UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

# EMERGENT MATHEMATICS IN A GRADE-TWO CLASSROOM: A SEARCH FOR COMPLEX RELATIONSHIPS

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

By

NOEL GEOGHEGAN
Norman, Oklahoma
1998

UMI Number: 9911857

EMERGENT MATHEMATICS
IN A GRADE-TWO CLASSROOM:
A SEARCH FOR COMPLEX RELATIONSHIPS

A Dissertation APPROVED FOR THE
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC
CURRICULUM

BY

## ACKNOWLEDGMENTS

The completion of this project has been made possible by the encouragement and support of many people. Especially, I wish to express my sincere gratitude to Dr. Anne Reynolds whom, with sensitive insight, patience, and faith provided the guidance and support I needed to accomplish the completion of this dissertation.

My other advisory committee members also proved inspirational in their contribution to the progress of my work. Dr. Jayne Fleener, Dr. Susan Laird, Dr. Neil Houser, and Dr. Frank McQuarrie have all been thoroughly supportive and key to my accomplishments at the University of Oklahoma.

I will be forever grateful to Eileen Lillard. Her unfailing optimism in children's thinking proved a key perspective in helping me make sense of the whole experience. I also wish to express my gratitude to all the children who so enthusiastically participated in the classroom during my visits.

In memory of Marg Fagan who passed away during the time of this study, I wish to pay respectful gratitude for her contribution to assisting in the timely completion of this project.

Finally, without the support and love of my dear family, Deborah, Patrick, Robert, and Thomas, who had their lives changed so dramatically by coming to Oklahoma, the dream would still be merely a challenge.

## TABLE OF CONTENTS

# LIST OF FIGURES

**Page**

**Figure**

## ABSTRACT

This study investigated the emergence of relationships in a grade-two classroom constituted by the contemporaneous implementation of the three theoretical perspectives of (1) problem-centered learning, (2) constructivism, and (3) positive discipline. The study explored the organic interconnectedness, evolution, and creative processes of learning mathematics framed by these three theoretical perspectives from a systems theory viewpoint. The study proceeded by accounting for the epistemological, ontological, methodological, and psychological characteristics reflected in a grade-two mathematics program addressing current reform agendas in mathematics education.

The study employed a multiple-methods ethnographic approach incorporating the use of action research and constant comparative models, and the development of grounded theory. The findings highlighted the reflexive complementarity of a mathematics program based in the theoretical perspectives of problem-centered learning, constructivism, and positive discipline. Furthermore, the study indicated that approaches to learning mathematics advocated by current reform agendas understate the significance of learning as being as much to do with control as it does with change.

In light of the organically interconnected relationships of learning mathematics, a theoretical heuristic was developed to exemplify the implications of the findings of the study. Called SEARCH, the heuristic highlighted learning

mathematics as a synergistic relationship constituted by an interconnectedness among social, emotional, physical, and cognitive development, each of which requires a balanced consideration in order to pursue the epistemological, ontological, and methodological paradigmatic frameworks embedded in current mathematics reform agendas. SEARCH is an acronym that stands for Social Emancipation, Active Referencing, and Creative Heuristics. SEARCH also metaphorically implies that learning mathematics is a "search" - i.e., what learners do as they endeavor to make mathematical sense. At the same time, a "search" is what teachers do in making sense of the mathematics each student constructs. It is upon such a foundation that the implications for pedagogical change engendered in contemporary mathematics education reform perspectives have been addressed.

While the SEARCH heuristic endeavors to provide a paradigmatic framework for addressing the current mathematics reform agenda, it brings to light the potential for tension in pedagogical pragmatism as teachers seek to locate themselves in and between "intervention" (qua behaviorist) approaches and "invention" (qua constructivist) perspectives.

# CHAPTER I

# INTRODUCTION

Mathematics education has been of considerable international concern over the last quarter century (Australian Education Council, 1991; Cockcroft, 1982; McKnight, et al., 1987; National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, (NCTM) 1980; 1989; 1991; National Research Council, 1989). Set against a backdrop of controversy and debate, efforts to reform mathematics education have ebbed and flowed to a variety of theoretical, philosophical and political issues. Increased attention to "better understand the nature of mathematical learning" (Kieran, 1994, p. 605), and attempts to "unravel the mystery of effective [mathematics] teaching" (Cooney, 1994, p. 622), however, have not generated the desired outcomes. The Third International Mathematics and Science Study (TIMSS) sounded a note of caution for many educators around the world. The results of the survey suggested that the mathematics standards of North American students, for example, were well below the international average and that the performance of twelfth-grade U.S. students was at the bottom of the standard.

The TIMSS results, along with other mounting evidence (Burton, 1994; National Commission on Teaching and America's Future, 1996; National Council of Teachers of Mathematics, 1989; National Research Council, 1989; Schmidt, McKnight, & Raizen, 1996; Tall, 1995), have emerged as an indictment on mathematics education in U.S. classrooms. In a follow-up report from the TIMSS results called, *Splintered Vision*, Smith, McKnight, and Raizen (1996) concluded that mathematics curricula in the USA are a "mile wide and an inch deep."

> *The claim is based on both the curriculum analyses and video study of instruction in three countries, where researchers documented that, on average, American students are introduced to more topics in math . . . than most of the other countries studied. In Minnesota, for example, a typical eighth-grade math lesson spanning a single class period includes an average of 3.5 topics (SciMath, 1997). The average U.S. mathematics curriculum, it seems, gives little sustained attention to any one aspect of a content area. The number of topics introduced in math . . . classrooms also has implications for how teachers present the material. It seems that in an attempt to cover a wide range of topics, U.S. teachers tend to lecture rather than allowing time for student groups to engage in problem solving. According to student surveys in Minnesota, the most frequent classroom activities are teacher demonstration of problems, worksheets, and individual work (SciMath, 1997). TIMSS showed that American students spend considerably more hours in math classes than their German and Japanese counterparts. Thus, poor U.S. performance on the math test cannot be explained by time, but how that time is spent. (North Central Regional Educational Laboratory, 1997, p. 1)*

In addition, U.S. textbooks mirror this trend by including a great variety of topics. Teachers emphasize coverage of content over understanding content matter. Ironically, according to the North Central Regional Educational Laboratory (1997), the mathematics principles included in the standards developed by the National Council of Teachers of Mathematics (1989), "appear to be more consistently applied in Japanese classrooms than in the U.S." (p. 1).

TIMSS researchers also found U.S. teachers believe they are integrating the new mathematics standards (which are considered the flagship of the contemporary reform movement) into their lessons, giving rise to a concern among academics that there might be a significant gap between the goals of standards-based reform and the reality of how standards are being implemented in the classroom (North Central Regional Educational Laboratory, 1997). As a consequence, mathematics education continues to attract intensifying demands for explicating the current reform agenda in practical terms.

Along with the evolution of the current mathematics education reform movement has been the establishment of a research community which seeks to challenge the legitimacy of the pedagogical hegemony implicated by recent test results, i.e., pedagogy which emphasizes procedural and instrumental knowledge (Skemp, 1978), decries epistemic inventiveness (Glasersfeld, 1996a), promotes heteronomy and absolutism (Ernest, 1991; Hersh, 1993), and promulgates a logic of domination (Fleener & Laird, 1997). Considered endemic in American classrooms (Hersh, 1993) such pedagogical hegemony is deemed to have been perpetrated and perpetuated by behavioristic assumptions about teaching and learning. However, a strong challenge continues to emerge and question the counter-productivity of traditional mathematics education. As retiring editor of *FOCUS*, the journal of the *Mathematical Association of America*, Keith Devlin (1997) argued that:

> *The aim of mathematics education should be to produce an educated citizen, not a poor imitation of a $30 calculator. . . . The justification for [this] goal is simply this: A human life is the richer for having greater understanding of the nature of that life. The more ways we have to know our world and ourselves, the richer are our lives . . . any university mathematics instructor will tell you that the present high school mathematics curriculum does not prepare students well for university level mathematics. Nor is success at high school mathematics a good predictor of later success in mathematics. The reason is simple. School mathematics is largely algorithmic: To succeed, the student needs only to learn various rules and procedures and know only when and how to apply them. In contrast, university level mathematics is highly creative, requiring original thought and the ability to see things in novel ways . . . our present system of school mathematics education probably turns off a significant number of students who have the talent for later mathematical greatness. (pp. 2-3)*

Educational practices steeped in the linear and procedural thinking of the industrial and other bygone eras are no longer considered relevant to the needs of society. Betts (1992) claims that while the rest of society has moved into a post-Industrial era, schools remain firmly rooted in the paradigm of the industrial age. Accordingly, Steffe (1992) avers that "one of the most urgent problems of education today" (p. 1) is the reconstitution of mathematics education. As a reconstitution of classroom practice continues to set the focus for mathematics education research, in order to revitalize and advance mathematics education, the National Council of Teachers of Mathematics (1993) insists that "instruction in mathematics must be significantly revised" (p. 1) and, that "we must go beyond how we were taught and teach how we wish we had been taught" (p. 3).

However, despite efforts to achieve reform in mathematics education, an anomaly has emerged. Teachers' approaches and teaching methods are not in line with the latest research. It appears that we know more about how children learn than we do about how to apply this knowledge. "Reports that are disseminated to teachers appear to have little impact on classroom practices, and anecdotal evidence from teachers who have seen the TIMSS teaching videos [for example] attribute differences in instruction to the culture of the school and the behavior of the students" (North Central Regional Educational Laboratory, 1997, p. 1).

Despite the abundance of research on intellectual and social development, and learning that is rich with implications for the kind of curriculum that we should be providing for young children, classroom teaching is conspicuously slow to change. Remonstrations about classroom practices being way behind what we know about learning are becoming increasingly audible. In his concluding remarks as final contributor in the *Handbook of Research on Mathematics Teaching and Learning*, Robert Davis (1992) noted that all the "ingredients seem to be in place" (p. 732), however he, along with "a number of other observers" (p. 730), was convinced that the classroom mathematics "situation is worse now than it was in 1958" (p. 730). Wood (1995) notes that, for example, the "activity of children's learning from a constructivist perspective has been well researched, but attempting to understand teaching from this perspective has been neglected" (p. 204).

## Cognitive Preoccupation

Mathematics education research has historically tended to focus on the foundations of human learning from a purely cognitive perspective emphasizing aspects such as IQ, intellectual milestones, and isolated psychological phenomena such as memory strategies and task-specific cognitive strategies. The continuation of such a focus has generated considerable disagreement as to what should constitute the locus of reform in mathematics education. Rogoff (1990), for example, contends that the emphasis on the isolation of individual cognitive processes "in vacuo" (Rommetveit, 1979), as if mental activities existed in a cultural, social, emotional, and historical vacuum, has been detrimental to the dialogue between educational researchers, practitioners, and the general public. Correspondingly, Csikszentmihalyi (1995) contends that after decades of translation of educational research in the classroom, a severe "limitation of schools [these days] is that they concern themselves almost exclusively with the development of cognitive skills" (p.107).

Proponents of educational research who assert that it is possible, even desirable, to emphasize independent, or specific areas of mental functioning in the individual in isolation, justify this approach by claiming that we must simplify the problems we address if we are to get concrete research underway. Only then, it is argued, can we go on "to understand how cultural, historical, or institutional 'variables' enter into the picture" (Wertsch, 1991, p. 2). Crawford (1996) alleges that Western psychologists' traditional limitation of study of human development "to the skull, or skin, of an individual" (p. 133) has positioned cognitivism as the paradigmatic mother-lode for educational research. Eisenhart (1988) has even suggested with considerable justification that as a consequence of established psychological orientations, mathematics educators "are accustomed to assuming that the development of cognitive skills is central to human development, [and] that these skills [are accepted as appearing] in a regular sequence regardless of context or content" (p. 101).

Lerman (1996) argues that in mathematics education in general, "the 'pure' abstract cognitive functioning that appears to emanate from [academic activity] is reified as the exemplar of the highest level of intellectual activity" (p. 145). But Lave (1988) challenges that reification process by stating:

> *It is not at the level of cognitive processes that the unique, the non-routine, the crisis, the exception, the creative novelty, the scientific discovery, major contributions to knowledge, ideal modes of thought, the expert and the powerful, are brought into being and given significance and experienced as such. These are all matters of constitutive order in the broadest and most complex sense, and they*

*are constructed in dialectical relations between the experienced lived-in world and its constitutive order - in practice. (p. 190)*

Cartesian and Newtonian Roots

The deification and reification of cognitive analyses in mathematics education research is rooted in a Cartesian split between the body and the mind and a Newtonian image of a mechanistic universe.

> *Rene Descartes created the metaphor of analytic thinking, which consists in breaking up complex phenomena into pieces to understand the behavior of the whole from the properties of its parts. Descartes based his view of nature on the fundamental division between two independent and separate realms - that of mind and that of matter. The material universe, including living organisms, was a machine for Descartes, which would in principle be understood completely by analyzing it in terms of its smallest parts. (Capra, 1996, pp. 19-20)*

Following Descartes, Newton's grand synthesis of a "machine model" reality became the established paradigm of science for nearly three centuries. The "machine model" assumption established the scientific approach to human research, that by pulling apart and comprehending the workings of each piece (qua of the individual), the whole can be put back together without any significant loss and understood in its entirety (qua the norm). In general, according to Sarason (1981), mathematics education as part of the American educational research tradition, has been for the last 100 years quintessentially an analysis of the individual as an independent cognizing organism.

The Cartesian and Newtonian views which have been the backbone of American educational research are referred to as "modernist" thought, or "modernism." The Cartesian/Newtonian "machine" paradigm characterized modernist thought by particularism, materialism, and reductionism, and emphasized a linear approach to explaining how things work. As a result "learning" was cast as a lock-step procedure of putting knowledge ("the building blocks of knowledge") into place according to sets of predictable (and in the case of mathematics, immutable) rules. Translated into pedagogical terms "learning" consisted of mastering a series of predetermined mental concepts identified as being of appropriate rigor and distinguished by designated cognitive steps and stages. The ensuing concept of curriculum as "autonomous but interconnected is ubiquitous" (Doll, 1993, p. 38). Even today, from first grade on, curriculum is considered in terms of units of knowledge arranged in linear order based on hierarchical sequences and organized by cognitive stages of development.

The modernist desire to reduce all learning to a well-defined prescription of mental functioning has been criticized by recent theorists claiming that such an approach has the pervasive goal of control, "control of teachers, of students, of content" (Noddings, 1992, p. 9). Dewey (1938), as an ardent critic of Cartesian/Newtonian educational perspectives (which he referred to as "traditional" education), described the institutionalized patterns of traditional education as "imposition from above and outside" (p. 18).

> *The subject-matter of education consists of bodies of information and of skills that have been worked out in the past; therefore the chief business of the school is to transmit them to the new generation. In the past, there have also been developed standards and rules of conduct; moral training consists in forming habits of action of conformity with these rules and standards. (pp. 17-18)*

However, despite Dewey's and others' aspirations to evoke reform in education by revoking modernist educational approaches, pedagogy in today's classrooms is still dominated by the atomistic, mechanistic, and reductionist perspectives reflective of a Cartesian/Newtonian paradigm.

### Post-modern Developments

Recently, the relevance of modernism and its "machine model" paradigm as the foundation for educational as well as political, economic and social decision making is being increasingly questioned. The epistemological, ontological, and methodological issues resident in the modernist Cartesian/Newtonian perspectives are emerging as anachronisms within a sea of late 20th-century scientific, social, and philosophical change. Known as "post-modernism," new worldviews emanating from the currents of 20th-century science (Gleick, 1987) are redefining the scientific, social, and philosophical landscapes of our times.

Post-modernist thinking has developed as an extension and response to modernism and has been influential in moving the world (particularly Western society) towards a 21st-century of New Science infused with concepts of self organization, indeterminacy, order emerging from chaos, and creative making of meaning. Post-modern thinking heralds a new paradigm of holism that contrasts with the modern paradigm of reductionism. Holism seeks to understand the system as a system and gives primary value to the relationships that exist among seemingly discrete parts. Post-modern thinking is providing new perspectives with which to address the reform agenda in mathematics education. For example, Cobb and Bauersfeld (1995) note that a "growing trend to go beyond purely cognitive analyses is indicated by an increasing number of texts that question an exclusive focus on the individual learner" (p. 2).

In consonance with early 20th-century education theorists (cf. Dewey, 1938 and other pragmatists such as Mead, 1934) a zeitgeist of systemic ideology, advocating systemic rather than reductionist educational imperatives, has continued to develop through to the present time. Reflecting the world of relationships to be found in quantum physics, a paradigm of systemic holism and self-organization is gaining significant pedagogical reform currency as we approach the 21st century (Csikszentmihalyi, 1995; Doll, 1993; Kohn, 1996; Noddings, 1992; Rorty, 1989).

Systemic and holistic perspectives identify human development and learning with the process of life itself (Capra, 1996; Maturana & Varela, 1980; Varela, Thompson & Rosch, 1991) and in doing so, seek to transcend prevailing behaviorist views ensconced in a sufferance for quantifying an objective reality in terms of cognitive development, and vice versa. Mandelbrot's (1977) seminal fractal exercises (e.g. measuring the fractal coastline of Britain) and Heisenberg's "Indeterminacy Principle" clearly imply that it is impossible to ever know any precise measurement. As Favre, Guitton, Guitton, Lichnerowicz, and Wolff (1995) advise,

> *It is . . . clear that mathematical [quantified] representation must be used cautiously to avoid overly simplified and reductive formulations, which risk being artificial and irrelevant. . . . With or without mathematical models, science never arrives at absolute truths - merely at truths approximated with greater or lesser degrees of precision. (p. 1)*

Wheatley (1994) argues that if we continue to focus on a quantifiable reality we will ignore the qualitative features of systems - in their complexity - and will always be frustrated by the incomplete and never-ending information we receive. Measurement of cognitive development will never be a conclusive basis for explaining and informing classroom instruction. In short, the myopia of the behaviorist vision extends from a mistaken belief that the physics of the modern era is the ultimate science and that by reducing all to physics and quantifiable causes, one is dealing with the basic principles underlying reality.

The origin of our educational dilemma, according to Capra (1996),

> *. . . lies in our tendency to create the abstractions of separate objects, including a separate self, and then to believe that they belong to an objective, independently existing reality. To overcome our Cartesian anxiety, we need to think systemically, shifting our conceptual focus from objects to relationships. Only then can we realize that identity, individuality, and autonomy do not imply separateness and independence. (p. 295)*

Doll (1993) suggests that "in the modern paradigm, stability, external control, and an *a priori* aboriginal reality . . . were all considered self-evident . . . [h]owever in the post-modern paradigm . . . contingency abounds [and it] is common to say that in post-modernism that nothing is foundational, all is relational" (italics in original, p. 158). What appears to be self-evident and natural in one paradigm becomes absurd in the other.

In an era of educational flux precipitated by post-modern and New Science revelations (Hargreaves, 1994), classroom teachers are beginning to find themselves caught within and between significantly different paradigms. Making substantial changes in classroom practice is proving to be a challenging and complex affair. Fleener and Fry (1998) argue that reform in mathematics education could draw considerable inspiration from the discoveries of New Science and perspectives of post-modern philosophical thought. However, as Voigt (1995) points out, "if we want to reform classroom life, we should not only know what we want to do, but we also need the means to understand what happens in fact" (p. 198).

In recent years, research in mathematics education has been attempting to explain how emerging theories about how people learn link with classroom pedagogy. Increasingly it is argued that critical to understanding student outcomes is the relationship between what is taught and how it is taught (Kroeze & Johnson, 1997). Such an orientation gives countenance to Bauersfeld's (1995) view that "teaching and learning are intimately related issues" (p. 287).

However, the articulation of the complexities engendered in the relationship between the teacher's role (qua teaching) and learning, from post-modern and New Science perspectives, in the mathematics classroom is proving to be an imposing challenge. "Connecting and transforming modernism with 'post' thinking will not be easy. Modernism is so ensconced in our language and thought that its most basic assumptions seem self-evident" (Doll, 1993, p. 157). (Further elaboration of the post-modern point of view is provided in APPENDIX G.)

## Dynamical Systems Theory

Recent developments in cognitive science have made it clear that human intelligence develops in a very different manner from the machine model depicted over the centuries. The human nervous system does not process information in the sense of discrete ready-made elements waiting to be absorbed, but rather interacts with the environment by continuously modulating its structure (Capra, 1996). The process of continuous modulation is a highly complex phenomenon and in recent decades has been explored and developed under the guise of "self organization." The key ideas of self organization have been elaborated from the work of many different researchers working on many different systems and in several countries - Ilya Prigogine in Belgium, Hermann Haken and Manfred Eigen in Germany, James Lovelock in England, Lynn Margulis in the United States, and Humberto Maturana and Francisco Varela in Chile.

The theories and models of self organization deal with highly complex systems involving thousands of interdependent elements. Gradually, a coherent framework is emerging for dealing with the enormous complexity involved. While various names are being used to describe the associated field of research, including terms such as "the mathematics of complexity," "complex dynamics," and "nonlinear dynamics," probably the most widely used one is "dynamical systems theory" (Capra, 1996; Gleick, 1987).

New high speed computers have played a crucial role in gaining insight into dynamical systems and the mastery of complexity. As scientific insights of the 20th-century unfold, new ways to express the characteristics of complex relationships are emerging. In this way, complexity has become a valuable aspect and important scientific and philosophical tool for interpreting the world. For example, in Maturana and Varela's (1980) terms regarding a living system, the process of circular organization (which they called "autopoiesis") is identical to the process of cognition - the entire network of the living system continually makes itself. "In a living system," the authors explain, "the product of its operation is its own organization" (p. 82). Also, in Lovelock's (1991) representation of the Gaian system which he called

"Daisyworld" it was shown how self organization becomes more and more stable as the model's complexity increases.

New qualitative patterns of the behavior of complex systems is revealing "a new level of order underlying the seeming chaos" (Capra, 1996, p. 113).

> *Chaotic behavior has two important characteristics . . . at one level it is inherently unpredictable, while at another it displays a 'hidden pattern'. Chaos in its scientific sense is not utter confusion. . . . It is a combination of order and disorder in which patterns of behavior continually unfold in irregular and similar forms. (Stacey, 1992, pp. 62-63)*

As part of the perspectives of New Science, "chaos" and "complexity" are presenting new ways in which to develop meaning within human and physical systems. Organized complexity has become the very subject of systems theory.

Early systems thinkers recognized the existence of different levels of complexity with different kinds of laws operating at each level. "At each level of complexity observed phenomena exhibit properties that do not exist at the lower level. For example . . . the taste of sugar is not present in the carbon, hydrogen, and oxygen atoms that constitute its components" (Capra, 1996, p. 28). It was C. D Broad, as a philosopher in the early 1920s, who coined the term "emergent properties" for those properties that emerge at a certain level of complexity but do not exist at lower levels.

From a systems point of view, a system, like the classroom, cannot be understood by its pieces. Such a system needs to be understood through the interactions taking place at many levels. Complex interactions are often not discernible yet underpin the order, or our perceptions of the order, within the classroom. Rather than using hierarchies and simple causal relationships reminiscent of behaviorist perspectives to bestow order on a classroom, a dynamical systems view seeks to highlight organization as governed by interconnections and relationships of complex systems. "In the new systems thinking, the metaphor of knowledge as a building is being replaced by that of a network" (Capra, 1996, p. 39).

This complex view contrasts the closed, mechanistic, Newtonian approach to the relationship of causality. Systems thinking in the classroom setting portrays an open dynamic system whereby "complex interactions and mutual connections prevent attribution of simple causes to classroom events" (Pourdavood, 1996, p. 3). Recognized as one of the most important events in 20th-century science (Capra, 1996), the idea of systemic relationships in complexity theory is providing a new perspective from which to articulate mathematics education reform. Doll (1993) contends that if complexity theory has as strong an influence on teaching and learning as it has had on scientists' views of physics and the universe, there will be unequivocal transformation in the mathematics classroom. Furthermore, Doll contends that mathematics curricula designed within the purview of New Science and post-modern perspectives are going to be qualitatively different from ones based on behavioristic measures of the past. In the former, indeterminacy, unpredictability, relationship and spontaneous self-organization are key features whereas the latter reflect an emphasis on the transmission of authoritatively-prepackaged, absolute knowledge.

Just as New Science emerged from ones that reached dead ends (Wheatley, 1994), the constitutive aspects of post-modern mathematics education portending classroom transformation imbued with new vitality and purpose will require extensive exploration and discussion in order to achieve the "critical-mass paradigm shift" needed to synthesize the implicit changes embedded in current reform agendas. As Fennema, et al. (1996) note, there is "little agreement and even less evidence about what knowledge will enable teachers to teach so that students learn mathematics with understanding. Thus, a major question that faces us is what knowledge will enable teachers to modify their instruction so that it becomes more in line with current recommendations" (pp. 403-404).

A clear message has emerged: reform in mathematics education will only be realized when mathematics education is viewed from a completely different perspective (Hersh, 1979, 1986; Kilpatrick, 1992; Noddings, 1992; Wheatley, 1991). The challenge has become not only the articulation of the differentiation between the old and new paradigmatic perspectives but also the translation of the new ones into pragmatic educational frameworks.

Engendered in such a challenge is the expectation that a profound transformation amongst teachers, administrators, and governments needs to take place. The urgency, however, lies in ameliorating teachers' attempts as they struggle to come to terms with the visions of such a transformation. A major perspective that could assist teachers in comprehending the mathematics reform movement is the explication of the New Science and post-modern notion of relationship in the mathematics classroom. Indeed, as Cobb, et al. (1991) argue, an appreciation of relationship between teaching and learning in the classroom is needed in order to give empirical support for educational reforms.

## RATIONALE FOR THE STUDY

The notion that teaching and learning are inextricably tied together is one of the implicit ideas embedded in emerging post-modern philosophical and theoretical educational orientations (Kieran, 1994). Within and between the dimensions of late 20th-century social, cultural, emotional, and cognitive research, a zeitgeist of "relationship" is unfolding. For example, "pluralistic sensitivities" (Senge, 1990), "synergetic functioning" (Haken, 1996), "systemic ecology" (Capra, 1996), "interdependent connectedness" (Wheatley, 1994), "reflexive complementarity" (Cobb & Yackel, 1995; Ernest, 1996), "complementary harmony" (Doll, 1993), "transient synchronization" (Varela, 1995), "co-evolutionary processes" (Steinbring, 1991), "fundamental relativism" (Bauersfeld, 1988), "synchronicity" (Peat, 1987), "mitigated relativism" (Code, 1991), "socio-autonomy" (Fleener & Rodgers, 1998), and, "synergistic coalescence" (Geoghegan, Reynolds & Lillard, 1997), are all contemporary perspectives resonating with the post-modern paradigmatic view that scientific thinking has moved away from atomistic and reductionist principles and towards complex relationships as the focus for explaining "how the world wags."

Complex Relationships

Because of what has been shown to happen in the quantum world, the ideal of scientific objectivity no longer holds. Nothing in the quantum world happens without something encountering something else. Nothing is independent of relationships and everything is in a constant flux of dynamic processes. This is a world of process, not of things. Complex systems are in a spontaneous state of becoming and self-organizing. The future is unpredictable. It is a world where everything is open and susceptible to change. In this view, complex systems constantly change, the environment changes, and some scientists argue, even the rules of evolution change.

"Evolution is the result of self-transcendence at all levels. . . . [It] is basically open. It determines its own dynamics and direction. . . . By way of this dynamic interconnectedness, evolution also determines its own *meaning*" (Jantsch, 1980, p.14, italics in original). The whole universe is operating like a network of networks where everything is connected to something else through relationship.

As a result of discoveries in the quantum world, "relationship" has become the hallmark of 20th-century New Science while at the same time providing new conceptions of knowledge, teaching and learning. For example, Wheatley (1994) asserts,

> *. . . we are beginning to recognize [classrooms] as systems, construing them as "learning organizations" and crediting them with some type of self-renewing [coherent, evolving and interactive] capacity (p. 13) . . . we talk of quantum interconnectedness, of a deep order that we are only beginning to*

*sense . . . a constant weaving of relationships, of energies that merge and change, of constant ripples that occur in a seamless fabric [of learning]. (p. 20)*

Understanding relationship as the key determiner of social, political, economical and scientific knowledge is the exigent preoccupation of New Science. In mathematics education a similar imperative is emerging as foundational to the idea of the post-modern mathematics classroom (Forman, 1996; Greeno, 1997; Elkind, 1998). Indeed, Doll (1993) believes that with the new emphasis on systemic relationships a new sense of educational order is emerging between teachers and students, and teaching and learning, and will culminate in a new concept of curriculum. The traditional positivist systems which dominate mathematics education today could give way to more complex, pluralistic, unpredictable systems. This new and subtler form of order will drastically change relations between teachers and students in the classroom (Fleener & Laird, 1997; Elkind, 1998).

Wheatley (1994) asserts that as we begin to examine systemic relationships, things increase in number or detail, the span of control stretches out elastically, and, suddenly we are snapped into unmanageability. But, she advises, there is a way to overcoming our fear of the associated complexity, and we find it as we step back and refocus our attention on the relationships that constitute the whole. When we give myopic attention to details and stand far enough away to observe the function of the total system, we develop a new appreciation for what is required to work with and within a complex relationship (qua teaching and learning mathematics).

Understanding this whole-system approach requires a very different set of expectations and analytic processes. Rather than creating a model that forecasts the future of a system, nonlinear models encourage the modeler to play with them and observe what happens. Different variables are tried out in order to learn about the system's critical points and its homeostasis. An important premise of the present study was that controlling the model was neither a goal nor an expectation. As a researcher and analyst I sought to increase my intuitions about how the mathematics classroom worked as a system so that I could "interact with it more harmoniously" (Briggs & Peat, 1989, p. 175).

> *This is such a remarkably different approach to analysis, this sensing into the movement and shape of a system, this desire to be in harmony with it. The more we develop a sensitivity to systems, the more we redefine our role in managing the system. The intent is not to find the one variable or set of variables that will allow us to assert control. This has always been an illusion anyway. Rather, the intent becomes one of understanding movement based on a deep respect for the web of activity and relationships that comprise the system. (Wheatley, 1994, pp. 110-111)*

Physicist David Peat (1991) terms this "gentle action . . . involving extremely subtle actions that are widely disturbed over the whole system...The intent is not to push and pull, but rather to give form to what is unfolding" (p. 217). "A system's perspective, then can handle complexity because it does not need to deal with it in a linear fashion" (Wheatley, 1994, p. 111).

If we view the classroom as an unfolding evolving system determining its own dynamics and direction, and through dynamic interconnectedness determining its own meaning, then everything is free to adapt and open to change. The relationship between teaching and learning mathematics takes on a completely different perspective.

> *But change is not random or incoherent. Instead, we get a glimpse of systems that evolve to greater independence and resiliency because they are free to adapt, and because they maintain a coherent identity throughout their history. Stasis, balance, equilibrium - these are temporary states. What endures is process - dynamic, adaptive, creative. If an open system seeks to establish equilibrium and stability through constraints on creativity and local changes, it creates the conditions that threaten its survival. (Wheatley, 1994, p. 98)*

A classroom characterized by open, dynamic, creative, and adaptive processes alludes to a very different approach to mathematics education than is to be found in traditional linearly oriented settings. The translation of these non-traditional perspectives into classroom practice requires artful and coherent articulation. It is because our classrooms are dictated to by external scientific, economic and political Western hegemonies that educational change is slow to be forthcoming. Nonetheless, if there is going to be a significant change, dealing with educational reform will require "all of us to take a stand" (Routman, 1996, p. 16).

Even though the impetus of the reform initiatives appears to be floundering at the "chalk-face," New Science and post-modern thinking continues to draw us irrevocably into a new relationship with nature. In doing so, the foundations of education will be increasingly scrutinized from a new set of pedagogical perspectives. However, though the reform movement in mathematics education is enthusiastically endorsed at the research level, efforts to supplant traditional behaviorist methods with the perspectives of post-modernism and New Science are meeting with noticeable disquiet in the trenches of antiquarian mathematics pedagogy.

Therefore, the explication of what constitutes "systemic relationships" in teaching and learning mathematics in the classroom could provide a new perspective from which to reconsider classroom practices in a post-modern era. Research which explores and develops New Science and post-modern perspectives in the mathematics classroom could avail opportunities for deeper discussion and broader perspectives on the current reform initiatives.

The present study sought to provide an opportunity for broader discussion; to widen the circle of discourse among professionals (Wheatley, 1994). The value of contributing to the dialogic community (Pourdavood, 1996), as Senge (1990) notes, lies in the fact that a type of intellectual teamwork resting on philosophy and principle is encouraged, that the pooled experiences, ideas, and feelings of others makes a valuable contribution to how individuals perceive themselves during the process of reform.

In attempting to join ongoing critiques and explications that address limitations in traditional methodology and philosophy, the present study hoped to provide information that would further the dialogue on the methodological and theoretical import of systemic relationships in mathematics education. Such an effort aimed to "locate specific puzzles and their solutions . . . and reveal the interrelation of apparently separate and isolated areas of inquiry, both in their potential for mutual illumination and in their multiple effects" (Code, 1991, p. 159).

When eventually confronted by paradigmatic incongruencies, especially in mathematics education, teachers look to practical perspectives in rethinking their views and approaches. When provided with descriptions and explications of some of the professional and personal demands, and potential expectations that post-modern and New Science perspectives might hold for the mathematics classroom, teachers will be availed an opportunity to begin to deconstruct and reconstruct their own personal connections with, and ideas of the issues embedded in the kind of paradigm shift engendered in contemporary mathematics education reform agendas.

In summary, by exploring complex relationships within the mathematics classroom from post-modern and New Science perspectives, the present study sought to interpret the implications inherent in the expectations of current mathematics education reform initiatives. As research of such a nature is currently at a premium (M. J. Fleener, personal communication, July 23, 1998) the study sought to expand and inspirit that forum.

## PURPOSE OF THE STUDY

That a need exists to consider implications of contemporary scientific and philosophical perspectives, speaks to the existential nature of educational progress. The present study investigated the contemporary notion of relationship in a mathematics classroom constituted by the simultaneous implementation of three current educational theories, namely, (a) problem-centered learning (Wheatley, 1991; Murray, Olivier & Human, 1998; Reynolds & Wheatley, 1996), (b) constructivism (Cobb & Yackel, 1995; Ernest, 1991; Glasersfeld, 1996a, 1996b), and (c) a positive approach to discipline (Kohn, 1996; Nelsen, 1996; Noddings, 1984; 1992). By adopting a systems theory approach, the study explored the dynamic interconnectedness, evolution and creative processes of mathematics development in a classroom framed by such a multiple perspective. The study aimed to provide an account of the epistemological, ontological, and methodological dimensions that evolved in a grade-two classroom that had adopted the three theoretical perspectives of constructivism, problem-centered learning, and positive discipline to guide the mathematics program.

Classroom research traditionally has been conceived of and done by "outside" researchers. However, in consonance with the value of the dialogic community (Pourdavood, 1996), Schubert (1992) argues that dialogue among teachers willing to share their stories is a powerful way to add to our knowledge of teaching and learning. The present study sought to allow the teacher in the classroom to pursue her own questions as a practitioner as she addressed the cultural, historical, and social influences that contributed to her pedagogical choices and approaches. By candidly portraying the experiences that took place in the classroom, it was hoped that the outcomes of the study would benefit her and other teachers. It was hoped that the explication of the children's and the classroom teacher's experiences included in the present study might help to provide for a better understanding for other teachers about teaching and learning mathematics.

Geertz (1973) asserted that the use of illustrative narratives and "thick description" (p. 6) to depict actual classroom experiences has the potential to provide insight into how to develop purposeful decision making. Through the intimate contact in the present study of "living with the ideas" and elaborating upon the perspectives embedded in a grade-two mathematics classroom, pedagogical issues were made more transparent. By analyzing the implementation of perspectives appertaining to current agendas of mathematics education reform, the study sought to provide insight into the efficacy and constraints relevant to a post-modern classroom. By providing descriptive coverage of the teacher's pioneering effort to honor a multiple post-modern pedagogical posture in a grade-two mathematics classroom, the study hoped to avail information that would assist other teachers in comprehending and addressing the challenging perspectives engendered in adopting problem-centered learning, constructivism, and a positive discipline approach in mathematics education.

## CONTEXT OF THE STUDY

Sless (1986) has argued that before researchers ask "What am I studying?" an even more basic and important question is, "From where am I conducting my study?" Therefore, I would like to explain how I became interested in the present study and the lenses through which I viewed the project.

With the current state of mathematics education showing a monolithic tendency to elude the calls for reform, investigation into contemporary philosophical as well as methodological perspectives which might aid practical interpretations of the reform agenda is sorely warranted. In the ten years in which I have been involved with mathematics education as a lecturer and researcher, critical issues have emerged at international levels to challenge the status quo of teacher education and mathematics education in the primary grades. As part of prior research (Geoghegan, 1993) that examined the potential effects of music on children's early mathematical achievement, I developed a set of principles which sought to address issues of pedagogy encompassed in the mathematics education reform agendas of the last two decades.

The three principles were developed around one basic tenet, namely, that when young children's experiences engender (a) feelings of positive self-worth, (b) opportunities to reflect upon their own ideas when making sense of new concepts, and (c) opportunities for creative expression of the concepts with which they are exploring, then those children have the opportunity to develop holistically qua cognitively, socially, emotionally, physically, and spiritually. The three principles of this basic tenet were formulated into a model named SEARCH which was an acronym for Self-Esteem (SE), Active Referencing (AR), and Creativity Hierarchy (CH).

Upon arriving in Norman, Oklahoma to commence higher study at the University of Oklahoma, I began participating as a research assistant in a research project exploring children's development of spatial and number concepts in a local grade-two classroom. During this experience, aspects of the SEARCH model became discernible though not ostensibly formulated in perspectives adopted by the grade-two classroom teacher for the spatial/number research project. My involvement in the spatial/number project provided the opportunity for further discourse on the SEARCH idea with the classroom teacher.

The teacher became increasingly interested in the SEARCH principles and together she and I discussed aspects of her classroom practice and personal philosophy that reflected aspects of SEARCH in her classroom. Over two years she and I studied and collaborated together to crystallize the ideas inherent in SEARCH. From this collaboration a language of principles emerged. Embedded in the initial changes that the grade-two teacher had already made, namely towards a constructivist standpoint, problem-centered learning, and positive discipline, mutually identified principles relating to SEARCH became apparent; a refined framework of the teacher's philosophical and methodological perspectives was able to be developed with clearer language and clearer purpose.

It is in the context of the two years spent in collaboration, participation, and observation of the grade-two classroom exploring the SEARCH principles that the present study is formulated. A descriptive analysis of the way in which SEARCH emerged out of the teacher's efforts to implement a mathematics program based upon the theories of constructivism, problem-centered learning, and positive discipline is provided in the present dissertation.

## SIGNIFICANCE OF THE STUDY

The emerging perspectives of New Science and post-modernism are heralding new conceptions of learning. These new views are beginning to precipitate new approaches to mathematics education. Bauersfeld (1996) advises that the most promising trend in mathematics education at present appears to be a shift from specialization to integration, from applying limited monolithic theories toward a combining of insights from different disciplines, and

> *. . . consequently, we shall have to engage much more in interactions than in arranging for a set of tasks to be solved by the single child in competitive isolation . . . as teachers we will have to act much more carefully in all classroom interactions taking into account that our children actually learn along more fundamental paths, and actually learn deeper lessons than those that we think we are teaching them. (p. 6)*

An examination of recent major publications in mathematics education research testifies to the fact that there is increasing acknowledgment of the systemic aspects of the phenomena we wish to explain. The reconstitution of mathematics pedagogy based in systemic relationship invokes the notion that mathematics education research be conducted in natural contexts. Exploring contexts which reflect meaningful and authentic aspects of learners' lives, inclusive of their language, cultures, and everyday lives as well as their school-based experiences is, as Schoenfeld (1994) suggests, part of the recognition of the value of multiple perspectives and approaches that the field of mathematics education requires in order to understand the complexity of issues it faces. In short, researchers in mathematics education now find "themselves emerging from a methodological straightjacket into a Pandora's box of opportunities and problems" (Schoenfeld, 1994, p. 708).

A belief in complex relationships accords center-stage attention to the interdependence of all phenomena. The systemic nature of mathematics education is then exemplified by a classroom/(general) child/(specific) relationship. Such a notion has been highlighted by Vygotsky (1978) and Piaget and Inhelder (1969) and developed further through contemporary sociological (Lerman, 1996; Greeno, 1997) and biological notions (Luhmann, 1995; Haken, 1996; Maturana & Varela, 1980) that presuppose human learning as a communicative and social process by which children grow into the intellectual life of those around them (Wertsch, 1991; Berger & Luckman, 1966; Cobb & Bauersfeld, 1995).

Luhmann (1995), for example, has provided a perspective which portrays consciousness co-emerging with social structure within and through meaning relationships. Wood, Cobb and Yackel (1995) in suggesting what might be done to "reconstruct" what it means to "engage" in mathematics, recommend that analyses of teaching and learning mathematics should associate constructive activity with social location; the qualities of students' thinking being generated by or derived from the organizational features of the social activities in which they participate (Cobb & Yackel, 1995). Lerman (1996) implied that immersion of the individual in sociocultural relationships from birth is an acculturation which encapsulates the identification of meaning.

Dewey (1938) had much to say about the idea of relationship in education. His fundamental premise of the educative experience was formulated around a relationship of experience constituted by interaction and continuity; "the organic connection between education and personal experience (p. 25). Dewey's (1956/1990) four basic instincts of children, namely, "conversation or communication; inquiry or finding out things; . . . making things or construction; and . . . artistic expression" (p. 47) also emphasized the significance of addressing the notion of relationship in developing the whole child.

Zen perspectives portray exemplary systemic relationships:

> *While separating himself from Nature, Man is still part of Nature, for the act of separation itself shows that Man is dependent on Nature. We can therefore say this: Nature produced Man out of itself; Man cannot be outside of Nature, he still has his being rooted in Nature. Therefore there cannot be any hostility between them. On the contrary, there must always be a friendly understanding between Man and Nature. Man came from Nature in order to see Nature in himself; that is, Nature came to itself in order to see itself in Man. (Suzuki, 1956 cited in Noddings and Shore, 1984, p. 160)*

Individuals in the classroom cannot be separated from their context, and teaching cannot be separated from learning. The Indian philosopher Radhakrishnan, reflecting upon the entrenched dominant Western scientific paradigm, commented:

> *. . . the modern mechanistic societies lack the vision of self in man. They recognize only an external mechanistic universe reflected in the machines that man has devised. This is how disintegration becomes the key image of the modern world. (1967, p. 145)*

In order to adequately explore the complexity of systemic relationship in the classroom, Maher and Martino (1996) assert that the long-term case study enables us to better comprehend mathematical development as an interconnected and interdependent phenomenon rather than as an isolated set of skills that are directly taught and acquired in formal ways. In consonance, Freire (1985) argued that the valuing of thought divorced from action, and of decontextualized knowledge, mystifies learning and leads to oppression rather than empowerment.

By using multiple ethnographic approaches and techniques including close observation and intensive interviewing over a period of two years, the present research study explored the complex relationships underpinning major perspectives of the current reform movement in mathematics education. In doing so, perspectives that challenge the entrenched conventional approaches of mathematics education emerged. By employing interconnected processes of (1) careful recording of the teaching and learning activities, (2) thick description, (3) the hermeneutic cycle of interpretation, (4) informed reflection on related theories, (5) and grounded theory, insight into the pragmatics of current mathematics education reform emerged.

Action research has only recently been espoused as a legitimate methodology for educational research in the United States while it enjoys more acceptance in other countries such as Great Britain and Australia. Therefore, the action research approach of the present study sought to explore facets of North American early childhood mathematics classroom practice from a different perspective.

## FOCUS QUESTION

The study initially sought to answer the following question:

What are the emerging relationships among the sociocultural norms, sociomathematical norms, and instructional practices in a second grade classroom?

However, as the study developed, a new focal point emerged and the following became the focus question:

What are the emerging relationships in the evolution of a second grade teacher's attempts to implement reform in her mathematics program?

## EXPLANATION OF KEY TERMS

Several key terms will be explained in the following section in order to establish perspectives from which the present study derived its orientation.

*Constructivism* - Piaget's characterization of the individual as a dynamic, self-organizing and self-regulating organism was founded upon a premise that the human organism actively constructs knowledge through interaction with the environment. Piaget's theory of cognitive development has been portrayed as constituting a "dialectic" process of meaning making, and even referred to as "dialectic constructivism" (Garcia, 1992) which implies that, rather than accretion of knowledge, learning is a transformative process which reconciles new information with existing structures. "The dialectical relationship between the individual and the environment means that as a result of interaction, both are transformed or changed: the individual and the environment" (Fleener & Rodgers, 1998, p. 9).

Such a view implies that developing personal ideas, qua learning, is a creative and inventive affair; the reconciliation of new information generates new transformations of the individual and the environment. Piaget's notion of invention as the essence of how we make sense of our world was developed further by a movement which developed through the latter part of the 20th-century and became known as the "constructivist" movement. The constructivist movement portrayed the inventive process of meaning making as one of "constructing knowledge."

Ernest (1996) notes that the metaphor of construction is contained in the first principle of constructivism as expressed by Glasersfeld (1989), that knowledge "is not passively received but actively built up by the cognizing subject" (p. 182). This simple notion, as basic as it may seem, represents a very significant move away from naive empiricism or classical behaviorism, "for it recognizes that knowing is active, that it is individual and personal, and that it is based on previously constructed knowledge" (Ernest, 1996, p.336). (For a more comprehensive discussion of constructivism, see APPENDIX D.)

*Problem-centered Learning* - Problem-centered learning has been elaborated by Grayson Wheatley (1991), and it is his explanation that I will use to provide an overview of the methodology. According to Wheatley, problem-centered learning involves three components: tasks, groups, and sharing. Firstly, in preparing for class a teacher selects tasks that will have a high probability of being problematic for students, i.e., tasks which may cause students to find a problem. Secondly, the students work on these tasks in small groups. During this time the teacher attempts to convey collaborative work as a goal. Finally, the class is convened as a whole for a time of sharing. Groups present their solutions to the class, not to the teacher, for discussion. A goal of the discussion is consensus. The role of the teacher in these discussions is that of facilitator and every effort is made to be nonjudgemental and encouraging.

Problem-centered learning requires considerable restructuring of course materials as well as different teaching strategies. The core of the approach are the problematic tasks that focus attention on key mathematical concepts (or of any discipline involved) that will guide the students to construct effective ways of thinking about mathematics. As in preparing for any lessons, the goals must be identified. But, in this approach, the goal analysis is more critical because we are asking which ideas must be constructed, not what behaviors should be specified in objectives. We are concerned more with competence (what will they know) than behavior (what can they do).

An opposing view of mathematics education to that of the prevailing behaviorist perspective, problem-centered learning is about construction rather than instruction (DeVries & Kohlberg, 1990). It is to do with students making decisions, making inferences, asking questions about something of interest, and that it tends to be more mathematical when it deals with solving new and challenging problems (Murray, Olivier, & Human, 1998). It is a view that promotes the development of experiences wherein students are obliged to invent ways to deal with mathematical problems. It is a view which suggests that students come to problems without prior instruction in how to solve them in order to make "their own sense" of the mathematics.

From this view, mathematics education contrasts "filling pupils' heads with facts" with "evoking a way of knowing" (Bishop, 1988). Accordingly, Freudenthal (1991) suggested that problem solving in mathematics is more to do with mathematizing than mathematics; abstracting than abstractions; schematizing than schemes; formalizing than formulas; algorithmizing than algorithms; and verbalizing than language. From this perspective, mathematics education focuses more on the process of constructing knowledge rather than the process of absorbing it ready-made. (For a more comprehensive discussion of problem-centered learning, see APPENDIX E.)

*Positive Discipline* - According to Nelsen (1996), children do not develop responsibility when parents and teachers are too strict and controlling, nor do they develop responsibility when adults are too permissive. Children learn best when they have opportunities to participate in an atmosphere of kindness, dignity, and respect. We need to understand why the general approach towards discipline and the use of controlling methods, which were the norm when we were children, are questionable today. As adults and teachers, we need to reconsider our obligation to provide opportunities for children to develop responsibility and motivation by replacing ineffective discipline techniques with effective ones.

Nelsen (1996, p. xxiv) outlines the fundamental elements of a positive discipline framework. The overarching principles are:

- *Kindness and firmness at the same time*
- *Mutual respect*
- *Mistakes as opportunities to learn*
- *Social interest*
- *Class and family meetings*
- *Involving children in problem solving*
- *Encouragement, not blame.*

A positive discipline approach also employs the Significant Seven Perceptions and Skills necessary for developing capable people. They are:

> *1. Strong perceptions of personal capabilities ('I am capable')*
>
> *2. Strong perceptions of significance in primary relationships ('I contribute in meaningful ways and I am genuinely needed')*
>
> *3. Strong perceptions of personal power or influence over life ('I can influence what happens to me')*
>
> *4. Strong intrapersonal skills (the ability to understand personal emotions and to use that understanding to develop self-discipline and self-control)*
>
> *5. Strong interpersonal skills (the ability to work with others and develop friendships through communicating, co-operating, negotiating, sharing, empathizing, and listening)*
>
> *6. Strong systemic skills (the ability to respond to the limits and consequences of everyday life with responsibility, adaptability, flexibility, and integrity)*
>
> *7. Strong judgmental skills (the ability to use wisdom and to evaluate situations according to appropriate values). (Nelsen, 1996, p.6)*

(A more thorough development of a positive discipline approach is provided in APPENDIX F.)

*New Science* - At the beginning of the 20th century Werner Heisenberg formulated his Uncertainty or Indeterminacy Principle. "It can be regarded as a result of the search for a consistent means of linking the everyday world of the laboratory with the strange new world of the minuscule atom . . . [and] for the first time since the Scientific Revolution a leading physicist had proclaimed a limitation to scientific understanding" (Cassidy, 1992, p. 106). Heisenberg's most profound contribution was that uncertainty challenged causality, the principle which requires that every effect be preceded by a unique cause. The causality principle had served for over a century as a basic assumption of practically every form of rational research. The Cartesian method of analyzing the world into constituent parts and then arranging those elements according to causal laws had resulted in a deterministic picture of the universe, utterly logical and predictable. The discovery that nothing resembling strict causality operates on the atomic level generated decades of study eventually evolving as a set of new theories which reflected a shift from physics as a science of state to one of process, "of becoming rather than being" (Gleick, 1987, p. 5). Our world is now conceived as impossible to pin down, constantly changing and infinitely more dynamic than previously imagined. New Science is constituted by relationships between chaos and order. According to Gleick (1987) New Science explores how simple systems give rise to complex behavior and how complex systems give rise to simple behavior. Capra (1996) sees the universe as more like a great thought than like a great machine. He contends that:

> *. . . the ideas set forth by organismic biologists during the first half of the [20th] century helped give birth to a new way of thinking - systems thinking - in terms of connectedness, relationships, context. According to the systems view, the essential properties of an organism, or living system, are properties of the whole, which none of the parts have. They arise from the interactions and relationships among the parts. These properties are destroyed when the system is dissected, either physically or theoretically, into isolated elements. Although we can discern individual parts in any system, these parts are not isolated, and the nature of the whole is always different from the mere sum of its parts (p. 29) . . . systems thinking is always process thinking. (p. 42)*

Paradigm Shift - According to any argument that accounts for basic changes in scientific understanding, a given paradigm (from the Greek *paradigm*, pattern) or framework for thought "may be supplanted at a historical moment when anomalous findings no longer can be accommodated in or reconciled to the scientific model currently prevailing. Cumulative inconsistencies stress the existing structure to the point of collapse and a crisis ensues. If a newer, more comprehensive model offering greater explanatory power gains ascendancy, a consensus eventually forms on its behalf. The shift from one paradigm to another is then apt to be relatively abrupt" (Lucas, 1985, p. 165).

*Sociocultural Norms* - Sociocultural norms in the classroom focus upon equity and opportunity for participation. Sociocultural norms are defined as sensitivity and awareness within a classroom community to issues related to race, gender, class, ethnicity, culture, and handicap (Sleeter & Grant, 1988). The assumption in the mathematics classroom is that conflicts in individual students' mathematical interpretations give rise to internal cognitive conflicts, and that these precipitate mathematical learning. In this account, social interaction is viewed as a catalyst for autonomous mathematical development. Examples of sociocultural norms include explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a conflict in interpretations or solutions has become apparent.

Sociocultural norms constitute classroom roles and expectations, presence or absence of extrinsic rewards, emphasis on or neglect of intrinsic motivation, and differential role assignments to individuals or groups. Cobb & Yackel (1995) explain further:

> *In general, [sociocultural] norms can be seen to delineate the classroom participation structure. . . . a [sociocultural] norm is not a psychological construct that can be attributed to any particular individual, but is instead a joint social construction. As a consequence, we would object to accounts framed in individualistic terms in which the teacher is said to establish or specify [sociocultural] norms for students. To be sure, the teacher is necessarily an institutionalized*

*authority in the classroom. However, in our view, the most the teacher can do is express that authority in action by initiating and guiding the renegotiating process. The students also have to play their part in contributing to the establishment of [sociocultural] norms. One of our primary contentions is in fact that in making these contributions, students reorganize their individual beliefs about their own role, others', and the general nature of mathematics activity. . . . In this perspective . . . [sociocultural] norms are seen to evolve as students reorganize their beliefs and, conversely, the reorganization of these beliefs is seen to be enabled and constrained by the evolving [sociocultural] norms. ( p. 7)*

A key feature of sociocultural norms is their reflexive relationship between individual children's mathematical conceptions and classroom mathematical practices. To say that this relationship is reflexive means more than that they are mutually enabling or constraining.

*It means that one literally does not exist without the other. In terms of explanations, we would say that as children give explanations that they deem viable, they are both acting in accordance with the taken-as-shared normative understanding of what constitutes acceptability and also contributing to the ongoing negotiation of what is taken as normative. Thus, the explanations that individual children give and the* *normative understandings of what constitutes an acceptable explanation [qua sociomathematical norms] are mutually constitutive. (Yackel, 1998, p. 211)*

<u>Sociomathematical Norms</u> - Sociomathematical norms are different to sociocultural norms in that they apply to normative understandings in mathematics as distinct from other subject areas. Sociomathematical norms can be referred to as normative understandings of what constitutes acceptable mathematical explanations and justifications which are negotiated as the teacher and students interact in the classroom (Yackel & Cobb, 1996). Examples of sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an unacceptable mathematical explanation (Cobb & Yackel, 1995). "Sociomathematical norms are not obligations that students have to fulfill; they facilitate the students' attempts to direct their activities in an environment providing relative freedom for interpreting and solving mathematical problems" (Voigt, 1995, p. 196).

To further clarify the subtle distinction between sociocultural norms and sociomathematical norms Yackel and Cobb (1996) offer the following examples.

*The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. Likewise, the understanding that when* *discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm. . . . Further, . . . the construct of sociomathematical norms is pragmatically significant, in that it clarifies how students in classrooms that follow an inquiry tradition develop mathematical beliefs and values that are consistent with the current reform movement and how they become intellectually autonomous in mathematics. . . . Nevertheless, sociomathematical norms, such as what counts as an acceptable mathematical explanation and justification, are established in all classrooms regardless of instructional tradition.* (pp. 461-462)

The opposite of the above examples of sociomathematical norms can be characterized as an expectation or emphasis which highlights quick numeric answers, answers as more important than solution processes, and restatement of the teacher's or another child's explanation without judging or comparing the similarities or differences with other solutions and explanations.

The negotiation of sociomathematical norms gives rise to learning opportunities for teachers as well as learners. As teachers search to make sense of the wide variety of mathematical explanations and solutions offered by children, they are able to capitalize on the learning opportunities that arise for them as they begin to listen to their students' conversations. The increasingly sophisticated way teachers "select tasks and respond to children's solutions, shows their own developing understanding of the students' mathematical activity and conceptual development. The learning opportunities for the teachers are directly influenced by the sociomathematical norms negotiated in the classrooms" (Yackel & Cobb, 1996, p. 466).

Just as a teacher plays a significant role in initiating and guiding the development of sociocultural norms such as facilitating children's developing ability to engage in collaborative dialogue, the teacher also initiates and guides the development of norms that are specific to mathematical aspects of the children's activity. "These sociomathematical norms are critical to the children's understanding of what constitutes mathematical difference, mathematical sophistication, and mathematical elegance, and consequently influence the children's mathematical activity" (Yackel, 1995, p. 134).

There is a reflexive relationship between the sociocultural beliefs and values that students develop and their negotiation of sociomathematical norms. This reflexive relationship underpins the psychological constructs that constitute what the National Council of Teachers of Mathematics (1991) calls a mathematical disposition.

*Emergent Theory* - The theory of mathematics development which emanates from a combined constructivist and interactionist approach is referred to as emergent theory. In this approach, an account for the origin of psychological processes involves analyses of both the conceptual constructions of individual minds and the evolution of the local social worlds in which those minds participate (Cobb, 1995).

Whereas Vygotskian theorists tend to assume that cognitive processes are subsumed by social and cultural processes, and place emphasis on the homogeneity within a cultural group, emergent theorists emphasize the diversity of group members' activity. Emergent theorists typically focus on qualitative differences in individuals' activity.

> *The position from which [emergent theorists] analyze activity is, therefore, that of an observer located inside the group. . . Consequently, whereas analyses conducted in Vygotskian terms investigate the influence that mastery of a cultural tool has on individual thought, analyses conducted from the emergent perspective focus on the individual conceptual constructions involved in learning to use a cultural tool appropriately. From this latter perspective, it is the practices of the local community rather than the wider society that are taken as a point of reference. (Cobb, 1995, pp. 123-124)*

As a combined constructivist and interactionist approach, emergent theory highlights the contributions that actively interpreting individuals make to the development of local social and cultural processes (cf. Cobb, 1989). "In this view, the development of individuals' reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings" (Yackel & Cobb, 1996, p. 460).

## ORGANIZATION OF THE DISSERTATION

In Chapter I, the general nature of the present study has been developed. The rationale and purpose of the study, context and significance of the study, focus question, explanation of key terms, and organization of the study have all been provided. Chapter II will provide a description of the research design by discussing the methodological characteristics of the study, data collection procedures, method of data analysis, and a description of the classroom setting. Chapter III contains the analysis and discussion of the findings while Chapter IV focuses on conclusions and limitations of the study and makes recommendations for teacher education and further study.

# CHAPTER II

# RESEARCH DESIGN AND METHOD OF INVESTIGATION

In this chapter, the research design and methodology used in the present study will be discussed. Elaboration of the methodological characteristics of ethnography, action research, and grounded theory will be provided to highlight their contribution to the present study. The focus of the study, data collection procedures, method of data analysis, and description of the classroom setting in which the study took place will be discussed in order to establish the guidelines adopted for the investigation.

## RESEARCH PERSPECTIVE

By studying the interactions of the teacher and the children in the classroom in light of the three theoretical perspectives of constructivism, problem-centered learning, and positive discipline, the study aimed to provide insight into potential problems and/or opportunities associated with current reform agendas of mathematics education. The exploration of relationships associated with a diversity of classroom aspects such as the role of the teacher, the classroom culture, interaction patterns among students, the appropriateness of problems posed, the mathematical structure of the problem, sustained learning, the type of response elicited from the student, teacher awareness of co-operation, and, informing the larger community (Murray, Olivier &

Human, 1998), was considered most appropriately served by qualitative research methods rather than quantitative methodologies.

The present study employed an action research model using a multiple-methods ethnographic approach incorporating a constant comparative model, participant observation, interview techniques, and the development of grounded theory. The characteristics of these methodological dimensions will be discussed by looking individually at ethnographic study, action research, and then grounded theory in order to establish their relevance to the present study.

## Ethnographic Study

The ethnographic approach used in the present study involved first-hand, intensive study of the features of the culture, and the patterns of those features in a grade-two mathematics classroom. An ethnographic approach was adopted because the focus research question was considered to be of the kind which did not meet traditional positivistic frameworks (Gall, Borg & Gall, 1996). According to Shimahara's (1988) three major characteristics of ethnographic research, the present study: (1) focused upon discovering cultural patterns in human behavior and thus study members of the classroom culture in order to determine how their behavior reflected the values, beliefs, customs, and other aspects that were typical of their culture, (2) focused on the emic (i.e. the individual's own personal sense and definition of reality) perspective of members of the classroom culture, and, (3) focused on studying the natural settings in which the mathematics culture was manifested. "Culture is the central concept in ethnographic research" (Gall, Borg & Gall, 1996, p. 608).

The ethnographic examination of the classroom culture sought to "embrace what people do, what people know, and the things people make and use" (Spradley, 1980, p. 5). As an ethnographer, I sought to share in the meanings that the classroom cultural participants took for granted, and then depict new understanding for the reader. Spradley (1980) holds that ethnographic research has to do with learning from people, not from studying them. This then was articulated through "thick description" (Geertz, 1973, p. 6).

Mathematics educators and researchers in recent years have increasingly been drawn to consider ethnographic perspectives. However, while Eisenhart (1988) acknowledges that researchers in mathematics education are identifying questions for which an ethnographic approach is appropriate, he suggests that the tendency for these researchers is "to use case studies, in-depth interviews, or in-classroom observations without doing what most educational anthropologists would call ethnographic research" (p. 99). For the present study, ethnography with its basic methodologies of participant observation, ethnographic interviewing, searching for artifacts, and researcher introspection provided insight into the culture of the mathematics classroom from which to elaborate a grounded theory. Though such a multi-perspective methodology has been slow to gain wide acceptance in the mathematics education research community, researchers are now adopting it in order to approach the examination of mathematics education from new perspectives.

Brown, Cooney and Jones (1990) suggest that multi-perspective qualitative research possibilities (they refer to this as "adaptive" qualitative research) have the potential to influence mathematics education in several ways. It can provide a deeper understanding of meaning-making processes of teachers and students and thereby provide a basis for constructing education programs that are responsive to individuals' beliefs and needs. However, "to respond to the hidden beliefs and meanings that all classroom participants bring to the teaching and learning situation, they must first of all be identified" (Nickson, 1992, p. 109). Hiebert and Carpenter (1992) suggest that research in mathematics education that contributes to our awareness of identifying students' and teachers' needs,

> *. . . will be research that reveals students' and teachers' thinking as it occurs in classroom settings and as it changes over the course of instruction. Such a focus on research will take us well beyond earlier investigations based on simple achievement measures and will feed the development of increasingly useful theories of mathematics learning and teaching. (p. 92)*

The present study's use of an ethnographic approach also reflected the emergent nature of the project. An interest in complexity, in process over product, and in becoming rather than being, required a methodology which would accommodate a timeline and process upon which to formulate and provide thick description. Because the present study explored issues related to equity, gender, emotion, and feelings, an ethnographic methodology that incorporated first-hand study of these interrelated features of a classroom culture was also considered to appropriately reflect feminist theory and practice.

Feminist theory and practice have helped shape qualitative research by asking questions about gender, emotion, and feelings and their relevance to research. Some would even say the impact of feminist practice and methodology almost transformed "qualitative research" into "feminist research." Feminist scholarship is marked by an attitude of "inter-relatedness and interconnectedness, wholeness and oneness, inseparability of observer and observed, transcendence of the either-or dichotomy, dynamic and organic processes" (Perreault, 1979).

## Action Research

The action research methodology used in the present study involved the classroom teaching approach (Cobb, Wood, Yackel, & Wheatley, 1993) which included the classroom teacher as part of the research and development team. This approach extends on the theoretical and experimental aspects of Cobb and Steffe's (1983) research methodology to the pragmatic setting of the classroom by exploring aspects of children's learning in the context of the classroom setting rather than as a one-on-one experience removed from the dynamics of the classroom context. The intention of the present study was to create a situation that would offer a naturalistic setting in which as many of the classroom activities as possible could be recorded.

In order to record as much as possible of first-hand knowledge of the actual teaching and learning processes, I participated as an observer in situ interviewing children and the teacher, and by contributing to lesson planning and teaching episodes that helped to develop the project. The constant comparative model provided opportunity for informing action theory by continuous extrapolation and analysis during the course of the study (Bogdan & Biklen, 1992). The teacher and I reflected upon developments as they emerged and endeavored to address associated issues "on the spot." Such an approach, to ground theory in practice, reflects a view that the relationship between theory and practice is reflexive (Simon, 1995).

As theory developed, it fed back into the project to inform and guide practice. In this manner, theory and practice had a reciprocal relationship; theory informed practice and practice informed and provided for the revision of theory. A theory which is developed through this process is referred to as grounded theory. This approach can be contrasted with more traditional styles of presentation in which the basic principles or tenets of theoretical positions are stated, and then implications are deduced for practice (Cobb & Yackel, 1995).

As Schon (1983) observes, the rhetorical style of traditional quantitative approaches elevates theory over practice and enacts a positivist epistemology of practice, thereby devaluing the relation between theory and practice as it is lived by reflective practitioners. Furthermore,

> *. . . it positions researchers and practitioners in superior and subordinate roles as producers and consumers of theory. In contrast, alternative approaches that attempt to ground theory in practice tend to position researchers and practitioners in more collaborative roles*

*and to treat their areas of expertise as complementary. . . . Approaches of this type also acknowledge the importance of developing a basis for communication between researchers and practitioners. As a consequence, they seem to have greater potential to contribute to current reform efforts.* (*Cobb & Yackel, 1995, p. 4*)

## Grounded Theory

The grounded theory approach used in the present study followed the American pragmatist position associated with Dewey and Mead (Strauss and Corbin, 1994). Multiple perspectives to explain relationships within the classroom were systematically sought; the emerging perspectives and interpretations of those in the study were continuously analyzed, synthesized and incorporated back into the project. By constantly addressing the interpretations brought to bear by the teacher, another university professor, another early childhood teacher, the children and myself, the inductive nature of grounded theory became manifest. Regular and on-going discussions that reflected upon what was transpiring in the classroom produced enlightening insights which in turn empowered the theoretical conceptualizing which in turn enriched the decision making process for providing for children's mathematical experiences.

Thus, conceptualizing and cross-referencing various viewpoints was an intellectual process that extended throughout the entire course of the research project. Grounded theory methodology incorporates the assumption, shared with other but not all social science positions concerning the human status of actors to be studied

(Strauss and Corbin, 1994), that the multiple "voices" (Wertsch, 1991) of those involved are attended to.

The coding procedures used in the present study - including the important procedures of constant comparison, theoretical questioning, theoretical sampling, concept development, and their relationship - helped to guide interpretations away from accepting any of those "voices" on their own terms, and to some extent forced the teacher's and my voice, though questioning, to be also questioned and provisional. Strauss and Corbin (1994) contend that,

> *. . . in grounded theory, concepts are formulated and analytically developed, conceptual relationships are posited - but . . . emphasizing here that they are inclusive of the multiple perspectives of the actors. Thus grounded theories, which are abstractions quite like any other theories, are nevertheless grounded directly and indirectly on perspectives of the diverse actors toward the phenomenon studied. . . . Grounded theories connect this multiplicity of perspectives with patterns and processes of action/interaction that in turn are linked with carefully specified conditions and consequences. (p. 280)*

Feminist theory contributed substantially to sensitizing the present study towards making sense of the multiplicity of data. The procedures of the grounded theory approach, theoretical coding, theoretical sampling and constant comparison were particularly enhanced by theoretical sensitivity to issues of power and domination. Such a sensitivity consisted of interdisciplinary research and professional knowledge, as well as personal experience, brought to the inquiry by both teacher and researcher. The more theoretically "sensitive researchers are to the issues of class, gender, race, power, and the like, the more attentive they will be to these matters" (Strauss and Corbin, 1994, p. 280).

Grounded theory carries the obligation, while contributing to the knowledge of respective professions, to tell the stories of those studied to them and to others - to give them voice. In contrast to the conventional scientific approach, the use of grounded theory in the present study endeavored to construct a picture as it began to take shape, and as pieces were collected and examined. Through an analytical "dialogue about the subject that one cannot help but enter" (Bogdan and Biklen, 1992, p. 175), the inductive nature of the experience became manifest.

## FOCUS OF THE STUDY

The purpose of this study was to explore complex relationships in mathematics education first-hand in a grade-two classroom and to examine how the relationships could be articulated in light of proposed and desired changes identified in the current agendas of the reform movement in mathematics education. In order to do this a post-modern perspective of mathematics education was established through the teacher's efforts to (a) dispense with prevailing dominant positivist view of teaching and learning and, (b) create an environment wherein the individual student, the classroom community, the mathematics curriculum and the teacher's pedagogy harmonized in a relationship of systemic wholeness.

The present research project involved an ethnographic study that explored the actualization of constructivist perspectives, problem-centered learning, and a positive approach to discipline used as guiding perspectives in a North American grade-two mathematics classroom. The study analyzed critical issues related to the struggles and successes of all members of the classroom.

## DATA COLLECTION

My role as the researcher in the present study involved observing, participating in, and analyzing a grade-two classroom which had adopted a mathematics program grounded in a problem-centered learning approach (Wheatley, 1991) and constructivism (Ernest, 1991; Glasersfeld, 1996b) along with a positive discipline approach to classroom management (Nelsen, 1996; Nelsen, Lott, & Glenn, 1993).

My initial contact with the class and teacher of the present study was through my participation as an assistant researcher in a colleague's research project that focused on children's experiences as problem solvers in this classroom. It was through this encounter that I gained knowledge of the classroom culture and the teacher's aspirations. After visiting the classroom three days a week for one ten-week term in the role as an assistant, I became interested in the dynamics and relationships of the classroom. Because I participated in the children's activities, I was able to establish a good relationship with the teacher and her class. My initial experiences in the classroom availed the opportunity for conducting the present study.

For the present study. I attended the grade-two classroom for daily hour-long mathematics lessons and hour-long follow-up discussion sessions three days a week during seven school terms (each of 10 weeks) over two years. Along with time spent in video-taping children, over 400 hours were involved in collecting data. Field notes, written observations, samples of children's work, video-tape recordings of children, and audio-taped teacher interviews formed the basis of the data collection.

Each day, immediately following my observation of each one-hour classroom mathematics lesson, the teacher and I spent another hour during her weekly release time in postobservation reflective discourse analyzing each lesson, examining issues, reviewing developments. Further discussion and reflective analysis between the teacher and myself was conducted during a joint paper presentation at a major four-day North American mathematics education conference, informal weekend meetings and as needed telephone conversations.

Along with the teacher's personal notes and reports, data were compared applying a constant comparative method (Bogdan & Biklen, 1992). Data were collated, analyzed, coded and synthesized as part of a hermeneutic cycle of dialectic analysis (Guba & Lincoln, 1989) conducted between the teacher, a university colleague, another teacher of similar methodological and philosophical orientation and myself. The major purpose of this process was not to justify anyone's personal construction or to attack the weakness of the constructions offered by others, but to form a connection between a group of professionals that allowed for their mutual exploration by all interested parties; it was a process which educated because all parties reached new levels of information and sophistication, and were empowered because their constructions were given full consideration and because each individual had an opportunity to provide a critique, to correct, to amend, or to extend all the other parties' constructions (Guba and Lincoln, 1989).

Key Informants

In order to gauge the quality of the teaching experiment, each year, six key informants were monitored and assessed as to the depth and breadth of mathematics developed in the classroom. The key informants participated in scheduled half-hour video-taped interviews at the beginning of the study, in the middle and at the end to get an indication of the development of key mathematical concepts relevant to their grade level. Key informants were chosen to allow the opportunity to describe and explain individual's mathematical learning in more detail, and because they represented different types of participation based on the teacher's assessment of their learning. Questions used in the interview were drawn from items used by Cobb and Bauersfeld (1995) in a similar recent study. Please see APPENDIX C for a sample of interview questions.

## DATA ANALYSIS

In ethnographic terms, the design of the present study is a triangulating research design in which information is gathered in several ways and from various aspects rather than just one. In contrast to a quantitative research study which, according to Gall, Borg and Gall (1996), is a relatively straightforward process whereby the data can be entered into a computer without much difficulty for quickly performing the statistical analysis, the present qualitative study generated a great many pages of observational notes, interview transcripts, and documents obtained from the field for analyzing.

Lincoln and Guba (1985 cited in Gall, Borg and Gall, 1996, pp. 561-562) identify four criteria for determining when it is appropriate to end data collection (1) exhaustion of sources, (2) saturation of categories, (3) emergence of regularities, and (4) overextension. After two years of data collection it was felt that this stage had been reached.

Although qualitative research tends to be densely descriptive, it is important to organize the information in a way that people who have not been to the classroom setting can make sense of the learning environment and interactions. The data were organized using the interpretational approach which is a "process of examining data closely in order to find constructs, themes, and patterns that can be used to describe and explain the phenomenon being studied" (Gall, Borg and Gall, 1996, p. 562). The procedures were carried out manually.

The first step was to carefully read and reread the data with a plan to developing a set of categories that adequately encompassed and summarized the data. Categories emerged to indicate particular or certain types of phenomena which could be classified numerically as variables. The classification of variables included constructivist epistemological considerations, sociocultural classroom norms, sociomathematical norms, positive discipline considerations, and problem-centered learning dimensions.

"This process of category development is consistent with the principles of grounded theory. [Ethnographic] researchers who use these principles derive their categories directly from their data rather than from theories developed by other researchers. In other words, the categories are 'grounded' in the particular set of data collected. Furthermore, the categories seek to explain the phenomenon as well as describe them" (Gall, Borg and Gall, 1996, pp. 564-565).

After formulating the category system, the data from each session were coded. It was necessary to examine each recorded section and decide whether the phenomenon it described eventually did or did not fit one of the categories in the category system. When all coding was completed a grouping strategy was employed to bring together all the segments that were tagged with the same category code. Glaser and Strauss, the developers of the grounded theory approach, called this procedure constant comparison to refer to the continual comparing of segments within and across categories. By applying the method of constant comparison, a set of well-defined categories with clear coding instructions was developed.

Finally reflective analysis was incorporated. "Its use involves a decision by the researcher to rely on [personal] intuition and personal judgment to analyze the data rather than on technical procedures involving an explicit category classification system" (Gall, Borg and Gall, 1996, p. 570).

> *In doing reflective analysis from a hermeneutical perspective, the researcher carefully examines and then re-examines all the data that have been collected. As this process continues, certain features of the phenomena are likely to become salient. The researcher should then develop an understanding of these features by themselves and in relation to each other. In other words, the analysis should account for as much as possible of the phenomenon being studied. An interpretation or criticism that fits some of the data should not be contradicted by other data. (Gall, Borg and Gall, 1996, p.571)*

## THE CLASSROOM SETTING

The teacher in the project had 21 years teaching experience and was familiar, though not au fait, with the agendas of the current mathematics reform movement (NCTM, 1989, 1991). She was Caucasian, married with one teenage son and had lived and worked in the local community for 20 years, 5 years of which she worked as a middle school remedial reading teacher, then 15 years as an elementary school teacher in early childhood settings. Since her first teaching appointment, she had been involved in her specialization of reading, and had become committed to a whole language method of literacy development despite considerable turmoil and debate within her school over the effectiveness of such an approach.

Prior to the commencement of the present study, the teacher actively sought assistance in changing her conventional approach to mathematics instruction following a mathematics education inservice session at her school. The teacher requested help from a local university professor involved in pre-service elementary teacher mathematics education. The teacher requested assistance in incorporating a problem-centered approach to learning in her classroom. This was her initial point of departure from traditional mathematics education methods and was negotiated prior to the commencement of the school year in order to start her new approach at the beginning of first term.

Her voluntary implementation of a problem-centered approach to learning mathematics was the beginning of her involvement in the previously-mentioned study in which I acted as an assistant. That particular study was to run concurrently with the present study. Part of the teacher's decision to establish a "more effective approach to teaching mathematics" in her classroom was her pursuit, at the time, of a positive discipline approach to classroom management (cf. Nelsen, 1996). She admitted that authoritative traditional methods of teaching mathematics stood in stark conflict with the perspectives of classroom management based in positive discipline. This perspective was to unfold as a vital connection between establishing a new agenda for her new mathematics program and parity with a positive discipline approach.

Her new problem-centered approach to learning mathematics involved using visualization tasks (approximately for 10 minutes) (cf. Wheatley, 1996) to commence each one-hour mathematics lesson, a problem task for pairs of children to solve collaboratively over 30 minutes, and finally a whole-class sharing of solutions for the last 20 minutes.

The teacher was also at this time involved in a local professional educators study group who met monthly to discuss the implications of constructivist views on education. She admitted that these meetings had had an effect on her, and that she felt the need to address constructivist perspectives when implementing her mathematics program.

The school in which the teacher taught is in a middle-class district of a small (80,000 people) state-funded university town on the outskirts of a US state-capital city. The school could be described as typical of the thirteen elementary schools that belonged to the district.

The grade-two classroom comprised thirteen boys and nine girls in the initial year, and twelve girls and ten boys the next. The second cohort included one boy identified as having mild learning difficulties and another identified as having mild emotional problems. These boys were included in the teacher's class at her behest and as part of her belief in the advantages of integrating children with special needs. Both boys eventually moved out of town and left the class. Otherwise, the two cohorts constituted an average range of abilities, a variety of socio-economic backgrounds, and diversity of cultural backgrounds with approximately 70% being of Caucasian descent.

The research approach for the present study served as an opportunity for the classroom to become a "learning laboratory" (Cobb, et al., 1990, p. 131) where the teacher was able to integrate her intuitive knowledge with the research-based model and ascertain what the children knew in order to make instructional decisions that would influence children's learning (Fennema, et al., 1996).

All names used in this dissertation are pseudonyms to protect the anonymity of participants in the study.

## SUMMARY

Chapter II provided an overview of the methodological approach and setting for the present study. This description was an attempt to help establish a sense of the methodological perspectives, and build an image of the classroom used in the study, and in so doing increase the power of interpreting the discussion and analyses provided in the following chapters.

By examining complex relationships in a grade-two mathematics classroom, the present study explored an approach which contrasts prevailing traditional pedagogy constituted by a hegemony of Euro-centered classroom practice that continues to promote heteronomy, extols absolutism, decries epistemic inventiveness, and by and large as an indictment upon itself fails to "deliver the goods" to a large proportion of the student population.

The forthcoming analyses in Chapter III discuss issues faced by a grade-two teacher and her children as they attempted to address current mathematics education reform agendas that abound with explicit and implicit expectations. Accordingly, Lestienne's (1995) description of science becomes apt:

*More than the art of dominating nature and harnessing forces so as to use them to our profit through technology, science is first the art of progressively adapting our language to nature. (p.208)*

It was Robert May (1976) who suggested that "the most important applications" of dynamic systems relationships "may be pedagogical" (p.467). As Kahn (1995) advised, educational research needs to be initiated now that is very different from the present linear methodologies. He has encouraged educational researchers to accept and take the risks inherent in moving in new, largely unexplored directions, in order to explore whether or not chaotic patterns and underlying order within the dynamic systems of education can be comprehended. Then the lines from Robert Frost (1969) become relevant:

*Let chaos storm!*
*Let cloud shapes swarm!*
*I wait for form. (p. 308)*

# CHAPTER III

# ANALYSIS AND DISCUSSION

In this chapter, three descriptive frameworks will be employed to provide a discussion of the analyses of field notes and audio- and video-tape transcripts from the present study. Firstly, a framework of the teacher's modus operandi will situate her in relation to her theory and practice. Discussion of the teacher's framework will revolve around the principle of "learning to relinquish control."

Secondly, a framework that situates students' participation and their mathematical development in relation to their teacher's modus operandi will be discussed around the principle of "learning to take control." The third framework will be constituted by the circuitous interpretations of my involvement as the researcher, learning from the teacher's modus operandi and the children's participation in mathematical experiences. The related discussion will revolve around the principle of "learning to explain control."

Each of these three frameworks will establish the relevance of the theoretical perspectives of constructivism, problem-centered learning, and a positive approach to discipline within the present study, and thereby distill the complex relationships engendered in a grade-two mathematics classroom experience that endeavored to address current mathematics reform agendas.

# TOWARDS A PHILOSOPHY OF MATHEMATICS EDUCATION

Research into mathematics education continues to search for and understand processes and conditions that restrict and/or enhance practical classroom opportunities for learning and teaching. However, as Hersh (1979) cautions, there are controversies surrounding teaching and learning mathematics which "cannot be resolved without confronting problems about the nature of mathematics" (p. 34). Put another way, different conceptions of mathematics are causing controversies in the way it is taught (qua experienced by children). Dossey (1992) suggests, the conception of mathematics held by the teacher has "a great deal to do with the way in which mathematics is characterized in the classroom" (p. 42).

A study by Thompson (1984) indicated that "the observed consistency between the teacher's professed conceptions of mathematics and the way they typically presented the content strongly suggests that the teachers' views, beliefs and preferences about mathematics do influence their instructional practice" (p. 125). Each teacher teaches mathematics the way he or she conceives of it and whether anyone likes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (Thom, 1973). As Hersh (1986) argued:

> *One's conceptions of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it. . . . The issue,*

*then is not, What is the best way to teach it? but, What is mathematics really all about? (p. 13)*

Ernest (1991) suggests that a philosophy of mathematics education accounts for the basic tenets underpinning a teacher's approach to teaching. He suggests that a philosophy of mathematics should account for:

*(i) Mathematical knowledge: its nature, justification and genesis.*

*(ii) The objects of mathematics: their nature and origins.*

*(iii) The applications of mathematics: its effectiveness in science, technology and other realms.*

*(iv) Mathematical practice: the activities of mathematicians, both in the present and the past. (p. 27)*

However, as Audi (1995) notes in his preface to the "Cambridge Dictionary of Philosophy," there is no short definition adequate enough to define what a "philosophy" is, let alone what it means to have one. A good percentage of philosophers would agree that philosophy is "roughly the critical, normally systematic, study of an unlimited range of ideas and issues" (Audi, 1995, p. xxvi). A philosophy of education, for example, entails:

*. . . virtually every aspect of the educational enterprise. It significantly overlaps other, more mainstream branches (especially epistemology*

*and ethics, but even logic and metaphysics). The field [raises] such fundamental issues as whether virtue can be taught; what virtue is; what knowledge is; what the relation between knowledge of virtue and being virtuous is; what the relation between knowledge and teaching is; and how and whether teaching is possible. (Audi, 1995, p. 584)*

While such a statement speaks to the complexity of what it means to have a philosophy as a professional educator, it would appear that there has been a general neglect in the education of teachers in developing their philosophy of education, and much less their philosophy of mathematics education. Generally speaking, an individual teacher's philosophy of education (without mentioning mathematics) has been dictated to by the forces of natural evolution based on behaviorist practices.

For centuries, teaching has been based on common sense, trial and error, and opinions called philosophies, such as Rosseau's and Dewey's philosophies. However, when teachers claim they have a philosophy of mathematics education it is questionable whether they are referring to descriptive or explanatory theories, or an approach to education. This lack of attention and focus is traceable in some measure to the almost inevitable genetic perspectives ascribed to educational philosophy by analytic philosophy; the latter tending to decry the former as armchair science and as a "threat to the autonomy and integrity of proper philosophical inquiry" (Audi, 1995, p. 584).

John Dewey (1938) asserted that,

> *[t]he traditional school could get along without any consistently developed philosophy of education. About all it required in that line was a set of abstract words like culture, discipline, our great cultural heritage, etc., actual guidance being derived not from them but from custom and established routines. Just because progressive schools cannot rely upon established traditions and institutional habits, they must either proceed more or less haphazardly or be directed by ideas which, when they are made articulate and coherent, form a philosophy of education. (pp. 28-29)*

Dewey (1938) believed that the appropriate perspective for educational plans and projects, was a commitment to framing and adopting an intelligent theory which he called a "philosophy of experience" (p. 51). However, Kamii (1985) suggests that, despite Dewey's efforts from nearly a century ago to provide a clear account of what a "progressive" educational philosophy might be based upon, a theory of education still needs to be developed which unites the diversity of influential ideas that have emanated from the fields of psychology, epistemology, philosophy, biology, and so forth in more recent times. Kamii argues that if we want to win the battle against teacher's philosophical poverty, we have to have a scientific theory that is powerful enough to supplant the dominant lack of cohesion with a clear language of principles.

For example, in developing a philosophy of mathematics, one must heed the "ontological and epistemological problems raised by the content and practice of mathematics" (Audi, 1995, p. 594). The teacher in the present study invoked three guiding theoretical principles to underpin her philosophy and approach to her mathematics program. These three theories provided the foundation for the conceptual lenses through which she accounted for her practice. The three theoretical perspectives were constructivism, problem-centered learning, and positive discipline. While these three theoretical perspectives have been explained briefly at the end of Chapter I, a more comprehensive coverage of each is provided in Appendices D, E, and F.

## A FRAMEWORK OF THEORY AND PRACTICE: ENTER THE TEACHER

### Learning to Relinquish Control

The theoretical approach adopted by the teacher for the present study did not develop without considerable provocation and challenge to her traditionalist background. Justifying the outcomes and processes of her new classroom order caused her considerable consternation. Amongst all her aspirations, the teacher, whom shall be named Harriett for the sake of this study, constantly endeavored to adhere to one main principle; a perspective which was to become her abiding tenet for levering away the behaviorist shroud under which she had felt so disenfranchised and disenchanted for so long:

> Harriett: It's all about letting go, being willing and able to let go and not be so controlling [of the children].

How Harriett developed this basic principle, what it meant to her, and how she translated it in terms of current mathematics education reform, will become the basis of analysis and discussion of the following section.

### Learning What and How To Let Go

Harriett expressed from the beginning of the study that she felt a change in her approach to mathematics education was long overdue mainly because her old (behaviorist) "textbook methods" were not in keeping with the information she was receiving about educational reform in constructivist or positive-discipline terms. To her way of thinking, both constructivist and positive-discipline perspectives revolved around respect for the individual and the individual's attempts at making meaning. This, Harriett argued, was in stark contrast with the traditional (qua behaviorist) view of conveying standardized objective knowledge in order to regulate children's thinking as well as their behavior.

Harriett's involvement in her school's faculty development sessions and in professional study groups prior to her participation in the present study, had made her aware of constructivist theory. By the commencement of the present study, she had developed a sound commitment to translating the constructivist theory into classroom practice. With her philosophy of learning still evolving after more than 20 years, and with sound knowledge of international changes taking place in her specialization of reading instruction, she felt vindicated in making changes in her mathematics instruction; firstly, as she felt she had not changed her approach significantly in the last two decades, secondly, that that situation had not reflected anywhere near the pace of change evident in reading education, and thirdly, she felt a need to develop her mathematics program so that it more closely reflected her reconciliation with teaching and learning from a constructivist perspective.

Extensive experience in remedial reading had helped Harriett develop a sound appreciation and commitment to the theory of whole language. Marie Clay (1991), in her book "Becoming Literate: The Construction of Inner Control," describes the theory of whole language as "a theory of generic learning, that is learning which generates further learning. The generic competencies are constructed by the learner as he works on many kinds of information coming from the printed page. . . . It means an approach which looks at active learners changing over time within their contexts (pp. 1-2) . . . [it is] a questioning or problem-solving process in which we search for meaning" (p. 14).

Harriett knew her old mathematics program was "out of step" with the generic teaching approach with which she so strongly associated whole language theory. She felt a tension between the child-centered whole-language approach and the rote mechanical approach she had been laboring under in her mathematics program. She felt that mathematics should be taught in the same way as whole language but did not know what could be done to achieve this until, with the eventual aid of a local university professor in implementing a problem-centered approach to learning mathematics in her classroom, Harriett began making connections between her perceptions of the whole language approach and how they could be translated in her mathematics program.

Harriett's initial foray into reformulating her mathematics program began with experimenting with a problem-centered approach to learning. With her exploration came a consolidation of the constructivist perspective that knowledge cannot be transmitted to learners but that they must construct it themselves. She enthusiastically embraced the constructivist perspective and openly acknowledged that her background in whole language supported the connections she had made between the theoretical perspectives and practical implications in her mathematics program.

Constructivism proved to be an important orientation in the composition of a metaphor of learning as a meaning-making process in Harriett's classroom. The manner in which she embraced a constructivist view is summarily captured in Papert's (1988) quote:

> *Children don't conceive number, they make it. And they don't make it all at once or out of nothing. There is a long process of building intellectual structures that change and interact and combine. (p. 4)*

## Shifting The Sense of Control

One of the major steps Harriett took towards embracing a constructivist perspective was the reassessment and reformulation of her position of authority in the classroom. Harriett's effort to dispense with the authority bestowed upon her through her position of privilege as the classroom "teacher," was reflected in the way that she perceived learning as more important than teaching for her children. In quintessential constructivist terms she believed that she could not act as the dispenser of knowledge, qua the classroom authority and depository of knowledge, but instead saw her role as one of facilitating children's own knowledge-building experiences. As Piaget said, you can't teach "anything," nothing is teachable; and similarly, Carl Rogers (1969), in his book "Freedom to Learn," argued you can't "teach" anyone, people are not teachable.

In seeking ways to reorientate her own perceptions as well as the perceptions of her children towards her position as the classroom "teacher," Harriett pursued ways in which she could firstly, recant the manner in which she exercised control, and secondly, justify such action in terms of providing maximum opportunity for mathematics learning to take place. She consequently aspired to a position where being the "teacher" did not mean using her authority to determine and dictate standardized ways of thinking in the children, but rather meant being open and attentive to children's ideas as they struggled to make sense and negotiate their way around their own emerging ideas. In short, she was not there to impart her knowledge; the children's ideas were of more importance than hers.

By renouncing her preordained status as the class "expert," she consciously worked at relinquishing all tendency to tell children "answers," and instead encouraged them to construct their own meaning. She became committed to affording children the opportunity to develop their own procedures, algorithms, solutions, and personal approaches to answering a question. She found that by using a problem-centered approach as the basis of each mathematics lesson, it allowed her the opportunity to function in the manner of manager of children's mathematical ideas rather than confirmer of their solutions.

In doing so, she rejected the traditionalist approach that advocates "teaching" children standardized algorithms and stressing correct and only single correct answers. Instead, by using a problem-centered approach, Harriett felt in a better position to encourage children to negotiate and devise their own methods and solutions than previously, when she was compelled to follow the traditional textbook approach which by and large dictated the "best" qua conventional way to solve problems or present solutions.

Harriett's approach to problem-centered learning revolved around children being required to solve a variety of mathematical problems. For example, children would be provided with a problem such as "Seventeen boys and girls went to the library. There were three more boys than girls. How many boys went to the library? How many girls went to the library?" After working on the problem for about 10 minutes with a partner (or sometimes a small group), children would congregate for a whole-class sharing time during which a pair of partners (or their small group) would be asked to go to the chalkboard and present their solution to the class.

By referring to the mathematical problem cited above, the following dialogue segment indicates how Harriett sought to relinquish her authority as the determiner of correctness (the "knowledge de-terminator") and still optimize the children's potential for learning in a problem-centered environment. Shane was asked to present his solution:

Shane: (draws on the chalkboard)

| | | | | | | | | | | | | | $\boxed{|\,|\,|}$ (and after putting a box around the last 3 strokes, explains)

I put my pencil in the middle and counted 7 to the left and 7 to the right and that left 3 extra on one side. So it's 10 and 7.

Dillard: (was next to write his and his partner's solution on the chalkboard)

20 + 17 = 37

Class: No . . . that's not it . . . Nah! We disagree. . . . that's not right (etc.)

Duke: But there's only 17. There's only 3 more boys. (He comes to the board and writes)

10 boys and 7 G (with a back-to-front 7)

Shane: (snatches Dillard's paper out of his hands and begins to scrutinize the working)

Teacher: Careful. We must be respectful (as she reminded her class regularly).

Ali: We took 17. (and writes on the chalkboard)

$$\begin{array}{r} 1\,7 \\ + \\ \underline{1\,7} \\ 2\phantom{\,7} \end{array}$$

Ali had proceeded by adding the tens first to arrive at two tens for the first step in her computation. She then wrote beside her first "sum" what looked to be a duplicate representation of her initial idea. However, her first written "sum" was a record of step one. After establishing that there were two tens, her second step involved retaining the two tens and then addressing what was left, namely the two sevens. She wrote:

$$\begin{array}{r} 1\,7 \\ + \\ \underline{1\,7} \\ 2\phantom{\,7} \end{array} \qquad \begin{array}{r} 1\,7 \\ + \\ \underline{1\,7} \\ 2\phantom{\,7} \end{array}$$

> We got 14 in this column (pointing to the two sevens in the second "sum") and so put 4 in, and carry the ten to get 34.

Harriet understood full well Ali's probable misinterpretation of the question but declined to make judgment or halt proceedings to correct Ali's thinking. For one reason, while Ali may in fact have misconstrued the intent of the question, she and her partners had demonstrated a substantially high level of mathematical thinking in computing 17 + 17 to arrive at 34. Harriett was alert to the fact that what could be interpreted as an algorithm "gone wrong" was indeed indicative of worthwhile mathematical conceptualization. However, at this stage, Harriett had not asserted any authority by challenging the students' presentations but instead waited for a reaction from the class. One response came in the form of Ned and Megsie asking to show their solution.

Ned: We knew that the total is 17 and we know that 10 plus 7 is 17, and 10 is 3 more than 7.

Dillard: (returns to the front of the class and again tries, by rereading the question, to reiterate the logic of his solution. However, he suddenly stops and shrieks)

Oh! Oh! Oh! - I get it! Sean's right. I read it again. There are 3 *more* boys (emphasizing "more"). I see. I see!

Teacher: Dillard, do you know what Shane means?. . . how Shane did it?

Dillard: Yeah! I get it. I read it differently before. I now see that there were only 3 more boys than girls and only 17 altogether, and that's 10 and 7.

While Harriett knew some solutions were questionable, she refrained from intervening, allowing the class to reach a consensus of opinion as to the viability of the various solutions. She allowed for a variety of solutions to be presented in contrast to the operationalization of standardized procedures and rules characteristic of traditional approaches to mathematics education. It was of more importance for Harriett to leave some children's thinking in obeisance than assert her authority by telling them the answer or "solution."

As it is obvious, there were more ways than one to reach the solution, there were different ways of explaining the solution, there were different ways of interpreting the question, and for some children who had done some credible mathematical thinking, their outcome was disenfranchised from the logic of the problem only by their emerging language skills. Harriett's ordination of a standard procedure and compliance with a single solution would have jeopardized much of the distillation of mathematical thinking that was generated at the various levels of construction. Problem-centered learning had as much to do with the process as the product.

Harriett: It all goes back to control. [In my classroom] they really learn to take it [criticism from each other]. It's not like you're ruffling any feathers. It should always be positive, they should never hear that it's wrong.
And it depends on the environment you've set up. If you have established their trust, and they know it's OK to take risks, and it's OK to express your own point of view, even when you do something if it [your point of view] isn't what you thought it was, it's OK to be wrong; it's OK to make mistakes.

> By creating that type of environment, kids learn from that; we all learn from making mistakes. And if you're afraid to make mistakes you won't be a risk taker.
> You need to be able to take risks; to know a sense of real freedom of expression; like being confident in yourself as the author [to profess the authority] of your own ideas.

As Sawada (1997) points out, in the internationally recognized Japanese mathematics education system, mistakes are just another variable in the solution. Dewey (1938/1963) asserted that "the ideal aim of education is creation of power of self-control" (p. 64). He argued that when external authority is rejected,

> *. . . it does not follow that all authority should be rejected, but rather that there is a need to search for a more effective source of authority. The solution of this problem requires a well thought-out philosophy of the social factors that operate in the constitution of individual experience (p. 21). . . . The traditional scheme is, in essence, one of imposition from above and from outside. It imposes adult standards, subject matter, and methods upon those who are only growing slowly towards maturity. [In contrast] to imposition from above is opposed expression and cultivation of individuality; to external discipline is opposed free activity; to learning from texts and teachers, learning through experience; to acquisition of isolated skills and techniques by drill, is opposed acquisition of them as means of attaining ends which make direct vital appeal; to preparation for a more or less remote future is opposed making the most of opportunities of present life; to*

*static aims and materials is opposed acquaintance with a changing world. (pp. 19-20)*

Thus children in Harriett's class were encouraged to have their say, and to participate by presenting their solutions while at the same time feeling confident that they would be respectfully listened to during the presentation of their solutions. In the same process, a sociomathematical norm developed; educing personally viable mathematical propositions to be verbalized and communicated logically using their own personal language, symbols, and terms became a classroom expectation. Placing the onus on the children to make sense of their own mathematical thinking and be prepared to express it cogently, was Harriett's way of embedding a constructivist perspective in practice.

Harriett openly acknowledged the impact of the writings about mathematics education by Constance Kamii. Kamii (1989) suggested that, rather than focusing on the NCTM's (1989) goal that children should learn to reason mathematically, it would be better to focus upon children's confidence in their ability to think. Also, Kamii suggested in contrast to NCTM's agenda that children should learn to communicate mathematically, rather they should learn to exchange viewpoints with other people.

Kamii's viewpoints helped Harriett build a bridge between a whole-language perspective to learning and that of the mathematics reform movement. Kamii's (1989) four basic principles of learning mathematics also guided the development of Harriett's approach to her classroom instruction, namely:

*1. Encourage children to invent their own procedures rather than showing them how to solve problems.*

*2. Encourage children to invent many different ways of solving the same problem.*

*3. Refrain from reinforcing correct answers and correcting wrong ones, and instead encourage the exchange of points of view among children.*

*4. Encourage children to think rather than to write, and write on the chalkboard for them to facilitate the exchange of viewpoints. (p. 77)*

Harriett: Traditional schooling which perpetuates the assumption that there is just one answer; that there is just one way for everything, is so stifling. It shuts down children's disposition towards viewing learning as the having of wonderful ideas; like Duckworth says.

Harriett's reference in the preceding dialogue segment to Eleanor Duckworth's (1987) book, "The Having of Wonderful Ideas" reflected an appreciation and accommodation of the constructivist point of view. Harriett believed, in consonance with Duckworth, that having confidence to try out one's own ideas does not mean "I know my ideas are right"; it means "I am willing to try out my ideas." The following episode exemplifies how Harriett accommodated children's willingness to try out their ideas.

Harriett regularly asked children to compose problems for the rest of the class with the proviso that they could supply the answer. This in itself was an effort to encourage children to try out their own ideas. Dillard was not confident with mathematics but willingly presented his problem: "Joe had $18. Ali gave him $18. Mick gave him $18, and Duke gave him $18. How much does Joe have?"

| | |
|---|---|
| Caleb: | (Calling out) It's 74. |
| Mick: | Nah. It's, it's 7,000. |
| Teacher: | Why do you say 7,000? |
| Mick: | I dunno. |
| Duke: | It's 70 something. I counted all the eights, and altogether that's 8 + 8 + 8 + 8 |
| Nancy: | (comes to the chalkboard) I got 54. I took three tens (and writes three tens underneath each other and eight tally marks beside each ten)<br>10 IIIIIIII<br>10 IIIIIIII<br>10 IIIIIIII<br><br>I put three eights like this (pointing to the three rows of tally marks.) The three tens are thirty, so it's 31, 32, 33, 34, 35, 36, 37, 38, (pointing to each tally mark in the top row. She then continues on the next line to count from 39 to 46, and then counts 47 to 54 on the next line.) |
| Caleb: | (writes on the chalkboard)<br><br>10 + 10<br>+10 +10<br>+ 8 + 8 + 8 + 8<br><br>I added four tens. That's 40. Then I added 8 + 8 and that's 16. Then I added 10 and 40. That's 50. Then I did 50 and 10, and that's 60. Then 6 + 6 off the two sixteens is 12. So then, 60 and 12 is 72. |
| Mona: | (writes on the chalkboard) |

| | |
|---|---|
| 48 | 8 |
| 56 | 8 |
| 64 | 8 |
| 72 | 8 |

(She explains) 10 + 10 + 10 +10 is 40.
Then 40 plus 8 is 48. 48 and 8 is (counts by ones) 49, 50, 51...56. Then 56 and 8 is (again counts up by ones) 57, 58, 59...64. An then 64 and 8 is (counting by ones) 65, 66, 67...72

Other examples of the way in which Harriett afforded opportunities for children to express their own unique mathematical propositions are provided in APPENDIX H.

During the present study, Harriett persisted in finding opportunities for children to express their own ideas on the premise that through finding their "voice" and realizing that their "voice" was a respected aspect in the milieu of class interaction, children would increase their confidence as they struggled to invent mathematical meaning. Harriett knew that finding one's own voice was fundamental to the effectiveness of a whole-language approach but foreign to a traditional mathematics approach. She believed that traditional mathematics education methodologies focused too much upon emphasizing conformity, compliance and heteronomy, and had little to do with developing individual "voice," qua autonomy (Fleener & Rodgers, 1998).

Such an approach is in keeping with Cashdan's (1976) view of learning that there are two ways in which you can help children learn. One way is by attempting to teach them; the other is by facilitating their attempts to teach themselves. This approach infers that if teachers give children freedom to explore and to learn on their own, they will be self-stimulating and self-starting provided conditions are right for them. Through the implementation of a problem-centered approach to learning mathematics. Harriett found an instructional format in which to translate and consolidate her theoretical perspectives.

Harriett: I wanted something new in my class. I knew there had to be more to learning mathematics than what I was supplying [my children]. Now that I've tried this approach [a problem-centered approach] I can never go back to the way I used to teach. It seems so unjust on the children. Now they seem so much more empowered. There is more dignity in their learning.

## THE POWER OF PROBLEM-CENTERED LEARNING

In a Deweyan manner. Harriett opted to make her classroom a "learning laboratory." By establishing a problem-centered approach to learning. Harriett was not only able to concentrate on providing suitable experiences for maximizing meaning making but was also able to focus more and more upon actively withdrawing from her position of bestowed authority by avoiding the "dispenser of knowledge" role. In terms of her status, she sought to be the "guide on the side" and not "the sage on the stage." However, Harriett was not "slack at the back," either. Further, in Deweyan terms, she retracted her authority of control by relinquishing her "position of external boss or dictator [and took] that of leader of group activities" (Dewey, 1938/1963, p. 59).

Harriett was working with a clearly established mathematics agenda formulated by a semester-based program constituted by big math ideas, e.g., constructing units of ten; basic facts combinatorial strategies; using money; investigating properties, relationships and modeling of 3D and 2D shapes; modeling and sketching the position of objects; using graphs; formal and informal units of measurement; and organizing data. Within the scope of these big ideas she orchestrated developmentally appropriate problem-solving experiences that paid heed to children's emerging conceptualizations. Rather than dictate, prescribe, impose, direct, or instruct the manner in which these big mathematical ideas were to be internalized by the children, Harriett afforded every opportunity for children to explore, play with, investigate, negotiate, propose and formulate their own ideas of which they made sense as they related to their evolving notions of the basic big mathematical ideas. It is in this way that Harriet considered herself able to relinquish control to follow the children's thinking.

As Dewey (1965/1990) asserted:

> *The demand is to secure arrangements that will permit and encourage freedom of investigation; that will give some assurance that important facts will not be forced out of sight; conditions that will enable the educational practice indicated by the inquiry to be sincerely acted upon, without the distortion and suppression arising from undue dependence upon tradition and preconceived notions. (p. 98)*

Dewey's (1938/1963) argument that the ideal aim of education is creation of power of self-control (p. 64) helped Harriett focus her aspirations of establishing a classroom environment sponsored by children's own initiatives. Early in the year, children frequently urged Harriett to tell them the answer and often found her responses to be less than gratifying. Typical of such occasions was an episode that occurred when children were discussing numbers that came before zero. The following discussion was generated not from the teacher's decision to program or schedule negative numbers as part of the curriculum, but from the children's own decision making and inquisitive line of investigation.

| | |
|---|---|
| Lucy: | (after noticing a decreasing number pattern ended at zero) I think it's minus . . . hmm? (and pauses). |
| Tor: | I think it's 0.21. |
| Jason: | I think it's zero coz there are no numbers before zero. |
| Birt: | What do you think, Mrs. H? |
| Teacher: | (Politely shrugging her shoulders) I don't know. |
| Tor: | Ah come on Mrs. H. Just tell us the answer! |

Often finding herself in such a position, Harriett invariably declined to tell the children the answer and would rather leave the question unanswered and unresolved than provide "the correct" answer. Closure was not to be determined by the teacher's authoritative decree of what the answer must be. As a consequence, children increasingly became less reliant upon Harriett for their solutions and more reliant on each other. Children eventually gave up asking Harriett for "the answer."

Such a classroom expectation became a classroom sociocultural norm. The expectation that children would rely more upon each other and their own ingenuity, rather than the teacher for answers or closure, precipitated a community of collaborators and cooperative problem-solvers. Correspondingly, a classroom sociomathematical norm developed. Children increasingly had to rely on their own mathematical sense and personal points of mathematical reference (qua constructed knowledge) to imbue new mathematical propositions with meaning.

Rather than trying to make sense of the teacher's preordained or predetermined procedural dictates, qua "spoon fed" information, and rather than trying to predict, interpret and/or guess what was in the teacher's head, children were obliged to construct mathematical meaning based on their own predilection for making sense and finding resolution to the questions and problems posed. Their mathematics was their own, not the teacher's.

The following episode demonstrates how Harriett utilized the children's emerging conceptual frameworks as a basis for negotiating mathematical meaning. She refrained from being the determiner of knowledge, preferring to afford children the opportunity to make sense of the emerging mathematical propositions themselves. Harriett had placed a blank hundreds grid on the overhead projector with a blue transparent counter in the square for 59 and asked the class, "What number is covered by the blue counter?"

Dillard: I think it's 79. (He counts aloud) 10, 20, 30, 40, 50, 60, 70 then go back one, 79.

Class: (Several children disagree.)

Catlin: (counts aloud) 10, 20, 30, 40, 50, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69.

Catlin and Dillard proceeded to reiterate their procedures as if re-referencing their own ideas in light of the contradiction presented by their classmate's proposition. Their perturbation seemed to have stymied their ability to clearly express their thinking.

Dillard: (suddenly exclaims) We're both wrong.

Blinky: I think it's either 59 or 58. (He counts aloud) 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59.

Meanwhile, Catlin has proceeded to show her work by drawing on the blank one hundred's grid overhead transparency. She writes the numbers 10 to 50 down the last right hand column and numbers 51 to 59 across the sixth row.

Teacher: Class, how was Dillard doing it?

Mona: (superimposing her numbers over Catlin's work, writes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 across the first row on the overhead transparency.)
All of them in this column (points to the ones - the first column) will be ones, and twos, will all have a two. See, (points down the twos column) and the zero's column will always have a zero - 10, 20, 30, 40, 50, 60, 70, 80, 90 (points down the tens column). (She then writes 11, 21, ,31, 41) See, they all have one in this column (pointing to the first column).

Blinky: Well I disagree. It has to be 59, because this column (points to the nines column) is all nines - 19, 29, 39, 49, 59! (emphasizing 59).

Teacher: Is that different to what Mona said?

Blinky: Huh!? What did Mona get? Oh Yeah! (realizing that he had the same answer).

| | |
|---|---|
| Ned: | I did it differently. (She counts aloud) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (pointing across the first row - the ones. Then points down the right hand column and counts) 10, 20, 30, 40, 50 (then counts in single squares from the left hand side) 51, 52, 53, 54, 55, 56, 57, 58, 59. |
| Teacher: | Is that the same as Catlin's? |
| Ned: | No |
| Teacher: | Catlin, was that different to the way you did it? |
| Catlin: | Yes! Because Ned went 1, 2, 3, 4, 5, 6, 7, 8, 8, 9, 10, and I just started at 10. |
| Ned: | Yeah. It's different. |

The quality of the mathematical tasks was important. The developmental level of children was of paramount importance in devising the problems to be solved. All problem tasks were presented to the class on the premise that they would not intimidate the children by their complexity but be conceptually approachable. Also the problems had to be conceptually challenging at a variety of levels. Harriett was fortunate to be able to formulate her mathematics program in conjunction with the two mathematics education professors working concurrently in her classroom. However, she increasingly began to get the gist of what constituted appropriate mathematics problems and, within the framework of her semester-based mathematics program, began successfully formulating and assigning her own problems.

The provision of appropriate problems was not a lock-step textbook-driven approach. Instead, the program evolved out of the children's identified needs. Based on the progress (or lack of progress) each week, and reflecting overarching objectives encompassed by a set of big ideas appropriate for a grade-two curriculum. Harriett structured the classroom experiences to respond to and extend the mathematical ideas that emerged as children struggled to establish meaning. With the help of the University researchers working in her room, and based upon a program of key mathematical propositions. Harriett developed a program that regularly introduced new key mathematical ideas while at the same time remaining responsive to the children's emerging needs.

To further facilitate children's emerging mathematical concepts. Harriett ensured a ready supply of various materials, such as laminated blank 100 grid boards, plastic squares, colored cubes, counters, unifix blocks, dice, larger sized boards, tiny teddies, markers, dice, play money, blank paper, and index cards was easily accessible to the children. Children were availed time during math lessons to use any of the materials to explore with. She regularly asked the class to invent games which would reflect the particular mathematical concept which they were exploring. They were also often asked to write the rules of the game. Children readily found their own space in the room and would work in pairs or small groups.

One day, unexpectedly, a father arrived at the door and his daughter, Mona, called out, "Daddy! Daddy! Come and watch us do math." Mona's father was welcomed by Harriett and invited to sit with his daughter. Mona proceeded to explain the game to her father. She asked him, "Can you help us out?" and her father, who appeared to be intrigued replied, "This looks very interesting." Mona's father asked her to explain what to do as they jointly established the rules of her game. As they worked, her father consistently asked, "What do I do next?" to which Mona enthusiastically responded with confident and clear directions couched in substantial and cogent mathematical language.

She was clearly empowered in her mathematical experience as she cohesively articulated his and her roles, how to use the die, how to move, what strategies he could use, and how to make a decision in the game. For example, as they played, Mona often quipped, "Wait dad, don't do that; or, OK, you can do that; or, Do this dad!" Her father sat patiently with his arm around Mona allowing her to dictate the terms, and take charge of the experience, not unlike Harriett's efforts to empower her children by metaphorically "putting her arm around them" and letting them take responsibility and assert their authority over their learning experiences.

Eventually Mona and her father completed the game. Mona's father exclaimed, "Wow. That was a great game. I love it." and then took his leave. Once he had left, Mona immediately turned to another child and asked if she could show how to play her game, "Do you wanna play my game?" Not only did Mona have ownership of the game and control of the mathematical concepts embedded in it but her personally-generated mathematical thinking, developed of her own volition, had produced something exciting to be shared with others. She had generated and solved her own mathematical problem.

Mona, as were other class members, was given the opportunity to take control of her own learning and was empowered in the process. It is interesting to note that Mona's father's interaction with his daughter strongly reflected Harriett's approach to encouraging responsibility in the child. Of interest here is the fact that Mona is a child who was consistently able to demonstrate substantially sound mathematical thinking.

Movement around the classroom during mathematics lessons was free and unrestrained with children moving from group to group examining each other's work. Discussion was free flowing as was the exchange of ideas. It was not a classroom censured by strict behavior or restrained by coercive discipline. The entrance of a father did not cause any disruption but instead added to the dimension of the experience. The sense of authority divested to the children as with a sense of responsibility were evident by the children's continued attention and interest to the mathematical task at hand.

Not only were children motivated when regularly asked to invent a game but they were able to maintain their focus as they investigated possibilities related to their emerging mathematical concepts. By generating their own challenge and creating their own mathematical problems, the children were empowered as decision makers and mathematical problem solvers. They were regularly asked to function at the edge of, and within, their personal zone of potential construction (cf. Steffe and D'Ambrosio, 1995) and in doing so were consolidating previous mathematical ideas as well as exploring new ones.

This was one way in which Harriett was successfully able to divest the authority of control to the children, empowering them to be responsible for their own learning. While there was always a mood of fun and playfulness in the room, it was apparent that the children were actively engaged with their mathematics. Harriett's job then became one of monitoring the different pairs and small groups to ascertain their levels of mathematical meaning. She did this by circulating and sharing in the conversations of the children, asking questions and delving into the thinking of each child.

On another occasion, Blinky and Mick were working with small green triangular pattern blocks. They had each progressively built larger and larger triangles with the basic green triangular unit. In the spirit of the problem-centered approach, I sat with the two boys and joined in with their exploration.

| | |
|---|---|
| Researcher: | How many triangles do you have? |
| Mick: | Oh. One hundred. No, more! [there would be more]. |
| Blinky: | Yeah, over one hundred. |
| Researcher: | (builds the next unit size triangle using 4 triangles as shown in Figure 1; displays it in front of Mick and Blinky and asks) How many of these in your giant triangle? |

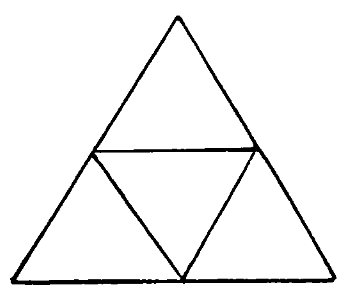


Figure 1. 4-unit triangle constituting the second triangular unit that can be generated using green pattern blocks.

| | |
|---|---|
| Mick: | (carefully taking his time to disembed the larger 4-unit triangles from his giant triangle, eventually says) I got 29. Hey, Blinky how many have you got? |
| Blinky: | Oh that's too hard. I can't do that. |

Researcher: I'll help you.

Blinky: (without my assistance) OK. (he is not as astute as Mick at disembedding the 4-unit triangles but manages) I got 43.

Meanwhile Mick has written 29 + ___ = on a sheet of paper.

Mick: (after Blinky finishes) How many have we got altogether? (He begins to compute an answer on the sheet of paper) 20 and 40 are 60, take one off here, that makes 70, and 2 more is 72. ("Take one off" referring to adding 9 and 3 where he took 1 away from or "off" the 3, compensating with the 9 to make another 10 to make 70, and then adding the 2 that was left.)

Teacher: (out of interest of what the boys had done) Class, Mick and Blinky have something to share with us.

Mick: (at the front of the room) Blinky got 43 and I got 29 (writes on the board 29 + 43 but the 9 is badly drawn and looks like a seven). So 20 and 40 is 60, take off 1 is 70, and 2 is 72. No, it should be 70 (and shows how 7 + 3 is 10 to which several children respond, mainly due to the badly drawn nine.) Nah! It's a nine, not a seven (to which several children retreat to alter their calculations.)

Mona: I agree (approaches the board and writes)

$$\begin{array}{r} 9\,+ \\ \underline{3} \\ 12 \end{array}$$

and 2 and 4 are 6. So, 6 and 12 are 72.

Caleb: (almost bounding out of his seat) What!? 6 and 12 aren't 72.

Mona: Yes it is. See 6 (and writes)

6
10 is 70 and 2 more is 72.

Such an episode highlights the potential for learning through a problem-centered approach to mathematics education. The two boys were initially involved in what early childhood educators refer to as "developmental play," creating and solving their own personally initiated problem by building and exploring an open-ended giant triangle. They willingly cooperated to meet the mathematical challenge embedded in the thinking they had already explored. They used their own unique method of translating and recording their thinking into mathematical symbols that they felt could be deciphered and understood by others. They were willing, when encouraged, to present their findings to the class, justify their methodology, and be prepared to address their peers' refutations or queries. They displayed a confidence and pride in presenting their work. In making their ideas publicly transparent, they confidently asserted the logic of their mathematical proposition for others to dispute. Their public participation stimulated other children's consideration of a new and novel mathematical proposition.

My contribution in extending their exploration was not meant to be an intervention per se. I did not seek to distract them from their task, but moreso develop it. I did not aim to dictate the outcome of their exploration but rather provide the possibility for extension of their thinking as they worked freely with the pattern blocks. The manner in which the boys cooperated with one another, shared their ideas, used manipulatives, recorded their computation in their own unique fashion, articulated their investigation to the class, addressed refutations to their solution, justified their approach, and precipitated further mathematical negotiation in the class, were all aspects constitutive of a problem-centered approach to learning mathematics used in their classroom.

Other examples of aspects constitutive of a problem-centered approach to learning mathematics used in Harriett's classroom are provided in APPENDIX H. Involved in these episodes is evidence of the range of sociocultural norms activated by Harriett's approach to problem-centered learning including working cooperatively, being respectful, listening attentively, encouraging a partner's participation, showing concern for the other person's point of view, planning together, taking responsibility, making decisions, sharing a solution publicly, and open mindedness to negotiating differing perspectives.

Also engendered in the same episodes is a range of sociomathematical norms including helping to clarify a partner's difference in mathematical reasoning by rephrasing or refiguring the mathematical proposition, judging the accuracy of each mathematical proposition as it arises, utilizing personal methods incorporating appropriate mathematical symbols for recording mathematical propositions, elaborating an efficient explanation in mathematical terms that can be comprehended by class members, deciding what constitutes a new or creative mathematical proposition, willingness to experiment with unusual or unconventional mathematical propositions, judging and diagnosing the mathematical accuracy, viability, and elegance of other's as well as personal computation, willingness to articulate a mathematical proposition either as an initial view or a contrasting view verbally and symbolically, separately and concomitantly, and deciding what constitutes meaningful or real mathematical propositions.

From this perspective, mathematical meaning was interactively constituted. The way in which the classroom mathematics curriculum emerged was largely precipitated by the way in which children participated as mathematical problem-solving thinkers. As Yackel (1998) explains, "Classroom mathematical practices themselves emerge as what is taken for granted evolves" (p. 216).

No Coercion, No Threats, No Bribes.

Harriett considered her approach to problem-centered learning as contingent upon a positive discipline policy and vice versa. Accordingly, she worked diligently to create a classroom community willing to solve problems and ready to participate as cooperative negotiators of mathematical meaning. Through encouraging a sense of mutual respect for one another, and for each person's contribution to the problem-solving experience, Harriett aimed to develop children's confidence, their readiness to try their own ideas, willingness to consider other's points of view, and a disposition towards contributing positively towards making sense of their own and other's mathematical propositions.

By dispelling any sense of blame or shame ("We only know what we know and there is no shame in that") during the problem-solving experiences, Harriett further sought to develop a community that could work together collectively, with pride, dignity, confidence and trust. One of her main approaches for developing such a community was to not only dispense with all manner of "authoritative strangleholding," but to eliminate all manner of intimidation, coercion, threat, and bribery. In consonance with her approach to positive discipline, Harriett developed a tone of mutual respect within her classroom.

Positive discipline is based on mutual respect and cooperation as is a problem-centered learning approach. Whereas both these theoretical perspectives adhered to the development of an inner locus of control, both punishment and reward come from an external locus of control. The following episode indicates an example of how Harriett's children developed a sense of confidence knowing that they had little to fear in terms of failure and shame, and everything to gain from personal endeavor. By fostering such a climate, Harriett sought to develop children's positive feelings and attitudes towards functioning as confident problem solvers, willing to take the risks of charting unknown territories in mathematics. Such an approach reflects a desk calendar I recently read which said, "Our students have a way of becoming what we encourage them to be or what we belittle them to be."

On one occasion, Dillard had lost concentration and had become disengaged from the task at hand. He had wandered over to my table at the back of the room, and when I asked him, "What do you have to do?" his response came as a surprise. It was indicative of the charter of his membership in a non-threatening and respectfully encouraging community. Whereas in a classroom setting in which coercion and punishment are the norm, it would not have surprised me that in response to such a question from a teacher or adult, a child might construe the interaction as a reprimand, a challenge or an attempt to "strike fear" into him; a fear of punishment or of being reproached for not doing what they "had" to, or were "expected" to do, or what they had been "told" to do.

However, in Harriett's classroom, where children are encouraged to be self-respecting problem solvers, where they are respected for being themselves, and nurtured as unique contributors to classroom activity, where there is no sense of threat, fear of public recrimination, and no attempt to blame. Dillard quickly regained his composure and perspective, and addressed my question without any sign of fear or concern for negative consequences. He provided an honest and frank answer. He explained clearly what he had been requested to do with his mathematics task.

His explanation reflected an attempt to respond not only out of courtesy but also as if he perceived a genuine interest by me for what he was thinking, as if I was genuinely concerned to take the time as to inquire of his activity. Rather than recoil in fear of being "caught out," Dillard enthusiastically began to speak openly with me about his mathematical thinking, urged more by my "apparent" display of interest than by any sense of veiled threat. It was as if he had transcended all sense of anxiety and trepidation and was calmly able to refocus on his work immediately.

The importance here is not so much what Dillard said but the way in which he responded. Not only was he confident to pick up from where he had left off with his mathematical thinking but he was also able to instantly reinstate himself in a mathematical framework of thinking in contrast to being temporarily distracted by an emotionally fearful one. Such a demonstration speaks to Harriett's conscientious efforts to liberate her children and establish confident self-directed mathematical thinkers rather than disenfranchised and apprehensive math-phobics.

Research has shown that children who experience a great deal of punishment become either rebellious or fearfully submissive (Nelsen, 1996). Harriett's approach did not include any blame, shame, or pain (physical or emotional) as motivators. In consonance with Harriett's stance, Lewis, Schaps and Watson (1995) ask:

*How could we create a caring community in the classroom when children's own needs - to make sense of the world, to be known and liked by others, to influence the environment - were being ignored by a skill-and-drill curriculum? A curriculum that holds little intrinsic interest for children forces teachers to use "motivators," "consequences," and competition to keep children on-task, thereby undermining community and demonstrating that some children are more valued than others. (p. 552)*

Letting Go But Holding On

Harriett believed that the process of constructing meaning in her classroom would require as conducive an environment as possible for "construction" to be optimized, with as little impedance to learning as possible. In other words, she sought to provide a classroom climate that maximized the knowledge-making process.

Harriett felt that in order to cultivate her capacity for relinquishing control, there had to be more than constructivism and problem-centered learning upon which to establish her program of classroom instruction. She felt that her commitment to effectuating an engaging, child-centered, problem-solving approach would have to be supported by synchronizing it with her management technique. Her management policy was framed around the establishment of a caring community within the classroom. It was her exploration of positive discipline which provided her with the cohesion she sought to ratify her new pedagogical stance. While Harriett was to eventually admit that she never really followed the positive discipline format (cf. Nelsen, 1996) "to the letter," she acknowledged how much she was influenced by aspects of the theory, primarily the Significant Seven Perceptions and Skills (see Chapter I).

Fundamentally, Harriett believed that the manner in which children are treated in the classroom, that is to say their discipline regimen, underpins the success of their learning environment. She maintained that "working together on being able to understand different people's points of view and trying to make sense out of things together" was paramount in developing an effective learning environment. From such a perspective, Harriett pursued a classroom management policy of positive discipline based in establishing a classroom community that would grow to respect each other.

According to Kohn (1996), children are "more likely to be respectful when important adults in their lives respect *them*. They are more likely to care about others if they know *they* are cared about" (p. 111, italics in original). The essence of such a view revolves around the principle of establishing a community that,

> *. . . rests on the knowledge of, and connections among , the individuals who are part of it. This knowledge, in turn, is deepened by helping students imagine how things appear from other people's points of view. What psychologists call "perspective taking" plays a critical role in helping children become generous, caring people . . . and activities designed to promote an understanding of how others think*

*and feel. . . have the added advantage of creating the basis for community. (Kohn, 1996, pp. 113-114)*

Harriett had the fortune of operating from a position of sound professional standing within her school faculty and local community, having established her reputation over many years of successful reform within her current school. With her reputation as an excellent teacher, she was able to persuade faculty and parents to bear with her as she strove to establish her new mathematics program. She was skilled in handling parents and, with their respect, was able to assure most of them that her new approach to mathematics would be beneficial for the children despite its unorthodox appearance.

Harriett: It's not like, some people think, well, you can do this and you can have them all [doing it]. If it works, the way you tell if it works, [is] if they're all in control and they're all totally respectful to each other. But I see it kind of like they're on a continuum, you know, and some kids are at a certain spot and need more work with it. But I have seen them all progress, and we still need [to do more]. I mean, even as adults we need to work on it. And I said in the faculty, when we had our faculty meeting, that I was talking about things, about the year; about what things, things that have been significant to you this year, and I talked about this, and I said, you know, what I've noticed is that it's really made a difference with my personal life. Because just looking, trying to really look at other people's points of view, and trying not to look for the blame but for the solutions, and trying to encourage; and, it just seems to go everywhere. And I hear, even the parents telling me, about how at home, they'll [the kids] talk about [it. The kids say]"Well, we need to solve this problem [like we do at school]" (laughs).

It must be noted that while Harriett's endeavors to reformulate her mathematics program were aided and abetted by respectfully sympathetic parents, this may not always be the case in other classroom settings.

Harriett's aspiration to establish a caring community as part of her positive discipline program was sustained by the following paragraph from the Child Development Project (1991):

> *To say that a classroom is a community is to say that it is a place where care and trust are emphasized above restrictions and threats, where unity and pride (of accomplishment and in purpose) replace winning and losing, and where each person is asked, helped, and inspired to live up to such ideals and values as kindness, fairness, and responsibility. [Such] a classroom community seeks to meet each student's need to feel competent, connected to others, and autonomous. . . . Students are not only exposed to basic human values, they also have many opportunities to think about, discuss, and act on those values, while gaining experiences that promote empathy and understanding of others. (cited in Kohn, 1996, p. 102)*

Reflecting a problem-centered approach and constructivist perspective, Harriett's notion of community was one that would be forged out of struggle, where children would be expected to disagree and challenge their peers. She often commented that forging such a community was "not easy and that students were not always going to agree on issues and topics, and there would always be disagreements and arguments." "The struggles, tears, and anger are the crucible from which a real community grows" (Christensen, 1994, p. 14).

Harriett's approach to positive discipline advocated the importance of children constituting the "voices" of the classroom community.

> *One of the most important concepts to understand about Positive Discipline is that children are more willing to follow rules that they have helped establish. They become effective decision makers with healthy self-concepts when they learn to be contributing members of a family and of society. (Nelsen, 1996, p. 16)*

A positive discipline approach excludes excessive control or permissiveness, and aims to develop self-discipline, responsibility, cooperation and problem-solving in children. To assist in achieving this, Harriett implemented classroom meetings. "The single most significant and multifaceted activity for the class as a whole [is] the class meeting" (Kohn, 1996, p. 114). According to Harriett, class meetings became "one of the best ways to give children an opportunity to develop in all the Significant Seven Perceptions and Skills." According to Nelsen (1996), classroom meetings provide the "best possible circumstances for adults and children to learn the democratic procedure of cooperation, mutual respect, and social interest" (p. 132).

Classroom meetings were held daily and usually in the morning to allow every child to speak her or his mind. Children would sit in a circle on the floor and pass a teddy-bear. The person with the teddy-bear could either say what had been worrying them or what was important to them, or simply pass the teddy to the next child without making any contribution. It was a time of respectful and attentive listening as well as enlightening and insightful interactions.

> Harriett: Those morning meetings are really very, very powerful. I think that's the heart of it; that you have to have that, to build and nurture all that. And I think it has helped with the math, because just building that community, I think it would be very difficult to all of a sudden say, OK, we're going to now [be respectful problem solvers]. It's all about patience, and I pray for patience every night. And it's not always easy. That's the thing, this is not easy at all, it's not, it's not. When the children said about the [substitute teacher], how, how "She just wouldn't even listen to what [we had to say]; to our point of view." And so much of it is what you listen to, what they're saying. It's not what you thought it was.
>
> We assume so much. We make so many assumptions all the time [about what children have said]. And I think that's another thing, because, with the math thinking and what I've had to really work with this year, is all the assumptions that I had formulated, that how kids learn math have just, you know, been blown to pieces, and just realizing that you can't assume anything, you know you really can't; and you need to listen to them, and I think you "need" to listen to them. Learning to listen to them in math has helped train me to listen to them in other areas and in other things. Traditionally, we get up there and we're the giver of all knowledge, and it's all about them listening to us. And so why should they be good listeners when we're not even modeling that - for them?

Harriet often related one of her favorite stories to demonstrate the power of classroom meetings and the resultant building of community. Harriett referred to a boy in her class, Jonah, who was an extremely capable mathematician but who had trouble assimilating himself into her classroom environment. Jonah often requested that he be given worksheets that he could do on his own, and indicated a tendency to avoid working collaboratively. He was having difficulty being part of the sociocultural and sociomathematical norms that had developed in his classroom.

Harriett's anecdote is paraphrased below as it was relayed to me over the telephone:

Harriett: Jonah was falling apart socially and emotionally and was becoming a problem for the school as well as our class. I kept wondering whether I just let him create chaos through his misbehavior, or punish him and run the risk of contradicting everything I believed to be important. Then, one day, Jonah spied a spider on the ceiling in our room and leapt across a desk to boldly alert the rest of the class. In his role as class ring leader, he soon had some other boys quickly following him over the desk towards the spider. I tried to remind these boys that they were not being respectful, but felt a total loss of control.

I was tempted to take away their hydro-rocket jet activity which they had been so looking forward to that day, but felt that this would be tantamount to revenge. I even told the class later in the day that I had seriously considered leveling such disciplinary action but had declined to do so. One boy commented that, "Yeah, that would be like using teacher power to punish us." I felt upset and wrote my concerns in the class meeting book for consideration at the next class meeting. I told them I needed to cool off.

Next day, in the class meeting, the boys admitted that they did it because they thought they could get away with it and that the teacher wouldn't punish them. The discussion began to revolve around whether we should be encouraging others to do the wrong thing and should we be helping Jonah. Jonah never really participated in class meetings and would always just pass the bear on.

However, on the last day, in the last 5 minutes of school, in a final class meeting, Jonah took the bear and spoke out. He said, "At the beginning of the year, I didn't think I had any friends but now I feel like I have friends. I think I have been

> encouraged." Jonah continues to visit with me now and we are good friends and he tells me of his fond memories of grade two. Jonah always makes me feel vindicated, that our class sought to encourage and not blame. It's a blaming society. By allowing the chaos to unfold during the spider incident, it helped the "conversation" focus on doing the right thing; and not doing the right thing because you'll be punished or because you'll be unpopular, but because it's the right thing.

Harriett was of the opinion that not enough teachers encourage this sort of reflection. Kohn (1996) suggests that "particularly in elementary schools, one often finds an aggressively sunny outlook, such that space is made only for happy feelings. . . . I once saw a poster near the door that read: *ONLY POSITIVE ATTITUDES ALLOWED BEYOND THIS POINT*. The message here might be restated as "Have a nice day - or else. Alas, feelings of anger or self-doubt do not vanish when their expression is forbidden" (p. 114).

A positive discipline approach (Nelsen, 1996) emphasizes the importance of classroom meetings as a time for children and the teacher to practice many communication and problem-solving skills together. Harriett felt that it was an important time for everyone in the class to reflect upon not only what they as children were involved in but also how she, as the teacher, fitted into the classroom picture. She found that children's integrity continued to grow with the consistent honoring of the items and issues covered in the classroom meetings. She was always interested in receiving feedback from the children to help her gauge the effectiveness of her own contribution.

Harriett: The very first day when we had a class meeting, the very first day, one of the kids complimented me - no, it was the second day, complimented me on not yelling at them the first day, because they had talked to a friend from another school and that friend had said that their teacher yelled the whole day, the first day. And they said, "I was really scared and I really did appreciate that you did not yell at us yesterday." You don't think about how scared they really feel that first day.

Harriett's struggle to sustain the same sense of safety, security, and respect for her children not only during mathematics lessons but all day long, every day, precipitated a coalescence across her entire school program. Not only did her approach to positive discipline become fundamental in the management of the daily routine but also became essential in implementing her mathematics program. In like manner, she discovered that the implementation of a problem-centered approach to learning mathematics proved to be consequential in all other lessons.

She claimed that, "The way children approach their math became more and more evident in other subject areas."

Harriett: When the children gave their [Social Studies] reports it was as if they were just doing the same thing they do in math, listening, challenging each other, expressing themselves being confident. It's all connected, one has to do with the other. Doing all this has helped me with everything I do. In spelling we write all the possible ways, phonetically, that we think the word can be spelled, then we talk about all the good things about that, then we talk about what the book spelling of the word is. The kids during.that period, they kind of got that connection, you know, with what they also did with the mathematics activities. It [the problem-solving approach] has filtered through the whole environment.

Harriett increasingly focused on what it meant to include the "social" dimension with the "constructing" dimension during mathematics meaning making. She commented that, "The math is so social that anything that's going on with them, at recess or at home in the neighborhood, it comes out, if they're having any kind of problem [in math time]."

Harriett: I felt like, so much of the math that we're doing, really just went hand in hand with this [positive discipline]. And I've not only seen that the positive discipline has helped the math, but I feel like the math has helped with the morning [classroom] meetings. In the math we talk a lot about different points of view, about how different people can see [different things], and there are different solutions, and different ways of getting at answers to problems. And they have been able to go back and use that with their problem solving [in every lesson]. In math, so much has focused on respecting each other, and respecting each other's view points, and accepting them, and it's OK to make mistakes, and it's OK if everyone doesn't have the same viewpoint.

And that's where I felt like positive discipline in the classroom was looking at the same objectives. Where I [think positive discipline] is so beneficial with the math [lessons] is, especially at the beginning of the year, whenever there would be conflicts that would come out of trying to deal with all the things they had to deal with, with trying to work together to solve a problem, which wasn't something they were used to, and all the problems that they were having socially with each other, that they could go write it down [in the classroom meeting book for later discussion]. It wouldn't stop what we were doing because they felt like they had an outlet for it.

And then we would come back and discuss a lot of that the next morning at the [classroom] meeting, and also, it was interesting how much of the problems, especially at first, that we discussed [in the classroom meetings] were problems that had come from the conflicts during math, [for example] where they were having trouble with their math partners. And they would want to discuss [issues like], so-and-so wouldn't

> listen, or the other person wanted to take over. That was a common thing. During the class meetings, some of the kids who were working well would tell the class what they thought they were doing that was helping them get along, and be able to solve their problems.
>
> [For example], someone said, "I let so-and-so do one problem and then I do the next problem and then we see if we agree." And then they [talked about] how one would take turns writing words; how one would draw the pictures and one would write the words. Like they just kind of assigned tasks for each other to do.

Harriett continued to be encouraged by the way in which the positive climate of the classroom flourished through the dialectic nature of children's interactive dialogue and their attempts to solve problems collectively. It is worthwhile to keep in mind that the "dialectical relationship between a complex adaptive system and its environment is inseparable. When we talk about the interaction of a system with its surroundings, we do not mean that the system is separated from its environment but rather we mean a coexistence partnership between the system and its surroundings. It is an inseparable unity in diversity" (Pourdavood, 1996, p.149).

Harriett relayed an incident that indicated the power of dialogue in establishing an effective learning community. She described how Dwayne, Tom and Spike had invented a chess game using the hundreds board and had asked to present it to the class:

Harriett: They had [their game] all set up and the dialogue started again, like I've been noticing in other subject areas, where they were asking questions like, asking them why they did things, like, "Why did you make the kings different colors?" and "Why were there different sizes?" And, actually, some of my boys

> and girls play chess on Fridays, and they know about that game much more than I do. And you could tell that their questions were much more focused. They had knowledge of how their game fit. Instead of normally being more passive about it [asking superficial questions]. You could tell they were really making sense of what [was being described].
>
> Then all of a sudden Birt goes, "Wait a minute, I have a question for you." And it's like they're not willing to just have someone get up there and just say their piece, and just accept it. They want to get into to it deeper and have an understanding. And I had noticed it when they were doing projects [too]. They had done projects on a pond unit, and they got up there and started sharing information, and they were challenging each other on the information. And with Spelling [too], when my niece came [to visit the classroom], she couldn't believe that they were so willing to give their words and not be afraid if they're not right.
>
> And I've noticed lately in Spelling that they will say, "I can really see why so-and-so decided to spell that word that way because of the rule, like why they spelled it "ph" because sometimes "ph" makes the "f" sound," even though they know it's not the correct spelling, they're trying to understand. It's like they're trying to really understand what the other person is saying, And I think it's all about working on being able to understand different people's points of view, and trying to make sense out of things.

A primary motive for Harriett's approach in linking constructivist perspectives, problem-centered learning and positive discipline was to find avenues to help her children create a sense of community in their classroom, to construct a place where they could feel trusted and respected, and a place where they would be empowered. Obviously, a key ingredient in effectuating mathematics lessons in such a classroom was the establishment of a community of learners of mathematics; a community where no one owned the truth and everyone had the right to be understood. As Kohn (1996) observes, "community is not enough; we need autonomy, too. In fact, when both of these are present, there is another way to describe the arrangement that results: it is called democracy" (p. 119).

Hence, Harriett's mathematics curriculum was one that emerged through the action and interaction of all the participants, which included her; it was not to be a static curriculum set in advance (except in broad general terms). As a teacher searching for her place in the classroom and her role in curriculum development, Harriett knew she had to be an equal participant but not *just* another participant. Her added responsibilities did not entail power over students, however it did carry an authority, an authority based not on subordination but on cooperation.

## A FRAMEWORK OF PARTICIPATION:
## ENTER THE CHILDREN

### Learning to Take Control

The idea of control as an external imposition is firmly rooted in modernist thought. Without a qualifier such as, "self- or internal-" control is contiguously assumed to mean external intervention, resonating with a *deus ex machina* connotation. Harriett's commitment to encouraging children to take control of their own situation reflected a view to be found in self-organization, chaos mathematics, Dewey's naturalism, Whitehead's process cosmology, Bruner's narrative, Piaget's phenocopy, and Gadamer's hermeneutics. All of these assume authority and control to lie within (not outside) the parameters of the situation. Further, they all assume control to be the self control that emerges from interactions within these situational parameters (Doll, 1993).

Harriett's efforts to empower her children included the establishment and development of a range of sociocultural and sociomathematical norms, all of which she argued were inseparably interconnected. "The richer the curriculum, the more the points of intersection, the more the connections constructed, and the deeper the meaning" (Doll, 1993, p. 162).

> Harriett: Years ago, I would never had said that mathematics is a social experience.

Harriett always felt that her children had to be first and foremost, sensitive and responsive to their social situation if they were to develop as reflective problem solvers. She interpreted reflective problem solving as "the capacity to pause, or even step back, examine, and be able to talk about past experiences in the light of other connections and alternatives. It's like taking a re-look at what we know and what it all means in light of what makes sense already." Dewey (1948/1957) said that reflection "is a method of reconstructing experience" (p. 141). It was in this sense that Dewey (1938/1963) fully elaborated his principle of continuity: "The principle of continuity of experience means that every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after" (p. 35).

The following episode which took place early in the first year demonstrates the import of social transactions in the mathematics problem-solving experience in

Harriett's classroom, and how personal reflection and social sensitivity underpinned the potential for learning mathematics.

Three boys, Jed, Chip and Chong-Mui, were working as a small group endeavoring to make sense of how to use a number balance beam to calculate with what 8 and 9 would balance. Jed and Chip were the most vocal, displaying some difficulty working with the teen numbers.

Jed: Woah. That's 8 and 9.

Chip: Just use touch dots.

Jed: OK.

Chip and Jed's initial reaction to the situation was fairly trite and indicated a predisposition for using procedural rather than conceptual thinking, and rote clues rather than problem solving. Touch dots incorporates corresponding a number of dots strategically positioned on the numeral to indicate its cardinality. However, both boys quickly became aware that touch dots was of no help in the mathematical context of balancing number bars.

Though Jed and Chip were actively engaged in trying to work out how to translate their thinking on the balance beam, they were unable to model how the two bars for 8 and 9 would balance with bars on the other side in any combination. Meanwhile, Chong-Mui who was a Chinese boy with little English language and unable to communicate verbally or articulate his thinking with the other two boys, was clearly able to conceptualize and solve the number sentence by appropriately using the ten and seven bars to balance the beam. He had done this unobtrusively while Jed and Chip struggled with their touch dots. His construction of teen numbers was considerably more developed than either Jed's or Chip's.

However, as Chong-Mui began to demonstrate to the other boys his proposition for solving the problem using the bars, he was politely intercepted and refused further access to the balance beam by the other two boys who seemed to assume, as if in a patronizing manner, that he could not possibly make sense of the question, let alone understand what was required to solve it. More to the point, Jed and Chip could not make sense of what Chong-Mui was trying to demonstrate, assuming him to be less mathematically as well as language literate as they.

Because of their possibly biased, immature, and underdeveloped sociocultural and sociomathematical norms, Jed and Chip forwent an opportunity to negotiate a mathematical proposition, disenfranchised from a potential learning opportunity by a limited sense of their roles as cooperative problem solvers. Chong-Mui had to forego an opportunity to negotiate a mathematical proposition with his peers as he submissively retreated. At this early stage of the year, these three boys were in the process of constructing what it meant to participate in their classroom mathematics experiences. Willingness to listen and negotiate was still emerging as a social norm as was a proclivity for respectful consideration of other's mathematical propositions

Guba and Lincoln (1989) assert, an environment in which constructivist perspectives prevail is one in which individuals are,

> *. . . required to confront and take account of the inputs from other[s]. It is not mandated that they accept the opinions or judgments of others*

*but it is required that they deal with points of difference or conflict, either reconstructing their own constructions sufficiently to accommodate the differences or devising meaningful arguments why the other's propositions should not be entertained. In this process a great deal of learning takes place . . . each stakeholder . . . comes to understand [his or her] own construction better, and to revise it in ways that make it more informed and sophisticated than it was prior to the . . . experience. (p. 56)*

On a different occasion, the power of cooperative, collaborative and socially constructive problem solving, melded through a reflexive dialogic interaction based upon the taken-as-shared classroom sociocultural and sociomathematical norms showed the potential for the children in Harriett's class to support and extend each other's learning. The figure shown in Figure 2 was displayed on the overhead projector for three seconds while children viewed it.

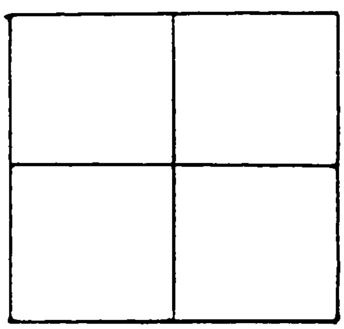


Figure 2. Shape projected by the teacher on the overhead projector for children to view and then draw.

The projector was then turned off and children were asked to draw what they saw. Next, Harriett addressed the class collectively.

| | |
|---|---|
| Teacher: | Tell me, what did you see and how did you draw it? |
| Norton: | I saw a window, and I drew a box and a cross in the middle. |
| Duke: | I saw a rifle scope; a scope used on BeeBee guns; like a laser guide for targets. |
| Caleb: | Yeah! It's like an infer-red sight. |
| Duke: | I drew a cross then a square. |
| Nancy: | I saw it as a package. |
| Teacher: | How many squares does this have? |
| Class: | 4. No 5. No 4!! 5! No 4, . . . 5, . . . 4, . . . 5! (They could not agree.) |
| Caleb: | Oh! Now I get it! I thought you meant 4 little squares. |
| Nancy: | I still think there are only 4 (with which several children flocked to her side to demonstrate where the fifth square was embedded in the design.) |
| Teacher: | OK. Let's try the next one. (See Figure 3.) |

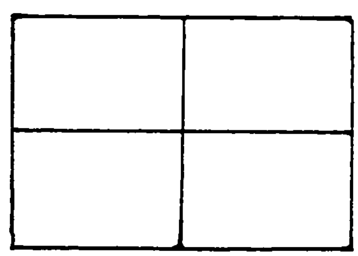


Figure 3. Shape projected by the teacher on the overhead projector for children to view and then draw as a follow up to Figure 2.

| | |
|---|---|
| Class: | OOOh! That's easy! |
| Dillard: | (jumping up and down in excitement) I got it! (For Dillard, mathematics is extremely challenging, yet he has opportunities such as this to join in as a successful participant and readily demonstrates his enthusiasm.) |
| Norton: | I saw a smaller, er, I mean bigger window; especially if you turn it over (indicating a 90-degree rotation). I drew the outside first. |
| Pedro: | I see another gunscope; for a gunnery. I drew the box and put the cross in it. |
| Duke: | Yeah! I see it. It's a different gunsight. |
| Nancy: | I saw 2 packages joined together. I drew the rectangle first then the inside lines. |
| Mona: | I drew the small rectangle first (indicating the bottom left-hand corner rectangle). Then I drew the small rectangle here (indicating the top right-hand corner rectangle). Then I drew this small rectangle next (indicating the top left-hand corner rectangle). Then I drew this small rectangle (indicating the bottom right-hand corner rectangle). |
| Teacher: | That reminds me. How many rectangles are there? |
| Class: | 4! No . . . 5, 5, 5 . . . yeah 5, . . . 5, . . . 5! There are 5. |
| Mona: | I think there are seven. |
| Class: | No, 5, . . . yeah 5, . . . 5, . . . 5! There are 5. |
| Ned: | (comes to the board and draws how she sees only the 5 rectangles.) There are 5. |
| Mona: | I disagree. There are seven (and draws the large outside rectangle). |
| Class: | What? I don't get it! What seven? What!? Where? |
| Mona: | (then draws a similar rectangle but with a horizontal line across the middle of the rectangle. She then writes 1 under her |

first rectangle, and 2 under the next and a plus sign in between.)

Ned: But that's counting the same rectangle twice!

Mona: (She now has Norton standing right beside her monitoring the discussion very closely and others stealing closer with a vested interest. She draws another similar large rectangle and draws the "cross" in the middle and writes 4 underneath, and another plus sign adjoining her third rectangle. Finally, she writes 7 at the end.)

Ned and several others were not convinced, and openly said so. However, several other children had clearly paused to reflect upon Mona's and their own thoughts as they considered and reconsidered the mathematical proposition engendered in what Mona had suggested. It was unfortunate that the children had to leave for their music lesson; the discussion was to be continued at a later date.

Nonetheless, it is apparent that the sociocultural and sociomathematical norms contributed to enlivening and supporting the developmental thinking of the children as well as the teacher, and vice versa; that is, the emerging developmental thinking in the classroom community enlivened and supported the establishment of the sociocultural and sociomathematical norms. This was the emergent nature of the classroom curriculum. From this perspective, mathematical meaning was not only a constructive activity but also an interactive one. With the children constantly laying open their formulations, sharing and negotiating meanings, and collectively reformulating varieties of mathematical propositions, Harriett was availed the opportunity to gain insight into children's needs. Their thinking became increasingly transparent through their social interactions. Further examples of how children's interaction underpinned their mathematical development are provided in APPENDIX I.

Oriented towards an interactive openness in the classroom, Harriett felt that all her children were more likely to express and defend their ideas and thus, she would be better positioned to evaluate and address her children's needs. By respecting their struggles and achievements, and by "close listening" to their conversations, Harriett aspired to a pedagogical orientation based in a democratic ethic. Such an approach accommodated what Sarason (1990) noted as the distinctly political characteristic of the school classroom. Sarason suggested that all educational reform is destined to fail because we fail to recognize schools and classrooms as political organizations. Consequently, in order to achieve reform, students must participate responsibly in negotiating and constructing the agreements under which they can continue to work, play, and share their interests, concerns and resources.

In this view, there is a dictum about conversation being the source of Harriett's guidance in liberating her children. In consonance with Rorty's (1990) hermeneutically-oriented neo-pragmatist view, it is conversation that Harriett took to be the "ultimate context." However, while Harriett struggled to embellish and sustain her new concept of curriculum, Rorty is dubious as to whether such curriculum reconceptualization can occur wholesale within the current frames of philosophy, social thought, and education. Harriett sought to establish a groundwork upon which Rorty's skepticism might be assuaged. Although, and it should be noted here, Rorty in fact does wish for such a reconceptualization to occur.

Doll (1993) accedes that in conversation resides the hope of developing for ourselves a true sense of nature.

> *As we begin to give up the 'false metaphysical comfort' Western philosophy and theology have provided us, we see that it is community which binds us together in and against the 'dark night of existence'. It is conversation which fuels this sense of community, that allows us through imagination and play (more than through rational or scientific analysis) to bring some light to our search. This hermeneutic view where we engage ourselves in conversation with our histories provides us with a concept where curriculum is not just a vehicle for transmitting knowledge, but is a vehicle for creating and re-creating ourselves and our culture. (p. 131)*

When interviewed at the end of the year children were asked, "What did you like best about math this year?" A common answer to the question was, "Listening to the other people's ideas." Harriett remained focused on liberating children from the confines of compliance and coercion. She consistently encouraged children to find their own "voice," and to use it appropriately to enter into a caring relationship with their peers. As Noddings (1992) states, a community which seeks to empower its constituents "proceeds by confirmation. It is concerned with raising the moral level of relations. When we remain in connection, we have opportunities to point out and nurture the best in others" (p. 120). However, Harriett asserted that "you can't force it to happen; it is a nurturing exploration based in dialogue, reflection and trust."

### Sociomathematical Norms Revisited

The following episode reiterates the sociomathematical norms mentioned earlier, but elaborates them further, including, helping to clarify a partner's difference in mathematical reasoning by rephrasing or refiguring the mathematical proposition in alternative mathematical language or symbols, judging the elegance and accuracy of a mathematical proposition when it is presented by another class member, utilizing a viable personal method which incorporates appropriate mathematical symbols for recording mathematical propositions, efficiently articulating a mathematical proposition in terms that can be comprehended by class members, deciphering what constitutes a new or creative mathematical proposition, willingness to explore and play with unusual or unconventional mathematical propositions, judging and diagnosing the accuracy, viability, and elegance of other's as well as personal mathematical propositions, willingness to articulate a mathematical proposition either as an initial view or a contrasting view verbally or symbolically, separately or concomitantly, and deciding what constitutes meaningful or real mathematical propositions.

It is not within the scope of this dissertation to provide a complete analysis of sociomathematical norms and their impact upon the learning of the children. However, three examples are provided below to indicate the import of their emergence and development.

*1) Efficiently articulating a mathematical proposition in terms that can be comprehended by class members:*

The double tens frame (see Figure 4) was displayed on the overhead projector for the children to view for three seconds. Children were asked to decide how many dots they saw and to explain how they decided.

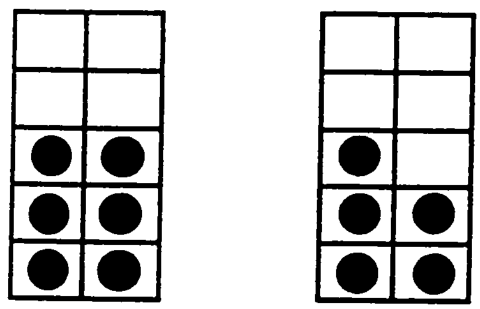


Figure 4. The double tens frame arrangement of dots shown to the children for three seconds.

| | |
|---|---|
| Asha: | I saw 11, and I saw it like Oklahoma. |
| Caleb: | It's like a backward Oklahoma. |
| Asha: | I knew the Oklahoma one was five (referring to the right-hand frame) and the other was 6. |
| Duke: | I saw the 4 missing and the 6 and the 5 right here (points to the two separate frames). But actually, I saw that there were 6 and another 6 but the second 6 was 1 less than 6 which is 5 and because 5 and 5 are 10, then 5 and 5 and one more would be 11. |
| Teacher: | OK. Let's try the next one (shows Figure 5). |

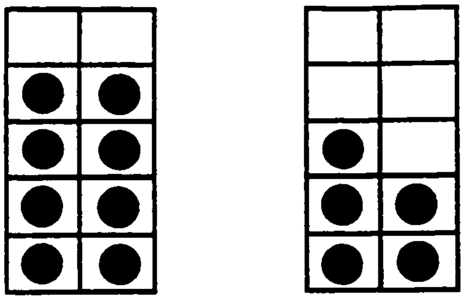


Figure 5. The second double tens frame arrangement of dots shown as follow up to the arrangement shown in Figure 4.

Ned: I saw 13. I counted on 5 to 8. I went 9, 10, 11, 12, 13 (using 8 fingers to show her starting point).

Jack: I saw the 8 then I saw the 5. First I saw the 2 fours and I know 3 plus 2 is 5, just like the other one of Oklahoma.

Shane: (is asked to explain how he thought Jack got 13.)
He saw 4 on top and 4 underneath to make 8 (to which Jack abruptly interjects).

Jack: No! No! I saw 4 over here (points to right-hand frame) and 4 over here (points to the bottom of the left-hand frame).

Caleb: (interjecting) Yeah! that's 4 and 4 is 8 and 4 again is 12 and 1 more.

Jack: I know 4 and 4 is 8 and 2 is 10 and 3 more is 13.

Mona: I put these bottom 2 (points to the bottom of the right-hand frame) up here with the 8 (points to the top of the left-hand frame) to make 10 and that left 3 more.

*2) Utilizing a viable personal method which incorporates appropriate mathematical symbols for recording mathematical propositions:*

During the first year, a group of three dots in the shape of a triangle was constituted by the class to be a "bear paw" and a basic unit for counting and representing the number (not numeral) three. Most class members regularly referred to the counting of three dots as a "bear paw." For example, "I saw it by bear paws, 3 and 3 and 3 and 1 is 10," and "I saw two things. This bear paw with 2 to get 5, and 2 more to get 7."

When interviewed at the end of the year and asked to define and explain how they saw similar dot patterns, some key informants still referred to the "bear paw" as constituting 3 despite having explored an extensive range of memorable counting patterns during the year. The "bear paw" was interactively constituted and had retained its appeal as an effective strategy for the process of coding, formulating, connecting and interpreting, qua counting, a pattern of random or organized dots.

*3) Willingness to articulate a mathematical proposition either as an initial view or a contrasting view verbally or symbolically, separately or concomitantly:*

After an open class discussion on symmetry, the whole-class had successfully negotiated the lines of symmetry as shown in Figure 6, describing them as one horizontal and one vertical. Unexpectedly, Birt chipped in claiming that there were 2 more. He demonstrated where he thought the 2 extra lines of symmetry should be drawn as shown in Figure 7. Both Harriett and the rest of class were unaware of these 2 other axes of symmetry.

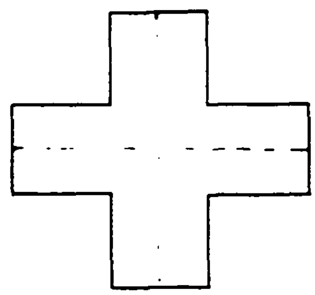


Figure 6. The shape shown to the class for establishing two lines of symmetry, one vertical and one horizontal.

It was Birt's insight that was to avail the class another perspective for considering a mathematical proposition as to what might constitute different lines of symmetry. However, the opportunity for the consideration of this proposition was availed not from the teacher's authoritative position of "knowledge giver" or "expert revelation-er" but from a willingness for children to participate in an open forum of dialogic interaction.

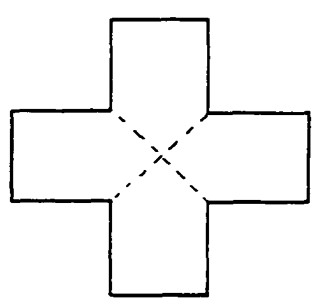


Figure 7. The two other lines of symmetry proposed by Birt.

The emerging curriculum was revealed through a reflexivity among sociocultural and sociomathematical norms (cf. Cobb & Yackel, 1995). Further examples of sociocultural and sociomathematical norms contributing to children's mathematical development are provided in APPENDIX I.

With constant encouragement to express their ideas, children in Harriett's classroom developed their confidence to speak out and be heard. In such an open and respectful climate, children were afforded the opportunity to contribute their personal insights which often availed instances when new mathematical ideas could be negotiated and considered. As it often happened, children's insights transcended the teacher's and brought new opportunity for discussion on a variety of new and enriching mathematical propositions.

It was out of respect for the children's attempts to make meaning that Harriett allowed the open dialogical forum to develop. Through problem-centered learning and positive discipline and an affinity with constructivist principles, she was constantly searching for opportunities for ensuring that children heard other's and used their own "voice" (cf. Wertsch, 1991); that they learned to respect the "voice" of others. Heidegger (1962, cited in Code, 1991) claims that "listening to" is fundamental to being open to and being with others, and that authentic listening is possible only when one understands. Gilligan (1988) also writes of conceiving identity formation in terms of respectful dialogue in that listening has both moral and epistemic dimensions.

Harriett related a story about one of her children's anecdotes:

Harriett: Mariah told me about her mom's boyfriend; and how he had come over the night before and had started screaming, and yelling, and had scared her. And Mariah said, "I told him he needed to come to our class to learn some respect." They [the children] learned to know that everyone deserves to be treated respectfully.

The Development of Substantial Mathematics

Interviews of key informants were conducted at the beginning, twice during, and at the end of the school year. Time, equipment, and space for conducting interviews with the children were shared with the other university researcher working in Harriett's classroom. These interviews indicated that children in Harriett's class were indeed creating substantial mathematical knowledge. Below are some episodes to indicate the thinking that emerged during some of the interviews held at the end of the year.

Interviewer: What's 12 and 18

Jed: (calculating mentally) 30, because 12 and 12 is 24, and 24 and 6 is 30. I know that because 2 and 6 are 8.

Jed was able to mentally calculate 12 plus 18 by addressing the tens first. This strategy was the most commonly used strategy used by key informants and ran counter to the convention for adding double-digit numbers, namely, that the ones "should be" done first. It must be emphasized that Harriett had not enforced, suggested, or hinted at using any conventional algorithmic approach during the year. Children had constructed, qua invented, their own computational strategies; strategies that best suited their needs and made the most sense to them for efficient and viable computational methods. Megsie used a similar but different approach.

| | |
|---|---|
| Interviewer: | What's 12 and 18? |
| Megsie: | (calculating mentally) 30, because I take the ten from the 12 and ten from the 18 and put them together. And that gives me 20. Then, I add 8 and 2 which gives me 10. So, 20 and 10 is 30. |

Megsie and Jed approached this question differently but similarly. They both used their own method for compensation yet addressed the tens first. Such was the standard approach of the majority of the key informants. Most key informants had clearly constructed units of ten and had developed efficient methods for computing double-digit calculations.

Another question involved deciding how to double 29 and then add 29 to that.

| | |
|---|---|
| Interviewer: | (showing two jars of pennies) This jar (jar one) has 29 pennies and this jar (jar two) has twice as many, that is, two times as many pennies as jar one. How many pennies are in jar two? |
| Jed: | (calculating mentally and taking her time) 58, because 9 and 9 are 18, and 20 and 20 are 40. So take 10 from the 18 and it goes to 40. Then 40 makes 50, and 8 left is 58. |
| Interviewer: | So, how many pennies are in both jars altogether? |
| Jed: | (calculating mentally again) Well 8 and 9 is 17. Fifty and twenty is 70. Then, 70 and ten from the 17 is 80. 87. |

All key informants indicated similar capacity to execute a calculation for mentally determining the number of pennies in both jars altogether. Other key informant's approach to solving the pennies in the jars problem mirrored Mona's which involved the following for computing double 29:

> 20 and 20 is 40
>
> Then 9 and 9 are needed. That's 40 and a 9 is 49
>
> So then 49 and the other 9 is, 49 and 1 to make 50
>
> That leaves 8, so then 50 and 8 is 58.

However, when asked to mentally put all the pennies together (mentally compute 29 + 58), Mona explained her reasoning behind her solution as one jar of 29 and the other jar as twice that many (29 x 2 = 58):

> 20 + 20 +20 is 60
>
> From the first 9 get 5 so that 60 + 5 is 65
>
> Then use the 4 so that 65 + 4 makes 69
>
> Then use the next 9 so that 69 and 1 is 70 and that leaves an 8
>
> So put the 8 with 70 is 78
>
> Then 2 more from the last 9 makes 78 into 80 with 7 left
>
> So put the 7 with the 80 to make 87.

Pedro and Megsie, other key informants, used similar but still different approaches for the same problem task. Jed used the following:

For 29 add 58 put aside the 8 and the 9 for a second

So then 50 and 20 will be 70.

Then bring back the extra 9 and the 8

Make the 9 with a one (from the 8) into a ten, so that only 7 is left

Then put 70 and the ten to make 80

And then 80 with 7 would be 87.

Megsie's approach was slightly different but viably effective:

58 and 29 is 20 and 50 to make 70

8 and 9 is 17 because 9 + 9 is 18, so it has to be one less, 17

Then 70 and 17 is 87.

As another indication of the level of their mathematical development, key informants were asked to devise a problem task with the following criteria: the answer needed to be about 50, the problem had to use a "plus" and a "minus," it had to be a problem that someone in their class could solve, and as the inventor of the problem they had to provide the solution. Mona's problem indicates that the level of mathematical development in Harriett's class had produced significant mathematical thinking. Though Mona did not couch her problem in the following symbolic manner, the mathematical proposition she invented involved the following computation:

$(20 + 30) \times 3 + 50 - 45 = 155$ (all done mentally).

Though Mona's solution did not meet the criteria for having a solution close to 50, it nonetheless indicates the confidence with which she was able to "play" with mathematics. Other key informants, on the other hand were confident but less imaginative, and produced problems such as "21 birds on the fence, 30 more came and 1 flew away. How many birds?"

Key informants were also asked to express their opinions about their mathematical experiences in Harriett's class.

Interviewer: Does it help to work with a partner in math?

Jed: Uh-huh. Because, like you and your partner can be thinking, so it sort of helps if you don't get the same way [answer/result], and you can explain it to your partner. Then you might realize it, that it's their way. It's [theirs] is the right answer.

Interviewer: Should your teacher always tell you the answer?

Jed: Ummm, yes. Because if I'm wrong I like to figure it out the right way and get the right answer.

Interviewer: Can you tell if you're wrong any other way besides the teacher telling you?

Jed: Yeah! If I can figure it out and then your partner tells you how she got it, if she got it, it would like, if it made sense to you, and yours wouldn't make sense, and so you'd get their way and they would have a different answer.

Interviewer: So, should your teacher always tell you if you are right?

Jed: No [she shouldn't]. Maybe she should tell you at the end, but she should give you time to figure it out. [To see for yourself] if you think you are right or wrong.

Consideration of the combined perspectives such as constructivism, problem-centered learning, and positive discipline has been suggested as a general theoretical perspective which Cobb (1995) refers to as "emergent" theory. Emergent theory "highlights the contributions that actively interpreting individuals make to the development of local social and cultural processes" (p. 124). The challenge for the future, Cobb suggests, is that of exploring ways in which the complementarity between such intersecting perspectives can be translated in classrooms so that children take more responsibility for control over their social interactions.

## A FRAMEWORK FOR SEARCHING FOR PEDAGOGICAL POSSIBILITIES: ENTER THE RESEARCHER

### Learning to Explain Control

Doll (1993) is one of few writers in education who has provided an examination and synthesis of systemic and complex relationships in schooling. In his attempt to connect and transform modernism with post-modern thinking he developed a pedagogic creed:

> *In a reflective relationship between teacher and student, the teacher does not ask the student to accept the teacher's authority; rather, the teacher asks the student to suspend disbelief in that authority, to join with the teacher in inquiry, into that which the student is experiencing.*

*The teacher agrees to help the student understand the meaning of the advice given, to be readily confrontable by the student, and to work with the student in reflecting on the tacit understanding each has. (p. 160)*

Such a creed goes some distance in focusing the role of the teacher in supporting a post-modern re-appraisal of pedagogy, and begins to articulate a pedagogy relevant to mathematics education. However, such a creed understates the significance of reflective relationship in a "student's" pursuit of learning. Such a creed could be enhanced by a pedagogical framework imbued more with a heuristic of relationships rather than a rhetoric, particularly when considering mathematics education. It should be noted that, while Doll has developed a curriculum matrix based in what he calls the "Four R's - Richness, Recursion, Relations, and Rigor" (1993, p. 176) as a way of addressing his post-modern agenda in education, he speaks only of a general approach to classroom pedagogy.

In the following section, I will elaborate upon a perspective that I eventually developed after participating in and analyzing Harriett's classroom. I will discuss what that perspective could mean in terms of a pedagogical heuristic that would complement the likes of Doll's matrix, while also reflecting the current agenda of the mathematics education reform movement.

## Relationships

As it transpired, several aspects of relationship emerged in Harriett's mathematics lessons. In a learning environment in which inquiry was established as a key ingredient in the mathematics sense-making process, sociocultural and sociomathematical norms encompassed a relationship among the social, emotional, cognitive, and physical needs of the children. Harriett sought to find how these aspects coalesced rather than remained separated, as they were all considered complementary and interdependent upon each other. Likewise, Harriett viewed a problem-centered approach to learning as inseparable from and complementary to a constructivist perspective and vice versa. There was a reflexivity that could not be ignored in the relationships among her theoretical and practical approaches.

Harriett was unable to consider using an inquiry-based approach without positive discipline, and her ideas about positive discipline could not have been implemented without using a problem-centered approach to learning mathematics. All her theoretical perspectives and practical applications were in systemic relationship and coalesced in a reflexive complementarity. While some might claim that this constitutes her classroom pedagogy, it is not enough to say that it is "the way" she taught. There are pedagogies and there are pedagogies. Harriett based her approach in democratic and ethical ideologies. This approach yielded a unique tone, mood and reflexive functionality to her classroom during mathematics lessons.

When Robert Haas (cited in Howard, 1990) commented that "we are at the center of a seamless web of mutual responsibility and collaboration . . . a seamless partnership, with interrelationships and mutual commitments" (p. 136), he was not talking about mathematics education, but about his highly successful business campaign with Levi Strauss. Whether we think about large corporations, microbes, seemingly inert chemical structures, or classrooms, there is an emergence of thinking that relates their organization to relationships. Among the images of the new thinking is that relationships are based upon understanding interdependence.

As Capra (1996) explains, "it requires the shifts of perception that are characteristic of systems thinking - from the parts to the whole, from objects to relationships, from contents to patterns. A sustainable human community [such as a classroom] is aware of the multiple relationships among its members. Nourishing the community means nourishing those relationships" (p. 298). New Science shows that nothing exists, or can be observed, at the subatomic level without engagement with another energy source. This participatory nature of reality has focused increasing attention on relationships.

Harriett's attempts to address the complex relationships of her mathematics classroom experiences were invoked by a responsibility for raising thinking, competent, healthy, and happy children. The achievement of her academic goals hinged upon the modulation of complex relationships in the learning environment.

On one occasion, Tor, who appeared to have been distracted and was not paying attention to his work, was challenged by Harriett with a question that she thought would, without coercing him, turn his attention back to the class discussion and the task at hand. While Harriett did not seek to cajole Tor into submissive compliance, nor make him feel threatened, she respectfully availed the opportunity for him to stop what he was doing and refocus his attention upon what was happening within the class at large and away from his personal state of apparent distraction.

Harriett: (pointing to a line drawn vertically down the middle of a square on the chalkboard). Do you agree that this shows a line of symmetry, Tor?

In many instances, such a directed and pointed question, obvious in its intent to stop a student's misbehavior, could provoke an automatic response of "Yes" or even, "I don't know," as the student quickly attempts to show compliance and avoid further reprehension. However, the children in Harriett's class were accustomed to states "far from equilibrium" and were comfortable with articulating personal points of view spontaneously and unexpectedly, and also being ready to defend, explain, refute, or challenge a mathematical proposition whenever one arose.

Tor: (after a momentary pause) Yes, and there are more.

At this stage, Tor not only stopped what seemingly had been something of a distraction, but within an instant, had begun to extend the mathematical conversation into the bargain, by walking to the chalkboard and showing three other lines of symmetry in the square. The question that emerges is, To what extent and at what level do children remain "tuned in" in spite of what might sometimes appear to be a flagrant lack of attention? Children of a multi-media post-modern era might just be capable of paying attention to several events at once while still retaining their composure for respectful participation. Are teachers suffering from generations of positivist and behaviorist thinking, enculturated by a need for authoritative regulation of discipline; believing that a fixation of mental as well as physical attention is an imperative for learning?

Langer (1997) suggests that for children to truly be able to sustain attention to something for any amount of time, the image or the focus of thought must be varied. A teacher's monotonal voice dispensing information is an example of a lack of variety and an unrealistic demand upon attention. Similarly, for children who have trouble paying attention, the problem may be that they are asked to follow rigidly inappropriate instructions and expectations. As was noted earlier, when I took the opportunity to query Dillard's apparent lack of attention, and here with Tor, both children maintained a spontaneity for functioning effectively as participants in the class experience, despite displaying what might be interpreted as misbehavior in more traditional classrooms.

In an environment characterized by a constant variety in views, opinions, ideas, and where coercion and punishment are excluded, the spontaneity of these two boys was not thwarted by fear and avoidance tactics. The children's ability to self-organize and self-regulate in a spontaneous and responsible manner was not compromised but instead was a vital dimension for a fully participative relationship with their environment.

The manner in which both boys displayed confident perceptivity and ability to spontaneously self-organize reflects Prigogine's (1984) theory of dissipative structures. His theory shows that the behavior of a dissipative structure far from equilibrium no longer follows any universal law but is unique to the system. Near equilibrium we find repetitive phenomena and universal laws. As we move away from equilibrium, we move from the universal to the unique, toward richness and variety (Capra, 1996). In other words, children in Harriett's class were used to being far from equilibrium by being constantly immersed in a context of irregularity, unpredictability, and openness rather than regulated by predictable states of equilibrium enforced through compliance, coercion, and punishment.

"To pay constant, fixed attention to a thought or an image may be a kind of oxymoron" (Langer, 1997, p. 39). Yet this is the very way in which behavior and attention is enforced in many traditional classrooms. Harriett's classroom, by contrast, was characterized by change and disequilibrium, constant reorganization of personal thought, novelty of expression, unexpected and unpredictable change, irregularity in routine, and playful variability. From such a perspective, learning mathematics in Harriett's classroom was reflective of what Prigogine's (1984) work on the evolution of dynamic systems demonstrated, that disequilibrium is the necessary condition for a system's growth.

Prigogine called such systems "dissipative structures" because they dissipate their energy in order to recreate themselves into new forms of organization. Faced with amplifying levels of disturbance within their problem-solving experiences, Harriett's children showed innate properties and tendencies to reconfigure themselves so that they could deal with new and unexpected information. For this reason, Harriett's classroom was indicative of a self-organizing or self-renewing system. As members of a system of reflexive organization, participation in her classroom frequently bore a strong resemblance to Jantsch's (1980) characterization of systems as self-referencing. As a system changes, it does so by referring to itself. Self-reference is what facilitates orderly change in turbulent environments. "The natural dynamics of simple dissipative structures teach the optimistic principle of which we tend to despair in the human world: the more freedom in self-organization, the more order" (Jantsch, 1980, p.40).

Harriett's classroom exhibited such tendencies with children independently and collectively self-organizing. There was a reflexivity between the organization of the individual, and the organization of the collective. There was a profound interaction between how the children responded to their experiences and the way the experiences unfolded. Form and function of the mathematical experiences were in a fluid process where the classroom, qua system, maintained itself and continued to evolve as a new order. Such a system "possesses the capacity for spontaneously emerging structures, depending on what is required. It is not locked into any one form but instead is capable of organizing information in the structure that best suits the present need" (Wheatley, 1994, p. 91).

As such, Harriett's children became stakeholders in the direction setting and leadership of the classroom. Wheatley (1992) asserts that leadership "is always dependent on the context, but the context is established by the relationships we value. We cannot hope to influence any situation without respect for the complex network of people who contribute to our organization" (p. 145).

Organizations and business systems outside the classroom are increasingly tapping into this property of self-organization or self-renewal. Some theorists have termed these "adaptive organizations," where the task or problem to be solved determines the organizational form (Dumaine, 1991). Such organizations are depicted as avoiding rigidity or permanent structures and instead develop a capacity to respond with great flexibility to external and internal change. Teams, action, knowledge, expectations, and norms emerge in response to needs. When a need changes, so does the organizational structure.

The children in Harriett's class participated in and created an organization that celebrated openness: openness to each other, and to the learning context. Wheatley (1992) contends that openness to information from the environment and context over time generates a firmer sense of identity, one that is less permeable to externally induced change. Though some fluctuations will always break through, what comes to dominate the system over time is not the external controls emanating from the environment but self-organizing dynamics of the system itself. "High levels of autonomy and identity result from staying open to information from the outside" (Wheatley, 1994, p. 92).

In contrast to traditional classrooms where externally imposed authority, isolation, and rigid boundaries are promulgated as the best way to achieve control, the self-organizational characteristic of Harriett's classroom meant that the complex classroom relationships coalesced to basically determine the dynamics and direction of the learning. In Harriett's room, the pattern of self-organization and "dynamic interconnectedness" was based on attending to relationships which would address the mathematics reform agenda.

From this perspective, Harriett's class fostered an integrative harmony across social, emotional, physical, and cognitive domains rather than pursuing a curriculum characterized by discreet and unrelated subjects and a classroom consisting of individuals isolated from each other by having to work independently. The interactive nature of classroom activity engendered dynamic interconnections among curriculum subjects and a holistic perspective of child development. Children's self-directed and self-regulating initiatives generated continuous mathematics learning as well as learning in other areas. The following description and analysis of an episode in Harriett's classroom from March in the second year indicates how diverse the learning experiences were and how integrated a simple paper-folding activity can be.

The whole class had just completed writing the steps to use for folding an envelope from a sheet of paper. Harriett asked for someone to come to the front of the room and share with the rest of the class his or her instructions for folding the envelope he or she had made. This was one way in which Harriett sought to integrate language arts (writing instructions) with mathematics. Norton was the only one willing to attempt the task; it was perceived as a hard one by the class. He began drawing his creation on the overhead projector but caused considerable confusion amongst class members as his drawing was ambiguous. His limited verbal and written instructions were unsatisfactory to the rest of the class who became frustrated with his explanation. However, all the children were attentive to his presentation and eagerly compared what they had done amongst themselves citing differences and similarities. Problem solving was a collective negotiation and collaborative effort for the whole class. While most children were engaged with the problem and Norton's presentation, some began to move on to the next origami task of completing a set of instructions for a shape that had become popular during the week - a snapping dragon.

Meanwhile, Norton was still struggling to articulate and illustrate his set of instructions when Catlin approached him to help. Catlin was normally disinclined to front the class to present her mathematical propositions out of a lack of confidence in her mathematical ability. However, she confidently began to converse with Norton in negotiating his procedures in front of the class. It was she who had been the instigator of the snapping dragon paper-folding activity, having brought an example from home to show the rest of the class following paper-folding activities during mathematics sessions that week. Harriett's constant attention to children's interests regularly precipitated child-initiated frameworks for the setting of problems. That is, she used their ideas for framing her mathematics program arguing that if the basic ideas came from the children they would have more of a sense of ownership and connection with their work in class. Such an approach sought to instill confidence in the children as was evidenced with Catlin's effort to assist Norton in a task that no-one else was willing to attempt.

More and more children began making the snapping dragon as Norton and Catlin's explanation bogged down. It wasn't long before the class was teaming with snapping dragons. Apparently, when Catlin had first brought her snapping dragon example to school, several class members had begun making their own with her assistance. Her expertise had made her the center of attention. Her apparent rise in status was also part of Harriett's conscious effort to improve each child's self-esteem which in turn was considered important in modulating self-image as a mathematical thinker.

Mick eventually made more than twenty snapping dragons of various sizes. Chicka put ears on hers, while Caleb and others added eyes and tongues, and others made families. With work on the snapping dragons, the level of noise and pace of activity rose noticeably as children moved back and forth across the room to compare and contrast their ideas, spoke expressively about their ideas, and excitedly played with their creations. Such a self-regulation of activity created what could be misconstrued as chaos. However, substantial mathematics began to emerge. As children's language became increasingly creative their mathematical ideas flourished. Concepts of size, proportion, dimension, position, transformation, magnitude, seriation, measurement, conservation, and two-dimensionality effervesced in a chaotic milieu of activity around the room. However, the chaos was not bedlam and all children were actively engaged.

It was not long before an array of creatures had invaded the classroom. Not only were children learning mathematical concepts but art and craft were also being integrated. Allowing children to freely explore and "play-fully" (cf. Geoghegan, Reynolds, & Lillard, 1998) with their emerging ideas was one way in which Harriett was able to encourage children's responsibility and self direction. Their learning became a social and collaborative effort. Children's creativity was unleashed by the opportunity to be self-directed. Some children established families of "Georgies," as the snapping dragons became affectionately known. Some children were using double-thickness paper to experiment with. Others were experimenting to see what was the smallest size possible, while others endeavored to join sheets of paper together to make a giant Georgie. Duke made one of his into a hat with string under his chin and proudly wore it around the room, while Norton experimented with double-thickness paper to see how much water his would hold.

Children were liberated to follow and set their own problem-solving agenda. They were delegated responsibility for assuming control of their own learning. Children were constructing mathematical meaning precipitated by their self-initiated experimentation and investigation. A fluidity in classroom movement allowed children the opportunity to cooperatively and collaboratively work together. Negotiation, comparison, and synthesis of mathematical propositions freely flowed in and between groups. The classroom experience was a portrayal of the interconnections between curriculum subjects of art, craft, language, science, and mathematics; interconnections between social, emotional, physical, and cognitive development; and interconnections between self-regulation, chaos, order, and learning. The reflexive nature of the learning process emerged as children displayed confidence, autonomy, and a creative disposition for making sense of new ideas. Harriett's class was given the opportunity to be self-organizing by actualizing interconnections among a variety of pedagogical, psychological, and sociological domains. This in turn generated a propensity for new interconnections and perspectives to be forged.

While the rest of the class was slowly depleting the room's supply of paper, I showed Catlin, who was evidently "in command" of the mathematics lesson with a willing band of followers advising her of their new suggestions for how to use their Georgies, how to make a paper cube. I called it a square balloon hoping to appeal to her sense of amusement. She immediately assumed ownership of it and began showing other class members how to fold it. She had apparently developed considerable spatial awareness and fine motor dexterity that translated effectively into paper folding.

Soon, other class members were showing interest in making square balloons to which Harriett gave her approval noting the level of engagement that was emerging from the change in activity, and its relevance to her mathematics objective of exploring relationships between 2D and 3D shapes during the lesson. Translations of the mathematical concepts engendered in the snapping dragon activity began to emerge with the square balloon. Giant and miniature square balloons were attempted; Georgies were matched and paired by size with square balloons, and Norton experimented with how much water his balloon would hold. He showed me saying, "You can drink out of them too [figuratively speaking]. I made a cup and it held water. I could drink out of it. My experiment was a success."

Asha, was a child who struggled with mathematics but was empowered to participate through the paper folding activities. She deftly made a family of eight seriated Georgies and a giant one, which she called her Georgie Holder, to place the others into in decreasing size. As the class ended, she presented me with a selection of Georgies to take home to my family and quickly made a card in which she wrote, "To you. From Asha." The connections emerging from the classroom activity were not merely mathematical. For example, while Asha's developing writing skills were given the opportunity for an outlet, connections with others, with feelings, emotions, and values also were being developed.

*The pattern of organization of any system, living or nonliving, is the configuration of relationships among the system's components that determines the system's essential characteristics. In other words, certain relationships must be present for something to be recognized. . . . That configuration of relationships that gives a system its essential characteristics is what we mean by its pattern of organization. (Capra, 1996, p. 158)*

"By way of this dynamic interconnectedness, evolution [qua learning ] also determines its own meaning" (Jantsch, 1980, p. 14).

SEARCH Emerges

From coding the events and episodes that transpired during my observations of Harriett's mathematics lessons and from discussions with her, particular facets of her approach were identified as consistent, encompassing, and enduring across time and context. The manner in which Harriett approached a constructivist perspective harmonized with and complemented both a problem-centered approach to learning and a positive discipline approach to management.

While particular constitutive facets were individually identifiable, their "synergistic coalescence" (Geoghegan, 1996; Geoghegan, Reynolds & Lillard, 1997) and interconnectedness precluded them from existing separately, and thus provided for a concept of three dimensionality in which all facets formed a multi-dimensional, complex but unified relationship. "As a meaning structure based on evolving and socially negotiated rules, values and signs, the social system which displays self-organizing properties becomes what Haken defines as a synergetic system (1996)" (Fleener & Rodgers, 1998, p. 18).

> *One of the most striking features of animals is the cooperation of many cells that manifest itself . . . in the coordination of muscles in locomotion and in other movements. . . . Such high coordination may also be observed in breathing, heartbeat and blood circulation. At a still higher level, in the human brain many cells cooperate in a purposeful manner to produce perception, thinking, speech, writing, and other phenomena, including emotions. In all these cases, new qualities emerge at a macroscopic level. . . . Synergetics can be considered as the most advanced theory of self-organization. (Haken, 1996, cited in Fleener & Rodgers, 1998, p. 18)*

A synergistic relationship emerged in the way Harriett sought to find the optimum curriculum with which to maximize learning. It was when I was in Harriett's classroom in early February of the second year when the connection between systems theory and her pedagogical approach truly registered. After watching the ice-skating championships at the Winter Olympics on television, I had been reflecting on what a commentator had remarked about the difference in the quality of the pairs figure skating: "The difference is that you can tell the partners who move together, flow together, act in unison as one, compared to those who have to work to be together."

During one of Harriett's mathematics lessons I was fascinated by the chaos but marveling at the harmony when the ice-skating metaphor struck me. Her class was basically acting as one fluid self-organizing entity, a complex adaptive system.

The focus of engagement was not on getting the groups or pairs to work as a class but rather getting the class to work as pairs or groups. The unity of the class was self-sustaining as if one cohesive but diversely disparate group. Harriett often commented how chaotic her room must have appeared to other teachers and parents. She noted that many adults felt restless and uncomfortable when visiting in her room. However, the chaotic milieu of apparent uncoordinated and disordered activity belied the underlying organization and learning taking place. What must have been interpreted as a dubious state of chaos by adult visitors was no doubt reflective of a superficial notion of what constitutes children's learning in mathematics, mirroring a set of values and attitudes reminiscent of a logic of domination and subordination.

The following interpretations of what was happening in the classroom at the time were recorded in my field notes in an attempt to describe and capture how the moment of realization had manifested itself. They included: "responsive to each other, comradeship, interest in others, discussion with others, drop ins and move ons, freedom, movement and interaction, focus then distraction, consonance with mindfulness, self-expression, body and mind, singing, teacher as participant, reflecting true self, shift and flow . . . the systems theory collides with practical application. It is easy to be prescriptive and judgmental of children according to "acceptable behavior." Being creative and expressive, spontaneous and individual is all too often frowned upon; openness to self is spurned. Freedom of choice to be, do, go, play, choose, say, explore, contemplate and consider are all too often controlled - and disciplined out of being a learner i.e., conform and follow, comply and avoid" (Field notes, February 1998).

From this perspective, Harriett was committed to finding ways to afford every opportunity for children to develop their full mathematical potential. This involved reflective and critical attention to herself, her children, the curriculum, and the complex relationships engendered in the process of learning mathematics. Concurrently with Harriett's search for effective teaching strategies, each child searched for meaning, not an objective meaning of something "out there," but the construction of their own sense of reasonableness and meaning. By participating in a child-centered community, children were making sense of what it meant to do mathematics. Thus, the unified relationship of what was happening in Harriett's classroom was portrayed as that of a "search" in which *all* participants struggled to find meaning in *all* they were doing.

During the present study I regularly reflected upon a personal theory which I had been developing for several years called SEARCH (see Geoghegan, 1993). I gradually began to assimilate what Harriett was endeavoring to achieve in her mathematics program into a unified framework based upon the metaphor of "searching." Searching befits the journey upon which teacher and child embark in endeavoring to make sense of the world around them in synergistic relationship. The teacher's function is to search for the best way to optimize a child's learning. By gauging a child's progress, and attempting to establish relevant learning experiences, the teacher commits to a search for providing developmentally appropriate experiences to meet each child's needs. Similarly, children are viewed as participants in a constant search for ways to make sense out of the experiences they encounter.

SEARCH was a theoretical model originally developed to explain why musical experiences might assist in the development of early childhood mathematical achievement. It consisted of three components: Self Esteem (SE), Active Referencing (AR), and Creativity Hierarchy (CH). However, the explanation provided by the original model was not enough and did not go far or deep enough to adequately explain or depict the complexity of the relationships that emerged in Harriett's mathematics lessons. Consequently, a reformulation and reconceptualization of the original model as a pedagogical heuristic with broader theoretical perspectives began to emerge.

Newell and Simon's (cited in Audi, 1995) influential work on problem solving and in cognitive science is closely linked to philosophical interests in the way they define "heuristic." They construed problem solving as a "search through a problem space and introduced the idea of *heuristics* [as] generally reliable but fallible simplifying devices to facilitate the search" (Audi, 1995, p. 130).

## The SE in SEARCH

One consistent and enduring dimension to Harriett's mathematics lessons was her attention to a child-centered approach. A child-centered approach is characterized by the encouragement of democratic, caring, ethical, respectful, self-directing, self-organizing, and socially-relational principles (cf. Arthur, Beecher, Dockett, Farmer, & Richards, 1993). Whereas the SE in SEARCH originally stood for the development of an individual's Self Esteem and to some extent represented the ideals of a child-centered approach, such a focus did not adequately represent what Harriett was achieving in her mathematics lessons.

Harriett's view of control and freedom in her classroom was constituted by a democratic outlook akin to Dewey's reform agenda from nearly 100 years ago which advocated self control as the aim of the educative process. Described below are instances which demonstrate how Harriett and her children searched together to establish freedom and independence as major facets of learning mathematics.

> Harriett: I felt that it was important that the children work with the same math partner all the time, you know, in order to foster a working relationship and to establish a familiarity between them, to develop a level of communication; to assist in establishing a sense of security and familiarity in their negotiation in the learning. But after trying to enforce these partnerships the kids let me know that some of them just weren't working. So I just let them choose their own partners, and now they are much happier, in their own choices. They're working much better. It's just another way of looking at how to solve problems together.

Each day, after Harriett's children had finished work on their specified mathematics problems in their small groups, they were encouraged to self-select a mathematical game or activity, such as setting up a shop, constructing paper shapes, or using math-games on the computer. A large variety of materials was readily available and easily accessible for children to select from. When asked to explain to the rest of the class their solution or what mathematics they had been doing, each child or group was provided ample opportunity to finalize their presentation and time to answer questions from the rest of the class.

Rigid classroom structures in the form of prescribed seats or seating arrangements were not imposed. Children regularly reorganized their desks and chairs to accommodate what they felt would provide a better arrangement for their working groups. Harriett was flexible in permitting children to make their own decisions about where they sat. However, she closely monitored their resultant activity. While she was keen for children to take responsibility for making decisions, there were limitations as to how the room could be functionally reorganized. Interestingly, the self-regulatory actions of the children produced a consistently viable order of organization. Harriett commented profoundly, "That's just another example of them doing what they want to do and ignoring me. They have to find their own way to do things that best suits them."

Another record from my field notes reads: "There is a level of tolerance of youthful playfulness that must be accommodated [by Harriett] during math time in order to allow the fluidity that underpins the freedom of their learning. [The teacher's] threshold of frustration, anger, impatience, and intolerance is suppressed and controlled. The focus is always on the children partnership in participating, and transcends [the adult's] aggravation or annoyance over individual's foibles" (Field notes, October 1997).

On many occasions, Harriett remarked that she felt like she was providing a climate of democratic cooperation that children were not used to or might not be receiving at home. "Teachers have the opportunity to establish an environment that many children do not get at home . . .clear and simple rules, consistent expectations, caring loving attention, respectful and developmentally appropriate experiences, and recognition of their aspirations in life" (Field notes, September 1997).

Harriett did not believe in the customary procedure of giving awards or emphasizing extrinsic motivation for good work. Children were not encouraged to think that they would get stickers, stars, or rewards if they worked hard on their mathematics. Accordingly, the class opted to withdraw from the "pupil of the month" program instigated by the local newspaper in conjunction with the school at large.

> Harriett: We [she and the children] discussed this during class meetings. We all agreed that everyone is special in our class, every month, and it just isn't fair to select just one person as being better than the rest of us. We all try hard and do our best. I always encourage the children to be themselves, value themselves and think for themselves, as being important people all the time. During class discussions [in mathematics as well as other subjects] each child is allowed to be heard and encouraged to express their own ideas with the expectation that all the other children will remain respectful of them. The expectation of respectfulness is consistently reinforced. I constantly encourage the children to feel secure in the fact that they can participate at their own level.

From fieldnotes, the following observation was recorded: "Some children wander freely around the classroom yet appear to be still "tuned in." Others who apparently seem "tuned out" are suddenly and unexpectedly "tuned in" just when one might think they were lost from the class conversation. Classroom dynamics are ebbing and flowing without coercion or fear of punishment. Instead of disrespectful or demeaning disciplinary threats, cooperative and unanimous respect for each other are being emphasized - there is a remarkable show of tolerance and patience by the teacher" (Field notes, November 1997).

On one occasion, a disruption erupted in the middle of the classroom over the ownership of a pencil. "Harriett and the class were trying to listen to Alice [a child presenting her mathematics work to the class] but could not hear her for the disruption. Harriett said, 'Wait a minute, Alice. Class, I am trying to listen to Alice. Homer [the child causing the trouble with the pencil], if you have a problem, please leave it till we can discuss it in our class meeting later.' Immediately the disruption was quelled and the class quickly became refocused upon Alice's work" (Field notes, February 1997). Harriett consistently respected children's ability to develop self control, and persistently appealed to them to act responsibly by being respectful to others at all times as she endeavored to be with them.

On another occasion, Harriett had been trying to get the class to clarify their different perspectives of line symmetry. Just when she felt the lesson, in her words, ". . . was about to disintegrate," her willingness to respect children's stake in their mathematics program proved how fruitful such a consideration can be.

Harriett: Suddenly, Norton jumped up and said he had made up a game using the hundreds board. I could have kept going with the symmetry lesson but felt this was one of those teachable moments. So I let Norton show the whole class how to play [his game] with a die and move along the board. The whole class responded so enthusiastically that we immediately started playing the game. The hundreds boards have just been sitting there on the shelf all year and no-one has touched them. Now I think they are ready for them.

In accordance with the projected goals of Harriett's semester-based mathematics program, it was planned that the children would be introduced to the hundreds board later that semester. However, Harriett's decision to follow the children's apparent readiness by acknowledging and responding to the children's enthusiastic and spontaneous involvement indicated her commitment to liberating children through their own decision making. By allowing them a stake in determining opportunities for their own mathematical development, Harriett endeavored to shift authority away from the prescriptive linearity of mathematics curricula by honoring the children's emerging interests. Within her mathematics program schedule, Harriett maintained a flexibility for making spontaneous program redirections which on this occasion precipitated months of productive number work on the hundreds board. By following the children's lead Harriett was able to reformulate her program to meet their needs rather than enforcing hers.

Instead of pursuing her "symmetry" lesson at the time knowing it was disintegrating, Harriett availed the opportunity for children to act as self-directed, self-regulating, creative and collective decision makers, and proactive stakeholders in determining their own learning opportunities. While the symmetry agenda was to be revisited later in other ways, a new item moved onto the classroom mathematics agenda in the form of the hundreds board. Code (1991) suggests that "in epistemic activity, 'personal knowledge' depends on common knowledge. Even the ability to change one's mind is learned in a community that trains its members in conventions of criticism, affirmation, and second thinking. Being self-conscious means knowing oneself to be a 'person among persons', and realization, if professed is essentially shared" (p. 84).

Code (1991) cites Baier's (1985) idea of "second personhood" to elaborate the significance of the emancipatory potential of systemic interconnectedness at the social, historical and consciousness levels, that persons are the creation of persons. "Persons are essentially 'second persons'. Implications of this claim . . . add up to a repudiation of individualism in its ethical and epistemological manifestations, which is less an explicit critique than a demonstration of the communal basis of moral and mental activity. It is possible to endorse Baier's 'second persons' claim without renouncing individualism: she shows that uniqueness, creativity, and moral accountability grow out of interdependence and continually turn back to it for affirmation and continuation" (p. 82).

Such was the focus by Harriett upon the social dimension of constructing mathematics knowledge. Her attention to sociocultural and sociomathematical norms precipitated a community engendered with a freedom based interdependence. From this perspective, an emancipatory purpose imbued with self-control and respect for others became fundamental to her approach to mathematics education. It is in view of such a purpose that Social Emancipation in the SEARCH heuristic was adopted as the SE instead of Self Esteem.

It should be added here, in contrast to Kant's edict that ethical responsibility to others is to be found only when we do our duty for the sake of doing our duty, Harriett sought to motivate her children to take control of their concern for others by transcending their own interests through reflecting upon feelings of benevolence and sympathy. "If we are to find meaning in our lives by working for a cause, that cause must be . . . a 'transcendent cause', that is, a cause that extends beyond boundaries of our self" (Singer, 1995, p. 218). Such a focus resonates with a post-modern intent to reassess an obsession with the self that has been construed as the characteristic psychological error of the generations of the seventies and eighties (Singer, 1995).

Harriett's focus on self-regulation was fashioned through an ecological perspective which sought to link each child in partnership.

> *In human communities partnership means democracy and personal empowerment, because each member plays an important role. Combining the principle of partnership with the dynamic of change and development, we may also use the term 'coevolution' metaphorically in human communities. As a partnership proceeds, each partner better understands the needs of others. In a true, committed partnership both partners learn and change - they coevolve. Here again we notice the basic tension . . . in the way in which our present societies are structures, between economics and ecology. Economics emphasizes competition, expansion, and domination; ecology emphasizes cooperation, conservation, and partnership. (Capra, 1996, p. 301)*

An attempt to interweave epistemological and ontological issues with moral, political and social dimensions during her mathematics lessons revealed the intricacy and ubiquity of the complex relationships imbued in emancipatory and ecologically sensitive goals. Collins (1991) contends that central to the knowledge validation process is an ethic of care comprising personal expressiveness, emotions, and empathy. She cites three interrelated components comprising an ethic of care, namely, the emphasis placed on individual uniqueness, the appropriateness of emotions in dialogues, and developing the capacity for empathy. Such components reflect that which Harriett's approach to mathematics education drew upon to develop children's sense of partnership in the learning experience.

Coalescence of epistemological, ontological, and methodological relationships and the emergent nature of her mathematics program proved to be the basis for children's empowerment. Empowerment from this perspective was a creative power used for the good of the classroom mathematics community. Such an approach overtly rejected theories based on domination and embraced a vision of power, qua control, based in self-actualization, self-definition, self-determination, that went to form the framework for establishing the SE in the SEARCH heuristic as Social Emancipation.

## The AR in SEARCH

The approach taken by Harriett to encourage children's mathematical thinking revolved around efforts not to tell them the answer, to let them negotiate meaning for themselves and amongst themselves, to build upon their previous knowledge, and to view each individual child as a unique learner with unique ideas. To achieve such ends, she selectively chose the mathematical experiences for her classroom. It was her aim to provide an environment in which children would be unencumbered in their search to construct mathematical thinking. The environment she facilitated was built upon sociocultural and sociomathematical norms which in turn determined the tenor of the classroom experiences.

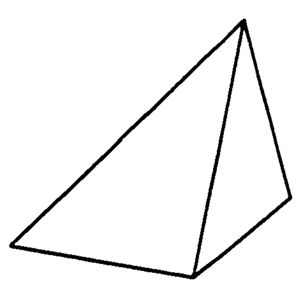
The emergent nature of such a reflexive relationship was also reflected in the perspective Harriett took towards actively engaging children in their mathematical experiences. The learning environment was characteristically activity-based, hands-on, participative, and interactive. A sociomathematical norm was constituted through an expectation that mathematics lessons would be an active use of time, space and thinking to develop mathematical ideas. Not only was cognitive growth addressed, but also social, emotional, and physical development activated. Harriett saw as her role not to teach her children but to facilitate experiences which provided for active engagement in the construction of mathematical ideas.

Harriett based her approach to active engagement in mathematics lessons on a working relationship and alliance with the principles of partnership and friendship, not just at a level of inter-activity but also in an intra-activity sense. Children were expected to utilize all their senses and capacities intra-actively, in relationship, in a holistic fashion to think mathematically. Code (1991) suggests that,

> *[o]nly by taking the trouble to know other people well, in 'their' circumstances, sensitive to what their circumstances mean to 'them', can people participate responsibly in each other's lives. . . . People have to be able to judge which 'expert' conversations merit confidence*

*and trust: they have to situate themselves within many of the ongoing conversations before they can become clear about what they need to reject. But such dependence need not be abject. [A] valuable contrast [can be drawn] between the practice of an expert who 'tells people' what is true and an exchange in which views are tried out, considered, and reconsidered. In the best forms of teaching, for example, people are introduced ' to ideas they can play with and use (or ignore) to create and correct their own views' - and to which they contribute their own ideas, from which the 'experts' are prepared also to learn. (pp. 312-313)*

In general, each mathematics lesson commenced with open-ended "mind stretching" (Gordon, 1961) tasks. Typical of such tasks is the following Quickdraw task (see Wheatley, 1996). The children were asked to draw the shape that they saw when it was shown for three seconds on the overhead projector, and then hidden. The following shape was used:



Whole class: OOO! That's easy.

| | |
|---|---|
| Teacher: | (After waiting several minutes for children to draw their shapes) What did you see and how did you draw it? |
| Dan: | I saw a video camera. It's like looking at a door right in the corner. It's like... . |
| Kisha: | (interjecting) Oh yeah. I know, it's like a video monitor. |
| Jimmy: | Yeah. I know. I get it (then turns to explain to Donald beside him). |
| Catalina: | I saw a race car. The middle line is the top. |
| Jack: | I saw it like an envelope. If you turn it. (He rushes out to turn the shape and another child calls out, "Yeah! I see an envelope too.") |
| Ronald: | I saw a ramp for a bike. |
| Steven: | I saw the Star Wars ship. |

During mathematics time students readily engaged and actively participated in open class discussions such as the above episode demonstrates. Each whole-class sharing time was a time of agitated activity when children's, rather than the teacher's, ideas were freely espoused and openly negotiated. Also, later in the lesson during independent activity time, when Harriett worked with separate pairs or in small groups, children approached their mathematical experiences with an enthusiasm and confidence that was clearly evident in the manner in which they could candidly articulate and concretely demonstrate personal levels of thinking, as the following episode demonstrates.

Jack and Dan were paired together. They were both considered to be less developed mathematically than most in the class. They were to do, for the first time ever, some balance beam multiplication, the first of which had 6 on one side and two equal mystery numbers on the other (see Figure 8).

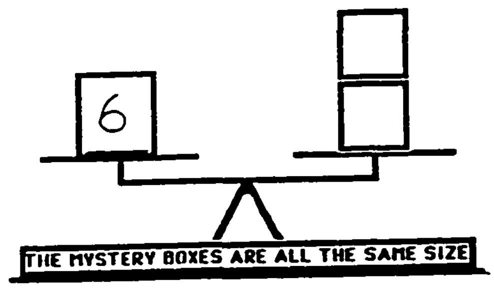



Figure 8. Balance beam task involving multiplication addressed by Jack and Dan.

| | |
|---|---|
| Jack: | This is too hard. It can't be done. |
| Dan: | You can't do this. This is impossible. |
| Teacher: | Why do you say that? |
| Jack: | Because the way we've been doing it is different. We don't know whether it's add or take away. |

The boys were confused by the graphical and symbolic representation of the multiplication process implied by the balance beam diagram. So Harriett asked them how it might look if they drew it. Their representation was accompanied by a convoluted description of addition and subtraction as being the only possible ways to "do this." So Harriett discussed with the boys some ideas of balance as being even, level, equal, and the same on both sides, and then asked the boys to consider the possibility of having two numbers the same size on one side of the balance beam to make the beam balance with the six.

Jack: Oh! I get it. (He promptly wrote in the two threes to balance the six.)

Harriett then asked Jack to explain what he had done and why to Dan. Dan nodded as if to say he understood Jack's explanation. They then moved onto the next question which involved 9 balanced by three identical mystery numbers. Jack cried, "Whoa! This is hard. Where's the calculator?" Harriett encouraged them not to get the calculator "just yet" but instead "to use the computer in their heads" to which Dan acceded "coz it is more powerful." Dan wrote 1, 8 and 4 in the boxes only to be quizzed by Jack that this would make 9 plus 4 and that was "too much." With their teacher, they discussed how the mystery boxes were the same size. "Oh! I get it." exclaimed Jack and wrote 3, 3, 3 to which Dan exclaimed, "Oh! I get it."

Their enthusiasm was growing with their engagement, and vice versa. In their personally constructed sense of the mathematical task they found knowledge, confidence, and security. They turned to the next question: 12 balanced by three identical numbers.

Jack: Oh! This is hard. These numbers get big.

Teacher: Are these big numbers? What are really big numbers?

Jack: Hundreds and millions.

| | |
|---|---|
| Teacher: | What would balance on this side (pointing to the left side showing 12) if the three boxes (pointing to the right side) had 5 in each of them? |
| Dan: | Um, that would be 20. |
| Jack: | No. It would be 15. (With this Jack promptly filled in the next problem which showed three equal boxes to balance 15.) |
| Dan: | (very excitedly) I know. I know! (and wrote 4, 4, 4 in the three boxes needed to balance 12.) |

But Jack looked puzzled and, presenting four fingers on each hand, began to count. Harriett promptly put out another four fingers and together Dan and Jack counted aloud, "8, umm, 9, 10, 11, 12. Oh Yeah." Now, it should be noted, that Jack was a child prone to bad temper spells, lack of patience, and often failed to complete his work, preferring to ask brighter students for the answers so that he could fill in his sheet. He often showed less knowledge of mathematics than his peers.

However, on this occasion, he promptly and enthusiastically turned over to continue with the next page of questions. His enthusiasm in this instance was contagious and Dan responded along with him. The next question was: 4 to be balanced by two identical numbers. Dan quickly and enthusiastically responded, "I know! I know!" and together they scrambled to write 2, 2 in the empty boxes. They were no sooner done when Jack wrote 2, 2, 2, in the next question requiring three numbers to balance 6.

"Whoa! These go up to thirty," Dan commented in surprise after he had looked ahead to the next question requiring three identical numbers to balance 30. He wrote 5 in the first box to which Jack promptly replied, "Nahh! That would be 5, 10, 15." Harriett noticed their connection and referred them to their previous example of three fives to equal 15 by turning back the page. It was now time to return to their desks for the whole class discussion but neither Jack nor Dan stopped their thinking, both absorbed and engrossed in their own ideas and the potential for more solutions. Harriett had to encourage them to return to their desks which they politely did, only to have Dan, smiling from ear to ear, rush back to her with the three tens correctly entered in the three boxes to balance 30. He said, "I worked it out and Jack wrote it."

The above episode demonstrates how Harriett helped children to make sense of new mathematical propositions by having them relate to their already constructed mathematical ideas. Rather than tell the boys what to do or how to think, qua teach them, she facilitated a child-centered discussion relating their emerging thinking to their already-established and familiar mathematical points of reference. A classroom sociomathematical norm required that the boys actively and reflectively discuss new mathematical propositions in their own words using their own mathematical ideas already developed.

The above episode also highlights a respect for each student's level of potential construction (cf. Steffe & D'Ambrosio, 1995). By encouraging self-initiated conjectures in the learning process, Harriett positioned the two boys to be actively engaged and developmentally challenged. Rather than "spoon feeding" them ideas that potentially could be too far removed from their constructed sense or level of comprehension, Harriett sought to work with them from their points of reference, rather than imposing hers (see Vygotsky, 1978). It should be noted here, that Harriett had expected Jack and Dan to complete only the first few problems on the worksheet.

Yet, of their own volition, excited by the way they were able to think through the mathematics of the more challenging questions, they confidently offered to present their solutions during the whole-class sharing time.

Later that afternoon, Harriett collected and examined the paper that Jack and Dan had been working on. She noticed that they had correctly completed every question on the sheet including the last one: three equal numbers (20, 20, 20) to equal 60. What is even more interesting, was that of all the other children's papers, no-one, not even the most mathematically able, had managed to correctly answer the same number of questions as Jack and Dan. The development of Jack and Dan's thinking had been supported through negotiation and sharing of ideas facilitated by an interested and sympathetic adult alert to <u>their</u> SEARCH for meaning. Another point of interest about Jack was that he was one of the key informants who, when asked during the interview what the best part of a mathematics was, replied, "Listening to other people's ideas."

Another episode demonstrates how attention to activating each child's social, emotional, physical, as well as cognitive development was important in forming a positive mathematical disposition. Mertle was an emotionally variable child, socially awkward, mathematically challenged, but an excellent reader. From my field notes, the following record portrays how Mertle's mathematical inadequacies and immature display of sociomathematical norms were transformed into a mature and positive mathematical predilection.

*Mertle has put it together. She is out front [of the class] leading the charge, challenging the others' thinking, and clearly in control of the mathematical discussion. She seems to have settled down and is less distracted by her own lack of "sensibility." Remember how violent she used to be, scribbling all over her papers, throwing tantrums, fussing over every little issue, using overt distractionary tactics to remove the focus from the "non-sense" she was struggling with. Was she frustrated by her own [mathematical] "non-sense?" Perhaps with [mathematical] thinking at a "non-sense" stage she manifested an agitated discomfort with such "non-sense" (e.g. babbling excessively, scribbling frenetically, doodling aimlessly, displaying moody shifts). Or are these strategies to bide her time as she fathoms her thinking, an opportunity to stall for time till she does (can) make sense? Now [as she confidently demonstrates her mathematical ideas in front of the class], the manifestation of her [mathematical] "non-sense" appears to have been displaced by a [mathematical] "sense" [or "sensibility"] and hence she has been acting less "non-sensibly" (no more frantic, agitated or unsecured nonsense behavior). She is more articulate and cohesive in her work - "things" [mathematical] seem to be making more sense - she's "making" more sense! Is she less disfunctionally distracted now that [the mathematics] is making more sense to her? She doesn't seem to need to folly with or be follied by* *"non-sense" She's out there "doing it" and doing it well." (Field notes, February 1997)*

In consonance with constructivist perspectives that knowing is active, and that it is based on previously constructed knowledge, Harriett rejected the structuring of her mathematics lessons upon the transmission of instrumental and procedural rule-oriented thinking. Instead she favored establishing interdependent learning wherein the construction of mathematical knowledge harmonized in a reflexive relationship between group interaction and individual sense making. This reflexive view of learning offered her a vantage point from which to address some of the complexities of prevailing theories of mathematics education.

The way Harriett structured her mathematics learning environment could be portrayed metaphorically as a "theater in which students perform their identities, choos[e] their roles and scripts, and us[e] the props available" to make sense of their own thinking (Damarin, 1995). Such an image implies that knowledge construction is not purely a personal experience but constitutive of an active participation in a dynamically volatile environment wherein effective engagement implies self-directed harnessing of contextual variables rather than submission to a blind faith of an imposed direction determined by the teacher.

Guba and Lincoln (1989) suggest "the major task of the [teacher] is to tease out the constructions that various actors in a setting hold and, so far as possible, to bring them into conjunction - a joining - with one another and with what ever other information can be brought to bear on the issues involved" (p. 142). Their idea is a suggestion of re-evaluating the metaphysics of the person that underpins the objectivity and subjectivity of the knowing being. They exemplify their point by quoting from Tom Wolfe's (1982) "The Purple Decades," "Only through the most persistent and searching methods of reporting. . . can the journalist's entree in point-of-view, the subjective life, inner voices, the creation of scenes and dialogue and so on, be justified" (p. ix). To understand a child's sense-making experience the teacher needs to be able to acknowledge and identify with the child's points of reference.

Thus, in consonance with Steffe and Wiegel's (1996) explicatory model of mathematical learning that implies "it isn't until we have constructed such schemes can we legitimately claim to understand the student's" (p. 488) and that learning "is a living activity and must be experienced to be understood" (p. 482), Harriett felt obliged to enter the child's theater of experience 1) to try to understand the complex web of connectivity of human aspiration and 2) to try to explore mathematical learning as both objective as well as subjective experiences. Such a dispositional commitment aligned learning with a metacognitive approach which builds knowledge from reflective practice.

Harriett's students were encouraged to resolve perturbations not only through reflection but also through feeling confident to search for resolutions (imperturbation). Thus student's sense making was portrayed as an active and confident foray into new conceptualizations but, and importantly, enmeshed by points of conceptual reference which were already constructed as "sensible" and meaningful. The building of meaning and confidence went hand in hand in Harriett's mathematics lessons.

From such perspectives, the SEARCH heuristic adopted the "active" from the notion of constructive inter- and intra-activity of the meaning-making process, and "referencing" from the perspective of reflective, metacognitive engagement. From these elements the AR in SEARCH was distilled as Active Referencing.

## The CH in SEARCH

In his search to explain human development Piaget's guiding question was How does knowledge grow? In his wisdom, he eventually remarked, "The essential functions of intelligence consist in understanding and in inventing. . . . It increasingly appears, in fact, that these two functions are inseparable" (1971, pp. 27-28). His book "To Understand is to Invent" (1973) is testimony to his revelations that invention is construction. Harriett consistently encouraged children to be creative and orchestrated opportunities for creative endeavor during mathematics lessons. She based her motive on the assumption that through creating, children would be constructing mathematics. She dedicated considerable time in her mathematics lessons for children to invent new ways and be uniquely creative in discussing, expressing, and describing their mathematical thinking.

As mentioned above when Norton spontaneously offered to explain his own invented game on the hundreds board, Harriett showed high regard, respect and commitment to fostering the creativity that he had brought to the situation. The following day Mona and Ali were endeavoring to explain to the class how they solved 35 + 11. Mona had written on the chalk board:

$$\begin{array}{r} \underline{35 +} \\ \underline{\ 6 =} \end{array}$$

Mona: 5 and 6 are 11 (She then erased it and rushed to the back of the room to get a hundreds board which she showed to the class.)

I used pennies all the way down to 35. Then I added another 5 and I knew it had to be 40. Then I added another 6 and that went to 46.

Mona's unique way of thinking not only entailed a creative way of mathematically encoding and decoding her calculation but also included working creatively with materials readily available in the environment to assist her in providing coherence to her thoughts; a coherence which she felt confident would be "sensible" to her peers. With Norton's demonstration of his hundreds board game the previous day came an opportunity for other children to consider other possibilities for incorporating the hundreds board. It should be remembered that Harriett had not "officially" used the 100's board at this stage. Her children had ventured of their own accord to find ways to express their mathematical ideas. This also became a sociomathematical norm, namely, that manipulatives or materials available in the classroom could and should be used freely in expressing and defining mathematical propositions.

Part of Harriett's conscious efforts to encourage children's personal mathematical sense making revolved around making the environment conducive to spontaneous and creative activity. Establishing such an environment engendered a patience and tolerance for children's freedom. In order to foster children's capacity to express unique ideas Harriett allowed children to roam the room with apparent autonomous flexibility. Their freedom was based upon a sociocultural norm that during group time, as long as they respected each other's attempts to complete the projects assigned, they were free to sit, cluster, or gather to work wherever they wished. Such a fluid environment often depicted a state of chaos during mathematics sessions. However, as discussed earlier (see Jantsch, 1980), the self-regulation of the class endured in systemic order; "the more freedom in self-organization, the more order" ( p. 40).

The driving force for learning, according to the emerging views of New Science, is to be found not in controlled or imposed structures, but in life's inherent tendency to "create novelty, in the spontaneous emergence of increasing complexity and order" (Capra, 1996, p. 228). Whitehead (1929/1978) believed that the "ultimate principle" of reality itself was a process of becoming and perishing. He contended, in contrast with a Newtonian view of an ultimately atomistic and mechanical reality, that reality was a set of relations. In consonance with Dewey and Piaget, Whitehead thought of "the pupil's mind is a growing organism" and that "the only avenue towards wisdom is by freedom in the presence of knowledge" (1929/1967, p. 30). For him, mathematical ideas "give power to create, to bring into actual existence an infinitude of possibilities. . . . For this reason it is not only good we, as teacher and students, throw 'ideas into every combination possible'; it is essential we do so. For in this 'throwness,' meaning, experience, reality are created" (Doll, 1993, p. 145).

A key element of Whitehead's idea is captured in an essence of creative opportunity; namely, that growth, development, knowledge, and wisdom occur when there is "balance between the creative opportunity freedom can give and the knowledge we acquire from discipline" (Doll, 1993, p. 147).

> *Thus freedom should live its existence in 'the presence of knowledge'. To balance these, Whitehead developed his 'rhythm of education' - romance (play), precision (mastery), and generalization (abstraction). While believing that these three should be integrated continually instead of ordered sequentially, Whitehead also believed that life's natural, developmental rhythms favored a predominance of romance or play of ideas in the elementary and lower high school grades, with the development of precision or mastery starting in the high school years, and abstraction or generalization being focused in the university years. To break away from this general plan, particularly to push precision and mastery before the student is psychologically ready for them, is to go against life's natural rhythm; it is to render the educational experience barren and boring. Here is denial of self-development and the opportunity for each individual to make 'ideas his own.' (Doll, 1993, p. 147)*

In Whitehead's process philosophy we see many attributes of Harriett's approach to mathematics education; firstly, the acknowledgment of freedom as an essential element for learning; second, creativity as the driving force behind knowledge development; third, the importance of developmentally appropriate experiences to meet the needs of the students; fourth, the value of playing, exploring, and negotiating conceptual propositions, and fifth, integration relationships such as partnership, respect, self-regulation, and construction of personal knowledge as interwoven rhythms of the learning experience.

"In this 'ferment', lie the possibilities to be actualized, to be created. The process of education like the process of life must work to order this ferment, not to impose a pre-set and nonmeaningful pattern on it" (Doll, 1993, p. 147). To do the latter is to render the process ineffectual, artificial, and sterile. Harriett's class appeared to be in ferment all the time as was evidenced by the children's seemingly chaotic participation. However, the freedom which permeated the learning was in reflexive relationship with their creative endeavor.

Langer (1997) claims that effective engagement in learning involves both mindful attention and seeking out novelty. From this perspective, Harriett's approach to establishing an effective mathematical learning environment involved more than a mathematical or psychosocial solution. With attention to the emotional, social, physical, and cognitive dimensions of freedom permeating all aspects of the meaning-making process, her children were availed opportunities to engage "play-fully" with their developing ideas; to be creative was part of their emerging sense of what it meant to "do" and "know" mathematics.

Harriett's children needed to play with what they were creating and create with what they playing. With respectful attention to the resolution of their needs, children appeared to become increasingly creative as if liberated from the bonds of conformity, compliance and objective heteronomy that typify "traditional" classroom practice.

Harriett also encouraged children to "make up" their own problems once they had completed their set tasks. The children began slowly with this but gradually started to develop creative ideas enthusiastically. Their attempts to create new problems emerged as a major assessment tool, providing valuable insight into their thinking for Harriett. Also their creative endeavors sparked considerable discussion and mathematical negotiation; novel ideas were constantly examined, refuted and reconsidered. In impromptu and spontaneous situations, children explored, conjectured, hypothesized, tested, and calculated with considerable enthusiasm. They were creatively disposed towards their emerging mathematical ideas.

During one week the children had been exploring a mathematical modeling technique in the genre of "How many ways can you make 28 cents with coins?" As these types of questions were open ended, children knew that several ways were possible. That there can be several answers to a problem was another revealing dimension for the children, of the creative potential in mathematics exploration. This also became a sociomathematical norm, i.e., to look for other possibilities for a solution. At the same time the class was working with their money tasks, Harriett was reading a series of stories by Australian authors, one of which by Mem Fox, "Night Noises" contained a problem about how many people came to the party.

Of his own volition, one of the boys, Jonah used the same mathematical problem-solving technique used in the coins problems to solve the people problem in

"Night Noises." He had initiated his own authentic problem-solving challenge, and in his own unique way had creatively fathomed his way through the requirements of the problem. Jonah announced that he had "worked out" the people problem at home, using dimes and nickels, and was able to clearly demonstrate to the class his approach and solution. "Such intriguing and challenging problems (especially the open ended variety) foster a disposition for searching for creative possibilities. A searching mind and a deliberate willingness to attempt to think of possibilities. . . an acceptance that there must be a range of possibilities. . . and so how many can we consider. . . it's like having a smorgasbord of mathematical ways of thinking to play with. Through creative endeavor, their sense of ownership is fostered" (Field notes, November 1997).

Further observations from my field notes showed the significance of diversity and novelty in Harriett's mathematics program. "The teacher provides a diversity of experiences, like items on a restaurant menu, or dishes on the dinner table. She provides a broad scope of opportunity for participatory engagement. Children are able to partake of a wide range of experiences like group discussion, personal reflection, developmentally appropriate activities, creative expression, hands-on activity, a range of manipulatives, self-initiated challenge, peer challenge, partner interaction, and free investigations; what a flexibly dynamic table of activities and opportunities to explore. This is what construction is - making sense of the possibilities provided. The broader the range of possibilities the more each child comes to realize what possibilities there are. They take from what is provided (family, environment, experience, interaction, etc.) and construct meaningful ideas. With their disposition creatively nurtured i.e., to realize how many different possible ways there are to portray the same idea, and that "my" idea is valued, then the way they construct is injected with a scope of an infinitude of possibilities" (Field notes, November 1997).

Such a comment reflects back to Whitehead's focus on becoming as a purposeful, self-creative process, in which freedom is fundamental, and "creativity is the principal criterion by which what is actual may be distinguished from what is merely abstract and derivative" (Lucas, 1979, p.15). According to Whitehead, the self- educative drive towards the liberation of self-knowledge is the story of creativity and the creativity of the spirit occurs through the dialectic of its self-movement (Rosner & Abt, 1970). What we want, therefore, asserts Doll (1993), is

> *. . . an appreciation of the infinite variety of vivid values achieved by an organism in its proper environment. It is this sense of vivid values - of intellectual variety that moves beyond the technically rational to introduce the artistic, the narrative, the intuitive, and the metaphoric - Whitehead's cosmology [forms] a basis for curricular thought. Developing vivid and diverse values into an integrative and relational frame is what makes Whitehead's curricular thought so post-modern. (pp. 146-147)*

Harriett provided opportunities for children to function creatively in a dynamically integrated environment by grounding their confidence in a freedom and willingness to take risks, to be inventive, and not be fearful of recrimination, shame or blame for making errors. Children were empowered through their creative endeavors to demonstrate in a variety of ways how they had constructed their knowledge. Through the teacher's nurturing and stimulation of creative possibilities, children's thinking was empowered through creative discourse, creative use of materials, and involvement in a continuously evolving variety of learning contexts precipitated by a reflexive and emergent relationship among materials, environment, class members, sociocultural and sociomathematical norms.

Such a focus on evolving and coevolving relationships reflected the self-regulating and self-creative nature of Whitehead's "becoming," and resonated with the principles of complex adaptive systems. Through exploratory experiences which honored inventiveness and open-ended outcomes, children were encouraged to be creative by expressing their own ideas in their own ways. Many unique suggestions were "given air" and while some were rejected and others modified, others were eventually accommodated as viably robust mathematical propositions. As Devlin (1997) asserted, "original thought and the ability to see things in novel ways" (p. 3) are exactly what mathematicians require.

> Harriett: It's all about control. The children have to develop a sense of ownership about their own learning. We are learning ways to solve problems together and respect each other's attempts at thinking. The broader the range of possibilities the broader the child's thinking. They must take control and be confident about taking risks.

The CH in SEARCH originally portrayed a notion of a creativity hierarchy and purported to demonstrate that the construction of knowledge is not only a constructive process but also entails a facet of creativity which differentiates different levels of knowledge construction; the pinnacle being the production and expression of knowledge which is altogether new and revealing and hitherto unexpressed and/or constructed within a given environment or context. The process of knowledge acquisition was seen as being the formulation (in a constructed sense) of a basic idea, for example, an A, B, C pattern repeats. Creativity was then seen as more than constructed reality because creation (creativity) can be of an unreality that exists as a momentary outburst, a fleeting instant of insight, or a impulsive and spontaneous reaction that defies explanation yet, for an instant makes sense or even works. In some sense the creation of knowledge is synonymous with the construction of knowledge but creativity was seen as a process which transcends construction in that it transfigures initial constructed conceptions in a way that a new experience is "had" but nothing necessarily from the experience remains except the previously constructed knowledge, however, the participant or executor is intrinsically better for the experience but is unable to rationalize why nor comprehend the significance or make sense of the revelation imbued within the fleeting moment of creative experience.

Upon considering the linearity of hierarchies and the possible ambiguity of their place in post-modern thinking, I left the development of a creativity hierarchy in obeisance. Nonetheless, inklings of how children construct knowledge in a multitude of ways, through their own creativity, qua unique inventions conceptually specific to their particular frame of reference, lingered in my mind. The present study provided an opportunity for a new interpretation of CH as part of the SEARCH heuristic.

> *A creative paradigm has major implications for education and curriculum. First, the teaching - learning frame switches from a cause - effect one where learning is either a direct result of teaching or teaching is at least in a superior - inferior relationship with learning. The switch is to a mode where teaching becomes ancillary to learning, with learning dominant, due to the individual's self-organizational abilities. Further, in this mode teaching changes its 'modus operandi', from the didactic to the dialogic.* (Doll, 1993, p. 101)

Such was the modus operandi of Harriett. Her focus upon dialogue and communication precipitated a learning community which evolved through creatively thinking about mathematics. The children demonstrated such a capacity by confidently inventing a variety of novel problem-solving strategies and using them in unique ways in their mathematical experiences. Hence the CH in SEARCH was adopted as Creative Heuristics, qua creative problem-solving strategies. In other words, the significance of searching for mathematical meaning relies upon a confidence and disposition to employ a variety of creative strategies. The broader and more complex the strategies the better the potential for making sense of new mathematical propositions.

## SUMMARY

Chapter III has provided a discussion about research conducted in a grade-two classroom which sought to implement the three theoretical perspectives of problem-centered learning, constructivism, and positive discipline. The data from the present study were analyzed and discussed by using three interpretative frameworks relating to the teacher's, the children's, and the researcher's locations in the classroom. Within and between these three frameworks, interconnections and relationships were analyzed and discussed in light of what was required to effectuate a grade-two mathematics education program. The ways in which sociocultural and sociomathematical norms constituted the classroom community and underpinned the evolution of mathematical learning were discussed. A reflexive relationship was shown to exist between the emergence of learning and the emergence of the classroom norms.

The consummation of mathematical meaning-making in a classroom addressing a problem-centered approach to learning, constructivist perspectives, and positive discipline was shown to be underpinned by freedom, democratic child-centered principles, active engagement in developmentally appropriate mathematical problems, and the opportunity to think creatively. The interconnections among all these perspectives were highlighted as being inseparably interwoven. Such an outlook was contrasted against well-entrenched traditional mathematics education programs based on procedural and instrumental knowledge frequently associated with logical positivism and behaviorism.

The analysis and discussion of the mathematics that took place in the grade-two classroom sought to reconstruct what it means to "know" and to "do" mathematics and thus what it means to learn mathematics. Sustaining the pedagogical dimensions of mathematical development was an interrelational perspective of democratic partnership based on (1) replacing competition with cooperation, (2) fostering collaborative cooperation for reaching solutions to social as well as academic problems, (3) respecting each individual's attempt to have an opinion, (4) rejecting coercive authority, and (5) promoting personal agency.

Deriving from these principles and relationships, mathematical learning based on respect (by teacher and students) for student intellectual, social, physical, and emotional development was shown to be akin to the self-regulating and self-creative basis of complex adaptive systems. Such an elaboration of the classroom environment resonated with what is portrayed in the writings of Dewey in that the classroom is a place "where the inhabitants - students and teachers alike - were invited to find personal fulfillment and social well-being in their daily activity, a place where the ultimate test of knowledge was to be its usefulness but where the useful was to include the aesthetic, the contemplative and what some would call the spiritual aspects of human experience" (Jackson, 1990, p. xxxvi).

The implementation of a problem-centered approach to learning mathematics, guided by constructivist principles, and positive discipline was shown to be effective in actively engaging children in mathematical tasks, increasing the range and quality of problems solved, developing the clarity of their mathematical thinking, and extending the diversity of their creative aspirations. The fruits of the children's mathematical endeavors exposed the depth of their conceptual ideas and provided a powerful evaluative insight into their thinking.

Data relating to the teacher's efforts to encourage and orchestrate opportunities for children to be independent and self-regulating mathematical thinkers were discussed. The data indicated that the approach constituted a complex set of emergent and reflexive classroom relationships. Fostering cooperation, freedom, and personal agency generated a positive disposition towards creative thinking, which in turn bolstered children's confidence in taking risks and developing ownership of their ideas. Similarly, by encouraging the children to generate novel suggestions and invent unique ways of explaining their mathematical thinking, a community of active mathematical negotiators was developed, which in turn fortified children's progress towards confidence to be even more creative and more adventurous with justifying and refuting mathematical propositions.

The SEARCH heuristic was formulated through recognizing the reflexive complementarities among three psychopedagogical dimensions, namely, Social Emancipation (SE), Active Referencing (AR), and Creative Heuristics (CH), and was developed as a grounded theory. An elaboration of the SEARCH heuristic was provided to highlight the emergent nature of relationships engendered in the mathematics lessons of the grade-two classroom observed in the present study. SEARCH was given form by the metaphor "learning as a search for meaning," not for an objective reality, but rather a personal and intersubjectively connected one. The metaphor was made more relevant by relating it not only to children seeking meaning in their mathematical endeavors, but also through the teacher's role of seeking to comprehend each child's cognitive, social, emotional, and physical needs in order to make developmentally appropriate decisions about what to include next in the curriculum.

As such, the search for mathematical meaning was portrayed as a complex reflexive relationship founded upon sociocultural and sociomathematical norms. As is the trend in the new generation of mathematicians and scientists who seek to explain complexity theory, the thinking engendered in the SEARCH heuristic "represents a shift from quantity to quality [and] is characteristic of systems thinking" (Capra, 1996, p. 135).

The analyses showed that negotiation of sociocultural and sociomathematical norms must be an ongoing process and it is crucial that students participate as part of the negotiation process. The execution of each mathematical experience was contingent upon children's emerging appreciation of their teacher's and their own roles during mathematics lessons. As classroom norms were negotiated new relationships were established as the class coevolved into new roles with new expectations. As the roles evolved, new norms emerged and so the cyclical and reflexively emergent nature of learning manifested a process of continuous self-regulation and self-organization.

# CHAPTER IV

# CONCLUSIONS AND RECOMMENDATIONS

The focus question of the present study asked: What are the emerging relationships among the sociocultural norms, sociomathematical norms, and instructional practices in a second grade classroom? In Chapter IV, the focus question will be addressed by discussing implications of themes emanating from the previous chapters, and by highlighting limitations of the study. Finally, recommendations for teacher education and future research will be made.

## CONCLUSIONS

Examination and interpretation of the data suggested that there are complex interwoven and interconnected relationships underpinning the implementation of a mathematics program which employs theoretical perspectives advocated by the current mathematics education reform movement. A sense of these relationships is imbued in the inseparability, interconnectedness, and convergence among the three theoretical perspectives of constructivism, problem-centered learning, and positive discipline. In the present study, the emergent nature of sociocultural and sociomathematical norms was constituted by a reflexive complementarity among them and the three theoretical perspectives adopted by the teacher. That is, as each theoretical and practical component emerged, it reflexively constituted other components and different dimensions within one, two, or three of the abiding theoretical perspectives.

The data implied that such a systemic perspective in mathematics education can be fostered through a set of classroom principles far removed from traditional behaviorist and positivist paradigms. Analyses showed that the children in the study were empowered with a disposition for thinking and "doing" mathematics with confidence. Children were able to accommodate a wide range of sociocultural and sociomathematical norms which in turn empowered their mathematical development. Analyses also indicated that the teacher became increasingly positive about her new approach especially in light of the benefits attributed to the children's participation as active mathematical problem solvers. By participating in a child-centered community, children were able to articulate a substantial level of sense of the mathematics with which they were exploring.

Derived from the data, an overriding and unifying relationship of classroom mathematical experiences was portrayed as that of a "search" in which *all* participants struggled to find meaning in *all* they were doing. The metaphor of searching was formulated into an acronym called SEARCH and presented as a grounded theory. SEARCH was suggestive of a pedagogical heuristic and was composed of three components, namely Social Emancipation (SE), Active Referencing (AR), and Creative Heuristics (CH).

Social Emancipation focused importance on the issues of freedom and control during a mathematics lesson. The democratic principles that adhere to social emancipation were promulgated during the mathematics lessons by rejecting theories based on domination, and embracing a vision of power and control based in self-regulation, respect, and partnership.

Active Referencing focused importance on child-centered and developmentally appropriate practice. Student's effective mathematical sense making was portrayed as an active foray into "uncharted mathematical waters," but by being anchored by points of conceptual reference already constructed as "sensible" and meaningful, new ideas could be confidently addressed. The building of meaning and confidence went hand in hand in learning mathematics.

Creative Heuristics focused importance on the significance of searching for meaning with a playful disposition. Exploring unusual propositions and employing a variety of novel strategies to solve problems implied that learning mathematics was a creative endeavor. The broader and more complex the strategies at hand, the better the potential for making sense of new mathematical propositions. Children's thinking was empowered through creative discourse, creative use of materials, and involvement in a continuously evolving variety of learning contexts precipitated by a spontaneously reflexive relationship among materials, environment, class members, sociocultural and sociomathematical norms.

Chapter IV continues with a set of implications derived from the present study, a look at limitations of the study, and recommendations for future research.

## IMPLICATIONS

The study sought to explore the complex relationships among three theoretical perspectives and their classroom implementation. The literature has addressed each theoretical perspective separately and in detail but consideration of the interconnections among the three in mathematics education is still emerging. Two perspectives are prominent in the current mathematics education reform literature, namely constructivism and problem-centered learning. However, the third, positive discipline is overlooked and underestimated in its role in providing coherence and continuity between theoretical and practical standpoints.

The present study provided a theoretically grounded, classroom-based consideration of the relationships among these three theoretical perspectives and the way in which their interconnectedness addresses contemporary mathematics education reform. Students involved in the classroom in the study displayed considerable capacity to participate as effective mathematical problem-solvers because they were immersed in an environment based in encouragement, respect, democratic freedom and partnership. The teacher's theoretical stance on the way children learn and the way a learning environment should be managed underpinned a climate that fostered children's willingness to actively and enthusiastically participate in problem-solving activities which in turn, provided the foundation for the development of substantial mathematical thinking. A reflexive relationship among the principles of constructed knowledge, problem-centered learning, and cooperative and respectful partnership coalesced and manifested as opportunities for valuable mathematical knowledge development.

The basic premise adopted by the teacher in the present study encapsulated constructivist perspectives which claim that knowing is active, personal, and based on previously constructed knowledge. Such a premise rejected the notion of learning mathematics as a transmission of instrumental and procedural rules, and in keeping with New Science and post-modern thinking, the ontological perspective was one that rejected an objective reality. In adopting a constructivist perspective, the teacher subscribed to the point of view that knowledge is not a reflection or representation of reality, but the result of the active construction by the knowing child. Characteristics of the knowing child "form an indiscriminable [sic] part of what is known about the world. Consequently, constructivism discards any claim (including scientific claims) regarding the possibility of a privileged, uniquely true representation of reality. Constructivism therefore legitimizes plurality, diversity, and difference" (Vandestraeten & Biesta, 1998, p.1 ).

The adoption of a problem-centered approach to learning mathematics provided the pedagogical foundation for the classroom mathematics program. Problem-centered learning advocates a shift of emphasis from rote procedures to the development of higher order thinking and acknowledges that favorable conditions for learning exist when a person is faced with a task for which no known procedure is immediately available, that is, when learners find themselves in a problematic situation. In the present study, the teacher sought ways to empower her students as mathematical thinkers by establishing a problem-centered learning environment. A synergy and coalescence of theory and practice was highlighted through the complementary relationship between the construction of meaning and a problem-centered approach to learning mathematics.

The teacher's adoption of a positive discipline approach sought to provide coherence to her philosophical and pedagogical stances by way of democratic and child-centered principles. Through a community based in child-centered practices dialogue was the ultimate principle of learning. Such a community is characterized by the encouragement of democratic, caring, ethical, respectful, self-directing, self-organizing, and socially-relational principles.

## Opportunities for Other Settings and Challenges to Traditions

A range of issues adhered to the classroom approach and mathematics education program implemented by the teacher in the present study. The issues and their implications will be elaborated in the following section with a view to providing a means for reassessing traditional mathematics educational practices, and an avenue for addressing current mathematics reform agendas. The relevance of the issues from the present study engenders a need to reassess the manner in which authority is asserted, power maintained, and control employed in mathematics education. The teacher's role as the "expert" and "font of wisdom from which all knowledge flows" implores reconsideration. Under a constructivist banner, it is impossible to "teach" children mathematics. Hence, the domination of the notion that the teacher's main role is teaching, must give way and be reconstituted by a perspective that focuses more upon children's learning as the primary objective. The teacher's focus moves from a content and subject orientation to a student-centered learning focus.

The teacher's dictum for the present study was "Let go of the control." At no time did this infer an abdication of a teacher's responsibility. On the contrary, it carried with it great responsibility. It meant empowering children with a sense of confidence in their ability to think about mathematics. Letting go of control meant reconstituting and reformulating the teacher's role in the mathematics classroom. Providing meaningful mathematics experiences no longer meant telling children what to do but rather encompassed the facilitation of a community and environment in which a climate of partnership availed opportunities for children to function as active mathematical thinkers rather than passive receptacles into which mathematics is "poured." Children's empowerment became the focus of the mathematics program.

The relevance of the present study in contributing to the development of mathematics education lies in the SEARCH heuristic. As a heuristic, SEARCH implies the implementation of systemic strategies for a reconstitution of mathematics education. The heuristic avails a "simplifying device to facilitate the search" for the reformulation of classroom practice. In order to apply the heuristic in a classroom setting, the basic principle behind the metaphor of "searching for meaning" and an ontological stance that there is no objective reality, require accommodation.

SEARCH implies that the mathematics teacher's role has less to do with being an authoritative transmitter of objectively-set mathematical information and more to do with empowering children in their search to construct viable mathematical meaning. By honoring constructivist principles that infer that mathematical knowledge cannot be transmitted but is personally and actively constructed, the mathematics program which incorporates the SEARCH heuristic would be constituted by a more child-centered and less teacher-directed imperative.

The implication of letting go and relinquishing power, means that teachers would have to consider the implications of a logic of domination manifested through using coercion and competition in mathematics lessons. Similarly, compassion, sensitivity, and receptivity to children's emotional, social, physical, and cognitive needs would need greater consideration. The principles of SEARCH infer a responsiveness to children's development in a holistic sense, and a rejection of a preoccupation with cognitive development. Such considerations give substance and significance to encouraging children's conversation and open communication and are necessary dimensions of the mathematics lesson in order to truly hear the "voices" and needs of the children. Such a view runs counter to many traditional practices in mathematics education that stress keeping control means keeping children quiet while they work.

In order to encourage children to find their "voices," the SEARCH heuristic stresses fostering open communication based in mutual respect, teamwork, dialogic and respectful partnerships. These components stand in stark contrast to traditional approaches that achieve control by keeping students passively silent and insisting that they work alone. Hence, the importance of group work that facilitates children's discussion in the mathematics lesson becomes paramount in the SEARCH heuristic.

As part of group activity, candid and lively conversation is considered a normative classroom expectation. The hermeneutic dialectic principle underpins the reflexive nature of dialogue and meaning making. Children's conversations and interactions would be central in facilitating the construction of mathematical meaning. Without the conversations and the dialogue, qua the "ultimate context" (Rorty, 1990), ultimate mathematical power would continue to reside in the teacher's authority and propensity for dispensing mathematical truths.

The SEARCH heuristic implies that the engagement of children in mathematical thinking involves honoring their attempts to make meaning as they struggle with their mathematical explorations. To this end, providing developmentally appropriate tasks and activities is a key component. If children are to be encouraged to explain and justify mathematical propositions, then appropriately designed and selected activities would be necessary in order to avert excessive and undue frustration in the mathematics-meaning process. The suitability of tasks and problems is derived from the children's activity. Their progress provides the signposts for what should constitute the developmental appropriateness of mathematical challenges. SEARCH implies that, in taking responsibility for solving problems, children will need to use a variety of methods to reach a solution. This in turn infers that, rather than relying on the teacher to provide prescriptive methods of thinking, or supplying the answers to their problems, children are capable of designing and constructing ways of making sense of the mathematics that they explore.

The edict that emerged from the study, that there is "no shame, no blame, only encouragement" during mathematics lessons, has ethical as well as pedagogical implications. Respect and encouragement were primary imperatives for establishing a community of confident and articulate mathematical thinkers. The SEARCH heuristic characterizes the development of mathematical thinking as the negotiation and renegotiation of sociocultural and sociomathematical norms moderated by democratic and emancipatory principles of freedom and autonomy.

Garofalo (1989) suggests, "The nature of the classroom environment in which mathematics is done strongly influences how students view the subject of mathematics, the way they believe mathematics should be done, and what they consider appropriate responses to mathematics questions" (p. 451). In short the type of learning is influenced by the type of learning environment.

The notion of the SEARCH heuristic presents a challenge to many prevailing approaches in mathematics education. A tolerance for functioning far from equilibrium becomes a key component in the mathematics lesson. Principles of complexity theory imply that the unpredictable disorder and creative actions of many individual parts emerge spontaneously as order. Tolerance on the part of teacher, and also the children for working in constant states of disequilibrium constitute a new dimension of controlling the mathematics classroom. The characteristics of self-regulation suggest a new pattern of responsibility and organization on the part of the children. Learning mathematics from this perspective would involve considerable flexibility on the part of all involved to accommodate unpredictable changes, respond spontaneously to contentious claims, justify or challenge unexpected irregularities in mathematical propositions, and be predisposed to creatively articulating mathematical views.

Emphasizing creativity in mathematics is another departure from traditional paradigms especially ones that portray mathematics as a set of immutable rules, sets of prescribed formulas, and rote recipes that need to be memorized. Children's creativity entails a proclivity for experimentation and a disposition for handling uncertainty. To be effective problem solvers and adaptive mathematical thinkers children need to be creatively disposed. The development of such an investigative nature implies affording children opportunities to employ a wide range of thinking strategies. Time and space to contemplate the many different mathematical propositions that emerge from the different and unexpected perspectives generated by their peers becomes important. Fostering a predisposition for creativity suggests that children need opportunities to develop confidence in confronting the irregular, coping with uncertainty, inventing, and sharing their newly-formed ideas publicly. Such a predisposition implies a tolerance for uncertainty. Accordingly, teachers support of children's attempts to delve into uncertainty with confidence constitutes part of the search metaphor in the SEARCH heuristic.

Langer (1997) contends that for some, uncertainty represents an absence of personal control, and for others, uncertainty creates the freedom to discover meaning. "If there are meaningful choices, there is uncertainty. If there is no choice, there is no uncertainty and no opportunity for control" (p. 130). Langer insists that uncertainty and the experience of personal control are inseparable. However, as Langer notes, despite the tendency of uncertainty to enhance creative thinking, students are still taught to view mathematics as an immutable set of facts and unconditional truths.

The SEARCH heuristic stands to contravene many well-entrenched pedagogies perpetuated under the positivist paradigm. The implications of working within a child-centered, activity-based, and self-regulating mathematics program are contradictory to the traditional role and position of the teacher as the dominant mathematics authority. A willingness and ability to foster children's self-regulation during mathematics lessons insinuates a willingness to delegate authority for decision making to the children. In the classroom in the present study, class meetings were the vehicle for initiating child-centered decision making. However, the principle was effectively reinforced during mathematics lessons, and vice versa. It is in this manner that the relationship of pedagogical interconnectedness flows from within and between classroom activity. The teacher's abiding theoretical perspectives cannot be separated as a stance relevant to mathematics education alone.

## Second Thoughts

The teacher in the present study often commented that she felt she had created a monster in "setting kids up to be on their own and at the mercy of the traditional mathematics system" after they left her class. She said that she often felt a sense of panic knowing how different her mathematics program was compared to most traditional approaches.

> Harriett: As you let go and give [authority] to the kids, well, at least [like] I did, I found myself kind of panicking at times, thinking sometimes I'd created this monster, and trying to decide what is the fine line between just having chaos [and control] And I think the thing that will pull it back together [each time], like I told [the children] the other day, "I know that all you hear from me sometimes is 'respect,' 'respect,' 'respect,' and what is respect?" [ And I sometimes wonder] are they just [feigning respect]. Do you want them to respect because they want to respect, and not because they feel like they're being forced [or cajoled] into it? And what is that fine line there, where, you know, this is something that is expected in [our classroom] that we will respect each other, and when they move on, [what happens to their sense of respect then?]

Dewey wrestled with a similar problem in the progressive education movement. His "Philosophy of Education" was an attempt to explain why progressive education needed to be more than just anti-traditional, and why progressive education had to have its own foundations and framework. In contrasting the "too romantic view" of overstated progressive education perspectives against the "too rigid view" of the established traditional perspective, he wrote, "This alternative is not just a middle course or compromise between the two procedures. It is something radically different from either" (1934/1964, p. 8). Dewey (1922) encouraged that "There must be change in objective arrangements and institutions. We must work on the environment not merely on the hearts of men. To think otherwise is to suppose that flowers can be raised in a desert or motor cars run in a jungle" (p. 22).

## The Intervention - Invention Tension

In the call for contemporary mathematics education reform there is demand for major change, in fact, a paradigm shift is clearly intimated. As was evidenced in the classroom mathematics program of the present study, major change was possible based on an appreciation of systemic relationships. The three theoretical perspectives adopted required accommodation of the expectations of the mathematics education reform agendas.

In endeavoring to address such a diversity of issues, an undercurrent of tension associated with making the transition from traditional mathematics education to a contemporary reform modus operandi emerged as part of the present study. Continued criticism is leveled at traditional mathematics education approaches for being overly interventionist and reflective of a logic of domination associated with positivist, Euro-centered, white male, and behaviorist paradigms. Teacher's "lust for intervention"

(Cockcroft, 1982) has been well documented and criticized for its anti-reform posture. While intervention stands as the trademark of traditional behaviorist pedagogical approaches, invention has become the trademark of the reform movement. New Science, post-modernism, problem-centered learning, positive discipline, and constructivist perspectives all have a stake in emphasizing the importance of the construction or invention of knowledge.

The extreme extension of the intervention characteristic portrays a teacher who does everything while the child does nothing. Obviously, this is an untenable posture in the teaching profession. Similarly, the extreme extension of the invention characteristic portrays a child who does everything while the teacher does nothing. This also is an untenable learning context for educators. As teachers adhere to variations and modulations of diverse theoretical perspectives they position themselves somewhere within the extremes of the two characteristics of invention and intervention. Differing conceptions of both paradigms make it impossible to definitively locate anyone in any one position. From this modulation of theoretical location a tension of adequacy emerges.

The two conditions of intervention and invention are basically philosophically opposed. As the teacher in the study sought to understand the implications of the reform movement, a tension derived from "shedding old habits" began to surface as part of the paradigmatic shift. As old habits die hard, other teachers will not simply fall into the reform paradigm. In fact, as there is no final resting place in the new paradigm to fall upon, accommodation of the new paradigm will take different forms and manifest in various guises. However, as teachers struggle to embrace the theoretical and philosophical dimensions of the new paradigm there will be confusion, doubt, skepticism, and tension as to what constitutes an adequate "embrace." Hence, a tension in mathematics education reform is beckoning from the post-modern horizons of educational transition.

*Developing the right amount of "essential tension" is the art I believe all curricularists, teachers, and learners need to develop - not to mention that special class: world leaders. This is an art born not of faith in the rightness of our ideologies but our ability to be playful with serious commitments. Such a paradoxical blending becomes key, if we are to make our future age better, not poorer, than the one in which we now live. In this frame curriculum becomes process, learning and understanding come through dialogue and reflection. Learning and understanding are made (not transmitted) as we dialogue with others and reflect on what we and they have said - as we "negotiate passages" between ourselves and others, between ourselves and our texts. (Doll, 1993, p. 156)*

Curriculum then becomes a process rather than a product. As Rorty (1980) has pointed out, no one discipline can act as a foundation for all learning. Nor is there one special methodology, scientific or otherwise, in which curriculum, learning or pedagogy can be packaged. Curriculum then, in a post-modern framework is not conceivable as a package; it is a process, dialectical and dialogical, transformative and transactional, reflective and responsive, and based in the particular peculiarities of each situation.

The SEARCH heuristic is a process device, qua strategy couched in a theory of systemic relationship relating cooperation, dignity, democracy, partnership, child-centeredness, dialogue, integrity, respect, interconnectedness, and self-regulation to a set of values and guidelines that might be useful for assisting other teachers as they address the ramifications of meeting the current mathematics education reform agenda. As such, SEARCH is an ecological heuristic. "The promise I see in an ecological model [qua heuristic] comes from the fact that it starts from a presumption of relationship in whose terms the analogical possibilities of reciprocity, respect - and friendship - between knowing other people . . . are clearly visible" (Code, 1991, p. 273). A number of educators are beginning to encourage the idea of relationships and relations extending beyond our personal selves to include the ecosystem. The SEARCH heuristic takes a step towards establishing a cosmic and interrelational consciousness by encouraging a closer look at one of the most isolating experiences on the face of the planet, namely mathematics education.

As a final word from the children, the following episode elucidates how the intervention - invention tension in the classroom of the present study was a constant challenge for the teacher.

Megsie and Ned were asked to present their solution to the following problem: "Tim had 17 nuts. He ate 7. Then he shared the rest equally with Sam. How many nuts did Sam get?"

Ned: (writes on the chalkboard as Megsie reads the problem)

$17 - 7 - 5 = 5$

Blinky: (comes to the chalkboard and writes) $17 - 7 = 10$ (but suddenly notices that, as his partner has not verified his work, that the solution was not complete, and rushes back to his partner).

Mona: (comments as she approaches the front of the room with Ali) We did... (and writes on the chalkboard...)

$17 - 7 = 10$ $10 - 5 = 5$ (and circles the last 5).
(She then writes underneath)

$$\begin{array}{r} \underline{17\ -} \\ \underline{7\ =} \\ 10 \end{array} \qquad \begin{array}{r} \underline{10\ -} \\ \underline{5} \\ 5 \end{array}$$

(Mona continues) But this is the long way. There is a short way. (and writes on the board)

$$\begin{array}{r} \underline{17\ -} \\ \underline{10\ -} \\ \underline{5\ =} \\ 5 \end{array}$$

Meanwhile Blinky is calling out and returns to the chalk board to point at the last 5 of Mona's work.

Blinky: I forgot to do that part!

Asha: (After being asked to share her solution, writes on the board) $17 - 7 =$ (hesitates, then explains) If Tim takes away 7 he'd only have 1 left. Then he'd give the one left to his friend, so he'd have 0 left.

Teacher: Do you agree or disagree with Asha, class? (Several children call out "We disagree!")

Teacher: OK. I know and can see that some of you disagree but can you explain Asha's thinking to explain what she has done?

Mona volunteered to explain Asha's thinking and began to deconstruct what she thought Asha had done. Meanwhile Asha's face had dropped and she eventually, slowly and despondently took some chalk and ruled a large X through her work, then sadly returned to her seat as Mona completed her explanation. Soon after, the class had to go to Physical Education, and as Asha left the room, she dejectedly said to her teacher, "I still think the answer is zero." The teacher was challenged as to how to meet Asha's apparent need to construct units of ten knowing that the rest of the class had constructed the concept and were moving on. How long would be adequate time to allow Asha to construct the concept? What happens if she doesn't construct it? What experiences would be appropriate to help Asha ? Such issues and challenges will underscore curriculum and pedagogical decisions in the new paradigm of mathematics education. The intervention-invention tension amongst teachers will not be easily resolved.

In exploring current reform perspectives of mathematics education, the present study sought to find ways to redress the hegemony of Euro-centered classroom pedagogy which emphasizes procedural and instrumental knowledge (Skemp, 1978) and promulgates a logic of domination (Fleener & Laird, 1997). In keeping with Linda Darling-Hammond's message in her presidential address at the 1996 AERA Annual Meeting, that "the problem of the next century will be the advancement of teaching . . . and its resolution will depend upon our ability to develop knowledge for a very different kind of teaching than what has been the norm for most of this century," the present study sought to explore the relationships embedded in applying the theoretical perspectives of constructivism, problem-centered learning, and positive discipline in the natural setting of a grade-two classroom. While the SEARCH heuristic presents one way to address current mathematics education reform, implicit in the implementation of such a broad-based set of theoretical perspectives is a set of broad-based pragmatic issues. Some of the perceived issues are discussed in the next section.

## LIMITATIONS

While the present study was conducted in a grade-two classroom and is limited to an interpretation based on the specific context of that situation, the findings might prove beneficial towards guiding other classroom attempts to accommodate theoretical perspectives engendered in current mathematics education reform agendas. Nevertheless, it is acknowledged that the findings of the present study need to be viewed with respect to the following limitations.

1) The fact that the class teacher had twenty-three years experience with many in early childhood education may have impacted upon the way in which the study unfolded. While it is conceivable that other teachers with that many years of experience would prove intransigent to change, the teacher in this study showed a commitment and flexible predisposition towards self-improvement. Consequently, such a transformation as was occasioned by the teacher in this study might be out of reach for teachers who are not similarly predisposed or experienced.

2) In the present study, the teacher's years of experience could also have combined with expertise in whole language to provide her with insights that transcend those of inexperienced and generalist classroom teachers.

3) Having two university professors assisting in the daily programming and evaluation would have had a consequential bearing on the implementation and outcomes of the mathematics program. The fact that the teacher was able to continue showing extraordinary levels of patience. tolerance. and calm during days of chaos and unpredictability may have been partly due to the regular support and reassurance provided by her research associates.

4) The social predilection of the class might prove significant in different situations when addressing the SEARCH principles. It should be noted that the cohort from the first year had a more difficult time adapting to the approach engendered in the teacher's new theoretical perspectives. Whereas the first cohort displayed considerable tension and social unease especially evident in their classroom meetings and mathematics lessons, by comparison the second cohort settled into their mathematics education schedule with consummate ease. This could have been a reflection of the teacher's enlightened position after one year of experimenting with her role in her new paradigm.

On the other hand, the second cohort may have just been a more socially adept group. However, it is coincidental that the first cohort had "math partners" imposed upon them as an essential part of the other study running concurrently in the classroom with the present one. Even though the teacher endeavored to reach democratic agreement amongst the first cohort as to which partners would prove the best combinations, there were regular outbursts of tension and social upheaval during mathematics lessons. In the second year however, the idea of "math partners" was dispensed with and children were allowed to choose their own partner, small group, or take the opportunity to work independently should they desire. It was noted how much more cooperation, focus, and social cohesion emerged from the second cohort during their mathematics lessons.

Conceivably, the imposition of "math partners" ran counter to and caused friction with the ideals of the democratic principles underpinning the classroom sociocultural norms. In fact, children in the second cohort occasionally commented on the inconsistency of enforcing math partners with the integrity of the decision-making process with which they were entrusted.

5) Class size is conceivably a factor which impacted upon the manner in which the classroom functioned. With a few children away on a regular basis, the class comprised 18 or fewer children. Not every teacher will have the opportunity to work with such a small group. The small group size may have contributed to the cohesion of the group and thus, the success of the project. Larger-sized classes might generate a different dynamic when implementing such a child-centered approach, especially if all children are being encouraged through a dialogic community to find their "voice." The level of demand emanating from a larger group of children could produce a different dynamic in accommodating a wider range of needs. For example, the simple request for children to return to their seats will be exponentially compounded in a large class.

6) Not every teacher will feel immediately comfortable or secure in the chaos and freedom of movement that ensues from such an approach. A flexible attitude to participatory and interactive behavior will be a challenge to many teacher's concept of classroom management. The adoption of a SEARCH heuristic for mathematics

lessons not only relies upon a tolerance for interactive and cooperative participation but also depends upon recognition of its reflexive and interdependent relationship with day-long classroom norms and regimens.

7) The use of a positive discipline approach will not work if it is employed just during mathematics lessons. It is to be seen as an essential component of the entire classroom day. As it forms and informs the general identity and conduct of the classroom it reconstitutes mathematical experiences. It was suggested to the teacher in the present study that she should teach mathematics to all the grade-two classes at her school. However, she declined emphasizing that her approach to mathematics was predicated upon a classroom climate of positive discipline, and it was within that climate that classroom norms were generated which in turn established the foundations of the mathematics lessons.

8) As a consequence of the preceding limitation, mathematics teachers in settings such as high schools, who only see their students for one-hour class periods are encumbered by a different set of dynamics. Whereas the present study suggests that the SEARCH heuristic was a tenable characteristic in a grade-two classroom setting, its translation into shorter class timeframes other than whole-day classes will give rise to a different set of challenges.

9) Encouraging intrinsic motivation in the place of extrinsic reward will prove a major challenge to the discipline policy of most teachers. A preoccupation with competition is unquestionably a part of our present-day pedagogy and cosmology. As the teacher in the present study repeatedly said, she wanted to "replace competition with cooperation and blame with encouragement." The translation of such a

philosophical stance in pedagogical terms will prove challenging for those teachers entrenched in a competition and rewards syndrome. The promulgation of an extrinsic reward system will run counter to the SEARCH principles.

10) Not every teacher will feel secure in his or her knowledge of mathematics for making decisions on how to respond to children's emerging conceptualizations. Knowing with what and how to respond to children's emerging mathematical constructions requires a sound knowledge of mathematics. In order to make appropriate decisions about child-centered and developmentally-appropriate experiences, the teacher needs to be "one step ahead" of each child's emerging ideas. Without a broad knowledge of mathematics, some teachers will feel inadequate in deciding how to follow and facilitate children's unfolding mathematical knowledge.

11) The teacher in the present study had forged a highly respected position as a formidable teacher within the school faculty and local family community after years of advocacy and success with whole language reform. It was with a sound reputation that she embarked upon her mathematics reform process. Not every teacher will commence from such an advantaged position and could consequently encounter challenges from parents who demand a drill and practice regimen for their children. With the SEARCH heuristic implying a dramatic departure from traditional mathematics education, teachers who seek to employ it could attract a range of challenging responses from parents and faculty unsympathetic to its principles and approach.

Such challenges will need to be met by an informed confidence if the transformation is to proceed. The transformative process for many teachers will entail mustering considerable knowledge and courage. The question is however, whether teachers are prepared to wade into such treacherous waters when there are much easier ways to execute a mathematics lesson.

## RECOMMENDATIONS

### Teacher Education

Current teacher education programs might consider the SEARCH heuristic. In doing so, attention to an integrated and relational perspective of what constitutes mathematics education would be required as the foundation to learning mathematics. While it is clear that broadened views of mathematics education which encompass problem-centered learning and constructivist perspectives are replete in the teacher education literature, more attention needs to be focused on the implications of overlooking and underemphasizing classroom management based in democratic and positive discipline principles. The complementarity engendered in the synergistic coalescence of three theoretical aspects of the SEARCH heuristic implores a balanced consideration in order to meet the reform agenda's current demands.

The dictum for young students of mathematics, "students construct not absorb mathematics" has powerfully salient implications about what it means to teach mathematics. Consequently, teacher education programs will need to be couched in contexts that adhere to the same principles they espouse. Exemplary classroom management and teaching in university mathematics education classes will need to be provided for teacher students to experience and develop a sense of the principles involved. Too many university professors are criticized too often for not teaching the way they want their students to teach. Too many practicum experiences convey contradictory messages through a lack of understanding in the workplace. The mathematics education reform messages conveyed during university classes need to be reinforced in the workplace. To achieve this, a major professional development program in mathematics education is implied. Would established teachers entrenched in traditional methods be amenable to this? Would governments consider funding such a move?

Such a formidable challenge requires a formidable starting point. To redress the potentially debilitating experiences for both teacher and student, predicated by prevailing traditional mainstream agencies of cognition, a new paradigm of mathematics education based upon relationships and the systemic nature of learning is required to accommodate changes implied in New Science and post-modern perspectives. The SEARCH heuristic is a strategy that avails such a viewpoint from which to continue the ongoing conversation of mathematics education reform.

## Future Research Directions

Because of the complexity of the relationships associated with mathematics education promulgated by current reform agendas, several directions for future research have emerged from the present study. First, further research into constructivism as a pedagogical orientation embedded in an ethical or political framework is required. Considerable literature points to the limitations of constructivism as an epistemological perspective. Its translation into classroom practice leaves many pragmatic questions unanswered. The extent to which constructivist perspectives can be embraced, distilled, modified, or adapted in mathematics education is still to be clearly articulated. The relationship between constructivism and practical classroom methodology needs to be further researched.

With the primary aim for every teacher to promote the growth of students as "competent" people, a humanistic dimension becomes part of the overall mathematics education project. Getting past the sloganized attempts to make our schools look democratic and egalitarian, when in fact they are continually struggling for tighter control, means instigating research that examines the agency of adult authority and control in the mathematics education classroom. Further research is required to understand what is entailed in teachers relinquishing control and fostering children's self-regulation in the mathematics classroom.

As New Science and post-modern perspectives emerge, research will be required to reflect their impact in education. Students are growing up in an era of unprecedented mathematics development accompanied by major transformations in social, economic, and political paradigms. The extent to which students cope with such changes can be reflected through research in mathematics education. One area in particular that will help with understanding how students learn to cope with constant change and handle spontaneity, irregularity, and unpredictability in their world is creative thinking, qua seeing things from different perspectives. To be successful in mathematics in the future, students will need to have a predisposition towards creative thinking. Further research is needed in determining the role creative thinking plays in learning mathematics and its contribution to the way in which traditional approaches to mathematics education might be reformulated in the classroom.

The integration of social, emotional, cognitive, and physical needs of young children learning mathematics also needs further research The complex relationships that interweave all the human domains together impacts upon all learning. How these relationships interconnect and the ramifications for classroom practices need further distillation. What it means to view and do mathematics as an integrated and holistic experience rather than merely a cognitive endeavor, requires further examination especially in early childhood where the foundations of mathematical disposition are by and large, established for life.

# REFERENCES


Adler, J. (1998). Alive and well: A review of reviews of mathematics education research in Australasia. Mathematics Education Research Journal, 10, (1), 75-79.

Airasian, P. W., & Walsh, M. E. (1997). Constructivist cautions. Phi Delta Kappan, 78, (6), 444-449.

Angell, A. V. (1991). Democratic climates in elementary classrooms: A review of theory and research. Theory and Research in Social Education, 19, 241-266.

Arthur, L., Beecher, B., Dockett, S., Farmer, S., & Richards, E. (1993). Programming and planning in early childhood settings. Sydney: Harcourt Brace Janovich.

Audi, R. (Ed.). (1995). The Cambridge dictionary of philosophy. Cambridge: Cambridge University Press.

Australian Education Council, (1991). A national statement on mathematics for Australian schools. Canberra: Curriculum Corporation.

Baier, A. (1985). Postures of the mind: Essays on mind and morals. Minneapolis: University of Minnesota.

Bakhurst, D. (1988). Activity, consciousness, and communication. Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 10, 31-39.

Battistino, R. M. (1985). Public schooling and the education of democratic citizens. Jackson: University Press of Mississippi.

Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws & T. J. Cooney (Eds.), Effective mathematics teaching (pp. 27-41). Reston, VA: National Council of Teachers of Mathematics.

Bauersfeld, H. (1995). "Language games" in the mathematics classroom: Their function and their effects. In P. Cobb & H. Bauersfeld (Eds.), The emergence of mathematical meaning (pp. 271-291). Hillsdale, NJ: Lawrence Erlbaum.

Bauersfeld, H. (1996). Social constructivism, classroom cultures, and other concepts - what can they mean for teachers? Keynote address: 18th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Tallahassee, FL: Florida State University.

Berger, P. L., & Luckman, T. (1966). The social construction of reality: A treatise in the sociology of knowledge. Garden City, NY: Doubleday.

Berman, S. (1990). The real ropes course: The development of social consciousness. ESR Journal, 1-18.

Betts, F. (1992). How systems thinking applies to education. Educational Leadership, 50 (3), 38-41.

Bishop, A. J. (1988). Mathematical enculturation: A cultural perspective on mathematics education. Dordrecht: Kluwer.

Bodin, A. (1993). What does to assess mean? In M. Niss (Ed.), Investigations into assessment in mathematics education: An ICMI study. Dordrecht: Kluwer.

Bogdan, R. C., & Biklen, S. K. (1992). Qualitative research for education: An introduction to theory and methods. Needham Heights, MA: Allyn and Bacon.

Briggs, J., & Peat, F. D. (1989). Turbulent mirror: An illustrated guide to chaos theory and science of wholeness. New York: Harper and Row.

Brown, S. I. & Walter, M. I. (1993). Problem posing: Reflections and applications. Hillsdale, NJ: Lawrence Erlbaum.

Bruner, J. S.(1996). The culture of education. Cambridge, MA: Harvard University Press.

Burton, L. (Ed.). (1994). Gender and Mathematics. London: Cassell.

Capra, F. (1996). The web of life: A new scientific understanding of living systems. New York: Doubleday.

Cassidy, D. C. (1992, May). Heisenberg, uncertainty and the quantum revolution. Scientific American, 106-112.

Child Development Project. (1991). Start the year. Unpublished manuscript. San Ramon, CA: Developmental Studies Center.

Christensen, L. (1994, Autumn). Building community from chaos. Rethinking Schools, 1, 14-17.

Clay, M. M. (1991). Becoming literate: The construction of inner control. New Zealand: Heinemann.

Cobb, P. (1989). Experiential, cognitive, and anthropological perspectives in mathematics education. For the Learning of Mathematics, 9, (2), 32-42.

Cobb, P. (1990). Multiple perspectives. In L.P. Steffe, & T. Wood (Eds.), Transforming children's mathematics education: International perspectives (pp. 200-215). Hillsdale NJ: Lawrence Erlbaum Associates.

Cobb, P. (1995). Mathematical learning and small-group interaction: Four case studies. In P. Cobb, & H. Bauersfeld (Eds.), The emergence of mathematical meaning (pp. 25-129). Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P., Jaworski, B., & Presmeg, N. (1996). Emergent and sociocultural views of mathematical activity. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning. New Jersey: Lawrence Erlbaum.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. Journal for Research in Mathematics Education, 14, 83-94.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. Davis, C. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education Monograph Series, Number 4 (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Wood, T., Yackel, E., & Wheatley, G. H. (1993). Introduction: Background of the research. In T. Wood, P. Cobb, E. Yackel, & D. Dillon (Eds.), Rethinking elementary school mathematics: Insights and issues. Journal for Research in Mathematics Education Monograph Series, Number 6 (pp. 1-4). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G., Trigatti, B., & Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics project. Journal for Research in Mathematics Education, 22, 3-29.

Cobb, P., & Bauersfeld, H. (1995). The emergence of mathematical meaning: Interaction in classroom cultures. Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P., & Yackel, E. (1995). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. In D. T. Owens, M. K. Reed & G. M. Millsaps (Eds.), Proceedings of the 17th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, (pp. 3-29). Columbus, OH: Ohio State University

Cockcroft, W. (Chair), (1982). Mathematics counts: Report of the committee of inquiry into the teaching of mathematics. London: Her Majesty's Stationary Office.

Code, L. (1991). What can she know? Feminist theory and the construction of knowledge. London: Cornell University Press.

Collins, P. H. (1991). Black feminist thought: Knowledge, consciousness, and the politics of empowerment. New York: Routledge.

Cooney, T. A. (1994). Research and teacher education: In search of common ground. Journal for Research in Mathematics Education, 25, 608-636.

Crawford, K. (1996). Cultural processes and learning: Expectations, actions, and outcomes. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 131-148). New Jersey: Lawrence Erlbaum.

Csikszentmihalyi, M. (1995). Education for the twenty-first century. Daedalus, 124, 107-108.

Damarin, S. K. (1995). Fairness in dealing: Diversity, psychology and mathematics education. In D. T. Owens, M. K. Reed, & G. M. Millsaps (Eds.), Proceedings of the 17th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 30-53). Columbus, OH: Ohio State University.

D'Amico, J. (1980, October). Reviving student participation. Educational Leadership, 44-46.

Davis, R. B. (1992). Reflections on where mathematics education now stands and on where it may be going. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 724-734). New York: Macmillan.

Davis, R. B. (1996). Cognition, mathematics, and education. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 285-299). New Jersey: Lawrence Erlbaum.

Davydov, V. V. (1988). Problems of developmental teaching (Part I). Soviet Education, 30, (8), 6-97.

De Lange, J. (1995). Assessment: No change without problems. In T. A Romberg (Ed.), Reform in school mathematics and authentic assessment (pp. 87-172). New York: State University of New York Press.

Devlin, K. J. (1997, December). Reduce skills teaching in the math class. FOCUS. (Editorial). (pp. 1-2). Washington, DC: Mathematical Association of America.

DeVries, R., & Kohlberg, L. (1990). Constructivist early education: Overview and comparison with other programs. Washington, DC: National Association for the Education of Young Children.

DeVries, R., & Zan, B. (1994). Moral classrooms, moral children: Creating a constructivist atmosphere in early education. New York: Teachers College Press.

Dewey, J. (1922). Human nature and conduct: An introduction to social psychology. New York: Modern Library.

Dewey, J. (1934/1964). Need for a philosophy of education. In R. D. Archambault (Ed.), John Dewey on education: Selected writings (pp. 3-14). New York: Random House.

Dewey, J. (1938/1963). Experience and education. New York: Collier.

Dewey, J. (1948/1957). Reconstruction in philosophy. Boston: Beacon Press.

Dewey, J. (1956/1990). The school and society and The child and the curriculum. Chicago: The University of Chicago Press.

Doll, W. E. (1993). A post-modern perspective on curriculum. New York: Teachers College Press.

Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 39-48). New York: Macmillan.

Duckworth, E. (1987). The having of wonderful ideas and other essays on teaching and learning. Columbia University, NY: Teachers College Press.

Dumaine, B. (1991, June). The bureaucracy busters. Fortune, 37-50.

Eisenhart, M. A. (1988). The ethnographic research tradition and mathematics education research. Journal for Research in Mathematics Education, 19, 99-114.

Elkind, D. (1998). School and family in the postmodern world. In E. N. Junn & C. J. Boyatzis (Eds.), Child growth and development 98/99. Guilford, CT: Dushkin/McGraw Hill.

Ernest, P. (1991). The philosophy of mathematics education. London: Falmer Press.

Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 335-350). New Jersey: Lawrence Erlbaum.

Favre, A., Guitton, H., Guitton, J., Lichnerowicz, A., & Wolff, E. (1995). Chaos and determinism: Turbulence as a paradigm for complex systems converging toward final states. (B. E. Schwarzbach, Trans.). Baltimore, MA: Johns Hopkins.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. Journal for Research in Mathematics Education, 27, 403-434.

Fleener, M. J., & Laird, S. (1997, October). Logic of domination and the mathematics curriculum: Postmodern process for reconstructing education. Paper presented at the annual meeting of the Conference on Curriculum Theory - sponsored by the Journal of Curriculum Theory. Indiana University, Bloomington, Indiana.

Fleener, M. J., & Fry, P. G. (1998). Adaptive teacher beliefs for continued professional growth during postmodern transitions. In D. J. McIntyre & D. M. Byrd (Eds.), Strategies for career-long teacher education. Teacher Education Yearbook VI. (pp. 154-167). Thousand Oaks, CA: Sage.

Fleener, M. J., & Rodgers, D. (in press). A systems approach to understanding: Transformation in learning communities. The Journal of Thought.

Forman, E. A. (1996). Learning mathematics as a participation in classroom practice: Implications of sociocultural theory for educational reform. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 115-130). New Jersey: Lawrence Erlbaum Associates.

Freire, P. (1985). The politics of education: Culture, power and liberation. South Hadley, MA: Bergin and Garvey.

Freudenthal, H. (1991). Revisiting mathematics education. Dordrecht: Kluwer.

Frost, R. (1969). Ten mills. V. Pertinax. The secret sits. In E. Lathem (Ed.), The poetry of Robert Frost (pp. 308, 362). New York: Holt, Rinehart & Watson.

Gall, M. D., Borg, W. R., & Gall. J. P. (1996). Educational research: An introduction. New York: Longman.

Garofalo, J. (1989). Beliefs, responses, and mathematics education: Observation from the back of the classroom. School Science and Mathematics, 89, 451-455.

Garcia, R. (1992). The structure of knowledge and the knowledge of structure. In H. Beilin, & P. Pufall (Eds.), Piaget's theory: Prospects and possibilities (pp. 21-38). Hillsdale, NJ: Lawrence Erlbaum Associates.

Geertz, C. (1973). The interpretation of cultures. New York: Basic Books.

Geoghegan, N. (1993). Possible effects of early childhood music on mathematical achievement. Masters dissertation. Sydney: Macquarie University.

Geoghegan, N. (1996). Emotional issues in mathematics education: Reflexivity in knowledge - a monanthous phenomenon. Paper presented at the Annual Meeting of the International Group for the Psychology of Mathematics Education, Social Aspects of Mathematics Education Working Group, Valencia, Spain.

Geoghegan, N., Reynolds, A., & Lillard, E. (1997). Problem-centered learning and early childhood mathematics. In J. A. Dossey, J. O. Swafford, M. Parmantie, & A. E. Dossey (Eds.), Proceedings of the 19th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 235-240). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Geoghegan, N., Reynolds, A., & Lillard, E. (1998, November). Creative thinking: A mental fund and fundamental of mathematics education. Paper presented at the 20th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Raleigh, NC: North Carolina State University.

Gilligan, C. (1988). In a different voice: Psychological theory and women's development. Cambridge, MA: Harvard University Press.

Glasersfeld, E. von. (1989). Constructivism in education. In T. Husen, & N. Postlethwaite (Eds.), International encyclopedia of education (supplementary vol., pp. 162-163). Oxford: Pergamon.

Glasersfeld, E. von (1995). Radical constructivism. London: Falmer Press.

Glasersfeld, E. von (1996a). Aspects of radical constructivism and its educational recommendations. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 307-314). New Jersey: Lawrence Erlbaum Associates.

Glasersfeld, E. von (1996b). Footnotes to "The many faces of constructivism." Educational Researcher, 25, (6), 19.

Glasser, W. (1986). Control theory in the classroom. New York: Harper & Row.

Gleick, J. (1987). Chaos: Making a New Science. New York: Penguin.

Goldin, G. A. (1996). Theory of mathematics education: The contributions of constructivism. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning. New Jersey: Lawrence Erlbaum.

Gordon, J. J. (1961). Synectics: The development of creative capacity. New York: Harper & Row.

Gravemeijer, K., McClain, K., & Stephan, M. (1998). Supporting students' construction of increasingly sophisticated ways of reasoning through problem solving. In A. Olivier & K. Newstead (Eds.), Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, (1) (pp. 194-209). Stellenbosch, South Africa.

Greeno, J. G. (1997). Participation as fundamental in learning mathematics. In J. A. Dossey, J. O. Swafford, M. Parmantie, & A. E. Dossey (Eds.), Proceedings of the 19th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 1-14). Columbus, OH: ERIC Clearinghouse for Science, Mathematica, and Environmental Education.

Greer, B. (1996). Theories of mathematics education: The role of cognitive analyses. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 179-196). New Jersey: Lawrence Erlbaum Associates.

Gregg, J. (1993, April). The interactive constitution of competence in the school mathematics tradition. Paper presented at the annual meeting of the American Educational Research Association, Atlanta.

Groves, S. (1998). Supporting students' reasoning through problem solving - implications for classroom practice: A reaction to Gravemeijer, McClain and Stephan. In A. Olivier & K. Newstead (Eds.), Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, (1) (pp. 210-217). Stellenbosch, South Africa.

Guba, E. G., & Lincoln, Y. S. (1989). Fourth Generation Evaluation. Newbury Park, CA: Sage.

Habermas, J. (1987). The philosophical discourse of modernity: Twelve lectures (F. Lawrence, Trans.). Cambridge: Polity

Haken, H. (1996). Principles of brain functioning: A synergetic approach to brain activity, behavior, and cognition. New York: Springer-Verlag.

Hargreaves, A. (1994). Changing teachers, changing times: Teacher's work and culture in the postmodern age. New York: teachers College Press.

Heidegger, M. (1962). Being and time. New York: Harper & Row.

Hersh, R. (1979). Some proposals for reviving the philosophy of mathematics. Advances in Mathematics, 31, 31-50.

Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Ed.), New directions in the philosophy of mathematics (pp. 9-28). Boston: Birkhauser.

Hersh, R. (1993). Humanistic mathematics and the real world. In A. M. White (Ed.), Essays in humanistic mathematics (pp. 15-18). Washington DC: Mathematical Association of America.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 65-97). New York: Macmillan.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. Educational Researcher, 25, (4), 12-21.

Howard, R. (1990, September). Values make the company: An interview with Robert Haas. Harvard Business Review, 133-144.

Jackson, P. W. (1990). Introduction. In J. Dewey, The school and society and The child and the curriculum. Chicago: The University of Chicago Press.

Jantsch, E. (1980). The self-organizing universe. Oxford: Pergamon Press.

Kahn, R. E. (1995, September). Chaos and complexity theory, dynamic systems models and educational research. Paper presented at the European Conference on Educational Research, Bath, England.

Kamii, C. (1985, September). Leading primary education toward excellence: Beyond worksheets and drill. Young Children, 3-9.

Kamii, C. (1989). Young children continue to reinvent arithmetic. New York: Teachers College Press.

Kamii, C. (1991). Toward autonomy: The importance of critical thinking and choice making. School Psychology Review, 20, 382-388.

Kieran, C. (1994). Doing and seeing things differently. Journal for Research in Mathematics Education, 25, 583-607.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 3-38). New York: Macmillan.

Kincheloe, J. L., & McLaren, P. L. (1994). Rethinking critical theory and qualitative research. In K. Denzin & Y. S. Lincoln (Eds.), Handbook of qualitative research (pp. 138-157). Thousand Oaks, CA: Sage.

Kohn, A. (1996). Beyond discipline. Alexandria, VA: Association for Supervision and Curriculum Development.

Kroeze, D., & Johnson, D. (1997). Achieving excellence: A report of initial findings of eighth grade performance from the TIMSS. Oak Brook, IL: North Central Regional Educational Laboratory.

Langer, E.J. (1997). The power of mindful learning. Reading, MA: Merloyd Lawrence.

Lave, J. (1988). Cognition in practice: Mind, mathematics and culture in everyday life. Cambridge, MA: Cambridge University Press.

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? Journal for Research in Mathematics Education, 27, 133-150.

Lestienne. R. (1995). The children of time: Causality, entropy, becoming. Urbana & Chicago: University of Illinois Press.

Lewis, C. C., Schaps, E., & Watson, M. (1995, March). Beyond the pendulum: Creating challenging and caring schools. Phi Delta Kappan, 547-554.

Lovelock, J. (1991). Healing Gaia. New York: Harmony Books.

Lucas, G. R. (1979). Two views of freedom in process thought. Missoula, MT: Scholars Press.

Lucas, C. (1985). Out at the edge. Notes on a paradigm shift. Journal of Counseling and Development, 64, 165-172.

Luhmann, N. (1995). Social systems. (J. Bednarz, Trans.). Stanford, CA: Stanford University Press. (Original work published 1984).

Maher, C. A., & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. Journal for Research in Mathematics Education, 27, 194-214.

Mandelbrot, B. (1977). The fractal geometry of nature. New York: Freeman.

Maturana, H. R., & Varela, F. J. (1980). Autopoiesis and cognition: The realization of the living. Dordrecht, Holland: D. Reidel Publishing Company.

May, R. M. (1976). Simple mathematical models with very complicated dynamics. Nature, 261, 459-467.

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Cooney, T. J. (1987). The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. Champaign, IL: Stipes Publishing.

McNeal, B. (1992, August). Mathematical learning in a textbook based classroom. Paper presented at the International Congress on Mathematical Education, Quebec City, Canada.

Mead, G. H. (1934). Mind, self, and society. Chicago: University of Chicago Press.

Murray, H., Olivier, A., & Human, P. (1998). Learning through problem solving. In A. Olivier, & K. Newstead (Eds.), Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, (1) (pp. 169-185). Stellenbosch, South Africa.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. Washington, DC: U.S. Government Printing Office.

National Commission on Teaching and America's Future. (1996). What matters most: Teaching for America's future. Summary report. New York, NY: Teachers College, Columbia University.

National Council of Teachers of Mathematics (NCTM). (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (1993). Curriculum and Evaluation Standards for School Mathematics. Addenda series, Grades K-6. Geometry and spatial sense. Reston, VA: NCTM.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington D.C.: National Academy Press.

Nelsen, J. (1996). Positive discipline. New York: Ballantine.

Nelsen, J., Lott, L., & Glenn, H. S. (1993). Positive discipline in the classroom. Rocklin, CA: Prima.

Noddings, N. (1984). Caring: A feminine approach to ethics and moral education. Berkeley, CA: University of California Press.

Noddings, N. (1992). The challenge to care in schools. An alternative approach to education. Columbia University, NY: Teachers College Press.

Noddings, N., & Shore, P. J. (1984). Awakening the inner eye. Columbia University, NY: Teachers College Press.

North Central Regional Educational Laboratory. (1997). Using the Third International Math and Science Study for Making Policy - Policy Analysis: What Does TIMSS Tell Us About U.S. Education? Available: http://www.ncrel.org/sdrs/pbriefs/97/97-msana.htm

Papert, S. (1988). The conservation of Piaget: The computer as grist to the constructivist mill. In G. Forman, & P. B. Pufall (Eds.), Constructivism in the computer age. Hillsdale, NJ: Lawrence Erlbaum Associates.

Peat, D. F. (1987). Synchronicity. The bridge between matter and mind. New York: Bantam Books.

Perreault, G (1979). Futuristic wolrd views: Modern physics and feminism. Columbus, OH: ERIC Clearning House #184016

Piaget, J. (1971). Biology and knowledge. (B. Walsh, Trans.). Chicago: University of Chicago Press.

Piaget, J. (1973). To understand is to invent. New York: Grossman.

Piaget, J., & Inhelder, B. (1969). The psychology of the child. New York: Basic Books.

Pourdavood, R. G. (1996). Dialogic community: Four elementary teachers' dialogue on mathematics teaching. Unpublished doctoral dissertation, University of Oklahoma.

Prigogine, I., & Stengers, I. (1984). Order out of chaos: Man's new dialogue with nature. New York: Bantam Books

Radhakrishnan, S. (1967). Religion in a changing world. London: George Allen & Unwin.

Reynolds, A., & Wheatley, G. (1996). How do social interactions among children contribute to learning? In H. M. Mansfield, N. A. Pateman, & N. Bednarz (Eds.), Mathematics for tomorrow's young children (pp. 186-197). Dordrecht: Kluwer.

Rogers, C. (1969). Freedom to learn. New York: Merrill.

Rogoff, B. (1990). Apprenticeship in thinking: Cognitive development in social context. New York: Oxford University Press.

Rommetveit, R. (1979). Deep structure of sentence versus message structure: Some critical remarks on current paradigms, and suggestions for an alternative approach. In R. Rommetveit, & R. Blakar (Eds.), Studies of language, thought and verbal communication. London: Academic Press.

Rorty, R. (1980). Philosophy and the mirror of nature. Princeton: Princeton University Press.

Rorty, R. (1989). Contingency, irony, and solidarity. Cambridge: Cambridge University Press.

Rosner, A., & Abt, L. E. (1970). The creative experience. New York: Grossman.

Routman, R. (1996). Literacy at the crossroads: Crucial talk about reading, writing, and other teaching dilemmas. Portsmouth, NH: Heinemann.

Sarason, S. B. (1981). An asocial psychology and a misdirected clinical psychology, American Psychologist, 36, (8), 827-836.

Sarason, S. B. (1990). The predictable failure of educational reform. San Francisco: Jossey Bass.

Sawada, D. (1997). NCTM's standards in Japanese elementary schools. Teaching Children Mathematics, 4, 1.

Saxe, G. B. (1991). Culture and cognitive development: Studies in mathematical understanding. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schmidt, W., McKnight, C., & Raizen, S., in collaboration with six others. (1996). A splintered vision: An investigation of U.S. science and mathematics education, executive summary. Lansing, MI: U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), Cognitive science and mathematics education (pp. 189-216). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1994). A discourse on methods. Journal for Research in Mathematics Education, 25, 697-710.

Schon, D. (1983). The reflective practitioner. New York: Basic Books.

Schubert, W. H. (1992). Readings as resources for teacher lore. In W. H. Schubert, & W. C. Ayres (Eds.), Teacher lore, learning from our own experience (pp. 140-147). White Plains, NJ: Longman.

SciMath. (1997). Minnesota TIMSS report: A preliminary summary of results. St. Paul, MN: SciMath.

Senge, P. (1990). The fifth discipline: The art and practice of the learning organization. New York: Doubleday/Currency.

Silberman, C. E. (1970). Crisis in the classroom: The remaking of American education. New York: Random House.

Silver, E. A. (1994). On mathematical problem posing. For the Learning of Mathematics, 14, (1), 19-28.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26, 114-145.

Singer, P. (1995). How are we to live? Ethics in an age of self-interest. New York: Prometheus Books.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. Arithmetic Teacher, 26, (3), 9-15.

Sleeter, C. E., & Grant, C. A. (1988). Making choices for multicultural education. New York: Macmillan.

Sless, D. (1986). In search of semiotics. Beckenham: Groom Helm Ltd.

Stake, R. E. (1995). The individuality of standardized testing for measuring mathematics achievement. In T. A Romberg (Ed.), Reform in school mathematics and authentic assessment (pp. 173-235). New York: State University of New York Press.

Stacey, R. D. (1992). Managing the unknowable: Strategic boundaries between order and chaos in organization. San Francisco, CA: Jossey-Bass Publishers.

Steffe, L. P. (1992). Schemes of action and operation involving composite units. Paper presented at 7th International Congress on Mathematics Education, Montreal, Canada.

Steffe, L. P., & D'Ambrosio, B. S. (1995). Toward a working model of constructivist teaching: A reaction to Simon. Journal of Research in Mathematics Education, 25, 146-159.

Steffe, L. P., & Wiegel. H. G. (1996). On the nature of a model of mathematical learning. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning. New Jersey: Lawrence Erlbaum Associates.

Steinbring, H. (1991). Another epistemology of school mathematics: Can teachers learn from their students? Mathematics Didactica, 14, (2/3), 69-99.

Tall, D. (1995). Cognitive growth in elementary and advanced mathematical thinking. In L. Meira, & D. Carraher (Eds.), Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (pp. 61-75). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Thom, R. (1973). Modern mathematics: Does it exist? In A. G. Howson (Ed.), Developments in mathematical education: Proceedings of the Second International Congress on Mathematics Education (pp. 194-209). Cambridge: Cambridge University Press.

Thompson, A. (1984). The relationship of teachers' conception of mathematics and mathematics teaching to instructional practice. Educational Studies in Mathematics, 15, 105-127.

Vanderstraeten, R., & Biesta, G. (1998). Education, diversity, and constructivism: A pragmatic point of view. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.

Varela, F., Thompson, E., & Rosch, E. (1991). The embodied mind. Cambridge, MA: MIT Press.

Varela, F. (1995). Resonant cell assemblies. Biological Research, 28, 81-95.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb and H. Bauersfeld (Eds.), The emergence of mathematical meaning (pp. 163-201). Hillsdale, NJ: Lawrence Erlbaum Associates.

Vygotsky, L.S. (1978). Mind in Society. The development of higher psychological processes. Cambridge, MA: Harvard University Press.

Vygotsky, L.S. (1979). Consciousness as a problem in the psychology of behavior. Soviet Psychology, 17, 3-35.

Wertsch, J. V. (1991). Voices of the mind: A sociocultural approach to mediated action. Cambridge, MA: Harvard University Press.

Wheatley, G. H. (1991). Constructivist perspectives on mathematics and science learning. Science Education, 75, (1), 9-21.

Wheatley, G. H. (1996). Quick draw: Developing spatial sense in mathematics. Tallahassee, FL: Mathematics Learning.

Wheatley, M. J. (1994). Leadership and the new science. San Francisco: Berrett-Koehler.

Whitehead, A. N. (1929/1967). The aims of education. New York: Free Press.

Whitehead, A. N. (1929/1978). Process and reality: An essay in cosmology. New York: Free Press.

Wood, T. (1995). An emerging practice of teaching. In P. Cobb & H. Bauersfeld (Eds.), The emergence of mathematical meaning (pp. 203-227). Hillsdale, NJ: Lawrence Erlbaum Associates.

Wood, T., Cobb, P., Yackel, E., & Dillon, D. (Eds.) (1993). Rethinking elementary school mathematics: Insights and issues. Journal for Research in Mathematics Education Monograph Series, Number 6. Reston, VA: National Council of Teachers of Mathematics.

Wood, T., Cobb, P., & Yackel, E. (1995). Reflections on learning and teaching mathematics in elementary school. In L. P. Steffe, & J. Gale (Eds.), Constructivism in education (pp. 401-422). Hillsdale, NJ: Lawrence Erlbaum Associates.

Yackel, E. (1995). Children's talk in inquiry mathematics classrooms. In P. Cobb & H. Bauersfeld (Eds.), The emergence of mathematical meaning (pp. 131-162). Hillsdale, NJ: Lawrence Erlbaum Associates.

Yackel, E. (1998). A study of argumentation in a second-grade mathematics classroom. In A. Olivier & K. Newstead (Eds.), Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, (4). (pp. 209-216). Stellenbosch, South Africa.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27, 458-477.

APPENDIX A

## STUDENT PARTICIPATION AGREEMENT

(Parent/guardian: Would you please read this to or with your child as needed)

Dear ....................................,

As a teacher of mathematics working at the University of Oklahoma I am interested in what students learn in mathematics. I would like to interview you about this to help me understand it better. I would like to ask you some questions that will help me to see how you are thinking about your mathematics. I am not wanting to see if you are right or wrong in the answers you give me, so the ideas you share with me will not be used in grading you. Instead, I want to understand what you are thinking about. Then I can share these ideas with other teachers so that they can help students like you learn mathematics. I will arrange to do this at a time that best fits other things you are doing in the classroom. Also, so that I can remember what you share with me better I want to video record what you say and do as you answer the questions. If at any time you wish to stop working with me or stop being taped all you have to do is say so. At all times you have the right to withdraw from participating. There is no penalty connected to withdrawing. I guess our interviews will take about 40 minutes each and I hope to do one early in the year and then one towards the end of the year.

Sometimes, when I talk with other teachers about the way students think about mathematics, it is helpful if I can show them what students do and say, like what you have done or what you have said. So I would like your permission to show parts of the video tape when I think it might help teachers to understand your ideas better. I will not use your real name when I do this. Instead I will ask you to make up a name for me to use.

I also plan to visit your classroom for about one or two hours, three days a week till the end of the school year. I hope to be able to watch you and talk with you occasionally in class for a few minutes or longer, depending on what you are doing when I come to visit your classroom.

If you have any questions about this please talk with me when I visit your classroom or call me at 325-1498.

I look forward to working with you.
Sincerely,

Noel Geoghegan.
Postgraduate Student, Mathematics Education.

Dear Parent/Guardian of ................................,

As a doctoral student in the College of Education, University of Oklahoma, Norman Campus, I am planning to conduct research into how students learn mathematics. Dr. Anne Reynolds from the College of Education, University of Oklahoma is assisting and guiding my work. As part of this research I would like permission to interview your child at school (early in the year and at the end of the year). The questions that I plan to ask your child during the interviews are designed to explore how students construct meaning for various mathematical ideas. The interviews are in no way intended to evaluate correctness of responses but rather to explore how students think mathematically. The interviews will be arranged in cooperation with your child's teacher at a time that will best fit your child's classroom routine with as little disruption as possible. Your child's responses will not be used in any way for grading purposes in her/his classroom. Rather it is hoped that by encouraging your child to explain how s/he is thinking about the tasks used in the interviews a deeper understanding of how students learn mathematics will be developed. I estimate the interviews will take about 40 minutes. To allow for later analysis by myself as researcher the interview will be video recorded.

I anticipate that this experience will not cause any harm or disruption whatsoever for your child. If your child in any way wishes to discontinue the interview or stop the recording s/he may do so at any time. Your child at all times has the right to withdraw from participating. There is no penalty connected to withdrawing from participation.
Also, in order to communicate the results of this research more effectively within the mathematics education community I would like your child's and your permission to show selected portions of the videotape. Your child's name will not be used and the tape will be edited to preserve his/her anonymity whenever possible. Please let me emphasize that the use of the tape and data from the observations are for professional purposes only. No public showing of the video will be considered and strict confidentiality of your child's participation will be observed.

I also plan to visit your child's classroom on a regular basis in order to observe class activity during mathematics time. I hope to visit your child's classroom for up to three days a week for about one to two hours each day till the end of the school year. I will be working closely with your child's teacher and wish to ask your permission to allow me to participate in classroom interactions with your child which could be as short as a few minutes or longer depending on the classroom activity.

If for any reason you have reservations about these requests please call me at 325-1498. I look forward to receiving your approval form and working with your child. I have enclosed a stamped addressed envelope for you to use in returning these forms.

Sincerely,

Noel Geoghegan (Postgraduate Student, Mathematics Education)

## APPROVAL FORM FOR INTERVIEW

I give permission for my child, __________________________, to be interviewed and observed as described above.

Signed: _________________________________ Parent/Guardian

Date: _________________________

## APPROVAL FORM FOR USE OF VIDEOTAPE

I give permission for a videotape made of my child, ______________________________, during interviews to be used as described above. I understand that I may withdraw this permission at any time.

Signed: _________________________________ Parent/Guardian

Date: ______________________________

## APPROVAL FORM FOR INTERVIEW

I, ________________________________, agree to being interviewed and observed as described in the letter I have just read or had read to me.

Signed: ________________________________ Student

Date: ______________________________

## APPROVAL FORM FOR USE OF VIDEOTAPE

I, ________________________________, agree that the videotape can be used to help teachers to understand better how students think about mathematics. I understand that if I later change my mind about the tape being shown to other teachers I can withdraw this permission.

Signed: ________________________________Student

Date: ______________________________

# APPENDIX B

## TEACHER PARTICIPATION AGREEMENT

Dear (Teacher)..................................
As a doctoral student in the College of Education, University of Oklahoma, Norman Campus, I am planning to conduct research into how students learn mathematics. Dr. Anne Reynolds from the College of Education, University of Oklahoma is assisting and guiding my work. As part of this research I would like permission to observe and interview your children and you in your classroom as part of my data collecting process.

I would like to interview six children in your class as key informants. I estimate each child's interview will take about 40 minutes. Each of the key informants will have an interview early in the year and at the end of the year. The interviews are in no way intended to evaluate correctness of responses but rather to explore how students think mathematically. The interview times will be arranged with your approval to best fit your classroom routine with as little disruption as possible. Your children's responses will not be used in any way for grading purposes. Rather it is hoped that by encouraging the children to explain how they are thinking about their tasks and activities and then analyzing their efforts, a deeper understanding of how students learn mathematics will be developed. To allow for later analysis by myself as researcher I would also like to video record the children's interviews.

I would like to visit your classroom for up to three days a week for about one to two hours each day till the end of the school year finishing May 1998. During this time I plan to observe you and your children during daily classroom interactions. I anticipate that this experience will not cause any harm or disruption whatsoever for you or any of the children. If any of the children in any way wish to discontinue the interviews or stop the recordings they may do so at any time. All children will be aware that they may cease participating at any time and without penalty. Similarly you may withdraw at any time.

Also, in order to communicate the results of this research more effectively within the mathematics education community I would like your permission to show selected portions of the videotapes. None of your children's or your name will be used and tapes will be edited to preserve complete anonymity whenever possible. Please let me emphasize that the use of these tapes and data from the observations are for professional purposes only. No public showing of the videos will be considered and strict confidentiality of all participants will be observed.

I would also like your permission to interview you on occasions. I would only interview you when it best suited you, your children and the school's schedule.

If for any reason you have reservations about this please call me at 325-1498. I look forward to receiving your approval forms. I have enclosed a stamped addressed envelope for you to use in returning these forms.
Sincerely,

Noel Geoghegan (Postgraduate Student, Mathematics Education)

## TEACHER'S APPROVAL FORM FOR INTERVIEW

I, ______________________________, agree to participate and allow the children in my class to be interviewed and observed as described in the letter above.

Signed: ________________________________ Classroom Teacher

Date: ______________________________

## APPROVAL FORM FOR USE OF VIDEOTAPE

I, ______________________________, agree that videotapes of me can be used to help teachers to understand better how students think about mathematics. I understand that if I later change my mind about the tapes being shown to other teachers I can withdraw this permission.

Signed: ________________________________Classroom Teacher

Date: ______________________________

APPENDIX C

SAMPLE QUESTIONS FROM KEY INFORMANTS INTERVIEWS

Number Conservation tasks - which line is longer or are they the same? Why?

Mental calculations - How much is 6 and 5? 11 take away 6? 21 take away 19? How did you do that?

Tangram activities - Draw the shape (after looking at it for a few seconds). Tell me what you saw and how you drew it.

Dot patterns - Tell me how many dots you saw and how you knew it was that many.

Screened counting - There are 5 chips under the cloth, how many are there altogether? How did you get the answer?

Tens task - How many dots can you see (as the screen cover progressively reveals collections of ten).

Conceptual association - Tell me what you think of when I say: Square; Busy; Red; Fast; Time

Context solving - It's 6 o'clock and no-one has arrived to attend your birthday party - why might that be?

Lateral thought - Tell me a story about a fox, a bottle and a circle.

What is math? What sort of things do you do when you do math? Does it help to work with a partner? Should your teacher tell you when you are right? Are you pretty good at math? What do you like about math? What don't you like about math?

Copy a pattern made up of 8 elements.

Continue the following pattern - 10347891034789

What comes before zero?

How far would a number line go if we just kept going? Where would we end up?

What is any easy way to show someone what 100 is? What about 200?

What is an easy way to find out how many pennies are in these jars?

(JAR 1= 25; JAR 2=15)

There were 68 pennies in this jar and I took some out and put them in here (JAR 2 hidden). There are 18 pennies left in the jar. How many did I take out?

I'm a jeweler who makes tie bars (Diene's longs=10) from diamonds (Diene's shorts=1); and then I make brooches (Diene's flats=100) from the tie bars. How many diamonds would I need to make 2 tie bars? 1 brooch? 5 tie bars? 2 brooches? If I broke off 6 diamonds from a set of 5 tie bars, what would be left?

If I have 3 tie bars and 6 diamonds, how can I make 6 tie bars? What will I need to do to complete the 6 tie bars?

## APPENDIX D

## CONSTRUCTIVISM

### A Radical Constructivist Perspective

The process of construction and reorganization as elaborated by Piaget was his way of explaining how people come to know about their world. Piaget viewed the human organism as being in a flux of dynamic cognitive activity creating structures that help us make sense of what we perceive. These structures grow in intellectual complexity as we mature and relate to the world through our experiences. Seen as a radical departure from the behaviorist paradigm, Piaget's ideas have been advanced in mathematics education under the auspices of "radical constructivism," a term coined by its founder and follower of Piaget, Ernst von Glasersfeld.

Radical constructivism is currently a major, if not the dominant, theoretical orientation in the mathematics education community (Lerman, 1996; Ernest, 1996). Radical constructivism has emerged from Piaget's epistemological work and is based on the fundamental assumption that people create knowledge from the interaction between their existing knowledge and the new ideas or experiences they encounter. In this sense, the radical constructivist view supports the need to foster interactions between students' existing knowledge and new experiences. This emphasis is perceived to be different from the more traditional (positivist and behaviorist) "transmission" model, in which teachers try to convey knowledge to students directly.

Radical constructivism is viewed as a philosophical explanation about the nature of knowledge, and is thus referred to as an epistemology. It is important to understand that radical constructivism is not an instructional approach; it is a theory about how learners come to know. Radical constructivism seeks to describe how a person develops and uses cognitive processes. In general, radical constructivists compare an "old" view of knowledge to a "new" one - the constructivist view. In the old view, knowledge is considered to be fixed and independent of the knower. There are "truths" that reside outside the knower, qua "objective truths." Knowledge is the accumulation of the "truths" in any subject area; the more "truths" one acquires, the more knowledge one possesses.

In contrast, a radical constructivist view,

> *. . . rejects the notion that knowledge is independent of the knower and consists of accumulating "truths." Rather, knowledge is produced by the knower from existing beliefs and experiences. All knowledge is constructed and consists of what individuals create and express. Since individuals make their own meaning from their beliefs and experiences, all knowledge is tentative, subjective, and personal. Knowledge is viewed not as a set of universal "truths," but as a set of "working hypotheses." Thus constructivists believe that knowledge can never be justified as "true" in an absolute sense. (Airasian & Walsh, 1997, p. 445)*

As Glasersfeld (1995) states in his book, "Radical Constructivism,"

> *Radical constructivism starts from the assumption that knowledge, no matter how it is defined, is in the heads of the persons. . . . What we make of experience constitutes the only world we consciously live in. . . . But all kinds of experiences are essentially subjective, and though I may find reasons to believe that my experience may not be unlike yours, I have no way of knowing that it is the same. (p. 1)*

International interest in the radical constructivist view has accelerated during the last quarter-century. With this burgeoning of interest have come critiques as well as refinements. Saxe (1991) has argued that within Piagetian and radical constructivist interpretations of student mathematical learning, the interplay between social life and cognitive development is, noticeably and importantly, not a core concern; "indeed for Piaget, the focus was on the formal properties of action without regard for the situatedness of actions in a socio-historically articulated web of meanings" (p. 6).

In a rejoinder to radical constructivists' paradigmatic portrayal of the learner as a rational, self-organizing, self-regulating, idiosyncratically personal, and separately subjective individual (Glasersfeld, 1996a), it was suggested by Cobb (1990) that analyses which focus solely on separate(d) individual's "construction of mathematical knowledge tell only half a good story" (p. 213).

Adler (1998) notes that current discourse on radical constructivist perspectives in mathematics education "connects with learner- or student-centered discourses prevalent in wider educational discourse" (p. 77). However, she asserts, radical constructivist perspectives are being increasingly scrutinized over "emerging cracks in what might mischievously be referred to as the uncritical and perhaps ideological embracing by the mathematics (and science) education research community of a perspective that privileges the learner and his or her private and diverse ways of knowing" (p. 77).

Kieran (1994) suggests that one of the reasons for the impediments to recent development of the new classroom orientations of research in mathematics education is related to a limitation of past radical constructivist, cognitively oriented work:

> *Researchers have been hard-pressed to reconcile the theory that all learning is individually constructed with the evidence of commonalities found across individuals. They could not explain how people came to develop 'taken as shared' meanings. Constructivists had to, in fact, admit to the social dimensions of learning . . . the impact on research of this theoretical response was the beginning of attempts to account for learning as it occurs in classroom environments. (p. 601)*

Critiques of the conflicts and redundancies within the radical constructivist viewpoint highlight issues related to solipsistic, instrumentalist and relativist positions, for example, sole reliance on personal meaning to justify constructions can lead to potentially biased, self-serving, and dishonest constructions (cf. Airasian &

Walsh, 1997; Code, 1991). Nonetheless, the ramifications emerging from the implications of its perspective on learning have had a pronounced effect on contemporary mathematics reform agendas. The NCTM Curriculum an Evaluation Standards (1989) state that:

> *In many classrooms, learning is conceived of as a process in which students passively absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. Research findings from psychology indicate that learning does not occur by passive absorption alone. . . . Instead, . . . individuals approach a new task with prior knowledge, assimilate new information, and construct their own meanings. . . . This constructive, active view of the learning process must be reflected in the way much of mathematics is taught. (p. 10)*

A Social-Constructivist Perspective

A second version of constructivism has emerged in more recent years to establish a philosophical beach-head in the deliberations of the education research community. Drawing upon the basic tenets of radical constructivism (i.e., students are active creative learners of their ways of knowing), and social interactionism (i.e., learning involves the interactive constitution of meanings in a culture) a form of constructivism called "social constructivism" has come to the fore, especially in mathematics education research (Greer, 1996). Reflecting a growing trend that seeks to transcend if not reject, the individualistic orientation of the radical constructivist view, social constructivism emphasizes the social construction of knowledge.

Emphasizing a sociocultural perspective, social constructivists argue that knowledge is constructed by the individual's interaction within a social culture or particular social context. Social constructivists believe that knowledge has a social component and cannot be considered to be generated by an individual acting independently of his or her social context. Because individual social and cultural contexts differ, the meanings people make may be unique to themselves or their cultures, potentially resulting in as many meanings as there are meaning makers (Airasian & Walsh, 1997).

Increasingly, social constructivism is being recognized in the mathematics community for its contribution to a shift from the "empirical, analytic school of carefully controlled studies to more studies dealing with classrooms with teachers and students doing things" (Kieran, 1994, p. 601). Contemporary social constructivists working in mathematics education argue that:

> *Mathematical learning is an interactive, as well as an individual, constructive activity. . . . This social interactionist perspective should be distinguished from the Neo-Piagetian view that social interaction merely serves as a catalyst for autonomous individual cognitive development. In the course of classroom social interactions, the teacher and students mutually construct taken-to-be-shared mathematical interpretations and understandings. This taken-to-be-*

> *shared mathematical knowledge, itself the product of prior interactive negotiations, both makes possible communication about mathematics and serves to constrain individual students' mathematical activity. In other words, students, in the course of their individual cognitive development, actively participate in the classroom community's negotiation and institutionalization of mathematical meanings and practices. It is by viewing students not only as individual learners but as members of a community that one can escape from the solipsism inherent in a purely psychological analysis of learning. (Cobb, et al., 1991, p. 6)*

Such a perspective highlights the learner as a member (specific) of a learning community (general), and in doing so implicates the experience of learning of mathematics as more than a purely personal activity.

The evolution of the social constructivist movement has been accompanied by, while at the same time helping to precipitate, the development of research approaches that emphasize the observation of the processes of learning rather than the measurement of their products. The theoretical framework that many mathematics educators are starting to explore in conjunction with this new kind of classroom research is a Vygotskian one. Vygotsky's theoretical perspectives ascribe greater weight to the role of social processes in the construction of knowledge than Piagetian cognitive perspectives. From a Vygotskian perspective, "the social dimension of consciousness is primary in fact and time. The individual dimension of consciousness is derivative and secondary" (Vygotsky, 1979, p. 30). From this, it follows that thought (qua cognition) "must not be reduced to a subjectively psychological process" (Davydov, 1988, p. 16). Instead, according to Bakhurst (1988), thought should be viewed as "something essentially 'on the surface', as something located . . . on the borderline between organism and the outside world. For thought . . . has a life only in an environment of socially constituted meanings" (p. 38).

For Vygotsky there was no separation between teaching and learning; the assumption being that the individual's action is mediated and cannot be separated from the milieu or the setting in which the teaching and learning transpires. Lerman (1996) acknowledges Vygotsky's view; there is no separation, "mediation by materials, tools, peers, and teacher are [all] constitutive of learning" (p. 147). Guba and Lincoln (1989) assert that an individual's "constructions are inextricably linked to the particular physical, psychological, social, and cultural contexts within which the constructions are formed and to which they refer" (p. 8), and Forman (1996) contends that by focusing on the classroom environment, "one is forced to recognize the connections between what a person does, feels, thinks, and believes: the constraints and supports provided by other people and artifacts in that particular setting; and cultural rules, norms and values" (p. 118).

Bruner (1966) remarked that he increasingly came to recognize that "most learning in most settings is a communal activity, a sharing of culture" (p. 127) and Piaget (1973) eventually concluded in his book, *To Understand is to Invent: The Future of Education*, that the social aspects of teaching and learning are "strictly necessary for the mental development that is called education" (p. 46).

The social constructivist perspective that emphasizes the individual dimensions of experience as being subsidiary to the social and cultural dimensions, has given rise to another group of theorists known as emergent theorists who argue that individuals jointly create interactional routines and patterns as they adapt to each other's activity (Cobb, Jaworski, & Presmeg, 1996). In contrast to Vygotsky's focus on the social and cultural basis of personal experience, emergent theorists have combined constructivist and interactionist analyses to highlight "the contributions that actively interpreting individuals make to the development of social and cultural processes" (Cobb & Bauersfeld, 1995, p. 124).

> *The emergent perspective places the students' and teacher's activity in social context by explicitly coordinating sociological and psychological perspectives. The psychological perspective is constructivist and treats mathematical development as a process of self-organization in which the learner reorganizes his or her activity in an attempt to achieve purposes or goals. The sociological perspective is interactionist and views communication as a process of mutual adaptation wherein individuals negotiate mathematical meaning. From this perspective, learning is characterized as the personal reconstruction of societal means and models through negotiation in interaction. Together, the two perspectives treat mathematical learning as both a process of active individual construction and a process of enculturation into the mathematical practices of wider*

*society. Analyses of individual students' activity are therefore coordinated with analyses of the collective or communal classroom processes in which they participate. (Gravemeijer, McClain, & Stephan, 1998, p. 194)*

However, along with the emergent theorists' view (cf. Cobb & Bauersfeld, 1995), on-going theoretical argumentation that addresses fundamental differences between the positions of the individual (specific) - for example, vis-à-vis radical constructivist views, and the social (general) - for example, vis-à-vis socioculturalist/emergent views, is proving slow to have an impact on mathematics pedagogy in the classrooms of North America. As the field of mathematics education attempts to craft a research approach that will provide pragmatic directions for classroom implementation, Ernest (1996) contends that the complementarity between the two perspectives of individual construction and social interaction will need to be clearly articulated for current reform agendas to be realized.

APPENDIX E

PROBLEM-CENTERED LEARNING

The theoretical perspective of problem-centered learning contends that through solving problems, students learn what mathematics really is, and what it means to "do mathematics." Problem-solving experiences readily avail opportunities for students to go further and, on their initiative and of their own volition, go on to pose additional problems, perhaps extending or modifying the original problem that they have just solved - or even posing their own problems to begin with (Brown & Walter, 1993; Reynolds & Wheatley, 1996; Silver, 1994).

From this viewpoint, students are expected to spend time solving problems, and after they invent solutions, reflect on what they have just done. Mathematics, then, according to Davis (1996), as an outcome of such problem-centered experiences, might be called "My Accumulated Collection of Ways That I Have Invented in Order to Do Things" (p.293). Duckworth (1987) in her book, "The Having of Wonderful Ideas," echoes Davis's perspective that pedagogy in mathematics education is more to do with inventing rather than absorbing.

> *I react strongly against the thought that we need to provide children with only a set of intellectual processes - a dry, contentless set of tools that they can go about applying. I believe that the tools cannot help developing once children have something real to think about; and if*

*they don't have anything real to think about, they won't be applying tools anyway. That is, there really is no such thing as a contentless intellectual tool. If a person has some knowledge at his disposal, he can try to make sense of new experiences and new information related to it. He fits it into what he has. By knowledge I do not mean verbal summaries of somebody else's knowledge. I am not urging textbooks and lectures. I mean a person's own repertoire of thoughts, actions, connections, predictions, and feelings. Some of these may have as their source something read or heard. But the individual has done the work of putting them together for himself or herself, and they give rise to new ways to put them together. . . . The greater the child's repertoire of actions and thoughts . . . the more material he or she has for trying to put things together in his or her own mind. (p. 13)*

Problem solving as the central focus of the mathematics curriculum commands considerable international currency within the mathematics education reform movement (Australian Education Council, 1991; Cockcroft, 1982; Groves, 1998; NCTM, 1989; Wheatley, 1991; Wood, Cobb & Yackel, 1995). Problem solving has been for the last decade, promulgated as "a primary goal of all mathematics instruction and an integral part of all mathematical activity" (NCTM, 1989, p. 23). A problem-solving approach to mathematics education portrays a methodology in stark contrast to traditional behaviorist approaches that emphasize skill development in a few specific "forms of symbol pushing . . . [that often result in ] surprisingly high levels of proficiency, [but] no concern for understanding" (Davis, 1996, p. 288).

As part of the criticism leveled at methodologies that emphasize learning by rote and following specific procedures and rules, Bodin (1993) points out that students can solve a given equation without being able to express the steps taken or to justify the results without knowing which type of problem it is connected to, or without being able to use it as a tool in another situation. As an example, Bodin observed children who were able to solve the following equation,

$7y - 3 = 13y + 15$

but who were unable to answer the question:

Is 10 a solution to the equation $7y - 3 = 13y + 15$?

The implication here is that the equation can be solved simply by following a procedure, but the latter requires judgment and reasoning (De Lange, 1995; NCTM, 1989).

Research (Gregg, 1993; McNeal, 1992; Schoenfeld, 1987) has indicated that students can appear mathematically competent in classrooms which have the quality of "instructions to be followed," but their symbol-manipulation acts do not necessarily carry the significance of acting mentally on mathematical objects past a level of superficial procedural thinking. Furthermore, "because there is nothing beyond the symbols to which the teacher and students publicly refer, a mathematical explanation involves reciting a sequence of steps for manipulating symbols. . . . Mathematics as it is constituted in these classrooms, therefore, appears to be a largely self-contained activity that is not directly related to students' out-of-school activities"

(Cobb & Bauersfeld, 1995, p. 2). The criticism continues; too much time is spent on learning to apply rules and memorizing facts, and the resultant disenfranchised feelings, and lack of judgment when working with mathematics, are too high (McKnight & Raizen, 1996).

A problem-solving approach to mathematics education in the classroom advocates a shift of emphasis from rote procedures to the development of higher order thinking (Brown & Walter, 1993; De Lange, 1995; Silver, 1994). A problem-solving approach acknowledges that favorable conditions for learning exist when a person is faced with a task for which no known procedure is available, that is, when learners find themselves in a problematic situation (Hiebert, et al., 1996; Murray, Olivier & Human, 1998).

What is problematic to one child is not necessarily so to another; problems are a function of the learner's conceptual level. A problem-solving approach requires that the teacher be cognizant of such contextually specific situations and, in order to identify potentially tenable problematic experiences, focuses on each student's understandings. Rather than trying to persuade students to see mathematics from the teacher's standpoint (qua the behavioristic "this is how it is" approach), the pedagogical task then becomes a search to understand the thought patterns of students so that tasks will be seen to be problematic by the students (Wheatley, 1991).

As Dewey (1945) says,

> *It is also essential that the new objects and events be related intellectually to those of earlier experiences, and this means that there*

*be some advance made in conscious articulation of facts and ideas. It thus becomes the office of the educator to select those things within the range of existing experience that have the promise and potentiality of presenting new problems which by stimulating new ways of observation and judgment will expand the area of further experience. He [the child] must constantly regard what is already won not as a fixed possession but as an agency and instrumentality for opening new fields which make new demands upon existing powers of observation and of intelligent use of memory. Connectedness in growth must be his constant watchword. The educator more than the member of any other profession is concerned to have a long look ahead. (p. 75)*

Jean Piaget's work was instrumental in providing ways for rethinking the legitimacy of behaviorist views about learning, and in promoting problem solving. Piaget's work has helped to advance an argument that implies, contrary to the empiricist view that learning is the absorption of objective knowledge transmitted by the teacher, that students construct knowledge. Such a view implies that through problem solving students stand to gain more by exploring and interacting with their own developing mathematical ideas. Piaget was by his own definition a genetic epistemologist, and concerned with how cognitive development and the "putting of things together to make sense" constituted the formation of knowledge. His research led him to conclude in the latter years of his career, that:

> *The current state of knowledge is a moment in history, changing just as rapidly as knowledge in the past has changed, and, in many instances, more rapidly. Scientific thought, then is not momentary; it is not a static instance; it is a process. More specifically, it is a process of continual construction and reorganization. (1971, pp. 1-2)*

However, Piaget's idea of learning as continual construction continues to struggle to secure a substantial foothold in the pedagogical psyche of Western education. With attempts to clarify and ratify the import of problem solving in Piagetian terms in mathematics education has come a different account of problem-solving in the classroom. The use of "problem-centered learning" as the essential perspective for mathematics education, rather than "problem-solving" (Cobb, et al., 1990; Murray, Olivier & Human, 1998; Reynolds & Wheatley, 1996; Wheatley, 1991) has emerged to place more emphasis upon learning than instruction. The differentiation is subtle but profound. Problem solving as an instructional approach was promulgated somewhat at the expense of the philosophical and theoretical basis underpinning its implementation.

Though the term "problem-solving" has been afforded considerable exposure in the literature, by and large, it has been ineffectual in its successor claim for pedagogical primacy. It is possible that problem solving has failed to achieve broad acceptance and critical mass because it has been too broadly defined and too loosely translated. Loose translation and poor articulation has been detrimental to the debate as problem-solving struggles for consummation in the workplace. With the concept of "problem-centered learning" comes a new slant and new promise to resolving the educational impasse under which a clear articulation of the practical issues behind the problem-solving approach has labored.

Wheatley (1991) explains that a constructivist theory is an essential perspective in working with children in a problem-centered approach. He argues that in a problem-centered approach, the teacher must seek to understand students' thinking; learn to look at the world through their eyes. Rather than being considered mistakes to be corrected, student errors are rich sources of information about children's thinking. They indicate the meaning children have given to associated ideas. The issue is not what procedures and knowledge have they amassed but what concepts have they constructed, the cognitive level at which they are operating, their motivations, their beliefs, and the sense they are making.

To understand a child's sense-making experience we need to be able to first, acknowledge and then, identify with each child's development. Steffe and Wiegel (1996, p. 493) refer to this by way of "close listening" or as "the act of decentering in order to imagine what the experience of the learner might be . . . the results of many hours of listening can yield a dynamic, living mathematics of students." Goldin (1996) expresses a similar view, that we are always building models and seeking to hear the "voice" of the child in the model. Effectual close listening infers a capacity on the part of the teacher to attend compassionately and subjectively to the cognizing child not only in cognitive terms but also emotionally, socially, physically and even spiritually.

As Wheatley (1991) asserts, it is important for the teacher to observe students closely in problematic situations in order to determine their level of competence. In order to assist a child learn we must try to look at the world as they do. Teachers and students are viewed as active meaning-makers who continually give contextually-based meanings to each others' words and actions as they interact. We might call this the teacher as researcher metaphor. The teacher-pupil interaction is viewed as a process of negotiating meaning rather than imposing fixed procedures.

Cobb, et al. (1991) described a classroom that used a problem-centered learning approach: children worked in pairs and engaged in problematic tasks, and after 25 minutes of collaboration, came together for a teacher-lead class discussion in which groups presented their solution methods and elaborations of methods. The role of the teacher was not to correct "wrong" answers but to leave them for students to discuss. In this manner, students did not look to the teacher as the authority for sanctioning but accepted responsibility for determining agreement. The teacher's role was to attempt to elicit as many different views from the children without imposing her view on the students. Through the exchange of ideas with the teacher and their peers, children developed shared meanings that allowed them as members of a responsible and self- and of-each-other-respecting group to communicate with one another.

However, Murray, Olivier, & Human (1998) point out that, critical to the success of the problem-centered learning approach, is the teacher's framework for the social dimension of the classroom; no matter how well-designed the problematic situations or tasks are, "the amount of quality of the learning which takes place depends on the classroom culture and on students' and teachers' expectations" (p. 172). Such a statement reflects squarely upon the philosophical framework a teacher holds for learning in a classroom when committed to a problem-centered approach.

While curriculum and policy makers have endorsed a problem-centered approach to learning in principle, neither universal nor popular articulation of this approach has been forthcoming, due in part to the fact that it "requires changing the entire system of instruction" (Hiebert, et al., 1996, p. 19). As we move towards the challenges of the next century, the North American mathematics curriculum stands out for its lack of depth compared to that offered by other industrialized nations (Smith, McKnight, & Raizen, 1996). Though often well-intended, political and administrative pressures to specify, standardize, and assess student learning "appear to drive the curriculum further toward the shallows. . . . Many mathematics teachers do not recognize the drift toward oversimplification" (Stake, 1995, p. 212).

Charges of oversimplification generated a storm of debate during the "California Math Wars" and considerable arguments over the problem-centered approach ensued. The foundations of the arguments were based in at least two distinct perspectives: Firstly, differing and contradicting views as to (a) what constitutes mathematics, (b) how children learn mathematics; and, (c) what a problem-centered approach entails. Secondly, very real concerns about how to implement an innovative approach for which teachers may not have the necessary mathematical and pedagogical qualifications and skilis, and for which there is a shortage of appropriate materials (Murray, Olivier, & Human, 1998).

The issues of implementing a problem-centered learning approach continue to challenge researchers and practitioners. Groves (1998), as the invited reactor to one of only two international papers on problem solving presented at the Research Forum for Learning through Problem Solving at the 22nd annual conference of the International Group for the Psychology of Mathematics Education held in Stellenbosch, South Africa, answered the questions, "How can such an approach [a problem-centered learning approach] be translated into a model of classroom practice which is accessible to the wider teaching community?" and, "What could a new, widely applicable model of mathematics teaching which adopts the aim of fostering student's constructions of powerful ideas through problem solving and inquiry look like?" by simply declaring, "I have no answers to these questions" (p. 213).

APPENDIX F

# A POSITIVE APPROACH TO DISCIPLINE

A fundamental aspiration of education systems in North America and most industrialized countries is the procreation of a democratic ideology. Within such an ideology it is assumed that democracy is founded upon freedom, and with freedom comes equity, with equity comes emancipation, and with emancipation comes liberation - and liberty means that "we are at liberty" to do as we please, because we are free. However, there is ironical twist to the canonical constitutional typology of democratic principles residing in present-day North American classrooms. With Berger and Luckman's (1966) warning that "man is capable paradoxically of producing a reality that denies him" (p. 89), the great promise of democracy purporting to deliver humans unto their own greatness through freedom has become a pedagogical paradox. Teachers and students are being relegated to positions of fear and intimidation born of unfettered change.

One only has to look into today's classrooms to see how hard teachers strive for excellence in their students, constantly aiming to equip their charges for a productive role in furthering the democratic ideology in their own lives (whether as mathematicians or citizens). Teachers work tirelessly buoyed by a belief that they can help make a difference in the lives of their students. However, as they seek ways to empower their students with the ideals of democracy, they increasingly find themselves in a paradoxical conundrum when confronted daily by situations that generate severe dissonance and discordant mismatches with the envisioned ideology of democratic principles.

Buffeted by such demands as the unpredictability of student needs and the irregularities of standardized curricula, teachers find themselves impulsively, though hesitantly, retreating from and retaliating against a barrage of psychological, social and emotional aberrations manifesting in their classroom. Feeling "trapped" and with a compulsion to survive, teachers find it easy to revert to "base-line minimalist teaching," and "common-sense, intuitive subsistence strategies."

Teachers are being increasingly compromised, having to move further and further away from the democratic principles they wish to develop. In developing a "minimalist" climate in their classrooms, mathematics teachers find themselves struggling to live up to the expectations of the democratic model; the impetus for liberating their students is lost through a perceived necessity for authoritative classroom management.

Whether one believes it to be an increasingly stark phenomenon or a persisting one, classroom teachers are faced with a milieu of personalities, attitudes, characters, feelings, and the incumbent social and emotional "needs" of students and colleagues which ultimately constrain democratic freedom in the classroom. In coping with the immense pressures of classroom diversity and sustained unpredictability, many teachers increasingly succumb to coercive and authoritative education practices, justified under the auspices of "making grades" and "making it through another day."

More often than not, the "fall-back" pedagogical and philosophical position is a style of teaching characterized by intimidation, oppression, forced compliance and subjugation (of both personal and the other). Authoritative discipline rears itself as the organizational solution to classroom frustration. And hence the paradox: control and authority pervade educational practice like omnipresent incubuses, while democracy remains its guiding principle.

Noddings (1992) highlights this authoritarian discipline-centered position as "shallow educational response to deep social change" (p. 1). In confronting the dictates of classroom control and authority, Kohn (1996) quotes Glasser (1986): "To focus on discipline is to ignore the real problem: we will never be able to get students (or anyone else) to be in good order if, day after day, we try to force them to do what they do not find satisfying" (p. 12). However, with the ideology of control so firmly entrenched in teacher's professional and personal lives, it is very difficult to move to an alternative approach.

Of such a pedagogical quandary, Dewey (1938) suggested that one of the main reasons why a "control" mentality emerges in the classroom is a fight for power, because the situation almost forces it upon the teacher to "keep order" (p. 55), but, he concluded, "the primary source of social control resides in the very nature of the work done as a social enterprise in which all individuals have an opportunity to contribute and to which all feel a responsibility" (p. 56).

Part of the quandary lies in how teachers are coping with the new social order as it emerges from generations of submissiveness; a post-modern society imbued with the values of autonomy and equality and children orientated towards new perceptions of responsibility. As individuals and groups claim their rights to autonomy and equality, new expectations for treating others with dignity and mutual respect are unfolding. DeVries and Zan (1994), along with Kamii (1991), drawing on the work of John Dewey and Jean Piaget, argue that students must actively participate as autonomous inventors (and reinventors) of what it means to participate in a responsible, respectful, and ethical manner. If a democratic ideal is to be sustained, children must construct ethical meaning, just as they must construct mathematical meaning.

In taking this position, DeVries, Zan, and Kamii emulate the work of Lawrence Kohlberg who spent his career applying notions of cognitive development in the moral domain. Collectively these researchers would agree that, emphasizing children to comply cannot be the teacher's primary goal (Kohn, 1996). Vanderstraeten and Biesta (1998) suggest that with the rising awareness of the complexity of teaching and learning mathematics, educators are increasingly emphasizing the ethical dimensions of education. They argue that out of respect for individual emancipation, teaching and learning are being reformulated and reconstituted in light of a continuity between student's own values, feelings, and expectations as members of a community of learners.

Kamii (1991) agrees that such a reconstitution is needed:

> *We cannot expect children to accept ready-made values and truths all the way through school, and then suddenly make choices in adulthood. Likewise, we cannot expect them to be manipulated with reward and punishment in school, and to have the courage of a Martin Luther King in adulthood. (p. 398)*

With the rising awareness of the complexity of mathematics education and increasing recognition of diversity in the classroom, educators are emphasizing the ethical dimensions of education. "To live ethically is to think about things beyond one's own interests" (Singer, 1995, p. 174).

> *I have a very strong sense of being responsible to the world, that I can't just live for my enjoyment, but just the fact of being in the world gives me an obligation to do what I can to make the world a better place to live in, no matter how small a scale that may be on. (Gilligan, 1988, p. 21)*

For mathematics teachers to genuinely embrace democratic and ethical dimensions of learning in their classrooms, Singer (1995) suggests the adoption of a broad perspective called "the point of view of the universe."

> *From this perspective, we can see that our own sufferings and pleasures are very like the sufferings and pleasures of others; and that there is no reason to give less consideration to the sufferings of others, just because they are 'other'. This remains true in whatever way 'otherness' is defined, as long as the capacity for suffering or pleasure remains. (p. 222)*

Such a perspective begs the question: Haven't children suffered enough in mathematics classrooms?

According to DeVries and Zan (1994) the reconstitution of mathematics classrooms depends on teachers pointedly declining to lay down the law and take control. By "refus[ing] to be all knowing or all powerful, they open the way for children to struggle with issues and not rely on adults for truths and values" (p. 193).

However, with the elevated belief in the immutability and infallibility of mathematics, the difficulty of relinquishing power will be evidently noticeable in mathematics teachers; too many mathematics classrooms remain committed to emphasizing prescribed instructions, respecting authoritative truth, and obeying rules (regardless of whether they make sense or for whatever reason we learn mathematics). Such an emphasis teaches a disturbing lesson.

Stanley Milgram's famous experiment, in which ordinary people administered what they thought were terribly painful shocks to hapless strangers merely because they were told to do so by someone in charge, "is not a just a comment about 'society' or 'human nature'. It is a cautionary tale about certain ways of teaching children. . . .To talk about the importance of choice is also to talk about democracy" (Kohn, 1996, p. 85). As Berman, (1990) noted, "[w]e teach reading, writing and math by [having students do] them, but we teach democracy by lecture" (p. 2).

Studies support such a view (Angell, 1991; Battistoni, 1985; D'Amico, 1980); students who were able to participate in making decisions at school were more committed to the processes of decision making and democracy in other contexts. In his powerful introduction to "Crisis in the Classroom" (1970) Charles Silberman corrected his own earlier view that emphasized cognitive goals as the educational "mother lode" by stating that "what tomorrow needs is not masses of intellectuals, but masses of educated [people]- [people] educated to feel and to act as well as think" (p.7). Noddings (1992) suggests that what is needed in education is a scheme that "speaks to the existential heart of life - one that draws attention to our passions, attitudes, connections, concerns and experienced responsibilities" (p. 47).

The moral is, according to Kohn (1996), if you don't attend to how children feel about school and each other, then you don't attend to education. "The only way to help students become ethical people, as opposed to people who merely do what they are told, is to have them construct moral meaning" (p. 67).

## APPENDIX G

## POST-MODERNISM

Post-modern views deny the existence of an objective reality, and challenge subject-object dualism. The epistemological and ontological notions of post-modernist thinking are linked with the nineteenth-century writings of Nietzsche and Kierkegaard, and later Wittgenstein, and are based on the idea that knowledge is what a rational self-organizing being creates as part of an interacting system; the basic premise being that social conditions and individual minds and selves are fundamentally influenced each by the other (Berger and Luckman, 1966; Mead, 1934).

Identified as the third epoch, (Antiquity and the Middle Ages constituting the first and second epochs respectively), the modern age, or "modernism," is considered to have preceded the post-modern era and a long time in the making, dating from medieval times. Modernism found unity in a revolt against autocracy, religion, government, science, art and education. As a mainly Western phenomenon, modernism emerged as a "celebration of the individual as opposed to established authority" (Elkind, 1998, p. 166) and owes much to Rene Descartes whose dictum "I think therefore I am" rooted authority in subjective thought and reasoning. The supremacy of reason, the individual, self expression, and individual freedom have been the abiding tenets of modernity. As modernity spread throughout Western society it established three basic beliefs that were the foundation for our modern perception and understanding of the world. The first of these was the belief in the concept of progress.

This vision was closely tied up with the idea of growth as the unveiling of scientific phenomena in order to benefit mankind. Such a view superseded the medieval view that knowledge was a fixed body of information provided by an authoritative text. Modernism determined that through the understanding and ensuing harnessing of nature, humankind could move toward a world in which every individual could enjoy the rights of liberty and life. A second fundamental truth of modernism was the belief in universals. A belief in universal "natural" laws was a repudiation of medieval laws promulgated on the basis of the divine right of kings or high church officials. It encouraged the grand scientific theories of Newton, Darwin, Marx, Freud, and Einstein all of whom believed they had discovered universal principles of nature. A third belief was that of regularity, that predictability of natural phenomena could counter the often arbitrary and willful dictates of premodern authorities. Modern science was based on a search for universal and regular natural laws that were believed to govern the physical and social worlds (Elkind, 1998).

The post-modern period in the Western world has emerged in response to modernist beliefs, not so much as a revolution but as a modification cum correction of the basic principles, and is considered to be the fourth and latest epoch. Modernist beliefs were considered not entirely wrong, however, were viewed as often idealized to the point of becoming blind to invisible assumptions and consequential inadequacies. Habermas (1987) suggested that modernistic principles maintained existing power relationships while disregarding the ways in which current sociopolitical relationships affect human life and he repeatedly called for the "unmasking of the human sciences" (p. 295).

He argued, in Kantian terms, that the "objectifying attitude in which the knowing subject regards itself as it would entities in the external world is no longer privileged" and that the Cartesian "paradigm of the philosophy of consciousness" be replaced with the "paradigm of mutual understanding" (p. 296). Foreshadowing a reconstitution of the modern normative paradigm of reasoning, a new orthodoxy of mutual understanding, or "interactive universalism" as Code (1991, p. 127) refers to it, is being advocated by feminist philosophers as worthy of succession and indicative of a post-modern era.

By drawing upon a range of metaphors and narratives to present a vision and re-vision of post-modernist perspectives, gender-, race-, and culture-sensitive perspectives have displaced the Platonic-rooted notion of autonomy and domination (sometimes described as a "removed" sense of knowing) by proposing a more artistic sense of knowing (qua seeing and being) that is closer to a "comprehensive awareness . . . a total imaginative experience . . . a more respectful relation with nature" (Code, 1991, p. 145; see also Capra, 1996).

Like modernism, post-modernism has been largely a Western phenomenon and has been developing not all at once but at different times, in different places and in different social, political, and cultural ways. Kincheloe and McLaren (1994) argue that:

> *post-modern theoretical trajectories take as their entry point a rejection of the deeply ingrained assumptions of Enlightenment rationality, traditional Western epistemology, or any supposedly "secure" representation of reality that exists outside of discourse itself. . . . Post-modern criticism takes as its starting point the notion that meaning is constituted by the continual playfulness of the signifier, and the thrust of its critique is aimed at deconstructing Western metanarratives of truth and the ethnocentrism implicit in the European view of history as the unilinear progress of universal reason. (p. 143)*

Post-modernism, according to Elkind (1998), venerates language, rather than thought, and honors human diversity as much as human individuality. He contends that the ascendancy of language over reason as the true groundwork of human existence owes much to the language games played out during the 19th century to demonstrate that there is no such thing as "pure" reason and that our thinking can never be abstracted from our language. Through the use of parody, irony, and satire, language was shown to be inherently ambiguous and,

> *that the truths of reason, which must employ language, must thus be ambiguous as well. When language, rather than reason, is taken as the fundamental model of how the world works, an alternative set of themes moves into prominence (Elkind, 1998, p. 28).*

First, sociocultural differences become more important than linear progress. Second, language is particular to a given culture at a given time and post-modernists are concerned more with domain-specific issues and discourses rather than with grand universals. Third, unlike reason, language is often as irregular as it is regular and accordingly post-modernism seeks to legitimize the irregular as worthy of exploration as the regular (Elkind, 1998; Hargreaves, 1994). The themes of difference, particularity, and irregularity are increasingly transforming science, the arts, industry, and education.

# APPENDIX H

## EXAMPLES OF THE TEACHER'S PEDAGOGICAL FRAMEWORK

Episode 1

Children were asked to create their own games based on an idea for a mathematics game that had been introduced to them as the Four-In-A-Row game.

Harriett: They started making their games yesterday and little by little they all wanted to [branch out] and make their own games. So they did. They are getting good at just ignoring me and doing their own thing.

The teacher was not implying that children had indiscriminate reign in following their own bent but rather highlighted the significance of allowing them the flexibility to create their own challenges within the parameters of her stated mathematical objectives.

Mona developed her game and called it Make-An-E-or-F game which required numbers 0 to 10, 20 and 11 to be added in combinations to make numbers 10 to 30.

Megsie and Ned developed a game they called 5-In-A-Row requiring numbers 4, 6, 7, 8, 9, 10, 11, 12, 13 to be added in combinations to make numbers between 11 and 25.

The teacher provided children with opportunities to make sense of their mathematics by playing with and creating new ideas based on their already constructed knowledge.

Episode 2

Figure 9 was presented to the first cohort of children. The teacher asked them, "What did you see and how did you draw it?"

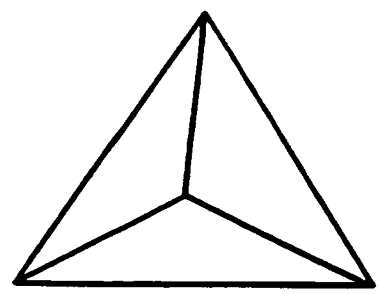


Figure 9. Shape projected on the overhead projector for children to view and then draw.

| | |
|---|---|
| Whole class: | Oh, that's easy. |
| Teacher: | What did you see and how did you draw it? |
| Brian: | I saw a pyramid. See, like here's the middle part; and here's the bottom part. |
| Teddy: | See the pyramid. It's like you're looking up at it in the sky [from underneath]. |
| Mertle: | It's like a kite; like a paper airplane. Then it's like a hat, too. And I saw a sad face too. |
| Fred: | I saw a triangular peace symbol. |
| Tracey: | I saw a teepee. If you turn it like this (tries to orient the overhead transparency but has difficult positioning it to effect). |

Episode 3

Figure 9 was also presented to the second cohort of children.

| | |
|---|---|
| Teacher: | What did you see and how did you draw it? |
| Pedro: | I saw the top of a pyramid. Looking down on it. |
| Jack: | I saw, that car company, Ford, no not Ford. Yeah. Honda; their card; their badge on cars. Yeah Honda. |
| Duke: | Yeah. I see it, and if you have a pyramid, if you see it moving around, like on my computer [at home] you can make a pyramid moving like this (and gestures a floating motion with his hands.) |
| Ali: | I saw a Georgie, like if you turn it sideways. I see the tiny section bit as inside the mouth. |
| Catlin: | If the top part is facing down, these, the 2 big triangles are the bottom and you can spin it this way. (She comes to the overhead projector and, pointing to the bottom corners indicates a clockwise spinning action.) It will be a top. |
| Mona: | I saw it as an airplane; like a paper airplane. |
| Chicka: | I see it as a petal falling off a flower; like the inside of a petal. |

Both Episodes 2 and 3 indicate the range of different views to be found within a group of young children's mathematical perspectives. The teacher encouraged children to express their various perspectives while at the same time fostering a responsiveness to the variety of other's viewpoints. She felt the variety of perspectives enhanced the way in which to construct mathematical ideas.

Episode 4

The teacher held regular and spontaneous discussion sessions to reinforce the children's responsibility for working together.

Teacher: How do we work with our math partners?

Mertle: I do two [questions] and my partner does two.

Birt: Sometimes I don't get it so Jonah helps me to work it out.

Pearl: Sometimes one of us figures out the answer and the other person writes it down

Fred: We just get along.

Brian: Dwayne and I get a balance scale and divide up the bits between us.

Erica: When I get the answer, and she doesn't agree with me, we figure out if it is my answer or her answer.

Episode 5

The tenor of the social climate being encouraged in the classroom reflected a respect for others and a rejection of competitive rivalry. I had been working with Jonah exploring the game of Nimm and asked him if he could figure out a way to always win. He was a bright, precocious but emotionally distracted boy. The attention I was providing him was being devoured as he was faced with severe family traumas and major homelife complications. He was causing considerable behavioral disruption at school, but not in this class. Nonetheless, after I asked him if he might figure out a way to always win, and we had tested his proposition only to find it unreliable, he left only to return after a whole-class sharing time to whisper in my ear, "I know how to win; always do your best." "His interpretation of success was 'always doing your best.' Is this classroom affecting his attitude?" (Field notes, December 1996)

Episode 6.

The teacher frequently used the overhead projector. On its moveable stand it was just the appropriate height for her to conduct her lesson. However, it became apparent that children were having difficulty in using the projector at that height. So the teacher moved the projector onto the floor where children were able to function more strategically with their discussions and demonstrations.

Episode 7.

Pearl wandered over to talk to a group after discussing the problem at hand with his partner Teddy. "You're just tricking me," he said to his math partner. "No," said Teddy, "it's 32." Pearl walked back to his desk followed by Teddy who repeated, "It's 32. It's 32." But Pearl insisted, "You're tricking me," to which Teddy replied, "Come. I'll show you." The two sat together as Teddy demonstrated the strategy he used to get 32 for the answer. With that done, Pearl ran back to the other group and assertively informed them, "32 is the answer," emphasizing "is."

This episode demonstrates how the teacher's modus operandi encouraged interactive and participatory cooperation to solve problems within the class. It also indicates how Teddy was willing to persist with his mathematical claim despite protests from his partner. Children in the class were motivated to be respectful and encouraging towards others.

Episode 8

The teacher used the technique of asking, "Who did it a different way?" during whole-class sharing times to encourage everyone to participate. At the same time, she sought to generate opportunities for viewing and considering a variety of mathematical propositions from different perspectives. "After a few presentations, others don't necessarily offer the original way they "did it" but instead, reformulate alternative and creatively endowed ways just to be able to have a turn and say that they did it a different way." (Field notes, February 1997.)

Figure 10 shows the random arrangement of dots projected on the screen by the teacher for 3 seconds as children endeavored to calculate or count how many there were.

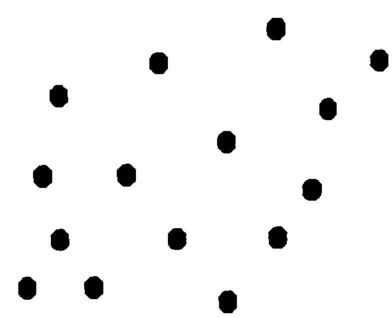


Figure 10. Arrangements of dots shown to the class for 3 seconds.

| | |
|---|---|
| Teacher: | How many dots did you see and how did you see them? |
| Dwayne: | 4 along the top and 3 [in the top right-hand side]. That's 7. Then I saw 4 and 3 again. That's 7. So that's, (unfinished sentence). |
| Fred: | I counted 5 on the bottom, then go to the 5 across the center. It was like a face. Then there's 3 [in the top right-hand corner] and 2 more. That's 15. |

Erica: I saw 4 [top right-hand corner] and a second set of 4 in the bottom [bottom right-hand corner]. I moved the top set [top right-hand corner] to show another 4, and so that's 12. And last there's a set of 3. That adds to 15.

Dwayne: I change mine to 15. I missed one at the bottom.

Erica: (having just grouped in fours) I can count by fives. There's five [bottom left-hand corner] and a middle row of 5. Then add the top curved five. So three fives is 15. I also see 5 sets of 3 is 15.

Episode 9.

Children were asked to solve 16 + 7 - 3 = ?

Tom and Pearl: We got 20. Because 16 and 4 is 20 We knew this.
So, 16 + 7 is really 16 + 4 + 3, and that's 23. 20 + 3.
Then we took away 3 to get 20.

Teddy and Trevor: We knew 4 + 3 is 7.
And we knew 6 + 4 is 10.
So add it together. That's 20. Then take away 3 equals 20.

Teddy and Trevor's explanation defied explanation at first. Their brief but assertive explanation confounded four adults present in the classroom at the time. An explanation follows: Decompose 7 as 4 + 3. Then, use the 4 (from the 7) with 16 to make 20 (as 6 + 4 is 10, 16 + 4 is 20) which can be concurrently seen as the 20 in 23 if 23 is composed of 20 and 3. Hence, the 3 is subtracted leaving only 20 of the 23. Children were encouraged to construct their own ways of thinking about their mathematics. Oftentimes, they proved how sophisticated their ways can be.

APPENDIX I

EXAMPLES OF THE CHILDREN'S PARTICIPATIVE FRAMEWORK

Episode 1

Children were encouraged to participate in their mathematical explorations by creating as well as constructing mathematical propositions. Also, the inclination to simply supply answers was discouraged.

| | |
|---|---|
| Teddy: | (writes for his math partner to solve) 5 x 5 = |
| Spike: | (writes) 25. |
| Teddy: | (writes for Spike to solve) 7 x 7 = |
| Spike: | Hmm. I don't know that one. |
| Teddy: | It's 40-something, and I'll give you three guesses. |
| Spike: | 45 |
| Teddy: | Nope. It's four more. |
| Spike: | (quickly responds) 49 (and writes 49). |
| Teddy: | (writes) 6 x 7 = (but as Spike does not know, he writes in 42) You do one [for me]. |
| Spike: | (writes) 10 000 x 0 = (and chuckles as he watches Teddy count the zeros as he knows Teddy is on the wrong track). |
| Teddy: | (writes) 10 000 (to which Spike chortles and rejects his answer). Sure it is! Because if you have that times zero, it's the same number. Hey, wait a minute. Oh oh! No it's not; it's zero. (Both boys grin with satisfaction.) |

Episode 2

Children's individual levels of mathematical development were addressed in a fluidly flexible framework in which the children worked. The class was given the task: "How many ways can you make 30 cents?" Such mathematics tasks are open-ended and seek to transcend the pedagogical stance that mathematics questions have only one answer. Teddy and Tracey wrote 5 x 6 on their worksheet and left it at that. They weren't compelled or inclined to do more. They were more interested in working with other mathematics propositions. Having met the obligation to the "open-ended" problem about 30 cents, Tracey began calculating the proportional mathematical proposition that if one second stands for 8 hours, how many seconds in a day. Tracey was thoroughly interested in a math game called Guess My Rule that had been introduced earlier. His computations included solving for the rule: 2n + 1; for example he figured 8 would give 17, 5 would give 11, and 4 would give 9. By couching such tasks in a game format, Harriett availed children opportunities for open-ended developmentally appropriate mathematizing. Children knew they had the flexibility to initiate self-challenging mathematical problems.

Episode 3

The teacher allowed children to determine viable methods rather than use pre-ordained and prescriptive ones. She encouraged children to make sense of their mathematics by creating and inventing mathematical propositions by playing with their own and other's ideas rather than hers.

Mertle was asked how to explain 20 + 20.

Mertle: (writes on the chalkboard with a line drawn down the middle between the tens and ones columns)

$$\begin{array}{c|c} 2 & 0 \\ \underline{2} & \underline{0} \\ & \end{array}$$

The line separated the columns to be added. She said this would make 40 with which the teacher asked her to show how she would do 28 + 28. Again she wrote the double-digit numbers with a line down the middle splitting the tens and ones into two columns.

Mertle: (writes on the board)

$$\begin{array}{c|c} 2 & 8 \\ \underline{2} & \underline{8} \\ & \end{array}$$

2 and 2 is 4 (and writes 4 in the tens column) Then 8 and 8 is sixteen. (She writes 6 in the ones house and explains) You take away the one from the sixteen.

With this the class erupted with children objecting to Mertle's approach. Tracey was asked to demonstrate his suggestion for 28 + 28.

Tracey: (writes on the board)

$$\begin{array}{r} 28 \\ \underline{+\,28} \end{array}$$

(He explains) 8 plus 8 is 16. Put down the six. Then 20 and 20 and 10 is 50. So it's 56.

Once again there was uproar from the class challenging Tracey's approach this time. Birt was then asked to demonstrate his suggestion for 28 + 28.

Birt: (writes on the board)

$$\begin{array}{r} 28 \\ \underline{+\,28} \end{array}$$

(He explains) 8 plus 8 is 16. Put down the 6 and one goes up the top (pointing above the top 2 in the tens column and writing in a 1 in place). 1 and 2 and 2 are 5, and that's fifty. So it's 56.

The class, by and large, found this method to be a viably reasonable proposition.

<u>Episode 4</u>

Children were provided with a wide variety of concrete, creative, structured and unstructured play-focused activities in the form of learning centers and work stations. Watching and listening to children in the various contexts afforded the teacher opportunities to gain insight into children's developing mathematics. Dwayne was asked how to do 20 + 20.

Dwayne: You have to count by twos. 2, 4. That's 40.

Later in the day, Dwayne was playing in the class shop (math activity center) and I asked if I could buy four lids at $10 each. He gave me the four lids and said, "That's forty dollars." I gave him fifty dollars which he checked then promptly found a ten dollar note for my change. Interestingly, while these two mathematical encounters would indicate that Dwayne has constructed units, an earlier encounter suggested otherwise.

I asked Dwayne how to do 19 + 21. He counted (19) from 21 by ones to 40 using his fingers showing no hint of employing a tens grouping strategy. By engaging with children one-on-one in a variety of classroom contexts the teacher was able to gauge the extent of each child's developing mathematics. To do this required creating an environment where children were actively involved with a range of mathematical materials and situations.

Episode 5

The teacher rejected traditional methods of drill and practice in favor of liberating children's capacity to create their own mathematics. Fundamental to such an approach entailed demonstration and justification of personal mathematical propositions to the rest of the class and a proclivity for negotiation and renegotiation.

Mona had written her explanation of how she arrived at 14 by subtracting 16 from 30. During her protracted explanation and explicit recording on the chalkboard, she was constantly challenged as the other children endeavored to make sense of her thinking. At one stage, Megsie called out, "But 4 and 6 don't make ten. Oh! OOPS! Yes it is." And later Pedro interjected saying, "I do not agree in any way. 4 take away 6 is not 10." With this, Mona stopped, reconsidered her work on the board and promptly clarified her error and corrected her recording appropriately, then continued with her explanation.

Episode 6

Rather than impose authority in the form of the knowledge determiner, the teacher allowed children to ratify and justify their own mathematical propositions.

Nancy was working on $7 + 7 + 7 + 7 =$

She said that it would be 21. The teacher was more interested in how she did it rather than the answer being incorrect.

| | |
|---|---|
| Teacher: | How did you do it? |
| Nancy: | I did 6 + 6; and I know this is 12. 12 plus 1 plus 1 or 2 is 13, 14. Then I just used my fingers to count the rest. |
| Teacher: | Show me. |
| Nancy: | (gets muddled and is confused.) |
| Teacher: | Can you show me a different way? |
| Nancy: | Yeah. With tally marks (and writes) |
| | IIIII II IIIII II<br>IIIII II IIIII II |
| | (She then counts) 5, 10, 15, 20...21, 22,...23, 24,...25, 26,...27, 28. Ooooh. I think it's 28. |

Episode 7

The teacher modulated her expectations with the children's propensity for self-regulated decision making. Rather than impose her standard or level of thinking, she encouraged children to diversify their activity in keeping with the projected mathematical goals. After reading the story of "Ping the Duck," the teacher asked children to use tangram pieces to make a picture from the story.

| | |
|---|---|
| Caleb: | (leans over and whispers to Norton) You're supposed to be making the bird. |
| Norton: | Well, I want to make the boy. (And he had. To Caleb's dismay, Norton showed how he had used four sets of tangram pieces to make the boy from the story in the same time Caleb had used only one set.) |

Episode 8

Children were liberated to explore, experiment, investigate, conjecture, trial, negotiate, and study mathematical propositions through participating in cooperative problem-centered experiences. Such experiences involved active interactions and self-regulated decision making. Subject boundaries were often crossed and integration often became the norm of mathematical activity.

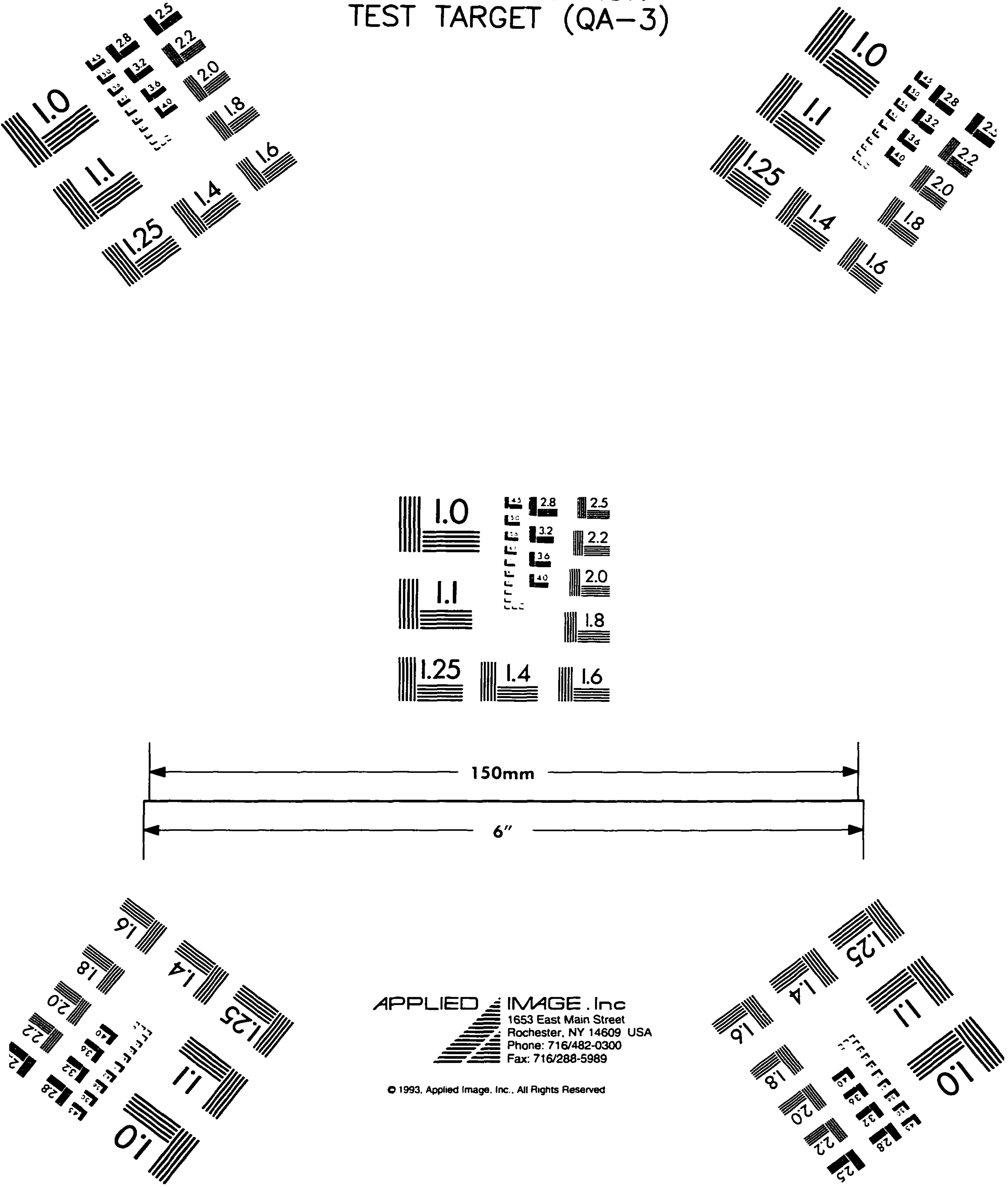
The whole class had been designing and folding paper planes to see which would fly the furthest. Catlin and Asha had been working together following instructions in a book (developing language skills). Catlin endeavored to show Asha how to start and then proceeded to finish hers with remarkable dexterity before Asha had reached step 3 in the instructions. As Asha realized that her paper folding was proving to be inadequate, she despondently retreated to her desk and began to cut and add extra sections using cellotape. She then made several unsuccessful attempts to get her plane to fly any distance. She continued to add sections until her plane looked like a flat fish kite with a long tail.

Meanwhile, as Caleb tested his three-tiered glider stapled together in the shape of a jet at the back of the room, Dillard approached Asha and said, "Come here. I'll help you make one" to which Asha replied, "No thanks." She was committed to making her own effort work and determined to succeed. Interestingly, Asha and Dillard both have poorly developed number skills but show well developed social intentions. Catlin, by this stage, had turned to another page in the book and was able to create a boomerang glider while Norton had made a double-winged aerodynamically designed dart.

Asha tried several times again but was unsuccessful in launching her plane. However, she noticed how her plane floated and announced, "I've made a kite!" Later, she made the comment, "Nobody knows how to make *my* kite," and later, "Mrs H., Mrs H. It did it! It did it!" as she heaved it skyward and watched her 'kite' glide unexpectedly gracefully in a spiraling movement. With this, Pedro sidled up to her and said, "We're supposed to be making one that flies the fartherest." Asha seemed unfazed and began decorating hers and singing repeatedly, "Nobody knows how to make mine." It was interesting to note that all the boys had made aerodynamically efficient darts while the girls had concentrated on large Saturn rockets, kites and more intricate designs and models.

Norton had now engineered a triple-decker triplane. His test proved surprisingly successful and he beamed his satisfaction with his creation. In the meantime, Pedro had persisted with a coiled ring design in the shape of a weighted tube. After many failed attempts, he resorted to a conical shape using paper clips to hold it together. His initial launch was a resounding success to which he added, "There you go. Now it's starting to work." (Field notes, April 1998.)

# IMAGE EVALUATION TEST TARGET (QA-3)



APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989