{clf})(0.67(1670q)^{0.08})}{\text{clf} + (1 - \text{clf})(1.08(1 - \exp(e^{1700}))} \right] \quad \text{Staley/Jurica (1972)} \quad (2.17)
\]

\[
\varepsilon = \left[ \frac{\text{clf} - K_{l} - \text{clf}(1.24(e/T)^{1/7})}{\text{clf} - K_{l} - \text{clf}(1.08(1 - \exp(e^{1700}))} \right] \quad \text{B75 (1972)} \quad (2.18)
\]

\[
\varepsilon = \left[ \frac{\text{clf} + (1 - \text{clf})(0.67(1670q)^{0.08})}{\text{clf} - K_{l} - \text{clf}(1.08(1 - \exp(e^{1700}))} \right] \quad \text{Satterlund (1979)} \quad (2.19)
\]

\( T \) is in degrees K and \( \varepsilon \) is in mb. In (2.17), \( q \) is the specific humidity, which is a function of \( \varepsilon \) and the barometric pressure. Equation (2.14) is a modification of the formulation originally developed by Brunt (1932), and has been shown to provide acceptable clear-sky estimates of \( \text{LW}_d \) (using the value of \( \varepsilon \) in (2.13)) in Oklahoma (Arnfield, 1979).

Calculations of \( \text{LW}_d \) using (2.13) and the six parameterizations of \( \varepsilon \) (2.14)-(2.19) were first compared to observed \( \text{LW}_d \) ARM CF data over four one-month periods: November 1995, February 1996, May 1996, and August 1996. By doing this, seasonal biases in the performance of any of the formulas can be detected. Since the clear-sky model can only be used during daylight hours, the total amount of data available is restricted. For fall and winter (spring and summer), data from 1400-2230 (1300-2330) UTC were used. Table 2.1 shows the results of these comparisons.

The MBE (mean bias error) is given by

\[
\text{MBE} = \frac{1}{n} \sum_{i=1}^{n} (\text{LW}(p)_{d,i} - \text{LW}(o)_{d,i}), \quad (2.20)
\]

where \( \text{LW}(p)_{d,i} \) and \( \text{LW}(o)_{d,i} \) are the parameterized and observed values, respectively, and \( n \) is the total number of half-hourly averaged observations for the month. A positive value means that the parameterization overestimates \( \text{LW}_d \). Only the Anderson (1954) and the B75 schemes have absolute MBEs that were consistently less than 12 W m\(^{-2}\)
<table>
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<td>best fit y-int. (W m⁻²)</td>
<td>14.3</td>
<td>-22.7</td>
<td>60.4</td>
<td>114.8</td>
</tr>
<tr>
<td>best fit slope</td>
<td>1.01</td>
<td>1.14</td>
<td>0.88</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>MBE (W m⁻²)</td>
<td>RMSE (W m⁻²)</td>
<td>best fit R</td>
<td>best fit y-int. (W m⁻²)</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------</td>
<td>--------------</td>
<td>------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>Brutsaert (1975)</td>
<td>0.2</td>
<td>14.9</td>
<td>0.95</td>
<td>-36.6</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>22.2</td>
<td>0.92</td>
<td>-48.7</td>
</tr>
<tr>
<td></td>
<td>-10.0</td>
<td>11.9</td>
<td>0.94</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td>-11.9</td>
<td>10.5</td>
<td>0.85</td>
<td>104.7</td>
</tr>
<tr>
<td>Satterlund (1979)</td>
<td>19.7</td>
<td>14.9</td>
<td>0.93</td>
<td>19.2</td>
</tr>
<tr>
<td></td>
<td>15.8</td>
<td>20.2</td>
<td>0.91</td>
<td>-19.6</td>
</tr>
<tr>
<td></td>
<td>14.9</td>
<td>13.4</td>
<td>0.91</td>
<td>65.4</td>
</tr>
<tr>
<td></td>
<td>10.3</td>
<td>10.9</td>
<td>0.84</td>
<td>108.6</td>
</tr>
</tbody>
</table>

Table 2.1. Comparative LW₄ statistics for the six formulations (2.14)-(2.19).

throughout the year.

The RMSE (root mean square error) is given by

$$\text{RMSE} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} [(LW(p)_{d,i} - LW(o)_{d,i}) - MBE]^2}$$

Assuming a normal error distribution means that 68% of the individual errors are within a range bounded by the MBE +/- RMSE. Values of the RMSE were consistently higher in fall and winter than in spring and summer for all schemes. The Anderson (1954) scheme had the lowest RMSEs over the course of the year. Statistics from a linear regression of the comparisons were also analyzed. The correlation coefficient is denoted by "best fit R". The B75 and Anderson (1954) schemes had the highest values throughout the year, and every scheme had its smallest value in the summer. The y-intercept and the slope of the regression lines are also given in Table 2.1. A perfect scheme would produce a y-intercept of zero and a slope of one. The Idso/Jackson (1969) scheme performed best in these two categories, but had the largest RMSEs of all the equations tested.
Based on their comparatively low MBE and RMSE values, the Anderson (1954) and the B75 formulations were considered to be superior to the other four. Scatterplots of the two schemes for November 1995 are depicted in Figs. 2.10 and 2.11. One can see that the distributions of data points about the linear fits are similar. Since the B75 equation has a physically based derivation while the Anderson (1954) equation is strictly empirical, the B75 scheme was chosen for further investigation.

Equation (2.18) was derived using Schwarzschild's radiative transfer equation, in which standard atmosphere vertical profiles of temperature and vapor pressure were used to calculate the leading coefficient of 1.24 (see (1)-(11) in B75). Using measured profiles of water vapor pressure and temperature in Niger, Culf and Gash (1993) were able to re-derive the original B75 equation and get a slightly different value of the leading coefficient. By doing this they were able to reduce the RMSE by 50%. They found that, during the dry season in Niger, the lapse rate of vapor pressure is significantly smaller than the standard atmospheric lapse rate assumed in B75. This means that for the given surface conditions there is more water vapor aloft, and thus higher emissivity, than expected using a standard atmosphere vapor pressure profile. Because of this, the leading coefficient was increased to 1.31 to properly represent \( LW_d \) in these conditions. On the other hand, for wet surface conditions, the vapor pressure lapse rate would be expected to be larger than in a standard atmosphere, and the B75 coefficient would have to be reduced to compensate. It is important to note that it is the magnitude of the lapse rate that determines the correct value of the leading coefficient, not the magnitude of the measured vapor pressure at the surface.

Since the Culf and Gash (1993) results showed a variation in the leading coefficient between the dry season and the wet season, it was hypothesized that this coefficient may
undergo an annual sinusoidal variation similar to that of other meteorological variables (temperature, solar radiation, vapor pressure). In Oklahoma, the lowest value was expected to occur in July (more humid, larger vapor pressure lapse rates) with the highest value in January (less humid, smaller vapor pressure lapse rates). Analysis of a 12-month period of ARM CF data (November 1995-October 1996) was performed in order to find the best-fit sinusoidal variation. The resultant leading coefficients range from 1.28 in January to 1.16 in July according to

\[ \text{LW}_d = [\text{clf} + (1-\text{clf})(1.22 + 0.06 \times \sin((\text{month} + 2)\pi/6))(e/T)^{1/7}]\sigma T^4, \]

(2.22)

where month is the numerical month (e.g., January = 1). The fact that the leading coefficient is largest in winter and smallest in summer may seem counterintuitive, since \( \text{LW}_d \) is least in winter and greatest in summer. However, the relatively low \( e \) during the winter months more than offsets the larger leading coefficient, with the opposite relationship during the summer months.

Table 2.2 reveals that using (2.22) rather than (2.18) resulted in fairly uniform and small absolute MBEs (less than 10 W m\(^{-2}\)) throughout the year. It is immediately apparent that the new scheme had a smaller MBE than the other five schemes (Table 2.1),

<table>
<thead>
<tr>
<th>MONTH</th>
<th>MBE (W m(^{-2}))</th>
<th>RMSE (W m(^{-2}))</th>
<th>R</th>
<th>y-int. (W m(^{-2}))</th>
<th>slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 1995</td>
<td>-1.6</td>
<td>15.0</td>
<td>0.94</td>
<td>-34.6</td>
<td>1.13</td>
</tr>
<tr>
<td>December 1995</td>
<td>-4.3</td>
<td>16.6</td>
<td>0.93</td>
<td>-3.6</td>
<td>1.00</td>
</tr>
<tr>
<td>January 1996</td>
<td>0.2</td>
<td>19.6</td>
<td>0.91</td>
<td>-9.2</td>
<td>1.04</td>
</tr>
<tr>
<td>February 1996</td>
<td>-0.5</td>
<td>22.3</td>
<td>0.91</td>
<td>-55.4</td>
<td>1.21</td>
</tr>
<tr>
<td>March 1996</td>
<td>-1.1</td>
<td>17.9</td>
<td>0.95</td>
<td>-13.6</td>
<td>1.05</td>
</tr>
</tbody>
</table>
Table 2.2. Statistics for the improved formulation, based on a sinusoidal variation of the leading coefficient.

with the exception of the Anderson (1954) scheme in August. RMSEs are similar to the original B75 scheme, while correlation coefficients are slightly smaller. The $y$-intercept and slope of the regression lines are also similar to B75. The new scheme appears to be valid for all seasons and sky conditions with RMSEs of less than 23 W m$^{-2}$ for 30-minute averages. This scheme will not work at night, since there is then no way to ascertain cloud fraction.

Because of the empiricism involved in fitting the sinusoid to the CF data, the broader applicability of (2.22) may be in question. For this reason, data from FIFE were acquired. These observations of $LW_d$ from the summer of 1987 were compared to the parameterization represented by (2.22), and the error statistics are shown in Table 2.3. The FIFE comparisons show larger MBEs but smaller RMSEs than the ARM CF data.
Table 2.3. Comparisons of ARM and FIFE LW\textsubscript{d} data sets using the sinusoidal variation in effective atmospheric emissivity in (2.22). ARM data are from 1996; FIFE data are from 1987. Units in W m\textsuperscript{-2}.

| September MBE | -8.7 | -11.9 | September RMSE | 16.2 | 10.8 |

Considering that the method was developed empirically for ARM data, the FIFE comparisons are very encouraging.

The results obtained using this new parameterization compare favorably to recent parameterizations of LW\textsubscript{d} as well. Sugita and Brutsaert (1993) were able to calibrate empirically the adjustable parameters in their formulation using 1987 FIFE data (summer and fall only) to reduce their RMSE to around 15-17 W m\textsuperscript{-2}, larger than the summer and fall errors found by our improved technique. Moreover, they used only 325 data points and did not test the effectiveness of their method in winter and spring. Culf and Gash (1993), by tuning the Brutsaert coefficient to 1.31 during the dry season in Niger in 1990, were able to get RMSE of 12 W m\textsuperscript{-2}, but again this is a limited data set from summer and early fall and was only valid in clear skies. Because of these encouraging results, this new technique will be used in this study to estimate LW\textsubscript{d}.

2.2.4 Upwelling Longwave Radiation (LW\textsubscript{u})

Outgoing longwave radiation (LW\textsubscript{u}) is a function of the temperature (T\textsubscript{u}) and emissivity (e\textsubscript{u}) of the surface:

$$LW_{u} = e_{u} \sigma T_{u}^{4}.$$  \hspace{1cm} (2.23)

Surface emissivities at all sites are set in the model to 0.98, which is representative of natural land surfaces in Oklahoma (Humes et al., 1994). T\textsubscript{u} is set to the temperature at z_{ob} (T_{ob}, roughness temperature). This temperature is calculated by integrating downwards from the measured air temperature at 1.5 m (T):
\[ T_s = T_{soil} = T + r_s H/c_p. \] (2.24)

The estimated value of \( H \) from the previous 5-minute interval is used in (2.24) to calculate \( T_s \).

### 2.3 Ground Heat Flux (\( G \))

The ground heat flux (\( G \)) describes the conduction of heat from the overlying atmosphere into the soil. Smith et al. (1992) summarized the difficulty in accurately measuring this term:

> The soil heat flux measurements are the least consistent of the four flux quantities but that is to be expected because of the heterogeneous nature of soils, soil moisture, and canopy cover.

The soil type at a given location significantly influences the surrounding climate. The texture and porosity determine the ability of the soil to conduct heat. And, as mentioned previously, the amount of soil moisture is often an important factor in determining the partitioning of the available energy into sensible and latent heat fluxes. Variations in soil type also significantly influence the transfer of heat from the surface into the ground.

For example, peat soil, which has properties between those of organic surface material and other soils, is highly porous (full of air). Since air has very low thermal conductivity, peat transfers very little heat from the surface into the soil. Because of this, a dry peat surface can heat up significantly, resulting in very large values of \( H \). On the other hand, moist sand or wet clay soils have relatively high conductivity, allowing large amounts of energy to be transferred into the soil. Because of this and the higher values of \( LE \) associated with wet ground, the diurnal temperature cycle is dampened (Oke, 1987).

The amount and density of native vegetation also modulate ground heat flux. For bare soils, \( G \) is generally much higher, since more net radiation can reach the surface.
The ad-hoc parameterization $G = 0.1 R_a$ is commonly employed in surface energy budget studies. Here, we will utilize the Mesonet soil temperature and soil moisture sensors to implement a more physically based parameterization for $G$.

If we assume that heat is conserved within the first 5 cm of soil, then the rate of change of temperature within that soil layer is determined by the heat flux gradient between the top and bottom of the layer (Garratt, 1992):

$$\frac{\partial T_s}{\partial t} = \frac{\partial G}{\partial z} = \frac{G - G_5}{0.05}.$$  

(2.25)

where $c_s$ is the heat capacity per unit volume of soil and $T_s$ is the 5-cm soil temperature, which will represent the layer temperature. $G$ represents the soil heat flux at the surface (ground heat flux), while $G_5$ represents the 5 cm soil heat flux:

$$G_5 = k_{hs} c_s \frac{\partial T_s}{\partial z} = \frac{T_s - T_{10}}{0.1}.$$  

(2.26)

Here, $k_{hs}$ and $\lambda$ are the thermal diffusivity and thermal conductivity of the soil, $T_s$ represents the soil temperature, and $T_{10}$ is the 10 cm soil temperature. The final form of $G$ follows from a combination of (2.25) and (2.26):

$$G = 0.05 c_s \frac{\partial T_s}{\partial t} + \frac{T_s - T_{10}}{0.1}.$$  

(2.27)

The first term on the RHS of (2.27) is referred to as the storage term, since it represents the energy that is "stored" in the topsoil as heat. $T_s$ is measured by the soil moisture sensors and $T_{10}$ is measured by the standard soil temperature sensors. Since $T_s$ is only measured every 30 minutes, linear interpolation is used to specify values at 5-minute intervals. To calculate $\lambda$ and $c_s$, the newly installed soil moisture sensors must be utilized. The second term is the flux term, since it represents the transfer of energy.
through the soil. Typically, the flux term is 5-10 times larger than the storage term (C. Marshall, NCEP, pers. comm.).

The temperature change in the soil moisture sensor is inversely related to \( X \). A large (small) temperature rise in the sensor implies that the soil cannot effectively conduct the heat away from the sensor due to high (low) \( \lambda \). It is well known that low (high) \( \lambda \) is associated with dry (wet) soil. Two methods of estimating \( \lambda \) were tested.

2.3.1 Calculating Thermal Conductivity - Physical Method

To calculate the thermal conductivity, we can assume that the sensor acts as an infinitely long line heat source in a homogeneous and isotropic sample of soil. Shiozawa and Campbell (1990) showed that the temperature rise (\( \Delta T \)) associated with the heat produced from an electrical current is

\[
\Delta T = \left( \frac{q}{4\pi \lambda} \right) \int_{r^2/4Dt}^{\infty} \frac{1}{u} \exp(-u) \, du,
\]

where \( q \) is the heat produced in the sensor per unit time, \( r \) is the radial distance from the source, \( D \) is the thermal diffusivity, \( t \) is the elapsed time since the current was switched on, and \( u \) is a variable of integration. For \( r^2/4Dt \ll 1 \) (since \( r \) is very small in this case),

\[
\Delta T \approx \left( \frac{q}{4\pi \lambda} \right) \left[ -\gamma - \ln\left( \frac{r^2}{4Dt} \right) \right] = \left( \frac{q}{4\pi \lambda} \right) \left[ c + \ln(t) \right],
\]

where \( \gamma \) is Euler’s constant (~0.577), and \( c \) is a separate constant. Equation (2.29) can be re-written in linear form:

\[
\Delta T = \frac{q \ln(t)}{4\pi \lambda} + \frac{qc}{4\pi \lambda} = q \frac{\ln(t)}{4\pi \lambda} + d,
\]

where \( d \) is a lumped constant.

If a plot of \( \Delta T \) vs. \( \ln(t) \) were constructed, the slope \( (m = q/4\pi \lambda) \) could be extracted through regression analysis (Fig. 2.12). Then \( \lambda \) can be calculated directly.
The quantity \( q \) is expressed as the ratio of the power produced by the electric current and the length of the heating wire (\( L, 28 \) mm) per unit time:

\[
q = \frac{I^2R}{L} = 2.86 \text{ W m}^{-1} \text{ s}^{-1}, \tag{2.32}
\]

where \( I \) and \( R \) are the current (50 mA) and resistance of the wire (32 \( \Omega \)), respectively (K. Fisher, formerly of Cooperative Institute for Mesoscale Meteorological Studies (CIMMS), pers. comm.).

The quantity \( m \) is the slope of the line that represents the temperature rise (\( \Delta T \)) as a function of the natural logarithm of the time interval (20 seconds):

\[
m = \frac{(\Delta T - 1.0)}{\ln 20}. \tag{2.33}
\]

The offset of 1.0 degree accounts for the start-up time after the electrical current has been switched on. Substituting (2.32) and (2.33) into (2.31) produces

\[
\lambda = \frac{0.682}{(\Delta T - 1.0)}. \tag{2.34}
\]

Equation (2.34) allows for the calculation of thermal conductivity from the raw \( \Delta T \) data available from the Mesonet sensors at a 5-cm depth. This value can then be used to represent \( \lambda \) in the top 10 cm of the soil.

To verify this technique, data from the newly installed 229-L probe at the Norman Mesonet site were examined. Typical \( \Delta T \) values range from 1.3 (wet ground) to 3.4 degrees (dry ground) (Fig. 2.13, J. Basara, personal communication). Using (2.34), this corresponds to a range in \( \lambda \) from 2.27 (wet) to 0.30 (dry). These values compare favorably to those quoted for dry and saturated soils in Oke (1987, p. 44).
Unfortunately, this technique is not universally applicable. The offset of 1.0 degree is an estimate from only one sensor, while individual sensors have exhibited a wide array of responses to a given set of conditions (K. Fisher, formerly of CIMMS, pers. comm.). Because of this, an alternative method must be employed to account for sensor-to-sensor differences.

2.3.2 Calculating Thermal Conductivity - Calibration Method

Twenty measurements were made with each of the 221 available 229-L probes in both “dry” and “wet” soils, and the averages were compiled. The results are shown in Fig. 2.14 (K. Fisher, formerly of CIMMS, pers. comm.). The average will be used as the “reference” or normalized response. The range of sensor responses for a given soil wetness is greater than 1°C. Because of this, adjustments were made to each of the sensor responses in order to normalize them. Each sensor has been assigned a set of two unique calibration coefficients ($b_1$ and $b_2$, see (2.36)) in order to adjust for sensor irregularity (K. Fisher, formerly of CIMMS, pers. comm.). Then, sensors were tested against known values of soil water potential ($\psi$) in order to establish the relationship between sensor response and potential. The soil water potential is the work required to extract water from the soil against surface tension, and is expressed as a negative value and in terms of work per mass of water. This value can then multiplied by the density of water (~1000 kg m$^{-3}$) and be expressed as a pressure. Larger absolute values imply drier soil, since it takes more work to extract the remaining water. Regression analysis produced the following equation:

\[
\psi = \frac{1}{a} \left[ \left( \frac{dT_w - dT_d}{dT_{ref} - dT_d} \right) - 0.9 \right]^{1/n},
\]

where $a = -0.01$ kPa$^{-1}$, $dT_w = 1.45 ^\circ C$, $dT_d = 4.0 ^\circ C$, $n = 0.77$, and
\[ dT_{ref} = b_1 \Delta T + b_2, \]  

(2.36)

where \( b_1 \) and \( b_2 \) are the two calibration coefficients mentioned above which are unique to each sensor.

The next step is converting these potentials into thermal conductivity. Al Nakshabandi and Kohnke (1965) concluded that this relationship is independent of soil type. Reece (1996) calibrated six 229-L sensors against standard potentials and found that the inverse of the thermal conductivity was linearly related to the logarithm of the potential:

\[ \lambda^{-1} = 0.134 + 0.529 \ln(-\psi). \]  

(2.37)

There are no direct measurements of thermal conductivity available to verify this technique. So, in order to test this formulation, ground heat flux and net radiation data from five enhanced Mesonet stations in Oklahoma were acquired (Fig. 2.15): Apache (APAC), Goodwell (GOOD), Marena (MARE), Norman (NORM), and Wister (WIST). Equation (2.27) was used to calculate \( G \). Since there are no direct measurements of \( T_e \), it must be estimated from \( T \). In a typical diurnal cycle, there are two times when \( T_e = T \), once after sunrise and once before sunset. As an estimate, the assumption has been made that \( T_e = T \) when the measured net radiation at the surface is zero, i.e., when the net incoming solar radiation equals the net outgoing longwave radiation from the Earth's surface. The net radiation data at the enhanced sites were used to determine when this occurs. Unfortunately, the measured ground heat flux at the five sites does not account for the storage term in (2.27) (C. Marshall, National Center for Environmental Prediction (NCEP), pers. comm.). However, the storage term is typically negligible in the early morning. Using (2.27), \( G \) can be calculated and compared to the directly measured
values. This was done at the five Mesonet sites using data from five clear mornings from spring 1997.

The comparisons at APAC are very favorable (Fig. 2.16). The individual errors are less than 5 W m\(^2\) except on 3 May. On that day, relatively strong 10 m winds near sunrise (6 m s\(^{-1}\)) likely resulted in stronger vertical mixing near the surface. This means that \(T_s\) was likely greater than \(T\) at this time, which would account for the larger error found on that day. Similar conclusions can be drawn from the GOOD comparisons (Fig. 2.17). The significant errors found in the MARE comparisons (Fig. 2.18) are consistent from day-to-day, and their cause is not apparent. The WIST data (Fig. 2.19) shows small errors except for 30 April, when strong warm air advection resulted in very warm early morning air temperatures (21° C). It is likely that the ground surface itself had not yet responded to the warm air above it. This would cause the significant overestimation of \(G\) found on this day. Data from the NORM site were missing for these analysis times.

Overall, the results are very encouraging. The changes in calculated ground heat fluxes qualitatively followed the changes in the observed ground heat flux at all four sites, though the magnitudes of the errors differed somewhat. This qualitative similarity breeds confidence in the utility of (2.37) in this study.

### 2.3.3 Calculating Heat Capacity

For this study, \(c_s\) is calculated using the following equation (de Vries, 1963):

\[
c_s = 1.93V_m + 2.51V_{om} + 4.19\eta,
\]

(2.38)

where \(V_m\) and \(V_{om}\) represent the fractional volume of solids and organic matter in the soil, respectively, and \(\eta\) is the fractional volume of water in the soil (volumetric water content). For practical purposes, the assumption \(V_m = 0.5\) and \(V_{om} = 0\) is a good one (K.
Fisher, formerly of CIMMS, pers. comm.). To calculate $\eta$, a calibration equation relating $\eta$ to $\psi$ was employed (K. Fisher, formerly of CIMMS, pers. comm.):

$$\eta = \eta_r + \frac{\eta_s - \eta_r}{[1 + (\alpha(-\psi/100))^n]^{1-1/n}},$$

(2.39)

where $\eta_r$ and $\eta_s$ are the residual and saturated water content, and $\alpha$ and $n$ are empirical constants. Each soil moisture sensor has unique values of $\eta_r$, $\eta_s$, $\alpha$, and $n$ which are representative of the soil at a given site.

2.4 Latent Heat Flux (LE)

Latent heat is energy associated with the evaporation of water. This energy is stored in the water vapor molecule, and can be released as heat upon condensation. Evapotranspiration from the surface is primarily due to transpiration from vegetated surfaces (except for within 1-2 days immediately following precipitation events, when evaporation is dominant). Pores on the surface of the vegetation (stomata) open during the daytime in response to sunlight in order to take in carbon dioxide. In doing so, the watery interior of the plant is exposed, allowing transpiration to occur. The latent heat flux, then, describes the vertical transfer of water vapor from the surface (vegetation and soil) to the overlying atmosphere. The magnitude of the flux is dependent upon the availability of water in the vegetation, the humidity of the overlying atmosphere, and the amount of turbulent vertical mixing just above the surface among other things. The latent heat flux is most significant during the daytime, but usually also continues at a reduced rate at night. It can be calculated using the PM formula (same as (1.17)):

$$LE = \frac{(R_n - G)\Delta r_s \gamma^{-1} + [e_s(T) - e] \rho c_p \gamma^{-1}}{r_s + (1+\Delta \gamma^{-1})r_s}.$$  

(2.38)
A crude scale analysis of (2.38) follows for typical warm season conditions in Oklahoma:

\( (R_a - G) \sim 500 \text{ W m}^{-2} \)
\( \Delta \sim 200 \text{ Pa K}^{-1} \text{ at } T = 25^\circ \text{C} \)
\( r_s \sim 50 \text{ s m}^{-1} \)
\( \gamma^{-1} \sim 0.015 \text{ K Pa}^{-1} \)
\( e_s(T) - e \sim 1000 \text{ Pa for 75\% RH at } T = 25^\circ \text{C} \)
\( \rho c_p \sim 1000 \text{ J K}^{-1} \text{ m}^{-3} \)
\( r_s \sim 50 \text{ s m}^{-1} \)

\[
\text{LE} \sim \frac{(500)(200)(50)(0.015) + (1000)(1000)(0.015)}{50 + (1 + (200)(0.015))50} \quad (2.39)
\]

\[
\text{LE} \sim \frac{75000 + 15000}{250} \sim 360 \text{ W m}^{-2}.
\]

It is apparent that the magnitude of LE is strongly forced by both the available energy \((R_a - G)\) and the humidity deficit \((e_s(T) - e)\) terms in the numerator of (2.38). The available energy dependence is rather simple to understand, since increases in the amount of available energy would automatically cause increases in both LE and \(H\) (1.2). The humidity deficit dependence can best be described using a simple analogy. If the streets are wet after a rainstorm, they will dry more quickly if the overlying air mass is dry (large humidity deficit) than if it is saturated (small humidity deficit). For typical cool season conditions in Oklahoma, the analysis is significantly different:

\( (R_a - G) \sim 300 \text{ W m}^{-2} \)
\( \Delta \sim 40 \text{ Pa K}^{-1} \text{ at } T = 5^\circ \text{C} \)
\( r_s \sim 50 \text{ s m}^{-1} \)
\( \gamma^{-1} \sim 0.015 \text{ K Pa}^{-1} \)
\( e_s(T) - e \sim 200 \text{ Pa for 75\% RH at } T = 5^\circ \text{C} \)
\( \rho c_p \sim 1000 \text{ J K}^{-1} \text{ m}^{-3} \)
\( r_s \sim 50 \text{ s m}^{-1} \)

\[
\text{LE} \sim \frac{(300)(40)(50)(0.015) + (200)(1000)(0.015)}{50 + (1 + (40)(0.015))50} \quad (2.40)
\]

47
LE \sim \frac{9000 + 3000}{130} \sim 92 \text{ W m}^{-2}.

This estimate may actually be too high, since \( r_s \) is usually significantly higher in cooler conditions.

The simplified analyses show that cooler and drier weather is associated with less evapotranspiration, even with growing, healthy vegetation (as manifested by low \( r_s \)). Comparing the Bowen ratios of the two cases, the cool season value of 2.26 is six times greater than the warm season value of 0.38. Intuition would suggest that the Bowen ratio is strongly related to \( r_s \), but this analysis demonstrates that the ambient atmospheric conditions play a significant role as well.

The two resistance terms in the denominator of (2.38) also strongly modulate the magnitude of LE. Since more turbulent vertical mixing (and thus more evaporation) occurs with stronger winds (smaller \( r_s \)), one would expect LE to be inversely proportional to \( r_s \). Similar logic applies to \( r_w \). If plant stomata are closed (high \( r_s \)), LE will be reduced. Mesonet observations provide the humidity deficit term directly, and parameterizations for the \((R_n - G)\) and \( r_s \) have been developed earlier in this dissertation. For the purposes of this study, a simple model relating \( r_s \) to available meteorological data is needed.

The biggest challenge in modeling the surface energy budget using the PM method is the proper representation of the biophysical behavior of the native vegetation. In the absence of evaporation, the vegetation directly controls the magnitude of the latent heat flux (and thus the sensible heat flux) through control of stomata, or small pores on the surface of the plant. By opening and closing these stomata, the plant dictates how much sub-surface water is lost to the atmosphere through transpiration.
Plants are driven by both self-preservation and the need for growth. To grow, the stomata must be open in order to take in sunlight and carbon dioxide so that photosynthesis can occur. Without an adequate water supply, however, the plant will wilt and die. By controlling the stomata, then, the plant is simply responding to its environment in order to maximize its productivity.

How can this biological process be modeled satisfactorily by a meteorologist? There are very few direct observations taken of $r_s$. It is possible to measure the average resistance of a vegetation canopy using an instrument called a porometer, but this is a rather time-intensive procedure (Oke, 1987, p. 387) and thus is not practical here. Obviously, this makes the development of a parameterization of $r_s$ troublesome.

Methods have been developed to extract $r_s$ and/or soil moisture from satellite data. Measurements of infrared radiation have been used to estimate soil moisture (Tarpley, 1988) based on the increase in satellite-estimated surface temperature during the morning. Drier (wetter) conditions result in large (small) increases in surface temperature. Since plant stomata open in response to light intensity (with sufficient soil moisture), many studies have tried to relate $r_s$ to solar radiation. Although observations have shown a correlation, it was also found that the surface resistance is highly sensitive (in a non-linear fashion) to the amount of soil moisture as well (Monteith, 1973). For wet surfaces, $r_s$ is zero since the stomata are not involved in the transfer of water vapor from the surface in this case and evaporation occurs freely. The Berkowicz and Prahm (1982) scheme relates $r_s$ to the humidity deficit, the net radiation, and the accumulated net radiation since the previous rainfall (a proxy for soil moisture). However, Galinski and Thomson (1995) showed that this simple surface resistance scheme resulted in only a small improvement over two other flux schemes that did not account for soil moisture at all. This may be due
to the strictly empirical nature of the Berkowicz and Prahm (1982) scheme, which was
developed based on data from Denmark, Sweden, and Holland.

Using FIFE data, Stewart and Verma (1992) described the dependence of surface
conductance \(1/r_s\) on solar radiation, specific humidity deficit, and extractable soil
moisture using "stress functions" (Fig. 2.20), whose values ranged between 0.0 and 1.0.
The magnitudes of these functions were then multiplied together to quantify the response
of the vegetation to a given set of conditions. This response ranged from maximum
transpiration (1.0) to complete stomatal closure (0.0). Finally, Betts and Ball (1995)
concluded that "above some soil moisture threshold, evapotranspiration depends
primarily on atmospheric parameters, rather than vegetative controls."

Typical diurnal cycles of \(r_s\) can be described (Fig. 2.21). For relatively moist
conditions, \(r_s\) will be small in the early morning, since dew often covers the vegetation.
When the dew evaporates, \(r_s\) will increase to some "equilibrium" level for a few hours, as
the water availability is usually able to match the transpiration demand. By late
afternoon, \(r_s\) may increase due to decreasing solar radiation (closing stomata) and
possibly limited water availability. Under very dry conditions however, the early
morning resistance would likely be very high, since the stomata remain closed without
solar radiation and there would likely be no dew. A mild decrease would be noted later in
the morning as the stomata respond to the sunlight, but the lack of available water near
the surface would regulate the stomatal opening, so that the daytime equilibrium
resistance would be much higher than in the wet case. In the late afternoon, the stomata
will close further as the sun begins to set.
A second-order approach must be used to model plant behavior since there are no direct regular observations of stomatal behavior available. Indirect "observations" can be obtained by rearranging the PM equation:

\[ r_s = \frac{(R_a - G)\Delta\gamma^{-1} + [e_e(T(z_s) - e(z_s)]pc_\gamma^{-1} - (1 + \Delta\gamma^{-1})r_s LE}{LE}. \] (2.41)

Many of the ARM sites provide observations of \( R_a \) and \( G \), as well as \( H \) and \( LE \) (through the Bowen ratio method). The data at these sites are sufficient to account for all the terms on the right-hand-side of the equation, except for \( r_s \). In order to solve for \( r_s \), the Monin-Obukhov length \( L \) is systematically varied from -10000 (10000) m to 0 for positive (negative) observed \( H \) until the calculated \( H \) (from inverting (1.19)) converges to the observed \( H \). Then \( r_s \) is calculated from (1.26), followed by the calculation of \( r_s \) from (2.41). In order to test this method for extracting \( r_s \), one full year of data (July 1996 - June 1997) at half-hourly intervals from the ARM CF were analyzed. In order to reduce noise in the data set, only clear days were examined. These days were qualitatively selected by viewing the observed SW\(_d\) data. Since \( R_a \) cannot be calculated at night, the calculations were limited to daytime hours. Seasonal averaging was performed; the spring results are presented in Fig. 2.22. Due to limited data availability, winter averages were not calculated. The general trend begins with a moderate 1-2 hour rise in the early morning likely associated with the evaporation of dew or the melting and subsequent evaporation of frost. This is followed by a slower increase throughout the course of the midday hours, which represents a state where the loss of water through transpiration is replenished by soil water. Finally, in the late afternoon, \( r_s \) increases significantly as the magnitude of solar radiation decreases. The character of the diurnal cycle of these indirectly observed values compare favorably to directly observed values taken over a barley crop in England (Fig. 2.23, Oke (1987), p. 135). The character of the summer and fall averages from the ARM site are similar to the spring average.
The next step was to try to correlate calculated values of $r_s$ to more commonly observed atmospheric variables. Jarvis (1976) concluded that $r_s$ was related to solar radiation, root zone soil moisture, vapor pressure deficit, and air temperature as well as three empirically-derived coefficients. Many other studies have tuned these relationships, based on various data sets. The parameterization of Deardorff (1978) relied on only solar radiation and soil water potential, but was less empirical than the Jarvis (1976) formulation. Federer (1979) developed a formulation similar to Jarvis (1976), but with more species-dependent empirical coefficients. Noilhan and Planton (1989) were able to avoid empirical coefficients while developing a formulation that maintained dependence upon the original four factors of Jarvis (1976). Avissar and Pielke (1991) showed that the magnitudes of the surface heat flux are highly sensitive to small changes in plant stress in dry conditions (low stomatal conductance), detailed in Fig. 2.24. They also developed relationships between plant stress and individual atmospheric variables, displayed in Fig. 2.25 (from Lee (1992)). Here, $d_i$ represents a modification to the maximum surface conductance (inverse of $r_s$), which is represented by $c_{\text{max}}$:

$$\frac{1}{r_{\text{surf}}} = c_{\text{max}} \prod_{i=1}^{4} d_i,$$

(2.42)

where $i$ represents each of the four environmental variables in Fig. 2.25. These results show that the minimum resistance occurs for solar radiation values greater than 200 W m$^{-2}$, air temperatures between 20-30 °C, a vapor pressure deficit of less than 100 kPa, and soil water potentials absolutely smaller than −200 kPa (relatively moist soil). Variations of any of these factors out of their "ideal range" will result in an increase in $r_s$ as the vegetation responds by partially or totally closing stomata.

Avissar and Pielke (1991) also stressed the importance of "leaf age." They hypothesized that the vegetation will have maximum resistance at the beginning and end of the growing season, while during the period of peak growth, this value will be minimized. However, all of their parameterizations were developed based on the
response of one tobacco plant (Avissar et al., 1985), and may not be applicable to either
different species or to a field of similar species (i.e., a tobacco crop).

To study the relationship between $r$, and the more common atmospheric variables,
data from the ARM CF in Lamont were acquired and analyzed. Data for one growing
season (22 April-24 October 1997) were acquired and the calculated values of $r$, were
correlated with the following variables: vapor pressure deficit (VPD), solar radiation
($SW_0$), air temperature ($T$), and soil water potential (SWP). Three of the four variables
were also measured at Lamont, while the fourth (SWP) was measured at Blackwell
(BLAC), which is the closest (~20 km away) Mesonet site. The soil moisture
measurements at Lamont were of insufficient quality for this study (K. Fisher, formerly
of CIMMS, pers. comm.). The comparisons were done at 30-minute intervals from 1600-
2000 UTC, and after bad data had been eliminated, a total of 1448 comparisons were
available for analysis. The analysis is limited to midday hours since the method used to
evaluate (2.41) would sometimes fail for small values of observed $H$.

Fig. 2.26a shows the time series of calculated $r$, for the period at Lamont. From
spring into early summer, there is little change in the mean value of $r$,. Around day 190,
a sharp increase in $r$, occurs, associated with an extended spell of dry weather. Summer
rains then reduce $r$, back to spring levels. Finally, in late summer, $r$, increases sharply
with a corresponding increase in variability. In Fig. 2.26b, the significant changes in $r$,
that can occur over the course of a given day are illustrated. In general, the larger values
of $r$, occur throughout the late summer and early fall. In Fig. 2.27, it is apparent that the
VPD increases significantly from spring to early fall as well, with more humid conditions
noted in early August, and again in parts of September and October. Since the saturation
vapor pressure increases exponentially with temperature (while the observed vapor
pressure does not), the VPD would be expected to be larger in the warmer months of the
year.
The temperature and solar radiation time series (not shown) both peak in summer as expected. Fig. 2.28 reveals only three periods of significant drying of the soil - in mid-May, early August, and late September. Surprisingly, there is no significant correlation between SWP and \( r_s \), except for the driest period in late September. However, Deardorff (1978) assumed a linear relationship between the "wetness function" and the soil moisture, but only when the amount of soil moisture was smaller than the field capacity. Otherwise, the soil evaporates at its full potential. Lee and Pielke (1992) concurred, stating that "when the soil-water content is greater than the field capacity, the soil should behave very close to the saturated soil"; they also cited the common practice of assuming the field capacity of all soil classes as -330 kPa. Figure 2.28 shows that the soil was drier than field capacity only twice throughout the period of interest, explaining the relative independence of \( r_s \).

Of the four observed variables, \( r_s \) is most strongly correlated with VPD (Fig. 2.29). Because of this, the VPD is considered the most significant, or first-order contributor to \( r_s \). This is an apparent contradiction, since one would expect there to be more evaporation (higher LE, lower \( r_s \)) under drier (high VPD) conditions. However, it is likely that the behavior of the vegetation as manifested by \( r_s \) is controlling VPD rather than vice versa. So, to say that VPD contributes to \( r_s \) may be misleading, but in a diagnostic model like the one being developed in this study, we can use the VPD values to estimate more accurately the behavior of the vegetation. It is apparent that the phase of the two time series is rather similar. The apparent correlation between \( r_s \) and VPD is detailed further in Fig. 2.30. A linear relationship was determined:

\[
 r_{s, VPD} = r_{s1} = \text{VPD} / 20. \tag{2.42}
\]

Both SW\(_d\) and T were well-correlated with \( r_s \), with the SW\(_d\) correlation slightly more significant (the second-order contributor). Again, a linear relationship was determined:

\[
 r_{s, SW_d} = r_{s2} = 85 - 0.1(\text{SW}_d), \text{only if VPD > 500 Pa}. \tag{2.43}
\]

From the scatterplots it was apparent that T was the third-order contributor:
\[ r_{st} = r_{s3} = -10(T - 30), \text{ only if } T > 30^\circ C. \] (2.44)

One final relationship is constructed with SWP (the fourth-order contributor):
\[ r_{swp} = r_{s4} = -2(SWP + 330)/5, \text{ only for } SWP < -330 \text{ kPa.} \] (2.45)

The \( r_i \) model has a simple form:
\[ r_{sw} = \sum_{i=1}^{4} r_{si}, \] (2.46)

where \( r_{sw} \) represents the \( r_i \) parameterization in western Oklahoma. However, due to uncertainties in both observations and the underlying theory, the model is not perfect (Fig. 2.31). The modeled values of \( r_i \) have an MBE of 5.9 s m\(^{-1}\) and an RMSE of 30.0 s m\(^{-1}\). This compares to a typical value of 100 s m\(^{-1}\).

Preliminary calculations revealed that use of this \( r_i \) parameterization caused large overestimates of \( H \) in the eastern part of Oklahoma. Satellite estimates of greenness (Loveland et al., 1991) and statistical analyses of rainfall patterns (Richman and Lamb, 1985) suggest that Oklahoma can be crudely divided into two land use types ("green" in eastern Oklahoma, "brown" in western Oklahoma). Because of this, it was determined that a second parameterization should be developed for eastern Oklahoma sites. Similar correlation analyses were performed using the \( r_i \) values at the Pawhuska (refer to Fig. 2.2 for location) ARM site (Fig. 2.32) for VPD, T, SW\(_w\), and SWP, with the following results:
\[ r_{se} = \sum_{i=1}^{4} r_{si}. \] (2.47)

Here, \( r_{se} \) represents the sinusoidal variation in the \( r_i \) values:
\[ r_{s5} = (80 - 115 \sin(\pi/day - 110)/170)), \] (2.48)

where day is the Julian day. This sinusoidal variation is the only difference between the \( r_i \) parameterizations at Pawhuska and Lamont. The reason for this dependence is unknown. Modeled values of \( r_i \) at Pawhuska have an MBE of 0.6 s m\(^{-1}\) and an RMSE of 44.1 s m\(^{-1}\), slightly higher than the Lamont comparisons.

2.5 Sensible Heat Flux (H)
Sensible heat is energy detected as a rise or fall in temperature. The surface sensible heat flux describes an energy transfer between the surface and the overlying atmosphere. It is manifested as a change in temperature at a given level due to turbulent vertical mixing of heat. Fast-response instruments have shown that most of this energy transfer occurs in 10-20 second intervals associated with rising thermals (Oke, 1987). Sensible heat is transferred from the surface to the atmosphere during the day and is usually returned to the surface at night. Once the latent heat flux is specified through a partitioning of the available energy \((R_a - G)\) using the PM equation, the sensible heat flux becomes a residual in the surface energy budget (same as 1.18):

\[
H = \frac{(R_a - G)(r_a + r_s) - [e_s(T) - e]pc_p\gamma^{-1}}{r_s + (1 + \Delta\gamma^{-1})r_a}.
\]  

(2.49)

However, since the aerodynamic resistance \(r_a\) is a function of \(L\), an iterative procedure must be employed to solve for \(H\). In this procedure, the Monin-Obukhov length \(L\) is initially set to a very large value (representing a neutral surface-layer), and values for \(r_a\) (from 1.26) and \(H\) are calculated. A new value of \(L\) is calculated from \(H\) (from 1.19), and the process continues until \(r_a\) and \(H\) converge. This final value of \(r_a\) is then used in (2.38) and (2.49) to calculate \(LE\) and \(H\), respectively. Surface temperature can then be obtained using the calculated sensible heat flux:

\[
T_g = T + r_sH/pc_p.
\]  

(2.50)

To start the flux calculations for a given day, we set \(T_g = T\) and solve for the surface fluxes. The resultant values of \(H\) and \(r_s\) are then used in the initial time step to solve for \(T_g\). This process continues throughout the series of flux calculations during the day. The computational process is outlined in Fig. 2.33. This technique works since, even if \(T_g\) is radically different than \(T\) initially, the surface energy balance constraint will quickly bring the value of \(T_g\) back to more realistic values.

A simple error analysis of the entire surface energy budget equation is shown in Appendix C.
CHAPTER 3
RESULTS AND DISCUSSION

In order to test the accuracy of the energy budget model, Mesonet data from July 1997 were used. July was chosen since significant synoptic weather fluctuations are climatologically rare then, and this allows for a relatively undisturbed environment in which to compare the modeled and observed fluxes. Since the parameterization for $R_n$ is only valid in the daytime, the estimates of $H$, $LE$, and $G$ are only useful at those times as well.

3.1 Net Radiation ($R_n$)

To verify the $R_n$ parameterization, data from the five enhanced Mesonet stations (Fig. 2.15) were used. These observations of $R_n$ were taken at 15-minute intervals, and were compared to the $R_n$ parameterization at the Mesonet stations at the corresponding interval. Three 5-minute observations were averaged to get the representative 15-minute value. All of the daytime data (measured $R_n > 0$) from July 1997 were analyzed at each of the five stations, and the results are shown in Table 3.1 and Figs. 3.1-3.5.

<table>
<thead>
<tr>
<th></th>
<th>MBE (W m$^{-2}$)</th>
<th>RMSE (W m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>APAC</td>
<td>-6.2</td>
<td>21.8</td>
</tr>
<tr>
<td>GOOD</td>
<td>-17.0</td>
<td>25.5</td>
</tr>
<tr>
<td>MARE</td>
<td>-8.6</td>
<td>24.3</td>
</tr>
<tr>
<td>NORM</td>
<td>1.0</td>
<td>18.0</td>
</tr>
<tr>
<td>WIST</td>
<td>-6.9</td>
<td>18.4</td>
</tr>
</tbody>
</table>

Table 3.1 Mean bias errors (MBE) and root-mean square errors (RMSE) of the $R_n$ parameterization applied at the five enhanced Mesonet sites at 15-minute intervals during the daytime in July 1997.

The average MBE over all five sites is negligible ($-5.1 \text{ W m}^{-2}$) while the average
RMSE is 21.6 W m\(^2\). This error is much smaller than the expected value as determined by error analysis (Appendix C). Considering that \( R_n \) is a combination of four separate terms, three of which are parameterized, these RMSE values are quite satisfactory. The y-intercepts of the regression lines are all below 15 W m\(^2\) (where 0 W m\(^2\) is a perfect fit) and the slopes are between 0.9 and 1.0 (where 1.0 is a perfect fit). It is apparent that the newly developed \( R_n \) parameterization performs quite well across the entire state. The ability to model \( R_n \) accurately is crucial, since it represents the input to the land-atmosphere system. Inaccurate estimates of \( R_n \) provide little chance to represent \( H \) or \( LE \) accurately.

3.2 Ground Heat Flux (G)

Though the five enhanced Mesonet sites purportedly measure G, they actually measure only the second term on the RHS of the equation for G (2.27), neglecting the storage term. The NORM site has additional instrumentation that does measure the storage term (C. Marshall, NCEP, pers. comm.). However, the parameterization developed for G requires the soil moisture observation, and the observations at the NORM site were inadequate for this purpose (see section 3.3 for details). Finally, there are two ARM sites (El Reno and Pawhuska) that are nearly co-located (within 100 m) with Mesonet sites. The ARM method for estimating G involves measurements at five different locations near the site. The object is to sample the area well, and then take the average of all the measurements to get a representative value.

Thirty-minute values of G from the ARM sites were compared with the parameterization of G using the Mesonet data at the 5-minute observation corresponding to the end of the averaging period. Comparisons were performed between 1400-2200 UTC. At El Reno (Fig. 3.6), the observed values generally peaked around 50 W m\(^2\), with
little significant variation throughout the month. The calculated values, however, varied from about 10% larger to up to three times as large as the observed values. Peak values varied significantly throughout the month, ranging from approximately 50-170 W m\(^2\). The Pawhuska/Foraker comparisons (Fig. 3.7) are of similar quality.

The relatively small observed values are likely due to the height of the vegetation at the ARM sites. Documentation reveals that the grass around both the El Reno and Pawhuska ARM sites was 1-2 m high during these observations. The simple assumption made for this study was that the native vegetation at all the sites was 10 cm, an order of magnitude smaller than the documented height at the ARM sites. Taller vegetation causes more of the net radiation to be transferred into the turbulent fluxes (H and LE) due to greatly increased surface roughness and decreased aerodynamic resistance. Further, tall vegetation more effectively shades the underlying soil, keeping it cooler and resulting in smaller observed values of G. Conversely, at sites with shorter grasses, such as NORM, the magnitude of G is much larger (Fig. 3.8). Here, grass was approximately 15-30 cm high during July 1997 (C. Marshall, NCEP, pers. comm.), allowing the soil to warm more readily.

An analysis of equations (1.26), (2.50), and (2.27) (not shown) reveals how increasing the height of the vegetation by an order of magnitude affects the modeled ground heat flux. Increasing the roughness length for momentum from 0.01 to 0.1 m (with corresponding changes in the roughness length for heat) will only decrease the roughness temperature by ~0.2 K and the ground heat flux by ~2 W m\(^{-2}\). However, this analysis is only valid if the temperature remains constant in the region between \(z_{ob}\) and the ground. Observations suggest that the temperature may instead decrease significantly beneath \(z_{ob}\) (Oke, 1987, p. 138). If this were the case, the actual ground temperature
would be significantly cooler under tall grass than under short grass (Fig. 3.9). A difference in ground temperature ($\Delta T$) of 5 K corresponds to a $-50 \text{ W m}^{-2}$ in G, which would account for most of the difference in observed ground heat flux between the El Reno ARM site and the NORM Mesonet site. The exact magnitude of the slope of the temperature profile beneath $z_{oa}$ is unknown, however, so only qualitative conclusions can be drawn.

It is apparent, though, that the instantaneous vegetation height must be known at a given location in order to properly estimate $G$. Since these data are not available for this study, the qualitative similarity of the modeled/observed values of $G$ in Figs. 3.6-3.7 will have to suffice.

3.3 Latent and Sensible Heat Fluxes (LE and H)

Once $R_{s}$ and $G$ are calculated, the quantity $(R_{s} - G)$ is partitioned into LE and $H$. First, LE is calculated using the Penman-Monteith equation; then $H$ is determined as a residual ($H = R_{s} - G - \text{LE}$). Unfortunately, there are no measurements of LE available at the Mesonet sites, and the two ARM sites that are co-located with Mesonet sites (El Reno and Pawhuska) had significant problems with the Bowen ratio instrumentation during July 1997. Because of this, the modeled latent heat fluxes could not be verified.

To begin to evaluate the modeled values of $H$, the calculated fluxes at the Norman site (NORM) and the observed fluxes from the Bowen ratio system co-located with NORM (Marshall et al., 1998) were compared for July 1997. Fig. 3.10 highlights the significant disagreement between the observed and calculated Bowen ratios. Measured Bowen ratios were generally below 1 until the last few days of the month. This is typical of a region of relatively unstressed vegetation. However, the modeled Bowen ratios are
typically much larger. Values are often well above 1, and even approach 5 towards the end of the month.

The cause of this discrepancy is found by examining a time series of 5 cm soil water potential at the NORM site (Fig. 3.11). Throughout July, the measured soil water potential remained below -400 J/kg, representative of very dry soil. A generally accepted value for the wilting point of vegetation is -330 J/kg. These observations appeared suspicious, since there were two significant rainfalls during the month (4th and 18th). This rain was apparently enough to keep evaporation rates fairly significant during the month, since observed Bowen ratios remained low. Though the 5 cm soil moisture sensors did not respond as expected to these rainfall events, the sensors at 60 and 75 cm responded immediately to both events (Fig. 3.12, J. Basara, OU, pers. comm.). Most notable is the sharp response on the 18th, during which time the upper sensors were relatively unaffected. These unusual observations likely represents a real measurement problem: that is, when the topsoil dries past a certain point some of the rainfall may drain down along the instrument wires to the lower sensors (J. Basara, OU, pers. comm.). It would seem that the rest of the rainfall is ponding on the surface, unable to seep into the soil (as manifested by the lack of response at the 5 cm sensor). Then the water evaporates slowly over the next few days, explaining the relatively low observed Bowen ratios. Unfortunately, by relying on the 5 cm soil water potential in the energy budget model, the flux calculations are susceptible to this previously unrecognized phenomenon.

To see how common this problem was, time series of soil water potential and rainfall were examined for all of the Mesonet sites that measure soil moisture. Of the 57 possible sites, 13 had problems similar to those outlined above during July 1997. To illustrate the problem, one can compare the NORM data to those from other sites that did
not experience this measurement problem. In Fig. 3.13, it is apparent that the 5 cm soil water potential responds more typically to rainfall events at El Reno (ELRE). The three significant rainfall events all force the potential back to a saturated value around -10 J/kg. The time series from the BOWL Mesonet site from July 1997 also show a significant correlation between 5 cm soil water potential and rainfall (Fig. 3.14). Owing to the two most significant rainfall events of the month, the 5 cm soil water potential returned to saturated values, unlike the NORM data.

Unfortunately the soil-moisture instrumentation problem renders the energy budget model useless at the affected sites. For these locations, it would make more sense to model the response of the vegetation using some form of antecedent precipitation index. As Fig. 3.15 shows, there is a more significant relationship between the measured Bowen ratio and the rainfall at NORM than there is between Bowen ratio and 5 cm soil water potential. The period of sharply increasing Bowen ratios in the middle of the month ends abruptly on the 18th with the significant rainfall, while the lack of rainfall the last half of the month was associated with steadily increasing Bowen ratios. As noted earlier, the 5 cm soil moisture did not "sense" this rain, which effected poor surface-flux calculations. However, since one of the unique aspects of this study is the incorporation of the new soil moisture observations into an operational energy budget model, surface fluxes were calculated at only those 44 sites without these observational problems. These sites had an average station spacing of approximately 60 km, and were more closely spaced across the western half of the state (Fig. 3.16).

The next concern was finding a good sensible heat flux data set to validate the model results. Eddy fluctuation data are most desirable to use as "ground truth" when evaluating the accuracy of the estimated sensible and latent heat fluxes. Special
instrumentation was installed at the El Reno ARM site during the summer of 1997 in order to get more accurate estimates of H and LE using the eddy fluctuation method. However, there have been many problems with this data set, and it currently appears as if "good" data won't be available until late 1998 (L. Mahrt, Oregon State University, pers. comm.). And, as mentioned earlier, the Bowen ratio estimates of H and LE from both the El Reno and Pawhuska ARM sites were bad or missing for much of the month.

However, Brotzge (1998) equipped nine Mesonet sites with matching temperature and wind sensors at two levels. Then, using a variation of an aerodynamic method developed by Halliwell and Rouse (1989), sensible heat fluxes were calculated at the Foraker site and were then compared to the ARM values from Pawhuska for a ten-day period in May 1997. The comparisons were very good with the exception of days with light winds, when the method overestimated afternoon sensible heat fluxes by 50-150 W m\(^{-2}\) due to a problem with radiative heating of the instrumentation. Since the Brotzge (1998) method compared favorably to the ARM data otherwise, we will use these estimates as verification for evaluating the sensible heat flux estimates in this study.

Of the nine specially equipped Mesonet sites, only five have soil moisture sensors, which is a necessary condition to evaluate the sensible heat flux estimates from this study. Of these five sites, three had the soil moisture observational problem mentioned earlier in this section. The two remaining sites available for verification are Beaver (BEAV) and Nowata (NOWA). Since BEAV is located in the semi-arid eastern panhandle while NOWA is located in the more green, humid eastern part of the state, these sites provided a good test of the versatility of the newly developed method.

For each site, wind speed data were analyzed in order to eliminate days where the Brotzge (1998) method would not work due to light winds (< 4 m s\(^{-1}\)). Also, days with a
significant amount of cloud cover were eliminated in order to reduce the amount of noise in the comparisons. There were five remaining "golden days" at BEAV during July 1997, which are detailed in Figs. 3.17-3.19 and Figs. 3.21-3.22.

On 2 July (Fig. 3.17) the phases of the time series of the modeled and observed values of H are similar, and the magnitudes are very close during the midday hours. The next day (Fig. 3.18) the model performed very poorly in the morning, but matched the observed values very well in the afternoon. It is apparent that the modeled values of H become positive earlier in the morning than the observed values. There are two possible causes for this problem. First, the Brotzge (1998) estimates of H are based on the air temperature gradient between 1.5 and 9 m, while the model uses the gradient between $z_{ob}$ and 1.5 m. In the early morning, the condition $T(z_{ob}) > T(1.5 \text{ m})$ will occur before the condition $T(1.5 \text{ m}) > T(9 \text{ m})$. This would explain the early morning lag between the modeled and observed H. The other cause is likely more significant. As mentioned in chapter 2, the parameterization for LW_d is only valid during the daytime. Since this is the case, pre-sunrise values of LW_d and thus R_a may be significantly in error. If R_a becomes positive prematurely due to this problem, then H will likely follow.

By 13 July (Fig. 3.19), modeled and observed values were similar in the morning with a ~100 W m$^{-2}$ model overestimation in the afternoon. On 21 July, a significant rainfall occurred at BEAV (Fig. 3.20), saturating the soil at 5 cm. As expected, there is a significant reduction in observed H between 13 July and 25 July (Fig. 3.21). The modeled H is also smaller, though, and follows the time series of observed H quite closely. Finally, the best comparison of the month occurs on 31 July (Fig. 3.22), when both the magnitude and character of the time series are remarkably similar.
The modeled and observed values of $H$ at BEAV were compared for the hours 1400-2200 UTC. The early morning data were removed due to the consistent positive bias found due to the problems mentioned earlier. The analysis of all five days (Fig. 3.23) shows a negligible MBE and a RMSE similar to that found for the $R_a$ verifications. The slope of the regression line suggests that the model slightly underestimates (overestimates) $H$ for small (large) magnitudes.

For the NOWA site, there were only two days that qualified for analysis. For both days (Fig. 3.24), the modeled $H$ was very similar to the observed $H$ during the morning hours. In the afternoon, the observed $H$ decreased rapidly while the modeled $H$ completed a nearly sinusoidal cycle. The reasons for this sharp drop-off in $H$ in the afternoon are not known, since there were no frontal passages and no clouds during the day. However, since $R_a$ is parameterized accurately, this implies that LE and/or G are underestimated during this time, since $H$ is calculated as a residual in the energy balance equation. Since $R_a$ typically peaks near 1800 UTC on a clear day, one would expect $H$ to peak then as well. The peak in the observed $H$ data occurs between 1600-1700 UTC instead. Still, the maximum values of $H$ are modeled well for these two days.

Based on the limited verification data available, it is apparent that the model estimates of $H$ are of reasonable quality. More verification data will become available when the OASIS project begins sometime in 1999. As part of this project, instrumentation will be installed at all of the Mesonet sites so that the Brotzge (1998) method can be used to calculate $H$.

Finally, all four of the energy fluxes were calculated at all 44 available sites at 5-minute intervals during July 1997. Then, monthly average maps were constructed for 1800 UTC. Fig. 3.25 shows the average $R_a$ values. Values range from 480 in north
central Oklahoma to 590 in the eastern panhandle. It is likely that these variations are mainly driven by differences in solar radiation over the course of the month. In Fig. 3.26, it is apparent that there is little variation in $G$ across the state. Magnitudes range from 40 in the north central part of the state to 120 in the southwest, where there was plentiful soil moisture all month. The pattern of sensible heat flux exhibits three distinct features (Fig. 3.27). First, the low values in the eastern half of the state are no surprise due to the density of green vegetation there. The large values in northwest Oklahoma are likely due to the winter wheat harvest. Once this harvest is completed in late May or early June, the heavily transpiring green wheat fields are transformed into piles of short brown wheat stubble. The harvest has been shown to produce drastic changes in day-to-day weather, specifically lower dewpoints and higher temperatures (Rabin et al., 1990). Finally, the minimum in southwest Oklahoma is most likely due to above-normal rainfall in that region.

Lastly, the patterns of $LE$ are, as expected, just the opposite of $H$ (Fig. 3.28). The largest values occur in the more humid eastern part of the state and in the southwest, with the smallest values in the north central near the harvested winter wheat fields. Overall, the patterns of modeled energy fluxes are quite reasonable and not unexpected considering the land use types and ambient atmospheric conditions during July 1997.
CHAPTER 4

SUMMARY AND CONCLUSIONS

The goal of this study was to develop a surface energy budget model in order to estimate surface fluxes. A method was developed for this task that requires only standard meteorological observations at one level. The inclusion of solar radiation and soil moisture observations (e.g., from the Oklahoma Mesonet) improve our ability to model surface fluxes. Of the four components of the surface energy balance, only $R_m$, $G$, and $LE$ are calculated explicitly; $H$ is computed as a residual.

To calculate $R_m$, four separate components were evaluated. The downwelling shortwave radiation is measured at the Mesonet sites, while the upwelling shortwave radiation is dependent upon a specification of albedo. The albedo parameterization was developed based on satellite estimates as well as a climatological analysis of a typical annual albedo variation. This method provides for realistic spatial and temporal variations across the state of Oklahoma.

To estimate downwelling longwave radiation, a new technique was developed. In this technique, $LW_d$ can be estimated based on measurements of near-surface air temperature, relative humidity, and solar radiation. The use of solar radiation observations allows for an estimate of fractional cloudiness, an important factor in modulating $LW_d$. In addition, an annual sinusoidal variation in clear-sky atmospheric emissivity was discovered. Estimates of $LW_d$ are unbiased with RMSEs generally between 10-20 W m$^{-2}$. This method is unique (Crawford and Duchon, 1998) and is an improvement over previous attempts to estimate $LW_d$. Finally, the upwelling longwave radiation is estimated using a typical value of surface emissivity and the calculated skin temperature. To get the skin temperature, estimates of $H$ from the previous 5-minute observation are needed.
Estimates of $R_n$ were compared to direct measurements at the five enhanced Mesonet sites. The accuracy of the estimates was quite good, with negligible mean bias errors and root mean square errors of 18-26 W m$^{-2}$. Accurate estimates of $R_n$ are the most crucial part of the energy budget model, since this quantity represents the total energy input to the surface.

Estimates of $G$ were more difficult to verify, since the observations at the enhanced Mesonet sites did not include the storage term. The values of $G$ measured at the ARM sites were significantly lower than the modeled values. However, the observations from the NORM site were of similar magnitude to the modeled values from the ARM sites. It is apparent that the height of the vegetation significantly modulates the amount of net radiation that can reach the soil. To estimate $G$ more accurately, instantaneous observations or estimates of vegetation height at the Mesonet sites are needed. Such observations are currently not available.

To get accurate estimates of $H$ and $LE$ with only one level of data, the Penman-Monteith (resistance) method must be used. An Ohm's Law analogue is implemented in order to provide a conceptual model of the energy transfer near the surface. Formulations for aerodynamic and surface resistance were also developed. The aerodynamic resistance, which is a measure of the potential for turbulent vertical motion near the surface, is a function of the roughness length for momentum, the wind speed, and the stability. Monin-Obukhov similarity theory is used to develop this formulation.

The parameterization for surface resistance is crucial to the success of the model, since it determines how much of the available energy ($R_n - G$) is partitioned into $LE$ and how much into $H$. Previous studies concluded that this quantity was related to many different atmospheric variables, including vapor pressure deficit, air temperature, solar radiation, and soil moisture. Since direct observations of this quantity are not readily available, indirect observations were acquired by rearranging the Penman-Monteith equation and using ARM observations.
One full growing season was analyzed, using data from the ARM sites in Lamont and Pawhuska. It was found that $r_e$ was linearly related to the vapor pressure deficit. This relationship is not surprising since large values of $r_e$ will result in dry near-surface conditions and large values of vapor pressure deficit. Besides the vapor pressure deficit dependence, $r_e$ was also linearly related to the magnitude of solar radiation, but only for dry near-surface conditions. This increase is also physically consistent, since a plant will typically close its stomata in conditions of low light.

The surface resistance was found to be independent of air temperature, except during very warm conditions ($T > 30^\circ C$), when $r_e$ will decrease. The opening of the stomata on hot days is likely a self-preservation mechanism similar to the sweating experienced by human beings. Finally, the relationship between $r_e$ and soil moisture was examined. The amount of soil water potential was shown to effect $r_e$ only in very dry conditions ($<-330$ kPa). For dry soils, $r_e$ increases significantly with the continued decrease in soil water potential. In addition to the four relationships described above, the Pawhuska site exhibited a significant sinusoidal variation in $r_e$, with minimum values found in early July. Since this behavior was not found in the Lamont data, separate parameterizations of $r_e$ were used for eastern and western Oklahoma. A simple linear combination of the four (five) relationships described above is used to parameterize $r_e$ for western (eastern) Oklahoma. These parameterizations were unbiased and had RMSEs of 30 s m$^{-1}$ and 45 s m$^{-1}$ for western and eastern Oklahoma, respectively. These values correspond to errors of $\sim 30$ W m$^{-2}$ (eastern) and $\sim 45$ W m$^{-2}$ (western) in the estimates of both LE and H.

Observed values of H during July 1997 at two Mesonet sites (BEAV and NOWA) were then used to verify the new model. Days with light winds were eliminated since the observed values of H were in error due to radiational heating of the temperature sensors. Partly or mostly cloudy days were also eliminated in order to avoid spurious values of solar radiation (due to large values of diffuse radiation). For BEAV, five days were
examined. The model values of $H$ were unbiased with RMSEs near 60 W m$^{-2}$. At the NOWA site, two days were examined. In both cases, the modeled $H$ was similar to the observed $H$ during the morning hours, but slightly larger in the afternoon.

Finally, sensible heat fluxes were calculated at 44 Mesonet stations during the daytime hours of July 1997. As expected, $H$ was typically larger in western Oklahoma, owing to less green vegetation. On 12 July, a sharp meridional gradient in $H$ was found in southwestern Oklahoma (Fig. 4.1), with values increasing from 150 W m$^{-2}$ to 325 W m$^{-2}$ over approximately 100 km. Assuming a hydrostatic atmosphere, air near the ground should accelerate from the region of higher density (low $H$) to the region of lower density (high $H$), as long as the gradient in $H$ is maintained. Since the larger values of $H$ were found north of the smaller values, an enhancement of the ambient southerly surface winds in western Oklahoma would be expected. Indeed, the theory was verified in this case by an area of enhanced southerly winds just downstream of the strongest $H$ gradient (Fig. 4.2).

It is possible that the locally stronger winds near the relative maximum of sensible heat may simply be a manifestation of a locally deeper boundary layer, i.e., vertical turbulent mixing of momentum from a greater distance aloft (which usually means higher wind speeds). Either way, however, it is apparent that the sharp gradient in $H$ is likely the cause of the significant convergence in northwestern Oklahoma. Having the ability to detect these gradients early in the day would provide the short-term forecaster ("nowcaster") with valuable information that can be used to improve forecasts of convective initiation.

Another interesting example showing the correlation between gradients in $H$ and the surface wind field occurred on 13 June 1998. On this day, a significant zonal gradient in $H$ was established in southern Oklahoma by 1800 UTC (Fig. 4.3). Here, a change in $H$ of 150 W m$^{-2}$ occurred over a distance of 50 km. This gradient is approximately twice as strong as that found in the 12 July 1997 case. According to Pielke (1984), the magnitude
of this gradient should cause a "minor but statistically observable effect" on local winds. This argument is supported by Ziegler et al. (1995), who determined through two-dimensional mesoscale model simulations that the surface wind field should be altered with a 100 W m$^2$/50 km horizontal gradient in $H$. An even stronger gradient is apparent in north-central Oklahoma. These zonal gradients in $H$ should cause the winds to back from the prevailing southerly direction to southeasterly. Figure 4.4 shows the wind direction two hours later at 2000 UTC. There are two areas where the winds have backed to less than 170°, and both correspond to the significant gradients in $H$ mentioned above.

Although these examples do not represent a thorough examination of the relationship between gradients of $H$ and perturbations in surface wind fields, it does provide at least one example of the "inland sea breeze" (Sun and Ogura, 1979). These variations in surface winds locally enhance surface convergence, favoring one region over another for convection initiation. The model developed in this study, then, may eventually be a useful tool in improving short-term forecasts of convection initiation.

To examine further the correlation between the sensible heat flux gradients and local perturbations in the surface wind field, the following steps should be taken. First, the magnitude of the perturbations would have to be determined. This can be done by calculating the monthly average $u$- and $v$-wind components at each 5-minute observation interval at each location for the month of July. Again, July should be chosen for the relative paucity of synoptic disturbances. The perturbation wind is then the difference between the observed wind and the average wind. Both $u$- and $v$-wind component perturbations should be calculated at each time at each station. These perturbation values and the calculated sensible heat flux values can then be gridded and the correlation between the sensible heat flux gradient and the perturbation winds can be quantified. If significant relationships can indeed be established between the sensible heat flux gradients and local wind perturbations, the next logical step would be to examine the correlation between these gradients and the location of convective initiation.
Convection-initiation cases in particular meteorological conditions should be examined. Often, a sharp-edged dense cirrus band traverses Oklahoma, providing for strong differential surface heating across the band. This may force isolated convection near the band. Dryline convection also provides a good opportunity to test the energy budget model since differential heating often enhances this boundary. Small variations along the dryline in surface fluxes may play a key role in determining exactly where the first storm will develop along the dryline.

The results from this study are already being utilized in the meteorological community. The newly developed parameterization for LW_d will be used to estimate the net radiation at Oklahoma Mesonet sites as part of the OASIS project (K. Humes, OU, pers. comm.). Also, the surface energy budget model will soon be used to calibrate remotely-sensed data in order to better understand the first appearance of spring foliage, or the "green wave" (M. Schwartz, University of Wisconsin-Milwaukee, pers. comm.).

Simple but important improvements in current operational and research models can also be made using the tools developed here. For example, the Eta model estimates of R_a exhibited a positive bias of ~100 W m^2 during May-July 1997 (C. Marshall, NCEP, pers. comm.), while the parameterization developed for this study is unbiased. This 100 W m^2 error in modeled R_a gets partitioned into the other three fluxes in the surface budget, eliminating any hope of accurately modeling sensible and latent heat fluxes. Further, this excess energy, once it is distributed to the other fluxes, results in warmer and/or moister surface conditions in the model. This condition likely results in poor forecasts of convection.

The Eta model vegetation classification considers most of the body of Oklahoma to have similar types of vegetation in determining the model-predicted surface resistance. However, analysis of the observed behavior of surface resistance in this study contradicts this assumption, with separate parameterizations for eastern and western Oklahoma. Finally, Pan (1990) assumed a minimum value of r_a of 60 s m^-1 for the scheme used in the
National Center for Environmental Prediction's medium-range forecast model (MRF). The results from this study suggest that this minimum value may vary considerably across a range of vegetation types.

Since the Mesonet measurements are made in open fields, they may not accurately represent conditions in the surrounding areas. In northwestern Oklahoma, for example, the spring land use pattern is dominated by green, heavily transpiring winter wheat. Since Mesonet sites are not located in these wheat fields, flux estimates made from these observations may not be representative. However, examination of temperatures and dewpoints does reveal that the effect of the wheat fields is detectable at nearby Mesonet sites. In this case, a swath of relatively lower temperatures and higher dewpoints is commonly observed on a clear, late spring afternoon in the "wheat belt." This implies that the Mesonet point observations may indeed accurately represent the surrounding areas.

There are several possible ways to improve model results. Real-time knowledge of vegetation height would allow for significant improvements in the estimation of G, and to a lesser extent, H and LE. Measurements of $T_a$, possibly using infrared thermometers, would also greatly improve estimates of G, H, and LE. Another way to improve the model would be to analyze a greater volume of ARM data so as to tune more finely the $r$, parameterization.

Finally, a working hypothesis has been developed: **Horizontal variations in the sensible and latent heat fluxes at the surface can portend small-scale changes in the surface-layer wind field.** Accordingly, the diagnostic model developed in this study will be used to test this hypothesis.
Fig. 1.1 The 114 stations of the Oklahoma Mesonet which record solar radiation, air temperature, relative humidity, wind speed and direction, and rainfall at five-minute intervals.
Fig. 1.2 Establishment of local circulation due to flux gradients in synoptically quiescent regime. Sensible heat flux is denoted by $H$ and latent flux by $LE$. $P_a$ represents a constant-pressure surface in the free atmosphere, while $P_s$ is a representative surface-layer pressure before sunrise. (a) Undisturbed conditions at 0700 local standard time (LST). (b) Conditions at 1200 LST. Note that the column of air over the dry surface has expanded vertically relative to that over the moist surface, creating a horizontal pressure gradient and subsequent air flow. (c) Conditions at 1500 LST. Establishment of surface pressure gradient results in return flow, and continuity considerations ensure a completed solenoidal circulation.
Fig. 1.3 Illustration of surface energy balance on a typical afternoon. The vegetation canopy is located between the thicker solid lines. The aerodynamic roughness length of heat and moisture is represented by $z_{th}$. Positive values of the fluxes refer to energy transfer in the direction of the arrow.
Fig. 1.4 (a) Representative diurnal clear-sky variations in $R_n$ and its components from data taken at Cedar River, Washington on 10 August 1972 (Oke, 1987, p. 148). Dashes represent gaps in data. (b) Representative diurnal clear-sky variations in the four components of the surface energy budget from data taken at Agassiz, British Columbia on 30 May 1978 (Oke, 1987, p. 124). SR and SS represent sunrise and sunset, respectively.
Fig. 1.5 Comparison between the daytime heat flux schemes of (a) Smith (1990), (b) Holtslag and Van Ulden (1983), and (c) Berkowicz and Prahm (1982), where $n = \text{number of observations}$, $\bar{x} = \text{mean observed value}$, $\bar{y} = \text{mean estimated value}$, $s_x = \text{standard deviation of the observed value}$, $s_y = \text{standard deviation of the estimated value}$, $\bar{y} - \bar{x} = \text{mean bias error in the estimated value}$, $\sigma = \text{root mean square error}$, and $r = \text{correlation coefficient}$.
Fig. 1.6 Illustration of Ohm's law. (a) Classic electrical representation, where $V$, $I$, and $R$ represent voltage, current, and resistance, respectively. (b) Sensible heat flux analogue, where $r_a$ is the resistance of the air to the transfer of sensible heat.

\[ I = \frac{(V_2 - V_1)}{R} \]

\[ H = \frac{\rho c_p (T_2 - T_1)}{r_a} \]
Fig. 1.7 Physical manifestation of Ohm's law analogy (from Lee, 1992).
Fig. 1.8 Saturation vapor pressure versus temperature diagram showing Penman's linearization (adapted from Oke, 1987). The assumption of a constant slope $\Delta = (e_{sg}(T_g) - e_{s}(T))/(T_g - T)$ is necessary to derive the Penman-Monteith equation (see Appendix A).
Fig. 1.9 Typical (a) wind and (b) temperature profiles in the surface layer based on Monin-Obukhov similarity theory, assuming $z_{zm}$ and $z_{ao}$ are 1 cm and $T_s = 300$ K. The quantity $L$ is a measure of surface layer stability. Positive (negative) values represent stable (unstable) conditions, and infinity represents neutral conditions. Appendix B provides more details on the derivation of the theoretical profiles.
Sensible heat fluxes
NORM Mesonet site
July 12, 1997

Fig. 1.10 Calculated H for $z_{oh}=z_{om}$ and $z_{oh}=z_{om}/e^2$ on 12 July 1997 at NORM.
Fig. 1.11 Typical vertical temperature profiles for (a) morning and (b) afternoon, where temperature increases to the right. The vegetation canopy is represented by the space between the thicker solid lines.
Overhead View of a Mesonet Station

Fig. 2.1 Spatial arrangement of the instrumentation suite at a Mesonet site (overhead view).
Fig. 2.2 (a) Locations of Mesonet soil moisture sensors. (b) Locations of ARM sites. EBBR sites are denoted with "EF" prefixes.
Campbell Scientific 229L Matric Potential Sensor

Fig. 2.3 Schematic of the Campbell Scientific 229-L Matric Potential Sensor.
Fig. 2.4 (a) Spectral response of the LI-COR 200SZ pyranometer (LI-COR, 1991). (b) Radiative energy spectrum of the sun (left) and Earth (right) (Wallace and Hobbs, 1977, p. 288). The y-axis represents the irradiance (E) normalized by wavelength (\(\lambda\)).
Fig. 2.5 (a) Diurnal clear-sky albedo cycle from half-hourly ARM data. (b) Relationship between clear-sky albedo and solar altitude (Oke, 1987, p. 133).
Fig. 2.6 Oklahoma statewide average of mean monthly precipitation, 1892-1990 (adapted from Johnson and Duchon, 1995).
Average albedo at Norman site (1990-1995)
Annual cycle accounts for 69% of variance in observed data

Fig. 2.7 Annual albedo variation at Norman radiation site.
Fig. 2.8 (a) Annual albedo variation at four ARM sites. (b) Annual albedo variation at FIFE site.
Fig. 2.9 Monthly mean albedo estimates from April 1996.
Fig. 2.10 Comparison of observed and calculated LW_{\text{d}} using the Anderson (1954) scheme from the daylight hours (1400-2330 UTC) of November 1995. The data were obtained from the ARM CF site. The solid line represents the results of the linear regression, while the dashed line represents a "perfect-fit" line. Mean bias errors and root mean square errors are given in W m^{-2}.
Fig. 2.11 Same as Fig. 2.10 except using the Brutsaert (1975) scheme.
Fig. 2.12 Response of a 229-L probe (ΔT) as a function of the natural logarithm of time after current is applied to the probe. Response shown over the typical range of water potentials, ranging from wettest (-3.4 kPa) to driest (-310.3 kPa). Regression equations provided, where R is the correlation coefficient. Data provided by Jim Bilskie of Campbell Scientific, Inc.
Fig. 2.13 Response of the 229-L probe ($\Delta T$) at the Norman Mesonet site at a depth of 5 cm from 5/28/96 to 7/5/96. The diurnal oscillation is likely due to vertical transport of soil moisture due to during the day due to evapotranspiration (Basara et al., 1998).
Fig. 2.14 Scatterplot of response of 221 229-L probes to both "dry" and "wet" soil. Each data point represents an average of 20 observations. Data is organized by instrument number along the x-axis.
Fig. 2.15 Locations and station ID's for 5 Mesonet sites that measure ground heat flux and net radiation.
Fig. 2.16 Comparisons between calculated and observed ground heat flux near sunrise at APAC from 29 April to 4 May 1997.
Fig. 2.17  Same as Fig. 2.16, but for GOOD.
Fig. 2.18 Same as Fig. 2.16, but for MARE.
Fig. 2.19 Same as Fig. 2.16, but for WIST.
Fig. 2.20 Relationship of surface conductance to (a) incoming solar radiation, (b) humidity deficit, and (c) soil moisture based on FIFE data. The solid (dashed) curves represent data from FIFE site 16 (26) (Stewart and Verma, 1992). The surface conductance stress functions are then multiplied together to quantify the response of the vegetation to given atmospheric conditions.
Hypothetical diurnal cycles of surface resistance

Fig. 2.21 Diurnal variation of $r_s$ for wet and dry conditions.
Fig. 2.22 Average calculated \( r_s \) for March-May 1997 at the Lamont CF from 0830-1700 LST. Average is comprised of 22 clear days.
Fig. 2.23 Surface resistance of a barley field at Rothamsted, England on 23 July 1963.
Fig. 2.24 The influence of relative stomatal conductance on the daily maximum surface heat fluxes simulated with a biosphere-atmosphere model (Avissar and Pielke, 1991).
Fig. 2.25 Schematic representation of the stomatal response to the environment. Adopted from Avissar and Pielke (1991).
Fig. 2.26 (a) Surface resistance time series at Lamont for Julian days 105-298. (b) Surface resistance time series at Lamont for Julian days 257-266.
Fig. 2.27 Vapor pressure deficit time series at Lamont.
Fig. 2.28 Soil water potential time series at BLAC.
Fig. 2.29 Surface resistance (solid) and vapor pressure deficit (dashed) time series at Lamont for Julian days 100-200 (a) and 200-300 (b) in 1997.
Fig. 2.30 Scatterplot of surface resistance and vapor pressure deficit at Lamont. Dashed line represents relationship described in (2.42).
Fig. 2.31 Time series of final residual.
Fig. 2.32 Time series of $r_i$ at Pawhuska ARM site during the growing season of 1997.
Fig. 2.33 Flow chart describing the computational process used to calculate surface fluxes.
Fig. 3.1 Comparison of observed and calculated $R_n$ at 15-minute intervals from the daylight hours ($R_n > 0$) of July 1997 at APAC. The solid line represents the results of the linear regression. $R$ is the correlation coefficient. Mean bias errors and root mean square errors are also given.

$y = 8.2694 + 0.99399x$  
$R = 0.99539$  
$MBE = 6.2 \text{ W m}^2$  
$RMSE = 21.8 \text{ W m}^2$
Fig. 3.2 Same as Fig. 3.1 except at GOOD.
Fig. 3.3 Same as Fig. 3.1 except at MARE.
Fig. 3.4 Same as Fig. 3.1 except at NORM.
Fig. 3.5 Same as Fig. 3.1 except at WIST.
Fig. 3.6 Time series of observed (thin) and calculated (thick) $G$ at El Reno ARM site and El Reno Mesonet site, respectively. Comparisons were made at 30-minute intervals during July 1997.
Fig. 3.7 Same as Fig. 3.6 except at Pawhuska ARM site and Foraker Mesonet site. Note the missing data from 6 July to 18 July.
Fig. 3.8 Measured G at the NORM site during July 1997.
Fig. 3.9 Idealized near-surface temperature profiles for small \( (z = z_{\text{low}}) \) and large \( (z = z_{\text{high}}) \) roughness lengths. The thick solid line represents the background temperature profile. The long-dashed (short-dashed) line represents the temperature profile beneath the roughness length for large (small) roughness length. The top (bottom) figure represents an assumption of constant (decreasing) temperature beneath the roughness length. \( \Delta T \) is the expected ground temperature difference between the large and small roughness lengths. Note that \( \Delta T \) is much larger for the case of decreasing temperature beneath \( z_{\text{cr}} \).
Fig. 3.10 Comparison of observed (thick) and calculated (thin) Bowen ratios at NORM Mesonet site at 1800 UTC during July 1997.
Fig. 3.11 Time series of rainfall (thick) and 5 cm soil water potential at 1800 UTC at NORM Mesonet site during July 1997.
Fig. 3.12 Time series of soil moisture sensor temperature response at 1800 UTC at NORM Mesonet site during July 1997. Larger (smaller) values imply drier (wetter) soil.
Fig. 3.13 Time series of soil water potential and rainfall at El Reno Mesonet site during July 1997.
Fig. 3.14 Same as Fig. 3.12 except for Bowlegs Mesonet site.
Fig. 3.15 Time series of rainfall and 1800 UTC Bowen ratio at NORM Mesonet site during July 1997.
Fig. 3.16 Locations of 44 Mesonet sites used in surface flux calculations.
Fig. 3.17 Time series of observed (dashed) and calculated (solid) $H$ at BEAV Mesonet site on 2 July 1997. Data are available at 5-minute intervals.
Fig. 3.18 Same as Fig. 3.17 except for 3 July.
Fig. 3.19 Same as Fig. 3.17 except for 13 July.
Fig. 3.20 Time series of soil water potential and rainfall at BEAV Mesonet site for July 1997.
Fig. 3.21 Same as Fig. 3.17 except for 25 July.
Fig. 3.22 Same as Fig. 3.17 except for 31 July.
Fig. 3.23 Comparison of observed and calculated $H$ at 5-minute intervals from the daylight hours (1400-2200 UTC) for five "golden days" in July 1997 at BEAV. The solid line represents the results of the linear regression, and $R$ is the correlation coefficient. Mean bias errors and root mean square errors are also given. The dashed line represents a perfect fit between the observed and calculated values.
Fig. 3.24 Time series of observed (dashed) and calculated (solid) H at NOWA Mesonet site on 12-13 July 1997. Data are available at 5-minute intervals.
Fig. 3.25 Average calculated $R_n$ for July 1997 at 1800 UTC. Contours are spaced at intervals of 25 W m$^{-2}$. 
Fig. 3.26 Same as Fig. 3.25 except for G.
Fig. 3.27 Same as Fig. 3.25 except for H.
Fig. 3.28 Same as Fig. 3.25 except for LE.
Fig. 4.1 Calculated H on 12 July 1997 at 1800 UTC. Values are contoured at 25 W m\(^2\) intervals.
Fig. 4.2 V-component of wind on 12 July 1997 at 1800 UTC in m s\(^{-1}\). Contours represent 8.0 m s\(^{-1}\).
Fig. 4.3 Calculated H on 13 June 1998 at 1800 UTC. Approximate position of dryline denoted by scalloped line.
Fig. 4.4 Wind direction on 13 June 1998 at 2000 UTC in degrees. Contours represent 170 degrees.
REFERENCES


Appendix A: Derivation of the Penman-Monteith Equation

Equation (1.16) can be rearranged:

\[ e_g = e_{ig}(T_g) - \frac{\gamma r_s LE}{\rho c_p} \]  

(A.1)

and substituted into (1.13) to produce

\[ LE = \left(1 + \frac{r_s}{r_a}\right) \frac{e_{ig}(T_g) - e}{\frac{\rho c_p}{r_a}} \]  

(A.2)

The following expansion is then employed:

\[ e_{ig}(T_g) - e = e_{ig}(T_g) - e_i(T) + e_i(T) - e \]

\[ = \frac{e_{ig}(T_g) - e_i(T)}{T_g - T} (T_g - T) + e_i(T) - e \]  

(A.3)

\[ = \Delta (T_g - T) + e_i(T) - e, \]

where (see Fig. 1.8)

\[ \Delta = \frac{e_{ig}(T_g) - e_i(T)}{T_g - T} \]  

(A.4)

and \( e_i(T) \) is the saturation vapor pressure at \( T \). From (1.12), we get

\[ T_g - T = \frac{r_s H}{\rho c_p} \]  

(A.5)

which can be substituted back into (A.3):

\[ e_{ig}(T_g) - e = \Delta \left( \frac{r_s H}{\rho c_p} \right) + e_i(T) - e, \]  

(A.6)

and from there into (A.2):

\[ \left(1 + \frac{r_s}{r_a}\right) \frac{\Delta \left( \frac{r_s H}{\rho c_p} \right) + e_i(T) - e}{\frac{\rho c_p}{r_a}} \]  

(A.7)
Equation (1.2) provides $H = R_a - G - LE$, which is substituted into (A.7). After some algebraic manipulation, the final form of the PM equation is obtained:

$$LE = \frac{(R_a - G)\Delta r_s \gamma^{-1} + [e_s(T) - e_p c_p \gamma^{-1}]}{r_s + (1 + \Delta \gamma^{-1})r_a}.$$  \hspace{1cm} (A.8)

Again, $H = R_a - G - LE$, and this along with (A.8) this produces the final form of the sensible heat flux:

$$H = \frac{(R_a - G)(r_s + r_s) - [e_s(T) - e_p c_p \gamma^{-1}]}{r_s + (1 + \Delta \gamma^{-1})r_a}.$$  \hspace{1cm} (A.9)
Appendix B: Similarity Theory

Similarity theory describes the behavior or the characteristics of a set of mutually similar entities. The simplest type of similarity to envision is probably geometric similarity. Two triangles are similar if the ratios of their sides are equal. For example, a triangle with sides of length 3-4-5 is similar to a larger triangle with sides of length 9-12-15. Similarity does not imply equality, just a change in scale. Picking one out of a group of similar triangles, one needs only to specify the length of any of the sides to know all the characteristics of the triangle. In an analogous way, dynamic similarity theory can be employed in order to understand the characteristics of turbulent flow in the surface layer. First, dimensionless quantities are constructed from the relevant atmospheric variables in a given situation. The relationship between these quantities remains constant as the scale of the turbulent eddies changes, just like the relationship between the sides of similar triangles. If only one of the dimensionless quantities is measured, the remaining characteristics of the flow can be revealed.

The derivation of the logarithmic wind profile is a typical example of applied similarity theory (Sorbjan, 1989). In a neutral surface layer, it is assumed that the vertical wind shear is a function of height and surface stress. The surface stress $\tau$ is parameterized as proportional to the square of the friction velocity, $u^*$,

$$\tau = -\rho u'w' = -\rho u^2, \quad (B.1)$$

where $\rho$ is the air density, and $u'$ and $w'$ are the perturbation horizontal and vertical velocities, respectively. The overbar represents a short-term time average. So, we start with

$$\frac{du}{dz} = f(z,u^*). \quad (B.2)$$

Note that there are two relevant quantities ($z,u^*$) and two dimensional units (length, time) involved. One of the tenets of dimensional analysis (Buckingham's Pi Theorem) is that
the number of independent dimensionless parameters which can be constructed is equal
to the difference between the number of quantities (2) and the number of dimensional
units (2). In this case, no independent dimensionless parameters can be constructed, so
the dependent dimensionless parameter is constant:

\[
\frac{du}{dz} = \frac{1}{k} z^a u^b, \quad (B.3)
\]

where \( k \) is called the von Kármán constant, which has been experimentally determined to
be approximately 0.4. Dimensional analysis gives \( a = -1 \) and \( b = 1 \):

\[
\frac{z}{u_*} \frac{du}{dz} = \frac{1}{k}. \quad (B.4)
\]

Integrating (B.4) from \( z_{0m} \) to \( z \), we get

\[
u(z) - u(z_{0m}) = u(z) = \frac{u_*}{k} \ln \frac{z}{z_{0m}}, \quad (B.5)
\]

which describes the vertical variation of the wind in the neutral surface layer. By
definition, \( u(z_{0m}) = 0 \). From (B.5), we can see that the wind at any level in the surface
layer can be predicted just from a knowledge of the surface roughness and surface stress.
In other words, for a given surface, all vertical wind profiles in the neutral surface layer
are similar and scale with \( u_* \).

Monin and Obukhov (1954) showed how surface layer flow is affected by non-
nearal stability by introducing another scale, \( L \), dubbed the Monin-Obukhov length.
This value represents the scale of turbulent eddies, whose size varies greatly with
stability. The stability modification alters the analysis:

\[
\frac{du}{dz} = f(z, u_*, L). \quad (B.6)
\]
Since there are three relevant quantities and only two-dimensional units, dimensional analysis produces one independent dimensionless parameter (specified as $\Phi_m$), which itself must be a function of a dimensionless quantity $(z/L)$:

$$\frac{du}{dz} = \frac{1}{k} \Phi_m \left( \frac{z}{L} \right) z^a L^b u^c,$$  \hspace{1cm} (B.7)

where $\Phi_m$ is called the universal function for momentum. From the dimensional analysis, $a = -1$, $b = 0$, and $c = 1$, so

$$\frac{kz \ du}{u_* \ dz} = \Phi_m \left( \frac{z}{L} \right).$$  \hspace{1cm} (B.8)

The exact form of $\Phi_m$ has been determined empirically for different stability regimes.

Integrating (B.8), we get

$$u(z) = \frac{u_*}{k} \left[ \ln \frac{z}{z_{om}} - \int_{z_{om}}^z \left( 1 - \Phi_m \left( \frac{z}{L} \right) \right) \ln \left( \frac{z}{L} \right) \right]$$

$$= \frac{u_*}{k} \left[ \ln \left( \frac{z}{z_{om}} \right) - \psi_m \left( \frac{z}{L} \right) + \psi_m \left( \frac{z_{om}}{L} \right) \right],$$  \hspace{1cm} (B.9)

where $\psi_m(z/L)$ represents a stability correction factor. The term $\psi_m(z_{om}/L)$ is a constant of integration, and insures that $u(z_{om}) = 0$. Since $\psi_m$ represents an integral form of $\Phi_m$, it is also of an empirical nature (see (1.23)). So, for the non-neutral regime, the form of the vertical wind profile is scaled by $u_*$ and $L$. The vertical temperature profile is derived in the same fashion, but it is scaled by $T_*$ and $L$:

$$T(z) = T_* + \frac{T_*}{k} \left[ \ln \left( \frac{z}{z_{om}} \right) - \psi_b \left( \frac{z}{L} \right) + \psi_b \left( \frac{z_{om}}{L} \right) \right].$$  \hspace{1cm} (B.10)

The forms of (B.9) and (B.10) are illustrated in Fig. 1.9.
Appendix C: Error Analysis

In this section, a simple error analysis of the energy budget equation is discussed.

C.1. Net Radiation

The four components of net radiation each have different sources of error. The SW_d term is bound by the error of the instrument, which is 5% of the reading. The SW_u term is controlled by the error in the parameterized albedo value, which is estimated to be 0.05. The new parameterization for LW_d has RMSEs of around 25 W m^{-2}. Finally, the LW_u term is mainly dependent upon the skin temperature estimate, which itself is dependent upon the previous estimate of sensible heat flux. Differentiating (2.50) with typical values of r_s and p, the expected error in skin temperature can be calculated:

\[
\frac{\partial T_s}{\partial H} = \frac{r_s}{\rho c_p} = \frac{50}{1000} = 0.05 \frac{K}{W m^{-2}}. \tag{C.1}
\]

For a error in estimated H of 50 W m^{-2}, this results in a skin temperature error of 2.5 K. This error can then be converted to the error in LW_u by differentiating (2.23) and assuming a mean skin temperature of 300 K and emissivity of 0.98:

\[
\frac{\partial L W_u}{\partial T_s} = 4\varepsilon\sigma T_s^3 \approx 6 \frac{W m^{-2}}{K}. \tag{C.2}
\]

For the estimated 2.5 K error in skin temperature, a 15 W m^{-2} error in LW_u would be expected. The errors are summarized in Table C.1. The combination of the four errors results in a total error of \sim 50 W m^{-2} for the net radiation term. If it is assumed that the errors are uncorrelated, then the total error is just the square root of the sum of the squares of the individual errors (Pythagorean addition).
C.2. Ground heat flux

The formulation for $G$ will be repeated here from (2.27):

$$G = 0.05c_s \frac{dT_s}{dt} + \lambda \frac{T_g - T_{10}}{0.1}. \quad \text{(C.3)}$$

The first term on the right, or storage term, is dominated by the heat capacity of the soil, $c_s$. This term has an estimated error of 25%, which for typical values of the storage term will result in a 2.5 W m$^{-2}$ error. The second, or flux term, can be expected to exhibit noticeable errors in both $\lambda$ and $T_g$. A 25% error in $\lambda$ alone results in a 12.5 W m$^{-2}$ error, while the 2.5 K error in skin temperature causes a 25 W m$^{-2}$ error. Since these two errors are multiplied, the total error in the flux term may be significantly large ($12.5 \times 25 = 312.5$ W m$^{-2}$). However, the errors in $\lambda$ and $T_g$ are inversely related (though not strongly), which will limit the maximum error of the entire term. For example, if $\lambda$ is overestimated (wetter soil) then both $G$ and LE will also be overestimated. This will result in $H$, and thus $T_g$, being underestimated. A summary of the errors is given in Table C.2.

### Table C.1. Summary of the error analysis for the net radiation term.

<table>
<thead>
<tr>
<th>how obtained?</th>
<th>typical value (W m$^{-2}$)</th>
<th>typical/specifed error (W m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SW_d$</td>
<td>measured</td>
<td>500</td>
</tr>
<tr>
<td>$SW_u$</td>
<td>albedo parameterization</td>
<td>100</td>
</tr>
<tr>
<td>$LW_d$</td>
<td>new parameterization</td>
<td>300</td>
</tr>
<tr>
<td>$LW_u$</td>
<td>use skin temp. estimate</td>
<td>400</td>
</tr>
</tbody>
</table>
Table C.2. Summary of the error analysis for the ground heat flux term.

C.3. Latent heat flux

The latent heat flux is calculated using the Penman-Monteith equation:

$$LE = \frac{(R_n - G) \Delta r_s \gamma^{-1} + [e_{s}(T) - e] \rho c_p \gamma^{-1}}{r_s + (1 + \Delta \gamma^{-1}) r_s}.$$  \hspace{1cm} (C.4)

To simplify the analysis, we will assume that the only errors of significance are in $R_n$ (50 W m$^{-2}$), $G$ (assume 50 W m$^{-2}$), and $r_s$ (30 s m$^{-1}$). Pythagorean addition of the $R_n$ and $G$ errors results in a total error in $(R_n - G)$ of ~70 W m$^{-2}$. To clarify further, it is important to note that the errors in $(R_n - G)$ and $r_s$ are correlated, e.g., if $r_s$ is overestimated then $H$, and subsequently $G$, are also overestimated. More succinctly, the errors in $(R_n - G)$ and $r_s$ are of opposite signs.

So, adding typical error values (-70 for $(R_n - G)$ and +30 for $r_s$) to the scale analysis values in (2.39) yields (see 2.39 and 2.40 for scaling values)

$$LE = \frac{(430)(200)(50)(0.015) + (1000)(1000)(0.015)}{80 + (1 + (200)(0.015))50} = 285 \text{ W m}^{-2}, \hspace{1cm} (C.7)$$
which is a 75 W m\(^{-2}\) underestimate of the typical value in (2.39), and is the same sign as the error in \((R_n - G)\).

C.4. Sensible heat flux

The sensible heat flux is calculated as a residual, simply \(H = R_n - G - LE\). In this analysis, errors of 70 W m\(^{-2}\) for \((R_n - G)\) and 75 W m\(^{-2}\) for \(LE\) were found. Since these errors are of the same sign, the error in \(H\) is only 5 W m\(^{-2}\), since practically all of the error in \((R_n - G)\) is absorbed by \(LE\) through the Penman-Monteith method. The result of this error analysis suggests that this method may not be effective in calculating \(LE\) but will perform well in calculating \(H\).